AN ANALYTICAL FOUNDATION
FOR THE
DESIGN OF AUTOMOTIVE VEHICLES

By

Louis Leslie Otto

A THESIS

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VITA

The author was born on October 25, 1910 on Long Island, in the State of New York. He received his elementary and high school education in the Sayville Union School and the Sayville High School at Sayville, New York. His undergraduate training was obtained in the Sibley School of Mechanical Engineering of Cornell University, from which he was graduated in 1933 with the degree of Mechanical Engineer after majoring in Automotive Engineering.

After graduation he spent several years as a junior production engineer with the United States Gypsum Company at its plant in Oakfield, New York, and as chief engineer of the Kaustine Company at Perry, New York, engaged in the design and production of truck bodies, fabricated steel products, and sewage disposal equipment.

In 1938 he returned to Cornell University as a graduate student, majoring in Automotive Engineering. In 1939 he became an instructor in the Experimental Engineering Department of the Sibley School of Mechanical Engineering. In 1943 he was awarded the degree of Master of Mechanical Engineering and was promoted to the position of Assistant Professor of Automotive Engineering. In July 1946 he was promoted to Associate Professor of Mechanical Engineering in the Department of Heat Power Engineering.

During the school year 1949-50 the author was granted a sabbatical leave from his work at Cornell. This period, together with the preceding and succeeding summers, was spent as a graduate student in residence at Michigan State College. In the fall of 1950 the author returned to his position at Cornell, and on July 1, 1951, was advanced to the rank of Professor of Mechanical Engineering.

The author is a member of Tau Beta Pi, Sigma Xi, Phi Kappa Phi, Society of Automotive Engineers, American Society for Engineering Education, American Association of University Professors, and is a licensed Professional Engineer.

The author is now a candidate for the degree of Doctor of Philosophy, majoring in Mechanical Engineering, with minors in Electrical Engineering and Agricultural Engineering.

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INTRODUCTION

The self-propelled wheeled vehicle has become an integral and indispensable part of the military and civilian life of our day. The engineer concerned with the design, operation, and application of the slef-propelled wheeled vehicle is confronted with problems in which a knowledge of the methods by which the basic principles of mathematics and physics are applied to vehicle design would be of great use.

Automotive vehicle design involves the detail consideration of many separate sections, followed by the combination of these sections into an harmonious whole. Since the functioning of each section is not completely independent of the functioning of other sections, the utility of the whole vehicle will be increased if the designer of each section is fully acquainted with the basic principles governing the design and operation of all other sections.

The student majoring in automotive engineering should have presented to him the methods whereby basic engineering principles are applied to automotive vehicle design, and both the student engineer and the practicing engineer should have available for reference a presentation of these methods.

It is the purpose of this thesis to present in sufficient detail the way in which the basic principles obtained from an engineering training are applied to several phases of automotive vehicle design.

Selected as the most appropriate fields for this purpose are: driving ability, vehicle performance, and brakes and braking ability.

These three fields cover the problems involved in getting the vehicle moving and keeping it moving, power requirements for a given performance level, or the performance level to be expected from a given installed power, and the problem of stopping the vehicle or controlling its speed of motion.

There exists in the literature devoted to the automotive vehicle much that has been written about each of these fields. Most of it is descriptive, with only an occasional illustration of the application of basic principles. These occasional items must be separated from the mass of material, correlated with one another, and used as a check on the validity of the results obtained from a theoretical treatment of known basic relations.

II

DRIVING

ABILITY

of

AUTOMOTIVE VEHICLES

DRIVING FORCE

The self-propelled wheeled vehicle is able to move itself about over the surface only by means of the frictional force which it is able to develop between its driving wheels and the surface which supports it. The magnitude of this frictional force is a function mainly of the nature of the supporting surface and the wheel surface. A low value of shear strength in either one will limit the frictional force which can be developed to the shearing area times the average unit shear strength of the weaker material. This behavior limits the frictional force which can be developed by a tire operating on gravel, sand, soft soil, snow, and mud. The use of cleats which penetrate the material often can increase the driving force, but only by increasing the area subjected to shearing stress.

Any combination of wheel surface and road surface which has sufficient shear strength to resist rupture of either of the surfaces can develop a true coefficient of friction. If an attempt is made to move one of two contacting surfaces relative to the other, more and more tangential force must be applied with no resulting movement, until finally movement begins, and with continued force application the movement will continue. Repeated observations of this process have shown that for a dry rubber tire contacting a dry dense concrete or bituminous paving, the force required to cause continued movement, or slipping of one surface relative to the other, is lower than the force required to cause the initial movement.

The coefficient of friction developed between two contacting surfaces is defined as the ratio of the force tangential to the two contacting surfaces, to the force normal to the two contacting surfaces. The tangential force is usually called the Friction Force, while the normal force is called the Adhesive Force, since the ability of a body to adhere frictionally to a surface is directly proportional to the force normal to the surface.

The coefficient of friction computed from the frictional force developed when two surfaces are at the point of impending slip of one relative to the other is the maximum value attainable and is called the coefficient of friction at impending slip, or more simply, the impending slip coefficient. The magnitude of the impending slip coefficient for pneumatic rubber tires with good non-skid tread will vary from a minimum of about 0.1, for snow-coated tires operating on smooth ice, to a maximum value of over 1.0 for rough open surfaces on which the tread rubber penetrates the open spaces of the surface and grips the projections from the surface (1). An average value for this coefficient for a smooth dry dense concrete or macadam surface is 0.8.

The maximum driving force which a self-propelled vehicle can develop occurs when all wheels carrying weight are brought simultaneously to impending slide. If the vehicle weight is signified by W, and the angle between the road surface and a level surface by θ, then the normal component of gravity force perpendicular to the road surface, or the Adhesive Weight, is equal to $W \cos \theta$. If the symbol "f" is used to indicate coefficient of friction then the maximum possible driving force which can be developed between vehicle and road surface is equal to

$$f_{impending} \cdot W \cos \theta.$$

RELATIVE DRIVING EFFECTIVENESS

The maximum driving force is obtained only when all weight carrying wheels are driving wheels and are brought simultaneously to impending slip. If only a portion of the weight carrying wheels are driving wheels, or if for some reason the wheels cannot be brought simultaneously to impending slip, then the maximum driving force which can be developed will be lower than the theoretical amount which could be developed, and the driving effectiveness will be less than it theoretically could be.

The driving effectiveness of a vehicle is a measure of its ability to move itself over the ground or road surface. To the civilian vehicle operating over dry concrete paving it is an academic matter, neither understood nor appreciated. To the civilian vehicle attempting to crawl up an ice coated hill it is something which his vehicle has in insufficient quantity. To the military vehicle, operating off the road and in rugged terrain, drive effectiveness during combat operations is often the difference between life and death for personnel.

A very useful term for comparing the driving effectiveness of various vehicles is the Relative Driving Effectiveness, defined by the following ratio:

$$R.D.E. = \frac{\text{Actual Drive Force Produced by Vehicle}}{\text{Maximum Theoretical Drive Force}}$$

This is equivalent to:

$$R.D.E. = \frac{\text{Actual Drive Force Produced by Vehicle}}{f_{(\text{impending slip})} \cdot W \cos \theta}$$

The Relative Driving Effectiveness can be expressed either as a decimal or as a percent.

There are several driving wheel combinations for a four-wheel vehicle which occur in practice and whose relative driving effectiveness should be investigated. The obvious driving wheel combinations are: rear wheel drive only, front wheel drive only, and all-four-wheel drive. A less obvious but equally important set of conditions is created by the use of differentials in the drive train. A differential gear set is a device which allows its two output shafts to turn relative to one another without affecting the average speed of rotation of the two units. One shaft can overspeed the average, but the other must underspeed the average by the same amount. The ordinary differential gear set consists of a pair of output shafts carrying a bevel gear at one end and so placed that their axes coincide with one another with the bevel gears face to face. The bevel gears mesh with two, three, or four differential pinion gears mounted in a differential carrier on axes which intersect the output shaft axis at ninety degrees at a common point. The differential carrier is so mounted that it is free to rotate about the axis of its output shafts, and carries on its periphery a bevel or spur gear by means of which it is driven from another shaft. The function of the differential gear set is to carry torque to its two output shafts while giving them the freedom to rotate with respect to one another.

When a four wheel vehicle is driven around a curve its outside wheels operate over a greater radius than its inside wheels and must turn at a different speed. Also, especially in short radius turns, the front wheels will operate over an appreciably shorter radius than the rear wheels and must turn at a different average speed than that of the rear wheels. If the two rear wheels, or the two front wheels, are mounted on a common shaft, the more lightly loaded wheel is forced to slip sufficiently during cornering operations to make up the difference in rotational speed, with consequent accelerated tire wear and augmented shaft stress. If the common shaft is split into two axle shafts, one shaft can over run the other and avoid the high shaft stress and the excessive tire wear. The ordinary bevel gear differential performs this part of its functions admirably, allowing relative rotation of the two shafts with only very slight frictional resistance, while still carrying torque from the differential carrier into both shafts.

The difficulty which the ordinary differential creates, insofar as relative driving effectiveness is concerned, is that it always splits the torque flow from the differential carrier into the two output shafts into exactly the same ratio, regardless of the ability of either shaft to absorb the torque which the differential wishes to impress upon it. In the cross-axle differential this ratio, except in very special circumstances, is one to one, and the differential gearing cannot transmit to the wheel havin good tractive ability any more torque than can be developed by the wheel having poor tractive ability. This means that the unfortunate circumstance which sets one wheel of a driving pair, connected by an ordinary differential, on ice or snow or mud, while the other remains on dry hard ground or paving, limits the driving force which can be developed by the pair of wheels to twice that developed by the wheel which slips first, instead of the sum of the driving forces which could be developed at impending slip of each wheel. This condition causes the utterly ridiculous situation which sometimes occurs when a vehicle with three wheels on bare dry paving is unable to move because the fourth wheel is on a patch of ice or slippery mud. To the ordinary passenger car it is a sufficiently rare occurrence to be only an annoyance. To the commercial vehicle operating off the paved road, it is sufficiently common to be a source of economic loss and an incentive to purchase remedial equipment. To the military vehicle it is an unthinkable condition, one which must be avoided at all costs.

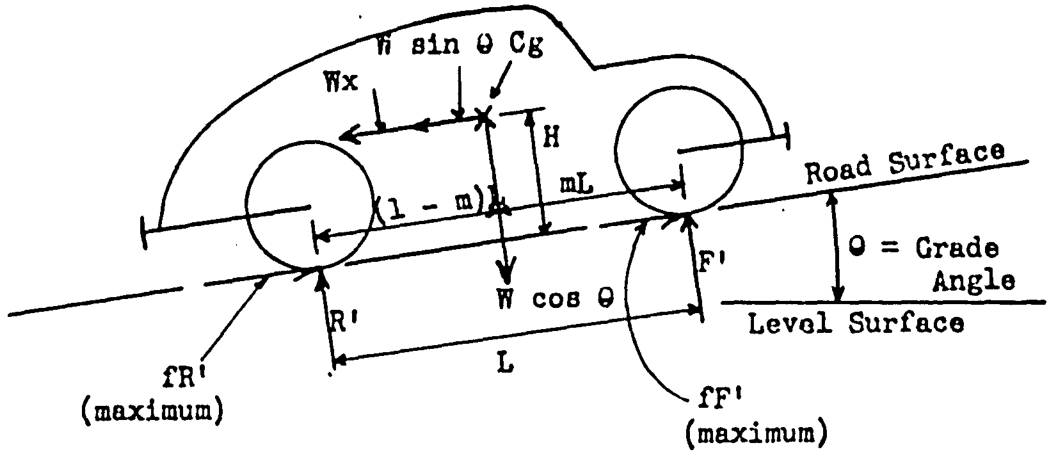
The difficulties caused by the torque splitting deficiency of the ordinary differential can be avoided by the use of a common axle shaft, with its accompanying shaft strain and tire wear, or by installing at extra cost a special type of self-locking "differential" which sends torque to each of its output shafts in an amount equal to the torque which that shaft can take without overspeeding its partner. These special "differentials" allow either shaft to overspeed the carrier during driving operations, but neither shaft to underspeed the carrier (2). Their initial cost is sufficient, and their benefits so little appreciated, that they are used on a very small proportion of the commercial vehicles in use, and not at all on passenger cars.

The use of the ordinary differential increases the number of possible drive cases for a four wheel vehicle to the following: (1) all four wheel positive drive, with either continuous shaft-gear power trains, or with self-locking differentials at all necessary points;

(2) rear wheel drive with front wheels not driven, using no differential or a self-locking differential; (3) front wheel drive with the rear wheels not driven, using no differential or a self-locking differential; (4) four wheel drive, with an ordinary differential between the shafts driving the front and rear axles, and with no differential or a self-locking differential in the cross shafts. Cases in which ordinary differentials are used in the shafts connecting front and rear wheels can be treated as special cases of one of the former types.

The Relative Driving Effectiveness for each of the four driving wheel combinations can be determined as a function of the weight distribution of the vehicle between front and rear axles, the coefficient of friction between tire and road, and the ratio of the height of the center of gravity to the wheel base. The method used in developing the relative driving effectiveness equations is based upon the consideration of the vehicle as a free body, with gravity, friction, and acceleration forces acting on it. Figure 1 shows the location and direction of these forces.

The weight of the vehicle will be shared by the front and rear wheels in a proportion determined by the distribution and mass of the various vehicle components. The symbol "m" is used to indicate the decimal of vehicle weight supported by the rear wheels when the vehicle is motionless on level road. Whenever the vehicle is subjected to a grade force (gravity component parallel to the road surface) or to an acceleration force parallel to the road surface there is a shift of weight towards one end or the other of the vehicle. The fore and aft weight distribution under dynamic conditions is designated by the



FORCES ACTING ON THE VEHICLE

Θ = Grade Angle = Angle between Road Surface and Level Surface
W = Vehicle Weight
W cos Θ = Adhesive Weight
W sin Θ = Grade Force = Gravity Component Parallel to Road Surface
x = Ratio of Linear Vehicle Acceleration to Gravity Acceleration
Wx = Acceleration Force Acting on Vehicle
Cg = Center of Gravity
H = Height of Center of Gravity above Road Surface
L = Wheelbase of Vehicle
m = Decimal of Adhesive Weight on Rear Wheels, Vehicle Motionless on a Level Surface
R = Adhesive Weight on Rear Wheels, Static Level Conditions
F = Adhesive Weight on Front Wheels, Static Level Conditions
m' = Decimal of Adhesive Weight on Rear Wheels under Dynamic Conditions
R' = m'W cos Θ
F' = (1 - m')W cos Θ
f = Friction Coefficient, Tire to Road, at Impending Slip
P_{max} = Maximum Driving Force
E = Relative Driving Effectiveness

Figure 1

symbol "m'". This weight shift occurs under the influence of driving forces, and is important to the relative driving effectiveness, but fortunately proper treatment of the equations obtained can determine the dynamic weight distribution as a function of friction coefficient and vehicle dimensions.

The Maximum Driving Force and the Relative Driving Effectiveness of the first driving wheel combination of the several systems listed in a previous paragraph is obtained as follows:

Case 1. All Wheel Positive Drive

(Self-Locking Differentials or No Differentials)

Referring to the diagram on Figure 1.

Summation of Forces Parallel to the Road:

$$P_{max} - Wx - W \sin \theta = 0$$

$$W(x + \sin \theta) = P_{max} = fR' + fF'$$

Summation of Forces Perpendicular to the Road:

$$R' + F' - W \cos \theta = 0$$

$$W \cos \theta = R' + F'$$

Also, by definition, $R' = m'W \cos \theta$, and $F' = (1 - m')\, W \cos \theta$

Summation of Moments about the Front Wheel to Road Contact Point

$$R'L - mL\,W \cos \theta - HW\,(x + \sin \theta) = 0$$

By substitution:

$$m'LW \cos \theta - mLW \cos \theta - Hf(R' + F') = 0$$

$$m'LW \cos \theta - mLW \cos \theta - Hfm'W \cos \theta - Hf(1 - m')W \cos \theta = 0$$

Divide by $LW \cos \theta$

$$m' - m - m'f\frac{H}{L} - f\frac{H}{L} + m'f\frac{H}{L} = 0$$

$$m'(1 - f\frac{H}{L}) = m$$

$$m'_{(max)} = \frac{m}{1 - f\frac{H}{L}}$$

$$P_{max} = f(R' + F') = fW \cos \theta \ (m' + 1 - m') = fW \cos \theta$$

By definition:

$$\text{Relative Driving Effectiveness} = \frac{\text{Actual Driving Force}}{fW \cos \theta}$$

In this case:

$$\text{Relative Driving Effectiveness} = \frac{fW \cos \theta}{fW \cos \theta} = 1.0$$

Since a vehicle with all wheel positive drive should develop the maximum driving force which could be produced by any driving wheel combination, this result is not unexpected.

The same system, with minor variations, can be used to determine the Maximum Driving Force and Relative Driving Effectiveness for Case 2, Rear Wheel Drive, and Case 3, Front Wheel Drive. The analysis for Case 4, All Wheel Drive with Interaxle Differential is complicated by the possibility of two different maximum drive conditions. If the weight distribution or friction coefficient is such that the front wheels slip first, then the maximum driving force for the vehicle is twice the amount produced by the front wheels. Similarly, if the rear wheels slip first, the maximum driving force is twice that developed by the rear wheels.

The equations for Maximum Driving Force P_{max}, and for Relative Driving Effectiveness E, for each of the four cases are given in Table I.

TABLE I

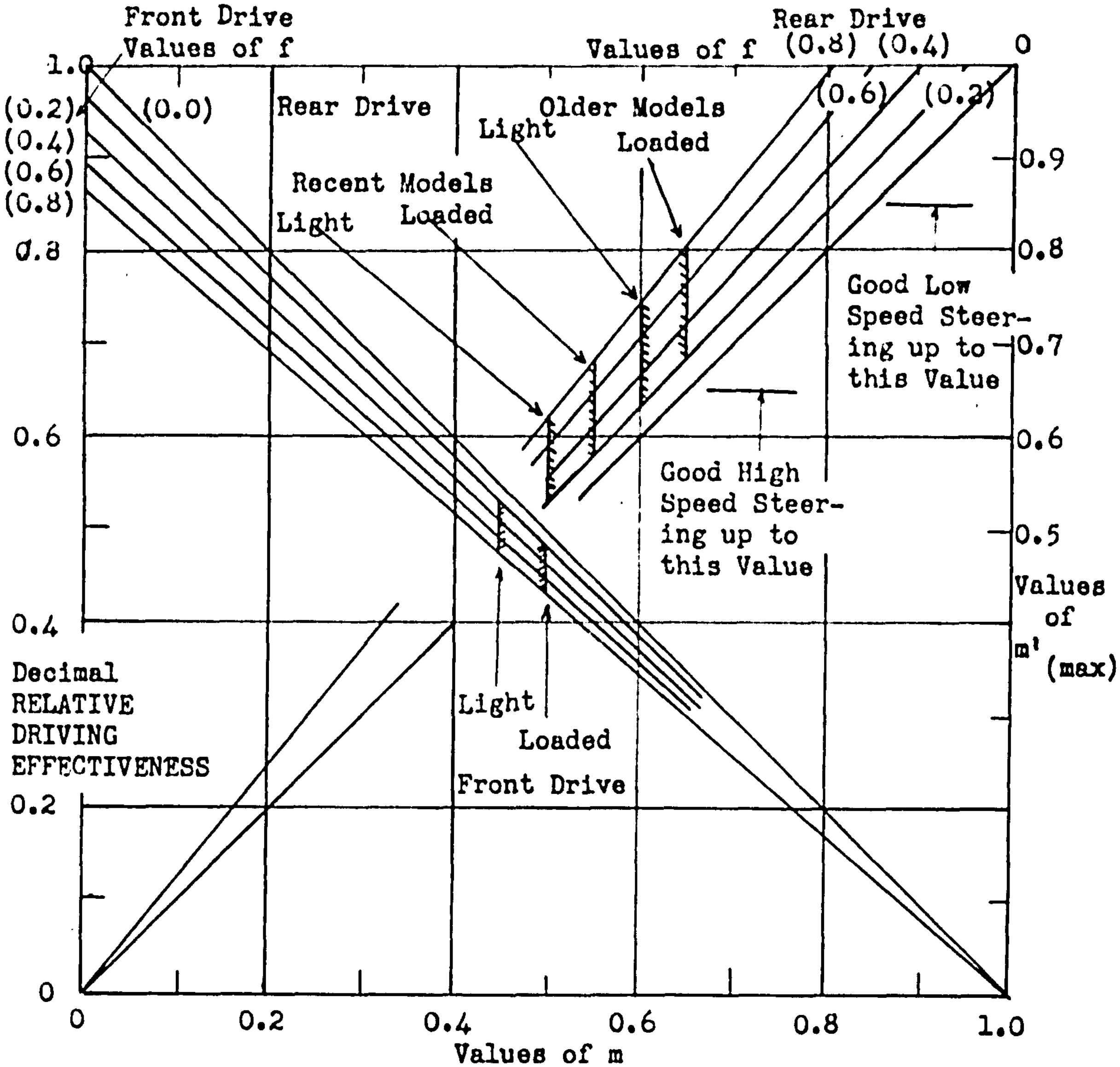
DRIVING ABILITY RELATIONS

Maximum Driving Force	Relative Driving Effectiveness
Case 1. All Wheel Positive Drive (All Wheels Simultaneously at Impending Slip)	
$P_{max} = fW \cos\theta$	$E = 1.00$
Case 2. Rear Wheel Drive (Both Rear Wheels Simultaneously at Impending Slip)	
$P_{max} = fW \cos\theta \dfrac{m}{1 - f\frac{H}{L}}$	$E = \dfrac{m}{1 - f\frac{H}{L}}$
Case 3. Front Wheel Drive (Both Front Wheels Simultaneously at Impending Slip)	
$P_{max} = fW \cos\theta \dfrac{1 - m}{1 + f\frac{H}{L}}$	$E = \dfrac{1 - m}{1 + f\frac{H}{L}}$
Case 4. All Wheel Drive with Interaxle Differential	
Action A (Rear Wheels at Impending Slip, Front Wheels Below)	
$P_{max} = fW \cos\theta \dfrac{2m}{1 - 2f\frac{H}{L}}$	$E = \dfrac{2m}{1 - 2f\frac{H}{L}}$
Action B (Front Wheels at Impending Slip, Rear Wheels Below)	
$P_{max} = fW \cos\theta \dfrac{2(1 - m)}{1 + 2f\frac{H}{L}}$	$E = \dfrac{2(1 - m)}{1 + 2f\frac{H}{L}}$

These equations do not readily yield any comparative information on the magnitude of the relative driving effectiveness produced for different combinations of the affected variables. Passenger cars at the present time are rear wheel driven exclusively. Front drive models have been built in the past, but were never very popular. To show graphically the effect of the two design types on the relative driving effectivenss produced, a series of solutions of the necessary equations have been combined to form Figure 2. The probable values of $\frac{H}{L}$ for the two types have been assumed and used. Also indicated is the effect of the passenger load. Since most of the passenger compartment is in the rear half of the vehicle, the addition of passengers to the vehicle increases the effective value of m. The magnitude of the change is about 0.5, between m for the light vehicle and m for the loaded vehicle. Adding passengers to the rear drive car increases its driving effectiveness. Adding passengers to the front drive car decreases its driving effectiveness.

Also shown on Figure 2 is an indication of the effect of weight shift on steering. The value of m chosen for the empty car should be such that even after the maximum weight shift due to acceleration grade forces has taken place, there will remain adequate weight on the front wheels to provide the necessary frictional force for steering the vehicle. Low speed operation does not demand high steering forces, and a minimum of fifteen percent of the vehicle's weight remaining on the front wheels should be adequate. High speed operation, however, requires greater steering forces for safety, and a minimum of thirty five percent of the vehicle's weight should remain on the front wheels

RELATIVE DRIVING EFFECTIVENESS FOR PASSENGER CARS



$\frac{H}{L}$ for Rear Drive Passenger Cars assumed to be 0.25

$\frac{H}{L}$ for Front Drive Passenger Cars assumed to be 0.20

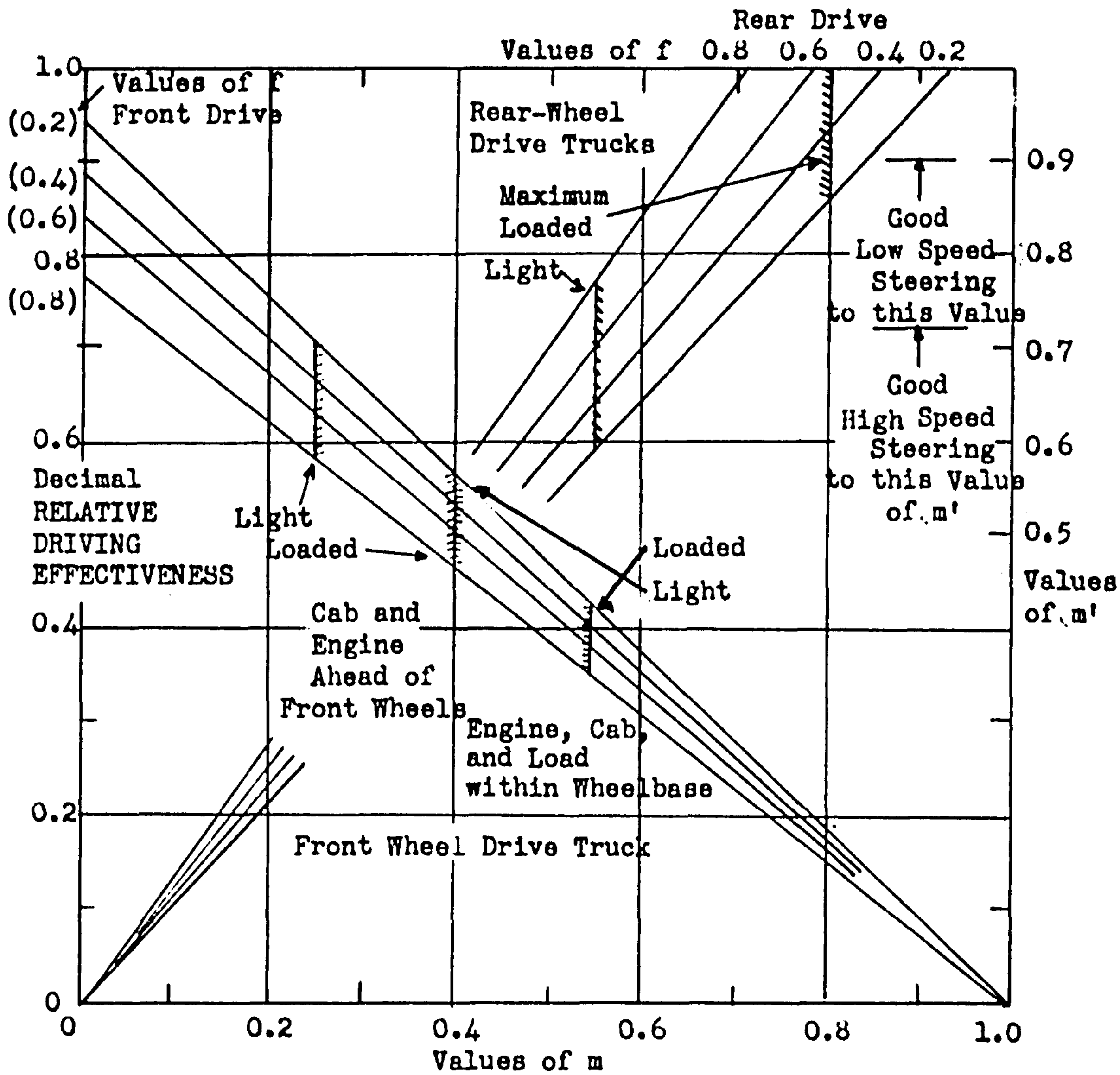
Figure 2

to guard against front wheel skid during a combination of maximum driving and maximum steering forces. This condition is approached or attained by our present rear drive passenger cars, and possible front end skid conditions could occur. Good driving practice here would indicate deceleration, or at least no acceleration, when taking a car into a sharp curve at high speed.

For most car driving conditions the difference between the relative driving effectiveness values for front drive and rear drive cars is academic. It is only when a high value of driving effectiveness is really needed that the difference becomes evident. Operation up a hill on snow or ice produces the marginal conditions required. Under these conditions the rear drive vehicle can often operate when the front drive vehicle is completely helpless. The driving ability of the rear drive vehicle under these conditions can be noticeably improved by a temporary increase in m, such as that produced by several hundred pounds of sand or iron added to the back compartment, or by several people standing on the rear bumper.

The use of front wheel drive on trucks is relatively much more frequent than its use on passenger cars. Special low loading platforms, unencumbered by driveshaft and axle housings, are sometimes of sufficient value to warrant the extra cost of the front wheel drive truck. A comparison of the relative driving effectiveness of the conventional and the special truck are shown on Figure 3. The use of special weight distributions such as those attained by mounting both engine and cab ahead of the front wheels, can produce a driving ability comparing favorably with that of a rear drive truck, but it

RELATIVE DRIVING EFFECTIVENESS
FRONT DRIVE and REAR DRIVE TRUCKS



$\frac{H}{L}$ for Rear Drive Trucks assumed at 0.36

$\frac{H}{L}$ for Front Drive Trucks assumed at 0.28

$m_{Loaded} - m_{Light}$ assumed equal to 0.15

Figure 3

is done at the expense of easy steering and good road handling, since the truck is decidedly nose heavy and would require power steering, plus a limitation on maximum operating speed.

Figure 3 shows up a condition which the rear drive truck sometimes encounters unintentionally. Adding too much of the load to that section of the loading platform which extends behind the rear wheels will increase the value of m to the point where it is possible to obtain one hundred per cent relative driving effectiveness, which is desirable, but when the vehicle is subjected to an appreciable driving force so much weight is transferred from the front wheels that the driver is no longer able to steer the truck, either because there is insufficient steering force developed, or because the front wheels have lifted completely off the road surface. Such a condition would be dangerous, and trucks with a long rear overhang should be loaded with care and then driven slowly to avoid loss of steering on hills or during maximum acceleration.

The four wheel drive truck with the interaxle differential was quite common before the availability of satisfactory self-locking differentials. Proper choice of m for a truck of this type can produce one hundred percent relative driving effectiveness for any given coefficient of friction. Inspection of the equations for P_{max} and E in Table I for this type of vehicle shows that when E = 1.0, the maximum drive force for front and rear axles will be equal, and the P_{max} relations for Action A and Action B can be set equal to one another.

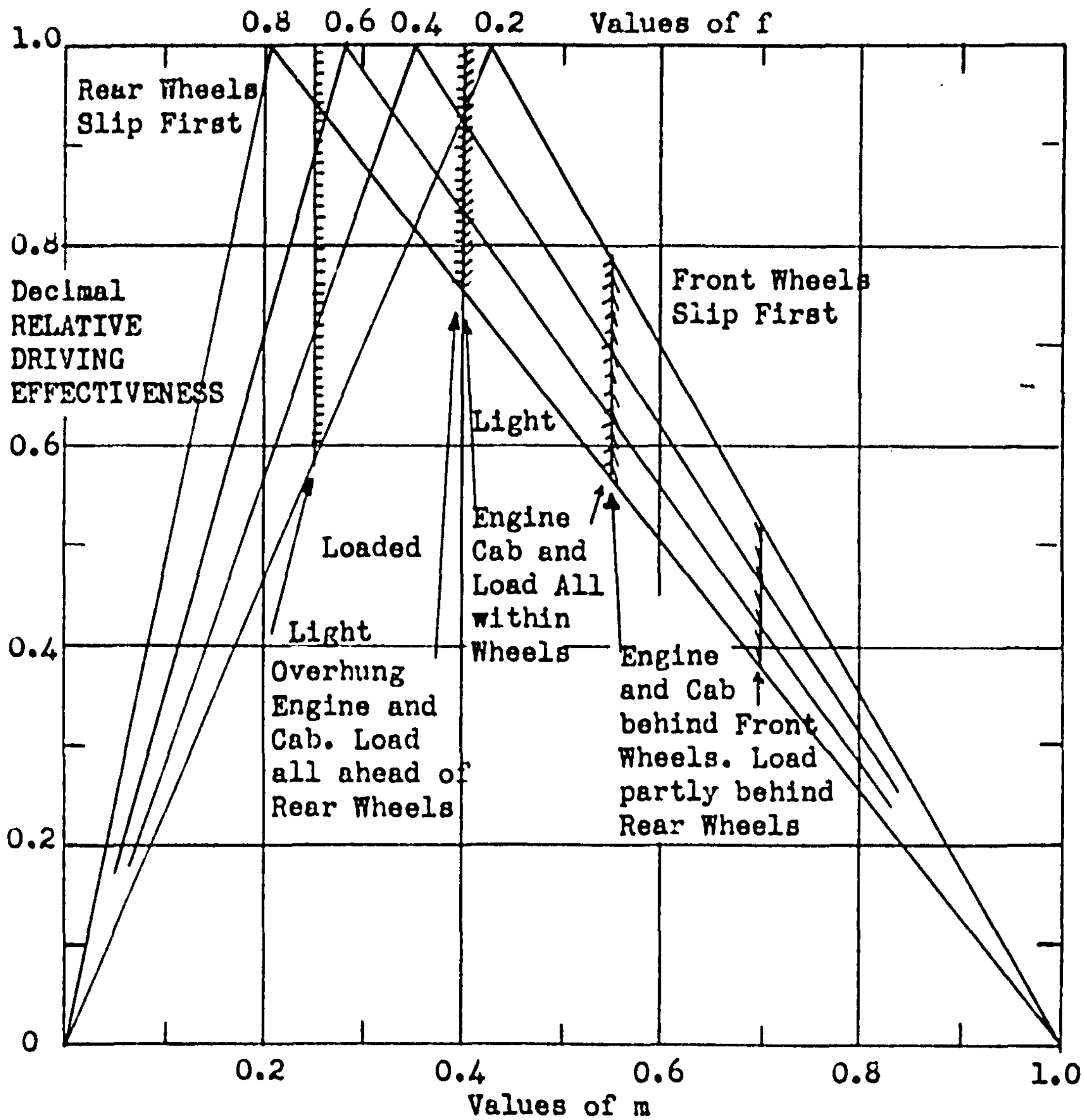
$$\frac{2m}{1 - 2f\frac{H}{L}} = \frac{2(1 - m)}{1 + 2f\frac{H}{L}}$$

$$m_{optimum} = 0.50 - f\frac{H}{L}$$

Using a value of $\frac{H}{L} = 0.36$, the value of $m_{optimum}$ will be 0.43 for $f = 0.2$, and 0.21 for $f = 0.8$. The required value of $m_{optimum}$ for low values of the friction coefficient can be attained in a truck without too much difficulty. The required value of $m_{optimum}$ for $f = 0.8$ can be approached, but not attained with any reasonable construction.

Figure 4 shows the relative drive effectiveness to be expected from three different types of trucks equipped with this drive system. The slope of the lines indicating operation at different friction coefficients indicates which pair of wheels will slip first and limit the maximum driving effectiveness. Too high a value of m always predisposes the vehicle to front wheel slip. By adopting a construction having the rear wheels at the extreme rear end of the loading platform, with both the engine and cab projecting in front of the front wheels, it is possible to get a vehicle with a relatively high driving effectiveness over the whole lower range of friction coefficients. The appearance of the vehicle is unusual, and it would be hard-steering and limited to low speeds of operation, but its utility would be high. The vehicle with the more conventional appearance, with engine and cab behind the front wheels and load partly behind the rear wheels would have a much lower driving ability, running into trouble from front wheel slip whenever maximum driving force was desired.

RELATIVE DRIVING EFFECTIVENESS
FOUR-WHEEL DRIVE TRUCKS WITH INTERAXLE DIFFERENTIALS



$\frac{H}{L}$ assumed to be 0.36

$m_{Loaded} - m_{Light}$ assumed equal to 0.15

Figure 4

The six wheel truck is not specifically covered in the foregoing analyses. Two variations of this type of truck are in use. A very common practice is to use one driving axle and one dead axle in the rear bogey or axle pair. The driving force which this axle would develop is equal to f $(\frac{1}{2}R')$. The maximum driving force and the relative driving effectiveness of this type of construction can be obtained by substituting $(\frac{1}{2}f)$ in the equations or charts wherever f appears. The other type of six-wheel construction puts driving torque in both of the rear bogey axles. This type is the exact equivalent of the conventional rear drive truck insofar as the equations and charts are concerned, and can be so treated.

Vehicles with ordinary cross-axle differentials which encounter a lower value of f under one drive wheel than under the other, will meet the requirements of the equations and charts if the lower of the two values of f is used.

SUMMARY ON DRIVING ABILITY

The ability of a self-propelled vehicle to produce sufficient driving force to meet its own needs for hill climb, acceleration, high speed, and towing operation is a function of the coefficient of friction between tires and road, the amount of the vehicle's weight on the driving wheels, and the presence or absence of an ordinary differential gear set.

The Maximum Driving Force which any vehicle can develop is equal to $fW \cos \theta$. Drive systems producing less driving force than this are compared with it by their Relative Driving Effectiveness, having a value of 1.0 as its maximum.

Rear drive vehicles have in general a higher relative driving effectiveness than front drive vehicles because of the difficulties, such as hard steering, associated with getting a sufficiently large proportion of the vehicle weight onto the front wheels.

Rear drive vehicles can, and occasionally do, concentrate so much of the vehicle's weight upon the rear wheels that they cannot steer adequately or at all when accelerating or ascending a hill.

Four-wheel drive trucks, using an ordinary interaxle differential, can attain high values of relative drive effectiveness over a wide range of f by making proper choice of m.

Maximum driving ability, important to the civilian vehicle and indispensable to the military vehicle, can be attained by driving all weight carrying wheels with a drive system containing either no differentials, or all self-locking differentials.

III

PERFORMANCE CALCULATIONS

for

AUTOMOTIVE VEHICLES

DEFINITION OF PERFORMANCE

Automotive vehicle performance means many things to many people. Some of the performance criteria are tangible quantities, measurable by the usual engineering methods. Others are relatively intangible, relying for their evaluation on the opinion of so-called experts. Included in this category are such quantities as "eye-appeal", and "style-rating", factors which are seized by the advertising departments and thrust upon the public in the hope that mere reiteration of a claim to superiority will sway the decision of a prospective buyer in the desired direction.

The performance factors in which this paper is interested are those factors in the road performance of the vehicle which are determined by the power output of the engine, and the matching of engine supply power to vehicle demand power. The factors involved are: maximum vehicle speed on level paved road, vehicle hill climb-ability, vehicle acceleration ability, and vehicle fuel economy. All of these quantities are measurable in the actual vehicle, and with proper handling of the factors involved are predictable for any vehicle. The prediction of all of these quantities is very useful to the design engineer when designing a new vehicle, to the test engineer as an indication of the magnitude of the factors he must measure, and to the quality supervisor as a measure of the performance he should expect from current production.

The performance calculation methods developed in this section are applicable to passenger cars especially, but with very minor modifications they can be made to apply to trucks and other wheeled vehicles (3)(4).

VEHICLE DEMAND HORSEPOWER

The horsepower demanded by a vehicle for its propulsion has been the subject of much testing and research. Its magnitude is one of the determining factors in all road performance results, and constant effort is being made to bring about a reduction in its value. It is sensitive to every frictional force which resists motion of the vehicle or any of its components, and in general any significant reduction in total demand horsepower is attained only as the result of a concerted effort to reduce all of the demand components.

For the purposes of road performance criteria the demand horsepower is measured at the engine flywheel, since it is here that the engine supply horsepower is delivered. All losses in the power system from the flywheel to the road are included in the chassis frictional loss.

Chassis Friction

The frictional losses in the chassis include the following: Clutch windage, clutch throw-out bearing friction, transmission shaft bearing friction, transmission lubricant churning friction, transmission gear tooth friction, universal joint friction, drive shaft windage drag, pinion bearing and seal friction, final drive gear tooth friction, differential carrier bearing friction, rear axle lubricant churning friction, wheel bearing friction, brake drag friction and tire rolling resistance.

Of this long list there are three items which can be singled out as having a sufficient magnitude when compared to the total resistance of the vehicle to warrant even individual mention. These three are tire rolling resistance, wheel bearing friction, and transmission churning friction.

The methods used to determine tire rolling resistance usually include in the measured quantity the resistance of the bearings upon which the wheel turns, and wheel bearing resistance is usually included in the values given for tire rolling resistance.

The transmission churning friction is a function of engine speed, the viscosity or temperature of the lubricant and the degree to which the gears dip into the lubricant. With present day transmissions, operating at normal running temperature, the magnitude of this quantity is not sufficient to warrant its separate treatment, and it is merged with the other items mentioned under the common term of chassis friction losses, which then excludes only tire rolling resistance and wheel bearing friction. The items making up this combined chassis friction vary either with vehicle speed (differential lubricant churning, seal friction), or vary with the first power of vehicle speed, or vary with the second power of vehicle speed (driveshaft windage), or vary with the torque being transmitted (gear tooth friction), or vary with a combination of transmitted torque and vehicle speed. The horsepower required to overcome these chassis losses would then be determined by an equation of the type:

$$HP = (K_1M + K_2M^2 + K_3M^3)\frac{1}{K_4}$$

in which M represents vehicle speed in miles per hour, and K_1 etc., represent appropriate constants.

The method adopted for including chassis frictional losses in the demand horsepower of the vehicle depends upon the method used to evaluate the major items of demand horsepower. In the commercial vehicle field the chassis friction horsepower is evaluated separately, and given by an equation of the type $FHP = K_1 + K_2N$, where N is engine speed in revolutions per minute and K_1 and K_2 are experimentally determined constants (3)(4).

In the passenger car field the method of measuring the major components of demand horsepower make it more convenient to represent the chassis friction as a percentage of the towing effort required at the drive wheel hubs, which is equivalent to adding a certain percentage to the horsepower demand created by losses external to the drive system. An appropriate value for this percentage for our present passenger cars with standard transmissions is five percent (5)(6)(7).

Tire Rolling Resistance

The magnitude of the resistance which a pneumatic rubber tire offers to rolling over the road has been, is, and will continue to be the subject of much testing and research. Tire manufacturers and vehicle manufacturers are both vitally concerned with the magnitude of this quantity. The resistance arises from two sources, the internal friction of the tire during deformation at the road contact area, and the scrubbing of the tread surface, or deformation of the tread blocks, as the toroidal surface of the tire accommodates itself repeatedly to the flat surface of the road.

The internal friction of the tire converts its portion of rolling resistance into heat energy, raising the temperature of the tire body and decreasing the strength of its components. Excessive heating leads to early tire failure, a major concern of the tire manufacturer.

The magnitude of tire rolling resistance at any given speed is a function of tire design and inflation pressure, and of the smoothness of the road surface over which the tire operates. Provided the surface is not so rough that excessive carcass deformation occurs, the increased rolling resistance noted on rough surfaces is due to greater tread deformation and greater tread scuffing. The magnitude of the driving force required to overcome tire rolling resistance (and wheel bearing friction) at low speeds on smooth paving varies from 0.010W to 0.165W, depending on the investigator (5)(6)(7)(8). The magnitude of this quantity also increases with vehicle speed. An average value which can be used for passenger car tires in the low and medium speed range is 0.015W, where W represents the load in pounds carried by the tire contact surface. However, the variation in the magnitude of the rolling resistance force with vehicle speed is sufficient to demand recognition in any evaluation of demand horsepower which extends over a wide speed range.

Vehicle Wind Resistance

During its travel over the surface of the road the automobile moves about a considerable quantity of air, some of which is compressed and expanded, and some of which is given an appreciable velocity in the direction of the car. In addition to the energy required to create this "wake" with its attendant eddies and vortices in the air body, there exist both viscous friction and induced drag resistances which absorb energy taken from the moving vehicle.

The viscous friction or skin friction occurs when the air body moves relative to a surface which bounds it. The greater the relative velocity, and the thinner the boundary layer or quiescent layer of air adjacent to the surface the greater will be the viscous drag produced. The conventional automobile has contours which force the air to attain in certain regions adjacent to the car a velocity relative to the car which is appreciably greater than the velocity of the car relative to the road, increasing the local viscous drag. Also there exist on many car bodies, articles which project from the general car surface through any conceivable boundary layer which might exist into the high velocity air streams, and act as "wind claws" to increase the resistance to motion.

The induced drag resistance occurs at the rear of the car, where the air attempts to return to its original condition prior to its disturbance by the passage of the vehicle. The shape of our sedan bodies creates a certain amount of aerodynamic lift as the air tries to follow the downward curve at the rear of the body. This lift is in a direction inclined backwards from a perpendicular to the road surface, and has a component, called the induced drag, in opposition to the motion of the vehicle. The magnitude of the lift produced on some vehicles is sufficient to change the effective weight at the rear of the car, leading to objectionable and unpredictable bobbing of the rear end at high speeds (8).

The direction in which car body design should go to reduce wind resistance has been indicated by many investigators (9)(10), but any sincere effort to create a low wind resistance body shape is thwarted by the demands of the stylists and the sales departments for "eye-appealing" shapes and shiny ornaments, by the limitations imposed on car length, and by the necessity for seating the passengers within the vehicle with reasonable comfort and visibility.

The magnitude of the force required to overcome wind resistance is given by the relation KAM^2, where A is the frontal area of the vehicle, M the speed in miles per hour, and K a constant dependent on the shape and surface smoothness (aerodynamic cleanliness) of the car body. To some investigators the shape of the rear end of the car is very important in determining the value of K (8), and to others the rear end shape is immaterial and only the frontal area is important (11). The magnitude of K varies with the car and with the source, with values ranging from 0.0012 to 0.0018.

The total force required to move the vehicle, known as the towing force or towing effort, will be the sum of the air drag force, the tire rolling resistance force, and the chassis friction force. These quantities must be evaluated individually or collectively for a given vehicle, and the results used to obtain the demand horsepower of that vehicle.

Measurement of Towing Force

Many methods have been used to measure the towing force or towing effort in the past, but at present the most popular and practical method is to use a torque meter located in the hub of one or both driving wheels (5)(6). The indications of this torque meter are obtained continuously and recorded on an indicating chart. The use of this method measures the wheel torque required to overcome wind resistance, tire rolling resistance, and the wheel bearing resistance of the front wheels but not of the rear driving wheels. This rear wheel bearing loss, plus the chassis friction loss in the drive system, must be added to the towing effort obtained in this way to determine the total demand which must be supplied by the engine. The results of the torque meter determinations can be plotted as a function of vehicle speed, and an empirical relation developed which fits the average demand curve most satisfactorily. This method is used by most of the passenger car manufacturers. The resulting empirical equation adopted by one of the larger companies after repeated tests on many makes of present-day passenger cars is as follows:

Towing Effort $T.E. = K_1W + K_2WM + K_3AM^2$

In this equation W represents the actual vehicle weight in pounds, M the vehicle speed in miles per hour, A the frontal area of the vehicle in square feet, and the K constants have the following values:

	Standard Size Cars	Small Cars
K_1	0.0112	0.0112
K_2	0.000097	0.00008
K_3	0.00147	0.0012

The use of this equation will determine the towing effort required. If the towing effort, increased by a factor of five percent to take care of the rear wheel bearing and drive system losses, is combined with vehicle speed in the proper manner, the horsepower demanded of the engine by vehicle may be obtained.

$$HP = \frac{\text{Force} \cdot \text{Velocity}}{375} = \frac{1}{.95}\ \text{T.E.}\ \frac{M}{375} = \frac{(\text{T.E.})M}{356}$$

This relation gives the horsepower demanded from the engine to move the vehicle at a speed of M miles per hour over smooth level paving with no natural wind present.

