

A MOLDED PAPER PULP PACKAGING DESIGN
USING TOPOLOGY OPTIMIZATION METHOD

By

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ABSTRACT

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Molded Paper Pulp (MPP) is a form of packaging that has become widely used because of the environmental and economic benefits. The most common MPP design method would be creating different prototypes based on the engineer's knowledge and experience and analyzing each model to determine which model has the best design. However, this trial-and-error design method is a costly and time-consuming process. To overcome these drawbacks, topology optimization will be looked at being incorporated into the MPP design process.

Topology optimization is a structure optimization method based on a Finite Element Analysis (FEA) and a sensitivity analysis. It has been widely used in various industries because of the design efficiency. However, the topology optimization method has not been applied into the packaging industry. The main reason is because of the limited design space of packaging and manufacturability. This thesis presents a methodology to find the optimal design of the MPP using the topology optimization method and post processing. First, the critical area is defined using a topology optimization result under a given boundary condition. Second, the topology optimization results will be superimposed to the MPP design space again. Lastly, the supporting rib structure will be added to finalize the MPP design process. Once the design is completed, the optimal design is compared to the original design to evaluate the improvement.

In this thesis, MPP wine shipper is used as an example. The proposed method was able to present a new design using the topology optimization method. To demonstrate the efficiency of the new design, FEA results are presented, and the new design shows a lower stress concentration compared to the typical MPP wine shipper design in the market.

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KEY TO ABBREVIATIONS

MPP	Molded Pulp Packaging
FEA	Finite Elemental Analysis
CAD	Computer Aided Design

CHAPTER 1 – INTRODUCTION

1.1 Introduction

Packaging is an integral part of society and with an ever-increasing demand for environmentally friendly solutions the increase in different materials has risen. For Molded Pulp Packaging (MPP) the move to environmentally friendly solutions has increased the demand [1]. This increase in demand has created new avenues of usage for MPP. The current design method for designing MPP is based on the design engineers experience and knowledge. These engineers follow the conventional design method which can be time consuming and costly.

To alleviate the time and cost issues in the conventional design method, topology optimization can be incorporated. Topology optimization is a method used to optimize a given object in a given design space. Topology optimization has seen widespread use in many aspects of engineering; however, it has yet to be applied to packaging. One of the main constraints that has limited topology optimization to be used for packaging is the small, limited design space available.

This thesis looks at the enhancement of MPP design through the application of topology optimization. The method presented, as an alternative to the conventional design method, is analyzed for effectiveness using MPP.

1.2 Objectives

This research is aimed at proving that the incorporation of the topology optimization method into the design step of MPP can be beneficial to the overall performance of the

package. To do this, current MPP was modeled into a 3D CAD model and analyzed through topology optimization method. The results were used to create new updated models. The new and original models will be compared and analyzed through Finite Elemental Analysis.

1.3 Structure of Thesis

The first chapter of this thesis covers the introduction to this research and why it was performed. The second chapter covers the background of the packaging material that will be studied. In the third chapter, the background of the optimization method, topology optimization, will be covered. From there, the fourth chapter will cover the proposed method and the fifth chapter will go over the results the proposed method. To finalize, the sixth chapter will cover the conclusions and future plans.

CHAPTER 2 – MOLDED PAPER PULP

2.1 Introduction

Packaging has been used for centuries to protect goods from damage. Molded Pulp Packaging (MPP) is a form of packaging that has seen an increase in demand due to its perception as an environmentally friendly material [2]. This continuously growing demand for MPP, along with bans for single-use plastic packaging, has allowed for different types and usages of MPP as a packaging solution [1]. Despite the continuous growth of the MPP market, research has been limited leaving it to be an undeveloped area [2].

2.2 History

MPP was first introduced in 1903 with a patent by Martin L. Keyes [4] for an apparatus to make pulp articles Figure 1. The machine described in the patent has a porous mold that is dipped in the pulp slurry. A sister mold is then used to compress the original mold to remove the excess liquid. The slurry is applied to the original mold using suction and ejected with compressed air. The mold is oven dried.

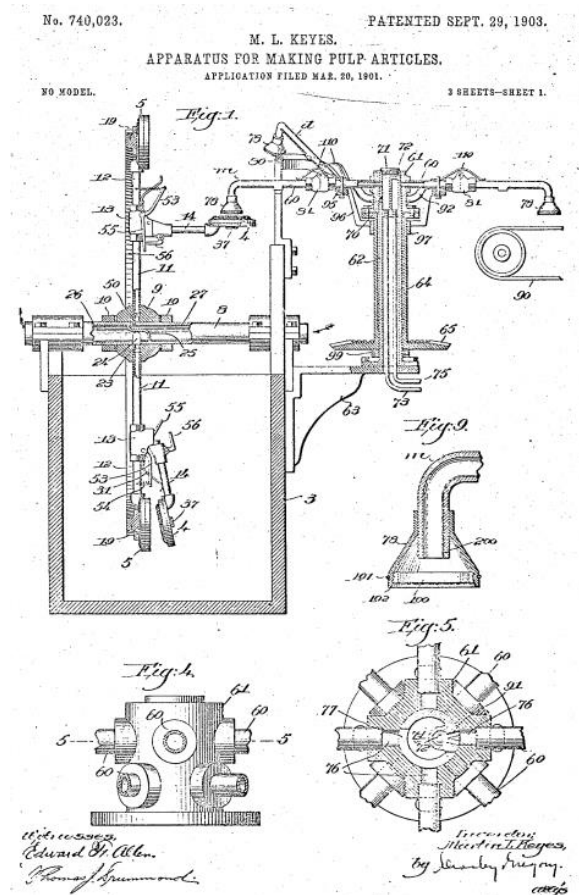


Figure 1 First Patent by Martin L. Keyes

The invention of an apparatus to make molded pulp articles did not have widespread implementation at the start. Early uses of the packaging material were designated for egg packaging. The first use of egg molded pulp packaging involved a combination of molded pulp and carboard in its design [2]. Egg packaging then continuously evolved until it became solely molded pulp in material and had the standard carton shape of today by World War Two.

MPP did not spread much from use besides egg packaging for years until a demand for environmentally friendly solutions drove the demand up. This new demand expanded MPP into

multiple markets such as, industrial packaging, food packaging, and disposable items. Expanding into the new markets also increased the variety of MPP solutions. These new solutions range from corner protectors, clamshell packaging, end caps and disposable items.

2.3 Process

MPPs are created, as the name suggests, by molding paper pulp into a desired shaped designed for protection. Molded pulp is created in a process that involves the vacuuming of a slurry of pulp over a wire mesh mold and heat dried [2] [5]. Figure 2 shows the conceptual process for making MPP.

The first step of the process begins with the mixing of raw materials with water to form a pulp. The raw materials being used come in the form of virgin or recycled wood fibers. Virgin and recycled fibers play a key role in determining the mechanical properties of the MPP. Virgin fibers are longer and stronger than those of recycled fibers but need to be sourced from trees. Recycled fibers are shorter and weaker than virgin fibers and are sourced from recycled wood products. Recycled fibers can only be recycled between 5 to 7 times [6]. Recycled fibers used in MPP are most commonly obtained from old newsprint [7] [8].

With the formation of the pulp, the next step in the process of making MPP is the forming stage. The forming stage begins by dipping a mold into the prepared pulp. These molds are a custom design based off the needs of the MPP. The molds are also porous to allow for excess water to be removed during the drying stage.

After the mold is dipped into the slurry during the forming stage, the mold is then transferred into a heated mold. These molds press and heat the dipped mold to remove any excess water and dry the mold.

With the completion of the heated mold step, the process can continue into the last stage of the process. The last stage is the wrap, stack, and pack stage. This stage involves taking the molds that are finished drying and nestling them into each other. Once nestled, these stacks can be packed and shipped out to their destinations. This nestling allows for the saving of space throughout the supply chain from storage and to shipping.

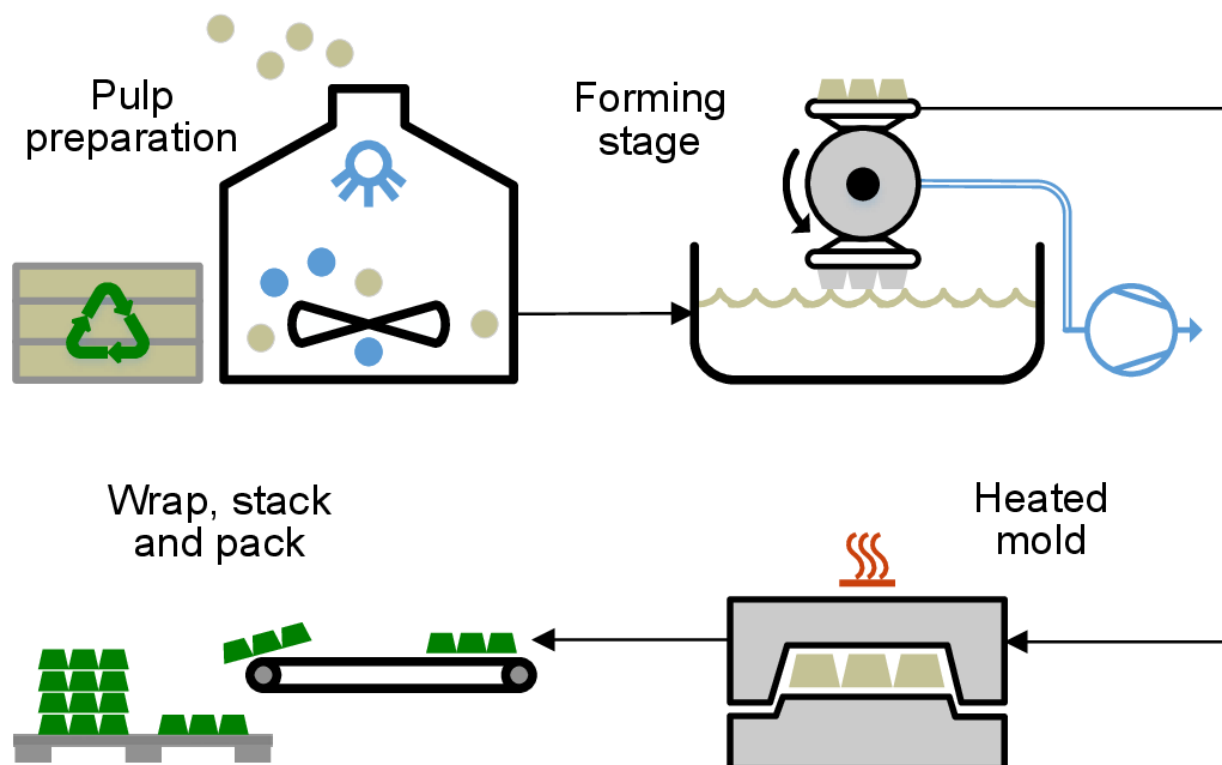


Figure 2 MPP Process (Howe) [5]



Figure 3 Example pictures of MPP process [9] [10] [11] [12]

Figure 3 above shows picture examples of the process for creating MPP. The images follow the same order of Figure 2. The top left picture is a vat of a prepared slurry of wastepaper and water. The next picture represents the forming stage showing an empty mold and a mold that has picked up some slurry. Below that picture on the bottom right, MPP can be seen leaving an open-air oven during the drying stage. The final picture on the bottom left shows how MPP can be nestled in each other during the wrap, stack and pack stage to save space.

During this process of making MPP, quality checks are used to assure accurate production. If a mold is to fail quality assurance, it can be thrown back into pulp and start over in the process at the first stage of pulp preparation. The water that is lost during the formation and drying stages of MPP is also recycled back into process [13].

MPP can be classified to four different types [14] [15].

1. Thick Walled: Produced using a single mold for a final thickness range of 5 to 10 millimeters (mm). Mainly used as packaging support for non-fragile, heavier items. One side is relatively smooth, with the other side being rough.
2. Thermoformed or Thin Walled: Use of multiple heated molds with a final thickness range from 2 to 4 mm. The MPP is dried in mold instead of oven drying. Both surfaces are smooth. Resembles thermoformed plastics.
3. Transfer Molded: The usage of a forming mold and transfer mold to create the MPP with a final thickness range of 3 to 5 mm. Smooth on one side. Mainly used for egg cartons and being used for electronic like cell phones and DVD players.
4. Processed: MPP that requires special treatments after being molded and cured. This can include printings, special slurry formations, die-cuts, additives, etc.

