AN ANALYSIS OF TWO CONCRETE THRU GIRDER BRIDGES

Thesis for the Degree of B. S.

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THESIS

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Foreword.

The writer undertook the analysis of the concrete thru girder bridge in order to gain a knowledge of the theory of design of this type of structure, being already familiar with their construction thru acting as bridge inspector for the Michigan State Highway Department in 1920 and 1921. A description of this type of bridge and its construction is given so that the analysis will be clearer.

Acknowledgement is made to Mr. C. E. Melick and Mr. Osans, Bridge Engineer and Assistant Bridge Engineer respectively of the Michigan State Highway Dept., for their assistance and advice in this analysis.

May 1922. H. K.

Outline.

A. Introduction.

I. Description of standard concrete thru girder bridge.

II. Construction of standard concrete thru girder bridge.

III. Method of analysis of these bridges.

B. Body.

I. State Highway Dept., specifications relating to the design of these bridges.

II. Computation for floor slab of the 35' bridge.

b. Computation for girder of the 35' bridge.

c. Computation for the abutment of the 35' bridge.

d. Computation for floor slab of the 45' bridge.

e. Computation for girder of the 45' bridge.

C. Conclusion.

I. Actual stresses as compared to the theoretical, and the economy of the designs.

Bibliography.

Concrete Engineers'Handbook----Hool and Johnson.

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Lectures on Reinforced Concrete----Dunn.

Michigan State Highway Dept.,Specifications for Steel and Concrete Highway Bridges.1920 Edition.

An Analysis of Two Concrete Thru Girder Bridges.

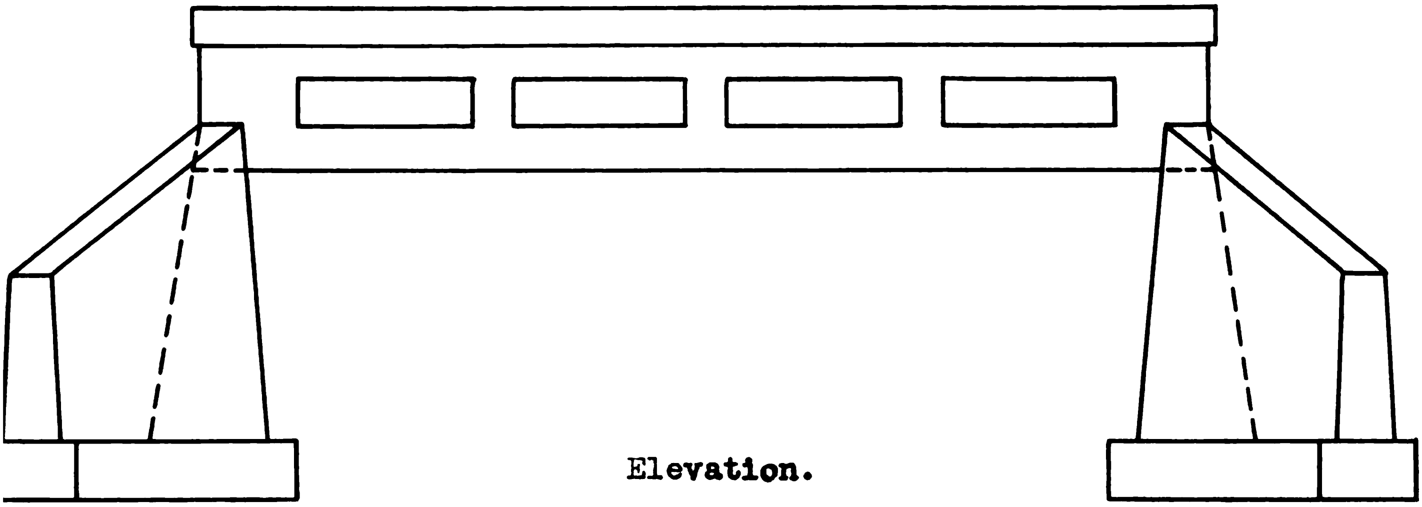
Description.

Figure No.1 is a drawing to scale of a standard 35' span concrete thru girder bridge, designed and built by the Michigan State Highway Department. A bridge of this type was built in 1920 over the Pine River in Standish, Michigan. Its description will apply as well to the second bridge analyzed, a 45' span structure. It consists of two plain concrete abutments with battered faces, two feet thick footings, and 45 degree wing-walls, and a superstructure consisting of two reinforced concrete girders supporting a reinforced floor slab. The girders also act as railings on the bridge. The method of constructing one of these bridges will now be given.