To illustrate the magnitude of the demand horsepower for a typical passenger automobile, and also the methods used in making the desired performance calculations, an evaluation of the developed equations will be made for an American passenger car, with certain variations introduced at some points in the development to illustrate the effect of vehicle variables.

Dimensional values for American passenger cars are available in several trade magazines (12). From one of these sources the following pertinent dimensions are taken: shipping weight 3500 pounds, height 64 1/16 inches, width 75 3/16 inches. To the shipping weight must be added 150 pounds for gas, oil, and water, and 350 pounds for two passengers and their luggage. This procedure, which is common in the industry, adds 500 pounds to the shipping weight of the vehicle to get its operating weight.

The block area of the vehicle, obtained from the product of height and width, is 33.4 square feet. This area must be modified by a blocking coefficient to take care of the area reduction from the enclosing rectangle caused by roof and side curvature and the space between the road and the underside of the body. A representative value for a blocking coefficient for our present cars is 0.8. Using this value produces a frontal area A equal to 26.7 square feet. Using the appropriate values of the K constants produces the following equation for the demand horsepower of this car:

$$\text{Demand HP} = \frac{(44.8 + .388M = .0393M^2)M}{356}$$

The results of the numerical evaluation of this relation are shown on Figure 5. Because of the effect of the wind resistance term the demand horsepower increases rapidly at high car speeds. At a speed of approximately 40 miles per hour the wind resistance is one half of the total resistance, and above that speed its relative importance increases rapidly.

The effect of vehicle weight on the steady-speed power demand is quite low, and is illustrated on Figure 5 by a curve of demand horsepower for a vehicle having the same frontal area but only half the weight of the standard vehicle. The effect of reducing the frontal area to one half that of the standard car is shown to be appreciable. Reducing both the weight and the frontal area to one half their value in the standard car produces a noticeable reduction in demand horsepower, with the pint-size vehicle having the same demand at eighty miles per hour that the standard vehicle has at sixty miles per hour. The demand horsepower of our stock American passenger cars at sixty miles per hour varies from thirty to thirty six.

PASSENGER-CAR DEMAND HORSEPOWER

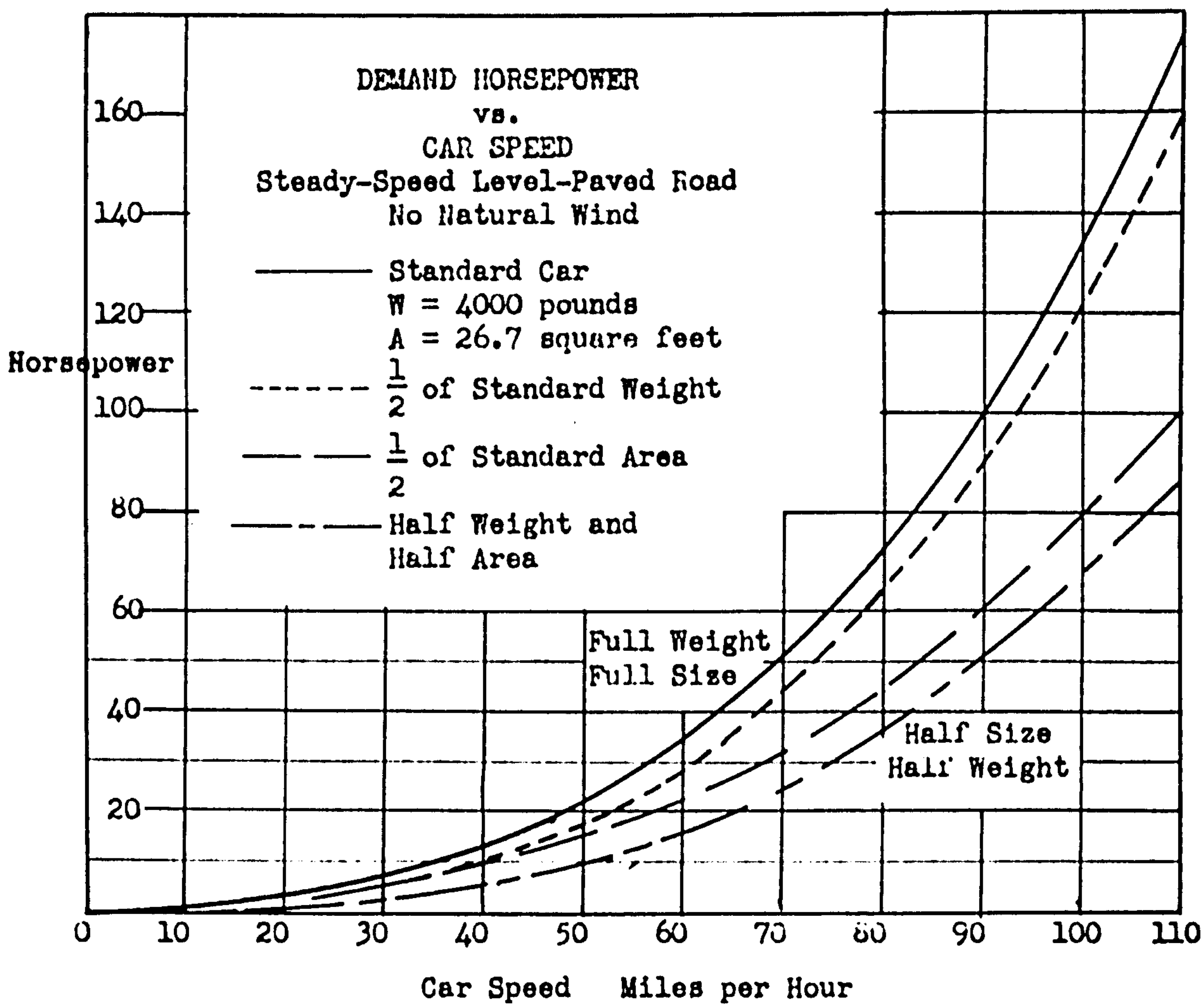


Figure 5

VEHICLE SUPPLY HORSEPOWER

Automotive vehicle power demands are supplied by internal combustion engines of various sizes, but of similar output characteristics. On Figure 6 are shown the typical full-throttle torque and horsepower curves for an automotive engine. As built and used for two decades these engines have shown a ratio between the speed of maximum horsepower and the speed of maximum torque of about 1.7 to 1.8. Also the maximum torque is usually about one and one quarter times as great as the torque developed at the speed of peak horsepower. Automotive engines are expected to operate well over an extreme range of speeds and loads, with great emphasis on freedom from noise and vibration.

One fundamental deficiency of the internal combustion engine as applied to automotive applications is the inability of the engine to develop any useable torque or power until it is rotating at an appreciable speed.

Also shown on Figure 6 are the affects on engine power output produced by partially closing the throttle. Proper proportioning of the throttle opening makes it possible to obtain continuously from the engine any power output between zero and the maximum which it can produce at the particular speed of operation.

PASSENGER-CAR ENGINE PERFORMANCE

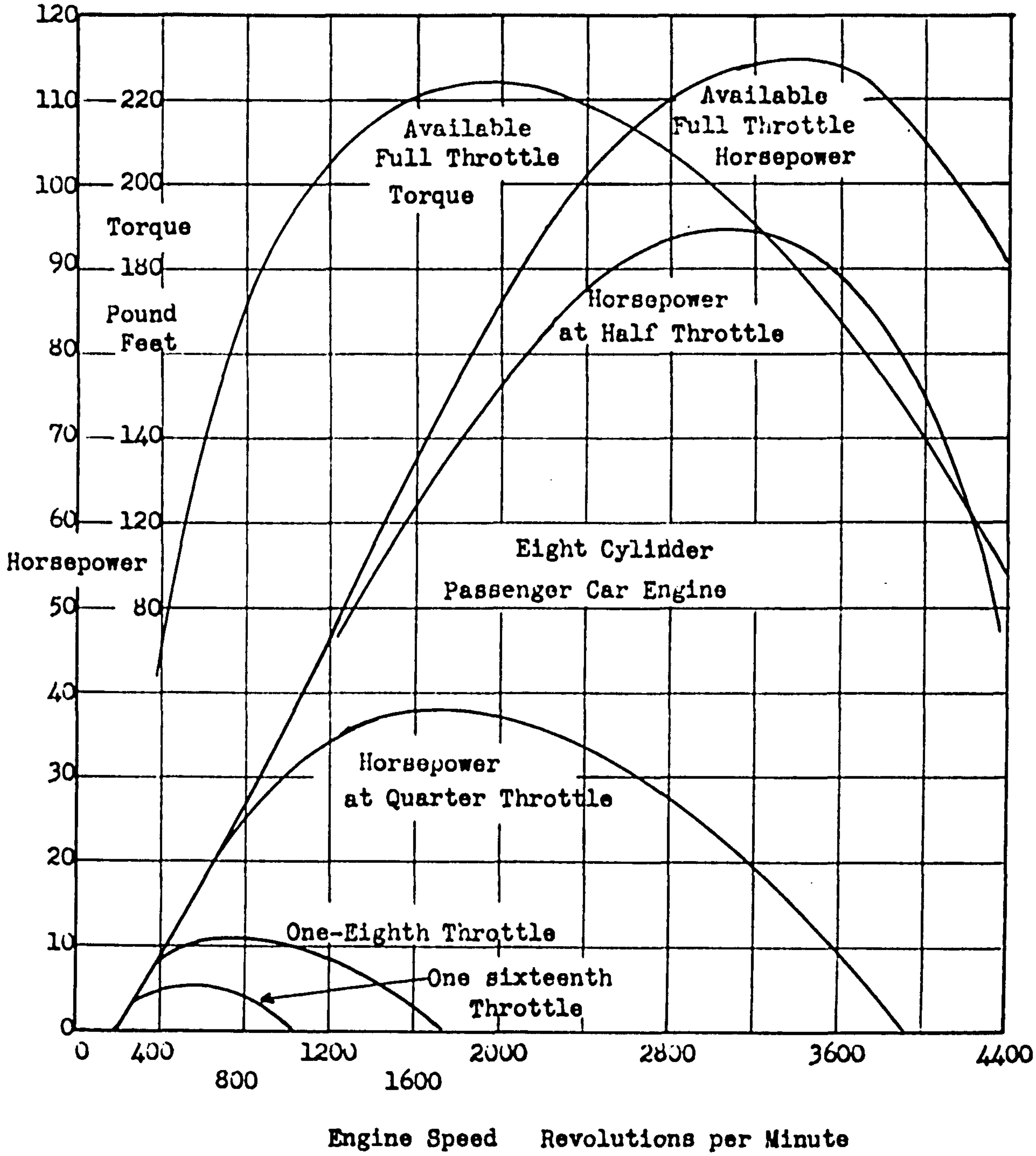


Figure 6

MATCHING OF ENGINE SUPPLY HORSEPOWER WITH VEHICLE DEMAND HORSEPOWER

Ideal matching of supply and demand horsepowers would require that maximum engine horsepower be made available to the vehicle whenever it was needed. To do this would require that the engine be allowed to operate at its maximum horsepower rpm at any vehicle speed, or, in other words, that the ratio between engine crankshaft speed and driving wheel speed be infinitely variable over whatever range was necessary. The so-called "ideal" transmission would allow this, but all practical transmissions marketed up to the present cannot allow it. The standard or conventional sliding-gear transmission and clutch combination imposes a rigid relationship between engine speed and wheel speed, with as many different numerical relations as there are gear combinations in the transmission. The newly popular fluid transmissions cannot maintain a rigid relationship between engine speed and wheel speed, but they are so constructed that at any appreciable wheel speed there is only a very limited degree of flexibility in the speed relationship.

Assuming a friction clutch-sliding gear transmission combination is used, the relationship between engine speed N and vehicle speed M is obtained as follows:

$$N = M \cdot \frac{5280}{60} \cdot \frac{1}{2\pi(\text{Tire Rolling Radius})} (\text{Overall Gear Ratio})$$

Overall Gear Ratio = Final Drive Ratio x Transmission Ratio

MATCHING OF SUPPLY AND DEMAND HORSEPOWER

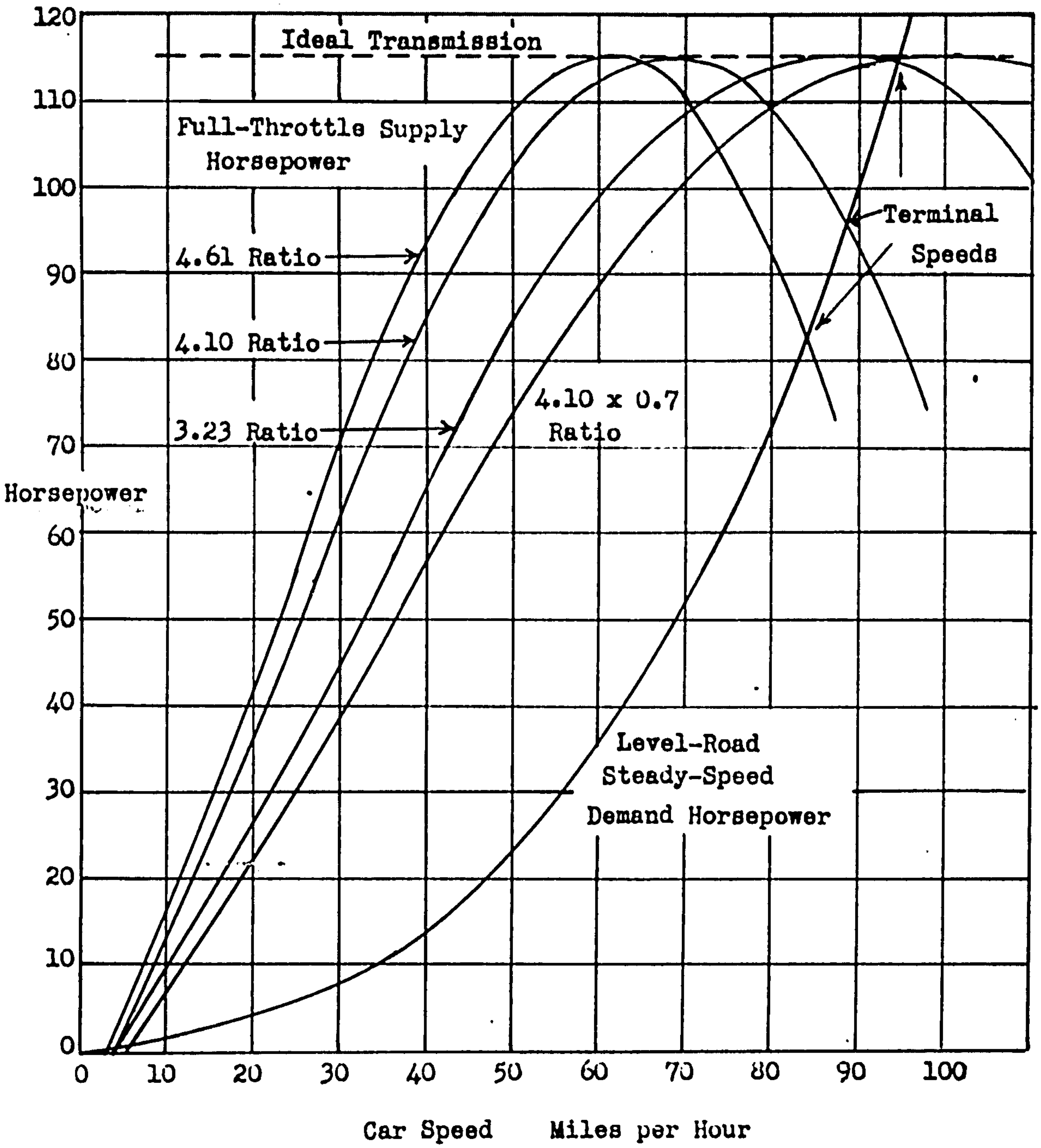


Figure 7

The transmission ratio in high gear is always one to one. The "overdrive" transmission ratio when used is usually 0.7 to one. The final drive gear ratio in combination with tire rolling radius is then the deciding factor in the matching of supply and demand horsepower, and the value of this ratio will to a large extent determine the performance characteristics of the vehicle.

Maximum Vehicle Speed

Figure 7 shows the matching obtained between supply and demand horsepowers for the example car chosen, for three different overall gear ratios, and for the ideal transmission. The 4.1 final drive ratio has been popular for about two decades, and produces a good compromise between the demands of high top speed and adequate medium-speed hill-climb and acceleration ability. The intersection of the full-throttle supply curve and the steady speed demand curve indicates the maximum speed which the vehicle will attain in continuous quiet-air level-paved-road operation. The ordinate difference at any given car speed between the full throttle supply curve and the steady speed demand curve is the excess horsepower (above the steady-speed requirement) which the engine can make available to meet the added requirements of acceleration or hill-climb.

The demand for high top speed can be met by choosing a final-drive gear ratio which allows the engine to produce its peak horsepower at peak vehicle speed. On Figure 7 are shown the supply curves produced by a 3.23 ratio and a 4.1 x 0.7 overdrive ratio. Both ratios produce practically the same top speed, six miles per hour higher than that produced by the 4.1 ratio. However both ratios have noticeably reduced the excess horsepower available in the intermediate speed range, correspondingly reducing the hill climb and acceleration ability of the vehicle.

The ideal transmission would obtain a top speed equal to or better than that obtained by any choice of final drive gear ratio, and would in addition obtain an average hill-climb and acceleration ability better than that attainable by any choice of final drive gear ratio combined with a selective ratio gear box.

Maximum speed can be attained only at the expense of an appreciable sacrifice in medium speed acceleration and hill climb ability.

Good medium speed acceleration and hill climb ability can be obtained at the expense of a slight sacrifice in top speed. This condition has been the most popular design choice. The use of an overdrive transmission has made it possible to get both conditions with a minimum of effort on the part of the driver.

The use of the 3.23 ratio shown is a compromise between the performance of overdrive and no overdrive. It obtains high top speed, at not too great a sacrifice in hill climb and acceleration ability. This ratio, or one very close to it, is used on some models with high output engines, capable of producing satisfactory acceleration and hill-climb abilities even with this low ratio, in an effort to get good fuel economy and low engine noise and vibration levels. It is also used on most models which offer overdrive in combination with a relatively small engine. With a small engine this ratio (3.23 = 4.61 x 0.7) will produce an unsatisfactory level of hill-climb and acceleration abilities, but do very well on fuel economy. Whenever the driver wishes to improve his medium-speed power performance he needs only to render the overdrive inactive by flooring the accelerator pedal, and the car then operates on an effective 4.61 ratio, producing ample excess horsepower from even a small engine. The relative gain in horsepower in the intermediate speed range by using this kick-down system is shown on Figure 7 by comparing the supply curves for the 3.23 and the 4.61 ratios. The gain for speeds up to sixty five is appreciable. This procedure is of no value for high speeds (above 71 for vehicle used in the illustrations) and could even be dangerous to the engine by allowing it to operate at excessively high speeds.

Starting Requirements and High-Torque Requirements

The use of horsepower demand and supply curves does not effectively illustrate two very important factors in the matching of vehicle demand and engine supply. If the demand torque in the driveshaft is plotted against vehicle speed in miles per hour and the driveshaft torque produced by the engine with the transmission in high gear is superimposed upon the demand torque graph, this deficiency is brought out more clearly. These curves for our example car, using a 3.23 final-drive gear ratio, are shown on Figure 8.

The complete inability of the reciprocating internal-combustion engine to produce any useful torque at speeds approaching zero rpm makes it imperative that some device be used between flywheel and transmission which will transmit full engine torque while allowing relative movement, or slip, between its two ends. Unless such a device is used it would be impossible for the engine to start the vehicle from a state of rest. The device used in practice to overcome this engine deficiency is a clutch, of either the dry-friction or fluid-friction type. The dry-friction clutch consists of a stamped sheet-steel clutch disc with an annular ring of heat-resisting high-friction lining riveted to each face near the periphery. This lined clutch disc is squeezed between the machined faces of the flywheel and a cast-iron pressure plate by the force of a clutch spring or springs. A clutch release lever actuated by a foot pedal makes it possible to control the magnitude of the torque transmitted by the clutch from complete release to complete engagement, or from one hundred percent slip to zero percent slip. By means of the clutch the engine can be run at a speed fast enough to develop sufficient torque to accelerate the vehicle from zero speed up to some speed at which the engine and clutch shaft can be directly coupled together and the engine made to drive the vehicle without any clutch slippage. For the illustration on Figure 8 the minimum speed for direct drive is six miles per hour (intersection of supply and demand curves).

The torque supplied by the engine with the transmission in high gear is sufficient to meet the level-road, steady-speed demands of the vehicle over the full speed range from six miles per hour to ninety four miles per hour. It is also sufficient to meet the demands of the vehicle on a ten percent uphill grade at any speed between twenty two and fifty six miles per hour. If the vehicle were never called upon to climb more than an eleven percent grade, a clutch is all that would be needed between engine and driveshaft. Although this arrangement would

TRANSMISSION AND CLUTCH ACTION

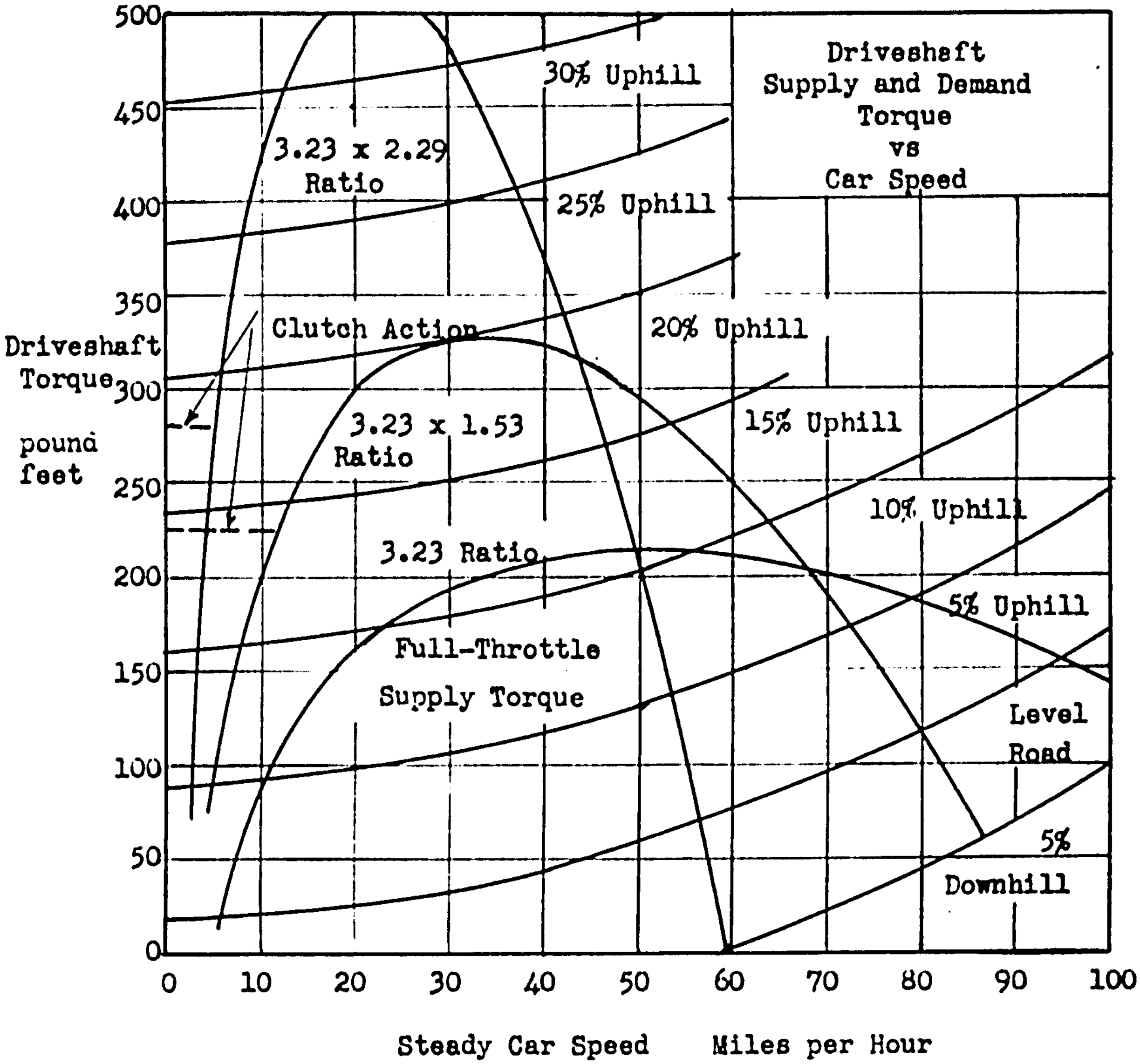


Figure 8

be mechanically very simple, it would not provide for reverse movement, nor would the acceleration from a standstill on level road be anything more than mediocre.

The demands of the public for adequate low-speed acceleration, and the actual need for reverse movement and for hill-climb ability far above that adequate for a ten percent grade makes it imperative that some means, in addition to the final-drive gears, be provided for multiplying the engine torque before sending it to the driving wheels. This means is usually provided by a gear box, popularly called a transmission, which acts as a torque converter in all gears except high gear, multiplying the engine torque and in some instances reversing its direction before sending it on to the driveshaft. Conventional American passenger-car practice is to provide a transmission with three forward speeds and one reverse speed plus a neutral. The top speed is always direct drive to minimize transmission losses. The two lower speeds are approximately a geometric ratio of one another, with the reverse ratio equal to or greater than the lowest forward ratio. The combination clutch-transmission installation is shown schematically on Figure 9. The conventional names of the various parts are given on the diagram. This type of unit is known as a selective sliding-gear transmission because the driver can select the gear ratio he desires by sliding gears (or jaw clutches) forwards or backwards on the tail shaft or main shaft of the transmission.

The effect of the addition of the transmission upon the matching of torque supply and torque demand is shown on Figure 8. Using a "second" gear ratio of 1.53, the driveshaft torque is increased above

DRY-FRICTION CLUTCH
and
SELECTIVE-SLIDING-GEAR
TRANSMISSION

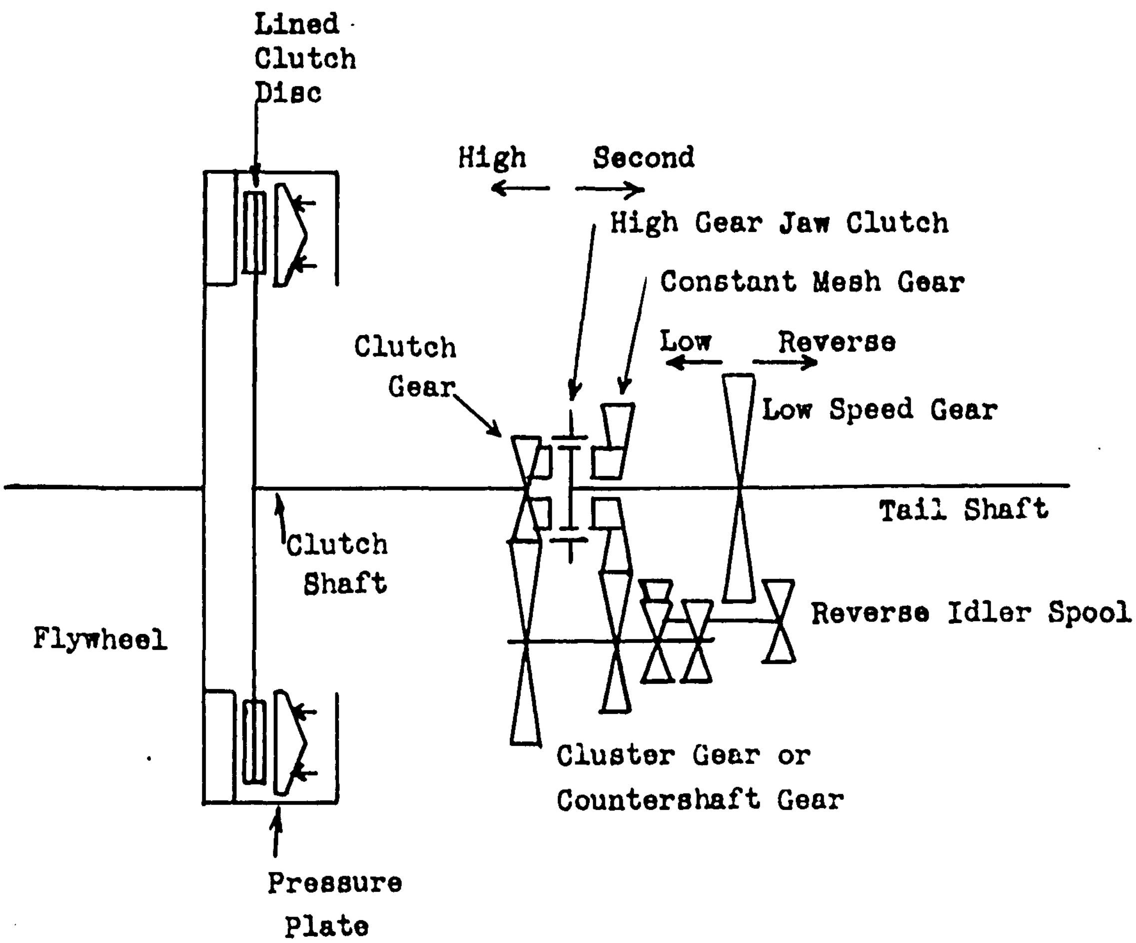


Figure 9

that available from high gear at all speeds below sixty eight miles per hour, with the maximum torque being sufficient to meet the demands of a twenty percent grade. Using a "low" gear ratio of 2.39, the low-speed torque is further increased, with the maximum torque being more than sufficient for a thirty percent grade, although the operating speed range is correspondingly reduced.

The friction clutch, plus the selective sliding-gear transmission, plus a minimum amount of intelligence on the part of the driver, can overcome the obvious deficiencies of the internal-combustion engine and make it a satisfactory power source for an automotive vehicle. Recently introduced automatic and semi-automatic transmissions have reduced even further the degree of skill which must be possessed by the driver.

STEADY SPEED HILL-CLIMB ABILITY

When a vehicle is proceeding at steady speed up an incline it is subjected to all of the retarding forces which oppose its level road operation, and in addition must overcome that component of the gravity force which is parallel to the road surface and to the path of travel of the vehicle. Under these conditions the horsepower available for overcoming the grade component is that part of the full-throttle horsepower, over and above the amount needed for steady-speed level-road operation, which the engine can develop at the particular car speed under consideration. For the "ideal" transmission with the infinitely variable gear ratio the maximum engine horsepower is available at all car speeds, and the horsepower available for hill-climb operations is the difference between the steady-speed level road demand horsepower and the maximum engine horsepower. For the conventional transmission the horsepower available for hill-climb operations is the ordinate difference between the level-road demand curve on Figure 7 and the full-throttle engine supply horsepower curve at the same speed. If the symbol HP_{XS} is used for this extra horsepower, the hill-climb ability of the vehicle may be computed by the following system.

The measure of the steepness of a grade is either the decimal grade of the percent grade. The decimal grade is defined as the tangent of the grade angle. For the grades normally encountered in automotive work (zero to ten percent on main highways) the tangent and the sine of the grade angle are substantially equal (one half of one percent error at ten percent grade) and it should be permissable to substitute the sine and tangent for each other without introducing a significant error into the computations.

$$\text{Grade Force} = W \sin \theta \doteq W \tan \theta \doteq WG$$

G here refers to the decimal grade, and W. to the vehicle weight.

$$\text{Excess Towing Effort Available} = \frac{356 \cdot HP_{XS}}{M} = WG$$

$$\%G = 100 \frac{356 \cdot HP_{XS}}{WM}$$

Using this relation the Hill-Climb Ability of a vehicle, expressed as a percent grade can be determined by finding the relationship between HP_{XS} and M.

Using the HP_{XS} determined from Figure 7 with additional information for second and low transmission ratios on a 3.23 final drive ratio, the hill climb abilities as a function of M for the example passenger car are shown on Figure 10.

The effect of a low overall gear ratio on power performance is graphically illustrated in Figure 10. An overdrive transmission compounded on a 4.1 final-drive ratio gives a maximum hill-climb ability of only nine percent. The 3.23 final-drive ratio is somewhat better with a maximum of twelve and one half percent. The ten percent grade is a significant boundary in hill-climb ability since it has been adopted by some states as the maximum acceptable grade for their state roads. A hill-climb ability of better than ten percent is necessary if main highways are to be negotiated without shifting from high gear, an

HILL-CLIMB ABILITY

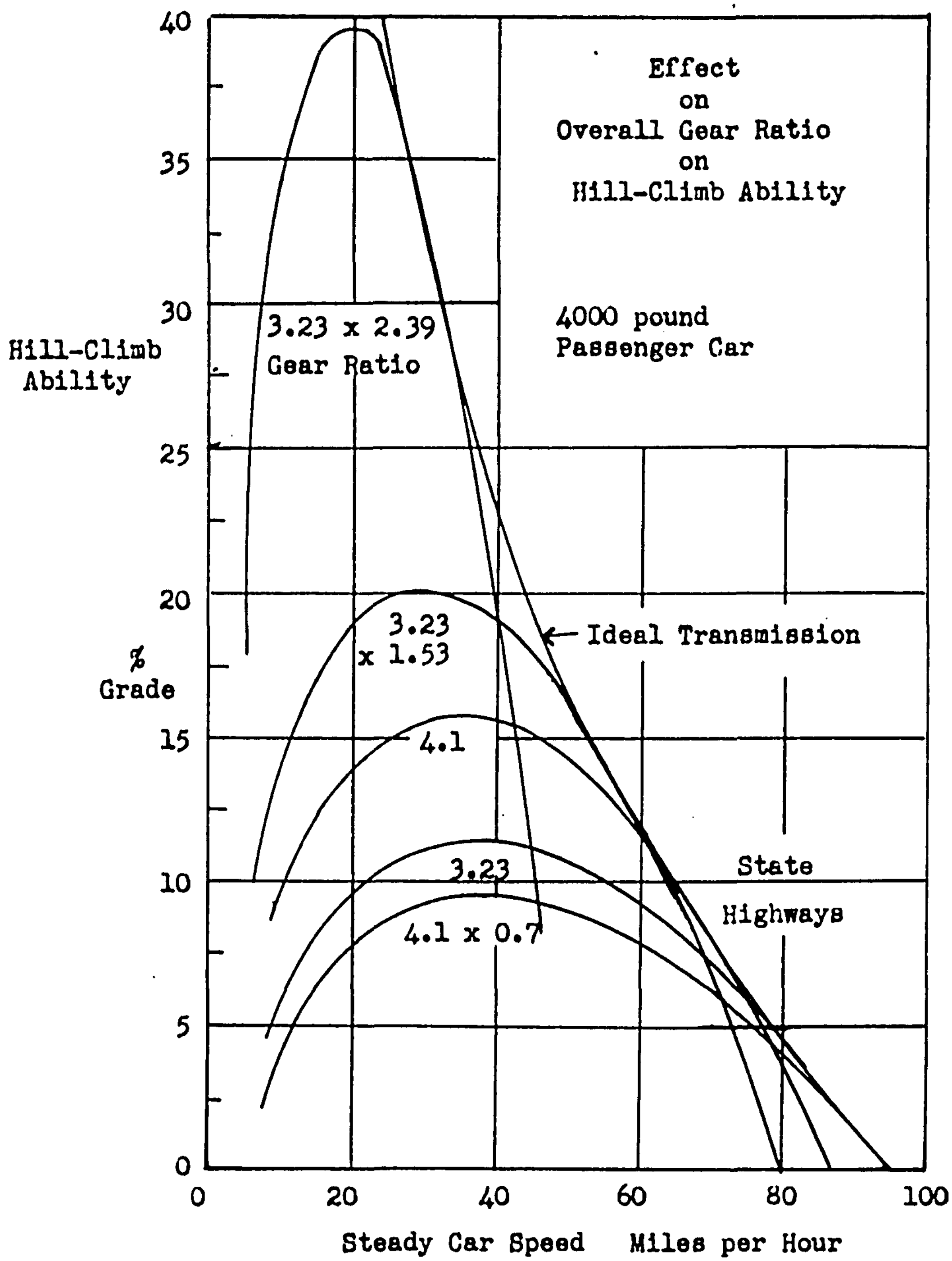


Figure 10

operation which the American public has been reluctant to do. The only way in which a low overall gear ratio can provide satisfactory hill-climb ability for the general public is to combine it with a large powerful engine, or to attain it by compounding a higher final-drive ratio with a semi-automatic two-speed overdrive transmission.

City streets and country roads on which the grades may reach twenty five percent can be traversed by shifting the transmission to the necessary low ratio. It has been the practice in our American passenger cars to make the maximum torque multiplication sufficient to climb a thirty to forty percent grade.

The ideal transmission provides a hill-climb ability which is the envelope curve of those curves produced by the several gear ratios. Its only superiority from the standpoint of hill-climb ability would be its maintenance of maximum ability at any car speed without the necessity of gear shifting.

ACCELERATION ABILITY

The acceleration ability of a vehicle is closely related to the hill-climb ability, since both are determined by the excess horsepower available above the steady-speed level-road requirements, but there is one important difference in the method of its computation.

During steady-speed hill-climb operation no energy is absorbed by the vehicle to change its kinetic energy level. During acceleration, on level road or on a grade, the kinetic energy level of the vehicle changes continually and energy must be imparted to the vehicle from the engine to raise this kinetic energy level. Not only is the entire vehicle given a linear acceleration, but in addition the rotating parts are given an angular acceleration. This latter addition increases the acceleration horsepower requirements by several percent, with the magnitude of the addition being a function mainly of the overall gear ratio.

The clearest way of deriving the relations necessary to determine acceleration ability from the excess horsepower available from the engine is to determine the towing effort required for each type of acceleration, and then combine them into a horsepower requirement.

For strictly linear acceleration:

Acceleration Force = Mass x Linear Acceleration

For strictly angular acceleration:

Acceleration Torque = Moment of Inertia x Angular Acceleration

All rotating parts of the vehicle are given an angular acceleration but the moment of inertia of many of these parts is so low the acceleration torque required is negligible. The wheel assemblies, including the tires, tubes, rims, wheel discs, hubs, and brake drums, are given an angular acceleration which is small in magnitude because of the relatively low wheel speed, but the fairly high mass of the four wheel assemblies, plus the fact that much of the mass is at an appreciable distance from the center of rotation, makes the acceleration torque requirements for the wheel assemblies important. Rotating at the same speed as the wheels are the driving axles and the differential carrier. The driving axles are solid, with an average diameter of less than one and one half inches and a length of about four and one half feet. This makes their mass comparatively small, with all of it concentrated very closely about the center of rotation. The torque required for axle shaft rotation is negligible. The same reasoning applies to the differential carrier. It is a hollow shell about five inches in diameter, mounting a ring gear of about eight inches in diameter. The entire assembly weighs only a few pounds and is of small diameter and runs at low speed, so its acceleration torque requirements are negligible.

All rotating elements between the differential carrier and the transmission rotate at wheel speed times the final-drive gear ratio, and the angular acceleration to which they are subjected is equal to the angular acceleration of the wheels times the final-drive gear ratio. The only items included in this group are the final-drive pinion, the universal joints, the driveshaft, and the transmission tail shaft. The pinion and the transmission tail shaft are both small-diameter short shafts, with little weight and less moment of inertia. The universal joints are short, compact, and skeletal in construction, with very little moment of inertia. The driveshaft is a hollow thin-walled tube about four feet long, two to three inches in diameter, and with a wall thickness less than one sixteenth of an inch. Its mass and moment of inertia are negligible. None of the items in this group have acceleration torque requirements sufficiently large to warrant their inclusion in acceleration computations.

The transmission countershaft runs at a speed about equal to or slightly higher than the driveshaft speed. It is a gear cluster about seven or eight inches long, of small diameter, and weighing only a few pounds. Its acceleration requirements are negligible.

The clutch shaft, clutch assembly, and engine assembly rotate at a speed that is equal to wheel speed times overall gear ratio, and their angular acceleration would equal overall gear ratio times the angular acceleration of the wheels. The crankshaft, flywheel, and clutch assemblies have fairly high moments of inertia and operate at substantial speed levels, so their effect on acceleration ability must be evaluated.

The torque required to accelerate the clutch and engine assembly ($T_{C\ \&\ E}$) is equal to the product of the moment of inertia of the clutch and engine assembly ($I_{C\ \&\ E}$) and the angular acceleration of the clutch and engine assembly ($\alpha_{C\ \&\ E}$). The angular acceleration of the clutch and engine assembly is equal to the overall gear ratio (GR) times the angular acceleration of the wheels (α_{Wh}). Using the indicated symbols:

$$T_{C\ \&\ E} = I_{C\ \&\ E} \cdot \alpha_{C\ \&\ E} = I_{C\ \&\ E} \cdot GR \cdot \alpha_{Wh}$$

The acceleration of the wheels is measured in radians per second per second, and is directly related to the linear acceleration rate of the vehicle (a), measured in feet per second per second. If the rolling radius (RR) of the wheels is measured in feet the following relations are true:

$$\alpha_{Wh} = 2\pi \cdot \frac{\text{revolutions}}{(\text{second})^2} = 2\pi \cdot \frac{\text{ft./sec.}^2}{\text{ft./rev.}} = 2\pi \cdot \frac{a}{2\pi(RR)}$$

$$\alpha_{Wh} = \frac{a}{RR}$$

$$T_{C\,\&\,E} = I_{C\,\&\,E} \cdot \frac{GR}{RR} \cdot a$$

Similarly the torque required to accelerate the wheels (T_{Wh}) may be found as a function of the linear acceleration rate, and the moment of inertia of the wheel assemblies (I_{Wh}).

$$T_{Wh} = I_{Wh} \cdot \alpha_{Wh} = I_{Wh} \cdot \frac{a}{RR}$$

The total angular acceleration torque which must be produced by an equivalent angular acceleration force acting at the tire rolling radius is the sum of $T_{Wh} + (T_{C\,\&\,E} \cdot GR)$.

$$\text{Angular Acceleration Force} = \frac{1}{RR}\left[T_{Wh} + (T_{C\,\&\,E} \cdot GR)\right]$$

$$= \frac{1}{RR}\left[I_{Wh} \cdot \frac{a}{RR} + I_{C\,\&\,E} \cdot \frac{GR}{RR} \cdot a \cdot GR\right]$$

$$= a\left[I_{Wh} \cdot \frac{1}{(RR)^2} + I_{C\,\&\,E} \cdot \frac{(GR)^2}{(RR)^2}\right]$$

The linear acceleration force required is equal to $\frac{W}{g} \cdot a$, where

W is the vehicle weight and g is the acceleration due to gravity.

$$\text{Total Acceleration Force} = \left\{ \frac{W}{g} + \left[I_{Wh} \cdot \frac{1}{(RR)^2} \right] + \left[I_{C\ \&\ E} \cdot \frac{(GR)^2}{(RR)^2} \right] \right\} a$$

(Linear + Angular)

The quantity in { } may well be called the Equivalent Mass Factor.

The horsepower required for acceleration must come from the engine through the drive system, so the regular horsepower relation is applicable.

$$\text{Acceleration HP} = \frac{M}{356} \cdot \text{Acceleration Force}$$

The acceleration rate which can be produced at any given speed on level road can then be determined from the excess horsepower available.

$$a = \frac{356\ HP_{XS}}{M} \cdot \frac{1}{\text{Equivalent Mass Factor}}$$

To illustrate the acceleration rates produced by various gear ratios requires the numerical evaluation of this equation for the chosen example car.

This car uses 7.60 x 15 tires on steel wheels with eleven inch brake drums. Experimental measurement of the moment of inertia of the wheel assembly on a torsional pendulum produced the value of 5.07 pound feet per second per second for the four wheel assemblies. The rolling radius of this tire under normal load is 1.177 feet. These figures can be combined to find the contribution of the wheels to the total equivalent mass.

The evaluation of the moment of inertia of the engine and clutch assembly is more complicated. The crankshaft, flywheel, and clutch assembly may be spun as a unit on a torsional pendulum and their moment of inertia determined experimentally. The moments of inertia of the camshaft, water pump, generator, oil pump, and distributor are comparatively so small they may be neglected. However, the kinetic energy level of the reciprocating parts is appreciable, and changes with engine speed, so its effect must be determined.

The connecting rod bearing inserts have pure rotational motion at crank pin radius, and their moment of inertia addition can be computed. In a short-stroke engine, two thirds of the connecting rod weight is considered as purely rotating weight with the crank pin as its radius, and the other third of the rod weight is considered as purely reciprocating weight. Other items of reciprocating weight are the pistons, piston rings, piston pins, and piston pin retainers.

The kinetic energy put into the reciprocating parts by the crankshaft varies from a maximum to zero twice every revolution, with the energy being returned to the crankshaft at top and bottom center piston positions. In a multi-cylinder engine there is always some kinetic energy stored in piston motion, and since the kinetic energy varies as the square of velocity, the average kinetic energy level of the reciprocating parts will vary as:

$$\frac{(\text{Maximum Instantaneous Piston Velocity})^2 - (\text{Minimum Instantaneous Piston Velocity})^2}{2}$$

The minimum instantaneous piston velocity is zero. The maximum instantaneous piston velocity is equal to the peripheral velocity of the crank pin times an <u>angle factor</u>. The angle factor equals $(\sin \theta + \frac{\sin 2\theta}{4n})$, where θ is the crank angle from top dead center, and n is the ratio $\frac{\text{connecting rod length}}{\text{stroke}}$.

When the kinetic energy level at any speed has been computed, an <u>equivalent mass</u>, rotating at crank pin radius may be computed, which will have the <u>same kinetic energy level as the reciprocating masses</u>.

Reciprocating Kinetic Energy =

$$(\text{Reciprocating Mass}) \frac{(\text{Crank Velocity})^2}{2} \frac{(\text{Maximum Angle Factor})^2}{2}$$

$$\text{Rotating Kinetic Energy} = \text{Rotating Mass} \frac{(\text{Crank Velocity})^2}{2}$$

$$\text{Equivalent Mass} = \text{Recip. Mass} \times \frac{(V_{cr})^2}{(V_{cr})^2} \times \frac{(\text{Maximum Angle Factor})^2}{2}$$

For the example car, using an eight cylinder engine with a three and three quarter inch bore and a three and seven sixteenths inch stroke, and a value of n = 1.93, the maximum angle factor occurs at $\theta = 76°37'$ and has a magnitude of 1.0313.

