

AGGREGATE PLANNING IN MANUFACTURING  
OF REUSABLE CONTAINERS

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## **ABSTRACT**

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Aggregate production planning (APP) is a method to make several decisions simultaneously on production, inventory, and workforce levels over a finite time horizon, aiming to maximize the profit or minimize the cost while meeting fluctuating demands. Building mathematical models that reflect real-world problems is often difficult, as the constraints are usually intricate and may interact with each other. Decomposing the interconnected system into a number of independent phases could simplify the problem; however, it may not guarantee the optimality of the best solutions due to the missed constraints between stages. In this study, two mixed integer programming models for the manufacturing of reusable plastic containers are presented. One is based on the flow of the material and the other is based on the level of the workforce at each period. The proposed models are able to (i) deal with varying demand, (ii) reflect various regulations and restrictions of public and private warehouses for storing materials, and (iii) identify the importance of subcontracting when demand increases dramatically. Both mathematical models are implemented in the case of packaging manufacturing. A comprehensive sensitivity analysis are conducted on different parameters of the problem to test the effect of their changes. To sum up, the general framework of the mathematical models not only can be used for the reusable container manufacturing but also the manufacturing of any type of product with a similar supply chain network.

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## **KEY TO ABBREVIATIONS**

APP: Aggregate Production Planning

MAPE: Mean Average Percentage Error

MAD: Mean Average Deviation

TS: Tracking Signal

SKU: Stock-Keeping-Unit

GAMS: General Algebraic Modeling System

## CHAPTER 1 - INTRODUCTION

It is generally a challenging task to manage a multiple-stage manufacturing process to meet customer needs while keeping costs as low as possible through mathematical modeling, as each process may have its considerations (e.g., particular machinery, process capacity, plant capacity, and trained workforce), and the complexity of the various constraints. Aggregating all steps such that there will not be any shortage or surplus in terms of material and/or workforce at each stage and period of manufacturing increase the size of the modeling to reflect the real-world situations. Modeling the phases separately could simplify the problems because it could reduce the number of constraints and variables involved. However, the result may not be accurate due to the lost connection between various segments of the processes.

Most of the existing researching papers mathematically model such problems based on material flow. In this work, two mixed-integer programming models are presented, one based on material flow and the other based on working hours, to determine the optimal material and working time flow between the stages of the manufacturing process and the optimal workforce assigned to each phase. The models are applied to a case of packaging manufacturing. The consistent results obtained from both models prove the feasibility of the model based on working hours. Besides, the models reflect various considerations for public and private storage. Finally, a comprehensive analysis is performed to examine the effect of various parameters, such as the length of the planning horizon, the number of available extruders, the annual increase in the raw material price, the labor costs, and the subcontracting cost on the optimal solution. The labor costs are proved to be the most sensitive factor, and the investment of extra extruders over the planning horizon is not so necessary in the test condition. The influence of subcontracting on optimal solutions during the

planning period is also signified. The general framework of the mathematical models can be used not only for the manufacturing of reusable containers but also for any type of product with a similar supply chain network.

## CHAPTER 2 - LITERATURE REVIEW

APP is a method of making multiple decisions about production, inventory, and workforce levels simultaneously over a finite time horizon (Pan and Kleiner, 1995; Wang and Yeh, 2014). The decisions can be made at the long-, intermediate-, and short-term levels (Sultana et al., 2014). The APP problem aims to minimize total costs while satisfying time-varying demand assuming fixed sales and production capacity (Nam and Logendran, 1992; Pan and Kleiner, 1995). Although APP is more appreciated when demand is fluctuating, or resources are scarce, it is not recommended in cases of excess capacity (Gansterer, 2015).

Nam and Logendran (1992) classified the existing APP methods into exact or heuristic methods based on the optimality of the solutions. The exact solution approaches include linear programming model, linear decision rule, lot size model, goal programming, etc. The category of heuristic (near-optimal) solutions consists of search decision rule, production switching heuristics, management coefficient model, and simulation model. Pan and Kleiner (1995) proposed a classification of APP models based on the solution techniques, including informal approaches, mathematical models, linear programming models, linear decision rules, heuristic techniques, management coefficients models, and search procedures using computer simulation.

Various approaches have been developed to solve the APP problems, but very few of them have been implemented on real-world problems. Nam and Logendran (1992) point out that these approaches have more theoretical, rather than practical, value. The models have assumptions, for example, the deterministic demands and workforce with the same level of expertise, that do not reflect on the actual situations (Gilgeous, 1987; DuBois and Oliff, 1991; Pan and Kleiner, 1995;

García et al., 2009). Also, frequent change in the level of the workforce may be unappreciated in reality. The indirect cost, such as human resources, marketing, and finance are not integrated into the formulation of the APP models (García et al., 2009). Lot size models incorporate scheduling issues associated with lot size indivisibilities into capacity planning decisions, but they require detailed information throughout the planning horizon, which is quite expensive to gather and process. Search decision rule methods incorporate a variety of cost functions that vary periodically as capacity levels changes, adapting to changes in operational conditions, and flexibly replicating multiple types of planning objectives. However, the cost of the methods is quite high. Moreover, particular expertise is required to accomplish such a complex task (Gilgeous, 1987; DuBois and Oliff, 1991; Pan and Kleiner, 1995). Dejonckheere et al. (2003) cited obstacles to applying APP methods, including formulating the model, interpretation of results, and disaggregating from the overall optimal results. The management coefficient models help to reduce the inconsistency of management decisions by eliminating the variability of managers' behavior. Eilon (1975) stated that simulation models can resolve some real scheduling issues and are well adapted to specific supply chains. However, this method is quite costly, and the results are not guaranteed to be optimal (Nam and Logendran, 1992).

Various models have been proposed to facilitate the use of APP in the industry. Ebert (1976) presented a method for the APP in a variable productivity setting. Apart from the administrative, initial investment, materials, and overhead costs, the planning costs are also considered in the model. Kamien and Li (1990) introduced a multi-period production planning model that integrates subcontracting as a production planning strategy. The authors also demonstrated the smoothing effect of outsourcing by reducing the fluctuation of production and inventory levels. Van Mieghem (1999) used a single-period, competitive stochastic investment game model in a stochastic demand

setting to examine the interaction between capacity, inventory, and pricing decisions. Dejonckheere et al. (2003) utilized the filter theory to connect the dynamics of order replenishment to production planning strategies. Techawiboonwong and Yenradee (2003) offered a multi-product APP model where the workforce can be exchanged between different production lines.

Jain and Palekar (2005) provided a configuration-based formulation, where a product line consisting of several stages is used for manufacturing various products at different rates. Moreover, machines at each stage are allowed to combine to form various production lines. Tian and AbouRizk (2010) developed a simulation-based model that modeled the dynamics and constraints of the production, storage, and distribution processes of the whole process. The model was applied successfully in searching for the best production plan for asphalt production operations; however, varying demand made the production planning quite challenging. Sillekens et al. (2011) built a mixed-integer programming model for the APP in the automotive industry. The model is focused on the adaption of the capacity of a single production line by adjusting the workforce and working times. Chinguwa et al. (2013) explored the APP problem for a specific furniture firm. The best solution was obtained using the informal trial and error method on spreadsheets. Sadeghi et al. (2013) developed a fuzzy grey goal programming model in which the grey numbers were adopted to deal with the uncertainty of parameters. The model could provide a range of APP scenarios with flexibility for planners.

In the APP model of a cable company regarding transportation, the concept of “dummy sources” or “dummy destinations” was developed innovatively to control the situation in which demand does not match with the supply (Sultana et al, 2014). Mendoza et al. (2014) developed a simulation modeling approach for the APP in a two-level intensive supply chain by applying system dynamics. Gongbing and Kun (2014) established a data envelopment analysis-based APP

model that dealt with the uncertainty of demand with the normal distribution. Wang and Yeh (2014) proposed a modified particle swarm optimization method for solving an integer linear programming APP problem. Davizón et al (2015) formed a mathematical model to achieve optimal control, which includes the level of production, inventory, capacity, as well as related costs of the workforce in the same formulation. Gholamian et al. (2015) built a fuzzy multi-objective mixed-integer nonlinear programming model of the APP problems under the context of some uncertain parameters, where multiple suppliers, manufacturers, and customers are involved. Modarres and Izadpanahi (2016) proposed a multi-objective linear programming model that integrates energy saving into the APP with uncertain product demand. The objective function of their model consists of various terms: operational cost, energy, carbon emission, and uncertainty related to demand and capacity. Rosero-Mantilla et al (2017) summarized the general process of applying the APP to solve a real-world problem. Entringer and Ferreira (2018) proposed a conceptual reference model of typical business planning modules that aimed to connect existing processes and aggregate planning. Yaghin (2018) presented a non-linear APP model to address the effect of varying prices and marketing expenditures in the setting of multi-site manufacturing systems and multiple demand classes. Mahmud et al. (2018) developed a multi-product and multi-period APP problem in the interactive probabilistic environment, in which some main costs, such as production, backorder, labor level, and demand are uncertain. Recently, Ruangngam and Wasusri (2019) constructed a mixed-integer linear programming model that incorporates setup time, setup cost, capacity restrictions, perishable product shelf life, and perishable supply restrictions in their formulation for a newly built fruit juice concentrated factory.

It is noted that all models in the literature are based on material flow, however, the working hour may be more practical as it can bring convenience for work scheduling and the possibility for

adding constraints. Also, the fixed cost at the private warehouse is not addressed yet in the existing research about APP, where contracts should be signed with the pre-determined rental area and usage period and payment.



## **CHAPTER 3 - FORECASTING METHODS**

### **Forecasting in Supply Chain Planning**

Forecasting customers' demands is fundamental to supply chain planning. Push and pull processes are two different ways of meeting customer needs. In the pull process, production actions are driven by customers' actual orders; However, the activities in the push process - a strategy used by most modern corporations - are based on a long-term prediction of customer needs before the real order arises.

Forecasting has some basic characteristics. One is that it does not always match the real data. This is why the forecast errors should be considered and measured. Another feature is that the long-term forecast is normally less accurate than the short-term prediction, this is because the longer the time, the more factors are assumed to emerge and influence the result. At last, the aggregate forecast typically has fewer errors than the disaggregate forecast, as it tends to have a smaller standard deviation for the errors.

Before selecting an appropriate forecasting method, it is necessary to conduct a thorough investigation of the factors including historical demand, lead time of product replenishment, planned promotion activities, economic situation, and competitors' strategies (Chopra, 2017). It also requires cooperation at the level of the entire supply chain. This is because the activities of each party in the supply chain are interrelated. Forecasting at an appropriate level of aggregation can effectively lower error since it is usually more precise than disaggregated forecasts. Forecasting must be monitored, and its error measured for further decision-making.

Forecasting methods are divided into two categories, qualitative and quantitative. Qualitative forecasting methods are primarily implemented when less historical information is available that only human judgment with expertise can be used for the forecasts. Time series, causal, simulation are the main methods that fall under the category of quantitative method. Time-series forecasting methods are suitable where historical demand implies its future trend well. Causal forecasting methods are established on the assumption that the demand forecast is highly correlated with certain environmental factors, such as policy and interest rate. Simulation forecasting is a method in which imitating the consumer choices that induce demand. Forecasting using a combination of several methods is deemed to have a better performance than forecasting with one method. The time-series methods are applied in this research.

### **Forecasting Techniques**

Any observed demand can be considered as a combination of the systematic- and random component. The goal of forecasting is to achieve the systematic part, instead of the random portion that is hardly predictable. Three factors are taken to define the predicting model of the systematic component: (i) level: the systematic element, (ii) trend: the change rate of demand for the next period, and (iii) seasonality: the predictable seasonal fluctuations in demand.

There are three common types of equations reflecting the relation between the systematic component and the factors: (i) multiplicative:  $\text{systematic component} = \text{level} \times \text{trend} \times \text{seasonal factor}$ ; (ii) additive:  $\text{systematic component} = \text{level} + \text{trend} + \text{seasonal factor}$ ; and (iii) mixed:  $\text{systematic component} = (\text{level} + \text{trend}) \times \text{seasonal factor}$ . The mixed equation is selected for the calculation of this project as it is considered the most accurate (Chopra, 2017).

Static and adaptive forecasting models are based on distinct assumptions of the factors. Static methods presume the estimated level, trend, and seasonality constant, whereas adaptive models integrate the varying effect of these parameters. Four common adaptive forecast techniques are listed and compared in Table 3.1.

Table 3.1: Comparison of adaptive forecasting methods

Forecasting methods	Application
Moving average	No trend, or seasonality
Simple exponential smoothing	No trend, or seasonality
Holt's model	Trend, but no seasonality
Winter's model	Trend and seasonality

Before presenting the forecasting methods, some basic definitions, as shown in Table 3.2, are necessary to be introduced.

Table 3.2: Definition of factors in the systematic component

L	The estimate of the level at $t = 0$ (the deseasonalized demand estimate during Period $t = 0$ )
T	The estimate of the trend (increase or decrease in demand per period)
$S_t$	The estimate of the seasonal factor for Period $t$
$D_t$	Actual demand observed in Period $t$
$F_t$	Forecast of demand for Period $t$
$E_t = F_t - D_t$	Forecast error in Period $t$

In the *static forecasting method*, the forecast demand in Period  $t + l$  is thus given as (Chopra, 2017):

$$F_{t+l} = (L + (t + l)T)S_{t+l} \quad (1)$$

, where  $t$  is the number of pieces of historical data available.

The first step is to estimate the level and trend for Period 0. This begins with deseasonalizing the demand data. That is, to reduce the seasonal fluctuations in the original demand data. Below are two equations for obtaining the deseasonalized demand ( $\bar{D}_t$ ), one when  $p$  is even and the other when  $p$  is odd. In which each historical data is given equal weight.

$$\bar{D}_t = \begin{cases} \frac{D_{t-(\frac{p}{2})} + D_{t+(\frac{p}{2})} + \sum_{i=t+1-(\frac{p}{2})}^{i=t+1+(\frac{p}{2})} 2D_i}{2p}; & \text{for } p \text{ is even} \\ \sum_{i=t-(\frac{p-1}{2})}^{i=t+(\frac{p-1}{2})} \frac{D_i}{p}; & \text{for } p \text{ is odd} \end{cases} \quad (2)$$

, where  $p$  is the periodicity which is the number of periods after which the seasonal cycle repeats (Chopra, 2017).

The level and trend for Period 0 can then be retained by applying a linear equation for the period ( $t$ ) and the deseasonalized demand data ( $\bar{D}_t$ ).

$$\bar{D}_t = L + Tt \quad (3)$$

The next step is to estimate seasonal factors with the following formula. The seasonal factor for Period  $t$  is the ratio of actual demand  $\bar{D}_t$  to the deseasonalized demand and is given as:

$$\bar{S}_t = \frac{D_t}{\bar{D}_t} \quad (4)$$

The seasonal factors are then averaged for each season:

$$S_i = \frac{\sum_{j=0}^{r-1} \bar{S}_{jp+i}}{r} \quad (5)$$

Where,  $r$ , is the seasonal cycles in the data, for all periods of the form  $pt + i$ , and  $1 \leq i \leq p$ .

In adaptive forecasting, the forecasting demand for Period  $t + l$  is expressed as follows:

$$F_{t+l} = (L_t + lT_t)S_{t+l} \quad (6)$$

Unlike in the static forecasting methods, an additional step of revising factors is required in adaptive forecasting techniques to compensate for the forecast value after minimizing the forecasting error of historic data. The error for Period  $t + 1$  is stated as Eq. (7).

$$E_{t+1} = F_{t+1} - D_{t+1} \quad (7)$$

Four adaptive forecasting methods are introduced in this portion. The *moving average method* is used when the trend and seasonality are absent. In this method, the level for Period  $t$  is estimated as the averaged demand over the most recent  $N$  periods. Since it is assumed that nearby observations in past are likely to be close to the future demand. The equation for the N-period moving average is presented as follows:

$$L_t = \frac{D_t + D_{t-1} + \dots + D_{t-N+1}}{N} \quad (8)$$

The forecast is evaluated as:

$$F_{t+1} = L_t, \text{ and } F_{t+n} = L_t \quad (9)$$

The new moving average is calculated by adding the latest observation of demand and dropping the oldest one. The revised moving average serves as the next forecast.

The *simple exponential smoothing method* is suitable when demand demonstrates no trend or seasonality. The initial estimate of level,  $L_0$ , is taken to be the average of all historical data with the following equation (Chopra, 2017).

$$L_0 = \frac{1}{n} \sum_{i=1}^n D_i \quad (10)$$

, where  $n$  is the total number of given demand data.

The current forecast for all future periods is given as:

$$F_{t+1} = L_t, \text{ and } F_{t+n} = L_t \quad (11)$$

After observing the demand ( $D_{t+1}$ ) for Period  $t + 1$ , the estimate of the level is revised as follows:

$$L_{t+1} = \alpha D_{t+1} + (1 - \alpha) L_t \quad (12)$$

, where  $\alpha$  is the smoothing factor for level,  $\alpha \in [0, 1]$ .

*Trend-Corrected Exponential Smoothing (Holt's model)*: The method runs a linear regression equation between historical demands ( $D_t$ ), and time (Period  $t$ ), so that from which the initial  $L_0$  and  $T_0$  could be obtained. Forecasting for Period  $t$ , is expressed as (Chopra, 2017):

$$F_{t+1} = L_t + T_t \text{ and } F_{t+n} = L_t + nT_t \quad (13)$$

After observing the demand for Period  $t$ , the estimates for the level and trend are revised as follows:

$$L_{t+1} = \alpha D_{t+1} + (1 - \alpha)(L_t + T_t) \quad (14)$$

$$T_{t+1} = \beta(L_{t+1} - L_t) + (1 - \beta)T_t \quad (15)$$

, where  $\alpha$  and  $\beta$  are the smoothing factors for the level and trend, respectively.  $\alpha, \beta \in [0, 1]$

*Winter's Model:* Initial estimates of the level ( $L_0$ ), trend ( $T_0$ ), and seasonal factors ( $S_1, \dots, S_p$ ) are obtained with the same procedure as those for static forecasting (Chopra, 2017).

