A MULTICASE STUDY OF THREE GRADUATE TEACHING ASSISTANTS PARTICIPATING IN THE MDISC TEACHING PROFESSIONAL DEVELOPMENT

By

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A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

Mathematics Education—Doctor of Philosophy

2022

ABSTRACT

A MULTICASE STUDY OF THREE GRADUATE TEACHING ASSISTANTS PARTICIPATING IN THE MDISC TEACHING PROFESSIONAL DEVELOPMENT

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In this multicase study (Stake, 2005), I studied a semester-long online implementation of the "Mathematics Discourse in Secondary Classrooms" (Herbel-Eisenmann, Cirillo, et al., 2017) (MDISC) teaching professional development (T-PD)—slightly modified for the university context—which I offered to three mathematics-teaching graduate teaching assistants (GTAs) (i.e., Finnegan, Valeria, and Alice) who were no longer in their first year of teaching. I sought to understand: (a) How does the classroom discourse change in each of the participant's classes over the course of their participation in the MDISC T-PD?; and (b) How do participants talk about their use of the teacher discourse moves (TDMs) (e.g., Cirillo et al., 2014; Herbel-Eisenmann et al., 2013)? To analyze the change in classroom discourse, I studied two sets of classroom discourse dimensions: (a) the TDMs (i.e., waiting, inviting student participation, revoicing, asking students to revoice, probing a student's thinking, and creating opportunities to engage with another's reasoning), which were a central set of practical tools offered by the T-PD; and (b) student discourse dimensions drawn from the discourse dimensions of the "Equity QUantified In Participation" (EQUIP) tool (Reinholz & Shah, 2018).

In line with a multicase approach, this study consists of one case report for each participant, with each report driven by an emic issue, as well as a multicase report thereafter in which the participants' similarities and differences with respect to the research questions are explored. Finnegan, Valeria, and Alice's respective emic issues were: (a) How can an instructor who wants students to participate in their class invite student participation?; (b) How does an

instructor who weighs wielding her authority to engage students against respecting students' agency (even if that agency is used to not participate) implement the TDMs?; and (c) How does an instructor who finds herself in a stage of "survival" (Beisiegel et al., 2019; Katz, 1972) implement and talk about the TDMs? Finnegan, who sought to invite student participation, especially took to TDMs that helped him do so: waiting (particularly wait time 1 [Rowe, 1986]), probing a student's thinking, and inviting student participation via cold-calling. Valeria, who did not want to force students to do things, felt more comfortable implementing TDMs that relied more on what she was doing than on what students were doing. She mainly used waiting (particularly wait time 1), revoicing, and inviting student participation via inviting further responses. Last, Alice, who found herself in a stage of survival, continued using those TDMs that she was already familiar with, that is, waiting and revoicing.

This study contributes to the emerging area of T-PD for GTAs beyond their first year of teaching. In addition, it offers a refinement of the TDMs and a detailed account of how and why participants used (or did not) each TDM. The TDM refinement includes: (a) a refinement of "waiting" that builds on work by Rowe (1986) and Ingram and Elliott (2016); (b) a refinement of "probing a student's thinking" that distinguishes between two types of probing (i.e., to clarify a student's turn or to go deeper with it) enacted in two ways (i.e., instructor- or student-centric) in response to student statements or questions; and (c) a refinement of "creating opportunities to engage with another's reasoning" by distinguishing between five types of such engagement (i.e., adding to, anticipating, comparing to, evaluating, and understanding). Further, the participants' uses for revoicing add to work on different uses of revoicing (e.g., Herbel-Eisenmann et al., 2009), and the participants' hesitations about asking students to revoice provide insight into the challenges of learning to use this TDM.

I dedicate this dissertation to all the graduate students who strive to create a meaningful experience for their students despite being overworked and underpaid, their teaching often overlooked and undervalued.

ACKNOWLEDGMENTS

Thank you, Danielle, for your love and support during this Ph.D. journey while being on two academic journeys yourself. I love you.

Danke Mama, Papa, Maya und Miriam—wie auch Oma Marie, Opa Norbert, Opa Erwin, Onkel Tommi und Ka—für eure Unterstützung aus der Ferne … und aus der Nähe, wenn ich dann mal zu Besuch war. Ich bin unendlich dankbar so eine wunderbare Familie zu haben und könnte mir keine bessere wünschen.

Thank you to my dissertation committee, Shiv, Jack, Monica, and Beth, each of whom possesses at least one academic superpower: (a) generosity of time, (b) feedback that is critical but makes you feel better about yourself, (c) attention to detail, and (d) critical questions. Perhaps more importantly, each of them also possesses at least one interpersonal superpower: (a) unwavering trust in students, (b) being *present* in conversations, (c) caring about and respecting students' experience, and (d) judgement-free honesty. I have tried to take notes. I am incredibly grateful that our paths crossed, and I hope they will cross many more times.

Superpowers aside, thank you, Shiv, for being the best advisor anyone could hope for.[1]

Thank you, Jack, for welcoming me into the T2P team five years ago and watching out for me.

Thank you, Monica, for creating better experiences for graduate students.

Thank you, Beth, for your guidance and generously sharing your MDISC materials with me. Without them, this dissertation would have looked very different.

Thank you, Freda, Kelly, and Lisa: Without you, I would not have made it this far.

[1] Onus.

Thank you, participants, for agreeing to participate in this dissertation research and giving me hope that university mathematics teaching is increasingly in good hands. I could not have done this work without you.

Thank you also to the MDISC guinea pigs, who generously shared their time so that I might gain experience facilitating the T-PD and offer the study participants a better experience.

Thank you, Manasi and Amit, for being my incredibly supportive adoptive parents. I can't wait for our next chai night and hope that the world will be a slightly better place.

Thank you, Jihye, for countless walks and conversations. I am glad I decided to give combinatorics a chance.

Thank you, Sofía, for sharing new ideas and comics with me. You make people around you better, and I hope to have picked up a thing or two.

Thank you, Isabel, for being a beacon of determination and out-of-the-box thinking.

Danke, Dimitris, für gute Gespräche und viel Gelächter. Ich hoffe, du kannst mir irgendwann verzeihen, dass das Wasser immer zu heiß war.

Thank you, Bob, Chuck, and Merve, for your support. Regardless of the questionable productivity of our check-ins that some have pointed out, they were important to me and kept me sane.

Thank you, T2P team, for being the most wonderful research group to meet each week.

Thank you, Tsveta and Andy, for inviting me to join the Center for Instructional Mentoring. We need more bridges between mathematics and mathematics education, and the work you do is invaluable.

Thank you, Brin, for encouraging me to run with scissors. I now encourage others to do the same.

Thank you to all who attended my dissertation defense: Your presence made the day that much more special.

Finally, thank you, PRIME students, alumni, staff, and faculty, for *being* PRIME. When deciding which Ph.D. program to join, the sense of community PRIME emanated was the deciding factor, and I am grateful for each and every one of you. You shaped my journey in countless ways, and I cannot be grateful enough to you. Thank you! Go Green!

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KEY TO ABBREVIATIONS

CGI	Cognitively Guided Instruction
CSPCC	Characteristics of Successful Programs in College Calculus
EQUIP	Equity QUantified In Participation
GTA	Graduate Teaching Assistant
IRE	Initiation-Reply-Evaluation
IRF	Initiation-Response-Feedback (later: Initiation-Response-Follow-up)
MDISC	Mathematics Discourse in Secondary Classrooms
PtC	Progress through Calculus
QUASAR	Quantitative Understanding: Amplifying Student Achievement and Reasoning
STEM	Science, Technology, Engineering, and Mathematics
TDM	Teacher Discourse Move
T-PD	Teaching Professional Development
(word)	best guess for a word the speaker used
((*description*))	description of, for example, a sound (e.g., laughter) or relevant information
[word]	a word that was missing or was implied through, for example, a deictic

CHAPTER 1: INTRODUCTION

The results of the *Progress through Calculus* (PtC) survey (Apkarian & Kirin, 2017) documented that U.S. mathematics departments offering graduate degrees in mathematics did not necessarily offer their graduate teaching assistants[2] (GTAs[3]) any teaching professional development (T-PD). Specifically, of the 215 responding institutions (out of all 330 U.S. institutions offering graduate degrees in mathematics), 31.2% did not offer a department-specific GTA T-PD.[4] Where a departmental GTA T-PD was offered, it was typically confined to GTAs' first year of teaching as indicated by almost 80% of responding institutions (Apkarian & Kirin, 2017). The short duration of such T-PDs seems problematic given how much there is to learn about teaching and given that many GTAs find themselves in a stage of survival during their first year of teaching (Beisiegel et al., 2019). GTAs may be more receptive to topics beyond classroom management in later years of their teaching, when they find themselves in the stages of consolidation, renewal, or maturity (Beisiegel et al., 2019; Katz, 1972). Thus, the purpose of this dissertation was to offer a T-PD to three GTAs—who were teaching undergraduate mathematics classes and were no longer in their first year of teaching—and study parts of it using a multicase study approach (Stake, 2005).

Choosing a T-PD

When choosing or designing a T-PD, the question of the T-PD's focus arises. To address this question, I want to highlight two core problems with the teaching of mathematics at the

[2] The term "graduate teaching assistant" is an umbrella term that encompasses various roles graduate students play in classrooms: instructors, recitation leaders, in-class instructional assistants, tutors, and graders.
[3] Unless specifically indicated otherwise, I will be speaking about mathematics graduate teaching assistants.
[4] The *Progress through Calculus* survey indicates that 47.9% of responding institutions offered a university-wide T-PD (Apkarian & Kirin, 2017), but it is unclear to which extent institutions with university-wide T-PD and institutions with department-specific T-PD were mutually exclusive.

undergraduate level: the widespread use of lecture in college[5] classrooms and the predominant narratives around who can and cannot do mathematics.

To this day, college mathematics instruction is frequently lecture-based and teacher-centered (Stains et al., 2018). Shifting away from lecture and towards active learning—as called for by, for example, the Mathematical Association of America (Abell et al., 2018)—requires instructors who can engage their students in mathematical practices (e.g., conjecturing, explaining, and justifying). For this purpose, instructors will need to know how to move beyond traditional lectures, as well as "interactive" Initiation-Reply-Evaluation (IRE) (Mehan, 1979) lectures, to genuinely open the classroom discourse to students.

Further, instructors are needed who recognize the myths surrounding mathematics and who is (and is not) good at it. Specifically, instructors should understand how the widely accepted "romance of mathematics"[6] (Lakoff & Núñez, 2000)—as well as the racialized (e.g., McGee & Martin, 2011; Shah, 2017) and gendered (e.g., Mendick, 2006; Walkerdine, 2012) narratives around who can (and cannot) do mathematics—are harmful to students. As has been documented, Science, Technology, Engineering and Mathematics (STEM) classrooms may be unwelcoming spaces to students—particularly to students of color and women—due to factors such as a cliquish culture of toxic competitiveness and emotionally harmful interactions with instructors (Holland, 2019).

Having identified these two core problems of undergraduate mathematics education, I chose to offer a T-PD based on the "Mathematics Discourse in Secondary Classrooms" (MDISC) T-PD materials, which were designed with two foci in mind: cultivating *productive* and *powerful*

[5] I use the words "college" and "university" interchangeably throughout this dissertation.
[6] The "romance of mathematics" (Lakoff & Núñez, 2000) describes the notion that mathematics is objective and provides access to universal truth.

discourse in the classroom. As the MDISC developers (Herbel-Eisenmann et al., 2017, p. xxxi) explained:

> We use "productive discourse" to mean "discourse that provides students with opportunities to make meaningful mathematical contributions toward particular mathematical learning goals" (Cirillo et al., 2014, p. 142) and "powerful discourse" to mean "discourse that positions students as people who are capable of making sense of mathematics and supports students' developing identities in terms of status, smartness, and competence in mathematics class" (Cirillo et al., 2014, p. 142).

I hoped that the MDISC T-PD's foci of cultivating productive and powerful discourse would begin to address the two problems with undergraduate mathematics education I outlined above.

Overarching Research Questions

Beyond the practical goal of offering the MDISC T-PD, I was interested in studying whether participating in this T-PD changed the classroom discourse in the participants' classes—especially with respect to the participants' use of "teacher discourse moves" (a central set of practical tools introduced in the T-PD)—using a multicase study approach (Stake, 2005). As part of a multicase study, a "quintain" is studied (i.e., "the entity having cases or examples" [Stake, 2005, p. vi]) with the help of individual cases that are driven by their own issues but connected by one (or more) overarching research questions. For the purposes of this dissertation, the quintain is the MDISC T-PD (slightly modified for the university context), which was studied with the help of three cases of mathematics-teaching GTAs participating in the MDISC T-PD (who were all no longer in their first year of teaching). The overarching research questions that guided this study were:

1. How does the classroom discourse[7] change in each of the participant's classes over the course of their participation in the MDISC T-PD?
2. How do participants talk about their use of the teacher discourse moves?

Overview of the Dissertation

This dissertation is presented in ten chapters. Following this introduction chapter, I will provide an overview of relevant literature before sharing in Chapter 3 information about my implementation of the MDISC T-PD. In Chapter 4, I summarize my conceptual framework with the help of the first three chapters and the subsequent Method chapter. Chapters 6, 7, and 8 are case reports for each of the three participants, that is, for Finnegan, Valeria, and Alice. Chapter 9 provides the multicase report before I conclude the dissertation with a discussion chapter. Multiple appendices provide additional context for the study.

[7] The dimensions of "classroom discourse" studied will be discussed in the Method chapter. In short, there are two sets of dimensions studied: (a) the "teacher discourse moves" and (b) student discourse dimensions drawing on discourse dimensions from the "Equity QUantified In Participation" (EQUIP) tool (Reinholz & Shah, 2018).

CHAPTER 2: LITERATURE REVIEW

In this chapter, I seek to review research about GTA T-PD and explore some takeaways from research on T-PD. I will then highlight the importance of student talk in the classroom and discuss some ways in which interactions in classroom discourse have been studied with an eye towards introducing the six teacher discourse moves (TDMs) appearing in the MDISC. Although this literature review will be brief and focused on introducing ideas I use in this dissertation, I wish to emphasize that there is a rich tradition of research on professional development and classroom discourse, particularly in the K–12 context.

GTA T-PD

GTA T-PD has taken many forms. Ellis (2015), for instance, described three models of GTA T-PD (*apprenticeship*, *coordinated-innovation*, *peer-mentor*) at four institutions[8] that took part in the *Characteristics of Successful Programs in College Calculus* (CSPCC) study. Taking a different approach to Ellis (2015), Bragdon et al. (2017) quantitatively identified nine distinct models of GTA T-PDs based on data[9] from the PtC survey (Apkarian & Kirin, 2017). The nine distinct models they identified varied along three dimensions: (a) the amount of interaction the GTAs had through their T-PD, (b) the amount of activities involved in the T-PD, and (c) the level of feedback given to GTAs involved with the T-PD. Combined, the works of Ellis (2015) and Bragdon et al. (2017) demonstrate that there is great variability in GTA T-PD offerings.

The three dimensions of variability that Bragdon et al. (2017) identified were based on institutions' responses to the PtC survey (Apkarian & Kirin, 2017), which included questions

[8] The four institutions were a subset of five Ph.D.-granting institutions that had been deemed particularly successful by the CSPCC-team. The fifth institution was excluded by Ellis (2015) as it did not employ GTAs in the teaching of Calculus I.

[9] Specifically, Bragdon et al. (2017) used data from the 120 U.S. mathematics department who had responded to the PtC survey (Apkarian & Kirin, 2017) that they had a GTA T-PD and that this T-PD was primarily intended for GTAs who served as primary instructors.

about, for example, when GTAs participated in a T-PD, what the format of the T-PD was, what activities the T-PD included, and what evaluation of GTAs the T-PD included. Particularly relevant to this study are that: (a) fewer than 20% of the responding institutions offered GTAs T-PD beyond their first teaching year, (b) T-PDs focused on teaching fundamentals (e.g., developing lesson plans and learning about assessment methods), and (c) T-PDs were overwhelmingly locally created (83.2%). Thus, responding GTA T-PD-offering departments offered their GTAs a locally created T-PD during GTAs' first year of teaching that covers teaching fundamentals.

The absence of further T-PD for GTAs represents a missed opportunity to improve undergraduate instruction, given that GTAs teach a large number of STEM courses that undergraduates take (Gardner & Jones, 2011). In general, STEM GTAs and their professional development appear to often be neglected by universities even though they represent future faculty (Gardner & Jones, 2011). This omission is particularly dissatisfying in light of the unique conditions that GTAs find themselves in. Halfway between undergraduates and faculty, STEM GTAs benefit from generally being perceived by STEM undergraduates as easier to approach, build working relationships with, and understand than faculty (Harper et al., 2019). Thus, several factors work in GTAs' favor that could contribute to very positive teaching experiences. But, without proper support, GTAs may find themselves feeling like they are "jumping out of an airplane" (Dudley, 2009, p. 8).

Beisiegel et al.'s (2019) work also highlights why the lack of T-PD beyond GTAs' first year of teaching may be problematic, for GTAs in their first year of teaching are similar to beginning teachers in that they are in a stage of *survival*—the first of four at least yearlong developmental stages: *survival*, *consolidation*, *renewal*, and *maturity* (Katz, 1972). Thus, at the

same time as GTAs are struggling to "survive" as teachers, they are likely to receive the only T-PD during their time as graduate students, focused on fundamentals. About a year later, once GTAs find themselves in the consolidation or renewal phase, they are left without teaching support. The lack of T-PD may lead GTAs to draw on the only resource that is easily available to them: their own extensive experience as students. Yet, GTAs drawing on their own learning experiences is unlikely to lead to the paradigm shift toward active learning widely called for in the field (Abell et al., 2018; Deslauriers et al., 2019; Freeman et al., 2014).

To support T-PD for GTAs, different resources exist (e.g., the Video Cases for College Mathematics Instruction [Hauk et al., 2013], the Resource Suite of the College Mathematics Instructor Development Source [CoMInDS] [Bookman et al., 2021]). Further, there are several efforts in the U.S. to develop and offer multiyear T-PD to GTAs. These efforts include: (a) the collaborative "Mathematics Graduate Teaching Assistant Professional Development Focused on Implementation of Evidence-Based Teaching Practices" study (NSF grants 2013422, 2013563, 2013590), which seeks to offer and evaluate a multiyear GTA T-PD based on evidence-based teaching practices that focus on active learning and inclusive teaching, and (b) the collaborative "Promoting Success in Undergraduate Mathematics Through Graduate Teaching Assistant Training" study (NSF grants 1821454, 1821460, 1921619), which also seeks to offer and evaluate a multiyear GTA T-PD as part of which GTAs are exposed to evidence-based teaching practices. In short, there is a growing set of resources for those wishing to provide GTA T-PD as well as a growing focus on offering and studying multiyear T-PD for GTAs.

In summary, GTAs are frequently underserved (in terms of T-PD) by their departments and universities—a missed opportunity to improve undergraduate education and change teaching culture. Even where a T-PD exists, it is unlikely to be offered beyond GTAs' first year of

teaching. The T-PDs that do exist for GTAs in their first year of teaching typically appear to be locally created and vary widely from institution to institution. Content-wise, T-PDs that go beyond teaching fundamentals seem to be atypical. That said, this dissertation ties in with a growing body of work that seeks to tackle this dearth of GTA T-PD particularly after GTAs' first year of teaching.

Takeaways From Research on T-PDs

Borko (2004) and Hill et al. (2013) have suggested that T-PD studies can be classified into several phases or stages, beginning with small-scale pilot studies at a single institution and ending with cross-institution studies that implement different T-PDs. As progression through these phases—when it occurs—is, at best, linear for a T-PD, it is perhaps unsurprising that a majority of T-PD research has been of the small-scale, single-institution nature (Borko, 2004). Given that this is also the nature of the T-PD I offered, I will focus on literature in this context.

Looking across small-scale T-PD studies and those that started out that way, Borko (2004) identified several takeaways: (a) T-PDs have been documented to bring about desired instructional change; (b) instructors benefit from learning in a supportive, collaborative group of fellow instructors; (c) documenting and analyzing classroom practices fosters instructor learning; and (d) T-PD researchers may benefit from analyzing their data using multiple frameworks and units of analysis. Below, I explore each of these takeaways in a little more depth.

Two examples of T-PDs that brought about at least some of their developers' desired changes are *Cognitively Guided Instruction* (CGI) (Carpenter et al., 1989) and *Quantitative Understanding: Amplifying Student Achievement and Reasoning* (QUASAR) (Silver & Stein, 1996). Although CGI emphasized teacher learning about student thinking and QUASAR focused on teachers engaging their students in mathematical tasks of high cognitive demand, both

projects also attended to classroom interactions. CGI, for instance, included four instructional practices to guide their participants, one of which explicitly encouraged classroom discourse through teachers "asking appropriate questions and listening to children's responses" (Carpenter et al., 1989, p. 505). The QUASAR project also sought to promote student discourse as well as foster student collaboration (Silver & Stein, 1996). In short, evidence exists that T-PDs can have positive instructional effects and that classroom interactions may be an integral focus of T-PDs.

The importance of building a supportive, collaborative teacher community during a T-PD was touched upon by researchers of CGI (Franke et al., 2001) and the QUASAR project (Silver & Stein, 1996) but significantly expanded upon by Grossman et al. (2001). Specifically, Grossman et al. (2001) argued that building a supportive teacher community is important because it provides teachers with a community to learn with and from, in which they can share their experiences and possibly model a culture of learning for their students. In summary, being part of a community is a vital support for teachers engaged in a T-PD.

Watching and analyzing recordings of other teachers' classrooms may allow teachers to learn more about teaching (e.g., by witnessing novel practices or different implementations of familiar practices) (Ball & Cohen, 1999; Sherin et al., 2009; Sherin & van Es, 2009). Particularly in conversation with other teachers, engagement with teaching recordings can lead to important discussions and the raising of crucial questions about teaching and learning (Ball & Cohen, 1999). Although these recordings need not be of the teachers' own classrooms, recording one's own classroom can be a powerful learning experience for a teacher. For example, as Rowe (1986) shared: Transcribing one's own classroom discourse can help teachers become aware of unused opportunities for building in wait time after a teacher's question or a student's response.

Finally, Borko's (2004) suggestion that T-PD researchers may benefit from analyzing their data using multiple frameworks and units of analysis is rooted in her own experience. But even beyond Borko's experience, the need for multiple frameworks and units of analysis is evident when one considers that a T-PD connects facilitators to teachers and teachers to students, with teachers being actively positioned as both teachers and learners (Prediger et al., 2019). Topic-wise, T-PD researchers may find themselves—like T-PD facilitators and participants—negotiating content and pedagogy. Grain size-wise, T-PD researchers' focus can range from a single teacher's utterance in a classroom to multiple teachers' changes along several dimensions over multiple years. In short, a T-PD spans many grain sizes of research—or "timescales" (Lemke, 2000)—and combines questions of learning, teaching, content, and pedagogy. Thus, understanding a T-PD's impact holistically, as much as that is possible, may require multiple frameworks and different units of analysis.

Beyond Borko's (2004) four key takeaways, I also want to emphasize two smaller takeaways she shared: (a) teacher learning does not happen overnight and varies across teachers; and (b) some issues in teaching are easier to learn than others. Expanding on the latter—and with an eye towards implementing a discourse-focused T-PD—I note Franke et al.'s (2001) comparison of two CGI participants, which illustrated that learning how to elicit student thinking may be easier to learn than drawing on student thinking as a resource.

The Importance of Student Talk in the Classroom

Pimm (1987) outlined two types of talk that can be useful in a classroom: talk for oneself and talk for others. In the former case, through expressing what one has heard in one's own ways (e.g., through revoicing), students are given the opportunity to make greater sense of the material. In the case of talking for others, one can distinguish between *message-oriented* and

listener-oriented speech: The former's intention is to communicate content, the latter's is to cultivate social relationships (Brown, 1982). Although the two types of speech may appear to be opposites, they are not mutually exclusive: Any talk will usually combine message- and listener-orientation.[10] Thus, the two types of speech represent a spectrum and suggest why more student talk is not inherently better (Brown, 1982; Pimm, 1987)—or: talking ≠ learning (Michaels et al., 2013). Specifically, outside of school, students engage in predominantly listener-oriented speech, which they may default to in school unless instructors are specific about and offer students opportunities to practice more message-oriented discourse practices (e.g., explaining, justifying) (Brown, 1982).

Apart from personal sense-making through communication and engaging in content-focused discursive practices, talk can also be useful to practice the "language" of mathematics (Pimm, 1987). Mathematics has its own register, that is, "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (Halliday, 1975, p. 65). This register includes, for example, vocabulary—often co-opted from everyday language and given new mathematical meanings (Pimm, 1987)—as well as frequent nominalizations[11] (Schleppegrell, 2004). When encouraged to speak mathematically, students are given the opportunity to familiarize themselves with the mathematics register. Although students may struggle to speak mathematically, the teacher can aid students' linguistic development by, for example, revoicing students' contributions using a more official mathematical register.

[10] That any talk typically combines message- and listener-oriented speech is similar to how in systemic functional linguistics one's lexical and grammatical choices constantly combine to form ideational, interpersonal, and textual meaning (Schleppegrell, 2004).
[11] Nominalizations involve taking a verb (e.g., add) or adjective (e.g., diagonal) and turning it into a noun (addition, diagonal) (Schleppegrell, 2004).

Students talking is also an idea fundamental to active learning, specifically cooperative or collaborative learning (Prince, 2004), which usually incorporates some form of groupwork. Students in active learning classrooms learn more (Deslauriers et al., 2019; Freeman et al., 2014) and active learning classrooms reduce "achievement gaps" for underrepresented students (Theobald et al., 2020). By de-centering the classroom from the instructor, students are given a greater number of opportunities to talk about mathematics and to talk mathematically.

Finally, it is also beneficial for instructors to hear students talk more. Listening to students can provide instructors with formative assessment to gauge how their students understand the content. Instructors can then use this information to better tailor their lesson to their students.

Studying Interactions in Classroom Discourse

In their literature review, Herbel-Eisenmann, Meaney, et al. (2017) found that studies of mathematics classroom discourse can be categorized into four theoretical heritages (the first two of which can be further broken down into subheritages): "(1) cultural, social, and discursive psychology; (2) sociolinguistics and discourse studies; (3) 'reconceptualists'; and (4) threads of work" (p. 724). Particularly relevant for this dissertation is work stemming from the subheritage of interactional sociolinguistics (e.g., Cazden, 2001; Mehan, 1979).

Researchers of interactional sociolinguistics have demonstrated that there is a near ubiquitous interaction that occurs in classrooms: the Initiation-Reply-Evaluation (IRE) interaction (e.g., "Angie, what do you think commutativity means?" "a plus b equals b plus a" "Good!") (e.g., Mehan, 1979). A focus of this dissertation will be on six teacher discourse moves (TDMs) (Cirillo et al., 2014; Herbel-Eisenmann et al., 2013), which are the central practical tools of the MDISC T-PD and can serve to break out of IRE interactions in the classroom. The six

TDMs are: (a) waiting, (b) inviting student participation, (c) revoicing, (d) asking students to revoice, (e) probing a student's thinking, and (f) creating opportunities to engage with another's reasoning. Below, I describe the IRE pattern and TDMs in greater depth.

IRE and TDMs

Classroom discourse—when not a one-way street called "lecture"—consists of instructor–student exchanges and, possibly, student–student exchanges. As many researchers have found (e.g., Lemke, 1990; Mehan, 1979; Sinclair & Coulthard, 1975), instructor–student exchanges often follow a similar pattern: (a) exchanges are initiated by the instructor with a request for information, (b) a student responds to the instructor's request, and (c) the instructor proceeds to evaluate the student's response. This pattern has been referred to as, for instance, *triadic dialogue* (Lemke, 1990), *Initiation-Reply-Evaluation* (IRE) (Mehan, 1979), and *Initiation-Response-Feedback* (IRF) (Sinclair & Coulthard, 1975).

Instructors' use of IRE—particularly the "E"—has been critiqued for reinforcing the instructor's position as the mathematical authority and undermining students' authority over their own reasoning. That said, an instructor may respond with a different type of follow-up—possibly why Sinclair later amended IRF's "F" from "Feedback" to "Follow-up" (e.g., Sinclair, 1992).[12] As Nassaji and Wells (2000) suggested, the "Follow-up" move need not be an evaluation and may instead serve other pedagogical purposes (e.g., asking students to justify or expand on their responses). Therefore, Follow-up moves can start a new sequence of interactions (i.e., serve as Initiation moves in their own right), leading Nassaji and Wells (2000) to observe that IRF sequences may consist of several linked interactional sequences.

[12] Henceforth, IRF will refer to Initiation-Response-Follow-up.

The six TDMs discussed in the MDISC T-PD (i.e., waiting, inviting student participation, revoicing, asking students to revoice, probing a student's thinking, and creating opportunities to engage with another's reasoning) can be thought of as tools to offer instructors ways of rethinking their initiations and follow-ups as well as fostering student responses. They can help instructors move from answer-givers to facilitators of student thought and reasoning (Chapin et al., 2003). A table summarizing definition, intended purposes, and examples for each TDM can be found in MDISC PD materials (Herbel-Eisenmann, Cirillo, et al., 2017, pp. liv–lvii). Below, I describe each TDM in more depth.

Waiting

There are two forms of wait time: wait time 1 (i.e., when the teacher waits after speaking) and wait time 2 (i.e., when the teacher waits after a student's turn) (Rowe, 1986). Rowe (1986) demonstrated that increasing wait time 1 and (especially!) wait time 2 to three seconds or more has remarkable effects on students and teacher, such as: (a) students speaking and reasoning more; (b) student participation, confidence, and achievement increasing; (c) teachers' discourse containing fewer—but more cognitively demanding—questions; and (d) teachers' expectations of students changing positively. Further, a teacher who waits may signal to students that their thinking matters and that speed is not what makes someone good at mathematics. Looking across all these effects, wait time appears to foster both productive and powerful classroom discourse.

Rowe (1986) also shared three findings related to the implementation of wait time. First, learning to wait is a process that takes time and teachers may revert to not waiting after only 3–4 weeks if they are not given the chance to discuss their experiences. Second, there are discursive habits that appear to be detrimental to the implementation of wait time, such as mimicry (cf. revoicing) and "Yes … but …" and "… though" constructions which may shut out students'

ideas. Last, teachers may fail to enact wait time when they believe it will lead to classroom chaos—a mix-up of content and behavioral management. These findings suggest that improving one's use of wait time can be supported by a T-PD in which: (a) wait time is introduced and continually discussed, (b) participants are able to share their thoughts and experiences, and (c) other discursive interactions (e.g., revoicing) are also discussed. Finally, it is relevant to this dissertation study that Rowe (1986) extended her conclusions and recommendations to the university setting.

Inviting Student Participation

Inviting student participation is a teacher discourse move that involves the teacher creating an opportunity for students—either students in general or specific students—to share their thinking with the class. Beyond shifting the focus from the instructor to the students, inviting student participation can allow the instructor to: (a) give less vocal students a chance to voice their thoughts, (b) position less vocal students as knowers and doers of mathematics, (c) position mathematics as a discipline in which there are multiple answers or multiple ways to get to an answer, and (d) initiate discussions between students.

In an ideal enactment, by inviting student participation: (a) the instructor gathers (or seeks to gather) multiple students' responses, (b) the instructor asks students to add on to a previous student's response, or (c) the instructor encourages students to respond to a previous student's response. (Thus, inviting student participation can overlap with another teacher discourse move: creating opportunities to engage with another's reasoning.) That said, inviting student participation can also result in an Initiation-Reply-Evaluation (IRE) interaction (Mehan, 1979) (e.g., "What do you, Angie, think commutativity means?" "a plus b equals b plus a" "Good!"). Such IRE interactions—although technically inviting student participation—are

unlikely to open up the classroom discourse (i.e., create space for many students to be heard and interact).

Revoicing

"Revoicing involves the reuttering of another person's speech through repetition, expansion, rephrasing, and reporting" (Forman et al., 1998, p. 531). Revoicing may also include a second step: providing the "revoicee" with a chance to react to the revoicing (O'Connor, 2009)—some have called this two-step revoicing *full revoicing* (Herbel-Eisenmann et al., 2013). There are many possible reasons for teachers to revoice: (a) to clarify what a student is saying to help the teacher and/or class understand the student's response; (b) to amplify what a student is saying to ensure the class heard the idea, the class has more time to process the idea, the student felt heard, and/or that a turn in the discussion is captured; (c) to introduce ideas or language to, for instance, build students' familiarity with the mathematics register (Pimm, 1987); or (d) to position different students' ideas with respect to one another for the purposes of a discussion (Chapin et al., 2003; Forman et al., 1998). (Further possible uses of revoicing are discussed by Herbel-Eisenmann et al. [2009].) In short, revoicing is a teacher discourse move that can create and direct classroom discussions. Further, through choosing a student response to revoice, the instructor can position students and their ideas.

Asking Students to Revoice

Asking students to revoice can allow the student to make sense of what they heard in their own words. Similar to revoicing, asking students to revoice can be useful to: (a) clarify what a student is saying to help the class understand the student's response; and (b) to amplify what a student is saying to ensure the class heard the idea, the class has more time to process the idea, and that the student felt heard (Chapin et al., 2003; Forman et al., 1998). Further, a

classroom in which asking students to revoice is the norm requires students to listen to one another. Such a classroom emphasizes that what students have to say is important. As with teacher revoicing, students can engage in full revoicing if either they or the teacher confirm with the original student that the revoicing was accurate.

Probing a Student's Thinking

"Probing a student's thinking" can be understood as a counterpart to "inviting student participation:" Rather than seeking to gather many students' responses, probing a student's thinking involves learning more about a particular student's response. A teacher may wish to probe a student's thinking to get the student to clarify (e.g., when the teacher is not quite sure that they or the class follows) or elaborate (e.g., when the teacher would like to hear justification or learn about the students' process). Both clarifying and elaborating are important practices for students to engage in: Clarifying allows students to practice speaking mathematically (Pimm, 1987), and elaborating can shift the focus from product to process and engage students in genuine mathematical practices (e.g., reasoning, justifying, proving).

Creating Opportunities to Engage With Another's Reasoning

Although touched upon in—and perhaps subsumed by—"Inviting Student Participation," "Creating Opportunities to Engage with Another's Reasoning" is a discourse move that merits attention in and of itself. It draws attention to the fundamental mathematical practice of reasoning, and—similar to "Asking Students to Revoice"—requires students to listen to one another. Such opportunities to engage with another's reasoning can involve students applying, responding to (with justification), building on, or revising said reasoning (Chapin et al., 2003; Herbel-Eisenmann et al., 2013).

CHAPTER 3: T-PD OVERVIEW

In this chapter, I provide an overview of the MDISC T-PD as well as ways in which I modified it. I then describe my prior experience facilitating the MDISC T-PD and ways in which I sought to learn from Borko's (2004) takeaways for small-scale T-PD studies. Finally, I reflect on balancing the roles of T-PD researcher and T-PD facilitator.

What Is Included in the MDISC T-PD?

As touched upon in Chapter 1, the MDISC T-PD (Herbel-Eisenmann, Cirillo, et al., 2017) seeks to help its participants cultivate productive and powerful discourse in their classrooms. To achieve these ends, the MDISC materials draw on theories and concepts like: (a) discourse and register from systemic functional linguistics (Halliday, 1978; Pimm, 1987) and constructs inspired by it (e.g., communication context), (b) Chapin et al.'s productive talk moves (2003), and (c) positioning theory (Harré & van Langehove, 1999). Structurally, the MDISC T-PD consists of an introduction, five core "constellations" of activities centered in artifacts (e.g., videos, transcripts, student work) from mathematics classrooms, and an action research capstone experience. The five core constellations are: (a) engaging students in mathematics classroom discourse, (b) teacher discourse moves and positioning, (c) planning for rich discourse, (d) setting up and gathering evidence of student work, and (e) concluding and contemplating evidence.

How Was the MDISC Modified?

Through correspondence with the first author of the MDISC materials, Herbel-Eisenmann (personal communication, November 22, 2020), I learned that: (a) the first three constellations are "the most substantial parts," (b) constellations 4 and 5 provide opportunities to explore the ideas from constellations 1–3 in more depth rather than introduce new content, and

(c) she omitted constellations 4 and 5 in some past MDISC implementations. Thus, when I offered the MDISC in spring 2021, I implemented an abridged version of the MDISC T-PD that omitted constellations 4 and 5. These omissions allowed me to implement the T-PD over the course of a single semester in the form of thirteen weekly 2-hour meetings that were held via Zoom due to the coronavirus pandemic. For a detailed timeline of my MDISC T-PD implementation alongside implementation notes, see Appendix A.

As its name suggests, the MDISC was originally designed with secondary classrooms in mind. Thus, where possible, I modified T-PD artifacts to be more relevant to undergraduate mathematics without changing the purpose of activities (e.g., using a transcript from an undergraduate mathematics course, working on an abstract algebra problem, working with undergraduate mathematics texts). Also, suspecting that participants would be more invested in the T-PD if their questions and concerns are considered, I added a few additional participant-driven topics, such as cold-calling.

What Is My MDISC Facilitation Experience?

Before implementing the MDISC T-PD in spring 2021, I had practiced facilitating MDISC T-PD sessions with a rotating number of volunteers in fall 2020. Thus, facilitating the MDISC for this study's participants was my second time facilitating the T-PD. It being my second time, I benefited from familiarity with the T-PD tasks and their facilitation (e.g., timing, potential participant responses).

Learning From Borko's (2004) Takeaways

As discussed in Chapter 2, Borko (2004) identified several takeaways for small-scale T-PD studies. I identified two of these takeaways as relevant for facilitating the MDISC T-PD: (a) instructors benefit from learning in a supportive, collaborative group of fellow instructors,

and (b) documenting and analyzing classroom practices fosters instructor learning. Below, I explore how I incorporated both takeaways into my facilitation of the MDISC T-PD.

First, I tried to build and foster a sense of community. To this end, I started each session with a check-in. There were also elements of the T-PD that helped foster a sense of community, such as participants working together on mathematics problems and participants generating a set of uniting goals. Especially the MDISC's introductory activities were meant to establish a supportive, collaborative learning environment. The community-building appeared to be somewhat successful: In the mid- and post-T-PD interviews, all participants reflected mostly positively on the T-PD group.

Second, I made sure to incorporate activities in which participants were able to study their own classroom data. These included activities in the T-PD sessions as well as little homework assignments participants were asked to complete, largely drawn from the MDISC's "connecting to practice" activities (see Appendix A for further detail). That said, the T-PD participants often did not find the time to do such homework assignments.

Balancing the Roles of Facilitator and Researcher

Being both the T-PD facilitator and the T-PD researcher, I held two roles with occasionally competing interests. That said, this tension was mitigated by my decision to prioritize my role as a researcher before the T-PD and my role as facilitator during the T-PD. By prioritizing the role of researcher before the T-PD, I decided to recruit a small number of T-PD participants to be able to capture all participants' entire discourse—a larger number of participants would have led to simultaneous breakout room discussions that I could not have simultaneously recorded (see "Participant Recruitment and Selection" in Chapter 5 for more detail). Had I prioritized the facilitator role instead, I might have recruited more participants to

make the T-PD more widely available to GTAs and expose the participants to a greater number of viewpoints. By prioritizing the role of facilitator during the T-PD, I sought to offer the participants the best possible T-PD experience I could provide. Had I prioritized my researcher role instead, I might have: (a) modified the MDISC T-PD materials less to make my implementation more comparable to others' implementations and spent less time following up with participants about their interests (e.g., cold-calling), and (b) encouraged participants more emphatically to use the TDMs in their classrooms.

CHAPTER 4: CONCEPTUAL FRAMEWORK

As Ravitch and Riggan (2012) noted, "For us, a conceptual framework is an argument about why the topic one wishes to study matters, and why the means proposed to study it are appropriate and rigorous" (p. 7). In this vein, I wish to summarize from the previous chapters why the topic of this dissertation matters and lay groundwork for discussing the appropriateness and rigor of the study's method. The method will be discussed in depth in the next chapter.

Why Does GTA T-PD Beyond the First Teaching Year Matter?

As outlined in the previous chapters, there is a need for GTA T-PD in general—for otherwise GTAs teach classes without any pedagogical education—and for GTA T-PD beyond GTAs' first year of teaching in particular—for not everything can be covered in a year especially when GTAs may find themselves in a stage of survival. Thus, with the need for GTA T-PD beyond GTAs' first year of teaching established, the question may move to why GTA T-PD beyond GTAs' first year of teaching matters to me in particular. Thus, I wish to share a few words about myself.

Despite having been very successful in mathematics throughout school and university, I have had mostly negative learning experiences at the undergraduate level. Through many conversations with other mathematics graduates and through reading research on teaching at the college level, I know that my experiences are indicative of a wide-spread problem with the teaching of mathematics at the college level. My disappointment with college mathematics teaching led me to pursue doctoral studies in mathematics education, the culmination of which is this dissertation. I share this story to highlight that the improvement of teaching and the opportunity to offer a T-PD are immensely important to me. Although my desire for teaching improvement might lead to speculation that I may be tempted to turn a blind eye towards aspects

of the T-PD that do not work, I want to highlight that it is precisely my desire for teaching improvement that will ensure that I will not blindly call the T-PD a success. I want to understand what can (and cannot) help GTAs be better instructors, and I understand that this goal is best served by being critical of the T-PD offered.

I also want to acknowledge that there are narratives around who can and who cannot do mathematics, and that these narratives are racialized (e.g., McGee & Martin, 2011; Shah, 2017) and gendered (e.g., Mendick, 2006; Walkerdine, 2012). Further, I acknowledge that there are relations between mathematics education and White supremacist capitalist patriarchy (Gutiérrez, 2017). Thus, as a White, heterosexual, middle class man—a combination of identities that has traditionally benefited from the narratives around mathematics—I understand those who might have suspicions about my involvement in seeking instructional change. Although such suspicions cannot be allayed through words alone, I want to be transparent about pedagogies and works that inspire me and that I strive to teach by to point the reader in the direction of the instructional change I am seeking. These pedagogies include: engaged pedagogy (hooks, 1994), culturally relevant pedagogy (Ladson-Billings, 2009), critical pedagogy (Freire, 2000), and feminist pedagogy (Burton, 1995).

In conclusion, there is a need for GTA T-PD beyond GTAs' first year of teaching: both a general need—rooted in its current absence and the potential for instructional improvement at the college-level—and a very personal need—rooted in the desire to improve mathematics instruction at the college-level. To meet this need, I decided to offer such a GTA T-PD for GTAs beyond their first year of teaching, specifically, the MDISC T-PD.

Why the MDISC T-PD?

As addressed in Chapter 1, the MDISC T-PD's two foci on productive and powerful discourse serve to address two core problems of undergraduate mathematics education: (a) the focus on lecturing, and (b) the myths around who is (and is not) good at mathematics and the devastating impact such racialized and gendered narratives have. Further, the MDISC materials (which include extensive documentation for facilitators) were available for purchase online—and are expected to be available again in the future—and the T-PD can therefore be probably offered by anyone else. Finally, beyond the MDISC materials' suitability and usual availability, they were also made available to me by the MDISC developers.

Why a Case Study Methodology?

As Borko (2004) and Hill et al. (2013) noted, there are several stages to professional development research ranging from one-site pilots to cross-institution studies that implement different T-PDs. This dissertation is a one-site pilot of the MDISC for GTAs. Using a case study methodology to study (parts of) the T-PD allowed me to: (a) attend to the complexity of GTAs implementing tools and ideas from the T-PD to change the classroom discourse in their classes, as well as to (b) bring participants' issues to the foreground that were not conceived of by myself a priori. That said, as Bassey (1999) and Schwandt and Gates (2018) highlighted with an assortment of case study descriptions, researchers have described case study in many different ways. I draw on Stake (1995, 2005), who is rooted "firmly within the interpretive paradigm" (Bassey, 1999, p. 27). Stake's work stands, for example, in contrast to Yin's work (e.g., Yin, 2013)—another body of work commonly cited by those employing a case study methodology—whose "writing tends toward the positivist (or scientific) paradigm" (Bassey, 1999, p. 27).[13] In

[13] For a more in-depth comparison of Stake's and Yin's (as well as Merriam's) works, see Yazan (2015).

short, I draw on Stake's work on case study (Stake, 1995) and multicase research (Stake, 2005), which is philosophically rooted in constructivism and holds that "knowledge is constructed rather than discovered. The world as we know is a particularly human construction" (Stake, 1995, pp. 99–100)—"a collective making" (Stake, 1995, p. 102).

Due to Stake's philosophical commitment to constructivism, he holds that the aim of research is not to discover some external reality but to construct a clearer experiential reality and a more sophisticated rational reality (Stake, 1995, p. 101). Thus, "[o]f all the roles, the role of interpreter, and gatherer of interpretations, is central" (Stake, 1995, p. 99) and so the researcher "ultimately comes to offer a personal view" (Stake, 1995, p. 42). Further, as Stake (2005) held: "Researchers should be encouraged to 'have a life' and to 'have a dream,' so their interpretations are enriched by personal experience. Comprehensive, idiosyncratic, irreproducible interpretations are a contribution to understanding and action" (p. 87). And as much as "[e]ach researcher contributes uniquely to the study of a case[,] each reader derives unique meanings" (Stake, 1995, p. 103). The reader's meaning-making is a crucial element of case studies: "Because the reader knows the situation to which the assertions might apply, the responsibility of making generalizations should be more the reader's than the writer's" (Stake, 2005, p. 90). Thus, researchers are encouraged to "provid[e] readers with good raw material for their own generalizing" (Stake, 1995, p. 102). Good raw material should also be supplemented with clear writing: "We seek to portray its cases comprehensively, using ample but nontechnical description and narrative. Each case report may read something like a story" (Stake, 2005, p. vii). In short, Stake (1995, 2005) recognized: (a) the value the researcher as an individual and interpreter brings to case study research, and (b) the importance of the reader and their interpretations to the value of a case study.

Multicase Research

In this dissertation, I engaged in multicase research: As Stake (2005) noted, "[t]he aim of multicase research […] is to come to understand the quintain better" (p. 14) (i.e., "the entity having cases or examples" [Stake, 2005, p. vi]). In this dissertation, the quintain was a modification of the MDISC T-PD, offered in the university context to three mathematics-teaching GTAs who were no longer in their first year of teaching. A quintain is studied with the help of individual cases that are driven by their own issues but connected by one (or more) overarching research questions. In this dissertation, the individual cases were of the three mathematics-teaching GTAs with multiple years of teaching experience participating in the MDISC T-PD. These individual cases were instrumental cases, instrumental to understanding the quintain better. Although these individual instrumental cases were studied with respect to the overarching research questions (i.e., etic issues chosen by me, the researcher), the guiding issues structuring each case were emic. As Stake (1995) noted, "[t]hese are the issues of the actors, the people who belong to the case. These are issues from the inside" (p. 20).

Etic Issues and TDMs

The first etic issue that guided my dissertation study was to understand whether and, if so, how the modified MDISC T-PD I offered led to changes of the classroom discourse in participants' classes—in particular the participants' use of the six TDMs, the central set of practical tools offered by the T-PD. As described in the literature review, the TDMs may be viewed as growing out of interactional sociolinguistics (Herbel-Eisenmann, Meaney, et al., 2017) in the tradition of works like Sinclair and Coulthard (1992), Mehan (1979), O'Connor & Michaels (1996), and Cazden (2001). Thus, I inherit from this line of the work the TDMs as well as discursive terminology such as "discourse," "move," and "utterance."

Although the TDMs will be the central concepts with respect to which I will study the classroom discourse in the participants' classes, I will also seek to understand whether the classroom discourse changed with respect to several "student discourse dimensions" taken from the "Equity QUantified In Participation" (EQUIP) tool—a tool to "support practitioners in reflecting on equity issues in their own practice, particularly with respect to implicit bias" (Reinholz & Shah, 2018, p. 142). The EQUIP tool was originally included in the design of this dissertation study to understand participants' implicit bias, but due to several uncircumventable human subjects research guidelines restrictions stemming from this study being conducted in an online context during the coronavirus pandemic, this goal became impossible. After a reevaluation of the tool, several of its dimensions were included as codes that could provide insight as to whether students' discourse changed over the course of the semester. As research on the TDMs shows, one might, for instance, expect students' turns becoming longer and more complex—changes that the EQUIP tool's dimensions account for.

The second etic issue I bring to this multicase research is the desire to understand how participants talked about their use of the TDMs. This etic issue is driven by my belief that participant discourse about the TDMs can provide important insight into why participants used TDMs (or not) the way they did (or did not). Further, participants' discourse about TDM use and the analysis of classroom discourse in the participants' classes can serve to triangulate[14] one another.

The two etic issues described above are encapsulated in my research questions:

[14] Since the goal is not to establish a universal truth or discover some external reality, I use triangulation to describe an attempt "to minimize misrepresentation and misunderstanding" (Stake, 1995, p. 109). Rather than seeking a single truth or meaning, such triangulation can help identify multiple interpretations (Stake, 1995, 2005).

1. How does the classroom discourse change in each of the participant's classes over the course of their participation in the MDISC T-PD?
2. How do participants talk about their use of the teacher discourse moves?

