INTERACTIONAL WORK OF ADVANCED PLACEMENT MATHEMATICS TEACHERS: TOWARD PRODUCTIVE AND EQUITABLE MATHEMATICS DISCUSSIONS

By

Sunghwan Byun

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

Mathematics Education—Doctor of Philosophy

2021

ABSTRACT

INTERACTIONAL WORK OF ADVANCED PLACEMENT MATHEMATICS TEACHERS: TOWARD PRODUCTIVE AND EQUITABLE MATHEMATICS DISCUSSIONS

By

Sunghwan Byun

In this dissertation, I explored the interactional work of three Advanced Placement (AP) mathematics teachers by applying ethnomethodology and conversation analysis (EMCA) as the primary methodological approach. The analyses focused on teachers' facilitation of mathematics discussions in the context of the sequential progression of classroom interaction. This study was situated in two public high schools located in suburban communities in the Midwestern region of the United States. The data included video and audio recordings of three consecutive lessons in the middle of the academic year from each class. Findings from this dissertation are threefold and organized as three chapters in this dissertation. Chapter 2 presents how the teachers initiated and supported minoritized students' participation in collective argumentation. Chapter 3 details teachers' use of partner talk and examines how the teachers facilitated a focused and accountable space for mathematics discussions. Lastly, Chapter 4 focuses on the context of the AP Exam and examines how the institutional context of high-stakes testing afforded the teachers to position their students as particular kinds of learners. The overall contributions of this dissertation are detailing some of the discourse practices in existing frameworks for mathematics classroom discourse, offering insights into when-questions as well as how-questions, and highlighting the importance of mutuality and reciprocity of social interactions to understand teachers' facilitation of classroom discussions. I conclude by suggesting other potential ways to further the understanding of mathematics classroom discourse from EMCA approaches.

To Nancy, Logan, and Elena

ACKNOWLEDGEMENTS

First, I thank participating teachers and students. My dissertation would not be possible without them welcoming me to their classrooms and allowing me to study their teaching and learning space. This dissertation is also motivated by my experience as a mathematics teacher in Phoenix Union High School District. I thank my former students, whose brilliance ignited my passion for research in mathematics education.

I cannot thank enough my advisor and the chair of the committee, Beth Herbel-Eisenmann, for her support throughout my entire Ph.D. career here at Michigan State University. She shaped me as a teacher educator, discourse analyst, and most importantly, a self-reflective life-long learner. Her keen insights amended many parts of this dissertation if not all. I am also grateful for the support from my dissertation committee by engaging with my ideas with genuine interest. Kristen Bieda helped me grow as a teacher educator when I was working on the UTEMPT project. I cherish the thoughtful conversations we had as we faced multiple challenges while preparing a cohort of mathematics teachers. Alexa Hepburn introduced me to the world of conversation analysis and the wonderful community of conversation analysts at the Rutgers University Conversation Analysis Lab. Shiv Karunakaran showed genuine curiosity about my thinking and treated me as an independent thinker. David Stroupe shared his insights on learning theories and broadened my view of science education. My committee's support went beyond completing this dissertation. I am grateful for their help when I was navigating academia and searching for my place.

I also sought support from outside of my committee. I am particularly grateful for the instrumental advice by Jenni Ingram when I was planning this dissertation. Her advice on data gathering and analyses allowed me to see things I did not notice before. I thank Jen VanDerHeide for introducing a range of discourse analytic approaches and supporting my practicum study, which became a foundation for this dissertation. I thank Dorinda Carter Andrews for helping me open my critical eyes to my own racialized experiences and existing systemic racism. I also thank Gail Burrill and Vince Melfi, who helped shape my emerging interest in Statistics Education. I also received tremendous support from the Program in Mathematics Education. Freda Cruél, Kelly Fenn, Lisa Keller, and Ralph Putnam played an essential role in completing my Ph.D. program.

The community of mathematics education graduate students at Michigan State University has been an important part of my Ph.D. program experience. Kevin Voogt supported my first year in the program. I enjoyed doing reading groups with Chris Dubbs, Jihye Hwang, Valentin Kuechle, and Luke Tunstall. I learned from co-facilitating the MLRG and Writing Group with Greg Beaudine, David Bowers, and Amy Scheerhorn. I also appreciated comradery among Korean graduate students with Younggon Bae, Sunyoung Park, and Wonjung Kim. Lastly, I thank the strong bonds with my BHE Family members, Jeff Craig, Michael Morissette, Molade Osibodu, Melvin Peralta, and the new addition to the family, Eli Claffey and Kristin Giorgio.

This dissertation also reflects my learning from two large research projects. My engagement with the A3IMS project team set the stage for my graduate study focusing on equity. I feel so fortunate to have heartfelt collaborations with Victoria Hand and Carlos López Leiva. They have demonstrated to me how I should work with graduate student colleagues and support them in the future. Joel Amidon, Tonya Bartell, Mary Foote, Courtney Koestler, Greg Larnell, and Anita Wager shared multiple perspectives on equity and social justice and helped me shape my own stances and views. I will remember numerous conversations I shared with Missy Cosby, Frances Harper, Brent Jackson, Durrell Jones, Ashley Scroggins, and Ayşe Yolcu about our own development as an equity-oriented researcher serving a marginalized community.

ICSAM project team helped me realize my interest in computing and related equity issues. I am grateful to Niral Shah, who opened multiple doors for me to precious experiences with the ICSAM project, AMSSI conference, and co-teaching a methods course. Danny Caballero and Paul Irving showed me the world of computation and physics education research. My conversations with Julie Christensen and Ai-Khanh Nguyen helped me further my thinking about equity-oriented teacher education.

Last but not least, I am grateful for my family and the Buddhist community's support. My partner, Nancy Byun, was the biggest supporter of my Ph.D. journey. Our children Elena and Logan brought us tremendous joy over the past five years. My accomplishment would not be possible without my parents, Sooduk Byun and Sunhee Lim, who devoted their lives to raising me and my brother with sincere prayers. My brother, Kyunghwan Byun, and his family made me feel like I am never too far from my family in Korea. My Hawaii parents, Guy and Shirley Yamashita, welcomed me to their family and helped me take new roots in the United States. My in-laws, Suegrita Reyna Garcia, Gilberto Yáñez, Armando Lopez, Elizabeth Lopez, David Garcia, Danny Garcia, Kenny Garcia, Steven Garcia, and Suegra David Garcia did not hesitate to fly to Michigan whenever we needed them. Finally, I thank the local Buddhist community members in Soka Gakkai International, who reminded me of our humanistic values and shared heartfelt life stories of ordinary people across the boundaries of culture, nationality, and race. I conclude this acknowledgment by reciting the last part of our daily silent prayers, hoping that this dissertation will serve as a vehicle for our prayer: *I pray for peace throughout the world and the happiness of all living beings*.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: INTRODUCTION

In a mathematics classroom, after Calvin[1] shared an idea, Ms. Gray asked Bill the following:

Ms. Gray: Bill, is there anything else you would want to add to that thinking?

Bill: No.

Similarly, just a few minutes later, after Calvin shared another idea, Ms. Gray asked Mia the following:

Ms. Gray: Mia, what would you like to add to that?

Mia: Translate the function, so it like top half does or like the axis of rotation …

Why did Bill say no? And why did Mia offer a lengthy addition of her thinking? Many mathematics educators I worked with often find the reason from the characteristics of the individual students. Maybe, Bill is a shy student, or Bill "really" did not have anything to add. Mia must be a mathematically competent student. This kind of folk psychology is a pervasive way of how educators and researchers make sense of everyday interaction, attributing the patterns of interaction to the psychological characteristics of the students (Edwards, 1993).

Perhaps, this folk psychology is not entirely wrong; I do not deny that there is a difference in people's dispositions. However, this commonly accepted way of explaining patterns of interaction overshadows other alternative perspectives, which is troubling to me for two main reasons. First, if there is any issue or trouble, the problem is in students. This deficit-oriented view on students often leads to intervention approaches that aim to fix what is in the student's mind. Second, it renders the teacher in a helpless situation because the issue is not something the teacher can change or work against. More so, I found this kind of folk psychology

[1] All names and locations are pseudonyms.

emerged when I shared my concern about disproportionate participation by a particular group of students. For example, one teacher responded, “but he is painfully shy.” There seemed not much the teacher could do if participation caused the student’s pain due to the unavoidable shyness.

This dissertation serves my broader goal of dispelling this folk psychology about student participation in the mathematics classroom. I describe how the details of what teachers do and say matter, to the extent that can change the patterns of students’ participation. To put it another way, I aim to show that teachers’ talks and gestures can purposefully shape interactional contexts that are influential to students’ participation. For instance, Bill’s above “No” was afforded by Ms. Gray’s question, which does *not* assume that Bill had something to add. Contrast this question with Ms. Gray’s question to Mia, which presupposes that Mia already had something to add to what had been shared. Ms. Gray’s differing formulations offer an alternative explanation to the folk psychology I discussed earlier.

I start with the assumption that the teacher is “an engineer of learning environment” (Stein et al., 2008, p. 315). This metaphor of an engineer highlights the teacher’s deliberate effort to design a desirable learning space. Although the engineered learning space does not guarantee certain kinds of student participation, it can add normative forces to make a kind of participation more likely to occur. The metaphor of an engineer, however, does not fully capture what teachers do. The teacher and students are also co-builders of the learning environment. Like Ms. Gray’s question formulations, the meaning and function of what teachers do and say cannot be understood without what has been done or said by students. In this dissertation, I refer to this engineering and building process as the teacher’s *interactional work*, which is the primary object of study in this dissertation.

Interaction:

mutual or reciprocal action or influence

Work:

activity in which one exerts strength or faculties to do or perform something:

- **a:** activity that a person engages in regularly to earn a livelihood
- **b:** a specific task, duty, function, or assignment often being a part or phase of some larger activity
- **c:** sustained physical or mental effort to overcome obstacles and achieve an objective or result

(Merriam-Webster, n.d.)

Interactional work concerns mutual and reciprocal aspects of teachers' work. What teachers do and say matters, but in what context they do or say matters as well. In a discussion-rich classroom, what students have shared and done offers resources for the teacher to formulate the following action, and vice versa, what teachers said and done creates a new context for students' subsequent actions. Facilitating interaction is also an important part of professional work. Not only do teachers earn a livelihood by interacting with students, but interaction is the primary medium to build relationships with students and communicate important disciplinary ideas. Interaction can be a space to problem-solve difficult issues in mathematics education, such as achieving equitable disciplinary engagement at the classroom level (Engle et al., 2014).

This dissertation investigates the interactional work of three Advanced Placement (AP) mathematics teachers. I focused on AP mathematics classrooms for three primary reasons. First, AP mathematics programs (AP Calculus and AP Statistics) offer opportunities for students to learn college-level mathematics and potentially qualify for college credits based on their scores on the AP Exam. Equitable distribution of these opportunities is an important social concern, and teachers' interactional work plays an important role in the distribution of such opportunities. Second, AP mathematics have a long history of standard-based course and exam development. Examining AP mathematics classroom can illuminate how such standard-based curriculum and testing may shape mathematics instruction. Lastly, AP courses are college-level courses, but they are taught by high school mathematics teachers in high school settings. This setup offers a contact zone between mathematics education research in K-12 settings and undergraduate settings. Examining AP mathematics teachers' facilitation of classroom interaction may illuminate how the decades of reform effort in the K-12 setting can inform teaching and learning college-level mathematics.

I entered these AP classrooms with a particular orientation to *productive* and *equitable* mathematics discussions. First, by the term *productive*, I mean the class's orientation toward students as a source of knowledge (i.e., a producer) as opposed to passive receivers of knowledge (see, Boaler & Greeno, 2000). To attend to the productive features of classroom interaction, I examine if and how the authority of students manifests during discussions. Second, I use the term *equitable* to refer to leveraging the participation of historically marginalized students in AP mathematics classrooms, such as Black, Indigenous, other students of color, and girls. Gutiérrez (2012) argued that "[e]quity means fairness, not sameness" (p. 18). In other words, equitable interaction does not mean treating all students equally; it rather means teachers' countering work against existing marginalization patterns in AP mathematics programs.

To be clear, the aim of this dissertation is *not* to make a generalized claim about any of these teachers nor their classes. The focus is on discursive practices—ways of facilitating classroom interaction—that can move the class *toward* the direction of productive and equitable mathematics discussions. These momentary efforts may not appear significant in a cursory view, and this dissertation aims to unveil the important role of such fine-grained work that often goes unnoticed. Before I present the broader design of this dissertation and introduce three independent chapters, I share my personal history of how I arrived at the current study.

How I Got Here

My journey to mathematics education started when I was deciding which major and university to choose as I was graduating from high school in South Korea. I grew up in a family with my father working as an electrical engineer at a telecommunication company and my mother as a stay-at-home mother. Growing up in a tech-privileged family, I played with the most up-to-date computers and electronics. I was initially inclined to becoming an engineer, but I chose to major in mathematics education in the Teachers College at Kyungpook National University, the flagship university in my province. At that time, South Korea was experiencing economic hardship. My father was on the verge of losing his job multiple times. We saw many of his colleagues lost their jobs, and I remember how stressful it was for my father. Becoming a teacher would let me have a more secure career, I thought. I felt raising the next generation would be more meaningful than working in the corporate world. Back then, I did not know that I was still signing up to become an engineer, a kind of engineer who engineers learning environment rather than machinery.

My interest in classroom interaction developed when I taught at Bostrom Alternative High School in Phoenix Union High School District in Arizona as a first-year mathematics teacher. The high school served students whom the other larger high schools in the district did not serve well. Most of my students were expelled or “counseled out” from their neighborhood high schools due to a range of reasons (e.g., number of absences, getting involved in fights), and my high school served as their “second-chance” in the public school system. As a new immigrant to the United States, I was still learning to communicate in English. I initially struggled with keeping my students engaged with mathematical tasks. During my teacher preparation program at Oregon State University in the year before, I learned how to plan lessons that are aligned with NCTM standards and reform-based teaching practices. Yet, none seemed to work at the time when there was a big cultural and linguistic barrier between my students and me. “You don’t get me,” a student said. “You don’t know us,” another student said.

It took me years of trials and errors, learning from my students and their parents to understand where they were coming from and what their needs are. What my students wanted was to interact with a human who has emotion, opinion, and insecurity. They wanted to talk to a responsive person rather than a boring wall, test and negotiate social boundaries, share funny stories and laughs, and become recognized as a capable and valuable person in the classroom. For my students, negotiating boundaries, bending the rules, engaging constant push and pull with me was a way to become an important participant in the classroom community. This sharply contrasted with rigid rule-based classroom management, standard-based lesson planning, and the I Do, We Do, You Do type of repetitive instruction. I had trouble teaching my students, but it was not difficult for me to relate to their feelings about my teaching. I chose to study mathematics, not because of its “beauty” nor “universal truth” but because of the people who taught me and made me feel valuable. The students showed me that somehow, I lost this humanness and became unrelatable as their mathematics teacher. This was a turning point in my perspective; I wanted to learn more about teaching mathematics as a *social and interpersonal* endeavor.

After three years at Bostrom Alternative High School, I took the opportunity to teach AP

Calculus at Metro Tech High School, a vocation-focused magnet high school in the same school district. I was inspired by Jaime Escalante, who was depicted in the movie *Stand and Deliver*. I admired the intimate bond and comradery he created with his students and his students' academic achievement shown on the AP Calculus Exam. I applied what I learned from my students at Bostrom Alternative High School to expand the AP Calculus program at Metro Tech High School. The program grew from one section of AP Calculus AB and a group of a handful of students for AP Calculus BC to two sections of AP Calculus AB and one section of AP Calculus BC. My students showed a strong commitment to learning calculus, often staying in my classroom until 6 p. m. to study with their peers. Their excellent AP Exam scores reflected their efforts.

During this time, I also pursued National Board Certification issued by National Board for Professional Teaching Standards. With my students' help, I regularly video recorded my lessons and reflected on how my actions shape my students' participation during classroom interaction. Preparing my teaching portfolio in a cohort of teachers allowed me to see beyond the four walls of my classroom. With this learning experience, I gained more interest in teachers' everyday interaction with their students rather than the movie scenes that tend to be dramatized to attract more viewers. I started yearning to learn how other experienced mathematics teachers work toward facilitating productive and equitable mathematics discussions. I wanted to systematically examine such responsive facilitation instead of going through a series of trials and errors.

To pursue my interest in classroom interaction in a more systematic manner, I started the Ph.D. program at Michigan State University. In the past five years, through my research, I entered and observed multiple mathematics teachers' classrooms, which I could not do as a classroom teacher. I explored and applied multiple discourse analytic approaches to capture what I learned to be important for facilitating classroom interaction that was taken for granted as common sense and not taught in my teacher preparation programs. Among many approaches, I chose ethnomethodology and conversation analysis (EMCA) in this dissertation as my primary lens to view and understand social interaction. The main affordance of EMCA approaches is to investigate and describe teacher actions situated in a co-operative[2] classroom interaction. I apply this research approach consistently throughout this dissertation. I further discuss my theoretical view of teaching and learning mathematics and its relation to EMCA in the following section.

Broader View on Teaching and Learning Mathematics

Throughout this dissertation, I consistently use the term *participation* as a proxy for learning. Here, I briefly present the theoretical underpinning of the term because the theory of *learning-as-participation*—the underlying goal of teaching in this dissertation—may not be familiar for some readers. Also, EMCA approaches are chosen to operationalize social interactions. I also present a few examples of applications of EMCA in mathematics education.

Situated Learning

To broaden how people think of learning, Sfard (1998) and Wenger (1998) juxtaposed two different ways to view learning. On the one hand, learning is often equated with the acquisition of one's knowledge that is assumed to be in their mind. The perspective of *learning-as-acquisition* (Lave & Wenger, 1991) is a familiar way to consider learning in current school systems. In mathematics classrooms, teachers carefully design and implement lessons that can

---

[2] Goodwin (2017) makes the distinction between *co-operative* and cooperative. Co-operative action does not assume that participants are working toward a common goal. For example, playing chess is a co-operative activity but not cooperative. Two players work together to achieve an activity of chess game, but they work toward competing goals, which is to win over the other player.

develop students' mathematical knowledge. Lessons are followed by quizzes and tests that assess how much knowledge and skills individual students can demonstrate. In universities, the body of knowledge is parsed into different fields, and departments and colleges are organized to efficiently transmit the knowledge in a piecemeal manner (Kennedy, 2016).

On the other hand, Wenger (1998) offered an alternative perspective of *learning-as-participation*. From this situated learning perspective, learning is participating in the practice of a particular community, which Lave and Wenger (1991, p. 29) refer to as *legitimate peripheral participation*. Apprenticeship models of education commonly adopt this view. Prospective teachers, for instance, participate in teaching internships under the supervision of mentor teachers. As prospective teachers participate in various teaching practices in the community of the teachers in the building and the profession, the prospective teachers become practicing teachers with more nuanced ways to notice and respond to important aspects of teaching with the necessary tools and resources.

Although learning-as-participation is often contrasted with learning-as-acquisition, these perspectives are not opposite ends of a spectrum (Wenger, 1998); they rather highlight different integral parts of the complexity of learning (Sfard, 1998). For instance, acquiring mathematical knowledge is an important part of the process of participating in the community of mathematicians. In turn, participating in the community of mathematicians offers opportunities for learners to acquire knowledge that is valued in the community. The difference between these two views is their theoretical foci: one on the individual mind, the latter on the process through which an individual becomes a social being. This dissertation aims to understand teachers' work as a social and interpersonal endeavor rather than individual thinking. Therefore, I take the perspective of learning-as-participation and its close relationship with the teacher's facilitation of classroom interaction.

EMCA and Social Interaction

> Social interaction is the primordial means through which the business of the social world is transacted, the identities of its participants are affirmed or denied, and its cultures are transmitted, renewed, and modified. Through processes of social interaction, shared meaning, mutual understanding, and the coordination of human conduct are achieved. (Goodwin & Heritage, 1990, p. 283)

> Classroom teaching is relational work: Working on learning in the classroom involves concerted action by at least two people, the teacher and a student. (Lampert, 2010, p. 22)

As Goodwin and Heritage (1990) and Lampert (2010) stated above, social interactions are inherent in teaching and learning. EMCA approaches are particularly powerful when researchers examine social interactions in and for teaching, co-construction of mathematical meaning, and participation in disciplinary practices (Ingram, 2018). EMCA also offers a systematic view on social conduct with a set of tools to theorize tensions and nuances related to social interactions in and for teaching (Waring, 2016).

The primary concern in EMCA is to understand normative practices (often referred to as *participants' methods*) of social conduct, particularly talk-in-interaction. As the term "talk-in-interaction" refers, EMCA attends to forms of talk (e.g., lexical, syntactic, speech features) and other visible social cues (e.g., gaze direction, gesture) in interactional contexts (Schegloff, 2007). EMCA views that talk-in-interaction as orderly and normative. Norms are an essential feature of talk-in-interaction for interlocutors to accomplish even basic features of social conduct such as turn-taking, designing one's action, and ascribing others' actions (Heritage & Clayman, 2010).

Applying EMCA to investigate teaching and learning is not new to mathematics education (Ingram, 2018). Krummheuer (1995, 2007, 2011) introduced *argumentation* as a way to view students' participation in everyday classroom interaction. Informed by ethnomethodology, Krummheuer (1995) defined argumentation as "techniques or methods of establishing the claim of a statement" (p. 232), and it is a kind of an "accounting practice" (Garfinkel, 1967, p. 1), which makes an event, setting, or action accountable (or intelligible) for participants and constructs a version of an intersubjective reality. Forrester and Pike (1998) applied conversation analysis to examine what it means to learn to estimate as manifested in classroom interaction. Their analysis of fine details of the speech showed how the teacher and students generate the discursive object of estimating, in contrast to measuring. They found that students sanctioned a group member's participation if the member did not comply with the negotiated meaning of estimating (e.g., not using a ruler). Application of EMCA commonly highlights that learning is a social process through which the teacher and students negotiate not only mathematical meanings but also what kinds of participants they are. In this dissertation, I applied EMCA approaches to attend to various features of classroom interaction, such as turn-taking, argumentation, changing the interactional setting, positioning of students, with an eye on productive and equitable mathematics discussions.

Participants and Data

In this dissertation, I worked within two public high schools and examined three AP mathematics teachers' facilitation of classroom interaction. Here, I briefly describe the community and school contexts (including information about how students tended to get to AP courses) and introduce these three teachers: Ms. Gray, Ms. Hill, and Mr. Robinson.

Both Oaktree High School (OHS) and Riverside High School (RHS) are located in suburban areas in the Midwestern region of the United States, but there are differences in the characteristics of the two communities. OHS is located near one of the main public universities in the region. Many members of the community are either employees or students in the university. Roughly 30% of students were qualified for the free or reduced lunch program, and OHS has a wider range of racial diversity compared to RHS (see Table 1). RHS is located in another suburban area. The community is mostly residential; many of the community members commute to the nearby urban center for work. The community members in this community are predominantly White, and this is reflected in the racial distribution in RHS. Roughly 23% of students are qualified for the free or reduced lunch program. Both schools offer all AP mathematics courses: AP Calculus AB[3], AP Calculus BC, and AP Statistics. At RHS, Mr. Robinson was the only AP Calculus teacher, and he taught two sections of AP Calculus AB and one section of AP Calculus BC. At the same school, Ms. Hill was the only AP Statistics teacher, and she taught two sections of AP Statistics. At OHS, Ms. Gray was one of the three AP Calculus teachers. She taught two sections of AP Calculus AB. Two other mathematics teachers taught additional sections of AP Calculus AB and BC.

[3] AP Calculus AB is equivalent to the first calculus course in college settings (e.g., Calculus I). AP Calculus BC includes the contents of AP Calculus AB in addition to the contents that appear in the second calculus course in colleges (e.g., Calculus II).

Table 1. Race and Ethnic Distributions

Schools/ Classes (# of students)	Asian	Black/ African American	Hispanic/ Latinx	White	Multiracial
Riverside HS	3%	5%	8%	79%	3%
Oaktree HS	7%	20%	8%	58%	6%
Class A (12) Mr. Robinson	8.3% (1)	0% (0)	16.6% (2)	75% (9)	0% (0)
Class B (20) Mr. Robinson	0% (0)	0% (0)	5% (1)	90% (18)	5% (1)
Class C (27) Ms. Hill	11.1% (3)	3.7% (1)	0% (0)	85.1% (23)	0% (0)
Class D (26) Ms. Hill	7.7% (2)	0% (0)	0% (0)	92.3% (24)	0% (0)
Class E (20) Ms. Gray	5% (1)	5% (1)	0% (0)	90% (18)	0% (0)
Class F (27) Ms. Gray	14.8% (4)	7.4% (2)	3.7% (1)	74% (20)	0% (0)

Note. The percentages of American Indian/Alaskan Native and Native Hawaiian/Pacific Islander were less than one percent in both schools, and they are not included in this table. The school level information is gathered from National Center for Education Statistics (https://nces.ed.gov/ccd/schoolsearch). The classroom level information was gathered from the participating teachers based on their knowledge about students' self-identification and school-provided demographic information.

Here, I briefly introduce the three teachers. First, I met Ms. Gray at OHS when I was looking for an opportunity to support historically marginalized students in their AP Calculus courses. Ms. Gray is a White woman with 13 years of teaching experience. In addition to the two sections of AP Calculus AB, she also taught three additional sections of Geometry. She had taught AP Calculus for five years, and she often collaborated with her colleague who was also teaching AP Calculus by sharing lesson materials (e.g., handouts, video lessons, slideshows). In addition, Ms. Gray had six years of history collaborating with university faculty members. Through her collaborations, she engaged in multiple action research projects to enhance her discourse practices and to make her classroom discussion more equitable. I approached Ms. Gray to facilitate after-school tutoring sessions in her classroom for students who would like to get additional support. As I worked with the students in the tutoring sessions, I had multiple opportunities to talk to Ms. Gray about her AP Calculus lessons and observe her lessons. I noticed a few discourse practices, such as her use of craft sticks to call on students after a warm-up practice randomly. I became curious about how her discourse-oriented professional development may have shaped the range of her discourse practices.

Second, I met Mr. Robinson, an AP Calculus teacher from RHS, at the AP Reading (national grading session for the official AP Calculus Exam). Mr. Robinson is a White man and, at the time, had four years of grading the AP Calculus Exam. He had taught AP Calculus for 16 years with a total of 24 years of teaching experience. Mr. Robinson shared his commitment to teaching his students to think and communicate mathematical ideas critically, which is an important aspect of the written portion of the AP Calculus Exam. I wanted to know how his facilitation of classroom interaction may differ from that of Ms. Gray, who never participated in the grading session. In addition to the two AP Calculus AB courses, he also taught one section of AP Calculus BC and another section of Algebra.

Third, I came to know Ms. Hill, a White woman and AP Statistics teacher at RHS, through my university colleague. My colleague recommended Ms. Hill for my study because of her commitment to making her instruction equitable across racial groups. When I met Ms. Hill, she shared with me that she was using Think-Pair-Share and looking into facilitating Fishbowl discussions to help students feel safe and support students' participation in her classroom. Ms. Hill also majored in communication for her undergraduate degree, and I wondered how her academic background might have shaped her facilitation of classroom interaction. She had taught

AP Statistics for seven years with a total of 13 years of teaching experience. She taught two sections of Probability and Statistics in addition to her two sections of AP Statistics. Prior to gathering data, I regularly visited both Mr. Robinson's and Ms. Hill's classrooms to become familiar with the students and the school.

I obtained consent from participants prior to any recording. The current study received Institutional Review Board approval (MSU Study ID: STUDY00003860), and I handled collected data confidentially following the approved protocol. The primary data source of the current study consists of three video-recorded lessons from each AP mathematics class. OHS had a regular semester system with varying lengths of lessons depending on the day of the week (50-60 minutes). RHS had a trimester system with a longer period of lessons (70 minutes). Figure 1 below illustrates a typical classroom setup of these classrooms and the placement of recording equipment. Capturing detailed speech features is crucial for an EMCA-informed analysis, so in the middle of the classroom, I placed four audio recorders to capture students' speech that two camera recorders on the opposite sides of the classroom may not be able to capture. Camera 1 captured the teacher's gestures and other bodily movements, and Camera 2, with a wide-angle lens, captured the students' gestures and gaze directions. I also collected additional data to supplement the video data during analysis. I gathered copies of task sheets or handouts and pictures of student work as necessary. These additional data provide contextual information that is relevant for social interactions. In addition, I maintained a data log to keep a brief record of classroom activities. This log allowed me to organize each lesson into smaller activities for further fine-grained analyses.

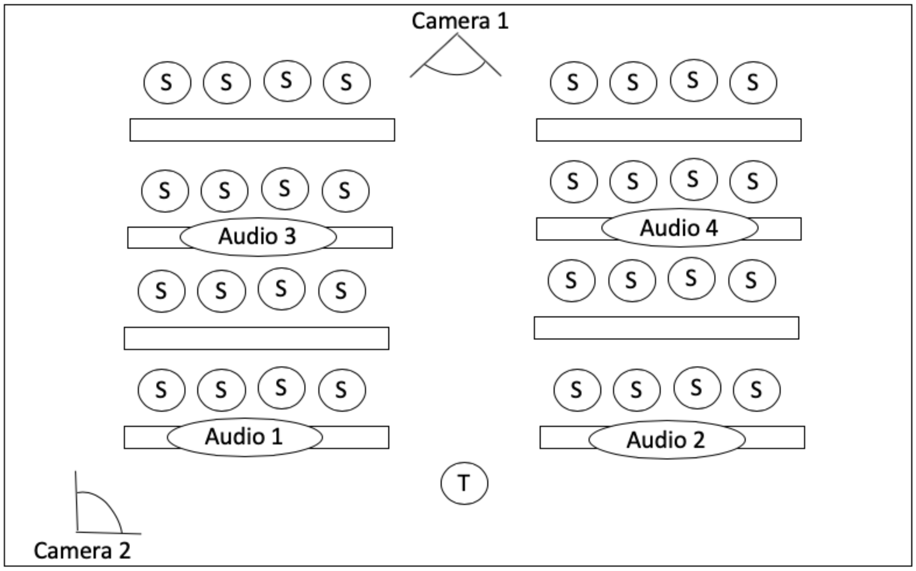


Figure 1. Camera and Audio Recorder Positions During Instruction

To decide on when to gather data, I asked the teachers for an opportunity to observe classroom discussions that advance students' thinking at the beginning of the spring semester in the year 2020. Ms. Gray and Mr. Robinson both said their lessons on finding volumes of a revolved region would be great lessons. Finding volumes of a revolved region is one way to apply integral calculus. This topic appears in the AP Calculus curriculum after students learn about fundamental concepts and skills of differential and integral calculus. Ms. Hill said her lessons on the central limit theorem would be a great fit. The central limit theorem appears in the middle of the AP Statistics curriculum after students learn about different types of data, methods of collecting data, and probability distributions. The central limit theorem later serves as a fundamental concept to explain inferential statistics using quantitative and categorical data. I gather data from three consecutive lessons that make a topical instructional unit. All teachers taught two identical sections each day, and I recorded both lessons.

Table 2. Sequences of Observed Lessons

Teacher / Classes	Day 1	Day 2	Day 3
Mr. Robinson / AP Calculus Class A & B	• Reviewing homework problem on finding area • Lesson on disc method (Mar 4, 2020)	• Lesson on washer method (Mar 5, 2020)	• Lesson on finding volume with known cross-sections (Mar 6, 2020)
Ms. Hill / AP Statistics Class C & D	• Reese's Pieces sampling activity (Jan 13, 2020)	• Lesson on central limit theorem (Jan 14, 2020)	• Continuing lesson on central limit theorem (Jan 15, 2020)
Ms. Gray / AP Calculus Class E & F	• Reviewing homework problems on disc method, • Lesson on washer method (Feb 28, 2020)	• Solving practice problems on disc and washer methods (Mar 2, 2020)	• Lesson on finding volume with known cross-sections (Mar 3, 2020)

Mr. Robinson's Class A and B occurred during the second (9:12-10:23) and fourth hours (12:18-1:28) of the typical daily schedule. Ms. Hill's Class C and D happened in the first (7:55-9:05) and third (11:00-12:11) hours. Ms. Hill's Class E and F were the fifth (12:33-1:30) and sixth (1:37-2:35) hours of the typical schedule. Throughout this dissertation, I marked each excerpt with class, day, and time in the lesson. For instance, "F1-20" means the excerpt came from Class F, Day 1, and 20 minutes into the lesson.

I initially treated these data through unmotivated looking (Psathas, 1995); I repeatedly watched the recordings while bracketing my assumptions about how the teachers and students interact with each other as much as possible. I did this initial analysis with an aim to understand how the teachers worked toward facilitating a productive and equitable mathematics discussion. For example, I noted the moments when minoritized students in the class shared their thinking and students offered justifications. In this initial stage, I started to notice subtle differences in minoritized students' participation (e.g., A student seemed to produce a response with or without delay) and some of the features of teachers' facilitation, such as the occurrences of partner talk in

Ms. Hill and Ms. Gray's lessons and the salient presence of the AP Exam in Mr. Robinson's lessons. I developed these initial noticing as formal analyses using the Jefferson Transcription System (see Appendix), and I further describe these analytic processes in the following three chapters.

Overview of Chapters

As I progress from Chapter 2 through Chapter 4, I consider different layers of classroom interaction (e.g., turn-taking, interactional settings, the institutional context of the AP Exam) to highlight multiple aspects of teachers' interactional work. I intended these chapters to be independent manuscripts that I will submit for journal publications. Therefore, these chapters speak to different audiences in the journals that I specify below.

Chapter 2 concerns minoritized students' participation in whole-class discussions. It used data when the three Black students participated in whole-class discussions in Ms. Gray's Class F and Ms. Hill's Class C. I draw from the literature on turn-taking and argumentation to examine how the participation of the three students started and how their turn-taking led to an intellectual contribution to the development of argumentation. I discuss the teachers' interactional work in relation to supporting minoritized students' participation. I prepared this chapter to submit to *Journal for Research in Mathematics Education*.

Chapter 3 attends to Ms. Gray and Ms. Hill's use of partner talk, which consistently appeared in their lessons. My analysis attends to discursive practices that facilitate partner talk based on what the teachers do and say before, during, and after the partner talk. The chapter highlights how the teachers skillfully shift between interactional settings (i.e., whole-class discussion, partner talk) to facilitate focused and accountable space for every student in the classroom. I plan to submit this chapter to *Linguistics and Education*.

Chapter 4 focuses on the moments when Ms. Gray and Mr. Robinson refer to the AP Exam during instruction. I examine how the institutional context of the AP Exam may have shaped the facilitation of classroom interaction. For this analysis, I also draw from positioning theory to consider broader storylines related to the AP Exam. I discuss the unintended consequence of the AP Exam in everyday classroom interaction and how the influence is mediated by the teachers' professional development backgrounds. I will submit this chapter to *Educational Studies in Mathematics*.

Chapter 5 concludes this dissertation with a summary of contributions of the three chapters to the field of mathematics education and education more broadly. I also reflect on my learning as a researcher through this dissertation and offer suggestions for further research.

APPENDIX

APPENDIX: TRANSCRIPTION NOTATIONS

. / , / ¿ / ?	falling / slightly rising / mid-rising / rising intonation
(.*x*)	pause in 1/10 sec
(.)	pause shorter than 0.3 second
[words]	onset and offset of overlapping talk
wo:rds	vowel elongation
words	emphasis
( )	unrecoverable speech
(words)	dubious hearings
↑ / ↓	rise / fall in intonation
=	latching turns
hh / .hh	out / in breathing
°words°	lower volume
WORDS	higher volume
>words<	faster speech
<words>	slower speech
£words£	smiley speech
((words))	transcriber comment
((words))	gestures and other movements

Note: These notations are adopted from Jefferson (2004). More detailed instruction on transcribing can be found in Hepburn and Bolden (2017)

REFERENCES

REFERENCES

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171–200). Ablex Publishing.