In Period  $t$ , given estimates of level,  $L_t$ , trend,  $T_t$ , and seasonal factors,  $S_t, \dots, i$ .

$$F_{t+1} = (L_t + T_t)S_{t+1} \text{ and } F_{t+l} = (L_t + lT_t)S_{t+l} \quad (16)$$

On observing demand for Period  $t + 1$ , the estimates for the level, trend, and seasonal factors are revised as follows:

$$L_{t+1} = \alpha \left( \frac{D_{t+1}}{S_{t+1}} \right) + (1 - \alpha)(L_t + T_t) \quad (17)$$

$$T_{t+1} = \beta(L_{t+1} - L_t) + (1 - \beta)T_t \quad (18)$$

$$S_{t+p+1} = \gamma \left( \frac{D_{t+1}}{L_{t+1}} \right) + (1 - \gamma)S_{t+1} \quad (19)$$

, where  $\alpha$ ,  $\beta$  and  $\gamma$  are smoothing constants for the level, trend, and the seasonal factor,  $\alpha, \beta, \gamma \in [0, 1]$ .

Measurement of forecast errors is essential to assessing the accuracy of forecasting methods. There are a variety of measures to assess the error.

One is mean squared error (*MSE*). The *MSE* penalizes large errors much more significantly than small ones as all errors are squared. Thus, it is more appropriate in situations where the cost of a large error is much larger than the gains from very accurate forecasts. It is appropriate to be exploited when forecast error has a distribution that is symmetric about zero.

$$MSE_n = \frac{1}{n} \sum_{t=1}^n E_t^2 \quad (20)$$

Another measurement is the mean absolute deviation (*MAD*), which refers to the average of the absolute deviation over all periods. It is expressed by the following equation:

$$MAD_n = \frac{1}{n} \sum_{t=1}^n |E_t| \quad (21)$$

The *MAD* can be employed to estimate the standard deviation of the random component assuming that the random component is normally distributed. In this case, the standard deviation of the random component is:

$$\sigma = 1.25MAD \quad (22)$$



The mean absolute percentage error (*MAPE*) is the average absolute error as a percentage of demand and is given by:

$$MAPE_n = \frac{\sum_{t=1}^n \left| \frac{E_t}{D_t} \right| 100}{n} \quad (23)$$

The *MAPE* can be considered as a good choice when the underlying forecast has significant seasonality, and demand varies considerably from one period to the next. However, it is not good as *MAD* if the forecast error is asymmetrically distributed.

In general, one needs a method to track and control the forecasting method. One approach is to use the sum of forecast errors to evaluate the bias, where the following holds:

$$bias_n = \sum_{t=1}^n E_t \quad (24)$$

The tracking signal (*TS*) is the ratio of the bias and the *MAD* and is given as:

$$TS_t = \frac{bias_n}{MAD_t} \quad (25)$$

## CHAPTER 4 - AGGREGATE PLANNING

Aggregate production planning plays an important role in industries. Managers want to fulfill as many customer orders as possible to make more profit; however, this is difficult because the volume of orders from customers is usually uneven, as well as there are always various resource and condition constraints. For example, lead times are typically long; manufacturers may need to start production before they receive orders; capacity costs often do not amount to outsourcing costs; hiring and layoff costs are often high; inventory can be expensive.

Aggregate production planning, as an approach to schedule a company's capacity, production, subcontracting, inventory, stockouts, and pricing over a finite time horizon at an overall level, can help planners achieve their goal of minimizing the total costs or maximizing the profits while meeting non-constant demands simultaneously. Specifically, it determines the levels of production, inventory, capacity (internal and outsourced), and any backlogs (unmet demand) for each period, that maximize the firm's profit over the whole planning horizon based on the forecast demands are fully met (Chopra, 2017).

The aggregate planning acts as a broad scheme for production management and builds the boundaries within which production and distribution decisions can be made. The aggregate plan enables the supply chain to adapt to the capacity distributions and business agreement. It is concentrated on solving problems at the aggregate level, rather than the detailed stock-keeping-unit (SKU) level decisions. It is usually applied in advance of 3 - 18 months. In such a period, determining production levels by SKU is unrealistic as it is too early, adding production capacity may be also too late. Therefore, aggregate planning is generally limited to searching for optimal production options based on existing facilities (Chopra, 2017).

It is critical to collaborate with other parties throughout the supply chain for the effective practice of aggregate planning, as other partners are important inputs for the planning (Chopra, 2017). Moreover, many constraints lie outside these companies. Such as the vendors or customers of their warehousing, logistics service, which are also crucial. If a manufacturing company has determined to adjust its production, its vendor, transportation, warehousing service must be informed of the plan and integrate the change into their schedules. Without engagement from upstream and downstream of the supply chain, the aggregation planning can hardly generate its complete power.

The planning horizon should be specified before starting the aggregate planning. It indicates a timeframe over which the aggregate plan produces a solution. Another element that ought to be specified is the duration of each period within the planning horizon, e. g., weeks, months, or quarters.

A variety of information should be gathered before employing the aggregate production planning: (i) production rate, (ii) workforce, (iii) overtime, (iv) machine capacity level, (v) subcontracting, (vi) backlog, (vii) inventory on hand. The planners should also identify other key information: (i) aggregate demand forecast,  $F_t$ , for each Period  $t$  in a planning horizon that extends over  $T$  periods, (ii) production costs; (iii) labor costs, regular time (\$/hour), and overtime costs (\$/hour), (iv) cost of subcontracting production (\$/unit or \$/hour), (v) cost of changing capacity, specifically, cost of hiring/laying off workforce (\$/worker) and cost of adding or reducing machine capacity (\$/machine), (vi) labor/machine hours required per unit (vii) inventory holding cost (\$/unit/period), (viii) stockout or backlog cost (\$/unit/period), and (ix) constraints on overtime, layoffs, capital, stockouts and backlogs, and from suppliers to the enterprise (Chopra, 2017).

The aggregate production planning can determine (i) the production quantity from regular time, overtime, and subcontracted time, (ii) inventory held, (iii) backlog or stockout quantity, (iv) workforce hired/laid off, and (v) machine capacity increase or decrease (Chopra, 2017).

The quality of the aggregate production planning affects profitability because the loss can be caused not only by insufficient or late supply but also by excess inventory and capacity. There are several noteworthy principles about implementing high-quality aggregate production planning.

First, aggregate units for production and time should be selected at a proper level. This is because the final schedule will be disaggregated at the product level, although the production planning is carried out in aggregation. Another notable point is the bottleneck of any manufacturing facility, as it is likely to be the most constraining area that may fail the aggregated planning. The setups and maintenance should also be considered in the model since it occupies capacity but results in no production. Otherwise, the aggregate plan will misjudge the production capacity available, resulting in a plan that cannot be achieved in practice (Chopra, 2017).

Trade-offs must be made among capacity, inventory, and backlog costs to achieve the best plan (Chopra, 2017). The chase strategy, the flexibility strategy, and the level strategy are three common tactics, which are generally combined or tailored in practices. The chase strategy deals with the demands with the adjustable machine or labor. The problem with this approach is that there is a high expense for the company and it hurts the employees due to the frequent hiring or laying-off of workers; thus, it is only useful when the inventory cost is higher than changing the level of machine and workforce. The flexibility strategy depends on the varying utilization rate of machines and of the workforce's working time to meet the fluctuating demand. This tactic avoids the issues associated with the chase strategy but presents a new problem of low machine utilization.

In the level strategy, machine capacity and workforce are kept at a constant output rate, while inventory is used as the lever. Backlogs and surpluses are the main challenges to be dealt with under this scheme.

Thinking beyond the firm to the entire supply chain may facilitate producing better results of aggregate production planning. This is due to many factors outside the enterprise that may have a significant impact on the optimal aggregate plan. Not only should the firm communicate with downstream partners for a better forecast of future demand, but also, they should work with upstream partners to review the constraints, and with other parts of the supply chain to improve the performance of the aggregate plan (Chopra, 2017).

Another key principle is that an aggregate planner must make the plan flexible enough as forecasts are always inaccurate. Aggregate planning is an overall blueprint in advance of a specified horizon before orders emerge. The firm should be prepared for the forecast error. A sensitivity analysis of the inputs is a recommended solution to the issue as it can evaluate how the varying parameters impact the optimal solution (Chopra, 2017).

The third rule is that the aggregate plan should be rerun as new data becomes available. This is because the updated inputs may have a radical influence on the previously obtained results. Therefore, it is important to use the latest input to run again the aggregate planning to check if any adjustments should be made (Chopra, 2017).

The final point is that a firm needs to perform aggregate planning as capacity utilization increases (Chopra, 2017). It may be unnecessary when the utilization rate is low since they can arrange production as order received. However, when the utilization rate is high and capacity is an

issue, it may be too late to fit the order into the busy production line. Thus, it is necessary to apply aggregate planning in production for a firm in situations of high utilization.

## **CHAPTER 5 - PROBLEM STATEMENT**

### **Problem Statement**

Polystyrene is widely used in the packaging industry because of its various advantages. The material is economical, transparent, easy to mold, rigid, recyclable, and with good dimensional stability. This research is mainly focused on polystyrene resins that are used for manufacturing plastic containers in the forms of black and clear. The raw material is purchased quarterly in the form of resin pellets. Extrusion and thermoforming are the main processes to convert polystyrene resin pellets into plastic containers.

Extrusion is a high-volume manufacturing process in which the raw plastic is melted and formed into a continuous profile through a die. The raw material is fed into a preheated extruder via a hopper. The material is then compressed to the exit side by a rotating conical screw. Heating devices surround the barrel, softening, and melting the polymer. The melted material pumping out of the die is cooled to a solid shape in the air or through a stream of water, which is finally cut into various shapes. The shape of the product is determined by the die at the end of the extruder. Dyes can be also added in the process to have colored products. Extrusion is generally applied to thermoplastics which refer to materials whose polymeric structure will not change drastically after multiple cycles of heating and cooling; such a character promotes its recycling. The extruded products can be further molded by other processes, such as blow molding or thermoforming, to expand their usages.

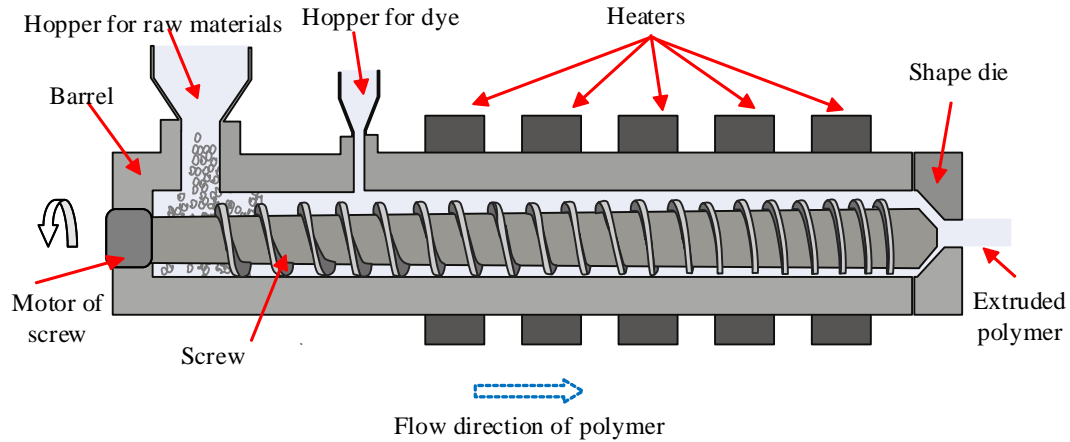


Figure 5.1: A principal scheme of an extruder

Thermoforming is a manufacturing process, where a thermoplastic material or preform is heated to a forming temperature, stretched to a specific shape in a mold, and trimmed to a finished product.

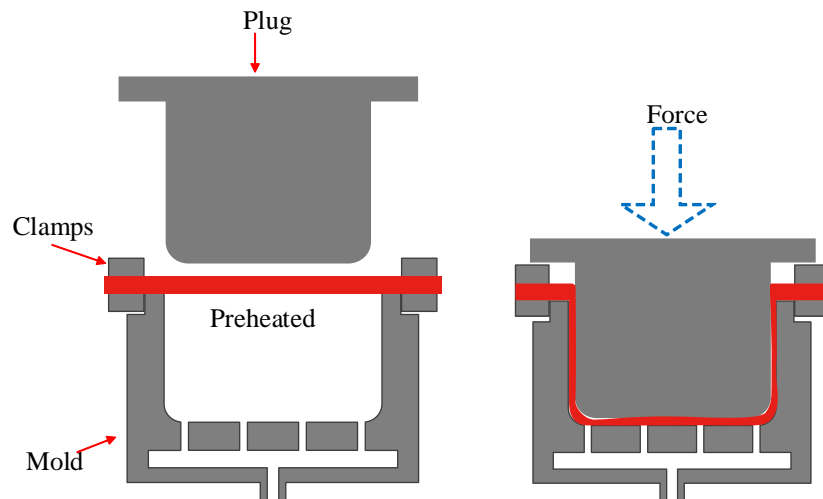


Figure 5.2: A scheme for mechanical thermoforming press

In this case, the black and clear plastic sheets from the extrusion process are wrapped into rolls. The option of subcontracting extruded sheets is available when the customer's needs exceed the extrusion capacity. The rolls are sent either to the thermoforming presses or the warehouses for



future use. Of note, two types of warehouses, i.g. public and private warehouse, are available in the setting.

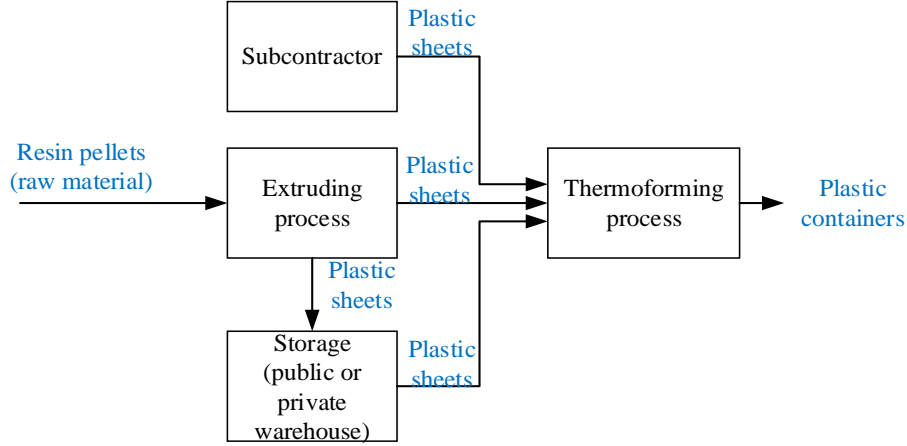


Figure 5.3: The manufacturing process of plastic containers

Suppose quarterly historical demand for plastic containers is given. Let  $d_i$  denote the demand forecast for containers in quarter  $i$  (see Table 5.1 for the summary of notations). The raw material is quarterly purchased at  $\$c_{pur}$  per 1,000 lb. to match the planned production. Extruders produce rolls of plastic sheets. There are  $n_e$  number of extruders in the facility. Each extruder has a processing capacity of  $Q_e$  '000 pounds per hour. Each extruder requires  $w_e$  workers. The amount required is passed forward to thermoforming presses, while the rest is stored at a public and/or private warehouse. There are  $n_t$  number of thermoforming presses in the plant. Each thermoforming press has a processing capacity of  $Q_t$  and requires  $w_t$  workers. Each worker is paid  $\$r$  per hour for a regular-time salary and  $\$o$  per hour for overtime. Workers are limited to  $O$  overtime hours per quarter. The training cost per person is  $\$t$ . During any quarter, extruders and thermoforming presses may be idled. In this case, the associated workers should be laid off. Laying off each worker costs  $\$l$ . If an idled extruder/thermoforming press is brought online, a training cost of  $\$t$  per worker is required.

It is assumed that the manufacturer has the option of subcontracting the production of plastic sheets to one of its supply chain partners. Sufficient production capacity is deemed always available by the subcontractor to make up for the shortage of plastic sheets for the thermoforming process. The manufacturer spends \$ $s$  per 1,000 lb. of the plastic sheet produced by a subcontractor.

Surplus plastic sheets are sent for storage. Transportation is needed to bring the sheets back from the warehouse (when they are needed) to feed the thermoforming presses. Let \$ $c_{tr}$  denote the total transportation cost of 1,000 lb. of plastic sheet. If the option of public warehousing is selected, material handling and storage charge the manufacturer \$ $c_{mh\&s}$  per 1,000 lb. at the end of each quarter. If the option of private warehousing is selected, two types of cost incur: (i) fixed cost: as a contract of a certain area must be signed before use and the least leasing period is three years. It means the leasing area must be paid whether it is used or not. Suppose one square foot is required per 1,000 lb. of plastic sheets in storage. Then, lease rates average \$ $c_f$  per square foot per quarter; (ii) variable cost: private warehousing charges the manufacturer a variable operating cost of \$ $c_v$  per 1,000 lb. of plastic sheet stored per quarter. The supply chain network is illustrated in Figure 5.3.

Table 5.1: Notations used for both mathematical programming models

Parameters	Definition
$d_i$	Demand forecast for plastic containers in quarter $i$ (in '000 pounds)
$c_{pur}$	Raw material cost per 1,000 pounds (in dollars)
$n_e$	Number of extruders
$Q_e$	Processing capacity of an extruder (in '000 pounds) per hour
$w_e$	Number of required workers to work on an extruder
$n_t$	Number of thermoforming presses
$Q_t$	Processing capacity of a thermoforming press (in '000 pounds) per hour
$w_t$	Number of required workers to work on a thermoforming press
$r$	Regular time salary (in dollars per hour)
$o$	Overtime salary (in dollars per hour)
$O$	Limited overtime hours per quarter (in hours)
$t$	Training cost per person (in dollars)
$l$	Laying off cost per person (in dollars)
$s$	Cost of subcontracting plastic sheets per 1,000 pounds (in dollars)

Table 5.1 (cont'd)

Parameters	Definition
$c_{tr}$	Transportation cost per 1,000 pounds of the plastic sheet from extruding plant to a warehouse (in dollars)
$c_{mh\&s}$	Cost of material handling and storage of plastic sheets per 1,000 pounds in a public warehouse (in dollars)
$c_f$	Fixed leasing cost (for three years) per square foot per quarter at a private warehouse
$c_v$	Variable leasing cost of 1,000 pounds of plastic sheet per quarter at a private warehouse
$u$	The utilization rate of private warehouses
$\phi_m^i$	Incidence matrix to relate quarter $i$ to $m^{th}$ 3-year leasing contract. The elements are either 0 or 1.

