TABLES FOR STRESS CALCULATION

THESIS

FOR THE DEGREE OF

CIVIL ENGINEER

JOHN R. LAMBERT

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1914

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AND

PROBLEMS IN DEFLECTIONS

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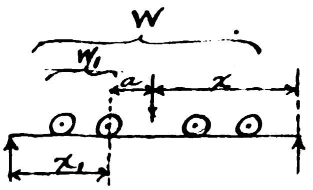
This thesis consists of a set of tables for the calculation of stresses in bridge trusses and girders due to Cooper's wheel loads. The tables are from original computations with the following exceptions: Table II was published in the Engineering News of June 21st, 1906, Tables V and VI were computed by Mr. Clifford Rowe, M.A.C. 1907, and the writer. In addition to the tables there is given a number of problems in which the writer has been interested at various times since graduation in 1906. The problems treat of deflections of beams and trusses and of methods of calculating stresses.

EXPLANATION OF THE TABLES.

Table I gives the static moments of the wheel loads and of the uniform load for every foot up to 233 feet from the head of the first engine. In all the tables the quantities are given for one rail of Cooper's E-50 loading, except where otherwise stated. This table was computed by means of the constant first difference between any two wheels and the constant second difference under the uniform load. Direct interpolation is correct between any two wheels and sufficiently exact between any two feet under the uniform load. The exact moment may be obtained by deducting $1.25(x-x^2)$ where x is the fractional part of the distance This correction is a maximum when $x = 0.5$ its value is then .3125

Table II shows the proper wheel to place over the common support of two adjacent spans in order to find the maximum concentration. Ifthe maximum moment at any point of a span is desired, the table is entered with the two segments of the span, in fact it may be used to determine the setting of the load for the maximum value of any function which can be represented by a triangular influence line.

Table III shows the position of the wheels for the absolute maximum moments in girders, and gives the equations for the moments up to a span of 70 feet, and the span limits over which each equation can be used.



W The total load on the span.

W_1 The load to the left of a given wheel.

"m" Moment of the loads to the left of any given load about that load.

"a" the distance from the centre of gravity of all the loads to any given load, "a" is measured from the centre of gravity and is negative when measured to the left.

The bending moment under any wheel $= \frac{Wx}{l} x_1 - m$.

This expression is a maximum when $x = x_1$ since the sum of x & x_1 is constant, $l + a$. substituting $\frac{l+a}{2}$ for x and x_1, $M = \frac{W(l+a)^2}{4l} - m$

If the moment under the n^{th} wheel is greater than that under the $(n+1)^{th}$ wheel, then

$$\frac{W(l+a_n)^2}{4l} - m_n > \frac{W(l+a_{n+1})^2}{4l} - m_{n+1}$$

Reducing the inequality by substituting for m_{n+1}, $m_n + W_1(a_{n+1} - a_n)$, there is obtained,

$$W_1 > W\left(\frac{1}{2} + \frac{a_n + a_{n+1}}{4l}\right)$$

This is the criterion by which to test for maximum moment. The maximum does not necessarily occur under a wheel which is adjacent to the centre of gravity of all the loads.

The equation for moments may be written, $M = C_1 l + \frac{C_2}{l} + C_3$ in which C_1, C_2, & C_3 are constants for any given wheel in a given set of wheels, $C_1 = \frac{W}{4}$, $C_2 = \frac{Wa^2}{4}$, $C_3 = \frac{Wa}{2} - m$

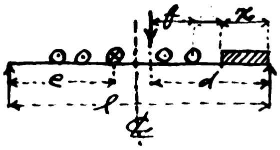
The equations are written for successive numbers of wheels and solved for the simultaneous values of l.

The limits obtained by solution may be restricted by the following conditions: The least span upon which a given set of wheels can be placed in correct position is equal to twice the distance from the mid-point of the distance a to the farthest load used in the calculation of the distance a, and thegreatest span upon which the wheels can be properly placed

is equal to twice the distance from the mid-point of a to the nearest wheel off the span.

With Cooper's loadings, the uniform load is on the span for lengths above 69.9 feet. The distance of the critical wheel from the centre of the span therefore changes for every span length. Equations are given for determining the length of uniform load on the spang and thus the other quantities can be found.

$d = e$ for a critical position



$$\frac{M + Wx + \frac{w}{2}x^2}{W + wx} = l - f - x$$

$$x = \frac{-C \pm \sqrt{C^2 - 6w[M - W(l-f)]}}{3w}$$

M is the moment of the wheels on the span about the head of the train, W is the weight of the wheels, w is the weight per foot of the uniform load, $C = 2W - w(l-f)$

The lengths e, f, l, and x are as shown in the figure.

The equations for length of uniform load given in Table III are written for a Cooper's loading having one pound on the driver. These equations give correct results for all of Cooper's loadings, since the spacing remains the same and the weights are proportional for each class. In no case does the absolute maximum moment for spans above 70 feet exceed the centre moment by one per cent, and the excess is generally much less. In nearly all cases the absolute maximum moment occurs under the wheel which causes maximum centre moment.

Table IIIa gives equations for a Mallet engine, the present standard of the Chesapeake & Ohio R.R.

Table IV gives the maximum moment, the end shear the quarter point shear, and the center shear for girders up to 129 feet in span. The shears at the intermediate points are

useful for computing rivet pitches. For end shears the first driver of the first engine is placed at the end of the span, except for spans 24 to 27 feet in length inclusive, where wheel 5 is placed at the end of the span with wheels 4, 3, 2, and 1 on the span; and except for spans 46 to 62 feet in length, where one engine followed by the uniform load is used. This arrangement governs for quarter point shears on spans 61 to 82 feet, and for centre shears on spans 91 to 124 feet. In figuring the end shears, the pilot wheel is 8 feet off the span to the left. To avoid multiplying the weight on the pilot by the length of the span plus 8 feet, in order to substract its moment about the right end of the span, the following formula was used;

End shear $= \dfrac{M_2 - M_1}{l} - 12{,}500$. M_2 is the moment of all the wheels up to the right end of the span, and M_1 is the moment of the wheel off the span about the left end of the span, equal to 100 in this case.

Shear for 100' foot span $= \dfrac{20{,}100 - 100}{100} - 12{,}500 = 187{,}500$

TABLE V gives the shears in trusses. These are used for finding web stresses in trusses with horizontal chords. They were figured by the usual method, for example the shear in the second panel of a 5 panel truss, the panels 25 feet in length is found with wheel 3 at the panel point.

$$
\begin{aligned}
M_{88} &= 13520 \\
5 \times M_{③} &= \underline{1437.5} \\
& 12082.5 \div 125 = 96{,}700
\end{aligned}
$$

TABLE VI gives the bending moment in trusses.

When the forward wheels of the loading are off the span, the

following method may be used: The moment will be found at the second panel point of a truss consisting of 5 panels of 20 feet. Entering Table II with the segments 40 and 60 feet, it is found that wheel 12 gives the maximum and that the longer segment is ahead.

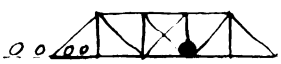
69 + 40 = 109 $\quad M_{109} = 20455 \times 3/5 = 12273.0$

$M_{60} = 8385.0$

$12273.0 - 8385.0 = 3888.0$

69 − 60 = 9 $\quad M_9 = 137.5 \times 2/5 = 55.0$

$3888.0 + 55.0 = 3943.0$



Bending moment.

An explanation of this method is given farther on.

TABLE VII gives the reactions of beams continuous over two equal spans, and the reaction of beams continuous over three spans when the middle span cannot carry shear. The equations for two equal spans can be derived as follows: The value of $EI\alpha$ over the centre support in fig. 1 is $\frac{P\ell^2}{6}(3k_1^2 - k_1^3 - 2k_1)$ where k_1 is measured from the centre support.

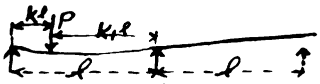


Fig. 1.

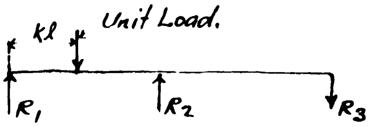
See page 44.

It is desired to measure k from the left support as shown. Substituting $1-k$ for k_1 gives $EI\alpha = -\frac{P\ell^2}{6}(k - k^3)$

Remove the right support. Then the deflection at the right end of the overhanging beam is $-\frac{P\ell^3}{6EI}(k - k^3)$.

With a single load of unity at the right end of the beam the deflection is twice as large as for a cantilever beam of length ℓ, that is, $\frac{2\ell^3}{3EI}$ Then if three supports are furnished and the load placed upon the beam, the reaction at the right end is;

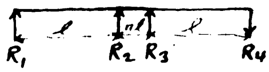
$$-\frac{P\ell^3}{6EI}(k - k^3) \div \frac{2\ell^3}{3EI} = -\frac{P(k-k^3)}{4}$$



For unit load $R_3 = -\frac{k - k^3}{4}$

Taking moments about R_2, $R_1 = 1 - k + R_3$

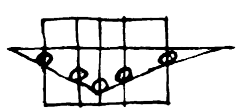
Taking moments about R_1, $R_2 = K - 2R_3$



In the case of three spans, partially continuous, it is assumed that no shear can be carried through the middle span nl; the moment in this span is therefore constant. The end deflection when a load is placed upon the left span and the support at R_4 removed, is as before $-\frac{Pl^3}{6EI}(k-k^3)$ The end deflection with a load placed at the end is greater than that for the two span beams, owing to the effect of the bending moment in the middle span nl. This moment equals l, and the deflection due to it $= l\int_0^{nl}\frac{M\,dx}{EI} = \frac{nl^3}{EI}$

R_4 (for unit load) $= -\frac{l^3}{6EI}(k-k^3) \div \left(\frac{2l^3}{3EI} + \frac{nl^3}{EI}\right) = -\frac{K-K^3}{4+6n}$. R_3, R_2, & R_1 have the values shown on Table VII.

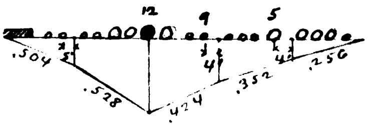
Tables VIII, IX, X, and XI give the bending moments and shears in eight and ten panel swing bridges, for panel lengths of 20, 25, and 30 feet. The influence lines were computed by means of the reactions given in Table VII. The trusses therefore are assumed to act like beams of constant moment of inertia. The moment influence lines are plotted for values of M/l and M/p for the eight and ten panel bridges respectively. The tangent of the angle that each segment makes with the horizontal is given, or numbers which are proportional to these tangents. The loading is set so that $\Sigma P \tan\alpha$ passes through zero. P is the load on any panel. The following mechanical idea is interesting in this connection.



If each wheel of the series is free to rotate and to slide on its vertical rod, while the horizontal distances between the wheels are maintained constant, and if the series is placed upon an influence line, then the position of the wheels when they come to rest is a position for max-

imum influence. It is evident that when an influence line is convex towards the base line, as those for positive moments, critical positions for maximum moment will be obtained only with a wheel at the largest ordinate; but when the influence line is concave towards the base line, as those for negative moments, then critical positions may occur with a wheel at any ordinate. The moments and shears were found by multiplying the panel concentrations by the corresponding ordinates.

The positive moment will be found at the second panel point of the ten panel bridge, using 25 foot panels. Referring to Table II



it is found that wheel 12 placed at the maximum ordinate would cause the greatest moment in a ~~single~~ simple span, the engines are headed toward the right. This position will be tried.

Movement	R & L	L	R	L	R	R & L	R & L
P	57.5	107.5	82.5	53.75	78.75	73.75	87.5
tan α	.504	.528		.424		.352	.256
P tan α	29.0	56.8	43.6	22.8	33.4	26.0	22.4

The rate of change of the moment at the instant the train moves toward the left is found by adding the positive and negative rates for each panel.

	INCREASE	DECREASE
1		29.0
2		56.8
3	22.8	
4	26.0	
5	22.4	
	71.2	85.8
		71.2
		14.6 Net Decrease.

Similarly if the loading moves to the right the rate of change of the moment is:

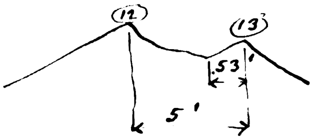
	INCREASE	DECREASE
1	29.0	
2	43.6	
3		33.4
4		26.0
5		22.4
	72.6	81.8 − 72.6 = 9.2 Net Decrease.

Wheel 12 is thus seen to give a maximum value of the bending moment. It is possible that wheel 13 may also give a maximum value, because more uniform load comes on the bridge. This may be determined by taking account of the wheels that pass panel points and of the additional uniform load coming on the bridge as the loading advances five feet.

Wheel 5	25 (.352 -.256)	=	2.40
Wheel 9	16.25(.424-.352)	=	1.17
Uniform load	5' x 2.5 x .504	:	6.30
			9.87 - 9.2 = .67

This shows that the decrease of 9.2 has been changed to an increase of .67 during the five foot advance of the loading. That is, a point of minimum moment exists between the two positions, but wheel 13 has nearly reached the panel point, having only .53 of a foot to go. $\frac{.67}{2.5 \times .504} = .53$. It is evident from this that wheel 12 gives the larger moment.

The change of moment can be represented thus:



Wheel 13 will be tested by $\Sigma P \tan\alpha$

Movement	L	R	L	R	L	R	R & L	R & L
P	70.0	53.75	82.5	73.75	62.5	87.5	65	112.5
tan α	.504		.578		.424		.352	.256
P tan α	35.3	27.1	43.6	38.9	26.5	37.1	22.9	28.8

Movement to Left.

	Increase.	Decrease.
1		35.3
2		43.6
3	26.5	
4	22.9	
5	28.8	
	78.2	78.9
		78.2
		.7 Net Decrease.

Movement to Right.

	Increase.	Decrease.
1	27.1	
2	38.9	
3		37.1
4		22.9
5		28.8
	66.0	88.8
		66.0
		22.8 Net Decrease.

With the loading headed towards the left, the smaller wheels come over the largest ordinate, and while critical positions can be obtained, there is no doubt but that the resulting bending moments would be less than those under wheels 12 and 13 with the loading.

headed toward the right, these moments will be computed.

1.	2.	3.	4.	5.	
19	687.5	2350	.256	601.7	
44	3725	1622.5	.608	986.6	
69	8385	2328.75	1.032	2403.3	
94	15373.75	1767.5	.504	890.9	
119	24130			4882.5	Bending Moment (12)→
24	1150	2247.5	.256	575.4	
49	4547.5	1640	.608	997.2	
74	9585	2363.75	1.032	2439.4	
99	16986.25	1673.75	.504	843.6	
124	26061.25			4855.6	Bending Moment (13)→

Column 1 contains the distance of the panel points from the head of the loading;column 2, the static moments of all the wheels in front of the panle points, these are read from Table I; column 3,the panel concentrations multiplied by the panel length that is $M_{n+1} - 2M_n + M_{n-1}$ where M_n is the static moment at the panel point; column 4 the values of M/p from Table X; and column 5 gives the products of the quantities in columns 3 and 4, which are the bending moments.

It is possible to determine the difference in the moments figured above without actually making both computations, it sometimes takes longer however, than to figure the moments. For example, start with wheel 12 at the largest ordinate and figure the change in the moment for a movement of 5 feet.

Wheels 1, 2, 3, and 4	87.5 × 5 × .256 =	112.0	
" 5	25 × 1 × " =	6.4	
	25 × 4 × .352 =	35.2	
" 6, 7, and 8	48.75 × 5 × " =	85.8	
" 9	16.25 × 1 × " =	5.72	
	16.25 × 4 × .424 =	27.56	
" 10, 11, and 12	62.5 × 5 × " =	132.50	
		405.18	Decrease.

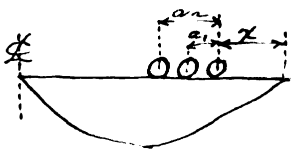
Wheels 13, 14, 15, and 16	82.5 × 5 × .528	217.80	
" 17, and 18	32.5 × 5 × .504	81.9	
Uniform Load - difference in area	$2.5\left[(15 \times .504)\frac{15}{2} - (10 \times .504)\frac{10}{2}\right]$ =	78.75	
		378.45	Increase.
		405.18	Decrease.
		26.73	Net Decrease.

Difference from computed moments = 26.9.

The maximum negative moments at all points in the left span are caused when the loading is upon the right span and in the proper position to cause the maximum uplift at the left support.

The exact placing of the loads upon these influence lines is a matter of more theoretical than practical interest, since with a little practice the loads can be placed by eye in positions which will give values close to the maximum, and moreover, these influence lines do not give true bending moments because the trusses do not follow the three-moment theorem exactly.

Table XII gives the uplift and centre reactions for deck plate girder draw spans. The derivations of the equation for setting the load for uplifts is as follows:



Influence Line for Uplift, R_3.

$$R_3 = \frac{k - k^3}{4}$$

This is the equation of the influence line.

$\frac{dR_3}{dk} = \frac{1-3k^2}{4}$ this is the tangent at any point of the influence line. For maximum influence make $\Sigma w\left(\frac{1-3k^2}{4}\right) = 0$ Substituting for K its value for each load, $K_1 = \frac{x}{l}$, $K_2 = \frac{x+a_1}{l}$, etc. and solving the equation for x, $x = \sqrt{\left(\frac{\Sigma wa}{\Sigma w}\right)^2 - \frac{\Sigma wa^2}{\Sigma w} + \frac{l^2}{3}} - \frac{\Sigma wa}{\Sigma w}$

or $x = \sqrt{\frac{l^2}{3} - A} - B$, where A and B are constants.

The values of A and B for Cooper's loadings are given in the table. For the centre reactions the loading is placed as far maximum reaction for two single spans. Both uplifts and reactions were scaled from plotted influence lines, shown in table XIII.

Table XIV gives the bending moment at the support for beams fixed at both ends.

$M_0 = -Pl\,K(1-K)^2$ This equation is treated similarly to the one of Table XII, and the condition for maximum moment obtained.

Table XIV gives also the bending moment at the support for beams fixed at one end. The conditianal equation is similar to that for uplift. If a beam is fixed at one end and partially fixed at the other, the moment at the fixed end can be interpolated between the two given values.

Tables XV and XVl give the exact equivalent uniform loads for Cooper's E-50 loading, and the curves plotted from the same. These curves are given in Johnson's "Modern Framed Structures" and their use in connection with influence lines is explained.

Table I. Moments for Cooper's E50 Loading - One Rail.

