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Analysis of Cedar Street
Reinforced Concrete Arch Bridge

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1913

THESIS

This thesis was contributed by

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under the date indicated by the department stamp, to
replace the original which was destroyed in the fire of
March 5, 1916.

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THEESIS IN CIVIL ENGINEERING.

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ANALYSIS OF THE CEDAR STREET REINFORCED CONCRETE
ARCH BRIDGE OVER CEDAR RIVER, LANSING, MICHIGAN.

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INVESTIGATED BY THE ELASTIC THEORY.

A. F. ^{U. S. patent} Zickgraf.

E. K. ^{U. S. patent} Lovelace.

-1913-

THESIS

This thesis was chosen with the idea of gaining more knowledge of the theory and practice in the design of reinforced concrete arches.

This bridge was built in 1909, by the Western Bridge Co. of Chicago, Ill., following the destruction of an old wooden bridge at that place. The bridge is 30 feet wide and consists of two equal spans of 70 feet, with the springing line at the pier 2 feet higher than that of the abutments at each end. It is on a skew of 30 degrees with the river which makes the problem of design and construction more difficult. The arch is 16 inches thick at the crown and has a rise of 10 feet. It is back-filled with earth on which rests the base of the brick pavement. The bridge seems well constructed and in good condition, no cracks of any magnitude being visible except in the railing.

The loadings and allowable stresses we used were those given in the specifications and for which the bridge was designed. A live load of 200 # per sq. ft. of roadway, 100 # per sq. ft. of walls, a concentrated moving load of a 28 ton steam roller, and a dead load of 90 # per sq. ft. for earth backfill and 150 # per sq. ft. for concrete. The stresses allowed are:

Modulus of Elas. of Concrete 1,800,000. lbs.

Modulus of Elas. of Steel 28,000,000. lbs.

Max. comp. per sq. in. on steel 10,000. lbs.

Max. comp. per sq. in. on concrete 500. lbs.

Max. shear per sq. in. on concrete (plain) 100. lbs.

Max. tension per sq. in. on concrete 50. lbs.

The theory followed in analyzing the design is on plates No. 1 and 2 and is a copy of the Elastic Theory of Arches by Prof. C. A. Melick. In our analysis most of the data for the horizontal forces, due to lateral earth pressure were calculated but owing to a lack of time only the vertical forces were considered.

The arching was divided into 20 radial parts symmetrically about the center and the loads applied at the points A, B, C, D, E, E, D, C, B, and A, the location of which being determined graphically as shown on diagram.

Tables No. 1 and 2 consist of values to be used in the further calculation and were constructed according to the properties of the arch. Plate 1 shows the general data that was computed and used in further work. The area of the arch ring where given is the equivalent area of concrete and is equal to $bh = 2pbh(n-1) = bh + A_s(n-1)$, where b and h are respectively the breadth and thickness of the ring at the section, A_s the area of the steel, p the percentage of steel in the section, and n the ratio of the Modulus of Elas. of steel to that of concrete and taken as 15. Table 3 was made of the summations of the quantities in Tables 1 and 2.

Table 4 was calculated from the formulae of the theory and from which Table 3 was made from the summations which were used to construct this table. The lead used was a 1,000 # load at

on Plates 2 and 3 were plotted and besides being a comparative check on the calculations up to that point give graphical values of the moment, thrust, and shear for every point on the arch ring. These values were scaled off and make up part of Table 5. The other values shown in Table 5 were used to compute Table 6.

From Table 5 the influence lines for thrust, shear, and moment were plotted (see Plate 4).

After the thrust was calculated it's affect upon the sections of the arch was calculated but was so small it could not be noticed and was neglected. The calculations for a change in H due to this thrust depends on the following theory:

$$\text{Average axial thrust} = \frac{\sum^L T}{n} = T.$$

$$M = Hy \text{ or } dx = \sum_o^L \frac{Hy^2 \Delta s}{EI} .$$

$$\text{If } H = 1,000 \text{ then } a = dx = \frac{1,000}{E} \sum_o^L \frac{y^2 \Delta s}{I} .$$

$$\text{Change in } dx \text{ due to axial thrust} = b = \frac{T \Delta x}{AE} = \frac{1}{E} \sum_o^L \frac{T \Delta x}{A} .$$

$$H \text{ due to axial thrust} = \frac{b}{a} \times 1000 = Q$$

$$\text{or } Q = \left[\frac{1}{E} \sum_o^L \frac{T \Delta x}{A} \quad \div \quad \frac{1000}{E} \sum_o^L \frac{y^2 \Delta s}{I} \right] .$$

$$Q = \sum_o^L \left\{ \frac{\sum_o^L \frac{\Delta x}{A}}{\sum_o^L \frac{y^2 \Delta s}{I}} \right\} .$$

$$\text{Actual } H = H_L - Q.$$

In Table 6 from values shown in Table 5, we have figured the stresses at the different points for the top and bottom of the arch ring, for a 1,000 # load at the loaded points. The upper and lower are tabulated respectively.

The upper is $\frac{T}{A} + \frac{Mc}{I}$ and the lower $\frac{T}{A} - \frac{Mc}{I}$, where A is the transformed area of that section, c one half the thickness of the arch ring at that point, and I the moment of inertia of the transformed section. Plates 5 and 6 show respectively the influence lines for the stresses in the top and bottom fibres for a 1,000 # load, (moving). The stresses for dead and live load and for combined loads are given for top and bottom in Tables 6 and 7 respectively and give the maximum stresses in the ring and where they occur.

In testing the arch for shear the value of the shear in Table 5 were used. The shear for dead load was first computed and then the shears for live load computed so as to give the maximum positive or negative values of the same sign as the dead load shear. These values of shear are given in Table 8. This shows the max. shear to be 25 # per sq. in. The bond strength of the shear bars did not have to be tested as this strength does not have to be figured where the shear is less than 40 # per sq. in. In Table 8 the Dead and Live load loadings are also given. A slider of the axle loads of the roller was used on the shear curves of Plate 4 to get the position for max. live load shear.

On Table 9 the diagrams and Tables of the stress in the steel are given. The reinforcement was placed at a uniform distance of 2.5 inches from the outside and is shown by the dotted lines. The top and bottom lines are the stress at the

top and bottom from Tables 6 and 7 and the distance apart is the ring thickness at that point.

From the specifications the pier is supported by piles, 12 inches in diameter at the large end and 8 in. in dia. at the ~~at the~~ small end and placed 3 foot c. to c. They were driven to such depth that the penetration under the last blow of a hammer weighing 2,000 # and falling 30 feet should not be greater than one inch.

From the Engineering New's Formula, the load one pile will support or $P = \frac{2WH}{s-1}$ where W = weight = 2,000 #, h = height of fall of hammer = 30 feet, and s = penetration = 1 inch. Then P is found equal to 60,000#. In this pier, considering that one pile takes care of 8 sq. ft. of pier-base, which is equal to a section 1 foot wide times the width of the pier which is 8 ft., the dead load on one pile equals

$$2(\text{Dead load on all points plus Pier weight}) = 2(1654 + 1946 + 2768 + 3965 + 7541 + 150[(8' \times 5.5' \times 1') + (5.5 \times 3 \times 1)])$$

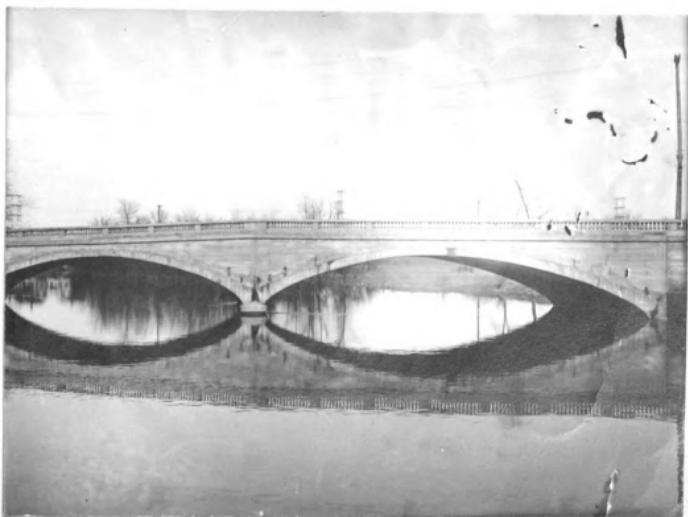
or equals $35,548 + 8,025 = 43,573\#$. This shows the pier to be safe from sinking under the dead load.

