PROBLEMS AND RESULTS IN THE USE OF FARM ACCOUNT RECORDS TO DERIVE COBB-DOUGLAS VALUE PRODUCTIVITY FUNCTIONS

By

LOUIS SCHNEIDER DRAKE

A THESIS

Submitted to the School of Graduate Studies of Michigan State College of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

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(AN ABSTRACT)

The purpose of this study is to test the usefulness of the Cobb-Douglas value productivity function for estimating the gross income of a farm by considering the inputs or factors used. This function permits estimating of elasticities of gross income with respect to factors and the marginal value productivities of factors. The hypothesis is set up that experimentation with the function will show that the latter estimates will be useful to farmers. The hypothesis is advanced that differences between recorded gross income and gross income estimated from Cobb-Douglas equations can be accounted for by variations between farms in prices received, yields and production rates, choice of enterprises, and size of business. Special attention is given to procedures which will make statistically determined value productivity equations of practical value to farmers.

The data are 194 farm account records for type-of-farming areas 5 and 6, Michigan, 1950. The farms are classified into two groups, dairy and other than dairy. This is done in order that the statistical techniques may be tried on a group of homogeneous farms (dairy), as well as on the more heterogeneous group of all farms. The statistical method consists of converting the farm account data into logarithms and solving for equations estimating gross income by least squares.

Use of the Cobb-Douglas function to estimate gross income and marginal returns to inputs rests on three primary assumptions. These are that the relationship between gross income and any particular input is linear in the logarithms, that gross income is a function of inputs, and that different farms are essentially trials with varying combinations of factors, all farms being on substantially the same value productivity function. These assumptions are considered.

It is shown that the Cobb-Douglas function gives a good estimate of gross income. The estimates of marginal value productivities of factors turn out to be about as should be expected _a priori_. For different equations estimating gross income the marginal value productivity of investment in land varies between 0.034 and 0.096 in its estimated value. The estimate of the marginal return to investment in machinery, including obsolescence and depreciation as well as interest, ranges from 0.23 to 0.35. These estimates and similar ones refer to the return to a marginal dollar of investment or charge.

The results of the work suggest that farm business analysis reports may profitably include the following information:

1. Estimates of gross income, so that a farmer would have a notion of what he should receive, considering his inputs in relation to the inputs of other farmers.
2. Estimates of average marginal value productivities of factors for all farms so that there should be an additional basis for recommendations for future expenditures on the average farm.
3. Estimates of marginal value productivities on individual farms to help farmers plan for the future.

4. Explanations of differences in gross income and net income from their estimated values according to the effects of yields, prices, choice of enterprises, and size of business.

In this study the estimates of gross income and marginal value productivities of inputs are based on all farms, as are the estimates of the effects of differences in yields and prices. The standard of comparison need not necessarily be the "average" farm but may be a group of high-profit farms.

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The author acknowledges his debt to Professor E. E. Peterson, in whose course in agricultural production economics he came on the application of the exponential gross income function to farming. Professor Peterson helped him set out the idea of the work in definite form, and to coordinate the method with the available data.

Teachers other than those directly concerned with the dissertation have helped the author to form a philosophy of an economic system and to realize the possibilities of service through study of the economics of agriculture. These professors are D. C. Cline, R. W. Lindholm, Leonard Rall, M. E. Cravens, and Raleigh Barlowe.

Thanks are due the farm account clerks for their willing aid with the computations, typing, and diagrams.

By his understanding cooperation in granting the author an extended leave of absence, Dean Fay L. Partlo of the Michigan College of Mining and Technology helped to make the work possible.

It is impossible to give specific credit--or enough--to the anonymous workers who, over the years, have developed the Michigan farm account system, compiled the data, and thus provided the foundation on which this work could be built.

VITA

Louis Schneider Drake was born in Leelenau County, Michigan, on June 11, 1912. He attended public schools in Traverse City, Bellaire, and Mancelona, Michigan, and graduated from the Mancelona High School in 1929. At Michigan State College he specialized in agricultural economics and farm management, and obtained the degree of Bachelor of Science in 1934 from that institution. He obtained a graduate assistantship in farm mangement at Cornell University and pursued graduate studies from 1934 to 1937. At that time he accepted a position as Land Use Specialist for the Resettlement Administration, and was later transferred to the Bureau of Agricultural Economics as Acting B. A. E. Representative for New York State. He received the degree of Master of Science from Cornell University in 1939. He was with the federal government as Assistant B. A. E. Representative for Michigan and B. A. E. Representative for Connecticut until 1942. He had meanwhile, from 1938 until 1942 been a partner in a commercial potato farm in Houghton County, Michigan, and continued in the farming business until 1948. In 1946 he accepted a position with the Michigan College of Mining and Technology in the department of engineering administration, where he was successively an instructor, an assistant professor, and an associate professor of economics, which position he now holds.

INTRODUCTION

A Brief of the Purpose, Materials, Methods and Results of the Dissertation

I. Purpose of the study and definition of terms: the purpose of this study is to show how certain types of formulas can aid farmers in determining

A. How their gross incomes compare with those of other farms when investments and expenses are considered; (estimate of gross income);

B. What increase in gross income farmers should expect if they increase any single kind of outlay by a given proportion (elasticities of gross income with respect to factors of production);

C. What the additional returns for additional outlays are (estimates of marginal returns to factors);

D. Why net income usually varies from its expected amount (effects on net income of yields, production rates, prices, and size of business).

II. Materials: Two types of materials are used in this study:

A. The analytic tools: the Cobb-Douglas gross income estimating equations--$P = Cx^{a} y^{b} \ldots z^{k}$. (Chapter I)

B. The data: observations of 194 farms in type-of-farming areas 5 and 6, Michigan, 1950 (Chapter II)

III. Methods (Chapters III and IV)

A. Classificatory methods

1. Inputs: factors of the farm business are classified into different categories with different degrees of refinement. Thus more insight is gained into the componental structure of gross income;

2. Farms: the farms are divided into two groups:

   a. 86 dairy farms

   b. 108 farms other than dairy.

B. Mathematical methods

1. The categories of factors and gross income are taken off the farm account records;

2. These data are converted into logarithms;

3. Regression equations are calculated by the Doolittle method;

4. Standard errors of estimate of single coefficients are computed in some cases;

5. The elasticities of gross income with respect to categories of factors are given by the coefficients of the terms on the right-hand side of each logarithmic equation;

6. Marginal value productivities of categories of factors are determined by taking the partial derivatives of the gross income

equations with respect to the categories of factors. These equations are given in numbers.

IV. Some Applications (Chapter V)

A. Some changes in methods of recording farm account data are suggested with regard to the value of

1. Land

2. Livestock, particularly cows.

B. Suggestions are made concerning the classification of factors of production in farm account records.

C. Farm business analysis reports can profitably include:

1. Estimates of income to show the average relationships of inputs to income on all farms;

2. Estimates of gross income for individual farms to show each farmer whether he is running ahead or behind the average relationships;

3. Statements of average marginal value productivities of classes of inputs so that there can be some indication to farmers in general whether they would be better off to invest more money in particular items, such as machinery rather than labor;

4. The same as 3, only the statements of

average marginal value productivities would indicate to the individual farmer comparative marginal returns on his own farm;

5. Estimates of marginal value productivity of a given single input at different levels, when other inputs are held constant. A farmer thus might have an idea of whether he should put more money into cows, for instance.

D. The effects of the following upon variations in gross income between farms can clearly be seen:

1. Yields,
2. Rates of production,
3. Prices,
4. And size of business.

Thus a specific explanation that gets down to crops, yields, *etc.*, can be given for differences between the net income recorded in the farm accounts and the average net income; or between the recorded net income and the net income of, say, the most successful third of the farms.

E. We can gain more insight into the true values of the various factors of a farmer's business.

F. We can also find out something of the different subjective values farmers put upon the same factors. Even if two farmers could earn identical labor incomes with identical combinations of factors, in practice they might still not earn the same amount because one might be afraid of becoming "land-poor," for instance.

The reader of this study should always bear in mind that the essence of the Cobb-Douglas method is comparative. Its value lies in the fact that a great number of relationships between various parts of the farm can be found out. For example, what bearing will an increased number of cows have on a decreased proportional expenditure for labor per cow?

The possibilities of the usefulness of this method for the analysis of farm accounts are infinite. Only a few of the applications of the method can be given in this study. But it will be a rewarding field for any one interested in pursuing further the delicate cross-influences between what a farmer puts into his farm, and what he gets out of it financially.

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PROBLEMS AND RESULTS IN THE USE OF FARM ACCOUNT RECORDS TO DERIVE COBB-DOUGLAS VALUE PRODUCTIVITY FUNCTIONS

CHAPTER I

THE GROSS INCOME OF A FARM AS A FUNCTION OF CATEGORIES OF INPUTS

1. An Estimate of Gross Income

This study is concerned with estimation of gross income of farms from the values of the categories of factors of production employed. An equation of the type

$$\underline{P} = \underline{C}\underline{x}^{\underline{a}}\underline{y}^{\underline{b}}\ldots\ldots\ldots\ldots\ldots\underline{z}^{\underline{k}}$$

is used to estimate gross income.[1] So used, this equa-

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1. Theoretically, value product functions of individual firms for categories of inputs in the economy, if known, could be used in connection with demand and supply curves of factors and products to solve the economic system. If estimates can be made of the mean values of the coefficients, they may have some social implications. See Joan Robinson, "Euler's Theorem and the Problem of Distribution," Economic Journal, V. 44 (1934) p. 398.
   This equation is sometimes called a "Cobb-Douglas" function when applied to a theoretical statement of the gross income of a single firm. Actually, the function antedates Cobb's and Douglas' work, the function appearing in the work of Wicksell. (See Knut Wicksell, Lectures in Political Economy, V. 1, pp. 121-3, 127-130; also Martin Bronfenbrenner, "Production Functions: Cobb-Douglas, Interfirm, Intrafirm," Econometrica, V. 12 (Jan., 1944) pp. 37,8). Furthermore, Douglas actually sought average functions for numbers of firms by statistical methods. His work was not with individual firms. See P. H. Douglas, The Theory of Wages.

tion is a value productivity function of categories of inputs. Categories of inputs, measured in dollars, are factors determining the gross income, which is likewise measured in dollars. Gross income is given by P; x, y, ...z are the values of different categories of inputs (factors) used on farms. C is a constant term, and a, b,...k are powers to which the respective categories of factors are raised. In words, the equation says that gross income equals some number times the investment in land (for example) raised to some power, times the investments other than in land (again, for example) raised to some power, and so on.

The terms inputs and factors (of production) will be used synonymously. On both sides of the equation the terms are expressed in dollars. This means that different physical products and different kinds of factors have been reduced to their dollar values and combined into dollars' worth of gross income on the left side, and dollars' worth of each category of factors on the right.

When equations are used to express the physical product of an enterprise or of a phase of a total enterprise in terms of physical quantities of inputs, the equation is called a production function.[1] Production then refers

1. See "Input-Output Relationships in Milk Production," U. S. D. A. Technical Bulletin No. 815 (May, 1942); W. J. Spillman, "Use of Exponential Yield Curves in Fertilizer Experiments," U.S.D.A. Technical Bulletin No. 318 (1933); Earl O. Heady and Carl W. Allen, "Returns from Capital Required for Soil Conservation Farming Systems," Iowa Research Bulletin 381 (May, 1951).

directly to "product." Thus, a production function could be derived in which milk could be a product of dairy cattle, hay, and concentrates. Or, the milk given by one cow could be stated as a function of silage, hay, and concentrates.

With a value productivity equation of the form given on page 1, together with data regarding inputs for a particular business, it is theoretically possible to obtain estimates of the following:[1]

1. The gross income

2. Elasticities of gross income with respect to categories of factors.[2]

3. Marginal incomes attributable to categories of factors.

4. Information as to whether the business operates according to increasing, constant, or decreasing returns to scale.

5. The net income, if the rates at which categories of inputs are charged against the business are given.

The exponents of the value productivity functions

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1. See Chapter VI for an interpretation of the meaning of each of the five items listed.

2. See Appendix A, and Gerhard Tintner and O. H. Brownlee, "Production Functions Derived from Farm Records," J. F. E., V. 26 (Aug., 1944) pp. 566-571.

in Chapter IV are the elasticities of gross income with respect to the categories of factors. By this elasticity is meant the ratio of the relative change in gross income to the relative change in a specified category of factors. Continuing with the symbols given on page 1, we find that this elasticity is, by formula:

$$E_x = \frac{\frac{\text{change in } P}{P}}{\frac{\text{change in } x}{x}} = \frac{\Delta P}{P} \cdot \frac{x}{\Delta x} \; ; \; E_y = \frac{\Delta P}{P} \cdot \frac{y}{\Delta y}, \text{ etc.}$$

The notion of elasticity is important in considering returns to scale. A business operates according to increasing returns to scale if, with an equal proportional change in all of the categories of factors, the gross income changes in a greater proportion. Thus, if the prices of the factors do not change as the firm employs more of them, it is apparent that the business must become increasingly profitable as its size is made larger. If returns to scale are constant, gross income changes in the same proportion as the employed factors are changed. If returns to scale are decreasing and the factors are changed in equal proportions, then the gross income will change in a smaller proportion.[1]

The sum of the elasticities of gross income with respect to the categories of factors indicates the nature of returns to scale. If the sum is greater than one, the business is operating under increasing returns

1. Knut Wicksell, op. cit., pp. 127-130.

to scale. If the sum is one exactly, then returns to scale are constant. And if it is less than one, returns to scale are decreasing. The mathematical proof of this proposition is given by Euler's theorem. That the proposition is true can be seen simply by considering first the meaning of elasticity, and then the meaning of the sum of the elasticities.[1] If the factor categories are all increased in the same proportion, then the elasticity of "*x*" category gives the ratio of the relative change in gross income to the relative change in *x*. The elasticity of *y* gives the ratio of the relative change in gross income to the relative change in *y*, and so on. If *x*, *y*, ...*z* are all increased in the same proportion, then the elasticity of gross income with respect to each category gives the proportional change in gross income ascribable to each category. If the increase in all categories of factors is uniformly ¼, for example, then the change in gross income will be ¼ if the sum of the elasticities is exactly 1; gross income will change by more or less than ¼ if the sum of the elasticities is greater than or less than 1, respectively. It should be pointed out here that the use of the sum of the elasticities of gross income with respect to factor categories to answer questions of returns to scale implies that all of the categories are included in the

1. *Ibid.*

value product equation. Obviously, if one of the categories is missing and the elasticities of gross income with respect to the others are given correctly, the sum of the elasticities times the proportional change in the respective categories will give a smaller proportional change in gross income than would be indicated should all of the factor categories be included. However, the degree of a function may be useful even though not the true measurement of "scale," if omitted factors are not easily changed in amount.

Estimates of the marginal income attributable to factors can be determined by differentiating the estimating equation with respect to the factors. By the marginal income of an input category is meant the incremental change in gross income which is associated with an incremental change in the specified group of inputs.

Estimates of net income can be made from the value productivity function provided the rates at which the categories of inputs should be charged are known. Labor income can be given thus:

$$\underline{L} = \underline{C}\underline{x}^{a}\underline{y}^{b}\ldots\ldots\underline{z}^{k} - (\underline{mx} + \underline{ny} + \ldots\underline{uz}).$$

Here labor income is indicated by $\underline{L}$; $\underline{m}$, $\underline{n}$, ...$\underline{u}$ are rates at which the categories of factors are charged in subtracting costs from gross income in the determination of labor income. For example, in the Michigan farm account project, total farm expenses are charged at a rate of 1; all investments are charged at a rate of 5%. The symbols $m, n, \ldots u$ do not refer to quantities of inputs. If the factor categories are given values in $Cx^a y^b \ldots z^k$ which exactly equal their used up costs, then in the net income equation $m, n, \ldots u$ would all equal 1. The cases of interest and depreciation will call for values of $m, n, \ldots u$ different from 1 if the factor categories are entered in $Cx^a y^b$ at their asset values. Then in the case of machinery depreciated at 10%, with a 5% interest charge against investment, the value of m would be 0.15. In the case of land, assuming it not depreciable, the value of n would be 0.05.

The value product function which has been discussed in the previous pages will indicate diminishing returns to a specified category of factors if the elasticity of gross income with respect to the category is less than 1. That is, if the value of a in the equation on page 1 is ½, then gross income will change as the square root of the category of factors called x, if other categories are held constant. If all other categories are held constant and x is increased, thus, the relative change in gross income will be smaller than the relative change in x. It is apparent that in the real economic world the values of the terms $a, b, \ldots k$ must generally be individually less than 1. From what has been said about returns to scale, it would appear that, for a purely competitive business in equilibrium, the sum of these terms must be near 1; it pays the business neither to expand nor to contract.

2. The Cobb-Douglas Statistical Function:[1]

The mathematical function $P = Cx^a y^b$ has been used in efforts to measure average marginal productivities of factors employed for industries within regions and national economies. Professor Paul H. Douglas has been associated with an extensive list of such studies.[2] The Cobb-Douglas production function itself involves two categories of factors of production, labor, and capital. Douglas' estimates of production functions for various industries

1. In some of the literature, the Cobb-Douglas function is taken to mean any value productivity or production function of the form $P = Cx^a y^b$, using the definitions of the symbols given on page 2. Why the names of Cobb and Douglas should refer to all uses of this type of function with regard to factors and product is not altogether clear. In this study the title Cobb-Douglas will be interpreted to mean the estimate of product (specifically, gross income) by multiple regression from categories of factors. See footnote 1, page 1.

2. The following list includes part of the studies of Cobb, Douglas, and associates: P. H. Douglas and C. W. Cobb, "A Theory of Production," A.E.R., V. 18, supplement, (1928), pp. 139-165. C. W. Cobb, "Production in Massachusetts Manufacturing, 1890-1928," J.P.E., V. 38 (1930), pp. 705-7. P. H. Douglas, Patricia Daly, and Ernest Olson, "The Production Function for Manufacturing in the U. S., 1904," J.P.E., V. 51 (Feb., 1943), pp. 61-5. P. H. Douglas and Grace T. Gunn, "Further Measurement of Marginal Productivity," Q.J.E., V. 54 (1940), pp399-428; also "The Production Function for American Manufacturing for 1914," J.P.E V. 30 (Aug., 1942), pp. 595-602.

were based on observations in which product varied according to differences in the way in which labor and capital were combined.[1] The original users of this method of study sought to obtain such observations by considering whole industries over a period of years, in which the records of an industry for each year would show different combinations of factors and product. In another method used by Douglas, groups of firms within an industry were treated as separate observations in the employment of factors over a common time period. One of Professor Douglas' principal aims was to determine whether various industries were competitive in factor markets as well as in the selling markets. Douglas indicated that manufacturing businesses were generally competitive.

In this study it is assumed that farm businesses are highly competitive. The degree of competition between farms is not the main issue here. However, by the use of Douglas' basic statistical method, it is hoped that estimates of the gross income of farms will be useful in the discovery of principles of efficient farm management.

The observations in this study are of the 194 farms in type-of-farming areas 5 and 6, Michigan, 1950. The observations are all for one year. This eliminates the necessity for year-to-year adjustment of factor and pro-

1. Objections to the Cobb-Douglas function which have been raised by other economists are considered later in this chapter, beginning on page 10.

duct values according to changes in prices. From these observations, value productivity functions will be estimated.

Exponential value productivity functions of the type given on page 1 have been used by Professor Heady of Iowa and others in the study of farm account records.[1] In this study we shall likewise try to determine the expected average values of the exponents and the constant terms of the value productivity functions which face individual farms. To attempt this by statistical methods implies the assumption that the farms in the survey all operate on essentially the same value productivity function. The farms represent different positions on the function according to the combinations in which farm operators employ categories of factors. The fact that the positions are different and that the gross incomes are different when they are buying and selling in essentially the same markets from one farm to another creates a problem of choice of factors.

Farm operators choose different combinations of factors in their individual efforts to maximize net income according to (1) their particular circumstances (with respect to the availability of family labor, for

1. Gerhard Tintner, "A Note on the Derivation of Production Functions from Farm Accounts," Econometrica, V. 12 (Jan., 1944), pp. 26-34. Gerhard Tintner and O. H. Brownlee, "Production Functions Derived from Farm Records," J. F. E., V. 26 (Aug., 1944), pp. 566-71. Earl O. Heady, "Production Functions from a Random Sample of Farms," J. F. E., V. 28 (1946), pp. 989-1004.

example), (2) their different concepts of net income,[1] and (3) their individual appraisals of the relation of gross income to different factors.[2] Thus, granting that all farms operate on essentially the same value productivity function, farm operators for several reasons will choose different combinations of factors.[3] When the factors are grouped into categories, the average relation between the categories of factors and gross income as stated by account records can be estimated by multiple regression. The farmer will find this average relation informative when he is faced with the problem of choosing factors. If there were one choice of factors which maximized net income according to a common definition held by all farmers, and if all farmers were completely free to choose this combination, then all choices of factors would be the same, and all gross and net incomes would be the same. There would be no set of observations of different choices, and there would be no statistical problem of estimating the average relationship between stated gross income and categories of factors.

Several rather serious theoretical objections have

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1. For example, individuals will have different feelings about owning land. One may subjectively demand only a 4% return on land, and another may demand 8%.

2. Farmers can not, for instance, be in complete agreement about the returns to machinery investment.

3. See pages 17 and 18 following.

been raised against the process of estimating value productivity functions from observations of competitive businesses. The equation which has been stated on page 1 for one farm can be given in logarithmic form for two factor categories, x and y, by

$$\log P = \log C + a \log x + b \log y.$$

If the gross incomes earned by different farms result from different combinations of factors with all operating on essentially the same value productivity function, then the value productivity function can be estimated by multiple regression from the linear equation

$$(\log P)' = \log C + a \log x + b \log y.$$

In this equation $(\log P)'$ is the estimate of the log of gross income. This logarithmic function can be interpreted in numbers as

$$P' = C x^{a} y^{b}.$$

In logarithms the equation forms a plane; in natural numbers it will form a curved surface.

3. The Dependence of Gross Income Upon Factors of Production:

Professor Mendershausen has questioned the justification of expressing gross income as a surface determined by categories of factors when the data do not themselves suggest a surface but rather a mass of points or a line.[1] If gross income should be expressed as a

1. Horst Mendershausen, "On the Significance of Professor Douglas' Production Function," Econometrica, V. 6 (1938) pp. 143-147.

function of total farm expense and total investment, for example, it is possible mathematically to determine a surface which gives a best estimate of gross income by assuming that these latter two are independent variables and that they are substitutable for one another. However, the variations in gross income not associated with changes in factors may be so great above and below the statistical "surface" that it can neither be said with certainty that a true gross income surface does exist, nor be said that it does not exist. In this case, if the sums of the squares of the differences between estimated and actual values of gross income are minimized in the directions of total farm expense and total investment, rather than in the direction of gross income, different "surfaces" will result. Simply stated, it is possible to get several estimates of gross income, different answers to the same problem, depending upon the direction of the minimization of the sums of the squares of the differences.

In this latter case a question is raised of the justification of minimization of the sum of the squares of the differences between estimated and observed values of P in the P direction rather than in some other direction. In the case of the use of multiple regression to study the response of gross income to changes in the dollar values of input categories, our justification is that the gross income of a farm can be rightly considered, generally, to be a function of the inputs.[1] In this case, the argument resolves itself into a question of whether the expenditure of resources on a farm arises out of the products sold from the farm, or whether the product results from the use of resources.[2] It is believed that product can generally be considered a result of factors.[3] There can be no product without fac-

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1. At the micro level, the level of the operation of the individual farm, this argument is particularly defensible. If the whole economy is considered (the macro level) then the demand curve facing the entrepreneurs taken as a group can not be assumed to be infinitely elastic. The causal relationship from factors to product is no longer straight-forward. An increase in total factors could result in a decrease in total gross income, even though, for any individual farm, an increase in factors might indicate an increase in gross income.

2. Determination of a regression equation by minimizing the sum of the squares of the differences between estimated and actual values of gross income in the gross income direction implies that gross income is a function of the other variables in the equation. It is recognized that the simple assumption that gross income is a function of categories has limitations. However, the data for this assumption may be adequate enough so that a meaningful and consistent set of relationships can be determined from a number of farms.

3. The Cowles Commission has been interested in developing a method of statistical analysis which rests on supposedly more defensible grounds than merely on the thesis that the variable of interest is a simple function of the "independent" variables. For one thing, when the "independent" variables are related to each other, they will take upon themselves portions of the total offered explanations of the dependent variable in an apparently erratic fashion. Partially to answer this objection, work (cont. p.15)

tors, but it is possible that there can be factors without product.[1]

4. Individual Value Product Functions and the Cobb-Douglas Function

Another objection to the statistical production function is possibly more serious on theoretical grounds. It has been pointed out by Reder and Bronfenbrenner that this function may not represent what it is supposed to represent.[2] They show that the statistical function is merely a surface which describes the relation of

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has been done on the "simultaneous equations" approach, in which there are as many equations as unknowns, and each equation is designed to take up some important phase of the whole set of interrelations. The use of simultaneous equations may not eliminate the problem which arises out of cross relationships between the independent variables. They may, however, come closer to a full explanation of the combined set of relationships than multiple regression. For the problem of estimation of the gross income of a farm from categories of factors possibly the advantages of simultaneous equations over multiple regression are not so definite as in the case of market relations. For discussions of the method and application of the simultaneous approach, see M. A. Gershick and Trygve Haavelmo, "Statistical Analysis of the Demand for Food: Examples of Simultaneous Estimation of Structural Equations," Econometrica, V. 15 (1947), pp. 79-111, particularly pp. 79-83. Also Wassily Leontief, "Introduction to a Theory of the Internal Structure of Functional Relationships," Econometrica, V. 15 (1947), pp. 361-372. Also A. R. Prest, "Some Experiments in Demand Analysis," Review of Economics and Statistics, V. 31 (1949), pp. 33-47.

1. See Appendix A for a more detailed analysis of the questions of the existence of a logarithmic plane expressing the relation of gross income to categories of factors and the effects of minimization of the sums of the squares of the differences in directions other than the direction of gross income.

2. M. W. Reder, "An Alternative Interpretation (cont. p.16)

gross income to the combinations of factors which the entrepreneurs actually employ. They do not assume that the entrepreneurs are operating on essentially the same production function. Thus, if the businesses are in equilibrium but all on essentially different production functions, then the Cobb-Douglas function is nothing more than an envelope giving the optimum combinations of factors for the various businesses and has little analytical significance for individual businesses. Each farm operator is simply maximizing his gross income according to his own particular production function. This could be altogether different in all cases from the function given by considering the earned gross incomes to be observations of results from the employment of different combinations of factors.[1] As far as the mathematics of the problem are concerned, the individual production functions could be sloping downward as the factors (in Bronfenbrenner's article, labor and capital) are increased. Yet the most favorable positions for the firms could be such that the *loci* of the coordinates for labor, capital, and product should tend to move upward with respect to product

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of the Cobb-Douglas Function," *Econometrica*, V. 11 (July-Oct., 1943), pp. 259-264. Martin Bronfenbrenner, "Production Functions: Cobb-Douglas, Interfirm, Intrafirm," *Econometrica*, V. 12 (Jan., 1944), pp. 39-42.

1. See pp. 11 and 12, *supra*.

as labor and capital are increased.

The objection which Reder and Bronfenbrenner raise to the estimation of a mean interfirm value productivity function from the values of the inputs and the gross incomes of a number of firms in an industry is based on the assumptions that the firms are in equilibrium and are actually operating on different functions. There is little question that firms in agriculture are not in equilibrium. On the contrary, the farm operators are more or less continuously in the process of changing the structure of their businesses in order to meet new technological and economic conditions. It is practically impossible to determine for a particular farm what its value productivity function as of a given moment will turn out to be in the light of changes in farming methods. The process of adjusting to a changing environment and even changing objectives of the farm operator himself is to a great extent one of trial and error. An assumption of this study is that the experiences of a number of farmers in the matter of gross income received from different combinations of factors will be useful to operators in planning their businesses. These experiences are condensed to an average in the exponential value productivity function.

Individually, farmers seek to maximize particular personal or family net utility functions.[1] This means that farmers will have different attitudes toward the risk of capital, the expenditure of their own and their families' labor, and so on.[2] Part of these differences will depend upon the resources, including unmeasured resources, which the farmer has at his command.[3] Neither will all farmers make the same estimate of the relationship between gross income and factors. Thus the fact that farm operators actually choose differing quantities and combinations of factors of production does not imply necessarily that these positions are different because the value productivity functions are different. The variations in employment of factors can occur because of differences in personal objectives alone, even should the farmers be operating on substantially the same value productivity function. There are, thus, three reasons why the value productivity functions facing the individual farms do not have to be different in order to have a result other than a convergence of all farms to the same set of coordinates of factor categories and value of product. To recapitulate: farms are

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1. Martin Bronfenbrenner, op. cit.,pp. 37-38.

2. D. B. Williams, "Price Expectations and Reactions to Uncertainty by Farmers in Illinois," J. F. E., V. 33 (Feb., 1951)no. 1, pp. 20-22.

3. M. Kalecki, "The Principle of Increasing Risk," Econometrica, V. 4 (New series, 1937) pp. 440-47, particularly, 440-42.

not in equilibrium. But even assuming that they were, the fact that they select different combinations of factors and earn different gross incomes does not prove that they are not on essentially the same value productivity function.

The value productivity function given on page 1 is simple. It does not contain all of the features which may be needed to express the true average value productivity of the categories employed on a particular farm. For one thing, marginal value product with respect to any input category is always decreasing provided the exponent of the category (a, b, ...or k) is less than 1, as it apparently must be. In reality there may be ranges in the use of factors in which this marginal productivity increases. On the other hand, according to this type of function, the total product will always increase with an increase in the employment of any single factor, holding other factors constant. In reality, the total product may in some circumstances decrease with the use of more of one of the factors. Consider the case of the number of cows on a 160-acre farm. With the number of cows carried by the farm continuously increased, a point would eventually be reached at which the total value of milk and beef produced would actually fall should still more cows be added.

Within the range of the use of factors in practical farm operation, these are perhaps not serious objections.[1] The form of the equation does behave according to several economic concepts. It is relatively easy to handle with the use of logarithms. If the value of the constant term C and the exponents of the factor categories, a, b, ...k are chosen by the method of least squares, the form of equation can be expanded to give a good estimate of gross income over the economic range of choices of factors.

1. A function of the type $P = Cx^a y^b$ could be fitted to any stage of the true porductivity function of a farm or of a group of farms provided the data were available. The values of C, a, and b would be these which applied to the stage. Data are available for the so-called second stage, in which total value productivity is increasing and marginal value productivities of categories of factors are decreasing. This is the stage of rational economic activity in conditions of approximate equilibrium. It is likely that the function could not be extended over all conceivable combinations of factor categories.

CHAPTER II

DATA FROM FARM ACCOUNT RECORDS IN TYPE-OF-FARMING AREAS 5 AND 6, MICHIGAN, 1950

This chapter presents an evaluation of data from farm account records as a basis for estimating gross income equations.