2.4 Applications

Today, MPP has seen constant growth as a packaging solution, which has allowed for its incorporation into four distinct markets; Food related, Industrial or engineered packaging, single use medical, and horticultural trays, and pots [15].

Food related MPP products are diverse in design and usage. The extensive designs used for food related packaging range from clamshells, dishware like cups, bowls and plates, and trays. Clamshell MPP is typically implemented as carryout containers from restaurants. MPP dishware is a convenient way to have disposable plates, bowls, or cups for times where single use dishware is needed. Utensils also have products created with MPP for single use items. MPP trays are used to protect fragile items such as eggs, and fruits. Examples of food related MPP can

be seen in Figure 4. Figure 4 shows the wide variety of MPP used in the food market. The top left picture shows a clamshell MPP. The top right picture is of a typical egg carton that can be found in stores. The bottom two pictures are used for single-use food related issues such as take-out and disposable plates.



Figure 4 Food related MPP examples [16] [17] [18] [19]

The next main market for MPP is the single use medical packaging. For medical packaging, especially single use packaging, remaining sterile is important to protect all the individuals the package may contact along its life cycle. Also, with the amount of waste

generated through the medical industry, having more environmentally friendly packaging is important. Single use medical MPP are used as bedpans, urine bottles, kidney dishes and disposable bowls. For the products designed to hold liquids, a special interior coating will be applied to prevent seepage and accidental contamination. Examples of single use medical MPP can be seen in Figure 5. Shown in the figure are bed pans, urine bottles and single-use trays.



Figure 5 Single use medical MPP examples [20]

The third distinct market for MPP is the industrial or engineered packaging market. MPP in this market is used as cushioning packages to protect products during shipment. These packages are used in markets such as in electronics, household items, automotive, and industrial parts. For electronics, MPP can be used as trays for products like routers, or cell phones and end cushions for larger products like televisions. Household items packaged with MPP range from kitchen items like toasters and coffee makers to furniture. Just like electronics, automotive parts

such as gears, headlights and vehicle parts can be packaged in trays and end caps. Figure 6 shows examples of the industrial and engineered market of MPP. These examples cover the electronic, household, and automotive industries and show the variety of designs available for similar functions of the packaging.

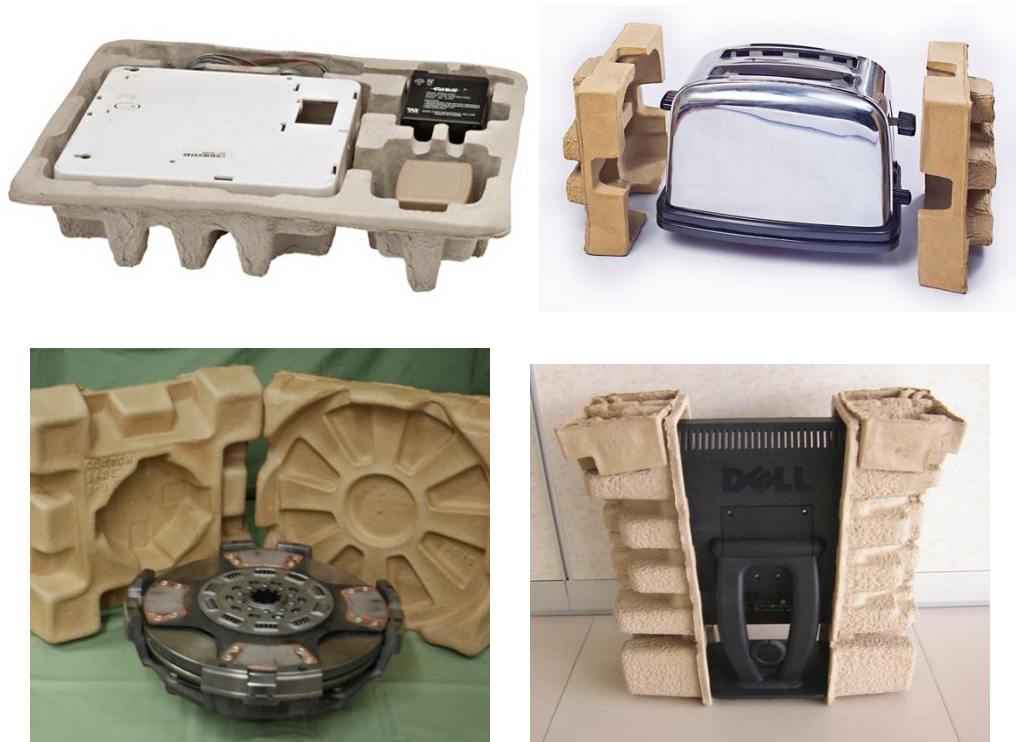


Figure 6 Industrial and engineered packaging MPP examples [21] [22] [23] [24]

The last distinct market for MPP is for horticultural trays and pots. For the horticultural industry and plant nurseries, these companies have a focus on the environment and sustainability. With a focus on sustainability having environmentally friendly packaging for their pots and trays is important. These products can be seen below in Figure 7.



Figure 7 Horticulture and pots MPP example [25] [25]

MPP has seen a widespread of implementation into multiple markets due to the increase in packaging environmental considerations. With the widespread implementations, the designs have also varied depending on their intended use.

2.5 Current Developments

With the increase in demand for MPP, research and developments have been done to expand MPP's usage.

While MPP has been used for more standard packaging examples, one company has created a molded pulp bottle to hold liquids [27]. While the molded pulp is formed to a bottle shape, its biggest weakness is that it is made from recycled paper thus making it susceptible to liquid damage. To combat this, a thin inner plastic liner is created and placed inside. This liner provides a protective barrier to protect the pulp while allowing for minimal plastic usage and waste. This bottle can be seen below in Figure 8. This company also offers bottles of different sizes to accommodate different uses from water bottles to laundry detergent.



Figure 8 Molded pulp bottle [27]

Another new use being pursued by another company is by offering a molded pulp cooler [28]. This cooler aims to target the same market as expanded polystyrene (EPS) coolers by promoting a more environmentally friendly solution. The company also states that unlike the EPS coolers, these MPP coolers have the ability to be reused. The cooler can be seen in Figure 9.



Figure 9 MPP cooler [28]

2.6 MPP Design Challenges

The problem with designing MPP is that the designs are based on the engineer's experience and knowledge. With designs based on an engineer's experience or knowledge, it creates designs that could be varied throughout the industry. These designs risk not being optimized. Non optimized designs can be costly through material usage or financial waste.

Designing a new product follows the conventional design method. The conventional design method follows three steps: Design, Test and Analyze.

1. Design: the design concept is formed with the product and final goal in mind.
2. Test: the conceptual design is tested to the specifications required.
3. Analyze: the results of the test are analyzed for any inconsistencies or room for improvement.
4. Repeat: the process is repeated until a final design meets the final requirements and tests needed.

While the conventional design method is effective in creating a sufficient design, the process is time consuming and costly. The final design can also not be optimized for the best performance. With non-optimized packaging, the issues of overpacking or under packing a product becomes an issue.

When it comes to packaging, trying to find the balance between protection and cost is important. If you package a product with more protection than necessary, the failure rate will decrease, but the cost per package increases. On the other side, under packing is the attempt to reduce cost of the packaging. Reducing the overall cost of the packaging, by reducing material, will increase the failure rate of the package. An increase failure rate of the package will result in

more products broken during shipping. The balance between over packing and under packing is what can be considered optimal packaging. Optimal packaging is packaging with the best performance and cost possible. The issue with optimal packaging is finding the optimal solution.

To try and aide in finding an optimal solution, Finite Element Analysis has been briefly studied with MPP applications. Finite Element Analysis was used to look at optimizing molded honeycomb molds [29] and molded pulp pallet drop testing optimization [30].

CHAPTER 3 – TOPOLOGY OPTIMIZATION

3.1 Introduction

To overcome the conventional design method, topology optimization will be incorporated into the first stage, the design stage. The addition of topology optimization aims to decrease the cost and time of the conventional design method [32]. Topology optimization is a tool that assists a designer in the selection of optimal structural designs by adding or removing material in the structure [33] [34].

With the increase in power of computers in the late 1980's, the idea of developing structures that are optimized started emerging [35] [36]. These more powerful computers increased the confidence in computer aided design and analysis.

3.2 Topology Optimization Method

The topology optimization method follows the steps outlined in Figure 10 below. The main steps involved are Finite Element Analysis, Sensitivity Analysis, Update the Material Density, Convergence, and Plot Out the Results.

The topology optimization method begins with the Finite Element Analysis. The FEA consists of defining the design space and applying the boundary conditions to perform the analysis. The design space is the area in which the topology optimization will be performed. With the design space defined, the boundary conditions can start being applied. Boundary conditions are the special limits placed upon the process to guide topology optimization to the desired results. The volume constraint, for example, can be set to 30% of the desired volume. With the volume constraint set, the material density of the object will be eventually distributed in the design space with each

element set to 0.3. Once the boundary conditions are set, the mesh can be applied. The mesh splits up the object into individual elements. These elements will be analyzed under the FEA using the chosen boundary conditions. The FEA can be performed once the boundary conditions and mesh are applied.

While the FEA is being performed, the displacement of the structure will be calculated with the use of Hooke's Law. The calculation of displacement is used to perform the sensitivity analysis. The sensitivity analysis calculates the importance of each element in relation to another. This calculation is then used to update the material density of each element.

With the sensitivity analysis performed and the subsequent redistribution of material between elements, the next step of the topology optimization process can begin. The next step, convergence, begins by checking to see if the topology optimization method has converged. Convergence is checked by looking at the difference between the value of the objective function of the current iterations and the previous iteration. If the difference is small enough, the topology optimization method has determined that the function has converged. After the topology optimization method determines convergence, a final result will be output.

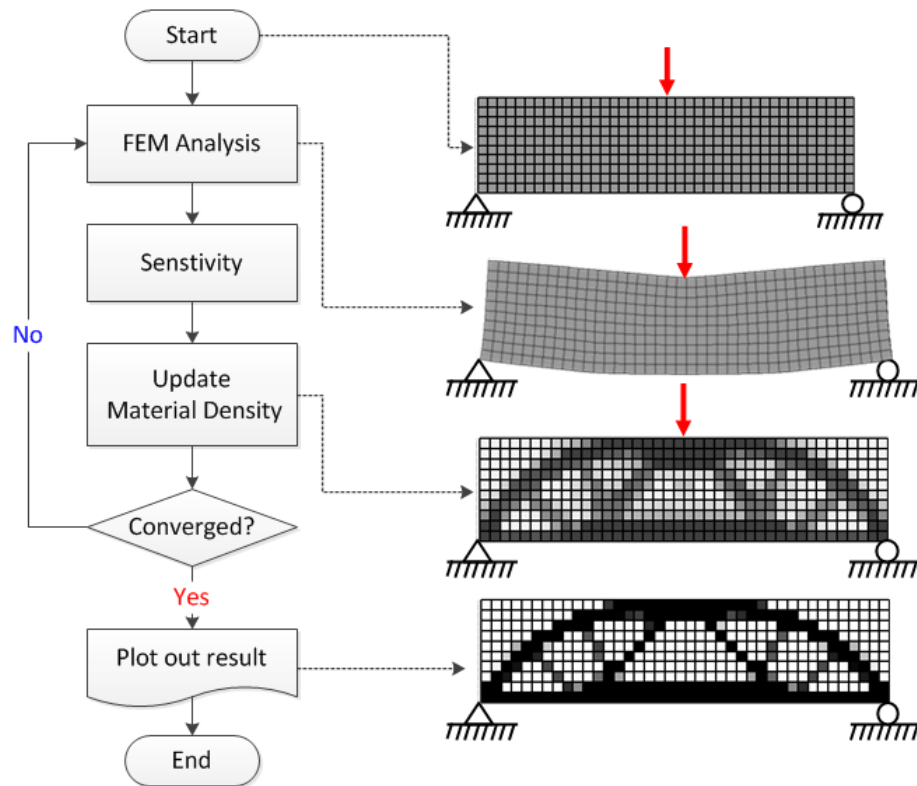


Figure 10 The Flow of Topology Optimization Method

Topology optimization has found widespread use in multiple industries due to its ability to output optimized results. Specifically looking at structural design topology optimization and packaging, integration of the two has yet to be widely applied due to the limited design space available in current packaging design.