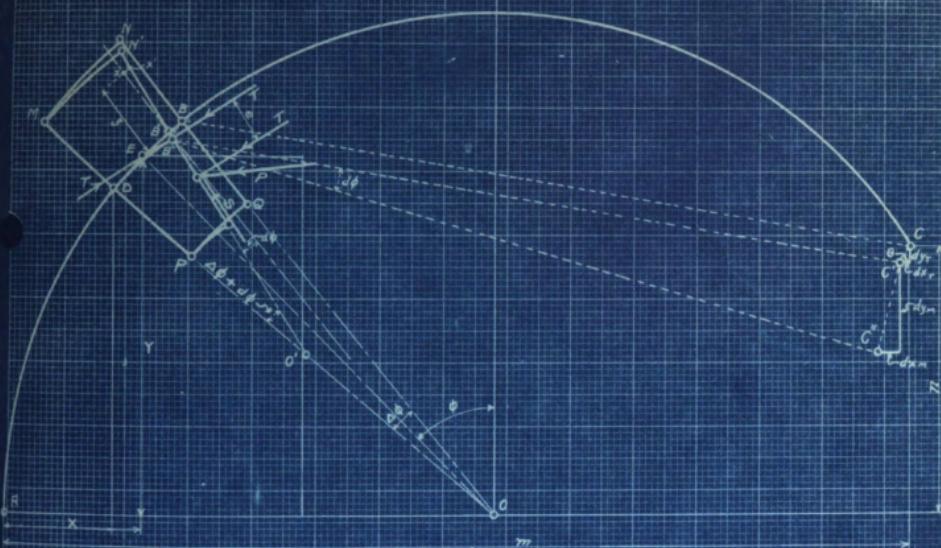
The one foot section of the arch analyzed was taken parallel to the face of the arch. For Voussoir Arches the thrust, (lines of pressure), in an oblique arch are assumed to act in planes parallel to it's faces ((S. Edward Warren on Stereotomy). A monolithic arch on a skew is considered in this same way. The section 1 foot wide analyzed in this arch was taken under that



assumption and therefore the effect of skew is not considered. No other references than that of Warren's book on Stereotomy on this subject could be found as the theory of skew arches is one not taken into consideration in most of the books on arches.

The bridge was found to be safe. The stresses determined are within the allowable limits with a factor of safty of about 2 for compression and about 4 for shear. The only tension found in the arch was less than 3 pounds per sq. in. No account of forces due to the slight incline of the arch, 2 ft. in 70 ft., were considered as they were too small to affect the stresses by any amount.





Theoretical Analysis of the Elastic Arch.

Fundamental Formulae -

Let $ADBG$ represent a portion of the linear arch, and let $MNPQ$ be any section of the arch ring. Let E , on the linear arch, be the center of this section with Coordinates (x, y) measured from A . Let the central angle $\Delta\theta$ be subtended by the arc DB of length Δs .

This section of the Arch Ring will be acted upon by some Resultant Arch Pressure P , which may be resolved into a Radial Shear S , a Normal Axial Thrust T , and a Bending Moment $T_{B.T.M}$. These Shears are Small in Ordinary Arches, and Arch Theories neglect them in them in the Theory of Stresses, just as done in the Common Theory of Flexure for Beams. This leaves, for consideration then, the two agents Thrust & Bending acting on the Section: these both vary with the different Sections of the Arch Ring. T shortens the section DB uniformly an amount $\frac{T \Delta s}{E}$. A rise of Temperature lengthens the Section DB uniformly an amount $\frac{E \Delta s}{T}$. The resultant Lengthening of the Section $DB = BB' + \frac{T \Delta s}{E} - \frac{E \Delta s}{T}$. A negative value of M will cause the Section DB to deform in the direction DB'' changing $\Delta\theta$ by an amount $d\theta$. θ is measured from the Vertical through the Crown. Positive to the left, Negative to the Right - hence the change $d\theta$ is a Negative one. At $x = 0$ the $\frac{\partial x}{\partial \theta} = M \frac{\partial s}{\partial \theta}$. But $x = z \tan \theta$ hence $\frac{\partial x}{\partial \theta} = \frac{z \sec^2 \theta}{E}$ and since when M is Negative $d\theta$ is Negative, the equation stands correct for sign. Now imagine that, for any Arch Loading, starting at A , each section is taken in order, and its effect found separately and in turn. Then MP may be regarded as firmly fixed to the last deformed section preceding, and the effect of the deformation of $MNPQ$, on the portion of the Arch to the Right determined. Let us find the effect of this deformation on the movement of the point G whose coordinates are (m, n) . The change in DB , DB' , due to Thrust & Temperature will produce an equal change CC' at G . Also the change in $\Delta\theta$ due to M will produce a change of C of CC'' such that $CC'' \cdot d\theta$. The increase in Span - m - due to Thrust & Temperature alone on this section alone $= -dx = -CC' \cos \phi = -BB' \frac{\cos \phi}{E} = \frac{E \cdot d\theta}{T}$. The increase in Rise - n - due to Thrust & Temperature on this section alone $= dy = -CC' \sin \phi = -BB' \frac{\sin \phi}{E} = \frac{E \cdot d\theta}{T}$. The increase in span - m - due to a Positive Bending Moment M will be dm . But $\frac{dm}{d\theta} = \frac{y-n}{E} = \frac{dy}{E}$. Hence $dm = (y-n)d\theta = \frac{M \Delta s}{E.I.} - \frac{M \Delta s}{E.T}$. The increase in Rise - n - due to a Positive Bending Moment M will be dn . But $\frac{dn}{d\theta} = \frac{x-m}{E}$. Hence $dn = (m-x)d\theta = -\frac{M \Delta s}{E.I.} + \frac{M \Delta s}{E.T}$.

The total increase in span m due to a thrust T , a rise in Temperature of $E.F$, & a Positive Bending Moment M , acting on this section alone will be $dm = \frac{E \cdot d\theta}{T} + \frac{M \Delta s}{E.I.} - \frac{M \Delta s}{E.T}$. The total increase in Rise n due to a Thrust T , a rise in Temperature of $E.F$, & a Positive Bending Moment M , acting on this section alone will be $dn = \frac{E \cdot d\theta}{T} - \frac{M \Delta s}{E.I.} + \frac{M \Delta s}{E.T}$.

In a Fixed Arch - Symmetrical the following Conditions are Assumed.

1. The Radial Lines of the Springing Remain Unchanged in Direction - Thus, summing up $d\phi$ for all Sections between Springing Points $\sum_{E,I}^t d\phi = \sum_{E,I}^t \frac{M ds}{E I} = 0$
2. The Supports are fixed an immovable distance apart i.e. are Rigid. Thus $\sum_{E,I}^t dx = \sum_{E,I}^t a + \Delta x - \sum_{E,I}^t \frac{T ds}{E I} + \sum_{E,I}^t \frac{M ds}{E I} = 0$ since $\sum_{E,I}^t \frac{M ds}{E I}$ becomes 0 if $\sum_{E,I}^t \frac{M ds}{E I} < 0$.
3. The Supports are Relatively Immovable in Elevation - Then $\sum_{E,I}^t dy = E_e a + \Delta y - \sum_{E,I}^t \frac{T dy}{E I} - \sum_{E,I}^t \frac{M x ds}{E I} = 0$ since the term $\sum_{E,I}^t \frac{M x ds}{E I}$ becomes $m \sum_{E,I}^t \frac{M ds}{E I}$ and since $\sum_{E,I}^t \frac{M ds}{E I} = 0$.

From the figure below $H = M_a + V_a \cos \phi - W_a(x-a) \sin \phi - W_a(y-b) \cos \phi$.

$$T = H \cos \phi + V_a \sin \phi + W_a \cos \phi + W_a \sin \phi \propto a. \text{ Substituting these values in Equations 1,2,6,3, they become}$$

$$1. M_a \sum_{E,I}^t \frac{dx}{I} + \sum_{E,I}^t \frac{x ds}{I} = H_a \sum_{E,I}^t \frac{y ds}{I} - W_a \sum_{E,I}^t \frac{(x-a) ds}{I} - W_a \sum_{E,I}^t \frac{(y-b) ds}{I} = 0.$$

$$2. E_e a + \sum_{E,I}^t \Delta x - H_a \sum_{E,I}^t \frac{\Delta x \cos \phi}{I} - W_a \sum_{E,I}^t \frac{\Delta x \sin \phi}{I} + W_a \sum_{E,I}^t \frac{\Delta x \sin \phi}{I} + M_a \sum_{E,I}^t \frac{x ds}{I} + \sum_{E,I}^t \frac{x y ds}{I} = 0.$$

$$3. E_e a \sum_{E,I}^t \Delta y - H_a \sum_{E,I}^t \frac{\Delta y \cos \phi}{I} - W_a \sum_{E,I}^t \frac{\Delta y \sin \phi}{I} - W_a \sum_{E,I}^t \frac{\Delta y \cos \phi}{I} + W_a \sum_{E,I}^t \frac{\Delta y \sin \phi}{I} - M_a \sum_{E,I}^t \frac{x ds}{I} = V_a \sum_{E,I}^t \frac{x ds}{I} + W_a \sum_{E,I}^t \frac{(x-a) ds}{I} + W_a \sum_{E,I}^t \frac{(y-b) ds}{I} = 0.$$

Rearranging the above equations they become

$$1. a M_a + b V_a + C_1 H_a + d_1 = 0 \quad \text{or} \quad H_a = -\frac{a}{C_1} M_a - \frac{b}{C_1} V_a - \frac{d_1}{C_1}$$

$$2. a_1 M_a + b_1 V_a + c_1 H_a + d_2 = 0 \quad \text{or} \quad H_a = -\frac{a_1}{c_1} M_a - \frac{b_1}{c_1} V_a - \frac{d_2}{c_1}$$

$$3. a_2 M_a + b_2 V_a + c_2 H_a + d_3 = 0 \quad \text{or} \quad H_a = -\frac{a_2}{c_2} M_a - \frac{b_2}{c_2} V_a - \frac{d_3}{c_2}.$$

Substituting 3 in 1,2,

$$M_a(\frac{a}{C_1} - \frac{a_1}{c_1}) = -W_a(\frac{b}{C_1} - \frac{b_1}{c_1}) \quad \text{or} \quad A_1 M_a = B_1 V_a + C_1 \quad \text{or} \quad M_a = \frac{B_1}{A_1} V_a + \frac{C_1}{A_1}.$$

$$M_a(\frac{a}{C_1} - \frac{a_2}{c_2}) = -W_a(\frac{b}{C_1} - \frac{b_2}{c_2}) \quad \text{or} \quad A_2 M_a = B_2 V_a + C_2 \quad \text{or} \quad M_a = \frac{B_2}{A_2} V_a + \frac{C_2}{A_2}.$$

Solving the last two equations gives

$$\frac{B_1}{A_1} V_a = -(\frac{B_2}{A_2} - \frac{B_1}{A_1}) \quad \text{or} \quad D_1 V_a = E_1 \quad \text{or} \quad V_a = \frac{E_1}{D_1}.$$

But for Symmetrical Arches $A=0$, hence $V_a = -\frac{E_1}{D_1}$.