Farm account records of 194 farms in type-of-farming areas 5 and 6, Michigan, for the year 1950, were used in the study. These types-of-farming areas are in the south-central part of the lower peninsula of Michigan. One hundred sixty-three farms in area 5 and 35 farms in area 6 kept records in 1950.

Equations estimating gross income will be given in Chapter IV. They will be derived from values of categories of inputs as these categories are set up by the farm accounting procedure. This chapter considers some of the stated values of input categories in the light of using them for making estimates of gross income from the categories of factors.

1. The Value of Land

In order to keep land values between farms comparable and thus tend to assure comparability of the measurement of total charges against the farms, the staff of Farm Management Extension at Michigan State College has adopted a policy of valuing the land of new account cooperators similarly to the land of farmers already in the project. The results given in Appendices D and E indicate that the policy has been effective. It is recognized that over a period of years the land values in the farm account project have drifted substantially below market values.

An objective of this study, as has been stated, is to estimate gross income functions from categories of factors of production. Some of the shortcomings of the estimates may arise simply from inconsistencies between farms in the value of land as stated in the farm accounts.

There appeared to be no way of uncovering errors in the valuation of land between farms which started records in the same year without a separate appraisal of each farm. Land values, as long as the records were started at essentially the same time, were thought to be as useful as could be obtained without prohibitive field work. The use of crop yields and types of crops grown as partial indicators of land values was ruled out. Such a procedure would involve the valuation of land after its value had been more or less proved. It would have the effect of arbitrarily assigning part of the residual of gross income to land, with no reason for doing so except that the residual exists. There appeared to be no workable method of separating the differences in land values due to inherent differences in the productivity of the soil from the variations due to such differences as in favorableness of the season, farming systems, and abilities of the individual operators as crop producers.

The hypothesis was set up that the year of starting records should have a tendency to bias stated land values. It was thought that a consistent bias with respect to the year of starting farm account records might be found and that it might appear to be worth eliminating by statistical means. Altogether 176 records for the whole state were used in tests to determine whether the year of starting accounts had a significant influence upon the level at which farms were valued.[1] The conclusion is reached that the year of starting farm account records had no significant influence upon the level of land values or upon the dispersion of the values around the means. In general, land values as stated in the farm account records for areas 5 and 6 for 1950 seem to be at levels prevailing in the late 1930's, even for farms on which records were started after the second world war.

1. See Appendices D and E for the analysis of these records.

2. The Valuation of Dairy Cows:

In the case of dairy cows there was a tendency for the stated values of animals in herds on farms coming into the accounting project in the years since the second world war to be higher than the stated values of animals in herds coming into the project earlier. The effect of this basis for differences in values of animals will be partially to invalidate the results of the analysis of marginal returns on investments in livestock. A test was made of the relations of the average stated value per cow to gross income received per cow. The relation was not significant at 5%.[1] The analysis of cow values does not establish the hypothesis that the stated value of a cow is related to the gross income which she produces for the farm. The effect of this will be to reduce the statistical response of gross income to dairy cattle investment. When the inconsistencies in the valuations of the factors are not similar, for example, comparing investment in cows or land with cash expenditures for fertilizers (with the latter given exactly) the effect will be to increase the relationship of gross income to fertilizer expenditures at the expense of the relationship of gross income to investment in dairy cows. Eliminating the influence of the time of starting farm account records on the stated value of dairy cows per

1. Appendix F.

head about doubled the significance of the relations obtained between cow values and the output of 3.5% fat-corrected milk.[1] Inconsistencies in the valuations of dairy cows represent one problem which should be solved if gross income estimating equations are to be calculated as a part of the regular procedure in farm business analysis.

3. The Measurement of Categories of Factors other than Land and Dairy Cows:

Inputs other than land could be classified broadly as labor or capital.

In the case of labor, we shall use the cost of hired labor, the charge for family labor, the operator's labor charge, and the total labor charge as they are reported. Total labor charge includes an allowance for the operator's labor, which was entered at $1560 for all farms (with minor exceptions), or $130 per month for the time that the farm operator worked on the farm. It is, admittedly, not a good measure of the operator's labor and management input. The operator's labor charge does not include an allowance for management. The dollar estimate of the total labor charge can be assumed generally to include a differential according to the quality of hired and unpaid family labor. That is, more expensive hired

1. Appendix F.

labor is probably more valuable than cheaper hired labor. The charge for unpaid family labor takes into consideration the age of each worker. An effort to charge for the operator's labor according to the gross income or the measure of net income would amount to a valuing of the operator's labor according to the total residual. In the case of labor income, the total residual to all factors above the stated total charges is assigned to the operator. Thus the operator is credited with windfall gains and losses. "Labor income" and the value of the labor and management input of the operator are not the same thing. In short, there is no satisfactory measure of the value of the operator to the farm.

There are several ways of measuring the capital input on a farm. All items can be considered together or they can be classified separately. Investments other than land can be taken from the farm account records in such categories as machinery, livestock, feeds, and crops. Such expenses as fuel, feed, and fertilizer can be taken all together as "total farm expense" or can be put in special categories.

As far as direct outlays for feed, fertilizers, and so on, as they appear in such items as feed expense and crop expense, there appears to be no reason to believe that these figures are biased between farms. The values of livestock other than dairy are relatively unimportant in the records included in the study.

A time error in valuation of machinery arising from differences in the years of starting records should be relatively unimportant. The most expensive types of equipment generally need to be replaced every few years either because of obsolescence or because of physical wear. When new or replacement items appear in the farm inventory, they are practically always, if not always, entered at cost. Since all of the farmers in this study are buying in essentially the same machinery market, there seems to be little justification for an assumption that machinery investments are not on substantially the same basis from one farm to another. Feed and crop inventories are comparable because these items are entered at market prices, which are essentially the same throughout the area.

4. The Data Concerning Gross Income:

Product is measured by gross income, dollars being the best least common denominator available in which to express the output of the farm. As the data are for a single year, price changes are not particularly important. From the point of view of the total operation of a farm, gross income seems to offer the most accessible and meaningful estimate of the volume of product. If the farms in the area or in the study produced one specific physical product only, say fluid milk, all of which was sold in the same market, it should be possible to estimate product simply by using fat-corrected milk. However, most farms in the study have a number of enterprises other than dairy. On each farm other enterprises absorb a share of the total inputs. Therefore the estimation of product from the inputs must account for the output of other enterprises and a least common denominator must be used.

Several types of error may be introduced by using gross income from farm records as the value product of a farm. (1) It may include income from off the farm, the pay for which may be on some different basis from the pay for operating a farm. (2) It may include inventory gains arising out of price changes when inventories are valued at market prices. (3) The organization of the farms in the survey is based on experienced relations between prices of alternative products and alternative inputs. If in any one year these relationships should be appreciably out of line, then gross income includes a windfall type of gain or loss which is not necessarily related to operational efficiency. (4) Gross income will include the effects of events such as windstorms, floods, unusual seasons, and so forth.

With these objections to the use of gross income as the measure of gross product, one could logically ask: "Why use gross income?" There are four main reasons: (1) In the first place, the objections arising out of the first preceding item can be partially answered by eliminating unusual cases. That is, the farmer who earns a large part of his income from doing custom work for other farmers can be excluded from the study. We can at least make the statement, then, that the results apply to the typical farm in the area of the survey, not to farmers who are primarily contractors, shop workers, or dealers in merchandise. (2) It is possible to eliminate the effects upon gross income of price changes of goods held in inventory and of changes from expected prices of crops sold. A study was made of a sample of farms in order to evaluate the importance of the effects of these variables upon gross incomes in the areas 5 and 6 for the year 1950.[1] (3) In the third place, it would be extremely difficult to find a satisfactory basis for adjusting values of product to meet the qualifications of the long run. No one can be sure, for instance, whether a relative shift in the price of one product in comparison to another is a temporary fluctuation from a long-run average, or whether it represents a trend which will continue. The relative fall of the market price of

1. See page 30.

cotton and potatoes in relation to farm products in general offers illustrations of cases in which "deviations" turn out to be secular trend. (4) The use of gross income permits study of the marginal returns to the factors under the conditions which face the farm operator. Farms are not operated in terms of long run averages; the costs and returns from farming consist of a series of short-run conditions in the buying and selling markets. If the returns for a certain type of operation are low in a given year, the farmers who have been adversely affected have no alternative except to take their losses. It is more pertinent that the returns to factors should be studied continuously in order that changes in markets and techniques can be noted and responses of farm operators to these changes observed. Regarding a specific area for a given year, with given economic conditions, technology, and climate, it is important to make a practical accounting of what actually happened, not of what would have happened if the weather and prices had been "normal." Thus the idea of a value productivity function or relation of categories factors to gross income has the advantages of being simple and realistic.

5. The Effects of Price Changes of Crops Held in Inventory and of Changes in "Expected" Prices Upon the Reliability of Estimates of Gross Income:

The hypothesis has been stated that price changes of goods held in inventory and changes in the market prices of farm products from "expected" values constitute a form of random error in gross income from its expected value. The farmer apparently can not count on variations in gross income from these sources in the process of developing the farm business. If this is true, the employment of factors should tend to be more closely related to a gross income which does not include these "random errors." If it should be practicable to eliminate annual variations in crop yields, apparently the relationship of gross income to factors should be improved even further. A farmer plans according to average experience, but receives returns according to particular conditions of prices and weather.

It was found that eliminating the effects of variations in prices from "expected" values (on goods sold and goods held in inventories) did not result in an increase in the relation of productive factors to gross income.[1] Neither did simply eliminating price changes of inventories result in an increase of the relation between factors and gross income. This means that, if a farmer typically does have a "planning function" in mind, the "planning function" in this case accommodated moderate changes from "expected" prices as well as it could have accommodated normal or expected prices. Obviously, the results of the work in Appendix G can

1. Appendix G.

be considered to bear only upon moderate variations in prices from expected values.[1]

6. Conclusion

In this chapter questions have been raised regarding the adequacy of data from farm account records for areas 5 and 6 for calculating equations estimating gross income. It has been found that the year of starting records did not bias stated values of land per acre as between farms. However, as a whole, the farms appeared undervalued. The year of starting records did not bias the values of dairy cows sufficiently to justify a systematic adjustment.

It is concluded that variations in the bases of valuation of factors from one farm to another are impractical to determine at this time. The factor, management, is not included in the data, but there does not appear to be much that can be done about it. In general, the input data from the farm accounts in the two areas are as suitable as could be obtained without extensive additional field work.

Unadjusted gross income values were as much related to factors as were adjusted gross income figures. Adjustments were for price changes in inventories and changes in prices from "expected" values. Thus there appeared to be no reason for a general adjustment of gross income to make allowance for shifts in prices from "normal."

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1. See Appendix C, p. 150.

CHAPTER III

THE METHOD OF CALCULATING GROSS INCOME EQUATIONS FROM FARM ACCOUNT DATA

This chapter contains two main headings: 1. a description based on farm account records of farms in areas 5 and 6; and 2. a statement of the methods of using these data in calculating gross income equations.

1. Farms in Type-of-Farming Areas 5 and 6

Areas 5 and 6 are in the milksheds of Detroit, Flint, Pontiac, Lansing, and Jackson. Table 1 (page 34) summarizes income and expense data for all farm account farms in areas 5 and 6, 1950.

Area 5 is described as "dairy and general farming." Area 6 is called "dairy and cash crops."[1] That the sale of dairy products makes up a somewhat larger proportion of the total income for farms in area 6 than in area 5 is evident. There is more emphasis on other types of livestock than dairy cows in area 5.

The main single product sold in both areas is whole milk. There are a few farms which maintain beef herds,

1. "Farm Business Analysis," Publication of the Department of Agricultural Economics, Michigan State College, A. Ec. 477, May, 1951.

Table 1. Income and Expense Items, Type-of-Farming Areas 5 and 6, 1950[1]

Averages for 198 Farm Account Farms

Item	Area 5	Area 6
Income		
Crops	$ 2,431	$ 2,773
Dairy products	4,517	5,582
Hogs	1,233	213
Cattle	2,147	1,835
Poultry and eggs	709	955
Sheep	361	179
Other	733	118
Gross Income	$12,131	$11,655
Expense		
Feed bought	$ 1,335	$ 1,049
Machinery	2,017	2,023
Hired Labor	632	983
Family labor	486	572
Crop expense	1,069	1,147
Improvements	637	605
Taxes	244	216
Other	384	706
Total*	$ 6,804	$ 7,301
Net Farm Income	$ 5,327	$ 4,354
Int. on invest. @ 5%	1,640	1,498
Labor Income**	$ 3,687	$ 2,856

* Does not include a charge for the operator's labor.

** Labor income is what the farmer has left for his labor and management after cash expenses, family labor, depreciation, and interest on investment are deducted from total income--it is not the total value productivity of the operator's labor input in a precise conceptional sense. It is the total of all residuals remaining when charges against the farm are made according to dollar costs or according to assumed rates.

1. Ibid., areas 5 and 6, p. 5.

and some which specialize in corn and hogs, particularly in the southern part of area 5. A few farms specialize in poultry. Important cash crops are wheat and, particularly in the northern parts of both areas, beans. A relatively small number of farms specialize in sugar beets, the center of the sugar industry being farther north in the Saginaw Valley. On muck lands onions, mint, potatoes, and celery are grown. There are relatively few muck farms. In 1950 the farm accounts were concerned with farms containing primarily upland soils. The farms in the study are somewhat larger than the "average" and possibly are better managed on the whole.

Investments and expenses are grouped into categories for areas 5 and 6 in tables 2 and 3. In these tables a classification is made according to labor income. For area 5 the low third and high third labor income farms are compared with the average of all farms (table 2). For area 6 the half of the farms with low labor incomes, the half of the farms with high labor incomes, and all farms are compared (table 3). In both areas the farms with greater investments in all categories and greater expenses in all categories tended to receive larger gross incomes and larger labor incomes.

It is a purpose of this study to take the invest-

Table 2. Income, Expense, and Investment Data Related to Labor Income[1]

Farms in Type-of-Farming Area 5
Michigan, 1950

	1/3 low income farms	Average of all farms	1/3 high income farms
Number of farms	54	163	55
Labor income	$ 746	$ 3687	$ 7068
Gross Income	7,992	12,131	18,020
Investments			
Total	28,456	32,662	41,114
Productive livestock	4,089	5,243	7,551
Machinery, equipment	5,485	6,266	7,720
Feed and crops	3,056	3,800	4,920
Improvements	7,904	8,578	10,800
Land	7,923	8,775	10,123
Expenses			
Total farm expense	7,436	8,422	10,428
Total labor charge	2,429	2,642	3,123
Hired labor	477	632	940
Family labor (unpaid)	445	486	651
Operator	1,507	1,524	1,532
Feed	1,020	1,335	2,020
Crop	755	890	1,107
Machinery net decreases	1,859	2,017	2,332
Improvements	593	637	808
Other	780	901	1,038

1. Compiled from tables in publication 477, cit. supra.

Table 3. Income, Expense, and Investment Data Related to Labor Income[1]

Farms in Type-of-Farming Area 6
Michigan, 1950

	½ low income farms	Average of all farms	½ high income farms
Number of farms	17	35	18
Labor income	$ 623	$ 2,856	$ 4,964
Gross income	8,741	11,655	14,406
Investments			
Total	32.911	33,394	33,849
Productive livestock	4,920	4,998	5,071
Machinery, equipment	6,358	6,569	6,771
Feed and crops	3,584	3,482	3,386
Improvements	8,746	8,603	8,470
Land	9,303	9,742	10,151
Expenses			
Total farm expense	7,923	8,625	9,288
Total labor charge	2,827	3,063	3,285
Hired labor	851	983	1,107
Unpaid family labor	500	572	639
Operator	1,476	1,508	1,539
Feed	959	1,049	1,135
Crop	735	990	1,231
Mach. net decreases	2,027	2,023	2,019
Improvements	528	605	677
Other	847	895	941

1. Compiled from "Farm Business Analysis, Area 6," Agricultural Economics Department, Michigan State College, A. Ec. 478, May, 1951.

ment and expense data by farms and calculate the average relationship between gross income and the outlay in each investment and expense category. This sort of relationship is not available when either labor income or gross income is taken as the basis of classification after which the average value of each investment and expense category is given for each labor income or gross income class.

In the following procedure the analytic process will be reversed, so to speak, and an explanation of differences in gross income according to each category of factors will be offered. This latter process is more meaningful because we proceed from factors to income rather than from income to factors. This method shows the structure of gross income from factors; the former implies a structure of factors from gross income.

2. Procedure for Calculating the Gross Income Estimating Equations

Of the 194 farms, 86 were classified as dairy farms. The others were called not-dairy farms.

The categories of factors used in this study are those from the farm account records which are shown in tables 1 to 3. These categories can be combined for simplicity or listed separately for detail (table 4). The categories in columns I, II, and III are headings

Table 4. Categories of Factors as given in the Michigan Farm Accounts

I	II	III
Total investments	Land and improvements	Land
		Improvements
	Investments other than land	Machinery investment
		Livestock investment
		Feeds and crops
Total farm expense	Total labor charge	Total labor charge
	Total farm expense	Machinery expense
		Feed expense
		Crop expense
		All other expense
Sum: All categories of factors	All categories of factors	All categories of factors

used in the Michigan system of farm accounts. Gross income equations can be calculated for any desired amount of detail within the limitations imposed by the detail of the basic data. (The calculated equations are given in Chapter IV).

The procedure at Michigan State College is to take the data from the farm account records and place them on summary sheets. The data on the summary sheets are then punched on IBM cards as part of the annual process of farm business analysis. The IBM cards and the summary sheets are then used in the preparation of type-of-farming area reports.

The calculations for this study began at this point. They are summarized as follows:

1. Equations of the form $P = Cx^{a}y^{b} \ldots z^{k}$ appear in log form as $\log P = \log C + a \log x + b \log y + \ldots k \log z$. The second equation is linear in the logarithms. By converting the basic data to logs, thus, it is possible by multiple linear regression techniques to determine a mean relationship between factor categories and gross income which is (a) linear in the logs and (b) exponential in the natural numbers. This is essentially the method of Douglas, Heady, Tintner and others as discussed in Chapter I.

The data were converted into logs in four steps: (a) An IBM log card was prepared containing the mantissas of the logs of 100 to 999. A card was made for each number, 100.00, 101.00,......999.00. Columns 1 to 5 were not used in the work with logs since these columns were needed for identification in subsequent sets of cards. In columns 6 to 45 the natural numbers were repeated 8 times. In columns 46 to 78 the mantissas of logs of the respective natural numbers were repeated 8 times. Columns 46, 50, and every fourth column thereafter were left blank for the characteristics of the logs. Three significant figures were given for the mantissas.

(b) Data from the IBM cards for the farm accounts were transferred to columns 6 to 45 of a new set of cards, hereinafter called "transfer cards." (Four transfer cards were used in recording information used in this study).

(c) An IBM collator was used with the log cards and the transfer cards to place the mantissas of the farm account data in columns 46 to 78 of the transfer cards. The characteristics were punched by mechanical means.

(d) Columns 46 to 78 of the latter group of cards were reproduced on a final set of cards on which the data were given in log form.

2. The least squares regression equations were solved by the Doolittle method. Standard errors of estimate and confidence limits of coefficients of elas-

ticity were computed in some cases. The logarithmic equations were restated in terms of natural numbers.

Once the above procedure had been carried through, the gross income equations and the information which is implied by these equations were ready for interpretation.

In Chapter IV the equations of gross income and the information which follows from them, such as elasticities of gross income with respect to categories of factors and the marginal value productivities of categories of factors, are stated and discussed.

CHAPTER IV

GROSS INCOME EQUATIONS AND THEIR DERIVATIVES

1. Gross Income Estimating Equations

Equations estimating gross income were calculated (tables 5 to 7). The equations are read thus: The coefficients of the factor categories for the different equations are given in the numbered columns. Therefore the first equation in table 5, for 86 dairy farms, reads:

$$\text{Log gross income} = 0.458 + 0.541 \text{ total farm expense} + 0.322 \text{ total investment.}$$

Translated into numbers, this is:

$$\text{Gross income} = 2.87(\text{total farm expense})^{0.541}(\text{total investment})^{0.322}$$

Altogether, 10 equations are given for the dairy farms, 6 for the not-dairy farms, and 3 for all farms.

Table 5. Ten Gross Income Estimating Equations for 86 Dairy Farms

Type-of-Farming Areas 5 and 6, 1950

	Equations									
	1	2	3	4	5	6	7	8	9	10
Constant term	0.458	0.436	0.422	0.400	0.531	0.149	0.363	0.522	0.119	0.094
Category of factors	(Coefficients)									
Feed expense					0.138			0.140		
Crop expense			0.432	0.387	0.403			0.370	0.273	0.271
Net decrease, machinery	0.541	0.480			-0.017	0.351	0.383	-0.069		
Total labor charge			0.145	0.130	0.100		0.125	0.088	0.090	0.086
Investment in land		0.082		0.088		0.050	0.031	0.108	0.061	0.020
Investment in improvements							0.088			0.062
Machinery and equipment investment	0.322		0.322		0.367	0.172			0.181	0.180
Productive livestock investment		0.328		0.328		0.150	0.322	0.356	0.154	0.153
Feed and crop investment						0.312			0.307	0.303
Sum of coefficients	0.863	0.890	0.909	0.933	0.991	1.035	0.949	0.983	1.066	1.075

Table 6. Six Gross Income Estimating Equations for 108 Farms Other Than Dairy

Type-of-Farming Areas 5 and 6, 1950

	Equations					
	1	2	3	4	5	6
Constant term	0.237	0.180	0.297	0.228	0.189	0.350
Category of factors	(Coefficients)					
Total farm expense, not labor	0.696	0.622	0.577	0.497	0.482	0.582
Total labor charge			0.159	0.160	0.150	0.164
Investment in land	0.247	0.002	0.239	-0.004	0.039	0.018
Investment in improvements					-0.006	
Machinery and equipment investment		0.350		0.351	0.346	0.184
Productive livestock investment						0.064
Feed and crop investment						-0.003
Sum of coefficients	0.943	0.974	0.975	1.004	1.011	1.009

Table 7. Three Gross Income Estimating Equations for 194 Farms

Type-of-Farming Areas 5 and 6, 1950

	Equations		
	1	2	3
Constant term	0.361	0.241	0.359
Category of factors	(Coefficients)		
Total farm expense, not labor	0.544	0.449	0.544
Total labor charge	0.149	0.157	0.124
Investment in land		0.101	0.070
Investment in improvements			0.001
Machinery and equipment investment	0.256	0.281	0.117
Productive livestock investment			0.040
Feed and crop investment			0.106
Sum of coefficients	0.949	0.988	1.002

2. Comparison of Coefficients of Elasticity and Their Confidence Intervals for Dairy and Not-Dairy Farms:

Dairy farms were analyzed separately from farms other than dairy for two reasons: (1) The separation of the dairy farms from all farms permits a more specific analysis of the structure of the farm business. The kinds of work done and the classes of inputs are more alike from farm to farm. Thus the equation of gross income has greater structural significance. And (2) the hypothesis was set up that the correlation of gross income with the factor categories should be greater for the more homogeneous group of dairy farms than for the not-dairy farms. From this it should follow that the confidence intervals of the coefficients of elasticity of gross income for the different categories of factors should tend to be narrower. Better estimates of the coefficients should be obtained by classification of farms according to type.

The coefficients themselves were not significantly different for dairy farms and not-dairy farms (tables 8 to 10). The hypothesis that the confidence intervals should be narrower for the supposedly more homogeneous group of dairy farms was not supported by the data. The relationships between categories of factors and gross income are estimated to be stronger in the case of the 108 not-dairy farms. Examination of the columns

Table 8. Gross Income Equations Based on Categories of Productive Factors
86 Dairy Farms and 108 Farms Not Dairy
Type-of-Farming Areas 5 and 6, 1950

Category of Factors	Symbol	Coefficient		10% Confidence Limits		50% Confidence Limits		Ratio 10% confidence interval dairy farms to 10% confidence other farms
		86 Dairy Farms	108 Farms Not Dairy	86 Dairy Farms	108 Farms Not Dairy	86 Dairy Farms	108 Farms Not Dairy	
Constant term	(c)	0.458	0.237	0.392-0.522	0.221-0.253	0.431-0.484	0.230-0.243	4.1
Total farm expense	(a)	0.541	0.696	0.269-0.813	0.561-0.652	0.431-0.652	0.641-0.751	2.0
Total investment	(b)	0.322	0.247	0.093-0.551	0.101-0.393	0.229-0.414	0.188-0.306	1.6
Sum of coefficients		0.863	0.943					
Correlation coefficient		0.833	0.914					
Standard error of estimate		0.112	0.094					

Table 9. Gross Income Equations Based on Categories of Productive Factors
86 Dairy Farms and 108 Farms Not Dairy
Type-of-Farming Areas 5 and 6, 1950

Category of Factors	Symbol	Coefficient		10% Confidence Limits		50% Confidence Limits		Ratio 10% confidence interval dairy farms to 10% confidence other farms
		86 Dairy Farms	108 Farms Not Dairy	86 Dairy Farms	108 Farms Not Dairy	86 Dairy Farms	108 Farms Not Dairy	
Constant term	(c)	0.422	0.297	0.358–0.486	0.292–0.302	0.398–0.446	0.295–0.299	12.8
Total farm expense, not labor	(a)	0.432	0.577	0.191–0.673	0.471–0.683	0.377–0.487	0.534–0.620	2.3
Total labor charge	(b)	0.145	0.159	0.021–0.311	0.019–0.299	0.096–0.194	0.103–0.215	1.2
Total investment	(d)	0.332	0.239	0.124–0.542	0.132–0.346	0.261–0.405	0.196–0.282	2.4
Sum of coefficients		0.909	0.975					
Correlation coefficient		0.878	0.916					
Standard error of estimate		0.106	0.089					

Table 10. Gross Income Equations Based on Categories of Productive Factors
86 Dairy Farms and 108 Farms Not Dairy
Type-of-Farming Areas 5 and 6, 1950

Category of Factors	Symbol	Coefficient		10% Confidence Limits		50% Confidence Limits		Ratio 10% confidence interval dairy farms to 10% confidence other farms
		86 Dairy Farms	108 Farms Not Dairy	86 Dairy Farms	108 Farms Not Dairy	86 Dairy Farms	108 Farms Not Dairy	
Constant term	(c)	0.400	0.228	0.380-0.421	0.213-0.243	0.392-0.409	0.222-0.234	1.4
Total farm expense, not labor	(a)	0.387	0.497	0.159-0.615	0.358-0.636	0.295-0.479	0.441-0.553	1.6
Total labor charge	(b)	0.130	0.160	-0.084-0.344	0.052-0.268	0.043-0.217	0.116-0.204	2.0
Investment in land	(d)	0.088	-0.004	-0.088-0.264	-0.108-0.100	0.017-0.159	0.041-0.047	1.7
Investment other than land	(e)	0.328	0.351	0.228-0.528	0.227-0.475	0.247-0.409	0.301-0.401	1.9
Sum of coefficients		0.933	1.004					
Standard error, estimate		0.107	0.089					

called <u>Ratio 10% confidence interval dairy farms to the 10% confidence interval other farms</u> (tables 8 to 10) shows this. Calculation of the confidence intervals was designed to take into consideration interdependence of the categories of factors; that is, interdependence among the "independent" or causal variables.[1]

3. Interpretation of the Gross Income Equations

The equations given in tables 5 to 10 can be considered as gross income estimating equations or as value productivity equations facing the individual farms. Knowing the values of the input categories on a particular farm, the gross income of the farm can be predicted within the limits implied by the standard error of estimate of the gross income. Farmers who combine categories of factors in one proportion are assumed generally to be able to combine them in different proportions and earn gross incomes accordingly. In other words, it is assumed that farmers in a particular area at a certain time period are operating on value productivity equations which are essentially comparable.

The objection implied in the work of Reder and Bronfenbrenner is borne in mind.[2] However, this ob-

1. See P. Cramér, <u>Mathematical Methods of Statistics</u>, pp. 545 to 548.

2. Melvin W. Reder and Martin Bronfenbrenner, <u>opera cit.</u>, p. 15, footnote 2.

jection could be made to the bulk of statistical research in farm management. For example, according to the long-established farm "success factors," farmers obtaining high crop yields and high rates of production of livestock also earn relatively high labor incomes. It is <u>theoretically</u> possible that certain farmers with low production records would earn even lower labor incomes should they seek to raise the level of productivity of their farms. But, while a theoretical possibility, this proposition is not ordinarily taken to mean that analysis of the experiences of a group of farms in use of resources is not valuable to the farm operator, and that, as a rule, he will increase his net return if he can increase his yields.

One of the basic premises in farm management research and extension work is that farm operators are able to learn by the experiences of others and are capable of duplicating these experiences to some degree. Naturally, not all individuals will get exactly equal results from the same procedures. In the absence of proof to the contrary, gross income will in general respond according to the values of the categories of inputs as specified in the equations. In this case, the functions can be called value productivity equations for the farms in the study. That is, should the responses of gross income to the values of categories of inputs be the same for all farmers, assuming that the effects of weather, prices, and so on could be averaged out as random variables, then the gross income equations are value productivity functions.

4. Reasons for Errors in the Estimate of Gross Income

Gross income is predicted by estimating equations within certain limits (tables 8 to 10). Clearly, the more accurately gross income can be predicted, the more useful will be the method of prediction. There are at least four causes of variations of actual gross income from predicted gross income. These differences may arise from the following causes:

(1) The method of statistical analysis may be inadequate. Inadequacies of this type include failure to use proper types of equations, failure to combine factors of production into appropriate categories, and multicollinearity. By multicollinearity is meant significant relationships between the independent variables (categories of factors) themselves. When the independent variables are correlated, the usefulness of multiple regression is impaired. In effect, when the independent variables are themselves correlated as well as each being correlated with the dependent variable, it is extremely difficult, if not impossible, to establish an hypothesis that regression analysis attributes a "proper" portion of the combined rela-

tionship to each independent variable.

(2) Variations in gross income may arise from windfall events. Increases or decreases in gross income from expected values because of weather, prices, new diseases of crops, accidents, and so forth, fall into this group.

(3) Gross income will be different from its predicted value if the factors of production are not valued on similar bases from farm to farm.

(4) There will be differences in gross income because of management. Management is not measured in farm account systems, and thus the quantity of management used is not recorded as an input.

To make the estimation of gross income useful, it is desirable that these sources of error be cut off where practicable. Statistical methods may be developed over a period of time which may give at least a partial answer to the first source of error in the preceding discussion. The shortcomings of estimation of gross income which arise from windfall types of gains and losses may be eliminated to some extent by the use of "normal" prices and yields for each farm. Furthermore, possibly a more detailed classification of farms by type and area will partially answer this problem. The use of gross income predicting equations with regard to (3) above implies a need for constant reevaluation of resources in order to keep average stated values in line with markets, and to keep valuations comparable from farm to farm. The differences in gross income arising from management will remain as residual unless a workable scheme for placing a value on the management input is devised.

5. The Effects of Overvaluation and Undervaluation of Categories of Factors

When a category of factors is undervalued in the farm account books, the effect on the elasticity of gross income with respect to the category and the effect on the marginal return is to increase both. Understatement of the value of land leads to a conclusion that the elasticity coefficient for land at market values is lower than the values given from the gross income equations as calculated from farm account data. The marginal return to land will be lower than stated.