Equivalent Reciprocating Mass =

$$\text{Actual Reciprocating Mass} \cdot \frac{(\text{Maximum Angle Factor})^2}{2}$$

For the example engine the various parts have the following magnitudes:

Clutch Assembly	348
Flywheel and Crankshaft	1280
Rotating Rods and Bearings	62.8
Equivalent Reciprocating Body	54.3
	1744.1 pound (inches)^2
	= 0.376 pound feet per (second)^2

The equivalent mass factor can now be evaluated, using 4000 pounds as the value of W.

TABLE II

ACCELERATION FACTORS

Linear Accel.	Wheel Accel.	Engine-Clutch Accel.	Acceleration Factor	Gear Ratio	Acceleration Ratio
124.2	3.66	2.24	130.1	4.1 x 0.7	.954
124.2	3.66	2.84	130.7	3.23	.951
124.2	3.66	4.56	132.4	4.1	.938
124.2	3.66	6.63	134.5	3.23 x 1.53	.924
124.2	3.66	16.2	144.0	3.23 x 2.39	.863
124.2	3.66	0	127.9	Ideal	.973

When the ideal transmission is used, the engine speed is held constant at the speed of maximum horsepower, and no energy need by consumed in accelerating the engine and clutch assembly.

The Acceleration Ratio is defined as the ratio:

$$\frac{\text{Actual acceleration rate produced}}{\text{Acceleration rate with no rotating parts}}$$

With no rotating parts the acceleration ratio would be equal to 1.0. As the gear ratio between wheels and engine is progressively increased, the horsepower required to accelerate the engine becomes greater and greater, progressively reducing the value of gear reduction as a factor in increasing acceleration ability.

The relation for determining acceleration ability, using the acceleration ratio is:

$$a = \frac{356\ HP_{XS}}{M} \cdot \frac{\text{Acceleration Ratio}}{\frac{W}{g}}$$

$$a = \frac{356\ g\ HP_{XS}}{WM}$$ Acceleration Ratio

The acceleration rates obtained by the example car with the several different gear ratios are shown on Figure 11. They reflect directly the effect of transmission gear ratio on excess horsepower. Acceleration rates above three feet per second per second are noticeable but not annoying. Acceleration at a rate above seven feet per second per second is objectionable to the majority of vehicle passengers (13).

The deviation between the acceleration produced by the ideal transmission and that produced by the selective sliding-gear transmission represents the loss due to the need for accelerating the engine. It is more noticeable in low gear than in second, and practically negligible in high gear.

ACCELERATION ABILITY

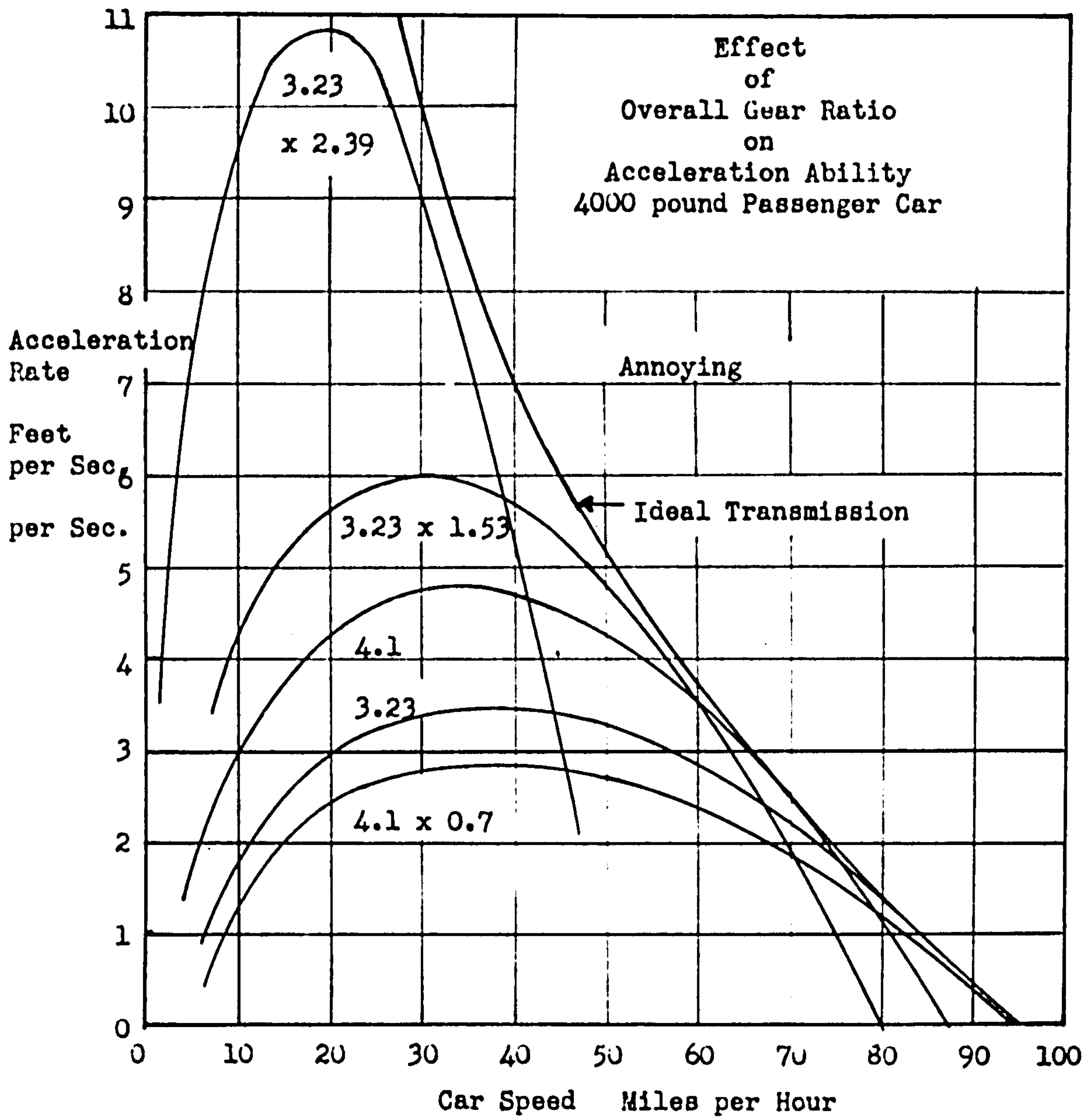


Figure 11

FUEL ECONOMY

The miles per gallon ability of a vehicle is an inverse function of vehicle power demand, and a direct function of engine thermal efficiency.

$$\text{Energy Demand Btu/Mile} = \frac{\text{Demand HP} \cdot 2545}{\text{Miles per Hour}}$$

$$\text{Btu/Mile} = \frac{2545 \cdot HP_{Demand}}{M}$$

The constant 2545 is the Btu equivalent of one horsepower hour. This energy demand by the vehicle is supplied from the engine by burning gasoline and converting a portion of the heat energy produced by combustion into mechanical energy at the flywheel. The efficiency with which this process is carried out is called the Brake Thermal Efficiency. It is defined as the ratio of the actual mechanical energy produced at the output shaft to the potential heat energy of the fuel taken into the engine. A high Brake Thermal Efficiency is essential to a high miles per gallon performance. The brake thermal efficiency of an engine can be improved by increasing its compression ratio or by decreasing its friction losses. Both methods have been used on recent new passenger-car engines with good results, but the most important single factor in the actual brake thermal efficiency produced by the engine in the vehicle is the relative load level at which the engine operates. The following relation for miles per gallon illustrates the importance of <u>Brake Thermal Efficiency (BTE)</u>.

$$\underset{\text{Btu/gallon of fuel}}{\text{Engine Output}} = \frac{\%BTE}{100} \left(\underset{\text{Btu/pound}}{19{,}200}\right) \left(\underset{\text{pounds/gallon}}{0.73 \times 8.34}\right)$$

$$= (1170)\ \%BTE$$

The figure of 19,200 is the Lower Heating Value of gasoline with a specific gravity of 0.73.

$$\frac{\text{Btu/gallon}}{\text{Btu/mile}} = \text{Miles per gallon}$$

$$\text{Miles /gallon} = \frac{(1170)\ \%BTE}{\frac{(2545)HP_{Demand}}{M}} = \frac{M(\%BTE)}{(2.17)HP_{Demand}}$$

The most convenient way to express %BTE for the evaluation of this relation is to show %BTE as a function of the % Full Load at which the engine is operating. The % Full Load is defined as follows:

$$\%\ \text{Full Load} = 100\ \frac{\text{HP actually being produced at given RPM}}{\text{Full-Throttle HP at same speed}}$$

The relationship between %BTE and % Full Load for a typical passenger car engine is shown on Figure 12. This shows directly the effect of engine loafing on the Brake Thermal Efficiency. At low outputs at any speed, too much of the energy of the fuel is used up in overcoming the internal losses of the engine. As the relative load level of the engine increases, the frictional losses, which are relatively insensitive to load, decrease in importance, causing an improvement in Brake Thermal Efficiency. This improvement would continue, in diminishing magnitude, up to 100% Full Load, but above about 60% Full Load the throttle movement opens a "Full Throttle Enrichening Valve", changing the mixture of air and gasoline produced by the carburetor from an "economy" mixture to a "power" mixture, and causing a drop in Brake Thermal Efficiency.

At low speeds the engine suffers increased jacket losses per cycle of operation, lowering the efficiency, and at high speeds it suffers in-

EFFECT OF ENGINE LOAD ON BRAKE THERMAL EFFICIENCY

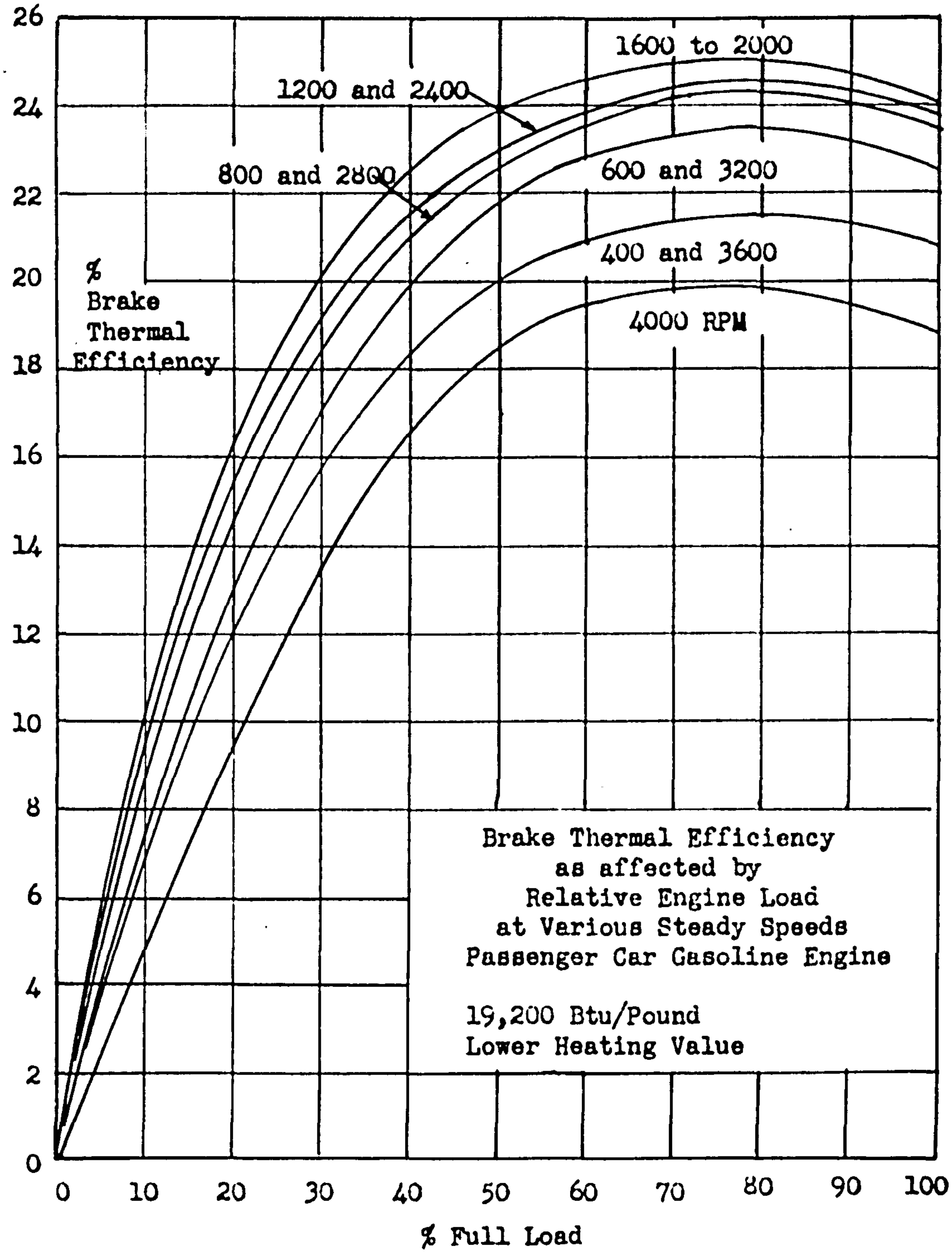


Figure 12

creased frictional losses, again lowering the efficiency. Best efficiencies occur at medium speed operation with the engine producing from seventy to eighty percent of its full-throttle power at those speeds. For steady-speed level-road operation the engine produces continuously the demand horsepower shown on Figure 7. It is capable of producing the full-throttle horsepower shown on Figure 7 for the various gear ratios. The ratio of the ordinate of the road load curve to that of the full load curve, times 100, is the percent Full Load at which the engine must operate for a particular car speed and final-drive gear ratio.

The effect of car speed and final-drive gear ratios on the % Full Load of the engine in the example car are shown on Figure 13. The relative engine load is low for all gear ratios with the sliding gear transmission, rising to acceptable values only at wastefully high speeds. Passenger car engines are forced to loaf through most of their life with a lamentable effect on their efficiency. Lowering the overall gear ratio by the use of overdrive brings some improvement, but not in drastic proportions.

Since the relation between car speed M and engine speed N was determined in the computations for Figure 7, these values of N, plus the value of % Full Load from Figure 13, may be used to determine from Figure 12 the % Brake Thermal Efficiency which the engine is producing. The % Brake Thermal Efficiency for the example car for all gear ratios used is shown on Figure 14. Also shown for comparison purposes is the effect of compounding an overdrive on a 3.23 final drive ratio. These curves show that the lower the overall gear ratio, the higher is the

RELATIVE ENGINE LOAD

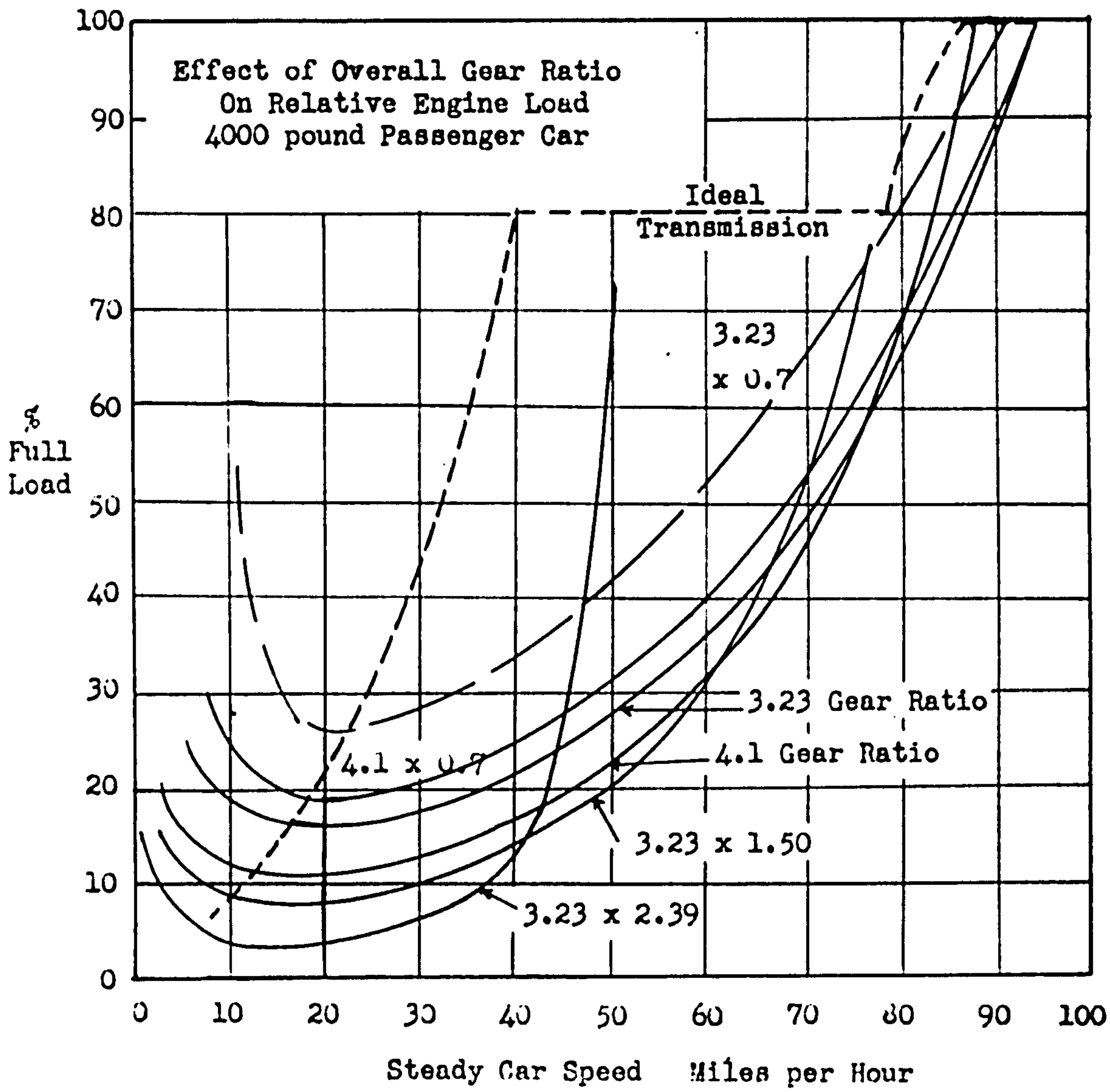


Figure 13

ROAD LOAD BRAKE THERMAL EFFICIENCY

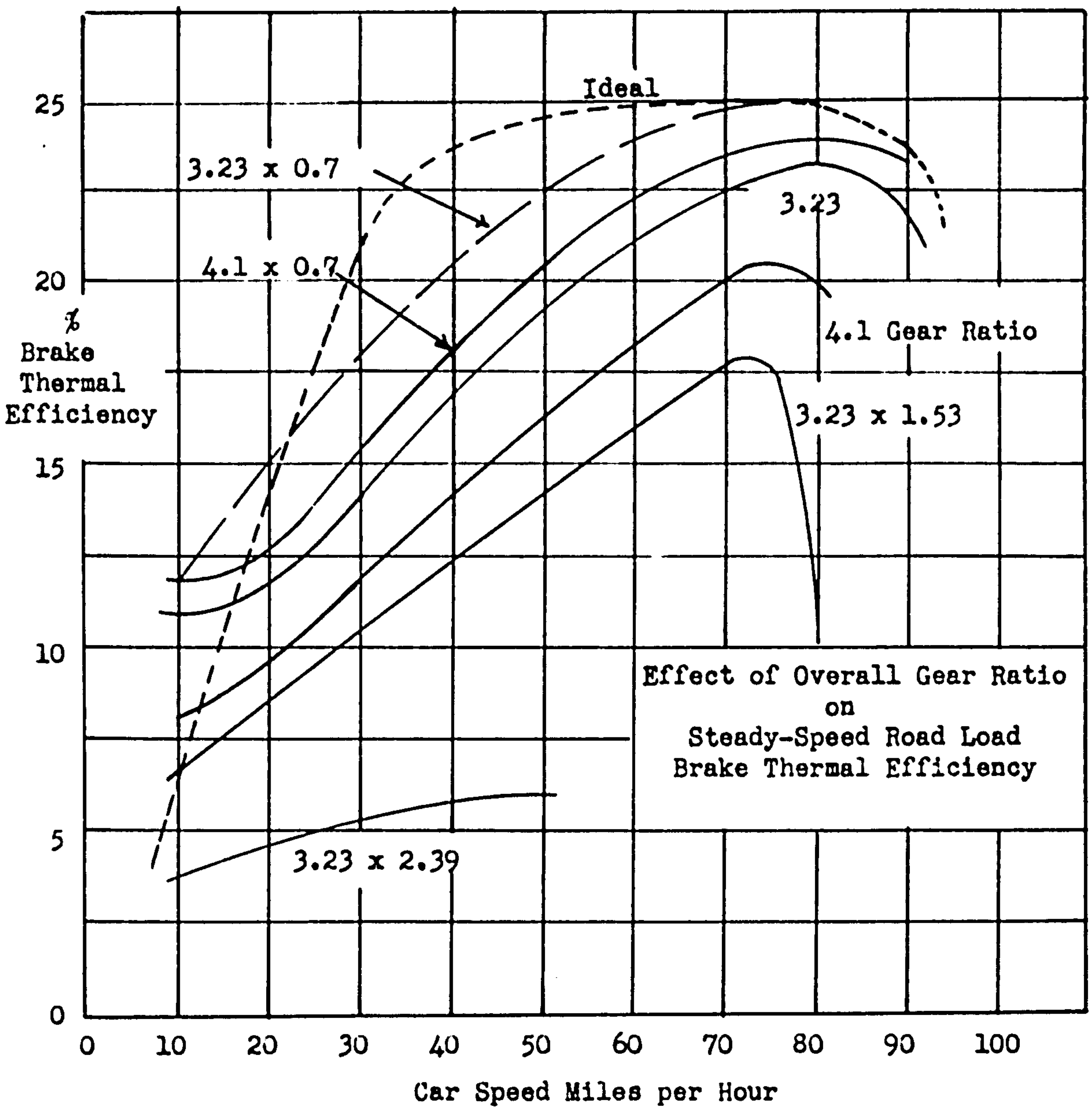


Figure 14

thermal efficiency of the engine throughout the whole speed range. High efficiency accompanies the combination of low engine speed and high engine load.

With the % Brake Thermal Efficiency known as a function of M on Figure 14, and the HP_{Demand} as a function of M shown on Figure 7, these quantities can be combined in the relation for miles per gallon to determine the fuel economy as a function of car speed. The results of this combination are shown on Figure 15. Noteable here is the fact that Miles per Gallon is highest at the low speeds of operation, and decreases progressively as car speed increases. The extremely low road load demand horsepower at low speeds more than counterbalances the very low engine efficiency. As car speed increases, more and more energy is dissipated in stirring air, and in spite of the constantly increasing thermal efficiency the fuel economy diminishes.

The curves of Figure 15 indicate that the use of an overdrive adds two to four miles per gallon to the steady speed fuel mileage over most of the speed range, a factor which should allow the overdrive to pay for itself if enough open highway driving is done. The advantages of the overdrive or the low final-drive ratio may be erased by the need for special fuels to counteract detonation difficulties brought on by high relative load levels. The required use of more expensive fuels would not eliminate the benefits of lower noise level, lower vibration level, and lower wear level which the overdrive offers, but it would remove the ability of the overdrive unit to repay its cost by saving fuel.

STEADY-SPEED FUEL ECONOMY

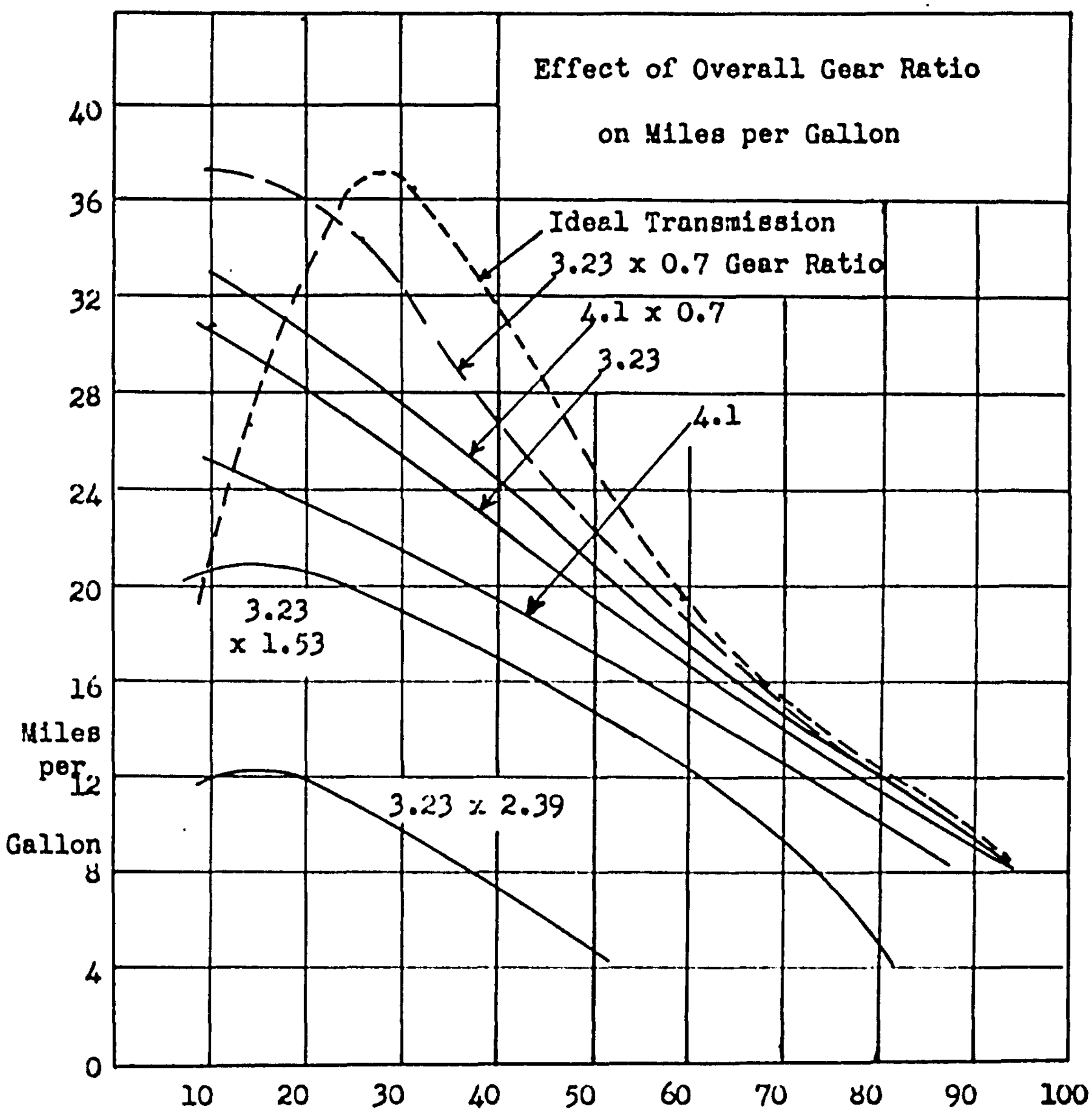


Figure 15

The Ideal Transmission

The ideal transmission acts in an entirely different manner when used for economy operation than when used for power operation. Economy considerations indicate that best economy is attained when the engine is operated at eighty percent of full load at the lowest speed at which it will produce the required demand horsepower. As the car speed varied the transmission ratio would be changed to maintain this condition.

Practical limitations prevent complete attainment of this ideal condition. The lowest speed at which an engine should be expected to run satisfactorily is somewhat above the maximum speed at which it will buck when running at a high load level. For the safety of the transmission and the engine, the transmission should not be allowed to pull the engine speed below this level, even though the % Full Load might need to be dropped to appreciably less than eighty percent. For the engine in the example car this "floor" on engine speed was assumed as 600 rpm. The effect of this floor on the % Full Load of the engine with the ideal transmission is shown on Figure 13. At car speeds below forty miles per hour it is necessary to hold the engine speed at the 600 rpm floor level, and to decrease the % Full Load accordingly. Between forty miles per hour and seventy nine miles per hour the load level is held at 80%, while the engine speed is increased from 600 to 2000 rpm. Between seventy nine and eighty seven miles per hour the engine speed is held at 2000 rpm while the load is increased to 100%, and then from eighty seven miles per hour to the maximum speed of ninety four miles per hour the load level is held at 100% while the engine speed is raised from 2000 to 3500 rpm.

This combination of operations produces the highest overall fuel economy. Figure 14 shows the % Brake Thermal Efficiency which results. An appreciable gain is evident over the entire speed range except for the extreme high and low speeds.

The miles per gallon produced by ideal transmission is shown on Figure 15. There is little gain in the extreme high speed range over the performance of the overdrive ratios. At the extreme low speed end the ideal transmission suffers from the imposition of the speed floor, a factor which would also affect the other curves although it is not shown on the chart. In the medium speed range the ideal transmission produces several more miles per gallon than fixed ratio transmissions.

The gear ratios which the ideal transmission would have to produce with a 3.23 final-drive ratio to obtain the performance indicated in these charts is shown on Figure 16. A satisfactory range of operation would require a variation from 10 to 1 at one extreme to 0.4 to 1 at the other. This is a ratio between extremes of twenty five to one, more than any practical transmission could attain. There is not now available any transmission which meets the necessary requirements. Up to the present time all ideal transmissions, whether mechanical, electrical, or hydraulic, have suffered from either high initial cost or excessive weight and bulk, or unacceptably low efficiency. The analysis of the ideal transmission is of value chiefly as a performance ideal with which other transmissions can be compared.

IDEAL TRANSMISSION RATIOS

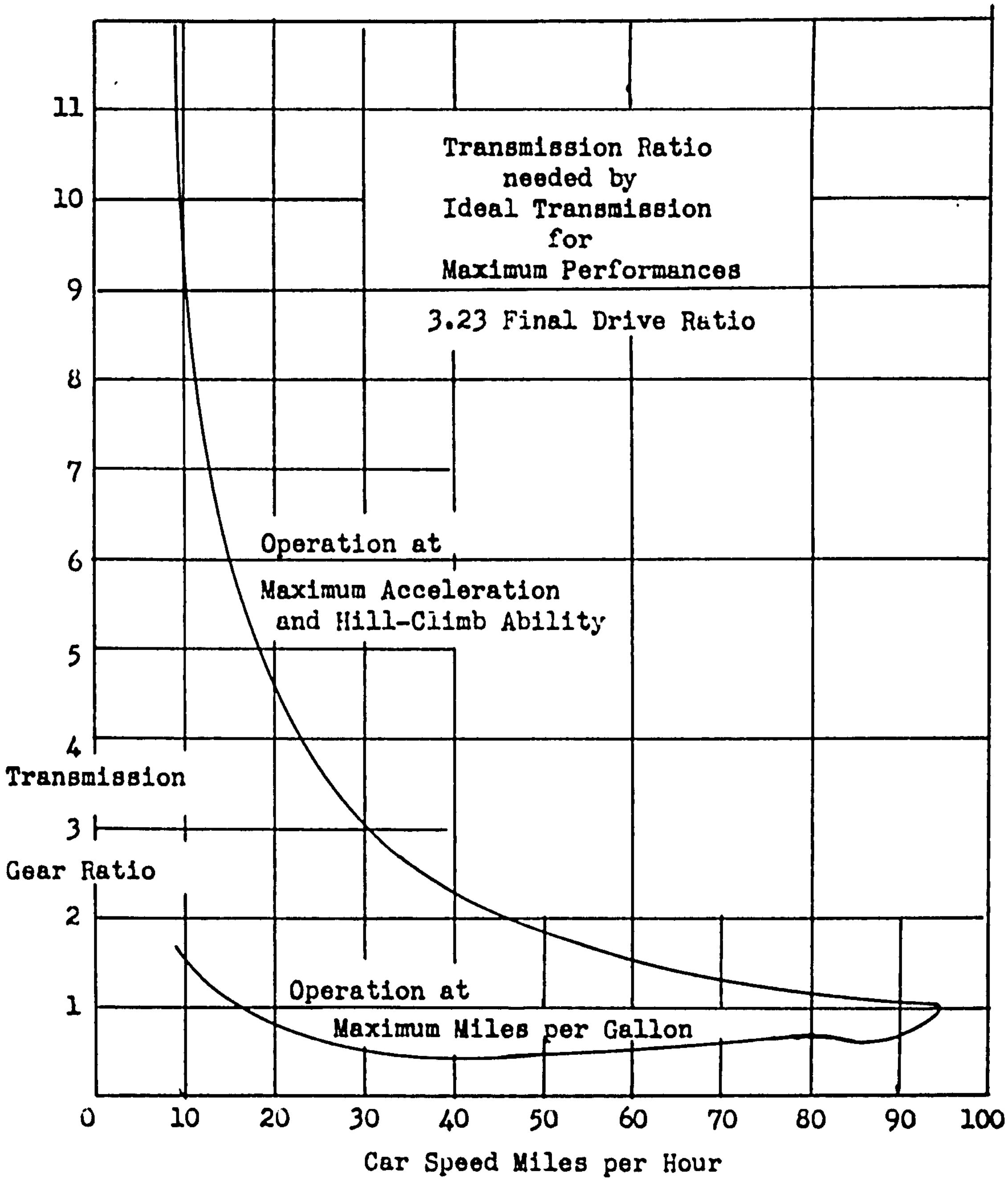


Figure 16

HYDRAULIC TRANSMISSIONS

The relatively new hydraulic transmissions which have been introduced in recent years make use of an hydraulic clutch or a combination of hydrokinetic torque converter and fluid clutch. Their chief virtue is that they simplify or reduce in number the operations which the driver must perform during normal driving. All of them increase the first cost of the vehicle, and all of them decrease the efficiency of operation to a slight degree. Once the vehicle has attained normal driving speed the flexibility between driveshaft and engine speeds is so low that the relations developed in this chapter apply to them as well as to the conventional transmission. During starting of the vehicle from rest the torque converter can take the place of the geared transmission to a limited extent, but at a decided drop in efficiency.

SUMMARY ON PERFORMANCE

An automotive vehicle offers an appreciable resistance to its forward motion, made up primarily of wind resistance, tire rolling resistance, and drive system friction. At speeds above forty miles per hour the wind resistance is the major factor. Its magnitude depends on frontal area, and on aerodynamic cleanliness. No significant reductions in air resistance can be expected as long as passenger capacities and style requirements remain as at present.

Vehicle demand is determined experimentally, with the results of the experiments expressed as empirical equations for either towing effort or demand horsepower. Typical American passenger cars demand thirty to thirty five horsepower at sixty miles per hour.

The power necessary to drive the vehicle comes from an internal-combustion engine having a definite relationship between torque, horsepower and rotational speed. The inability of the engine to produce any useful torque below a minimum speed demands the use of some type of clutch between engine and driveshaft. High top speed demands a gear ratio between engine and wheels which produces peak engine horsepower about at peak car speed, but which lowers acceleration and hill-climb ability in the medium speed range. A fair compromise between top speed and hill-climb ability must be made.

The use of a gear ratio producing a decent top speed prevents the engine alone from producing sufficient torque to meet all of the normal driving demands, such as starting on or driving up a steep hill. To meet these requirements a torque multiplier, popularly called a transmission, is placed between the clutch and driveshaft. The conventional transmission is a selective sliding-gear unit with three forward and one reverse speeds, plus a neutral.

The magnitude of the excess horsepower furnished by the engine at open-throttle, above the amount required for steady-speed level-road operation, can be used to determine the hill-climb or acceleration ability of the vehicle. Low overall gear ratios, attained either directly or by means of an overdrive gear, result in low values for hill-climb and acceleration ability. Ability to climb a ten percent grade in high gear is necessary for public acceptance.

Acceleration ability is reduced by the necessity for accelerating all rotating parts angularly as well as linearly. High gear ratios accentuate this condition. Acceleration rates over seven feet per second per second, obtainable only in the low gear of the transmission, are objectionable to most people.

The fuel economy of a vehicle is affected by gear ratio more than any other single factor. Low overall gear ratios reduce engine speed and raise engine relative load, both of which help to increase the brake thermal efficiency of the engine. High thermal efficiencies are necessary for high fuel economy. Overdrive is worth two to four miles per gallon on fuel economy.

The ideal transmission provides a basis for comparison for the performance produced by other transmissions. No ideal transmissions for passenger cars are now available.

IV

BRAKES

AND

BRAKING

BRAKES AND BRAKING

The safety of the passengers and driver, and the preservation of cargo, requires that the driver of a self-propelled vehicle be capable of either stopping the vehicle quickly when necessary, or of reducing its rate of travel from a high value to a lower value, or of preventing the occurrence of excessive speed when proceeding down a hill.

The natural viscous and mechanical friction losses of the moving vehicle will have some retarding or "braking" effect, but except in very mild cases this degree of braking effect is far from adequate. The driver must be provided with equipment with which he can create, under his control, a braking effect varying from zero to the maximum value which either the braking system can produce, or the mechanical construction of the vehicle can withstand without failure, or the passengers or goods can withstand without danger of injury. The last named condition is of importance in any situation involving standing passengers, such as railroads, street cars, and busses, but in most automotive vehicle applications the passengers are seated, and in an emergency stop the chances of passenger injury from imminent collision are so much greater than the discomfort created by maximum braking effect that the braking system is built to produce the maximum braking effect obtainable within practical limitations.

BRAKING FORCE REQUIREMENTS

In a wheeled vehicle which must depend for its braking effect on the frictional forces produced between its own wheels and the road surface, the maximum possible braking effect which can be produced is equal to the product of the adhesive weight of the vehicle and the coefficient of friction between tires and road.

If W signifies the weight of the vehicle, and θ the angle between the road surface and a level surface, then the adhesive weight of the vehicle (force normal to the road surface) equals $W \cos \theta$. If the symbol "f" is used to signify the coefficient of friction developed between the tire and the road surface at impending slide of the tire, the maximum braking effect which can be brought to bear on the vehicle is equal to $\underline{fW \cos \theta}$.

The coefficient of friction between tires and road surface at impending slip of the tire varies over wide limits (14). Apparently its lowest value occurs with snow-coated tires operating over glare ice at temperatures slightly below freezing and has a magnitude of about 0.1. The highest value of this coefficient can be over 1.0, when pneumatic rubber tires with a good non-skid tread are operated over an open-surfaced, angular-stone, asphalt-bound macadam road. Between these maximum limits the friction coefficient fluctuates with changes in such factors as tread design, tire contact area and inflation pressure, vehicle speed, moisture conditions, shear strength of road surface, continuity of road surface, etc., affecting it.

The average value of this coefficient for pneumatic tires with good non-skid tread operating over dry concrete at normal vehicle speeds is quoted by various authorities as ranging from 0.6 to 0.85, with 0.6 having been admittedly taken on the conservative side (15). After checking all references it seems that 0.8 can be used as the best average value for this coefficient for the conditions indicated above.

Using this value for f, observing that for the grades usually encountered by a vehicle the value of cos θ differs only slightly from 1.00, and admitting that vehicle braking systems seldom are able to produce theoretically perfect braking (braking force = $fW \cos \theta$), it is entirely permissible to say that, as a first approximation, the brake-actuating force exerted by the driver must be capable of producing a braking force at the road surface equal to three quarters of the weight of the vehicle, or in symbols 3/4W.

BRAKE ACTUATION

This vehicle-braking force of 3/4W must be produced by whatever brake-actuating force the driver can produce, either directly by pushing with his foot on a "brake pedal", or with assistance from vacuum-actuated or compressed-air-actuated pistons or diaphragms controlled by a valving system, or from electromagnet-operated brake actuators. The direct pedal actuation demands the most attention from the designer because a limit is set by the actual physical abilities of the driver on the maximum brake-actuating force which can be produced. The "assisted" braking systems have their troubles and limitations, but lack of actuating force is usually not one of them. The following development of brake-linkage design factors will be based on direct physical actuation by the driver of a foot-actuated brake pedal.

The force which can be exerted by the driver on the brake pedal varies from that produced by a petite 90 pound woman driver to that produced by an athletically-active 260 pound man. This variation is wide, and is complicated by the fact that under emotional stress, such as that produced by an impending crash, people can exert appreciably more than their normal muscular force.

Not all of the force exerted by the driver is effective in applying the brakes. Some of it must be used in overcoming mechanical and fluid friction in the brake-actuation system, and some in overcoming the force of the various springs which are required to release the brakes and return the brake pedal. In the following discussion the <u>net</u>

pedal force will refer to that portion of the pedal force actually used to apply the brakes, as opposed to the gross pedal force actually applied to the pedal. The magnitude of the difference between net and gross pedal forces can be of the order of twenty pounds.

To reduce the confusion over the net pedal force available, it is customary to assume a net pedal force of 100 pounds, and to compare braking abilities of various systems on the 100 pound net pedal force as a common base.

The actual braking force produced by the brakes does not act at the tire-road contact surface, but at the brake drum surface, at a radius which is appreciably smaller than the rolling radius of the wheel. Statistics of the physical dimensions of our passenger cars show that the average ratio of brake drum radius to tire rolling radius is about 0.4 (as car weight decreases the brake drum radius decreases more rapidly than does tire rolling radius, so small cars have values below this average, with large heavy cars above the average). This means that the driver's force on the pedal must produce a force at the brake drum which is $\frac{1}{0.4}$ or 2.5 times the force required at the wheel. Since the desired brake force at the road was approximately 3/4W, the force multiplication ratio in the brake system, or the ratio of total braking force at all drum surface to the Net Pedal Force must be $\frac{(2.5 \times .75)W}{NPF}$, or for a Net Pedal Force of 100 pounds, the Brake System Force Multiplication Ratio = .0188W.

For various significant values of W this ratio has the following magnitudes:

Type of Vehicle	W	Force Multiplication Ratio Pedal to Brake Drums
Small Passenger Car	3600 lb.	68
Heavy Passenger Car	4750 lb.	89
Light Truck	10000 lb.	188
Heavy Truck	40000 lb.	750

Presently available braking systems can produce the indicated ratio values for passenger cars, but trucks require "assisted" or "power" braking systems.

BRAKE SYSTEM FORCE MULTIPLICATION RATIOS

The force multiplication ratio which can be produced between brake pedal and brake drum is composed of two parts: the force multiplication ratio in the brake linkage (or hydraulic system) between the brake pedal and the points on the brake shoes which contact the brake-shoe-actuating cams or pistons and the force multiplication ratio produced between the brake-actuating-cams or pistons and the brake drum surfaces. The latter is actually the ratio of the brake drag force at the drum surface to the cam force which actuates the brake shoes. The product of these two individual force multiplication ratios equals the total brake system force multiplication ratio. To simplify the required analysis each of these individual means of brake force multiplication will be considered separately.

FORCE MULTIPLICATION IN BRAKE RIGGING

The force multiplication ratio which can be produced between the brake pedal and the brake-shoe-actuating cams is strictly a function of the relative distances through which the two ends of the system travel. If we neglect mechanical and fluid friction in the brake rigging (this is covered by the difference between gross and net pedal forces) the work done at the pedal must equal the total of the work done at each of the individual cam or piston faces. This is equivalent to:

$$\text{Average Net Pedal Force} \times \text{Pedal Travel} = \text{Average Total Cam Force} \times \text{Cam Travel}$$

or

$$\frac{\text{Average Total Cam Force}}{\text{Average Net Pedal Force}} = \frac{\text{Pedal Travel}}{\text{Cam Travel}}$$

In a strictly mechanical brake rigging as illustrated in Figure 17, there would be strict proportionality between net pedal force and the sum of the cam forces at all points in the pedal travel, so the average force values may be replaced by the instantaneous force values at any point in the pedal travel. We then have:

$$\frac{\text{Total Cam Force}}{\text{Net Pedal Force}} = \frac{\text{Pedal Travel}}{\text{Cam Travel}} = \text{Force Multiplication Ratio in Brake Rigging}$$

For an hydraulic brake system as illustrated in Figure 18, the result is the same but is arrived at somewhat differently.

MECHANICALLY-ACTUATED BRAKE RIGGING

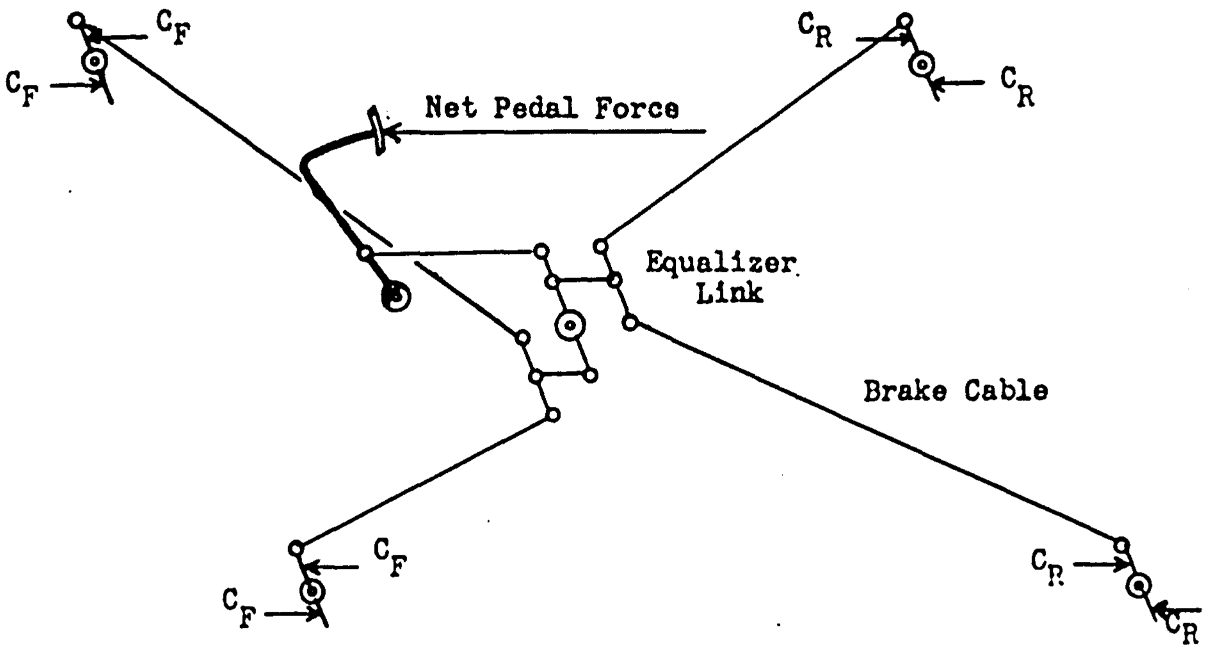


Figure 17

HYDRAULICALLY-ACTUATED BRAKE RIGGING

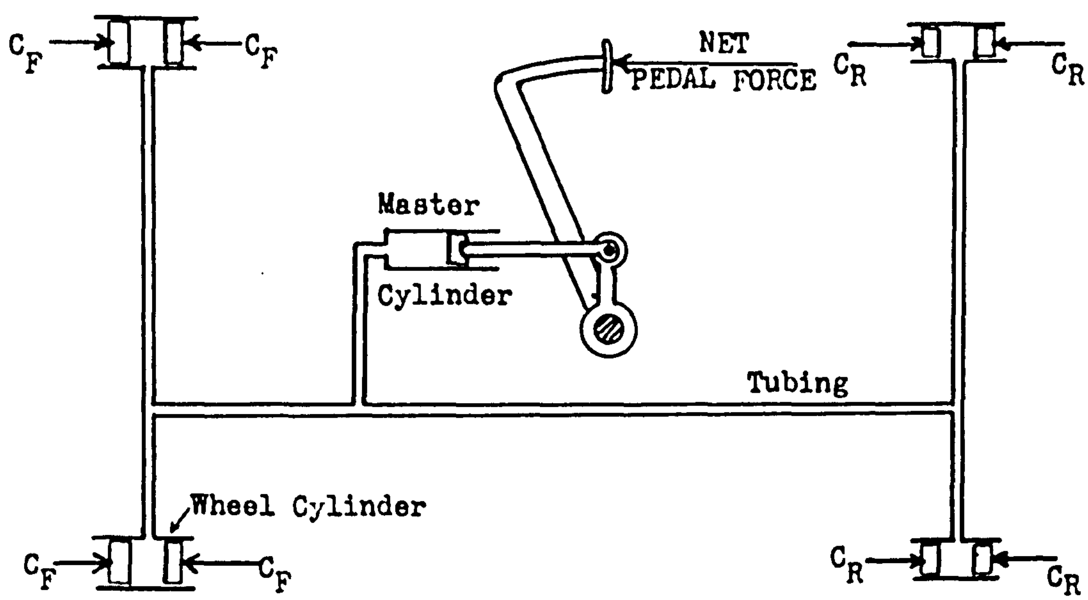


Figure 18

Net Pedal Force x Linkage Ratio between Pedal and Master Cylinder =

Force on Master Piston

$$\frac{\text{Force on Master Piston}}{\text{Area of Master Piston}} = \text{Hydraulic System Pressure}$$

Hydraulic System Pressure x Total Brake Piston Area = Total Cam Force available to actuate Brake Shoes

Then:

$$\frac{\text{Total Cam Force}}{\text{Force on Master Piston}} = \frac{\text{HSP x Total Brake Piston Area}}{\text{HSP x Master Piston Area}}$$

Or:

$$\frac{\text{Total Cam Force}}{\text{Net Pedal Force x Pedal Linkage Ratio}} = \frac{\text{Total Brake Piston Area}}{\text{Master Piston Area}}$$

Since the fluid in the hydraulic system is practically incompressible at the pressures used, and the bulk expansion of the system is very small, any change in enclosed volume caused by movement of a brake piston must be counteracted by a volumetrically equivalent movement of the master piston if the hydraulic system pressure is to be maintained.