Edwards, D. (1993). But what do children really think? Discourse analysis and conceptual content in children's talk. *Cognition and instruction*, *11*(3-4), 207-225.

Engle, R. A., Langer-Osuna, J. M., & McKinney de Royston, M. (2014). Toward a model of influence in persuasive discussions: Negotiating quality, authority, privilege, and access within a student-led argument. *Journal of the Learning Sciences*, *23*(2), 245-268.

Forrester, M. A., & Pike, C. D. (1998). Learning to estimate in the mathematics classroom: A conversation-analytic approach. *Journal for Research in Mathematics Education*, *29*(3), 334-356.

Garfinkel, H. (1967). *Studies in ethnomethodology.* Prentice Hall.

Goodwin, C. (2017). *Co-operative action*. Cambridge University Press.

Goodwin, C., & Heritage, J. (1990). Conversation analysis. *Annual review of anthropology*, *19*(1), 283-307.

Gutiérrez, R. (2012). Context matters: How should we conceptualize equity in mathematics education? In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in discourse for mathematics education: Theories, practices, and policies* (pp. 17–33). Springer.

Heritage, J., & Clayman, S. (2010). *Talk in action: Interactions, identities and institutions*. Wiley-Blackwell.

Ingram, J. (2018). Moving forward with ethnomethodological approaches to analysing mathematics classroom interactions. *ZDM*, *50*(6), 1065-1075.

Kennedy, M. (2016). Parsing the practice of teaching. *Journal of Teacher Education*, *67*(1), 6-17.

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures*. Lawrence Erlbaum.

Krummheuer, G. (2007). Argumentation and participation in the primary mathematics classroom: Two episodes and related theoretical abductions. *The Journal of Mathematical Behavior*, *26*(1), 60-82.

Krummheuer, G. (2011). Representation of the notion "learning-as-participation" in everyday situations of mathematics classes. *ZDM*, *43*(1), 81-90.

Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean? *Journal of teacher education*, *61*(1-2), 21-34.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge University Press.

Merriam-Webster. (n.d.). Interaction. In *Merriam-Webster.com dictionary*. Retrieved June 11, 2021, from https://www.merriam-webster.com/dictionary/interaction

Merriam-Webster. (n.d.). Work. In *Merriam-Webster.com dictionary*. Retrieved June 11, 2021, from https://www.merriam-webster.com/dictionary/work

Psathas, G. (1995). *Conversation analysis: The study of talk-in-interaction.* Sage Publications, Inc.

Schegloff, E. A. (2007). *Sequence orgnaization in interaction: A primer in conversation analysis.* Cambridge University Press.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational researcher*, *27*(2), 4-13.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical thinking and learning*, *10*(4), 313-340.

Waring, H. Z. (2016). *Theorizing pedagogical interaction: Insights from conversation analysis*. Routledge.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* Cambridge University Press.

CHAPTER 2: INTERACTIONAL WORK OF AP MATHEMATICS TEACHERS TO SUPPORT BLACK STUDENTS' PARTICIPATION IN ARGUMENTATION

Introduction

A group of teacher candidates[4] video recorded their facilitation of a Number Talk (Parrish, 2010) and analyzed which students participated. As they begin to reflect on racialized patterns in their classroom discourse, the following exchange happens:

> Molly: I was looking at like I was like, 'Oh no, my [classroom observation] data, only the people who were contributing are White.'
>
> Katie: Right! I had the same issue when I was giving my Number Talk, I was like, 'Kids, stop raising your hand.' Like Josh kept raising his hand in the third hour. (Byun, 2020)

Historically, Science, Technology, Engineering, and Mathematics (STEM) classrooms have been a *marked territory* of White and Asian[5] male students (Margolis et al., 2008). Like Molly and Katie did, with a race-conscious lens, mathematics teachers can not only become aware of this persistent racial inequality manifested in their teaching, but they can also realize their professional responsibilities to support minoritized[6] students' participation and to manage well-intended yet dominating students, such as Josh in the above scene. Although teachers alone would not be able to fully redress racial inequities in mathematics education, what teachers do in everyday classroom interaction can mitigate the racial marginalization at the classroom level

---

[4] All names and locations are pseudonyms.
[5] This does not include all Asian people since Asian as a racial category encompasses ethnicities with a wide range of representations in STEM fields (see Lam, 2015)
[6] Adopted from Gillborn (2010), I use *minoritized* people instead of racial minorities to highlight the social and historical process of racial marginalization. Other scholars also use Black, Indigenous, and people of color (BIPOC) to recognize severe systemic racial injustices faced by Black and Indigenous people.

(Gutiérrez, 2012). This chapter focuses on discourse practices that may work as a counterforce to such marginalization by supporting minoritized students to take up social space, which is often done by dominating students.

The current study is motivated by the invaluable insight on students' racialized experiences of learning mathematics based on minoritized students' narratives (Berry, 2008; Gholson & Martin, 2014; Joseph et al., 2019; Stinson, 2010; Larnell, 2016; Leyva et al., 2021). One of the common themes in the findings is that mathematics educators are highly influential in the students' experiences; their teaching practices can both facilitate and deny minoritized students' opportunities to participate in mathematical activity. Joseph and colleagues (2019), in particular, highlighted the importance of inclusive pedagogy within which the voices of minoritized students would be integrated with robust mathematics content knowledge. Despite the importance of everyday teaching practices with respect to minoritized students' learning experiences, Jackson and Wilson (2012) reported that there is very little research that focuses on the teaching practices that can better support minoritized students (especially Black students) in participating in mathematical activities.

To identify potential ways teachers may support minoritized students' opportunities to participate, I examined the social interaction between two Advanced Placement (AP) mathematics teachers and their three Black[7] students. By applying ethnomethodological approaches—namely, conversation analysis and interactionism (see, for an overview, Ingram, 2018)—the current study aims to "penetrate to the core interactional work of teaching and learning" that supports "African American students to participate in rigorous mathematical

---

[7] I use Black (rather than African American) following the students' self-identification.

activity" (Jackson & Wilson, 2012, p. 380). By applying conversation analysis (Sacks et al., 1974), I examined teaching practice that supported the students in taking up the conversation floor (i.e., taking a turn during a discussion). By applying interactionism (Krummheuer, 2007), I investigated how the teachers further supported the students in participating in collective argumentation. To do so, I used the following questions as a guide:

In these two AP mathematics classrooms,

1. how did the teachers allocate the turns taken by the three students during whole-class discussions?
2. how did the teachers support the three students during the process of collective argumentation?

These questions are informed by the literature on classroom interactions. First, I use the verb "allocating" to signal that the teachers are often in authority to facilitate turn-taking (who speaks and when) during whole-class discussions (Oyler, 1996). This teacherly authority, in turn, highlights the teachers' professional responsibility to counter the persistent racialized participation pattern in mathematics classrooms (Byun et al., 2020). Second, as suggested by Conner and colleagues (2014), the current study considered the teachers' role of supporting students in participating in collective argumentation, which offered a lens to examine how the voices of the students became integrated (or not) in the ongoing progression of the classroom discourse.

It is important to note that the goal of the current study is not to prove the brilliance of the three Black students. Following critical Black scholars (Gholson et al., 2012; Leonard & Martin, 2013), I take the three students' brilliance as an *axiom*, which is self-evident and needs not to be proven. The goal is rather to examine the teachers' role in bringing the students' brilliance to the fore in the whole-class discussion and integrate their voices in the process of collective argumentation. This is important because teaching practices may disrupt or reinforce pervasive racial narratives and stereotypes that undermine minoritized students' intellectual contributions (Shah & Leonardo, 2016; Leyva et al., 2021). I attended to the details of the talk to gain a more nuanced understanding of how the subtle differences in teachers' discourse practice might shape the students' participation. In doing so, this chapter contributes to the discussion about *how*-question; "How do I [a mathematics teacher] support African American learners in mathematics?" (Jackson & Wilson, 2012, p. 365).

I begin this chapter with a brief overview of the discussion on equity issues in calculus and statistics courses in both high school and college settings, with a particular focus on minoritized students' participation. In the review, I argue for the importance of detailed examinations of classroom interaction to better understand equity issues at the classroom level. Based on two ethnomethodological approaches in mathematics education (conversation analysis and interactionism) I present analyses of five episodes to illustrate how the teachers supported the participation of the three students and consider the advantages and limitations of different approaches. I conclude this chapter with implications for teachers who aim to better support minoritized students and the chapter's contribution to broader discussions on facilitation of classroom discourse.

Equity Issues and Classroom Interaction in AP Mathematics

Introductory calculus and statistics courses in colleges play a crucial role in many fields. For example, "calculus is a gatekeeper course for engineering majors" (Moore, 2005, p. 536), and statistics coursework is one of the core requirements to apply for nursing programs (Hayat et al., 2013). Given the significant status of these courses, ensuring equitable opportunity to learn calculus and statistics is an important social concern in relation to issues of access. Data show the historical and persistent underrepresentation of minoritized students in these courses (Champion & Mesa, 2017; Office for Civil Rights, 2018) and large portions of minoritized students being enrolled in non-credit-bearing remedial mathematics courses (Larnell, 2016). The disproportionate representation of minoritized students is not only unjust in itself, but it also adds an unjust psychological burden on the minoritized students to prove themselves academically while obscuring social and institutional barriers (Larnell, 2016; Leyva et al., 2021).

The disproportionate representation of minoritized students in university courses is, in part, a reflection of inequity issues that stem from the K-12 educational system. AP mathematics courses in high schools are becoming more important in equity discussions because an increasing number of students are taking university-level calculus and statistics courses in high school AP classrooms. Bressoud and colleagues (2015) estimated that "roughly three-quarters of all students who eventually study calculus take their first calculus course in high school" (p. vii). Moreover, many colleges and universities consider students' AP course enrollment in their admission decision-making (Geiser & Santelices, 2006), and AP mathematics programs significantly impact students' future learning trajectories and careers in STEM fields (Bressoud, 2021; Robinson, 2003).

Although there has been some progress in regard to increasing the number of minoritized students enrolling in AP courses, the progress does not reflect the better achievement of the students. In the case of Black male students, Tawfeeq and colleagues (2013) reported that "[w]hile the population of African American male test-takers increased (percentage-wise) at a faster rate, their national mean score from 1997-2009 is only about 2.17 and is slightly trending downward" (p. 92). In other words, despite the increased enrollment of Black male students, inequities in access to quality instruction still persist. To be clear, I do not locate the problem of enrollment and achievement with the students themselves. Multiple studies have reported, in fact, that Black students in AP courses experience low expectations from White teachers and classmates (Flowers & Banda, 2019; Diamond, 2006). Similarly, in the study by Hallett and Venegas (2011), students from low-income communities of color expressed that “nice,” “caring,” or “awesome” teachers often did not provide necessary learning opportunities and high expectations (p. 480). Although widening access to AP enrollment remains a critical issue in the field, examining social interactions in these classrooms is necessary to understand and address some of the equity concerns when minoritized students take these courses.

The ongoing reform efforts in these AP courses, which focus on interpersonal communication and justification, corroborates the need for examining social interactions. Boaler and Greeno (2000) compared two types of AP calculus instruction. Didactic teaching facilitated procedure-driven learning, whereas discussion-based teaching emphasized “relationships—between the different aspects of mathematics as well as the people in the class” (p. 178). Additionally, the recent emphasis on communication and justification in AP mathematics courses (College Board, 2019a, 2019b), which are well aligned with recommendations made by the National Council of Teachers of Mathematics (NCTM) and the Common Core State Standards Initiative, highlights the importance of equitable participation in classroom discourse. This push for interpersonal communication in teaching and learning mathematics makes the classroom interaction, in particular, an important avenue to examine racial equity. Research has shown that, through social interactions, the teacher and students (re)constitute relationships and treat each other in different ways (e.g., treat oneself or others as an authority) (Esmonde & Langer-Osuna, 2013; Herbel-Eisenmann, 2009; Lampert et al., 1996). These interactions, even without apparent racial undertone, influence the ways minoritized students perceive themselves and participate in everyday learning activities (Joseph et al., 2019; Leyva et al., 2021; Shah & Leonardo, 2016).

Researchers have conducted both qualitative and quantitative analyses of classroom interaction and made important progress in categorizing teaching practices that are associated with the increased achievement of minoritized students (e.g., Battey et al., 2016; Wilson et al., 2019). An important task, however, remains. Jackson and Wilson (2012) write:

> An important next step, in our view, is for researchers to move beyond general descriptions and detail, at the interactional level, how successful teachers negotiate productive relationships and establish norms with African American students (p. 373).

The current study takes this next step with detailed interactional analyses drawing from ethnomethodological approaches. I focus on three Black students in the two AP mathematics classrooms for a few reasons. First, equity research needs to go beyond the rhetoric of "Mathematics for All" (Martin, 2019), which does not "*focus solely on gains for the collective Black*" (Martin, 2015, p. 21, emphasis original). Martin's critique implies that, at the classroom level, teachers need to be equipped with discourse practices that can *prioritize* the participation of minoritized students to counter existing racial marginalization. Second, Black students are hyper-minoritized in AP mathematics programs (see Champion & Mesa, 2017). This is due to the broader societal practices (e.g., redlining and housing segregation) that funnel Black students to schools that are often under-resourced, and, at the school level, inequitable practices such as racialized patterns in tracking and course recommendations (Faulkner et al., 2014). Against this backdrop of the broader social context, equitable teaching requires redressing the way the disproportionate representation further marginalizes individual Black students in mathematics classrooms (Larnell, 2016; Leyva et al., 2021). This chapter takes an interactional approach at the classroom level to detail potential ways to support minoritized students toward opportunities to participate.

Existing frameworks of teacher's discourse practices (Chapin et al., 2009; Michaels et al., 2010; Herbel-Eisenmann et al., 2013) offer a starting point to examine how teachers can support minoritized students in classroom interaction. For example, one of the Teacher Discourse Moves *inviting student participation* (Herbel-Eisenmann et al., 2013) is relevant to the current study as a way to encourage minoritized students' participation. When the teacher works to invite students to participate *equitably*, however, the teacher needs to be able to prioritize minoritized students' participation over that of dominating students to counter existing patterns of marginalization. Equity-oriented teachers have worked to evenly distribute the opportunity to participate among a diverse group of students in small groups (Nasir et al., 2014) and whole class (Busby et al., 2017; Herbel-Eisenmann & Shah, 2019), but the field needs more detailed examinations to understand the teachers' facilitation of classroom interaction that allowed them to achieve their equity-related goals. For example, it is important to understand how a teacher can invite minoritized students to the conversational floor while managing well-intended dominating students; the exact issue that the two teacher candidates, Molly and Katie, faced in the opening to this chapter.

The current study advances the understanding of mathematics teachers' discourse practices by detailing their facilitation of whole-class discussions that support minoritized students' participation in AP classrooms. While centering on the three Black students' participation, I attend to turn-taking practice and collective argumentation, two complementary features of classroom interaction. Through the lens of turn-taking, I attended to the interactional process that initiated the students' participation. With collective argumentation, I examined how the students contributed to the co-operative process of developing mathematical arguments.

Combining these two lenses together, I focused on the facilitation of classroom interaction, in which the three students take up the conversational floor and participate in developing mathematical thinking (Engle et al., 2014; Hand, 2012). Next, I discuss how I approach these two aspects of classroom interaction from ethnomethodological approaches.

Ethnomethodological Approaches

In mathematics education, researchers have used ethnomethodological approaches (e.g., conversation analysis, discursive psychology, interactionism) for fine-grained analyses of classroom interaction (Ingram, 2018). Ethnomethodological approaches focus on social processes (often referred to as *participants' methods*), which reflexively reveal the local social order that participants are constructing (Garfinkel, 1967). For example, researchers used conversation analysis and discursive psychology to reveal socially negotiated meanings of estimation and measurement (Forrester & Pike, 1998) and knowing, remembering, and understanding (Barwell, 2013; Ingram, 2020). Researchers also applied interactionism to examine the development of arguments (Krummheuer, 1995) and norms that mediate the process (Yackel & Cobb, 1996). The current study applied both conversation analysis and interactionism to examine the constructs of turn-taking and collective argumentation. In this section, I briefly present these two constructs.

Turn-Taking: A Conversation Analytic Approach

Goffman (1981) refers to a *turn* as "an opportunity to hold the floor, not what is said while holding it" (p. 23). In other words, attending to turn-taking allows the analyst to examine how participants distribute the opportunity to talk rather than the content of the talk. The analysis of turn-taking (Sacks et al., 1974), details different methods of turn-allocation and attends to the effects of these methods on students' participation in the discussion. Attending to turn-taking is one way to operationalize the social process of distributing learning opportunities during discussions on a fine-grained scale (Engle et al., 2014).

Turn-taking practices are often taken for granted, and the primary task of conversation analysis is to unveil such seen-but-unnoticed practices (Waring, 2016). For example, in language education, scholars attended to how teachers distribute turns evenly across the classroom. These studies showed that students negotiate for more or less participation than their fair share of opportunities to participate in classroom discussions (Allwright, 1980), and provide some ways teachers can invite turn-taking from a wider range of students than the students who tend to respond quickly (e.g., Waring, 2013; Waring, 2014). These studies illustrate how turn-taking is a co-operative process in which both the teacher and students negotiate who will be the next speaker. In a whole-class discussion, for example, students display their (un)willingness to speak with both their talks and gestures (e.g., "Josh kept raising his hand"), which in turn influences the teacher's turn-allocation.

McHoul (1978) and Mehan (1979) made early contributions to turn-taking practices in whole-class discussions. Their studies showed the unique features of turn-taking in a classroom setting compared to everyday conversation. In the classroom, the institutional identities of being a teacher or a student constrain or allow ways to participate in turn-taking practice. In particular, Mehan (1979) identified the teacher's three basic turn-allocation methods, which are (a) individual nomination (naming a student to respond), (b) invitation to bid (asking to raise hand), and (c) invitation to reply (inviting the whole class to respond in unison). In this chapter, I refer to these methods as *conventional* methods. Teacher educators presented other methods of allocating turns, although they are not as common as conventional methods. Michaels and colleagues (2010), for example, discussed having the student who is currently taking a turn determine the next speaker. As Michaels and colleagues (2010) stated, each turn-allocation method has both advantages and limitations, and the current chapter mainly focuses on its uses to initiate minoritized students' turns.

Collective Argumentation

While turn-taking relates to the students' opportunity to talk, collective argumentation is about the content of the talk: the students' contribution to the co-operative development of mathematical thinking. Within the tradition of interactionism, Krummheuer (1995) introduced Toulmin's (2003) framework of argumentation to organize the mathematical content of talks that emerged in the classroom interaction. From an ethnomethodological perspective, argumentation is an "accounting practice" (Garfinkel, 1967, p. 1) to achieve an intersubjective meaning of mathematical objects and processes between participants in everyday mathematics classroom situations (Krummheuer, 1995). In other words, argumentation is a building block of unfolding negotiation on the mathematical meaning among participants. Thus, participation in argumentation is a critical part of learning mathematics from a sociocultural perspective (Krummheuer, 2007).

Argumentation often happens in subtle ways by multiple people in everyday classroom interaction (as opposed to an interview or experiment setting). Therefore, following Conner and colleagues (2014), I use the term *collective argumentation* "very broadly to include any instance where students and teachers make a mathematical claim and provide evidence to support it" (p. 404). Also, I do not attend to the mathematical correctness of the students' contributions since my goal is not to assess their mathematical ability. The focus is on the students' participation and associated teachers' support. The simplest organization of argumentation includes four functional parts: claim, data, warrant, and possibly backing (see Figure 2; Toulmin, 2003). A *claim* is supported by *data* (given or assumed information functions as a foundation for the claim). A *warrant* is a proposition that connects the data with the claim. A *backing* is an added account to strengthen the warrant. Throughout the process of argumentation, as Conner and colleagues (2014) have shown, the teachers play an important supportive role in the development of collective argumentation. The current study attends to how the teachers specifically offer support for the three students.

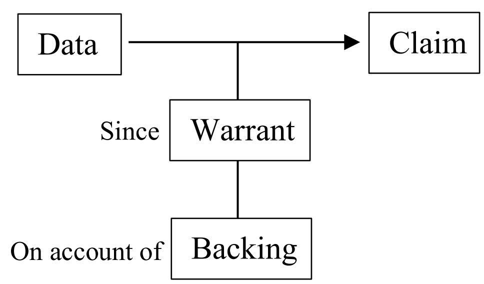


Figure 2. Diagram of an Argument (adapted from Toulmin, 2003)

Turn-taking and collective argumentation are distinct features of classroom interaction. Yet commonly rooted in ethnomethodology, they both are situated in the temporal progression of classroom interaction. Heritage (1984) stated that interaction is "doubly contextual," meaning an action is shaped by the context of the prior interaction, and the same action also renews the context for the next action (p. 242). This theoretical stance has an important implication for my analysis. I examine the turns taken by the students while referring to the context shaped by the prior interactions—especially the immediately preceding action and vice versa—I examine the subsequent turn to examine the effect of the current turn. This turn-by-turn analysis can offer a way to examine how particular teacher actions can support minoritized students' participation in collective argumentation. I further detail this analytic process in the following section.

Methods

Before describing the research context and how I attended to turn-taking and collective argumentation, I briefly discuss how I arrived at the current study. I explain how my past experiences informed the choice of ethnomethodological approaches to examine the interaction between the two White teachers and three Black students.

Researcher Positionality

In the current study, I present myself as an Asian man and former AP Calculus teacher who taught students with a different racial background than mine. Prior to my graduate studies, I spent seven years teaching predominantly Hispanic/Latinx students (95% of the school population) from a low-income community of color in the Southwest region of the United States. My experience as an Asian teacher teaching Hispanic/Latinx students shaped my interest in how mathematics teachers can better serve minoritized students across racial boundaries (see for a critical analysis of Asian American mathematics teachers, Kokka & Chao, 2020). My experience of preparing White prospective teachers and prospective teachers of color also strengthened my interest because mathematics teachers face increasingly more racially diverse classrooms than ever before.

I am neither Black nor White, and I do not know the Black students' racialized experiences or the White teachers' experiences. This lack of access to racialized experiences beyond my own racial identity informed my research focus on the *interaction* between the teacher and students rather than their perceptions. Ethnomethodological studies tap into the seen-but-unnoticed features of interaction (Heritage, 1984), and they require detail-oriented analyses of speech and gestures to understand the processes of interaction that are often taken as granted yet still visible or hearable to the teachers and students during the interaction. This analytic approach resembles what I was doing as a teacher when I engaged in classroom interaction with my students across racial boundaries. I had to initiate and recalibrate my course of actions based on what I saw and heard from the students in the moment-be-moment interaction. Understanding how other mathematics teachers accomplish this contingent, spontaneous interaction with their students has been my primary research agenda.

Participants and Data

My study is situated in two public high schools, Riverside High School (RHS) and Oaktree High School (OHS), located in the Midwest region of the United States. Both schools are located in suburban areas that comprise a predominantly White population. Both Ms. Gray (a calculus teacher from OHS) and Ms. Hill (a statistics teacher from RHS) are White women with over 10 years of teaching experience. The current study attends to the social interactions facilitated by these experienced AP teachers with different professional development experiences. One of the relevant professional development is Ms. Gray's participation in a practitioner-research study group focusing on classroom discourse and equity for six years. Although I do not make a causal claim, the variability in teachers' prior professional development might have added richness to the data to find a variety of interactional features. For example, in the Findings section, I will present Ms. Gray's non-conventional methods compared to that of Ms. Hill.

The racial and ethnic distributions within these two schools and classrooms are presented in Table 1. Indigo was the only Black student in Ms. Hill's AP Statistics class, and Tiana and Calvin were the only Black students in Ms. Gray's AP Calculus class. This racial and ethnic pattern reflects the hyper-minoritized Black students in AP mathematics courses that I discussed earlier.

Table 3. Race and Ethnic Distributions of Two Classes

Race/ Ethnicity	Ms. Gray's AP Calculus Class (27 Students)	Oaktree High School	Ms. Hill's AP Statistics Class (27 Students)	Riverside High School
Asian	14.8% (4)	7%	11.1% (3)	3%
Black or African American	7.4% (2)	20%	3.7% (1)	5%
Hispanic or Latinx	3.7% (1)	8%	0% (0)	8%
White	74% (20)	58%	85.1% (23)	79%
Multiracial	0% (0)	6%	0% (0)	3%

Note. The percentages of American Indian/Alaskan Native and Native Hawaiian/Pacific Islander were less than one percent in both schools, and they are not included in this table.

The primary data source of the current study consists of three consecutive video-recorded lessons from each AP mathematics class. One video camera captured the teacher's gestures (e.g., pointing, nodding), and another video camera captured students' gestures (e.g., pointing, nodding). Capturing detailed speech features is crucial from a conversation analytic approach; thus, I placed four additional audio recorders in the middle of the classrooms to ensure capturing students' speech just in case the two video cameras did not capture it due to their locations.

Analysis

Prior to the fine-grained analysis, I identified a total of five episodes from whole-class mathematics discussions in which the three students took turns (see Table 4). I did not include the turns about a nonmathematical topic or the turns taken during partner talk in the analysis because the current study focuses on the teacher's turn-allocation methods and the students' participation in collective argumentation during whole-class discussions. I determined the boundary of the identified episodes as topical chunks to narrow the data pragmatically (e.g., a whole group discussion initiated by teacher question about predicting the shape of the distribution). I then transcribed episodes using the Jefferson System (Hepburn & Bolden, 2017; Jefferson, 2004), following the tradition of conversation analysis. The transcript system includes a range of speech features and gestures. These are important because, in interaction, participants use various speech features and gestures (e.g., delays in response, emphasized words, pointing, gazing, raising hand) to negotiate speakership.

Table 4. Turns taken by the three Black students

Data	Student	Class, Activity	Turn-allocation methods
Excerpt 2.1	Indigo	Ms. Hill' AP Statistics (Day 1), comparing sampling distributions	Nomination with bidding
Excerpt 2.2	Indigo	Ms. Hill' AP Statistics (Day 2), continued discussion on sampling distribution	Nomination without bidding
Excerpt 2.4	Tiana	Ms. Gray's AP Calculus (Day 1), review on mean value theorem	Random selection
Excerpt 2.6	Calvin	Ms. Gray's AP Calculus (Day 1), disk and washer methods	Pre-arranged nomination
Extract 2.8	Calvin	Ms. Gray's AP Calculus (Day 1), disk and washer methods	Self-selection

With the collection of transcribed episodes, I first examined turn-taking practices based on Mehan's (1979) three conventional turn-allocation methods, which are (a) individual nomination, (b) invitation to bid, and (c) invitation to reply. Ms. Hill used the individual nomination method to initiate the two turns taken by Indigo. These two episodes differed because Indigo raised her hand to bid for her turn in the first episode, but she did not do so in the second episode. To make this distinction, I referred to them as *nomination with bidding* and *nomination without bidding*. Ms. Gray, on the other hand, employed two distinct methods that Mehan (1979) did not capture; these are *random selection* and *pre-arranged nomination*. In addition, the last episode includes Calvin's turn that is initiated by himself without Ms. Gray's nomination. I adopted the term *self-selection* from Ingram (2021). I will present turn-by-turn analyses to detail the different effects of turn-allocation methods on the students' participation in the finding section.

I also examined collective argumentation in these episodes and the teachers' support during that process. As Simosi (2003) discussed, identifying the functional elements of argumentation is not straightforward, and the analyst needs to consider the particularities of the context in which arguments are produced (see, e.g., Bieda et al., 2015). To reflect the common question and answer sequences during the whole-class discussion, I considered *claim* as a response to the question, *data* as the information given in the question, and *warrant* as an account that participants provide to justify the response. Occasionally, participants may add additional mathematical reasoning (i.e., *backing*) to support the provided warrant.

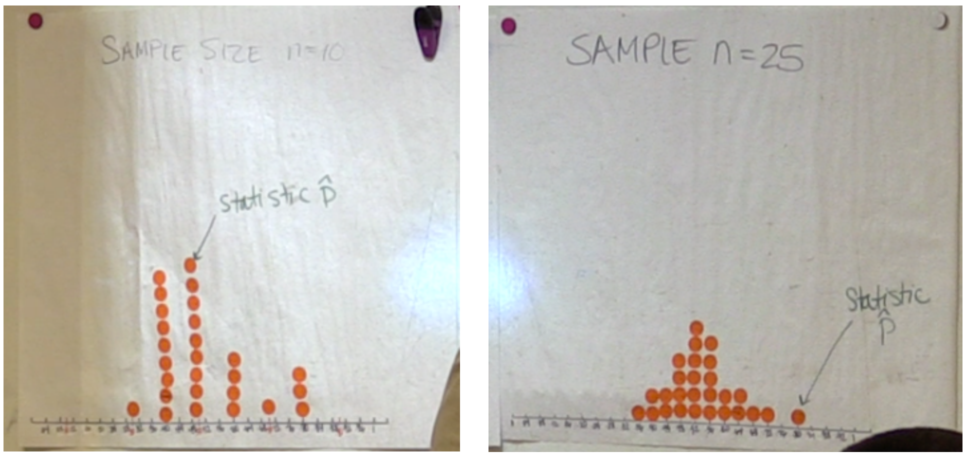
The focus of the analysis is how the teachers supported the students' participation in collective argumentation. Informed by Conner and colleagues (2014), I attended to how the teachers' questioning, repeating, or using gestures might have shaped the students' participation. In particular, Joseph and colleagues (2019) highlighted the importance of empowering minoritized students' voices in the process of constructing knowledge. To examine the effects of teachers' supportive actions in that regard, I attended the *epistemic stance* of the students (Heritage, 2012). Meaning, I examined how the students present themselves as a less-knower or a more-knower based on fine details of their speech, such as syntax, intonation, delay in speech. For instance, a declarative syntax with falling intonation is often aligned with the stance of a more-knower, whereas an interrogative syntax with rising intonation is often aligned with uncertainty or less-knower. Importantly, an epistemic stance is not an enduring characteristic of a student; it is fluid and shaped by the ongoing interaction. This analysis focuses on how the teachers' supportive action may have shaped the way the students present themselves as a more-or less-knower.

Findings

Here, I present detailed analyses of the five episodes that I listed in Table 2 above. I organized the current section based on the identified turn-allocation methods. I support my claims on the effects of different turn-allocation methods (some of their advantages and limitations) based on the details of speech as the three students take up the conversation floor (e.g., delays in speech, emphasized words). Similarly, I describe teachers' support for collective argumentation and its effect on the students' contribution with the details of speech and gestures. I consistently use the terms—data, claim, warrant, and backing—to mark such teachers' support in the context of collective argumentation. The two illustrative diagrams of collective argumentation present when and how the teacher's support is provided in the process of argumentation. I begin with the most commonly seen turn-allocation method from Ms. Hill's lesson.

Nomination with Bidding

As Mehan (1979) described, the conventional method of *nomination with bidding* entails the teacher's invitation to bid and subsequent students' bids for the floor prior to the teacher selecting a student to speak. Ms. Hill often employed this method and encouraged students to raise their hands by raising her hand when she invited students to bid. Excerpt 2.1 illustrates how Indigo took the floor with this conventional method. The class was engaged in a whole-class discussion about comparing and contrasting two sampling distributions that the class constructed. Earlier in the lesson, each student took two samples of Reese's Pieces candies with sample sizes of 10 and 25. Each student computed the proportion of orange candies within each sample, and then they put orange dots on the chart papers in front of the room based on their proportion values. The final artifacts are shown in Figure 3 below. Indigo's bid and turn-taking emerged when the class discussed the differences between these two distributions in terms of shape, center, and spread. Excerpt 2.1 begins as Ms. T asks Nihan what aspects of the distribution with the sample size 25 make the distribution more trustworthy.



Plot A Plot B

Figure 3. Class-Constructed Sampling Distrubtions

Excerpt 2.1: C1-54 Roughly Symmetric[8]

```
33  TCH:    Okay. I'm gonna push you. (0.7) Wha:t (.) th:ing (.)
34          shape center spread do you notice. (.) That's
35          different about that graph that makes you trust it
36          more [than this one.
37               [((WLM & JSH raise hands. JSH holds his
38                hand up until he gets a turn in line 53))
39          ((“that graph” refers to plot with n=25
40            and “this one” refers to plot with n=10))
41          (0.9)
42  NIH:    [U:h it's unimodal, (.) On the: s- uh: when the
43          [((WLM puts his hand down))
44          sample is twenty five?
45  TCH:    Okay it's more unimodal when the sample size's
46          twenty five? [Okay that's goo:d, (.) Okay what
47                       [((Indigo raises hand))
48          else:.
49  IDG:    It's roughly symmetric.
50  TCH:    It's roughly symmetric. This one is not quite as
51          symmetric right guys?
52          (0.7)
53  TCH:    Okay now (I'm-) what else.
54  JSH:    (                           )
55  TCH:    Yea:h it looks a little bit like a bell curve maybe?
56          So- and we like tha:t? Cause we trust those types of
```

[8] In this excerpt, Ms. Hill's gesture is missing due to a technical difficulty during data collection.

57 (.) distributions better.

Once Ms. Hilll confirms Nihan’s response by repeating it, Indigo raises her hand (line 47). Ms. Hill then selects Indigo as the next speaker (lines 46-48). Note that prior to this moment, William and Josh raised their hands as well (line 37), and Josh kept his hand up till he gets his turn in line 54. This indicates that Ms. Hill is *prioritizing* Indigo’s bid over Josh’s bid. Given Josh is one student who frequently raises his hand, Ms. Hill’s selection of Indigo distributes the speakership more evenly across the classroom. Ms. Hill’s turn-allocation method aligns with Sahlström’s (2002) finding that “[b]y selecting ‘lasts’ as next speakers, the teacher can reward late hand-raisers with turns—and thus increase participation from the students” (p. 54). This method also extends wait time, allowing students to formulate their responses and likely elicits responses from a wider range of students in the mathematics classroom (Herbel-Eisenmann et al., 2017; Ingram & Elliott, 2014).

During her turn, Indigo presents herself as “a knowing-and-willing-answerer” (McHoul, 1978, p. 201). Her response comes with no delay, and the declarative syntax and falling intonation display her certainty about her answer (line 49). An important part of this turn-allocation method is the pre-sequence of Ms. Hill’s inviting students to bid, and Indigo’s bidding with her raised hand. This pre-sequence achieves the mutual alignment between the teacher and Indigo that Indigo is prepared to answer the question before Ms. Hill selects Indigo as the next speaker. This pre-sequence removes the chance that Indigo is called by surprise when she is not fully prepared with a formulated response. As Mortensen (2008) argues, this mutual alignment before nominating a student is “an intrinsic aspect of the way in which the lesson is socially organized” (p. 62).

Although the conventional method of *nominating with bidding* appears as a benign way to organize turn-taking in the classroom, it has a significant equity-related implication. When the teacher selects the next speaker who is bidding for a turn, the teacher limits the range of possible next speakers to the students who raise their hands. This limitation becomes an issue when frequent and persistent bidders are students who come from dominant social groups in STEM fields, which was the case for both Ms. Hill and Ms. Gray's classes. For example, in this excerpt, Josh raised his hand and held his hand up until he took his turn (lines 37-38). With this conventional method, teachers may inadvertently reproduce the existing social hierarchies within the classroom. One way to avoid the danger of social reproduction is to nominate a student who is not bidding to take the floor. Next, I show that Ms. Hill nominates Indigo without her bidding, and as the analysis shows, it leads to interactional trouble.

Nomination without Bidding

Excerpt 2.2 occurred the next day during a whole-class discussion on the nature of sampling distributions as the sample size increases. Continuing the discussion from the prior lesson in Excerpt 2.1, Ms. Hill prompts students to consider what the shape of the sampling distribution would be when the sample size increases to 100.

