ESSAYS IN INTERNATIONAL BANKING

By

Giacomo Romanini

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ABSTRACT

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In the first chapter, we present a model that can capture the effects of banking networks and the potential for spillovers from the concentration of claims at financial hubs. In particular, we gauge the implications from indirect international flows and the role of financial hubs with respect to intermediation costs. Using the variation from direct and indirect banking flows provided by the Consolidated and Locational banking statistics from the Bank for International Settlements, we use the model to recover the bilateral and expected edge-specific costs through minimal distance estimation, thus providing a theoretical estimation of the unobserved cost usually proxied by a combination of observable distance measures. A network model defines the extensive margin. Then, we embed the Allen and Arkolakis (2019) transportation model in a Eaton and Kortum (2002) setting to allow for endogenous financial hub formation and incorporate the choice of path. The intensive margin of lending flows arises from banks choosing the path within a network through which they allocate their corporate lending, while decentralized bilateral shocks determine the interbank network equilibrium.

In the second chapter, we investigate the role of intermediation networks in propagating shocks. We study a dynamic multicountry gravity model where banks can send loans to firms either directly or indirectly, through network paths across countries. To the best of our knowledge, it is the first DSGE that allows for endogenous network paths. We show how our model can be used in several settings. First, the model can show how shocks to a node (country) transmit international through a network. Second, we present the first attempt to include *edge* shocks in a DSGE model. Following an increase in the intermediation shock between two counties, the presence of a country with network centrality implies negative network effects on all bilateral flows, and positive effect on domestic loan issued in non-central countries. Third, we explore changes in node centrality following a shock to multiple edges in the network, as in Brexit. Finally, we show how networks and

indirect flows amplify the role of agglomeration forces, where financial hubs emerge by exploiting information-based scale economies.

How does the expansion of multinational banks influence the business cycle of host countries? In the third chapter, we study an economy where multinational banks can transfer liquidity across borders through internal capital markets but are hindered in their allocation of liquidity by limited knowledge of local firms' assets. We find that, following domestic banking shocks, multinational banks moderate the depth of the contraction but slow down the recovery. A calibration to Polish data suggests that multinational banks reduce the average depth of recessions by about 5% but increase their duration by 10%. The predictions are broadly consistent with evidence from a large panel of countries.

To Giulia, my wife, you never left me.
To family and friends I did leave in Italy, you had been a bright reminder of love during dark times.
To friends I shared joy and despair in Michigan, and had to leave in the midst of the pandemic, we will share happy tunes again.
To Shengpan, you left us way too soon, may your infinite love linger in our hearts forever.

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CHAPTER 1

INTERMEDIATION COSTS AND BANKING HUBS IN THE GLOBAL NETWORK

1.1 Introduction

Firms acquire international funding in different ways. In one case, firms directly look for loans from abroad to diversify their funding. This case can be related to an Eaton and Kortum (2002) setting, where bilateral flows occur because of direct lending. Niepmann (2015) develops a theoretical framework to further refine this category, such that global, international and foreign banking emerge from the different way banks acquire deposits. The second option is related to the literature that focuses on indirect trade flows, which emerge either because of global value chains (e.g. Antràs and De Gortari (2020)), or because of transport hubs (Ganapati et al., 2020). Similarly, lending flows might be intermediated between source and destination: among many, the double-decker banking model in Bruno and Shin (2015) presents a setting where regional banks face loan demand by firms, and diversify away regional shocks by borrowing from global banks.

Given global demand and supply of loans, it is crucial to understand the geography of banking flows. Modeling bilateral trade in goods has been the object of an intense research agenda. Reduced form gravity estimation was complemented by an effort in using heterogeneity to obtain a microfunded and closed-form expression for bilateral trade. The banking literature has followed a similar path: the theoretical foundations laid by Martin and Rey (2004) was empirically explored, among many, by Buch (2005), Portes and Rey (2005) for equity in addition to papers focusing on both trade in goods and assets, e.g. Aviat and Coeurdacier (2007). The baseline gravity equation relies on a measure of distance between the country pairs. While this is a natural assumption in trade, it raises concerns for asset flows, which are not per se constrained by physical shipping and hence distance. A possible reconciling approach is to interpret geographical distance as a proxy for cultural, information, and transaction costs. While the link between distance can be captured by the use of data on time zones or presence of subsidiaries, as in Brei and von Peter (2018), the estimation still relies on finding a variable that can meaningfully proxy the relationship. Moreover, while empirically using proxy variables is a feasible option to quantify the effect of frictions, it cannot capture the structural microeconomic foundations.

How close are countries in banking? While geographical distance, cultural, or institutional observed measures have been used as a proxy for trade costs in both trade and banking, this paper is the first to use (Allen and Arkolakis, 2019) transportation model to structurally estimate the international banking cost between countries. We provide a theory of determination of banking flows[1] and endogenous information costs. As in Proost and Thisse (2019): "The spatial distribution of activities is the outcome of a trade-off between different types of scale economies and costs that are generated by the transfer of people, goods, and information." Total information costs are spatially dependent: they are a combination of bilateral information costs and the information costs on the other pairs. In other words, the network structure of banking flows, as pictured in figure 1.1, emerges as banks find the best path to reach corporations around the world, while looking for funds in the interbank market.

It is useful to introduce here the network terminology used throughout the paper[2]. A graph G is an ordered pair of disjoint sets (V, E) such that E is a subset of the set $V^{(2)}$ of unorderd pairs of V, the set of vertices, or nodes, and E is the set of edges, where an edge $\{x_1, x_2\}$ joins the nodes x_1 and x_2. We assume V and E finite. A path is a graph P which consists of an ordered sequence of nodes and a set of edges that connects the nodes, following the sequence, i.e. $V(P) = \{x_0, x_1, \ldots, x_N\}$, and $E(P) = \{x_0x_1, x_1x_2, \ldots, x_{N-1}x_N\}$. The network structure of our model allows us to tackle the complexity of this economy. On one hand, we face an "edge problem": a change in the cost on one edge of the network has an effect on the entire E set, since flows are indirect, such that trade occurs over multiple edges, i.e. paths. On the other hand, there is a "node problem": the change in one edge will have complex general equilibrium effects on non-banking sectors of the economies of the countries, which will in turn feed back into the edges of the banking network. This paper

[1] "Households' cross-border deposits can be important, however, and are determined in part by non-resident nationals placing deposits with banks in their home country." Luna and Hardy (2019)