Packaging design can come in a variety of solutions ranging from shell type packaging, end caps, trays, and cushion support. If viable current packaging designs can be identified and have topology optimization incorporated into their design processes, optimized packaging designs can be created. Optimized packaging is a balance between performance and material usage.

3.3 Definition

Topology optimization method works by taking the material within a given design space and optimizing its material layout [37]. Topology optimization is guided by three major parts: the objective function, constraints, and design variables. The three major parts can be expressed as:

$$\begin{aligned} \min_x \quad & \Pi(X) = \frac{1}{2} U^T K U \\ \text{s.t.} \quad & V(X) = \sum_{i=1}^N x_i v_i \leq \bar{V}_0 \\ & KU = F \end{aligned} \tag{1}$$

$$X = \{x_1, x_2, \dots, x_N\}, \quad \underline{x} \leq x_i \leq \bar{x}$$

U is a displacement vector of the structure and K is a stiffness matrix which characterizes the rigidity of the structure. Multiplying K and U together gets the loading vector, F , which is applied to the structure using the equilibrium equation $KU=F$. x_i is the design variable which is the material density in the topology optimization problem and v_i is the volume of each element. For the objective function, minimizing the total strain energy will be used. Strain energy is the energy that is stored during displacement [38]. Minimizing the strain energy will minimize the displacement, while higher strain energy means higher displacement. To calculate strain energy stored in the structure, the force vector (F) and displacement (U) will be multiplied together.

Along with the objective function, there are two constraints being used. The volume constraint is used to limit the material usage as an inequality constraint, and the equality constraint, Hooke's Law. For the volume constraint, the lower volume constraint reduces the material usage, but at the same time can increase the objective function values. To limit the increase of the objective function, the volume constraint needs proper settings to obtain the optimized structure shape. The equality constraint, Hooke's Law, theorizes that the deformation, or strain, of an elastic object or material is directly proportional to the stress applied to it. Understanding this relationship, minimizing the strain should minimize the deformation of the object. For structural optimization, material density will be used for the design variable. The design variable has a value that varies from 0 to 1 for each element of the structure. The design variable will be incorporated into the topology optimization through the use of the Solid Isotropic Microstructure with Penalty Model (SIMP). The SIMP model defines the elastic tensor E_{ijkl} as:

$$E_{ijkl}(x) = x^p E_{ijkl}^0, \quad p > 1 \quad (2)$$

The Elastic tensor E is used as the function for material density (x). To increase the efficiency of the topology optimization process, the penalty power, p , is used. The effect of the penalty value to the relative stiffness (E/E^0) is shown below. As the penalty value increases, the function begins to act like a step function. This allows the optimization process to be able to converge the material density to either a void ($x=0$) or solid ($x=1$). Therefore, the topology optimization design consists mostly of void and solid elements. The stiffness matrix K is calculated by applying the elastic tensor E which results in the stiffness matrix being defined as a function of the material density x .

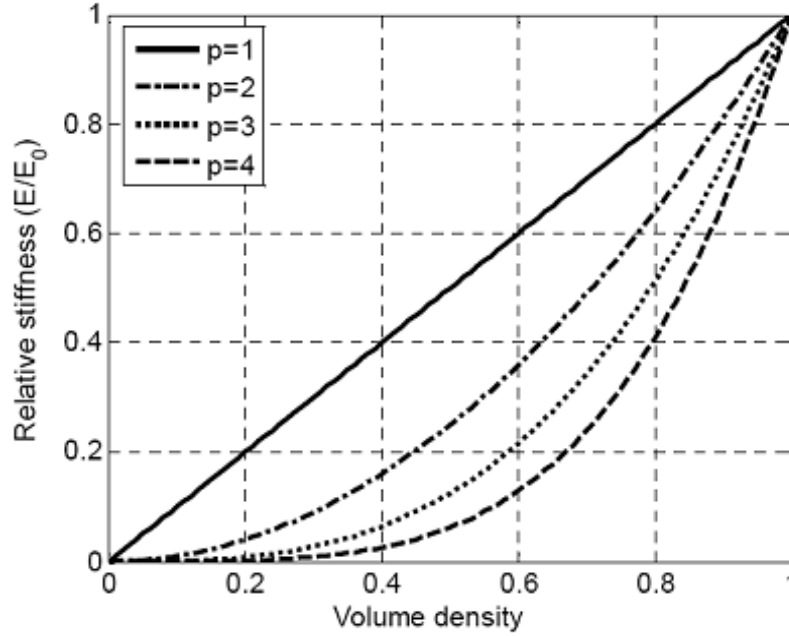


Figure 11 The Penalty Function in SIMP Model

3.4 Sensitivity Analysis

A main part of the topology optimization process involves the increasing or decreasing the material density of each element, and sensitivity analysis is the key process for that. The sensitivity analysis is the process of calculating the gradient of the objective function with respect to each element. The results of the sensitivity analysis to help determine whether and element should have an increase or decrease in material. To do this in mathematics, the rate of increase is represented by calculating the gradient of the objective function.

$$\frac{d\Pi_i}{dz_i} = \frac{\partial \Pi_i}{\partial y_i} \frac{dy_i}{dz_i} \quad (3)$$

Using equation 3 and substituting in the relation $dy_i=dx_i$ and $dz_i=v_idy_i$ the equation is converted into equation 4

$$\frac{d\Pi_i}{dz_i} = \frac{\partial \Pi_i}{\partial x_i} \frac{dy_i}{dz_i} = \frac{\partial \Pi_i}{\partial y_i} \frac{1}{v} = \frac{\partial}{\partial x_i} \left(\frac{1}{2} U^T K U \right) \frac{1}{v} \quad (4)$$

Understanding that the stiffness matrix K and displacement vector V are both dependent on the design variable, the chain rule can be applied to equation 3 which then extends as:

$$\frac{\partial}{\partial x_i} \left(\frac{1}{2} U^T K U \right) \frac{1}{v} = \frac{1}{2v} \left(2U^T K \frac{\partial U}{\partial x_i} + U^T \frac{\partial K}{\partial x_i} U \right) \quad (5)$$

Deriving $KU=F$, the equilibrium equation, now becomes:

$$\begin{aligned} \frac{\partial K}{\partial x_i} U + K \frac{\partial U}{\partial x_i} &= 0 \\ \frac{\partial K}{\partial x_i} U &= -K \frac{\partial U}{\partial x_i} \end{aligned} \quad (6)$$

Which allows for equation 5 to now be written as:

$$\frac{\partial}{\partial x_i} \left(\frac{1}{2v} U^T K U \right) = -\frac{1}{2v} U^T \frac{\partial K}{\partial x_i} U \quad (7)$$

Equation 7 can be expressed at the element level by using the element stiffness matrix, k_e , which assembles the global stiffness matrix, and that each element stiffness matrix is a function of the material density of each element.

$$\frac{\partial}{\partial x_i} \left(\frac{1}{2v} U^T K U \right) = -\frac{1}{2v} u^T \frac{\partial k_e}{\partial x_i} u \quad (8)$$

u is the element displacement vector.

Now assuming that the material stiffness is a function of the design variable, the element stiffness matrix can be defined as:

$$k_e^i = f(x_i) k_e^0 \quad (9)$$

k_e^0 is the stiffness matrix with the full material. Taking equation 9, the derivative can be written as:

$$\frac{dk_e^i}{dx_i} = \frac{\partial(f(x_i)k_e^0)}{\partial x_i} = \frac{f'(x_i)}{f(x_i)} f(x_i) k_e^0 = \frac{f'(x_i)}{f(x_i)} k_e \quad (10)$$

Now taking the derivative of the element stiffness and plugging it back into equation 8, the sensitivity of the total strain energy with respect to the design variable is now derived as follows:

$$\frac{\partial}{\partial x_i} \left(\frac{1}{2} U^T K U \right) = - \frac{f'(x_i)}{2vf(x_i)} u_e^T k_e u_e \quad (11)$$

Equation 11 allows for the calculation of the gradient of each element which will be used to base the updated material density on. [37]

Equation 11 allows for the calculation of each element's sensitivity with simple vector and matrix operation. The calculation allows for the determination of importance of each element's sensitivity in relation to each other. Sensitivity analysis allows for material redistribution from less sensitive elements to more sensitive elements.

3.5 Applications

Topology optimization has a history of a wide variety of applications ranging from structural design, complaint mechanism, energy absorbing, heat conduction, micro-electromechanical systems, and material design.

Structural design is the process of understanding the stability, strength, and rigidity of structures. [39] Topology optimization used for structural design is used to aid in optimizing the material within the design domain. Optimizing the material can remove volume that is unnecessary for support. [40] Structural design topology optimization examples can be seen in Figure 12. In the figure, the left image is a representation of a 2D cantilever beam example. A cantilever beam

is a beam that is fixed, or supported, on one end and free at the other. [41] The picture is split into three parts: a, b, and c. For the top picture, labeled a, is the design domain where the beam is fixed on its left side while a force is applied downward to the right side. The middle picture, labeled b, is the deterministic topology optimization and the bottom picture c, is the corresponding reliability-based topology [42]. The image on the right is the topology optimization results of a high-rise building. This example shows the load being applied to one side, both sides, and then concentrated applied force.

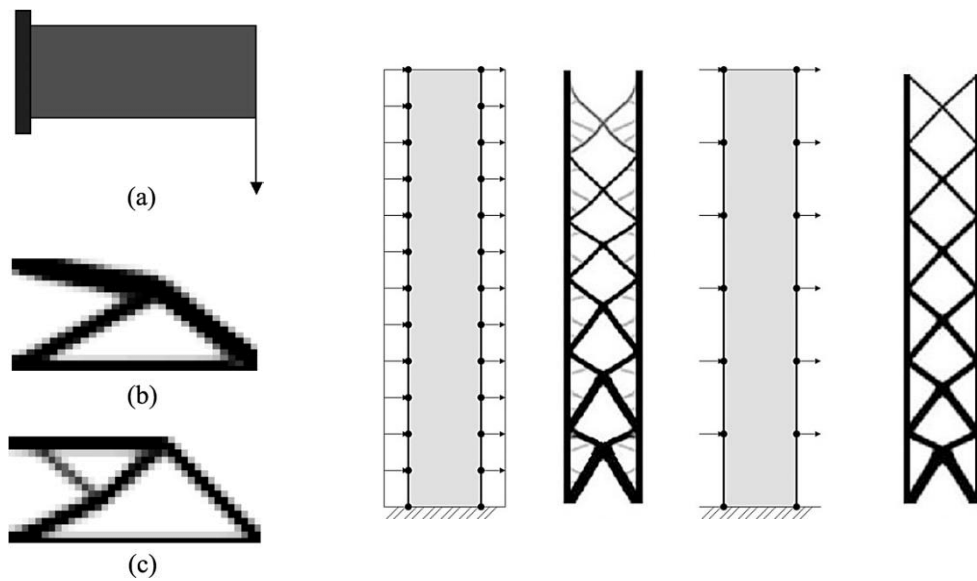


Figure 12 Topology Optimization Structural Design [42] [43]

The next market used for topology optimization is in compliant mechanisms. Compliant mechanisms are a mechanical engineering mechanism that uses the deformation of its body to achieve force and motion transmission [44]. Compliant mechanisms can decrease the overall part count required for a specific task. With the decrease in parts, the mechanisms can become more predictable and easier to manufacture. Due to the deformation of the mechanisms to perform a

specific action, topology optimization is used to find the optimal material layout. Compliant mechanisms can be seen in Figure 13.