"A"	Position Wheel	"B"	Moment.	"A"	Position Wheel	"B"	Moment.	"A"			Moment.
0	1		0	78		4	10645	156	47	473.75	39901.25
1		1	12.5	79	14	5	10910	157	48	476.25	40375
2		2	25	80		1	11200	158	49	478.75	40851.25
3		3	37.5	81		2	11490	159	50	481.25	41330
4		4	50	82		3	11780	160	51	483.75	41811.25
5		5	62.5	83		4	12070	161	52	486.25	42295
6		6	75	84		5	12360	162	53	488.75	42781.25
7		7	87.5	85		6	12650	163	54	491.25	43270
8	2	8	100	86		7	12940	164	55	493.75	43761.25
9		1	137.5	87		8	13230	165	56	496.25	44255
10		2	175	88	15	9	13520	166	57	498.75	44751.25
11		3	212.5	89		1	13826.25	167	58	501.25	45250
12		4	250	90		2	14132.5	168	59	503.75	45751.25
13	3	5	287.5	91		3	14438.75	169	60	506.25	46255
14		1	350	92		4	14745	170	61	508.75	46761.25
15		2	412.5	93	16	5	15051.25	171	62	511.25	47270
16		3	475	94		1	15373.75	172	63	513.75	47781.25
17		4	537.5	95		2	15696.25	173	64	516.25	48295
18	4	5	600	96		3	16018.75	174	65	518.75	48811.25
19		1	687.5	97		4	16341.25	175	66	521.25	49330
20		2	775	98		5	16663.75	176	67	523.75	49851.25
21		3	862.5	99	17	6	16986.25	177	68	526.25	50375
22		4	950	100		1	17325	178	69	528.75	50901.25
23	5	5	1037.5	101		2	17663.75	179	70	531.25	51430
24		1	1150	102		3	18002.5	180	71	533.75	51961.25
25		2	1262.5	103		4	18341.25	181	72	536.25	52495
26		3	1375	104	18	5	18680	182	73	538.75	53031.25
27		4	1487.5	105		1	19035	183	74	541.25	53570
28		5	1600	106		2	19390	184	75	543.75	54111.25
29		6	1712.5	107		3	19745	185	76	546.25	54655
30		7	1825	108		4	20100	186	77	548.75	55201.25
31		8	1937.5	109	0	5	20455	187	78	551.25	55750
32	6	9	2050	110	1	356.25 358.75	20811.25	188	79	553.75	56301.25
33		1	2178.75	111	2	361.25	21170	189	80	556.25	56855
34		2	2307.5	112	3	363.75	21531.25	190	81	558.75	57411.25
35		3	2436.25	113	4	366.25	21895	191	82	561.25	57970
36		4	2565	114	5	368.75	22261.25	192	83	563.75	58531.25
37	7	5	2693.75	115	6	371.25	22630	193	84	566.25	59095
38		1	2838.75	116	7	373.75	23001.25	194	85	568.75	59661.25
39		2	2983.75	117	8	376.25	23375	195	86	571.25	60230
40		3	3128.75	118	9	378.75	23751.25	196	87	573.75	60801.25
41		4	3273.75	119	10	381.25	24130	197	88	576.25	61375
42		5	3418.75	120	11	383.75	24511.25	198	89	578.75	61951.25
43	8	6	3563.75	121	12	386.25	24895	199	90	581.25	62530
44		1	3725	122	13	388.75	25281.25	200	91	583.75	63111.25
45		2	3886.25	123	14	391.25	25670	201	92	586.25	63695
46		3	4047.5	124	15	393.75	26061.25	202	93	588.75	64281.25
47		4	4208.75	125	16	396.25	26455	203	94	591.25	64870
48	9	5	4370	126	17	398.75	26851.25	204	95	593.75	65461.25
49		1	4547.5	127	18	401.25	27250	205	96	596.25	66055
50		2	4725	128	19	403.75	27651.25	206	97	598.75	66651.25
51		3	4902.5	129	20	406.25	28055	207	98	601.25	67250
52		4	5080	130	21	408.75	28461.25	208	99	603.75	67851.25
53		5	5257.5	131	22	411.25	28870	209	100	606.25	68455
54		6	5435	132	23	413.75	29281.25	210	101	608.75	69061.25
55		7	5612.5	133	24	416.25	29695	211	102	611.25	69670
56	10	8	5790	134	25	418.75	30111.25	212	103	613.75	70281.25
57		1	5980	135	26	421.25	30530	213	104	616.25	70895
58		2	6170	136	27	423.75	30951.25	214	105	618.75	71511.25
59		3	6360	137	28	426.25	31375	215	106	621.25	72130
60		4	6550	138	29	428.75	31801.25	216	107	623.75	72751.25
61		5	6740	139	30	431.25	32230	217	108	626.25	73375
62		6	6930	140	31	433.75	32661.25	218	109	628.75	74001.25
63		7	7120	141	32	436.25	33095	219	110	631.25	74630
64	11	8	7310	142	33	438.75	33531.25	220	111	633.75	75261.25
65		1	7525	143	34	441.25	33970	221	112	636.25	75895
66		2	7740	144	35	443.75	34411.25	222	113	638.75	76531.25
67		3	7955	145	36	446.25	34855	223	114	641.25	77170
68		4	8170	146	37	448.75	35301.25	224	115	643.75	77811.25
69	12	5	8385	147	38	451.25	35750	225	116	646.25	78455
70		1	8625	148	39	453.75	36201.25	226	117	648.75	79101.25
71		2	8865	149	40	456.25	36655	227	118	651.25	79750
72		3	9105	150	41	458.75	37111.25	228	119	653.75	80401.25
73		4	9345	151	42	461.25	37570	229	120	656.25	81055
74	13	5	9585	152	43	463.75	38031.25	230	121	658.75	81711.25
75		1	9850	153	44	466.25	38495	231	122	661.25	82370
76		2	10115	154	45	468.75	38961.25	232	123	663.75	83031.25
77		3	10380	155	46	471.25	39430	233	124		83695

Table II. Wheel which gives Maximum Concentration for two spans or Maximum Moment at a given point of a span. Cooper's Loadings.

Span Lengths	10	15	20	25	30	35	40	45	50	55	60	65	70	80	90	100	110	120	130	140
300-290 280-270-260	2	3	3	4	4	5	5	6	7	7	8	9	10	11	12	13	14	15	17	18
250-240-230 220-210-200	2	3	3	4	4	5	5	6	7	8	8	9	10	11	12	13	14	15	17	18
190-180 170-160-150	2	3	3	4	4	5	5	6	7	8	9	9	11	12	12	13	14	15	17	18
140	3	3	3	4	4	5	5	6	7	8	9	10	11	12	12	13	14	15	17	18
130	3	3	3	4	4	5	5	6	7	8	9	10	11	12	12	13	14	15	17	
120	3	3	3	4	4	5	5	6	7	8	9	10	11	12	13	13	14	15		
110	3	3	3	4	4	5	6	7	7	8	9	10	11	12	13	13	14			
100	3	3	3	4	5	5	$\overline{14}$	$\overline{14}$	$\overline{14}$	$\overline{14}$	$\overline{13}$	$\overline{13}$	11	12	13	13				
90	3	3	4	4	5	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	12	13					
80	3	3	4	4	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	12						
70	3	3	4	4	$\overline{13}$	$\overline{13}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{11}$	$\overline{11}$	11							
65	3	3	4	4	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{11}$	$\overline{11}$	11								
60	3	3	4	4	5	$\overline{13}$	$\overline{12}$	$\overline{11}$	$\overline{11}$	$\overline{11}$	11									
55	12	12	12	4	12	$\overline{13}$	$\overline{12}$	$\overline{12}$	$\overline{13}$	11										
50	12	12	12	12	12	$\overline{13}$	$\overline{13}$	$\overline{13}$	12											
45	3	12	12	12	12	$\overline{13}$	$\overline{13}$	13												
40	3	3	3	12	12	$\overline{13}$	13													
35	3	3	4	4	13	13														
30	3	3	4	4	13															
25	3	3	4	4																
20	$\overline{4}$	3	4																	
15	3	3																		
10	3																			

The shorter span or span segment is ahead except where the wheel is overlined.

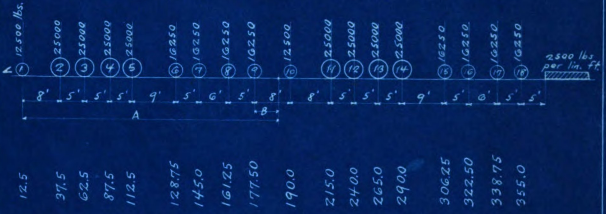
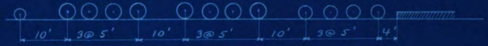


Table III. Equations for Absolute Maximum Moments.

Position of Loads.	Span Limits.	Equations for Cooper's E50. One Rail.
5/4 '	8.5 to 11.1	$12{,}500\,l + \frac{78125}{l} - 62{,}500$
	11.1 to 18.7	$18750\,l - 125{,}000$
5/4 '	18.7 to 27.6	$25{,}000\,l + \frac{156{,}250}{l} - 250{,}000$
7/18 '	27.6 to 35.0	$28{,}125\,l + \frac{17{,}014}{l} - 331{,}250$
99/103 '	35.0 to 38.7	$32{,}187.5\,l + \frac{118944}{l} - 476{,}250$
49/232 '	38.7 to 48.3	$36{,}250\,l + \frac{6468}{l} - 630{,}625$
187/129 '	48.3 to 53.5	$40{,}312.5\,l + \frac{338847}{l} - 833{,}750$
9/71 '	53.5 to 58.5	$44375\,l + \frac{2852}{l} - 1{,}045{,}000$
213/155 '	58.5 to 63.4	$48437.5\,l + \frac{365879}{l} - 1{,}288{,}750$
23/336 ' ; 5' 8' 8' 5' 5' 5' 9' 5' 6' 5'	63.4 to 69.9	$52{,}500 + \frac{984}{l} - 1{,}540{,}625$

Spans		Wheels on Span.	Equations for length of uniform load on span.
70 to 73	.08' -70' ; .49' -73' ; (13)	8 to 18	$x = \frac{\sqrt{C^2 + 0.6[8.4(l-35) - 292.85]} - C}{0.3}$; $x = .08$ to 1.99 ; $C = 16.8 - 0.1(l-35)$
74 to 82	1.07' - 74' ; .01' - 82' ; (13)	7 to 18	$x = \frac{\sqrt{C^2 + 0.6[9.05(l-35) - 339.65]} - C}{0.3}$; $x = .93$ to 5.99 ; $C = 18.1 - 0.1(l-35)$
83 to 99	99' - .63' ; 1.55' - 83' ; (13) (13)	6 to 18	$x = \frac{\sqrt{C^2 + 0.6[9.7(l-35) - 389.70]} - C}{0.3}$; $x = 4.95$ to 15.13 ; $C = 19.4 - 0.1(l-35)$
100 to 107	2.69' - 100' ; 1.73' - 107' ; (12)	4 to 18	$x = \frac{\sqrt{C^2 + 0.6[11.7(l-40) - 566.70]} - C}{0.3}$; $x = 7.31$ to 11.77 ; $C = 23.4 - 0.1(l-40)$
108 to 120	4.44' - 108' ; 2.86' - 120' ; (11)	2 to 18	$x = \frac{\sqrt{C^2 + 0.6[13.7(l-45) - 763.70]} - C}{0.3}$; $x = 4.56$ to 12.14 ; $C = 27.4 - 0.1(l-45)$
121 to 129 and over	3.92' - 121' ; 2.85' - 129' ; (11)	1 to 18	$x = \frac{\sqrt{C^2 + 0.6[14.2(l-45) - 818.20]} - C}{0.3}$; $x = 11.58$ to 16.65 ; $C = 28.4 - 0.1(l-45)$

Table IIIa. Equations for Chesapeake and Ohio R.R. Standard Loading, Specifications of 1911.



Equations are for One Track.

5'/4	8.5 11.1	$28000\ell + \frac{175000}{\ell} - 140000$
	11.1 18.7	$42000\ell - 280000$
5'/4	18.7 30.6	$56000\ell + \frac{350000}{\ell} - 560000$
1'/2	30.6 37.3	$70000\ell + \frac{70000}{\ell} - 980000$
25'/12	37.3 41.7	$84000\ell + \frac{1458333}{\ell} - 1540000$
15'/14	41.7 46.4	$98000\ell + \frac{450000}{\ell} - 2100000$
5'/2	46.4 53.7	$112000\ell + \frac{2800000}{\ell} - 2800000$
155'/122	53.7 61.3	$122000\ell + \frac{787705}{\ell} - 3300000$
5'/132	61.3 66.5	$132000\ell + \frac{758}{\ell} - 3900000$
185'/142	66.5 71.3	$142000\ell + \frac{964084}{\ell} - 4580000$
5'/76	71.3 78.1	$152000\ell + \frac{2632}{\ell} - 5280000$

Table IV. Bending Moments and Shears for Cooper's E50 - One Rail.

Span	abs. Max. Moment.	Shears. End	Shears. Quarter Point.	Shears. Centre.	Span	abs. Max. Moment.	Shears. End.	Shears. Quarter Point.	Shears. Centre.
10'	70.3	37.5	25.0	12.5	70'	2134.4	138.1	82.6	38.4
11	82.1	40.9	26.1	13.6	71	2187.1	139.8	83.4	38.9
12	100.0	43.8	27.0	14.6	72	2240.2	141.7	84.1	39.2
13	118.8	46.2	27.9	15.4	73	2293.8	143.5	84.9	39.6
14	137.5	48.2	29.4	16.1	74	2349.8	145.3	85.6	40.0
15	156.3	50.0	31.2	16.7	75	2406.9	147.1	86.3	40.4
16	175.0	53.1	32.8	17.2	76	2464.7	148.8	87.1	40.8
17	193.8	55.9	34.2	17.6	77	2522.8	150.5		
18	212.5	58.3	35.4	18.0	78	2581.3	152.1	88.5	41.5
19	233.2	60.5	36.5	18.0	79	2640.2	153.7		
20	257.8	62.5	37.5	17.9	80	2699.5	155.2	90.0	42.1
21	282.4	64.3	39.3	18.5	81	2759.3	157.0		
22	307.1	65.9	40.9	19.0	82	2819.5	158.6	91.5	43.0
23	331.8	67.4	42.4	19.4	83	2884.2	160.2		
24	356.5	69.3	43.7	19.9	84	2945.5	161.9	93.0	43.8
25	381.3	71.0	45.0	20.3	85	3009.8	163.4		
26	406.0	72.6	46.1	20.7	86	3074.4	165.1	94.8	44.5
27	430.8	74.1	47.2	21.1	87	3139.5	166.8		
28	456.9	75.5	48.2	21.4	88	3205.0	168.4	96.4	45.2
29	485.0	76.9	49.1	21.7	89	3270.9	170.0		
30	513.1	78.8	50.0	22.1	90	3337.1	171.5	98.4	45.9
31	541.2	80.5	50.9	22.7	91	3403.8	173.1		
32	569.3	82.2	51.9	23.4	92	3470.8	174.7	100.3	46.6
33	597.4	83.7	52.9	24.1	93	3538.4	176.4		
34	625.5	85.1	53.9	24.6	94	3606.3	177.9	102.2	47.3
35	653.7	86.5	54.8	25.2	95	3674.5	179.5		
36	685.8	88.2	55.6	25.7	96	3743.2	181.0	104.2	47.9
37	717.9	89.9	56.3	26.2	97	3812.2	182.7		
38	750.0	91.4	56.9	26.6	98	3881.7	184.4	106.2	48.6
39	783.3	92.9	57.5	27.1	99	3951.8	185.9		
40	819.5	94.3	58.4	27.5	100	4029.0	187.5	108.2	49.3
41	855.8	96.0	59.4	27.9	101	4107.0	189.0		
42	892.0	97.6	60.3	28.2	102	4184.8	190.5	110.1	49.9
43	928.3	99.2	61.1	28.6	103	4263.6	192.1		
44	964.5	100.7	61.9	29.0	104	4344.0	193.6	111.9	50.6
45	1000.8	102.1	62.7	29.3	105	4422.0	195.1		
46	1037.0	103.5	63.4	29.6	106	4501.0	196.6	113.7	51.2
47	1073.3	104.9	64.3	29.9	107	4582.0	198.1		
48	1109.5	106.3	65.1	30.2	108	4680.0	199.5	115.5	51.9
49	1148.5	107.7	66.0	30.6	109	4769.0	201.0		
50	1188.7	109.0	66.8	31.1	110	4858.0	202.5	117.4	52.6
51	1228.8	110.4	67.6	31.5	111	4947.0	204.0		
52	1269.0	111.8	68.4	31.9	112	5035.0	205.5	119.2	53.2
53	1309.2	113.1	69.2	32.2	113	5125.0	206.9		
54	1351.3	114.5	70.1	32.6	114	5216.0	208.3	120.9	53.9
55	1395.7	115.8	71.0	33.0	115	5306.0	209.8		
56	1440.1	117.2	71.9	33.3	116	5398.0	211.3	122.8	54.5
57	1484.4	118.5	72.8	33.6	117	5488.0	212.7		
58	1528.8	119.8	73.6	33.9	118	5581.0	214.2	124.6	55.2
59	1575.3	121.2	74.4	34.4	119	5673.0	215.7		
60	1623.6	122.5	75.1	34.8	120	5768.0	217.1	126.4	55.8
61	1671.9	123.9	75.9	35.2	121	5858.0	218.5		
62	1720.3	125.2	76.6	35.6	122	5954.0	220.0	128.1	56.5
63	1768.6	126.6	77.4	36.0	123	6050.0	221.4		
64	1819.4	128.3	78.1	36.4	124	6147.0	222.9	130.0	57.2
65	1871.9	129.8	78.9	36.7	125	6244.0	224.3		
66	1924.4	131.2	79.6	37.1	126	6341.0	225.6	131.7	57.9
67	1976.9	133.0	80.4	37.4	127	6439.0	227.1		
68	2029.4	134.7	81.1	37.8	128	6537.0	228.5	133.4	58.6
69	2081.9	136.5	81.9	38.1	129	6636.0	229.9		

Table V. Shears in Trusses. E50 Loading - One Rail.

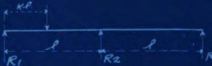
Where wheel 11 is given, the first engine is omitted.
Shears are given in Thousands of Pounds.

No. of Panels	Panel Point	Panel Lengths in Feet. 15	16	17	18	19	20	21	22
4	1	[3] 83.7	[3] 87.3	[3] 90.6	[3] 94.5	[3] 98.4	[4] 103.1	[4] 108.2	[4] 113.2
	2	[2] 40.7	[3] 42.8	[3] 45.0	[3] 47.2	[3] 49.4	[3] 51.3	[3] 53.1	[3] 54.9
	3	[2] 10.6	[2] 11.7	[2] 12.7	[2] 13.5	[2] 14.3	[2] 15.0	[2] 15.6	[2] 16.2
5	1	[3] 105.4	[3] 111.8	[3] 118.3	[3] 124.6	[3] 130.4	[4] 136.6	[4] 142.9	[4] 149.0
	2	[2] 63.4	[3] 66.3	[3] 69.1	[3] 72.4	[3] 75.7	[3] 79.1	[3] 82.6	[3] 86.1
	3	[2] 31.2	[2] 32.9	[2] 34.3	[2] 35.8	[2] 37.3	[2] 38.7	[2] 40.2	[2] 41.6
	4	[2] 7.2	[2] 8.2	[2] 9.0	[2] 9.7	[2] 10.4	[2] 11.0	[2] 11.6	[2] 12.0
6	1				[3] 153.9	[3] 161.2	[3] 168.1	[4] 175.2	[4] 182.2
	2				[3] 101.2	[3] 106.2	[3] 111.1	[3] 116.0	[3] 120.7
	3				[11] 58.9	[11] 61.1	[3] 63.5	[3] 66.6	[3] 69.6
	4				[2] 28.9	[2] 30.2	[2] 31.4	[2] 32.7	[2] 33.9
	5				[2] 7.2	[2] 7.8	[2] 8.3	[2] 8.8	[2] 9.3
7	1						[3] 197.7	[3] 205.6	[4] 213.7
	2						[3] 142.0	[3] 147.9	[3] 153.6
	3						[3] 93.1	[3] 97.5	[3] 101.6
	4						[11] 53.6	[2] 55.5	[3] 57.8
	5						[2] 26.2	[2] 27.4	[2] 28.4
	6						[2] 6.4	[2] 6.9	[2] 7.3
8	1								
	2								
	3								
	4								
	5								
	6								
	7								

No. of Panels	Panel Point	Panel Lengths in Feet. 23	24	25	26	27	28	29	30
5	1	[4] 154.9	[4] 160.5	[4] 166.0					
	2	[3] 89.9	[3] 93.4	[3] 96.7					
	3	[11] 42.9	[3] 44.2	[3] 45.6					
	4	[2] 12.5	[2] 12.9	[2] 13.4					
6	1	[4] 189.1	[4] 195.8	[4] 202.5	[4] 209.0	[4] 215.4	[4] 221.7		
	2	[3] 125.4	[3] 130.1	[3] 134.5	[4] 139.0	[4] 143.5	[4] 148.0		
	3	[3] 72.9	[3] 75.9	[3] 78.6	[3] 81.5	[3] 84.3	[3] 87.0		
	4	[11] 35.0	[11] 36.1	[11] 37.2	[11] 38.2	[11] 39.2	[11] 40.2		
	5	[2] 9.7	[2] 10.1	[2] 10.5	[2] 10.9	[2] 11.3	[2] 11.7		
7	1	[4] 221.7	[4] 229.7	[4] 237.4	[4] 245.1	[4] 252.7	[4] 260.2	[4] 267.6	[4] 275.0
	2	[3] 159.2	[3] 164.8	[3] 170.2	[4] 175.8	[4] 181.5	[4] 187.0	[4] 192.5	[4] 197.9
	3	[3] 105.7	[3] 109.8	[3] 113.6	[3] 117.4	[3] 121.1	[3] 124.7	[3] 128.3	[3] 131.8
	4	[3] 60.7	[3] 63.3	[3] 65.8	[3] 68.3	[3] 70.7	[3] 73.1	[3] 75.4	[3] 77.8
	5	[11] 29.4	[11] 30.4	[11] 31.3	[11] 32.2	[11] 33.1	[11] 34.0	[11] 34.9	[11] 35.7
	6	[2] 7.7	[2] 8.0	[2] 8.5	[2] 8.8	[2] 9.2	[2] 9.5	[2] 9.8	[2] 10.2
8	1	[4] 253.4	[4] 262.5	[4] 271.5	[4] 280.3	[4] 289.1	[4] 297.8	[4] 306.4	[4] 315.0
	2	[3] 191.7	[3] 198.3	[3] 204.9	[4] 211.6	[4] 218.3	[4] 225.0	[4] 231.6	[4] 238.2
	3	[3] 137.8	[3] 142.7	[3] 147.6	[3] 152.3	[3] 157.0	[3] 161.5	[3] 166.2	[3] 170.7
	4	[3] 91.0	[3] 94.6	[3] 98.0	[3] 101.3	[3] 104.6	[3] 107.8	[3] 111.0	[3] 114.2
	5	[2] 52.1	[2] 54.2	[2] 56.4	[2] 58.4	[2] 60.5	[2] 62.7	[2] 64.8	[2] 66.8
	6	[11] 25.2	[11] 26.1	[11] 26.9	[11] 27.7	[11] 28.5	[11] 29.3	[11] 30.1	[11] 30.8
	7	[2] 6.2	[2] 6.5	[2] 6.9	[2] 7.2	[2] 7.6	[2] 7.9	[2] 8.2	[2] 8.5

Table VII. Reactions of Beams on Three Supports

K	R_1	R_2	R_3
0	1.00000	.00000	.00000
1/10	.87525	.14950	-.02475
1/9	.86146	.16597	-.02743
1/8	.84424	.18652	-.03076
1/7	.82215	.21284	-.03499
1/6	.79282	.24769	-.04051
1/5	.75200	.29600	-.04800
2/9	.72497	.32784	-.05281
1/4	.69141	.36718	-.05859
2/7	.64869	.41691	-.06560
3/10	.63175	.43650	-.06825
1/3	.59260	.48147	-.07407
3/8	.54443	.53614	-.08057
2/5	.51600	.56800	-.08400
3/7	.48397	.60349	-.08746
4/9	.46640	.62276	-.08916
1/2	.40625	.68750	-.09375
5/9	.34842	.74760	-.09602
4/7	.33236	.76385	-.09621
3/5	.30400	.79200	-.09600
5/8	.27979	.81542	-.09521
2/3	.24074	.85185	-.09259
7/10	.21075	.87850	-.08925
5/7	.19825	.88921	-.08746
3/4	.16797	.91406	-.08203
7/9	.14540	.93142	-.07682
4/5	.12800	.94400	-.07200
5/6	.10301	.96065	-.06366
6/7	.08601	.97084	-.05685
7/8	.07373	.97754	-.05127
8/9	.06447	.98217	-.04664
9/10	.05725	.98550	-.04275
1	.00000	1.00000	-.00000

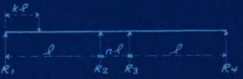


$$R_3 = -\frac{K-K^3}{4}$$

$$R_2 = k - 2R_3$$

$$R_1 = 1 - K + R_3$$

Four Supports, Partially Continuous.