Thus:

Brake Piston Travel x Total Brake Piston Area =

Master Piston Travel x Master Piston Area

and:

$$\frac{\text{Total Brake Piston Area}}{\text{Master Piston Area}} = \frac{\text{Master Piston Travel}}{\text{Brake Piston Travel}}$$

By substitution in a previous equation:

$$\frac{\text{Total Cam Force}}{\text{Net Pedal Force x Pedal Linkage Ratio}} \qquad \frac{\text{Master Piston Travel}}{\text{Brake Piston Travel}}$$

Because of the simple mechanical lever system which exists between the Brake Pedal and the Master Piston connection point, the following relationship holds:

$$\text{Master Piston Travel} = \frac{\text{Brake Pedal Travel}}{\text{Pedal Linkage Ratio}}$$

If this is substituted into the previous relation we obtain as a result:

$$\frac{\text{Total Cam Force}}{\text{Net Pedal Force}} = \frac{\text{Brake Pedal Travel} \times \dfrac{\text{Pedal Linkage Ratio}}{\text{Pedal Linkage Ratio}}}{\text{Brake Piston Travel}}$$

or

$$\frac{\text{Total Cam Force}}{\text{Net Pedal Force}} = \frac{\text{Pedal Travel}}{\text{Cam Travel}} = \text{Force Multiplication Ratio in Brake Rigging}$$

In order to obtain any indication of the order of magnitude of the Force Multiplication Ratio in the brake rigging it is necessary to determine the probable values of both Pedal Travel and Cam Travel.

The maximum value of Pedal Travel through which a short-legged driver can effectively apply full force to the brake pedal cannot safely be set at more than five inches. This value cannot well be increased without sitting the driver so close to the pedal that he is badly cramped for leg room, and rapidly becomes uncomfortable and tired while driving. If physical limitations of the driver prevent the effective use of a long pedal travel, then the only possibility of obtaining a high Force Multiplication Ratio in the Brake Rigging lies in keeping the Cam Travel to a very low value. As will be shown in the subsequent mathematical analyses of the various brake types, the cam travel necessary to move the brake shoes from their released to their fully applied positions is a function mainly of the amount of clearance required between a shoe and an out-of-round drum to avoid dragging, plus the amount of cam travel which must be allowed to compensate for natural wear of the brake linings and brakedrums between successive brake adjustments. If adjustments can be frequent, the wear allowance can be small, and the necessary cam travel can also be small, in some types of brakes as low as 1/10 of an inch. If brake adjustments must be less frequent, the minimum cam travel must be as much as 1/5 of an inch.

With a maximum effective pedal travel of five inches, and a minimum cam travel of one-fifth to one-tenth of an inch, the Force Multiplication Ratio in the Brake Rigging will vary from 25 to 50. For passenger cars this is a fair share of the Brake Force Multiplication Ratio requirements, but for trucks it is far from adequate.

FORCE MULTIPLICATION BETWEEN CAM AND DRUM

The magnitude of this ratio varies appreciably with the type of brake used. The basic types of brakes now in use on vehicles include the following: external-contracting band brakes, separately pin-anchored internal-expanding shoe brakes, separately sliding-block-anchored internal-expanding shoe brakes, separately articulated-lever-anchored internal-expanding shoe brakes, duo-servo self-energizing internal-expanding shoe brakes, and self-energizing internal-expanding disc brakes (16).

The external-contracting band brake is now nearly obsolete in automotive practice, having been replaced by the more effective and more easily-shielded internal expanding brake. Moreover, the theory of the band brake is well developed and available in many textbooks and handbooks, so a discussion of the theoretical aspects of the band brake will not be included here.

CAM MOVEMENT AND SHOE CLEARANCE

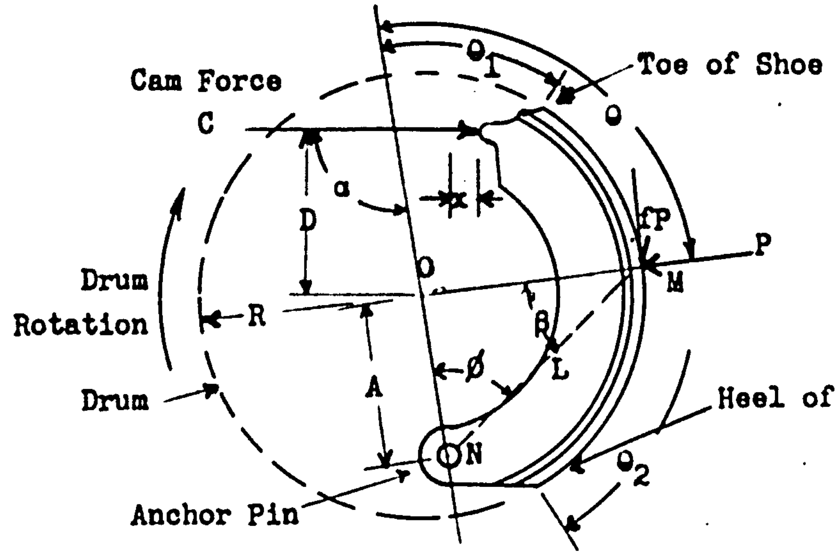
The various shoe arrangements used in contemporary internal-expanding brakes are shown in Figure 19, with a complete notation of the symbols and dimensions necessary for a theoretical analysis of the relationship which exists between cam movement and the radial clearance between brake shoe and brake drum. The detail dimensioning is shown on a pin-anchored shoe, but in each of the other brake types the shoes act as pin-anchored shoes until contact is made with the drum. Thus an analysis of the cam movement versus shoe to drum clearance relationship can be made for the pin-anchored shoe, and will apply equally well to the other shoe types.

The articulated-link shoe of Figure 19 must have a high-friction joint at the junction of the link and the brake shoe flange if it is to be able to retract the shoe from the drum when the brakes are released. Thus until the shoe contacts the drum it acts as a rigid shoe pivoting about anchor pin N.

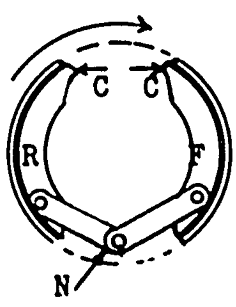
The Duo-Servo brake as shown in Figure 19 consists of a primary shoe and a secondary shoe. The secondary shoe is pin anchored at N_S and swings about this point when moving. The primary shoe is anchored to the toe of the secondary by a spring and an adjustable-length floating link. Until contact occurs between shoe and drum the primary shoe pivots about one end of the floating link and acts the same as a pin-anchored shoe.

The sliding-block anchor system shown on Figure 19 allows the brake shoes to center themselves within the drum after contact has been made

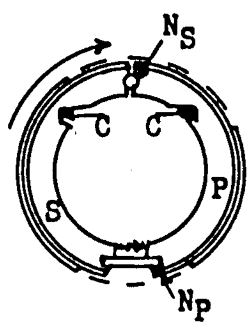
INTERNAL EXPANDING SHOE BRAKES



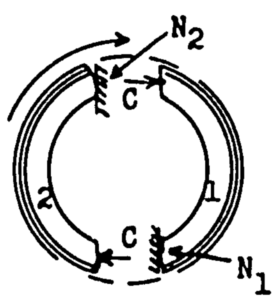
A = Anchor Distance, Pin Center to Drum Center
C = Cam Force
D = Cam Distance, Cam Force to Drum Center
f = Coefficient of Friction, Lining to Drum
L = Distance, Anchor Pin Center to Lining-Drum Contact Point
M = Contact Point, Lining to Drum
N = Anchor Point, attached to Brake Backing Plate
O = Drum Center
P = Radial Force of Lining on Drum
R = Drum and Shoe Radius
α = Angle, Cam Force to Anchor Diameter
β & ϕ = Angles of Triangle NOM
θ_1 = Angle, Anchor Diameter to Toe End of Shoe Lining
θ_2 = Angle, Anchor Diameter to Heel End of Shoe Lining
θ = Angle, Anchor Diameter to Infinitesimal Lining Segment at M
x = Cam Movement Distance, Released to Applied



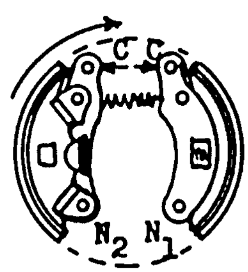
Articulated Link Shoe Brake High Friction Link to Shoe Joint



Duo-Servo Shoe Brake Pin-Anchored Secondary Shoe Link-Anchored Primary Shoe



Sliding-Block Anchor Shoe Brake



Double Primary Balanced Brake

Sliding-Block Anchor Plates

Figure 19

between the shoe and drum. Until contact is made between shoe and drum the shoe pivots about some point of contact between shoe and anchor block and acts as a pin-anchored shoe.

The Timken DP Brakes as shown on Figure 19 have the cam force transmitted to the midpoint of the length of the shoe by a pair of pin-anchored levers. The shoes are held against a semicylindrical pressure block on the levers by a retainer spring, and until the shoe contacts the drum the brake shoe and actuating lever combination acts as a unit and pivots about the anchor pin of the lever.

Thus any shoe combination used in present internal-expanding shoe brakes can be analyzed as a pin-anchored shoe for that portion of its movement between the released position and the initial shoe-drum contact.

The first step in this analysis is to find the relationship between cam movement and change in radial clearance between shoe and drum. With reference to Figure 19 with x representing cam movement and Δ R representing the change in radial clearance between shoe and drum at any point around the periphery of the shoe, the procedure is as follows:

For any point M on the periphery of the shoe, let L = length of line MN, β = angle OMN and ϕ = angle ONM. Then $\Delta\phi$ = the angle of swing of the shoe about the anchor pin N due to the cam movement x.

$$\text{Then } \Delta\phi = \frac{\text{arc}}{\text{radius}} = \frac{x}{D + A \sin (180 - a)}$$

where x = arc swing and D + A sin (180 − α) equals the radius to the arc x.

Then: $\Delta R(\text{at point } M) = L \cdot \Delta \phi \cdot \sin \beta$

In the triangle OMN, $\beta = 180 - \phi - (180° - \theta) = \theta - \phi$

By substitution:

$$\Delta R = L \cdot \frac{x}{D + A \sin (180 - \alpha)} \cdot \sin (\theta - \phi)$$

but: $\sin (\theta - \phi) = \sin \theta \cos \phi - \cos \theta \sin \phi$

and: $\sin \phi = \frac{R \sin \theta}{L}$ $\qquad \cos \phi = \frac{A + R \cos \theta}{L}$

$\sin (180 - \alpha) = \sin \alpha$

so: $$\Delta R = \frac{x}{D + A \sin \alpha} \left\{ \frac{LA \sin \theta + LR \sin \theta \cos \theta}{L} - \frac{LR \sin \theta \cos \theta}{L} \right\}$$

By cancellation this reduces to:

$$\Delta R = \frac{Ax \sin \theta}{D + A \sin \alpha} = x \frac{A}{D + A \sin \alpha} \cdot \sin \theta$$

Thus the change in radial clearance between shoe and drum at any point along the shoe becomes a function of the sine of the angle between the anchor diameter and the point involved. The clearance is then lowest at the largest and smallest values of θ, or at the heel and toe of the shoe. Due to drum warpage during use, some clearance must be left at these points when brakes are adjusted. In practice this clearance is set at 0.010 inches and the angle θ_1 to the toe of the shoe is approximately 30°. This is equivalent to 0.020 inches clearance at $\theta = 90°$.

Since the radial movement of the shoe varies with sin θ, and if the shoe is very rigid compared to the drum, the force between shoe and drum and the wear rate of the shoe lining will also follow the sin θ relation. If in a brake of this type $A = D$, α = approximately

90°, and the amount of brake lining wear between adjustments is either 0.030 inches or 0.080 inches at $\theta = 90°$, the necessary cam movement is as follows:

$$x = \Delta R \cdot \frac{A \sin (90) + D}{A \sin (90)} = \Delta R \cdot \frac{2A}{A} = 2\Delta R_{90}$$

$$\Delta R_{90} = (.020 + .030) \text{ or } (.020 + .080)$$

Thus: $x = 2 \cdot (.050) = 0.100 = 1/10$ of an inch

or $x = 2 \cdot (.100) = 0.200 = 1/5$ of an inch

This is a rough check on the previously stated figures for cam movement magnitude.

FORCE MULTIPLICATION RATIO - CAM TO DRUM

This portion of the analysis must be presented in several parts, one part for each distinctly different type of shoe anchoring method. The first and simplest type to be considered is the pin-anchored shoe. For its analysis Figure 19 will be used, and the length of the lining which contacts the drum will be limited to a very short element at point M. This lining will press against the moving drum with a force P, and will generate a frictional force fP, where f is the coefficient of friction between lining and drum. The pin-anchored shoe can be either a "forward" shoe or a "reverse" shoe. A "forward" shoe is one in which the drum rotates past the shoe toward the anchor pin. A "reverse" shoe is one in which the drum rotates past the shoe away from the anchor pin. The action of the two shoes is appreciably different.

By taking moments about the anchor pin of all of the forces acting on the shoe we can determine the ratio $\frac{P}{C}$ and then $\frac{fP}{C}$, which is the force multiplication ratio between drag force and cam force.

Forward Shoe:

$$C\ (D + A \sin \alpha) - PA \sin \theta + fP\ (R + A \cos \theta) = 0$$

$$C\ (D + A \sin \alpha) = P \quad A \sin \theta - f(R + A \cos \theta)$$

$$\frac{P}{C} = \frac{D + A \sin \alpha}{A \sin \theta - f\ (R + A \cos \theta)}$$

$$\frac{fP}{C} = \frac{f\ (D + A \sin \alpha)}{A \sin \theta - f\ (R + A \cos \theta)}$$

The numerator of this equation must always be finite in any practical brake (both A and D could not simultaneously be reduced to zero), but the denominator could be made to approach zero by a high value of f, or by reducing either A or Θ or both. As the denominator approaches zero the ratio $\frac{fP}{C}$ approaches ∞ , and the brake will become self-locking, with the shoe needing only to be touched to the drum to cause the brake immediately to stop all relative drum to shoe movement, and to stay that way until the drum rotation is reversed.

Reverse Shoe:

$$C\ (D + A \sin \alpha) - PA \sin \Theta - fP\ (R + A \cos \Theta) = 0$$

This equation is identical with the previous one except for an interchange of + and - signs on the last term. It would yield the following equation:

$$\frac{fP}{C} = \frac{f\ (D + A\sin \alpha)}{A \sin \Theta + f(R + A \cos \Theta)}$$

The denominator of this equation cannot go to zero for any finite value of R, and a reverse shoe is never in danger of becoming self-locking.

The foregoing analysis applies only to shoes with an elemental length of lining. The practical brake must have a finite lining length (from Θ_1 at toe to Θ_2 at heel) and the analysis must be expanded to cover this situation. The moment equations must be set up as differential equations with Θ as the main variable, and then integrated between Θ_2 and Θ_1 to determine the proper relationships. Before this can be done the relationship between C and P must be more realistically established.

In a previous section the radial movement of a pin-anchored brake shoe relative to the drum was shown to vary as sin Θ. If the shoe were much more rigid than the drum, and if the lining would compress according to a linear stress-strain relationship then the pressure between lining and drum (and the force per unit of arc length of the constant width shoe) would also vary according to a sin θ law with the maximum force occurring at θ = 90°. However, the brake shoes are not infinitely rigid, and if lightly built they may deflect under cam and anchor forces to produce a pressure and force variation approximated by $\sin^n\theta$, with "n" varying from greater than one to less than one depending on shoe design and illustrated in Figure 20.

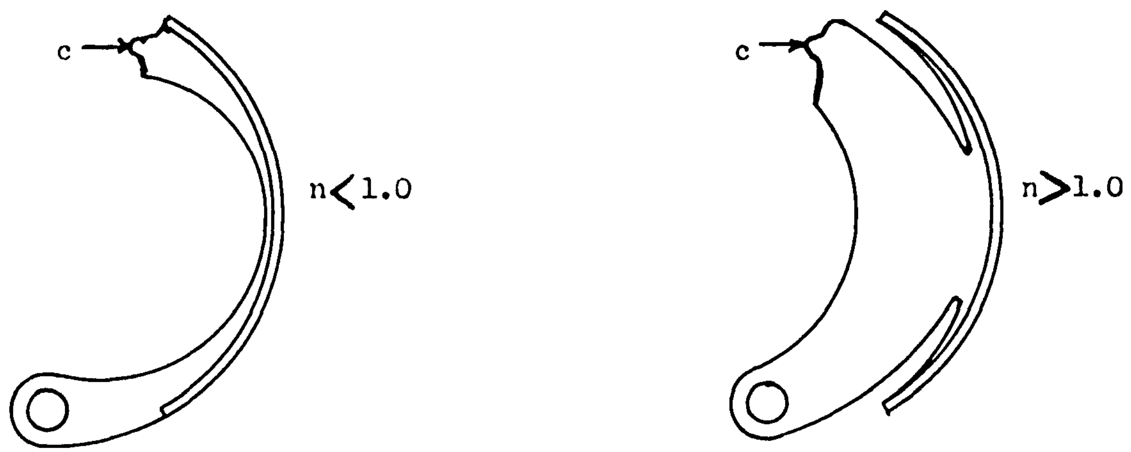


Figure 20. Brake Shoe Stiffness

Also, the maximum radial force location may shift from the 90° position, due to the design of the shoe, or to the interaction of cam, anchor, and friction forces. This is especially true in swinging-link or sliding-block anchored shoes. Present day brake shoes are sufficiently rigid so the pressure variation will follow a sine relation either side of the maximum pressure point. To make the analysis general the relationship may be expressed as $\cos^n(\theta_m - \theta)$ or $\cos^n(\theta - \theta_m)$. For properly designed and constructed shoes the value of "n" is practically 1.0, and this value will be used in the following developments to avoid unnecessary mathematical complications. When the center of pressure moves far enough toward one end of the shoe to unload the other end, then the limits of integration should be taken as θ_1 to $(\theta_m + 90°)$ or $(\theta_m - 90°)$ to θ_2. The use of θ_1 and θ_2 as integration limits when either end of the shoe unloads would lead to fictitious results, as it is impossible to develop a negative pressure between shoe and drum.

To make this relation useful in the analysis, assume that the maximum lining to drum pressure at $\theta_m = p_m$, and the width of the lining is b. Then the area of lining in the arc $d\theta$ is equal to $bRd\theta$, and the force dP on this area is:

$$dP = bRd\theta \cdot p_m \cos(\theta_m - \theta)$$

or $$dP = p_m bR \cos(\theta_m - \theta)d\theta$$

Returning now to Figure 19 and the moment equations for the forward and reverse shoes, but using differentials in place of the variables C, P, and θ, the result is:

<u>Forward Shoe</u>: $$dC = dP\left[\frac{A \sin\theta - f(R + A\cos\theta)}{D + A\sin\alpha}\right]$$

<u>Reverse Shoe</u>: $$dC = dP\left[\frac{A \sin\theta + f(R + A\cos\theta)}{D + A\sin\alpha}\right]$$

Now replace dP by its equivalent equation above, simplify, and integrate between limits of θ_2 and θ_1:

<u>Forward Shoe</u>:

$$dC = p_m bR \cos(\theta_m - \theta)\, d\theta\left[\frac{A \sin\theta - f(R + A\cos\theta)}{D + A\sin\alpha}\right]$$

$$\cos(\theta_m - \theta) = \cos\theta_m \cos\theta + \sin\theta_m \sin\theta$$

$$dC = \frac{p_m bR}{D + A \sin \alpha} \Big[A \sin \theta \cos \theta_m \cos \theta + A \sin^2 \theta \sin \theta_m - fR \cos \theta_m \cos \theta$$

$$- fR \sin \theta_m \sin \theta - fA \cos^2 \theta \cos \theta_m - fA \cos \theta \sin \theta_m \sin \theta \Big] d\theta$$

$$C = \int dC = \frac{p_m bR}{D + A \sin \alpha} \int_{\theta_1}^{\theta_2} \Big[\sin \theta \cos \theta \, (A \cos \theta_m - fA \sin \theta_m)$$

$$+ (A \sin \theta_m) \sin^2 \theta - (fA \cos \theta_m) \cos^2 \theta - (fR \cos \theta_m) \cos \theta$$

$$- (fR \sin \theta_m) \sin \theta \Big] d\theta$$

This may be integrated, treating θ_m as a constant.

$$C = \frac{p_m bR}{D + A \sin \alpha} \Big[(A \cos \theta_m - fA \sin \theta_m) \frac{\sin^2 \theta}{2} + (A \sin \theta_m) \, 1/2(\theta - \sin \theta \cos \theta)$$

$$- (fA \cos \theta) \, 1/2 \, (\theta + \sin \theta \cos \theta) - fR \cos \theta_m) \sin \theta$$

$$- (fR \sin \theta_m)(- \cos \theta) \Big]_{\theta_1}^{\theta_2}$$

Evaluating between θ_2 and θ_1, there appears:

$$C = \frac{p_m bR}{D + A \sin \alpha} \Big[\frac{A}{2}(\cos \theta_m - f \sin \theta_m)(\sin^2 \theta_2 - \sin^2 \theta_1)$$

$$+ (A \sin \theta_m) \, 1/2(\theta_2 - \theta_1 - \sin \theta_2 \cos \theta_2 + \sin \theta_1 \cos \theta_1)$$

$$- (fA \cos \theta_m) \, 1/2 \, (\theta_2 - \theta_1 + \sin \theta_2 \cos \theta_2 - \sin \theta_1 \cos \theta_1)$$

$$- (fR \cos \theta_m)(\sin \theta_2 - \sin \theta_1) - (fR \sin \theta_m)(- \cos \theta_2 + \cos \theta_1) \Big]$$

Since the desired quantity is the ratio $\frac{P}{C}$ of $\frac{fP}{C}$, P must be evaluated between θ_2 and θ_1 and then the two quantities simplified to a useable relation.

$$P = \int_{\theta_1}^{\theta_2} dP = p_m bR \int_{\theta_1}^{\theta_2} \cos\,(\theta_m - \theta) d\,\theta$$

$$P = p_m bR \int_{\theta_1}^{\theta_2} \left[(\cos \theta_m \cos \theta + \sin \theta_m \sin \theta) \right] d\theta$$

$$P = p_m bR \left[\cos \theta_m \sin \theta - \sin \theta_m \sin \theta \right]_{\theta_1}^{\theta_2}$$

$$P = p_m bR \left[\cos \theta_m (\sin \theta_2 - \sin \theta_1) - \sin \theta_m (\cos \theta_2 - \cos \theta_1) \right]$$

In a pin-anchored shoe which is functioning in its normal manner the maximum pressure occurs at $\theta_m = 90°$. Then $\cos \theta_m = 0$ and $\sin \theta_m = 1.0$. Using these values the ratio of P to C develops as follows:

$$\frac{P}{C} = \frac{p_m bR}{\dfrac{p_m bR}{D + A \sin \alpha}} \quad \frac{\cos \theta_1 - \cos \theta_2}{\frac{A}{2}(\theta_2 - \theta_1) + \frac{A}{4}(\sin 2\theta_1 - \sin 2\theta_2) - f\left[R(\cos \theta_1 - \cos \theta_2) + \frac{A}{2}(\sin^2 \theta_2 - \sin^2 \theta_1) \right]}$$

$$\frac{P}{C} = \frac{(D + A \sin \alpha)\ (\cos \theta_1 - \cos \theta_2)}{\frac{A}{2}(\theta_2 - \theta_1) + \frac{A}{4}(\sin 2\theta_1 - \sin 2\theta_2) - f\left[R(\cos \theta_1 - \cos \theta_2) + \frac{A}{4}(\cos 2\theta_1 - \cos 2\theta_2) \right]}$$

This relationship must be investigated for indications of self-locking tendency ($\frac{P}{C} \rightarrow \infty$). The numerator cannot approach zero for any finite length of lining, nor can it exceed R as a maximum value. The denominator can approach zero if either A is made too small, or f is made too large, or θ_1 is made too small. To show the effect of these variables, typical values for a brake of this type will be used and the effect of each one determined.

A typical two-shoe, hydraulically-actuated brake using pin-anchored shoes shows the following dimensions: $R = 5.5"$, $A = 4.00"$, $D = 4.125"$, $\alpha = 110.4°$, $\theta_1 = 42°$, $\theta_2 = 158°$. The simultaneous effect of f and θ_1 can be obtained by setting the denominator of the $\frac{P}{C}$ ratio equal to zero, and finding the value of f which makes it true for various values of θ_1. This procedure produces the relations

$$f = \frac{\frac{A}{2}(\theta_2 - \theta_1) + \frac{A}{4}(\sin 2\theta_1 - \sin 2\theta_2)}{R(\cos\theta_1 - \cos\theta_2) + \frac{A}{4}(\cos 2\theta_1 - \cos 2\theta_2)}$$

If this is evaluated, using the values of R, A, D, α, and θ_2 (lining placed at values of θ_2 in excess of 150° has very little braking effect, and θ_2 for a pin-anchored shoe is usually between 150° and 160°) given in this paragraph, the result obtained is shown in Figure 21. The extensive numerical tabulation necessary for this solution is omitted.

Figure 21 indicates the value of friction coefficient between lining and drum necessary to cause self-locking of this brake shoe for any given value of θ_1 between zero and forty five degrees. If the highest probable value of f is known, the necessary minimum value of θ_1 for safety from self-locking tendency can be chosen from this figure. In practice the value of f usually lies between 0.2 and 0.4, so the brake cited as an example would be safe from self-lock with any normal value of θ_1. Occasionally however, after periods of disuse in very humid conditions the surface of the brake drum will become roughened by rusting and raise the value of f temporarily to very high values, leading

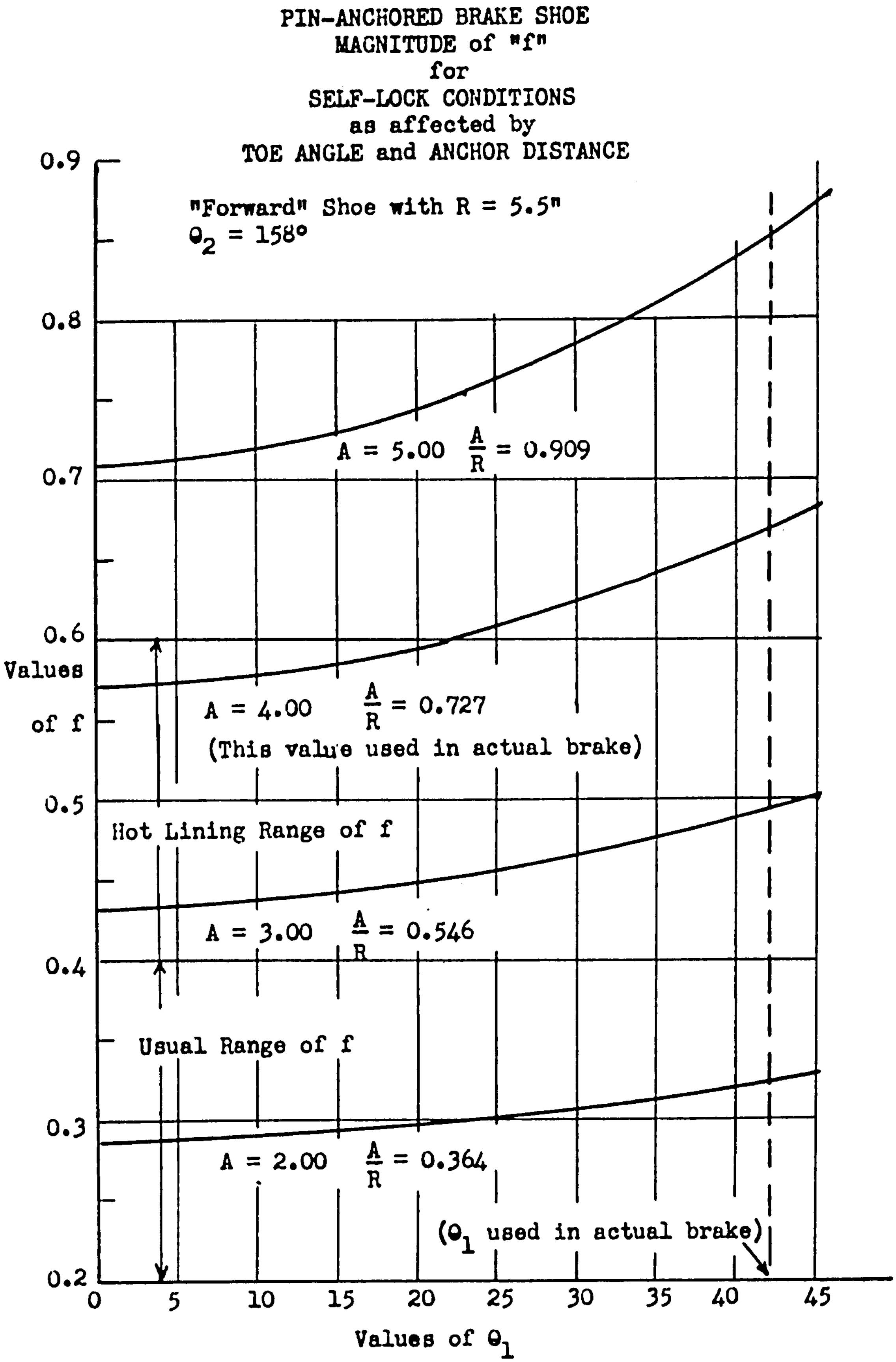

PIN-ANCHORED BRAKE SHOE
MAGNITUDE of "f"
for
SELF-LOCK CONDITIONS
as affected by
TOE ANGLE and ANCHOR DISTANCE
"Forward" Shoe with R = 5.5"
Θ2 = 158°
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
Values of f
A = 5.00 A/R = 0.909
A = 4.00 A/R = 0.727
(This value used in actual brake)
Hot Lining Range of f
A = 3.00 A/R = 0.546
Usual Range of f
A = 2.00 A/R = 0.364
(Θ1 used in actual brake)
0
5
10
15
20
25
30
35
40
45
Values of Θ1

Figure 21

to self-locking conditions on initial brake application. The value of θ_1 (42°) chosen for the example brake shoe would assure freedom from self-lock for any value of f up to 0.670.

The same analysis for several values of A shows the effect of anchor distance upon the tendency towards self-locking. Increasing the value of A relative to R reduces the tendency, while decreasing the value of A (moving the anchor point towards the drum center) increases the tendency to the point where self-lock would occur even with normal values of f and θ_1.

The previous analysis was made for a forward shoe only, but careful inspection of the differential equations for <u>Forward Shoe</u> and <u>Reverse Shoe</u> on page 107 will show that the two are identical with the exception of an interchange of plus and minus signs on one term. The final $\frac{P}{C}$ equation for the forward shoe can then be converted into the proper equation for the reverse shoe by interchanging plus and minus signs on all terms containing f. Thus for a <u>Reverse Shoe</u>:

$$\frac{P}{C} = \frac{(D + A \sin \alpha)\ (\cos \theta_1 - \cos \theta_2)}{\frac{A}{2}(\theta_2 - \theta_1) + \frac{A}{4}(\sin 2\theta_1 - \sin 2\theta_2) + f \left[R(\cos \theta_1 - \cos \theta_2) + \frac{A}{4}(\cos 2\theta_1 - \cos 2\theta \right.}$$

Because of this interchange of sign the <u>Reverse Shoe</u> cannot be self-locking for any positive value of A, f, and θ_1. The length of lining on a reverse shoe is then determined only by manufacturing or service reasons.

DRAG FORCE RELATIONS - PIN-ANCHORED SHOES

The ratio $\frac{fP}{C}$ is the Force Multiplication Ratio between Cam Force and Drum Drag Force. A high value of this ratio is desirable, if it can be obtained without danger of self-lock conditions and without too greatly reducing the degree of control exercised by the driver over the magnitude of the braking force produced. The final choice of A, f, and θ_1 will be made to produce the best compromise between high force multiplication and good control. The values of A, D, α, θ_1 and θ_2 cited on page 110 represent the choice of one manufacturer, and will be used to illustrate the magnitude of the force multiplication ratios which may be produced by pin-anchored brakes.

Figure 22 illustrates the variation of $\frac{fP}{C}$ which is obtained for both Forward Shoes and Reverse Shoes as the coefficient of friction f is varied. The results shown are obtained by a numerical evaluation of the $\frac{P}{C}$ relations just derived, but the extensive numerical work is omitted. For any except the lowest values of f the Forward Shoe is much more effective than the Reverse Shoe, and in a brake containing one forward and one reverse shoe it will do the major portion of the braking work, and suffer the major portion of lining wear. To improve the effectiveness of the Reverse Shoe and to even up the rate of wear of the lining on the two shoes the reverse shoe is often provided with a short lining, concentrated towards its anchor end. On the brake cited as an example the value of θ_1 for the reverse shoe was 78.8°, and the effectiveness of the short lining was slightly better than that of the

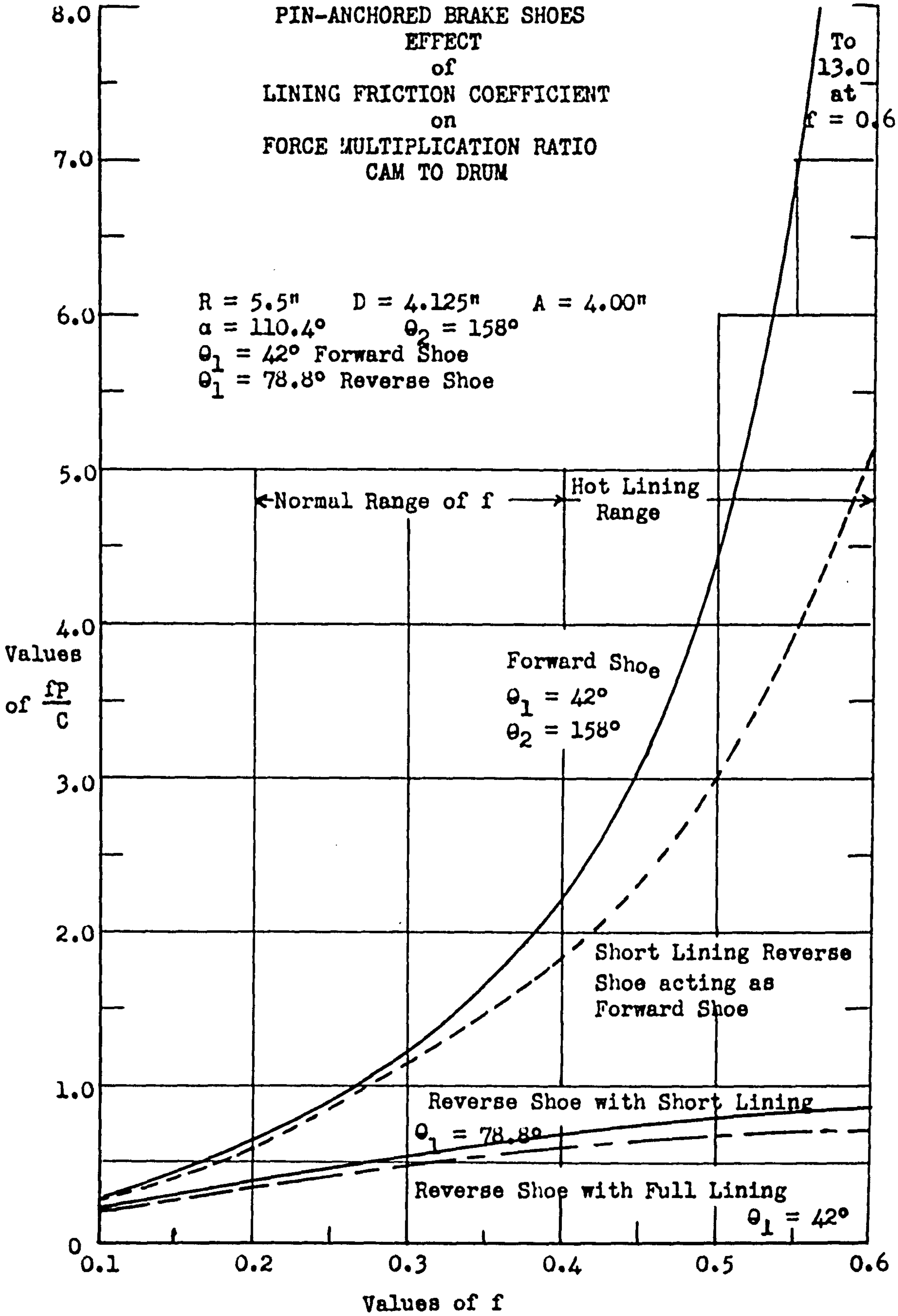

PIN-ANCHORED BRAKE SHOES
EFFECT
of
LINING FRICTION COEFFICIENT
on
FORCE MULTIPLICATION RATIO
CAM TO DRUM
To 13.0 at f = 0.6
R = 5.5" D = 4.125" A = 4.00"
α = 110.4° θ2 = 158°
θ1 = 42° Forward Shoe
θ1 = 78.8° Reverse Shoe
Normal Range of f
Hot Lining Range
Forward Shoe
θ1 = 42°
θ2 = 158°
Short Lining Reverse Shoe acting as Forward Shoe
Reverse Shoe with Short Lining
θ1 = 78.8°
Reverse Shoe with Full Lining
θ1 = 42°
Values of fP/C
Values of f
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
0.1
0.2
0.3
0.4
0.5
0.6

Figure 22

full lining, as shown on Figure 22. The length of this lining and its position on the shoe relative to the anchor pin must be chosen with due regard to reverse rotation of the brake drum, during which the reverse shoe becomes a forward shoe, and might have so high a value of $\frac{fP}{C}$ as to be savage in its action. Figure 22 shows the relative $\frac{fP}{C}$ value obtained by the short-lining shoe when acting as a forward shoe. The value is not unduly high.

In practice a brake consisting of one forward and one reverse pin-anchored shoe often has the ability of the reverse shoe improved by providing a larger hydraulic brake piston to operate the reverse shoe than to operate the forward shoe. In the brake cited as an example the reverse shoe piston had an area 1.21 times as great as the forward shoe piston, improving the braking ability of the reverse shoe.

The two-shoe brake can be made more effective in one direction of travel by making both shoes forward shoes. This method is used for front-wheel brakes on many passenger cars, with the rear-wheel brakes being a forward-reverse shoe combination. The two-forward-shoe brake also is a "balanced" brake, since the radial forces produced by one shoe are substantially balanced by those of the other shoe, leaving no unbalanced radial load which must be borne by the wheel bearings, as is the case in the forward and reverse shoe brake.

If the Force Multiplication Ratio of fifty produced by the <u>Brake Rigging</u> is now combined with the Force Multiplication Ratio produced by the <u>Brake Shoes</u>, the total ability of this type of brake can be evaluated for comparison with other brake types. For this comparison a value of f = 0.35 is used for all brakes, and the total braking effort at the drum surface produced by a net pedal force of one hundred pounds is determined. Four wheel brakes are assumed, with sixty percent of the total braking force developed at the front wheels.

Two different braking combinations using pin-anchored shoes are evaluated. Combination A uses two forward shoes in each front wheel brake and one forward and one reverse shoe in each rear wheel brake. Combination B uses one forward and one reverse shoe in each of the four brakes. In all cases the reverse shoe has a short lining, and its actuating piston has an area that is 1.21 times as large as the area of the piston actuating the forward shoe. The two combinations are analyzed in the same way, with the method of analysis shown for combination A, and the results only given for combination B.

Combination A

Total Cam Force = four front wheel forward-shoe cams
+ two rear wheel forward-shoe cams
+ two rear wheel reverse-shoe cams

Total Cam Force = $4C_{FA} + 2C_{RA} + 2(1.21)C_{RA}$

$= 4C_{FA} + 4.42C_{RA}$

Total Cam Force = 100# Net Pedal Force x 50 (Brake-Rigging Force Multiplication Ratio) = 5000#

$4C_{FA} + 4.42C_{RA} = 5000$

$\frac{fP}{C}$ for Forward Shoe at $f = 0.35 = \underline{1.66}$

$\frac{fP}{C}$ for Reverse Shoe at $f = 0.35 = \underline{0.62}$

Total Brake Force = $4C_{FA}(1.66) + 2C_{RA}(1.66) + 2.42C_{RA}(0.62)$

$= 6.64C_{FA} + 4.82C_{RA}$

To obtain sixty percent of the total brake force on the front wheels the following relation must be true:

$$6.64C_{FA} = 0.6\ (6.64C_{FA} + 4.82C_{RA})$$

or: $2.66C_{FA} - 2.89C_{RA} = 0$

This provides two equations in C_{FA} and C_{RA}. Their solution produces the following values for C_{FA} and C_{RA}:

$$8.07C_{FA} = 5000\#$$

$C_{FA} = 620\#$ $\qquad\qquad$ $C_{RA} = 571\#$

Total Braking Effort = $4120_{Front} + 2750_{Rear} = \underline{6870\#}_{Combination\ A}$

A similar solution for Combination B produces the following:

Total Braking Effort = $3000_{Front} + 2000_{Rear} = \underline{5000\#}_{Combination\ B}$

This analysis shows the advantage gained by the use of the two-forward-shoe brake for forward travel. This brake suffers some disadvantage for reverse travel, but reverse-travel braking occurs so seldom it does not require much consideration.

There are also available automatic brake-shoe-clearance adjusters for pin-anchored shoe brakes. If these are used, no wear allowance need be made in designing the brake rigging, and a larger force multiplication ratio may be built into the rigging for the same pedal travel, or a shorter pedal travel obtained with the same force multiplication ratio.

BRAKE SHOES WITH ARTICULATED-LINK ANCHORS

If the method of anchoring the brake shoe to the brake backing plate is changed from a rigid pin anchor by placing between the shoe and the anchor pin a short rigid link free to swing about both the anchor pin and a pin in the shoe web, the action of the brake shoe is changed materially (16). If a single shoe retracting spring at the toe end of the shoe is to be able to withdraw the entire shoe from contact with the drum there must be a minimum amount of frictional resistance to relative shoe-link motion at their pin joint. This resistance must not be so great that it prevents some adjustment of the shoe-link angle after the shoe contacts the drum.

The brake lining at the heel of a rigid pin-anchored shoe is not very effective, since the anchoring conditions prevent it from swinging over to make heavy contact with the drum. The use of a swinging pin-jointed link between the shoe and anchor pin changes this condition, since that component of the friction force between shoe and drum which is perpendicular to the link axis will cause the heel end of the shoe to swing over against the shoe until sufficient radial force is generated at the heel to balance this swinging tendency. This makes more effective use of the lining at the heel and makes the shoe partially self-centering in the drum.

The theoretical analysis of this link-anchored brake is similar to that of the pin-anchored shoe, but is complicated by the fact that the point of maximum pressure between shoe and drum is no longer constrained by anchor forces to remain at $\Theta_m = 90°$, but can shift back and forth along the shoe until an equilibrium is established among cam force, link force, radial force, and drag force. The angle to the point of maximum pressure is now a variable, a function of all of the physical dimensions of the shoe, and especially of the friction coefficient between shoe lining and drum. If this angle is determined as a function of f, for any given combination of shoe dimensions, then the force multiplication ratio produced between cam force and drum drag force can be determined.

The theoretical analysis is based on the physical relations shown in Figure 23. Since the shoe is free to swing about the anchor pin at a radius equal to the link length, a summation of forces acting on the shoe, perpendicular to the link axis, must be equal to zero. The relation thus obtained, plus another obtained by taking a summation of moments about the anchor pin, produce a pair of equations containing all of the dimensional factors in the brake shoe assembly, and can be solved to obtain the desired $\frac{fP}{C}$ ratio.