Excerpt 2.2: C2-08 More Symmetric

```
01  TCH:    The only thing we haven't really talked about is the
02          shape you guys,(0.5) What do you think that the shape
03          is gonna do. [The shape over here looks a little bit
04                       [((TCH points to Plot A))
05          like slightly right skewed ↓this way Nihan said
06          yesterday he said slightly right skewed.↓
07          (0.7)
08  TCH:    [This one looks more unimodal and symmetric but there
09          [((TCH points to Plot B))
10          was like [weirdness going on over here. (0.6) What do
11                   [((TCH points to the orange dot in the far
12                      right of Plot B))
13          you imagine gonna happen. (0.5) If we start using n
14          values of hundred what would the shape start to look
15          like.
16          (0.5)
17  TCH:    How would the shape change.
18          (2.0)
19  TCH:    Do you think it's gonna become skewed?
20          (1.0)
21  TCH:    Bimodal?
22          (.)
```

```
23  TCH:    Unimodal? It's gonna be symmetric?
24          (.)
25  TCH:    Pretty close to perfectly symmetric? What do you
26          expect to see in there.
27          (5.2)
28  TCH:    [Indigo what do you think would happen.
29          [((TCH open her R palm forward))
30          (0.6)
31  TCH:    To the shape.
32          (2.5)
33  IDG:    U:::m
34          (5.6)
35  IDG:    [(Would it-)
36  TCH:    [That's the lowest sample size (and [this is) mid
37          [((TCH points to Plot A))           [((to Plot B))
38          sample size and [this is sample size of one hundred.
39                          [((TCH turns her hand in the air))
40          How do you think it would change.
41  IDG:    <Would it still be: like mo:re (1.0)
42          s:ym[metric > if you're-
43              [((TCH nods))
44          [cause it's not gonna spread (1.0) out it's
45          [((IDG puts her R palm up))
46          [((TCH nods until IDG finishes her turn))
47          gonna [get smaller.
48                [((IDG pinches))
49          (.)
50  TCH:    >Yes< (0.5) so she says (.) she says would it be more
51          symmetric. It's not gonna spread o:ut as much. (.)
52          Yes. (0.3) Do you guys think that would be: (0.5)
53          har↑der (.) to get a weird answer? If you have a
54          hundred pieces of candy?
```

Despite the five-second wait time after Ms. Hill poses the question (line 27), no one bids to speak. In line 28, Ms. Hill nominates Indigo without Indigo's raised hand (i.e., without Indigo bidding). Contrarily to the nomination with bidding, nomination *without* bidding gives Ms. Hill the autonomy to select anyone in the room. Yet, there are at least a couple of issues with this method evident in this excerpt. First, it brings the class's attention to Indigo without Ms. Hill's knowledge of Indigo being prepared to speak. After Ms. Hill nominates Indigo, there is silence (lines 30-32). When Indigo starts to speak, her elongated "Um" (noted with ":") and the subsequent five-second silence is a noticeable delay in her response (lines 33-34). This is significantly longer than the common delay of 0.7 seconds in the whole-class discussion (Ingram, 2012). Ms. Hill manages this issue of delay in Indigo's response by rephrasing the question (lines 36-38). This action gives additional time for Indigo to formulate her response and treats the delay in Indigo's response as a communication issue (e.g., not hearing the question well) rather than Indigo's lack of knowledge.

The second issue with the nomination without bidding is the sudden, unannounced nomination creates an interactional context that leads to Indigo's initial epistemic stance as a less-knower. Note that Indigo's speech is slower (noted with "< >") and she uses interrogative syntax (line 41). Indigo displays her uncertainty about her response and seeks confirmation from the teacher. Ms. Hill manages this issue by nodding, which shows her affirmation as soon as Indigo utters the first syllable of her response, "sym" (line 42), without interrupting Indigo's turn. As Ms. Hill nods, Indigo's speech becomes faster, and she shifts her epistemic stance from less-knower to more-knower with the falling intonation and large gestures toward the end of her turn (lines 42-48).

Figure 4 below shows how Indigo's formulation of her argument is supported by Ms. Hill's contingent affirming actions. When Ms. Hill rephrases the earlier question for Indigo, Ms. Hill points to two distributions on the chart, highlighting the data for the argument (line 36). Additionally, Indigo's epistemic stance changes concurrently with Ms. Hill's nodding. This use of gesture is timely because Ms. Hill did not wait until the end of Indigo's turn to show her affirmation. Ms. Hill's nodding functioned as a go-ahead sign and propelled Indigo's ongoing talk with an added affirmation from the teacher. Note Ms. Hill orients to Indigo's ownership of the argument by saying "she says …" (line 50). Although the legitimacy of the claim and the warrant were supported by the authority of Ms. Hill, Indigo remains the sole person who produced the claim and the warrant without Ms. Hill's verbal contribution.

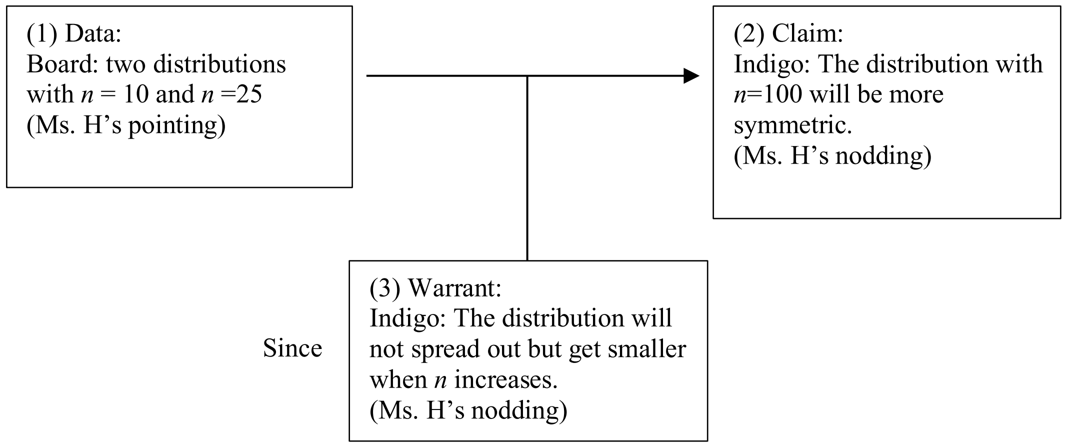


Figure 4. Indigo's Argument with Ms. Hill's embodied support

With multiple responsive moves, Ms. Hill successfully supports Indigo to produce her response with a claim and warrant with Indigo's more-knowing stance. Nonetheless, as Indigo's significantly delayed response signals, nomination *without* bidding places Indigo within the conversational floor without any prior alignment between Indigo and Ms. Hill. Indigo being the only Black student in the classroom, the stake can be even higher. Researchers reported that Black students have to manage anti-Black stereotypes and feel obligated to prove themselves academically capable (e.g., Leyva et al., 2021; McGee & Martin, 2011; McGee, 2018). This unanticipated interactional context may heighten these socio-psychological barriers, especially if a student is uncertain of their response. Table 5 presents a comparison of the two conventional turn-allocation methods. As I discussed above, solely relying on the nomination with bidding also leads to reproducing existing inequitable patterns within the classroom. Both conventional methods have advantages and limitations.

Table 5. Advantages and Limitations of Conventional Methods

	Nomination with Bidding	Nomination without Bidding
Advantage	Constituting the student as a knowing-and-willing-answerer	Opportunity to prioritize minoritized students' participation
Limitation	Reproducing existing inequitable patterns	Interactionally challenging space that may exacerbate minoritized students' racialized psychological burden

In the rest of the findings section, I discuss Ms. Gray's non-conventional turn-allocation methods, *random selection* and *pre-arranged nomination*. I will illustrate how these two methods shape different opportunities for Tiana and Calvin to participate than the conventional methods.

Random Selection

In Ms. Gray's AP Calculus classroom, the beginning routine is working on a warm-up question, a multiple-choice problem that prepares students for the AP Calculus exam. When the class debriefs on the warm-up question, Ms. Gray uses a *random selection* method using craft sticks labeled with a letter and a number. In her classroom, all rows and seats are labeled with letters and numbers, respectively, to allow for quick reference to particular seats in the room. Prior to Excerpt 2.3 below, Ms. Gray surveyed the class and announced that the correct answer was A. Ms. Gray then asked students to talk to their partners about why someone might have chosen the incorrect answer D (see Chapter 3 in this dissertation for an elaborated discussion on partner talk). After a few minutes, Ms. Gray gets the attention of the class (line 1).

Excerpt 2.3: F1-09 Why Answer D

```
TCH:   Oh↑kay. (0.7) check ↑your row. (.) Cause I moved some
       of you. (.) Check your sea:t number. (0.7) [I wanna
                                                  [((TCH
                      reaches the cup with name sticks))
       hear your thoughts around why: answer D WHY: NO:T
       answer D basically right,=Why might someone have
       answered (.) D: (.) and have been incorrect.
       (1.0)/((TCH picks a stick))
       >S:O< (0.3) no one's in B.
       (.)
       Let's try again.
       (1.5)/((TCH picks a stick))
       G.
       (1.5)/((TCH picks a stick))
       °In have three people in row G°
       (.)
       G one?
       (0.5)
ISB:   >Nope< ((There is no one seating on seat one))
       (.)
STs:   Heheheheh
       (1.5)/((TCH picks a stick))
TCH:   G ↑Two:.
       (.)
ISB:   Okay.
TCH:   'kay.=
ISB:   U:m (0.7) so we said the mean va↑:lue theorem↑ would
       state that the average slope has to exist somewhere
       along the interval, (.) A:nd fi:ve (.) is (.) the
       average slope. So: (0.7) it has to exist somewhere.
       But that's not the intermediate value theorem.
       (1.3)
TCH:   Okay,
       (1.2)
```

This excerpt presents how the teacher and students orient to the random selection methods. As Ms. Gray picks a stick, she announces the letter or the number with an emphasis and increased pitch (e.g., line 23). When the nomination progresses, participants display game-like excitement. The students laugh (line 21) as Ms. Gray's random selection continues to select seats that are empty. This scene resembles that of a raffle announcement, in which all participants have a chance to be nominated and also not nominated. Isabella gets the first pick (line 25), and once she completes her turn, in Excerpt 2.4 below, Ms. Gray initiates Tiana's turn with the second round of random selection.

Excerpt 2.4: F1-09 Why Answer D (Continued)

```
35  TCH:    Thank you, [NEXT PERson. Just gonna add to her
36                     [((TCH picks a stick))
37                  thinking or maybe explain in a different (.) way. Row
38                  ↑C:
39                  (1.0)/((TCH picks a stick))
40  TCH:    ↑SEAT O:NE↑
41  STs:    heheh (0.7) heheh/((STs in row C laugh while gazing
42                                  at TNA))
43  TCH:    Tiana whaddid you and Sophia talk about.
44  TNA:    We were like more general about we were talking about
45                  how they are just thinking of the: (.) mean va↑:lue
46                  theorem↑ instead of like the intermediate (.) °value
47                  theorem.°
48  TCH:    Okay?
49                  (1.7)
50  TCH:    So if- you are proposing that if that if the question
51                  prompt had been just replaced with M V T instead of I
52                  V T then D would have been the correct answer?
53                  (.)
54  TNA:    Yea:[s::  ]
55  TCH:        [‘kay.] (.) [one more person. A:h row B:?
56                          [((TCH picks a stick))
57                  (3.5)/((TCH picks a stick))
58                  Sea:t FOUR.
59                  (2.8)
60  SEN:    Oh
61  CVN:    Oh is that (the way)? /((Gazing at Sean))
62  SEN:    Yeah:
63  TCH:    Is that [you Sean?]
64  CVN:            [ Y e : s ]
65  SEN:    U:m (1.7) I guess like (1.2) I am not sure exactly
66                  what (.) I V T is? (.) anymore but I think it doesn’t
67                  have to be uh: (2.0) differentiable? (.) maybe?
68                  (0.5)
69  SEN:    [  S  o  :  ]
70  TCH:    [Is he right] ↑guys?
```

Note that Tiana begins her turn without any delay (line 44). I offer two interpretations for her immediate response despite the sudden, unpredictable nature of the random selection method. First, Ms. Gray’s question is designed to be answered in multiple ways. Ms. Gray’s prompt, before selecting Tiana, asks the next speaker “to add to her [Isabella’s] thinking or maybe explain in a different way” (lines 35-37). This prompt opens more than one way the selected student can respond to and contribute to the ongoing discussion. After selecting Tiana, Ms. Gray reformulates the question as, “What did you and Sophia talk about” (line 43). A notable feature of this reformulation is that the question offers a blank canvas and allows Tiana to report what

Tiana and Sophia have discussed. This reporting of the earlier partner talk allows Tiana to share the accountability of her response with Sophia. Tiana's response aligns with Ms. Gray's reformulated question by beginning her turn with "we" (line 44). Ms. Gray's reformulation lowers the social barrier, and Tiana takes the floor immediately.

Once Tiana's turn completes, Ms. Gray frames Tiana's response as a proposal for which the class will continue to engage. In lines 50-52, Ms. Gray rephrases Tiana's response and seeks Tiana's confirmation. By doing so, Ms. Gray orients to Tiana as a more-knower. Furthermore, Ms. Gray facilitates an interactional context in which other students can engage in Tiana's proposal since a proposal makes accepting or rejecting as the next relevant action, which is *creating opportunities to engage with another's* (Tiana's) *reasoning* (Herbel-Eisenmann et al., 2013). This is, however, when the shortcoming of the random selection emerges. Sean, the next randomly selected speaker, says that he does not remember the intermediate value theorem anymore, and the discussion derails away from Tiana's proposal to reminding Sean about the theorem. As Ingram (2012) discussed, the difficulty of controlling topics is a downside of random selection. The method puts a normative expectation to respond on a randomly selected student. Still, the selected student may not be prepared to address the presented question, and the discussion can be fragmented and not lead to a cohesive argument (Ingram, 2012).

Sometimes called "equity sticks" by educators (Safir, 2015), the random selection method provides an alternative way to organize turn-allocation while evenly distributing speakership among students. The game-like excitement, often with laughs, may lower the social barrier for randomly selected students to take the floor. As with the case of Tiana's proposal, however, this method can make the whole-class discussion less cohesive, leaving the teacher with the responsibility to manage a wide range of unanticipated responses. Contrarily, the next turn-allocation method *pre-arranged nomination* allows teachers to both intentionally distribute speakership among a diverse group of students as well as sequence the topic of responses in a way that can build on each other. Moreover, the random selection method presented above initiated students' turns without mutual alignment about speakership between the teacher and students. As I illustrated in Excerpt 2.1, this mutual alignment is often negotiated by nominating students with raised hands, which establishes the student's status as "a knowing-and-willing-answerer" (McHoul, 1978, p. 201). The pre-arranged nomination method offers an alternative way to negotiate this speakership status with a student prior to placing the student in the center of the floor.

Pre-Arranged Nomination

Here, I present an episode of Ms. Gray's use of *pre-arranged nomination* to allocate a turn to Calvin. In contrast to prior turn-allocation methods, this method begins with a pre-arrangement of speakership before the onset of whole-class discussion. This method is made possible by the teacher's employment of partner talk in their lessons. When students engage in discussions with their peers, the teacher can negotiate speakership with particular students for the upcoming whole-class discussion (see Chapter 3 in this dissertation). Excerpt 2.5 below includes the scene when Ms. Gray pre-arranges with Calvin to be one of the speakers during the upcoming whole group discussion when she visits Calvin's small group. The class was engaging in partner talk on the given question, "Why is the disk method appropriate for EX #2 but not HW #10?" (see Figure 5 for the two problems). Prior to visiting Calvin's small group, Ms. Gray already visited two other groups. At the beginning of the partner talk, Calvin and Sean formed a pair, then later, Natalie and Nora in the front row turned back and joined the discussion.

EX #2
Region bounded by $y = 3 - x^2$ and $y = 1$, revolved around $y = 1$.

HW #10
Region bounded by x= $\sqrt{3 - y}$, $x = 2$, and $y = 0$, revolved around $x = 0$.

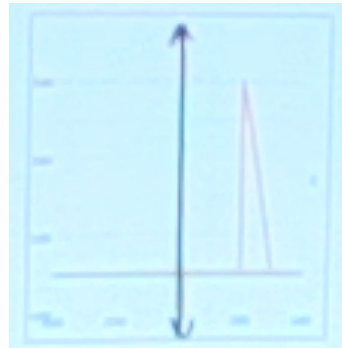


Figure 5. Two Problems for Disk and Washer Methods

Excerpt 2.5: F1-40 A Cylinder in the Middle

```
01   TCH:    Whaddid you guys discuss over here.
02           (0.9)
03   CVN:    [U:h]
04   NTL:    [U: ]:h=
05   CVN:    =[Cause there's a ga:p (in the) middle [of this one,
06            [((CVN points to paper until line 10)) [((TCH moves
07                     from the behind of CVN to the side of CVN))
08   TCH:    Ok[ay,]
09   CVN:      [a  ] cylinder in the middle and this one is jus:t
10           like (.) in (that's) (.) touchy,=
11   NOR:    =And that one's (       ).
12   CVN:    I guess.
13   TCH:    Will you share that?/((TCH gazes at Calvin))
14           (.)
15   CVN:    To the class?
16           (0.3)/((TCH nods))
17   CVN:    I guess?
18   TCH:    >Okay.<
19           (0.7)/((TCH walks to the front of the room))
20   TCH:    ALRIGHT. ((TCH addresses to the class))
21           ((TCH claps her hands))
22           (1.5)
23   TCH:    ↑SO::, (0.7) ↑let's debrief.↑
24           (3.5) /((STs' chatter continues))
25   TCH:    let's debrief.
26           (2.6) /((STs' chatter continues))
27   TCH:    [So: I asked (2.0) Calvin?
28           [((TCH walks to the center of the room with two index
29           fingers pointing forward, and points her left finger
30           to CVN))
31           (.)
32   CVN:    Yes.
33           (.)
34   TCH:    To share [he's gonna] get started,
35   CVN:             [ o : h.   ]
36   SPH:    Yea. Cal↑vi:n
37           (0.5)/((CVN rubs his hands))
```

```
38  SPH:    Whoo
39  TCH:    Calvin, Bill, kinda heard the same thing from Calvin
40          and Bill so [Calvin and Bill, (.) and then I asked
41                      [((CVN shakes his hand gazing at BLL))
42  TCH:    someone from [this team to share.
43                       [((TCH points to a row with Isabella))
44  TCH:    [Okay,]
45  SPH:    [Wo:w ]
46  TCH:    And they're all like ((TCH points her thumb toward
47                                   ISB))
48  STs:    Heheheh
```

When Ms. Gray requested an update from his small group, Calvin starts to explain what is on his paper. Ms. Gray uses this opportunity to negotiate Calvin's speakership as one of the speakers during the subsequent whole-class discussion. As soon as Calvin starts to explain (line 5), Ms. Gray moves from behind Calvin to his left-hand side to make eye contact with Calvin. While looking at Calvin, Ms. Gray asks him to share with the class. Calvin's initiation of repair (line 15) and the subsequent response "I guess?" display Calvin's surprise and hesitation about sharing with the whole class. Nonetheless, Ms. Gray closes the sequences of talk with the group with a quick "okay" (line 18), and then she gets the attention from the whole class to begin a debrief.

There are multiple pieces of evidence that show how Ms. Gray draws Calvin to the center of the whole-class discussion. First, Ms. Gray completes the pre-arrangement with Calvin and moves to the whole-class discussion as soon as she hears "I guess?" from Calvin. Parallel to Sahlström's (2002) finding of selecting last hand-raisers as next speakers, closing the pre-arrangement with Calvin's confirmation shows that Ms. Gray's search for speakers becomes practically complete once Calvin—who has not raised a hand over the course of the three lessons—was selected as one of her pre-arranged speakers. The second piece of evidence occurs when Ms. Gray publicly announces the speakership by designating Calvin as the first speaker (line 34). Ms. Gray "heard the same thing from Calvin and Bill" (line 40), and she heard "the same thing" from Bill prior to hearing from Calvin during the partner talk. Nominating Calvin as the first speaker over Bill shows Ms. Gray's prioritization of Calvin over Bill, who is a frequent participant in Ms. Gray's class. Excerpt 2.6 below illustrates how Calvin's turn unfolds.

Excerpt 2.6: F1-40 A Cylinder in the Middle (continued)

```
49  TCH:    Okay Alright Calvin. You wanna get started?
50  CVN:    YEAH. Alright. SO:: (1.3) fo:r example number two:
51          there's not like a: (.) gap in the middle like so
52          fo:r number ten there's a cylinder in the middle?
53          when you (0.9) [rotated around y equals zero, (1.6)
54                         [((TCH draws a cylinder))
55  CVN:    a::nd (0.4) for example number two since as you
56          rotate it around (0.5) u:h y equals: (.)
57          [one? (.5) there's no uh (0.5) gap °in the middle.°
58          [((TCH draws a bottom half of the rotated shape))
59  TCH:    Okay, (.) Bill. is there anything else you wanna
60          add to [that thinking?
61                 [((TCH points to CVN))
62          (0.8)
63  BLL:    No:,
64          (0.3)
65  TCH:    [°Okay°]/((TCH smiles with slight nodding))
66  STs:    [ heh ] HEH heh
67  TCH:    ↑A:lrhighthh. (0.5) Isabella?=
68  ISB:    =So (0.3)a:h (0.3) our group said that the region
69          isn't up against the axis you're rotating around,
70          (0.3) so that's why you can't use the disc method,
71          [and when you rotate it- it creates the shape that's
72          [((TCH shades the region with red))
73          (.) similar to more of (       ) donut (.) instead
74          of (0.5) A (0.5)
75  TCH:    'kay,=
76  ISB:     =sphere (.) like shape.
```

Calvin begins his turn with "YEAH" with a loud volume (noted with uppercase) without delay (line 50). While Calvin is taking a turn, Ms. Gray draws two figures on the whiteboard to support Calvin's warrant. Similar to Ms. Hill's nodding, without interrupting Calvin's turn, Ms. Gray's drawing supports Calvin's warrant that the gap in HW #10 generates a cylinder in the middle. Although Ms. Gray does not confirm the correctness of Calvin's warrant, her act of recording may provide Calvin and other students with assurance about the shared idea. Ms. Gray's stance is also visible when she asks, "Is there anything else you wanna add to that thinking" to Bill (lines 59-60). The question design does not assume that Bill has something to add to that thinking compared to another possible formulation, "Bill, what do you want to add?"

The question also includes the word "anything," instead of "something," which less likely leads to an affirming response (Heritage & Robinson, 2011). Therefore, the question does not imply that Calvin's warrant is incomplete or needs to be augmented. Bill's blunt "no" without an account indicates Bill's alignment with Ms. Gray's stance on what Calvin has shared. After Bill's "no," Ms. Gray moves on to Isabella, the last speaker in the pre-arranged sequence. Isabella adds the idea of different cross-sectional shapes between EX #2 and HW #10. For HW #10, the cross-sectional shape is no longer a circle, but a new shape looks like a "donut," which necessitates the new washer method.

Excerpt 2.7: F1-40 A Cylinder in the Middle (continued)

```
TCH:    So (you) talked about the ↑region sits right up
        against [the axis here. (.) and over here the [red
                [((TCH puts her hand on y = 1))       [((TCH
                                   shades the rotated shape))
        region (1.2) it's not physically [touching it's not
                                         [((TCH puts her hand
                                            on x = 0))
        sitting right up against the axis and
        [because of that we end up with this like (.)
        [((TCH points to Calvin))
        cylinder Calvin mentioned that it's kinda getting
        drilled out of the center, (0.5) Okay,
```

After Isabella's turn, Ms. Gray cements the significance of Calvin's contribution to the class with equity-oriented teacher moves, which White (2003) may categorize as "valuing [minoritized] students' ideas." While Ms. Gray is revoicing Isabella's contribution, she points to Calvin as she says, "because …" (lines 85-86). The precisely-timed pointing and the word "because" highlight the logical connection between what Calvin said earlier and what Isabella shared next. In addition, in line 87, Ms. Gray explicitly mentions Calvin, thereby drawing the class's attention to Calvin's ownership of the idea.

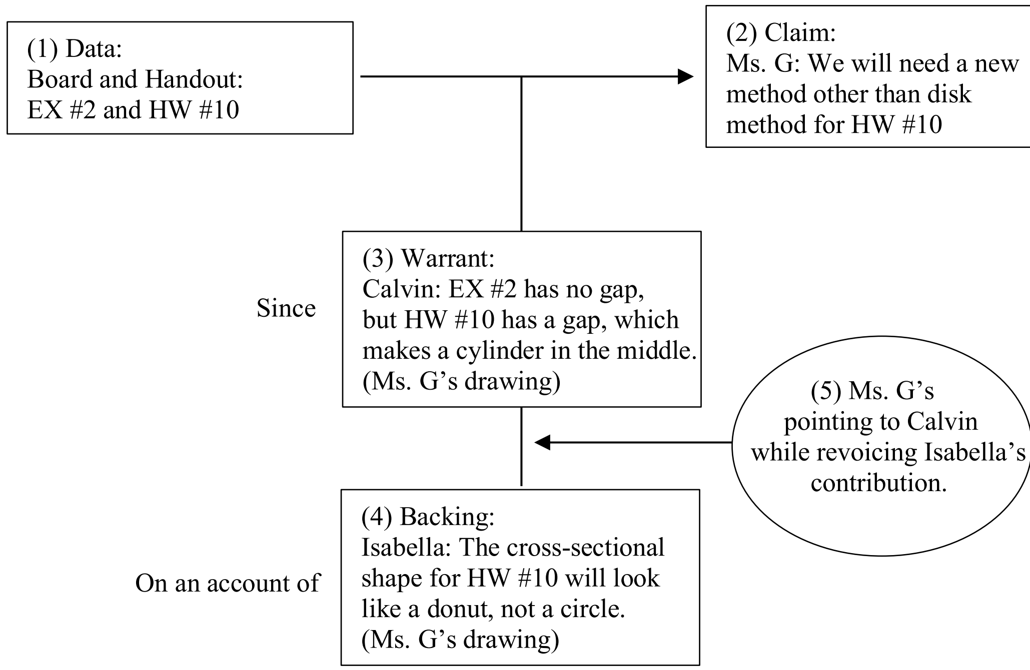


Figure 6. Collective Argument on the Necessity for a New Method

In contrast to Tiana's proposal in Excerpt 2.5, which did not gain ground in the earlier whole-class discussion, Calvin's idea was furthered by the continuing discussion, including Isabella's backing (see Figure 6). The sequencing from Calvin's idea of "a cylinder drilled out of the shape" to Isabella's idea of a "donut" like cross-sectional shape highlights the logical flow between these two ideas. Furthermore, Ms. Gray strengthened this connection between the two contributions with her pointing to Calvin when she uttered the word "because." Connecting seemingly disparate ideas for the learning goal of the lesson is a challenging yet important task for mathematics teachers (Stein & Smith, 2011). This episode shows how a teacher can achieve such a challenging task while centering the class's attention on Calvin's contribution.

One of the key features of the *pre-arranged nomination* method is that the method allows the teacher to nominate a student as "a knowing-and-willing-answerer" (McHoul, 1978, p. 201). As Mortensen (2008) shows, "teachers orient to finding a student who displays availability to be selected as next speaker," and "students display whether or not they are willing to be selected as next speaker.” The *pre-arrangement* part of this method allows the teacher to negotiate minoritized student’s speakership in a small group setting rather than the whole-class setting (e.g., nomination with bidding). As Excerpt 2.5 illustrates, arranging speakership in a small group setting affords the teacher to elevate minoritized students’ idea that is less likely shared in a whole-class setting without such arrangement and to encourage the student to be a speaker without getting attention from the whole class. This negotiated mutual alignment between the teacher and the student constitutes the student’s status as a knowing-and-willing-answerer, which is preferable for both the teacher and the student. The most apparent constraint of the pre-arranged nomination is that it requires prior partner talk (or groupwork), which may not always be feasible.

Student Self-Selecting

Taking up the conversational floor can be an empowering experience for minoritized students and may momentarily reshape the learning environment as an affirming space for the student. Although I do not make a causal claim, I draw a connection between the earlier Calvin’s participation initiated by pre-arranged nomination and his later turn-taking with the method of *self-selecting*. Excerpt 2.8 below highlights Calvin’s momentarily broadened “access to the conversational floor” (Engle et al., 2014, p. 252) to offer a warrant without Ms. Gray’s support or encouragement. The excerpt begins with Nora’s question why “we” subtract one from the given function in EX #2 to represent the radius of the cross-sectional circle (line 100).

Excerpt 2.8: F1-40 Why Do We Subtract One?

```
91  TCH:    Okay? So think about what you guys did here right?
92          (.) y over here (0.3) were we allowed to subtract
93          the- away the one, to determine a radius over here we
94          weren't allowed to subtract away two: to determine
95          radius it seems like in theory those are the same
96          ↑thing. (2.5) Why is this a radius and why this is
97          not a radius.
98          (2.4)/((ALX, NOR raise hands))
99  TCH:    Yeah. Nora.
```

```
NOR:    Why do we↑ subtract one from the first one.
TCH:    Okay. Who can answer Nora's question. Why are we
        subtracting o:ne (0.7) ↑fro::m (1.0) >to determine
        the radius in example two.<
        (1.0)/((Bill raises his hand until he gets a turn.))
CVN:    Is it [because it's: sitting above like it's at y
              [((Mia raises her hand.))
        equals one,
CVN:    [(          )]
TCH:    [y equals one] is the axis of revolution, Mia what
        would [you like to add to that.]
MIA:          [  Does    it      like  ]
        (.)
MIA:    Translate the function, so it (.) like (.) top half
        does (0.3) or like (0.3) the axis of rotation, (0.4)
        like matches a x: I don't know, (0.8) like does it
        translate the function down one, so:. (0.3) the (.)
        axis of rotation is the same as a (1.2) x axis?
```

After Ms. Gray repeats Nora's question and a pause, Calvin self-selects as the next speaker. His interrogative syntax (i.e., Is it … , line 106) indicates his less-knowing stance, and Ms. Gray also treats Calvin's utterance as seeking confirmation by her confirming the warrant of the location of the rotational axis (line 109), which deviates from the canonical *x*- or *y*-axis. Nonetheless, Calvin's self-selecting himself as the next speaker shows a greater "socially negotiated degree of access to the conversational floor" for Calvin since he did "self-select to speak without censure by others" (Engle et al., 2014, p. 252). His contribution is also furthered by Ms. Gray's question to Mia, the subsequent speaker. Ms. Gray asks, with an emphasized speech, what she can "add" to Calvin's idea (line 110).

As Mehan (1979) showed, teachers often reprimand student self-selecting in a conventional classroom. Student self-selecting can initiate turns from multiple students at the same time; thus, it is prone to producing overlapping speech (see also Ingram, 2021). These overlaps can undermine clear communication during the whole-class discussion. In the above episode, however, Ms. Gray allows Calvin to initiate his turn despite the fact that Bill (a frequent bidder) is raising his hand on the backside of the classroom. This prioritization of minoritized students over frequent participants, in turn, shows how flexibly the teacher needs to organize turn-taking in the classroom to support minoritized students' participation.

Discussions and Implications

Based on the above findings, I engage with two broad discussions in mathematics education. The first one is the discussion on equity issues, especially on potential ways mathematics teachers can support minoritized students' (in this case the three Black students') participation. To be clear, the goal of the current study was not to make a generalized claim about how to support minoritized students. It was rather to show the complexity of such interactional work while highlighting the important contingencies, such as managing well-intended dominating students. The second part of the discussion is about the current study's contribution to the existing frameworks related to classroom discourse, such as turn-taking in classroom interaction, facilitating whole-class discussions, and teacher support for collective argumentation. I focus my discussion on the possibilities of furthering these frameworks with an eye on equity, based on what the current study has shown.

Detailing Potential Ways to Support Black Students' Participation

The current study examined teachers' discourse practices (turn-allocation and supportive actions in collective argumentation) concerning the participation of the three Black students in AP mathematics courses. The analysis highlighted subtle features of the teachers' talk and gestures that encouraged and facilitated spaces for the students to take the conversation floor and to make integral contributions to the ongoing discussion. Researchers examined Black students' narratives and showed that these subtle features of classroom interaction play an important role in their development of robust mathematical identities (Gholson & Martin, 2014; Larnell, 2016; Martin, 2000; Stinson, 2010). The current study adds detailed texture to our understanding of interactions that may shape the students' identity by examining moment-by-moment classroom interaction. The analysis highlighted the teacher's role in prioritizing the participation of the three Black students over well-intended students who end up dominating the classroom interactions. This effort to foster the students' participation is crucial because of the broader social context that hyper-minoritizes these students. My analysis, however, only focused on the change in the students' epistemic stance in a micro-timescale (i.e., turn-by-turn interactions); this warrants further studies to understand if and how compounding experiences over time as a more-knower positively influence the enduring positionings and identities of the students.

Leyva and colleagues (2021) found that the teachers who explicitly acknowledge racial and gendered inequities, as opposed to avoiding discussions about race or gender, can foster equitable learning opportunities. Such race-explicit efforts will be critical, but the findings in the current study illustrate that opening up the potential for fostering equitable learning opportunities need not always be done with explicit discussion about race or gender. Neither the teachers nor students in the current study invoked race or race-related terms. Due to race-neutral ideologies, such explicit orientation to race during classroom interaction may be rare (Pollock, 2009). Moreover, invoking students' racial identities during mathematics instruction may inadvertently make minoritized students feel tokenized and add unnecessary sociopsychological burdens (McGee, 2018). Racialization of the minoritized students, however, is always operating in the mathematics learning space through mundane, seemingly race-neutral practices (Martin, 2000; Shah & Leonardo, 2016; Stinson, 2013). The details of the interaction presented in the current chapter (e.g., how the speakership is ordered, what the design of question assumes) also appear race-neutral, and I do not claim that these are race-conscious efforts of the teachers. Yet, the discourse practices presented in the current chapter may offer important insights to teachers who are intentionally aiming to counter racial marginalization and to support their Black students' participation in their everyday teaching (e.g., Busby et al., 2017; Herbel-Eisenmann & Shah, 2019).

The detailed turn-by-turn analysis in the current study also showed that supporting the Black students' participation was done through negotiation, not by rigid, rule-based coercion. As Allwright (1980) stated, "the management of participation, by teachers and by learners, is a negotiated process" (p. 166). A simplified categorization scheme of teaching practices may not adequately describe such negotiation processes that are contingent on students' moment-by-moment responses. In other words, the participation of the Black students in the above excerpts were joint accomplishments between the teacher and students, and what the students do or say influenced how the participation of the students is negotiated. Even when Ms. Gray selected Tiana to take a turn with the random selection method, Ms. Gray offered multiple ways for Tiana to contribute to the ongoing discussion. As critical scholars showed, when minoritized students choose not to participate, it can be a sign of resistance to the existing educational system that denies their humanity (Malagon, 2010; Martin, 2019). Although the data set in the current study did not include an episode of the minoritized students' participation by expressing their choice not to participate, further studies on this topic remain critical. It is important to know how the teacher can not only support minoritized students' political expressions of choosing not to participate but also how the teacher can bring the students' critical voices to the fore to reshape the classroom community (Hand, 2012).

Furthering Existing Frameworks Through an Equity Lens

Another contribution of the current chapter is furthering broader discussions on the facilitation of classroom discourse, such as turn-taking practices, facilitation of the whole-class discussion, and teachers' support for collective argumentation. I offer some points while recognizing that the number of contributions of the three Black students during my observations was few compared to White students. Thus, the intent of this discussion is not to make generalized claims about discourse practices but to offer potential places for more extended investigation based on the detailed accounts that I presented in the above analyses. First, the current study illustrated a variety of turn-allocation methods and their advantages and limitations (see also Ingram, 2021; Michaels et al., 2010). The current study contributes to the discussion by illustrating each method's advantages and limitations in regards to *initiating minoritized students' participation*. In particular, the analysis showed critical limitations of conventional methods (*nomination with or without bidding*) when the teacher tries to allocate turns to the students. The nomination with bidding only gives the teacher the opportunity to nominate students who are raising hands, which often reflects existing status hierarchies that are often racialized and gendered (Shah & Crespo, 2018). The nomination without bidding may add an unjust burden on minoritized students by putting them at the center of the class's attention when they are not prepared to talk. The difficulty faced by Molly and Katie in the opening scene reflects this dilemma when teachers over-rely on the hand-raising practice for their turn-allocation.

