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THESIS

AN ANALYSIS OF A CONCRETE ARCH
FOR KENT COUNTY HIGHWAY

J. VAN BUREN

T. B. DIMMICK

1916

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AN ANALYSIS
of a
CONCRETE ARCH
for
KENT COUNTY HIGHWAY



A Thesis Submitted to
The Faculty of
THE MICHIGAN AGRICULTURAL COLLEGE

By

J. Van Buren

and

T.B. Dimmick

Candidates for the Degree of
Bachelor of Science

JUNE 1916

THESIS

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FOREWORD

The purpose of this thesis is to enable us to become more familiar with the theory and analysis of a concrete arch.

INTRODUCTION

THE WRITERS OF THIS THESIS LEARNED OF THIS ARCH BRIDGE THROUGH THE AGENCY OF THE MICHIGAN STATE HIGHWAY DEPARTMENT. THE BRIDGE IS TO BE CONSTRUCTED THIS YEAR ACROSS PLASTER CREEK ON THE GRANDVILLE AVE ROAD IN WYOMING TOWNSHIP KENT COUNTY, MICH.

A DESIGN OF AN ARCH BRIDGE WAS SUBMITTED BY AN INDEPENDENT COMPANY, BUT THE ENGINEERS OF THE STATE HIGHWAY DEPARTMENT SAW FIT TO DISCARD THAT DESIGN AND CHOOSE THE ARCH HEREINAFTER ANALYZED.

IN ORDER TO DETERMINE THE STRESSES ACCURATELY AND TO BECOME FAMILIAR WITH THE ANALYTICAL METHOD OF SOLUTION, THIS METHOD WAS CHOSEN IN PREFERENCE TO THE GRAPHICAL METHOD OF SOLVING STRESSES IN AN ELASTIC ARCH. THE REACTIONS WERE DETERMINED ANALYTICALLY. HAVING FOUND THE REACTIONS, THE THRUSTS, MOMENTS AND SHEARS WERE DETERMINED GRAPHICALLY. UNIT STRESSES WERE COMPUTED BY THE USE OF INFLUENCE LINES.

WE WISH TO THANK THE STATE HIGHWAY DEPARTMENT, MR. L. C. SMITH, MR. C. V. DEWART AND MR. C. H. HALL, ESPECIALLY, FOR THEIR KIND ASSISTANCE IN PROVIDING DRAWINGS AND INFORMATION. WE ARE DEEPLY INDEBTED TO PROF. C. R. MELICK FOR THE METHODS USED AND THE CARE TAKEN IN CHECKING THE WORK.

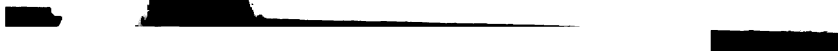
J. VAN BUREN.
T. B. DIMMICK.

Moment of Inertia of Transformed Section

point	t	A_c	A_s	nA_s	A_t	I_c	nI_s	I_t
1	49.3"	49.9	0.06545	18326	51.7	1293.3	965.20	2258.50
3	35.0	35.0	"	"	36.8	446.61	440.26	886.87
5	28.5	28.5	"	"	30.3	241.14	274.99	516.13
7	24.0	24.0	"	"	25.8	144.0	183.26	327.26
9	21.75	21.75	"	"	23.6	107.18	144.33	251.51
11	20.75	20.75	"	"	22.6	93.06	128.53	221.59
13	18.9	18.9	"	"	20.7	70.33	101.71	172.04
15	17.4	17.4	"	"	19.2	54.88	82.26	137.14
17	16.50	16.5	"	"	18.3	46.79	71.58	118.37
19	16.0	16.0	"	"	17.8	42.67	65.97	108.64
21	16.0	16.0	"	"	17.8	42.67	65.97	108.64
23	16.50	16.5	"	"	18.3	46.79	71.58	118.37
25	17.4	17.4	"	"	19.2	54.88	82.26	137.14
27	18.9	18.9	"	"	20.7	70.33	101.71	172.04
29	20.75	20.75	"	"	22.6	93.06	128.53	221.59
31	21.75	21.75	"	"	23.6	107.18	144.33	251.51
33	24.0	24.0	"	"	25.8	144.0	183.26	327.26
35	28.5	28.5	"	"	30.3	241.14	274.99	516.13
37	35.0	35.0	"	"	36.8	446.61	440.26	886.87
39	49.9	49.9	"	"	51.7	1293.3	965.20	2258.50

Calculations for ΔS for $\frac{1}{2}$ Spon. (spon. symmetrical)
Values of X & Y Squared

$\Delta X'$	11.1	1	13.8	20.9	11	21.1
$\Delta Y'$	18.6		16.8	56		49
$\Delta X'^2$	123.21		190.44	436.81		445.21
$\Delta Y'^2$	345.96		282.24	31.36		24.01
Sum	469.17		472.68	468.17		469.22
Log Sum	2.6713302		2.6745672	2.6704036		2.6713765
Log $\Delta S'$	1.3356651		1.3372836	1.3352018		1.3356882
$\Delta S'$	21.660		21.741	21.636		21.661
Total ΔS			43.401			43.293
$\Delta X'$	15.6	3	16.9	21.2	13	21.2
$\Delta Y'$	15.0		13.7	4.4		3.6
$\Delta X'^2$	243.36		285.61	449.44		449.44
$\Delta Y'^2$	225.00		187.69	19.36		12.96
Sum	468.36		473.30	468.80		462.40
Log Sum	2.6705798		2.6751365	2.6709876		2.6650178
Log $\Delta S'$	1.3352899		1.3375682	1.3354938		1.3325083
$\Delta S'$	21.641		21.755	21.652		21.504
Total ΔS			43.396			43.156
$\Delta X'$	17.8	5	18.4	21.3	15	21.4
$\Delta Y'$	12.4		11.1	3.1		2.8
$\Delta X'^2$	316.84		338.56	453.69		457.96
$\Delta Y'^2$	153.76		123.21	9.61		7.84
Sum	470.60		461.77	463.30		465.80
Log Sum	2.6726519		2.6644257	2.6658623		2.6681995
Log $\Delta S'$	1.3363259		1.3322129	1.3329312		1.3340097
$\Delta S'$	21.693		21.484	21.524		21.582
Total ΔS			43.177			43.106
$\Delta X'$	19.0	7	19.9	21.5	17	21.5
$\Delta Y'$	10.0		9.0	2.0		1.35
$\Delta X'^2$	361.00		396.01	462.25		462.25
$\Delta Y'^2$	100.00		81.00	4.00		1.8225
Sum	461.00		477.01	466.25		464.0725
Log Sum	2.6637009		2.6785275	2.6686188		2.6665858
Log $\Delta S'$	1.3318505		1.3392637	1.3343094		1.3332929
$\Delta S'$	21.471		21.840	21.593		21.543
Total ΔS			43.311			43.136
$\Delta X'$	20.0	9	20.9	21.6	19	22.0
$\Delta Y'$	7.9		6.5	7.5		7.5
$\Delta X'^2$	400.00		436.81	466.56		484.00
$\Delta Y'^2$	62.41		42.25	56.25		0.0625
Sum	462.41		479.06	467.125		484.0625
Log Sum	2.6650275		2.6803899	2.6690308		2.6849015
Log $\Delta S'$	1.3325138		1.3401949	1.3345154		1.3424508
$\Delta S'$	21.504		21.888	21.603		22.001
Total ΔS			43.392			43.604



Coordinates of Linear Arch

Point	X	Y	ΔX	ΔY	$\angle \phi$	$\sin \phi$	$\cos \phi$
1	11.1"	18.6"	24.9"	+35.4"	16° 30'	.2840	.9588
3	40.5	50.4	32.5	+28.7	8° 50'	.1536	.9881
5	75.2	76.5	36.2	+23.5	7° 30'	.1305	.9914
7	112.6	97.6	38.9	+19.0	6° 00'	.1046	.9945
9	152.5	114.5	40.9	+14.4	6° 50'	.1190	.9929
11	194.3	126.6	42.0	+10.5	4° 10'	.0727	.9974
13	236.6	135.9	42.4	+8.0	2° 30'	.0436	.9991
15	279.1	142.6	42.7	+5.9	3° 00'	.0523	.9986
17	322.0	147.4	43.0	+3.35	3° 00'	.0523	.9986
19	365.1	149.5	43.6	+1.0	3° 20'	.0581	.9983
21	409.1	149.5	43.6	-1.0	3° 20'	.0581	.9983
23	452.2	147.4	43.0	-3.35	3° 00'	.0523	.9986
25	495.1	142.6	42.7	-5.9	3° 00'	.0523	.9986
27	537.6	135.9	42.4	-8.0	2° 30'	.0436	.9991
29	579.9	126.6	42.0	-10.5	4° 10'	.0727	.9974
31	621.7	114.5	40.9	-14.4	6° 50'	.1190	.9929
33	661.6	97.6	38.9	-19.0	6° 00'	.1046	.9945
35	699.0	76.5	36.2	-23.5	7° 30'	.1305	.9914
37	733.7	50.4	32.5	-28.7	8° 50'	.1536	.9881
39	763.1	18.6	24.9	-35.4	16° 30'	.2840	.9588

Arch Properties

Log 63-16374997 " I-3358201 " $\frac{a^2}{I}$ 82836796 = 0.019217 " $\frac{a^3}{I^2}$ 93290026 = 0.21330 " $\frac{a^{4/3}}{I^{1/3}}$ 03743256 = 2.3677 " $\frac{xy}{I^{3/2}}$ 63985155 = 3.0675 " $\frac{y^2}{I^2}$ 35531925 = 0.35793 " $\frac{y^{4/3}}{I^{1/3}}$ 48227054 = 6.6482	(1)	16364779 23455502 9.2909277 - 0.19590 15794005 - 37.967 3.8678733 - 7376.9 3.6818342 - 4806.6 1.3933614 - 24.738 3.4957957 - 3131.8	(11)	16395263 20359898 9.6035365 - 0.40136 2.2153660 - 164.20 4.8271955 - 67173.1 4.3900072 - 245475 1.7781777 - 60004 3.9528189 - 8970.5	(21)	163774097 24005553 9.2368544 - 0.17253 2.0304363 - 107.26 4.8240162 - 66683.2 4.0892408 - 12281.2 1.2956599 - 19754 3.3544654 - 2262.4	(31)
Log 63-16374997 " I-29978600 " $\frac{a^2}{I}$ 8689589740 = 0.048332 " $\frac{a^3}{I^2}$ 02870447 = 19817 " $\frac{a^{4/3}}{I^{1/3}}$ 19044997 = 80.260 " $\frac{xy}{I^{3/2}}$ 43594752 = 99.879 " $\frac{y^2}{I^2}$ 03920202 = 2.4662 " $\frac{y^{4/3}}{I^{1/3}}$ 20944507 = 12.429	(3)	16350412 22356294 9.3994118 - 0.25085 1.7734265 - 59.351 4.8744412 - 14042.5 3.9064960 - 8066.8 1.5326313 - 34.091 3.6658508 - 4632.9	(13)	16348399 2.0732416 9.5615983 - 0.36442 2.2169289 - 164.79 4.8722595 - 74577 4.3854264 - 24283.9 1.7300958 - 53715 3.8985933 - 7917.6	(23)	16365982 25148929 9.1217053 - 0.13234 1.9423008 - 87.559 4.7628963 - 57829.0 3.9317506 - 8545.8 1.1111551 - 12.917 3.1006049 - 1260.7	(33)
Log 63-16352525 " I-27127591 " $\frac{a^2}{I}$ 89224934 = 0.083635 " $\frac{a^3}{I^2}$ 07987112 = 62908 " $\frac{a^{4/3}}{I^{1/3}}$ 26749290 = 47308 " $\frac{xy}{I^{3/2}}$ 26823726 = 48125 " $\frac{y^2}{I^2}$ 08061548 = 63996 " $\frac{y^{4/3}}{I^{1/3}}$ 26898162 = 489.63	(5)	16345377 2.1371641 9.4973736 - 0.31432 1.9431334 - 87.727 4.3888932 - 24484.6 4.0972529 - 12509.9 1.6514931 - 44822 3.8056126 - 6391.7	(15)	16345377 2.1371641 9.4973736 - 0.31432 2.1020665 - 155.62 4.8067594 - 77047.63 4.3461860 - 22191.5 1.6514931 - 44822 3.8056126 - 6391.7	(25)	16352525 2.7127591 8.9224934 - 0.083635 1.7669706 - 58.475 4.6114478 - 40874.1 3.6506320 - 4473.3 0.8061548 - 6.3996 2.6898162 - 489.63	(35)
Log 63-16365982 " I-25188929 " $\frac{a^2}{I}$ 91217053 = 0.13234 " $\frac{a^3}{I^2}$ 11732437 = 14902 " $\frac{a^{4/3}}{I^{1/3}}$ 32247821 = 16780 " $\frac{xy}{I^{3/2}}$ 31626935 = 14594 " $\frac{y^2}{I^2}$ 1.1111551 = 12.917 " $\frac{y^{4/3}}{I^{1/3}}$ 31006049 = 1260.7	(7)	16348399 20732416 9.5615983 - 0.36442 2.0494542 - 117.34 4.5773101 - 37784.2 4.2379517 - 17294.2 1.7300958 - 53715 3.8985933 - 7917.6	(17)	16350412 2.2356294 9.3994118 - 0.25085 2.1298711 - 134.86 4.8603304 - 72498.7 4.2630906 - 18327.0 1.5326313 - 34.091 3.6658508 - 4632.9	(27)	163774997 2.9478600 8.6895897 - 0.048332 1.5551082 - 35.901 4.4206267 - 26340.7 3.2575387 - 1809.4 0.3920202 - 24662 2.0944507 - 124.29	(37)
Log 63-16374997 " I-24005553 " $\frac{a^2}{I}$ 92368544 = 0.17253 " $\frac{a^3}{I^2}$ 14201242 = 26.310 " $\frac{a^{4/3}}{I^{1/3}}$ 36033946 = 4012.3 " $\frac{xy}{I^{3/2}}$ 39789297 = 3012.5 " $\frac{y^2}{I^2}$ 12956599 = 19754 " $\frac{y^{4/3}}{I^{1/3}}$ 33544654 = 2262.4	(9)	16395263 2.0359888 9.6035365 - 0.40136 2.1659483 - 146.54 4.7283601 - 53500.8 4.3405894 - 2190.3 1.7781777 - 60004 3.9528189 - 8970.5	(19)	16364779 2.3455502 9.2909277 - 0.19540 2.0542008 - 113.32 4.8176339 - 65710.4 4.1567145 - 14345.4 1.3933614 - 24.738 3.4957957 - 3131.8	(29)	16374997 3.3538201 8.2836796 - 0.019217 1.1662611 - 14.664 4.0488426 - 11190.3 2.4357740 - 27276 9.5531925 - 0.35793 0.8227054 - 6.6482	(39)