Similarly, the overvaluation of a category of factors, such as labor (as this study seems to show) leads to a low coefficient of elasticity. Should labor be valued in the farm account books at the rates which are subjectively attached to it, apparently, by the farm operators, the stated value will be lower; the coefficient of elasticity will be higher, and the marginal return for a dollar of labor charge will approach 1, as explained in the next section.

6. The Constant Term in the Estimating Equation

The constant term (C in $P = Cx^a y^b$) may be in a sense regarded as a regulator of the extent to which it pays to expand the business. Thus C can be interpreted to indicate, particularly, the capacity of the fixed inputs profitably to accommodate the inputs which can be varied. Whatever may be the exponents of x and y (x and y designating the categories of factors employed), gross income is some number times the $x^a y^b$ part of the expression. If returns to scale are decreasing and the subjective rate of charge against stated dollar costs by the operator remain constant, the larger C is, the more x and y it will pay to use. If returns to scale are constant or increasing, it may be expected that considerations of risk and uncertainty in connection with charges (entered in the accounting sense) by x and y will eventually bring expansion to a close in any case. But the larger C is in this case also, the greater will it pay to expand the business.

From the point of view of the individual farm, in the long run the really fixed factor is management. The quantity of this fixed factor, in turn, determines in considerable degree how much of the other factors it pays to employ. Thus there is an analogy between the constant term C and management; and management is the prime factor which is not included in the categories x, y, ...z. As improvements in techniques cause the optimum size of farm to become larger, the average value of C. should become greater.

7. Elasticities of Gross Income with Respect to Categories of Factors[1]

The sum of the elasticities is not significantly different from 1 in any of the equations (tables 5 to 7). Thus returns to scale for both dairy farms and not-dairy farms in areas 5 and 6 for 1950 are indicated as constant.

The tendency for the sum of the elasticities to increase with more detailed breakdowns of categories is noted. The sum of the coefficients increases fairly constantly from 0.863 to 1.075 as the number of categories (detail of breakdown) is increased. A test showed that the probability that the sum of the coefficients would by chance increase as consistently with an increase in the number of categories as it did was 0.025, or one in forty.[2] This possibly can be explained on the basis that the more detailed breakdowns of factor categories imply more nearly proportional changes in real inputs. A more exactly proportional change in all factors would elicit a greater total response in gross income than would a change

1. For a discussion of the theoretical meaning of these elasticities and their sum, see pp. 3 to 6.

2. The test is analagous to a coin-tossing experiment. Each time the sum of the coefficients increased when the number of factor categories increased, the outcome was considered as a "head" and vice-versa.

which was concentrated relatively heavily in certain factors.

If not all of the categories to which elasticity of gross income can be properly assigned have been included, then the included factors will take credit for elasticity of gross income from the excluded factor or factors. If important factors are not taken into account, and included factors are themselves less important but are strongly correlated with the omitted factors, it is possible seriously to overestimate the elasticities of the included factors. If the ignored factors are particularly important, gross income may appear to have more or less elasticity with respect to the factors used in the estimate of gross income than really exists for all factors taken together. All of the estimated elasticity will be credisted to only the factors which are included.

Thus the question of the adequacy of the function in the absence of a statement of the value of management appears again. If management is a fixed factor on any farm, but varies between farms, then the factors which are used in making the estimates of gross income may be acquiring apparent influence upon gross income from management. Thus the effect of the exclusion of a value of management may be to bias upward the elasticities of gross income with respect to the other factors. Bronfenbrenner expresses the same idea, but in a different way, when he writes of different firms' being on different "production functions." If all of the factors other than management are consistently evaluated from farm to farm, then the elasticities of gross income with respect to each factor are the same for all farms. But differences in the management input imply different values of C (the constant term) and different points of equilibrium.[1]

If returns to scale are constant or increasing, there are two possible answers to the question of why some farm businesses do not become large enough to influence the market price: (1) Increasing returns to scale (from the standpoint of the farmer, prices and costs favorable to expansion) may be regarded as a temporary phenomenon. Farmers may not try to capitalize upon increasing returns to scale because of the risks involved. That is, after a business becomes very large, particularly in agriculture, the operator will tend to value commitments of capital and expenditures at greater than their dollar amounts and at progressively higher rates in reckoning costs. (2) As a business becomes very large, the type of function

1. Martin Bronfenbrenner, "Production Functions: Cobb-Douglas, Interfirm, Intrafirm," Econometrica V. 12 (Jan., 1944), pp. 37-8.

used here probably becomes inadequate. For example, as a dairy farm becomes very large, even the time which the cows spend going to and from pasture will increase, or the need for delegation of authority will increase. Thus returns to factors may even finally be zero at the margin.

8. Marginal Value Productivities[1]

The average values of the different categories of factors for the 86 dairy farms, the 108 not-dairy farms, and the 194 farms were calculated from the farm account data (table 11). An estimate of the marginal value productivity of each category of factors when all categories are at their mean values is given by the first derivative of gross income with respect to the particular category. These derivatives for all categories were calculated for 13 equations (tables 12 to 14).

The marginal value productivity of each category of factors can be estimated by farms if the values of the categories applying to each farm are inserted in the marginal value productivity equation. This was done for the 86 dairy farms, using as the equation of estimate of gross income the first equation in table 9. This

1. For estimates of marginal productivities on Iowa farms from farm account records (1939) and survey records (194), see Appendix J.

For reservations concerning the use of the Cobb-Douglas function in estimating marginal value productivities see chapter 1, parts 3 and 4, and appendices C and I.

equation is, in log form:

$$\text{Log gross income} = 0.422 + 0.432 \cdot \text{log total farm expense not labor} + 0.145 \cdot \text{log total labor charge} + 0.332 \cdot \text{log total investment.}$$

The average return on investment at the margin was 11.5% by the latter method (table 15). Average returns for total labor charge and total farm expense not labor at the margin were \$0.566 and \$0.874, respectively.

Estimates of marginal returns by different gross income equations for both the dairy and the not-dairy farms are shown in tables 12 to 14. The marginal return to investment in land is estimated between 0% and 10%. The marginal return to labor on the dairy farms was estimated at about 0.5, which means 0.5 for the marginal labor dollar. Thus, the estimated return on labor for the dairy farms is \$/0.50 at the margin. The return at the margin is estimated to be about \$0.95 an hour for the not-dairy farms. It is generally recognized that the marginal return to labor for dairy farms is comparatively low, but the regularity of employment is an offsetting attraction. The marginal return of \$0.50 to about \$0.70 for a dollar of labor charge for all farms indicates that at the margin labor does not earn as much as is shown by the charges entered in the farm account books. This also is as one might well expect. Even in a relatively good year, farmers will work at some phases of their business for substandard returns to piece out an acceptable income.

Table 11. Average Values of Categories of Factors
Type-of-Farming Areas 5 and 6, 1950

	86 Dairy Farms	108 Farms not-dairy	194 Farms
Feed expense	$ 1,008	$ 1,552	$ 1,311
Crop expense	812	1,017	926
Machinery net decreases	1,949	2,147	2,060
Miscellaneous expense	1,600	1,553	1,575
Total farm expense not labor	5,369	6,259	5,872
Total labor charge	2,735	2,802	2,769
Total farm expense	8,104	9,061	8,637
Investment, land	8,615	9,657	9,195
Investment, improvements	8,414	9,007	8,744
Investment, land and improvements	17,029	18,664	17,939
Machinery and equipment investment	6,475	6,430	6,450
Feed and crop investment	3,411	4,148	3,821
Productive livestock investment	5,177	5,214	5,197
Investment, not land and improvements	15,087	15,879	15,528
Total investment	32,116	34,543	33,467
Gross Income	11,084	13,400	12,373
Net Farm Income	4,515	5,894	5,211
Labor Income	2,909	4,167	3,506

Table 12. Estimates of Marginal Value Productivities of Categories of Inputs

86 Dairy Farms, Type-of-Farming Areas 5 and 6, 1950

Each category taken at mean value for all farms[1]

Category of Input	Equations 1	2	3	4	5
Total farm expense not labor			0.855	0.778	0.773
Total labor charge			0.564	0.513	0.493
Total Farm expense	0.705	0.635			
Investment land					0.039
Investment improvements					0.103
Investment land and improvements		0.052		0.056	
Machinery and equipment investment					
Productive livestock investment					
Investment feed and crops					
Total investment not land and buildings		0.232		0.235	0.231
Total investment	0.106		0.110		

1. The arithmetic mean is used because the values of the factors at which the marginal value productivities are estimated can then be taken from the farm business analysis reports as they stand. Use of the arithmetic means of the factors tends to bias the marginal value productivities of factors downward. The arithmetic mean is larger than the geometric mean in all cases and the data (factors) are subject to diminishing returns.

Table 13. Estimates of Marginal Value Productivities of Categories of Inputs

108 Not-Dairy Farms, Type-of-Farming Areas 5 and 6, 1950

Each category taken at mean value for all farms

Category of Input	Equations 1	2	3	4	5
Total farm expense not labor			1.216	1.143	1.081
Total labor charge			0.942	0.952	0.894
Total farm expense	0.999	0.981			
Investment land					0.034
Investment improvements					0.005
Investment land and improvements		0.002		0.003	
Machinery and equipment investment					
Productive livestock investment					
Investment feed and crops					
Total investment not land and buildings		0.290		0.318	0.242
Total investment	0.093		0.091		

Table 14. Estimates of Marginal Value Productivities of Categories of Inputs

194 Farms, Types-of-Farming Areas 5 and 6, 1950

Each category taken at mean value for all farms

Category of Input	Equations 1	2	3
Total farm expense not labor	1.112	0.916	1.150
Total labor charge	0.681	0.700	0.573
Total farm expense			
Investment land			0.100
Investment improvements			0.002
Investment land and improvements		0.069	
Machinery and equipment investment			0.228
Productive livestock investment			0.060
Investment feed and crops			0.340
Total investment not land and buildings		0.217	
Total investment	0.092		

Table 15. Average Marginal Value Productivities of Categories of Factors

86 Dairy Farms, Type-of-Farming Areas 5 and 6, Michigan, 1950

Marginal value productivity of each category calculated separately for each farm

Category of Factors	Average Marginal Value Productivity of a Dollar of	
Labor	Cash outlay or charge	$0.566
Total farm expense, not labor	Cash outlay or charge	$0.874
Total investment	Investment	$0.115

Estimates of marginal returns to factors other than land and labor are not much different from what one should expect. For total farm expense, and for the components of total farm expense, marginal returns appear to approach a ratio of 1 to charges. This could be interpreted to mean that the farmers in the study have been able to equate direct outlays to returns at the margin. If the estimates of elasticity coefficients are biased upward, then the marginal value productivity coefficients will actually be less than stated.

Some rates of return per dollar of outlay, which theoretically should be 1 at the margin, are estimated to be less than 1; i. e., total farm expense not labor. There are two possible reasons for this: (1) There are charges included in total farm expense which are fixed, such as depreciation on machinery, equipment, and improvements. The marginal return per dollar of stated charge on these items may tend to be low; that is, the stated charges are too high. The reason for this is that conservative bookkeeping demands that the rates be fully high enough to cover losses. (2) Because of federal income taxes farmers may reason that a dollar of cash outlay for running the business actually costs something less than a dollar. Expense items can be charged to the farm business and the amount deducted from taxable income. Thus a dollar spent for feed, fertilizer, or fuel conceivably could cost only $0.80 or $0.90, depending upon the per cent of the marginal personal income which is taken by income taxes.

The marginal returns to categories of factors show a general tendency to agree in the different formulations from the same data (tables 12 to 14). The marginal value productivity of labor is always less than the comparable figure for total farm expense not labor. Part of this may be due to a comparatively inadequate evaluation of the labor input. The charge for labor and the response of gross income to changes in the labor input are poorly correlated. If gross income is more highly correlated with the total stated farm expense not labor, then the expense not labor will take credit for part of the influence of the labor input on gross income. Thus the difference may lie in a poor evaluation of labor, not in a real difference between the marginal value productivities of the two categories of factors.

The estimated marginal returns to machinery investment are higher than called for by considerations of interest and depreciation. The return to machinery investment at the margin is estimated for 194 farms at 23% (table 14). If 10% is allowed for depreciation, then the interest return to machinery and equipment is high in comparison to other categories of factors. This could be interpreted in these ways:

(1) Greater uncertainty associated with the ownership of machinery and equipment than with the ownership of other assets such as land may account in part for the high marginal return to machinery and equipment.

(2) Because of risk and uncertainty, capital may be short relative to labor. In this case there is rationing of capital. The estimates of marginal returns to land and livestock, however, are not higher than would be expected, considering prevailing interest rates.

(3) The return to investment in machinery, because

of correlation with machinery expenses and net decreases, may be taking part of the statistical "credit" for the latter.

(4) The returns to investment in machinery at the margin may actually be high, as the estimates show. In this case the inference is that further increases in the relative importance of machinery and equipment may be expected. According to this interpretation, machinery and equipment as part of the assets on farms are below their equilibrium quantities, and adjustment continues.

9. Estimates of Net Income

The accuracy of the estimate of gross income is shown by the correlation coefficients between factor categories and gross income and the standard errors of estimate of gross income (tables 8 to 10). Estimates of gross income may be close enough that part of the variations in measures of net income of a farm can be explained on the basis of the way in which the factors are combined. If this is true, then one step in increasing the net income in the usual case would be to change the combination of resources employed on the farm. Practically, this could mean that there could be too much invested in land in relation to the investment in livestock and the charge for labor, and so forth.

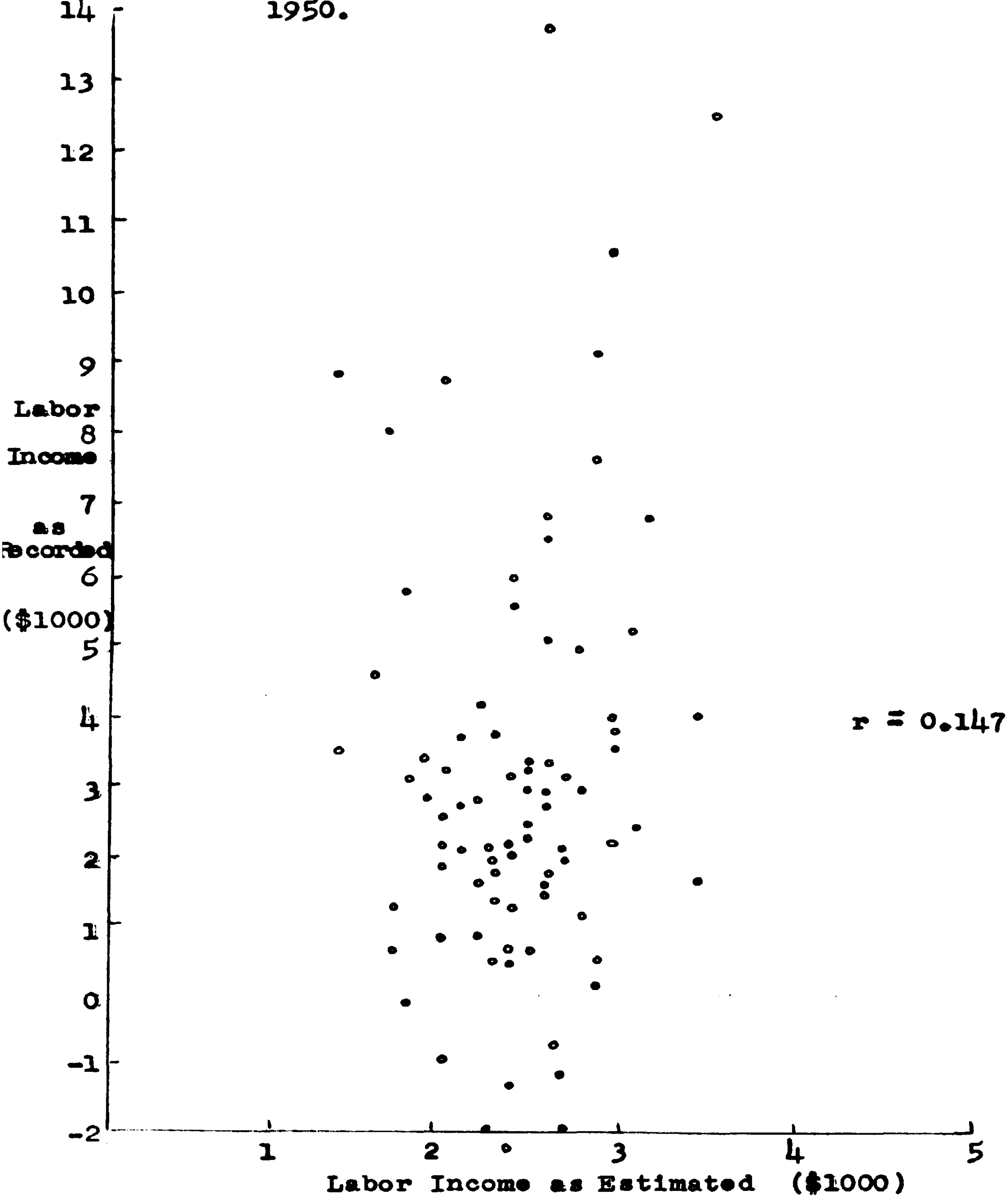
"Net farm income" is estimated by subtracting from the estimate of gross the "total farm expense" other than the operator's labor. This estimate of net farm income for the 86 dairy farms was correlated with the net farm income given by the farm account books. The simple correlation coefficient was 0.49. This correlation does not tell much. It states that, if one should choose to earn a larger return to both the operator and the farm, he should increase the variable factor, the farm. Nothing is said about whether the adjustment will bring in enough additional income to cover the additional interest on the increased investment in farm.

If one will accept "labor income" as a measure of what the farmer seeks to maximize, the following question may be raised:[1] Is the estimate of gross income close enough that, when charges are deducted, a useful estimate of labor income is also obtained? This hypothesis is rejected for the 86 dairy farms when labor income as given in the farm accounts is used as the standard of comparison (figure 1). The variations in gross income which are not accounted for by the manner of the combining factors are so great that the residual of the estimated gross income less charges by the definition of labor income bears no significant relation to the labor income given in the farm accounts.

1. Labor income is gross income less total farm expense other than the operator's labor less interest on investment at 5%.

Figure 1. The Relation between Labor Income as Recorded and Labor Income as Estimated from a Value Productivity Function.

86 Dairy Farms, Type-of-Farming Areas 5 and 6, 1950.



The failure of this relationship may arise from one of at least three sources: (1) The estimate of gross income is not good enough for the purpose implied by the hypothesis. (2) Windfalls tend to exaggerate differences in the farm account labor income between farms, and in erratic ways. (3) Some standard charges which are given by the definition of labor income and by accounting procedures are not appropriate. For example, the charge against the investment is 5%. Possibly this charge should vary for the average of farms between types of investment, and vary again between farms for investment of the same types if a more accurate measure of what the farmer seeks to maximize is to be obtained.

"Labor income" is conditioned by the three items above. The effect of the definition is to charge items other than the operator's labor at cost, or at standard or assumed rates, and then to assign all of the difference between charges and gross income to the farm operator. In competitive equilibrium, the return to labor would be its marginal return times the quantity employed. "Profit" would be zero. The returns to all factors, the others also being valued at marginal value productivity times quantity used, would absorb the whole gross income. Factors would all be charged at their market prices as the factors would be used in such quantities and proportions that the marginal value productivities would equal the respective prices.

The business of farming is competitive, but it is never in equilibrium from the standpoints of weather, prices, technology, and biology. Thus, the marginal returns to some factors on a farm are greater or less than implied by their prices or stated costs. All of the residuals resulting from disequilibrium are thrown together in "labor income."

This is not to say that labor income is not an extremely useful concept. It is the pay of the operator plus the residuals from the actual costs and returns attributable to all other factors. Management is an unmeasured input. That some farmers are able to earn high labor incomes over a period of years suggests that the factor of management often is important enough as a contributor to all of the elements in labor income that there is actually a high correlation between labor income and the returns to the operator in the theoretical sense.

Let it be assumed, for the moment, that labor income fairly represents the concept which farmers try to maximize. Then we are interested in whether differences in labor income can be explained in part by the "structure" of the farm business. By structure in this case is meant the quantities and proportions in which categories of productivity factors have been combined. It has just been shown that the estimate of gross income for the 86 dairy farms was not close enough, generally, that, when costs were subtracted according to the definition of labor income, the "structure" actually helped to explain part of the differences in labor income. The failure of correlation, however, can not be assumed to be caused altogether by shortcomings in the estimate of gross income from the gross income function. In other words, there is error in the estimate of labor income by the function <u>and</u> by the statement of labor income according to the farm account record.

Estimates of the labor incomes of the 108 <u>not-dairy</u> farms were made from the gross income equation given in the second column of table 6. These estimates <u>were</u> correlated at the 5% level of confidence with the recorded labor incomes, the simple coefficient of correlation being 0.38. Thus, part of the differences in the labor income of the 108 not-dairy farms can be explained on the basis of the manner in which the categories of factors are combined. It is to be noted in this connection that the correlation coefficients between the recorded gross incomes and the estimates of gross income are slightly higher for the not-dairy farms than for the dairy farms (tables 8 to 10).

10. Estimates of Labor Income When Categories of Factors are Charged at Marginal Value Productivities

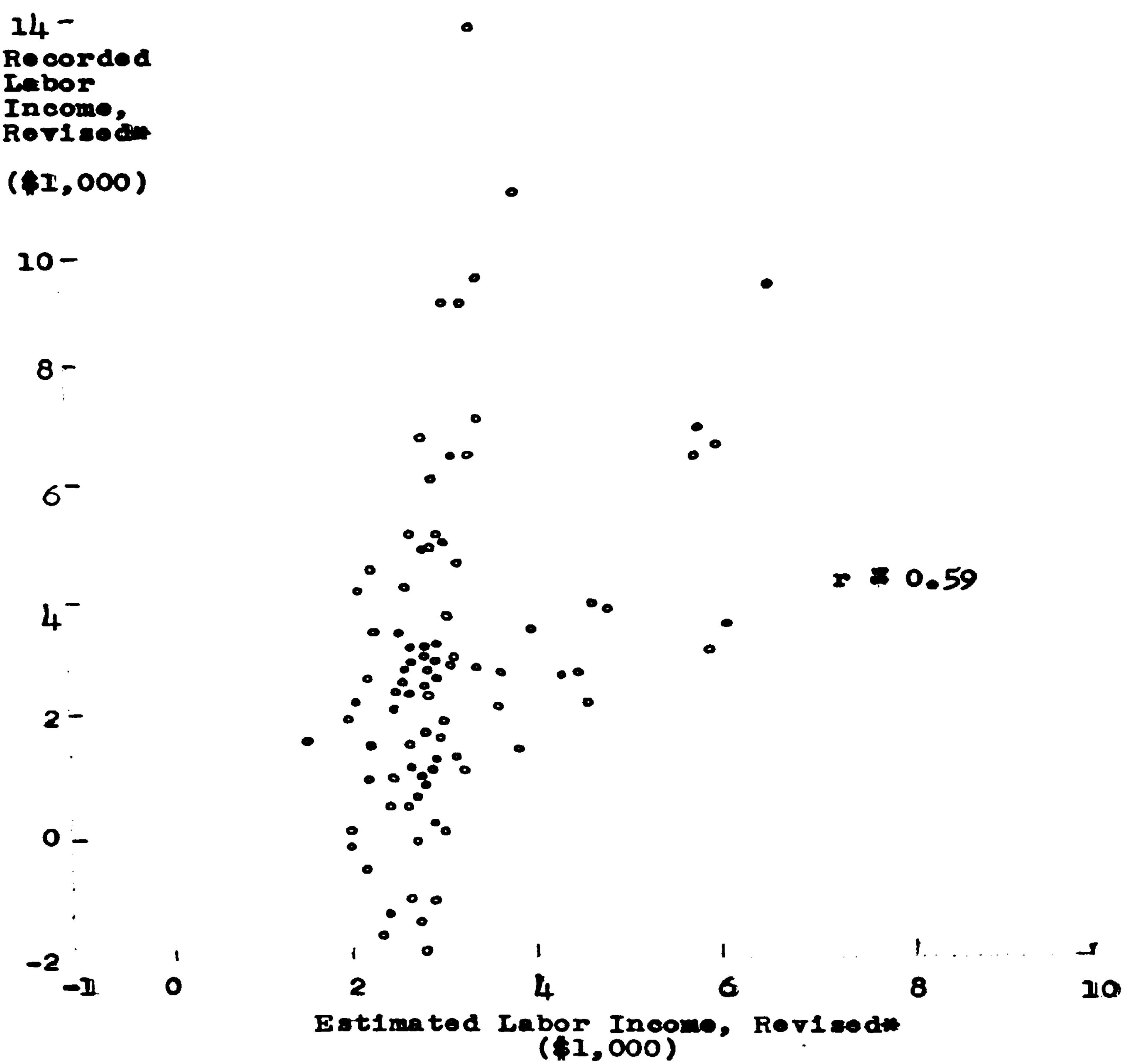
The hypothesis was set up that farmers have different concepts of marginal costs depending on personal objectives and circumstances. This should result in the use of resources in ways which will tend not to maximize labor income as defined by the standards of farm accounting. That is, farmers will have different concepts of the marginal costs of the same factors. The marginal costs for categories in the following estimating equation were calculated for each of the 86 dairy farms:

$$\text{Log } \underline{P} = 0.422 + 0.432 \text{ log total farm expense not labor} + 0.145 \text{ log total labor charge} + 0.332 \text{ log total investment (table 5).}$$

This is the same equation as used in the previous estimate of labor income. When labor income was calculated by charging factors against the business according to their estimated marginal value productivities applying to the particular farm, the relationship between stated labor income and estimated labor income was significant at 5% (figure 2). By charging factors according to their estimated marginal value productivities rather than at standard rates, the estimates of costs are changed. Costs are thus subtracted at different rates for each farm from both the farm-account recorded gross income and the estimate of gross income from the function. When costs are valued in this way, the differences between recorded and estimated gross income are not so great but

Figure 2. The Relation between Recorded Labor Income and Estimated Labor Income when Factors of Production Are Charged against the Business According to Their Marginal Value Productivities.

86 Dairy Farms, Type-of-Farming Areas 5 and 6, 1950.



* See title.

that the two measures of gross less costs are completed. The average value productivity function thus explains part of the differences in this measure of net between farms.

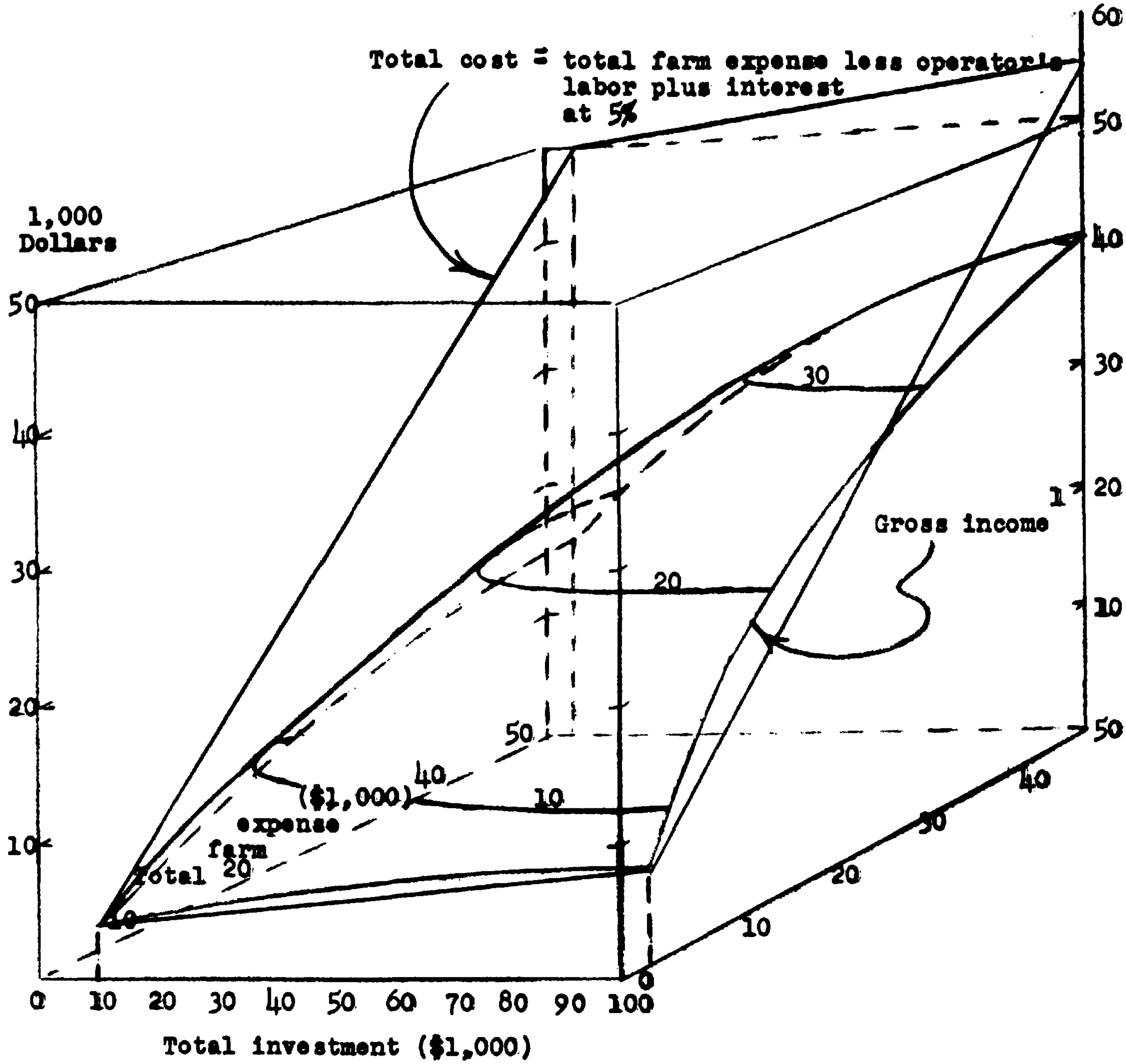
11. Graphic Presentation of the Estimate of Gross Income and Costs

If the number of categories of factors is two, the gross income estimating equation can be shown in three dimensions (figure 3). The equation of the curved surface is the first equation in table 5 expressed in natural numbers. Deductions from gross income for the estimate of labor income are given by the total cost plane. Part of the area on the curved surface lies above the plane; gross income is larger than total charges, including the operator's labor. At the $100,000 level of investment, for example, income lies above expense between the $5,000 and approximately the $30,000 levels of total farm expense.

This concept can be seen more clearly on a contour map (figure 4). Gross income is shown as a function of total farm expense and total investment. The contours indicate different levels of gross income, and the parallel lines indicate levels of total recorded costs. Total recorded costs are total farm expense plus 5% on the total investment. The farm operator who has a given amount of total resources is interested in using them in such forms as to maximize the difference between the

Figure 3. Gross Income Estimated from Total Farm Expense and Total Investment, and Total Costs, Including Interest on the Investment at 5 % but Not Including Operator's Labor.