Non-conventional turn-allocation methods offer potential ways for teachers to navigate such difficulty. In particular, *random selection* can be a tool to distribute the speakership across the room with an equal chance for every student, including Tiana and Calvin. This method, however, should be introduced and used with caution. I emphasize Ms. Gray's effort to make this random selection a game-like, routine part of the warm-up activity for every lesson. Ms. Gray also facilitated partner talk prior to the random selection to offer abundant opportunities to talk to their peers before the whole-class discussion. The other non-conventional method, *pre-arranged nomination*, seems to bring the best of both worlds. The pre-arrangement part allows the teacher to negotiate the speakership with minoritized students, and this negotiation also allows the student to prepare for the arranged turn in the upcoming whole-class discussion. The data, however, only included one episode of pre-arranged nomination, and examining multiple episodes may reveal other affordances or limitations of this turn-allocation method. Pre-arranged nomination also requires using instructional time for either groupwork or partner talk. Therefore, learning to use the pre-arranged nomination method involves learning to facilitate groupwork and/or partner talk. In other words, this turn-allocation method should not be seen as a stand-alone method but as an integral part of a larger set of teacher's discourse practices. In general, turn-taking practices are a seen-but-unnoticed feature of social interaction. Making these methods explicit can help mathematics teachers, such as Molly and Katie in the opening scene, who intend to disrupt existing inequitable patterns of participation.

Second, the current study shows how the existing frameworks of teaching practice can be expanded with an eye on supporting minoritized students' participation. The turn-by-turn analysis of the initiation of the three students' participation detailed the teachers' refined interactional work of *inviting student participation*, one of the Teacher Discourse Moves (Herbel-Eisenmann et al., 2013). As Stein and Smith (2011) discussed, mastering such a new interactional craft (e.g., Ms. Gray's non-conventional turn-allocation methods) will take time and effort to develop. Notably, Ms. Gray had participated in six years of professional development, in which she conducted a series of action research projects to refine her discourse practices, examine her implicit bias (Herbel-Eisenmann & Shah, 2019), and work on equitably distributing opportunities for students to participate (see, Herbel-Eisenmann et al., 2017). The current study illustrated an instantiation of her refined craft, especially to facilitate an affirming space for Calvin and Tiana. Thus, this chapter confirms the important role of discourse-oriented professional development to help teachers better support minoritized students' participation.

As presented in the excerpts, the range of ways of inviting student participation (i.e., turn-allocation methods) shaped different interactional contexts for the student's participation, and it ultimately shaped how the students presented themselves as knowers. More importantly, with the focus on inviting the three minoritized students, the analysis illustrated how subtle details of speech prioritize the students' participation (e.g., selecting Indigo over other two students who were raising their hands, selecting Calvin to speak first prior to another student who had a similar idea). The findings in the current study are not exhaustive. Further studies that detail other Teacher Discourse Moves (e.g., revoicing, creating opportunities to engage with another's reasoning) may offer important features of facilitating classroom interaction that can foster an affirming space for minoritized students.

Lastly, the current study's examination of collective argumentation showed that the teachers supported the students in subtle yet consequential ways. The framework set by Connor and colleagues (2014) offers a broader overview of how mathematics teachers provide support for collective argumentation (e.g., direct contributions, questions). The turn-by-turn analysis in the current study revealed that by formulating questions in different ways, the teacher could balance between the contribution of minoritized students and dominating students to collective argumentation. As Hayano (2013) noted, "questions are a powerful tool to control interaction: they pressure recipients for response, impose presuppositions, agendas and preferences, and implement various initiating actions" (p. 395-396). For instance, the question in Excerpt 2.6, "Bill, is there anything else you wanna add to that thinking?" opened an opportunity for Bill to contribute his idea, but it did not presuppose that Bill's additional contribution was necessary after Calvin's contribution. Thus, applying the understanding of *question design* from a conversation analytic approach (see, for an overview, Hayano, 2013) to collective argumentation may reveal subtle relational work that teachers are doing with questions. This relational work is important from an equity standpoint since question designs may either promote or impede minoritized students' contribution.

Another category in Conner and colleagues' (2014) framework is *other supportive actions*, which includes directing, promoting, evaluating, informing, and repeating (p. 418). By attending to the temporal progress of collective argumentation, the current study showed that teachers' gestures are an important resource to enact those other supportive actions. For instance, in Excerpt 2.2, Ms. Hill's nodding, which began when Indigo uttered "sym" can be categorized as *evaluating* (Conner et al., p. 421). Ms. Hill's nodding, without a word, confirms the correctness of Indigo's projectible contribution "symmetric." Notably, the support through gestures also allows the student to hold the ongoing turn and to self-correct their contribution as necessary. Furthermore, the sequential analysis of the interaction illustrated the effect of such use of gesture. The change in Indigo's epistemic stance within a single turn while Ms. Hill was nodding showed the effect of Ms. Hill's supportive action. The use of gestures remains largely under-examined for teachers' facilitation of mathematics discussion, and the application of an ethnomethodological approach offers a tool to explore this less-known area of use of gesture.

Conclusion

To conclude, I return to Ms. Mackey's question in Jackson and Wilson's (2012) study: "*How* do I [a mathematics teacher] support African American learners in mathematics?" (p. 365, emphasis original). Her question implies that teacher's intent to support Black students is not enough; teachers need methods and professional development to move from intent to actions (Jackson & Wilson, 2012). There may be no silver bullet to support minoritized students in the mathematics classroom, but a part of the answer is in the details of how the classroom operates every day. The detailed analyses of turn-taking and collective argumentation suggest that supporting the three Black students is contingent acts, initiated by the teachers yet responsive to what the students do or say. It requires reshaping the taken-for-granted teaching practices (e.g., conventional turn-allocation methods) that likely reproduce existing status hierarchies.

Martin (2019) showed that equity discussions in mathematics education largely remain within the rhetoric of "Mathematics for All." Likewise, researchers too often examined and categorized teachers' discourse practices without considering which student is benefiting from the identified practices. If not distributed equitably, teachers' discourse practices and associated opportunities to participate can exacerbate existing inequities. The fine-grained analysis of the classroom interaction while centering on the three Black students highlighted the importance of understanding how the opportunities get distributed and to whom. Their participation did not happen by coincidence; the teachers prioritized the three students' participation amid the multiple bids coming from the well-intended dominating students. Given mathematics classrooms are often claimed as a marked territory of White and Asian males, knowing and being able to provide necessary support specifically to Black students (and other historically marginalized students) is a crucial aspect of teaching mathematics. Future research that builds on the detailed accounts presented in the current study may illuminate many more subtle yet crucial ways a mathematics teacher can push against the existing marginalization in the mathematics classroom.

REFERENCES

REFERENCES

Allwright, R. (1980). Turns, topics, and tasks: Patterns of participation in language teaching and learning. In D. Larsen-Freeman (Ed.), *Discourse analysis and second language acquisition* (pp. 165-187). Newbury House.

Barwell, R. (2013). Discursive psychology as an alternative perspective on mathematics teacher knowledge. *ZDM*, *45*(4), 595-606.

Battey, D., Neal, R. A., Leyva, L., & Adams-Wiggins, K. (2016). The interconnectedness of relational and content dimensions of quality instruction: Supportive teacher–student relationships in urban elementary mathematics classrooms. *The Journal of Mathematical Behavior*, *42*, 1-19.

Berry, R. Q. (2008). Access to upper-level mathematics: The stories of successful African American middle school boys. *Journal for Research in Mathematics Education*, *39*(5), 464-488.

Bieda, K. N., Sela, H., & Chazan, D. (2015). "You Are Learning Well My Dear" Shifts in Novice Teachers' Talk About Teaching During Their Internship. *Journal of Teacher Education*, *66*(2), 150-169.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171–200). Ablex Publishing.

Bressoud, D. M. (2021). The strange role of calculus in the United States. *ZDM–Mathematics Education*, *53*(3), 521-533.

Bressoud, D. M., Mesa, V., & Rasmussen, C. L. (Eds.). (2015). *Insights and recommendations from the MAA national study of college calculus*. MAA Press.

Busby, L., Goff, C., Hanton, D., Herbel-Eisenmann, B., Jones, L., Loeffert, C., Pyne, E. & Wheeler, J. (2017). Supporting powerful discourse through collaboration and action research. In A. Fernandes, S. Crespo & M. Civil. (Eds.) *Access and Equity: Promoting high quality mathematics in grades 6-8* (pp. 59-76). Reston, VA: NCTM.

Byun, S. (2020). *Developing the equity literacy of prospective teachers in their practice-based experience: An application of EQUIP* [Conference presentation]. Annual Conference of the Association of Mathematics Teacher Educators, Phoenix, AZ.

Byun, S., Peralta L. M., & Herbel-Eisenmann, B. (2020). A case of shared authority during a student demonstration in a mathematics classroom. In A. I. Sacristán, J. C. Cortés-Zavala, & P. M. Ruiz-Arias (Eds.), *Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the*

Psychology of Mathematics Education (pp. 1975–1983). Mexico: Cinvestav & AMIUTEM

Champion, J., & Mesa, V. (2017). Factors affecting calculus completion among U.S. high school students. In D. Bressoud (Ed.), *The role of calculus in the transition from high school to college mathematics*. Washington, DC: MAA and NCTM.

Chapin, S. H., O'Connor, C., O'Connor, M. C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, Grades K-6*. Math Solutions.

College Board (2019a). *AP Calculus AB and BC Courses and Exam Description.* Retrieved from https://apcentral.collegeboard.org

College Board (2019b). *AP Statistics Courses and Exam Description.* Retrieved from https://apcentral.collegeboard.org

Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics*, *86*(3), 401-429.

Diamond, J. B. (2006). Still separate and unequal: Examining race, opportunity, and school achievement in" integrated" suburbs. *The Journal of Negro Education*, 495-505.

Engle, R. A., Langer-Osuna, J. M., & McKinney de Royston, M. (2014). Toward a model of influence in persuasive discussions: Negotiating quality, authority, privilege, and access within a student-led argument. *Journal of the Learning Sciences*, *23*(2), 245-268.

Esmonde, I., & Langer-Osuna, J. M. (2013). Power in numbers: Student participation in mathematical discussions in heterogenous spaces. *Journal for Research in Mathematics Education*, 44(1), 288-315.

Faulkner, V. N., Stiff, L. V., Marshall, P. L., Nietfeld, J., & Crossland, C. L. (2014). Race and teacher evaluations as predictors of algebra placement. *Journal for Research in Mathematics Education*, *45*(3), 288-311.

Flowers III, A. M., & Banda, R. M. (2019). An Investigation of Black Males in Advanced Placement Math and Science Courses and Their Perceptions of Identity Related to STEM Possibilities. *Gifted Child Today*, *42*(3), 129-139.

Forrester, M. A., & Pike, C. D. (1998). Learning to estimate in the mathematics classroom: A conversation-analytic approach. *Journal for Research in Mathematics Education*, *29*(3), 334-356.

Garfinkel, H. (1967). *Studies in ethnomethodology.* Prentice Hall.

Geiser, S., & Santelices, M. V. (2006). The role of Advanced Placement and honors courses in college admissions. In P. Gandara, G. Orfield, & C. Horn (Eds.), *Expanding opportunity in higher education: Leveraging promise* (pp. 75–114). State University of New York Press.

Gholson, M. L., Bullock, E. C., & Alexander, N. N. (2012). On the brilliance of Black children: A response to a clarion call. *Journal of Urban Mathematics Education, 5*(1), 1–7.

Gholson, M., & Martin, D. B. (2014). Smart girls, Black girls, mean girls, and bullies: At the intersection of identities and the mediating role of young girls' social network in mathematical communities of practice. *Journal of Education*, *194*(1), 19-33.

Goffman, E. (1981). *Forms of talk*. University of Pennsylvania Press.

Gutiérrez, R. (2012). Context matters: How should we conceptualize equity in mathematics education? In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in discourse for mathematics education: Theories, practices, and policies* (pp. 17–34). Springer.

Hallett, R. E., & Venegas, K. M. (2011). Is increased access enough? Advanced placement courses, quality, and success in low-income urban schools. *Journal for the Education of the Gifted*, *34*(3), 468-487.

Hand, V. (2012). Seeing culture and power in mathematical learning: Toward a model of equitable instruction. *Educational Studies in Mathematics*, *80*(1-2), 233-247.

Hayano, K. (2013). Question design in conversation. In J. Sidnell & T. Stivers (Eds.), *The handbook of conversation analysis* (pp. 395–414). Blackwell Publishing Ltd.

Hayat, M. J., Eckardt, P., Higgins, M., Kim, M., & Schmiege, S. J. (2013). Teaching statistics to nursing students: An expert panel consensus. *Journal of Nursing Education*, *52*(6), 330-334.

Hepburn, A., & Bolden, G. B. (2017). *Transcribing for social research*. Sage.

Herbel-Eisenmann, B. (2009). Negotiation of the ‘‘presence of the text’’: How might teachers’ language choices influence the positioning of the textbook? In J. Remillard, B. Herbel-Eisenmann & G. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 134–151). Routledge.

Herbel-Eisenmann, B., Cirillo, M., Steele M. D., Otten, S., and Johnson, K. R. (2017). *Mathematics Discourse in Secondary Classrooms: A Practice-Based Multimedia Resource for Professional Learning*. Math Solutions.

Herbel-Eisenmann, B., & Shah, N. (2019). Detecting and reducing bias in questioning patterns. *Mathematics Teaching in the Middle School*, *24*(5), 282-289.

Herbel-Eisenmann, B. A., Steele, M. D., & Cirillo, M. (2013). (Developing) teacher discourse moves: A framework for professional development. *Mathematics Teacher Educator*, *1*(2), 181-196.

Heritage, J. (1984). *Garfinkel and ethnomethodology*. Polity Press.

Heritage, J. (2012). Epistemics in action: Action formation and territories of knowledge. *Research on Language & Social Interaction*, *45*(1), 1-29.

Heritage, J., & Robinson, J. D. (2011). "Some" vs. "any" medical issues: Encouraging patients to reveal their unmet concerns. In C. Antaki (Ed.), *Applied conversation analysis: Changing institutional practices* (pp. 15–31). Palgrave MacMillan.

Ingram, J. (2012). Whole class interaction in the mathematics classroom: A conversation analytic approach (PhD thesis). Institute of Education, University of Warwick.

Ingram, J. (2018). Moving forward with ethnomethodological approaches to analysing mathematics classroom interactions. *ZDM*, *50*(6), 1065-1075.

Ingram, J. (2020). Epistemic management in mathematics classroom interactions: Student claims of not knowing or not understanding. *The Journal of Mathematical Behavior*, *58*, 100754.

Ingram, J. (2021). *Patterns in Mathematics Classroom Interaction: A Conversation Analytic Approach*. Oxford University Press.

Ingram, J., & Elliott, V. (2014). Turn taking and 'wait time' in classroom interactions. *Journal of Pragmatics*, *62*, 1-12.

Jackson, K., & Wilson, J. (2012). Supporting African American students' learning of mathematics: A problem of practice. *Urban Education*, *47*(2), 354-398.

Jefferson, G. (2004). A sketch of some orderly aspects of overlap in natural conversation. In G. H. Lerner (Ed.), *Conversation analysis: Studies from the first generation* (pp. 43-62). John Benjamins.

Joseph, N. M., Hailu, M. F., & Matthews, J. S. (2019). Normalizing Black girls' humanity in mathematics classrooms. *Harvard Educational Review*, *89*(1), 132-155.

Kokka, K., & Chao, T. (2020). 'How I show up for Brown and Black students': Asian American male mathematics teachers seeking solidarity. *Race Ethnicity and Education*, *23*(3), 432-453.

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures*. Lawrence Erlbaum.

Krummheuer, G. (2007). Argumentation and participation in the primary mathematics classroom: Two episodes and related theoretical abductions. *The Journal of Mathematical Behavior*, *26*(1), 60-82.

Lam, K. D. (2015). Theories of racism, Asian American identities, and a materialist critical pedagogy. *Journal for Critical Education Policy Studies, 13*(1), 83–102.

Lampert, M., Rittenhouse, P., & Crumbaugh, C. (1996). Agreeing to disagree: Developing sociable mathematical discourse. In D. R. Olson & N. Torrance (Eds.),*The handbook of education and human development: New models of learning, teaching and schooling*(pp. 731–764). Blackwell.

Larnell, G. V. (2016). More than just skill: Examining mathematics identities, racialized narratives, and remediation among black undergraduates. *Journal for Research in Mathematics Education*, *47*(3), 233-269.

Leonard, J., & Martin, D. B. (Eds.). (2013). *The brilliance of Black children in mathematics*. Information Age Publishers.

Leyva, L. A., Quea, R., Weber, K., Battey, D., & López, D. (2021). Detailing racialized and gendered mechanisms of undergraduate precalculus and calculus classroom instruction. *Cognition and Instruction*, *39*(1), 1-34.

Malagon, M. C. (2010). All the losers go there: Challenging the deficit educational dis- course of Chicano racialized masculinity in a continuation high school. *Educational Foundations, 24*(1–2), 59–76.

Margolis, J., Estrella, R., Goode, J., Holme, J. J., & Nao, K. (2008). *Stuck in the shallow end: Education, race, and computing*. MIT Press.

Martin, D. B. (2000). *Mathematics success and failure among African-American youth: The roles of sociohistorical context, community forces, school influence, and individual agency*. Erlbaum

Martin, D. B. (2015). The collective Black and principles to actions. *Journal of Urban Mathematics Education*, *8*(1). 17-23.

Martin, D. B. (2019). Equity, inclusion, and antiblackness in mathematics education. *Race Ethnicity and Education*, *22*(4), 459-478.

Mehan, H. (1979). *Learning lessons: Social organization in the classroom*. Harvard University Press.

McGee, E. O., & Martin, D. B. (2011). “You would not believe what I have to go through to prove my intellectual value!” Stereotype management among academically successful Black mathematics and engineering students. *American Educational Research Journal*, *48*(6), 1347-1389.

McGee, E. (2018). “Black genius, Asian fail”: The detriment of stereotype lift and stereotype threat in high-achieving Asian and Black STEM students. *AERA Open*, *4*(4), 1–16.

McHoul, A. (1978). The organization of turns at formal talk in the classroom. *Language in Society, 7*(2), 183–213.

Michaels, S., O’Connor, M. C., Hall, M. W., & Resnick, L. B. (2010). *Accountable Talk sourcebook: For classroom conversation that works*. University of Pittsburgh.

Moore, J. (2005). Undergraduate mathematics achievement in the emerging ethnic engineers programme. *International Journal of Mathematical Education in Science and Technology*, *36*(5), 529-537.

Mortensen, K. (2008), Selecting next-speaker in the second language classroom: How to find a willing next-speaker in planned activities'. *Journal of Applied Linguistics,* 5(1), 55-79.

Nasir, N. S., Cabana, C., Shreve, B., Woodbury, E., & Louie, N. (Eds.). (2014). *Mathematics for equity: A framework for successful practice.* New York, NY: Teachers College Press; Reston, VA: National Council of Teachers of Mathematics.

Office for Civil Rights. (2018). *STEM course taking.* Retrieved from https://www2.ed.gov/about/offices/list/ocr/docs/stem-course-taking.pdf

Oyler, C. (1996). *Making Room for Students: Sharing Teacher Authority in Room 104*. Teachers College Press.

Parrish, S. (2010). *Number talks: Helping children build mental math and computation strategies, grades K-5*. Math Solutions.

Pollock, M. (2009). *Colormute: Race talk dilemmas in an American school*. Princeton University Press.

Robinson, M. (2003). Student enrollment in high school AP sciences and calculus: How does it correlate with STEM careers?. *Bulletin of Science, Technology & Society*, *23*(4), 265-273.

Sacks, H., Schegloff, E. A., & Jefferson, G. (1974). A simplest systematics for the organization of turn-taking for conversation. *Language, 50*(4), 696–735.

Safir, S. (2015, December 1) 3 Practices to Promote Equity in the Classroom. *Edutopia.* https://www.edutopia.org/blog/practices-promote-equity-in-classroom-shane-safir

Sahlström, J. F. (2002). The interactional organization of hand raising in classroom interaction. *The Journal of Classroom Interaction*, 47-57.

Shah, N. & Crespo, S. (2018) Cultural narratives and status hierarchies. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas & D. Wagner (Eds.), *Mathematical discourse that breaks barriers and creates space for marginalized learners* (pp. 23–37). Sense Publishers.

Shah, N. & Leonardo, Z. (2016). Learning discourses of race and mathematics in classroom interaction: A poststructural perspective. In I. Esmonde & A. Booker (Eds.), *Power and privilege in the learning sciences: Critical and sociocultural theories of learning* (pp. 50-69). Routledge Press.

Simosi, M. (2003). Using Toulmin's framework for the analysis of everyday argumentation: Some methodological considerations. *Argumentation*, *17*(2), 185-202.

Smith, M. S., & Stein, M. K. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions*. National Council of Teachers of Mathematics.

Stinson, D. W. (2013). Negotiating the "White male math myth": African American male students and success in school mathematics. *Journal for Research in Mathematics Education*, *44*(1), 69-99.

Tawfeeq, D.A., Hakim, I. A., & Greene, P. (2013). The Case of African American Males' AP Calculus Test Scores: A Snapshot of the District of Columbia as Panoramic View of the Natio*n. Journal of Education and Social Justice*, *1*(1), 86-101.

Toulmin, S. E. (2003). The uses of argument (updated ed.). New York: Cambridge University Press. Original work published 1958.

Waring, H. Z. (2013). Managing Stacy: A case study of turn-taking in the language classroom. *System*, *41*(3), 841-851.

Waring, H. (2014). Turn allocation and context: Broadening participation in the second language classroom. In J. Flowerdew (Ed.), *Discourse in context: Contemporary applied linguistics* (Vol. 3, pp. 301–320). Bloomsbury.

Waring, H. Z. (2016). *Theorizing pedagogical interaction: Insights from conversation analysis*. Routledge.

White, D. Y. (2003). Promoting productive mathematical classroom discourse with diverse students. *The Journal of Mathematical Behavior*, *22*(1), 37-53.

Wilson, J., Nazemi, M., Jackson, K., & Wilhelm, A. G. (2019). Investigating teaching in conceptually oriented mathematics classrooms characterized by African American student success. *Journal for Research in Mathematics Education*, *50*(4), 362-400.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for research in mathematics education*, *27*(4), 458-477.

CHAPTER 3: "POWER OF TWO HEADS": TEACHERS' SPONTANEOUS USE OF PARTNER TALKS IN AP MATHEMATICS CLASSROOMS

Introduction

After posing a question, the teacher shouts, "Turn and talk. Go!" With this signal, students talk to their partners for less than two minutes on the given topic. This interactional setting of *partner talk*, also called Think-Pair-Share and Turn and Talk, has been widely promoted amongst mathematics educators under the goal of moving away from teacher-centered classroom interaction. These terms, such as Think-Pair-Share, now became an integral part in many instructional recommendations, even in policy reports (e.g., College Board, 2019a, 2019b; MAA, 2018). Its simplistic image of a step-by-step teaching strategy (as represented by think, pair, and share), however, can obscure teachers' intricate work of facilitation. From an interactional view, the employment of partner talk marks a significant shift from other interactional settings because momentarily, the teacher cedes the control over turn-taking, and students instead sustain interactions with a smaller audience—typically two students, depending on classroom seating—compared to the entire class or small groups. Examining how teachers take advantage of shifting interactional settings to advance the ongoing lesson would give a fuller picture of partner talk as a situated teaching practice. This chapter attends to two teachers' nuanced interactional work before, during, and after partner talk to facilitate a focused and accountable space for mathematics discussions.

Despite its common use, facilitating partner talk has rarely been examined empirically as a teaching practice, whereas whole class discussion has been the primary focus of classroom discourse research and professional development (Chapin et al., 2009; Herbel-Eisenmann et al., 2017; Ingram, 2021; Smith & Stein, 2011). Similarly, researchers have theorized and documented the groupwork setting (i.e., groups of three or more students with cooperative roles) to a great extent (Esmonde & Langer-Osuna, 2013; Langer-Osuna, 2016), but these efforts rarely reached the moments when two or more students speak to each other in a relatively short timeframe. What defines the partner talk, distinguished from groupwork, is its spontaneous employment without any pre-designed tasks and roles. The prevalence of partner talk suggests exploring its various purposes and uses.

In this chapter, I examine this commonly used, yet under-examined, interactional setting: partner talk. Without understanding the underlying workings of partner talk, mathematics educators may overlook its key affordances that can facilitate focused and accountable mathematics discussions. A fine-grained study sensitive to teacher's spontaneous enactment of partner talk can inform mathematics educators of subtle details of facilitation that are consequential to student participation. To do so, I apply understandings of social interactions from ethnomethodology and conversation analysis (hereafter EMCA) (Garfinkel, 1967; Schegloff, 2007) to data from two Advanced Placement (AP) mathematics classrooms. EMCA approaches concern unveiling seen-but-unnoticed discursive practices[9] that organize partner talk that is situated in the sequential progression of classroom interaction.

Exemplified by Goodwin's (1994) work on *professional vision*, researchers have applied EMCA to examine discursive practices used by members of a profession. I draw on Goodwin's (1994) notion of *highlighting* to reconfigure the simplistic notion of partner talk into a set of discursive practices for teachers to: 1) highlight important mathematical features before partner

---

[9] I consistently use the term *discursive practice* as the teachers' methods of achieving the social activity of partner talk. Garfinkel (1967) refers to discursive practices as *participants' methods*, from which the name ethno-methodology originates. Ethnomethodology views social order (e.g., the way doing partner talk) as reflexive of discursive practices employed by the participants within the local activity rather than transcendental rules or social structures.

talk, 2) negotiate speakership and norms during partner talk, and 3) facilitate a whole-class discussion that holds every student accountable for speaking after partner talk. I illustrate how this set of discursive practices are mutually sustaining and how it generates a focused and accountable space for students.

This chapter begins with a brief review of the literature related to partner talk. I include discussions on Think-Pair-Share, Pair Share, Turn and Talk, paired problem-solving, and so on to capture a wide range of findings. Adopted from Chapin and colleagues (2009), I purposely use the term *partner talk* throughout the current chapter to reflect my primary focus on talk, or to be more specific, the change in norms of talk-in-interaction as the class moves from other interactional settings (e.g., individual work, whole class discussions) to partner talk and vice versa. I then present turn-by-turn analyses of excerpts from two AP classes to illustrate the discursive practice of highlighting and how the teachers utilize the changing norms as they employ partner talk. I end this chapter by discussing the current chapter's contribution to the literature in mathematics education about partner talk and classroom interaction more broadly, and I also make a few suggestions about facilitating partner talks.

Partner Talk in the Literature

Partner talk is a mode of classroom interaction in many instructional recommendations for mathematics teachers. For example, both the College Board and the Mathematical Association of America (MAA) suggest mathematics educators include partner talk (which they call Think-Pair-Share) to engage students with active learning or collaborative learning (College Board, 2019a, 2019b; MAA, 2018). It is described as "a good first step" for teachers to turn around their lecture-based instruction toward cooperative learning (MAA, 2018, p. 9) without the need for modifying the course structure or classroom setup (Barkley et al., 2014). Partner talk is presented as a quick and easy antidote to the persistent and pervasive overreliance on lectures as a dominant mode of teaching.

Prior to these recommendations, there have been many versions of advocacy for partner talk among mathematics educators. Thornton (1991) introduced Think-Tell-Share, which is a sequenced activity that requires students to think individually, tell their partner, and share with the whole class. Tyminski and colleagues (2010) also came up with an enhanced version that emphasizes the element of play in teaching children through their researcher-practitioner collaboration. More recently, Walter (2018) lamented the common absence of the "think" portion before mathematics teachers shout, "Turn and talk!" She made an additional recommendation of accompanying writing activity that requires students to fill in a graphic organizer as they move through the steps of think, pair, and share. Other suggestions also include Write (instead of think) -Pair-Share and Think-Pair-Square (talk in pairs followed by talk in groups of four) (Barkley et al., 2014; Lyman, 1981). There seem to be endless variants of partner talk in work written to mathematics educators.

Contrarily to the strong presence of partner talk in aforementioned practitioner-oriented literature, research that supports such recommendations has been very scarce. There are only a few supportive findings in undergraduate science education settings. The earliest finding is that partner talk (pausing a lecture with a peer discussion) led to better long-term memory (Di Vesta & Smith, 1979). More recently, Smith and colleagues (2009) reported partner talk's positive effect on responding to conceptual multiple-choice questions in an undergraduate genetics course with clickers. These studies provided promising evidence of the use of partner talk for enhancing student performance on a particular set of multiple-choice questions. Without any detailed accounts of what actually took place during the partner talk, however, they could only speculate how the partner talk might have supported the increased student performance.

Detailed investigations on student interaction during paired problem-solving sessions add cautionary tales to the discussion. Sfard and Kieran (2001) and Kieran (2001) examined interactional features of paired problem-solving episodes with 13-year-olds. They found that the student-to-student communication that took place as two students collaboratively worked on a solution to a given task were surprisingly ineffective and unproductive. The students, in general, had difficulty communicating their mathematical thinking to their partner and did not take full advantage of the paired problem-solving setting to co-construct solution paths. Bishop (2012) also illustrated concerning patterns of student interactions during partner talk. The two students in her study developed and enacted hierarchical identities between them (e.g., smart one, dumb one) over the course of partner talk. These findings revealed equity issues associated with partner talk, even though instructional recommendations for mathematics educators univocally promote partner talk as a way to make mathematics classrooms more equitable by broadening student participation.

This inconsistency in findings, in turn, hints at the complexity in teachers' work of facilitating partner talk: there is more to it than just asking students to think, pair, and share; or asking paired students to work on a given task. The primary focus of the aforementioned studies (Bishop, 2012; Sfard, & Kieran, 2001; Kieran, 2001) were on student-to-student interaction; these studies did not show what possible roles teachers could play in the process of partner talk. Turpen and Finkelstein (2009), however, showed that there were large variations in how the instructors employed partner talk in undergraduate physics classrooms. For instance, they contrasted how two instructors facilitated the subsequent whole-class discussions differently.

One instructor focused on presenting a correct answer, whereas the other instructor focused on students' sense-making using both correct and incorrect answers. Langer-Osuna (2016) presents the important role of the teacher during partner talk. In a grade five classroom, the teacher's evaluation of student ideas and behaviors during partner talk influenced how the pair of students treated each other with more or less authority in the subsequent partner talk. These findings highlight what teachers say or do during and after partner talk can shape how students participate in partner talk as well as the subsequent discussion.

Mathematics educators and teacher educators also seem to be keenly aware of the important role of teacher facilitation of partner talk. Chapin and colleagues (2009) encouraged teachers' spontaneous use of partner talk when the class faces an impasse or when there is need for eliciting more student responses. They also presented partner talk as a social primer for a whole-class discussion to invite students who may seem reluctant to share their thinking. Their insights align with the reflective commentary below by Krusi (2009), a veteran teacher who conducted action research on her own discourse practices:

> This strategy [Think-Pair-Share] allowed students time to think about their answers and to test them before sharing with the whole group. It also increased the number of students who had a voice and a stake in the discussion. It allowed me to eavesdrop and get an idea of the different approaches students took and the difficulties they were having. I could use this information to sequence responses in the whole-class discussion. (p. 122)

Chapin and colleagues and Krusi seem to concern not with the step-by-step implementation of partner talk but with its potential situated purposes in the unfolding classroom interaction. In other words, the above accounts shed light on partner talk as a spontaneous practice situated in the temporal progression of the lesson rather than a stand-alone learning activity. To take advantage of the versatility of partner talk, teachers need to decide when, how, and why to incorporate partner talk to serve various purposes based on the flow of the ongoing lesson.

In the current chapter, I examine this situated use of partner talk as a set of discursive practices based on fine-grained analysis of data from two AP mathematics classrooms. As a guide, I respond to the following research questions, addressing discursive practices before, during, and after partner talk, respectively:

1. How and in what context do two AP teachers initiate partner talk?
2. How do the teachers support students during partner talk?
3. How do the teachers use partner talk to shape the subsequent whole-class discussion?

An employment of partner talk typically involves making a shift from an ongoing whole-class discussion to partner talk and then bringing the class back to a whole-class discussion. The overarching goal of the current study with the research questions above is to encapsulate what kind of interactional environment teachers might facilitate by switching the interactional setting back and forth. The shifting interactional setting, in essence, represents changing norms of interaction. Thus, the above research questions explore teachers' discursive practices that put the changing social norms to use for a focused and accountable mathematics discussion.

An application of an EMCA approach offers a theoretical foundation and analytic methods to examine discursive practices and social norms. In the following, I present how I apply the understandings of social interaction from EMCA to the context of the current study.

Methods

EMCA provides a theoretical ground to investigate normative ways people conduct social interactions (i.e., social norms). These norms are not rule-governed but mutually achieved by participants with discursive practices. One of the simplest examples would be a greeting sequence. As a person says "Hi," there is a normative expectation that the recipient of the greeting would immediately respond "Hi." When the norm is breached, this orderliness of interaction becomes explicit in interaction because participants orient to the breaching moment as significant. For example, if the recipient does not return "Hi" or returns "Hi" after a long delay, the speaker may mark such occasion by requesting an account saying, "What's the matter?" In other words, this kind of greeting sequence holds people accountable to each other as a part of a particular social order, through which both parties maintain a certain kind of social relation. In the case of a greeting, the social relation is one of friendly, affiliative relations. EMCA scholars have identified discursive practices that achieve a variety of kinds of action (e.g., greeting) with speech features, linguistic features, gestures, and so on. In this current study, I apply such understanding to my analysis to describe intricate discursive practices of the teachers before, during, and after partner talk.

Key Theoretical Constructs

One of the important constructs in EMCA is *accountability* (Robinson, 2016) because of its close relationship with norm. Although the term has been used with slightly different meanings within EMCA, I consider "accountability as responsibility" (Robinson, 2016) in the current chapter to examine the participant's moral responsibility to adhere to the norms of interaction. Returning to the greeting example, receiving a greeting from someone makes the recipient accountable for reciprocating the greeting. When people breach this norm (i.e., not fulfilling the responsibility), they often engage in the practice of *accounting* for the event, such as giving a reason why they failed to adhere to the norm. For example, they may say, "I didn't hear you." Researchers use EMCA to study classroom interactions to reveal norms of classroom interaction and the associated participants' accountability. For instance, Mortensen (2008) showed that teachers are accountable for finding a willing next speaker as they negotiate turn-taking during a whole-class discussion with students (see Chapter 2 in this dissertation). This norm explains, in part, why the discursive practice of hand-raising is ubiquitous in classrooms. As teachers narrow the scope of turn-allocation only to the students with raised hands, they can allocate the next turn to a willing student (Sahlström, 2002). The current study considers how such accountability of the teacher and students get reshaped by employing partner talk.

As mentioned earlier, a teacher's use of partner talk involves shifting interactional settings between a whole-class discussion and partner talk. Goffman (1956) offered the constructs of *frontstage* and *backstage* to understand this kind of shifting interactional settings. Briefly put, frontstage represents highly public social settings, in which, participants manage their and other's public presentations. Backstage, on the other hand, represents a more private setting (i.e., less number of audiences) where participants can lower their guards and rehearse for their acts on the frontstage. This distinction is not necessarily made by physical settings. Goffman's example of the newsroom illustrates how such distinction can be made by discursive practices. When the camera for the live news is on, the newscaster is on frontstage. When the camera is off, or the newscaster is out of the view, the newscaster is on backstage. Embarrassing moments happen when a newscaster did not know the camera was in fact on and the newscaster acted as if the camera was off (e.g., stretching arms, rolling eyes). These breaching moments, in turn, reveal the differing social norms in two different social settings. Hersh (1991) discussed this role of changing interactional settings in the practices of mathematicians:

> In this sense of the term, the "front" of mathematics is mathematics in "finished" form, as it is presented to the public in classrooms, textbooks, and journals. The "back" would be

> mathematics as it appears among working mathematicians in informal settings, told to one another in an office behind closed doors (p. 128).