Based on this premise, this investigation aims to answer the following questions:

1. How many pounds of the plastic sheet should be produced by regular time/overtime working at each quarter? i.e., how many regular/overtime hours should extruders work each quarter?
2. How many extruder workers should be laid off/hired at each quarter?
3. How many extruders should work at each quarter?
4. How many pounds of plastic containers should be produced by regular/overtime time working at each quarter? i.e., how many regular/overtime time hours should the thermoforming presses work at each quarter?
5. How many thermoforming workers should be laid off/hired at each quarter?
6. How many thermoforming presses should work at each quarter?
7. How many pounds of plastic sheets should be sent to public or private warehousing or should be subcontracted? How many square feet of the private warehouse should be leased if it is involved?

To address these questions, two mixed-integer programming models, based on the material flow and the level of the workforce, along each segment of the manufacturing process are presented.

## Production Planning Optimization

### *A mathematical model based on the flow of material*

The mixed-integer programming model corresponding to the aggregate planning of reusable container manufacturing based on the material flow is shown in this sector (see Table 5.2 for notations).

$$\min \sum_i \left[ c_{pur}(x_i^R + x_i^O) + sy_i^{Sub} + \frac{rw_e}{Q_e} x_i^R + \frac{rw_t}{Q_t} z_i^R + \frac{ow_e}{Q_e} x_i^O + \frac{ow_t}{Q_t} z_i^O \right. \\ \left. + w_e tx_i^H + w_t tz_i^H + w_e lx_i^L + w_t lz_i^L + (y_i^{Pu} + y_i^{Pr}) C_{tr} \right. \\ \left. + C_{mh\&s} y_i^{Pu} + \sum_m c_f \phi_m^i y_m' + y_i^{PrC} \right] \quad (1)$$

$$x_i^R = 504 Q_e x_i^W \quad \forall i \quad (2)$$

$$x_i^O \leq 0 Q_e x_i^W \quad \forall i \quad (3)$$

$$z_i^R = 504 Q_t z_i^W \quad \forall i \quad (4)$$

$$z_i^O \leq 0 Q_t z_i^W \quad \forall i \quad (5)$$

$$x_i^W \leq n_e \quad \forall i \quad (6)$$

$$z_i^W \leq n_t \quad \forall i \quad (7)$$

$$x_i^W = x_{i-1}^W + x_i^H - x_i^L \quad \forall i \quad (8)$$

$$z_i^W = z_{i-1}^W + z_i^H - z_i^L \quad \forall i \quad (9)$$

$$x_i^R + x_i^O + y_i^{Sub} + y_{i-1}^{Pu} + y_{i-1}^{Pr} - (y_{i-1}^{Pu} + y_{i-1}^{Pr}) = z_i^R + z_i^O \quad \forall i \quad (10)$$

$$z_i^R + z_i^O = d_i \quad \forall i \quad (11)$$

$$uy_m' \geq \phi_m^i y_i^{Pr} \quad \forall i, \forall m \quad (12)$$

Objective function (1) includes several terms:  $c_{pur}(x_i^R + x_i^O)$  is the raw material purchasing cost,  $sy_i^{Sub}$  is the cost of subcontracting,  $\frac{rw_e}{Q_e} x_i^R$  is the labor cost during the regular time working at the extruding plant,  $\frac{rw_t}{Q_t} z_i^R$  is the labor cost during the regular time working at the thermoforming plant,  $\frac{ow_e}{Q_e} x_i^O$  is the labor cost during overtime working of the extruding plant,  $\frac{ow_t}{Q_t} z_i^O$  is the labor cost during overtime working of the thermoforming plant,  $w_e tx_i^H$  is the cost of training workers

when a new extruder is brought online,  $w_t t z_i^H$  is the cost of worker training when a new thermoforming press is brought online,  $w_e l x_i^L$  is the cost of laying off workers when one extruder is idle,  $w_t l z_i^L$  is the cost of laying off workers when a thermoforming press becomes idle,  $c_{tr}(y_i^{Pu} + y_i^{Pr})$  is the cost of transporting plastic sheets to a public/private warehouse,  $c_{mh\&s} y_i^{Pu}$  is the cost of material handling and storage cost of plastic sheets at a public warehouse,  $c_f \varphi_m^i y'_m$  is the fixed leasing cost of a private warehouse, and  $c_v y_i^{Pr}$  is the variable leasing cost of a private warehouse. It is noted that  $i \in \{1, 2, \dots, 12\}$  is related to  $m = 1$  (the first 3-year leasing contract),  $i \in \{13, 14, \dots, 24\}$  is related to  $m = 2$  (the second 3-year leasing contract), and so on. Parameter  $\varphi_m^i$  is 1 if  $i$  is related to  $m$ , and 0 otherwise.

Consider that there are 8 working hours per day and 63 working days per quarter (a total of 504 hours per quarter). Constraint (2) shows the connection between the plastic sheets in '000 pounds produced by regular time working of extruders and the total number of working extruders at each quarter. Constraint (3) shows the connection between the plastic sheets in '000 pounds produced by overtime working of extruders and the total number of working extruders at each quarter. Constraint (4) shows the connection between the containers in '000 pounds produced by regular time working of the thermoforming process and the total number of working thermoforming presses at each quarter. Constraint (5) shows the connection between the containers in '000 pounds produced by overtime working of thermoforming presses and the total number of working thermoforming presses at each quarter. Constraint (6) guarantees that the total number of working extruders does not exceed the total number of extruders. Constraint (7) guarantees that the total number of working thermoforming presses does not exceed the total number of thermoforming presses. Constraint (8) guarantees that the total number of working extruders at quarter  $i$  is equal to the total number of working extruders at quarter  $i - 1$  plus the total number

of newly hired extruders at quarter  $i$  minus the total number of laid-off extruders at quarter  $i$ . Similarly, constraint (9) guarantees that the total number of working thermoforming presses at quarter  $i$  is equal to the total number of working thermoforming presses at quarter  $i - 1$  plus the total number of newly hired thermoforming presses at quarter  $i$  minus the total number of laid-off thermoforming presses at quarter  $i$ . Constraint (10) ensures the flow balance of materials between extruding plant, storage warehouses, and thermoforming plant. Finally, constraint (11) guarantees that the containers (in '000 pounds) produced by thermoforming presses at quarter  $i$  are equal to the demand of quarter  $i$ . Constraint (12) ensures that the level of inventory at the private warehouse at each period is less than the preset inventory level leased at the beginning of the corresponding 3-year leasing contract. The utility rate of the private warehouse is also considered.

Table 5.2: Variables used for the mathematical modeling of the aggregate planning based on the material flow

Variables	Definition
$x_i^R$	'000 pounds of the plastic sheet produced by regular time working at quarter $i$
$x_i^O$	'000 pounds of the plastic sheet produced by overtime working at quarter $i$
$x_i^L$	Number of laid off (idled) extruders in quarter $i$
$x_i^H$	Number of newly hired extruders in quarter $i$
$x_i^W$	Number of working extruders in quarter $i$
$z_i^R$	'000 pounds of containers produced in regular time in quarter $i$
$z_i^O$	'000 pounds of containers produced in overtime in quarter $i$
$z_i^L$	Number of laid off (idled) thermoforming presses in quarter $i$
$z_i^H$	Number of newly hired thermoforming presses in quarter $i$
$z_i^W$	Number of working thermoforming presses in quarter $i$
$y_i^{Pr}$	'000 pounds of plastic sheets stored in a private warehouse in quarter $i$
$y_i^{Pu}$	'000 pounds of plastic sheets stored in a public warehouse in quarter $i$
$y_i^{Sub}$	'000 pounds of plastic sheets subcontracted in quarter $i$
$y'_m$	Area (proportional to '000 pounds of plastic sheets) leased at the private warehouse for $m^{th}$ 3-year leasing contract

*A mathematical model based on the working hours of machines.*

The mixed-integer programming model corresponding to the aggregate planning of plastic container manufacturing based on the working hours of the equipment is presented (see Table 5.3 for notations).

$$\begin{aligned} \text{Min } \sum_i \{ & c_{\text{pur}} Q_e(x_i^R + x_i^O) + sy_i^{\text{Sub}} + rw_e x_i^R + rw_t z_i^R + ow_e x_i^O + ow_t z_i^O + \\ & w_e tx_i^H + w_t tz_i^H + w_e lx_i^L + w_t lz_i^L + c_{\text{tr}}(y_i^{\text{Pu}} + y_i^{\text{Pr}}) + c_{\text{mh\&s}} y_i^{\text{Pu}} + \end{aligned} \quad (13)$$

$$\sum_m c_f \phi_m^i y_m' + c_v y_i^{\text{Pr}} \} \quad \forall i \quad (14)$$

$$x_i^R = 504 x_i^W \quad \forall i \quad (15)$$

$$x_i^O \leq 0 x_i^W \quad \forall i \quad (16)$$

$$z_i^R = 504 z_i^W \quad \forall i \quad (17)$$

$$z_i^O \leq 0 z_i^W \quad \forall i \quad (18)$$

$$x_i^W \leq n_e \quad \forall i \quad (19)$$

$$z_i^W \leq n_t \quad \forall i \quad (20)$$

$$x_i^W = x_{i-1}^W + x_i^H - x_i^L \quad \forall i \quad (21)$$

$$z_i^W = z_{i-1}^W + z_i^H - z_i^L \quad \forall i \quad (22)$$

$$Q_e(x_i^R + x_i^O) + y_i^{\text{Sub}} + y_{i-1}^{\text{Pu}} + y_{i-1}^{\text{Pr}} - (y_{i-1}^{\text{Pu}} + y_{i-1}^{\text{Pr}}) = Q_t(z_i^R + z_i^O) \quad \forall i \quad (23)$$

$$Q_t(z_i^R + z_i^O) = d_i \quad \forall i \quad (24)$$

$$uy_m' \geq \phi_m^i y_i^{\text{Pr}} \quad \forall i, \forall m \quad (24)$$

Objective function (13) includes several terms:  $c_{\text{pur}} Q_e(x_i^R + x_i^O)$  is the raw material purchasing cost,  $sy_i^{\text{Sub}}$  is the cost of subcontracting,  $rw_e x_i^R$  is the labor cost during the regular time working at the extruding plant,  $rw_t z_i^R$  is the labor cost during the regular time working at the thermoforming plant,  $ow_e x_i^O$  is the labor cost during overtime working of the extruding plant,  $ow_t z_i^O$  is the labor cost during overtime working of the thermoforming plant,  $w_e tx_i^H$  is the cost of training workers when a new extruder is brought online,  $w_t tz_i^H$  is the cost of training workers when a new thermoforming press is brought online,  $w_e lx_i^L$  is the cost of laying off workers when an extruder becomes idle,  $w_t lz_i^L$  is the cost of laying off workers when a thermoforming press becomes idle,  $c_{\text{tr}}(y_i^{\text{Pu}} + y_i^{\text{Pr}})$  is the cost of transporting plastic sheets to a public/private warehouse,  $c_{\text{mh\&s}} y_i^{\text{Pu}}$  is the cost of material handling and storage cost of plastic sheets at a public

warehouse,  $c_f \phi_m^i y'_m$  is the fixed leasing cost of a private warehouse, and  $c_v y_i^{Pr}$  the variable leasing cost of a private warehouse.

Constraint (14) shows the connection between the regular time working hours of extruders and the total number of working extruders at each quarter. Constraint (15) shows the connection between the overtime working hours of extruders and the total number of working extruders at each quarter. Constraint (16) shows the connection between the regular time working hours of the thermoforming process and the total number of working thermoforming presses at each quarter. Constraint (17) shows the connection between the overtime working hours of thermoforming presses and the total number of working thermoforming presses at each quarter. Constraint (18) guarantees that the total number of working extruders does not exceed the total number of extruders. Constraint (19) guarantees that the total number of working thermoforming presses does not exceed the total number of thermoforming presses. Constraint (20) guarantees that the total number of working extruders at quarter  $i$  is equal to the total number of working extruders at quarter  $i - 1$  plus the total number of newly hired extruders at quarter  $i$  minus the total number of laid-off extruders at quarter  $i$ . Similarly, constraint (21) guarantees that the total number of working thermoforming presses at quarter  $i$  is equal to the total number of working thermoforming presses at quarter  $i - 1$  plus the total number of newly hired thermoforming presses at quarter  $i$  minus the total number of laid-off thermoforming presses at quarter  $i$ . Constraint (22) ensures the flow balance of materials between extruding plant, storage warehouses, and thermoforming plant. Finally, constraint (23) guarantees that the containers (in '000 pounds) produced by thermoforming presses at quarter  $i$  are equal to the demand of quarter  $i$ . Constraint (24) ensures that the level of inventory at the private warehouse at each period is less than the preset inventory level leased at



the beginning of the corresponding 3-year leasing contract. The utility rate of the private warehouse is also considered.

Table 5.3: Variables used for the mathematical modeling of the aggregate planning based on the working hours of the equipment

Variables	Definition
$x_i^R$	Number of regular time working hours of extruders at quarter i
$x_i^O$	Number of overtime working hours of extruders at quarter i
$x_i^L$	Number of laid off (idled) extruders in quarter i
$x_i^H$	Number of newly hired (brought online) extruders in quarter i
$x_i^W$	Number of working extruders in quarter i
$z_i^R$	Number of regular time working hours of thermoforming presses at quarter i
$z_i^O$	Number of overtime working hours of thermoforming presses at quarter i
$z_i^L$	Number of laid off (idled) thermoforming presses in quarter i
$z_i^H$	Number of newly hired (brought online) thermoforming presses in quarter i
$z_i^W$	Number of working thermoforming presses in quarter i
$y_i^{Pr}$	'000 pounds of plastic sheets stored in a private warehouse in quarter i
$y_i^{Pu}$	'000 pounds of plastic sheets stored in a public warehouse in quarter i
$y_i^{Sub}$	'000 pounds of plastic sheets subcontracted in quarter i
$y_m'$	Area (proportional to '000 pounds of plastic sheets) leased at the private warehouse for $m^{th}$ 3-year leasing contract

Finally, it is noted that one can evaluate various storage strategies, i.e., using either public or private storage (but not both at the same time), and a combination of both warehouses. The following constraints provide such flexibility to the model:

$$\sum_i y_i^{pu} \leq M\alpha \quad (25)$$

$$\sum_i y_i^{pr} \leq M\beta \quad (26)$$

$$\alpha + \beta = b \quad (27)$$

, where  $\alpha$  and  $\beta$  are binary variables and  $M$  is a large number. Constraints (25), (26), and (27) with  $b = 1$  guarantee that using either public or private storage is used and not both. Constraints (25), (26), and (27) with  $b = 2$  guarantee that using a combination of each storage is acceptable.

## CHAPTER 6 - COMPUTATIONAL EXPERIMENTS

Historical demand data and the value of some parameters are adopted from Chopra (2017).

Historical demands for black and clear plastic containers from the year 2005 to 2009 are presented in Figure 6.1 and Table 6.1.

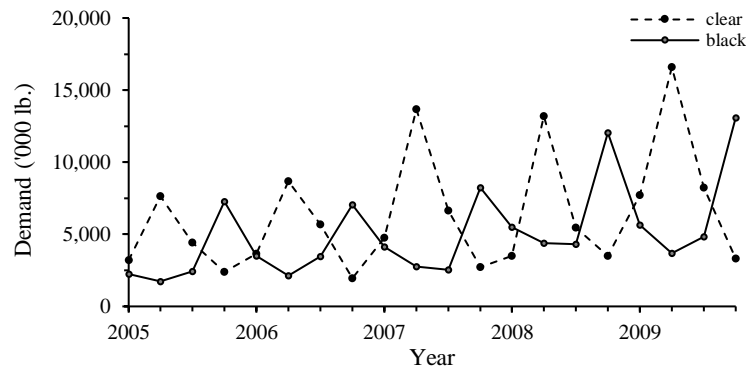


Figure 6.1: Historical demand data

Table 6.1: Historical demand data

Year	Quarter	Black Plastic Demand ('000 lb)	Clear Plastic Demand ('000 lb)
2005	I	2,250	3,200
	II	1,737	7,658
	III	2,412	4,420
	IV	7,269	2,384
2006	I	3,514	3,654
	II	2,143	8,680
	III	3,459	5,695
	IV	7,056	1,953
2007	I	4,120	4,742
	II	2,766	13,673
	III	2,556	6,640
	IV	8,253	2,737
2008	I	5,491	3,486
	II	4,382	13,186
	III	4,315	5,448
	IV	12,035	3,485
2009	I	5,648	7,728
	II	3,696	16,591
	III	4,843	8,236
	IV	13,097	3,316

In Figure 6.1, the trend and seasonality can be observed for the demands of both plastics. The demands for clear plastic containers peak every summer which is assumed to be relevant to the demand for cold drinks in the season, and the demands for black ones reach the highest-level during winters. The demands for both containers have an increasing trend. It is assumed the demands will continue to increase in the following three years at historical rates. Winter's model, which considers trend and seasonality, is expected to give the best forecast among all other forecasting methods.

### **Forecasting Demands with the Static Method**

In the static forecasting method, level, trend, and seasonal factors are assumed to be constant. The first step of forecasting with the static method is to deseasonalize the historical data series. It is observed that periodicity  $p = 4$  which is an even number. Therefore, the first row of Eq. (2) is taken to calculate the deseasonalized demand for quarter 3 to quarter 19 ( $\bar{D}_3$  to  $\bar{D}_{19}$ ).

The second step is to obtain a linear equation between deseasonalized demands ( $\bar{D}_3$  to  $\bar{D}_{19}$ ) and quarter number ( $t_3$  to  $t_{20}$ ) Which can be obtained by running a linear regression analysis in Excel or adding a linear trendline to the data series. The equations of the obtained linear trendlines are shown in Figure 6.2 and 6.3.

