A GRAPHICAL SOLUTION OF THE SHORT TRANSMISSION LINE PROBLEM.

THESIS FOR DEGREE OF E. E.

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SUPPLEMENTARY
MATERIAL
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A GRAPHICAL SOLUTION OF THE SHORT TRANSMISSION LINE PROBLEM.

The writer has felt for some time that there is need of a comprehensive treatment of the short transmission line problem (by short distance lines are meant those in which the capacity effect is negligible) in such a way that the average individual who has to deal with it , may, without the use of involved calculations and formulae, be able to ascertain the performance of lines of different characteristics, and under different operating conditions such as power-factor, and to readily make comparison of these things. To this end, a set of curves showing voltage drop for the various sizes of wire at a standard spacing arbitrarily chosen as 24 inches, for various power-factors, has been plotted. This constitutes the groundwork for the method and is chart A. A means is given for applying a correction for other frequencies and spacings. In order to show the limitations of the method, and that they may not be exceeded in its application, curves are given showing the error under various conditions, and the effect of capacity at no load, the latter giving an idea of the limits beyond which it can no longer be neglected.

THE LINE DROP CHART.

The line drop chart is drawn from points located by the formula, $D = IR\cos\phi + IX\sin\phi$, I=line currrent, R=resistance in ohms, X=reactance in ohms, ϕ=the angle of lag or power-factor angle at the receiver end of the circuit, and D=line drop in volts. These quantities are of course per single con-

ductor or to neurtal. This formula was published in an article in the General Electric Review of June, 1913, entitled, "A Graphical Mehtod for the Calculation of Short Transmission Lines", by Fredrik Waern. It will be readily understood by reference to Figure 1, which is really nothing more than the vector diagram of the voltages in an alternating current circuit consisting of resistance and inductance.

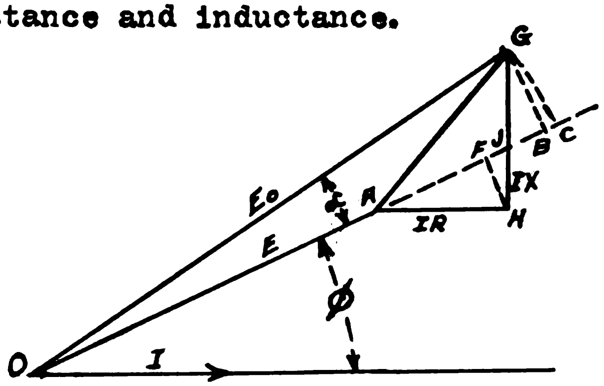

Figure 1.

Let E=receiver volts.

Let I=line current.

Let R=total resistance of circuit.

Let X=total reactance of circuit.

Let cosφ=power-factor of receiver.

Then φ=angle of lag, or power-factor angle.

It is then evident from an inspection of Figure 1, that E_0 is the generator voltage, this being the vector sum of the receiver voltage, the resistance drop, and the reactance drop.

Now let the E vector be produced and BG erected perpendicular to it from G.

Also let HF be erected perpendicular to E from H.

Since HF and BG are perpendicular to E, the angles FHJ and JGB are each equal to ϕ.

Angle FAH also equals ϕ.

Therefore, $IR\cos\phi + JH\sin\phi + JG\sin\phi = AB$,

or $$IR\cos\phi + (JH+JG)\sin\phi = AB,$$

or $$IR\cos\phi + IX\sin\phi = AB. \qquad (1)$$

If an arc, GC, be drawn with O as a center,

$$E_0 - E = AC,$$

or, taken numerically, AC equals the voltage drop.

It will be noticed that AB very nearly equals AC, and we may accordingly write with negligible error in most cases,

$$AB = \text{voltage drop (numerically)}.$$

and from (1), $$D = IR\cos\phi + IX\sin\phi, \qquad (2)$$

which is the formula used in this method for the calculation of line drop.

The error in this formula is represented by BC, the difference between Ac and AB. This error is fully discussed in a subsequent section.

In the construction of chart A, the Line Drop Chart, formula (2) was used to compute the voltage drop for the various sizes of wire from No.10 solid to 500,000 c.m. stranded, for power-factorsfrom unity to .6, lagging. Curves were then drawn, one for each size of wire, with volts drop as abscissae and powerfactor as ordinates. This was all done on the basis of drop per single conductor, per 1000 feet, per ampere, at 60 cycles, with twenty-four inch spacing. Twenty-four inch spacing was arbitrarily chosen as being a spacing (equivalent)

commonly used in low and medium voltage distribution.

