LESSON PLANNING AND RESPONDING TO STUDENT ERRORS AND UNCERTAINTIES IN MATHEMATICS CLASSROOMS

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A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

Educational Psychology and Educational Technology—Doctor of Philosophy

2021

ABSTRACT

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In this study, I worked with five secondary mathematics preservice teachers (PSTs) in their final year of their undergraduate teacher education program at a large, midwestern university. I examined written lesson plans, observed enacted lessons, and interviewed PSTs about planning and teaching. The purpose of this study was to better understand how their planning practices influenced their classroom discourse practices, particularly when responding to students' errors and uncertainties. Despite using a robust planning framework, 64% of anticipated student thinking was instrumental or low level, focused on facts or procedures. The data strongly showed that PSTs were challenged to anticipate student thinking and were likely to over- or underestimate students thinking. Additionally, there were considerable differences in the quality and quantity of PSTs' planned and enacted discourse, particularly in response to errors and uncertainties. PSTs planned 41% of their discourse moves to be low level, but 50% of their enacted moves were low level and 58% of their responses to errors and uncertainties also were. However, their high level moves only declined from planned to enacted, not from enacted to responding to errors and uncertainties. PSTs identified challenges in maintaining high cognitive demand, time management, communicating the purpose of the lesson, scaffolding, and in-the-moment decision making. These results support the known data that indicate PSTs struggle to plan for student thinking and need support, and with support can learn to engage in productive discourse practices, particularly in response to errors and uncertainties.

I dedicate this dissertation to Taylor, Carson, Rowan, Reed, and Lian.
May you make the world a better place, one student, one plan, and one lesson at a time.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter 1:

Introduction and Overview

The mathematics education community has encouraged more productive classroom discourse practices in recent years (Herbel-Eisenmann & Cirillo, 2009; Kazemi & Hintz, 2014; National Council of Teachers of Mathematics, 2012; M. S. Smith & Stein, 2011). In such classrooms, teachers use their knowledge of student thinking to drive planning and instruction, students have a variety of opportunities to engage in numerous types of mathematical communication, and students have opportunities to work with authentic and challenging tasks. Although the work in this area is leading to a better picture of how teachers can create classroom environments in which student thinking is the central tenet for planning and instruction, these practices still prove to be difficult for teachers–particularly preservice teachers (PSTs)–to enact. This is particularly true when it comes to ways in which teachers respond to students' errors and uncertainties (statements, questions, or behaviors that indicate the student is struggling or in the process of understanding) in mathematics classrooms (Santagata, 2005; Tulis, 2013).

One reason PSTs may struggle to enact more productive discourse practices could be their reliance on lower-level discourse techniques such as the Initiate-Respond-Evaluate (IRE) pattern (Mehan, 1979) in which teachers asks a question with a right or wrong answer, a student responds, and the teacher acknowledges the student's response as correct or incorrect. This technique rests on the notion that mathematics is a set of explicit, factual rules and procedures with no ambiguity and the learning of mathematics comes from the teacher only. IRE does not allow students to engage with each other over the complexities of mathematics and the uncertainty and errors that are a natural part of learning. Reliance on IRE-related discourse patterns limits the ability of teachers and students to have productive mathematical discourse in mathematics classrooms.

The ideal of good mathematics teaching is shifting away from such lower-level techniques. Research and trends in mathematics education encourage teachers to use student thinking to make curricular and instructional decisions, including engaging students in meaningful mathematics discourse (Hughes, 2006; Putnam, 1992; M. S. Smith et al., 2007; M. S. Smith & Stein, 2011). Supporting teachers to lead productive and discourse-rich classrooms can happen at different stages. Some scholars have focused on lesson planning and preparation (Carpenter et al., 1988; Clark & Peterson, 1986; Peterson et al., 1988; M. S. Smith et al., 2007; M. S. Smith & Stein, 2011). Others have focused on attending to student thinking during enactment of lessons (Carpenter & Fennema, 1991; Jacobs et al., 2011; Knapp & Peterson, 1995; M. G. Sherin et al., 2011). And still others have considered classroom discourse patterns and the ways in which teachers facilitate student discussion and engage in classroom discourse with students (Ghousseini, 2015; Herbel-Eisenmann & Cirillo, 2009; Hufferd-Ackles et al., 2004; Kazemi & Hintz, 2014; M. S. Smith & Stein, 2011).

However, the essence of richer mathematics practices and discourse is rooted in teachers' planning. Richardson (1994) observed that "teachers do not lesson plan in the linear manner prescribed in many teacher education programs" (p. 6). Suggesting that teachers develop personal planning methods and styles after leaving teacher education. Warren (2000) explained that PSTs and novices often rely heavily on how they were taught to lesson plan which can often be a step-by-step experience. However, Warren found experienced teachers reflect on prior lessons and pull successful aspects of those lessons into plans for future lessons. Like many things in teaching and other professions, previous experience provides a rich body of knowledge.

Then, in a study about early career teachers, Morine & Valence (1973), gathered data on practicing teachers' lesson plans. Results showed that teachers wrote minimal information about their lesson plans in their plan books. One might consider this alarming, but in interviews after the lessons, Morine and Valence found that teachers had much more detailed *mental images* of the class than the plans themselves. This suggests that at some point, teachers develop some sort of shorthand or personal code that is used to jog memory and small amounts of text may actually represent a larger amount of thought and ideas. This likely comes from experience and practice and is challenging for PSTs.

Hughes (2006) and other scholars (e.g., Clark & Peterson, 1986; Reusser, 2000) found that for some novice teachers, written plans underrepresented the thinking that teachers used during the planning process. However, having trained these particular novice teachers with the TTLP, Hughes (2006) found what was missing in the written lesson plans were things such materials and tools, grouping, time for parts of lesson, and student prior knowledge and experience. When taught to attend to student thinking, novice teachers still have lesson plans that are not fully reflective of their thinking during planning, but the part that is lacking is not anticipation of student thinking.

As teachers become more adept at planning many things start to change in lesson plans. Leinhardt (1993 as cited in Hughes, 2006) found that experts anticipate difficulties students might have with content, attend to student thinking with more ease, and can think along two dimensions–teachers' thoughts and students' thoughts. Livingston & Borko (1989) found that experienced teachers' planning was brief and efficient and teachers could elaborate fully when asked to do so. Berliner (2002) found experts to be experts are "more flexible, more opportunistic planners, and can change their representations faster when it is appropriate to do so" (p. 464).

Shroyer (1981) worked with three teachers, Ralph, Martha, and Zelda, who each presented with different planning practices and styles. Ralph was a "typical math teacher," Martha was the "opposite of Ralph/closest to NCTM reform teaching," and Zelda was a mix between the traditional and reform. Ralph planned his unit based on the textbook and used pages and problem sets as his organizing structure. He read the textbook before teaching each lesson. Every activity was planned. He did not assign the more challenging questions. On the other hand, Martha began with required objectives, determined what objectives students were already familiar with, and then used that information to guide her planning. Martha did not use a textbook. She had specific plans for about half the class activities and the other half was dedicated to responding to student suggestion and things that needed to change. Her plans were brief comments about tasks students were to perform. Finally, Zelda listed topics she intended to cover. She used ideas from the textbook, but did not use the book when she felt the book was confusing or not helpful for the students to learn. Sometimes Zelda's plans were incredibly detailed, others they were much less so. Although these three scenarios do not provide all possible ways of planning, but do offer conceptions of different ways in which teachers plan for lessons.

Although Shroyer (1981) described how little teachers anticipate unpredictable student performance before teaching a lesson, in more recent work about how teachers might leverage the potential of student error, Bray (2013) identified three steps that teachers should use. Bray suggested that teachers first identify the flawed and correct solutions that will be focused on in the public discussion, then determine the order and format the solutions will be shared, and finally determine the mathematical points to be made with each. This is similar to TTLP protocol and illustrates the movement in current educational thinking to capitalize on student thinking which includes student mistakes.

Despite these advances in the field, how lesson planning influences teachers' enactment of productive discourse practices–specifically how teachers respond to students when students make mistakes or show signs of uncertainty–is still not fully understood. It is likely that what teachers plan for greatly influences what they notice and what they are intellectually prepared to respond to, but research to show this is limited. Additionally, the connections between teacher planning, student thinking, mathematics classroom discourse, and responses to student errors and uncertainties are complex. For example, it is likely that when teachers do not prepare for student errors and those errors surface in a lesson, teachers might default to IRE-type responses because they have not considered ways in which they could leverage the error into a more productive discourse opportunity. Responding to student errors and uncertainties is both challenging and an inevitable classroom experience. Thus, it is crucial for teachers to know how to respond. In addition, the teacher's plan for responding to errors and uncertainties are not fully understood. If we have a better understanding of how teachers plan for student errors and uncertainties, we are better posed to empower teachers to enact richer, more productive discourse practices, which elevates and improves student thinking and understanding.

Purpose

The purpose of this study is to examine and better understand how PSTs' lesson planning practices influence their ability to "respond professionally rather than just react" (Mason, 2011, p. 8) to student errors and uncertainties. As the focus of mathematics education has moved away from rote skill building to student thinking and understanding (e.g. M. S. Smith & Stein, 2011), it is important for teachers to plan for and develop their ability to respond to students' errors and uncertainties. Different ways of responding to students–IRE (Mehan, 1979), math-talk learning community (Hufferd-Ackles et al., 2004), productive mathematics discussions (M. S. Smith & Stein, 2011), or Teacher Discourse Moves (TDM) (Herbel-Eisenmann et al., 2013, 2013, 2015)–communicate different messages about learning, mathematics, and students.

Research has shown that teachers can learn how to improve their noticing and discourse practices, including how they respond to students (Bautista et al., 2014; Jacobs et al., 2010; Santagata & Yeh, 2015; Wagner, 2014). However, it is unclear exactly how this learning can best be supported for PSTs, or what role lesson planning plays in their enactment. This study examined how PSTs anticipate student thinking during lesson planning and how their lesson planning practices relate to the ways they respond to errors and uncertainties in the classroom. It is important for PSTs to plan for and develop their ability to respond to students' errors and uncertainties to help elicit and elevate student thinking. It is also important for researchers and teacher educators to understand how PSTs develop their lesson planning and classroom discourse practices.

Overview of the Study

I worked with five PSTs to examine their lesson planning and classroom discourse practices, particularly their responses to student errors and uncertainties, across two semesters. In the fall semester the PSTs planned and taught lessons in an undergraduate remedial mathematics course that served as a microteaching lab; the spring teaching took place in the PSTs' field placement classrooms. The PSTs allowed me access to their written lesson plans and written reflections from after teaching. I interviewed them each six times over the course of Spring semester and observed them teaching in field placements as well as videotaped lessons from their university. I developed a coding scheme built on previous research and frameworks and relied on open coding and cross-comparative methods. These codes provided data around lesson planning and enacted teaching as well as PSTs connections between them. Findings indicate that PSTs are challenged to anticipate student thinking while planning and their enactment of plans decreases the cognitive demand for students without PSTs being aware of their planning and discourse tendencies.

Chapter 2:

Review of Literature

In recent years, mathematics educators have urged those in the mathematics education community to reconceptualize the mathematics classroom (Ball et al., 2005; Gower, 2015; Herbel-Eisenmann & Cirillo, 2009; Hufferd-Ackles et al., 2004; Lampert, 2010; Putnam, 1992; Shulman, 1986a, 1986b). This work has challenged less productive mathematics classrooms–learning spaces dominated by teacher- or textbook-centered lessons in which learning is assumed to occur through a transmission process, with students engaging with drill and practice, such as worksheet-based activities on isolated tasks that are typically free from context. Instead, views of more productive mathematics classrooms rely more on student-to-student collaboration, mathematics in context, rich discourse, and beliefs that all members of the classroom bring valuable knowledge and experience.

A key aspect of this shift is the role of student thinking as a guide for instruction. Scholars have argued for the importance of positioning student thinking at the center of mathematics teaching and learning (Bautista et al., 2014; Hughes, 2006; Peterson et al., 1988; Putnam, 1992; M. S. Smith et al., 2007). Putnam (1992) posited two reasons for the importance of using student thinking. From a constructivist perspective, students create their own knowledge, and knowing what students are thinking, is a central part of teachers' ability to shape learning. Additionally, if mathematics involves thinking about patterns found in the world–not a collection of explicit rules and processes with no ambiguity–then how students think is a central component of the instruction and learning process.

Making students' thinking public is one important component related to its development. Putnam (1992) suggested this in the first of his five components of high-quality mathematics teaching–(a) get students' thinking out, (b) establish norms of sense-making, (c) use concrete embodiments, (d) have students justify and explain, and (e) choose mathematics tasks that afford divergent solutions. Once thinking has been made public, it can take on new roles. Public student thinking allows the focus of learning to move away from correctness of answers and toward the ideas and understandings students have. To understand how to help this become more mainstream, it is essential to examine the practices teachers engage in to create environments in which this type of teaching is possible.

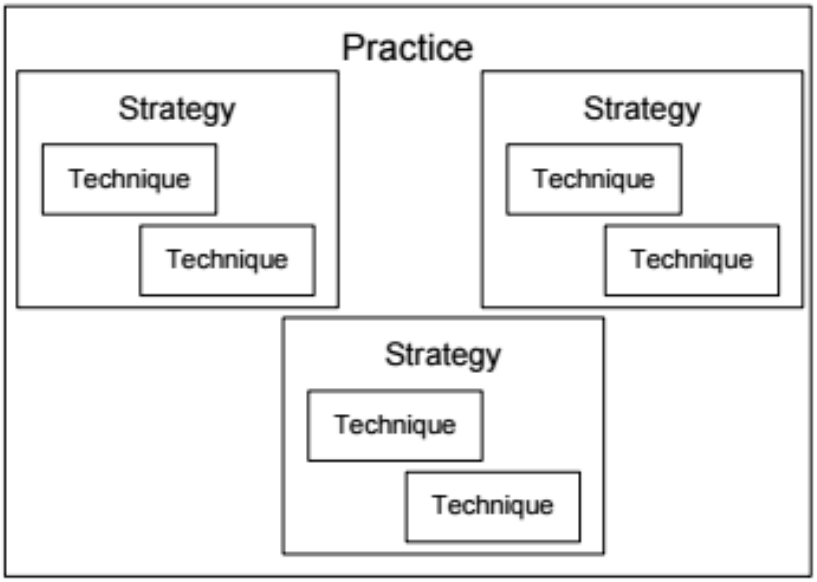
Practices

Lampert (2010) analyzed different uses of the construct *practices* in scholarship on teaching and learning. One conception considers practices as a collection of things that teachers do, habits or routines. It is this conception of practice I use in this study: practices are habits or routines. Because practices do not stand alone, strategies and techniques are subcomponents that can be examined to provide a finer grain with which to describe and measure teaching practices (Lampert, 2010). Strategies make up practices and answer the broad question of "how" a teacher would meet a learning goal. Techniques make up strategies and are enacted behaviors or teacher moves that illustrate how a strategy is enacted (See Figure 2.1). A teacher could have a large number of practices, composed of a large number of strategies and techniques. A particular strategy may occur within multiple practices and a particular technique may occur within multiple strategies.

In this study, a classroom discourse practice might be eliciting student thinking. It is a broad routine or habit in which teachers engage. How a teacher enacts this practice can vary and might come with eliciting solutions or eliciting a process or explanation. These are different strategies. If a teacher wants to elicit an explanation, they could use any number of techniques.

Figure 2.1:

Dillman's Conception of the Relationship of Practice, Strategy, and Technique



Note. This illustration is not intended to imply that there are specific or limited number of practices, strategies and techniques or that they are mutually exclusive or unique to a certain path. Teachers could have a nearly endless number of practices, strategies, and techniques which could exist in countless arrangements.

For instance, they could directly ask a student, engage students in a think-pair-share, or ask a student to share or re-voice what another student had explained.

Classroom Discourse

Classroom discourse is a critical component of classroom culture, instruction, and learning. The mathematics education community has called for increases in focus on and attention to the discourse in mathematics classrooms (Cazden, 1986; Cazden & Beck, 2003; Ghousseini, 2015; Herbel-Eisenmann et al., 2013; Herbel-Eisenmann & Cirillo, 2009; Hufferd-Ackles et al., 2004). In *Principles and Standards for School Mathematics* (PSSM), the NCTM

(2000) laid out guidelines to shape the discourse of more productive mathematics classrooms. For elementary grades, NCTM encouraged teachers to make mathematical conjectures, ask questions, share thinking, and justify ideas. In middle school, teachers are encouraged to "continually provide opportunities for students to experience mathematics as a coherent whole through the curriculum used and the questions teachers and classmates ask" (p. 274). By the high school years, students should be able to "express themselves coherently and clearly, listen to the ideas of others, and think about their audience when they write or speak" (p. 348); communication should be a "vehicle for assessment" (p. 351); and teachers should use wrong answers from students to guide lesson planning. More recent documents such as the *Common Core State Standards of Mathematics* (Common Core State Standards Initiative (Common Core State Standards Initiative (CCSSI), 2010) introduced practice standards which support the idea of mathematics being a context-laden subject with real world tasks and applications. Specifically, Practice 3 describes the importance of students being able to "construct viable arguments and critique the reasoning of others."

Many have found that teaching environments with rich discourse led to rich student learning, but enacting meaningful and rich discourse is not without challenges (Ghousseini, 2015; Herbel-Eisenmann & Cirillo, 2009; Hufferd-Ackles et al., 2004). Specific challenges include how much teachers should push student thinking (Putnam, 1992), who determines correctness (Herbel-Eisenmann et al., 2013, 2015; Hufferd-Ackles et al., 2004; Putnam, 1992), and what to do with student thinking (Putnam, 1992). Researchers have described these difficulties in individual cases and across groups of teachers. Ms. Meadows (Putnam, 1992) was successful in eliciting student thinking, but then did not know what moves to employ next.

Franke et al. (2009) found many teachers were able to ask initial questions to elicit student thinking, but their follow up techniques employed varying levels of effectiveness.

These discourse challenges are not as prevalent in less productive classrooms. When supporting students to master procedural and low-level skills, it works for teachers to rely on IRE (Mehan, 1979). If factual learning is the goal, teachers can quickly evaluate and correct students' answers and move on. For more productive mathematics teaching and learning, this is not sufficient. A reliance on using evaluating (the E from IRE) as a go-to strategy, falls short when student thinking is central, and teachers are positioned to respond from the mindset that students' mathematical thinking is interesting, important, and valuable. As this change of philosophy has emerged, the discourse needs of mathematics classrooms have also changed, yet practice lags behind.

IRE

In an IRE discourse pattern (Mehan, 1979), a teacher initiates or asks a question (I), a student responds (R), and then the teacher evaluates the student's statement (E). Some scholars refer to the IRE pattern as Initiate-Response-Feedback (Kutz, 1997), Initiate-Response-Follow up (Wells, 1999), or Initiate-Reply-Evaluate (Poole, 1990). Lemke (1989) referred to it as Triadic Dialogue. The important similarities are: (a) the pattern has three parts, (b) the teacher takes parts one and three, (c) the teacher evaluates the student, (d) the process involves low-level information exchange (Cazden & Beck, 2003), and (e) the process occurs at a rapid or hurried pace (Garcia, 2015; Kutz, 1997).

The prevalence of the IRE pattern in classrooms over time is overwhelming. Several scholars noted the IRE pattern was the most prevalent classroom discourse pattern (Capraro et al., 2010; Garcia, 2015; Kaya et al., 2014; Kutz, 1997; Neal, 2008; Poole, 1990). Wells (1999) found that IRE patterns accounted for up to 70% of secondary classroom student-teacher discourse interaction. The data show this trend in IRE's overwhelming presence in classrooms for decades. Discourse relying heavily on IRE stays on a shallow level, values answers and speed over rationale and thinking, and positions the teacher as the central-knowledge figure. Kutz (1997) argued that IRE does not provide students the opportunity to talk in open-ended ways to grapple with mathematical thinking. Garcia (2015) argued that if students were only exposed to IRE, the quality of instruction would be minimized because basic facts and short answers limit opportunities to expand on thinking and that IRE limits students' potential to speak and can prevent academic learning. These concerns are echoed in Brantlinger (2014)–who observed remedial geometry students in a night-school program engaged in mathematics with critical mathematics (CM) discourse–and Moreno (2015)–who argued that different learning can result from different discourse patterns and that IRE focuses on procedural knowledge in her study of remedial mathematics students at a community college.

Because many practitioners and scholars believe mathematics should challenge students, it follows that IRE patterns limit the potential to challenge students and that other patterns may be more fitting to engage students and elevate their thinking (M. S. Smith & Stein, 2011). Thus, if IRE had been found to be a pedagogically strong discourse pattern, these results would be something to celebrate. However, because IRE is problematic (Cazden & Beck, 2003; Culican, 2007; Kutz, 1997; Neal, 2008; Poole, 1990; Wells, 1999), this prevalence leads the mathematics education community to try to find more productive discourse practices and ways to support teachers in enacting these patterns.

Alternative Discourse Practices

In response to the problematic nature of IRE and less productive discourse patterns, numerous scholars (Herbel-Eisenmann et al., 2013; Kaya et al., 2014; Kazemi & Hintz, 2014; Munter, 2014; M. S. Smith & Stein, 2011) have presented alternative discourse practices for increasing quality and complexity of classroom discourse. They've found that helping teachers move out of IRE practices and into richer discourse practices is challenging, but possible.

Teacher Discourse Moves

Herbel-Eisenmann et al. (2013) developed the Teacher Discourse Moves (TDM) framework as a way of measuring and describing discourse within classrooms. This framework includes waiting, inviting student participation, revoicing, asking students to revoice, probing a student's thinking, and creating opportunities to engage with another's reasoning. These discourse moves help examine possibilities beyond IRE and allow teachers and researchers to consider different ways of interacting with mathematics.

Teacher Moves for Supporting Student Reasoning

Ozgur, Reiten, and Ellis (2015; 2016) developed a framework for analyzing discourse practices: Teacher Moves for Supporting Student Reasoning (TMSSR). This framework includes four practices–eliciting student reasoning, responding to student reasoning, facilitating student reasoning, and extending student reasoning–as shown in Figure 2.2. These four practices are partitioned into between seven and 11 strategies each. Reiten et al. (2016) have used this framework to analyze and compare teachers teaching the same lessons and to help explain differences in student learning from the various classrooms. See Appendix L for details and definitions for each of the strategies.

Figure 2.2:

TMSSR Framework

Eliciting Student Reasoning	
Eliciting Answer	Eliciting Ideas
Eliciting Facts or Procedures	Eliciting Understanding
Asking for Clarification	Pressing for Explanation
Figuring Out Student Reasoning	
Checking for Understanding	

Responding to Student Reasoning	
Validating a Correct Answer	
Re-voicing	Re-representing
Encouraging Student Re-voicing	
Correcting Student Error	Prompting Error Correction

Facilitating Student Reasoning		
Cueing	Providing Guidance	
Topaze Effect[a]	Building[b]	
Funneling		
Providing Procedural Explanation	Providing Summary Explanation	Providing Conceptual Explanation
	Providing Information	
	Encouraging Multiple Solution Strategies	
	Providing Alternative Solution Strategy	

Extending Student Reasoning	
Pressing for Precision	Encouraging Reasoning
Encouraging Evaluation[c]	Encouraging Reflection
Topaze for Justification[d]	Pressing for Justification
	Pushing for Generalization

Math-talk Community

Hufferd-Ackles et al. (2004) described the classroom changes that a third-grade teacher, Ms. Martinez, and her 25 English Language Learning (ELL) students experienced over the course of an academic year during which Ms. Martinez attempted to improve the math-talk community in her classroom. The goal of implementing a math-talk community is “to understand

and extend one's own thinking as well as the thinking of others in the classroom" (p. 82). This study examined Ms. Martinez's changes in four categories: questioning, explaining mathematical thinking, the source of mathematical ideas, and responsibility for learning. These categories were measured using a four-level rubric in which level zero described a traditional mathematics classroom and level three described classrooms in which collaboration and student thinking were central factors of the classroom environment. Ms. Martinez made substantial improvement in creating a math-talk community in all four categories. Two categories of this framework–questioning and source of mathematical ideas–deeply connect with teacher responses to student errors and uncertainties.

Learning to Lead Classroom Discussions

In another study that examined a teacher's journey to improve her classroom discourse patterns, Ghousseini (2015) described the experience of a student teacher, Linda, as she attempted to learn how to better orient students to each other's thinking in an eighth-grade mathematics class. During her internship, Linda focused on improving her classroom discourse. Ghousseini argued that facilitating classroom discussions is difficult for teachers–especially novices–because teachers need to make quick decisions, honor students and their ideas, and encourage students to listen to each other, all while focusing on mathematics.

As Linda worked on learning to facilitate discussions that relied on patterns other than IRE, she was successful in establishing other discourse patterns, but did so with some rigidity that also became problematic. For example, when a student said a correct answer, she frequently asked a different student to revoice. However, when a student gave a wrong answer, she often asked other students to indicate agreement or not. Although these patterns are more complex than the IRE pattern, they were found to be somewhat limiting because she used them so consistently and rigidly.

Throughout her teaching, Linda asked many questions that were designed to help students to share thinking and reasoning–not the IRE structure. Linda showed both growth and potential for growth in her facilitating skills. This article, similar to Hufferd-Ackles et al. (2004), illustrates that changing practice proved to be challenging, but possible, and that teachers' ability to change their practice is likely connected to the beliefs and the conceptions they hold.

Teachers' Knowledge and Beliefs

To support teachers in enriching their mathematics classroom discourse, scholars have examined teachers' knowledge, beliefs, conceptions, and identities. By understanding teacher thinking, researchers can better understand the learning teachers experience when trying to change teaching practices. Teachers' beliefs and conceptions shape the decisions they make in enacted lessons by acting as a filter (Wood et al., 1991).

Cognitively Guided Instruction (CGI; Fennema et al., 1992; Peterson & Clark, 1978) was a program developed to help bridge the gap between research on children's thinking and research on teaching. CGI is a philosophy of teaching elementary mathematics that stems from the constructivist belief that children construct their own knowledge and bring knowledge into formal schooling. CGI builds on the idea that children know, and their teachers learn, about diverse strategies for solving addition and subtraction problems. When teachers are educated to use this program and way of thinking to make curricular and instructional decisions, children are often able to solve problems that they have not been "taught" how to solve (Carpenter et al., 1996). The CGI research team concluded that teachers' beliefs could change with support, mostly in the form of sustained professional development (Knapp & Peterson, 1995; Medrano, 2012; Peterson & Clark, 1978). This work helped teachers change and better their ability to notice and use student thinking in classroom decisions.

Other work with teachers' knowledge and beliefs and pedagogical content knowledge (PCK; Shulman, 1986b) concluded that change can happen, but it is often slow, inconsistent, and sometimes nonexistent. Shulman's notion of PCK, the specialized knowledge that teachers possess that bridges the content and subject matter with the knowledge of how to teach and what instructional practices work for students, helps describe how complicated the knowledge of teaching is and, thus how challenging it is to teach and change. The cases of Mrs. Oublier (Cohen, 1990) and Ms. Meadows (Putnam, 1992) illustrate these ideas. Mrs. Oublier self-reported change than researchers observed. Ms. Meadows changed some practices (e.g., eliciting student thinking), but did not have solid practices for using the thinking once it was made public. Because research has shown that teacher change is possible, but is difficult and needs sustained support, considering what mathematics teachers attend to and make sense of is critical for supporting more effective responses to errors and uncertainties.

Mathematics Teacher Noticing

Teacher noticing is the active "process through which teachers manage the 'blooming, buzzing confusion of sensory data' with which they are faced" (M. G. Sherin et al., 2011, p. 5). Scholars consider three aspects of teacher noticing: what teachers attend to, what they make sense of, and their responses. Teacher noticing is interrelated and cyclical, thus cannot easily be isolated into these segments which are inherently connected (M. G. Sherin et al., 2011). Some scholars use only the first two components to define and study noticing (B. Sherin & Star, 2011; van Es, 2011). Responding has been argued to be the most complex and an important component (Ding & Domínguez, 2016; Jacobs et al., 2010, 2011; Kazemi et al., 2011) and is a critical aspect in this study, where responses to students are the central focus. Lesson planning practices intersect with mathematics teacher noticing because what teachers plan for–and do not plan for–will influence what they do (and do not) notice. So better understanding the connection between lesson planning and noticing can help support teachers increase their intentional responses to student errors and uncertainties

Lesson Planning

The preparation and planning that teachers do is central to what happens in enacted lessons, specifically how they respond to errors and uncertainties. Planning for lessons provides teachers an opportunity to be directed in their noticing and intentional in their discourse practices. If teachers have prepared for instances of student errors and uncertainties, the responses they give can be less dependent on in-the-moment thinking and reacting, which are likely to be IRE-based. This shift to being more planned and mindful of discourse practices is likely to manifest in responses to errors and uncertainties that are better able to elevate, not evaluate, student thinking.

Clark and Peterson (1986) defined lesson planning as a "set of basic psychological processes in which a person visualizes the future, inventories, means, and ends, and constructs a framework to guide his or her future action" (p. 260). How teachers psychologically prepare for lessons is both complex and individualized. When teachers plan, they consider many components of the lesson–curriculum, resources, students, etc. The "most obvious function of teacher planning in American schools is to transform and modify curriculum to fit the unique circumstances of each teaching situation" (Clark & Peterson, 1986, p. 262). Lesson planning is considered a high leverage practice (Morris & Hiebert, 2017) and is important because of the adaptive nature of this practice. Lesson plans play a vital role in customizing curriculum to particular students and have been found to be a practice that varies greatly from teacher to teacher (Peterson et al., 1978). Lesson planning that focuses on student thinking and teacher response should allow teachers to more easily capitalize on student thinking and rely less on IRE.

Historical Overview of Lesson Planning Models

Up until the mid-1980's the dominant lesson planning model (Tyler, 1950) involved forward, linear planning. This model did not emphasize goals or learning objectives and focused more attention on what students and teachers were doing. Student thinking took a backseat to student doing. When Madeline Hunter's (1985) lesson planning guide became mainstream, it helped change teacher thinking, classroom discourse, and the role of student thinking. Hunter's lesson planning model included checks for student understanding and started to centralize student thinking. Another significant shift in lesson planning philosophy and practice was backward design (Wiggins & McTighe, 1998, 2005). Backward design is predicated on the idea of starting with learning outcomes and connecting other elements of lessons and units toward that. As teachers design with the end in mind, student thinking plays an even more central role in planning.

Microadaptations and Changes

Corno and Snow (1986) studied microadaptations–teachers' in-the-moment decisions that aim to tailor instruction to the needs of different learners which are similar to Shroyer's (1981) critical moments, moments in which teachers had to experience "cognitive difficulty or emotional discomfort" (p. 115) about "unpredictable student difficulties or insights" (p. 113). Teachers make microadaptations in a variety of ways–organizational structures, ways information is presented, materials to guide problem solving, support materials, time, and feedback, to name a few. The prerequisite to making adaptations is having something to adapt: the lesson plan, even if that plan is just a mental image of the lesson. Because curriculum and textbooks cannot possibly be designed to engage every child, teachers use their knowledge of students to bridge the gap between student and content (Ball et al., 2005; Dewey, 1902; Hill et al., 2005; J. P. Smith III & Girod, 2003).

Lesson Plays

In work with PSTs in a mathematics methods course, Zazkis, Sinclair, and Liljedahl (2012) developed a model of planning that they call lesson plays. In this model, lessons are written in the style of the script of a play. Line-by-line dialogue is written out. The intent is that teachers can use this format to examine their beliefs and assumptions as well as student thinking. This combination of planning and analysis helps teachers move from teacher-centered and IRE-based discourse toward student-centered and diverse discourse. This strategy offers teachers a specific tool to address this study's focus: how teachers respond.

5 Practices for Orchestrating Productive Mathematics Discussions

Smith & Stein (2011) continued the argument of centering student thinking through their *5 Practices for Orchestrating Productive Mathematics Discussions*. In this framework, specific and high-level lesson goals lay the foundation for lessons that engage students in substantial discourse. This model encourages teachers to find an open-ended task that can be accessed and solved in a variety of ways and then solve the problem in as many ways as they can consider. This helps teachers consider the thinking and ideas that students may have and use in their work. When students are working, teachers take notes of what strategies students are using and use all this information to decide how the solution paths will be shared in the lesson. This is one example of how teachers can better prepare for responding to students by considering possible student solutions and potential benefits and pitfalls of the variety of anticipated student solutions. By using these specific steps and broader practices, teachers should become less dependent on IRE.

Thinking Through a Lesson Protocol

In related work from the same scholars (Hughes, 2006; M. S. Smith et al., 2007) the *Thinking Through a Lesson Protocol (TTLP)* was designed to help mathematics teachers plan for high-level cognitive tasks and to help them move to deeper considerations of student thinking and how to extend it. TTLP is partitioned into three sections: (a) selecting and setting up a mathematical task, (b) supporting students' exploration of the task, and (c) sharing and discussing the task. Within TTLP, questions prompt and drive teachers to focus on student thinking and their responses (i.e., likely mistakes students make, places where students should have prior knowledge, and questions students might ask). This model lays significant groundwork for teachers to be well-equipped to respond to student errors and uncertainties. It is the model from which PSTs in this study planned their lessons. See Appendix I for the version PSTs used in Fall Semester. See Appendix J for the version PSTs used in Spring Semester.

To help teachers best prepare and position themselves to engage with and help their students through rich, classroom discourse, their planning should center on student thinking, classroom discourse opportunities, and the discourse practices they intend to use. Lesson planning is at the heart of teacher beliefs and conceptions and is a strong entry point when trying to support teachers improve their discourse practice.

Chapter 3:

Research Questions and Frameworks

The purpose of this study was to examine the relationship between the lesson planning practices and classroom discourse practices of PSTs, specifically focusing on PSTs responses to students' errors and uncertainties. I did this by examining the planning and teaching of five PSTs across two semesters.

Research Questions

Three research questions (RQ) guided my inquiry:

1. How do PSTs anticipate student thinking in their lesson planning?
    a. To what aspects of student thinking do PSTs attend while planning lessons?
    b. What resources do PSTs draw on and use as they attend to student thinking?
2. What role do errors and uncertainties play in the enacted mathematics lessons?
    a. What is the context for the lesson, practices, and norms that make student thinking public?
    b. What teacher moves precede instances of errors and uncertainties?
    c. How do PSTs respond to instances of errors and uncertainties?
3. What is the relationship between PST's anticipation of student thinking in lesson planning and the role played by errors and uncertainties in the enacted lessons?

The first research question entailed examining the written lesson plans PSTs created and interviews with the PSTs about their planning for evidence of the aspects of student thinking they attended to in their planning. The goal here was to document how PSTs addressed student thinking in the planning and the resources they drew upon in that consideration of thinking. RQ 2 focused on the enacted (taught) lessons, drawing on video recordings of the lessons and post-lesson interviews with the PSTs to examine teacher discourse moves that preceded and followed instances of student error or uncertainty. RQ 3 brought together the planning and the enacted lessons to examine how the PSTs' plans for student thinking compared and contrasted with how they responded to errors and uncertainties in their teaching.

Conceptual Framework

In this research, I examined PSTs' lesson planning and classroom discourse practices, focusing on anticipation of student thinking and on the role of errors and uncertainty in the lessons. I begin presenting the conceptual framework for the study by defining a number of key constructs.

Student Thinking

By student thinking, I refer to a variety of mental processes that students may engage in around mathematics. Synonyms include *student understanding* (Anthony et al., 2015; Jacobs et al., 2010) and *reasoning* (Anthony et al., 2015; Baker, 1994; Wood & McNeal, 2003). More specific categories of student thinking include: strategy creation and use (Baker, 1994; Carpenter et al., 1989; Jacobs et al., 2010; Kazemi & Franke, 2004), problem solving (Baker, 1994; Carpenter et al., 1989; Fennema et al., 1996; Schoenfeld, 1992), working with models and representation (Kazemi & Hintz, 2014), errors, misconceptions, and how students work through errors (Baker, 1994; Bray, 2011, 2013; Kazemi & Hintz, 2014), student justifications (Steinberg et al., 1994; Stockero, 2014; Stockero et al., 2014; Stockero & Van Zoest, 2012), and the communications around student thinking (Carpenter et al., 1989; Franke et al., 2009; M. S. Smith & Stein, 2011; Steinberg, 2013; Wood & McNeal, 2003).

In this study, I focused my attention primarily on *student understanding* (what they are correctly making sense of), *misconceptions* (mistakes and errors), and *communication*, based on

Carpenter et al.'s (1989) argument that "the most effective way to analyze children's thinking is by asking appropriate questions and listening to children's response" (p. 505).

Practices

Teachers' practices play an important role in this research. Overall, I used Lampert's (2010) second conception of practice: Practices are a collection of things that teachers do, habits, customs, or routines with subcategories of strategies and techniques. For example, if a teacher's goal for part of a lesson is to elicit student thinking (practice), then the teacher might elicit an answer (strategy) or ask for clarification (strategy). To elicit the answer, the teacher might use think-pair-share (technique) or directly call on a student (technique). This conception of practice aligns with Smith and Stein's (2011) five practices for orchestrating mathematics discussions and the TMSSR (Ozgur et al., 2015; Reiten et al., 2016). The four TMSSR categories–elicit, respond, facilitate, extend–are practices which are made up of strategies such as ask for clarification or press for justification.

Lesson Planning Practices

Lesson planning practices are processes and routines that PSTs use to prepare for a lesson. Features of lesson plans include who the PST plans with (individual or in teams); the use of materials and resources (textbooks, colleagues, or the Web); and the structure of the planning (between systematic and organized to spontaneous).

Mathematics Classroom Discourse Practices

Mathematics classroom discourse practices are the "routines and patterns that take place within a communication system" (Cazden, 1986, p. 432), which include many "ways of representing thinking, talking, agreeing and disagreeing" (National Council of Teachers of Mathematics, 1991, p. 34). Discourse practices are situated in the social context of mathematics classrooms (Barwell, 2008; Cobb et al., 1993; Wood et al., 1991) and facilitate students' learning of mathematics. Teacher discourse moves around student errors and uncertainties are a significant component of mathematics classroom discourse because of its importance and challenge.

I define a discourse *teacher move* as something the teacher says (or intentionally does not say as in the case of waiting or monitoring) in an enacted lesson. I used the TMSSR Framework (Reiten et al., 2016) to code, categorize, and explain the classroom discourse data. I relied on the four TMSSR practices–elicit, respond, facilitate, extend–to group the discourse moves which are the strategies in the aforementioned relation of practices, strategies, and techniques.

Teacher's Knowledge and Beliefs

Teacher practices are shaped by teachers' knowledge, beliefs, conceptions, and identities. Knowledge encompasses mathematical, pedagogical, curricular, technological, and instructional understandings. These categories of knowledge influence the curricular and instructional decisions that teachers make. By teacher beliefs or conceptions, I mean the ideas and philosophies teachers have that inform and shape their professional decision making. The beliefs could be about tools (e.g., whether calculators should or should not be used in various settings or tasks), how students learn best (e.g. when working with others, when given multiple chances on assessments, with high-stakes testing), or what mathematics is and is not (e.g. patterns to be explored, procedures to be memorized). Beliefs and conceptions teachers hold may not be ideas that teachers readily or intentionally think about. These ideas may come from previous experiences as a student, their education, and their teaching experience. Teacher identities are the aspects of the teachers' self, gender, race, ethnicity, age, abilities, and life experiences that make them who they are. Teachers' knowledge and beliefs influence planning and discourse practices via the enactment of strategies and techniques.

In this work, the primary beliefs I explored were those about student thinking, errors, and uncertainty. I conceptualized these categories of beliefs as having substantial overlapping and blurry boundaries. PST beliefs and conceptions influence the way mistakes are used or not, leveraged or not, and the ways teachers respond (Bray, 2011, 2013). What teachers believe the role of errors, their responsibility around errors, and students' responsibility around errors influence the classroom error climate (Steuer et al., 2013) and the subsequent decisions that teachers make based on these beliefs.

Teachers have a variety of beliefs around errors. For some teachers, particularly those from less productive views of mathematics or those who use IRE (Mehan, 1979) as a primary discourse pattern, errors likely show where "faulty" thinking lies and provides teachers the opportunity to "fix" or correct such thoughts. Some teachers may hold the idea that people can learn from mistakes and thus, mistakes are positive (Bray, 2013, 2013; Seifried & Wuttke, 2010). Some teachers may see mistakes as simply another thought that students have and not have much value attached to them one way or the other. These beliefs are all likely to influence the culture and philosophy of the classroom through the students' emotional and cognitive experience making errors and seeing others do so, too.

As an overall framework, I drew on Remillard's (1999) model of curriculum. Remillard posited three connected parts of curriculum: the *design arena*, the *construction arena*, and the *improvisation arena* (see Figure 3.1). I incorporated the model into my framework to examine the connections between lesson planning, enacted lessons, and responses to student errors and uncertainties (see Figure 3.2).

Figure 3.1:

Remillard's (1999) Model of Curriculum

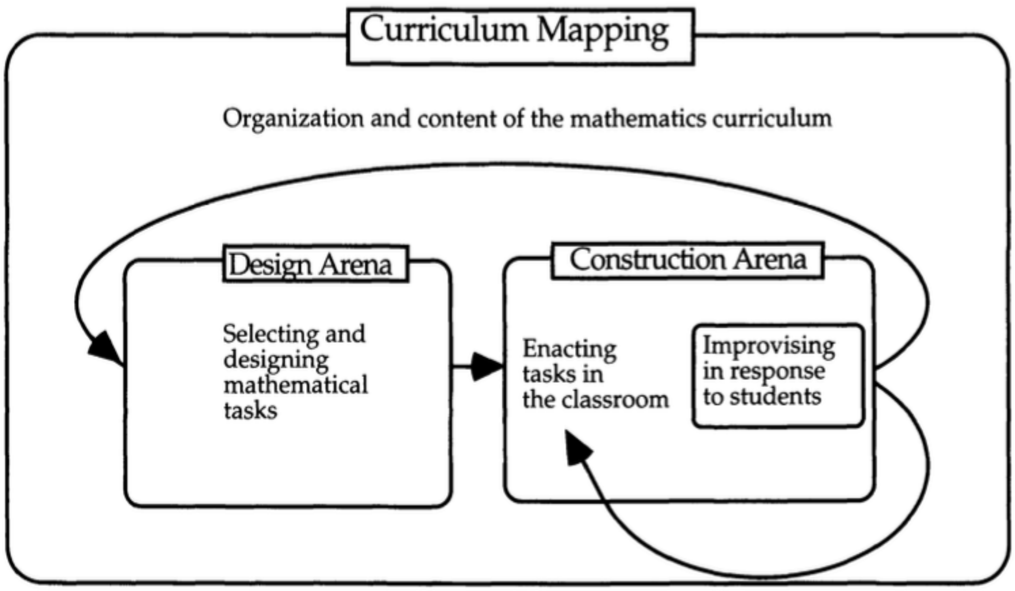
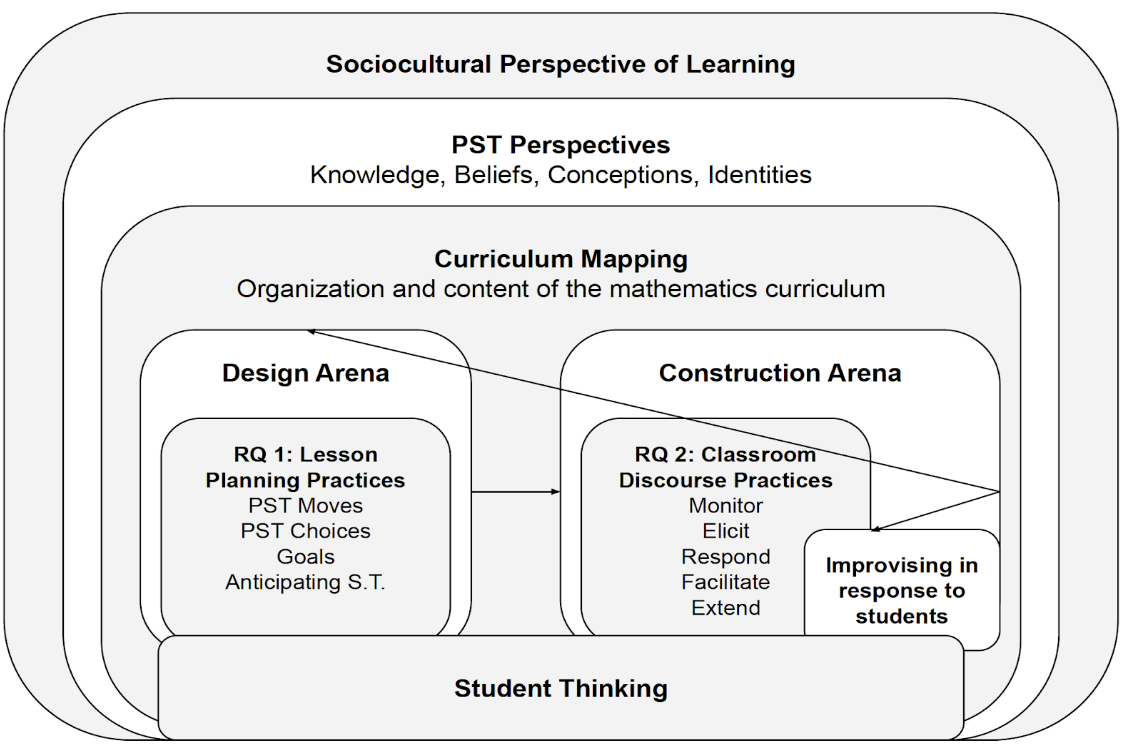


Figure 3.2:

Framework for Planning, Classroom Discourse, and Student Thinking



In this framework, Remillard's design arena maps to the lesson planning practices and preparation work that teachers do before the lesson. The construction arena includes the enacted lesson components such as classroom discourse practices, teacher noticing, and planned responses to student errors and uncertainties. Teachers' responses to unplanned or unexpected student errors and uncertainties constitute the improvising space. The framework centers on student thinking as it exists in and interacts with practices in the Design and Construction arenas of Curriculum Mapping. PSTs' knowledge and beliefs provide a broader context that shapes teachers' practices.

RQ 1 focuses on the design arena–PSTs' lesson-planning practices for anticipating student thinking. RQ 2, focuses on classroom discourse practices in the construction arena in terms of how PST responded to student errors and uncertainties. Finally, RQ 3 focuses on the relationship between lesson planning practices and classroom discourse practices.

Chapter 4:

Method

This study examined five PSTs' lesson planning practices, classroom discourse practices, and beliefs about teaching to better understand the relationship between planning for student thinking and responses to students' errors and uncertainties during enacted lessons. I used written lesson plan and interview data to consider what types of student thinking (e.g. procedural, conceptual, metacognitive) PSTs planned for, whether they considered the student thinking as expected to occur or goals for the lesson, and how error played a part in the planning around student thinking. I also examined the resources PSTs used to plan for student thinking. I observed and video recorded lessons, noting classroom and school norms and culture as they situate the teaching and learning in this study. I analyzed what events occur prior to moments of errors and uncertainties and the ways in which PSTs responded to those moments as we all overall PST discourse moves. Finally, I used interview data to draw connections between the planning and classroom discourse practices.