Regarding the first research question, and as described above, the changes in classroom discourse I attend to are with respect to the TDMs as well as several student discourse dimensions taken from the EQUIP tool.

In sum, I use Stake's (2005) multicase approach (philosophically rooted in constructivism) in order to understand how and why the classroom discourse in each of the T-PD participant's classes changed (or not) with respect to the TDMs (a set of tools rooted in interactional sociolinguistics) and several student discourse dimensions (a set of relevant dimensions inherited from the EQUIP tool).

CHAPTER 5: METHOD

In this chapter, I begin by providing an overview of the three T-PD participants. This overview will be brief as each participant will be focused on in detail in their respective chapter. I will then describe the data sources collected and how I analyzed the data.

Participants

Three doctoral students from a large public university in the Midwest participated in my implementation of the MDISC T-PD: Alice, Finnegan, and Valeria (all pseudonyms). Alice and Finnegan were both mathematics doctoral students, whereas Valeria was a mathematics education doctoral student. I had never interacted with Alice and Finnegan before I contacted them to see if they would be interested in participating; Valeria was a friend. All three had multiple years of teaching experience, be it as teaching assistants at universities (Finnegan and Valeria) or as a schoolteacher (Alice). None received any compensation for their participation. Despite being very willing to compensate them for their time, I feared that compensation might create the wrong impression and environment. I wanted the motivation for joining the T-PD to be purely their interest in the T-PD and not a monetary incentive.

Participant Recruitment and Selection

My goal was to have a group of three (possibly four) participants to be able to hear and capture all group discussions via Zoom. (A larger number of participants would have had to be broken up during activities into groups that I would have had to rotate between and would not have been able to record simultaneously). Further, I felt that a T-PD with fewer than three people would have been a bad experience for the participants as well as myself. I also wanted to bring mathematics and mathematics education students together, so I recruited among both groups.

Valeria, a mathematics education doctoral student and friend, was the first person whom I spoke with, and after a week of deliberation she agreed to join. She noted in the first interview that our friendship had played a factor in her decision to join, but that "it wasn't like, the main one." Instead, she expressed that she had joined because she wanted to learn more about positioning theory, become a better teacher, and see "what's being talked about." Being cognizant that she and I would be positioned as mathematics educator researchers and not wanting to have mathematics education researchers outnumber mathematics researchers, I stopped recruitment of mathematics education doctoral students and proceeded to recruit mathematics doctoral students. After consulting with staff familiar with the mathematics doctoral students, a list of mathematics doctoral students was generated who might be interested in the T-PD. Before the start of the semester, I e-mailed the people on this list one-by-one. By the time the semester began, two had agreed to participate in the T-PD (i.e., Alice and Finnegan). In total eight recruitment e-mails were sent out to mathematics GTAs.

Data Sources

The data sources consisted of: (a) audio-recordings of all participants' classes, (b) anonymized chat histories from the classes, (c) three semi-structured 60-minute interviews with each of the participants, (d) video-recordings of all thirteen T-PD meetings, and (e) extensive reflections written by me after every T-PD meeting. Below, I describe each data source as well as how it proved useful in addressing my research questions.

Classroom Audio-Recordings & Chat Histories

All participants had been instructed by their course coordinators to video-record their Zoom classes for absent students. The audio-recordings that were created as byproducts were then shared with me and transcribed by otter.ai. Since I was interested in understanding whether

the classroom discourse of the participants changed, I first needed to decide which classroom recordings to code. Alice and Finnegan taught (and recorded) 26 classes of 80 minutes each. Given the time-intensive coding I intended to do, I decided to narrow down the number of classroom recordings to nine for both Alice and Finnegan. Of the nine classroom recordings, three were from the start of the semester (after the syllabi had been discussed), three from the middle of the semester (after the teacher discourse moves had been introduced), and three from the end of the semester (before exam reviews). The nine classroom recordings were selected with three thoughts in mind: (a) to capture a "typical" day of classroom discourse (i.e., excluding the first day of class in which the syllabus was covered and the final day(s) of class when an exam review took place), (b) to capture discursive change across the semester, particularly with respect to the usage of TDMs, and (c) to have some consecutive classroom recordings which could provide relevant contextual information for one another. For Valeria, whose audio-recordings were typically around 60 minutes long and who taught only once a week, I decided to code all eleven classroom recordings. Valeria taught two sections of the same course, but rather than code parts of both, I decided to code one section's classroom recordings wholly. I picked the section in which Valeria felt more students participated, thinking that it would be more interesting to understand a potential better-case scenario than a worse-case scenario.

After participants reported to me that their students were fond of communicating via chat, I also asked them to download the chat histories. I then wrote a Python program that allowed the participants to anonymize their chat histories before sharing them with me. Specifically, the program: (a) replaced all students' names with a number, (b) replaced private messages between students with "Private message not involving the instructor," and (c) replaced private messages between a student and the instructor with a placeholder noting that a private message had been

sent between the student and the instructor (e.g., “Private message sent from person #4 to the instructor”). Unfortunately, the code contained a tiny bug that clipped around 0.60% of the analyzed public chat messages. Next, for the selected classroom audio-recordings, I merged the chat histories with the transcripts generated by otter.ai. The alignment of transcripts and chat histories was done to an accuracy of around 1–2 seconds. I then listened to each selected audio-recording while reading the corresponding merged transcript to improve the transcript’s accuracy, in general, and to ensure that the transcript was correctly chunked into student and instructor turns, in particular. I then proceeded to code the merged transcripts while listening to the audio-recordings using the coding methods described under “Data Analysis” to understand how the classroom discourse changed in each of the participant’s classes.

Semi-Structured Interviews

Each participant was interviewed before the start of the T-PD and semester, after 7–8 T-PD sessions in the latter half of the semester (after the TDMs had been introduced), and 2–4 weeks after the end of the T-PD (i.e., 1–3 weeks after the end of the semester). The first interview served as an opportunity to get to know each other before the start of the T-PD, to learn about the participant’s beliefs about teaching, learning, and mathematics, and to learn about the participant’s expectations for the T-PD. The mid-semester interview served to check in with participants, to learn how they were making sense of the teacher discourse moves, and to learn about their teaching and any changes to it. The post-T-PD interview was intended to allow the participants to reflect on the T-PD and their teaching and think about the future. All interviews were transcribed “clean verbatim” using GoTranscript (i.e., speech errors, false starts, and filler words were excluded). The original interview protocols can be found in Appendix B. The mid-T-PD and post-T-PD interview protocols were supplemented with participant-specific questions

that emerged for me from relistening to prior interviews or from reflecting on their participation in the T-PD. The semi-structured interviews were then coded using coding methods described under "Data Analysis" to capture how the participants talked about the TDMs.

T-PD Video-Recordings and Reflections

Since the T-PD was facilitated on Zoom, I video-recorded the T-PD sessions through Zoom. Further, immediately after the end of each T-PD session, I wrote a reflective memo to capture: (a) everything I could remember happening in the session in chronological order, (b) moments that stood out to me from the T-PD, (c) my feelings towards and analysis of the implementation of the day's T-PD activities, and (d) the modifications I had made to T-PD activities. The memos ranged in length from 1,103 to 3,580 words with a mean of 2,309 and a median of 2,218 words. I then drew on the memos and video-recordings to build the cases as described under "Data Analysis," specifically under "Writing the Case Reports."

Data Analysis

As discussed in depth in the previous chapter, I used Stake's (2005) multicase approach. The quintain that I studied was the MDISC T-PD modified for the university context, offered to three mathematics-teaching GTAs with multiple years of teaching experience. To understand the quintain, I developed three (instrumental) cases of the mathematics-teaching GTAs with multiple years of teaching experience participating in the MDISC T-PD modified for the university context. Each case was structured according to its own, emic issue that was a priori unknown. Yet, the three cases were connected by etic issues that I brought to the table, and which were encapsulated in my research questions:

1. How does the classroom discourse change in each of the participant's classes over the course of their participation in the MDISC T-PD?

2. How do participants talk about their use of the teacher discourse moves?

As Stake (1995) elaborated, in order to analyze cases, a researcher may make use of "direct interpretation of the individual instance and […] aggregation of instances until something can be said about them as a class" (p. 74). Yet, "[w]ith instrumental case studies, where the case serves to help us understand phenomena or relationships within it, the need for categorical data and measurements is greater" (Stake, 1995, p. 77). Further, Stake (2005) described:

> The methods actually used in the cases may be quite similar from case to case, or may be quite different. The more the multicase analysis is to be quantitative, the more useful it may be to use the same methods for all cases. (pp. 29–30)

To strike a balance between (a) being able to easily look across the cases and compare with respect to the etic issues of classroom discourse and the participants' discourse about the TDMs, and (b) respecting the individuality of each case, I used some coding methods—all conducted in MAXQDA 2020 (VERBI Software, 2019)—across all participants but also performed some participant-specific analyses.

To further balance the answering of my etic research questions with the arising of emic case-specific issues, my coding started off driven by my etic research questions but evolved to account for participants' emic issues. For example, although I began by coding for instances of "probing a student's thinking," I later refined the code inspired by the ways in which the participants used and talked about different types of probing. Thus, my coding changed to help drive the individual cases and mirror participants' issues.

In the following two subsections, I outline how I sought to attend to my overarching etic issues and answer each research question. I then discuss how I wrote the case reports.

Analyzing How the Classroom Discourse Changed in Each of the Participant's Classes Over the Course of Their Participation in the MDISC T-PD

To answer the question "How does the classroom discourse change in each of the participant's classes over the course of their participation in the MDISC T-PD?", I selected classroom recordings from the participants and coded them along two sets of dimensions: the TDMs and several student discourse dimensions drawn from the discourse dimensions of the EQUIP tool (Reinholz & Shah, 2018). Below, I describe in detail the coding methods used to analyze the classroom recordings with respect to these two sets of dimensions.

Coding for TDMs

The TDMs are a set of practical tools offered by the MDISC T-PD: They are not introduced in the T-PD materials as ready-to-be-coded concepts for researchers. Thus, to code for the TDMs (except wait time and revoicing[15]), I first needed to decide what the unit size of a "move" was. For this, I drew on work by Sinclair & Coulthard (1992). The idea of a "move" can be traced back to Bellack et al. (1963) who drew on Wittgenstein's (1953/2009) notion of language-games to "view[] classroom discourse as a kind of language game" (p. 5) and classified teachers' and students' discourse into "pedagogical moves." Inspired by Bellack et al.'s (1963) terminology, Sinclair & Coulthard (1992) adopted the term "move" as part of their classroom discourse rank scale. Explaining the idea of a rank scale, Sinclair and Coulthard (1992) noted,

> The basic assumption of a rank scale is that a unit at a given rank, for example, *word*, is made up of one or more units of the rank below, *morpheme*, and combines with other units at the same rank to make one unit at the rank above, *group* (Halliday, 1961). (p. 2)

[15] The unit size of wait time, and to some extent revoicing, does not fit with the size of a "move" as espoused by Sinclair & Coulthard (1992). Their respective unit sizes will be described in their respective sections.

Sinclair and Coulthard's (1992) found the following rank scale to be appropriate for classroom discourse (in descending order): lesson, transaction, exchange, move, and act. Thus, moves consist of acts and combine to form exchanges. A type of exchange is the Initiation-Response-Feedback interaction. This interaction, for example, breaks down into an Opening move, an Answering move, and a Follow-up move. An Opening move, in turn, may break down into acts like a starter, an elicitation, and a nomination (e.g., "Some of you seemed to notice something about the rectangle. What is it that you noticed? Jerome?). It is in this sense that I understood the size of a "move" to code for four of the TDMs: inviting student participation, asking students to revoice, probing a student's thinking, and creating opportunities to engage with another's reasoning. An important consequence of adopting this grain size for TDM-coding was that when a TDM-elicitation received no student response and the instructor re-initiated the TDM, the re-initiation was counted as a separate move. At the same time, a chain of elicitations without room for a student response was considered a single move. When a TDM was interrupted by a student but the instructor chose to talk over the student or finish the TDM before responding to the student, the TDM was counted as a single instance. Finally, I also borrowed the term *utterance* from Sinclair and Coulthard (1992)—"everything said by one speaker before another beg[ins] to speak" (Sinclair & Coulthard, 1992, p. 2)—which can consist of multiples moves.

In coding for TDMs, I restricted my coding to discourse about mathematics. In other words, I excluded all TDM-candidates that were explicitly tied to class logistics (e.g., probing a student's question about the exam date, waiting after a student's question about exam scores). This distinction was easy to make, with one exception: When instructors invited student participation (e.g., "Does anybody have any questions?") and students chose to speak about class logistics. Given that the invitation opened the potential for discourse about mathematics, I chose

to code such instances. Below, I describe in detail my coding for each of the TDMs. A summary of my coding scheme for the TDMs can be found in Table 5.1.

Waiting. Coding for wait time was complicated by students being able to communicate via Zoom's chat. Since the alignment of each audio-recording and chat history was only accurate to 1–2 seconds, the possible errors in wait time coding led me to focus only on spoken student responses. In coding for wait time, I used MAXQDA 2020's (VERBI Software, 2019) multimedia browser which displays a wave form for audio-files and allowed me to code pauses between utterances to within an error of 0.2 seconds. Given the very time-intensive nature of this type of coding, I only analyzed two classroom recordings for each of Finnegan and Alice (one from the start and one from the end of the semester) and four for Valeria (two from the start and two from the end of the semester). I coded more classroom recordings for Valeria as her classes were shorter.

To code for wait time, I drew on Rowe's (1986) distinction between wait time 1 (i.e., the pause after a teacher's question) and wait time 2 (i.e., the pause after a student's response). Coding for wait time 1 and wait time 2 alone, however, proved insufficient. Consider finding that an instructor waited 2 seconds after they posed a question. Was this because a student responded within 2 seconds or because the instructor moved on after 2 seconds? The former merely signifies that a student quickly responded, whereas the latter could point to the instructor not leaving enough time for students to answer. Thus, I drew on Ingram and Elliott's (2016) work, which broke down both wait times into two subcategories. Specifically, the four different types of wait time are:

- Wait time 1a (or wait time I-S[16]): pause between an Instructor's question and a Student's response to said question;
- Wait time 1b (or wait time I-I): pause between an Instructor's question and the Instructor continuing to speak;
- Wait time 2a (or wait time S-I): pause between a Student's utterance and the Instructor responding;
- Wait time 2b (or wait time S-S): pause between a Student speaking and then continuing their utterance.

I then broke down the wait times even further, accounting for possible differences in wait time norms for different types of instructor questions or student turns. This was based on two suspicions: (a) instructors might find it easier to remind themselves to wait after asking for questions (and when a question they asked was not immediately answered, they might be more easily tempted to repeat or rephrase the question), and (b) instructors might find it more difficult to wait after students posed questions (because they might want to immediately "help" students by answering quickly). Thus, the refined coding scheme became:

- Wait time 1a (I-S) W?: wait time 1a after the instructor asked a What/hoW/Why-type question[17]
- Wait time 1a (I-S) Q?: wait time 1a after the instructor asked for Questions
- Wait time 1b (I-I) W?: wait time 1b after the instructor asked a What/hoW/Why-type question
- Wait time 1b (I-I) Q?: wait time 1b after the instructor asked for Questions

[16] The "I-S" (and "I-I", "S-I", "S-S") terminology is my own and intended to help the reader distinguish between the different wait times: "I" is short for instructor, "S" is short for student.
[17] These types of questions will be defined later. For now, they should be thought of as instructor questions that are not asking for questions or whether something made sense.

- Wait time 2a (S-I) R!: wait time 2a after a student gave a Response
- Wait time 2a (S-I) Q?: wait time 2a after a student posed a Question
- Wait time 2b (S-S)[18]

It should be noted that wait time 2b (S-S) is ambiguous[19], and I therefore decided to only code pauses between a student speaking and then continuing their utterance if the pause was greater or equal to 3.0 seconds—Rowe's (1986) recommended minimum wait time.

To represent the different types of wait time, I used boxplots. Boxplots were created with Microsoft Excel, version 16.60, and "exclusive median." Although boxplots can be created with as few as five data points, I only created boxplots for wait time types with at least ten data points.

Inviting Student Participation. The MDISC materials describe "inviting student participation" in the following way:

> Inviting student participation takes on multiple forms and addresses a variety of goals. For example, a teacher may wish to solicit multiple solution processes or strategies for the same answer. Or a teacher may want to determine how students arrived at their answers. One main goal of *Inviting* is to make diverse solutions available for public consideration, a key practice related to orchestrating productive discussions (M. S. Smith & Stein, 2011). Other goals could be more social nature, such as including multiple students in the discussion. (Herbel-Eisenmann, Cirillo, et al., 2017, p. liv)

Based on this description, I define "inviting student participation" as follows:

[18] Wait time 2b (S-S) was not further broken down as not a single instance of it was found. It could have, however, also been broken up into Wait time 2b (S-S) R! and Wait time 2b (S-S) Q?.

[19] Given that wait time 2b (S-S) is the pause between a student speaking and then continuing their utterance, one could argue that even miniscule pauses between syllables are wait time 2b. Coding this way is counter to the spirit of wait time 2b, which is for instructors to give a student time to add to their utterance in case they were not finished expressing their line of thought. Thus, the ambiguity: Where does one draw the line to determine which pauses within a student's utterance are coded as wait time 2b (S-S)?

Inviting student participation includes any form of request made by the instructor to students for information. These requests can be made to a specific subset of students (*specific-student(s)-inviting*) (e.g., "Danielle, what do you think the answer should be?," "What does group four think?") or to the whole class (*anyone-inviting*) (e.g., "Does anybody have an idea for what we should do next?").

Both specific-student(s)-inviting and anyone-inviting can be done in an *explicit* or an *implicit* fashion. Consider the difference between *explicit specific-student(s)-inviting* and *implicit specific-student(s)-inviting:* "How did you get those x-intercepts, Shiv?" (i.e., using a student's name or "you") versus following up with a student without addressing them explicitly (e.g., "Why to infinity?" in response to "Would it be three to infinity?"). Furthermore, "Does anyone have an idea for what we should do next"? is an instance of *explicit anyone-inviting,* whereas "What is the answer to problem 3?" is an example of *implicit anyone-inviting*.

I decided to code for explicit specific-student(s)-inviting to understand whether instructors called on single students (i.e., *explicit specific-student-inviting*) and groups of students (i.e., *explicit specific-students-inviting*). To not skew the numbers for explicit specific-student(s)-inviting, I only counted the initial inviting move of an exchange. Imagine an exchange that begins as follows:

Instructor: Ben, what is the area of the triangle?

Ben: 25.

Instructor: How did you get that answer, Ben?

Rather than counting these two instructor utterances as two instances of explicit specific-student(s)-inviting, I only count the first since I believe repeated counting of such explicit

specific-student(s)-invitations for the same student as part of an exchange skews the picture of how often an instructor invites participation from specific students.

Apart from differing in who is being addressed, inviting student participation can also differ in terms of what is being asked of the addressee(s). One specific purpose that I wish to foreground—in line with the MDISC's description of "inviting student participation—is *inviting further responses*, that is, soliciting a second (or third, etc.) response to an instructor's question. All such instances were coded for.

Finally, to get an understanding of the types of questions instructors asked—and therefore the types of mathematical activity that instructors asked their students to engage in—I coded all instructor solicitations using the following coding scheme:

- *What:* the instructor asks students to make a claim but justification for said claim is not explicitly requested (e.g., "What is the derivative of f(x) at x=3?");
- *How:* the instructor asks students to discuss how they arrived at a result or how to perform a mathematical process (e.g., "How should we proceed?", "What is the next step in the process?");
- *Why:* the instructor asks students to provide justification alongside a claim or for a given claim (e.g., "Why is the sum of two even numbers even?", "How would you convince your friend that the sum of two odd numbers is even?");
- *Questions?:* a code to capture instructors' ways of asking for questions (e.g., "Do you have any questions?", "What questions do you have?", "Questions, anyone?");
- *Make sense?:* if a student has no questions and feels they understand the material, they would respond "No" to "Any questions?" but respond "Yes" to "Does that make sense?"

Given this contrast, I chose to distinguish instructor solicitations of the type "Does that make sense?" from solicitations of the type "Any questions?";

- *Other:* any other types of instructor solicitations.

The coding scheme was adapted from the EQUIP tool's (Reinholz & Shah, 2018) "Teacher Solicitation Type" code, a code discussed under "Coding Student Discourse" below. As is evident from the examples, this coding scheme is non-trivial as the English language allows, for example, the asking of why-type questions using a "how"-construction.

Moreover, I added a weight to each instructor solicitation code in MAXQDA 2020 (VERBI Software, 2019) to signify the number of student responses it received. To keep myself from overlooking any instructor solicitations, I set the default weight of codes to 100.

In summary, my coding for "inviting student participation" attended to whether instructors explicitly invited participation (i.e., engaged in explicit specific-student(s)-inviting), whether they engaged in inviting further responses, what types of instructor solicitations they made, and how many students responded to each solicitation.

Revoicing. As is described in the MDISC materials,

> Revoicing occurs when a teacher restates or rephrases a student's contribution. More specifically, revoicing has been defined as "the reuttering of another person's speech through repetition, expansion, rephrasing, and reporting" (Forman, Larreamendy-Joerns, Stein, & Brown, 1998, p. 531). An essential ingredient of what we call full revoicing lies in the second part of the teacher's contribution (O'Connor, 2009). "Full revoicing" occurs when the teacher checks back with the original speaker and offers an explicit opportunity to respond to questions such as "Did I get that right?" (Herbel-Eisenmann, Cirillo, et al., 2017, p. lv)

Using Forman et al.'s (1998) understanding of revoicing and O'Connor's (2009) notion of full revoicing, I coded each student turn[20] by whether it had been fully revoiced, simply revoiced (i.e., revoiced, but not fully), or not revoiced. I also coded the corresponding revoicing by the instructor.

Asking Students to Revoice. "Asking students to revoice" involves the instructor asking a student to engage in the "reuttering of another person's speech through repetition, expansion, rephrasing, and reporting" (Forman et al., 1998, p. 531). Students could be asked to revoice another student or the instructor. Thus, I coded all instances of the instructors asking a student (or students) to revoice, distinguishing between "asking students to revoice another student" and "asking students to revoice the instructor." (That said, as there was no instance in which the instructor asked students to revoice something the instructor had said, this subcode is omitted from further discussion.) In deciding where to draw the line for "asking students to revoice another student," I decided to err on the side of caution. Specifically, after wondering whether to include instructor prompts for students to share their group's findings as instances of this subcode (i.e., interpreting the prompt as reporting on the group's discussion), I decided not to for two reasons: (a) I knew at the start of coding that in many of the participants' classes, group members had not communicated with one another, and "representatives" from such groups would not be reporting on another group member's speech, and (b) from personal experience, even "representatives" from communicative groups do not necessarily report on the group's discourse but on their own thoughts. Thus, I decided to exclude instructor requests for students to share their group's findings from "asking students to revoice another student."

[20] A student "turn" will be defined more carefully in the section on "Coding Student Discourse."

Probing a Student's Thinking. The MDISC provides the following description of *probing a student's thinking*:

> This move is about following up with an individual student's solution, strategy, or question. The goal here is to have the student elaborate on or clarify his/her ideas. For example, the teacher might ask how or why, or invite the student to come up to the front of the room to provide additional information such as a diagram. Probing may stem from a teacher's genuine desire to know more about the student's thinking, or it could be used to make a student's thinking explicit for the benefit of the other students. (Herbel-Eisenmann, Cirillo, et al., 2017, p. lvi)

Based on the MDISC's description, I define "probing a student's thinking" as follows:

> *Probing a student's thinking* means following up with an individual student, with the goal of having them elaborate on a previous utterance.

As suggested by the MDISC's description of "probing a student's thinking"—and as also became evident from the data—there appear to be at least two reasons for why an instructor may ask a student to elaborate on a previous utterance. First, the instructor may seek to *clarify* a student's previous utterance. This may be because the instructor believes the meaning of the utterance was unclear (e.g., "What do you mean by 'the function swings wide'?") or incomplete (e.g., "Where in this equation should we 'plug in 2'?"). Second, the instructor may seek to *go deeper* with the content of a student's previous utterance. This may be because the instructor would like to know how a student arrived at their answer, why they believe their statement is true, or why they think something is a significant piece of information. I refer to these two types of probing as *clarifying-probing* and *deepening-probing* respectively.

Further, as I observed while coding for these two types of probing, clarifying-probing and deepening-probing can be enacted in a *student-centric* or an *instructor-centric* way, that is, the responsibility for clarifying or deepening a student's previous utterance can rest on the student or the instructor. For clarifying-probing, a student-centric version might be "What do you mean by […]?", whereas an instructor-centric version is "Is […] what you mean by […]?" For deepening-probing, a student-centric version is "How did you get […]?", whereas an instructor-centric version might be "Is this how you got […]?"

Clarifying-probing is both a tool for clarifying students' answers as well as questions. Suspecting that instructors may use clarifying-probing in an instructor-centric way in response to questions more than in response to student statements or responses, I created further subcategories.[21] In the end, I coded instances of "probing a student's thinking" as follows:

- **CP**: Clarifying-probing
    - **CP-SC**: Student-centric clarifying-probing
        - **CP-SC-S!**: Student-centric clarifying-probing after a student statement
        - **CP-SC-Q?**: Student-centric clarifying-probing after a student question
    - **CP Ins**: Instructor-centric clarifying-probing
        - **CP-IC-S!**: Instructor-centric clarifying-probing after a student statement
        - **CP-IC-Q?**: Instructor-centric clarifying-probing after a student question
- **DP**: Deepening-probing

[21] One reason I suspect that instructors may be more likely to instructor-centric clarifying-probe a student's question is because instructors may fear that student-centric clarifying-probing a student's question may make students feel like they are being quizzed and discourage them from asking questions.

- **DP-SC**: Student-centric deepening-probing
- **DP-IC**: Instructor-centric deepening-probing

Creating Opportunities to Engage With Another's Reasoning. In the MDISC materials, "creating opportunities to engage with another's reasoning" is described in the following manner:

> This move involves asking students to engage with another student's idea. For example, the teacher might ask the class to use a particular student's strategy to solve a similar problem or to agree or disagree with a solution. Another form that this move [can take] might be to ask students to add on or revise another student's explanation or conjecture. Effective use of this discourse move could be enhanced by the prerequisite use of other discourse moves and works best when students are actively listening to each other. (Herbel-Eisenmann, Cirillo, et al., 2017, p. lvii)

With this in mind, I coded for any instances in which an instructor asked students to engage with another student's idea. Distinguishing between different types of "creating opportunities to engage with another's reasoning" in the coded data—as well as after studying different examples of this TDM in the T-PD materials—I ended up with the following coding scheme for different types of "creating opportunities to engage with another's reasoning:"

- Creating opportunities to *add* to another's reasoning (e.g., "Does anyone want to add to what Axel said?", "Claudia got us off to a great start. Can someone continue with her argument?")
- Creating opportunities to *anticipate* another's reasoning (e.g., "What do you think two common student mistakes are when solving this type of problem?")

- Creating opportunities to *compare* to another's reasoning (e.g., "What is similar about Maya's and Miriam's solutions?", "How is Manasi's solution similar or different from your own?")
- Creating opportunities to *evaluate* another's reasoning (e.g., "Do you agree with what Amit said?")
- Creating opportunities to *understand* another's reasoning (e.g., "Looking at her work, can you tell me how Jihye got from […] to […]?", "What questions do you have for Sofía about the way she solved the problem?")

Since the coded observations only included participants creating opportunities to add to, evaluate, and understand another's reasoning, creating opportunities to anticipate and compare to another's reasoning do not appear in the tables of the participants' TDM use.

Although group work does not inherently create a space in which students engage with other students' ideas, the T-PD participants expressed that they saw group work as creating opportunities to engage with another's reasoning. Thus, although not a move per se, I also coded for instances of group work under the umbrella of "creating opportunities to engage with another's reasoning."

Table 5.1

Coding Scheme for the Teacher Discourse Moves

Teacher Discourse Move Codes

Wait time
• Wait time 1
  ○ Wait time 1a
  ▪ Wait time 1a (I-S) W?: wait time 1a after the instructor asked a What/hoW/Why-type question
  ▪ Wait time 1a (I-S) Q?: wait time 1a after the instructor asked for Questions
  ○ Wait time 1b
  ▪ Wait time 1b (I-I) W?: wait time 1b after the instructor asked a What/hoW/Why-type question
  ▪ Wait time 1b (I-I) Q?: wait time 1b after the instructor asked for Questions
• Wait time 2
  ○ Wait time 2a
  ▪ Wait time 2a (S-I) R!: wait time 2a after a student gave a Response
  ▪ Wait time 2a (S-I) Q?: wait time 2a after a student posed a Question
  ○ Wait time 2b (S-S)

Inviting student participation
1. Explicit specific-student(s)-inviting
  • Explicit specific-student-inviting
  • Explicit specific-students-inviting
2. Inviting further responses
3. Instructor Solicitation Type
  • What
  • How
  • Why
  • Questions?
  • Make sense?
  • Other

Revoicing • Simply revoiced • Fully revoiced • Not revoiced
Asking students to revoice • Asking students to revoice another student • Asking students to revoice the instructor

Table 5.1 (cont'd)

Teacher Discourse Move Codes

Probing a student's thinking
• Clarifying-probing (CP)
  ○ Student-centric clarifying-probing (CP-SC)
  ▪ Student-centric clarifying-probing after a student statement (CP-SC-S!)
  ▪ Student-centric clarifying-probing after a student question (CP-SC-Q?)
  ○ Instructor-centric clarifying-probing (CP-IC)
  ▪ Instructor-centric clarifying-probing after a student statement (CP-IC-S!)
  ▪ Instructor-centric clarifying-probing after a student question (CP-IC-Q?)
• Deepening-probing (DP)
  ○ Student-centric deepening-probing (DP-SC)
  ○ Instructor-centric deepening-probing (DP-IC)

Creating opportunities to engage with another's reasoning • Creating opportunities to add to another's reasoning • Creating opportunities to anticipate another's reasoning • Creating opportunities to compare to another's reasoning • Creating opportunities to evaluate another's reasoning • Creating opportunities to understand another's reasoning

Note. There were multiple ways in which I coded for "inviting student participation," signified by the numbered list.

Coding Student Discourse

To learn about possible changes in student discourse, I decided to draw on the dimensions of the EQUIP tool (Reinholz & Shah, 2018). As aforementioned, I felt that the dimensions of the EQUIP remained relevant to this dissertation, despite not being cross-referenced with aspects of students' identities. In Table 5.2, I provide an overview of the Student Discourse Codes with which I coded the data as well as rationales for including them. Of the seven codes the EQUIP tool uses, I used six: I excluded the wait time code since my wait time analysis (as described above as part of the TDM analysis) was more fine-grained. In addition to the six EQUIP codes I used, I introduced an additional code: "mode of discourse."

Table 5.2

Student Discourse Codes and Their Rationales

Student Discourse Code	Rationale for the Code

Mode of Discourse
• Speech
• Writing
  ○ Public Chat
  ○ Public Chat (clipped)
  ○ Private Chat
• Body Language
• Intermodal
	"Mode of Discourse" was included to keep track of the mode in which discourse was communicated—particularly relevant in the online context which easily allowed written communication.

Discourse Type • Discourse About Class Logistics • Discourse About Mathematics	"Discourse Type" was included to distinguish between "Discourse About Class Logistics" and "Discourse About Mathematics." It served as a filter to focus my attention on discourse about mathematics.
Student Talk Length • 1–4 words • 5–20 words • 21+ words • n/a	"Student Talk Length" was included to have a rough sense of whether students' turns grew longer over the course of the semester. The expectation was that use of TDMs—particularly wait time, asking students to revoice, probing a student's thinking, and creating opportunities to engage with another's reasoning—might lead to longer student turns.
Student Talk Type • what • how • why • question • other	"Student Talk Type" was included to get a sense of the type of mathematical discourse students engaged in. The expectation was that greater use of probing a student's thinking might lead to more why-type student discourse.
Teacher Solicitation Method • Not called on • Called on • Random selection	"Teacher Solicitation Method" was included to get a sense of whether participants called on students as a means of inviting student participation.
Teacher Solicitation Type • what • how • why • other • n/a (unsolicited)	"Teacher Solicitation Type" was included to get a sense of the types of (mathematical) invitations participants made (which were taken up by students).

Table 5.2 (cont'd)

Student Discourse Code	Rationale for the Code
Explicit Evaluation • Yes (by instructor) • Yes (by TA) • No • n/a	"Explicit Evaluation" was included as an IRE-proxy, that is, to gauge whether participants may have evaluated fewer student turns by the end of the semester (e.g., by instead asking other students to react to a student's solution).

For the purposes of the EQUIP, a student's turn is any number of student utterances that are not interrupted by another student (but possibly interrupted by the instructor). Since the EQUIP tool's focus is on implicit bias, it makes sense to combine a student's utterances, even when they are neither thematically nor temporally linked for it captures the lack of different speakers. Yet, since, for the purposes of the analysis, I was more interested in the types of (mathematical) opportunities offered and seized than who seized them, I modified the unit size to define a student turn as any number of student utterances that are linked topically by responding to the same question or by elaborating on a previous utterance (even when interrupted by the instructor).

As the EQUIP merely served as the starting point for my analysis, I tailored its codes to my needs and adjusted them during my analysis. Below I describe the adjustments I made as well as the extra code I added (i.e., "mode of discourse").

Mode of Discourse. To capture the different ways in which students communicated, I included the following four *mode of discourse* codes: (a) *speech*, (b) *writing* (with subcodes *public chat* and *private chat*), (c) *body language*, and (d) *intermodal* (with several subcodes for various combinations). Below, I expand on each of these four codes.

Any student turn that was communicated entirely orally was coded as *speech*. I acknowledge that this coding decision may oversimplify the modality of a student's turn as students with their cameras on probably also used their bodies to communicate and therefore

used multiple modes of communication. Yet, I was unable to ascertain which students had their cameras on while speaking and thus err on the side of caution by counting all fully oral student turns merely as *speech*.

There were two ways in which students could communicate in *writing*: by sending a *public chat* message or a *private chat* message. As explained earlier, the contents of private chat messages were unknown to me, and so my coding of private messages typically stopped with the acknowledgement of their existence. That said, in a few select instances, instructors chose to revoice students' private chat contributions, which enabled me to code some of the private chat messages with further EQUIP codes. As aforementioned, around 0.60% of analyzed public chat messages were clipped due to a programming error, and I kept track of these clipped message with the "Public Chat (clipped)" code.

Given that I only had access to the audio-recordings and chat histories, *body language* only captured instances in which the instructor's discourse implied that the students were using their bodies to communicate, typically by nodding or shaking their heads.

Turns were coded as *intermodal* if there was a shift between modes of communication, the most common form involving a switch from writing to speech. These instances typically involved a student responding to an instructor question via chat, the instructor asking a follow-up question, and the student responding orally.

Discourse Type. The EQUIP tool distinguishes between two types of discourse: (a) *logistics* and (b) *content*. I kept this binary but chose to relabel it as: (a) *discourse about class logistics* and (b) *discourse about mathematics*.

Any discourse that was coded as *discourse about class logistics* was not coded further with any EQUIP code. I made this decision given my interest in studying students' (opportunities for making) mathematical contributions.

By changing "content" to "discourse about mathematics," I sought to capture the multiple ways in which mathematics was communicated (about). Consider the following three examples: "I think the function is continuous," "What? I don't understand that," and "Aaaah, okay. Thank you!" Although the latter two may feel "less" mathematical in nature, I argue that they are still referring to and are *about* mathematics. In short, I took a broad view of what constitutes discourse about mathematics. I chose to err on the side of overcoding for *discourse about mathematics* as any turn coded as *discourse about class logistics* would not be coded further.

Student Talk Length. In addition to the EQUIP's codes of *1–4 words*, *5–20 words*, and *21+ words*, I added a *n/a* code to account for (a) turns coded as body language and (b) the few private chat messages whose contents I could reasonably infer and code. I decided to count filler words (e.g., uhm, like) as words as they represent the speaker holding the floor and add to the length of a student's turn.

Student Talk Type. The student talk type code was meant to capture different types of talk students engaged in: *what*, *how*, *why*, *question*, and *other*. In line with the distinction between what, how, and why discussed under "inviting student participation," I define the codes as follows: (a) students engage in *what*-type discourse when they are making a claim without providing justification, (b) students engage in *how*-type discourse when they explain how they arrived at a result or how to perform a mathematical process, (c) students engage in *why*-type discourse when they provide justification alongside a claim or for a given claim, (d) students engage in *question* discourse when posing a question, and (e) any discourse not captured by the

above four codes is captured in *other* (the possibility for "other" discourse stems from my expansive coding of "discourse about mathematics"). I drew on the different types of student talk type from the EQUIP tool, which inherited it from Braaten and Windschitl's (2011) distinction between types of explanation in science education. Given the differences between mathematics and other science education subjects as well as Braaten and Windschitl's (2011) focus on explanations, the EQUIP's meanings for what-, how-, and why-talk were unclear to me. Thus, I wish to emphasize that the above definitions for the student talk type subcodes are my own.

Teacher Solicitation Method. This code was meant to capture whether (and how) instructors called on their students and consisted of three possible subcodes: *not called on*, *called on*, and *random selection*. Since none of the participants used random selection, I omit this code from further discussion as explicit specific-student(s)-inviting already captures when students were called on.

Teacher Solicitation Type. This code sought to capture the different types of talk instructors asked their students to engage in. It is very similar to the code discussed in "inviting student participation" and contains the following subcodes: *what*, *how*, *why*, *other*, and *n/a (unsolicited)*. These subcodes are defined as follows: (a) *what:* the instructor asks students to make a claim but justification for said claim is not explicitly requested; (b) h*ow:* the instructor asks students to discuss how they arrived at a result or how to perform a mathematical process; (c) w*hy:* the instructor asks students to provide justification alongside a claim or for a given claim; (d) *other*: any other form of solicitations, such as asking for questions or asking whether something made sense; and (e) *n/a (unsolicited)*: the student turn was not in response to an instructor solicitation.

The main difference between this code and "instructor solicitation type" (previously described under "inviting student participation") is that this code is only applied to student turns. Whereas the coding of "instructor solicitation type" captures all questions asked by the instructor—and therefore also documents which questions students did not respond to—this present code allows me to easily cross-reference whether "teacher solicitation type" and "student talk type" match up for a given student turn—or whether, for instance, a student did not provide justifications (e.g., what-type student talk) when asked for justification (e.g., why-type teacher solicitation).

Explicit Evaluation. For the purposes of this code, evaluation refers to evaluating the mathematical correctness of a statement. In other words, instructor responses like "Thank you so much! That was a super interesting answer!" are insufficient to be called an evaluation as "super interesting" may be a judgement but not of the mathematical correctness of the statement. Given that it is highly likely that any student statement will eventually be evaluated by the instructor—even though it might take several turns—I only coded a student turn as being evaluated if the instructor evaluated the student turn within their next turn. Note that the student's turn and the instructor's next turn could be separated by several student responses. Thus, the instructor's turn could simultaneously evaluate multiple students' turns.

I also decided to code questions as having been mathematically evaluated if the instructor responded with the answer in their next turn—as opposed to asking the asking student what they thought or posing the student's question to the class. Since Finnegan and Alice had teaching assistants who were monitoring the chat for questions, I also distinguished between who evaluated, that is, whether the instructor or the TA evaluated a student's response or question.

Finally, in addition to the *Yes (by instructor)*, *Yes (by TA)*, and *No* codes, I also had an *n/a* code. As aforementioned, even though I only further coded student turns that were "Discourse about Mathematics," I took a very expansive view of what constitutes discourse about mathematics. Thus, there were some student turns that did not consist of any mathematical statement to evaluate, hence the n/a code.

Analyzing the Participants' Talk About Their TDM Usage

To capture what the participants had to say about each of the TDMs, I used structural coding (Saldaña, 2009) to identify the relevant interview segments. In addition to the mid- and post-T-PD interviews, I also coded the pre-T-PD interviews in case participants were already familiar with a subset of the TDMs. To condense the relevant interview segments, I wrote summaries for each participants' discourse about each TDM. The summaries were written in response to two focus questions: (a) Did the participant try this TDM? Why or why not?; and (b) If the participant tried this move, how did they use it? The resulting summaries formed a basis for the case reports.

Writing the Case Reports

Through (a) analyzing the classroom discourse of a selection of each participant's classes with respect to TDM use and student discourse dimensions, and (b) summarizing participants' discourse about the TDMs, a guiding emic issue for each participant crystalized for me. I then began writing the case reports with the goal of centering participants' respective emic issues, and I used the classroom discourse analyses and the summaries of the participants' discourse about the TDMs as the foundation for each case report. To explore the emic issues in greater depth, some case-specific further analyses were conducted (e.g., an analysis of chat use in Finnegan's class, an analysis of non-evaluated student turns in Valeria's case, an analysis of the frequency of

Alice's what/how/why-solicitations). Further, I drew on the T-PD reflections and T-PD video-recordings to add details to the cases and explore emic issues (e.g., participants' talk about their beliefs and the TDMs).

While writing the individual case reports, I sought to follow Stake's (1995) approach for writing a case report: to write "a narrative that makes the case comprehensible" (p. 124) "using ample but nontechnical description" (Stake, 2005, p. vii). As encouraged by Stake (1995), I included raw data in the case reports, for example, through quotes from the participants or transcripts of classroom interactions. Further, of the three paths for a case report Stake (1995) described, I sought to develop the cases chronologically. Finally, as discussed in the chapter on my conceptual framework, I kept in mind the centrality of the researcher's role as interpreter (Stake, 1995) and did not shy away from offering my interpretations, particularly in the conclusion of each case report.

Although my case studies were instrumental and not intrinsic (i.e., [emic] issues were centered and not the case itself), context remained important: "Issues are not simple and clean, but intricately wired to political, social, historical, and especially personal contexts" (Stake, 1995, p. 17). Thus, in each case report, I sought to include context such as what course the participants taught, their beliefs about mathematics and the teaching and learning thereof, their motivations for joining the T-PD, their expectations for the T-PD, and any teaching goals they shared. That said, to protect the participants' identities, I erred on the side of caution and shared less information about the participants' (course) context than interested readers may wish for.

An important contextual factor was the coronavirus pandemic during which the T-PD was offered and the participants taught. An acknowledgement of the pandemic is found in all cases in as far as the participants spoke of it and the effects of technology on their TDM use.

That said, the pandemic had been ongoing for a year and two of the participants (i.e., Finnegan and Valeria) had already gained experience teaching synchronously online. Thus, although the context of the pandemic is present in the individual case reports, it is not front and center because not all participants described it playing such a central role.

After finishing the writing of the individual case reports, I proceeded to write the multicase report. For writing a multicase report, Stake (2005) outlined three possible tracks (i.e., emphasizing case findings, merging case findings, and providing factors for analysis), and I sought to merge case findings. In other words, I returned to my research questions and structured the multicase report into seven main sections—six sections for the six TDMs and one section for the student discourse dimensions. In each section, I then looked across the three cases with respect to the participants' use of and discourse about the relevant TDM or student discourse dimension with an eye towards merging findings as well as highlighting what was unique about each case. The multicase report ends with a conclusion in which I sought to answer the two research questions.

CHAPTER 6: THE CASE OF FINNEGAN

Overview of the Case

Finnegan is a case of a mathematics-teaching graduate teaching assistant who was no longer in his first year of teaching and who participated in the MDISC professional development over the course of the semester, with the MDISC slightly abridged to fit into a semester and slightly modified to account for the undergraduate mathematics context. The central emic issue that arose while developing Finnegan's case was the question: How can an instructor who wants students to participate in their class invite student participation? (For the purposes of this case report, I distinguish between a more general, colloquial "inviting student participation" and the TDM of "inviting student participation^{TDM}".[22]) Thus, Finnegan's case—by itself—should be understood as an instrumental case study that seeks to answer this question by detailing how Finnegan wrestled with the issue of inviting student participation. Three different facets of the issue of student participation are raised by this case: the type of invited participation (e.g., the types of questions asked, group work), the manner in which one invites participation (e.g., cold-calling, wait time), and how teaching synchronously online may affect the inviting of participation. Finnegan's case is split into five sections: an introduction, a chronology of his semester in three parts, and a final section in which I interpret his case with an eye towards the central issue. For the reader's convenience, Table 6.1 below summarizes when which data were collected.

[22] Wait time, for example, is a facet of inviting student participation (in its broad, colloquial sense) as waiting longer after posing a question is likely to increase (up to a point) the likelihood of a student responding. That said, wait time is *not* an aspect of the TDM of inviting student participation.

Table 6.1

Finnegan's Data Collection Timeline

Week	Classroom Recordings	T-PD Session	Interview
1 (reading week)			Pre-T-PD interview
2		Session 1 (Intro part 1)	
3	CR1, CR2	Session 2 (Intro part 2)	
4	CR3	Session 3 (Discourse part 1)	
5		Session 4 (Discourse part 2)	
6		Session 5 (TDMs part 1)	
7		Session 6 (TDMs part 2)	
8	CR4		
9	CR5	Session 7 (Positioning part 1)	
10	CR6	Session 8 (Positioning part 2)	Mid-T-PD interview
11		Session 9 (EQUIP, Recap part 1)	
12	CR7	Session 10 (Recap part 2)	
13	CR8	Session 11 (Capstone part 1)	
14	CR9	Session 12 (Capstone part 2)	
15		Session 13 (Capstone part 3)	
16 (Exam week)			
Post-semester week 1			Post-T-PD interview

Meeting Finnegan

Interviewer: [Thinking ahead to this semester], what would you want that space [your classroom] to look like and feel like, and be like?

Finnegan: I'd want the students to be comfortable asking questions and comfortable being wrong. Especially, if it's not a quiz or an exam, I think it's very important to know that it's okay to be wrong at times because that's how you learn. With that in mind, I want students to be helping each other out. When they're working on problems together in group work, I would want every student to actually be involved, whether that's they're understanding it or are trying to explain it, or they're not understanding it so they're asking questions, or they're in the middle of the road and they're giving guesses out. I want all of the students to feel like they can speak up and that's not going to be a bad thing.

Six days before he was going to start teaching an algebra course with around 60 students, Finnegan—a doctoral student in his final year who had already been involved in the teaching of over a dozen courses—shared his hopes and fears for the upcoming semester with me. As the excerpt above from the pre-T-PD interview shows, he had several hopes for how his students would participate. In particular, he hoped that students would be: comfortable asking questions, comfortable being wrong, helping each other out (in group work), and comfortable speaking up. At the same time, he feared that teaching the class on Zoom (twice a week, synchronously) would make fostering participation difficult as "it's much more difficult to have students work together on math when they're not in the same room" and students may not necessarily have the right materials for participating in class online (e.g., a webcam, good internet, a tablet). Given his

concerns about students' access to stable internet and a webcam—and despite a directive from his course coordinator to enforce a camera-on policy—he shared that he would not worry about the camera-on policy too much as long as most students' cameras were on.

Course Context

Finnegan's algebra course was coordinated and introduced students to various families of functions. The course was taught entirely synchronously on Zoom (twice a week for 80 minutes), and he had a TA during class who answered students' questions in the chat. The Zoom classes were recorded and made available to students online. Around 60 students were registered for the class. The course used standards-based grading.

Finnegan's Motivation, Expectation, and Beliefs

In the pre-T-PD interview, Finnegan shared what motivated him to join the T-PD in the upcoming semester:

> I love teaching. There are times where I think I just want to be a high school teacher because I love teaching and I want to be better at it. I want students to want to learn from me because I want to help them. And if there's any way I can get better at that is something I want to pursue.

Expanding on what he enjoyed about teaching, he added, "I love seeing that moment of things clicking for people and I want to help more people get to that." In line with his hopes for the semester, he also identified a specific goal for the semester: not coming off as uninterested in students. Finally, he also spoke about his expectations for the T-PD:

> I'm hoping that it'll help me improve my teaching in some way of, at least, making me a bit better at being able to teach different types of students. As long as it helps my teaching, that's really all I care about.

Finnegan also elaborated on the roles of mathematics instructors, noting that, "one of the most important things for an instructor is to not discourage students and to realize that just because a student isn't good at math, does not mean that they're stupid." He pointed to the importance of instructors encouraging their students and helping their students "understand that math is difficult, but you can still do it if you put your mind to it." He recognized that "no student is going to learn the same way" and tried to give his students a variety of resources they could use. Speaking about himself, he shared that he considered himself a "stock standard learner," someone who learned by building up to things: starting small and then doing slightly harder and harder problems. As he shared, "a lot of [mathematics] is problem-solving."

In the second T-PD session, Finnegan expanded on his beliefs about mathematics and the teaching and learning thereof after participating in the T-PD activity of constructing a "beliefs map" (see Figure 6.1). When explaining his map to the group, he noted:

> What I was noticing is there's sort of three core beliefs for me that sort of drive most of my thinking, which is, for me, at least math is about the process of solving things, as opposed to worrying too much about the exact details. Also think math should be accessible that it shouldn't, that anybody can do at least some math. […] And finally, and sort of, almost paradoxically, as a mathematician, I don't think math should ever be touted as the end all be all, that it shouldn't be like you need to know math or else. I think it should just sort of be a step in the process or something people want to learn. And like most of these seem to drive my other beliefs. […] I also mostly for me marked some of the notes that I noticed I struggle with the most. Those are the red stars. Because I noticed with what you ((*referring to the facilitator*)) were talking about how our actions and our beliefs don't always work together. I was sort of thinking about which one of these am I

really am I good at and which one of these do I maybe I try and just don't succeed at or which ones have I just not been comfortable putting into action.