An example of one calculation is given for illustrative purposes, using No.4 solid wire at 85% power-factor.

R=Resistance per 1,000' No. 4 wire=.2530 ohms.

X=Reactance per 1,000' No. 4 wire, 24" spacing=.1309 ohms. (at 60 cycles)

$\cos\phi = .85.$

$\sin(\cos^{-1}.85) = .527.$

$R\cos\phi = .2150.$

$X\sin\phi = .0690.$

$D = R\cos\phi + X\sin\phi = .2840$, which value is the one plotted.

The wire data used was taken from the Standard Hand-Book for Electrical Engineers, Fourth Edition. Resistance values were taken at 25 degrees, Centigrade.

THE FREQUENCY AND SPACING CORRECTION CHART.

The principal drawback to the line drop chart is that it provides for only one frequency and spacing. To overcome this objection, the Frequency and Spacing Correction Chart, Chart F, has been drawn.

This chart is practically the same as that published as a supplement to the article above refered to in the General Electric Review of June, 1913, but is applied in a different way. By means of it, the correction to be applied to the values of Chart A, for other frequencies and spacings can readily be determined.

Chart F consists essentially of set of diagrams by means of which the last member of formula (2), $X\sin\phi$, can be determined.

A resistance scale was laid off along the Y axis, and the various sizes of wire up to No. 4 inclusive, were plotted on this scale according to their respective resistances. The scale was shortened for numbers 6, 8, and 10, to bring them within the limits of the chart.

A scale of ohms reactance was laid off along the negative X axis, the divisions being given the same value as used for the resistance scale. A set of curves of reactance per 100 cycles was then plotted, one for each spacing. Then the projection of the intersection of any one of these curves with a size of wire line, upon the X axis, gives the reactance per 100 cycles for that size of wire and spacing.

Radial lines were then drawn from the origin, in the left hand quadrant, at such an angle for each frequency, that the ordinate of any point is in the same proportion to the distance along the radial line from the origin, as the frequency is to 100 cycles. Arcs of circles with the origin as the center were also drawn between the 100 cycle line, or X axis, and the upper radial frequency line, to facilitate the transferring of the reactance per 100 cycles to the other frequency lines. The values of reactance for any frequency and spacing can then be read from the Y axis, direct.

Radial lines were next drawn from the origin, in the right hand quadrant, one for each power-factor from .95 to .60, changing by .05, in such a way that the vertical projection of any point on one of these lines, upon a horizontal scale of ohms, equal to the scale used for resistance and reactance, will equal

the horizontal projection of that point upon the Y axis, multiplied by sinϕ. Values for Xsinϕ can thus be read from the chart.

For instance, let it be desired to determine X sinϕ for No. 2 solid wire , at 5 ft. spacing, 25 cycles, and 80% power-factor. Project vertically from the intersection of the No. 2 size line with the 60 inch spacing line, to the X axis, which will give a value of .244 ohms reactance at 100 cycles. Follow the circle from this point to the 25cycle line and project horizontally to the 80% power-factor line in the right hand quadrant. This will be at.063 on the Y axis. Then project vertically to the X sinϕ scale, which will give the value, X sinϕ equal to .038, the value for which we are solving.

It will be noticed that the spacing curves break where the change is made from solid to stranded wire, all values larger than No. 00 being stranded.

It will be observed in the use of the formula, $D=Ir\cos\phi+IX\sin\phi$, that when the frequency and spacing are changed, only the term, IXsinϕ changes. Therefore to correct the curves of the line drop chart for any different frequency and spacing, it is only necessary to determine the difference in the term, IXsinϕ, due to the change., and apply that difference to the values of chart A by addition or subtraction as the case may be.

For convenience, a table of values of Xsinϕ, for 24 inch spacing at 60 cycles, is given on chart F, so that to make a correction, it is only necessary to take the value for the different frequency and spacing from the chart, using the tabular value for the other term of the correction equation.

THE LINE LOSS CHART.