The following terms are constants for any given arch and are independent of the loading.

$a, a_1, a_2, b, b_1, b_2, c_1, c_2, c_3, A_1, B_1, B_2$. Their values are - from above.

$$a, a_1, a_2, b, b_1, b_2, c_1, c_2, c_3, A_1 = 0, \quad B_1 = \frac{b_2}{c_1} - \frac{b_1}{c_1}.$$

$$a_2, a_1, b_2, b_1, c_2, c_3, A_2 = \frac{a_2}{c_2} - \frac{a_1}{c_1} - \frac{\sum_{E,I}^t \Delta x \sin \phi}{I}, \quad C_1 = -\frac{\sum_{E,I}^t \frac{x ds}{I}}{I} - \frac{\sum_{E,I}^t \Delta x \cos \phi}{I}, \quad B_2 = \frac{b_2}{c_2} - \frac{b_1}{c_1}, \quad B_2 = \frac{b_2}{c_2} - \frac{b_1}{c_1}.$$

$$a_2, a_1, b_2, b_1, c_2, c_3, A_2 = \frac{a_2}{c_2} - \frac{a_1}{c_1} - \frac{\sum_{E,I}^t \Delta x \sin \phi}{I}, \quad C_2 = \frac{\sum_{E,I}^t \frac{x ds}{I}}{I} - \frac{\sum_{E,I}^t \Delta y \cos \phi}{I}, \quad C_2 = \frac{d_1}{c_2} - \frac{d_2}{c_1}.$$

The following terms are dependent on the loading - d_1, d_2, d_3, C_1, C_2 & C_3 . Their values are - from above -

$$d_1 = -W_a \sum_{E,I}^t \frac{(x-a) ds}{I} - W_a \sum_{E,I}^t \frac{(y-b) ds}{I}$$

$$d_2 = E_e a + \sum_{E,I}^t \Delta x - W_a \left[\sum_{E,I}^t \frac{(x-a) ds}{I} - \sum_{E,I}^t \frac{\Delta x \sin \phi}{I} \right] - W_a \left[\sum_{E,I}^t \frac{(y-b) ds}{I} + \sum_{E,I}^t \frac{\Delta x \cos \phi}{I} \right], \quad C_1 = \frac{d_1}{c_2} - \frac{d_2}{c_1}.$$

$$d_3 = E_e a \sum_{E,I}^t \Delta y + W_a \left[\sum_{E,I}^t \frac{(x-a) ds}{I} + \sum_{E,I}^t \frac{\Delta y \sin \phi}{I} \right] - W_a \left[\sum_{E,I}^t \frac{(y-b) ds}{I} + \sum_{E,I}^t \frac{\Delta y \cos \phi}{I} \right], \quad C_2 = \frac{d_2}{c_2} - \frac{d_3}{c_1}.$$

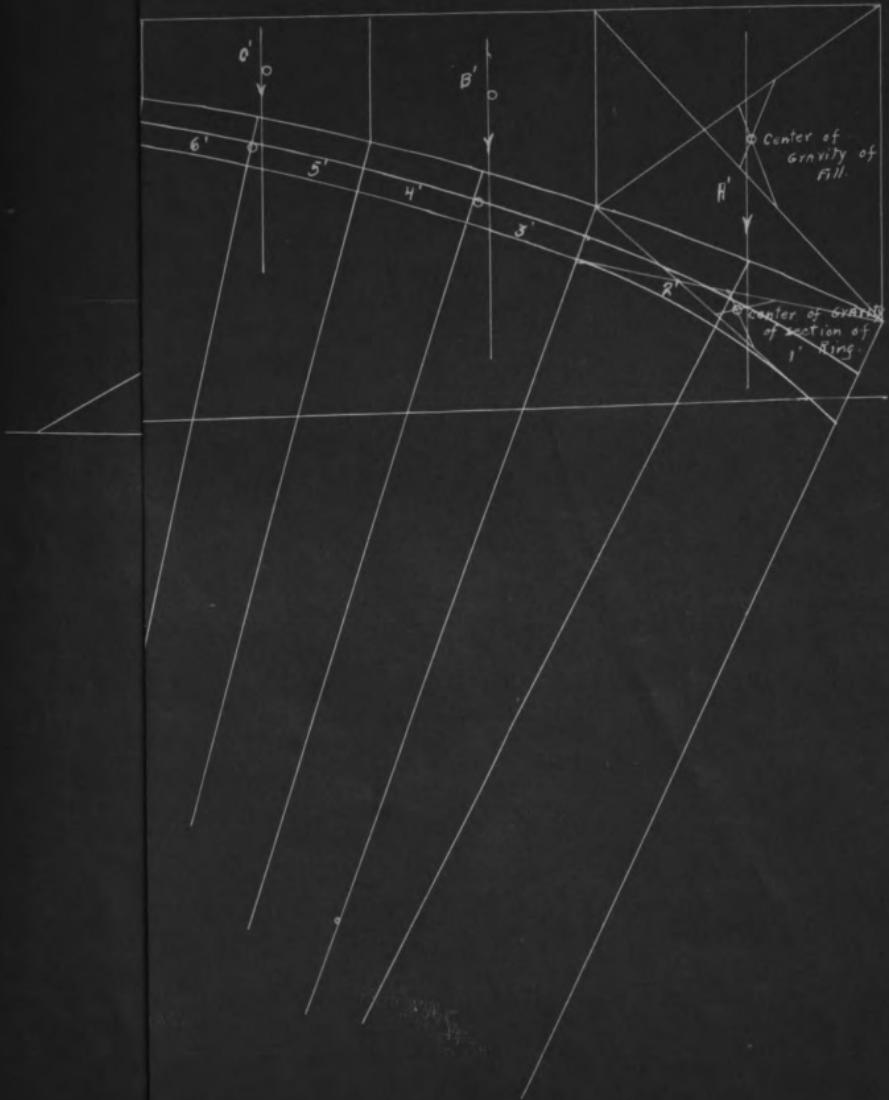




TABLE I.