Figure 6.1

Finnegan's Beliefs Map From T-PD Session 2

Note. Finnegan constructed this beliefs map in the second T-PD session. The stars denote beliefs Finnegan felt he struggled to implement.

In summary, Finnegan joined the T-PD because he loved teaching and wanted to get better at it. He had a lot of experience teaching undergraduate mathematics (over a dozen courses) and, in a beliefs mapping activity during the early stages of the T-PD, Finnegan identified many beliefs he held—among them three core beliefs and five beliefs he felt that he struggled to implement. Some of these connected to Finnegan's hopes and goals for the upcoming semester: students being comfortable asking questions, students being comfortable being wrong, students helping each other out (in group work), students comfortable speaking up, and him not coming off as uninterested in students. He also shared his worries about how the

online context of teaching synchronously on Zoom might make fostering student participation difficult.

Inviting Student Participation: The Start of the Semester

In this section on Finnegan's inviting of student participation at the start of the semester, I focus on two subissues: the types of invited participation and the role of technology in inviting student participation. The manner of invitation—that is, attending to cold-calling and wait time—will be left for subsequent sections in line with how this subissue emerged over time.

Types of Invited Participation

In this section, I draw attention to the participation Finnegan invited in two different contexts: whole-class time (i.e., time in which the entire class was together) and group work.

Whole-Class Time

Finnegan: Yeah, so it's going to be a, remember, if we're raising something to a fractional power, we're going to be taking some sort of root. When we're raising it to the *half* power, we take the *square* root. So we get log of base 10 of square root of 25, times 200. Ah. And then, as I saw people say, we can simplify that, the square root of 25 is just five. And we multiply that by 200. Well, what is five times 200?

Jake: 1000.

George: 1000.

Finnegan: It's 1000. Ah, that looks more like a six, so I'll rewrite it as a 1000. What is log base 10 of 1000?

George: 10?

Finnegan: Not quite.—

Jake: —Is it one hundred?

Finnegan: Not quite. So maybe here's a better question. Can we rewrite 1000 as a power of 10?

Tyler (in chat): cubed

George: No.

This excerpt from Finnegan's second day of class shows Finnegan inviting his students to participate by asking "What is five times 200?", "What is log base 10 of 1000?", and "Can we rewrite 1000 as a power of 10?". (Weeks later—during the mid-T-PD interview—he would describe these types of questions as "asking for quick responses" and being part of IRE interactions.) Looking at Table 6.2, which describes the number and type of Finnegan's solicitations, one can make several observations in line with the above excerpt: (a) Finnegan frequently asked for student input at a rate of around one solicitation per minute, (b) of the solicitations Finnegan made, "what"-type solicitations were most frequent (i.e., the type of solicitation that frequently asks students to apply some known mathematical procedure to find some mathematical object or property thereof), (c) Finnegan made sure to ask his students whether they had questions or if the content made sense (and students responded with questions at least half of the time), and (d) Finnegan almost never asked students to justify any claims (his or their own).

Table 6.2

Finnegan's Solicitations at the Start of the Semester

Instructor Solicitations	Classroom Recording (Day of Class) [# of min. spent together as a class]		
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]
What	25	34	38
How	18	8	12
Why	0	1	1
Questions? (taken up by student(s))	15 (14)	13 (7)	21 (11)
Make sense?	2	3	3
Other	0	1	1
Total	60	60	76

The instructor's types of solicitations are mirrored in the student discourse data, and the short responses given by Jake, George, and Tyler in the transcript above are indicative of the student discourse at the start of the semester. In each of the first three classroom recordings (i.e., CR1–CR3), over 50% of student turns consisted of fewer than 5 words (see Table 6.3). Conversely, when looking at the 21+ words category (corresponding roughly to multiple sentences), one can see that at best 15.1% (in CR1) of student turns were longer than 20 words. Mirroring the types of questions asked, one can also note that the three most common student talk types for all three classroom recordings were "what," followed by "question" and "how". Specifically, we can note: (a) what-type student talk was most common (similar to how Finnegan's what-type solicitations were most common), (b) students frequently asked questions, and (c) students hardly engaged in any why-type talk, mirroring Finnegan's low number of why-solicitations.

Table 6.3

Student Talk Length and Student Talk Type in Finnegan's Class at the Start of the Semester

Student Discourse (percentage out of total)	Classroom Recording (Day of Class) [# of min. spent together as a class]		
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]
Student Talk Length			
• 1–4 words	65 (.516)	50 (.562)	52 (.515)
• 5–20 words	42 (.333)	34 (.382)	37 (.366)
• 21+ words	19 (.151)	5 (.056)	12 (.119)
• n/a	0 (.000)	0 (.000)	0 (.000)
Student Talk Type			
• what	53 (.421)	44 (.494)	55 (.545)
• how	26 (.206)	13 (.146)	15 (.149)
• why	0 (.000)	3 (.034)	3 (.030)
• question	40 (.317)	21 (.236)	23 (.228)
• other	7 (.056)	8 (.090)	5 (.050)
Total # of coded student turn segments	126	89	101

In short, at the start of the semester, Finnegan frequently solicited responses and students frequently responded with very short turns. That said, the types of solicitations were often what-type solicitations and rarely why-type solicitations. This distribution is mirrored in the student discourse data, which show students most commonly giving what-type responses and few why-type responses. Finally, in line with Finnegan's valuing of students' questions, he frequently asked for questions and students frequently asked questions.

Group Work

Finnegan made time for group work during the first five classes, implementing group work on the first, second (CR1), and third (CR2) day of class. To understand how he set up group work, consider the transcript below from the first day of class. After 32 minutes of being together as a class, Finnegan mentioned group work for the first time and set up group work in the following manner:

I'm actually going to split you up into groups into breakout rooms. And we're going to have each breakout room work on a different property, just trying to figure out what exactly it's going to be. Ah, so that is, if you're looking at your notes, they're going to be page two, and three, will have four different properties. I'm going to split us up into, let's see how many people are in this class, about 60. So, I'm gonna split us up into about, I don't want to do this. Despite being a mathematician, I can never do math off the top of my head. So, give me a second. Okay, I'm going to split us up into about 15 different groups. The first five groups, groups one through five, are going to work on the first half of page two. That is, if you're looking at the notes, right here, I want you to just be working on—don't worry about that—I want you to be working on this part I've circled here. This properties of logarithms with all of the additions. If you're going to be in groups six through 10, you're going to be working on the second half of this page. Which is going to have all of these negatives. Ah, this is, yes, yeah. It's gonna have all of the negatives. And finally, if you're in the last five groups, groups 11 through 15, you're going to be on page three, working on this top part with the multiplication stuff. Yes, yeah. And as people are saying in the chat, you can find this under lesson 10. There're the notes in the chat. If somebody has a way to write on a tablet, that'd be great. Otherwise, you can sort of talk these through and figure them out as you go. Ah, so I will create 15 breakout rooms to assign automatically. If the TAs could just stay out so we can just talk a bit. Ah, and feel free to join them once you get there. And yeah, we'll be around talking about this with you guys. ((*Finnegan proceeds to send students to breakout rooms*))

From this, we can learn that Finnegan created 15 (randomized) groups in Zoom, creating groups of four, and that different groups were tasked with working on different problems. He

also indicated that he and the TA would eventually be going around the breakout rooms to talk with the students. The group work lasted almost 22 minutes, during which Finnegan visited different breakout rooms. Afterwards, he went through the problems, asking the corresponding groups about their work. For example, he began the post-group work discussion by asking: "Anybody in the first five groups, the groups that worked on this, uh, product rule, were you able to figure out the connection between the left- and the right-hand side?"

On the second day of class (CR1), Finnegan implemented group work 15 minutes before the end of class, forming 20 random breakout rooms and asking them to work on four problems for 10 minutes. He added, "Yeah, so yeah, please work on these as groups." No post-group work discussion was recorded, so it is unclear whether any such discussion took place. It was recorded, however, that some students did not join their breakout rooms and had their names taken down. The taking down of names was announced, making it unclear whether students chose not to join groups (knowing their names would be taken down) or whether the students were not paying attention to the happenings of the class.

In the middle of the third day of class (CR2), Finnegan placed students in breakout rooms again and asked the groups to do an investigation on a worksheet together. He also noted that he would be collecting copies of their worksheets as parts of their grade. The group work lasted around 16 minutes before he reconvened the class and asked them for their observations. During the group work, he visited different groups asking them how it was going and whether they had questions. Once the class had reconvened, Finnegan asked questions about the worksheet, with the first three questions being: "So for example, what happens as we change 'a'?", "So when we looked at our graph as 'a' increased or decreased, what happened to the graph itself?", and

"What did 'c' do? So we have this x plus c inside the logarithm? Was anybody able to figure out what happened when we change 'c'?"

Finnegan did not implement group work in the fourth (CR3) and fifth classes. Reflecting on these two classes during T-PD session 3, Finnegan shared:

> Yeah, I mean, I had a similar problem with like the fitting things in. I wanted to do group work this week. Because like, I wanted them to, like, try to figure out […] ((*Finnegan discusses algebraic concepts*)). And there's just no time to let them explore that because I have to get through everything else.

Earlier in the session, Finnegan had explained why he felt he had no time:

> So I know I found this week actually pretty difficult. My students are gearing up for the first exam next week. They had their first, they had their first quiz on Monday as well and they have a second quiz tomorrow on WeBWorK. So a lot is happening, and we have to cover a lot. And we've covered everything within the allotted time but students always feel like we're moving way too fast when it's like, this is, this is just the speed at which this course moves. And it sucks 'cause I want to slow down, but I can't slow down because we're going as slow as the course really allows us to.

In short, Finnegan implemented group work for short periods in some classes during the first two weeks and had some assessment tied to participating in group work (i.e., possibly participation points in the second class [CR1] and a worksheet in the third [CR2]). The only instance I noted of Finnegan expressing his expectations for group work was, "Yeah, so yeah, please work on these as groups." Groups were randomly created in Zoom, with some students not joining their groups starting in the second class. During group work, Finnegan and his TA checked in with the groups. After group work, no individual groups were called on to report.

Specifically, in the first class, Finnegan asked sets of groups for responses, and, in the third class, he posed generic questions that did not reference any group(s). (No post-group work discussion was recorded in the second class.) By the third week of classes, he found that there was too little time for group work despite wanting to do group work.

The Role of Technology in Inviting Participation

By the second class (CR1), "a lot of" students already had their cameras off, and Finnegan reminded them of the camera-on class policy: "And also remember, because I'm seeing a lot of people with them off, your camera should be on during class." Other than this one request, Finnegan was not recorded asking his students to turn their cameras on in CR1–CR3. Thus, as suggested by his pre-T-PD interview remarks, he did not consistently enforce a camera-on policy.

Although "a lot of" students may not have made use of their cameras, the Zoom chat function was used frequently. Consider the following transcript from CR2:

Finnegan: ((*Finnegan finishes discussing an answer a student gave him*))
And then, what's the range? What values can we possibly get when we plug something into a logarithm? ((*20 second pause*))
What are your thoughts, Vincent?

Gabriel (in chat): Inf, inf

Vincent: Negative infinity to infinity.

Elias (in chat): -inf, inf

Joy (in chat): -inf,inf

As the transcript suggests, students made use of the chat—even when a specific student had been addressed. Broader evidence of this can be found in Table 6.4 below, which shows that the chat

was increasingly taken up and used by the students. Furthermore, students appear to have gravitated towards the public chat feature, that is, they let their fellow students see their answers.

Table 6.4

Mode of Discourse Used by Students in Finnegan's Class at the Start of the Semester

"Mode of Discourse" used in a student turn (percentage out of total)	Classroom Recording (Day of Class) [# of min. spent together as a class]		
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]
Spoken	56 (.421)	37 (.346)	33 (.250)
Public chat	76 (.571)	69[a] (.645)	99[a] (.750)
Private chat	0 (.000)	0 (.000)	0 (.000)
Other (i.e., body, intermodal)	1 (.008)	1 (.009)	0 (.000)
Total	133	107	132

Note. The numbers in this table reflect *all* student turns: both those about mathematics and those about logistics. For a table focused only on the mode of discourse in student turns about mathematics, see Table C.1.

[a] Counts marked with a superscript "a" include a single instance of a clipped message.

As the above transcript with Finnegan, Gabriel, Vincent, Elias, and Joy also suggests, the chat feature allowed multiple students to answer the same question. Indeed, as can be seen in Table 6.5 below, what/how/why/other-solicitations (i.e., all solicitations excluding asking for questions and whether something made sense), frequently received more than one reply. Thus, it appears as though the availability of the chat may have opened the door for several students to simultaneously respond to a solicitation by Finnegan.

Table 6.5

Overview of Number of Times a Given Number of Responses Occurred to a "What," "How," "Why," or "Other" Solicitation Across the First Three Classroom Recordings

Number of responses to a "what," "how," "why," or "other" solicitation	Number of times in CR1–CR3 that a given number of responses to a "what," "how," "why," or "other" solicitation occurred
0	16
1	39
2	28
3	9
4+	5

Note. This table shows how many times a given number of responses occurred to a "what," "how," "why," or "other" solicitation. For example, across CR1–CR3, 28 times a "what," "how," "why," or "other" solicitation received two student responses.

In addition to being a way to respond to Finnegan's solicitations, the chat also proved to be useful to students for asking questions: Across CR1–CR3, students asked 46 questions aloud and typed 39 questions in the chat. Typing questions in the chat had the added benefit of sometimes being answered by Finnegan's teaching assistant whose job it was to answer questions in the chat and check in with breakout rooms during group work time.

To determine whether the uptake of the chat translated to wide-spread chat usage by the entire class, consider Table 6.6. This table shows that some students used the chat substantially more than others. Specifically, the top five chat-using students of each day contributed about half the mathematical chat turns of the class—more than that in CR2. Yet, despite this distribution of mathematical chat turns, it is also evident that at least a third of the students used the chat at least once—although this proportion is likely to have been higher. (There were approximately 60 students registered for the class, but the class was online and students had access to recordings of

the class. Furthermore, by the middle of the semester, Finnegan described that "you'll have like 40 to 45 people show up, but on a bad day, like when we were going over exam review, I think I had 25 people show up." Thus, attendance during CR1–CR3 probably did not reach 100%.)

Table 6.6

Chat Diversity Across CR1–CR3

Chat Diversity	Classroom Recording (Day of Class) [# of min. spent together as a class]		
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]
# (math'l chat turns)	70	54	71
# (students who contributed a math'l chat turn)	24	21	27
# (math'l chat turns by five most frequent chat users of the day)	32	33	31

Note. If students changed their Zoom names during the class or accessed the class with two different devices, their chat responses would have been attributed to different people by the anonymization program.

In summary, already by the second class (CR1), many students in Finnegan's class had their cameras off and Finnegan did not rigorously enforce the camera-on course policy. Between a third to a half of the class did, however, use the chat to respond to and ask questions, and the classroom discourse appeared to shift from spoken to written discourse across CR1–CR3. Furthermore, the existence of the chat led to two opportunities: (a) for multiple students to easily reply to the same question, and (b) for students to ask questions that Finnegan's TA could answer in the chat without Finnegan's input.

Inviting Student Participation: The Middle of the Semester

In this section on Finnegan's mid-semester inviting of student participation, I will focus on all three subissues: types of invited participation, the manner of the invitation, and the role of technology in inviting student participation. In particular, I will discuss Finnegan's stance towards IRE, TDMs, and group work (see "types of invited participation"), cold-calling (see "the manner of the invitation"), and the access to and participation in the online context (see "role of technology in inviting student participation").

Types of Invited Participation

Below, I discuss both how Finnegan invited student participation during whole-class time as well as group work.

Whole-Class Time

In T-PD sessions 5 and 6, halfway through the semester, the six teacher discourse moves were introduced as a way to move away from IRE interactions. Finnegan's initial response to being introduced to the idea of IRE in session 5 included seeing the benefits of IRE:

> I don't think it's all bad. Because I mean, there's, there is some, like there's some necessity to just getting short and short, quick answers to make sure everyone's on the same page. But when you sort of start to see it overused, is when I think you get into a lot of these things.

In session 6, when he was asked to read his own classroom transcript from the start of the semester with an eye towards IRE or TDM use, he focused on his IRE use and made the following observations:

> I noticed I was using a lot of the IRE stuff. And part of it might have just been the material, we were just learning logarithms. So I was still trying to get them comfortable

> with actually taking a logarithm. And IRE sort of worked best for that. But it was just sort of a lot of that, going back and forth of asking a question getting a simple response. There were some places where I, it didn't just have that and you actually could see some like them asking questions, but, or you could see them actually answering questions and showing some knowledge but even then, maybe because of the IRE, you still have like the question marks and I can remember when it happened, sort of them asking "I think it's this?" The question marks after their answers.

In session 6, after analyzing a T-PD transcript for TDMs, the group also discussed the possible merits of different TDMs.

A TDM that resonated with Finnegan was "probing a student's thinking." In the mid-T-PD interview he described:

> I've started to ask students to more explain their reasoning. I used to be like I would get an answer and then I would essentially rephrase it for them. More sort of assuming that they got the right answer and then trying to put it in more mathematical terms and now I'm trying to dig a bit deeper and just like say, "Okay, that sounds right but let's make sure we're on the same page." Which I feel like students have responded well to, but I guess we'll see in scores how they actually respond to. […] It might be something like, "Okay, what is the slope? What does that tell us?" And someone says, "It's the change." And, I'm like, "Yeah, exactly, it's that change in height over the change in width. Or change in Y over change in X." Which they probably knew that if they said "the change," but it doesn't really show that they know it, it shows that they know one of the keywords there. What I've started to do now is using that same situation being like, "Okay, change in what? What sort of change?"

In addition to using probing to clarify students' turns (i.e., clarifying-probing), Finnegan described "asking deeper questions" (i.e., deepening-probing): "I'm asking, 'Okay, how did you get this answer?' and having them explain their reasoning or asking them to intuit that like the square root function looks like a logarithmic function and figure out information from that." Reflecting on his use of probing, he noted that it made his teaching more difficult but also more rewarding:

> I would actually say the PD has made teaching a bit harder because I'm actually putting a lot more thought into what I'm saying and into some of the things I'm asking students to do. It's a lot less essentially that IRE stuff, I'm not just asking for quick responses, I'm asking for more in-depth questions, which does make teaching harder but also a bit more rewarding. […] Because I'm actually asking deeper questions […] it's a lot easier to see that students are making progress in that learning. It makes it easier to see that students are making those connections between these different topics which makes it feel like I'm actually doing my job and actually teaching them well.

Thus, Finnegan used "probing a student's thinking" in two different ways (i.e., clarifying-probing and deepening probing) and found that probing made teaching more difficult for him because he spent more time thinking about his teaching. At the same time, probing made teaching more rewarding and made him feel like he was doing his job.

Moving to TDMs that resonated less with Finnegan, Finnegan expressed that "asking students to revoice" might position the student who is revoiced as someone who did a bad job explaining and that being revoiced could feel condescending. He re-expressed this view in the mid-T-PD interview and added a second concern: asking a student to revoice may make the

revoiced student feel like something was taken away from them. He recognized, however, that asking students to revoice could be done in other ways.

During session 6, Finnegan also noted that "inviting further responses" (an important way of "inviting student participation[TDM]") is great, because it allows students to see multiple solutions and can include people who did not answer first, but he felt he did not have the time for it. In the mid-T-PD interview he added, "Somebody answered a question and I asked, 'Can somebody else tell me how they got there?' It definitely worked. It allowed for more people to participate, but I'm not sure how comfortable I am using it all the time."

Finally, Finnegan was not recorded as sharing his thoughts on "creating opportunities to engage with another's reasoning" in session 6. During the mid-T-PD interview, he shared that he thought that he had not used this TDM. His perspective on "wait time" will be discussed in a subsequent section and his use of and views on "revoicing" will be largely omitted as they fall outside the scope of the central issue of this case.

The frequencies in Table 6.7 suggest that Finnegan indeed did not use "asking students to revoice," "inviting further responses," and "creating opportunities to engage with another's reasoning" particularly often. Yet, his use of probing increased after it was introduced in the T-PD, particularly his use of deepening-probing. How his students responded to Finnegan's probing will be covered in the end-of-semester section (and Table 6.11).

In summary, Finnegan saw positive and negative features of IRE interactions and, after analyzing a transcript of his, noted a lot of IRE interactions in the transcript. He appeared to take to the TDM of "probing a student's thinking," using it both to clarify students' responses and dig deeper into students' responses. He did not appear to take to "asking students to revoice," "inviting further responses," and "creating opportunities to engage with another's reasoning."

Table 6.7

Finnegan's Use of "Probing a Student's Thinking," "Asking Students to Revoice," "Inviting Further Responses," and "Creating Opportunities to Engage With Another's Reasoning" Across CR1–CR6

Teacher Discourse Move	Classroom Recording (Day of Class) [# of min. spent together as a class]					
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]	CR4 (12) [60]	CR5 (14) [64]	CR6 (15) [49]
Probing a Student's Thinking	1	3	2	6	13	14
• Clarifying-Probing	1	2	0	3	4	1
• Deepening-Probing	0	1	2	3	9	13
Asking Students to Revoice	0	0	0	0	0	0
Inviting Further Responses	1	0	0	2	0	0
Creating Opportunities to Engage With Another's Reasoning (excluding gp. work)	0	0	0	0	1	3

Group Work

At times, Finnegan continued engaging his students in group work: Although he did not implement group work in CR4 and CR5, he implemented group work in CR6. He faced problems implementing group work by this point, noting in the mid-semester interview that "both group participation with each other, as well as just participation in the class has dropped dramatically from what I would hope." He elaborated on this situation later in the interview:

Valentin: How are things working out with group work this semester?

Finnegan: They're not. Just because once again, students don't want to turn their webcams on, they don't want to talk, they're not doing group work. It's essentially just that they don't want to do group work and because we're not in-person, because I have to join each breakout room individually, I

> can't force them all to do it because I'm going to only be spending a minute out of 15 minutes in group work.

In short, although Finnegan appeared to continue implementing group work every now and then, by the middle of the semester, he faced challenges tied to the technological constraints of teaching via Zoom.

The Manner of the Invitation

In sessions 4 and 5, after watching some T-PD classroom videos together, the T-PD participants made numerous observations, some of which included the teachers calling on specific students. Feeling that calling on students had become an issue of interest within our group, I asked the T-PD participants in session 6 about the ways in which they sought to invite student participation^{TDM}. Valeria and Alice voiced a general hesitance towards cold-calling noting that they did not want to force student participation (Valeria) and felt nervous about cold-calling (Alice). Finnegan noted that he had used it a bit at the start of the semester:

> So I definitely did do some cold-calling in this transcript ((*referencing his own classroom transcript from the beginning of the semester*)) just because it's, it's usually like I wait 20 seconds, I haven't gotten a response back. So I'll call on someone till we get a response to talk about. But for the most part, just by looking at (chatter), I would, for the most part by the sort of rephrasing questions I was able to get most of the questions answered in some capacity.

After hearing Valeria and Alice's concerns, Finnegan noted that when he called on someone, "it also wasn't me just saying, 'Okay, what's the answer, this person?' It's, 'Okay, what thoughts do you have on this, Elias?' or something like that." He felt that this made it a lot less stressful for his students. Reflecting on his current teaching, he noted that he had to call on students "a lot less

often," which he put down to students becoming more comfortable speaking out loud or responding via the chat. "I haven't had to call on anybody in a couple of weeks, so, maybe, I don't know, maybe I scared them into realizing if nobody answers, someone's going to be called on." Looking at Table 6.8, one can see confirmation that Finnegan cold-called on two students at the start of the semester (i.e., "What are your thoughts, Vincent?", "Elias, what are your thoughts on this?") and did not cold-call in any of the mid-semester classroom recordings. Further, as shown in Table 6.9, which shows Finnegan's what/how/why-type solicitations and the number of these that were taken up students, at least three quarters of Finnegan's what/how/why-type solicitations received responses by students. In summary, one can see that despite a few instances of cold-calling early in the semester, Finnegan found that he did not need to call on students by the middle of the semester as they were responding to his questions.

Table 6.8

Instances of Cold-Calling in Finnegan's Class (CR1–CR6)

	Classroom Recording (Day of Class) [# of min. spent together as a class]					
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]	CR4 (12) [60]	CR5 (14) [64]	CR6 (15) [49]
# of times cold-called	0	2	0	0	0	0
# of times student answered to cold-call	0	2	0	0	0	0

Table 6.9

Finnegan's Instructor Solicitations and Their Uptake (CR1–CR6)

Instructor Solicitations (taken up by student(s))	Classroom Recording (Day of Class) [# of min. spent together as a class]					
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]	CR4 (12) [60]	CR5 (14) [64]	CR6 (15) [49]
what	25 (21)	34 (28)	38 (32)	37 (32)	38 (28)	26 (22)
how	18 (16)	8 (6)	12 (9)	1 (1)	7 (6)	10 (10)
why	0 (0)	1 (1)	1 (1)	3 (3)	7 (5)	7 (7)
Total	43 (37)	43 (35)	51 (42)	41 (36)	52 (39)	43 (39)

Note. In instances where no student responded to a question by Finnegan, it may have been that a student responded to the same question once it was re-iterated after some wait time had passed.

The Role of Technology in Inviting Participation

In T-PD session 6, Finnegan expressed how the online context made his teaching more difficult because he could not read the room and could not come up with ways of staying away from IRE-interactions:

Finnegan: […] There's so much I wish I could be in person for, because it's just a ton of things like, I wish I could do differently, but we're not in person. So it's impossible.

Valentin: Like reading the room better, or …?

Finnegan: Reading the room. But also, just like I realized a lot of maybe it's just knowing more now, but a lot of the IRE stuff. I'm like, I can't think of a way to do this over Zoom without doing that call and response questions. There's, there's ways to do it in class, with like having students come up and draw something, but you can't just easily have other people share your screen. So it becomes a bit more difficult.

Three weeks later, in the mid-T-PD interview, I asked Finnegan about the two worries he had expressed about teaching online: access to the right technologies and student participation. He responded by explaining how the former was not a substantial problem but the latter was:

> I think one of the problems I thought would happen, the access problem, luckily is not quite as much of a problem. I have had students, like their Wi-Fi has cut out or their cameras don't work quite as well or something like that, but for the most part, they're able to get through it. I was told they'll be able to use all this technology stuff perfectly fine by [*name*], the course coordinator, which is not true. A lot of students are still struggling to use things like Gradescope or D2L but they're getting through it. The student participation however is definitely a big problem I've seen. At the beginning of the class, or at the beginning of the semester I said, "Hey, try to leave your web cameras on, just sort of to hold yourself accountable." I get like five students who have their web cameras on during class. And some of them are probably still paying attention just don't want the webcams on, but some of them, or probably quite a few of them, are definitely checked out, and just have it on so that it shows they're attending class. I have found participation, both group work participation with each other, as well as just participation in the class has dropped dramatically from what I would hope.

He then shared that despite reminding his students to turn their cameras on the first few weeks of the semester, he had "let it slide" explaining that a camera-on policy was impossible to enforce and that at some point students needed to take responsibility for their own learning.

Asking about whether his motivation to teach had been affected by not seeing people, he answered:

> It has not actually affected my motivation because I still have students asking questions, I have students that definitely do seem like they want to learn. There are six or seven students that are constantly answering questions or asking questions in class so that's enough to keep up motivation.

Instead, he identified a different problem: Not being able to identify his students (and their voices) because he had not seen them much. "I hear someone's voice and I'm like, I know someone's talking, but I cannot put a name to it." Another issue that Finnegan noted was dropping attendance: Although around 60 students were registered for the course, "you'll have like 40 to 45 people show up, but on a bad day, like when we were going over exam review, I think I had 25 people show up."

In summary, by the middle of the semester, Finnegan identified multiple challenges with teaching online: (a) not being able to read the room, (b) finding it difficult to implement alternatives to IRE interactions, (c) student participation dropping (both within whole-class and group work time), (d) not being able to identify students by their voices, and (e) student attendance dropping. Despite all these challenges and many students keeping their cameras off, he related that his motivation had not dropped because the presence of several students who seemed willing to learn kept his motivation up. Furthermore, his initial fears about students lacking the technologies to access and engage with his class were largely not realized.

Inviting Student Participation: The End of the Semester

In this section on Finnegan's end-of-semester inviting of student participation, I will again focus on all three subissues: types of invited participation, the manner of the invitation, and the role of technology in inviting student participation. In particular, I will discuss Finnegan's stance on and use of TDMs and group work (see "types of invited participation"), cold-calling

and wait time (see “the manner the of the invitation”), and the participation in the online context (see “role of technology in inviting student participation”).

Types of Invited Participation

As can be seen in Table 6.10, the patterns of Finnegan’s TDM usage observed at the middle of the semester continued to hold true until the end of the semester: (a) Finnegan probed student’s thinking more than he had done at the beginning of the semester (both clarifying- and deepening-probing), and (b) Finnegan had not worked “asking students to revoice,” “inviting further responses,” or “creating opportunities to engage with another’s reasoning” into his teaching repertoire. Although Finnegan may not have worked “creating opportunities to engage with another’s reasoning” into his teaching repertoire, it appears that he used one form of it (i.e., “creating opportunities to add to another’s reasoning”) a few times in the second half of the semester.

Table 6.10

Finnegan's Use of "Probing a Student's Thinking," "Asking Students to Revoice," "Inviting Further Responses," and "Creating Opportunities to Engage with Another's Reasoning" Across CR1–CR9

Teacher Discourse Move	Classroom Recording (Day of Class) [# of min. spent together as a class]								
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]	CR4 (12) [60]	CR5 (14) [64]	CR6 (15) [49]	CR7 (20) [58]	CR8 (21) [81]	CR9 (24) [82]
Probing	1	3	2	6	13	14	11	17	9
• Clarifying	1	2	0	3	4	1	3	8	3
• Deepening	0	1	2	3	9	13	8	9	6
Asking Students to Revoice	0	0	0	0	0	0	0	0	0
Inviting Further Responses	1	0	0	2	0	0	1	0	1
Creating Opportunities to Engage With Another's Reasoning (excluding gp. work)									
• Adding	0	0	0	0	1	3	0	1	0
• Evaluating	0	0	0	0	0	0	1	0	0
• Understanding	0	0	0	0	0	0	0	0	0

Finnegan's use of probing was notably student-centric, particularly when using deepening-probing. Even when he used clarifying-probing, the onus was mostly on the student to explain their previous utterance. When Finnegan used probing in an instructor-centric fashion, it often involved him trying to make sense of students' questions. (For a full breakdown of Finnegan's use of "probing a student's thinking," see Table C.3.)

The online context—particularly the possibility to communicate via chat—appeared to increase the relevance of clarifying-probing as Finnegan explained in the post-T-PD interview:

> [S]ometimes we'll get a student that says like, "Oh, yeah. It's that." or "Yeah, like four over two." I'm like, "Well, four over two what? You can't just say four over two." If I'm

not reading the chat or making sure students actually explain themselves in the chat, because I'm not sure if you've experienced it, but whenever I text friends or post on Discord or something, it tends to be very brief, or a lot of people tend to be very brief and give sentence fragments, which works fantastically when you're just talking conversationally because it doesn't matter. When you're in a classroom, you need to reinforce, "No, we're in a classroom. We need to have complete sentences and make sure we're saying everything what we mean to be saying." Being cognizant of that helped a lot of making things run smoothly.

Given that Finnegan placed the onus on the students for justifying and explaining themselves when he posed probing questions, one might expect that the type of talk students engaged in changed to include more why-type talk. As Table 6.11 shows, this seems to indeed have been the case and can be directly traced back to his probing questions.

Table 6.11

Student Talk Type in Finnegan's Class CR1–CR9

Student Talk Type	Classroom Recording (Day of Class) [# of min. spent together as a class]								
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]	CR4 (12) [60]	CR5 (14) [64]	CR6 (15) [49]	CR7 (20) [58]	CR8 (21) [81]	CR9 (24) [82]
what	53	44	55	39	42	21	21	41	53
how	26	13	15	2	7	7	12	18	13
why (post-probing)	0 (0)	3 (1)	3 (0)	6 (3)	7 (6)	8 (5)	6 (5)	13 (11)	4 (3)
question	40	21	23	9	8	14	12	18	6
other	7	8	5	2	9	4	1	11	10
Total	126	89	101	58	73	54	52	101	86

Note. The "post-probing why" counts include why-turns of students who were not addressed by the instructor but chose to answer anyway. (For example, if Finnegan probed Luke for justification for his answer but Leia responded, her turn is counted as a post-probing why.)

In the post-T-PD interview, Finnegan also elaborated on how he felt the online setting shaped which TDMs he thought were useful. He highlighted the utility of probing (also as a way to move away from IRE), waiting, inviting student participation^{TDM} (through cold-calling and asking more open-ended questions), and revoicing students' chat responses aloud. At the same time, he noted the difficulty of inviting student participation^{TDM} when he could not see many of the students—or even know if they were sitting in front of their computers. Two TDMs he described as probably working much better in person than online were "asking students to revoice" and "creating opportunities to engage with another's reasoning." With regard to the former, he noted, "it wouldn't feel as natural to me asking students like, 'Well, okay, what do you think they said?' or that sort of thing when they can't turn and talk to the student." With regard to the latter, he focused on group work as a setting in which students could possibly engage with another's reasoning and described the difficulties of implementing group work online by comparing it to his in-person experiences with implementing group work:

> I really enjoy group work. It allows students to work together. It's incredibly informal. Students can talk with each other. It gives a nice break for the whole learning thing. They're still learning, but it gives a nice time to just talk with people. During that time, as you walk around, you can see somebody struggling with something while another person in the group has it, so you talk to the struggling person, like, "Well, it seems like this person has it. Why don't you guys talk to each other and figure out what's going on?" which wasn't really possible in Zoom because even when I did group work, I couldn't see everyone doing group work all at once. They were all off in their Zoom rooms and even then a lot of them were just doing individual work.

Lastly, given Finnegan's concern for students being comfortable asking questions, it should be noted that Finnegan continued asking for students' questions throughout the semester—although with decreased success by the middle of the semester. (For a breakdown of Finnegan's solicitations across CR1–CR9, including his asking for questions, see Table C.2.) That said, even though students may have posed fewer questions in response to Finnegan's requests for questions, students continued to ask questions throughout the semester as can be seen in Table 6.11.

In summary, Finnegan's use of TDMs continued the trend of the mid-semester: he increasingly used "probing a student's thinking," but largely stayed away from "asking students to revoice," "inviting further responses," and "creating opportunities to engage with another's reasoning." His use of probing was very student-centric and allowed students to engage in justifying and explaining their responses, which lead to an increase in why-type student talk. Further, the online context shaped Finnegan's thinking about the TDMs and affected his implementation and experience with group work. Lastly, in line with his belief that "students should feel comfortable asking questions," Finnegan asked consistently for student questions throughout the semester—although with decreasing success. Nevertheless, students continued to ask questions in his class.

The Manner of the Invitation

In this subsection, I will share Finnegan's end-of-semester cold-calling practice as well as his discourse about and use of wait time.

Cold-Calling

At the start of T-PD session 10, Finnegan shared that participation had decreased in his class and that he had recently experienced situations where even simple questions like "What's zero squared times two?" received no response. He added that his students appeared exhausted and were upset that there was not enough time for reviews before quizzes. The lower participation, did, however, have an upside for Finnegan:

> So the students being quiet actually did have one benefit of I was actually able to call on students. […] But it actually did get them talking and the students I called on, they either knew the answer, or they at the very least they were able to keep us moving in the class.

After feeling that cold-calling had often come up as a topic of conversation during T-PD sessions but few strategies for cold-calling had been shared, I decided to provide some cold-calling resources in this session (i.e., *Cold Calling*, 2019; Lemov, 2010; Rush, 2018). The resources were not intended to be instructions for the participants; they were merely intended to serve as examples of how other instructors managed the tensions between the benefits of cold-calling (e.g., giving students a voice who might not otherwise get to speak, creating a culture of participation) and the downsides (e.g., putting students on the spot). For example, the following bullet points from Rush (2018) were shared with the participants:

- Use "softball" questions at the beginning of the semester to get everyone talking. Content-relevant but low-stakes questions like: *Have you ever experienced a food aversion?* (psych) or *How much would you pay for this beaded bracelet?* (econ) help students find their voice.

- Give students time to answer questions in a journal before sharing aloud. They have time to think through their response and don't feel pressured. This strategy opened up much better discussions in Civil Liberties—and put a pause on the fast responders.
- Let students think-pair-share before answering aloud. That way no one is on the spot to provide their own answer—and the risk of being wrong is diluted to the pair or group.
- Allow students to say "pass" with no repercussions. Period.
- Say the student's name first, then pose the question, so there's never a "gotcha" moment.
- Finally, don't use cold-calling for challenging questions. I always give a warning—*"this is a tough one"*—and seek volunteers. The fact that I've called it "tough" automatically lowers the risk of being wrong and rewards the brave soul who volunteers.

Although Valeria and Alice shared that the bullet points resonated with them, Finnegan made no comments about them. Despite this lack of reaction, session 10 may have contributed to Finnegan embracing cold-calling (see Table 6.12). (For reference, CR7 took place a day before session 10, whereas CR8 and CR9 took place in the fortnight thereafter.)

Looking at the way Finnegan cold-called during CR7–CR9, I observed two patterns: (a) Finnegan said the name of the student before asking (or repeating) a question, and (b) he appeared to be purposeful in his phrasing of cold-calling, frequently using formulations such as "[name], do you have an idea for […]" or "[name], do you want to explain/recontextualize […]".

Finnegan also noted in T-PD session 10 how he tried to use cold-calling to increase the participation of female students:

> And I did try to make that active effort to calling on in particular female students more than male students. Like, and it was difficult because it always felt like, one, now I'm picking the female students more, but I guess that's part of, partially the point. […] And I

> also sort of noticed that in times where I didn't call on someone, maybe 'cause someone just answered right away, it tended to be males that were answering right away. So the calling definitely seemed to help get more female interaction in the course.

Finnegan's comments about calling more on female students were rooted in a discussion that had begun in session 9. In that session, he had noted—after analyzing one of his transcripts with the EQUIP tool—that men responded aloud more than women, that is, the ratio of men to women responding aloud to his questions were "at best, it's equality." After the use of cold-calling was discussed in the session as a way of bringing more female students into the conversation, Finnegan noted that cold-calling "it's going to probably have something I have to get, I'm going to have to work on, because it does seem like that's one of the better ways that help, help increase equity in the classroom. So I'll work on it." As the comments from session 10 and the entries in Table 6.12 show, Finnegan did work on cold-calling female students.

Table 6.12

Instances of Cold-Calling in Finnegan's Class (CR1–CR9)

	Classroom Recording (Day of Class) [# of min. spent together as a class]								
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]	CR4 (12) [60]	CR5 (14) [64]	CR6 (15) [49]	CR7 (20) [58]	CR8 (21) [81]	CR9 (24) [82]
# of times cold-called	0	2	0	0	0	0	1	3	7
# of times female student cold-called	0	0	0	0	0	0	1	2	5
# of times student answered to cold-call	0	2	0	0	0	0	0	3	3

Note. Students did not have the opportunity to disclose their preferred pronouns (as part of protecting their anonymity). Therefore, who might identify as female (and who might not) is based on my positioning of these students' gender identities from listening to the classroom audio-recordings.

In the post-T-PD interview, Finnegan identified cold-calling as one of the most useful practices in the context of online teaching that he had picked up from the T-PD. In thinking about what he wanted to do differently the next time he taught, he shared:

> I want to actually do more cold-calling right from the start, of actually having students raise hands and calling on them, just so that sort of an expectation of "this is something that can happen in the classroom" and continue doing it throughout the whole class so that I actually get those responses. Depending on the class I teach, I would like to do more of the probing into students' reasoning, even if it's me probing a student or getting a student to expand on what they've said, or getting another student to expand on what they said. I would really like to do that a bit more, which the cold-calling would help because then I can start with one student say, "What do you think?" and then turn to another and say, "Why do you think they think that?" which isn't really something that I established as something I would do in this class, so it felt uncomfortable to do that sort of stuff.

In other words, Finnegan had worked cold-calling into his repertoire and saw it as a way to create a more complex interactional pattern that included cold-calling and then "probing a student's thinking" or "creating opportunities to engage with another's reasoning."

In summary, Finnegan took up cold-calling towards the end of the semester as a way to increase participation in general and participation of female students in particular. By the end of the semester, Finnegan thought very positively of cold-calling, decided that it was something he wanted to do more in the future, and considered ways of combining it with other TDMs.

Waiting

Three weeks after wait time 1 and wait time 2 had been introduced in the T-PD, Finnegan noted in the mid-T-PD interview that he used wait time in his teaching. He described that prior to his participation in the T-PD, he was already good at waiting for students to respond after posing a question (i.e., wait time 1); now he was also willing to pause after making a statement to give students extra time to ask questions or ask for clarification. It should be underlined that this latter form of wait time is neither wait time 1 nor wait time 2 but a separate form of wait time which I shall refer to as wait time 3.

By the mid-PD interview, Finnegan believed not to have used wait time 2, expressing that he was uncomfortable with waiting after a student's turn due to his discomfort with silences—especially those he had not initiated. He explained that it takes conscious effort to stop himself from filling silences. (In session 6, he had shared that he used his fingers to count to five to make sure he actually waited.) But, by the post-T-PD interview, he expressed that he had tried to incorporate wait time 2 into his teaching. He still "hate[d]" silences but had come to embrace them more and wait despite his discomfort because he realized their usefulness.

Finally, he noted that he gave "a lot of time"—"five to seven seconds"—after asking students for their questions. In summary, Finnegan describes using waiting in four different ways during the semester: (a) waiting after asking questions (wait time 1), (b) waiting after making statements (wait time 3), (c) waiting after a student's response (wait time 2), and (d) waiting after asking for students' questions (wait time 1).

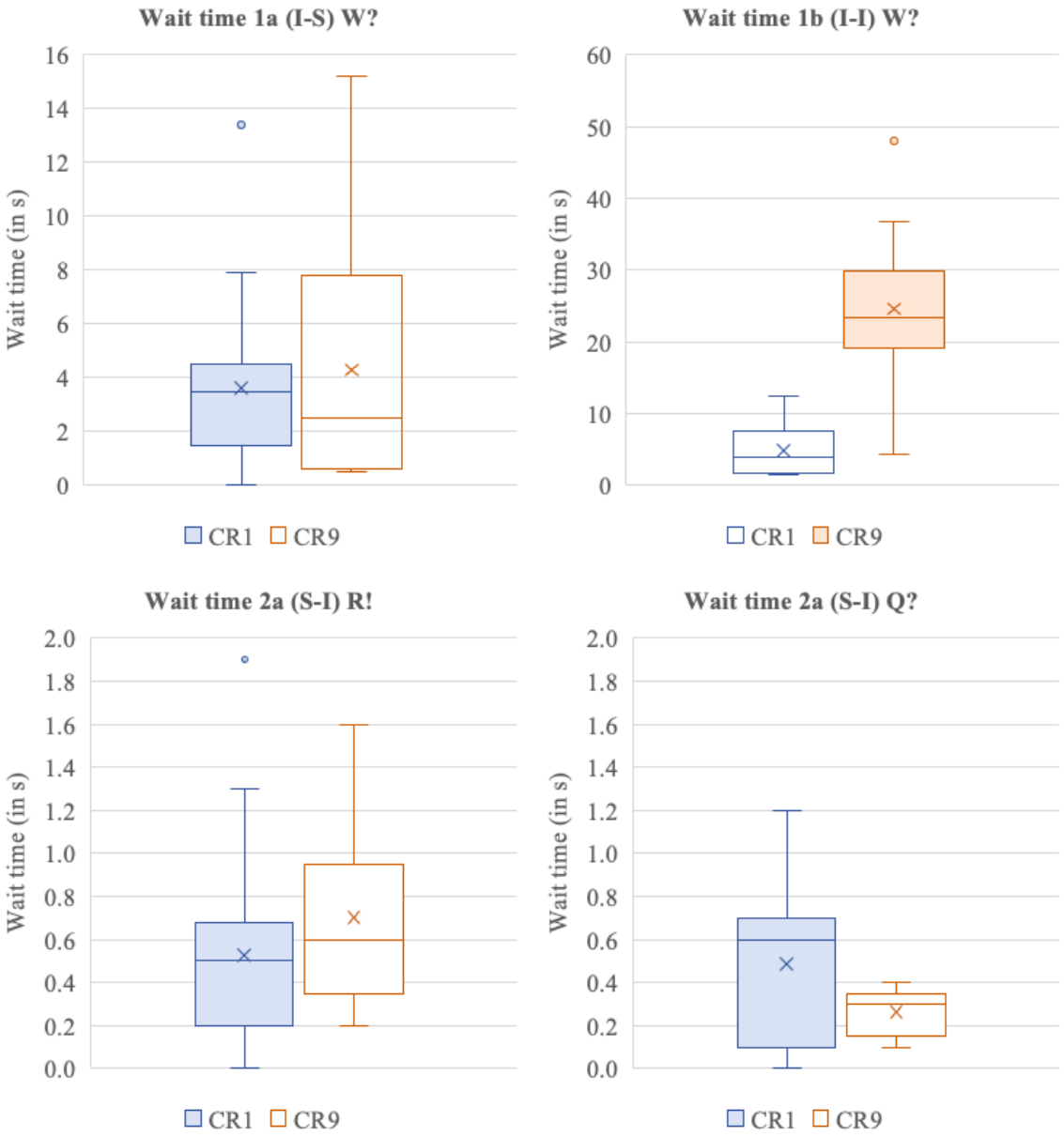
Looking at the wait time comparison between CR1 and CR9, Figure 6.2 provides support for at least some of the above claims. In particular, Figure 6.2 suggests that Finnegan's wait time mean exceeded 3 seconds—Rowe's (1986) threshold for wait time—both at the start and the end

of the T-PD after asking students what, how, or why-type questions. Furthermore, it suggests that Finnegan may have increased his wait time 1b by the end of the semester after asking students what, how, or why-type questions. (His wait time 1b (I-I) Q? also appears to have increased, with the shortest such wait time in CR9 exceeding the longest such wait time in CR1. That said, there were only five such data points for each of CR1 and CR9, leading to the omission of wait time 1b (I-I) Q? from Figure 6.2.) Figure 6.2 also provides confirmation of Finnegan's expressed struggles with waiting after students' responses and questions, with no single instance of wait time 2a of over 2 seconds recorded. In addition, no single instance of wait time 2b was recorded in CR1 and CR9. Coding for Finnegan's wait time 3 was not feasible—for deciding what makes a pause in someone's speech purposeful for them is a tall task—but I noted several instances in both CR1 and CR9 in which Finnegan appeared to make a purposeful pause after covering some material before launching into new material or asking for questions.

In short, the analyzed data seem to suggest the following with an eye towards Finnegan's description of his wait time use: (a) Finnegan waited (on average longer than 3 seconds) after asking questions, (b) Finnegan appears to have increased the time he was willing to wait after asking a question by the end of the semester, (c) Finnegan waited only very briefly after a student's response or question, and (d) Finnegan sometimes purposefully paused after making statements.

Figure 6.2

Wait Time Comparison: CR1 Versus CR9



Note. The cross on a boxplot marks the mean; the line splitting the interquartile range marks the median. During the semester, students increasingly took to communicating via chat, and so there were far fewer wait time data points in CR9 than in CR1. Consequently, no single type of wait time had 10 or more data points in both CR1 and CR9. Thus, I chose to include boxplot comparisons for those wait times which had 10 or more data points in one classroom recording

and at least 5 in the other. Unshaded boxplots represent those boxplots with fewer than 10 data points. The mean and median of the boxplots, given in the format ((mean$_{CR1}$, median$_{CR1}$),(mean$_{CR9}$, median$_{CR9}$)) and read left-to-right, top-to-bottom, are: ((3.6, 3.5),(4.3, 2.5)), ((4.9, 4.0),(24.6, 23.4)), ((0.5,0.5),(0.7,0.6)), and ((0.5, 0.6),(0.3, 0.3)) (given to the nearest tenth of a second). The number of data points each boxplot reflects (from left to right, top to bottom) are: 27, 9, 8, 15, 36, 9, 39, and 5.

The Role of Technology in Inviting Participation

As noted in the end-of-semester subsection on "Types of Invited Participation," Finnegan expressed in the post-T-PD interview that he was able to use certain TDMs in the online context (i.e., waiting, probing, revoicing [students' chat contributions], and inviting student participation^{TDM} [via cold-calling and asking open-ended questions]). Clarifying-probing, he explained, was particularly valuable due to the differing norms of chat and spoken communication, with chat messages often being terser than spoken turns, increasing the likelihood of students omitting key words and information. Clarifying-probing could therefore serve as a way for students to elucidate their thinking.