The line loss chart, chart B, has been drawn to show the line loss for the various sizes of wire at different power-factors. From it can be read directly, the watts loss per single conductor, per ampere at unity power-factor, at any desired power-factor above .60. The expression, per ampere at unity power-factor, needs some explanation. The values of curves at the point of power-factor equal to 1 were calculated for a current of one ampere per conductor. The values at the other power-factors were computed on the basis of the power remaining constant at the value due to the one ampere current at unity power-factor, as the power-factor decreases. The current in the line will thereby increase, with consequent increase of loss, the power component of current however remaining constant. This current will vary inversely as the power-factor, and since the loss varies as the square of the current, it will also vary inversely as the square of the power-factor.

The line loss chart then shows the variation of loss under constant kilowatts load, as the power-factor varies. In its application, if the real amperes-resultant of power and wattless components- are known, the loss is read from the intersection of the size of wire line with the unity power-factor line. If the power component of current is known, the loss is read from the intersection of the size line with the line corresponding to the power-factor at which the calculation is being made. To determine the loss for other loads than for that of one ampere at unity power-factor, it is of course necessary to multiply by the number of amperes flowing in the line.

Chart B then gives a ready comparison of the losses for the various sizes of wire at different power-factors, and a quick means of determining the loss for any condition.

THE ERROR IN THE LINE DROP CHART.

In the discussion of the line drop chart, the statement was made that in most cases, the error involved by the use of formula (2) is negligible. That this is really so is shown by Chart C, which consists of curves of error and size of wire for power-factors from 1.0 to .60, with D equal to 10% of E_0. The sizes of wire are laid off along the X axis so that the error can be found for any one. The values of error were determined by means of the formula,

$$\%\ \text{Error} = 100\left[1 - 1/E_0\sqrt{E_0^2 - \tan^2(\tan^{-1}X/R - \phi)\ D^2}\right] \quad (3)$$

Referring again to Figure 1, and remembering that

AB=D=Drop,

E_0=generator voltage,

E=receiver voltage,

it is readily seen that $\text{error} = E_0 - (E+D)$. (4)

$$(E+D)^2 = E_0^2 - BG^2 = E_0^2 - \tan^2(\text{angle } GAB)D^2 \quad (5)$$

Angle GAB=angle GEH - angle JAH.

Angle $GAB = \tan^{-1}X/R - \phi$.

Substituting in (5), $(E+D)^2 = E_0^2 - \tan^2(\tan^{-1}X/R - \phi)D^2$.

$$E+D = \sqrt{E_0^2 - \tan^2(\tan^{-1}X/R - \phi)\ D^2}.$$

Substituting in (4), $\text{error} = E_0 - \sqrt{E_0^2 - \tan^2(\tan^{-1}X/R - \phi)D^2}$.

$$\%\ \text{error} = \text{error}/E_0(100).$$

$$\%\ \text{error} = E_0/E_0(100) - 100/E_0\sqrt{E_0^2 - \tan^2(\tan^{-1}X/R - \phi)D^2}.$$

$$\%\ \text{error} = 100\left[1 - 1/E_0\sqrt{E_0^2 - \tan^2(\tan^{-1}X/R - \phi)\ D^2}\right].$$

Solutions of this formula were made for the various sizes of wire and power-factors used in chart C, and the results were plotted as shown by the curves. All calculations were made for the standard conditions of spacing, frequency, etc. used in chart A, and for a drop equal to 10% of the generator voltage. To show the application of the formula, an example of the calculation of error for No. 6 wire at a power-factor of .70, with generator voltage equal to 100 and drop equal to 10 volts, is given.

$$\% \text{ error} = 100\left[1 - 1/100\sqrt{100^2 - \tan^2(\tan^{-1}.1362/.403 - \cos^{-1}.7)10^2}\right].$$
$$= 100\left[1 - 1/100\sqrt{10{,}000 - \tan^2(\tan^{-1}.338 - \cos^{-1}.7)100}\right].$$
$$= 100\left[1 - 1/100\sqrt{10{,}000 - \tan^2(18.68^\circ - 45.58^\circ)100}\right].$$
$$= 100\left[1 - 1/100\sqrt{10{,}000 - \tan^2(-26.90^\circ)100}\right].$$
$$= 100\left[1 - 1/100\sqrt{10{,}000 - .5073^2(100)}\right].$$
$$= 100\left[1 - 1/100\sqrt{10{,}000 - 25.75}\right].$$
$$= 100\left[1 - 1/100\sqrt{9974.25}\right].$$
$$= 100\left[1 - 1/100(99.875)\right].$$
$$= 100(1 - .99875).$$
$$= 100(.00125).$$
$$\% \text{ error} = .125.$$