Figure 13 Topology Optimization Compliant Mechanisms [45]

Energy absorption is defined as the surface below the load-displacement curve [47]. A load-displacement curve is a curve that measures the properties of an object for which can sustain a certain load without breaking. Energy absorption structures are structures that are designed to absorb and withstand the energy of an impact. Car crashworthiness is an example of an energy absorption structure. Having optimal design that can absorb the energy of an impact can protect the product that the structure is a part of. In Figure 14 the topology optimization of a car's chassis can be illustrated.

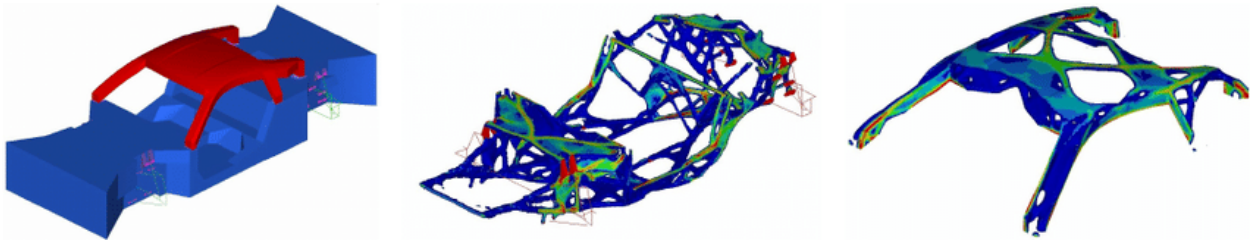


Figure 14 Energy absorption topology optimization - car crashworthiness [46]

Heat Conduction topology optimization is used to optimize the thermal loading, or the maximization of temperature diffusivity in the structure, to reduce operating temperatures to increase durability [48]. Heat conduction topology optimization is used to help electronic equipment from controlling high heat temperatures to allow for optimum performance [49]. An example of topology optimization heat conduction can be seen in Figure 15. This image shows the design space with an initial point selected for the heat source. The image on the right is the heat spread which can be used to aid in controlling the temperature of a product. The spread of heat looks similar to that of plant roots.

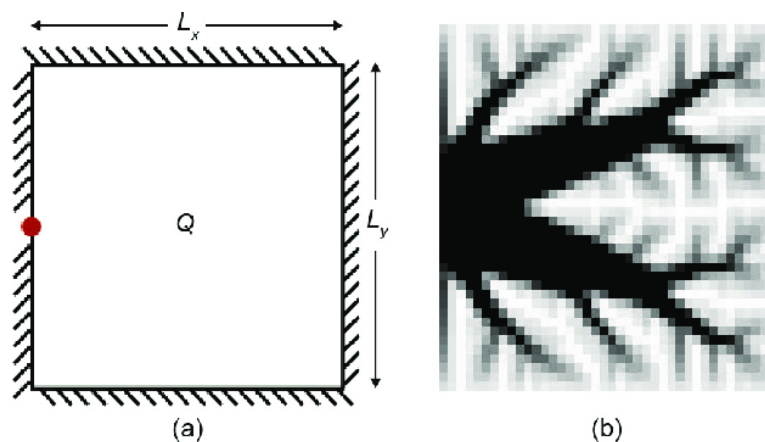


Figure 15 Heat conduction topology optimization [50]

Micro-electromechanical systems, or MEMS, are a miniature machine with both electrical and mechanical components. MEMS are incorporated in a multitude of applications from accelerometers to inkjet printer heads. Topology optimization is used to optimize the material layout of these miniature machines. In Figure 16 an example of MEMS topology optimization results on an actuator.

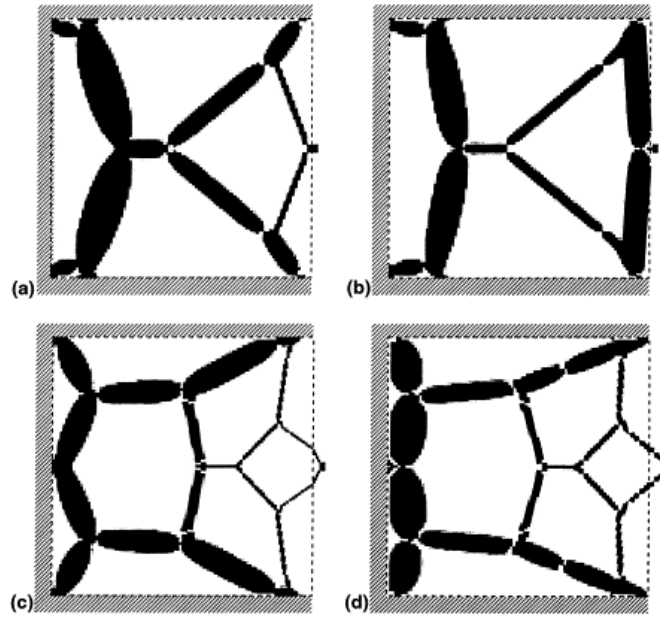


Figure 16 MEMS topology optimization example [51]

CHAPTER 4 – METHODOLOGY

The proposed method to optimize packaging has six different processes. The design space will be defined with a shell shape model to perform the topology optimization. The results of the topology optimization will be overlayed, or superimposed, onto the original shell model. The superimposed method will reveal the critical areas needed for support. These critical areas, with some post processing to simplify the results, will be used to create new leg supports for the model. The new model will be compared to the original package design model through Finite Elemental Analysis (FEA).

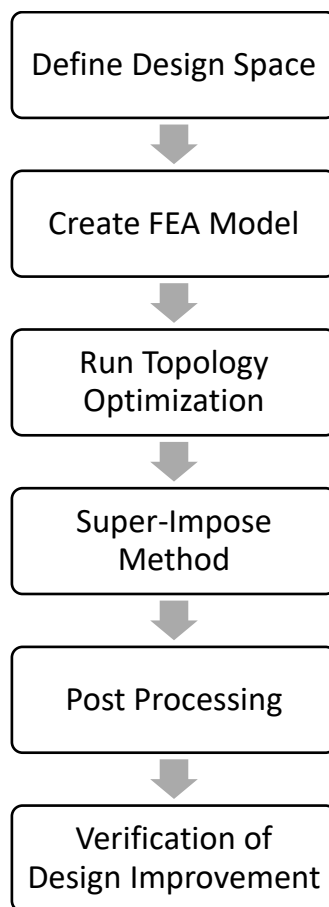


Figure 17 Methodology

4.1 Define Design Space

The design space is the 3D geometrical area that the structural optimization will be performed. Defining this design space is the first step of the proposed method. The issue with applying topology optimization to packaging is caused by the limited design space in packaging. This limited design space can be assisted by creating a shell model once the design space is defined.

For MPP, the design space will be defined as the contact area of the product and package that needs structural support. With the design space determined, a shell model will be created of the MPP. The shell model is used to simplify the MPP to increase the calculation efficiency of the topology optimization process. This simplicity also helps retain the design objective of the original MPP design.

4.2 Create FEA Model

With the design space defined, a FEA model will be created. The FEA model consists of the CAD model, the mesh, and the applied boundary conditions. During the defining of the design space, the CAD model will be created. The mesh and the boundary conditions are then applied onto the CAD model. The mesh, when applied to the CAD model, breaks the model into smaller elements, or nodes. Controlling the mesh size will help control the efficiency and accuracy. Boundary conditions are designed to simulate the actual environment in the computer simulation.

Two boundary conditions are used in the FEA: the geometry constraint, and the loading constraint. The geometry constraint fixes certain aspects of the CAD model in place to not be affected by the simulation. The loading constraint is the addition of an external load that represents the weight of the products.

For this methodology, the geometry constrain was set up to fix the corners of the CAD model. The corners were fixed to maintain the overall dimensions of the shell model to obtain an accurate result. The loading condition was applied in two cases. The first case had the force applied to the center of mass of the CAD model, and the second case had the force applied to the center of faces of the neck and body of the model. The force was applied to the center of faces to mimic where the bottle would rest inside the package. Mimicking where the force of the product would be felt by the package would give an accurate representation of the critical areas needed for support. The center of mass location was chosen to centralize the force to the center of the package. Centralizing the force into the center of the package will show results of an ideal world with the force felt by a package.

4.3 Run Topology Optimization

With the completed FEA modeling, the topology optimization method can begin. For the objective function, minimizing strain energy will be used. To determine the upper bound of material usage, a volume constrain will be used. The volume constrain that is used is first set to 25% of the overall volume of the model. The topology optimization process was run four times starting with that first setting of 25% reduction and then was reduced by 5% each time. The new volume constrains had a topology optimization performed at 20%, 15% and 10%. The reasoning for performing four topology optimizations at different volume constrain values was to determine if there were any trends or major differences at different volume reductions. The topology optimization method is processed iteratively until the convergence of the objective function. The topology optimization will end once the objective function converges.

4.4 Superimpose Method

By themselves, the results of the 90% reduction in volume topology optimization method do not assist much in designing packaging. The results of the topology optimization method can reveal the critical areas needed for support once they are overlayed or superimposed onto the original shell model. For both cases, the results of the topology optimization were superimposed onto the shell model which revealed the critical areas needed for structural support. With the critical areas needed for support highlighted, to design and create these support structures, post processing will be used.

4.5 Post Processing

While the topology optimization method outputs optimized results, the results are not optimal for manufacturing and practical package use. To make the results more realistic for the real world, post processing should be applied. Post processing is the process of editing, or fine tuning, the results to match real world applications.

For the results of the topology optimization performed on the MPP shell model, post processing will be used to not only smooth out the results, but also to build a new support structure. Smoothing out the results will help highlight the critical areas needed for support. Using these areas as guidelines, new support structures are created for the MPP shell model. These support structures are created during the post processing step.

For the two cases used in the topology optimization, center of faces and center of mass, two super-impose and post processing steps will be performed. Through these processes, two new models will be created: the Center of Faces model and the Center of Mass model. Both

models have new support structures in the shape of “U” bridges as opposed to the original model’s standard rectangular support structures.

4.6 Verification of Design Improvement

After the completion of the topology optimization and post processing processes, the new design will be verified using FEA and compared to the original design for benefit. Both models are analyzed using the same boundary conditions and their von misses stress distributions are compared. Von misses stress values are calculated and used to determine how much a given structure will yield [52].

To set up the FEA of the two new models compared to the original model, similar boundary constrains will be applied to that of the topology optimization. In setting up the geometry constrains; two different cases were used. The first case had the corners fixed to keep the dimensions the same during the FEA. The second case had the bottom of the support structure fixed in space. Fixing the bottom of the support structure simulates the package being shipped on its side. For the force being applied, the force was applied to the faces of where the product would touch the package. The force being applied in this way is to simulate the force felt by having the product in during shipping.

Once the FEA analysis is performed, the results will be analyzed and compared to each other to determine performance. To get the most accurate reading from the results, the Von Mises stress distributions graph will be set to the same scale. With the scale set to the same for all three models, a visual comparison between the three model for stress felt.

CHAPTER 5 – RESULTS

5.1 Single-use Wine Shipper

To demonstrate the topology optimization design process for MPP, a single-use wine shipper was chosen Figure 18. The chosen wine shipper has two symmetrical halves; one half is chosen as the design space. The single-use wine shipper was chosen due to its larger available design space and simple design. To determine the critical areas needed for support, the design space is modeled without any leg supports Figure 19.



