This thesis was contributed by

Mr. W. H. Urquhart

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THESIS

HYDRAULIC FORMULAE

for

ORIFICES and CHANNELS

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THESIS

PREFACE.

The object of this thesis is to investigate the laws which govern the flow of water through orifices and in open channels. It was intended to investigate also the flow of water over weirs and through pipes of various kinds, but the time at our disposal proved to be insufficient. No original experiments were made but instead we used the data of such standard authorities as, Darcy and Bazin, Kutter, Fteley and Stearns, Cunningham, Rittinger, Korf, Legler, LaNicca, Leveille, M.Poirre, Humphreys and Abbott, and the Missouri River Commission for channels, and Hamilton Smith Jr., Ellis, Judd and King, for orifices.

In order to facilitate the work it was divided into the investigation of channels and investigation of orifices. The former part of the work was carried on by Mr. Urquhart and the latter by Mr. Powell. The problem of circular orifices was so large that time was not found for the consideration of square and rectangular orifices and therefore they do not appear in this work. In the case of both orifices and channels the idea was to determine, first the factors which controlled the discharge, and then to derive an empirical formula which would give the discharge as a definite function or functions of these factors. In the case of channels, innumerable formulas have been proposed at various times and the problem was to obtain a better one if possible. The case was somewhat different in the matter of orifices as all statements of the law of flow which are at all tenable seem to be in the form of tables.

The computations in this work were facilitated by the use of Burkhardt's Arithmometer.

BIBLIOGRAPHY

Title	Author
Hydraulics	Hamilton Smith, Jr.
Flow of Water in Rivers and Other Channels	Ganguillet and Kutter
Hydraulics of Great Rivers	J. J. Révy
Flow of Water	Schmeer
Experiments at Roorkee	Cunningham
Applied Mechanics	Goodman
New Hydraulics	Sullivan
Hydraulic Tables, Coefficients and Formulae	Neville
Least Squares	Merriman
Treatise on Hydraulics	Merriman
Hydraulics	Bovay
Discussion of Kutter's Formula	A.S.C.E., Vol. 9, p. 326
Flow of Water in Rivers	A.S.C.E., Vol. 8, p. 173
Hydraulic Experiments with Large Apertures	A.S.C.E., Vol. 5, p. 19
Experiment on Frictionless Orifice	Eng. News, Vol.56, p. 326
Coefficients of Discharge Circular Orifices	Eng. News, Vol. 60, p. 49
Contraction of Jets	Engineering, Mar. 11, 1904.

The works just mentioned have been consulted and much of the data included in them has been used in the work which follows.

INVESTIGATION OF UNIFORM FLOW OF WATER IN OPEN CHANNELS.

The investigation of the movement of water in open channels has been before hydraulicians for many years. The first investigators sought to express the laws of flow by means of mathematical principles but were unsuccessful in obtaining accurate results. Galileo is said to have been the first investigator upon the flow of water in rivers, but some of his statements have been proved erroneous, such as the irregularities of rivers caused no retardation of flow. Torricelli discovered that, except for resistances, the velocity of jets of water issuing from small orifices was equal to that of bodies falling the same distance in space.

In order to make an investigation of the various formulae for the flow of water in open channels, it will be necessary to consider briefly the conditions under which water flows, the elements which should enter into the computations of mean velocities, and the circumstances which affect the rate of flow. The inclination or slope is generally considered to be one of the chief elements which affect the velocity and rate of flow. In a long, straight, uniform channel, the accelerating force due to the slope serves only to maintain a uniform velocity; the head in a given length being expended in overcoming the resistances. If the supply of water is increased and slope diminished the velocity is increased, but the area of the cross-section will be decreased until the resistances equal the force due to the head in unit length. From this we see that the velocity and discharge depends upon the area of the channel and the resistances together with the inclination. There is a great uncertainty as to the manner in which the area and resisting forces enter into the formulae for computing mean velocity. The more common way being to use some function of the hydraulic radius, or area of cross-section divided by the wetted perimeter.

The first attempt to discover the law by which the velocity of flow depends upon the fall or slope and cross-section was made by Brahms, who observed that the water in streams acquires a constant velocity. He points to the friction of the water against the wetted perimeter as the force which opposes the acceleration and assumes that the resistance is proportional to the hydraulic radius. Brahms and Chezy are to be regarded as the authors of the formula, $v = c\sqrt{RS}$ where

v = mean velocity

c = a constant

R = hydraulic radius

S = inclination or slope.

It became very evident to the earlier investigators that the laws must be derived from experimental data, and after work with that principle involved, we have a formula by Prony; $RS = a\,v + b\,v^2$ in which "a" and "b" are coefficients of friction deduced from experiments.

Eytelwein used the same form but found different values of "a" and "b". Many others derived formula but in every case the coefficients were constant values.

The many formulae up to this time did not recognize the influence of roughness of wetted perimeter, or degree of slope upon the coefficients. A new understanding of the subject was opened up by the investigations of Darcy and Bazin. They constructed an experimental canal over 1800' feet long, varying in cross-section, slopes and lining material. All measurements were made with the greatest care and have been the basis of many computations. The principal facts derived from Bazin's results are

I. The coefficient "c" varies with the degree of roughness of wetted perimeter.

II. The coefficient "c" varies with R.

Bazin's formula has the following form $V = \sqrt{\frac{RS}{A+\frac{B}{R}}}$ in which "A" and "B" are constants varying with the surfaces.

Bazin divided the experiments into five categories and the following is a table of results.

Bazin's Coefficients

Category	Channel	A	B
I	Cement Carefully Planed Board	0.000046	0.0000013?7
II	Smooth Ashlar Brick Unplaned Board	0.000058	0.00000405
III	Rubble	0.000073	0.0000183
IV	Earth	0.000085	0.000107
V	Carrying gravel	0.000122	0.000214

This formula is not universally applicable although it could be made so if the number of categories were increased and variations considered for all possible cases.

The formula next in importance for practical use is that derived by Ganguillet and Kutter, better known as Kutter's Formula. In the development of their formula they used Bazins as basis and endeavored to embody in it the effects of the slope as well as a relation between the coefficients of roughness. The coefficient "c" in the form $v = c\sqrt{RS}$ was put equal to $\frac{y}{1 - \frac{x}{\sqrt{R}}}$, where "y" and "x" are variables depending upon the slope and roughness. The variation of "y" is expressed by "y" = $(a + \frac{L}{n} + \frac{m}{s})$ and variation of "x" by "x" = $(a + \frac{m}{s})n$.

Formula $$v = \frac{a + \frac{L}{n} + \frac{m}{s}}{1 + \frac{n}{\sqrt{R}}(a + \frac{m}{s})}$$

The determination of the various constants in this formula will be outlined. The form $c = \frac{y}{1 + \frac{x}{\sqrt{R}}}$ can be expressed as $\frac{1}{c} = \frac{1}{y} + \frac{x}{y} \cdot \frac{1}{\sqrt{R}}$ which is a straight line by plotting values of "$\frac{1}{c}$" and $\frac{1}{R}$. A number of experiments were so plotted and straight lines drawn through points. Different lines being drawn for different slopes. These experiments so plotted were some of Humphreys and Abbot and some of Bazin. According to Kutter, the various lines drawn through the plotted points intersect in a common point. This point is where $\sqrt{R} = \sqrt{1 \text{ meter}} = 1.811$ feet. This is the constant "L" in the general formula. We purpose to show by diagram of plotted values of "$\frac{1}{c}$" and $\frac{1}{\sqrt{R}}$ that these lines have no common intersection and our statement is borne out by the statement of Herschal in his book on Flow of Water. Assuming that the point of intersection is $\sqrt{R} = 1.811$ feet, Kutter makes the following statements,

I. Values of "c" increase with decrease of inclination when R is greater than 3.28 feet.

II. Values of "c" increase with increase of slope when R is less than 3.28 feet. From the plotted results of "$\frac{1}{c}$" and "$\frac{1}{\sqrt{R}}$" the values of "y" are measured on the vertical axis.

By plotting the values of "y" as ordinates and "$\frac{1}{s}$" as abscissa a number of points were obtained which were assumed to lie upon a straight line. From where this line so drawn intersected the axis of ordinates, the distance to the origon was measured as $(a + \frac{l}{n})$ and the constant "m" = 0.00281 was the tangent made by the line and a horizontal through point of intersection. The values of $(a + \frac{l}{n})$ and $(\frac{l}{n})$ being known, the value of "a" is determined = 41.6. The values of "n" were obtained by a graphical method, as follows, plot values of "$\frac{1}{\sqrt{R}}$" as abscissae and those of "$\frac{1}{c}$" as ordinates. Assume a series of values of "n" and plot them upon the ordinate for "$\frac{1}{\sqrt{R}}$" = 1.811 feet. Plot derived values of $(a + \frac{l}{n})$ upon the axis of ordinates and connect the points with those of "n" by straight lines. For each guaging read the preliminary value of "n" indicated by the position of the point with relation to these lines, and taking known value of "s" solve for "y" $= a + \frac{l}{n} + \frac{m}{s}$. Take reciprocal of "y" and plot on axis of ordinates and draw a line through it and gauging. The point where line so drawn intersects the series of "n" plotted on ordinate $\frac{1}{\sqrt{R}}$ = 1.811 gives value of "n" to be used.

The conclusions of Kutter are as follows concerning the coefficient "c".

The coefficient increases

1. With the increase of the hydraulic radius.

2. With decrease of roughness of wetted perimeter.

3. With decrease of inclination when R is greater than 3.2 [illegible]

4. With increase of inclination when R is less than 3.28 feet.

The following is a table of values of "n" as assigned to different surfaces:

N = 0.009 for well planed timber

N = 0.010 for neat cement

N = 0.011 for cement one-third sand

N = 0.012 for unplaned timber

N = 0.013 for ashlar and brick

N = 0.015 for sewers and conduits

N = 0.017 for canals in firm gravel

N = 0.025 for canals and rivers free from stones

N = 0.030 for canals and rivers with some stones

N = 0.035 for canals and rivers in bad order.

The values of "n" must be assumed by "guess and allow" method, which may be all right for experienced man but is poor for a novice. Take for example the flow of the Mississippi river using "n" = 0.03, we obtain a velocity of 5.60 feet per second for a specific case and then using "n"= 0.010 we obtain a velocity of 11.35 feet per second. This gives more than double the discharge for different surfaces which is very hard to believe. Very few prominent engineers believe that if we should line the Mississippi with cement the discharge would be double for the same area and slope which goes to show that a separate formula is necessary for large rivers.

In all the formulae so far discussed, we find that the velocity is assumed to vary as the hydraulic radius or as the square root of the hydraulic radius and the same holds true regarding the variation due to slope. Kutter's own investigation shows that a great and uncertain error is involved in assuming that the velocity varies as the $\sqrt{R}$.

The following diagram and computation is now introduced to substantiate the statement previously made concerning the intersection of lines drawn through plotted values of "$\frac{1}{c}$" and "$\frac{1}{\sqrt{R}}$" and to disprove the statement of Kutter that it is at a point when $\frac{1}{\sqrt{R}} = 1.811$. The data used being taken from that used in derivation of Kutter's formula.

The following data was used in the preparation of Plate I illustrating the effect of change of slope upon the value of coefficient "c".

Experiment	Slope										
Neat Cement	0.0015	$\frac{1}{\sqrt{R}}$	1.65	1.41	1.28	1.21	1.15	1.11	1.07	1.04	1.02
Bazin #24		$\frac{1}{c}$	.0077	.0073	.0072	.0069	.0068	.0067	.0068	.0067	.0065
Neat Cement	0.0049	$\frac{1}{\sqrt{R}}$	2.44	1.99	1.76	1.63	1.52	1.45	1.39	1.34	1.29
Bazin #2		$\frac{1}{c}$	.0086	.008	.0079	.0076	.0075	.0074	.0074	.0073	.0073
Smooth Boards	0.0081	$\frac{1}{\sqrt{R}}$	5.13	4.66	4.26	3.86	3.58	3.13			
Bazin #30		$\frac{1}{c}$	.0245	.0217	.0190	.0176	.0168	.0153			
Smooth Boards	0.0152	$\frac{1}{\sqrt{R}}$	5.77	4.82	4.34	4.05	3.68				
Bazin #29		$\frac{1}{c}$	.0114	.0110	.0106	.01016	.0094				
Smooth Boards	0.0047	$\frac{1}{\sqrt{R}}$	5.87	4.38	3.89	3.65	3.45	3.32	3.28		
Bazin #28		$\frac{1}{c}$	.0131	.0121	.0112	.0106	.0103	.0098	.0097		
Cement and Sand	0.0015	$\frac{1}{\sqrt{R}}$	1.62	1.37	1.25	1.19	1.13	1.09	1.05	1.03	1.01
Bazin #25		$\frac{1}{c}$	.0083	.0082	.0079	.0075	.0074	.0074	.0073	.0072	.0071

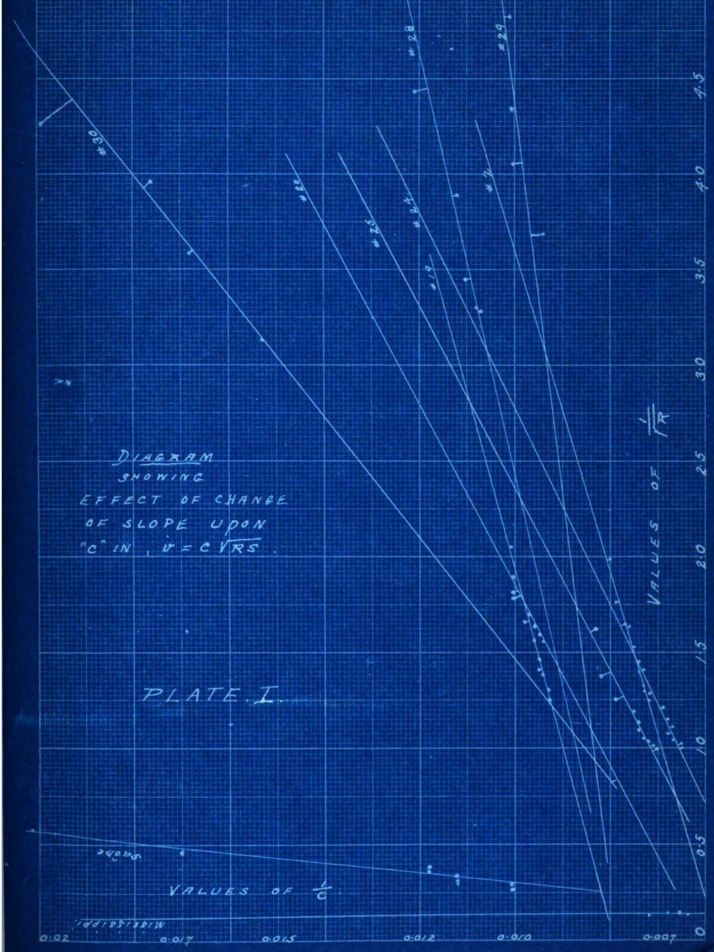
DIAGRAM
SHOWING
EFFECT OF CHANGE
OF SLOPE UPON
"C" IN, v = C√RS.
PLATE I.
VALUES OF 1/C
VALUES OF 1/√R
#30
#20
#25
#24
#19
#28
#29
#2
0.02
0.017
0.015
0.012
0.010
0.007
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5

Experiment	Slope										
Unplaned Boards	0.006	$\frac{1}{R}$	2.05	1.89	1.81	1.76	1.70	1.64	1.59	1.56	1.52
Bazin #20		$\frac{1}{C}$	.0106	.0103	.0102	.0103	.0097	.0096	.0095	.0094	.0093
Unplaned Boards	0.0043	$\frac{1}{R}$	2.16	1.83	1.66	1.56	1.47	1.42	1.37	1.33	1.27
Bazin #19		$\frac{1}{C}$	.0103	.0101	.0098	.0096	.0095	.0095	.0094	.0093	.0093
Mississippi River	0.0000112	$\frac{1}{R}$	1.29	1.15	1.05	1.00					
		$\frac{1}{C}$	.0072	.0068	.0065	.0066					
Saone River	0.00004	$\frac{1}{R}$	.507	.457	.376	.334	.303	.294	.291	.274	.261
		$\frac{1}{C}$	.0222	.070	.0118	.012	.0112	.0102	.0102	.0102	.0102

Thrupp's Formula for the Flow of Water.

This formula has been derived from a wide range of experimental data and it is said to be the best of the known formulae by many hydraulic engineers. It is a modification of Hagen's formula and was published in a paper read before the Society of Engineers in England in 1887.

Notation.

V = velocity of flow in feet per second.

R = hydraulic radius.

L. = length of channel.

H = fall in length "L"

S = cosecant of angle of slope ($\frac{L}{H}$)

X,O,N, are coefficients depending upon the nature of the surface.

Then $V = \frac{R^x}{O\sqrt[N]{S}}$.

For small values of "R" more accurate results are obtained by substituting for the index "x" the value $(X + Y\sqrt{\frac{Z - R}{R}})$.

The following table is used in connection with this formula.

Thrupp's Formula.

Surface	N	O	X	Y	Z
Wrought iron pipe	1.80	0.004787	0.65	0.018	0.07
Riveted sheet "	1.825	0.005674	0.677	———	———
New Cast Pipe	{1.85 {2.00	{0.005347 {0.006752	{0.67 {0.65	———	———
Lead Pipe	1.75	0.005224	0.62	———	———
Pure Cement	{1.74 {1.95	{0.00400 {0.006429	{0.67 {0.61	———	———
Brickwork(smooth)	2.00	0.007746	0.61	0.01224	0.50
Brickwork(rough)	2.00	0.008845	0.625	0.01224	0.50
Unplained boards	2.00	0.008451	0.615	0.03349	0.50
Small gravel in cement	2.00	0.01181	0.66	0.03958	0.60
Large gravel in cement	2.00	0.01415	0.705	0.07590	1.00
Hammered masonry	2.00	0.01117	0.66	0.07820	1.00
Earth (no vegetation)	2.00	0.01536	0.72	———	———
Earth (stoney,rough)	2.00	0.02144	0.78	———	———

Thrupp recognizes the fact that the velocity varies as different powers of "R" and "S" for different surfaces. The relation existing between Thrupp's "S" and that generally used is

Thrupp's "S" $= \frac{1}{S}$ as generally used.

Working with the same idea that "V" varies differently for various surfaces, a number of coefficients for "S" and "R" have been derived and a comparison is submitted.

Surface	Coefficients for Radius		Coefficients for Slope	
	Thrupp	Urquhart	Thrupp	Urquhart
Pure Cement	{0.67 {0.61	0.61	{1.74 {1.95	1.72
Brickwork (smooth)	0.61	0.612	2.00	2.00
Brickwork (rough)	0.625	0.76	2.00	2.00
Unplaned Boards	0.615	0.675	2.00	1.82
Small gravel in cement	0.66	{0.66 {0.73	2.00	2.60

Surface	Coefficients for Radius		Coefficients for Slope	
	Thrupp	Urquhart & Powell	Thrupp	Urquhart & Powell
Large gravel in cement	0.705	0.75	2.00	2.50
Hammered masonry	0.66	0.75	2.00	2.58
Earth(no vegetation)	0.72	0.65	2.00	2.00
Earth (rough, stony)	0.78	0.75	2.00	2.00

DERIVATION OF FORMULA

The derivation of a new general formula was now undertaken after a study of the various formula herein outlined and discussed. The first work to be undertaken was to find if possible how the velocity varies with the hydraulic radius. The velocities and radii as data taken with constant slope were used in the computation. The form of expression as decided upon was

$\frac{V_1}{V_2} = \frac{R_1^z}{R_2^z}$ and from this we have

that, $Z = \frac{\log V_1 - \log V_2}{\log R_1 - \log R_2}$.

Values of "Z" must be derived for various surfaces and shapes of conduit as was found in working this out. On the following page is given a sample of the work necessary. This includes, radius, logarithum of radius, velocity, logarithum of velocity, and computed value of "Z" for neat cement conduit or channel. Computation for "Z" in form $Z = \frac{\log V_1 - \log V_2}{\log R_1 - \log R_2}$ for neat cement:-

Radius	log radius	Velocity	log. velocity	Z
0.366	9.563481	3.02	0.480007	
0.503	9.701568	3.72	0.570543	0.655
0.605	9.781755	4.16	0.619093	0.605
0.682	9.833784	4.60	0.662758	0.730
0.750	9.875061	4.87	0.687529	0.600
0.809	9.907449	5.12	0.709270	0.672
0.867	9.938019	5.29	0.723456	0.465
0.915	9.961421	5.51	0.741152	0.754

Radius	log Radius	Velocity	log. Velocity	Z
0.949	9.977266	5.75	0.759668	
0.168	9.225309	3.34	0.523746	
0.251	9.399674	4.39	0.642465	0.680
0.322	9.507856	5.04	0.702431	0.434
0.375	9.574031	5.68	0.754348	0.784
0.430	9.633468	6.08	0.783904	0.498
0.474	9.675778	6.51	0.813581	0.702
0.518	9.714330	6.83	0.834421	0.531
0.558	9.746634	7.12	0.852480	0.560
0.595	9.774517	7.41	0.869818	0.621
0.632	9.800717	7.63	0.882525	0.578
0.665	9.822822	7.86	0.895423	0.583
0.696	9.842609	8.07	0.906874	
			Average	0.610

The preceding method as outlined and shown was used for finding values of the exponent "z" for various cases.