Similarly, for the teacher and students in a mathematics classroom, partner talk serves as an example of a backstage as opposed to a whole-class discussion as a frontstage. The current study attends to what teachers do to utilize such change in interactional settings.

During mathematics discussions, students often work with multiple objects, contexts, and representations. An important part of teacher's facilitation of a discussion is to draw students' attention to important mathematical features. From the view of EMCA, Goodwin (1994) named the term *professional vision*, referring to how members of a profession shape events to their professional scrutiny in work settings. Professional vision, for example, represents how archaeologists co-shape fine features of the ground into archaeological phenomena with their competent deployment of situated practices. In the context of calculus and statistics classrooms, the professional vision represents how the teacher and students co-configure particular situations or problems as disciplinary phenomena of mathematics and statistics. Goodwin (1994) named three discursive practices for participants to achieve such vision: coding, highlighting, and producing and articulating material representations. About *highlighting*, Goodwin (1994) writes:

> Human activity characteristically occurs in environments that provide a very complicated perceptual field. A quite general class of cognitive practices consists of methods for highlighting this perceptual field so that relevant phenomena are made salient. … Practices such as highlighting link relevant features of a setting to the activity being performed in that setting. (p. 628)

Thus, by highlighting, mathematics teachers link relevant mathematical features to the ongoing mathematics discussion. In the Findings section, I will show how initiating partner talk serves as highlighting situated in ongoing discussions.

In research on mathematics teaching and learning, in particular, Yackel and Cobb (1996) distinguished the notion of *sociomathematical norm* from social norms. With its roots in ethnomethodology, the term highlights the norms that are specific to participating in mathematical discussions. For example, sociomathematical norms concern what mathematical explanations or justifications are treated by the participants as adequate or valid *in* classroom discussions. Just as social norms, the sociomathematical norms are seen as a co-operative achievement negotiated by the participants rather than a set of rigid rules. This construct has contributed to the discussion on social processes of teaching and learning mathematics. Likewise, the current study also considers sociomathematical norms when there is salient negotiation over them.

Context

The current study is situated in two different public high schools located in suburban areas in the Midwest region of the United States. The larger study included three teachers, but the current analysis did not include one teacher since the teacher never employed partner talks. Ms. Hill (AP Statistics teacher) and Ms. Gray (AP Calculus teacher) are included in the analysis. Both teachers are White females and have at least ten years of experience teaching mathematics. Ms. Gray has participated in a professional development focusing on classroom discourse and equity for six years, which is relevant to the current study's topic—facilitation of classroom interaction. This variability in teachers' prior professional development might have added richness to the data to find a variety of interactional features associated with the partner talk.

Data

The theoretical stance of EMCA toward social interaction is also reflected in what and how I gathered data. EMCA studies strictly use naturally occurring data; that is, recordings from everyday interactional settings. Data from interview or experiment settings are not suitable from an EMCA perspective since they are participants' accounts formulated in another interactional setting. As such, I video and audio recorded four consecutive lessons by each teacher over the first three months in 2020 before the COVID-19 pandemic. Video recordings came from two stationary cameras on the opposite ends of the classroom to capture both teacher's and students' gestures and other bodily movements. There were four additional audio recorders placed in front of focus pairs of students in the classroom to capture their partner talks. Ms. Gray's lessons focused on finding volumes of revolved shape by applying integral calculus. Ms. Hill's lessons included a Reese's Pieces sampling activity (Rossman & Chance, 2001) on the first day and follow-up lessons about sampling distribution and central limit theorem in statistics. From the recordings, I identified a total of 26 instances of partner talks with an average duration of 1 minute and 45 seconds. Ms. Hill employed nine partner talks during the first lesson with the Reese's Pieces activity, which included a series of discussion prompts. In other lessons, both teachers employed 2-4 partner talks in each lesson. To attend to the situated use of partner talk in the analysis, I determined the boundaries of these episodes as inclusively as possible with neighboring whole-class discussions that took place before and after the partner talk.

Analysis

Analysis began by transcribing the identified episodes of partner talks with the notations developed by Gail Jefferson (Hepburn & Bolden, 2017; Jefferson, 2004), which includes fine-grained details of intonations, silence, and gestures, in addition to textual data. To answer the three research questions, I conducted turn-by-turn analysis while focusing on teachers' actions and orientations in the three parts of each episode: before, during, and after partner talk. In other words, as I examined the transcripts turn-by-turn, I asked, why this now? (Schegloff & Sacks, 1973). Why did the teacher say just this at this specific moment? What has been said before that makes the teacher say this? What does the teacher's talk accomplish? To answer these questions, I attended to a range of features of talk, such as referencing (e.g., use of pronoun), turn-taking (e.g., pause), accounting (e.g., justifying what has been just said) as the teachers initiate partner talk, talk to students during partner talk, and bring the class to the subsequent whole-class discussion. Based on identified teachers' actions and orientations, I identified three main discursive practices: 1) highlighting important mathematical features before partner talk, 2) negotiating speakership and norms during partner talk, and 3) holding every student accountable for speaking after partner talk. I validated these claims based on how the students respond to what the teachers do (i.e., *next-turn proof procedure*). In the Findings section below, I present the analysis in detail with illustrative excerpts from the data.

Findings

The findings that I report here correspond to the three research questions, describing three discursive practices mentioned above. First, I describe how the teachers initiate partner talk to achieve the discursive practice of highlighting. The analysis focuses on how the teacher marks the departure from the ongoing whole-class discussion to partner talk, and the finding underlines the spontaneity of partner talk tailored to meet the pedagogical needs at the moment. Second, the analysis focuses on what teachers accomplish during partner talk. The finding presents a parallel between backstage (Goffman, 1956) and partner talk in which the teacher can negotiate with students with less need for face-saving than in a whole-class discussion (i.e., frontstage). Lastly, the analysis attends to the momentarily changed norms when the class transitions into a subsequent whole-class discussion. I discuss how the teachers coordinate between partner talk and the subsequent whole-class discussions to establish mutually sustaining accountability for students to participate in both settings of discussion. I begin with focusing on highlighting before partner talk.

Highlighting Important Mathematical Features

The following two excerpts illustrate how the teachers *highlight* important mathematical features as they initiate partner talks. I describe what and how the teachers tell students (i.e., highlighting) shapes the context for subsequent partner talk by students. Excerpt 3.1 begins as Ms. Gray confirms the answer for the warm-up problem: *What is the total distance traveled by the particle with its velocity* $v(t) = 6t - t^2$ *from* $t = 0$ *to* $t = 3$, which is 18. Consider how she initiates a partner talk as she makes the distinction between the change in position (i.e., displacement) and total distance traveled despite the fact that they have the same numerical value. The transcript follows the talks by Luke, Min, and Tom[10] who were seated in the first row in the room after the onset of the partner talk.

Excerpt 3.1: F3-06 Total Distance Traveled

```
TCH:    Mo↑:s:t people are answering D¿
        (4.0)
TCH:    So when I work through this problem I also got D¿ (0.7) U:m
        (1.3) but I approa↑ched as the way I was doing as change in
        posi↑tion problem↑? (4.2) But this is asking us for total
        distance t↑RAveled. (7.2) S:o I am curious: (.) to hear
        your explanations for WHY:- we ↑KNOW that these are not (.)
        always equal right? (.) Change in position is no:t always
        equal to total distance traveled. So .hh (0.5) What did you
        know about this problem.=What within the problem allo:wed
        you to conclude that (0.3) these are one in the same. (1.2)
        Right? How do we KNO:W by computing change in
        position.=Cause this was my approach right,=I said well
        this is v of t:¿ (3.0) Then position would be the
```

[10] Typically partner talk involves two students. In this case, there was an odd number of students in this row of tables, and these three students voluntarily formed a group to include everyone.

```
          antiderivative of velocity¿ (5.5)And then I used my
          position function¿ (0.2) Right? (0.5) What does time of
          zero and time of three and I found the change in position.
          Which (.) for me was eigh↑tee:n. So WHAT ABOUT this PROBlem
          .hhh TO:LD you that change in position (0.3) WAS total
          distance traveled. Turn and check in with your table
          partner.=Expla:in that.
          (1.0)
    MIN:  Hm.
          (0.4)
    TCH:  Ok↑ay turn [and TALK.]      [>GO.<]
    LKE:             [Velocity ] is ne[ver  ] (.) u::m (0.3) never
          zero.
          (5.5)
    LKE:  That never start to going backward.
          (0.9)
    TOM:  It's never less than [(zero)
    MIN:                       [(Can't) go negative.=It's not gonna
          go negative.=
    TOM:  [=It's never negat[ive.]
    LKE:  [                 [Yeah] (something) like that.
          [((TCH arrives at the front of the table of MIN, TOM, LKE))
```

As Ms. Gray initiates the partner talk, she "highlights" relevant mathematical features and let other features "fade into the background" (Goodwin, 1994, p. 628). On the one hand, she tells the class that the answer to the warmup question is 18 and that change in position is not always the same as total distance traveled. By stating this information, Ms. Gray is settling any potential disputes about what the correct answer is or whether the change in position is the same thing as total distance traveled, thereby removing possible confusion about these given facts. On the other hand, Ms. Gray highlights the specificity of the problem for the students to pay close attention to. This is done so with a great extent of emphasis when Ms. Gray iterates the prompt. Note the increased volume (marked with capital letters) for "WHAT ABOUT this PROBlem," "TO:LD," and "<u>WAS</u>" in lines 18 and 19; Ms. Gray narrows the domain of inquiry to the not-yet-seen important detail of the given problem that makes the change of position equal to the total distance traveled.

This highlighting is consequential in students' subsequent talk. After the onset of the partner talk, marked by "turn and check in …" (line 20), the students start to exchange ideas.

Note that the substance in their talk represents the particle and its velocity in the context of the problem (i.e., velocity, that, and it). In other words, the focus of the partner talk remains on the mathematical objects situated in the problem that Ms. Gray highlighted earlier. Furthermore, the interactional norms of partner talk (e.g., student-initiated turns without teacher's nomination, overlapping talks) affords the spontaneous interactions through which the students can rapidly negotiate mathematical meanings with opportunities to self-correct, which in turn lessen the burden of formulating a definitive statement. For example, Luke's first account is "velocity is never zero" (line 26), but his talk does not lead to any uptake from Min and Tom (line 28). Luke spontaneously responds by repairing his earlier account by replacing "never zero" with "never start to going backward" (line 29). Tom and Min then follow up with Luke's talk and formulate modified descriptions of "It's never less than zero" (line 31) and "can't go negative" (line 32). The modified descriptions indeed better match with the context of the problem since the change of position equals the total distance traveled when the velocity of a particle remains positive or zero—hence not negative. This series of talk—Luke's self-correction and the subsequent modification made by Min and Tom—shows how partner talk can allow the students to test their thoughts with their partners and come to reformulations as needed within the highlighted topical area; thus, partner talk can lessen the social burden of responding with a complete or correct answer.

Although my analysis does not attend to Ms. Gray's deliberate decision-making process of choosing what to highlight, I note that the highlighted information matters. A particle's displacement and total distance traveled (and their relationship with velocity and speed) are one of the "essential knowledge" and "required course content" (College Board, 2019a, p. 150). Ms.

Gray's initiation of partner talk highlights the distinction between these two concepts that are often confused by students.

The partner talk in the following Excerpt 3.2 also touches on "essential knowledge" in the AP Statistics curriculum, which includes checking for necessary assumptions (e.g., independence in data collection) before making claims using inferential statistics (College Board, 2019b, p. 231). Ms. Hill highlights this issue of statistical assumption by initiating two partner talks back-to-back. Excerpt 3.2 begins with the whole-class discussion after the first partner talk about the question presented in Figure 7 below. Here, I attend to the initiation of the second partner talk to show that multiple partner talks can be employed to highlight mathematical features progressively based on what students bring up from an earlier partner talk. The transcript follows the talk by the pair of Alice and Leah once the partner talk begins.

Example 1: Of all cars on the interstate, 80% exceed the speed limit. We take a sample of 50 randomly selected cars on the interstate. d) If your friend says that 49 of the 50 cars they saw on the interstate when they are driving were speeding, do you believe them? What questions might you ask them?

Figure 7. Example for Sample Distribution Model for Proportion

Excerpt 3.2: D3-36 Will You Trust Your Friend's Comment?

```
TCH:    Yes or no do you trust it or no.
STs:    [No. ]
ST?:    [Yes.]
TCH:    Woo I'm hearing yeses and noes.
CTR:    No.
TCH:    If you do:n't trust it why::.
        (1.0)/((CTR raises his hand))
TCH:    [Tell me.]
CTR:    [  Very  ] unlikely causes it's three standard
        deviations away from the mea[n.]
TCH:                                >[VE]ry GOOD.< So this is
        very unlikely to occu:r .hh it's three standard
        deviations above average. (1.1) Now (0.4) I have
        a new question for you. For those of you guys that
        think it's possible that you trusted it. (1.0)
        [Do you think your friend's p hat belongs in this
        [((TCH points to the given sampling distribution
              under the document camera.))
        distribution. (1.7) [Did it meet these requirements.
                            [((TCH circles the list of
```

```
94                    assumptions for the Central Limit Theorem))
95           (1.6)
96  ALC:     No[:.     ]
97  TCH:       [I want] you to talk to your table partner about it
98           [if (        ) think met those requirement.]
99  ALC:     [No        it         didn't       because ] q
100          (woun't like [(    )won't]
101 LEH:                  [(    )  q  ] would not equal ten,=
102 ALC:     =tsh >yeh<
```

After hearing the mixed responses from students, Ms. Hill solicits Carter's justification for why they should not trust the comment. Note that Carter's comment later would be shown to be incorrect, but Ms. Hill fully accepts his justification with her quick, emphasized "very good." Ms. Hill even repeated the justification (lines 84-86). Then, Ms. Hill turns to the opposite side of responses by posing a question if the given comment meets the necessary assumption. At this moment, Ms. Hill initiates the second partner talk (line 97). This partner talk, together with Ms. Hill's approval of Carter's earlier comment, facilitates open—yet focused—space for the class to approach the disagreement between "yes" and "no." While keeping Carter's justification (and the final answer as no) as a viable option, Ms. Hill draws students' attention to the statistical assumption on which Carter's justification stands. Ms. Hill, thus, offers an opportunity for students to critically assess Carter's justification with a particular disciplinary lens that is marked by both her talk and mutually elaborating gestures of pointing (line 90) and circling (line 93). This highlighting is, however, done so without highlighting Carter's ownership of the idea (e.g., What do we think about Carter's reasoning?). This allows the class to engage with Carter reasoning while saving Carter's face.

The above excerpt also shows the importance of treating partner talk as a mutually construed practice that relates to other classroom interactions in its vicinity. The students' mixed responses were not evenly distributed between yes and no (lines 75-76). "Yes" was hearable from only one student, whereas "no" dominated the classroom. Sequentially, the above partner talk is positioned as a response to Carter's justification for "no," which is analogous to one of the common misconceptions among statistics learners (Sotos et al., 2007). The spontaneity of partner talk afforded Ms. Hill to highlight the line between what was commonly (and wrongly) seen (i.e., "no") and the not-yet-seen by the majority of the students (i.e., "yes").

Partner Talk as Backstage for Negotiations

One of the features partner talk shares with groupwork is smaller social spaces compared to the whole-class discussion. Although the physical size of the classroom remains the same, students and the teacher treat each other as if they are in smaller rooms, engaging in conversations with the reduced number of audiences. The current analysis shows that this change in social setting leads to an important change in social interaction that broadens the opportunity to communicate mathematical thinking. As discussed above, Goffman's (1956) distinction between backstage and frontstage can be revealing. With the following excerpts, I show how Ms. Gray uses partner talk as backstage to prepare her students for the acts on the subsequent frontstage of the whole-class discussion.

Negotiating Speakership with Students

Ms. Gray's turn-allocation method that I named *pre-arranged nomination* (see Chapter 2 in this volume for a range of methods) illustrates a case where a teacher utilizes the backstage setting of partner talk to negotiate norms. In the below excerpt, Ms. Gray approaches a small group of four (Calvin, Nora, Natalie, and Sean), formed by two initial pairs coming together in the middle of the partner talk[11]. The analysis focuses on how Ms. Gray negotiates the speakership of Calvin for the upcoming whole-class discussion.

---

[11] This resembles Think-Pair-Square (Lyman, 1981), but the difference is the students in this episode made a group of four on their own without teacher's initiation.

Excerpt 3.3: F1-40 The Washer Method

```
185 TCH:    Whaddid you guys discuss over here.
186         (0.9)
187 CVN:    [U:h]
188 NTL:    [U: ]:h=
189 CVN:    =[Cause there's a ga:p (in the) middle  [of this one,
190          [((CVN points to paper till line 194)) [((TCH moves
191                 from the behind of CVN to the side of CVN))
192 TCH:    Ok[ay,]
193 CVN:      [a  ] cylinder in the middle and this one is jus:t
194         like (.) in (that's) (.) touchy,=
195 NOR:    =And that one's (      ).
196 CVN:    I guess.
197 TCH:    Will you share that?/((TCH gaze at Calvin))
198         (.)
199 CVN:    To the class?
200         (0.3)/((TCH nods))
201 CVN:    I guess?
202 TCH:    >Okay.<
203         (0.7)/((TCH walks to the front of the room))
      ((lines 204-232 omitted; TCH gets the attention of the
      class and announces who will share and in what order.))
233 TCH:    Okay Alright Calvin. You wanna get started?
234 CVN:    YEAH. Alright. SO:: (1.3) fo:r example number two:
235         there's not like a: (.) gap in the middle like so
236         fo:r number ten there's a cylinder in the middle?
237         when you (0.9) [rotated around y equals zero, (1.6)
238                        [((TCH draws a cylinder))
239 CVN:    a::nd (0.4) for example number two since as you
240         rotate it around (0.5) u:h y equals: (.)
241         [one? (.5) there's no uh (0.5) gap °in the middle.°
242         [((TCH draws a bottom half of the rotated shape))
```

After Ms. Gray hears the group's—mostly Calvin's—explanation, she asks, "Will you share that?" while looking at Calvin. Calvin initiates a repair that sharing, in fact, means speaking to the entire class, which may be interpreted as an act of surprise rather than an issue with not understanding the request. Then, Calvin responds with "I guess?" (line 201) with rising intonation. His response does not display a strong willingness to speak to the class, which may have suggested he would have been unlikely to raise his hand to share if this was in a whole-class discussion setting. Ms. Gray closes the sequence with a quick "okay" (line 202), then she brings the entire class back to the whole class. This negotiation for Calvin's speakership establishes Calvin's mutually ratified status as a soon-to-be-nominated speaker until his turn arrives, and there is a noticeable shift in his displayed willingness. Note how Calvin responds to

Ms. Gray's initiation of his turn in the subsequent whole-class discussion. He starts with "YEAH." (line 234) with a loud voice (noted with capitalized letters), and he takes up the conversational floor with no delay, which contrasts with his earlier stance shown with "I guess?" (line 201).

This type of negotiation with specific students illustrates the merit of having a casual, backstage setting of partner talk, compared to that of whole-class discussion in which the entire class's attention is focused on the teacher and the speaking student (i.e., frontstage). This different social setting provides teachers with a distinct opportunity to engage in fine negotiation with students that they otherwise may have difficulty engaging in during instruction. This affordance is not limited to negotiating future speakership; this backstage setting of partner talk can also aid the teachers in negotiating sociomathematical norms, which I show in the following excerpt.

Negotiating the Sociomathematical Norm for Justifying

Earlier in Excerpt 3.1, I showed that Ms. Gray highlighted the not-yet-seen detail of the given problem that makes the values of change of position equal to total distance travel equal. During the earlier part of the partner talk, the three students, Luke, Min, and Tom, arrived at the consensus that those two values are equal because the velocity is never less than zero. In Excerpt 3.4, I return to this scene when Ms. Gray also joins the partner talk. The analysis focuses on how Ms. Gray poses a series of questions to Luke.

Excerpt 3.4: F3-06 Total Distance Traveled

```
34  TOM:    [=It's never negat[ive.]
35  LKE:    [                  [Yeah] (something) like that.
36          [((TCH arrives at the front of the table of MIN, TOM, LKE))
37          (0.8)
38  TCH:    Hoddid- how can you be sure.
39          (0.5)
40  TCH:    >When you say< [IT what's it.
41                         [((TCH stretches her R hand toward LKE))
42          (.)
```

```
43  LKE:   The velocity is never negative over the °interval zero to
44         three.°
45  TCH:   How can you be sure.
46  LKE:   I plugged in three.
47         (1.7)
48  TCH:   Is that enough?
49  LKE:   Yes.
50         (1.0)
51  TCH:   Plugging in just three? (.) °Is enough?°
52  LKE:   Yes.
53         (0.3)
54  TCH:   How so.
55         (0.7)
56  MIN:   Plug in three (          [           ) positive]
57  TOM:                            [Plug in zero too and then use M V
58         [T there.   ]
59  ISB:   [Plug in any]thing le:ss than th↑ree: [then the t square
60                                               [((TCH nods till ISB
61                                                finishes her turn))
62         would have to be less than six times t:.
63         (0.5)
64  TCH:   'kay.
65         (1.1)/((TCH moves to the front center of the room))
66  TCH:   >Okay.< SO I'm gonna go fr:om (1.3) [Lu↑:ke (1.5) to
67                                             [((TCH points to LKE))
68         [Isabella (2.2) did I ask you to share?
69         [((TCH points to ISB then to TOM))
70  TOM:   ↑No.
71  TCH:   No. Ohheh [no. ]
72  LKE:             [Am I] sharing?
73  TCH:   Okay,
74         (0.3)
75  LKE:   I am sharing?=
76  TCH:   YEA:H. Come on. Luke¿
77         (0.7)
78  TCH:   Okay¿ Tell'm- tell me a (0.3) what you and Min and Tom
79         talked about.
80         (3.0)
81  LKE:   The velocity of the interval zero to three (.) is never (.)
82         less than zero.
83         (1.0)
84  TCH:   And my question to him was how can you be sur:e, (0.5) and
85         then [Isabella kinda jumped in ↑and what were you saying,
86              [((TCH points to ISB with a pencil.))
87         (1.0)
88  ISB:   Well: (0.3) Luke said that he plugged in three:? (0.5) so
89         then (0.3) u:m (1.0) t squared would be three times three
90         and sixteen would be six times three, (0.7) since (0.6) six
91         times three is larger than three times three (0.5) the
92         whole (0.5) thing would be (1.0) positive anything smaller
93         than three would also has to be that way because then (.) t
94         squared would have to be °smaller than six times three°
```

At first glance, Ms. Gray's series of questioning (lines 38, 45, 48, 51, 54) may appear as benign probing of Luke's thinking. The word "probing," however, does not adequately describe what Ms. Gray is accomplishing here. These repeated forceful questions highlight negotiating *sociomathematical norm* (Yackel & Cobb, 1996; Yackel et al., 2000) related to what is a mathematically acceptable justification. On the one hand, with a series of questions, Ms. Gray holds the stance that *just* substituting three is not sufficient to show the claim's mathematical validity. Luke, on the other hand, with an immediate yes with a falling intonation, takes an opposing stance that substituting three alone is enough.

Comparing the current scene with another scene from a whole-class discussion (i.e., frontstage) can be telling. In Excerpt 3.6, Ms. Gray asked a question with a similar formulation, "How do we know …" When the student could not immediately produce a justification, Ms. Gray apologized to the student by saying, "Sorry." In the current scene (i.e., backstage), however, the delay in Luke's response (line 42) is followed by Ms. Gray's persistent pursuit for an adequate justification. In this backstage of partner talk, Ms. Gray can interact with students with less worry about saving students' face compared to the frontstage setting, and therefore, she can negotiate the sociomathematical norm in a less apologetic manner.

Isabella, who was sitting behind the three students, joins the talk (lines 59-62). Without sanctioning Isabella's participation in another partner talk, Ms. Gray nods and accepts Isabella's justification once she hears "anything less than three." This exchange, in turn, shows that Ms. Gray's orientation to advancing the discussion on the highlighted topic rather than enforcing rigid rules of interaction during partner talk. In the subsequent whole-class discussion, Ms. Gray asks Luke and Isabella to report the conversation (lines 78-79, 84-85). As the conversation gets re-enacted for the entire class to hear, the talk is reformulated to adhere to the norms of frontstage; that is, the re-enactment is "in 'finished' form, as it is presented to the public in classrooms" (Hersh, 1991, p. 128). Luke offers his claim as he did during the partner talk (lines

81-82), which is followed by Isabella's justification, while Ms. Gray coordinates the progression. There is no longer a sign of disagreement between Ms. Gray and Luke over the adequacy of just substituting three. This frontstage re-enactment appears as a seamless process of building a mathematical argument.

Returning to Frontstage After Partner Talk

So far, I reported on how Ms. Gray used partner talk as backstage based on its different norms of interaction from the frontstage norms. This last part of the Findings section attends to the subsequent whole-class discussion after the partner talk in more detail than the previous sections. The analysis focuses on how its norms deviate from other whole-class discussions because of the employment of partner talk. I argue that such differences in norms offer distinctive opportunities for teachers to hold every student in the class accountable for participating in the discussion. The following excerpts show how such changed norms are visible when the class returns to the whole-class discussion. Excerpt 3.5 illustrates how Ms. Hill and her students orient to the differences in accountability after a partner talk. Prior to this excerpt, the students were asked to put down their own definitions of population and sample. The class was casually using these terms, but with the increasing complexity of statistical distributions, they needed clearly defined (or conceptually distinguishing) meanings of distributions of population, sample, and statistics. The excerpt below begins as Ms. Hill tells the whole class that she will call on people once the partner talk concludes (lines 32-34). After spending more time in partner talk, Ms. Hill returns to the whole-class discussion by nominating Lily to share her response (line 55).

Excerpt 3.5: D2-35 Population and Sample

```
32  TCH:    I'm gonna ca:ll on people to respond so makes sure
33          you have an answer on your paper that you feel
34          comfortable with.
35          (0.2)
```

```
36  TCH:    It helps (to) bounce language off of your partner to
37          make sure that you worded things how you wanted to.
38          That's why you have this person here be your support
39          person.
      ((Lines 40-54 omitted; STs resume their partner talk.))
55  TCH:    Alright (2.0) Lily whaddid you put.
56          (1.1)
57  LLY:    U:m entire group that you're studyi:ng?
58          (0.7)
59  TCH:    ↑Good. [The entire group (5.4) you are studying.
60                 [((TCH write on the note as she speaks))
      ((Lines 61-77 omitted; TCH further elaborates the
      definition.))
78  TCH:    U::M let's go fo:r s:ample: Haily whaddid you put for
79          sample:.
80          (.)
81  HLY:    U:m the smaller group of people drawn from the
82          population that you're studying?
83          (0.8)
84  TCH:    Good a smaller group (2.3) of >I'm gonna say
85          individuals.< (3.2) dra:wn (1.7) fro:m (0.5) the
86          population °good.°
```

Nominating Lily (line 55) and Hailey (lines 78-79) after the partner talk may not seem much different from a typical scene of a whole-class discussion, but there is a clear deviation from Ms. Hill's typical norms of turn-taking in a whole-class discussion. In this scene, Ms. Hill nominated both Lily and Hailey *without* their bidding; that is, Lily and Hailey did not raise their hands to volunteer. This interactional pattern shows that students are accountable for speaking even when they did not show their willingness to speak. Based on this accountability of students, Ms. Hill can nominate any student in the room to speak—thus not limited to only a handful of students who are raising their hands. This momentarily reshaped norm of turn-taking can be a resource for teachers to leverage more equitable participation among students (see Chapter 2 in this dissertation). It is also important to note that students' accountability for speaking is bounded by the particular context of partner talk. Consider how Ms. Hill formulates the question, "whaddid you put." This question does not seek what the student knows or thinks; it specifically asks for what she wrote as an answer during the partner talk, as Ms. Hill spoke to the whole class earlier (lines 32-34). Students' accountability is, thus, a context-specific norm that has been temporally shaped by the earlier partner talk.

Ms. Gray's AP Calculus class also shows a consistent pattern. For example, she asked students, "What did you and your partner talk about?" after bringing students back to the whole-class discussion (see Excerpt 3.8). This formulation shows Ms. Gray's orientation toward what the students talked about during the partner talk rather than what they know or what the answer should be. In Ms. Gray's case, the nomination is done by randomly picking a stick with a student's seat number. This method is different from Ms. Hill's nomination without a student's raising their hands, but it still provides an opportunity to nominate any student in the room. The student to be called on is unnamed and who is called on is a surprise, which is the same across both types of nominations. For both teachers, the turn-allocation method is accompanied by a prior partner talk, which plays a role in broadening the speakership within the classes.

I present a contrastive episode with no prior partner talk to show that having partner talk is necessary for the turn-taking norm that I discussed above. Prior to Excerpt 3.6, Ms. Gray asked the whole class how the curve defined as $x = -y^2 + 4y$ looks, which will determine the region for a volume of revolution. Ms. Gray nominates Tom even though he does not raise his hand. I will show how this method of nominating without partner talk may lead to interactional trouble.

Excerpt 3.6: F1-27 Parabola

```
15  TCH:    What does: (.) [this curve look like. Graphically.
16                         [((TCH put L hand on the equation 8))
17          (1.5)/((TOM moves R hand following a parabola that
18                opens to L in the air))
19          [Tom- Tom (just went) like this.
20          [((TCH moves L hand following a parabola that opens
21                to R mirroring Tom's gesture))
22  ST?:    heheheh
23  TCH:    So it's a- it's a [PARAbola shape, (1.0) HOW do we
24                            [((TCH repeats the same gesture))
25          know that it's not opening [upward or downward Tom?
26                                     [((TCH makes U facing up
27                               and facing down with L hand))
28          (0.7)
```

```
29  TOM:    HHm.
30          (1.2)
31  TCH:    Sorry./((TCH smiles))
32          (1.1)
33  TOM:    Because it is x equals (.) with a y squared rather
34          than (it-) uh y equals x squared.
```

After Ms. Gray asks the class how the curve looks like, Tom uses his left hand to show a parabola that opens to his left with his hand (lines 17-18). Ms. Gray mirrors Tom's hand motion, and then she asks, "How do we know that it's not opening upward or downward?" (lines 23-27). In this scene, Tom did not raise his hand to indicate that he wants to be called on, nor has the class engaged in a partner talk regarding this question. Ms. Gray nominates Tom as the next speaker before closing her turn, and what follows is a noticeable silence and absence of an explanation with Tom saying "HHm" in the middle (lines 28-30). Ms. Gray apologizes to Tom lightheartedly, saying, "Sorry." This apology indicates that Ms. Gray treats Tom's delay in responding as her fault, that she nominated Tom when he was not ready to share. This exchange shows that Tom was not accountable for speaking, but instead the teacher was accountable for locating and nominating a willing student, which the common hand-raising practice precisely achieves (Sahlström, 2002). This contrasting case shows that employing partner talk disrupts this normative organization of accountability between the teacher and students.

I have presented so far how employing a partner talk can reshape the norm for the subsequent discussion, but an important question still remains. Why with partners? That is, allowing time for each individual student to think about a question *without* their partners may achieve the functions of highlighting and establishing students' accountability for sharing what they thought about. The necessity of partners (at least one) is presented in Ms. Hill's talk, "at least power of two heads," in the below excerpt. This illustrates how partner talk may shape how the teacher and students treat each other as knowers, their epistemic stances. Excerpt 3.7 begins as Ms. Hill brings the class back from partner talk about guessing the most prevalent color of Reese's Pieces candies.

Excerpt 3.7: C1-05 What Color Does Show Up the Most?

```
75  TCH:    What did you guys (0.2) What did you guys start to
76          think about. (.) In terms of (0.3) which color (1.0)
77          do you think shows up the mo:st. (0.4) So I wanna see
78          some volunteers. Volunteers hands up.
79          (0.3)/((only LDS raises hand))
80  TCH:    How many of you gu:ys (1.0) [have an answer you and
81                                      [((TCH raises her hand.))
82          your partner agree on.
83          (1.5)/((Eleven Ss raise hand))
84  TCH:    Okay,
85          (0.7)
86  TCH:    So you and your table partner agrees so that's at
87          least power of two heads:.
88          (0.3)
89  TCH:    [Whaddid you put?
90          [((TCG reaches her hand toward WLM))
91  WLM:    Orange,
```

The class returns to the whole-class discussion as Ms. Hill looks for volunteers to share. This leads to only one student raising her hand. Ms. Hill then reformulates her question to encourage more hands to raise their hands. In lines 80-82, she asks, "How many of you guys have an answer you and your partner agree on." The question no longer seeks what the students think about the question, but it seeks which partners had consensus in their thinking. Eleven students raise their hands instead of one, and Ms. Hill asks William to share what he put. The account that Ms. Hill offers right before nominating William is revealing (lines 86-87). The agreement between the two partners represents the "power of two heads" (line 87); that is, Ms. Hill is treating the pair of students as collective knowers rather than individual students.

This orientation toward collective knowers is not only evident in the teachers' talk but also in the ways students present themselves. When a student is nominated by the teacher, they often use the subject pronoun "we" or refer to themselves as, for example, "Lucy and I" instead of "I." This self-representation as collective knowers, however, can be unstable and may change as the interaction unfolds. The below Excerpt 3.8 presents a case where "we" gets modified by a partner as “I.” The excerpt begins as the class returns to the whole-class discussion after they had a partner talk about why the answer choice C ($\lim_{x=4} f(x)$ exists is a false statement) makes sense (see Figure 8). During the earlier partner talk, there was not much talk between Nora and Samantha.

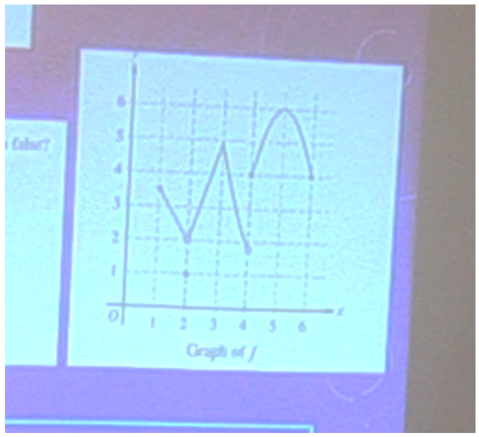


Figure 8. Graph of *f*

Excerpt 3.8: F2-06 Why Does C Makes Sense?

```
25  TCH:    ↑ROW A:↑ (1.3) seat tw:o (0.5) Is that you Nora?
26  NOR:    Yes.
27  TCH:    Alright Nora what did you guys talk about.
28  NOR:    U:m we talked about ho:w .hh um the sl↑o:pe (0.7) doesn’t
29          (0.3) it’s like <chan↑ges> so that it can’t (0.5) the limit
30          °doesn’t exist.°
31  TCH:    Okay (0.3) the s:lope changes.[=What do you mean we say
32                                        [((SMT raises hand))
33          this. >YEAH< Samantha.
34          (.)
35  SMT:    U:m I said that (0.3) the limit at x equals AS x (        )
36          exist because .hh your (.) limit from the le↑:ft (0.3)
37          as you approach four doesn’t equal your limit .h from the
38          right °as you approach 4,°
```

Nora gets nominated when her seat number was randomly called (line 25), and she begins her turn with the subject pronoun “we” (line 28). Once Ms. Gray repeats what Nora said, Samantha, Nora’s partner, raises her hand (line 32). Note here that Samantha starts with “I,” a singular pronoun, which marks her distancing from being the collective knowers, “we.” Samantha, instead of responding to Ms. Gray’s question (lines 31-33) about what Nora said earlier, restarts with her own response and claims her sole ownership. This case shows that the status of being collective knowers is *discursively* constructed and performed based on the common activity of partner talk, and students adhere to (or reject sometimes) such status as the discussion unfolds. In other words, Nora did not necessarily use "we" because she and Samantha actually reached a consensus, but because that she is accountable for doing so due to the earlier partner talk. Whether the partner talk cognitively enhanced students' knowledge, the partner talk nonetheless shaped students' status as collective knowers, and such status is fluid and up for further negotiation.

