A QUADRILATERAL SURVEY CONTROL FOR THE CITY OF EAST LANSING

APPLIN

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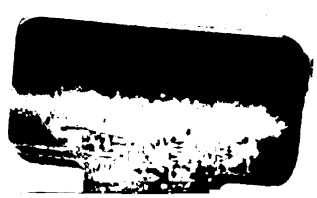

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A QUADRILATERAL SURVEY CONTROL
for the
CITY OF EAST LANSING, MICHIGAN

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Michigan Agricultural College

1911.

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A Quadrilateral Survey Control.

A quadrilateral survey of a city is a very practical problem for civil engineers. The work on the following pages is the establishment of a system of quadrilaterals covering the city of East Lansing, which will be of future use to the Civil Engineering Department of M. A. C. for checking other city surveys and in tying to topographical maps. In this system a base line and the adjoining quadrilateral will afford a control for the classes in higher surveying (geodesy).

The thesis was begun in the latter part of March, 1911, by R. J. Van Winkle and C. A. Hamilton, but owing to the inclement weather very little was accomplished except the reconnoissance and the location of several stations.

Several elevated points such as the U. S. Weather Bureau Observatory, the Engineering Hall, Prof. Snyder's residence, and the water towers were visited and their practicability as stations determined. The ground north of the Red Cedar River was also examined with a view to using the base line parallel to the first lane on the west side of the drive. No suitable quadrilateral could be formed on this, hence the line was abandoned. An apparently suitable line was then found south of the Red Cedar on the farm of C. D. Woodbury, which promised to be feasible as a base line and was intended to be used as such. This with points

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on the steeple of Pres. Snyder's residence and the Engineering Hall was to form the first quadrilateral. (See Map).

After reading the [illegible] work it was found that this proposed base was outside the city limits. Since it was intended to have line but was retained as a base of the first quadrilateral as described above. For [illegible] ability, in order to establish a base which could be cross [illegible], a line of fully equivalent length on the property line between the College Athletic Field and that of Mr. McCaffery was chosen. The line as finally selected runs from the three inch iron rod on the south, terminating at the south end of a forty foot path south of the River Drive.

After considering the [illegible] proposed base line for its intended purpose, the line was successfully extended into a quadrilateral of which it forms a diagonal; the other diagonal being on outer side of the quadrilateral adjoining the [illegible] base line.

After one of the quadrilaterals had been measured J. W. Aplin and C. A. Hamilton took up the [illegible] and R. J. Van Wagenen was appointed to leave college on account of illness.

On [illegible] ability was experienced in measuring angles on account of the very much wind [illegible] time was lost in waiting for suitable weather.

Formation of Quadrilaterals.

Five quadrilaterals were formed as follows:-

Quadrilateral No. 1 ABCD (sketch). This was laid out by having a preliminary base line AB, mentioned above, on Mr. Woodbury's farm, the other two points C & D being on Prof. Snyder's residence and the Engineering Hall, respectively.

Quadrilateral No. 2 CDEF. Points C & D were as mentioned above and E & F were points on the east and west water towers respectively.

Quadrilateral No. 3 AKLN. AB is the preliminary base line as given and KL is a line cutting AB.

Quadrilateral No. 4 BGHK. This was formed by constructing HK with the two lines, BC, which runs along the boundary of the Campus adjoining Mr. Woodbury's land, being between the rail fence and the cobble road of Washington.

Quadrilateral No. 5 BGIJ. B & G are the same points as designated in quadrilateral No. 4. "I" is a point in Lot No. 4, plan in Plot, on Canal River Avenue and J is a station in the southern corner of the poultry yards.

For the exact location of all stations, see note book, pages 41 to 47.

In quadrilateral surveying the most advisable angle

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is 30 degrees. In this survey the pursuit terrels were laid out to conform with this rule as nearly as possible. It so happened that pursuit terrel No. 2 CDEF had four angles less than 30 degrees. It would have been impracticable to change these stations since no other elevated points could be used to advantage. Pursuit terrels No. 3 and No. 4 had four angles less than 30 degrees when they were formed at first, but they were again placed so that no angle was less than 30 degrees.

Description of Instruments.

The transit used in this survey was the Fauth transit, No. 16, owned by the C. E. Department. The instrument has three leveling screws, inverting telescope, seven inch horizontal circle with least reading of ten seconds.