Summation of Arch Properties

Point	$\frac{\Delta S}{F}$	$X \frac{\Delta S}{F}$	$X^2 \frac{\Delta S}{F}$	$XY \frac{\Delta S}{F}$	$Y \frac{\Delta S}{F}$	$Y^2 \frac{\Delta S}{F}$	$\frac{\Delta S}{F} \sum \sin \phi$	$\frac{\Delta S}{F} \sum \cos \phi$	$\frac{\Delta S}{F} \sum \phi$
1	0.019217	0.2133	2.37	3.97	0.3574	6.648	+0.137, 0.000	0.195	1.284
3	0.048932	1.9817	80.26	99.88	2.4662	124.29	+0.135,	0.120	0.874
5	0.088655	6.2909	473.08	481.25	6.3996	489.63	+0.155, -0.272	0.097	0.965
7	0.13234	14.802	1678.0	1454.4	12.917	1266.7	+0.158,	0.077	1.500
9	0.17253	26.310	4012.3	3012.5	19.754	2262.4	+0.206, -0.385	0.073	0.295
11	0.19540	37.967	7376.9	4806.6	24.738	3181.8	+0.136,	0.034	1.850
13	0.25085	59.351	14042.5	8065.3	34.091	4632.9	+0.087, -0.327	0.017	0.688
15	0.31432	87.727	24484.6	12509.9	44.822	6391.7	+0.116,	0.016	2.220
17	0.36442	117.340	37784.2	17259.6	53.715	7917.6	+0.123, -1.130	0.010	0.653
19	0.40136	146.540	53500.8	21907.3	60.004	8970.5	+0.142,	0.003	2.346
21	0.40136	164.200	67173.1	24547.5	60.004	8970.5	-0.142, -1.395	0.003	0.642
23	0.36442	164.790	74517.7	24289.9	53.715	7917.6	-0.123,	0.010	2.346
25	0.31432	155.620	77047.65	22191.5	44.822	6391.7	-0.116, -1.130	0.016	0.629
27	0.25085	134.860	72498.7	18327.0	34.091	4632.9	-0.087,	0.017	2.020
29	0.19540	113.320	65710.4	14345.4	24.738	3181.8	-0.136, -0.327	0.034	0.596
31	0.17253	107.260	66683.2	12281.2	19.754	2262.4	-0.206,	0.073	1.850
33	0.13234	87.559	57929.0	8545.8	12.917	1260.7	-0.158, -0.385	0.077	1.725
35	0.088655	58.475	40874.1	4473.3	6.3996	489.63	-0.155,	0.097	1.500
37	0.048932	35.901	26340.7	1809.4	2.4662	124.29	-0.135, -0.272	0.120	0.315
39	0.019217	14.667	1119.03	272.76	0.3574	6.648	-0.137,	0.195	0.874
Σ	3.96048	1535.2749	70399.86	200721.56	518.5284	70376.336			0.461
									0.658
									0.000

Arch Constants

$$a_1 = \sum_0^L \frac{\Delta S}{T} = 3966048 \quad \text{Log}(a_1) = 0.5983580$$

$$a_2 = \sum_0^L y \frac{\Delta S}{T} = 518.5284 \quad \text{Log}(a_2) = 2.7147726$$

$$a_3 = -\sum_0^L x \frac{\Delta S}{T} = -1535.2749 \quad \text{Log}(a_3) = 3.1861861$$

$$b_1 = \sum_0^L x \frac{\Delta S}{T} = +1535.2749 \quad \text{Log}(b_1) = 3.1861861$$

$$b_2 = \sum_0^L x y \frac{\Delta S}{T} - \sum_0^L \frac{\Delta x S \sin \phi}{\bar{n}_c} = 200721.56 \quad \text{Log}(b_2) = 5.3025940$$

$$b_3 = -\sum_0^L x^2 \frac{\Delta S}{T} - \sum_0^L \frac{\Delta y S \sin \phi}{\bar{n}_c} = -703401.44 \quad \text{Log}(b_3) = 5.8472081$$

$$C_1 = \sum y \frac{\Delta S}{T} = -518.5284 \quad \text{Log}(c_1) = 2.7147726$$

$$C_2 = -\sum_0^L y^2 \frac{\Delta S}{T} - \sum_0^L \frac{\Delta x C_{21} \phi}{\bar{n}_c} = 70409.592 \quad \text{Log}(c_2) = 4.8476318$$

$$C_3 = \sum_0^L x y \frac{\Delta S}{T} - \frac{\Delta y C_{31} \phi}{\bar{n}_c} = 200721.56 \quad \text{Log}(c_3) = 5.3025940$$

$$a_1/c_1 = 0.0076487 \quad \text{Log}(a_1/c_1) = 7.8835854 - 10$$

$$b_1/c_1 = -2.9608 \quad \text{Log}(b_1/c_1) = 0.4714135$$

$$a_2/c_2 = -0.0073645 \quad \text{Log}(a_2/c_2) = 7.8671408 - 10$$

$$b_2/c_2 = -2.8508 \quad \text{Log}(b_2/c_2) = 0.4549622$$

$$a_3/c_3 = -0.0076488 \quad \text{Log}(a_3/c_3) = 7.8835921 - 10$$

$$b_3/c_3 = -3.5044 \quad \text{Log}(b_3/c_3) = 0.5446091$$

$$A_1 = (a_1/c_1 - a_3/c_3) = 0.0000001 \quad \text{call it zero}$$

$$A_2 = (a_2/c_2 - a_3/c_3) = +0.0002843 \quad \text{Log}(A_2) = 6.4537769 - 10$$

$$B_1 = (b_2/c_2 - b_3/c_3) = -0.5436 \quad \text{Log } B_1 = 9.7352794 - 10$$

$$B_2 = (b_3/c_3 - b_2/c_2) = -0.6536 \quad \text{Log } B_2 = 9.8153120 - 10$$

$$B_1/A_1 = \infty$$

$$B_2/A_2 = -2.298.98 \quad \text{Log} = 3.3615351$$

Arch Properties Due to Loading

Load at 1			5			9			13		
$(x-0)$	23.4	3	37.4	1	41.8	11	42.5	15	42.5	15	
$L_{23}(x-0)$	19683473		1,5728716		1,6211763		1,6283889		1,6283889		
$"(x-0) \times \frac{1}{2}$	0,1579370		0,9465769		0,9121040		0,16772		0,16772		13,3586
$"(x-0) \times \frac{1}{4}$	1,7653920		557,3348		3,2005768		1,5870206		3,5775223		3726,3975
$"(x-0) \times \frac{3}{4}$	12603675		483,0958		3,0145377		10,34,6484		3,2786820		1904,9370
$(x-0)$	64.1	5	77.3	9	84.1	13	85.4	17	85.4	17	
$L_{64}(x-0)$	1,8068580		1,8881795		1,9247960		1,9314579		1,9314579		
$"(x-0) \times \frac{1}{2}$	0,7293514		1,1250339		1,3336569		2,1096485		1,4930282		3,1121468
$"(x-0) \times \frac{1}{4}$	2,6055692		3,3083037		2,033763		4,9914191		4,2009121		10,020,836
$"(x-0) \times \frac{3}{4}$	2,6130128		3,1838394		1,526,9842		3,45744273		3,2615337		4,587,2610
$(x-0)$	101.5	7	119.1	11	126.6	15	128.5	19	128.5	19	
$L_{101}(x-0)$	2,0064660		2,0759118		2,1024537		2,1089631		2,1089631		
$"(x-0) \times \frac{1}{2}$	1,1281713		1,3646395		2,3,27214		1,5996073		1,7124396		5,1579476
$"(x-0) \times \frac{1}{4}$	3,1797097		3,6553123		4,421,8697		4,04557671		4,2748574		18030,39
$"(x-0) \times \frac{3}{4}$	3,1176211		3,4452732		2,946,2358		3,7539268		5,674,4632		7710,5140
$(x-0)$	141.4	9	161.4	13	169.5	17	172.5	21	172.5	21	
$L_{141}(x-0)$	2,1504494		2,2079035		2,2291697		2,2367891		2,2367891		
$"(x-0) \times \frac{1}{2}$	1,3873038		1,6079153		4,0,42719		1,7907680		1,8403256		6,82346
$"(x-0) \times \frac{1}{4}$	3,5705736		3,9613300		9,579,2594		4,2986239		4,4721551		28324,3
$"(x-0) \times \frac{3}{4}$	3,4461003		3,7405346		5,302,2287		3,5392653		4,0149568		10300,69
$(x-0)$	183.2	11	203.9	15	212.6	19	215.6	23	215.6	23	
$L_{183}(x-0)$	2,2029255		2,309472		2,3275633		2,3336488		2,3336488		
$"(x-0) \times \frac{1}{2}$	1,5358532		1,8067908		64,069848		1,9310988		1,8552471		7,8568872
$"(x-0) \times \frac{1}{4}$	3,842360		4,525306		1,7887,5353		4,493,5716		4,5505777		35528,724
$"(x-0) \times \frac{3}{4}$	3,6562869		3,9609103		9,139,2038		4,1057410		4,0637446		1150,854

Arch Properties Due To Loading (Continued)

Arch Properties Due to Loading (Continued)