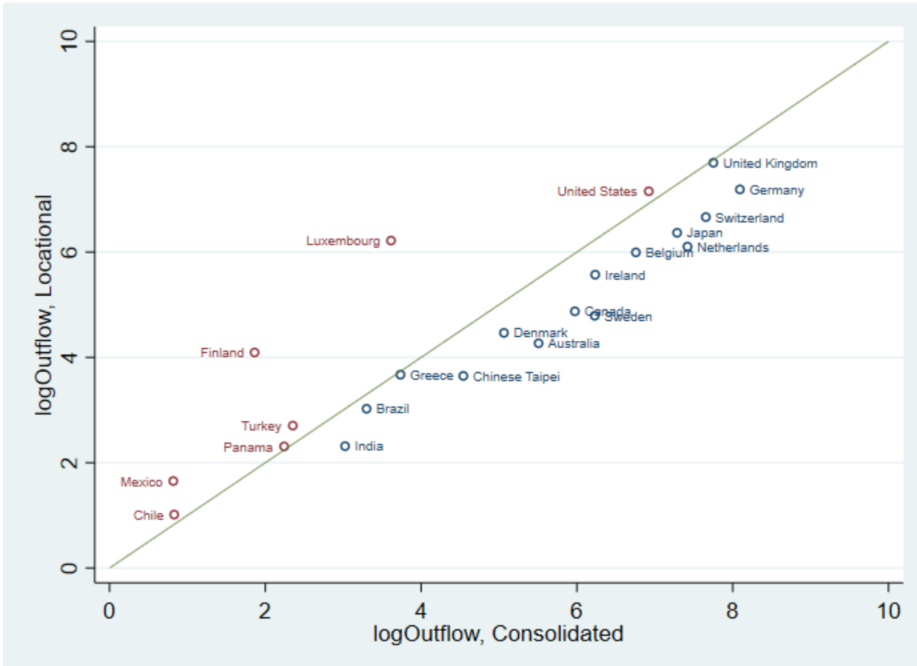
[2] See Bollobás (2013) for a general reference on graph theory.

Figure 1.1: BIS CBS Bilateral Banking, aggregated by origin, 2010

addresses both concerns by developing a model with a rich network structure which leads to a structural estimation of the information frictions. The intuition is the following: a country with a high level of outstanding direct and indirect flows must be appealing to both. Even more so, if the amount of direct flows is significantly less then the indirect flows, it must be that the country presents cheaper bilateral information costs. Figure 1.2 helps visualize the concept: plotting total banking flows for each country, it is possible to see how countries like Luxembourg present more indirect trade, while countries like the UK are characterized by a high amount of both direct and indirect flows.

We will use two data sources from the Bank of International Settlements (BIS): the consolidated banking statistics (CBS) which we will consider as direct trade, and the locational banking statistics (LBS), which corresponds to the indirect trade, or traffic. The model will deliver a closed-form expression of the traffic flows between an arbitrary pair of nodes (countries). The expression relies on the intermediation frictions and the amount of trade between origin and destination of the entire path. We use the transportation model by Allen and Arkolakis (2019) to describe the complexity

Figure 1.2: Consolidated (direct) vs Non-consolidated (indirect) flows



of international lending, where flows might happen directly between a bank and a firm, but also indirectly, through the presence of affiliates and complex financial routes, as recently shown by Coppola et al. (2021). The assumption we make is that lenders in country i minimize the cost of sending a loan to a firm in country j, and that firms minimize the cost of the loan. Heterogeneity in the types of banks and the max-stability property of the Fréchet distribution allow us to obtain an closed-form expression of the information frictions, taking into account intermediate steps at other nodes k and l. Summing over all the origin destination pairs, we obtain the probability that we observe an indirect flow between k and l. By the law of large numbers, we obtain the amount of flows, as in the gravity literature. These frictions will be then estimated as the values that rationalize the difference between observed direct and predicted flows. This is the core mechanism of the paper, augmented by two refinements: the aforementioned analysis implicitly relies on the existence of a network, assumed exogenous. We present a way to study the extensive margin through the lens of an endogenous network formation model, mainly based on the work by Mele (2017). Finally, we introduce an interbank market, which represents a relevant portion of international banking flows. Here, we relax the path assumption we make in the corporate sector, and instead let the network to be defined by bilateral optimization decisions. As in Antràs and De Gortari (2020), we obtain an equivalency result, which allows us to estimate the informational costs under this weaker assumption. As a final remark, we allow the intermediation costs is broadly capture all costs associated with sending a loan from one country to another, including banking commissions and fees, exchange rate risk, taxation differentials, differences in accounting and legal standards, and information-specific costs, which is our preferred interpretation.

1.1.1 Prior Literature

This paper speaks to a wide range of topics in the literature. The first strand is gravity in finance. Martin and Rey (2004) introduce a two-county model for equity flows. Brüggemann et al. (2011) use an Eaton and Kortum (2002) framework to obtain a banking gravity equation where frictions are interpreted as monitoring cost, and proxied with geographical distance. Okawa and Van Wincoop (2012) introduce a gravity model for assets where information frictions affect the variance of equity payoffs. Niepmann (2015) focuses on different global banking financing. Empirically, Buch (2005) uses BIS LBS data and reduced-form panel estimation to find that the role of distance is relevant and stable through the 80s and the 90s; Papaioannou (2009) confirms the relevance of geographical distance and adds the role of institutional quality as a key variable to predict cross border bank lending. Aviat and Coeurdacier (2007) point out the relationship between trade and equity flows trying to solve for the "correlation" puzzle, further explored by Brei and von Peter (2018). Lane and Milesi-Ferretti (2008) and Buch et al. (2013) include several other proxies: time zone difference, common language, colonial relationship, currency union, the existence of an investment tax treaty, and common origins to the legal system. While the latter find a significant role for geographical distance, this is not the case in the former work. Olivero and Yotov (2012) find instead a significant role played by geographical distance in a dynamic framework. This paper also relates to the literature on hubs and scale economies in finance and trade. Restricting the scope of the review, the work of Martin and Rey (2004) remains an important starting point: transaction costs generate scale effects, since smaller economies, that rely on foreigners to finance their projects, are more heavily subject to the frictions. More specifically, our work finds its trade counterpart in Ganapati et al. (2020), who use the path structure developed by Allen and Arkolakis (2019) to understand the role of scale economies in containerized trade. Finally, this paper contributes to network models, which have a long tradition in the peer effect literature (Calvó-Armengol et al., 2009), and has seen relevant advances in combining strategic and random endogenous formation (Mele, 2017), and in production chains, either from a industrial point of view (Oberfield, 2018), or related to trade and global value chains (Antràs and De Gortari, 2020).

1.2 The Environment

There is a finite number of countries. In each country $i \in I$ a representative household supplies labor in the goods market and in the banking sector and consumes a single final good, produced by firms. Firms pay labor and buy capital from capital producers. Capital and labor are immobile across countries. Capital is produced by borrowing internationally from the banking sector. Capital producers have a taste for diversity over bank heterogeneity, e.g. differences in the type of financial products, to finance the production of capital for the domestic economy. In other words, banking loans are the equivalent of intermediate goods, aggregated via CES by the capital producers.

In every country, there is a continuum of banks along the type of financial products they offer, indexed by $\omega \in \Omega$. The overall banking system operates in three stages. In the first stage of the model, a set of global intermediaries, one per country, generate the international network i.e. they choose the set of countries where they place subsidiaries, branches, form interbank partnerships. In the second stage individual banks take the network as given, generate loans as in Goodfriend and McCallum (2007). They extend loans internationally to capital producers. For a given ω, the price paid by the capital producers equals to the marginal cost, times an intermediation costs:

$$p_j(\omega) = \min_{i \in I, r \in G} \{c_i \tau_{ij}(\omega, p)\} \tag{1.1}$$

where $\tau_{ij}(\omega, p)$ is the realized trade cost from i to j and p is the path chosen by lenders. Countries differ in their geography, as captured by a $N \times N$ matrix of deterministic iceberg trade coefficients,

τ_{ij}, and idiosyncratic shocks. Importantly, per each type of product, capital producers in country i choose the cheapest option globally available. The price will depend on the deterministic geography and on path and type specific shocks. Hence, we do not assume that each country borrows from all the other countries. This will happen in equilibrium: the law of large numbers will determine the fact that there will be a non-zero trade between countries, aggregating over all the types.