Discussions and Implications

The above analysis showed that facilitating partner talk can involve a set of discursive practices that can highlight relevant mathematical features, coordinate accountability for participating in partner talk and the subsequent whole-class discussion, and utilize different interactional settings to tactfully support student participation. In the current section, I discuss the contribution of the current study to the existing literature on partner talk, and classroom interaction more broadly. Further, I discuss implications for teacher education to better support mathematics teachers in incorporating partner talk or refining their facilitation practices.

Broadening the Conceptualization of Partner Talk

As I noted earlier, the most common way the partner talk has been presented in professional literature is Think-Pair-Share (e.g., College Board, 2019a, 2019b). These recommendations often focus on telling students to think, pair, then share in a recommended timeframe without much consideration of how, when, and why the teacher can purposeful use partner talk. The current study, however, shows that what teachers achieve with partner talk is more complex than a step-by-step student activity. With partner talks, teachers can *highlight* a specified domain of inquiry, through which teachers provide an accountable space for students to further their mathematical thinking. To be fair, focusing on the structure of Think-Pair-Share offers a blueprint for what students do in partner talk, and such recommendation could be helpful for mathematics educators to begin to use partner talk. I, however, add a caution to the proliferating recommendations that focus on different ways of controlling student behavior without much empirical evidence, such as mandating students to write before talk (Walter, 2018). The effort will be more meaningful if teacher educators draw teachers' attention to their own situated use of partner talk while considering how their students respond, so that the teachers can purposefully assess and refine their discursive practices (see, for example, an edited volume by Herbel-Eisenmann & Cirillo, 2009).

This study also showed the use of partner talk situated in the ongoing progress of the lesson. That is, partner talk is shaped by teacher's prior highlighting of mathematical features, and partner talk also offers a space for the teachers to shape the subsequent whole-class discussion. Similarly, the 5 Practices (Smith & Stein, 2011) and complex instruction (Cohen & Lotan, 1997) attend to the temporal progression of the whole lesson, such as launch, explore, discuss and summarize (Smith & Stein, 2011, p. 1). The current study shows that partner talk can serve a similar purpose but in a much smaller scale than the entire lesson with more versatility. Initiating partner talk, for example, does not require pre-designing or selecting a high-level groupworthy task. The episodes presented here show that partner talk allows the teachers to take advantage of the emerging needs and opportunities for a more focused discussion such as an important mathematical feature that students seem to overlook (e.g., Excerpt 3.1) and a disagreement between yes's and no's (e.g., Excerpt 3.2). The findings from this study suggests that facilitating partner talk is as complex as facilitating groupwork or whole-class discussion with multiple moving parts that require teachers' spontaneous attention. Further studies on wider range of episodes of partner talk may illuminate other purposes and functions that partner talk can serve.

Importance of Shifting Interactional Settings

My analysis on how the teachers participated in some of the partner talks shows how teachers can use the social setting to negotiate interactionally and mathematically important matters with the students. For example, Excerpt 3.4 illustrated that partner talk can offer a backstage social space for the teacher to negotiate sociomathematical norms in an unapologetic manner. In other words, the teachers and students are sensitive to the social setting when they engage in the process of negotiating sociomathematical norms. Although Yackel and Cobb (1996, p. 461) presented the sociomathematical norm (e.g., "what counts as an acceptable mathematical explanations and justification") as distinctive to social norms (e.g., when to explain or justify), negotiating such sociomathematical norms is inherently co-operative social endeavor, which is organized by broader norms of interaction. Just attending to sociomathematical norms for the sake of teaching and learning mathematics would limit our view on how its negotiation emerges to the surface of classroom discussion.

In the discussion on the teacher's role in the process of negotiating sociomathematical norms, the notion of professional obligations (teachers' obligation to the discipline, to the individual students, to the classroom space and community, and to the institution of schooling) offers a helpful lens to illuminate its complexity (Herbst et al., 2011). Ms. Gray's series of questioning in Excerpt 3.4 can be viewed as a teacher's fulfilling the role as "a representative of the mathematical community" (Yackel & Cobb, 1996, p. 474). That is, communicating to Luke that just substituting 3 is not mathematically sufficient for showing the validity for all values in the interval of [0,3] is her professional obligation to the discipline of mathematics. Asking a series of questions in front of the whole-class, however, can be face-threatening to Luke (see, for a similar concern, Bills, 2000; Brandt & Tatsis, 2009; Lampert et al., 1996), which could lead to a conflict with her obligation to the individual student. Ms. Gray's case illustrates a way a teacher can skillfully manage such conflictive professional obligation by using the momentarily reshaped interactional setting with partner talk. The backstage setting reduced Luke's social burden of receiving disapproval from the teacher, yet the combination of Luke's claim and Isabella's (acceptable) justification during the subsequent whole-class discussion offered a legitimized version of justification to the whole class. Considering how people interact in different social settings (e.g., frontstage vs. backstage) can further our understanding of when and how the issue of sociomathematical norms emerge and get negotiated in everyday classrooms with multiple competing priorities and obligations.

Furthering the Discussion on Classroom Interaction

The inconsistent findings in the literature about partner talk bring up an important question to consider: Does partner talk afford worthwhile opportunities for students to engage in mathematical discussions? Although evaluating the content of partner talks was not the focus of the current study, just as Bishop (2012) and Sfard and Kieran (2001) reported, I could see a wide range of mathematical communication during partner talk. Nonetheless, the subsequent whole-class discussions, which the aforementioned studies did not attend to, affirms the value of the partner talk suggested by the practitioners and teacher educators. Krusi (2009), for example, stated that the partner talk "increased the number of students who had a voice and a stake in the discussion" (p. 122). The current analysis provides empirical evidence that suggests such workings of partner talk. The employment of partner talk provides a backstage space for students to co-operate with their partners and simultaneously establishes all students' accountability for reporting their partner talk to the whole class.

This change in accountability constitutes, and was constituted by, Ms. Gray's random turn-allocation method and Ms. Hill's nomination without students' bidding (i.e., no hand-raising). These turn-allocation methods coupled with the prior partner talk broaden the current understanding of teachers' turn-allocation practice. Contrarily to the teacher's accountability for locating and nominating a knowing-and-willing next speaker (Mehan, 1979; Mortensen, 2008), the two teachers' turn-allocation methods do *not* consider students' willingness to speak. This norm can be a useful resource, in particular, to make the whole-class discussion more equitable. Teachers can purposely nominate students who seemed marginalized in classroom discussions and invite them to take up the conversational floor. As Excerpt 3.3 illustrated, teachers could even go further in their effort by pre-arranging speakership with a specific student during partner talk. This negotiation of speakership may redress some of the inequitable patterns of interaction within pairs by imbuing the student with additional accountability for the subsequent whole-class discussion (see Chapter 2 in this volume). As Langer-Osuna (2016) showed, what teachers say during partner talk can influence which student's idea gets valued during partner talk. Similar to "assigning competence" (Cohen & Lotan, 1997), an explicit invitation to speak to the whole class can convey that the student's idea is worthy of sharing with the entire class. As Gardner (2013) points out, prior research on turn-taking in the classroom has primarily been in traditional, teacher-centered classrooms (e.g., McHoul, 1978). The current study shows the glimpse of how such normative organization (e.g., turn-allocation) may change as teachers move toward facilitating a less teacher-centered discussion.

Implications for Facilitating Partner Talk and Teacher Education

Based on the findings in the current study, I make a few suggestions to mathematics teachers and teacher educators. First, I ask teacher educators to reconsider their heavy emphasis on the “think” part of Think-Pair-Share. This emphasis, in turn, reflects the common absence of the structured think time when teachers employ partner talk (Walter, 2018). Likewise, both teachers in the current study never asked students to have a think time before talking to their partners despite the fact that Ms. Hill referred to her use of partner talk as *Think*-Pair-Share. I contend that it would be wrong to treat this absence of the “think” time as the teachers’ bad habit or negligence that needs to be fixed. Such treatment may underestimate the value of trial-and-error processes these teachers went through to attain the refined mastery of the discursive practice presented in this chapter; there must be unknown pragmatic reasons why the “think” part does not “stick” with teachers’ everyday practice. What Kieran (2001) reported about partnered problem-solving is illuminating for this matter. Kieran (2001) discusses that once the students generate a solution path, student talks are often directed to self rather than to the partner. In other words, students are more likely to exchange their thinking, rather than just talking to themselves, when their thinking is not fully refined and organized. This finding implies that having structured think time can be counterproductive for the goal of facilitating generative student-to-student communication. As Kieran (2001) showed, once students are set on their own way of thinking, they tend to argue for their own stances rather than considering other’s thinking and negotiating for a mutual path.

Another basis of my argument for reconsidering the structured “think” time is the emerging understanding of the relationship between conversation and cognition. Discursive perspectives on communication (e.g., conversation analysis, discursive psychology) offer an alternative view on talk, contrarily to the taken-for-granted theory of communication; that is to say, people's talk reflects people's thinking. The discursive approaches highlight that human cognition is situated in the moment-to-moment interaction, meaning our cognition (or its representation in our talk) is interactionally generated (e.g., Drew, 2005; Edwards, 1993). This understanding implies that thinking should not be treated as an isolated mental activity. Excerpt 3.1 illustrates such a point. When Luke shared his initial thought, there was no uptake from the other two students. This led to Luke's reformulation of his earlier thinking. Without this interactional context (i.e., the absence of uptake), Luke's reformulation (i.e., his revised thinking) may not have emerged. Allowing students to talk to their partners freely may enhance their thinking since such an interactional context offers a ground on which new thinking can emerge.

Second, I suggest the consideration of using more spontaneous partner talk in between whole-class discussions, rather than as a separate partnered problem-solving time on a given task. Excerpt 3.1 and Excerpt 3.2, for example, show when partner talk can be suitable in the unfolding classroom interaction. The partner talk in Excerpt 3.1 highlights a particular mathematical feature that requires further investigation, and the partner talk in Excerpt 3.2 highlights an important statistical feature to resolve the mixed responses of yeses and noes. This seemingly improvised partner talk can also be planned. For example, in Excerpt 3.2, Ms. Hill might have anticipated that the majority of the students would say "yes," mindlessly following a statistical procedure that she presented in earlier lessons. This, in turn, brings up a great opportunity to employ partner talk for students' more careful consideration.

My last suggestion is based on how Ms. Gray participated in partner talks to build up for the subsequent whole-class discussion. Similar to the suggestion by Smith and Stein (2011), teachers can plan for the subsequent whole-class discussion during partner talk by selecting and sequencing mathematical content to be presented after the partner talk. What the current study adds is that such planning of selecting and sequencing is not entirely teachers' cognitive mental activity but also a social process of negotiation. Partner talks offer a backstage space, in which the teachers can openly negotiate the speakership with students while listening to students' talk, which in turn shapes particular students' status (e.g., a soon-to-be-nominated student). This opens more opportunities for teachers to support marginalized students' participation in classroom discussions and better utilize the "power of two heads."

Conclusion

Partner talk momentarily offers a different interactional setting compared to the common teacher-led whole-class discussions. The current chapter presented how the teachers facilitated focused and accountable space for mathematics discussions by using partner talk. The pedagogical value of partner talks is not only the student-to-student communication that occurs during the partner talk but also the changing norms of interaction, which generates different interactional settings of backstage and frontstage. The fine-grained analysis showed how the backstage of partner talk and the subsequent frontstage of whole-class discussion mutually sustain all students' accountability for their participation in a discussion. The purpose of partner talk, or utilizing the "power of two heads," was not entirely cognitive phenomena that putting two minds together leads to a greater degree of knowledge, but more so *social* phenomena through which the status and moral responsibility of students get reshaped. Spontaneous use of partner talk (i.e., skillfully shifting the interactional settings between front and backstage) offers a helpful resource to facilitate an equitable discussion while holding students accountable for social and sociomathematical norms. This chapter provided an empirical account for such refined mastery of discursive practices for facilitating partner talk, which is often hidden under the simplistic notion of Think-Pair-Share.

REFERENCES

REFERENCES

Barkley, E. F., Cross, K. P., & Cross, K. P. (2014). *Collaborative Learning Techniques: A Handbook for College Faculty.* John Wiley & Sons

Bills, L. (2000). Politeness in teacher-student dialogue in mathematics: A socio-linguistic analysis. *For the Learning of Mathematics*, *20*(2), 40-47.

Bishop, J. P. (2012). “She's always been the smart one. I've always been the dumb one”: Identities in the mathematics classroom. *Journal for Research in Mathematics Education*, *43*(1), 34-74.

Brandt, B., & Tatsis, K. (2009). Using Goffman's concepts to explore collaborative interaction processes in elementary school mathematics. *Research in Mathematics Education*, *11*(1), 39-55.

Chapin, S. H., O'Connor, C., O'Connor, M. C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, Grades K-6*. Math Solutions.

Cohen, E. G., & Lotan, R. A. (1997). *Working for equity in heterogeneous classrooms: Sociological theory in practice*. Teachers College Press.

College Board (2019a). *AP Calculus AB and BC Courses and Exam Description.* Retrieved from https://apcentral.collegeboard.org

College Board (2019b). *AP Statistics Courses and Exam Description.* Retrieved from https://apcentral.collegeboard.org

Di Vesta, F. J., & Smith, D. A. (1979). The pausing principle: Increasing the efficiency of memory for ongoing events. *Contemporary Educational Psychology*, *4*(3), 288-296.

Drew, P. (2005b). Is confusion a state of mind? In H. te Molder & J. Potter (Eds.), *Conversation and cognition* (pp. 161–183). Cambridge University Press.

Edwards, D. (1993). But what do children really think? Discourse analysis and conceptual content in children's talk. *Cognition and instruction*, *11*(3-4), 207-225.

Esmonde, I., & Langer-Osuna, J. M. (2013). Power in numbers: Student participation in mathematical discussions in heterogeneous spaces. *Journal for Research in Mathematics Education*, *44*(1), 288-315.

Gardner, R. (2012). Conversation Analysis in the Classroom. In J. Sidnell & T. Stivers (Eds.), *The handbook of conversation analysis* (pp. 593-611). Wiley-Blackwell.

Garfinkel, H. (1967). *Studies in ethnomethodology.* Prentice Hall.

Goffman, E. (1956). *The presentation of self in everyday life*. Doubleday Anchor.

Goodwin, C. (1994). Professional vision. *American Anthropologist*, *96*(3), 606-633.

Hepburn, A., & Bolden, G. B. (2017). *Transcribing for social research*. Sage.

Herbel-Eisenmann, B., & Cirillo, M. (Eds.). (2009). *Promoting purposeful discourse: Teacher research in mathematics classrooms*. National Council of Mathematics Teachers.

Herbel-Eisenmann, B., Cirillo, M., Steele M. D., Otten, S., and Johnson, K. R. (2017). *Mathematics Discourse in Secondary Classrooms: A Practice-Based Multimedia Resource for Professional Learning*. Math Solutions.

Herbst, P., Nachlieli, T., & Chazan, D. (2011). Studying the practical rationality of mathematics teaching: What goes into "installing" a theorem in geometry? *Cognition and Instruction*, *29*(2), 218-255.

Hersh, R. (1991). Mathematics has a front and a back. *Synthese*, *88*(2), 127-133.

Ingram, J. (2021). *Patterns in Mathematics Classroom Interaction: A Conversation Analytic Approach*. Oxford University Press.

Jefferson, G. (2004). "At first I thought": A normalizing device for extraordinary events. In G. H. Lerner (Ed.), *Conversation analysis: Studies from the first generation* (pp. 131-167). John Benjamins.

Kieran, C. (2001). The mathematical discourse of 13-year-old partnered problem solving and its relation to the mathematics that emerges. *Educational Studies in Mathematics*, *46*(1), 187-228.

Krusi, J. (2009). Revoicing: The good, the bad, and the questions. In B. Herbel-Eisenmann & M. Cirillo (Eds.), *Promoting purposeful discourse: Teacher research in mathematics classrooms* (pp. 117-135). National Council of Mathematics Teachers.

Lampert, M., Rittenhouse, P., & Crumbaugh, C. (1996). Agreeing to disagree: Developing sociable mathematical discourse. In D. R. Olson & N. Torrance (Eds.),*The handbook of education and human development: New models of learning, teaching and schooling*(pp. 731–764). Blackwell.

Langer-Osuna, J. M. (2016). The social construction of authority among peers and its implications for collaborative mathematics problem solving. *Mathematical Thinking and Learning*, 18(2), 107-124.

Lyman, F. (1981). The responsive classroom discussion. In A. S. Anderson (Ed.), *Mainstreaming digest*. University of Maryland College of Education.

Mathematical Association of America (MAA) (2018). *MAA Instructional Practices Guide*. Retrieved from https://www.maa.org/programs-and-communities/curriculum%20resources/instructional-practices-guide.

Mehan, H. (1979). *Learning lessons: Social organization in the classroom*. Harvard University Press

McHoul, A. (1978). The organization of turns at formal talk in the classroom. *Language in society*, 183-213

Mortensen, K. (2008). Selecting next-speaker in the second language classroom: How to find a willing next-speaker in planned activities. *Journal of Applied Linguistics,* 5(1), 55-79.

Robinson, J. D. (2016). Accountability in Social Interaction. In J. D. Robinson (Ed.), *Accountability in social interaction* (pp. 1-44). Oxford University Press

Rossman, A. J., and Chance, B. L. (2001), Workshop Statistics: Discovery with Data (2nd ed.), Key College Publishing.

Sahlström, J. F. (2002). The interactional organization of hand raising in classroom interaction. *The Journal of Classroom Interaction*, 47-57.

Schegloff, E. A. (2007). *Sequence orgnaization in interaction: A primer in conversation analysis.* Cambridge University Press.

Schegloff, E. A., & Sacks, H. (1973). Opening up closings. *Semiotica*, *8*(4), 289-327.

Sfard, A., & Kieran, C. (2001). Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture, and activity*, *8*(1), 42-76.

Sotos, A. E. C., Vanhoof, S., Van den Noortgate, W., & Onghena, P. (2007). Students' misconceptions of statistical inference: A review of the empirical evidence from research on statistics education. *Educational research review*, *2*(2), 98-113.

Smith, M., & Stein, M. K. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions.* National Council of Teachers of Mathematics.

Smith, M. K., Wood, W. B., Adams, W. K., Wieman, C., Knight, J. K., Guild, N., & Su, T. T. (2009). Why peer discussion improves student performance on in-class concept questions. *Science*, *323*(5910), 122-124.

Thornton, C. (1991). Think, Tell, Share—Success for Students. *The Arithmetic Teacher, 38*(6), 22-23.

Turpen, C., & Finkelstein, N. D. (2009). Not all interactive engagement is the same: variations in physics professors' implementation of peer instruction. *Physical Review Special Topics-Physics Education Research*, *5*(2), 020101.

Tyminski, A., Richardson, S., & Winarski, E. (2010). Enhancing think-pair-share. *Teaching Children Mathematics, 16*(8), 451-455.

Walter, H. A. (2018). Beyond Turn and Talk: Creating Discourse. *Teaching Children Mathematics, 25*(3), 180-185.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for research in mathematics education*, *27*(4), 458-477.

Yackel, E., Rasmussen, C., & King, K. (2000). Social and sociomathematical norms in an advanced undergraduate mathematics course. *The Journal of Mathematical Behavior*, *19*(3), 275-287.

CHAPTER 4: "GUESS WHAT THEY WOULD MAKE YOU DO ON THIS ONE": THE AP CALCULUS EXAM AND POSITIONING OF STUDENTS

Introduction

In the United States, nearly a half-million students take the Advanced Placement Calculus Exam (hereafter, the AP Exam) every year. Like government-mandated exams and college entrance exams, the AP Exam is high-stakes testing. Based on the scores students earn, roughly 65% would qualify for college credits in most post-secondary institutions in the United States (College Board, 2019), and the number of participating students has been continuously increasing (Bressoud, 2021). Many scholars, however, raise issues related to high-stakes testing, such as the AP Exam, due to the potential damage involved in the narrowing of the curriculum and exacerbating the existing educational inequality (Au, 2011; Berliner, 2011; Boaler, 2003; Horn, 2018). These are critical concerns because high-stakes testing is supported by policies or programs that are intended to broaden and ensure students' access to quality instruction (e.g., No Child Left Behind).

One of the concerns is how high-stakes testing influences everyday instruction, such as the effect of teaching to the test (Sonnert et al., 2020). Too often, however, these alarming messages on the high-stakes testing are based on data gathered outside of the classroom instruction (e.g., student interview, teacher survey, teacher meeting). What remains underexamined is how high-stakes testing might permeate the moment-by-moment instruction and how it ultimately shapes ways of engaging in mathematics. Attending to the classroom interaction, a primary site for teaching and learning mathematics, may offer an insight into how such institutional influence operates in everyday interactions in mathematics classrooms and offer ways to inform mathematics educators to ameliorate some of the unintended harms.

The current chapter addresses this missing site for investigation by attending to the authority of the AP Exam as manifested in the classroom interactions in two AP Calculus classrooms. Authority has been a central issue for reform efforts in mathematics education, especially toward inquiry-based instruction (e.g., Ju & Kwon, 2007) and discussion-based instruction (e.g., Boaler & Greeno, 2000). Facilitating a learning environment in which students can participate in mathematical activities as autonomous thinkers is a critical part of the work of teaching (Ball, 1993). By combining positioning theory and conversational analysis, the current study revealed contrasting social realities that are constructed in the classrooms related to the AP Exam and different ways such realities shape the ways of doing calculus in these classrooms. Before presenting these findings, I first offer a brief review related to issues of authority in the mathematics classroom and an overview of positioning theory as a theoretical frame.

Authority in Mathematics Classrooms and High-Stakes Testing

In mathematics education, the issues of authority have been one of the central topics in discussions on pedagogical reform (Ball, 1993; Cohen, 1990; Hamm & Perry, 2002; Lampert, 1990). In traditional classrooms, mathematics teachers often act and are seen as arbiters of mathematical truth who dispense their knowledge in mathematics to their students. Mathematics educators have been striving to reform such patterns of mathematics instruction toward discussion-based (e.g., Boaler & Greeno, 2000; Wilson & Lloyd, 2000) and inquiry-based (e.g., Gerson & Bateman, 2010; Ju & Kwon, 2007) instruction. One of the challenges in this pedagogical reform is reshaping the authority relations in the classroom to facilitate a learning environment in which students can participate in mathematical activities as autonomous thinkers (Ball, 1993; see, for an example in science education, Stroupe, 2014). To that end, researchers have examined authority in mathematics classrooms from multiple perspectives of students and teachers.

Amid and Fried (2005) offered a detailed account of authority relations based on students' perceptions. From student interviews, they found that students seek authority from the teachers, peers, and sometimes parents, which functions as "a web of sources of guidance" for students (Amit & Fried, 2005, p. 154). The authority relations are hierarchical and often endorsed by the students. The authority of teachers, in particular, is more desirable among students and cannot be easily challenged. When the teacher is not available during an individual or group task, the students turn to the authority of peers instead. Langer-Osuna (2016) adds an interactional perspective to this discussion by attending to the student-to-student interactions in a collaborative learning setting. One of the notable findings is that teachers' evaluation of student ideas and behaviors influence the authority patterns in the subsequent student-to-student interactions. What is evident from both what students say in interviews and what students do during collaborative activities is that the authority of the teacher plays a central part in the web of authorities.

Teachers' perception of authority broadened the conceptual scope of the authority relations beyond teachers, peers, and other actors in students' lives. Wagner and Herbel-Eisenmann (2014b), for example, examined teachers' visual representations of authority relations. Although the teacher, students, and families still took a substantial part in the teachers' representations, the study showed that the teachers included broader institutional agencies and personnel such as administrators and the Department of Education. In addition, the teachers included classroom objects such as textbooks (or curriculum more broadly) as a source of authority. Their findings align with what Ball and Bass (2000) stated:

> By default the book has epistemic authority: Teachers explain assignments to pupils by saying, "This is what *they* want you to do here," and the right answers are found in the answer key (p. 204, emphasis original).

This broadened view on authority relations locates teachers in an even more complex web of authority, interacting with not only students but also multiple external influences (e.g., curriculum and high-stakes testing). For instance, Herbel-Eisenmann (2009) presented that teachers and textbooks have varying authority relations based on teachers' language use. During instruction, sometimes the authority of the teacher is privileged over that of the textbook, and vice versa. Other times, the authority pattern showed the alignment between the teacher and the textbook, or between the teacher and students. This finding illustrates that authority relations are fluid and often change based on the context of teaching (see also, Wagner & Herbel-Eisenmann, 2014a).

An important context of the current study is that AP Calculus students are expected to take the AP Exam—a high-stakes standardized test—close to the end of the academic year. The issue of high-stakes testing is critical because there have been numerous discussions on how high-stakes testing interferes with the reform effort in mathematics education (e.g., Boaler, 2003). Many studies based on teacher surveys and interviews showed that high-stakes testing narrows the scope of curriculum and modes of instruction (e.g., Au, 2011; Berliner, 2011). High-stakes testing, together with accountability policies (e.g., No Child Left Behind), shapes teachers' instructional sensemaking during teacher meetings (Horn, 2007, 2018). And teachers, in images they produce, recognize high-stakes testing as a part of authority relations in the mathematics classroom (Wagner & Herbel-Eisenmann, 2014b).

To date, however, it is largely unknown how the authority of high-stakes testing may shape ways of doing mathematics *in* classroom interaction. Even though Boaler and Greeno (2000) examined how the students were positioned by either didactic teaching or discussion-based teaching in AP Calculus classrooms, the issue of the AP Exam was surprisingly absent in their discussion. Examining authority relations in the classroom while keeping an eye on the institutional context of the AP Exam can offer a complementary look at how high-stakes testing may shape the opportunities for students to act as doers of calculus.

The current study examines two AP Calculus classrooms to attend to the issue of authority related to the AP Exam. Informed by the literature, I see authority as fluid and manifested in mundane moment-by-moment interactions. To examine such patterns of authority in interaction, I apply the theory of *positioning*, which has been used to examine authority relations in multiple studies (e.g., Herbel-Eisenmann, 2009; Langer-Osuna, 2016; Wagner and Herbel-Eisenmann, 2014a). In doing so, I use the following research questions as a guide:

1. How are students positioned based on the AP Exam during instruction?
2. How might such positioning relate to authority relations?

Before I get into the analysis, I elaborate on positioning theory as a theoretical framework for the current study.

Positioning and Authority

The construct of *position* has been proposed as an alternative to the static notion of role (Davies & Harré, 1999). Institutional roles are often associated with a fixed set of rights and obligations, such as how the teacher and students ought to conduct themselves in the mathematics classroom. This notion of role may explain the authority relations in the classroom from a broader social view, but it often fails to capture how such images of self and others are negotiated or how the participants use such images to carry themselves in social interaction. The notion of position, on the other hand, elucidates the social dynamics through which participants construct self and others, and it also provides discursive resources for participants to perform social actions. This theoretical orientation is well reflected in its immanentist stance on language, highlighting the use of language situated in contexts (Wagner & Herbel-Eisenmann, 2009). The functions of language are not seen as inherent in the language itself; the functions are rather achieved by mobilizing discursive resources (i.e., storylines) that construct the present moment of interaction in a particular way. Positioning theory attends to how the social meaning of self and others are constructed in unfolding social interactions, which in turn reveals how rights and obligations are fluidly assigned and negotiated among participants.

Positioning theory offers three mutually supportive constructs: storylines, positions, and communication acts (Harré & van Langenhove, 1999; Herbel-Eisenmann et al., 2015). Here, I briefly present these key ideas in relation to the issue of authority (see, for an elaborated discussion, Wagner & Herbel-Eisenmann, 2009; Herbel-Eisenmann et al., 2015). First, a storyline represents "ongoing repertoires that are already shared" (Herbel-Eisenmann et al., 2015, p. 188). These storylines can be culturally familiar scenes, such as seeing mathematics as a rule-bounded game (Hersh, 1979), or they can also be scenes from prior interactions that the participants share. Storylines offer multiple, reciprocal positions to which the speaker implicitly invites other participants to adopt. For example, the storyline of a rule-bounded game may include the positions of players, teammates, coaches, opponents, and so on, and each position is associated with particular rights and obligations. By invoking the storyline of a rule-bounded game, for instance, the speaker can position the self as a coach and others as players (Wagner & Herbel-Eisenmann, 2014a), and thereby shape the range of roles and communication acts that might be possible (which is addressed further below). The positioning is this social dynamic through which the speaker tries to establish local and imminent moral order in the ongoing interaction.

It is important to note that positioning happens pragmatically for a speaker to employ social actions. By establishing a particular moral order, the speaker influences what the participants can and should do in an interaction. The notion of *communication acts* represents what the speaker achieves with the social dynamic of positioning. For instance, by positioning the teacher as a coach and students as players, the teacher may shape students' obligation to comply with the teacher's directions, thereby heightening the teacher's authority. The teacher may tell students, "trust the process!" This implies that the students should not doubt the legitimacy of the program and the direction their teacher is giving them. Of course, this positioning can be either accepted or resisted by the students. In mathematics classrooms, examining different teachers' positioning can reveal what and how authority patterns are negotiated and how various positionings may be accepted or contested by students (Wagner & Herbel-Eisenmann, 2009). The current chapter focuses on storylines, positionings, and communication acts that may be associated with *the AP Exam*. In doing so, I show how the institutional context of the AP Exam permeates authority relations in mundane classroom interaction and ultimately shapes the ways of doing mathematics in the classroom.

Methods

Context

The current study is situated in two public high schools located in suburban areas in the Midwest region of the United States. I purposely chose two experienced AP Calculus teachers with varying professional development backgrounds to capture a range of ways of positioning students in AP Calculus classrooms. One teacher, Mr. Robinson[12] served as an official grader for the annual AP Calculus Exam for four years, which afforded him detailed knowledge of the scoring process. The other teacher, Ms. Gray, taught AP Calculus for five years and has participated in action-research-oriented professional development to facilitate classroom discourse through a six-year partnership with university-based researchers (see, e.g., Herbel-Eisenmann & Shah, 2019). Each teacher taught two sections of AP Calculus AB[13] classes each day; a total of four AP Calculus classes are included in the current study. The teachers independently chose lessons on finding volume using integration when I asked them for an opportunity to observe classroom discussions that advance students' thinking. The following table shows the observed sequences of the lessons.

Table 6. Sequence of Lessons

Teacher / Classes	Day 1	Day 2	Day 3
Mr. Robinson / Class A & B	• Reviewing homework problem on finding area • Lesson on disc method	• Lesson on washer method	• Lesson on finding volume with known cross-sections
Ms. Gray / Class E & F	• Reviewing homework problems on disc method, • Lesson on washer method	• Solving practice problems on disc and washer methods	• Lesson on finding volume with known cross-sections

Data

The data consists of audio and video recordings of the above lessons. I placed one video camera in the front corner of the room and another camera in the back of the room to capture both talks and gestures of the students and the teacher, respectively. I placed four supplementary

[12] All names are pseudonyms.

[13] AP Calculus AB is equivalent to the first calculus course in college settings (e.g., Calculus I). The other one is AP Calculus BC, which includes the contents of AP Calculus AB in addition to the contents that appear in the second calculus course in colleges (e.g., Calculus II).

audio recorders evenly distributed in the classroom because sometimes, two video cameras do not capture the details of student talk. The number of video cameras and their placement were necessary to support my fine-grained analyses of storylines, positions, and communication acts in classroom interaction. To examine the communication acts, I applied a conversation analytic approach, detailed in the next section, which required audio and video recordings of talks and gestures of participants.

Analysis

I analyzed the data in two interrelated phases. One phase focused on identifying a storyline and potential corresponding positions related to the AP Exam by attending to participants' word choices. Another phase moved my attention to how the participants use the storylines to enact communication acts. I moved between these phases as necessary to attend to the mutual relationship among storylines, positionings, and communication acts. Here, I describe these phases of analysis in more detail.

I began by selecting episodes when the teachers implicitly or explicitly invoked "AP Exam" or "College Board." I also included the word "they" when it refers to the AP Exam developers (e.g., "Guess what they would make you do on this one."). After a thorough review of the entire data set, I identified 24 episodes from Mr. Robinson's lessons and three episodes from Ms. Gray's lessons for further analysis. The fact that there were eight times as many episodes in Mr. Robinson's class, as compared to those from Ms. Gray's class, informed the further direction of the analysis. I narrowed the scope of this phase to identifying a storyline related to the AP Exam from Mr. Robinson's class, and an alternative storyline from Ms. Gray's class. I attended to Mr. Robinson's and his students' word choices to identify the storyline. Sometimes the teacher's words were part of a phrase (e.g., *ahead of the game*), and they were an adjective (e.g., *smug*), a verb (e.g., *dominate*), or a pronoun (e.g., *we*). Based on these words as communication acts, I identified the storyline of *the Competitive Game of the AP Calculus Exam* in Mr. Robinson's class, which I elaborate on in the Findings section. The lack of invocation of AP Exam in Ms. Gray's classes, on the other hand, led to my identification of an alternative storyline. Informed by Yamakawa and colleagues (2005), I identified "a reform storyline" based on the patterns of interactions (p.197). In Ms. Gray's classroom, students shared multiple mathematical ideas, the teacher sought confirmation from a student, and students made some of the instructional decisions through negotiation. In the following, I present how I examined communication acts in the other phase while also reminding the reader that I moved back and forth between these phases.

Informed by a conversation analytic approach, I conducted a fine-grained analysis of communication acts associated with the above storylines. Herbel-Eisenmann and colleagues (2015) stated that mathematics education researchers often make claims about positioning without much attention to communication acts. Application of conversation analysis can ground the discussion of socially meaningful actions on the sequential progression of social interaction. The analysis began with transcribing the selected episodes using Jefferson System (Hepburn & Bolden, 2017; Jefferson, 2004). This transcript system allowed me to capture a range of speech features and gestures (e.g., delays in response, elongated pronunciation, intonation changes, pointing, hand raising), with which participants formulate and ascribe actions. Using these transcripts, I analyzed authority manifested in classroom interaction by applying two primary ideas from conversation analysis: adjacency pair (Schegloff, 2007) and epistemic authority (Heritage & Raymond, 2005). First, I used the notion of *adjacency pair* to examine the sequential progression of communication acts. From the view of conversation analysis, social actions are sequentially paired. The first pair part of interaction makes a limited set of actions relevant for the second pair part of the interaction. For example, asking a question makes answering the question relevant for the second pair part. If silence ensues after a question, the silence is "negative observation," a noticeable absence of an answer (Schegloff, 2007, p. 19). That is, what has been said immediately before shapes an important context for interpreting what is just said or what is left unsaid. Second, *epistemic authority* means a speaker's relative right to talk about a particular matter (Heritage & Raymond, 2005; Stivers, 2005). Findings from conversation analysis illustrate multiple discursive practices either to assert or to cede epistemic authority. For example, speakers may use tag questions[14] to mark their ceding of epistemic authority (Heritage & Raymond, 2005) or utters an uninvited confirmation to assert epistemic authority (Stivers, 2005). By attending to the sequential progression of interactions and discursive practices related to epistemic authority, I examined how the positioning of the students may have shaped the authority patterns in the classroom.