Another TDM that Finnegan used in a novel way due to the online context was revoicing. Although revoicing has been almost completely omitted from discussion so far (as it is a TDM less explicitly geared towards inviting student participation), Finnegan noted using revoicing to bring chat messages into the spoken classroom discourse, allowing people who were not following the chat to follow the classroom conversation. Both the clarifying-probing of chat messages and the revoicing of chat messages may have gained prominence in Finnegan's teaching in light of the shift from spoken to written student discourse as the semester progressed (see Table 6.13 and Table C.1).

Other TDMs, Finnegan felt, worked less well online (i.e., asking students to revoice and creating opportunities to engage in another's reasoning [with respect to group work]). He expressed that "asking student to revoice" and "inviting student participation^{TDM}" would work much better in-person. With regard to implementing group work on Zoom, he observed that it "wasn't really possible" because he could not walk around to easily check in with groups and students were not collaborating in their breakout rooms. He also reiterated the challenges of inviting student participation^{TDM} when one could not see students.

In the post-T-PD interview, Finnegan also noted difficulties that arose from being able to rely less on body language and tone of voice for communication:

> [B]ody language is I think incredibly important in teaching. One of the things I like to do, I pace back and forth a lot when I teach, which I've been told is a good thing because it makes students pay attention to the board. I don't know if it is or not, I still do it. But one of the nice things then you can do is you're pacing and then when you want students to focus on something, you can stop and you'd look at them while you're doing it. I've gotten very good at being able to look at students while writing on a board and I don't really have that same ability on OneNote. Or group work. I like kneeling down to the student's level so that we're on the same eye level. We're talking about something, I can point things out. You can't do that on Zoom and there's no kneel down function on Zoom. I think like a lot of it is body language is missing a lot and you really can't replicate that online. […] [E]ven telling tones online is difficult because there's that audio translation thing. It's very difficult to pick up a rising tone when students are asking questions or saying answers. There's that. It's incredibly difficult to interact with students when you can't see them or you can't see their whole body or their audio is garbled, or

even worse, they're talking in chat and you just don't know what they need. Are they certain, are they unsure? All that sort of stuff.

In short, he highlighted how important using and seeing body language as well as hearing the students' tone of voice were to his usual in-person communication with students.

Table 6.13

Mode of Discourse Used by Students in Finnegan's Class Across CR1–CR9

"Mode of Discourse" used in a student turn (percentage out of total)	Classroom Recording (Day of Class) [# of min. spent together as a class]								
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]	CR4 (12) [60]	CR5 (14) [64]	CR6 (15) [49]	CR7 (20) [58]	CR8 (21) [81]	CR9 (24) [82]
Spoken	56 (.421)	37 (.346)	33 (.250)	17 (.283)	19 (.241)	35 (.574)	11 (.196)	9 (.086)	9 (.102)
Public chat	76 (.571)	69[a] (.645)	99[a] (.750)	43[a] (.717)	53[a] (.671)	25[a] (.410)	41 (.732)	93 (.886)	79 (.898)
Private chat	0 (.000)	0 (.000)	0 (.000)	0 (.000)	0 (.000)	0 (.000)	0 (.000)	0 (.000)	0 (.000)
Other (i.e., body language, intermodal)	1 (.008)	1 (.009)	0 (.000)	0 (.000)	7 (.089)	1 (.016)	4 (.071)	3 (.029)	0 (.000)
Total	133	107	132	60	79	61	56	105	88

Note. The numbers in this table reflect *all* student turns: both those about mathematics and those about logistics. For a table focused only on the mode of discourse in student turns about mathematics, see Table C.1.

[a] Counts marked with a superscript "a" include a single instance of a clipped message.

As Table 6.13 indicates, the start-of-semester trend away from spoken turns and towards chat participation persisted throughout the semester. The proportion of students who participated at least once in the chat each class may have dropped to as low as a quarter, but this proportion is difficult to calculate given the absence of attendance data. (The only relevant information about class size that Finnegan shared [in the mid-T-PD interview] was that although around 60 students

were registered for the course, “you’ll have like 40 to 45 people show up, but on a bad day, like when we were going over exam review, I think I had 25 people show up.”)

> An observation Finnegan made in T-PD session 9 regarding diversity and chat use was:
>
> I will say like with the chat, this is completely anecdotal. But I have noticed that almost all of my chat questions are from women. I’m not sure if they’re just more comfortable sharing through chat or if something else, but anecdotally, anecdotally, that’s what I’ve noticed.

Looking at Table 6.14 the analyzed classroom recordings do not corroborate Finnegan’s perception. Instead, the table suggests that at best half the questions in the chat (and half the turns) were contributed by female students.

Table 6.14

Chat Diversity Across CR1–CR9

Chat Diversity	Classroom Recording (Day of Class) [# of min. spent together as a class]								
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]	CR4 (12) [60]	CR5 (14) [64]	CR6 (15) [49]	CR7 (20) [58]	CR8 (21) [81]	CR9 (24) [82]
# (math'l chat turns)	70	54	71	41	48	18	37	90	77
# (students who contributed a math'l chat turn)	24	21	27	17	14	11	12	23	13
# (math'l chat turns by five most frequent chat users of the day)	32	33	31	22	35	12	24	50	60
# (math'l chat turns by female students)	31-36	24-29	29-39	20-22	21-27	5-7	18	39-42	47-48
# (math'l chat turns: questions)	14	12	12	4	4	5	4	10	3
# (math'l chat turns: questions by female students)	7-8	6-7	7-10	1-2	0-2	3	1	3	2

Note. In instances where multiple students are tied for the place of fifth most frequent chat user, only one of these students' chat messages are counted. That is, the "# (math'l chat turns by five most frequent chat users of the day)" is always based on exactly five students. Some of the table entries are given as ranges since I was sometimes unable to make a guess at a chat user's gender identity from the surrounding audio-recording.

In summary, Finnegan felt that certain TDMs (i.e., waiting, probing, revoicing [students' chat contributions], and inviting student participation^{TDM} [via cold-calling and asking open-ended questions]) were more useful in the online context than other TDMs (i.e., asking students to revoice and creating opportunities to engage in another's reasoning [in the context of group work]). Clarifying-probing and revoicing students' chat contributions in particular may have

even been more useful *given* the online context. Finnegan also observed that the limited use and seeing of body language as well as the limited hearing of tone of voice made communication more difficult. Finally, looking across the entire semester, it can be observed that student participation moved from mainly spoken participation to mainly written participation. Although typically five students or so were responsible for at least half of the chat turns, it is likely that between a quarter and half of the class participated in the chat in some form.

My Interpretation of Finnegan's Case

The entire case has already been suffused with my interpretation through my selecting of data and the telling of the case. That said, in the case thus far, I have sought to provide a narrative that sticks close to data. In this section, I will look across the entire case and offer interpretations.

The Type of Invitation

The two contexts in which the types of invitation were considered were whole-class time and group work. I expand on the types of invitation in both contexts below.

Whole-Class Time

The most notable change Finnegan made with respect to the use of teacher discourse moves was his use of probing. After having almost no probing interactions with students at the start of the semester, by the middle and the end of the semester, probing was part of his teaching repertoire—both clarifying-probing and deepening-probing. Furthermore, the way in which he used probing largely placed the onus of clarifying and justifying on the students. Consequently, probing served as a way for Finnegan to learn more about what his students meant as well as increase his students' engagement in why-type discourse (i.e., justifying). This focus on students'

thinking stands in contrast to how at the start of the semester he used to just "assum[e] that they got the right answer."

A reason Finnegan may have taken to probing was that it was rewarding and made him feel like he was doing his job as a teacher because the students demonstrated that they could justify their answers. Furthermore, clarifying-probing may have been particularly relevant in the online context for clarifying what students meant by their chat messages—messages that are typically briefer and opaquer than a spoken message.

Other TDMs that could have provided ways of inviting different types of student participation (i.e., asking students to revoice, creating opportunities to engage with another's reasoning) as well as "inviting further responses" were not seriously taken up by Finnegan. In particular, he expressed two fears about "asking students to revoice:" (a) that it positions the student who is revoiced as someone who did a bad job explaining, and (b) that it may make the revoiced student feel like something was taken away from them. Furthermore, he felt that the online context made the move unnatural as students could not easily turn around and talk to each other.

With respect to "inviting further responses" (an important way of "inviting student participation^{TDM}") he noted it was great—because it allows students to see multiple solutions and can include people who did not answer first—but that he did not have the time for it. There may have also been less of a need for "inviting further responses" as the chat offered students the opportunity to reply near simultaneously and defy the traditional consecutive turn-taking structure of spoken discourse. In other words, multiple responses could be given by students more naturally in the online context without the need for Finnegan to invite further responses.

Finally, Finnegan thought of group work as a way of "creating opportunities to engage with another's reasoning," which will be discussed in the subsection below.

Finnegan did, however, highlight the utility of two other TDMs: waiting (as discussed above) and revoicing (for reuttering students' chat responses aloud to bring them into the spoken discourse). Furthermore, throughout the semester, Finnegan asked for students' questions, and students posed questions. Yet, as the semester progressed, students formulated fewer questions in response to Finnegan's requests for questions.

Group Work

Although Finnegan espoused group work, he experienced setbacks with his online group work implementation and appeared to no longer incorporate it into his teaching by the end of the semester. Specifically, he observed as the semester progressed that: (a) students were not really engaging in group work, (b) he could not easily look across all breakout rooms to see whether students were engaging in group work or not, and (c) he had no time to implement group work.

Although implementing group work was not a topic of discussion in the T-PD, the importance of group work as a context for student learning was considered. I suspect that discussions about the establishment of group work norms (in an online setting) would have been useful for Finnegan. As noted in the start-of-semester section, Finnegan's only recorded statement of group work expectations was "Yeah, so yeah, please work on these as groups." Given that organizing and enforcing group work is more difficult in an online context, this single statement may have been insufficient to create a culture of collaboration. Furthermore, students may have found it disheartening to be placed in randomized groups. Last, students may have seen little need to work together at the start of the semester because groups were not always individually called on to report on their group's findings.

The Manner of the Invitation

The two subissues with respect to which the manner of invitation was considered were cold-calling and waiting.

Cold-Calling

For the first half of the semester, Finnegan saw little need to cold-call on students as they responded to his questions. During the middle of the semester, the benefits and downsides of cold-calling were brought up by the T-PD participants across multiple sessions, particularly after observing classroom recordings of teachers who cold-called on their students. Noting that this was a topic of interest for the T-PD participants, I shared some information towards the end of the semester on how some instructors cold-called on their students. Cold-calling was also mentioned in the T-PD as a way to disrupt inequitable participation patterns (e.g., male students responding first to an instructor's questions).

By the end of the semester, student participation was down in Finnegan's class, and he began to cold-call on students. Furthermore, after having noted that men responded aloud more to his questions than women, he used cold-calling to increase female participation in the spoken classroom discourse. By CR9, he cold-called more on students than in CR1–CR8 combined—mostly on female students. His cold-calling followed two patterns: (a) Finnegan said the name of the student before asking (or repeating) a question, and (b) he appeared to be purposeful in his phrasing of cold-calling, frequently using formulations such as "[name], do you have an idea for […]" or "[name], do you want to explain/recontextualize […]".

He ended the semester feeling that cold-calling had been an important addition to his teaching repertoire and one of the most useful practices he had picked up from the T-PD. He

planned to continue using it in the future and combine it with "probing a student's thinking" and "creating opportunities to engage with another's reasoning."

In short, there appear to be several explanations for why Finnegan took to cold-calling: There was a two-fold need (i.e., students were not responding and when students responded they were mostly male), his skepticism of cold-calling decreased (after discussing its benefits and different ways of cold-calling), and it was convenient (i.e., it worked well together with probing).

Waiting

The T-PD discussions around wait time—particularly wait time 1—seemed to resonate with Finnegan: On average, he waited longer than 3 seconds after asking questions (both at the start and end of the semester). Further, by the end of the semester, Finnegan appeared to have increased the time he was willing to wait after asking questions. This longer wait time appeared to be in line with his desire for students to be comfortable asking questions. Furthermore, the shift in student discourse towards written communication may have necessitated longer wait times as typing a response may take students longer than responding in speech.

That said, Finnegan shared that he struggled with silences, "hate[d]" them, and had to use his fingers to keep himself from speaking up too early. His struggle with keeping silent was seemingly exacerbated when waiting after students spoke; he struggled with wait time 2 throughout the entire semester. Finnegan's longest observed instance of wait time 2a was 1.9 seconds. He did, however, use wait time in a third way: pausing during his turns to give students a chance to let things settle and formulate questions.

The Role of Technology

Finnegan had started the semester with two big worries about teaching online: (a) that students lacked access to adequate technological resources, and (b) that participation was going

to be down. By the middle of the semester, he felt that the former had not come to pass but that the latter indeed posed a problem. Specifically, student participation (in group work and the class) dropped after the start of the semester. His motivation had, however, not been affected since he had a core of six or seven students who were engaged and kept up his motivation.

Teaching synchronously to a large class via Zoom appeared to have a wide-ranging impact on Finnegan's class. One of the most prominent differences to an in-person setting was the availability of the chat feature. By the end of the semester, most of the student discourse was written (and public). There appeared to be several benefits for instructor and students in using the chat: (a) many students could respond to the same question (i.e., the first student to respond does not take space away for other students), (b) students could ask questions without disrupting the flow of class, and (c) the TA could answer students' questions. Although the existence of the chat was not enough to change the pattern of a few students having most of the turns (i.e., the top five chat-using students of each class day contributed about half the mathematical chat turns), it appeared as though in each class about a quarter to half the students used the chat at least once.

There were also some ways in which the chat changed Finnegan's teaching: (a) clarifying-probing became more relevant since the norms for chat responses were less formal than for spoken responses, (b) since not everyone could be counted on to read the chat, Finnegan revoiced students' chat contributions, and (c) Finnegan may have had to wait longer for student responses (given that typing mathematics is slower than speaking mathematics).

Yet, downsides (particularly for Finnegan) to the prevalence of chat communication were: (a) the loss of the speaker's tone as an aspect of communication, (b) decreased clarity of contributions because the norms for chat messages and spoken discourse are different, and (c) that Finnegan could not easily identify his students by voice when they chose to speak up.

These chat-specific downsides were complemented by other challenges that Finnegan identified to teaching online: (a) not being able to use body language in the same way as he would in an in-person setting (e.g., pacing, kneeling down to talk with students, (b) not being able to quickly glance across the room to read it, and (c) group work being more difficult to implement successfully.

The fact that the class was online (and recorded) may have also contributed to low attendance in the class. Furthermore, the number of students who had their cameras on quickly plummeted, particularly since Finnegan chose not to rigorously enforce the class's camera-on policy out of concern for students' access to the class. Although he noted in the mid-T-PD interview that his fear that students did not have access to the appropriate technologies had not really manifested itself, he continued not to rigorously enforce the camera-on policy for the rest of the semester.

Finally, Finnegan felt that the online context made it difficult for him to stay away from IRE-interactions as more student-centric ideas for his teaching (e.g., having students come to the board) seemed more difficult to do in an online context. And although he found certain TDMs to work well in the online context (i.e., waiting, probing a student's thinking, inviting student participation^{TDM} [via cold-calling and asking more open-ended questions], and revoicing [chat responses aloud]), he felt others worked less well in an online context (i.e., asking students to revoice, inviting student participation [given technology mediating the interaction], and creating opportunities to engage with another's reasoning).

Conclusion

Over the course of his participation in the T-PD, Finnegan invited different forms of student participation in different manners in an online context. His case provides insight into

why an instructor may use certain TDMs (and not others) while also documenting new possibilities and challenges of teaching synchronously online via Zoom. Finnegan's case adds to discussions around how instructors may use wait time, cold-calling, and "probing a student's thinking" and confirms that "probing a student's thinking"—particularly student-centric deepening-probing—can engage students in justifying.

CHAPTER 7: THE CASE OF VALERIA

Overview of the Case

Valeria is a case of a mathematics-teaching graduate teaching assistant who was no longer in her first year of teaching and who participated in the MDISC professional development over the course of the semester, with the MDISC slightly abridged to fit into a semester and slightly modified to account for the undergraduate mathematics context. The emic issue that arose while developing Valeria's case was the question: How does an instructor who weighs wielding her authority to engage students against respecting students' agency (even if that agency is used to not participate) implement the TDMs? Thus, Valeria's case—by itself—should be understood as an instrumental case study that seeks to answer this question by detailing how Valeria experienced this balancing effort and gravitated towards some TDMs while not taking up others. Valeria's case is split into four sections: an introduction, the classroom discourse in Valeria's class before the introduction of the TDMs, the classroom discourse in Valeria's class after the introduction of the TDMs, and a final section in which I offer my interpretation of the case. For the reader's convenience, Table 7.1 below summarizes when which data were collected.

Table 7.1

Valeria's Data Collection Timeline

Week	Classroom Recordings	T-PD Session	Interview
1 (reading week)			Pre-T-PD interview
2	CR1	Session 1 (Intro part 1)	
3	CR2	Session 2 (Intro part 2)	
4	CR3	Session 3 (Discourse part 1)	
5	CR4	Session 4 (Discourse part 2)	
6	CR5	Session 5 (TDMs part 1)	
7		Session 6 (TDMs part 2)	
8	CR6		
9	CR7	Session 7 (Positioning part 1)	
10	CR8	Session 8 (Positioning part 2)	Mid-T-PD interview
11	CR9	Session 9 (EQUIP, Recap part 1)	
12	CR10	Session 10 (Recap part 2)	
13	CR11	Session 11 (Capstone part 1)	
14		Session 12 (Capstone part 2)	
15		Session 13 (Capstone part 3)	
16 (Exam week)			
Post-semester week 3			Post-T-PD interview

Note. In weeks 7 and 14, Valeria's students completed exams during the class. The class in week 15 was cancelled due to a delayed spring break.

Meeting Valeria

Interviewer: What is something about your teaching that you think you would like to improve?

Valeria: I think definitely confidence and being more active in motivating my students, to talk to other students because I go by a lot by how I felt when I was a student. I didn't like teamwork and stuff like that, so I always leave it open so that people don't participate with other people, if they don't want to. I feel like at times, maybe, I'm, I don't know. I do have conflicting feelings because I know they'll have to do it anyways, and maybe, they should start getting used to that. Maybe, I need to push myself too because I'm obviously more comfortable with not pushing myself because, I don't know. I just feel like there's just so many things that I could handle. Sometimes, that's the one that I let go and not pay too much attention to it. Maybe, some of my students do suffer for that in a big class setting.

Valeria was a mathematics education graduate student with several years of university teaching experience. As the above interaction from the pre-T-PD interview suggests, a struggle she faced in her teaching was determining the balance between respecting a student's desire for non-participation (rooted in her own bad experiences with group work) and taking a more active role as an instructor in encouraging students to engage with one another as "they'll have to do it anyways." This issue of balancing her respect for students' agency (even if they choose to not participate) with the authority of her instructor position to ask students to act in certain ways (because it may be good for them) will be at the center of Valeria's case.

Course Context

In the semester Valeria chose to participate in the professional development, she was teaching two sections of a coordinated content course for preservice elementary teachers—a course new to her. Her classes met once a week for 80 minutes, had around 30 students, and were taught online. For the rest of the case, I will be focusing on the section Valeria identified as going better (i.e., had more student participation) to understand what a potential best-case scenario looks like.

Valeria's Motivation, Expectation, and Beliefs

Interviewer: […] What motivated you to join this professional development?

Valeria: Good question. Part of it, it's a good opportunity to learn about positionality. I've heard it a lot, but I don't think I've gone into it deeply. Also, just from a curiosity standpoint, like seeing what's being talked about and all of that was one motivation. Another one is that I always want to be a better teacher. If I can get something from this, I'm more than happy. I also wanted to help you, but that one, it wasn't the main one.

As this interaction from the pre-T-PD interview suggests, reasons for Valeria joining the T-PD were to learn about positionality, to become a better teacher, and wanting to help me. As the latter suggests, Valeria was (and is) a friend of mine—although this friendship did not appear to be the main reason for her joining.

Further, as the start of this section shows, Valeria also had thought about aspects of her teaching she wanted to improve: "definitely confidence and being more active in motivating my students to talk to other students." Yet, Valeria also had some worries about the T-PD: "I'm worried, but earlier, I think I shared this earlier, this is more my insecurities of overthinking it,

and then trying to use those tools in class, makes me self-aware and stuff like that." On top of worrying that she may feel self-aware implementing T-PD tools, she simultaneously wondered about improving her teaching in the online context: "I don't know how long should I be thinking about improving my teaching online. Then, once you're good at it, then you go back to person, and then you weren't ready for improving in person and stuff like that."

Two weeks after the pre-T-PD interview, as part of session 2 of the T-PD, Valeria created a beliefs map (see Figure 7.1) in which she drew heavily on bell hooks's feminist teaching practices and noted, for instance, "trying to not reinforce oppressive practices" and being "compassionate" and "understanding." She described the relationship between her beliefs as "very circular" and "inform[ing] each other," which she represented via the circles in the diagram. In line with this beliefs map, she had explained in the pre-T-PD interview that she saw her teaching role as a helper:

> I've seen it as a helper. I do not see myself as the person that will give anything to another person, it's more like, I am going to help you in whatever it is that you—not in whatever way you want, I'm not your family member—but in terms of what I can do within the structure of the class, that also, I see myself as someone that helps them in math, in particular, but I don't know how to explain it.

In the T-PD, Valeria presented her beliefs map to the group and shared that she had really become interested in Indigenous epistemologies:

> [I]t's very fluid and dynamic and against all of these categorizations that we have and, that's, it's more like, knowledge lives within the experience, so it's fluid. It's not like one knowledge that you got from one experience will like be able to be applicable to something else. So things that like that that even have like made changes in my life

> outside of the classroom and that's why I again, feel like this kind of like fluidity and never like rigidity of like, "Oh, I'm in the classroom, so now I'm a teacher. I'm outside the classroom, I stop being a teacher." So it's still like, within the classroom, I'm still myself and yes, maybe some other ways of being might be a little bit more present, but the others are still there. So because of that, I think for me, it's very important to like, understand myself as well and, just be present. It sounds kind of cliché, but I realized that sometimes we assume that every classroom is the same when it's not. So if you're not present, you don't see what is in front of you, who is in front of you. And that's why also, it's important for me to be open to change.

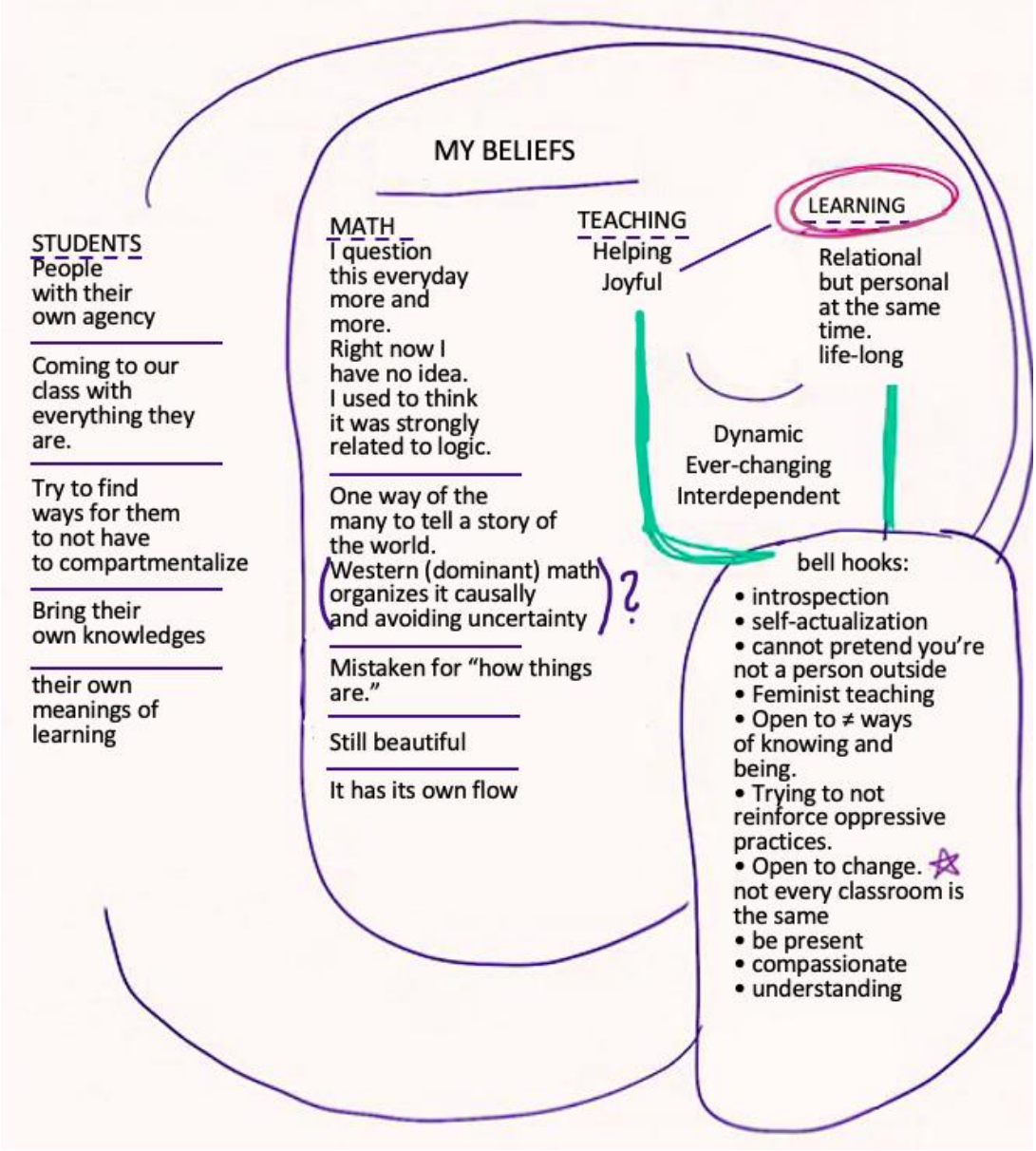
While discussing the belief "Open to change. not every classroom is the same", Valeria highlighted it with a star, thereby denoting it as a belief she struggled to implement. As she explained, "I, honestly, I grew up with ((*laughs a bit*)) a fear of change so that's one that it's been the hardest for me."

Finally, she noted experiencing an existential crisis with mathematics after "reading a lot about what math could be:"

> I just think it's everchanging. It's a way to tell a story about the world. And I do think, though, that now, it's been mistaken for how things are, which I think can be problematic, at times, still beautiful. And it has its own flow.

Figure 7.1

Valeria's Beliefs Map From T-PD Session 2



Note. The beliefs map was handwritten and hand-drawn by Valeria, but I replaced the handwriting with typed writing to protect her anonymity. The typed writing respects Valeria's spelling as well as spacing of words and merely overlays her original handwriting. When sharing her beliefs map with the group, Valeria added the star next to "open to change" in the spirit of Finnegan, marking it as the belief she struggled to implement. She described the relationship

between her beliefs as “very circular” and “inform[ing] each other”, which she represented via the circles in the diagram. The “≠” should be read as “different.”

In summary, Valeria found herself torn. On one hand, she wanted to improve her confidence as an instructor and become better at being open to change. She also identified motivating her students to talk to one another (e.g., in group work)—possibly despite student objections—as a possible aspect of her teaching to improve. At the same time, she did not want to “reinforce oppressive practices” but rather be “compassionate” and “understanding.” In the past, she had made student engagement with other students optional—due to her own negative experiences. She respected that students had their own agency “and their own meaning of what learning is” and saw her role as a “helper.” Concurrently, Valeria was experiencing an “existential crisis” about the ontology of mathematics.

Wielding Authority Versus Respecting Student Agency: CR1–CR5

In this section, I will describe the classroom discourse in Valeria’s class during CR1–CR5, that is, before the TDMs were introduced in the T-PD. I begin by describing her use of TDMs before moving on to the students’ discourse.

Valeria’s Use of Teacher Discourse Moves

Although the TDMs had not been introduced in the T-PD yet, Valeria made use of several TDMs during CR1–CR5. In particular, she used all TDMs except for “asking students to revoice,” which she did not use a single time. In this section, I describe her use of “inviting student participation,” “waiting,” “revoicing,” “probing a student’s thinking,” and “creating opportunities to engage with another’s reasoning.”

Inviting Student Participation

In the pre-T-PD interview, Valeria expressed her hope for student participation: "Participation is a thing that I'm hoping the most for because otherwise I just end up giving a lecture. I cannot stop myself. I hate silence." She noted that she hoped her students would feel comfortable asking questions as "questions are the things that make me the happiest because they imply that they're paying attention, they're thinking about things […] at least they're curious about something and that also helps me to know what they're thinking about." She also hoped that students would answer her and each other's questions.

Asked about how to create a classroom environment in which students feel comfortable asking and answering questions, Valeria was conflicted about wanting to create her desired classroom environment but also not wanting to overthink it. In particular, she worried about ending up like the millipede who forgot how to dance when asked how it dances—her analogy, not mine. She highlighted the importance of being in the present:

> [I]nstead of thinking of what's probably going to happen, or preparing for certain things, it's more like seeing what you have there and work with what you have there. […] I try to come in with the smallest amount of expectations, not in a bad way that I don't expect anything from them, but in a way that, "Let's see what you bring to the table and what we can do with that."

At the same time, she shared that to help her students feel comfortable in her classroom she: (a) tries to tell students who apologize—presumably for getting something wrong or asking a "stupid" question—that there is no need, and (b) shares her own mistakes and struggles.

Table 7.2 shows that Valeria asked for questions (and whether things made sense) multiple times a class, but students rarely responded. Looking at Table 7.7, one can observe that

Valeria's students did not ask many questions in general, with at most three student turns being labeled as "question" (i.e., in CR2 and CR3). In CR1 and CR5, no student turn was labeled as "question." Thus, at the start of the semester, students did not appear to ask many questions during whole-class time. They may, however, have asked their questions during group work time in the breakout rooms, which were not recorded.

Of the seven student turns coded as "question" across CR1–CR5, six were completely answered by Valeria. One student question, however, received a response from a fellow student after Valeria appeared to not immediately know an answer. Group work time in breakout rooms may have offered more opportunities for students to respond to one another's questions.

In terms of Valeria's hope that students would be answering her questions, one can see from Table 7.2 that most of Valeria's what/how/why/other-solicitations received at least one student response. Looking at Table 7.2 one can also note that: (a) in 4 out of 5 classes, what-type solicitations were the most common, (b) the numbers of what, how, and why solicitations varied a lot from class to class, and (c) the total number of what, how, why, and "other" solicitations also varied a lot from class to class.

Table 7.3 shows that Valeria did not engage in explicit specific-student(s)-inviting, that is, she did not call on people by name and she did not call on specific groups. Despite not calling on any specific group, she did ask students during CR2 and CR5 to pick someone to share their respective group's findings. After group work was over, Valeria would typically ask "Does anyone want to share what they […]?" Finally, she also engaged in "inviting further responses" multiple times.

In summary, Valeria created many opportunities for students to ask questions, but students rarely used whole-class time to ask questions. That said, the students responded to most

of Valeria's what, how, and why-type solicitations. In line with Valeria's respect for students' agency, she did not call on any of her students. At the same time, she did invite further responses.

Table 7.2

Valeria's Types of Instructor Solicitations Across CR1–CR5

Types of Instructor Solicitations	Classroom Recording (# of min. spent together as a class)				
	CR1 (63)	CR2 (61)	CR3 (51)	CR4 (54)	CR5 (65)
What	10 (5)	12 (9)	2 (1)	7 (6)	25 (22)
How	1 (1)	11 (6)	4 (4)	3 (2)	2 (2)
Why	0	9 (6)	7 (6)	1 (1)	3 (1)
Questions?	9 (0)	12 (0)	10 (2)	13 (1)	12 (1)
Make sense?	0	7 (2)	4 (2)	4 (2)	2 (2)
Other	0	0	13 (13)	1 (1)	3 (2)

Note. Numbers in parentheses indicate how many of a given instructor solicitation received at least one student response. For example, 5 out of 10 "what"-solicitations received at least one student response in CR1.

Table 7.3

Valeria's Types of Inviting Student Participation Across CR1–CR5

Types of Inviting Student Participation	Classroom Recording (# of min. spent together as a class)				
	CR1 (63)	CR2 (61)	CR3 (51)	CR4 (54)	CR5 (65)
Specific-student(s)-inviting (explicit)	0	0	0	0	0
• Specific-student-inviting	0	0	0	0	0
• Specific-students-inviting	0	0	0	0	0
Inviting further responses	0	3	2	1	5

Probing a Student's Thinking

As can be seen in Table 7.4, Valeria already used "probing a student's thinking" before this TDM's introduction in the T-PD. In particular, she used student-centric deepening-probing once or twice each class after CR1. All her instances of clarifying-probing were recorded on one

day (i.e., CR3). In CR3, Valeria used instructor-centric clarifying-probing extensively to verify that she was following students' constructions of geometric arguments correctly. (Six of the nine instances of clarifying-probing can be traced to a single sequence of interactions with a student about their geometric argument.) The entries in Table 7.7 suggest that probing contributed to students engaging in why-type discourse, but students also found other ways to engage in why-type discourse by responding to why-type solicitations directly or providing a warranted response without prompting.

Table 7.4

Valeria's Instances of Probing a Student's Thinking Across CR1–CR5

Probing a student's thinking	Classroom Recording (# of min. spent together as a class)				
	CR1 (63)	CR2 (61)	CR3 (51)	CR4 (54)	CR5 (65)
Clarifying-probing (CP)	0	0	9	0	0
• CP Instructor-centric (CP-IC)	0	0	9	0	0
○ CP-IC Statement	0	0	8	0	0
○ CP-IC Question	0	0	1	0	0
• CP Student-centric (CP-SC)	0	0	0	0	0
○ CP-SC Statement	0	0	0	0	0
○ CP-SC Question	0	0	0	0	0
Deepening-probing (DP)	0	2	2	2	1
• DP Instructor-centric	0	0	0	0	0
• DP Student-centric	0	2	2	2	1

Waiting

In the pre-T-PD interview, Valeria noted that she used wait time although she "hate[s] silence." She expanded that she does not mind waiting for students to respond—although that took her a while to be okay with. Instead, she struggles with silences in the online setting, which she does not know how to interpret and cause her "a lot of anxiety."

The wait time data (see Figure 7.2) shows that Valeria used wait time 1. One can summarize the findings with respect to wait time 1 as follows:

- Wait time 1a (I-S) W?: Valeria's students responded to her W?-type solicitations after a mean of 4.9 seconds and a median of 2.1 seconds. That said, student responses were also found to come as long as 13.7 seconds after Valeria had made her W?-solicitation.
- Wait time 1a (I-S) Q?: The only recorded instance of this wait time in CR1 & CR2 was 0.3 seconds long. The absence of this type of wait time can be explained by students almost never asking questions in response to Valeria's asking for questions.
- Wait time 1b (I-I) W?: Valeria waited a mean of 6.0 seconds, a median of 4.5 seconds, and a minimum of 2.3 seconds before speaking again after making a W?-solicitation. In other words, in 14 out of 16 instances (i.e., except for when she waited 2.3 and 2.9 seconds), Valeria waited for 3 or more seconds before speaking again—the minimum recommended waiting time suggested by Rowe (1986).
- Wait time 1b (I-I) Q?: Valeria waited a mean of 7.0 seconds, a median of 7.0 seconds, and a minimum of 1.4 seconds. Apart from the two smallest instances of 1.4s and 1.6s, all other 22 instances of Valeria using wait time 1b (I-I) Q? were at least 4.2 seconds long. Thus, in almost all instances, Valeria waited for 3 or more seconds before speaking again—the minimum recommended waiting time suggested by Rowe (1986).

Valeria did not, however, use wait time 2. One can summarize the findings with respect to wait time 2 as follows:

- Wait time 2a (S-I) R!: In all her responses to students' responses, Valeria responded in under 2 seconds. In particular, her mean time for responding to a student's response was 0.8 seconds, her median time was 0.7 seconds, and her maximum time was 1.7 seconds.

- Wait time 2a (S-I) Q?: The results for wait time 2a (S-I) Q? are similar to wait time 2a (S-I) R! with a mean response time to students' questions of 0.5 seconds, a median time of 0.3 seconds, and a maximum time of 1.3 seconds.
- Wait time 2b (S-S): No instance of Valeria using wait time 2b (S-S) was recorded.

In summary, Valeria appeared to make use of wait time 1 (i.e., she was mostly willing to wait for longer than 3 seconds for a student response or question)—sometimes waiting for student responses and questions for over 10 seconds. She did not appear to use wait time 2: All instances of wait time 2a were under 2 seconds and no instances of wait time 2b were recorded.

Revoicing

Valeria used revoicing at the start of the semester. Looking at Table 7.5, one can see that she revoiced a majority of student turns in some way and used full revoicing on three occasions.

Table 7.5

Valeria's Instances of Revoicing Across CR1–CR5

Revoicing	Classroom Recording (# of min. spent together as a class)				
	CR1 (63)	CR2 (61)	CR3 (51)	CR4 (54)	CR5 (65)
Turn simply revoiced	16	24	11	12	40
Turn fully revoiced	0	0	2	0	1
Turn not revoiced	2	9	9	5	7

Creating Opportunities to Engage With Another's Reasoning

As can be seen in Table 7.6, Valeria used "creating opportunities to engage with another's reasoning" in multiple ways. For example, she asked students to evaluate another student's reasoning (i.e., whether they agreed or disagreed). Furthermore, she also once asked the students to make sense of the ideas of a student character from the course textbook. (In the pre-T-PD interview, she had shared that in her previous semester teaching a similar course, students had struggled particularly with tasks asking them to engage with [hypothetical] students' work—

especially if the work did not follow the algorithms her students were familiar with.) Finally, she also implemented group work multiple times each class.

Table 7.6

Valeria's Instances of Creating Opportunities to Engage With Another's Reasoning Across CR1–CR5

Creating Opportunities to Engage With Another's Reasoning	Classroom Recording (# of min. spent together as a class)				
	CR1 (63)	CR2 (61)	CR3 (51)	CR4 (54)	CR5 (65)
Adding to Another's Reasoning	0	0	0	0	0
Evaluating Another's Reasoning	0	2	1	1	2
Understanding Another's Reasoning	0	1	0	0	0
Implementing Group Work	2	3	2	3	2

Group Work. In the pre-T-PD interview, Valeria had expressed her hope that she could figure out how to implement group work via Zoom's breakout rooms. In thinking about breakout rooms, she observed that: (a) students need a prompt for starting good conversations, (b) figuring out how much time to give groups is important, (c) figuring out how much time she should spend in breakout rooms is important (she felt she spent too much time in breakout rooms in the past), (d) figuring out a way for students to show their breakout room work is important (otherwise she might not know whether they are engaged in the work or not), and (e) some groups in the last semester had their cameras and microphones off (despite some people wanting to talk to their breakout rooms). Valeria also noted that "being in the breakout room it's just awkward" and that she is "bad at just starting normal conversation." In the upcoming semester, she intended to follow her course coordinator's recommendation to (a) give groups in breakout rooms a prompt, and (b) have breakout rooms for shorter periods of time. Valeria was also debating with herself whether to send groups into (different) pairs each time as her coordinator recommended. She worried that it might be stressful to talk to someone when there is only one other group member.

She also considered letting students opt out of group work a couple of weeks into the semester. After initially not offering this option in the past (in line with other instructors' implementations), she had had a positive experience with a student who had appreciated and made use of this option after they had stopped attending class because of the social anxiety the group work caused. Valeria described being comfortable offering the opt-out option since she felt not a lot of people chose it, and she did not worry about people taking advantage of the option. Valeria also described second-guessing herself after being in conversation with other GTAs teaching the same course and worrying whether she was the worst.

As already seen in the previous section, Valeria shared that she "go[es] by, a lot by how I felt when I was a student" and had negative experiences with teamwork (to the extent of dropping a class because of it), which is why she thought she might be open to students not participating with one another. To this day, she shared, "I still hate it a bit" because she needs time to understand math (i.e., first do it on her own) and "cannot hear math." Yet, thinking about her students, she felt a tension:

> I do have conflicting feelings because I know they'll have to [participate with other people] anyways, and maybe, they should start getting used to that. Maybe I need to push myself too because I'm obviously more comfortable with not pushing myself […] Sometimes that's the one that I let go and not pay too much attention to it. Maybe, some of my students do suffer for that, like in a big class setting.

Yet, Valeria wondered how long she should be thinking about improving her teaching online given that classes would be in-person again in the future. She also pointed to how little time there was in a semester to create a positive dynamic (of students talking to one another).

Although I could determine how much time was spent in group work in each class (i.e., between 15 and 29 minutes [out of an 80-minute class]), I did not know how much time was spent in each instance of group work during a given class. Calculating an estimate by dividing the entire time spent in group work by the number of group work instances, one can see that group work lasted on average 8:50 minutes.

Aspects of Valeria's group work implementation included asking students to document their work on Jamboards and giving students the option to stay in the main room instead of joining a breakout room in CR2 (i.e., offering an opt-out option for group work). Before she sent groups into breakout rooms, she would announce the activity or question she wanted the students to work on, sometimes providing motivation (e.g., "So now, let's see how we can use [the diagrams] to, to use, to work on the categories that we worked on for the slides") or more specific information (e.g., "For now focus just on number one and number two. If you're still in your breakout room and you are done with those, then you can move on to three and four").

She randomized breakout rooms during CR1 and CR2, but after discussing her implementation of group work with the students in CR2, several students suggested keeping groups fixed. Valeria appeared to take this suggestion to heart, noting:

> Okay, perfect, then definitely, I think. So today will be the last day that you'll have a random breakout room. And then after that, it'll be the same one, at least until well, the semester is really short, I think maybe until the end of the semester. So yeah, definitely. I'll do that.

It is unclear how she formed groups during CR3–CR5.

As aforementioned in the discussion of inviting student participation, Valeria asked students during CR2 and CR5 to pick someone to share their respective group's findings. Yet,

after group work ended, she did not call on any specific group, preferring to typically ask "Does anyone want to share what they […]?"

Student Discourse in Valeria's Class

Table 7.7 summarizes the student discourse dimension codes across CR1–CR5. From these frequencies, one can several observations about the discourse in Valeria's class during whole-class time.

Students communicated both via speech and writing, but there is no clear pattern whether students mostly communicated via one or the other. Students made use of both the public and the private chat features, with the majority of chat messages being sent publicly.

Although several student turns were about logistics, most classroom discourse was about mathematics. Setting aside the first day of class (in which 25 minutes of class time were dedicated to discussing the syllabus), the number of student turns coded as "discourse about mathematics" varied widely from 18 (i.e., CR4) to as many as 48 (i.e., CR5) (or from an average of one student turn every 3 minutes to one student turn every 1:22 minutes).

Student talk length was quite varied, with some classes having very short student turns (i.e., CR1) to some having longer student turns (i.e., CR3) and some in-between (i.e., CR2, CR4, CR5). Apart from CR1, at least 2 student turns were longer than 20 words each class.

Looking at the student talk type, one can notice that looking across CR1–CR5, most student discourse was of the what-type. That said, students also engaged in why-type discourse each class. How-type discourse appeared to be limited to CR1–CR3, and, as touched upon in an earlier section, students asked a small number of questions—if at all (during whole-class time). Further, the teacher solicitation type codes suggests that most student turns came in response to what-type solicitations.

Finally, it can be noted that Valeria evaluated almost all student turns. Looking across the ten turns she did not immediately evaluate, one can note that these occurred when she instead invited further responses (6 times), asked students if they agreed or not with a given response (2 times) (i.e., “creating opportunities to evaluate another’s reasoning), and did not know the answer or did not respond (4 times). (The numbers do not add up to ten since the two instances of “creating opportunities to evaluate another’s reasoning” were also coded as “inviting further responses.”)

Table 7.7

Valeria: Student Discourse Dimensions Across CR1–CR5

Student Discourse Dimensions	Classroom Recording (# of min. spent together as a class)				
	CR1 (63)	CR2 (61)	CR3 (51)	CR4 (54)	CR5 (65)
Mode of Discourse	39	53	24	24	60
• Speech	4	27	14	10	20
• Writing	35	25	8	10	38
○ Public chat	21	14	6	10	31
○ Private chat	14	11	2	0	7
• Body	0	1	2	3	2
• Intermodal	0	0	0	1	0
Discourse Type	25	42	22	24	53
• Discourse about mathematics	18	33	22	18	48
• Discourse about logistics	7	9	0	6	5
Student Talk Length	18	33	22	18	48
• 1–4 words	15	14	1	6	28
• 5–20 words	3	9	9	4	16
• 21+ words	0	9	9	5	2
• n/a	0	1	3	3	2
Student Talk Type	18	33	22	18	48
• what	15	16	4	10	45
• how	2	5	4	0	0
• why (post-probing)	1 (0)	9 (1)	5 (0)	4 (2)	2 (1)
• question	0	3	3	1	0
• other	0	0	6	3	1
Teacher Solicitation Type	18	33	22	18	48
• what	15	15	1	11	40
• how	3	6	3	1	2
• why	0	7	4	2	1
• other	0	0	4	4	2
• n/a (unsolicited)	0	5	10	0	3
Explicit Evaluation	18	33	22	18	48
• Yes	18	29	13	14	38
• No	0	1	2	1	6
• n/a	0	3	7	3	4

Wielding Authority Versus Respecting Student Agency: CR6–CR11

> I noticed that the things that I grab onto the most are things that I can do, or like, that rely more on what I'm doing than on what my students are doing. So, for example, waiting times, I can do that. I can wait a little bit longer, and I can wait after my students, respond. And then for example, I can also revoice my students' answers, so that I have been trying. I think it's when it involves me asking someone else to do something when I'm like a little bit insecure of trying that out, because of the fear of how it, how they might respond. [...] I think I have a very hard time, finding ways, to make people do things, without it sounding like I'm forcing them.

As this quote from the mid-T-PD interview with Valeria illustrates, she felt more comfortable with implementing TDMs that were centered on her (e.g., waiting and revoicing) than TDMs that asked things of students. In line with her pre-T-PD interview, she identified her own emotions (i.e., fear) and her teaching philosophy (i.e., not using force) as reasons that made using TDMs that asked students to do things more difficult.

In the following subsections, I will discuss Valeria's implementations of the TDMs during CR6–CR11 after the TDMs had been introduced in the T-PD. Echoing the distinction Valeria made, I will first discuss those TDMs that Valeria identified as being more centered on her before discussing those that she felt relied on asking things of students. I will end with a subsection on the student discourse in Valeria's class.

Valeria-Centered TDMs

Valeria identified three TDMs as being more centered around her: waiting, revoicing, and probing a student's thinking. Below, I discuss each of these.

Waiting

In session 6 of the T-PD, the T-PD participants had the opportunity to listen to their classroom audio-recordings and study their classroom transcripts. Valeria's observations about the classroom discourse included the following:

> I did notice that the waiting time that I thought I was giving felt like way longer, like when I'm like, like waiting during class. And now that I'm hearing it is not as long as I thought. So maybe I just need to like, just live in silence. I don't know, I feel like I thought that I had it longer. And now, I don't know if maybe it was just because it was the first the first weeks. But I did notice that. […] But, yeah, I also noticed that I don't wait after someone answers. For sure. I always just, I just jump in, it's like, "Great. Yes, that's exactly, what I wanted you to say." So I think I need to work on that.

As this quote indicates, Valeria felt that she did not use wait time 1 for as long as she thought and that she did not use wait time 2.

At the mid-T-PD interview, Valeria spoke more about wait time and identified it as one of the TDMs she gravitated towards since waiting was something she could do, rather than something that relied more on what students were doing. She reported having used wait time and having found it useful. She noted the importance of being comfortable with wait time because she did not want her students to feel like she was uncomfortable waiting. She also highlighted wanting her students to not feel anxious about nobody talking but, instead, realize that they had time to answer.

With respect to wait time 2, she noted that it was a new idea for her and that she liked it. Yet, she added that it would take her some time to figure out how to use wait time 2. She noted that wait time 2 gave her the opportunity to not jump in immediately after a student's response

(the pattern she had noticed in session 6 while analyzing a classroom audio-recording of hers) and instead take time and ask "Okay, does anyone want to add to that?"

In the post-T-PD interview, Valeria noted that wait time helped even if it appeared not to. That is, even when she felt she waited for "too long," the second time she asked a question, someone would answer. Indeed, of the twelve what/how/why-type questions she asked during CR1 and CR2 that received no answer, ten were answered after being re-asked—the other two received responses after being re-asked several times. Similarly, of the nine what/how/why-type questions Valeria asked during CR10 and CR11 that received no answer, eight were answered after being re-asked—the final question received a response after being asked a third time.