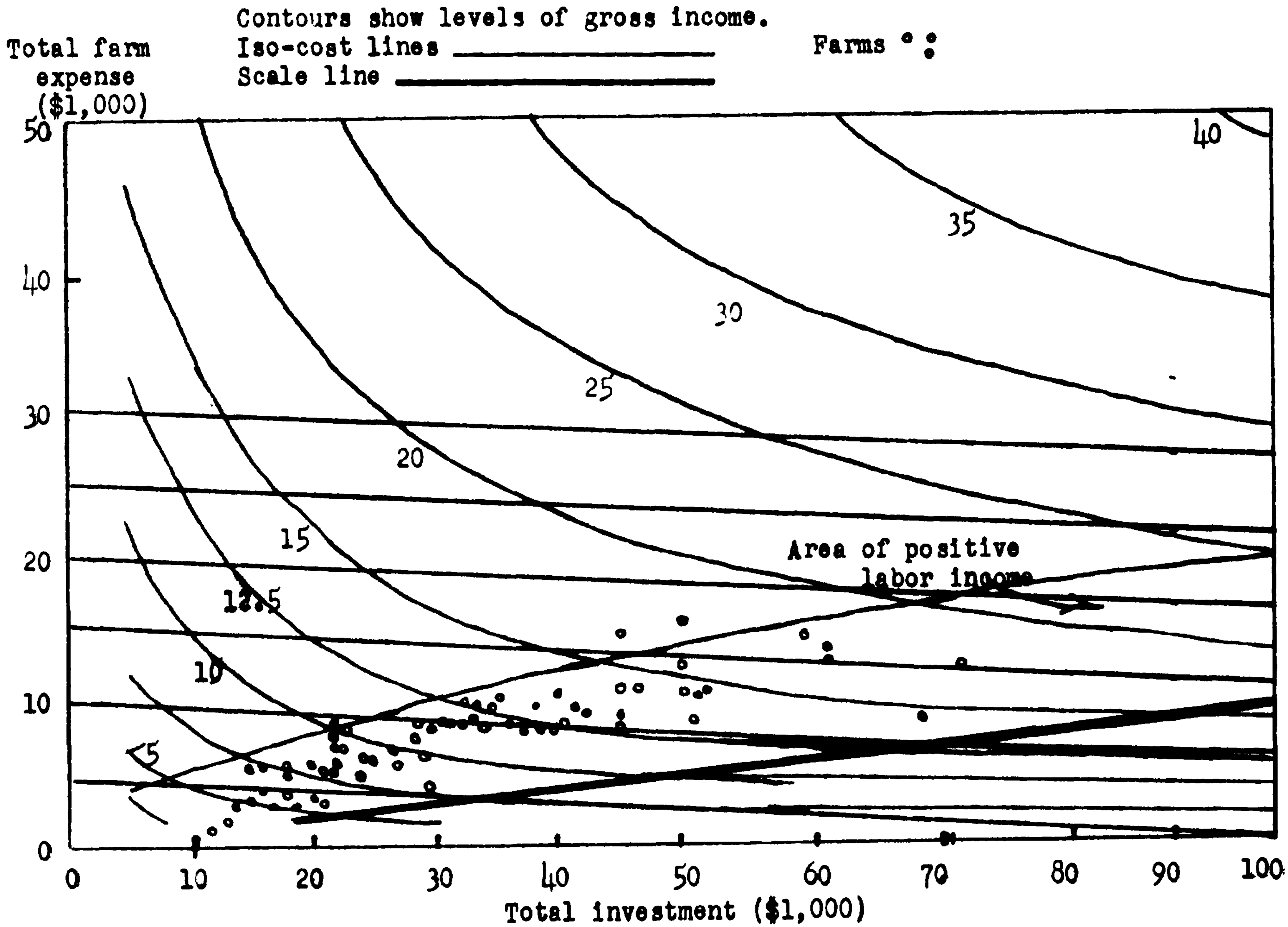
86 Dairy Farms, Type-of-Farming Areas 5 and 6, 1950.



1. Gross income is estimated by 2.87(total farm expense)$^{0.541}$ times (total investment)$^{0.322}$. The contours show $10,000, $20,000, and $30,000 levels of gross in come.

Figure 4. Gross Income Estimated from Total Farm Expense and Total Investment

86 Dairy Farms
Type-of-Farming Areas 5 and 6, Michigan, 1950



curved gross income surface (contours) and the parallel lines (the plane of total costs). The "scale line" indicates the amounts of "total farm expense" and "total investment" which would on an average maximize the net return to the farm and the operator for different values of total outlays plus charges. The positions of the farms with respect to investment and total farm expense are plotted. They do not follow the "scale line" which is developed from taking "total farm expense" as charged, and taking interest on investment at 5%. Instead, they emphasize expense more, and investment less.

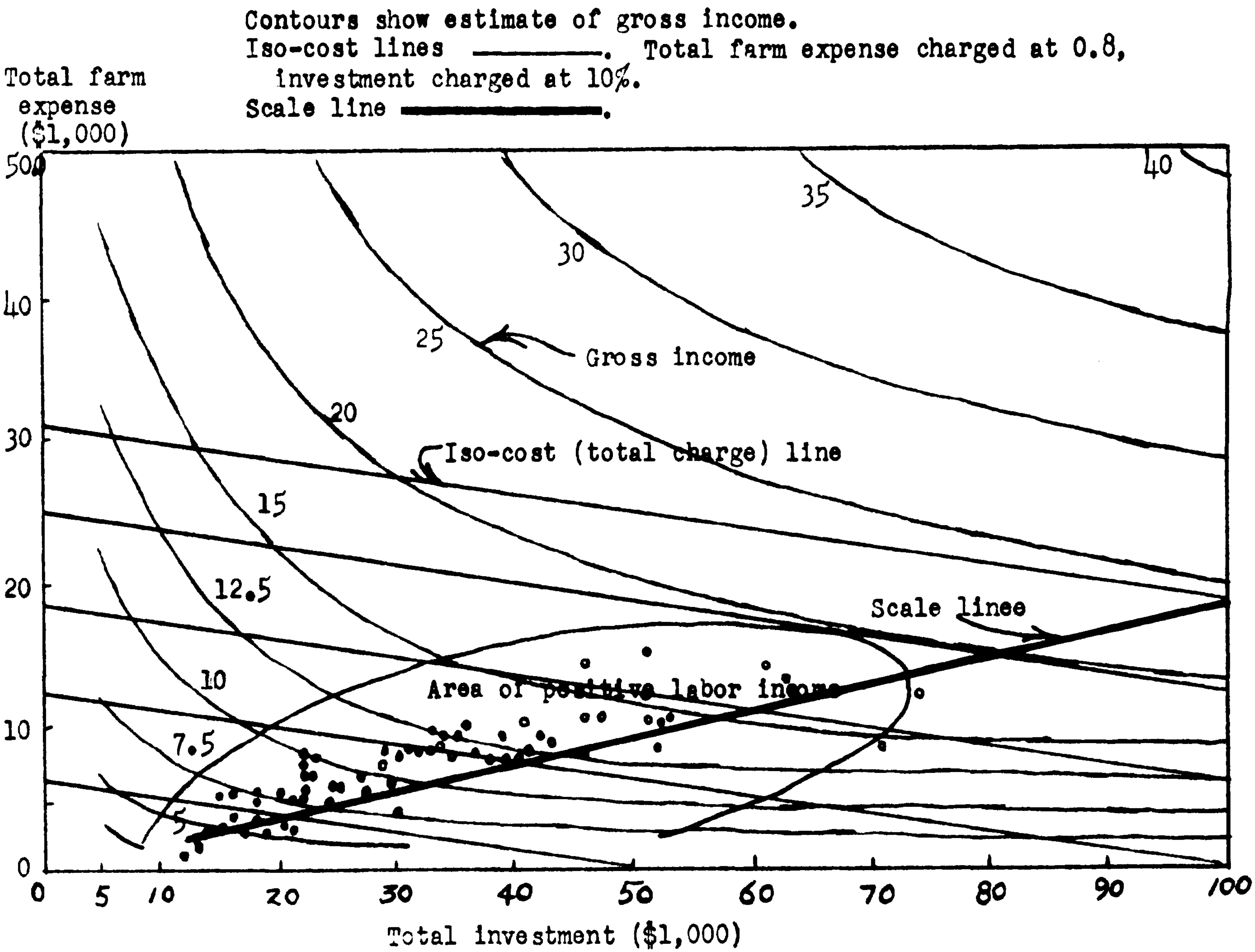
Furthermore, at the $100,000 level of investment, the area of net return to both the farm and the operator has hardly, if at all, begun to diminish. If the figure were to be continued, in fact, it would show that the area of a positive net return would not be closed off altogether with respect to investment until a level of $300,000 was reached. This result is not consistent with common observation. Either the gross income estimating equation or the definition of charges or both are in error.

It is recognized that farmers may not necessarily have the charges for total farm expense items which are entered in the farm account books in mind when they undertake outlays for factors in the total farm expense category. Nor are they necessarily equating the marginal return on invested capital to the standard rate of 5%.

The evidence on marginal value productivities of these two categories of factors indicates that at the margin the return to total farm expense is in the neighborhood of 80¢ for a dollar of outlay or charge entered in the farm account books (table 14). For all investments taken together the return at the margin is about 10%. If farmers equate subjective costs and returns at the margin, then these marginal rates of return may be interpreted as measures of the proportions of the stated values of the factor categories which they have in mind in the organization and operation of the farm business.

If total farm expense is subjectively valued at only 0.8 of the amount entered in the farm account books at the margin and total investment is charged at a rate of 10% at the margin, a new set of iso-cost lines appears, with new points of tangency with the gross income contours. Therefore a new "scale line" or line of best allocation of total resources between permanent investment and outlays (including charges for non-cash expense items) appears (figure 5). On figure 5 it appears that under average conditions the area of net return above all costs is closed off in the neighborhood of $70,000 of total investment. Plotting of positions of the 86 dairy farms with respect to total farm expense and total investment shows that they more closely follow the new scale line than the scale line arising from taking expenses at dollar value and

Figure 5. Gross Income Estimated from Total Farm Expense and Total Investment
86 Dairy Farms
Type-of-Farming Areas 5 and 6, Michigan, 1950

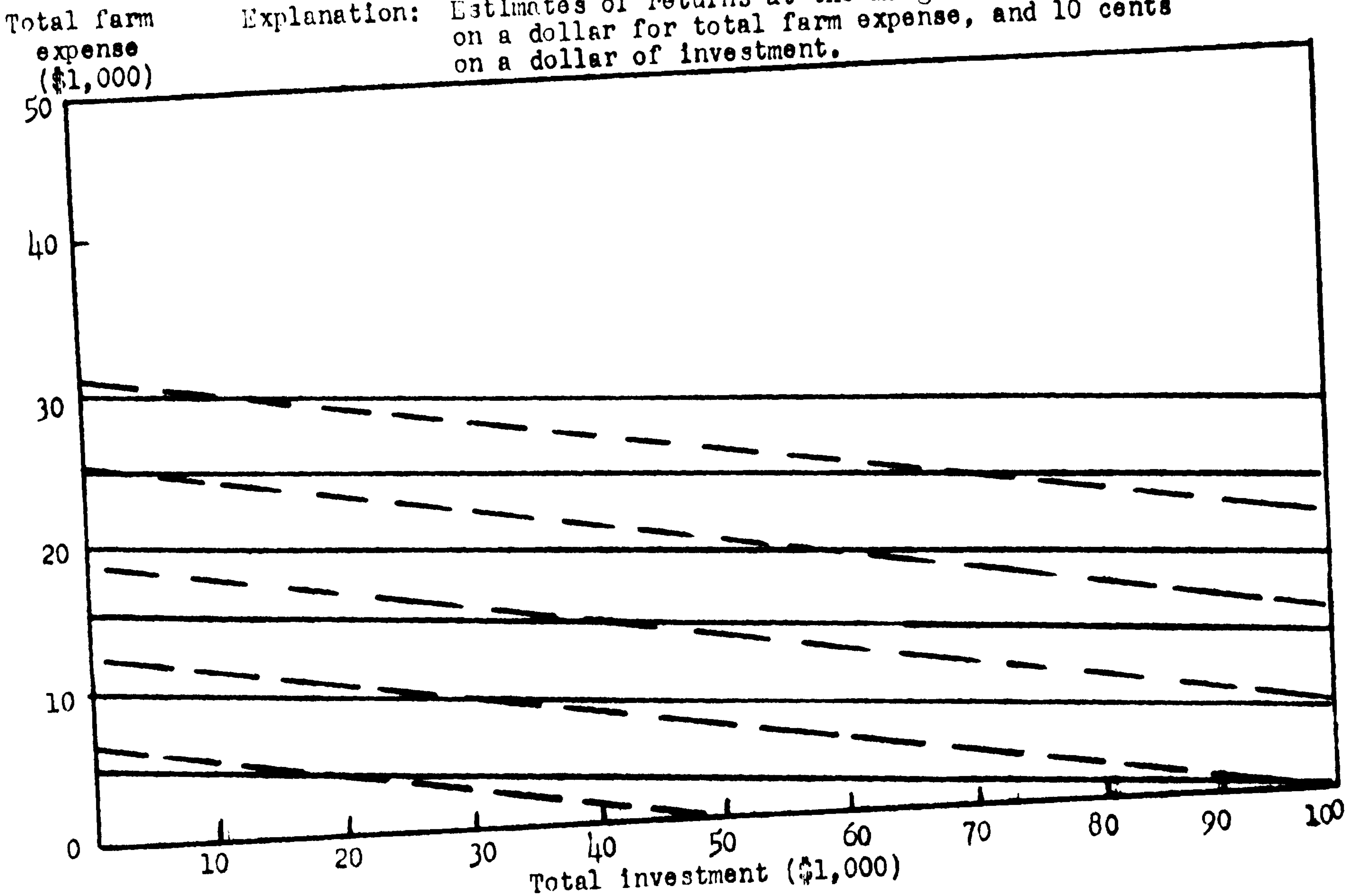


charging interest at 5% (figure 4). In other words, charging total farm expense at 0.80 on the dollar and charging interest on the investment at 10% are consistent with the actual business behavior of farmers. It may be clarifying to show the positions of iso-cost lines under the two hypotheses in a diagram which is not complicated by the presence of the gross income contours. This is done in figure 6.

As was stated earlier in this chapter, two of the factors which cause variations in gross income (and hence in labor income) are windfalls and management. Gross income and labor income as defined by the standard procedure include the former as a contribution to gross. Theoretically the residual due to management appears in both measures also. If the estimate of gross income is derived from a gross income equation, the windfall gains and losses and the difference due to management are both eliminated. Management may be regarded as a random element affecting the value of gross income as calculated by accounting. When gross income is estimated from a regression equation, the influence of management upon gross income is lost. Possibly the definition of labor income that is commonly accepted includes irrelevant elements which tend to cause the labor income figures between farms to show greater variations than in a real sense exist.

Figure 6. Effect on Iso-Cost Lines of Charging Total Farm Expense at 0.8 of Reported Amount and of Charging Total Investment at 10%

Explanation: Estimates of returns at the margin are 80 cents on a dollar for total farm expense, and 10 cents on a dollar of investment.



11. Summary of Chapter IV

The information in this chapter can be summarized under six headings, which are as follows:

1. By using categories of factors given in the farm accounts, the log of gross income of a farm is estimated with a standard error of about 0.1.

2. The farms are estimated to have been operating at constant returns to scale. The sums of the elasticities of gross income with respect to categories of factors are slightly more than one when the number of categories of factors is larger than four.

3. The confidence limits of the coefficients of elasticity are relatively wide, even at the 10% level of reliability. The ranges of the confidence limits of the coefficients are not narrower for 86 dairy farms than they are for all other farms, regardless of type.

4. Marginal returns to categories of factors are estimated. The estimates are generally reasonable, although the confidence intervals of the elasticity coefficients are wide.

5. Labor income as defined by farm accounting procedure can not be predicted from the gross income equations, subtracting total farm expenses and interest on the investment at 5%. The estimate of gross income is useful in explaining part of the differences in income between farms when charges are made at the marginal rates implied on each farm by the general gross income equation. From this

it is inferred that individual farm operators seek to maximize different forms of "income," and these are to an extent revealed by the marginal returns demanded from the use of the categories of factors.

6. In the matter of the choice between total farm expense and investment, the farms are operated according to charges for categories of factors at calculated marginal rates of return. That is, the disposition of total resources between investment and expense on actual farms agrees better with charges against total farm expense at 0.8 on the dollar, and a charge on total investment of 10%.

CHAPTER V

USE OF GROSS INCOME FUNCTIONS IN THE FARM MANAGEMENT EXTENSION PROGRAM

The work in the preceding chapters shows that consistent regression equations estimating gross income can be derived from farm account data. It follows that gross income estimates could be included as a phase of farm account work. But the preceding part of this study has shown that there are pitfalls of the regression analysis which may lead to wrong conclusions. Thus, the first part of this chapter will be concerned with methods of increasing the reliability of estimates and confidence in the meaning of estimates. The second part will take up uses which might be made of gross income equations in the farm account program.

1. Suggested Changes in the Data and Method of Analysis

It has been shown that there may be biases in the valuations of factors used in computing gross income equations. These biases enter into the calculation of labor income and the cross sectional analysis of labor income. By the usual cross sectional analysis, labor income is related to crop yield, efficiency of man labor, size of business, and other factors of success.

The valuations should be as free from biases as practicable if greatest use is to be made of gross income equations. In the cross sectional analysis explaining differences in labor income according to farm success factors, it is ordinarily taken as conclusive if a general relationship between labor income and success factors is shown. In the computation of gross income equations, however, and in the estimation of marginal value productivities of factor categories, biases lead to wrong numerical estimates of definite concepts. Thus, the undervaluation of land, for example, has the effect of overstating the marginal value productivity of land at its market value in direct proportion to the undervaluation. The undervaluation of land ordinarily would not affect the conclusions indicated by cross sectional analysis. But the bias is more serious in the case of analysis by the gross income function.

In addition to general biases in the data, there may be random inconsistencies between farms. These inconsistencies have the effect of causing the confidence intervals of regression coefficients to be wider than is actually called for by the nature of the underlying data. Thus, there are two reasons for a general consideration of absence of bias and consistency of valuations of factors in the farm account books. It is recognized that this is not an easy problem. In the case of land, elimination of bias would call for periodic revisions in the values of farms. The main purpose in avoiding such changes is to prevent changes in land value from contributing to labor income. Feeds and crops are regarded as more liquid, and changes in their prices in inventories are permitted to affect labor income.

As an alternative to the keeping of land and livestock values more or less in line with markets, the values in the farm account books could be specially adjusted for computing gross income equations. This procedure has the disadvantage of implying two schemes of values and would result in some loss of confidence in the whole analytic work. Furthermore, the procedure of adjustment of the data for one purpose (gross income equations) and not another (analysis of labor income by success factors) would call for a complicated interpretation of both results.

<u>2. Selection of Categories of Factors</u>

Consideration could be given to the types of data obtained by the farm account records and to the manner of summarizing the data. The categories of factors used in computing gross income equations in this study are not the "factors" of economic theory, land and capital. <u>Land</u> and <u>capital</u> in economic theory must be considered as factor categories themselves, as well as items such as <u>machinery</u> which appear in farm account books. The question of interest concerns the manner of combining factors into

into categories to obtain meaningful equations of gross income. The two broad categories, for example, total farm expense and total investment, are concerned with a practical problem--the problem of the ratio of the permanent investment in the business to the outlay in one period of operation. In order to avoid combining expense and investment factors, this two-way breakdown should be maintained as a fundamental division of factors into classes. Within these two classes expenses and investment can be broken down into further detail.

Within each of the two broad classes of factors there can be several <u>categories</u> of factors. These categories should be designed so that the factors <u>within</u> each are nearly perfect complements or nearly perfect substitutes. When factors are complementary, the use of one implies a value in the use of the other. For example, gasoline and oil are complements. Factors are substitutes when one can be used to replace the other. Family labor and hired labor are substitutable and could be included in the same category, labor. Livestock feed and labor are neither complements nor substitutes and should be in different categories.[1] The Michigan farm account system factors are generally combined into categories which thus consist largely of complements and substitutes. When gross income is estimated from categories of factors, it is important that this be so. This kind of scheme of

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1. At the level at which factors ordinarily are combined for accounting purposes.

classification makes it possible to determine elasticities of gross income and marginal value productivities by factor categories which are easily understood. Furthermore, it implies mathematical results which should distinguish between categories of factors according to elasticity of gross income and marginal value productivities.

The estimates of marginal returns to investment and expenses are sometimes confused or duplicated. Consider improvements and machinery and equipment. Depreciation on both items is included at standard rates in "improvement expense" and "machinery and equipment net decreases" in the farm accounts. Thus depreciation is included as an expense for which the return at the margin would theoretically be one in equilibrium. That is, the farmer will equate marginal costs and marginal returns of machinery and improvements at the margin. Marginal costs will include depreciation.

At the same time the farmer will equate marginal costs and marginal returns of investment in machinery and improvements. The marginal costs of these items include depreciation. Therefore, when depreciation is included as an expense item of machinery in the estimation of gross income, it is included twice, or duplicated. Thus, depreciation on investment items should not be included in expense in the computing of gross income equations, as was done in the previous work. There are uses of gross income equations which call for data outside of that necessary for pure accounting of income and expense. These data include acreages and yields of crops, production rates of livestock, and average prices of products sold. Some of this information is already available. This type of information would be useful in connection with gross income equations when the equations are used to account for differences between estimated and stated gross income and labor income by enterprises. The place of physical units of production, average yields or rates of production, and prices of prdducts sold will be made evident in part three of the second section of this chapter.

3. Suggested Uses of Gross Income Functions in Farm Business Analysis Reports

This section will show how analysis of gross income as a function of factors can be included in publications based on farm account records.

Part I. Estimates of gross income and labor income

One of the main purposes of farm business analysis is to explain differences in the profitableness of farms. When such explanations are based on groups of farm account records, a long-used procedure has been to show correlations between farm success factors and labor income. This procedure is based upon the assumption that labor income as defined is usually a good estimate of what farmers seek to maximize. It was pointed out in Chapter IV that labor income is not a perfect measure of income, and furthermore, that it is a measure of the sum of all the residuals above recorded costs, including, of course, the residual above the recorded charge for the operator's labor. Thus, its use as an absolute measure of the return to the operator can be questioned. However, labor income has long been used, is widely understood, and is easily defined. If labor income is to be treated as if it were the measure of returns to the operator, gross income equations can be considered from the viewpoint of contributing to the explanation of differences in labor income.

If the estimate of gross income is precise enough, part of the differences in labor incomes can be explained on the basis of the way in which factors are combined in the operation of the farm. Following this, further differences can be explained on the basis of the farm success factors. The logic of this order is that first the efficiency of the organization of the farm business as a whole (the general layout) is considered. Then more detailed questions are taken up. These questions are concerned with crop yields, efficiency of labor, and so on. This does not imply that one phase of the problem of differences in labor income is more important than another. It implies that first there must be a general plan (proper combination of resources) of a farm; secondly, the effectiveness of the plan will depend in part upon how efficiently the combination of resources performs, once given.

Specifically, a high crop yield, for example, if the farm is to be profitable, must be obtained subject to the restriction imposed by a practical limit to the ratio of the outlay for growing the crop to the investment in the land upon which it is grown. In order to increase labor income, crop yields are not increased regardless of the direct outlay involved. The balance between land investment and crop expense which is most favorable to labor income in the general case can be determined by the use of a gross income function based on factors of production. The factors include land and crop expense.

As another illustration, a gross income function may be used to define how the number of productive man work units per man will reach an optimum. The investment in a farm and the outlays other than for labor obviously can not be increased without limit while holding the amount of labor constant. That is, productive man work units per man are subject to diminishing returns, if labor is held constant. The available labor is spread too thin in relation to other factors. The gross income function will show how, for most farms, resources should be divided between labor, land, machinery, livestock, and so forth, in order to maximize labor income.

It is recognized that the earning of a high labor income is contingent upon the farm's having generally favorable ratios in regard to most or all of the farm success factors. It is also recognized that the farm success factors are in some ways in conflict with one another. To illustrate, a large number of productive man work units per man may be achieved at the expense of the care with which the work is done. The gross income estimating equation will lay down the general conditions for the concurrent maximization of labor income with respect to all of the farm success factors.

All of this is a way of pointing out to farm account cooperators how one of the reasons for a high labor income or a low one may be associated with the balance between the factors of production. This means that there may be too little or too much labor in relation to the investment in land, too little or too much outlay for direct operating expenses in relation to the total investment in the farm, and so on.

The results of the foregoing analysis could be presented in a table, as on page 96. (table 16).

Notwithstanding crop yields, production rates of livestock, and so on, the labor income of this farm is estimated to be $307 higher than the average labor income which could be expected from $16,092 of all costs, simply because of the way in which total farm expense and investment are combined. If the relationship between the estimated labor income and the stated labor income is statistically significant, the table on the next page (table 16) will be of interest to the cooperating farmers.

Table 16. Analysis of Recorded Labor Income According to Estimate of Gross Income[1]

Your Farm

Item	Recorded	Estimated	Gross income at average returns per dollar of total farm expense plus interest @ 5%
Gross income	$22,362	$19,820	$19,513
Total farm expense	12,602	12,602	16,092
5% interest on investment	3,490	3,490	
Labor income	7,830	3,728	3,421
Difference from expected return			
Gross income	2,849	307	
Labor income	4,409	307	

1. The estimating equation is the first one in table 6. The farm is in Jackson County.

The explanation of difference in labor income by the combination of resources admittedly is small--$307 of a total difference of $4,409 ($7,830 minus $3,421). This explanation, however, is legitimate in that it does not rest upon crop yields, production rates, dollars income per dollar of expense, and so on, which are themselves prejudicial of gross income and hence of labor income. This table (16) is most important, however, because it gives a statement of the gross income and of the labor income which the farmer <u>should have expected</u>, considering his investments and outlays. For farmers with low labor incomes in particular, the table should stimulate inquiry into <u>why</u> gross income and labor income do not measure up to standard. This phase of the study is developed in Part III of this section.

In Chapter IV it is shown that for the 86 dairy farms the correlation between recorded labor income and labor income from the estimate of gross income is not significant at 5%. For the 108 not-dairy farms the coefficient is 0.38, which is significant at 5%. This is a low correlation, as is expected. However, it is a meaningful correlation. It shows that <u>part</u> (albeit a small part) of the differences in recorded labor income <u>can</u> be explained from equations estimating gross income.

Part II. Estimates of Marginal Value Productivities

Estimated marginal value productivities can be calculated for all farms for categories of factors used in the gross estimating equation. One purpose of estimates of marginal value productivities of categories of factors is that they show conditions of imbalance in the use of resources on the individual farm. Consider a farm which uses too much land in relation to the amount of machinery and equipment, according to the gross income function. This will show up in the form of a high marginal value productivity for machinery and equipment and a relatively low marginal value productivity for land. The implication is that the farm is underequipped for its size, or is too large for the amount of equipment.

Estimates of marginal value productivities at the mean values of factors for farms classified according to labor income can be shown (table 17). The breakdown is made by labor income groups for this reason: marginal returns to factors *may* vary with labor income because of variations in the quantities of factors employed at different levels of labor income. It is to be noted that marginal value productivities tend to fall as labor income (and the expense and investment categories) become larger. This warns the person whose labor income and outlays are already high that he can expect relatively smaller returns at the margin. Table 17 indicates the lines of investment and expense which, according to

Table 17. Amounts and Mean Marginal Value Productivities of Categories of Factors for Farms Classified According to Labor Income[1]

All Farms, Area 5, 1950

Category of Factors	1/3 lowest farms		All Farms		1/3 highest farms	
	Amt.	MVP[2]	Amt.	MVP	Amt.	MVP
Land and improvements	$15,827	7%	$17,446	7%	$20,923	7%
Total labor charge	2,429	$0.83	2,641	$0.70	3,123	$0.75
Total farm expense other than labor	5,007	0.91	5,730	0.92	7,305	0.88
Investments other than land and improvements	12,629	23%	15,215	22%	20,191	21%
Labor income	746		3,687		7,068	

1. The estimating equation is 2 in table 7, Chapter IV.

2. Marginal Value Productivity

average experiences should be expanded or contracted. The average marginal returns to machinery, equipment, livestock, feed and crops as a group are larger for all labor income groups than called for by accepted interest and depreciation rates.

The marginal principle can be demonstrated by a series of tables in each one of which all factors but one are held constant. Such tables will show how the marginal value productivities of the constant factors increase as the amount used of the varied factor becomes larger. The marginal value productivity of the varied factor will decrease as its quantity is increased (table 18). The example given in table 18 was selected at random.

Tables 17 and 18 show how estimates of marginal value productivities can be expressed without computations farm by farm. If the clerical help is available, computations can be reduced to a routine which can be handled readily by clerks with no statistical training. The writer's experience has been that the clerks, if carefully instructed as to the meaning of the work, are most cooperative because they are eager to get to the results which they themselves can anticipate. The possibility that cooperating farmers might calculate the estimates of marginal value productivities for their farms may be considered. This might be too difficult from the standpoint of the mathematics involved. However, the proportion of farm operators who have been through high school is constantly increasing, and most of these people have had some train-

Table 18. Estimated Marginal Value Productivities of Categories of Factors for Different Investments in Productive Livestock[1]

163 Farms, Type-of-Farming Areas 5 and 6, 1950

(Other Categories of Factors Held Constant at Their Average Values)

Category of Factors	Charge or Investment (average)	Investment in Productive Livestock			
		$1,000	$2,500	$5,000	$7,500
Land	$8,775	0.008	0.091	0.094	0.096
Improvements	8,578	0.001	0.001	0.001	0.001
Total labor charge	2,641	0.525	0.545	0.560	0.569
Total farm expense other than labor	5,780	1.064	1.104	1.135	1.153
Productive livestock investment	(varied factor)	0.257	0.107	0.055	0.036
Machinery and equipment investment	6,266	0.209	0.217	0.223	0.227
Investment in feed and crops	3,607	0.306	0.318	0.327	0.332

1. The estimating equation is 3 in table 8, Chapter IV, for 194 farms.

ing in algebra. Furthermore, the Smight-Hughes program in the high schools could well afford to devote considerable time to teaching boys how to estimate what their farms ought to earn considering their inputs.

Computations of the estimated marginal value productivities of categories of factors for individual farms can be shown as in table 19 (page 103). This farm is short on land, machinery, and feed and crop inventories. Its marginal returns to these factors are comparatively high. "Expenses" and investment in productive livestock are high, and the marginal returns are lower than average.

It is not necessary that the average of all farms be used as the standard of comparison. The one-third farms with the highest labor incomes can be used; or farms can be classified according to size, and standards established for the various size groups.

Part III. Effects of Yields, Rates of Production, Prices, and Size of Business on Variations in Gross Income Between Farms

The recorded gross incomes of farms vary above and below their estimated values. It is of interest to account specifically for these variations by yields and rates of production, prices received, and the size of business.