Construction.

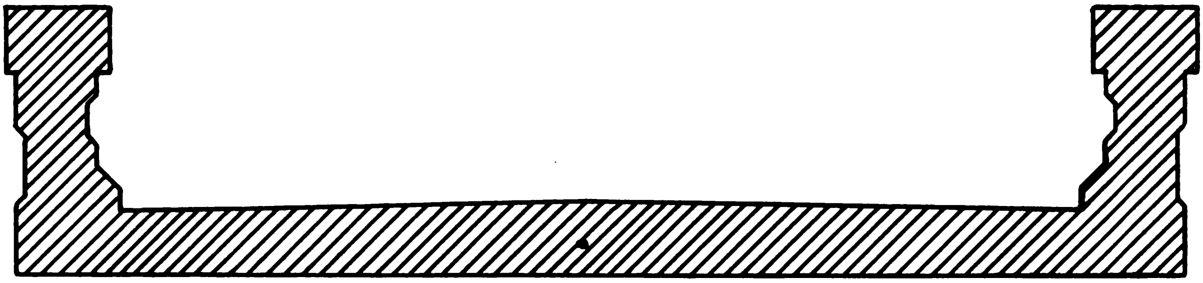
The lines and grades are established and then the excavations are made for the abutments. Test borings are made to determine the quality of the supporting soil. If it is found necessary, piles are ordered driven according to the plan. Then the footings are poured to the required elevation, with an extra 6" of concrete if piles are used - the extra concrete being below the pile heads. This concrete is plain and of a 1:2 1/2:5 mix with the aggregate passing the Highway Dept., tests as to the proportions of the various sizes of stone. After the footings have seasoned for several days the abutment forms are built on them and wired and braced. The abutments are then poured, using the same mix as in the footings, and pouring each one continuously. After the abutments have set for three days, more or less, according to the weather, the forms are removed and the surface is rubbed down with a carborundum brick and any holes are filled with a grout consisting of one part cement and two parts sand.

The falsework for the support of the floor and girders is next constructed. Rows of eight piles are driven at specified intervals between the abutments and at right angles to the road. These are cut off just above the ground and heavy timbers are laid on them. Short timbers are set vertically on the horizontal ones, one over each pile head, and another long timber is laid horizontally across. Floor joists are laid on this falsework and the flooring is nailed to them. Wedges are used for leveling and the floor is given a slight camber, both for appearance and to provide for slight settlement of the superstructure. The outside girder forms and panels are now erected, nailed, and braced in place. The steel reinforcing for the floor and girders is placed and wired; then the inside girder form is erected and wired to the outside form, with spreaders to hold the proper dimensions. Formwork is placed at each end of the bridge floor, and 1/4" of tarred felt is placed over each abutment to provide for expansion and contraction of the concrete. After a thoro inspection one half of the floor and one girder is poured one day and the other floor half and girder is poured on the succeeding day. The floor construction joint is in the center of the slab and parallel to the direction of traffic. The concrete mix



Elevation.

Fig. 1.



Cross-Section.

Fig. 2.

Note: These figures apply to both the 35' and 45' spans.

is 1:2:4 with tested materials. The girder forms are removed in two or three days, and the surface is finished with a carborundum brick and grout, if necessary, as was the abutment. Sometimes a coat of tar is placed on the floor after the concrete is dry. Traffic is allowed on the bridge after ten days, but the falsework and floor form is not removed for a month or six weeks.

The method of analysis of these structures will now be given.

Method of Analysis.

The floor slab was first taken and analyzed as a reinforced concrete beam with a concentrated live load of as many 18 ton trucks as could be placed on it. This is the loading used in the 1920 specifications of the Highway Dept., and it gives a load on any cross-section of the bridge of a maximum of 24 tons, spaced as shown in fig.6 and spread over the slab in the direction of traffic according to specifications. The total moment was next computed, and standard formulas for the design of reinforced concrete members were used to determine the stresses and the amount of steel required. The actual stresses were compared to the allowable and deductions drawn as to the strength and economy of the design. The web reinforcement for diagonal tension near the ends of the slab was computed next. This was compared with the actual steel area.

Then the girder was analyzed as a simply supported beam with the same truck loading as before. The total moment was found, and the amount of steel of actual use in the girder was compared to the theoretical amount required. The web reinforcement for diagonal tension was investigated, both the vertical stirrups and the inclined bars, and the actual amount of steel used was compared to the theoretical amount required. As before, deductions were drawn as to the strength and economy of the design.