Figure 18 Single-use Wine Shipper

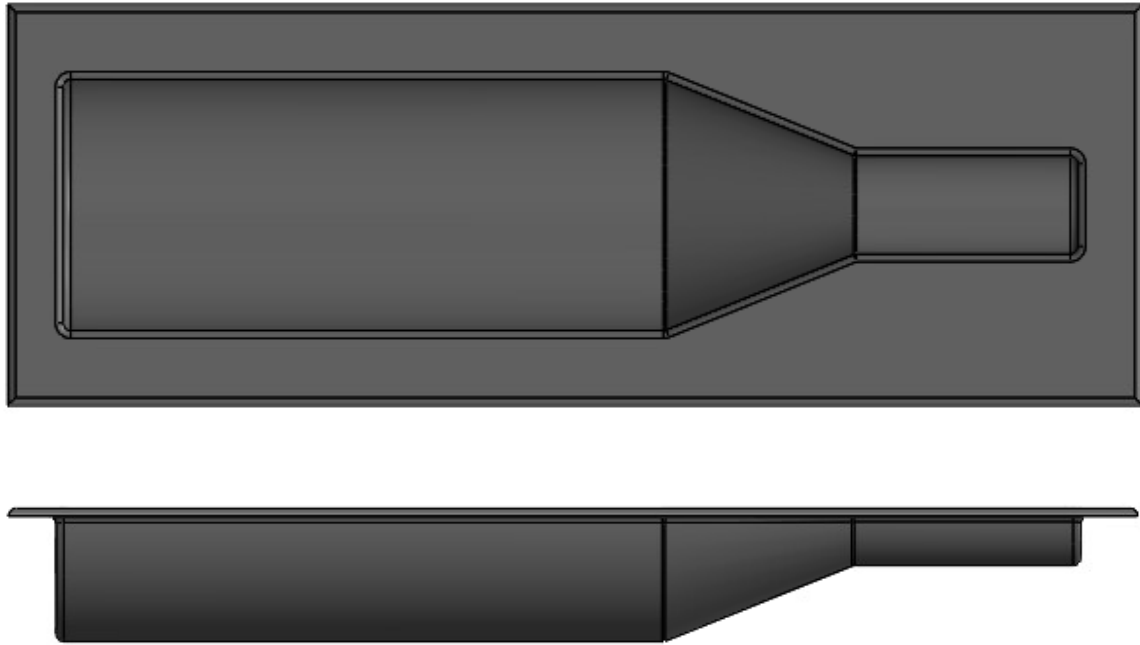


Figure 19 Shell Model: Top view (top), Side View (Bottom)

With the design space defined, the FEA modeling is created. The CAD model is created during the defining of the design space. The mesh and boundary conditions will be applied to the CAD model to create the FEA model. The mesh had an average size of 0.190825 inches with a total mesh number of 17,349. For the boundary conditions, fixed geometry and external force is applied. The corners of the MPP will be fixed. The external force that was applied was applied in two cases. The first case had the force applied to the center of the body of the model, and the second case had the force applied to the center of faces of the neck and body of where the bottle would rest.

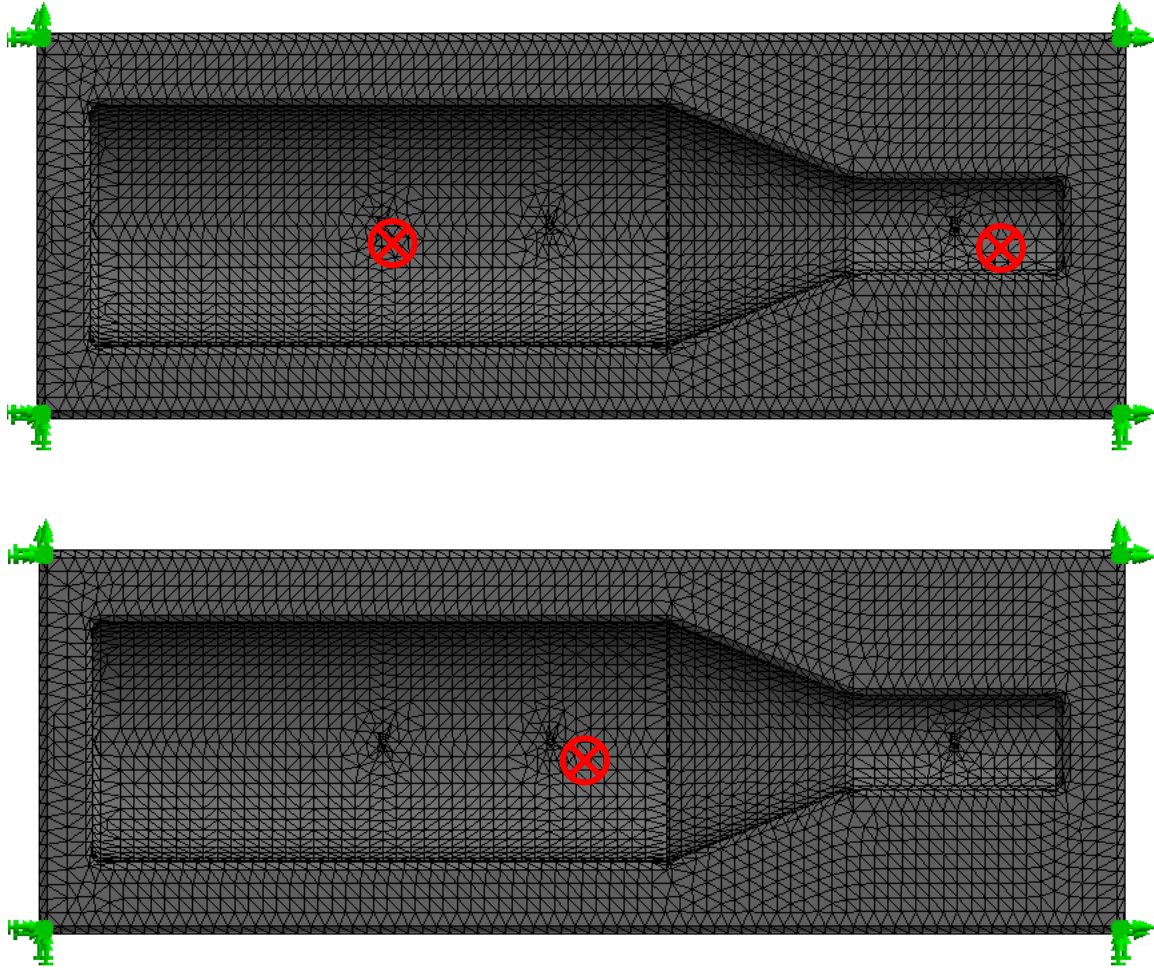


Figure 20 Mesh with Boundary Conditions: Center of Faces (Top) Center of Mass (Bottom)

To guide the topology optimization, the goal for the best stiffness to weight ratio, which is equivalent to minimizing the strain energy under volume constraints, will be applied. For the single-use wine shipper, the upper bound of the volume constraint is set to 25% of the volume at the start and in 5% increments decreased to 10% of the volume. The resulting topology optimization was used to create two new models.

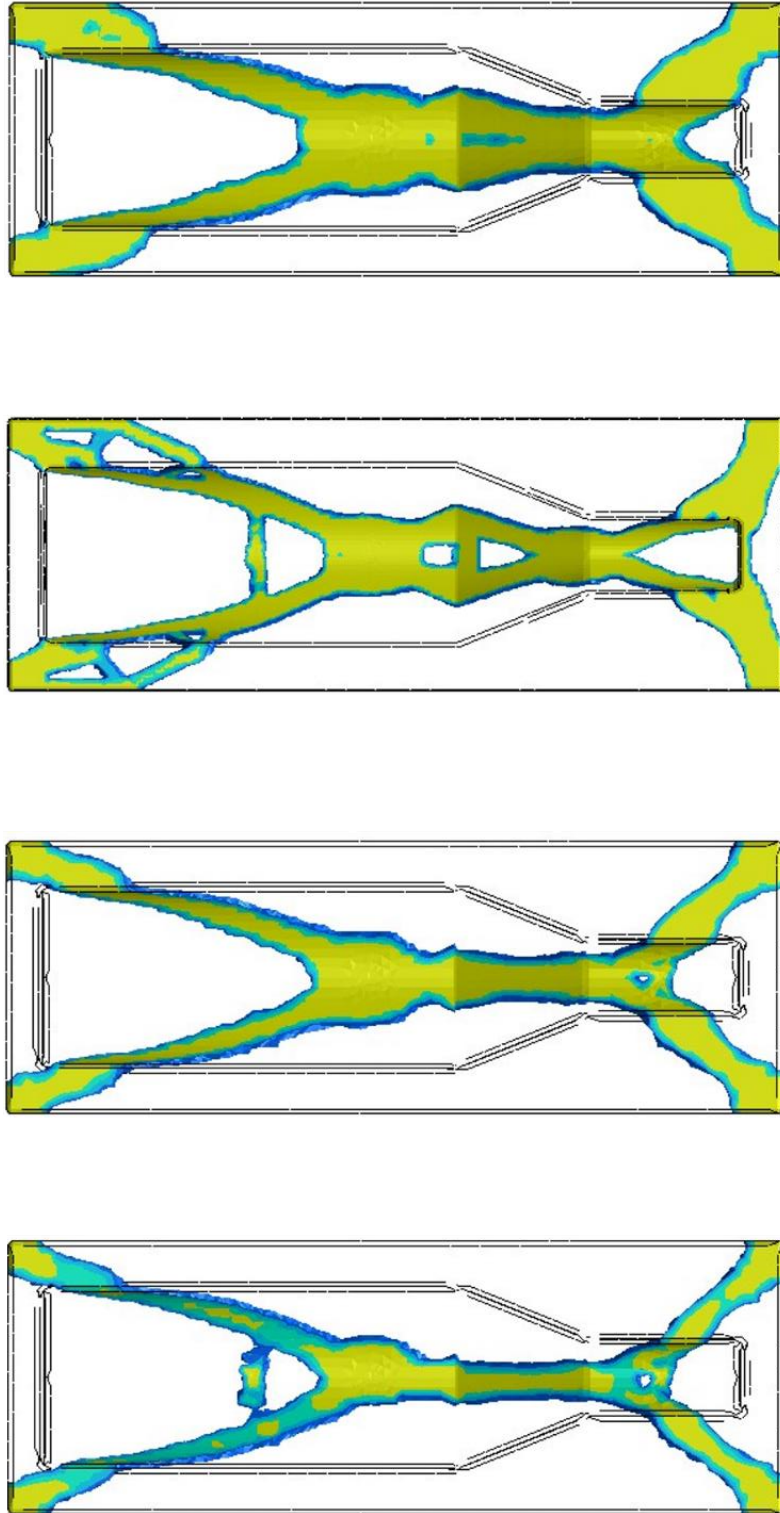


Figure 21 Center of Mass: from top to bottom: 25%, 20%, 15%, and 10% of volume.