The base line was measured with a five hundred (500) foot engineer's steel tape made by Kaufel and Esser Co. The entire length was permanented into .01 of a foot.

The angle readings at the pursuit stations were comparatively easy to take, tripod circles furnished by the C. E. Department being used. The tripod consist of a tripod which supports the instrument which is centrally turned in a bushing and clamped vertically by a milled nut. The upper part of the instrument is about five inches in diameter and is threaded in one end so receive the lower part which is

about two inches in diameter. In nearly every case the bottom of the vertical signal near the top of the tripod could be seen. In setting the signals great care was taken to have the centre over the point but it was usually found necessary to plumb below the line. The signal is held in place by three guy lines and these were adjusted so as to bring the signal vertical. The signal was plumbed by holding the plumb bob between the eye and the station.

The stations were selected on other device but to be employed for signals. Station C which is located on Pine. Observer's residence could be easily seen from all the other stations only a broom stick they might be used. It was held vertical by means of guy lines. At station D a range pole was placed on the point and carefully fixed in.

At stations E & F ranging poles were used on the outer towers. At station E, the water tower east of the People's Church, the pole was placed on the station in position sufficient to set the transit over. At station F the station was so near to the tower it impossible to have the station within the tower. The station was located over the outer edge of the tower the ranging pole securely fixed in place. In ranging the signal at station "F" some error was likely to have been made in ranging out the transit. A piece of oak two by four about five feet long was employed to the platform, the ranging out marks to the transit. In

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this was a hole about five-eighths of an inch in diameter in which one of the tripod legs rested. The other two legs were placed in similar holes in the ceiling. This afforded a suitable means of adjusting the telescope accurately level. The transit was prevented from falling to the floor by placing it in a heavy wooden box on the wall. A piece of rubber was placed between the box and the wall.

Instruments.

Instruments of various sorts were used in the ground to locate stations. The principal instrument was that of a good 3 to 1 picture of Gurley level and Fauth solar attachment. They were six inches square at the base and four inches square at the top. The length was thirty-six inches and the top was provided with a steel bar about four inches in length.

The monuments set under the outer towers were located by means of a heavy lead plumb bob suspended from the station on the platform above. The plumb bob used was carefully turned on a lathe to a sharp point about two inches long. The points were found with great satisfaction as the bob settled very quickly. The monuments set directly under the towers were very little used.

The other monuments were set at stations I, J, R and S. A tripod signal was set over the station and a plumb bob suspended. Four tubes were set about the station in such positions that the two lines drawn over the tops of the tubes would intersect over the station. The point

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where the lines were drawn over the stakes were carefully marked and the lines removed. The holes were dug to such a depth that the top of the monument was even with the ground. By using the plumb bob and the intersecting lines a double check could be had in locating the monuments. After the monument was set, preferably in place of a stake accurately located, it was then set in the ground and filled around it with earth and rammed in the ground firmly.

Turning of Angles.

The angles were usually repetition. In each case the [illegible] of the original [illegible]. The sum of the two angles at each point was checked by subtracting the entire angle.

In turning the angle there were [illegible] with the telescope normal and three with the telescope reversed. This theoretically would bring the lines to zero but owing to the [illegible] of location, inaccuracies of the vertical cross wire, etc., they do not check. The [illegible] and better results were obtained if they do not check. To simplify computations the instrument was set ten (10) minutes to start with so if part of the final turning would always be positive.

The program of reading the angles is as follows:-

Telescope Normal.

1. Set on left station, read both verniers.
2. Unclamp sleeve and set on right station; read verniers.
3. " lower " " " left "
4. " sleeve " " " right " read verniers.
5. " lower " " " left "
6. " sleeve " " " right " read verniers.

Telescope reversed.

1. Set on right station.
2. Unclamp sleeve and set on left station; read verniers.
3. " lower " " " right "
4. " sleeve " " " left " read verniers.
5. " lower " " " right "
6. " sleeve " " " left " read verniers.

The mean value of the vernier readings is found from the difference of two consecutive readings taken. The mean value of these differences is taken as the observed value of the angle.

