THESIS

A THEORETICAL METHOD OF DESIGNING THE PORTS OF A TWO-CYCLE INTERNAL COMBUSTION ENGINE

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A THEORETICAL METHOD
OF
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INTERNAL COMBUSTION ENGINE.

THESIS

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INTRODUCTION.

The subject of cylinder port design for two-cycle engines is seeming ly avoided by all writers on internal combustion motors. The author of this thesis has searched thru innumerable standard works for data on this subject, without success.

The question arises, "How are the port sizes of our modern two-cycle engines determined ?" The author has endeavored in this thesis to present a method of procedure from a theoretical viewpoint. The problem is one of flow of gases thru an orifice of variable area, and under variable conditions of temperature, pressure, specific volume, etc.

A general statement of the functions and operations of the ports will help in showing the problem in hand.

Near the end of the expansion stroke of a two-cycle engine, (in general practise about 45° to 60° crank revolution from outer dead center) the piston starts to uncover a belt of exhaust slots in the cylinder wall. The burned gases in the cylinder at this point are in the neighborhood of 2000° F.abs., and under a pressure of 30 to 70 #/sq. in. abs. They rush out thru these uncovered ports into the exhaust duct, until the pressure in the cylinder has dropped to atmospheric.

When the pressure has dropped to perhaps 18# abs., the piston uncovers a second belt of ports for inlet of the new charge, which has been pre-compressed in the crank case, or other wise, to perhaps 21 # abs. This inlet port in general practise starts to open at about 25° to 45° before dead center, and is of course entirely closed at the same angle after dead center. During this time of perhaps 70° crank revolution, the inlet charge is entering and forcing out the burned products thru the larger exhaust ports. The exhaust ports are of course still open for a time after the inlet port is closed.

Here then is the problem of design:- The inlet port area varien from zero, to its maximum at dead center, and back to zero again. During this time, the inlet charge is entering the cylinder, and the crank case pressure drops accordingly to atmospheric. How large must the inlet port be made, and how soon must it open, for the above events to take place ? Also, having the inlet port designed, how seon must the exhaust port open, and how large must it be, so the pressure in the cylinder shall have dropped below the crank case pressure before the inlet port is uncovered?

NOTATION:-

For the sake of uniformity, and to avoid confusion, the following system of notation will be adhered to thruout the work.

T - abs. temp. in Fahr. degrees.

P - abs. pressure in pounds per sq. ft.

p - abs. pressure in pounds per sq. in.

R - crank radius in inches.

α - angular motion of crank in degrees.

t - time in seconds.

F - port area in sq. ft.

A - port area in sq. in.

l - length of port around the cylinder in inches.

h - mean height of port opening in inches, during revolution of crank between any two given positions.

o - mean height in inches, of port opening from angle at which port starts to open, to any other given angular position.

s - time in seconds for one degree of crank revolution.

W - weight in pounds of total cylinder capacity of gas.

M- weight in pounds of gas escaping from the port per second.

G - weight in pounds of gas escaping in any interval "t".

v - specific volume in cu. ft. per pound of gas.

THERMODYNAMICS OF THE EXHAUST.

The characteristic factors and constants of the gases will vary somewhat with the fuel used, method of governing, fuel-mixture, etc. The gas flow is also affected by a multitude of other disturbing factors, such as:- Shape of ports, conditions of the rounded approach, back pressure in the exhaust pipe, volume of gas behind the port, friction inthe orifice, altitude of the place, varaitions in load, speed, etc. It is easy to see then, that no general formula covering all cases can be derived.

The calculations to follow are based on specific values of the gas constants previously obtained by the author in designing an alcohol engine. While not applying strictly to any other fuel, yet the results thus obtained may safely be used as a guide in designing any ordinary two-cycle motor.

Following are the thermodynamic relations upon which the conclusions later are based.

Change of specific heat with temperature is given by Langen's formulae : $C_p = C_{p_0} + zT$, and $C_v = C_{v_0} + zT$

The relation $Pv = BT$ also holds.

$n = \frac{C_p}{C_v} = 1 + \frac{B}{J\,C_v}$ and $B/J = C_p - C_v = C_{p_0} - C_{v_0}$

The flow of gas thru the ports will follow the adiabatic law. With specific heat varying as above, this adiabatic relation is given thus : * $\log P = K \text{ (constant) } - \frac{C_{p_0}}{C_{v_0}} \log v - .4343 \frac{z\,P\,v}{B \cdot C_{v_0}}$

The following values apply to the exhaust gases of an alcohol engine:

P - at point of release - 7200 #/sq.ft. abs.
v - at point of release - 15.9 cu. ft. per pound.
B - 55.95
B/J - .072
C_{p_0} - .2403
C_{v_0} - .1683
z - .0000495
T - at release, from $Pv = BT$, - 2045 F. abs.