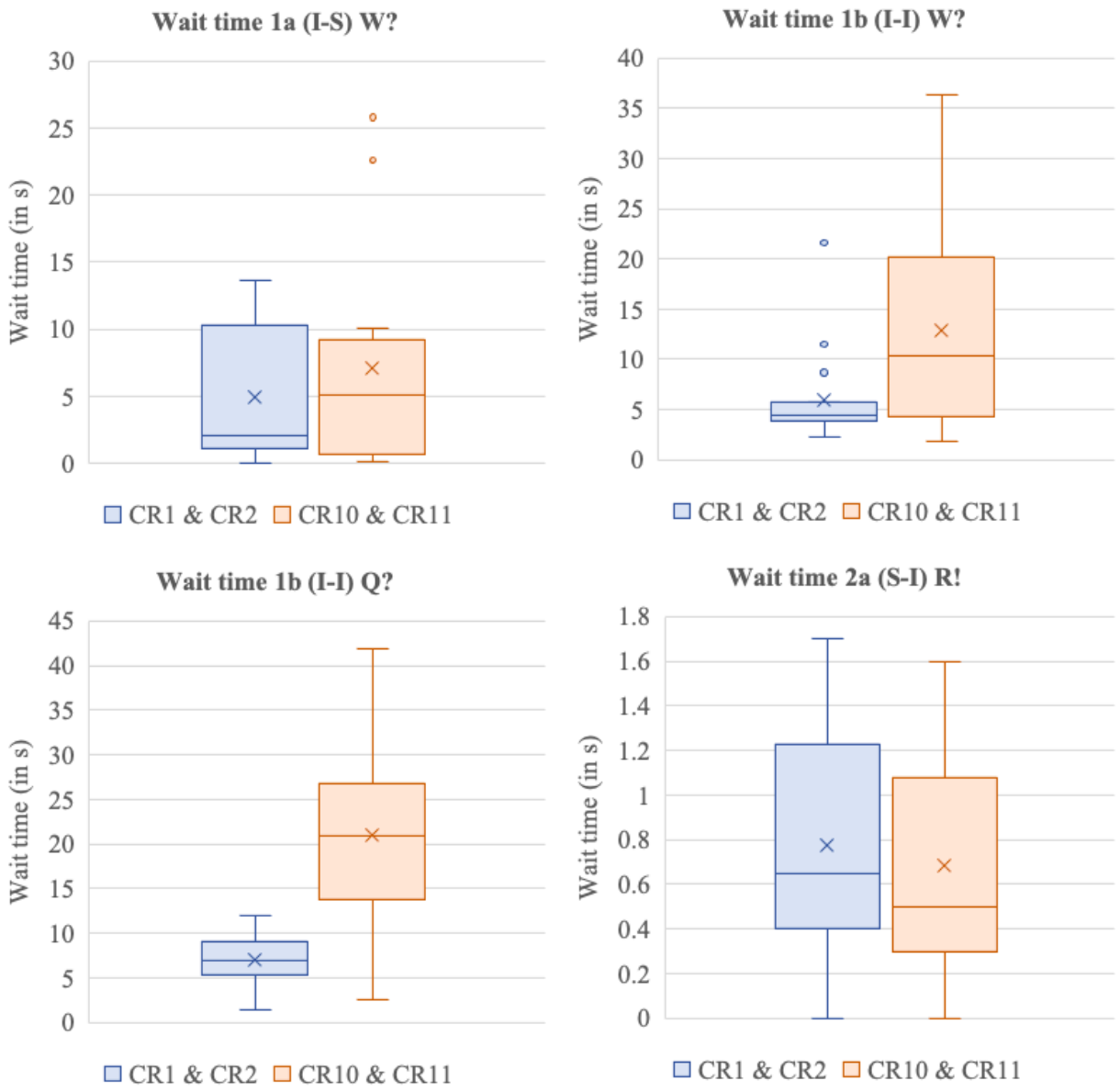
Figure 7.2 shows that Valeria appeared to use wait time 1 for longer at the end of the semester. In particular, studying the graphs for wait time 1b (I-I) W? and wait time 1b (I-I) Q?, one can notice the increase in mean, median, and maximum wait times. In other words, the graphs for wait time 1b suggest that, overall, Valeria waited longer for students' responses and questions at the end of the semester than at the start.

In the post-T-PD interview, Valeria also reaffirmed liking wait time 2, noting she had sometimes not realized that students were not done talking and that wait time 2 made things "less abrupt." Yet, it is unclear from Figure 7.2 whether Valeria's wait time 2 increased over the course of the semester. All instances of wait time 2a (S-I) R! in CR10 and CR11 lasted 1.6 seconds or less and, accounting for errors, the start- and end-of-semester boxplots for wait time 2a (S-I) R! are near identical. Furthermore, no instance of wait time 2a (S-I) Q? was recorded in CR10 or CR11, meaning a start- and end-of-semester comparison of wait time 2a (S-I) Q? is not possible. Finally, like at the start of the semester, no instance of Valeria using wait time 2b was

recorded. Thus, studying Valeria's wait time during CR10 and CR11 was insufficient to document her reported use of wait time 2.

Figure 7.2

Wait Time Comparison: CR1 & CR2 Versus CR10 & CR11



Note. The cross on a boxplot marks the mean; the line splitting the interquartile range marks the median. The mean and median of the boxplots, given in the format

((mean$_{CR1\&CR2}$, median$_{CR1\&CR2}$),(mean$_{CR10\&CR11}$, median$_{CR10\&CR11}$)) and read left-to-right, top-to-bottom, are: ((4.9, 2.1),(7.1, 5.2)), ((6.0, 4.5),(12.8, 10.3)), ((7.0,7.0),(21.0,20.9)), and ((0.8, 0.7),(0.7, 0.5)) (given to the nearest tenth of a second). Boxplots for other forms of wait time are omitted due to insufficient data. The number of data points each boxplot reflects (from left to right, top to bottom) are: 20, 14, 16, 19, 24, 26, 20, and 20.

Revoicing

During the T-PD session 7 check-in, Valeria shared the following:

> I've been practicing also a lot revoicing what students said before, and I always remember what [Alice] said that maybe she should like revoice what is written on the chat, so I've been reading those too. And I feel like it gives voice to like the people that do want to participate but, they do it like in a written form, and that way they can hear that, yes, this is a contribution and yes, it's part of the class. So I've been doing that.

Less than a week after T-PD session 7, in the mid-T-PD interview, Valeria identified revoicing as a TDM that she had been trying out and that she grabbed onto more than other TDMs because it relied more on what she was doing than on what her students were doing. She added that the helpfulness of revoicing depended on the day for her, since if she was feeling "spacey that day," her revoicing may have just ended up confusing students more. She reported using revoicing both in her office hours and in class and explained several ways in which she revoiced: (a) revoicing while drawing or taking tiny notes and (b) using (full) revoicing to make sure she understood what a student said. She also identified two reasons for revoicing: (a) to ensure she knows what a student said, and (b) to keep students from feeling the pressure of having to repeat their answers.

In the post-T-PD interview, Valeria noted that she had used revoicing during the semester. She reflected that she liked the checking-in aspect of full revoicing and found it useful in two regards: (a) it allowed her to give credit to students' ideas, and (b) it ensured that she confirmed what students meant and did not do a disservice to their ideas.

The entries in Table 7.8 indicate that as the semester progressed, Valeria may have engaged in more revoicing and used full revoicing twice across CR6–CR11.

Probing a Student's Thinking

In the mid-T-PD interview, Valeria noted that she "love[d]" probing. She expressed that this TDM was easier for her to use because she felt it centered more on her since it involved her trying to understand what a student was saying—possibly indicating that she was thinking of clarifying-probing in particular. She shared that "probing questions help me a lot to figure out how to continue with things." She described probing a student's thinking for two reasons: "I usually try to, probing questions, one, to know what they're thinking, and then two, also to make sure that they keep remembering what is it that we want, don't forget. Or to emphasize important things." She added:

> Probing is very helpful for me, very useful, yes. I find questions very helpful because I don't want to assume anything, so I like asking questions to make sure. Probing usually, well not usually, but a lot of times shows me wrong. I sometimes have an assumption of what might be the misunderstanding, so once I probe it then I'm like, "Oh, that was not it at all." I prefer to do that over assuming things.

During the post-T-PD interview, Valeria alluded to having used probing a student's thinking (in a clarifying manner). She appeared to have several positive experiences with using it

(i.e., students responded by "say[ing] it again or explain[ing] it in a different way"). Valeria also described having used full revoicing—in a way akin to instructor-centric clarifying-probing.

The data on Valeria's use of "probing a student's thinking" in Table 7.8 show some isolated uses of clarifying-probing (all but one instructor-centric) and occasional uses of deepening-probing (mostly student-centric). In CR6–CR11, the amount of deepening-probing appeared to slightly decrease in comparison to CR1–CR5 and, for the first time, Valeria used deepening-probing in an instructor-centric manner—three of Valeria's six instances of using deepening-probing in CR6–CR11 were instructor-centric.

Student-Centered TDMs

In this subsection, I will discuss Valeria's use of TDMs that she identified as being more centered on students, that is, TDMs that asked things of students: inviting student participation, asking students to revoice, and creating opportunities to engage with another's reasoning. As will be seen below, Valeria expressed struggling with asking students to revoice and creating opportunities to engage with another's reasoning, as well as with aspects of inviting student participation.

Inviting Student Participation

In the mid-T-PD interview, Valeria shared that she invited student participation "in a very passive way" by giving students the option to participate rather than forcing them, and she described forcing someone to participate, for example by calling on them, as "something that is not me." In line with the observations made in CR2 and CR5, Valeria noted that before groups joined breakout rooms, she would ask them to pick someone to share each group's findings. Yet, she added, she would not call on them later, instead preferring to ask, "Who wants to share what

they talked about?" which she called "a more open invitation." She noted, "I have a very hard time finding ways to make people do things without it sounding like I'm forcing them."

Valeria also described the complexities of inviting student participation in the mid-T-PD interview, observing that asking students questions requires not just thinking about the words you use:

> […] it's also the tone, the face, the way that you say it, if it seems that you're feeling comfortable saying it or if you're just like kind of tense. I think that all of those things are very hard for me. That's what I mean that I go into my mind I become self-conscious like, "Oh my God, what do I do with my hands? What do I do with my eyes? Are my eyebrows too high? Should I like start?" ((*Valeria laughs*)) I don't know. "Am I smiling too much?" That's where I need to practice so that it's just something that does come naturally and it's not like, "Okay, now I should ask, 'What do you think?'"

Finally, Valeria reflected that "the PD has made me more aware of whether I do truly, like ask students to participate or if I give up too quickly" citing a difference between her experienced reality and rational reality: "[I]n my mind, I'm like, 'Oh my God, I asked so many questions and nobody answered,' but I just asked like one and waited for one second or something."

In the post-T-PD interview, Valeria reflected about the semester that she "wasn't as active in making more students talk or feel comfortable in talking." Furthermore, although she noted trying out some things during the T-PD, "I definitely knew that I was not going to try others like calling on people or things that I know that I am not comfortable with […] that I wouldn't know how to handle with my way of being." Accordingly, looking at Table 7.9, one can see that Valeria did not use explicit specific-student(s)-inviting at all during the semester.

Similar to her discourse from CR1–CR5, this lack of specific-student(s)-inviting also meant that although she asked groups to select a group member to present their group's findings (i.e., CR2, CR5, CR6, CR8), she did not call on them. Valeria noted that instead of calling on students, she had invited student participation in a "very general" way. A way in which this general inviting may have expressed itself is in Valeria's consistent use of inviting further responses (see Table 7.9), which she used slightly more often in the second half of the semester than the first.

Reflecting on the semester, Valeria noted that although she tends to start semesters strong and tries to get students to participate, as the semester goes on, her energy drops, and she starts to not go outside her comfort zone:

> If nobody speaks, then I'll just talk. If there's no participation, then yes, I'll just make sure that they know what they have to know for the quiz, and then that'll be the class. That's when I start feeling like a very crappy teacher.

That said, during the semester in question, one can see from studying Table 7.9 that most what/how/why-type solicitations received at least one student response without Valeria repeating or rephrasing them. Combining this with the earlier observation (from the wait time subsection) that students typically answered any unanswered question after the question was repeated or rephrased (possibly multiple times), one can note that students in Valeria's class answered her what/how/why-type solicitations eventually—mostly without Valeria needing to repeat or rephrase her solicitation. Thus, perhaps because student participation did not drastically change (in terms of responding to Valeria's solicitations), neither did the average number of what/how/why-type solicitations Valeria asked. Although there was substantial fluctuation from class to class, comparing the average number of what/how/why-type solicitations asked across CR1–CR5 to those asked across CR6–CR11 (adjusting for class length), one can see that in the

first half of the semester (i.e., CR1–CR5), Valeria made a what/how/why-type solicitations on average every 3:02 minutes and in second half of the semester (i.e., CR6–CR11) on average every 3:03 minutes.

Table 7.9 also shows that the numbers of what, how, and why-solicitations substantially fluctuated from class to class. Thus, no pattern emerged except a decline in why-type solicitations towards the end of the semester. What Table 7.9 does highlight is that Valeria frequently asked for students' questions throughout the semester. These requests were, however, mostly met with silence (see Table 7.9). Further, one can see that students did not simply ask questions unprompted—Table 7.11 shows that across CR6–CR11, only two student turns were questions. Thus, students rarely asked questions during whole-class time and had fewer question-turns across CR6–CR11 (i.e., two) than across CR1–CR5 (i.e., seven).

In summary, Valeria invited student participation in a "very general" way. As part of this, she did not call on students. Neither did she call on groups to share their findings—even when she had asked each group to select a group member to be ready to share the group's findings. Instead, she used inviting further responses. She highlighted the complexities of inviting student participation and shared that the T-PD had helped her be more aware of whether she truly invited student participation or gave up too quickly. Looking at the classroom data, one can see that: (a) the average number of solicitations stayed roughly the same across the two halves of the semester, (b) students responded to most of Valeria's what/how/why-type solicitations—and to virtually all what/how/why-type solicitations eventually, (c) Valeria consistently asked for questions to which students typically did not respond, and (d) even fewer student turns were questions in the second half of the semester than in the first.

Asking Students to Revoice

At the start of T-PD session 7, I asked Valeria whether she had asked students to revoice. She shared:

> No, I've been, I've been thinking about it. ((*sighs*)) I just I don't want to feel that silence yet, so I'm preparing myself for ((*laughs*)) for no one to answer on that one, but I'll, I'll try it next week.

A week later, during the mid-T-PD interview, Valeria shared not knowing how to use "asking students to revoice" and worried about: (a) it feeling similar to calling on someone (which she described as something that was not "her"), (b) having never asked students to revoice one another before, and (c) not knowing what tone to use when asking. Despite these worries, she reiterated that she intended to try this TDM in the week's class. In the post-T-PD interview, Valeria reported that she used "asking students to revoice" and that it "definitely failed" because she was met with silence. She noted that she gave up on trying to ask students to revoice, observing that "it was already hard to make them say what they were thinking, let alone talking about what somebody else said."

Looking at Table 7.8, one can note that Valeria tried to use "asking students to revoice" a total of three times: twice in CR9 and once in CR11. Of these three times, two received no student response—despite ample wait time (i.e., 12.4 and 24.3 seconds respectively). The one instance in which Valeria received a response was the following interaction:

> **Valeria:** No, it makes, ah well, at least for me it makes all the sense. Yeah. Uhm. Does, uhm, yeah does anyone wanna like, say what they understood, uh, from Caitlyn's, uh, strategy, which I really, I really like. ... Then I, I wrote some like guiding points here, if, in case you forgot. Uhm.

Janet (in chat): I used the same strategy

Thus, we can see that even in this one instance in which Valeria received a response to "asking students to revoice," the responding student (Janet) did not actually engage in revoicing the previous student's (Caitlyn's) strategy. Despite the move not having the intended effect, Valeria still saw something positive about this interaction in the following T-PD session: "[I]n a sense, they were like, 'I heard what they said, and I did the same thing.' So it was kind of like, okay, at least I know someone is listening to what other students are saying."

Creating Opportunities to Engage With Another's Reasoning

In the mid-T-PD interview, Valeria reported struggling with "creating opportunities to engage with another's reasoning." She noted that her version of this TDM was having breakout rooms. Looking at Table 7.8, one can see that in line with this, she implemented group work throughout the semester between once and thrice each class.

During the interview, Valeria also agreed with my assessment that the course contained tasks in which students were asked to try to understand hypothetical students' thinking. Yet, "[students] don't like it, but we are trying to incorporate it more." Valeria then drew a connection to the "perfect no," an idea introduced by one of her professors (i.e., showing a student's answer and then asking the students to grade it). Although she had not tried out using the perfect no, she had the intention. Table 7.8 shows that Valeria used "creating opportunities to understand another's reasoning" sporadically—possibly a bit more after the TDMs were introduced.

In the mid-T-PD interview, Valeria also shared "tr[ying] in the discussion to say after someone participates, I was like, 'Does anyone want to add to that, or has any thoughts about what was just said?'" which was "usually" met with silence. "I have incorporated that question, but there hasn't been responses yet." She highlighted that wait time 2 helped her with this TDM

since it offered her the possibility to pace herself and respond to a student's response with "Okay, does anyone want to add to that?" instead of jumping into "'yes' or 'almost,' or whatever I say after that." In the post-T-PD interview, Valeria called "creating opportunities to engage with another's reasoning" her "pitfall":

> I don't think I did that very well. That one was also met with silence. If a student shared their thinking and then I asked, "Does everyone agree with it or does anyone want to add to that or have any ideas to share?" It was just silence I think. That one didn't go very well.

She then amended that "it probably worked once or twice." Although she couldn't remember specific instances where it worked, she suspected that it may have worked "like when there was a mistake or something." Again, from looking at Table 7.8, one can see that Valeria tried "creating opportunities to add to another's reasoning" after the TDMs were introduced and used it a total of eight times in the semester spread across three class days (i.e., CR6, CR7, CR11). In line with her experience of "usually" being met with silence, students only made use of this opportunity to add on to another's reasoning twice (once in CR7 and CR11 each).

In addition to the above three observations (i.e., Valeria's continual use of group work, a sporadic use of "creating opportunities to understand another's reasoning," and mostly-met-with-silence uses of "creating opportunities to add to another's reasoning"), Table 7.8 also shows that Valeria continued to use "creating opportunities to evaluate another's reasoning" once or twice each class as she had done starting with CR2.

Group Work. As mentioned above, Valeria noted in the mid-T-PD interview that having breakout rooms served as her way of "creating opportunities to engage with another's reasoning." Table 7.8 shows that she implemented group work between once and thrice each

class in the semester. Further, although she asked groups—before sending them to breakout rooms—to pick someone who could share the group's findings, she reported that she would not call on them, instead preferring to ask, "Who wants to share what they talked about?" which she called "a more open invitation." Table 7.9 shows that Valeria did not call on groups and their representatives a single time during the semester.

Reflecting on her start-of-semester intentions for following her coordinator's group work suggestions, Valeria reported in the mid-T-PD interview that she had tried giving her breakout rooms prompts, which had "made the class flow." She intended to "try randomizing groups now because a lot of students were saying that not everyone talked in the breakout rooms and that was rough." Further, she had chosen to offer the opt-out option for group work, which "in one class, a couple and in the other one, nobody" made use of.

Valeria also expressed how teaching online made implementing group work more challenging for her. Specifically, Valeria observed that "Zoom has been hard for me" and that she feels she has been helping less than she used to:

> Maybe I'm idealizing what I used to do when I was in person, but I think that if we were in a classroom, I would definitely approach groups and like try to help them and all of this. But now I have this huge anxiety when I go into breakout rooms that I just either avoid them or just stay there for like a second and then leave (or) I become really awkward. I think that helping part is lacking now for me, and that's been rough because I'm not used to being like that.

She elaborated that she views help "as just knowing where my students are and what I can do to ((*Valeria's voice trails off*))" and does not feel like she knows a lot about her students which means she does not know what to do.

In the post-T-PD interview, after being asked about things she was still thinking about regarding her teaching and might want to try the future, Valeria noted, "[s]omething that has always been hard for me is like the group work, so maybe if it's still something that I feel like could be beneficial, maybe I would try to work on that." She added:

> [M]ost of the things that I want to work on in my teaching is more about myself, which sounds self-centered, but it's more like being comfortable, because I feel like if I'm comfortable then […] so many things open up in how you can like relate to your students, like the things that you could do in the moment instead of having to plan. […] Mindset, I feel is so important in teaching. I feel like I would definitely want to work on being less awkward with group work, because I don't think it helps that your teacher seems very uncomfortable with group work. Yes. That would be, yes, something I would want to work on.

Student Discourse in Valeria's Class

Table 7.10 and Table 7.11 summarize the student discourse dimension codes across the entire semester. These frequencies support a handful of observations about the discourse in Valeria's class during whole-class time.

Students continued to communicate both via speech and writing. In the second half of the semester, there were typically more written student turns than spoken student turns. Students made use of both public and private chat features, with the majority of student turns continuing to be sent via public chat.

As in the first half of the semester, despite some student turns being about logistics, most student turns were about mathematics. Although the number of turns that were about

mathematics continued to fluctuate from class to class, there was less fluctuation than in the first half of the semester.

Student talk length continued to be quite varied, but student turns may have been longer on average in the second half of the semester (with respect to the categories of student talk length excluding “n/a”)—both written and spoken turns.

Looking at the student talk type, one can note a substantial drop in why-type talk in the second half of the semester as well as student questions growing even rarer. In line with the former, there was a substantial drop in why-type teacher solicitations in the second half of the semester. Patterns with regard to what- and how-type student talk or what- and how-type instructor solicitations were not evident to me.

Finally, Valeria continued to evaluate almost all student turns—possibly slightly more so in the second half of the semester

Table 7.8

Valeria's Use of TDMs Across CR1–CR11 (Part 1)

Instructor Discourse Dimensions	Classroom Recording (# of min. spent together as a class)										
	CR1 (63)	CR2 (61)	CR3 (51)	CR4 (54)	CR5 (65)	CR6 (58)	CR7 (74)	CR8 (57)	CR9 (64)	CR10 (70)	CR11 (61)
Turn simply revoiced	16	24	11	12	40	21	18	12	16	9	26
Turn fully revoiced	0	0	2	0	1	0	0	1	0	1	0
Turn not revoiced	2	9	9	5	7	4	1	3	0	0	1
Clarifying-probing (CP)	0	0	9	0	0	0	0	1	0	2	0
• CP Instructor-centric (CP-IC)	0	0	9	0	0	0	0	1	0	1	0
○ CP-IC Statement	0	0	8	0	0	0	0	1	0	1	0
○ CP-IC Question	0	0	1	0	0	0	0	0	0	0	0
• CP Student-centric (CP-SC)	0	0	0	0	0	0	0	0	0	1	0
○ CP-SC Statement	0	0	0	0	0	0	0	0	0	1	0
○ CP-SC Question	0	0	0	0	0	0	0	0	0	0	0
Deepening-probing (DP)	0	2	2	2	1	0	0	3	1	2	0
• DP Instructor-centric	0	0	0	0	0	0	0	1	0	2	0
• DP Student-centric	0	2	2	2	1	0	0	2	1	0	0
Creating opportunities to engage with another's reasoning											
• Adding	0	0	0	0	0	3	3	0	0	0	2
• Evaluating	0	2	1	1	2	2	2	2	1	2	1
• Understanding	0	1	0	0	0	1	0	0	3	0	1
• Group work	2	3	2	3	2	2	1	2	2	1	2

Table 7.9

Valeria's Use of TDMs Across CR1–CR11 (Part 2)

Instructor Discourse Dimensions	Classroom Recording (# of min. spent together as a class)										
	CR1 (63)	CR2 (61)	CR3 (51)	CR4 (54)	CR5 (65)	CR6 (58)	CR7 (74)	CR8 (57)	CR9 (64)	CR10 (70)	CR11 (61)
Types of Instructor Solicitations	20	51	40	29	47	39	35	37	31	33	42
• What	10 (5)	12 (9)	2 (1)	7 (6)	25 (22)	17 (13)	6 (5)	12 (7)	3 (3)	7 (6)	21 (13)
• How	1 (1)	11 (6)	4 (4)	3 (2)	2 (2)	0	14 (9)	9 (5)	5 (5)	5 (5)	4 (3)
• Why	0	9 (6)	7 (6)	1 (1)	3 (1)	8 (6)	2 (1)	0	2 (2)	1 (1)	0
• Questions?	9 (0)	12 (0)	10 (2)	13 (1)	12 (1)	13 (0)	10 (0)	10 (0)	12 (2)	15 (0)	14 (2)
• Make sense?	0	7 (2)	4 (2)	4 (2)	2 (2)	0	3 (0)	6 (1)	7 (3)	0	0
• Other	0	0	13 (13)	1 (1)	3 (2)	1 (1)	0	0	2 (1)	5 (3)	3 (0)
Specific-student(s)-inviting (explicit)											
• Specific-student-inviting	0	0	0	0	0	0	0	0	0	0	0
• Specific-students-inviting	0	0	0	0	0	0	0	0	0	0	0
Inviting further responses	0	3	2	1	5	5	7	5	3	4	3
Asking students to revoice	0	0	0	0	0	0	0	0	2	0	1

Table 7.10

Valeria: Student Discourse Dimensions Across CR1–CR11 (Part 1)

Student Discourse Dimensions	Classroom Recording (# of min. spent together as a class)										
	CR1 (63)	CR2 (61)	CR3 (51)	CR4 (54)	CR5 (65)	CR6 (58)	CR7 (74)	CR8 (57)	CR9 (64)	CR10 (70)	CR11 (61)
Mode of Discourse	39	53	24	24	60	25	22	25	16	16	36
• Speech	4	27	14	10	20	15	10	3	6	8	2
• Writing	35	25	8	10	38	8	12	21	7	8	34
○ Public chat	21	14	6	10	31	8	9	14	7	4	31
○ Private chat	14	11	2	0	7	0	3	7	0	4	3
• Body	0	1	2	3	2	2	0	0	3	0	0
• Intermodal	0	0	0	1	0	0	0	1	0	0	0
Discourse Type	25	42	22	24	53	25	19	18	16	12	33
• Discourse about mathematics	18	33	22	18	48	24	19	16	16	10	27
• Discourse about logistics	7	9	0	6	5	1	0	2	0	2	6
Student Talk Length	18	33	22	18	48	24	19	16	16	10	27
• 1–4 words	15	14	1	6	28	11	7	6	1	1	15
• 5–20 words	3	9	9	4	16	7	8	5	7	5	9
• 21+ words	0	9	9	5	2	5	4	5	5	4	3
• n/a	0	1	3	3	2	2	0	0	3	0	0

Table 7.11

Valeria: Student Discourse Dimensions Across CR1–CR11 (Part 2)

Student Discourse Dimensions	Classroom Recording (# of min. spent together as a class)										
	CR1 (63)	CR2 (61)	CR3 (51)	CR4 (54)	CR5 (65)	CR6 (58)	CR7 (74)	CR8 (57)	CR9 (64)	CR10 (70)	CR11 (61)
Student Talk Type	18	33	22	18	48	25	19	16	16	10	27
• what	15	16	4	10	45	16	9	8	1	5	25
• how	2	5	4	0	0	0	10	5	4	5	2
• why	1	9	5	4	2	7	0	0	5	0	0
• question	0	3	3	1	0	1	0	0	1	0	0
• other	0	0	6	3	1	1	0	3	5	0	0
Teacher Solicitation Type	18	33	22	18	48	25	19	16	16	10	27
• what	15	15	1	11	40	16	7	7	3	5	25
• how	3	6	3	1	2	0	10	6	4	4	2
• why	0	7	4	2	1	7	0	0	2	1	0
• other	0	0	4	4	2	1	0	1	7	0	0
• n/a (unsolicited)	0	5	10	0	3	1	2	2	0	0	0
Explicit Evaluation	18	33	22	18	48	25	19	16	16	10	27
• Yes	18	29	13	14	38	23	19	11	10	10	23
• No	0	1	2	1	6	0	0	2	2	0	3
• n/a	0	3	7	3	4	2	0	3	4	0	1

My Interpretation of Valeria's Case

As this case report sought to show, over the course of Valeria's participation in the T-PD, she was weighing the wielding of her authority to engage students against her respect for students' agency (even if students used their agency to not participate). This balancing act was mediated by numerous emotions: a desire to be more confident and open to change, anxiety and awkwardness in certain classroom situations (e.g., when entering breakout rooms, when the class is silent), self-awareness while implementing T-PD tools, self-consciousness about asking students questions (e.g., about her tone, face, gestures, way of saying the question), and a skepticism of the long-term utility of improving online instruction. Given that the balancing of, on one hand, deciding for students what is right for them and, on the other, letting them decide for themselves is by no means unique to Valeria, it is important to understand which TDMs she took up and which she did not. Below, I offer summaries of Valeria's use of and discourse about each TDM alongside my interpretations.

Waiting

Valeria came into the T-PD knowing about—and reportedly using—wait time 1. Yet, she also admitted to "hat[ing]" silence and struggling with it a lot in the online setting as it caused her "a lot of anxiety." The start-of-semester data showed Valeria using wait time 1 (i.e., she was willing to wait on average over 3 seconds for student responses or questions) but not wait time 2.

Listening to her classroom audio-recording in the T-PD appeared to be a powerful experience for her as part of which she noticed that her wait time 1 was not as long as she thought and her wait time 2 not present. Combined with the T-PD discussions, this observation may have contributed to her using wait time 1 longer (on average) by the end of the semester. That said, her use of wait time 2 did not appear to change, that is, she did not appear to make use

of wait time 2 by the end of the semester (i.e., all instances of wait time 2a were under 3 seconds, no instance of wait time 2b was recorded).

Although the analyzed data (i.e., 4 out of 11 classes) revealed no use of wait time 2, Valeria reported using it and finding it useful as it (a) helped make things less abrupt, (b) allowed her students to finish thoughts—she had sometimes not realized that students were not done talking, and (c) allowed her to take time and ask whether anyone wanted to add to the response. Most of the reasons Valeria cited for finding wait time broadly useful were focused on student experience: (a) she viewed wait time as something she could do rather than something that required asking her students to do things, (b) she wanted students to not feel anxious about nobody talking and know they had enough time to answer, and (c) she did not want students to feel like she was uncomfortable waiting. In addition to these reasons, Valeria also made the pragmatic observation that wait time helped even when it appeared not to: Even when students did not immediately respond, someone would answer the second time the question was asked—a pattern clearly observable in the data.

In short, Valeria gravitated towards wait time as it was a TDM that relied more on what she did rather than asking things of her students. Although there were aspects of the TDM that were unpleasant for Valeria, she identified many aspects of the TDM that would be good for students, particularly related to their classroom experience. Listening to her classroom audio-recording was a powerful experience for Valeria that put her use of wait time into perspective for her and may have contributed to her increased use of wait time 1 and reported-but-not-observed increased use of wait time 2.

Revoicing

At the start of the semester, Valeria already used revoicing, that is, she revoiced a majority of student turns and occasionally used full revoicing. After the TDM was introduced in the T-PD, Valeria shared that revoicing was a TDM she grabbed on to more than some other TDMs because it relied more on what she was doing than on what her students were doing. By the end of the semester, Valeria may have engaged in more revoicing, but there was no notable rise in instances of full revoicing in the second half of the semester.

Valeria discussed multiple ways in which she revoiced. First, she discussed revoicing while drawing or taking tiny notes. Although she did not elaborate why she did so, this type of revoicing lends itself to assisting or working in new aspects to a student's answer (e.g., a diagram to accompany a student's response). Second, Valeria used (full) revoicing to make sure she understood what a student said and did not do a disservice to their ideas. Further, she noted that full revoicing allowed her to give credit to students' ideas. Third, inspired by Alice, Valeria explained that she began to read students' chat messages out loud to give voice to students in the chat so they could hear that their chat messages were a contribution and part of the class. More broadly, Valeria noted that she used revoicing to ensure she knew what a student said and to keep students from feeling the pressure of having to repeat their answers. Valeria also shared how the utility of revoicing depended on the day for her, since if she was feeling "spacey," her revoicing would probably confuse students. Looking across Valeria's ways of and reasons for revoicing, one can see how Valeria's use of revoicing was centered around respect for students' discourse and a care for how students felt.

Probing a Student's Thinking

Even before "probing a student's thinking" was introduced in the T-PD, Valeria used this TDM: She typically used deepening-probing once or twice each class. Furthermore, she used clarifying-probing in one class period in the first half of the semester. After the TDMs had been introduced in the T-PD, Valeria noted that she "love[s]" probing a student's thinking and that this TDM was easier for her to use as it centered more on her since it involved her trying to understand what a student was saying. From Valeria's discourse about probing a student's thinking, it appeared that she mainly thought of using probing in a clarifying manner to understand a student's discourse and to not assume, for example, what misunderstanding there might be. This allowed her to "figure out how to continue with things." Further, she shared that she had several positive experiences using (clarifying-)probing. By the end of the semester, more isolated instances of Valeria using clarifying-probing were recorded, but Valeria's initially relatively consistent use of deepening-probing once or twice each class decreased to an occasional use of deepening-probing questions once or twice a class. All but one of Valeria's instances of clarifying-probing were instructor-centric, whereas most of Valeria's deepening-probing was student-centric. That said, in the second half of the semester, half of Valeria's instances of deepening-probing were instructor-centric. In short, one can note that the type of "probing a student's thinking"—a TDM that can be very student-centric via student-centric deepening-probing—Valeria was particularly drawn to was, in line with her beliefs, (instructor-centric) clarifying-probing.

Inviting Student Participation

Valeria wanted students to participate in her class but not at the price of forcing student participation. In this vein, throughout the semester she did not use explicit specific-student-

inviting. Further, although she asked each group to select a group member to share the group's findings, she did not call on groups or their representatives. Instead, she described inviting student participation "in a very passive way." This "very general" way included Valeria engaging in inviting further responses. Valeria highlighted that the T-PD helped her become more aware of whether she actually invited student participation or just thought she did.

Valeria also described the complexities of inviting student participation, explaining how asking students questions is not just about words: "it's also the tone, the face, the way that you say it, if it seems that you're feeling comfortable saying it or if you're just like kind of tense." She shared that thinking about all these complexities made her self-conscious, and she concluded that she needed to practice so that question-asking came more naturally to her.

With regards to Valeria's solicitations, one could observe the following: (a) the average number of solicitations stayed roughly the same across the two halves of the semester, (b) students responded to most of Valeria's what/how/why-type solicitations—and to virtually all what/how/why-type solicitations eventually, (c) Valeria consistently asked for questions to which students typically did not respond, and (d) even fewer student turns were questions in the second half of the semester than in the first.

In short, what Valeria's engagement with this TDM demonstrates is that there were aspects of it that worked for Valeria and others that did not. On one hand, she believed in the importance of student participation and created a space in which students could participate and ask questions. On the other hand, she did not want to call on students (not even students who had been chosen by their groups), and she felt very self-conscious about asking questions. As her analysis of the complexities of asking questions demonstrated, Valeria thought deeply about inviting student participation. At the same time, she tried not to think too much about these

complexities for fear of feeling self-conscious—like the millipede who forgot how to dance when asked how it dances.

Asking Students to Revoice

The TDM of "asking students to revoice" was new for Valeria, and she was initially hesitant to try it as she was not ready to be met with silence. She also voiced many concerns that reflected uncertainties she felt and how the TDM clashed with her beliefs: (a) the TDM felt like calling on someone, which she described as something that was not "her", (b) she had never asked students to revoice one another before, and (c) she did not know what tone to use when she asked. Yet, despite all these worries, Valeria tried using this TDM thrice—possibly in line with her belief to be open to change. Out of these three times, she twice received no response and once received an incommensurate response. Although she initially found that the one student response she received—although not a revoicing—was a sign that the student had listened, she ultimately concluded that asking students to revoice had "definitely failed." She gave up on this TDM because she had found it difficult enough to get students to talk about their own thoughts. Given the student responses Valeria received (i.e., silence and a non-revoicing), I suspect that the students may have been confused by this TDM, particularly since it was only used in the latter half of the semester and was therefore not part of the discursive norms and culture that had formed in the class.

Creating Opportunities to Engage With Another's Reasoning

Throughout the semester, Valeria used "creating opportunities to evaluate another's reasoning"—once or twice each class (except for in CR1). She also sporadically used "creating opportunities to understand another's reasoning" and "creating opportunities to add to another's reasoning," with the latter being used by her after the TDM was introduced in the T-PD.

Valeria shared that she struggled with "creating opportunities to engage with another's reasoning": When she used it—specifically "creating opportunities to add to another's reasoning"—she was "usually" met with silence (i.e., 75% of the time). Despite concluding that the TDM "didn't go very well," she added that "it probably worked once or twice." Valeria also drew a connection to wait time, noting that wait time 2 helped her pace herself and instead of jumping into an evaluation, it allowed her to ask whether anyone wanted to add to the given response. I suspect that Valeria was willing to try out "creating opportunities to add to another's reasoning" as it allowed more students to participate—if they wanted to.

Group Work

Valeria called having breakout rooms her version of "creating opportunities to engage with another's reasoning," and she implemented group work between once and thrice each class. Coming into the semester, she had many ideas for how to improve group work on Zoom, based on her experience teaching online the previous semester as well as on recommendations from her course coordinator.

Despite the pro-group work picture presented by her frequent use of it in the course, Valeria had a fraught relationship with group work, particularly in the online context. Valeria's struggles with group work included feeling anxiety about joining breakout rooms, feeling awkward in breakout rooms, feeling like she is "bad at just starting normal conversation," and her own negative experiences with group work as a student. To this day, "I still hate it a bit," citing her need to first figure things out on her own. At the same time, she recognized that students would need to interact with others in the future "and maybe, they should start getting used to that." She also signaled that she wanted to work on being less awkward with group work.

In line with her desire to respect students' agency, she allowed students to opt out of group work, and although she asked each group to pick someone to share the group's findings, she did not call on these representatives. Further, in response to a request by the students, she stopped randomizing groups. She kept groups fixed until later in the semester when students bemoaned their group members' silence, which led to her randomizing groups again.

Conclusion

As this case has demonstrated, for an instructor like Valeria, who weighed respecting students' agency against wielding instructor authority to foster students' learning, the TDMs are not straightforward to implement. Although some of the TDMs may ask less of students and are centered more on the instructor (e.g., wait time, revoicing), others ask more of students (e.g., asking students to revoice, inviting student participation). This case shines light on: (a) TDMs (or aspects of them) that instructors who want to respect students' agency and fear reinforcing oppressive practices might embrace (e.g., clarifying-probing, inviting further responses) as well as (b) those TDMs (or aspects of them) that such instructors might steer away from (e.g., inviting student participation via calling on students).

CHAPTER 8: THE CASE OF ALICE

Overview of the Case

Alice is a case of a mathematics-teaching graduate teaching assistant who was no longer in her first year of teaching and who participated in the MDISC professional development over the course of the semester, with the MDISC slightly abridged to fit into a semester and slightly modified to account for the undergraduate mathematics context. The emic issue that arose while developing Alice's case was the question: How does an instructor who finds herself in a stage of "survival" (Beisiegel et al., 2019; Katz, 1972) implement and talk about the TDMs? Thus, Alice's case—by itself—should be understood as an instrumental case study that seeks to answer this question by detailing how Alice used the TDMs while in survival mode. Alice's case is presented in three sections: an introduction, Alice's use of and discourse about TDMs over the course of the semester, and my interpretation of the case. For the reader's convenience, Table 8.1 below summarizes when which data were collected.

Table 8.1

Alice's Data Collection Timeline

Week	Classroom Recordings	T-PD Session	Interview
1 (reading week)			Pre-T-PD interview
2		Session 1 (Intro part 1)	
3	CR1, CR2	Session 2 (Intro part 2)	
4	CR3	Session 3 (Discourse part 1)	
5		Session 4 (Discourse part 2)	
6		Session 5 (TDMs part 1)	
7		Session 6 (TDMs part 2)	
8	CR4		
9	CR5	Session 7 (Positioning part 1)	
10	CR6	Session 8 (Positioning part 2)	Mid-T-PD interview
11		Session 9 (EQUIP, Recap part 1)	
12	CR7	Session 10 (Recap part 2)	
13	CR8	Session 11 (Capstone part 1)	
14	CR9	Session 12 (Capstone part 2)	
15		Session 13 (Capstone part 3)	
16 (Exam week)			
Post-semester week 2			Post-T-PD interview

Meeting Alice

> I have experience teaching as a high school teacher. I've taught asynchronously in the [last two semesters], so I was nervous about teaching this semester because it was a different experience. It's my first time teaching synchronously. First time teaching on Zoom, for real, instead of just making videos. My first time really teaching a college class, because the other ones were making videos and then kind of answering questions and stuff. But I didn't, I didn't anticipate that I was just going to feel, it almost felt like it was my first time teaching ever when I started. It was like, I didn't feel as confident as ((*laughs*)) I was hoping I would. Yes, it was a little bit weird at the beginning, and I think a lot of that has to do with what you said, there are just so many other things going on right now that are different. Yes, it's been weird. It's felt like I'm a completely new teacher at times. Not that we're not always learning, not that you should ever feel like, "Ah, I've made it. I know exactly what to do." Yes, it's been interesting. I do feel better than I did at the beginning of the semester. At the beginning of the semester I was like, "I'm the worst teacher ever." ((*laughter*)) "Everything I'm doing is wrong." ((*laughs*)) I'd call my mom and I'd just cry like, "I'm so horrible."

As the above excerpt from the mid-T-PD interview documents, Alice—despite several years of experience as a high school teacher—felt like "a completely new teacher at times." In particular, she wrestled with synchronously teaching a college class for the first time—a class she had to teach via Zoom due to the coronavirus pandemic. This case report will focus on how Alice implemented and talked about the TDMs during this semester in which she found herself in a stage of survival—language borrowed from Katz (1972) to describe the experiences of

(preschool) teachers in their first year of teaching, a stage that experienced GTAs have been documented to sometimes return to in novel teaching contexts (Beisiegel et al., 2019).

Course Context

Alice taught a coordinated algebra class with around 50 students twice a week for 80 minutes. Not all 80 minutes of class time were filled with covering content: Every couple of weeks, students also had to complete quizzes during class time for 30–35 minutes. On days without quizzes, the coordinator asked instructors to do group work. Classes were held synchronously on Zoom, and Alice had a teaching assistant whose role it was to answer any unanswered questions in the chat.

Alice's Motivation, Expectation, and Beliefs

> Alice had two reasons for joining the T-PD, which she shared in the pre-T-PD interview: One is I wanted to push myself to develop myself as an instructor more and not just be like, "Teaching on Zoom again. This is what I'm doing." Two, I have had some experience teaching at the secondary level and learning about how to teach the secondary level. Obviously, a lot of that transfers but I haven't really had, other than the first-year orientation, an opportunity to do any professional development, specifically for teaching at a college level. That was probably the biggest thing that drew me to it and, like we talked about before, the goals of the PD really closely align with a lot of the things that I'm passionate about and things that I thought I could get better at the college level, and just in general.

Although Alice's motivations for joining the T-PD were already laced with expectations, Alice formulated the following set of expectations moments later:

> Something that I hope to be able to do after this professional development is better understand how to help the students that I'm actually teaching and strategies for doing that, and how to, since it's all about positioning—or since it's partly about positioning—how to position myself as an instructor to help students find their mathematical voices and feel more confident.

Finally, after being asked about which aspects of her teaching she sought to improve, Alice shared:

> I would definitely say the discussion piece […] because that's been something that I've been wondering about, since I started the Ph.D. is like, how am I going to incorporate like group work and mathematical discussions at the college level, because that was never modeled for me in college and really hardly at all in high school. I don't remember any math classes actually that I've been in, other than during my Master's, where I've engaged in mathematical discussion with other students. I learned a little bit about how to do it at the high school secondary level, but really don't have a good basis for how it works at the college level, but that's definitely probably the biggest thing for me, is how to facilitate good mathematical discussion in my classroom.

As part of T-PD session 2, two weeks after the pre-T-PD interview in which she had voiced her motivation for joining the T-PD and her expectations for it, Alice created a beliefs map (see Figure 8.1) where she presented twenty-three "beliefs about students." Below, I share the five beliefs she chose to present to the T-PD group.

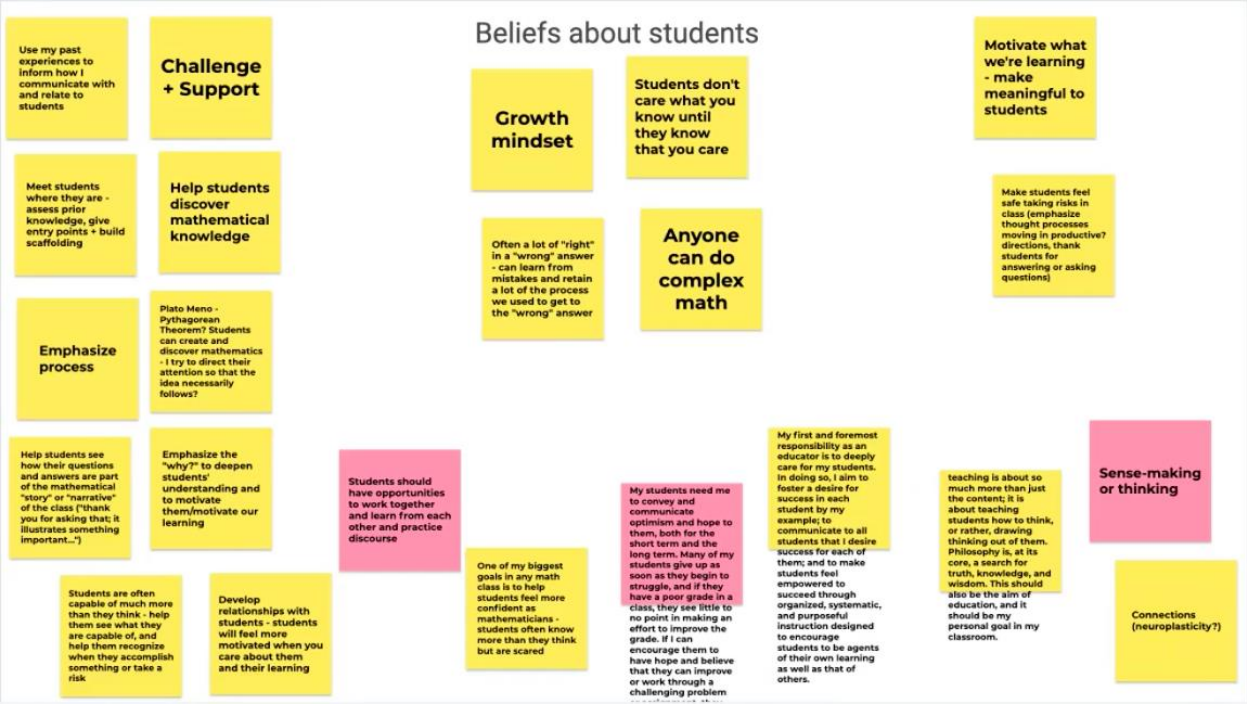
The first belief Alice spoke of was the idea of "growth mindset," a concept she "really like[d]" and "tr[ied] to emphasize." She shared that she recognized the responsibility and opportunity to help students change their story and recognize that they are capable of more than

they think (i.e., belief #2). Recognizing the fear students might have of being wrong or looking stupid, she liked to thank students for their participation to reinforce her appreciation of their participation and to make them feel more comfortable speaking up and taking risks. Fourth, she believed that "even when students give wrong answers, right, there is a lot of right in there" that one can learn from and build on. The last belief Alice spoke of was the post-it note: "Students don't care what you know until they know that you care." As she explained, "I feel like my, my biggest responsibility is to show students that I care about them as people, you know, and then I care about their learning also." This belief appeared especially important to Alice as it was something she had not felt when she was in college.

In summary, Alice had two reasons for joining the T-PD: (a) to grow as an instructor, and (b) to receive a professional development at the college level that closely aligned with things she was passionate about. She hoped that the T-PD would enable her to better help her students, particularly with respect to positioning herself "as an instructor to help students find their mathematical voices and feel more confident." Further, she identified the aspects of her teaching she sought to improve as, first, becoming better at facilitating good mathematical discussions in her classroom and, second, incorporating group. Finally, she compiled many beliefs in a beliefs map of which she explained five to the T-PD group. Running through the five beliefs she shared with the T-PD group is the fundamental theme that the instructor's role is to care for and support students to help them achieve goals they may never have thought possible.

Figure 8.1

Alice's Beliefs Map From T-PD Session 2



Note. To help the reader read the beliefs map, Figure C.1 is accompanied by the individual beliefs typed out in a larger font size. Alice did not explain whether differently colored post-its represented different types of beliefs. She noted that she had only organized her beliefs "a little bit" and was not done organizing.

Alice's Use of and Discourse About TDMs in a Semester of Survival

As the quote in the beginning of the case report demonstrated, Alice felt like a first-time teacher at the start of the semester. In particular, she felt like "the worst teacher ever" and would call her mother in tears. Yet, by the time of the mid-T-PD interview, she felt a bit better than at the beginning of the semester after having graded the first exam with several other instructors (each instructor grading a question). Despite her fears that "my students are going to be doing so poorly and everybody else's students are going to be doing great because I'm just a terrible

teacher," she realized that everyone's students were doing about the same and that "I'm not the worst teacher." Furthermore, towards the end of the semester she was able to connect with a few students "which really helped" and made her feel more positive about her teaching.

As will be seen, Alice's use of and discourse about the TDMs were colored by her experience teaching in survival mode and struggling particularly with managing class time and teaching online. During the weekly T-PD check-ins, Alice frequently brought up the struggle of covering enough content in the given amount of time. Only by T-PD session 11 (i.e., week 13 of the semester) did Alice start feeling better, citing that students would be re-exposed to her end-of-course materials in the follow-up course. Keeping in mind the narrative of this negative-slowly-turned-positive teaching experience, I focus below on how she used and spoke about each of the six teacher discourse moves.