At station "C" an additional mark is set up. One angle is read in place by the instrument in one half, while the rest of lines are taken in reverse order, one half from their position. This necessitates the rearrangement of the instrument, following carefully the above rules relating to the level. Care must be taken to have

the vertical cross hair perpendicular to the axis of the instrument to mark the line of sight perpendicular to the horizontal axis of the transit scope.

Location of grade line.

The type for locating the grade line was a five hundred (500) foot Engineer's standard type. It was standardized by combining it with a recovery stake and its coefficient of distance in the grade book, page 61. The type was designed to be used throughout, one hundred (100) foot intervals from the line.

In the location of the grade line a transit was set over one station and backsighted on the line every hundred feet. A level was used to get the elevation and grade. The height of the grade above or below the ground at any point. The other tools necessary to locate elevation and grade are given in the following type. The entire type can be used for this application, but the limit is one hundred (100) feet, ordinarily the first one hundred (100) feet. The reason for this is short measurements are necessary because of the great difference in elevation of grade.

In locating grade lines, it is not necessary that the entire cut be full, generally a spring line, parallel to the type.

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The [illegible] are fully discussed in Johnson's Surveying, pages 467 to 510. The [illegible] have to be applied, since in surveying, the type are found to have definite [illegible] with small and [illegible].

Adjustment of Quadrilaterals.

When a quadrilateral is formed there results [illegible], but in each case, [illegible] triangle. The [illegible] conditions to be fulfilled are:

(1) The [illegible] in each triangle formed must be 180 deg., [illegible] the angles at the intersection of the diagonals must be equal.

(2) The computed length of any [illegible] from any other side through two independent sets of triangles [illegible] the same in both cases.

The [illegible] quadrilateral [illegible].

The method used in adjusting the quadrilaterals was that described in "The Theory and Practice of Surveying" by Johnson (1908 edition) on pages 470 to 474. Only the right [illegible] was used. The logs. since were taken from [illegible] in Trigonometry Tables.

[illegible] taking into account [illegible] all [illegible] [illegible] [illegible] [illegible].

Computation of Final.

In the case of [illegible] the line bh is corrected to the [illegible] which is computed through the aid of [illegible] previously [illegible].

The only [illegible] on the base line. From this line the [illegible] by [illegible] successively using the law of sines. [illegible] logarithmic tables were used in the computations.

Probable Errors.

The errors of observations in [illegible] of three kinds: (1) [illegible]; (2) [illegible] errors; (3) [illegible] errors. By [illegible] errors is meant errors [illegible] which [illegible] in the triangles and by [illegible] errors have been eliminated. These accidental errors are [illegible] by the theory of least squares as given in Chapter IX of the "Manual of Field and Office Methods", by [illegible].

The probable error in [illegible] is computed by the formula:-

$$P = .674 \sqrt{\frac{\Sigma d^2}{n(n-1)}} \qquad \text{(See Page 6 Notebook page 11)}$$

Where P is the probable error of the mean of all closing lines.

n = the number of observations.

d = the difference between any observation and the mean of all observations.

Conclusion.

In the preceding pages of this report a survey and description of the triangulation in Wyoming has been given. It is a well known fact in all extensive surveys, that in order to obtain reliable and suitable work with great economy, the triangulation work must have training and skilled supervision from beginning to end. The training and knowledge received from this work has an intrinsic value to the young engineer.

By adjusting the probable errors of the angles of all the observations in the triangulation, this ranges from ±1.0 second to ±1.4 seconds, it can be seen that a very accurate survey in this state. The quadrilateral No. 1 AMCD, for example, is not as accurate as the remaining parts of the survey, having the greatest probable error, that is, ±1.4 seconds. This quadrilateral is to be regretted that it was the first to be adjusted, but another method of adjustment was not then known.

This survey is not complete in the section covered for other surveys may be made in the future. The ground covered by the triangulation does not extend over the entire county, hence a continuation of the survey is necessary.

A resurvey of this quadrilateral is not necessary, but it could be strengthened if the azimuth of any one of the lines could be determined with a transit and solar attachment in connection with the general survey.

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Inc[illegible] Linear, C[illegible] with Time and Log Time, of [illegible].