A simplified example will be considered first. Suppose that the business of a farm consists of one enterprise, the production of whole milk. The essential data

Table 19. Categories of Factors and Their Estimated Marginal Value Productivities

Category of factors	163 Area 5 Farms		Your Farm[1]	
	Average amount	Estimated marginal value productivity at average amount	Amount	Estimated marginal value productivity
Land	$8,775	0.096	$10,645	0.138
Improvements	8,578	0.001	16,094	0.001
Total labor charge	2,641	0.561	4,075	0.524
Total farm expense other than labor	5,780	1.134	10,983	0.857
<u>Investment in:</u>				
Productive livestock	5,243	0.053	19,279	0.036
Machinery and equipment	6,266	0.223	6,180	0.440
Feed and crops	3,607	0.327	3,197	0.569

1. A Calhoun County Farm.

of the farm are given in the table below:

Table 20. Data Needed to Account for Difference in Recorded Labor Income from Estimated Labor Income on a One-product Farm

This Farm

Recorded gross income	Recorded total farm expense plus 5% on investments	Production of milk per cow (cwt.)	Price of milk per cwt.	Average number of cows	Labor income
$4,000	$3,000	50	$4.00	20	$1,000

All Farms

Estimated gross income	Total farm expense plus 5% on investments[1]	Production of milk per cow (cwt.)	Price of milk per cwt.	Average number of cows	Labor income
$5,000	$3,100	60	$5.00	16 2/3	1,900
Difference in labor income on this farm from expected value considering inputs:					$ -900

1. From the average relationship (for all farms) between average gross income and total farm expense plus 5% on investment. The ratio expressing this relationship is 100/62. Because of the arrangement of factors on this farm, the gross income is estimated at $5,000. According to the average relationship between gross income and total farm expense plus 5% on investment, a farm with $5,000 gross income is estimated to have a total cost (total farm expense plus 5% on the investment) of $3,100. In other words, structurally this farm is somewhat "better" than average.

With average milk per cow at 6,000 pounds and the average price of milk at $5.00 per hundredweight, in general a farm would need an average of 16 2/3 cows to earn the gross income estimated for this farm. It is observed that the total farm expenses plus five percent on investment are different in order to obtain equal estimates of gross income for the average of all farms and for this farm. The particular farm is in a more favorable position on the gross income function from the standpoint of maximizing labor income than is the average of all farms.

There is a difference of $1,000 in gross income to be accounted for. The average rates of production, prices, and number of cows which a farm would need in order to earn the estimated gross income can be taken as the standard of comparison. Then the difference in gross income is given by:

1. Number of cows (other farms) times average price times difference in production rate for this farm plus

2. Average production times number of cows (other farms) times difference in price for this farm plus

3. Average production times average price times difference in number of cows for this farm plus

4. Three cross-products involving two differences and one average value plus the cross-product involving only the three differences.[1]

Numerically this is given as follows:

1. See Appendix K.

16 2/3 (cows)x$5.00x(-10(cwt.) plus

60(cwt.)x16 2/3(cows)x$1.00 plus

60(cwt.)x$5.00x3 1/3(cows) plus

(-10)x(-$1.00)x16 2/3 plus (-10)x3 1/3x$5.00 plus (-$1.00)x

3 1/3x60 plus 3 1/3x(-10)x(-$1.00)

which equals

1. -$833, the rate of production effect plus
2. -$1000, the price effect plus
3. $1000, the size of business effect plus
4. -$167, the cross effects.

The sum is -$1000. Thus the difference in gross income from its expected value is accounted for by the price received, the rate of production, and the size of business. The procedure can be simplified and a fair approximation of the full difference can be obtained by considering yields, prices, and numbers of units at the midpoints between values given for the farm and the averages for all farms. In this case there are the three primary effects listed above. The cross-effects can be ignored.[1]

The difference in labor income from its expected value is analyzed according to the effects of combination of factors, production rate, price, and size of business (table 21).

1. Appendix K.

Table 21. Explanation of Difference of Labor Income from its Estimated Value, According to Effects of Production Rate, Price, Size of Business, and Combination of Factors, on a One-Product Farm

Item	This farm	Expected, based on all farms	Average of all farms
Gross income	$4,000	$5,000[1]	
Total farm expense plus 5% interest	3,000	3,100[2]	
Labor income	1,000	1,900	
Milk per cow (cwt.)	50		60
Price of milk per cwt.	$4		$5
Number of cows	20	16 2/3	
Difference of gross income from expected, consisting of the following:	$-1,000		
Price effect (60x 16 2/3x($5-$4))	-1,000		
Rate of production effect (16 2/3x$5 x10)	-833		
Size of herd effect (60x$5x(20-16 2/3))	1,000		
Cross-effects	-167		
Difference in labor income from expected value because of difference in gross income			$-1,000
Difference in labor income from expected value because of combination of factors to earn an estimated gross			100
Net difference in labor income from its expected value			-900

1. The $5,000 is obtained from estimating the gross income for this farm from $P=Cx^ay^b$.

2. The $3,100 is given by the average relationship for all farms between gross income and total farm expense plus 5% interest on the investment.

Part IV. Extension of Analysis of Differences in Gross and Labor Income to Two or More Enterprises

In practical work it would not be feasible to include all of the enterprises on the average farm. However, the bulk of differences in gross income and labor income can be accounted for by considering the major enterprises. The analysis consists of two principal operations, with two parts each:

1. Differences from expected expenses are calculated.
   a. Gross income is estimated from the employment of factors, using a gross income equation derived from all farms.
   b. The total of cash outlays plus implicit charges which would be incurred on an average to obtain the estimated gross are determined. This is the next to the last item (table 21). Thus, altogether the first step shows how much labor income is gained or lost by the manner in which factors are combined.
2. Differences from expected gross are calculated.
   a. Using the same enterprises in the same proportions, and using average yields and selling prices, the acres of crops and the numbers of livestock needed to obtain the estimated gross are computed.
   b. The differences between expected and recorded gross are divided into production rate, price, and size of business effects.

Tables 20 and 21 were for a hypothetical farm, producing only whole milk. Tables 22-25 give the complete analysis for an actual farm with several enterprises.

Table 22. Estimates of Gross Income and Labor Income from the Combination of Factors on a Michigan Dairy Farm[1]

Gross income			$23,960
Total farm expense	$15,058		
Less: operator's labor	1,430	$13,628	
Interest on invest. $46,341 @ 5%		2,317	
Total charges less operator's labor			15,945
Labor income			$ 8,015
Estimate of gross income[2]			17,420
Expected total charges[3] $17,420 ÷ 1.1415	15,262		
Less operator's labor	1,523		
Expected total charges less operator's labor			13,739
Expected labor income from $17,420 of gross at average relation of charges to gross			3,681
Labor income from $17,420 of gross with charges other than operator as of this farm			1,475
Labor income			8,015
Part of labor income to be explained by yields, prices, size of business, and choice of enterprises			6,540

1. A farm in Calhoun County. In dealing with complex enterprises, it is necessary to include one more factor which affects gross besides yield, price, and size of business--the selection of enterprises. It is important that, when this type of analysis is used, the farms in the study be comparable regarding kinds of products sold. Then the forces affecting gross income are practically the same as for a one-product situation.
2. Estimated from equation 1, table 6, for 86 dairy farms.
3. The average relationship between gross income and total charges for the 86 dairy farms is 1.1415 to 1.0000.

Table 23. Analysis of Yields, Prices, and Units of Major Enterprises on a Michigan Dairy Farm Earning a Higher Gross Income than Estimated

Enterprise	Expected values for all farms			Recorded values for this farm		
	Yield	Price	Units[1] needed to earn estimated gross	Yield	Price	Units
Dairy cattle						
Cattle income[2]	($139.80)		28.9	($150.80)		34.0
Milk sales	65.1	$4.52	28.9	106.7	$3.54	34.0
Poultry[3]	13.0	0.40	191.0	12.2	0.40	225.0
Wheat	28	1.95	30.6*	28.4	1.92	36.0
Corn	42	1.57	7.6*	44.4	1.60	9.0
Oats	43	0.84	14.7*	40.0	0.91	17.3

*Acres sold

Gross income by enterprises (yield x price x units):

Enterprise	Expected gross income	Recorded gross income	Difference
Dairy cattle			
Cattle income	$4,040	$5,126	+ $1,086
Milk sales	8,490	12,858	+ 4,368
Poultry	1,000	1,099	+ 99
Wheat	1,670	1,965	+ 295
Corn	500	642	+ 142
Oats	530	630	+ 100
Total	$16,230	$22,320	+ $6,090

1. Units refer to milk cows for dairy, acres for crops, etc. Expected values of units are calculated by multiplying the units for this farm by 0.849. This establishes expected units so that when multiplied by average prices and average yields, the gross income is equal to the gross estimated for this farm.
2. Cattle income consisted of sales of animals of different types from the dairy herd. It is impracticable to attempt to separate yield and price effects.
3. Average yields and prices of these crops are estimated.

How this farm earns a gross income of $22,230 from its five major enterprises is compared with how an "average" farm would earn the <u>estimated</u> gross ($16,228) from five identical enterprises (table 23). The "average" farm would receive average yields and average prices for products sold. It would have in production of output for sale either more or less units than this farm in each enterprise.[1] The five major enterprises account for $6,090 of the $6,540 of gross income to be explained by yields, prices, size of business, and choice of enterprises (lower right-hand corners, tables 22 and 23). Thus, $7,565 of a labor income of $8,015 are accounted for by the combination of factors and the yields, price, and volume of the five major enterprises.

The effects upon gross income by the five enterprises are separated according to <u>yield</u>, <u>price</u>, and <u>size of business</u>. (table 24). It was impracticable with the data to attempt to show a separation between price and yield effects for dairy cattle income. Dairy cattle income consists of sales of calves, heifers, bulls, and cows from

1. The expected number of units producing for sale is computed as follows: Multiply each unit-figure for this farm by a common factor. Compute the factor in this way: (x) times cows this farm times average sales per cow times average price of milk, plus (x) times cows this farm times average cattle income per cow, plus (x) times hens this farm times average eggs per hen times average price of eggs,.......equals estimated gross income. Thus a factor of size is derived which will fix the expected numbers of units in such a way that the sum of average yields times average price times expected units will equal the estimate of gross.

Table 24. Analysis of Difference of Gross Income from Its Estimated Value According to Yield, Price, and Size of Business Effects on a Michigan Dairy Farm, 1950

	Effects accounting for difference in gross[1]			
Enterprise	Yield	Price	Size of business[2]	Total
Dairy Cattle	....$345.....		$741	$1,086
Milk	$5,260	$-2,640	1,748	4,368
Poultry (eggs)	-70	0	169	99
Wheat	30	-280	545	295
Corn	30	10	102	142
Oats	40	50	10	100
Total, not including cattle	$5,290	$-2,860		
Unseparated between yield and price Cattle	$345			
Total	$2,775		$3,315	$6,090

1. The effects are calculated as follows:
   Yield effect = difference in yield from average times price for this farm plus average price, the sum divided by two, times units for this farm plus expected units, the sum divided by two. (See table 23 for data).
   For the yield effect of milk, thus:

$$\text{Yield effect} = (106.7 - 65.1) \times \frac{\$4.52 + \$3.54}{2} \times \frac{28.9 + 34.0}{2}$$

$$= \$5,260$$

2. The size of business and choice of enterprise effects are together called size of business in this case. The comparison is with a group of 86 dairy farms. This farm is typical. Thus it is impossible for gross income to be affected by unusual prices in any line. Should the group of farms be heterogeneous the meaning of the size of business column should be expanded to include selection of enterprise effects. The column can be computed as in (1), preceding or as a residual.

the herd. There is no common denominator of output. Most of the price and yield effects are accounted for by the dairy enterprise, which is dominant. The yield of milk is more than adequate to offset the adverse price, $5,260 compared with $-2,640. Altogether prices and yields account for $2,775 of the difference in gross. The size of business accounts for $3,315.

The preceding analysis will contain information of interest aside from the estimate of the causes of variation in gross income from expected values. Average yields of crops and production rates of livestock for all farms and for the particular farm will be compared directly. Average selling prices will also appear alongside prices received by this farm. As in other analyses calling for the comparison of a particular farm with some standard, there is no compelling reason here why the comparison should necessarily be with the mean of all farms. The comparison could be with the high third labor income farms, in which case the majority of the farm account cooperators would have an opportunity for direct comparisons of their farms with the more successful units, enterprise by enterprise.

The analysis does not depend upon a significant relationship between expected labor income and recorded labor income. Even if the correlation in this matter is not significant, the analyst can account for the differences in expected from recorded gross income and labor income according to the factors which in this case must

must account for the differences in labor income. These factors are yields, prices, size of business, and choice of enterprises. The computations involve no mathematics other than arithmetic once the estimate of gross income is obtained.

4. The Valuation of Farm Businesses

It will turn out in some cases that it will not pay a farmer to increase the yields of crops or perhaps the production rates of livestock in order to eliminate adverse yield effects shown by the preceding analysis. Possibly because of the location of a farm or because of the effect of certain types of soil on the quality of crops grown, it may not pay to seek to eliminate unfavorable price effects. In the case of plans designed for the stabilization of the price and quantity of milk, it may not pay a farmer to reduce the production of milk in the surplus season even though he could raise his average price by doing so.

Thus the price effects, yield effects, and possibly the size of business effects may all be adverse, and yet it may not pay to do anything about them. The conclusion in this case must be that the farm and the factors employed upon it are overvalued in relation to other farms. It is difficult to find any way by which the influence of the farm operator can be separated from the effects of the farm itself and its appurtenances. In the case of the farm which consistently returns more than it should according to the gross income estimating function, some or all of the primary factors are undervalued. A reduction in the valuation in the first instance, and an increase in the valuation in the second case will tend to cause the estimates of gross income to agree more nearly with a general gross income estimating equation. No specific suggestions are made here regarding how the implications of persistent non-conformity to gross income estimating equations should be considered in connection with stated values of factor categories.

5. Subjective Rates of Charge for Categories of Factors

The estimates of marginal returns to categories of factors could be used to make revisions of rates of charge against the farm business. In this study, gross income estimating equations were calculated for 194 farms. The sample was broken into two parts, dairy farms and not-dairy farms, two independent samples from the universe of farms. The statistical instability of the coefficients of elasticity of gross income with respect to factor categories is shown in tables 8-10 of Chapter IV. If similar analyses should be conducted by type-of-farming areas, the coefficients of elasticity and the marginal value productivities of categories of factors will cluster around central values. Then tables of marginal returns to categories of factors for the state as a whole could be prepared with confidence that the marginal rates would be nearly what farmers received for the use of the factors.

Thus the marginal return to investment in land could be shown to be, say, five per cent; the marginal return to investment in productive livestock could be another figure, and so on. These marginal rates of return should be indicative of the charges against the factors which farmers actually had in mind in the organization of farm businesses. If the factor categories were charged according to marginal rates of return, the effect should be to approxi/amte more nearly a type of net income which farmers in practice seek to maximize.

It is recognized that various disequilibria can account in part for marginal rate of return to categories of factors. For instance, the proportion of investment in machinery and equipment on Michigan farms has been increasing for some time. The results of this study indicate that the marginal returns for machinery and equipment continue to be relatively high. The increase in investment in this category can thus be expected to continue. Nevertheless farmers have not bid up the prices of available machinery and equipment to a point where the marginal returns would be similar to those for land and improvements, for example. In general, the subjective rate of interest return demanded for machinery and equipment remains high, and farmers appear to maximize a labor income which calls for a differential between the interest rate on machinery and equipment and the interest return on land investment. It is possible that the definition of labor income may come a little closer to conformity with the behavior of the entrepreneurs in the field if the rates of charge are adjusted to be more in the line with marginal value productivities. Theoretically, of course, all entrepreneurs should borrow money and invest money in the factors of production up to where the marginal rate of return is equal to the rate of interest. However, if farmers do not actually follow such a procedure with respect to all categories of factors, it is likely that there are valid reasons for their not doing so, and a more accurate report of earnings above charges is possible if charges are made according to practice.

A summary of this chapter has already been made in the Introduction, page vii.

APPENDIX A

COEFFICIENTS OF ELASTICITY OF GROSS INCOME WITH RESPECT TO CATEGORIES OF FACTORS

Proof that a, b, ... k are the elasticities of P with respect to x, y, ...z, respectively, in

$$P = cx^a y^b \ldots z^k$$

For an increment of x the change in P is

$$\frac{\partial P}{\partial x} = acx^{a-1} y^b \ldots z^k$$

The elasticity of P with respect to x, η_x, is

$$\eta_x = \frac{\Delta P}{P} \cdot \frac{x}{\Delta x} = \frac{\Delta P}{\Delta x} \cdot \frac{x}{P}$$

$$= \frac{acx^{a-1} y^b \cdot x}{P}$$

$$= \frac{acx^a y^b}{cx^a y^b}$$

$$= a$$

Appendix B

Illustration of Returns to Scale

If there is an equal proportional change in all of the factors then the proportional change in product will be greater than, equal to, or less than the change in the factors. Let the equation for product be $\underline{P} = 4\underline{x}^{1/4}\underline{y}^{3/4}$. The sum of the elasticities of product with respect to the factors is 1. Returns to scale are constant. If $\underline{x}$ and $\underline{y}$ are assumed to be 16 each, and are then raised to 81 each it will be found that product will increase from 64 to 324. The ratios of change are equal.

If the equation is given by $\underline{P} = 4\underline{x}^{1/4}\underline{y}^{1/2}$ and both x and y are equal to 16, product will be 32. If they are both raised to 81 the new product will be 108. The ratio of change of product is less than the ratio of change of factors. If the exponents of $\underline{x}$ and $\underline{y}$ are more than 1 in their sum the increase in product is relatively larger than the increase in factors.

Appendix C

The Relation of the Mean Value Productivity Function to the Estimate of Gross Income by Least Squares

Following is a study of the behavior of the statistical "value productivity" function, assuming that the actual value productivity functions are known. For this purpose three arbitrary value productivity functions are chosen:

Farm 1. $P_1 - c_1 = 6x_1^{1/4} y_1^{1/2} - (x_1 + y_1)$

Farm 2. $P_2 - c_2 = 4x_2^{1/3} y_2^{1/3} - (x_2 + y_2)$

Farm 3. $P_3 - c_3 = 6x_3^{1/2} y_3^{1/4} - (x_3 + y_3)$

Returns to scale are given as less than 1 in order that optimum inputs can be determined without specifying a risk function. The three farms are operated for the purpose of earning as large a net income as possible. To determine what quantities of *x* and *y* will be employed the partial derivatives of *P* with respect to *x* and *y* are set to zero. The supply curves of factors and the demand curve of product are assumed to be infinitely elastic. As these are assumptions of perfect competition they are not unreasonable for most farm operations.

The next problem is to solve for the plane which will result if the coordinates expressing the optimum positions for the three farms are to be joined, as is inherent to the idea of the statistical value productivity function. This plane turns out to be, in logs:

$$\log \underline{P} = 0.486 + 0.488 \log \underline{x} + 0.488 \log \underline{y}$$

The coefficients of $\underline{x}$ and $\underline{y}$ are equal because of the values of the exponents chosen in the three value productivity functions.

There are now four planes altogether (figure 7). All of the individual value productivity planes cut through the statistical plane from above. Furthermore, it follows from the construction of the latter that the optimum position of each firm must lie upon the line where the individual plane cuts below the plane common to the three businesses. This will be true if the average value productivity is greater than the marginal value productivity. The average will be greater than the marginal if the exponent of the factor in the function is less than 1. If average value productivity equals marginal value productivity in the function then value productivity in total for the case of constant returns to scale can equal the value of only one factor. This is a useless case for analysis.

The statistical plane is fixed by the condition that the derivatives of net income with respect to the factors are zero. As long as the plane for the firm lies above the statistical plane these derivatives are larger than called for by the conditions of equilibrium. As soon as the plane for the firm drops below the general plane the marginal value productivities of the factors become less than their costs.

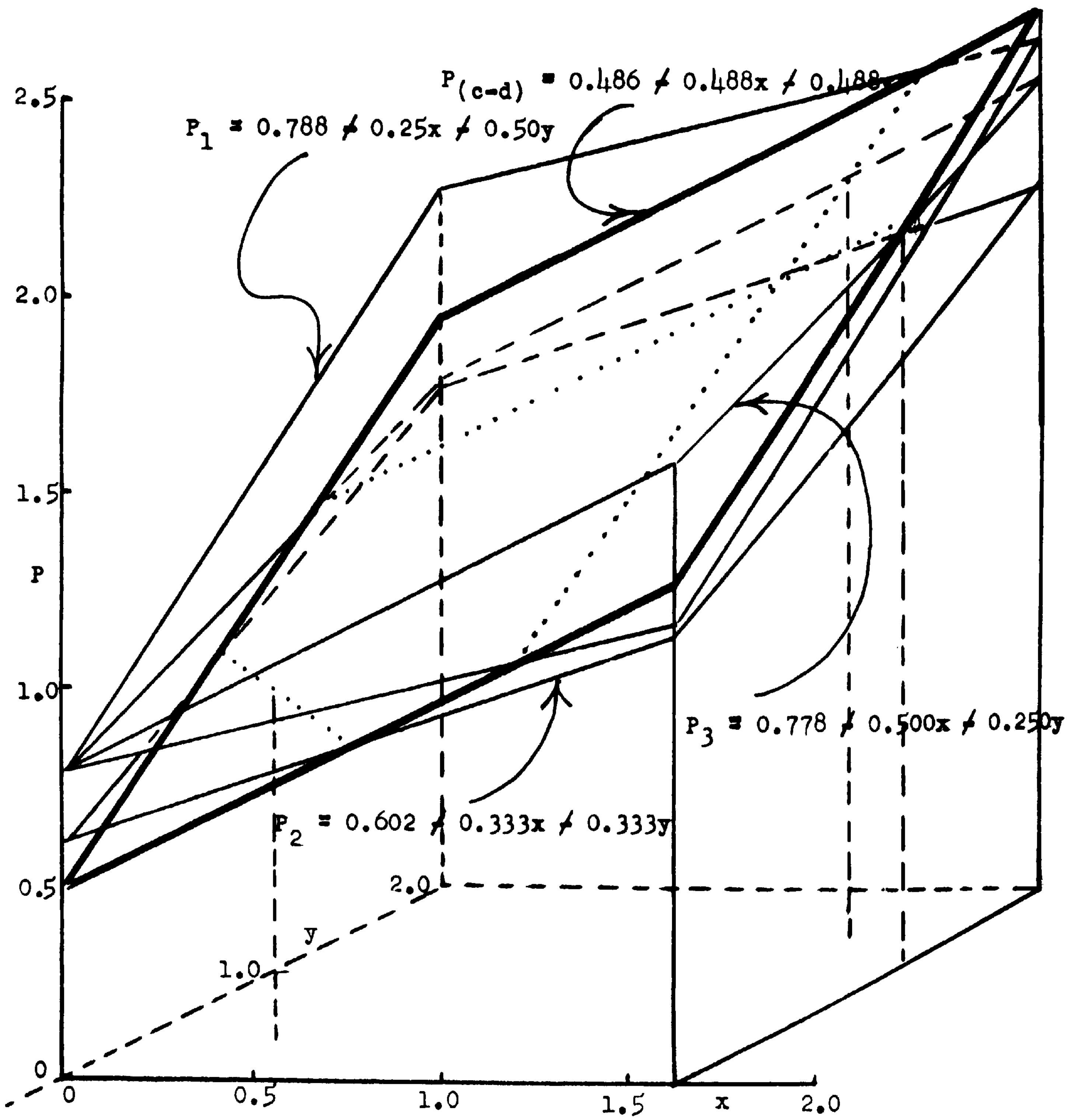
It is worthwhile to express the gross income equations for the three farms in their natural numbers and then determine marginal value productivities of factors according to (1) the value productivity plane on which each farm is assumed to operate and, (2) the statistical plane. This amounts to differentiating two functions using the optimum values of the factors according to one of the functions. The results are given below:

Farm	Factor	Marginal value productivity according to the "true" value productivity function of the firm	Marginal value productivity according to the statistical function
1	x	1.00	1.94
	y	1.00	0.97
2	x	1.00	1.46
	y	1.00	1.46
3	x	1.00	0.97
	y	1.00	1.94

The errors in the estimation of marginal value productivity are biased upward. This is to be expected as the slope of the common plane is greater than the slopes of any of the individual planes (figure 7)

The above difficulties do not apply if one is interested only in an estimate of gross income from the employment of factors. The trouble starts when the function describing the positions of the firms is interpreted to be an average

Figure 7. The Relation of Theoretical Individual Production Functions to the Cobb-Douglas Function



P: log of gross income, for Cobb-Douglas function, farms 1, 2, and 3.

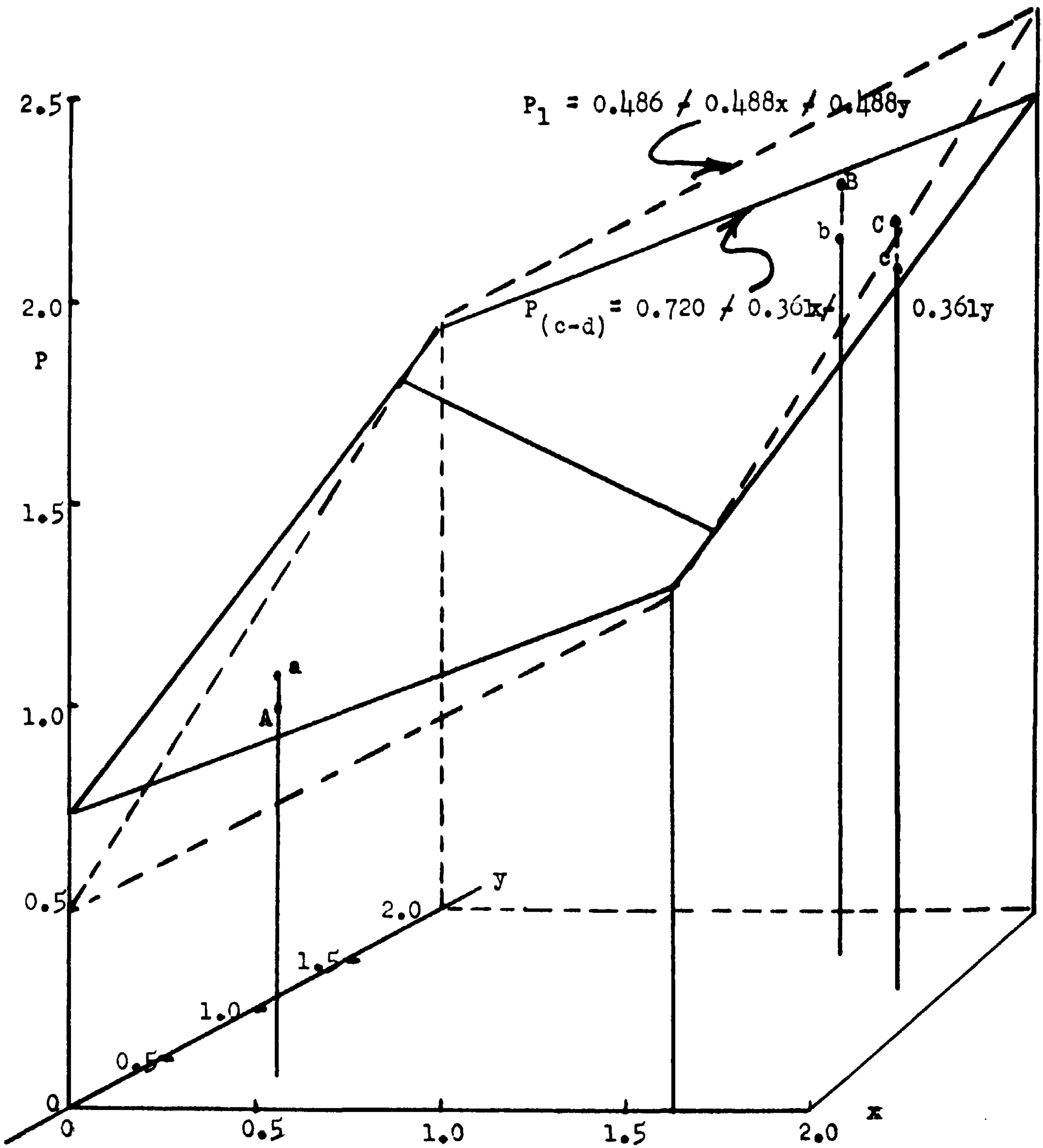
x and y: logs of the two factors of production.

of the value productivity planes facing all of the firms. At worst the Cobb-Douglas function can at least provide a measure or estimate of gross income considering the employment of factors.

In agriculture there are many firms in the industry and the firms are confronted with substantially similar conditions in a homogeneous type-of-farming area. Furthermore, who is to say what are the value productivity functions for the different farmers? This would involve an appraisal of the worth of the management contribution of the individual farmer. A procedure for such an evaluation has not been developed. Perhaps the best that can be done in the matter is to use a function such as the Cobb-Douglas and qualify its use with the statement that it does show the results obtained when different <u>measurable</u> factors are used in different combinations. That not all of the farmers will be able to take full advantage of the function is recognized.

In the most favorable light the function can be considered as one which corrects erroneous impressions of the nature of response of gross income to employment of factors. This is shown in figure 8. The three points <u>A</u>, <u>B</u>, and <u>C</u> represent the positions which farmers believe that they should reach in order to maximize net incomes. The points <u>a</u>, <u>b</u>, and <u>c</u> represent the gross incomes according to relationships between factors and product tested by experience.

Figure 8. The Cobb-Douglas Production Function as a Measure of the Relation of Product to Inputs



P: log of gross income.

x and y: logs of factors

P_1: for plane formed by farmers' estimates of optimum positions.

$P_{(c-d)}$: for plane formed by the Cobb-Douglas function derived from gross incomes.

It cannot be claimed for the Cobb-Douglas function that theoretically and in equilibrium that it gives an average of the value productivity functions facing the individual firms. However, farms are not in equilibrium. The farm operator will be interested in the outcomes of combinations of factors on other farms.

Appendix D

The Valuation of Land

It was expected that stated land values in 1950 would be more or less in line with the level of land prices prevailing when records were started. According to this hypothesis, account cooperators who began records in 1935 and maintained them continuously should tend to give lower stated values of land per acre than farmers who began records in, say, 1948.

The values of farm land in the farm account books are not changed to conform with changing land prices. This procedure avoids the inclusion of changes in land values in measures of net income.