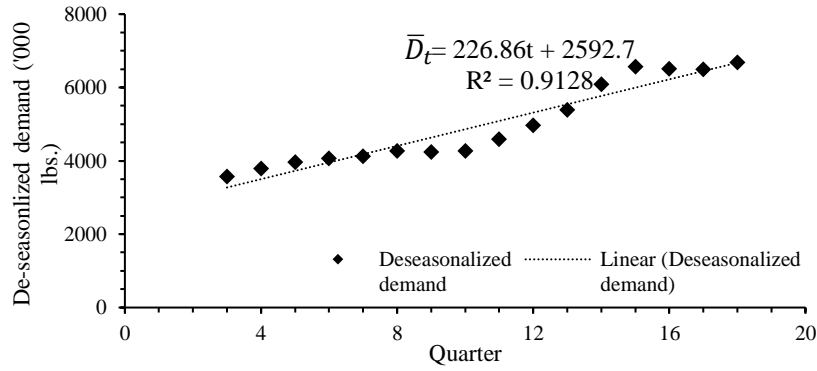


Figure 6.2: Linear regression for the deseasonalized demands of black plastics and the quarter number

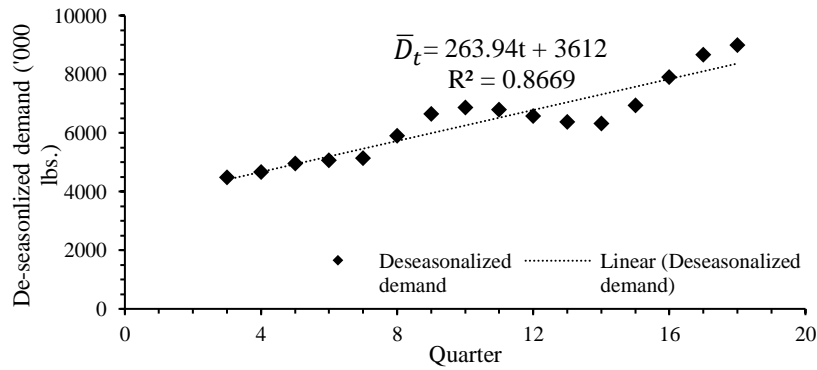


Figure 6.3: Linear regression for the deseasonalized demands of clear plastics and the quarter number

Table 6.2: Input range for obtaining trendline

Series	Values
x	B4:B19
y	D4:D19

As the coefficients of the regression equation are round, the equations for deseasonalized demand data of both containers are as follows.

$$\bar{D}_t = 227t + 2593 \quad (3A)$$

$$\bar{D}_t = 264t + 3612 \quad (3B)$$

The deseasonalized demands for both plastics during all quarters can be calculated with the above equations.

All seasonal factors are calculated by Eq. (4) (in Column F) and then averaged by Eq. (5) (in Column G). Originally, there are 20 values of seasonal factors, where they are considered five cycles (each year is considered a cycle). Seasonal factors at the quarter I, II, III, IV, are averaged respectively because the four seasonal factors are assumed to be repeated every year. For example,  $S_1 = S_5 = S_9 = S_{13} = S_{17}$ ,  $S_2 = S_6 = S_{10} = S_{14} = S_{18}$ , etc.

At last, the forecast is calculated by Eq. (1). Results for both black and clear containers are presented in Figure 6.4 and Figure 6.5. The formulas involved are also listed in Table 6.3. Note that the symbol “\$” in the middle of the cell number is used to lock the referenced cell so that it will not be changed automatically.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Year, quarter	Period t	Historical demand $D_t$	Deseasonalized demand $\bar{D}_t$	Calculated deseasonalized demand $\bar{D}_t$	Seasonal factor $\bar{S}_t$	Averaged seasonal factor $\bar{S}_t$	Level $L_t$	Trend $T_t$	Forecast $F_t$	Error $E_t$	MAPEt (%)	Sqared MSEt	MADt	$\sigma$	bias	TSt
2	2005, I	1	2250		2820	0.80	0.90	2593	227	2535	285.18	12.67	81325.59	285.18	557.59	285.18	1.00
3	2005, II	2	1737		3047	0.57	0.60	2593	227	1824	87.17	8.85	44462.23	186.17		372.35	2.00
4	2005, III	3	2412	3575	3274	0.74	0.70	2593	227	2283	-128.93	7.68	35182.25	167.09		243.42	1.46
5	2005, IV	4	7269	3784	3501	2.08	1.80	2593	227	6301	-968.41	9.09	260843.5	367.42		-724.99	-1.97
6	2006, I	5	3514	3965	3728	0.94	0.90	2593	227	3351	-162.53	8.20	213958.2	326.44		-887.53	-2.72
7	2006, II	6	2143	4070	3955	0.54	0.60	2593	227	2368	224.77	8.58	186718.8	309.50		-662.76	-2.14
8	2006, III	7	3459	4119	4182	0.83	0.70	2593	227	2916	-542.75	9.60	202126.9	342.82		-1205.50	-3.52
9	2006, IV	8	7056	4272	4409	1.60	1.80	2593	227	7935	878.67	9.95	273368.7	409.80		-326.83	-0.80
10	2007, I	9	4120	4237	4636	0.89	0.90	2593	227	4168	47.76	8.98	243247.8	369.57		-279.08	-0.76
11	2007, II	10	2766	4274	4863	0.57	0.60	2593	227	2911	145.37	8.60	221036.3	347.15		-133.71	-0.39
12	2007, III	11	2556	4595	5090	0.50	0.70	2593	227	3549	993.43	11.35	290660.8	405.91		859.73	2.12
13	2007, IV	12	8253	4969	5317	1.55	1.80	2593	227	9569	1315.76	11.74	410706.7	481.73		2175.48	4.52
14	2008, I	13	5491	5390	5544	0.99	0.90	2593	227	4984	-506.95	11.54	398883.1	483.67		1668.53	3.45
15	2008, II	14	4382	6083	5771	0.76	0.60	2593	227	3455	-927.03	12.23	431776	515.34		741.50	1.44
16	2008, III	15	4315	6575	5998	0.72	0.70	2593	227	4183	-132.39	11.62	404159.4	489.81		609.11	1.24
17	2008, IV	16	12035	6509	6225	1.93	1.80	2593	227	11203	-832.16	11.33	422180.1	511.20		-223.05	-0.44
18	2009, I	17	5648	6490	6452	0.88	0.90	2593	227	5800	152.34	10.82	398711.1	490.09		-70.71	-0.14
19	2009, II	18	3696	6688	6679	0.55	0.60	2593	227	3999	302.57	10.67	381646.5	479.68		231.86	0.48
20	2009, III	19	4843		6906	0.70	0.70	2593	227	4816	-27.21	10.14	361598.8	455.86		204.65	0.45
21	2009, IV	20	13097		7,133	1.84	1.80	2,593	227	12837	-260.08	9.73	346,901	446.07		-55.42	-0.12
22	2010, I	21					0.90	2,593	227	6,617							
23	2010, II	22					0.60	2,593	227	4,542							
24	2010, III	23					0.70	2,593	227	5,449							
25	2010, IV	24					1.80	2,593	227	14,471							
26	2011, I	25					0.90	2,593	227	7,433							
27	2011, II	26					0.60	2,593	227	5,086							
28	2011, III	27					0.70	2,593	227	6,082							
29	2011, IV	28					1.80	2,593	227	16,105							
30	2012, I	29					0.90	2,593	227	8,249							
31	2012, II	30					0.60	2,593	227	5,629							
32	2012, III	31					0.70	2,593	227	6,715							
33	2012, IV	32					1.80	2,593	227	17,739							

Figure 6.4: Forecast demands for black plastic containers using the static method

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Year, quarter	Period t	Historical demand $D_t$	Deseasonalized demand $\bar{D}_t$	Calculated deseasonalized demand $\bar{D}_t$	Seasonal factor $\bar{S}_t$	Averaged seasonal factor $\bar{S}_t$	Level $L_t$	Trend $T_t$	Forecast $F_t$	Error $E_t$	MAPEt (%)	Sqared MSEt	MADt	$\sigma$	bias	TSt
2	2005, I	1	3200		3876	0.83	0.76	3612	264	2951	-248.54	7.77	61773.3	248.54	820.38	-248.54	-1.00
3	2005, II	2	7658		4140	1.85	1.90	3612	264	7862	204.02	5.22	51699.7	226.28		-44.52	-0.20
4	2005, III	3	4420	4472	4404	1.00	0.95	3612	264	4175	-245.22	5.33	54510.2	232.59		-289.73	-1.25
5	2005, IV	4	2384	4657	4668	0.51	0.41	3612	264	1936	-448.41	8.70	91150.3	286.55		-738.14	-2.58
6	2006, I	5	3654	4944	4932	0.74	0.76	3612	264	3756	101.57	7.51	74983.5	249.55		-636.57	-2.55
7	2006, II	6	8680	5049	5196	1.67	1.90	3612	264	9867	1187.41	8.54	297477	405.86		550.84	1.36
8	2006, III	7	5695	5132	5460	1.04	0.95	3612	264	5176	-519.18	8.62	293487	422.05		31.66	0.08
9	2006, IV	8	1953	5892	5724	0.34	0.41	3612	264	2373	420.46	10.24	278900	421.85		452.12	1.07
10	2007, I	9	4742	6634	5988	0.79	0.76	3612	264	4560	-182.32	9.53	251604	395.24		269.80	0.68
11	2007, II	10	13673	6850	6252	2.19	1.90	3612	264	11873	-1800.20	9.89	550517	535.73		-1530.40	-2.86
12	2007, III	11	6640	6791	6516	1.02	0.95	3612	264	6177	-463.14	9.63	519970	529.13		-1993.54	-3.77
13	2007, IV	12	2737	6573	6780	0.40	0.41	3612	264	2811	74.33	9.05	477100	491.23		-1919.21	-3.91
14	2008, I	13	3486	6363	7044	0.49	0.76	3612	264	5364	1877.79	12.50	711639	597.89		-41.41	-0.07
15	2008, II	14	13186	6308	7308	1.80	1.90	3612	264	13878	692.18	11.98	695030	604.63		650.77	1.08
16	2008, III	15	5448	6932	7572	0.72	0.95	3612	264	7178	1729.90	13.30	848198	679.65		2380.67	3.50
17	2008, IV	16	3485	7887	7836	0.44	0.41	3612	264	3249	-235.79	12.89	798660	651.90		2144.87	3.29
18	2009, I	17	7728	8662	8100	0.95	0.76	3612	264	6168	-1560.09	13.32	894850	705.33		584.78	0.83
19	2009, II	18	16591	8989	8364	1.98	1.90	3612	264	15884	-707.43	12.82	872940	705.44		-122.65	-0.17
20	2009, III	19	8236		8628	0.95	0.95	3612	264	8179	-57.07	12.18	827167	671.32		-179.72	-0.27
21	2009, IV	20	3316		8,892	0.37	0.41	3,612	264	3687	371.08	12.13	792,694	656.31		191.36	0.29
22	2010, I	21					0.76	3,612	264	6,972							
23	2010, II	22					1.90	3,612	264	17,889							
24	2010, III	23					0.95	3,612	264	9,180							
25	2010, IV	24					0.41	3,612	264	4,125							
26	2011, I	25					0.76	3,612	264	7,776							
27	2011, II	26					1.90	3,612	264	19,894							
28	2011, III	27					0.95	3,612	264	10,181							
29	2011, IV	28					0.41	3,612	264	4,563							
30	2012, I	29					0.76	3,612	264	8,580							
31	2012, II	30					1.90	3,612	264	21,900							
32	2012, III	31					0.95	3,612	264	11,182							
33	2012, IV	32					0.41	3,612	264	5,001							

Figure 6.5: Forecast demands for clear plastic containers using the static method

Table 6.3: Formulas for forecasting demands using the static method

Cell	Formula	Eq. No.	Copied down to
D4	=(C2+C6+2*(C3+C4+C5))/(2*4)	2	D19
E2	=2593+227*B2	3A	E21
F2	=C2/E2	4	F21
G2	=AVERAGE(F2,F6,F10,F14,F18)	5	G5
G6	=G2	5	G21
H2	2593	coefficient from 3A	H21
I2	227	coefficient from 3A	I21
J2	=(H2+B2*I2)*G2	1	J33
K2	=J2-C2	7	K21
L2	=SUMPRODUCT(ABS(\$K\$2:K2), POWER(\$C\$2:C2,-1))*100/B2	23	L21
M2	=SUMSQ(\$K\$2:K2)/B2	20	M21
N2	=SUM(ABS(\$K\$2:K2))/B2	21	N21
O2	=1.25*N21	22	-
P2	=SUM(\$K\$2:K2)	24	P21
Q2	=SUM(\$K\$2:K2)/N2	25	Q21

### Forecasting Demand with Adaptive Methods

In this section, various adaptive methods which have been mentioned in Chapter 3, are used to forecast the demands for black and clear plastics in the next three years. The values of forecasting error, *MAPE*, Squared *MSE*,  $\sigma$ , *bias*, and *TS* are accordingly calculated. Values of *MAPE* and  $\sigma$  are compared among different methods to evaluate the accuracy of each method.

#### *Forecasting demands with moving average method*

The moving average method does not consider trend or seasonality. Eq. (8) is used to calculate the level from period  $t = 5$  to  $t = 20$  by taking the average of the previous four periods. Eq. (9) is then used to calculate the forecasting demand which is equal to the level of the previous quarter. The forecasting results for the demand for black and clear plastics are presented in Figure 6.6 and Figure 6.7, respectively. Formulas used to forecast demands are presented in Table 6.4.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Year, quarter	Period t	Historical demand Dt	Level Lt	Forecast Ft	Error Et	MAPEt (%)	Sqared MSEt	MADt	$\sigma$	biast	TSt
2	2005, I	1	2,250									
3	2005, II	2	1,737									
4	2005, III	3	2,412									
5	2005, IV	4	7,269	3,417								
6	2006, I	5	3,514	3,733	3,417	-97	2.76	9,409	97	2,587	-97	-1.00
7	2006, II	6	2,143	3,835	3,733	1,590	38.48	1,268,755	844		1,493	1.77
8	2006, III	7	3,459	4,096	3,835	376	29.27	892,836	688		1,869	2.72
9	2006, IV	8	7,056	4,043	4,096	-2,960	32.44	2,859,657	1,256		-1,091	-0.87
10	2007, I	9	4,120	4,195	4,043	-77	26.33	2,288,912	1,020		-1,168	-1.15
11	2007, II	10	2,766	4,350	4,195	1,429	30.55	2,247,528	1,088		260	0.24
12	2007, III	11	2,556	4,125	4,350	1,794	36.21	2,386,358	1,189		2,055	1.73
13	2007, IV	12	8,253	4,424	4,125	-4,129	37.94	4,218,627	1,556		-2,074	-1.33
14	2008, I	13	5,491	4,767	4,424	-1,067	35.88	3,876,449	1,502		-3,141	-2.09
15	2008, II	14	4,382	5,171	4,767	385	33.17	3,503,588	1,390		-2,757	-1.98
16	2008, III	15	4,315	5,610	5,171	856	31.96	3,251,614	1,342		-1,901	-1.42
17	2008, IV	16	12,035	6,556	5,610	-6,425	33.74	6,420,431	1,765		-8,326	-4.72
18	2009, I	17	5,648	6,595	6,556	908	32.38	5,989,937	1,699		-7,418	-4.37
19	2009, II	18	3,696	6,424	6,595	2,899	35.67	6,162,384	1,785		-4,519	-2.53
20	2009, III	19	4,843	6,556	6,424	1,581	35.47	5,918,091	1,771		-2,939	-1.66
21	2009, IV	20	13,097	6,821	6,556	-6,542	36.38	8,222,662	2,069		-9,480	-4.58
22	2010, I	21			6,821							
23	2010, II	22			6,821							
24	2010, III	23			6,821							
25	2010, IV	24			6,821							
26	2011, I	25			6,821							
27	2011, II	26			6,821							
28	2011, III	27			6,821							
29	2011, IV	28			6,821							
30	2012, I	29			6,821							
31	2012, II	30			6,821							
32	2012, III	31			6,821							

Figure 6.6: Forecast demands for black plastic containers using four-period moving average method



	A	B	C	D	E	F	G	H	I	J	K	L
1	Year, quarter	Period t	Historical demand Dt	Level Lt	Forecast Ft	Error Et	MAPEt (%)	Squared MSet	MADt	$\sigma$	biast	TSt
2	2005, I	1	3,200									
3	2005, II	2	7,658									
4	2005, III	3	4,420									
5	2005, IV	4	2,384	4,416								
6	2006, I	5	3,654	4,529	4,416	762	20.84	579,882	762	4,047	762	1.00
7	2006, II	6	8,680	4,785	4,529	-4,151	34.33	8,905,342	2,456		-3,390	-1.38
8	2006, III	7	5,695	5,103	4,785	-911	28.22	6,213,231	1,941		-4,300	-2.22
9	2006, IV	8	1,953	4,996	5,103	3,150	61.49	7,140,942	2,243		-1,150	-0.51
10	2007, I	9	4,742	5,268	4,996	254	50.26	5,725,606	1,845		-896	-0.49
11	2007, II	10	13,673	6,516	5,268	-8,406	52.13	16,546,744	2,939		-9,302	-3.17
12	2007, III	11	6,640	6,752	6,516	-124	44.95	14,185,128	2,537		-9,426	-3.72
13	2007, IV	12	2,737	6,948	6,752	4,015	57.67	14,427,016	2,721		-5,411	-1.99
14	2008, I	13	3,486	6,634	6,948	3,462	62.29	14,155,730	2,804		-1,949	-0.70
15	2008, II	14	13,186	6,512	6,634	-6,552	61.03	17,033,027	3,179		-8,501	-2.67
16	2008, III	15	5,448	6,214	6,512	1,064	57.26	15,587,536	2,986		-7,437	-2.49
17	2008, IV	16	3,485	6,401	6,214	2,729	59.02	14,909,309	2,965		-4,708	-1.59
18	2009, I	17	7,728	7,462	6,401	-1,327	55.80	13,897,844	2,839		-6,034	-2.13
19	2009, II	18	16,591	8,313	7,462	-9,129	55.74	18,858,227	3,288		-15,164	-4.61
20	2009, III	19	8,236	9,010	8,313	77	52.09	17,601,407	3,074		-15,087	-4.91
21	2009, IV	20	3,316	8,968	9,010	5,694	59.56	18,527,671	3,238		-9,393	-2.90
22	2010, I	21			8,968							
23	2010, II	22			8,968							
24	2010, III	23			8,968							
25	2010, IV	24			8,968							
26	2011, I	25			8,968							
27	2011, II	26			8,968							
28	2011, III	27			8,968							
29	2011, IV	28			8,968							
30	2012, I	29			8,968							
31	2012, II	30			8,968							
32	2012, III	31			8,968							
33	2012, IV	32			8,968							

Figure 6.7: Forecast demands for clear plastic containers using four-period moving average method

Table 6.4: Formulas for forecasting demands using the moving average method

Cell	Formula	Eq. No.	Copied down to
D5	=AVERAGE(C2:C5)	8	D21
E6	=D5	9	E21
E22	=\$D\$21	7	E33
F6	=E6-C6	7	F21
G6	=SUMPRODUCT(ABS(\$F\$6:F6),POWER(\$C\$6:C6,-1))*100/(B6-4)	23	G21
H6	=SUMSQ(\$F\$6:F6)/(B6-4)	20	H21
I6	=SUM(ABS(\$F\$6:F6))/(B6-4)	21	I21
J6	=1.25*I21	22	-
K6	=SUM(\$F\$6:F6)	24	K21
L6	=SUM(\$F\$6:F6)/I6	25	L21

### ***Forecasting demands with the simple exponential smoothing method***

The simple exponential smoothing forecasting method does not take trend or seasonality into consideration.