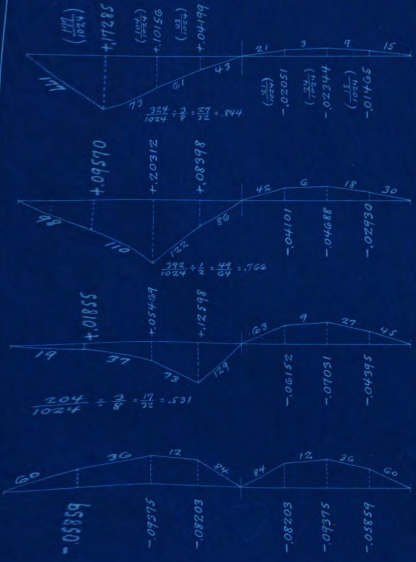
$$R_4 = -\frac{k-k^3}{4+6n} \qquad R_2 = k - 2R_4$$

$$R_3 = -R_4 \qquad R_1 = 1 - k + R_4$$

Table VIII. Bending Moments in Eight Panel Swing Bridge

Moments are given in thousands of foot-pounds for Cooper's E50, one ro[...]
Conditions of loading: Any length of uniform load following one or tw[...] engines, no uniform load in front of engines, loading continuous, the whee[...] of the tender are not separated from the drivers. These conditions ap[...] also to the three tables following.

Influence are for values of $\frac{M}{\ell}$.



20	+ 1716	(12)→	1	.8
	− 307	(15)→	4	
25	+ 2678	(13)→	1	.8
	− 481	←(8)	5	
30	+ 3760	←(5)	1	.8
	− 682	(14)→	4	

20	+ 2092	←(13)	2	.7
	− 613	(15)→	4	
25	+ 3107	←(12)	2	.7
	− 962	←(8)	5	
30	+ 4419	←(11)	2	.7
	− 1363	(14)→	4	

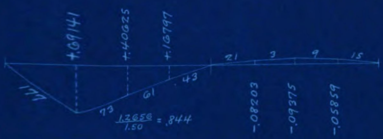
20	+ 1150	(4)→	3	.5
	− 920	(15)→	4	
25	+ 1719	(4)→	3	.5
	− 1443	←(8)	5	
30	+ 2394	(5)→	3	.5
	− 2045	(14)→	4	

20	− 2123	←(10)	2
25	− 3377	←(12)	1.8' left of 3
30	− 4836	←(14)	3

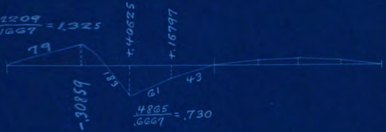
The ordinates to these influence lines may be written as fractions wi[...] the denominator 1024. The tangents given are the numerators of [...] fractions whose denominators are 256.

Table IX. Shears in Eight Panel Swing Bridge.

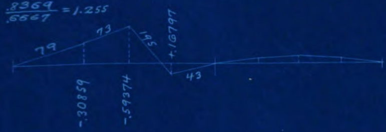
Shears are given in thousands of pounds for Cooper's E50 loading - one rail.



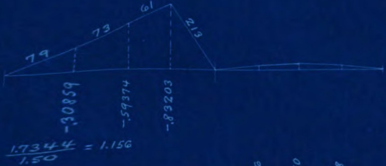
20	+ 85.8	(12)→	1	.833
	− 15.3	(15)→	4	
25	+ 107.1	(13)→	1	.847
	− 19.2	←(8)	5	
30	+ 125.3	←(5)	1	.847
	− 22.7	(14)→	4	



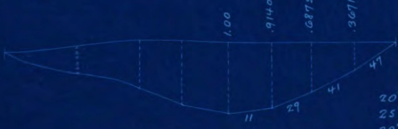
20	+ 38.1	←(3)	2	.743
	− 18.9	(3)→	1	1.26
25	+ 45.1	←(3)	2	.755
	− 23.3	(3)→	1	1.31
30	+ 47.5	←(4)	2	.691
	− 26.9	(3)→	1	1.33



20	− 63.5	(3)→	2	1.238
	− 64.7	←(5)	2	
25	− 73.2	(3)→	2	1.226
	− 72.0	←(6)	2	
30	− 83.3	(4)→	2	1.212
	− 79.8	←(8)	2	



20	− 118.3	(4)→	3	1.147
	− 128.1	←(13)	3	
25	− 145.8	(4)→	3	1.153
	− 159.9	←(13)	3	
30	− 170.6	(5)→	3	1.153
	− 188.0	←(14)	3	



20	301.4	←(5)	2
25	359.9	(14)	Centre
30	417.7	←(2)	1

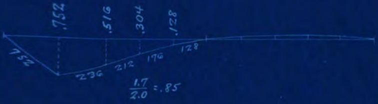
Maximum Concentration.

20' - 248.6 × 1.25 × .970 = 301.4
25' - 296.4 × 1.25 × .972 = 359.9
30' - 340.1 × 1.25 × .983 = 417.7

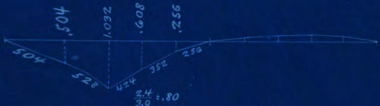
Table X. Bending Moments in Ten Panel Swing Bridge.

Cooper's E50 Loading - one rail.

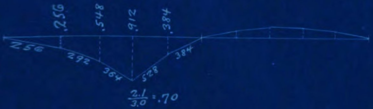
Influence Lines are for values of $\frac{M}{P}$.



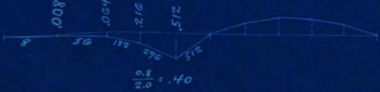
20	2255	←(4)	1	.825
25	3463	←(4)	1	.835
30	4935	←(5)	1	.857



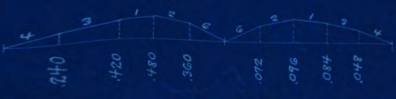
20	3166	(12)→	2	.803
25	4883	(12)→	2	.801
30	6802	(13)→	2	.803



20	2790	←(12)	3	.708
25	4293	←(12)	3	.705
30	5984	←(13)	3	.706



20	1090	(4)→	4	.399
25	1655	(4)→	4	.399
30	2400	(5)→	4	.417



20	3449	←(10)	3
25	5342	←(4)	1
30	7539	←(13)	3

Ordinates for $\frac{\text{Centre M}}{P}$. Ordinates for Uplift.

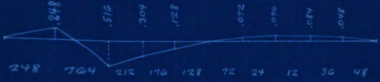
20	19.7	←(7)	6
25	24.1	←(4)	5
30	28.0	←(4)	5

Table XI. Shears in Ten Panel Swing Bridge.

Cooper's E50 Loading - one rail.

Approximate Uplift
= {Moment for Simple Span at 2} × $\frac{1}{10p}$

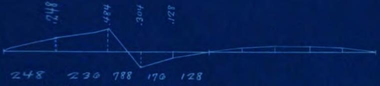
20	+ 57.0	←(3)	2	.721
	− 19.7	(7)	3	$\frac{1}{200}$
25	+ 74.3	←(3)	2	.768
	− 24.1	(4)	4	$\frac{1}{250}$
30	+ 85.7	←(4)	2	.752
	− 28.0	(4)	4	$\frac{1}{300}$



$\frac{.1642}{.125} = 1.314$ $\frac{.8642}{1.125} = .768$

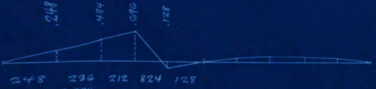
↙ 1 Engine

20	+ 27.0	←(2)	3	.698
	− 49.2	(3)→	2	1.271
25	+ 31.9	←(2)	3	.701
	− 57.5	(3)→	2	1.264
30	+ 36.3	←(2)	3	.689
	− 66.0	(3)→	2	1.252



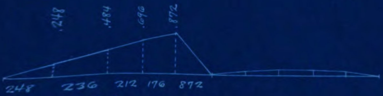
$\frac{.6386}{.5} = 1.277$ $\frac{.3386}{.5} = .677$

20	− 95.2	(3)→	3	1.204
	− 104.3	←(13)	3	
25	− 113.5	(4)→	3	1.174
	− 130.0	←(13)	3	
30	− 134.7	(4)→	3	1.182
	− 153.4	←(14)	3	

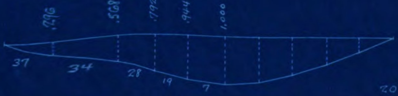


$\frac{1.374}{1.125} = 1.221$

20	− 151.7	(4)→	4	1.110
	− 170.1	←(13)	4	
25	− 186.6	(4)→	4	1.124
	− 203.4	←(14)	4	
30	− 219.8	(5)→	4	1.145
	− 229.9	←(14)	4	



$\frac{2.3}{2.0} = 1.15$



20	361.8	(14) Centre
25	433.7	(17) Centre
30	505.9	←(11) .7' l/ft. Ctr.

Maximum Concentration.

20' - 296.4 × 1.25 × .977 = 361.8

25' - 351.2 × 1.25 × .988 = 433.7

30' - 406.7 × 1.25 × .995 = 505.9

Table XII. Reactions for Deck Plate Girder Swing Bridges.

Cooper's E50 Loading - One Rail.

For Maximum Uplift:-

$$x = \sqrt{\left(\frac{\Sigma wa}{\Sigma w}\right)^2 - \frac{\Sigma wa^2}{\Sigma a} + \frac{l^2}{3}} - \frac{\Sigma wa}{\Sigma w}$$

$$x = \sqrt{\frac{l^2}{3} - A} - B$$

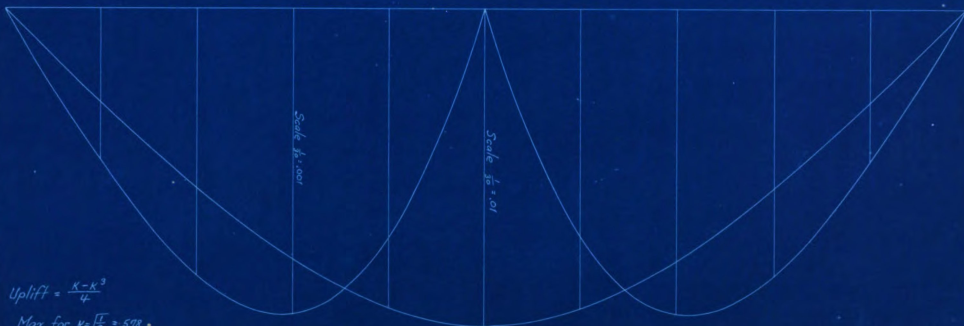
No. of Wheels.	A	B
5	51.5	9.2
6	81.8	15.9
7	116.1	18.6
8	159.3	22.1
9	205.8	24.6
10	257.6	30.5
11	380.1	34.0
12	482.3	34.9
13	572.9	36.2
14	656.9	37.6
15	731.3	44.1
16	809.8	46.7
17	898.3	50.1
18	989.5	52.6

Span.	Wheels.	x.	Uplift.
30'	5	6.5'	8000 lbs.
40'	7	1.8	9800
50'	8 9	3.8* .5	11500
60'	9 10	6.9 .2*	13100
70'	10 11	6.6* 1.4	14300
80'	13 14	3.4 .8	16200

Span.	Centre Reaction.		Max. Conc. Two Simple Spans.				
30'	129,000	=	107800	×	$1\frac{1}{4}$	×	.957
40'	162,500	=	135000	×	"	×	.963
50'	195,000	=	161000	×	"	×	.969
60'	237,000	=	191500	×	"	×	.990
70'	274,500	=	221300	×	"	×	.992
80'	306,000	=	248600	×	"	×	.985

Table XIII.

Influence Lines for Centre Reaction and Uplift.
Deck Plate Girder Draw Spans.

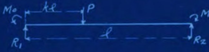


Uplift = $\frac{K-K^3}{4}$

Max. for $K=\sqrt{\frac{1}{3}}$ = .578

Uplift = .09623

Table XIV. Bending Moments for Beams with Fixed Ends.

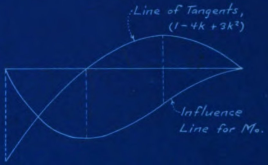


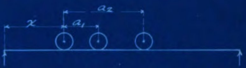
$$M_0 = -P\ell\, k(1-k)^2$$

$$M_1 = -P\ell\, k^2(1-k)$$

$$R_1 = P(1-k)^2(2k+1)$$

$$R_2 = P k^2 (3-2k)$$





To find the position of a group of concentrated loads for maximum M_0 make $\Sigma P(1-4k+3k^2) = 0$.

ℓ	N	x	E50-1 Rail Moment M_0	Percent. of Absolute Max. Moment. Simple Beam.
8'	2	1.90'	30,900	61.6
9	2	1.84	39000	66.5
10	2	1.96	47800	68.0
11	2	2.15	56700	69.0
12	2	2.38	65600	65.6
13	3	2.21	75100	63.2
14	3	2.07	86600	63.0
15	3	2.11	99400	63.6
16	3	2.23	112700	64.4

For N equal loads solve the equation:

$$3Nx^2 + (6\Sigma a - 4N\ell)x + N\ell^2 - 4\ell\Sigma a + 3\Sigma a^2 = 0$$

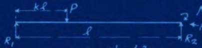
$a = 5'$ $N = 2$

$$6x^2 + (30-8\ell)x + 2\ell^2 - 20\ell + 75 = 0$$

$a_1 = 5'$ $a_2 = 10'$ $N = 3$

$$9x^2 + (90-12\ell)x + 3\ell^2 - 60\ell + 375 = 0$$

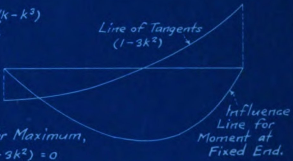
Bending Moments for Beams with One End Fixed.



$$M = -\frac{P\ell}{2}(k-k^3)$$

$$R_1 = P\,\frac{2-3k+k^3}{2}$$

$$R_2 = P\,\frac{3k-k^3}{2}$$



ℓ	N	x	E50-1 Rail. Moment at Fixed End.
8	2	1.38	45800
9	2	2.05	58400
10	2	2.70	70500
11	2	3.34	82200
12	2	3.96	93700
13	3	1.30	110900
14	3	1.98	129900
15	3	2.64	148500
16	3	3.29	166700

Condition for Maximum,

$$\Sigma P(1-3k^2) = 0$$

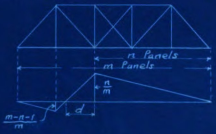
Equal Loads.

$N = 2$ $a = 5'$ $x = \sqrt{\frac{\ell^2}{3} - 6.25} - 2.50$

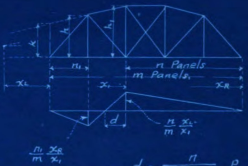
$N = 3$ $a_1 = 5'$, $a_2 = 10'$ $x = \sqrt{\frac{\ell^2}{3} - \frac{50}{3}} - 5.00$

Table XV. Equivalent Uniform Loads. Cooper's E50 Loading - One Rail.

Span	End	.1 Point	.2	.3	.4	Centre.
30'	5252	4940	4620	4667	4587	4556
40'	4715	4410	4220	4190	4115	4095
50'	4360	4084	3972	3896	3828	3784
60'	4083	3817	3730	3697	3643	3597
70'	3946	3617	3563	3497	3483	3486
80'	3882	3590	3492	3385	3385	3375
90'	3811	3551	3460	3353	3304	3296
100'	3750	3512	3416	3340	3286	3220
110'	3682	3473	3385	3335	3282	3189
120'	3618	3415	3343	3295	3263	3192
130'	3560	3364	3295	3235	3231	3180
150'	3456	3290	3200	3118	3141	3136
200'	3263	3123	3027	2981	2991	2965
250'	3132	3004	2916	2896	2872	2809
300'	3039	2914	2840	2824	2766	2711
350'	2966	2847	2786	2757	2695	2655
400'	2916	2794	2751	2700	2648	2619
450'	2873	2753	2718	2658	2617	2594
500'	2838	2723	2686	2627	2595	2576

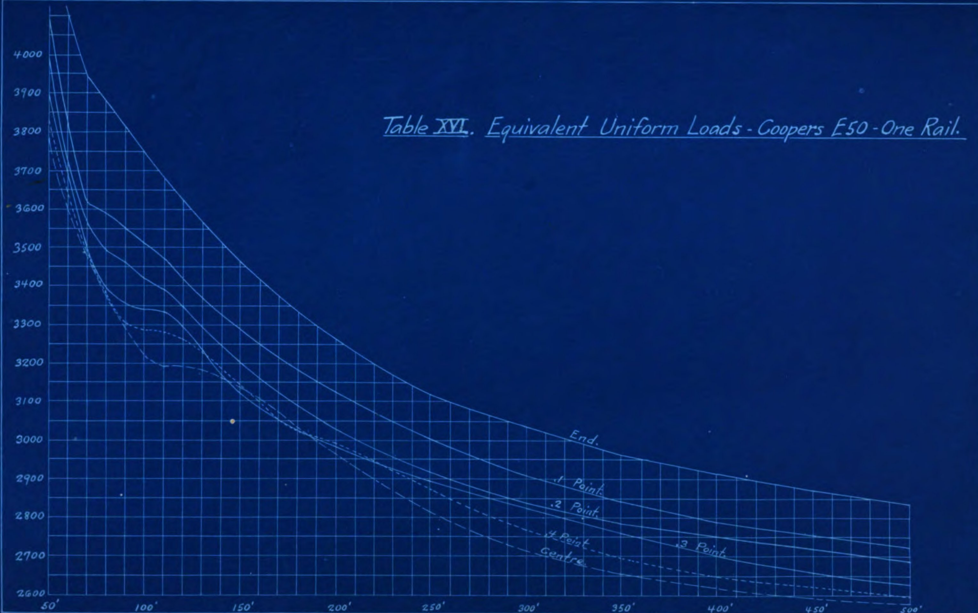


$$d = \frac{n}{m-1} P$$



$$d = \frac{n}{m\frac{h}{k} - 1} P$$

Table XVI. Equivalent Uniform Loads - Coopers E50 - One Rail.