α	X	K	γ	ΔS	ϕ	$\sin \phi$	$\cos \phi$	ΔX	ΔY	θ	A	I	$\frac{\Delta S}{I}$	$\frac{X \Delta S}{I}$	$\frac{Y \Delta S}{I}$	$\frac{X + Y \Delta S}{I}$			
1	+ 21° 46' 44"	0.246398	+ 13° 57' 47"	50.8785	50.8785	.51400	.85785	41.646/	26.515/	35.533	142.146	92.835.5	.0010209	.051926	.013332	.170322	.299411 / .9013		
2	+ 66° 44' 2	0.76418	+ 37° 44'	"	1191.50	.961420	.901238	.761.585	.23.6/18	.26.531	331.182	21.446.5	.0021723	.151570	.088821	.10.4622	.5.8388	.3.32554	
3	+ 10° 06' 17	1.66650	+ 55° 27' 3	45.8290	1227.50	.34918	.93624	.41.0608	.15.3130	.21.652	275.574	11.936.8	.0036539	.402.120	.2.01334	.44.260/	.82.2263	.11.1614	
4	+ 15° 06' 1	1.73820	+ 68° 47' 6	"	1063.83	.30427	.95221	.41.7432	.13.3476	.20.898	266.526	10.844.3	.0020446	.6.0350	.2602794	.9.2.2273	.42.4072	.19.5084	
5	+ 19° 27' 21	2.21737	+ 61° 809	"	900.16	.25687	.96523	.44.3357	.11.3460	.20.1418	257.430	3.773.08	.0019486	.861.870	.366880	.166.53	.70.7081 / .0.0441		
6	+ 23° 59' 9	.870313	+ 96° 035	"	736.50	.21261	.97914	.42.9147	.9.31848	.19.3912	248.444	8.903.61	.00194226	.1.1541	.553050	.271.662	.106.43302	.11.6364	
7	+ 27° 35' 6	.319374	+ 97° 336	"	572.33	.16587	.98615	.43.2219	.7.66992	.18.637	239.339	7.580.02	.00205828	.1.55302	.544080	.430.312	.50.9603	.52.9385	
8	+ 32° 14' 58	.368890	+ 103° 541	"	403.16	.11874	.99293	.42.5791	.5.20428	.17.884	230.338	6.979.33	.00162798	.201.297	.650217	.674.253	.208.4249	.67.3241	
9	+ 36.3° 17	.418680	+ 107° 644	"	245.50	.07136	.99745	.43.7172	3.12763	17.1304	281.350	6.712.38	.00170008	.2.58303	.76.4558	.933.362	.218.047	.82.2785	
10	+ 40° 66' 67	.466374	+ 109° 717	"	81.830	.02381	.99972	.43.8167	.1.04357	16.3768	212.274	5.448.28	.0020445	.3.21947	.882.334	.336.93	.352.803	.966.3229	
10'	+ 45° 06' 65	.493220	+ 109° 714	"	81.630	.03381	.99972	.43.8167	.1.04357	16.3768	212.274	5.448.28	.0020445	.3.62025	.882.334	.62.9248	.397.225	.968.3229	
9'	+ 49.3° 9.5	.56833	+ 107° 644	"	275.50	.07136	.99745	.43.7172	3.12763	17.1304	281.350	6.712.38	.00170008	.3.50719	.764.558	.1733.15	.377.528	.84.2885	
8'	+ 53.7° 18.7	.61812	+ 103° 541	"	109.16	.11874	.99293	.43.51591	.58.045	.17.884	230.338	6.979.33	.00162798	.3.33791	.6.50217	.1817.14	.319.2886	.67.3241	
7'	+ 58.0° 76	.66759	+ 97° 336	"	572.33	.16587	.98615	.43.2219	.7.66992	.18.637	239.339	7.630.82	.00205828	.3.871.3067	.544080	.881.35	.375.6667	.52.9385	
6'	+ 62.4° 81.5	.716651	+ 92° 035	"	736.50	.21261	.97914	.42.9147	.9.31848	.19.3912	248.444	8.903.61	.00194226	.3.04586	.4.53050	.19.045	.202.166	.11.6364	
5'	+ 66.5° 01.1	.76207	+ 81.809	"	300.16	.23887	.96593	.42.3357	.11.3460	.20.1418	257.430	3.773.08	.0019486	.2.982.308	.3.66880	.198.327	.273.580	.72.0141	
4'	+ 70.6671	.81344	+ 69° 476	"	1063.83	.30454	.93241	.41.7432	.13.3476	.20.898	266.526	10.844.3	.0020446	.2.85603	.2.807294	.204.8.31	.98.449	.12.3084	
3'	+ 74.1° 66.5	.860374	+ 55.223	"	1227.50	.31948	.93684	.41.0608	.15.3130	.21.652	275.574	11.936.8	.0036539	.2.731.52	.201.934	.204.2.26	.150.929	.11.1614	
2'	+ 79.7° 15.9	.941.59	+ 36.056	57.8330	.572.00	.472.88	.96383	.52.2712	.24.7156	.27.700	318.150	2.427.65	.0023824	.1.901.78	.085889	.15.0.84	.68.5689	.30.9717	
1'	+ 84.5° 09.7	.912.466	+ 8.168	"	.941.5	.53521	.87472	.48.8327	.30.3528	.59.010	467.230	.65.504.9	.00206829	.7.462.30	.0072216	.6.0.637	.6.03521	.058904	
Sum														.0493829	.304.6719	.0483.677	.2089.2	.363.575	.81.2203





8

TABLE 2.

P_{out}	$\text{Load at } E'$			$\text{Load at } D'$			$\text{Load at } C'$		
	$(x-a)\Delta s$	$\frac{(x-a)\Delta s}{I}$	$\frac{(x-a)\Delta s}{I}$	$\frac{(y-b)\Delta s}{I}$	$\frac{(y-b)\Delta s}{I}$	$\frac{(x-a)\Delta s}{I}$	$\frac{(y-b)\Delta s}{I}$	$\frac{(x-a)\Delta s}{I}$	$\frac{(y-b)\Delta s}{I}$
10									
10									
9'	+1474.4	-69.8757	-15.2822	-0.067884	-3.55287	-2074.6			
8	+59.6783	+243.746	-411.0633	-0.41769	-0.70661	-3.58915			
7	+59.3502	-399.136	-57.76824	-0.062963	-61.2352	+0.0873	+63.7948	+0.5944	-0.693246
6'	+73.2597	-1456.240	-67.71437	-0.01545	-50.7866	-2.30977	+30.6644	+0.30282	-0.620073
5'	-61.7623	+562.634	-10.1728	-120.417	-79.8391	-9.83908	+6.68000	-0.311624	-0.535259
4'	+90.0263	+647.527	-6.53326	-158.124	-111.741	-0.3658	-5.906684	-0.417112	-0.13382
3'	+98.9008	-717.521	-55.2623	-194.825	-145.664	-0.7685	-0.683715	-0.511190	-0.377949
2'	+76.3762	-609.473	-475.5271	-172.029	-137.938	-6.23146	-5.977236	-4.978866	-20.0951
1'	+2276.55	-2716.94	-2.76620	0.088672	-74.9457	-7.24275	-2194.771	-185.996	-1.72264
Sum	+5.1095	-3953.62	+492.3697	-9.7661	-657.394	-56.1926	-2.93632	-2.12539	-1779.086

P_{out}	$\text{Load at } E'$			$\text{Load at } D'$			$\text{Load at } A'$		
	$(x-a)\Delta s$	$\frac{(x-a)\Delta s}{I}$	$\frac{(x-a)\Delta s}{I}$	$\frac{(y-b)\Delta s}{I}$	$\frac{(y-b)\Delta s}{I}$	$\frac{(x-a)\Delta s}{I}$	$\frac{(y-b)\Delta s}{I}$	$\frac{(x-a)\Delta s}{I}$	$\frac{(y-b)\Delta s}{I}$
6									
5'									
4'									
3'	+0.59723	-14.7822	+3.28496	-0.60318	-19.7675	-1.43916			
2'	+150.252	-19.921	+5.57671	-0.627669	-50.0989	-2.66292			
1'	+10.089	-8.4897	+8.93977	-0.72682	-10.4695	-3.9109	+0.477703	+0.46397	-0.07398
Sum	+3.00339	-293.935	+9.8211	-1.76682	-10.03539	-5.93799	+0.77713	+0.4697	-0.070796



TABLE I Cont'd.

$\frac{\Delta X \cos \phi}{A}$	$\frac{\Delta Y \cos \phi}{A}$	$\frac{\Delta X \sin \phi}{A}$	$\frac{\Delta Y \sin \phi}{A}$
Σ_0	Σ_0	Σ_0	Σ_0
1 .0846681	.050739	.050739	.0304014
2 .125332	.0614205	.0614205	.0320249
3 .139587	.0220379	.0220379	.0134979
4 .149166	.0476366	.0476366	.0152512
5 .158815	.0423686	.0423686	.0114068
6 .169130	.0367249	.0367249	.0079744
7 .178046	.02997483	.02997483	.0050371
8 .187583	.0224722	.0224722	.0026826
9 .197025	.0149357	.0149357	.0010084
10 .206358	.0049147	.0049147	.00011705
10' .206358	.0049147	.0049147	.20011705
9 .197025	.00490957	.00490957	.0010084
8' .187583	.0224722	.0224722	.0026826
7' .178046	.02997483	.02997483	.0050371
6' .169130	.0367249	.0367249	.0079744
5' .158815	.0423686	.0423686	.0114068
4' .149166	.0476366	.0476366	.0152512
3' .139587	.0220379	.0220379	.0134979
2' .135701	.0642418	.0642418	.0304126
1' .0852216	.05339959	.05339959	.0349215
Sum .32202456	.7333969	.7333969	.45363246

TABLE 3.



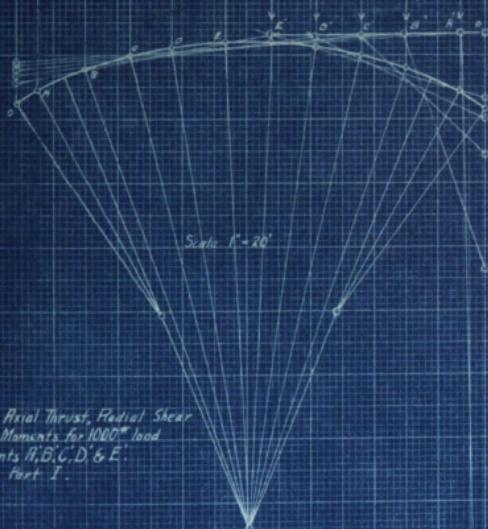
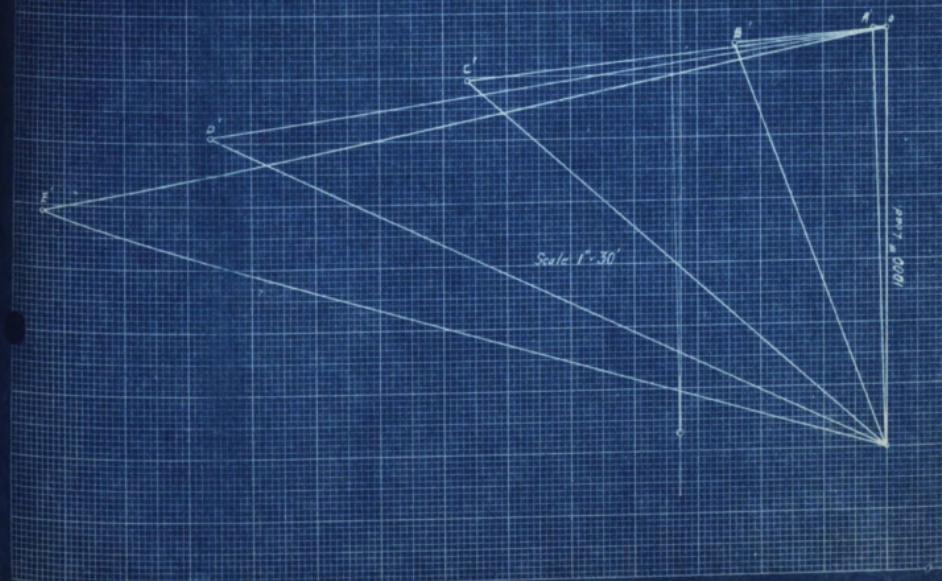


PLATE 2.