Findings

I organized this section into three parts. First, I show how Mr. Robinson constructed and represented the storyline of *the Competitive Game of AP Calculus Exam* through his use of language (word choices, to be specific) and thus offered different positions for the AP Exam provider, the teacher, the students, and other exam-takers. The second part of the findings focuses on the communication acts associated with the storyline (i.e., social actions that Mr. Robinson employs while invoking the storyline). This finding shows how such positioning with the storyline may have shaped the authority patterns in the classroom interaction. Lastly, I

[14] A tag question consists of a declarative statement and an added interrogative component (e.g., It's nice weather, isn't it?). They are often used to request confirmation. See Heritage (2012) for its relationship with epistemics in interaction.

compare episodes from Ms. Gray's classroom to those from Mr. Robinson's classroom. Comparing different patterns of interaction can be a powerful way to illuminate the subtle differences in the positioning of students (e.g., Herbel-Eisenmann, 2000; Esmonde, 2014). Through the comparison, I highlight how the two classes deal with the same mathematical issues, yet do so in contrasting ways. The differences that are highlighted through this comparison reveals different patterns of epistemic authority in the classroom. Putting these findings together, I argue that invoking the AP Exam during instruction shapes a hierarchical authority pattern, which can hinder students' participation in mathematical activities as autonomous thinkers. The contrasting episodes also suggest that the extent of such institutional influence can vary significantly. In this case, the variations may be the result the teacher's professional development experiences. Before the comparison, I lay the groundwork by illustrating the storyline in Mr. Robinson's classes that relates to the AP Exam.

Storyline: The Competitive Game of AP Calculus Exam

Participants' word choices reflect what kind of social reality is being constructed as well as how participants orient to such reality. With the following two excerpts, I illustrate how Mr. Robinson and his students talked in terms of the storyline of the *Competitive Game of AP Calculus Exam*. The analysis shows that the storyline is taken-as-shared without any sign of contention and how the storyline provides a context for positioning of the students, and reciprocally, the teacher.

I open with a scene from Excerpt 4.1 in which the teacher and students discuss one of the homework problems (Figure 9). At the beginning of the lesson, Mr. Robinson asked students if they needed any explanation in addition to his written solution to the assigned homework problems. Prior to Excerpt 4.1, Bella asked, "I just don't understand how you know how to graph it" regarding Problem 11. The problem required students to graph $x = \sin y$ and find the area bounded by the graph, y-axis, and two horizontal lines, $y = \pi/4$ and $y = 3\pi/4$. The challenge lies in treating y and x as independent and dependent variables, respectively, which is the opposite of treating x and y as independent and dependent variables. Mr. Robinson explained to the class that the students needed to reflect the graph $y = \sin x$ in $y = x$, or to rotate 90 degrees for the practical purpose of finding the area. Here, the excerpt begins as Mr. Robinson discussed the importance of such skill. The analysis focused on his word choices and description of "they," which constructed the storyline.

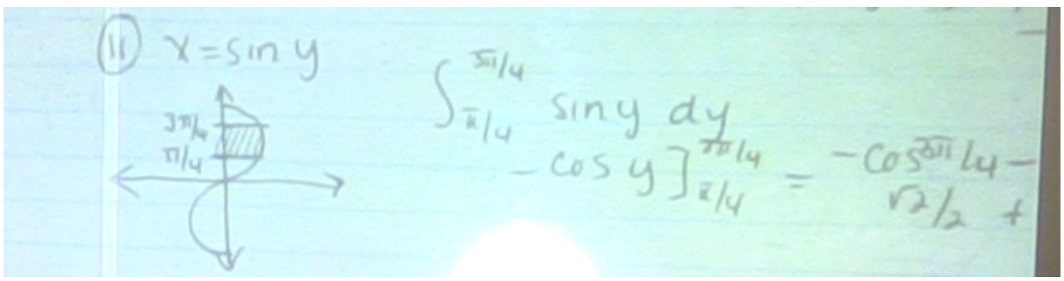


Figure 9. Homework Problem 11

Excerpt 4.1: A1-27 Ahead of the Game

```
TCH:     How many times do you actually have to do it though?=
         I am making these like- these are harder. °Like the on
         the exam I don't ah-eh:° I am trying to remember that
         I don't remember too much rotating it making it
         harder on you. And the- they got the problem hard
         enough as it is in a sense you know what I mean?=They
         don't need to- £You got enough to worry about.£
         They're not gonna try to get you too much on there.
         (.) I don't think (anyway). (2.3) (Ah they're) good
         questions. What- Anything else we- Ah Jack.
JCK:     I was gonna ask the same thing- the same exact thing
         that Bella just asked.
TCH:     Yeah.
JCK:     So I'm good.
         (1.2)
TCH:     Yes. (.) A- and my point aga- same thing and just
         keep remembering (.) I'm like woo [this would be hard
                                           [((TCH points to
                                       Q11 on the whiteboard))
         like [I am making] har- picking ones I know
JCK:          [  Mmm.     ]
TCH:     that would be harder [to graph] by hand. A lo:t of
JCK:                          [ Yeah.  ]
TCH:     times they just give you those (.) pictures
         [they don't] always make you
```

JCK: [Mm.]
TCH: do those by hand so. U:m (0.7) If you ↑can do these
you are like ahead of the game ↑I [think.] You know
JCK: [Mmhm.]
TCH: what I mean?=They usually don't (.) try to trick you
with the graph part because [that's] not the goal of-
JCK: [Yeah.]
TCH: Like the goal is to see if you can [set it up] right.
JCK: [Mmhm.]
TCH: Not to say ah-ha you don't know how to draw. You know
Heheheh
JCK: Yeah.
TCH: And that's- you know so I don't- I don't see: that- I
↑don't see that ending up being a big problem.
JCK: Okay.
TCH: To be honest. I- I'm kind of making it harder on you
here. (0.9) A little bit.

In line 28, Mr. Robinson uses the word "game." The teacher says they are "ahead of the game" if the students can do the graphing with inverted independent and dependent variables. This phrase portrays doing calculus not as an individual activity but as a competitive game for the students to win over others. The cultural image of a sports game (e.g., gymnastics meet, figure skating match) also comes with multiple positions, such as the players of the game, the opponents whom the players play against, the coach that supports the players, the judges with the authority to evaluate the performances and determine who wins the game, and so on. The culturally shared images of a game thus provide the boundaries of the storyline and somewhat determine the range of positionings that are made available to participants in the discussion.

The entirety of the storyline with those positions is reflected in the details of word choices in Mr. Robinson's speech. First, "they," the College Board or the AP Exam provider, work as judges. "They" are to "try to get" students (line 8); that is, they are the ones who evaluate students' performances and see if the students can do a certain task or not. They will determine who is "ahead of the game" or behind so that they can rank the students accordingly. "They," the judges, are portrayed as with high standard and integrity without being punitive. "They got the problem hard enough" (line 5), and "[t]hey usually don't try to trick" the students with the graph "because that's not the goal" (lines 30-31). This portrayal implies that the game that "they" facilitate is not entirely random but predictable to an extent.

The predictability of the game heightens the importance of another position, the coach. In the above scene, Mr. Robinson positions himself as the coach who prepares the players with insider knowledge about strategies. This positioning is done so by displaying his detailed knowledge about what "they" are to do and what is the expectation of the "game." For example, on the exam, "[t]hey just give" the students pictures instead of asking them to graph (line 24). Therefore, not knowing how to graph the curve would not end "up being a big problem" (line 39). These descriptions not only inform the students about the game but also display Mr. Robinson's epistemic authority on the matters related to the game (i.e., the AP Exam), which is an important characteristic of being a coach.

The distinction between the judges and the coach plays an important role in shaping the social solidarity between the teacher and the students. The coach helps students win the game but is not an ultimate evaluator of student performance. Note here that Mr. Robinson's epistemic authority is subordinate to that of "they" on the matter of the game. The appropriateness of the choice of homework problem is relative to the expectation of the AP Exam, which sets *the* standard. For example, in the above scene, Mr. Robinson downgrades the importance of knowing how to graph $x = \sin y$. His choice of Problem 11 is him being "harder on" the students (line 41). Hence, the comparison of "harder" is relative to what "they" will ask on the AP Exam. In other words, the adequateness of his instructional choice is measured up against the expectation of the AP Exam. This separation and hierarchical relation between the teacher and "they" is important because Mr. Robinson, as a coach, is not the ultimate evaluator. What he does is to help students to be “ahead of the game,” which will be judged by the AP Exam provider. I elaborate more on this point about social solidarity with another excerpt.

The second scene in Excerpt 4.2 was on the next day when Mr. Robinson introduced the washer method to find the volume of the revolved shape to the class. Prior to this excerpt, he introduced a template for a “setup” as $\pi\int_{()}^{()}(\ \ )^2-(\ \ )^2d(\,)$, which the students were to fill in the empty spaces with expressions and values. He then demonstrated doing the example of finding the volume of shapes generated by rotating the given region, which is bounded by $y=\sqrt{x}$ and $y=x^2$, rotated around (1) x-axis, (2) y-axis, (3) $y=3$, and (4) $y=-2$. Figure 10 shows the written work on the whiteboard. The excerpt began once the teacher filled the spaces for the big radius and small radius for (3) $y=3$. I focus on how Mr. Robinson marked the significance of (3) $y=3$ compared to (1) the x-axis and (2) the y-axis, and how one student, Aiden, elaborated on Mr. Robinson’s talk.

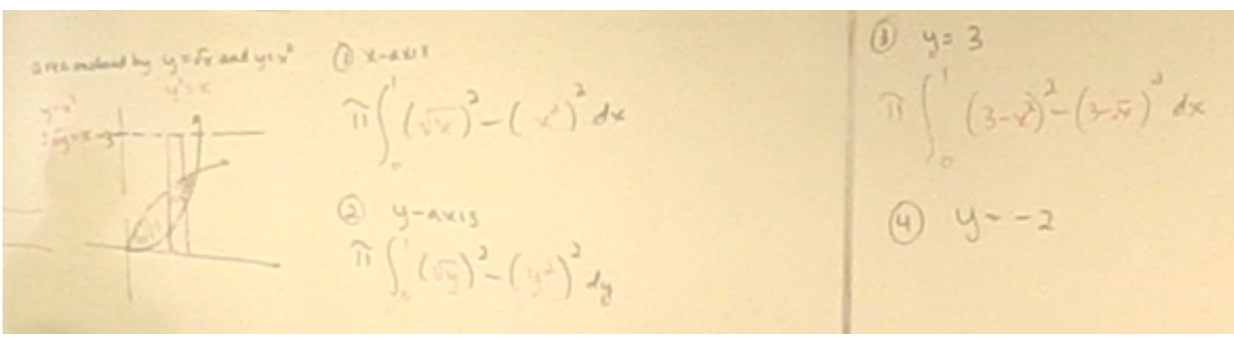


Figure 10. Example of Washer Method

Excerpt 4.2: B2-37 We Gotta Dominate That

```
TCH:     You see HOW ALL the sudden it’s like wo:↑w we gotta
         do some thinking? (0.5) So what ends up happening
         fine students it’s like [everyone gets these? (0.3)
                                 [TCH points to response for
                     rotating around (1) x- and (2) y-axis
         And they [put this one on there
                  [TCH points to response for
                     rotating around (3) y = 3
         [that’s where (and that-)] and
ADN:     [And  then  we’re  s:mug.]
TCH:     we gotta dominate that. (0.7) Right? We gotta
```

dominate that. Did you see how (0.3) MOVing that
(0.3) that makes it if- you are right that it's x
squared but there's mo:re (.) because of- so the idea
fine students is because functions are given to you
relative to the axis (0.4) and I am no:t going to
relative to the axis I gotta dea:l with that
properly.

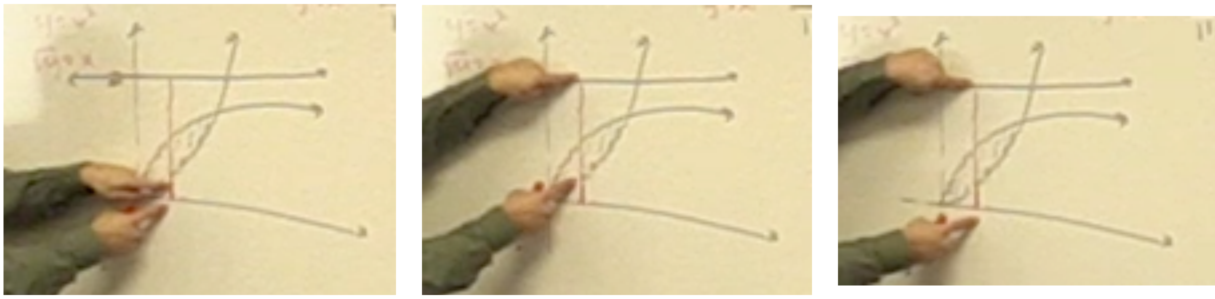
This scene further illustrates the storyline of the Competitive Game of AP Calculus Exam in at least two ways: The AP Exam is a competitive game, and the storyline is mutually elaborated by the teacher and a student named Aiden. In lines 1-6, consider how Mr. Robinson compares (1) and (2) to (3). Question (3) requires the students to "do some thinking" (line 2). He then points to (1) and (2) and says, "everyone gets these." This comparison implies that Question (3) is what differentiates the players in the game.

This storyline is elaborated in collaboration with a student, Aiden. Once the teacher points to (3) and says, "they put this one on there" (line 6), then Aiden completes the teacher's talk by saying, "and then we're smug" (line 10). In other words, when "they" place a problem like (3), the students can show their superiority against their opponents. Moreover, Aiden's use of "we" (line 10) displays that the students in the class as a whole compete against other AP Exam takers *outside* of Mr. Robinson's class. The classmates, then, are not positioned as opponents but as teammates. This position of opponents in the storyline, thus, does not undermine the solidarity among the students in the class although the students are participating in a competitive game.

Mr. Robinson further elaborates on Aiden's talk by repeatedly saying, "We gotta dominate that" (lines 11-12). The word "dominate" is common in the headlines of sports game reports (e.g., Simone Biles dominates the competition.), which maintains the storyline of "game." Note again his use of "we" instead of "you." Mr. Robinson includes himself in the team, which displays the solidarity between the teacher (i.e., the coach) and the students, in addition to the students themselves (i.e., players in the team). The above scenes in two excerpts overall illustrate how the teacher, together with the students, constructs the storyline with multiple positions: the players (the students), the coach (the teacher), the judge (AP Exam provider), and the opponents (other AP Exam takers outside of the class).

Positioning of Students as Players of the Competitive Game

According to positioning theory, a storyline provides a context in which participants employ situated actions with *communication acts*. Examining communication acts can, in turn, reveal how the storyline influences the patterns of interaction. This section attends to such actions that the storyline of the Competitive Game of AP Calculus Exam affords during the instruction. Both Mr. Robinson and Ms. Gray dealt with two common issues in the topic of finding the volume of revolved shapes: 1) dealing with a non-canonical axis of revolution other than *x*- and *y*-axis and 2) deciding the use of a calculator to compute the integral. The following analysis focuses on what communication acts Mr. Robinson performs while positioning the students as players of the game. Just as the class presented in Excerpt 4.2, in the below Excerpt 4.3, Mr. Robinson is walking through an example of applying the washer method when the region bounded by $y = \sqrt{x}$ and $y = x^2$ is rotated around (1) *x*-axis, (2) *y*-axis, (3) $y = 2$, and (4) $y = -2$. The below scene includes Mr. Robinson's demonstration of his procedure to find the big radius when the rotational axis is $y = 2$.



Length A Length B Length C

Figure 11. Lengths A, B, and C

Excerpt 4.3: A2-30 To get what I want

```
TCH:     But NO:W is it’s going to the axis and I don’t ↑WANT
         going to the axis. I want this way. Draw it so you
         can see how everything compares. .hh [x squared
                                              [((TCH points to
                                                   Length A))
         actually gives me this. (0.5) I [want this length.
                                         [((TCH points to
                                             Length B))
         How do I [u:se what I know to get what I [want?
                  [((TCH points to Length A))     [((TCH
                                       points to Length B))
         (2.7)
TCH:     Isn’t (.) [the whole thing? (.) Which is
                   [((TCH points to Length C till line 18))
ST?:     °(      )°
TCH:     Two:, (0.7) What’s that length.
         (1.3)
STs:     Two:.
TCH:     You see how it’s two, and [what do I know,
                                   [((TCH points to Length A))
ST?:     °(x squared)°
TCH:     And if I get rid of x squared what does that leave me
         with.
         (.)
ELE:     [The thing you want=
         [((TCH points to Length B))
TCH:     =That’s what I want.
         (5.5)/((TCH writes 2-x^2 as big radius in integrand))
TCH:     You see how that’s way different? And tons of people
         are going to get that wrong if it’s asked of them.
         (3.0) If you can do this that’s gonna be a big
         advantage to you. (1.5) Okay? (0.7) You see why it’s
         gotta be two minus this time.
         (1.7)
TCH:     [The order matters. You can’t have x squared ↑plus
         [((TCH points to 2-x^2 that he just wrote))
         two you know what I mean. It’s gotta be cor- a-
         written correctly. (.) Okay. (.) So that’s what I
         want.
```

The above scene began as Mr. Robinson demonstrated his procedure to find the big radius, which is the vertical distance between the rotational axis ($y = 2$) and the curve ($y = x^2$) at a fixed x-value. His gestures (see Figure 11) mutually elaborated and were precisely timed with his speech (i.e., “x squared,” line 3; “want,” lines 6 & 9; and “the whole thing,” line 13). While showing heavy clues to the students with his gestures, he walked through a series of “cued elicitations” (Edwards & Mercer, 1987, p. 142), which ultimately arrived at the conclusion that the desired length (Length B in Figure 11) is $2 - x^2$ (Length C - Length A). The whole sequence came to an end when he wrote the expression as the solution on the whiteboard (line 28). This was the moment when the storyline of the competitive game appeared (lines 29-32), and the positioning of the students as players in the game took place.

Looking at the sequential progression of Mr. Robinson’s talk revealed what he achieved with the positioning of his students (i.e., communication act). By positioning students as players in the game, Mr. Robinson marked the just-shown procedure as a critical part of the lesson. That is, knowing how to do the procedure was so critical that it would be “a big advantage” to the students (emphasis noted by underline) since “tons of people are going to get that wrong” (lines 29-32). Note here that the origin of the significance was based within the characteristics of the game; that is, the opponents in the game would likely get this particular kind of task wrong, which was far from the mathematical aspects of doing such a task. In other words, the teacher’s positioning of the students as players drew the students’ attention away from the mathematical aspects of doing the task and brought their attention to the just-shown procedure’s critical role in winning the competitive game.

The subsequent talk by Mr. Robinson added another layer to the positioning. The just-shown procedure was to be followed *strictly* by the students. He said, “[t]he order matters. You can't have x-squared plus two" (lines 35-36). It has "gotta be written correctly" (lines 37-38). This talk elaborated that the obligation of the students (i.e., the players) is to reproduce the teacher's (i.e., the coach's) procedure without an error when they played the game of the AP Exam. Since the teacher knew the game better than the students in the classroom, the teacher brought the relative epistemic authority on the best way to prepare for the competitive game. Once the students accept Mr. Robinson's positioning of them as players, there seemed to be little room for students to reason differently from the teacher in this interaction.

Mr. Robinson's positioning of the students did not stop there. After Excerpt 4.3, Mr. Robinson finished Question (3) $y = 2$ and wrote $\pi\int_0^1 (2 - x^2)^2 - \left(2 - \sqrt{x}\right)^2 dx$ on the whiteboard. He then positioned a student named Ravenna when she asked a question. The Excerpt 4.4 presents the scene of that positioning.

Excerpt 4.4: A2-31 Calculator

```
01  TCH:    What do you think about that.
02          (2.7)/((Ellie & Sophia put thumbs up))
03  TCH:    Again one- one thing is for you to kind of see it for
04          the first time but [those are the ones that are gonna
05                             [((Raveena raises her hand))
06          cause you the challenge. (0.3) Is trying to piece
07          that together. (.) That's what we are gonna have to
08          work on. [Go ahead (dear).
09                   [((TCH points to RVN))
10  RVN:    [You would simplify that whole thing though right?
11          [((RVN points to the setup for (3) y = 2))
12          [Af]ter you
13  TCH:    [So]        [here's- here's the thing.
14                      [((TCH points to the same setup))
15          >Guess< what they would make you do on this one.
16          (0.5)
17  SRH:    °Calcu[lator:°]
18  TCH:          [ Calcul]ator,
19  RVN:    Uhoh. Okay.
20  TCH:    Think about it on the exam they want you to foil all
21          this out and do all (     ), Probably not. (0.5)
22          Probably not. Um (.) [usually it's: calculator or
23                               [((RVN erases her written work
24                                  on her paper till line 28))
25          I've seen it lots of time where it says set up but do
26          not evaluate. So that whole question is you get that
27          and call it good.
28          (3.3)
29  TCH:    That's why I am not even doing them right now. The
```

30 set up is by far most important here. We can later
31 work on like you know making sure we're pu- Isn't
32 this? Couldn't it this be one that you could easily
33 get messed up in your calculator because <all the
34 parenthesis and the> You know you gotta make sure you
35 type it in. So we will practice that because we want
36 them- We don't wanna throw points away by no:t- You
37 know like hitting a button wrong on a calculator or
38 something like that right? I want you to get full
39 points but. Um. (1.0) But yeah.

Ravenna asked the question, "You would simplify that whole thing though right?" (line 10). With this tag question[15], Ravenna sought confirmation from the teacher about *his* next step after the setup with some degree of her certainty. Right at this moment, Mr. Robinson positioned Ravenna by saying, ">Guess< what they would make you do on this one" (line 15). The faster speech (noted with > <) highlighted "guessing" as the action Ravenna should take instead of seeking confirmation from the teacher. Also, it was guessing what *they* would make her do rather than what the teacher would do, which positioned the AP Exam provider as the ultimate evaluator at the peak of the hierarchical authority pattern. The teacher's subsequent talk, "[t]hink about it on the exam they want you to foil[16] all this out and do all ( )," again, drew Ravenna's attention to the AP Exam. This talk positioned Ravenna as a player who should decide one's next action based on what would be expected and valued in the game.

This positioning was consequential to what Ravenna did next. She erased her work on her paper until the teacher said, "That's why I am not even doing that right now" (line 29). Although the video data did not show what exactly Ravenna erased, the earlier talk made it relevant that Ravenna was probably erasing her work of simplifying the integrand, for which she sought confirmation from the teacher. What was notable here was that this course of interactions did not

[15] See Heritage (2012) about the use of tag questions in interaction.
[16] FOIL stands for First, Outer, Inner, and Last, which is a common acronym for remembering how to expand a squared binominal in US schools.

involve much mathematical reasoning either of the teacher or the student. The interaction mainly involved guessing what "they" would ask the students in the competitive game on which the teacher holds the relative epistemic authority.

Despite the fact that the students would be able to compute the definite integral *without* using a calculator, the reciprocal positioning of the teacher as a coach and students as players made such a discussion irrelevant in the interaction. Overall, the storyline of the Competitive Game of AP Calculus Exam offered a common frame of reference among the teacher and students, which determined the value of mathematical work based on what the students would likely face on the AP Exam. The positionings of students that I presented above showed how such taken-as-shared social reality shaped the ways of doing calculus in the classroom. In the following section, I turn to an alternative positioning of students as *collective doers of mathematics* in *the reform storyline* evidenced in the excerpts from Ms. Gray's classroom. The lessons are on the same topic (i.e., finding the volume of revolved shapes), and the analysis focuses on how Mr. Gray's class deals with the same issues (i.e., a non-canonical axis of revolution and use of a calculator) but in a different way from Mr. Robinson's class. I offer this alternative positioning for two main reasons. First, comparing and contrasting with another kind of positioning can illuminate important aspects that the analysis of a single kind may not show. Second, alternative positioning can also reveal possible ways to move beyond one kind of positioning and potential challenges to do so.

An Alternative Positioning of Students as Collective Doers of Mathematics

To provide context for the following, Ms. Gray's class just learned both the disc and washer methods to find the volume of a revolved region. Similar to Mr. Robinson's class, dealing with a non-canonical rotational axis (i.e., not *x*- nor *y*-axis) became a central issue within this topic, and the following excerpts illustrate how Ms. Gray's class interacted around such problems. My analysis focused on how the students in Ms. Gray's class were positioned differently from Mr. Robinson and how such positioning shaped the classroom interaction. Prior to Excerpt 4.5, the class worked on the problem: *Find the volume generated by revolving the region bounded by* $y = \sqrt{x}$*;* $x = 0$*;* $y = 2$ *around* $y = 2$. The class concluded that the disc method was applicable for this problem and that the radius is "$2 - \sqrt{x}$." Excerpt 4.5 below began right after the teacher finished her summary of the method of finding the radius with a non-canonical rotational axis, $y = 2$.

Excerpt 4.5: F2-36 Squaring It

((Alex raises hand))
TCH: Yes.
(1.5)
ALX: I- I am so what uh would you get the same answer by
um (0.3) that uh um the square root of x minus two,
(1.0)
TCH: [°Two minus square root of x.° ↑Woo. That's
[*((TCH fills "*$2-\sqrt{x}$*" in* $\pi\int_0^4(2-\sqrt{x})^2\,dx$ *on the*
whiteboard))
interesting? (.) What makes you think. So this is
what Alex's just- is proposing,
(2.7)/*((TCH writes* $\pi\int_0^4(\sqrt{x}-2)^2\,dx$ *under* $\pi\int_0^4(2-\sqrt{x})^2\,dx$*))*
ALX: Well I am not prop- I am not proposing anything I am
just asking [a question.]
TCH: [You're ques]tioning it. What do you guys
think. Will those produce the same volume.
(0.8)
ST?: Maybe [square it.]
ST?: [Think]so::?
ST?: Probably no:t.
HNA: [Well if you square it] [it's just negative but then-
MIN: [Probably not.] [
ISB: [I think it is because it-
(0.5)
ISB: [You are finding the] difference
MIN: [oh you oh]
ISB: and then [squaring it so it should be the same.]
MIN: [Squaring it Yeah squaring it so]
MIN: it should be the same.
HNA: It would be the same. (0.5) But if didn't square it
wouldn't be.
(0.7)
TCH: Because of the squaring [component?]
HNA: [Yea:h] makes it
positive.
ALX: I- I mean just thinking about like um (0.5) it (2.0)

```
37          o- on the function um you can translate it down two
38          (and) to get the area. or you could've instead
39          revolving around the axis or you can also (0.5)
40          translate up then flip it [over. ]
41  TCH:                              [Coo:l.]
42  HNA:                              [It's like] what I was
43          saying last time when I was like you just minus
44          [whatever the thing is]
45  TCH:    [     Ye:↑::s.         ] I think we heard Mia is this
46          kind of what you were bringing up on friday too?
47  MIA:    Mmhm/((MIA nods))
48  TCH:    So Mia was imagining that you can shift the axis
49          down? And imagine this [same problem positioned here,
50                                 [((TCH draws the same region
51                                        translated down by 2))
52  TCH:    Right?
53  MIN:    Yeah yeah.
54  TCH:    Did the VO:LUME change?
55  ST?:    No.
56  TCH:    [We just displaced the volume. We moved it elsewhere.
57          [((TCH moves her L hand up and down))
58          (0.7)
59  TCH:    So sometimes more than one integral could produce the
60          same volume.
61          (0.7)
62  TCH:    Coo:l.
63          (1.0)
64  TCH:    That is cool. Okay.
```

After Alex brought up the idea of switching $2-\sqrt{x}$ with $\sqrt{x}-2$, Ms. Gray named Alex's action as "proposing" (line 11). This overt naming of Alex's action positioned Alex and the rest of the class as collective doers of mathematics because the class's next relevant action after proposing was to engage in a negotiation to either accept or reject the proposal. Simultaneously, the talk also constructed a reform storyline in which students actively engage in communicating their thinking. Alex, however, did not take up Ms. Gray's positioning and renamed the action as "asking a question" (line 14), trying to position Ms. Gray as the evaluator of the proposed idea. The subsequent interaction nonetheless shows Ms. Gray and the rest of the class rejecting Alex's positioning because everyone focused on the veracity of Alex's idea. The students initially showed mixed responses to Alex's question (lines 18-22), but as the discussion progressed, more students took up the idea that squaring the expressions would make

$(2-\sqrt{x})^2 = (\sqrt{x}-2)^2$. For example, Min, a student who initially said, “probably not” (line 22), says, “oh you oh” (line 26). This “change of state token” (Heritage, 1984) displayed that Min became convinced that those expressions were, in fact, equal. During this interaction, the students seemed to hold relative epistemic authority. For example, when Ms. Gray asked, “because of the squaring component?” (line 33), Henna treated the teacher’s talk as *seeking confirmation* when she said, “Yeah… makes it positive,” using a declarative syntax and falling intonation.

This interaction sharply contrasted with that in Mr. Robinson’s class (Excerpt 4.3) in at least two ways. First, the conclusions were opposite. Recall that Mr. Robinson said, “the order matters” (line 35 in Excerpt 4.3). In the current scene, however, Ms. Gray’s class showed that the order did *not* matter. In the case of Mr. Robinson’s class, the order mattered because the order $2-x^2$ (thus, not x^2-2) followed Mr. Robinson’s procedure of finding the radius (Length C - Length A). The storyline and positioning indicated that his procedures should be strictly followed when the students worked on tasks. Ms. Gray’s classroom interactions, however, suggested proposals were open to different methods and representations insofar as the class, as collective doers of mathematics, could justify them.

The second point of deviation was the contrasting origins of epistemic authority. In Mr. Robinson’s class, the teacher held relative epistemic authority based on his knowledge about what “they” would ask on the AP Exam. In Excerpt 4.4, Ravenna, a *student*, sought confirmation from the teacher, which in turn determined her next action. In contrast, in Ms. Gray’s class, even when Alex tried to seek confirmation, the *teacher* turned to the students to discuss the idea and produce a justification based on the squaring component. This move allowed Alex to share another idea that $y=\sqrt{x}-2$ can be represented as a curve generated by translating $y=\sqrt{x}$ two units downward with the corresponding rotational axis of the *x*-axis (lines 36-40). This idea led to the conclusion: “So sometimes more than one integral could produce the same volume” (lines 59-60) based on students’ use of vertical translation.

Positioning students as collective doers of mathematics added complexity to the authority relations in the classroom. Such positioning works against the hierarchal authority among the AP Exam provider, the teacher, and the students, which makes room for students to negotiate the authority in the classroom. The negotiation, however, was not always smooth as positioning theory suggests that people do not have to take up a positioning that is offered to them. Excerpt 4.6 below reveals the possible challenge associated with Ms. Gray’s positioning of students. At the moment, Ms. Gray’s class was dealing with the issue of choosing between computing the integral either with a calculator or by hand, just as Mr. Robinson’s class did. The excerpt begins as Ms. Gray prompted the class about the use of a calculator after she wrote $\pi \int_0^4 (2-\sqrt{x})^2\,dx$ on the whiteboard.

Excerpt 4.6: E2-33 But Why

TCH: Raise your hand if you wanna do this one by hand.
(2.7)/*((Nick slightly raises hand))*
TCH: Raise your hand if you wanna plug n’ chu:g on a
calculator for this one.
(0.7)/*((3 STs slightly raise hand))*
TCH: >SHOW me ↑show me show me<
(0.3)/*((5 STs raise hand, Owen puts hand up high))*
TCH: Pick one or the other. .hh Where are the ha:nds for I
wanna do this one by hand,
(1.5)/*((2 STs raise hand including NCK))*
TCH: [WITH A CALculator] ha:nds up,=For with a calculator.
NCK: [This is easy.]
(0.5)/*((7 STs raise hand including OWN))*
TCH: [Okay. (0.5) practice this] one
NCK: [°Why this one is easy.°]
TCH: on a calculator.
TCH: [Accurate to the thousandth place.]
OWN: [(I would) do them a:ll on my calculator.]
[*((Owen turns back and looks at Nick))*
NCK: Huh?
OWN: ( ) cal[culator]
NCK: [Yeah] but you can’t.
(1.0)
TCH: We could totally have done this by hand Nick.

```
25  NCK:    ↑I kno::w.↑=
26  TCH:    =We cou:ld have.
27  OWN:    Let’s [do it  by   hand   then.]
28  NCK:          [  It’s    ea:sy.        ]
29  TCH:          [(But) we didn’t practice] with a calculator.
30  NCK:    But why:.
31  TCH:    Okay? Try this one with a calculator,
```

Ms. Gray surveyed how many students wanted to compute the integral with a calculator or by hand, and then she determined that the class, as a whole, computed the integral with a calculator. This method resembles plurality voting by which decisions are made based on the preference most students in the collective; thus, the teacher positioned the students as *collective* doers of mathematics. This decision process was not smooth. One student, Nick, protested during the process, repeatedly saying, “This is easy” (lines 12 & 15) and displayed his dissatisfaction with the decision of using a calculator. The teacher acknowledged that the class could do it by hand, but she also stated that the class did not practice with a calculator (line 29). Nick asked for more justification (line 30), but the teacher did not return any additional justification but reiterated the earlier decision.

Recall when Mr. Robinson said, “Guess what they would make you do on this one,” in Excerpt 4.4. The positioning of the student as a player in the competitive game led to the single decision, “calculator,” without much room for negotiation. Mr. Robinson asserted epistemic authority with his vast knowledge about the AP Exam. In Ms. Gray’s classroom, on the other hand, students were instead deciding the course of action, but the issue was that students did not always agree with each other. Like in Excerpt 4.5, with adequate time and the teacher’s skillful facilitation, different ways of thinking among students could be a great starting point for a rich mathematical discussion or debate. Deciding between using a calculator versus computing by hand, however, can be “arbitrary” (Hewitt, 1999) without much need for mathematical justification. The challenge associated with positioning students as collective doers of mathematics was that reaching consensus may not be guaranteed or the negotiation process may not always be worthwhile. It may instead add a burden on the teacher to deal with multiple preferences among the students.

The comparison between the earlier positioning by Mr. Robinson with positioning by Ms. Gray showed how different positionings could alter the authority pattern in the classroom. The students in Ms. Gray's class drew from their prior learning and presented multiple ways to make sense of the problems with a non-canonical axis of revolution. The way how the class handled the use of a calculator, however, showed the added complexity when students are positioned as collective doers of mathematics. Since the class did not solely rely on the authority of the teacher, the disagreement among students could emerge, and facilitating meaningful negotiation of such disagreement can be a challenging task for the teacher.

Discussions and Implications

The above findings suggested that the authority of the AP Exam (or "they," the provider of the AP Exam) *can* shape a hierarchical web of authority in which the AP Exam is at the top of the hierarchical order followed by the teacher and then the students. As shown in the episodes from Mr. Robinson's classes, one key ingredient for the teacher to construct this kind of hierarchical reality is the teacher's vast knowledge on the AP Exam, which affords the epistemic authority of the teacher on the matter of the AP Exam. The contrasting episodes from Ms. Gray's classes suggest that the institutional influence of the AP Exam is not uniform across the AP Calculus classrooms and deserves further investigation in the field. The diminished authority of the AP Exam in Ms. Gray's classroom suggests that the institutional influence of the AP Exam is, in part, mediated by the teachers' professional development backgrounds. Her skillful facilitation of a discussion and the immediate take-up of students to explore the idea illustrated in

Excerpt 4.5 shows a promising sign that students can be positioned as collective doers of mathematics despite the institutional context of the AP Exam.

These findings, in turn, show the importance of professional development on discourse practices (see, Herbel-Eisenmann et al., 2013) for which Ms. Gray was involved for almost six years. College Board coordinates various professional learning opportunities for AP Calculus teachers across the United States (see, College Board, n.d.). These workshops and summer institutes, however, often focus on the alignment between the AP courses and AP Standards and the scoring process of the AP Exam; they rarely address how teachers may facilitate a learning environment in which students can thrive as autonomous doers of mathematics. Based on my own experiences of participating in two summer institutes, the facilitators of these professional learning opportunities often had expertise in the scoring process of the AP Exam (e.g., years of being in a leadership position for the AP Exam grading), not necessarily in classroom discourse or thinking about epistemic authority. Of course, making the scoring process transparent and accessible to all teachers is crucial to ensure equal access to the information about the AP Exam, yet solely focusing on the content in the AP Exam can amplify an unintended outcome of shaping doing calculus as a guessing game with arbitrary rules set by the teacher and the AP Exam.