Participants

I recruited participants at the end of November 2017 and into early December 2017 during two visits to the Fall Semester secondary mathematics methods class. During the first visit I introduced myself and the project and dispersed informational handouts, consent forms, and envelopes (see Appendix A for materials). I told the PSTs that I was looking to better understand how lesson planning impacts classroom discourse. I answered a few questions in the class and received no emailed questions. All 14 students in the methods course indicated their choice to participate or not on the consent form, which they enclosed in an envelope to help protect anonymity and confidentiality. On the second visit I collected all envelopes. From this process, five PSTs self-selected to participate, and I chose to work with all of them. All five participated in the entirety of the study.

All participants were fourth year, secondary mathematics majors completing their undergraduate course work. They all planned to participate in a yearlong teaching internship the next academic year. See Table 4.1 for teaching partners and mentor teachers.

Research Setting: The Methods Courses

During Fall Semester, all PSTs were members of the same secondary mathematics methods course–the first of a two-semester sequence–in a large, research university in the midwestern United States. The class was taught by one university faculty member and two graduate student TAs. The faculty and TAs led the methods courses and were present during the accompanying micro teaching lab which occurred in an undergraduate remedial mathematics course. In these methods courses, the teaching labs (the lessons PSTs taught to become more familiar with the planning and teaching process) were central to the curriculum and PSTs' experiences.

Table 4.1:

PST Fall and Spring Semester Teaching Partners and Mentor Teachers

Participant	Fall PST Partner	Spring PST Partner(s)	Spring Field Placement Mentor Teacher
Taylor	*Elizabeth	Carson, *Matt	Mrs. Tharp
Carson	Rowan	Taylor, *Matt	Mrs. Tharp
Rowan	Carson	*Peter, *Sheri	Mrs. Langston
Reed	Lian	*Adam	Mrs. Ebb
Lian	Reed	*Elizabeth	Mrs. Milton

Note. * denotes the PST was not a participant in this study.

During Fall Semester, PSTs planned and taught in the undergraduate course three times in teams of two based on the number of PSTs and lessons during the semester. Lesson topics were determined by the curriculum and pacing of the undergraduate course. For their first lesson, two PSTs led a Number Talk (Boaler, 1999) and the TA taught the remainder of the lesson (the launch, explore, and summary). Because PSTs' led only the Number Talk, this lesson was not included in the study. For their second Fall Semester teaching (*Teaching 1* in this study), PSTs planned for and led a Number Talk and launch which was 40-45 minutes, then the graduate student TA taught the rest of the lesson. For their third Fall Semester teaching (*Teaching 2*), the PSTs planned and led the entire 110-minute lesson. (See Table 4.2 for summary.) PSTs wrote individual reflections about their experiences after Teachings 1 and 2.

In Spring Semester, all PSTs took the same second-semester mathematics methods course. The course was taught by a different faculty member and two graduate student TAs. During Spring Semester, PSTs taught twice in their field placements–middle school, junior high, and high school classrooms within a 45-minute drive of the university. Per their methods course requirements, the first Spring Semester teaching (*Teaching 3*) was a test review with an activity

Table 4.2:

Summary of Four Teachings and Lesson Plan Requirements

Teaching	Date	Methods Course Requirement	Lesson plan template
1	October	Formal written lesson plan	Abbreviated TTLP
2	November	Formal written lesson plan	Abbreviated TTLP
3	March	None	None
4	April	Formal written lesson plan	Different Abbreviated TTLP

based on students using stations, planned and enacted in pairs. For Teaching 4, PSTs individually planned, enacted, and reflected upon a lesson that fit into the curriculum of their field placements. (See Appendix C for a summary of the topics and tasks for all lessons.)

Data Collection

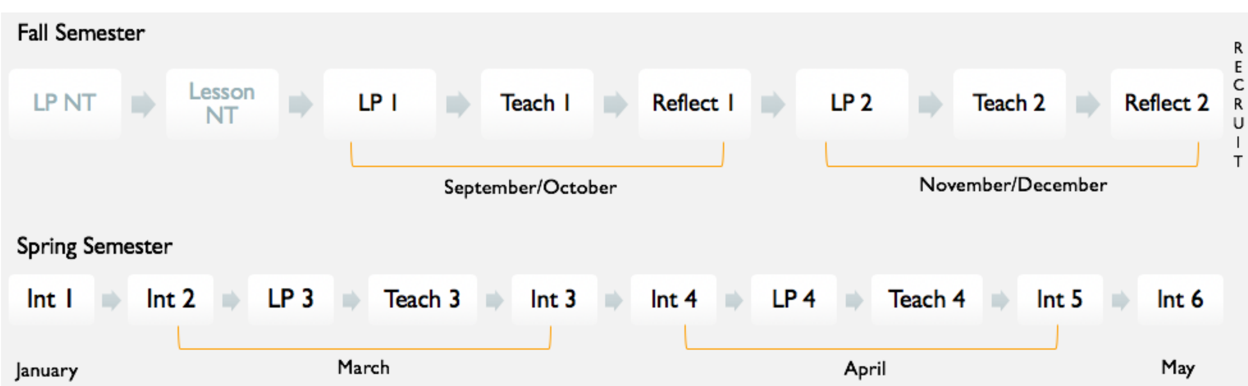
Data for the study included written lesson plans, written reflections, interviews, and video recorded lessons. From Fall Semester, I had access to written lesson plans, videotaped lessons, and written reflections. During Spring Semester, I conducted six interviews with each PST, observed and videotaped their enacted lessons (two each), took field notes, and collected their written lesson plans. See Figure 4.1 for a pictorial view of the data and timeline of the lesson plans (LP), lessons, written reflections (Reflect), and interviews conducted in this study.

Lesson Plans

The lesson plan documents were required components of their methods courses and the structure and layout was determined by their methods instructors. For Fall Semester lesson plans, I was given permission to access the course website where all work was submitted. For Spring lesson plans, PSTs shared their lesson plans with me in the most convenient way for them–

Figure 4.1:

Pictorial View of the Study's Data Collection Timeline



emailing me the documents or sharing them with me via Google Drive. Upon receipt of lesson plans, I changed the participants' names to their pseudonyms, removed any other names of their colleagues or students, and saved the document on my password protected computer.

Lesson plans for Teachings 1 and 2 had the same routine and structure as each other. Through this process, PSTs ultimately created three versions of their lesson plan. First, PSTs created and submitted an initial lesson plan prior to their enacted lessons. TAs provided feedback on this submission. After receiving feedback, PSTs revised the lesson plan and submitted version two. Finally, after teaching, PSTs revised and submitted the third lesson plan which was amended to match what had happened during the enacted lesson. For analysis, I used the second iteration of these lesson plans.

During Spring Semester for Teaching 3, PSTs did not have a required lesson plan to turn in for their methods course. Three PSTs' (Reed, Rowan, Lian) lesson plans consisted of the document they printed with questions for the students' stations. The other two PSTs (Taylor and Carson, who taught together) had pictures of plans they had made with their third PST partner on chalkboards from a university classroom.

The lesson plans for Teaching 4 consisted of an initial draft that was written by the PSTs individually and a revised draft that was edited based on methods instructor feedback. I used the version of the lesson plan that had received instructor feedback and was created prior to the enacted lesson for this analysis. Several PSTs had additional documents and information (copies of textbook pages and worksheets for students) that they also shared with me.

Interviews and Reflections

I interviewed each PSTs six times during Spring Semester (January – May). Most interviews were conducted in private conference rooms within the College of Education building. Two of my interviews with Taylor (after Teaching 4 and the final interview) were conducted on Zoom due to the fact that Taylor was unable to meet in person at that time. All interviews were audio recorded and transcribed prior to coding and analysis. Audio recordings, transcriptions, and coded documents were saved on my computer. Written documents were backed up on a password protected, external hard drive. Audio and video files were saved in a private section of a university provided repository for videos.

The first round of interviews occurred in late January. The purpose of these interviews were to better understand the PSTs and their conceptions around lesson planning, errors, uncertainties, and learning. These interviews were structured and provided introductory information. I asked about the PSTs' backgrounds, what drew them into teaching, and their philosophies and beliefs about errors and uncertainties. The interviews lasted about 15 to 20 minutes (see Appendix D for all interview protocols).

I interviewed each PST in March before they enacted Teaching 3 and in April before they enacted Teaching 4. The focus of these pre-lesson interviews was the lesson planning practices that the PST engaged in prior to teaching. I asked about general planning practices, specific elements for the upcoming lesson, and about the resources they used for planning for student thinking. These semi-structured interviews lasted about 20 to 30 minutes and occurred one or two days prior to the enacted lesson. I had a series of questions that I asked all participants and followed up with probing questions when I needed clarification or an idea seemed pertinent to this study. Taylor and Carson planned and taught Teaching 3 together, but I interviewed them separately and did not discuss the other participant or participant's responses with them.

I interviewed all PSTs one or two days after they enacted Teachings 3 and 4. During these interviews, which tended to last around 40 minutes, I asked about the alignment between the enacted lesson and the planned lesson. I chose three instances from the lesson, and using stimulated recall, showed the PST these clips. I asked them to tell me about the student thinking and their responses to these instances.

I conducted a final round of interviews in May. During this round, I asked similar questions to the January interviews, in a semi-structured format. These interviews took about 30 to 40 minutes. These interviews provided an opportunity for PSTs to revisit topics they had discussed in initial interviews.

I do not have interviews from Fall, because I did not recruit for the study until the end of Fall Semester. However, their written reflections captured some of the same ideas that spring pre- and post-lesson interviews did.

Video and Audio Recorded Lessons

Teachings 1 and 2 were video recorded as part of a separate study that was being conducted in the methods course. I was given access to those videos from the Principal Investigator of that study. During Spring Semester, I attended, observed, video or audio recorded, and took field notes on most of the Teachings 3 and 4. See Table 4.3 for a summary. With permission from field placement districts and teachers and consent from the PSTs, I brought the video and audio recording equipment to the field placement classrooms. I set up a camera in the back of the room as directed by the mentor teacher. I sat near the video camera and moved it to ensure the PST was in the frame. These recordings were saved to my computer and backed up to a private location in the university provided media repository. After Teaching 4, I shared the recordings with the PSTs via flash drive to help them with a methods course task.

When coding, I considered each PST as their own entity, despite the fact that Teachings 1, 2, and 3 were cotaught. Because each PSTs wore an individual lapel mic, I captured individual

Table 4.3:

Notes About Recordings of Teachings 3 and 4

Participant	Teaching 3	Teaching 4
Taylor	Had 2 video cameras. Captured audio and video.	Captured audio and video.
Carson	Had 2 video cameras. Captured audio and video.	Captured audio and video.
Rowan	Captured only audio via lapel mic based on mentor's request.	Captured only audio via lapel mic based on mentor's request.
Reed	Captured only intermittent video. Audio recording did not work.	Captured only intermittent video. Audio recording did not work.
Lian	Captured audio and visual. Minutes 12-17 did not have Lian's audio.	Captured audio and video from Lian's cell phone that she recorded. Field placement school was on lockdown and I was not allowed to enter the building.

audio for each PST. In the vast majority of instances, the same PST whose move preceded the error or uncertainty also responded. In the few cases in which one PST was involved in the preceding moment and a different PST was involved in the response, I coded those for the different PSTs. There were also some situations in which a mentor teacher or PST not within this study initiated the instance. Those are not included as preceding utterances, but I did include responding utterances if the PSTs was in this study. I had technical issues with Reed's lapel mic for Teachings 3 and 4. The video recorder's microphone picked up some audio, but Reed's lapel microphone did not. Because the audio was incomplete, I have not included Reed's data from Teachings 3 and 4.

Data Analysis

I conducted data analysis to develop understandings of each PST so that I could create descriptions of each PST (see Appendix G for full description) and describe them as unique individuals. I continually compared and contrasted within and between the PSTs themselves and with relevant literature. These comparisons helped illustrate similarities in lesson planning and classroom discourse practices. The contrasting helped highlight the uniqueness of each PST related to their planning and classroom discourse. I grounded my analysis and thinking in context and relationships. I considered broad issues such as community and school culture, as well as more specific issues such as whether students regularly had the opportunity to share thinking or were only asked to explain things if they made mistakes. I also acknowledge that my presence in the classroom during the lessons impacted the experiences of the PSTs and their students.

Analysis Process

I coded and analyzed data using Google Sheets online spreadsheets. Each segment of data that received a code was put on its own row. This often involved more than one code per utterance, so a single utterance was frequently on several rows, separated by different codes it received. To get data into Google Sheets, I used the following processes. First, for written lesson plans and reflections, I examined the document and when I found key text that needed to be coded, I copied and pasted the text into the Google Sheet. I entered the code in the cell to the right of the lesson plan data. Second, for interview data, I used Express Scribe to manually transcribe interviews. When I found text that needed to be coded, I copied and pasted the text into my Google Sheet and put the code in the next cell. Finally, for video data, I used iMovie to compile and synchronize video and audio files. I transcribed the lessons into Google Sheets and coded key text. Particularly within transcribed discourse data from enacted lessons, I tracked for patterns of language that precede and was in response to student errors and uncertainties. However, this is an imperfect system as much classroom discourse and conversation is iterative and circular and it was challenging to consider what was a response from a PST if the response prompted another student utterance.

Quantitative and numerical data helped illustrate the themes and patterns in the data. These data indicated places that needed further scrutiny or examination by qualitative data.

Coding

I developed a coding scheme (see Appendix F) for all data sources that was built on previous research and frameworks: Teacher Discourse Moves (TDM) Framework (Herbel-Eisenmann et al., 2013), TMSSR (Ozgur et al., 2015; Reiten et al., 2016), Tulis (2013), Santagata (2005), lesson plan coding structure from Bieda, Dillman, Voogt, & Gundlach (2017), *5 Practices for Orchestrating Productive Mathematics Discussions* (M. S. Smith & Stein, 2011) and the coding scheme from my practicum. I relied on open coding to capture ideas that emerged from the data that were not included in previous research. I used cross-comparative methods to build and organize codes, comparing my current data with the coding system that I was building (see Appendix H for a detailed process of the code development). I started with a sampling of interviews, then lesson plans, then enacted lessons to build the code book. When I noticed a new code or the codes changed, I revisited all previously coded data to cross check against updates. I continued to encounter the challenge of the interconnected nature of the codes. My coding process involved grouping and regrouping many times. My analysis was both ongoing and iterative. As I was going through this process, I ended up with roughly 200 codes. The final version of my codebook contained 43 codes, many of which contained subcodes.

Chapter 5:

Introducing the PSTs

To provide context for addressing my research questions, I begin by introducing the five PSTs. By sharing who they are, their backgrounds, their beliefs about errors and uncertainties, and lesson planning practices--as shared--in Interviews 1 and 6, I hope to establish who they are as humans as well as establish ways they are similar and different.

The PSTs

Taylor is a white female from a large urban area and school district. She has academic interests in both mathematics and computer science. Taylor reported that she is "very emotional" (Int 1) and is the only participant who mentioned social and emotional components of teaching. Taylor's teaching philosophy centers around the idea of guiding students through the big ideas of mathematics and learning rather than telling them answers. Taylor believes that student thinking is how students interpret, interact with, digest, and process material. Taylor explained that student thinking is not always visible, but it can be revealed by using probing questions. Taylor believes that errors are good and "fruitful" (Int 1). When errors arise, she tries to figure out where the errors are coming from, having students explain their thinking so she can understand what is happening. In January, Taylor said that uncertainty was more of a problem than mistakes students need to be confident to make mistakes and uncertainty is the opposite of confidence. In May, Taylor reported that vocalized uncertainty was good. She argued that if a student is struggling, they are likely not alone. Taylor described her lesson planning as a straightforward, linear progression (similar to Tyler, 1950) in which after she got her topic, she would consider what she knew, develop an activity that connect to what students already knew and then consider student responses, “right or wrong” (Int 1). In May, she emphasized the importance of picking a topic or lessons to allow students to have multiple points of entry.

Carson is a white male who grew up in a rural town and school district. Carson has tutored mathematics for several years as part of his undergraduate work at the university he attends. He has a strong history in scouting and is an Eagle Scout (Int 1). Carson is quite nuanced in his thinking; his answers to interview questions and lesson planning practice were specific and context driven. Carson’s knowledge of pedagogy and educational research was evidenced by use of phrases such as IRE (Int 4, 6) and self-efficacy (Int 1, 6). No other PSTs mentioned either of these concepts in this way. Carson’s teaching philosophy stems from his belief that mathematics has many approaches and many ways to be useful because it is a broad subject. When describing student thinking, Carson emphasized that student thinking comes in a variety of forms. He described it as “what goes through students’ heads” (Int 1) and prior experiences, background, and identity influence student thinking. Student thinking includes the ways students process, strategies, and ways they relate to mathematics content. Carson characterized errors as “misconceptions” (“lack of understanding”) or “algebra mistake” (e.g., “dropping a sign”) (Int 1). He is more concerned with long term impacts of students’ errors compared to them arising in the present. In May, Carson explained more that errors are not necessarily good or bad, but it is how teachers respond and what learning happens as a result of the error is the important part. In January, he attributed uncertainty to a lack of either “confidence” or “student self-efficacy.” In May, Carson described uncertainty as a natural reaction to learning because when things are unfamiliar, it is normal to be uncertain.

Rowan is a white male who moved around a lot as a child. Rowan was heavily concerned about honoring his students’ backgrounds and “building competence to build confidence.” In his teaching, he tries to guide rather than tell students answers, and he thinks that probing questions are "the ultimate tool for teachers." It was sometimes difficult to follow Rowan's train of thought and explanations; he tended to get himself sidetracked. Rowan is hearing impaired and wears a cochlear implant. He did not bring this up or talk about it, but it significantly impacts his cadence and pronunciation. There were a number of times during his teaching when he did not hear what students were saying or when students were trying to get his attention. Rowan's teaching philosophy is centered on building relationships with students to establish respect (Int 6) and making mathematics accessible to all (Int 1). When talking about student thinking, Rowan mentioned students' prior knowledge, their interaction with other students and their teacher, how students conceptualize, process, approach, think they can solve, actually solve, and their work. For Rowan, learning from mistakes is how people learn. Errors are good and uncertainty is also positive and part of the learning process. In January, he reported that his lesson planning process stemmed from the main concept and then he created a task with "lots of approaches to it, so it can be done many different ways." His lesson planning practice was influenced by the TTLP.

Reed is a white female who came from a lower middle-class family and a large school system. Reed is passionate about mathematics not necessarily because she thought she was "the best at it," but because it was something that she "actually struggled with" (Int 1). Reed's teaching philosophy was not clearly articulated and seemed to draw from perspectives that are at odds to each other. Reed reported that her teaching philosophy centers on making mathematics "accessible" (Int 1), approachable (Int 1), and interesting to students who are marginalized from the mathematics community such as "minority students, students whom English is not their first language, and women" (Int 6). In January, she named Culturally Relevant Pedagogy and in May, she named Teaching for Social Justice as movements and frameworks that influence and inspire her. On the other hand, she also described elements of her teaching philosophy that focused on grades and college. In Interview 1, she explained

> I want to try to make mathematics at least fun to learn. I don't necessarily want it to be everyone's favorite subject, but I want everyone to be able to pass the class–I'm not talking A, I'm talking C. I want all students to be able to get at least a C in a class, but also enjoy being there even if they don't enjoy the math. I kind of want to learn how to do that.

For Reed, student thinking is “any thought behind a process of solving something, answering a certain way” (Int 1, 6), misconceptions, preconceived notions (Int 6). Reed believes that errors are good because if “you pay attention to the fact that you made a mistake, you're probably not going to make it again.” She described that uncertainty is normal and expected, however if uncertainty prevents students from trying, then it can have a negative impact. Reed’s lesson planning involves a lot of her thinking about things, even though she does not necessarily write them down. She relied on her fiancé (Int 1) (who has a degree in writing) and his (lack of) mathematical content knowledge to represent struggling students (Int 1).

Lian is a Chinese female who came to the United States in 12th grade. Lian focused heavily on the connections and intellectual relationships that her students make within mathematics. For Lian, student thinking is how students view a question, how they interpret or translate it, and the process or strategies they use to come up with an answer (Int 6). The key word that she consistently used when asked about student thinking was “connections” (Int 1, 4, 6). For Lian, errors are normal or good (Int 1, 6). Errors provide an opportunity to learn from others and make mistakes prior to assessments. While Lian considers errors normal to positive, she considers uncertainty as a bad thing (Int 1, 6). To her, uncertainty was when students “cannot (make) the connection of what we learned to the question" and this lack of connection implies that the student "cannot make mistakes to improve or find the right steps to get closer to the answer" (Int 1). Lian reported that she planned for lessons with her PST colleague (Reed) during Fall Semester. They would work in the library, look at the problem, try to solve it, and try "to think of any situations that might come up, and other strategies we can use to solve the problem" (Int 1).

Knowledge and Beliefs

The PSTs' knowledge, beliefs, and world views influenced their thinking and lesson planning. In the lesson planning process and interviews with me, PSTs revealed knowledge and beliefs relevant to their planning and teaching. I coded the lesson plans and interviews to identify occasions on which the PSTs referred to their knowledge (of content, of self, of students) or their beliefs. About 300 occasions, PST's knowledge and beliefs emerged in the lesson plans and interviews. These occasions clustered around their knowledge of mathematical content, their beliefs about mathematics, the role of accuracy, creativity, mindsets, and their knowledge of students. I share these because the PSTs' knowledge and beliefs influence the relationships they have with students, the type of lessons and tasks they create, plan, and enact, the student thinking they anticipate, the way they teach, and how they respond to students.

Knowledge of Students

PSTs' knowledge of their students emerged 133 times throughout their plans. This knowledge clustered in several ways. PSTs shared how they knew their students as humans (student identity). PSTs also shared how they knew their students as mathematicians (students' mathematical knowledge). Knowledge of students was not a discrete collection of isolated facts. Instead, this knowledge interacts with other curricular knowledge and instructional moves in both the design and construction arenas as well as microadaptations in the construction arena. Knowledge of students underscores PSTs practices, strategies, and techniques.

Knowledge of Student Identity

All PSTs mentioned their students' identities and this knowledge emerged 48 times in total. PST described a variety of different aspects that encompass their students' identities. For example, they described the context of the schools and classrooms they were in. Rowan shared with me the school he would be teaching in "which houses from seventh to twelfth grade students. So, there's six grades of students in one building. There's a lot of cultures–student cultures" (Int 2). In Interview 4, Carson explained,

> This class is a lab and it is reserved for students who have been recommended for extra support in their math course. They end up in here either based off of testing scores in the previous grade and recommendation from prior teachers. Their continued enrollment in this course is based off of my mentor teacher's thoughts after they're placed there.

In sharing about the same classroom (but in her own, individual interview), Taylor noted "The students in this particular hour often review some basic ideas as well" (Int 4).

Timing, the calendar, and the typical daily schedule for secondary education (five to seven "hours" a day in which a teacher teaches multiple sections of the same course) impacted PSTs knowledge and awareness of students. Three PSTs were placed in secondary schools in which they observed two hours that were the same course, but different students (e.g., 3rd hour and 4th hour). Lian explained, "(the later) hour and (the earlier) hour are not the same. (The earlier) hour has more special needs students" (Int 4) and Lian also described the impact that spring break was having on her students and their focus. In describing Teaching 4, Reed said, I will begin the launch by asking students about what they know about hourly wages...I know that a lot of students are working part time jobs so this is a context that some students can connect with and explain. (Int 5)

PSTs also talked about what their students liked, were motivated by, and not motivated by. Taylor and Carson both mentioned Fortnite (a popular video game) in their interviews prior to Teachings 3 and 4. Reed acknowledged "my students don't care about temperatures…[if] I was like, oh, 'we'll convert from Kelvin to Celsius,' they'll be like, 'so….'" (Int 5). Lian mentioned that one way she was thinking about her students was "and also what activity they are interested in" (Int 5).

PSTs described the learning strengths, challenges, and supports of their students. Specifically, Rowan, Taylor, and Carson mentioned Individualized Education Plans (IEPs). Rowan said, "some of them are English learners and some of them have IEPs. Their IEPs are for reading competence" (Int 2). Carson reported "thanks to our student case study I know of the three students in the class with IEPs" (Int 4) and Taylor said, "the students in the class that have IEPs have 'frequent check-ins' as an accommodation" (Int 4).

Each PST had approximately 10 occasions of acknowledging student identity, the vast majority occurring in interviews, not written lesson plans. During Teaching 3 (March) and 4 (April), all PSTs were teaching in field placement classrooms that they had been in them twice a week since at least January. Overall, I was struck with what they knew about students. Within the scope of this project, I do not have the ability to verify whether their knowledge of students was accurate, so I am using the assumption that PSTs' knowledge of student identities was accurate.

Knowledge of Students' Mathematical Knowledge

In addition to knowing their students as humans, there were 85 occasions of how PSTs know their students as mathematicians. These statements occurred for every PST in preparation for every lesson. Several themes emerged from these statements, which ranged from general ("students will have seen vertex form of an equation" (Taylor, LP 1)) to specific ("students have discussed the growth of exponential functions via graphical, algebraic, and tabular models" (Carson, LP 2)).

First, all five PSTs identified the significance and importance of knowing what students have experienced, done, and said prior to the given lesson. Since PSTs had been in their field placement classrooms for several months at the time of these interviews, they talked about the norms of their classrooms and students' action–activities students do, routines they have, and expectations of students. Taylor called this "subconscious knowledge" (Int 4). There were times they talked about disrupting these norms or routines, too.

Secondly, in Reed's case–and Rowan's, to a lesser extent–knowledge of students' mathematical knowledge emerged intertwined with a narrative about the students. Both Reed and Rowan were teaching in urban schools with large numbers of students of color. Reed and Rowan both made numerous statements about students being behind (Rowan, Int 2) or having learned things but forgotten them (Reed, Int 2). These ideas did not emerge from Taylor, Lian, and Carson who were each working in suburban/rural schools with primarily white students.

Third, PSTs relied on having multiple hours of the same course throughout the day to influence their knowledge of students' mathematical knowledge. Of the second of two hours Rowan observed and participated with, he remarked that they were "a lot more competent than [the other] hour in terms of students' performance and getting the problems done accurately...but they do finish their work more consistently...It's pretty obvious between the two." Being able to compare and contrast impacted how Rowan interpreted and made sense of students' mathematical thinking.

Fourth, as prompted by their TTLP framework, PSTs considered students' mathematical knowledge as they made choices about goals, tasks, and activities. Lian (Int 4) described a goal for her lesson as "basically the students know [the content]. We just need to help them remember what they've learned and help them simplify the expressions." PSTs considered things students knew, such as when Carson (Int 4) brought up that "students have learned how to use a protractor to measure and draw angles," and Carson also kept in mind that students might not be familiar with "use of an actual navigational compass" (Carson, Int 4). Lian used her knowledge of students' knowledge in how she would "introduce basic design elements in the butterfly first because [rotational symmetry] is a new term," (Lian, Int 4).

More broadly, PSTs demonstrated how students' mathematical knowledge related to the course and curriculum. Rowan (Int 2) said "but it's hard at this point because they're behind in the curriculum" and he later described how content and lessons fit together "especially since we've been working with slope" (Int 4). Reed (Int 2) said, "we're looking more at what questions we could ask because definitely this is something that I think [mentor teacher] went over on Friday, but it's not like they'd been working with this for two or three weeks." Lian (Int 2) broadly described the importance of students' mathematical knowledge when she said, "It's more about how much they know about the content."

Finally, PSTs drew on their knowledge of students' mathematical knowledge to consider and plan for things students would not know, had not seen, or might struggle with based on previous experiences. Taylor (Int 4) claimed "order of operations is always something they need to refresh." Lian used her knowledge of the previous days' lesson when she said, "yesterday we observed that the students are not quite doing well for the negative exponentials" (Int 4) to inform the lesson she was planning. Reed (Int 4) drew on her previous knowledge from conducting "a curriculum topics study on inverses" and knew that students would likely struggle particularly because "they haven't done compositions, yet" and "don't know compositions." Carson (Int 2) noted that students

> seem to do just fine with theoretical probability and "oh, this is what should happen!" But when it becomes actual values, they try to relate the experimental probability to the theoretical probability rather than just taking their data and going from there.

Despite all the ways that PSTs mentioned and acknowledged their knowledge of students' mathematical knowledge, there were also times when PSTs did not have a good understanding of students' mathematical knowledge. In one case, Taylor (LP 2) answered both TTLP questions "What are your mathematical goals for the lesson?" and "In what ways does the task build on students' prior knowledge?" with the same answers, indicating that she did not interpret these as different ideas or did not know or understand students' prior mathematical knowledge or experiences and how the lesson they were planning would build upon that. PSTs knowledge of students' mathematical knowledge informed their anticipated student responses in a few cases. In Interview 4, Lian said students "might come up with some questions we didn't even think of because it's the first time they really get into rotational symmetry" and Carson acknowledged that "familiarity makes it a little bit different because I feel like in prior lessons, I've had to do a lot more anticipation" (Int 4).

Content Knowledge

Another cluster of PST knowledge that emerged as supporting and impacting their planning for and anticipating student thinking was the PSTs' own content knowledge (CK). I am using CK to group data that would fall into Common Content Knowledge (CCK), Horizon Content Knowledge (HCK), and Specialized Content Knowledge (SCK) from the Mathematical Knowledge for Teaching (MKT) Framework (Ball, 1990; Ball et al., 2001, 2008, 2009). These CK occasions manifested from PSTs' indications of their understanding and knowledge of mathematics. CK occurred about 135 times across the 20 lessons. Reed, Lian, Rowan, and Carson each had about 30 occasions of CK and Taylor had about 20. All PSTs had some occasions of CK in preparation for Teachings 1, 2, and 4. Taylor, Lian, and Carson mentioned CK in before Teaching 3, though Rowan and Reed did not. CK came up in several ways in the lesson plans. It came up as PST's knowledge of mathematics, as related to student thinking, and connected to curricular knowledge and lesson planning.

First, PSTs included their own factual, procedural, and conceptual knowledge in their planning. Examples of factual CK were when Carson (Int 4) described that "perimeter of circles is circumference" and Taylor (Int 4) explained how "Scratch doesn't use the equal sign. It uses words like set, so it'll be like set solution as or set solution to or set this whatever variable name you have to this." Procedural CK arose with written comments such as "the problem of inverse functions can be solved a number of ways such as, reverse steps, guess and check, 'swapping' x and y and solving for y, the inverse" (Rowan & Carson, LP 1) and "the 3 solution methods are drawing diagrams, tables values with x, y coordinates, or graphs" (Rowan, Int 4). Several occasions of conceptual CK arose and were related to goals for the lessons they were planning. Lian (LP 4) wrote "there are more than one lines of symmetry because you can trace one side and see it’s congruent on the other side with different lines of symmetry” and Taylor (LP 2) wrote “X values change by addition for both. Y values change by multiplication for exponential and addition for linear.” At times, PSTs shared deeper and more nuanced knowledge of mathematics with utterances such as when Reed noted, “but undoing isn't necessarily the right term because to get the real undoing you hit the compose them” (Int 4).

Second, occasions of CK often bridged their own knowledge of mathematics to that of their students and what students were likely to do. Based on the TTLP prompt of “what are all the ways the task can be solved?” PSTs used their CK and knowledge of their students to plan, such as generically saying “There are many ways students can create a scale model and choose to go about the activity” (Carson, Int 4). Lian and Reed (LP 1) wrote that potential solutions were “set up a table; manipulate blocks to make the structure ‘calculable’ using algebra; draw cross-section representations; and rearrange the structures on paper to make them calculable.” The TTLP prompts often generated ideas more deeply connected to anticipated student thinking such as “the origin may depend on how the students define time” (Rowan, LP 4) and

> they kind of brought up this whole thing of “what's the purpose of switching x and y?” I'm kind of like “(you) technically don't have to, it's a convention, you can actually have a function of y equals x.” ...switching it to f inverse of x…. the composition thing where I was thinking of it more as you have a function of x equals y and then you have a function of y equals x. Then those are inverses of each other. And I was thinking, “I'm going to want to get into why you need to switch x and y. (Reed, Int 4)

Finally, PSTs used their CK to make choices for their lesson planning. They described specific mathematics their students would focus on practicing and learning during the lesson, “for example: x^a * x^b = x^(a+b)” (Lian, LP 3). They used their knowledge of mathematics to describe how they designed lessons such as when Taylor (Int 2) said, “we have one [station] on simple probability; theoretical and experimental probability; independent and dependent events; and combinations and trees.” And related to the same lesson, but separate interview, Carson (Int 2) described how they grouped the topics and ideas

> Theoretical probability and experimental probability were separate. But we know those subjects kind of go together...so, we decided to group them together. We made groupings based on what makes sense...theoretical and experimental with independent and dependent went together and then family trees and the combination theorem also went together. So that's how we grouped them.

Beliefs About Accuracy

All PSTs mentioned accuracy in lesson planning, with a total of 23 mentions. Carson’s statements both occurred in Teaching 4 and centered around the idea of the accuracy and precision of the compasses he was using in the lesson. Taylor’s statements centered around students knowing if they were right and working with each other to determine their accuracy. Prior to Teaching 3, when talking about the lesson and his goals, Rowan said “I hate to say it, but it's kind of like getting them to get the answers correct. I hate the notion of only focusing on correctness” (Int 2). However, in Rowan’s Teaching 4, he explained that he was “trying to stay away from getting correctness” and focusing on the thinking and process. Lian and Reed each had four occasions that came from their cotaught Teaching 2. These focused around having students check their own work and choosing a student who had gotten the work correct to be the one who explains to the class and Lian’s individual Teachings 3 and 4 statements followed those two themes. Reed’s individual Teachings 3 and 4 also focused on the use of accuracy for selecting students to present and how much energy students will put into the activity and how that is likely related to their accuracy.

Chapter Summary

The five PSTs in this study had a variety of backgrounds and a diverse set of prior life experiences. Although they all experienced the same methods courses, their unique backgrounds, Fall semester lessons, and Spring semester field placement experiences impacted the types of planning and discourse practices they engage in.

Chapter 6:

Anticipating Student Thinking

In Chapter 6, I examine how PSTs anticipated student thinking in their lesson planning, beginning broadly with goals, tasks, and how they group students, followed by a deeper examination of their anticipation of student thinking and resources used. In Chapter 7, I describe overall discourse patterns and focus on those that preceded and followed instances of student errors and uncertainties. Finally, in Chapter 8, I will describe the ways in which PSTs' lesson planning practices and discourse practices relate to each other based on post-lesson interview data and stimulated recall reflections.

In examining how PSTs anticipated student thinking, I drew from the 20 written lesson plans and eight interviews prior to their teaching in which PSTs planned for their lessons. These two data sources produced 1,837 occasions of anticipating student thinking. The occasions ranged from anticipated student errors, predictions of things students would likely get stuck on to what PSTs knew about the students' lives and prior knowledge.

Lesson Goals

PSTs described their goals for lessons in both written lesson plans and pre-lesson interviews. PSTs mentioned lesson goals related to student thinking (content goals) 275 times. I examine content goals below. There, I consider other goals for students that were not specific to student thinking. Most mentions of all these goals came in response to specific prompts in the lesson planning templates or specific questions I asked about goals for the lesson.

Process goals were statements about things PSTs wanted students to do that did not include specific mathematics, such as "and write it in different forms" (Lian and Reed, LP 1) or "to get some experience using tools…" (Carson, Int 4). Process goals came up 10 times and only from Carson, Reed, and Lian.

Social goals were utterances and written statements about the way students would interact, engage with others, persevere, or have fun. In Interview 4, several PSTs told me about learning about social goals in their Spring Semester methods course; I saw social goals 10 times in planning for Teaching 4 from all PSTs except Reed. Lian wrote "SWBAT [students will be able to] listen to others respectfully and improve their own reasoning" (LP 4) and Taylor wrote "SWBAT build confidence in their math and coding abilities" (LP 4). Carson (Int 4) described that one of his goals was "to try and re-engage them and re-kickstart that enthusiasm."

Tasks and Activities

Also important in PSTs' lesson planning were the tasks and activities in their plans. I define tasks to be the mathematical questions or problems that PSTs found, created, and used for students to engage with. Activities were the ways in which students engaged in the tasks, for example, whole class, small groups, individuals, stations, worksheets, or whiteboards. Tasks and activities set up the context in which student thinking, and errors and uncertainties, would arise and then be responded to by PSTs. The tasks in all lessons are summarized in Appendix C. Here I highlight three tasks and activities to illustrate how different types of tasks and activities have the potential to impact student thinking, error, uncertainty, and PST responses. These three represent the variety of tasks and activities within the study.

In Rowan's Teaching 3, he and his partners created seven stations (activity), each with basic arithmetic tasks (e.g., simplifying an expression with exponents, dividing two mixed numbers) and "challenge" tasks that were more difficult tasks using the same procedural skills. Students were expected to solve the given tasks. Students rotated every five minutes with a small group of peers that they were encouraged to work with. Students were asked to rate their competence on each station at the end of the five minutes.

In Teaching 3, Carson and Lian created 4 stations (activity) to help their students review for a test on probability later that day. Each station had different topics–theoretical and experimental probability, basic counting principle, tree diagrams, and chance (none, unlikely, equally likely, more likely, certain). Students were encouraged to start at the station with the topic for which they needed the most help. At each station, students were required to complete one mandatory question at each station that was open-ended and conceptual (task). There were numerous computational tasks within the same topic on which students could work. Students moved from station to station on their own and often worked with preselected peers and others who were at the same station when a PST or mentor teacher came to help.

In Teaching 4, Carson drew on his background and experience as a boy scout to create a lesson in which students worked in groups of three to four students, hid a marker, and took bearings from objects in the room (activity) to be able to give to another group so their peers could find the marker they had hidden by taking their own bearings and locating the hidden marker (task).

In Rowan's lesson, all tasks were computational and devoid of context. Student thinking was limited to factual, procedural, and computational. Carson and Taylor's lesson involved both computational and conceptual tasks, but were separated into those categories, thus providing students opportunities to engage in more conceptual and connection-based thinking, but those were separated from the computation and procedural tasks and thinking. In Teaching 4, Carson's lesson was predominantly applied mathematics, students were required to use a variety of thinking and problem solving–factual, procedural, conceptual, metacognitive–and group-based problem solving. Each of these tasks and activities impacts the type of thinking that students are expected to engage in, and thus, how PSTs are likely to respond to the students.

Grouping

In addition to what students were asked to do, who students would work with came up 61 times in lesson plans and interviews. For example, in Teaching 1, in response to “will students be partnered in a specific way?” Reed and Lian responded with “No, just table groups for the Number Talk and launch. However, if there is an uneven table grouping (2:4 split) we will have one person from the larger table volunteer to work with the other group” (LP 1). This can be contrasted with Reed’s planning for Teaching 3, in which Reed reported spending lots of time with her MSU partner thinking about and planning the grouping with the students. Reed described how she was thinking about grouping:

> Because we know that some people work better with others, we know that student really has a lot of initiative but also takes over the work, and this student will let this person take over the work. And so, we're kind of like, “do we just group up in their table groups because they're familiar that way? Do we mix them up?” Because I know some students, if you pair them together, they'll get off task, but they'll also work and if you pair one of those students with somebody else, they'll never work. (Reed, Int 2)

In Interview 4, Carson described his newness of thinking about grouping students. He said, “I think grouping is something that I didn't use to think about up until about maybe a quarter of the way through my senior year–in the middle of [my fall semester methods course]. Starting to think about it then. It's been similar since [my fall semester methods course] through now.” As he became more aware and practiced in grouping, in planning for Teaching 4, Carson considered the size of his groups and how that impacted students' opportunity to interact with the compasses, but not have too many groups.

Thus far, I have described PSTs' knowledge of students, student identity, and content as well as beliefs that situate student thinking broadly in planning. Then, I considered lesson planning, goals, tasks, and activities to provide a more detailed picture of the lessons PSTs are planning. I now turn to the more specific analysis of student thinking in lesson planning.

Anticipating Student Thinking

In this section, I consider the 489 statements about student thinking that emerged in PSTs lesson plans and interviews. Reed mentioned student thinking 79 times; Lian, 83; Carson, 100; Taylor, 109; and Rowan, 118. I considered all occasions of student thinking in planning from two perspectives. The first was whether the PST was (a) anticipating how students would think (*expected*, 215 of the 489 statements), or (b) specifying the thinking they wanted to happen, a content *goal* of the lesson (274 statements). The second perspective was based on the type of student thinking mentioned, (e.g., factual or conceptual). In the sections that follow, I examine PSTs' statement from each of the two perspectives–goals versus expected and types of thinking–followed by considering the two perspectives together

Perspective 1: Goal Occasions of Student Thinking in Lesson Plans

When PSTs wrote or talked about student thinking in their planning, they described the thinking they hoped would occur, content goals for the lesson. For example, in LP 2, Taylor wrote that students will "determine a missing value if the same given data was exponents." PSTs made 224 such content goal statements in written lesson plans and 46 in Interviews 2 and 4, for a total of 274 goals statements. In written lesson plans, the content goals statements occurred primarily in response to prompts and questions directly related to goals for the lesson, the way they would launch the lesson, and what they would hear and see letting them know students understand. During interviews, content goal statements usually came up when I asked PSTs about their goals for the lesson.

PSTs' written responses to lesson plan prompts about student thinking content goals were straightforward. Many statements started with "students will" or "students will be able to" (SWBAT), followed by what they hoped students would be thinking about during and at the end of the lesson. In LP 1, Lian and Reed wrote that "Students can find the different forms of quadratic equations" and "they will see the correct symbolic representations are the same quadratic formula just in different forms….and justify whether they are correct" was how they would know their students understood. Rowan and Carson (LP 2) wrote "students will consider how exponential functions model real situations," "students are expected to come up with an exponential function...and explain their process," and when they heard students say "the paper's thickness doubles every time it's folded, thus base 2" they would know students were understanding. Rowan was the only PST who included content goals in his written planning for LP 3. He planned to ask students to reflect and think metacognitively by scoring their confidence on each station, which students did during the lesson. In LP 4, Taylor described "SWBAT observe patterns involving the order of operations through reasoning with solving the puzzle" and that she wanted them "to reason with the order of operations" and "have the students who shared explain why the order of their pieces that were different didn't matter" as part of her exploration and sharing.

Interview responses contrasted in tone to the written plans. When I asked PSTs about their lesson and their goals, they said things such as: "So the main goal is to let them know something about rotational symmetry, the important ideas, angle rotation and the center of rotation and also maybe some differences between reflectional symmetry and rotational symmetry" (Lian, Int 4) and "I want them to get that, that idea 'If I have three bearings, I might not get a perfect intersection. I might get a triangle'" (Carson, Int 4). Written goal statements in lesson plans averaged 14.3 words compared to 24.8 words per response for the same codes of interview statements. When PSTs talked with me about their goals, they considered their students as holistic learners, shared longer and more nuanced responses than what they provided for their written lesson plan that they had to submit for a university methods course.

Goal statements, beyond the list of standards they were expected to cover, that PSTs included in their lessons plans indicated PSTs know the learning outcomes they want from their students for a given lesson. Lack of goal statements from a template that has asked for goals, such as the TTLP, could indicate that PSTs do not know the purpose of the lesson or how it fits in with the broader unit or curriculum. One challenge with measuring goal statements is that they may come from a textbook or curriculum in which PSTs are given a set of learning objectives or goals and they are just including them in a written lesson plan without much thought as to the meaning of the statements. This challenge does occur for the PSTs in this study and will be explained in more depth in the results of Chapter 8. Now, I turn to expected student thinking in lesson planning.

Perspective 1: Expected Occasions of Student Thinking in Lesson Plans

PSTs also included expected student thinking in their planning. Expected student thinking statements were those in which PSTs anticipated a particular thought or idea to emerge in the lesson. For example, Lian and Reed expected students to answer "y=x^2 (x≥0)" to a question in LP 2. Expected student thinking statements occurred 215 times. PSTs expected correct student thinking in 117 occasions and student thinking that had misconceptions or errors in 98 occasions.

All PSTs expected correct student thinking statements (though Rowan and Lian did not include this in planning for Teaching 3), the way they did varied in terms of specificity and focus. Sometimes PSTs made broad, generic statements such as, “I've been thinking about how they'll look at the lesson” (Carson, Int 4). Other statements were more specific such as when Carson and Rowan (LP 1) expected “students to use some version or combination of all three of these steps” as they referred to “reverse steps...guess and check...and swapping x and y and solving for y, the inverse.” PSTs also anticipated what students would notice, such as “For graphing questions, they see that the ‘a’ term tells them if the graph is increasing or decreasing” (Taylor, LP 2).

Every PST in each lesson planned for challenges, misconceptions, and errors students might have, making up the 98 occasions with errors. In LP 2, Reed and Lian relied on their knowledge of students’ mathematical thinking and prior lessons when they wrote that students “might say it’s exponential for the Number Talk task if they recognize exponential functions from previous classes.” Taylor anticipated “students will be graphing equations in scales that are quite large (max y on one is 30,000). We definitely graphed wrong a couple of times, so we expect students to do the same” (LP 2). Lian anticipated student thinking when she reflected that she “also needed to think ‘how might the students interpret this question?’ Will they look carefully to see it's counterclockwise or are they going to be misunderstanding the smallest rotation in degrees?” (LP 4). Rowan demonstrated his knowledge of students’ responses as he explained “I often get ‘I don't know’ responses” (Int 4). Taylor shared how she and Carson encouraged metacognitive thinking and choice to help differentiate instruction and help students’ misconceptions when they designed their Teaching 3. They wanted to prompt the students to start with the one that they feel they need the most work with. So that if they don't get to all four stations, they at least get to the ones they want to see before they take their exam later. (Int 4)

Several times, Carson anticipated student thinking that things would not happen. In LP 1, Rowan and Carson wrote “although we don’t anticipate them necessarily realizing that they are taking these steps” and in Interview 4, Carson explained “and another misconception that I kind of want to address is when we cover triangles and angles, I don't think a lot of students realize that you can have an angle greater than 180 degrees.” This showed an awareness of student thinking and students’ knowledge in ways that other PSTs did not articulate. Knowing what things were unlikely to appear would likely help Carson consider the bounds of his students’ thinking.

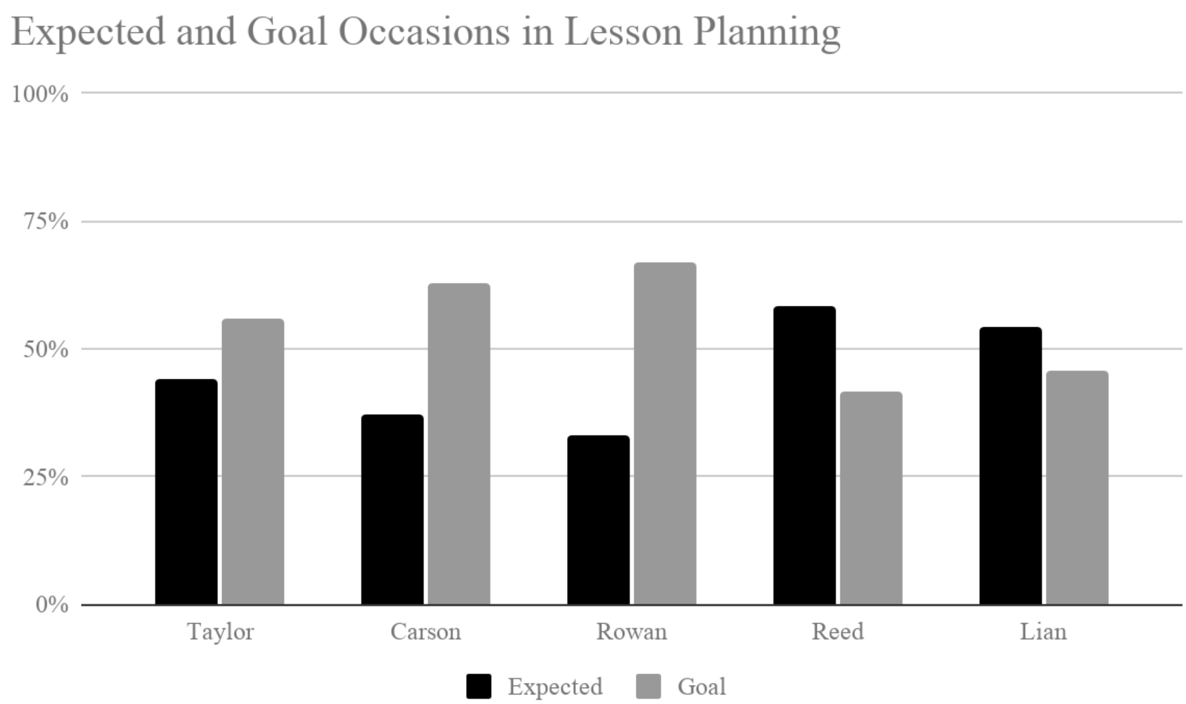
Expected statements demonstrate that PSTs know their specific learners or typical learners and what those learners will likely bring in (prior knowledge) and think about (including errors and misconceptions) in the lesson. A lack of expected statements from a template that has asked for expected student thinking, such as the TTLP, could indicate PSTs do not know what their students are likely to think, do, or say in the lesson. This theme also emerged in data presented in Chapter 8. For now, I shift to examining both goal and expected student thinking.