Substituting these values in the adiabatic formula :

$$\log 7200 = K - \frac{.2403}{.1683} \log 15.9 - .4343 \frac{.0000495 \times 7200 \times 15.9}{55.95 \times .1683}$$

Whence $K = 5.5965$

* Wimperis- Internal Combustion Engines.

From twelve values obtained from a variety of sources:- indicator cards, and statements of various authorities, it appears that a mean value of pressure at release is 50 #/sq,in.abs., which is the same as that assumed above.

* Levin says, under proper conditions, the temperature at release should not exceed 1900° F. (2260° F.abs.)

† From Poole's values, the ordinary value of release temperature is 1500° F. (1960° F.abs.) with the maximum for ordinary engines not over 2340° F.abs. These instances are given to show the calculated values preceding to be about those obtained in practise.

Conditions at other points of the release curve may be obtained by a method of successive approximations, from the adiabatic equation before given, which for this case becomes :

$$\log P = 5.5965 - 1.426 \log v - .00000228\, P\, v$$

Thus : assume $v = 33$, find P corresponding. For the first approximation, disregard the last term of the formula, which is small. Then, $\log P = 5.5965 - 1.426 \log 33 = 3.431$, $P = 2695$. Substitute this approximate value of P in the last term of the formula, and solve again'

$$\log P = 5.5965 - 1.426 \log 33 - .00000228 \times 2695 \times 33 = 3.412 ,$$

whence, $P = 2580$ as a closer approximation. Introducing this value in the last term, we find the true value of P to be nearer to 2590 .

The temperature is obtained from the relation : $Pv = BT$,

$$2590 \times 33 = 55.95 \times T , \text{ or } T = 1525 \text{ F. abs.}$$

C_v is obtained from the formula before given, which for this case becomes : $C_v = .1683 + .0000495 \times 1525 = .2438$

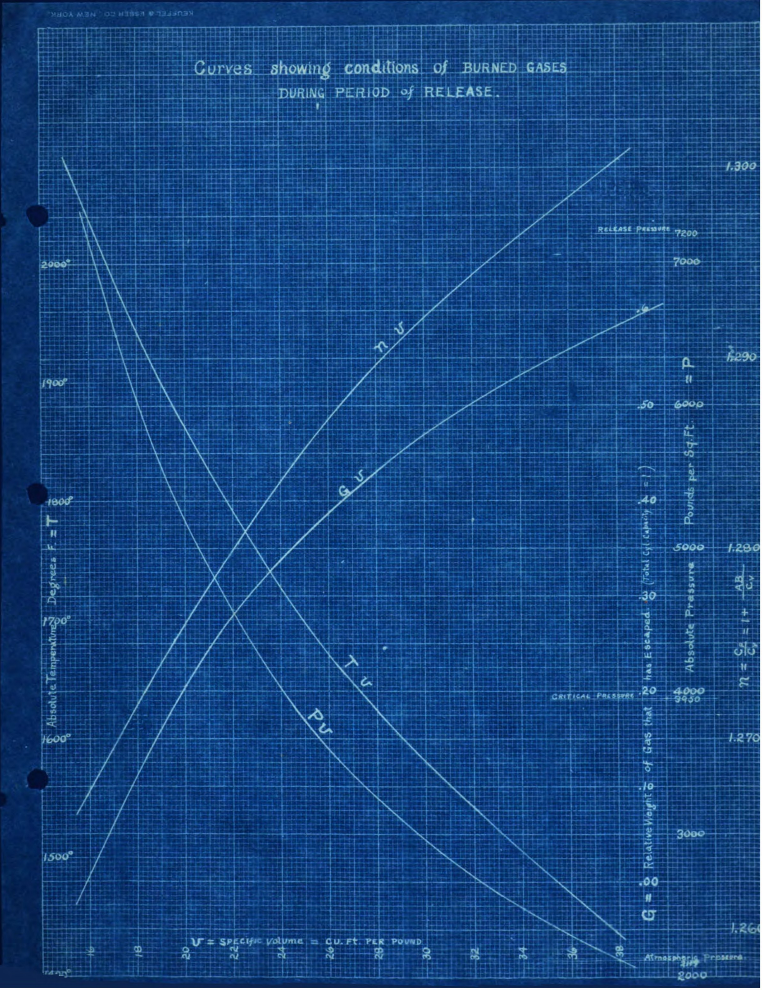
n is obtained from the equation : $n = 1 + \frac{.072}{.2438} = 1.2954$

Let W = total piston displacement in pounds, then G , the weight of gas discharged between $v = 15.9$ and $v = 33$, is given by the relation : $G = W\left(\frac{33 - 15.9}{33}\right) = .518\, W$

The curves following were plotted from values calculated by the method just outlined. All the conditions for any given point of the release period can be found on the curves, if one value is known.