The abutments were computed with Rankine's formulas using a level surcharge. All stresses were computed as for retaining walls. Drainage and scour were also investigated.

All measurements were taken from blueprints of the the Highway Dept.

Michigan State Highway Dept.Specifications
Used In The Analysis.
1920 Edition.

Sect.200 Reinforced Webs.For reinforced webs five-sixths of the average vertical shear shall be considered as being taken by the stirrups,diagonal web pieces,and bent-up bars.

Sect.201 Vertical Web Reinforcement.Shall be looped around the horizontal reinforcement.

Sect.202 Inclined Web Reinforcement.Shall be securely attached to the longitudinal rods to prevent slipping.

Sect.243 Stress in Reinforcing Steel.The tensile stress shall not exceed 16,000 lbs.per sq.in.

Sect.244 Modulus of Elasticity of Concrete.Shall be taken as 1/15 that of steel when the strength of the concrete is taken as 2,200 lbs.or less,per sq.in.

Sect.252 LiveLoad.Whether concentrated or uniform,shall be considered as moving and shall be placed in the position which will give the maximum stresses for each member considered.

Sect.253 The loading for the floor system and its supports shall consist of 18 ton trucks,concentrated as shown in sketch,and distributed as described in 258.

Sect.258 Distribution of Live Loads.

a.Floor slabs with main reinforcement transverse to the direction of traffic.One or more rear truck axles,as the slab span may permit,shall be placed in such a position as to give the maximum moment in the slab,considered as a simple span. An equivalent concentrated load at the center of the span producing the same maximum moment shall then be calculated. This equivalent load shall then be considered as concentrated transverse to the direction of traffic, but as distributed uniformly in the direction of traffic over a length equal to one and one-half feet plus sixty percent (60%) of the span. The resulting equivalent load (concentrated) per inch width of slab shall be used in calculating the live load bending moment, and shall also be considered as moving load which shall be placed so as to produce the maximum live load shear.

Sect. 265 Impact. An impact load of 25% of the specified concentrated loading shall be added to said concentrated loads for the design of all slabs, stringers, floor-beams, and hangers.

Standard Notation.

f_s = tensile unit stress in steel.

f_c = compressive unit stress in concrete.

E_s = modulus of elasticity of steel.

E_c = " " " " concrete.

n = E_s/E_c = 15 in this analysis.

M = bending moment.

A_s = steel area.

b = breadth of beam.

d = depth of beam to center of steel.

D = total depth of beam.

k = ratio of depth of neutral axis to depth d.

K = M/bd^2.

z = depth below top to resultant of the compressive stresses.

j = ratio of the lever arm of resisting couple to depth d.

p = steel ratio = A_s/bd.

V = total shear.

v = shearing unit stress.

u = bond stress per unit area of bar.

o = circumference of bar.

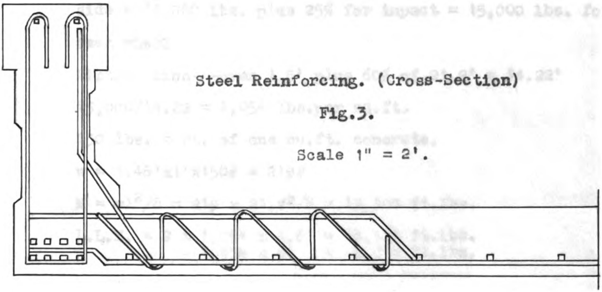
s = horizontal spacing of reinforcing members.

D.L. = dead load.

L.L. = live load.

w = weight.

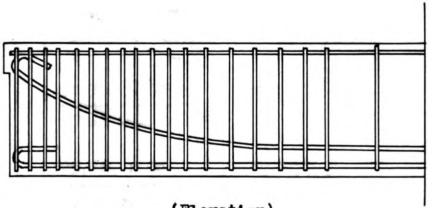
l = length.



Steel Reinforcing. (Cross-Section)

Fig.3.

Scale 1" = 2'.



(Elevation)

Fig.4.

Scale 1" = 4'.

Analysis of a 35' Concrete Thru Girder Bridge.

Slab span---clear span plus depth of beam = 20' plus 1.2' = 21.2'

Loading---rear axle loads of two 18 ton trucks side by side = 12,000 lbs. plus 25% for impact = 15,000 lbs. for each wheel.

Distribution---over 1.5' plus 60% of 21.2' = 14.22'

15,000/14.22 = 1,054 lbs.per sq.ft.