Perspective 1: Goal and Expected Occasions

As I coded each occasion of student thinking as goal or expected, I found across all lesson plans and interviews, 44% of the occasions of student thinking were expected (215 times) and 56% were goals (274 times). Overall, Carson, Rowan, and Taylor had more occasions of goal statements than expected statements contrasted to Reed and Lian, who had more expected statements (See Figure 6.1). This distribution is impacted by the assignment of teaching pairs

Figure 6.1:

Distribution of Student Thinking Statements by Perspective 1

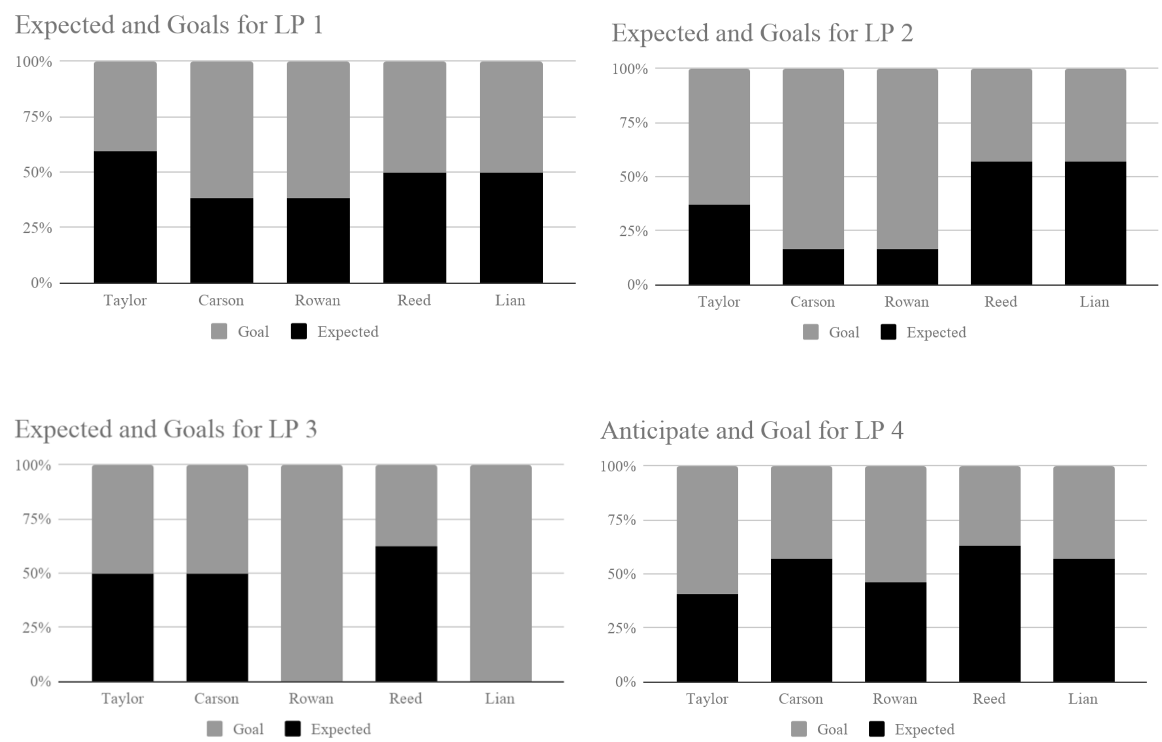


who taught together during Fall Semester (Rowan and Carson, Lian and Reed) have similar breakdowns when all four lessons are considered. This is likely because their LP 1s and LP 2s and data are identical. Fall Semester partners do have individual distributions in their goal and expected student thinking occasions in their planning data for Teachings 3 and 4 which are not with their Fall Semester co PST. Taylor (whose Fall Semester co PST was not in this study) has an overall breakdown that is different from both Rowan and Carson or Lian and Reed.

To examine patterns across individual PSTs and individual lessons, I examined the distribution of expected student thinking compared to goal statements per PST per lesson. See Figure 6.2. The distribution was fairly consistent across PSTs in LP1.There was greater variation

Figure 6.2:

Distribution of Expected and Goal Planned Statements by PST by Lesson



of distribution in LP 2 with Rowan and Carson having a much higher percentage of goal statements than Reed, Lian, and Taylor. Because there was not a written lesson plan required for LP 3, smaller numbers of planned statements and coded occasions led to greater variation in percentages. For LP 3, Taylor and Carson have the same percentage of goal and expected student thinking and Reed has slightly more anticipated than goal. Rowan and Lian only have goal occasions and no expected occasions. In LP 4, the PSTs had the largest percentage of expected student thinking statements which likely comes from the lesson plan template in which they were asked to describe multiple ways students might complete tasks–both correct and incorrect–which produced more occasions of expected student thinking.

Each PST has a unique pattern of change in the percentage of their expected and goal statements across the lessons. Reed is the only PST whose percentages of expected student thinking grew across the LPs. In LP 1, she had 50% expected occasions, 57% in LP 2, and 63% in both LP 3 and LP 4. Taylor, Carson, Rowan, and Lian all had changes from each LP to the other. Next, I will shift to the second perspective of planned student thinking: types of student thinking PSTs considered during planning.

Perspective 2: Types of Student Thinking

In addition to considering whether student thinking statements were expected or goals, I examined the types of thinking that PSTs considered in their planning. I began my coding with types of students' mathematical thinking from the literature, adding to and refining the categories to seven: *factual, procedural, connections, conceptual*, *justification*, *representation,* and *metacognition*. (See Appendix H.) These categories fit within Skemp's (1977) notion of instrumental and relational understanding Instrumental thinking involves procedural and tool-centric views and approaches of mathematics, which aligns with the factual and procedural in my data. Relational thinking considers mathematics as a series of ideas that connect to each other with a more conceptual and connected way of thinking. In this study, I saw planned, relational student thinking that I categorized into connections, conceptual thinking, justification, representation, and metacognition.

Instrumental Thinking in Lesson Planning

Instrumental understanding-based statements were those in which PSTs focused on specific facts or procedures. PSTs included 310 such statements across their planning. Anticipated instrumental thinking made up the majority of planned student thinking occasions.

Factual. Factual statements were those that involved finite, discrete elements of mathematics such as a definition, key points on graphs, or properties such as, “that the pattern cannot be linear” (Taylor, LP 1) or “the paper’s thickness doubles every time it’s folded, thus base 2” (Rowan and Carson, LP 2). Sometimes PSTs planned for a collection or series of facts such as when Taylor described how “Students will try to match based on vertex, zeroes, and the sign of ‘a’” (LP 1). PSTs also considered possible factual questions that students might ask such as “is this right?” (Reed and Lian, LP 1), though this only occurred a few times. Carson described how collaboration with his mentor teacher helped reveal some factual misconceptions when he told me “she thinks that a lot of students have the misconception that once you have two intersecting lines, that's the angle, it's fixed and can't move” (LP 4). One of Rowan’s goals from LP 4 was “by the end of the launch, students should understand that we can write speed as a ratio of distance and time.” About half of the planned factual thinking were responses from lesson plan template prompts about what students would do and how they would solve tasks.

Procedural. Statements about procedural thinking involved PSTs planning for processes in mathematics. For example, “Students will be able to find the next term in the pattern” (Taylor, LP 1) or “not rotating correctly around the center of rotation” (Lian, LP 4). Reed said, “I'm trying to get them to see those switch and then maybe connect that to the procedural thing that we do in the actual functions” (Int 4). Rowan and Carson explained that “Students bringing up the idea of ‘undoing’ or reversing/doing the steps backwards” would let them know students are understanding (LP 1) and Taylor (LP 2) indicated that “We will hear them discussing how to find the ‘a’ and ‘b’ terms to find an equations from their tables” as one way she would know her students understood.

Relational Thinking in Lesson Planning

Relational understanding statements were those that involved students understanding why things work in mathematics. This included conceptual thinking (broadly) and a collection of more specific codes: connection, justification, metacognition, representation, and reasoning.

Conceptual. Conceptual student thinking was the broadest category of relational thinking. Carson (Int 4) exemplified conceptual thinking when he described how he wanted to "get them thinking about the dynamics of an angle change and if we kept that angle constant, what does that do to our coordinate system when we write our things?" When I asked Taylor how she was thinking about her students, she described how students "might be able to do the computation" but she was "thinking about overall topics so she can help students more than just for today" (Int 4), indicating that she was trying to help students thinking in more relational ways than simply computation. A focus on conceptual thinking also emerged as potential questions students might ask. For example, Lian and Reed predicted that students might ask "Why isn't there a third equation for this specific graph?" in LP 1. When planning for conceptual thinking, PSTs drew on their knowledge of students' mathematical knowledge such as when Carson wrote, "Many of the students don't understand that angles can be formed through fluid rotations as well" in his LP 4.

Connections. Connections are a subset of conceptual thinking which occurred in several ways. In the most specific to their classroom learning experiences, sometimes PSTs wanted students to notice similarities between mathematical ideas and representations. Reed and Lian (LP 2), wrote that "verbal connections between what they see on the page and what they have talked about with inverse functions in class" would let them know their students were thinking about the mathematical ideas. Taylor wanted her students to "make connections about how the ‘b’ value affects their graphs” in LP 2 and for students to “see the connection between how your calculator views your input as what you're doing with the pieces” in LP 4. Lian wrote “students can see how rotations are connected to circles and 360 degrees from talking about the basic design element” as a way to explain her choice of task (LP 4).

In a few cases, PSTs wanted students to make connections between mathematics and the real world. Before Teaching 3 (Int 2), Carson said “I was thinking about the things that they might relate angles to.” Carson and Rowan had a goal that “Students will consider how exponential functions model real situations” (LP 2). Rowan planned to “come back to the list we made in the launch and…talk about how we can relate to slope because slope will probably come up in their solutions a little bit or rate of change” (Int 4).

Justification. Justification was mentioned in lesson plans, but was typically referred to as a mathematical practice of sharing ideas more than a particular type of student thinking. The way PSTs referred to justification was often of using justification to equal the action of voiced reasoning. Lian and Reed (LP 1) wrote “and justify whether they are correct” and Taylor who wrote “They need to be able to explain why they are correct” (LP 2) and “explain to your partner why you think the pieces belong together” (LP 4). In LP 2, Carson and Rowan want to students to share “a reasoning for their function;” Lian and Reed will know students understand if students “explain the reason behind their work or why they are doing a certain procedure” and “give an explanation why;” and Taylor has a goal for her students to support their opinions with reasoning.” In LP 1, Lian and Reed considered that students might make errors by “not accounting for the unseen blocks behind and under the visible ones” which addresses reasoning more of a thinking process than action. Reasoning is only mentioned once in the interviews prior to Teaching 3, by Carson, who was “thinking about how to best get students to think critically.”

Lian and Taylor are the only PSTs to mention representation prior to Teaching 4. Lian included reasoning in her lesson as a goal for students in her written plan to “construct viable arguments about their reasoning for their answers” and interview “back up with their reasonings and listen to others to improve their own reasonings.” Taylor described that a goal was for students “to reason with the order of operations” and how she would be listening “for student explanations of the pieces they are placing” and “for students explaining their reasoning for placing pieces together.” This planning for higher-level thinking (e.g., justification and representation) more as mathematical practices than as types of thinking likely came from PSTs having a more difficult time anticipating this type of student thinking and learning how these practices and types of thinking were important.

Representation. PSTs included elements of how students would consider and use representation in their lessons. I coded for representation when PSTs specifically mentioned representation, described how students would interact or think about models or physical manipulatives, discussed multiple forms of a function (graph, equation, table), or referred to tools such as protractor or compass. In LP 1, Lian and Reed explained how “to find that there are multiple ways to represent the same equation” was one way their task built on prior knowledge and they had goals that “students can find a representation of a visual pattern” and “students can find the different forms of quadratic equations.” In LP 2, Taylor described her goals that students will be able to “compare the growth of linear and exponential functions; create exponential functions based off data; and decide which option to take.” All of Carson’s representation occasions came in LP 4 in his triangulation lesson. He wrote “I anticipate students struggling with the use of the compass (i.e., how to take a bearing)” and wanted them to understand “if I have three bearings, I might not get a perfect intersection...I might get a triangle,” and Carson “considered letting students use their iPhones because I know on an iPhone you have different compasses as well, but I know that the iPhone will just tell you an angle if you pointed in the direction.” These exemplify how representational thinking intersects knowledge of students’ identity and mathematical knowledge for Carson.

Metacognition. Some of PSTs planning involved planning for and around metacognition–students’ thinking about their own thinking. Rowan and Carson included “having the students think about their thinking process” and explained that “what makes sense in the context of a real application” as a way of explaining how the task builds on students’ prior knowledge (LP 2). In preparation for Teaching 3, Taylor and Carson both shared how they considered metacognition in their planning. Taylor said,

> We want students to start with the one that they feel they need the most work with. So that if they don't get to all four stations, they at least get to the ones that think they want to see before they take their exam later that day. (Int 2)

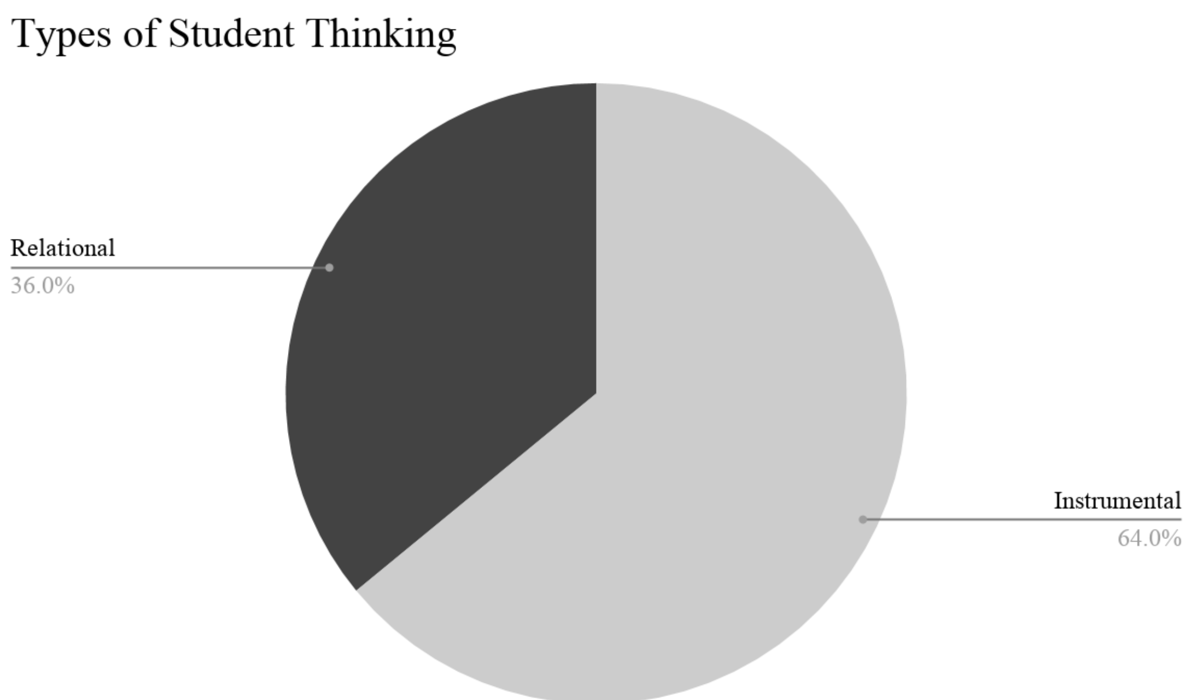
In his own (separate) interview prior to that same lesson, Carson said “we're having them start at the station they feel they struggle with most in case we run out of time” (Int 2). In Teaching 3, Rowan asked students to rate their confidence on each station’s set of questions. In Interview 4, Taylor told me she was planning on providing an opportunity for students to “see if they want to make any changes to their own solution.” Also, in Interview 4, Carson told me that he “might ask students if they struggle with actual applications.”

Quantitative Results of Student Thinking

When I examined the distribution of these types of thinking across all occasions of student thinking in lesson plans, I found that 64% of student thinking occasions were instrumental and 36% were relational (See Figure 6.3). Procedural student thinking occasions

Figure 6.3:

Distribution of Types of Student Thinking in Planning

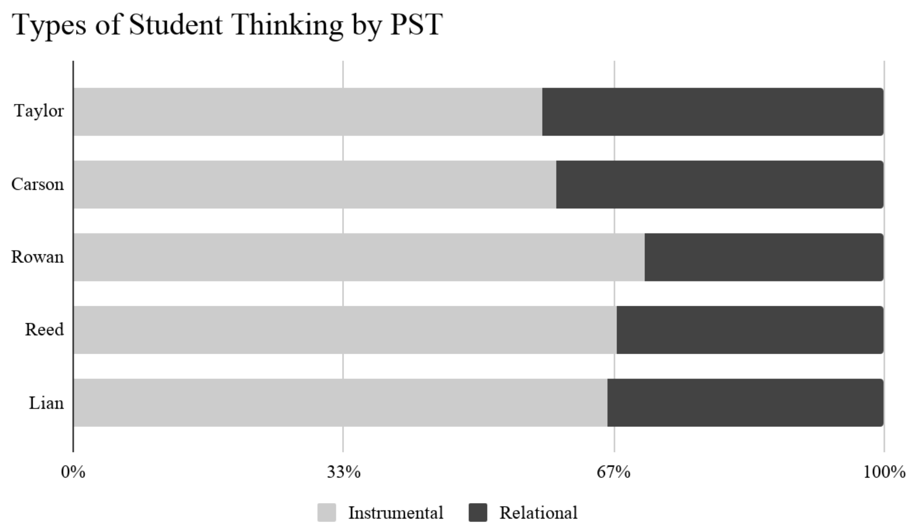


made up 35% of all student thinking occasions and factual occasions accounted for 29% of all student thinking occasions. Relational thinking emerged in three groups: conceptual thinking (15% of all student thinking occasions), connections (11% of all student thinking occasions), and the remaining types of thinking (justification, representation, reasoning, and metacognition), which made up 10% of all student thinking occasions when combined. Of the 54 occasions of connection student thinking, 50 were goal statements–things PSTs wanted students to be able to do–and only 4 were examples of connections PSTs expected students to make in the lessons.

Individual PSTs' Type of Student Thinking in Planning. I saw fairly consistent patterns across PSTs with regard to their individual distribution of planned instrumental and relational thinking (See Figure 6.4) with slightly higher percentages of relational thinking for

Figure 6.4:

Types of Student Thinking by PST Across All Lessons



Carson and Taylor. However, when I considered this data from the perspective of individual type of thinking, each PST has their own, unique distribution (See Figures 6.5 and 6.6).

Taylor's most common type of student thinking was procedural, and she had the most occasions of all PSTs. She is also the only one who had conceptual student thinking planning statements in the top two most common, but barely. Taylor had the smallest percentage of instrumental thinking across all PSTs. Carson had the largest inclusion of metacognition and on toward the larger amounts of representation and conceptual student thinking and was the only PST who did not have justification. Rowan had the largest number of factual occasions, the most of all PSTs. He also had the largest percentage of instrumental thinking and was the only PST to not include representation. Reed had the smallest number of student thinking occasions in her planning and her most common types of student thinking were procedural and factual. Lian's most common type of thinking was factual and she had the smallest percentage of procedural occasions across all PSTs. Lian also had the largest percentage of connections across all PSTs.

Figure 6.5:

Individual Types of Thinking by PST Across All Lessons (Count)

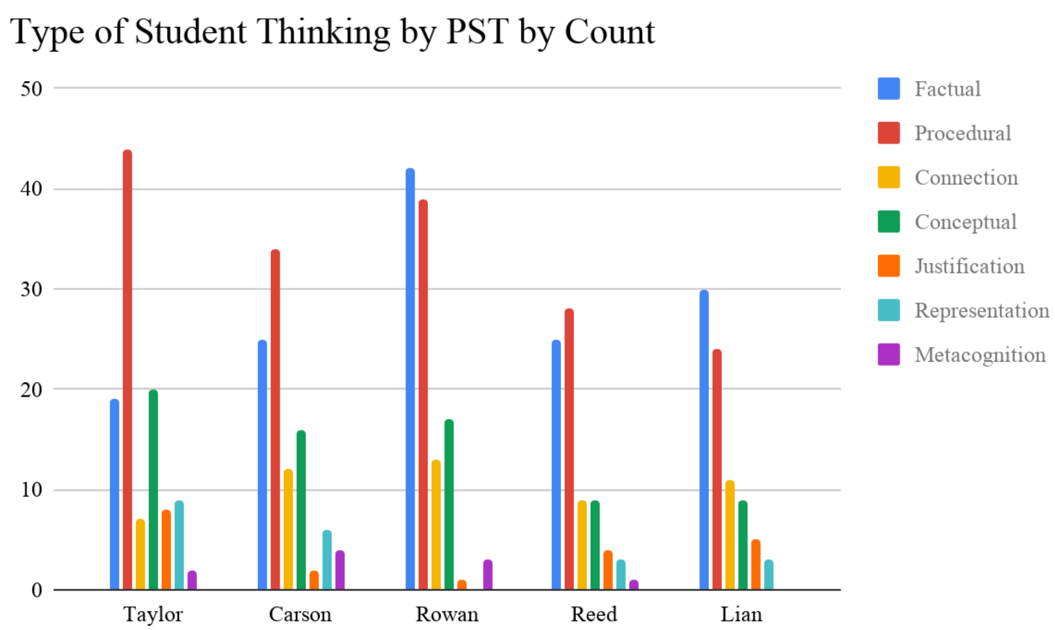
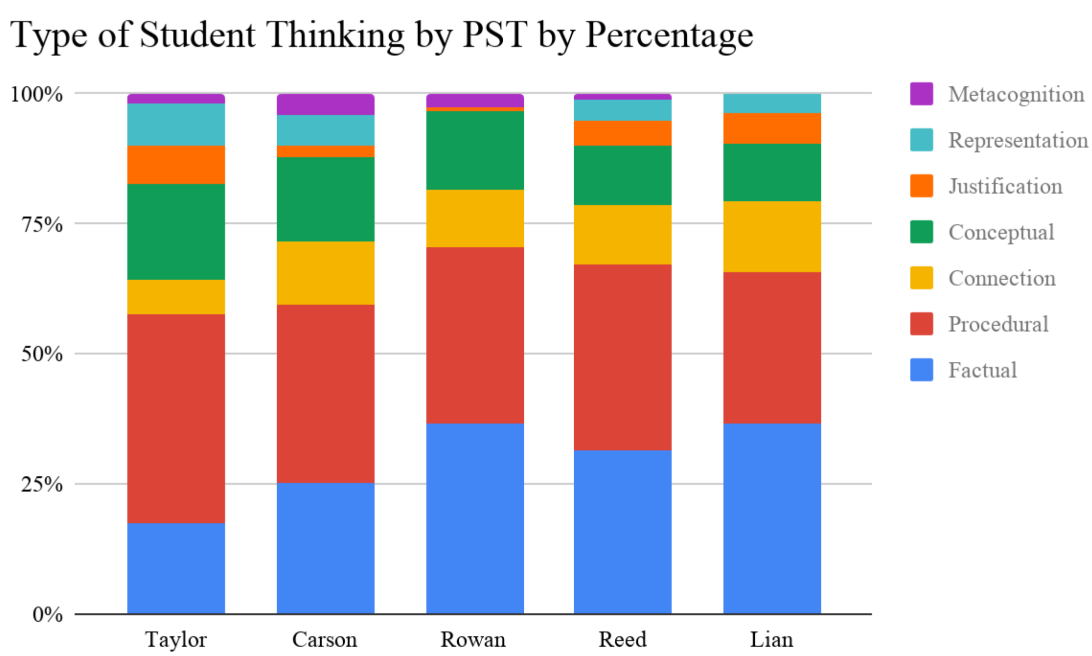


Figure 6.6:

Individual Types of Thinking by PST Across All Lessons (Percentage)



Specificity of Types of Thinking. In addition to considering what type of thinking emerged in lesson plans, I noticed differences in the ways that PSTs described the various types of student thinking. I decided to investigate this and coded for the mathematical specificity of the occasions. I assigned a score of zero if the occasion had no mathematics mentioned, such as "and back up with their reasonings and listen to others to improve their own reasonings" (Lian, LP 4). I assigned a score of one if the occasion mentioned mathematics, but broadly or generally, such as "asking them to think about what they know about 30, 60, 90 or supplementary angles" (Reed, Int 2). Finally, I scored an occasion a two if the mathematics was specific and detailed, such as

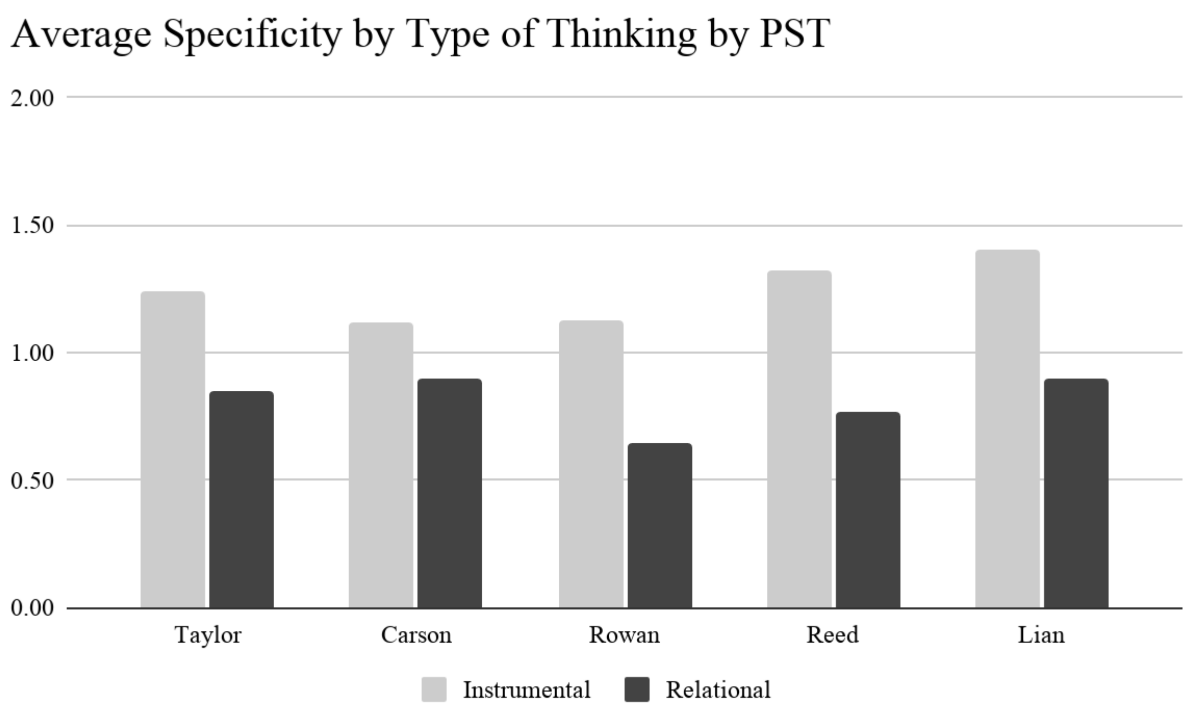
> Student might try finding patterns between the numbers, for example 10-3 = 7, 21-10 = 11. Maybe noticing that 11-7 is 4? This would show to add 4 to 11 to get 15 and then 15 added to 21 is the next term: 36. (Taylor, LP 1)

When I looked at individual PSTs' specificity for instrumental student thinking occasions compared to relational occasions, all PSTs had higher (more specific) instrumental thinking compared to relational (See Figure 6.7). Carson had the smallest difference 0.22 (out of two), Reed had the largest, 0.55 (out of two), and the average difference across all five PSTs was 0.43 (out of two). When I consider these results in addition to the amount of relational thinking, the story begins to unfold that approximately 1 out of 3 planned statements about student thinking is relational in nature and those statements are likely to be less specific about the mathematics when included.

When I examined this across PSTs and all four lesson plans, I found three times in which planned relational student thinking was more specific than instrumental. In LP 4, Taylor's instrumental specificity average was 0.58 and her relational was 0.68 meaning that Taylor was able to be more specific about higher levels of student thinking. The two other cases that show

Figure 6.7:

Average Mathematical Specificity by Type of Thinking by PST



relational thinking being more specific are for Rowan and Carson's LP 1 in which their average instrumental specificity is 1.19 and their average relational is 1.20 for a difference of 0.01 out of two. This difference of 0.5% is likely not significant or meaningful.

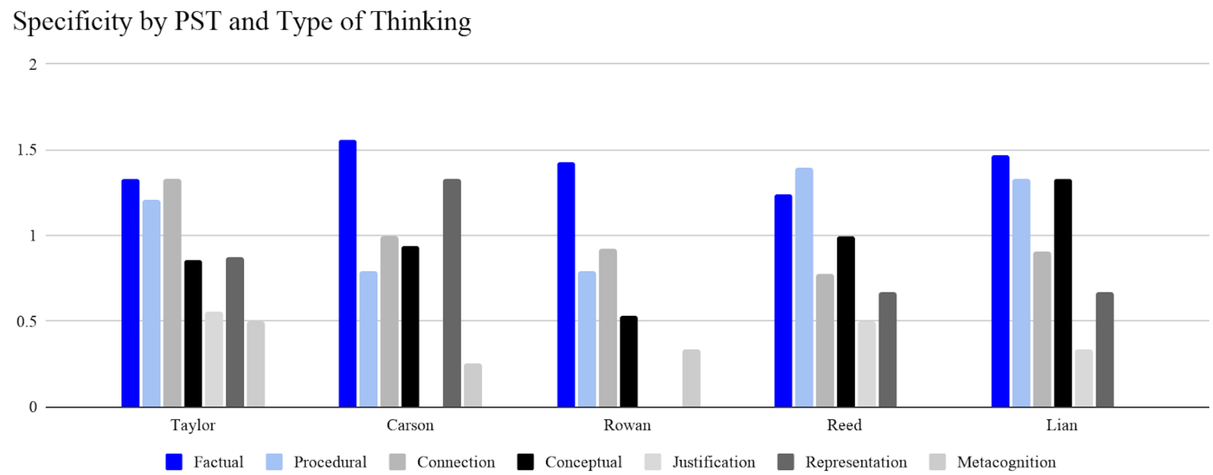
When I considered the same data broken down by the specific type of instrumental and relational thinking (factual, procedural, connections, etc.), there appeared to be an inverse relationship between the complexity of thinking and the specificity as shown in overall decreases in specificity as thinking becomes more complex (See Figure 6.8).

PST Challenges with Planning and Types of Thinking

Based on the lesson planning template the PSTs used, I expected many planned occasions of student thinking to be relational. The TTLP-based planning templates are focused on

Figure 6.8:

Specificity of Anticipated Thinking by Type of Thinking by PST



prompting teachers to consider student thinking rather than a series of lesson activities (first we do a warmup, then an activity, etc.). So, I was surprised at the large number and percentage of instrumental thinking that emerged in written lesson plans and interviews prior to teaching because the TTLP is oriented toward higher-level thinking and conceptual ideas, yet lower-level thinking still dominated the design arena. This speaks to how challenging it is, especially for PSTs who are early in their teaching careers, to consider student thinking more than what factual information students know and what steps they will do or should do. In fact, four of the five PSTs mentioned in pre-lesson interviews that lesson planning is a challenging process. This likely relates to PSTs building their PCK and MKT as early learners of teaching and I will explore this further in Chapter 8.

Considering Both Perspectives: Anticipated or Goal and Type of Thinking

Now that I have laid out each perspective, I consider the interaction of goal occasions and expected occasions. I also break expected occasions into two groups, occasions that PSTs expect students to understand or know and occasions in which PSTs expect students to struggle with, make mistakes on, or not know. I analyzed and represented this with a seven (type of thinking) by three (goal, expected error, expected knowing) table per PST per lesson. Then, I combined those data, and the following table represents the totals for all PSTs across all lessons as percentages.

Prominent Patterns of Planned Student Thinking

In the next section, I describe the eight most common patterns in student thinking as I consider both perspectives. These are procedural goals, factual goals, expected procedural thinking, connection goals, conceptual goals, expected factual errors, expected factual thinking, and expected procedural errors. These data come from the cells in the goal, expended error, and expected knowledge columns in Table 6.1. These data include all PSTs across all lessons.

Table 6.1:

Type of Thinking by Expected or Anticipated for All Lessons (Percentage)

	Type of Thinking	Total	Goal	Expected Error	Expected Know
Instrumental	Factual	29%	14%	8%	7%
	Procedural	35%	16%	7%	12%
Relational	Conceptual	15%	8%	4%	3%
	Connection	11%	10%	0%	1%
	Justification	5%	4%	1%	0%
	Representation	4%	3%	0%	1%
	Metacognition	2%	1%	0%	1%
Total		100%	56%	19%	26%

Procedural Goals. The most prevalent type of planned student thinking was procedural goals. These occurred 76 times and represent 16% of all planned student thinking occasions. I coded as procedural goals when PSTs referred to processes and algorithmic-type tasks that students would be able to do at the end of a given lesson. Procedural goals were statements such "students are able to work the tasks out" (Lian, LP 4) that is one way she would know that students understand and when Reed (Int 4) told me that one of her goals for Teaching 4 was "to see the switching of x and y of the points." All PSTs mentioned procedural goals (Taylor 20, Carson 19, Rowan 21, Reed 8, Lian 8). Procedural goals occurred in all lesson plans, but Lian and Reed did not have any in LP 1 and Rowan, Lian, and Taylor did not have any in LP 3.

Factual Goals. PSTs described their goals of students learning factual mathematical content 66 times (14% of all planned student thinking occasions) throughout the lessons plans and interviews. Factual goals were similar to procedural goals, but instead of a process, students would know by the end of the lesson, it was a fact or definition students would know. Rowan and Carson described a factual goal when they wrote "students will understand the idea of inverse functions through differentiating the inputs from the outputs and 'switching' the input and output" (LP 1). Taylor (Int 4) shared a factual goal that she was "just looking to review some basic math skills" (Int 4). All PSTs mentioned factual goals (Taylor 12, Carson 16, Rowan 24, Reed 5, Lian 9). Factual goals occurred in all lesson plans, but Lian and Reed did not have any in LP 1; Rowan, Carson, and Taylor did not have any in LP 3; and Carson did not have any in LP 4.

Expected Procedural Thinking. The third most common planned student thinking was occasions of expected procedural thinking which came up 57 times (12% of all planned student thinking occasions). I coded for expected procedural thinking when PSTs made or wrote about processes that they expected students to do or know. These were often responses to the question “what all the ways the task can be solved?” in their written plans and occurred as a result of the PST completing the task in multiple ways as prompted by their planning templates. Taylor provided eight occasions of expected procedural thinking in LP 2 (the largest for any PST in any lesson plan). Her occasions of expected procedural thinking included “matching graphs by plugging in points” and “noticing that (1/b) ^ x is the same as b ^ (-x) and that is why we flip over the y-axis;” All PSTs mentioned expected factual goals (Taylor 18, Carson 11, Rowan 12, Reed 12, Lian 4). Expected procedural thinking occurred in all lesson plans, but Taylor, Lian and Rowan did not have any in LP 3.

Connection Goals. Occasions of connection goals were those in which PSTs mentioned connections between mathematical concepts or between mathematics and the real world that PSTs hoped students would make. Lian planned that, “students will be able to make connections between rotational symmetry and reflectional symmetry” (LP 4). Reed and Lian’s response to the prompt “What will you hear that lets you know students understand?” was “the original function should pass the horizontal line test because the inverse needs to pass the vertical line test to be a function” (LP 1) which I coded as a connection goal. Carson described how he came up with the topic of “orienteering or navigation” which would help students, “make connections between angles and the real world” (Int 4). Connection goals came up 48 times (10% of all planned student thinking occasions) and were mentioned by each PST (Taylor 7, Carson 10, Rowan 13, Reed 8, Lian 10). Connection goals occurred in all lesson plans, but Taylor, Carson and Rowan did not have any in LP 3.

Conceptual Goals. PSTs had 41 occasions of conceptual goals (8% of planned student thinking occasions). Conceptual goals were mathematical concepts that PSTs wanted students to understand at the end of the lesson. PSTs wrote things such as “students will understand the idea of inverse functions" (Carson & Rowan, LP 1); "students will be sharing their ideas with their table partner" will let Reed and Lian know that students are understanding (LP 2); and "our other goal would be that they do at least one of the optional questions from each (station)" (Taylor, Int 2) which is a conceptual goal because the tasks Taylor referred to was conceptual and without that knowledge would be challenging to intuit from her statement alone. Each PST mentioned conceptual goals (Taylor 8, Carson 10, Rowan 14, Reed 6, Lian 3). All PSTs mentioned conceptual goals in LP 2 and LP 4. Lian and Reed did not mention conceptual goals in LP 1, Lian and Carson did not mention conceptual goals in LP 3.

Expected Factual Errors. PSTs expected factual errors 39 times across the lesson plans and interviews (8% of planned student thinking occasions). In LP 1, Taylor expected "students might think its linear." In LP 2, Reed and Lian expected that students might erroneously think "a function cannot have a restricted or changed domain." In Interview 2, Carson explained that "there's a lot of misconceptions about...computing the theoretical and experimental probability." In Interview 4, Rowan concluded that "students get confused [because] slope has many, many names." Each PST anticipated factual errors (Taylor 4, Carson 7, Rowan 8, Reed 9, Lian 9). All PSTs anticipated factual errors in LP 1 and LP 2. Carson is the only one who did in LP 3. Taylor is the only one who did not in LP 4.

Expected Factual Thinking. PSTs mentioned expected factual thinking 36 times (7% of all planned student thinking occasions). Every PST mentioned expected factual thinking, but there were two clusters. Taylor and Carson had a small number to Reed, Carson, and Lian, who had larger numbers. Taylor expected factual thinking three times and Carson anticipated it once. Taylor was the only PST who expected factual thinking in LP 1 when she mentioned it twice: "students will try to match based on vertex, zeroes, and the sign of 'a'" and "we anticipate them to say the vertex on a graph/equation, how to find the intercept on a graph/equation, and the importance of ‘a’ on a graph/equation.” Taylor's third occasion was in LP 2 when she expected students “taking the $1 million, because you get it right away.” Carson’s only expected factual thinking occurred in Interview 4 when he said, “what I'm looking for there is not where can we see angles ‘I see the TV has a 90-degree angle’.”

Rowan included nine occasions of factual thinking and Reed and Lian each mentioned it 12 times. Rowan’s nine examples of expected factual thinking all occurred during planning for Teaching 4. He predicted things such as “because I'm sure ‘fast’ will come up when we talk about speed” and “they may define 0 for 7:00 or actually use the time 7:00 for the initial time.” Reed and Lian each had five of their 12 times come from LP 2 (which they taught together) when they expected students to answer, “it should be a function”; the “original function is one-to-one”; “y=1”; and “y=x.” In LP 4, Lian had five solutions she expected students to come up with from the textbook-based activity she was planning. When I asked her how she was thinking about her students, she responded with “Are they going to just find the smallest in number of rotation?” and in her written lesson plan, she noted that “For [questions] 3 and 4, students might have used different wording so we would go more in depth with those.” Reed anticipated four solutions during LP 4.

Expected Procedural Errors. PSTs expected occasions of student thinking with procedural errors 36 times (7% of all student thinking occasions). PSTs predicted that students “would miscount the blocks” (Lian and Reed, LP 1); “might distribute in the A values” (Taylor, LP 2); “may not include ‘final’ distance of 1.2 miles in the table of values or graph” (Rowan, LP 4); or “use the compass without rotating the bezel” (Carson, LP 4). Each PST expected procedural errors in student thinking (Taylor 6, Carson 4, Rowan 6, Reed 8, Lian 12). All PSTs expected procedural errors in LP 1 and LP 2. No PSTs anticipated procedural errors in LP 3. Taylor is the only one who did not in LP 4.

Summary. Overall, these patterns highlight a few key aspects of PST planning. First, the most frequent patterns are within instrumental thinking (Skemp, 1977) – factual and procedural thinking. Though the TTLP is designed to help PSTs notice and plan for higher levels and relational mathematical thinking, they are challenged to do so. Secondly, when they do anticipate higher levels of thinking (conceptual and connections, in this case), they are instances of what PSTs want students to be able to do and PSTs still do not anticipate when students will have these types of thinking. Finally, anticipating errors or places students will likely struggle represents less than 20% of the overall anticipated of student thinking, making it difficult for PSTs to respond to error and uncertainties.

Differences Among PSTs

When I examined patterns across PSTs to see how planned student thinking occasions differed by PST, there were patterns that were unique to each PST. See Table 6.2. So, even when PSTs use the same lesson planning templates and engage in the same interview protocol prior to teaching a lesson, they do have their own unique perspectives that impact what they notice and bring into the design arena.

Goal or Expected Occasions Across Types of Thinking

When I considered the distribution of expected and goal occasions of student thinking for each type (See Figures 6.9 and 6.10), I noticed the only types of planned student thinking that PSTs expected more than half of the time are the instrumental types of thinking–procedural and factual. This seems to indicate that PSTs are most able to expect lower levels of thinking and have more goals in higher levels of thinking. Although the anticipation percentage for

Table 6.2:

Common and Uncommon Planned Thinking Patterns by PST

PST	Uncommon Patterns	Common Patterns
Taylor	Expected connection understanding (0%)	Expected conceptual challenges (47%) Expected accurate procedural think (32%) Expected conceptual understanding (31%)
Carson	Expected procedural challenges (11%) Expected accurate factual think (3%)	Expected accurate connections (50%) Expected conceptual challenges (29%)
Rowan	Expected connection understanding (0%)	Factual goals (36%) Conceptual goals (34%)
Reed	Factual goals (8%) Expected conceptual challenges (0%)	Expected accurate factual thinking (31%)
Lian		Expected accurate factual thinking (33%) Expected procedural challenges (33%) Expected conceptual understanding (31%)

Figure 6.9

Goal and Expected Breakdown Across Type of Thinking (Quantity)

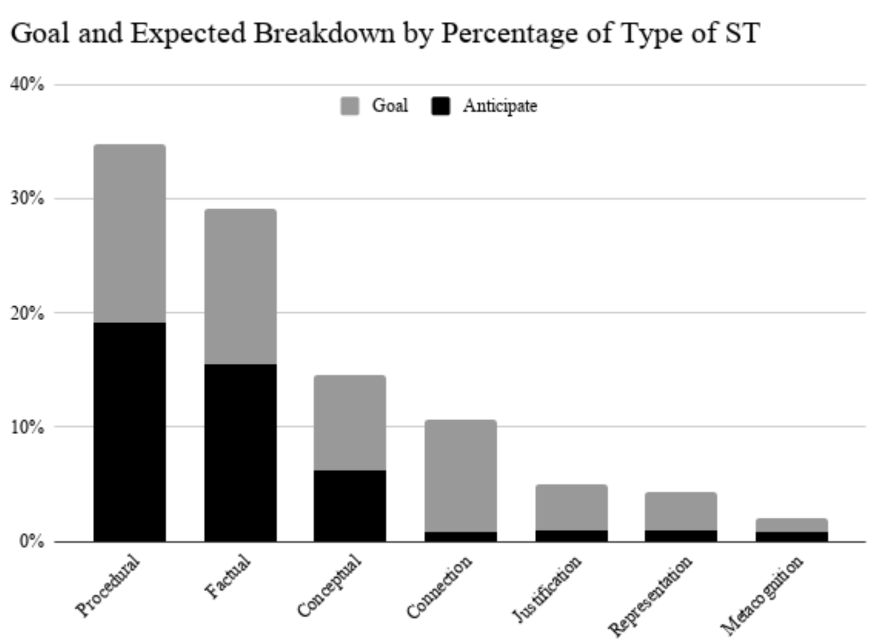
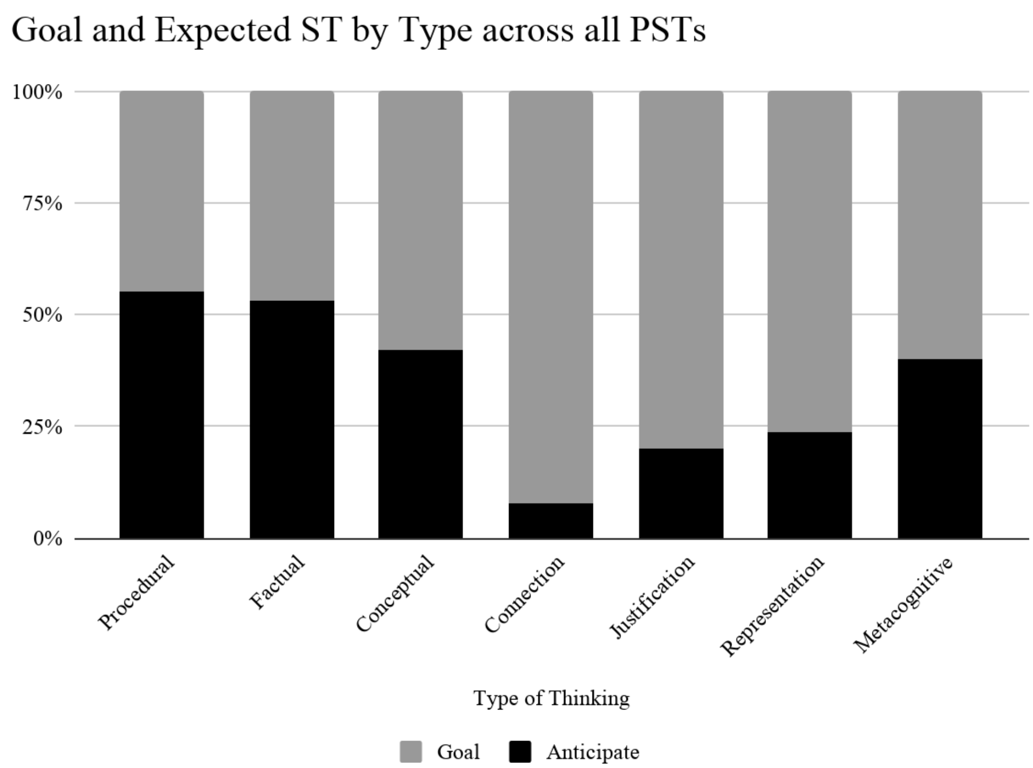


Figure 6.10:

Goal and Expected Breakdown Across Type of Thinking (Percentage)

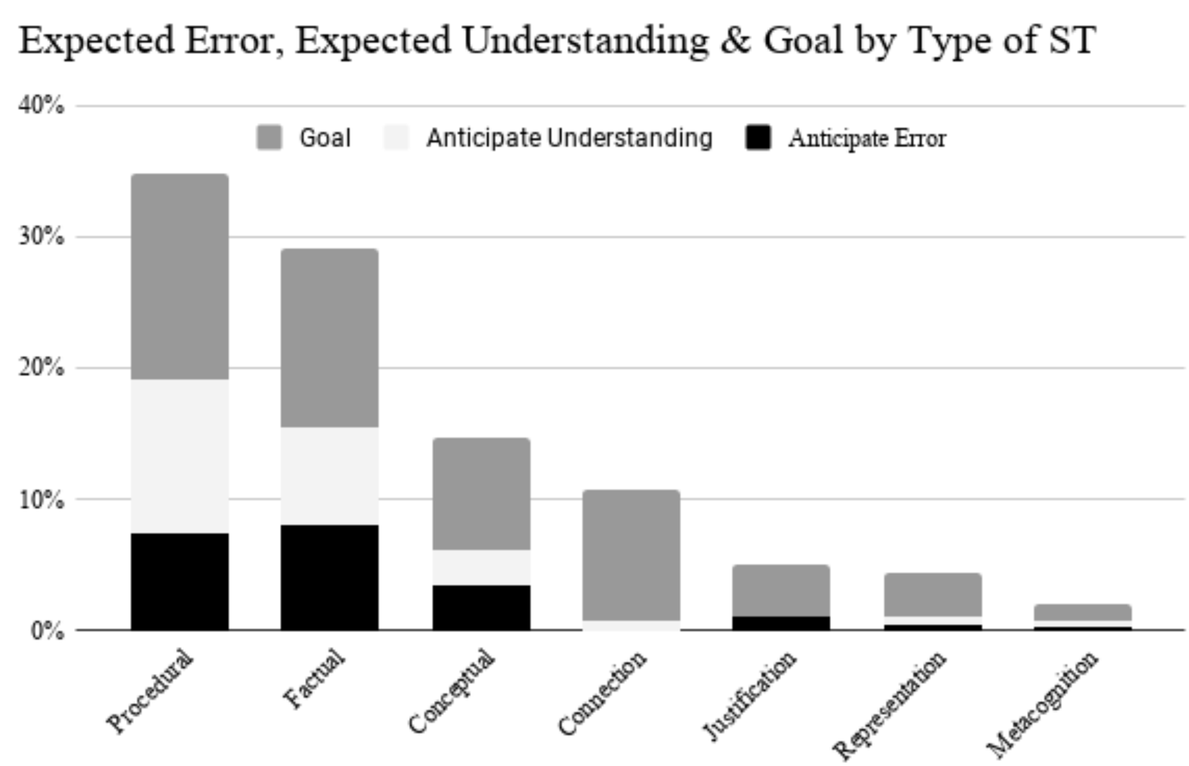


metacognition percentage is 40%, metacognition only occurred 10 times across all PSTs and all lessons. In relational thinking, PSTs have more goal statements and fewer expected statements around. This seems to indicate that PSTs want students to be doing this type of thinking, but PST perceive that students are not likely to engage in higher order thinking in the lessons. This is most prominent in connection and justification. With occasions of planned connections, the vast majority (92%) of the statements (48 out of 52) were goals and only 8% of connection statements were expected student thinking. With justification, (which only had four occasions in total) all occasions were goals, and none were expected. And although the distributions may be favorable to the type of planning that led to more productive discourse, the data suggests that PSTs struggle to include relational thinking in their plans. In the next section, I will explain the resources PSTs relied on and used when lesson planning.