Finally, banks lend and borrow liquidity in the interbank market. As in the corporate case, borrowers choose the minimum price available internationally. However, we relax the assumption that lenders optimize over the overall path, allowing for a decentralized network equilibrium to arise.

The timing is the following: global intermediaries play their best responses and endogenously determine the existence of binary edges between pairs of nodes in the network. To produce, firms need to invest in capital. Capital producers acquire funds from abroad. The idiosyncratic shocks realize, path forms, and banks allocate funds internationally in the corporate market. At the same time, banks acquire liquidity in the international interbank market. Lenders minimize trade cost and borrowers choose the cheapest price: bilateral flows and price indexes endogenously arise for both the corporate and the interbank market.

A few notes on iceberg costs are worth mentioning. First, iceberg cost implies that the elasticity of demand for an asset with respect to its price is the same whether the transaction cost is paid or not, that is, whether the asset is a domestic or a foreign one. Second, we model trade costs as iceberg cost on the level of price. Alternatively, Okawa and Van Wincoop (2012) introduce a gravity model for assets where information frictions change the variance of equity payoffs. While explicitly modeling portfolio allocation better relates to the information in assets return, the modeling cost is that demand for loans in this case depends on the covariance structure of the loans, making it harder to meet the condition laid by Anderson and Van Wincoop (2004).

1.3 Global intermediation

There is a finite set I of global intermediaries, simultaneously deciding who to link with, under complete information. There are infinitesimally short periods. In each period, pairs of representative financial intermediaries are drawn (e.g. USA and Italy); then, given the existing network, they decide if they take some actions. Based on the payoffs on the action, they decide want to change the bilateral link, until no pair of countries wants to change the network. We first describe the payoff structure played on the network, and subsequently characterize how countries form bilateral links. Given the existing network $\mathbb{G}$, each intermediary $i \in I$ chooses an individual action $a_i = (x_i, z_i)$ in order to maximize her payoff. The payoff structure, as in Bala and Goyal (2000), consists of $h(g_i)$, the benefit that i obtains from the links $\mathbb{G}$, and $c(g_i)$, the cost associated with forming and maintaining the links. g_i is the adjacency matrix of country i, i.e. an I x 1 vector where each element is one if it's connected with the jth country, zero otherwise.

We can equivalently express the payoff as a function of endogenous and exogenous actions: $U_i : A \times \mathbb{G} \rightarrow \mathbb{R}$, which depends on both individual action z_i, on the action of the other players z_{-i}, and matrix of choice and exogenous variables $\mathbb{X}$:

$$\Pi_i \left[h_i(g), c_i(g) \right] = U_i(z_i, z_{-i}; \mathbb{G}, \mathbb{X}) \tag{1.2}$$

We interpret the meaning of forming international links in a broad sense: this includes the physical establishment of subsidiaries and branches, as well as M&A operations or relationship with financial intermediaries. In what follows, benefits and cost shifters are both included in the matrix $\mathbb{X}$.

We assume that $U(\cdot)$ is additively separable, as in Mele (2017), and comprises of four components. First, u is the network endogenous component of the utility: each intermediary chooses action z_i, given the network interaction of the action profile, in the spirit of Ballester et al. (2006). Second, v represents the network and pre-determined transitivity effect: country i utility is affected by the existence of a link with country j through the existence of a node k linked to both i and j. The transitivity effects is a cost shifter whose sign is ambiguous: on one side, country i might benefit from country j if they have a lot of friends in common, e.g Graham (2016). On the other side, the existence of an intermediate node k reduces the need of i to form a link j, since it can reach the market in j through k. In other words, k act as a platform[3]. The effect depends on the potential violation of the triangular inequality related to τ_{t-1}, the pre-determined expected banking costs[4]. Third, w represents the idiosyncratic and exogenous component, bilateral benefits and cost which do not depend on network effects, e.g. standard gravity variables such as language or trade. Fourth, agents' receive an idiosyncratic shock to their preferences which the econometrician does not observe. The error is assumed i.i.d. among links and across time, drawn from a Gumbel(0,1) distribution. Hence, the matrix $\mathbb{X}$ can be split into three components: an exogenous matrix x, the first moment of the distribution of the unobserved costs τ and an endogenous choice vector z, so that the individual utility is[5]:

$$U_i(z_i, z_{-i}; \mathbb{G}, \mathbb{X}) = \underbrace{\sum_j g_{ij} u(z_i, z_{-i})}_{endogenous} + \underbrace{\sum_j g_{ij} \sum_{k \neq i,j} g_{ik} g_{kj} v_{ikj}(\tau_i)}_{transitivity} + \underbrace{\sum_j g_{ij} w_{ij}(x_i)}_{exogenous} + \epsilon_{i,\mathbb{G}} \tag{1.3}$$

The network formation process arises in two stages: in the first stage the agents play their best replies in the endogenous game, taking the network as given[6]. Second, links will create if agents do not have an incentive to add new edges, or remove existing ones:

$$U_i(z^*, \mathbb{G}) - U_i(z^*, \mathbb{G} \setminus \{ij\}) + \epsilon_{i,j,\mathbb{G}} > 0$$

The next two subsections will provide details of both steps.

1.3.1 Nash Equilibrium

In this section, we provide an example of the endogenous game played on the vector of actions z over the network $\mathbb{G}$. While the example helps building an intuitive microfundation for the following

[3] Among others, see Tintelnot (2017).

[4] This paper does not model the dynamics of the network formation, hence this term will be statically captured by fixed effects.

[5] We could express the utility as a potential utility to obtain the same result, see Mele (2017) for details.

[6] As in Calvó-Armengol et al. (2009), we can omit the exogenous part of the game, the details are provided in appendix D.

equilibrium, it does not exclude alternative options. Indeed, the link between the optimal decision by the global intermediaries and the decision made by the banks generally capture global financial decisions that lead to the emergence of an adjacency matrix. Intermediaries of each countries simultaneously choose z_i, the net long position of their total asset inventory:

$$u_i(z, \mathbb{G}) = p_i z_i - c(\mathbb{G}) z_i$$

We assume the following inverse demand relationship for the price of the asset:

$$p_i = 1 - \gamma z_i$$

and the marginal cost function:

$$c(\mathbb{G}) = c_0 - \lambda \sum_j g_{ij} z_j$$

where $c_0 > 0$ is the intermediary's marginal cost in absence of links, $\lambda > 0$ captures the cost reduction induced by the connection with other countries' intermediaries, as in Cohen-Cole et al. (2015). We normalize $\gamma = 1$. The utility function is therefore:

$$u_i(z, \mathbb{G}) = \mu z_i - z_i^2 + \lambda \sum_{j \neq i} g_{ij} z_i z_j$$

where $\mu = 1 - c_0$.