One hundred seventy-six accounts were used in a study of the effect of the year of starting records upon stated values of land per acre. Regardless of when started, these records had to be carried straight through to a recent year (1949) in order that conclusions could be drawn. Table 25 gives the number of accounts beginning in scattered years since 1929, the original values of improved land and total land per acre, and the values of land and improvements per acre. The standard deviations of the means of samples were estimated by average range in subsamples of two.[1]

There was no definite tendency for farmers beginning

1. See E. L. Grant, Statistical Quality Control, pp. 103-112.

Table 25. Average Values of Land, and Land and Improvements per Acre: Farm Accounts Beginning in Selected Years, 1929-1949

Year of starting records	Number of farms	Improved land per acre Mean value	$2\hat{\sigma}_{\bar{x}}$[1]	Total land per acre Mean value	$2\hat{\sigma}_{\bar{x}}$	Land and improvements per acre Mean value	$2\hat{\sigma}_{\bar{x}}$
1929	13	$62	$7.3	$60	$10.3	$92.7	$10.2
1930	14	58	10.4	50	9.9	75	16.8
.							
1934	9	45	9.6	42	9.5	73	19.3
1935	16	43	9.1	37	6.9	62	12.6
.							
1939	16	38	4.2	36	2.9	53	6.4
.							
1942	16	37	4.3	34	3.6	62	9.1
.							
1944	16	42	10.7	37	10.2	60	21.3
.							
1947	17	44	6.5	39	7.0	67	10.4
.							
1949	17	40	8.5	34	6.6	65.	17.1

1. Two standard deviations (estimated) of the mean.

<-- accounts in recent years to give higher initial figures per acre than farmers beginning accounts in the earlier years. The ranges of two standard deviations of the mean indicate that the differences between years are not significant.

The hypothesis is set up that farmers beginning records since the second World War can be divided into two groups: those who value land at market prices and those who value it according to some earlier scale. In this case there should be a tendency toward greater dispersion of values per acre in later years. The farm real estate values obtained in this study were converted to an index, with 1929 taken as 140. Land values reported by beginning account cooperators did not rise comparably with the Michigan farm real estate index (figure 9).[1] There was not a significant tendency for land values to vary more about the average in the books started in recent years (figure 10).[2]

The primary concern is with the influence of year of starting records on real estate values as of January 1, 1950. Neither the average value of land and improvements per acre nor the average value of land per acre was appreciably affected by the year of starting records. That is, the figures for 1950 tend to be comparable regardless of when records were started (figures 11 and 12).

1. See table 26, p. 132.
2. Table 27, p. 133.

Figure 9. Stated Values of Farm Land and Buildings of Beginning Account Coop-
Compared with the Michigan Farm Real Estate Index

Selected years, 1929-1949

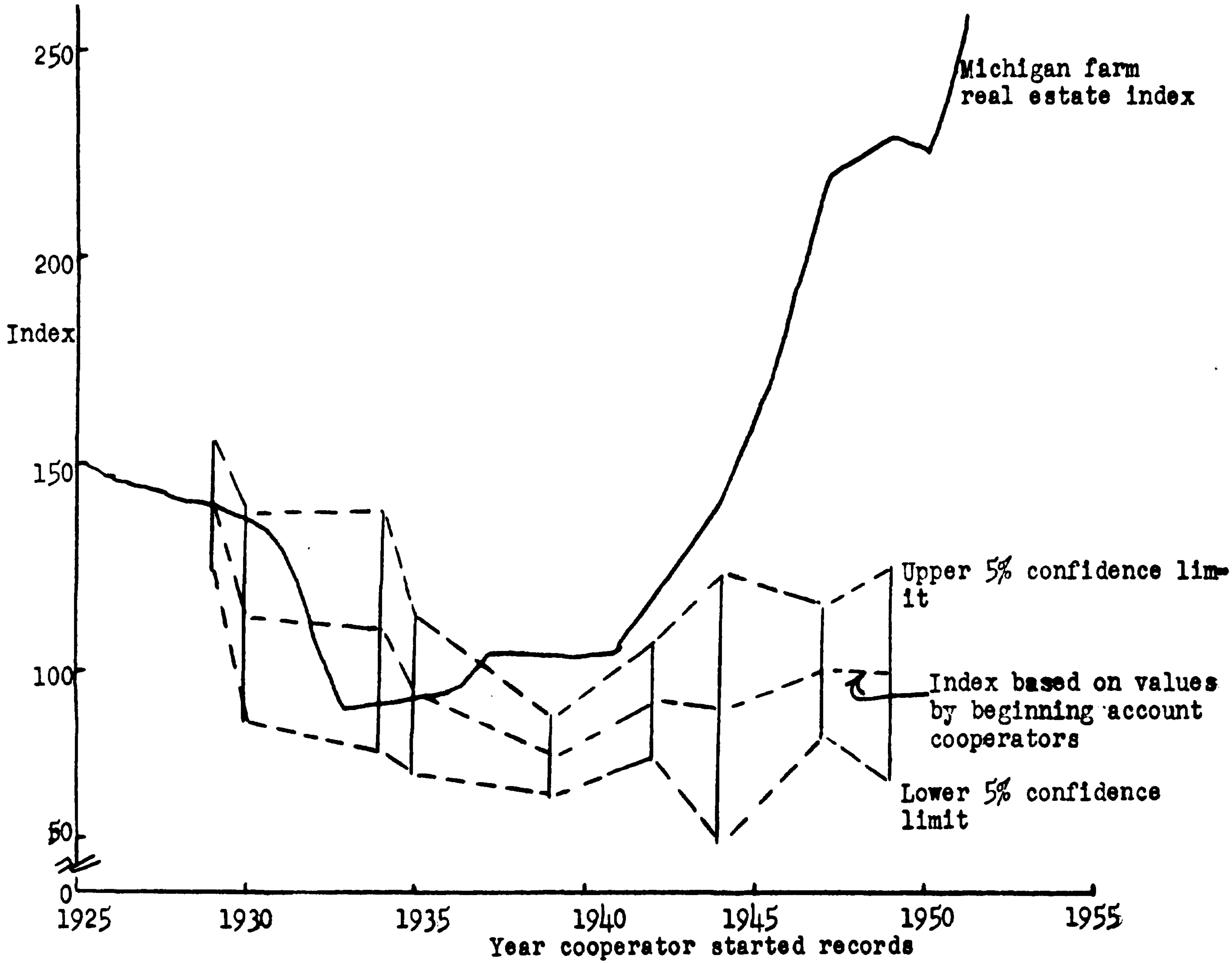
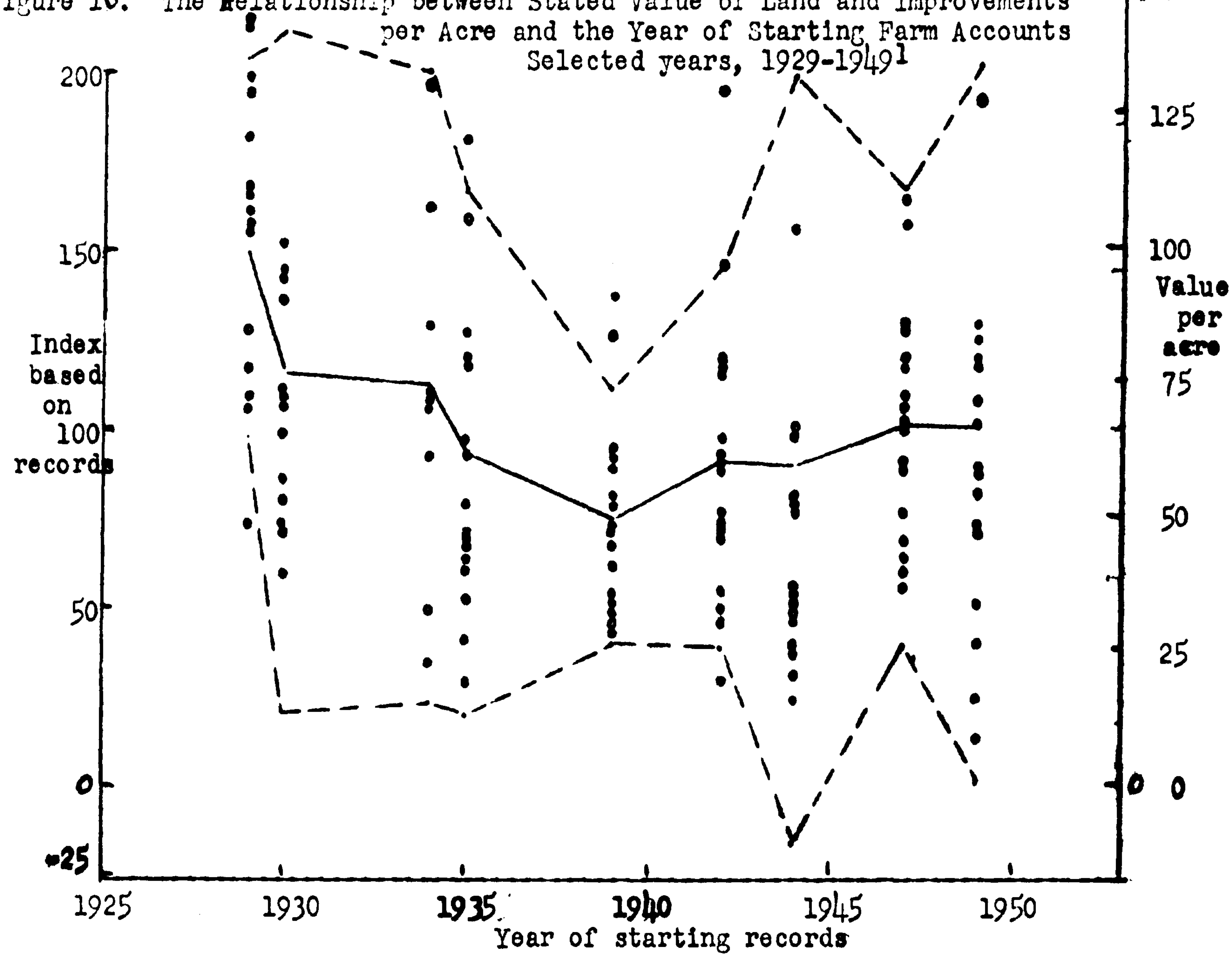


Figure 10. The Relationship between Stated Value of Land and Improvements per Acre and the Year of Starting Farm Accounts Selected years, 1929-1949[1]



1. Values as of year records were started.

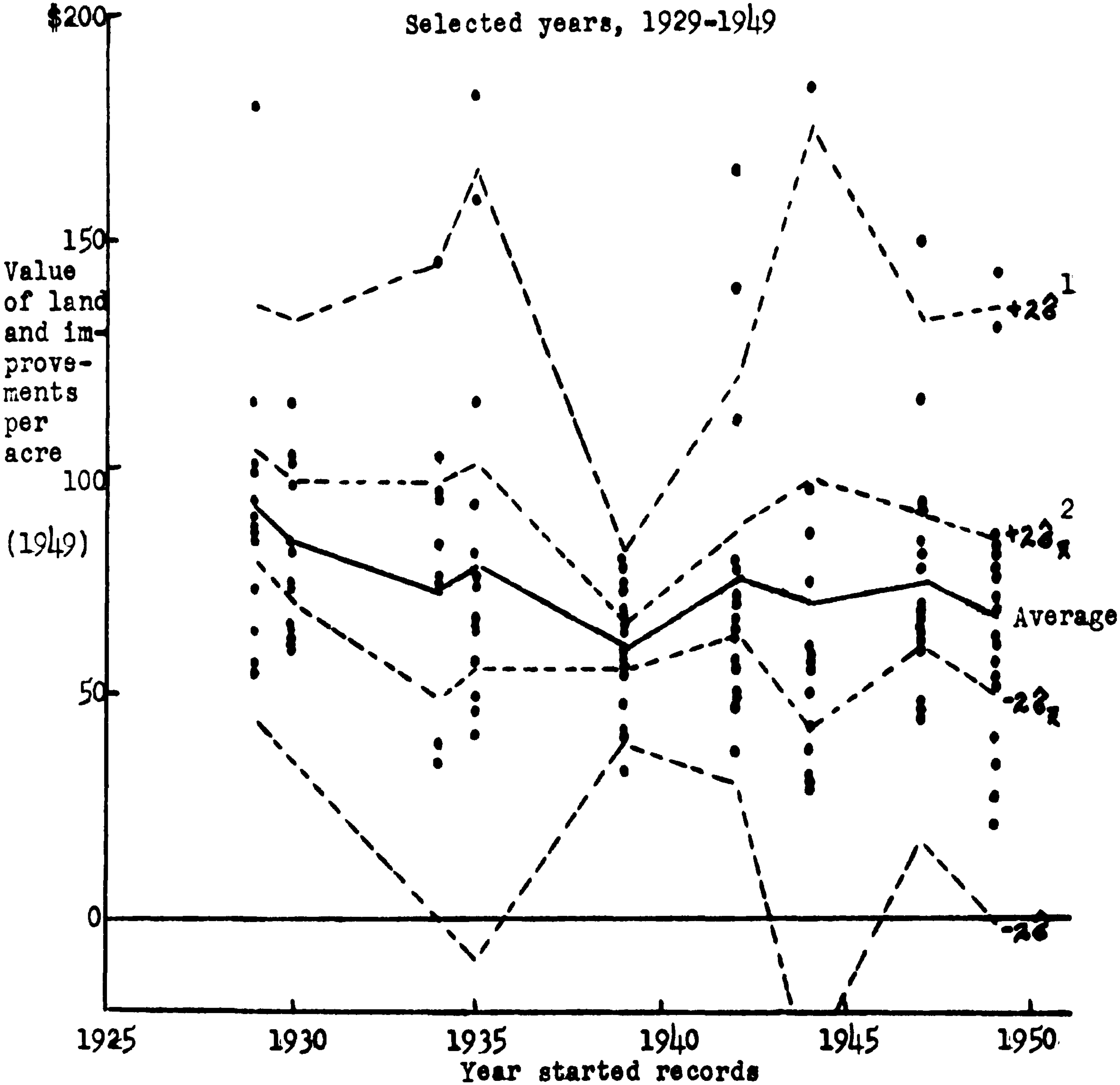
Table 26. The Michigan Farm Real Estate Index and an Index of Values of Land and Improvements per Acre of Beginning Farm Account Cooperators

Year	Michigan index (1935-39 = 100)	Index of values of land and improvements of beginning farm account cooperators	5% confidence intervals of farm account index
1929	140	140	125 - 155
1930	137	113	88 - 139
1931	130	---	--- - ---
1932	109	---	--- - ---
1933	91	---	--- - ---
1934	93	110	81 - 139
1935	94	94	75 - 113
1936	95	---	--- - ---
1937	104	---	--- - ---
1938	104	---	--- - ---
1939	104	80	70 - 90
1940	103	---	--- - ---
1941	106	---	--- - ---
1942	119	93	79 - 107
1943	130	---	--- - ---
1944	152	91	69 - 123
1945	164	---	--- - ---
1946	190	---	--- - ---
1947	219	100	85 - 115
1948	224	---	--- - ---
1949	229	100	74 - 126
1950	225	---	--- - ---
1951	258	---	--- - ---

Table 27. The Relation Between the Year of Starting Farm Account Records and the Dispersion of Reported Land Values, Michigan Farm Account Farms 1929-1949

		Land and Improvements			
Year	Number of farms	Average value per acre	Index, 1929 = 140	Standard deviation	
				Terms of dollars	Terms of index
1929	13	93	140	19	27
1930	14	75	113	42	47
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
1934	9	73	110	40	44
1935	16	62	94	40	38
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
1939	16	53	80	24	18
.	.	.	.	.	.
.	.	.	.	.	.
1942	16	62	93	29	28
.	.	.	.	.	.
1944	16	60	91	49	60
.	.	.	.	.	.
.	.	.	.	.	.
1947	17	67	100	31	31
.	.	.	.	.	.
1949	17	66	100	52	52

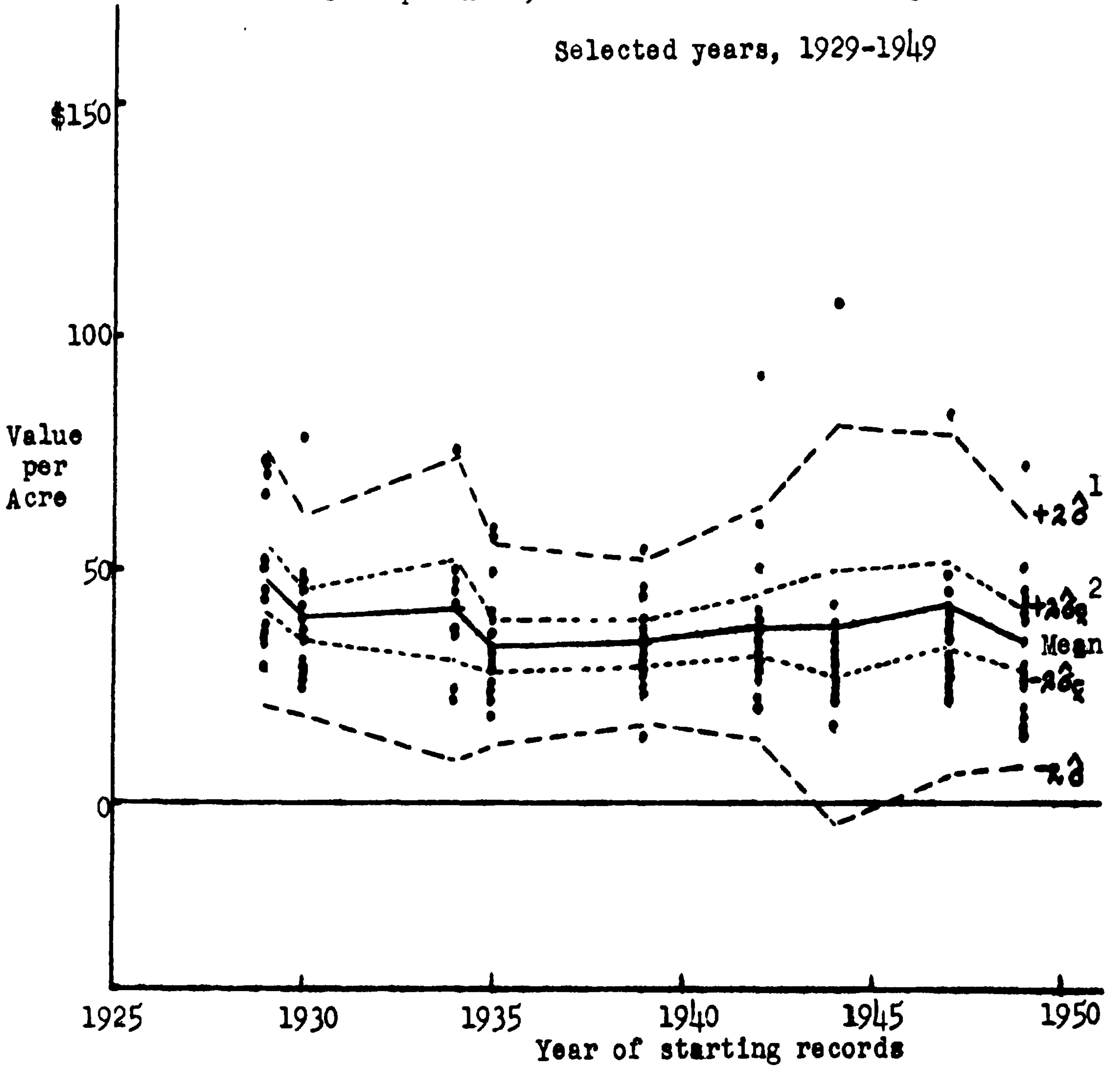
Figure 11. The Relationship between the Stated Value of Land and Improvements per Acre as of 1949, and the Year Farm Accounts Were Started



1. Two standard deviations. 2. Two standard deviations of the mean.

Figure 12. The Relationship between the 1949-Stated Value of Land per Acre, and the Year of Starting Farm Accounts

Selected years, 1929-1949



1. Two standard deviations. 2. Two standard deviations of the mean.

Appendix E

A Test of Differences in Recorded Values of Land and Improvements

The principal conclusion of appendix D, that the year of starting farm account records had little or no effect upon recorded values per acre of land and improvements, was tested independently. The hypothesis was set up that within a particular county the 1949-given values would not be affected by whether records began in the period 1932 to 1942 or in the period 1943 to 1949. In order to minimize the number of observations a sequential probability ratio test was used.[1] The counties were chosen at random. A pair of observations was chosen at random for each county. Ond of the observations began records in the earlier period and one began in the later period.

The sub-hypothesis that 0.5 of the records in the matched pairs beginning in the 1932-42 period should show higher values per acre than that 0.7 should show higher values was accepted with 20 observations. The probability of being correct is 9/10ths. This initial hypothesis concerned values of land per acre, not land and buildings. Twenty-four observations were needed to establish with 9/10ths probability of being right a similar hypothesis with respect to land and improvements.

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1. See Paul G. Hoel, Introduction to Mathematical Statistics, pp. 124, 125.

A second sub-hypothesis was set up. This was that 0.5 was a better estimate than 0.3 of the proportion of comparisons of values within the same county in which the farms beginning records in the earlier period would show values per acre which were higher than values for the farms beginning in the later period. This hypothesis was accepted with 12 observations, with respect to land alone, and land and buildings together.

The results of the sequential probability ratio test show that as far as the period of starting records is concerned, whether per acre values are higher or lower is essentially similar to coin-tossing.

Appendix F

The Valuation of Dairy Cows

In a random sample of 32 farms in areas 5 and 6 there is some relation between the year of starting records and the average value of dairy cows per head (figure 13). The sample shows little or no relationship between average value per head and dairy sales per cow (figure 14). Average value per head is determined on the basis of the beginning inventory. Dairy sales per cow is determined on the basis of "cow units", a figure obtained when the farm account books are checked in.

One source of error in using sales as a measure of productivity of cows is that not all farmers have equally good markets. Therefore, for 26 farms the microfilmed records at Michigan State College were used to convert pounds of milk sold over into pounds of fat-corrected milk. The simple correlation coefficient between average value per cow and pounds of fat-corrected milk sold per cow is 0.22. This coefficient was tested according to its theoretical distribution should a series of similar trials be made.[1] If it is assumed that the actual correlation coefficient is 0.00, it turns out that the standard deviation of similar trials is estimated at 0.21. Thus an absolute value of the correlation coefficient equal to or greater than the 0.22

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1. See Paul G. Hoel, Introduction to Mathematical Statistics, pp. 88-90.

Figure 13. The Relation between Inventory Value of Dairy Cows and the Year of Starting Farm Accounts

32 Random Farms, Areas 5 and 6
1950

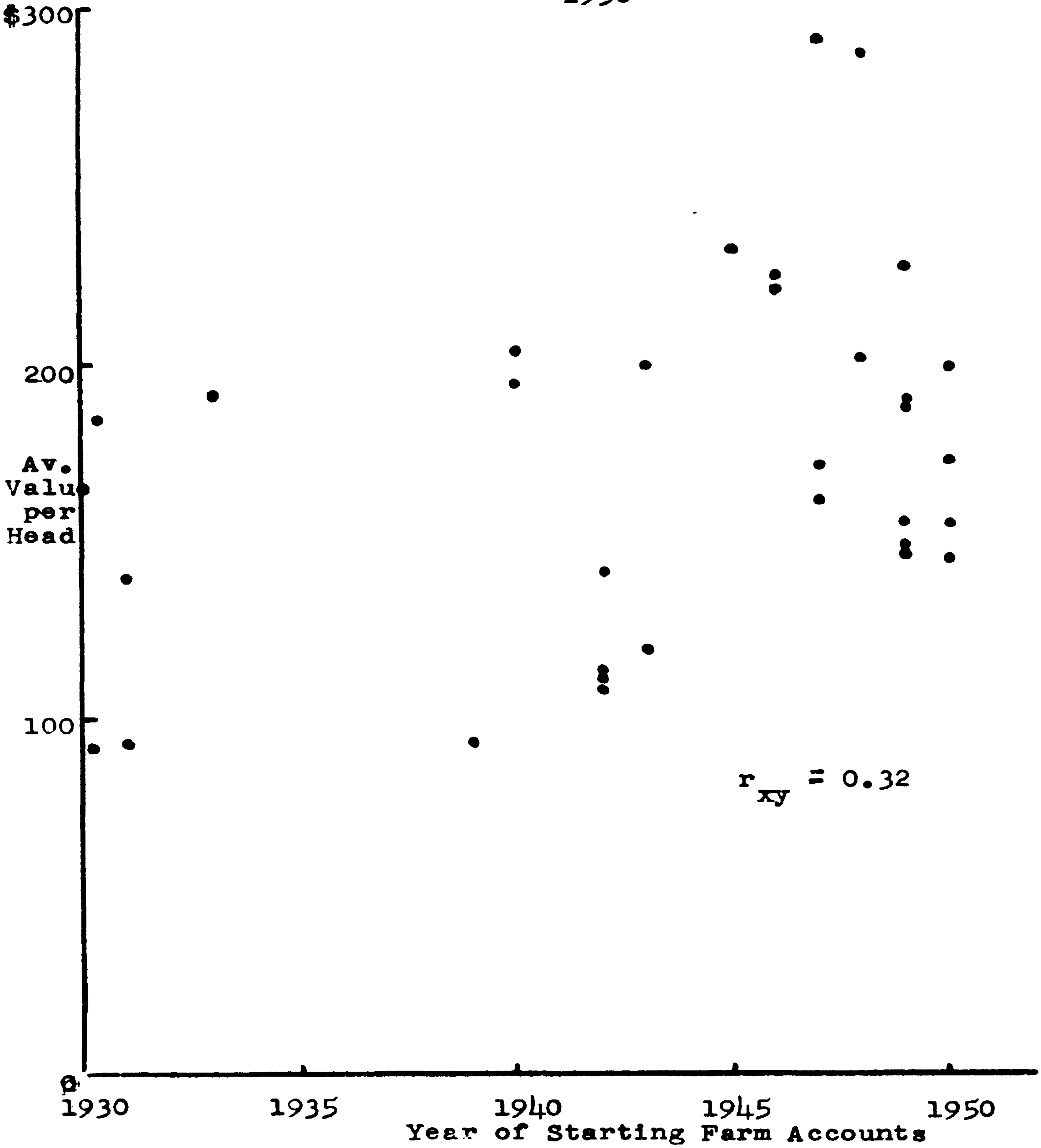
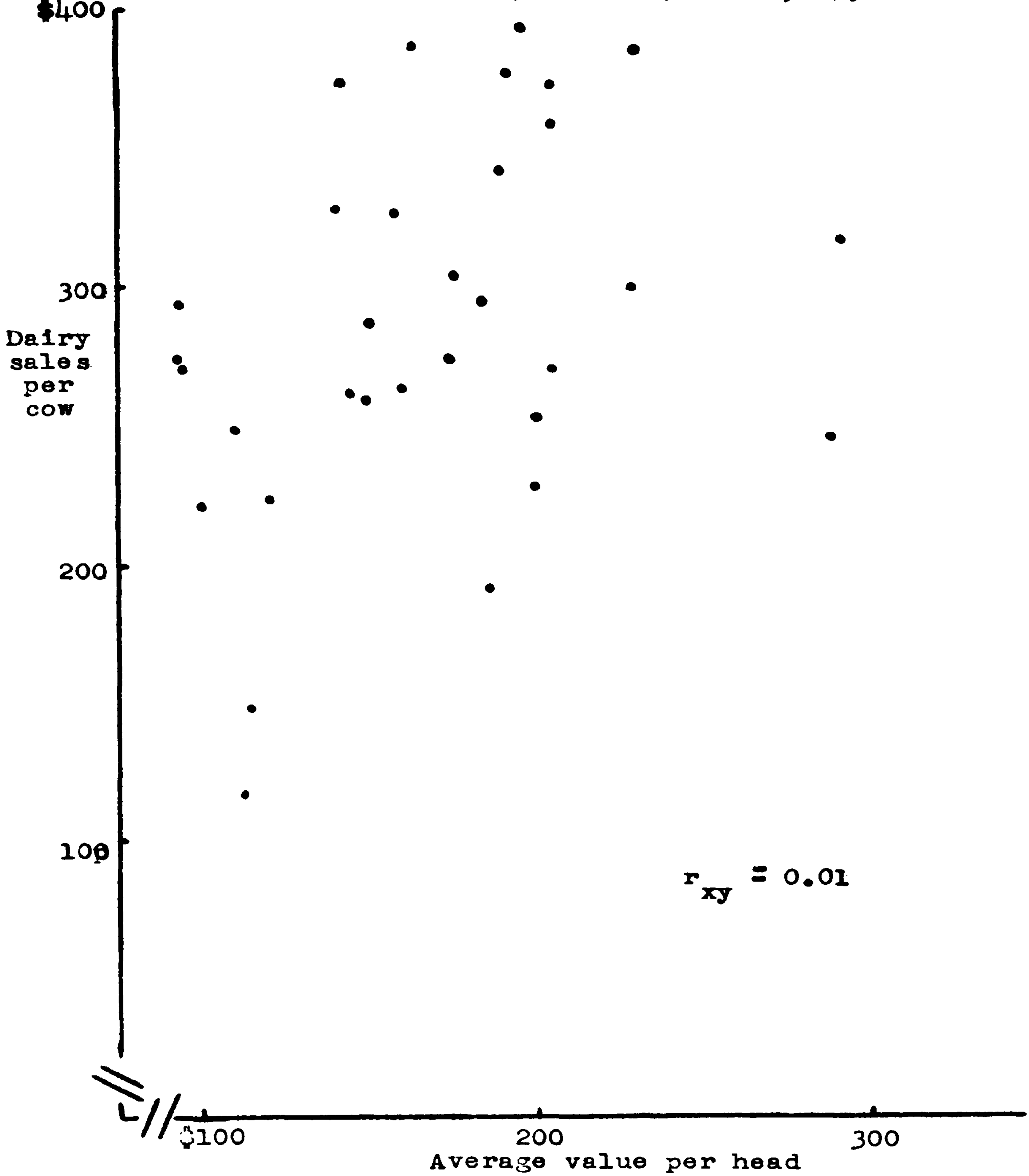


Figure 14. Dairy Sales per Cow and Average Value of Cows per Head at Beginning Inventory 32 Random Farms, Areas 5 and 6, 1950



obtained in this trial could be expected to occur about one time in three by chance. The results of the test of the relationship between fat-corrected milk sold per cow and the average value of cows in the herd are shown in figure 15.

The hypothesis was set up that if the influence of the time of starting records upon the value of cows per head should be eliminated, that the relation between fat-corrected milk per cow and average value per head should be statistically significant. The partial correlation coefficient implied by the above hypothesis is 0.29.[1] Thus the degree of relationship between productivity and reported value is increased somewhat. However, the significance test still shows that a value of $r_{xy \cdot z}$ as large as 0.29 could be expected about one time in seven by chance.