When I coded anticipate student thinking comments, I further broke those down into statements that PSTs anticipated students would know things or be correct (anticipated knowledge) and those in which PSTs anticipated students would not know, make mistakes, or run into challenges (anticipated misconceptions). When I considered how these two groups of anticipation statements cut across the types of thinking, I noticed that procedural goals represented the largest section, anticipated procedural knowledge was the next most frequent, and factual goals were third. (See Figure 6.11).

Figure 6.11:

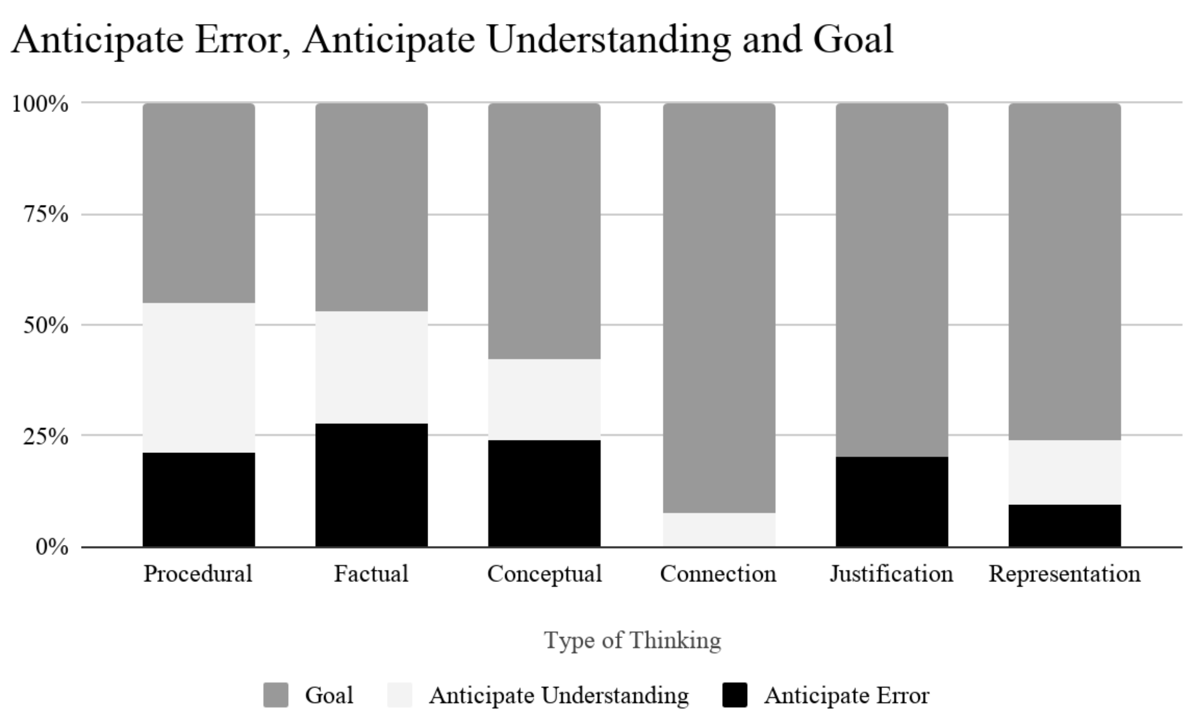
Expected Error, Expected Understanding, and Goal by Type of Thinking



When I considered each type of thinking and broke down the goal, anticipated knowledge, and anticipated misconceptions, I noticed PSTs tended to anticipate slightly more occasions of student knowledge (ratio of about 3:2) than misconceptions. All metacognitive anticipated student thinking is knowledge as metacognition often centers around what learners do know, not what they do not. PSTs also predicted more conceptual errors than knowledge (ratio of about 3:2). PSTs did not anticipate any correct reasoning and a number of incorrect predictions of reasoning. PSTs predicted some accurate representation and connection and no misconceptions. For those categories. (See Figure 6.12).

Figure 6.12:

Perspective 1 Compared to Perspective 2 for All Lessons (Percentage)



Summary of Anticipated Student Thinking

Although PSTs in this study did anticipate student thinking, they struggled to do so in ways that prioritized higher level and relational (Skemp, 1977) thinking. Nearly 2/3 of anticipating student thinking was low level; of the anticipated higher-level thinking, the vast majority were things PSTs wanted to have happen (goals), not what they actually anticipated students would know or not know (expected). Anticipating errors and uncertainties, the precursor to responding to errors and uncertainties, happened even less. In order to better understand how PSTs anticipate student thinking, I turn to an examination of the resources–sources of support–PSTs rely on to help with their planning.

Resources

As research and practice demonstrate that learning to lesson plan–and particularly learning to anticipate student thinking–is challenging, in addition to what student thinking PSTs anticipated I was also curious about the resources the PSTs used to anticipate student thinking. The resources I describe in this section are those that PSTs used for planning–textbooks, methods instructors, peers, etc. I am not referring to resources that students used in lessons (e.g., pencils, protractors, calculators, etc.). In this section, I explain those results which came from their written lesson plans and asking PSTs what resources they used in pre-lesson Interviews 2 and 4.

In total, PSTs mentioned resources 118 times in these interviews, 41 occasions came from Interview 2 and 77 came from Interview 4. Each PSTs mentioned resources between 18 and 27 times. See Table 6.3. All five PSTs mentioned peers, general digital sources (i.e., the internet), specific digital sources (i.e., GeoGebra), their mentor teacher, and their methods instructor. Reed was the only PST who mentioned friends which was her fiancé and she

Table 6.3:

Resources Used in Planning for Student Thinking by PST

Resource	Taylor	Carson	Rowan	Reed	Lian	Total	Number of PSTs who used
assessment	1	4	2	0	0	7	3
digital - general	2	2	3	3	1	11	5
digital - specific	4	1	1	6	1	13	5
friends	0	0	0	3	0	3	1
mentor teacher	2	4	1	4	10	21	5
methods instructors	2	4	7	4	3	20	5
professor	1	0	0	0	0	1	1
PST peers	1	2	3	2	3	11	5
rehearsal	1	0	3	1	1	6	4
self	3	2	2	0	0	7	3
standards	0	0	1	0	0	1	1
textbook	0	3	1	4	5	13	4
TTLP	1	1	1	0	1	4	4
Total	18	23	25	27	25	118	---

mentioned him three times. Taylor was the only PST who referenced a university professor. Rowan was the only one who mentioned the Common Core State Standards. All other resources were mentioned by three or four PSTs.

Perspective 1: Location of Resource

I considered resources from two perspectives: their location and type of resource. When I grouped resources by location they clustered into university, field placements, and personal contexts. (See Figures 6.13 and 6.14). First, the university context included the resources related to their methods courses and experiences. This included peers, methods instructors, university

Figure 6.13:

Quantity of Resource Location by PST

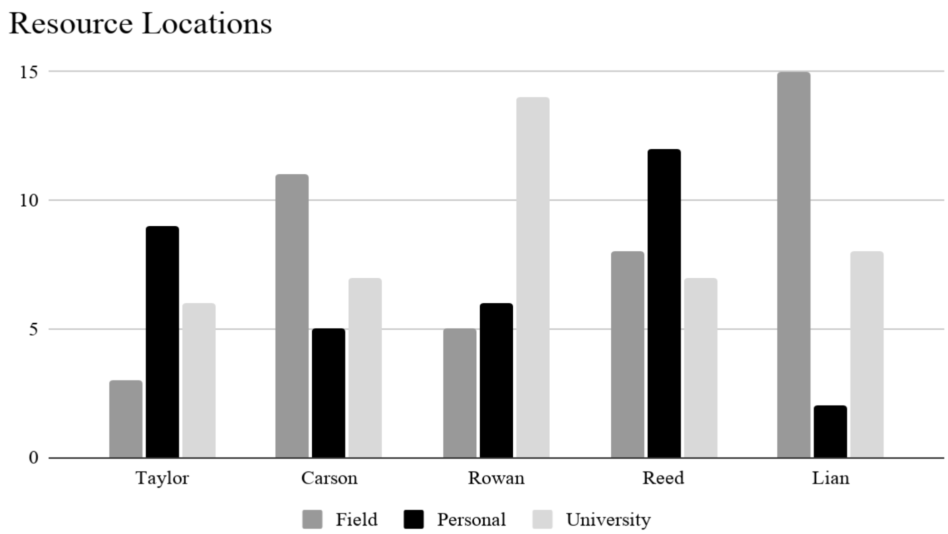
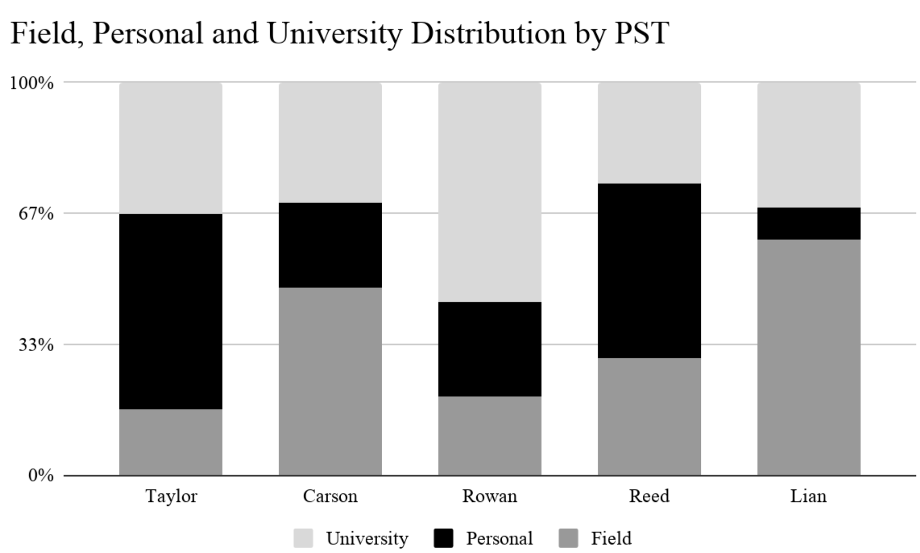


Figure 6.14:

Distribution of Resource Location by PST



instructors, rehearsal (practicing their lesson with peers and methods instructors), and lesson plan templates. University resources accounted for 36% of all resources. Second, field placements included the resources PSTs encountered and had access to as a result of their secondary mathematics classrooms in schools local to their university. This included their mentor teachers, assessments, standards, and textbook. Field placements also made up 36% of the total number of resources mentioned. The final cluster was personal and included resources personal to each PST in their life outside of their university and field placements. This cluster included general digital sources (i.e., Google), specific digital sources (i.e., GeoGebra), friends (including fiancé), and self (the PST). This perspective provides some insight into where the PSTs are going for support for lesson planning and anticipating student thinking. Each PST has at least two resources in each location, but the individual breakdowns are quite varied. Personal resources comprised 29% of all resources.

Taylor mentioned the fewest number of resources (18) compared to the other four PSTs (each mentioned resources between 23 and 27 times). Half of Taylor's resources were personal which was the largest percentage for personal resources across PSTs. She mentioned using general digital resources six times (second most to Reed) and of those mentions, two were general (blogs and email) and four were specific (Google Draw once and Scratch three times). She also referred to herself three times with "I created the task myself" (LP 4), "I made the pieces myself" (Int 3), and "we made them up" (Int 2). Carson relied on field placement resources for 48% for supporting planning for student thinking. He mentioned assessment four times, his mentor teacher four times and the textbook three times. Rowan relied primarily on university-based supports (56% of his resource mentions) which was the most of any PST. He mentioned his methods instructors seven times. He mentioned the lesson rehearsal three times (50% of all mentions). He also mentioned peers three times. Reed had the largest number of references to personal resources. She accounted for nine of the 24 (38%) references to digital resources across all PSTs as well as six of the 13 (46%) related to specific digital resources. Reed was the only PST who mentioned people outside the university and field placement location and referred to her fiancé three times. Lian accounted for the highest number and percentage of field-based resources with 15 mentions which was 60% of her resources. She mentioned her mentor teacher 10 times (48% of all mentions of mentor teacher across PSTs) which was more than double any other PST. Lian mentioned five of the 13 (38%) of all mentions of the textbook as a resource. Lian only mentioned two personal resources (one digital specific and one digital general) which was the smallest number and percent of all PSTs.

Perspective 2: Type of Resource

I also considered resources by their type: human or nonhuman (which I call resources). First, human resources were the people that played a role in PSTs planning for student thinking. This group was composed of peers, methods instructor, university instructors, friends, self, mentor teacher, and rehearsal. On the other hand, resources were those such as lesson plan templates, general digital sources, specific digital sources, assessments, standards, and textbooks. All PSTs relied more on humans (58%) than resources (42%). See Figures 6.15 and 6.16.

Considering Both Perspectives

When I considered resources from both location and type, the most prevalent type of resource was university humans. University humans were the people PSTs relied on connected to them by their university experiences. This group included methods instructors, peers, and rehearsals in methods classes and was mentioned 38 times which represents 32% of all

Figure 6.15:

Quantity of Human and Resources by PSTs

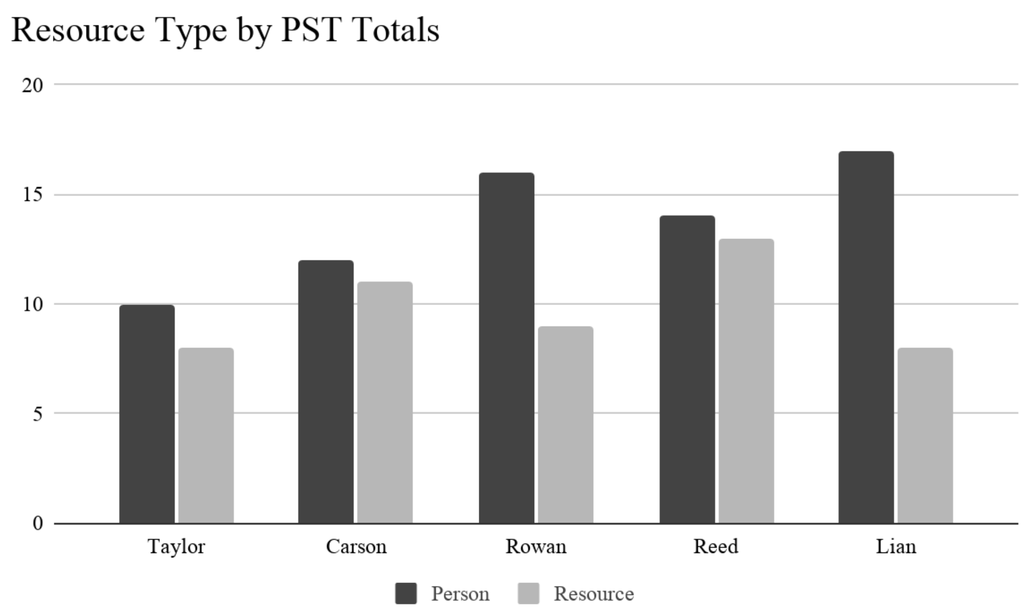
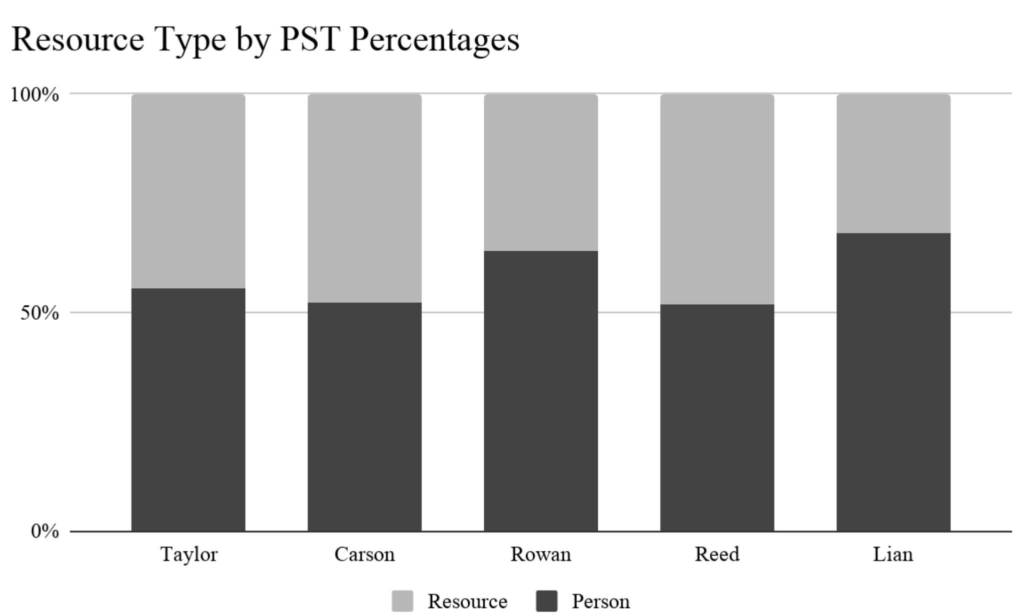


Figure 6.16:

Percentage of Human and Resources by PSTs



resources. (See Figure 6.17). The second most prevalent crossover category was personal resources. These were digital resources, both general and specific. Personal resources were mentioned 24 times (20% of all resources). In field-based resources, human and resources were each mentioned 21 times (18% of the total number of resources). In the less relied on types of resources, PSTs mentioned humans in their personal lives (their friends and selves) 10 times (8% of all resources). And finally, the least mentioned group was university resources (namely the TTLP) which they mentioned four times (3% of all resources).

Each PST also shared their own, unique distribution of types of resource by location. (See Figure 6.18). Across all PSTs, field placement humans and resources occurred the same number of times, but this distribution differed by PST. Rowan relied the most on personal resources, Taylor and Lian relied the most on humans, and Carson and Reed were in the middle of that range. Humans from PST's personal lives represented 30% of the personal resources. This distribution is consistent across all PSTs except Lian who did mention any people in her personal life that helped her lesson plan.

Figure 6.17:

Percentage of Human and Resources by Locations

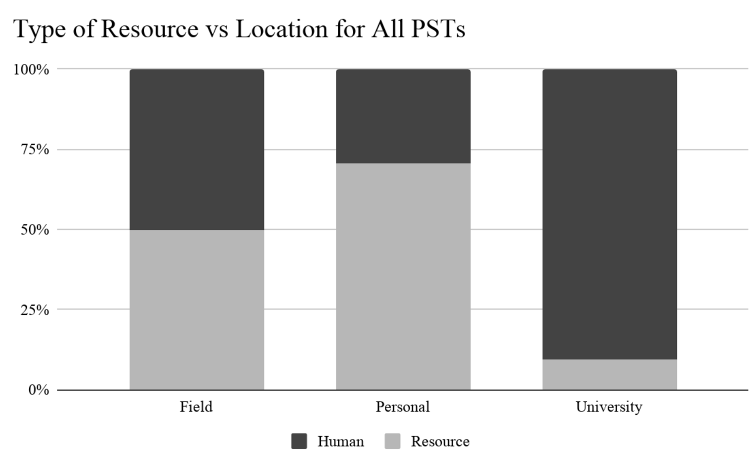
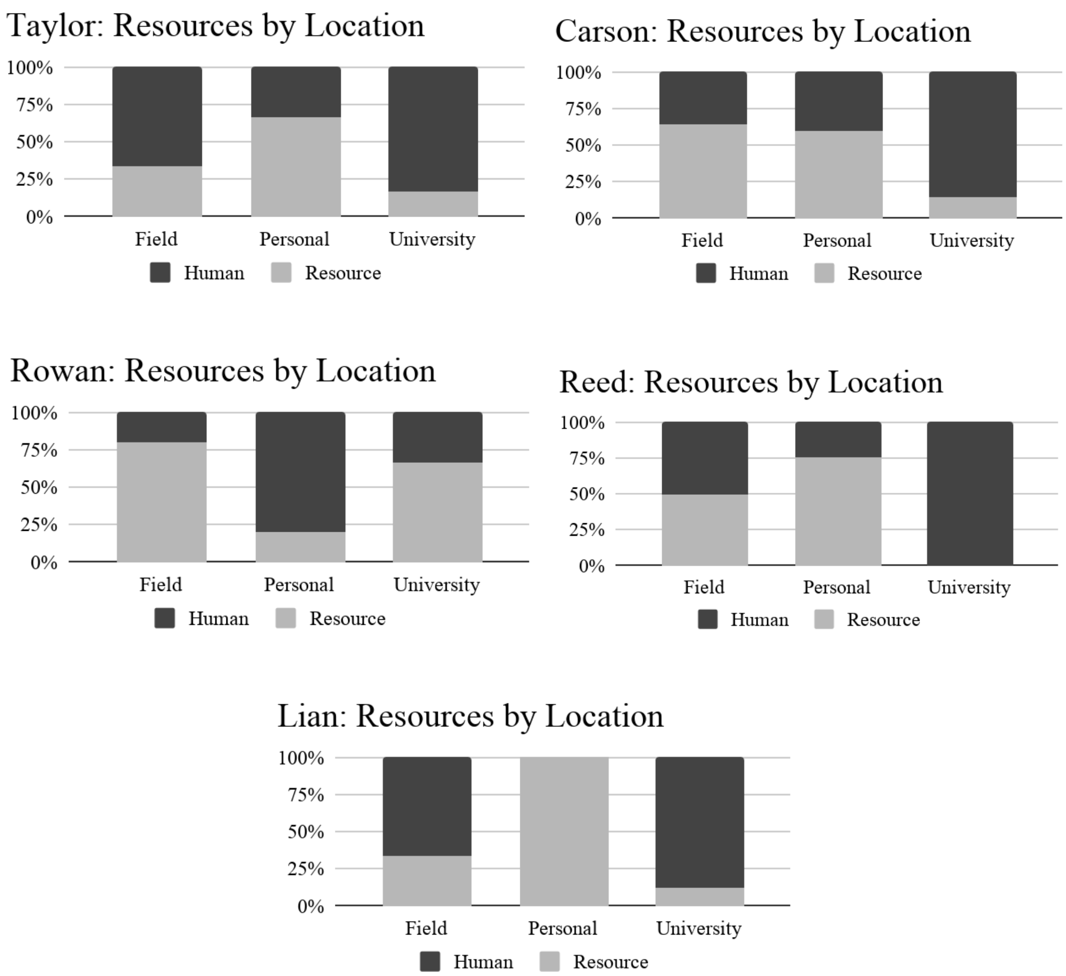


Figure 6.18:

Percentage of Human and Resources by Locations by PST



Resources by Lesson

When I examined the number of times PSTs mentioned resources from the planning phase for Teachings 3 and 4 (combining pre-lesson interview and written lesson plan), I noticed that every PST increased the number of times resources were mentioned. (See Table 6.4). Taylor increased by one (from eight to nine), but everyone else nearly doubled the number of times they mentioned resources in the interview and lesson plan for Teaching 4 compared to Teaching 3.

Table 6.4:

Number of Mentions and Resources Across Teaching 3 and Teaching 4

	Number of Mentions		Number of Resources	
PST	T3	T4	T3	T4
Taylor	8	9	6	6
Carson	7	15	4	8
Rowan	9	15	6	8
Reed	8	18	6	8
Lian	9	15	3	6

Additionally, when I considered the number of different resources that PST mentioned, everyone except Taylor increased the number of different resources they got support from when thinking about student thinking about the planning before Teaching 3 to Teaching 4. PSTs used the aforementioned resources in various ways as they were planning for lessons. They used them most broadly for generic planning, for activities and lesson structure, and most specifically for support with anticipating student thinking.

First, PSTs used resources that helped them generically in planning, such as when they said that "the teachers' guide" (Lian, Int 4) or "to work with our mentor teacher" (Reed, Int 2) were resources they had relied on These generic statements occurred nine times (out of 118 resource occasions) and did not point to a particular lesson, activity, or topic. Lian mentioned generic resources five times, Reed twice, Carson and Rowan once each, and Taylor did not mention any.

Secondly, PST used resources to help them plan for activities or tasks in lessons. This occurred 98 times in total and occurred from 17 to 25 times for each PST. These were statements such as when Lian (Int 4) said “yesterday we had a launch practice and we got feedback on the lesson” and Taylor (Int 4) said “Scratch, which is a coding language that uses like block pieces.”

The third way was when they used resources to help plan for specific student thinking and this only occurred 11 times. Taylor, Reed, and Rowan each had one mention, Lian had three, and Carson had five. All 11 resources that helped PSTs think about student thinking came in their planning for Teaching 4. The source of the student thinking planning resources were: mentor teacher (five times), methods instructor (two times), lesson plan template (two times), peer (one time), and practicing the lesson’s launch in the methods class (one time). When considering this by location (See Table 6.5), six mentions came from university, five from field placements, and none from self. When considered from the human or resource source, nine mentions were from humans and two resources. When I examine these mentions by both location and type, five were humans from their field placements, four were university humans, and two were university resources. No resources that PSTs used to anticipate student thinking came from their field placement resources, personal resources, or humans in their lives outside of field placements and university.

Table 6.5:

Number of Mentions That Supported Anticipating Student Thinking

		Type	
		Resource	Human
Location	Field	0	5
	Personal	0	0
	University	2	4

Lian described how her methods instructor “gives feedback on our planning. Like she asked for the clarifying questions...but our methods instructor asked if we could add something to that to make the students think more than yes or no” (Lian, Int 4). Carson (who taught in the same field placement as Taylor) described how he relied on Taylor and their third PST, Matt (not in this study). Carson explained how he described his task ideas at different stages of creation and asked Taylor and Matt, “Is this something you could see them not having issues with or do you think that they could do this task fine?” (Int 4). Carson also relied on his mentor teacher who “wanted to push students’ understanding to the ‘rotating a ray’ perspective, as she thinks it will help them understand angles conceptually better” (Int 4).

Reed’s single resource to help with student thinking was before Teaching 4 when she said the following about her lesson planning

> I talked with my mentor teacher...she said something about “I don't really care if you get like that explicit procedural switch x and y, but if you can get them to see that x and y switch and (are) inverses that'd be good. So even just showing them tables of inverses that sort of thing.” (Int 4)

Rowan mentioned how in his “launch practice...both words [distance and time] came up, so that was really nice” (Int 4). Taylor described how the lesson plan template prompted her to “go through and fill out all these details of like preparing for student thinking and all that kind of stuff” (Int 4).

Summary of Resource Use

PSTs drew on resources to support their lesson planning and attending to student thinking. The majority of their resource use was for task or activity choice and came from a wide variety of types and locations of resources. However only about 9% of the resource use was for anticipating student thinking and these came from a smaller, more select collection of people and places. When PSTs did use resources for anticipating student thinking, they were most likely to rely on mentor teachers and methods instructors which positions these folks and their knowledge and experiences central to PSTs' learning about planning for student thinking. Knowing that methods instructors and mentor teachers are the key resources for helping PSTs learn to anticipate student thinking has the potential to impact the ways in which teacher education programs structure their learning experiences for PSTs. In Chapter 7, I turn to the enacted lessons that emerged as a result of this planning and examine contexts, patterns, and responses to errors and uncertainties.

Chapter 7:

The Role of Errors and Uncertainties in Enacted Mathematics Discourse

To explore the role that errors and uncertainties played in enacted mathematics lessons, I coded and analyzed teacher moves in videotaped lessons, paying particular attention to moves preceding and following instances of students' errors and uncertainties. I used the TMSSR Framework (Reiten et al., 2016) to code and categorize teacher moves within the four TMSSR practices–elicit, respond, facilitate, and extend. (See Appendix F for full list of codes.) I also considered how the moves encouraged different types of student thinking along Skemp's (1977) notion of instrumental and relational thinking. (See Table 7.1).

In the videotaped lessons, I also coded students' *errors* and *uncertainties*. An error is something a student says or does that is incorrect. Examples of errors are computational mistakes such as coming up with the wrong result or answer to a multiplication calculation, procedural mistakes such as performing the order of operations incorrectly, or misconceptions such as misstating the meaning of a proportion and how it connects to an example. By uncertainty, I mean student questions, quizzical looks, or statements that indicate a student is confused, not sure what to do, or not understanding the mathematics. Uncertainty instances could be initiated by the student or teacher. If a student initiates the uncertainty, it might be via the student asking a question of what to do next or making a statement such as "I don't get it." If a teacher initiates the uncertainty, it is likely from something the teacher noticed about the student, their written work, or their spoken words.

The coding of lessons for teacher moves yielded 3,512 coded utterances from PSTs throughout the lessons. Within this total, 326 discourse moves preceded instances of errors and uncertainties, PSTs used 2,049 moves in responding to errors and uncertainties, and the

Table 7.1:

Discourse Moves by Practice and Level of Thinking

Practice	Instrumental	Relational	Context-based
Monitor			Monitor
Elicit	Elicit answer Elicit fact/procedure	Invite Elicit ideas Elicit understanding Pressing for explanation	Ask for clarification Figure out reasoning Check for understanding
Respond	Validate correct response Correct error	*Encourage revoice Prompt error correction	Wait Revoice Re-present
Facilitate	Cue Topaze Funnel Explain procedure	Guidance Build on Explain conceptually Summary explanation Providing information Encourage mult. strat. Provide alt. strategy	
Expand	Topaze for justification	Probing Press for precision Encourage reasoning Encourage reflection Press for justification Push for generalization	

Note: No PST used encourage student to revoice

remaining discourse moves were around instances of non-error and non-uncertainty and were not included in this study for individual analysis. I coded each PST individually (despite most lessons being co-taught) there were a few instances that involved multiple PSTs, such as if one asked the initial question and another had some follow up moves. In a few cases, a mentor teacher was part of the preceding or response moves.

First, I describe the context and nature of the lessons for each PST which include descriptions of students and classrooms. I share an overview of each of the PST's four lessons and their primary discourse moves within each. I, then, present an overview of each PST's general discourse patterns across lessons, preceding, and following instances of errors and uncertainties to illustrate different ways in which PSTs engaged in discourse, particularly around errors and uncertainties. Finally, I consider overall patterns across PSTs, collectively, to tell the story of what discourse across several PSTs is.

Contexts

To help situate the ways in errors and uncertainties play a role in classrooms, I first describe contextual aspects such as classrooms, students, PST's world views, content, and activities are substantial factors that influence the construction arena. First, I describe contextual factors that impacted Teachings 1 and 2. These factors applied to all PSTs who taught in this situation. Students' ages, grade level, current mathematics course, and relationship with the PSTs played immediate contextual roles. Students' prior mathematics courses and experience, broader education experiences, mindsets, race, gender, home life, family income, and languages spoken more broadly influenced these classroom experiences. Then, I describe each of the lessons the PSTs taught, the topic, the primary activities, and the PSTs' primary discourse moves within the lesson as these impacted the role of errors and uncertainties, as well as PSTs' discourse around them. To wrap up this section, I provide an overview of each PST with unique discourse patterns they used.

Fall Teaching Contexts

For Teachings 1 and 2, the students were university undergraduates in a mathematics lab course that accompanied a remedial mathematics course. I was not in the classroom and watched videotaped lessons after. Despite coding (and thus, spending many hours with the) six lessons taught to the same students for Teaching 1 and 2, and because of the distance between the camera and students (who were not wearing a lapel microphone and faced away from the camera), I do not have any knowledge of the individual students. For Teachings 3 and 4, I observed Taylor and Carson's students three times, Rowan's students twice, Reed's students twice, and Lian's students once. Thus, I only have a surface level knowledge of the students from what I could see and hear in the few lessons I observed. Plus, the fact that I was in those classrooms influenced the behaviors of the students who did not know me. PSTs did offer some insight and information about particular students in interviews.

Teachings 1 and 2 took place in a well-resourced university classroom with less than 10 students and a large number of PSTs who rotated teaching and observing lessons. During the lessons, PSTs who were not teaching sat around the outside of the classroom observing and taking notes on the lessons. Teachings 3 and 4 took place in several secondary schools within a 30-minute drive of the university, but no schools were affiliated with the university. Appendix C includes summaries of all the lessons.

Taylor

For Teachings 3 and 4, Taylor taught in a suburban junior high school, substantially smaller than the school district she attended. She was placed with two other interns–Carson and Matt (not in this study). The classes I observed her teach were for a first hour, 7th grade, mathematics lab class. The students in this class were placed in for mathematics support. Taylor's mentor teacher, Mrs. Tharp, is the 7th grade Algebra teacher for all of the students in class later in their school day. Mrs. Tharp is a mid-career, white female.

Of the 20 students enrolled in the class, one appeared to be a Latinx male, seven appeared to be white females, and 12 appeared to be white males. In the three lessons I observed, students were actively engaged, working with others, moving around the room. Mrs. Tharp and the PSTs spoke quietly and politely with them. There were times that students were active (as middle school students tend to get in places where they can move and interact with others), all teachers (PSTs and Mrs. Tharp) focused primarily on mathematics and not behaviors.

Taylor's Lessons

In Teaching 1, Taylor taught with Emily (not in this study) about equivalencies between tables, graphs and equations. They facilitated a Number Talk about the number of helmets in the next term of a pattern, and then launched an activity that involved students being given a card with a function or graph and finding their matching partner. In Teaching 1, Taylor used 118 discourse moves, 13 moves preceded error and uncertainty and 46 were responses to error and uncertainty. To elicit ideas, Taylor relied primarily on *eliciting facts and procedures* ("What should the shape of our graph be if it's quadratic?) and *inviting* ("What else is important when we're looking at our equation and graph?"). Her primary response move was to *validate correct responses* which often came in the form of nodding or repeating a students' correct idea (which she used differently from *revoicing,* which she used for the sake of sharing ideas without the added evaluation of correctness). For facilitating, Taylor frequently relied on *conceptual explanations*, *cueing*, and *procedural explanations*. *Conceptual explanations* were those such as, "We're looking at this and it's positive, so it's gonna be the opposite because we want it to be a negative so we could just re-write this as negative or minus negative 2," that provided students with information about why things were happening. *Cueing* involved her drawing students' attention to particular features, in this case of quadratic functions such as, "In our form, we have this -h and +k." *Procedural explanations* were those in which she focused on doing the steps involved in the task without focusing on any reasoning as to why, such as "we want to take whatever's next to this minus sign, which is negative 2." Taylor did not use any extending moves during Teaching 1.

In Teaching 2, Taylor taught about exponential functions, again with Emily. They facilitated a Number Talk about receiving one penny on day one, doubled every day for a month compared to receiving one million dollars on day one. Their launch was guiding students through matching tables, graphs, and equations. Their exploration was a worksheet and discussions around exponential functions. In Teaching 2, Taylor used 269 discourse moves and was involved in 38 instances of student errors and uncertainties and used 142 moves in response to errors and uncertainties. Taylor relied on *monitoring* 53 times which involved her walking around and looking at students' work and listening to them when they talked with peers. Her primary elicit moves were *eliciting answers* ("Are they smaller or bigger than our negative values?") and *eliciting procedures or facts*. Similar to Teaching 1, Taylor primarily relied on *validating correct responses*. Also similar to Teaching 1, her primary facilitate move was *cueing*, but the majority of her *cueing* (17 out of 22 occasions) came in the form of her pointing to direct students' attention. Taylor did not use any extending moves during Teaching 2.

In Teaching 3, Taylor taught with Carson and Matt. They created and facilitated a station activity to support students' conceptual and computational understanding of the probability topics students had a test on later that day. Students were asked to complete the open-ended, conceptual, mandatory question at each station and then choose what other computational questions they needed to do to practice. Taylor used 191 discourse moves and was involved in 13 instances of student errors and uncertainties in which she used 106 response moves. Taylor relied on *monitoring* 30 times. Her primary elicit move was to *check for understanding*–often asking "How's it going?" or "Does that make sense?"–which was a change from Teachings 1 and 2. This was striking since this was a lesson to help students prepare for a test rather than introduce new content. Her primary response move was, again, to *validate correct responses*. Taylor used four facilitate moves–*cueing*, *conceptual explanations*, *funneling*, and *Topaze*–each with nearly the same frequency. *Conceptual explanations* were those such as when she provided an example about using the basic counting principle over tree diagram in which she explained why the basic counting principle was a more efficient method when large numbers of choices were involved. *Funneling* is a move in which PSTs ask a series of questions to help a student complete a procedural task two (non-related) examples are "Right. And what's half called?" and "26 out of how many?" *Topaze* moves involve PSTs breaking tasks down into smaller and easier pieces. Taylor's *Topaze* moves in Teaching 3 were exemplified by "The outcomes are right here. How many of these have five first?" Taylor used one extending move, *pressing for justification,* during the lesson when she asked, "can you imagine drawing a tree diagram with 250 combinations on the bottom?" to help the student think about the reasoning over choosing the basic counting principle over tree diagrams for large combinations of numbers.

In Teaching 4, Taylor individually taught a lesson about order of operations and programming using paper manipulatives based on the coding language, Scratch. The launch was a Taylor-led whole class "discussion" about order of operations which involved lots of Taylor asking IRE-based questions and writing responses on the board, providing most information. To explore, students worked in pairs to arrange the "puzzle pieces" to represent the three numerical expressions Taylor provided. Taylor attempted to have students share out in the summary, but ended up running the summary primarily herself. In Teaching 4, Taylor used 237 discourse moves and there were 33 instances of student errors and uncertainties with 84 responses. Taylor’s primary elicit move was to *check for understanding*, her primary response move was to *validate correct responses,* and she relied on *procedural explanations* as her most frequently used facilitating move. She used *encouraging reflection* (an extend move) twice with comments such as “Think about how you solved B.” In this lesson, the combination of *checking for understanding* and *procedural explanation* strategies likely came from the fact that many students were confused throughout the exploration part of this lesson.

Taylor’s Discourse

Across Taylor’s teaching, she used a total of 815 discourse moves, the second largest number of moves. Of those discourse moves, 97 preceded instances of errors and uncertainties (the most) and 378 followed (second smallest). (See Table 7.2 for highlights of Taylor’s discourse moves and Appendix M for a full comparison across PSTs). Taylor’s overall noticing and discourse pattern involved her first observing–which I call *watch and praise* for Taylor–as she relied heavily on *monitoring* and *validating*. Then, she was likely to make sense of by *checking for understanding* and *eliciting facts or procedures*. Finally, her responses were most likely to *explain procedures* and *cue*, drawing students’ attention to particular elements. Most of these moves are low level, but Taylor also used the most *pressing for explanation*, *conceptual explaining*, and was the most likely to *encourage reflection*, all of which are high-level moves. Taylor’s moves that preceded error and uncertainties also included a lot of the *monitoring*, *checking for understanding*, and *eliciting facts and procedures* as happened overall.

She provided opportunities for errors and uncertainties by *eliciting answers* (low level) and *inviting student ideas* (high level). Taylor was most likely to have errors and uncertainties arise after she monitored and elicited low-level information from students. Taylor’s responses to

Table 7.2:

Highlights of Taylor's Enacted Discourse Moves

	Enacted	Preceded	Response
Number	815	97	378
Frequent	Validate correct Monitor Elicit facts or procedures Check for understanding Procedural explain Cue	Monitor Check for understanding Elicit facts or procedures Invite Elicit answer	Validate correct Correct incorrect Cue Procedural explain Conceptual explain Elicit facts or procedures Monitor
Most	Elicit facts or procedures Press for explain (tied) Correct incorrect answer Conceptual explain Encourage reflection	Monitor Elicit answer Elicit facts or procedures Topaze Procedural explain (tied) Classroom manage (tied)	Validate correct Correct incorrect Conceptual explain Encourage reflection

Note: Number refers to the number of moves she used in each phase. Frequent refers to Taylor's 5 to 6 most used moves, greatest to least. Most refers to moves in which Taylor had more uses of the move than other PSTs. They are listed in order of the TMSSR framework, not by quantity.

error and uncertainties were mostly focused on correctness (using validating correct responses and correcting incorrect responses). And though she continued to use lower-level moves in her responses, she also employed *conceptual explanations* the most of all PSTs.

Carson

Carson's spring placement for Teachings 3 and 4 was a suburban junior high school similar in size to the school district that he attended. He was placed with two other PSTs–Taylor and Matt (not in this study). See *Taylor's Spring Classroom* section for details.

Carson's Lessons

In Teaching 1, Carson taught with Rowan about inverse functions. They facilitated a Number Talk in which they described a number and students needed to use inverse functions to determine the number. Their launch activity was an extension of their Number Talk with a discussion of the symbolic representation and solving process. In Teaching 1, Carson used 88 discourse moves and was involved with eight instances of student errors and uncertainties and used 41 response moves. Carson's primary elicit move was *inviting*, in which he asked open ended questions to generate students' ideas such as "Did anyone else do it differently?" and "What do we think of that?" His primary response move was to *validate correct responses*. He most frequently used *summary explanations* which were higher-level facilitating moves such as, "You're saying that x is representing my number, abbreviate my number with the symbol and then y represents our solution, I'm going to say -70 in quotation marks because clearly y can be anything" and also *cueing* to facilitate. Carson did not use any extend moves in Teaching 1.

In Teaching 2, Carson taught with Rowan about exponential equations and logarithms. They facilitated a Number Talk in which they showed four graphs and had students determine which one did not belong. In their launch, they explained how to solve equations with exponents and logarithms. Their exploration was a worksheet-driven activity in which groups of students folded a paper in half and answered questions about it, working toward exponential functions to model the mathematics. In Teaching 2, Carson used 229 discourse moves and was involved with 19 instances of student errors and uncertainties with 153 response moves. Similar to Teaching 1, Carson's primary elicit move was *inviting*, his primary response move was to *validate correct responses*, and he did not use any extending moves. For facilitating, Carson used *procedural explanations* and *cueing* most frequently.

In Teaching 3, Carson taught with Taylor and Matt. They created and facilitated a station activity that was designed to support students' conceptual and computational understanding of the probability topics they had a test on later that day. Students were asked to complete the open-ended, conceptual mandatory question at each station and then choose what other questions they needed to do to practice. In Teaching 3, Carson used 173 discourse moves and was involved in 13 instances that preceded student errors and uncertainties and 117 responses. Carson used *monitoring* 29 times, the most of any lesson for him. Carson's primary elicit move was *checking for understanding* (which was the same as Taylor and pedagogically aligned with a test review lesson). He, again, relied on *validating correct responses* for responding moves as well as providing *procedural explanations* and *cueing* for facilitating moves. He did not have any extending moves.

In Teaching 4, Carson individually taught a lesson about triangulation. His launch was a whole group "discussion" in which Carson elicited uses and knowledge about angles. Then, students worked in groups of four to hide a marker and find bearings from objects in the classroom to the marker so another group could find their marker using their bearings. In Teaching 4, Carson used 235 discourse moves of which 31 instances preceded errors and uncertainties and 82 were responses. Carson's primary elicit move was to *elicit ideas* which primarily occurred in the launch, when he elicited ideas of how students were familiar about angles and their use ("Can you think of an example?"), and summary, when he was eliciting the ways in which students completed the task ("How else could you use angles?"). His primary response move was *revoicing* (repeating what students had said). He relied on *procedural explanations* while facilitating, and had one extending move, *pressing for justification*, when he asked, "Why's it obtuse?"

Carson's Discourse

Carson had a total of 725 discourse moves throughout the four lessons, the median value of all PSTs. Of those discourse moves, 71 (second highest) preceded instances of errors and uncertainties and 393 followed (second highest). (See Table 7.3 and Appendix M). Carson's overall noticing and discourse pattern involved observing–also used *watch and praise*–as he relied heavily on *monitoring* and *validating*, similar to Taylor. Carson was most likely to make sense of student thinking by making it public using *inviting*, *figuring out reasoning*, and *eliciting ideas*, all high-level elicit moves. As the first part of a response, he was the PST most likely to *revoice* and *re-present* ideas making them heard and known publicly with additional information and structuring by him. Then, he would typically use *procedural explanations*. He was the PST

Table 7.3:

Highlights of Carson's Enacted Discourse Moves

	Enacted	Preceded	Response
Number	725	71	393
Frequent	Validate correct Monitor Elicit facts or procedures Elicit ideas Procedural explain Revoice	Monitor Invite Elicit ideas Elicit facts or procedures Check for understanding	Validate correct Procedural explain Cue Elicit facts or procedures Correct incorrect Revoice
Most	Invite Figure out reasoning Elicit ideas Revoice Re-present Summary explain Provide information	Invite Figure out reasoning Elicit ideas Re-present Summary explain	Invite Elicit facts or procedures Figure out reasoning Press for explain (tied) Re-present Summary explain Provide information

most likely to provide general mathematics information, not just about the particular topic, and summary explanations to round out his responses.