Load of 1		5		9		13	
(x-a)	23	462.4	2.7	469.2	3.1	462.4	3.5
Log (x-a)	444.1	2.6650178		2.6713580		2.6650178	
" (x-a) $\frac{1}{2}$	2.2061354	2.0644296	115.99304	1.9002124	8.0851076	1.5875112	38.682072
" (x-a) $\frac{1}{3}$	4.8614660	4.7948808	62359.264	4.7017933	50326.392	4.44319884	27038.84
" (x-a) $\frac{1}{4}$	4.3746329	4.1976491	15763.6784	57192.604	9268.5768	3.4711726	2059.17504
(x-a)	25	504.7	2.9	509.1	3.3	497.1	3.7
Log (x-a)	404.0	2.7030333		2.7068031		2.6964438	
" (x-a) $\frac{1}{2}$	2.1822190	1.9939610	9861838	1.8285084	67.374294	1.3860335	243240972
" (x-a) $\frac{1}{3}$	4.8769179	4.7573141	57192.604	4.6491039	44576.2869	4.2515520	17846.3871
" (x-a) $\frac{1}{4}$	4.3363385	4.0863947	12405.2686	38179582	6576.0447	3.0884640	1225.94802
(x-a)	27	546.5	3.1	546.5	3.5	526.5	3.9
Log (x-a)	526.5	2.7375902		2.7375902		2.7213984	
" (x-a) $\frac{1}{2}$	2.1208102	1.9744446	94.887645	1.6600836	45.7174575	1.0050780	10.1177505
" (x-a) $\frac{1}{3}$	4.8512695	4.7680255	58817.590	4.5045608	31956.5875	3.88765395	7720.596
" (x-a) $\frac{1}{4}$	4.2540297	4.0332501	10793.561	3.5437450	3497.3814	2.27475909	188106895
(x-a)	29	586.4	3.3	681.2	3.7	43.1	19
Log (x-a)	568.8	2.7681940		2.76443256		Load of 17	
" (x-a) $\frac{1}{2}$	2.0458873	1.8898993	71.604176	1.4539153	28439278	1.2380138	17.298616
" (x-a) $\frac{1}{3}$	4.8092404	4.7104948	51344.5976	4.3194338	20865.6612	3.8004256	631.5874
" (x-a) $\frac{1}{4}$	4.1485210	3.8793491	7574.5288	3.1563458	1433.35544	3.7126550	2586.1724
(x-a)	31	623.0	3.5	610.6	3.9	87.1	21
Log (x-a)	610.6	2.7950454		2.78575568		1.9400182	
" (x-a) $\frac{1}{2}$	2.0226112	1.7175388	52.183989	1.0694364	11.7339002	1.5435547	34.958456
" (x-a) $\frac{1}{3}$	4.8161921	4.6620160	36476.705	3.9520179	8953.8384	4.1553842	14301.82
" (x-a) $\frac{1}{4}$	4.0814167	3.6012002	3992.07046	2.3894993	218.246758	3.7181959	5826.3484

Arch Properties Due to Loading (Continued)

Load of	5	21	23	17
(X-0) 650.5	35	43.1	1302	23
Log(x-0)	2.8132473	1.6344773	21146110	42447484
"(x-0) ^{1/2}	1.9349526	1.5706502	1.6762033	21455638
"(x-0) ^{1/3}	1.3543481	1.29640262	43313799	6993.693
"(x-0) ^{1/4}	1.0244024	1.0244024	3.647068	
(X-0) 6879	35	86.0	2.5	23
Log(x-0)	2.8375253	1.9344985	173.1	2.2382971
"(x-0) ^{1/2}	1.7600167	1.4318721	2.2382971	54408792
"(x-0) ^{1/3}	1.46044959	1.265630	4.4303636	26937822
"(x-0) ^{1/4}	1.3643601	1.23907178	3.854692	7758.6882
(X-0) 7228	37	128.5	2.7	27
Log(x-0)	2.8588980	2.1089031	215.6	2.3336488
"(x-0) ^{1/2}	1.5484877	1.5083149	1.730606	5408326
"(x-0) ^{1/3}	1.4140062	1.2387742	4.4635199	29075816
"(x-0) ^{1/4}	1.32509182	1.1616202	3.8662801	7350.0196
(X-0) 752.0	39	170.8	2.9	29
Log(x-0)	2.8762178	2.2324879	257.9	2.4114513
"(x-0) ^{1/2}	1.598974	1.5234156	1.7023790	5039366
"(x-0) ^{1/3}	1.40424789	1.2867687	4.4657321	29225228
"(x-0) ^{1/4}	1.32429403	1.16258493	3.8048127	63795302
(X-0) 418	31	12.6	3.1	31
Log(x-0)	1.6211763	2.1024337	2.097	2.4766867
"(x-0) ^{1/2}	1.1580307	1.3392881	1.5644177	51707241
"(x-0) ^{1/3}	1.056116	1.328690	4.3579986	32145882
"(x-0) ^{1/4}	1.0168362	1.3380936	3.622232	5920.2738



Arch Properties Due to Loading (Continued)

Load at 29			25			21			17		
(X-a)	81.7	33	166.5	33	252.5	33	339.6	33	339.6	33	33
$L_{09}(X-a)$	1.912221		2.2214142		2.4022614		2.5309697		2.5309697		
" (X-a) $\frac{1}{2}$	1.0339274	10.812178	1.3431195	22.03461	1.5239667	33.41585	1.6524730	44.8942664	1.6524730	44.8942664	
" (X-a) $\frac{1}{3}$	3.8545229	7133.5703	4.1637150	14578.573	4.3445622	22108.6475	4.4732485	29735.0364	4.4732485	29735.0364	
" (X-a) $\frac{1}{4}$	3.0233772	1053.3189	3.3325693	2150.6805	3.5134165	32.615425	34.421228	4386.6132	34.421228	4386.6132	
(X-a)	119.1	35	203.9	35	289.9	35	377.0	35	377.0	35	35
$L_{09}(X-a)$	2.0759118		2.3094172		2.4622482		2.5763414		2.5763414		
" (X-a) $\frac{1}{2}$	0.9984052	9.9633105	1.2319106	1705.72545	1.3847416	24.2515848	1.49883448	31.537935	1.49883448	31.537935	
" (X-a) $\frac{1}{3}$	3.8428824	6964.3725	4.0763878	11923.0525	4.2292188	16931.9025	4.3433120	22.045075	4.3433120	22.045075	
" (X-a) $\frac{1}{4}$	2.8820666	762.19236	3.1155720	1304.8784	3.2684030	1855.24404	3.3824962	2412.6492	3.3824962	2412.6492	
(X-a)	1538	37	258.6	37	324.6	37	411.7	37	411.7	37	37
$L_{09}(X-a)$	2.1869563		2.3776704		2.5113485		2.6145809		2.6145809		
" (X-a) $\frac{1}{2}$	0.8765460	752.57416	1.0672601	11675.7152	1.2003382	15.8833272	1.3041706	20.1453044	1.3041706	20.1453044	
" (X-a) $\frac{1}{3}$	3.7420645	552.15738	3.9327786	8345.9786	4.0664567	11653.4646	4.1695891	14780.447	4.1695891	14780.447	
" (X-a) $\frac{1}{4}$	2.5789765	379.30156	2.7696906	588.43532	2.9033687	800.52852	3.0066011	1015.33454	3.0066011	1015.33454	
(X-a)	1832	37	268.0	37	354.0	37	444.0	37	444.0	37	37
$L_{09}(X-a)$	2.2628255		2.4281348		2.5490033		2.6445371		2.6445371		
" (X-a) $\frac{1}{2}$	0.5466051	3520.5344	0.7118144	5150.156	0.8326823	6802818	0.9282167	8.4765187	0.9282167	8.4765187	
" (X-a) $\frac{1}{3}$	3.4291866	2.6864448	3.5843959	3969952	3.7152644	5191.056	3.8107382	6468.2904	3.8107382	6468.2904	
" (X-a) $\frac{1}{4}$	1.8161180	65.48176	1.9813273	95.79124	2.1021958	126.53022	2.1977296	157.662373	2.1977296	157.662373	
(X-a)	Load at 33	35	72.4	35	101.5	35	129.4	35	129.4	35	35
$L_{09}(X-a)$	1.5728716		1.8579353		2.0064660		2.099456		2.099456		
" (X-a) $\frac{1}{2}$	0.4953650	31286970	0.5475250	35279972	0.299456	1.9505255	97320269	0.5649798	97320269	0.5649798	
" (X-a) $\frac{1}{3}$	3.3398462	2186.965	3.4430435	2588.4621	3.1727271	1488.396	2.6346084	431.216	2.6346084	431.216	
" (X-a) $\frac{1}{4}$	2.3790264	239.34504	2.4498555	177.81302	1.5596585	36.279145	1.0215398	10.508442	1.0215398	10.508442	



Summation of Arch Properties Due to Loading

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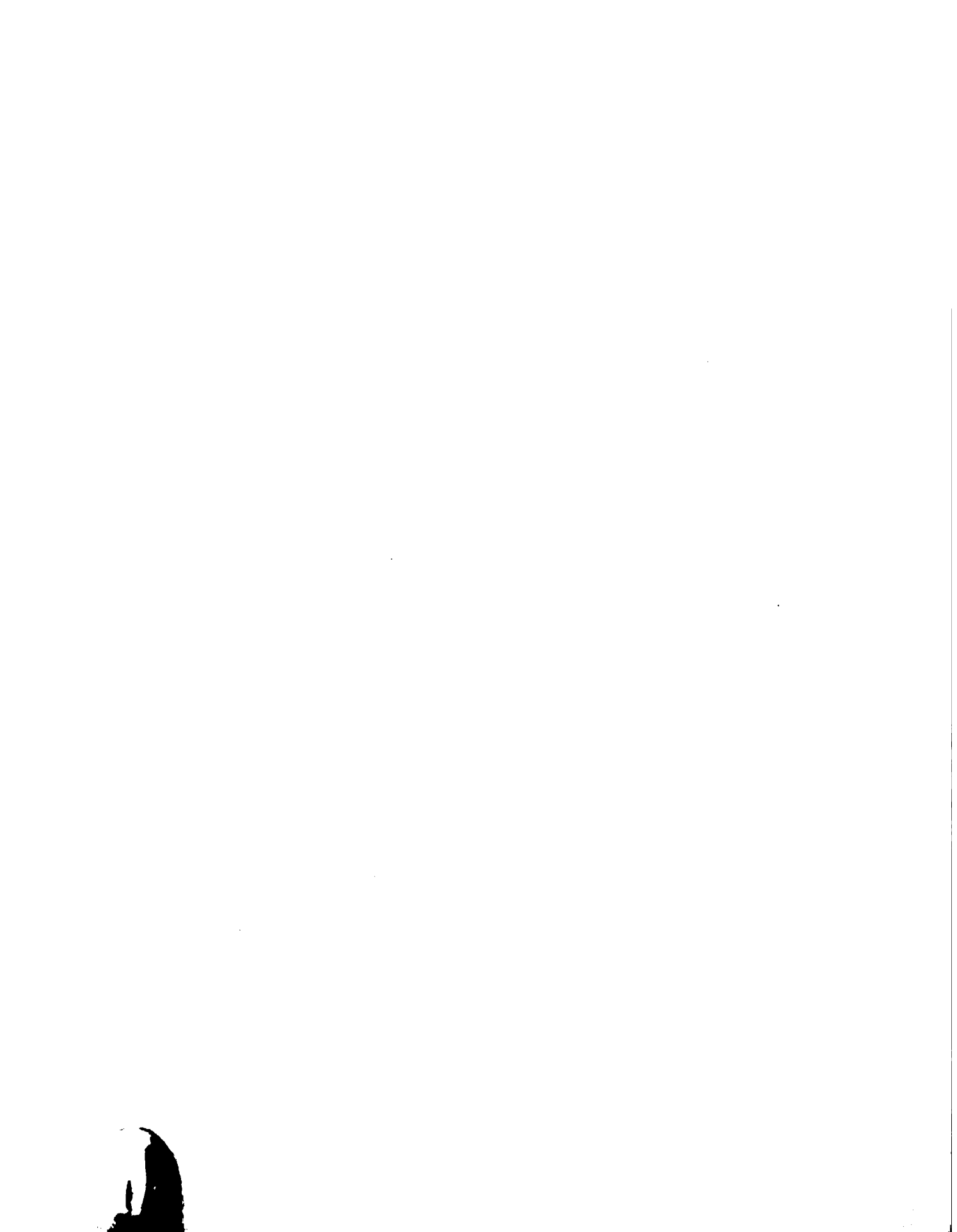
Summation of Arch Properties Due to Loading (Continued)