THE CALCULATION OF WHEEL LOAD STRESSES FROM INFLUENCE LINES.

———0———

Let a,b,c,d,e, be an influence line consisting of a number of straight segments. Let M_1, M_2 etc. be the static moment of all the loads to the left of a,b,c, etc. respectively. The load is headed toward the left. The tangents of the angles which the segments make with the horizontal are T_1, T_2, T_3, etc.

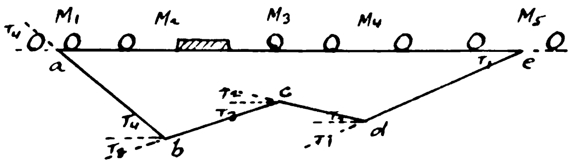


Fig. 1.

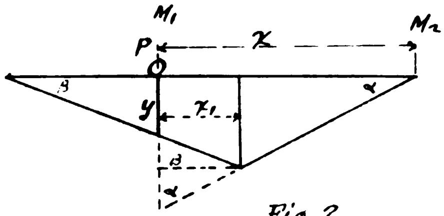


Fig. 2.

In fig.2 the influence of the load is Py

$$Py = Px \tan\alpha - Px_1(\tan\alpha + \tan\beta)$$

$$= M_2 \tan\alpha - M_1(\tan\alpha + \tan\beta)$$

Similarly the total influence in fig.1. is

$$M_5 T_1 - M_4(T_1 + T_2) + M_3(T_2 + T_3) - M_2(T_3 + T_4) + M_1 T_4$$

The form of the influence line determines whether the succeeding products are to be added or subtracted.

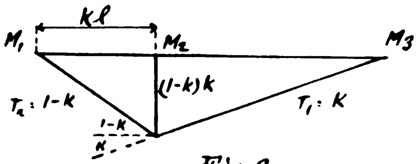


Fig. 3.

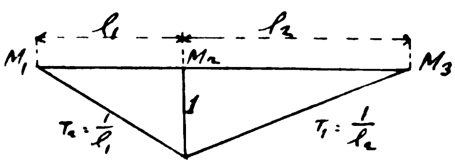


Fig. 4.

The bending moment at any point of a beam is

$$M_3 k - M_2(k + 1 - k) + M_1(1 - k) = M_3 k - M_2 + M_1(1 - k)$$

If no loads are off the span to the left, the bending moment is, $M_3 k - M_2$

The pier concentration for two unequal spans is

$$\frac{M_3}{l_2} - M_2\left(\frac{1}{l_1} + \frac{1}{l_2}\right) + \frac{M_1}{l_1} = \frac{M_3 - M_2}{l_2} - \frac{M_2 - M_1}{l_1}$$

When $l_1 = l_2$ the concentration is $\frac{M_3 - 2M_2 + M_1}{l_1}$

Fig.5. shows one arm of a single track swing bridge. Let it be required to find the stresses due to E-50 loading in the chord 3 and in the diagonal T2 when the arm is acting as a simple span. Use the panel length as the unit for calculating the tangents of the influence lines

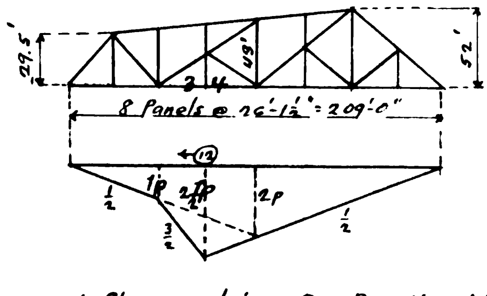
Wheel 12 at the panel point with the train headed towards the left is the critical position for the bending moment. Some of the wheels are off the bridge to the left.

E50 $M_{199.625} = 62893$ per rail.

$M_{(12)} = 8385$

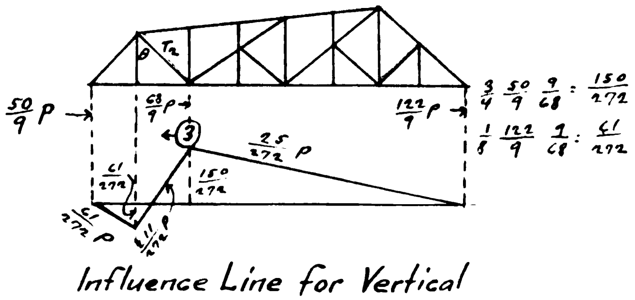
$M_{42.875} = 3545.6$

$$\text{Stress in Chord 3} = \frac{62893 \times \frac{1}{2} - 8385 \times \frac{4}{2} + 3545.6 \times \frac{2}{2}}{43'} = 423,800 \text{ lbs.}$$



Influence Line for Bending Moment.

Fig. 5.



Influence Line for Vertical Component of Stress in T_2.

The maximum stress in the diagonal T_2 occurs when wheel 3

stands at the panel point.

$M_{169.75} = 46635$ Secant $\theta = \frac{39.41}{29.5}$

$M_{(3)} = 287.5$

$$\text{Stress in } T_2 = \left[46635 \times 25 - 287.5 \times 236\right] \frac{39.41}{29.5 \times 26.125 \times 272} = +206{,}400 \text{ lbs.}$$

STRESSES IN TRUSSES AND FLOOR DUE TO LOADS ON A CURVED TRACK.

-----0-----

Centrifugal force stresses in the trusses and lateral system can be found directly from the vertical load stresses in the trusses. Stresses due to the eccentricity of the load on a curved track are usually found from the panel loads computed from an equivalent uniform load, using the average eccentricity in each panel.

A method by which these stresses can be more easily found follows.

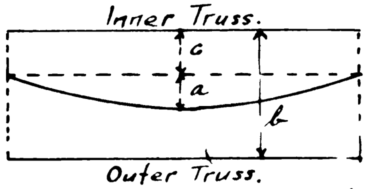


Fig. 1.

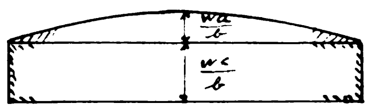


Fig. 2.

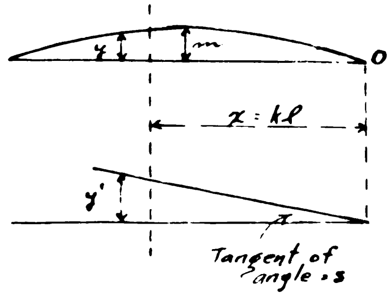


Fig. 3.

Assume that the path of the centre of gravity of the loads is a parabola. This is perhaps as true as to assume that it is circular. Figure 2 shows the portion of a uniform load of w pounds per lineal foot, which rests upon the outer truss, the curve is a parabola. Figure 3 shows a parabola whose middle ordinate is m, its Equation referred to O as an origin is; $y = \frac{4mx}{l^2}(l - x)$, and an influence area bounded by two straight lines of indefinite length, the tangent of whose included angle is s. The effect of the parabola upon the influence area over the distance x is,

$$\int_0^{k\ell} y y' dx = \int_0^{k\ell} \frac{4s w x^2}{\ell^2}(\ell - x) = s w \ell^2\left(\frac{4}{3}k^3 - k^4\right)$$

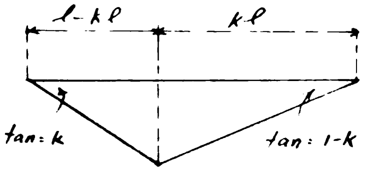


Fig. 4.

The moment factor for the outer truss is found from figure 4 which shows the influence line for bending moment.

Moment from parabolic load $= (1-k)\frac{wa}{b}\ell^2\left(\frac{4}{3}k^3 - k^4\right) + k\frac{wa}{b}\ell^2\left[\frac{4}{3}(1-k)^3 - (1-k)^4\right]$

$$= \frac{1}{3}(k - 2k^3 + k^4)\frac{wa\ell^2}{4} \qquad (A.)$$

Moment from load $\frac{w}{2}$, that is, for straight track $= \frac{1}{2}(k-k^2)\frac{w\ell^2}{2}$ (C)

Moment from uniform load in Figure 2. $= \frac{wc}{2b}k\ell(\ell - k\ell)$

$$= \frac{1}{2}(k-k^2)\frac{wc\ell^2}{b} \qquad (B)$$

Factor for Bending Moment $= \frac{A+B}{C} = \frac{2}{b}\left[\frac{2a}{3}(1+k-k^2) + c\right]$

This is the factor by which to multiply the moment due to loads on a straight track, to obtain the moment at any point in the outer truss when the track is on a curve. The variable part of the factor $(1+k-k^2)$ is a maximum for $k = \frac{1}{2}$. When k is zero the factor becomes $\frac{2}{b}\left(\frac{2a}{3} + c\right)$, this is the factor for end shear.

For $k = \frac{1}{2}$, the moment factor $= \frac{2}{b}\left(\frac{5a}{6} + c\right)$

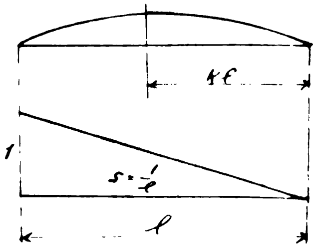


Fig. 5.

Shear factor $= \dfrac{\frac{1}{\ell}\frac{wa}{b}\ell^2\left(\frac{4}{3}k^3 - k^4\right) + \frac{wc}{b}k\ell\frac{k\ell}{2\ell}}{\frac{w}{2}k\ell\frac{k\ell}{2\ell}}$

$$= \frac{2}{b}\left[2a\left(\frac{4}{3}k - k^2\right) + c\right]$$

When $k = 1$, factor $= \frac{2}{b}\left(\frac{2a}{3} + c\right)$

The factor for end shear in trusses is

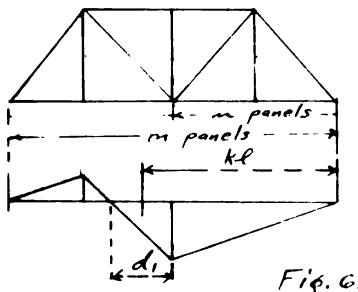


Fig. 6.

$d_1 = \frac{m}{m-1} p$

the same as that for moment at the first panel point. For intermediate shear factors figure k to the middle of the distance d_1.

Figures 7 & 8 shows the values of $\frac{2}{3}(1+k-k^2)$ and $2(\frac{4}{3}k-k^2)$.

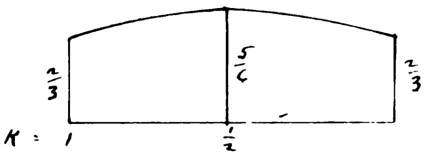


Fig. 7.

Both curves are parabolas.

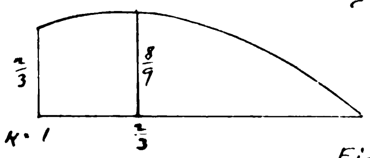


Fig. 8.

For an outer stringer, the left reaction is $\frac{wp}{4}\left(\frac{3b+4e_1+2e_2}{3b}\right)$,

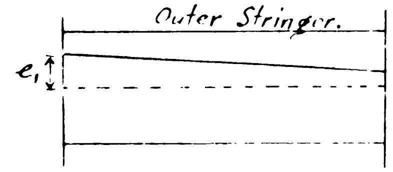


Fig. 9.

and the right reaction is

$$\frac{wp}{4}\left(\frac{3b+2e_1+4e_2}{3b}\right).$$

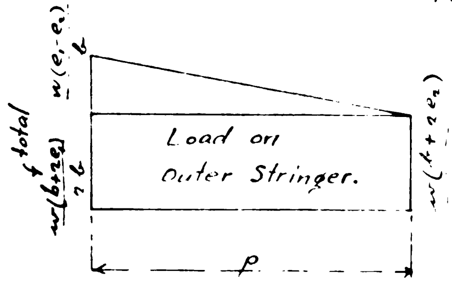


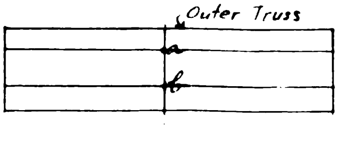
Fig. 10.

The terms in parentheses are the shear factors to be used when the stresses are calculated from wheel loads.

The centre moment in the outer stringer is

$$\frac{wp}{4}\left(\frac{3b+2e_1+4e_2}{3b}\right)\frac{p}{2} - \frac{wp(b+2e_2)p}{4b \times 4} - \frac{wp(e_1-e_2)p}{2\times 2b \times 2\times 6} = \frac{wp^2}{16}\left(\frac{b+e_1+e_2}{b}\right)$$

Panel loads on truss.



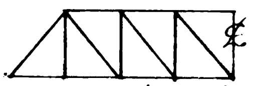
Load at "a" = $\frac{wp}{4}\left(\frac{6b+2e_1+8e_2+2e_3}{3b}\right)$

Load at "b" = $\frac{wp}{4}\left(\frac{6b-2e_1-8e_2-2e_3}{3b}\right)$

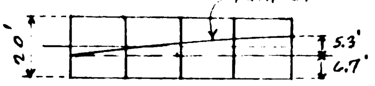
The outer panel load $= \frac{wp}{6d}(3d + e_1 + 4e_2 + e_3)$

Symbols:- w = load per lineal foot of track
p = panel length
d = distance centre to centre of trusses
b = distance centre to centre of stringers
e_1, e_2, e_3 = eccentricities of centre line of load, measurdd positive outwards

The following illustrates the method of computing the corrections in stresses due to wheel loads.



8 Panels @ 25' = 200'



Middle ordinate for 6° curve, 200' chord = 5.3'

This problem is treated on page 279, Johnson's Structures 9th Edition Rewritten Part I.

Factors for Chord Stresses in Outer Truss.

$$\frac{2}{b}\left[\frac{2a}{3}(1+k-k^2)+c\right]$$

	$\frac{2}{3}(1+k-k^2)$	a							
$k = \frac{7}{8}$	.740	× 5.3 =	3.92	+ 6.70 =	10.62	× $\frac{2}{20}$ =	1.062	1st Panel Pt.
$\frac{6}{8}$	.792	do	4.20	do	10.90	do	1.090	2nd do
$\frac{5}{8}$	.823	do	4.36	do	11.06	do	1.106	3rd do
$\frac{4}{8}$	.833	do	4.41	do	11.11	do	1.111	Center.

Factors for Web Stresses in Outer Truss

$$\frac{2}{b}\left[2a\left(\frac{4}{3}k-k^2\right)+c\right]$$

	$2\left(\frac{4}{3}k-k^2\right)$							
k = .938	.742	× 5.3 =	3.93	+ 6.70 =	10.63	× $\frac{2}{20}$ =	1.063 (1.062, should be.)	1st Panel.
.804	.852	do	4.52	do	11.22	do	1.122	2nd do
.670	.888	do	4.71	do	11.41	do	1.141	3rd do
.536	.856	do	4.54	do	11.24	do	1.124	4th do
.402	.748	do	3.96	do	10.66	do	1.066	5th do

MAXIMUM TENSION IN VERTICALS OF COUNTER BRACED TRUSSES WITH CURVED UPPER CHORDS.

0

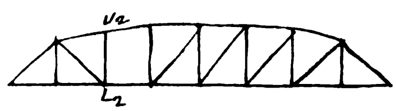
In order to determine the proper position of the loading to get the maximum tension in a vertical adjacent to a counter, it is convenient to trace the cycle of stress in the vertical as it passes through its maximum tension. The truss chosen has eight panels of 25 feet, the depth at the hips is 28 feet, at the second vertical 33 feet, and at the centre 36 feet. The dead load assumed is 18000 lbs. on the lower panel points and 7000 lbs. on the upper panel points. The influence lines, and dead load stresses are given on page and the curves of stress on page The cycle of stress for U_2L_2 is as follows; the train, Cooper's E-50 enters the bridge from the left. When the pilot wheel is 33.82 feet from the left support, the dead load compression in U_2L_2, $-21,800$ is reduced to zero:

$M_{33.82} = 2,285,000 \times .00955 = +21,800$ live load stess.

When the pilot is 39.77 feet from L_0 the dead load tension in U_2L_3, vertical component $= +21,900$ is reduced to zero:

$3,095,000 \times .00708 = -21,900$ live load vertical component.

The truss members acting are as shown,



The stress in U_2L_2 at this instant may be found from influence lines (2) or (3), in the former case a dead load compression of $-21,800$ must be added and in the latter case a dead load tension of $+2100$ must be added.

(2) $3,095,000 \times .00955 = +29,500 - 21,800 = +7700$

(3) $3,095,000 \times .00982 = +5600 + 2100 = +7700$

As the live load advances the counter U_3L_2 is called into action until the pilot wheel is 74.0 feet from L_0, the truss is again in the condition shown in the figure above.

$$M_{74} = 9585 \qquad 9{,}585{,}000 \times .00708 - 1150000\,\frac{79+17}{96 \times 25} = -21{,}900$$
$$M_{24} = 1150$$

The stress in U_2L_2 may be found from (2)or (3)

(2) $9{,}585{,}000 \times .00955 - 1{,}150{,}000 \times .0461 = +38{,}500 - 21{,}800 = +16{,}700$

(3) $9{,}585{,}000 \times .00182 - 1{,}150{,}000 \times .00242 = +14{,}600 + 2100 = +16{,}700$

The stress may be found from the bending moments at U_2L_2 or U_3L_3 in which case the upper panel load of 7000 lbs. must be taken into account.

$9585 \times 5/8$	Live load Bending Moment at L_3	= 5990.6
$7\frac{1}{2} \times 25000 \times 25$	Dead Load " "	4687.5
		10678.1

$$\frac{10678.1 \times 2}{36 \times 25} = +23{,}700 - 7{,}000 = +16{,}700$$

This is the maximum tension that can occur in U_2L_2.

As the load advances U_4L_3 comes into action, the stress in U_2L_2 quickly falls to zero and passes into compression and remains in compression during the remainder of the passage of the train.

If U_2L_3 could take compression and U_3L_2 were omitted the maximum tension in U_2L_2 would occur with wheel 3 at L_2.

$$7{,}120{,}000 \times .00955 - 287{,}500 \times .0461 = +54{,}800 - 21800 = +33{,}000$$

The curves of stress shown are made up of short straight lines, a break occurs whenever a wheel passes a break in the influence lines, or whenever a wheel comes on the span.

The cycle of stress for U_3L_3 is as follows; the train enters the bridge from the left, when the pilot is 17.2 feet from L_0 the dead load compression in U_3L_3, -3900 is reduced to zero.

$550,000 \times .00709 = +3900$ live load.

When the head of the train is 35.5 feet from L_0, U_3L_4 passes out of action,

$2,500,000 \times .005 = -12500$ live load shear in panel L_3L_4.

At this instant the stress in U_3L_3 may be found from influence lines (2a) or (4a)

(2a) $2,500,000 \times .00709 = +17,700 - 3900 = +13800$

(4a) $2,500,000 \times .00208 = +5,200 + 8600 = +13800$

at 39.77 feet U_2L_3 goes out of action.

(3a) $3,095,000 \times .005 = -15,500 + 30,500 = +15000$

(4a) $3,095,000 \times .00208 = +6400 + 8600 = +15000$

The stress in U_3L_3 now decreases to a minimum and then increases. It is possible for this ~~difference~~ dip in the stress curve to go below the zero line. If this happens, U_3L_3 has the following changes during a single passage of the load; compression, tension, compression, tension and compression. Thus the necessity for careful detailing of the pin joints of such verticals is shown.

At 74.0 feet U_3L_2 goes out of action. The stress in U_3L_3 is

(3a) $9,585,000 \times .005 - 1,150,000 \times .04 = -1900 + 30,500 = +28,600$

(4a) $9,585,000 \times .00208 = +19,900 + 8600 = +28,600$

The stress in U_3L_3 now increases at a nearly uniform rate until U_4L_3 goes out of action. This occurs when the head of the train is 110.6 feet from L_0.