In Carson's moves that preceded errors and uncertainties, he was commonly *monitoring* and eliciting both low (*answers, facts, procedures*) and high (*inviting*) levels of thinking as well as *checking for understanding* to see how students were doing. Similar to his overall discourse patterns, Carson was the PST most likely to *invite*, *figure out reasoning*, and *elicit ideas*, all high-level eliciting moves. He was also most likely to precede instances of error and uncertainty with *re-presenting* and *summary explanations*. As he synthesized ideas that students had shared and summarized them overall, he made space for students to ask questions or reveal errors.

Carson's common response moves to errors and uncertainties were mostly lower-level moves which illustrate a trend in declining cognitive demand when errors and uncertainties arose. This trend is commonly reported in literature and practice. Carson was the PST who was most likely to *invite*, *press for explanations*, *provide summary explanations,* and provide *general mathematics information* to support broader understandings. So, though his common moves were more likely to be low level, he also did engage with higher-level moves.

Rowan

For Teachings 3 and 4, Rowan was placed in a large, urban secondary school (grades 7-12) with a veteran Black female teacher, Ms. Lansford. This school is racially diverse, and many students come from homes with low incomes. Rowan cotaught with two PSTs (not in this study) during Teaching 3. The classroom–long and narrow–was covered with friendly mathematics posters and was a pleasant and welcoming space to be in. The 20 students were in an 8th grade mathematics class, but Rowan mentioned that much of the work they did was quite remedial and "behind." Based on my observations, the 20 students consisted of seven black males, five white males, three east Asian males, two white females, two black females, and one Arab female. I did not video tape in this class because Ms. Lansford was concerned about the identity and privacy of her students–many of whom come from low-income families and worry about their clothes and social status. During Teaching 3, Ms. Lansford seemed pleasant and friendly with PSTs and students. During Teaching 4, Ms. Lansford was imposing, yelling at several students, removing several students from class, and talking over Rowan. In our interview before Teaching 3, Rowan said "My mentor is an older woman. I wouldn't say strict or easy teacher. She's her own person. There is no teacher like her. And I actually like her."

Rowan's Lessons

In Teaching 1, Rowan taught with Carson about inverse functions. They facilitated a Number Talk in which they described a number and students needed to use inverse functions to determine the number. Their launch activity was an extension of their Number Talk with a discussion of the symbolic representation and solving process. In Teaching 1, Rowan used 80 discourse moves and he was involved in six instances of student errors and uncertainties with 41 response moves. Rowan's primary elicit move was *inviting*. He relied mostly on *validating students' correct responses* for his responding moves. *Procedural explanations* were his primary facilitating moves. He did not use any extending moves.

In Teaching 2, Rowan taught with Carson about exponential equations and logarithms. They facilitated a Number Talk in which they showed four graphs and had students determine which one did not belong. In their launch, they explained how to solve equations with exponents and logarithms. Their exploration was a worksheet-driven activity in which groups of students folded a paper in half and answered questions about it, working toward exponential functions to model the experience. In Teaching 2, Rowan used 233 discourse moves and was involved in 19 instances of student errors and uncertainties and 148 response moves. Rowan's primary elicit move was *checking for understanding*, he relied heavily on *validating correct responses* to respond, and *cueing* to facilitate. He used five extending moves which was the most of any PST in any single lesson (and he used the most overall). He relied on *pressing for justification* three times and *pushing for generalization* twice in which he *encouraged students to test* to see whether the question worked.

In Teaching 3, Rowan taught with two PSTs not in this study. They created and facilitated a station activity that was designed as a review of operations with mixed numbers, repeated multiplication and exponents, and solving linear equations. Students rotated based on time and were encouraged to work with each other and use each other as support. After completing the work at each station, students were asked to rate their confidence level from one to 10. At the conclusion of the stations, PSTs facilitated going through two questions as a whole class. Rowan used 298 discourse moves and was involved in 21 instances preceded student errors and uncertainties with 174 responding moves. Similar to Teaching 2, Rowan's primary elicit move was *checking for understanding* and his primary responding move was to *validate correct responses*. While facilitating, Rowan relied most often on *funneling*. Rowan used the extending moves of *probing* and *encouraging reasoning* once each.

In Teaching 4, Rowan individually taught a lesson about speed. His launch was a whole group "discussion" in which he elicited uses and knowledge about speed and words and phrases they associated with speed. Then, students worked on a step-by-step worksheet that took them through a scenario of two students walking to school, leaving at different times, arriving at different times. They were asked to complete charts and determine speeds. In Teaching 4, Rowan used 374 discourse moves and there were 23 instances preceded student errors and uncertainties with 194 responding moves. Rowan's primary elicit move was to *check for understanding*. His primary response was *validating correct responses*. He most frequently relied on *procedural explanations* while facilitating and did not use any extending moves.

Rowan's Discourse

Rowan used 985 discourse moves throughout his four lessons, more than all other PSTs by 170 moves. Rowan had 69 preceding moves (the median) and 563 response moves (the maximum also by 170). (See Table 7.4 and Appendix M). Rowan's overall noticing and discourse pattern involved him *checking for understanding* directly by asking them (compared to monitoring) and *validating correct responses*. I suspect this is partly due to Rowan's hearing loss which makes hearing individual students in classroom environments challenging. Rowan would frequently *cue* students to draw their attention to certain aspects, *provide procedural explanations*, and *elicit answers* as a way of checking for understanding. Rowan's students' tasks were primarily computational, arithmetic, and devoid of context; Rowan was the only PST who relied on *task clarification and direction* as a common discourse move. Rowan also used monitoring and waiting the least across all PSTs. Rowan had the largest number of discourse moves across all PSTs and the most of many individual moves. Because he used so many moves, I do not report out on the ones in which he used the most.

When Rowan was engaged with students prior to instances of error and uncertainties, his common patterns were similar to Taylor and Carson in terms of moves used. However, they manifested differently. Because Rowan *checked for understanding* as frequently as he did, it provided many opportunities for students to reveal and share uncertainties. Interestingly, when Rowan did *monitor*, he also provided ample opportunities for errors and uncertainties to come

Table 7.4:

Highlights of Rowan's Enacted Discourse Moves

	Enacted	Preceded	Response
Number	985	69	563
Frequent	Validate correct Check for understanding Cue Procedural explain Elicit answer Task	Check for understanding Monitor Invite Elicit facts or procedures Elicit answer	Validate correct Cue Procedural explain Check for understanding Funnel
Most	Elicit answer Ask for clarification Check for understanding Elicit understanding Push for explain (tied) Validate correct Prompt error correction Topaze Funnel Procedural explain Build on Encourage mult strategies Probe Press for justification Push for generalization Classroom manage Positive emotions Task	Check for understanding Elicit understanding Push for explain Revoice Cue Procedural explain (tied) Push for generalization Task (tied)	Elicit answer Ask for clarification Check for understanding Elicit ideas Elicit understanding Validate correct Revoice Prompt error correction Topaze Funnel Build on Encourage mult strategies Press for justification Classroom manage Positive emotions Task

up. Rowan also preceded many instances of errors and uncertainties with *inviting* and *eliciting answers, facts, and procedures*.

When Rowan responded to instances of errors and uncertainties, he relied on *validating correct responses*, *cueing* students into particular aspects of the mathematics or task, and *providing procedural explanations*, all common, low-level moves. The ways in which Rowan's response patterns stand out is via his unique use of large numbers of both *checking for* *understanding* and *funneling* in which he would ask questions to lead students down the solution path with prompts and questions. Thus, Rowan asked a lot of questions of students in response to errors and uncertainties, but primarily, they were guiding students down procedural paths.

Reed

Reed's spring placement for Teachings 3 and 4 was a large, urban secondary school. She worked with Mrs. Ebb, a white woman who was in her 7th year of teaching. Reed cotaught in Teaching 3 with another PST (Adam) who was not part of this study. The lessons I observed Reed teach were in a first hour Algebra II class that had about 24 (or more) students enrolled, but the two times I visited had approximately 12 students present. Reed reported this level of absenteeism was normal. Many students had their heads on their desk or slept. During the two lessons I observed, all teachers present (one or two PSTs and Mrs. Ebb) would work with students who asked questions and sought support but seemed to allow sleepy or disengaged students to be. Of the 12 students, based on my observations there were five black males, two white males, three white females, one black female, and one Latinx female.

Reed's Lessons

In Teaching 1, Reed taught with Lian about recognizing equivalencies between tables, graphs, and equations. They facilitated a Number Talk in which they students were asked to find the next, 50th, and nth term of a pattern. Their launch activity was for students to match different forms of the same question with graphs of the equation. In Teaching 1, Reed used 99 discourse moves and she was involved in 8 instances of student errors and uncertainties with 70 responding moves. Reed used *monitoring* 13 times and her primary elicit move was asking *eliciting facts or procedures*. Her primary response move was to *validate correct responses*. She used *cueing* most frequently to facilitate. Reed did not use any extending moves in Teaching 1.

In Teaching 2, Reed taught with Lian about exponential graphs and inverse functions. They facilitated a Number Talk in which they showed two graphs (an exponential function and its inverse) and asked students what they noticed. In their launch, they described properties of inverse functions. Their exploration was a worksheet-driven activity in which students worked in small groups to complete it. There was also an exam review activity (stations) in which students moved around the room and worked on various tasks. During that time, Reed and Lian were joined by other PSTs as instructors supporting and interacting with students. In Teaching 2, Reed used 331 discourse moves and was involved in 16 instances of student errors and uncertainties and 262 response moves. Similar to Teaching 1, Reed's primary elicit move was *eliciting factual or procedural information*, *validating correct responses* to respond, and *cueing* to facilitate. She used *pressing for justification* once as an extending move.

In Teaching 3, Reed taught with Adam (not in this study). They created and facilitated a scavenger hunt (wall hunt) activity that was designed as an Algebra II review for the upcoming American College Testing (ACT) test that their students would be taking. The tasks involved algebraic equations that modeled geometric relationships that included the Triangle Sum Theorem, supplementary, and complementary angles. This wall hunt involved students working to solve a given problem in one location and then finding a second location in which the answer was displayed and then completing the question at that station.

In Teaching 4, Reed individually taught a lesson about inverse functions. She launched the lesson with a whole group "discussion" in which she elicited ideas students had and knew about hourly wages and hours needed to work to make a particular amount of money. Then, students worked on a step-by-step worksheet that took them through a scenario of a person at a job who had a set wage and questions and questions related to that scenario.

Reed's Discourse

Reed used 430 discourse moves in Teachings 1 and 2 which was the minimum of all PSTs when the others' totals are included. Of those discourse moves, 24 preceded instances of errors and uncertainties (the minimum) and 332 followed (the minimum). (See Table 7.5 and Appendix M). Though Reed shared many common moves with the other PSTs, her combination of letting students know when they were correct and focusing their attention on particular aspects of mathematics or tasks as her primary moves was unique. In her collective common discourse moves, she emphasized correctness, *elicited facts or procedures*, *monitored* students' thinking, and focused on procedures and focusing their attention. However, Reed was also the PST with the most uses of *guidance* in which she provided prompts and supports without giving solutions away. This is particularly noteworthy considering this came from only two lesson's worth of data and moves because I did not include Teachings 3 and 4 due to lack of consistent audio recording.

Table 7.5:

Highlights of Reed's Enacted Discourse Moves

	Enacted	Preceded	Response
Number	430	24	332
Frequent	Validate correct Cue Elicit facts or procedures Procedural explain Monitor Correct incorrect	Monitor Invite Task Ask for clarification Elicit answer	Validate correct Cue Elicit facts or procedures Procedural explain Correct incorrect Guidance
Most	Guidance	Ask for clarification Task	Elicit facts or procedures Guidance

Note: These data come from Teachings 1 and 2 compared to Teachings 1-4 for the other PSTs.

In moves that preceded errors and uncertainties, Reed was likely to *monitor* students' thinking. She also relied on *inviting* students to share which is a high-level eliciting move, suggesting that the use of inviting helped provide spaces for errors and uncertainties to arise. Reed also used *asking for clarification* and *task clarification* moves frequently (for her) and the most (compared to all PSTs) in response to errors and uncertainties. Indicating that when she asked students to clarify their thinking, errors were likely to emerge. She often started explaining the task to students, a technique that led toward students sharing uncertainties or making errors.

When Reed responded to errors and uncertainties, she relied on nearly the same moves as she did overall in her discourse: a focus on correctness, drawing students' attention to particular things, asking procedural questions, and providing procedural explanations. However, she also used guidance, providing hints and prompts without giving away solutions strategies frequently and more than all other PSTs despite only having two lessons. Her guidance was her most frequently used high-level thinking strategy and the only one she used frequently.

Lian

For Teachings 3 and 4, Lian was placed in a rural middle school. Lian's mentor teacher was Ms. Milton, a mid-career white woman. The class Lian worked in was an 8th grade mathematics class. Two other university PSTs taught the same classes on the opposite days as Lian and her teaching partner (not in this study). She had 30 students, all of whom appeared to be white. There were 17 males, 13 females, and one paraprofessional in the classroom. Their classroom was quite small for 30 students and four adults, nicely decorated with mathematics and inspirational decor. Lian used the library during Teaching 3. The students seemed to be used to working in groups and had assigned seats in groups with designated numbers over them. Lian told me there were a large number of special education students and several students who exhibited low levels of motivation and disengagement. Her students seemed to be more distracted by my presence than the other students in this study.

Lian's Lessons

In Teaching 1, Lian taught with Reed about recognizing equivalencies between tables, graphs, and equations. They facilitated a Number Talk in which they students were asked to find the next, 50th, and nth term of a pattern. Their launch activity was for students to match different forms of the same question with graphs of the equation. In Teaching 1, Lian used 76 discourse moves and she was involved in 9 instances of student errors and uncertainties with 52 responding moves. Lian relied on monitoring 17 times. Her primary elicit move was *eliciting facts and procedures*. Her most frequent response move was to *wait*. She used a variety of facilitating moves with *guidance* being her most prominent. One of Lian's guidance moves was when she said "but this one is not that" in response to a student error. Lian did not use any extending moves during Teaching 1.

In Teaching 2, Lian taught with Reed about exponential graphs and inverse functions. They facilitated a Number Talk in which they showed two graphs (an exponential function and its inverse) and asked students what they noticed. In their launch, they described properties of inverse functions. Their exploration was a worksheet-driven activity in which students worked in small groups to complete it. There was also an exam review activity (stations) in which students moved around the room and worked on various tasks. During that time, Reed and Lian were joined by other PSTs as instructors supporting and interacting with students. In Teaching 2, Lian used 166 discourse moves, 18 of which preceded student errors and uncertainties with 107 response moves. Lian used *monitoring* 48 times. Her primary elicit moves were *inviting* and *eliciting ideas*. Her common responding moves were *waiting* and *validating correct responses*. She relied heavily on *cueing* to facilitate and did not use any extending moves.

In Teaching 3, Lian created and facilitated a scavenger hunt (wall hunt) activity that included simplifying algebraic and numerical expressions with exponents using rules for exponents. This wall hunt involved students working to simplify a given expression in one location and then finding a second location in which the answer was displayed and completing the question at that station. In Teaching 3, Lian used 140 discourse moves and was involved in 15 instances of student errors and uncertainties and 111 response moves. Lian used *monitoring* 43 times. She used a variety of elicit moves, using eight different moves between two and five times each. She relied heavily on *validating correct responses* for her responding moves. Once again, her primary facilitation move was *cueing*, and she did not use any extending moves.

In Teaching 4, Lian individually taught a lesson about rotational symmetry. Her launch was a whole group "discussion" in which she showed pictures of a butterfly and a pinwheel and elicited ideas about ways in which their symmetry was similar and different. The exploration was a textbook based activity in which students worked through the five parts of a task that asked them specific questions about a pinwheel and similar questions. Lian used 175 discourse moves and there were 23 instances of student errors and uncertainties and 113 responses. Lian relied on *monitoring* 21 times. Her primary elicit move was to *elicit facts and procedures*. *Validating correct responses* and *cueing* were, once again, her most often used responding and facilitating moves. Lian used *encouraging reasoning* three times as an extending move in Teaching 4.

Lian's Discourse

Overall, Lian used 557 discourse moves, the second smallest number of enacted moves. Of those discourse moves, 65 preceded instances of errors and uncertainties (the median) and

383 followed (the median). (See Table 7.6 and Appendix M). Lian's discourse involved the most monitoring and waiting across all PSTs and Lian's monitoring was initial, thoughtful, and focused. Lian spent 28% of her discourse moves not talking, *monitoring* and *waiting*. This quiet may come from her background of growing up outside the United States, her schooling, her personality, the fact that English is not her native language, some combination of these, or other reasons. Whatever the reasons, her quiet was substantial. When she did use discourse moves for talking, Lian frequently engaged in similar moves that the other PSTs did, drawing attention to particular aspects of mathematics or task, focusing on correctness, asking procedural questions, and providing procedural explanations.

In the process of creating opportunities for errors and uncertainties to arise, Lian used the same common moves as her peers, but relied on *monitoring* 45% of the time which, again, gave

Table 7.6:

Highlights of Lian's Enacted Discourse Moves

	Enacted	Preceded	Response
Number	557	65	383
Frequent	Monitor Cue Validate correct Elicit facts or procedures Correct incorrect Procedural explain	Monitor Check for understanding Elicit facts or procedures Elicit ideas Invite	Cue Monitor Validate correct Wait Correct incorrect Procedural explain
Most	Monitor Wait Cue	Classroom manage (tied)	Monitor Press for explain (tied) Wait Cue Encourage reasoning

her overall discourse a substantially different personality than the other PSTs. When she used words for her discourse, they were *checking for understanding*, *eliciting facts or procedures*, *eliciting answers*, and *inviting ideas*. Lian (tied with Taylor) also used the greatest number of classroom management moves preceding error and uncertainty which likely came from her solo taught lesson (Teaching 4) in which she had a number of students who were challenged to be working on the mathematics task at hand.

Lian used *cueing*, focusing students on mathematics or the task, more than *monitoring* when responding moves to errors and uncertainties (but only by three moves) and still relied heavily on *monitoring* and *waiting*. Lian used the most *monitoring* and *waiting* in response to errors and uncertainties of all PSTs. And although her more frequent moves focused on correctness and procedural work, she used the most *encouraging reasoning* (an extending move) and *pressing for explanation* (a high-level elicit move, tied with Carson) than any other PSTs, thus using some high-level thinking combined with the procedural thinking.

I have described how each PST presented and enacted their own unique discourse patterns. Taylor's prototypical routine was to observe, elicit low-level information or ask how students were doing, praise accurate responses and explain procedures. Carson relied on a combination listening to students, asking them to share ideas, validating the ideas, and then re-voicing them. Rowan used a large number of moves, mostly low-level, and frequently would check for understanding. Reed used many low-level moves and focused on procedures in her discourse. Lian's overall pattern was to quietly observe and notice and then *cue* students to focus on particular elements. Next, I turn to overall patterns, across all PSTs, to explain collective patterns across the PSTs and share more about the ways in which they were similar in their discourse.

Discourse Moves

To describe the overall discourse patterns for the PSTs across these lessons, first I present findings about planned discourse moves that emerged from written lesson plans and Interviews 2 and 4. Then, I describe the combined, collective PST discourse across all lessons, moves that preceded errors and uncertainties, and response moves (except Reed's Teachings 3 and 4). Then, I compared and contrasted planned, enacted, preceding, and response discourse moves more broadly and make connections between the design and construction arenas. For all data, I report findings of discourse moves that come from the TMSSR framework (See Appendix L) and *monitoring*. I also share findings about three moves outside the TMSSR framework that had frequent uses: *classroom management*, *positive emotions*, and *task-based moves*.

Planned Discourse Moves

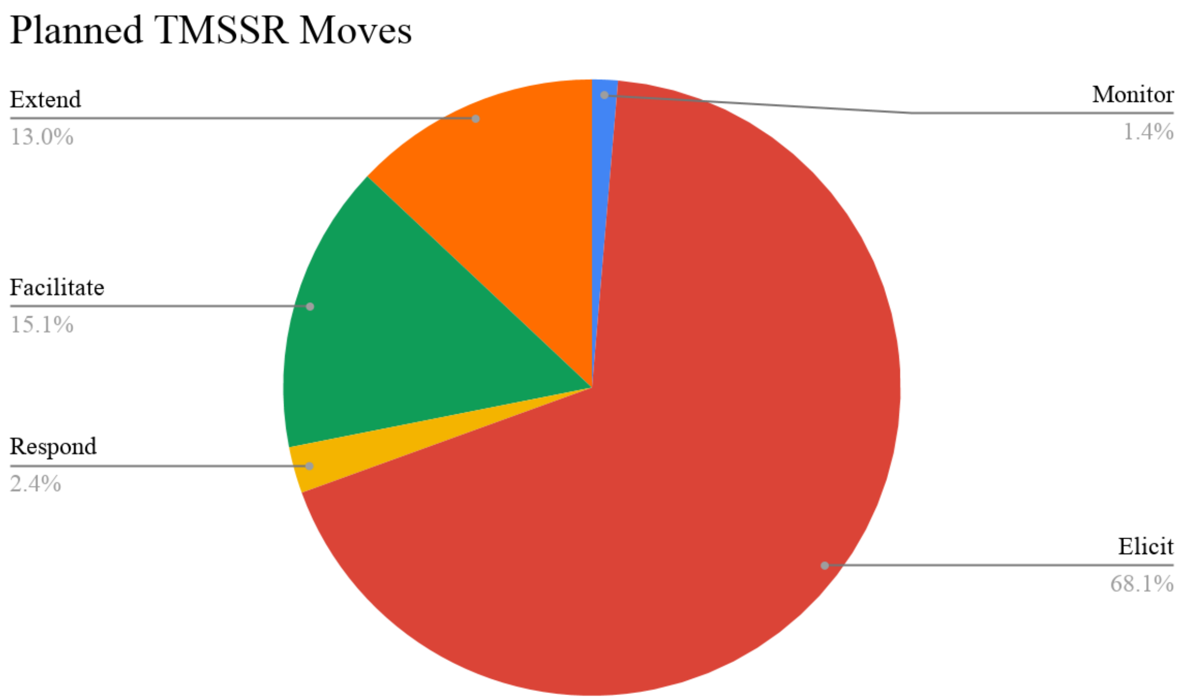
From their lesson plans and interviews, PSTs mentioned planned or intended discourse moves 370 times. (See Appendix K for a summary of planned, enacted, preceding and response discourse moves). The majority of planned discourse moves (about ⅔ of all planned moves) were eliciting moves (See Figure 7.1). The TTLP-based lesson plan template prompts, from which PSTs considered ways in which they would elicit student thinking, substantially plays a role in this frequency of eliciting moves. Monitoring and responding were practices that PSTs rarely planned. Facilitating and extending moves each accounted for about 15% of planned discourse moves.

Planned Monitoring Moves

In total, PSTs planned for five monitor moves in their lesson planning. The five planned monitor moves all came in LP 4 from Carson (once), Rowan (twice) and Reed (twice). Taylor and Lian did not include any monitoring in any planning. This is noteworthy because I did notice

Figure 7.1:

Distribution of TMSSR and Monitor Moves Across All Planned Lessons



different quality of monitoring during the enacted lessons. Sometimes PSTs were focused on students and their work and thinking. PSTs were close enough and to listen and were focused on what students were doing. Other monitoring took on more of a or test-proctoring quality in which the PST was seemingly giving students time to complete the given task. I wonder if planning for monitoring, which rarely happened, could involve PSTs engaging more explicitly in what they are expecting and hoping for and if that could support PSTs engaging in more focused and detailed monitoring.

Planned Eliciting Moves

The TTLP prompts and questions focused heavily on eliciting student thinking. This occurred consistently across the launches, summaries, and prompts throughout explorations such as: "What questions will you ask to help students access their prior knowledge? And "follow up questions that probe understanding." This resulted in PSTs planning for 252 occasions of elicit moves in their lesson planning practices. The most commonly planned elicit moves were *eliciting facts or procedures* and *eliciting answers*. The distribution of planned eliciting moves were fairly evenly distributed across PSTs with Taylor having the least (23 occasions) and Reed having the most (82 occasions). Of these planned eliciting moves, 48% were low-level thinking moves, 20% were contextually-based moves, and 32% were high-level thinking moves.

Planned Responding Moves

Considering the focus of this study, I was surprised to see so few response moves included in the lesson plans and interviews. The theme of "anticipating student thinking is challenging" emerged in interviews and this supports those experiences. Of the nine planned responses, two came from prompts about accommodations to support Emergent Bilingual Students or struggling students, two came from launch scripts, two came from summary prompts, two came from interviews when I asked about the goals for their lesson, and one came from an interview question about the lesson in general. Eleven of the 12 response plans came from LP 4 and the 12th came from LP 3. None came from LP 1 or LP 2 despite using a more robust version of the TTLP. From the nine planned responses, 22% were low-level thinking, 44% were context based, and 33% were high-level thinking responses.

Planned Facilitating Moves

PST planned for 56 facilitating moves throughout the preparing phase. Of all of these moves 75% of them (42 occasions) occurred in preparation for Teaching 4. These moves were fairly evenly distributed across PSTs. Reed had the least (five occasions) and Lian had the most (17 occasions). Planned facilitating moves came from prompts about accommodating a variety of learners, anticipating student solutions, how PSTs are thinking about their students, launch scripts, choosing what solutions are presented. Half of the facilitating moves supported low-level thinking and the other half supported high-level thinking.

Planned Extending Moves

PSTs planned to enact 48 extending moves. Lesson planning for Teaching 4 accounted for 81% of the planned extending moves (39 occasions). The distribution of planned extending moves was a bit more varied. Reed (two times) and Rowan (five times) had noticeably smaller numbers than Taylor (nine times) and Carson (12 times) and even smaller than Lian (20 times). Most planned extending moves of these came from prompts “Follow up question that extends or advances student thinking about this solution,” “Follow up question that extends thinking,” “Follow up question that helps students make connections,” and “Follow up question that probes understanding.”

Overall, analyzing PSTs’ planned discourse moves paints a clear picture. PST’s rarely planned to monitor or use response moves. The majority of their planning was in eliciting moves and they had substantial amounts of facilitating and extending moves planned. Their most planned for moves were *eliciting facts or procedures* (77 times), *eliciting ideas* (36 times), *eliciting understanding* (29 times), and *figuring out student reasoning* (28 times). Overall, they planned for about 41% of moves to be low-level thinking (instrumental), 43% were high-level thinking (relational) and 16% would be context-based moves. However, these findings live completely in the design arena. In the next section, I consider the moves that occurred within the construction arena, which are substantially different.

Enacted Discourse Moves

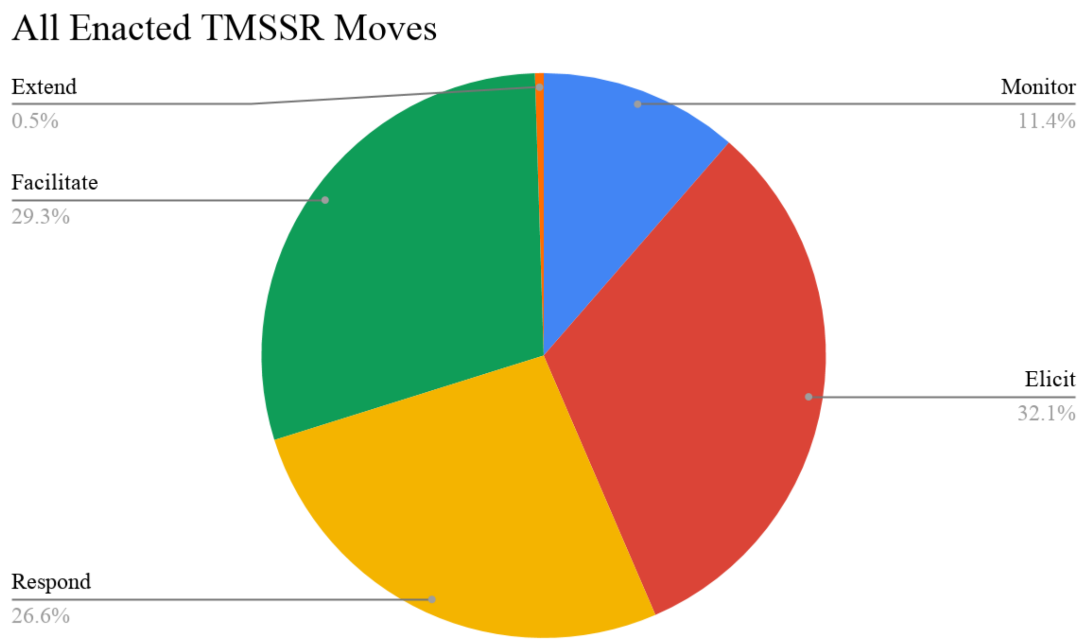
Across all lessons, PSTs uttered 3,512 discourse moves throughout their enacted lessons. The most common overall moves were *validating correct responses*, *monitoring*, *cueing*, *eliciting facts or procedures*, *checking for understanding*, and *explaining procedures*. (See Appendix K for details). In the construction arena, PSTs used monitoring 11% of the time, eliciting 32% of the time, responding 27% of the time, facilitating 29% of the time and extending less than 1% of the time. See Figure 7.2. This distribution offers a sharp contrast to what occurred in the design arena. PSTs monitored and used responding moves with substantially more frequency than they had planned. They elicited about half the frequency with which they planned. The used facilitating moves about twice as frequently as they had planned and used extending moves substantially less frequently than they had planned. This lack of alignment further supports the known challenge of lesson planning. In the following sections I share more specific findings about PSTs enacted discourse.

Enacted Monitoring Moves

Overall, PSTs used *monitoring* 327 times across the lessons (11% of total discourse moves). When *monitoring*, PSTs observed students working. The purpose of monitoring was for PSTs to know and better understand student and formative assessment, knowing how students were doing. This frequently involved PSTs slowly walking around, stopping behind students so they could see what students were writing, or pausing to listen to students if they were working with others. *Monitoring* may also have helped PSTs with classroom management and lesson

Figure 7.2:

Distribution of TMSSR and Monitor Moves Across All Enacted Lessons



pacing or timing. I am unable to know all the things that were going on in the heads and minds of PSTs during these *monitoring* moves.

Enacted Eliciting Moves

Eliciting moves made up 32% of all discourse moves. The two most frequently used elicit strategies were ones most likely to draw out instrumental thinking: *eliciting facts or procedures* (213 utterances) and *eliciting answers* (138 utterances). *Eliciting facts or procedures* were utterances such as "What points did you pick?" (Carson, Teaching 2) or "What is 0.7?" (Lian, Teaching 2). Compared to *eliciting facts or procedures*, *eliciting answers* encouraged less challenging student thinking. *Eliciting facts* involved PSTs asking students questions that only required them to share an answer, such as "So, let's look at number 2. What answer do you have?" (Reed, Teaching 2) or that PSTs phrased into a multiple choice situation in which the correct answer was given and just needed to be identified by a student such as, "8.193...is that in inches, feet, miles?" (Rowan, Teaching 2). In total, these low-level elicit moves accounted for 37% of all elicit moves.

PSTs also used elicit moves that encouraged relational, higher-level student thinking. They used *inviting* 96 times with utterances such as "What do you notice?" They *elicited ideas* 94 times in which they tried to draw out student thinking about a solution strategy such as "What do you think about inverse points and the original points?" (Lian, Teaching 2) or "How do you know that she's faster than Anthony?" (Rowan, Teaching 4). They *pressed for explanation* 76 times which often occurred as a follow up to an answer a student gave and usually were a form of "how?" or "why?"–such as "How'd you get 64?" (Carson, Teaching 3)–which prompted students to provide more details and information. They also *elicited understanding* 29 times in which they drew out the ways in which students were reasoning such as "What does that number mean?" (Rowan, Teaching 3). Enacted elicit moves to encourage relational thinking occurred in 17% of all eliciting moves, substantially lower than the planned 43% of eliciting moves.

PSTs enacted context-based moves in 26% of all eliciting moves, more than they had planned to do. They *checked for understanding* 207 times, *asked for clarification* 65 times, and *figured out student reasoning* 48 times. These moves supported PSTs in figuring out what students were thinking and how they were making sense. Considering the challenge of planning for student thinking, it makes sense to see an increased frequency of these moves compared to planning as PSTs relied on these moves in the construction arena to help them better understand the student thinking they struggled to anticipate.

Enacted Responding Moves

Responding moves made up 27% of enacted discourse moves despite only making up 2% of planned discourse moves. *Validating correct student responses* was the most frequent responding move and the most common enacted move overall with 406 uses. PSTs relied on *correcting student errors* 124 times. These two moves align with the evaluation part of the IRE triad and are lower-level responding moves. PSTs relied on them for 66% of responding moves which was a substantially larger fraction of the time then they had planned for low-level respond moves (22%). These two moves are common moves in classrooms. The frequency of *validating correct responses* likely comes from a twofold perspective of wanting students to know when they are on the right path and creating a positive classroom environment. Correcting student errors is also common (as the IRE literature shows) and likely part of the schema of what a teacher is, despite PSTs sharing with me in interviews that they try to avoid telling students if they are correct or not.

When it came to higher level responding moves, PSTs rarely (only eight times total) engaged in responding moves that supported relational thinking. PSTs *prompted error correction* eight times. In this move, they encouraged students to address errors themselves. Although PSTs planned to *encourage students to revoice* three times, they never used the move in enacted teaching. This supports Ghousseini's (2015) claim of how challenging it is for PSTs to have students engage with each other's reasoning. Collectively, these high-level thinking moves represent less than 1% of all enacted responding moves compared to 33% of planned moves.

PSTs enacted the context-based moves of *revoicing* (131 times), *waiting* (73 times), and *re-presenting* (61 times). Re-presenting was a move in which PSTs revoiced what a student said, and then added more detail or reorganized it to continue to increase student understanding. For example, when one of Taylor's students responded to a question with "multiply," Taylor's re-presenting response was "so, you multiply by the same thing each time." Context-based moves made up 33% of all enacted responding moves. This is less than the 44% planned.

Enacted Facilitating Moves

Facilitating moves made up 29% of all discourse moves which was nearly double the planned frequency. Low-level facilitating moves were the most prevalent. *Cueing* was the single most frequent enacted facilitating move with 246 occurrences. Other low-level moves that occurred frequently were *explaining procedures* (201 times), *funneling* (98 times), and *Topaze* (68 times). These accounted for 69% of all enacted facilitating moves, more than 50% that PSTs had planned.

PSTs used higher-level facilitating moves of *explaining conceptually* 91 times, *building on* 67 times, *providing summary explanations* 54 times, and *providing guidance* 47 times. *Building on* occurred when PSTs used prior contributions from students to connect and build knowledge or encouraged students to do so. Rowan did this in Teaching 4 when he said, "Remember from our discussion at the beginning of class?" when helping students about halfway through the lesson. PSTs rarely used *providing information* (five times) and *encouraging multiple strategies* (six times). PSTs did not enact any occasions of *providing alternative strategies*. High-level facilitating moves comprised 31% of all enacted facilitating moves, less than the 50% they had planned.

Enacted Extending Moves

Despite planning for extending moves to account for about 15% of all discourse, enacted extending moves only occurred 15 times which was less than 1% of all enacted discourse moves.

All PSTs used at least one and Rowan used the most with seven. This continues to support the need to plan for extending moves since the decline from planned to enacted is substantial.

Other Common Enacted Moves

In addition to monitoring and TMSSR Framework moves, PSTs used 20 other moves in their enacted lessons. Because of their frequency, I report on the three most common of those–*classroom management*, *tasks*, and *positive emotions*–in this section. The other 17 moves comprise 6% of all enacted teaching moves. PSTs used *classroom management* moves 120 times across lessons. These were utterances such as when Taylor said, “We're going to have you spin around” (Teaching 2) to a student to make their planned groups and when Carson said, “Do you need a card to work on? Why don't you grab one of those and start working on that” (Teaching 3). *Classroom management* moves were those that help get students focused on doing the expected mathematical task. PSTs also relied on moves focused on describing the *task* 109 times. These moves helped explain the task that students were supposed to be working on and likely came from students being unsure about what they were to be doing, either behaviorally or mathematically. These were statements such as, “And then we can copy down the definitions” (Lian, Teaching 4) or “So, for this one, A is looking for end behavior” (Reed, Teaching 2). *Classroom management* and *task* moves may both be student uncertainty (students not knowing what to do), but students’ behavior may have influenced the PST response. If students directly asked a question, it was likely that the PST’s response was a *task* compared to if the student was doing something else because they did not know what to do, the PSTs likely responded with (and/or I coded as) *classroom management*. PSTs also used *positive emotion* discourse moves 73 times. These were comments such as “I like how you explained it” (Rowan, Teaching 2) and when Taylor fist bumped a student in Teaching 3. These helped create positive learning environments for students and created spaces where errors and uncertainties were welcomed.

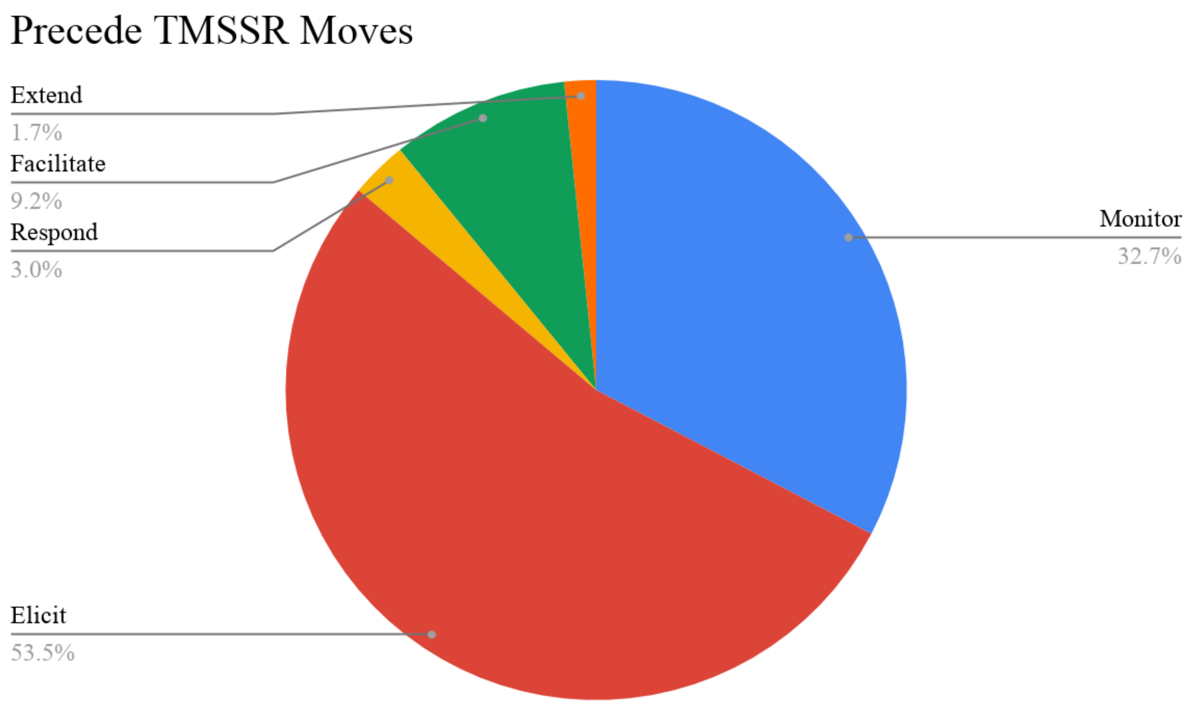
Preceding Moves to Errors and Uncertainties

To examine what preceded instances of errors and uncertainties, I looked at the approximately 326 instances of errors and uncertainties throughout the enacted lessons and considered the PST's move prior to it. There were a handful of instances that were preceded by moves from mentor teachers or PSTs not part of this study. I do not include those. Preceding moves to errors and uncertainties are a subset of the total enacted discourse moves discussed in the previous section. (See Appendix K for details). Preceding moves are also an unusual group. They are not moves that PSTs can think of as "preceding" because PSTs would not be able to know when an instance of error or uncertainty would occur. Thus, in this section I intend this describe the phenomena and not draw conclusions about what PSTs could or should have done to make this discourse or lesson better or different.

Among moves that preceded errors and uncertainties, monitoring accounted for 33%, a much larger percentage than the total enacted (11%) and an even larger percentage than the planned 1%. (See Figure 7.3) Eliciting made up 54% of preceding moves, a larger proportion than all enacted moves (32%), but less than planned (68%). It seems logical for PSTs to have used monitoring and eliciting prior to instances of error and uncertainty, as these are precursors to student discourse. Responding moves occurred about 3% of the time before students' error and uncertainty, substantially less than overall (27%), but nearly the same as planned (2%). Facilitating moves preceded error and uncertainty about 9% of the time, much less than overall discourse moves (29%) and planned (15%). Extending moves made up about 2% of all preceding moves, more than the less than 1% overall, but also substantially less than the 13% planned.

Figure 7.3:

Distribution of TMSSR and Monitor Moves That Preceded Instances of Errors and Uncertainties Across All Lessons



Monitoring Moves That Preceded Errors and Uncertainties

Monitoring preceded instances of errors and uncertainties 99 times. This was the single most common move to precede instances of errors and uncertainties and accounted for 30% of preceding moves. This is logical given the purpose of *monitoring* was to help PSTs understand what students were doing and thinking. *Monitoring* as a preceding move was likely to happen when a PST was near a student, creating an opportunity for the student to reveal uncertainty or the PST to notice an error.

Eliciting Moves That Preceded Errors and Uncertainties

Eliciting moves preceded errors and uncertainties 162 times (54% of preceding moves). This is also logical as the purpose of eliciting moves is to draw out ideas, some of which are bound to be errors or uncertainties during the learning process. The most prevalent elicit moves that preceded error and uncertainty was when PSTs *checked for understanding* (49 times) with questions such as, “How’s it going?” or “Does that make sense?” PSTs used similar moves–*figuring out student reasoning* and *asking for clarification*–less frequently (three times each), but those also functioned to help PSTs understand student thinking around errors and uncertainties. *Figuring out reasoning* came in the form of direct questions such as when Taylor asked, “Why did you put them in that order?” (Teaching 4), and also as questions such as when Carson asked, “What are some ways we can confirm that is true?” (Teaching 2). When it came to *asking for clarification*, both before instances of error and uncertainty and across all situations, it was not always clear to me whether these inquiries were from genuine lack of clarity from the PSTs’ perspectives or as a technique to get student thinking out and public. For instance, when Reed asked a student “What is opposites?” (Teaching 2) and Rowan asked a student “Can you repeat that?” (Teaching 1), both indicated the PST wanted more clarification, but the use and tone is different. In total, these context-based moves made up 34% of elicit moves that preceded error and uncertainty which is slightly more than the use of context-based elicit moves in all discourse (26%) and context-based planned elicit moves (20%).

PSTs also used lower-level eliciting moves prior to errors and uncertainties. They *elicited facts or procedures* (31 times) and *elicited answers* (17 times). These low-level moves preceded 30% of instances of errors and uncertainties which is slightly less than the 37% of lower-level eliciting moves in total and lower than the 48% of planned low-level eliciting moves.

PSTs engaged in high-level eliciting moves 59 times preceding errors and uncertainties. PSTs relied on *inviting* 31times. These moves were used to help keep ideas coming and implied a variety of potential answers were welcome and acceptable. They were questions such as, "What else is important when we're looking at our equation and graph?" (Taylor, Teaching 1) and "What did you guys notice about using these compasses?" (Carson, Teaching 4). PSTs *elicited ideas* 19 times, *pressed for explanations* five times, and *elicited understanding* four times. *Eliciting understanding* moves were those in which PSTs asked questions around how students were understanding or reasoning. These were statements such as, "Why might theoretical and experimental probability be different?" (Taylor, Teaching 3) and "How would you tell if something's increasing or decreasing?" (Reed, Teaching 2). High-level elicit moves made up 36% of all eliciting moves that preceded instances of errors and uncertainties which was in line with the planned proportion (32%) and higher than the overall enacted proportion (17%) of high-level eliciting moves.

Responding Moves That Preceded Errors and Uncertainties

PSTs relied on TMSSR responding moves nine times preceding instances of errors and uncertainties (3% of preceding moves). Though responding moves logically follow students' utterances, due to the nature of discourse being a back-and-forth experience, when PSTs said something in response to a previous student utterance and the following utterance was an error or uncertainty, it ended up preceded an error or uncertainty. The most common responding move to precede errors and uncertainties was *validating correct student responses* which happened three times. *Correcting students* was the next most common and happened twice. PSTs used context-based responding moves four times preceding errors and uncertainties: *waiting*, *revoicing*, and *re-presenting*. Overall, PSTs' low-level responding moves accounted for 56% of all responding moves that preceded errors and uncertainties, 44% of responding moves that preceded errors and uncertainties were context based. PSTs did not use any high-level responding moves to precede instances of errors and uncertainties.

Facilitating Moves That Preceded Errors and Uncertainties

PSTs used facilitating moves preceding error and uncertainty 28 times. Low-level facilitating moves–*cueing*, *funneling*, *Topaze*, and *explaining procedures* combined–preceded errors and uncertainties 20 times and all share the characteristic of trying to help move the student down a procedural path toward an answer. These moves accounted for 71% of the facilitating moves that preceded error and uncertainty, more than both planned (50%) and total enacted (69%) facilitating moves.

High level moves–*explaining conceptually*, *building on*, and *providing guidance*–preceded errors and uncertainties a total of eight times and were done to help students consider the reasons, ideas, or concepts behind the tasks they were doing. High-level facilitating moves made up 29% of all facilitating moves preceding error and uncertainty, less than the planned (50%) and overall enacted (31%) proportions.

Extending Moves That Preceded Errors and Uncertainties

Extending moves proceeded five instances of errors and uncertainties and represent 2% of all preceding moves. These moves came from Rowan (twice), Reed (once), Lian (once), Carson (once), and none from Taylor. Rowan used two *pushing for generalization* questions "You can test it. Did it work?" and "You guys can test this question to see if it works" (Teaching 2.) Reed used *pressing for justification* with "How do we know that 1 and 2 the inverses are functions, but 3 and 4, the inverses are not functions?" (Teaching 2). Lian used *encouraging reasoning*, "And why not A to G?" (Teaching 4). Carson also *pressed for justification* when he asked, “Why’s it obtuse?” (Teaching 4). I expected extending moves to represent a larger proportion of preceding moves as these practices are ones related higher-level thinking and extension and could spark errors and uncertainties. However, extending moves did precede instances of errors and uncertainties with greater proportion than overall discourse and responses to error and uncertainties which I discuss next.