Inspection of the formula shows that the per-cent error depends upon the size of the angle GAB, the angle between voltage drop vector and the receiver voltage vector, and upon the relation of D to E. When the ratio D/E_0 is small, the error is small. When the angle GAB is zero, that is, when the impedence or voltage drop vector is in line with the receiver voltage vector, the error is zero. the curves show this latter point where they are tangent to the line of zero error.

The curves of chart C show that for most cases, the error is less than 1%. At unity power-factor, for sizes larger than No. 00, and at a power-factor of .95 for sizes larger than 350,000 circular mils, the error exceeds this value, and judgement will accordingly have to be used in the application of the method by taking this limitation into consideration.

In order to give some idea of the variation in error for other drops than 10%, calculations have been made by the same method for drops from 0 to 15%, for No. 10 wire at .7 power-factor, for No. 00 wire at the power-factor of 1.0, and for 300,000 circular mil cable at .9 power-factor. These sizes and power-factors were chosen as representing a range of error at 10% drop up to about 1%. These results were put into curves as shown by chart D. It will be noticed that for the first few per-cent drop below 10, the error decreases quite rapidly.

THE EFFECT OF CAPACITY.

Another thing necessary to consider in transmission line problems is the effect of line capacity. In most cases, where the line is short, this effect can be neglected. The capacity effect causes the voltage to rise at the receiver end of the line, especialy at no load, where there is no lagging component of voltage due to inductive drop to counteract the leading component due to the line capacity. This rise in voltage at the end of the line at no load may be taken as a measure of the capacity effect. The per-cent increase in voltage depends upon the length of the line, the frequency, and the spacing.

The per-cent rise in voltage has been computed for numbers 4 and 0 wire, and for 250,000 and 500,000 circular mil cable, at 25 and 60 cycles, and for five foot equivalent spacing, and for distances up to slightly above 300 miles. Curves have been plotted showing these results in chart E.

The formula used in these calculations was published by F. W. Peek in the General Electric Review of June, 1913, in the article entitled, "Practical Calculations of Long Distance Transmission Line Characteristics." The formula is,

$$\underset{\cdot}{E}_g = \underset{\cdot}{E}_r(1+ZY/2)+Z\underset{\cdot}{I}_r(1+ZY/6) \ .$$

The nomenclature is as follows.

$\underset{\cdot}{E}_g$=generator volts to neutral.

$\underset{\cdot}{E}_r$=receiver volts to neutral.

$\underset{\cdot}{I}_r$=amperes per conductor at receiver end.

Z=impedence of one conductor=r - jx.

r=resistance of one conductor in ohms.

x=inductive reactance of one conductor to neutral, ohms.

C=capacity in farads per single conductor to neutral.

f=frequency in cycles per second.

$Y = -2\pi fCj$.

The dot beneath a symbol denotes a vector value. j is the operator used to denote vector addition, being chosen to represent lag when positive and lead when negative.

Since in this case, application is made of the formula at no load only, $\underset{\cdot}{I}_r=0$, and the last term of the second member=0.

The formula then becomes,

$$\dot{E}_g = \dot{E}_r(1 + ZY/2),$$

or $$\dot{E}_g/\dot{E}_r = 1 + ZY/2,$$

or $$\dot{E}_g/\dot{E}_r - 1 = ZY/2,$$

or $$\dot{E}_g/\dot{E}_r - \dot{E}_r/\dot{E}_r = (\dot{E}_g - \dot{E}_r)/\dot{E}_r = ZY/2,$$

or $(\dot{E}_g - \dot{E}_r)/\dot{E}_r(100) = 100ZY/2 = \%$ increase in receiver volts,

or % increase in receiver volts $= 100ZY/2 = 100/2(r + jx)(-2\pi fC)$.

An example of the calculation is given for No. 0 wire.

r per mile $= .528$. ohms.

x per mile $= .746$ ohms, with 5' spacing, at 60 cycles.