The results of this work is shown in the following tabulation:

Surface of channels	Value of coefficient "z"
Neat cement (semicircular)	0.610
Neat cement (rectangular)	0.610
Planed boards (semicircular)	0.750
Planed boards (rectangular)	0.750
Unplaned boards	0.675
Brick masonry (smooth)	0.612
Brick masonry (rough)	0.760
Small gravel in cement (semicircular)	0.730
Small " " " (rectangular)	0.600
Large gravel in cement	0.750
Rubble masonry (clean)	0.750
Rubble masonry (dirty)	0.750
Earth, masonry sidewalls	0.700
Small Rivers (regular)	0.650
Irregular Rivers (rough)	0.750

More than 350 values of the coefficient "z" were computed and the mean value found was "Z" = 0.7512. From a comparison of

4. With increase of inclination when R is less than 3.28 feet.

The following is a table of values of "n" as assigned to different surfaces:

N = 0.009 for well planed timber

N = 0.010 for neat cement

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Bazin #24		$\frac{1}{c}$	.0077	.0073	.0072	.0069	.0068	.0067	.0068	.0067	.006
Neat Cement	0.0049	$\frac{1}{\sqrt{R}}$	2.44	1.99	1.76	1.63	1.52	1.45	1.39	1.34	1.29
Bazin #2		$\frac{1}{c}$	.0086	.008	.0079	.0076	.0075	.0074	.0074	.0073	.0073
Smooth Boards	0.0081	$\frac{1}{\sqrt{R}}$	5.13	4.66	4.26	3.86	3.58	3.13			
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Smooth Boards	0.0047	$\frac{1}{\sqrt{R}}$	5.87	4.38	3.89	3.65	3.45	3.32	3.28		
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Bazin #25		$\frac{1}{c}$	.0083	.0082	.0079	.0075	.0074	.0074	.0073	.0072	.0071

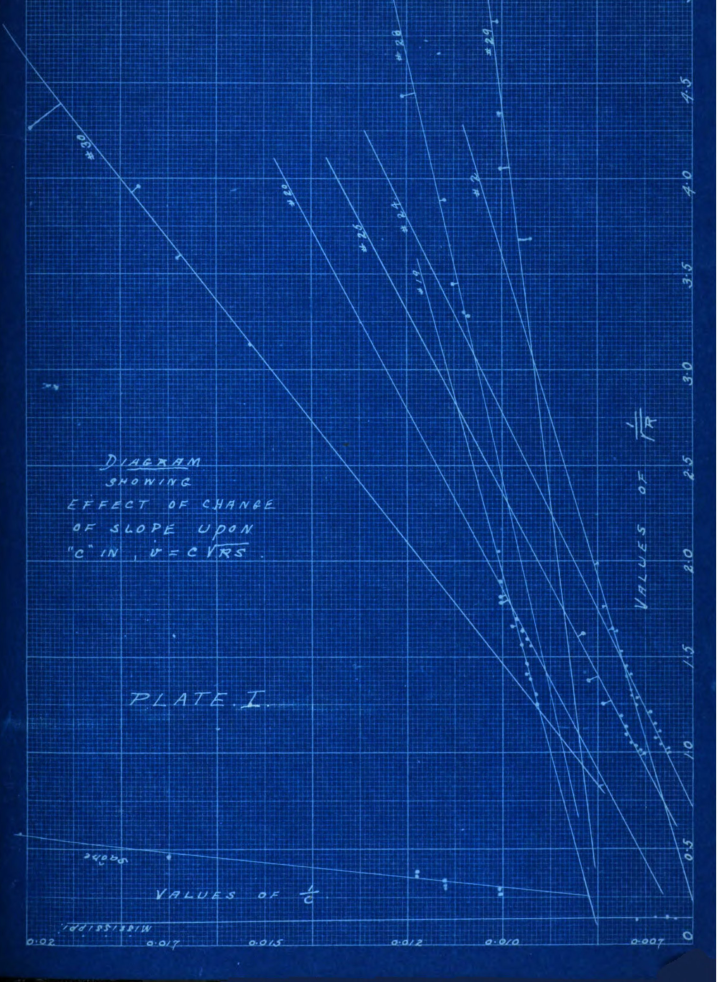


DIAGRAM
SHOWING
EFFECT OF CHANGE
OF SLOPE UPON
"C" IN , v = C√RS
PLATE I.
VALUES OF 1/C
VALUES OF 1/√R
#30
#20
#25
#24
#28
#29
#2
#10
0.02
0.017
0.015
0.012
0.010
0.007
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5

Experiment	Slope										
Unplaned Boards	0.006	$\frac{1}{R}$	2.05	1.89	1.81	1.78	1.70	1.64	1.59	1.56	1.52
Bazin #20		$\frac{1}{C}$	.0106	.0103	.0102	.0103	.0097	.0096	.0095	.0094	.0093
Unplaned Boards	0.0043	$\frac{1}{R}$	2.16	1.83	1.66	1.56	1.47	1.42	1.37	1.33	1.27
Bazin #19		$\frac{1}{C}$	.0103	.0101	.0098	.0096	.0095	.0095	.0094	.0093	.0093
Mississippi River	0.0000112	$\frac{1}{R}$	1.29	1.315	1.305	1.300					
		$\frac{1}{C}$	.0072	.0068	.0065	.0063					
Saone River	0.00004	$\frac{1}{R}$	.507	.457	.376	.354	.303	.294	.291	.274	.261
		$\frac{1}{C}$	.0222	.0170	.0118	.0112	.0112	.0102	.0102	.0102	.0102

Thrupp's Formula for the Flow of Water.

This formula has been derived from a wide range of experimental data and it is said to be the best of the known formulae by many hydraulic engineers. It is a modification of Hagen's formula and was published in a paper read before the Society of Engineers in England in 1887.

Notation.

V = velocity of flow in feet per second.

R = hydraulic radius.

L.= length of channel.

H = fall in length "L"

S = cosecant of angle of slope $(\frac{L}{H})$

X, O, N, are coefficients depending upon the nature of the surface.

Then $V = \dfrac{R^x}{O\sqrt[N]{S}}$.

For small values of "R" more accurate results are obtained by substituting for the index "x" the value $\left(X + Y\sqrt{\dfrac{Z - R}{R}}\right)$.

The following table is used in connection with this formula.

Thrupp's Formula.

Surface	N	O	X	Y	Z
Wrought iron pipe	1.80	0.004787	0.65	0.018	0.07
Riveted sheet "	1.825	0.005674	0.677	———	———
New Cast Pipe	{1.85 {2.00	{0.005347 {0.006752	{0.67 {0.63	———	———
Lead Pipe	1.75	0.005224	0.62	———	———
Pure Cement	{1.74 {1.95	{0.00400 {0.006429	{0.67 {0.61	———	———
Brickwork(smooth)	2.00	0.007746	0.61	0.01224	0.50
Brickwork(rough)	2.00	0.008845	0.625	0.01224	0.50
Unplained boards	2.00	0.008451	0.615	0.03349	0.50
Small gravel in cement	2.00	0.01181	0.66	0.05958	0.60
Large gravel in cement	2.00	0.01415	0.705	0.07590	1.00
Hammered masonry	2.00	0.01117	0.66	0.07620	1.00
Earth (no vegetation)	2.00	0.01536	0.72	———	———
Earth (stoney, rough)	2.00	0.02144	0.78	———	———

Thrupp recognizes the fact that the velocity varies as different powers of "R" and "S" for different surfaces. The relation existing between Thrupp's "S" and that generally used is

Thrupp's "S" $= \dfrac{1}{S}$ as generally used.

Working with the same idea that "V" varies differently for various surfaces, a number of coefficients for "S" and "R" have been derived and a comparison is submitted.

Surface	Coefficients for Radius		Coefficients for Slo[pe]	
	Thrupp	Urquhart	Thrupp	Urquhart
Pure Cement	{0.67 {0.61	0.61	{1.74 {1.95	1.72
Brickwork (smooth)	0.61	0.612	2.00	2.00
Brickwork (rough)	0.625	0.76	2.00	2.00
Unplaned Boards	0.615	0.675	2.00	1.82
Small gravel in cement	0.66	{0.66 {0.73	2.00	2.60

Surface	Coefficients for Radius		Coefficients for Slope	
	Thrupp	Urquhart & Powell	Thrupp	Urquhart & Powell
Large gravel in cement	0.705	0.75	2.00	2.50
Hammered masonry	0.66	0.75	2.00	2.50
Earth(no vegetation)	0.72	0.65	2.00	2.00
Earth (rough, stony)	0.78	0.75	2.00	2.00

DERIVATION OF FORMULA

The derivation of a new general formula was now undertaken after a study of the various formula herein outlined and discussed. The first work to be undertaken was to find if possible how the velocity varies with the hydraulic radius. The velocities and radii as data taken with constant slope were used in the computation. The form of expression as decided upon was

$$\frac{V_1}{V_2} = \frac{R_1^z}{R_2^z}$$ and from this we have

that, $$Z = \frac{\log V_1 - \log V_2}{\log R_1 - \log R_2}.$$

Values of "Z" must be derived for various surfaces and shapes of conduit as was found in working this out. On the following page is given a sample of the work necessary. This includes, radius, logarithm of radius, velocity, logarithm of velocity, and computed value of "Z" for neat cement conduit or channel. Computation for "Z" in form $Z = \frac{\log V_1 - \log V_2}{\log R_1 - \log R_2}$ for neat cement:-

Radius	log radius	Velocity	log. velocity	Z
0.366	9.563481	3.02	0.480007	
0.503	9.701568	3.72	0.570543	0.655
0.605	9.781755	4.16	0.619093	0.605
0.682	9.833784	4.60	0.662758	0.730
0.750	9.875061	4.87	0.687529	0.600
0.809	9.907449	5.12	0.709270	0.672
0.867	9.938019	5.29	0.723456	0.465
0.915	9.961421	5.51	0.741152	0.754

Radius	log Radius	Velocity	log. Velocity	Z
0.949	9.977266	5.75	0.759668	
0.168	9.225309	3.34	0.523746	
0.251	9.399674	4.39	0.642465	0.680
0.322	9.507856	5.04	0.702431	0.434
0.375	9.574031	5.68	0.754348	0.784
0.430	9.633468	6.08	0.783904	0.498
0.474	9.675778	6.51	0.813581	0.702
0.518	9.714330	6.83	0.834421	0.551
0.558	9.746634	7.12	0.852480	0.560
0.595	9.774517	7.41	0.869818	0.621
0.632	9.800717	7.63	0.882525	0.578
0.665	9.822822	7.86	0.895423	0.583
0.696	9.842609	8.07	0.906874	
			Average	0.610

The preceding method as outlined and shown was used for finding values of the exponent "z" for various cases.

The results of this work is shown in the following tabluation:

Surface of channels	Value of coefficient "z"
Neat cement (semicircular)	0.610
Neat cement (rectangular)	0.610
Planed Boards (semicircular)	0.750
Planed Boards (rectangular)	0.750
Unplaned Boards	0.675
Brick Masonry (smooth)	0.612
Brick Masonry (rough)	0.760
Small gravel in cement (semicircular)	0.730
Small " " " (rectangular)	0.600
Large gravel in cement	0.750
Rubble Masonry (clean)	0.750
Rubble Masonry (dirty)	0.750
Earth, Masonry sidewalls	0.700
Small Rivers (regular)	0.650
Irregular Rivers (rough)	0.750

More than 350 values of the coefficient "z" were computed and the mean value found was "Z" = 0.7512. From a comparison of the values of the coefficient we find that for small channels it varies as the degree of roughness and also increases with increase in roughness of wetted perimeter.

THE COEFFICIENT "C" IN FORM $V = C\sqrt{RS}$

The loss of head in a channel due to resistances may be expressed by the formula, $H = f\,\frac{LV^2}{R2g}$ (where "H" is lost head, "f" a friction factor.

From this can be derived

$$f\frac{V^2}{2g} = \frac{HR}{L} = RS \quad \text{or} \quad V = \sqrt{\frac{2g}{f}}\sqrt{RS} = C\sqrt{RS}.$$

This is assuming that the resistances are proportional to the $\sqrt{R}$ but it has been found that they are more nearly proportioned to $R^{\frac{3}{4}}$ and formula, $V = p\,R^{\frac{3}{4}}\sqrt{S}$ will be used as a basis for further investigation. This can also be written, $V = p\,R^{\frac{1}{4}}\sqrt{RS}$ where the value of, "c" $= p\,R^{\frac{1}{4}}$. The liberty is taken of introducing into the formula for "c" a constant "m" which will vary with different degrees of roughness of wetted perimeter, and the formula as it now stands is, "c" $= p\sqrt[4]{R}(1+m)$.

A trial value of "p" is now assumed as 50 and the following data in metric measure used as a basis for further calculation.

$R = 1.0$ meters, $C = 112{\cdot}0$; $R = 0{\cdot}0208$ meters, $C = 80{\cdot}4$

now, $\frac{112{\cdot}0}{50\sqrt[4]{1}} = 2{\cdot}24$, hence $(1 + m) = 1 + 1{\cdot}24$

Also, $\frac{80{\cdot}4}{50\sqrt[4]{.0208}} = 4{\cdot}21$, hence $(1 + m) = 1 + 3{\cdot}21$

If we divide $\frac{1{\cdot}24}{3{\cdot}21} = 0{\cdot}386$ which equals $\sqrt[4]{.0208}$ and therefore "c" can be expressed by the following "c" $= 50\sqrt[4]{R}(1 + \frac{m}{\sqrt[4]{R}})$ for metric measure

and as

"c" $= 66\sqrt[4]{R}(1 + \frac{m}{\sqrt[4]{R}})$ for English measure.

From this we can find a value for "m" which is , $\dfrac{\dfrac{C}{66\sqrt[4]{R}} - 1\cdot 0}{\dfrac{1}{\sqrt[4]{R}}}$ the tangent when values of "$\left(\dfrac{C}{66\sqrt[4]{R}}\right)$" and $\dfrac{1}{\sqrt[4]{R}}$ are plotted.

The 350 gaugings used in the computation of the variation of "R" were used in finding values of "m" for the various surfaces of the channels. A sample page of this computation is inserted here showing gauged values of "R", computed values of "C" from $V = C\sqrt{RS}$, the "$\sqrt[4]{R}$," the reciprocal of "$\sqrt[4]{R}$", "C" divided by $(p\sqrt[4]{R})$ and the values of "m" with the final average.

Following this computation is a table giving all values of "m" as derived for the various surfaces.

Computation of "m" $= \dfrac{\dfrac{C}{66\sqrt[4]{R}} - 1\cdot 0}{\dfrac{1}{\sqrt[4]{R}}}$ for (Neat Cement)

p	C	R	$\sqrt[4]{R}$	$\frac{1}{\sqrt[4]{R}}$	$\frac{C}{66\sqrt[4]{R}}$	m
66	138.0	0.605	0.882	1.132	2.366	1.208
	146.2	1.863	1.170	0.372	1.890	1.360
	128.9	0.366	0.778	1.285	2.505	1.172
	155.1	1.034	1.008	0.991	2.330	1.341
	152.5	0.949	0.988	1.012	2.340	1.325
	120.5	0.379	0.785	1.273	2.320	1.035
	116.5	0.168	0.640	1.562	2.675	1.200
	140.1	0.985	0.997	1.002	2.128	1.215
	131.3	0.787	0.942	1.060	2.113	1.051
	135.3	0.839	0.957	1.045	2.140	1.090
	141.7	1.022	1.005	0.995	2.135	1.140
	125.1	0.251	0.709	1.410	2.670	1.165
	126.9	0.322	0.754	1.326	2.545	1.165
	132.4	0.430	0.810	1.235	2.474	1.194
	135.5	0.518	0.850	1.177	2.415	1.203
	137.2	0.632	0.892	1.120	2.330	1.187
	137.8	0.665	0.905	1.105	2.310	1.185
	138.2	0.696	0.915	1.094	2.285	1.175
	147.9	2.048	1.195	0.836	1.872	1.040
	153.6	2.111	1.205	0.830	1.930	1.120
					Average	1.152

Derived values of "m" for all surfaces used in this work.

Surface	The Value of "m"
Neat Cement (semicircular)	1.155
Neat Cement (rectangular)	1.150
Planed Boards (semicircular)	0.925
Planed Boards (rectangular)	0.920
Unplaned Boards	0.866
Brick Masonry (smooth)	0.820
Brick Masonry (rough)	0.800
Small Gravel in cement (semicircular)	0.375
" " " " (rectangular)	0.290
Large Gravel in cement	0.250
Rubble Masonry (clean)	0.100
Rubble Masonry (dirty)	-0.100
Earth, Masonry walls	-0.220
Small Rivers regular	-0.240
Irregular rivers	-0.420

The effect of wrong selection of the value of "m" does not have a very great effect upon the computed velocity as derived from the final formula.

The variation in velocity due to variation of the slope of channel.

After trying several methods it was found that if we designated the slope as S_1 with corresponding velocity V_1, and another by S_2 with velocity V_2, we obtained for each pair of two cases with equal values of R and the same surface material but different slopes the constant relation

$$\frac{V_1}{V_2} = \frac{S_1^x}{S_2^x}.$$

In other words, in all comparable cases with equal mean radii but with different slopes, the velocity varies as a certain power of the slope. About 200 cases were found where two gaugings could be combined with equal values of "R" and varying slopes and

from those the powers to which the slopes must be raised were deduced. The form used was

$$x = \frac{\log V_1 - \log V_2}{\log S_1 - \log S_2}$$ and the values

are given in a table following.

Variation of Slope.

Surface	Coefficient "x"
Neat cement (semicircular)	0.58
Neat cement (rectangular)	0.58
Planed Boards (semicircular)	0.57
Planed Boards (rectangular)	0.57
Unplaned Boards	0.55
Brick Masonry (smooth)	0.50
Brick " (rough)	0.50
Small Gravel (semicircular)	0.385
Small Gravel (rectangular)	0.385
Large Gravel	0.40
Rubble Masonry (clean)	0.68
Rubble " (dirty)	0.42
Earth, Masonry Walls	0.55
Small Rivers, regular	0.50
Irregular Rivers	0.50

The greatest uncertainty in any one is in the value for Rubble Masonry as there was very little data obtainable.

The form of the general formula has now changed from

1. $V = C\sqrt{RS}$, - to

2. $V = p R^{\frac{1}{4}}\sqrt{S}$ - to

3. $V = 66(\sqrt[4]{R} + m)\sqrt{S}\sqrt{R}$, - to

4. $V = 66(R^y + m)\sqrt{R}\sqrt{S}$, - where $y = (Z - g)$ - to -

5. $V = 66(R^y + m)\sqrt{R}\, S^x$,

Now after determining the coefficients to be applied to R and S, it is very evident that the value 66 is not a constant, so that it now devolves upon us to find out what should be put in its place.

The formula must now be expressed as $V = C(R^y + m)\sqrt{R}\,S^x$ where "C" is either a constant for each kind of surface or a variable, if a variable how does it vary? The method used was to compute the value of "C" for each kind of surface using the values obtained by former work forthe coefficients of "R" and "S" and also value of "m",

$$C = \frac{\text{mean velocity}}{(R^y + m)\sqrt{R}\,S^x}.$$

Over 300 values of "C" were computed and it was found that in every case "C" was a variable. The problem now was to find how "C" varied and after several trials the form decided upon was, $C = (a + bR)$. This seemed to fit the case quite well and the most probable values for each case were computed by means of Least Squares.

The observation equation is

$c = (a + bR)$ and thenumber of equations for each set or surface is designated by "n".

The normal equations are

I. $C = a + bR$

II. $RC = aR + bR^2$

but as there are "n" observation equations they now have the form

I. $\Sigma C = na + b\Sigma R$

II. $\Sigma RC = a\Sigma R + b\Sigma R^2$

in order to solve for the constants "a" and "b" we must multiply

#1 by ΣR giving ; $\Sigma R\,\Sigma C = n\Sigma Ra - b\Sigma R\,\Sigma R$

#2 by n giving; $n\Sigma RC = n\Sigma Ra - nb\Sigma R^2$

#3 therefore, $b = \dfrac{n\Sigma RC - \Sigma R \Sigma C}{n\Sigma R^2 - \Sigma R \Sigma R}$.

#4 and $a = \dfrac{\Sigma C - b\Sigma R}{n}$.

This made it necessary to compute values for ΣR, $\Sigma R \Sigma R$, ΣC, ΣR^2, ΣRC and $\Sigma R \Sigma C$ and solve equations #3 and #4 for "a" and "b".

A sample page of this computation is given, also work for solving equations for "a" and "b" for the case of "Unplaned Boards."

Solving for c in $V = C(R^{.175} + .866)\, S^{0.55} \sqrt{R}$

$C = (a + bR)$.

Unplaned Boards.

R	$R^{0.175}$	$R^{0.175} + .866$	S	$S^{0.55}$	$\sqrt{R}$	V	C	
.214	.7633	1.6295	.0043	.04994	.4626	2.85	75.71	
2.99	.8095	1.6755	"	"	.5468	3.47	75.84	
.412	.8563	1.7223	"	"	.6418	4.54	82.24	
.499	.8854	1.7514	"	"	.7064	5.12	82.86	
.618	.9192	1.7852	"	"	.7861	5.92	84.47	
.700	.9395	1.8055	"	"	.8366	6.48	90.67	
.314	.8284	1.6944	.0049	.05365	.5839	4.43	83.27	
.498	.8852	1.7512	"	"	.7056	5.54	83.57	
.612	.9176	1.7836	"	"	.7823	6.26	83.62	
.703	.9402	1.8062	"	"	.8384	6.76	83.21	
.808	.9633	1.8293	"	"	'8988	7.42	84.12	
.500	.8858	1.7158	.00839	.07212	.7071	7.57	84.63	
.621	.9200	1.7860	"	"	.7880	8.74	83.15	
.255	.7873	1.6533	.0059	.05942	.5049	3.98	80.24	
.623	.9205	1.7865	"	"	.7893	7.24	86.83	
.686	.9405	1.8065	"	"	.8282	7.71	86.73	
.341	.8284	1.6944	.00824	.07141	.5839	5.83	82.52	
.466	.8749	1.7409	"	"	.6826	7.17	84.49	
.506	.8876	1.7536	"	"	.7113	7.44	83.52	
.630	.9223	1.7883	"	"	.7937	8.57	83.15	

Solving for "a" and "b"

in

$$b = \frac{n\Sigma RO - \Sigma R\Sigma O}{n\Sigma R^2 - \Sigma R\Sigma R} = \frac{155.309}{10.4004} = 14.9$$

$$a = \frac{\Sigma O - b\Sigma R}{20} = \frac{1512.9}{20} = 75.64$$

$n = 20$

$\Sigma R = 10.332$

$\Sigma R\Sigma R = 106.7502$

$\Sigma O = 1666.84$

$\Sigma R^2 = 5.857532$

$\Sigma RO = 868.855$

$\Sigma R\Sigma O = 17221.7908$

R^2	RO
0.045796	16.20194
0.089401	22.67616
0.169744	33.88268
0.249001	41.34714
0.381924	52.20246
0.490000	63.46900
0.116281	28.39507
0.248004	41.61786
0.374544	51.17544
0.494209	58.49663
0.652864	67.96896
0.250000	42.31500
0.385641	51.63615
0.065025	20.46120
0.388129	54.09509
0.470596	54.49678
0.116281	28.13932
0.217156	39.37234
0.256036	42.26112
0.396900	53.64450

The method just outlined was used to find the following constants.