Inviting Student Participation

In the pre-T-PD interview, Alice shared that she wanted to learn how to incorporate group work and facilitate mathematical discussions at the college level, which is part of what appealed to her about the MDISC T-PD—and something she had wondered about since starting her Ph.D. She hoped that "since [class is] online […] that students will participate ((*laughs*)) especially in breakout rooms and in groups." She proceeded to describe both how she wanted to invite student participation during whole-class and group work time. Although I will touch on how Alice sought to invite student participation during group work, group work will be discussed in more depth in a separate subsection later.

Alice shared being nervous about getting her students to participate in the online context, particularly in group work. She mentioned having no experience with implementing group work in a college classroom or online—only in person in high school—and having not had group work

and discussions modeled for her in college and hardly in high school. She feared that students would not talk with one another in groups—although her hope was that they would talk as well as ask and answer each other's questions. She added:

> It's so easy to go under the radar online. I want to make sure that I make a connection with all the students and encourage them to talk, especially in the smaller settings where maybe it feels a little bit safer.

Alice's ideas for inviting student participation in breakout rooms included asking students to build off one another's responses and answering one another's questions, which is:

> way easier to do in the classroom than online, but I do want to try to prompt students to do that and ask students, "Hey, does anybody else want to add to that?" Like have them revoice it and stuff like that.

She planned to get students used to communicating in their breakout rooms before asking them to build off one another's responses and answering one another's questions in the "full lecture." Given her large class size (of 40+ students), she felt unsure about whether students would feel comfortable engaging with these prompts in the "full lecture"—she reflected that she, as a student, would not speak up in a large class.

Alice noted that she loved explaining things and needed to remind herself to be "more of a facilitator." She wanted "students to feel like my classroom, virtual classroom is […] a safe space for them to make mistakes and to feel supported as they learn and to feel more confident […] in their voice" and use best practices for facilitating discussion. To encourage participation during whole-class time, Alice mentioned two ideas: (a) having all students unmute themselves at the start of the semester and say something to make students feel comfortable unmuting themselves, and (b) waiting a long time for students to respond. The former had been an idea

suggested to her by an instructor in the mathematics department. The latter she felt was particularly relevant in the online setting (which required more "buffer time" and whose awkwardness students might "take advantage of" by staying silent) and "I learned in my Master's […] that we can show students that you're giving them enough time to formulate a thought, and that you don't expect them to have the answer right away." She felt "on the fence" about calling on students because she "never want[s] students to feel like they've been put on the spot," but she considered prompting students once she had gotten to know them better. To foster student participation, Alice intended to ask students in breakout rooms whether they were willing to share something in the full class—a strategy she had picked up during her Master's.

By the time of the mid-T-PD interview, Alice reflected that she "should be more purposeful about [inviting student participation]." She explained two ways in which she had invited student participation thus far: (a) asking students "Can someone share what they got for this?", and (b) asking students in their breakout rooms whether they would be comfortable sharing their group's work with the whole class. She added that when she had done the latter at the start of the semester, students shared their group's work and "did really well." Thus, she wanted to do more of this type of inviting—as well as "more meaningful group work."

Alice also shared that it has been "really difficult" for her to do group work well online. She explained that the online context limited her ability to encourage students to participate in group work and she bemoaned the lack of visual feedback.

Speaking about whole-class time, Alice reported feeling nervous cold-calling students and was not sure if she had used cold-calling. She reflected that "inviting student participation" instead involved "asking somebody who I've previously checked with" (or a group) to share or a

more “open” invitation like “Can anyone share?” She added, “I know I shouldn’t do that quite so much because then the same students tend to answer.” At the same time, Alice observed:

> Even though there are students who tend to respond first, sometimes still students will give more of an explanation in the chat or they’ll speak, I guess, usually in the chat. Even so, I think I could do a better job of not just being really open-ended with my inviting students participate [*sic*] to make sure that more people get a chance to talk.

When asked about whether she had asked “Can someone share who maybe hasn’t shared yet?” in class, Alice shared her reservations about this prompt. She recalled her experience as a student who frequently responded to a teacher’s questions, felt horrible once her teacher used this prompt, and then worried that the teacher was mad at her. Although she recognized that this question could be perceived differently, “it’s difficult for me to figure out how to say that in a way that doesn’t make the students who participate a lot feel bad.” Instead, she noted, she “sometimes, when one student answers, I’ll ask for another answer maybe, ‘Anybody else wanna, have another answer?’” She added she “definitely” could invite further responses more.

Reflecting on the online modality, Alice wondered whether it was easier or harder for students to reach out on Zoom: “[B]ecause maybe their camera’s not on, people aren’t like looking at them? Then again, it’s like, their name is attached to it in the chat and it stays there.” Alice also shared that she sometimes received private chat messages—both questions and answers. She explained that she shared correct student answers with the class verbally—although she was unsure if this was “horrible” of her. Finally, she reported not having tried to ask the students to unmute themselves at the start of the semester to say something.

In the post-T-PD interview, Alice reflected that all TDMs could have been used on Zoom, but that she found some of them easier to apply in the online setting than others. She intended to incorporate the TDMs more into her teaching in a typical semester. She added:

> I don't think that I laid enough of the groundwork for group work at the beginning of the semester to try to bring [some of the TDMs] in later on, or I could have but it would have been difficult.

She noted that she: (a) found inviting student participation useful, (b) had used wait time and revoicing, but (c) did not think she used asking students to revoice. Alice shared that she felt awkward participating online and used revoicing to "mak[e] sure that everybody knows what was said" and "really honor students' participation since again, it's like, I feel it's more difficult to participate online." She highlighted that the chat gave students "an interesting opportunity […] to participate" if they did not feel comfortable speaking up in class—something they did not have in in-person settings.

Alice also reflected on group work:

> [P]retty much everything that we talked about with regards to group work, I didn't really get to incorporate a whole lot this semester, but I definitely would think it would be really, really useful in future semesters when I do a better job of establishing group work early on.

Further, she explained:

> [A] lot of the things we talked about, I'd like to try in a better group work setting than just like breakout rooms. Like talking to students and being like, "Hey, would you be comfortable sharing?" or having them share just like in the group work setting where it's maybe like lower stakes for them.

She had felt unsure at the start of the semester about what group work in the online and college context would look like and was busy "just trying to keep my head above water." Now, however, she felt "a lot more confident moving forward with more tools for group work and knowing that that's something that, at least [the university] seems to encourage, or the teaching support team does."

Studying the data from Table 8.2, one can observe that Alice's nervosity about cold-calling students is evident in that she was not recorded using specific-student-inviting. She did, however, use specific-students-inviting, that is, after two of her four group work implementations, she called on groups of students after they returned from group work (i.e., on CR2 and CR8). Studying Alice's instances of using "inviting further responses," one can see that she only rarely used this type of invitation: only once on each of CR6 and CR9.

Regarding the types of instructor solicitations she used, one can observe that Alice frequently asked for questions (and sometimes whether things made sense) and received student responses most of the time. Further, what-type solicitations were most common overall, followed by how-type, and then why-type solicitations. In eight classroom recordings (counting a tie in CR3), what-type solicitations were most frequent. In most recordings, Alice used a few instances of how-type solicitations. She also made a few why-type solicitations each recording in the beginning of the semester, but in the second half of the semester, she appeared to nearly cease making why-type solicitations. Most of Alice's what, how, and why solicitations received responses. Finally, there were massive differences in the rate at which what, how, and why solicitations were made per class. For instance, whereas in CR7 Alice made only one what, how, or why solicitation in 54 minutes, she made one what, how, or why solicitation every 2:30 minutes in CR2. Studying the three days with the highest average time between what, how, and

why solicitation (accounting for the length of whole-class time), one can see that they are the three quiz days amongst the nine classroom recordings (i.e., CR3, CR5, CR7). Thus, it appears that on quiz days, Alice may have been less willing to make many what, how, or why solicitations. That said, CR6 was almost on par with CR3—the quiz day with the lowest average time between what, how, and why solicitations (i.e., one what, how, or why solicitation on average every 7:24 minutes instead of 7:30 minutes). Thus, there was also at least one non-quiz day (i.e., CR6) on which Alice made relatively few what, how, or why solicitations.

In summary, Alice wanted her students to participate and thought about participation happening in two places: breakout rooms and whole-class time. Breakout rooms appeared to be particularly important to her as a site for inviting student participation as they were a safer space than the large class of 40+ students. Drawing on her own experiences, she did not like the idea of cold-calling students, preferring to instead make open invitations for anyone in the class to share or ask students during breakout room time whether they would feel comfortable sharing their work with the class. In line with this, she was not observed using explicit specific-student-inviting, but she was observed using explicit specific-students-inviting. She also had reservations about the prompt "Can someone share who maybe hasn't shared yet?" preferring to instead invite further responses (e.g., "Anybody else wanna, have another answer?"). Yet, she was only recorded using "inviting further responses" twice. With respect to the types of instructor solicitations Alice used, one can observe: (a) Alice frequently gave students opportunities to ask questions, to which they responded, (b) Alice mostly made what-type solicitations, some how-type solicitations, and some—but increasingly fewer—why-type solicitations, and (c) Alice appeared to make fewer what, how, and why solicitations on quiz days. Finally, she used some of the other TDMs (i.e., waiting and revoicing) to help her invite student participation.

Probing a Student's Thinking

Although Alice did not discuss probing a student's thinking in the pre-T-PD interview, she did speak about reasoning and justification. Specifically, she shared that as a high school teacher she had had the experience of students just wanting "the answer" and not caring about why things worked the way they did. She had the sense that university students cared a bit more about why things worked the way they did. She also speculated that students—both in a one-on-one and a classroom setting—might find it easier and less risky to include an explanation with their answer in case their answer was incorrect. Finally, she explained that getting students to engage in justification takes practice: practice being wrong and practice communicating mathematics. She noted the importance and utility of instructor and peer feedback—both affirmative and constructive feedback—to foster students' engagement with justification.

In the mid-T-PD interview, Alice reflected that she had done a little bit—but not much—of probing students' thinking. She related a specific instance in which she used (student-centric deepening-)probing (i.e., "Oh, can you tell me what you did to get your answer?") after someone shared their group's answer with the whole class, describing this instance as probing of the type "Can you tell me more about that?"

Alice also shared that she "probably feel[s] weird about [probing] online," specifically, putting students on the spot. She reflected that she should not necessarily feel this way and that it should be possible for her to communicate in a caring way to make students not feel put on the spot. Later in the interview, she added that probing—like revoicing and wait time and unlike "asking students to revoice"—happened naturally for her "to the cadence of going back and forth with students."

In the post-T-PD interview, Alice shared:

> [Probing] is again something that I think I wish I would have done a little bit more. It's one of those that I felt maybe a little uncomfortable doing on Zoom, or not uncomfortable, but unsure because I don't know. Like cold-calling, it's like you don't know what's going on there. I don't know, not that it was impossible, but it's something that I think I was maybe a little bit more nervous to do because I couldn't see my students and everything.

Using Table 8.3, one can see that in line with Alice's discourse about probing, some isolated uses of probing were documented in CR1–CR9. A total of four instances of deepening-probing and nine instances of clarifying-probing were coded. Of the four deepening-probing instances, three were student-centric; of the nine clarifying-probing instances, all were instructor-centric. In other words, when using clarifying-probing, Alice always made the interpretive effort of figuring out whether students meant a specific thing rather than asking them to elaborate. Looking at the breakdown by statement and question, one can see that Alice used instructor-centric clarifying-probing both in response to student statements (three times) and student questions (six times).

Asking Students to Revoice

In the pre-T-PD interview, Alice spoke of wanting to ask her students to revoice, yet it was unclear whom she wanted them to revoice (i.e., her or other students). She reflected that it might be uncomfortable for students to speak up in class (given the larger-than-usual number of 40+ students) and that she would instead give a prompt like revoicing in breakout rooms. Possibly, once students were used to this prompt, she might use it when the class was together.

In the mid-T-PD interview, Alice explained:

> Asking students to revoice. I don't think I've done that this semester. I think I could easily weave that in somehow. ((*chuckles*)) It's not something that comes naturally I feel. Like the revoicing, probing and wait time I feel like they naturally happen to the cadence of going back and forth with students, but asking students to revoice, I think it's something that I don't naturally do. I need to be more cognizant of that.

In the post-T-PD interview, Alice reflected that some TDMs were more difficult for her to use on Zoom but that she would want to incorporate those more in a typical semester. She explained that all TDMs could have been used on Zoom, but she singled out not laying enough groundwork for group work as an obstacle. With regards to asking students to revoice, she noted not having used this TDM but wanting to "definitely" try it in some regard, adding that she could maybe use it in an in-person classroom. Alice was not observed using "asking students to revoice" (see Table 8.2).

Waiting

In the pre-T-PD interview, Alice explained that it "was really easy for students to take advantage of […] the fact that teaching online is kind of awkward" and make instructors share the answer by making them uncomfortable with silence. Thus, she wanted "to be really deliberate about […] waiting a really long time" for students to respond out loud or via chat. She identified two additional reasons for wanting to use wait time: (a) she wanted to show students that she was giving them enough time to formulate a thought and that she did not expect them to have the answer right away, and (b) she pointed out that one has to wait longer in an online space due to "the buffer time of like, 'Oh, is anyone else going to talk? Like I don't want to talk over somebody.'"

In the mid-T-PD interview, Alice observed that waiting—like revoicing and probing and unlike asking students to revoice—"naturally happen[s] to the cadence of going back and forth with students." At the same time, Alice reflected that she had used wait time 1 "definitely" more than wait time 2 and that that she had not "used wait time two much, if at all, but I want to, because it seems like it could be very beneficial, in class." She felt she had not been purposeful enough in her use of wait time, specifically waiting long enough and using it at strategic times. She added that she had never thought of wait time 2 before, in part because she feels the need to immediately reassure her students. At the same time, she found it interesting to think about how to implement wait time 2 and the opportunities it created for students to follow up with one another, to add to one another's responses, and to ask questions to one another or the instructor. Alice also shared using the facilitator's (i.e., my) trick of having something to drink while waiting. Alice concluded:

> I've definitely been trying to use wait time more and use wait time better. That's something, again, that I was aware of, but didn't really know about the different ways in which you can use it and how that can be useful.

In the post-T-PD interview, reflecting on the utility and applicability of the TDMs in in-person and online settings, Alice noted that "wait time, you know, is ((*laughs*)), is obviously something that you can always do." She added:

> I'm pretty sure that I used wait time more frequently and for longer periods of time than I would in person or than I have in person 'cause again I feel a lot more pressure in a room of people just staring.

The online context allowed her to wait longer than she would have been comfortable in person, and she hoped that she could transfer her more frequent and longer use of wait time to in-person

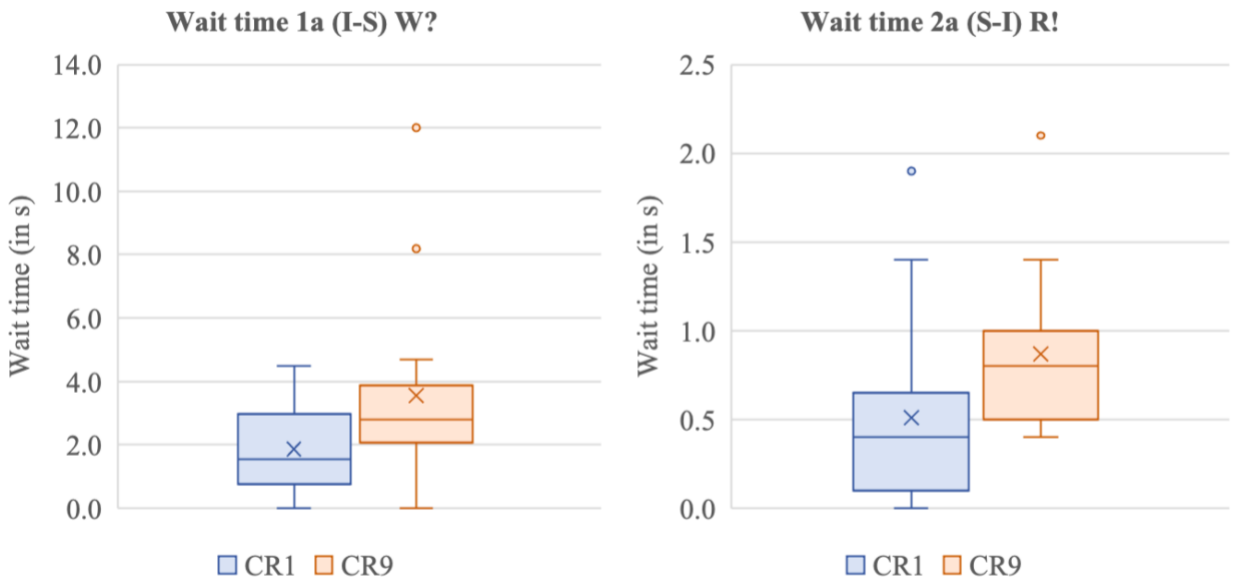
classes. She reiterated liking the "trick" of having something to drink while waiting so she did not have "to just be sitting there, like on Zoom looking at the camera," and she had begun to keep a water bottle next to her. Despite the online context and the water bottle "trick," "sometimes the wait time was uncomfortable" because "sometimes it just feels like you're just like talking to the void like nobody's there." With respect to wait time 2, Alice reflected it had been a novel concept that was a useful tool. That said, she shared she did not use wait time 2 enough "because […] I was always just thrilled to have participation at all" and always felt she needed to provide students with affirmation for their participation

Figure 8.2 shows that the data are inconclusive as to whether Alice's wait time 1a (I-S) W? increased from CR1 to CR9. Yet, two instances of wait time 1a (I-S) W? exist in CR9 which were notably longer than any in CR1, that is, two instances occurred in CR9 in which Alice waited for a longer than usual time and received responses. There were too few data points to make any claims about wait time 1a (I-S) Q?, wait time 1b (I-I) W?, and wait time 1b (I-I) Q?. In short, the data are inconclusive as to whether Alice used wait time 1 for longer periods of time by the end of the semester.

Looking at Figure 8.2 again, one can note that the boxplot for wait time 2a (S-I) R! suggests a likely increase from CR1 to CR9. That said, the likely increase is small and not a single instance of wait time 2a (S-I) R! exceeded 3 seconds—all but one instance did not exceed 2 seconds. The few data points that exist for wait time 2a (S-I) Q? paint a similar picture. Further, no instance of wait time 2b was recorded. Thus, although there may have been a slight increase in Alice's use of wait time 2 from CR1 to CR9, she only waited very briefly—generally less than 2 seconds—after students' responses before speaking.

Figure 8.2

Wait Time Comparison: CR1 Versus CR9



Note. The cross on a boxplot marks the mean; the line splitting the interquartile range marks the median. The mean and median of the boxplots, given in the format ((mean$_{CR1}$, median$_{CR1}$),(mean$_{CR9}$, median$_{CR9}$)) and read left to right, are: ((1.9, 1.6), (3.6, 2.8)) and ((0.5, 0.4), (0.9, 0.8)) (given to the nearest tenth of a second). Boxplots for other forms of wait time are omitted due to insufficient data. The number of data points each boxplot reflects (from left to right) are: 14, 14, 17, and 19.

Revoicing

During the pre-T-PD interview, Alice demonstrated that she was familiar with the term "revoicing," noting that she wanted to ask her students to "revoice it," possibly referring to other students' answers and questions.

In the mid-T-PD interview, Alice shared that revoicing was probably the TDM she most frequently used. She knew this TDM coming into the T-PD but had found it really interesting to learn about full revoicing—"which I definitely don't do enough" even though "that can be really

useful and really helpful." Alice expressed that revoicing—like probing and wait time and unlike asking students to revoice—"naturally happen[s] to the cadence of going back and forth with students."

Alice spoke about one type of revoicing in particular: revoicing of correct private chat responses. Although she did not specify whether she named the student when sharing their answer, she shared that she was unsure whether sharing students' correct but private answers "if that's horrible of me." Alice explained that she had become more cognizant of revoicing students' chat messages (not just private ones) since reading her transcript during the T-PD and observing that she did not know what she was talking about—eventually, she realized that she must have been referring to a chat message that had caught her eye. Noting that students may not see a chat message, particularly if they access Zoom from their phones, she has "definitely been, tried to be more aware of saying what's been asked in the chat, instead of just answering it and assuming that everybody saw it."

In the post-T-PD interview, reflecting on the TDMs she used during the semester, Alice noted, "[a] lot of the revoicing I tried to do." She appeared to imply that revoicing was one of the TDMs "fairly easily applicable in this [online] setting." Alice reflected that she had used more revoicing while teaching online than in her past in-person teaching. She reasoned, "I think some of it's for practicality's sake, making sure that everybody knows what was said. Also, I think I wanted to, again, really honor students' participation since again, it's like, I feel it's more difficult to participate online." She explained two ways in which she revoiced: revoicing students' questions and revoicing chat messages. She highlighted that revoicing students' private messages offered a way for students to participate in a way they could not in in-person settings. At the same time, she shared, she felt unsure about revoicing private messages. She clarified that

she "was never like, 'Oh, so and so messaged me this privately' […] Sometimes it was a really helpful question and I would just say, 'Oh, I have a question, blah blah.'"

Alice noted she had tried "to use more revoicing as the semester went on" and hoped to use more revoicing in her future in-person teaching but admitted that "it would look different." Although she remembered sometimes checking in with students, she felt she did not use enough full revoicing on Zoom: "I feel like I should have, because […] I feel like it's maybe easier for things to get misunderstood on Zoom."

Table 8.3 shows that Alice frequently used revoicing in CR1–CR9. In particular, in each of CR1–CR9 the majority of students turns was revoiced—in most of CR1–CR9, the vast majority. Further, Alice was only once recorded fully revoicing a student turn. Thus, the data from Table 8.3 suggest that revoicing was a frequently used TDM by Alice but that she mainly used simple revoicing rather than full revoicing.

Creating Opportunities to Engage With Another's Reasoning

As was true for the other T-PD participants, Alice viewed group work as a form of creating opportunities to engage with another's reasoning. That said, I will first discuss whole-class prompts like "Do you agree or disagree with Isabel? Why?" before talking about group work in a dedicated subsection.

In the pre-T-PD interview, Alice shared:

> One thing that I do want to make sure I do in online teaching is ask other students to build off of other students, answers and questions and things, because that's, I think, way easier to do in the classroom than online, but I do want to try to prompt students to do that and ask students, "Hey, does anybody else want to add to that?" Like have them revoice it and stuff like that.

Alice added that she was thinking of first asking prompts like these in breakout rooms before asking them in the large lecture. Given her large class size (of 40+ students), she felt unsure about whether students would feel comfortable engaging with these prompts in the "full lecture"—she reflected that she, as a student, would not speak up in a large class.

Other ways in which Alice touched on "creating opportunities to engage with another's reasoning" included Alice explaining that peer feedback is important for getting students used to justifying their reasoning. She also noted that she loves explaining things and needs to remind herself to be "more of a facilitator" of discussions.

In the mid-T-PD interview, Alice reflected that she "definitely" needed to do more to create opportunities for students to engage with another's reasoning and explained:

> I mean, I have tried to be like, "Why does someone think that so and so gave this answer?" At least I think I tried to say something like that yesterday. It's been difficult to find time and space to do that, I guess.

She described that students, however, engaged with one another's responses of their own volition: "[E]ven though there are students who tend to respond first, sometimes still students will give more of an explanation in the chat or they'll speak, I guess, usually in the chat." Alice also made a connection between wait time and creating opportunities to engage with another's reasoning, reflecting that she had never thought about wait time 2 before the T-PD but had found it interesting to think about wait time 2 as an opportunity "that can help other students have a chance to follow up with that. Or add to that. Or ask questions to the student or to me."

In the post-T-PD interview, reflecting on the utility and applicability of the TDMs in in-person and online settings, Alice noted "[creating opportunities for students to engage with other's reasoning] was really useful. I don't know that I did too much of that one on Zoom, but I

enjoyed seeing examples of how that was done in other classes." Although she felt that creating opportunities to engage with another's reasoning could have been used on Zoom, she appeared to imply that this TDM was more difficult for her to apply on Zoom because she did not lay "enough of the groundwork for group work at the beginning of the semester" to try to bring this TDM in later on. She further appeared to imply that she would want to try to incorporate this TDM more in a typical semester.

Table 8.3 shows that Alice only used this TDM twice. Both times that she used it took place towards the end of the semester (i.e., CR6 and CR9), and both times she used the same form of creating opportunities to engage with another's reasoning, namely, asking students to evaluate another student's reasoning. In short, group work aside, Alice appeared to rarely use "creating opportunities to engage with another's reasoning."

Group Work

In the pre-T-PD interview, Alice shared her hope that students would participate in group work in their breakout rooms, asking and answering one another's questions rather than sitting in silence. She was, however, "really nervous" about implementing group work online as she had never done so online or in a college class and had not seen group work modeled for her in college and hardly in high school. She only had experience with facilitating group work in an in-person high school setting. Learning how to incorporate group work was something she had wondered about since starting her Ph.D. and had appealed to her about the MDISC T-PD. At this point, she felt: "I don't have a good concrete idea of what I want [group work] to look like."

At the same time, to foster student comfort with unmuting themselves and talking as well as to show students she wanted them to speak, Alice considered following a suggestion she received from a university teaching mentor half a year ago: asking all students at the start of the

semester to unmute themselves and say something. In the breakout rooms, she intended to ask students to build off one another's responses and answer one another's questions. Given her large class size (of 40+ students), she felt unsure about whether students would feel comfortable asking and answering one another's questions in the "full lecture." Thus, she planned to get the students used to communicating in their breakout rooms before asking them to build off one another's responses and answering one another's questions in the large lecture. She reflected that she, as a student, would not speak up in a large class.

Alice also touched upon why she found group work important. First, she noted that getting students to engage in justifying their reasoning takes practice: practicing being wrong and practicing how to communicate. She highlighted the importance and utility of instructor and peer feedback—both affirmative and constructive feedback. Second, she wanted to make sure that:

> both marginalized students and students who are naturally less inclined to participate in class, I want to make sure that I still communicate with those students, especially online. It's so easy to go under the radar online. I want to make sure that I make a connection with all the students and encourage them to talk, especially in the smaller settings where maybe it feels a little bit safer.

With respect to some of the technicalities of group work, Alice noted being nervous about figuring out how to group students online. She had discussed different ways of grouping students in her first-year GTA PD but felt "really nervous" about grouping and that "there's never a right answer." In particular, she worried about not getting a good sense in the online environment of whom to group together. She concluded:

> I do want to make sure that I group students in a way that doesn't further marginalize them, if that makes sense. Encourage students to talk and show them that I'm someone that they can trust, I guess, mathematically and in class.

Alice also shared feeling "on the fence" about calling on students because she "never want[s] students to feel like they've been put on the spot." She considered prompting students to respond once she knew them better. To foster student participation at the start of the semester, Alice intended to go to breakout rooms and ask a student in the breakout room whether they were willing to share something in the full class—a strategy she had picked up during her Master's.

In the mid-T-PD interview, Alice reflected:

> I haven't done any meaningful group work. I've been really frustrated with myself […] I feel like I'm focusing so much on trying to move the class forward that I'm not spending as much time as I would like to on creating meaningful group work and meaningful experiences for students to interact with the material.

Alice felt pressured to constantly move the class forward because—particularly at the start of the semester—she underestimated how long things would take and fell behind. She reflected:

> [I]n person I'd be able to see everything at once and I'd be able to go around and whatever, but now it's like I have to go to each breakout room and it just takes so long and the students have to wait for me if they have questions.

She bemoaned the online context limiting her ability to interact with and encourage students to participate in group work and the lack of visual feedback as formative assessment. Yet, she hoped:

> [N]ow that I've at least got a little bit of a better handle on my prep for the class and like how I envision each Zoom session going that I can be a little bit more purposeful about more of the things that we've been talking about.

Alice remembered being pleasantly surprised at the beginning of the semester by the conversations students were having in their breakout rooms and wanted to try "to get them to do that again." Further, at the start of the semester she had asked students in breakout rooms whether they would be comfortable sharing their group's work with the whole class and students had done "really well." Thus, she also wanted to do more of this type of inviting. Yet, unlike the start of the semester, Alice reported that "there's been a lot less discussion recently in the group work" and that "yesterday, I don't think anybody was talking to each other in the groups that I went to. Mikes are all off" and students responded that they had no questions for her. Further, in the previous class, a student in a breakout room had complained to Alice that none of her group members responded to her question. Alice identified three potential reasons why participation in group work may have decreased. First, she shared that she had not made her expectations for group work very explicit because at the beginning of the semester group work was going surprisingly well and she had concluded the students knew what to do. Second, she wondered if students were just exhausted and burned out at this point in the semester. Last, she observed that students could do a lot of the group work tasks on their own and "didn't need to talk to each other at all." Alice suspected that she could set the activities up better or that in an in-person setting, she could get students to work together on the tasks, but online, "it's just so much easier to be like, 'Well, I'll just do this on my own.'" She wanted to make sure that going ahead, "I'm doing group work where they actually need to talk to each other," but she had not "figured out a

good way" to do so yet. Alice concluded, "probably the main thing that I'm thinking about right now is just constructing better group work."

In the post-T-PD interview, Alice shared:

> [P]retty much everything that we talked about with regards to group work, I didn't really get to incorporate a whole lot this semester, but I definitely would think it would be really, really useful in future semesters when I do a better job of establishing group work early on.

Because she felt she had not laid "enough of the groundwork for group work at the beginning of the semester," she implied it was difficult for her to use "asking students to revoice," "probing a student's thinking," and "creating opportunities to engage with another's reasoning." Further, she explained,

> a lot of the things we talked about, I'd like to try in a better group work setting than just like breakout rooms, talking to students and being like, "Hey, would you be comfortable sharing?" or having them share just like in the group work setting where it's maybe like lower stakes for them.

She remembered that early in the semester, she had felt unsure about what group work in the online and college context would look like and was busy "just trying to keep my head above water." She described herself as clueless coming into the semester because most of her educational experiences did not involve group work and her teaching observations at university were cut short due to the pandemic. Now, however, she felt "a lot more confident moving forward with more tools for group work and knowing that that's something that, at least [the university] seems to encourage, or the teaching support team does."

Table 8.3 shows that Alice implemented group work throughout the semester but not in every class. (Of the five coded classes in which Alice did not implement group work, three were quiz days [i.e., CR3, CR5, CR7], that is, days on which there was no expectation to implement group work.) Further, when Alice implemented group work in one of the classroom recordings, she implemented one instance of group work, not several. These instances of group work were all under 10 minutes (i.e., 3 min in CR1, 6 min in CR2, 8 min in CR6, and 7 min in CR8). During CR1, the group work took place in the first 15 minutes, but during CR2, CR6, and CR8 group work roughly took places between the 50th and 70th minutes of the class. During group work, she and her TA would go around the breakout rooms checking in with the groups.

In summary, Alice came into the semester wanting to implement group work—and being expected to by the course coordinator. She felt she had little experience with group work—both as a student and as an instructor—and was not sure what she wanted group work to look like. After being initially pleasantly surprised by students' conversations in breakout rooms, student participation in breakout rooms decreased. She identified three possible reasons for this decrease: (a) not making her group work expectations clear at the start, (b) students being exhausted, and (c) the group work tasks not actually requiring group work. Alice's group work implementation was also seemingly hampered by the online context. That said, Alice was not discouraged by her experience during the semester, feeling more confident moving forward and wanting to do a better job of implementing group work in the future.

Student Discourse in Alice's Class

Table 8.4 and Table 8.5 summarize the student discourse dimensions codes assigned to Alice's CR1–CR9. These frequencies support several observations about the discourse in Alice's class during whole-class time.

First, students communicated both via speech and writing. In six of the nine classroom recordings (i.e., CR1, CR2, CR4, CR5, CR6, CR7), spoken turns were more common than written ones. In the final two classroom recordings (and in CR3), written turns were more common. Alice's students communicated via chat both publicly and privately.

Although student turns addressing logistics were observed, most student turns involved discourse about mathematics. That said, due to many logistics questions about the final exam and grades, the number of student turns about logistics shot up in CR8 and CR9. The number of student turns about mathematics varied drastically from class to class with as few as 8 and as many as 53 recorded. Adjusting for the different amounts of whole-class time, this means that whereas on some days student turns about mathematics took place on average every 1:29 minutes, on other days they occurred on average only every 6:44 minutes. As will be discussed below alongside "student talk type," this fluctuation might partially be explained by whether a class was a quiz day or not.

With respect to student talk length, students' turns about mathematics were a mix of lengths: (a) short student turns (i.e., 1–4 words) were the most frequent on five occasions (counting a tie with "21+ words" student turns in CR5); (b) medium-length student turns (i.e., 5–20 words) were the most frequent on three occasions; and (c) longer student turns (i.e., 21+ words) were the most frequent on two occasions (counting a tie with 1–4 words in CR5). During each class recording, at least two turns were longer than 20 words.

Studying the student talk type, one can note that of what, how, and why-type talk, what-type talk was the most common student talk type in each of CR1–CR9—followed in most recordings by how-type talk. With one exception, students asked at least three questions in each classroom recording (i.e., students asked no questions in CR6). There were also large

fluctuations from class to class with respect to the total number of what, how, and why-type talk. Especially on quiz days (i.e., CR3, CR5, CR7), very few student turns of what, how, or why-type talk occurred, with only one being recorded on CR7. That said, on CR6, a non-quiz day, the adjusted-for-class-length total number of what, how, and why-type talk was on par with CR3. Thus, although quizzes cannot entirely explain the low total numbers of what, how, and why student turns on certain days, they may have contributed. For the "question" talk type, one can note that the proportion of student turns that were questions spiked on quiz days.

Studying the teacher solicitation type codes and cross-referencing them with the student talk type codes, one can see that most what, how, and why teacher solicitations that received responses were taken up in the intended fashion (i.e., 92.9% of what-type solicitations that received responses were taken up as what-type talk, 80.5% of how-type solicitations that received responses were taken up as how-type talk[23], and 92.3% of why-type solicitations that received responses were taken up as why-type talk). Further, one can observe that most student turns were solicited. That said, on two days (i.e., CR3, CR7)—both of them quiz days—the number of unsolicited responses spiked to over half of student turns. All but two of the combined 21 unsolicited turns during CR3 and CR7 were student questions.

Finally, one can observe that throughout the semester Alice evaluated most of her students' turns. In four classroom recordings, she evaluated all evaluable student turns. In the other five, she evaluated at least 88.8% of evaluable student turns. Her TA was not recorded evaluating a student turn.

[23] 17.1% of how-type solicitations that received responses were taken up as what-type talk and 2.4% as why-type talk.

Table 8.2

Alice's Use of TDMs Across CR1–CR9 (Part 1)

Instructor Discourse Dimensions	Classroom Recording (# of min. spent together as a class)								
	CR1 (77)	CR2 (75)	CR3 (45)	CR4 (79)	CR5 (44)	CR6 (74)	CR7 (54)	CR8 (74)	CR9 (79)
Types of Instructor Solicitations	29	37	13	24	10	16	11	39	46
• What	11 (10)	27 (21)	3 (1)	9 (8)	2 (2)	7 (6)	0	17 (15)	18 (15)
• How	4 (3)	2 (2)	0	4 (4)	1 (1)	3 (3)	1 (1)	2 (2)	13 (13)
• Why	4 (3)	1 (1)	3 (3)	3 (2)	0	0	0	2 (1)	0
• Questions?	9 (4)	4 (2)	5 (3)	6 (3)	6 (5)	6 (0)	4 (3)	10 (7)	10 (7)
• Make sense?	0	1 (1)	2 (2)	0	0	0	2 (2)	1 (1)	3 (3)
• Other	1 (1)	2 (2)	0	2 (0)	1 (1)	0	4 (4)	7 (7)	2 (1)
Specific-student(s)-inviting (explicit)									
• Specific-student-inviting	0	0	0	0	0	0	0	0	0
• Specific-students-inviting	0	3	0	0	0	0	0	7	0
Inviting further responses	0	0	0	0	0	1	0	0	1
Asking students to revoice	0	0	0	0	0	0	0	0	0

Table 8.3

Alice's Use of TDMs Across CR1–CR9 (Part 2)

Instructor Discourse Dimensions	Classroom Recording (# of min. spent together as a class)								
	CR1 (77)	CR2 (75)	CR3 (45)	CR4 (79)	CR5 (44)	CR6 (74)	CR7 (54)	CR8 (74)	CR9 (79)
Turn simply revoiced	24	36	14	21	5	10	8	29	44
Turn fully revoiced	0	0	0	0	0	0	1	0	0
Turn not revoiced	5	7	4	3	3	1	6	12	9
Clarifying-probing (CP)	1	1	0	0	1	0	4	2	0
• CP Instructor-centric (CP-IC)	1	1	0	0	1	0	4	2	0
○ CP-IC Statement	0	1	0	0	0	0	0	2	0
○ CP-IC Question	1	0	0	0	1	0	4	0	0
• CP Student-centric (CP-SC)	0	0	0	0	0	0	0	0	0
○ CP-SC Statement	0	0	0	0	0	0	0	0	0
○ CP-SC Question	0	0	0	0	0	0	0	0	0
Deepening-probing (DP)	1	0	0	0	0	2	0	1	0
• DP Instructor-centric	0	0	0	0	0	0	0	1	0
• DP Student-centric	1	0	0	0	0	2	0	0	0
Creating opportunities to engage with another's reasoning									
• Adding	0	0	0	0	0	0	0	0	0
• Evaluating	0	0	0	0	0	1	0	0	1
• Understanding	0	0	0	0	0	0	0	0	0
• Group work	1	1	0	0	0	1	0	1	0

Table 8.4

Alice: Student Discourse Dimensions Across CR1–CR9 (Part 1)

Student Discourse Dimensions	Classroom Recording (# of min. spent together as a class)								
	CR1 (77)	CR2 (75)	CR3 (45)	CR4 (79)	CR5 (44)	CR6 (74)	CR7 (54)	CR8 (74)	CR9 (79)
Mode of Discourse	35	48	18	36	11	18	19	58	70
• Speech	26	30	6	24	9	13	10	26	31
• Writing	9	17	12	12	2	5	9	32	38
○ Public chat	9	13	12	0	0	0	0	23	38
○ Private chat	0	4	0	12	2	5	9	9	0
• Body	0	0	0	0	0	0	0	0	0
• Intermodal	0	1	0	0	0	0	0	0	1
Discourse Type	35	45	18	28	10	15	15	55	70
• Discourse about mathematics	29	43	18	24	8	11	15	41	53
• Discourse about logistics	6	2	0	4	2	4	0	14	17
Student Talk Length	29	43	18	24	8	11	15	41	53
• 1–4 words	16	26	1	6	3	8	1	15	31
• 5–20 words	7	10	14	9	2	0	4	18	20
• 21+ words	6	7	3	5	3	2	5	6	2
• n/a	0	0	0	4	0	1	5	2	0

Table 8.5

Alice: Student Discourse Dimensions Across CR1–CR9 (Part 2)

Student Discourse Dimensions	Classroom Recording (# of min. spent together as a class)								
	CR1 (77)	CR2 (75)	CR3 (45)	CR4 (79)	CR5 (44)	CR6 (74)	CR7 (54)	CR8 (74)	CR9 (79)
Student Talk Type	29	43	18	24	8	11	15	41	53
• what	14	30	4	10	4	8	0	20	28
• how	3	4	0	4	1	3	1	7	18
• why	5	1	3	4	0	0	0	2	0
• question	7	5	11	4	3	0	13	10	5
• other	0	3	0	2	0	0	1	2	2
Teacher Solicitation Type	29	43	18	24	8	11	15	41	53
• what	14	31	3	12	3	8	0	21	21
• how	4	3	0	4	1	3	1	2	23
• why	4	1	3	3	0	0	0	2	0
• other	2	2	2	2	3	0	3	5	5
• n/a (unsolicited)	5	6	10	3	1	0	11	11	4
Explicit Evaluation	29	43	18	24	8	11	15	41	53
• Yes	24	40	16	18	5	10	9	34	48
• Yes (by TA)	0	0	0	0	0	0	0	0	0
• No	3	0	1	2	0	1	0	0	1
• n/a	2	3	1	4	3	0	6	7	4

My Interpretation of Alice's Case

As Alice shared in the mid-T-PD interview, "It's felt like I'm a completely new teacher at times." In particular, she highlighted that she struggled because she: taught her course for the first time, "really" taught a college course for the first time (i.e., synchronously), taught on Zoom for the first time, and felt that there was not enough time to do everything. As her use of and discourse about the TDMs demonstrated, these challenges seemed to play a large role in whether and, if so, how Alice used TDMs. Below, I summarize and interpret her use of and discourse about the TDMs while attending to issues that contributed to her struggles, in particular technology and the scarcity of time.

As previously summarized, Alice wanted her students to participate and thought about participation happening in two places: breakout rooms and whole-class time. Breakout rooms appeared to be particularly important to her as a site for inviting student participation as breakout rooms were a safer space than the large class of 40+ students. Drawing on her own experiences, she did not like the idea of cold-calling students, preferring instead to make open invitations for anyone in the class to share or to ask students during breakout room time whether they would feel comfortable sharing their work with the class. Accordingly, she was not observed using explicit specific-student-inviting, but she was observed using explicit specific-students-inviting. She also had reservations about the prompt "Can someone share who maybe hasn't shared yet?" preferring to instead invite further responses (e.g., "Anybody else wanna, have another answer?"). Yet, she was only recorded using "inviting further responses" twice. With respect to the types of instructor solicitations Alice used, one could observe: (a) Alice frequently gave students opportunities to ask questions, which they seized, (b) Alice mostly made what-type solicitations, some how-type solicitations, and some—but increasingly fewer—why-type

solicitations, and (c) Alice appeared to make fewer what, how, and why solicitations on quiz days. Finally, she used some of the other TDMs (i.e., waiting and revoicing) to help her invite student participation. Alice's use of various forms of inviting student participation appears to reflect at least in part her struggles with the online context and the amount of content to cover. In particular, although she generally seemed to dislike the idea of cold-calling, she noted that she was uncomfortable using it over Zoom since she did not know "what's going on there" on students' ends. Further, the pressure she felt to cover a certain amount of material may have manifested itself in the very few instructor solicitations she used on quiz days and the very few instances of "inviting further responses."

Although Alice felt that probing happened naturally for her "to the cadence of going back and forth with students," she did not often use "probing a student's thinking:" Across CR1–CR9 she used clarifying-probing nine and deepening-probing four times. All of her instances of clarifying-probing were instructor-centric, mostly in response to student questions. Thus, Alice made the interpretive effort of trying to figure out what a student meant rather than asking the student to elaborate. Three out of her four instances of deepening-probing were student-centric. As Alice explained, she felt weird and uncomfortable probing online while not seeing the students and likened it to cold-calling. She was also worried about making students feel put on the spot. Thus, one can see how teaching online (and a care for how students might feel) appeared to affect Alice's stance towards and uptake of probing a student's thinking.

"Asking students to revoice" was the TDM Alice took to the least with no instances of her using the TDM recorded, and she explained that it did not come naturally to her. She also implied that the online context made the TDM more difficult to use, particularly since she had not laid enough groundwork for group work—she had wanted to use breakout rooms to get

students used to and comfortable with certain prompts before posing them during whole-class time. That said, Alice wanted to "definitely" try this TDM in some regard in an in-person classroom. Thus, in addition to the TDM not coming naturally to her, Alice again cited the online context and insufficient group work as reasons for not using the TDM during the semester.

"Waiting" was a TDM Alice knew she wanted to use going into the semester (a) to show students they had enough time to answer, (b) to outawkward students who thought they could get her to share the answer by staying silent, and (c) because she felt there was a "buffer time" in the online space during which students figured out whether they could talk or someone else was going to talk. Halfway through the semester, she observed that waiting happened naturally for her "to the cadence of going back and forth with students." That said, the data were inconclusive as to whether Alice used wait time 1 for longer periods of time, and although there may have been an increase in Alice's length of wait time 2a, not a single instance of Alice using wait time 2a (or 2b) for 3 seconds or more was recorded. Inconclusiveness of Alice's wait time 1 data aside, Alice felt she had used wait time 1 "definitely" more than wait time 2. Although she had wanted to use wait time 2 (and appeared to find it interesting and useful), she had not used it much, citing her felt need to immediately reassure students and provide them with affirmation for their participation since she was thrilled to have online participation at all. Alice also brought up several times how useful it was to have something to drink while waiting—a "trick" I had shared with the T-PD group. Finally, Alice felt that she was more comfortable waiting longer in the online context than in person. Looking across Alice's uptake of and discourse about wait time, again technology stands out—both as an enabler and a hindrance.

Revoicing was a TDM Alice was already familiar with before the T-PD, and Alice frequently used revoicing, often revoicing the vast majority of student turns in CR1–CR9. Alice

found that revoicing happened naturally "to the cadence of going back and forth with students," and, as she shared, she had tried to use revoicing more as the semester went on. She found that revoicing was a TDM that was "fairly easily applicable in this [online] setting." That said, she was only recorded using full revoicing once and felt like she should have used it more. Alice explained four ways in which she used revoicing: (a) revoicing correct private chat messages, (b) revoicing public chat messages to make sure everyone knows what was said in the chat, (c) revoicing student turns to honor students' participation in the online context, and (d) revoicing students' questions. Thus, one can see that revoicing did not prove a struggle for Alice, perhaps because she was already familiar with simple revoicing and found it fairly easy to use online. It is unclear why, apart from possibly its novelty, she did not take up full revoicing even though she thought it could be "really useful and really helpful."

Although Alice came into the semester wanting students to build on each other's responses, Alice ended up rarely using "creating opportunities to engage with another's reasoning." The two times she was recorded using this TDM, she used "creating opportunities to evaluate another's reasoning." A struggle for Alice seemed to be that she felt uncomfortable asking prompts along the lines of "creating opportunities to engage with another's reasoning" during whole-class time and wanted to instead get students more comfortable engaging with such prompts in groups. Yet, because she felt she had not laid "enough of the groundwork for group work at the beginning of the semester," she implied it was difficult for her to use "creating opportunities to engage with another's reasoning" (as well as "probing a student's thinking" and "asking students to revoice"). Thus, despite finding "creating opportunities to engage with another's reasoning" "really useful," she was never comfortable using it during whole-class time because she implemented group work less frequently and less meaningfully than she would have

liked. As she remarked: “It’s been difficult to find time and space to [use creating opportunities to engage with another’s reasoning].” She made several further observations: (a) students engaged with one another’s reasoning in the chat without being prompted to, (b) she thought about wait time 2 as an opportunity for students to engage with another’s reasoning, and (c) she would want to try using this TDM in a future “more typical” semester. In short, the barrier to Alice’s use of this TDM seemed to be, as she put it, not laying “enough of the groundwork for group work at the beginning of the semester.” As described in the group work subsection, a lack of time (due to the amount of content she needed to cover) and the online context made group work less frequent and less meaningful, and, consequently, Alice did not appear to go through with her plan to eventually use this TDM during whole-class time.

Alice viewed group work as a form of creating opportunities to engage with another’s reasoning, and she came into the semester wanting to implement group work—and being expected to by the course coordinator. She felt she had little experience with group work—both as a student and as an instructor—and was not sure what she wanted group work to look like. After being initially pleasantly surprised by students’ conversations in breakout rooms, student participation in breakout rooms decreased. She identified three possible reasons for this decrease: (a) not making her group work expectations clear at the start, (b) students being exhausted, and (c) the group work tasks not actually requiring group work. She reflected that she did not get to implement group work for as long as she wanted (because everything took longer on Zoom) and that her group work implementations were not very meaningful (since discussions had decreased and a lot of the assigned tasks were not group-worthy). Indeed, her recorded group work implementations were very short—on average 6 minutes during a class with group work. Alice’s group work implementations were also, she explained, hampered by the online context (e.g., not

being able to walk around the classroom and see everyone at once). That said, Alice was not discouraged by her experience during the semester, feeling more confident moving forward and wanting to do a better job of implementing group work in the future.

Conclusion

In conclusion, there were TDMs that came to Alice more naturally and that she was able to weave into her teaching, such as wait time 1 and (simple) revoicing—TDMs she already knew coming into the T-PD. Other TDMs, like "asking students to revoice," "creating opportunities to engage with another's reasoning," and "deepening-probing," Alice implemented rarely or not at all—despite expressing no fundamental objections. Instead, for these moves which required asking students to act, Alice had imagined getting students used to them in a group work setting as she worried that initially using them in the whole-class environment would be intimidating. Yet, due to issues including technology and time, she found her group work implementations not to be very meaningful and appeared to abandon the thought of eventually using TDMs like "asking students to revoice," "creating opportunities to engage with another's reasoning," and "deepening-probing" in the whole-class environment. Her use of "inviting student participation" also seemed to suffer from the challenges of technology and time since she avoided cold-calling, rarely used "inviting further responses," and made very few instructor solicitations on quiz days. That said, in the post-T-PD interview Alice looked to a future in which she was not in survival mode and could weave the TDMs into her teaching:

> I think, maybe some of [the TDMs] were, at least for me, fairly easily applicable in this setting. Some of them I think were more difficult for me to apply on Zoom, but definitely, absolutely, will want to try to incorporate those more in maybe ((*laughs*)) a more typical semester.