150 lbs. = wt. of one cu.ft. concrete.

$w = 1.46' \times 1' \times 150\# = 219\#$

$M = wl^2/8 = 219 \times 21.2^2/8 = 12,300$ ft.lbs.

$$\begin{aligned} L.L.M. &= 2 \times 1,054 \times 8.6' &&= 18,130 \text{ ft.lbs.} \\ &\quad - 1,054 \times 5' &&= -5,270 \text{ ft.lbs.} \\ \hline &\text{Total } M &&= 25,160 \text{ ft.lbs.} \end{aligned}$$

$D = 17.5"$ $d = 15"$

$$K = M/bd^2 = \frac{25,160 \times 12}{12 \times 15^2} = 111.7$$

$A_s = 1.41$ sq.in. actual.

$p = A_s/bd = .00787$

$$k = \sqrt{2pn \text{ plus } (pn)^2} -- pn = .382$$

$$f_c = \frac{50,320 \times 12}{.382 \times .863 \times 12 \times 15^2} = 678 \text{ lbs.}$$

$$f_s = \frac{25,160 \times 12}{1.41 \times .863 \times 15} = 16,450 \text{ lbs.}$$

Allowable f_c is 650 lbs. and f_s is 16,000 lbs. so the slab is seen to be slightly overstressed.

Alternate loading is 125 lbs. per sq.ft.Using the same formula for M as before and w = 125 plus 219 lbs.= 344 lbs. we have $M = 344 \times 21.2^2/8 = 19{,}340$ ft.lbs.This is smaller than the other M so is not used.

Web Reinforcement.

Inclined bars to take diagonal tension.

A_s = 1.14 sq.in.

$$V = \frac{3}{2} \times \frac{A_s f_s jd}{s} = \frac{3}{2} \times \frac{1.14 \times 16{,}450 \times .87 \times 15}{21} = 17{,}500\#$$

$$A_s = \frac{5}{6} \times \frac{.7Vs}{f_s jd} = \frac{5}{6} \times \frac{.7 \times 17{,}500 \times 21}{16{,}000 \times .87 \times 15} = 1.027 \text{ sq.in.needed.}$$

We have 1.14 sq.in. and need 1.027 sq.in.so is safe.

Region where no web reinforcement is needed:

x_1 is distance from slab end.

$x_1 = 1/2 - v_1 bjd/w = 21.2/2 - 40 \times 12 \times .87 \times 15/344 = 7.6'$

The actual distance is 7.6' so that is correctly designed.

Bond Stress.

$u = v/ojd = 17{,}500/14 \times .87 \times 15 = 21.2\#$

The allowable is 80# so that too is safe.

Note: $j = 1 - 1/3\,k$, in all computations.

Girder of 35' Bridge.

Span---33.3' center to center of supports.

Average b = 23" or 1.9'

D = 5'7 1/2" or 5.625'

d_1 = depth to reinf.steel$_1$ = 4.96'

d_2 = " " " steel$_2$ = 5.30'

Average d = 5.13' or 61.6"

w = 1.9' x 5.625' x 150# = 1,603 lbs.

Each girder carries 1/2 of the floor slab and 1/2 of the live load.

$M = wl^2/8 = 1{,}603 \times 33.3^2/8 = 222{,}000$ ft.lbs.

L.L.M. = 33.3 x 12,580 = 419,000 ft.lbs.

Total M = 641,000 ft.lbs.

641,000 x 12 = 7,670,000 inch lbs.

$7{,}670{,}000/107.4 = 71{,}400 = bd^2$

$d^3 = 1.6 \times 71{,}400 = 114{,}200$ and d = 48.5" or 4.041'

We have 5.13' and need 4.041' so this actual depth is seen to be very uneconomical.

A_s = 8 x 1.5625 sq.in. = 12.5 sq.in. actual steel.

$$p = A_s/bd = \frac{12.5}{23 \times 61.6} = .00883$$

$$k = \sqrt{30 x .0088 - (15 x .0088)^2} - 15 x .0088 = .400$$

$$f_c = \frac{2 \times 7{,}670{,}000}{.4 \times .87 \times 23 \times 61.6^2} = 506 \text{ lbs.}$$

$$f_s = \frac{7{,}670{,}000}{12.5 \times .87 \times 61.6} = 11{,}450 \text{ lbs.}$$

The allowable f_c is 650 lbs. and f_s is 16,000 lbs. so it can be seen that the design is uneconomical.