Surface	a	b
Neat Cement (semicircular)	130.0	20.0
Neat Cement (rectangular)	92.7	13.0
Planed Boards (semicircular)	99.8	9.0
Planed Boards (rectangular)	90.0	18.0
Unplaned Boards	75.6	14.9
Brick Masonry (smooth)	52.5	15.4
Brick Masonry (rough)	41.3	7.5
Small Gravel in cement (semicircular)	31.3	2.9
Small " " " (rectangular)	28.2	8.7
Large " " "	29.5	4.1
Ribble Masonry (clean)	153.2	16.5
Rubble Masonry (dirty)	21.0	14.6
Earth Masonry Walls	79.4	2.9
Small Rivers (regular)	71.5	1.4
Irregular Rivers	71.5	1.3

The final form of the formula as derived now is

$$V = (a + bR)(R^y + m)\sqrt{R}\, S^x$$ in which

V = mean velocity

R = hydraulic radius

S = slope

a,b,m,x,y, are constants for different classes of surfaces and are all found in table I on page(   ) of this volume.

The meanvelocity can be computed by this formula with use of tables II and III with comparative ease and rapidity.

No claim is made for the accuracy of this formula other than is shown from the comparison that follows but we find that it gives results as accurate and sometimes more accurate than Kutter's formula. Lack of time alone has prevented a more thorough investigation of the laws of the flow of water in channels.

COMPARISON OF FORMULA.

In order to show in how far the general formula as derived in this work accords with the results obtained by the use of the formulas of Bazin and Kutter, the following comparisons are made. The values of "n" used in computation of velocity by Kutter's formula were the means of values of "n" as found by solving Kutter's formula for "n". This gives Kutter an undue advantage, because in applying his formula to an unknown channel a guess must be made for "n". M Bazin's values for his four categories were used in computing velocity by his formula.

A series of columns of differences show how much the computed values differ from the observed and the error in per cent is given in another series of columns. A mean value of the percentage of error is taken in each series. Symbols at head of columns are; B, for Bazin; G & K, for Ganguillet and Kutter; and U & P, for Urquhart and Powell.

Comparison of Formulae.

Basin 24

		Velocity				Difference			Error Percent		
R	S	Mea-sured	B	G&K	U&P	B	G&K	U&P	B	G&K	U&P
.366	0.0014	3.021	2.998	2.982	3.049	0.023	0.039	0.028	0.76	1.29	0.92
.503		3.723	3.618	3.723	3.721	0.105	0.000	.002	2.82	0.00	0.01
.605		4.156	4.032	4.229	4.193	0.124	0.073	.037	2.98	1.76	0.88
.682		4.596	4.432	4.583	4.538	0.164	0.013	.058	3.57	0.28	1.28
.750		4.865	4.547	4.882	4.826	0.318	0.017	.029	6.55	0.35	0.60
.809		5.124	4.740	5.134	5.090	0.384	0.010	.034	7.50	0.19	0.66
.867		5.288	4.927	5.347	5.342	0.361	0.059	.054	6.82	1.11	1.02
.915		5.515	5.075	5.570	5.556	0.440	0.055	.041	7.97	0.99	0.75
.949		5.754	5.173	5.708	5.697	0.581	0.046	.054	10.10	0.80	0.99
.992		5.915	5.305	5.879	5.874	0.610	0.037	.041	10.32	0.62	0.69
1.029		6.059	5.410	6.020	6.047	0.649	0.039	.012	10.70	0.64	0.20
1.034		6.109	5.423	6.040	6.072	0.686	0.069	.037	11.22	1.13	0.61
									6.78	0.76	0.71

Basin No.2

R	S	Mea-sured	B	G&K	U&P	B	G&K	U&P	B	G&K	U&P
.291	0.0049	4.39	4.38	4.07	4.37	0.01	0.32	.02	0.23	7.29	0.45
.332		5.04	5.13	4.67	5.11	0.09	0.17	.07	1.78	3.37	1.39
.375		5.68	5.63	5.42	5.60	0.05	0.26	.08	0.88	4.57	1.41
.430		6.08	6.11	5.98	6.08	0.03	0.10	.00	0.49	1.65	0.00
.474		6.51	6.46	6.40	6.44	0.05	0.11	.07	0.77	1.69	1.07
.518		6.83	6.81	6.81	6.81	0.02	0.02	.02	0.29	0.29	0.29
.558		7.12	7.11	7.16	7.13	0.01	0.04	.01	0.14	0.56	0.14
.595		7.41	7.38	7.49	7.42	0.03	0.08	.01	0.41	1.08	0.13
.632		7.63	7.64	7.80	7.70	0.01	0.17	.07	0.13	2.23	0.92
.665		7.86	7.85	7.99	7.96	0.01	0.18	0.10	0.12	2.26	1.27
									0.52	2.59	0.71

Comparison
Bazin 26

R	S	Velocity Measured	Velocity B	Velocity G&K	Velocity U&P	Differences B	Differences G&K	Differences U&P	Percent of Error B	Percent of Error G&K	Percent of Error U&P
.390	.001424	2.61	2.55	2.58	2.64	0.06	0.03	0.03	2.30	1.15	1.1
.537		3.23	3.14	3.24	3.25	0.09	0.01	0.02	2.79	0.31	0.6
.652		3.71	3.49	3.64	3.63	0.22	0.07	0.08	5.93	1.89	2.1
.717		4.04	3.78	3.98	3.96	0.26	0.06	0.08	6.44	1.48	1.9
.796		4.25	4.03	4.27	4.25	0.22	0.02	0.00	5.17	0.47	0.0
.856		4.51	4.22	4.48	4.48	0.29	0.03	0.03	6.42	0.66	0.6
.921		4.64	4.41	4.72	4.72	0.23	0.08	0.08	4.96	1.76	1.7
.964		4.87	4.52	4.86	4.87	0.35	0.01	0.00	7.18	0.21	0.0
1.015		5.00	4.67	5.03	5.04	0.43	0.03	0.04	8.61	0.60	0.8
1.054		5.18	4.77	5.16	5.19	0.41	0.02	0.01	7.90	0.38	0.1
									5.77	0.89	0.9

Bazin 29

R	S	Velocity Measured	Velocity B	Velocity G&K	Velocity U&P	Differences B	Differences G&K	Differences U&P	Percent of Error B	Percent of Error G&K	Percent of Error U&P
.030	.0152	1.87	1.53	1.64	1.94	0.34	0.23	0.07	18.15	12.30	3.7
.043		2.30	2.08	2.21	2.38	0.22	0.09	0.08	9.56	3.92	3.4
.053		2.68	2.48	2.43	2.57	0.20	0.25	0.11	7.46	9.33	4.1
.061		3.00	2.78	2.94	2.94	0.22	0.06	0.06	7.34	2.00	2.0
.074		3.56	3.24	3.42	3.30	0.32	0.14	0.26	9.00	3.93	7.3
									10.30	6.29	4.1

Bazin 28

R	S	Velocity Measured	Velocity B	Velocity G&K	Velocity U&P	Differences B	Differences G&K	Differences U&P	Percent of Error B	Percent of Error G&K	Percent of Error U&P
.029	.0047	0.90	1.01	0.87	0.97	0.11	0.03	0.07	12.21	3.00	7.7
.052		1.30	1.36	1.43	1.37	0.06	0.13	0.07	4.62	10.00	5.3
.066		1.58	1.62	1.73	1.58	0.04	0.15	0.00	2.53	9.50	0.0
.075		1.74	1.80	1.91	1.71	0.06	0.17	0.03	3.45	9.77	1.7
.084		1.94	1.98	2.07	1.84	0.04	0.13	0.10	2.06	6.70	5.1
									4.97	7.79	4.0

Comparison

Bazin 6

R	S	Velocity				Differences			Per. Error		
		Measured	B	G&K	U&P	B	G&K	U&P	B	G&K	U&P
.240	.00221	2.08	2.16	1.98	2.18	0.08	0.10	0.10	3.85	4.81	4.81
.363		2.69	2.91	2.68	2.85	0.22	0.01	0.16	8.18	0.37	5.95
.453		3.16	3.38	3.16	3.30	0.22	0.00	0.14	6.96	0.00	4.43
.528		3.53	3.74	3.54	3.67	0.21	0.01	0.14	5.95	0.29	3.96
.601		3.78	4.07	3.86	4.00	0.29	0.06	0.22	7.68	2.12	5.82
.648		4.13	4.28	4.08	4.21	0.15	0.05	0.08	3.63	1.21	1.94
.704		4.34	4.51	4.33	4.47	0.17	0.01	0.13	3.92	0.23	2.98
.759		4.51	4.72	4.55	4.72	0.21	0.04	0.21	4.66	0.89	4.66
.801		4.72	4.88	4.72	4.91	0.16	0.00	0.19	3.39	0.00	4.03
.846		4.88	5.04	4.81	5.10	0.16	0.07	0.22	3.28	1.43	4.50
									5.15	1.13	4.30

Bazin 7

.272	.0049	3.697	3.533	3.585	3.703	0.164	0.112	0.006	4.44	3.03	0.15
.342		4.347	4.146	4.235	4.286	0.201	0.112	0.061	4.62	2.58	1.41
.402		4.852	4.655	4.757	4.767	0.197	0.095	0.085	4.06	1.96	1.75
.453		5.288	4.973	5.193	5.162	0.315	0.095	0.126	5.95	1.79	2.38
.504		5.613	5.403	5.590	5.548	0.210	0.023	0.065	3.74	0.41	1.16
.547		5.931	5.705	5.931	5.877	0.226	0.000	0.054	3.81	0.00	0.91
.587		6.226	5.964	6.226	6.154	0.262	0.000	0.072	4.28	1.00	1.17
.628		6.453	6.220	6.532	6.454	0.233	0.079	0.001	3.60	1.23	0.01
.662		6.709	6.430	6.709	6.699	0.279	0.000	0.010	4.16	0.00	0.15
.698		6.896	6.666	7.027	6.957	0.230	0.131	0.061	3.34	1.90	0.88
									4.20	1.29	0.99

Bazin 8

.147	.008163	3.523	2.933	3.077	3.347	0.590	0.446	0.176	16.75	12.65	5.0
.231		4.422	4.032	4.314	4.420	0.390	0.108	0.002	8.82	2.44	0.0
.289		5.229	4.780	5.104	5.099	0.449	0.125	0.130	8.60	2.44	2.4
.341		5.826	5.354	5.767	5.671	0.472	0.059	0.155	8.10	1.01	2.6
.393		6.240	5.598	6.377	6.224	0.322	0.137	0.016	5.16	2.19	0.2
.431		6.735	6.295	6.745	6.610	0.440	0.010	0.125	6.54	0.15	1.8
.466		7.171	6.633	7.204	6.969	0.538	0.033	0.202	7.51	0.45	2.8
.506		7.440	7.007	7.637	7.366	0.433	0.197	0.074	5.82	2.65	0.9
.541		7.732	7.319	8.005	7.719	0.413	0.273	0.013	5.34	3.53	0.1
.572		8.028	7.585	8.638	8.018	0.443	0.610	0.010	5.51	7.60	0.12
									8.81	3.51	1.64

Bazin 18

R	S	Measured	B	G&K	M&P	B	G&K	M&P	B	G&K	M&P
			Velocity			Difference			Percent error		
.341	.0049	4.43	4.15	4.13	4.28	0.28	0.30	0.15	6.31	6.76	3.38
.428		5.05	4.86	4.87	4.97	0.19	0.18	0.08	3.76	3.56	1.58
.498		5.54	5.37	5.44	5.50	0.17	0.10	0.04	3.07	1.81	0.72
.558		5.94	5.78	5.90	5.95	0.16	0.04	0.01	2.70	0.67	0.17
.612		6.26	6.14	6.29	6.34	0.12	0.03	0.08	1.92	0.48	1.28
.661		6.50	6.45	6.63	6.69	0.05	0.13	0.19	0.77	2.00	2.82
.703		6.76	6.70	6.81	6.99	0.06	0.05	0.23	0.89	0.74	3.40
.741		7.00	6.92	7.18	7.25	0.08	0.18	0.25	1.14	2.57	3.57
.777		7.20	7.12	7.41	7.49	0.08	0.21	0.29	1.11	2.92	4.03
.808		7.42	7.30	7.62	7.70	0.12	0.20	0.32	1.62	2.70	4.20
									2.33	2.42	2.51

Bazin 3.

.284	.0050	3.664	3.687	3.379	3.625	0.023	0.285	0.039	0.63	7.82	1.06
.365		4.179	4.563	4.074	4.261	0.304	0.095	0.102	9.20	2.28	2.42
.424		4.724	4.878	4.547	4.705	0.154	0.177	0.019	3.26	3.75	0.40
.481		5.101	5.308	4.986	5.124	0.207	0.115	0.023	4.06	2.26	0.45
.540		5.334	5.738	5.416	5.550	0.404	0.062	0.216	7.57	1.54	4.05
.582		5.679	6.016	5.711	5.848	0.337	0.032	0.169	5.84	0.57	2.97
.620		6.007	6.256	5.974	6.119	0.249	0.033	0.112	4.15	0.55	1.87
.668		6.148	6.558	6.286	6.457	0.410	0.138	0.309	6.67	2.24	5.02
.697		6.473	6.742	6.482	6.657	0.269	0.009	0.184	4.15	0.14	2.84
.739		6.600	6.984	6.742	6.950	0.384	0.142	0.350	5.81	2.15	5.30
									5.13	2.33	2.63

Bazin 39

0.41	.0081	5.728	6.026	5.610	5.850	0.298	0.118	0.122	5.20	2.06	2.13
.57		7.522	7.542	7.158	7.330	0.020	0.364	0.192	0.27	4.84	2.55
.68		8.185	8.431	8.097	8.300	0.246	0.088	0.115	3.01	1.07	1.41
.77		8.746	9.077	8.792	9.100	0.331	0.046	0.354	3.78	0.53	4.04
									3.06	2.12	2.53

Bazin 32 Rough Masonry Brick

.324	.10076	12.293	11.299	11.955	12.280	0.994	0.338	0.013	12.21	2.75	[illegible]
.467		16.177	15.327	15.839	15.800	0.850	0.338	0.377	5.25	2.09	[illegible]
.580		18.680	18.162	18.556	18.550	0.518	0.124	0.130	2.78	0.66	[illegible]
.662		21.092	20.098	20.445	20.400	0.994	0.547	0.692	4.70	2.59	[illegible]
									6.23	2.00	[illegible]

Basin 33

R	S	Velocity: Measured	Velocity: B	Velocity: G&K	Velocity: U&P	Difference: B	Difference: G&K	Difference: U&P	Error percent: B	Error percent: G&K	Error percent: U&P
.424	.0036856	9.045	8.562	8.881	8.905	0.483	0.164	0.140	5.35	1.81	1.55
.620		11.463	11.577	11.771	11.713	0.114	0.308	0.250	0.99	2.69	2.18
.745		13.552	13.241	13.480	13.447	0.311	0.072	0.105	2.30	0.53	0.77
.852		15.075	14.796	14.862	14.918	0.279	0.213	0.157	11.85	1.41	1.04
									2.62	1.61	1.30

Fteley & Stearns

SMOOTH BRICK.

R	S	Velocity: Measured	Velocity: B	Velocity: G&K	Velocity: U&P	Difference: B	Difference: G&K	Difference: U&P	Error percent: B	Error percent: G&K	Error percent: U&P
1.016	.000014	.443	.286	.510	.468	0.157	0.067	0.025	35.45	15.15	5.64
0.858	.0000246	.550	.381	.321	.543	0.169	0.229	0.007	30.70	41.60	1.27
1.008	.0000383	.789	.726	.480	.768	0.053	0.309	0.021	6.71	39.20	2.66
0.957	.0000746	1.064	.999	1.022	1.033	0.065	0.042	0.031	6.11	3.94	2.91
0.778	.0000983	1.098	1.010	1.134	1.012	0.088	0.036	0.086	8.01	3.28	7.84
0.850	.0001115	1241	1.135	1.241	1.150	0.106	0.000	0.091	8.54	0.00	7.34
0.577	.0001396	1.149	1.066	1.120	1.036	0.083	0.029	0.113	7.22	2.52	9.83
0.673	.0001633	1.298	1.188	1.315	1.171	0.110	0.017	0.127	8.47	1.31	9.80
0.493	.000164	1.079	.975	1.051	.942	0.104	0.028	0.137	9.65	2.60	12.70
0.891	.000170	1.569	1.442	1.645	1.474	0.127	0.076	0.095	8.10	4.85	6.05
									12.89	11.44	7.20

Bazin 27

Small Gravel in Cement (Semicircular)

R	S	Velocity: Measured	Velocity: B	Velocity: G&K	Velocity: U&P	Difference: B	Difference: G&K	Difference: U&P	Error percent: B	Error percent: G&K	Error percent: U&P
.454	.0025	2.17	2.62	1.93	2.173	0.45	0.24	0.003	20.70	11.06	0.13
.546		2.50	3.16	2.22	2.474	0.66	0.06	0.026	26.40	2.50	1.04
.619		2.69	3.42	2.43	2.706	0.73	0.26	0.016	26.20	9.66	0.59
.681		2.93	3.63	2.61	2.898	0.70	0.32	0.032	23.90	10.92	1.09
.731		3.05	3.79	2.75	3.050	0.74	0.30	0.000	24.30	9.84	0.00
.784		3.22	3.96	2.90	3.210	0.74	0.32	0.010	23.00	9.94	0.31
.826		3.33	4.08	3.01	3.370	0.71	0.32	0.040	21.32	9.60	1.20
.900		3.54	4.32	3.19	3.554	0.78	0.35	0.014	22.00	9.90	0.39
.968		3.73	4.48	3.38	3.749	0.75	0.35	0.019	20.10	9.40	0.51
1.012		3.95	4.62	3.47	3.881	0.67	0.28	0.069	16.98	7.08	1.75
									22.49	8.99	0.70

Bazin 27
Small Gravel in Cement (Rectangular)

R	S	Velocity Mea-sured	Velocity B	Velocity G&K	Velocity U&P	Difference B	Difference G&K	Difference U&P	Percent Error B	Percent Error G&K	Percent Error U&P
.250	.0049	2.16	2.16	2.14	2.274	0.00	0.02	0.114	0.00	0.93	5.27
.357		2.95	2.84	2.82	2.876	0.11	0.13	0.074	3.73	4.41	2.51
.450		3.40	3.39	3.37	3.372	0.01	0.03	0.028	0.29	0.88	0.82
.520		3.84	3.83	3.76	3.734	0.01	0.08	0.106	0.26	2.08	2.76
.588		4.14	4.15	4.12	4.081	0.01	0.02	0.059	0.24	0.48	1.42
.644		4.43	4.42	4.41	4.362	0.01	0.03	0.068	0.23	0.45	1.53
.700		4.64	4.63	4.69	4.644	0.01	0.05	0.004	0.21	1.08	0.08
.746		4.88	4.87	4.90	4.875	0.01	0.02	0.005	0.20	0.41	0.10
.785		5.12	5.12	4.98	5.068	0.00	0.14	0.052	0.00	2.74	0.01
.832		5.26	5.16	5.30	5.224	0.01	0.04	0.036	1.90	0.76	0.68
									0.61	1.46	1.62

Bazin 5
Large Gravel in Cement.