* Levin- Modern Gas Engine & Gas Producer.

† Kent- Mechanical Engineer's Pocket Book.


KEUFFEL & ESSER CO. NEW YORK.
Curves showing conditions of BURNED GASES
DURING PERIOD of RELEASE.
nv
Gv
Tv
Pv
Absolute Temperature Degrees F. = T
2000°
1900°
1800°
1700°
1600°
1500°
1400°
v = SPECIFIC volume = CU. FT. PER POUND
16
18
20
22
24
26
28
30
32
34
36
38
G = Relative Weight of Gas that has Escaped. (Total cyl. capacity = 1)
.00
.10
.20
.30
.40
.50
.6
Absolute Pressure Pounds per Sq. Ft. = P
RELEASE PRESSURE 7200
7000
6000
5000
4000
CRITICAL PRESSURE 3950
3000
Atmospheric Pressure 2117
2000
1.300
1.290
1.280
1.270
1.260
n = Cp/Cv = 1 + AB/Cv

The flow of gas thru the ports may be calculated from the following commonly used formulae, as given in all standard texts and handbooks.

The so-called "Critical Ratio" of $\frac{P_2}{P_1}$ is given by the relation : ✳ $\frac{P_2}{P_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$ This critical ratio shows the pressure relations necessary for maximum discharge for a given value of the reservoir pressure.

Assuming the back pressure in the exhaust pipe to be about 15 #/sq.in.abs., or 2160 #/sq.ft.abs., then the value of P_1 corresponding to the critical ratio, is found from the foregoing formula by transposition :

$P_1 = 2160 \left(\frac{n+1}{2}\right)^{\frac{n}{n-1}}$ which for an approximate mean value of $n = 1.27$, becomes $P_1 = 3920$ #/sq.ft.abs.

For all values of P_1 above 3920 , the following formula applies : ‡

$$M = \left(\frac{2}{n+1}\right)^{\frac{1}{n-1}} \times F \frac{P_1}{\sqrt{T_1}} \sqrt{\frac{2\,g\,n}{B(n+1)}}$$

For all values of P_1 less than 3920 , the formula becomes :

$$M = F\sqrt{2g \times \frac{n}{n-1} \times \frac{P_1}{v_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}} - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right]}$$

In both formulae the notation is as follows :

- M - pounds of gas flowing per second.
- F - effective area of orifice or port in sq. ft.
- n - mean value of C_p/C_v between P_1 and P_2
- P_1 - pounds per sq.ft.abs. on high pressure side of the port.
- P_2 - pounds per sq.ft.abs. on low pressure side of the port.
- T_1 - F.abs. temperature on high pressure side of port.
- v_1 - specific volume or cu.ft.gas per pound on high pressure side.
- B - characteristic constant for the gas, in $Pv = BT$.

✳ Goodenough- Principles of Thermodynamics. Pg. 253.

‡ Ditto - Pg. 255 & 256.

Let G = weight of gas discharged in "t" seconds, and let s = time of one degree of crank revolution in seconds. Then $G = Mt$, and while the crank revolves thru $\alpha°$, $M = \frac{G}{s \times \alpha}$

Let R = crank radius in inches.

l = length of port around the cylinder in inches.

h = mean height of port opening in inches during a crank revolution of $\alpha°$.

Then $F = \frac{l\,h}{144}$, and the formulae for gas flow may be re-stated thus :

(Above the critical point) $\frac{h\,\alpha}{R} = \frac{144\,G}{lsR}\left(\frac{n+1}{2}\right)^{\frac{n+1}{2n-2}}\sqrt{\frac{v_1}{g\,P_1}}$

(Below critical point) $\frac{h\,\alpha}{R} = \frac{144\,G}{lsR\sqrt{2g \times \frac{n}{n-1} \times \frac{P_1}{v_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}} - \left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right]}}$

Divide the pressure range during release into a number of convenient sections, and find mean values of P_1, n, v_1, G, etc. for each section by use of the curves pg. 5a.

Apply the above formula to each section in turn. The results are expressions in terms of l, s, R, and W, all of which depend upon the size, speed, and mechanical design of the engine, and are known approximately or are easily found from data given for a preliminary design.