$21,026,500 \times .005 - 2,513,500 \times .04 + 197,500 \times .04 = -12500$

The stress in U_3L_3 is;

(2a) $21,026,500 \times .00709 - 2,513,500 \times .04339 + 197500 \times .04005 = +47900 - 3900 = +44,000$

(4a) $21,026,500 \times .00208 - 2,513,500 \times .00333 = +35,400 + 8,600 = +44000$

This is the maximum tension which $U_3 L_3$ can receive.

$U_3 L_3$ now takes compression as the load advances and remains in compression during the remainder of the passage of the load. If the diagonals are stiff members, $U_3 L_3$ gets a much larger tension, as shown in the stress curve.

Most specifications add an impact stress to the live load stress, this is equivalent to increasing the weight of the live load, and this may be done in finding the tension in a vertical, taking the impact factor as constant over any panel. Less load will be on the bridge when the vertical gets its maximum stress.

By dividing the load a somewhat larger tension can be obtained, but as this complicates the question of impact, it is not generally done. For $U_2 L_2$ the calculation is as follows:

The static moment of panel loads 1 and 2 about L_0 must be a maximum.

Wheel 4 at L_2 gives this condition.

Live Load Bending Moment at 2 $= \frac{6}{8}\, 8170 - 600 = 5527.5$

Dead " " " " 3750

Total 9277.5

Total Moment at 3 $= 9793.75$

Now enough load must be added on the right end of the bridge to make $\frac{M_2}{h_2} = \frac{M_3}{h_3}$ so that no diagonals will be acting in panel $L_2 L_3$.

$$\frac{9793.75 + \frac{3}{8}M}{36} = \frac{9277.5 + \frac{2}{8}M}{33}$$

$M = 3199$ This requires $\sqrt{\frac{2M}{2500}} = 50.6$ feet of uniform load.

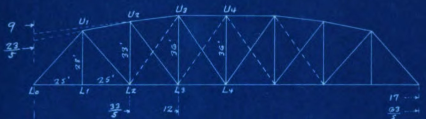
and the maximum tension is 17,400 .

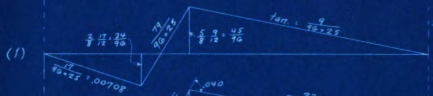
Similarly with wheel 4 at L_3 and 79 feet of uniform load, maximum tension in $U_3 L_3 = 47,800$.

This is Merriman's method, given in Part II of his bridge texts.

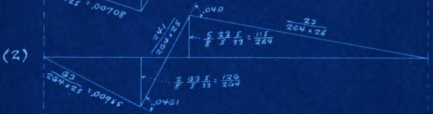
The rule for placing an undivided load is; have as much load on the bridge as possible, without calling into action the main

diagonal adjacent to the vertical and between the vertical and the centre of the span.

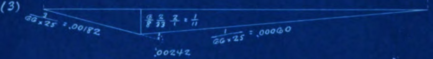




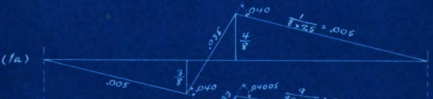
Influence Line for the vertical component of the Stress in U_2L_3.
D.L. + 21900



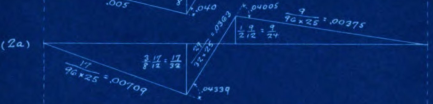
Stress in U_2L_2
U_2L_3 acting.
D.L. -21800



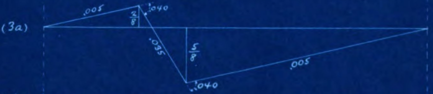
Stress in U_2L_2
U_2L_3 not acting.
D.L. +2100



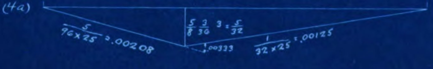
Shear in Panel L_3L_4.
D.L. Shear = +12500



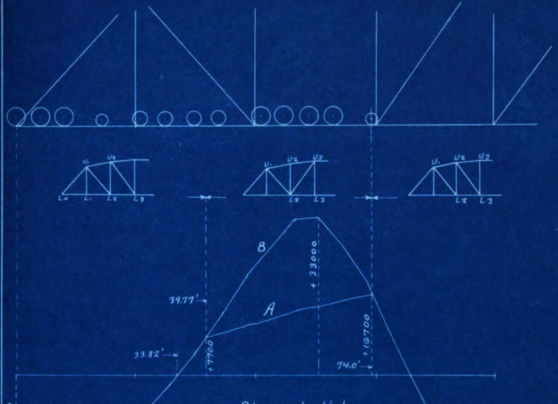
Stress in U_3L_3
U_2L_3 and U_3L_4 acting.
D.L. = -3900



Stress in U_3L_3
U_3L_2 and U_4L_3 acting.
D.L. = +30500

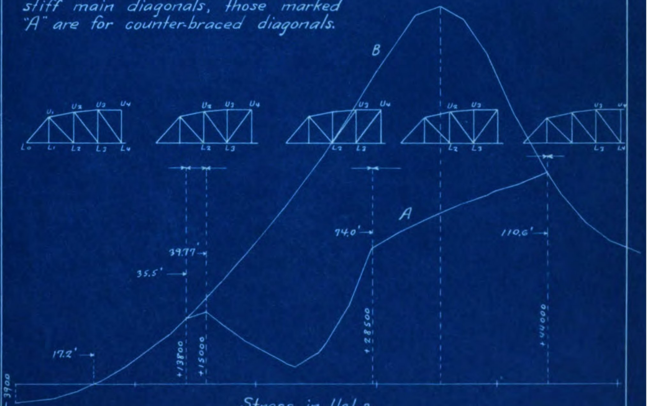


Stress in U_3L_3
U_2L_3 and U_4L_3 acting.
D.L. +8600



Stress in U_2L_2.

The curves marked "B" are for stiff main diagonals, those marked "A" are for counter-braced diagonals.

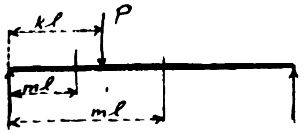


Stress in U_3L_3.

DEFLECTION OF A ~~SINGLE~~ SIMPLE BEAM

Supporting a number of Loads.

———————0———————



The deflection at any point of a beam loaded with any number of loads is equal to the sum of the deflections due to each load.

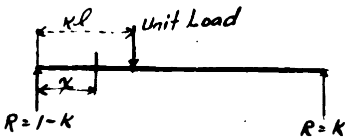
The deflection at any point whose distance from the left support is ml dueto a load P whose distance is kl when ml is less than kl is

$$y = \frac{Pl^3}{6EI}\left[(2k - 3k^2 + k^3)m - (1-k)m^3\right] \qquad (A)$$

And when ml is greater than kl,

$$y = \frac{Pl^3}{6EI}\left[(2k + k^3)m - 3km^2 + km^3 - k^3\right] \qquad (B)$$

These equations are derived as follows:



1. $EI\frac{d^2y}{dx^2} = (1-k)x$ for left segment.

2. $EI\frac{dy}{dx} = (1-k)\frac{x^2}{2} + C_1$ $x = 0$, $C_1 = EI\alpha$

Where α is the slope of the elastic line at the left support.

3. $EIy = (1-k)\frac{x^3}{6} + EI\alpha x + C_2$ for $x = 0$, $y = 0$, $\therefore C_2 = 0$.

4. $EI\frac{d^2y}{dx^2} = (1-k)x - (x - kl)$ for right segment.

5. $EI\frac{dy}{dx} = (1-k)\frac{x^2}{2} - \left(\frac{x^2}{2} - klx\right) + C_3$

Make $x = kl$ in 2 and 5 since these equations give identical slopes at the load.

6. $EI\alpha = -\left(\frac{k^2l^2}{2} - k^2l^2\right) + C_3$ $C_3 = EI\alpha - \frac{k^2l^2}{2}$

7. $EIy = (1-k)\frac{x^3}{6} - \left(\frac{x^3}{6} - \frac{klx^2}{2}\right) + EI\alpha x - \frac{k^2l^2x}{2} + C_4$

Make $x = k\ell$ in 3 and 7 since these equations give identical deflections at the load, and $c_4 = \frac{k^3\ell^3}{6}$.

To determine $EI\alpha$ make $x = \ell$ and $y = 0$ in 7, and

$$EI\alpha = \frac{\ell^2}{6}(3k^2 - k^3 - 2k)$$

Then the equation for deflection in the left segment, that is, when $m\ell$ is less than $k\ell$, is;

$$EIy = (1-k)\frac{x^3}{6} + \frac{\ell^2 x}{6}(3k^2 - k^3 - 2k)$$

Make $x = m\ell$, $y = \frac{\ell^3}{6EI}\left[-(2k - 3k^2 + k^3)m + (1-k)m^3\right]$

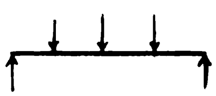
Changing the signs, in order that the deflections will appear positive, and multiplying by P gives equation A.

In the same way equation B is found from 7.

In equations A and B, m and k are interchanged.

This is proof of Maxwell's Reciprocal Theorem for beams.

The Theorem is, that if a beam supports two unit loads, at any two points as a and b, the deflection at a due to the load at b equals the deflection at b due to the load at a, or $D_{ab} = D_{ba}$.



Equations A and B can be used to find the maximum deflection of a beam like that shown in the figure, as follows:

Select the section which probably contains it, write the coefficients of m, m^2, and m^3 which apply to that section, and add the like coefficients. Let C_1, C_2, and C_3 be the sums of the coefficients of m, m^2, and m^3 respectively. Then for a Maximum, $\frac{d(\text{sum of } y\text{'s})}{dm} = C_1 + 2C_2 m + 3C_3 m^2 = 0$

Solve for m, and if $m\ell$ falls in the section assumed, the maximum

deflection is at that point. Find the deflections due to each load at point ml from equations A and B and add them. A full uniform load may be combined with the concentrated loads, the deflection is; $y = \frac{Wl^3}{6EI}\left(\frac{m}{4} - \frac{m^3}{2} + \frac{m^4}{4}\right)$

W = total uniform load.

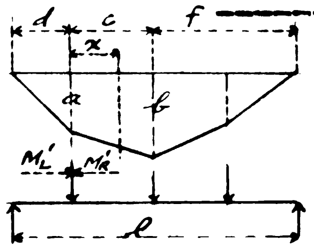
In this a cubic equation must be solved.

$$C_1 + 2C_2 m + 3C_3 m^2 + 4C_4 m^3 = 0$$

$$C_4 = \frac{W}{4}$$

The following table gives values of the bracketed parts of equations A and B. Maximun deflections for combinations of concentrated loads can be found approximately by use of this table without solving any eqations.

DEFLECTION OF A BEAM SUPPORTING A NUMBER OF LOADS. GEOMETRICAL METHOD.

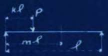


To find the point of maximum deflection and its amount.

Plot the moment diagram. Working from the supports call the static moments of the portions of the moment diagram up to any point, about the left and right supports M_L & M_R. Locate the panel at the right end of which $M_L > M_R$ and at the left end of which $M_R > M_L$. To make $M_L = M_R$ the condition for a horizontal tangent, the moment of the part "x" about the left support must be added to M_L' and the moment of "x" about the right support must be substracted from M_R'.

$$ax\left(d + \frac{x}{2}\right) + \frac{(b-a)x^2}{2c}\left(d + \frac{2}{3}x\right) + M_L' = M_R' - \left[ax\left(f + c - \frac{x}{2}\right) + \frac{(b-a)x^2}{2c}\left(f + c - \frac{2}{3}x\right)\right]$$

$$alx + \frac{b-a}{2c}lx^2 = M_R' - M_L' \qquad d + c + f = l$$



The deflection at any point of a beam supporting a concentrated load $= \frac{P\ell^3}{EI} \times K$, in which K is a constant whose value is given in the table for values of m and k varying by .05.

m or k	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
.05	.00075	.00141	.00195	.00238	.00272	.00296	.00311	.00319	.00319	.00311
.10	.00141	.00270	.00379	.00467	.00534	.00583	.00615	.00630	.00630	.00617
.15	.00195	.00379	.00542	.00675	.00778	.00853	.00902	.00926	.00928	.00909
.20	.00238	.00467	.00675	.00853	.00994	.01097	.01165	.01200	.01205	.01183
.25	.00272	.00534	.00778	.00994	.01172	.01305	.01395	.01444	.01455	.01432
.30	.00296	.00583	.00853	.01097	.01305	.01470	.01584	.01650	.01671	.01650
.35	.00311	.00615	.00902	.01165	.01395	.01584	.01725	.01811	.01845	.01830
.40	.00319	.00630	.00926	.01200	.01444	.01650	.01811	.01920	.01971	.01967
.45	.00319	.00630	.00928	.01205	.01455	.01671	.01845	.01971	.02042	.02053
.50	.00311	.00617	.00909	.01183	.01432	.01650	.01830	.01967	.02053	.02083
.55	.00298	.00591	.00872	.01136	.01378	.01592	.01772	.01913	.02008	.02053
.60	.00279	.00553	.00818	.01067	.01296	.01500	.01674	.01813	.01913	.01967
.65	.00255	.00506	.00748	.00977	.01189	.01378	.01541	.01674	.01772	.01830
.70	.00227	.00450	.00666	.00870	.01059	.01230	.01378	.01500	.01592	.01650
.75	.00195	.00386	.00572	.00748	.00911	.01059	.01189	.01296	.01378	.01432
.80	.00160	.00317	.00469	.00613	.00748	.00870	.00977	.01067	.01136	.01183
.85	.00122	.00242	.00358	.00469	.00572	.00666	.00748	.00818	.00872	.00909
.90	.00082	.00163	.00242	.00317	.00386	.00450	.00506	.00553	.00591	.00617
.95	.00041	.00082	.00122	.00160	.00195	.00227	.00255	.00279	.00298	.00311

m or k	.55	.60	.65	.70	.75	.80	.85	.90	.95	
.05	.00298	.00279	.00255	.00227	.00195	.00160	.00122	.00082	.00041	
.10	.00591	.00553	.00506	.00450	.00386	.00317	.00242	.00163	.00082	
.15	.00872	.00818	.00748	.00666	.00572	.00469	.00358	.00242	.00122	
.20	.01136	.01067	.00977	.00870	.00748	.00613	.00469	.00317	.00160	
.25	.01378	.01296	.01189	.01059	.00911	.00748	.00572	.00386	.00195	
.30	.01592	.01500	.01378	.01230	.01059	.00870	.00666	.00450	.00227	
.35	.01772	.01674	.01541	.01378	.01189	.00977	.00748	.00506	.00255	
.40	.01913	.01813	.01674	.01500	.01296	.01067	.00818	.00553	.00279	
.45	.02008	.01913	.01772	.01592	.01378	.01136	.00872	.00591	.00298	
.50	.02053	.01967	.01830	.01650	.01432	.01183	.00909	.00617	.00311	
.55	.02042	.01971	.01845	.01671	.01455	.01205	.00928	.00630	.00319	
.60	.01971	.01920	.01811	.01650	.01444	.01200	.00926	.00630	.00319	
.65	.01845	.01811	.01725	.01584	.01395	.01165	.00902	.00615	[illegible]	
.70	.01671	.01650	.01584	.01470	.01305	.01097	.00853	[illegible]		
.75	.01455	.01444	.01395	.01305	.01172	[illegible]				
.80	.01205	.01200	.01165	.01097	[illegible]					
.85	.00928	.00926	[illegible]							
.90	[illegible]									

Compute x, complete the static moment about either support up to the section and find the deflection.

$$\text{Deflection} = \frac{\text{Static Moment}}{EI} = \int \frac{Mx\,dx}{EI}$$

GENERAL EQUATION FOR THE DEFLECTION OF A BEAM SUPPORTING A NUMBER OF LOADS.

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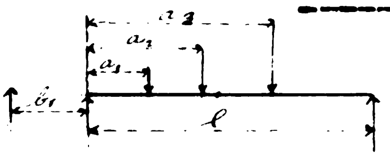


R is the left reaction of all the loads.
R_1 is the resultant of R and P_1.
R_2 is the resultant of R, P_1 and P_2.
b_1 is the distance of the line of action of R_1 to the left of of R. $b_1 = \frac{P_1 a_1}{R - P_1}$ b_2 is the distance of the line of action of R_2 to the left of R. $b_2 = \frac{P_1 a_1 + P_2 a_2}{R - P_1 - P_2}$

Let α be the inclination of the elastic line at the left support, a negative quantity. E and I are Young's modulus for the material and the moment of inertia of the beam respectively.

$$EI\alpha = \frac{P_1 l^2}{6}(3k_1^2 - k_1^3 - 2k_1) + \frac{P_2 l^2}{6}(3k_2^2 - k_2^3 - 2k_2) - \text{etc.} = \frac{l}{6}\left(\frac{3\Sigma Pa^2}{l} - \frac{\Sigma Pa^3}{l^2} - 2\Sigma Pa\right)$$

The summations cover the whole span. This equation was derived on page 44.

The series of constants was established by integrating for a beam supporting two loads which gives the constants C_1 to C_3 & K_1 to K_3.

$$C_1 = EI\alpha \qquad K_1 = 0$$
$$C_2 = C_1 - \frac{P_1 a_1^2}{2} \qquad K_2 = \frac{P_1 a_1^3}{6}$$
$$C_3 = C_2 - \frac{P_2 a_2^2}{2} \qquad K_3 = K_2 + \frac{P_2 a_2^3}{6}$$
$$C_n = EI\alpha - \sum_0^{n-1} \frac{Pa^2}{2} \qquad K_n = \sum_0^{n-1} \frac{Pa^3}{6}$$

First Section: $EI\frac{d^2y}{dx^2} = Rx \qquad EI\frac{dy}{dx} = \frac{Rx^2}{2} + C_1$

$$EIy = \frac{Rx^3}{6} + C_1 x + K_1$$

Second Section: $EI\frac{d^2y}{dx^2} = R_1(x + b_1) \qquad EI\frac{dy}{dx} = R_1\left(\frac{x^2}{2} + b_1 x\right) + C_2$

$$EIy = R_1\left(\frac{x^3}{6} + \frac{b_1 x^2}{2}\right) + C_2 x + K_2$$

All other sections are similar to this. The general equations are

then as follows:

$$EI\frac{d^2y}{dx^2} = (R-\Sigma P)\left(x+\frac{\Sigma Pa}{R-\Sigma P}\right) = (R-\Sigma P)x + \Sigma Pa$$

$$EI\frac{dy}{dx} = (R-\Sigma P)\frac{x^2}{2} + x\Sigma Pa + EI\alpha - \Sigma\frac{Pa^2}{2}$$

$$EIy = (R-\Sigma P)\frac{x^3}{6} + \frac{x^2}{2}\Sigma Pa + \left[EI\alpha - \Sigma\frac{Pa^2}{2}\right]x + \Sigma\frac{Pa^3}{6}$$

The summations include the loads to the left of the section. This equation is general and may be applied to a simple beam loaded in any manner with uniform and concentrated loads; in the case of uniform loads covering a part or all of the span, the summation signs must be replaced by integral signs. The deflection at the quarter-point of a beam covered with uniform load will be found, this will furnish a check on the correctness of the general equation, since the equation for uniform load,

$$y = \frac{wl^4}{6EI}\left(\frac{m}{4} - \frac{m^3}{2} + \frac{m^4}{4}\right)$$

gives, $y = \frac{19wl^4}{2048EI}$, positive in this case, since the equation was arranged to give deflections positive downward.

$$EI\alpha = \frac{l}{6}\left(\frac{3w}{l}\int_0^l x^2dx - \frac{w}{l^2}\int_0^l x^3dx - 2w\int_0^l xdx\right) = -\frac{wl^3}{24}$$

w = the load per linear unit.