No.	Final Corrected Value	Log Variable Time.	Incl[illegible] Linear Coef[illegible]. [illegible].
			±
A_1 [illegible]	7 - 7=[illegible]	[illegible]	4.6
A_8 CAD	[illegible] - 1.15	[illegible]	0.7
A_0 [illegible]	11[illegible]	[illegible]	
C_4 [illegible]	[illegible]	[illegible]	0.3
C_6 [illegible]	[illegible]	[illegible]	
C_0 [illegible]	[illegible]	[illegible]	
D_4 [illegible]	[illegible]	[illegible]	
D_7 [illegible]	[illegible]	[illegible]	
D_0 [illegible]	[illegible]	[illegible]	
D_2 [illegible]	[illegible]	[illegible]	
E_3 [illegible]	[illegible]	[illegible]	
E_0 [illegible]	[illegible]	[illegible]	
	Dupl. N. S.		
C_7 [illegible]	[illegible]	[illegible]	
C_8 [illegible]	[illegible]	[illegible]	0.4
C_0 [illegible]	134-[illegible]	[illegible]	
D_2 [illegible]	[illegible]	[illegible]	
E_3 [illegible]	[illegible]	[illegible]	
E_0 [illegible]	[illegible]	[illegible]	
E_9 [illegible]	[illegible]	[illegible]	
E_5 [illegible]	[illegible]	[illegible]	0.1
F_0 [illegible]	[illegible]	[illegible]	
F_7 [illegible]	[illegible]	[illegible]	
F_0 [illegible]	[illegible]	[illegible]	
	Dupl. Pl. J.		
A_1 [illegible]	[illegible]	[illegible]	
A_8 [illegible]	[illegible]	[illegible]	1.3
A_0 [illegible]	[illegible]	[illegible]	
B_2 [illegible]	[illegible]	[illegible]	
B_3 [illegible]	[illegible]	[illegible]	
B_0 [illegible]	[illegible]	[illegible]	
B_9 [illegible]	[illegible]	[illegible]	
E_5 [illegible]	[illegible]	[illegible]	
D_0 [illegible]	[illegible]	[illegible]	
F_6 [illegible]	[illegible]	[illegible]	
F_7 [illegible]	[illegible]	[illegible]	
F_0 [illegible]	[illegible]	[illegible]	

Test Air-Error, [illegible] Values [illegible] Deg. Time of [illegible].

([illegible])

[illegible] No. 4.

D_1	[illegible]	[illegible]	[illegible]	[illegible]
I_8	[illegible]	[illegible]	[illegible]	[illegible]
P_0	[illegible]	[illegible]	[illegible]	[illegible]
D_2	[illegible]	[illegible]	[illegible]	[illegible]
I_3	[illegible]	[illegible]	[illegible]	[illegible]
P_0	[illegible]	[illegible]	[illegible]	[illegible]
I_4	[illegible]	[illegible]	[illegible]	[illegible]
I_6	[illegible]	[illegible]	[illegible]	[illegible]
P_0	[illegible]	[illegible]	[illegible]	[illegible]
C_6	[illegible]	[illegible]	[illegible]	[illegible]
C_7	[illegible]	[illegible]	[illegible]	[illegible]
C_0	[illegible]	[illegible]	[illegible]	[illegible]

[illegible] No. 5.

D_1	[illegible]	[illegible]	[illegible]	[illegible]
D_8	[illegible]	[illegible]	[illegible]	[illegible]
I_0	[illegible]	[illegible]	[illegible]	[illegible]
J_2	[illegible]	[illegible]	[illegible]	[illegible]
J_3	[illegible]	[illegible]	[illegible]	[illegible]
J_8	[illegible]	[illegible]	[illegible]	[illegible]
I_4	[illegible]	[illegible]	[illegible]	[illegible]
I_5	[illegible]	[illegible]	[illegible]	[illegible]
I_0	[illegible]	[illegible]	[illegible]	[illegible]
I_6	[illegible]	[illegible]	[illegible]	[illegible]
I_7	[illegible]	[illegible]	[illegible]	[illegible]
D_0	[illegible]	[illegible]	[illegible]	[illegible]

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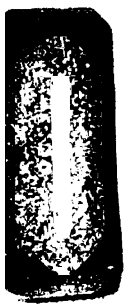
Length and Logarithm of Sides.