Theorem 1 in Ballester et al. (2006)

If and only if $\lambda\rho(\mathbb{G}) < 1$, where $\rho(\cdot)$ is the spectral radius, then the matrix $[\mathbb{I} - \lambda\mathbb{G}]^{-1}$ is well defined and non-negative, such that there is a unique and interior Nash equilibrium:

$$z^*(\mathbb{G}) = \frac{\mu}{\lambda} \text{Ɓ}(\mathbb{G}, \lambda)$$

where $\text{Ɓ}(\mathbb{G}, \lambda) = (\mathbb{I} - \lambda\mathbb{G})^{-1}(\lambda\mathbb{G}\mathbf{1})$ is the Katz-Bonacich centrality measure. The condition $\lambda\rho(\mathbb{G}) < 1$ requires that the own-concavity is high enough to counter the payoff complementarity, in order to prevent the positive feed-back loops triggered by such complementarities to escalate without bound. Equilibrium utilities are:

$$u_i(z^*, \mathbb{G}) = (z_i^*)^2 = \frac{\mu^2}{\lambda^2} \text{Ɓ}_i(\mathbb{G}, \lambda)^2 \tag{1.4}$$

where $Ɐ_i$ can be written as $1 + \sum_j Ɐ_j$.

1.3.2 Network Formation Process

At the beginning of each period, agent i meets agent j according to a matching technology, a stochastic sequence $m = \{m^t\}_{t=1}^{\infty}$, such that the probability that the two agents meet is given by:

$$\Pr(m^t = ij|\mathbb{G}^{t-1}, \mathbb{X}) = \rho(\mathbb{G}^{t-1}, Z_{ij}) \tag{1.5}$$

with $\sum_i \sum_j \rho(\cdot) = 1$. Given the static nature of the model, the distributional assumption of the matching probability function is not crucial[7]. The two agents form a link if it improves utility under the existing network structure: we say that $\mathbb{G}$ is a pairwise stable equilibrium if there is no player that has an incentive to delete an existing link or to establish a new link:

$$U_i(z^*, \mathbb{G}, g_{ij} = 1, \mathbb{X}) + \epsilon_{i,\mathbb{G},g_{ij}=1} > U_i(z^*, \mathbb{G}, g_{ij} = 0, \mathbb{X}) + \epsilon_{i,\mathbb{G},g_{ij}=0} \tag{1.6}$$

or equivalently:

$$U_i(z^*, \mathbb{G}, g_{ij} = 1, \mathbb{X}) - U_i(z^*, \mathbb{G}, g_{ij} = 0, \mathbb{X}) + \epsilon_{i,\mathbb{G}'} > 0 \tag{1.7}$$

where $\epsilon_{i,\mathbb{G}'}$ is the difference of two Gumbel random variables and hence follows a logistic distribution. The difference in the utility is given by the difference of each deterministic component of the utility function: endogenous U_i^u, transitivity U_i^v, and exogenous U_i^w[8]:

$$\begin{cases} U_i^w(\mathbb{G}, g_{ij} = 1) - U_i^w(\mathbb{G}, g_{ij} = 0) = u_{ij} = f(X_i, X_j, X_{ij}) = Z'_{ij}\beta \\ U_i^v(\mathbb{G}, g_{ij} = 1) - U_i^v(\mathbb{G}, g_{ij} = 0) = \sum_k g_{ik} g_{kj} v(\tau) + \sum_{j' \neq j} g_{ij'} \sum_j g_{jj'} v(\tau) \\ U_i^u(\mathbb{G}, g_{ij} = 1) - U_i^u(\mathbb{G}, g_{ij} = 0) = \frac{\mu^2}{\lambda^2} \left[Ɐ_i(\mathbb{G}, g_{ij} = 1, \lambda)^2 - Ɐ_i(\mathbb{G}, g_{ij} = 0, \lambda)^2 \right] \end{cases} \tag{1.8}$$

Summarizing, the link between i and j depends on: direct and homophily effects (Z), transitivity (where δ can be positive or negative) and connectivity, and network centrality. The network $\mathbb{G}$

[7] The sequentiality is important in leading to a unique equilibrium. See Gualdani (2020) for multiple equilibria.

[8] See, respectively, Joshi et al. (2020), Mele (2017), Calvó-Armengol et al. (2009).

is a pure strategy Nash equilibrium if it is robust to multi-link deviations by each player. Under additively separability the network is PSNE if and only if (Gualdani, 2020):

$$g_{ij} = \mathbf{1}\left\{ Z'_{ij}\beta + \delta\left(\sum_{k=1}^{N} g_{ik}g_{kj}\right) + A_i + A_j + \epsilon_{i,\mathbb{G}'} \geq 0 \right\} \tag{1.9}$$

Overall, the model is providing a microfundation for an exponential random graph model which, given the distribution of the error, we can related to an empirical expression of the form:

$$\Pr(g_{ij}|y, z, X) = \frac{1}{1 + \exp(\psi_{ijt})} \tag{1.10}$$

where

$$\begin{aligned} \psi_{ij} =& \beta_0 + X'_i\beta_1 + X'_j\beta_2 + X'_{ij}\beta_3 + && \text{direct and homophily effect} \\ &+ \delta r_{ij} + \gamma_1 \text{Ƀ}_i + \gamma_2 \text{Ƀ}_j + && \text{transitivity and centrality} \\ &+ \kappa_1 + \kappa_2 && \text{unobserved degree heterogeneity} \end{aligned}$$

We do not assume $\delta > 0$: while transitivity has a positive effect is several social networks, i.e. the more friends we have in common, the more likely we are connected, it might not be the case in this context. In fact, if banks might place affiliates in countries that have a centrality node in the entire network, or a cluster of the network. Hence, if there is a node k which has a high clustering coefficient hence connected to many neighbours, nodes i might not need to connect to node j, since k is closely linked to j. In other words, k act as a platform, a way for the intermediary in country i to extend its operations to a larger subset.

1.4 Path gravity and prices

In the previous section countries has sequentially formed links according to their utility, such that in every period the network is a PSNE. Given the existence of the network, i.e. the extensive margin of international lending, lenders and borrowers trade. Trade form i to j is a loan from a bank in country i that is used for production from a firm in country j, through the CES aggregation of capital producers. There are four major ways such that the production of funds by the bank and the production of the good will be related. First, the bank in i and the firm in j might arrange a contract in a direct way, according to an international finance model where firms diversify their funding. However, international finance can be indirect, along several dimensions. Second, the bank in i might decide to conduct operations in j through a subsidiary or a branch located in country j (or in a third country k)[9]. Third, the bank in i might provide liquidity to a local bank in j, which will then lend money to the firm. Fourth, international interbank liquidity might be tied to a specific corporate project only after going through k, l, m, and so on[10]. We assume that, in order to send funds abroad, banks face informational asymmetries that prevent a frictionless allocation of resources. Absent information frictions, the interest rate equals the marginal cost of extending the loan c. The observed interest rate p_{ij} is the product of the marginal cost and the intermediation friction $\tau_{ij} > 1$[11]:

$$p_{ij}(p,\omega) = c_i \tau_{ij}(p,\omega) \tag{1.11}$$

where c_i is the marginal cost of generating the loan.