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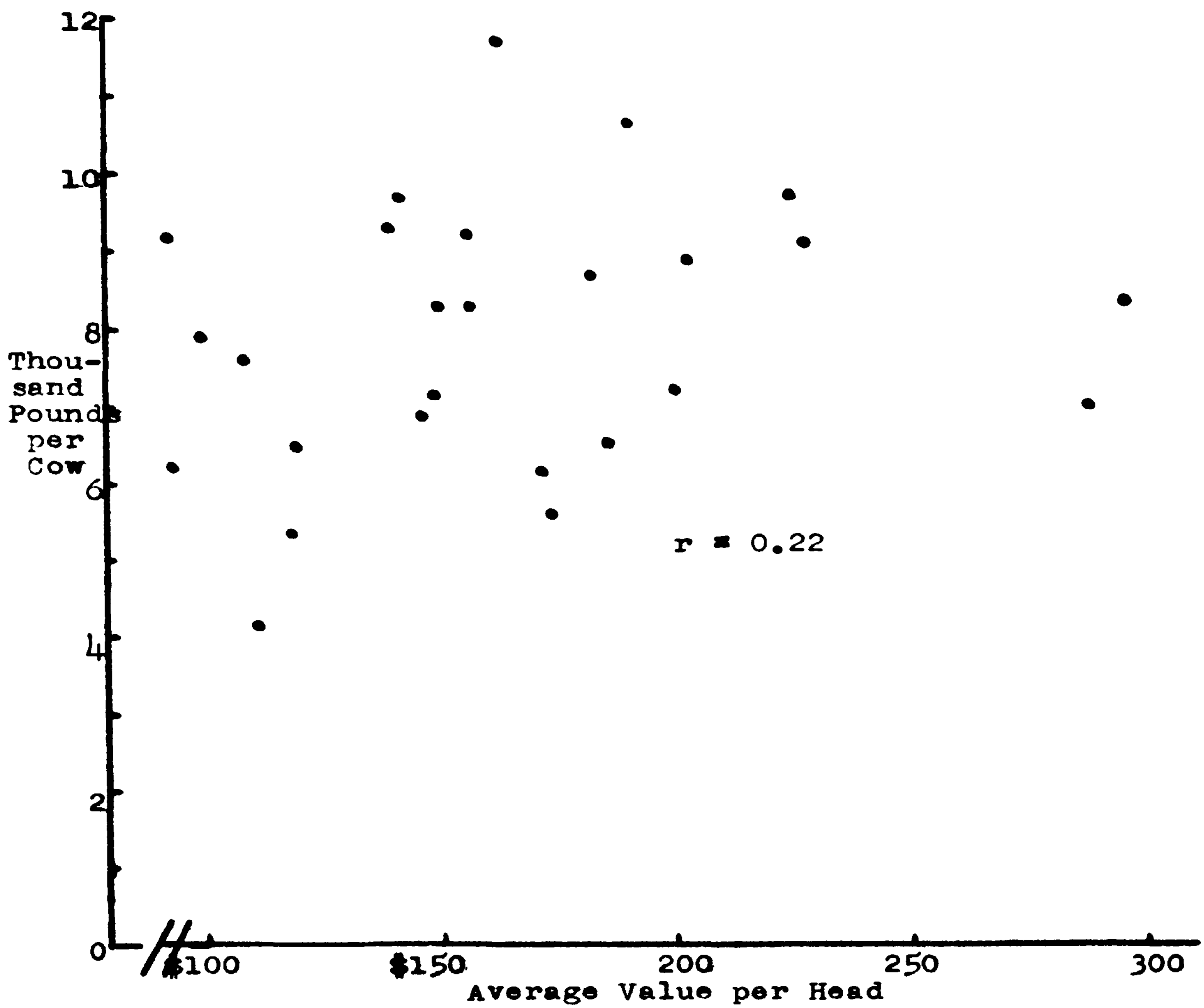
1. Let <u>y</u> be the value of dairy cows per head; let <u>x</u> be the pounds of 3.5% fat-corrected milk sold per cow; let <u>z</u> be the year of starting records. Then $r_{xy} = 0.22$, $r_{yz} = 0.32$, $r_{xz} = -0.16$, and $r_{xy \cdot z} = 0.29$. See Paul G. Hoel, op. cit., pp. 110-118.

Figure 15. The Relation between Average Value of Dairy Cows per Head, Beginning Inventory, and Pounds of 3.5% Fat-Corrected Milk Sold per Cow

26 Random Farms, Type-of-Farming Areas 5 and 6, 1950



Appendix G

Effects of Changes in Prices of Inventories and of Relative Changes in Selling Prices on Gross Income

Changes in Prices of Inventories

A study was made of effects on gross income of price changes which could presumeably not be anticipated and their effects on gross income. One such price change is concerned with goods held in inventories. Simple changes in the values of inventories were considered first (table 28).

Table 28. Summary of Inventory Changes for 45 Farms, Type-of-Farming Areas 5 and 6, 1950

Item	Average change per farm, Jan. 1-Dec. 31	Greatest change for any one farm	
		Increase	Decrease
Crops	$272	$2,810	$1,776
Dairy cattle	723	3,360	700
Other livestock			
Beef cattle	42	3,100	3,293
Hogs	81	1,245	700
Sheep	131	2,070	102
Poultry	48	628	190
Total livestock	1,025	5,900	3,293
Total crops and livestock	1,296	5,204	1,809

The range of changes among farms is large relative to the average effect. The average gross income for the 45 farms in table 28 is $13,324. Thus the total inventory change accounted on an average for about 10% of the gross income.

Inventory changes of crops were analyzed for a different group of 50 farms. The average beginning crop inventory was $2,860 (table 29). At the end of the year the average inventory was $2,907. When ending inventories were assigned beginning prices the average ending inventory was $2,448. Thus on an average the increases in prices of crop inventories accounted for $459 of gross income, and the same amount of labor income.

Table 29. Ending Crop Inventories Valued at Beginning Inventory Prices on 50 Farms, Areas 5 and 6, 1950

Crops	Average beginning inventory	Average ending inventory	Ending inventory valued at beginning prices	Gross income accounted for by	
				Price changes	Quantity changes
Corn	$1,227	$1,193	$896	$297	$-331
Oats	371	503	412	91	41
Wheat	349	250	222	28	-127
Hay	744	873	813	60	69
Beans	75	30	27	3	-48
Potatoes	42	16	24	-8	-18
Sugar beets	21	25	32	-7	11
Barley	25	15	21	-6	-4
Soybeans	6	2	1	1	-5
Total	$2,860	$2,907	$2,448	$459	$-412

It has been shown that the average increase in the dairy cattle income for 45 farms was $723. By use of film strips (microfilm) on which sections of the farm account books for Michigan for 1950 were recorded it was possible to analyze changes in dairy cattle inventories for the effects of changes in prices. The greater part of the increase in dairy cattle inventories is accounted for by larger numbers and values per head of dairy calves and heifers (table 30). A comparison of average values of dairy cows at beginning and ending inventories showed no appreciable difference.

Table 30. Beginning and Ending Inventories of Different Kinds of Livestock, 38 Farms, Type-of-Farming Areas 5 and 6, 1950

Class of livestock	Average inventory				Gain in inventory
	Beginning		Ending		
	Number	Value	Number	Value	
Dairy cattle					
Cows	20.1	$3,463	19.1	$3,544	$81
Heifers	7.2	718	8.7	1,037	319
Bulls	0.9	226	0.9	268	42
Calves	9.3	411	11.4	716	305
Other	1.9	221	2.3	278	57
Beef cattle	1.2	189	1.0	138	-51
Hogs					
Sows	2.1	116	2.0	122	6
Gilts	1.5	67	1.4	71	4
Boars	0.3	21	0.2	18	-3
Other	17.8	334	19.2	420	86
Sheep					
Ewes	3.0	58	2.9	55	-3
Rams	0.3	11	0.2	6	-5
Other	0.4	8	0.9	14	6
Poultry	208.0	261	222.0	302	41
Total		$6,104		$6,989	$885

The prices of animals other than dairy cows of comparable age, weight, kind, etc. were not significantly different in the ending inventories from the beginning inventories. Therefore, as far as price changes in all crop and livestock inventories are concerned, the problem is reduced to one of changes in the prices of crops.

For 43 farms gross income was recalculated to determine what it would have been had there been no price changes in the crop inventory (table 31).

Table 31. Summary of Gross Income, Change in Gross Income and Change in Labor Income because of Price Changes in Feed and Crop Inventories, 43 Farms

Type-of-Farming Areas 5 and 6
1950

Number of farms	Increase in gross income because of price changes in feed and crop inventories	Range of gross income	Range of labor income
2	$1,500 and more	$19,785 to $23,780	$7,830 to $9,842
6	$1,000 to $1,500	$8,393 to $20,523	$2,762 to $7,672
9	$500 to $1,000	$6,313 to $15,894	$1,359 to $5,739
26	Less than $500	$3,501 to $23,811	$281 to $6,980
		Averages	
43	$532	$11,000	$3,482

The tendency for farms with larger gross incomes to show greater effects of price changes in inventories suggests that the relationships of gross income to factors will not be increased by eliminating price changes in inventories. An average of $532 of the labor income for the 43 farms is accounted for by price differences between beginning and ending inventories of crops. The 5% confidence intervals are $367 and $697. In the state of Michigan as a whole labor incomes increased from 1949 to 1950. In type-of-farming area 5 labor incomes increased, on an average, $1,040. In area 6 the average increase was $322. The sample of 43 farms included 37 farms in area 5 and 6 farms in area 6. According to this evidence price gains in feed and crop inventories can not explain all of the increase in the average labor income. Price gains of inventories of livestock have been shown to be negligible.

The hypothesis was set up that price changes in inventories constitute a random type of contribution to gross income. The random contribution should not be included in planning the operations of the farm. Therefore, a revised figure for gross income, a figure from which this type of change or variation has been eliminated, should bear a stronger relation to the employment of productive factors. Revised gross income figures were correlated with total farm expense and other classes of factors. The gross income as reported was similarly correlated. None of the differences is significant at the 5%

level of confidence (table 32).

Table 32. Correlation Coefficients between Categories of Factors and Reported Gross Income Compared with Correlation Coefficients between Categories of Factors and Revised Gross Income[1]

34 Farms, Type-of-Farming Areas 5 and 6
1950

Categories of factors correlated with gross income	Total correlation coefficient with gross income as reported	Total correlation coefficient with gross income as revised
Total farm expense and total investment	0.855	0.778
Total labor charge, total farm expense not labor, and total investment	0.840	0.900
Total farm expense, investment in land and investments other than in land	0.852	0.857

1. Gross income is revised by valuing ending inventories at beginning inventory prices, for crops.

From the fact that none of the correlations is significantly improved by eliminating price gains from crop inventories, it follows that there can be no general narrowing of the confidence limits of the coefficients in the gross income equation. Other elements which cause variations of gross income from its predicted value are so important that the effects, if there are any, of changes in inventory prices are obscured.

The hypothesis was set up that if products sold and held in inventories should be valued at "normal" prices the relationship between factors of production and gross income should be stronger. As a first approximation to normal values the average prices for different crops and livestock products for the period 1946-1949 were considered. A ratio was calculated, expressing the relationship of the 1946-1949 prices to the prices for 1950. This ratio has the effect of bringing the prices of the different farm products into line with each other in accordance with the 4-year period, from 1946 to 1949. (See table 33).

It would be simple enough merely to value the quantities of products sold at the Michigan average season farm prices for the years 1946 to 1949. However, equal pricing for all farms would imply no differences in quality of products, per cent butterfat of milk, and marketing practices. For example, the valuation of the physical quantity of milk sold at the Michigan season average price (even if corrected for butterfat) would deny to a farmer who delivers milk the part of gross income arising from the delivery service. Income differences caused by marketing practices should remain in the data after "normalizing" prices. In order to accomplish this, and at the same time make an allowance for the possibility that some prices may be "out of line", the dollar sales of crops and livestock products and inventory changes of crops were multiplied by factors which converted 1950 sales and inventory changes over to 1946-1949 "expected" sales and inventory changes. Beginning and ending inventories were both valued according to average year-end figures for 1946 to 1949.

There is little doubt that the length of the period needed to develop a set of expectations varies between enterprises. Furthermore, the responses of farmers to price changes of a single product will vary, depending upon the length of run considered. For these and other reasons it is admitted that the four years preceding 1950 do not form a perfect basis for the development of a set of expectations. The hypothesis was, however, that the price relationships which prevailed in those four years should be more related to the organization of the average farm than the prices and their relationships which happened to occur in 1950.

The hypothesis was not supported by the data. The correlation coefficients between categories of factors and reported gross income were not significantly different from the coefficients with respect to "normalized" income.

It can be concluded that the reliability of the value productivity functions would not be increased by adjustment of the farm account data in this study, to

Table 33. Weighted Average Annual Michigan Farm Prices of Crops, Livestock, and Livestock Products, and Ratios of Average 1946 to 1949 Prices to the Average Prices of 1950.[1]

(Farm products important on farm account farms in areas 5 and 6)

Farm product	Unit	Weighted average annual prices 1946 to 1949	Weighted average annual prices 1950	Ratio of average of 1946 to 1949 to average price for 1950
Corn	bu.	$1.63	$1.65	0.99
Wheat	"	2.09	2.01	1.04
Oats	"	0.84	0.84	1.00
Barley	"	1.44	1.20	1.20
Rye	"	1.68	1.27	1.32
Sugar beets	T.	13.12	11.40	1.15
Potatoes	bu.	1.38	1.00	1.38
Field beans	cwt.	8.78	6.90	1.27
Soybeans	bu.	2.54	2.50	1.02
All hay	T.	18.45	20.70	0.89
Veal calves	cwt.	23.78	28.80	0.83
Hogs	"	20.98	18.10	1.16
Beef cattle	"	18.67	21.80	0.86
Sheep	"	8.15	10.50	0.80
Lambs	"	20.78	25.40	
Chickens	lb.	0.30	0.25	1.19
Milk cows	each	169.18	208.58	0.81
Milk, wholesale	cwt.	4.17	3.64	1.15
Eggs	doz.	0.44	0.37	1.21
Wool	lb.	0.46	0.50	0.92

1. Adapted from Michigan Agricultural Statistics, 1950, Michigan Separtment of Agriculture

eliminate price changes in inventories, or to eliminate relative changes in the prices of farm products. Under circumstances comparable to those of this study, in other words, the manipulation of farm account data in order to obtain greater reliability of gross income estimating equations is not worthwhile. If a planning function based on previous experience is implied in the layout of each farm, nothing has been accomplished in this case by inventory and price adjustment in discovering what the function is.

APPENDIX H

Linearity of Relationships

Forty farms were selected to study the relationships (in logs) of the factors to gross income. The farms were chosen at random with the condition that the gross incomes should themselves form a logarithmic distribution. This condition was needed because, if the farms should be chosen at random, the gross incomes would be distributed around the average. There would then be less opportunity to study the behavior of gross income over its range. Figure 16 shows the relation between the log of investment in land and the log of gross income. The relationship appears to be approximately linear. In figure 17 the deviations of the estimate of the log of the gross income (figure 16) from the logs of gross income are plotted against logs of the total labor charge. This second relation appears to be linear.

There is a high degree of intercorrelation between the independent variables in the general equation $\underline{P} = \underline{f}(\underline{x}, \underline{y}, \underline{z})$. Thus it can be expected that the relationship between the present residuals and additionally introduced factors will disappear. In figure 18 the residuals from figure 17 are related to the log of the investments other than in land. There is no significant relationship. The order of plotting residuals against additional factors

Figure 16. Relation between Investment in Land and Gross Income, in Logarithms, 40 Farms

Type-of-Farming Areas 5 and 6, 1950

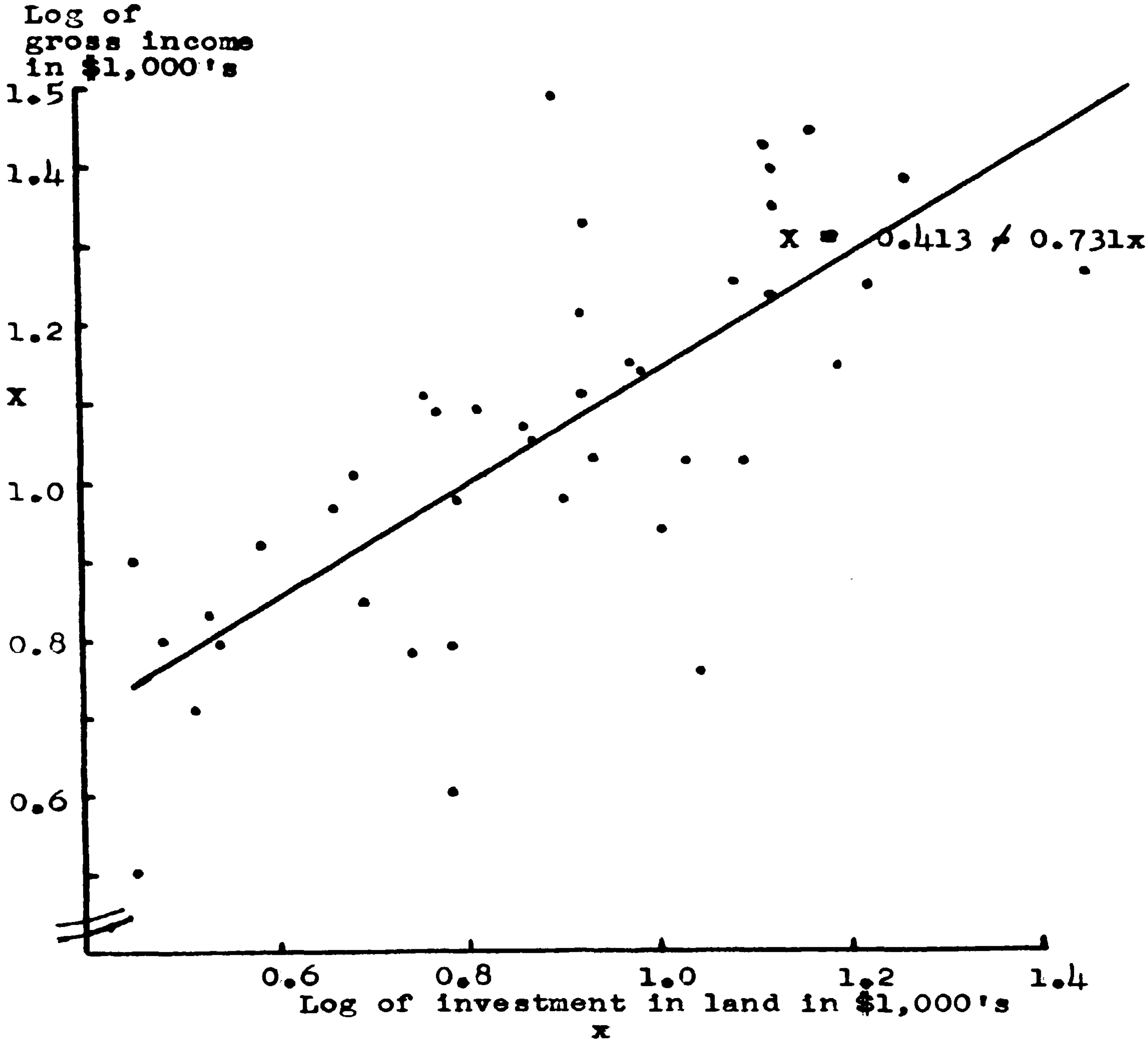


Figure 17. Relation between the Log of Total Labor Charge and Residual from Estimate of Gross Income from Investment in Land, 40 Farms

Type-of-Farming Areas 5 and 6, 1950

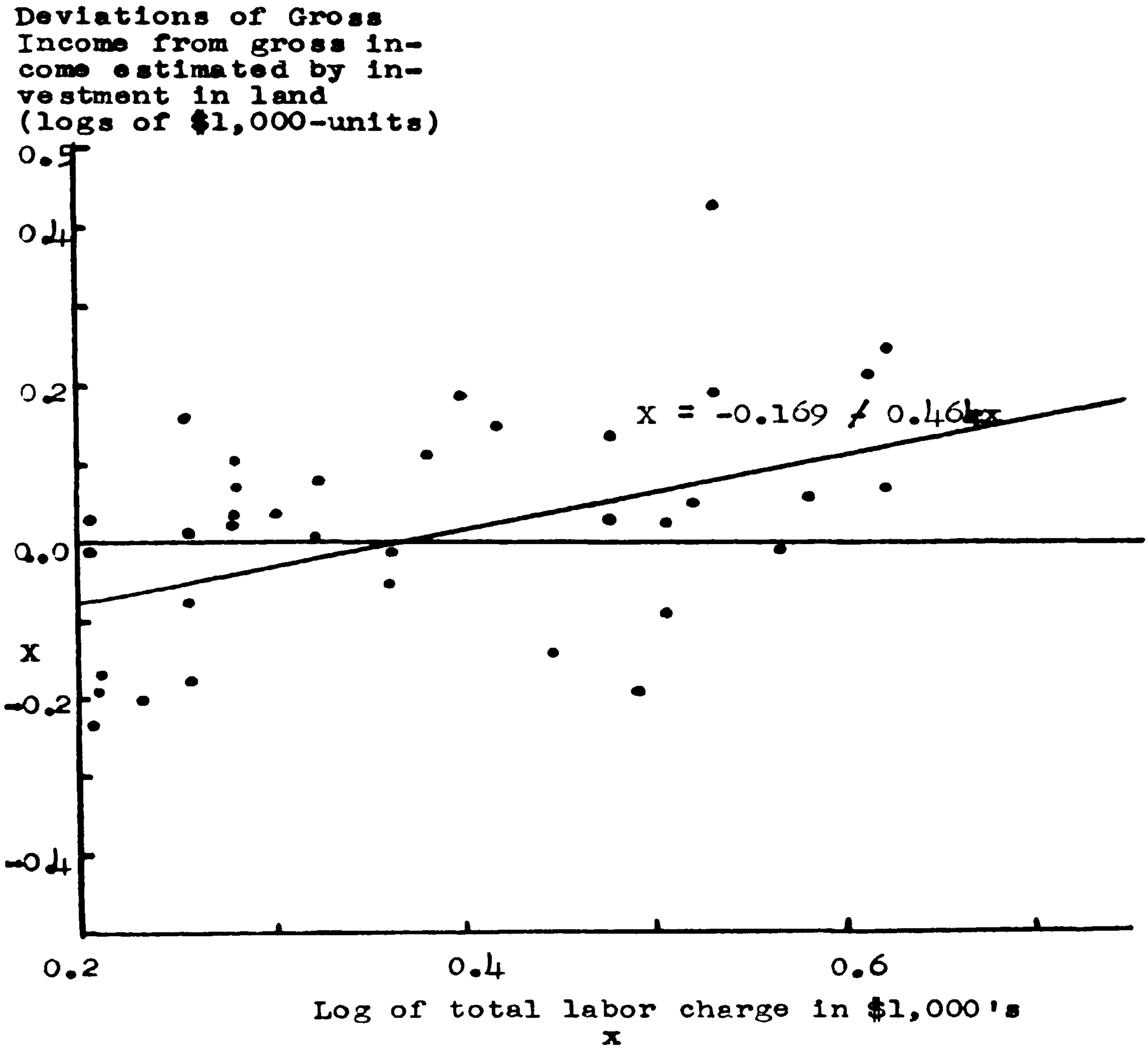
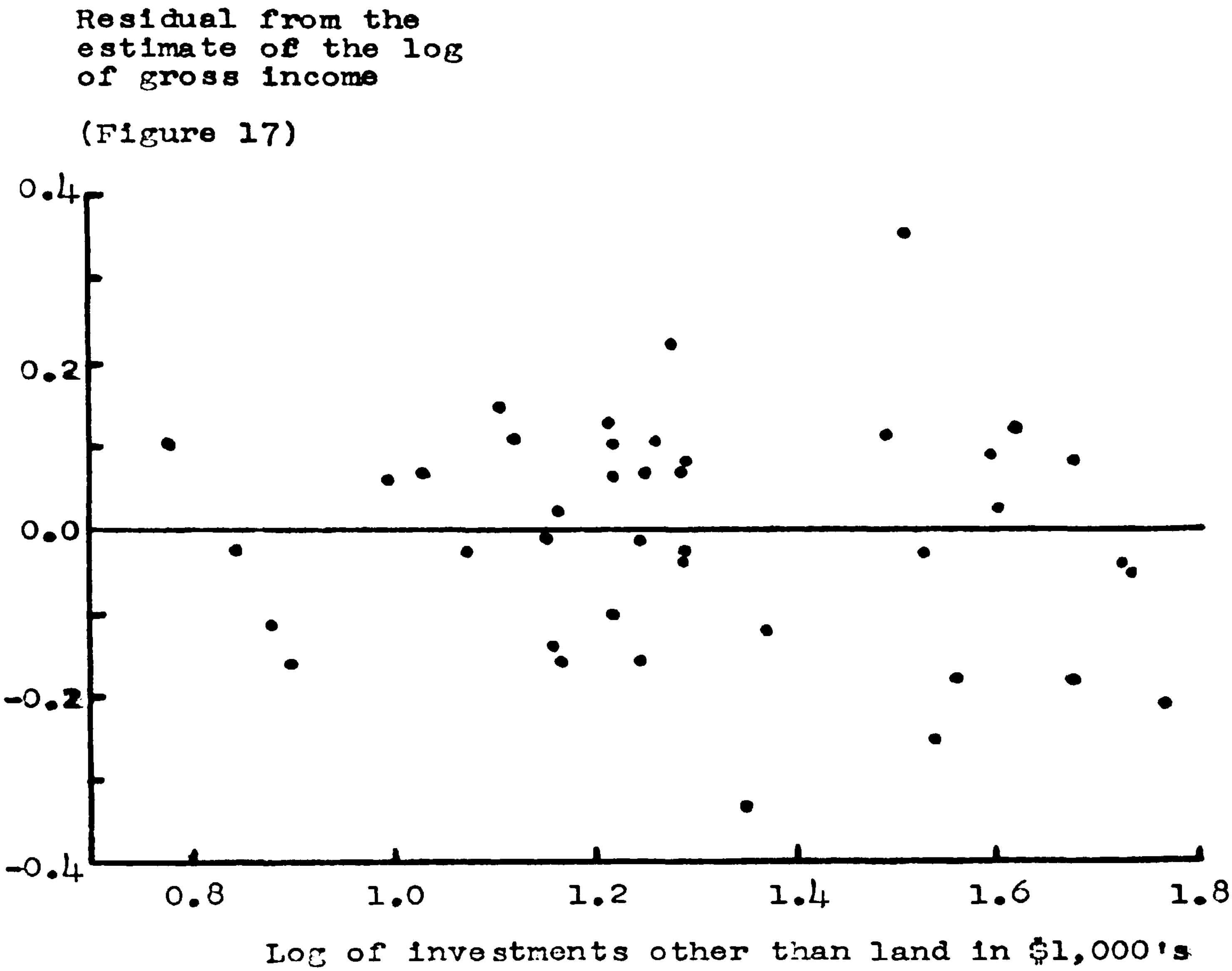


Figure 18. Relation of Log of Investments other than Land to the Residual from the Estimate of the Log of Gross Income from the Logs of Total Labor Charge and Investment in Land

40 Farms, Type-of-Farming Areas 5 and 6, 1950



can be changed with different results. In figure 19 the log of investments other than land is related directly to the log of the gross income, and, as expected, there is a good linear relation. In figure 18 this relationship is not apparent. This is because in figures 16 and 17 two other independent variables, which are themselves related to investments other than land, have taken about all of the relation of the three categories to the gross income.

In figure 20 the residuals from the estimate of the log of gross income from the log of investments in land (figure 16) are plotted against the log of the investments other than land. In figure 21 the log of the total labor charge is plotted against residuals in the log of the gross income which were (1) not explained by the simple relation between the log of the investment in land and the log of the gross income, and (2) were not explained by the relation between the residuals of (1) previous and the log of the investment other than in land, against the log of the total labor charge. In figure 21, which shows the residual errors from figures 19 and 20 plotted against the log of the total labor charge, an observable relationship still remains.