R	S	Velocity Mea-sured	Velocity B	Velocity G&K	Velocity U&P	Difference B	Difference G&K	Difference U&P	Percent Error B	Percent Error G&K	Percent Error U&P
.510	.0049	2.90	3.67	2.865	2.941	0.77	0.035	0.041	26.60	1.21	1.41
.587		3.27	4.06	3.200	3.275	0.79	0.070	0.005	24.20	2.14	0.15
.656		3.56	4.39	3.630	3.570	0.83	0.070	0.010	23.30	1.97	0.35
.712		3.85	4.65	3.705	3.807	0.80	0.145	0.043	20.75	3.77	1.11
.772		4.03	4.90	3.942	4.057	0.87	0.088	0.027	21.60	2.18	0.67
.823		4.23	5.12	4.130	4.270	0.89	0.100	0.040	21.00	2.36	0.94
.867		4.43	5.23	4.297	4.454	0.90	0.133	0.024	20.30	3.00	0.54
.909		4.60	5.48	4.450	4.624	0.88	0.150	0.024	19.12	3.26	0.52
.946		4.78	5.71	4.580	4.779	0.93	0.200	0.001	19.45	4.18	0.02
.987		4.90	5.80	4.730	4.946	0.90	0.170	0.046	18.35	3.46	0.94
									21.46	2.75	0.66

Dry Rubble (clean)

R	S	Velocity Mea-sured	Velocity B	Velocity G&K	Velocity U&P	Difference B	Difference G&K	Difference U&P	Percent Error B	Percent Error G&K	Percent Error U&P
.213	.0045	1.324	1.082	1.228	1.446	0.242	0.096	0.122	18.30	7.25	9.20
.439		2.396	1.610	2.210	2.280	0.786	0.186	0.096	31.60	7.76	4.01
.486		2.432	1.690	2.410	2.665	0.742	0.012	0.233	30.50	0.49	9.58
.278	.0036	1.502	1.112	1.522	1.482	0.390	0.010	0.020	25.90	0.66	1.33
.351		1.928	1.270	1.838	1.770	0.658	0.090	0.158	33.20	4.68	8.20
.403		2.108	1.360	2.050	1.980	0.748	0.058	0.138	35.40	2.75	6.54
									29.1	4.00	6.47

Dirty Rubble, Grosbois Canal

R	S	Velocity Mea-sured	Velocity B	Velocity G&K	Velocity U&P	Difference B	Difference G&K	Difference U&P	Percent Error B	Percent Error G&K	Percent Error U&P
1.40	.000683	2.34	3.32	2.52	2.28	0.98	0.18	0.06	41.90	8.04	2.68
1.50	"	2.78	3.46	2.64	2.48	0.68	0.14	0.30	24.42	5.03	10.80
1.07	.0003	1.12	1.89	1.30	1.15	0.77	0.18	0.03	68.60	16.10	2.68
1.38	.00033	1.69	2.37	1.70	1.68	0.68	0.01	0.01	40.30	0.59	0.59

Dirty Rubble, Grosbois Canal, Cont'd.

R	S	Mea-sured	Velocity B	G&K	U&P	Difference B	G&K	U&P	Percent Error B	G&K	U&P
1.57	.00033	1.92	2.48	1.81	1.93	0.56	0.11	0.01	29.20	5.92	0.52
1.71	.0003	2.18	2.52	1.83	2.08	0.34	0.35	0.10	15.60	16.05	4.58
									54.00	6.00	3.64

Regular Channels Linth Canal.

5.14	.00029	3.414	3.126	3.441	3.158	0.288	0.027	0.256	8.70	0.79	
5.93	.00030	3.830	3.513	3.850	3.500	0.317	0.020	0.250	8.28	0.52	
6.48	.00031	4.152	3.841	4.192	3.930	0.311	0.040	0.222	7.47	0.96	
7.12	.00032	4.418	4.123	4.471	4.290	0.295	0.053	0.128	6.66	1.20	2.90
8.09	.00034	4.920	4.625	5.006	4.900	0.295	0.086	0.020	6.01	1.75	0.40
8.28	.00034	5.058	4.711	5.085	5.020	0.347	0.027	0.038	6.85	0.53	0.75
8.62	.00035	5.225	4.894	5.282	5.255	0.331	0.057	0.030	6.45	1.11	0.57
8.87	.00036	5.392	5.055	5.455	5.430	0.337	0.063	0.038	6.26	1.17	0.70
9.18	.00037	5.530	5.245	5.646	5.670	0.285	0.116	0.140	5.14	2.10	2.53
									5.34	1.11	1.12

Regular River. Sallaz.

n = .025

4.50	.000291	2.820	3.050	2.785	2.810	0.230	0.035	0.010	8.15	1.24	0.35
4.37	.000297	2.790	3.025	2.780	2.820	0.235	0.010	0.030	8.42	0.35	1.07
4.18	.000304	2.740	2.920	2.705	2.750	0.180	0.035	0.010	6.56	1.28	0.36
4.07	.000306	2.710	2.980	2.680	2.730	0.270	0.030	0.020	9.62	1.11	0.74
3.50	.00045	3.706	4.450	3.930	3.750	0.744	0.214	0.044	20.02	5.76	1.18
									10.59	1.92	0.74

Canal du Jard

1.68	.0000362	0.449	0.442	0.438	0.475	0.007	0.011	0.026	1.56	2.45	5.79
1.94	.0000362	0.479	0.496	0.492	0.537	0.017	0.013	0.058	3.75	2.71	12.12
2.05	.0000458	0.607	0.588	0.588	0.636	0.019	0.019	0.029	3.13	3.13	4.78
2.58	.0000651	1.069	0.845	0.852	0.890	0.224	0.207	0.079	21.00	19.40	7.40
									7.61	6.42	7.

Bayou Lafourche

		Velocity				Difference			Percent Error		
R	S	Measured	B	G&K	U&P	B	G&K	U&P	B	G&K	U&P
12.80	.0000365	2.807	2.200	2.864	2.340	0.607	0.057	0.467	21.60	2.03	16.60
13.04	.0000373	2.843	2.038	2.710	2.440	0.805	0.133	0.403	28.30	4.67	14.18
12.47	.0000438	2.789	2.086	2.780	2.520	0.703	0.009	0.269	25.20	0.32	9.65
15.71	.0000446	3.376	2.550	3.380	3.140	0.526	0.304	0.064	17.12	9.80	2.08
									23.05	4.20	10.60

Regular Rivers   Swarz Weser

5.96	0.000183	1.410	2.758	2.916	2.810	1.348	1.506	1.400	95.50	106.60	99.20
8.70	0.0001917	3.470	4.220	4.410	3.920	0.750	0.940	0.450	21.60	27.10	12.95
6.31	0.000398	4.087	4.310	4.515	4.350	0.223	0.428	0.263	5.46	10.50	6.44
6.75	0.000410	4.950	4.492	4.695	4.675	0.458	0.255	0.275	9.25	5.15	5.55
6.49	0.000411	5.182	3.635	3.825	4.530	1.547	1.357	0.652	29.85	26.18	12.60
9.44	0.0002	4.064	3.932	4.080	4.280	0.132	0.016	0.216	3.24	0.39	5.32
9.98	0.0002	4.389	4.055	0.275	4.470	0.334	0.114	0.081	7.61	2.60	1.84
10.52	0.000216	4.756	4.375	4.595	4.900	0.381	0.161	0.144	8.00	3.32	2.97
11.06	0.000216	5.186	4.515	4.730	5.080	0.671	0.456	0.106	12.95	8.80	2.04
12.61	0.00053	7.924	7.700	7.880	8.740	0.224	0.044	0.816	2.82	0.55	10.30
13.55	0.00055	7.902	8.100	8.300	9.640	0.098	0.398	1.738	1.23	5.03	21.90
									17.96	17.84	16.46

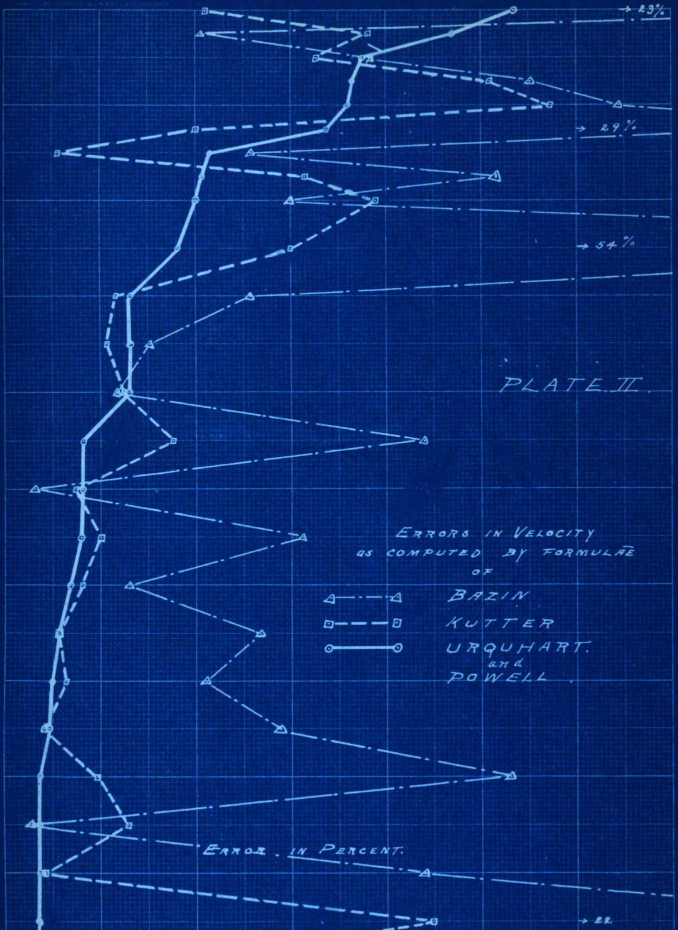
Regrular River   Seine @ Poissy.

7.10	.00009	2.310	2.200	2.040	2.252	0.110	0.270	0.058	4.78	11.68	2.51
7.68	.000087	2.313	2.260	2.120	2.360	0.053	0.213	0.047	2.29	9.22	2.02
11.24	.000057	2.362	2.345	2.310	2.580	0.017	0.052	0.218	0.72	2.22	9.24
12.43	.000060	2.359	2.563	2.532	2.920	0.204	0.173	0.561	8.65	7.33	23.78
									4.11	7.61	9.39

Irregular River La Nicca Rosa*

R	S	Velocity: Measured	Velocity: B	Velocity: GAK	Velocity: U&P	Difference: B	Difference: G&K	Difference: U&P	Percent Error: B	Percent Error: G&K	Percent Error: U&P
0.99	.011875	3.867	5.160	4.700	4.560	1.293	1.033	0.693	33.40	26.70	17.92
1.20	"	5.540	6.130	5.500	5.450	0.590	.040	0.100	10.62	0 .72	1.99
1.53	"	7.587	7.580	6.580	6.880	0.007	1.007	0.707	0.09	13.25	9.32
									11.03	10.17	7.31

1.25	.00965	6.002	5.750	5.950	5.120	0.252	0.052	0.882	4.20	0.86	14.66
2.33	"	9.988	9.810	9.160	9.130	0.178	0.828	0.858	1.79	8.30	8.60
3.48	"	10.194	13.420	12.420	13.160	3.226	2.226	2.966	31.70	21.80	29.30
3.58	"	13.579	14.520	12.670	13.450	0.941	0.901	0.129	6.84	6.70	0.09
3.59	"	13.943	13.820	12.710	13.480	0.123	1.233	0.463	0.88	8.85	3.33
									22.20	9.32	11.20


→ 23%.
→ 29%
→ 54%
PLATE II.
ERRORS IN VELOCITY
as COMPUTED BY FORMULAE
OF
BAZIN
KUTTER
URQUHART.
and
POWELL.
ERROR IN PERCENT.
→ 22

Explanation of Plate II.

The results of the comparison of twenty-five series of gaugings are shown by means of a diagram. The mean percentage of error is found from each series is plotted as abscissa. The results as derived from the three formula are plotted in the same horizontal line for each series and the series are at equal spaces vertically. The various points so plotted by connected by lines,

Derived formula, heavy full line
Kutter's " , dotted "
Bazin's " , dot anddash.

This method of demonstration was used as it was thought to make the results clearer than any other method.

DATA.

The data used in the derivation and computation of formula was selected from a great mass of experiments. Only such data as had apparently been taken with the greatest care being used. All information concerning the character of the bed, the number of experiments and methods of measurement being investigated whenever possible. Slope measurements were also taken with great care. The various methods of measurement were by measuring previously the quantity of water, Pitot tube, floats, and current meters. The data includes, authority, mean hydraulic radius, slope and mean velocity.

Data used in the derivation of formula.

Location and Description	Authority	Mean Hydraulic Radius. ft.	Slope	Mean Velocity. feet	Coefficient $c = \frac{V}{\sqrt{RS}}$
Test channel	"Darcy and Bazin	0.366	0.00150	3.02	128.9
Neat cement	#24	0.503	"	3.72	135.6
semicircular	"Recherches Hydrauliques"	0.605	"	4.16	138.0
		0.682	"	4.60	143.7
		0.750	"	4.87	145.1
		0.809	"	5.12	147.1
		0.867	"	5.29	146.7
		0.915	"	5.51	148.8
		0.949	"	5.75	152.5
		0.992	"	5.91	153.3
		1.029	"	6.06	154.2
		1.034	"	6.11	155.1

Location and Description	Authority	Mean Hydraulic Radius in feet	Slope of Surface	Mean Velocity feet per sec.	Coefficient $C = \frac{V}{\sqrt{RS}}$
Test Channel	Darcy	0.166	0.0049	3.34	116.5
Neat Cement	and	0.251	"	4.39	125.1
Rectangular	Bazin	0.322	"	5.04	126.9
	#2	0.375	"	5.66	132.4
		0.430	"	6.06	133.4
		0.474	"	6.51	135.1
		0.516	"	6.85	135.5
		0.558	"	7.12	136.2
		0.595	"	7.41	137.2
		0.632	"	7.63	137.2
		0.665	"	7.86	137.6
		0.696	"	8.07	138.2

Location and Description	Authority	Mean Hydraulic Radius in feet	Slope of Surface	Mean Velocity feet per sec.	Coefficient $C = \frac{V}{\sqrt{RS}}$
Test Channel	Darcy	0.030	0.0152	1.87	87.5
Carefully planed	and	0.043	"	2.30	90.0
Boards	Bazin	0.053	"	2.66	94.4
Rectangular	#29	0.061	"	3.00	96.5
		0.074	"	3.56	106.4

Location and Description	Authority	Mean Hydraulic Radius in feet	Slope of Surface	Mean Velocity feet per sec.	Coefficient $C = \frac{V}{\sqrt{RS}}$
Ditto	#25	0.029	0.0047	0.90	76.5
		0.052	"	1.30	83.0
		0.066	"	1.58	89.4
		0.075	"	1.74	92.7
		0.084	"	1.94	97.6
		0.091	"	2.11	102.1
		0.093	"	2.16	103.2
Sudbury	Fteley	1.863	0.0001606	2.529	146.2
		2.048	0.0001596	2.672	147.9
Conduit	and	2.111	0.0001580	2.805	153.6
Plaster of Pure Cement	Stearns				
Test Channel Planed Boards Semicirculars	"Darcy and Bazin" #26	0.390	0.0015	2.61	107.8
		0.537	"	3.23	113.8
		0.632	"	3.71	120.6
		0.717	"	4.04	123.0
		0.796	"	4.25	123.2
		0.856	"	4.51	125.8
		0.921	"	4.64	124.7
		0.964	"	4.87	128.2
		1.015	"	5.00	128.2
		1.154	"	5.18	130.3
		1.096	"	5.29	130.4
		1.129	"	5.45	132.3
		1.148	"	5.54	133.5
Test Channel Planed Boards Rectangular	"Darcy and Bazin" #26	0.029	0.0047	0.90	76.5
		0.052	0.0047	1.30	83.0
		0.066	"	1.58	89.4
		0.075	"	1.74	92.7
		0.084	"	1.94	97.6
		0.091	"	2.11	102.1
		0.093	"	2.16	103.2

Location and Description	Authority	Mean Hydraulic Radius	Slope	Mean Velocity	Coefficient $C = \frac{V}{\sqrt{RS}}$
Test Channel	"Darcy	0.235	0.0049	3.37	99.1
Unplaned Boards	and	0.341	"	4.43	108.3
Rectangular	Bazin"	0.428	"	5.05	110.2
		0.496	"	5.54	112.3
	#18	0.558	"	5.94	113.7
		0.612	"	6.26	114.3
		0.661	"	6.50	114.2
		0.703	"	6.76	115.3
		0.741	"	7.00	116.1
		0.777	"	7.20	116.7
		0.808	"	7.42	118.0
		0.839	"	7.59	118.4
Test Channel	"Darcy	0.147	0.00824	3.52	101.4
Unplaned Boards	and	0.231	"	4.42	101.4
Rectangular	Bazin"	0.289	"	5.83	109.8
		0.393	"	6.24	109.7
	#8.	0.431	"	6.74	113.1
		0.466	"	7.17	115.8
		0.506	"	7.44	115.2
		0.541	"	7.73	115.8
		0.572	"	8.03	116.9
		0.604	"	8.26	117.1
		0.630	"	8.57	119.0
Test Channel	"Darcy	0.188	0.0048	2.71	8.93
Unplaned Boards	and	0.272	"	3.70	101.2
Rectangular	Bazin"	0.342	"	4.35	106.2
		0.402	"	4.85	109.4
	#7	0.452	"	5.29	112.2
		0.504	"	5.61	113.0
		0.547	"	5.93	114.5
		0.587	"	6.23	116.1
		0.628	"	6.45	116.4
		0.662	"	6.71	117.8
		0.698	"	6.90	117.9
		0.727	"	7.15	119.8

Location and Description	Authority	Mean Hydraulic Radius	Slope	Mean Velocity Feet.	Coefficient $C = \frac{V}{\sqrt{RS}}$
Test Channel	"Darcy	0.245	0.00208	2.08	93.2
Unplaned boards	and	0.363	"	2.69	97.8
Rectangular	Bazin."	0.453	"	3.16	102.8
		0.528	"	3.53	106.5
	#6	0.601	"	3.78	106.9
		0.648	"	4.13	112.5
		0.704	"	4.34	113.5
		0.759	"	4.51	115.5
		0.801	"	4.72	115.8
		0.846	"	4.88	116.3
		0.880	"	5.09	119.0
		0.922	"	5.21	118.9
Chazilly Canal	"Darcy	0.41	0.0081	5.73	100.0
Ashlar masonry	and	0.57	"	7.52	111.0
smoothly dressed	Bazin"	0.68	"	8.19	110.0
		0.77	"	8.75	111.0
	#39				
Spillway Grosbois	"Darcy	0.324	0.101	12.29	67.9
Ashlar masonry	and	0.467	"	16.18	74.5
slimy deposit	Bazin"	0.580	"	18.68	77.2
		0.662	"	21.09	81.6
	#32				
Test Channel		0.192	0.0049	2.75	89.7
Brickwork	"Darcy	0.284	"	3.66	98.3
Smooth	and	0.365	"	4.18	98.8
Rectangular	Bazin	0.424	"	4.72	103.7
		0.481	"	5.10	105.1
	#3	0.540	"	5.33	103.7
		0.582	"	5.88	106.3
		0.620	"	6.01	109.0
		0.668	"	6.15	107.4
		0.697	"	6.47	110.8
		0.759	"	6.60	109.7
		0.779	"	6.72	108.7

Location and Description	Authority	Mean Hydraulic Radius	Slope	Mean Velocity	Coefficient $C = \frac{V}{\sqrt{RS}}$
Sudbury Conduit Smooth Brick well made joints	"Fteley and Stearns" 1880	1.016	0.000014	0.443	117.3
		0.858	0.0000246	0.550	119.8
		1.008	0.0000383	0.789	126.9
		0.937	0.0000746	1.064	125.9
		0.778	0.0000963	1.088	125.6
		0.850	0.0001115	1.241	127.5
		0.577	0.0001596	1.149	119.7
		0.673	0.0001633	1.298	123.9
		0.493	0.0001640	1.079	120.0
		0.891	0.0001701	1.569	127.4
		0.885	0.0001715	1.577	126.0
		0.762	0.0001742	1.423	123.6
		0.752	0.0001803	1.459	123.6
Grosbois Reservoir Ashlar masonry, nearly rectangular	"Darcy and Bazin" #33	0.424	0.037	9.04	72.2
		0.620	"	11.46	75.7
		0.745	"	13.55	81.6
		0.852	"	15.06	84.9
Solanis right aqueduct smooth brick	Cunningham Roorkee 1880	2.52	0.000151	2.20	112.6
		2.72	0.000145	2.54	127.9
		2.94	0.00020	2.51	103.5
		2.94	0.000208	2.79	112.8
		2.99	0.000253	3.20	116.4
		3.65	0.000473	4.83	116.2
		4.20	0.000025	1.24	121.0
Test Channel small gravel semicircular	"Darcy and Bazin" #27	0.454	0.0015	2.17	78.0
		0.546	"	2.50	82.0
		0.619	"	2.69	82.0
		0.681	"	2.93	84.0
		0.731	"	3.05	84.0
		0.784	"	3.22	85.0
		0.826	"	3.33	84.0
		0.900	"	3.54	85.0
		0.966	"	3.73	85.0
		1.012	"	3.95	88.0

Location and Description	Authority	Mean Hydraulic Radius	Slope	Mean Velocity	Coefficient $C = \frac{V}{\sqrt{RS}}$
Test Channel Small gravel Rectangular	"Darcy and Bazin" #4	0.250	0.0049	2.16	61.7
		0.357	"	2.95	70.5
		0.450	"	3.40	72.5
		0.520	"	3.64	76.1
		0.588	"	4.14	77.2
		0.644	"	4.43	78.8
		0.700	"	4.64	79.3
		0.746	"	4.88	80.7
		0.785	"	5.12	82.6
		0.832	"	5.26	82.4
		0.871	"	5.43	83.1
		0.910	"	5.57	83.4
Test Channel Large gravel Rectangular	"Darcy and Bazin" #5	0.291	0.0049	1.79	47.5
		0.417	"	2.43	53.8
		0.510	"	2.90	58.0
		0.587	"	3.27	61.1
		0.656	"	3.56	62.8
		0.712	"	3.85	65.2
		0.772	"	4.03	65.5
		0.823	"	4.23	66.6
		0.867	"	4.43	68.0
		0.909	"	4.60	69.0
		0.946	"	4.78	70.3
		0.987	"	4.90	70.4
Tail Race at Staukan Hungary dry rubble semicircular	Kittinger 1855	0.289	0.0025	1.257	46.8
		0.359	"	1.471	49.8
		0.419	"	1.643	50.8
Aqueduct at Libeth Hungary dry rubble rectangular	Kittinger 1855	0.213	0.0045	1.324	42.8
		0.439	"	2.376	53.9
		0.486	"	2.432	52.0
Head Race at Kapnikbany a Hungary dry rubble Trapezoidal	Kittinger	0.213	0.0038	1.369	48.1
		0.344	"	1.829	50.6
Tail Race Ditto	Kittinger	0.279	0.0036	1.502	47.5
		0.351	"	1.928	54.2
		0.403	"	2.104	55.2