The unobservable intermediation friction consists of two components: $\tilde{\tau}_{ij}(p)$ which captures the deterministic bilateral, gravity aspects of the route, as common language or geographical distance, and a banking type-specific idiosyncratic element $\xi_{ij}(p,\omega)$:

$$\tau_{ij}(p,\omega) = \frac{\tilde{\tau}_{ij}(p)}{\xi_{ij}(p,\omega)} \tag{1.12}$$

We place no structure on the edge costs, allowing them to be a flexible function of exogenous and endogenous variables: $t = f(X\_exog; X\_endog)$. The deterministic component, $\tilde{\tau}_{ij}(p)$, is the cost of going from i to j along route $p \in G$ of length:

$$\tilde{\tau}_{ij}(p) = \prod_{k=1}^{K_p} t_k \tag{1.13}$$

where $t_k = x_{k-1}x_k$ is the edge specific cost, i.e. the cost between the nodes $x_{k-1}x_k$ along the route p. K, the length of the route, is route specific.

[9] An example of the latter situation is the presence of a subsidiary of a US bank in the UK, which finances projects in the EU. Hence, acquiring information on institutional details in EU, while staying in a country with similar institutions, language, etc.

[10] There is a fifth option, which we cannot identify from the data: the possibility that the bank finances a firm in country j while the production is in country k, see Coppola et al. (2021).

[11] Note that this implies the standard no arbitrage condition $\frac{p_{ij}(p,\omega)}{p_i} = \tau_{ij}(p,\omega)$, which however allows for profitable intermediation opportunities when the triangle inequality is not satisfied.

The second component, $\xi_{ij}(p,\omega)$, the type-specific idiosyncratic shock, is assumed to be realizations of draws from a Fréchet distribution such that:

$$F_{ij}(\xi) = \exp\left\{-T_i \xi^{-\theta}\right\} \tag{1.14}$$

where T_i is an origin-specific banking technology parameter which capture absolute advantage, while $\theta > 0$ is the shape parameter which governs comparative advantage through the degree of heterogeneity in across loans[12]. Intermediation cost τ depends on the potential paths across the network available to the lender, for each product ω. As in Ganapati et al. (2020), the idiosyncratic route draws are generating the stochastic price dispersion usually assumed to be idiosyncratic TFP. Lenders choose the least cost path, or route[13], to send their type ω to capital producers around the world, such that:

$$\tau_{ij}(\omega) = \min_{p \in G} \tau_{ij}(p,\omega) \tag{1.15}$$

We assume $\tau_{ij}(\omega) = 1$ for $j = i$, and that $\tau_{ij} < \infty$ even in the case of no trade: it is possible that the friction from i to j is finite, but the benefits of connecting to j are less than the cost paid to form a link, as specified in section 1.3.

Given the stochastic nature of the ex-ante price, we want to know the probability that any given good ω is shipped from i to j on a specific route p. Lenders choose the lowest-cost route r from i to j for ω from all routes $p \in G$ and borrowers in j choose the lowest-cost supplier of good ω from all countries $i \in I$. In other words, the probability that a country i provides loans to country j at the lowest price[14].

Proposition 1 *The probability that, given that lenders choose the lower cost route, the cost is below τ is given by:*

$$H_{ij\omega}(\tau) = 1 - \exp\left\{ -\tau^{\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \right\}$$

[12]The interpretation of the latter, as pointed out by Allen and Arkolakis, is that "θ can be considered as capturing the possibility of mistakes and randomness in the choice of routes, with higher values indicating greater agreement across traders. In the limit case of no heterogeneity, $\theta \to \infty$, all traders choose the route with the minimum aggregate trade cost".

[13]We will use the term route to match the letter r, since p refers to price.

[14]A priori, each borrower could receive funds from multiple lenders. However, given the stochastic nature of interest rate determination, the probability that any borrower faces two lenders with the same price is zero.

Proof. *See Appendix A.0.1.*

Proposition 2 *The probability that the, given borrowers minimize price, the price is below r is:*

$$G_{j\omega}(r) = 1 - \exp\left\{ - r^{\theta} \sum_{i' \in I} c_i^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \right\}$$

Proof. *See Appendix A.0.2.*

We now can combine the two sides of the market:

Proposition 3 *The probability that a bank in country j with liquidity need υ chooses to borrow from a bank in country i, and that the route from country i to j is the minimal cost route:*

$$\pi_{ijp\omega} = \frac{\left[c_i \tilde{\tau}_{ij}\right]^{-\theta}}{\sum_{i' \in I} c_i^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}} \tag{1.16}$$

Proof. *See Appendix A.0.3.*

Summing across routes and heterogeneity, the probability that the banking sector of country i provides loans to the non-financial sector in country j at the lowest price is:

$$\pi_{ij} = \frac{\left(c_i \tau_{ij}\right)^{-\theta}}{\gamma P_j^{-\theta} a^{\theta}} \quad \text{by the price derivetion, see appendix equation A.14} \tag{1.17}$$

Therefore more productive countries, countries with lower marginal costs, and countries with relatively lower bilateral trade costs will account for a larger fraction of the loans given to country j. This is also the share of the loans sold from i to j, and so the fraction of the total loans acquired in j, $X_j = \sum_i X_{ij}$ that originate from country i:

$$\begin{aligned} X_{ij} &= \pi_{ij} X_j \\ &= a^{-\theta} \gamma^{-1} \tau_{ij}^{-\theta} (c_i)^{-\theta} P_j^{\theta} X_j \end{aligned} \tag{1.18}$$

As the bilateral trade costs rise, the origin country is the least cost provider in fewer goods. The greater θ, the less heterogeneity in a country's productivity across different goods, so there are a greater number of goods for which it is no longer the least cost provider. In other words, the greater the heterogeneity in productivity, the lower the density of these marginal producers that are indifferent between exporting and not exporting.

Proposition 4 *The expected intermediation costs is given by:*

$$
\begin{aligned}
\tau_{ij} \equiv& \mathbb{E}_p\left[\tau_{ij}(\omega)\right] \\
=& \Gamma\left(\frac{1+\theta}{\theta}\right)\left[\sum_{p\in G_{ij}} \tilde{\tau}_{ijp}^{-\theta}\right]^{-1/\theta} \qquad \textit{from equation A.7} \\
=& \gamma^{-\theta} b_{ij}^{-1/\theta}
\end{aligned} \tag{1.19}
$$

Proof. See Appendix A.0.4.

Hence the expected bilateral price:

$$
p_{ij} \equiv \mathbb{E}_p\left[p_{ij}(\omega)\right] = c_i \mathbb{E}\left[\tau_{ij}(\omega)\right] = c_i \gamma^{-\theta} b_{ij}^{-1/\theta} \tag{1.20}
$$

Following the derivations in Appendix F we can obtain an expression for the price index in a given country, aggregated over all industries and origin countries:

$$
\begin{aligned}
P_j =& \left(\int_{\omega\in\Omega} p_j^{1-\sigma}(\omega)\, d\omega\right)^{\frac{1}{1-\sigma}} \\
=& a\left(\sum_{i\in I}\left[c_i \tau_{ij}\right]^{-\theta}\right)^{-1/\theta}
\end{aligned} \tag{1.21}
$$

where the marginal cost of producing a unit of corporate loans X_i:

$$
c_i^X = w_i \tag{1.22}
$$

1.5 The rest of the economy

The model is presented in the form of a dynamic model, to provide an anchor to reader familiar with the macroeconomic literature. However, the nature of the model and the estimation is static, hence the economy will be taken at its steady state.