Steel actually needed for $f_c = 650, f_s = 16,000, b = 23"$, and $d = 61.6"$ is $A_s = pbd = .0077 x 23 x 61.6 = 10.91$ sq.in. There are 12.5 sq.in. so there is enough steel area.

Web Reinforcement.

Actual shearing stress for inclined bars is:

$$V = \frac{3}{2} x \frac{4.69 x 11,450 x .87 x 61.6}{12} = 360,000 \text{ lbs.}$$

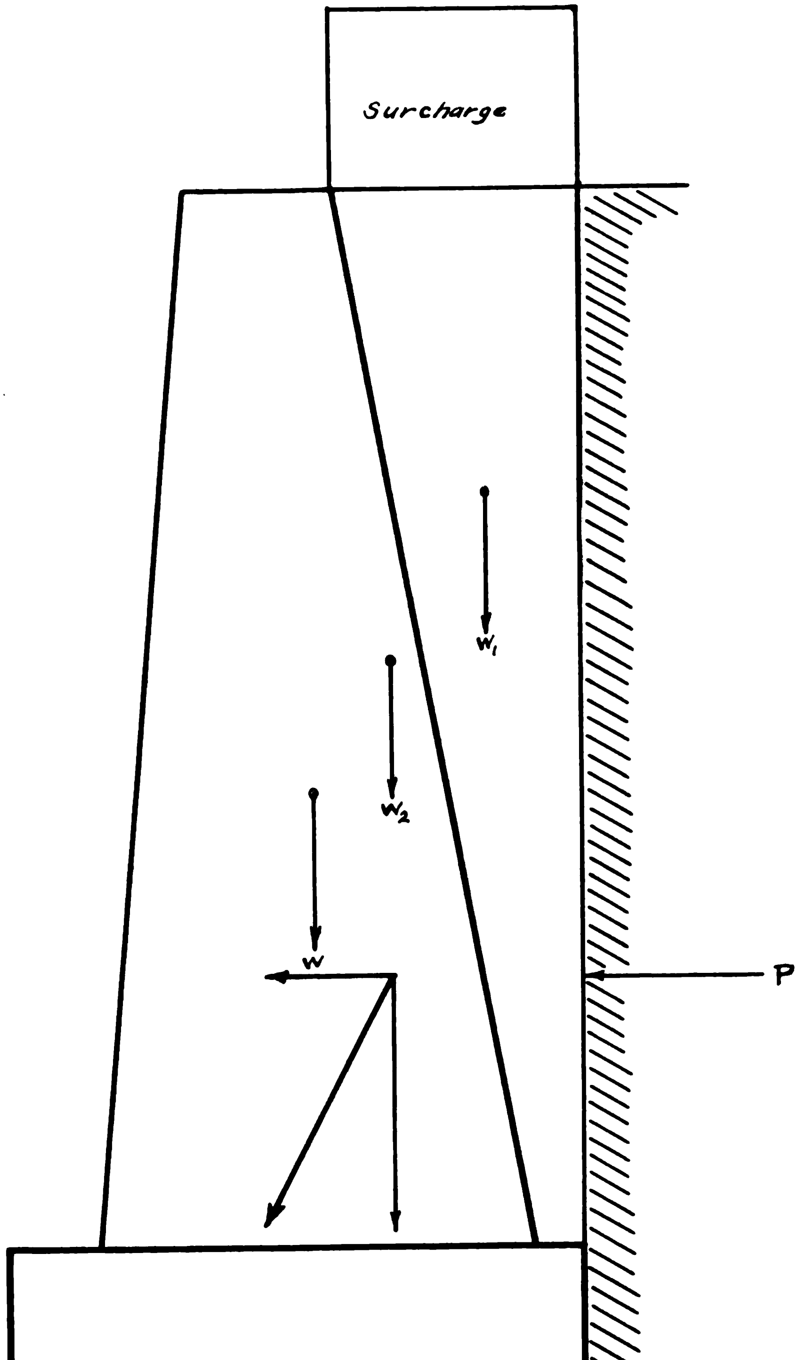
$$A_s = \frac{5}{6} x \frac{.7 x 360,000 x 12}{16,000 x .87 x 61.6} = 2.94 \text{ sq.in.}$$

There are 4.69 sq.in. and we need 2.94 sq.in. so this part of the web reinforcing is safe but uneconomical. Actual shearing stress for vertical stirrups is:

$$V = \frac{3}{2} x \frac{.65 x 11,450 x .87 x 61.6}{12} = 49,800\#$$

$$A_s = \frac{5}{6} x \frac{49,800 x 12}{16,000 x .87 x 61.6} = .582 \text{ sq.in.}$$

There is .65 sq.in. and we need .582 sq.in. so it is safe. The tension reinforcement bars, vertical stirrups, and inclined bars in the girders all have hooked ends, thus preventing slippage and increasing bond. All steel is thoroly wired besides. All this is good practise.