Initial level ( $L_0$ ) is calculated by averaging all the historical demands by Eq. (10). The levels over the periods from 1 to 20 ( $L_1$  to  $L_{20}$ ) are calculated by Eq. (12). Demands for the historical periods are then calculated by Eq. (11), and forecasting demands for the whole forecasting horizon is equal to the value of the last observed level ( $L_{20}$ ). Value of smoothing constant  $\alpha$  in Cell M2 is obtained by minimizing the  $MAD_{12}$ . The results are presented in Figure 6.8 and Figure 6.9. The formulas involved in the calculation are listed in Table 6.5.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Year, quarter	Period t	Historic Demand Dt	Level Lt	Forecast Ft	Error Et	MAPEt (%)	Sqared MSEt	MADt	$\sigma$	biast	TSt	$\alpha$
2		0		5,052									0.144
3	2005, I	1	2,250	4,648	5,052	2,802	124.54	7,851,764	2,802	2,790	2,802	1.00	
4	2005, II	2	1,737	4,229	4,648	2,911	146.07	8,163,674	2,857		5,713	2.00	
5	2005, III	3	2,412	3,967	4,229	1,817	122.49	6,542,626	2,510		7,530	3.00	
6	2005, IV	4	7,269	4,443	3,967	-3,302	103.22	7,632,896	2,708		4,228	1.56	
7	2006, I	5	3,514	4,309	4,443	929	87.86	6,278,847	2,352		5,157	2.19	
8	2006, II	6	2,143	3,997	4,309	2,166	90.07	6,014,256	2,321		7,323	3.15	
9	2006, III	7	3,459	3,919	3,997	538	79.42	5,196,396	2,066		7,861	3.80	
10	2006, IV	8	7,056	4,371	3,919	-3,137	75.05	5,776,707	2,200		4,724	2.15	
11	2007, I	9	4,120	4,335	4,371	251	67.39	5,141,869	1,984		4,975	2.51	
12	2007, II	10	2,766	4,109	4,335	1,569	66.32	4,873,894	1,942		6,544	3.37	
13	2007, III	11	2,556	3,885	4,109	1,553	65.82	4,650,065	1,907		8,097	4.25	
14	2007, IV	12	8,253	4,515	3,885	-4,368	64.74	5,852,377	2,112		3,730	1.77	
15	2008, I	13	5,491	4,655	4,515	-976	61.13	5,475,524	2,025		2,753	1.36	
16	2008, II	14	4,382	4,616	4,655	273	57.21	5,089,752	1,899		3,026	1.59	
17	2008, III	15	4,315	4,573	4,616	301	53.86	4,756,473	1,793		3,327	1.86	
18	2008, IV	16	12,035	5,648	4,573	-7,462	54.37	7,939,676	2,147		-4,135	-1.93	
19	2009, I	17	5,648	5,648	5,648	0	51.17	7,472,637	2,021		-4,135	-2.05	
20	2009, II	18	3,696	5,367	5,648	1,952	51.26	7,269,174	2,017		-2,183	-1.08	
21	2009, III	19	4,843	5,291	5,367	524	49.13	6,901,020	1,938		-1,659	-0.86	
22	2009, IV	20	13,097	6,416	5,291	-7,806	49.66	9,602,476	2,232		-9,465	-4.24	
23	2010, I	21			6,416								
24	2010, II	22			6,416								
25	2010, III	23			6,416								
26	2010, IV	24			6,416								
27	2011, I	25			6,416								
28	2011, II	26			6,416								
29	2011, III	27			6,416								
30	2011, IV	28			6,416								
31	2012, I	29			6,416								
32	2012, II	30			6,416								
33	2012, III	31			6,416								
34	2012, IV	32			6,416								

Figure 6.8: Forecasting results for black plastic containers using the simple exponential smoothing method

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Year, quarter	Period t	Historic Demand Dt	Level Lt	Forecast Ft	Error Et	MAPEt (%)	Sqared MSEt	MADt	$\sigma$	biast	TSt	$\alpha$
2		0		6,346									0.077
3	2005, I	1	3,200	6,105	6,346	3,146	98.30	9,894,799	3146	3,941	3,146	1.00	
4	2005, II	2	7,658	6,224	6,105	-1,553	59.29	6,153,461	2349		1,592	0.68	
5	2005, III	3	4,420	6,086	6,224	1,804	53.13	5,186,802	2167		3,396	1.57	
6	2005, IV	4	2,384	5,802	6,086	3,702	78.67	7,315,785	2551		7,098	2.78	
7	2006, I	5	3,654	5,638	5,802	2,148	74.69	6,775,809	2471		9,246	3.74	
8	2006, II	6	8,680	5,871	5,638	-3,042	68.08	7,188,735	2566		6,204	2.42	
9	2006, III	7	5,695	5,857	5,871	176	58.80	6,166,190	2224		6,380	2.87	
10	2006, IV	8	1,953	5,559	5,857	3,904	76.44	7,300,938	2434		10,285	4.22	
11	2007, I	9	4,742	5,496	5,559	817	69.86	6,563,818	2255		11,101	4.92	
12	2007, II	10	13,673	6,122	5,496	-8,177	68.85	12,593,558	2847		2,924	1.03	
13	2007, III	11	6,640	6,161	6,122	-518	63.30	11,473,099	2635		2,406	0.91	
14	2007, IV	12	2,737	5,899	6,161	3,424	68.45	11,494,259	2701		5,831	2.16	
15	2008, I	13	3,486	5,715	5,899	2,413	68.51	11,058,135	2679		8,244	3.08	
16	2008, II	14	13,186	6,286	5,715	-7,471	67.67	14,255,374	3021		773	0.26	
17	2008, III	15	5,448	6,222	6,286	838	64.18	13,351,882	2876		1,611	0.56	
18	2008, IV	16	3,485	6,013	6,222	2,737	65.08	12,985,689	2867		4,349	1.52	
19	2009, I	17	7,728	6,144	6,013	-1,715	62.56	12,394,871	2799		2,634	0.94	
20	2009, II	18	16,591	6,943	6,144	-10,447	62.58	17,769,493	3224		-7,813	-2.42	
21	2009, III	19	8,236	7,042	6,943	-1,293	60.11	16,922,184	3122		-9,106	-2.92	
22	2009, IV	20	3,316	6,757	7,042	3,726	62.73	16,770,371	3153		-5,380	-1.71	
23	2010, I	21			6,757								
24	2010, II	22			6,757								
25	2010, III	23			6,757								
26	2010, IV	24			6,757								
27	2011, I	25			6,757								
28	2011, II	26			6,757								
29	2011, III	27			6,757								
30	2011, IV	28			6,757								
31	2012, I	29			6,757								
32	2012, II	30			6,757								
33	2012, III	31			6,757								
34	2012, IV	32			6,757								

Figure 6.9: Forecasting results for clear plastic containers using the simple exponential smoothing method

Table 6.5: Formulas for forecasting with simple exponential smoothing method

Cell	Formula	Eq. No.	Copied down to
D2	=AVERAGE(C3:C22)	10	D22
D3	=\$M\$2*C3+(1-\$M\$2)*D2	12	E22
E3	=D2	11	E22
E23	=D\$22	11	E34
F3	=E3-C3	7	F22
G3	=SUMPRODUCT(ABS(\$F\$3:F3),POWER(\$C\$3:C3,-1))*100/B3	23	G22
H3	=SUMSQ(\$F\$3:F3)/B3	20	H22
I3	=SUM(ABS(\$F\$3:F3))/B3	21	I22
J3	=1.25*I22	22	-
K3	=SUM(\$F\$3:F3)	24	K22
L3	=SUM(\$F\$3:F3)/I3	25	L22

### ***Forecasting demands with Holt's model***

Holts Model includes the element of trend. The initial level ( $L_0$ ) and trend ( $T_0$ ) is acquired through running a linear regression between historical demands (from  $D_1$  to  $D_{20}$  ) and quarter No. ( $t_1$  to  $t_{20}$ ). The results for the black plastic containers are  $L_0 = 2043$ ,  $T_0 = 287$ , while  $L_0 = 4134$ ,  $T_0 = 211$  for the clear ones.

The level and trend for Quarters 1 to 20 are calculated by Eq. (14) and Eq. (15), respectively. Finally, forecasting demands during historical periods and future time are calculated. Two smoothing constants are set as  $\alpha$  and  $\beta$  in Cell N2 and Cell O2 by minimizing the  $MAD_{12}$ . Formulas applied in these calculations are listed in Table 6.6. The results are shown in Figure 6.10 and Figure 6.11

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Year, quarter	Period t	Historical demand Dt	Level Lt	Trend Tt	Forecast Ft	Error Et	MAPEt (%)	Squared MSEt	MADt	$\sigma$	biast	TSt	$\alpha$	$\beta$
2		0		2,043	287									0	0.155
3	2005, I	1	2,250	2,330	287	2,330	80	3.56	6,400	80	2,544	80	1.00		
4	2005, II	2	1,737	2,617	287	2,617	880	27.11	390,400	480		960	2.00		
5	2005, III	3	2,412	2,904	287	2,904	492	24.87	340,955	484		1,452	3.00		
6	2005, IV	4	7,269	3,191	287	3,191	-4,078	32.68	4,413,237	1,383		-2,626	-1.90		
7	2006, I	5	3,514	3,478	287	3,478	-36	26.35	3,530,849	1,113		-2,662	-2.39		
8	2006, II	6	2,143	3,765	287	3,765	1,622	34.57	3,380,855	1,198		-1,040	-0.87		
9	2006, III	7	3,459	4,052	287	4,052	593	32.08	2,948,111	1,112		-447	-0.40		
10	2006, IV	8	7,056	4,339	287	4,339	-2,717	32.88	3,502,358	1,312		-3,164	-2.41		
11	2007, I	9	4,120	4,626	287	4,626	506	30.60	3,141,656	1,223		-2,658	-2.17		
12	2007, II	10	2,766	4,913	287	4,913	2,147	35.30	3,288,451	1,315		-511	-0.39		
13	2007, III	11	2,556	5,200	287	5,200	2,644	41.49	3,625,022	1,436		2,133	1.49		
14	2007, IV	12	8,253	5,487	287	5,487	-2,766	40.83	3,960,500	1,547		-633	-0.41		
15	2008, I	13	5,491	5,774	287	5,774	283	38.08	3,662,007	1,450		-350	-0.24		
16	2008, II	14	4,382	6,061	287	6,061	1,679	38.10	3,601,795	1,466		1,329	0.91		
17	2008, III	15	4,315	6,348	287	6,348	2,033	38.70	3,637,215	1,504		3,362	2.24		
18	2008, IV	16	12,035	6,635	287	6,635	-5,400	39.09	5,232,389	1,747		-2,038	-1.17		
19	2009, I	17	5,648	6,922	287	6,922	1,274	38.11	5,020,076	1,719		-764	-0.44		
20	2009, II	18	3,696	7,209	287	7,209	3,513	41.28	5,426,804	1,819		2,749	1.51		
21	2009, III	19	4,843	7,496	287	7,496	2,653	41.99	5,511,625	1,863		5,402	2.90		
22	2009, IV	20	13,097	7,783	287	7,783	-5,314	41.92	6,647,974	2,036		88	0.04		
23	2010, I	21				8,070									
24	2010, II	22				8,357									
25	2010, III	23				8,644									
26	2010, IV	24				8,931									
27	2011, I	25				9,218									
28	2011, II	26				9,505									
29	2011, III	27				9,792									
30	2011, IV	28				10,079									
31	2012, I	29				10,366									
32	2012, II	30				10,653									
33	2012, III	31				10,940									
34	2012, IV	32				11,227									

Figure 6.10: Forecasting results for black plastic containers using Holt's model

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Year, quarter	Period t	Historical demand Dt	Level Lt	Trend Tt	Forecast Ft	Error Et	MAPEt (%)	Sqared MSEt	MADt	$\sigma$	biast	TSt	$\alpha$	$\beta$
2		0		4,134	211									0.0	0.168
3	2005, I	1	3,200	4,345	211	4,345	1,145	35.78	1,311,025	1,145	3,622	1,145	1.00		
4	2005, II	2	7,658	4,556	211	4,556	-3,102	38.14	5,466,715	2,124		-1,957	-0.92		
5	2005, III	3	4,420	4,767	211	4,767	347	28.05	3,684,613	1,531		-1,610	-1.05		
6	2005, IV	4	2,384	4,978	211	4,978	2,594	48.24	4,445,669	1,797		984	0.55		
7	2006, I	5	3,654	5,189	211	5,189	1,535	46.99	4,027,780	1,745		2,519	1.44		
8	2006, II	6	8,680	5,400	211	5,400	-3,280	45.46	5,149,550	2,001		-761	-0.38		
9	2006, III	7	5,695	5,611	211	5,611	-84	39.17	4,414,908	1,727		-845	-0.49		
10	2006, IV	8	1,953	5,822	211	5,822	3,869	59.04	5,734,190	1,995		3,024	1.52		
11	2007, I	9	4,742	6,033	211	6,033	1,291	55.51	5,282,244	1,916		4,315	2.25		
12	2007, II	10	13,673	6,244	211	6,244	-7,429	55.39	10,273,024	2,468		-3,114	-1.26		
13	2007, III	11	6,640	6,455	211	6,455	-185	50.61	9,342,224	2,260		-3,299	-1.46		
14	2007, IV	12	2,737	6,666	211	6,666	3,929	58.35	9,850,125	2,399		630	0.26		
15	2008, I	13	3,486	6,877	211	6,877	3,391	61.35	9,976,953	2,475		4,021	1.62		
16	2008, II	14	13,186	7,088	211	7,088	-6,098	60.27	11,920,428	2,734		-2,077	-0.76		
17	2008, III	15	5,448	7,299	211	7,299	1,851	58.51	11,354,146	2,675		-226	-0.08		
18	2008, IV	16	3,485	7,510	211	7,510	4,025	62.08	11,657,051	2,760		3,799	1.38		
19	2009, I	17	7,728	7,721	211	7,721	-7	58.43	10,971,345	2,598		3,792	1.46		
20	2009, II	18	16,591	7,932	211	7,932	-8,659	58.08	14,527,286	2,935		-4,867	-1.66		
21	2009, III	19	8,236	8,143	211	8,143	-93	55.09	13,763,147	2,785		-4,960	-1.78		
22	2009, IV	20	3,316	8,354	211	8,354	5,038	59.93	14,344,062	2,898		78	0.03		
23	2010, I	21				8,565									
24	2010, II	22				8,776									
25	2010, III	23				8,987									
26	2010, IV	24				9,198									
27	2011, I	25				9,409									
28	2011, II	26				9,620									
29	2011, III	27				9,831									
30	2011, IV	28				10,042									
31	2012, I	29				10,253									
32	2012, II	30				10,464									
33	2012, III	31				10,675									
34	2012, IV	32				10,886									

Figure 6.11: Forecast demands for clear plastic containers using Holt's model

Table 6.6: Formulas for forecasting demands using Holt's model

Cell	Formula	Eq. No.	Copied down to
D2	4134	Linear Regression	-
D3	=N\$2*C3+(1-N\$2)*(D2+E2)	14	D22
E2	211	Linear Regression	-
E3	=O\$2*(D3-D2)+(1-O\$2)*E2	15	E22
F3	=D2+E2	13	F22
F23	=D\$22+(B23-\$B\$22)*E\$22	13	F34
G3	=F3-C3	7	G22
H3	=SUMPRODUCT(ABS(\$G\$3:G3),POWER(\$C\$3:C3,-1))*100/B3	23	H22
I3	=SUMSQ(\$G\$3:G3)/B3	20	I22
J3	=SUM(ABS(\$G\$3:G5))/B5	21	J22
K3	=1.25*J22	22	-
L3	=SUM(\$G\$3:G8)	24	L22
M3	=SUM(\$G\$3:G8)/J8	25	M22

### ***Forecasting demands with Winter's model***

Winter's model takes not only trend but also seasonality into consideration. Initializing values obtained from the static method are used in this method. The coefficients obtained from Eq. (3A) and (3B) represent the initial level and trend. For black plastic,  $L_0 = 2593$ ,  $T_0 = 227$ ; for clear plastic,  $L_0 = 3612$ ,  $T_0 = 264$ . Four seasonal factors obtained by Eq. (4) and Eq. (5) for each plastic, are used as initializing values of seasonal factors. For black plastic  $S_1 = 0.90$ ,  $S_2 = 0.60$ ,  $S_3 = 0.70$ ,  $S_4 = 0.80$ ; and for clear plastic  $S_1 = 0.76$ ,  $S_2 = 1.90$ ,  $S_3 = 0.95$ ,  $S_4 = 0.41$ .

Eq. (17), Eq. (18) are used to estimate the level, trend, forecast of demand in Cells D3, E3, and F7 for the historical periods. Eq. (19) is applied to calculate seasonal factors for period  $t = 5$  to  $t = 24$  as the initial seasonal factors for the first cycle have already been obtained. Therefore, the forecasting values of demand for historical periods can be calculated by Eq. (16).