1.5.1 Banks

In each country $i \in I$ there is a continuum of banks, which differ by type of financial product, indexed by $\omega \in \Omega$[15]. In this section, we omit the subscript ω to simplify notation. Within each

[15]See Niepmann (2015) for an alternative approach, where banks are heterogeneous on the quality of monitoring.

country, the market for each type of financial product is contestable, in the sense that a large number of firms have access to the same production technology and can enter any sector without any entry barriers. This will ensure that equilibrium profits are always equal to zero. Banks expend effort and require borrowers to post collateral, but in equilibrium there is no default. Banks extend loans needed to produce capital. The core activity of the representative bank, as in Goodfriend and McCallum (2007) is the generation of loans to the firms. The loan generation technology is a function of the monitoring effort of the loan office, and the capital pledged by firms through the aggregating role of capital producers. Banks make decisions on deposit taking D_t, monitoring labor demand M_t, and loan supply to capital producers X_t, to maximize their value[16]:

$$V_{i,t} = \max_{\{D_{t+s}, X_{t+s}, M_{t+s}\}_{s\geq 0}} \mathbb{E}_0 \sum_{s=0}^{\infty} \Lambda_{t,t+1} N_{t,t+s+1}$$

$$s.t. \quad X_{i,t} = D_{i,t} + N_{i,t} - w_{i,t} M_{i,t} \tag{1.23}$$

$$N_{i,t+1} = P_t^X X_{i,t} - R_t^D D_{i,t} \tag{1.24}$$

$$X_{i,t} = (M_{i,t})^{\zeta} (K_{i,t})^{1-\zeta} \tag{1.25}$$

where R_i^D is the interest rate paid on deposits, and P_i^X is the price of loans to the corporate sector. Equation 1.23 captures the balance sheet identity, equation 1.24 is the law of motion of net worth. Equation 1.25 describes the firm loan generating process: banks grant loans based on the collateral provided by the borrower (capital) and the bank's ability to monitor.

1.5.2 Capital Producers

Loans are costlessly assembled to produce an aggregate nontraded good which is only used to finance capital, as in Connolly and Yi (2015). Since it is not crucial for our story, we abstract from further microfundation on the portfolio decision of capital producers. This could arise because the project that they finance are imperfectly correlated Martin and Rey (2004), but we posit, as in Brei and von Peter (2018), that capital producers have a diversification motive over the types of financial product. Diversification as the only motive has been criticized by Niepmann (2015) for the lack

[16]The model is dynamic, the trade analysis is done assuming steady state values.

of empirical support. A possible reconciliation is provided by Craig and Ma (2020): "Moreover, borrowing banks delegate their borrowing from lending banks to a small subset of large and well diversified intermediary banks". The role of capital producers is therefore merely providing firms a diversified access to international finance through a CES aggregator function:

$$I_t = \left(\int_{\omega \in \Omega} X_t(\omega)^{\rho}) \; d\omega \right)^{1/\rho}$$

1.5.3 Households and Firms

Households in country i maximize their lifetime utility by choosing consumption C_t, deposits D_t, labor supply in the goods market H_t, and labor supply in the banking monitoring activity M_t:

$$\max_{\{C_t, D_t, H_t, M_t\}_{t\geq 0}} \qquad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - \frac{(H_t+M_t)^{1+\epsilon}}{1+\epsilon} \right)^{1-\gamma} - 1}{1-\gamma}$$

$$s.t. \quad C_t + D_t = R^D_{t-1} D_{t-1} + w_t(H_t + M_t) + \Pi_t + N_t \tag{1.26}$$

Firms produces output using labor H and predetermined capital stock K, via an increasing and concave production function F. Firms cannot issue equity nor use internal financing; there is no default. The representative firm in country i solves the following:

$$\max_{\{H_t, I_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{t,t+1} \left[F(K_t, H_t) - w_t H_t - P_t I_t \right]$$

$$s.t. \quad K_{t+1} = (1-\delta) K_t + I_t \tag{1.27}$$

where $\Lambda_{t,t+1} = \beta^t \frac{u'_{C_{t+1}}}{u'_{C_t}}$ is the stochastic discount factor and P_t is the price of capital.
As we exposed in detail in section 1.4, the price at which firms finance their investment is a stochastic object whose distribution is known by the agents.
The equilibrium of the model is provided in appendix C.

1.6 Interbank and Financial Institutions

In this section, we extend the model by adding a network microfundation of the international interbank market, which provides a theoretical fundation for the resulting interbank gravity equation. The international interbank market was first established as an "informal market of short-term placements of deposits at fixed rates between banks in different countries", improving FX risk management. avoid the costs imposed by domestic regulations., and helping participants to gain information on foreign counterparties (Bernard and Bisignano, 2000). The international interbank market has been historically characterized by strong scale economies (BIS, 1983). In recent years, interbank lending has overall declined in share over total cross-border flows after the global financial crisis, but the concentration has remained stable[17] (Aldasoro and Ehlers, 2019), feature that has been captured in recent theoretical models, such as Craig and Ma (2020).

In our derivations on the corporate network, we assumed that agents optimize over the entire path, in other words, controlling the entire route that the tradable good will take. A similar approach has been adopted by Babus and Hu (2017) in an effort to model an over-the-counter market. In this section we relax this *centralized* path assumption: we assume instead that the interbank network emerges in a decentralized fashion, through bilateral relationships between lenders and borrowers. The latter might use the funds received to provide liquidity, hence becoming lenders, and so the initial liquidity is traded in the network. However, there is no centralized optimization over the entire path. Using the distributional assumptions by Oberfield (2018), we can obtain a closed-form expression for the probability of observing a bilateral trade flow, *conditional* on the path, even if agents do take the path into account when forming decisions. This will lead to a gravity equation that is close to our previous one, and therefore will allow us to use the same estimation procedure to retrieve the trade costs.

[17] France, Germany, Japan, the United Kingdom and the United States account for 70% of the volume of the largest bilateral links.

1.6.1 Model

Most of the work on interbank lending focuses on the overnight trade, e.g. the fed funds rate market. However, as described by Kuo et al. (2014), banks use term wholesale borrowing as a part of their structural funding. Different maturities help hedge against interest rate, liquidity, and rollover risk. Moreover, term borrowing serves the traditional purpose of insulate banks against a sudden inability to borrow, and against unexpected liquidity withdrawals. Finally, while secured transactions represents a relevant part of the interbank market, our model best captures the unsecured market, which is important especially for banks whose assets are opaque, i.e. loans that are difficult to use as collateral, especially domestically, and therefore search counterparties abroad. Within a country, an opposite mechanisms shapes the need for interbank and deposits based funding: as in Craig and Ma (2020) "in the presence of costly monitoring between banks, it is optimal for banks with more transparent assets to borrow from the interbank market and for those with more opaque assets to invest in retail deposits and become interbank lenders". This consensus on the fact that there is a strong link between the type of assets and the type of liabilities allows us to assume that the heterogeneity in the interbank market is the same as the one assumed for the corporate market, such the index $\omega \in \Omega$ fully characterizes the type of each bank.