Abutment.

Fig.5.

Abutment of 35' Bridge.

ϕ = angle of repose of filling.

h = vertical height of wall in feet.

h' = vertical height of surcharge in feet.

w = weight of filling per cu.ft.

W = " " masonry per ft.of length.

W_1 = " " earth wedge one ft. long.

W_2 = " " W plus W_1.

P = resultant earth pressure per ft.length of wall.

y = vertical distanse from base of wall to point where the resultant strikes the wall.

The abutment must be safe against overturning, sliding, and crushing the masonry or foundation.

Rankine's formula was used for a wall with a loaded surcharge.

ϕ = 1 1/2 to 1 or 33 42'

w = 110 lbs.

h = 13'.

$h' = 1.84'$ from $h' = \frac{T}{bw} = \frac{72,000}{25 \times 110 \times 14.22}$

P = 1/2 wh(h plus 2h')x1-sinθ/1-sinθ

P = .143x110x13(13-3.68) = 3,410 lbs.

$$y = \frac{h^2 \text{ plus } 3h'h}{3(h \text{ plus } 2h')} = 4.82'$$

W = 4,280 lbs. from 28.6 x 1 x 150#

W_1 = 2,402 lbs. from 21.834 x 1 x 110#

W_2 = 6,682 lbs.

From fig.5 the resultant comes within the middle third of the footing.This is where it should come.The construction lines of this figure are not shown.

The coefficient of masonry on sand for friction is .4

The safety factor is derived from the formula:

$f_s = F/P \times .4$

P = 3,410 lbs.

F = 15,262 lbs. from 1/2 x 66 cu.yds.x 27 cu.ft.x 150#

= 133,500 lbs.load on one abutment.(D.L.)Plus 36,000# for one truck as live load = 169,500 lbs.total load.This equals 6,780 lbs.per one ft.of width.

6,780+6,682+1,800# for footing = 15,262 lbs.= F.

$f_s = 15{,}262/3{,}410 \times .4 = 1.8$

The wall is safe against sliding.

For stability against crushing the masonry or foundation:

15,262-1,800 = 13,462 lbs.

d = width of base of wall.

b = " " " " footing from center of footing to point where the resultant strikes the base.

d = 4.5' and b = 1.4'

$p_1 = F/d = 13{,}462/4.5 = 3{,}000.$

$p_2 = \pm 6Fb/d^2 = \pm 5{,}630$ lbs.

p = 5,630-3,000 = 8,630 lbs.per sq.ft.

The allowable is from 4 to 6 tons per sq.ft. so this is safe.

For maximum pressure at toe of wall:

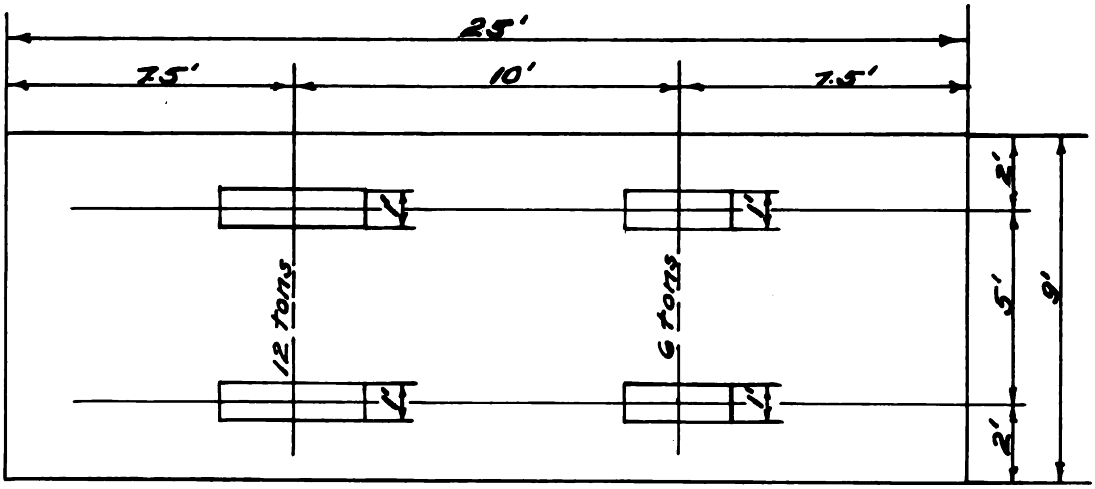
$p_1 = (4l-6a)F/l^2$ where l = width of base = 6' and a = 6/2 - 1.5' = 1.5' and F = 13,462 lbs.