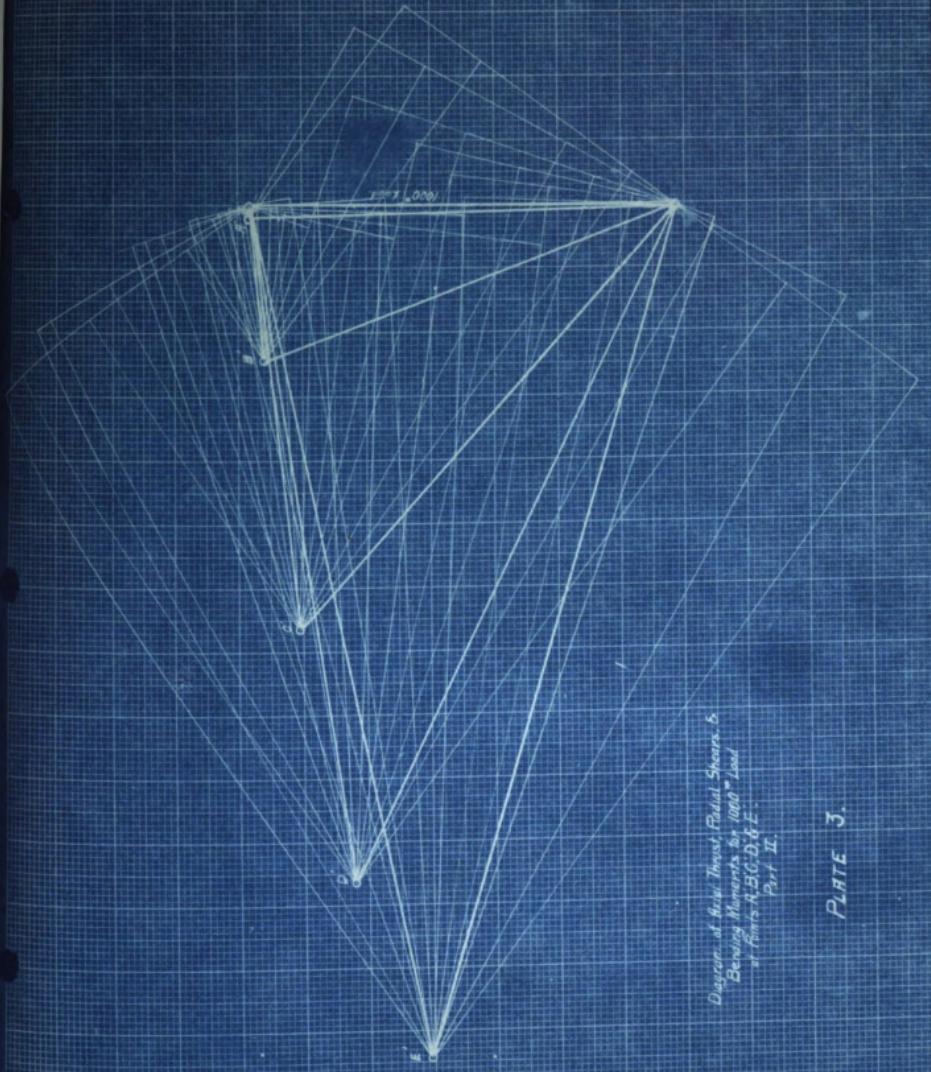
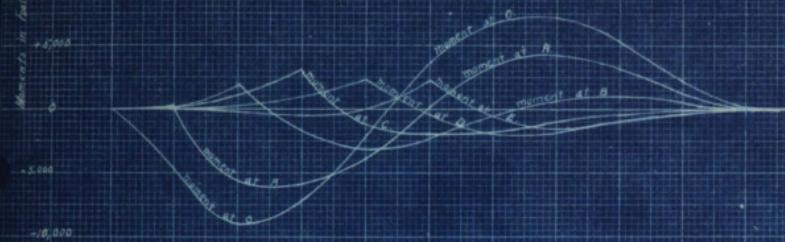


Diagram of Three-Hinged Portal Frame &
Bending Moments by 1000th Load
of Forces A, B, C, D, E,
A₁, F, II.

PLATE J.



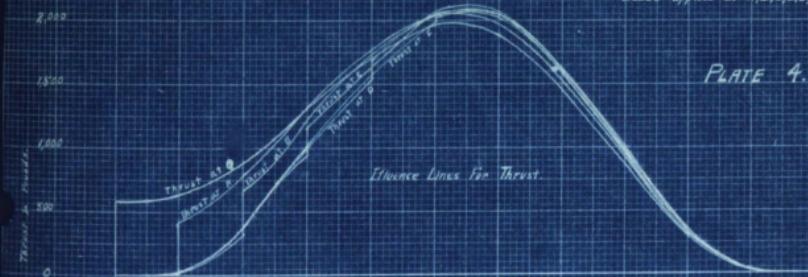
PLATE 4.
Influence Lines For Moments.



Influence Lines For Cedar Street Arch
for a Moving Load of 1000^t
For 1 Ft. of Barrel of
Arch.
Loads applied at H,B,C,D,E,D,C,B,&A.

PLATE 4.

Influence Lines For Thrust.



Influence Lines For Radial Shear.

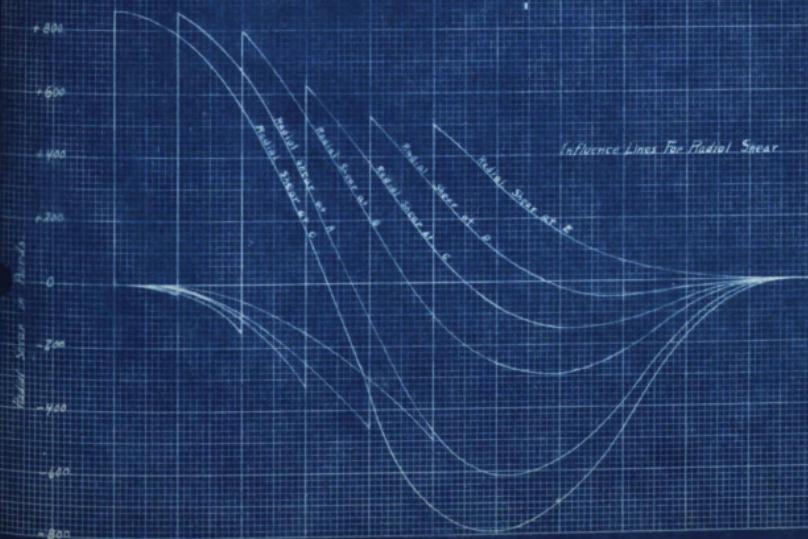


TABLE 4.

$\frac{d_1}{d_2}$	d_1	d_2	d_3	d_4/c_3	$-d/c_1$	c_1	$\frac{d_1}{c_3}$	$-d/c_2$	c_2
E'	+ 5.710.95	+ 402.369.	+ 3.953.620.	+ 1.087.61	- 673.17	414.44	1087.61	496.62	590.99
D'	+ 2.936.2	+ 177.968.	+ 2.185.340.	+ 587.666	- 346.11	238.556	384.666	219.65	365.01
C'	+ 1.252.71	+ 58.145.	+ 932.295.	+ 256.467	- 144.185	112.342	256.467	71.764	148.703
B'	+ 310.04	+ 9.521.	+ 249.193.	+ 68.551	- 36.55	32.001	68.551	11.75	56.801
A'	+ 14.779	+ 120.	+ 12.489.	+ 34.356	- 1.74	1.695	3.4356	.146	3.876

$\frac{Load}{wt}$	V_L	$\frac{B_2}{R_2} V_L$	c_2/c_1	M_L	$\frac{a_1}{c_1} M_L$	$-\frac{L_2}{c_1} M_L$	$-d/c_1$	H_L	M_H
E'	411.183	- 755.120.	+ 824.437.	+ 69.317.	+ 775.335	+ 1.971.03	- 673.17	+ 2.073.195	+ 31.600.
D'	236.671	- 434.120.	+ 509.193.	+ 75.073.	+ 839.718	+ 1.134.50	- 346.11	+ 1.628.108	- 27.786.
C'	111.459	- 204.447	+ 257.662.	+ 53.815.	+ 595.246	+ 534.287	- 144.125	+ 98.5.388	- 72.425.
B'	31.749	- 58.240.	+ 79.288.	+ 20.998.	+ 234.870	+ 152.190	- 36.550	+ 350.510	- 89.070.
A'	1.646	- 3.019.	+ 4.586.	+ 1.567.	+ 17.527	+ 7.890	- 1.740	+ 23.677	- 37.703.