CHAPTER 9: MULTICASE REPORT

In this chapter, I present the multicase report, structured with respect to the etic issues I chose at the beginning of the dissertation research. These etic issues were to understand (a) how the classroom discourse in each participant's class changed with respect to the TDMs as well as several student discourse dimensions, and (b) how participants talked about their use of the TDMs (so that I might learn more about how and why participants took up the TDMs in the ways they did). The chapter is structured by each TDM and the student discourse dimensions, and the participants' discourse about the TDMs will be woven into the respective sections. In each section, I will seek to share both commonalities and differences across cases.

Waiting

All participants shared that they used wait time 1, or were aware of its importance, before the T-PD. Finnegan even described himself as being good at waiting for students to respond pre-T-PD. In line with this awareness for wait time 1, the data from Figure 6.2, Figure 7.2, and Figure 8.2 suggest that even at the start of the semester, Finnegan and Valeria appeared to wait on average more than 3 seconds after asking a question before resuming their turn—the wait time 1 data for Alice were inconclusive. By the end of the semester, the data suggest that Finnegan and Valeria may have used wait time 1 for longer periods of time.

The concept of wait time 2 appeared to be new for all participants, and the participants were only recorded using wait time 2 for brief periods of time in the analyzed classroom recordings. In particular, no instance of wait time 2a exceeding 3 seconds was recorded (most instances did not exceed 2 seconds) and no instance of wait time 2b was recorded. Alice may have used wait time 2 for slightly longer periods of time by the end of the semester.

The participants all identified reasons for struggling with implementing wait time 2. Finnegan shared being especially uncomfortable with wait time 2 since it was a silence he had not initiated. Yet, in the end, he expressed that he had tried to incorporate wait time 2 into his teaching. Valeria and Alice both shared being eager to provide students with affirmation and therefore struggling to wait after students' responses.

Finnegan described himself also engaging in a third type of wait time, called wait time 3 in this dissertation: waiting after making a statement to give students extra time to ask questions or ask for clarification. Although this wait time could not be reliably coded for, I believe to have observed Finnegan using wait time 3.

All participants expressed that wait time was useful. On top of giving students more time to respond, Valeria and Alice identified pedagogical (and possibly meta-mathematical) reasons for wait time's utility. Valeria noted that by being comfortable with wait time, the students could feel that she was not uncomfortable waiting. Further, they could feel less anxious about nobody talking and realize they had time to answer. Similarly, Alice wanted to show students that she was giving them time and that she did not expect them to have the answer right away. Valeria also noted that wait time helped even when it appeared not to, that is, even when she felt she waited for "too long." The second time she asked a question, someone would answer—a pattern also observed in her classroom recordings coded for wait time. She also observed that she had sometimes not realized that students had not finished talking and that wait time 2 made things "less abrupt." Valeria and Alice also identified ways in which wait time 2 could serve as the starting point for more complex student–instructor interactions: (a) Valeria noted she could use wait time 2 and then ask "Okay, does anyone want to add to that?", and (b) Alice noted she could

use wait time 2 to create opportunities for students to follow up with one another, to add to one another's responses, and to ask questions to one another or the instructor.

In addition to finding wait time useful, Alice and Valeria identified wait time as part of a set of TDMs—alongside revoicing and probing a student's thinking—that "naturally happen to the cadence of going back and forth with students" (Alice) and "rely more on what I'm doing than what my students are doing" (Valeria).

Although all participants found wait time useful, they "hated" silences (Finnegan and Valeria) or found them "uncomfortable" (Alice). To keep themselves from talking, different strategies were used: Finnegan used his fingers to count to five and Alice began to keep a water bottle next to her—a strategy I shared during a T-PD session.

For Valeria, listening to her classroom audio-recording during T-PD session 6 was a powerful experience that led her to conclude that she did not use wait time 2 and did not use wait time 1 for as long as she thought. She decided she needed to work on wait time. Although Finnegan and Alice also had access to their audio-recordings during T-PD session 6, they chose to analyze other aspects of their transcripts and made other kinds of observations about their teaching.

Furthermore, the online context appeared to change the nature of wait time for Valeria and Alice, albeit in opposite ways. For Valeria, waiting in the online context caused her "a lot of anxiety" because she did not know how to interpret silence in the online space. Alice instead found:

> I'm pretty sure that I used wait time more frequently and for longer periods of time than I would in person or than I have in person 'cause again I feel a lot more pressure in a room of people just staring.

Finally, Alice made several other observations about using wait time online: (a) she wanted to be "really deliberate about […] waiting a really long time" online to not let students make her feel uncomfortable and give away answers; (b) she did not use wait time 2 enough because she felt the need to honor student participation in the online context since she was thrilled to have online participation at all; (c) she felt that the online context required longer wait times due to "the buffer time of like, 'Oh, is anyone else going to talk? Like I don't want to talk over somebody'"; and (d) wait time was sometimes particularly uncomfortable in the online context as it felt "like talking to the void."

Inviting Student Participation

The "inviting student participation" section will be split into three subsections: (a) explicit specific-student(s)-inviting, (b) inviting further responses, and (c) instructor solicitation type.

Explicit Specific-Student(s)-Inviting

Looking at the corresponding tables (i.e., Table C.2, Table 6.12, Table 7.9, Table 8.2), one can observe that the participants differed in their usage of explicit specific-student-inviting and explicit specific-students-inviting. Whereas only Finnegan used explicit specific-student-inviting, both he and Alice used specific-students-inviting.

With respect to explicit specific-student-inviting, Valeria shared "I definitely knew that I was not going to try others like calling on people or things that I know that I am not comfortable with […] that I wouldn't know how to handle with my way of being." Accordingly, Valeria did not use explicit specific-student-inviting at all. Like Valeria, Alice did not use explicit specific-student-inviting. She shared feeling nervous cold-calling students particularly due to teaching on

Zoom—"it's like you don't know what's going on there […] I couldn't see my students and everything."

In contrast to Alice and Valeria, Finnegan grew to embrace cold-calling in the second half of the semester following a discussion about the practicalities of cold-calling in T-PD session 10—he initially had little need for it because students were initially responding to his solicitations. In addition to increasing student participation, cold-calling allowed him to address inequities that had surfaced in his class (i.e., male students participating more in the spoken classroom discourse) by calling on female students. His cold-calling followed two patterns: (a) Finnegan said the name of the student before asking (or repeating) a question, and (b) he appeared to be purposeful in his phrasing of cold-calling, frequently using formulations such as "[name], do you have an idea for […]" or "[name], do you want to explain/recontextualize […]". Finnegan concluded that cold-calling had been an important addition to his teaching repertoire and one of the most useful practices he had picked up from the T-PD. He planned to continue using it in the future and combine it with "probing a student's thinking" and "creating opportunities to engage with another's reasoning."

Both Finnegan and Alice used explicit specific-students-inviting. Specifically, on some days in which they implemented group work, they asked whether someone from a certain group of students could share (e.g., "Can someone from group 9 tell me …?"). Finnegan, who implemented group work on CR1, CR2, and CR6, used explicit specific-students-inviting on CR6. Alice, who implemented group work on CR1, CR2, CR6, and CR8, used explicit specific-students-inviting on CR2 and CR8. Alice shared that she had—at the beginning of the semester—asked someone "Would you be comfortable sharing what you guys did in your group with the whole class?" Thus, some of her explicit specific-students-inviting might have been

done in conjunction with giving groups advance notice in their breakout rooms. Finnegan appeared to pursue a different strategy for providing students with advance notice by requesting in CR6—before sending students into their breakout rooms—that "at least one person is going to have to answer questions." Similar to Finnegan, Valeria was observed in CR2, CR5, CR6, and CR8 asking students (before sending them into breakout rooms) to select a group member to present their group's findings. That said, Valeria never actually called on groups (or their representatives), preferring to instead opt for more "general," "open," "passive" invitations like "Does anyone want to share what they […]?"

Inviting Further Responses

Looking at the corresponding tables (i.e., Table 6.10, Table 7.9, Table 8.2, Table C.2), one can see that "inviting further responses" was a TDM used by all T-PD participants—although to varying degrees (Finnegan: 5 times, Valeria: 38 times, Alice: 2 times). Alice's two uses of inviting further responses came in CR6 and CR9, and Finnegan's uses were sprinkled throughout the semester (i.e., CR1, CR4, CR7, CR9). Thus, in neither Alice's nor Finnegan's case there is compelling—if any—evidence that the T-PD changed their use of inviting further responses. In slight contrast, Valeria used inviting further responses more frequently in the second half of the semester, but only slightly so.

Their frequency of use was mirrored in the participants' discourse about "inviting further responses." Finnegan noted that it was great (because it allows students to see multiple solutions and can include people who did not answer first) but that he did not have the time for it. Although Alice made no remarks about "inviting further responses" in the interviews, her case report illustrated that Alice felt under time pressure while teaching and, thus, there may have been a connection between the lack of time she perceived and her rarely using inviting further

responses. In addition to time constraints, Finnegan shared that despite having used this TDM and finding that "it definitely worked," he was not sure how comfortable he was "using it all the time." Like Alice, Valeria did not directly speak about inviting further responses in her interviews, but she spoke about inviting student participation in a "very general" way. As her case report suggests, Valeria may have found inviting further responses a non-coercive alternative to calling on students.

Finally, Alice made a remark in the mid-T-PD interview that might explain a decreased need to invite further responses: "Something that is nice is, there are a couple students like even after someone's answered, they'll still type their answer in the chat or they'll follow up and say, you know, maybe give more of an explanation." (Some evidence for this observation—albeit from Finnegan's class—can be seen in Table 6.5.) Thus, in an online context where the chat offered students the opportunity to reply near simultaneously and defy the traditional consecutive turn-taking structure of spoken discourse, there may have been less of a need to use inviting further responses to receive further responses.

Instructor Solicitation Type

Studying the corresponding tables (i.e., Table C.2, Table 7.9, Table 8.2), I make observations about: (a) the rate at which participants made what, how, or why-type solicitations, (b) which what, how, or why solicitations participants preferred to ask, (c) whether students responded to the instructors' what, how, and why solicitations, (d) whether instructors asked for questions and whether things made sense, and (e) whether instructors received responses to their requests for questions.

Looking at Table 9.1, one can see that the three instructors varied in terms of the rate at which they made what, how, or why solicitations in their classrooms. In particular, whereas

Finnegan was remarkably consistent in making a what, how, or why solicitation between on average every 1:08 and 1:36 minutes, Alice and Valeria appeared to make what, how, or why solicitations at slower rates, which varied a lot from classroom recording to classroom recording. This variation is especially clear in the case of Alice, who appeared to make far fewer what, how, or why solicitations on quiz days and at one point (i.e., CR7) made only one what, how, or why solicitation in 54 minutes.

For all participants, what-solicitations were the most-posed solicitations in most classroom recordings (i.e., for Finnegan in nine out of nine, for Valeria in eight out of eleven, and for Alice in eight out of nine). All participants also appeared to make how-solicitations relatively consistently, but less frequently. There were, however, some differences in terms of why-solicitations: Whereas Finnegan appeared to make more why-type solicitations as the semester progressed—recall that he took to probing a student's thinking—Valeria and Alice appeared to make fewer why-type solicitations as the semester went on. All participants received student responses to most of their what, how, and why solicitations.

Finally, all participants very consistently asked whether students had questions or whether things made sense. That said, there were differences in student uptake. Whereas Valeria's students rarely responded with questions, Alice's students usually responded with questions around half of the time. (Valeria's students also rarely asked questions unprompted. That said, Valeria's students had more opportunities to ask questions in their breakout rooms as Valeria more frequently implemented group work than Finnegan and Alice.) Finnegan's students started out responding with questions at least half of the time (i.e., in CR1–CR3) but later responded with fewer questions as the semester went on.

Table 9.1

Participants' Average Times Between What, How, or Why Solicitations by Classroom Recording

Participant	Average Time Between What, How, or Why Solicitations by Classroom Recording										
	CR1	CR2	CR3	CR4	CR5	CR6	CR7	CR8	CR9	CR10	CR11
Finnegan	1:33[b]	1:17[b]	1:36	1:28[a]	1:14[a]	1:08[b]	1:23[a]	1:17	1:17	n/a	n/a
Valeria	5:44[b]	1:54[b]	3:55[b]	4:55[b]	2:10[b]	2:19[b]	3:22[b]	2:43[b]	6:24[b]	5:23[b]	2:26[b]
Alice	4:03[b]	2:30[b]	7:30[a]	4:56	14:40[a]	7:24[b]	54:00[a]	3:31[b]	2:33	n/a	n/a

[a] Counts marked with a superscript "a" were quiz days.

[b] Counts marked with a superscript "b" were group work days.

Revoicing

Looking at the corresponding tables (i.e., Table C.3, Table 7.8, Table 8.3), one can see that (simple) "revoicing" was used frequently by participants. In fact, in each classroom recording that was coded, half or more student turns were simply revoiced. Full revoicing was also used by the participants but only very rarely: Finnegan was recorded using it twice, Valeria five times, and Alice once. Looking at the percentages of turns that were either simply or fully revoiced, one can note some differences between the participants: Finnegan simply or fully revoiced 68.5% of student turns, Valeria 83.7%, and Alice 79.3%. Looking at each participants' use of revoicing across the semester, no pattern stands out for Finnegan or Alice. (That said, Alice felt that she tried "to use more revoicing as the semester went on.") For Valeria, after an initial drop in revoicing from CR1 to CR3, there may have been a trend towards more revoicing.

In the interviews, all participants described different ways in which they used revoicing (see Table 9.2). Several uses the participants described related particularly to teaching in the online context. Both Finnegan and Alice, for instance, described using revoicing to bring chat messages into the spoken classroom discourse since not all students may read the chat. Alice had come up with this form of revoicing after engaging with her classroom transcripts during T-PD session 6: She observed while reading the transcript that she did not know what she was talking about, before eventually realizing that she must have been referring to a chat message that had caught her eye in the moment. This observation prompted her "to be more aware of saying what's been asked in the chat, instead of just answering it and assuming that everybody saw it." Alice shared her observation with the other participants in the T-PD session, and this novel use of revoicing appeared to resonate with Finnegan, who shared in the mid- and the post-T-PD interview that he had incorporated this use of revoicing into his teaching.

Valeria and Alice also spoke more about why they took to revoicing. Valeria identified revoicing as a TDM that she had been trying out and that she grabbed onto more than other TDMs because it relied more on what she was doing than on what her students were doing. Alice expressed that revoicing—like probing and wait time and unlike asking students to revoice—"naturally happen[s] to the cadence of going back and forth with students." She also appeared to imply that it was "fairly easily applicable in this [online] setting." Finally, Valeria added an interesting caveat, noting that if she was feeling "spacey that day," her revoicing may have just ended up confusing students more, that is, the utility of revoicing depended on the day for her.

Table 9.2

Uses of Revoicing Described by the Participants in the Interviews

Participant	Uses of Revoicing Described in Interviews
Finnegan	• Revoicing a student's correct answer to make it more in line with the mathematical register. (He tried to make fewer "leaps" than in the past.) • Responding to students' requests for re-explanation of something by revoicing his past explanation in a different mathematical context (e.g., algebraic, numeric, graphic) to expose his students to as many viewpoints as possible. • *Revoicing a student's chat contribution since "not everyone's reading the chat."*
Valeria	• Revoicing while drawing or taking tiny notes. • Using (full) revoicing to make sure she understood what a student said and did not do a disservice to their ideas. • Keeping students from feeling the pressure of having to repeat their answers. • Using full revoicing to give credit to students' ideas.
Alice	• Revoicing students' questions. • *Revoicing correct private chat messages.* • *Revoicing chat messages because students may not see a chat message.* • *Honor students' participation in the online context.*

Note. Italicized bullet points reflect uses of revoicing participants mentioned in conjunction with the online context.

Asking Students to Revoice

Looking at the corresponding tables (i.e., Table 6.10, Table 7.9, Table 8.2), one can see that "asking students to revoice" was not embraced by the T-PD participants. Finnegan and Alice were not recorded using this TDM a single time, and Valeria was only recorded using it a total of three times—Valeria's three instances all occurred during her last three classes. Of Valeria's three instances, two were met with silence and one received an incommensurate response. Valeria concluded that trying this TDM had "definitely failed."

"Asking students to revoice" appeared to not be taken up by the participants due to a mix of individual concerns and a common reflection that the TDM did not feel natural. Despite this non-uptake, Finnegan and Alice noted that they would be interested in trying this TDM in an in-person setting where Finnegan felt it would work very well. Table 9.3 presents a list of hesitations which the participants shared in their interviews about "asking students to revoice."

Table 9.3

Participants' Hesitations About Asking Students to Revoice

Participant	Hesitations About Asking Students to Revoice
Finnegan	• The TDM might position a revoicee as someone who did a bad job explaining (i.e., in need of revoicing). In this vein, being revoiced could feel condescending. • Using this TDM might make a revoicee feel like something was taken away from them. • This TDM does not feel as natural in the online context because students cannot turn to each other and talk. Finnegan speculated it would work much better in an in-person context.
Valeria	• This TDM feels similar to calling on someone, which is not "her." • Valeria does not know how to use this TDM as it is new for her. • Valeria does not know what tone she should use with this TDM. • Valeria noted that "it was already hard to make [students] say what they were thinking, let alone talking about what somebody else said."
Alice	• This TDM does not come naturally to Alice.

Probing a Student's Thinking

From the start of the semester, all three participants used probing—both clarifying-probing and deepening-probing—in their teaching. That said, there were marked differences in how often the participants used probing and how they used probing. Comparing Table C.3, Table 7.8, and Table 8.3, there appear to be several takeaways:

1. After "probing a student's thinking" was introduced in the T-PD, Finnegan took to using it in both a clarifying and a deepening manner. The introduction of "probing a student's thinking" did not lead to a noteworthy increase in probing by Alice and Valeria. In fact, Valeria's use of probing may have decreased.
2. Finnegan's use of clarifying-probing was mostly student-centric, whereas Alice and Valeria used clarifying-probing in a more instructor-centric way.
3. All three participants appeared to mainly use deepening-probing in a student-centric way.

In understanding the rationale for these patterns, recall the participants' stances and experiences with probing. For Finnegan, (deepening-)probing became a way to feel like "I'm actually doing my job and actually teaching them well" as it was "a lot easier to see that students are making progress in that learning […] and making those connections between these different topics." Thus, although he found that it made his teaching harder—"because I'm actually putting a lot more thought into what I'm saying and into some of the things I'm asking students to do"—it made teaching more "rewarding" for him. Depending on the class he taught, he wanted to use more probing in the future.

Valeria noted that she "love[d]" probing and described that this move had to do with her because it was about her understanding what a student was saying—which made the move easier for her. (Recall Valeria's discomfort with asking things of students.) In addition to helping her

understand what a student was saying, “probing questions help me a lot to figure out how to continue with things.” She also brought up that she used probing “to make sure that they keep remembering what is it that we want.” She appeared to have positive experiences with using probing. That said, from the way she spoke of probing and the examples she gave, she seemed to speak of clarifying-probing in particular.

Last, Alice shared seemingly conflicting thoughts on probing. On one hand, she felt that probing “naturally happen[s] to the cadence of going back and forth with students.” On the other hand, she felt “weird” and “unsure” about using it on Zoom as it felt to her like putting students on the spot—likening it to cold-calling—and she was nervous to do so when she could not see her students. Yet, she recognized that she should not necessarily feel this way and that it should be possible for her to communicate in a caring way to make students not feel put on the spot. She wished after the end of the semester that she would have “done a little bit more” probing.

In short: (a) Finnegan had positive experiences with probing that made teaching more rewarding for him; (b) Valeria may have only considered using clarifying-probing; and (c) Alice struggled with probing in the online context.

Creating Opportunities to Engage With Another’s Reasoning

Looking at the corresponding tables (i.e., Table 6.10, Table 7.8, Table 8.3), one can see that Alice and Finnegan did not really use this TDM (especially before this TDM was introduced in the T-PD), whereas Valeria used this TDM in all classroom recordings, except the first.

Studying how the participants used this TDM, one can note that the two times Alice used it, she used it to “create opportunities for students to evaluate another’s reasoning.” Finnegan used this TDM in an evaluating manner once as well as five times to “create opportunities for students to add to another’s reasoning.” Both Alice’s and Finnegan’s uses of this TDM came in

the second half of the semester after the TDMs had been introduced in the T-PD (i.e., CR6 and CR9 for Alice, CR5–CR8 for Finnegan). After the first class, Valeria used "creating opportunities to evaluate another's reasoning" reliably once or twice every class. In the latter half of the semester, she also tried out "creating opportunities to add to another's reasoning"—a total of eight times split across three classes. Finally, she also occasionally used "creating opportunities to understand another's reasoning"—once in the first half of the semester (i.e., CR2) and five times in the second half of the semester (i.e., CR6, CR9, CR11).

The numerical patterns observed are reflected in the participants' discourse. Finnegan, for instance, noted in the mid-T-PD interview that he thought he had not used this TDM. In the post-T-PD interview, however, he described:

> Depending on the class I teach, I would like to do more of the probing into students' reasoning, even if it's me probing a student or getting a student to expand on what they've said, or getting another student to expand on what they said.

In short, he described wanting to use "probing a student's thinking" as well as "creating opportunities to add to another's reasoning," and, as aforementioned, five of his six uses of "creating opportunities to engage with another's reasoning" were of the "adding to"-subtype.

In the mid-T-PD interview, Alice had shared that students in her class engaged with one another's responses of their own volition, usually in the chat. Further, she noted that wait time 2 could be used as an opportunity for students to add to or understand another's reasoning. In her post-T-PD interview, Alice noted not having used "creating opportunities to engage with another's reasoning" a lot. And, as with several other TDMs, Alice noted that the online context made this TDM more difficult as she was not able to lay "enough of the groundwork for group work at the beginning of the semester," which she had viewed as necessary before using the

TDM during whole-class time. That said, Alice implied that she wanted to try to incorporate the TDM more in a typical semester.

In both the mid- and the post-T-PD interview, Valeria described trying to "create opportunities to add to another's reasoning," but often being met with silence. Similar to Alice, Valeria identified wait time 2 as an opportunity to pace herself and respond to a student's response with "Okay, does anyone want to add to that?" instead of jumping into "'yes' or 'almost,' or whatever I say after that." She concluded in the post-T-PD interview that this TDM was her "pitfall"—although she suspected that the TDM had probably worked "when there was a mistake or something."

In short, even though there were no fundamental objections the participants had to "creating opportunities to engage with another's reasoning," the participants—Alice and Finnegan especially—did not often use this TDM. Alice identified the online context as a reason for finding the TDM more difficult to use, and Valeria reported little success with receiving student responses. That said, both Finnegan and Alice wanted to try using the TDM in the future.

Group Work

Looking at the corresponding tables (i.e., Table C.3, Table 7.8, Table 8.3), one can see that all participants implemented group work: Finnegan in three classroom recordings (i.e., CR1, CR2, CR6), Alice in four classroom recordings (i.e., CR1, CR2, CR6, CR8), and Valeria in all classroom recordings. That said, when Finnegan and Alice implemented group work, they only implemented one instance of group work, whereas Valeria implemented one to three instances of group work every class—only during two classes did Valeria implement only a single instance of group work. From the instances of group work whose length is known, one can see that Alice implemented group work for very brief periods of time (i.e., 3–8 minutes), Finnegan for longer

periods (i.e., 10–22 minutes), and Valeria for slightly even longer periods (i.e., on average two implementations of group work of 8:50 minutes each).

Both Finnegan and Alice experienced similar challenges with group work as the semester progressed: (a) students engaged less with their groups members, and (b) there was little time to implement group work. Alice identified three possible reasons for decreased student participation in groups: (a) not making her group work expectations clear at the start, (b) students being exhausted, and (c) the group work tasks not actually requiring group work. As described in Finnegan's case report, Finnegan's group work implementation may also have suffered from him not appearing to make his expectations for group work explicit beyond stating on the second day of class (i.e., CR1): "Yeah, so yeah, please work on these as groups." All participants also pointed to technological issues relating to the implementation of group work: (a) Valeria mentioned having "huge anxiety" going into breakout rooms and feeling like she was helping less, (b) Alice expressed how long it took to go from group to group and how long students with questions had to wait for her to arrive, (c) Finnegan observed that it was difficult for students to work together on mathematics online, especially when not everyone had a tablet, and (d) Finnegan and Alice observed that they could not easily see all students at once, which meant they could not easily encourage students to interact.

For Alice, her implementation of group work not going the way she wanted became a barrier to trying many of the TDMs. Because she felt she had not laid "enough of the groundwork for group work at the beginning of the semester," she implied it had been difficult for her to use "asking students to revoice," "probing," and "creating opportunities to engage with another's reasoning."

As previously discussed under "Inviting Student Participation," both Finnegan and Alice were recorded engaging in specific-students-inviting. Alice shared that she at least once gave a group advance notice that she would call on them, whereas Finnegan and Valeria were recorded giving their students advance notice before sending them to breakout rooms. Valeria, however, did not call on the groups' chosen representatives. Valeria was also the only one to allow students to opt out of group work. (Several students in Alice and Finnegan's classes probably also opted out of group work, but they did not do so with their instructor's express permission.)

Finally, it should be noted that there were some differences in terms of "initial conditions." Whereas Valeria had gained experience implementing group work online in the previous semester and had come up with a list of possible improvements, Alice implemented group work online for the first time during the T-PD semester. Despite being unhappy with her group work implementation during the semester, Alice felt "a lot more confident moving forward with more tools for group work and knowing that that's something that, at least [the university] seems to encourage, or the teaching support team does."

Student Discourse

Below, with the help of Table C.4, Table C.5, Table 7.10, Table 7.11, Table 8.4, and Table 8.5, I discuss the six student discourse dimensions: mode of discourse, discourse type, student talk length, student talk type, teacher solicitation type, and explicit evaluation.

In Finnegan's, Valeria's, and Alice's respective classrooms, students communicated both via speech and writing and there seemed to be a trend towards more written student turns as the semester progressed. That said, the proportion of written to spoken turns varied across the three participants: Whereas by the end of the semester, over 80% of student turns were written in Finnegan's class, Alice's class never exceeded 66% of written student turns—and usually stayed

well under this number. Valeria's proportion of written to spoken turns was more erratic across the semester and although she exceeded Finnegan's proportion of written student turns once, she generally stayed well below it. There was also some variation with respect to private chat use: (a) in Finnegan's class students were not recorded using the private chat; (b) in Valeria's class the percentage of private chat turns (out of all written turns) never exceeded 50% and was usually well below that; and (c) in Alice's class, the private chat was rarely used at the start of the semester, then used almost exclusively in the middle (compared to the public chat), and then used less again towards the end of the semester.

For all three participants, most student turns were discourse about mathematics rather than logistics. On the days with the fewest turns about mathematics, the percentages were: 76.5% (Finnegan), 72.0% (Valeria), and 73.3% (Alice). Most days, 80–100% of students turns were discourse about mathematics.

For all participants, short 1–4 word turns were most frequent on most days (i.e., on 7 out of 9 days for Finnegan, on 7 out of 11 days for Valeria, and on 5 out of 9 days for Alice). 21+ word turns were usually the least frequent. For Valeria and Finnegan, there may have been a slight increase in 5–20 word turns in the second half of the semester compared to the first, and student turns may have gotten longer in Valeria's class (with respect to the student talk length categories, excluding subcode "n/a").

For all participants, what-type student talk was most frequent on most days (i.e., on 9 out of 9 days for Finnegan, on 8 out of 11 days for Valeria, and on 8 out of 9 days for Alice). In Finnegan's class, the number of why-type student talk turns appeared to increase as the semester progressed, whereas the numbers decreased for Alice and Valeria. Both Alice and Valeria recorded zero instances of why-type student talk turns in four of their last five classroom

recordings. That said, Alice's number of how-type student talk turns appeared to increase as the semester progressed. Looking across the entire semester, one can see that the distribution of what:how:why percentages per participant were rather similar: 69:21:9 (Finnegan), 69:17:15 (Valeria), and 68:24:9 (Alice). Finally, with respect to "question" turns, one can note that students' questioning patterns were quite different in the three participants' classes. In Valeria's class hardly any student turns were questions. On 6 out of Valeria's 11 class days, not a single student turn was coded as a question. The situation in Alice's classroom recordings was a bit more mixed, ranging from no question turn to one question turn roughly every 4 minutes. Finally, there were many question turns in Finnegan's class, but the rate at which students asked questions slowed over the semester. Whereas students asked a question on average every 1:41 minutes in CR1, they only asked a question on average every 13:40 minutes in CR9.

Since instructor solicitations were already covered in a prior subsection (i.e., "Instructor Solicitation Type"), I will limit my attention here to two issues: (a) to what extent "teacher solicitation type" and "student talk type" aligned, and (b) whether students asked questions without being prompted. Looking at the alignment between "teacher solicitation type" and "student talk type," the patterns are very similar. For all participants, over 90% of the given what-solicitations that received responses ended up in student what-talk—similarly for why-solicitations and why-talk. For how-solicitations and how-talk, the alignment was less strong: For all participants, between 80 and 85% of how-solicitations that received responses ended up as how-talk. The exact percentages of what/how/why-solicitations (that received responses) that ended up as what/how/why-talk (in the format [what][how][why]) are: [92.7][84.2][90.5] (Finnegan), [95.2][80.5][95.8] (Valeria), and [92.9][80.5][92.3] (Alice). Finally, looking at the "question" student turns that were unsolicited, one can see that whereas it was rare for students

to ask an unsolicited question in Valeria's class (i.e., it only happened 5 times across the entire semester), it was more common in Alice's (41 times) and Finnegan's (68 times). These numbers represent 55.6%, 70.7%, and 45.3% out of the total number of "question" student turns in the participants' respective classes.

Studying the percentages of student turns not evaluated (out of all applicable student turns, that is, subcode "n/a" aside), one can see that in all participants' classes most applicable student turns were evaluated. Studying this data in more depth, one can see that in Alice's class at most 11.1% of student turns were not evaluated, and in four out of nine classroom recordings, all student turns were evaluated. Alice may have evaluated student turns slightly more often as the semester progressed. Valeria's percentages of non-evaluated student fluctuated from class to class, ranging from 0% to 16.7%. Finally, Finnegan may have engaged in slightly less evaluating as the semester progressed, his percentages of non-evaluated student turns ranging from 5.4% to 22.4%.

Conclusion

As this chapter thus far has shown, the participants shared a great number of similarities while also each going their own way and wrestling with their own issues. Below, I summarize some of these similarities and differences while trying to address the overarching etic issues.

Change in Classroom Discourse in Each Participant's Class Over the Course of Their Participation in the MDISC T-PD

As this multicase report demonstrated, there were many similarities between the classroom discourses in the participants' respective classes. Yet, idiosyncrasies also became apparent. I will discuss the changes by TDM and in terms of student discourse below.

Finnegan and Valeria may have used wait time 1 for longer periods of time by the end of the semester. The data for Alice are inconclusive. With respect to wait time 2, neither Finnegan's nor Valeria's wait time appeared to increase. That said, there may have been a slight increase in wait time 2 for Alice—although none of her instances of wait time 2a crossed Rowe (1986) 3-second threshold.

All participants frequently used simple revoicing throughout the semester. Full revoicing was used by all participants, but only very rarely. Some differences were seen in how often the participants revoiced: Finnegan revoiced 68.5% of student turns, Valeria 83.7%, and Alice 79.3%. For Valeria, after an initial drop in revoicing from CR1 to CR3, there may have been a trend towards more revoicing. No pattern stood out for Finnegan or Alice.

All participants largely stayed away from "asking students to revoice." Finnegan and Alice were not recorded using this TDM even once, and Valeria only used it a total of three times—all three occurring after the TDM's introduction in the T-PD. Valeria's three instances of asking students to revoice were met with silence twice and an incommensurate response once.

Only Valeria used "creating opportunities to engage with another's reasoning" regularly. That said, Finnegan's and Alice's use of the TDM, irregular as it may have been, came after its introduction in the T-PD. Looking across the three subtypes of this TDM used by the participants, one can see that "creating opportunities to evaluate another's reasoning" was more common than "creating opportunities to add to another's reasoning," with "creating opportunities to understand another's reasoning" a distant third. All participants used group work. The implementing of group work appeared to largely stay the same throughout the semester for each participant.

"Probing a student's thinking" was used to some extent by all participants. After its introduction in the T-PD, Finnegan used this TDM more in both a clarifying- and a deepening-probing manner. Its introduction did not lead to a noteworthy increase in probing by Alice and Valeria—possibly the opposite in Valeria's case. Whereas Finnegan used clarifying-probing in a student-centric way, Alice and Valeria used clarifying-probing in a more instructor-centric way. When participants used deepening-probing, they mostly used it in a student-centric way.

With respect to explicit specific-student(s)-inviting, one could observe that Finnegan took to cold-calling after it was discussed in more depth in T-PD session 10, whereas Valeria and Alice did not. Both Finnegan and Alice were recorded using specific-students-inviting at several points in the semester, and although Valeria asked students to choose representatives of their groups to share their group's findings, she did not call on groups. Valeria did, however, use "inviting further responses" regularly and slightly more so in the second half of the semester. Alice only used "inviting further responses" twice, but she did so in the second half of the semester. Finally, Finnegan's five uses were sprinkled throughout the semester from CR1 to CR9, that is, his use of inviting further responses did not seem to change.

Moving to instructor solicitations, one can see that with respect to the rate at which what, how, or why solicitations were made by the participants, there were changes across the semester for Alice and Valeria. That said, these did not appear to be linked to the T-PD. For Alice, they seemed to be linked to quiz days. In line with the findings about probing a student's thinking, Finnegan's use of why-type solicitations increased as the semester progressed, whereas Valeria and Alice appeared to make fewer why-type solicitations as the semester went on. I believe this shows the close connection between deepening-probing and why-solicitations and how deepening-probing can serve to increase instructors' why-solicitations. Finally, as the semester

went on, Finnegan's students started responding with fewer questions when asked for questions. It is unclear whether any connection exists between this change and the T-PD.

Regarding the student discourse, one can also observe several changes. First, there seemed to be a trend towards more written students turns as the semester progressed in all three participants' classes. Second, in Valeria's and Finnegan's classes, there may have been a slight increase in 5–20 word turns in the second half of the semester compared to the first, and student turns may have gotten longer in Valeria's class (with respect to the student talk length categories, excluding subcode "n/a"). Third, for Finnegan the number of why-type student talk increased as the semester progressed, whereas the numbers decreased for Valeria and Alice. A connection to Finnegan's increased use of deepening-probing seems likely (as explored in Table 6.11). Fourth, for Alice, the number of how-type student talk turns appeared to increase as the semester progressed. Fifth, the rate at which "question" student turns occurred in Finnegan's class slowed over the course of the semester. Last, whereas in Finnegan's class fewer student turns were evaluated as the semester progressed, the opposite appears to have been the case in Alice's class.

In short, the most pronounced changes that occurred in the semester were: (a) Finnegan and Valeria's possible increase in wait time 1, (b) Finnegan's uptake of probing a student's thinking and the likely ensuing increase in why-solicitations and student why-talk, and (c) Finnegan's uptake of cold-calling. Minor changes that occurred over the semester were: (a) Alice's possible increase in wait time 2a, (b) Valeria using revoicing slightly more often, (c) Valeria trying out asking students to revoice towards the end of the semester, (d) Finnegan and Alice trying out "creating opportunities to engage with another's reasoning," (e) Valeria using probing slightly less in the second half of the semester, (f) Valeria using "inviting further responses" slightly more in the second half of the semester, (g) Alice trying out "inviting further

responses" in the second half of the semester, (h) a decreased use of why-type solicitations by Valeria and Alice as the semester progressed and a decrease in student why-type talk, (i) Finnegan's students responded with fewer questions when asked for questions and posed fewer questions in general, (j) a trend towards more written student turns as the semester progressed in all three participants' classes, (k) possibly slightly longer student turns in the second half of the semester in Valeria's class (with respect to the student talk length categories, excluding subcode "n/a"), (l) a slight increase in how-type student talk in Alice's class, and (m) a decreased percentage of evaluated student turns in Finnegan's class but an increased percentage of evaluated student turns in Alice's class.

Participants' Talk About Their Use of the TDMs

Participants' talk about their use of the TDMs has been woven throughout this multicase report, bringing to light: (a) how participants spoke of each TDM (e.g., whether it was useful or not), (b) how participants described using each TDM (or not), and (c) how participants explained why they used TDMs (or not) in the way they did. In the individual case reports, I highlighted one emic issue per participant that I felt strongly affected the participants' engagement with and talk about their use of the TDMs: (a) for Finnegan, the issue of how to invite student participation, (b) for Valeria, the balancing of respecting students' agency versus wielding instructor authority to foster students' learning, and (c) for Alice, the engagement with the T-PD during a semester of survival. Inevitably, these emic issues will resurface below, where I share some of the highlights of participants' talk about their use of the TDMs.

All participants found "waiting" to be a useful TDM but "hated" silences (Finnegan and Valeria) or found them "uncomfortable" (Alice). That said, Alice and Valeria identified wait time as part of a set of TDMs—alongside revoicing and probing a student's thinking—that

"naturally happen to the cadence of going back and forth with students" (Alice) and "rely more on what I'm doing than what my students are doing" (Valeria). For Valeria, listening to her classroom audio-recording proved a powerful experience that may have contributed to an increase in wait time 1 but did not lead to an increase in wait time 2. More broadly, all participants appeared to struggle with wait time 2, with Valeria and Alice citing an eagerness to provide students with immediate affirmation. Finnegan introduced a third type of wait time: waiting after making a statement to give students extra time to ask questions or ask for clarification.

The participants described many different uses of revoicing (see Table 9.2), with several related directly to teaching in the online context. One use stands out to me due to its reported uptake by all participants: revoicing students' chat messages to make sure everyone is aware of what is being said in the chat. Alice came up with this version of revoicing after engaging with a classroom transcript of her class and observing that at one point she was not sure what she was talking about. This version of revoicing resonated with Finnegan and Valeria, who both spoke of incorporating this form of revoicing into their teaching.

All participants had hesitations about "asking students to revoice," which are captured in Table 9.3. For Finnegan and Alice, these concerns were so strong that they were not recorded using the TDM. As previously described, Valeria used it three times, albeit to little success, and concluded that trying this TDM had "definitely failed."

Finnegan verily took to "probing a student's thinking" after its introduction in the T-PD and shared that although it made his teaching harder, it also made it more rewarding. From Valeria's discourse, it seemed that she thought mostly of clarifying-probing when she spoke of

probing, which she noted she "love[d]". Alice shared that she felt "weird" and "unsure" using probing in the online context and drew a connection to cold-calling.

Finnegan and Alice felt like they had not really used "creating opportunities to engage with another's reasoning" and identified the online context as an obstacle. Valeria described trying to use this TDM but often being met with silence.

The T-PD participants considered group work a way of creating opportunities to engage with another's reasoning, which is why I included group work in all discussions of this TDM. Both Finnegan and Alice ran into similar issues with group work as the semester progressed: (a) students engaged less with their groups members, and (b) there was little time to implement group work. Alice identified three possible reasons for decreased student participation in groups: (a) not making her group work expectations clear at the start, (b) students being exhausted, and (c) the group work tasks not actually requiring group work. All participants also pointed to technological issues relating to the implementation of group work: (a) Valeria mentioned having "huge anxiety" going into breakout rooms and feeling like she was helping less, (b) Alice expressed how long it took to go from group to group and how long students with questions had to wait for her to arrive, (c) Finnegan observed that it was difficult for students to work together on mathematics online, especially when not everyone has a tablet, and (d) Finnegan and Alice observed that they could not easily see all students at once, which meant they could not easily encourage students to interact.

After initially sharing a negative outlook on cold-calling with Valeria and Alice, Finnegan grew to embrace cold-calling and sought to increase student participation in general and the participation of female students in particular. He found cold-calling to be an important addition to his teaching repertoire and one of the most useful practices he had picked up from the

T-PD. Valeria and Alice remained uncomfortable with the thought of cold-calling all semester. Alice, like Finnegan, did, however, call on groups of students to present their group work findings. Valeria, on the other hand, shared that although she asked students to choose a representative to share their respective group's findings, she felt uncomfortable calling on them.

Instead of specific-student(s)-inviting, Valeria instead preferred what she called a more "general," "open," "passive" way of inviting student participation. Evidence of this can be seen in her use of "inviting further responses." Finnegan thought that "inviting further responses" was great, but he shared that he did not have the time for it. Although Alice did not link her rare instances of "inviting further responses" to time pressure, Alice's case report demonstrated that Alice decidedly felt under time pressure.

As the above shows, the emic issues explored in the individual case reports resurfaced. That said, I will briefly summarize the connections I see. First, Finnegan, who sought to invite student participation, found TDMs extremely useful that helped him invite student participation. In particular, cold-calling allowed him to increase student participation (in general and of female students in particular), probing allowed him to go deeper with and clarify students' responses, and wait time 1 gave students a longer period of time to respond. Finally, Finnegan, in speaking about the future, also thought about how he could create opportunities to engage with another's reasoning by first calling on a student and then calling on a second student to engage with the first student's response by asking a prompt like "Why do you think they think that?"

Second, Valeria, who weighed wielding her authority to engage students against respecting students' agency, found TDMs useful that were more centered on what she could do rather than on what her students were doing. Thus, she gravitated towards wait time, revoicing,

(clarifying-)probing, as well as "open" invitations like "inviting further responses" and "creating opportunities to add to another's reasoning."

Finally, Alice found herself in survival mode during the semester and mostly used the TDMs she was already familiar with coming into the semester: waiting and revoicing. Time constraints and the online context seem to have contributed to Alice's experience of survival during the semester and appeared to thwart any plans she had to try out the TDMs. A big obstacle to implementing many of the TDMs—in particular, "asking students to revoice," "probing a student's thinking," and "creating opportunities to engage with another's reasoning"—she identified was that she first wanted to use them during group work time to familiarize students with them before using them during whole-class time. Since group work participation decreased during the semester and Alice felt that she had not "done any meaningful group work," she concluded that she did not lay enough groundwork at the beginning of the semester to later bring in the aforementioned TDMs into the whole-class discourse.

CHAPTER 10: DISCUSSION

As aforementioned, T-PD for GTAs beyond their first year of teaching is rare and research on such T-PD is only just beginning. Thus, this dissertation contributes to an emerging area of research and offers many lessons. In this chapter, I first explore some takeaways: takeaways from studying the quintain, takeaways from the three cases, takeaways about each TDM, and takeaways about sequencing the TDMs. I then share some of the limitations I see for this study as well as some future directions for this line of work.

Takeaways

I begin by sharing takeaways from studying the quintain. I then proceed to offering takeaways from the individual cases before moving to takeaways about each TDM and takeaways about sequencing the TDMs.

Takeaways From Studying the Quintain

The goal of this dissertation study was to better understand a quintain, in particular, the modification of the MDISC T-PD, offered in the university context to mathematics-teaching GTAs who were no longer in their first year of teaching. Looking across the three individual cases, I conclude that with respect to TDM uptake, the T-PD was a mixed success. As Cirillo et al. (2014) had observed:

> Teachers tend to identify closely with the [waiting, inviting student participation, and revoicing], recognizing them in their existing practice. The teachers we have worked with, however, often discover new, more purposeful uses for the moves. [Asking students to revoice, probing a student's thinking, and creating opportunities to engage with another's reasoning] require different levels of student participation than what teachers

> and their students might be used to. In particular, students need to actively listen to each other for teachers' intended purposes to be fully realized. (p. 144)

These observations largely held true for the three participants. Wait time—at least wait time 1—was not a novelty to participants, and they already engaged in wait time 1, inviting student participation, and revoicing at the start of the semester. Yet, apart from these commonalities, each participant made slightly different choices in their practice. Finnegan took to probing a student's thinking and tried out inviting student participation in a way new to him (i.e., cold-calling). Valeria, despite pedagogically-rooted hesitations, tried out TDMs new to her (e.g., asking students to revoice), but there was little uptake by the students. Finally, Alice had a hard time working in novel TDMs as she was in survival mode and felt very constrained by the amount of content that needed to be covered as well as the online environment.

Although each case's emic issue provides insight into why the participants used the TDMs in the way they did, I should also mention the possibility of gender influencing the participants' uptake and use of the TDMs. As noted by MacNell et al. (2015):

> On the one hand, students expect female instructors to embody gendered interpersonal traits by being more accessible and personable. However, these same traits can cause students to view female instructors as less competent or effective. On the other hand, female instructors who are authoritative and knowledgeable are violating students' gendered expectations, which can also result in student disapproval. (p. 294)

Thus, female instructors—like Valeria and Alice—have a difficult balancing act to engage in if they do not want to be met with poor student ratings of teaching—ratings which have been shown to be influenced by gender bias (e.g., Basow, 1995; MacNell et al., 2015; Young et al., 2009). Consequently, gender bias may have made it more difficult for Valeria and Alice than

Finnegan to use more authoritative TDMs like inviting student participation via cold-calling, student-centric forms of probing, and asking students to revoice for fear of violating gendered expectations and being met with student disapproval.

In summary, the three T-PD participants ran the gamut from taking up TDMs novel to them (Finnegan), to trying out TDMs novel to them (Valeria), to largely forgoing TDMs novel to them (Alice)—a pattern that raises the question of the influence of gender bias on the participants. That said, although their stories end for the reader here, all T-PD participants expected to teach again in the future and wanted to try at least some of the TDMs in the future—a future in which teaching had returned to in-person instruction and they could build norms around the TDMs right from the start of the semester.

Takeaways From the Three Cases

Many of the takeaways from the cases are takeaways about TDMs, which will be discussed in subsequent subsections. Thus, in this subsection, I seek to share a few takeaways from the cases that are less tied to the TDMs.

Research on learning very clearly shows that to foster student learning instructors need to move away from traditional lecturing and toward active learning (e.g., Deslauriers et al., 2019; Freeman et al., 2014; Lew et al., 2016; Theobald et al., 2020). In this spirit, the very first practice in the Mathematical Association of America's "Instructional Practices Guide" (i.e., Abell et al., 2018) is "fostering student engagement." Finnegan's case shines light on the closely related issue of "inviting student participation." It unflattens this term and highlights the complexity of inviting student participation by drawing attention to nuances of inviting, such as: the types of invited participation, the manner of the invitation (e.g., calling on students, using wait time), and the role of technology in inviting participation. Further, Finnegan's uptake of "probing a

student's thinking" and cold-calling may be of interest to practitioners who are interested in these practices or seek to reflect on their own.

Valeria's case serves as a rare example of bringing into conversation bell hooks's engaged pedagogy (1994) with mathematics education. Further, compared to the only other such work I am aware of (i.e., Chahine, 2013), Valeria's case actually captures how an instructor is seeking to live their theory, that is, how theoretical commitments find themselves enacted in a classroom. It raises questions whether certain ways of enacting TDMs (e.g., inviting student participation via cold-calling) might rob students of agency and, if so, whether such ways can be in concordance with engaged pedagogy or are fundamentally antithetical to it.

Valeria's case also demonstrates which TDMs may be easier to use for instructors who want student participation to be more voluntary than mandated (i.e., waiting, revoicing, inviting student participation via inviting further responses, and [instructor-centric] clarifying-probing). Further, her case also documents the power and importance of engaging T-PD participants with recordings of their own classes. For Valeria, listening to her classroom audio-recording appeared to be an important moment that led to reflections on her use of wait time. That said, Valeria's use of wait time 2 did not appear to increase and thus a single engagement with one's own class recordings may be insufficient for sparking more difficult change like implementing wait time 2.

Finally, Alice's case documents a GTA back in "survival" stage despite years of teaching experience. This situation is in line with work by Beisiegel et al. (2019) who found that GTAs "sometimes return to the survival stage if their new teaching assignment varied significantly from their prior teaching assignments" (p. 494). As this quote is the only nod by Beisiegel et al. (2019) to such GTAs, this dissertation offers a deeper look at this phenomenon. It showed that there were two interconnected sides to Alice's survival experience: an affective dimension (i.e.,

feeling like a horrible teacher, feeling overwhelmed) and a practical dimension (i.e., not having enough time to cover the desired amount of content in the desired amount of depth). Further, Alice's case provided examples of what might help GTAs in survival mode overcome at least the affective struggles: (a) seeing empirical evidence that one's students are doing no worse than other instructors' students, and (b) connecting with (a few) students. With respect to the TDMs, it appeared that Alice did not take to any TDMs that she was not already familiar with—except for possibly slightly increasing her wait time after students' statements (i.e., wait time 2a).

Takeaways About Waiting

As part of this dissertation, I built on Rowe (1986) and Ingram and Elliott (2016) to develop a more fine-grained wait time coding scheme that also accounts for differences in terms of the type of instructor and student turn (i.e., the instructor asking a question versus asking for questions, students making a statement versus asking for questions) (see Table 5.1). As I explained in the Method chapter, I had two suspicions about wait time that I wanted to capture with my coding scheme that Rowe's (1986) and Ingram and Elliott's (2016) coding schemes did not. First, I suspected that instructors might find it easier to remind themselves to wait after asking for questions; I correspondingly suspected that when instructors asked a "regular" question, they might be tempted to repeat or rephrase the question (a possibility also noted by, for example, [Sinclair & Coulthard, 1992])—a temptation that seems less likely in the case of asking for questions. Second, I also suspected that instructors might find it more difficult to wait after a student posed a question because they might be tempted to immediately respond to the student's question.