$p_1 = 5{,}615$ lbs.

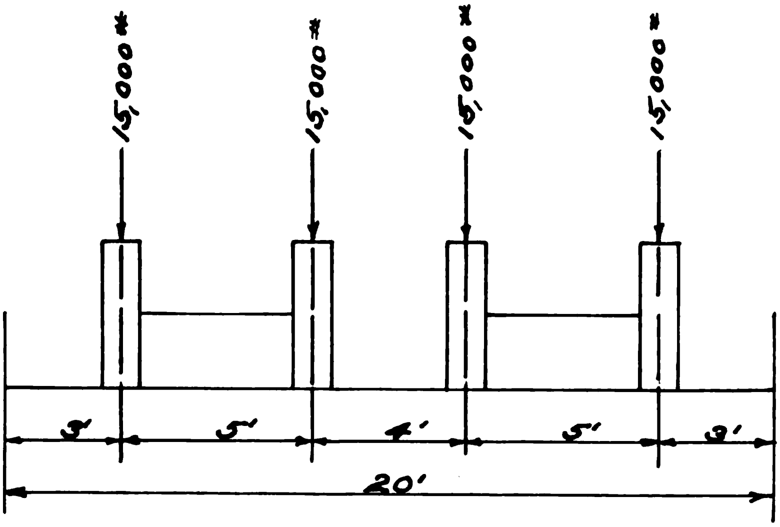
The allowable is from 4 to 6 tons so the actual p_1 is well within the allowable.

Fig.6.

Loading Diagram.



Space and weight distribution of 18 ton truck.



Loading for Maximum Moment.

Analysis of a 45' Concrete Thru Girder Bridge.

Span---clear slab span plus depth of beam = 18' plus .9' = 18.9'

Loading---rear axle loads of two 18 ton trucks side by side = 12,000 lbs. plus 25% for impact = 15,000 lbs. for each wheel.

Distribution---over 1.5' plus 60% of 18.9' = 12.84'

wt. of one cu.ft. of concrete = 150 lbs.

15,000/12.84 = 1,168 lbs.per sq.ft.

$w = 1.125' \times 1' \times 150\# = 168.75$ lbs.

$M = wl^2/8 = 168.75 \times 18.9^2/8 = 7{,}530$ ft.lbs.

L.L.M.	= 2 x 1,168 x 7.45'	=	17,400 ft.lbs.
	- 1,168 x 5'	=	-5,840 ft.lbs.
	Total M	=	19,090 ft.lbs.

D = 13.5" d = 11."

$$K = M/bd^2 = \frac{19{,}090 \times 12}{12 \times 11^2} = 157.7$$

$A_s = 1.333$ sq.in.actual.

$p = A_s/bd = .0101 \qquad n = 15.$

$$k = \sqrt{2pn \text{ plus } (pn)^2} - pn = .420$$

$$f_c = \frac{19{,}090 \times 2 \times 12}{.42 \times .86 \times 12 \times 121} = 874 \text{ lbs.}$$

$$f_s = \frac{19{,}090 \times 12}{1.333 \times .86 \times 11} = 18{,}180 \text{ lbs.}$$

As the allowable f_c is 650 and f_s is 16,000 lbs., the floor slab is seen to be considerably overstressed.

Alternate loading is 125 lbs. per sq.ft.

$w = 168.75$ plus $125 = 293.75$ lbs.

$M = wl^2/8 = 293.75 \times 18.9^2/8 = 13,130$ ft.lbs.

This is less than the maximum moment used so is not used.

Web Reinforcement.

For inclined bars:

$A_s = .556$ sq.in.

$$V = \frac{3}{2} \times \frac{.556 \times 18,180 \times .86 \times 11}{12} = 11,940 \text{ lbs.}$$

$$A_s = \frac{5}{6} \times \frac{.7 \times 11,940 \times 12}{16,000 \times .87 \times 11} = .546 \text{ sq.in.}$$

There is .556 sq.in. and we need .546 sq.in. of steel so it is safe.