Referring to Figure 23, and based on the Forward Shoe, the following relations are true:

Summation of Forces perpendicular to link axis:

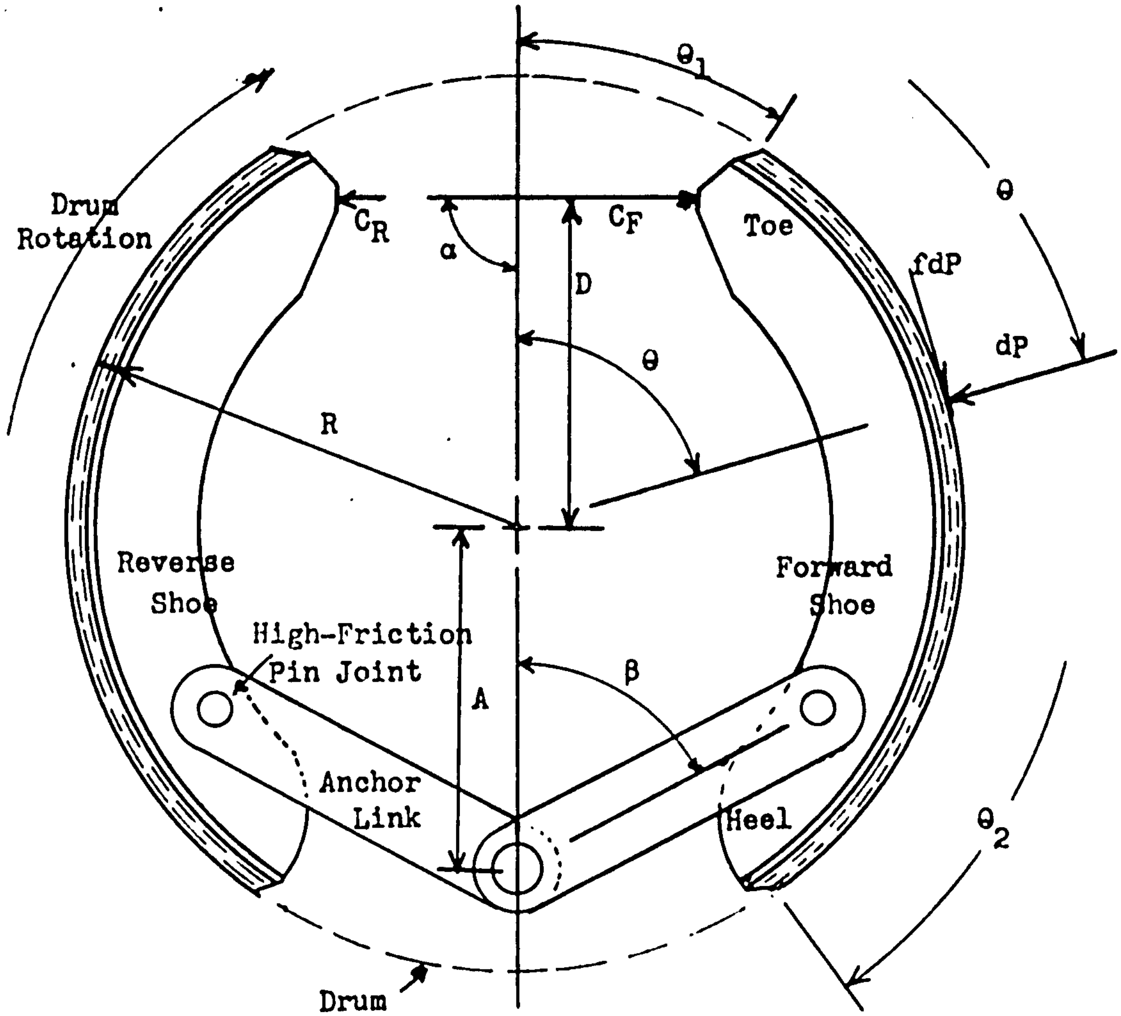
$$0 = C \sin(\alpha - \beta) - dP \cdot \sin(\Theta - \beta) + fdP \cos(\Theta - \beta)$$

Summation of Moments about the anchor pin (clockwise positive):

$$0 = C (D + A \sin \alpha) - dP \cdot A \sin \Theta + f \cdot dP \cdot (R + A \cos \Theta)$$

As in the previous analysis the relation for the differential dP is:

$$dP = p_m bR \cos(\Theta_m - \Theta)\, d\Theta$$



BRAKE SHOES WITH AN ARTICULATED-LINK ANCHOR

θ = Angle from Anchor Diameter to Point on Brake Shoe
θ_1 = Angle to Toe of Brake Shoe Lining
θ_2 = Angle to Heel of Brake Shoe Lining
R = Radius of Brake Shoe Lining Surface and of Brake Drum Surface
A = Anchor Distance
D = Cam Force Distance
C = Cam Force
α = Angle from Anchor Diameter to Cam Force
β = Angle from Anchor Diameter to Link Center Line
dP = Incremental Radial Force at θ
f = Coefficient of Friction - Lining to Drum Surface
fdP = Incremental Drag Force on Drum Surface
L = Link Force (Anchor Force)

Figure 23

If this relation is substituted for dP in the moment equation, and the indicated integration is performed the following results are obtained:

$$-\int dP \cdot A\sin\theta = -p_m bRA\int_{\theta_1}^{\theta_2}\cos(\theta_m-\theta)\sin\theta\cdot d\theta$$

$$= -p_m bRA\int_{\theta_1}^{\theta_2}(\cos\theta_m\cos\theta+\sin\theta_m\sin\theta)\sin\theta\cdot d\theta$$

$$= -p_m bR\left[\frac{A}{4}\cos\theta_m(\cos 2\theta_1-\cos 2\theta_2)+\frac{A}{4}\sin\theta_m(\sin 2\theta_1-\sin 2\theta_2)\right.$$
$$\left.+\frac{A}{2}\sin\theta_m(\theta_2-\theta_1)\right]$$

$$f dP\cdot(R+A\cos\theta) = fp_m bR\int_{\theta_1}^{\theta_2}(\cos\theta_m\cos\theta+\sin\theta_m\sin\theta)(R+A\cos\theta)d\theta$$

$$= fp_m bR\left[\frac{A}{4}\sin\theta_m(\cos 2\theta_1-\cos 2\theta_2)+\frac{A}{4}\cos\theta_m(\sin 2\theta_1-\sin 2\theta_2)\right.$$
$$+\frac{A}{2}\cos\theta_m(\theta_2-\theta_1)+R\cos\theta_m(\sin\theta_2-\sin\theta_1)-R\sin\theta_m$$
$$\left.(\cos\theta_2-\cos\theta_1)\right]$$

If these elements of the moment equation are now combined into the full relation, and this relation is solved for the cam force C, the following equation is obtained:

$$C = \frac{p_m bR}{D+A\sin\alpha}\left[\text{Moment Bracket}\right]$$, in which the Moment Bracket represents the following group of terms:

$$\left[-\frac{fA}{4}\sin\theta_m(\cos 2\theta_1-\cos 2\theta_2)+\frac{fA}{4}\cos\theta_m(\sin 2\theta_1-\sin 2\theta_2)\right.$$
$$-\frac{fA}{2}\cos\theta_m(\theta_2-\theta_1)-fR\cos\theta_m(\sin\theta_2-\sin\theta_1)+fR\sin\theta_m$$

$$(\cos \theta_2 - \cos \theta_1) + \frac{A}{4} \cos \theta_m(\cos 2\theta_1 - \cos 2\theta_2) + \frac{A}{4} \sin \theta_m$$

$$\left.(\sin 2\theta_1 - \sin 2\theta_2) + \frac{A}{2} \sin \theta_m(\theta_2 - \theta_1)\right] = \underline{\text{Moment Bracket}}$$

The perpendicular component equation is now treated in the same way.

$$C \sin (\alpha - \beta) = p_m bR \int_{\theta_1}^{\theta_2} \sin (\theta - \beta) \cos (\theta_m - \theta) d\theta - f p_m bR \int_{\theta_1}^{\theta_2} \cos (\theta - \beta) \cos (\theta_m - \theta) d\theta$$

$$\int_{\theta_1}^{\theta_2} \sin (\theta - \beta) \cos (\theta_m - \theta) d\theta = \int_{\theta_1}^{\theta_2} (\sin \theta \cos \beta - \cos \theta \sin \beta)(\cos \theta_m \cos \theta$$

$$+ \sin \theta_m \sin \theta) d\theta = \int_{\theta_1}^{\theta_2} (\cos \theta_m \cos \beta \cos \theta \sin \theta + \sin \theta_m \cos \beta \sin^2\theta$$

$$- \cos \theta_m \sin \beta \cos^2\theta - \sin \theta_m \sin \beta \cos \theta \sin \theta) d\theta$$

$$= \Big[(\cos \theta_m \cos \beta - \sin \theta_m \sin \beta) \frac{1}{4}(\cos 2\theta_1 - \cos 2\theta_2) + (\sin \theta_m \cos \beta$$

$$+ \cos \theta_m \sin \beta) \frac{1}{4}(\sin 2\theta_1 - \sin 2\theta_2) + (\sin \theta_m \cos \beta - \cos \theta_m \sin \beta)$$

$$\left.\frac{1}{2}(\theta_2 - \theta_1)\right] = \underline{\text{Radial Bracket}}$$

This bracket will be used in subsequent relations as the "Radial Bracket".

$\int_{\theta_1}^{\theta_2} \cos (\theta - \beta) \cos (\theta - \theta)\, d\theta$ treated in the same way produces the following:

$$= \Big[(\sin \theta_m \cos \beta + \cos \theta_m \sin \beta) \frac{1}{4}(\cos 2\theta_1 - \cos 2\theta_2) - (\cos \theta_m \cos \beta -$$

$$\sin \theta_m \sin \beta) \frac{1}{4}(\sin 2\theta_1 - \sin 2\theta_2) + (\cos \theta_m \cos \beta + \sin \theta_m \sin \beta)$$

$$\left.\frac{1}{2}(\theta_2 - \theta_1)\right] = \underline{\text{Tangential Bracket}}$$

This bracket will be used in subsequent relations as the "Tangential Bracket". The perpendicular component equation then produces the following relation for cam force C:

$$C = \frac{p_M bR}{\sin(\alpha - \beta)} \left[\text{Radial Bracket} - f \text{ Tangential Bracket} \right]$$

These two relations for C were developed from the same free body diagram and must equal one another. Equating them and solving for f provides a means of determining the mutual relationship between f and θ_m, which must be known before the $\frac{fp}{C}$ relation can be determined. Equating these two relations for C produces the following equation:

$$\sin(\alpha - \beta) \left[\text{Moment Bracket} \right] = (D + A \sin a) \left[\text{Radial Bracket} - f \text{ Tangential Bracket} \right]$$

This equation contains all of the dimensional quantities involved in the analysis, and further evaluation can be greatly simplified by adopting representative values for the dimensions. To illustrate the effect of toe angle θ_1 and link angle β on the location of the maximum pressure point this equation is first evaluated for the following dimensions: R = 5.0", D = 4.0", A = 4.0", $\alpha = 90°$, $\theta_1 = 30°$ or $10°$, $\theta_2 = 150°$, $\beta = 45°$ or $60°$ or $70°$. This evaluation produces the following equations:

$\theta_1 - 30°$
$\theta_2 = 150°$
$\beta = 60°$

$$f = \frac{2.95 \sin \theta_m - 4.23 \cos \theta_m}{5.89 \sin \theta_m + 1.23 \cos \theta_m}$$

$\theta_1 = 10°$
$\theta_2 = 150°$
$\beta = 60°$

$$f = \frac{2.989 \sin \theta_m - 7.11 \cos \theta_m}{5.312 \sin \theta_m + 3.015 \cos \theta_m}$$

$\theta_1 = 30°$
$\theta_2 = 150°$
$\beta = 45°$

$$f = \frac{4.03 \sin \theta_m - 3.30 \cos \theta_m}{2.09 \sin \theta_m + 1.564 \cos \theta_m}$$

(To be used only when θ_m is over 60°)

$\theta_1 = 30°$
$\theta_2 = 150°$
$\beta = 70°$

$$f = \frac{2.03 \sin \theta_m - 4.60 \cos \theta_m}{8.14 \sin \theta_m + 8.42 \cos \theta_m}$$

The numerical solution of these equations is omitted, and the results of the solution shown on Figure 24 as a graph of θ_m vs f. Figure 24 shows pictorially how the center of pressure on the shoe shifts as the coefficient of friction changes. For a normal forward shoe the center of pressure stays near to the center of the shoe over the whole useable range of f, but for the forward shoe with $\beta = 45°$ and for the reverse shoe, the center of pressure moves so far toward the toe of the shoe that the heel of the shoe has no actual radial force acting upon it. Since no negative force between shoe and drum can be developed, and since our integration process for radial force assumes that the pressure between shoe and drum follows a definite cosine law, the limits of integration must here be reduced to cover only that portion

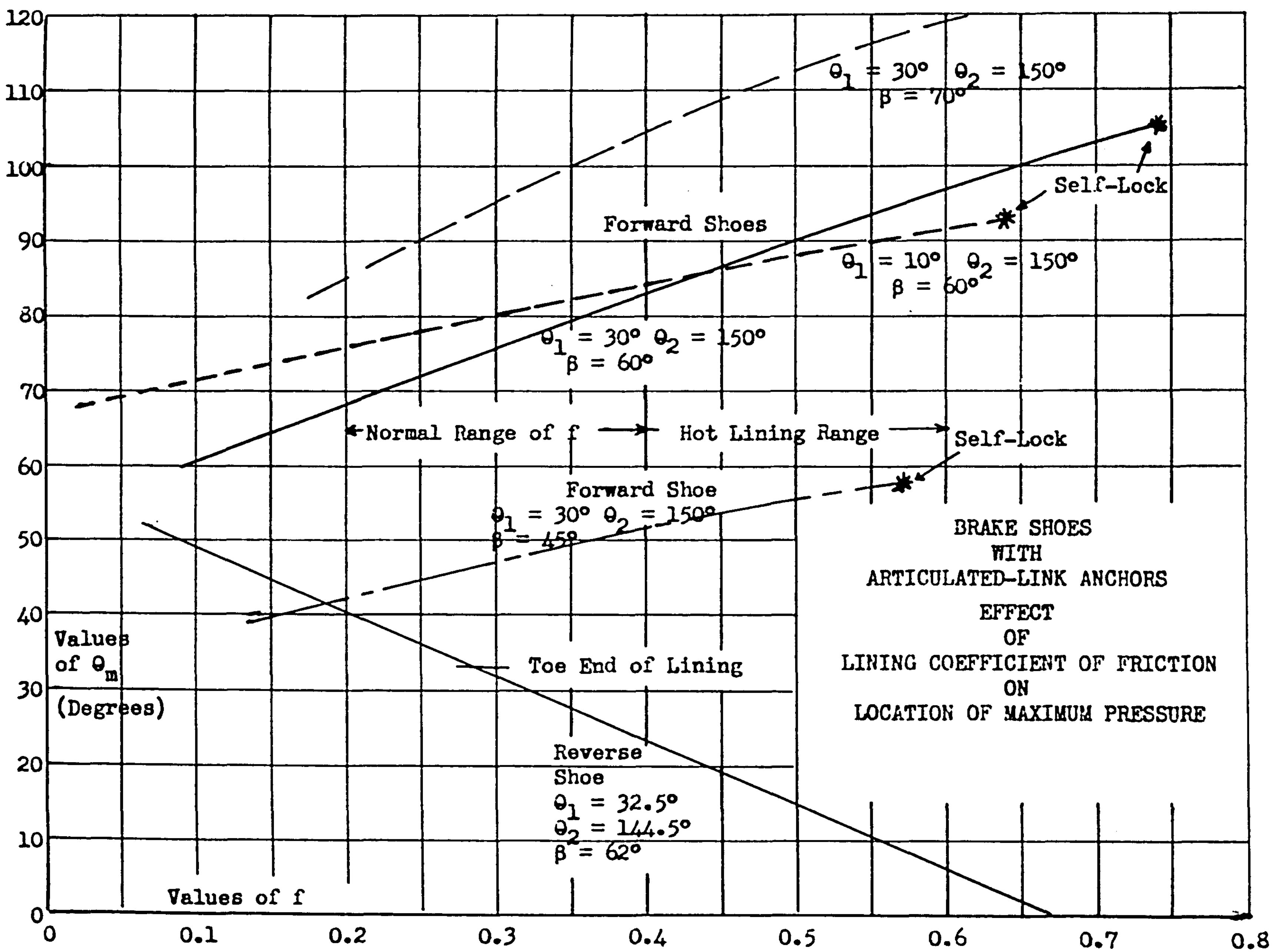

BRAKE SHOES
WITH
ARTICULATED-LINK ANCHORS
EFFECT
OF
LINING COEFFICIENT OF FRICTION
ON
LOCATION OF MAXIMUM PRESSURE
θ1 = 30° θ2 = 150°
β = 70°
Self-Lock
Forward Shoes
θ1 = 10° θ2 = 150°
β = 60°
θ1 = 30° θ2 = 150°
β = 60°
Normal Range of f
Hot Lining Range
Self-Lock
Forward Shoe
θ1 = 30° θ2 = 150°
β = 45°
Values
of θm
(Degrees)
Toe End of Lining
Reverse
Shoe
θ1 = 32.5°
θ2 = 144.5°
β = 62°
Values of f
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
10
20
30
40
50
60
70
80
90
100
110
120

Figure 24

of the shoe carrying actual load. The differential equations must then be integrated between limits of ($\theta_m + 90°$) and θ_1. Actually this does not change the resulting equations at all, but merely replaces θ_2 with $\theta_m + 90$. This complicates the evaluation of the integrals for the determination of f and $\frac{fP}{C}$ since each one must be treated as a special case.

For most effective utilization of the lining the center of pressure should occur at the center of the lining length. For lining angles symmetrical with the anchor diameter this would place the desired center of pressure position at 90°, which can be obtained in the normal range of f by choosing a link angle β slightly greater than 60°.

The use of an unusually long toe on the shoe ($\theta_1 = 10°$) has little effect on the center of pressure location, but does slightly increase the tendency of the brake toward self-lock, as shown by the values of f for self-lock for the normal shoe (f = 0.74) and the long toe shoe (f = 0.64). This agrees with the action of the pin-anchored shoe.

The effect of the link angle β on the center of pressure location is shown in the comparison of the three lines for β 45°, 60°, and 70°. High values of β load the heel of the shoe excessively, low values of β load the toe of the shoe excessively. Also, low values of β make the shoe more sensitive to self-lock. The actual value of β chosen for a shoe of this type should keep the center of pressure location near to the center of the lining to make the best use of the lining.

For comparison purposes the θ_m versus f curve for a reverse shoe of similar dimensions is shown. The center of pressure is either at or beyond the toe for any appreciable value of f, completely unloading the heel end of the shoe and preventing it from doing any work. This causes very ineffective use of the reverse shoe, and is a distinct disadvantage of this type of brake shoe anchor.

The effect of the Anchor Distance to Drum Radius ratio on the self-locking tendency of the brake is not specifically shown, but its effect would be the same as for the pin-anchored shoes. As the anchor distance is reduced the link angle must become greater, and the moment action of the friction component greater, until soon the brake would be self-locking in the normal range of f.

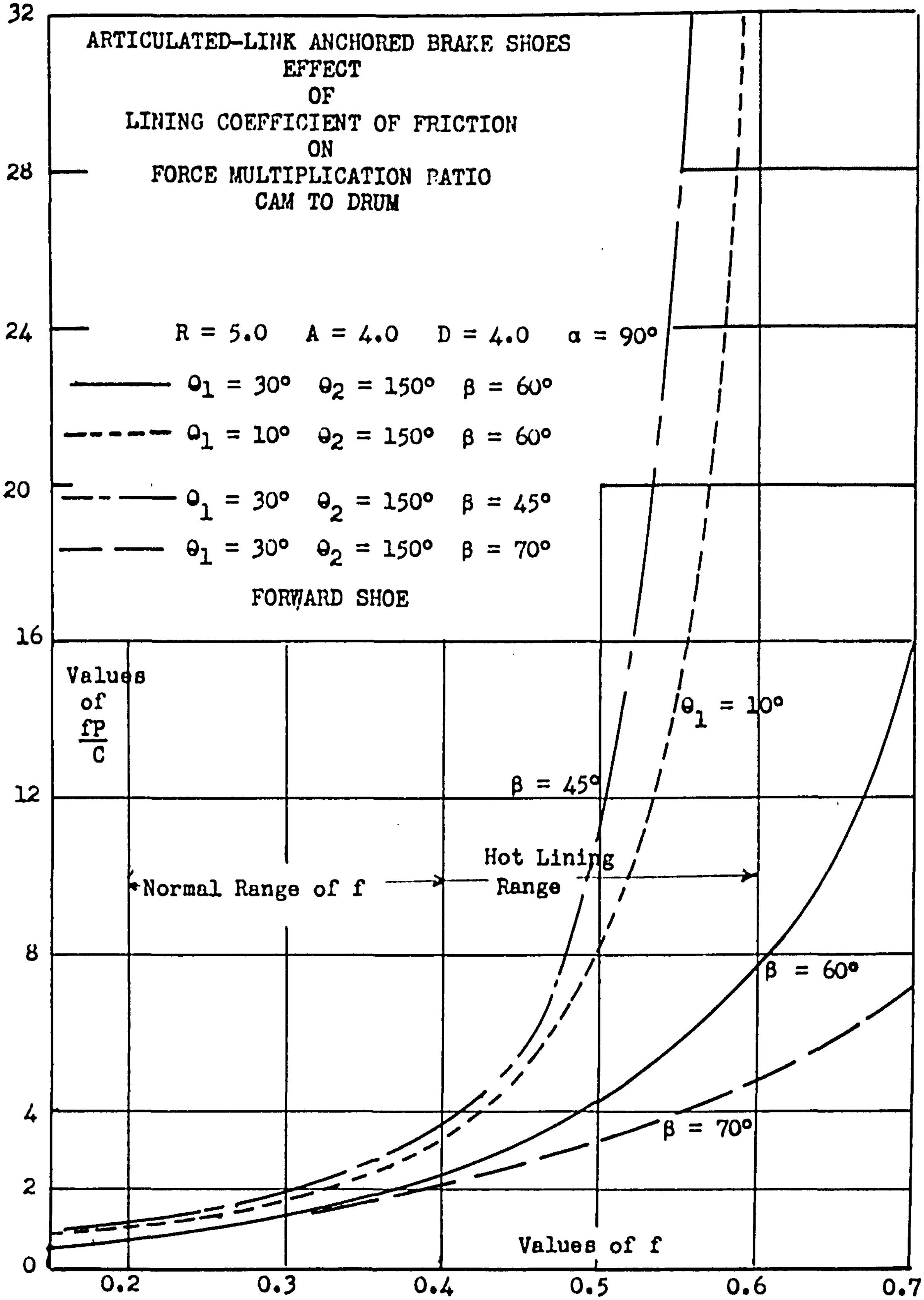
With the mutual relation for f and θ_m definitely established the desired ratio $\frac{fP}{C}$ can now be obtained. If the equation for dP is expanded and integrated from θ_1 to θ_2 the result is:

$$P = p_m bR \left[\cos \theta_m(\sin \theta_2 - \sin \theta_1) - \sin \theta_m(\cos \theta_2 - \cos \theta_1)\right]$$

This can be combined with either of the two relations for C previously determined to produce $\frac{fP}{C}$. For example:

$$\frac{fP}{C} = \frac{f\,(D + A \sin \alpha)\left[\cos \theta_m(\sin \theta_2 - \sin \theta_1) - \sin \theta_m(\cos \theta_2 - \cos \theta_1)\right]}{\left[\text{Moment Bracket}\right]}$$

This equation can now be evaluated for a particular brake shoe, using for f and θ_m the compatible values obtained from the previous equation of f and θ_m. The results of an evaluation of the $\frac{fP}{C}$ relation for the four forward brake shoes of Figure 24 are shown on Figure 25. These results are comparable with those obtained for the pin-anchored shoe, and are presented here as a group to show the effect of f on the ratio $\frac{fP}{C}$, and of β and θ_1 on the self-locking tendency of the shoe. Lowering the value of θ_1 by putting a long toe on the shoe noticeably lowers the value of f at which the $\frac{fP}{C}$ ratio increases to very high values. Low



ARTICULATED-LINK ANCHORED BRAKE SHOES
EFFECT
OF
LINING COEFFICIENT OF FRICTION
ON
FORCE MULTIPLICATION RATIO
CAM TO DRUM
R = 5.0 A = 4.0 D = 4.0 α = 90°
θ1 = 30° θ2 = 150° β = 60°
θ1 = 10° θ2 = 150° β = 60°
θ1 = 30° θ2 = 150° β = 45°
θ1 = 30° θ2 = 150° β = 70°
FORWARD SHOE
Values of fP/C
θ1 = 10°
β = 45°
Hot Lining Range
Normal Range of f
β = 60°
β = 70°
Values of f
32
28
24
20
16
12
8
4
2
0
0.2
0.3
0.4
0.5
0.6
0.7

Figure 25

values of β have the same effect. Any of these four shoes are sufficiently far from self-lock conditions to give adequate response control in the low range of f, but the long-toe shoe and the one with $\beta = 45°$ would be unpredictable in their action if anything (such as heat, dampness, drum roughness) caused the value of f to approach the value of 0.5. More reliable and predictable operation would be obtained from the normal shoes with $\beta = 60°$ to $70°$, with abnormal conditions leading to self-lock much less apt to occur.

For the purposes of comparison with a pin-anchored shoe brake, the braking ability of a link-anchored shoe brake of comparable dimensions is analyzed and presented on Figure 26 and in the accompanying discussion. The dimensions of a brake of this type in actual use are as follows:

$R = 5.5''$	$\alpha = 90°$	$\theta_1 = 32.5°$	Forward and Reverse Shoes are Identical
$A = 4.32''$	$\beta = 62°$	$\theta_2 = 144.5°$	
$D = 4.17''$			

The relation for f and θ_m for this brake is:

$$f = \frac{2.64 \sin \theta_m - 3.82 \cos \theta_m}{6.57 \sin \theta_m + 0.22 \cos \theta_m} \qquad \text{Forward Shoe}$$

For a value of $f = 0.35$ this produces a value of $\theta_m = 85°$, quite close to the middle of the lining. This should assure even lining wear.

A similar equation for the reverse shoe cannot be presented. The center of pressure is at or beyond the toe of the lining, and the complete equation:

$$\sin(\alpha - \beta)\left[\text{Moment Bracket}\right] = (D + A \sin \alpha)\left[\text{Radial Bracket} + f \cdot \text{Tangential Bracket}\right]$$

must be evaluated for successive values of θ_m to determine the f versus θ_m relation (shown on Figure 24). A value of f = 0.35 for the reverse shoe corresponds to $\theta_m = 27.5°$, which is five degrees off the end of the lining. Fortunately the reverse shoe is not worked as hard as the forward shoe, so this unsymmetrical loading should not lead to wear troubles of the reverse shoe before similar difficulty is imminent on the forward shoe.

The $\frac{fP}{C}$ ratios for these shoes have the following form:

Forward Shoe:

$$\frac{fP}{C} = \frac{8.49f\,(.0434 \cos \theta_m + 1.6575 \sin \theta_m)}{6.23 \sin \theta_m + 0.105 \cos \theta_m - f(9.23 \sin \theta_m + 2.469 \cos \theta_m}$$

Reverse Shoe:

$$\frac{fP}{C} = \frac{8.49f \left\{ \cos \theta_m \left[\sin (\theta_m + 90) - \sin \theta_1 \right] - \sin \theta_m \left[\cos (\theta_m + 90) - \cos \theta_1 \right] \right\}}{\left[\text{Moment Bracket, with f replaced by } (-f) \right]}$$

Figure 26 presents in graphic form the results of these two relations. For a value of f = 0.35, taken as the normal value, the forward shoe does over three times as much braking work as the reverse shoe. The values shown for these link-anchored shoes are substantially the same as those for the pin-anchored shoes. The forward shoe would become self-locking at a value of f = 0.71, a relatively high value.

As normally used these brakes have a forward-reverse shoe combination in each wheel, with the same size piston actuating each shoe. With f = 0.35, the value of $\frac{fP}{C}$ for the forward shoe is 1.68, for the reverse shoe 0.48. Using these values, and combining them with a 50 to 1 force multiplication ratio in the brake rigging and a 60% front to 40% rear

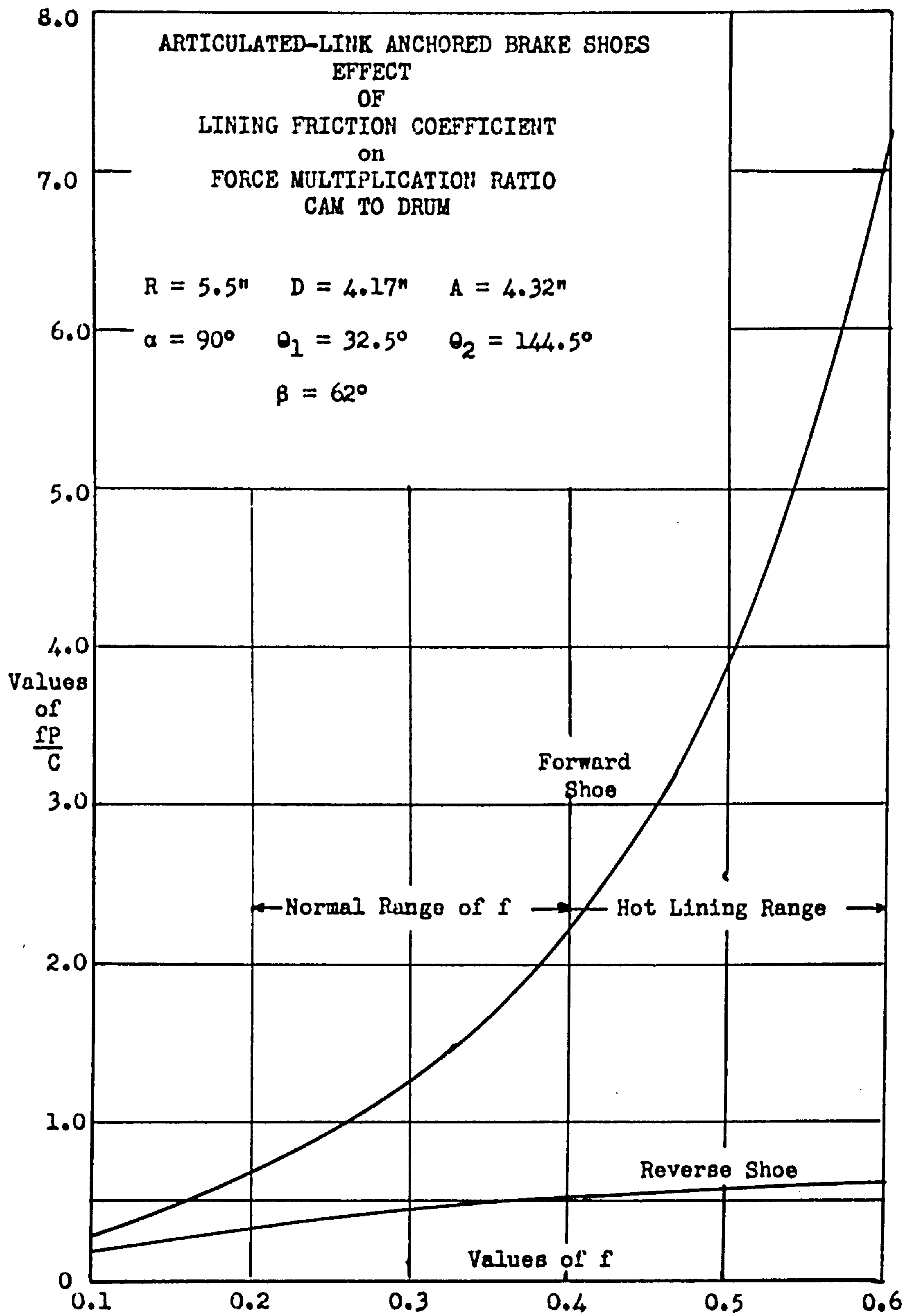

ARTICULATED-LINK ANCHORED BRAKE SHOES
EFFECT
OF
LINING FRICTION COEFFICIENT
on
FORCE MULTIPLICATION RATIO
CAM TO DRUM
R = 5.5" D = 4.17" A = 4.32"
α = 90° θ1 = 32.5° θ2 = 144.5°
β = 62°
Values of fP/C
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
Forward Shoe
Normal Range of f
Hot Lining Range
Reverse Shoe
Values of f
0.1
0.2
0.3
0.4
0.5
0.6

Figure 26

brake force distribution, the following drag force could be produced at the drums of a four-wheeled vehicle by a net pedal force of 100 pounds:

100 x 50 = 5000 pounds Total Cam Force

$$\text{Total Drag Force} = 2 \times 1.68 \times C_F + 2 \times 0.48 \times C_F + 2 \times 1.68 \times C_R + 2 \times 0.48 \times C_R$$

Since the front and rear brakes are identical in their $\frac{fp}{C}$ relations the required brake force distribution value can be obtained simply by using different piston sizes at the two places. Sixty percent of the total cam force will act on the four pistons in the front brakes, or 750 pounds per piston. The other forty percent will act on the rear pistons, or 500 pounds per piston.

$$\text{Total Drag Force at Drums} = (1500 \times 1.68) + (1500 \times 0.48) + (1000 \times 1.68) + (1000 \times 0.218)$$

$$= 2520 + 720 + 1680 + 480$$

$$= 5400 \text{ pounds}$$

This is equivalent to an overall force multiplication ratio between pedal and drums of 54, which is low even for a light car.

The link-anchored shoe does not offer any advantage over the pin-anchored brake from the standpoint of force multiplication ratio, but has one advantage which appears after hard continuous brake application. During continuous braking, such as occurs in lowering a heavy vehicle down a very long steep grade, the heat generated at the outer lining surface is transmitted in significant amounts to the flange of the brake shoe. This expands the flange with respect to the web of the shoe and the shoe "curls" inward or assumes a smaller radius. In the pin-anchored brake this pulls the toe of the shoe away from the drum and moves the center of pressure down towards the anchor or heel end of the shoe. Continued braking under these conditions wears away the lining at the center and heel of the shoe, without corresponding wear of the lining at the toe. After the braking period is over the shoe cools to normal temperature and resumes its correct radius. At the next application the lining at the toe end of the shoe contacts the drum, but because of the additional wear which the center and heel lining has had, it contacts the drum either lightly or not at all. With little or no radial force at the center and heel of the shoe to resist the moment created by the frictional force at the toe of shoe, the toe friction swings the shoe forcibly against the drum and the brake becomes either self-locking or very savage in its action. This condition continues until the toe lining has worn back sufficiently to allow the center and heel lining to contact the drum firmly enough to produce the radial force necessary to counteract the moment caused by the frictional force at the toe.

The link-anchored brake is subject to shoe curl in the same way as the pin-anchored brake, but because of the ability of the shoe to pivot on the link, the heel of the cold shoe is allowed to swing down and make contact with the drum, generating sufficient radial force to avoid more than a slight displacement of the center of pressure and maintaining practically normal brake operation. This is a definite advantage of the link-anchored shoe over the pin-anchored shoe.

BRAKE SHOES WITH SLIDING-BLOCK ANCHORS

In several types of shoe brakes the anchor forces are taken by a sliding block, which will permit shoe movement parallel to its face but not perpendicular to its face. Theoretically and actually this is only a variation in physical form of the articulated-link type of anchor, and although the physical appearance of the brake is much different, the theoretical analysis is accomplished in the same way.

A typical brake using this method of anchoring the shoes is shown in Figure 27. The two halves of the brake are identical in dimensions and in action. A double-ended hydraulic cylinder at each end of the brake is used to provide both cam force and anchoring force. For the normal direction of drum rotation the drag force on a shoe forces a slightly crowned face at the heel end of the web against a flat hardened button in the cap of the hydraulic cylinder. This cap bottoms against the cylinder to provide anchoring force. The cylinder cap at the opposite end of the shoe moves outward to provide the cam force necessary to actuate the shoe. When the direction of drum rotation is reversed, the two cylinder caps merely interchange functions. Both shoes are thus forward shoes for each direction of rotation, and if θ_1 were made equal to 180 - θ_2 their action would be identical for both forward and backward rotation. In installations in which one direction of motion is predominant, θ_1 is usually made somewhat greater than 180 - θ_2 to keep the center of pressure nearer to the center of the lining and thus promote more uniform lining wear around the shoe.

In practice this type of brake usually has both α and β equal to 90°, although there are examples of other angles in use. A value of β less than 90° increases the effectiveness of the brake, moves the center of pressure nearer to the 90° position, and increases the sensitivity of the shoe to self-lock conditions for high lining to drum friction coefficients. A value of β more than 90° would increase the force multiplication ratio, or brake effectiveness (actually reduces the effective value of A, leading progressively toward self-lock conditions), but would force the center of pressure so far towards the heel that the toe of the shoe would be ineffective and the lining wear would be all at the heel.

The use of a link angle of 90° allows the shoe to shift back and forth until an equilibrium is established among cam force, anchor force, radial force, and drag force. The value of θ_m to the center of pressure will vary with all of the dimensional factors of the shoe, and especially with f. To determine the magnitude of θ_m as affected by f, and the magnitude of the Force Multiplication Ratio $\frac{fP}{C}$ as affected by f, the following procedure can be used.

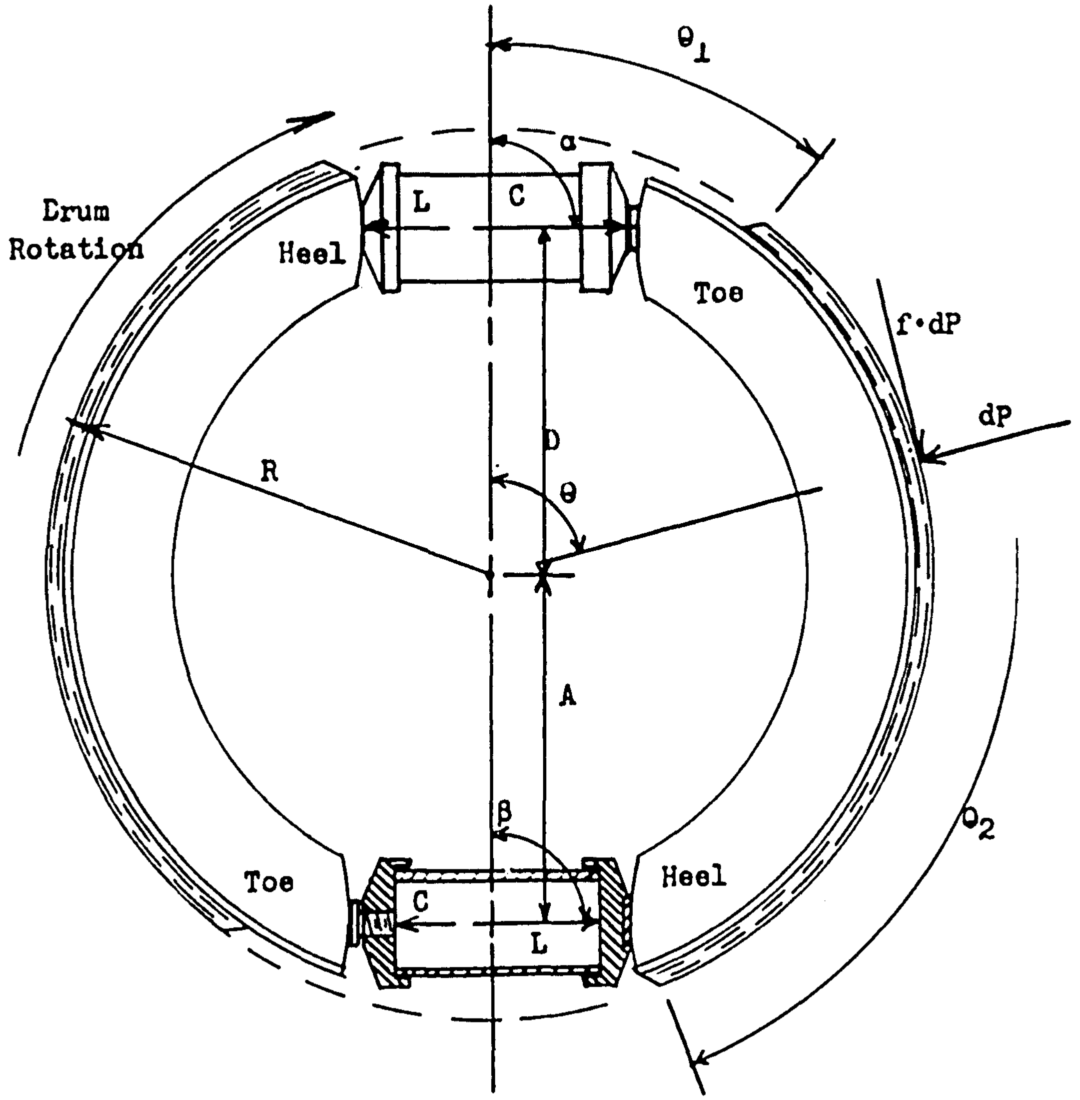
From Figure 27 the following equations relating the forces acting on the brake shoe can be written:

Summation of Forces Perpendicular to Link Axis (Vertical Forces)

$$0 = \int dP \cdot \cos \theta \quad + \int f dP \cdot \sin \theta$$

Summation of Moments about Intersection of Link Axis and Anchor Diameter

$$C(D + A) - \int dP \cdot A \sin \theta \quad + \int f \cdot dP \cdot (R + A \cos \theta) = 0$$



BRAKE SHOES WITH A SLIDING BLOCK ANCHOR

θ = Angle from Anchor Diameter to Point on Brake Shoe
θ_1 = Angle to Toe of Brake Shoe Lining
θ_2 = Angle to Heel of Brake Shoe Lining
R = Radius of Brake Shoe Lining Surface and of Brake Drum Surface
A = Anchor Distance
D = Cam Force Distance
C = Cam Force
α = Angle from Anchor Diameter to Cam Force
β = Angle from Anchor Diameter to a Normal at Center of Sliding Block
L = Link Force (Anchor Force)
f = Coefficient of Friction, Lining to Drum
dP = Incremental Radial Force at θ
$f \cdot dP$ = Incremental Drag Force at θ

Figure 27

If the distribution of radial forces is assumed to be the same as before, or:

$$\int dP = p_m bR \int_{\theta_1}^{\theta_2} \cos\,(\theta_m - \theta) \cdot d\theta$$

the moment and force equations can be evaluated to produce the desired results.

For the summation of perpendicular forces:

$$\int dP \cos\theta = p_m bR \int_{\theta_1}^{\theta_2} (\cos\theta_m \cos^2\theta + \sin\theta_m \sin\theta\cos\theta)\, d\theta$$

$$= p_m bR \left[\frac{1}{2}\cos\theta_m(\theta_2 - \theta_1) + \frac{1}{4}\cos\theta_m(\sin 2\theta_2 - \sin 2\theta_1) + \frac{1}{4}\sin\theta_m(\cos 2\theta_1 - \cos 2\theta_2) \right]$$

$$\int f \cdot dP \cdot \sin\theta = f p_m bR \int_{\theta_1}^{\theta_2} (\cos\theta_m \sin\theta\cos + \sin\theta_m \sin^2\theta)\, d\theta$$

$$= f p_m bR \left[\frac{1}{2}\sin\theta_m(\theta_2 + \theta_1) - \frac{1}{4}\sin\theta_m(\sin 2\theta_2 - \sin 2\theta_1) + \frac{1}{4}\cos\theta_m(\cos 2\theta_1 - \cos 2\theta_2) \right]$$

The addition of these two equations produces the summation of perpendicular forces, which is equal to zero.

$$0 = p_m bR \left[\frac{f}{4}\cos\theta_m(\cos 2\theta_1 - \cos 2\theta_2) - \frac{f}{4}\sin\theta_m(\sin 2\theta_2 - \sin 2\theta_1) + \frac{f}{2}\sin\theta_m(\theta_2 - \theta_1) + \frac{1}{4}\sin\theta_m(\cos 2\theta_1 - \cos 2\theta_2) + \frac{1}{4}\cos\theta_m(\sin 2\theta_2 - \sin 2\theta \right.$$
$$\left. + \frac{1}{2}\cos\theta_m(\theta_2 - \theta_1) \right]$$

Since the quantity $p_m bR$ cannot be equal to zero, the quantity in the bracket must equal zero. This provides the necessary relation for determining the mutual relation between f and θ_m. If this is solved for f the following equation results:

$$f = \frac{\sin \theta_m(\cos 2\theta_1 - \cos 2\theta_2) + \cos \theta_m(\sin 2\theta_2 - \sin 2\theta_1) + 2 \cos \theta_m(\theta_2 - \theta_1)}{- \cos \theta_m(\cos 2\theta_1 - \cos 2\theta_2) + \sin \theta_m(\sin 2\theta_2 - \sin 2\theta_1) + 2 \sin \theta_m(\theta_2 - \theta_1)}$$

This is exactly the same as the equation which would have been obtained by substituting a value of 90° for both α and β in the equation relating f and θ_m for link-anchored brake shoes (see page 123).

The summation of moments equation is exactly the same as the moment equation for the link-anchored brake (page 119) and would be developed in exactly the same manner. Integration of the moment equation produces the following results:

$$C = \frac{p_m bR}{D + A} \left[\text{Moment Bracket as on page 121}\right]$$

If the relation for dP as a function of θ is integrated between θ_1 and θ_2 the following result is obtained:

$$P = p_m bR \left[\cos \theta_m(\sin \theta_2 - \sin \theta_1) - \sin \theta_m(\cos \theta_2 - \cos \theta_1)\right]$$

These equations can now be combined to produce the Force Multiplication Ratio, or $\frac{fP}{C}$ for this type of brake.

$$\frac{fP}{C} = \frac{(D + A) \cdot f \left[\cos \theta_m(\sin \theta_2 - \sin \theta_1) - \sin \theta_m(\cos \theta_2 - \cos \theta_1)\right]}{\left[\text{Moment Bracket}\right]}$$

If the relationship between f and θ_m is known, the corresponding value of $\frac{fP}{C}$ can be obtained.

To illustrate the capabilities of a brake of this type, and to compare its abilities with other brake types of comparable size, the results of a numerical evaluation of the f and $\frac{fP}{C}$ relations are given on Figure 28. The brake analyzed has the following dimensions:
$R = 5.5"$, $A = 4.26"$, $D = 4.26$, $\theta_1 = 38.5°$, $\theta_2 = 158°$, $\alpha = 90°$, $\beta = 90°$.

The curve for θ_m on Figure 28 shows that this type of shoe is loaded more heavily at the heel of the shoe than at the toe. In fact for friction coefficient over 0.4 the center of pressure moves so far toward the heel that the toe of the shoe is completely unloaded. This would lead to more rapid lining wear at the heel than at the toe.

The effectiveness of $\frac{fP}{C}$ value for this type of brake shoe is slightly lower than that of a pin-anchored or inclined-link-anchored shoe, but its stability, or resistance to self-lock conditions is excellent. The fact that both shoes act as forward shoes for both directions of rotation should give it a high average force multiplication ratio.

The comparative braking effect produced by 100 pounds Net Pedal Force on a car equipped with four of these brakes would be determined in the following manner. Since the front and rear brakes would be identical, the front to rear brake force distribution would be determined by relative hydraulic piston diameters. The Force Multiplication Ratio in the Brake Rigging would be the same for these brakes as for the pin- or lock-anchored brakes.