Side.	Logarithm.	Length. In feet.
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
MA	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
AD	[illegible]	[illegible]
AB	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
AC	[illegible]	[illegible]
DC	[illegible]	[illegible]
CD	[illegible]	[illegible]
CD	[illegible]	[illegible]
CF	[illegible]	[illegible]
DF	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
CE	[illegible]	[illegible]
FE	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
DI	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
IJ	[illegible]	[illegible]
IJ	[illegible]	[illegible]



Quadrilateral No. 1-ADCB

Angles	Observed Values	Corrections for Angle equations	First Corrected Values	Log. Sines.	Tab Diff.	Corrections for Side equations.	Final corrected Angles.	Check Log. Sines.
A_1	72° 07′ 48.″3/6	− 9.″15	07′ 39.″18	9.9785192	6.8	+ 2.″81	72° 07′ 41.″99	9.9785198
D_3	34 − 50 − 46	− 4.60	50 − 40.40	9.7569037	30.3	+ 3.93	34 − 50 − 44.33	9.7569156
C_5	36 − 16 − 00 5/6	− 1.66	15 − 59.18	9.7719848	28.7	+ 2.01	36 − 16 − 01.19	9.7719906
B_7	37 − 17 − 18 2/6	− 6.19	17 − 12.14	9.7823320	27.6	+ 3.86	37 − 17 − 16.00	9.7823427
				.2897397				.2897687
D_2	24 − 36 − 54 1/6	− 9.14	36 − 45.03	9.6195934	45.9	− 4.17	24 − 36 − 40.86	9.6195742
C_4	48 − 25 − 00	− 4.60	24 − 55.40	9.8738879	18.7	− 2.57	48 − 24 − 52.83	9.8738831
B_6	60 − 28 − 26 4/6	− 1.66	28 − 26.02	9.9395136	11.9	− 3.37	60 − 28 − 21.65	9.9395796
A_8	45 − 58 − 29 5/6	− 6.19	58 − 23.65	9.8567381	20.3	− 2.50	45 − 58 − 21.15	9.8567330
	360 − 00 − 43 1/6	43.17	00 − 00.00	.2898030		00.00	360 − 00 −00.00	.2897699
				l_4 = −6.33				l_4 = −12.

l_3 = +43.17
l_2 = + 3.17
l_1 = −15.00

C_0 = − 41.2
C_1 = + 52.7
C_2 = + 49.0
C_3 = + 40.6
C_4 = + 47.9

$\frac{1}{4}C_0^2$ = 424
$\frac{1}{4}C_2^2$ = 2777
$\frac{1}{4}C_3^2$ = 2401
$\frac{1}{4}C_4^2$ = 1648
$\frac{1}{4}C_1^2$ = 2294

$$\frac{4\,l_4}{4\,l_2} = -\frac{l_4}{\frac{C_0^2}{4} + \Sigma\left(\frac{C^2}{4}\right)} = \frac{6.33}{95.45} = .0663$$

x_0 = − 6.8
x_1 = + 3.49
x_2 = + 3.25
x_3 = + 2.69
x_4 = + 3.18

Quadrilateral No. 2—CDEF

Angles	Observed Values	Corrections for Angle equations	First Corrected Values	Log. Sines.	Tab. Diff.	Corrections for Side equations	Final corrected Angles.	Check Log. Sines.
C_1	56° 01′ 26″ 5/6	− 7.1	01′ 19.″7	9.91868874	14.2	+ 0.″70	56° 01′ 20.″40	9.91868884
D_3	28 − 22 − 32 3/6	+ 0.4	22 − 32.9	9.67692466	39.0	+ 1.82	28 − 22 − 34.72	9.67693171
E_5	54 − 05 − 58 3/6	− 0.1	05 − 58.6	9.90850552	15.2	+ 0.60	54 − 05 − 59.20	9.90850611
F_7	24 − 21 − 20	− 7.5	21 − 12.5	9.61528115	46.5	+ 1.96	24 − 21 − 14.45	9.61529061
				.11939887				.11941688
D_2	20° 58′ 33″ 2/6	− 7.1	58 − 26.2	9.55381143	64.9	− 2.36	20 − 58 − 23.84	9.55380114
E_4	74 − 37 − 40 5/6	+ 0.4	37 − 41.1	9.98417786	5.7	− 0.16	74 − 37 − 40.94	9.98417785
F_6	82 − 53 − 47 2/6	− 0.1	53 − 47.4	9.59002522	49.8	− 2.26	82 − 53 − 45.14	9.59000139
C_8	78 − 39 − 09 1/6	− 7.6	39 − 01.6	9.99142322	4.2	− 0.29	78 − 39 − 1.31	9.99914231
			00 − 00.00	.11944413		00.00	360 − 00 − 00.00	.11941689
				$l_4 = -426$				$l_4' = -1$