Point	$(x-o) \frac{d^2 y}{dx^2}$	$(x-o) \frac{dy}{dx}$	$(x-o) \frac{d^2 y}{dx^2}$	$(x-o) \frac{dy}{dx}$	$(x-o) \frac{d^2 y}{dx^2}$	$(x-o) \frac{dy}{dx}$	$(x-o) \frac{d^2 y}{dx^2}$	$(x-o) \frac{dy}{dx}$
	$\frac{Load}{\sigma^2}$	$\frac{Load}{\sigma^2}$	$\frac{Load}{\sigma^2}$	$\frac{Load}{\sigma^2}$	$\frac{Load}{\sigma^2}$	$\frac{Load}{\sigma^2}$	$\frac{Load}{\sigma^2}$	$\frac{Load}{\sigma^2}$
35	3,12870	2,100,365	239,345	7,21175	825,717	10,66113	5731,550	14448,868
37	3,52800	2,508,462	177,813	10,81218	1055,319	17,65692	9,609,536	2,097,782
39	1,95053	1,488,386	36,279	9,96331	762,192	21,84230	13,579,116	2,500,856
Σ	8,60723	6,263,823	443,437	7,52574	379,302	52,03461	14,578,574	2,150,681
15	13,35860	3728,398	1904,935		308,811	11,67518	8,565,979	588,445
17	31,12147	10020,836	4587,261			31,5016	3,029,952	95,791
19	51,57476	18830,390	7710,514			104,99055	67917,760	10187,291
21	69,23460	28324,500	10350,690					
23	78,56895	35528,724	11580,854					
25	81,25172	40227,770	11586,487					
27	75,50585	40592,860	10261,391					
29	67,08082	38902,756	8492,555					
31	66,44130	41305,826	7607,265					
33	56,24450	37212,575	5489,725					
35	38,68207	27038,840	2959,175					
37	24,32410	17846,387	1225944					
39	10,11775	7720,596	168,187					
Σ	663,50649	347280,458	83945,087					



Computations for $V_2 - M_2 - H_2 - M_2$

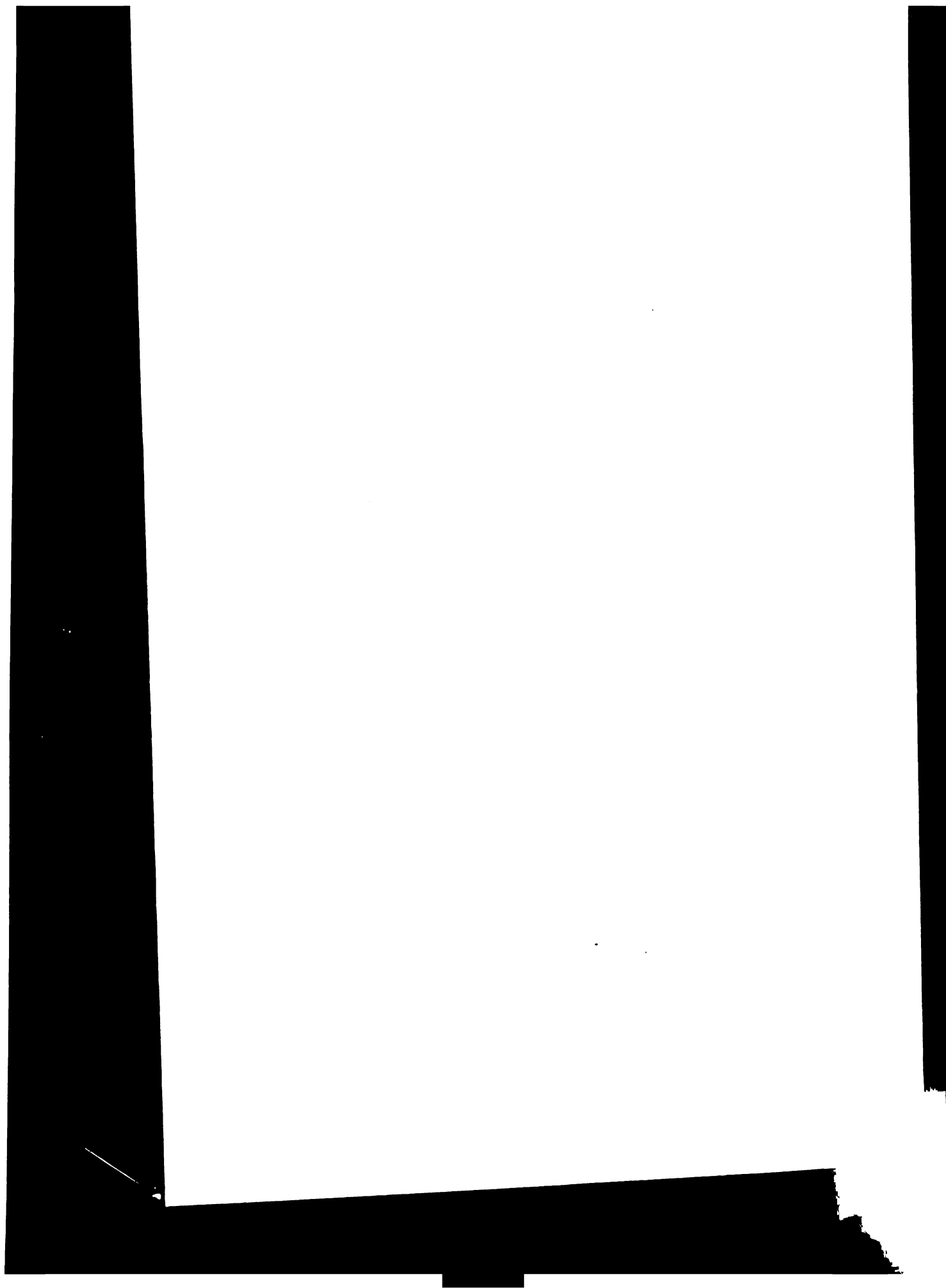
P_{min}	d_1	d_2	d_3	$\log d_1$	$\log d_2$	$\log d_3$	$\log d_4$	$\log d_5$	$\log d_6$	$\log d_7$	$\log d_8$
1	1.19123404	1.94966701	6.96366312	3.1735447	5.2899604	5.8365539	6.4587721	2.87590	0.442286		
5	1.12934002	1.61837183	5.98038165	3.0934607	5.2090783	5.7634055	6.3786281	2.39127	0.3614465		
9	1.30237943	1.22383022	4.10612022	2.9776970	5.6898452	5.8726621	6.2631244	1.83284	0.2422144		
13	1.61350349	1.01944160	3.47281146	2.8218452	4.9238005	5.5466812	6.1070726	1.27960	0.0765087		
17	1.41544003	1.50186250	2.97437558	2.6754465	4.7005845	5.3663997	5.9036939	0.80111	9.8529350		
21	1.21308002	1.25017904	1.95879524	2.1529154	4.3882409	4.8319938	5.3063766	0.20246	9.1603708		
25	1.04480035	1.01861161	6.7318389	2.0211492	4.0080106	4.4282971	4.8766652	0.0752775	8.6419166		
29	1.1902153	1.3057084	2.6810026	1.5319378	3.4695424	3.7968732	4.2200908	0.0165993	7.8083245		
33	1.840213	1.452252	6.264312	0.9348634	2.6553563	2.6349174	7.0372605	6.061066	6.1624385		
37	1.054488	1.0236	4.314879	9.220371	1.010303						
P_{min}	d_9	d_{10}	d_{11}	d_{12}	$C_1 = \frac{d_9}{d_{11}} - \frac{d_{10}}{d_{12}}$	$C_2 = \frac{d_9}{d_{11}} - \frac{d_{10}}{d_{12}}$	$\log C_1$	$\log C_2$	$\log C_3$	C_3/A_2	C_3/R_2
1	1.769036	0.5319539	3.410497	0.543597	0.650461	5.7352792	9.832206	3.3594437	2.2879387		
5	2.298510	0.4468115	2.929621	0.538351	0.631111	9.7310574	9.8001057	5.3463282	2.2198768		
9	1.746680	0.3700681	2.344601	0.511761	0.597921	9.7090663	9.7764310	3.3228662	2.1031339		
13	1.922226	0.2380912	1.730163	0.450563	0.537937	9.6597526	9.7307338	3.2769574	1.8921455		
17	0.712776	0.0638059	1.1582589	0.3571469	0.4454829	9.526507	9.6468282	3.1950513	1.5669465		
21	0.355319	3.8365601	0.5769553	0.2420057	0.3216343	9.38493713	9.5073701	3.0533932	1.313271		
25	0.144670	9.3283998	0.1383711	0.358911	0.1937611	9.1331875	9.2871296	2.8333527	6.8132638		
29	0.04384479	1.257031	0.1335662	0.052907	0.0897235	8.7656015	8.9528046	2.4940277	3.1559443		
33	0.10443168	8.4942732	0.03202836	0.01460966	0.02477288	8.1646502	8.3840487	1.9402718	0.7151877		
37	0.000145377	7.3323234	0.00214543	0.0005593	0.00004457	7.0252650	7.3013080	0.8361211	7.0490635		

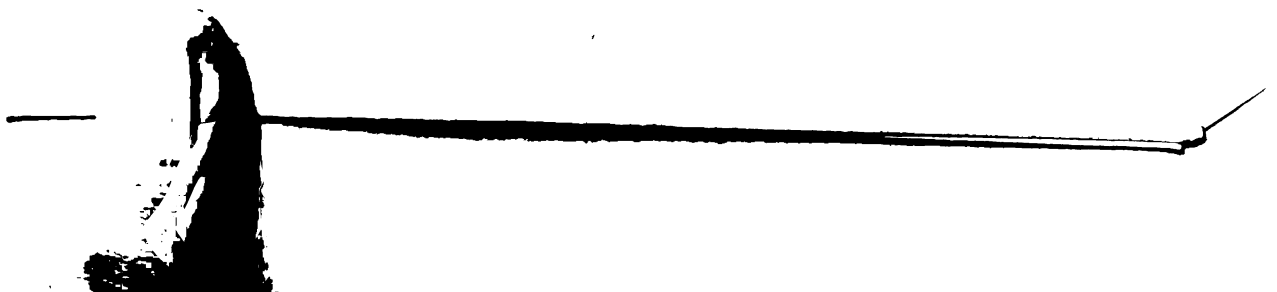


Computations for $V_1 - M_1 - H_1 - M_2$ (Continued)