Location and Description	Authority	Mean Hydraulic Radius	Slope	Mean Velocity	Coefficient $C = \frac{V}{\sqrt{RS}}$
Conduit at		0.212	0.021	3.305	49.5
Negyou Hungary	Kittinger	0.356	"	6.150	70.9
Dry Masonry		0.335	"	7.560	71.3
Grosbois Canal	"Darcy	0.88	0.000648	1.47	62.
Masonry in bad	and	1.23	0.000671	2.02	70.
order.	Bazin"	1.40	0.000683	2.34	76.
	#46	1.50	0.000683	2.78	67.
Same	Same	1.07	0.00030	1.12	62.
	#34	1.38	0.00035	1.69	77.
		1.57	0.00033	1.92	84.
		1.71	0.00030	2.18	96.
Spillway Grosbois	"Darcy	0.86	0.0146	4.19	37.5
Canal	and	1.09	0.0146	5.75	45.7
Hammered masonry	Bazin"	1.38	"	7.20	50.7
Slime and mud	#34	1.59	"	8.27	54.3
		1.69	"	8.99	57.2
Grümbach schale		0.36	0.08285	11.808	68.6
Merligen	Kutter	0.38	0.09927	13.323	68.4
Dry Rubble	1876	0.39	0.106775	13.746	67.3
		0.58	0.08285	15.537	70.6
		0.61	0.09927	18.283	72.8
		0.65	0.106775	19.168	72.8
Mill Race at	Kittinger	0.233	0.002	0.468	21.7
Göllintz Hungary	"Civil	0.296	"	0.854	35.1
rubble walls,	Ingenieur"	0.365	"	1.144	41.2
earth bottom.					
Mill Race at					
Pricbram, Hungary	Kittinger	0.316	0.0022	0.389	14.8
rubble walls		0.336	"	0.508	21.6
earth bottom		0.472	"	0.953	29.6
		0.546	"	1.135	32.7
		0.560	"	1.190	33.9
		0.566	"	1.269	36.0

Location and Description	Authority	Mean Hydraulic Radius	Slope	Mean Velocity	Coefficient $C = \frac{V}{\sqrt{RS}}$
Mill Race		0.242	0.005	0.782	22.5
Bezbanya, Hungary		0.282	"	1.191	31.7
Masonry Wall	Rittinger	0.407	"	1.956	43.4
Sand and gravel		0.483	"	2.134	43.4
bed		0.561	"	3.475	65.6
Solani Embankment		1.69	0.000090	0.44	35.7
ment	Cunningham	2.26	0.000148	0.87	45.9
Sides of masonry	1880	3.86	0.000068	1.35	73.2
Bed of clay		4.07	0.000215	1.79	60.5
		5.39	0.000155	2.40	83.0
		6.18	0.000171	3.05	93.8
		6.78	0.000221	3.39	87.5
		7.26	0.000214	3.22	81.7
		7.84	0.000215	3.43	83.6
		8.42	0.000217	3.58	83.6
		8.96	0.000227	3.71	82.3
		9.34	0.000227	4.02	87.3
Hockenbach, Bac,		0.866	0.000778	1.440	55.2
Creek	Grebenau	0.879	0.000797	1.463	55.0
Grosbois Canal		0.96	0.00025	0.89	57.
March	"Darcy	1.32	0.000275	1.34	70.
	and	1.57	0.000246	1.36	69.
	Bazin" #49	1.78	0.000275	1.47	66.
Saalach, Bav.	Koff. 1854	1.54	0.000875	2.073	56.5
River, detritus	Grebenau	1.31	0.00110	2.240	58.8
		1.91	0.001242	3.077	63.0
		1.98	0.00124	3.385	68.2
		2.16	0.00360	5.474	64.3
Canal du Jard		1.68	0.0000362	0.449	57.6
France	Dubuat	1.94	0.0000362	0.479	57.0
Earth,		2.05	0.0000458	0.607	62.6
		2.58	0.0000651	1.069	82.5
Solani,	Cunningham	4.50	0.000291	2.82	78.8
Earth		4.37	0.000297	2.79	77.4
		4.18	0.000304	2.74	78.3
		4.07	0.000306	2.71	76.8

Location and Description	Authority	Mean Hydraulic Radius	Slope	Mean Velocity	Coefficient $C = \frac{V}{\sqrt{RS}}$
Linth Canal at Grynau Earth	Legler	5.14	0.00029	3.414	88.4
		5.93	0.00030	3.830	90.8
		6.46	0.00031	4.152	92.6
		7.12	0.00032	4.418	92.6
		7.52	0.00033	4.753	95.4
		8.09	0.00034	4.920	93.8
		8.28	0.00034	5.058	95.3
		8.62	0.00035	5.225	95.1
		8.87	0.00036	5.392	95.5
		9.18	0.00037	5.530	94.9
Mösa in Misox Course detritus	La Nicca	0.99	0.011875	3.867	35.7
		1.20	0.011875	5.540	46.3
		1.53	"	7.587	56.1
Grosbois Canal Stony earth	"Darcy and Bazin" #47	1.09	0.000464	0.82	36.
		1.38	0.000450	1.32	53.
		1.63	0.000479	1.43	51.
		1.71	0.000493	1.68	58.
Ditto	#41	1.04	0.000445	0.96	45.
		1.38	0.00045	1.27	51.
		1.57	0.000455	1.40	52.
		1.71	0.000441	1.51	55.
Plessur na Chur Coarse gravel	La Nicca	1.25	0.00965	6.002	54.7
		2.33	"	9.988	66.4
		3.48	"	10.194	66.4
		3.58	"	13.579	72.8
		3.59	"	13.943	74.8
		4.58	"	13.746	65.4
River Salzach Bavaria Detritus	Kutter and Grebenau	3.68	0.000662	3.543	71.7
		3.53	0.00094	3.480	60.3
		4.20	0.00094	4.034	63.9
		7.39	0.00112	5.736	63.4
		3.51	0.00155	4.100	55.4
		4.64	0.00155	4.671	67.5
		3.87	0.001796	4.448	53.4
		5.26	0.001796	5.150	58.8

Location and Description	Authority	Mean Hydraulic Radius	Slope	Mean Velocity	Coefficient $C = \frac{V}{\sqrt{RS}}$
Aar, at Berne Irregular bed	Kutter	4.22	0.000461	2.821	63.7
		7.07	0.0008	5.150	68.6
		7.78	0.000993	7.511	85.3
Weser Earth	Schwarz	5.96	0.0001834	1.410	42.7
		8.70	0.0001917	3.470	84.7
		6.31	0.0003986	4.087	81.3
		8.75	0.0004107	4.950	93.6
		6.49	0.0004110	5.182	101.0
		9.44	0.000200	4.064	93.2
		9.98	0.000200	4.389	97.9
		10.52	0.0002167	4.756	99.2
		11.06	0.0002167	5.186	105.5
		12.61	0.0005316	7.924	96.5
		13.55	0.0005504	7.902	91.9
Saone at Raconnay	Léveillé	3.88	0.00004	0.564	45.3
		4.77	"	0.814	58.9
		7.06	"	0.988	58.7
		8.92	"	1.601	84.8
		10.87	"	1.854	88.9
		11.61	"	1.910	88.6
		11.81	"	1.942	89.3
		13.27	"	2.254	97.8
		14.64	"	2.369	98.0
		15.83	"	2.379	94.5
Seine at Paris	Villevert m.Poirée	5.66	0.000127	2.093	78.1
		7.08	0.000133	2.264	73.7
		8.43	0.000135	2.418	71.7
		9.48	0.000140	3.370	92.5
		10.92	0.000140	3.740	95.6
		12.19	0.000140	3.816	92.4
		14.50	0.000140	4.231	94.0
		15.02	0.000140	4.511	98.3
		15.93	0.000172	4.682	89.5
		16.85	0.000131	4.800	102.1
		18.39	0.000103	4.689	107.6

Location and Description	Authority	Mean Hydraulic Radius	Slope	Mean Velocity	Coefficient $C = \frac{V}{\sqrt{RS}}$
Seine at Poissy	Emmery	7.10	0.00009	2.310	91.3
		7.68	0.000087	2.313	89.5
		11.24	0.000057	2.362	93.3
		12.43	0.000060	2.359	86.4
		13.57	0.000050	2.372	91.1
		14.20	0.000054	2.595	93.6
		15.86	0.000062	2.910	92.7
		16.85	0.000067	3.101	92.4
		17.87	0.000075	3.330	91.1
Bayou LaFourche fine gravel	Humphreys and Abbott	12.80	0.00003655	2.807	129.7
		13.04	0.00003731	2.843	128.8
		12.47	0.00004384	2.789	119.3
		15.71	0.00004468	3.076	116.1
Missouri River Sand	Missouri River Commission	5.65	0.0001137	3.01	118.8
		6.80	0.0001109	2.97	108.2
		8.40	0.0001132	2.83	91.8
		8.15	0.0001150	3.39	110.7
		8.07	0.0001165	3.25	106.0
		8.05	0.0001170	3.10	101.0
		11.50	0.0001170	3.78	103.1
		10.70	0.0001183	3.63	102.0
		8.35	0.0001196	3.02	95.5
		8.05	0.0001210	2.96	95.0
		12.60	0.0001371	4.46	107.3
		12.10	0.0001518	3.97	92.6
		11.60	0.0001532	3.90	92.5
		11.60	0.0001540	3.85	91.1
		7.72	0.0001558	3.11	89.7
		14.10	0.0001615	4.72	98.9
		15.40	0.0001627	5.14	102.7
		13.0	0.0001672	4.22	90.5
		14.7	0.0001673	4.84	97.6

Tables For Practical Use.

The following Tables will facilitate the use of the Formula for the uniform flow of water in rivers and smaller channels as derived in this work viz.

$$V = (a + bR)(R^y + m)\ S^x \sqrt{R}.$$

in which

V = mean velocity

R = mean hydraulic radius

S = slope

m = a coefficient of roughness of perimeter.

a, and b, are numberical constants depending upon condition and material of wetter perimeter

y = variable power of R

x = variable " " S

Table I, contains a list of materials and constants to be used. Knowing the material and shape of channel all values of "a", "b", "y", "m", and "x" can be selected from this table and velocity computed by single multiplication.

Table II consists of values of R^y. The left hand column contains values of R and all other columns values of R^y, the value of "y" appearing at the top of the column. Values of R^y are given for every 0.05' and other values may be found by interpolation.

Table III consists of values of S^x. The left hand column contains values of S from 0.00001 up to 0.01. The other columns contain values of S^x, the value of "x" appearing at the top of the column. Values not appearing in the table can be found by interpolation as in case of values of R^y.

TABLE I

Material	a	b	y	m	x
Neat Cement (semicircular)	103.0	20.0	0.110	1.155	0.58
Neat Cement (rectangular)	92.7	13.0	0.110	1.150	0.58
Planed Boards (semicircular)	99.8	9.0	0.250	0.925	0.57
Planed Boards (rectangular)	90.0	18.0	0.250	0.920	0.57
Unplaned Boards	45.6	14.9	0.175	0.866	0.55
Brick Masonry (smooth)	52.5	15.4	0.112	0.820	0.50
Brick Masonry (rough)	41.3	7.5	0.260	0.800	0.50
Small Gravel(semicircular)	31.3	2.9	0.230	0.375	0.385
Small Gravel (rectangular)	28.2	8.7	0.100	0.290	0.385
Large Gravel in cement	29.5	4.1	0.250	0.250	0.40
Rubble Masonry (clean)	153.2	16.5	0.250	0.100	0.68
Rubble Masonry (dirty)	21.0	14.6	0.250	-0.100	0.42
Earth, Masonry walls	79.4	2.9	0.200	-0.220	0.55
Small Rivers, (rectangular)	71.5	1.4	0.150	-0.240	0.50
Irregular Rivers	71.5	1.3	0.250	-0.420	0.50

Table II

R = Hydraulic Radius R^Y = ______
Y = a variable power

R \ Y =	0.100	0.110	0.112	0.150	0.175	0.20	0.23	0.25	0.26	0.50
0.10	0.794	0.776	0.773	0.708	0.668	0.631	0.589	0.562	0.549	0.316
0.15	0.827	0.812	0.807	0.752	0.717	0.684	0.646	0.622	0.611	0.387
0.20	0.851	0.838	0.835	0.786	0.754	0.725	0.691	0.669	0.658	0.447
0.25	0.871	0.859	0.856	0.812	0.785	0.758	0.727	0.707	0.697	0.500
0.30	0.887	0.876	0.874	0.835	0.810	0.786	0.758	0.740	0.731	0.548
0.35	0.900	0.891	0.889	0.854	0.832	0.812	0.786	0.769	0.761	0.592
0.40	0.912	0.904	0.902	0.872	0.852	0.833	0.810	0.795	0.788	0.632
0.45	0.923	0.916	0.914	0.887	0.870	0.852	0.832	0.819	0.813	0.671
0.50	0.933	0.927	0.925	0.901	0.886	0.871	0.853	0.841	0.835	0.707
0.55	0.942	0.936	0.935	0.914	0.903	0.887	0.872	0.861	0.856	0.742
0.60	0.950	0.954	0.944	0.926	0.914	0.903	0.889	0.880	0.876	0.775
0.65	0.958	0.954	0.953	0.937	0.927	0.917	0.905	0.898	0.894	0.906
0.70	0.965	0.962	0.961	0.948	0.939	0.931	0.921	0.915	0.911,	0.938
0.75	0.972	0.969	0.968	0.958	0.951	0.944	0.936	0.931	0.928	0.866
0.80	0.978	0.976	0.975	0.967	0.962	0.956	0.950	0.946	0.944	0.895
0.85	0.984	0.982	0.981	0.976	0.972	0.968	0.963	0.960	0.958	0.923
0.90	0.990	0.989	0.988	0.984	0.982	0.979	0.977	0.974	0.973	0.949
0.95	0.995	0.994	0.993	0.992	0.990	0.989	0.988	0.987	0.986	0.975
1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.05	1.005	1.006	1.006	1.007	1.009	1.010	1.011	1.012	1.013	1.024
1.10	1.009	1.010	1.011	1.014	1.017	1.019	1.022	1.024	1.025	1.049
1.15	1.014	1.015	1.016	1.021	1.025	1.028	1.033	1.035	1.037	1.072
1.20	1.018	1.020	1.021	1.028	1.032	1.037	1.043	1.046	1.048	1.082
1.25	1.022	1.025	1.027	1.034	1.040	1.045	1.052	1.057	1.060	1.118
1.30	1.026	1.029	1.030	1.040	1.047	1.054	1.062	1.068	1.071	1.140
1.35	1.031	1.033	1.035	1.046	1.054	1.062	1.071	1.078	1.081	1.162
1.40	1.034	1.038	1.039	1.052	1.061	1.070	1.081	1.088	1.091	1.976
1.45	1.038	1.042	1.043	1.057	1.067	1.077	1.089	1.097	1.101	1.204
1.50	1.041	1.046	1.047	1.063	1.073	1.084	1.098	1.107	1.111	1.225
1.55	1.045	1.049	1.050	1.068	1.080	1.092	1.106	1.116	1.128	1.245
1.60	1.048	1.053	1.054	1.073	1.086	1.099	1.114	1.125	1.130	1.265
1.65	1.051	1.057	1.058	1.078	1.089	1.105	1.122	1.133	1.139	1.284
1.70	1.055	1.060	1.061	1.083	1.097	1.112	1.130	1.142	1.148	1.304
1.75	1.058	1.064	1.065	1.087	1.103	1.118	1.137	1.150	1.157	1.323
1.80	1.060	1.067	1.068	1.092	1.108	1.125	1.145	1.158	1.165	1.342
1.85	1.063	1.070	1.071	1.096	1.114	1.131	1.152	1.166	1.173	1.357
1.90	1.066	1.073	1.074	1.101	1.119	1.137	1.159	1.174	1.181	1.378
1.95	1.069	1.076	1.077	1.105	1.124	1.143	1.166	1.182	1.190	1.396
2.00	1.072	1.079	1.080	1.109	1.129	1.149	1.173	1.189	1.197	1.414
2.10	1.077	1.085	1.086	1.118	1.139	1.160	1.186	1.204	1.213	1.449
2.20	1.080	1.091	1.092	1.126	1.148	1.171	1.199	1.218	1.228	1.483
2.30	1.087	1.096	1.097	1.133	1.157	1.181	1.211	1.232	1.242	1.517
2.40	1.092	1.101	1.103	1.140	1.166	1.192	1.223	1.245	1.256	1.549
2.50	1.096	1.106	1.108	1.147	1.174	1.201	1.235	1.257	1.268	1.581

Table II. Continued

Y =	0.100	0.110	0.112	0.150	0.175	0.20	0.23	0.25	0.26	0.50
R										
2.60	1.101	1.111	1.113	1.154	1.182	1.210	1.246	1.270	1.282	1.612
2.70	1.105	1.115	1.117	1.161	1.190	1.220	1.257	1.282	1.295	1.643
2.80	1.108	1.120	1.122	1.167	1.197	1.229	1.267	1.294	1.307	1.673
2.90	1.112	1.124	1.127	1.173	1.205	1.237	1.278	1.305	1.319	1.703
3.00	1.116	1.128	1.131	1.179	1.218	1.246	1.287	1.316	1.331	1.732
3.20	1.123	1.136	1.139	1.191	1.226	1.262	1.307	1.336	1.253	1.789
3.40	1.130	1.144	1.147	1.201	1.239	1.277	1.325	1.358	1.374	1.844
3.60	1.137	1.154	1.157	1.212	1.251	1.292	1.343	1.377	1.395	1.897
3.80	1.143	1.158	1.161	1.222	1.263	1.306	1.359	1.395	1.415	1.949
4.00	1.149	1.165	1.168	1.231	1.275	1.320	1.375	1.414	1.434	2.000
4.20	1.154	1.171	1.175	1.240	1.285	1.332	1.391	1.428	1.452	2.054
4.40	1.160	1.177	1.181	1.249	1.296	1.345	1.406	1.448	1.470	2.098
4.60	1.165	1.183	1.187	1.257	1.306	1.357	1.521	1.464	1.487	2.145
4.80	1.170	1.188	1.192	1.265	1.316	1.368	1.434	1.480	1.504	2.191
5.00	1.175	1.194	1.198	1.273	1.325	1.380	1.448	1.495	1.523	2.237
5.50	1.186	1.206	1.210	1.292	1.348	1.406	1.480	1.531	1.558	2.345
6.00	1.196	1.218	1.222	1.308	1.368	1.431	1.510	1.565	1.594	2.450
6.50	1.206	1.229	1.234	1.327	1.388	1.454	1.538	1.597	1.627	2.550
7.00	1.215	1.239	1.244	1.339	1.406	1.476	1.564	1.627	1.659	2.646
7.50	1.223	1.248	1.253	1.353	1.423	1.495	1.590	1.655	1.689	2.739
8.00	1.231	1.257	1.262	1.366	1.439	1.516	1.613	1.682	1.717	2.829
8.50	1.239	1.265	1.270	1.379	1.454	1.534	1.636	1.707	1.745	2.916
9.00	1.246	1.273	1.279	1.390	1.469	1.552	1.658	1.732	1.771	3.000
9.50	1.252	1.281	1.287	1.402	1.483	1.569	1.678	1.756	1.796	3.082
10.00	1.259	1.289	1.294	1.413	1.497	1.585	1.699	1.779	1.820	3.163
15.00	1.311	1.347	1.354	1.501	1.606	1.719	1.864	1.968	2.022	3.873
20.00	1.350	1.390	1.399	1.567	1.689	1.821	1.992	2.115	2.179	4.472
25.00	1.380	1.425	1.434	1.620	1.757	1.903	2.097	2.236	2.309	5.000

Table III

S = slope
X = variable power

S^x

X =	0.385	0.40	0.42	0.50	0.55	0.57	0.58	0.68
S								
.00001	.01189	.0100	.00794	.00316	.00178	.00141	.00126	.000398
.00002	.01552	.01320	.01064	.00448	.00260	.00209	.00188	.000638
.00003	.01814	.01551	.01260	.00548	.00325	.00264	.00238	.000843
.00004	.02027	.0174	.01422	.00633	.00381	.00311	.00281	.00101
.00005	.0221	.0190	.0156	.00708	.00431	.00355	.00320	.00119
.00006	.0237	.0205	.0168	.00775	.00476	.00392	.00356	.00134
.00007	.0251	.0218	.0179	.00837	.00518	.00428	.00389	.00147
.00008	.0264	.0230	.0190	.00885	.00558	.00462	.00421	.00164
.00009	.0277	.0241	.0199	.00950	.00596	.00493	.00440	.00177
.0001	.02885	.02512	.02090	.0100	.00639	.00525	.00478	.00190
.0002	.0377	.0331	.0279	.0141	.00924	.00779	.00716	.00266
.0003	.0440	.0389	.0331	.0173	.0115	.00981	.00901	.00402
.0004	.0491	.0437	.0374	.0200	.0135	.0115	.0105	.00489
.0005	.0536	.0478	.0411	.0223	.0153	.0131	.0121	.00569
.0006	.0575	.0514	.0443	.0245	.0169	.0146	.0135	.00645
.0007	.0610	.0547	.0473	.0265	.0184	.0159	.0148	.00699
.0008	.0642	.0501	.0577	.0283	.0198	.0172	.0160	.00784
.0009	.0656	.0591	.0513	.0300	.0205	.0178	.0166	.00817
.0010	.06999	.0631	.05495	.0316	.0224	.0195	.0182	.00912
.0015	.0818	.0742	.0658	.0388	.0279	.0245	.0230	.00999
.0020	.00914	.0661	.0558	.0448	.0328	.0289	.0272	.0146
.0025	.0996	.0910	.0807	.0500	.0370	.0329	.0309	.0170
.0030	.0168	.0979	.0872	.0548	.0410	.0365	.0344	.0192
.0035	.1130	.1038	.0926	.0592	.0444	.0396	.0374	.0212
.0040	.1192	.1099	.0984	.0633	.0460	.0430	.0407	.0234
.0045	.1249	.1151	.1033	.0671	.0512	.0460	.0435	.0254
.0050	.1301	.1201	.1060	.0708	.0542	.1488	.0463	.0272
.0055	.1349	.1248	.1124	.0742	.0572	.0515	.0489	.0291
.0060	.1395	.1292	.1165	.0775	.0599	.0541	.0515	.0309
.0065	.1439	.1334	.1206	.0806	.0627	.0567	.0539	.0325
.0070	.1479	.1374	.1244	.0837	.0653	.0591	.0562	.0334
.0075	.1520	.1412	.1281	.0866	.0678	.0615	.0586	.0359
.0080	.1559	.1450	.1316	.0885	.0703	.0638	.0606	.0375
.0085	.1595	.1485	.1350	.0922	.0726	.0660	.0630	.0391
.0090	.1631	.1519	.1382	.0949	.0750	.0682	.0651	.0406
.0095	.1665	.1555	.1415	.0975	.0772	.0704	.0671	.0422
.0100	.1699	.1585	.1446	.1000	.0794	.0724	.0692	.0436

CIRCULAR ORIFICES

NOTATION

All linear distances are in English feet. All measures of area are in English square feet. All measures of capacity are in English cubic feet.