Region where no web reinforcement is needed:

$$x_1 = \frac{l}{2} - \frac{v_c bjd}{w} = 18.9/2 - 40 \times 12 \times .87 \times 11/294 = 5.19'$$

distanse from end.

Actual distance is 5.4' so is safe.

Girder of 45' Bridge.

Span---42' center to center of supports.

Average $b = 24.8"$ or $2.07'$

d_1 = depth to reinf.steel$_1$ = $4.96'$

d_2 = " " " steel$_2$ = $5.30'$

Average $d = 5.13'$ or $61.5"$

$D = 5.625'$ or $67.5"$

$w = 2.07' \times 5.625' \times 150\# = 1,748$ lbs.

Each girder carries 1/2 of the floor slab and 1/2 of the live load.

$M = wl^2/8 = 1,748 \times 42^2/8 = 385,000$ ft.lbs.

L.L.M. $= 42 \times 19,090/2 \quad = 401,000$ ft.lbs.

Total M $= 786,000$ ft.lbs.

$12 \times 786,000$ ft.lbs. $= 9,430,000$ inch lbs.

$9,430,000/107.4 = 87,800 = bd^2$

$d^3 = 1.6 \times 87,800 = 140,500$ and $d = 52"$ or $4.333'$

There is 5.13' and we need 4.333' so the design is safe but uneconomical.

$A_s = 10 \times (1.25")^2 = 15.625$ sq.in.of actual steel area.

$$p = A_s/bd = \frac{15.625}{24.8x61.5} = .01025$$

$n = 15$ as before.

$$k = \sqrt{2pn \text{ plus } (pn)^2} - pn = .423$$

$$f_c = \frac{2 \times 9,430,000}{.423x.86x24.8x61.5^2} = 552 \text{ lbs.}$$

$$f_s = \frac{9,430,000}{15.625x.86x61.5} = 11,450 \text{ lbs.}$$

As the allowable f_c is 650 and f_s is 16,000 lbs.,the design is seen to be safe but uneconomical.

The steel area actually required for $f_c = 650$ and $f_s = 16,000$ is $A_s = pbd = .0077 \times 24.8 \times 61.5 = 11.75$ sq.in. There are 15.625 sq.in.of steel and we need 11.75 sq.in. so this tee is safe.

Web Reinforcement.

For inclined bars:

$A_s = 15.625/2 = 7.813$ sq.in.

$$V = \frac{3}{2} \times 7.813 \times 11,450 \times .86 \times 61.5/12 = 591,000 \text{ lbs.}$$

$$A_s = \frac{5}{6} \times \frac{.7 \times 591,000 \times 12}{16,000 \times .87 \times 61.5} = 4.84 \text{ sq.in.}$$

There are 7.813 sq.in. and we need 4.84 sq.in. so this is all right.

Vertical stirrups:

$A_s = pbd = .61$ sq.in. actual steel area.

$$V = \frac{3}{2} \times .61 \times 11,450 \times .86 \times 61.5/12 = 46,250 \text{ lbs.}$$

$$A_s = \frac{5}{6} \times \frac{46,250 \times 12}{16,000 \times .87 \times 61.5} = .541 \text{ sq.in.}$$

There is .61 sq.in. and we need .541 sq.in. so this part of the design is safe.The ends of the steel are hooked and all the steel is well wired.This is good practise.

Conclusion.

In summarizing the results of the analysis of these two bridges the reader's notice is called to the fact that these bridges were designed in 1915 under no printed specifications, and with a smaller loading than now used. These bridges were built in 1920 and 1921, however, so the 1920 specifications were used in the analysis.

In the 35' bridge the floor slab is found to be so slightly overstressed that the amount is negligible, while the web reinforcement is within the allowable. The girder stresses are so far under the allowable that the design is uneconomical in all details. The abutment is safe and of economical design.

The floor slab of the 45' bridge is considerably overstressed and shows poor design in the light of present day knowledge. It is safe as the safety factor is between 3 and 4, and the maximum loading will probably never come upon it. The web reinforcement is within the allowable. The stresses in the girders are so far under the allowable that their design, like the girders of the 35' bridge, are uneconomical. The girder web reinforcement is all right. The floors of both bridges are well drained by weep holes.

The abutments of both bridges are safe and protected from scour, but are insufficiently drained. Several 3" or 4" pipes or tile should be used to drain each abutment instead of the 3/4" or 1" pipes used.

The latest designs of the State Highway Dept., show improvement in all respects over the above standards.