Table 5.

	<i>T</i>	<i>S</i>	<i>M</i>	<i>T</i> / <i>A_r</i>	<i>M</i> / <i>C_r</i>																			
<i>P</i> 0.834	-792	+7.3	8.840	1.858	5.691	1.982	1.950	-607	+7.9*	2.924	1.967	1.839	-957	684	-910	+7.5*	1.781	-7.5	1.781	-7.5	1.781	-7.5	1.781	-7.5
<i>R</i> 1.965	-610	+1.200	5.250	1.679	6.901	3.550	1.512	-561	+7.6*	4.052	2.020	6.028	-2.012	3.28	-3.81	+7.7*	2.468	-1.611	4.079	-1.611	4.079	-1.611	4.079	-1.611
<i>B</i> 2.040	-660	-261.2	7.500	-1.861	7.314	7.686	1.587	-295	+5.8*	5.835	.5912	6.46	5.244	.982	-2.20	+7.8*	3.610	.8285	4.438	.8285	4.438	.8285	4.438	.8285
<i>C</i> 2.052	-76	-810.8	6.645	-2.182	6.463	10.847	1.607	-193	-53	6.765	-1.155	5.60	7.920	.995	-185	-5.9*	4.95	-7.82	3.470	4.95	-7.82	3.470	4.95	-7.82
<i>D</i> 2.053	+98	-810.8	9.150	-2.436	6.110	11.568	1.612	-1752.8	7.180	-2.363	4.817	3.573	10.4	-98	-99.0	4.975	-1.340	3.135	5.815	-1.340	3.135	5.815	-1.340	
<i>E</i> 2.051	+318	-261.2	9.330	-2.889	9.111	9.569	1.604	-160	-59.0	7.380	-2.292	5.088	9.672	.998	+66	-39.5	4.595	-1.428	3.167	6.023	-1.428	3.167	6.023	
<i>E'</i> 1.983	+5.9*	-51.130	5.465	+2.885	12.886	5.885	1.580	+308	0	7.270	0	7.270	2.270	.987	+15*	-29.3	5.539	-5.483	4.111	4.111	-5.483	4.111	4.111	
<i>D</i> 2.023	-4.928	+441.12	3.335	+1.865	3.130	5.886	1.630	-5359	+7.5*	4.660	4.670	3.225	3.625	.970	+252	+109.5	4.320	-1.473	5.793	2.847	-1.473	5.793	2.847	
<i>D</i> 2.040	-303	-261.2	9.083	-2.707	8.817	9.559	1.630	-5359	+7.5*	4.660	4.670	3.225	3.625	.970	+252	+109.5	4.320	-1.473	5.793	2.847	-1.473	5.793	2.847	
<i>C</i> 2.092	-103	-810.8	8.015	-2.748	6.387	11.247	1.626	-397	-111.6	6.850	-1.374	2.816	5.502	9.4	-348	+3.7	3.600	6.573	3.737	3.600	6.573	3.737		
<i>B</i> 2.094	+98	-261.2	7.750	-2.098	5.682	9.776	1.750	-204	-1.96	6.435	-2.955	3.48	9.390	1.629	-5.6	-1.6	4.520	-1.638	2.862	6.178	-1.638	2.862		
<i>G</i> 2.044	+765	+7.17	5.213	-0.935	5.360	3.780	1.762	+105	-399.5	2.654	-1.845	2.693	3.809	1.02	-2.97	-590.0	3.358	-2.760	5.98	6.118	-2.760	5.98	6.118	
<i>G</i> 1.966	+717	+261.2	3.560	+810	4.370	2.750	1.686	+327	-2899.0	3.053	-551	2.502	3.604	1.330	-128	-2045.5	2.407	-1.628	2.783	4.031	-1.628	2.783	4.031	

	<i>T</i>	<i>S</i>	<i>M</i>	<i>T</i> / <i>A_r</i>	<i>M</i> / <i>C_r</i>																		
<i>G</i> 3/17	-778	+610	6.539	6.080	1.974	1.951	1.95	-23	+6.6	1.913	-1.043	0.5545	1.945	-0.388	4.996	2.3	71.500	0.00	20.00	0.00	20.00	0.00	
<i>R</i> 335	-150	+147.6	.632	.711	1.603	.031	2.2	-19	+7.8	.0638	.0628	.0628	.0628	.0628	.0628	.0628	3.75	30.555	0.00	15.00	0.00	15.00	0.00
<i>B</i> 353	-90	+7.6	1.297	.536	1.633	.761	.28	-10	+6.7	.0519	.0519	.0519	.0519	.0519	.0519	.0519	27.07	1.574	.97	10.68	.00	10.68	.00
<i>G</i> 360	-55	0	1.516	0	1.516	1.516	.29	-9	+6	.1241	.000749	.1248	.1214	.2375	.1214	.1214	7.665	.80	9.24	.00	9.24	.00	
<i>D</i> 365	-27	-233.4	1.626	-2.411	1.385	1.867	3.0	-5	-6	.1339	.00494	.1315	.1315	2.845	.1315	.1315	6.460	.13	8.70	.00	8.70	.00	
<i>F</i> 361	+40	-1.6	1.660	-0.521	1.139	2.081	30	0	-3.0	.1300	.003065	.1337	.1423	2.173	.1423	.1423	5.852	.95	8.40	.00	8.40	.00	
<i>E</i> 358	-53	-1.6	1.647	-2.60	1.587	1.907	.31	+3	-1.0	.1426	.00458	.1400	.1452	2.173	.1452	.1452	5.852	.95	8.40	.00	8.40	.00	
<i>D</i> 352	+87	+3.66	1.367	-4.87	2.054	1.080	.30	+7	+1.0	.1336	.005825	.1364	.1307	2.245	.1364	.1364	6.460	.13	8.70	.00	8.70	.00	
<i>C</i> 345	+142	+10.36	1.452	1.221	2.613	.31	.27	+9	+7.8	.1317	.00668	.1824	.1050	2.375	.1824	.1824	7.655	.80	9.24	.00	9.24	.00	
<i>B</i> 320	+800	+7.52	1.240	1.792	2.494	.57	.25	+11	+4.7	.1317	.010305	.1879	.1217	2.07	.1879	.1879	11.541	.97	10.68	.00	10.68	.00	
<i>H</i> 187	-617	-5792	2.030	-2.675	4.705	.27	*13	-860	+1.0	.1393	.01446	.10717	.1053	3.878	.10717	.10717	15.50	.62	20.0462	.00	20.0462	.00	
<i>O</i> 867	-562	-9242	1.367	-5.668	-2.097	-5.629	3.665	6.00	-790	-9770	1.2410	-2.818	-2.818	-2.818	-2.818	-2.818	5.523	.32	22.35	.00	22.35	.00	



TABLE 6. TOP STRESSES.

Load	Stress at O' at 1000^{st}	Stress at H'		Stress at H'		Stress at B'		Stress at C'		Stress at D'		Stress at E'				
		Dead	Live	1000 st	Dead	Live	1000 st	Dead	Live	1000 st	Dead	Live	1000 st	Dead	Live	
A'	.024	.0806	.0345	.0077	.057	.044	.0077	.057	.044	.0077	.057	.044	.0077	.057	.044	
B'	.763	.762	.615	.565	.565	.565	.565	.597	.625	.625	.625	.625	.625	.625	.625	
C'	.763	.762	.615	.565	.565	.565	.565	.597	.625	.625	.625	.625	.625	.625	.625	
D'	.2578	.1649	.598	.598	.598	.598	.598	.438	.395	.395	.395	.395	.395	.395	.395	
E'	.437	.6795	.10200	.5766	.8345	.3655	.3748	.6775	.5014	.6775	.5014	.6775	.5014	.6775	.5014	
E	.5698	.8865	.35480	.6901	.0730	.4295	.7314	.1136	.6763	.1030	.6763	.1030	.6763	.1030	.6763	
D	4.851	9.805	11.470	6.028	18.871	14.111	6.428	12.88	14.99	5.610	11.24	13.09	8.817	9.66	6.84	
C	3.256	9.335	7.590	4.019	11.70	9.52	4.438	18.73	10.35	3.470	9.95	8.10	3.185	8.39	4.53	3.167
B	1.2477	4.810	1.797	1.603	6.26	2.149	1.833	7.16	10.30	1.516	5.92	2.182	1.385	5.41	3.08	4.56
A	0.498	3.49	0.72	0.76	5.33	1.049	1.082	.759	1.093	.888	.86	1.177	.1375	.923	.1377	.938
Σ <i>Sum</i>	45.101	70.475	61.253	36.363	85.616	110.736	88.603	99.255	95.141	134.026	145.977	85.136	143.977	85.136	143.977	85.136
<i>Total</i>																

TABLE 7. BOTTOM STRESSES.