$R = \frac{wl}{2}$ $\quad\quad \Sigma P = \frac{wl}{4}$

$x = l/4$ $\quad\quad \Sigma Pa = \int_0^{\frac{l}{4}} wxdx = \frac{wl^2}{32}$

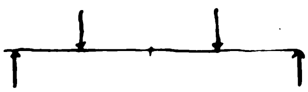
$\Sigma Pa^2 = \int_0^{\frac{l}{4}} wx^2dx = \frac{wl^3}{192}$ $\quad\quad \Sigma Pa^3 = \int_0^{\frac{l}{4}} wx^3dx = \frac{wl^4}{1024}$

$$EIy = \left(\frac{wl}{2} - \frac{wl}{4}\right)\frac{l^3}{384} + \frac{l^2}{32}\times\frac{wl^2}{32} + \left[-\frac{wl^3}{24} - \frac{wl^3}{384}\right]\frac{l}{4} + \frac{wl^4}{6144}$$

$$= \frac{wl^4}{1536} + \frac{wl^4}{1024} - \frac{17wl^4}{1536} + \frac{wl^4}{6144} = -\frac{19wl^4}{2048}$$

If there are no loads to the left of the section, the general equation becomes: $EI\cdot y = \frac{Rx^3}{6} + EI\alpha x$; thus for a beam with a single load at the centre: $EI y = \frac{Px^3}{12} - \frac{Pl^3}{16}x$
For $x = \frac{l}{2}$, $EIy = -\frac{Pl^3}{48}$

If equal loads are balanced about the centre of the beam, the general equation becomes; $EI y = \frac{x^2}{2}\Sigma Pa + [EI\alpha - \Sigma\frac{Pa^2}{2}]x + \Sigma\frac{Pa^3}{6}$ for middle sections. Thus for loads at the quarter-points



the equation of the middle section is:

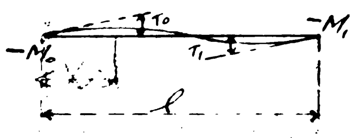
$$EI\cdot y = \frac{Pl}{4}\,\frac{x^2}{2} + \left[-\frac{9Pl^2}{96} - \frac{Pl^2}{32}\right]x + \frac{Pl^3}{384}$$

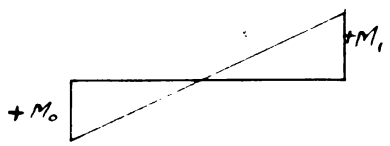
$$= \frac{Plx^2}{8} - \frac{Pl^2x}{8} + \frac{Pl^3}{384}$$

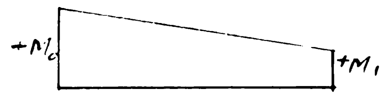
$$= -\frac{11}{384}Pl^3 \quad \text{for } x = \frac{l}{2}.$$

SECONDARY STRESSES IN THE RIVETED TRUSSES of a RAILROAD BRIDGE.

The method of solution is that of MOHR with an adaption of the Williot displacement diagram for finding certain angles. The development of the equation of the elastic line for a bar acted upon by moments at the ends is as follows:







$$EI\frac{d^2y}{dx^2} = -M = (M_0 + M_1)\frac{x}{\ell} - M_0 \qquad (1)$$

$$EI\frac{dy}{dx} = (M_0 + M_1)\frac{x^2}{2\ell} - M_0 x + C \qquad (2)$$

$$EIy = (M_0 + M_1)\frac{x^3}{6\ell} - M_0\frac{x^2}{2} + Cx + C_1 \qquad (3)$$

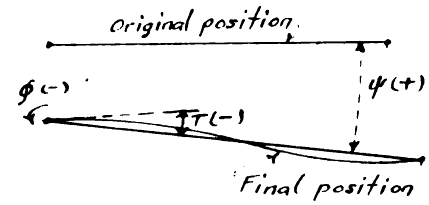
When $x = 0$, $y = 0$, and when $x = \ell$, $y = 0$.

$\therefore$ $C_1 = 0$ and $C = \dfrac{(2M_0 - M_1)\ell}{6}$

Put $x = 0$ in (2) $\quad \dfrac{dy}{dx} = T_0 = \dfrac{\ell}{6EI}(2M_0 - M_1) \qquad (4)$

Put $x = \ell$ in (2) $\quad \dfrac{dy}{dx} = T_1 = \dfrac{\ell}{6EI}(2M_1 - M_0) \qquad (5)$

When a truss deflects, all members, in general, revolve through some angle, and every panel point rotates through some angle. Calling the former angles ψ and the latter ϕ and substituting $\phi_0 - \psi$ for τ_0 and $\phi_1 - \psi$ for τ_1,



$$M_0 = \frac{2EI}{l}(2\phi_0 + \phi_1 - 3\psi) \qquad (6)$$

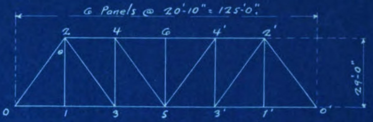
$$M_1 = \frac{2EI}{l}(2\phi_1 + \phi_0 - 3\psi) \qquad (7)$$

The value of ψ for each member can be obtained most easily by scaling from the displacement diagram the net movement of one end of the member with respect to the other end and at a right angle to the member, and dividing the movement by the length of the member. On pages 54 and 55 these diagrams are given, on page 53 the data from which the diagrams were constructed is given. On page 56 are shown the values of the moments at the ends of all the truss members expressed in terms of ϕ and ψ, and also the final values found by substituting the values of ϕ solved from the equations on page 57. These equations are formed by equating the sum of the bending moments around each panel point to zero. The influence lines for secondary stress and the ratio which the secondary stress bears to the primary stress are given on page 58. For example, the end bottom chord has a maximum primary stress of $+1.80$ for full load. The secondary bending moment at the end "o" for this loading is -1.522, the negative sign indicates the direction of the moment, in this case it causes tension on the top of the bottom chord. The unit stress on the extreme fibre equals

$M\frac{e}{I} = 1.522 \times .012$. The unit primary stress equals $\frac{1.80}{area} = \frac{1.80}{19.42}$. Then the ratio sought is ,

$$\frac{1.522 \times .012 \times 19.42}{1.80} = .20$$

sec θ = 1.23
tan θ = .718



Member	Length	Gross Area	Load at ① Stress	Load at ① $\frac{Pl}{a}$	Load at ① ψ	Load at ③ Stress	Load at ③ $\frac{Pl}{a}$	Load at ③ ψ	Load at ⑤ Stress	Load at ⑤ $\frac{Pl}{a}$	Load at ⑤ ψ
02	428"	39.32""	− 1.02	− 11.1	+.091	−.82	− 8.9	+.107	− .62	−6.8	+.087
0'2'	"	"	− .20	− 2.2	−.029	−.41	− 4.5	−.056			
01	250"	19.42	+ .60	+ 7.7	+.204	+.48	+ 6.2	+.136	+ .36	+4.6	+.110
0'1'	"	"	+ .12	+ 1.5	−.032	+.24	+ 3.1	−.071			
12	348	18.02	+ 1.00	+19.3	+.049	±0	±0	+.074	± 0	± 0	+.063
1'2'	"	"	± 0	± 0	−.022	± 0	±0	−.040			
13	250	19.42	+ .60	+ 7.7	−.068	+.48	+6.2	+.136	+.36	+4.6	+.108
1'3'	"	"	+ .12	+ 1.5	−.038	+.24	+3.1	−.070			
23	428	21.76	−.20	− 3.9	+.021	+.82	+16.1	+.083	+.62	+12.2	+.070
2'3'	"	"	+ .20	+ 3.9	−.023	+.41	+ 8.1	−.045			
24	250	38.31	− .48	− 3.1	−.004	−.96	−6.3	+.112	−.72	−4.7	+.144
2'4'	"	"	− .24	− 1.6	−.048	−.48	− 3.1	−.094			
34	348	19.42	+ .17	+ 3.0	+.019	+.33	+5.9	+.037	−.50	−9.0	+.036
3'4'	"	"	− .17	− 3.0	−.013	−.33	−5.9	−.021			
35	250	32.26	+ .48	+ 3.7	−.026	+.96	+7.4	−.052	+.72	+5.6	+.124
3'5'	"	"	+ .24	+ 1.9	−.040	+.48	+3.7	−.079			
45	428	19.42	− .20	−4.4	±0	−.41	−9.0	±0	+.62	+13.7	+.044
4'5'	"	"	+ .20	+4.4	−.015	+.41	+9.0	−.027			
46	250	38.31	− .36	−2.4	−.014	−.72	−4.7	−.029	−1.08	−7.1	+.088
4'6'	"	"	− .36	−2.4	−.030	−.72	−4.7	−.056			
56	348	13.52	±0	±0	+.001	±0	±0	+.003	±0	±0	±0

	Stress.	$\frac{Pl}{a}$	I	$\frac{2I}{l} = N$	ψN		Depth.	e/I
02	−3.08	−33.5	1932	9.04	+3.32	×2"	18 11/16"	.0038} .0059}
01	+1.80	+23.2	625	5.00	2.78		15"	.012
12	+1.00	+19.3	119	.684	.167		12 1/4"	.052
13	+1.80	+23.2	625	5.00	1.38		15"	.012
23	+1.85	+36.4	662	3.09	.732		15"	.011
24	−2.87	−18.7	1585	12.69	4.97	×2"	18 5/8"	.0046} .0072}
34	− .50	− 9.0	625	3.59	.442		15"	.012
35	+2.87	+22.2	788	6.30	1.03		15"	.0095
45	+ .62	+13.7	625	2.92	.242		15"	.012
46	−3.23	−21.1	1585	12.69	1.60	×2"	18 5/8"	.0046} .0072}
56	±0	±0	119	.684	±0		12 1/4"	.052

Williot Diagrams for Unit Loads.

Diagram 1.

Load at ①.
Scale 30.

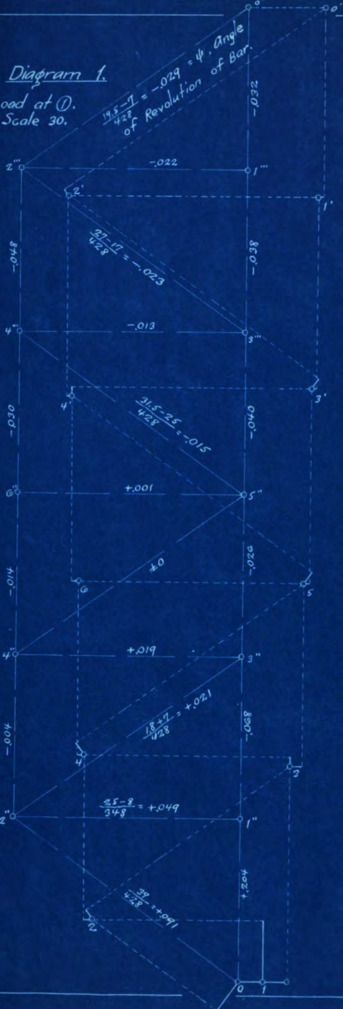
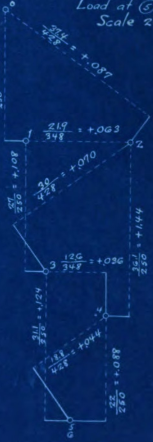


Diagram 2.

Load at ⑤.
Scale 20.



Williot Diagrams for Unit Loads.

Diagram 3. Load at ③. Scale 30.

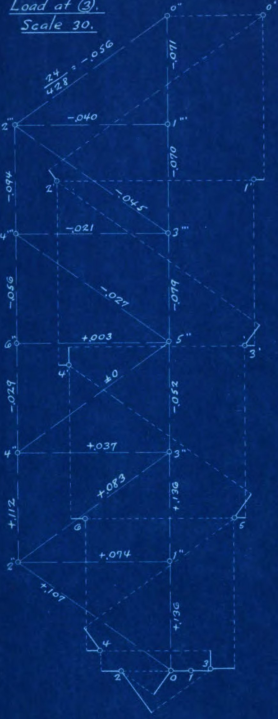
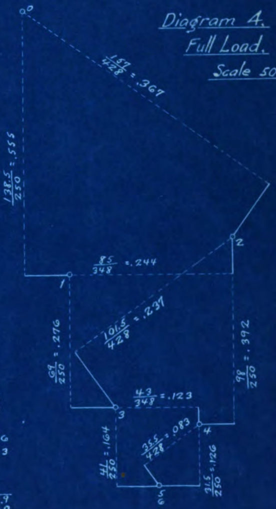


Diagram 4. Full Load. Scale 50.



Deflection at panel point,

Load at	⑤	③	①	②	④
5	85.6	54.5	27.5	27.5	63.6
3	55	68	34	34.4	62.3
3'	55	35.3	17.7	17.4	41
1	27.5	34	51	32	31
1'	27.5	17.5	8	8	20
	250.6	209.3	138.2	119.3	217.9
Diag. 4	248.5	207.5	138.5	119.0	217.0

	$-3\psi N$ ①	③	⑤
$M_{02} = 18.08\,\phi_0 + 9.04\,\phi_2$	−2.47	−2.90	−2.36
$M_{02}' =$ " ϕ_0' + " ϕ_2'	+.787	+1.52	
$M_{01} = 10.00\,\phi_0 + 5.00\,\phi_1$	−3.06	−2.04	−1.65
$M_{01}' =$ " ϕ_0' + " ϕ_1'	+.48	+1.065	
$M_{12} = 1.368\,\phi_1 + .684\,\phi_2$	−.1005	−.152	−.129
$M_{12}' =$ " ϕ_1' + " ϕ_2'	+.0452	+.0821	
$M_{13} = 10.00\,\phi_1 + 5.00\,\phi_3$	+1.020	−2.04	−1.62
$M_{13}' =$ " ϕ_1' + " ϕ_3'	+.570	+1.05	
$M_{23} = 6.18\,\phi_2 + 3.09\,\phi_3$	−.195	−.770	−.649
$M_{23}' =$ " ϕ_2' + " ϕ_3'	+.213	+.417	
$M_{24} = 25.38\,\phi_2 + 12.69\,\phi_4$	+.152	−4.264	−5.482
$M_{24}' =$ " ϕ_2' + " ϕ_4'	+1.827	+3.579	
$M_{34} = 7.18\,\phi_3 + 3.59\,\phi_4$	−.205	−.399	−.388
$M_{34}' =$ " ϕ_3' + " ϕ_4'	+.140	+.226	
$M_{35} = 12.60\,\phi_3 + 6.30\,\phi_5$	+.492	+.984	−2.344
$M_{35}' =$ " ϕ_3' + " ϕ_5'	+.757	+1.492	
$M_{45} = 5.84\,\phi_4 + 2.92\,\phi_5$	±0	±0	−.385
$M_{45}' =$ " ϕ_4' + $2.92\,\phi_5$	+.131	+.237	
$M_{46} = 25.38\,\phi_4 + 12.69\,\phi_6$	+.534	+1.104	−3.350
$M_{46}' =$ " ϕ_4' + " ϕ_6	+1.142	+2.131	
$M_{56} = 1.368\,\phi_5 + .684\,\phi_6$	−.002	−.006	±0

Bending Moments.

	$-3\psi N$ ①	③	⑤	From left equations. ①	③	⑤	From right equations. ①	③	⑤
$M_{20} = 9.04\,\phi_0 + 18.08\,\phi_2$	−2.47	−2.90	−2.36	+.932	+.217	+.249	−.443	+.325	+.517
$M_{20}' =$ " ϕ_0' + " ϕ_2'	+.787	+1.52		−.051	−.177	−.249	−.169	−.408	−.517
$M_{10} = 5.00\,\phi_0 + 10.00\,\phi_1$	−3.06	−2.04	−1.65	−.960	−.145	−.232	−1.500	+.085	−.108
$M_{10}' =$ " ϕ_0' + " ϕ_1'	+.48	+1.065		+.032	+.153	+.232	−.020	+.050	+.108
$M_{21} = .684\,\phi_1 + 1.368\,\phi_2$	−.1005	−.152	−.129	+.009	+.084	+.102	−.021	+.124	+.106
$M_{21}' =$ " ϕ_1' + " ϕ_2'	+.0452	+.0821		−.018	−.074	−.102	−.034	−.078	−.106
$M_{31} = 5.00\,\phi_1 + 10.00\,\phi_3$	+1.020	−2.04	−1.62	+1.510	−.732	+.005	+.980	−.860	−.035
$M_{31}' =$ " ϕ_1' + " ϕ_3'	+.570	+1.05		+.052	+.035	−.005	+.083	+.137	+.035
$M_{32} = 3.09\,\phi_2 + 6.18\,\phi_3$	−.195	−.770	−.649	−.164	+.113	+.386	−.366	−.145	+.346
$M_{32}' =$ " ϕ_2' + " ϕ_3'	+.213	+.417		−.125	−.242	−.386	−.097	−.163	−.346
$M_{42} = 12.69\,\phi_2 + 25.38\,\phi_4$	+.152	−4.264	−5.482	+.098	−.600	−.990	+.330	−1.615	−.914
$M_{42}' =$ " ϕ_2' + " ϕ_4'	+1.827	+3.579		+.348	+.745	+.990	+.376	+.940	+.914
$M_{43} = 3.59\,\phi_3 + 7.18\,\phi_4$	−.205	−.399	−.388	−.496	+.039	+.789	−.377	+.037	+.858
$M_{43}' =$ " ϕ_3' + " ϕ_4'	+.140	+.226		−.212	−.392	−.789	−.238	−.429	−.858
$M_{53} = 6.30\,\phi_3 + 12.60\,\phi_5$	+.492	+.984	−2.344	−.122	+1.060	−1.046	−.018	+.392	−1.695
$M_{53}' =$ " ϕ_3' + " ϕ_5	+.757	+1.492		+.238	+.393	+1.046	+.295	+.314	+1.695
$M_{54} = 2.92\,\phi_4 + 5.84\,\phi_5$	±0	±0	−.385	−.092	+.056	+.327	−.140	−.090	−.029
$M_{54}' =$ " ϕ_4' + " ϕ_5	+.131	+.237		−.149	−.333	−.327	−.103	−.340	+.029
$M_{64} = 12.69\,\phi_4 + 25.38\,\phi_6$	+.534	+1.104	−3.350	+.140	+1.483	−.264	−.063	+.236	−1.802
$M_{64}' =$ " ϕ_4' + " ϕ_6	+1.142	+2.131		−.074	−.203	+.264	+.134	−.094	+1.802
$M_{65} = .684\,\phi_5 + 1.368\,\phi_6$	−.002	−.006	±0	−.042	−.135	±0	−.041	−.128	±0

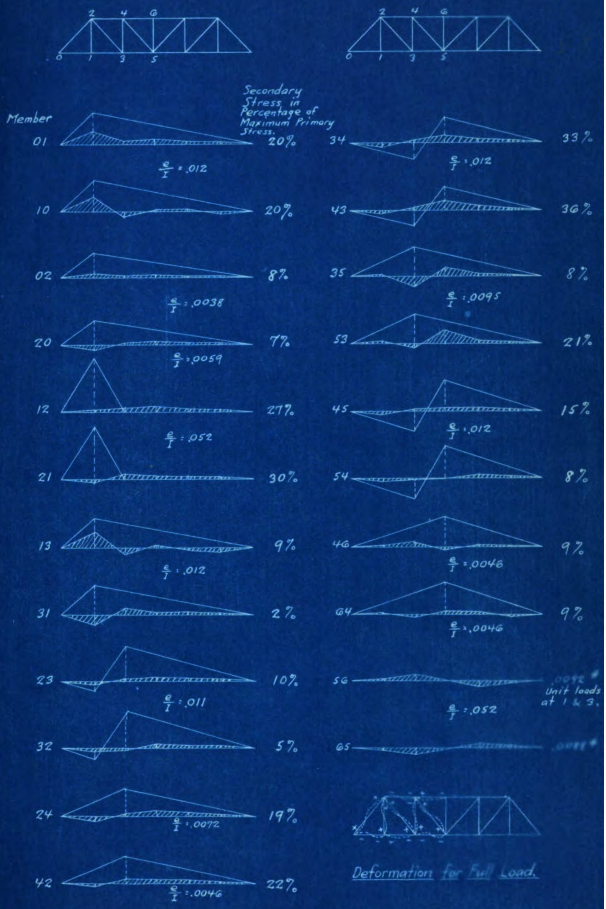
	Equation	①	③	⑤
0	$28.08\phi_0 + 5.00\phi_1 + 9.04\phi_2$	−5.53	−4.94	−4.01
0′	" ϕ_0' + " ϕ_1' + " ϕ_2'	+1.267	+2.585	
1	$5.00\phi_0 + 21.37\phi_1 + .68\phi_2 + 5.00\phi_3$	−2.141	−4.232	−3.399
1′	" ϕ_0' + " ϕ_1' + " ϕ_2' + " ϕ_3'	+1.095	+2.197	
2	$9.04\phi_0 + .68\phi_1 + 51.01\phi_2 + 3.09\phi_3 + 12.69\phi_4$	−2.614	−8.086	−8.620
2′	" ϕ_0' + " ϕ_1' + " ϕ_2' + " ϕ_3' + " ϕ_4'	+2.872	+5.598	
3	$5.00\phi_1 + 3.09\phi_2 + 35.96\phi_3 + 3.59\phi_4 + 6.30\phi_5$	+1.112	−2.225	−5.001
3′	" ϕ_1' + " ϕ_2' + " ϕ_3' + " ϕ_4' + " ϕ_5'	+1.680	+3.185	
4	$12.69\phi_2 + 3.59\phi_3 + 63.78\phi_4 + 2.92\phi_5 + 12.69\phi_6$	+.481	−3.559	−9.605
4′	" ϕ_2' + " ϕ_3' + " ϕ_4' + " ϕ_5 + " ϕ_6	+3.240	+6.173	
5	$6.30\phi_3$ / $6.30\phi_3'$ + $2.92\phi_4$ / $2.92\phi_4'$ + $38.25\phi_5 + .68\phi_6$	+1.378	+2.707	±0
6	$12.69\phi_4$ / $12.09\phi_4'$ + $.68\phi_5 + 52.13\phi_6$	+1.674	+3.229	±0

	①	③	⑤
ϕ_0	+.176	+.111	+.0863
ϕ_0'	−.0265	−.0540	
ϕ_1	+.0680	+.157	+.111
ϕ_1'	−.0367	−.0745	
ϕ_2	+.0240	+.123	+.116
ϕ_2'	−.0396	−.0796	
ϕ_3	−.0380	+.0395	+.103
ϕ_3'	−.0304	−.0540	
ϕ_4	−.0050	+.0428	+.122
ϕ_4'	−.0374	−.0042	
ϕ_5	−.0214	−.0666	±0
ϕ_6	−.0210	−.0556	±0

0	$M_{02} + M_{01} = 0$	$28.08\phi_0 + 5.00\phi_1 + 9.04\phi_2 - 18.30 = 0$	$\phi_0 = .454$
1	$M_{10} + M_{12} + M_{13} = 0$	$5.00\phi_0 + 21.37\phi_1 + .68\phi_2 + 5.00\phi_3 - 12.98 = 0$	$\phi_1 = .444$
2	$M_{20} + M_{21} + M_{23} + M_{24} = 0$	$9.04\phi_0 + .68\phi_1 + 51.01\phi_2 + 3.09\phi_3 + 12.69\phi_4 - 27.57 = 0$	$\phi_2 = .378$
3	$M_{31} + M_{32} + M_{34} + M_{35} = 0$	$5.00\phi_1 + 3.09\phi_2 + 35.96\phi_3 + 3.59\phi_4 - 10.75 = 0$	$\phi_3 = .178$
4	$M_{42} + M_{43} + M_{45} + M_{46} = 0$	$12.69\phi_2 + 3.59\phi_3 + 63.78\phi_4 - 21.76 = 0$	$\phi_4 = .256$

	ϕ_0	ϕ_1	ϕ_2	ϕ_3	ϕ_4
①	+.176	+.0680	+.0240	−.0380	−.005
①′	+.0265	+.0367	+.0396	+.0304	+.0374
③	+.111	+.157	+.123	+.0395	+.0428
③′	+.0540	+.0745	+.0796	+.0540	+.0042
⑤	+.0863	+.111	+.116	+.103	+.122
	+.4538	+.4472	+.3822	+.1889	+.2614
Check Values from full load	+.454	+.444	+.378	+.178	+.256

Influence Lines for Primary and Secondary Stresses.