Star	Log V_1	$V_1 - S_1$	$\log B_1 V_1$	$(B_1 V_1) V_1$	$M_1 - \frac{2}{5} K_1 + \frac{1}{5} V_1$	$\log M_1$	$\log \frac{2}{5} M_1$	$\frac{2}{5} M_1$	$\log \frac{2}{5} K_1$
1	9.533333	3993342	3.1747557	-2298.9665	-11.0279	10.424320	9.924282	+0.084330	0.4714133
5	9.9957820	9903438	3.1616402	-2276.7806	-86.9030	1.7551420	9.6361262	+0.435240	0.4612015
9	9.9132659	9917232	3.1371782	-2164.3263	-61.1930	1.7667010	9.6702871	+0.466597	0.4452004
13	9.3164732	8288500	3.0922269	-190.55096	-13.3641	1.1253365	9.0035219	+0.112210	0.36983067
17	9.6175813	6570466	3.0103633	-151.04452	-66.5013	1.7520561	9.6356415	+0.432164	0.2889948
21	9.6493039	94579481	2.6629052	-1024.7515	-106.5756	2.0270016	9.9105970	-0.8151648	0.1205054
25	9.3979081	24992357	2.6466647	-574.7023	-106.6241	2.0278387	9.9114241	-0.8155357	9.8693216
29	9.0303221	10723064	2.3173397	-246.5216	-69.0720	1.8393712	9.7229566	-0.5283171	9.5017356
33	8.4293708	06287578	1.7553836	-61.766879	-25.36507	1.4042349	9.2878203	-0.1940038	8.9007843
37	7.2893856	00342634	0.6634431	-4.422629	-2.566498	0.4033243	8.2923037	-0.0962299	7.6614031
$\frac{2}{5} V_1$	$H_1 - \frac{2}{5} K_1 - \frac{1}{5} V_1$	$\log K_1$	LVC	$K_1 H_1 (L - 0)$	M_R	M_L / H_L	M_R / H_R		
1	2.9607828	0.0065338	2.0888512	763.1	+0.676	-20680	+126.7		
5	2.5322090	0.1056998	2.6846379	766.7242	+10.8204	-538.4	+102.4		
9	2.7873636	0.4064960	2.6284567	728.5449	62.17	+45.0519	+125.7	+93.9	
13	2.4540590	1.072241	2.8073283	641.6957	5376	+90.7516	-12.46	+84.5	
17	1.9448263	1.5763167	2.7064230	508.6546	4522	+112.9559	+36.85	+71.7	
21	1.3197523	1.7002671	2.5379364	345.0933	3651	+86.5689	+62.75	+50.9	
25	0.7901714	1.3532071	2.2869876	193.5373	2791	+21.0614	+78.8	+15.56	
29	0.3174691	0.7705287	1.9191228	83.0181	1943	-42.2091	+89.6	-54.8	
33	0.07957371	0.2569842	1.3182120	20.8072	112.6	-66.42773	+98.7	-258.8	
37	-0.00577369	0.02431337	0.1788505	1.50956	40.5	-36.424	+105.5	-149.70	







Moments, Shears & Thrusts for 1 Kip Load

Moments, Shears & Thrusts for 1 Kip Load (Continued)

Point	Load at 17			Load at 21		
	Thrusts	Shear	Mom Arm	Thrust	Shear	Mom Arm
0	1.2792 Kips	-1.1336 Kip	+36.0"	1.144 Kip	-1.3356 Kip	+62.7"
1	1.4284	-0.9380	+22.2	1.3248	-1.158	+47.0
5	1.6652	-0.3792	-8.5	1.6448	-0.620	+6.2
9	1.7082	+0.0516	-14.6	1.7500	-0.1876	-11.6
13	1.6668	+0.3800	-1.0	1.7532	+0.1524	-11.0
17	1.6160	+0.5516	+23.2	1.7248	+0.3457	+0.1
20	1.5763	-0.4472	+6.5	1.7006	+0.4457	+14.7
21	1.5828	-0.3436	+2.0	1.702	+0.4808	+19.1
25	1.6076	-0.1332	-10.5	1.7576	-0.3248	-0.2
29	1.6120	+0.0488	-12.5	1.7836	-0.1220	-0.34
33	1.5640	+0.0936	-1.5	1.768	+0.2608	-9.3
37	1.4128	+0.7848	+30.0	1.6444	+0.7076	+14.4
40	1.0228	+1.2492	+71.8	1.2648	+1.2648	+51.4

Point	Load at 25			Load at 29		
	Thrust	Shear	Mom Arm	Thrust	Shear	Mom Arm
0	0.8164 Kip	-1.1084 Kip	+78.8"	0.4320 Kip	-0.6456 Kip	+89.7"
1	0.968	-0.792	+62.25	0.5212	-0.5764	+72.6
5	1.248	-0.578	+6.1	0.6884	-0.358	+23.7
9	1.354	-0.2468	-7.4	0.7568	-0.172	-3.4
13	1.3768	+0.0188	-13.2	0.7768	-0.024	-13.2
17	1.3668	+0.1684	-8.9	0.7768	+0.060	-12.8
20	1.3532	+0.2500	+1.0	0.7705	+0.1072	-6.2
21	1.3480	+0.2780	+5.4	0.71672	+0.122	-2.9
25	1.4184	+0.4284	+28.0	0.7488	+0.208	+13.8
29	1.4924	-0.3988	-2.5	0.8876	+0.283	+43.9
33	1.5436	-0.0696	-18.5	1.0852	-0.460	-21.4
37	1.5108	+0.3304	-11.5	1.1672	-0.164	-58.1
40	1.2808	+0.8640	+15.9	1.1448	+0.2768	-54.6



Moments, Shears & Thrusts for 1 Kip Load

Point	Load of 33			Load of 1 kip			Load at 37		
	Thrusts	Shear	Mom. Arm	Thrust	Shear	Mem. Arm	Thrust	Shear	Mem. Arm
0	0.1364 _{Kip}	-0.2168 _{Kip}	+98.8"	+25.39 _{Kip}	0.012 _{Kip}	+105.7"	+25.7 _{Kip}	-0.021 _{Kip}	+105.7"
1	0.1680	-0.1943	+81.4	+20.92	0.0196	+88.0	+2.14	-0.024	+88.0
5	0.2204	-0.0240	+30.3	+7.78	0.028	+35.3	+0.8385	-0.016	+35.3
9	0.2484	-0.0640	+0.5	+0.128	0.032	+3.6	+0.0876	-0.008	+3.6
13	0.2572	-0.0144	-12.3	-3.16	0.0322	-11.3	-0.2749	-0.0036	-11.3
17	0.2568	+0.0124	-14.7	-3.778	0.0324	-16.0	-0.3892	+0.0004	-16.0
20	0.2570	+0.0262	-10.2	-2.62	0.0243	-12.9	-0.3137	+0.0019	-12.9
21	0.2552	+0.0280	-7.7	-1.979	0.0322	-11.0	-0.2676	+0.0038	-11.0
25	0.2500	+0.0632	+8.0	+2.056	0.0312	+2.8	+0.0681	+0.0084	+2.8
29	0.2480	+0.0900	+33.2	+8.532	0.0296	+26.0	+0.6312	+0.012	+26.0
33	0.2460	+0.1458	+71.0	+18.25	0.0272	+61.7	+1.50	+0.0172	+61.7
37	0.2520	-0.1288	-156.7	-40.27	0.0208	+114.7	+2.90	-0.0236	+114.7
40	0.9828	-0.2148	-243.7	-62.6	0.9016	-149.9	-36.4	-0.4252	-149.9



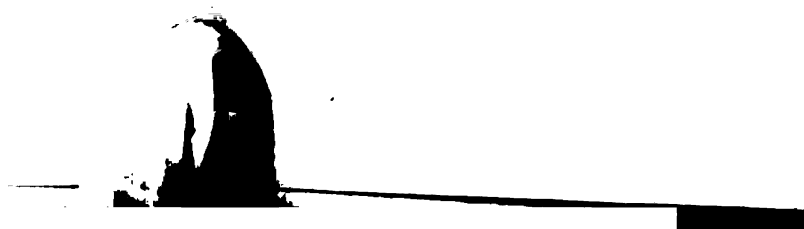
Unit Stress Due to a Load of 1000# in $\frac{1}{32}$ in.

Point	Load of 1			Load of 5		
	f_c	f_s	f_t	f_c	f_s	f_t
0	-68.5 $\frac{1}{16}$ " + 16.37	-95.0 $\frac{1}{16}$ " + 24.9 $\frac{1}{16}$ "	+ 136.0 $\frac{1}{16}$ " + 30.5 $\frac{1}{16}$ "	-335.5 $\frac{1}{16}$ " - 418.5	+ 424.5 $\frac{1}{16}$ " + 552.5	+ 598.0 $\frac{1}{16}$ " + 764.0
1	+ 16.37 + 1.276	- 1.250 + 16.48	- 16.121 + 11.35	- 243.8 + 209.56	+ 348.0 + 245.7	- 338.0 - 245.5
5	+ 0.972	+ 11.95	- 11.35	+ 98.57	- 88.63	+ 119.5
9	- 0.0625	+ 0.621	+ 1.381	- 19.33	+ 30.75	- 198.7
13	- 0.540	+ 0.598	+ 6.03	- 86.55	+ 98.45	+ 113.0
17	- 0.62	+ 0.68	+ 7.56	- 101.56	+ 113.44	- 119.7
21	- 0.6996	+ 0.7604	+ 8.66	- 93.08	+ 104.12	- 105.7
25	- 0.405	+ 0.461	+ 5.43	- 27.53	+ 36.73	- 31.6
29	+ 0.0864	- 0.0392	- 6.406	+ 62.83	- 55.07	+ 79.6
33	+ 0.8136	+ 0.574	- 7.13	+ 117.96	- 113.24	- 150.0
37	+ 0.820	- 0.796	- 10.54	+ 81.36	- 79.64	- 112.0
40	+ 0.5035	- 0.5025	- 7.07			
Point	Load of 9			Load of 13		
	f_c	f_s	f_t	f_c	f_s	f_t
0	-437.5	+ 470.5	+ 662.0	- 71.9	+ 109.3	+ 155.2
1	- 631.6	+ 692.4	+ 930.0	- 238.1	+ 287.9	+ 400.0
5	- 707.9	- 906.0	+ 1000.0	- 835.2	+ 924.8	- 1065.0
9	+ 1187.5	+ 1446.8	- 1388.0	- 334.0	+ 444.0	+ 560.0
13	+ 654.6	- 609.4	- 702.0	+ 2187.0	- 2680.3	- 2430.0
17	+ 93.7	- 41.1	- 372	+ 79.0	- 67.4	- 745.0
20	- 273.7	+ 328.3	+ 379.0	- 17.1	+ 29.1	+ 350.0
21	- 360.7	+ 415.3	+ 476.0	- 45.4	+ 57.4	+ 66.0
25	- 422.6	+ 473.4	+ 555.0	- 86.1	+ 97.5	+ 114.0
29	- 473.6	+ 216.4	+ 268.0	- 50.0	+ 79.6	+ 736.0
33	+ 200.6	- 165.0	- 202.0	+ 23.0	- 15.0	- 177.0
37	+ 476.0	- 454.0	- 601.0	+ 88.1	- 83.1	- 1098.0
40	+ 346.2	- 337.8	- 475.0	+ 686.0	- 666	- 935.0



Unit Stress Due to a Load of 1000#

Point	Load at 17			Load at 21		
	f_c	f'_c	f'_s	f_c	f'_c	f'_s
0	+464.4 $\frac{W}{D^2}$	-40.4 $\frac{W}{D^2}$	+6260 $\frac{W}{D^2}$	+816 $\frac{W}{D^2}$	-772 $\frac{W}{D^2}$	+11490 $\frac{W}{D^2}$
1	+414	-358	+5750	+906	-855	+12550
5	-317	+427	-3980	+347	-237	+4590
9	-925	+1069	-11140	-779	+927	+11550
13	-6	+166	+180	-943	+1113	-10870
17	+2620 +2620 +2620	-2433 -2433 -2433	430200	+1061	-82.3	+1550
20	+844.6	-667.4	+9740	+1926	-1734	+22300
21	+322	-144	+3960	+2485	-2295	+28350
25	-936	+1104	-10500	+69.9	+113.1	+1124
29	-845	+996	-10130	-853	+1011	-10100
33	-26	+140	+1995	-573	+650	-6230
37	+972	-896	+12950	+528	-438	+7100
40	+860	-828	+12040	+670	-632	+9290
Point	Load at 25			Load at 29		
	f_c	f'_c	f'_s	f_c	f'_c	f'_s
0	+807.9 $\frac{W}{D^2}$	-782.9 $\frac{W}{D^2}$	+1163 $\frac{W}{D^2}$	+521.1 $\frac{W}{D^2}$	-507.9 $\frac{W}{D^2}$	+7330 $\frac{W}{D^2}$
1	+950.7	-913.3	+13136	+629.4	-607.9	+8701
5	+642.7	-560.1	+8327	+529.3	-483.8	+6872
9	-376.3	+491.2	-4448	-81.1	+145.3	-904
13	-914.5	+1047.5	-10603	-521.0	+595.9	-6044
17	-765.0	+915.0	-8420	-645.7	+730.3	-7180
20	+175.6	-472	+226	-308.0	+394.3	-3305
21	+624.0	-472	+1190	-121.3	+207.5	-1205
25	+2448 +2448 +2448	-2348 -2348 -2348	14820	+713.8	-635.8	+8376
29	-92	+224	-931	+162.3	-154.4	+1923
33	-880	+980	-1580	-562.9	+647.1	-6930
37	-266	+348	-3470	-851.5	+914.9	-11260
40	+180	-1406	+2550	-295.9	+331.1	-4138