Total Cam Force = 50 x 100 pounds Net Pedal Force

Cam Force per Front Brake Piston = $\frac{0.6 \times 5000}{4}$ = 750 pounds

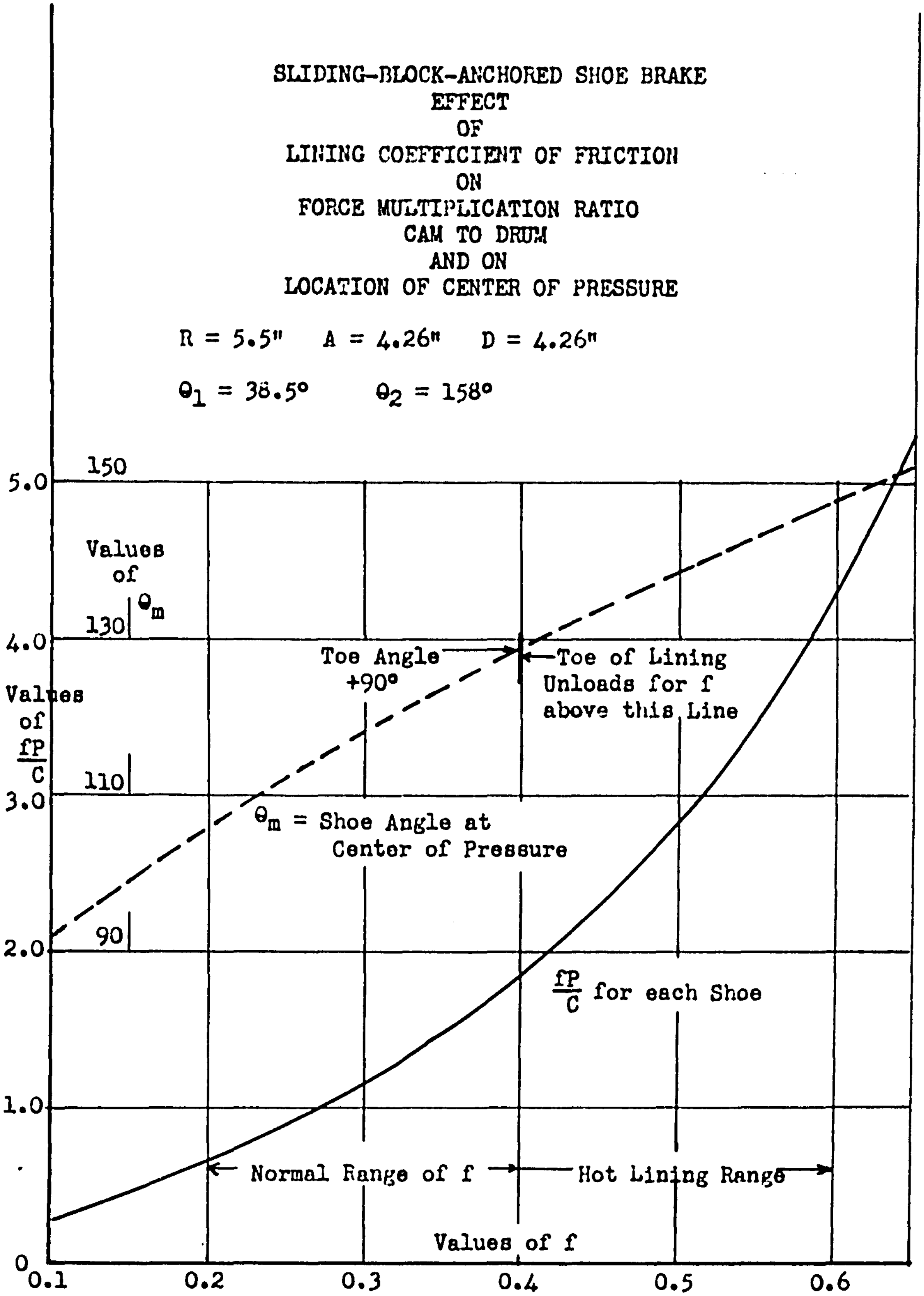

SLIDING-BLOCK-ANCHORED SHOE BRAKE
EFFECT
OF
LINING COEFFICIENT OF FRICTION
ON
FORCE MULTIPLICATION RATIO
CAM TO DRUM
AND ON
LOCATION OF CENTER OF PRESSURE
R = 5.5"   A = 4.26"   D = 4.26"
θ1 = 38.5°   θ2 = 158°
Values of θm
150
130
110
90
Values of fP/C
5.0
4.0
3.0
2.0
1.0
0
Toe Angle +90°
Toe of Lining Unloads for f above this Line
θm = Shoe Angle at Center of Pressure
fP/C for each Shoe
Normal Range of f
Hot Lining Range
Values of f
0.1
0.2
0.3
0.4
0.5
0.6

Figure 28

Cam Force per Rear Brake Piston $= \frac{0.4 \times 5000}{4} = 500$ pounds

At $f = 0.35$ the $\frac{fP}{C}$ per shoe equals 1.50

Total Drag Force $= (2 \times 750 \times 1.50)^2 + (2 \times 500 \times 1.50)^2$

$= 4500$ pounds front $+ 3000$ pounds rear

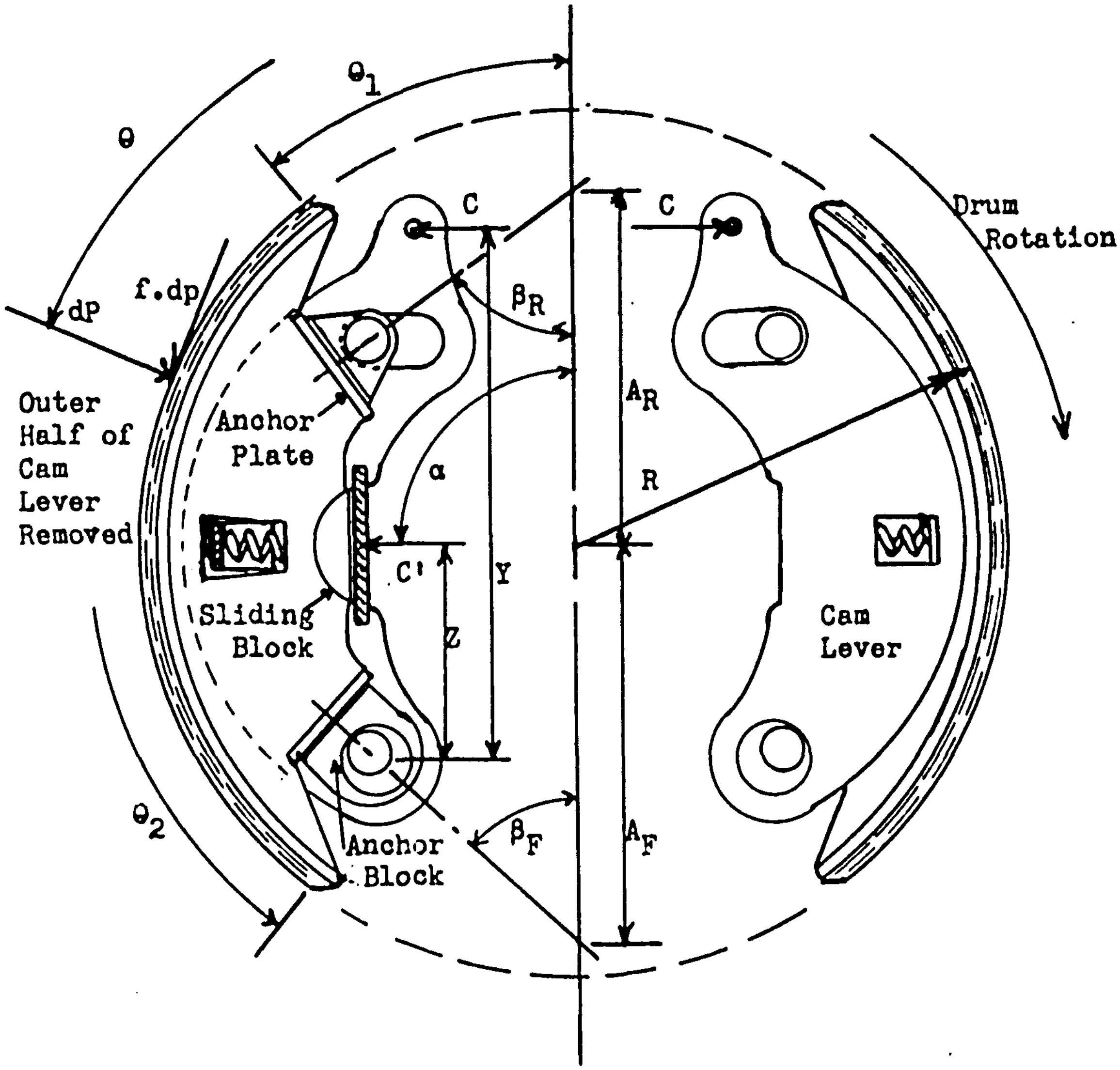
$= 7500$ pounds Total Drag Force

This braking system produces a total Force Multiplication Ratio between Pedal and Drum of 75 to 1, which is a fair value. In addition to this fair effectiveness value, this brake would be stable against variations in lining coefficient and against localized wear due to shoe "curl", and would cause a minimum of wheel bearing loading when applied.

DUAL-PRIMARY SHOE BRAKE

The dual-primary shoe brake is really a variation of the link-anchored shoe brake, but its physical appearance and its method of shoe actuation are sufficiently different to warrant a separate analysis (17).

Figure 29 shows the physical arrangement of the brake. Two lined, T section brake shoes which are mirror images of each other are held by the force of a retaining spring on a double section cam lever. Two anchor pins fastened to the backing plate extend through both halves of each cam lever. The lower pin carries an eccentric on which the cam lever pivots. The upper pin extends through a slot in both halves of the cam lever. The slot is sufficiently elongated to allow the cam forces C to swing the cam levers about the lower anchor pin eccentric without contacting the upper pin. The web of each shoe has a partial circle milled into the center of its inner edge. A sliding block, flat on one edge and nearly semi-circular on the other, bears against the milled section of the shoe web on one side, and on the other side against a flat plate which joins the two halves of the cam lever. This construction allows the shoe to slide endwise with respect to the cam lever, and to rotate with respect to the cam lever. Both actions are slightly restrained by the friction created by the retainer spring force, but not unduly so, and both actions are very necessary if the shoe surface contour is to be able to adapt itself to the drum surface contour. The ends of the shoe web carry anchor plates which butt



DUAL-PRIMARY SHOE BRAKE WITH SLIDING BLOCK ANCHORS

θ = Angle from Anchor Diameter to Point on Brake Shoe
θ_1 = Angle to Toe of Brake Shoe Lining
θ_2 = Angle to Heel of Brake Shoe Lining
R = Radius of Brake Shoe Lining Surface and Drum Surface
A = Anchor Distance F = Forward R = Reverse
D = Cam Force Distance (Zero in this Brake)
C = Cam Force Produced by Brake Rigging
C' = Shoe Actuating Force = CY + Z
α = Angle from Anchor Diameter to Shoe Actuating Force
β = Angle from Anchor Diameter to Anchor Plate Normal through Anchor Pin Center
L = Link Force (Anchor Force)
dP = Incremental Radial Force on Shoe
f·dP = Incremental Drag Force on Shoe

Figure 29

against one or the other of the anchor blocks attached to each anchor pin.

The result of this unique construction is a link-anchored shoe which is a forward shoe for both directions of rotation, and which is actuated by an augmented cam force at its center. Changing direction of rotation slides the shoe from contact with one anchor block to contact with the other anchor block. Pivoting of the anchor blocks allows the shoe to assume the angle necessary for complete contact with the drum, and relative sliding of the anchor block and anchor plate simulates the action of a link, allowing the heel of the shoe to contact the drum with whatever force is necessary to bring about the required equilibrium among cam force, link force, radial force, and drag force. Once the shoe has been applied to the drum, its action is exactly that of a link-anchored shoe, but unlike the ordinary link-anchored shoe it has no grossly ineffective reverse shoe action.

The anchor distance of this shoe is the distance from the shoe surface center to the point of intersection of the anchor diameter and a normal to the shoe anchor plate surface which passes through the pin center about which the anchor block pivots. The location of the anchor diameter is not rigidly fixed, but for the sake of convenience it should be taken perpendicular to the shoe actuating force to make the angle α equal to 90°.

The cam force distance for this brake is then zero for either direction of rotation, a fact which makes the brake more sensitive to Anchor Distance (Anchor Plate Angle in this brake) and which keeps the center of pressure near the middle of the shoe for either direction of rotation.

The method of analysis of this brake follows that of the link-anchored brake exactly, producing the following significant equation:

$$\sin(\alpha - \beta)\left[\text{Moment Bracket}\right] = A \sin\beta\left[\text{Radial Bracket} - f\ \text{Tangential Bracket}\right]$$

$$\frac{fP}{C'} = \frac{Af \sin\alpha\left[\cos\theta_m(\sin\theta_2 - \sin\theta_1) - \sin\theta_m(\cos\theta_2 - \cos\theta_1)\right]}{\left[\text{Moment Bracket}\right]}$$

In these equations C' represents the augmented cam force applied at the center of the shoe, and equals the actual cam force C times the quotient of distance Y divided by distance Z. The Moment Bracket is that group of terms shown on page 121 and the Radial Bracket and Tangential Bracket each represent other groups of terms shown on page 122.

For a comparison of the effectiveness of this brake with that of other types, the results of a numerical evaluation of these equations for an actual brake are given graphically on Figure 30. The brake analyzed had the following dimensions: $R = 5.5$, $\theta_1 = 38°$, $\theta_2 = 142°$, $\alpha = 90°$, $D = 0$, and for "Forward" shoe operation $A = 5.08"$, $\beta = 47°$, while for "Reverse" shoe operation $A = 4.47"$, $\beta = 55°$.

The different anchor distances and link angles produced two different sets of equations.

"Forward" Shoe Action

$$f = \frac{-2.115 \cos\theta_m}{\sin\theta_m}$$

$$\frac{fP}{C} = \frac{8.01\ f \sin\theta_m}{-8.67\ f \sin\theta_m - 2.145\ f \cos\theta_m + 7.075 \sin\theta_m}$$

Actually there is no reverse shoe action as occurs in the pin-anchored and conventionally link-anchored shoes because of the double

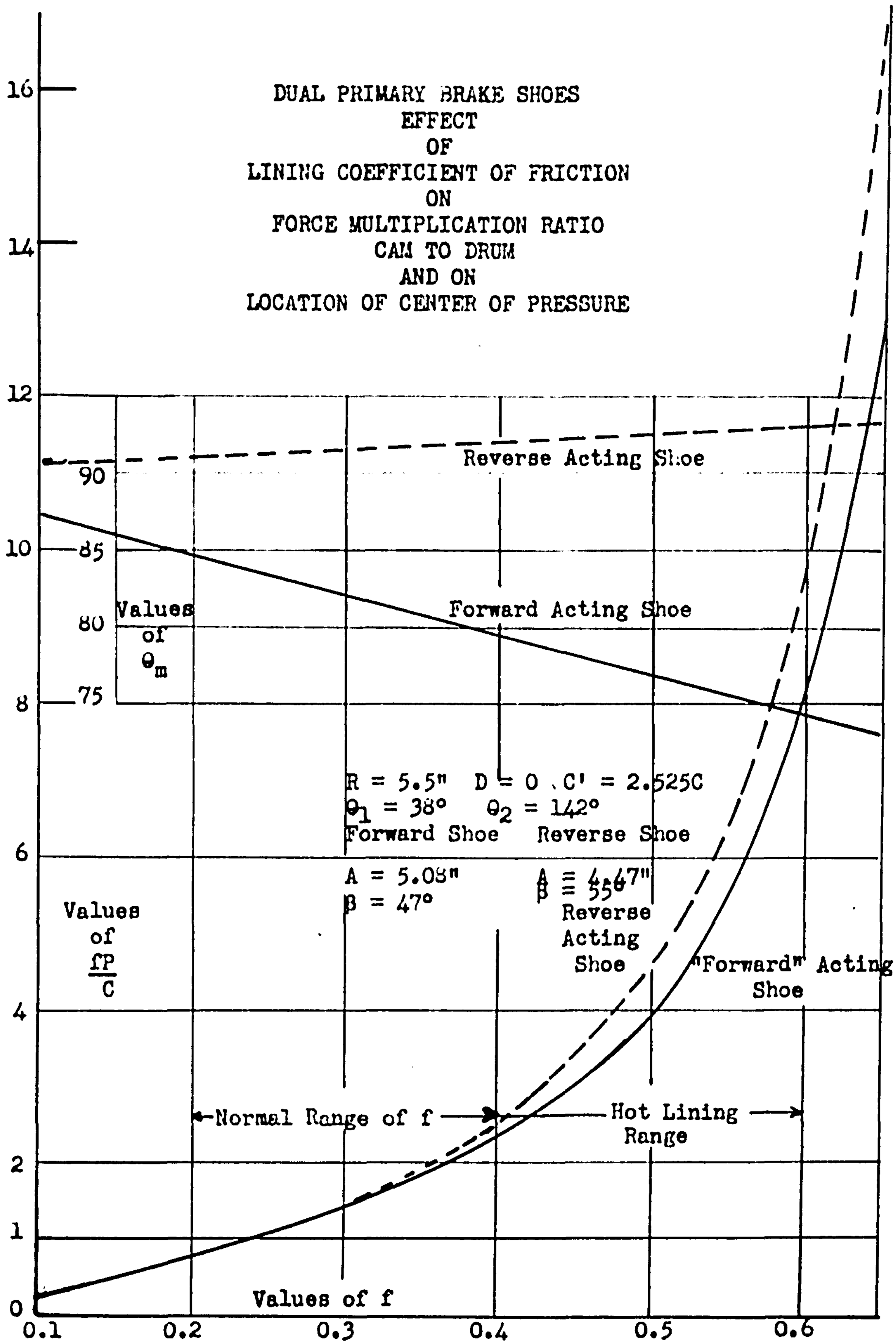

DUAL PRIMARY BRAKE SHOES
EFFECT
OF
LINING COEFFICIENT OF FRICTION
ON
FORCE MULTIPLICATION RATIO
CAM TO DRUM
AND ON
LOCATION OF CENTER OF PRESSURE
Reverse Acting Shoe
Forward Acting Shoe
Values of θ_m
R = 5.5" D = 0 C' = 2.525C
θ_1 = 38° θ_2 = 142°
Forward Shoe Reverse Shoe
A = 5.08" A = 4.47"
β = 47° β = 55°
Reverse Acting Shoe
"Forward" Acting Shoe
Values of fP/C
Normal Range of f
Hot Lining Range
Values of f
0.1 0.2 0.3 0.4 0.5 0.6
0 1 2 4 6 8 10 12 14 16
75 80 85 90

Figure 30

anchor provided. The difference between the action in the two directions occurs because of the slight difference in effective link angle which is built into the shoe anchor plates.

Figure 30 shows the combined effect of anchor distance and link angle on brake effectiveness. The increased link angle of the reverse acting shoe would normally reduce the sensitivity of the shoe to changes in f. However with this construction an increase in link angle causes a decrease in anchor distance, which increases sensitivity to f. In this case the anchor distance effect was stronger, and the reverse acting shoe is slightly more sensitive to f than the forward acting shoe.

Both shoes are relatively stable within the range of f to be expected, even with a hot lining surface. The Force Multiplication Ratio is good for both shoes, and since both act as forward shoes the average effectiveness of the complete brake will be as good as that of any type so far considered. The freedom of the shoe to move when necessary to adapt itself to the drum makes it free from difficulty caused by shoe "curl".

The braking effort which can be produced by four of these brakes, actuated by a 100 pound Net Pedal Force is appreciable. Since front and rear brakes are identical, the necessary brake force distribution will be attained by varying the cam piston size. The action of the brake in taking up clearance from the released position is identical with that of the pin-anchored or link-anchored brakes, so the Force Multiplication Ratio in the brake rigging will be the same. This gives us a Cam Force at front brake pistons of 750 pounds each, at rear brake pistons of 500 pounds each. For a friction coefficient of $f = 0.35$, the forward acting shoe has an $\frac{fP}{C}$ value of 1.83, and the reverse acting shoe has an $\frac{fP}{C}$ value of 1.92.

$$\text{Total Drag Force} = (2 \times 750 \times 1.83) + (2 \times 750 \times 1.92) + (2 \times 500 \times 1.83) + (2 \times 500 \times 1.92)$$

$$= (2745 + 2880) + (1830 + 1920)$$

$$= 5625 \text{ pounds Front} + 3750 \text{ pounds Rear}$$

$$= 9375 \text{ pounds Total Drag Force}$$

This is an overall Force Multiplication Ratio between pedal and drum of 93.75, which is quite good.

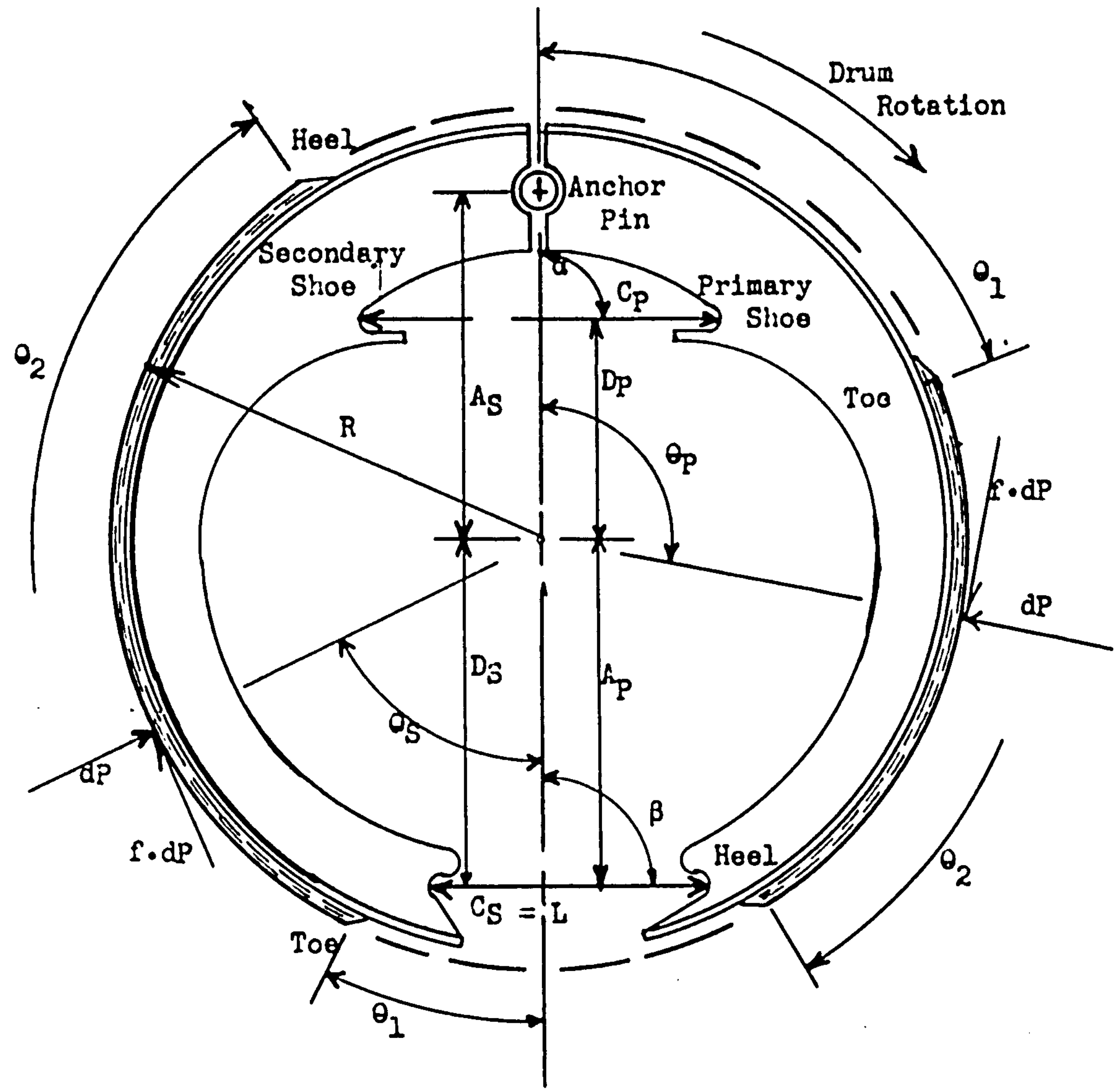
DUO-SERVO SHOE BRAKES

The very popular Duo-Servo type of shoe brake is a combination of one link-anchored shoe with one pin-anchored shoe, with the link force or anchor force of the link-anchored shoe acting as the cam force for the pin-anchored shoe. The link-anchored shoe is called the primary shoe, the pin-anchored shoe is called the secondary shoe.

Figure 31 shows the elements of the Duo-Servo brake as it is usually constructed. Two brake shoes in their released position are held tightly against a cylindrical anchor pin by a release spring (not shown). When a cam force C_p is applied to the Primary Shoe by an hydraulic cylinder, the shoe pivots about the link at its lower end until it contacts the drum, and then rides around with the drum until the link force L, acting as a cam force for the pin-anchored secondary shoe, has taken up the initial clearance of the secondary shoe and applied its surface to the drum. This movement releases the contact between the primary shoe and the anchor pin, leaving the shoe free to find its natural equilibrium position. This equilibirium condition requires a certain magnitude of link force L, which is transmitted without change to the toe of the secondary shoe.

Reverse drum rotation simply interchanges the functions of the two shoes, so the action of the brake for either direction of rotation is substantially the same.

The name Duo-Servo applied to this brake indicates that one shoe is used to actuate or serve the other, and that this action is avail-



DUO-SERVO SHOE BRAKE

θ = Angle from Anchor Diameter to Point on Brake Shoe
θ_1 = Angle to Toe of Brake Shoe Lining
θ_2 = Angle to Heal of Brake Shoe Lining
R = Radius of Brake Shoe Lining Surface and Brake Drum Surface
A = Anchor Distance P = Primary S = Secondary
D = Cam Force Distance
C = Cam Force
α = Angle from Anchor Diameter to Cam Force
β = Angle from Anchor Diameter to Link Axis
L = Link Force or Anchor Force
dP = Incremental Radial Force
f·dP = Incremental Drag Force

Figure 31

able in a dual direction.

The Force Multiplication Ratio which this type of brake can produce can be determined by an analysis first of the primary shoe action, and then of the secondary shoe action. The primary shoe is a link-anchored shoe, with both cam angle and link angle equal to ninety degrees. Its analysis is exactly the same as that of the shoe with the sliding-block anchor, with the addition of a determination of the Link Force L which is produced by the shoe.

The following equations, based on Figure 31, may be written for the Primary Shoe.

Summation of Forces Perpendicular to the Link Axis:

$$0 = \int dP \cdot \cos\theta + \int f \cdot dP \cdot \sin\theta$$

Summation of Moments about the Link Axis-Anchor Diameter Intersection:

$$0 = C\,(D + A) - \int dP \cdot A\sin\theta + \int f\,(R + A\cos\theta)\,dP$$

Summation of Forces Parallel to Link Axis:

$$0 = C + L - \int dP \cdot \sin\theta + \int f \cdot dP \cdot \cos\theta$$

The first two of these equations are exactly the same as those for the Sliding-Block anchored shoe, and the following general solutions for f, θ_m, and $\frac{fP}{C}$ are taken from the section dealing with that type of brake.

f =

$$\frac{\sin\theta_m(\cos 2\theta_1 - \cos 2\theta_2) + \cos\theta_m(\sin 2\theta_2 - \sin 2\theta_1) + 2\cos\theta_m(\theta_2 - \theta_1)}{-\cos\theta_m(\cos 2\theta_1 - \cos 2\theta_2) + \sin\theta_m(\sin 2\theta_2 - \sin 2\theta_1) - 2\sin\theta_m(\theta_2 - \theta_1)}$$

$$\frac{fP}{C} = \frac{(A + D)\, f \left[\cos\theta_m(\sin\theta_2 - \sin\theta_1) - \sin\theta_m(\cos\theta_2 - \cos\theta_1)\right]}{\left[\text{Moment Bracket on page 121}\right]}$$

Since the Link Force produced by the primary shoe is used as the Cam Force for the secondary shoe, its magnitude, preferably as a function of the Cam Force C, must be known. The relation for the summation of forces parallel to the link axis can be solved for C + L to yield the following:

$$C + L = p_m bR\left[\frac{1}{4}(\cos\theta_m - f\sin\theta_m)(\cos 2\theta_1 - \cos 2\theta_2) + \frac{1}{4}(\sin\theta_m + f\cos\theta_m)\right.$$

$$\left.(\sin 2\theta_1 - \sin 2\theta_2) + \frac{1}{2}(\sin\theta_m - f\cos\theta_m)(\theta_2 - \theta_1)\right]$$

To eliminate the quantity ($p_m bR$) and to introduce the anchor and cam distances, the C + L relation should be divided by C, using for C the relation derived from a solution of the moment equation. This results in the following relation:

$$1 + \frac{L}{C} =$$

$$\frac{(D + A)\left[\frac{1}{4}(\cos\theta_m - f\sin\theta_m)(\cos 2\theta_1 - \cos 2\theta_1) + \frac{1}{4}(\sin\theta_m + f\cos\theta_m)(\sin 2\theta_1 - \sin 2\theta_2) + \frac{1}{2}(\sin\theta_m - f\cos\theta_m)(\theta_2 - \theta_1)\right]}{\left[\text{Moment Bracket on page 121}\right]}$$

The secondary shoe is a standard pin-anchored shoe, with the center of pressure constrained by the shoe geometry to remain at $\theta_m = 90°$. The $\frac{fP}{C}$ relationship for this type of shoe is given on page 109 of the section on pin-anchored brake shoes as:

$$\frac{fP}{C} =$$

$$\frac{(D + A)\ f\ (\cos\theta_2 - \cos\theta_1)}{\frac{A}{2}(\theta_2 - \theta_1) + \frac{A}{4}(\sin 2\theta_2 - \sin 2\theta_1) - f\left[R(\cos\theta_1 - \cos\theta_2) + \frac{A}{4}(\cos 2\theta_1 - \cos 2\right.}$$

In evaluating this relation the C for the secondary shoe must be replaced by the L term of the primary, and the actual drag force of the secondary shoe obtained from the product of $\frac{fP}{L} \times \frac{L}{C}$. The total drag force of the two shoes is then $fP_{secondary} + fP_{primary}$.

The use of the anchor force of the primary shoe as a cam force for the secondary shoe is a potent way of increasing the effectiveness of a two-shoe brake. Unfortunately it also compounds the sensitivity of the brake to the effects of a change in friction coefficient, making it less stable in its action.

For purposes of comparison a numerical evaluation of the previous equations is presented for a typical brake of this type. The brake dimensions are as follows:

Primary Shoe	Secondary Shoe
R = 5.5"	R = 5.5"
A = 4.44"	A = 4.44"
D = 2.80"	D = 4.44"
$\theta_1 = 70°$	$\theta_1 = 27°$
$\theta_2 = 150°$	$\theta_2 = 146°$
$\alpha = 90°$	$\alpha = 90°$
$\beta = 90°$	

The results of this evaluation are shown in graphical form on Figures 32 and 33. The very large change in the location of the center of pressure with change in f is the effect of the very short lining used on the primary shoe. With low friction coefficients the relatively large moment arm of the cam force moves the center of pressure well up towards the heel of the shoe. The use of a longer lining by lowering

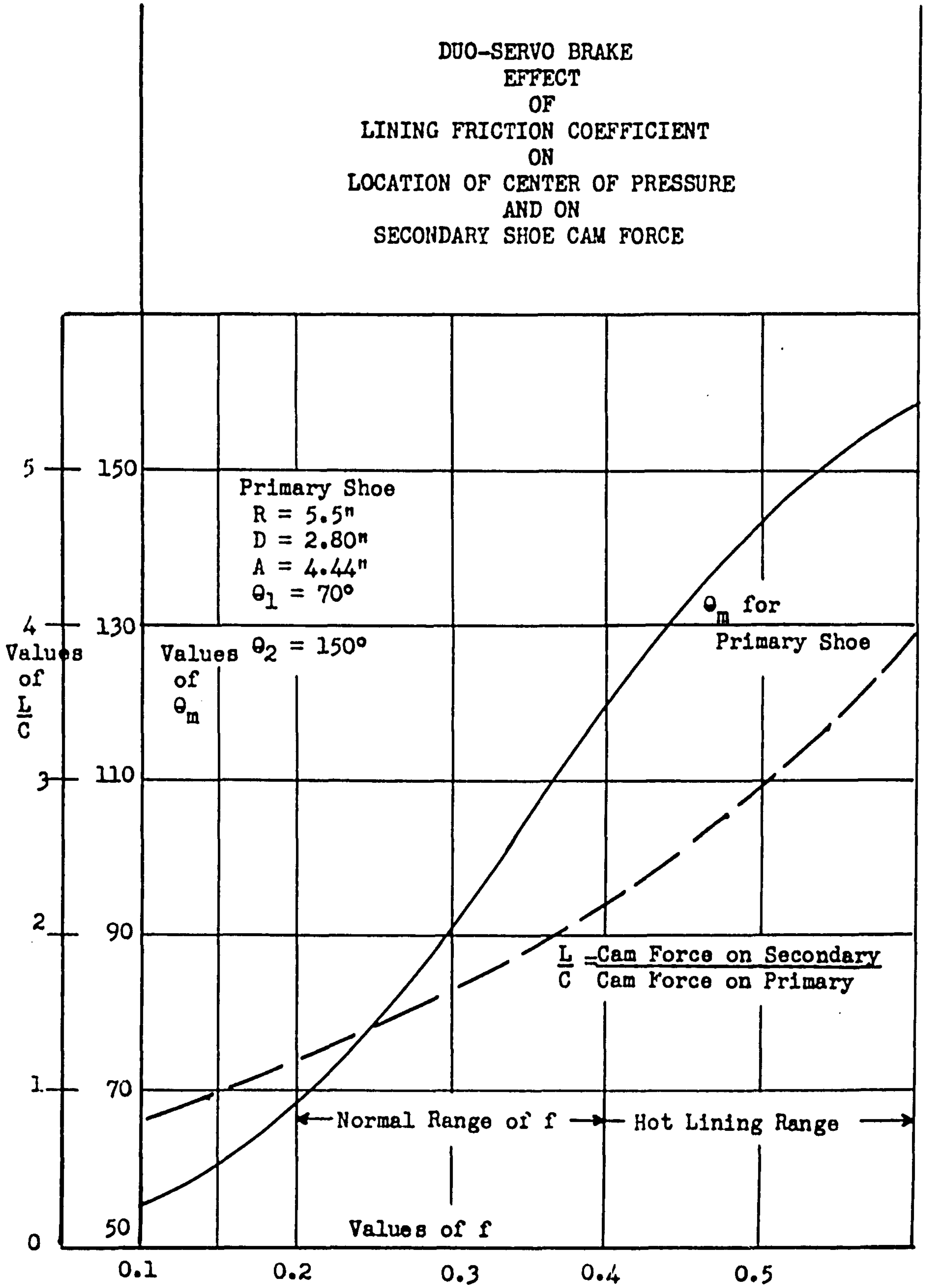

DUO-SERVO BRAKE
EFFECT
OF
LINING FRICTION COEFFICIENT
ON
LOCATION OF CENTER OF PRESSURE
AND ON
SECONDARY SHOE CAM FORCE
Primary Shoe
R = 5.5"
D = 2.80"
A = 4.44"
θ1 = 70°
θ2 = 150°
Values of L/C
Values of θm
θm for Primary Shoe
L/C = Cam Force on Secondary / Cam Force on Primary
Normal Range of f
Hot Lining Range
Values of f
0
1
2
3
4
5
50
70
90
110
130
150
0.1
0.2
0.3
0.4
0.5

Figure 32

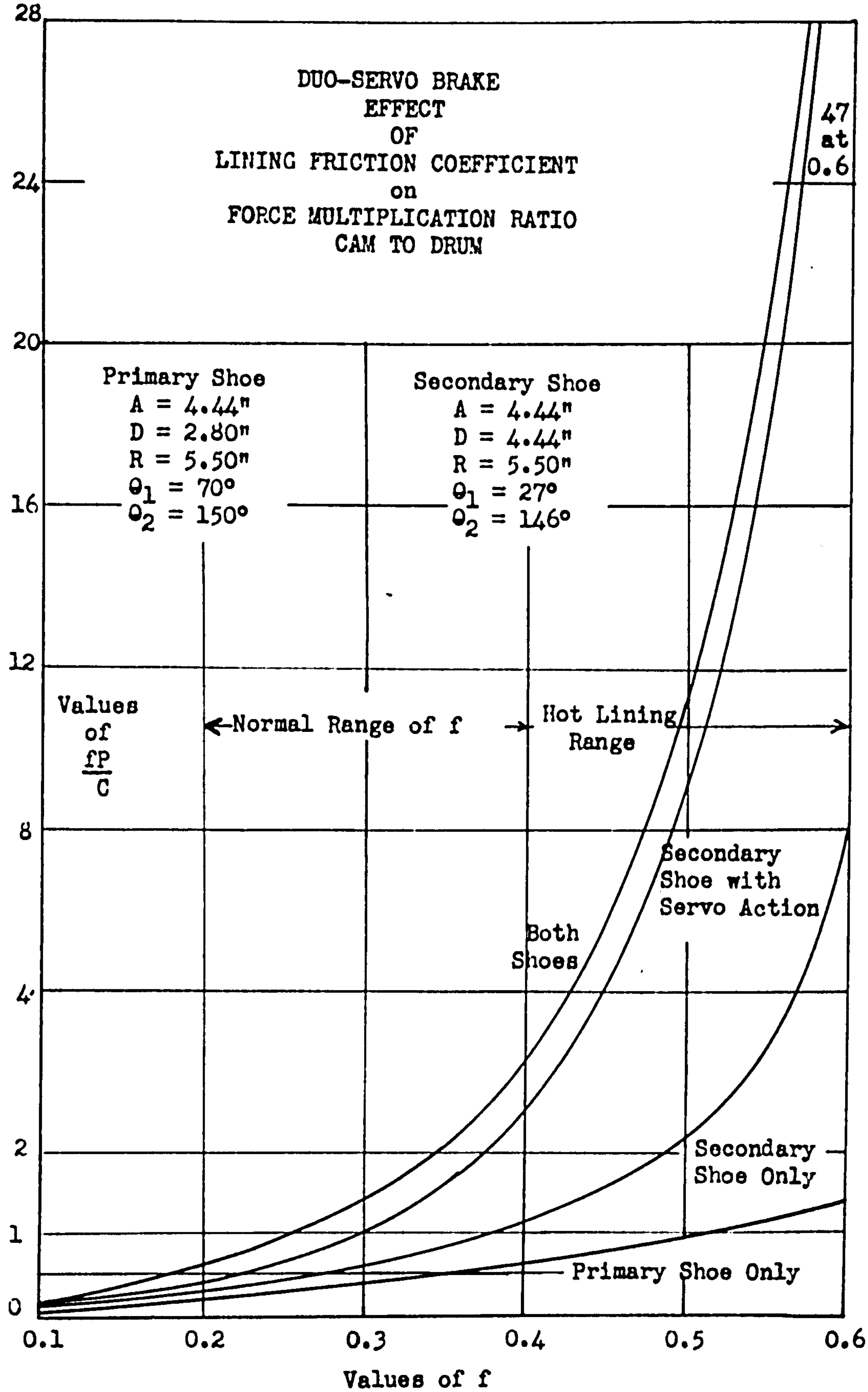

DUO-SERVO BRAKE
EFFECT
OF
LINING FRICTION COEFFICIENT
on
FORCE MULTIPLICATION RATIO
CAM TO DRUM
47 at 0.6
Primary Shoe
A = 4.44"
D = 2.80"
R = 5.50"
θ1 = 70°
θ2 = 150°
Secondary Shoe
A = 4.44"
D = 4.44"
R = 5.50"
θ1 = 27°
θ2 = 146°
Values of fP/C
Normal Range of f
Hot Lining Range
Secondary Shoe with Servo Action
Both Shoes
Secondary Shoe Only
Primary Shoe Only
28
24
20
16
12
8
4
2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
Values of f

Figure 33

the value of θ_1 would only increase the shift of the pressure center towards the heel due to the greater friction effect produced. Since space limitations limit any increase in heel angle θ_2, the use of 70° for θ_1 puts the center of pressure midway along the length of the shoe for a normal coefficient of friction, a condition conducive to even lining wear.

The link or anchor force of the primary shoe increases materially over the range of f investigated. A value of $\frac{L}{C}$ greater than one is desirable to produce a highly effective brake, but an appreciable change in this ratio with change in f is undesirable, since it reduces the stability of the brake, or in other words, it increases the probability of radical and unexpected changes in brake effectiveness with unintentional changes of f.

The Force Multiplication Ratio produced by this brake is shown on Figure 33. The primary shoe alone is somewhat less effective, due to its short lining, than the ordinary pin-anchored forward shoe, but the secondary shoe when actuated by the link force of the primary shoe becomes quite effective and very sensitive to change in f. The value of $\frac{fP}{C}$ for this augmented secondary shoe alone is about 47 for a value of f = 0.6. The two shoes together will produce a drag force forty eight times as great as the cam force applied to the toe of the primary shoe whenever the friction coefficient gets as high as 0.6, as it does occasionally at the end of a hard stop.

The ability of four brakes of this type to create drag force at the drums when actuated by a net pedal force of 100 pounds is appreciable. Although only one cam piston actually moves when this brake is applied, it must move far enough to take up the clearance on two shoes, so no significant changes in brake rigging force multiplication ratio can be expected. Since both front wheel and rear wheel brakes are identical, the front and rear brake force distribution must be accomplished by differences in piston sizes. As in the previous examples, this produces a cam force of 750# for the front wheels, and 500# for the rear wheels. The $\frac{fp}{C}$ ratio for the two shoes with f = 0.35 equals 4.20.

Total Drag Force = (2 x 750 x 4.20) + (2 x 500 x 4.20)

= 6300 Front + 4200 Rear

= 10,500 pounds

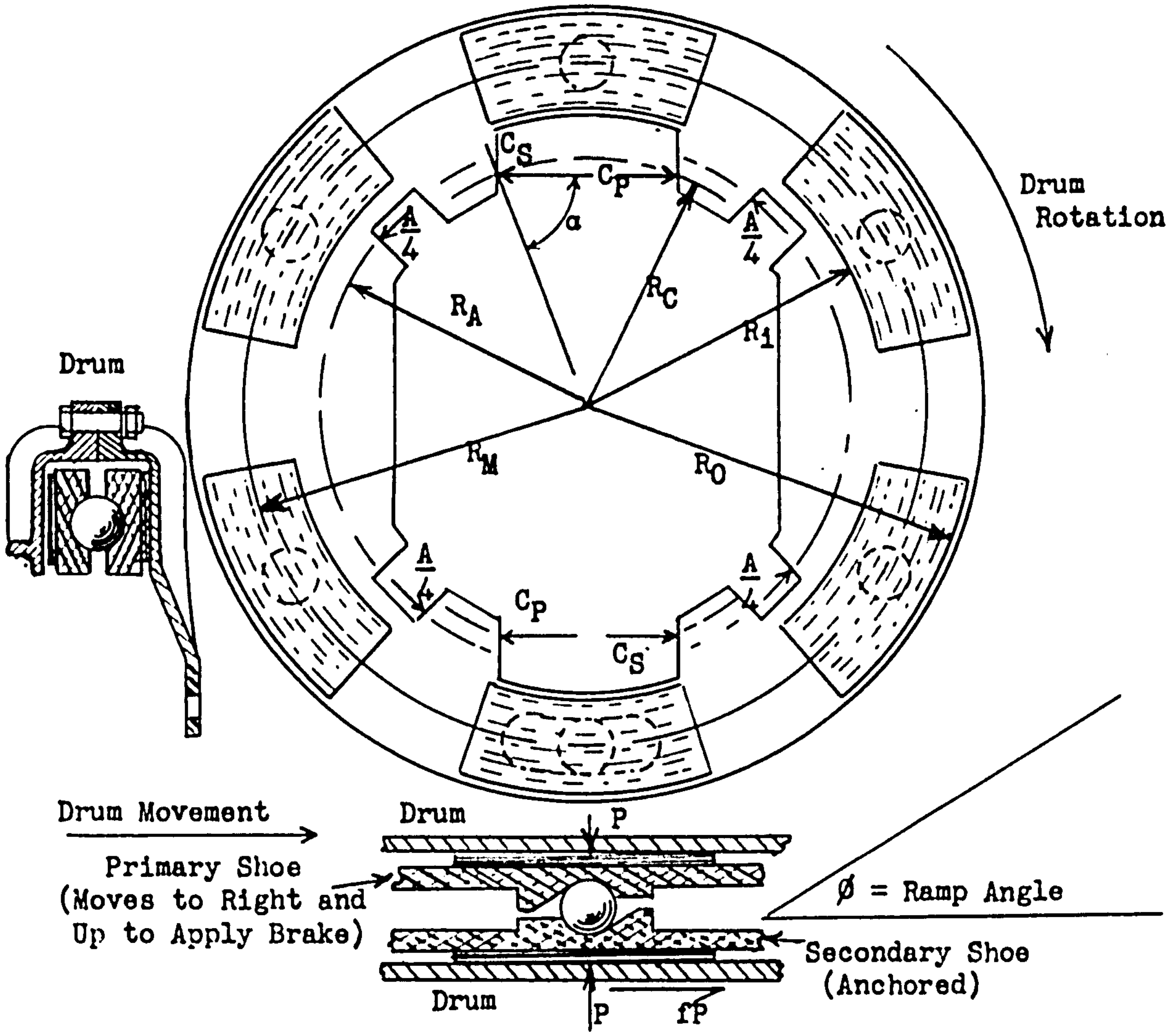
This is an overall Force Multiplication Ratio between Pedal and Drum of 105. This brake type is the most effective of all of the shoe brake types, but is also the most sensitive to changes in lining friction coefficient.

ANNULAR DISC SHOE BRAKES

Disc type brakes have been used for some time on industrial equipment, airplane landing wheels, and railroad cars (18). Their adoption for automotive vehicles is relatively recent, with only one important manufacturer having adopted them as standard or optional equipment. Of the many types of disc brakes proposed or in use, the type adopted by this company seems best adapted to passenger car and truck use, and is the one which will be analyzed in this section.

To provide sufficient braking effectiveness for adequate control of heavy vehicles, and do it without requiring excessive pedal force from the driver or without resorting to power braking requires some degree of servo action or shoe compounding as illustrated in the Duo-Servo brake.

The general construction of an annular disc brake incorporating this servo action and with good heat-dissipation ability is illustrated on Figure 34. The drum, shown in cross section, is formed from two ribbed circular castings bolted together around their periphery, and presenting two parallel machined wear surfaces to the brake lining. The brake lining is made in the form of short annular sections, cemented to one face of two annular ring aluminum shoes. The opposite faces of these shoes contain six carefully positioned semicircular grooves whose depth varies according to a carefully controlled ramp angle. The inner periphery of each shoe contains four radial slots to contact the four ears of an anchor spider which passes the braking



ANNULAR DISC BRAKE

R_O = Radius to Outer Edge of Lining Segments
R_i = Radius to Inner Edge of Lining Segments
R_M = Mean Effective Radius of Drag Forces
R_A = Effective Radius of Shoe Anchor Forces
R_C = Radius at which Cam Forces are applied to the Shoes
α = Angle between Cam Force and Radius to the Application Point
C_P = Cam Force - Primary Shoe
C_S = Cam Force - Secondary Shoe
A = Anchor Force
ø = Ramp Angle of Inclined Grooves in Shoes

Figure 34

forces to the axle housing, two wide slots to accommodate two single-ended hydraulic pistons for shoe actuation, and two wide webs for mounting two automatic brake adjusting mechanisms which adjust the shoe positions to eliminate the effect of lining wear on pedal position. Six steel balls are placed in the milled grooves and the two shoes placed back to back, held against the balls by several retracting springs. Angular movement of one shoe with respect to the other caused by the cam force C_p, rolls the balls up the inclined grooves, forcing the shoes apart from one another and bringing the lining segments into contact with the machined faces of the enclosing drum. The secondary shoe contacts the anchor ears, transferring its drag force directly to the axle housing. The anchor slots are sufficiently wide, however, to prevent the slot faces of the primary shoe from contacting the anchor ears, and the drag force on the primary tends to rotate the shoes still further with respect to one another, causing the balls to force the shoes against the drum faces with augmented force, increasing the amount of drag force produced. The drag force on the primary shoe thus assists the cam force in forcing both shoes against the drum, producing the servo-action necessary for a high degree of braking effectiveness.