C per mile $= (.01511)(10^{-6})$farads, with 5' spacing.

$$\% \text{ increase} = 100/2(.528 + .746j)(-2\pi 60j.01511)10^{-6}.$$

$$= (.528 + .746)(-j5.71)10^{-4}.$$

$$= (-3.015j - 4.26)10^{-4}.$$

$$= 10^{-4}\sqrt{(-3.015)^2 + (-4.26)^2}\,.$$

$$= 10^{-4}\sqrt{27.25}\,.$$

$$= (5.22)10^{-4}.$$

$= .000522$, for one mile.

Since Z and Y each vary as the distance and since the per-cent rise in voltage varies as their product, the latter will vary as the square of the distance.

Therefore the per-cent rise in voltage for 100 miles in the above example for No. 0 wire is, $.00052(100)^2 = 5.22$.

The other values used in the plotting of the curves were determined in the same way.

The curves of chart E then show that with the larger sizes of wire and the lower frequencies, the capacity effect is the least and can be neglected for longer distances. The actual

distances for which this may be neglected can be judged from chart E, depending upon the per-cent increase in voltage at no load which is allowable without taking the capacity calculations into consideration. Also, if the equivalent spacing is greater than five feet, as used in the calculation of the chart, the capacity effect will be correspondingly less and may be neglected for longer distances.

THE APPLICATION TO A PROBLEM.

In order that the application of the method to a practical problem may be fully understood, an example is worked out completely.

Let it be desired to determine the regulation and loss of a three phase line consisting of three number 0 solid wires, horizontally spaced with three and six feet between wires on a standard ten foot transmission cross-arm. The line is 100 miles long, operates at 25 cycles, at a voltage of 70,000 at the receiver end, at 80% power-factor, under a load of 6,000 kilowatts.

Equivalent spacing $= \sqrt[3]{3x6x9} = 5.45$ ft.

(Timbie & Higbie, A.C. Electricity, Course II, P. 343).

Amp. per conductor $= 6,000,000/70,000x1.73x.80 = 62$.

Refering to chart A, the drop for No. 0 wire at 60 cycles and 24" spacing is .1522 volts per ampere paer 1,000 ft. It is necessary to apply a correction to bring this to the conditions of the problem, which is done by means of chart F.

Refering to the table accompanying chart F, $X\sin\phi$

per 1,000 ft. for No. 0 wire at 24" spacing, 60 cycles, and .80 power-factor, is .0722. Refering to the diagram, the intersection of the No. 0 solid wire line with the 5.45 ft. spacing line (taken between the 60" and 72" lines) falls at .24 minus ohms reactance per 100 cycles. Following the circle from this point to the 25 cycle radial line and projecting the intersection across horizontally to the 80% power-factor line, by projecting vertically to the Xsinø scale, we read for Xsinø per 1,000 ft. at 25 cycles, 5.45 ft. spacing and .80 power-factor, .0364.

The correction to be applied then is

.0358 - .0722=.0364,

and the drop per ampere per 1,000 ft. under the conditions of the problem is .1523 - .0364=.1159 volts.

Applying the factors for length of line and load,

drop=.1159 x 100 x 5.28 x 62=3790 volts,

and % regulation=3790 x 1.73 x 100 / 70,000=9.38.

Refering to chart B, the loss per 100 ft. per ampere for No. 0 wire is .102 watts. The loss for the conditions of the problem then will equal per conductor,

$100 \times 5.28 \times 62^2 \times .102 = 20.7$kw.

or 20.7 x 3 x 100 / 6,000=10.36,

or % loss=10.36.

Now refering to chart C, it is seen that for No. 0 wire at .8 power-factor, the error is less than .03% and can therefore be neglected. This error is of course for 10% drop, which is very nearly that of the problem.

Refering next to chart C, it is seen that for No. 0

wire, the capacity effect for 100 miles would cause a rise in voltage at the receiver end at no load (25 cycles) of about 2%, which may be considered negligible.

CONCLUSION.

In conclusion, it may be said that while these curves and charts may not in every case provide the best solution of the problem in hand, it is thot that they will offer an easily applied method for the solution of a large number of the problems met with in every day practice.

The general survey afforded of the field of application is considered a valuable asset of these charts for the reason that one can often obtain from them by inspection, the relative characteristics of different lines, and in many cases, where accurate information is not required, the direct reading from the chart will suffice even tho the conditions may differ somewhat from those for which the charts were drawn.

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