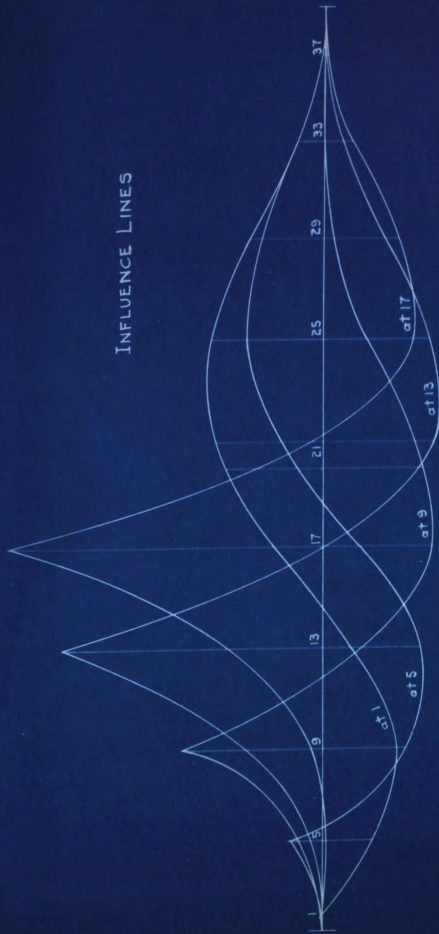


Unit Stress Due to a Load of 1000#

Point	Load of 33			Load of 37		
	f_c	f'_c	f_s	f_c	f'_c	f_s
0	+191.2 $\frac{lb}{in^2}$	-187.0 $\frac{lb}{in^2}$	+2689 $\frac{lb}{in^2}$	+191.7 $\frac{lb}{in^2}$	-19.3 $\frac{lb}{in^2}$	+272 $\frac{lb}{in^2}$
1	+234.4	-228.0	+3240	+240	-23.3	+335
5	+222.0	-207.4	+2078	+246	-22.8	+319
9	+161	+5.0	+226	+51	-2.4	+67
13	-161.0	+165.8	-1865	-13.5	+16.6	-155
17	-2493	+2773	-2782	-25.3	+28.9	-282
20	-1783	+2071	-1953	-21.7	+24.4	-239
21	-131.4	+160.1	-1426	-17.9	+21.5	-194
25	+143.4	-117.4	+1701	+59	-2.7	+74
29	+410.5	-388.5	+4995	+30.9	-28.3	+337
33	+6781	-661.3	+8900	+56.1	-53.9	+704
37	-771.4	+816.6	-10218	+55.7	-54.5	+740
40	-457.1	+481.3	-6328	+73.6	-36.8	+1009
				-257.4	+2850	-3603
						+4019



INFLUENCE LINES

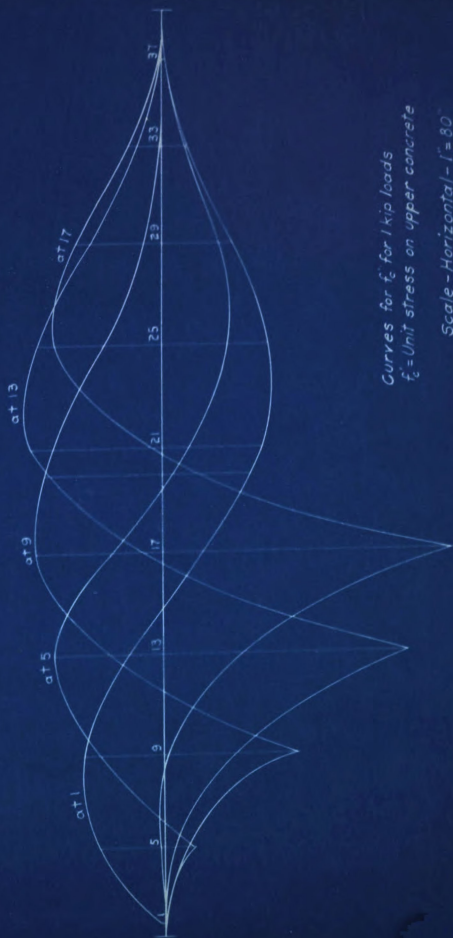


Curves for f_c for 1 kip loads
 f_c = Unit stress for lower concrete

Scale - Horizontal - $1" = 80"$
 - Vertical - $1" = \frac{4}{5}$ kips



INFLUENCE LINES

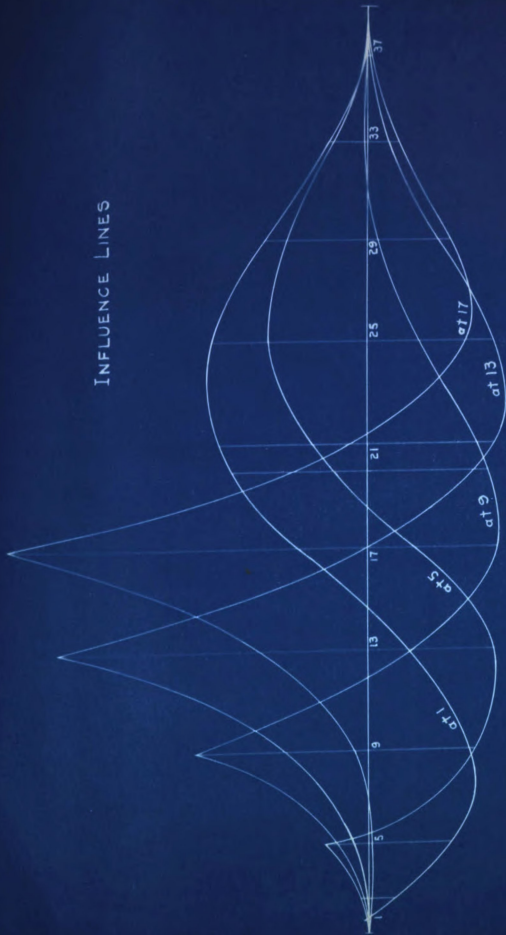


Curves for f_c'' for 1 kip loads
 f_c'' = Unit stress on upper concrete

Scale - Horizontal - $1'' = 80'$
 - Vertical - $1'' = \frac{4}{5}$ kips



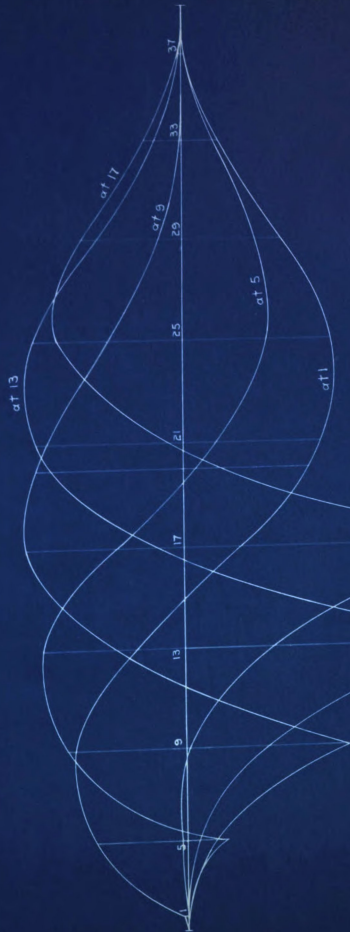
INFLUENCE LINES



Curves for f_s for 1 kip loads
 f_s = Unit stress on lower steel

Scale - Horizontal - $l' = 80''$
 - Vertical - $l = 8$ kips

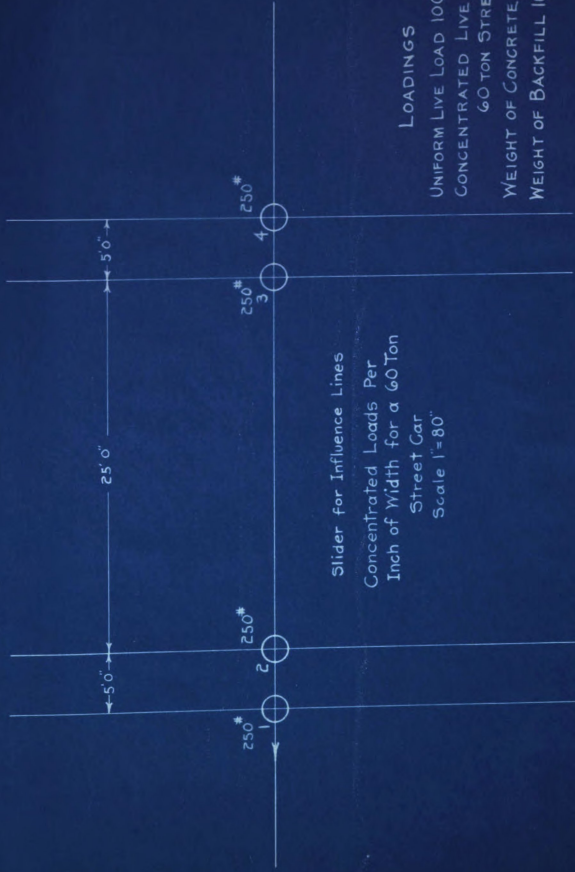
INFLUENCE LINES



Curves for f'_s for 1 kip loads
 f'_s = Unit stress on upper steel

Scale - Horizontal - $1" = 80'$
 - Vertical - $1" = 8 \text{ kips}$





Slider for Influence Lines
Concentrated Loads Per
Inch of Width for a 60 Ton
Street Car
Scale $1''=80'$

LOADINGS

UNIFORM LIVE LOAD $100^{\#}/\text{sq. ft.}$
CONCENTRATED LIVE LOAD
60 TON STREET CAR
WEIGHT OF CONCRETE $150^{\#}/\text{cu. ft.}$
WEIGHT OF BACKFILL $100^{\#}/\text{cu. ft.}$



Stresses Due to Dead Load in lbs./sq.in.