D = diameter of orifice.

A = area of orifice. $= D^2\frac{\pi}{4}$.

q = quantity of water discharged in one second.

Q = theoretical discharge $= \int_{+r}^{-r} 2dy\sqrt{r^2-y^2}\sqrt{2g(h-y)}$ * .

Q' = approximate theoretical discharge $= A\sqrt{2gh}$.

c = correct cofficient of discharge $= \frac{q}{Q}$.

C = approximate cofficient of discharge $= \frac{q}{Q'}$.

C_v = cofficient of velocity (assumed as unity) .

C_c' = actual cofficient of contraction = area of contracted section divided by area of orifice.

C_c = theoretical cofficient of contraction.

C_a = cofficient of approach $= \frac{1}{1-\frac{C_c A}{A_a}}$ .where

A_a = area of cross-section of water as it approachs the orifice.

C_x = "cofficient of loss" $= \frac{c}{C_c C_a}$.

n = diameter of approaching body of water divided by the diameter of the orifice.

h = vertical height between center of the orifices and the level of the comparatively still water, or in case of enclosed tanks, the hydrostatic pressure.

* See Merriman's Hydraulics page 112.

INTRODUCTION

THE PROBLEM. What would seem to be one of the most simple and obvious problems of hydraulics is that ofdeterming the discharge from a standard circular orifice in either the side or bottom of a tank when the diameter and head are given. By standard orifices we mean one through which the water flows so as to touch the sides in what is practically a geometrical line. But though the problem may be easily stated it has never been solved in anything more than an approximate manner. Although the formula which follows is for the most part empirical and therefor only approximate,we have found no other which even attempts to give the discharge in the terms of all the factors which determine it,and without the use of constants which are constants only in name,and are really variables whose values can be found only in a table.

THE CONTRACTED SECTION. The quantity discharged is of course the product of the velocity and the area of the contracted section or section of least area(if the jet be directed horizontally or upward)Of course if it is directed downward it will continue to contract because of the increased velocity due to the acceleration of gravity. The position of the contracted section has generally been said to lie at a distance from the plane of the orifice equal to one-half the diameter,but the experiments of Horace Judd and Roy S.King at Ohio State University(Eng. News Vol.56,page 327)show that the distance varies from one-half the diameter for large orifices to eight-tenths for a ¾ inch orifice and by inference at least,it would appear to be more for smaller

orifices.

THE COFFICIENT OF VELOCITY. At the contracted section the velocity has been repeatedly shown to be practically equal to the velocity which would be given by a body falling freely through a distance equal to the head on the center of the orifice. The ratio of the actual velocity to the theoretical velocity, called the cofficient of velocity has been placed by various authorities between .97and unity or even above. Neville in his tables published in I875 gives the value .974 . Merrimans Hydraulics I889 gives .98 as an average of the results obtained at McGill University. In fact as better experimental methods have come in use and the orifices employed have become more nearly "frictionless", the values have grown near to unity. This adds weight to the results obtained in the Ohio State experiments refered to above in which Cv was found to have a range between .9996 and .99999 or for all practical purposes, unity. It has been assumed in what follows that the cofficient of velocity, Cv, is unity for a standard (that is a perfectly frictionless orifice). As will be seen however if this assumption is incorrect it does not effect the validity of the formula since it would be taken care of in the cofficient C_x .

THE COFFICIENT OF CONTRACTION. The ratio of the area of the contracted section to the area of the orifice is called the coefficient of contraction, or, C_c . It is the one whose determination has caused the most difficulty, as the direct measurement of the contracted section with sufficient accuracy for scientific purposes is almost impossible. The earlier writers gave fixed values to this constant, some of them being as follows; --

Newton	.707
Poleni	.667
Borda	.643
Michellotti	.640
Bossut	.663
DuBuat	.667
Venturi	.637
Eytelwein	.640
Bayer	.617

It is interesting in this case to notice the decrease in value of the coefficient as the orifices became more perfect. The last value is interesting because it is founded on a rational basis by assuming that "the velocities of the particles of water which approach the orifice from all sides, are inversely as the squares of their distances from its center." From this he derived (although we were unable to find his work) that the ratio between the diameter of the orifice and of the contracted section was 1:.7854 and .7854 gives us .617. If a fixed coefficient of contraction were to be used this is probably as good as any, but it has been repeatedly shown by experiment that C_c is a variable, which as we shall show depends upon three governing conditions.

RANKINE'S FORMULA. The coefficient c depends not only on these three factors, but upon a fourth, namely the ratio of the area of the channel of approach to the area of the orifice. It is apparent that the water in approaching the orifice through the tank or other vessel must have some velocity, and that this velocity enters into the actual velocity through the orifice and contracted section. Rankine recognised this when he gave as an empirical formula:-

Fig. 1.

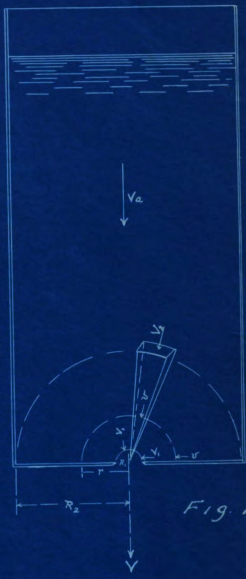


Fig. 2.

$\frac{1}{K} = \sqrt{2.618 - 1.618\frac{A^2}{A_a^2}}$, Which in our notation would be:- $\frac{C_v}{C} = \sqrt{2.618 - 1.618\frac{A^2}{A_a^2}}$ This formula leaves Cv to be determined independantly, and does recognise the other three factors which influence the value of c, namely h, D, and n.

THE VALUE OF C_c . Prof. John Goodman of the University of Leeds has shown that the theoretical coefficient of contraction depends on the factor n alone and has (in Engineering Mar. II, I904) accomplished what was long considered impossible by giving us a rational expression for C_c in terms of n for standard orifices. His derivation is substantially as follows. Assume a cylindrical tank (fig. 2) of radius R_2 and a circular orifices of radius of R_1 and area A in the center of the bottom. Then let a = the theoretical area of jet at contracted section and w= the weight of a column of water I foot high and I square inch in section. When the orifices is opened " the weight of the tank and contents is reduced by the weight of the column of water over the orifices (plus)the total pressure on the bottom of the tank equivalent to the kinetic energy of the approaching water as it flows over the orifices plate, (i.e. the bottom of the tank.)" If h is large it may be assumed that the water approaches the orifices in pyramidal passages, thes pyramids having their center at the center of the orifices. Then particals having equal velocities will lie in hemispheres having the center of the orifice as a center. The notation which follows will be understood from the figure, P_1, p, and P_2 being the pressures at the points having velocities V_1, v, and V_2. The area of the innermost shell is $2\pi R_1^2$ and that of the orifices is πR_1^2, therefore ;

$$v = \frac{V}{2} \quad ------(1)$$

Also since the area of the hemispheric shells vary as the squares of their radii, the velocity of flow across the shells will vary inversely as the square of the radii, or $\frac{V_2}{V_1} = \frac{r^2}{R_2^2}$. (2)

Also by Bernioulli's theorem the pressure head plus the velocity head is constant at each section, therefore ;

$$\frac{P_2}{w} + \frac{V_2^2}{2g} = \frac{p}{w} + \frac{v^2}{2g}$$ and hence from (2)

$$P_2 - p = \frac{w}{2g}(v^2 - V_2^2) = \frac{wV_2^2}{2g}\left(\frac{R_2^4}{r^4} - I\right) \quad \text{---------(3).}$$

The key to the whole derivation is the fact that when the orifice is opened and the water flows inward along the bottom of the tank, the pressure at any point on the bottom is reduced by an amount equal to the change in velocity head between the edge and the point in question. If we consider a ring on the bottom with its inner radius = r and its outer radius = r + dr, its area will be $2\pi r\,dr$ and the unit pressure is reduced by $(P_2 - p)$ which is gained while the water is flowing from the outer edge of the tank to the circle of radius r, therefore the whole loss is

$$\int_{r=R_1}^{r=R_2} (P_2 - p)\, 2\pi r\, dr$$ which from (3) gives

$$\frac{2\pi w V}{2g}\int_{r=R}^{r=R}\left(\frac{R_2^4}{r^4} - I\right) r\, dr = \frac{\pi w V}{g}\int_{r=R}^{r=R}(R^4 r^{-3} - r)\, dr =$$

$$\frac{\pi w V_2^2}{g}\left(\frac{R_2^4 R_1^{-2}}{-2} - \frac{R_2^4 R_2^{-2}}{-2} - \frac{R_2^2}{2} + \frac{R_1^2}{2}\right)$$ and since $R_2 = nR_1$ this equals

$$\frac{\pi w V_2^2 R_1^2}{g}\left(-\frac{I}{2n^2} - \frac{I}{2n^4} - \frac{I}{2n^2} + \frac{I}{2}\right) = \frac{\pi w V_2^2 R_1^2}{2gn^4}(n^4 - 2n^2 + I)$$ which, from (I)

equals $\frac{\pi w V_1^2 R_1^2}{8gn^4}(n^4 - 2n^2 + I)$ ------------------- ---(4).

There is also the loss of weight of the column of water over the orifice $= w\pi R_1^2 h = \frac{w\pi R_1^2 v^2}{2g}$ ----------------(5).

The loss of weight must equal the pressure which the escaping jet would exert on a plate across its path, that is,

$$\frac{(wa\ V)\ V}{g} = \frac{waV^2}{g} \text{ --------------(6).}$$

Then (4) + (5) must equal (6) or

$$\frac{wV^2\pi R_1^2}{8gn^4}(n^4 - 2n^2 + I) - \frac{wV^2\pi R_1^2}{2g} = \frac{waV^2}{g} \quad .$$

If for πR_1^2 we put A, this becomes

$$\frac{wV^2A}{2g}\frac{(n^4 - 2n^2 + I)}{4n^4} = \frac{waV^2}{g} \quad \text{or}$$

$$\frac{A}{2}\left(I + \frac{n^4 - 2n^2 + I}{4n^4}\right) = a$$

Therefore, $C_c = \frac{a}{A} = \frac{I}{2}\left(I + \frac{n^4 - 2n^2 + I}{4n^4}\right)$ which is the expression sought.

THE VALUE OF C . The work above has disregarded the velocity of approach for, V' the actual velocity is equal to $V + Va$. The result so far is correct however, since the difference between V', the actual velocity, and V, the velocity considered, is equal to V_a, the velocity in the upper part of the tank, both of which were neglected. To find C_a we proceed as follows:-

$$V_a\ A_a = a\ V' = C_c'\ A\ V'$$

Therefore:- $V_a = \frac{C_c' A\ V'}{A_a}$

Then $V = V' - V_a = V' - \frac{C_c'\ A\ V'}{A_a} = V'\left(I - \frac{C_c'\ A}{A_a}\right)$

Therefore:- $C_a = \frac{V'}{V} = \frac{I}{I - \frac{C_c' A}{A_a}}$

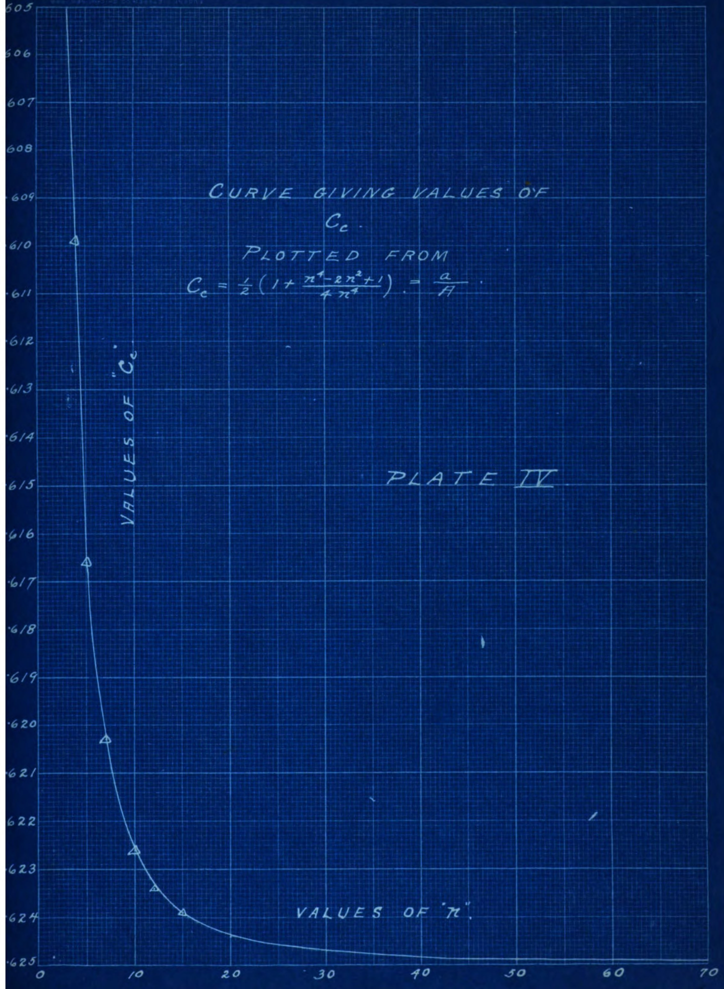
Since as will be shown later $C_c' = C_c$ within a few %, and since $\frac{A}{A_a}$ is usually very small, we may for all practically good use write this $C_a = \frac{I}{I - \frac{C_c A}{A_a}}$

THE COEFFICIENT OF LOSS. If then we assume that $C_v = I$ it would seem that $c = C_c C_\omega$ But experiments have shown that this is not correct but that we must instead write $c = C_o^1 C_a$ or if $C_o^1 = C_e C_x$ we may put $c = C_e C_a C_\omega$ where C_x is a variable which we may call the coefficient of loss, and which depends on both the head and the diameter of the orifice. Its presence is undoubtedly due to the fact that water is not a perfect fluid and does not move toward the orifice with the mathematical precision which is assumed in the derivation of the formula for C_ω. Prof. Hele-Shaw (in Engineering Jan. 6, I899.) has shown by actual photographs that in reality the water does not move as rapidly from the sides of the the tank toward the orifice as it does from the region directly behind the orifice. thus the formula gives too great a loss of weight due to velocity and therefore C_c will be larger than C_o^1, so that in general C_x will be less than unity. Also since C_v is not exactly equal to unity, C_x will be still smaller. On the other hand the theory is based on a relatively large value of h so that we are not sure of its truth in the case of small heads. Exeriments show that for these C_c is smaller than C_o^1, and therefore for small heads C_x will be larger. While the whole derivation has been for a horizontal orifice experiments have shown that the coefficients are the same for a vertical as for a horizontal orifice where the head is relatively large. Where h is small we must remember that c is not equal to C so that for small heads all observed values of C have been reduced in the work that followes by the table given on page 22 of Smith's Hydraulics.

THE VALUE OF n. Another difficulty arises from the fact that most tanks are not circular and therefore R_1 and hence n, are not easily determined. Prof. Goodman says to take the distance from

the orifice to the nearest edge of the "orifice plate" but in the experiments considered the bottom of the tank was close to the orifice and it was thought wise to average the distances to the bottom and top for a value of R_2. In evaluating C_c the curve on plate IV may be used with sufficient exactness for values of n greater than 8 or 10. The table below is copied from Goodman's article and will be useful when $\frac{A_a}{A} = n$, i.e., for a circular approach. The values from Rankine's formula are as follows - The notation is changed to our own. When $C_x = .97$ the second column gives values of c.

n	Goodman		Rankine	
	C_c C_a	.97 C_c C_a	c/C_v	.97 c/C_v
2	.665	.645	.672	.652
3	.641	.622	.640	.621
4	.634	.615	.631	.612
5	.631	.612	.626	.607
6	.629	.610	.624	.605
8	.628	.609	.622	.603
10	.627	.608	.620	.601
100	.625	.606	.618	.600


CURVE GIVING VALUES OF
C_c.
PLOTTED FROM
$C_c = \frac{1}{2}\left(1 + \frac{n^4 - 2n^2 + 1}{4n^4}\right) = \frac{a}{A}$.
PLATE IV
VALUES OF "C_c"
VALUES OF "n".
·605
·606
·607
·608
·609
·610
·611
·612
·613
·614
·615
·616
·617
·618
·619
·620
·621
·622
·623
·624
·625
0
10
20
30
40
50
60
70

DERIVATION OF THE VALUE OF C_x.

METHOD. The general method in the derivation of a formula for the value of C_x was first to find records of reliable experiments which gave sufficient data for finding e, C_a, and C_c. Then

$$C_x = \frac{e}{C_c \; C_a}.$$

Then values of h and C_x were plotted and a search made for the equation of a curve which would pass through these points. After the form of the equation was determined upon the values of the constants in the equation which would make it pass through the largest number of points, i. e. the one which would give the most probable values, was found by the method of least squares.

DATA. The records of only three sets of experiments were found in complete enough form to be available for this work. The first were the Holyoke experiments carried on by Hamilton Smith Jr. in I884-5 and recorded in his hydraulics. There were three orifices used, .02, .05, and .I0 feet in diameter respectively. These experiments were very accurate but the heads did not go above 5 feet and the approach was rectangular making a determination of the value of n a little uncertain. The second series were performed by Horace Judd and Roy S. King at the Ohio State University in I906 and recorded in Eng. News Vol. 56, page 327. Five orifices were used with diameters of ¾, I, I½, 2, and 2½ inches respectively. The heads varied from 4 to 93 feet and the approach was circular. This was the most satisfactory group of data as n was absolutely determined. The third series was carried on by Theodore G. Ellis C.E. at Holyoke, Mass. in I874 and recorded in the Transactions of the A.S.C.E. Vol.5, page I9. Three

orifices were used with diameters of .5, 1.007, and 2.00 feet respectively. The heads varied from 1.15 feet to 17.7 feet. This was the least satisfactory series as the heads were most of them very low compared with the diameter of the orifices, and the orifice was placed quite close to the bottom of the tank making "n" quite uncertain. More than this the discharge was measured over a weir, thus introducing another chance for error.

COMPUTATION OF C_x. The table on page shows in columns h and c the values as given by Smith. These are all means of from two to six observations. n was found by dividing the distance from the bottom of the tank to the surface of the water by the diameter of the orifice. C_c was then taken from the curve on PLATE IV. A_a is the cross-sectional area of the tank up to the level of the water by substitution in the formula:- $C_a = \dfrac{1}{1 - \dfrac{C_c A}{A_a}}$, C_a is determined. Then dividing c by the product of C_c and C_a gives C_x.