Each of the expressions for $\frac{h\,\alpha}{R}$ thus found, is a value for "Mean equivalent height of exhaust port opening (per unit crank radius), multiplied by the angular duration of that opening", necessary to produce the pressure drop indicated during that section of the curve.

The summation of these quantities gives the "total $\frac{h\,\alpha}{R}$", (for distinction called $\frac{o\,\alpha}{R}$) necessary to get the pressure in the cylinder down to atmospheric.

The following table of values were taken from the curves on pg. 5a, and calculated by the formulae above.

v_1		P_1		n	G	$\frac{h\ \alpha}{R}$	$\Sigma \frac{h\ \alpha}{R} = \frac{o\ \alpha}{R}$
initial	mean	mean	final	mean			
15.9	16.95	6620	6030	1.264	.112W	.2175 $\frac{W}{IsR}$	.2175 $\frac{W}{IsR}$
18.0	19.5	5370	4860	1.267	.132W	.344 "	.5615 "
21.0	22.7	4360	4020	1.270	.103W	.3215 "	.8830 "
24.4	26.2	3550	3250	1.273	.083W	.2735 "	1.1565 "
28.0	29.0	3100	2950	1.275	.040W	.1563 "	1.3128 "
30.0	31.5	2760	2590	1.276	.045W	.190 "	1.5028 "
33.0	34.5	2435	2300	1.277	.043W	.2885 "	1.7913 "
36.0	37.0	2200	2117	1.279	.024W	.413 "	2.2043 "

The mean height of port opening "o", during a revolution of the crank from α_1 when the port starts to open, to any other position α_2, is a quantity not easily found. It will depend on the ratio of $\frac{L}{R} = \frac{\text{Length of connecting rod}}{\text{Length of crank radius}}$, and upon the values of α_1 and α_2. The relation is very closely expressed thus:

$$\frac{o(\alpha_1-\alpha_2)}{R} = \sin\alpha_1 - \sin\alpha_2 + (\alpha_1-\alpha_2)\left(\frac{R}{4L} - \cos\alpha_1 - \frac{R\sin^2\alpha_1}{2L}\right) + \cdots$$
$$+ \frac{R}{8L}(\sin 2\alpha_2 - \sin 2\alpha_1)$$

Note:- α_1 and α_2 are expressed in "radian measure". The derivation will be found in appendix.

This formula is too complex for common usage, and for convenience the following charts have been plotted from values computed by the formula. The use of the curves is explained there-on. Also, the value of the actual height of port at any given crank angle may be found from the curves on pg. 8c, values on which were computed from the formula :

$$\frac{h}{R} = \text{vers}\,\alpha - \frac{R\sin\alpha}{2L} \quad \text{(also derived in appendix.)}$$

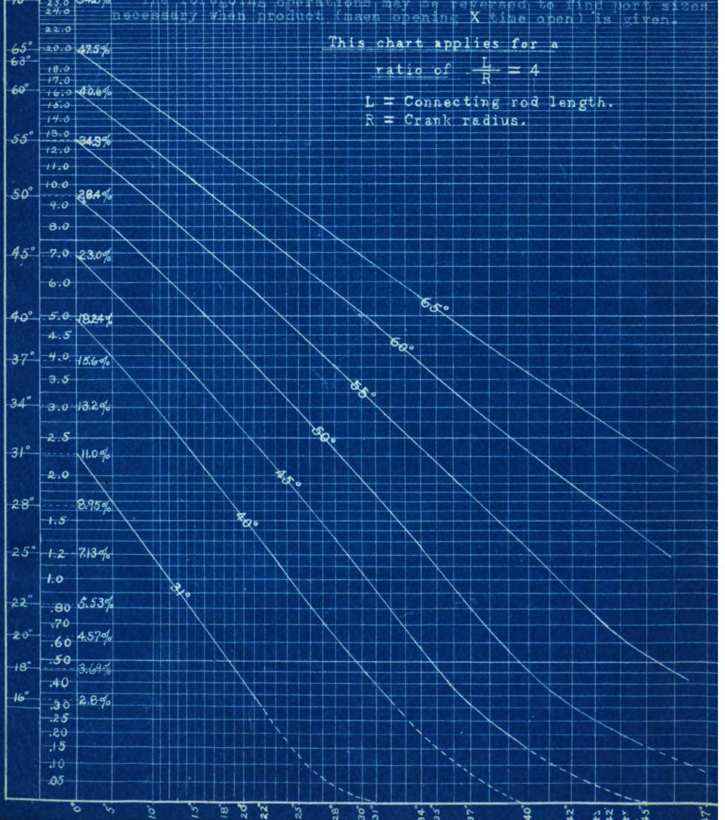
h is the actual height of a port which starts to open $\alpha°$ from the outer dead center. By substituting given values for l, s, R, and W, in the above table, we get a numerical value for $\frac{o\ \alpha}{R}$ from which the required height of port can be quickly found on charts pg. 8a&b.