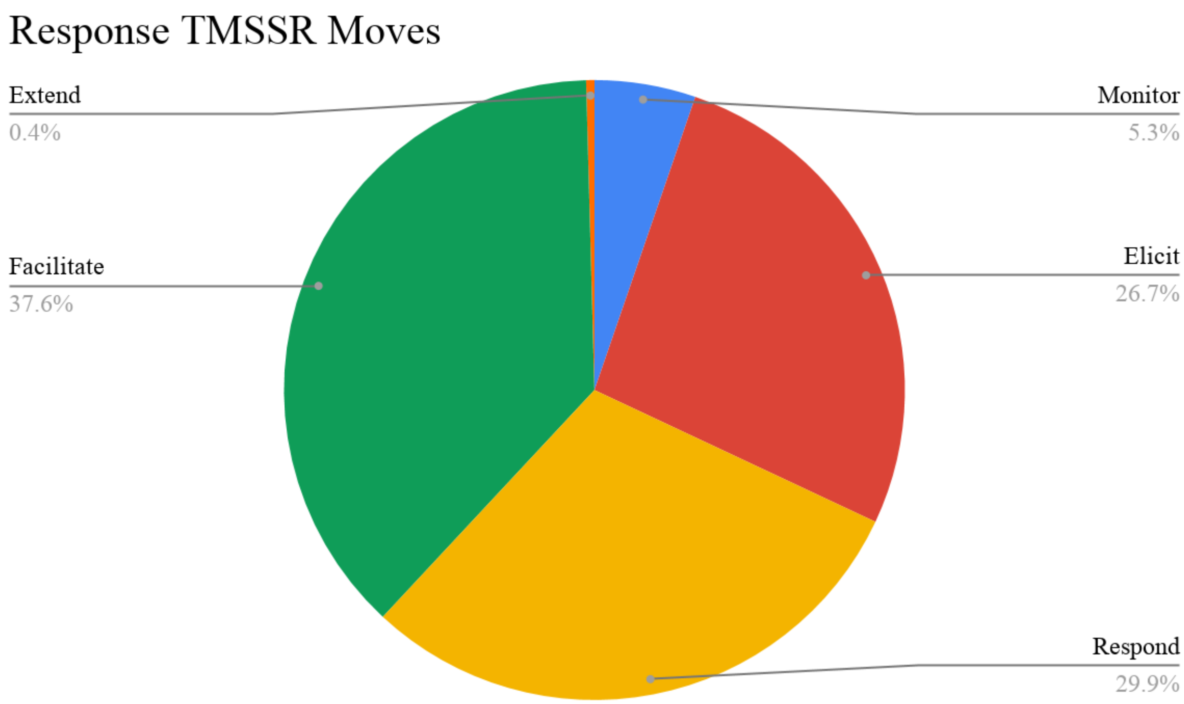
Responses to Errors and Uncertainties

To examine what followed instances of errors and uncertainties, I considered the 2,049 moves that PSTs uttered after the initial error or uncertainty arose through the end of the instance which typically ended with some indication that the student understood or the task was completed. Unlike moves that preceded instances of error and uncertainties, PSTs were (on some level) aware that an error or uncertainty had occurred, and their responses were likely to be more intentional or expected.

PSTs used monitoring 5% of the time in responding to error and uncertainty (See Figure 7.4). They used eliciting moves 27% of the time in response to error and uncertainty. The most common elicit move used was *eliciting facts or procedures*. They used TMSSR respond moves 30% in response to error and uncertainty. *Validating correct responses* was the most commonly used move in response to error and uncertainty. (See Appendix K for details). Facilitating moves made up 38% of PST’s responding moves and *cueing* was the most frequent facilitating move and second most move overall. PSTs used extending moves less than 1% of the time when responding to errors and uncertainties.

Figure 7.4:

Distribution of TMSSR and Monitor Moves That Followed Instances of Errors and Uncertainties Across All Lessons



Monitoring Moves That Followed Errors and Uncertainties

Monitoring followed instances of errors and uncertainties 98 times (5% of all response moves). *Monitoring* after an instance of error or uncertainty involved a PST staying quiet, not voicing anything, and often stepping in or moving closer and looking at student work, as if to understand what the student was thinking. *Monitoring* is similar to *waiting*, which happened 53 times in response to errors and uncertainties, but *waiting* had more of a stepping back and moving away action and did not include the PST seemingly trying to make sense of written work. Because only two *waiting* moves were planned (once by Reed and once by Rowan each in Teaching 4), most of this use likely came when a PST was uncertain about how to respond.

Monitoring in response to error and uncertainty happened with less frequency than monitoring that preceded errors and uncertainties (33%), with less frequency than all discourse (11%), but more frequency than planned monitoring (1%).

Eliciting Moves That Followed Errors and Uncertainties

The 497 eliciting moves that followed student errors and uncertainties made up 27% of all response moves. The most frequent eliciting move in response to errors and uncertainties was *eliciting facts or procedures* (122 times) such as when Carson said, “So, how do we find B...if we just found A?” (Teaching 2). PSTs *elicited answers* 73 times in response to errors and uncertainties such as when Rowan asked, “What is Anthony's speed?” (Teaching 4). These two moves represent lower-level thinking elicitations and accounted for 39% of eliciting moves that followed errors and uncertainties. This frequency was similar to low-level elicit moves that preceded (30%) and across all moves (37%), but was lower than planned low-level eliciting (48%) indicating that PSTs enacted proportionally lower low-level elicit moves than planned which is a powerful step toward more productive mathematics classroom discourse.

In higher-level thinking elicitations that followed instances of error and uncertainties, PSTs *pressed for an explanation* 52 times. These moves were characterized by a PST eliciting a student to explain how or why they got an answer they did, such as, “How'd you do that?” (Carson, Teaching 3) or “Why do you say it's exponential?” (Rowan, Teaching 2). Often this came as simply the question “Why?” PSTs *elicited ideas* 41 times which came in the form of questions such as, “What do we know about reflectional symmetry?” (Lian, Teaching 4) and “How can we compare them?” (Taylor, Teaching 2). PSTs *invited* 26 times and *elicited understanding* 19 times. In all, high-level eliciting moves followed errors and uncertainties 28%

of the time which was less frequent than preceding (36%) and planned (32%), but with more frequency than overall discourse moves (17%).

In the context-based moves that followed errors and uncertainties, PSTs *checked for understanding* 74 times, *asked for clarification* 53 times, and *figured out student reasoning* 37 times. These discourse moves were all delivered sincerely in the lessons. As an observer, I was unable to tell whether PSTs used these moves to increase their own understanding of student thinking in the moment or as strategies to make student thinking public for the student–and other students–to more easily address the errors and uncertainties.

Responding Moves That Followed Errors and Uncertainties

PSTs responded to errors and uncertainties using TMSSR's responding strategies 557 times, approximately 30% of all responses to errors and uncertainties. The most prevalent move was somewhat ironically given that the student utterances were errors and uncertainties: *validating correct responses*. PSTs used *validating* 264 times. *Validating* occurred verbally with a PST saying things such as "yeah," "yes," "exactly," "right," "yes," "mmm hmmm," or repeating the correct answer. Nonverbal examples of *validating* involved PSTs writing a correct answer on the board or nodding their head. The second-most used responding move in response to errors and uncertainties was PSTs *correcting incorrect student answers* which occurred 119 times and involved PSTs telling students the correct answer. These two responding moves represent low-level thinking and account for 69% of all responding moves in response to errors and uncertainties. This is a higher frequency than PSTs planned for (22%), enacted in total (66%), or that preceded errors and uncertainties (55%), further supporting how the evaluation part of the IRE triad is frequently used in response to errors and uncertainties.

PSTs engaged in context-based moves such as *revoicing* (78 times), *waiting* (53 times) and *re-presenting* (34 times) which made up 166 responding moves and was 30% of responding moves that followed errors and uncertainties.

PSTs engaged in *prompting error correction*, a high-level responding move, eight times (1% of the responding moves that followed errors and uncertainties). High-level responding moves were only used in response to errors and uncertainties. They represent less than 1% of overall discourse and none of the preceding moves. Yet, they were planned to happen 33% of all responding moves. Although it is good to see some high-level responding moves used in response to errors and uncertainties, this small number illustrates the challenges of engaging in high-level responding moves.

Facilitating Moves That Followed Errors and Uncertainties

Facilitating moves made up 38% of all responses to errors and uncertainties and occurred 700 times. *Cueing* was the most frequent facilitating move and occurred 204 times with *procedural explanations* happening 151 times, *funneling*, 83, and *Topaze* 61 times. These low-level facilitating moves comprised 72% of the facilitating moves that followed error and uncertainty and represent a series of interactions that were focused on helping students complete tasks with little attention to reasons why things were happening. Low-level responding moves occurred with about the same frequency as low-level moves that preceded errors and uncertainties (71%), and more than overall low-level facilitating moves discourse (69%) and those that were planned (50%).

PSTs also engaged in higher-level thinking facilitation moves that followed instances of errors and uncertainties. They engaged in *conceptual explanations* 69 times in which they focused on the ideas and reasons behind procedures in their explanations. They *built on previous* *ideas* 45 times, connecting to things students had said earlier in the lesson or previous lessons and also *provided guidance*, offering hints or prompts, but without telling students answers 45 times. PSTs also used *summary explanations* 31 times. Far less frequently, PSTs used *providing information* five times. In this move, PSTs gave general and new information related to mathematics more broadly such as when Reed said, “we can think of this as a grid, a pattern of potential solutions” (Teaching 2). PSTs also encouraged *multiple strategies* three times. These high-level facilitating moves in response to error and uncertainties represent 28% of all facilitating moves, similar to the 29% that occurred in preceding moves, but less than planned (50%) and overall enacted (31%) facilitating moves.

Extending Moves That Followed Errors and Uncertainties

Extending moves occurred eight times in response to errors and uncertainties, which was less than 1% of all response moves. Lian used two (Teaching 4), Rowan used three (Teachings 2 and 3), and Taylor used three (Teachings 3 and 4). Reed and Carson did not use any extending moves in response to errors and uncertainties. Lian used *encouraging reasoning* twice when she asked two different groups of students, “Why A to G, not A to H?” (Teaching 4). Rowan also *encouraged reasoning* when he asked, “Why not?” (Teaching 3). Rowan and Taylor each *encouraged students to engage in reflection* when they asked things such as, “Why'd you change that?” (Rowan Teaching 2) and “Can you imagine drawing a tree diagram with 250 combinations on the bottom?” (Taylor, Teaching 3). Taylor used *encouraging reflection* twice when she said, “It's going to look similar to, I think, solution B. So, think about how you can make it look similar to B using your pieces” and “Think about how you solved B” (Teaching 4).

Other Common Moves That Followed Errors and Uncertainties

PSTs used *task* focus and clarification moves 39 times in responses to errors and uncertainties as well as *classroom management* moves 27 times. These are consistent with the frequency of planned uses. PSTs employed *positive emotion* moves 35 times in response to errors and uncertainties, which, though under 2% was more frequent than many other TMSSR moves and shows a commitment to encouraging students to share errors and uncertainties.

Comparison of Discourse Moves Across Phases

I now describe broader patterns in the discourse across planned moves and enacted moves. I compare and contrast TMSSR moves and monitoring by phases (plan, enact, precede, respond) and then I compare and contrast levels of thinking across phases. These data are the same data from the previous sections and combined for a broader perspective. In this section I focus on the connection between data from planned lessons and enacted lessons. I do not address contexts and factors I have considered earlier such as PST knowledge and beliefs, classroom context, lesson topic, and tasks. These contribute to the findings in this section, but in order to be able to consider the relationship between planned and enacted discourse, I focus on those.

Types of Moves

As I considered the combined discourse data from all PSTs across arenas and phase, the distributions were different from the design arena (planned) to the construction arena (enacted, preceding, and response moves). See Table 7.7 and Figure 7.5 for details. It is not my argument that planning and enacted moves should be completely identical, but I argue that a closer alignment between the two arenas could help PSTs support students in engaging in higher-level thinking. I also acknowledge that it is logical, and even necessary, for there to be differences in distribution across preceding and responding moves to errors and uncertainties.

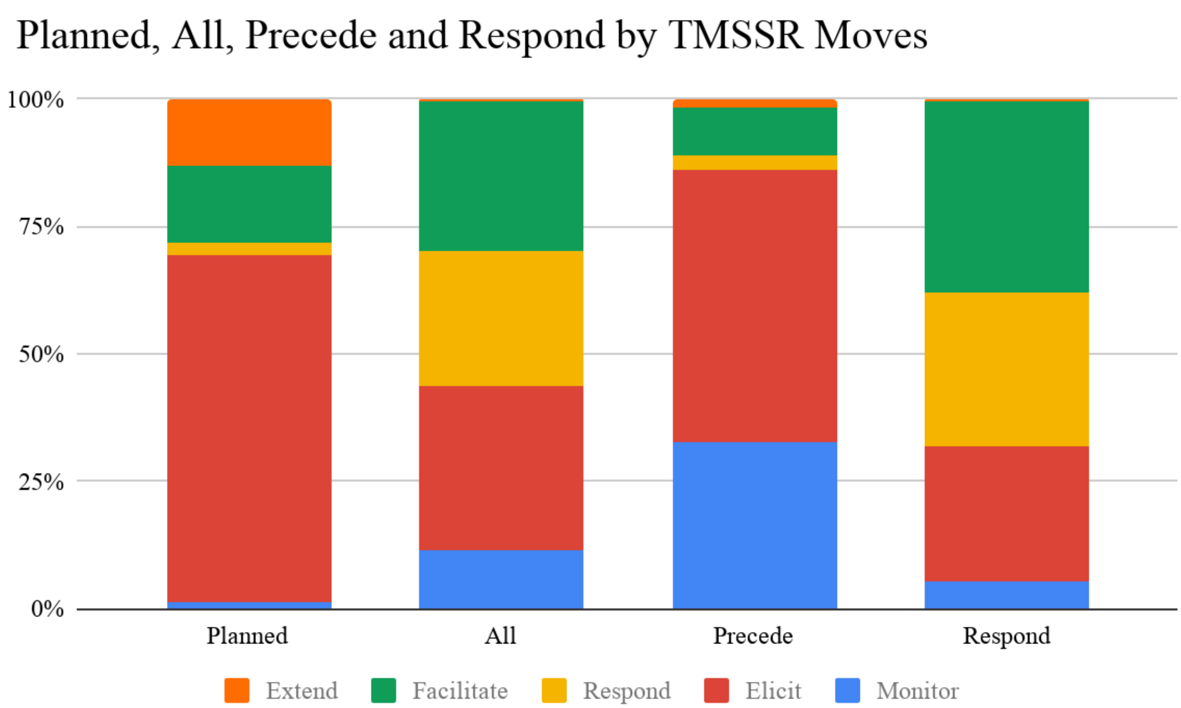
Table 7.7:

Distribution of TMSSR and Monitor Moves Across Categories of Discourse

	Planned	All	Precede	Respond
Monitor	1%	11%	33%	5%
Elicit	68%	32%	53%	27%
Respond	2%	27%	3%	30%
Facilitate	15%	29%	9%	38%
Extend	13%	<1%	2%	<1%

Figure 7.5:

Distribution of Total TMSSR and Monitor Moves Across Categories of Discourse



First, regarding monitor moves. Perhaps it is logical than monitor moves would not be planned for in the same way or with the same frequency that other moves would, however from the perspective that being able to plan for student thinking is a known challenge for teachers, especially early career teachers., I could see how planning to monitor and planning for what aspects of mathematics a PST is looking for, could help bridge that gap between the challenge of planning for student thinking and being able to enact high-level thinking practices.

Second, eliciting moves were heavily influenced by the TTLP format and a critical aspect of teaching and learning, especially when making student thinking visible is of high importance. However, the fact that eliciting made up more than ⅔ of all planned moves, yet only made up ⅓ of enacted moves indicates that helping PSTs also consider other moves during planning (and with structured template support to do so) could help PSTs more thoughtfully and intentionally engage in responding, facilitating, and extending.

Third, only 2% of planned moves were responding moves, yet during all enacted lessons 27% of moves were responding moves. This was the largest under planned type of move (by 25%) and helping PSTs prepare for using responding moves is a critical component of lesson planning, and the focus of this study. Planning for responding moves is more difficult than planning for eliciting moves. Learners are unpredictable and even despite a PSTs knowing their students well, students are likely to say and do things that surprise the PST. In addition, when framed by teacher noticing, observing and making sense of come between eliciting and responding and much can happen in this space.

Fourth, PSTs also under planned for facilitating moves, but the difference is less than in responding and PSTs did spend 15% of their planning moves on facilitation, giving them more moves to have considered what they would say to support facilitating. Facilitating functions similar to responding and is dependent on student thinking, a PST observing, making sense of it and then responding using a facilitation move.

Finally, extending moves tell a different story. During planning, 13% of PSTs' intended moves were extending, yet overall, less than 1% of enacted moves were extending. This misalignment continues to support the claims by PSTs in this study and in previous research about how challenging planning and responding to student thinking is (e.g. Franke et al., 2009).

Level of Thinking

When I consider the level of thinking across the different phases and arenas, a similar theme emerges that came up when considering the types of moves. Overall, PSTs planned for 43% of their moves to be high-level thinking that would lead to relational-mathematics thinking. However, when they actually enacted their plans, high-level thinking moves comprised 20% of all moves. See Table 7.8 and Figure 7.6. Based on the extensive nature of the TTLP, I was not surprised to see this distribution in the design arena. However, these findings continued to support the pattern that, in addition to being challenged to anticipate student thinking, PSTs also struggled to ensure their enacted moves align with the high-level thinking they had planned.

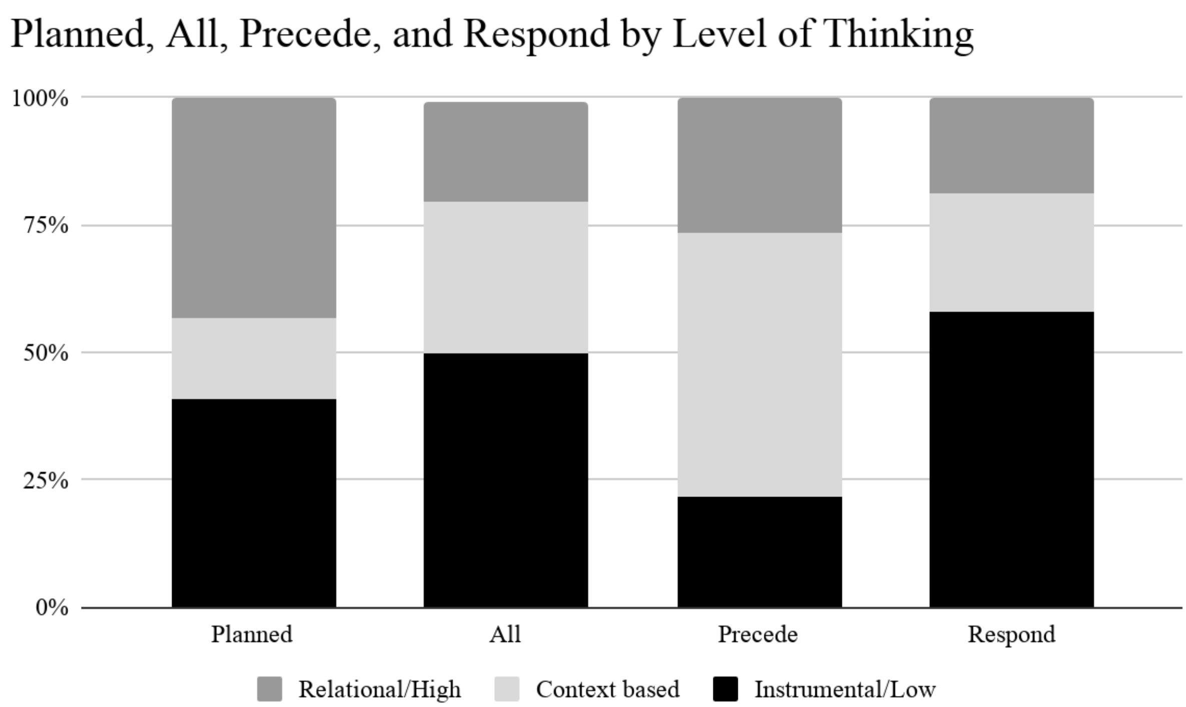
Table 7.8:

Distribution of Level of Thinking on Moves Across Categories of Discourse

	Planned	All	Precede	Respond
Instrumental/Low	41%	50%	22%	58%
Context based	16%	30%	52%	23%
Relational/High	43%	20%	26%	19%

Figure 7.6:

Distribution of Level of Thinking on Moves Across Categories of Discourse



Two other lessons emerged from this data. First, I expected larger percentages of moves that preceded errors and uncertainties would come from higher-level thinking prompts as those moves were more likely to be pushing or extending student thinking and thus lead to student errors and uncertainties. It turned out to be that 26% of moves that preceded errors and uncertainties were high-level thinking. This is higher than the overall, 20%, of high-level mores in the lessons and substantially less than the frequency of planned high-level moves (43%). Secondly, less of a surprise, but still a concern is the large percentage (58%) of low-level moves that were used in response to errors and uncertainties. Through literature and experiences, we know that teachers are challenged to maintain high cognitive demand for students in lessons, and this data demonstrates that is particularly impacted after students indicate uncertainties or demonstrate errors.

Summary of The Role of Errors and Uncertainties in Enacted Mathematics Discourse

Despite the same methods course, same planning templates, and in many cases, teaching the same lesson, PSTs exhibited unique discourse patterns in the type and number of moves used as described previously. These discourse profile differences are likely impacted by a variety of factors such as the PSTs' knowledge and beliefs, lesson topic, activity structure, students, classroom, and lesson goals. These factors all work together to impact the ways in which a PSTs' classroom discourse is unique and distinct.

However, there were also patterns that suggest the PSTs had many aspects in common with each other. All PSTs experienced a lack of alignment from planning to enacting suggesting that anticipating student thinking is a challenging, complex, and nuanced practice. Despite using a template that emphasized high level thinking, struggled to plan for and enact high level thinking moves. In addition to PSTs relying on many low-level moves, there was clear evidence of a decline in cognitive demand when errors or uncertainties arise. One particular pattern that came up across the data was the challenge of both anticipating student thinking and responding to it. In the next chapter, I explore this further.

Chapter 8:

The Relationship Between PSTs' Planning and Enactment

In Chapter 8, I describe the relationships between planning and enactment that emerged from interviews conducted after Teachings 3 and 4 (Interviews 3 and 5) as well as written reflections that PSTs wrote after Teachings 1 and 2. In these reflections and interviews about the lessons they had just taught, PSTs highlighted the important connection between lesson planning and classroom discourse, "I see how planning and anticipating student thinking in planning can help better serve as a guide for what to expect throughout a lesson" (Carson, Ref 1). They described the ways in which their planning influenced their teaching, challenges they had with anticipating student thinking, and the resulting difficulty in responding to errors and uncertainties. "I think a lot of the phrasing in my responses to student thinking was influenced by my anticipations, and because I was unable to see many solutions, I was resistant to differing ideas" (Carson, Ref 1). The boundaries between themes are blurry as many aspects in the design arena and construction arenas are connected. I used these reflections to describe their individual experiences and thematic experiences that emerged across PSTs.

Taylor's Planning and Enactment Connections

In her interviews and written reflections, Taylor addressed a variety of ideas that connected lesson planning and responding to students' errors and uncertainties. Broadly, she made statements that connected to her teaching philosophy around guiding students, but not giving them answers. She said,

> When I listen to the students, I want to make sure that I'm not giving away that answer … and then I'm like, "okay, this is what I want to say to give you a hint without giving you

the answer" and so carefully and I want to make sure I'm understanding them. So, I carefully think about what they said. (Int 3)

She expressed her happiness when "there was still some good struggle with the students needing more than one way to prove that their graphs and equations matched" (Ref 1) as further evidence of the importance of guiding, not telling, students. However, she also indicated that she did intervene at times when she, "had to progress the conversation to how to tell what kind of data it was instead of getting to talk about how other people saw the pattern" (Ref 1). Despite not actually engaging in this discourse pattern during the lessons (when she more commonly focused on correctness), Taylor saw this idea of providing guidance and support emerge in contrast to tutoring experiences she had. She explained that teaching groups of students was different when students encountered errors and uncertainties.

> In class, you can have students rely on others/group members…and it's always something that's so hard to remember because I'm used to tutoring and so just like one-on-one interaction, so there's not anyone else to point them to. (Taylor, Int 5)

Several times, Taylor mentioned her knowledge of students as significant to the connection between her lesson planning and responding to errors and uncertainties. During Fall Semester, she wrote the following regarding her experiences with her university students:

> Over the semester I have been able to get to know the students. Knowing the students has added to the level of comfort I have in front of the class. The lessons tend to go smoother when I am more comfortable because I am able to think clearly about the order I want to do things in and what changes to make on the spot that will benefit the students. (Ref 2)

Then, in reference to her Spring Semester, middle school students, Taylor reflected, "We [she, Carson, and Matt] expected their right answers and expected their wrong answers because we had already worked with them in those types of activities" (Int 3). However, later in the same interview, when she talked about how the students struggled with the dice rolling station, she said "I don't know if that was the way we worded things or if it was just something that they hadn't done before so that was new" (Int 3), thus illustrating that despite knowing her students in some ways, there were also times in which her planning failed to support her anticipating student thinking and led to challenges in responding to errors and uncertainties. This tension of knowing her students and also still being surprised emerged in Reflection 2 when Taylor wrote how a student surprised her "by making the connection to the Number Talk table we made with the table in the explore activity." But she later wrote

> We felt the most comfortable during this round [the third of three teaching rounds during Fall Semester], but somehow also the most unprepared. There were so many ways that the lesson could go, but trusting each other [her coPST] and knowing that the activity was a good walkthrough discovery of exponentials helped us to be able to lead a relatively smooth lesson. (Taylor, Ref 2)

So, for Taylor, the connection between planning and responding to errors and uncertainties centers on knowing students and providing spaces to guide them through challenges without giving them answers.

Carson's Planning and Enactment Connections

Carson–who frequently considers small differences and nuances in things and sees people and experiences as individualized and unique–often seemed skeptical of the role of planning, particularly in anticipating student thinking. In Reflection 1, Carson described how he has

> never been a person who very thoroughly planned any of my projects. In the past I have generally preferred to leave my planning more open so that I can adapt to student

> thinking through a lesson and explore ideas more freely. I used to think more along the lines of, “if you can’t anticipate student thoughts, don’t anticipate at all so that you can explore and be open to student thinking as it emerges without any prejudice of what they may be thinking.” In the past this has worked semi-effectively.

In the same reflection, he wrote “In the future when I plan for lessons, I think I’ll spend more time fleshing out possible tracks of student thinking and anticipating student thinking” (Carson, Ref 1). Later, in Interview 3, Carson articulated,

> When something you don't expect to happen happens, it becomes a bigger focus in your mind than you intend it to be and that’s really the big thing. When you do plan, you put forward all this effort to say, “this is what’s going to happen, here’s how it’s going to work” and hope that it goes to your plan. And when it starts to not go to your plan, you just latch on to that thing and go, “oh, I need to fix this so I can get back on plan”

In the same interview, Carson described how the lesson planning helped him know the goal and purpose of the lesson, so he knew what he wanted his students to get. This illustrated a small shift in thinking between the importance of planning in terms of being able to anticipate and respond to student errors and uncertainties. As he was doing that planning, Carson relied on his knowledge of his students

> as far as anticipating student thinking–a lot of that was done implicitly just through “what should I expect to happen based off of what they've done already? I knew to expect there to be some confusion with experimental versus theoretical. I knew to expect there to be some confusion with replacement vs non replacement. (Int 3)

He recognized that broadly “how we phrase our responses [to students] is something to consider and work on to develop as professionals” (Ref 1) and more specifically

> the miscommunication for the ordered pairs of dice rolling wasn't something that we had necessarily anticipated. As we [Carson, Taylor, and Matt] wrote the question, we were like “oh yeah! That makes sense” It’s like when you write an essay and you write it out and you're reading it in your head and it makes sense, but then you hand it to someone else who asks “what are you saying here? The grammar’s all off. I don’t get what you’re saying.” So, I don’t think that was something we were expecting or anticipating (Int 3).

Overall, Carson’s planning helps him stay on track (Putnam, 1987) and know where he is in the lesson and if needs to get “back on track.” Carson is aware that there are many ways students may approach a given task, but seemed to not put much value in anticipating as he was of the belief that there would always be more ways and so it was more valuable to be able to go with the flow in the moment of the lesson.

Rowan’s Planning and Enactment Connections

In alignment with his philosophy of honoring students’ backgrounds, building competence and guiding students instead of telling them answers, Rowan shared that he did not want his students to

> get frustrated because they get frustrated, they're not going to learn anything. It’s okay to be frustrated for a little bit, but as long as they can figure it out. But if they’re just stuck on something you just get frustrated, it’s not going to be beneficial for anyone. (Int 3)

Thus, a number of the ways that Rowan’s planning and enactment connected were focused on supporting students and scaffolded when they got stuck. In Teaching 2, he planned to, and then remembered to, “provide the piece of paper to show them what’s happening every time we fold it when students were struggling” (Ref 2). In Teaching 4, he planned to have students “use their group mates as a resource first before asking me and actually followed through with that really well" (Int 5). Additionally, in Teaching 4, he encouraged students to "go back through and underline or circle anything that you think is important." He shared with me that "It's a rule that expires, but I do that with myself still" (Int 5). Rowan relied on planning to help him "recognize different strategies they can use, so when I see one, I wouldn't be as surprised" (Rowan, Int 5).

As important as supporting students and scaffolding were to Rowan in theory, he struggled to anticipate student thinking and reported he frequently overestimated or underestimated his students. In Reflection 1, Rowan explained how he had underestimated students because he was surprised when "seemed to completely understand inverses." After Teaching 2, Rowan described that he was not expecting "one of the students kept questioning how would the function work" (Ref 2). In his next lesson, Rowan and his teaching colleagues "either didn't think enough of what type of problem it was, or we overthought some problems" (Int 3). Then, after Teaching 3, Rowan explained

> an example of me not thinking enough was with repeating decimals. I was thinking "I don't want to make the problems too challenging, but I want to make it different enough" so I had 125 over 50 and compared that to the next number. That was actually a really simple conversion. I should have had 621 over 43. (Rowan, Int 3)

Thus, Rowan exemplifies a PST who cares deeply about his students and supporting them, but struggled in how he both planned for and enacted moves to support student errors and uncertainties.

Reed's Planning and Enactment Connections

Reed's planning reflections did not reflect much of her philosophy of helping make mathematics accessible, approachable, and interesting, nor did it mention her other philosophy theme of grades and assessment. Instead, Reed explained how lesson planning broadly "helped me think about questions that I might have to ask students" (Int 3) and one time "really helped me with this because I was expecting it to be very challenging to get the inverse" (Int 5). Her purpose of lesson planning is for her to have a "general overall feel of what the content is and what students might have trouble going through and going 'oh I know where students might go awry'" (Int 3). In reflecting on Teaching 3, Reed shared that "it was very interesting to see a lot of second guessing" (Int 3) which she indicated in a positive way, that students were productively struggling and thinking and that "it was cool to see somebody's certainty turn into uncertainty" (Int 3).

Reed also reported struggling with overestimating and underestimating student thinking during planning. "My anticipated student responses were much different from the actual student responses that I got which is just an experience thing" (Int 5). She shared "at the same time it's kind of hard to know where they're at with the content" (Int 3). "I was expecting a lot of the responses I got in the launch about wages, yet I didn't get a lot of the ideas that I was expecting" (Int 5). Reed expected "there to be more ways to solve it because I saw a lot of people doing different things in the exam review, but I didn't see too many people doing the task in different ways" (Int 5).

Reed described several times how lessons did not go as she expected (Ref 1, Int 3, Int 5). Specifically, Reed shared that planning for when students would experience challenges was difficult. In Teaching 4, Reed "was not anticipating students to either guess and check or use multiplication as division" (Int 5) and in Teaching 3, she "was also surprised that students didn't really know where to start with the triangles" (Int 3). And even when she was aware of the challenge of anticipating student thinking and worked to be more prepared, she shared that she was feeling "more prepared than I actually was because I already thought about certain anticipated student responses and then I got one totally out of left field and didn't even think about" (Int 5). So, even amidst the challenge anticipating student errors and uncertainties and working to plan even more thoroughly, Reed exemplifies that students still came up with thinking that surprised her.

When she thought about, and watched videos of, her lessons, Reed reflected about things she might do things differently in the future. After Teaching 1, she wrote, "this experience encourages me to think closer about my wording and what specific questions do I want to ask where" (Ref 1). After Teaching 2, Reed wrote

> When teaching this lesson again, something that I would change if the opportunity was brought up would be when [student] said that the two graphs looked like inverses. In the moment, I did not want to expand on the idea, just because we got the answer we were hoping for. (Ref 2)

Thus, Reed exemplifies a PST who finds anticipating student thinking challenging, let alone responding to student errors and uncertainties, but is engaged in reflection on her learning journey to strengthen the planning-response connection in future teaching.

Lian's Planning and Enactment Connections

Lian, whose philosophy centered on the importance of connections and helping students make them, considered several key elements in her reflections and interviews. She acknowledged that she was surprised that students were confused about aspects of simplifying algebraic expressions with exponents. This surprise points to the challenges of anticipating student thinking in the design arena and responding to uncertainties in the construction arena. She also addressed the interesting and important issue of planning "for students forgetting what they've learned even if we know exactly about what the course has covered" (Ref 1). This lesson arose in a discussion in a debriefing session with her methods course colleagues and instructors after Teaching 1.

Lian considered several specific elements of language in her planning and enactment that might support students during moments of errors and uncertainties. In Reflection 2, she wrote

> In the Number Talk, we asked “Does everyone agree with that?” a lot at first, then it suddenly came to my mind that maybe this question is too “directing” for the students. So, I added “does anyone have a different idea?” but I think we could do better by asking “how does everyone else think?” which I think was on our TTLP but we didn’t follow it more precisely (like last round, but we are doing a better job this time).

In addition, she noted that she “would prepare more language that can be used for probing questions” (Ref 2). After Teaching 4, she shared that she “was able to ask them questions like ‘Why does A go to H? Or to E?’ from probing questions” (Int 5). She also told me, “Although we didn’t use them, we had some extended questions if the students are finished early” (Int 5).

Lian highlighted a known PST tension of multiple expectations at play for the lesson planning and enacting. She acknowledged the tension between her needs, her methods instructors’ needs, and her mentor’s needs when she reflected on

> …introducing the term “segment.” Here is the line segment. It means this part, everything between these points...and the definitions–maybe do less definitions. I really didn’t find the points of copy down mathematical definitions because the students can just look them up, but Mrs. Milton wants them to keep a journal so they can reference or refer back.

This represents both a philosophical and practical tension between these contexts that play a role in PST experiences. It also illustrates Lian’s views of mathematics and ways for students to interact with it that differ from her mentor’s.

Lian also was challenged to anticipate student thinking. She wrote "students were more knowledgeable in inverse function than I thought" (Ref 2) and reported that she "was surprised that students didn't know how to simplify" (Int 3). In Interview 5, she explained how inaccurate anticipation of student thinking caused her to forget high-level probing questions she had planned. She shared how she was "surprised" that students "knew it's 'all lines through the center'" were lines of symmetry "because I expected to have students say only one of them." Lian explained that she had "prepared the probing questions 'Are there other lines of symmetry? If so, where and why? If not, why not?' But, the students were just able to say 'all lines going through is the line of symmetry.' Pretty interesting." Thus illustrating how challenges in the design arena impact experiences in the construction arena.

In reflection to Teaching 4, Lian described surprise to some student thinking around the "path of rotation." When she asked, "what should the path of the rotation be?" students answered incorrectly and Lian explained that she "didn't think the students would answer that because it is not a path to me [her]...but maybe it makes sense in the students' minds because they know it's from A to E and they just find a path on that graph that's how the A goes to E–go down and go left." So, this showed that she was challenged to empathize with and predict student thinking based on their point of views, knowledge, and understandings. Lian exemplifies how high-level extending and probing moves could be planned for, but the student thinking surprised her or aspects did not go to her plan, she did not enact those "saved" higher-level moves.

Themes Across PSTs

In addition to each PST having a unique pattern in their own connection between planning and responses, they also shared experiences in connections and relationship between planning and enactment. The most prevalent and substantial theme that emerged was that lesson planning, broadly, and anticipating student thinking, more specifically, are difficult. Since PSTs are engaging in one difficult practice–planning for student thinking–it follows that responding to anticipated student thinking–particularly errors and uncertainties–is even more difficult. Carson summarized the experience that all PSTs described, “some of the miscommunications that came up during the lesson were unplanned for. That was something I wasn't expecting” (Int 3) and there are a number of ways that these challenges manifested.

Tension Between High Cognitive Demand and Scaffolding

PSTs described the tension between planning for and enacting tasks that were high in cognitive demand and also what they should do when students made errors or experienced uncertainty. PSTs all indicated their desire to not give answers or “take away the cognitive demand completely from their group by explaining through my questions what they were doing” (Taylor, Int 5). This came up 95 times in the enacted lessons and 13 times in post-lesson interviews. Sometimes PSTs described it broadly, relating to their philosophical stance and others it emerged as a specific response strategy during enacted lessons. Rowan (Int 3) described how he supported a student who was incorrectly computing with mixed numbers as he

> was just trying to scaffold her a little bit with the ‘ice cream cone’ [mental trick for the process of converting mixed numbers to fractions and the reverse] ...I try to avoid telling her it's right or wrong off the bat. I try to get her to question her work so she can do it on her own way to question her own work and see if it makes sense.

As PSTs reflected on the tension between challenging tasks and supporting students, they explained what a delicate and complex balance this is in teaching and learning. “I feel like I led him a little bit more than probably necessary,” Carson explained in Interview 3 and others shared similar sentiments.

Increase Scaffolding. Considering the complexity of this tension between high cognitive demand and supporting students, the ways in which PSTs reflected on how they might address this in future lessons showed both their learning and dedication to their future students. Rowan (Int 5) illustrated this when he named many things that he saw as ways to alter a future lesson about speed. He said he would "make instructions more explicit; provide a table, but I wouldn't put the cells in; get rid of one sheet of the worksheet; have the whole class help me make a model in the beginning just as a standard." And though these statements indicate many changes, the number of changes shows how challenging this is and Rowan's increased awareness in planning and discourse practices. Other PSTs shared related reflections.

Beginning the Lesson. There were many statements about how PSTs might have introduced or begun the lesson differently to address this tension between challenge and scaffolding that often emerged later in the lesson when students were working on their own or in groups.

> I think we should have added a real example of an equation in both forms so that we could actually find all the things we were saying you could find in each form. I think that a concrete example like we did at around 3:30 in the recording is a good example of what I think we should add for the introduction to make it more interactive. (Taylor, Ref 1)

Carson reflected on how this tension may have impacted the explore section of Teaching 4–and potentially the whole trajectory of the lesson–when he described an instructional choice he made. He said,

> I addressed the idea of going from the object to the marker or from the marker to the object. I feel like had I given them an example of what I meant to take a bearing from one object to another, the explore might have gone a bit more smoothly. (Int 5)

Reed (Int 3) described that "when introducing the manipulatives in the future, I [she] would possibly use different instructions to guide students' explorations of these structures" and Rowan reflected that "going over how to formulate the expressions from the graphs in the Number Talk may have helped with the explore task" (Ref 2).

Specific Language. Some reflections involved specific language that PSTs planned to use in future lessons as amendments to their enacted lessons to support students during moments of errors and uncertainties, perhaps in both proactive and reactive ways. Reed (Ref 1) reported

> I explained it as "play around with them and see if you can make a rule" to see in higher patterns how many blocks there will be. I would change this to be something more along the lines of "finding a way of manipulating each of these structures to illustrate the differences in each iteration."

Regarding the same lesson, Lian reflected that she

> should add "it really helps" after "you may rearrange those blocks to another structure which help you count or calculate (the number of blocks)," so they know I'm not just saying that they play with it without getting closer to solving the pattern (Ref 1).

Rowan shared that he "should have made that really explicit, 'take distance from their house'" (Int 5) in response to students being confused about calculating distance and time on a task about two students traveling to schools. Rowan reflected that in future lessons he would

> clarify the first part of the launch activity, which has directions for writing my number as the input and the given number as the output and vice versa. Although the directions also define x and y for us, I feel that it's not clear enough. Next time, I would change the directions to be more simple and clear, such as: Write an equation where x = "my number" is the input and y = "given number" is the output...So, maybe for the second set

> of instructions: Now reversing it, write an equation where x = "given number" is the input and y = "my number" is the output. Note that I explicitly said "reversing" in the beginning, which gives an implication for students to better grasp the idea of inverses. (Ref 1)

Wait Time and Sharing Answers. Two other ideas came up around this idea of reflecting and future lessons plans. Taylor (Int 5) described her thinking of when she wanted to wait, intentionally using the strategy of waiting, instead of correcting a student in Teaching 4. She said, "I think just like being more patient...having my thought of 'no, you have time like take a step back and let them do the work.'" Lian described how she

> would write down the answers, maybe upside down, on the corner of the page of the exam review questions. The answers would help the students check their answers, so they can be more confident for the exam or they can ask more specific questions at the review station. (Ref 2)

These reflections all support for the idea that lesson planning and responding to student errors are challenging, but can become less so with time and practice. PSTs also described what happened for them when they did not accurately anticipate student thinking or plan for responding to errors and uncertainties.

Time Estimation

Another concrete manifestation of the challenges of lesson planning and anticipating student thinking resulted in PSTs struggling to predict how long activities and tasks would take students to complete. Carson articulated this broadly when he wrote "wait time can feel like eternities when only seconds have gone by, whereas when you are doing something it can feel like seconds when minutes have already flown past" (Ref 1). Time–and the challenge of estimating it–played a role for more than just wait time throughout the lessons.

> The first thing we didn't expect is that the Number Talk took a lot longer than we planned for. I kept looking at the clock hoping one of the students could shout out the answer to release me from there. I didn't expect that they didn't have the idea of factored form, because it's written above. (Lian, Ref 1)

When PSTs struggled to know how long things would take, it was likely that they would run out of time, not get to summarizing or other concluding activities to help students make connections and relate content to other ideas. Thus, when they struggled with time management it made it difficult for them to know how long to work on things when their students struggled and resulted in a trickledown effect of them struggling to their lesson goals.

Purposing

For all the reasons described above, another struggle PSTs shared was ensuring the goals or purpose of the lesson were achieved. Although the goal of the lesson was required in all written lesson plans, ensuring that those goals were met was challenging for PSTs. Taylor described this as *purposing*. In explaining that her students struggled in Teaching 4, Taylor said, "from the lack purpose, students were just uncertain about why they were doing this. I think the biggest thing is like the setup of it…it was lack of purposing" (Int 5). In Reflection 1, Taylor wrote that "the students have done Number Talks every day and I think that they could have used a little reminder of the purpose of the Number Talk before we got started." Carson described how the purpose of the lesson was critical to the student thinking and his responses to students. He did this by "keeping in mind 'these are my 3 learning goals. This is what I'm shooting for. This is what I really want to hammer home' and just like keeping that in mind throughout the lesson" (Int 5). Lian said "I don't think we achieved our goal because we stopped at introducing and reminding them of the different forms of quadratics, while our goal was to lead to a deeper understanding of the information each form gives us directly, with practice and reasoning to others" (Ref 1). And Reed highlighted how purposing is related to the tension between cognitive demand and student challenge.

> Our initial goal with the n=5 question was to have students start thinking about how the structure changes with each iteration. However, some students took the question in more of a computational way, which means that we should consider rewording or omitting this question in the future. (Ref 1)

PSTs described purposing as having multiple uses for their lessons. Carson described how he used the lesson's purpose as a way of making sure he was on track in his lesson similar to Putnam's (1987) notion of curriculum scripts. He said, "in my head, having a checkbox of like 'yeah, we get all of our definitions,' 'yes, we've got some examples of where we can use angles'" (Int 5). Carson also described that when he lost sight of "those 2 or 3 goals, I just saw so many opportunities for discussion and kind of didn't know where to go in the summary discussion" (Int 5). In Reflection 2, Reed described a response she and Lian made when a student "stated that the red curve looks like a parabola, we did not expand on the idea … I think in the moment I was trying to stray away from explicit functions and focus on properties." These illustrate the challenge of relying on in-the-moment thinking and how the purpose of the lesson can help PSTs end and wrap up lessons in meaningful ways, thus making sure the lesson meets the learning goals and purpose which are likely more conceptual ideas.

PSTs also described how the purpose of the lesson plans would allow them to realize and capitalize on opportunities. In Reflection 2, Reed described how

> if I were to teach this again, I would try to explore this further and maybe spend less time on how to get the plus/minus square root of x and focus more on what happens when we graph these functions. I would spend less time on the 'figuring out' aspect of the square root functions for the sake of time and instead focus more on what makes a function a function.

Carson (Int 5) described how if he had "relied on my lesson planning more and ended thinking 'oh you know here are my goals,' I [he] would have realized that was an opportunity." And Taylor explained that when she kept the purpose in mind, it helped her prioritize her time, energies, lesson focus, and lesson trajectory.

Reliance on Noticing and In The-Moment Decision Making

With all of this in consideration, when PSTs struggled to anticipate student thinking, they were inevitably surprised with an unanticipated turn of events in their lessons. This surprise caused them to need to rely on their in-the-moment thinking, which–as reported by them–turned out to be less optimal than their planned moves. Rowan described how when a student said something he had not expected "it threw me off because I didn't think about it that way" (Int 5) and Lian wrote how "by that time I didn't think up a good reply for his answer. I just nodded and said 'yea' while I think he just read it from the board" (Ref 2). Taylor described how student thinking played a

> a really, really big role especially because I had a hard time anticipating how students were going to think. So once it [an unexpected student thought] was actually in motion I had a lot like to analyze in the moment. There was literally so much going on in my head during that whole entire time that I was forgetting the things that I even wanted to do

when I wasn't in the room and then adding in all the things that I wanted to do because of things were happening in the room was just like [gesture of poof] Gone! (Int 5)

This feeling of being caught off guard, unsure, and concerned that their facial expressions and body language were sending negative messages to students were prevalent themes for all PSTs.

Enacting Plans: Forgetting and Microadaptations

Another theme that arose was the tension between following lesson plans, forgetting plans, and making changes–microadaptations (Corno & Snow, 1986)–in enacted lessons. Reed wrote about this after Teaching 2. She described the challenge of

> being able to let certain ideas slide or know when to stop pressing and instead explain. There were a few places in the lesson that we seemed to get hung up on a few ideas, which didn't help us achieve our learning goals.

In this reflection, Reed also addresses the challenges and importance of timing, purposing, and in-the-moment decision making. Rowan noted a time he forgot a part of his plan when he did not address "the possible misconception about the notation for inverses being mistaken for being the reciprocal of the function f-1(x) = 1/f(x), which is not true)" (Ref 1). Lian described a microadaptation that she and Reed made in Teaching 2,

> the only one we didn't get to as a whole group was restricting the domain of the original function so that the inverse is a function for the sake of reserving more time for them to review for the upcoming exam by going to the stations. (Ref 2)

In Interview 5, Rowan reported about his use of Teacher Discourse Moves (TDM), "well obviously, then I didn't use them. Here, I tried" which offers a glimpse into his learning process. By Teaching 4, he knew of TDMs and had planned to use them, but did not enact them in the lesson.