VARIATION OF C_x WITH h. The values of C_x and h were then plotted on PLATE V. It is readily seen that the simplest curve of this form is the equilateral hyperbola. The vertical asymptote seems to be the C_x axis but the horizontal one is raised above the h axis. We may therefore write as a trial equation $h(C_x - b) = a$ or $C_x = b - \frac{a}{h}$. To apply the method of least squares let $hC_x - hb = a$ be a typical observation equation. Then for the unknown a the normal equation will be formed by adding the various equations of the form $hC_x - hb = a$ or if there are n equations we will have:-

$$\Sigma hC_x - b\Sigma h = na \quad \text{-------------------------(1)}$$

For the unknown b we will have the sum of the equations

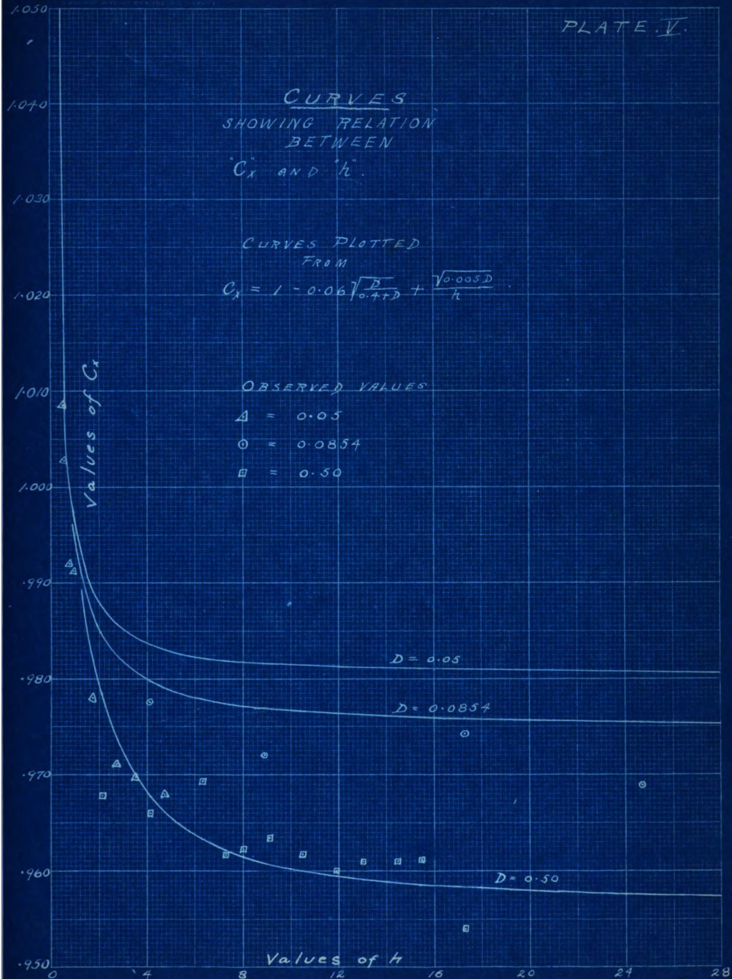


PLATE. V.
CURVES
SHOWING RELATION
BETWEEN
"Cx" AND "h".
CURVES PLOTTED
FROM
Cx = 1 - 0.06 √(D/(0.4+D)) + √(0.005D)/h
OBSERVED VALUES
△ = 0.05
⊙ = 0.0854
⊡ = 0.50
D = 0.05
D = 0.0854
D = 0.50
Values of Cx
Values of h
1.050
1.040
1.030
1.020
1.010
1.000
.990
.980
.970
.960
.950
0
4
8
12
16
20
24
28

h	e	n	C_m	A_e	C_a	C_x
			diameter = .02 feet			
0.739	.6425	74	.6249	4.47	I.0000	I.0394
2.430	.6298	I59	.6250	9.54	I.0000	I.0077
3.I90	.6264	I97	.6250	II.82	I.0000	I.0022
			Diameter = .05 feet			
0.477	.630I	24	.6246	3.56	I00003	I.0085
0.536	.6263	26	.6246	3.96	I.0003	I.0017
0.720	.6I99	29	.6247	4.4I	I.0003	0.9920
0.9I0	.6I60	33	.6248	4.98	I.0002	0.9858
0.929	.6I94	34	.6248	5.04	I.0002	0.99I2
I.740	.6II3	50	.6249	7.47	I.0002	0.978I
2.730	.6070	70	.6249	8.44	I.000I	0.97I2
3.570	.6060	86	.6249	I2.96	I.000I	0.9696
4.630	.605I	I08	.6250	I6.I4	I.000I	0.9680
			Diameter = .I0 feet			
0.66I	.6II9	I4	.6238	4.23	I.00I2	0.9798
0.900	.6096	I7	.624I	4.95	I.00I0	0.9758
I.730	.6042	25	.6246	7.44	I.0007	0.9667
3.I80	.6025	39	.6248	II.79	I.0004	0.9640
4.600	.60I3	54	.6249	I6.05	I.0003	0.96I9

of the form:-

$$h^2C - h^2b = ha \text{ or } \Sigma h^2C_x - b\Sigma h^2 = a\Sigma h \quad --(2)$$

Then from (I) $\Sigma h\Sigma hC_x - b(\Sigma h)^2 = na\Sigma h$

and from (2) $n\Sigma h^2C_x - nb\Sigma h^2 = na\Sigma h$

Therefore $\Sigma h\Sigma hC_x - n\Sigma h^2 C_x = b\left[(\Sigma h)^2 - n\Sigma h^2\right]$

Therefore $b = \dfrac{\Sigma h\Sigma hC_x - n\Sigma h^2C_x}{(\Sigma h)^2 - n\Sigma h^2}$

also from (1) $\Sigma h^2 \Sigma h C_x - b\Sigma h^2 \Sigma h = na\Sigma h^2$

also from (2) $\Sigma h \Sigma h^2 C_x - b\Sigma h^2 \Sigma h = a(\Sigma h)^2$

therefore $\Sigma h^2 \Sigma h C_x - \Sigma h \Sigma h^2 C_x = a\,(n\Sigma h^2 - (\Sigma h)^2)$

therefore $a = \dfrac{\Sigma h \Sigma h^2 C_x - \Sigma h^2 \Sigma hC}{(\Sigma h)^2 - n\Sigma h^2}$

Inthe following table the work for finding the quantities $\Sigma h, \Sigma h^2$, and $\Sigma h^2 C_x$, is given for the first orifice.

Diameter = .02

h	C_x	h^2	hC_x	h^2C_x
0.739	1.0394	0.5461	0.7681	0.5676
2.430	1.0077	5.9049	2.4487	5.9504
3.190	1.0022	10.1761	3.1970	10.1993
6.359		16.6271	6.4138	16.7173

Therefore $b = \dfrac{6.359\times 6.4138 - 3\times 16.7173}{6.359^2 - 3\times 16.6271} = \dfrac{40.8023-50.1519}{40.4369-49.8813} = \dfrac{-9.3494}{-9.4444}$

$= .9899$

Also $a = \dfrac{6.359\times 16.7173 - 16.6271\times 6.4138}{-9.4444} = \dfrac{106.3053-106.6429}{-9.4444}$

$= \dfrac{-.3376}{-9.4444} = .0357$

The following pages give in a condensed form the computation for the values of b. The value of a is not given, for as will be shown later, it is not used.

Diameter = .05

h	C_x	h^2	$h\ C_x$	h^2C_x
0.437	1.0083	.1910	.4407	.1926
0.536	1.0027	.2873	.5374	.2881
0.720	.9920	.5184	.7142	.5143
0.929	.9912	.8630	.9208	.8554
1.740	.9781	3.0276	1.7019	2.9613
2.730	.9712	7.4529	2.6514	7.2383
3.570	.9696	12.7449	3.4615	12.3575
4.630	.9680	21.4369	4.4818	20.7509
15.292		46.5220	14.9097	45.1878

$$b = \frac{227.9991 - 361.2624}{233.8453 - 372.1760} = \frac{133.2633}{138.3307} = .9634$$

Diameter = .10

h	C_x	h^2	$h\ C_x$	h^2C_x
0.661	.9798	.4369	.6496	.4281
0.900	.9758	.8100	.8782	.7904
1.730	.9667	2.9929	1.6724	2.8932
3.180	.9604	10.1124	3.0655	9.7484
4.600	.9619	21.1600	4.4247	20.3538
11.071		35.5122	10.6884	34.2139

$$b = \frac{171.0695 - 118.331}{177.5670 - 122.567} = \frac{52.7382}{54.9940} = .9590$$

<u>Judd and King's Experiments.</u> Diameter = .0626'

The diameter of the approach was 2 feet therefore ;-

$$n = \frac{1}{.0313} = 31.9 \text{ , and therefore } C_c = .62475 .$$

$$C_a = \frac{1}{1 - \frac{.62475 \times .0626^2}{2^2}} = 1.0006 \text{ ------- } C_aC_c = .62512$$

and $C_x = \frac{c}{.62512}$

Diameter =.0626

h	c	C_x	h^2	$h\,C_x$	h^2C_x
4.I0	.6085	.9734	I6.8I00	3.99I0	I6.3629
8.99	.6I02	.976I	80.8202	8.775I	78.8884
I6.75	.6I03	.9763	280.5625	I6.3530	273.9I32
23.60	.6I05	.9766	556.9600	23.0478	543.927I
35.33	.6I20	.9770	I248.2089	34.588I	I22I.9965
43.85	.6III	.9776	I922.8225	42.8678	I879.75I3
54.25	.6II9	.9789	2943.0625	53.I053	2880.9639
66.80	.6I04	.9764	4462.2400	65.2235	4356.93II
84.70	.6I50	.9838	7I74.0900	83.3279	7037.8697
338.37			I8685.5765	33I.2795	I83I0.604I

$$b = \frac{I64795.44 - II2095.04}{I68I70.I8 - II4494.26} = \frac{52700.40}{53675.92} = .98I8$$

Diameter = .0854

$$n = \frac{I}{.0427} = 23.4\ ,\quad C_f' = .62458\ ,\quad C_\alpha = I.00II\ ,\quad C_x = \frac{c}{.62527}\ ,$$

h	c	C_x	h^2	$h\,C_x$	h^2C_x
4.02	.6I22	.9776	I6.I604	3.9300	I5.7984
8.87	.6078	.9720	78.6769	8.62I6	76.4739
I7.35	.6092	.9743	30I.0225	I6.904I	293.2262
24.65	.6059	.9690	607.6225	23.8859	588.7862
35.80	.6II7	.9782	I28I.6400	35.09I6	I253.7002
5I.85	.6II8	.9784	2688.4225	50.7300	2630.3526
58.20	.6I08	.9768	3387.2400	56.8498	3307.6560
70.75	.6084	.9730	5005.5625	68.8400	4870.4I23
92.45	.6096	.9749	8547.0025	90.I295	8332.4727
363.94			2I9I3.3498	354.9I65	2I369.9385

$$b = \frac{I92329.45 - I29I66.I3}{I97220.I5 - I32452.32} = \frac{63I63.32}{64767.83} = .9752\ ,.$$

Diameter = .I255

$n = \frac{I}{.06275} = I5.9$, $C_e = .6240$, $C_a = I.0024$, $C_x = \frac{e}{.62550}$

h	e	C_x	h^2	$h\,C_x$	h^2C_x
4.07	.6084	.9726	I6.5649	3.9585	I6.IIIO
8.62	.6084	.9726	74.3838	8.3838	72.2685
I6.53	.6057	.973I	273.2409	I6.0853	265.8907
22.25	.6086	.9730	495.0635	2I.6493	48I.6958
35.66	.6086	.9730	I27I.6356	34.6972	I237.30I4
43.6I	.6086	.9730	I90I.832I	42.4325	I850.4826
58.I0	.6086	.9730	3382.2400	56.6286	3295.7845
70.56	.6I80	.9880	4978.7I36	68.6549	4844.2883
9I.I0	.6084	.9726	8299.2I00	88.6039	807I.8II6
360.60			20697.8040	34I.0940	20I35.6344

$b = \frac{I8I220.7I - II987.56}{I86280.24 - I22920.36} = \frac{6I633.I5}{63359.88} = .9727$

Diameter =.2083

$n = \frac{I}{.I04I5} = 9.6$, $C_e = .6227$, $C_a = I.0068$, $C_x = \frac{e}{.62695}$

h	e	C_x	h^2	hC_x	h^2C_x
6.78	.5960	.9506	45.9684	6.446I	43.6976
II.54	.5956	.9500	I33.I7I6	I0.9684	II3.6976
I7.59	.5938	.9467	309.408I	I6.7I58	294.0303
23.I7	.5960	.9506	536.8489	22.0254	510.3285
34.77	.5950	.9490	1208.9529	32.9967	1147.2963
46.27	.5960	.9506	3140.9129	43.9843	2985.5718
57.81	.5955	.9498	3341.9961	54.9079	3174.2279
69.42	.5955	.9498	4817.1364	65.9351	4577.2158
93.00	.5952	.9493	8649.0000	88.2849	8210.4957
360.35			22185.4030	342.2562	21056.2200

$b = \frac{21056.22 - 342.26}{22185.40 - 360.35} = \frac{66173.24}{69816.1} = .9478$,

Diameter = .1664

$n = \frac{1}{.0832} = 12$, $C_c = .62333$, $C_a = 1.0043$, $C_x = \frac{e}{.62601}$,

h	e	C_x	h^2	$h\,C_x$	$h^2 C_x$
5.00	.6084	.9718	25.0000	4.8590	24.2950
[illegible]	.6083	.9716	82.4414	8.8221	80.1049
17.79	.6080	.9712	316.4841	17.2777	307.3694
23.24	.6083	.9716	540.0976	22.5800	524.7588
36.12	.6082	.9715	1304.654	35.0906	1267.4417
47.02	.6088	.9724	2210.8804	45.7222	2149.8601
57.70	.6081	.9713	3329.2900	56.0440	3233.7394
69.99	.6080	.9712	4898.6001	67.9743	4757.5204
92.01	.6080	.9712	8465.8401	89.3601	8222.0239
357.95			21173.2931	347.7299	20567.1436

$$b = \frac{185104.293 - 124469.912}{190559.638 - 128128.202} = \frac{60634.381}{62431.44} = .9712 ,$$

ELLIS EXPERIMENTS .

The value of n was determined in the same manner as for the Smith experiments. For values of $\frac{h}{D}$ less than 10 the value of C was corrected to e .

The values in brackets were then averaged since these were determined by fewer observations than the first three.

Tables on page following;-

Diameter = 0.5

h	e	n	C_c	A_a	C_a	C_x
2.1516	.60049	6.4	.6192	51.2	1.0022	.9678
4.1558	.60230	10.4	.6228	83.2	1.0014	.9658
6.3476	.60534	14.8	.6239	118.4	1.0010	.9693
7.3000	.60076	16.7	.6241	133.6	1.0009	.9617
8.0100	.60117	18.1	.6243	145.0	1.0008	.9622
9.0600	.60191	2022	.6244	161.8	1.0007	.9634
10.5150	.60114	23.1	.6246	185.0	1.0007	.9618
11.9700	.59996	26.0	.6246	214.3	1.0006	.9599
12.9600	.60102	28.0	.6247	224.5	100005	.9616
14.4700	.60064	31.0	.6 47	248.3	1.0005	.9610
15.4600	.60077	33.0	.6248	264.2	1.0005	.9611
15.8500	.60535	33.8	.6248		1.0005	.9684
17.2650	.59626	36.6	.6248	293.0	1.0004	.9540

Diameter = 0.5

h	C_x	h^2	$h\,C_x$	h^2C_x
2.1516	.9678	4.6294	2.0823	4.4803
4.1558	.9658	17.2707	4.0137	16.6800
6.3476	.9693	40.2920	6.1527	39.0549
7.6550	.9620	58.5990	7.3641	56.3722
10.5150	.9617	110.5652	10.1123	106.3308
14.3030	.9612	204.5758	13.7480	196.6316
16.5570	.9612	274.1342	15.9146	263.4980
61.6850		710.0663	59.3877	683.0538

$$b = \frac{4781.3766 - 3663.3302}{4970.4641 - 3805.0392} = \frac{1118.0464}{1165.4249} = .9594$$

Diameter = 1.0007

h	e	n	C_c	A_a	C_a	C_x
1.1473	.56829	2.19	.5784	35.0	1.0132	.9868
2.3607	.58939	3.41	.6043	54.6	1.0069	.9672
4.8091	.59032	5.86	.6186	93.9	1.0052	.9494
7.9705	.58318	9.02	.6223	144.1	1.0034	.9340
7.9172	.58912	8.97	.6221	143.5	1.0034	.9438
10.8819	.59431	11.93	.6233	191.0	1.0026	.9510

Diameter = 1.0007

h	C_x	h^2	$h\,C_x$	h^2C_x
1.1473	.9868	1.3163	1.1322	1.2990
2.3607	.9672	5.5729	2.2833	5.3902
4.8091	.9494	23.1274	4.5658	21.9574
7.9705	.9340	63.5209	7.4444	59.3356
7.9172	.9438	62.6821	7.4723	59.1597
10.8819	.9510	118.4157	10.3487	112.6135
35.0867	.	274.6433	33.2467	259.7554

$$b = \frac{259.7554 - 33.2467}{274.6433 - 35.0867} = \frac{392.0154}{416.7833} = .9406$$

Diameter = 2.000

h	e	n	C_c	A_a	C_a	C_x
1.7677	.59[illegible]7	1.41	.5322	45.1	1.0328	1.0715
4.4735	.6041	2.76	.5959	88.4	1.0220	.9920
2.5958	.5963	1.82	.5610	58.2	1.0332	1.0789
5.8335	.6107	3.44	.6018	110.0	1.0175	.9923
6.2333	.6120	3.99	.6098	127.4	1.0153	.9865
8.3432	.6125	4.70	.6137	150.1	1.0129	.9854
9.6381	.6155	5.34	.6170	170.5	1.0115	.9862

Diameter = 2.000

h	C_x	h^2	$h\,C_x$	h^2C_x
1.7677	1.0715	3.1248	1.8941	3.3482
2.5958	1.0289	6.7382	2.6701	6.9310
4.4735	.9920	20.0122	4.4377	19.8521
5.8335	.9923	34.0297	5.7886	33.7678
6.9333	.9885	48.0706	6.8536	47.5181
8.3432	.9854	69.6090	8.2214	68.5928
9.6381	.9862	92.8930	9.5051	91.6111
39.5851		274.4775	39.3706	271.6211

$$b = \frac{271.6211-39.3706}{274.4775-39.5851} = \frac{342.8886}{354.3624} = .9675$$

VARIATION OF b WITH D . The values of b obtained above were then plotted with the values of D, in the upper curve of PLATE VI. The points do not lie very close to any regular curve but it will be seen in general that for small diameters the value of b approaches unity, and its variation with D becomes less as D decreases. It was assumed that the curve approaches a horizontal asymptote at a value of b which is greater than zero, apparently about .930 or .940 . It is also assumed that the curve becomes tangent to the vertical axis at the point $D = 0$ and $b = 1$. The simplest curve of this form which could be was of the form $x + f = \frac{f^3}{f^2 - y^2}$ and which with these variables and moved to the proper axis, gives: $-d + f = \frac{f^3}{f^2 - (1-b)^2}$. But since b is an abstract number and d is in feet a coefficient must be applied to d to make it in the proper scale. Then we

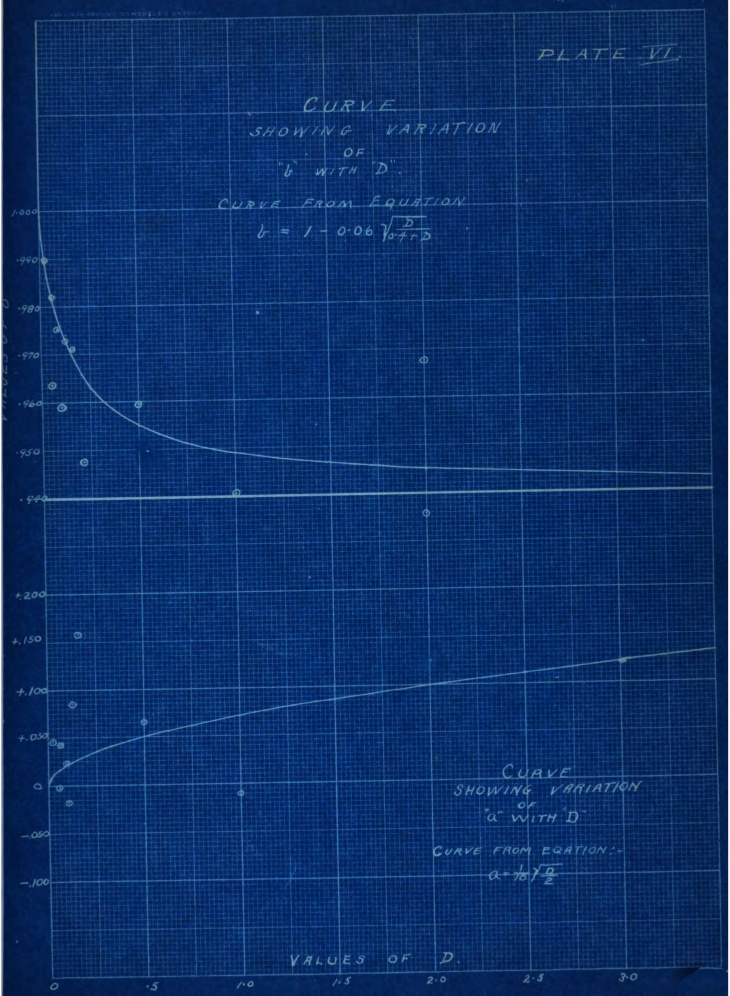


PLATE VI.
CURVE
SHOWING VARIATION
OF
"b" WITH "D".
CURVE FROM EQUATION
b = 1 - 0·06 √(D/(0·4+D))
1.000
·990
·980
·970
·960
·950
·940
+.200
+.150
+.100
+.050
0
-.050
-.100
CURVE
SHOWING VARIATION
OF
"a" WITH "D"
CURVE FROM EQATION:-
a = 1/10 √(D/2)
VALUES OF D.
0
·5
1·0
1·5
2·0
2·5
3·0

may write an equation: $-\ kd + f = \dfrac{f^3}{f^2 - (1 - b)^2}$ or

$$f^2 kd - (1 - b)^2 kd + f^3 - f(1 - b)^2 = f^3$$

therefore;- $(1 - b)^2 = \dfrac{f^2 kd + f^3 - f^3}{kd + f}$

therefore;- $(1 - b) = \sqrt{\dfrac{f^2\ kd}{f + kd}}$, (notice);- $d = D$.

therefore ;- $b = 1 - f\sqrt{\dfrac{D}{\frac{f}{k} + D}}$, and also

$$kD\left[f^2 - (1 - b)^2\right] = f(1 - b)^2$$

Since f occurs in the second power in this equation, to solve directly by least squares would be a very tedious process (it is described on page 183 of Merrimans Least Squares). The method employed was topass the most probable hyperbola through the poinys for the Judd and King experiments (which seem to be the most regular).

Hyperbola $jD(b - g) = 1$.

D	b	D f	D	D b
.0626	.9818	.06146	.00392	.00389
.0854	.9752	.08328	.00729	.00711
.1255	.9727	.12207	.01575	.01532
.1664	.9712	.16161	.02769	.02689
.2084	.9474	.19734	.04329	.04101
.6482		.62576	.09794	.09422

$$g = \frac{.62576 - .09422}{.6482 - .09794} = \frac{.06548}{.06954} = .9416\ .$$

Assuming that the asymptotes of the hyperbola and the cubic are the same, f will equal 1 - g = .0584 and f^2 = .0034106. Then our equation is :- $kD\left[.0034106 - (1-b)^2\right] = .0584(1-b)^2$.