USE OF THE CHART: Given the sizes of ports, for example - exh. port = 34% of crank radius, (17% of stroke) and inlet = 20% of crank radius, (10% of stroke). Find other data as follows:

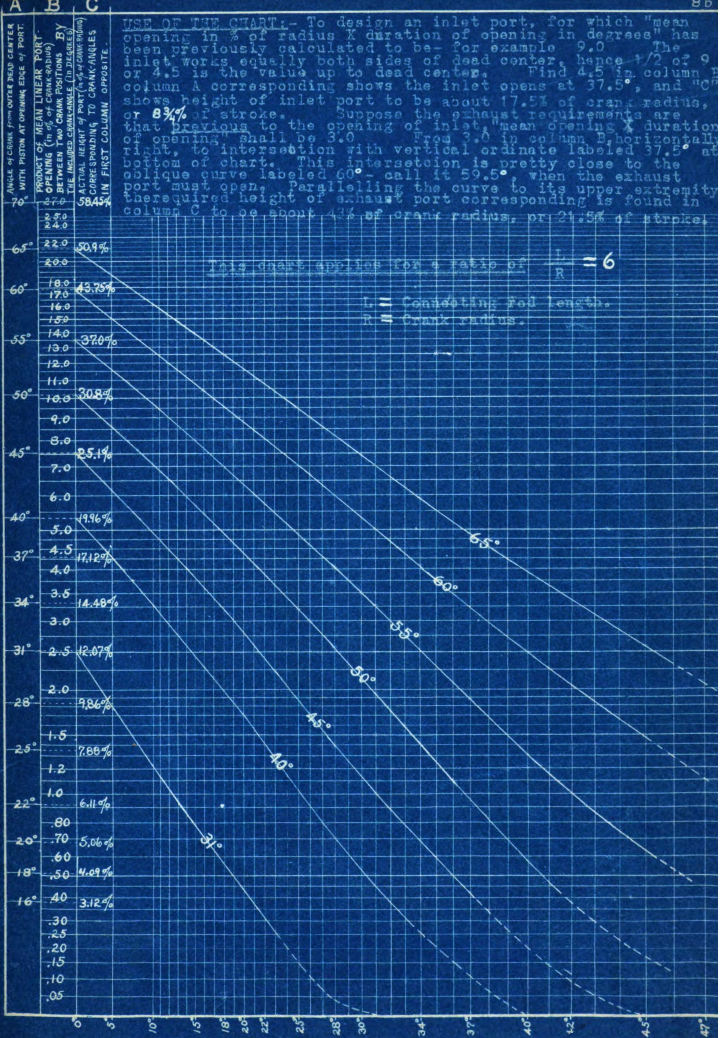
In column labeled "C", find "34%". Horizontally opposite, in column "A" find exh. port opens at 55°. Likewise, opposite "20%" column "A" shows the inlet opens at 42°, and column "B" shows a value about 5.7 - This is mean inlet opening (% of radius) multiplied by the angle during which it is open (42°), whence mean opening, if desired may be easily obtained.

To find mean exhaust opening up to the time when inlet starts to open:- From abscissa "42°" at bottom of chart, up vertically to intersection with exhaust curve labeled "55°", thence horizontally to column "B", which gives .9 as the mean opening multiplied by the angle (13°) during which it is open.

The foregoing operations may be reversed to find port sizes necessary when product (mean opening X time open) is given.



USE OF THE CHART:- To design an inlet port, for which "mean opening in % of radius X duration of opening in degrees" has been previously calculated to be- for example 9.0 The inlet works equally both sides of dead center, hence 1/2 of 9 or 4.5 is the value up to dead center. Find 4.5 in column B column A corresponding shows the inlet opens at 37.5°, and "C" shows height of inlet port to be about 17.5% of crank radius, or 8¾% of stroke. Suppose the exhaust requirements are that previous to the opening of inlet, "mean opening X duration of opening" shall be 3.0 From 3.0 in column B, horizontally right, to intersection with vertical ordinate labeled 37.5° at bottom of chart. This intersetcion is pretty close to the oblique curve labeled 60°- call it 59.5°, when the exhaust port must open. Parallelling the curve to its upper extremity the required height of exhaust port corresponding is found in column C to be about 43% of crank radius, or 21.5% of stroke.