		Stress at 1		Stress at 9		Stress at 17	
Load at	Load	ordinate	f/c	ordinate	f/c	ordinate	f/c
1	689#	+16.69*	+11.5*	+0.97*	+0.67	-0.54	-0.37*
5	678	-718.5	-285.0	+209.5	+142.0	-19.33	-13.12
9	544	-631.6	-343.0	+1187.5	+646.0	+93.7	+60.5
13	408	-238.1	-97.7	-334	-137.0	+79.0	+32.4
17	339	+414.0	+140.0	-925	-313.0	+2628	+890
21	305	+906.0	+286.0	-779	-238.0	+106	+32.4
25	343	+950.7	+326.0	-376.3	-129.0	-765	-262
29	413	+629.4	+260.0	-81.1	-33.5	-645.7	-267
33	541	+234.4	+126.8	+16.1	+8.7	-249.3	-134.5
37	660	+24.0	+15.85	+5.1	+3.37	-25.3	-16.7
total			+441.15		-47.26		+603.2
Load at	Load	ordinate	f/c	ordinate	f/c	ordinate	f/c
1	689	+15.31	+10.6	-0.931	-0.64	+0.598	+0.41
5	678	+552.5	+375.0	-201.4	-139.0	+30.75	+20.9
9	544	+672.4	+366.0	-1136.7	-620.0	-41.1	-22.3
13	408	+287.9	+118.0	+444.0	+182.0	-674.0	-276.0
17	339	-358.0	-121.3	+106.9	+362.0	+82.3	+25.1
21	305	-855.0	-261.0	+927.0	+283.0	+915.0	+312.0
25	343	-913.3	-313.0	+491.2	+168.0	+730.3	+302.0
29	413	-607.9	-251.0	+145.3	+60.2	+277.3	+149.5
33	541	-228.0	-123.0	+5.0	+2.7	+28.9	+19.1
37	660	-23.3	-15.4	-2.4	-1.58		-298.29
total			-353.5		+298.68		
Load at	Load	ordinate	f/s	ordinate	f/s	ordinate	f/s
1	689	+249.5	+172.0	+11.95	+8.25	-6.03	-4.16
5	678	-7120	-484.0	+2577	+1750	-198.7	-135.0
9	544	-8680	-4740	+14620	+7970	+1160	+632.0
13	408	-3276	-1340	-3950	-1617	+920	+377.0
17	339	+575.0	+1950	-11140	-3770	+30200	+10200
21	305	+12550	+3830	-9340	-2850	+1550	+474
25	343	+13136	+4510	-4448	-1525	-8420	-2890
29	413	+8701	+3600	-904	-374	-7180	-2970
33	541	+3240	+1760	+226	+122	-2782	-1505
37	660	+335	+221	+67	+44.3	-282	-186
total			+5113		-24145		+3992.84
Load at	Load	ordinate	f/s	ordinate	f/s	ordinate	f/s
1	689	+230.5	+159.0	-11.35	-7.85	+6.90	+4.76
5	678	+7640	+5190	-2455	-1670	+370	+251.5
9	544	+9300	+5060	-13850	-7550	-372	-202.0
13	408	+4000	+1640	+5600	+2290	-7450	-3050
17	339	-4900	-1660	+13300	+4500	-27600	-9350
21	305	-11760	-3590	+1550	+3530	+1277	+390
25	343	-12574	-4320	+6172	+2120	+10660	+3660
29	413	-8378	-3470	+1867	+773	+8450	+3490
33	541	-3144	-1700	+90	+49.6	+3203	+1732
37	660	-324	-214	-26	-17.1	+335	+221
total			+7576		+4017.65		-2852.74



Stresses Due to Uniform Live Load in #/sq. in.

Point	Loads +	Loads -	+ Area -	f _c +	f _c -
1	272"-774"	156"-272"	4.13 ^{sq} 1.55 ^{sq}	184 #/sq	68.9 #/sq
9	0"-212"	212"-774"	1.19 3.35	53.4	149.
17	114"-416"	416"-774"	4.33 2.30	192.5	102.1
				f' _c +	f' _c -
1	111"-276"	276"-774"	1.95 3.87	86.7	172
9	206"-774"	0"-206"	4.1 1.15	182.	51.1
17	404"-774"	142"-404"	2.75 3.55	122	158
				f' _s +	f' _s -
1	276"-774"	156"-276"	5.77 2.35	2560	1045
9	0"-214"	214"-774"	1.7 4.0	755	1760
17	116"-416"	416"-774"	4.8 2.5	2130	1110
				f' _s +	f' _s -
1	111"-280"	280"-774"	2.7 4.6	1200	2040
9	205"-774"	0"-205"	5.2 1.5	2310	666
17	402"-774"	145"-402"	3.2 4.0	1420	1760

Stresses Due to Concentrated Live Load #/sq. in.

Point	Σ+ordinates	Σ-ordinates	f _c +	f _c -
1	1920	1140	480	285
9	1580	1900	395	475
17	3980	1200	995	300
			f' _c +	f' _c -
1	1310	1800	327.5	450
9	2100	1520	525.0	380
17	2260	3660	565	915
			f' _s +	f' _s -
1	26200	16800	6550	4200
9	20000	21700	5000	5425
17	45600	16000	11400	4000
			f' _s +	f' _s -
1	17700	25000	4425	6250
9	26300	18900	6575	4725
17	20700	40800	5175	10200





Stress Due to Temperature Change

Assume a max fall of 40°F

$$d_2 = 1,500,000 \times 0.0000055 = 255.486$$

$$d_2/c_2 = -3.6101$$

$$M_L = 12703 \text{ " #}$$

$$H_L = 97,161 \text{ #}$$

Point	Thrust	Shear	Moment Arm	Moment
1	54.2 #	80.8 #	+ 111.4"	+ 10810 " #
9	91.0	34.0	+ 15.5	- 1505
17	97.1	4.0	- 17.4	+ 1688

Stresses

	f_c	f_t	f_s	f_s
1	- 108.75 #/D"	+ 120.85 #/D"	- 1636 #/D"	+ 1666 #/D"
9	- 61.34	+ 69.06	- 660	+ 775
17	+ 123.0	- 112.4	+ 1415	- 1256

Max Stress Due to Dead + Live Concentrated

Point	f_c	f_t	f_s	f_s
1	+ 921.15 #/D"	- 803.5 #/D"	+ 11663 #/D"	- 13826 #/D"
9	- 522.26	+ 823.68	- 5666	+ 10593
17	+ 1598.21	- 1213.29	+ 12393	- 12053

Max Stress Due to Dead & Live Loads & Temperature

Point	f_c	f_t	f_s	f_s
1	+ 812.4 #/D"	- 682.55 #/D"	+ 10027 #/D"	- 12160 #/D"
9	- 583.6	+ 892.74	- 6326	+ 11368
17	+ 1721.2	- 1325.69	+ 13708	- 13309

Position of Wheels for Max Stress (Car going ←)

Point	f_c	f_t	f_s	f_s
1	1 at 21	2 at 25	2 at 25	2 at 25
9	1 at 25	1 at 17	1 at 17	1 at 17
17	4 at 17	4 at 17	4 at 17	4 at 17



CONCLUSION

It is to be noted that the maximum allowed unit stress of 650 pounds per square inch as specified in the Michigan State Highway Specifications is exceeded throughout the arch. For the maximum conditions possible, the safe stress at Point 1 is exceeded 57%, at Point 9 it is exceeded 372%, and at Point 17 it is exceeded 165%. Due to a misunderstanding the stresses at the crown and spring line were not determined, but they should have been.

Although this arch will be overstressed considerable at various points, especially at the crown, no doubt it will carry all ordinary traffic; but it shows a poor type of design. If the arch rise were increased or the arch thickness increased from the haunch to the spring line so as not to change the direction of the linear arch so abruptly, the arch would undoubtedly carry the loadings specified without being overstressed.

The specified 24 ton auto truck was disregarded because no authority for, or specifications requiring such a loading could be found.

The effect of shortening due to thrust is fully considered in the analysis. Due to lack of time and the fact that shear stress does not govern, the unit shearing stress at the crown has not been determined.

Since piling may have to be used in the abutment foundation the effect of unequal settlement should have been considered, but time did not permit further investigation.



APPENDIX



OUTLINE FOR ELASTIC ARCH ANALYSIS



1. Draw the full arch section and abutments to a large scale, and carefully divide the intrados into 20 equal parts.

2. Locate the center of gravity of each radial transformed section

and carefully draw the linear arch through these points.

3. Now divide the linear arch into 40 equal parts, thus establishing the centers of 20 equal sections. Calculate the arch properties at the crown, springing, and each of these centers for one lineal inch of arch, if possible. Take all units in inches, if possible.

4. Scale x and y coordinates of each of the points on the linear arch as found under 3. From these values calculate segments of length of the linear arch, and segments of Δx and Δy . Add adjacent sections thus establishing values of Δs , Δx , and Δy for each of the 20 sections.

5. Scale values of ϕ . Calculate and tabulate the values of x , y , Δs , Δx , Δy , ϕ , A_t , I_t , $\sin \phi$, and $\cos \phi$.

6. Calculate and tabulate values of $\frac{\Delta s}{I_t}$, $x \frac{\Delta s}{I_t}$, $x^2 \frac{\Delta s}{I_t}$, $xy \frac{\Delta s}{I_t}$, $y \frac{\Delta s}{I_t}$, $y^2 \frac{\Delta s}{I_t}$, $\frac{\Delta x}{A_t} \sin \phi$, $\frac{\Delta y}{A_t} \sin \phi$, $\frac{\Delta x}{A_t} \cos \phi$, and $\frac{\Delta y}{A_t} \cos \phi$.

7. Calculate and tabulate coefficients of solution equations for reactions under single concentrated loads of one kip each placed at various points on the arch.

8. Calculate and tabulate values of $(x-a) \frac{\Delta s}{I_t}$, $(x-a) x \frac{\Delta s}{I_t}$, $(x-a) y \frac{\Delta s}{I_t}$, for each loading specified in 7.

9. Calculate and tabulate values of coefficients dependent on the loadings in solution equations.

10. Calculate horizontal thrust, vertical reaction, and bending moment at the left and right supports for the loadings in 7.

11. Lay off a load line of 1000 pounds (vertically) and plot values of horizontal thrust and vertical reactions for the left and right supports, and construct force triangles for each loading specified in 7.

12. Calculate y_0 and y_1 for each loading specified in 7.



13. Construct equilibrium polygons for each loading.

14. Scale from force polygon and tabulate thrust and radial shear for the points specified in 7 for all the loads used.

15. Scale and tabulate the values of the moment ordinates for the points specified in 7 for all the loads used.

16. Tabulate the products of the horizontal thrust times the ordinates in 15.

17. Tabulate the values of f_c , f_c' , f_s , and f_s' for the points specified in 7 for all the loads used.

18. On lines corresponding to the points specified in 7, erect ordinates and plot influence curves for f_c , f_c' , f_s , and f_s' for one kip moving loads. Plot the stresses at the crown, haunch, and springing points. [Use large scales.]



OUTLINE FOR ELASTIC ARCH ANALYSIS CONTINUED

19. Estimate actual concentrations at points of loading for Arch Ring, Backfill [Including Roadway], Uniform Live Load, and Special Moving Load [Make slider for this].

20. Tabulate the products of the ordinates times the loads [in kips] for f_c , f_c' , f_s , and f_s' curves for loadings of arch ring, backfill, uniform live load, and concentrated live load. [Place the concentrated live load so as to obtain the maximum in two positions.] Sum up these results giving the total maximum stress of each kind. [For the uniform live load a planimeter may be used.] Note amount and position of live loading giving the maximum stresses.

21. For a rise of 50°F . and a fall of 30°F . compute the thrust [Neglecting the effect of thrust on stress action] and tabulate the values of the thrust, radial shear, and bending moment at the points specified in 7.

22. Tabulate the values of f_c , f_c' , f_s , and f_s' for the temperature changes in 21, and draw influence curves for the same.

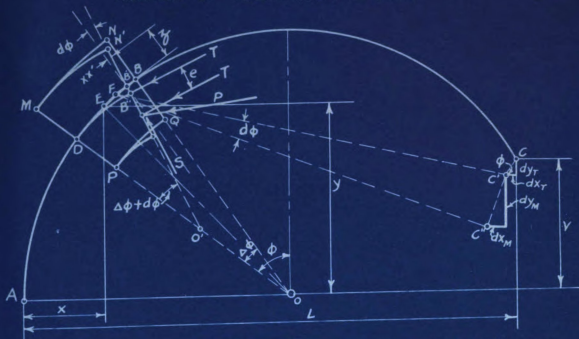
23. Tabulate the total maximum and minimum stresses of f_c , f_c' , f_s , and f_s' at the points specified in 7 with a key diagram showing the position and amount of live load for each case.

24. Using the moment, thrust, and shear, at the springing point, producing the maximum and minimum f_c and f_c' combine graphically with the abutment weight and earth pressure and test the foundations.

25. Summary—Give suggestions as to how the linear arch should be altered or arch thickness changed or reinforcement increased to improve the arch design. Mention points of weakness.



THEORY OF THE ELASTIC ARCH



Let ADBC represent a portion of the Linear arch and let MNPQ be any section of the Arch Ring. Let E, on the linear arch, be the center of this section with coordinates (xy) measured from A. Let the central angle $\Delta\phi$ be subtended by the arc DB of length Δs .