The smoothing constants  $\alpha$ ,  $\beta$ , and  $\gamma$  are decided by minimizing the  $MAD_{12}$ . The results are shown in Figure 6.12 and Figure 6.13. The detailed formulas to build the worksheet are shown in Table 6.7.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Year, quarter	Period t	Historical demand Dt	Level Lt	Trend Tt	Seasonal factor St	Forecast Ft	Error Et	MAPEt (%)	Sqared MSET	MADt	$\sigma$	biast	TSt	$\alpha$	$\beta$	$\gamma$
2		0		2593	227										0.00	0.00	0.00
3	2005, I	1	2,250	2820	227	0.90	2,535	285	12.67	81,326	285	558	285	1.00			
4	2005, II	2	1,737	3,047	227	0.60	1,824	87	8.85	44,462	186		372	2.00			
5	2005, III	3	2,412	3,274	227	0.70	2,283	-129	7.68	35,182	167		243	1.46			
6	2005, IV	4	7,269	3,501	227	1.80	6,301	-968	9.09	260,843	367		-725	-1.97			
7	2006, I	5	3,514	3,728	227	0.90	3,351	-163	8.20	213,958	326		-888	-2.72			
8	2006, II	6	2,143	3,955	227	0.60	2,368	225	8.58	186,719	309		-663	-2.14			
9	2006, III	7	3,459	4,182	227	0.70	2,916	-543	9.60	202,127	343		-1,206	-3.52			
10	2006, IV	8	7,056	4,409	227	1.80	7,935	879	9.95	273,369	410		-327	-0.80			
11	2007, I	9	4,120	4,636	227	0.90	4,168	48	8.98	243,248	370		-279	-0.76			
12	2007, II	10	2,766	4,863	227	0.60	2,911	145	8.60	221,036	347		-134	-0.39			
13	2007, III	11	2,556	5,090	227	0.70	3,549	993	11.35	290,661	406		860	2.12			
14	2007, IV	12	8,253	5,317	227	1.80	9,569	1,316	11.74	410,707	482		2,175	4.52			
15	2008, I	13	5,491	5,544	227	0.90	4,984	-507	11.54	398,883	484		1,669	3.45			
16	2008, II	14	4,382	5,771	227	0.60	3,455	-927	12.23	431,776	515		742	1.44			
17	2008, III	15	4,315	5,998	227	0.70	4,183	-132	11.62	404,159	490		609	1.24			
18	2008, IV	16	12,035	6,225	227	1.80	11,203	-832	11.33	422,180	511		-223	-0.44			
19	2009, I	17	5,648	6,452	227	0.90	5,800	152	10.82	398,711	490		-71	-0.14			
20	2009, II	18	3,696	6,679	227	0.60	3,999	303	10.67	381,647	480		232	0.48			
21	2009, III	19	4,843	6,906	227	0.70	4,816	-27	10.14	361,599	456		205	0.45			
22	2009, IV	20	13,097	7,133	227	1.80	12,837	-260	9.73	346,901	446		-55	-0.12			
23	2010, I	21				0.90	6,617										
24	2010, II	22				0.60	4,542										
25	2010, III	23				0.70	5,449										
26	2010, IV	24				1.80	14,471										
27	2011, I	25				0.90	7,433										
28	2011, II	26				0.60	5,086										
29	2011, III	27				0.70	6,082										
30	2011, IV	28				1.80	16,105										
31	2012, I	29				0.90	8,249										
32	2012, II	30				0.60	5,629										
33	2012, III	31				0.70	6,715										
34	2012, IV	32				1.80	17,739										

Figure 6.12: Forecast results for black plastic containers using Winter's model

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Year, quarter	Period t	Historical demand Dt	Level Lt	Trend Tt	Seasonal factor St	Forecast Ft	Error Et	MAPEt (%)	Sqared MSEt	MADt	$\sigma$	bias	TSt	$\alpha$	$\beta$	$\gamma$
2		0		3612	264										0.00	0.00	0.00
3	2005, I	1	3,200	3876	264	0.76	2,951	-249	7.77	61,773	249	820	-249	-1.00			
4	2005, II	2	7,658	4,140	264	1.90	7,862	204	5.22	51,700	226		-45	-0.20			
5	2005, III	3	4,420	4,404	264	0.95	4,175	-245	5.33	54,510	233		-290	-1.25			
6	2005, IV	4	2,384	4,668	264	0.41	1,936	-448	8.70	91,150	287		-738	-2.58			
7	2006, I	5	3,654	4,932	264	0.76	3,756	102	7.51	74,984	250		-637	-2.55			
8	2006, II	6	8,680	5,196	264	1.90	9,867	1,187	8.54	297,477	406		551	1.36			
9	2006, III	7	5,695	5,460	264	0.95	5,176	-519	8.62	293,487	422		32	0.08			
10	2006, IV	8	1,953	5,724	264	0.41	2,373	420	10.24	278,900	422		452	1.07			
11	2007, I	9	4,742	5,988	264	0.76	4,560	-182	9.53	251,604	395		270	0.68			
12	2007, II	10	13,673	6,252	264	1.90	11,873	-1,800	9.89	550,517	536		-1,530	-2.86			
13	2007, III	11	6,640	6,516	264	0.95	6,177	-463	9.63	519,970	529		-1,994	-3.77			
14	2007, IV	12	2,737	6,780	264	0.41	2,811	74	9.05	477,100	491		-1,919	-3.91			
15	2008, I	13	3,486	7,044	264	0.76	5,364	1,878	12.50	711,639	598		-41	-0.07			
16	2008, II	14	13,186	7,308	264	1.90	13,878	692	11.98	695,030	605		651	1.08			
17	2008, III	15	5,448	7,572	264	0.95	7,178	1,730	13.30	848,198	680		2,381	3.50			
18	2008, IV	16	3,485	7,836	264	0.41	3,249	-236	12.89	798,660	652		2,145	3.29			
19	2009, I	17	7,728	8,100	264	0.76	6,168	-1,560	13.32	894,850	705		585	0.83			
20	2009, II	18	16,591	8,364	264	1.90	15,884	-707	12.82	872,940	705		-123	-0.17			
21	2009, III	19	8,236	8,628	264	0.95	8,179	-57	12.18	827,167	671		-180	-0.27			
22	2009, IV	20	3,316	8,892	264	0.41	3,687	371	12.13	792,694	656		191	0.29			
23	2010, I	21				0.76	6,972										
24	2010, II	22				1.90	17,889										
25	2010, III	23				0.95	9,180										
26	2010, IV	24				0.41	4,125										
27	2011, I	25				0.76	7,776										
28	2011, II	26				1.90	19,894										
29	2011, III	27				0.95	10,181										
30	2011, IV	28				0.41	4,563										
31	2012, I	29				0.76	8,580										
32	2012, II	30				1.90	21,900										
33	2012, III	31				0.95	11,182										
34	2012, IV	32				0.41	5,001										

Figure 6.13: Forecast results for clear plastic containers using Winter's model

Table 6.7: Formulas for forecasting demands using Winter's model

Cell	Formula	Eq. No.	Copied down to
D2	2593	3A	-
D3	= $\$O\$2*(C3/F3)+(1-\$O\$2)*(D2+E2)$	17	D22
E2	227	3A	-
E3	= $\$P\$2*(D3-D2)+(1-\$P\$2)*E2$	18	E22
F3	0.899	from the static method	-
F4	0.5987	from the static method	-
F5	0.6973	from the static method	-
F6	1.7997	from the static method	-
F7	= $\$Q\$2*(C3/D3)+(1-\$Q\$2)*F3$	19	F22
G3	= $(D2+E2)*F3$	16	G22
G23	= $(\$D\$22+(B23-B22)*\$E\$22)*F23$	16	G26
H3	= $G3-C3$	7	H22
I3	= $\text{SUMPRODUCT}(\text{ABS}(\$H\$3:H3), \text{POWER}(\$C\$3:C3,-1))*100/B3$	23	I22
J3	= $\text{SUMSQ}(\$H\$3:H3)/B3$	20	J22
K3	= $\text{SUM}(\text{ABS}(\$H\$3:H3))/B3$	21	K22
L3	= $1.25*K22$	22	-
M3	= $\text{SUM}(\$H\$3:H3)$	24	M22

Table 6.7 (cont'd)

Cell	Formula	Eq. No.	Copied down to
N3	=SUM(\$H\$3:H3)/K3	25	N22
O2	0.1	-	-
P2	0.2	-	-
Q2	0.05	-	-

The mean absolute percentage error (*MAPE*), which represents the average absolute error as a percentage of demand, is selected to evaluate the forecast error. This is because the demand data have seasonality and relatively high variations from one quarter to the next.

### Error Measurement for the Forecasting

The forecasting demand for black and clear containers over the past five years using the aforementioned methods is presented in Figure 6.14 and 6.15. Smoothing constants in the simple exponential smoothing method, Holt's model, and Winter's Model are determined by minimizing the value of *MAD*.

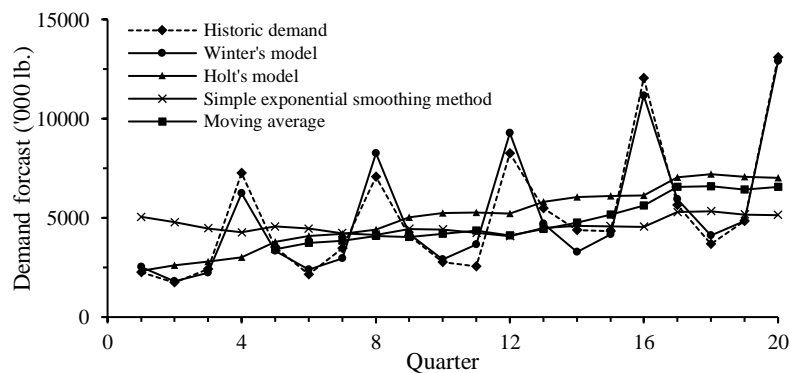


Figure 6.14: Comparison of different adaptive methods for black containers

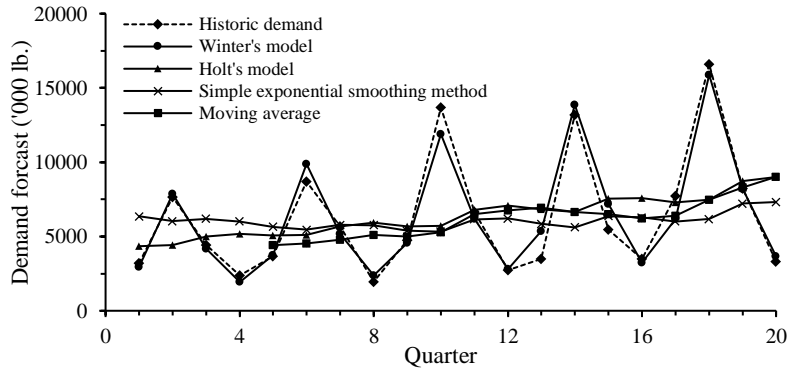


Figure 6.15: Comparison of different adaptive methods for clear containers

From Figure 6.14 and 6.15, it is clear that the demand forecast of Winter's model fits best with the historical data comparing to the results obtained through other methods. Table 6.8 also presents *MAD* and *MAPE* of different adaptive methods and confirms the aforementioned claim.

Table 6.8: Estimation of errors using adaptive methods

Forecasting method	Black plastic demand			Clear plastic demand		
	MAD	MAPE (%)	TS Range (%)	MAD	MAPE (%)	TS Range (%)
Four-period moving average	2069	36	-4.72 to 2.72	3238	60	-4.91 to -0.49
Simple exponential smoothing	1949	50	-3.91 to 4.98	3109	59	-3.13 to 4.62
Holt's model	2159	44	-2.27 to 3.00	3096	63	-1.95 to 2.04
Winter's model	445	10	-3.52 to 4.52	656	12	-3.91 to 3.5

The demand forecast for black and clear containers using Winter's model for the coming 3 years are presented in Figure 6.16 and 6.17, as well as in Table 6.9. Of note, the forecasting horizon is extended to 18 years, since three scenarios with various planning horizons as long as 18 years will be tested in the segment of sensitivity analysis. The results are presented in APPENDIX A.

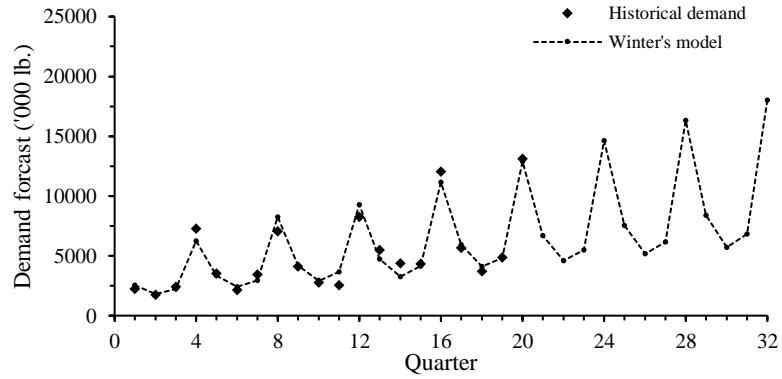


Figure 6.16: Estimated historical and forecasting future demands for clear plastic containers using Winter's model (for the coming 3 years)

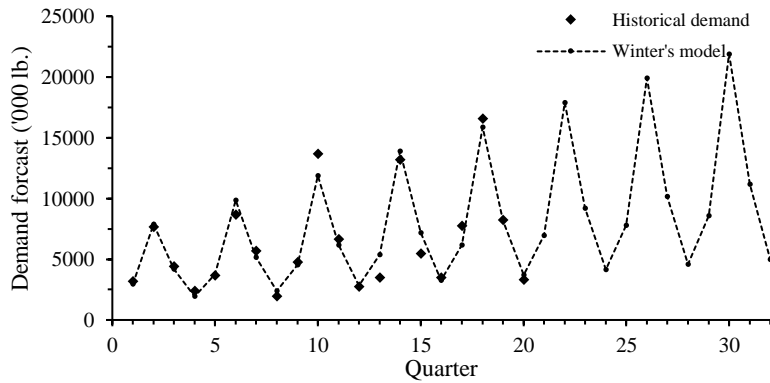


Figure 6.17: Estimated historical and forecasting future demands for black plastic containers using Winter's model (for the coming 3 years)

Table. 6.9: Forecasting demands using Winter's model

Year	Quarter	Black plastic demand ('000 lbs.)	Clear plastic demand ('000 lbs.)
2010	I	6,693	6,972
	II	4,599	17,889
	III	5,492	9,180
	IV	14,625	4,125
2011	I	7,543	7,776
	II	5,164	19,894
	III	6,148	10,181
	IV	16,321	4,563
2012	I	8,393	8,580
	II	5,730	21,900
	III	6,804	11,182
	IV	18,016	5,001

## Computational Result

The parameters adopted from Chopra (2017), and the predicted demands using Winter's model, are shown in Table 6.10. Of note, the demands for both colors of plastic containers are combined since they are compatible with each other at the same production line. The utilization rate of the area at the private warehouses is taken 80%, which is a common value.

Table 6.10: Parameters used in both models

Parameter	Value	Parameter	Value	Parameter	Value
$d_1$	13665	$d_{11}$	17986	$o$	22.5
$d_2$	22487	$d_{12}$	23017	$O$	60
$d_3$	14672	$c_{pur}$	10	$t$	3000
$d_4$	18750	$n_e$	14	$l$	2500
$d_5$	15319	$Q_e$	2.85	$s$	60
$d_6$	25059	$w_e$	6	$c_{tr}$	2
$d_7$	16329	$n_t$	25	$c_{mh\&s}$	16
$d_8$	20884	$Q_t$	2	$c_f$	4
$d_9$	16973	$w_t$	1	$c_v$	4
$d_{10}$	27630	$r$	15	$u$	0.8

A high-level modeling system for mathematical programming and optimization, e. g. General Algebraic Modeling System (GAMS), of distribution 30.3.0, is used to solve the problem due to its large size. The code is enclosed in APPENDIX C and D. Given the forecasted demands for two types of plastic containers over the coming three years, the optimal solutions of the proposed APP models (one based on the flow of materials and the other based on working hours) are obtained. The results from both models are consistent, which confirms the achievability of the model based on the working hour. The details of planning outcomes are included in APPENDIX B and an illustration of each in Figure 6.18-6.21. The first column at each quarter represents the forecasted demands, and the second column represents the production amount in '000 pounds (or equivalently working hours). Regular and overtime working hours are distinguished by two types of filled

patterns. Subcontracting is also demonstrated in the second column. Finally, the information related to the storage (public and private) is illustrated in the third column.