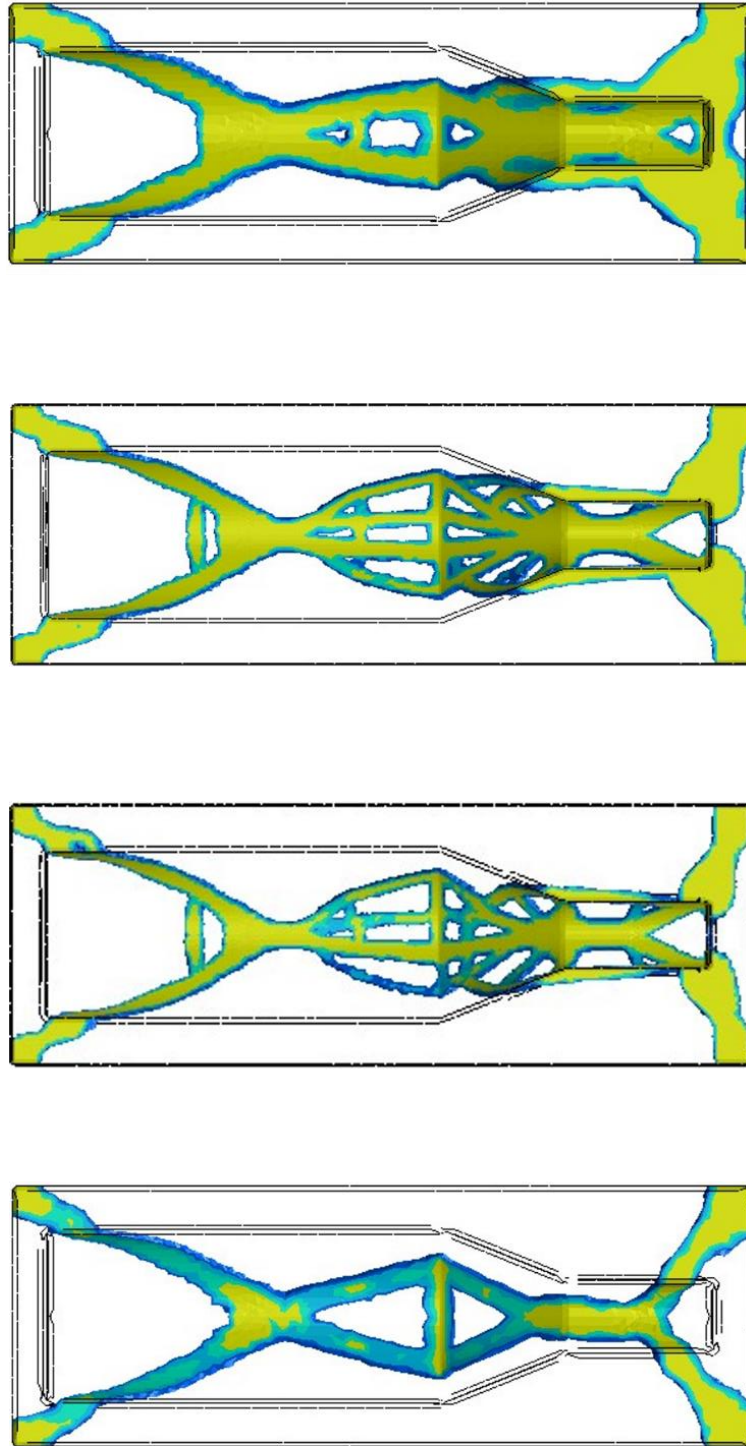


Figure 22 Center of Faces: from top to bottom: 25%, 20%, 15%, and 10% of volume

Four different topology optimizations were performed with different volume reductions for each of the two new models. The difference in volume reductions were used to look for any noticeable trends or patterns in the structural optimization. For both models, the 10% final volume optimization results highlighted the critical areas needed for support better than the other results. The two 10% volume results can be seen in Figure 23.

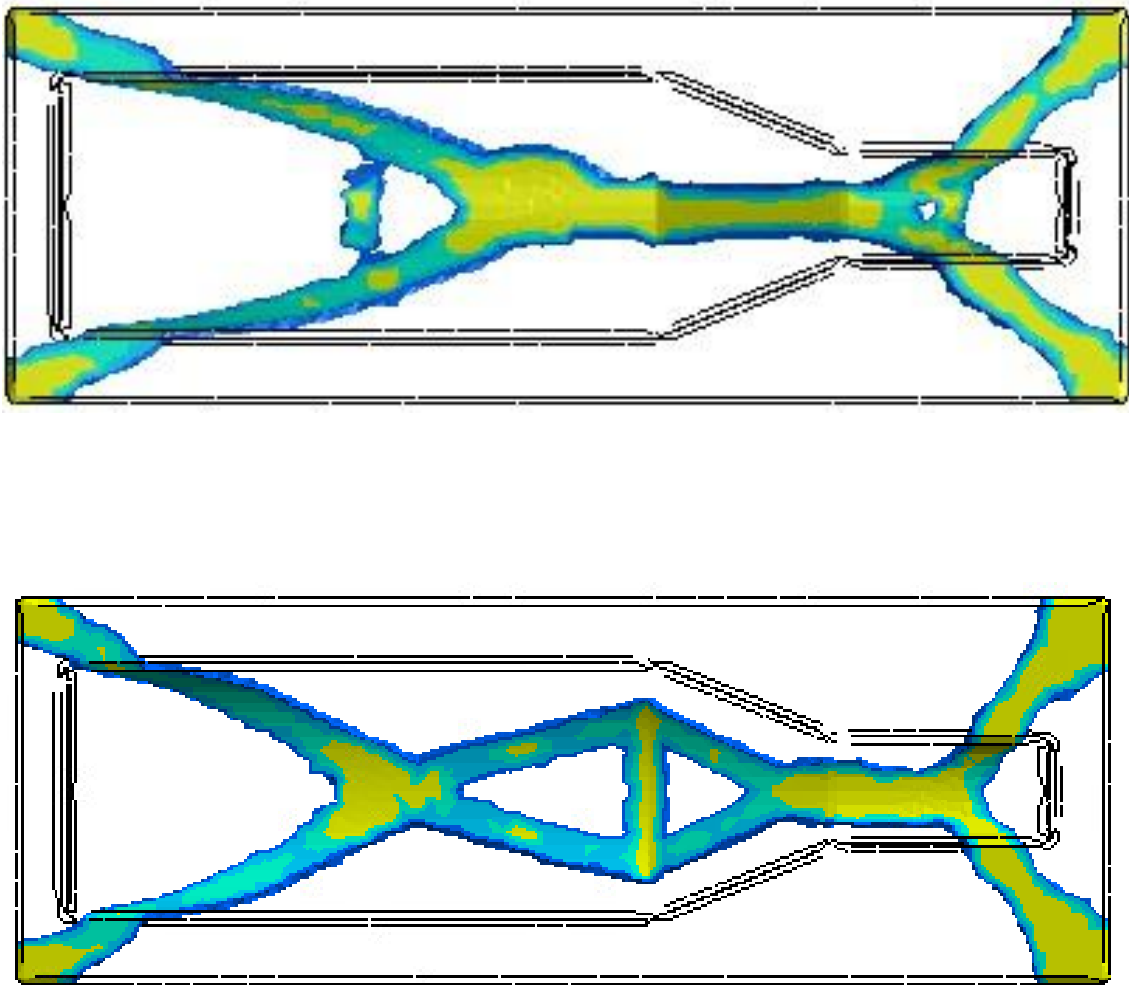


Figure 23 Topology Optimization Results: Center of Mass (Top) Center of Faces (Bottom)

To create the two new models, the results will be superimposed onto the original shell model to reveal the critical areas needed for support. The superimposed method can be seen below.

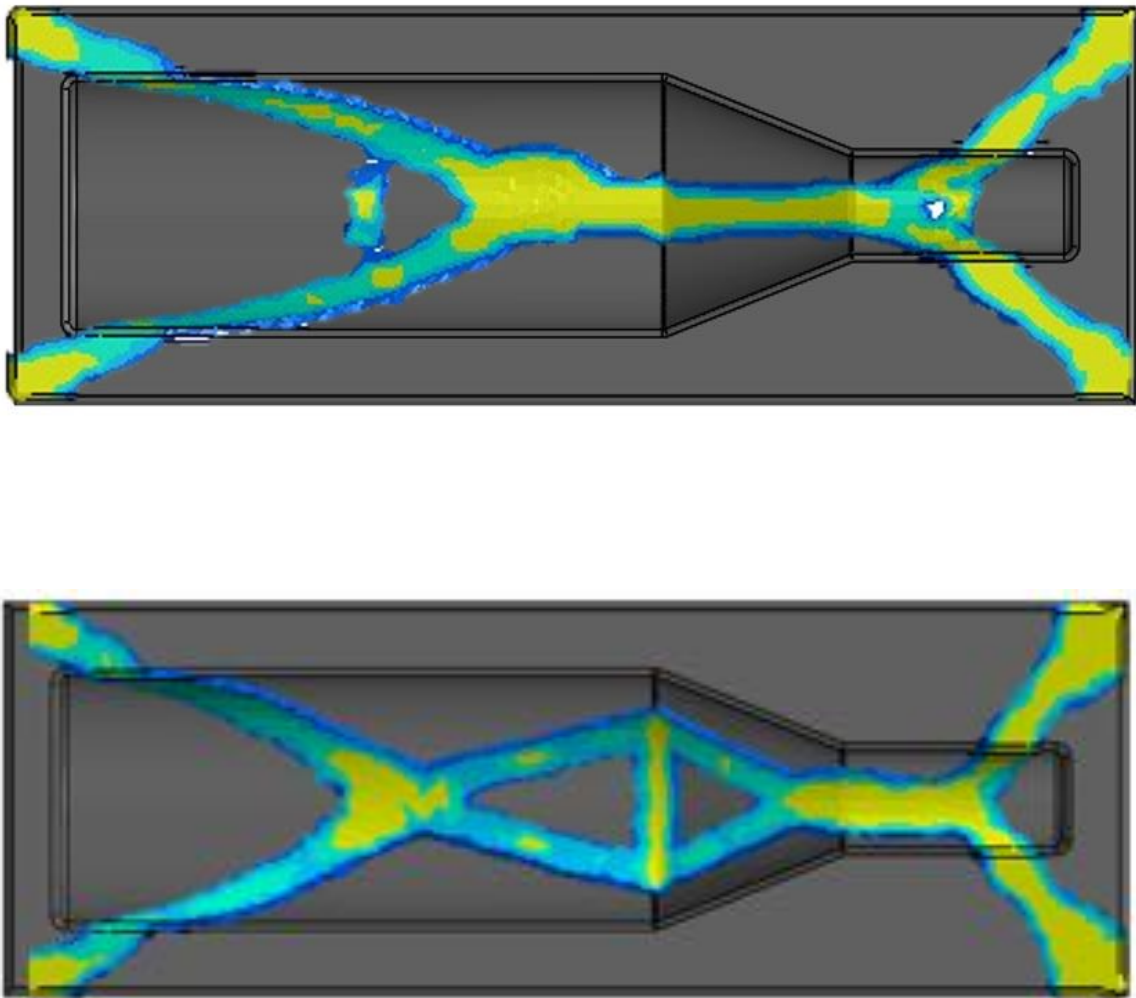


Figure 24 Super-Imposed Method: Center of Mass (Top) Center of Faces (Bottom)

With the topology optimization and the superimposed method, two new models will be created using post processing. For the two cases of the topology optimization results, two distinct critical areas that originated from the corners were formed to reveal two triangular bridge shapes.

The center had a reduced middle leg that was shifted upwards. These new models, named “Center of Faces Model” and “Center of Mass Model” can be seen below in Figure 25 and Figure 26 respectively.