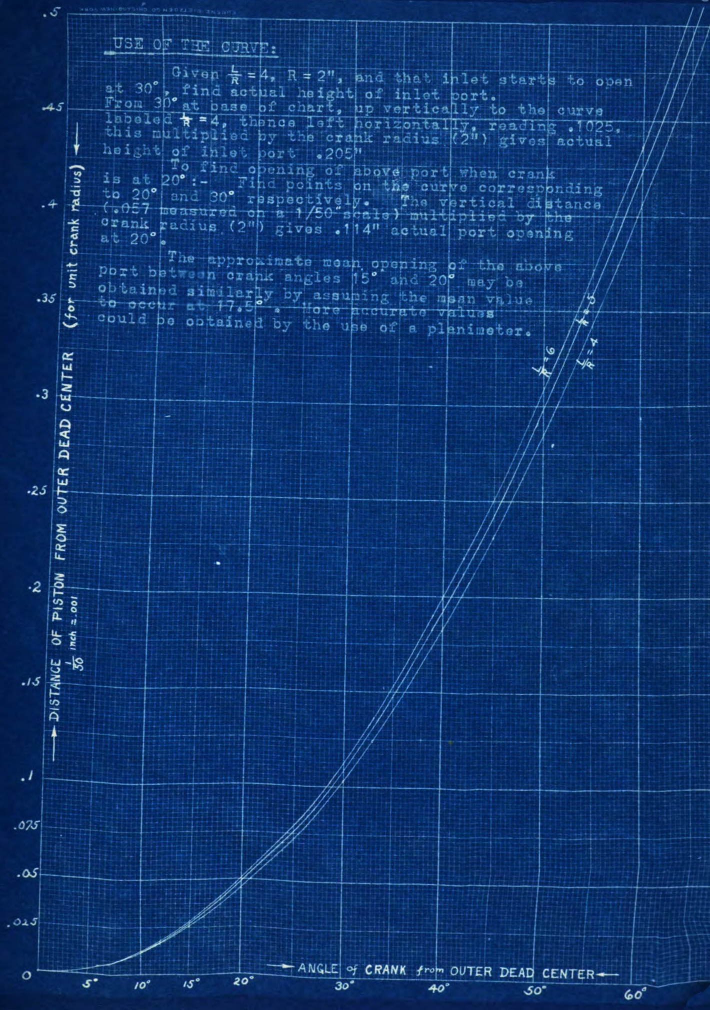


USE OF THE CURVE:

Given $\frac{L}{R} = 4$, R = 2", and that inlet starts to open at 30°, find actual height of inlet port.
From 30° at base of chart, up vertically to the curve labeled $\frac{L}{R} = 4$, thence left horizontally, reading .1025, this multiplied by the crank radius (2") gives actual height of inlet port .205"

To find opening of above port when crank is at 20°:- Find points on the curve corresponding to 20° and 30° respectively. The vertical distance (.057 measured on a 1/50" scale) multiplied by the crank radius (2") gives .114" actual port opening at 20°.

The approximate mean opening of the above port between crank angles 15° and 20° may be obtained similarly by assuming the mean value to occur at 17.5°. More accurate values could be obtained by the use of a planimeter.



INLET PORT CONSIDERATIONS.

The inlet may be a cylinder port, similar to the exhaust port, uncovered by the piston, or it may be a poppet valve, either automatic or mechanically operated.

The design of poppet valves and their mechanism is covered in most standard works on internal combustion engines. In this investigation, the common two- or three-port engine type, with pre-compression in the crank case, will be followed out more especially.

In some types the fuel mixture is pre-compressed, and in others air alone enters the crank case, the fuel being injected later. In either case, the laws applying may be assumed without sensible error to be those applying to air.

Fliegner's equation for air can be used for the inlet flow where the crank case pressure does not exceed 28 #/sq.in.abs.

Let P_c = max. crank case pressure in pounds per sq. in. abs. This rarely exceeds 21# abs. in practise.

P_c', P_c'', etc. = crank case pressure at any given point of the inlet period.

P_a = abs. pressure in the cylinder. This is practically constant at about 15 #/sq.in.abs.

T_c = abs. temperature in Fahr. degrees in the crank case.

Then Fliegner's equation may be stated :

$$M = 1.06\ 1h\sqrt{\frac{P_a\left[\frac{P_c' + P_c''}{2} - P_a\right]}{T_c}}$$

for the mean flow in pounds per second between P_c' and P_c''.

The conditions in the crank case during inlet are approximately isothermal, with T_c probably 530 F. abs. Then Pv is constant,

or, $\frac{P_c - P_c'}{P_c} = \frac{v_c' - v_c}{v_c'} = \frac{G'}{W}$ = percentage of total gas which is discharged during the interval from P_c to P_c'.