$l_3 = +28.50$
$l_5 = +16.00$
$l_7 = -14.00$

C_0 =	− 150.9
C_1 =	+ 69.1
C_2 =	+ 44.7
C_3 =	+ 65.0
C_4 =	+ 50.7

$\frac{1}{4}C_0^2$ =	5693
C_1^2 =	4775
C_2^2 =	1998
C_3^2 =	4225
C_4^2 =	2570

$$\frac{426}{150.9} = \frac{l_4}{\frac{C_0^2}{n} + \Sigma(C^2)}$$

$$= \frac{426}{19261}$$

$$= .02212$$

$x_0 = -.83$
$x_1 = +1.53$
$x_2 = +.99$
$x_3 = +1.43$
$x_4 = +1.12$

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Quadrilateral No. 3 – ANBM

Angles	Observed Values	Corrections for Angle equations	First Corrected Values	Log. Sines.	Tab. Diff.	Corrections for Side equations	Final corrected Angles	Check Log. Sines.
H_1	55° 23′ 07″	− 1.″17	23′ 05.″82	9.91539930	14.5	+ 2.″94	55° 23′ 08.″76	9.91539973
N_3	44 46 33 5/6	− 1.92	46 31.96	9.84777769	21.2	+ 2.71	44 46 34.67	9.84777827
B_5	30 26 07 3/6	− 4.33	26 03.00	9.70462206	36.8	+ 3.48	30 26 06.48	9.70463331
M_7	44 55 31 3/6	− 3.58	55 27.75	9.84891 10	21.1	+ 2.71	44 55 30.46	9.84891168
				.3167015				.3167299
N_2	46 26 32 2/6	− 1.17	25 31.15	9.86002 43	20.1	− 1.50	46 25 29.65	9.86002 13
B_4	33 24 53	− 1.92	24 51.07	9.74090 51	32.0	− 4.15	33 24 46.92	9.74088 19
M_6	71 22 38 3/6	− 4.33	22 34.00	9.97766 412	7.1	− 2.04	71 22 31.96	9.97766 398
H_8	33 15 58 0/6	− 3.58	15 55.26	9.73971902	32.1	− 4.15	33 15 51.10	9.73971769
	360 00 22 1/6		00 00.00	.3167827		00.00		.3167297
				$L_4 = -593$				$\Sigma' = 0$

$l_3 = +22.17$
$l_2 = +3.33$
$l_1 = +6.33$

$C_0 =$	+ 44.9
$C_1 =$	+ 34.6
$C_2 =$	+ 53.2
$C_3 =$	+ 42.9
$C_4 =$	+ 63.2

$\frac{1}{4}C_0^2 =$	504
$C_1^2 =$	1197
$C_2^2 =$	2829
$C_3^2 =$	1839
$C_4^2 =$	2529