There is a limit to the degree of self-energizing action or servo-action which can be built into this brake without seriously reducing the stability of the brake and rendering it unduly sensitive to self-lock conditions. The degree of servo-action produced is a function mainly of ramp angle and lining coefficient of friction. The following theoretical analysis shows the extent of this limitation, and the degree of brake effectiveness which should be expected from a disc brake of this type.

The theoretical analysis is based on Figure 34, and the symbols used are those defined on that page, plus others defined in the following text. All tangential forces acting on the brake shoes must be related to R_M as a base, and their magnitude or effect modified by the ratio of the radius at their point of action to the base radius R_M.

The effective Cam Force C_E is:

$$C_E = C \cdot \sin \alpha \cdot \frac{R_C}{R_M}$$

Friction forces and release spring forces are neglected here, since we are dealing with the Cam Force produced by the Net Pedal Force.

The effective Anchor Force A_E is:

$$A_E = A \cdot \frac{R_A}{R_M}$$

The Mean Effective Radius R_M is the equivalent radius at which a single total drag force fP would produce the same moment as is produced by the actual drag force fP distributed between R_i and R_o. This Mean Effective Radius can be found by integration of the drag torque relation produced by considering the shoe as a free body. If the shoe lining is a continuous circle or a series of circular arcs with radial edges, the unit pressure between lining and drum is equal to

$$\frac{P}{\pi \left(R_o^2 - R_i^2\right) \dfrac{\Theta}{360^\circ}}$$

where Θ is the total lining angle in degrees. The frictional force on an elemental ring or series of ring sectors at a radius R and of a width dR is equal to

$$f \cdot \frac{P}{\pi (R_o^2 - R_i^2)\frac{\theta}{360^\circ}} \cdot 2\pi R \cdot \frac{\theta}{360^\circ} \cdot dR$$

The frictional torque produced is:

$$R \cdot \frac{2fP}{(R_o^2 - R_i^2)} \cdot R \cdot dR$$

By definition of the Mean Effective Radius this friction torque is equal to $R_M \cdot fP$, so:

$$R_M fP = \int_{R_i}^{R_o} \frac{2fP}{(R_o^2 - R_i^2)} \cdot R^2 dR$$

or: $$R_M = 2/3 \; \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$$

This relation applies to a complete ring lining or to lining sectors having radial edges. If sectors with edges other than radial are used, another relation including the sector end configurations would have to be developed.

The normal forces P acting on the surface of the lining segments must be entirely sustained by the forces acting between the balls and the groove surfaces. If this combination of forces is to have a minimum tendency to cause curling of the shoes the radius to the center of the ball grooves should also be equal to R_M.

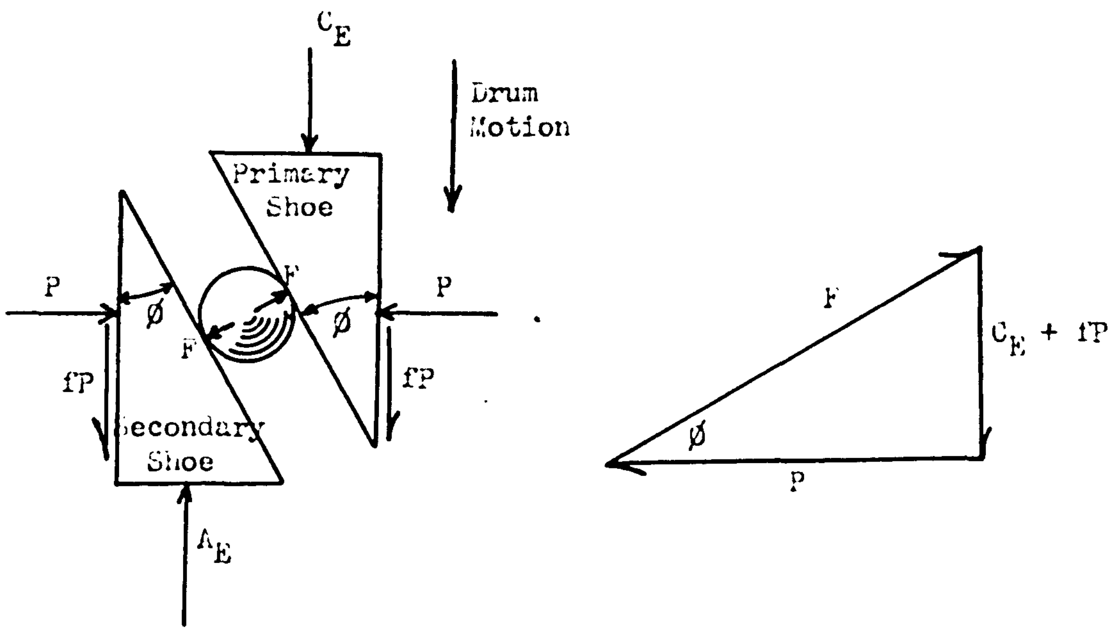


Figure 35. Annular Disc Shoe Forces

Figure 35 illustrates the way in which the various forces act on the primary and secondary shoes. The ball is able to exert a force F, perpendicular to the ramp or groove surface only. This fact allows a determination of the desired relation among f, P, C_E, and ø. From the force triangle at the right of Figure 35:

$$\tan \phi = \frac{C_E + fP}{P}$$

$$P \tan \phi = fP = C_E$$

$$\frac{P}{C_E} = \frac{1}{\tan \phi - f}$$

$$\frac{fP}{C_E} = \frac{f}{\tan \phi - f}$$

Substitution of the relation defining C_E produces:

$$\frac{fP}{C} = \frac{f}{\tan \phi - f} \cdot \frac{R_c \sin \alpha}{R_M}$$

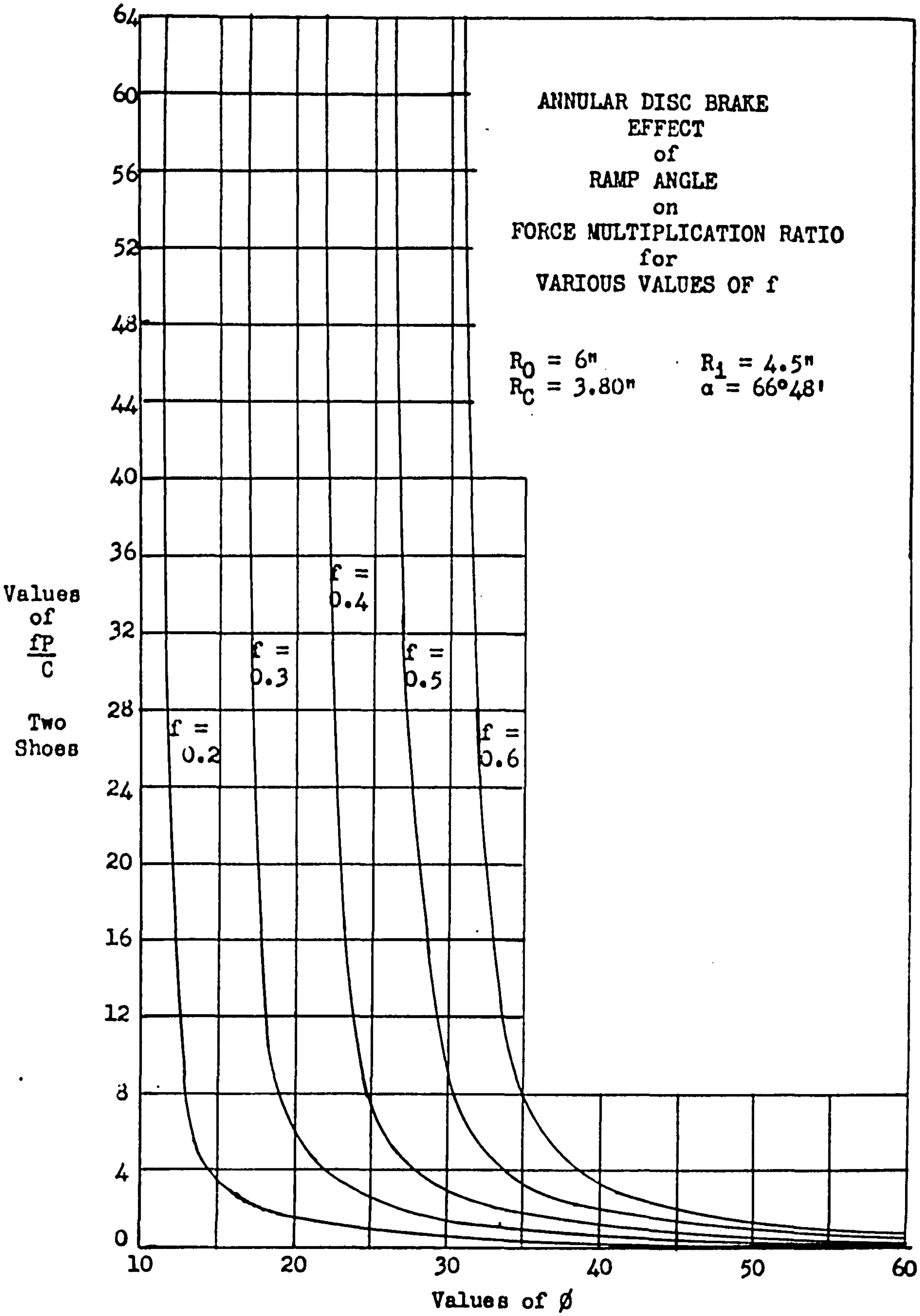

ANNULAR DISC BRAKE
EFFECT
of
RAMP ANGLE
on
FORCE MULTIPLICATION RATIO
for
VARIOUS VALUES OF f
R_0 = 6"
R_1 = 4.5"
R_C = 3.80"
α = 66°48'
Values of $\frac{fP}{C}$
Two Shoes
f = 0.2
f = 0.3
f = 0.4
f = 0.5
f = 0.6
Values of ø
0
4
8
12
16
20
24
28
32
36
40
44
48
52
56
60
64
10
20
30
40
50
60

Figure 36

This is the Force Multiplication Ratio, or brake effectiveness of this disc type brake. For a given physical size, the effectiveness of this type of brake can be varied over wide limits by changing f or ϕ or both. However, any combination of ϕ and f which makes the denominator $\tan \phi - f$ approach zero will make the force multiplication ratio approach infinity, or in other words produce a self-locking brake.

When the value of f is variable over reasonable limits, the value of ϕ should be chosen to avoid self-lock conditions at the highest expected value of f. This limitation on ramp angle imposed by variation in friction coefficient is illustrated graphically on Figure 36. The ramp angle value at which the curve for any given friction coefficient starts a nearly vertical ascent is the minimum safe ramp angle for that friction coefficient. Since the linings now in use do show values of f as high as 0.6 during hard usage (19), the ramp angle chosen for any actual brake must provide freedom from self-lock action at that coefficient. This limits the minimum ramp angle to some value above thirty one degrees. The value chosen should not be too far above this figure, however, or the effectiveness of the brake will be reduced. In the production brake from which these dimensions were taken the ramp angle was chosen as 32.5°, a value giving a slight margin of safety from self-lock for $f = 0.6$.

A disc brake of the type analyzed which is now in use has an outer lining radius R_o of six inches, an inner lining radius R_i of 4.5 inches, a cam force radius R_c of 3.8 inches, and a cam angle α of 66.8°, an anchor radius R_A of 4.0 inches, a ramp angle of 32.5°, and a total lining area of 66 square inches on two shoes. The Force Multiplication

Ratio which could be produced by this brake over the range of f is shown on Figure 37. The $\frac{fP}{C}$ value for our chosen comparison friction coefficient is only 1.6 for two shoes. The cam force C applied to these two shoes is the sum of the forces produced by two brake pistons, both acting on the same pair of shoes.

The size of the brake actuating pistons relative to the master piston determines the force which they will exert relative to the force applied to the master piston, or the Force Multiplication Ratio of the Brake Rigging. Since the construction of this brake differs so markedly from that of the drum brake, the possible value of the Brake Rigging Force Multiplication Ratio should be determined. As in the drum brake, the maximum value of this quantity is obtained by the ratio between effective pedal travel and necessary brake piston travel. The amount of clearance which must be maintained between the shoe linings and the drum can be assumed as .010 inches. The amount of shoe movement which must be allowed for lining wear depends upon the sensitivity of the automatic wear adjusting mechanism. If this is assumed to be .010 inches per shoe, the total maximum axial movement which the two shoes must make is equal to .040 inches.

The linear travel of the brake piston is converted into relative rotation of the shoes, and then into axial movement of the shoes as the balls roll up the ramps. By referring to Figures 34 and 35 the interrelation of these quantities can be determined.

$$\text{Tangential Shoe Travel at } R_c = \frac{\text{Piston Travel}}{\sin \alpha}$$

$$\text{Tangential Shoe Travel at } R_M = \frac{\text{Piston Travel}}{\sin \alpha} \times \frac{R_M}{R_C}$$

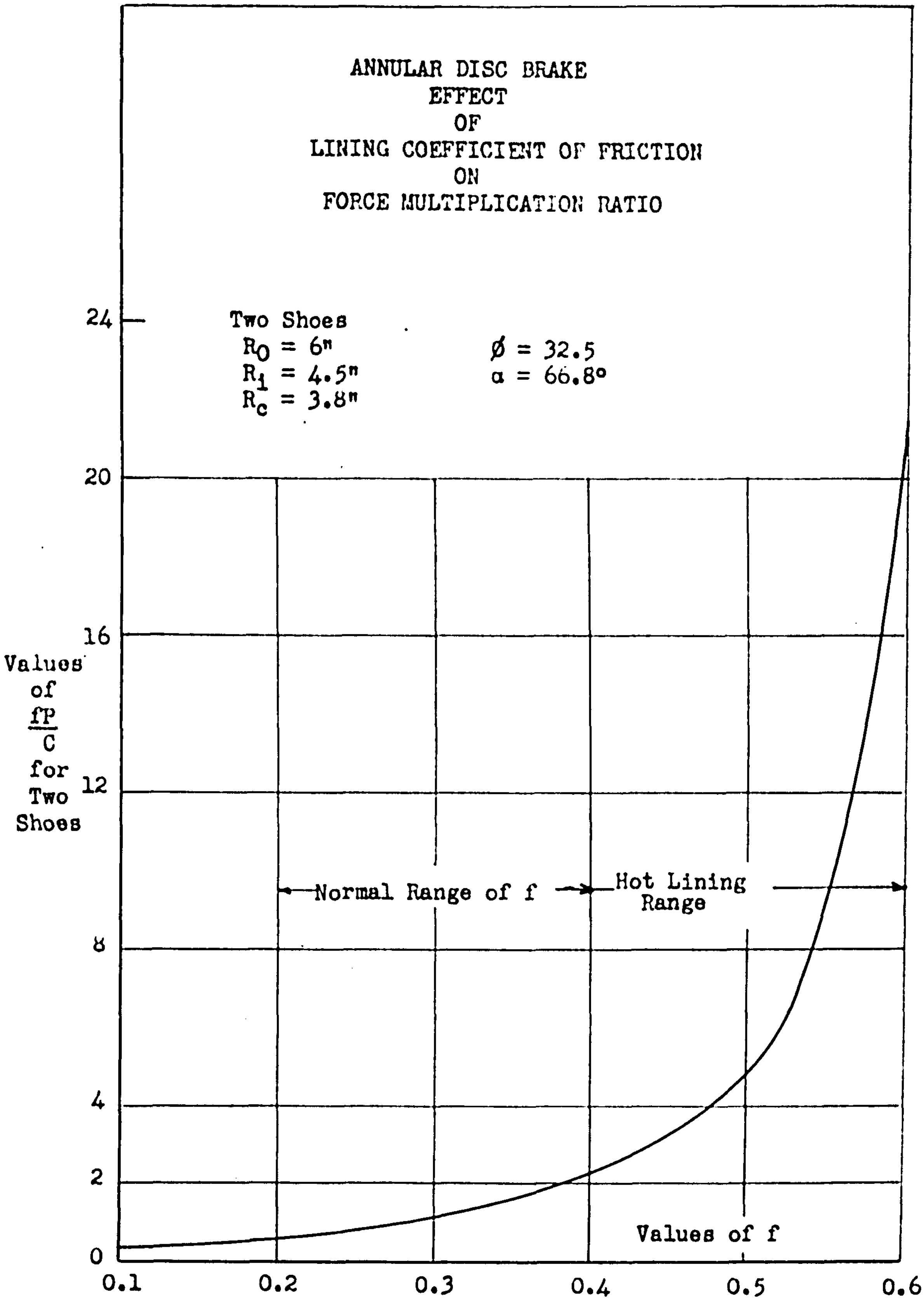

ANNULAR DISC BRAKE
EFFECT
OF
LINING COEFFICIENT OF FRICTION
ON
FORCE MULTIPLICATION RATIO
Two Shoes
R_0 = 6"
R_i = 4.5"
R_c = 3.8"
ø = 32.5
α = 66.8°
24
20
16
12
8
4
2
0
Values of fP/C for Two Shoes
Normal Range of f
Hot Lining Range
Values of f
0.1
0.2
0.3
0.4
0.5
0.6

Figure 37

Axial Shoe Movement = Tangential Shoe Travel at $R_M \cdot \tan \phi$

$$\text{Piston Travel} = \text{Axial Shoe Movement} \times \frac{R_C \sin \alpha}{R_M \tan \phi}$$

Using the value of .040 inches as determined above, and setting α = 66.8°, ϕ = 32.5°, R_C = 3.80", R_M = 5.28" the following relation develops:

$$\text{Piston Travel} = .040 \cdot \frac{3.80\ (.9192)}{5.28\ (.6371)} = .0415 \text{ inches}$$

If the maximum effective Pedal Travel is assumed to be five inches as before, then the maximum Brake Rigging Force Multiplication Ratio would be:

$$\text{Brake Rigging Force Multiplication Ratio} = \frac{5}{.0415} = 120$$

As installed on one passenger car, these brakes use one and one quarter inch diameter pistons on the front brakes, one inch diameter pistons on the rear brakes, and a one inch diameter master cylinder with a ratio of 8.2 between pedal travel and master cylinder piston travel. These dimensions produce a brake force distribution pattern of sixty one percent at the front brakes and thirty nine percent at the rear brakes. They also set the Brake Rigging Force Multiplication Ratio at 84, indicating either a greater clearance and wear allowance, or a shorter maximum effective pedal travel than was assumed in the previous calculations.

The disc brake as used thus can compensate for a smaller Cam to Drum Force Multiplication Ratio than the shoe brake by being able to use, with the help of an automatic wear adjuster, a higher Brake Rigging

Force Multiplication Ratio. Based on the values of 84 to 1 for the brake rigging, and an $\frac{fP}{C}$ of 1.60 for two shoes with f = 0.35, four of these brakes actuated by a net pedal force of 100 pounds would produce the following braking effect at the drums.

Total Brake Piston Force = 84 · 100 = 8400 pounds

8400 (0.60) = 5040 pounds Piston Force on Four Front Pistons

8400 (0.40) = 3360 pounds Piston Force on Four Rear Pistons

Total Braking Force = 5040 (1.60) [Front] + 3360 (1.60) [Rear]

= 8060 pounds Front + 5380 pounds Rear

= 13,440 pounds

This is a Total Brake Force Multiplication Ratio of 134.4, higher than that attained on any of the eleven inch diameter drum brakes. However, this disc brake was actually used to replace a 12-inch drum brake. For a more fair comparison of braking ability this value should be reduced by approximately the ratio of $\frac{4.80}{5.28}$, the ratios of the mean effective radii of eleven and twelve inch disc brakes, to make the brakes of comparable size, and then by the ratio of $\frac{4.80}{5.5}$, to put all brakes on a comparative effective brake torque radius. This operation produces an Equivalent Total Drag Force of 10,680 pounds. This puts the disc brake, <u>with</u> an automatic wear adjuster, at the same braking ability level as the Duo-Servo brake <u>without</u> an automatic wear adjuster.

Actual braking tests of the disc brake in comparison with a conventional pin-anchored shoe brake have indicated that the disc brake is capable of severe repeated braking efforts with much less loss of braking effectiveness than the shoe brake. On a high-speed heavy vehicle this ability would be of great importance.

SHOE BRAKE COMPARISONS

The main points of comparison among shoe brakes from the theoretical viewpoint are the Force Multiplication Ratio produced by the entire system, and the relative freedom from or sensitivity to the effects of change in the friction coefficient of the lining. The relative sensitivity of the shoe construction to the effects of excessive heating can be predicted in a general way from the shoe and anchor geometry, but no valid numerical comparison system can be deduced.

A comparison of the relative Force Multiplication Ratios for the several different types of shoe brakes is presented on Figure 38. The comparison is based on the drag force produced on an eleven inch diameter brake drum or its equivalent by a Net Pedal Force of one pound transmitted in its proper proportion to one complete brake. This method includes any ability a particular type may possess to utilize a high force ratio in the brake rigging.

The most obvious relationship shown by this comparison method is the difference in the degree to which the different brake types respond to high friction coefficients. The shape of the curves shown in the region between $f = 0.4$ and $f = 0.6$ are an indication of the stability of the brake against lining friction coefficient variations. Good vehicle control during braking depends on uniform action at all brakes, and a brake type which is highly sensitive to minor changes in lining friction coefficient could so unbalance the brake force picture during brake application that vehicle control would be adversely affected, if not lost completely (20). A highly stable brake, such as the double-forward, sliding-block anchored shoe brake is a desirable type.

One brake stability feature is somewhat masked by this graphical presentation. Brakes composed of one forward shoe and one reverse shoe get little braking effort from their reverse shoe. Most of the braking effort is obtained from the forward shoe, and its relative stability may be lower than that of the complete brake. A better indication of the stability of a brake type would be the ratio of the Force Multiplication Ratio of its more highly sensitive shoe at f = 0.2 to this same ratio at f = 0.6. When expressed as a percentage the magnitude of the numbers obtained is more easily comparable. If this quantity is christened Stability Factor, its definition would be:

$$\text{Brake Stability Factor} = \frac{\frac{fP}{C} \text{ at } f = 0.2}{\frac{fP}{C} \text{ at } f = 0.6} \times 100$$

For the various brake types analyzed, this Stability Factor has the following values.

Brake Type			Brake Stability Factor
Sliding-Block Anchor	$\frac{9.75\ (100)}{64.5}$	=	15.1
Articulated-Link Anchor (Forward Shoe)	$\frac{5.02\ (100)}{54.0}$	=	9.3
Pin-Anchor (Forward Shoe)	$\frac{4.47\ (100)}{88.2}$	=	5.1
Dual-Primary Brake	$\frac{6.00\ (100)}{134.6}$	=	4.5
Annular Disc Brake	$\frac{12.0\ (100)}{428}$	=	2.8
Duo-Servo Brake	$\frac{9.38\ (100)}{375}$	=	2.5

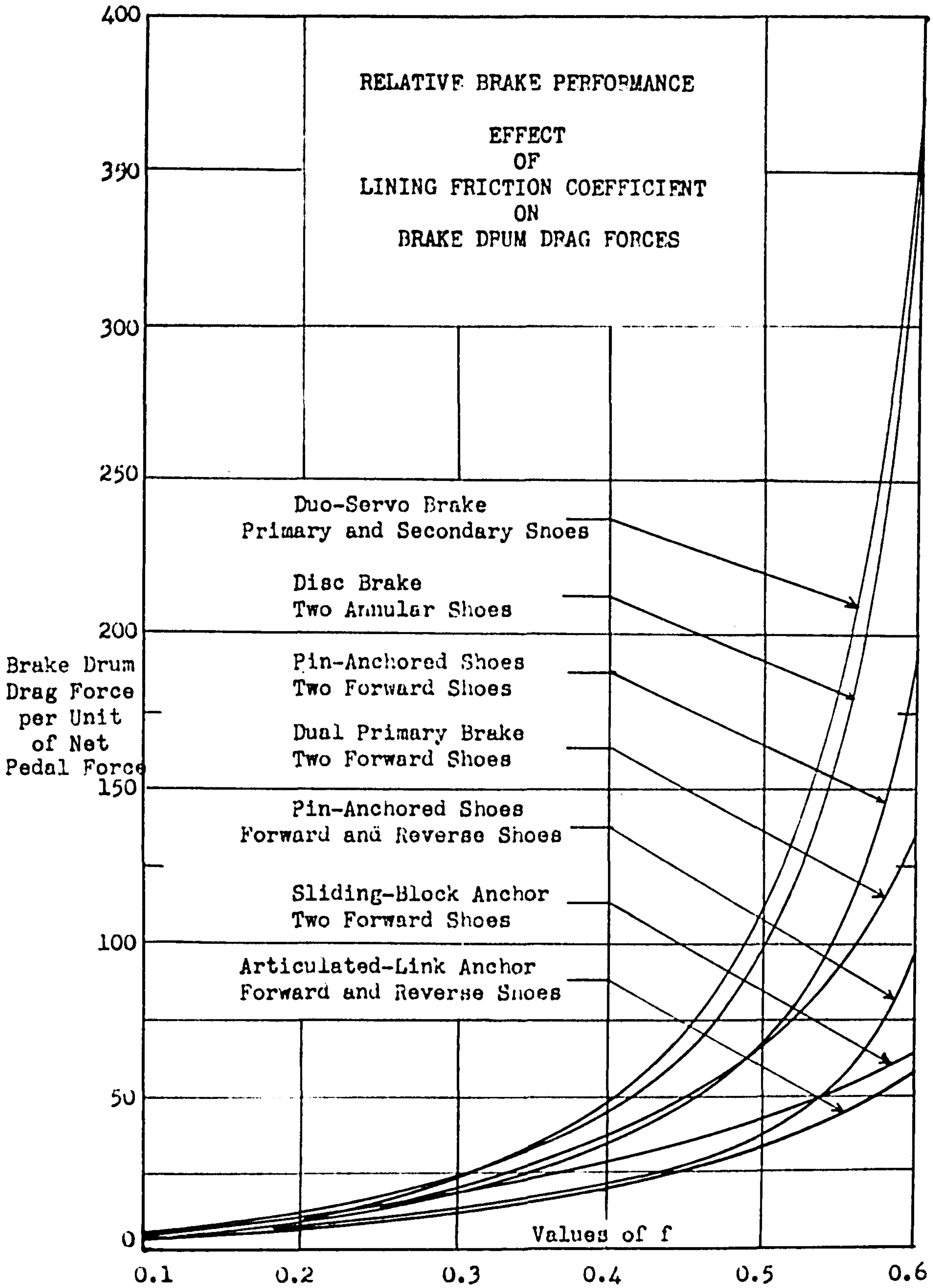

RELATIVE BRAKE PERFORMANCE
EFFECT OF LINING FRICTION COEFFICIENT ON BRAKE DRUM DRAG FORCES
Duo-Servo Brake Primary and Secondary Shoes
Disc Brake Two Annular Shoes
Pin-Anchored Shoes Two Forward Shoes
Dual Primary Brake Two Forward Shoes
Pin-Anchored Shoes Forward and Reverse Shoes
Sliding-Block Anchor Two Forward Shoes
Articulated-Link Anchor Forward and Reverse Shoes
Brake Drum Drag Force per Unit of Net Pedal Force
400
350
300
250
200
150
100
50
0
Values of f
0.1
0.2
0.3
0.4
0.5
0.6

Figure 38

The Brake Stability Factor would indicate a high degree of superiority in this respect for the sliding-block anchored shoe with the block angle at 90°. The Force Multiplication Ratio of this type of brake is not too high, but its superior stability under variable friction operation, and the ability of its shoes to center themselves during and after shoe "curl", should make this a desirable brake type for heavy duty power brakes. The low relative value of this factor for the Duo-Servo brake shows the effect of compounding shoe actions.

One factor which is not adequately brought out by the curves of Figure 38 is the relative braking ability of the several brake types in the intermediate friction coefficient range. The curves are closely bunched at this point with little actual difference in drum drag force. However, the numerical values are all so low that a slight numerical difference can represent an appreciable difference in relative braking ability. For instance, at f = 0.35, the drum drag force of the forward and reverse shoe, pin-anchored or link-anchored brake is seventeen, while that of the Duo-Servo brake is thirty two. This is a difference numerically of only fifteen, but it makes the brake ability of the Duo-Servo brake 1.88 times as great as that of the pin or link anchored brake. This fact is the main reason for the use of the Duo-Servo brake on our heavier passenger cars and on light trucks, rather than the more stable pin or link anchored brake types.

In the preceding analysis the width of the lining on a shoe brake was represented by the symbol "b". It entered the equations at various points, but in every case was cancelled from the relations before any significant result was obtained. Lining width on a shoe brake has no effect on Force Multiplication Ratio, Anchor Force, Drag Force, or Radial Force. Its importance lies in its effect on lining area, lining to drum pressure, and lining wear. Within limits set by drum and shoe distortion, an increase in lining width will reduce the wear rate of the lining for a given amount of service.

BRAKE FORCE DISTRIBUTION

Friction brakes acting on drums attached to the wheel hubs can resist rotation of the wheels, and, if they have sufficient braking ability, can actually stop the wheels from rolling and cause them to slide until the vehicle is brought to a halt. The frictional force between the wheel and the road surface actually stops the vehicle, and the optimum in stopping ability can be obtained only by proper distribution of braking torque among the weight carrying wheels of the vehicle.

The maximum amount of braking force which can be developed between a wheel and the road surface over which it travels is a function of the actual force which that wheel exerts normal to the road surface, and the coefficient of friction which can be developed between wheel surface and road surface. If a rolling wheel is subjected to a gradually increasing braking torque it develops a retarding force at the road surface which increases linearly with braking torque up to a maximum point and then drops to a lower value. The maximum retarding force occurs just before the wheel begins to slip over the road surface, and the coefficient of friction between wheel surface and road surface for this condition is called the coefficient of friction at impending slip, or the impending slip coefficient. The coefficient of friction computed from the retarding force produced when the wheel actually slides over the road surface is called the sliding coefficient of friction.

The numerical relation which exists between the impending slip coefficient and the sliding coefficient is not rigidly fixed. The retarding force which can be developed between a wheel and a road surface has been the subject of investigation by many parties, working with methods and conditions not always compatible with one another. The published data is voluminous, and must be carefully sorted to obtain a fair degree of agreement on methods and results. The coefficient of friction at impending slip for a pneumatic rubber tire with good non-skid tread operating over a dry concrete road surface varies with the smoothness and continuity of the road surface. It can be easily made greater than one on an open porous surface into which the tread rubber can penetrate. A good average value for a dense smooth concrete or macadam surface is 0.8 for the impending slip coefficient.

When a rubber tire is forced to slide over a road surface, the resulting coefficient of friction is a function mainly of the presence or absence of any material which could form a lubricating film between the tire and the road. When the material over which the tire is sliding is an unbound granular material, then the actual shear strength of the road material is the limiting factor for both the sliding and impending slip coefficients of friction. On high-shear-strength road surfaces, and in the absence of a lubricating film the ratio of

$$\frac{\text{sliding coefficient of friction}}{\text{impending slip coefficient of friction}}$$

has an average value of 0.9 for high friction surfaces, and 0.5 for low friction surfaces. The transition zone between these two values is not clearly defined, but a fair approximation is to apply the 0.9 value to surfaces with f = 0.7 or more, and the 0.5 value to surfaces with f = 0.6 or less.

If the maximum braking force which a wheel can develop occurs at impending slip of the wheel, then the maximum braking force for the whole vehicle is developed when every wheel which carries some of the weight of the vehicle is brought simultaneously to impending slip(21). To accomplish this requires that the braking force on the drums attached to these wheels must be carefully proportioned according to the weight being carried on each particular wheel. If the distribution of braking torque at the drums is anything other than this, then the maximum braking force will not be developed and the braking ability of the vehicle will be reduced to some value below its maximum level.

The distribution of a vehicle's weight among its several wheels is not a constant quantity, even if the positions of vehicle components and passengers remain fixed, but varies with the acceleration of gravity forces to which the vehicle is subjected. Thus the inclination of the road surface, and the deceleration of the vehicle produced by the brakes, causes the weight distribution pattern to change from the pattern produced when the vehicle is at a standstill on level road. Fortunately the magnitude of the weight shift which occurs during braking can be accurately predicted, and the effect of various braking torque distribution patterns on the stopping ability of the vehicle determined (22).

A very good way of showing the effect of braking torque distribution on vehicle stopping ability is to determine the magnitude of the quantity "Relative Braking Effectiveness", which is defined as the ratio of the actual braking force developed at the wheel to road surface, to the braking force which would be developed if all wheels were brought simultaneously to impending slip. The latter quantity is equal to fW cos θ, where f is the tire to road coefficient of friction at impending slip, W is the weight of the vehicle, and θ is the angle which the road surface makes with a level surface. The quantity W cos θ is actually the component of the gravity force on the vehicle which acts perpendicular to the road surface, and is properly known as the Adhesive Weight, since it is this force which helps the vehicle to adhere to the road surface.

If the actual braking force which can be produced by the wheels of a vehicle is known or can be computed, and is then compared with fW cos θ, the Relative Braking Effectiveness of the combination is obtained. Relative Braking Effectiveness is most conveniently expressed as a percentage.

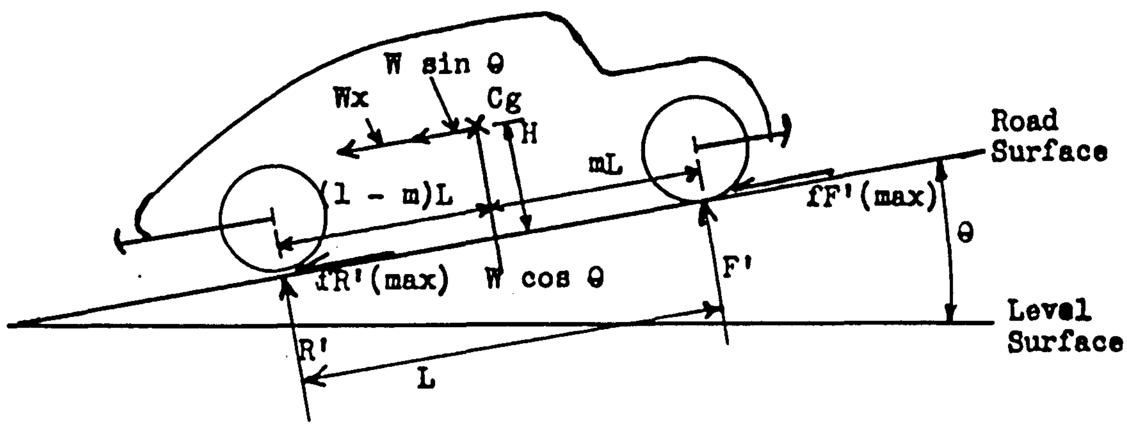
The vehicle factors most actively affecting the Relative Braking Effectiveness are the friction coefficient, the fore and aft weight distribution of the vehicle, the side to side brake drum torque distribution. The side to side weight distribution of our vehicles under static conditions, and also when the vehicle is subjected to forces parallel to its fore and aft axis, is usually uniform. In order to obtain a minimum tendency toward deviation and rotation during braking, the side to side brake drum torque distribution should follow the side to side weight distribution. Adopting even proportioning for side to side distribution of both weight and brake drum torque leaves us with three quantities as the determining variables for Relative Braking

Effectiveness, namely, coefficient of tire to road friction, or f, fore and aft weight distribution, or m, and fore and aft brake drum torque distribution, or r.

To simplify the mathematical manipulation of the equations relating these three quantities, the symbols on Figure 39 are used.

The fore and aft brake drum torque distribution can be controlled by the designer by using different hydraulic piston sizes. The quantity r, defined on Figure 39 as the decimal of the total brake drum torque produced by the rolling rear wheels, is fixed by the designer in this way, and once it has been built into the vehicle it cannot be conveniently changed. With varying vehicle loads and varying friction coefficients the chosen value may not always be the best one. Too high a value of r will cause premature skidding of the rear wheels, lowering the Relative Braking Effectiveness, while too low a value of r will cause premature skidding of the front wheels, again lowering the Relative Braking Effectiveness.

The following analysis shows the inter-relation among f, m, r, P_{max}, and E. There are six possible braking conditions which must be investigated. They are: (1) rear wheels at impending slide, with front wheels below impending slide and controlled by r; (2) front wheels at impending slide, with rear wheels below impending slide and controlled by r; (3) rear wheels sliding, front wheels at impending slide; (4) front wheels sliding, rear wheels at impending slide; (5) all wheels sliding; (6) all wheels at impending slip. By using the sketch on Figure 39, and treating the vehicle as a free body, equations can be written for the summation of forces parallel to the road,



VEHICLE BRAKING FORCES

W = Vehicle Weight
Θ = Grade Angle
W cos Θ = Adhesive Weight (Normal Component of W)
W sin Θ = Grade Force (Tangential Component of W)
x = Ratio of Linear Vehicle Acceleration to Gravity Acceleration
Wx = Acceleration Force Acting on Vehicle
Cg = Center of Gravity
H = Height of Cg Above Road Surface
L = Wheelbase of Vehicle
m = Decimal of Adhesive Weight on Rear Wheels, Vehicle Standing on Level Road
R = Adhesive Weight on Rear Wheels, Static Level Conditions
F = Adhesive Weight on Front Wheels, Static Level Conditions
m' = Decimal of Adhesive Weight on Rear Wheels under Dynamic Conditions
R' = m'W cos Θ
F' = (1 - m')W cos Θ
f = Coefficient of Friction, Tire to Road, at Impending Slip
P_{max} = Maximum Braking Force
E = RELATIVE BRAKING EFFECTIVENESS
r = Decimal of Total Brake Drum Torque Produced by Rolling Rear Wheels
μ = Ratio of Sliding Friction Coefficient to Impending Slip Friction Coefficient

Figure 39

summation of forces perpendicular to the road, and summation of moments about the front wheel road contact point. These equations can be manipulated to provide relations for Relative Braking Effectiveness (E) and Maximum Braking Force (P_{max}) in terms of determinable quantities and dimensions.

For the first case, in which the rear wheels are at impending slide, with the front wheels below impending slide and controlled by r, the equations are developed as follows:

Summation of Forces Perpendicular to Road

$$R' + F' = W \cos \theta$$

By Definition: $R' = m'W \cos \theta$ $\qquad F' = (1 - m')W \cos \theta$

Rear Wheel Braking Force = fR'

Front Wheel Braking Force is related to the rear wheel braking force by the r factor built into the brake rigging, which by definition produces an r portion of the total braking force at the rolling rear wheels, and a 1 - r portion at the rolling front wheels. The front wheel braking force will then be related to the rear wheel braking force by the ratio $\frac{1 - r}{r}$.

$$\text{Front Wheel Braking Force} = fR'\left(\frac{1 - r}{r}\right)$$

$$\text{Maximum Braking Force } P_{max} = fR'\left(1 + \frac{1 - r}{r}\right) = fR' \cdot \frac{1}{r}$$

By definition the Relative Braking Effectiveness is the ratio of P_{max} to the braking force which would be produced with all wheels simultaneously at impending slip.

$$\text{Relative Braking Effectiveness} = \frac{fR'\frac{1}{r}}{fW \cos \theta} = \frac{R'}{rW \cos \theta} = \frac{m'}{r}$$

The quantity m' must now be obtained in terms of the physical dimensions of the vehicle.

Summation of Forces Parallel to Road.

$$0 = P_{max} + W(x + \sin \theta)$$

$$= P_{max} = -W(x + \sin \theta)$$

Summation of Moments about Front Wheel Contact

$$0 = R'L - HW(x + \sin \theta) = mLW \cos \theta$$

By substitution:

$$0 = m'LW \cos \theta + H \cdot P_{max} - mLW \cos \theta$$

Divide by LW cos θ, and substitute $P_{max} = \frac{f}{r} \cdot m'W \cos \theta$

$$0 = m' + m'\frac{fH}{L} \cdot \frac{1}{r} - m$$

$$m'(1 + \frac{1}{r} \cdot \frac{fH}{L}) = m$$

$$m' = \frac{mr}{r + f\frac{H}{L}}$$

Then: $P_{max} = \frac{f}{r} \cdot \frac{mr}{r + f\frac{H}{L}} W \cos \theta = \frac{fmW \cos \theta}{r + f\frac{H}{L}}$ $\qquad E = \frac{m}{r + f\frac{H}{L}}$

The second set of conditions, with front wheels at impending slip and rears controlled by r, can be solved in the same way and will produce a similar set of equations. The third condition, with rear wheels sliding and front wheels at impending slip, must be handled in a slightly different manner.

Let $\mu = \dfrac{\text{sliding coefficient of friction}}{\text{impending slip coefficient of friction}}$

The Rear Wheel Braking Force is now $\mu fR'$ and the Front Wheel Braking Force is fF'.

$$P_{max} = \mu fR' + fF'$$

The Summation of Moments equation is:

$$0 = R'L - HW\,(x + \sin\theta) \quad - \quad mLW\cos\theta$$

$$0 = m'LW\cos\theta + H\mu fm'W\cos\theta + Hf(1 - m')W\cos\theta - mLW\cos\theta$$

$$0 = m'(1 + \mu f\frac{H}{L} - f\frac{H}{L}) + f\frac{H}{L} - m$$

$$m' = \frac{m - f\frac{H}{L}}{1 - f\frac{H}{L} + \mu f\frac{H}{L}} = \frac{m - f\frac{H}{L}}{1 - (1 - \mu)f\frac{H}{L}}$$

$$P_{max} = f\left[\mu m'W\cos\theta + (1 - m')W\cos\theta\right]$$
$$= fW\cos\theta\left[1 - (1 - \mu)m'\right]$$

$$P_{max} = fW\cos\theta\,\frac{1 - f\frac{H}{L} + \mu f\frac{H}{L} - m + f\frac{H}{L} + \mu m - \mu f\frac{H}{L}}{1 - (1 - \mu)f\frac{H}{L}}$$

$$P_{max} = fW\cos\theta\,\frac{1 - (1 - \mu)m}{1 - (1 - \mu)f\frac{H}{L}}$$

$$E = \frac{1 - (1 - \mu)m}{1 - (1 - \mu)f\frac{H}{L}}$$

The fourth set of conditions is treated in a similar manner and produces a similar but slightly altered set of relations for P_{max} and E. The fifth set of conditions, all wheels sliding, produces the following relations:

$$P_{max} = \mu fR' + \mu fF' = \mu fW\cos\theta(m' + 1 - m') = \mu fW\cos\theta$$

$$E = \frac{P_{max}}{fW\cos\theta} = \frac{\mu fW\cos\theta}{fW\cos\theta} = \mu$$

The sixth set of conditions, with all wheels at impending slip, is the condition which produces maximum braking force.

$$P_{max} = fR' + fF' = fW \cos \theta$$

$$E = \frac{fW \cos \theta}{fW \cos \theta} = 1.0$$

The resulting equations for P_{max} and E for all six sets of equations are now presented in a group for easier comparison.

TABLE III

BRAKING ABILITY RELATIONS

Maximum Braking Force P_{max}	Relative Braking Ability E

Case 1. Rear wheels at impending slip, front wheels rolling and controlled by r.

$$P_{max} = \frac{fmW \cos \theta}{r + f\frac{H}{L}} \qquad E = \frac{m}{r + f\frac{H}{L}}$$

Case 2. Front wheels at impending slip, rear wheels rolling and controlled by r.

$$P_{max} = fW \cos \theta \frac{1 - m}{1 - r - f\frac{H}{L}} \qquad E = \frac{1 - m}{1 - r - f\frac{H}{L}}$$

Case 3. Rear wheels sliding, front wheels at impending slip.

$$P_{max} = fW \cos \theta \frac{1 - (1 - \mu)m}{1 - (1 - \mu)f\frac{H}{L}} \qquad E = \frac{1 - (1 - \mu)m}{1 - (1 - \mu)f\frac{H}{L}}$$

Case 4. Front wheels sliding, rear wheels at impending slip.

$$P_{max} = fW \cos \theta \frac{\mu + (1 - \mu)m}{1 + (1 - \mu)f\frac{H}{L}} \qquad E = \frac{\mu + (1 - \mu)m}{1 + (1 - \mu)f\frac{H}{L}}$$

Case 5. All wheels sliding.

$$P_{max} = \mu fW \cos \theta \qquad E = \mu$$

Case 6. All wheels at impending slip.

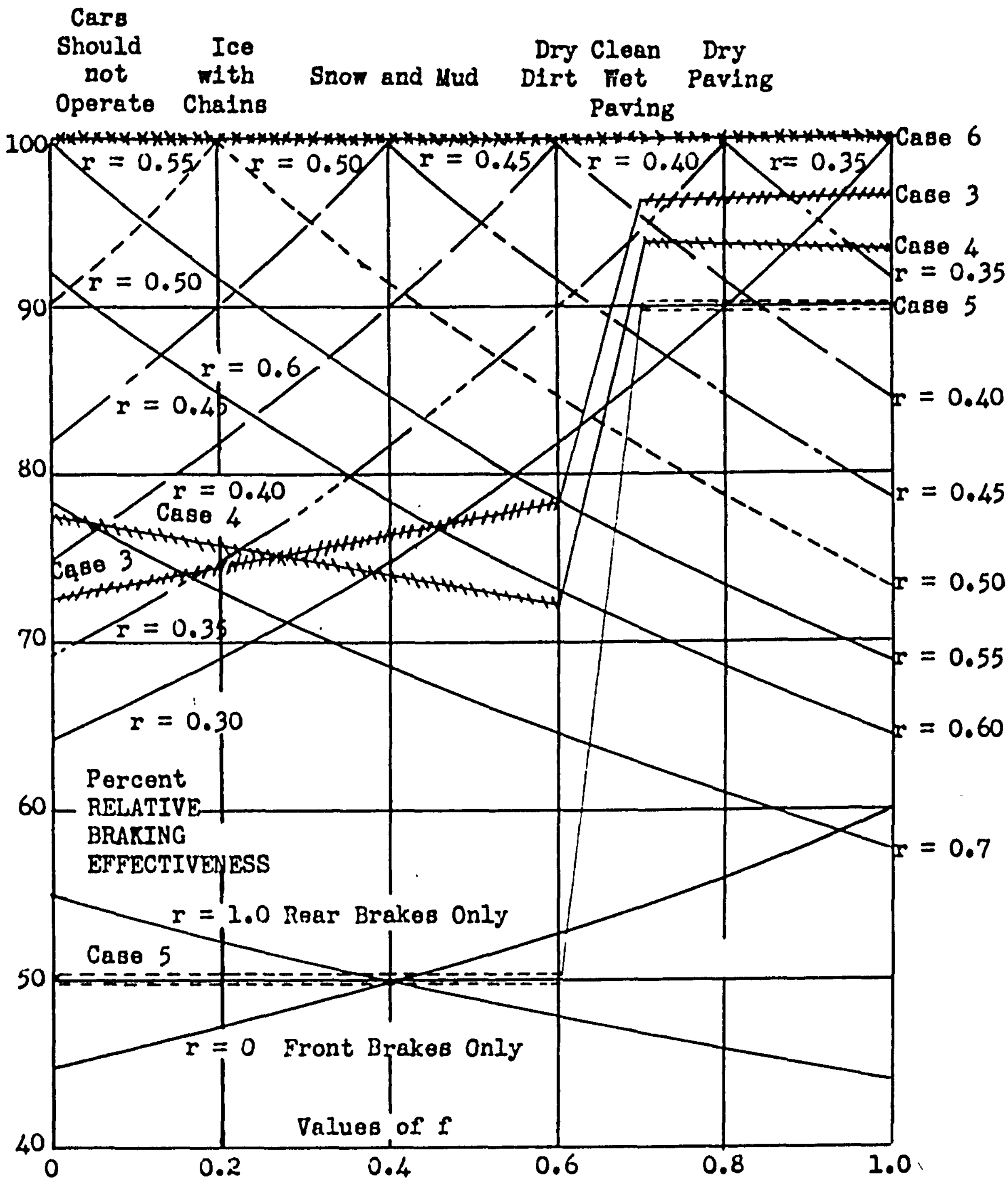
$$P_{max} = fW \cos \theta \qquad E = 1.0$$

If each individual brake on a vehicle operates as it is expected to operate, and there is no sidewise shift of weight in the vehicle and the road surface under all wheels is uniform, then the actual braking condition produced must fall into one of the six cases analyzed. While the equations are correct and complete, it is difficult to grasp the full significance of the relative effect of each variable on the braking performance. To bring out more clearly the effect of variation in friction coefficient f, and variation in braking torque distribution r, a numerical solution of these equations for a modern passenger car is presented in Figure 40.