Likewise,- $\frac{P_c - P_c''}{P_c} = \frac{G''}{W}$, whence, $\frac{G'' - G'}{W} = \frac{G}{W} = \frac{P_c' - P_c''}{P_c}$, which is the percentage of the total gas which escapes during the interval from P_c' to P_c''.

Then $M = \frac{G}{s\ \alpha} = \frac{W}{s\alpha\ P_c}(P_c' - P_c'') = 1.06\ lh\sqrt{\frac{P_a\left[\left(\frac{P_c' + P_c''}{2}\right) - P_a\right]}{T_c}}$

From which $\frac{h\ \alpha}{R} = \frac{W(P_c' - P_c'')}{1.06\ Rsl\ P_c}\sqrt{\frac{T_c}{P_a\left(\frac{P_c' + P_c''}{2} - P_a\right)}}$

Substituting $P_c = 21$, $T_c = 530$, $P_a = 15$, and denoting mean pressure $\frac{P_c' + P_c''}{2}$ by P_m, the formula reduces to the form :

$$\frac{h\ \alpha}{R} = \frac{.267\ W(P_c' - P_c'')}{R\ s\ l}\sqrt{\frac{1}{P_m - 15}}$$

In this case, "h" is the equivalent mean opening of port, and α is the total angular duration of that opening (before and after dead center) or twice the angle from dead center. Whence, $\frac{o\ h}{R} = 2\left(\frac{o\ \alpha}{R}\text{ to dead center}\right)$ on chart pg. 8a or 8b .

Now divide the inlet curve into sections, each including a small drop of pressure in the crank case, P_m for each section being $\frac{P_c' + P_c''}{2}$. Apply the formula to each section in turn. Sum up these sectional values of $\frac{h\ \alpha}{R}$ getting the total necessary for the entire curve. One half of this total is the value of $\frac{o\ \alpha}{R}$ to dead center for the inlet.

From this, for given values of W, R, s, and l, the height of port and angle of inlet can be quickly found from the charts pg. 8a or 8b.

P_c'	P_c''	P_m	$\frac{h\ \alpha}{R}$
21.00	20.00	20.50	.114 $\frac{W}{Rsl}$
20.00	19.00	19.50	.126 "
19.00	18.00	18.50	.143 "
18.00	17.00	17.50	.169 "
17.00	16.50	16.75	.101 "
16.50	16.00	16.25	.113 "
16.00	15.50	15.75	.154 "
15.50	15.25	15.375	.109 "
15.25	15.15	15.20	.060 "
15.15	15.05	15.10	.084 "
15.05	15.00	15.025	.060 "
			1.233 $\frac{W}{Rsl}$ $\times \frac{1}{2} = .617 \frac{W}{Rsl}$

$$\frac{o\ \alpha}{R} = .617\ \frac{W}{Rsl}$$

FINAL CONSIDERATIONS AND DISCUSSION.

In all the foregoing work, no account has been taken of friction losses, excessive back pressure from the exhaust pipe, vortex action in the cylinder, poor condition of the rounded approach to the ports, anh other disturbing factors. All these will seriously affect the flow of gas, and the designer can only use his best judgement regarding them in the problem.

Probably the easiest way to apply such a factor to the foregoing results, is to add a percentage to the value of W used in the formulae, which will thus increase the size of port obtained.

The contraction due to poor approach to the ports may amount to considerable, according to various authorities.

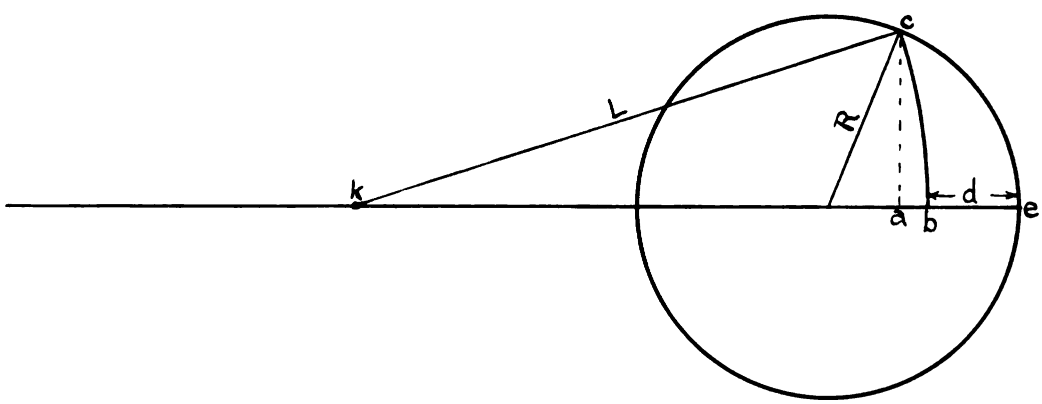
Kent says- 3 to 8% is lost for short well rounded mouthpieces.
20 to 44% " " with a sharp cornered approach.