PSTs did indicate that they had support in their growth with enacted planned lessons. Taylor described relying on was the rehearsal during a methods course prior to Teaching 1. She described how during the lesson they "tried to mix up the cards, especially after the practice when something similar also happened" (Ref 1) and how she adapted to students, as needed, in response to them. Reed explained that she relied on watching the video of her enacted lesson for reflection and learning. After Teaching 2, Reed wrote that she noticed

> how I explicitly do and don't follow the TTLP and some places where I can improve on my implementation of the 5 practices that we've talked about in [methods] class. One thing that I did notice is that we were able to stay closer to the TTLP planning compared to last lesson [Teaching 1]. (Ref 2)

Taylor articulated one connection between adaptation in lessons and time when she wrote

> I think that we were able to adapt relatively well, though I wish we had more time to really make a good introduction and change our activity a little more, so it didn't feel as much like we just repeated what we saw in the slides in the mini discussion before we started the activity. (Ref 1)

This, again, highlights the interconnectedness of the tension between high cognitive demand and scaffolding, timing, and planning for responding to students when they are uncertain.

Summary of The Relationship Between PST's Planning and Enactment

PSTs struggled to anticipate the student thinking that occurred in the lessons they taught. They over- and under-planned. They expected too much; they expected too little. They were challenged by planning for high cognitive demand and maintaining it. They struggled with time estimation which impacted the ways in which student thinking could play a role in lessons. They struggled to articulate, for themselves and their students, the purpose of the lessons. And when they had to rely on in-the-moment noticing (because of these challenges), it was harder for them.

However, as I conclude this chapter, I highlight the reflective and learning-from-experience mindset all PSTs shared. These emerged in their reflections after Teaching 1 when Carson wrote, “as far as teaching practices go, there are definitely some things to take from this initial Number Talk to develop myself as a teacher” (Ref 1) and Taylor wrote, “I think that that was a section when we were talking too much and that the students should have been more involved in that section” (Ref 1). These ideas remaind through the last interviews when Rowan said, “if I were to continue this lesson, I would’ve brought slope up the next day” (Int 5) and Lian reflected, “it depends on how well I know the students...I’ll introduce points, lines, rays, segments before these geometries–so they have the basic ideas of Euclidean geometry” (Int 5). All five PSTs embraced learning and struggling, reflecting and receiving support, and strengthening both their lesson planning and teaching practices.

Chapter 9:

Discussion and Challenges

The purpose of this study is to examine and better understand how PSTs' lesson planning practices influence their ability to respond to student errors and uncertainties. Data suggested that PSTs did plan for student thinking in their lesson plans, but often struggled to accurately anticipate student thinking while planning. When planning, PSTs relied on their philosophy and ideas about teaching, their knowledge of students and mathematics, their beliefs about learning, and the support of key resources. PSTs drew on support from humans and resources from the university, field placement, and their personal lives. The resources that most support PSTs in their planning for student thinking were field placement humans (e.g., mentor teachers) and university humans (e.g., methods instructors). In this chapter I address the broad challenges and successes PSTs experienced around the practice of anticipating student thinking in lesson planning as well as the challenges and successes PSTs experienced enacting classroom discourse, particularly in responding to students around issues of errors and uncertainties.

Using their assigned lesson planning templates and engaging in pre-teaching interviews, PSTs engaged in copious work to prepare for their enacted lessons. Their planning included decisions about lesson goals, tasks, activities students would engage in, student thinking, and discourse moves they would use. When it came to the planned student thinking, 56% of planned statements were what PSTs wanted students to think or know and 44% of planned student thinking statements were what PSTs expected students to think. During planning, PSTs focused on seven types of thinking (factual, procedural, conceptual, connections, justification, representation, metacognition) and 64% of the planned student thinking was low-level or instrumental in nature.

PSTs taught a range of content using a variety of activity structures; errors and uncertainties played a substantial role in the lessons the PSTs taught. Errors and uncertainties arose at least 326 times across the lessons. Overall, PSTs were most likely to *validate correct responses*, *monitor* students, and *cue* them to focus on a particular aspect of the task or solution. Across all lessons, 50% of PSTs' discourse moves were low level in nature and 20% were high level (30% were context-based). When PSTs responded to errors and uncertainties, 58% of their responses were low level and 19% were high (23% were context-based) as they were most likely to *validate correct responses*, *focus students' attention*, and *provide procedural explanations*. Each PST had their own, unique discourse patterns despite using the same templates to plan and often coteaching.

The relationship between PST's anticipation of student thinking in lesson planning and the role played by errors and uncertainties is complex and challenging. The results suggest that PSTs find lesson planning, anticipating student thinking, and responding to students difficult. This theme came up directly in every interview and indirectly in the ways in which their plans and enacted lessons did not always align. In sharing about their planning and discourse practices, PSTs described challenges in maintaining cognitive demand, time estimation during the lesson, setting a clear purpose of the lesson, noticing, in-the-moment decision making, and forgetting plans and making microadaptations. However, even with these challenges, PSTs demonstrated a clear and consistent commitment to reflecting, learning, and improving their planning and discourse practices.

The Challenges and Successes of Planning

The most prominent theme across the data was anticipating student thinking during lesson planning in the design arena is difficult. PSTs do not know much about students' likely thinking on the topics they are going to teach; they struggle to anticipate student thinking. Specific data came up more than 50 times to provide direct evidence to this theme. All PSTs mentioned something to the effect of "My anticipated student responses were much different from the actual student responses that I got" (Reed, Int 5). Indirect data also supported this main finding when planned and enacted discourse moves did not align and when PSTs were surprised by student thinking in enacted teaching.

The lesson planning templates in this study were based on the TTLP, created by prominent mathematics education scholars, research based, researched, and used in a top teacher education program. The protocol is framed to get PSTs thinking about student thinking, which it did. However, much of their thinking, nearly ⅔ of it, was low level and instrumental in nature, focused on facts and procedures. So, although this template and methods course process produced a large quantity of statements about student thinking, the level of student thinking named was often low.

Several PSTs attributed the difficulty planning to being new in the field and lacking experience. This certainly has value and known differences in novice and experienced teachers have been shown to exist (e.g., B. Sherin & Star, 2011; Star et al., 2011). However, it is dangerous for the field to rely on when PSTs get enough experience to better anticipate student thinking as research has also shown that teacher change is slow and challenged, often with support (e.g., Cohen, 1990; Franke et al., 2009; Putnam, 1992). In addition, I argue that planning for new content, material, lessons, or activities frequently takes educators with any level of experience back to their novice mindset because predicting learners' behavior is difficult. So, ways in which PST lesson planning practices can be supported can help empower an entire career.

PSTs will likely gain insights and information about and improve their anticipating of student thinking throughout their careers with experience and future professional development. However, relying only on time to help early career teachers develop these skills is problematic. It is not helpful for students to only be able to engage in rich discourse practices with relational student thinking from later career teachers. As the PSTs in this study indicated, methods and mentor teachers are the primary resources for them to rely on for anticipating student thinking. If these teacher educators know this, it may provide them opportunities to focus their time and energy on helping PSTs anticipate student thinking and encouraging PSTs to use other resources for other aspects of lesson planning–classroom management, tasks, activities, grouping strategies, etc.

The Challenges and Successes of Enacted Teaching

Building on the argument that lesson planning and anticipating student thinking are difficult, the data also support a second prominent theme: classroom discourse is challenging, particularly responding to errors and uncertainties. When I examined the low-level thinking moves, they made up 41% of planned discourse moves, increased to 50% for all enacted moves, and increased to 58% in response to error and uncertainty. Logically, this increase of low-level moves from enacted to the response moves could be argued by a thought of "my student does not understand, I need to simplify." However, if a goal of productive discourse is to maintain high cognitive demand, this is a challenge and may not be the type of sense making that is ideal.

The challenge of enacted discourse is further complicated by the encouraging and positive results that from enacted to response to error and uncertainty as high-level moves only dropped from 20% to 19%. This provides support that PSTs were using nearly the same frequency of high-level moves after errors and uncertainties arose which is strong evidence to celebrate. Most of the low-level increase came from context-based moves such as *monitoring*, *asking for clarification*, *figuring out reasoning*, and *checking for understanding*. This is logical as PSTs would need, for their own sake and that of their students, to know what students are thinking when errors and uncertainties occur. However, my understanding of the push for productive mathematics classroom discourse is to work on increasing the frequency of high-level moves beyond 20%.

Finally, I wish to highlight a substantial victory in enacted discourse that occurred for these five PSTs in this study. Data from IRE literature suggests that IRE is the most commonly used discourse pattern (Capraro et al., 2010; Garcia, 2015; Kaya et al., 2014; Kutz, 1997; Neal, 2008; Poole, 1990) and may be used upwards of 70% in classroom discourse (Wells, 1999). The PSTs in this study, unequivocally surpassed these milestones in the quality of their enacted discourse. Although I did not track IRE patterns explicitly, from the responding moves of *correcting incorrect responses* and *validating correct responses* (the TMSSR codes for the Evaluation portion of IRE), I estimate that the PSTs in this study used IRE somewhere between 35% and 45% of their overall discourse, approximately half of known literature norms. So, although PSTs were challenged in their classroom discourse practice in many ways, they clearly relied on IRE less than typical classrooms.

Intellectual Empathy

The last theme that emerged was an underlying or foundational construct that was not in my initial conception of this study. I did not ask about it directly or code for it during data analysis, but found that it seemed to be present around most of the challenges PSTs experienced within their planning and discourse practices. My conception of this construct is the intellectual "cousin" of empathy. Empathy is our human ability to take perspective and understand someone else's emotions. I argue there seems to be a construct, similar to empathy, but more the ability to understand someone else's thinking, on an intellectual level while considering their emotional and human status. This idea aligns with the *make sense of* part of noticing and deeply connects to PSTs' MKT (Ball, 1990; Ball et al., 2001, 2008, 2009).

PSTs made reference to this idea of empathy as it relates to the *make sense of* part of noticing 21 times in post-lesson interviews 3 and 5. For example, Lian wanted to "think more careful and think about what they said before I answer" (Int 3) to better understand students' perspectives. Reed reflected, "I get where he's coming from. 'I have a 4x, I need to get rid of 4x, I'm going to subtract 4 from everything'" (Int 3). PSTs relied on their MKT, particularly what they knew had happened previously such as "when [student] said she was going to use [specific strategy], I was like, 'it's probably just because you never had to work with order of operations in this way'" (Taylor, Int 5). Relying on their knowledge of content and curriculum, they said things such as, "I was like, 'Ok so he's doing with replacement and without replacement'" (Carson, Int 3). PSTs showed evidence of using intellectual empathy to help address error and uncertainties "I see where you got the positive from because you're thinking '2 negatives make a positive' so I was trying to directly address that misconception" (Rowan, Int 3).

Using their intellectual empathy was far from a perfect process as Carson described a time when "it's like 'what is he talking about?'" (Int 5). However, if this knowledge functions similarly to empathy, it is a skill that can be learned. It may be a crucial construct to better understand and teach PSTs to support their abilities to plan for and respond to errors and uncertainties in mathematics classrooms.

Challenges

This study supports the known phenomenon that lesson planning and anticipating student thinking are challenging skills, particularly so for PSTs. However, the data suggest that with lesson planning support, such as with high-quality methods instructors and a robust template such as the TTLP, PSTs can learn to engage in discourse patterns that are more productive and robust than the IRE-centric patterns as found in the literature. So, when the field calls for more productive discourse practices in the construction arena, PSTs need continued support from teacher education educators in their lesson planning practices in the design arena.

I wonder if there are additional supports that PSTs need during planning or reflection phases of teaching that could strengthen their planning practices. For example, I imagine a methods course experience based on the idea behind Zazkis, Sinclair, and Liljedahl's (2012) lesson plays. With support of methods instructors, PSTs could take anticipated instances of errors and uncertainties, plan multiple responses, and then consider the type and level of thinking that each of those responses is likely to elicit. In addition, similar to the PSTs in this study, I wonder what power can come from PSTs analyzing their own lesson plans and enacted lessons for levels of student thinking and discourse moves to help them detect patterns and notice alignment and misalignment between teaching philosophy and practice as well as between planning and discourse practices.

Theoretically and practically, I argue that more work is needed to consider the role of intellectual empathy as a way of supporting both lesson planning practices and mathematics classroom discourse practices. I believe this to be a powerful construct and skill that could help better understand and support PSTs do as Lian suggested: change their minds from "we are planning for our lesson" to "we are planning for the students" (Ref 1).

APPENDICES

APPENDIX A: Information Handout Given to All Potential PSTs

Information about Brittany Dillman's Dissertation

My name is Brittany Dillman and I am a doctoral candidate in MSU's Educational Psychology and Educational Technology (EPET) program. For my dissertation, I hope to work with 2-5 preservice teachers, looking at how you plan lessons and respond to students in class. The research will involve me examining your lesson plans and teaching videos from Fall and Spring Semester methods courses and interviewing you a few times next semester.

Saying yes to will mean that I give Brittany permission to:

- Use my completed Fall Semester lesson plans
- Use my completed Fall Semester teaching videos
- Use my required Spring Semester lesson plans
- Video tape my required Spring Semester teachings/lessons and use those videos
- Observe my experiences/interactions during Spring Semester methods class
- Interview me at the beginning of Spring 2018, before & after each of the lessons I teach for spring semester methods course, and at the end of Spring 2018

Questions you might have

- What would I have to do?
    - The only activity outside your required work for Fall Semester and Spring Semester is to allow Brittany to interview you a few times throughout the semester. This will be about 6 hours total.
- Will I be evaluated by Brittany?
    - No. Brittany will be documenting your learning process. She will not be evaluating you.
- Will this compromise the privacy of my students?
    - No. All your work or participation will remain confidential.
- Will this cost me anything?
    - No.
- How much time will this entail?
    - About 6 hours throughout Spring 2018. Expect a 1-hour interview at the beginning and end of the semester and a 30-minute interview before and after each lesson you teach.
- What is Brittany's experience?
    - Brittany taught middle school mathematics for 10 years, taught undergraduate and master's level education classes for 4 ½ years, and 4 ½ years of doctoral study in education at MSU.
- What do I get for doing this?
    - Support in your teaching and practices by providing opportunities for reflection.
    - A chance to advance research in education.
- What do I do if I have more questions? Email Brittany at dillmanb@msu.edu.

APPENDIX B: Consent Forms

Parent & Student Consent Form for Research Participation
(originally on university letterhead)

March 2018

Dear Parent or Guardian,

This semester your child's math teacher intern is working with Brittany Dillman, a doctoral student at Michigan State University, to learn more about ways to improve the dialogue and discourse in mathematics classrooms. This research will not interrupt the normal classroom instruction and will not affect your child's interactions with the teacher intern. Your child will not be asked to participate in any activities that are different than the normal daily activities.

Several of the activities around this study may include the research team video recording your child's teacher intern and classroom. The focus of the video recording will be on your child's teacher intern, but your child might be in the video, too. No personal identifying information will be included in the videos.

We would like to request your consent for your child to be included in the video recording. Our research team will analyze the videos for research purposes. We might talk about this study in classes, meetings, and/or conferences. We might also communicate the results in publications and/or presentations. In these cases, we will always keep your child's information private.

It will not cost your child anything to be in a classroom that is involved in this study. The videos will be used to help improve teaching and learning for your child's teacher and future teachers. All videos will be kept securely by our team at MSU. You reserve the right to withdraw consent at any time.

If you do not provide permission, we will not intentionally videotape your child and will edit any footage containing your child.
If you have any questions or concerns regarding your rights as a study participating, or are dissatisfied at any time with any aspect of this study or concerned about a conflict of interest, you may contact Brittany Dillman, doctoral candidate at Michigan State University (dillmanb@msu.edu or 517-410-2154).

Thank you for your consideration and for filling out the information on the backside of this paper.

Printed Name of Student: ____

Signature of Student: ____

I, (printed name of parent/guardian), ____ am the parent/legal guardian of the child named above. I have received and read your letter regarding the MSU research team in my child's classroom and agree to the following: (Please check ONE blank below)

____________ I DO give permission for my child to appear on a video recording and understand my child's name will not appear in any material written accompanying the recording.

____________ I do NOT give permission for my child to appear on a video recording and understand he/she will be seated outside of the recorded activities or edited from video recordings.

____
Printed Name of Parent/Guardian

_______________________________________ ____________________
Signature of Parent/Guardian Date

We may wish to use video and/or photos with other educators, in presentations and publications. Actual names will NOT be used with the photos of videos.

I give my consent for photos and videos of my child to be used for educational purposes. I understand that real names will NOT be used with the photos and videos.

____
Printed Name of Student

____
Printed Name of Parent/Guardian

______________________________________ ____________________
Signature of Parent/Guardian Date

Preservice Teacher/Participant Consent Form for Research Participation
(originally on university letterhead)

Dear Secondary Mathematics Preservice Teacher,

For my dissertation, I am studying the learning process that preservice teachers experience when they learn to plan mathematics lessons and teach those lessons. I hope you will grant your permission to include your work in my study.

I will be studying lesson plans completed for Fall Semester and to be completed in Spring Semester. I am also interested in the lessons you taught/will teach from these lesson plans (for Fall Semester and Spring Semester). During Spring Semester, I would observe your methods courses and interview you about your learning experiences, lesson planning, and taught lessons.

Participating in this research gives me permission to include your data in this study. Data will include your written lesson plans, my observations/notes and video recordings of your enacted teaching, my observations and notes during your 408 methods course, audio recordings of individual interviews with you, and reflective journals.

This study does not include any evaluative components. Information will not be shared with instructors or colleagues without your consent and knowledge.

Your data and contributions in the study will be communicated without identifying you in any way. Your privacy will be protected. Your confidentiality as a participant in this study will remain secure. I will never use your actual name in any reports. Data collected will be kept in confidence in our secure facility.

Your participation in this research is completely voluntary. You have the right to say no. At any time during the study, you may change your mind and withdraw without giving a reason and with no negative consequences.

You must be at least 18 years old to participate.

Participating in this study will not cost you anything. All materials needed will be provided for you. For participating in this study, you will receive an Amazon gift card at the completion.

I will use what I learn through this research to support teachers and I may share what I am learning in publications and presentations.

If you have any questions or concerns regarding your rights as a study participating, are dissatisfied at any time with any aspect of this study, or are concerned about a potential conflict of interest, you may contact Brittany Dillman, doctoral candidate, at Michigan State University (dillmanb@msu.edu or 517-410-2154).

If you have questions or concerns about your role and rights as a research participant, would like to obtain information or offer input, or would like to register a complaint about this study, you may contact, anonymously if you wish, the Michigan State University's Human Research Protection Program at 517-355-2180, Fax 517-432-4503, or e-mail irb@msu.edu or regular mail at 4000 Collins Rd, Suite 136, Lansing, MI 48910.

I voluntarily agree to participate in this study investigating lesson planning and response to errors and uncertainties in secondary mathematics classrooms. I grant permission for the researchers to use all information collected for research and educational purposes. I understand that all information will remain confidential and that individual identities of participants will not be revealed in any study reports.

____
Printed Name

____
Signature

______________________________ _______________________________
Date Email

In sharing what we are finding from this work or in future work with educators, we may want to use videos and/or photos collected during our study. We may include these in presentations and publications. Actual names will NOT be used with the photos of videos.

I give my consent for photos and videos to be used for educational purposes. I understand that real names will NOT be used with the photos and videos.

____
Printed Name

____
Signature

____
Date

APPENDIX C: Summary of all PSTs, Their Lesson Topics, and Tasks

Table A.1:

Summary of All PSTs, Their Lesson Topics, and Tasks

PST	Teaching 1	Teaching 2	Teaching 3	Teaching 4
Taylor	Topic: Recognizing equivalencies between tables, graphs, and equations	Topic: Exponential functions	Topic: Probability	Topic: Order of operations with coding
	Primary Tasks: NT: How many helmets in next term? Launch: Each student receives either function or graph on an index card. Match cards without showing them.	Primary Tasks: NT: One penny doubled daily for a year vs one million dollars. Launch: match filled tables and graphs with equations. Explore: Worksheet and discussions	Primary Task: Stations with one mandatory (open ended) question and several procedural questions	Primary Tasks: Launch: PST-led discussion about order of operations. Explore: Partner exploration with paper puzzle pieces to teach coding on Scratch for order of operations
Carson	Topic: Inverse functions	Topic: Exponential equations and logarithms	Topic: Probability	Topic: Triangulation
	Primary Tasks: NT: Guess my number (using inverse functions to determine an original number). Launch: More in-depth extension of NT with symbolic	Primary Tasks: NT: Provide four graphs, have students identify one that does not belong. Launch: Explain solving equations with exponentials and logarithms.	Primary Task: Stations with one mandatory (open ended) question and several procedural questions	Primary Tasks: Launch: Whole group "discussion" of where angles exist in life and properties of angles. Explore: Small group task to explore triangulation and compasses

Table A. 1 (cont'd)

	representation and solving.	Explore: Create an equation based on folding a paper in half and answer questions.		
Rowan	Topic: Inverse functions	Topic: Exponential equations and logarithms	Topic: Procedural arithmetic	Topic: Speed
	Primary Tasks: NT: Guess my number (using inverse functions to determine an original number). Launch: More in-depth extension of NT with symbolic representation and solving.	Primary Tasks: NT: Provide four graphs, have students identify one that does not belong. Launch: Explain solving equations with exponentials and logarithms. Explore: Create an equation based on folding a paper in half and answer questions.	Primary Task: Review stations for small groups of students to practice procedural arithmetic	Primary Tasks: Launch: Whole group "discussion" about what students knew about speed and words they associated with speed Explore: A step-by-step guided worksheet to have students experience rate, time, and speed
Reed	Topic: Recognizing equivalencies between tables, graphs, and equations	Topic: Exponential graphs and inverse functions	Topic: Algebra II review for ACT	Topic: Inverse functions
	Primary Tasks: Number Talk (NT): Find a rule to represent number of blocks, 50th term, nth term	Primary Tasks: NT: Put two graphs up (exponential and inverse) and ask students what they notice.	Primary Task: Scavenger hunt	Primary Task: Worksheet with step by step procedural tasks regarding inverse functions

Table A. 1 (cont'd)

	Launch: match different forms of the same equation with graphs of equation	Launch: Properties of inverse functions Explore: Inverse functions worksheet. Stations: review for exam.		
Lian	Topic: Recognizing equivalencies between tables, graphs, and equations	Topic: Exponential graphs and inverse functions	Topic: Exponent rules and simplifying expressions	Topic: Rotational Symmetry
	Primary Tasks: NT: Find a rule to represent number of blocks, 50th term, nth term Launch: match different forms of the same equation with graphs of equation	Primary Tasks: NT: Put two graphs up (exponential and inverse) and ask students what they notice. Launch: Properties of inverse functions Explore: Inverse functions worksheet. Stations: review for exam.	Primary Task: Scavenger hunt	Primary Tasks: Launch: Compare and contrast of a butterfly and pinwheel regarding symmetry Explore: Textbook-based activity (do a, b, c, d, e)

APPENDIX D: Interview Protocols

Introduction: Interview 1

Introduction
- Tell me about yourself
- Background
- Motivations
- Education

Tell me about your teaching
- What is your philosophy?
- What are your goals?

Describe the typical process you go through to prepare for a lesson:
- When?
- Where?
- How long?
- With whom?
- What resources?
- What do you write?
- What do you focus on?

Student Thinking
- What is "student thinking"?
- How do you use student thinking in lesson planning?
- How do you use student thinking in your lessons?

Error/Uncertainty
"A student error is when a student makes public (to the teacher or whole class) something that is incorrect (verbal wrong answer, written procedural mistake, etc)."
- In general, when students make an error in a lesson, what is your reaction?
- When students make an error, what is your response?
- If you consider error on a spectrum from "good" to "bad" – where would you place it?

"Student uncertainty is when a student communicates (makes public) being unsure of content, what to do next, how to proceed, etc. This could be a quizzical look, asking a question, not working, making a statement of "I don't know." This could be initiated by a student or teacher."
- In general, when you notice student uncertainty, what is your reaction?
- When students are uncertain, what is your response?
- If you consider uncertainty on a spectrum from "good" to "bad", where would you put it?

Overall
- Tell me about the connection between lesson planning and your interaction with students in the classes you have taught.

Pre-lesson: Interviews 2 and 4

Context

- Describe your classroom, the norms, and practices that I need to be aware of to understand your teaching.
- What is the lesson you are going to teach? Where does the lesson fit in the unit?
- What are your goals for this lesson?

Planning

- Describe your lesson planning or preparation for this lesson.
- Where/when/how?
- What resources?
- How was this similar to what you typically do for planning?
- How was it different?
- When you were planning this lesson, in what ways were you thinking about your students?
- What else is important for me to know when I observe this lesson?

Post-lesson: Interviews 3 and 5

Student thinking – general

- Overall, how did student thinking play a role in this lesson?
- Was it what you expected? Why? Why not?
- How did you make student thinking public/visible?

Errors

- What errors arose during this lesson?
- How did those errors occur?

Uncertainty

- What uncertainty arose during this lesson?
- How did the uncertainty arise?

Student thinking – specific instance (using stimulated recall)

- What did you notice about the thinking?
- Tell me about your response.
- Likely to be probed with specific questions

Reflections

- If you retaught this lesson to this group of students, what (if anything) would you do differently?
- When you teach this lesson next (next class, next year), what would you do differently?
- How did your preparation/planning help you during class?
- How did your preparation/planning hurt you during class?

End of Semester: Interview 6

Tell me about your teaching.

- What is your philosophy?
- What are your goals?

Planning

- As you move forward in your teaching career (internship), how will you plan/prepare for lessons?
- What resources?
- What will you think about?

Student Thinking

- What is "student thinking"?
- How do you use student thinking in lesson planning?
- How do you use student thinking in your lessons?

Error

"A student error is when a student makes public (to teacher or whole class) something that is incorrect (verbal wrong answer, written procedural mistake, etc)."

- In general, when students make an error in a lesson, what is your reaction?
- When students make an error, what is your response?
- If you consider error on a spectrum from "good" to "bad" – where would you place it?

Uncertainty

"Student uncertainty is when a student communicates (makes public) being unsure of content, what to do next, how to proceed, etc. This could be a quizzical look, asking a question, not working, making a statement of "I don't know." This could be initiated by a student or teacher."

- In general, when you notice student uncertainty, what is your reaction?
- When students are uncertain, what is your response?
- If you consider uncertainty on a spectrum from "good" to "bad", where would you put it?

Overall

- Tell me about the connection between lesson planning and your interaction with students in the classes you have taught.

Changes/Reflection

- Consider yourself at the beginning of Fall Semester through now (end of Spring Semester). How have you grown/changed:
    - In your planning?
    - Anticipating student thinking?
    - Responses to student thinking?

Looking forward, what are you most excited about for your internship?

APPENDIX E: Observation Protocol

Observation protocols will consider:

- What activity and task students were engaged in during the error/uncertainty instance
- Who started the instance
- When the instance occurred
- How long the instance was
- The teachers' response
- The connection of the instance to other instances

Figure A.1:

Observation Notes Document

Lesson Code: ________ Teacher: ____________ Date: _____________Topic: ___________

Instance 1 ______ : _________

Classroom activity:

Start:

Student:

Notes:

Instance 2 ______ : _________

Classroom activity:

Start:

Student:

Notes:

Instance 3 ______ : _________

Classroom activity:

Start:

Student:

Notes:

Instance 4 ______ : _________

Classroom activity:

Start:

Student:

Notes:

Instance 5 ______ : _________

Classroom activity:

Start:

Student:

Notes:

Instance 6 ______ : _________

Classroom activity:

Start:

Student:

Notes:

Instance 7 ______ : _________

Classroom activity:

Start:

Student:

Notes:

Instance 8 ______ : _________

Classroom activity:

Start:

Student:

Notes:

APPENDIX F: Code Book

I created and used the following code book to code interviews, lesson plans, and enacted lessons. The majority of the codes that I used in the final data analysis were those from the TMSSR framework (elicit, respond, facilitate, and evaluate) and monitoring (from TDM).

Table A.2:

Code Book for Reflections, Interviews, and Enacted Lessons

Code	Sub codes	Description and Keywords
Accuracy	general	awareness of accuracy, high, low
Accuracy	process	"she did the process correctly", check their work to make sure they did it correctly
Assessment	summative	summative, grades, pass the class, get credit, get points
Assessment	formative	seeing student thinking, checking in, measure
Attention		focus, multitask, monotask, get distracted, student engagement
Body language		including facial expressions, "surprise" face, confused look
Build relationships		get to know students
Choice		"they can choose 2 or 3", references to students choosing task, activity, order
Challenge		for students, difficulty, ease, cognitive demand of task
Classroom Management		physical proximity, general, on task, off task behaviors, timing
Connect		teacher practice to help students make connections
Control		reference to who has control or power
Creative		comments about students' creativity or curiosity
Effort		effort on the student or PST's part
Emotions	negative	frustration
Emotions	positive	enjoy, joy, interest, positive experience, feel comfortable, when teachers say "I like..."
Empathy		I was trying to figure out what student was thinking

Table A.2 (cont'd)

Error	computation	"students might make an algebra mistake...forgetting to change the sign"
Error	corrected	by student
Error	definition	knowing what a word means/doesn't know, exchanging domain and range
Error	general	general reference to, wrong answers, mistakes, noticeable
Error	negative	errors cause more misconceptions
Error	normal	
Error	occurred	a student made an error in class
Error	positive	errors are part of learning process, to be learned from, help learn other things, prevent future mistakes
Error	process	
Error	public	
Error	range of	different types of errors
Error	unnoticed	error that Brittany notices that PST doesn't notice/respond to
Goal	content	content goal, specific mathematics, mathematical purpose, inclusion of specific CCSSM standards in lesson plan, understand, know
Goal	general	overall, broad, purpose
Goal	process	what students will do, compare
Goal	social	affective, mindset, fun, social, groups
Group	community	with others, rely on, with math, participate, build a community of learners, learn from others
Group	group worthy tasks	specific roles each student would have, multiple entry points
Group	grouping	decisions about how students will be grouped
Inclusion		Accessible (make mathematics accessible)
Knowledge of content		references to examples of mathematics, specific answers, patterns or ideas
Knowledge of self	identity	style, personality, teaching style, strengths

Table A.2 (cont'd)

Knowledge of self	learn	the learning experience of teachers, reflection, improvement, newness to something, metacognition, reading, references to what PST is learning
Knowledge of students	identity	Most general: not directly connected to academic, things like: background, unique, culture, language, ELL, learning style, status, age, needs, marginalized, motivation (high/low), competence, confidence, self-esteem, self-efficacy, challenges, barriers
Knowledge of students	mathematical	Middle: academic/math, what students know, general/generic knowledge, strengths, challenges, what students should know, what students DO know
Knowledge of students	prior	Most specific: knowledge, lesson planning (previous grade, prior lesson), experience, activities, specific references to prior knowledge, what they should know
Language	discussion	class discussion, students discussing with each other, "I would lead a discussion"
Language	general	general references to language, key words, vocabulary, terminology, "I'm thinking right now, I'm thinking a lot about the language and discourse we use with the students"
Language	question	general references to questions, "I would ask a question"
Language	say	what teacher will say
Lesson plan	activity	reference to what students will do or what teacher will do...an planned behavior/activity
Lesson plan	adapted	microadaptations, deviate from lesson plan, changes in lesson plan path, off track lesson (?), or changed lesson plans from those found/used by others (e.g. I found a lesson plan online, but I tweaked it to meet my lesson's needs)
Lesson plan	backward design	when end goals are used to drive planning
Lesson plan	challenge	descriptions of how difficult it is to plan lessons, challenge, iterations, revisions
Lesson plan	follow	when plan was implemented/followed
Lesson plan	forget	PST/teacher, forgetting to enact parts of lesson plan,
Lesson plan	future	next grade, next unit
Lesson plan	multiple	reference to multiple tasks, strategies, solutions
Lesson plan	organize	lesson planning helped organize thinking, time, activities, order of lesson or activities

Table A.2 (cont'd)

Lesson plan	process	descriptions of the process, steps, decisions, questions that PSTs use to prepare for teaching. For example "I feel like most of my lessons start around my goals and then I work towards what should the task be reflected around those goals."
Lesson plan	rely on	relied on planned choices/decisions made before class
Lesson plan	structure	references to structure, requirements of lesson plan
Lesson plan	unit	references to how lesson fits into unit, mini unit, what students are expected to learn from a given unit
Lesson plan	yearly	Yearly lesson planning, reference to curricular understanding of what topics are introduced at what levels
Lesson Plan	TTLP	whole or parts, reference to launch, explore or summary (as made in conjunction with lesson planning)
Mindset	fixed	when things get hard students shut down, students/PSTs want to look good for grades
Mindset	growth	grow through challenge, persevere
Monitor	0.0 Monitor	walk and watch, observe.
Norms		establish, use, other classes, prior classes, school is community of learners, specific utterances of norms, general reference to norms, common language
Notice	observe	
Notice	make sense of	
Notice	act on	
Positionality		"How the students place themselves, rank each other," positioning
Resource	assessment	use of test or quiz
Resource	calculator	use of calculator, no calculator?
Resource	Colleague–fellow teacher	overall, broad, purpose
Resource	colleague–peer	When PSTs mention using classmates or other PSTs as resources
Resource	community	reference to the "mathematics community" as a whole
Resource	curriculum	

Table A.2 (cont'd)

Resource	digital–general	reference to Google or Twitter, not specific web places
Resource	digital–specific	name of website, app, blog, or program
Resource	friends	when PSTs talk about friends, partner, boyfriends, fiancee, roommate as resources
Resource	mentor	in school district, prior mentor (used as resource)
Resource	methods instructors	university instructors and graduate TAs who teach methods courses
Resource	pencil + paper	reference to using pencil and paper for lesson planning
Resource	prior	prior experience, prior lessons as reference, resource for lesson planning
Resource	professor	university teacher other than methods instructor, as resource for lesson planning, teaching
Resource	rehearsal	practicing part(s) of lesson, getting feedback from peers and instructor, as lesson planning resource
Resource	self	"I made the pieces myself"
Resource	standards	CCSSM, standards
Resource	teachers' textbook	teacher version of textbook (with answers and resources)
Resource	textbook	books used by students
Resource	template	when structured template is referenced as part of lesson planning process or assignment for class
Resource	visual aid	hands on activities, cards, puzzle pieces, drawings, manipulatives, models, illustrations, graph paper
Resource	whiteboard	whiteboard, chalk board
Resource	Word	reference to Microsoft Word as a resource
Resource	worksheet	reference to a worksheet (as a resource, or in class)
Scaffold		when PST does things to help students
Select		from 5 practices, things related to who presents
Sequence		references to the sequence or order of task, parts of lesson
Speed		references to how fast can a student do something (or slow)
Student Thinking	anticipate– challenge of	difficulty of anticipating student thinking

Table A.2 (cont'd)

Student Thinking	anticipate–general	anticipate response, anticipate approach, predict student thinking, what students will say
Student Thinking	anticipate–not	unexpected things students said, surprised, things students did/said were different than what was planned
Student Thinking	anticipate–overestimate	thinking that students can do more than they show they can
Student Thinking	anticipate–underestimate	Thinking that students can do less than they show they can
Student Thinking	approach	how students might/will start a task
Student Thinking	challenge	Misconceptions, mistake, common wrong answers, misunderstanding, ideas that students hold that are incorrect, difficulties, challenges
Student Thinking	communications	explain, talk, hear, say, explanation, heard, interact, response, translate?, write, record answers, translate to others, student asks a question
Student Thinking	connections	to self, to math, to context, relationships, similarities between things, similarities, translate (for self)
Student Thinking	elevate	mentioning the idea of students thinking more deeply
Student Thinking	general	general (unspecific) references to "student thinking", thoughts (this is the default if I can't tell anything more specific)
Student Thinking	justification	public sharing of rationale, backing up, supporting
Student Thinking	metacognition	students will think about their own thinking, teachers encourage
Student Thinking	problem solving	think they can solve, actually solve, use a technique
Student Thinking	representation	models, image, illustration, mental image, what things look like, visualize, manipulative use
Student Thinking	solutions	answers, idea of only knowing what students are thinking by their product of thinking
Student Thinking	strategy	references to strategies students might use, process, steps, procedures

Table A.2 (cont'd)

Student Thinking	understanding	references to understanding, conceptualize, high level thinking, definitions, differences, interpret, original thinking, digest, "students will 'see' that..."
Student Thinking	variety	so much, so many things, references to the broad amount of possible answers/responses/strategies
Task		mathematical task that students will engage in
TMSSR: 1Elicit	1.0 TDM invite	Broader use of "invite" to show work, progress, thinking, etc
TMSSR: 1Elicit	1.1 elicit answer	Elicit or Invite student thinking/ideas: Elicit answers
TMSSR: 1Elicit	1.2 elicit facts/procedures	Elicit or Invite student thinking/ideas: Elicit facts, procedures, Talk them through the steps/process
TMSSR: 1Elicit	1.3 ask for clarification	Teacher asks question to clarify student thinking because teacher genuinely doesn't understand
TMSSR: 1Elicit	1.4 figure out student reasoning	Elicit or Invite student thinking/ideas: Probing/figure out/understand student reasoning
TMSSR: 1Elicit	1.5 check for understanding	Teacher asks question to assess students' understanding
TMSSR: 1Elicit	1.6 elicit ideas	Teacher asks question geared at eliciting students' idea for a solution strategy
TMSSR: 1Elicit	1.7 elicit understanding	Teacher asks questions geared at how students understand and how they are justifying, connections
TMSSR: 1Elicit	1.8 pressing for explanation	Teacher asks students to elaborate on thinking, explain their thinking, or reflect and share their explanations
TMSSR: 2Respond	2.0 improvised	when PST has not planned for student utterance, actual response (not anticipated)
TMSSR: 2Respond	2.0 neutral	normal, not surprise, fluid, PST responds to student in neutral way despite (maybe) being surprised
TMSSR: 2Respond	2.0 planned	when PST has planned for student utterance
TMSSR: 2Respond	2.0 same	the idea/desire to respond similarly to students despite the accuracy of their responses
TMSSR: 2Respond	2.0 no action	PST did not follow up with, ignored, dismissed
TMSSR: 2Respond	2.0 wait	when PST pauses (intentional/accidental) before/after question/answer. May include TDM Waiting/wait time

Table A.2 (cont'd)

TMSSR: 2Respond	2.1 validates	when teacher brings notice/value to the student's correct response
TMSSR: 2Respond	2.2 revoicing	teacher repeats student ideas to make them public
TMSSR: 2Respond	2.3 encouraging a student to revoice	teacher encourages student to revoice other student ideas or solutions
TMSSR: 2Respond	2.4 corrects	when student is incorrect or when PST tells/gives answer
TMSSR: 2Respond	2.5 re-presenting	form of revoicing. Teacher provides own interpretation as a way to publically share students work, idea, or strategy. Teacher may organize, reframe, or formalize the student's statement or work.
TMSSR: 2Respond	2.6 prompting error correction	rather than correcting the student, the teacher prompts the student to address an error herself.
TMSSR: 2Respond	2.7 other	some response that doesn't fit in the other categories.
TMSSR: 3Facilitate	3.1 cueing	teacher cues students so they focus on certain aspects of problem, task, idea, or solution
TMSSR: 3Facilitate	3.2 topaze	teacher breaks task into smaller parts, reducing the complexity of the task by asking easier and easier questions, therefore reducing students' opportunities to engage in authentic problem solving tasks
TMSSR: 3Facilitate	3.3 funnel	teacher asks questions that move students down a specific path (leading questions)
TMSSR: 3Facilitate	3.4 explain procedure	teacher provides procedural explanation for how to solve a problem. This may include telling students a priori how to solve or by outlining the solution structure.
TMSSR: 3Facilitate	3.5 guidance	teacher provides hints, ideas, or potential strategy or another type of conceptual scaffolding of the problem, WITHOUT outlining the solution structure or otherwise shutting down students' opportunity to think on their own. "I don't give them the answer." Teacher giving sample/example question that's related.
TMSSR: 3Facilitate	3.6 build on	Teacher builds on students' earlier contributions to support new understanding, or encourages students' to build on one another's contributions. Or TDM Engage with others' reasoning
TMSSR: 3Facilitate	3.7 explain conceptually	Teacher provides an explanation that has a conceptual basis, often focused on explaining why something works. This move can also be thought of as demonstrating logic.

Table A.2 (cont'd)

TMSSR: 3Facilitate	3.8 summary explanation	Teacher summarizes for the class final thoughts of the task or problem, or summary about information or discussion of a task.
TMSSR: 3Facilitate	3.9 providing information	The teacher provides new information relevant to doing mathematics generally rather than informing about a specific problem or task. Teacher brings up counterexample.
TMSSR: 3Facilitate	3.10 multiple strategies	Teacher encourages a proliferation of solution strategies, including pressing students to come up with a different way for students to solve a problem.
TMSSR: 3Facilitate	3.11 providing alternative strategy	Teacher initiates a new or different way of solving a problem after students have shared their solution strategies or solutions.
TMSSR: 4Extend	4.0 TDM probing	
TMSSR: 4Extend	4.1 pressing for precision	Teacher encourages students to provide exact rather than vague answer, to check work for accuracy, or to quantify a qualitative statement.
TMSSR: 4Extend	4.2 topaze for justification	Teacher initially pushes for justification, but then immediately downgrades her to a less sophisticated why by heavily leading students into justification via easier questions.
TMSSR: 4Extend	4.3 encouraging reasoning	Teacher encourages students to think about the task conceptually, for instance why an strategy makes sense by thinking about where numbers connect to the quantitative situation, etc.
TMSSR: 4Extend	4.4 encouraging reflection	Teacher asks students to reflect on provided answers or explanations (either from teacher or another student).
TMSSR: 4Extend	4.5 pressing for justification	Teachers ask students to explain why something works or to justify (logically, conceptually) their idea, strategy or solution.
TMSSR: 4Extend	4.6 pushing for generalization	Teacher encourages students to generalize their thinking, either through formulating a rule, describing a process in general terms, or making connections across problems, numbers, cases or events.
Transfer knowledge		apply knowledge in different settings, applications, real life
Transmit		idea of giving knowledge

Table A.2 (cont'd)

Uncertainty	different types	
Uncertainty	negative	references to uncertainty being problematic, negative
Uncertainty	normal	uncertainty is between bad and good, neutral, part of learning process
Uncertainty	occurred	an instance of uncertainty occurred in a lesson
Uncertainty	positive	references to uncertainty being a good thing
Uncertainty	public	
View of math	negative	mathematics is hard, rigid, not useful
View of math	positive	mathematics is important part of life, fun, interesting, flexible)

APPENDIX G: Introducing the PSTs With Extended Detail

Introducing Taylor

Taylor is a white female from a large urban area and school district. She has academic interests in both mathematics and computer science. She was attentive to the emotional and social needs of her students. Taylor self-reported that she is "very emotional" (Int 1) and is the only participant who mentioned the social/emotional component of teaching. There were many times where I got glimpses that Taylor is hard on herself. Her high standards show through in the dedication she has to learning and improving, yet those emerged as self-critical several times. In her student teaching internship, she is most looking forward to starting the year from day one and, in general, having learning experiences and opportunities that she has not, yet, had.

Taylor's Teaching Philosophy

Taylor's teaching philosophy centers around the idea of guiding students through–not telling them answers–big ideas of mathematics and learning. She emphasized the idea of guidance, not forcing and directing. She focused in both interviews on the idea of the big ideas of math, not memorizing specific formulas. She situated this in the greater idea that the idea of learning how to learn is central and can be applied in all content areas, in particular making better people and citizens.

Taylor's Beliefs About Student Thinking

Taylor believes that student thinking is how students interpret, interact with, digest, and process material. Student thinking is not always visible and can be revealed through probing questions, then it can be seen and heard. She asserted that "it's very hard without having training on how to get the students to say what they're thinking" (Int 1). She relies on the discourse technique of mirroring so she does not misinterpret student thinking. Her goal in using student thinking is to help students make connections in their way, not her way as the teacher.

Taylor's Beliefs About Errors

Taylor believes that errors are good and "fruitful" (Int 1). She explained that errors were the root of inventions such as Velcro and silly putty. She explained that errors can help people continue down interesting, even if not the intended, path. In classroom settings, she argued that one student's public error is likely an error that others would make, so that it is beneficial to the entire class for errors to be shared and learned from.

When errors arise, she tries to figure out where the errors are coming from, having students explain their thinking so she can understand what is happening. She sometimes addresses errors immediately, but if she is planning on the issue coming up later in a lesson, she may wait to address it. This comes with the potential pitfall of forgetting to do so, which happened in Teaching 4 for her. The outcome of her practices is often students finding the errors themselves.

Taylor's Beliefs About Uncertainty

In January, Taylor reported that uncertainty was more problematic than mistakes. She explained this was because students need to be confident to make mistakes and uncertainty is the opposite of confidence. In May, Taylor reported that vocalized uncertainty was good. She argued that if a student is struggling, they are likely not alone. By sharing publicly, it provides students the opportunity to correct the mathematics and establishes a classroom norm that "it's okay to be wrong or uncertain" (Int 6).

To support students when they are having uncertainty, Taylor described encouraging students to start or try something (Int 1). She explained that many students have been "hit by math" and have internalized the idea that they are likely to be wrong so they do not want to try and may have a fixed mindset (Int 1). She also explained that in trying something, students can often get some credit or points. In May, she discussed how she may address uncertainty immediately or choose to address it later in the lesson depending on if she is planning on addressing it later or not.

Taylor's Lesson Planning Practices

Taylor described her lesson planning, as a straightforward, linear progression (i.e. Tyler, 1950) (Int 1). With her assigned teaching partner, she would get her topic, ask what she already knew, determine an engaging task to help students to connect to the topic, and then consider the student responses she was likely to encounter. She highlighted differences between coplanning compared to solo planning (for Teaching 4), but described how she still used other people in the process–her peers "who know the same things I [she] do[es]" (Int 6).

In May, she emphasized the importance of picking a topic or lessons to allow students to have multiple points of entry. She focused on the importance of what students say as well as coming up with responses and different ways of thinking about the task to help students. Being prepared to make changes–microadapations (Corno & Snow, 1986)–when students were struggling, not understanding the lesson, was prevalent in her May response and she also described the importance of "coming up with ways to assess their learning as well" (Int 6).

Introducing Carson

Carson is a white male who grew up in a rural town and school district. Carson has tutored for several years as part of his undergraduate work at the university he attends. He has a strong history in scouting and is an Eagle Scout (Int 1). He was able to infuse this interest and experience in Teaching 4. Carson is quite nuanced in his thinking–life is about shades of grey, not black or white. His answers to many of my questions were incredibly specific and context-driven. Overall, he believes that what is true in one situation does not necessarily hold for all situations. He talked about many types of student errors and the differences he would use with public compared to private student uncertainty. No other PST mentioned these things. Carson's knowledge of pedagogy and educational research was evidenced by use of phrases such as IRE (Int 4, 6) and self-efficacy (Int 1, 6) during our interviews. No other PSTs mentioned either of these in this way. Additionally, Carson has a subtle and intellectual sense of humor.

Carson's Teaching Philosophy

Carson's teaching philosophy stems from his belief that mathematics has many approaches and many ways to be useful because mathematics is a broad subject. His goals are to be adaptive and individualize the learning experiences of students by getting to know them as learners to help them in the ways that make most sense to them. This is an iterative process to best figure out how to help students by getting to know them so they can best relate to mathematics.

Carson's Beliefs About Student Thinking

When describing what student thinking is, Carson emphasized that student thinking comes in a variety of forms. He described it as "what goes through students' heads" (Int 1) and how prior experiences, background, and identity influence student thinking. Student thinking includes the ways students process, strategies, and ways they relate to mathematics content. Aligned with his teaching philosophy, he noted that student thinking "becomes a bit more evident as you get to know the students" (Int 6) and that as you understand the thinking, it explains the way they relate to mathematics.