There seems to be some indication that my suspicions may have been warranted by studying my largest two relevant data sets—note, however, that these are both small data sets.

First, Valeria's use of wait time 1b (I-I) W? (16 start-of-semester data points, 19 end-of-semester data points) compared to her use of wait time 1b (I-I) Q? (24 start-of-semester data points, 26 end-of-semester data points) shows higher means, medians, and interquartile ranges for wait time 1b (I-I) Q?. Second, Alice's use of wait time 2a (S-I) R! (17 start-of-semester data points, 19 end-of-semester data points) and wait time 2a (S-I) Q? (7 start-of-semester data points, 6 end-of-semester data points) show a lower-or-equal mean and median for wait time 2a (S-I) Q?. (The boxplot for Alice's wait time 2a (S-I) Q? was not included in her case report due to the low number of data points.)

In addition to the wait times included in my coding scheme, Finnegan alerted me to a third type of wait time, that is, waiting after making a statement to give students extra time "to ask questions […] or ask for clarifications." Thus, the dissertation offers a more fine-grained break-down of the two established types of wait time as well as a third type of wait time.

Studying the wait time data for the three participants, one can note that Finnegan and Valeria appeared to, on average, increase the length of their wait time 1 by the end of the semester. And although the data for Alice's use of wait time 1 were inconclusive, she reported trying to use wait time more. For Valeria in particular, listening to her classroom recording was a powerful experience that alerted her to not using wait time 1 for as long as she thought and to not using wait time 2. That said, all three participants did not use wait time 2 for 3 seconds or longer. This is potentially concerning as, "Based on research it is clear that wait time 2 is more important than wait time 1 in many of its effects" (Rowe, 1986, p. 47). Thus, the MDISC T-PD on its own might not be sufficient for helping participants increase their wait time 2. Instead, Rowe's (1986) procedure for increasing wait time could complement the T-PD: "transcribing 10-minute segments of tape recording from three teach-reteach cycles using groups of four students"

(Rowe, 1986, p. 46). That said, Rowe (1986) found this procedure to be "aversive" and taking long (i.e., 6–12 hours).

Rowe (1986) also observed that mimicry was the main inhibitor for using wait time 2. Although I did not break down the types of revoicing in this dissertation, I suspect that given the MDISC's inclusion of revoicing as a move, a more nuanced discussion of types of revoicing—mimicry (or repeating) included—could serve to avoid this inhibitor to wait time 2 use.

Finally, the online context appeared to change the nature of wait time for Valeria and Alice, albeit in opposite ways. Valeria found it harder to wait online whereas Alice found it easier. Reflecting on the online context, Alice also noted that the online context required longer wait times than in person.

Takeaways About Inviting Student Participation

The TDM of "inviting student participation" is perhaps the most ambiguous of the TDMs, and this dissertation provided a way to attend to different aspects of this TDM with the help of three sets of codes: (a) explicit specific-student(s)-inviting, (b) inviting further responses, and (c) instructor solicitation type.

Regarding explicit specific-student-inviting, this dissertation adds to the conversation about cold-calling. As Dallimore et al. have found (2006, 2013, 2019), cold-calling appears to have a positive impact on the number of voluntary student responses, and it does not appear to make students uncomfortable—despite instructor fears. Further, their research suggests that cold-calling "may make the classroom environment more equitable for women" (Dallimore et al., 2019, p. 14). Looking across the three participants, one can see that the reservations of Valeria and Alice about cold-calling were so strong that they were not recorded using it a single time. Finnegan, on the other hand, started using cold-calling towards the end of the semester to

increase participation in general and female students' participation in particular, and he found it to be one of the most useful practices he had picked up from the T-PD. Thus, there seemed to be a world of difference between, on one hand, the experiences of Alice and Valeria with cold-calling (recall that, for example, Valeria found calling on people incompatible with her way of being) and, on the other hand, the cold-calling experiences of Finnegan and Lemov (2010) who shared, "When it's done right, it's an extremely powerful and positive way to reach out to kids who want to speak but are reluctant to be hand raisers" (p. 113). To me, the key appears to be the conditional in Lemov's quote and suggests that *if* we want instructors to use cold-calling, there is much more to it than telling them to call on people because fears of cold-calling are well founded when cold-calling is done in harmful, vindictive ways.

Regarding explicit specific-students-inviting, this dissertation attended to whether instructors called on groups to report (e.g., "Can anyone from group four tell us …"?). Other researchers may wish to extend this code in their contexts to account for other ways in which a specific set of students might be called on (e.g., "Can anyone other than [name] tell me …?", "Can someone from the back tell me …?").

Regarding "inviting further responses," the question arises whether it is less relevant in the online context. Given a chat feature, all students can reply near simultaneously and defy the traditional consecutive turn-taking structure of spoken discourse. That said, for spoken discourse, "inviting further responses" might serve as a non-forceful alternative to cold-calling when an instructor wishes to hear from more than just the usual suspects. Then again, cold-calling is faster and thus instructors who find themselves under time pressure might steer away from "inviting further responses."

Last, "instructor solicitation type" was a useful code, especially to keep track of how often instructors asked students to justify. Yet, a more fine-grained coding might help understand what forms of discourse and practices instructors ask their students to engage in (see "Future Directions" below).

Takeaways About Revoicing

The participants often used simple revoicing and rarely used full revoicing.[24] That said, as Herbel-Eisenmann et al. (2009) found and Krusi (2009) reported, there are many functions of revoicing. The teachers in Herbel-Eisenmann et al.'s (2009) study generated at least 28 such functions, which each had mathematical and/or social functions. As captured in Table 9.2, the T-PD participants also discussed different uses of revoicing in their respective interviews. A particular contribution of this dissertation study is the uses of revoicing described by the participants that relate to teaching in the online context: (a) revoicing a student's chat contribution to ensure all students hear the contribution in case they missed the chat message, (b) revoicing (correct) private chat messages, which enables a type of participation typically unseen in in-person settings, and (c) revoicing (in general) to honor students' participation in the online context since it is more difficult to participate online.

In addition to the above uses of revoicing in the online context, the participants offered two other novel uses of revoicing: (a) keeping students from feeling the pressure of having to repeat their answer and (b) using full revoicing to not do disservice to a student's idea. (The latter is similar to Herbel-Eisenmann et al.'s (2009) use #8 of revoicing—"to clarify"—but has a dimension of respect for students not captured in use #8.) Furthermore, the participants'

[24] Although I did not analyze the three different revoicing categories (i.e., simple revoicing, full revoicing, not revoiced), I suspect from working with the data that most of the non-revoiced student turns were mathematically incorrect, whereas correct student responses were rephrased, expanded on, or repeated by the instructors.

discourse about uses of revoicing highlighted that a single use of revoicing they identified could span several of Herbel-Eisenmann et al.'s (2009) 28 uses of revoicing. For example, Finnegan described that he responded to students' requests for re-explanation of something by revoicing his past explanation in a different mathematical context (e.g., algebraic, numeric, graphic) to expose his students to as many viewpoints as possible. Thus, he simultaneously used revoicing in at least three known ways: "for explication of reasoning" (use #3), "to summarize or repeat or add" (use #4), and "to make ideas accessible" (use #27). Simultaneously, he offered a refinement of these three uses through the idea of changing mathematical context for the revoicing. For my thoughts on potential future avenues for research on participants' use of revoicing, see the "Future Directions" section below.

Takeaways About Asking Students to Revoice

Finnegan and Alice were not recorded trying this TDM, and although Valeria was recorded trying this TDM three times, her prompts were twice met with silence and once with an incommensurate response. The participants reflected that the TDM did not feel "natural" and shared several individual concerns (see Table 9.3). That said, Finnegan and Alice noted that they would be interested in trying this TDM in an in-person setting. Online context aside, given the poor uptake of the TDM by the participants, T-PD facilitators of future iterations of the MDISC T-PD to GTAs might need to: (a) better justify the TDM to participants, (b) better address participants' concerns about the TDM, (c) expose participants to more examples of the TDM, and (d) better encourage participants to use the TDM in their classrooms.

I am also left with a suspicion that this TDM might require more norm-building than other TDMs. Although college students are not much older than high school students, most of them are adults who have willingly chosen to go to college. Work on adult education and

andragogy, like Knowles et al. (2012), suggests that adult learners are different from children in that they are more often intrinsically motivated to learn. (That is not to say that all students in college mathematics come into the classroom with an intrinsic desire to learn mathematics—especially if they had no say in choosing said mathematics course.) Yet, I wonder whether someone who is more intrinsically motivated to learn expects that everyone in the classroom pays attention and is therefore put off by a prompt to revoice someone else: Why repeat what someone else said if the expectation is that everyone is quiet and listening? Thus, I believe college students need to understand why they are being asked to revoice someone (e.g., to make sure everyone gets to hear a really important point made in two slightly different ways).

Takeaways About Probing a Student's Thinking

As documented in Table 5.1, this dissertation offered a substantial refinement of "probing a student's thinking" as it was conceived of by the developers of the TDMs (e.g., Cirillo et al., 2014; Herbel-Eisenmann et al., 2013). In particular, I highlighted two very distinct purposes of "having the student elaborate on his or her ideas" (Cirillo et al., 2014, p. 144): clarifying-probing and deepening-probing. Furthermore, I introduced the distinction between enacting these two types of probing in instructor-centric and student-centric ways. This distinction is important to understand whether students are receiving the opportunity to explain (or justify) their words or if instructors are engaging in that meaning-making (or justifying) process for them. Finally, understanding that instructors might feel different obligations for meaning-making depending on whether a student made a statement or asked a question, my coding scheme also accounted for instructor- and student-centric clarifying-probing of both statements and questions.

Rather than one type of probing being better than another, different types of probing might be appropriate in different contexts. Consider the following three examples. First, if an

instructor wishes to engage students in why-type discourse, they may wish to use student-centric deepening-probing. Deepening-probing might also serve to engage students in justifying their reasoning without starting an interaction with a "why"-question. Thus, students who would not respond to an immediate "why"-question might respond to deepening-probing. Second, if an instructor wishes to give a student the opportunity to re-formulate a response more in accordance with the mathematical register or learn more about the student's thinking, the instructor may wish to use student-centric clarifying-probing after a student statement. Third, if an instructor wishes to show a student that they understood the student's question (while still checking in with the student), an instructor might wish to use instructor-centric clarifying-probing after a student question. In short, different types of probing a student's thinking can have important, distinct uses.

Looking at the participants' uptake of probing, one can see that only Finnegan's use of probing increased, especially his use of deepening-probing. Deepening-probing allowed Finnegan to engage his students in why-type discourse, and although he found that asking "more in-depth questions" made teaching harder, it also made teaching more rewarding: "It makes it easier to see that students are making those connections between these different topics which makes it feel like I'm actually doing my job and actually teaching them well."

Although Alice and Valeria also used probing occasionally, their use of probing did not appear to noticeably change throughout the semester. Their occasional uses of deepening-probing were mostly student-centric and they used clarifying-probing almost exclusively in an instructor-centric way. For Valeria, probing a student's thinking fell under the umbrella of TDMs that were asking things of students, which made her feel uncomfortable. Similarly, for Alice,

probing in the online context felt weird and uncomfortable while not seeing students, and she likened it to cold-calling. She worried about making students feel put on the spot.

Takeaways About Creating Opportunities to Engage With Another's Reasoning

As documented in Table 5.1, this dissertation offered a substantial refinement of "creating opportunities to engage with another's reasoning" as it was conceived of by the developers of the TDMs (e.g., Cirillo et al., 2014; Herbel-Eisenmann et al., 2013). In particular, I highlighted five distinct types of engagement with another's reasoning that instructors can create opportunities for: (a) adding to another another's reasoning, (b) anticipating another's reasoning, (c) comparing to another's reasoning, (d) evaluating another's reasoning, and (e) understanding another's reasoning. Of these five types, two were not used by the participants (i.e., "creating opportunities to anticipate another's reasoning," and "creating opportunities to compare to another's reasoning"). Of the remaining three, "creating opportunities to evaluate another's reasoning" was more common than "creating opportunities to add to another's reasoning," with "creating opportunities to understand another's reasoning" a distant third. Thus, it may be that "creating opportunities to evaluate another's reasoning" and "creating opportunities to add to another's reasoning" are more easily woven into one's teaching practice than the other forms of this TDM.

An issue that surfaced while coding the data was the ambiguity of the phrase "engaging with another's reasoning." To engage with someone's reasoning, does one need to (a) engage with their justification for a claim, or (b) can one simply engage with their claim (that was presumably reached through reasoning)? Since the T-PD materials described the TDM as "involv[ing] asking students to engage with another student's idea," I sided with the second option and used "reasoning" synonymously with "idea." That said, future researchers of this line

of work may be interested in narrowing the scope of this TDM by defining "reasoning" in a way that centers justifying. For instance, the TDM could be amended by incorporating Stylianides's (2009) notion of "reasoning-and-proving" which encompasses four activities: identifying patterns, making conjectures, providing non-proof arguments, and providing proofs.

Even though group work is not an instance of the TDM as envisioned by the MDISC's creators but rather a "communication context," the participants spoke of group work as a way of creating opportunities to engage with another's reasoning. One takeaway from the T-PD participants' group work implementations was how difficult it was to implement group work online. Further, this dissertation provides further evidence of possible differences in group work implementation as described by J. P. Smith et al. (2020).

Takeaways About Sequencing the TDMs

As Cirillo et al. (2014) put forth:

> Although the TDMs were introduced as six distinct moves, the set of moves, together, has important interconnections that can be used in purposeful combinations. For example, waiting might be used in preparation for other moves that require significant student contribution, such as inviting, asking, or creating. Probing might be used so that one student's explanation is well understood and, thus, the basis for a creating move. (p. 147)

This dissertation provides further support for this observation. In particular, the participants noted the following combinations of TDMs: (a) Valeria and Alice observed that wait time 2 could serve to create opportunities to engage with another's reasoning, particularly creating opportunities to add to another's reasoning; and (b) Finnegan saw potential in using cold-calling (i.e., a form of inviting student participation) before probing a student's thinking or creating opportunities to engage with another's reasoning.

Limitations

There are two main limitations of this study that I wish to address.

First, due to the coronavirus pandemic, the participants taught online. To protect the identities of the participants' students (under the Family Educational Rights and Privacy Act [FERPA]), the Institutional Review Board (IRB) of the participants' institution did not permit video-recordings of classes, only audio-recordings. The IRB's decision had several consequences for this dissertation study: (a) a positioning analysis and the use of the EQUIP tool (as originally intended) was unfeasible, and (b) chat histories had to be aligned with audio-recordings after the fact. The former consequence led to this study focusing on changes in the classroom discourse in each participant's class, particularly with respect to the TDMs. Thus, other important aspects of the T-PD, particularly related to positioning theory, took a backseat in this dissertation study. Although no study of a T-PD and its implementation can fully attend to all aspects of the T-PD, I believe a future study in which the focus is placed on the T-PD's contribution to GTAs' understanding and use of positioning theory would be important work. The consequence of the chat alignment limitation (which meant that chat histories and audio-recordings were only aligned to within 2 seconds) was that the wait time analysis was only based on wait time relative to spoken discourse. Due to many potential instances of wait time being omitted from consideration this way, there were not always enough instances for each type of wait time to make meaningful observations. A more accurate data set or a larger number of coded observations could address this issue.

Second, a study of T-PD participants' implementation of T-PD tools—like this dissertation—benefits from teaching data from participants after the end of the T-PD. Research about T-PD participants after the T-PD's end, like Dubbs and Herbel-Eisenmann's (2021) article

about a former MDISC participant, can be very informative for both T-PD developers and researchers. Further, in the case of the MDISC T-PD, which encourages participants to conduct action research in their classrooms after the T-PD's completion, a post-T-PD follow-up with the GTAs would have been particularly valuable to learn whether they chose to engage in action research and, if so, what aspects of their classrooms they had chosen to study and possibly change. Cognizant of this, I asked all participants whether I was allowed to contact them again in the subsequent semester to enquire about their teaching, and all participants agreed. Upon following up with the participants, only Valeria and Alice responded and neither of them were assigned teaching assistantships for at least the next two semesters. Thus, although I intended to get a sense of the T-PD's impact beyond the semester of its implementation, I was unable to.

I believe my experience with this second limitation leads to two important takeaways. First, it might be beneficial for future studies to immediately recruit participants for multiple semesters to increase the likelihood of participants participating in a multi-semester study. Second, the lack of teaching assistantships for Valeria and Alice for the entire year after their participation in the T-PD points to the need for coordination between T-PD providers and assistantship-assigners. Through a better coordination between these two groups, the T-PD experience could be enhanced by: (a) assigning T-PD participants a course they are already familiar with during the T-PD to avoid "survival mode," and (b) ensuring T-PD participants teach in subsequent semesters so they can implement T-PD ideas they feel need to be introduced from the start of a semester.

Future Directions

There are many ways in which this dissertation's data could still be analyzed or the study modified in future iterations. Below, I share several such ways.

Future Directions With This Study's Data

First, although I distinguished between not revoiced, simply revoiced, and fully revoiced, the revoicing data could be analyzed on a more fine-grained level. As Forman et al.'s (1998) definition of revoicing suggests—that is, "revoicing involves the reuttering of another person's speech through repetition, expansion, rephrasing, and reporting" (p. 531)—there are (at least) four ways in which revoicing is enacted. These four ways could serve as a starting point for analyzing how participants revoiced and help distinguish between, for instance, frequent rephrasing and more rare, targeted instances of repeating. Such an analysis might also reveal a pattern of frequent repeating, which as Rowe (1986) noted, can make using wait time 2 difficult and "cut[] off extended wait times and reduce[] the quantity and quality of student responses" (p. 46). On top of analyzing the ways in which participants revoiced, one could also analyze the participants' purposes for revoicing. Herbel-Eisenmann et al.'s (2009) study demonstrated the many purposes revoicing can serve, and Table 9.2 contains the different purposes the participants of this dissertation study identified for revoicing. Understanding how different uses of revoicing may be tied to different ways of revoicing could prove to be a fruitful endeavor for research.

Second, although the distinction between what, how, and why-type solicitations encouraged by the EQUIP was useful, particularly for studying how (deepening-)probing impacted students' engagement in justifying discourse, refining the distinction could be an interesting avenue for further research. Particularly the "what"-category was a very large category that included a range of prompts from asking students to apply a mathematical procedure to asking students to generate examples. Thus, a more fine-grained analysis of instructor solicitations could reveal instructors' use of more cognitively demanding solicitations that may have thus far been subsumed by the "what"-category.

Third, although the case reports were written with the intention of disclosing the participants' motivations for joining the T-PD, their expectations for the T-PD, and relevant beliefs, a more systematic analysis of participants' decision-making using Schoenfeld's (2010, 2011) Resources–Orientations–Goals (ROG) framework could be done to better understand the instructors' decision-making.

Future Studies

First, a similar study could be conducted when GTAs have returned to teaching in person. As has become evident from conducting this study, the online context was important for understanding the participants' use and uptake of the TDMs. Thus, studying participants' use and uptake of the TDMs in an in-person context would be a worthwhile extension of this dissertation.

Second, a future study examining GTAs' implementation of the TDMs could also attend to the curricular materials used by the participants. When GTAs are given teaching materials in which procedural fluency is valued over engagement with mathematical practices like justifying, the classroom discourse is likely to be affected—as suggested by, for example, the QUASAR project (e.g., Silver & Stein, 1996; Stein et al., 1996).

APPENDICES

APPENDIX A: Timeline of My MDISC T-PD Implementation With Implementation Notes

Table A.1

Timeline of My MDISC T-PD Implementation With Implementation Notes

Session No.	Activities	Implementation Notes
1	Intro.1	
	Intro.2	• *Intro.2.1: participants collaborated on awwapp for "Betty's Bakery Task"*
	Intro.3	• Intro.3.3: discussion of Herbel-Eisenmann (2002) cut short to around 5 min. • Intro.3.4: omitted
	HW	• HW: participants asked to pay attention to their use of language in class and whether they use bridging language
2	HW check-in	• HW check-in: I checked in with participants about the HW given in session 1
	Intro.4	• Intro.4.4: optional reading activity omitted • Intro.4.5.1: participants were not asked to take the beliefs survey but were given access to it • *Intro.4.5.2: participants constructed their beliefs maps using Jamboard*
	C2P Intro.A	• C2P Intro.A: participants introduced to activity
3	C2P Intro.A	• C2P Intro.A: discussion of possible survey questions
	1.1	• 1.1.1: participants were given a short time to reflect (i.e., only 2–3 min.) and were not asked to take notes in their MDISC journals
	1.2	• *1.2.1: participants collaborated on awwapp for "Area and Perimeter Tasks"* • 1.2.1: participants got stuck on version 1 task 3 and version 2 task 4 and were asked to move on
	1.3	• 1.3.1: participants were only given the (reduced) set #1 of student solutions • *1.3.1: participants sorted student solutions on awwapp* • 1.3.2: table of solution characteristics was constructed communally rather than individually • 1.3.3: I summarized Touchstone Document 2 instead of asking participants to read it

Table A.1 (cont'd)

Session No.	Activities	Implementation Notes
3 (cont'd)	C2P 1.A	• C2P 1.A: I introduced participants to the optional modified activity C2P 1.A (i.e., focus on one student or more instead of three) and told them there would be a check-in in a fortnight
	HW	• HW: I asked participants to bring in "a page your students (might) have to read (e.g., from your course textbook or your course notes) and a problem they (might) have to solve" for an activity in the next T-PD session
4	C2P Intro.A	• C2P Intro.A: participants were informed of two survey tools (i.e., Qualtrics and Google Forms); I shared I was going to send out my own class survey the following week
	C2P 1.A	• C2P 1.A: participants were reminded that there would be a check-in about this optional activity the following week
	1.4	• 1.4.1: I re-summarized Touchstone Document 2 • 1.4.1: participants analyzed excerpts from their textbooks (or course notes) instead of the two given textbook excerpts • 1.4.3 & 1.4.4: participants analyzed the excerpts from their textbooks (or course notes) instead of the given excerpts
	1.5	• 1.5.2: second part of transcript B skipped
5	C2P 1.A	• C2P 1.A: check-in with participants about their observations
	C2P Intro.A	• C2P Intro.A: check-in with participants about their survey results
	2.1	
	2.2	• *2.2.1: participants drew triangles for the "Hidden Triangle Exploration Task" on awwapp* • 2.2.3: positioning discussion focused on role of director
	2.3	• 2.3.2 and 2.3.3: discussions merged • 2.3.4: only a very brief introduction of TDMs due to time constraints

Table A.1 (cont'd)

Session No.	Activities	Implementation Notes
5 (cont'd)	C2P 2.A or C2P 2.B	• C2P 2.A or C2P 2.B: participants informed of upcoming task to analyze their first two classes' recordings and transcripts; participants asked to suggest transcripts for different classes—if they would like—by the next day
6	2.4	• 2.4.1: I re-summarized the TDMs before starting this activity • 2.4.2: the discussion centered around waiting, inviting student participation, revoicing, and asking students to revoice
	C2P 2.A or C2P 2.B	• C2P 2.A or C2P 2.B: participants were given 30 minutes to analyze their own classroom data either with respect to IRE use (C2P 2.A) or TDM use (C2P 2.B); the analysis was followed by a group discussion
	C2P 2.C	• C2P 2.C: participants were asked to try out the TDMs in the coming two weeks; the C2P 2.C activity sheet was provided as optional guidance
7	C2P 2.C	• C2P 2.C: check-in with participants about their TDM use over the past week
	2.5	
	2.6	• 2.6.3: omitted
8	C2P 2.C	• C2P 2.C: check-in with participants about their TDM use and positioning in the past week
	Positioning activity	• Positioning activity part 1: participants drafted a set of "goals that unite us" • Positioning activity part 2: I gave a short presentation on positioning theory • Positioning activity parts 3, 4, & 5: participants were asked to watch, analyze (with respect to positioning), and discuss the "Facilitating Group Work" videos from the "Video Cases for College Mathematics Instruction" (Hauk et al., 2013)

Table A.1 (cont'd)

Session No.	Activities	Implementation Notes
8 (cont'd)	Implicit bias activity	• Implicit bias activity part 1: I gave a short presentation on stereotypes/narratives in mathematics classrooms and implicit bias • Implicit bias activity parts 2 & 3: participants were given the time to take one (or multiple) implicit bias tests, after which there was a discussion • Implicit bias activity part 4: I shared strategies for combatting implicit bias
	EQUIP	• EQUIP activity: I introduced the group to the EQUIP tool's 7 dimensions, the EQUIP website (and online tool), and to an example of calculating equity ratios manually using one of the group work videos from the positioning activity
9	EQUIP	• EQUIP activity parts 1 & 2: the participants watched a video of a presentation from former MDISC T-PD participant Dean Hanton (Herbel-Eisenmann et al., 2020) about his experience using the EQUIP tool and then discussed the video • EQUIP activity part 3: as in session 8, I went over the EQUP tool; I also pointed out weaknesses of the EQUIP tool • EQUIP activity part 4: participants were given 30 minutes to use the EQUIP tool (either on the EQUIP webpage or manually) to analyze any of their classroom recordings for a demographic dimension of their choice and with respect to one or two EQUIP dimensions • EQUIP activity part 5: participants were asked to share their findings—if they felt comfortable—and a discussion of the EQUIP tool and the participants' findings ensued
	3.1	• *3.1.1: participants solved the "Intersecting Lines Task" on awwapp* • 3.1.2 & 3.1.4: omitted
	C2P 3.A	• C2P 3.A: activity given as an optional homework for the next session
10	C2P 3.A	• C2P 3.A: check-in about TDM use

Table A.1 (cont'd)

Session No.	Activities	Implementation Notes
10 (cont'd)	Resource-sharing	• Resource-sharing part 1: I shared three articles related to bias (i.e., Herbel-Eisenmann & Shah, 2019; Su, 2015, 2016) and we had a brief discussion • Resource-sharing part 2: I shared three resources related to cold-calling (i.e., *Cold Calling*, 2019; Lemov, 2010; Rush, 2018) and we had a brief discussion about how to implement cold-calling
	3.2	• 3.2.3 and 3.2.4: omitted
	Abstract algebra task	• *Abstract algebra task: participants solved this task on awwapp* • Abstract algebra task part 1: The participants were given an excerpt from a set of abstract algebra course notes that included a definition of two elements being "associated" and a problem which asked, given a ring $R=\mathbf{Z}_{18}$, to find all the units in the ring and to determine the equivalence classes of the associate-relation on R; participants were asked to solve the problem • Abstract algebra task part 2: participants were asked to discuss: What mathematical and social goals might this task accomplish? To which communication context might it be suited? On which representation context does it (or could it) rely? • Abstract algebra task part 3: participants were asked to discuss: Could we change the task to accomplish different mathematical and social goals? Could we change the task to make it suitable for a different communication context? Could we change the representation context?

Table A.1 (cont'd)

Session No.	Activities	Implementation Notes

10 (cont'd)	David Kung video task	• David Kung video task part 1: participants were given 2 minutes to familiarize themselves with the Wason selection task
		• David Kung video task part 2: participants were asked to watch the "Leading Whole Class Discussion" video from the "Video Cases for College Mathematics Instruction" (Hauk et al., 2013)
		• David Kung video task part 3: participants were asked to discuss the video with the help of the following questions:
		  ○ What might be the mathematical and social goals that the instructor intended?
		  ○ What TDMs do you notice?
		  ▪ How does his use of the TDMs provide opportunities for students to engage in productive or powerful discourse?
		  ○ Where do you see evidence of classroom norms?
		  ▪ What does it mean to know and do mathematics in this classroom?
		  ▪ What does it mean to participate in this classroom?
		  ▪ Who participates/gets to participate and how?
		  ▪ How do the norms of his classroom provide opportunities for students to engage in productive or powerful discourse?
		(after the first question, participants were given 10 minutes to analyze the video for TDM use with the help of a transcript)

Table A.1 (cont'd)

Session No.	Activities	Implementation Notes

10 (cont'd)	HW	• HW part 1: participants were asked to reflect on their classroom norms and the actions that might have led to (i.e., reified) these norms by thinking about:
		  ○ What do you notice about the ways in which your students participate? What do you notice about the ways you are participating?
		  ○ When you choose to talk with your students about your expectations, do you focus on social norms or mathematical norms?
		• HW part 2: participants were reminded of C2P 3.A

11	HW check-in	• HW check-in part 1 (C2P 3.A): participants discussed their TDM use and observations related to positioning • HW check-in part 2: participants discussed (mathematical and social) norms in their classrooms and how these became norms
	CAP.1	• *CAP.1.1: participants were invited to use awwapp to take notes*
	CAP.3	• CAP.3.2: reading time cut short • CAP.3.3: discussion was brief due to time constraints
	HW	• HW: for the next session, participants were given the prompt: "What are topics or questions related to your classroom discourse that you might consider pursuing via action research? (Revisiting your beliefs mapping may be useful.)"
12	Addendum	• Addendum: after mentioning bell hooks in discussion last time and receiving the impression that only one participant was familiar with her name and work, I shared the titles of several of her works with the group (i.e., "Teaching to Transgress," "Teaching Community," "Teaching Critical Thinking," "The Will to Change: Men, Masculinity, and Love," "Feminism is for Everybody")
	HW check-in	• HW check-in: participants were given 5 minutes to write down areas of interest before sharing them with the group ("areas of interest" were recorded in a communal Google Slides deck)

Table A.1 (cont'd)

Session No.	Activities	Implementation Notes
12 (cont'd)	CAP.5.3	• CAP.5.3: after reading the T-PD document on "Writing Researchable Questions," participants were given time to write down their "research question(s)" in the communal Google Slides deck
	CAP.4.1 (modified)	• CAP.4.1 (modified) part 1: I gave a brief presentation about IRB, FERPA, and consent • CAP.4.1 (modified) part 2: participants were each made responsible for two forms of data collection—I for one—from the T-PD document on data collection, and in a communal Google Slides deck we compiled "things to keep in mind for data collection" and "limitations" for our assigned forms of data collection; the compilation began with our brainstorming and was then supplemented by points from the T-PD document on data collection • CAP.4.1 (modified) part 3: participants were asked to think about which forms of data collection might align with their areas of interest and research questions
13 (half-session)	Reflection	• Reflection: I asked participants to think about what the challenges and highlights of the semester had been for them, and then we went around sharing our challenges and then our highlights
	Reminder	• Reminder: participants were asked to remind themselves of their draft action research plans from last time (i.e., area of interest, research question(s), data collection)
	CAP.4.1 (modified)	• CAP.4.1 (modified) part 4: participants were asked to read the T-PD document on data analysis • CAP.4.1 (modified) part 5: I said a few more words about research methods based on my experience • CAP.4.1 (modified) part 6: the participants shared their reactions to the data analysis T-PD document • CAP.4.1 (modified) parts 7 & 8: participants were asked to think about what kind of data analysis might align with their action research plans before presenting them to the group

Table A.1 (cont'd)

Session No.	Activities	Implementation Notes
13 (half-session) (cont'd)	Resources	• Resources: I shared some resources with the T-PD participants that I thought they might find useful for their action research projects: o the T-PD document "Resources for Continued Investigation" o two papers on mathematics education journals for reference (i.e., Nivens & Otten, 2017; Williams & Leatham, 2017) from which I highlighted IJRUME and PRIMUS o a list of undergraduate mathematics education conferences: RUME, INDRUM, and some MAA and AMS sessions

Note. Activities without implementation notes were implemented in line with the MDISC materials' descriptions. Italicized implementation notes describe changes made due to the T-PD being implemented online. Each T-PD session started with a check-in (which sometimes took as long as 30 minutes) that included how the participants were doing, how their teaching was going, and—especially in the second half of the semester—if they had used any T-PD tools or made any observations or connections related to the T-PD.

APPENDIX B: Interview Protocols

Pre-T-PD Interview Protocol

Interview Purpose

The goals of the 60-minute pre-T-PD interview are threefold: (1) for the participant and me to get to know each other, (2) for me to learn a bit about the participants' beliefs about teaching, learning, and mathematics, and (3) for me to learn about the participants' expectations for the T-PD. The discussion about beliefs does not need to be comprehensive as participants will be talking about their beliefs in the introduction of the T-PD. If during the interview participants steer in the direction of discourse or positioning—not necessarily in name, but in spirit—I would follow up with them and ask them to elaborate.

Guiding Questions

1. What do you think it means to "do mathematics"?
2. What do you think "doing mathematics" means to your students?
3. In a perfect world, how would you want the class you will be teaching this coming semester to go?
    a. Picturing this perfect class, what do you imagine yourself doing? What do you imagine your students doing in this class?
    b. [*If participant mentions something related to student discourse, for example, "encouraging students to talk" or "students talking:"*]
    Can you say more about that?
        i. [*some appropriate follow-up:*] What kind of student talk are you imagining?
    c. [*If participant mentions something related to positioning, for example, "I want all of my students to feel like they can succeed in the class"*]
    Can you say more about that?
        i. [*some appropriate follow-up:*] How would you try and foster this feeling?
4. What do you think your role is as an instructor/teaching assistant?
    a. What does "teaching" mean to you?
        i. What do you see as your role and responsibilities?
    b. How do you think your students learn (best)?
        i. [How do you as an instructor see yourself being able to foster that?]
5. What are your expectations for this T-PD?
    a. Do you have any questions about the T-PD? Anything you're wondering or worried about?
6. What motivated you to join this T-PD?
    a. Is there a specific aspect of your teaching that you would like to improve?

Mid-T-PD Interview Protocol

Interview Purpose

The goals of the 60-minute mid-T-PD interview are threefold: (1) to check in with participants to determine how the T-PD is going for them, (2) to learn how they are making sense of the two pillars of the T-PD, namely the TDMs and positioning (if the T-PD has gotten that far), and (3) to hear about their teaching and learn whether they have sought to change their classroom discourse.

Guiding Questions

1. How is the professional development going for you?
    a. What is going well? What isn't?
    b. Do you have any questions about the T-PD? Anything you're wondering about?
2. How is your teaching going?
    a. What is going well? What isn't? (What would you like to see improve?)
3. What are your thoughts about the teacher discourse moves?
    a. Which teacher discourse moves have been easier to use? Which ones harder?
    b. Which teacher discourse moves have been more useful? Which ones less useful?
    c. Are there any teacher discourse moves you haven't tried? Why? (e.g., intimidated to use them, not seeing an appropriate time)
4. What are your thoughts about positioning?
    a. Have you tried changing how you position mathematics in your class?
        i. How do you think mathematics is positioned in your class by you and your students?
    b. Have you tried changing how you position yourself in your class?
        i. How do you think you position yourself and students position you?
    c. Have you tried changing how you position your students in your class?
        i. How do you position your students and your students position themselves?

Post-T-PD Interview Protocol

Interview Purpose

The goals of the 60-minute post-T-PD interview are threefold: (1) to give the participant an opportunity to reflect on the T-PD, (2) to give the participant an opportunity to reflect on their teaching, and (3) for the participant to share how they would teach future classes.
After the end of the interview, I will ask the participant if they are all right with receiving a follow-up e-mail during the fall semester. (Agreeing to receive a follow-up e-mail does not commit them to participating in any future interviews or observations.)

Guiding Questions

1. Before I get started asking specific questions, is there anything you'd like to get off your chest? Anything that's been weighing on your mind regarding the PD?
2. Looking back at the professional development, what are your takeaways?
    a. Were there any things you learned that surprised you?
3. What parts of the professional development felt relevant (or were useful)? What parts didn't? (*Figure out what the useful parts of the T-PD were while teaching online during the pandemic versus what might be potentially useful in future in-person semesters.)*
    a. Were there any parts of the PD that were easier to implement than others in your classroom? Any that were harder?
        i. Why do you think that is?
    b. If we could go back in time, what do you think we should (or could) have done differently?
    c. If you were to run this PD, what would you do differently? (e.g., content, format, facilitation)
    d. Would you participate in a PD like this again?
        i. Do you feel like additional semesters of T-PDs would be useful (if so, how many) or did you feel this was a one-time thing?
4. You said at the start of the semester that [*insert their goal(s)/expectation(s) for the T-PD*]. How close (or not close) did we get with the MDISC PD this semester to achieving that goal?
5. If we asked your students right now what it means to do mathematics, what do you think they would say?
    a. *Have their past statements ready to engage them with.*
6. What about you? At the start of the semester, you shared that to you doing mathematics means [*short description*]. Has that changed?
7. If you had a conversation with your younger self from [*month of the start of the semester*], what are things you would agree on about teaching? About learning? What are things you think you might disagree on?
    a. Are there any things you would do differently teaching your next class? [*Maybe because they felt that it was too late to make changes this semester after classroom norms had been established.*]
        i. Has your picture of your perfect class changed?
    b. What are questions or issues about teaching and learning that you are still left with?

APPENDIX C: Additional Tables and Figures

Table C.1

Mode of Discourse Used by Students in Finnegan's Class Across CR1–CR9 in Turns About Mathematics

"Mode of Discourse" used in a student turn (percentage out of total)	Classroom Recording (Day of Class) [# of min. spent together as a class]								
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]	CR4 (12) [60]	CR5 (14) [64]	CR6 (15) [49]	CR7 (20) [58]	CR8 (21) [81]	CR9 (24) [82]
Spoken	55 (.437)	34 (.382)	30 (.297)	17 (.293)	19 (.257)	35 (.648)	11 (.212)	7 (.070)	9 (.105)
Public chat	70 (.556)	54[a] (.607)	71 (.703)	41[a] (.707)	48[a] (.649)	18 (.333)	37 (.712)	90 (.900)	77 (.895)
Private chat	0 (.000)	0 (.000)	0 (.000)	0 (.000)	0 (.000)	0 (.000)	0 (.000)	0 (.000)	0 (.000)
Other (i.e., body language, intermodal)	1 (.008)	1 (.011)	0 (.000)	0 (.000)	7 (.095)	1 (.019)	4 (.077)	3 (.030)	0 (.000)
Total	126	89	101	58	74	54	52	100	86

Note. The numbers in this table only reflect student turns whose discourse is about mathematics (i.e., student turns that are solely discourse about class logistics are excluded).

[a] Counts marked with a superscript "a" include a single instance of a shortened message.

Table C.2

Finnegan's Use of TDMs Across CR1–CR9 (Part 1)

Instructor Discourse Dimensions	Classroom Recording (Day of Class) [# of min. spent together as a class]								
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]	CR4 (12) [60]	CR5 (14) [64]	CR6 (15) [49]	CR7 (20) [58]	CR8 (21) [81]	CR9 (24) [82]
Types of Instructor Solicitations	60	60	76	60	76	59	59	94	75
• What	25 (21)	34 (28)	38 (32)	37 (32)	38 (28)	26 (22)	26 (20)	33 (26)	53 (37)
• How	18 (16)	8 (6)	12 (9)	1 (1)	7 (6)	10 (10)	10 (9)	21 (14)	9 (8)
• Why	0	1 (1)	1 (1)	3 (3)	7 (5)	7 (7)	6 (6)	9 (8)	2 (2)
• Questions?	15 (14)	13 (7)	21 (11)	11 (3)	12 (3)	11 (4)	12 (5)	17 (6)	5 (0)
• Make sense?	2 (1)	3 (3)	3 (2)	7 (5)	8 (2)	5 (3)	5 (2)	12 (8)	6 (5)
• Other	0	1 (1)	1 (1)	1 (1)	4 (1)	0	0	2 (2)	0
Specific-student(s)-inviting (explicit)									
• Specific-student-inviting	0	2	0	0	0	0	1	3	7
• Specific-students-inviting	0	0	0	0	0	5	0	0	0
Inviting further responses	1	0	0	2	0	0	1	0	1
Asking students to revoice	0	0	0	0	0	0	0	0	0

Table C.3

Finnegan's Use of TDMs Across CR1–CR9 (Part 2)

Instructor Discourse Dimensions	Classroom Recording (Day of Class) [# of min. spent together as a class]								
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]	CR4 (12) [60]	CR5 (14) [64]	CR6 (15) [49]	CR7 (20) [58]	CR8 (21) [81]	CR9 (24) [82]
Turn simply revoiced	87	52	76	43	50	32	35	67	62
Turn fully revoiced	0	0	1	0	0	0	1	0	0
Turn not revoiced	37	37	24	15	23	22	17	34	24
Clarifying-probing (CP)	1	2	0	3	4	1	3	8	3
• CP Instructor-centric (CP-IC)	0	1	0	0	1	0	1	4	1
○ CP-IC Statement	0	0	0	0	0	0	1	1	1
○ CP-IC Question	0	1	0	0	1	0	0	3	0
• CP Student-centric (CP-SC)	1	1	0	3	3	1	2	4	2
○ CP-SC Statement	0	1	0	3	3	1	1	4	2
○ CP-SC Question	1	0	0	0	0	0	1	0	0
Deepening-probing (DP)	0	1	2	3	9	13	8	9	6
• DP Instructor-centric	0	0	0	0	1	1	0	1	0
• DP Student-centric	0	1	2	3	8	12	8	8	6
Creating opportunities to engage with another's reasoning									
• Adding	0	0	0	0	1	3	0	1	0
• Evaluating	0	0	0	0	0	0	1	0	0
• Understanding	0	0	0	0	0	0	0	0	0
• Group work	1	1	0	0	0	1	0	0	0

Table C.4

Finnegan: Student Discourse Dimensions Across CR1–CR9 (Part 1)

Student Discourse Dimensions	Classroom Recording (Day of Class) [# of min. spent together as a class]								
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]	CR4 (12) [60]	CR5 (14) [64]	CR6 (15) [49]	CR7 (20) [58]	CR8 (21) [81]	CR9 (24) [82]
Mode of Discourse	133	107	132	60	79	61	56	105	88
• Speech	56	37	33	17	19	35	11	9	9
• Writing	76	69	99	43	53	25	41	93	79
○ Public chat	76	69[a]	99[a]	43[a]	53[a]	25[a]	41	93	79
○ Private chat	0	0	0	0	0	0	0	0	0
• Body	0	0	0	0	0	0	0	0	0
• Intermodal	1	1	0	0	7	1	4	3	0
Discourse Type	133	107	132	60	79	61	56	105	88
• Discourse about mathematics	126	89	101	58	74	54	52	100	86
• Discourse about logistics	7	18	31	2	5	7	4	5	2
Student Talk Length	126	89	101	58	74	54	52	100	86
• 1–4 words	65	50	52	38	44	17	20	54	40
• 5–20 words	42	34	37	14	18	27	23	42	39
• 21+ words	19	5	12	6	10	10	9	4	7
• n/a	0	0	0	0	2	0	0	1	0

[a] Counts marked with a superscript "a" include a single instance of a clipped message.

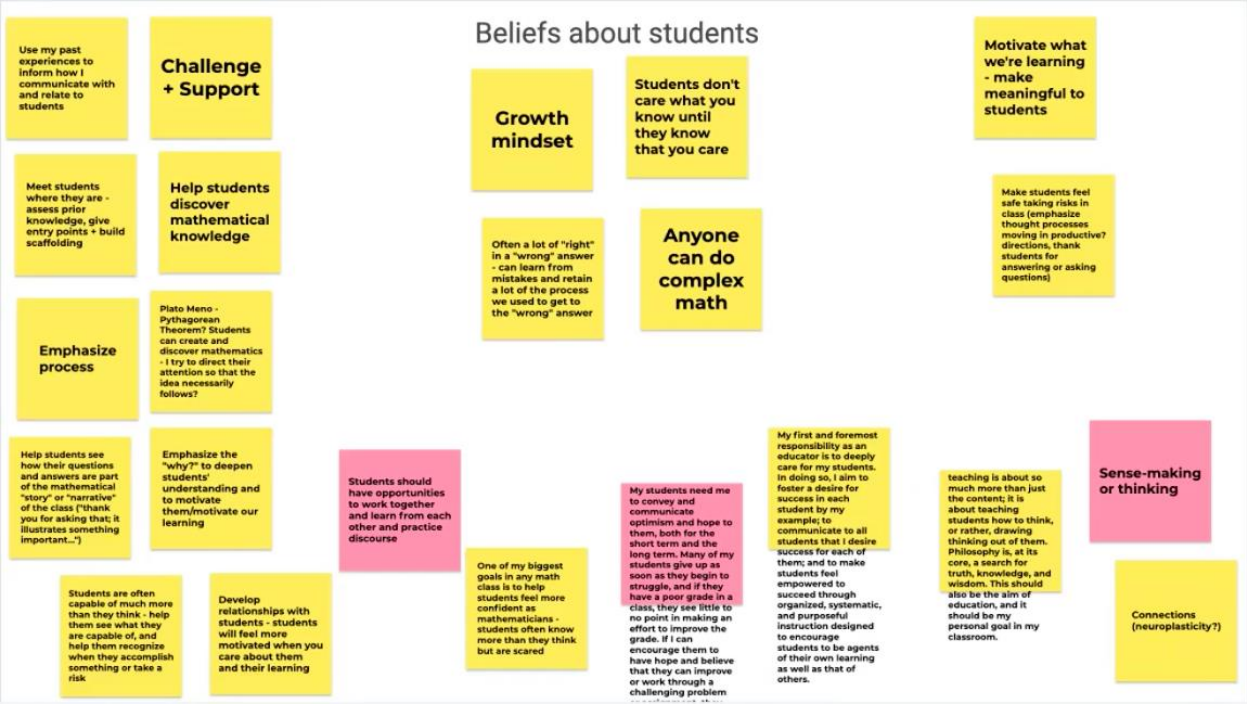
Table C.5

Finnegan: Student Discourse Dimensions Across CR1–CR9 (Part 2)

Student Discourse Dimensions	Classroom Recording (Day of Class) [# of min. spent together as a class]								
	CR1 (2) [67]	CR2 (3) [55]	CR3 (5) [82]	CR4 (12) [60]	CR5 (14) [64]	CR6 (15) [49]	CR7 (20) [58]	CR8 (21) [81]	CR9 (24) [82]
Student Talk Type	126	89	101	58	74	54	52	100	86
• what	53	44	55	39	43	21	21	41	53
• how	26	13	15	2	7	7	12	18	13
• why	0	3	3	6	7	8	6	13	4
• question	40	21	23	9	8	14	12	17	6
• other	7	8	5	2	9	4	1	11	10
Teacher Solicitation Type	126	89	101	58	74	54	52	100	86
• what	47	44	61	39	41	20	21	43	56
• how	27	10	12	3	8	8	12	24	10
• why	0	1	2	5	7	7	6	11	3
• other	19	16	9	6	4	3	8	16	6
• n/a (unsolicited)	33	18	17	5	14	16	5	6	11
Explicit Evaluation	126	89	101	58	74	54	52	100	86
• Yes	97	58	79	53	57	35	40	73	60
• Yes (by TA)	7	8	5	0	1	3	4	2	1
• No	12	6	11	3	4	11	4	10	12
• n/a	10	17	6	2	12	5	4	15	13

Figure C.1

Alice's Beliefs Map From T-PD Session 2 With Accompanying Notes



Note. The post-it notes (read roughly from left-to-right, top-to-bottom) say:

- Use my past experiences to inform how I communicate with and relate to students
- Challenge + Support
- Growth mindset
- Students don't care what you know until they know that you care
- Motivate what we're learning – make meaningful to students
- Meet students where they are – assess prior knowledge, give entry points + build scaffolding
- Help students discover mathematical knowledge
- Often a lot of "right" in a "wrong" answer – can learn from mistakes and retain a lot of the process we used to get the "wrong" answer
- Anyone can do complex math

- Make students feel safe taking risks in class (emphasize thought processes moving in productive? directions, thank students for answering or asking questions)
- Emphasize process
- Plato Meno – Pythagorean Theorem? Students can create and discover mathematics – I try to direct their attention so that the idea necessarily follows?
- Help students see how their questions and answers are part of the mathematical "story" or "narrative" of the class ("thank you for asking that; it illustrates something important…")
- Emphasize the "why?" to deepen students' understanding and to motivate them/motivate our learning
- Students are often capable of much more than they think – help them see what they are capable of, and help them recognize when they accomplish something or take a risk
- Develop relationships with students – students will feel more motivated when you care about them and their learning
- Students should have opportunities to work together and learn from each other and practice discourse
- One of my biggest goals in any math class is to help students feel more confident as mathematicians – students often know more than they think but are scared
- My students need me to convey and communicate optimism and hope to them, both for the short term and the long term. Many of my students give up as soon as they begin to struggle, and if they have a poor grade in a class, they see little to no point in making an effort to improve the grade. If I can encourage them to have hope and believe that they can improve or work through a challenging problem […] ((*the rest of the text was not visible on Alice's shared screen and was not discussed*))

- My first and foremost responsibility as an educator is to deeply care for my students. In doing so, I aim to foster a desire for success in each student by my example; to communicate to all students that I desire success for each of them; and to make students feel empowered to succeed through organized, systematic, and purposeful instruction designed to encourage students to be agents of their own learning as well as that of others.
- teaching is about so much more than just the content; it is about teaching students how to think, or rather, drawing thinking out of them. Philosophy is, at its core, a search for truth, knowledge, and wisdom. This should also be the aim of education, and it should be my personal goal in my classroom.
- Sense-making or thinking
- Connections (neuroplasticity?)

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