The current study also contributes to the broader discussions related to high-stakes testing and authority relations. In particular, the above analysis offers empirical evidence of how the phenomena of *curriculum narrowing* (Au, 2011) manifest in classroom interactions. For example, Mr. Robinson's talk, "That's why I am not doing them right now" (line 29 in Excerpt 4.4), shows how narrowing the curriculum is justified based on what is valued on the AP Exam. More importantly, this narrowing process occurs at the cost of devaluing student's work.

Ravenna's erasure of her work when she heard "Calculator" was not just her physical movement of erasing; it was a discursive action that signified her internalization of the devaluation of her legitimate approach to the problem. I also note that the authority of the AP Exam could be used in the opposite way; that is, the curriculum may be expanded based on what is included in the AP Exam (e.g., a problem with a non-canonical rotational axis). Thus, high-stakes testing may narrow or expand the curriculum. The point is that what is believed to be on the exam can demarcate the line between what is perceived as valuable and what is not.

Sonnert and colleagues (2020) found both the short-term positive effect and long-term negative effect of teaching to the AP Exam for the students with "weaker" mathematics preparation. The detailed empirical accounts from the current study provide an explanation for that pattern. Mr. Robinson's more frequent talk about the AP Exam constructed the storyline of the competitive game in which the students can learn to win the game (i.e., short-term advantage on the AP Exam). The game of the AP Exam, however, ends once the students take the AP Exam. Their situated learning of playing the game of the AP Exam may not be applicable in their college coursework. On the other hand, learning mathematics as collective doers of mathematics requires communicating one's thinking and convincing other peers. For example, Excerpt 4.5 presented that such positioning of the students led to applying their prior learning from geometry (e.g., vertical translation) while considering multiple representations. This type of situated learning may not optimize students' performance for the AP Exam, but it may offer a richer foundation for the later coursework in mathematics (i.e., positive long-term effect). Sonnert and colleagues (2020) suggested that the "weaker students" might have benefited more from learning "mathematical fundamentals" and "basic mathematical skills" instead of preparing for the AP Exam (p. 15). The comparison in the current study, however, offers a different view. When the authority of the AP Exam is pronounced (e.g., teaching to the test), the authority of students can be diminished. This finding suggests that what "weaker students" might need more is not passive practices of basic skills but opportunities to communicate and justify their mathematics thinking (see also, Bieda & Staples, 2020).

Lastly, the current study contributes to the discussion on external, institutional authorities in the mathematics classroom. Scholars have already examined the fluid position of the written curriculum within the authority relation in detail (Herbel-Eisenmann, 2009; Wilson & Lloyd, 2000). This chapter brings the important role of the authority of *high-stakes testing* to the discussion. The sequential analysis informed by conversation analysis revealed how teachers' positioning of the AP Exam is consequential to the ongoing interaction and, ultimately, the ways of doing mathematics. The varying extent of the presence of the authority of the AP Exam, however, warrants further studies on its fluid characteristic. For example, examining cases when the hierarchical authority relations among the AP Exam, the teacher, and the students are disrupted could offer important insights into ameliorating the unintended harms of high-stakes testing. Such reconfiguration of authority relation, as exemplified in Excerpt 4.6, may bring a new set of challenges to the teachers (see also, Ball, 1993; Wilson & Lloyd, 2000). Studies that focus on managing such challenges are needed to support mathematics educators' reform efforts in the context of high-stakes testing.

Conclusion

The AP Calculus program, coupled with the AP Exam, has been increasingly growing in high schools across the U.S. with the promise of making college-level calculus learning more accessible. This chapter provided an empirical account of how the AP Exam shapes, to a varying extent, ways of doing mathematics *in* classroom interaction. The storyline of the Competitive

Game of AP Calculus Exam represents the social reality constructed by the teacher and students on the basis of what is (and is not) believed to be on the AP Exam. The analysis revealed that this reality is constructed at the cost of the diminished authority of students, which is necessary for students to participate in mathematical discussions as autonomous doers of mathematics. The unintended harms of high-stakes testing are widely documented. The primary contribution of the current article is the unintended consequence of high-stakes testing *in* classroom interaction, narrowing the discursive ways of doing mathematics. The findings from the current study, therefore, make mathematics educators reckon with an important issue amid the concerted effort to make the college-level calculus more accessible: What kind of mathematics learners is the AP Calculus program making?

REFERENCES

REFERENCES

Amit, M., & Fried, M. N. (2005). Authority and authority relations in mathematics education: A view from an 8th grade classroom. *Educational Studies in Mathematics*, 58, 145-168.

Au, W. (2011). Teaching under the new Taylorism: High-stakes testing and the standardization of the 21st century curriculum. *Journal of Curriculum Studies*, *43*(1), 25-45.

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. Elementary School Journal, 93(4), 373–397.

Ball, D. L., & Bass, H. (2000). Making believe: The collective construction of public mathematical knowledge in the elementary classroom. In D. Phillips (Ed.), *Yearbook of the national society for the study of education, constructivism in education* (pp. 193–224). University of Chicago Press.

Berliner, D. (2011). Rational responses to high stakes testing: The case of curriculum narrowing and the harm that follows. *Cambridge journal of education*, *41*(3), 287-302.

Bieda, K. N., & Staples, M. (2020). Justification as an equity practice. *Mathematics Teacher: Learning and Teaching PK-12*, *113*(2), 102-108.

Boaler, J. (2003). When learning no longer matters: Standardized testing and the creation of inequality. *Phi Delta Kappan*, *84*(7), 502-506.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171–200). Ablex Publishing.

Cohen, D. K. (1990). A revolution in one classroom: The case of Mrs. Oublier. *Educational Evaluation and Policy Analysis*, 12(3), 311-329.

Bressoud, D. M. (2021). The strange role of calculus in the United States. *ZDM–Mathematics Education*, *53*(3), 521-533.

College Board (2019). *Student score distributions AP Exams – May 2019.* Retrieved from https://secure-media.collegeboard.org/digitalServices/pdf/research/2019/Student-Score-Distributions-2019.pdf

College Board (n.d.). *Professional Learning.* Retrieved from https://apcentral.collegeboard.org/professional-learning

Davies, B., & Harré, R. (1999). Positioning and personhood. In R. Harré & L. van Langenhove (Eds.), *Positioning theory: Moral contexts of intentional action* (pp. 32–52). Blackwell Publishers.

Edwards, D., & Mercer, N. (1987). *Common knowledge: The development of understanding in the classroom.* Methuen.

Esmonde, I. (2014). "Nobody's rich and nobody's poor… it sounds good, but it's actually not": Affluent students learning mathematics and social justice. *Journal of the Learning Sciences*, *23*(3), 348-391.

Gerson, H., & Bateman, E. (2010). Authority in an agency-centered, inquiry-based university calculus classroom. *The Journal of Mathematical Behavior*, *29*(4), 195-206.

Hamm, J. V., & Perry, M. (2002). Learning mathematics in first-grade classrooms: On whose authority? *Journal of Educational Psychology*, 94(1), 126-137.

Harré, R., & van Langenhove, L. (1999). The dynamics of social episodes. In R. Harré & L. van Langenhove (Eds.), *Positioning theory: Moral contexts of intentional action* (pp. 1–13). Blackwell Publishers.

Hepburn, A., & Bolden, G. B. (2017). *Transcribing for social research*. Sage.

Herbel-Eisenmann, B. A. (2000). *How discourse structures norms: A tale of two middle school mathe- matics classrooms*. Unpublished doctoral dissertation, Michigan State University, East Lansing, MI.

Herbel-Eisenmann, B. (2009). Negotiation of the ''presence of the text'': How might teachers' language choices influence the positioning of the textbook? In J. Remillard, B. Herbel-Eisenmann & G. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 134–151). Routledge.

Herbel-Eisenmann, B. & Shah, N. (2019). Detecting and mitigating bias in our questioning patterns. *Mathematics Teaching in the Middle School, 24*(5), 282-289.

Herbel-Eisenmann, B. A., Steele, M. D., & Cirillo, M. (2013). (Developing) teacher discourse moves: A framework for professional development. *Mathematics Teacher Educator*, *1*(2), 181-196.

Herbel-Eisenmann, B. A., Wagner, D., Johnson, K. R., Suh, H., & Figueras, H. (2015). Positioning in mathematics education: Revelations on an imported theory. *Educational Studies in Mathematics*, *89*(2), 185-204.

Heritage, J. (1984). A change-of-state token and aspects of its sequential placement. In J. M. Atkinson & J. Heritage (Eds.), *Structures of social action* (pp. 299–345). Cambridge University Press.

Heritage, J., & Raymond, G. (2005). The terms of agreement: Indexing epistemic authority and subordination in talk-in-interaction. *Social psychology quarterly*, *68*(1), 15-38.

Hersh, R. (1979). Some proposals for reviving the philosophy of mathematics. *Advances in mathematics*, *31*(1), 31-50.

Hewitt, D. (1999). Arbitrary and necessary part 1: A way of viewing the mathematics curriculum. *For the Learning of Mathematics*, *19*(3), 2–9.

Horn, I. S. (2007). Fast kids, slow kids, lazy kids: Framing the mismatch problem in mathematics teachers' conversations. *The Journal of the Learning Sciences*, *16*(1), 37-79.

Horn, I. S. (2018). Accountability as a design for teacher learning: Sensemaking about mathematics and equity in the NCLB era. *Urban Education*, *53*(3), 382-408.

Jefferson, G. (1985). An exercise in the transcription and analysis of laughter. In T. A. van Dijk (Ed.), *Handbook of discourse analysis* (Vol. 3, pp. 25–34). Academic Press.

Ju, M. K., & Kwon, O. N. (2007). Ways of talking and ways of positioning: Students' beliefs in an inquiry-oriented differential equations class. *The Journal of Mathematical Behavior*, *26*(3), 267-280.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27(1), 29-63.

Langer-Osuna, J. M. (2016). The social construction of authority among peers and its implications for collaborative mathematics problem solving. *Mathematical Thinking and Learning*, 18(2), 107-124.

Schegloff, E. A. (2007). *Sequence organization in interaction: A primer in conversation analysis.* Cambridge University Press.

Sonnert, G., Barnett, M. D., & Sadler, P. M. (2020). Short-term and long-term consequences of a focus on standardized testing in AP calculus classes. *The High School Journal*, *103*(1), 1-17.

Stivers, T. (2005). Modified repeats: One method for asserting primary rights from second position. *Research on language and social interaction*, *38*(2), 131-158.

Stroupe, D. (2014). Examining classroom science practice communities: How teachers and students negotiate epistemic agency and learn science-as-practice. *Science Education*, *98*(3), 487-516.

Wagner, D., & Herbel-Eisenmann, B. (2009). Re-mythologizing mathematics through attention to classroom positioning. *Educational Studies in Mathematics*, *72*(1), 1-15.

Wagner, D., & Herbel-Eisenmann, B. (2014a). Identifying authority structures in mathematics classroom discourse: A case of a teacher's early experience in a new context. *ZDM*, 46(6), 871-882.

Wagner, D., & Herbel-Eisenmann, B. (2014b). Mathematics teachers' representations of authority. *Journal of Mathematics Teacher Education*, 17(3), 201-225.

Wilson, M. S., & Lloyd, G. M. (2000). Sharing Mathematical Authority with Students: The Challenge for High School Teachers. *Journal of Curriculum and Supervision*, *15*(2), 146-69.

Yamakawa, Y., Forman, E., & Ansell, E. (2009). Role of positioning: the role of positioning in constructing an identity in a third grade mathematics classroom. In K. Kumpulainen, K. Hmelo- Silver, & M. César (Eds.), *Investigating classroom interaction: methodologies in action* (pp. 179– 202). Sense Publishers.

CHAPTER 5: CONCLUSION

At the time of writing this concluding chapter, my daughter, Elena, is five months old. Since her birth, I have been spending a lot of time interacting with and caring for her. During the writing process, she reminded me of two central ideas in this dissertation: *mutuality* and *reciprocity* of social interaction. I begin this chapter by sharing my lesson from Baby Elena.

It was springtime when Elena was about three months old. I was placing her in a car seat to take her and my five-year-old son, Logan, for a walk along the Red Cedar River at Michigan State University. As I was fastening her seat belt, I smiled at her, and then she showed me a big smile on her face. At that moment, I felt something significant happened between Elena and me. "Elena's smile" or "smiling Elena" does not adequately represent what I saw nor the connection I felt with her. It was the whole process: me smiling at her, then her smiling back at me. The mutual and reciprocating process constituted the situated meaning of her smile. Her smile meant that she recognized my smile and shared what I was expressing with my smile. Our reciprocating smiles showed the mutuality of our joy, happiness, and love.

Note that Elena was only three months old. This experience shows how early we, humans, start to communicate feelings and form relationships. More importantly, it also shows that mutuality and reciprocity are fundamental to how humans interact with each other. The sequentially ordered smiles by me and Elena momentarily tied us together. Attending to Elena's smile alone would overlook these fundamental aspects of social interaction.

This lesson from Baby Elena encapsulates the contribution of the current dissertation. Researchers often develop categorization schemes for teacher actions based on what teachers say or do. This dissertation has shown that understanding the situated meaning of teachers' action requires attending to the *interaction* between the teacher and students. What students do before the teacher's action offers crucial contextual information about the teacher's action. What students do after the teacher's action shows how the teacher's action is shaping the immediate subsequent action. The last three chapters attended to teacher actions situated in this mutual and reciprocal progression of classroom interaction. Here, I provide a summary of the three chapters, and then I share my reflections, implications, and suggestions for future research based on this dissertation.

Interactional Work Presented in Three Chapters

Chapter 2 attended to the social processes of distributing opportunities to speak and negotiating mathematical meanings while centering on the participation of the three Black students in two AP mathematics classrooms. Drawing from ethnomethodology and conversation analysis (EMCA), two features of classroom interaction (turn-taking and argumentation) allowed me to detail these social processes and fine-grained interactional work of the teachers to move the class toward productive and equitable discussions. In particular, teachers' subtle features of talk (e.g., sequencing speakers, question design) prioritized the minoritized students' participation and their intellectual contribution to the ongoing classroom discussion. Moreover, the teachers utilized a range of gestures (e.g., nodding, pointing) to support the minoritized students in the argumentation. This interactional work shows what a teacher can do to counter the existing social marginalization of minoritized students in the AP courses.

My application of EMCA showed the mutuality and reciprocity between teachers' support and the minoritized students' participation. The participation of the minoritized students was not coerced. These teachers engaged in subtle negotiations to establish mutual alignment with the minoritized students about their participation. The teachers also supported the students' contribution by reciprocating the students' talk with gestures (e.g., drawing, pointing) to bring the students' ideas to the fore of the discussion. This mutuality and reciprocity highlighted the intricate relationship between the teacher's facilitation of mathematics discussion and the minoritized students' participation.

Chapter 3 attended to two teachers' use of partner talk by zooming out from the turn-by-turn progression to the changes in broader interactional settings. The analyses showed that as teachers changed from the whole-class discussion to partner talk and vice versa, they shaped a focused and accountable space for mathematics discussions. Partner talk was a *focused* space because the teachers highlighted an important mathematical idea as they initiated partner talk. It was also an *accountable* space because every student was responsible for speaking to the entire class during the subsequent whole-class discussion.

Notably, partner talk provided a backstage setting for the teacher and students to sort out multiple aspects of classroom interaction prior to the upcoming discussion on the frontstage. For instance, teachers could negotiate speakership and sociomathematical norms with students worrying less about face-saving. It allowed the teacher to demand an adequate justification and leverage for the participation of minoritized students, thereby shaping the discussion to be more productive and equitable. The teachers used partner talk to identify mutual pathways to a solution based on what students had shared and what is considered as important in the AP curriculum. The teachers skillfully moved from the frontstage of whole-class discussions to the backstage of partner talk to promote these negotiations.

Chapter 4 focused on how the institutional context of the AP Exam shaped the everyday classroom interaction. Drawing from positioning theory, the analyses identified a storyline, *the Competitive Game of AP Calculus Exam,* that is associated with the AP Exam. Mr. Robinson positioned students as players of the game and reciprocally himself as a coach. This reciprocal positioning reflected a hierarchical authority pattern; the AP Exam providers were at the top of the hierarchy while the students were at the bottom. The teacher played a mediating role. The positioning narrowed the scope of discussions into what is believed to be on the AP Exam and the most time-efficient way to solve the problems with the diminished authority of students.

The reform storyline in Ms. Gray's classroom offered an alternative social reality in an AP Calculus classroom. The teacher positioned students as collective doers of mathematics who propose ideas and engage in discussions to reach an agreement. The students had opportunities to influence instructional decisions, such as whether the class should use a graphing calculator for a particular problem. The scenes of the reform storyline also showed that the social processes of negotiation could be messy, and students may not always reach an agreement. The messy social process, however, can offer rich opportunities for students to participate in producing multiple pathways to approach mathematical problems.

Reflection as a Researcher

The current dissertation has been an opportunity for my own learning as a researcher as much as an opportunity to examine the interactional work of the three mathematics teachers. As a part of this chapter, I reflect on my own development through the research process.

My first lesson is experiencing how much my choice of a theoretical approach shaped my view on the researched phenomena. Research on teaching practices often focuses on the visible conduct of teachers captured by a single camera. Informed by EMCA approaches, I used two video cameras to capture both the teacher's and students' visible conducts because these are available resources for the teacher and students to design and ascribe social actions during the interaction. Watching the data, I initially did not notice the significance of using two cameras for data gathering. When I started to transcribe gestures along with fine details of speech, however, I was immediately able to see the importance of attending to gestures of both the teacher and students. For instance, as I noted in Chapter 2, there is a difference between nominating a student when raising a hand and not raising a hand. In many episodes, teachers' gestures describing a shape (e.g., parabola) was also mirroring the gesture shown by a student. Using a single camera would not have captured this mutuality and reciprocity of the interaction between the teacher and students.

The second lesson I learned from this dissertation is that the research process is not linear nor predictable, although I may plan it in a linear and predictable manner. A few months after the launch of data gathering, the COVID-19 pandemic erupted, and it shut down every classroom. It required me to change the course of this research and come up with a plan for analyses based on the data already gathered. My original plan was to come back to these teachers' classrooms over the academic year and seeing how the teacher and students negotiate norms and patterns of interaction over time. This change of plan made me focus on describing interactional patterns that were captured in a relatively short timeframe of three consecutive lessons.

Despite the changes in the research process, the primary goal of the dissertation did not deviate from the original plan. The initial research questions in my dissertation proposal were as following:

RQ1: What actions of the teacher and other students make the students' participation in argumentation relevant for the interaction?

RQ2: How do social norms and relative positioning of the participants in regard to knowledge and emotion make those actions interactionally consequential?

RQ3: What association do the identified interactional patterns have with the students' gender and race identification and prior history of participation?

The last three chapters mainly focused on RQ1 and RQ2. I attended to turn-taking, argumentation, partner talk, and positioning to understand social processes and norms related to initiating and supporting students' participation in a mathematics discussion. Attending to the change in epistemic stances allowed me to see the effect of some of the teacher's actions.

There are, however, a few parts of the research questions that have not been answered. For example, my analyses did not include the emotional aspect of classroom interaction. This does not mean that emotional features were not important in classroom interaction. I found that analyzing emotional features required a better understanding of transcribing and analyzing speech features. I left the topic of emotion in classroom interaction for my future research. Another aspect that this dissertation did not reach is RQ3. Identifying statistical association between students' gender or racial identification and interactional patterns requires a large set of episodes across multiple gender and racial groups. The students in this dissertation were predominantly White, and the gathered data only included three lessons from each class. Given the narrow scope of the data, conducting a statistical analysis may not validate broader claims about gender and race in classroom interaction. I instead focused on three Black students' participation in Chapter 2 to highlight this dissertation's implication to racial inequity in AP mathematics programs. Gathering further data in mathematics classrooms may lead to an opportunity to investigate RQ3 in the future.

Lastly, examining the three teachers' facilitation of classroom interactions also allowed me to reflect on my past self as a teacher. In other words, examining these teacher's interactional work, I was able to see myself in my own classroom. The three teachers I observed were committed to their students' learning with different stances on what is best for their students. I note that these stances may change and do not show the innate characteristics of these teachers.

Although I might have portrayed these teachers as different kinds of teachers in this dissertation, a range of interactional work presented in this dissertation could possibly come from a single teacher. In particular, I could relate to Mr. Robinson's positioning of his students as players of the competitive game. As an AP Calculus teacher, I focused on increasing my students' AP Exam scores to justify expanding the AP Calculus program in the school I was teaching. I could also relate to Mr. Gray's positioning her students as collective doers of mathematics. When I was preparing for my teaching portfolio for the National Board Certification, I needed to provide evidence of student discussions through which I advanced my students' mathematical thinking. I could relate to Ms. Hill's spontaneous use of partner talk. When I learned that merely demanding student contribution does not lead to a productive and equitable discussion, I needed to facilitate a different interactional space in which students can talk to each other with less concern about how others might see them. This dissertation is based on the data gathered through camera lenses, but examining the data, sometimes, I felt as seeing myself through a mirror.

Implications and Suggestions for Future Research

Although the last three chapters are intended to be three independent manuscripts with different foci and implications, this dissertation as a whole offers broader implications for the understanding of mathematics classroom discourse. Based on Herbel-Eisenmann and colleagues' (2017) review of discourse analytic traditions in mathematics education, this dissertation mainly fits in the tradition of interactional sociolinguistics, which includes conversation analysis. This dissertation also drew from Toulmin's model of argumentation to organize mathematical contents (Chapter 2) and positioning to examine how the AP Exam appears in classroom interaction (Chapter 4). While drawing on multiple research traditions, I consistently applied EMCA approaches with the focus on the temporal progression of classroom interaction. This dissertation thus brings to light applying EMCA with other research traditions to understand mathematics classroom discourse. I conclude this dissertation by discussing the broader implications and possibilities based on EMCA approaches for research in mathematics education.

Furthering the Understanding of Mathematics Classroom Discourse

One of the main contributions of this dissertation is detailing some of the discourse practices of mathematics teachers in existing frameworks (e.g., Chapin et al., 2009; Conner et al., 2014; Michaels et al., 2010; Herbel-Eisenmann et al., 2013). The existing categorization schemes offer an outline of what teachers do when they facilitate mathematics discussions. However, detailing these practices remains crucial because there are multiple ways to implement each discourse practice with different affordances and limitations. In Chapter 2, I delineated ways of *inviting student participation* (Herbel-Eisenmann et al., 2013) through the lens of turn-taking. The analysis showed various turn-allocation methods for inviting students to the conversation floor and how the different methods facilitated different interactional contexts for student participation. For instance, using only the conventional turn-allocation methods may inadvertently place the student in an interactionally troubling situation. The illustrated details of discourse practices showed the complexity of achieving the intent of each practice.

These details of practices may offer insights to mathematics teachers who are working to implement discourse practices to achieve particular goals. Teachers and researchers noted that each discourse practice could serve different purposes based on the details of talk (e.g., Krusi, 2009; Waring, 2016). Examining multiple ways to enact a particular discourse practice can enrich understanding of the identified practice. In Chapter 2, for instance, I showed that teachers could support argumentation using embodied resources (Conner et al., 2014). This dissertation complements Conner and colleague's work by situating teachers' support in the temporal progression of classroom interaction in developing collective argumentation. The findings highlighted the importance of timing of these gestures and other nonverbal supportive actions (e.g., Ms. Hill's nodding when Indigo uttered the first syllable of her response). In Chapter 3, I also illustrated the refined work of facilitating partner talk by the two teachers. The set of discursive practices before, during, and after partner talk facilitated a focused and accountable space for classroom discussions. This finding complements the simplistic representation of partner talk as Think-Pair-Share (e.g., MAA, 2018) by adding details of why and how teachers may use partner talk to facilitate mathematics discussion. By examining how and to what ends teachers facilitate classroom interaction, I presented a more nuanced understanding of the commonly seen teaching strategies.

Another broader contribution of this dissertation is offering an alternative way to examine the effect of using particular discourse practices. Scholars often examined the effectiveness of teaching practices by finding statistical associations between the frequency of the use of the practice and improvement of achievement data of a targeted group of students (e.g., Battey et al., 2016; Wilson et al., 2019). The interactional analyses in this dissertation allowed me to examine how the employment of discourse practices leads to changes in students' epistemic stances that were displayed in interaction. For instance, in Chapter 2 and Chapter 4, I illustrated how teachers' actions shaped students' epistemic stances and epistemic authorities (Heritage, 2012; Heritage & Raymond, 2005). As I noted in Chapter 1, my analyses did not show students' learning through the lens of learning-as-acquisition, which the aforementioned studies addressed by analyzing achievement data. My interactional analyses rather showed the changes in how the students treated the ongoing classroom activity as the students presented themselves as a more-or less-knower. These changes represent social and interpersonal aspects of learning, learning-as-participation in a micro-timescale. Although the findings did not address enduring characteristics of the students (e.g., beliefs, identities), they showed how classroom interaction momentarily shaped students' self-presentation as knowers, which can serve as evidence for positioning students as producers of knowledge or leveraging for the participation of minoritized students. Of course, the epistemic stance is not the only way students display their orientations. In the following, I further present other potential approaches informed by EMCA.

EMCA Approaches and Future Research

This dissertation adopted EMCA as a primary methodological lens to analyze discursive practices in mathematics classrooms. The application of EMCA is not common and "so far played only a small part in mathematics education research" (Ingram, 2018, p. 1065). This contrasts with the wide contributions of EMCA-oriented research in various institutional settings, such as language classrooms, medical settings, and courtroom settings. Therefore, I highlight some of the unique features of EMCA approaches and their affordances that are exemplified in this dissertation. I hope this discussion invites readers to consider applying EMCA in research in mathematics education. While discussing these methodological features, I also suggest future lines of research informed by EMCA.

One of the central features of EMCA is its epistemological stance based on participants' orientation to examine discursive practices. EMCA approaches focus on participants' orientations *in situ*; that is, attending to how the participants are treating the ongoing interaction based on the details of speech, gesture, postures, and other visible and hearable means of communication. These details are essential not only for the analyst but more so for the participants. Participants use these discursive resources to design actions with their talk and gesture in a way that can be ascribed as such by other participants. For example, Ms. Hill's timely nodding in Chapter 2 showed Ms. Hill's orientation to Indigo's response. This nodding was significant for Indigo since Indigo's epistemic stance related to the topic under discussion changed from less-knowing to more-knowing stance within a single turn. EMCA approaches can illuminate the significance of teachers' actions based on how the actions are consequential to the immediate interaction.

Research findings from this theoretical approach bring as much practical value as theoretical value. The analyses focus on identifying social processes (often referred to as *participants' methods*) by offering detailed accounts of how teachers accomplish co-operative activity with students. These identified social processes may offer insights for those working to implement a particular discourse practice. For instance, the detailed accounts of partner talk in Chapter 3 illustrated how the teachers held students accountable for focused mathematics discussions during partner talk. Similar to Herbel-Eisenmann and Cirillo's (2009) collaboration with a group of middle grades mathematics teachers, the set of discursive practices may help teachers identify important missing elements of their facilitation when they face challenges with their implementation. This dissertation, however, illustrates additional details of talk (e.g., word choice, emphasis) and gesture (e.g., pointing, gaze) and shows how they are important in this process. As I argued in Chapter 1, when teachers are facing challenges, the focus on the details of talk may support teachers in avoiding deficit-oriented views on their students (e.g., This does not work with my students! He never raises his hand!) and drawing their attention to further refining their facilitation. Earlier work shows similar promise. For example, Cavanna and colleagues' (2015) examination of how one teacher-researcher's appraisals of her own discourse practices became more critical at the same time her appraisals of student contributions shifted toward more asset-based views.

Although EMCA approaches highlight the significance of details of talk and gesture, this does not suggest that a teacher alone can determine the course of interaction. As I noted earlier, mutuality and reciprocity are fundamental aspects of social interaction. In this reciprocating process, students shape the interaction as much as the teacher does. EMCA approaches afford to situate teachers' actions in this turn-by-turn progression of interaction. This affordance makes EMCA a powerful tool to consider answers to *when*-questions as well as how-questions. For instance, Chapter 3 showed the pedagogical value of partner talk when there was an important idea students seem to overlook or when students' responses did not agree. That is, the chapter highlighted that facilitating partner talk did not happen in a vacuum. The spontaneous facilitation allowed the teachers to take advantage of elicited student thinking as a resource to advance the mathematics discussion.

Peräkylä and Vehviläinen (2003) presented that EMCA findings may advance existing theories and models of interaction in three ways: falsifying or correcting existing assumptions, providing a more detailed picture of identified practices, and adding a new dimension to the current understanding of interaction. The findings in this dissertation largely align with the second part of the contribution, providing more detailed pictures of some of the discourse practices. Based on this dissertation, I suggest future EMCA studies to advance understanding of mathematics classroom discourse in multiple ways.

First, this dissertation identified non-conventional discursive practices from Ms. Gray's classroom, and those practices offer alternative ways to facilitate productive and potentially equitable mathematics discussions. For instance, prior research on turn-taking in classroom settings has primarily been in traditional, teacher-centered classrooms (Gardner, 2013), and the findings from this dissertation suggest further research on classroom interaction facilitated by the teachers who have participated in discourse-oriented professional development. These teachers may not be representative of the national or international community of experienced mathematics teachers. Still, findings may offer important clues and insights to overcome common challenges when teachers move from teacher-centered classroom interaction to more productive mathematics discussions.

Second, examining some of the emotional features shown in the data may contribute to the emotional dimension of understanding of mathematics classroom discourse. For instance, there were a few moments of laughter during discussions in Ms. Gray's classroom, but interestingly, there was no laughter observed in the other two classrooms. Conversation analysts showed how participants use laughter (e.g., Jefferson, 1985) in everyday conversation. Applying related CA findings to classroom interaction may illuminate an emotional dimension of classroom interaction (e.g., Tainio & Laine, 2015). In particular, participants' emotional stances displayed by prosody, gesture, and body posture (Goodwin et al., 2012) may help understand how the teacher and students (dis)affiliate with each other and negotiate social boundaries. Attending to the emotional stance may complement the construct of the epistemic stance that I applied in this dissertation.

Lastly, this dissertation can be furthered by looking at changes in patterns of classroom interaction over a more extended time (Mercer, 2008). This dissertation examined three consecutive lessons in the middle of the academic year. The study's timeframe suggests that the findings from this dissertation represent interactional patterns that were largely negotiated over the first half of the academic year. Examining interactional patterns at the beginning of the academic year may highlight social processes of negotiating norms and add more episodes of deviant cases (episodes that deviate from identified normative patterns). These variations of interaction could enrich the findings of this dissertation by detailing the historical process of how the teacher and students came to interact with a particular set of norms.

Toward Productive and Equitable Mathematics Discussions

Rawls (2002) stated, “social change requires, first and foremost, an understanding of social processes” (p. 19). In this dissertation, I explored social processes of classroom interaction with an eye toward social change for productive and equitable mathematics discussions. I presented moments of teachers’ interactional work that facilitated space for students to become producers of knowledge and leveraged minoritized students’ participation. Teachers’ work went beyond what is commonly seen in contemporary mathematics classrooms, such as non-conventional turn-taking methods, facilitating partner talks to move away from teacher-centered whole-class discussions, and positioning students as collective doers of mathematics in the institutional context with high-stakes testing. These moments showed the possibility of reconfiguring AP mathematics classrooms despite the existing social and institutional contexts.

At this moment of concluding this dissertation, however, I cannot stop asking myself the following questions: What does it take to move from understanding social processes to actualize social change in a sustainable manner? How can I use the insights and possibilities presented in this dissertation to inform mathematics teacher preparation and professional development? What are ways for teachers to engage with their classroom data, as I did by applying EMCA? These questions suggest that the end of this dissertation marks another new beginning, and my inquiry toward productive and equitable mathematics discussions will continue.

REFERENCES

REFERENCES

Battey, D., Neal, R. A., Leyva, L., & Adams-Wiggins, K. (2016). The interconnectedness of relational and content dimensions of quality instruction: Supportive teacher–student relationships in urban elementary mathematics classrooms. *The Journal of Mathematical Behavior*, *42*, 1-19.

Cavanna, J. M., Herbel-Eisenmann, B., & Seah, W. T. (2015). Investigating teachers' appraisal of unexpected moments and underlying values: an exploratory case in the context of changing mathematics classroom discourse. *Research in Mathematics Education*, *17*(3), 163-182.

Chapin, S. H., O'Connor, C., O'Connor, M. C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, Grades K-6*. Math Solutions.

Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics*, *86*(3), 401-429.

Gardner, R. (2012). Conversation Analysis in the Classroom. In J. Sidnell & T. Stivers (Eds.), *The handbook of conversation analysis* (pp. 593-611). Wiley-Blackwell.

Goodwin, M. H., Cekaite, A., & Goodwin, C. (2012). Emotion as stance. In A. Peräkylä & M.-L. Sorjonen (Eds.), *Emotion in interaction* (pp. 16–41). Oxford University Press.

Herbel-Eisenmann, B., & Cirillo, M. (Eds.). (2009). *Promoting purposeful discourse: Teacher research in mathematics classrooms*. National Council of Mathematics Teachers.

Herbel-Eisenmann, B., Meany, T., Pierson Bishop, J., & Heyd-Metzuyanim, E. (2017). Highlighting heritages and building tasks: a critical analysis of mathematics classroom discourse literature. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 722–765). National Council of Teachers of Mathematics.

Herbel-Eisenmann, B. A., Steele, M. D., & Cirillo, M. (2013). (Developing) teacher discourse moves: A framework for professional development. *Mathematics Teacher Educator*, *1*(2), 181-196.

Heritage, J. (2012). The epistemic engine: Sequence organization and territories of knowledge. *Research on Language and Social Interaction*, *45*, 25–50.

Heritage, J., & Raymond, G. (2005). The terms of agreement: Indexing epistemic authority and subordination in talk-in-interaction. Social Psychology Quarterly, 68(1), 15-38.

Ingram, J. (2018). Moving forward with ethnomethodological approaches to analysing mathematics classroom interactions. *ZDM*, *50*(6), 1065-1075.

Jefferson, G. (1985). An exercise in the transcription and analysis of laughter. In T. A. van Dijk (Ed.), *Handbook of discourse analysis* (Vol. 3, pp. 25–34). New York: Academic Press.

Krusi, J. (2009). Revoicing: The good, the bad, and the questions. In B. Herbel-Eisenmann & M. Cirillo (Eds.), *Promoting purposeful discourse: Teacher research in mathematics classrooms* (pp. 117-135). National Council of Mathematics Teachers.

Peräkylä, A., & Vehviläinen, S. (2003). Conversation analysis and the professional stocks of interactional knowledge. *Discourse & Society*, *14*(6), 727-750.

Mathematical Association of America (MAA) (2018). *MAA Instructional Practices Guide*. Retrieved from https://www.maa.org/programs-and-communities/curriculum%20resources/instructional-practices-guide.

Mercer, N. (2008). The seeds of time: Why classroom dialogue needs a temporal analysis. *The journal of the learning sciences*, *17*(1), 33-59.

Michaels, S., O'Connor, M. C., Hall, M. W., & Resnick, L. B. (2010). *Accountable Talk sourcebook: For classroom conversation that works*. University of Pittsburgh.

Rawls, A. W. (2002) *'Introduction' to ethnomethodology's program: Working out Durkheim's aphorism, by Harold Garfinkel*. Rowman and Littlefield.

Tainio, L., & Laine, A. (2015). Emotion work and affective stance in the mathematics classroom: the case of IRE sequences in Finnish classroom interaction. *Educational Studies in Mathematics, 89*(1), 67-87.

Waring, H. Z. (2016). *Theorizing pedagogical interaction: Insights from conversation analysis*. Routledge.

Wilson, J., Nazemi, M., Jackson, K., & Wilhelm, A. G. (2019). Investigating teaching in conceptually oriented mathematics classrooms characterized by African American student success. *Journal for Research in Mathematics Education*, *50*(4), 362-400.