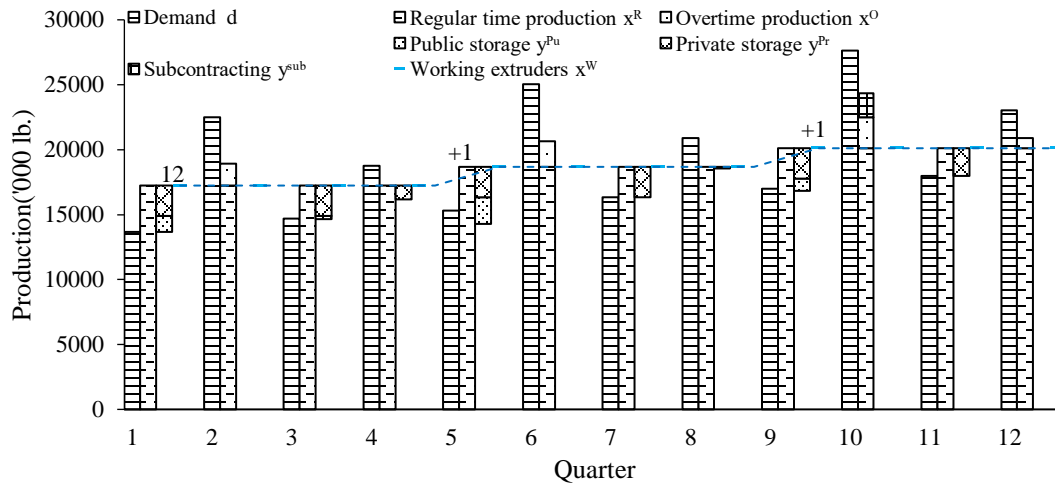


Figure 6.18: The optimal solution for the extruding and warehousing processes obtained from solving the APP model based on the flow of the material

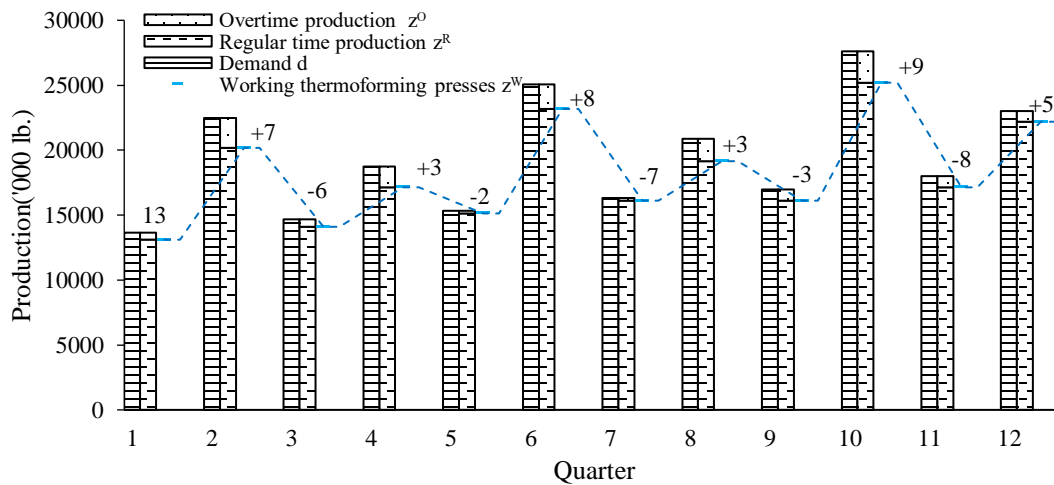


Figure 6.19: The optimal solution for the thermoforming process from solving the APP model based on the flow of the material

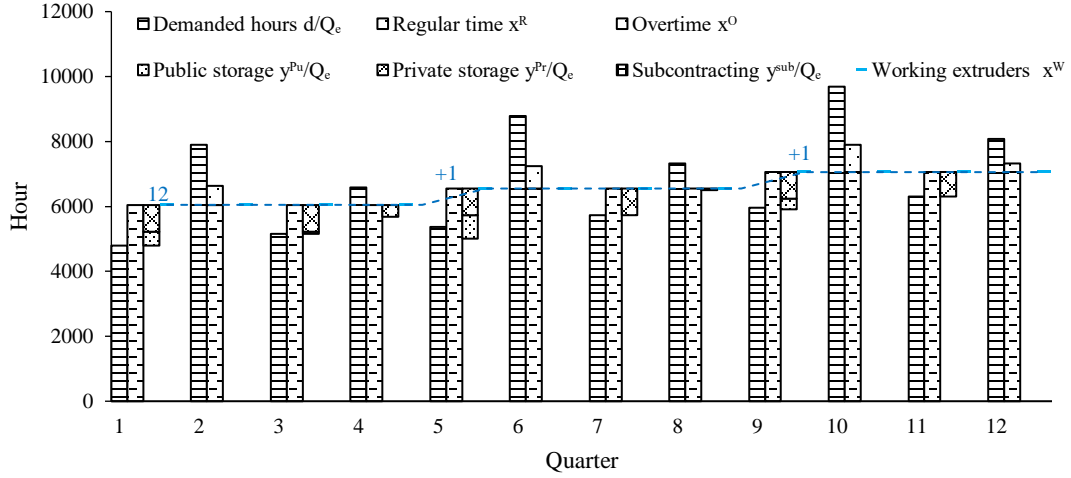


Figure 6.20: The optimal solution for the extruding and warehousing processes obtained from solving the APP model based on the working hour

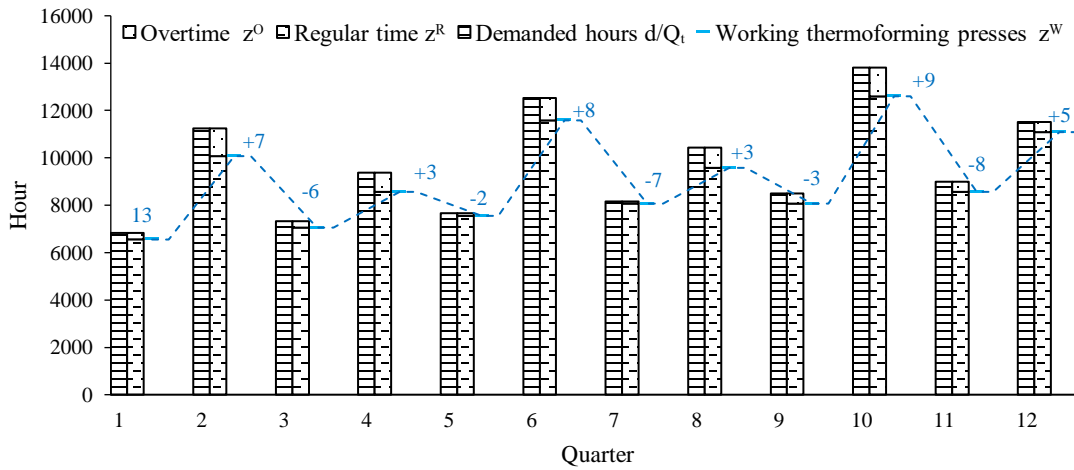


Figure 6.21: The optimal solutions for the thermoforming process from solving the APP model based on the working hour

In the experiments, the regular time production can meet the demand for most of the periods, while for several periods with relatively-high-demand that cannot be met by regular time capacity and storage from the previous quarter, overtime working become essential. Subcontracting is the last option to fill the gap between capacity and customers' needs due to its relatively high cost. Note that it is only used in Period 10 when the elevated demand cannot be satisfied by the full running of the current machines, i.e., the full capacity of regular and overtime working, plus the



inventory from period 9. Overtime and outsourcing are great options to provide high leveled flexibility to meet customers' demands (Mendoza et al., 2014). It is also noteworthy that the sheets extruded during overtime hours have neither been stored in the public nor the private warehouses. This is because a cheaper alternative, i.e., subcontracting, is assumed always available in the problem setting. The sheets produced during the regular working time are still appreciated to be stored for their competitive price (compared to subcontracting). The occurrence is because of the cost difference of products from the sources.

The private warehouse is more preferred than the public warehouse because of its low price, despite the fixed leasing area needs to be confirmed ahead of a leasing period. However, the unstable demands lead to low utilization of the public warehouse during low-demand seasons. The situation promotes adopting both types of warehousing to deal with the fluctuating demand for storage. Taking advantage of the flexibility from the public warehouse can reduce the storage cost during high inventory seasons, as it requires no fixed cost, despite it has a higher unit price. In other words, the surplus sheets should be sent to the private warehouse in priority unless it becomes full.

The workforce level varies from one period to another to adapt to the varying demand. The workforce level in the thermoforming process fluctuates more drastically than in the extruding process. This is because only one worker will be laid off or trained when idling/initiating a thermoforming press, while the cost becomes six times when the same activities occur for the extruding process because each extruder requires six operators. Therefore, it is less appreciated to lay off workers in the extruding department.

## CHAPTER 7 - SENSITIVITY ANALYSIS

In this part, the effect of a range of parameter variations on the optimal solution is analyzed. The factors being examined are demonstrated in Figure 7.1: (i) forecasting horizon (i.e., 3 years, 6 years, 9 years, and 18 years), (ii) number of extruders (i.e., 14, 16, 18, and 20), an annual increase of (iii) raw material price, (iv) labor costs, and (v) subcontracting cost with the following rates: 0%, 5%, 10%, 20%, 50% and 100%. The baseline scenario is a 3-year planning horizon, 14 number of extruders, 0% annual increase rate of raw material price, labor cost, and subcontracting price.

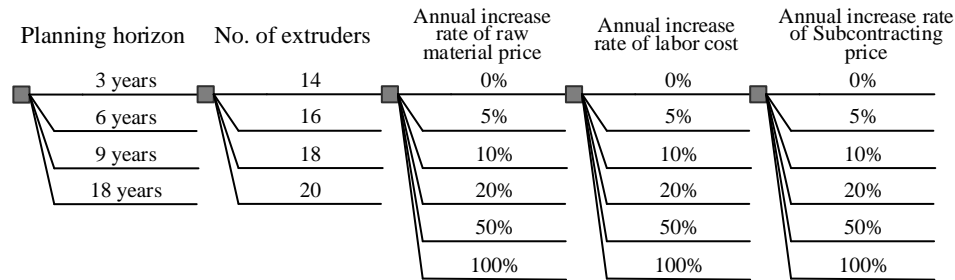


Figure 7.1: Sensitivity analysis over various parameters

The results of the sensitivity analysis are summarized in Table 7.1. Of note, to obtain the best possible solutions, the number of thermoforming presses is modified to be 32, 39, and 59, respectively, in scenarios where the length of horizons is 6-, 9-, and 18-year as the demands are assumed to keep increasing and exceed the offered capacity. It is not considered that purchasing or any other costs regarding the new machine in the objective function.

Table 7.1: Results of sensitivity analysis.

Parameters	Value	m <sup>th</sup> 3-year leasing contract	Total cost (\$)	Subcontracting ('000 lb.)	Public warehouse	Private warehouse
Planning horizon (Years)	3	1	12,390,594.15	1,856.40	✓	✓
	6	1	28,768,443.72	1,856.40	✓	✓
		2		43,653.60	✓	✗
	9	1	50,040,452.47	1,856.40	✓	✓
		2		43,653.60	✓	✗

Table 7.1 (cont'd)

Parameters	Value	m <sup>th</sup> 3-year leasing contract	Total cost (\$)	Subcontracting ('000 lb.)	Public warehouse	Private warehouse
Planning horizon (Years)	9	3		107,011.80	✗	✗
	18	1	143,471,482.47	1,856.40	✓	✓
		2		43,653.60	✓	✗
		3		107,011.80	✗	✗
		4		179,151.80	✗	✗
		5		251,293.80	✗	✗
		6		323,432.80	✗	✗
Number of extruders	14	1	12,390,594.15	1,856.40	✓	✓
	16	1	12,390,594.15	1,856.40	✓	✓
	18	1	12,390,594.15	1,856.40	✓	✓
	20	1	12,390,594.15	1,856.40	✓	✓
Increasing rate of raw material price	0%	1	12,390,594.15	1,856.40	✓	✓
	5%	1	12,516,618.27	4,486.20	✓	✓
	10%	1	12,642,701.26	4,486.20	✓	✓
	20%	1	12,901,574.51	7,664.00	✓	✓
	50%	1	13,758,080.51	14,196.20	✓	✓
	100%	1	15,973,673.75	232,771.00	✗	✗
Increasing rate of labor cost	0%	1	12,390,594.15	1,856.40	✓	✓
	5%	1	12,890,133.61	7,664.00	✓	✓
	10%	1	13,388,795.16	9,887.00	✓	✓
	20%	1	14,415,659.16	14,196.20	✓	✓
	50%	1	17,212,612.50	232,771.00	✗	✗
	100%	1	18,816,635.00	232,771.00	✗	✗
Increasing rate of subcontract price	0%	1	12,390,594.15	1,856.40	✓	✓
	5%	1	12,417,768.09	0.00	✓	✓
	10%	1	12,417,768.09	0.00	✓	✓
	20%	1	12,417,768.09	0.00	✓	✓
	50%	1	12,417,768.09	0.00	✓	✓
	100%	1	12,417,768.09	0.00	✓	✓

From Table 7.1, a combination of public and private storage is indicated as the most favorable option in most scenarios. However, there are some exceptions. In the second 3-year of the 6/9-year planning horizon, public storage is solely recommended. This is because the demands are more volatile over the 2<sup>nd</sup> or 3<sup>rd</sup> leasing period within which the demand is assumed to keep increasing. Private storage then becomes less attractive because it requires a minimum 3-year lease, which causes more losses due to the unused area during low-demand seasons than it would have saved by not using the public warehouse. During the third leasing period (only under the scenario of the 9-year planning horizon) where the demand is relatively high, subcontracting becomes an

appealing strategy due to the significant insufficiency of the capacity. There would be no need for storage, neither in public nor in private storage.

Increases in raw material prices or labor costs also make subcontracting very appealing, because it is assumed that the subcontracting price remains the same. In other words, the extruding facility will be idle, and the supply completely relies on the subcontracting, so neither public nor private storage will be needed. Once subcontracting becomes costly, manufacturing the product at the facility becomes more appealing, that is why both private and public warehouses are important in such cases. Labor cost is the most sensitive parameter in the examination, as it is the dominating component of the production costs.

## CHAPTER 8 - CONCLUSION

This study addresses an important optimization problem of aggregate production planning for the case of manufacturing reusable plastic containers. Such a problem aims to coordinate various segments of the supply chain such as production, inventory, and workforce levels together. Operations planning for these segments separately would be much less complex since fewer variables and constraints that connect these segments will be dealt with; however, it does not guarantee that resources (e.g., raw materials, storage space, machines, workforce) are used optimally. The manufacturing of reusable containers involves two main processes: extruding plastic sheets and thermoforming. Besides these processes, one can decide to store extra sheets extruded from the first phase (extrusion) to use for the second phase (thermoforming) in future periods. This can significantly reduce the concern of shortage when demand increases during a season. In the experiment, there are two options for storage: public and private warehouses. Each has its regulations. In the meantime, the option of subcontracting with unlimited production capacity exists to make up the limited production capacity.

This complicated production planning problem is mathematically modeled in two different ways: one based on the flow of materials and the other based on the level of the workforce. Both models produce the same results. The problem is coded with GAMS, distribution 30.3.0, and a comprehensive sensitivity analysis under various scenarios is carried out. In the sensitivity analysis, the impacts of various factors are examined, including the length of the planning horizon, the number of extruders, the annual increase rate of raw material price, annual labor costs, and subcontracting cost on the optimal solution. The proposed framework can be used not only for

reusable container manufacturing but also for the manufacturing of any type of product with a similar supply chain network.

Future exploration can be directed toward case studies in which various constraints for manufacturing phases, as well as limitations and regulations on subcontracting/third-party logistics and warehousing, be reflected on the APP models. This investigation provides a better understanding of the complicated APP models and presents a great tool for practitioners who would like to apply such decision support systems for their production lines.

## **APPENDICES**

## APPENDIX A: Forecasted demand for Black and Clear Containers

Table A: Forecasted demand for black and clear containers over 18 years

Year	Quarter	Black container demand forecast ('000 lb.)	Clear container demand forecast ('000 lb.)	Year	Quarter	Black container demand forecast ('000 lb.)	Clear container demand forecast ('000 lb.)
2010	I	6,693	6,972	2019	I	15,192	15,013
	II	4,599	17,889		II	10,258	37,943
	III	5,492	9,180		III	12,050	19,190
	IV	14,625	4,125		IV	31,581	8,504
2011	I	7,543	7,776	2020	I	16,042	15,817
	II	5,164	19,894		II	10,824	39,948
	III	6,148	10,181		III	12,706	20,191
	IV	16,321	4,563		IV	33,277	8,942
2012	I	8,393	8,580	2021	I	16,892	16,621
	II	5,730	21,900		II	11,390	41,954
	III	6,804	11,182		III	13,362	21,192
	IV	18,016	5,001		IV	34,972	9,379
2013	I	9,243	9,384	2022	I	17,742	17,425
	II	6,296	23,905		II	11,956	43,959
	III	7,460	12,183		III	14,017	22,193
	IV	19,712	5,439		IV	36,668	9,817
2014	I	10,093	10,188	2023	I	18,592	18,230
	II	6,862	25,910		II	12,522	45,964
	III	8,115	13,184		III	14,673	23,195
	IV	21,408	5,876		IV	38,363	10,255
2015	I	10,943	10,993	2024	I	19,442	19,034
	II	7,428	27,916		II	13,088	47,970
	III	8,771	14,185		III	15,329	24,196
	IV	23,103	6,314		IV	40,059	10,693
2016	I	11,793	11,797	2025	I	20,292	19,838
	II	7,994	29,921		II	13,653	49,975
	III	9,427	15,186		III	15,985	25,197
	IV	24,799	6,752		IV	41,755	11,131
2017	I	12,643	12,601	2026	I	21,142	20,642
	II	8,560	31,927		II	14,219	51,981
	III	10,083	16,187		III	16,641	26,198
	IV	26,494	7,190		IV	43,450	11,569
2018	I	13,493	13,405	2027	I	21,992	21,446
	II	9,126	33,932		II	14,785	53,986
	III	10,739	17,188		III	17,296	27,199
	IV	28,190	7,628		IV	45,146	12,007



## APPENDIX B: The Optimal APP Solutions

Table B1: The optimal solution for the APP model based on the flow of materials

Period i	Extruding process					Subcontract	Warehousing		Thermoforming process				
	'000 lb. of sheets		Extruders				'000 lb. of sheets					Thermoforming presses	
	$x_i^R$	$x_i^O$	$x_i^W$	$x_i^H$	$x_i^L$	$y_i^{Sub}$	$y_i^{Pu}$	$y_i^{Pr}$	$x_i^R$	$x_i^O$	$x_i^W$	$x_i^H$	$x_i^L$
1	17,236.80	-	12	12	-	-	1,228	2,344	13,104.00	561.00	13	13	-
2	17,236.80	1,678.40	12	-	-	-	-	-	20,160.00	2,327.00	20	7	-
3	17,236.80	-	12	-	-	-	221	2,344	14,112.00	560.00	14	-	6
4	17,236.80	-	12	-	-	-	-	1,052	17,136.00	1,614.00	17	3	-
5	18,673.20	-	13	1	-	-	2,062	2,344	15,120.00	199.00	15	-	2
6	18,673.20	1,980.00	13	-	-	-	-	-	23,184.00	1,875.00	23	8	-
7	18,673.20	-	13	-	-	-	-	2,344	16,128.00	201.00	16	-	7
8	18,673.20	-	13	-	-	-	-	133	19,152.00	1,732.00	19	3	-
9	20,109.60	-	14	1	-	-	926	2,344	16,128.00	845.00	16	-	3
10	20,109.60	2,394.00	14	-	-	1,856	-	-	25,200.00	2,430.00	25	9	-
11	20,109.60	-	14	-	-	-	-	2,124	17,136.00	850.00	17	-	8
12	20,109.60	783.80	14	-	-	-	-	-	22,176.00	841.00	22	5	-
Selection of public warehouse $\alpha$			1				Fixed leasing area for private warehouse (ft <sup>2</sup> )				2,930		
Selection of private warehouse $\beta$			1				Total costs (				12,390,594.15		