DEFLECTION OF A TRUSS

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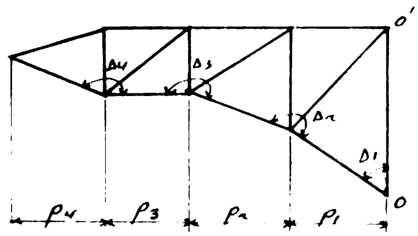
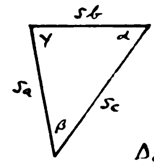




$$\Delta_\alpha = \frac{(S_a - S_b)\cot\gamma}{E} + \frac{(S_a - S_c)\cot\beta}{E}$$

The following method is sometimes convenient for finding the deflections of all the panel points of a truss.

Let Δ_1, Δ_2, etc. be the changes in the angles between the successive lower chord members. These changes are computed by means of the equation shown, in which S_a, S_b, and S_c are the unit stresses in the members due to the given system of loads. Let v_1, v_2, etc. be the vertical projections of the chord members p_1, p_2, etc. The vertical deflections, considering member oo' fixed are

$$d_1 = p_1 \Delta_1 + \frac{v_1 S_1}{E}$$

$$d_2 = p_2 (\Delta_1 + \Delta_2) + \frac{v_2 S_2}{E} + d_1$$

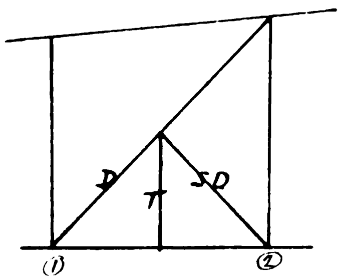
$$d_3 = p_3 (\Delta_1 + \Delta_2 + \Delta_3) + \frac{v_3 S_3}{E} + d_2$$

etc.

When the lower chord is horizontal and the panel lengths equal, the equations become very simple. Relative deflections, which will suffice when reactions of indeterminate structures are sought, can be found by dropping the factor E, and using unity as the panel length. In this case the deflections can be tabulated as follows:

Δ	$\Sigma\Delta$	Deflection.
Δ_1	Δ_1	$d_1 = \Delta_1$
Δ_2	$\Delta_1 + \Delta_2$	$d_2 = \Delta_1 + \Delta_2 + d_1$
Δ_3	$\Delta_1 + \Delta_2 + \Delta_3$	$d_3 = \Delta_1 + \Delta_2 + \Delta_3 + d_2$

etc.



In case the truss has sub-divided panels as in the figure, and the chords are straight between the main panel points, the deflections of the sub-panel points, due to a load at the end of the truss, can be found as follows; omit the sub-bracing, members T and SD. Now the deflection at the sub-panel point is $\frac{d_1 + d_2}{2}$. Replace the sub-members and find the stresses caused in the main truss members by a unit load at the sub-panel point. These stresses occur in the lower part of the main diagonal and in the bottom chord 1 - 2. Find $\Sigma \frac{pul}{E}$ due to these two stresses, where p is the unit stress due to the load at the end of the truss, u is the stress in the main truss member acting as a truss to transfer the unit load at the sub-point to the main panel points and l is the length of the member, measured in whatever units are being used for the problem.

Correct the deflection $\frac{d_1 + d_2}{2}$ by $\Sigma \frac{pul}{E}$.

TRUE REACTIONS OF SWING BRIDGES.

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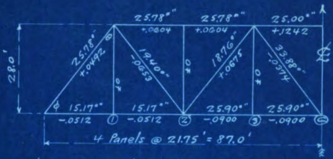
Swing bridges are usually designed in accordance with the stresses obtained by the use of the three moment theorem. The moment of inertia of the trusses is considered constant and the effect of deflections due to the stresses in the web members is neglected.

After the bridge is designed, the stresses may be found by means of deflections and such corrections made in the sections of the truss members as seem necessary. The greatest differences in stresses will be found near the centre pier of the bridge. Three trusses will be analyzed, of three different shapes, in order to get an idea of how the shape of the truss affects the reactions. The first one has a horizontal top chord throughout It is 174 feet long, for single track railroad, amd is designed for Cooper's E-40 loading. The second is 246 feet long and has a horizontal top chord, except in two centre panels where the chords are inclined. It is designed for E-60 loading, and is for single track railroad. The third is 436 feet long, and has inclined top chords. It is designed for E-50 loading, and is for single track railroad.

The results are shown on the three blue-prints following.

Centre Bearing Draw Bridge. 174 Feet Long.

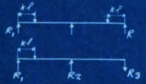


Symmetrical about Centre Line.

Gross Areas and Unit Stresses due to a load of one pound at each end.

cot. θ = 1.287 cot. ϕ = .777

			αE	ΣαE	Defl. × E / Panel length	Reactions for Balanced Loads.	
c	+.1616 × .777 +.0374 × 1.287		+.1253 +.0482 +.1735	+.1735	.1735	.1138	k = 3/4
3	+.0526 × .777 -.0374 × 1.287 +.0675 × 1.287 +.1575 × .777	+.2101 × .777 = +.0301 × 1.287 =	+.1632 +.0388 +.2020	+.3755	.5490	.3599	k = 1/2
2	-.0675 × 1.287 -.0071 × .777 +.1257 × .777 +.0653 × 1.287	+.1186 × .777 = -.0022 × 1.287 =	+.0922 -.0028 +.0894	+.4649	1.0139	.6649	k = 1/4
1	-.0141 × .777 -.0653 × 1.287 +.0492 × 1.287 +.1004 × .777	+.0863 × .777 = -.0161 × 1.287 =	+.0670 -.0207 +.0463	+.5112	1.5251		



R_3 for a single load on the left span =

$$\frac{R_1 \text{ (for balanced loads)} - (1-k)}{2}$$

Reactions by Elastic Analysis.

	R_1	R_2	R_3
k = 1/4	+.708	+.335	-.0425
k = 1/2	+.430	+.641	-.0705
k = 3/4	+.182	+.886	-.0681
			-.1811

Reactions by the Beam Formula.

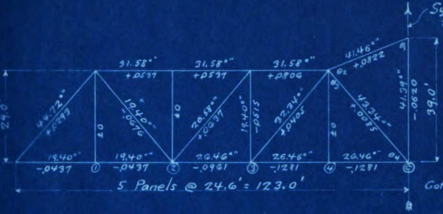
R_1	R_2	R_3
+.69141	+.36718	-.05859
+.40625	+.68750	-.09375
+.16797	+.91406	-.08203
		-.23437

$$\frac{.23437 - .1811}{.1811} = 29.4\%$$

The Uplift and Centre Moment by the Beam Formula are 29.4% larger than the true values.

Centre Bearing Draw Bridge. 246 Feet Long.

Symmetrical about the Centre Line.



Gross Areas and Unit Stresses due to a load of one pound at each end.

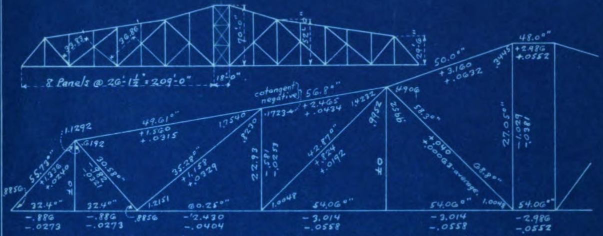
Cot. θ_1 = .4065
θ_2 = .3238
θ_3 = 1.1788
θ_4 = .8484

		αE	$\Sigma\alpha E$	Defl. × E / Panel length	R_1 Balanced Loads	R_3 Single Load	R_3 Beam Formula
α_5	+.1442 × .4065 = +.0586 +.0737 × .3238 = +.0239 -.0085 × 1.1788 = -.0100	+.0725	.0725	.0725	.0293	-.0853	-.0720
α_4	+.1366 × .8484 = +.1159 +.0085 × 1.1788 = +.0100 +.0405 × 1.1788 = +.0477 +.1686 × .8484 = +.1430	+.3166	.3891	.4616	.1865	-.1067	-.0960
α_3	-.0405 × 1.1788 = -.0477 +.0401 × .8484 = +.0340 +.1152 × 1.1788 = +.1358 +.1598 × .8484 = +.1356	+.2577	.6468	1.1084	.4478	-.0761	-.0840
α_2	-.1152 × 1.1788 = -.1358 -.0100 × .8484 = -.0085 +.1213 × .8484 = +.1029 +.0676 × 1.1788 = +.0797	+.0383	.6851	1.7935	.7247	-.0376	-.0480
						-.3057	-.3000
α_1	-.0239 × .8484 = -.0203 -.0676 × 1.1788 = -.0797 +.0293 × 1.1788 = +.0345 +.0730 × .8484 = +.0619	-.0036	.6815	2.4750			

$$\frac{.3057 - .3000}{.3057} = 1.9\%$$

The Uplift and Centre Moment by the Beam Formula are 1.9% too small.

Rim Bearing Draw Bridge, 436 Feet Long.



Gross Areas, Cotangents, Stresses and Unit Stresses due to 1# Load at Ends.

.0622 × 9 = .5598
.0552 × 9 = .4968
1.0566 / 70 = +.01509

+.1013 × .3445 = +.03490
+.0626 × .4906 = +.03071
−.0006 × .9952 = −.00063
+.08007 × 52.25 = 4.1836

+.0006 × .9952 = +.00060
+.0564 × 1.0048 = +.05666
+.0192 × .9952 = +.01911
+.0750 × .0048 = +.07536
+.15173
+.08007
+.23180 × 52.25 = 12.1115

−.0192 × .9952 = −.0191
+.0242 × 1.4232 = +.0344
+.0687 × −.1723 = −.0118
+.0582 × .8230 = +.0479
+.0733 × 1.2151 = +.0891
+.1405
+.2318
+.3723 × 52.25 = 19.4527

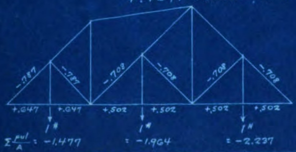
−.0582 × .8230 = −.0490
−.0014 × 1.7540 = −.0025
+.0636 × .6192 = +.0394
+.0321 × 1.1292 = +.0362
+.0241
+.3723
+.3964 × 26.125 = 10.3558

−.0048 × .8856
−.0321 × 1.1292
+.0240 × 1.1292
+.0513 × .8856
+.0418
−.0091
+.0327
+.3964
+.4291 × 26.125 = 11.2102

	Reactions for Balanced Loads	R4 for Single Load	R4 Beam Formula	
.6146	.0107	.0571	.0454	7/8
4.1836	.0730	.0885	.0727	6/8
8.2753	.1444	.1153	.0843	5/8
16.2951	.2843	.1079	.0830	4/8
23.7844	.4144	.1053	.0714	3/8
35.7478	.6237	.0632	.0519	2/8
46.1036	.8044	.0353	.0272	1/8
		.5726	.4359	
57.3138				

$\frac{.5726 - .4359}{.5726} = 23.9\%$

The Uplift and Centre Moment by the Beam Formula are 23.9% less than the true values.



DEFLECTION OF BEAMS OF VARIABLE MOMENT OF INERTIA

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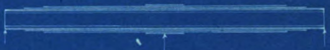
The deflection of any point of a beam from the tangent at any other point equals $\int \frac{Mx\,dx}{EI}$, the origin being taken at the point whose deflection is required. If the tangent is horizontal, then the vertical deflection of the point is found. The $\int Mx\,dx$ is the static moment of the moment diagram about the point whose deflection is sought, up to the tangent. With moment diagrams consisting of triangles and parabolas, this integral can be easily evaluated by this method. In the case of plate girders with constant depth of web and varying number of cover plates, the moment of inertia, I, is constant over definite lengths of the girder, and the exact value of the static moment divided by E I can be found. When the web plate varies in depth and the number of cover plates also varies, the moment of inertia of the girder does not follow any regular law, and an approximate value of the deflections must be used. The change in angle between the tangents to the elastic line at any two sections is $\int \frac{M\,dx}{EI}$. Having found these angular changes for sections taken at short distances apart along the beam, the deflections may be found by summing up the angular changes as in the case of a truss. The method used will depend also upon whether the deflection of every point of the beam is wanted, or whether the end deflection only is wanted. These methods are used on the following two blue prints to find the true reactions of a plate girder draw span and to find the deflection of an engine turntable.

In the case of the draw-span, the direct solution by means of the equation shown at the top of the page seems to be simpler.

Plate Girder Draw Bridge - 100 Feet Long.



1 Web Pl. 58 × 9/16
4 Ls 6×6 × 23#
2 Cov. Pl. 14 × 9/16 - F.L.
2 do do - 88'
2 do 14 × 1/2 - 76'
2 do 14 × 3/8 - 12'

Loading - E50

$$M = -\frac{3wl}{2}\,\frac{\sum_0^l \left[\frac{x^3}{I}\left(\frac{l}{3}-\frac{x}{4}\right)\right]_{x_1}^{x_2}}{\sum_0^l \left(\frac{x^3}{I}\right)_{x_1}^{x_2}}$$

x_1	x_2	I	x^3	$\left[\frac{x^3}{I}\right]_{x_1}^{x_2}$		$\frac{l}{3}-\frac{x}{4}$	$\left[\frac{x^3}{I}\left(\frac{l^3}{3}-\frac{x}{4}\right)\right]_{x_1}^{x_2}$	
0	6	1.00	216	216		15.17	3277	
			0	0	216	16.67	0	3277
6	12	1.32	1728	1309		13.67	17893	
			216	164	1145	15.17	2488	15405
12	44	1.63	85184	52260		5.67	296280	
			1728	1060	51200	13.67	14490	281790
44	50	1.86	125000	67210		4.17	280250	
			85184	45800	21410	5.67	259650	20600
					73,971			321,072

$M = -\frac{3}{2} wl \frac{321,072}{73,971}$

$= -6.51\ wl$

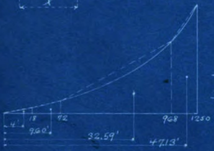
$M = -\frac{1}{8} wl^2 = -6.25\ wl$

with constant I.

Error $= \frac{.26}{6.25} = 4.16\ \%$

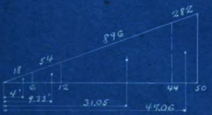
Moment of Inertia of Girder at end = 43,440.

$x = \frac{l}{3}\,\frac{b+2a}{b+a}$



54	÷	1.00	×	4	=	216
270	÷	1.32	×	9.60	=	1964
16640	÷	1.63	×	32.59	=	332680
6654	÷	1.86	×	47.13	=	168580
23618						503440
2784.7						47542
20833.3						455,898

18	÷	1.00	×	3	=	54
18	÷	1.32	×	9	=	123
2730.7	÷	1.63	×	28	=	46910
18	÷	1.86	×	47	=	455
2784.7						47542



$\frac{455,898}{24,656} = 18.49$ Reaction.

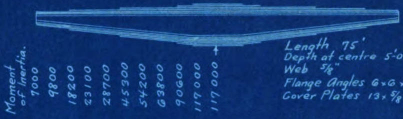
18	÷	1.00	×	4	=	72
54	÷	1.32	×	9.33	=	382
896	÷	1.63	×	31.05	=	17066
282	÷	1.86	×	47.06	=	7136
						24656

50 × 25 = 1250
18.49 × 50 = 924.5

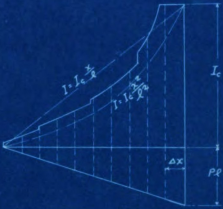
325.5
312.5 = $\frac{1}{8} wl$
13.0

$\frac{13.0}{312.5} = 4.19\ \%$

Deflection of an Engine Turntable.



Length 75'
Depth at centre 5'-0½", at ends 2'-1¼"
Web ⅝"
Flange Angles 6 × 6 × ⅝
Cover Plates 13 × ⅝ - 5 at centre.

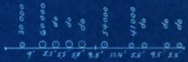


If $I = I_c \frac{x}{l}$ End Defl. $= \frac{Pl^3}{2EI_c}$

If $I = I_c \frac{x^2}{l^2}$ End Defl. $= \frac{Pl^3}{EI_c}$

Pl = 4,500,000 inch lbs.
(10000 lbs. at end.)

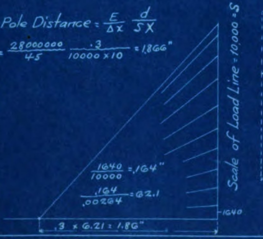
M	I	$\frac{M\Delta x}{I}$	$\Sigma\frac{M\Delta x}{I}$	$\frac{D\times E}{\Delta x}$	Defl.
4275	117	1640	1640	1640	.00264"
3825	104	1660	3300	4940	.008
3375	77	1970	5270	10210	.016
2925	59	2230	7500	17710	.029
2475	50	2230	9730	27440	.044
2025	37	2460	12190	39630	.064
1575	26	2730	14920	54550	.088
1125	21	2410	17330	71880	.116
675	14	2170	19500	91380	.147
225	8.4	1210	20710	112090	.180
		20710	112090		

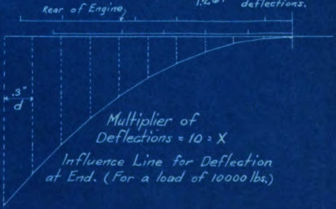


.123 × 1.5 = .185"
.060
.035
.016
.005
.116 × 3.0 = .348
.003 × 2.7 = .008
.029
.052
.110
.160
.351 × 2.05 = .720
1.261" Sum of end deflections.