As before, the technology generating supply of interbank loans Z^S follows Goodfriend and McCallum (2007). Inputs are deposits collected from the households and the liquidity previously acquired in the interbank market. The roundabout structure of the interbank financing and its role will be extensively discussed in the next section.

$$
\begin{aligned}
V_{i,t} = & \max_{\{D_{t+s}, X_{t+s}, Z^D_{t+s}, M_{t+s}\}_{s\geq 0}} \mathbb{E}_0 \sum_{s=0}^{\infty} \Lambda_{t,t+1} N_{t,t+s+1} \\
s.t. \quad & X_{i,t} + Z^S_{i,t} = D_{i,t} + Z^D_{i,t} + N_{i,t} - w_{i,t} M_{i,t} \\
& N_{i,t+1} = P^X_t X_{i,t} - R^D_t D_{i,t} + P^Z_t Z^S_{i,t} - P^Z_t Z^D_{i,t} \\
& X_{i,t} = (M_{i,t})^{\zeta} (K_{i,t})^{1-\zeta} \\
& Z^S_{i,t} = (D_{i,t})^{\eta} (Z^D_{i,t})^{1-\eta}
\end{aligned}
\tag{1.28}
$$

The marginal cost to produce a unit of interbank loans Z_i is[18]

$$c_i^Z = (R_i^D)^\eta (p_i^Z)^{1-\eta} \tag{1.29}$$

where P_i^Z is the price of liquidity paid in the interbank sector.

The market is organized in a probabilistic network structure. At each country/node k, banks produce liquidity using node-related inputs (deposits), and input from other nodes (international liquidity), giving rise to a recursive production structure, as in Oberfield (2018)[19]. Each bank at node $k+1$ meets a certain amount of potential liquidity suppliers from node $k \in I \setminus \{j\}$. As in the full path case, in order to send funds abroad, banks face informational asymmetries that prevent a frictionless allocation of resources. The bilateral intermediation cost can be separated into two components, both pair-specific: the bilateral deterministic cost t, and an industry-related shock $\xi_{kj}(\omega)$:

$$\tau_{k,k+1}(\omega) = \frac{t_{k,k+1}}{\xi_{k,k+1}(\omega)} \tag{1.30}$$

where the idiosyncratic cost ξ follows an iid Pareto distribution with shape parameter θ and lower bound ξ_0. Hence, as in the full-path case, the price is a combination of the marginal cost and the intermediation friction:

$$\begin{aligned} p_{k,k+1}^Z(\omega) &= c_k^Z \ \tau_{k,k+1}(\omega) \\ &= (R_k^D)^\eta (p_{k-1,k}^Z)^{1-\eta} \ \tau_{k,k+1}(\omega) \qquad \text{by equation 1.29} \end{aligned} \tag{1.31}$$

[18]We normalize the production function to avoid carrying the constant.

[19]Johnson and Moxnes (2019) provide an alternative formulation, which relies on numerical convergence.

1.6.2 The probability of using a particular link

We borrow some terminology from the global value chain literature, since the production occurs through a number of stages. At a given stage, the steps that happened before are said upstream, the ones that will occur later, or the final buyer, are located downstream relative to the current stage. In this section, we derive the probability that, conditional on a certain origin-destination node pair $\{ij\}$, trade occurs between the edge that connects nodes l and m. The novelty arises from the fact that that we do not assume that the lender in i chooses over the entire path; instead, at each intermediate stage, interbank liquidity producers take the upstream price as input cost, and sell the tradable good to the next node, forming a chain. Equation 1.31 introduces the main challenge we face: the marginal cost depends on the upstream distribution, embedded into $p^Z_{k-1,k}$. We proceed as follows: first, we derive $F(p)$, the distribution of prices at a given node, conditional on a the bilateral lender/borrower shock. Second, we derive $G_{k,k+1}(p)$, the distribution of the prices that characterize trade between an arbitrary country pair k and $k+1$, given the probability of upstream prices $F(p)$ previously derived[20]. The distributional assumptions and the recursive structure of the problem allow us to obtain a closed-form expression for the price distribution at an arbitrary node. There are two sources of randomness. The first is a bilateral shock $\xi_{k,k+1}(\omega)$ we introduced in equation 1.30. Moreover, each liquidity borrower does not know the number of stages of the current trade is based of, and this number, i.e. the position in the path, is stochastic. Given the recursive structure of the production, the position in the path determines how many bilateral shocks feed into the input price that the borrower faces. We now define the price distribution for a *bilateral* trade at a given node μ in the path, for a given type of financial product ω. The probability $F(p)$ is obtained by taking $G(p,\xi)$, the joint probability of downstream price and the upstream shock ξ[21],

[20] Given the realization of ξ, the bilateral deterministic costs and individual production optimization determine the price that borrowers face. While the price that each borrower faces varies across realizations of the economy, the cross-sectional distribution of prices does not.

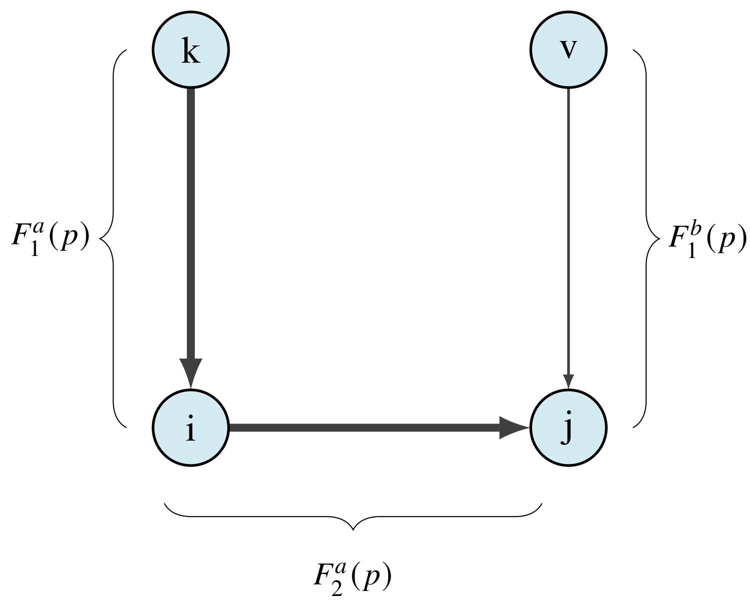
[21] Let G(p) be the fraction of lenders with price no greater than p. The law of large numbers assures that $G_{ijp\omega}(p) = \Pr(p_{ij}(p,\omega) \leq p)$.

and integrating out the bilateral shock:

$$F_{ij}(p) \equiv \int_{\xi_0}^{\infty} G_{ij}\left(p,\xi\right)\ dH(\xi) \tag{1.32}$$

To visualize the problem, consider a case with four nodes, k, i, v, and j, as in figure 1.3.

Figure 1.3: Decentralized path choice, an example



At j the choice is made between sourcing from i or from v, regardless of the fact that i sourced from k and so implicitly the overall path a comprises of two steps, and has origin k and destination j *through* i.