Figures 16 through 21 bring out two facts: (1) Visually it appears that the gross income of a farm is related logarithmically to the factors of production. Simple

Figure 19. Relation of Log of Investments other than Land to Log of Gross Income, 40 Farms

Type-of-Farming Areas 5 and 6, 1950

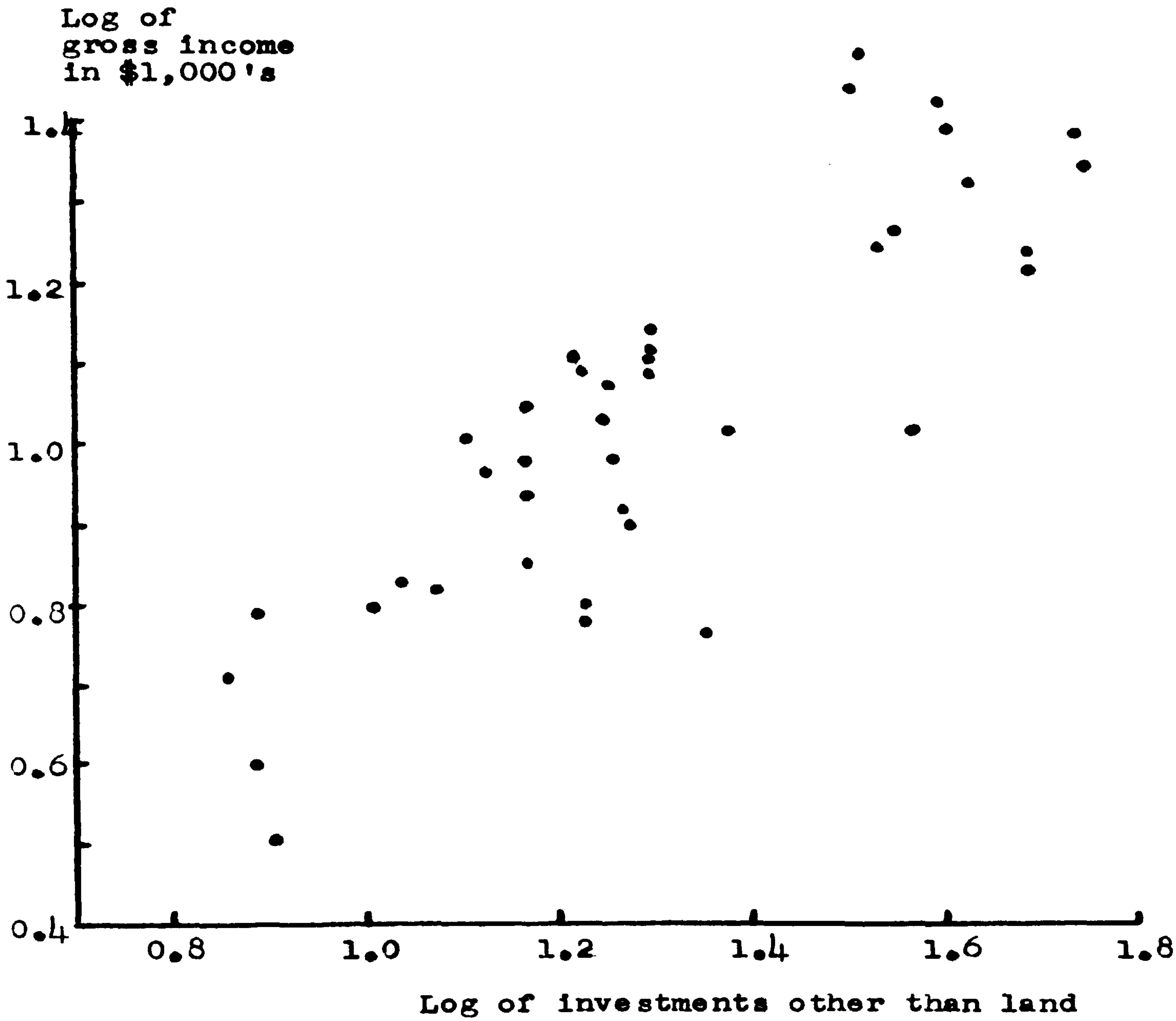


Figure 20. Relation of Log of Investments other than Land to Residual from Estimate of Log of Gross Income from Log of Investment in Land

40 Farms, Type-of-Farming Areas 5 and 6, 1950

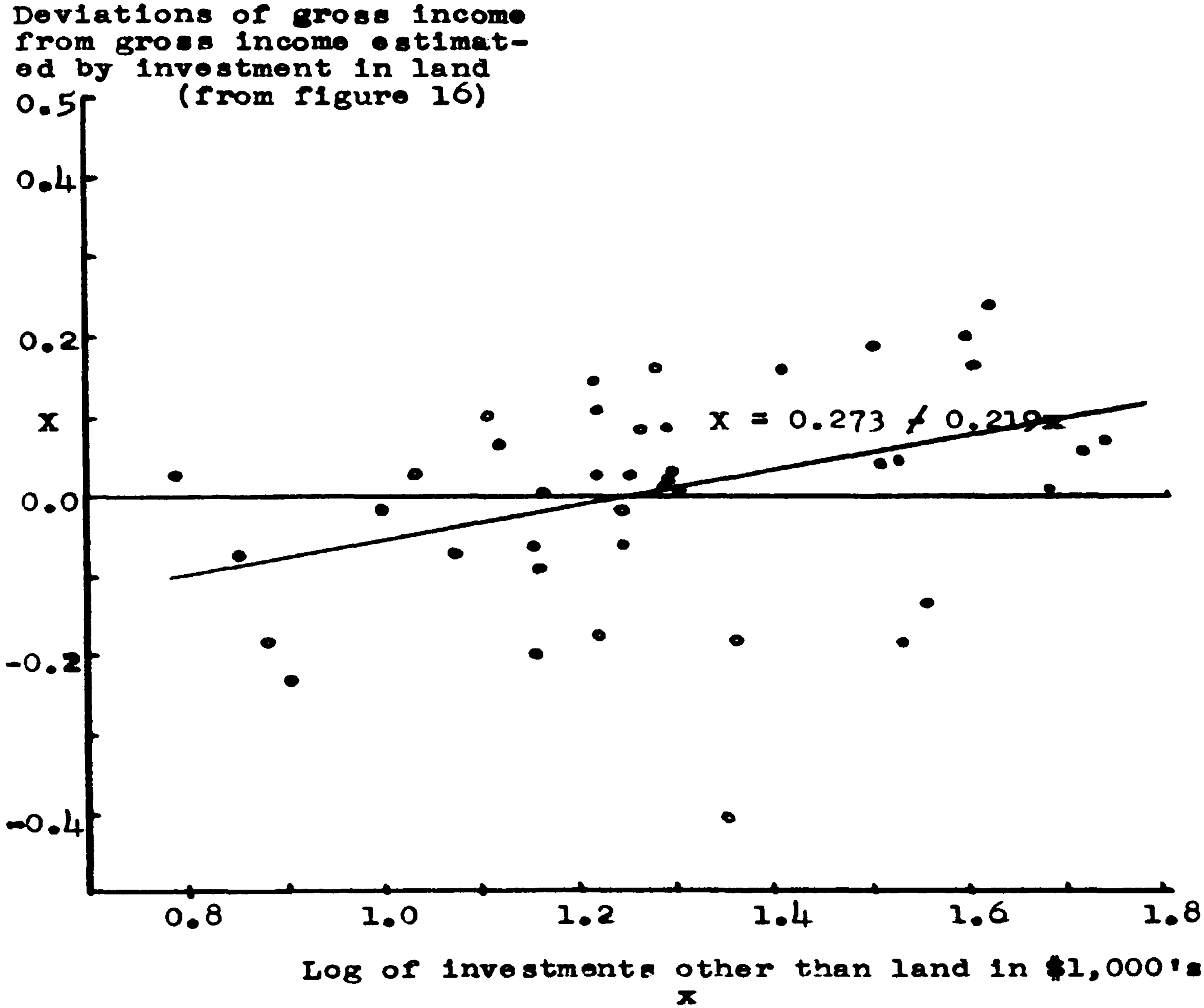
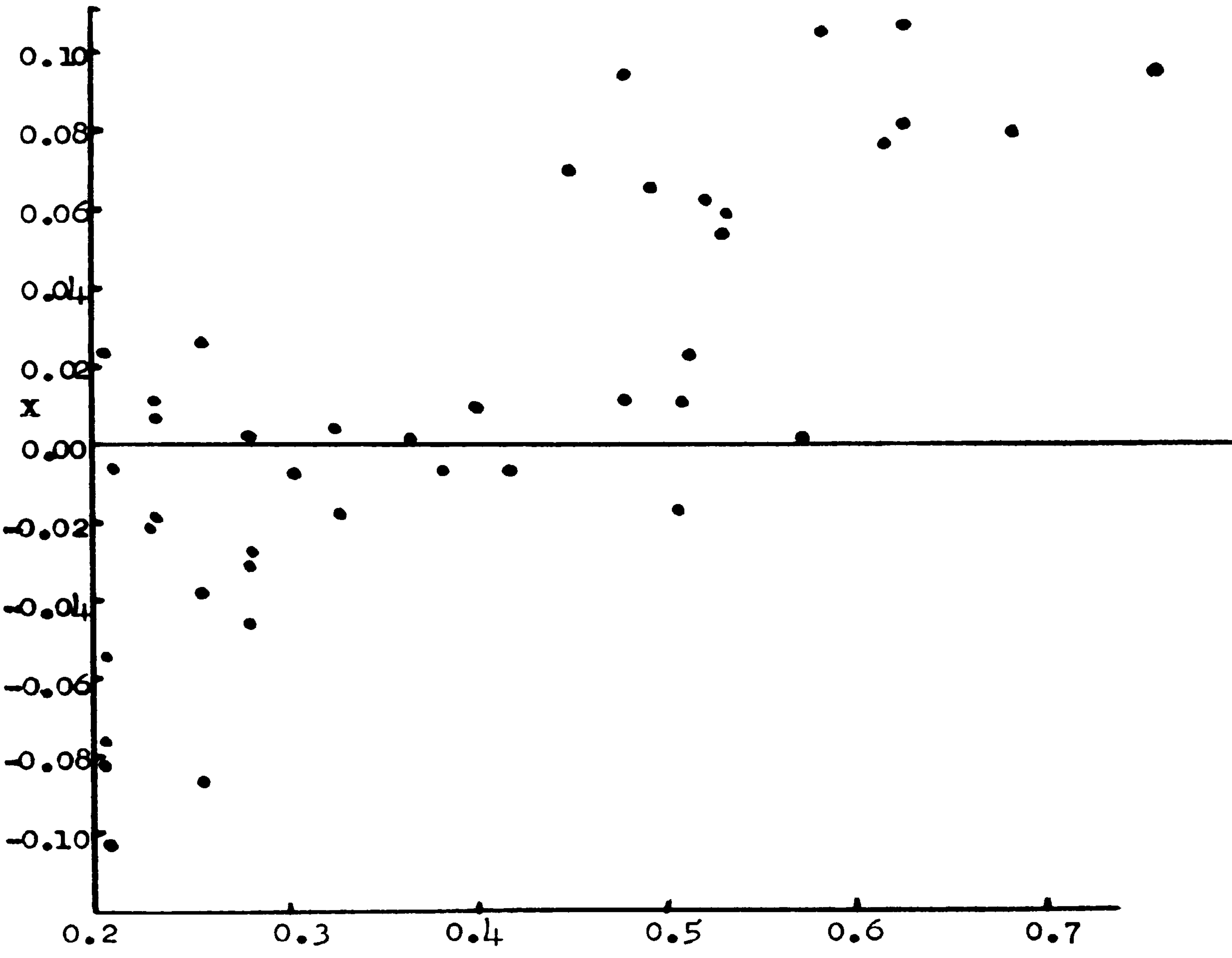


Figure 21. Relation of Log of Total Labor Charge to Residual of Estimate of Log of Gross Income from Log of Investment other than Land and Log of Investment in Land, 40 Farms

Type-of-Farming Areas 5 and 6, 1950

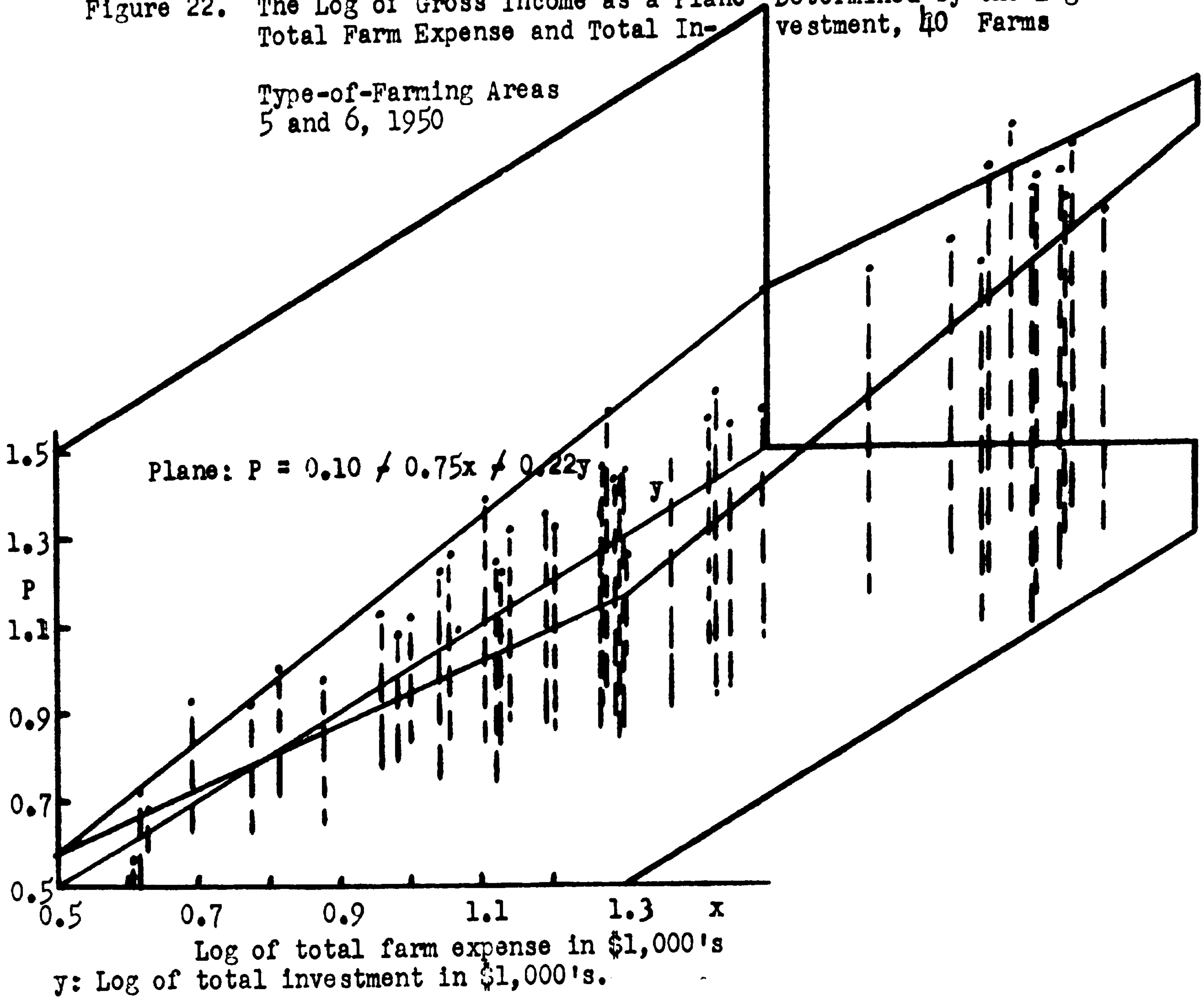


X: Residual from the estimate by the log of investment other than land of residual from the estimate of the log of gross income by the log of the investment in land. (from figure 20).

analysis of variance tests shows logarithmic linearity of the relations. That is, the lines of best fit will more closely estimate the value of gross income than will the mean of gross income, and (2) the factors affecting gross income are intercorrelated. In figure 22 gross income is taken as a function of the total farm expense and total investment. The 40 farms do not in these cases fall on a line as far as the relative amounts of the two categories are concerned. The points of intersection of the "causal" variables, investment in land and total labor charge, do not fall on a line in the horizontal plane. It can be concluded that there is some selection of categories of factors between farms.

Figure 22. The Log of Gross Income as a Plane Determined by the Logs of Total Farm Expense and Total Investment, 40 Farms

Type-of-Farming Areas
5 and 6, 1950



APPENDIX I

The Problem of Multiple Solutions of the Gross Income Equation

One of the criticisms of the work of Douglas is that he did not have statistically different observations of the employment of factors.[1] Mendershausen has shown that the regression surfaces which Douglas calculated are not significant since the observations in three dimensions form a line rather than a surface. Douglas' answer to this criticism was that in repeated studies the conclusions (values of C--the constant term--and of a, b,--the exponents in our work) tended to be consistent. The consistency of conclusions from one study to another was offered as a vindication of the method.[2]

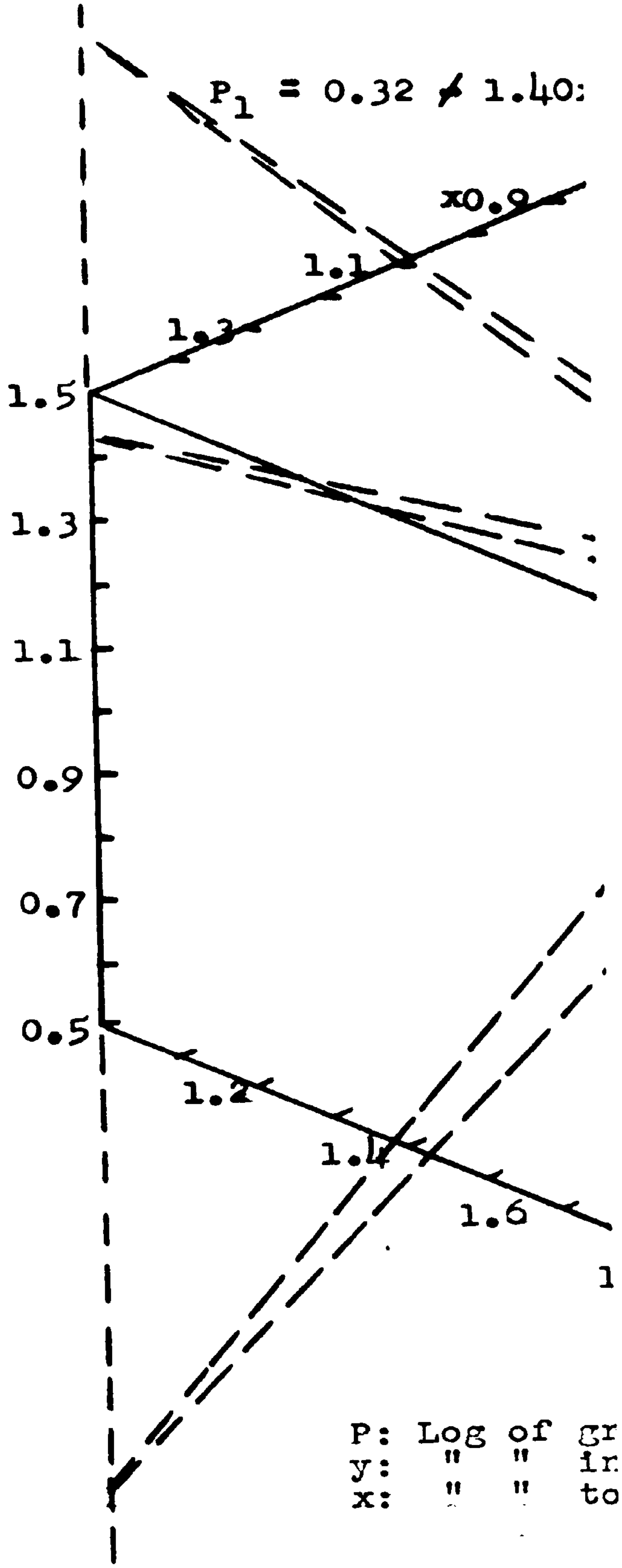
Figure 23 gives another opportunity to evaluate the degree to which the 40 farms selected for the study of logarithmic linearity (Appendix H) can be considered to represent independent observations in combinations of categories of factors. The dispersion of the observations in the directions of the total farm expense and the total investment axes indicates that within the range of prac-

1. Horst Mendershausen, "On the Significance of Professor Douglas' Production Function," Econometrica, V. 6 (1938) pp. 143-53.

2. See footnote 2, p. 8, supra.

Figure 23. Three S[…]
come, T[…]

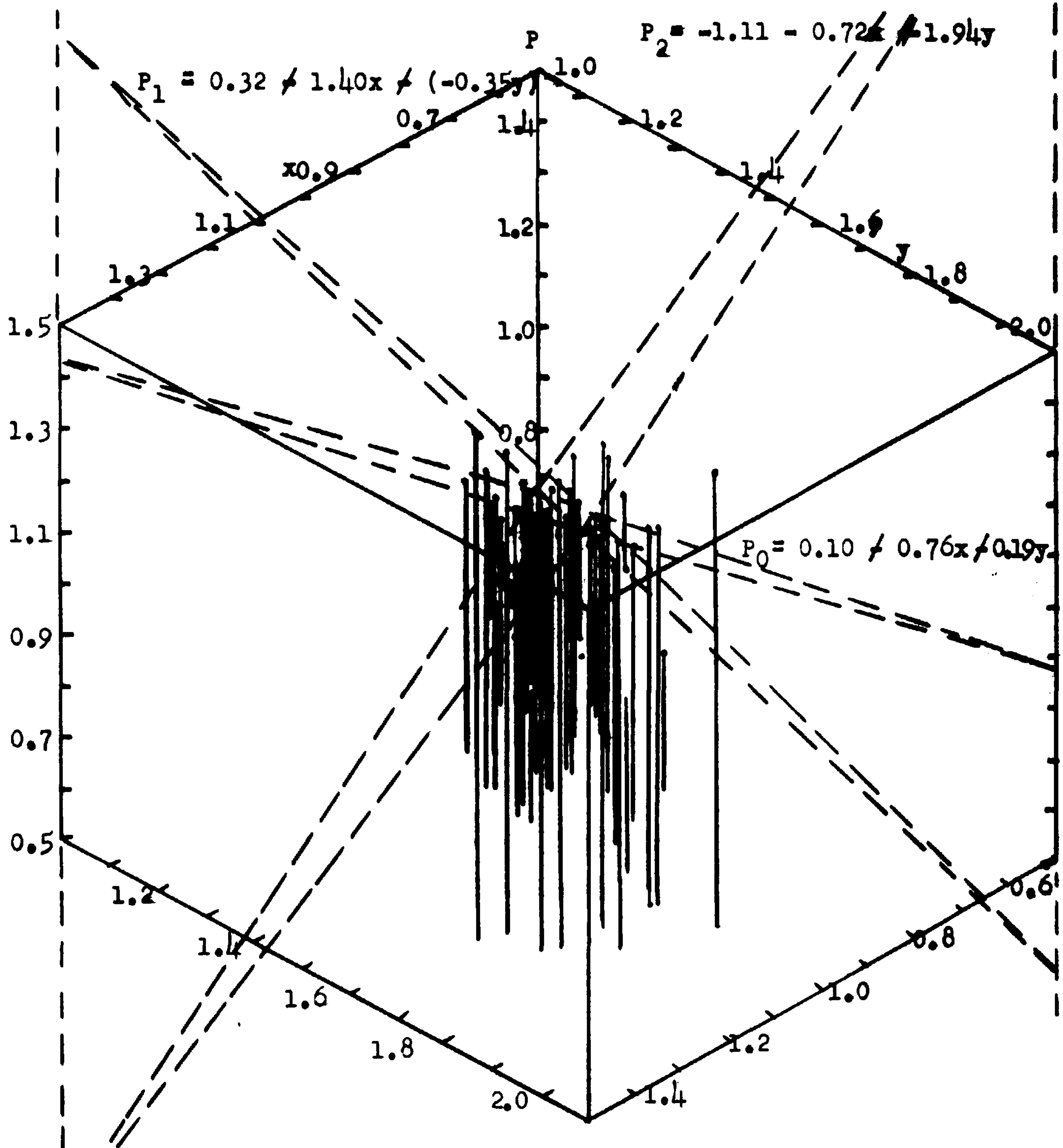
40 Farm[…]



P: Log of gr
y: " " in
x: " " to

Figure 23. Three Solutions of the Relationship[1] between Gross Income, Total Farm Expense, and Total Investment

40 Farms, Type-of-Farming Areas 5 and 6, 1950



P: Log of gross income in $1,000's
y: " " investment " "
x: " " total farm exp." "

ticability of farm operations there is some substitution between factors.

However, the explanation of gross income from total farm expense alone is practically as complete as the explanation of gross income by both total farm expense and total investment. The following table gives the correlation coefficients between the variables shown in figure 9:

Expression, r_{12}, *etc.*	Value of r
r_{xy}	0.926
r_{xz}	0.851
r_{yz}	0.856
$r_{x.yz}$	0.931

If a true plane existed, however, it should be possible to minimize the sums of the squares of the differences in all three directions and get approximately the same equation on the same plane. However, when this is attempted in figure 23 it is seen that divergent equations result. The meaning of the differences in the regression planes on figure 23 can be resolved into the question of what are the dependent and the independent variables in an equation relating the gross income of a farm with the categories of factors used on the farm. Bronfenbrenner is inclined to agree with Douglas that the gross income is a function of outlays, in effect.[1] In this case the

1. See footnote 2, p. 15, *supra*.

plane which minimizes the sums of the squares of the differences between actual and estimated values of gross income in the gross income direction is the relevant plane. This plane is the one referred to on page 163, and it is the plane of special interest in this study.

Practically, this means that it is assumed that product is a result of factors. The assumption is discussed in part 3 of chapter I.

...ucial Limits (At the 5 Percent Level).

	Land A	Labor B	Improve-ments C	Liquid assets D	Working assets E	Cash operating expenses F
Total						
Mean	0.098	0.087	0.044	0.179	0.158	0.280
Upper limit	0.118	0.123	0.092	0.243	0.289	0.394
Lower limit	0.077	0.051	-0.002	0.115	0.027	0.167
Hogs						
Mean	0.062	0.079	0.037	0.265	0.119	0.341
Upper limit	0.090	0.130	0.102	0.370	0.285	0.498
Lower limit	0.033	0.027	-0.026	0.159	-0.047	0.184
Beef feeders						
Mean	0.134	0.123	0.081	0.198	-0.115	0.082
Upper limit	0.179	0.202	0.188	0.298	-0.437	-0.110
Lower limit	0.089	0.044	0.027	0.099	-0.437	-0.110
Dairy						
Mean	0.083	0.030	0.008	0.325	0.164	0.901
Upper limit	0.163	0.124	0.214	0.685	0.558	1.580
Lower limit	0.003	-0.063	-0.197	-0.033	-0.230	0.223
Crops						
Mean	0.138	0.154	0.007	-0.141	0.190	0.440
Upper limit	0.202	0.257	0.021	0.094	0.570	0.866
Lower limit	0.074	0.059	-0.006	-0.378	-0.189	0.012
General						
Mean	0.132	-0.073	0.010	0.366	0.298	0.578
Upper limit	0.221	0.128	0.226	0.641	0.850	1.092
Lower limit	0.042	-0.275	-0.205	0.091	-0.252	0.063

Taken from Gerhard Tintner and O. H. Brownlee, "Production Functions Derived from Farm Records," Journal of Farm Economics, V. 26 (1944). **The data are for Iowa farms, calendar year 1939.**

Table 35. Marginal Productivities and Fiducial Limits at the Five Percent Level of Probability (Per Dollar of Input)

	Mean Upper} Lower} limits	Land A	Labor B	Equip- ment C	Live- stock, feed D	Misc. operat. expense E
All farms	M.	.046	.079	.201	.839	.393
	U.L.	.057	.232	.285	1.077	.432
	L.L.	.035	-.074	.117	.600	.354
Northeast Dairy Area	M.	.033	.097	.148	.658	.378
	U.L.	.051	.335	.285	1.072	.449
	L.L.	.014	-.141	.001	.245	.307
Cash Grain Area	M.	.061	.106	.180	.417	.369
	U.L.	.086	.146	.357	.679	.466
	L.L.	.036	.067	.004	.155	.272
Western Meat Area	M.	.038	.030	.241	.713	.403
	U.L.	.079	.320	.393	1.207	.473
	L.L.	-.002	.260	.089	.218	.333
Southern Pasture Area	M.	.018	-.109	.313	2.641	.402
	U.L.	.048	.196	.563	3.812	.516
	L.L.	-.011	-.414	.063	1.471	.298
Eastern Meat Area	M.	.039	.068	.214	.501	.340
	U.L.	.065	.089	.447	.973	.415
	L.L.	.014	.047	-.017	.028	.265
Crop	M.	.044	-.249	.179	.263	.623
	U.L.	.073	.213	.503	.786	.873
	L.L.	.156	-.713	-.145	-.260	.373
Hog	M.	.011	.059	-.229	.789	.545
	U.L.	.030	.390	-.017	1.189	.637
	L.L.	-.007	-.272	-.431	.388	.454
Dual and Dairy	M.	.020	.021	.138	.538	.463
	U.L.	.043	.310	.298	.944	.531
	L.L.	-.003	-.268	-.028	.133	.360
General	M.	.020	.281	.340	.991	.546
	U.L.	.056	.572	.498	1.601	.482
	L.L.	.006	.009	.182	.380	.242
Special	M.	.075	-.020	.040	1.593	.445
	U.L.	.111	.371	.069	2.631	.585
	L.L.	.040	-.413	.011	.555	.341
"Large"	M.	.051	.040	.259	.441	.416
	U.L.	.071	.294	.434	.677	.471
	L.L.	.031	-.293	.083	.205	.361
"Small"	M.	.046	.103	.208	1.469	.375
	U.L.	.065	.267	.312	1.986	.432
	L.L.	.028	-.061	.103	.952	.318

Taken from Earl O. Heady, "Production Functions from a Random Sample of Farms," Journal of Farm Economics, V. 28 (1946), pp.989-1004. **The data are for Iowa farms, calendar year 1939.**

APPENDIX K

An analysis of Differences Between Estimated and Reported Gross Income and Labor Income

The method of accounting for differences betweenreported and estimated gross income can be extended to a business involving two or more enterprises. Let the data for a given farm and for the average of all farms be given by the following table:

Enterprise	Averages, all farms			This Farm		
	Yield	Price	Acres	Yield	Price	Acres
1	a	b	c	$(a \neq \Delta a)$	$(b \neq \Delta b)$	$(c \neq \Delta c)$
2	d	e	f	$(d \neq \Delta d)$	$(e \neq \Delta e)$	$(f \neq \Delta f)$
'	'	'	'	'	'	'
'	'	'	'	'	'	'
	'	'	'	'	'	'

For two enterprises, using a numerical example:

1	15	1	(9.6)	$(15 \neq 5)$	$(1 \neq 0.2)$	(10)
2	20	1	(28.8)	(20- 8)	$(1 \neq 0.0)$	(30)

The gross income on this farm is $600. Suppose that the estimated gross income is $720. The average farm would require 12 units of enterprise 1, and 36 units of enterprise 2 in order to earn $720. The "expected" values of the number of units are inserted in the table preceding.

The difference between expected and reported gross inome is approximated by three primary effects as follows:

$$\frac{2a + \Delta a}{2} \cdot \frac{2b + \Delta b}{2} \cdot \Delta c = 17.5 \cdot \$1.1 \cdot 0.4 = \$7.70$$

$$\frac{2d + \Delta d}{2} \cdot \frac{2e + \Delta e}{2} \cdot \Delta f = 16.0 \cdot \$1.0 \cdot 1.2 = \$19.20$$

$$\frac{2a + \Delta a}{2} \cdot \frac{2c + \Delta c}{2} \cdot \Delta b = 17.5 \cdot 9.8 \cdot \$0.2 = \$34.30$$

$$\frac{2d + \Delta d}{2} \cdot \frac{2f + \Delta f}{2} \cdot \Delta e = 16.0 \cdot 29.4 \cdot \$0.0 = \$\ 0.00$$

$$\frac{2b + \Delta b}{2} \cdot \frac{2c + \Delta c}{2} \cdot \Delta a = \$1.1 \cdot 9.8 \cdot 0.5 = \$53.90$$

$$\frac{2e + \Delta e}{2} \cdot \frac{2f + \Delta f}{2} \cdot \Delta d = \$1.0 \cdot 29.4 \cdot (-8.0) = \$-235.20$$

he total of size of business effects (c) is \$26.90; the otal price and yield effects are \$34.30 and \$-181.30 re- pectively. The price and yield effects have significance hen considered with respect to individual enterprises. he size of business effect is most meaningful when the arm as a whole is considered.

The analysis of the difference between expected and ecorded gross income can be used in connection with the osts of operating the farm in an explanation of the differ- nces of estimated labor income from recorded labor income.

There will be a relation of total farm expense plus nterest on the investment at 5% and gross income for all arms. Suppose that this is given by:

Gross income = 5/4 (total farm expense + 5% on investment).

he gross income of each farm is estimated by a function:

$$\text{Gross income} = \underline{C}(\text{total farm expense})^{a}(\text{interest on investment})^{b}.$$

In this model let the equation be:

$$\text{Gross income} = 0.988(\text{total farm expense})^{3/4}(\text{interest on investment})^{1/4}.$$

Let it be assumed that the total farm expense and the total investment for the particular farm being considered are $455 and $4096, respectively. Then the following information is available:

Total costs, that is, total farm expense plus 5% interest on the investment for the average farm with a gross income of $720 and $540. The comparable figure for the farm being analyzed is $660. It is now possible to present in tabular form the sources of the differences between expected and recorded gross income and expected and recorded labor income.

Table 36. Explanation of Difference Between Recorded and Expected Values of Gross Income and Total Costs

	This farm		Mean of all farms
Gross Income			
Recorded	$600		$720
Expected	720		720
Difference in gross income from expected values		-$120	
Costs			
Total farm expense	455		
Interest on investment--5% x $4096	205		
Total costs	660		576
Expected total costs	576		576
Difference in costs from expected amount		84	

	This farm (cont.)		Mean of all farms (cont.)	
Labor Income				
Recorded	$ -60		$144	
Expected	144		144	
Difference		-204		0
Difference in Labor Income due to cost structure;		-84		0
due to difference in gross income		-120		0

Table 37. Analysis of Difference in Gross Income from its Expected Value by Enterprises, Yield Effects, Price Effects, and Size of Business Effect

	Gross Income		Difference Attributable to:			
Enterprise	Recorded	Estimated	Yield	Price	Size of business	Total Difference
(1)	$240	$144	$ 54	$34	$ 8	$ 96
(2)	360	576	-235	0	19	-216
Total	$600	$720	-$181	$34	$27	-$120

The preceding tables have accounted for differences from expected values of gross income and labor income according to the structure of the use of resources, yields, prices, and size of business. The difference in gross income attributable to yields and prices are accounted for by enterprises.

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