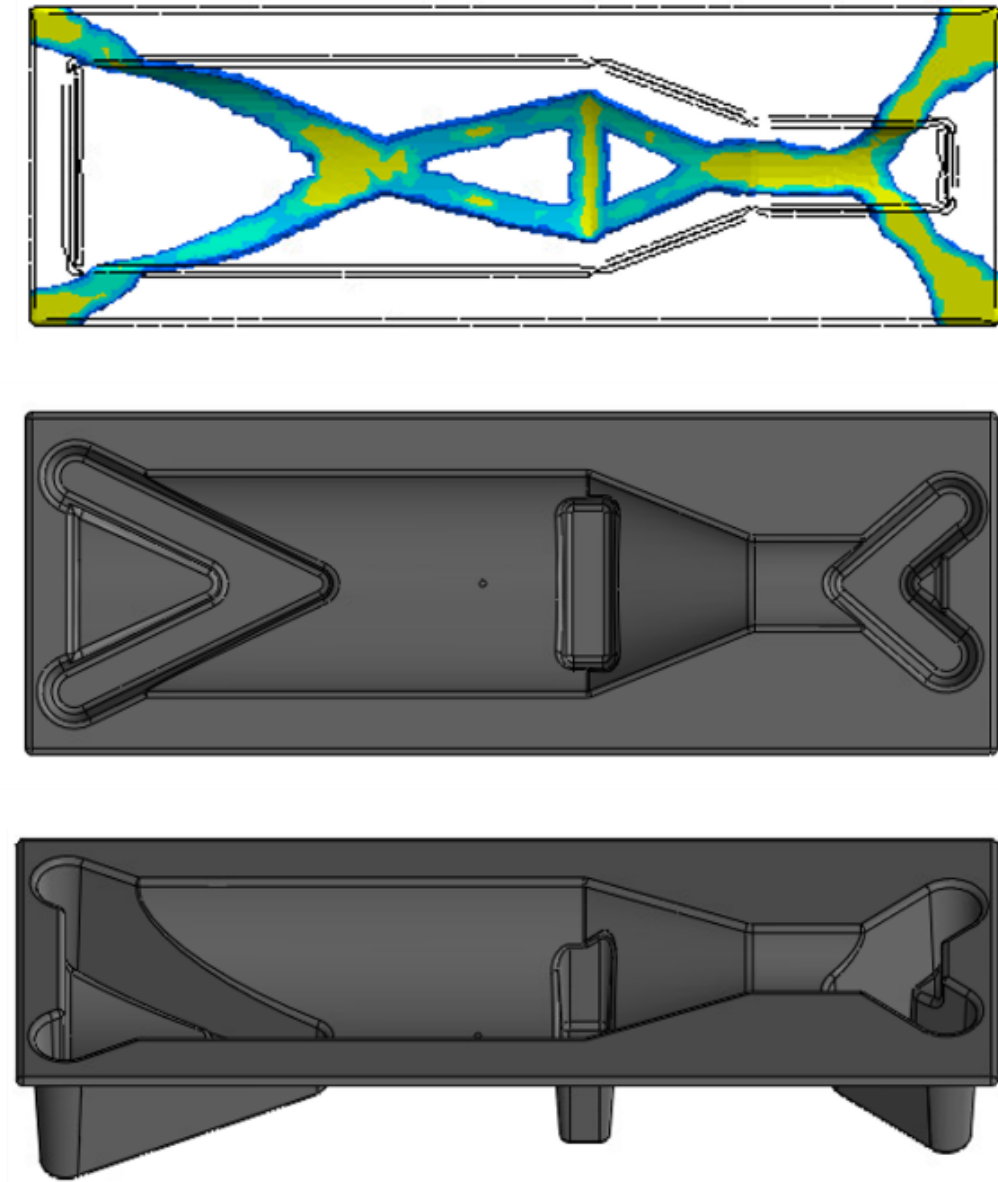


Figure 25 New Model: Center of Top

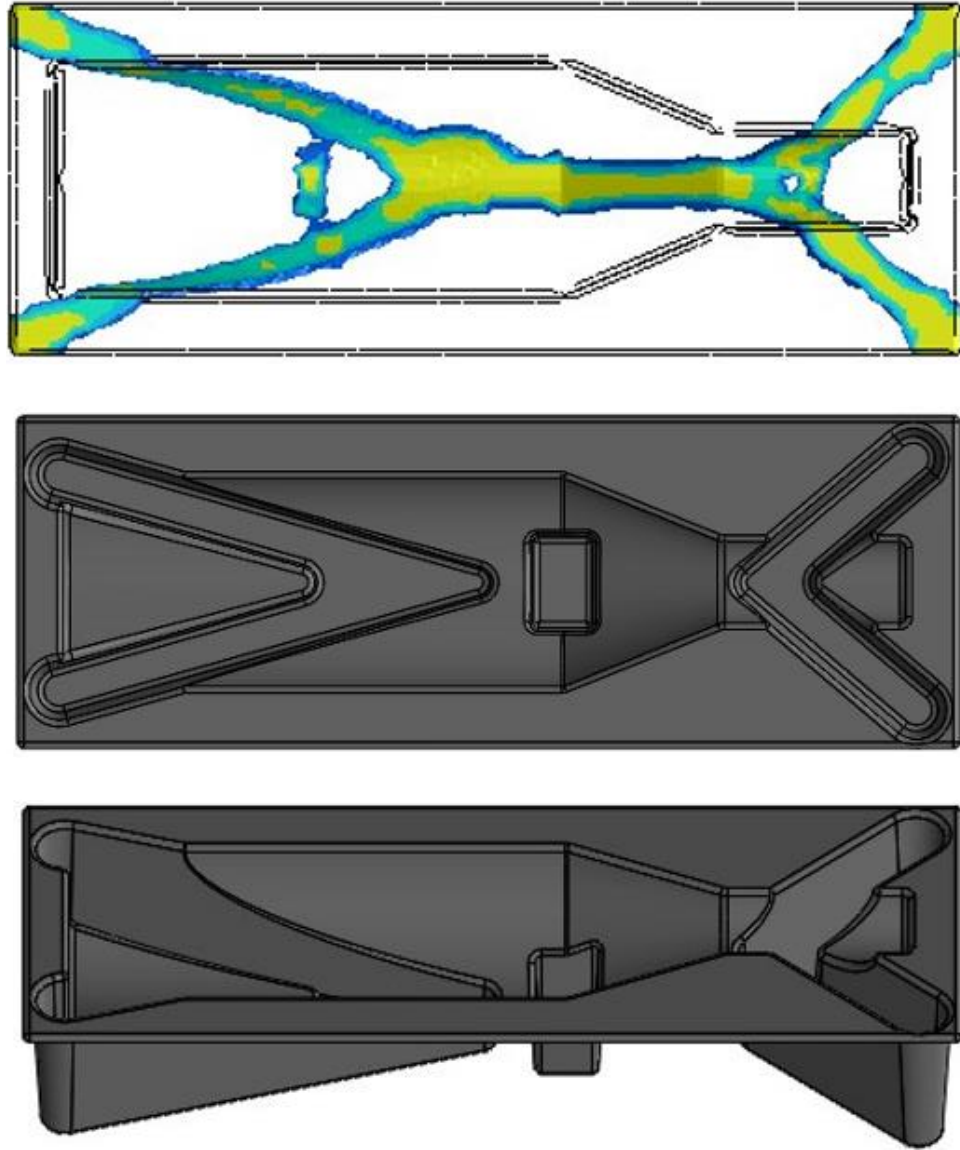


Figure 26 New Model: Center of Mass

Verification of design improvement between the two new models and the original model will be performed through a FEA. The FEA will compare the stress and deformation due to force from where the wine bottle would rest in the MPP. Fixing the corners and looking at the stress concentrations, the original model had the highest stress in the corners with a von Mises value of

1.860e+06 N/m² and the support area near the neck of the MPP. The two new models have similar results to each other. Looking at both two new models, a small concentration of stress can be seen in the corners and a spread-out concentration from the bottom of the “U” bridge located on the neck of the bottle to the top of the body.