$$\frac{440}{44.9} = -\frac{L_4}{\frac{C_0^2}{4} + \Sigma(C^2)}$$

$$= \frac{593}{9198}$$

$$= .0646$$

$x_0 =$	+ .722
$x_1 =$	+ 2.22
$x_2 =$	+ 3.43
$x_3 =$	+ 2.76
$x_4 =$	+ 3.43

Quadrilateral No. 4–RNMS

Angles	Observed Values	Corrections for Angle equations	First Corrected Values	Log. Sines.	Tab Diff.	Corrections for Side equations	Final corrected Angles.	Check Log. Sines.
R_1	30° 18′ 15″ 5/6	− 17.7	17′ − 58″.1	9.7028781	36.0	− 0″.15	30° − 17′ − 57″.95	9.7028775
N_3	86 − 08 − 58 2/6	− 7.2	08 − 51.1	9.9990176	1.4	− 0.06	86 − 08 − 51.04	9.9990175
M_5	30 − 06 − 37 3/6	+ 8.9	06 − 46.4	9.7004487	36.3	− 0.15	30 − 06 − 46.25	9.7004482
S_7	35 − 30 − 10 5/6	− 1.6	30 − 9.3	9.7639815	29.6	− 0.06	35 − 30 − 09.25	9.7639814
				.1663259				.1663246
N_2	32 − 05 − 31 1/6	− 17.7	05 − 14.0	9.7252659	33.6	+ 0.13	32 − 05 − 14.13	9.7252664
M_4	31 − 28 − 05	− 7.2	27 − 57.8	9.7176649	34.4	+ 0.08	31 − 27 − 57.88	9.7176652
S_6	32 − 16 − 16 1/6	+ 8.9	16 − 25.6	9.7275231	33.3	+ 0.13	32 − 16 − 25.73	9.7275136
R_8	82 − 06 − 39 1/6	− 1.5	06 − 37.7	9.9958696	2.9	+ 0.08	82 − 06 − 37.78	9.9958697
	360 − 00 − 36	36.00	00 − 00.00	.1663235		00.00	360 − 00 − 00.00	.1663249
				$2_4 = 24$				$2'_4 = -3$

$l_3 = +35$
$l_2 = -11.33$
$l_1 = -63.83$

$C_0 = +11.7$
$C_1 = +69.6$
$C_2 = +35.8$
$C_3 = +69.6$
$C_4 = +32.5$

$\frac{1}{4}C_0^2 = 36$
$\frac{1}{4}C_1^2 = 4844$
$\frac{1}{4}C_2^2 = 1281$
$\frac{1}{4}C_3^2 = 4844$
$\frac{1}{4}C_4^2 = 1056$

$$\frac{4x_0}{11.7} = -\frac{l_4}{\frac{C_4^2}{4} + \Sigma\left(\frac{C^2}{4}\right)} = \frac{24}{12061} = .00199$$

$x_0 = -0.01$
$x_1 = -0.14$
$x_2 = -0.071$
$x_3 = -0.14$
$x_4 = -0.065$

Quadrilateral No. 5-0d1E

Angles	Observed Values	Corrections for Angle equations	First Corrected Values	Log. Sines.	Tab. Diff.	Corrections for Side equations	Final corrected Angles.	Check Log. Sines.
Π_1	42°-39'-16".4/6	−8.95	39'-07".71	9.8309386	22.8	+ 5".47	42°-39'-13".18	9.8309510
d_3	34-57-35	− 5.63	57-29.37	9.7581380	30.1	+ 9.79	34-57-39.16	9.7581675
I_6	57-26-21.4/6	− 1.87	26-19.80	9.9267334	13.4	+ 7.10	57-26-26.90	9.9267430
E_7	39-54-30.5/6	− 5.21	54-26.62	9.8072271	25.2	+ 9.36	39-54-34.98	9.8072506
				.3220371				.3221121
d_2	40-43-45.9/6	− 8.96	43-36.87	9.8145502	24.4	− 10.77	40-43-26.10	9.8145238
I_4	61-39-51.7/6	− 5.63	39-46.04	9.9445662	11.4	− 4.49	61-39-41.56	9.9445611
E_6	25-56-26.4/6	− 1.87	56-24.80	9.6409117	43.3	− 12.40	25-56-12.40	9.6408581
Π_8	56-42-55	− 5.21	42-49.79	9.9221760	13.8	− 4.06	56-42-45.73	9.9221694
	360-00-43.3/6	43.33		.3222031			360-00-00.00	.3221124
				$l_4 = -1660$				$l_4' = -3$

$l_2 = +43.33$
$l_2 = -0.83$
$l_1 = -14.17$

$C_0 = -61.6$
$C_1 = +47.2$
$C_2 = +41.5$
$C_3 = +56.7$
$C_4 = +39.0$

$\frac{1}{4}C_0^2 = 949$
$\frac{1}{4}C_1^2 = 2228$
$\frac{1}{4}C_2^2 = 1722$
$\frac{1}{4}C_3^2 = 3215$
$\frac{1}{4}C_4^2 = 1521$

$$\frac{470}{61.6} = -\frac{l_4}{\frac{C_0^2}{4} + \Sigma\left(\frac{C^2}{4}\right)}$$

$$= \frac{1660}{9635}$$

$$= .172$$

$x_0 = -2.65$
$x_1 = +8.12$
$x_2 = +7.14$
$x_3 = +9.75$
$x_4 = +6.71$