Load	Stress at O' at 1000^{st}	Stress at H'		Stress at B'		Stress at C'		Stress at D'		Stress at E'		
		Dead	Live	1000 st	Dead	Live	1000 st	Dead	Live	1000 st	Dead	Live
A'	.028	.0806	.0345	.0077	.057	.044	.0077	.057	.044	.0077	.057	.044
B'	16.7	3.193	1.916	.0686	.388	.1517	.0189	.595	.114	.005	.732	.151
C'	14.53	5.2778	4.705	.0520	.665	.6175	.332	.665	.954	.231	.916	.332
D'	11.15	5.8005	6.116	.0686	.697	8.813	6.187	.771	.890	.1340	.3715	1.931
E'	3.604	5.188	6.389	.0207	.935	9.207	.0390	.1826	.1532	.5520	.1070	.1283
E	2.750	6.420	5.160	.055	.0208	9.718	.0280	.1520	.1745	.0207	.0207	.0207
D	1.982	3.068	1.733	.0521	.0208	7.686	.0195	.4780	.10.887	.16.84	.25.30	11.588
C	.957	1.918	2.232	.0065	.0065	.6955	.5244	.10.51	.12.22	.7.92	.15.87	18.46
B	.306	.818	.707	.0457	.0457	.1.938	.2.782	.7.98	.6.492	.9.72	.14.10	7.005
A	.031	.051	.045	.045	.045	.761	.761	.096	.516	.516	.2.183	1.867
Σ <i>Sum</i>												
<i>Total</i>												

Stress at O'	A'	B'	C'	D'	E'
T_{Op}	T_{Op}	B_{Op}	B_{Op}	B_{Op}	T_{Op}

$11.5 \cdot 516^{1/2}$	$117.616^{1/2}$	$201.94^{1/2}$	$225.163^{1/2}$	$234.57^{1/2}$	$229.15^{1/2}$
------------------------	-----------------	----------------	-----------------	----------------	----------------

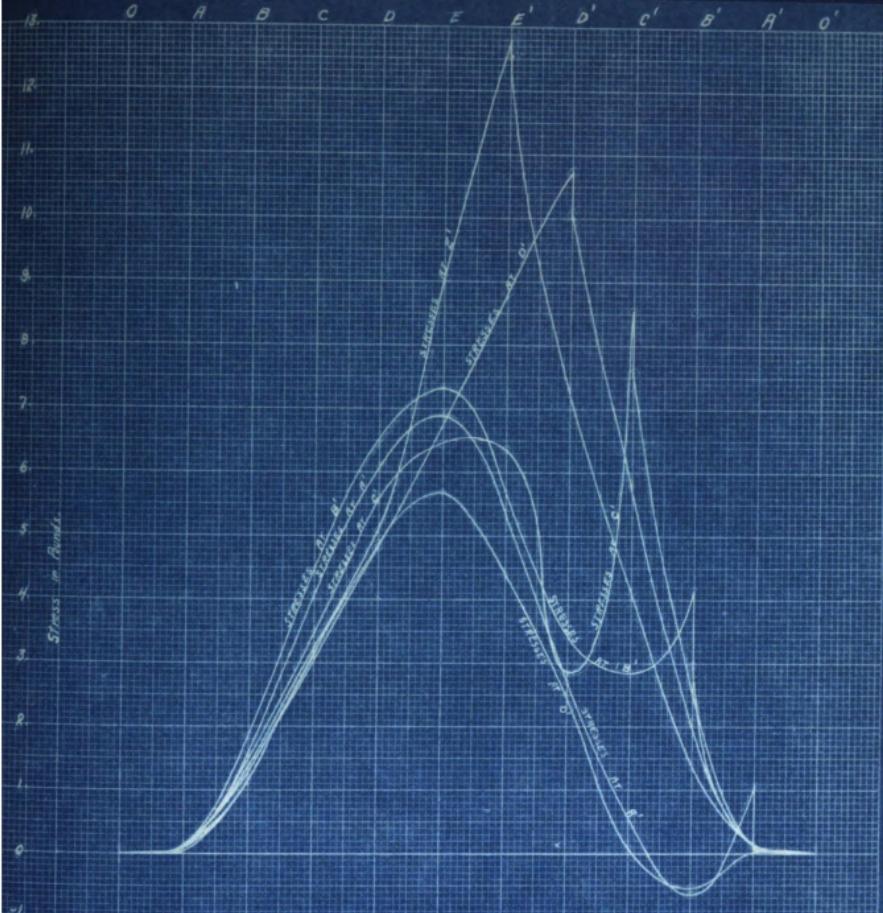


Plate
Curves of Stresses in Top of
Arch Ring for
1000^t Load.

PLATE 5.



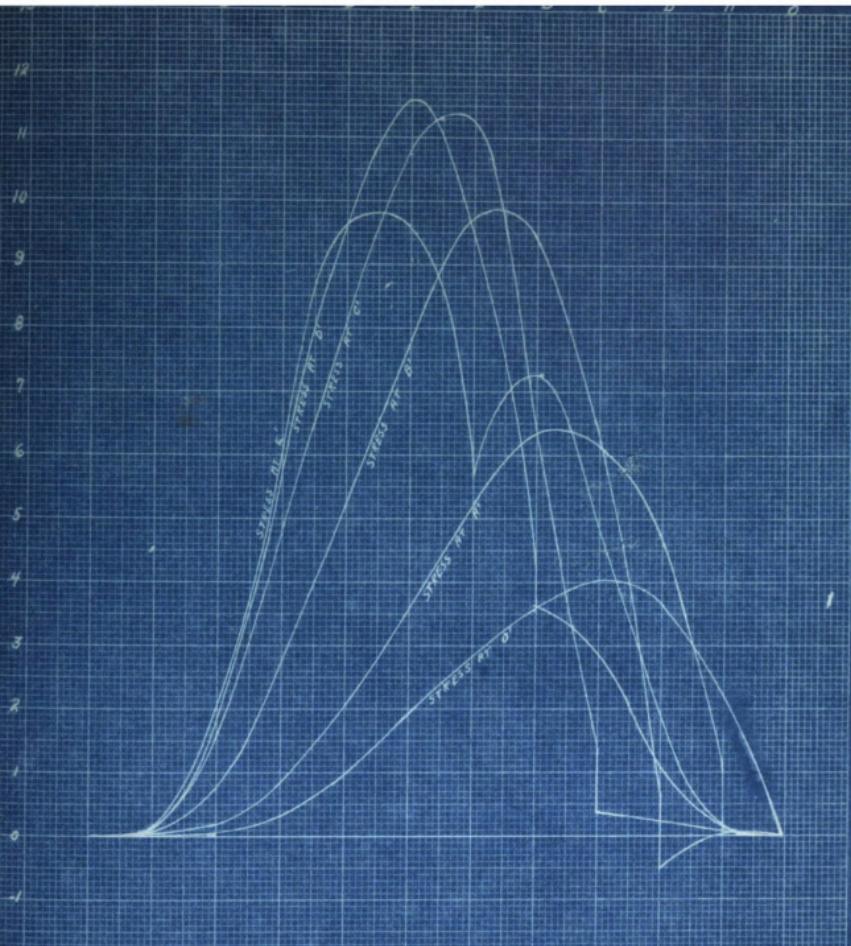
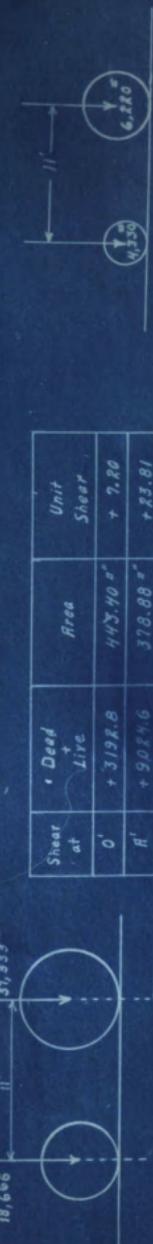


Plate
Curves for Stresses in Bottom
of Arch Ring for a
1000 lb Load

PLATE 6.

TABLE 8.

Load	Shear at O'	Shear at H'	Shear at B'	Shear at C'	Shear at D'	Shear at E'
	Dead Live	1000 ^a Dead Live	1000 ^a Dead Live	1000 ^a Dead Live	1000 ^a Dead Live	1000 ^a Dead Live
B'	-23 -703	-292 -150	-30 -365	-15.7 -15.4	-9 -66.2	-5 -35.6
C'	-170 -1500	-771 -620	-620 -620	-301.5 -125	-72 -84.6	-35.1 -29.3
D'	-607 -160	-990 -921	-921 -921	-57.9 -57.9	-175 -175	-132.7 -67.2
E'	-392 -830	-1300 -610	-610 -918	-1000 -260	-118 -118	-106.4 -106.4
F'	-717 -114	-177 -465	-74.5 -74.5	-152 -152	-10.3 -16.2	-15.6 -15.6
G	-357 -656	-105 -536	-211 -72.2	+204 -470	+242 -242	+242 -242
H	+362 +738	+297 +864	+69.4 +528	+120.7 +120.5	+90.2 +83.9	+92.5 +84.7
I	+653 +735	+672 +650	+156.5 +156.5	+282 +282	+86.6 +86.6	+169.7 +169.7
J	+550 +590	+550 +560	+51.0 +51.0	-11.1 -11.1	-13.7 -8.6	-15.2 -15.2
Sum	+1763.5	+1729.3	+5264.0	+3749.6	+1946.6 +5354.6	+1591.1 -1591.1
Total				+9424.6	+65002	+17992 -17992

18.66^a = J₁, J₂, J₃ = 11'

Load on Roller Wheels for a section
of Foot Wide.

Diagram of roller and loads.