Carson's Beliefs About Errors

Carson characterized errors as "misconceptions" (which he later referred to as a "lack of understanding") or "algebra mistake" (e.g. "dropping a sign") (Int 1). He explained that algebra mistakes are more minor than misconceptions because internalized misconceptions will lead to more errors in the future. Carson is more concerned with long-term impacts of students' errors compared to them arising in the present. In May, Carson explained more that errors are not necessarily good or bad, but it is how teachers respond and what learning happens as a result of the error is the important part. For example, if an error being shared allows for students to examine a component of mathematics and determine accurate information from it, that is a valuable experience. If the sharing of an error leads or causes other students to have misconceptions, then errors are problematic.

In January, Carson reported that when he encounters student errors when he is teaching, he records them and then asks for other possible answers and then guides students through the problem or task. In this process, he finds that students often realize the error when going through the task. In May, he reported that he wished he tried to understand students' thinking so he could correct it, but he said he realized he actually "spoon feeds" students counterexamples that go against the given error. He would prefer to ask why students think the way they do first or have students talking with others. He identified his use of IRE and the pattern of keeping things teacher-centric. He said "I think a lot of that's just like going to be practice for me, recognizing when a student presents that information to me and switching to a different discourse move other than just responding to that" (Int 6).

Carson's Beliefs About Uncertainty

In January, Carson attributed uncertainty to a lack of either "confidence" or "student self-efficacy." By confidence, he explained a typical student who frequently requests affirmation, but is almost always correct. By self-efficacy, he referred to a student who might say "I have no idea what's going on. I am really unsure what's going on in this class right now and I'm kinda lost." He explained that this "efficacy" problem is a little worse than the confidence problem and also worse than a gap in knowledge. In May, Carson described uncertainty as a natural reaction to learning because when things are unfamiliar, it is normal to be uncertain. In particular, he described that this was likely to happen with mathematics, as mathematics is often perceived as a "big scary subject" (Int 6). Carson talked about the potential for a student to grow from an experience of uncertainty. He described this in a growth mindset type way of thinking, but did not use the phrase.

His response to uncertainty, similar to errors, is context-dependent. He presented the following categorization of uncertainty. For students with "confidence" issues, his response is to make sure the student knows the procedure, likely by explaining it himself to the student, and then asking if the student is "comfortable" (Int 1). For students with self-efficacy challenges, he talked about the difficulty of building their confidence up (Int 1). If students are in individual work–"private uncertainty" (Int 6) which Carson said happens "so often"–he might ask "how are things going?" "Is this all making sense?" The student would respond "you know, I'm really having an issue with this" and Carson would work with them on that. In this case, his goal would be to bolster their mathematical thinking and confidence by assigning competence. Another possibility is when students say "yeah, it's going fine" and Carson can clearly see it is not going fine (Int 1). Then, he would ask the student "okay, what are you doing?" and ask them for details of their problem solving methods to see if there is a student error or it is actually going fine and the student is just bored or otherwise disengaged. Carson explained that public uncertainty is when Carson is talking to the class and a student raises their hand and says something such as "I'm not sure about this spot" or "I'm not sure how to go from here" (Int 1). In that case, likely in the middle of a lesson, Carson would reassure the student that it is okay not to know and encourage the student to work through it with him, using phrases such as “let's figure that out” and “this is a learning process” (Int 1).

Carson’s Lesson Planning Practices

Carson’s lesson planning is dependent on context. He described that it depends on the lessons, the students, the content. This context drives his decision for resources, too. He described his lesson planning practice as a “web” or “tree” that starts with his learning goals (Int 1). Activities or problems follow and he works through them in five or six ways to help decide what to include based on what criteria and situations he wants to show students. He considers possible misconceptions and responses to misconceptions. His web/tree analogy explains how he has multiple routes (my word) for different scenarios and if he runs into something that is not making sense for students, he “might backtrack a branch and then try going down a different branch” (Int 1).

In May, Carson focused heavily on the idea of unit planning as a driving force for his lesson planning. The unit planning he referred to showed a deeper understanding of planning for single lessons in conjunction with the topics before and after and the connection between those topics. He heavily described and connected to the backward lesson planning model (Wiggins & McTighe, 1998, 2005) and explained how he would start with learning goals and his exam (assessment) and the standards to see how those all fit together. His description of lesson planning placed great importance on students' prior knowledge (Int 6). His goals are to help students reach the highest level: conceptual thinking. He also had some push back and criticism of trying to plan for multiple ways students can solve tasks. He explained that he can be "expecting x, y, and z to happen and x and y happen, but maybe not z" so he thinks "okay, well I prepared for this. This is how I am going to respond to that" (Int 6). He continued to explain "then you get that thing where h happens and then you're like, 'shit!'" (Int 6). So, he decided it is more effective for him to "'focus more on what my students already know rather than what my students will probably do" (Int 6).

Introducing Rowan

Rowan is a white male who moved around a lot as a child. Rowan was inspired by an "awesome" 6th grade teacher. Rowan was heavily concerned about honoring his students' backgrounds and "building competence to build confidence." In his teaching, he tries to guide rather than tell students answers, and he thinks that probing questions are "the ultimate tool for teachers." It was sometimes difficult to follow Rowan's train of thought and explanations, he tended to get himself sidetracked. His responses were often amongst the shortest, but I often had a hard time following his story. Rowan is hearing impaired and wears a cochlear implant. He did not bring this up or talk about it, but it significantly impacts his cadence and pronunciation. There were a number of times during his teaching when he did not hear what students were saying or when students were trying to get his attention.

Rowan's Teaching Philosophy

Rowan's teaching philosophy is centered on building relationships with students to establish respect (Int 6) and making mathematics accessible to all (Int 1). Rowan explained that part of relationship building was "acknowledging" students' backgrounds and "learning patterns" because those are all unique (Int 6) and it is teachers' jobs to include everyone. In January, Rowan mentioned the importance of inspiring students to stay in school and "not go other worse possible avenues." I think this stemmed from his early experiences in his spring placement. This placement was in a large, urban secondary school with a large number of students with incredibly low family incomes (Ms. Lansford, personal communication).

Rowan's Beliefs About Student Thinking

When talking about student thinking, Rowan mentioned students' prior knowledge, their interaction with other students and their teacher, how students conceptualize, process, approach, think they can solve, actually solve, and their work. He explained that this is primarily for "in the classroom because their thinking might be different in a classroom versus at home." He referenced "blank slate" and that people have biases, prejudices, and experiences (Int 1). He described the difficulty with getting students to explain their reasoning and how important it is for teachers to push for that so that teachers know why students do things (Int 6).

Rowan's Beliefs About Errors

For Rowan, learning from mistakes is how we learn and errors are good. Errors are part of the learning process; "We make errors when we learn, and we just learn from" them (Int 6). The only time he would not consider errors positive is if students "can't come up with an explanation for their why" (Int 1). When students make mistakes, Rowan was adamant to not make the experience negative and to respond in ways that are consistent despite the accuracy of the students' responses. His responses are designed to not embarrass students or take away from their status by helping build competence (Int 6). A norm he hopes to establish is to "always push them further, even if their answer is correct" but "the goal is to get the student to see their mistake on their own...because if you can find your own mistakes you do learn more from it because you actually see it and you know not to do that next time. Instead of me pointing it out…it doesn't stick as well as that” (Int 6).

Rowan’s Beliefs About Uncertainty

In Rowan’s eyes, uncertainty is also positive and part of the learning process. Uncertainty indicates students are “trying to understand new subjects and they're coming to a road block...trying to find a way around it. That's how we navigate mathematics working things around and rearrange stuff” (Int 1). In May, Rowan described two types of uncertainty: a) questioning and asking “why does this work?” and b) “I just don't know what to do. I don't understand any of this at all.” The former is good–students are taking initiative and curious. The latter is an indicator of students giving up and “It's really hard to come back from that” (Int 1).

Rowan’s response to student uncertainty is to ask questions and go back through the process which is more important than the answer. “I usually won't tell them if their answer is right or wrong. I'll try to say, "doesn't your answer make sense?" How can we check your answer?" without telling them the answer is right or wrong” (Int 1). In May, Rowan’s response was “I always go back to probing questions. It's the ultimate tool.” His probing questions seemed to be intended to help students communicate their process, emphasizing how they got started. He talked about the importance of their peers who are likely able to help and his desire to have a classroom where the norm is to help others with learning. Rowan believes uncertainty and error are highly contextual, based on the situation and student, and “there's no one right super solution to it” (Int 6).

Rowan’s Lesson Planning Practices

In January, Rowan reported that his lesson planning process stemmed from the main concept and then he created a task with “lots of approaches to it, so it can be done many different ways. It could possibly have different answers, just depending on the argument.” He described the TTLP as a document that you can fill out or write. “It's about anticipating students’ answers and what processes they might use. That will help when you're writing a lesson because you can try some multiple (strategies), not just one. You want to try to get the ones that pop up to sequence it in class discussion, so it makes more sense to the students. Sometimes a certain sequence can help students' understandings” (Int 1).

He mentioned many resources–mathematics teachers, peers, mentor teachers, and the internet–that support his lesson planning practices (Int 1). In May, Rowan described the role that his future internship mentor teacher will likely have on his planning and he would likely use the same template his mentor used, but “the point is the more you do it becomes more automatic. You don't need to write everything down.” He cited the at a glance (this is part of the lesson planning process that Rowan has experienced in his two-course methods instruction) which was most useful for him in lesson planning (Int 6). In May, he also talked about using reliable sources and blogs. This seemed to come from a class discussion about this topic. Overall, his lesson planning practice was influenced by the TTLP.

Introducing Reed

Reed is a white female who is an avid animal lover. She has several unusual pets, including a pig. Her love of mathematics comes through in her description of teaching and the care she has for students. Reed came from a lower middle-class family and a large school system. Her passion for teaching comes from her mother who works in higher education. Reed is passionate about mathematics not necessarily because she thought she was “the best at it,” but because it was something that she “actually struggled with” (Int 1). As a person, Reed gravitates “toward things that are harder to get” (Int 1) She’s “grown to love the general feel of mathematics–how it relates to everything, the abstract and logical nature of it–that you didn't necessarily see back in high school" (Int 1).

Reed's Teaching Philosophy

Reed's teaching philosophy was not clearly articulated and seemed to draw from perspectives that are at odds to each other. Reed reported that her teaching philosophy centers on making mathematics "accessible" (Int 1), approachable (Int 1), and interesting to students who are marginalized from the mathematics community such as "minority students, students whom English is not their first language, and women" (Int 6). In January, she named Culturally Relevant Pedagogy and in May, she named Teaching for Social Justice as movements and frameworks that influence and drive her. Because of her financial background and drive for challenge, she connects with struggling through something (e.g. low family income, difficulty in mathematics). Reed also described elements of her teaching philosophy that focused on grades and college. In Interview 1, she explained

I want to try to make mathematics at least fun to learn. I don't necessarily want it to be everyone's favorite subject, but I want everyone to be able to pass the class–I'm not talking A, I'm talking C. I want all students to be able to get at least a C in a class, but also enjoy being there even if they don't enjoy the math. I kind of want to learn how to do that

In May, she also described the importance of students having "the tools to attend college and be mathematically literate citizens, in their lives, which can help even if you don't go to college."

In my observation of Teachings 3 and 4 and those interviews, Reed's embodiment of her philosophy seemed to mirror the multiple perspectives of her philosophy. Sometimes, she showed great empathy and understanding of her spring field placement students–many worked jobs, including the night shift, were tired at school, slept through class, Emergent Bilingual students. However, she also talked about what students ‘know’ compared to what students ‘should know”–the idea that students “should” have learned particular information, but had not. This difference was articulated with little empathy and consideration of the students’ contexts. I am not sure if these two aspects of her philosophy are indicative of a changing mind or the tension between ideas she holds and what she thinks people want to hear.

Reed’s Beliefs About Student Thinking

For Reed, student thinking is “any thought behind a process of solving something, answering a certain way” (Int 1, 6), misconceptions, preconceived notions (Int 6). She included the idea that student thinking does not need to be logical–meaning that even if students give wrong answers that they cannot explain, she would still consider that student thinking. She also includes process and steps in her inclusion of student thinking (Int 1, 6)

Reed’s Beliefs About Errors

In January, Reed said that errors are good because if “you pay attention to the fact that you made a mistake, you're probably not going to make it again.” She went on to apply that idea to assessment:

> If you never make a mistake on any of the homework or any of the quizzes, when you get to the exam, it's completely possible that you will make a mistake that someone else has made before. Only you don't know that's a mistake, you won't notice, so somebody else who's maybe got more wrong on the homework or the quizzes will do better on the exam because they've made those mistakes in the past.

She said something almost the same in May when she explained that errors are between good and neutral.

She believes that students view errors as negative, despite the best intentions of teachers to make classroom norms to the contrary and that this is likely from the messages that students have received in previous classes as much as what is happening in the current class (Int 1, 6). As a student herself, when she makes a mistake in mathematics class she feels as if she is the "stupidest person in the world" and tells herself "I don't know math!" (Int 1).

In May, she had a more robust answer from the teacher's point of view of errors. She explained how, on one hand, errors reveal student thinking. "If a student makes an error where they're solving linear equations and they always subtract to move a variable to the other side, then that means that they memorized a pattern that they saw because they probably just saw a bunch of "x + 7" or "x + 8." They did not actually know why we move that over–that subtraction is actually going, "OK, I can subtract x from both sides and when I do that, this cancels out because x - x = 0." On the other hand, she explained that errors can take lessons "off track" and the challenge they present the teacher about helping students understand mathematical ideas that they are supposed to already know.

When errors arise, Reed asks students how they got the answer to better understand their thinking. When students explain their thinking, they often figure out their own mistakes and can lead to classroom discussions about the problem (Int 1, 6). She nuanced this response with the situation when a student is unaware that what they said is incorrect. For that scenario, she leads a discussion or directly explains what should be happening (Int 1).

In May, Reed was aware that there were differences in errors based on whether she expected or planned for them or when she did not. For unplanned errors, she did not have specific follow up strategies. Reed described her process of asking for students' explanation if she was sure or pretty sure where the error came from (Int 6). She felt the process of talking about mistakes can allow other students to learn more aspects of mathematics.

Reed's Beliefs About Uncertainty

In January, Reed described that uncertainty is normal and expected, however if uncertainty prevents students from trying, then it can have a negative impact. She described how, through her teacher lens, uncertainty is connected to student thinking–revealing "where students are" (Int 6). Similar to errors, she described the challenge of keeping lessons on track with uncertainty of things that students "should know" (Int 6).

When Reed encounters student uncertainty, her first step is to see what students are uncertain about. She talked about physical proximity ("standing awkwardly near") and gently checking in with students and asking, "Do you have any questions?" (Int 1). Her strategy is to have students start the task and see how far that can get–his helps her determine what they are not understanding. In May, she talked about "leading" students through the process, not telling them what to do so they can keep moving in the task.

Reed's Lesson Planning Practices

Reed's lesson planning involves a lot of her thinking about things, even though she does not necessarily write them down. She explained that she was "unorganized" and has "a lot of anxiety" (Int 1). She types her documents, but when she is trying to find ways to solve, she uses paper and pencil. She relied on her teaching partner or her fiancé (Int 1). Her fiancé has a degree in writing and helps her with her grammar. He is not as familiar in mathematics; Reed relied on his (lack of) mathematical content knowledge to represent struggling students (Int 1). She described the importance of having all student resources (e.g. worksheets) done ahead of time, and her plans to "jot down" things students might say, common mistakes she expects, important questions (Int 6). In her future teaching, she intends to have some written lesson plans, but not to the extent of what she was asked to do through her methods courses (Int 6). Reed talked about the TTLP and how it helps her think about things–structure and accountability. Though, she did not like the length of the template and assignment, it was the impetus for her to think about some important aspects of teaching that she may not have done without the assignment requirement.

Introducing Lian

Lian is a Chinese female who came to the U.S. in 12th grade. Her motivation for being a teacher was a powerful middle school mathematics teacher who was both a strong teacher in mathematical content, and was also motivating for students of all abilities (Int 1).

Lian's Teaching Philosophy

Lian focused heavily on the connections and intellectual relationships that her students make within mathematics. This theme was strongly prevalent throughout all of our interviews. She drew on her experience and inspiration from the wonderful mentor teacher she had in middle school who "categorized similar problems" and taught "in categories" (Int 1). Lian believes that connections will hopefully allow students to "understand" rather than "memorize things" (Int 1). In January, Lian's focus on connections was primarily between various mathematical concepts. In May, Lian included connections to students' lives and prior knowledge as important aspects of the thinking and learning they do. Lian focused more about classroom discourse and the importance of students engaging with each other's reasoning and thinking in class in the May interview.

Lian's Beliefs About Student Thinking

For Lian, student thinking is how students view a question, how they interpret or translate it, and the process or strategies they use to come up with an answer (Int 6). The key word that she consistently used when asked about student thinking was "connections" (Int 1, 4, 6).

Lian's Beliefs About Errors

For Lian, errors are a normal (Int 1) or good thing (Int 1, 6). Errors provide an opportunity to learn from others and make mistakes prior to assessments. When students make errors, she asks them questions so that they can explain how they got the answer they did (Int 1, 6). She observed that this explanation process often allows students to realize their own errors, but if it does not, she might ask others about the students' process. In May, she had some additional responses of engaging more students to elicit more responses and engage students in discussion with multiple solution strategies and rationale.

Lian's Beliefs About Uncertainty

Lian considers errors normal or positive though she considers uncertainty as a bad thing (Int 1, 6). To her, uncertainty was when students "cannot (make) the connection of what we learned to the question" and this lack of connection implies that the student "cannot make mistakes to improve or find the right steps to get closer to the answer" (Int 1). In May, she added that "it's either they have a difficulty in listening, or they just don't want to listen to you." This statement seems to be connected to her experience with several students in her spring placement who I observed to be off task, in class without their books and supplies, and would disrupt others.

Lian's response to uncertainty is to try to help students make connections, by asking questions of the student directly or the student's group members (Int 1). In May, she described how talking with a fellow PST colleague she got the idea of having the student do one thing–a "baby step" such as take their book out or refer to the previous question–as a way to support uncertainty.

Lian's Lesson Planning Practices

Lian reported that she planned lessons with her PST colleague (Reed) during Fall Semester. They would work in the library, look at the problem, try to solve it, and try "to think of any situations (that) might come up, and other strategies we can use to solve the problem" (Int 1). Lian and Reed would ask their roommates or boyfriends to help identify things that were unclear by seeing how the roommate or partner interpreted it.

For her future lesson planning, Lian intends to use the launch-explore-summarize format of the TTLP as her structure (Int 6). She explained that she will consider what she expects from students and the responses she can give to those to help students make connections. She will consider her expectations during the explore section and what she will say for the summary. She also (and she was the only one) will consider homework and mentioned that was not something that she had practiced this year. She also mentioned the learning objective, but did so as almost an aside/after the fact (Int 6).

APPENDIX H: Code Development Process

Though prior frameworks and experience helped start the structure of my code development, I went through many versions of my codes and codebook. This process was slow and arduous. I would transcribe and code an interview, lesson plan, or enacted lesson and when new, important ideas emerged, I added the code, potentially removing and rearranging others. I then went back to all previously coded work to recode based on the new structure. I started with a sampling of interviews, then lesson plans, then enacted lessons to build the code book. Then I revisited all previously coded data to cross check against updates.

One challenge that continually arose was the interconnected nature of the codes. Early in the process, I categorized codes broadly into teacher, student, lesson planning, and enacted teaching. The challenge was that codes had a primary category designation, but they also related to other categories. I struggled for many months with how these ideas fit together and what codes should be, what subcodes should be and how they fit together. For example, instead of having the code of "student–background" and "teacher–background," what if I switched them and had "background–student" and "background–teacher"? How would that help me make sense of this data and share it?

A mid-journey categorization included teacher, student, student thinking, resources, lesson plan, enacted teaching as the larger categories. Overlap was still a challenge and it seemed that many codes fit into multiple categories. Upon realigning with my research questions, I came up with thinking/cognitive resources, actions or moves, goals, and assumptions. The next week, I took my codes (via many small pieces of paper) to my mathematics education research writing group. Their ideas brought up a number of interesting questions and ways to consider the partitioning. I continued to work through this process. It was stressful and anxiety producing–two steps forward, one step back. I sorted via computer. I printed codes out, cut them into individual papers and moved them around, as if they were puzzle pieces. This was challenging because regardless as to how I grouped the codes, some things fit, some things did not and some things could easily land in multiple categories depending on how they were referred to or referenced.

A few months later, as I continued to refine the code book, I looked for places where codes were similar or could be combined. This allowed me to combine a number of similar codes. The grain size of the code and category continued to be a challenge. Around this point, I realized that it was much easier for me to think from small and literal–what was said and what specific code that should be–into larger and more thematic.

Throughout this process, I needed to rename and restructure some of my identifying and categorizing components–for example phase and category. I worked to make those more explicit and unique. Phase became one of three things: plan, enact, reflect. This matches Remillard's (1999) model of curriculum.

A month or so later, after two weeks of intense teaching and not working on coding, I was able to see some of the codes and categories in a fresh light and was able to make some better clustering of codes and group things in ways that seem more logical. At the same time, I removed some sub codes. For example, at that point I had multiple classroom management codes that I collapsed into the larger code of classroom management. At this point, I had 43 codes which could branch into approximately 150 designations when I included subcodes. These codes clustered into eight categories: teacher knowledge, beliefs about learning, preparation, culture and relationships, mathematics, affect, student thinking, teacher action or moves.

Student Thinking Codes

Toward the end of June, I realized that I needed a more systematic breakdown of student thinking. I did a literature search of about 30 articles to examine how scholars were defining and categorizing student thinking. This led to the following components of student thinking: student understanding; reasoning; strategy creation and use; problem solving; working with models and representation; errors, misconceptions, and how students work through errors; student justifications; and communications around student thinking. I added connections to this list as it was a prevalent theme in my interview and lesson plan data.

APPENDIX I: Fall Semester Lesson Planning Template

This is the Microsoft Word document template PSTs used to plan for Teachings 1 and 2. I changed the text size and spacing for space use here. All content and formatting beyond text sizing and spacing is the same. Each section is separated by a black outline.

Figure A.2:

Images of the Fall Lesson Planning Template

Part 1: How can the task be solved?
How would *students* engage in the task to solve the problem?
What materials are needed? What resources or tools might you want to make available for students to use?
What might be written down or recorded?
What solution strategies might students use? (See Explore pg. 2 and complete the bubbles for each activity)
What misconceptions, mistakes, or errors may emerge. (See Explore pg. 2 and complete the bubbles for each activity).

Part 2: What is the purpose of the task (or tasks) of the lesson?
What *mathematical ideas* will the activity develop?
What NCTM process standards or Standards for Mathematical Practices (from the Common Core) might be relevant?
Will there be connections to other related ideas?

High Cognitive Demand Task Brief Description: Number Talk: Launch:	
Some good general probing questions for this task might be: 1) 2)	
Solutions	**Potential Follow-Up Questions**
Some good questions that help students engage in connecting these solutions are:	

Lesson Planning

Your Names **Lesson DATE:**

Overall Purpose: This lesson-planning document will provide a template to think through the lesson planning process for facilitating student-centered mathematics tasks.

MATH TASK OUTLINE (choose preferred format)

Part 1: How can the task be solved?
How would *students* engage in the task to solve the problem?

STAGE 1: Pre-Planning (Due 1 week (Rounds 1 and 2) or 2 weeks (Round 3) before you teach):

Purpose: You will consider different possible solutions to a variety of problems and propose possible learning goals for those specific tasks. This will increase your understanding of mathematics and the ways in which students will approach problems in your future class (and even when you are in the field now). "Unpacking" the mathematical work will help you write concrete, measurable learning goals and teaching strategies to effectively promote learning.

Anticipating student solutions, potential errors, mistakes, or misconceptions students might make.
For each task, try to come up with 2-3 ways students might solve the problem, anticipating possible misconceptions, and draft a few questions to encourage students to think more deeply about the mathematical ideas connected to specific solution strategies.

Answer the **pre-planning questions** below and complete the **math task outline** (Next two pages: Use either the Chart OR Table) for each task you plan to include in the lesson.

Pre-Planning Questions:

Part 1: How can the task be solved?
How would *students* engage in the task to solve the problem?
What materials are needed? What resources or tools might you want to make available for students to use?
What might be written down or recorded?
What solution strategies might students use? (See Explore pg. 2 and complete the bubbles for each activity)
What misconceptions, mistakes, or errors may emerge. (See Explore pg. 2 and complete the bubbles for each activity).

Part 2: What is the purpose of the task (or tasks) of the lesson?
What *mathematical ideas* will the activity develop?

Figure A.2 (cont'd)

STAGE 2: Lesson Planning (Due 1 DAY before you teach):

Purpose: The main purpose of the Thinking Through a Lesson Protocol is to prompt you in thinking deeply about a specific lesson that you will be teaching (based on a cognitively challenging mathematical task). The At a Glance page at the beginning of the lesson plan provides an overview of the lesson, which you should refer to as you teach.

Using your pre-planning work and the ideas that you discussed with your TA, complete the lesson plan sections below, adding additional details as requested.

You may find it helpful to complete the "at a glance" first, but we recommend revising it after completing the full "thinking through a lesson protocol" to reflect your deeper thinking about this lesson.

AT A GLANCE LESSON PLAN

Learning Goals	**Skills and Understanding**
Lesson Tasks	**Formative Assessment Opportunities**
Activity Flow **Task 1:** **Task 2:**	
Summary: *What are the big "take-away" ideas of the lesson? How do these connect to what you will do in the next lesson?*	

THINKING THROUGH A LESSON PROTOCOL

Part 0: Selecting and Setting up a Mathematical Task

A) What are your mathematical goals for the lesson (i.e., what is it that you want students to know and understand about mathematics as a result of this lesson)? (See Math Prep, #3 & At-a-Glance Lesson Plan) What are the mathematics standards this lesson teaches? (http://www.corestandards.org/Math/)	**Learning Goals** **Standard(s) Addressed:**
B) In what ways does the task build on students' previous knowledge? (See Math Prep #4 & 5 and At-a-Glance Lesson Plan)	**Skills and Understandings**
C) What definitions, concepts, or ideas do students need to know in order to begin to work on the task? (See Math Prep, #5)	
D) What questions will you ask to help students access their prior knowledge? (Potentially See At-a-Glance, use this in Part 1: Launch Script)	
E) What are all the ways the task can be solved? (See Math Prep, #1 to begin with) (1) Which of these methods do you think your students will use? (2) What misconceptions might students have? (3) What errors might students make? Note: Connect the ways the task may be solved to *Part 2: Supporting Student Exploration of the Task* as well as *Part 3: Sharing and Discussing the Task*	
F) What are your expectations for students as they work on and complete this task? (1) What resources or tools will students have to use in their work? (2) How will the students work -- independently, in small groups, or in pairs -- to explore this task? (3) How long will they work individually or in small groups/pairs? (4) Will students be partnered in a specific way? If so, in what way? (5) How will students record and report their work?	
G) How will you introduce students to the activity so as to set high expectations, maintain the cognitive demand of the task? Note: Connect this section to *Part 1: Lesson Launch Script* below.	
H) What will you hear that lets you know students understand the task? Note: Connect this section to *Part 1: Lesson Launch Script* below.	

Figure A.2 (cont'd)

Part 1 LAUNCH: Lesson LAUNCH Script

The launch of a lesson or a task is important because it sets up the mathematics and the tone for the whole class period. Use this space to plan/script exactly what you plan to say to launch the lesson or task. Also use this space to anticipate what you expect to hear from students. What are the questions they might have about the task? How will you respond to those questions? Note: Connect the questions you plan to ask students to **Part 0**, G. and H.	

APPENDIX J: Spring Semester Lesson Planning Template

This is the Microsoft PowerPoint template PSTs used to plan for Teaching 4. I removed the dots for ease of reading and copied and pasted notes that the methods instructor had in the notes section of PowerPoint on to the slides. Each slide is separated by a black outline.

Figure A.3:

Images of the Spring Lesson Planning Template

Plan-Teach-Reflect

Template for Modified TTLP

Name

Lesson Date:

NOTE: You need to record your lesson, so please make arrangements with the TIES to check out equipment if you need to do so. Also, check with your mentor about any policies for recording in their school/district. I assume this won't be a problem since you had to record your number talk earlier in the semester… but want to flag this in case you need to check on it.

Slides 2-15 must be completed at least a week before you will teach and sent to Kevin and I for feedback. You will not be allowed to teach if this requirement is not met and the modified TTLP is not up to the programs' expectations. You will replace the bullets with relevant information to your particular lesson and can use the notes section for any additional information or rationale you see as necessary for us to understand what you will do and why you will do it. After we have approved the plan, you will upload the final version to D2L.
Slides 16-17 will be completed after you teach. Reflecting, Part 1 (prompts on Slide 16) is due within 72 hours so you don't forget anything. Reflecting, Part 2 (prompts on Slide 17) is due within a week of teaching your lesson. These will be uploaded to D2L within the required time frame.

Slides that have dots in the background like this one are transition or explanatory slides and do not require information be typed into them.

Figure A.3 (cont'd)

Identifying goals

- Learning Goals:
- Content Standards
- Standards for Mathematical Practice

Finding an appropriate task

PST response would be here

Methods Instructor Notes: You can find appropriate tasks in the NCTM journals *Teaching Mathematics in the Middle School* and *Mathematics Teacher.* These are available both in print and electronically through the MSU library. Other resources include any of the NCTM Yearbooks (or Annual Perspectives on Mathematics Education is the more recent version of this), the Faces series published by NCTM, the Navigating mathematics series, or activities from the Math Solutions website (www.mathsolutions.com) or from the book written by Van de Walle. You also may search through Google or a mathematics-centric sites and blogs such as mathforum.org, MathTwitterBlogosphere, or use other search engines (e.g., ERIC) available through the library. The most important criterion for selecting your pieces is that it has been **peer-reviewed**. You may not use things from Pintrest nor from "teachers pay teachers" because there is no quality control on these sites.
Here are some places that I shared back when you were designing a groupworthy task, too:

- Broadly Comprehensive Resources
  - *Inside Mathematics:* http://www.insidemathematics.org/
  - *Shell Center for Mathematics:* http://map.mathshell.org/
  - *You Cubed:* https://www.youcubed.org/
- **Math Tasks**
- *Adapting tasks: https://solvingworldproblems.wordpress.com/2015/01/02/adapting-rich-mathematical-tasks-guiding-questions-resources/*
- *Spatial visualization: https://www.youtube.com/watch?v=qum9kvxR9K8*
- *Reasoning and sense making task library:* http://www.nctm.org/rsmtasks

Materials & Resources

Launch script

PST response would be here

Methods Instructor Notes: (Address how you will: discuss key contextual features; discuss key mathematical ideas; develop common language to describe key features; & maintain cognitive demand (*from Jackson et al, 2012)*)

Between the launch and monitoring

Anticipating student solutions & planning probing, extending and advancing questions

Information to use while students Explore

Anticipating student solutions

- Please include the actual high cognitive demand or groupworthy task(s) you intend to use in the lesson. You may not use a low cognitive demand task.
- Make sure you solve every task you will give to students and provide multiple solution strategies for each problem, including errors or partial thinking that you believe may come up during the lesson. Please do not just give generic solutions but *solve the actual tasks* you will be using for your lesson.
- For each solution, provide questions that allow you to probe student thinking, help them articulate connections, and provide justification. The questions must include both ones that relate to specific solutions you have anticipated as well as ones that would be useful, more generally, and ones that help students make connections or extensions.

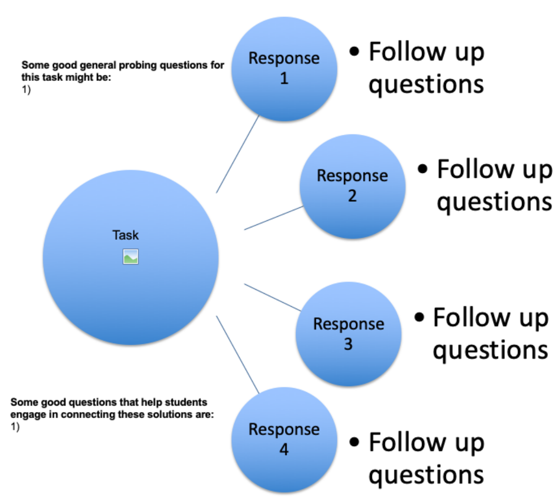


Figure A.3 (cont'd)

Monitoring, Selecting, Sequencing

Summarize

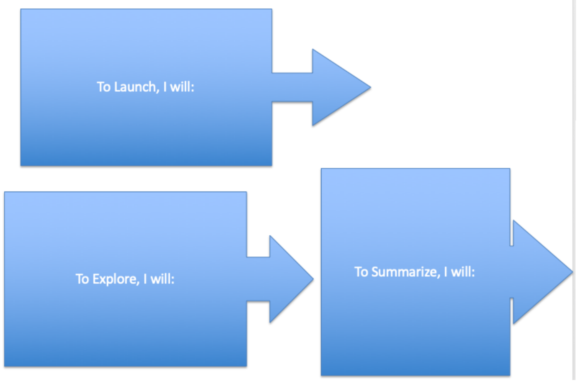
PST response would be here

Methods Instructor Notes:

Some things to consider here:

- How will you orchestrate the class discussion so that students:
  - make sense of the mathematical ideas being shared?
  - expand on, debate, and question the solutions being shared?
  - make connections between their solution strategy and the one shared?
  - look for patterns and form generalizations?

Abbreviated TTLP for Plan-Teach-Reflect



Assessment

- Formative Assessments:
- Summative Assessment:

Reflecting on your Teaching, Part 1

- Make sure you record your lesson!
- Within 24 hours of teaching, create a timeline for what you remember happening during the launch, the explore, and the summarize. Try to include things that went as you expected/anticipated and things that did not go as you expected/anticipated.
- Within the next two days, listen to your recording at least twice and make a notice/wonder chart.
  - Note when you notice yourself using the IRE pattern and when you notice yourself using any of the TDMs.
  - Record as many details as possible about what students said/did to help you write something about what you think students understood or accomplished, in relationship to your learning goal(s).
  - Pick 10 minutes to transcribe for Reflecting on your Teaching, Part 2
- Write a reflection that focuses on your discourse patterns/practices and what you think students learned (be specific in terms of using evidence to back up any claims)
  - Please return to your goals for the lesson and assess the extent to which those goals were met.
- Briefly describe what you would plan the next day to build on your students' learning in this lesson.
- Submit this on D2L within 72 hours of teaching

Reflecting on your Teaching, Part 2

- Choose 10 minutes of your lesson to transcribe
  - Explain why you selected that 10 minutes
  - Analyze the clip using the 5 dimensions included in the notes below
  - After completing the analysis, you will write up a **2-3 page, double-spaced summary of the analysis** you did of the clip. The summary should include: 1) a very brief description of the clip (what is being discussed, when the clip occurred in the lesson), 2) a description of the main findings from your analysis, and then 3) a reflection upon ways that you could improve your facilitation of discussion during the clip. The goal of part 3 is to identify moves you could have made to open up the mathematics, to provide every student with opportunities to participate in cognitively-demanding ways, and to make more space for student participation.
- Due within a week of teaching your lesson.

Figure A.3 (cont'd)

Methods Instructor Notes for Part 2

- Dimension 1: How many minutes do you talk during the clip? How many minutes do students talk during the clip? How many times do you talk? How many times do students talk? What is the ratio of teacher-spoken-words to students-spoken-words?
- Dimension 2: Record how many instances you notice yourself using the IRE pattern. Record how many instances you notice where you ask a question, one student responds, and then you ask another question. Record how many instances you notice where a student immediately questions or responds to another student's comment without a question or comment from you in between. In addition to these three, record any other patterns of talk that are prevalent in the clip.
- Dimension 3: Take a closer look at your turns in the clip. Where do you see yourself using the Teacher Discourse Moves [TDM], including what kind of TDM it was and the response that happened as a result of that TDM.
- Dimension 4: Although we cannot know how students would self-identify, typically bias can play out based on your perception of students' identities. How many times did you call on girls? How many times did you call on boys? How many times did you call on white students? How many times did you call on students of color? How many times did you call on students who are primarily/only English speaking? How many times did you call on students who are emergent bilingual and/or whose first language is not English?
- Dimension 5: After counting the different opportunities to participate in Dimension 4, consider the kinds of questions you asked these various groups of students: who did you ask "what" questions of? Who did you ask how questions of? Who did you typically ask why questions of?

This slide includes information about what you need to put your second reflection, which you will upload to D2L

APPENDIX K: Summary of all Planned and Enacted Discourse Moves

Table A.3:

Summary of All Planned and Enacted Discourse Moves

Practice	Teacher Move	Planned	Used	Preceded	Respond	Remain
Monitor	Monitoring	5	343	99	98	146
Elicit	Inviting	5	99	31	26	42
	Elicit answers	44	138	17	73	48
	Eliciting facts or procedures	77	213	31	122	60
	Asking for clarification	6	63	3	53	7
	Figuring out student reasoning	28	48	3	37	8
	Checking for understanding	16	207	49	74	84
	Eliciting ideas	36	94	19	41	34
	Eliciting understanding	29	29	4	19	6
	Pressing for explanation	11	76	5	52	19
Respond	Waiting	2	72	2	53	17
	Validating	0	406	3	264	139
	Revoicing	2	131	1	79	51
	Encouraging student to revoice	3	0	0	0	0
	Correcting	2	124	2	119	3
	Re-presenting	0	61	1	34	26
	Prompting error corrections	0	8	0	8	0
Facilitate	Cueing	9	246	6	204	36
	Topaze	0	68	4	61	3
	Funneling	4	98	2	83	13
	Explaining procedure	15	201	8	155	38
	Providing guidance	14	47	0	45	2
	Building on previous ideas	7	67	3	45	19
	Explaining conceptually	1	91	4	68	19
	Summary explanation	1	54	1	31	22
	Providing information	0	5	0	5	0

Table A.3 (cont'd)

Facilitate	Encouraging multiple strategies	5	6	0	3	3
	Providing alternative strategy	0	0	0	0	0
Extend	Probing	3	1	0	0	1
	Pressing for precision	0	0	0	0	0
	Topaze for justification	0	0	0	0	0
	Encouraging reasoning	8	4	1	3	0
	Encouraging reflection	4	2	0	2	0
	Pressing for justification	16	6	2	3	1
	Pushing for generalization	17	2	2	0	0
Non-TMSSR	Classroom Management	24	120	8	27	85
	Positive Emotion	0	73	2	35	36
	Describing the task	47	109	8	39	62
Other	Number of moves	12	17	4	15	---
	Total times used	1138	200	5	88	---

APPENDIX L: TMSSR Framework and Details for Strategies

Figure A.4:

TMSSR Framework

<table>
<tr><th colspan="4">Eliciting Student Reasoning</th><th colspan="2">Responding to Student Reasoning</th></tr>
<tr><td colspan="2">Eliciting Answer</td><td colspan="2">Eliciting Ideas</td><td>Validating a Correct Answer</td><td></td></tr>
<tr><td colspan="2">Eliciting Facts or Procedures</td><td colspan="2">Eliciting Understanding</td><td>Re-voicing</td><td rowspan="2">Re-representing</td></tr>
<tr><td colspan="2">Asking for Clarification</td><td colspan="2" rowspan="2">Pressing for Explanation</td><td>Encouraging Student Re-voicing</td></tr>
<tr><td colspan="2">Figuring Out Student Reasoning</td><td>Correcting Student Error</td><td>Prompting Error Correction</td></tr>
<tr><td colspan="2">Checking for Understanding</td><td colspan="2"></td><td></td><td></td></tr>
<tr><th colspan="4">Facilitating Student Reasoning</th><th colspan="2">Extending Student Reasoning</th></tr>
<tr><td colspan="2">Cueing</td><td colspan="2">Providing Guidance</td><td>Pressing for Precision</td><td>Encouraging Reasoning</td></tr>
<tr><td colspan="2">Topaze Effect[a]</td><td colspan="2" rowspan="2">Building[b]</td><td>Encouraging Evaluation[c]</td><td>Encouraging Reflection</td></tr>
<tr><td colspan="2">Funneling</td><td>Topaze for Justification[d]</td><td>Pressing for Justification</td></tr>
<tr><td rowspan="2">Providing Procedural Explanation</td><td colspan="2">Providing Summary Explanation</td><td rowspan="2">Providing Conceptual Explanation</td><td></td><td>Pushing for Generalization</td></tr>
<tr><td colspan="2">Providing Information</td><td rowspan="3"></td><td rowspan="3"></td></tr>
<tr><td rowspan="2"></td><td colspan="2">Encouraging Multiple Solution Strategies</td><td rowspan="2"></td></tr>
<tr><td colspan="2">Providing Alternative Solution Strategy</td></tr>
</table>

Figure A.5:

Practice 1: Elicit Moves

Table 1: Teacher Moves for Eliciting Student Reasoning

Eliciting Answer: Teacher asks a question geared at eliciting the answer to a given task or problem.	***Eliciting Ideas***: Teacher asks a question geared at eliciting students' ideas for a solution strategy.
Eliciting Facts or Procedures: Teacher asks questions geared at eliciting students' recitation of facts or procedures.	***Eliciting Understanding:*** Teacher asks questions geared toward assessing what students understand and how they are reasoning.
Asking for Clarification: Teacher asks a question to clarify the student's meaning because teacher genuinely does not know what the student meant.	***Pressing for Explanation:*** Teacher asks student(s) to elaborate on their thinking, explain their reasoning, or reflect on and share their reasoning.
Figuring Out Student Reasoning: Teacher is trying to figure out a student's solution, or understand a student's explanation or reasoning.	
Checking for Understanding: Teacher asks a question to assess students' understanding of the mathematical ideas that are currently under discussion.	

Figure A.6:

Practice 2: Respond Moves

Table 2: Teacher Moves for Responding to Student Reasoning

Validating a Correct Answer: Teacher actively confirms the student's idea by re-voicing, or re-wording in her own words, or adding a bit to the student's idea or response.	
Re-voicing: Teacher repeats student ideas (verbally or written) in order to make those ideas public.	***Re-representing:*** A form of re-voicing in which a teacher provides her own representation as a way to publicly share a student's idea, work, or strategy. The teacher may organize, re-frame, or formalize the student's statement or work.
Encouraging Student Re-voicing: Teacher asks students to re-voice other student ideas or solutions.	
Correcting Student Error: Teacher corrects a student error or supplies the correct answer more generally.	***Prompting Error Correction:*** Rather than correcting the student, the teacher prompts the student to address an error herself.

Figure A.7:

Practice 3: Facilitate Moves

Table 3: Teacher Moves for Facilitating Student Reasoning

<table>
<tr><td colspan="2">Cueing: Teacher cues students' attention by indicating that they should focus on a particular aspect of a problem, task, idea, solution, etc.</td><td colspan="2">Providing Guidance: Teacher provides hints, ideas, a potential strategy, or another type of conceptual scaffolding of the problem without outlining the solution structure or otherwise shutting down students' opportunities to reason on their own.</td></tr>
<tr><td colspan="2">Topaze Effect: Teacher breaks a task into smaller parts, reducing the complexity of the task by asking easier and easier questions, thereby reducing students' opportunity to engage in authentic problem solving.</td><td colspan="2" rowspan="2">Building: Teacher builds on students' earlier contributions to support new understanding, or encourages students to build on one another's contributions.</td></tr>
<tr><td colspan="2">Funneling: Teacher asks questions that move students down a specific path (e.g., through leading questions).</td></tr>
<tr><td rowspan="2">Providing Procedural Explanation: Teacher provides a procedural explanation for how to solve a problem. This move includes telling students a priori how to solve the problem by outlining the solution structure (or some other way).</td><td colspan="2">Providing Summary Explanation: Teacher summarizes for the class final thoughts about a task or problem, or a summary of information or discussion about the task.</td><td rowspan="2">Providing Conceptual Explanation: Teacher provides an explanation that has a conceptual basis, often focused on explaining why something works. This move can also be thought of as demonstrating logic.</td></tr>
<tr><td colspan="2">Providing Information: The teacher provides new information relevant to doing mathematics generally rather than information about a specific problem or task.</td></tr>
<tr><td></td><td colspan="2">Encouraging Multiple Solution Strategies: Teacher encourages a proliferation of solution strategies, including pressing students to come up with a different way to solve a problem.</td><td></td></tr>
<tr><td></td><td colspan="2">Providing Alternative Strategy: Teacher initiates a new or different way of solving a problem after students have shared their solution strategies or solutions.</td><td></td></tr>
</table>

Figure A.8:

Practice 4: Extend Moves

Table 4: Teacher Moves for Extending Student Reasoning

***Pressing for Precision**:* Teacher encourages student(s) to provide an exact rather than vague answer, to check his or her work for accuracy, or to quantify a qualitative statement.	***Encouraging Reasoning :***Teacher encourages students to think about the task conceptually, for instance by thinking about why a strategy makes sense, by thinking about where the numbers connect to the quantitative situation, etc.
	Encouraging Reflection: Teacher asks students to reflect on provided answers or explanations (either from the teacher or from another student).
Topaze for Justification: Teacher initially pushes for justification, but then immediately downgrades her question to a less-sophisticated why question by heavily leading students into justification via easier questions.	***Pressing for Justification**:* Teacher asks students to explain why something works or to justify (logically, conceptually) their idea, strategy, or solution.
	Pushing for Generalization: Teacher encourages students to generalize their reasoning, either through formulating a rule, describing a process in general terms, or making connections across problems, numbers, cases, or events.

APPENDIX M: Breakdown of Moves by TMSSR Practices by PST

Table A.4:

Total Number of Discourse Moves for Each PST by Type of Discourse

	Enacted	Preceded	Response
Taylor	815	97	378
Carson	725	71	393
Rowan	985	69	563
Reed	430	24	332
Lian	557	65	383
Totals	3,512	326	2,049

Note: Reed's data come from Teachings 1 and 2 only.

Table A.5:

Planned Discourse Moves

Code	Taylor	Carson	Rowan	Reed	Lian	Sum
Monitor	0	1	2	2	0	5
Elicit	23	37	44	82	66	252
Respond	0	2	2	5	0	9
Facilitate	14	11	9	5	17	56
Extend	9	12	5	2	20	48
Totals	46	63	62	96	103	370

Table A.6:

Enacted Discourse Moves

Code	Taylor	Carson	Rowan	Reed	Lian	Sum
Monitor	96	52	37	29	129	343
Elicit	224	232	301	92	118	967
Respond	199	175	204	114	110	802
Facilitate	173	178	256	141	135	883
Extend	3	1	7	1	3	15
Other	120	87	180	53	62	502
Totals	815	725	985	430	557	3512

Table A.7:

Moves That Preceded Errors and Uncertainties

Code	Taylor	Carson	Rowan	Reed	Lian	Sum
Monitor	34	19	10	7	29	99
Elicit	46	41	40	9	26	162
Respond	3	1	2	1	2	9
Facilitate	8	6	9	1	4	28
Extend	0	1	2	1	1	5
Other	6	3	6	5	3	23
Totals	97	71	69	24	65	326

Table A.8:

Moves That Followed Errors and Uncertainties

Code	Taylor	Carson	Rowan	Reed	Lian	Sum
Monitor	19	13	5	7	54	98
Elicit	71	110	153	82	81	497
Respond	125	101	136	96	99	557
Facilitate	118	137	194	129	122	700
Extend	3	0	3	0	2	8
Other	42	32	72	18	25	189
Totals	378	393	563	332	383	2049

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