Then the normal equation is :-

$$k\,\Sigma\,D^2\left[.0034106 - (1-b)^2\right] = .0584\,\Sigma\,(1-b)^2 D\left[.00341-(1-b)^2\right]$$

Therefore:- $k = \dfrac{.0584\,\Sigma\,(1-b)^2\,D\left[.0034106 - (1-b)^2\right]}{\Sigma\,D^2\left[.0034106 - (1-b)^2\right]}$

The computation follows:-

VALUE OF k .

D	1-b	.0034-(1-b)² = z	z (1-b)² D	D² z²
.02	.0101	.0033086	.00000000675	.00000000438
.05	.0366	.0020706	.00000013873	.00000001072
.10	.0410	.0017296	.00000006375	.00000002992
.0626	.0182	.0030796	.00000006375	.00000003718
.0854	.0248	.0027956	.00000021840	.00000005699
.1255	.0273	.0026656	.00000012944	.00000011190
.1664	.0288	.0025616	.00000035613	.00000018455
.2083	.0526	.0006436	.00000037098	.00000001793
.5000	.0406	.0017626	.00000145236	.00000077675
1.0009	.0594	.0001174	.00000041446	.00000001402
(2.[illegible]00)	(.0325)	(.0023516)	(.000004887)	(.000022176)
			.0000043079	.0000016979

Since the experiments by Ellis were much less certain thanthose of the other experiments the first eight values were given a weight of two, the next two of one, and the last ones were omitted althgether because of the uncertainty of a large orifice under so low a head. The weighting was accomplished by simply by turning the crank of the adding machine once or twice.

as the case might be, when the columns were added. By this method $k = \frac{.000004307 \times .0584}{.0000016979} = .14817$

then $\frac{f}{k} = \frac{.0584}{.14817} = .39414$ and $b = 1 - .0584\sqrt{\frac{D}{.39414 + D}}$

Since this value is somewhat approximate at any rate we may write it $b = 1 - .06\sqrt{\frac{D}{.4 + D}}$.

NEW VALUES OF a . The values of b computed by this formula will in general be different from the most probable values computed from the separate experiments, that is the hyperbolas will have different asymptotes. The most probable values of a will then have to be computed for each size of orifice on this basis. The equations were of the form $h(C - b) = a$ and if b be known, a is the only variable and can be easily computed. The work is as follows, b being the constant computed in each case from $b = 1 - .06\sqrt{\frac{D}{.4 + D}}$ and then substituted in the normal equation:--

$$a = \frac{\Sigma h C_x - b \Sigma h}{n}$$, values of "a" will be found on the next page .

VALUES OF "a".

D	b	$h\,\theta_x$	h	n	a
0.0200	.9863	6.4138	6.359	3	+.0460
0.0500	.9800	14.9097	15.292	8	-.0010
0.1000	.9732	10.6884	11.071	5	-.0172
0.0626	.9779	331.2795	338.370	9	+.0431
0.0854	.9746	354.9105	363.940	9	+.0238
0.1255	.9707	341.0740	350.600	9	+.0052
0.1664	.9[illegible]	347.7299	357.950	9	+.1570
0.2083	.9648	342.2582	360.350	9	+.6008
0.5000	.9553	59.3877	61.685	7	+.0659
1.0007	.9493	33.2467	35.0867	6	-.0102
2.0000	.9452	39.3706	39.585	7	+.2793

These values of "a" were then plotted on the lower part of PLATE VI. The curves which seemed to fit them most nearly was a parabola of the form:- $a = l\sqrt{D}$. From this the normal is $\Sigma a\sqrt{D} = l\Sigma D$, therefore $l = \frac{\Sigma a\sqrt{D}}{\Sigma D}$. The values for D = .2083 and D = 2.000 were not used because they were so far off the curve.

The computation follows on the next page.

COMPUTATION OF "1".

D	a	$\sqrt{D}$	$a\sqrt{D}$
0.02	.0460	.1414	.00650
0.05	.0010	.2236	.00022
0.10	.0172	.3162	.00544
0.0626	.0431	.2502	.01078
0.0854	.0238	.2922	.00695
0.1255	.0852	.3544	.03019
0.1664	.1570	.4079	.06404
0.5000	.0659	.7071	.04649
1.0007	.0102	1.0003	.01020
2.1106			.14911

$$1 = \frac{.14911}{2.1106} = .07065 = \frac{1}{10}\sqrt{\frac{1}{2}}$$

therefore:- $a = 1\sqrt{D} = .07065\sqrt{D} = \frac{1}{10}\sqrt{\frac{D}{2}}$

THE FINAL FORMULA. Then from the above derivation

$C_x = b + \frac{a}{h}$ with $b = 1 - .06\sqrt{\frac{D}{.4+D}}$ and

$a = \frac{1}{10}\sqrt{\frac{D}{2}}$

Therefore:-

$$C_x = b + \frac{a}{h} = 1 - .06\sqrt{\frac{D}{.4+D}} + \frac{\sqrt{\frac{D}{2}}}{10\ h}.$$

The thesis had for its purpose the determination of the discharge in the terms of the determining factors. This has now been accomplished as is shown by the following formulae:----

$$q = c\,Q$$

$$Q = \int_{+r}^{-r} 2\,dy\,\sqrt{r^2 - y^2}\,\sqrt{2g\,(h - y)} = \frac{C}{U} A\sqrt{2gh}\ .$$

For ordinary heads this is equal to $A\sqrt{2gh}$, and for any case it can be determined with the aid of the table which follows this paragraph.

$$c = C_c\,C_a\,C_x$$

$$C_c = \frac{1}{2}\left(1 + \frac{n^4 - 2n^2 + 1}{4n^4}\right)$$ and may be taken from PLATE IV.

$$C_a = \frac{1}{1 - \frac{C_c A}{A_a}}$$ and is usually nearly unity.

$$C_x = 1 - .06\sqrt{\frac{D}{.4 + D}} + \frac{\sqrt{\frac{D}{2}}}{10\,h}\ .$$

The second term may be taken from the upper curve on PLATE VI. and the expression $0.1\sqrt{\frac{D}{2}}$ from the lower curve.

Then :-

$$D = A\,\frac{\sqrt{2gh}\quad C_c}{1 - \frac{C_c A}{A_a}}\left(1 - .06\sqrt{\frac{D}{.4 + D}} + \frac{\sqrt{\frac{D}{2}}}{10\,h}\right)$$

where $C_c = \frac{1}{2}\left(1 + \frac{n^4 - 2n^2 + 1}{4n^4}\right)$.

Ratio $\frac{C}{c}$ in terms of the ratio $\frac{h}{D}$.

$\frac{h}{D}$	$\frac{C}{c}$	$\frac{h}{D}$	$\frac{C}{c}$	$\frac{h}{D}$	$\frac{C}{c}$
.5	.9604	1.25	.9948	2.2	.9983
.6	.9753	1.3	.9952	2.3	.9984
.625	.9774	1.4	.9960	2.4	.9986
.7	.9823	1.5	.9965	2.5	.9987
.75	.9849	1.6	.9969	3.0	.9991
.8	.9867	1.7	.9973	3.5	.9994
.875	.9892	1.8	.9976	4.0	.9995
.9	.9897	1.9	.9978	4.5	.9996
1.0	.9918	2.0	.9980	5.0	.9997
1.1	.9933	2.1	.9982	10.0	1.0000
1.2	.9944				

Taken from page 22 of Smith's Hydraulics.

COMPARISON OF FORMULAE.

The values of C_x obtained by the formula are compared with those observed, also the values of c given by Hamilton Smith in his table with the observed values of c.

D = .02

h	obser: C_x	comp. C_x	error form.	%	$(\%)^2$	Obs. c	c tab.	error tab.	%	$(\%)^2$
.739	1.0394	1.0004	.0390	3.85	14.82	.650	.650	.000	.00	.00
2.430	1.0077	0.9910	.0167	1.66	2.76	.630	.630	.000	.00	.00
3.190	1.0022	0.9900	.0122	1.22	1.49	.626	.627	.001	.20	.04
			aver. =	2.24	19.07			aver. =	.07	.04

Probable error = $.6745\sqrt{9.535}$ = 2.61 Prob. err. = 0.10

D = .05

h	obser: C_x	comp. C_x	error form.	%	$(\%)^2$	Obs. c	c tab.	error tab.	%	$(\%)^2$
.437	1.0085	1.0[illegible]	.0077	0.76	0.578	.630	.630	.000	.00	.00
.536	1.0027	1.0095	.0068	0.68	0.[illegible]	.627	.626	.001	.20	.04
.720	.9920	1.0020	.0100	0.99	0.980	.620	.622	.002	.30	.09
.929	.9912	.9970	.0058	0.58	0.336	.619	.618	.001	.20	.04
1.740	.9781	.9891	.0110	1.12	1.254	.611	.611	.000	.00	.00
2.740	.9712	.9858	.0146	1.50	2.250	.607	.607	.000	.00	.00
3.570	.9696	.9844	.0148	1.53	2.341	.606	.606	.000	.00	.00
4.630	.9680	.9834	.0154	1.59	2.528	.605	.605	.000	.00	.00
			aver. =	[illegible]	10.729			aver. =	.09	.17

Probable error = $.6745\sqrt{1.041}$ = .69 Prob. err. = .11

D = .10

h	obser. C	comp. C	error form.	% error	$(\%)^2$	observ. c	c tab.	err. tab.	% err.	$(\%)^2$
.661	.9798	1.0070	.2722	2.78	7.726	.612	.612	.000	.00	.00
.900	.9758	.9980	.0122	1.25	1.563	.610	.609	.001	.20	.04
1.730	.9667	.9861	.0194	2.00	4.000	.604	.604	.000	.00	.00
3.180	.9640	.9802	.0162	1.68	2.822	.603	.603	.000	.00	.00
4.600	.9619	.9781	.0162	1.68	2.822	.601	.601	.000	.00	.00

aver. = 1.88 18.935 aver. = .04 .01

Probable error = .6745$\sqrt{4.734}$ = 1.47 Prob. err. = 0.06

D = .0626

4.10	.97 1	.9822	.0088	0.90	0.810	.609	.604	.005	.80	.64
8.99	.9761	.9799	.0038	0.39	0.152	.610	.601	.009	1.50	2.25
16.75	.9763	.9790	.0027	0.28	0.078	.610	.598	.012	2.00	4.00
23.60	.9767	.9787	.0021	0.21	0.044	.611	.597	.014	2.3	5.29
35.33	.9784	.9784	.0006	0.06	0.001	.612	.596	.016	2.6	6.76
43.85	.9783	.9783	.0007	0.07	0.005	.611	.595	.016	2.6	6.76
54.25	.9782	.9782	.0007	0.07	0.005	.612	.594	.018	2.9	8.41
66.80	.9782	.9782	.0018	0.18	0.032	.610	.594	.016	2.6	6.76
84.70	.9781	.9781	.0057	0.58	0.336	.615	.593	.022	3.6	12.96

aver. = 0.30 1.166 aver. = 2.32 53.8

Probable error = .6745$\sqrt{.146}$ = .26 Prob. err. = 1.75

D = 1.0007

1.15	.9868	1.0109	.0241	2.44	5.954	.578	.592	.014	2.40	5.7
2.36	.9672	.9793	.0121	1.25	1.563	.589	.596	.007	1.2	1.4
4.81	.9494	.9640	.0146	1.54	2.372	.590	.596	.004	0.7	0.5
7.97	.9340	.9583	.0243	2.60	6.670	.583	.596	.013	2.2	4.8
7.92	.9438	.9583	.0043	0.48	0.230	.589	.596	.007	1.2	1.2
10.88	.9510	.9558	.0048	0.58	0.281	.594	.595	.001	0.2	0.1

aver. = 1.47 17.070 aver. = 1.32 13.[illegible]

Prob. error = [illegible]

D = .0[illegible]54

h	obs. C_x	com. C_x	err. form.	% err.	$(\%)^2$	obs. c	c tab.	tab. err.	% err.	$(\%)^2$
4.02	.9776	.9797	.0021	0.22	0.048	.612	.602	.010	1.6	2.56
8.87	.9720	.9769	.0049	0.50	0.250	.608	.600	.008	1.3	1.69
17.35	.9743	.9758	.0015	.15	0.023	.609	.597	.012	1.7	2.89
24.65	.9690	.9754	.0064	.56	0.314	.606	.596	.010	1.6	2.56
35.80	.9782	.9752	.0030	.31	0.096	.612	.595	.017	2.8	7.84
51.85	.9784	.9750	.0034	.24	0.058	.612	.594	.018	3.0	9.00
57.20	.9768	.9750	.0018	.18	0.032	.611	.594	.017	2.8	7.84
70.75	.9730	.9749	.0019	.19	0.036	.608	.593	.015	2.5	6.25
92.45	.9749	.9748	.0001	.01	0.000[illegible]	.610	.593	.018	3.0	9.00
			aver.=	0.25	0.857		aver.		2.26	49.67

Prob. error = $.6745\sqrt{.107}$ = 0.22   Prob. err. 1.68

D = .1255

h	obs. C_x	com. C_x	err. form.	% err.	$(\%)^2$	obs. c	c tab.	tab. err.	% err.	$(\%)^2$
4.07	.9726	.9769	.0043	.44	0.194	.608	.600	.008	1.3	1.69
8.62	.9726	.9736	.0010	.10	0.010	.608	.599	.009	1.5	2.25
16.53	.9731	.9722	.0009	.09	0.008m	.609	.597	.012	2.0	4.00
22.25	.9730	.9718	.0012	.12	0.014	.609	.596	.013	2.1	4.41
33.66	.9730	.9714	.0016	.16	0.026	.609	.595	.014	2.3	5.29
43.61	.9730	.9713	.0017	.17	0.029	.609	.594	.015	2.5	6.25
58.20	.9720	.9711	.0019	.19	0.031	.609	.594	.015	2.5	6.25
70.56	.9880	.9711	.0179	(1.81)	(3.276)	.618	.593	.025	(4.0)	(16.00)
91.10	.9726	.9710	.0016	.16	0.026	.608	.592	.016	2.6	6.76
			aver. -	.18	0.338		aver.		2.10	36.90

Prob. error = $.6745\sqrt{.048}$ = 0.15   Prob. error = 1.55

D = .1664

h	obser. C_x	comp. C_x	error	α	$(α)^2$	obser. c	tab	error	α	$(α)^2$
5.00	.9733	.9718	.0015	.15	.026	.608	.599	.009	1.50	2.25
9.08	.9707	.9716	.0089	.09	.008	.608	.597	.011	1.80	3.24
17.79	.9691	.9712	.0021	.22	.048	.608	.596	.012	2.0	4.00
23.24	.9687	.9716	.0029	.30	.090	.608	.596	.012	2.0	4.00
36.12	.9683	.9715	.0032	.33	.109	.608	.595	.013	2.1	4.41
47.02	.9681	.9724	.0043	.44	.194	.608	.594	.015	2.5	6.25
57.71	.9680	.9713	.0033	.34	.116	.608	.594	.014	2.3	5.29
69.99	.9679	.9712	.0033	..34	.116	.608	.593	.015	2.5	6.25
92.01	.9678	.9712	.0034	.35	.123	.608	.592	.016	2.6	6.79
		aver. =		0.28	0.830		aver.	=	2.14	42.52

Prob. error = .6745$\sqrt{.104}$ = 0.22 Prob. err. = 1.56

D = .2063

h	obser. C_x	comp. C_x	error	α	$(α)^2$	obser. c	tab	error	α	$(α)^2$
6.78	.9506	.9696	.0190	1.99	3.960	.596	.598	.002	.3	.09
11.54	.9500	.9676	.0176	1.85	3.423	.596	.597	.001	.2	.04
17.59	.9503	.9666	.0163	1.71	2.924	.596	.596	.000	.0	.00
23.17	.9506	.9662	.0156	1.64	2.690	.596	.596	.0	.0	.00
34.77	.9490	.9657	.0167	1.76	3.098	.595	.595	.000	.0	.00
46.27	.9506	.9655	.0149n	1.57	2.465	.596	.594	.002	.3	.09
57.81	.9496	.9654	.0158	1.64	2.690	.596	.594	.002	.3	.09
69.42	.9498	.9653	.0155	1.63	2.660	.596	.593	.003	.5	.25
93.00	.9493	.9651	.0158	1.66	2.756	.595	.592	.003	.5	.25
		aver. =		1.72	26.666		aver.		0.23	.81

Prob. error = .6745$\sqrt{3.33}$ = 1.23 Prob. err. = 0.21

D = .5

h	obser. Q_x	comp. Q_x	error	%	$(\%)^2$	obser. e	e tab.	err.	%	$(\%)^2$
2.15	.9778	.9785	.0107	1.11	1.231	.600	.598	.002	0.3	0.09
4.16	.9658	.9673	.0015	.15	.023	.602	.597	.005	.8	.64
6.35	.9693	.9632	.0061	.63	.397	.605	.597	.007	1.2	1.44
7.30	.9617	.9621	.0004	.04	.002	.601	.597	.004	.7	.49
8.01	.9622	.9615	.0007	.07	.005	.601	.596	.005	.8	.64
9.86	.9634	.9608	.0026	.27	.073	.602	.596	.006	1.0	1.00
10.51	.9618	.9601	.0019	.20	.040	.601	.596	.005	.8	.64
11.97	.9599	.9595	.0004	.04	.002	.600	.596	.004	.7	.49
12.98	.9616	.9592	.0024	.25	.063	.601	.596	.005	.8	.64
14.47	.9610	.9588	.0022	.23	.053	.601	.596	.005	.8	.64
15.46	.9611	.9585	.0026	.27	.073	.601	.596	.005	.8	.64
15.85	.9684	.9584	.0100	1.03	1.061	.605	.596	.009	1.5	2.25
17.26	.9540	.9582	.0042	.44	.194	.596	.596	.000	.0	.00
		aver.		.36	3.217		aver.		.79	9.60

Probable error = $.6745\sqrt{.268}$ = 0.35 Prob. err. = 0.6[illegible]

D = 2.00

1.77	1.0715	1.0017	.0698	6.50	42.250
2.60	1.2089	.9836	.0453	4.41	19.448
4.47	.9920	.9677	.0243	2.45	6.003
5.83	.9923	.9627	.0296	2.99	8.940
6.93	.9885	.9597	.0288	3.00	9.000
8.34	.9854	.9572	.0282	2.94	8.644
9.64	.9862	.9556	.0306	3.20	10.240
		aver.		3.64	104.525

Probable error = $.6745\sqrt{17.42}$ = 2.82

SUMMARY OF ERRORS IN %.

	Error from formula.			Error from table.		
D	ave.err.	Pro.err.	(P.E.)	ave.err	Pro. err.	(P.E.)
.02	2.24	2.61	6.812	0.07	0.10	0.010
.05	1.09	.69	.476	.09	.11	.012
.10	1.88	1.47	2.161	.04	.06	.004
.062	.30	.26	.068	2.32	1.75	3.063
.085	.25	.25	.063	2.26	1.68	2.822
.125	.18	.15	.023	2.10	1.55	2.403
.166	.28	.22	.048	2.14	1.56	2.434
.208	1.72	1.23	1.513	.23	.21	.044
.50	.36	.35	.123	.79	.60	.360
1.00	.147	1.25	1.573	1.32	1.12	1.254
	0[illegible].[illegible]8	= aver. error	12.850	1.14 =	aver. error	12.406

Prob. error = $.6745\sqrt{1.428}$ = .81 Pro.err. = 0.79

CONCLUSION.

The table by Hamilton Smith Jr. (page 59 of his Hydraulics) has been widely quoted and is the chief basis, at the present time, of estimates of the discharge of circular orifices. As will be seen from the comparisons above, our formula agrees more closely with the results of experiment than the table, although derived partly from the same experiments (i.e. Smiths' and Ellis'). We do not, of course, believe that our formula is perfect, but it is better than most emperical formulae in that it gives values at the extreme values of the variables which are at least possible, if not absolutely correct. For an orifice of zero diameter the cofficient of loss is unity. For heads infinitly large , the cofficient of loss varies between .940 and 1.000 with the diameter. The head can never be less than one-half the diameter or ,what is the same thing , the diameter more than twice the head, and for this condition the cofficent of loss has a maximum value of 1.689 . But with the surface of the water at the top of the orifice the factor n would be unity and the coefficient of contraction .500 . If we assume the cofficient of approach as unity this makes the cofficient of discharge .8445 , which is probably to high but is very possible. For orifices of ¼ inch diameter and upward , we believe that this formula gives a little better results than any table yet published. As stated in the introduction , it is , as far as we have been able to find , the only formula so far published which gives the discharge from a circular orifice in terms of all the factors which influence it and without any variable numerical cofficients, to be determined from the tables.

Head in feet.	1/4" = .0208'	1/2" = .04
0.1	1.091	
0.2	1.038	1.04
0.3	1.021	1.0
0.4	1.012	1.0
0.5	1.007	1.0
0.6	1.004	1.0
0.7	1.001	1.0
0.8	.999	1.0
0.9	.998	.9
1.0	.997	.9
1.1	.996	.9
1.2	.995	.9
1.3	.994	.9
1.4	.994	.9
1.5	.993	.9
1.7	.993	.9
2.0	.992	.98
2.2	.991	.98
2.5	.991	.98
3.0	.990	.98
3.5	.990	.98
4.0	.989	.98
4.5	.989	.98
5.0	.989	.98
5.5	.989	.98
6.0	.988	.98
6.5	.988	.98
7.0	.988	.98
7.5	.988	.98
8.0	.988	.98
10.0	.988	.98
15.0	.987	.98
20.0	.987	.98
30.0	.987	.98
50.0	.987	.98
70.0	.987	.98
100.0	.987	.98
Head	1/4" .0208'	1/2" = .04