Church gives- 44% lost with sharp cornered orifice in thin plate.
25% " " " " short tube.
2% " " well rounded mouthpiece.

Probably for most instances in gas engine practise, from 20% to 40% of the efficiency of the ports is lost thru not having rounded edges on the approach side. This fact can be entered in the preceding calculations by giving W in the formulae a value 1/4 to 1/2 larger than the actual piston displacement.

The value given W in the inlet calculations should be somewhat larger than for the exhaust, in order to compensate for the extra friction losses in the "passover port" or connecting passages from the crank case to the inlet port proper. The exact amount to add for this purpose depends entirely upon the mechanical design. Good judgement alone can be used in this connection.

For rough average use, the value of $\frac{o\,\alpha}{R}$ for both inlet and exhaust may be about $2 \cdot \frac{W}{lsR}$.



In the above figure, representing kinematically the working parts of an engine, $ae - ab = d$, $ae = R \text{ vers } \alpha$, $ab = L - ka$, $ka = \sqrt{L^2 - \overline{ca}^2}$, $\overline{ca}^2 = R^2 \sin^2\alpha$, whence $d = R \text{ vers } \alpha - L + \sqrt{L^2 - R^2\sin^2\alpha}$

d = distance of piston from end of stroke when corresponding crank angle is $\alpha°$.

The radical term in the above expression is of a form which cannot easily be integrated.

Expanding the radical by the binomial theorem :

$$\sqrt{L^2 - R^2\sin^2\alpha} = R\sqrt{\left(\frac{L}{R}\right)^2 - \sin^2\alpha} = R\left[\frac{L}{R} - \frac{R\sin^2\alpha}{2L} - \frac{R^3\sin^4\alpha}{8L^3} - \cdots\cdots\right]$$

The third term in this series will be very small for any instance in gas engine practise. Substituting values for $\frac{L}{R}$ and α such as to make the term a maximum within the ordinary limits of gds engine practise, we get a value for $\frac{R^3\sin^4\alpha}{8L^3} = .0026$

The error entailed by dropping all terms after the second inthe above series will then be negligible.

Then $d = R \text{ vers } \alpha - \frac{R^2\sin^2\alpha}{2L}$

The piston movement "m" from k to any later position k_x is given by the difference in values of d for the two positions.

$$m = d_1 - d_x = R \text{ vers } \alpha_1 - R \text{ vers } \alpha_x - \frac{R^2\sin^2\alpha}{2L} + \frac{R^2\sin^2\alpha_x}{2L}$$

or replacing vers α by $(1 - \cos\alpha)$, and simplifying :

$$\frac{m}{R} = \cos\alpha_x - \cos\alpha_1 - \frac{R\sin^2\alpha}{2L} + \frac{R\sin^2\alpha_x}{2L}$$

To find the mean equivalent port opening "o", during revolution from α_1 when the port starts to open, to any other position α_2 :

Mean value of "o" between α_1 and α_2 $= \int_{\alpha_2}^{\alpha_1} \frac{m \, d\alpha}{\alpha_1 - \alpha_2}$

Note :- The limits of integration are interchanged to avoid negative signs, since in this expression α_1 is greater than α_2 .

Substituting value of m :

$$\text{mean } o = \frac{1}{\alpha_1 - \alpha_2} \cdot \int_{\alpha_2}^{\alpha_1} \left(R \cos \alpha_x d\alpha - R \cos \alpha_1 d\alpha - \frac{R^2 \sin^2 \alpha_1 d\alpha - R^2 \sin^2 \alpha_x d\alpha}{2 L} \right)$$

In this expression, α_1 is constant and α_x is variable between the limits α_1 and α_2 .

Integrating, replacing $(\cos \alpha \sin \alpha)$ by its equal $\frac{\sin 2\alpha}{2}$ and simplifying : $\frac{\text{mean } o \, (\alpha_1 - \alpha_2)}{R} = \cdots\cdots$

$$= \sin \alpha_1 - \sin \alpha_2 - (\alpha_1 - \alpha_2) \left(\frac{R}{4 L} - \cos \alpha_1 - \frac{R \sin^2 \alpha_1}{2 L} \right) + \frac{R}{8 L} (\sin 2\alpha_2 - \sin 2\alpha_1)$$

α_1 and α_2 in this expression are of course in radian measure.