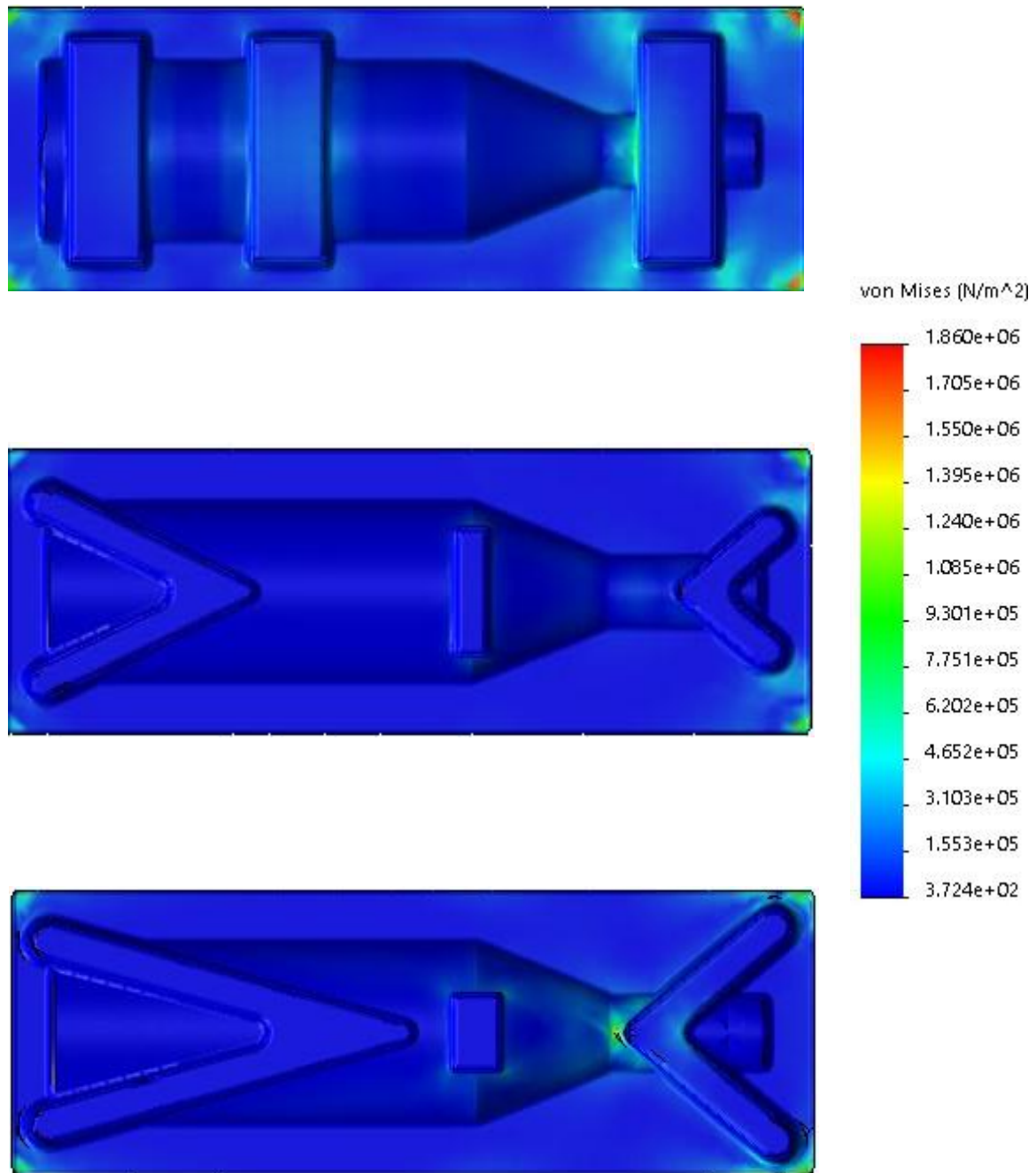


Figure 27 Finite and Elemental Analysis Results: Fixed corners

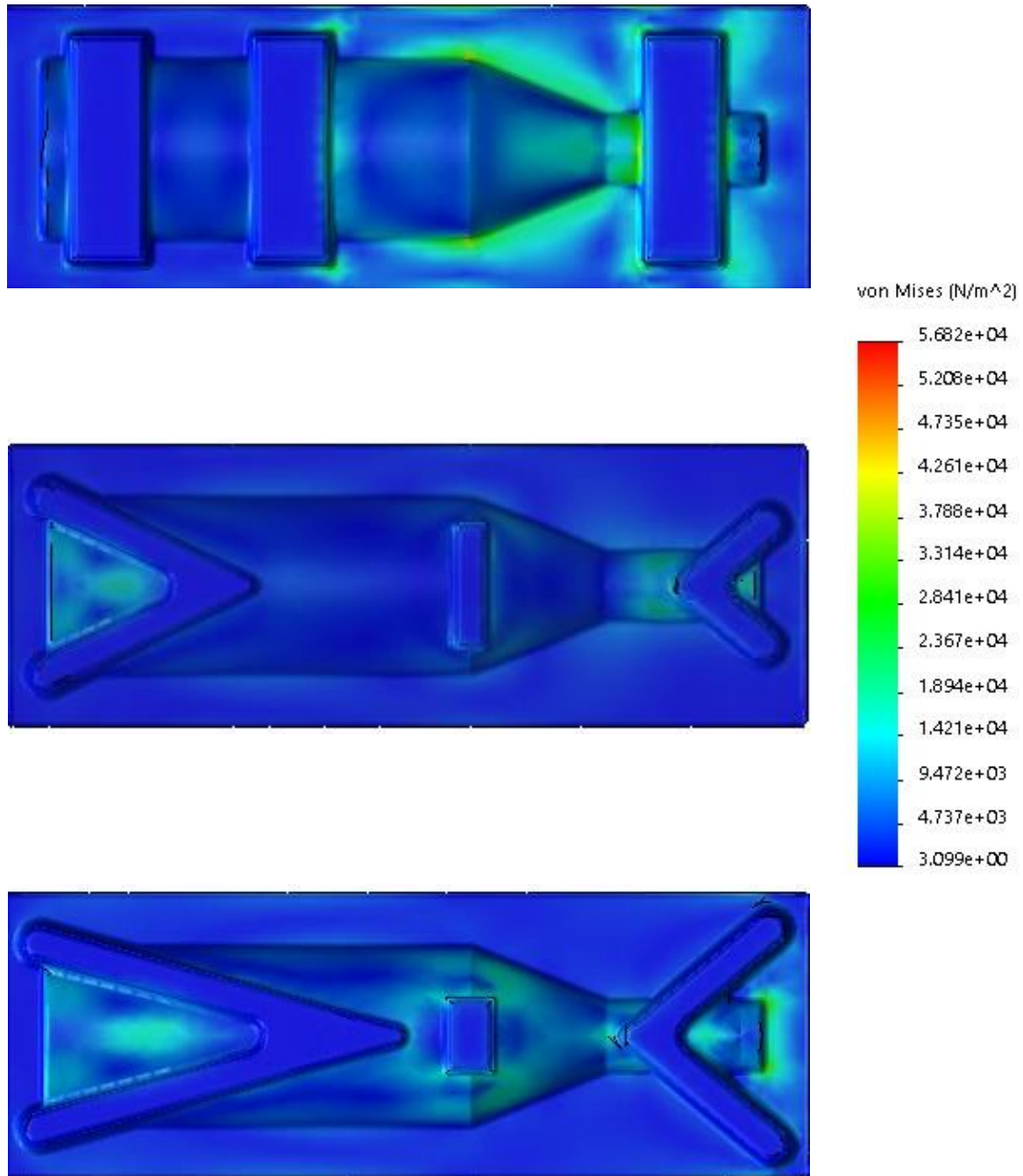


Figure 28 Finite Elemental Analysis Results: Fixed Legs

A second FEA comparison was run with the bottom of the leg supports fixed instead of the corners. For all three models, the results were more comparable; however, the original model has slightly higher concentrations of stress around the neck and top of the model with a von Mises value of $5.682 \times 10^4 \text{ N/m}^2$, while the two new models have slightly higher concentrations on the

bottom in between their triangular leg support. The center of mass model also exhibits stress concentrations around the shortened center support structure.

With the new supporting structures shown to exhibit less stress concentrations than that of the original design the structural layout is optimized. Looking further into the design of the MPP, the thickness can be adjusted using an exhaustive search method optimization. An exhaustive search method optimization is an optimization process that through brute force and based on the set boundary conditions, analyzes every possibility, and outputs an optimized result.

To further optimize the new models with the addition of an exhaustive search method, the CAD model will undergo the process to reduce its overall thickness. With the goal to reducing overall thickness of the model, material will also be reduced. Reducing the material will decrease the overall weight and increase the stress levels of the model. With the goal of the topology optimization to optimize material layout and reduce stress levels, increasing the stress levels through the exhaustive search method allows for the removal of material to further optimize the design.

The exhaustive search method is set up by first defining the goal of the process as minimizing the mass of the model. With the goal defined, the max stress is applied as the constraint. Max stress is applied as the constraint to limit the results to have a max stress that is as close to the original model's stress value without surpassing the original models stress value. For the exhaustive search, the thickness will be the chosen variable. Starting at a thickness of 0.1 inches for each model, the exhaustive search method was performed with a minimum thickness of 0.5 inches to a maximum of 0.1 inches. Each iteration of the exhaustive search method had a thickness increase of 0.005 inches.

The maximum stress results of the exhaustive search method for both models can be seen below in Figure 29. The original models max stress value can be seen as the red line with a value of 1.8 N/mm^2 . Looking at this line, any thickness value that is below this line is a viable result for reducing thickness with the value that is closest to the bottom of the line as the optimized result. First looking at the center of faces model, the optimal thickness given by the exhaustive search method was 0.65 inches for a stress value 1.714 N/mm^2 . The center of faces model had a thickness reduction of 35%. The center of mass model had an optimal thickness of 0.65 inches for a maximum stress value of 1.4781 N/mm^2 . The center of mass model also had a 35% thickness reduction.

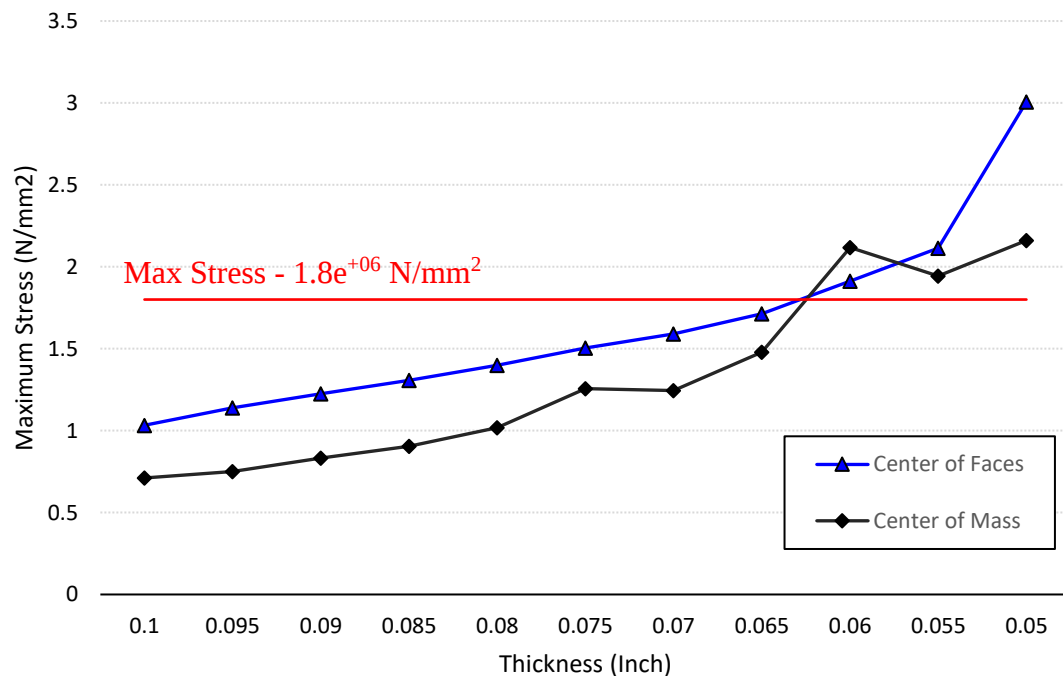


Figure 29 Exhaustive Search Method Results

With their updated reduced thicknesses, a new stress plot is prepared for comparison with the original model. The new stress plot can be seen below in Figure 30. Reducing the thickness of the two optimized models has made the stress plot between all three more comparable. Looking at each individually, the original model still exhibits the highest concentration of stress in the corners and with more stress concentrations around the support structure placed at the top of the MPP. The center of faces model shows its stress concentrations at the top of the model in the corners and at the end of the “U” shaped bridges. For the center of mass model, the stress concentrations are at their highest where the “U” shaped bridge support structure meets the bottom of the neck of the wine bottle cut out.

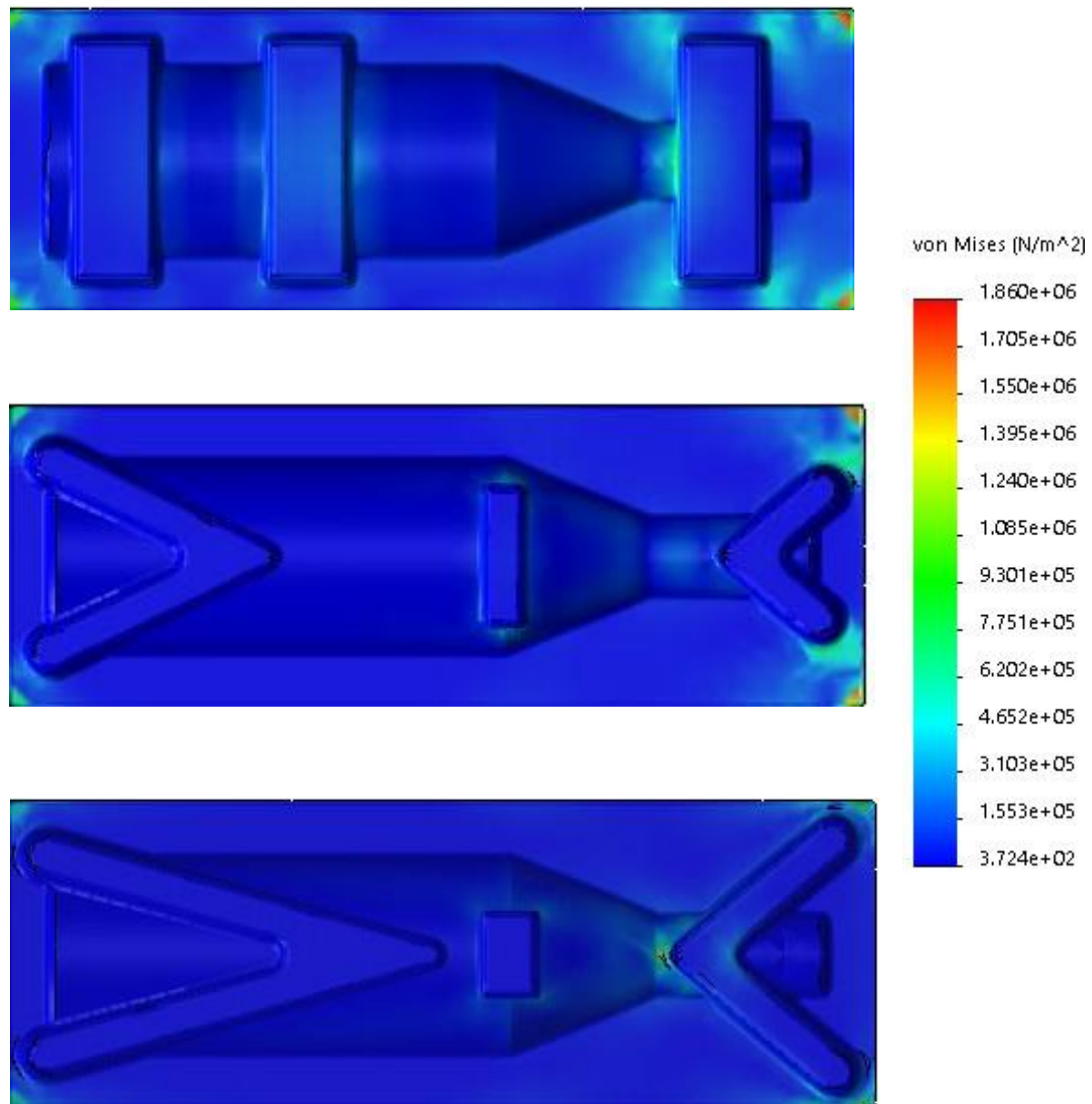


Figure 30 Updated Stress Plot: Original (Top), Center of Faces (Middle), Center of Mass (Bottom)

CHAPTER 6 – CONCLUSION AND FUTURE PLAN

6.1 Conclusion

The MPP single-use wine shipper used today has three rectangular support structures placed from the body to the neck of the where the wine bottle would rest. With the designs of MPP being based on the design engineers experience and knowledge, topology optimization can be employed to create an optimized MPP. Topology optimization results showed the critical areas needed for support once they were super imposed onto the shell model. Using the highlighted critical areas, two new models were created based on the topology optimization results. The three models were visually compared after performing a finite elemental analysis on them. From the inspection, the two newly created models had similar results. Compared to the original model, based on the current in use design, the two new optimized designs showed better resistance to deformation from stress than that of the original. An exhaustive search method can be used to reduce the thickness of the models. Reducing the thickness of the models increases the stress concentrations back towards that of the original model's stress value. The exhaustive search method further optimizes the MPP design by light weighting through thickness reduction.

6.2 Future Work

This research looked at the potential application of the topology optimization method into the design step of MPP. As shown, the results of this new design method have shown that the topology optimized assisted designs feel less stress compared to the original current in use design. This incorporation would generate better designs while shortening the time and cost associated with the conventional design method. Performing a design study on the models to determine the ideal thickness for performance vs weight. Also studying the potential of topology

optimization to be applied to more varied MPP design or different materials will be worth considering.

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