Table B2: The optimal solution for the APP model based on workforce level

Period i	Extruding process					Subcontract	Warehousing		Thermoforming process				
	Working hours		Extruders			'000 lb. of sheets			Working hours		Thermoforming presses		
	$x_i^R$	$x_i^O$	$x_i^W$	$x_i^H$	$x_i^L$	$y_i^{Sub}$	$y_i^{Pu}$	$y_i^{Pr}$	$x_i^R$	$x_i^O$	$x_i^W$	$x_i^H$	$x_i^L$
1	6,048.00	-	12	12	-	-	1,228	2,344	6,552.00	280.50	13	13	-
2	6,048.00	588.91	12	-	-	-	-	-	10,080.00	1,163.50	20	7	-
3	6,048.00	-	12	-	-	-	221	2,344	7,056.00	280.00	14	-	6
4	6,048.00	-	12	-	-	-	-	1,052	8,568.00	807.00	17	3	-
5	6,552.00	-	13	1	-	-	2,062	2,344	7,560.00	99.50	15	-	2
6	6,552.00	694.74	13	-	-	-	-	-	11,592.00	937.50	23	8	-
7	6,552.00	-	13	-	-	-	-	2,344	8,064.00	100.50	16	-	7
8	6,552.00	-	13	-	-	-	-	133	9,576.00	866.00	19	3	-
9	7,056.00	-	14	1	-	-	926	2,344	8,064.00	422.50	16	-	3
10	7,056.00	840.00	14	-	-	1,856	-	-	12,600.00	1,215.00	25	9	-
11	7,056.00	-	14	-	-	-	-	2,124	8,568.00	425.00	17	-	8
12	7,056.00	275.02	14	-	-	-	-	-	11,088.00	420.50	22	5	-
Selection of public warehouse $\alpha$			1					Fixed leasing area for private warehouse (ft2)			2,930		
Selection of private warehouse $\beta$			1					Total costs (\$)			12,390,594.15		

## APPENDIX C: GAMS Programming Code for the APP Model Based on the Material Flow

1 Aggregate planning in packaging manufacturing based on production amount, all production units are in 1000 pounds

Compilation

2

3 Options MIP = LINDO;

4 set i Quarter number/1\*12/;

5 Parameter Value\_i(i) Value of i

6 /

7 1 1

8 2 2

9 3 3

10 4 4

11 5 5

12 6 6

13 7 7

14 8 8

15 9 9

16 10 10

17 11 11

18 12 12

19 /;

20 parameter d(i) Demand in quarter i

21 /

22 1 13665

23 2 22487

24 3 14672

25 4 18750

26 5 15319

27 6 25059

28 7 16329

29 8 20884

30 9 16973

31 10 27630

32 11 17986

33 12 23017

34 /;

35

36

37 Parameter N\_e 'Number of extruders available' /14/;

38 Parameter Q\_e 'Production capacity of each extruder per hour, in 1000 pounds' /3/;

39 Parameter R\_ef 'The efficiency rate of production capacity of each extruder' /0.95/;

40 Parameter Q\_ef 'The efficient production capacity of each extruder per hour';

41 Q\_ef = R\_ef \* Q\_e;

42 Parameter R 'Workers salary in regular time' /15/;  
 43 Parameter O\_rate 'Rate of overtime salary to regular time' /1.5/;  
 44 Parameter O 'Workers salary in overtime';  
 45  $O = O\_rate * R$ ;  
 46 Parameter O\_m 'Max overtime working hours per quarter' /60/;  
 47 Parameter W\_e 'Number of workers needed for each extruder' /6/;  
 48 Parameter C\_h 'Training cost of a new worker' /3000/;  
 49 Parameter C\_l 'Laying off cost of an existed worker' /2500/;  
 50 Parameter N\_t 'Number of thermoforming presses available' /25/;  
 51 Parameter W\_t 'Number of workers needed for each thermoforming press per hour' /1/;  
 52 Parameter Q\_t 'Production capacity of each thermoforming press per hour' /2/;  
 53 Parameter C\_sub 'The subcontracting price of 1000pounds of plastic sheet (\$)' /60/;  
 54 Parameter C\_rm 'Price of 1000 pounds of raw materials' /10/;  
 55 Parameter C\_tr 'Transportation cost of 1000 pounds of plastic sheet  
from public warehouse to thermoforming presses' /2/;  
 56 Parameter C\_mh 'Unloading cost of 1000 pounds of plastic sheet at public warehouse' /5/;  
 57 Parameter C\_st 'Storage cost of 1000 pounds of plastic sheet at public warehouse' /11/;  
 58 Parameter C\_f 'Fixed leasing cost of 1 square foot per quarter in private warehouse' /4/;  
 59 Parameter C\_v 'Variable operating 1000 pounds of plastic stored per  
quarter in private warehouse' /4/;  
 60 Parameter M 'Constant of 1 billion' /1000000000/;  
 61 Parameter R\_Pr Utilization rate of private warehouse /0.8/;  
 62  
 63  
 64 Variable z objective function;  
 65  
 66 Binary variables  
 67 a Whether public warehousing is chosen  
 68 b whether private warehousing is chosen;  
 69 Nonnegative variables  
 70  $x\_R(i)$  Amount of extruded produced during regular working hours in period i  
 71  $x\_O(i)$  Amount of extruded produced during overtime in period i  
 72  $y\_pu(i)$  Amount of plastic sheets stored in public warehouse at the end of period i  
 73  $y\_pr(i)$  Amount of plastic sheets stored in private warehouse at the end of period i  
 74  $y\_sub(i)$  The amount of plastic sheet produced by subcontractor (in 1000 pounds) in  
period i  
 75  $y\_max$  Fixed leased area in private house  
 76  $z\_R(i)$  Amount of thermoformed products during regular working hours in period i  
 77  $z\_O(i)$  Amount of thermoformed products during overtime in period i ;  
 78 Integer variables  
 79  $x\_w(i)$  Number of working extruders in period i  
 80  $x\_H(i)$  Number of newly hired extruders in period i  
 81  $x\_L(i)$  Number of laid off extruders in period i  
 82  $z\_w(i)$  Number of workers in period i  
 83  $z\_H(i)$  Number of newly hired workers in period i  
 84  $z\_L(i)$  Number of laid off thermoforming workers in period i;

85 Equations

86 obj\_fn Objective function  
87 e\_reg(i) Capacity of extruders working time in regular time in period i  
88 e\_over(i) Capacity of extruders working time on overtime in period i  
89 t\_reg(i) Capacity of thermoforming presses working time in regular time in period i  
90 t\_over(i) Capacity of thermoforming presses working time on overtime in period i  
91 max\_e(i) Max number of working extruders in period i  
92 max\_t(i) Max number of working thermoforming presses in period i  
93 num\_e\_w(i) Number of working extruders in period i  
94 num\_t\_w(i) Number of working thermoforming presses in period i  
95 flow\_balance(i) Flow balance of materials in period i  
96 demand\_sat(i) Demand satisfaction in period i  
97 one\_time\_lease(i) Once y\_max is decided in period 1 it should be fixed for the whole 3 years  
98 pu\_lmt Public warehousing limit  
99 pr\_lmt Private warehousing limit  
100 a\_b Sum of a and b should be equal to 2;  
101  
102 obj\_fn .. z=e= sum(i,(x\_R(i)+x\_O(i)) \* C\_rm  
103 + y\_sub(i) \* C\_sub  
104 + W\_e \* x\_R(i)/Q\_ef \* R  
105 + W\_t \* z\_R(i)/Q\_t \* R  
106 + W\_e \* x\_O(i)/Q\_ef \* O  
107 + W\_t \* z\_O(i)/Q\_t \* O  
108 + x\_H(i) \* W\_e \* C\_h  
109 + z\_H(i) \* W\_t \* C\_h  
110 + x\_L(i) \* W\_e \* C\_l  
111 + z\_L(i) \* W\_t \* C\_l  
112 + y\_pu(i) \* C\_tr  
113 + y\_pu(i) \* C\_mh  
114 + y\_pu(i) \* C\_st  
115 + y\_pr(i) \* C\_tr  
116 + y\_pr(i) \* C\_v  
117 + y\_max \* C\_f);  
118 e\_reg(i) .. x\_R(i)/Q\_ef=e= x\_W(i) \* 504;  
119 t\_reg(i) .. z\_R(i)/Q\_t=e= z\_W(i) \* 504;  
120 e\_over(i) .. x\_O(i)/Q\_ef=l= x\_W(i) \* O\_m;  
121 t\_over(i) .. z\_O(i)/Q\_t=l= z\_W(i) \* O\_m;  
122 max\_e(i) .. x\_W(i)=l= N\_e;  
123 max\_t(i) .. z\_W(i)=l= N\_t;  
124 num\_e\_w(i) .. x\_W(i)=e= x\_W(i-1)+x\_H(i)-x\_L(i);  
125 num\_t\_w(i) .. z\_W(i)=e= z\_W(i-1)+z\_H(i)-z\_L(i);  
126 flow\_balance(i) .. (x\_R(i)+x\_O(i)) + y\_sub(i)-(y\_pr(i) + y\_pu(i)) + (y\_pr(i-1) + y\_pu(i-1))=e= (z\_R(i) + z\_O(i));  
127 demand\_sat(i) .. (z\_R(i)+z\_O(i))=e= d(i);  
128 one\_time\_lease(i).. y\_max \* R\_Pr=g= y\_pr(i);

```

129 pu_lmt .. sum(i,y_pu(i)) =l= M * a;
130 pr_lmt .. sum(i,y_pr(i)) =l= M * b;
131 a_b .. a+b =l= 2;
132
133
134 Model aggregate_planning /ALL/;
135 Solve aggregate_planning using MIP minimizing z;
136 Display z.l, x_R.l, x_O.l, x_W.l, x_H.l, x_L.l, y_max.l,y_sub.l, y_pu.l, y_pr.l, z_R.l,
z_O.l, z_W.l, z_H.l, z_L.l;

```

## APPENDIX D: GAMS Programming Code for the APP Model Based on the Flow of Workforce Level

1 Aggregate planning in packaging manufacturing based on working hours, all production units are in 1000 pounds;

Compilation

2 Options MIP = LINDO;

3 set i 'Quarter number'/1\*12/;

4 Parameter Value\_i(i) 'Value of i'

5 /

6 1 1

7 2 2

8 3 3

9 4 4

10 5 5

11 6 6

12 7 7

13 8 8

14 9 9

15 10 10

16 11 11

17 12 12

18 /;

19 parameter D(i) 'Demand in quarter i'

20 /

21 1 13665

22 2 22487

23 3 14672

24 4 18750

25 5 15319

26 6 25059

27 7 16329

28 8 20884

29 9 16973

30 10 27630

31 11 17986

32 12 23017

33 /;

34

35

36 Parameter N\_e 'Number of extruders available' /14/;

37 Parameter Q\_e 'Production capacity of each extruder per hour, in 1000 pounds' /3/;

38 Parameter R\_ef 'The efficiency rate of production capacity of each extruder' /0.95/;

39 Parameter Q\_ef 'The efficient production capacity of each extruder per hour';

40 Q\_ef = R\_ef \* Q\_e;

41 Parameter R 'Workers salary in regular time' /15/;  
 42 Parameter O\_rate 'Rate of overtime salary to regular time' /1.5/;  
 43 Parameter O 'Workers salary in overtime';  
 44  $O = O\_rate * R$ ;  
 45 Parameter O\_m 'Max overtime working hours per quarter' /60/;  
 46 Parameter W\_e 'Number of workers needed for each extruder' /6/;  
 47 Parameter C\_h 'Training cost of a new worker' /3000/;  
 48 Parameter C\_l 'Laying off cost of an existed worker' /2500/;  
 49 Parameter N\_t 'Number of thermoforming presses available' /25/;  
 50 Parameter W\_t 'Number of workers needed for each thermoforming press per hour' /1/;  
 51 Parameter Q\_t 'Production capacity of each thermoforming press per hour' /2/;  
 52 Parameter C\_sub 'The subcontracting price of 1000pounds of plastic sheet (\$)' /60/;  
 53 Parameter C\_rm 'Price of 1000 pounds of raw materials' /10/;  
 54 Parameter C\_tr 'Transportation cost of 1000 pounds of plastic sheet from public  
 warehouse to thermoforming presses' /2/;  
 55 Parameter C\_mh 'Unloading cost of 1000 pounds of plastic sheet at public warehouse' /5/;  
 56 Parameter C\_st 'Storage cost of 1000 pounds of plastic sheet at public warehouse' /11/;  
 57 Parameter C\_f 'Fixed leasing cost of 1 square foot per quarter in private warehouse' /4/;  
 58 Parameter C\_v 'Variable operating 1000 pounds of plastic stored per quarter in private  
 warehouse' /4/;  
 59 Parameter M 'Constant of 1 billion' /1000000000/;  
 60 Parameter R\_Pr Utilization rate of private warehouse /0.8/;  
 61  
 62 Variable z 'objective function';  
 63  
 64 Binary variables  
 65 a 'Whether public warehousing is chosen'  
 66 b 'whether private warehousing is chosen';  
 67 Nonnegative variables  
 68 x\_R(i) 'Regular working hours for all working extruders in period i'  
 69 x\_O(i) 'Overtime working hours for all working extruders in period i'  
 70 y\_pu(i) 'Amount of plastic sheets stored in public warehouse at the end of period i'  
 71 y\_pr(i) 'Amount of plastic sheets stored in private warehouse at the end of period i'  
 72 y\_sub(i) 'The amount of plastic sheet produced by subcontractor (in 1000 pounds) in  
 period i'  
 73 y\_max 'Fixed leased area in private house'  
 74 z\_R(i) 'Total working hours of regular for thermoforming in period i'  
 75 z\_O(i) 'Total working hours of overload for thermoforming in period i';  
 76 Integer variables  
 77 x\_w(i) 'Number of working extruders in period i'  
 78 x\_H(i) 'Number of newly hired extruders in period i'  
 79 x\_L(i) 'Number of laid off extruders in period i'  
 80 z\_w(i) 'Number of workers in period i'  
 81 z\_H(i) 'Number of newly hired workers in period i'  
 82 z\_L(i) 'Number of laid off thermoforming workers in period i';



83 Equations

84 obj\_fn Objective function

85 e\_reg(i) Capacity of extruders working time in regular time in period i

86 e\_over(i) Capacity of extruders working time on overtime in period i

87 t\_reg(i) Capacity of thermoforming presses working time in regular time in period i

88 t\_over(i) Capacity of thermoforming presses working time on overtime in period i

89 max\_e(i) Max number of working extruders in period i

90 max\_t(i) Max number of working thermoforming presses in period i

91 num\_e\_w(i) Number of working extruders in period i

92 num\_t\_w(i) Number of working thermoforming presses in period i

93 flow\_balance(i) Flow balance of materials in period i

94 demand\_sat(i) Demand satisfaction in period i

95 one\_time\_lease(i) Once y\_opt is decided in period 1 it should be fixed for the whole 3 years

96 pu\_lmt Public warehousing limit

97 pr\_lmt Private warehousing limit

98 a\_b Sum of a and b should be equal to 2;

99

100 obj\_fn .. z=e= sum(i,(x\_R(i)+x\_O(i)) \* Q\_ef \* C\_rm

101 + y\_sub(i) \* C\_sub

102 + W\_e \* x\_R(i) \* R

103 + W\_t \* z\_R(i) \* R

104 + W\_e \* x\_O(i) \* O

105 + W\_t \* z\_O(i) \* O

106 + x\_H(i) \* W\_e \* C\_h

107 + z\_H(i) \* W\_t \* C\_h

108 + x\_L(i) \* W\_e \* C\_l

109 + z\_L(i) \* W\_t \* C\_l

110 + y\_pu(i) \* C\_tr

111 + y\_pu(i) \* C\_mh

112 + y\_pu(i) \* C\_st

113 + y\_pr(i) \* C\_tr

114 + y\_pr(i) \* C\_v

115 + y\_max \* C\_f);

116 e\_reg(i) .. x\_R(i)=e= x\_W(i) \* 504;

117 t\_reg(i) .. z\_R(i)=e= z\_W(i) \* 504;

118 e\_over(i) .. x\_O(i)=l= x\_W(i) \* O\_m;

119 t\_over(i) .. z\_O(i)=l= z\_W(i) \* O\_m;

120 max\_e(i) .. x\_W(i)=l= N\_e;

121 max\_t(i) .. z\_W(i)=l= N\_t;

122 num\_e\_w(i) .. x\_W(i)=e= x\_W(i-1)+x\_H(i)-x\_L(i);

123 num\_t\_w(i) .. z\_W(i)=e= z\_W(i-1)+z\_H(i)-z\_L(i);

124 flow\_balance(i) .. (x\_R(i)+x\_O(i)) \* Q\_ef + y\_sub(i)-(y\_pr(i) + y\_pu(i)) + (y\_pr(i-1) + y\_pu(i-1))=e= (z\_R(i) + z\_O(i)) \* Q\_t;

125 demand\_sat(i) .. (z\_R(i)+z\_O(i)) \* Q\_t=e= d(i);

126 one\_time\_lease(i).. y\_max \* R\_Pr=g= y\_pr(i);

```

127  pu_lmt  .. sum(i,y_pu(i)) =l= M * a;
128  pr_lmt  .. sum(i,y_pr(i)) =l= M * b;
129  a_b     .. a+b =l= 2;
130  Model aggregate_planning /ALL/;
131  Solve aggregate_planning using MIP minimizing z;
132  Display z.l, x_R.l, x_O.l, x_W.l, x_H.l, x_L.l, y_max.l,y_sub.l, y_pu.l, y_pr.l, z_R.l,
z_O.l, z_W.l, z_H.l, z_L.l;

```

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