A modern five passenger sedan body with its full complement of passengers will have a value of m of approximately 0.55. The center of gravity will be approximately at the level of the wheel tops, giving a value of $\frac{H}{L} = \frac{30}{120} = 0.25$. Cases 1 and 2 create a series of sloping lines for different values of r. Cases 3 and 4 create a pair of lines which have a discontinuity between f = 0.6 and f = 0.7, due to the change in the value of μ in this range of friction coefficients. Case 5 likewise is discontinuous due to the change in μ . Case 6 is the ideal toward which all braking systems are designed.

Figure 40 may be used to indicate the desirable choice of r for a vehicle of this type. Rear wheel brakes only, r = 1.0, or front wheel brakes only, r = 0, are undesirable in that they produce a very low relative braking effectiveness. With brakes on all four wheels, a value of r = 0.35 (65% front, 35% rear) gives about the best dry-paving braking effectiveness, but the effectiveness on the lower coefficients of friction is not as good as a value of r = 0.4 or 0.45. With any normal

RELATIVE BRAKING EFFECTIVENESS
OF A FOUR-WHEELED VEHICLE



Loaded Passenger Car with m = 0.55, $\frac{H}{L} = 0.25$

m = Decimal of Car weight on Rear Wheels, Level Static Conditions
r = Decimal of Braking Torque at Rear Wheels, Wheels Rolling
Cases 1 and 2. All Wheels Rolling, Controlled by r
Case 3. Rear Wheels Sliding, Front Wheels at Impending Slip
Case 4. Front Wheels Sliding, Rear Wheels at Impending Slip
Case 5. All Wheels Sliding
Case 6. All Wheels at Impending Slip

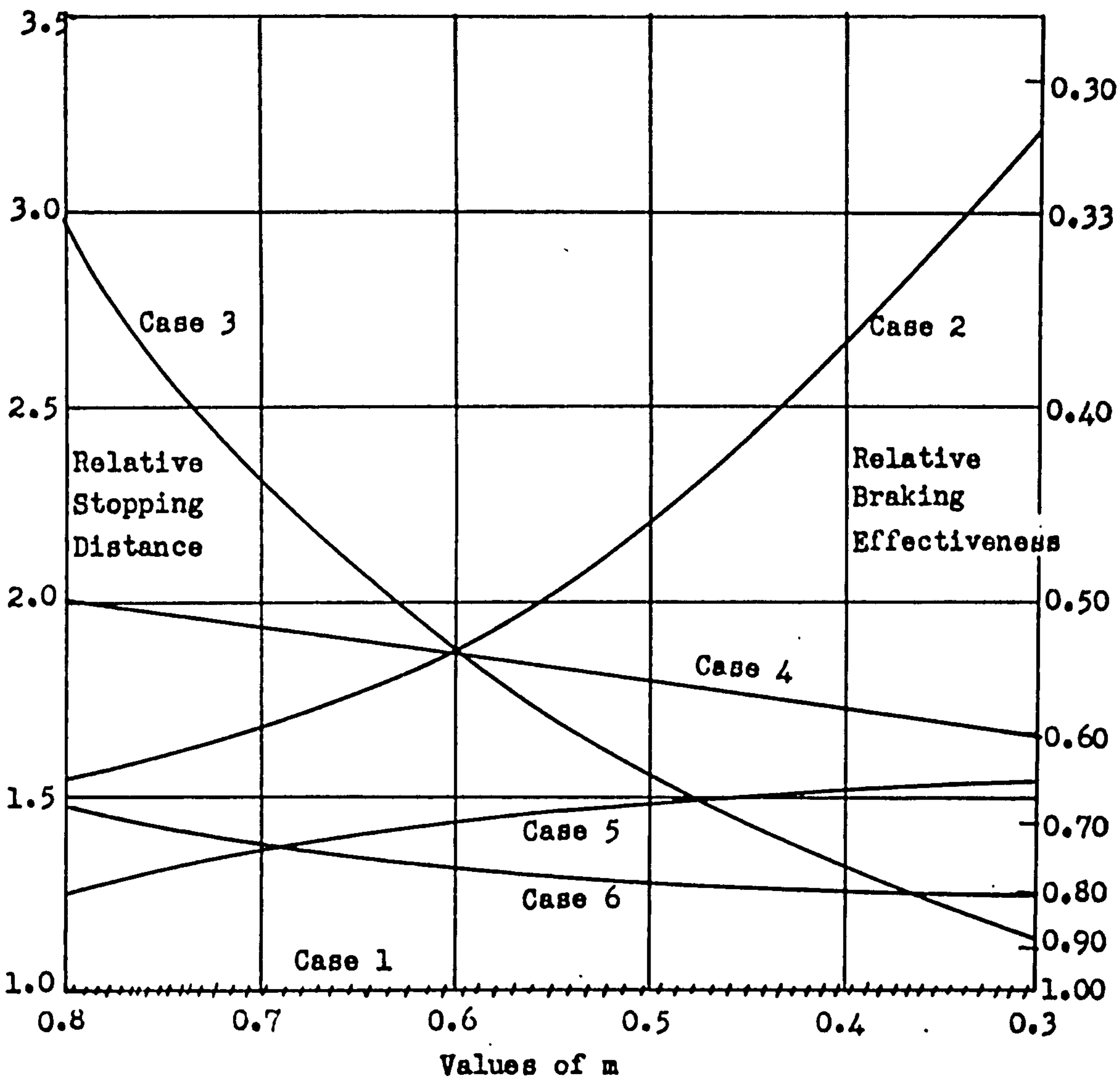
Figure 40

value of r, pushing hard enough on the brake pedal to lock up and slide front wheels or rear wheels or all wheels will produce a Relative Braking Effectiveness of 90% or better on dry paving. This is sufficient to produce really acceptable stopping ability. However, locking wheels on low friction coefficient paving causes a drop in Relative Braking Effectiveness, under conditions in which the low frictional force should be counterbalanced by a high braking effectiveness to produce even a moderately short stop. The designer may thus choose to use a value of r in the vicinity of 0.4 or 0.45 rather than 0.35, knowing that careful handling of the brake pedal on low friction coefficient surfaces can produce high braking effectiveness, while vigorous pushing of the pedal can get equally high braking effectiveness on high friction coefficient surfaces by locking and sliding the wheels. Once chosen and built into the brake rigging the value of r cannot easily be changed, and the designer must use his judgment to obtain the best overall performance of which the braking system is capable. Typical passenger car values for r vary from 0.39 to 0.45.

The effect of m is not directly shown by Figure 40. An increase in the value of m puts more weight on the rear wheels and less on the front, and to maintain the same relative braking effectiveness as shown on Figure 40, r should be changed in about the same proportion as m.

The effect of m on the Relative Stopping Distance of a car with various combinations of sliding and freely rolling wheels was investigated on models by the National Physical Laboratory (23). The results of their work are shown on Figure 41, with a scale of Relative Braking Effectiveness added. This figure shows very effectively the value of

RELATIVE BRAKING EFFECTIVENESS



Four Wheeled Model with Wheels Locked and Sliding or Free and Rolling

Case 1. All Wheels Locked

Case 2. Both Rear Wheels Locked

Case 3. Both Front Wheels Locked

Case 4. One Front and Diagonally Opposite Rear Locked

Case 5. One Front and Both Rear Wheels Locked

Case 6. One Rear and Both Front Wheels Locked

m = Decimal of Vehicle Weight on Rear Wheels, Static Level Conditions

Figure 41

having brakes on all weight carrying wheels, and of putting the brakes on those wheels carrying the most weight if not all wheels are to be braked. The shift of weight from rear wheels to front wheels under the impetus of braking deceleration makes the front wheels more effective than the rears in all cases except where the weight is predominantly on the rears. The superiority of Case 3 (front wheels locked, rears rolling) over Case 2 (rear wheels locked, fronts rolling) up to a value of m = 0.6 indicates this. The superiority of Case 6 (one rear and both front wheels locked, one rear rolling) over Case 5 (one front and both rear wheels locked, one front rolling up to a value of m = 0.68 also shows this.

Valuable information on the Relative Tendency toward Deviation and Rotation of a braked vehicle as affected directly by m was obtained from this same work, and the results are shown on Figure 42. Once again the superiority of brakes on all wheels is apparent. Also clearly shown is the noticeably greater deviation and rotation which are produced whenever the rear wheels are caused to slide, as in Cases 2 and 5, and to a lesser extent Case 4. A sliding or skidding wheel has no directional sense, and will slide in one direction as easily as another. A rolling wheel will develop a Cornering Force, or resistance to motion perpendicular to its plane of rotation, if any effort is made to move it in a direction other than parallel to its plane of rotation. Rolling rear wheels will resist deviation of the vehicle from a straight path, either by skidding sidewise or by rotating about a vertical axis through the center of gravity. Sliding rear wheels generate no more resistance to either deviation or rota-

RELATIVE DEVIATION AND ROTATION

Four Wheel Model, with Wheels either Locked and Sliding or Free and Rolling

Case 1. All Four Wheels Locked

Case 2. Both Rear Wheels Locked

Case 3. Both Front Wheels Locked

Case 4. One Front and Diagonally Opposite Rear Locked

Case 5. One Front and Both Rear Wheels Locked

Case 6. One Rear and Both Front Wheels Locked

m = Decimal of Vehicle Weight on Rear Wheels
Static Level Conditions

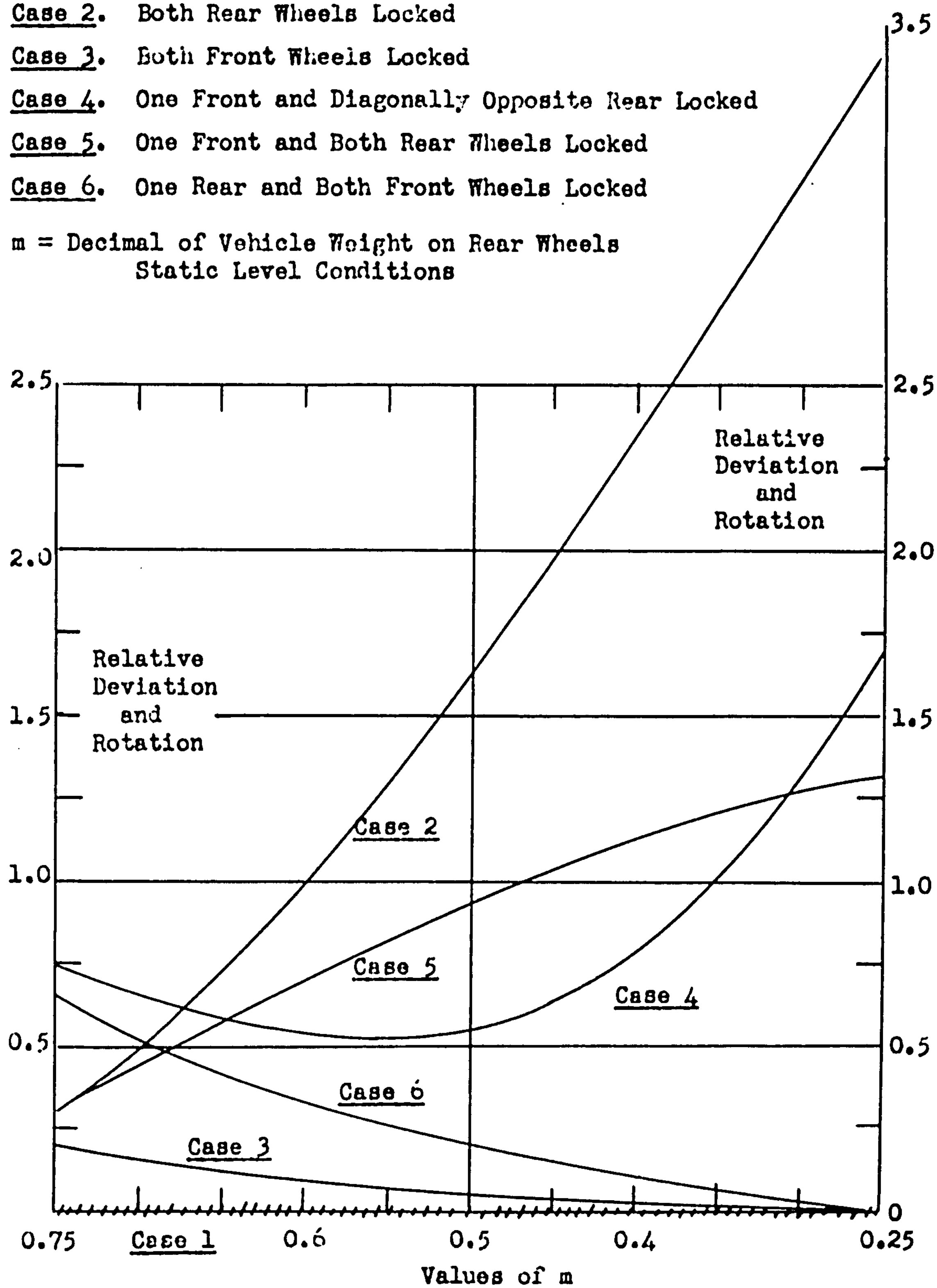


Figure 42

tion than they do to motion straight ahead, and a skid once started becomes progressively worse. Additional information presented here was obtained by braking various wheel combinations of trucks operated on the ice surface of a frozen Wisconsin lake just recently (24). The evidence against skidding rear wheels is so conclusive that any brake force distribution pattern which would lead to repeated locking of the rear wheels should be considered very carefully before being adopted.

STOPPING DISTANCE

According to nomenclature adopted by a technical committee of the Society of Automotive Engineers the Stopping Time or Stopping Distance is the time elapsed or distance travelled between the instant or point at which the driver has an opportunity to perceive a demand for braking and the instant or point at which the vehicle comes to rest. This time or distance is then separated into several component parts, the Driver Perception-Reaction Time or Distance, and the Vehicle Stopping Time or Distance. The latter quantity is composed of the Brake System Application Time or Distance and the Braking Time or Distance.

A simple equation which will give with a fair degree of accuracy the Stopping Distance of a braked automotive vehicle is a great convenience if not a necessity. It is possible, by progressive simplification and approximation to produce such a Stopping Distance equation with an order of accuracy about the same as that with which the various factors affecting stopping distance can be determined.

The moving vehicle has a certain amount of kinetic energy due to its velocity. This kinetic energy, plus any change in potential energy due to change in elevation during the stop, must be dissipated by the brakes and by vehicle losses in bringing the vehicle to a stop.

The kinetic energy of the moving vehicle is the sum of the translational kinetic energy of the whole vehicle and the rotational kinetic energies of the wheels and the engine-clutch-transmission assembly. The latter two quantities amount to approximately three percent and three and one half percent respectively of the translational kinetic energy. During an emergency stop the brakes themselves absorb the rotational kinetic energy of the wheels without any assistance from the braking force at the tire road contact surface. The rotational kinetic energy of the engine-clutch-transmission assembly is either absorbed by the driving-wheel brakes, or if the clutch is released the energy is dissipated in overcoming internal friction. This leaves the braking forces at the tire-road surface only the translational kinetic energy of the vehicle to overcome.

$$\text{Translational Kinetic Energy} = \frac{1}{2} mv^2$$

or using speed in units of miles per hour, designated by the symbol M:

$$KE = m\frac{(1.467M)^2}{2} = WM^2 \frac{2.15}{2(32.2)} = \frac{WM^2}{30}$$

In addition to the tire-road braking force, the motion of the vehicle will also be decreased by tire rolling resistance, wheel bearing losses, and vehicle wind resistance. If the brakes are applied hard enough to lock up and slide the wheels, the tire rolling resistance and wheel bearing losses disappear completely. If the wheels continue to roll, then the magnitude of these two factors in comparison with the braking force must be investigated.

Measurements of tire rolling resistance and wheel bearing loss on good smooth road produce figures varying from 0.012 W cos θ to 0.015 W cos θ.

Wind resistance of the vehicle is approximately $0.04M^2$ for our present passenger cars.

If the distance in feet which the vehicle travels to a stop during brake application is designated by S then the energy dissipated is as follows:

$$\text{Dissipated Energy} = S\left[EfW\cos\theta + W\sin\theta + .015W\cos\theta + \frac{1}{3}(.04M^2)\right]$$

The first term, $EfW \cos \theta$, is the braking force produced at the road surface. The second term, $W \sin \theta$, is the grade force due to an increase in potential energy as the vehicle moves up a grade. If the vehicle goes down hill during a stop, $\sin \theta$ is negative and the dissipated energy is decreased. The third term, $.015\ W \cos \theta$ is the tire rolling resistance and wheel bearing force. The fourth term, $\frac{1}{3}(.04M^2)$ is the distance-averaged wind resistance force during the stop.

The energy to be dissipated is $\frac{WM^2}{30}$. By equating these two quantities, the relative magnitudes of the factors involved can be determined:

$$S\left[EfW\cos\theta + W\sin\theta + .015\ W\cos\theta + \frac{1}{3}(.04M^2)\right] = \frac{WM^2}{30}$$

The magnitude of Ef will be from a minimum of ten to a maximum of seventy times as great as the coefficient .015 of the third term. If this term is not removed completely by locking and sliding the wheels, it can be neglected without greatly damaging the accuracy of the equation.

The wind resistance term must in some way be removed from the equation if it is to be appreciably simplified. Tests for friction coefficients between a sliding tire and the road have repeatedly shown that the friction coefficient decreases as relative tire-road speed increases (25). The magnitude of the decreases in Braking Force due to this drop in f with speed will be approximately the same as the magnitude of the increase in $\frac{1}{3}(.04M^2)$ with speed. If the low speed value of f is used in stopping distance calculations for all speeds, the error in f will be just about counteracted by the omission of the wind resistance term. If these approximations are allowed the equation becomes:

$$S\ (EfW \cos\theta + W \sin\theta) \doteq \frac{WM^2}{30}$$

$$\text{or } S \doteq \frac{M^2}{30(Ef \cos\theta + \sin\theta)}$$

For the grades encountered in highway work cos θ is practically equal to 1.0, and sin θ $\doteq$ tan θ. Since the definition of <u>Decimal</u> <u>Grade</u> <u>G</u> is the tangent of the grade angle θ, the equation can be written in final form.

$$S \doteq \frac{M^2}{30(E\ f + G)}$$

This gives the braking distance in feet after maximum brake application, as a function of initial speed M, decimal grade G, tire to road friction coefficient f, and Relative Braking Effectiveness E. It is admittedly an approximate equation, but its order of accuracy should be as good as the ability of an observer to determine f and E.

Various authorities have released empirical braking distance equations. One, by the Bureau of Standards, based upon many tests of actual car braking performance is $S = \frac{M^2}{22}$ for braking on level dry paving. Compared with the equation just presented this means that 22 = 30(Ef + 0).

If f is taken as 0.8 for dry paving operation then $F = \frac{22}{.8 \cdot 30} = \frac{22}{24} = 0.917$. Reference to Figure 39 shows that this value of F at f = 0.8 can be attained easily by several combinations of brake action. With careful choice of E and f, this equation should give a quick, easy, and reasonably accurate indication of the stopping idstance after brake application.

The human body does not respond immediately when its eyes perceive some emergency requiring action. There is a measurable lag between the perception of a condition requiring brake application and the actual application of the brakes by the driver's foot. This period, known as the Perception-Reaction Time, or usually just Reaction Time, varies from person to person, and even with a given individual it varies widely with physical condition with such factors as fatigue, intoxication, inattention, etc., increasing the reaction time appreciably above its normal value. Many series of tests have shown that the overall average reaction time for adult drivers is 0.75 seconds. This means that it takes the average driver three-quarters of a second to begin applying the brakes after he sees a condition requiring brake application.

During the reaction time the vehicle is proceeding with unchecked speed, rapidly decreasing the distance between it and trouble. Based on an average reaction time of three-quarters of a second, the reaction distance is as follows:

Reaction Distance = 0.75 (1.467M) = 1.1M feet.

The complete Stopping Distance equation is then:

$$\text{Stopping Distance } S = 1.1M + \frac{M^2}{30(Ef + G)}$$

To illustrate the relative magnitudes of Stopping Distance, Braking Distance, and Reaction Distance for a typical passenger car with m = 0.55, $\frac{H}{L}$ = 0.25, and r = 0.4, on level paving (f = 0.8) and on level ice with chains (f = 0.2) the Table IV is presented.

TABLE IV

STOPPING DISTANCE AS AFFECTED BY SPEED

Initial Speed M	10	20	40	60	80	100
Level Dry Paving						
Reaction Distance	11	22	44	66	88	110
Braking Distance	4.6	18.2	73	164	291	454
Stopping Distance	15.6	40.2	117	230	379	564
Level Ice with Chains						
Reaction Distance	11	22	44	66	88	110
Braking Distance	20.3	81	324	730	1298	2025
Stoppin Distance	31.3	103	368	796	1386	2135

The Stopping Distance in this table is the clear sight distance which a driver should maintain ahead of his vehicle at all times if he expects to be able to stop before hitting an obstacle in his path. At low speeds on clean dry paving the Reaction Distance is the major part of the total Stopping Distance required. The vehicle designer can do little about reaction time except to provide the driver with good visibility, comfortable seating, convenient brake pedal location, and an adequate supply of fresh air at the proper temperature. These things will help to keep the driver alert and awake and to ward off fatigue, thus helping the driver to display the lowest reaction time of which he is physically capable.

The Braking Distance is more susceptible to control by the designer, who should provide a braking system which produces adequate braking effectiveness, rapid response, light pedal operation, and proper force distribution. When in proper operating condition our present braking systems do a good job of this, with 90 to 100% Relative Braking Effectiveness being normal performance.

BRAKING WITH THE ENGINE

The service brakes on a vehicle are designed with the intention of providing sufficient braking ability to produce an emergency stop. Emergency stops on a properly operated vehicle are rare, but brake application to a level lower than that required for an emergency stop is frequent. Any vehicle driven at other than a snail's pace in traffic requires repeated brake applications to reduce the vehicle speed to avoid hitting the vehicle ahead, or to stop for traffic lights. These brake applications are usually mild in nature and sufficient time elapses between them for the drums and linings to cool to normal temperatures. This type of brake operation produces normal wear on linings and drums, and usually requires only infrequent replacement of the brake linings to keep the brakes in good operating condition.

In hilly country a vehicle descending a grade undergoes a continuous conversion of potential energy into kinetic energy, and unless this source of kinetic energy is counterbalanced by some device which converts kinetic energy into heat energy and dissipates it from the vehicle at the same rate at which it is generated the speed of the vehicle will rise to a dangerous level on a long steep grade. The service brakes can convert kinetic energy to heat energy at the necessary rate, but their heat dissipation ability is low due to their position within the wheel rim. Long continuous application of the brakes, or repeated short periods of application without adequate rest periods in between will cause drum temperatures, lining temperatures, and shoe temperatures to rise to excessive levels, leading to such troubles as drum cracking, drum distortion, shoe curl with uneven lining wear, bleeding of the lining, burning of the lining, and accelerated lining wear.

To avoid excessive annoyance and expense in the maintenance of service brakes, it would be desirable to have some other less troublesome means of dissipating the heat energy arising from descending a grade. The engine which powers the vehicle offers to a limited degree a means of dissipating this energy. The engine has a fair amount of internal frictional loss when operated at medium or high speeds which is converted into heat energy and dissipated by the engine's regular cooling system. During driving operation, enough power is produced in the cylinders to meet the friction losses and have enough left over to produce useful power. When the throttle of a quantity-governed gasoline engine is closed, very little power is developed in the cylinders, and the engine slows down until power production matches power demand.

The frictional losses in an engine come from many different sources, such as bearing friction, piston ring friction, piston skirt friction, camshaft friction, valve train friction, oil pump drag, water pump drag, generator drag, fan drag, internal windage, etc. Some of these items are substantially independent of engine speed, some vary with engine speed raised to the first power, some with engine speed to the second power. The sum of all of these quantities can be represented by a three term equation of the type $AN^0 + BN^1 + CN^2$, where N is engine speed in revolutions per minute, and A, B, and C are empirical constants.

For our present day passenger car engines a typical equation would be: Friction Mean Effective Pressure = $6 + .0005N + 1.0 \cdot 10^{-6}N^2$.

In addition to overcoming its own friction losses, an engine being pushed at high speed on closed throttle must do a certain amount of pumping work to get in a little fresh charge and get rid of a little exhaust gas. The intake manifold vacuum against which the pistons must work on the intake strokes is of the order of 20 to 24 inches of mercury. The exhaust back pressure is negligible, but there is an excess of compression work over expansion work on these strokes due to heat transfer. All together these amount to a Pump Mean Effective Pressure of about ten psi. Adding together the Pump Mean Effective Pressure and the Friction Mean Effective Pressure produces the Drag Mean Effective Pressure of an overspeeded closed-throttle engine. The equation for this is as follows: Drag Mean Effective Pressure = $16 + .005N + 1.0 \cdot 10^{-6}N^2$.

If the engine is geared to the driving wheels in such a way that the vehicle is forced to drive the engine at a speed higher than the engine would normally run itself at closed throttle, the Indicated MEP falls to zero because of extreme dilution effects, and the Drag MEP from the equation above is available to retard the progress of the car. To determine how effective this braking by the engine can be, a sample computation for a typical passenger automobile will be made and the results shown in graphical form.

The force which is urging the vehicle down the hill is the gravity component parallel to the road surface, called the Grade Force, and equal to $W \sin \theta$. This grade force is resisted by the natural resistances to motion of the vehicle, mainly tire rolling and wheel bearing resistance, drive train friction, and wind resistance, and in addition by any resisting torque offered by the engine. At an equilibrium speed the grade force is equal to the sum of these two latter quantities and an equation can be written to determine the equilibrium speed for a given grade.

$$\text{Grade Force} = \text{Towing Effort at Wheels} + \frac{\text{Equivalent Engine Drag Force}}{\text{Drive Train Efficiency}}$$

The Towing Effort at the Wheels is best measured experimentally, and expressed as an empirical relation including vehicle weight, vehicle speed, frontal area and appropriate constants. Such an equation is given in the preceding section on Vehicle Performance.

The Equivalent Engine Drag Force is the Drag Force produced at the drive wheel to road contact point by the Drag MEP of the engine. This would be obtained as follows:

$$HP = \frac{PLANk}{33,000} = \frac{2\pi TN}{33,000}$$

$$T = \frac{PLAk}{2\pi} \qquad \text{Drag Torque} = \text{Drag MEP}\ \frac{LAk}{2\pi}$$

Drag Torque x Overall Gear Ratio = Axle Drag Torque

$$\frac{\text{Axle Drag Torque}}{\text{Rolling Radius}} = \text{Equivalent Engine Drag Force}$$

or:

$$\text{Equivalent Engine Drag Force} = \frac{\text{Drag MEP}\ (\frac{LAk}{2\pi})\ (GR)}{RR}$$

Symbols are as indicated, with L = piston stroke in feet, A = piston area in square inches and k = the number of working piston strokes per crankshaft revolution.

The Drive Train Efficiency is a function mainly of the type of transmission used. For a standard sliding-gear transmission the drive train efficiency is approximately 95%.

Using symbols wherever possible the force equilibrium equation can now be rewritten and solved for sin θ.

$$W \sin \theta = TE + \frac{\text{Drag MEP } (LAk)(GR)}{.95 \ (RR) \ 2\pi}$$

$$\sin \theta = \frac{TE}{W} + \frac{\text{Drag MEP } (LAk)(GR)}{5.96 \ W \ (RR)}$$

For the grades encountered in any road work sin θ and tan θ are substantially equal, so tan θ = Decimal Grade G may be substituted for sin θ without introducing appreciable error.

$$G = \frac{TE}{W} + \frac{\text{Drag MEP } (LAk)(GR)}{5.96 \ W \ (RR)}$$

The overall Gear Ratio in this equation has several different values which may be chosen at will by the vehicle driver to obtain approximately a desired speed down a given grade.

The passenger car chosen to provide a numerical illustration has the following dimensions: W = 4000 pounds, Frontal Area = 26.7 square feet, $L = \frac{3.437}{12}$, $A = \frac{\pi}{4}(3.75)^2$, k = 4, Rolling Radius = 1.177 feet, Gear Ratio = 3.23 - (3.23 x 1.53) - (3.23 x 2.29), $TE = 44.8 + 0.388M + 0.0393M^2$. The results of a numerical evaluation of the equilibrium grade equation for this passenger car are shown on Figure 43.

Figure 43 shows the equilibrium or terminal speed which this vehicle would attain if allowed to coast down a long uniform grade without using the service brakes. If the car were allowed to coast down a ten percent

ENGINE BRAKING ABILITY
EFFECT
OF
ENGINE BRAKING AT CLOSED THROTTLE
ON
EQUILIBRIUM SPEED ATTAINED
ON LONG UNIFORM GRADE

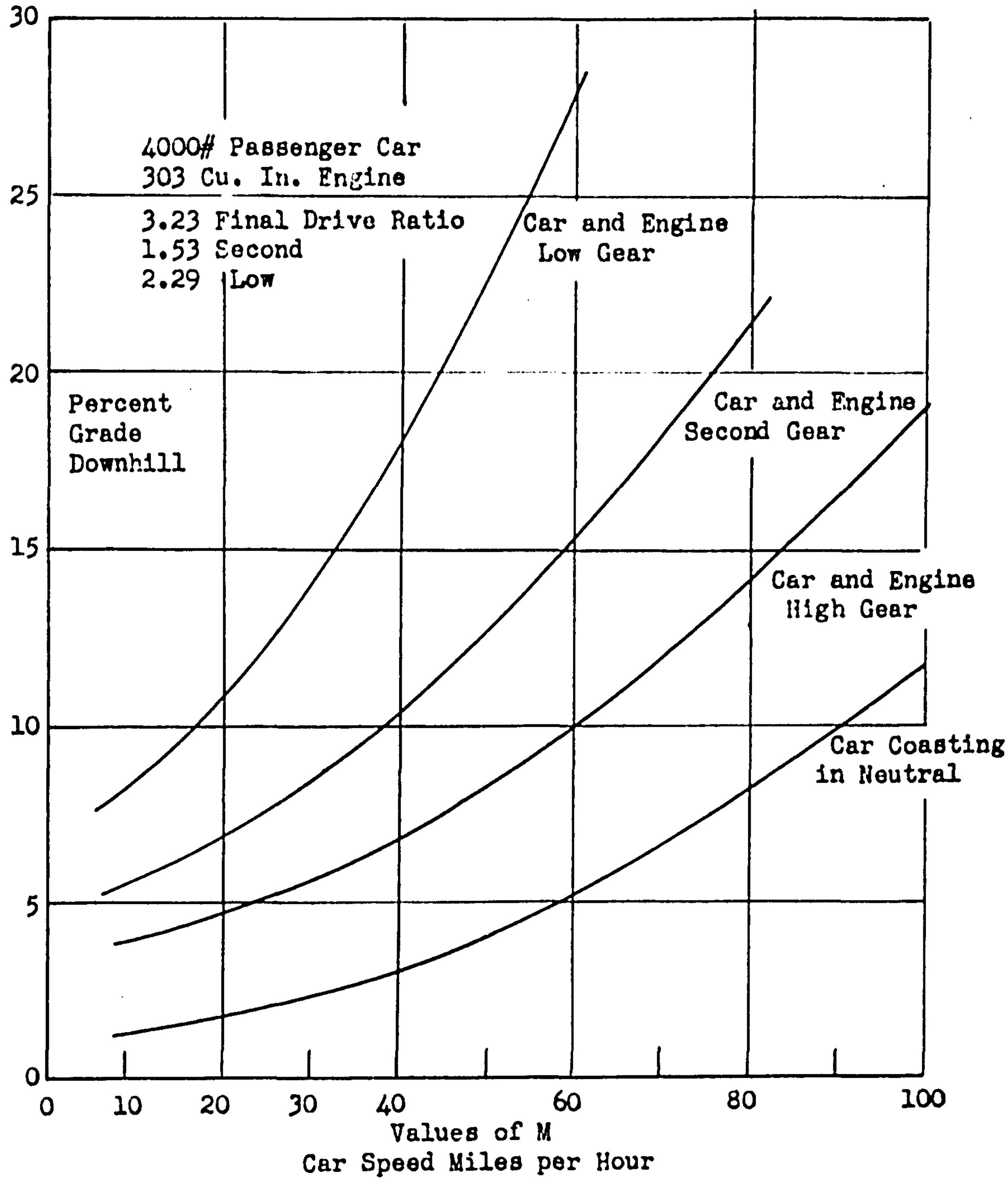


Figure 43

grade in neutral, it would reach a speed of ninety miles per hour before wind resistance and other losses stopped further acceleration. Using the closed throttle engine as a brake in high gear of the transmission would add sufficient braking effect to reduce the speed at equilibrium on the ten percent grade to 60 miles per hour. If this is too fast, the engine braking effect can be increased by causing it to turn faster with respect to vehicle speed. This is done by shifting the transmission into second or low gear. Using second gear on this car will cut the equilibrium speed on a ten percent grade to 38 miles per hour. If traffic or road conditions indicate that even this is too fast, shifting to low gear will cut the equilibrium speed to 17 miles per hour.

By using the engine as a brake in this way the vehicle can be lowered down a long grade at a controllable speed without any wear or any heating of the service brakes, and without any measurable increase in engine wear or fuel consumption. Operation on especially steep grades will require some assistance from the service brakes to hold engine and vehicle speeds within reasonable limits.

The closed-throttle engine cannot take the place of the service brakes for emergency stops, but it does serve admirably as an energy dissipator for downhill operation.

AUXILIARY BRAKING DEVICES

Engine braking provides a satisfactory means of lowering a passenger car down a long grade, and continues to do so for vehicles weighing up to as much as ten thousand pounds gross vehicle weight. For heavy commercial vehicles there is just insufficient braking power available in the engine at any feasible engine speed, and the service brakes must be relied upon almost completely to hold speeds to a safe level during long descents. Because of the limited heat dissipation ability of wheel mounted brakes, the allowable speed down the grade must usually be held to a value far below that at which the driver would negotiate a level road with equivalent curvature, and the schedule of the vehicle is prolonged unduly on trips with hilly terrain (26).

Any attempt to operate at higher speeds down hill than these experience-determined safe braking speeds results in such high lining and drum temperatures that either cumulative or immediate brake trouble results. Cumulative troubles, appearing in either a few hundred or a few thousand miles are less spectacular and less drastic, but nevertheless expensive and annoying. They include such things as premature tire failure due to overheating by radiation to the rims, brake drum cracking and checking, and excessively frequent brake lining replacement (27). The immediate troubles are either softening of the lining at high surface temperatures and its very rapid disintegration leading quickly to no braking ability, or charring of the lining with a drastic drop in its coefficient of friction, causing almost no braking ability. Unless the driver is lucky enough to be at the bottom of the grade when this occurs the vehicle goes careening down the grade at increasing speed, all too often stopping with disastrous results to persons and property. In hilly country with considerable heavy truck traffic this latter difficulty happens with distressing frequency.

If wheel-mounted brake drums are the only means available to brake the vehicle, there is no other answer than to hold the speed of descent to a value at which the brakes can dissipate the generated heat without attaining excessive temperatures. Some idea of the required heat transfer rates can be obtained from a calculation of known gross vehicle weights and ruling highway grades.

Intercity express trucks operating over the western mountain roads have gross vehicle weights for trucks up to 42,000 pounds and trailers up to 45,000 pounds, with the weight of tractor-trailer combinations limited to 72,000 to 79,000 pounds. These are the legal limitations imposed by the states through which these vehicles operate. In coming down out of the mountain passes they repeatedly descend hills ranging from ten to fifty-five miles in length, with grades varying from six per cent up to a maximum of twelve percent (28).

Using a gross vehicle weight of 45,000 pounds for a single vehicle and a nominal grade of ten percent, Table V shows the rate at which energy must be dissipated to maintain a certain speed. The grade force acting on the vehicle is $W \sin \theta$, and the relations used in developing the table are:

$$\text{Horsepower} = \frac{\text{Force x Miles per Hour}}{375}$$

$$\text{Truck Demand Horsepower} = (7.6 + 0.09M)\frac{M}{8.33} + (1 + 0.36M) + \frac{0.225M^3}{375} \qquad (28)$$

TABLE V

SPEED EFFECT ON ENERGY DISSIPATION RATE

45,000 Pound Vehicle 10% Downhill Grade

Speed	Grade Force	Energy Dissipation Rate in Horsepower Total	By Truck	By Braking
5	4500	60	8	52
10	4500	120	15	105
15	4500	180	24.5	155
20	4500	240	35.5	204
30	4500	360	65	295
40	4500	480	108	372
60	4500	720	236	484
80	4500	960	480	480

Trucking experience has shown that safe practice for this truck would be to lower it down the grade at speeds of between ten and fifteen miles per hour, using engine braking to its maximum safe extent and the service brakes to whatever extent is necessary to hold the vehicle to this speed range. Higher speed would necessitate shifting the transmission from "two under direct" to "one under direct" or to "direct" in order to avoid excessive engine speed, with proportionate loss in engine braking. This loss is engine braking plus the additional horsepower generated by the higher speed would have to be handled by the wheel brakes. Bitter experience shows that they cannot do it safely.

The only way in which higher speeds of descent can safely be used is to provide some auxiliary braking system of relatively high horsepower capacity, capable of dissipating thermal energy at high rates. Two distinct types of energy dissipators have found limited acceptance as auxiliary braking devices for trucks. One is a water brake, or water dynamometer. The other is an air-cooled eddy current electric brake. Both are driveshaft mounted and driven through gearing from a live axle, the regular live axle on a truck, or a specially installed live axle on a trailer.

The water brake is connected by means of quick dump valves to the engine cooling system of the truck, and dissipates the energy generated within itself by circulating water up through the cooling radiator. Normal or even oversized radiators are incapable of dissipating energy at the rate at which it must be generated by this auxiliary braking device, and the extra energy is dissipated by boiling away some of the water.

The eddy-current unit is air-cooled, and can be equipped with automatic governing devices which will apply it to whatever extent, up to its maximum, needed to maintain a chosen speed down a grade.

Properly installed and maintained these devices would permit increased speeds with safety on downhill stretches. Their adoption by the highly competitive commercial trucking industry is impeded by an economic situation which at present leads the industry to remove front wheel brakes completely in order to add a few hundred pounds of pay load, even though this may reduce their braking ability by about ten percent.

SUMMARY ON BRAKES AND BRAKING

The preceding sections have shown that the need for vehicle speed reduction must be met by some energy dissipation device called a brake. On relatively light vehicles, such as passenger cars or the lightest trucks, continuous braking need can be handled within certain limits by engine braking. Continuous braking need on heavy commercial vehicles can best be handled by special auxiliary energy converters.

Intermittent braking of major magnitude, and all braking for emergency stop conditions must be handled by special friction brakes installed usually on the wheel hubs. These friction brakes are either of the annular disc, or cylindrical drum and shoe type.

The braking forces which must be developed at the drums of these friction brakes is shown to be of the order of magnitude of three quarters of the vehicle weight. In order to develop this force magnitude from the rather limited force which the driver can exert on a brake pedal, a force multiplying system must be used. The force multiplication ratio which this system can produce is split into two parts. One part is the brake rigging, including those portions of the system between brake pedal and brake-shoe-actuating cams or pistons. The other part is the brake shoe to brake drum combination.

The limiting values of brake rigging force multiplication ratio are analyzed, and shown to depend on the magnitude of shoe to drum clearance in the released position, plus the allowance to be made for lining wear between brake adjustments.

The obtainable values of shoe to drum force multiplication ratio for six different types of brakes are analyzed by developing the basic equations which govern the operation of each type of brake, illustrating the limitations imposed by certain design and operational variables, and evaluating, for brakes of the same size, the numerical magnitude of the force multiplication ratio to be obtained for each type.

The probable stability of operation of each type of brake is determined from their basic equations, and the importance of brake stability indicated.

The effect of braking force distribution on the Relative Braking Effectiveness is illustrated, and equations for determining this quantity for various conceivable braking conditions are developed. The use of the relative braking effectiveness is then illustrated by developing an equation for Stopping Distance, in which the relative braking effectiveness is one of the key factors.

V

SUMMARY

SUMMARY

Certain phases of automotive vehicle construction and operation are quite well suited to an illustration of the application of the principles studied in engineering education. The phases used in this thesis are driving ability, performance, and braking ability.

Section II on driving ability illustrates how a self-propelled vehicle generates a driving force at the road surface to move itself about. Using a free-body diagram and the principles of mechanics and mathematics, a series of relations are developed which give the maximum driving force which the vehicle can produce as a function of the friction coefficient, the fore and aft weight distribution, and the ratio of the height of the center of gravity to the wheel base. These equations are developed for four different drive-wheel combinations, and for certain variations of these combinations. Maximum driving force is shown to be equal to the product of friction coefficient and adhesive weight, and is produced when all weight-carrying wheels are brought simultaneously to impending slip by driving forces. When the driving force produced by other driving wheel combinations is compared with the maximum driving force obtainable, the Relative Driving Effectiveness of the combination is obtained. Rear-wheel drive passenger cars show a relative driving effectiveness between 0.55 and 0.60, with front-wheel drive passenger cars showing values between 0.43 and 0.52. Except on snow and ice the difference in relative driving effectiveness between the two types is of little consequence. Special weight distri-

butions, such as hanging the cab and engine ahead of the front wheels, can make a front drive truck have fair driving effectiveness, and can make a four-wheel drive truck with an interaxle differential have a uniformly high driving effectiveness.

Section III on self-propelled vehicle performance is concerned with the top speed, hill-climb ability, acceleration ability, and fuel economy which can be produced by intelligent matching of the vehicle horsepower demand with the engine horsepower supply.

The various factors which enter into the vehicle demand horsepower are discussed, and then an experimentally-determined empirical relation is presented for both demand towing effort and demand horsepower.

By the use of illustrative graphs, the effect of overall gear ratio between engine and road on the matching between supply and demand horsepower is illustrated. The need for a clutch and the need for a torque-multiplying transmission are shown.

Using known or determinable quantities equations are developed for hill-climb ability, and representative values of hill-climb ability for an actual car are illustrated graphically.

The factors which enter into the acceleration ability of an automotive vehicle are enumerated, their relative magnitude discussed, and an equation derived for computing acceleration ability. The acceleration ability of an example vehicle is computed for several different conditions and illustrated in graphical form.

The problem of fuel economy, or miles per gallon performance, is attacked by determining the major factors which affect it and developing methods of determining their magnitude and effect. The effect of relative engine load on the engine's efficiency, and on the car's fuel economy is demonstrated by illustrating the effect caused on both of these quantities by various overall gear ratios.

For comparison purposes the performance which the ideal transmission would produce in the same vehicle is determined and is included on the performance curve sheets with the values produced by the conventional transmission. The ideal transmission does not produce any startling improvement over the performance of the standard types of transmissions, and for various reasons cited, there is no ideal transmission marketed today. A curve showing the ratio range over which the ideal transmission of the illustrative vehicle would have to operate is shown, with a maximum range of twenty five to one obtained.

Section IV is an analysis of brake types and braking ability. The braking requirements of an automotive vehicle are determined and the magnitude to which the driver's force on the foot pedal must be multiplied on its way to the brake drums is analyzed. The method by which this force multiplication ratio may be obtained is split into two parts and each part analyzed separately.

Force multiplication in the brake rigging, between foot pedal and shoe-actuating cams, is determined by developing the relation between cam movement and shoe to drum clearance, and matching this quantity with the available effective pedal travel.

Force multiplication between brake shoe and brake drum is then analyzed for five different types of shoe brakes and for one annular disc type brake. The shoe brake types covered are: pin-anchored shoes, link-anchored shoes, shoes with sliding block anchors, the dual-primary shoe brake, and the duo-servo shoe brake. Each type is analyzed by treating the shoe as a free body acted upon by the forces present. The relations for radial force and the ratio of radial force to cam force are derived in general form by integration of the free-body equations. The effect of various design and operating factors on the stability and performance of each type are analyzed, and a comparison of braking ability obtained by evaluating the equations for each type for a common brake size.

The relative ability of each brake type to resist self-locking tendencies is analyzed and a Brake Stability Factor developed to obtain a numerical comparison of this brake characteristic.

The Relative Braking Effectiveness of different braking arrangements and car weight distributions is derived and illustrated. An equation for the approximate stopping distance required by a vehicle, as a function of speed, friction coefficient, and relative braking effectiveness is derived and the effect of these factors on the stopping distance magnitude presented in tabular form.

The degree of the braking effect which can be expected from an engine operating with a closed throttle is determined, and the limitations of engine braking illustrated. The magnitude of the continuous braking effect required by a heavy vehicle coasting down a long grade is determined, and the use of auxiliary braking devices discussed.

The design and operation problems analyzed in these three thesis sections have illustrated by example the use of several engineering methods of attack on a problem. Pertinent design and operational equations are evolved from known or determinable forces acting on a body. Involved equations or groups of equations are made more understandable by two-dimensional graphs with families of curves. The equations for brake force multiplication ratio illustrate the derivation of a general equation for a situation involving distributed forces of unknown magnitude. The stopping distance equation and the acceleration ability equation represent the use of approximation and simplification to eliminate quantities of negligible magnitude and to obtain an equation simple enough to avoid unwieldiness and yet accurate enough for many engineering uses.

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