Point	H	B	C	D	E	E'	D'	C'	B'	A'
Dead Load	7008 ^a	3903 ^a	2480 ^a	2010 ^a	1534 ^a	1534 ^a	1946 ^a	2762 ^a	3265 ^a	3265 ^a
Live Load/ ^a 1600 ^a	1600 ^a	1370 ^a	1400 ^a	1420 ^a	1440 ^a	1440 ^a	1480 ^a	1400 ^a	1370 ^a	1800 ^a

Longings.



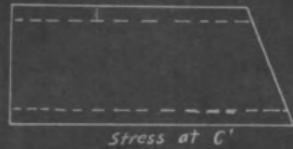
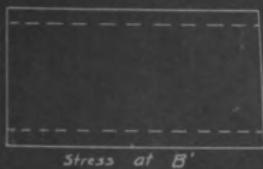
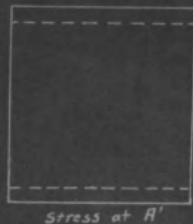


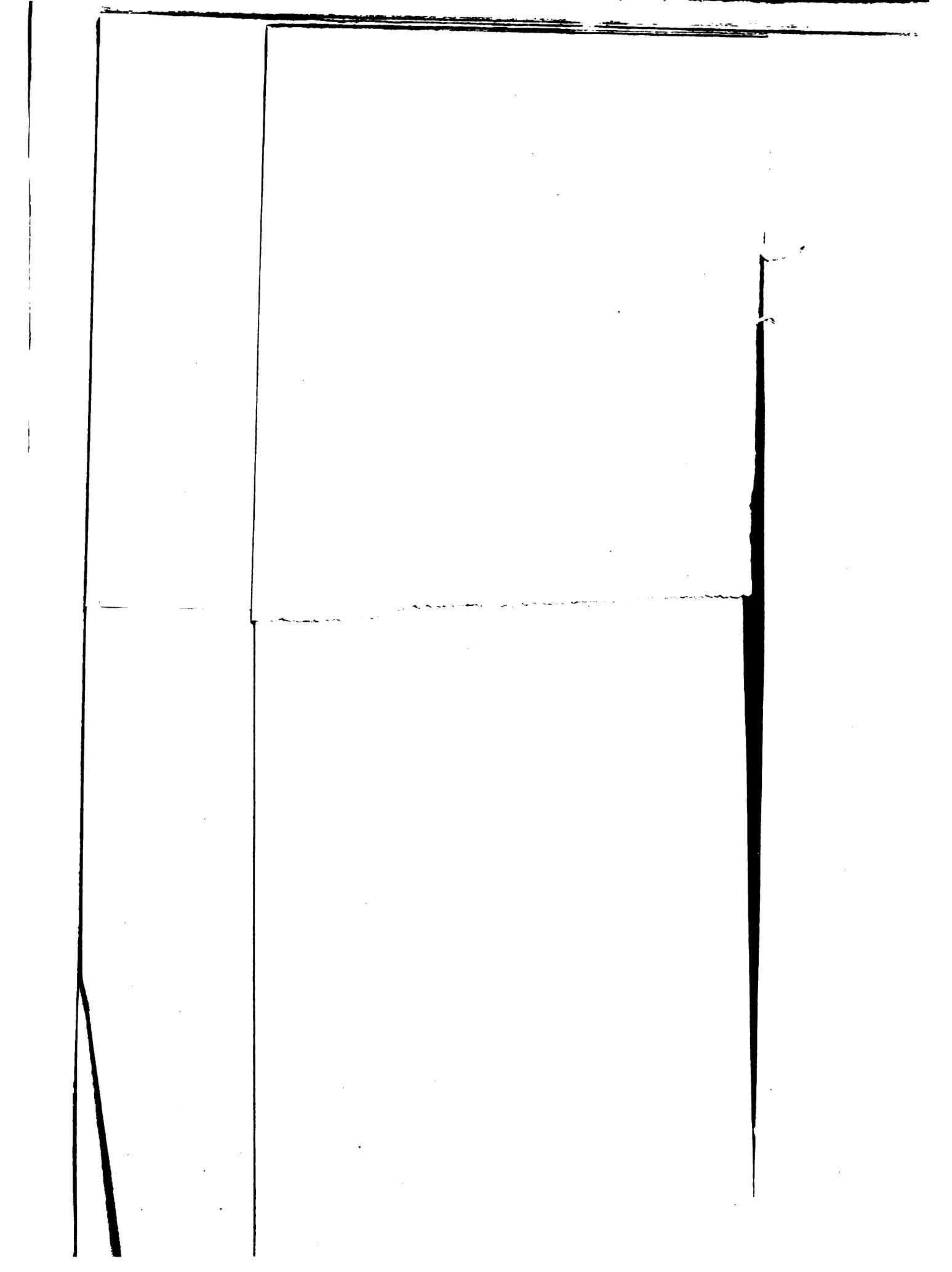
Plate 9.

Maximum Stresses in Steel
Reinforcement of Arch Ring.

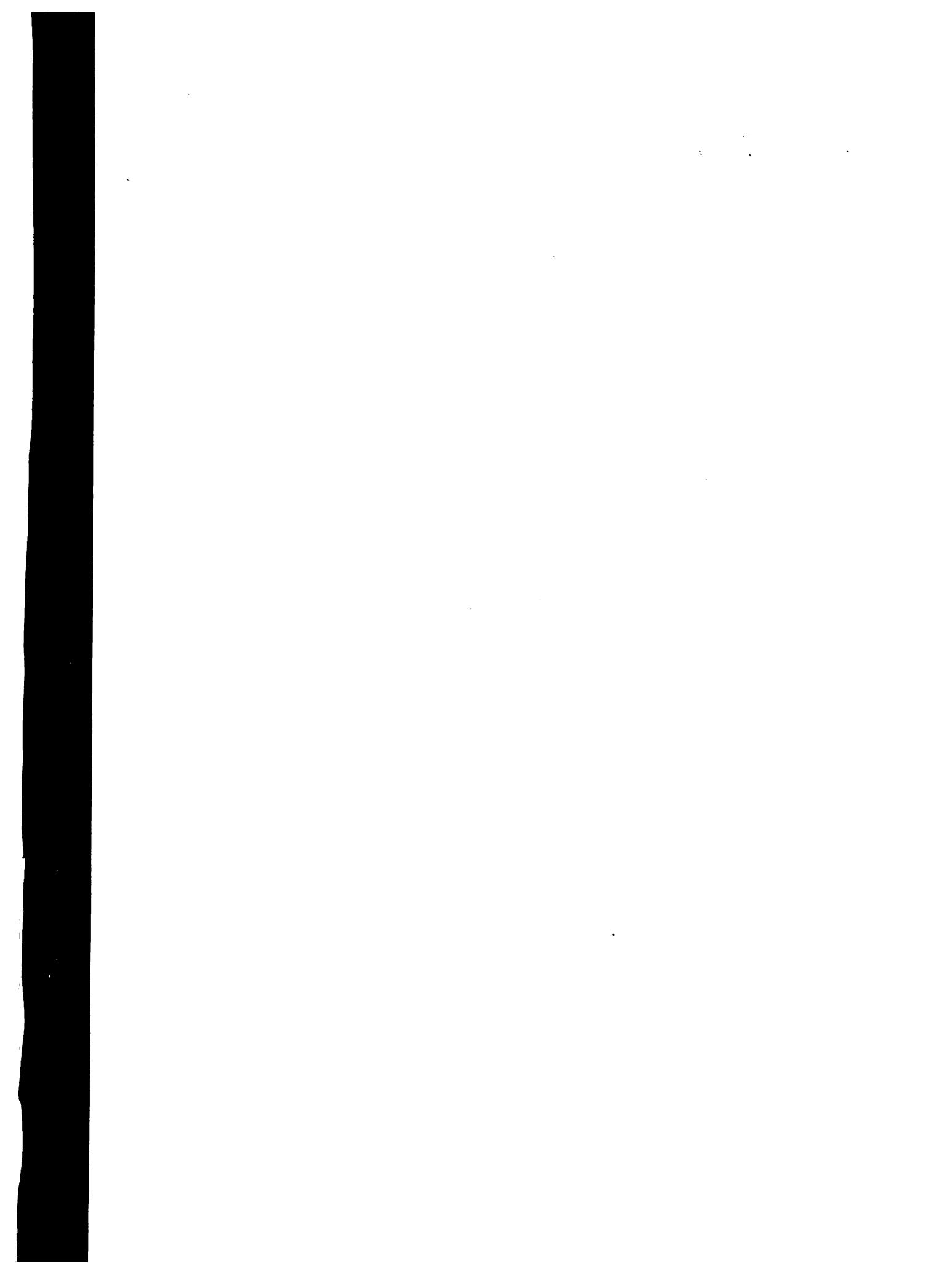
Scales : 1" = 20", 1" = 100#

Stress at	Max. Steel Stress	
	Top	Bottom
O'	64.7 [#] _{lb}	57.1 [#] _{lb}
A'	83. [#] _{lb}	81.5 [#] _{lb}
B'	110.9 [#] _{lb}	113.7 [#] _{lb}
C'	108. [#] _{lb}	122.6 [#] _{lb}
D'	129.9 [#] _{lb}	133.3 [#] _{lb}
E'	188.5 [#] _{lb}	187.7 [#] _{lb}





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