This section of the arch ring will be acted upon by some Resultant Arch Pressure P, which may be resolved into a Radial shear S, a Normal Axial Thrust T, and a Bending Moment $T_e = M$. These shears are small in ordinary arches and Arch Theories neglect them in the Theory of Stresses, just as is, done in the Common Theory of Flexure for Beams. This leaves then, for consideration, the two agents Thrust and Bending acting on the Section: these both vary with different Sections of the Arch Ring. T shortens the section DB uniformly an amount $\frac{T\Delta s}{E}$. A rise of temperature lengthens the Section DB uniformly an amount $e\Delta s$. The Resultant Lengthening of the Section DB = $-BB' = e\Delta s - \frac{T\Delta s}{E}$. A Negative Value of M will cause the Section DB to deform in the AE direction DB changing $\Delta\phi$ by an amount $d\phi$. ϕ is measured from the Vertical through the Crown. Positive to the Left, Negative to the Right - hence this change $d\phi$ is a negative one. At xx' the $\frac{\Delta s}{E} = \frac{M\Delta s}{ET}$. But $xx' = r d\phi$, hence $d\phi = -\frac{M\Delta s}{ET}$ and since when M is Negative $d\phi$ is Negative, the Equation stands correct for sign. Now imagine that, for any Arch Loading, starting at A, each section is taken in order and its effect on the arch found separately and in turn. Then MP may be regarded as firmly fixed to the last deformed section preceding and the deformation of MNPQ on the portion of the arch to the Right determined. Let us find the effect of this deformation on the movement of the Point C whose coordinates are (L, y) . The change in $\Delta s, BB'$ due to Thrust and Temperature will produce an equal change CC' at C. Also the change in $\Delta\phi$ due to M will produce a change at C of $C'C''$ such that $C'FC'' = d\phi$, where F is the intersection of tangents at E and B. The increase in span - L due to Thrust and Temperature on this section alone = $-dx_r = -CC' \cos \phi = -BB' \cos \phi = -\Delta s \left(\frac{E - T}{E} \right) = e\Delta s - \frac{T\Delta s}{E}$. The Increase in Rise - v, due to Thrust and Temperature on this section alone = $-dy_r = CC' \sin \phi =$



THEORY OF THE ELASTIC ARCH-CONTINUED

$\delta B' \frac{\Delta y}{\Delta s} = e \Delta y - \frac{T \Delta y}{AE}$. The Increase in span $-L-$ due to a positive Bending Moment M will be dx_M . But $\frac{dx_M}{ds} = \frac{y-v}{B'C} = \frac{dx_M}{B'C} \frac{dy}{dx}$. Hence $dx_M = (y-v) dy = \frac{My \Delta s}{EI} - \frac{v \Delta s}{EI}$. The Increase in C in $B'C = B'C' dy$ rise $-y-$ due to a Positive EI Bending Moment M will be dy_M . But $\frac{dy_M}{ds} = \frac{dy_M}{B'C' dy} = \frac{L-x}{B'C}$ or $dy = (L-x) d\phi = -\frac{Mx \Delta s}{EI} + \frac{L \Delta s}{EI}$.

The total Increase in span $-L-$ due to a thrust T , a rise in temperature of $t^\circ F$, and a positive bending moment M acting on this section alone will be $dy = e \Delta y - \frac{T \Delta y}{AE} - \frac{Mx \Delta s}{EI} + \frac{L \Delta s}{EI}$.

In a fixed arch-Symmetrical- the following Conditions are assumed.

1. The Radial Lines at the Springing Remain Unchanged in Direction- Thus, summing up $d\phi$ for all Sections between Springing Points

$$\sum_0^L d\phi = \sum_0^L \frac{\Delta s}{EI} = 0$$

2. The Supports are fixed an Immovable Distance apart- i.e. are Rigid. Thus $\sum_0^L dx = \sum_0^L e \Delta x - \sum_0^L \frac{T \Delta x}{AE} + \sum_0^L \frac{Mx \Delta s}{EI} = 0$ since $\sum_0^L \frac{v \Delta s}{EI}$

$$\text{becomes } 0 \sum_0^L \frac{L \Delta s}{EI} = 0$$

3. The Supports are Relatively Immovable in Elevation- Then $\sum_0^L dy = \sum_0^L e \Delta y - \sum_0^L \frac{T \Delta y}{AE} - \sum_0^L \frac{Mx \Delta s}{EI} = 0$ since the term $\sum_0^L \frac{L \Delta s}{EI}$ becomes $L \sum_0^L \frac{\Delta s}{EI}$ and since $\sum_0^L \frac{L \Delta s}{EI} = 0$.

From the figure below $M = M_1 + V_1 x - H_1 y - W_1(x-a) x + a - W_1(y-b) x + a$
 $T = H_1 \cos \phi + V_1 \sin \phi + W_1 \cos \phi x + a - W_1 \sin \phi x + a$

Substituting these values in Equations 1, 2, & 3 they become

$$1. M_1 \sum_0^L \frac{\Delta s}{I_1} + V_1 \sum_0^L \frac{x \Delta s}{I_1} - H_1 \sum_0^L \frac{y \Delta s}{I_1} - W_1 \sum_0^L \frac{(x-a) \Delta s}{I_1} - W_1 \sum_0^L \frac{(y-b) \Delta s}{I_1} = 0$$

$$2. e \sum_0^L \frac{\Delta x}{A_1} - H_1 \sum_0^L \frac{\Delta x \cos \phi}{A_1} - V_1 \sum_0^L \frac{\Delta x \sin \phi}{A_1} - W_1 \sum_0^L \frac{\Delta x \cos \phi}{A_1} + W_1 \sum_0^L \frac{\Delta x \sin \phi}{A_1} + M_1 \sum_0^L \frac{y \Delta s}{I_1} + V_1 \sum_0^L \frac{x y \Delta s}{I_1} - H_1 \sum_0^L \frac{y^2 \Delta s}{I_1} - W_1 \sum_0^L \frac{(x-a) y \Delta s}{I_1} - W_1 \sum_0^L \frac{(y-b) y \Delta s}{I_1} = 0$$

$$3. e \sum_0^L \frac{\Delta y}{A_1} - H_1 \sum_0^L \frac{\Delta y \cos \phi}{A_1} - V_1 \sum_0^L \frac{\Delta y \sin \phi}{A_1} - W_1 \sum_0^L \frac{\Delta y \cos \phi}{A_1} + W_1 \sum_0^L \frac{\Delta y \sin \phi}{A_1} - M_1 \sum_0^L \frac{x \Delta s}{I_1} - V_1 \sum_0^L \frac{x^2 \Delta s}{I_1} + H_1 \sum_0^L \frac{x y \Delta s}{I_1} + W_1 \sum_0^L \frac{(x-a) x \Delta s}{I_1} + W_1 \sum_0^L \frac{(y-b) x \Delta s}{I_1} = 0$$

Rewriting the above equations they become

$$1. a_1 M_1 + b_1 V_1 + c_1 H_1 + d_1 = 0 \text{ or } H_1 = -\frac{a_1}{c_1} M_1 - \frac{b_1}{c_1} V_1 - \frac{d_1}{c_1}$$

$$2. a_2 M_1 + b_2 V_1 + c_2 H_1 + d_2 = 0 \text{ or } H_1 = -\frac{a_2}{c_2} M_1 - \frac{b_2}{c_2} V_1 - \frac{d_2}{c_2} \text{ or } H_1 = -\frac{a_2}{c_2} M_1 - \frac{b_2}{c_2} V_1 - \frac{d_2}{c_2}$$

$$3. a_3 M_1 + b_3 V_1 + c_3 H_1 + d_3 = 0$$

$$\text{Substituting 3 in 1 and 2, } M_1 \left(\frac{a_1}{c_1} - \frac{a_2}{c_2} \right) = -V_1 \left(\frac{b_1}{c_1} - \frac{b_2}{c_2} \right) - \left(\frac{d_1}{c_1} - \frac{d_2}{c_2} \right) \text{ or } A_1 M_1 = B_1 V_1 + C_1$$

$$\text{or } M_1 = \frac{B_1}{A_1} V_1 + \frac{C_1}{A_1}$$

$$M_1 \left(\frac{a_2}{c_2} - \frac{a_3}{c_3} \right) = -V_1 \left(\frac{b_2}{c_2} - \frac{b_3}{c_3} \right) - \left(\frac{d_2}{c_2} - \frac{d_3}{c_3} \right) \text{ or } A_2 M_1 = B_2 V_1 + C_2 \text{ or } M_1 = \frac{B_2}{A_2} V_1 + \frac{C_2}{A_2}$$

$$\text{Solving the last two equations gives } \left(\frac{B_1}{A_1} - \frac{B_2}{A_2} \right) V_1 = - \left(\frac{C_1}{A_1} - \frac{C_2}{A_2} \right)$$

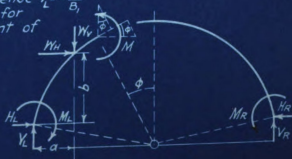
$$\text{But, for Symmetrical Arches } A=0 \text{ hence } V_1 = -\frac{C_1}{B_1} \text{ or } D_1 V_1 = E_1 \text{ or } V_1 = \frac{E_1}{D_1}$$

The following terms are constants for any given arch and are independent of the loading.

$a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3, A_1, B_1, B_2$

Their values are - from above

$$a_i = \sum_0^L \frac{\Delta s}{I} \quad b_i = \sum_0^L \frac{x \Delta s}{I} \quad c_i = \sum_0^L \frac{y \Delta s}{I}$$



THEORY OF THE ELASTIC ARCH-CONTINUED

$$\begin{aligned} a_2 &= \sum_0^L \frac{y \Delta S}{I} & b_2 &= \sum_0^L \frac{xy \Delta S}{I} - \sum_0^L \frac{\Delta x \sin \phi}{A} & c_2 &= -\sum_0^L \frac{y^2 \Delta S}{I} - \sum_0^L \frac{\Delta x \cos \phi}{A} \\ a_3 &= -\sum_0^L \frac{x \Delta S}{I} & b_3 &= -\sum_0^L \frac{x^2 \Delta S}{I} - \sum_0^L \frac{\Delta y \sin \phi}{A} & c_3 &= \sum_0^L \frac{xy \Delta S}{I} - \sum_0^L \frac{\Delta y \cos \phi}{A} \\ A_1 &= 0 & A_2 &= \frac{a_2}{c_2} - \frac{a_3}{c_3} & B_1 &= \frac{b_3}{c_3} - \frac{b_1}{c_1} & B_2 &= \frac{b_3}{c_3} - \frac{b_2}{c_2} \end{aligned}$$

The following terms are dependent on the loading d_1, d_2, d_3, C_1 & C_2 .
Their values are from above.

$$\begin{aligned} d_1 &= -W_V \sum_a^L \frac{(x-a) \Delta S}{I} - W_H \sum_a^L \frac{(y-b) \Delta S}{I} & C_1 &= \frac{d_3}{c_3} - \frac{d_1}{c_1} & C_2 &= \frac{d_3}{c_3} - \frac{d_2}{c_2} \\ d_2 &= Ee t \sum_0^L \Delta x - W_V \left[\sum_a^L \frac{(x-a) y \Delta S}{I} - \sum_a^L \frac{\Delta x \sin \phi}{A} \right] - W_H \left[\sum_a^L \frac{(y-b) y \Delta S}{I} + \sum_a^L \frac{\Delta x \cos \phi}{A} \right] \\ d_3 &= Ee t \sum_0^L \Delta y + W_V \left[\sum_a^L \frac{(x-a) x \Delta S}{I} + \sum_a^L \frac{\Delta y \sin \phi}{A} \right] + W_H \left[\sum_a^L \frac{(y-b) x \Delta S}{I} - \sum_a^L \frac{\Delta y \cos \phi}{A} \right] \end{aligned}$$

$$H_L = H_R = -\frac{a}{c_1} M_L - \frac{b}{c_1} V_L - \frac{d_1}{c_1} \quad M_R = M_L + V_L L - 1000(L-a)$$

Unit Stresses Assuming no Tension on the Concrete

$$f_c = \frac{T}{A_t} + \frac{M d(1-k)}{I_t} \quad f_s = n \left[\frac{T}{A_t} + M \frac{(1-k-g)d}{I_t} \right]$$

$$f_c' = \frac{T}{A_t} - \frac{M k d}{I_t} \quad f_s' = n \left[\frac{T}{A_t} - M \frac{(k-g')d}{I_t} \right]$$

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