Pole Distance $= \frac{E}{\Delta x}\,\frac{d}{SX}$

$= \frac{28000000}{45}\,\frac{.3}{10000\times 10} = 1.866"$





Influence Line for Deflection at End. (For a load of 10000 lbs.)

INFLUENCE LINES FOR THREE HINGED ARCH.

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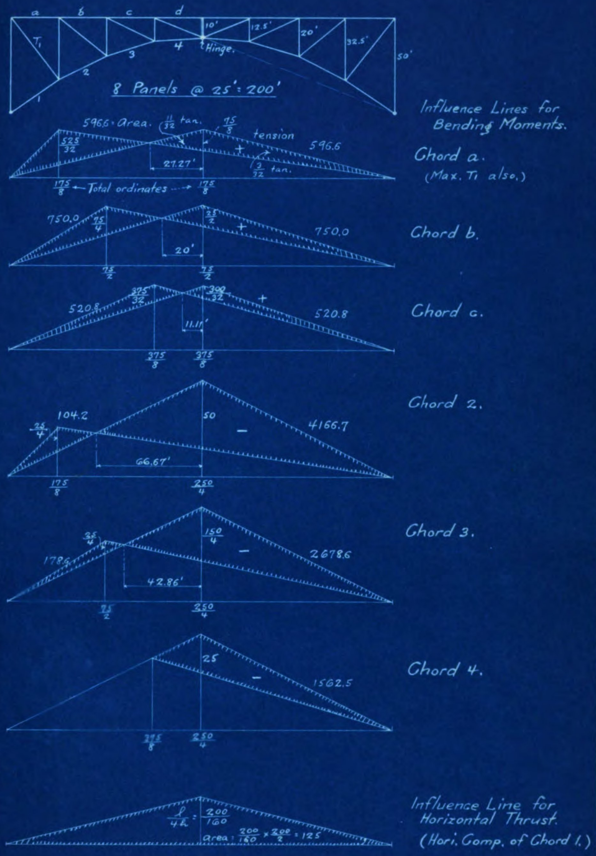
The two blue prints following give a set of influence lines for a three-hinged arch of 200 feet span and 40 feet rise. They are made by combining the effects of the vertical and horizontal components of the reactions. The curve of the lower chord is a parabola which causes the following relations between various parts of the influence lines. The maximum ordinates of the vertical components influence line and of the horizontal component influence line are equal for the upper chord members. The compression and tension areas are equal for the upper chord and diagonal members. The compression area of the verticals exceeds the tension area by the amount of the load at the top of the vertical in question. The stresses due to Cooper's Loadings may be easily found by means of the equivalent loads of Table XVI

Chord a	Equi. Load.	Area	Moment	Stress
$\frac{27.3}{127.3} = .21$	3290	596.6	1,962,600	+ 60,400
$\frac{25}{72.7} = .35$	3450	do	2,058,000	− 63,300

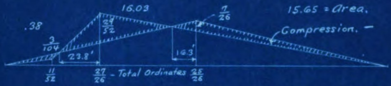
Or the stress may be figured directly from the influence lines using the static moments of the wheel loads and the tangents of the influence lines.

Wheel (4) $\quad 23751.25 \times \frac{3}{32} - 600 \times \frac{3+11}{32} = 19642 \div 32.5 = +60,440$

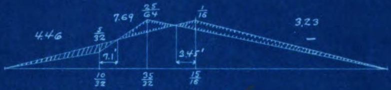
Influence Lines for 200 Foot Three Hinged Arch.



Influence Lines for 200 Foot Three Hinged Arch.

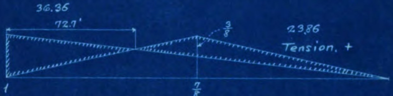


Influence Lines for Vertical Component of Diagonal T2.

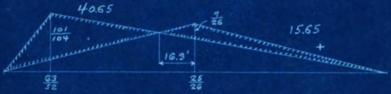


Diagonal T3.

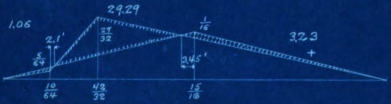
Diagonal T4



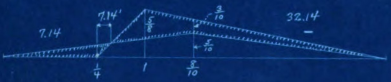
Post P1.



Post P2.



Post P3



Post P4.

On Tables VIII, IX, X, and XI near the influence lines are given the ratios of the areas of the continous bridge influence lines to the areas of the same influence lines drawn for simple spans. To the right of the moments and shears are given the ratios of the wheel load stresses for the two cases. The ratios of the moment areas are seen to be very nearly the same as the ratios of the actual wheel load moments, and can be used to find moments. The ratios are given on the following blueprint for paneled and unpaneled continuous bridges and for a paneled partially continuous bridge. As an example of their use the stresses due to the positive moments in a 316' partially continuous swing bridge having 12 panels of 25' and a centre panel of 16' will be found from the stresses for a simple span. These stresses are given on page 85 of Johnson's Structures.

Ratios $1 - \frac{1}{8+12r}\,\frac{n^2-1}{n(n-x)}$ $\qquad 12r = \frac{12\times16}{150} = 1.28$

$1 - \frac{1}{9.28}\,\frac{35}{6}\,\frac{1}{5} = 1 - .125 = .875$

" " " $\frac{1}{4} = 1 - .157 = .843$

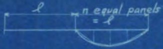
" " " $\frac{1}{3} = 1 - .209 = .791$

" " " $\frac{1}{2} = 1 - .314 = .686$

" " " $1 = 1 - .628 = .372$

Simple Span.			Stresses scaled from influence lines.
270900	× .875	= 237,000	237,800
412500	.791	= 326,200	326,900
210600	.372	= 78,400	81,600
388500	.843	= 327,400	326,200
343500	.686	= 235,600	234,000

The ratios for shears do not give such good results but may be used to get approximate stresses or to check stresses found by some other method.



Uplift.

Area of Influence Line for Uplift $= \frac{1}{16}\ell$

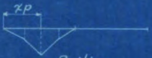
For a Paneled Bridge, Area $= \frac{1}{16}\ell\,\frac{n^2-1}{n^2}$

Positive Moments.

Area for simple span $= \frac{x(\ell-x)}{2}$

Area for continuous span $= \frac{x(\ell-x)}{2} - \frac{\ell x}{16}$

Ratio $= 1 - \frac{1}{8(1-k)}$



Positive Moments. Paneled Bridge.

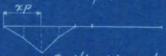
Area for simple span $= \frac{p^2x(n-x)}{2}$

Area for continuous span $= \frac{p^2x(n-x)}{2} - \frac{nxp^2}{16}\,\frac{n^2-1}{n^2}$

Ratio $= 1 - \frac{1}{8}\,\frac{n^2-1}{n(n-x)}$

Partially Continuous. Uplift.

Area $= \frac{\ell}{16+24r}\;\frac{n^2-1}{n^2}$



Positive Moments.

Area $= \frac{p^2x(n-x)}{2} - \frac{nxp^2}{16+24r}\,\frac{n^2-1}{n^2}$

Ratio to area for simple span $= 1 - \frac{1}{8+12r}\,\frac{n^2-1}{n(n-x)}$

Negative Shear.

Area for simple span $= -\frac{k^2\ell}{2}$

Area for continuous span $= -\left(\frac{5k^2}{8} - \frac{k^4}{16}\right)\ell$

Ratio $= \frac{5}{4} - \frac{k^2}{8}$

Positive Shear.

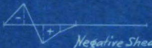
Area for simple span $= \frac{(1-k)^2\ell}{2}$

Area for continuous span $= \frac{7-16k+10k^2-k^4}{16}\,\ell$,

or = (shear under full load on left span) − neg. shear

$= \left\{\frac{7}{16} - k + \frac{5k^2}{8} - \frac{k^4}{16}\right\}\ell$

Ratio $= \frac{7-2k-k^2}{8}$



Negative Shear.

Area for simple span $= -\frac{px^2}{2(n-1)}$

Area for cont. span $= -\frac{px^2}{2(n-1)} - \left\{\frac{\left(\frac{x}{n-1}\right)^2}{8} - \frac{\left(\frac{x}{n-1}\right)^4}{16}\right\}pn$

Ratio $= 1 + \frac{n}{4(n-1)} - \frac{nx^2}{8(n-1)^3}$

Positive Shear $= \left(\frac{n-1}{2} - x\right)p - \frac{n^2-1}{16n}\,p$ − negative shear.

Divide by $\frac{p(n-x-1)^2}{2(n-1)}$ for ratio.

Partially Continuous, area for negative shear $-\frac{px^2}{2(n-1)} - \left\{\frac{\left(\frac{x}{n-1}\right)^2}{8+12r} - \frac{\left(\frac{x}{n-1}\right)^4}{16+24r}\right\}np$

Positive Shear $= \left(\frac{n-1}{2} - x\right)p - \frac{n^2-1}{(16+24r)n}\,p$ − negative shear

Approx. Formulas. Error about 1%.

- STRESSES IN RINGS AND HOOKS -

—-0—-

Two blue prints following give the derivation of the equations for bending moments and axial stresses in a ring supporting two loads as shown, and the application of the equation to some special cases. The solution is made according to the ordinary theory of bending as applied to straight pieces. The more exact theory of bending in curved pieces gives bending moments very nearly the same as the approximate method, but the distribution of the stress over the cross-section is different.

The following is Bach's equation for the stress at any point in the cross-section of a curved bar :

$$s = \frac{P}{F} + \frac{M}{Fr}\left(1 + \frac{y}{z(r+y)}\right)$$

P is the axial force at the section, positive when causing tension.

F is the area of the section.

M is the bending moment, positive when causing compression on the inside edge of the bar.

Y is any ordinate from the gravity axis of the section, positive when measured outward.

Z is $-\frac{1}{F}\int\frac{y\,dF}{r+y}$ and may be found by approximate

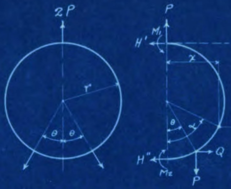
integration using Simpson's rule or by actual integration

in the case of regular Sections. For a ~~similar~~ circular section

$Z = \frac{2r^2}{a^2} - 1 - \frac{2r}{a^2}\sqrt{r^2 - a^2}$ where a is the radius of the

cross section and r is the radius of curvature of the bar. The development of the above equation for stress in a curved bar can be found in Bulletin #18 of the University of Illinois Engineering Experiment Station.

The third blue-print following gives the calculations of the stresses on the dangerous section of a 10 ton wrought iron crane hook designed by Towne's formula. The solution is, made both by the above formula and by the ordinary method. The maximum tension on the inside of the hook is found to exceed that given by the ordinary method by 39 per cent. On page 508 of Unwin's "Machine Design" it is stated that the stresses as usually determined are 40 or 50 per cent, too small.



$$\Sigma V \quad P = P$$

$$\Sigma H \quad H' + H'' = Q = P\tan\theta$$

$$\Sigma M \quad M_2 = M_1 + 2H'r - Q(r - r\cos\theta) - Pr\,[\sin\theta$$

$$\int_0^\pi \frac{M\,ds}{EI} = 0 = \int_0^\pi \frac{Mr\,d\alpha}{EI} \quad \therefore \int_0^\pi M\,d\alpha = 0$$

$$\int_0^\pi \frac{My\,ds}{EI} = 0 = \int_0^\pi \frac{Mr^2(1+\cos\alpha)\,d\alpha}{EI}$$

$$\therefore \int_0^\pi M\,d\alpha + \int_0^\pi M\cos\alpha\,d\alpha = 0$$

$$x = r\sin\alpha$$

$$y = r(1+\cos\alpha) \qquad \therefore \int_0^\pi M\cos\alpha\,d\alpha = 0$$

$$\int_0^\pi M\cos\alpha\,d\alpha = \int_0^\pi (M_1 - Px + H'y)\cos\alpha\,d\alpha + \int_0^\theta \left[-Qr(\cos\alpha - \cos\theta) - Pr(\sin\theta - \sin\alpha)\right]\cos\alpha\,d\alpha$$

$$\int_0^\pi M_1\cos\alpha\,d\alpha - Pr\sin\alpha\cos\alpha\,d\alpha + H'r\cos\alpha\,d\alpha + H'r\cos^2\alpha\,d\alpha =$$

$$\Big[_0^\pi\ M_1\sin\alpha - Pr\frac{\sin^2\alpha}{2} + H'r\sin\alpha + H'r\frac{\alpha}{2} + H'r\frac{\sin 2\alpha}{4} = H'r\frac{\pi}{2}$$

$$\int_0^\theta -Qr\cos^2\alpha\,d\alpha + Qr\cos\theta\cos\alpha\,d\alpha - Pr\sin\theta\cos\alpha\,d\alpha + Pr\sin\alpha\cos\alpha\,d\alpha =$$

$$\Big[_0^\theta\ -Qr\left(\frac{\alpha}{2} + \frac{\sin 2\alpha}{4}\right) + Qr\cos\theta\sin\alpha - Pr\sin\theta\sin\alpha + Pr\frac{\sin^2\alpha}{2} =$$

$$-Qr\frac{\theta}{2} - Qr\frac{\sin 2\theta}{4} + Qr\cos\theta\sin\theta - Pr\sin^2\theta + Pr\frac{\sin^2\theta}{2} =$$

$$-Qr\frac{\theta}{2} - Qr\frac{\sin\theta\cos\theta}{2} + Qr\cos\theta\sin\theta - Pr\frac{\sin^2\theta}{2} =$$

$$-P\tan\theta\, r\frac{\theta}{2} + P\tan\theta\, r\frac{\sin\theta\cos\theta}{2} - Pr\frac{\sin^2\theta}{2} = -P\tan\theta\, r\frac{\theta}{2}$$

$$\therefore \int_0^\pi M\cos\alpha\,d\alpha = H'r\frac{\pi}{2} - P\tan\theta\, r\frac{\theta}{2} \qquad \therefore H' = P\tan\theta\frac{\theta}{\pi}$$

$$\int_0^\pi M\,d\alpha = \int_0^\pi (M_1 - Px + H'y)\,d\alpha + \int_0^\theta \left[-Qr(\cos\alpha - \cos\theta) - Pr(\sin\theta - \sin\alpha)\right]d\alpha$$

$$\int_0^\pi M_1\,d\alpha - Pr\sin\alpha\,d\alpha + H'r\,d\alpha + H'r\cos\alpha\,d\alpha = \Big[_0^\pi\ M_1\alpha + Pr\cos\alpha + H'r\alpha + H'r\sin\alpha$$

$$= M_1\pi - 2Pr + H'r\pi$$

$$\int_0^\theta -Qr\cos\alpha\,d\alpha + Qr\cos\theta\,d\alpha - Pr\sin\theta\,d\alpha + Pr\sin\alpha\,d\alpha =$$

$$\Big[_0^\theta\ -Qr\sin\alpha + Qr\cos\theta\,\alpha - Pr\sin\theta\,\alpha - Pr\cos\alpha =$$

$$-Qr\sin\theta + Qr\cos\theta\,\theta - Pr\sin\theta\,\theta - Pr\cos\theta + Pr =$$

$$-Pr\tan\theta\sin\theta + Pr\sin\theta\,\theta - Pr\sin\theta\,\theta - Pr\cos\theta + Pr =$$

$$-Pr\tan\theta\sin\theta - Pr\cos\theta + Pr$$

$$\therefore \int M\,d\alpha = M_1\pi - Pr + H'r\pi - Pr\tan\theta\sin\theta - Pr\cos\theta$$

$$\therefore M_1 = \frac{Pr}{\pi} - H'r + \frac{Pr}{\pi}\tan\theta\sin\theta + \frac{Pr}{\pi}\cos\theta$$

$$= \frac{Pr}{\pi}(1 + \cos\theta + \tan\theta\sin\theta) - H'r$$

Substituting $\frac{P}{2}$ for P, the equations given on the next page are obtained.

Bending Moments in Rings.

Axial Stress at top $= H = \frac{P}{2}\tan\theta\,\frac{\theta}{\pi}$

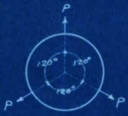
Moment at top $= M_1 = \frac{Pr}{2\pi}(1+\cos\theta+\tan\theta\sin\theta) - Hr$

This is the max. positive moment for $\theta < 60°$.

Max. negative moment occurs where $\tan\alpha = -\frac{P}{2H}$

$M = M_1 + Hr(1+\cos\alpha) - \frac{Pr}{2}\sin\alpha$

Axial stress at this section $= \frac{P}{2}\sin\alpha - H\cos\alpha$



Axial stress at top $= \frac{P\sqrt{3}}{6}$

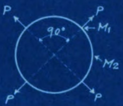
Moment at top $= \frac{Pr}{2}\left(\frac{3}{\pi} - \frac{\sqrt{3}}{3}\right)$

Axial stress at bottom $= \frac{P\sqrt{3}}{3}$

Moment at bottom $= \frac{Pr}{2}\left(\frac{3}{\pi} - \frac{2\sqrt{3}}{3}\right)$

$M_1 = \frac{Pr}{\pi}$ $\quad$ $M_2 = \frac{Pr}{2}\left(\frac{2}{\pi} - 1\right)$

Axial Stress $= \frac{P}{2}$



$M_1 = Pr\left(\frac{2}{\pi} - \frac{1}{2}\right)$ $\quad$ $M_2 = Pr\left(\frac{2}{\pi} - \frac{1}{2}\sqrt{2}\right)$

Axial Stress $= \frac{P}{2}$ $\quad$ Axial Stress $= \frac{P}{2}\sqrt{2}$

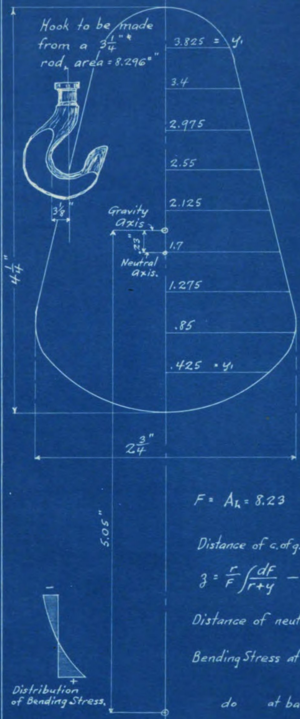
$M_1 = Pr\left(\frac{2}{\pi} - \cos\theta\sin\theta\right)$

$M_2 = Pr\left(\frac{2}{\pi} - \cos\theta\right)$

Axial Stress $= P\sin\theta\cos\theta$

or $P\cos\theta$

Calculation of the Stresses in a 10 ton Wrought Iron Crane Hook, designed by Towne's formula.



h	hy_1	$r+y$	$\frac{h}{r+y}$	hy_1^2
.00	.00	7.375	.00	.00
.68	2.60	6.950	.098	9.95
.80	2.72	6.525	.122	9.25
.90	2.68	6.100	.147	7.97
1.00	2.55	5.675	.176	6.50
1.10	2.34	5.250	.210	4.97
1.20	2.04	4.825	.249	3.47
1.30	1.66	4.400	.296	2.11
1.36	1.16	3.975	.342	.98
1.10	.47	3.550	.310	.20
.00	.00	3.125	.00	.00

Areas for whole cross section by Simpson's Rule.

$F = A_h = 8.23 \qquad A_{hy_1} = 15.85 \qquad A_{\frac{h}{r+y}} = 1.706 \qquad A_{hy_1^2} = 40.0$

Distance of c. of g. from front edge $= \frac{15.85}{8.23} = 1.925$"

$$z = \frac{r}{F}\int\frac{dF}{r+y} - 1 = \frac{5.05}{8.23} \times 1.706 - 1 = .047$$

Distance of neutral axis from gravity axis $= \frac{-zr}{1+z} = -.23$"

Bending Stress at front $= \frac{-101{,}000}{8.23 \times 5.05}\left(1 + \frac{-1.925}{.047 \times 3.125}\right) = +29{,}400$

do at back $= \frac{-101{,}000}{8.23 \times 5.05}\left(1 + \frac{+2.325}{.047 \times 7.375}\right) = -18{,}700$

Max. tension $= +29400 + \frac{20000}{8.23} = +31{,}800$

Max. comp. $= -18700 +$ do $= -16{,}300$

Ordinary theory for straight pieces.

$I_g = 40.0 - 8.23 \times \overline{1.925}^2 = 9.5$

Stress at front $= 101{,}000 \times \frac{1.925}{9.5} = +20{,}500 + 2400 = +22{,}900$

do back $= 101{,}000 \times \frac{2.325}{9.5} = -24{,}700 + 2400 = -22{,}300$