The price p_{ij} faced by borrower j depends on two parts: the bilateral shock ξ and the position in the network μ[22]. Borrowers at j will decide whether to choose i or v as a lender. The probability that they choose one of the two options depends on the bilateral shocks that affect the iceberg cost, as in the centralized case. However, the probability that j chooses to borrow from i depends on the price distribution that i faced in turn. There is indeed a path, indexed a in the figure, that characterizes the flow of funds from k to j. In the example, path a comprises of two steps, indexed as 1 and 2, while path b, the one that connects v and j, consists of one step only. Node j which is

[22]In the case where the position of the node is known, then integrating out the bilateral shock leads to a degenerate distribution.

at stage $\mu + 1$ chooses the lender from set of upstream nodes $V(\mu \in p)$ that offers the lowest price. In other words, $F(p)$ captures the distribution of a single bilateral stage, by removing the chained effect that the ξ shocks have on the price distribution.

We now consider the entire network. Interbank liquidity origins from an arbitrary node i and reaches another arbitrary node j through a chain of n steps, or number of edges. Let the position in the sequences be indexed by μ. The position of each node in the path is a random number which follows a Poisson distribution with mean M. Since we have integrated out the randomness of the bilateral shock at each node pair, the probability that the price is less than p is given by the probability of being at a certain position μ in the path p, times the probability that the price is lower than p at a each of the n steps in the path[23]. Formally, this is given by:

$$
\begin{aligned}
G_{ijp\omega}(p) &= \Pr(p_{ij}(p,\omega) \leq p) \\
&= \sum_{n=0}^{\infty} \Pr\left(\text{length} = n\right) \cdot \Pr\left(p_{ij} = \underset{v \in V(\mu \in p)}{\text{argmin}} \; p_{vj}\right) \\
&= \sum_{n=0}^{\infty} \underbrace{\Pr\left(\text{length} = n\right)}_{\text{Poisson}} \cdot \prod_{\mu=1}^{n} F(p)_{\mu} \\
&= \sum_{n=0}^{\infty} \frac{M^n e^{-M}}{n!} \left[F(p)\right]^n \\
&= e^{-M} \sum_{n=0}^{\infty} \frac{\left[MF(p)\right]^n}{n!} \\
&= e^{-M[1-F(p)]} \qquad (1.33) \\
&= \exp\left\{-M\left[1 - \int_{\xi_0}^{\infty} G\left(p,\xi\right) \; dH(\xi)\right]\right\}
\end{aligned}
$$

where the second step describes the price minimization of the borrowers, for a given set of available upstream lenders V through a path of length $|r| = n$, taking into account the fact that paths might be infinitely long. The third step can be obtained by the fact that we assume that shocks are

[23]Note that the buyer-seller productivity draws at stage-μ are independent of the upstream draws. Even if the idiosyncratic shocks are uncorrelated, the upstream price affects the marginal cost.

pairwise uncorrelated. Finally, we use the definition of e.

From equation 1.33, following Oberfield (2018), we obtain the following result:

Proposition 5 *Let M be the mean of the Poisson distribution. Let m satisfy $M = m\xi_0^{-\theta}$, and assume that ξ follows a Pareto distribution with shape parameter θ and cutoff ξ_0: $H(\xi) = 1 - \left(\frac{\xi}{\xi_0}\right)^{-\theta}$. Then, the price distribution is Fréchet, and given by:*

$$G_{ijp\omega}(p) = \exp\left\{-\kappa p^{-\theta}\right\} \tag{1.34}$$

Proof. See Appendix B.

We assume that m reflects the marginal price in the interbank market, as in 1.31. Using the recursive structure of the problem, we can obtain that, as in Antràs and De Gortari (2020), the probability that loans go from i to j assuming that the optimization happens at a bilateral level is directly related to the probability obtained if the optimization happens along the entire path:

$$\begin{aligned} G_{ijp\omega}(p) &= \exp\left\{-p^{-\theta}\prod_{k=1}^{K} t_{k-1,k}^{-\theta}\prod_{k=1}^{K}\left(c_k^{-\theta}\right)^{\eta_k\eta_k'}\right\} \\ &= \exp\left\{-p^{-\theta}\tilde{\tau}_{ij}^{-\theta}c_i\right\} \end{aligned}$$

where $\prod_{k=1}^{K}\left(c_k^{-\theta}\right)^{\eta_k\eta_k'} \equiv c_i$, and $\prod_{k=1}^{K} t_{k-1,k} \equiv \tilde{\tau}_{ij}$ by equation 2.2.5.1.
Since the idiosyncratic shocks are independent, by the law of total probability we obtain the probability that the chain starts at i, goes through K steps for a given path p and reaches j:

$$\pi_{ijr} = \frac{\tilde{c}_i\tau_{ij}^{-\theta}}{\sum_{i'\in I} c_{i'}\sum_{p\in G}\left[\tilde{\tau}_{i'j}(p)\right]^{-\theta}} \tag{1.35}$$

Relating this equation to equation 1.16, we can obtain, as in Antràs and De Gortari (2020), that the decentralized solution leads to an equivalency result: the probability of funds going from origin i to destination j can be obtained as the outcome of a bilateral decision process. To keep the estimation tractable, we assume that the marginal along the path do not differentially impact the choice of borrower, so that the aggregation of the two markets, the corporate and the interbank market, can be expressed as the simple sum of the flows. Alternative but stronger assumptions would be to let the interbank flows to match one-to-one the corporate flows, or that lenders optimize over the entire path.

1.7 Information Friction Estimation

This section takes the model and to the data, in order to estimate the unobserved bilateral costs t_{ij} and the expected bilateral cost τ_{ij}. We will first show how to use the probability previously derive to obtain the model prediction for the indirect flow, conditional on observing the direct flows. Second, we will use data on direct and indirect banking flows to estimate the unobserved costs. We start by defining two matrices:

$$[A]_{ij} = t_{ij}^{-\theta} \tag{1.36}$$

$$B = (\mathbb{I} - A)^{-1} \tag{1.37}$$

where t_{ij} is the deterministic edge friction between i and j. Recall from section 1.4 that $\tau_{ij} = \gamma^{-\theta} b_{ij}^{-1/\theta}$, where γ is a constant. Therefore, we can rewrite the probability of a banking flow to go from i to j (see equation 1.16) as:

$$\pi_{ij} = \frac{\left[c_i \tau_{ij}\right]^{-\theta}}{\Phi_j} \tag{1.38}$$

where $\Phi = \sum_{i \in I} c_i^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} = \sum_{i' \in I} c_{i'}^{-\theta} \, b_{i'j}$ by using the definition of b_{ij}. Intuitively, Φ is a multilateral resistance term, in the spirit of Eaton and Kortum (2002).

We can now obtain the probability that a loan starts origins in i with destination in j, going through nodes k and l. Following Allen and Arkolakis (2019), we first replace the expected cost τ with equation B.8. Second, we sum over all the possible origin destination routes that go through the edge kl, and further sum over the length of the path K. Third, we replace $\tilde{\tau}$ using its definition.

Fourth, we apply the definition of the A and B matrices:

$$
\begin{aligned}
\pi_{ij}^{kl} =& \frac{\gamma^{-\theta}}{\sum_{i\in I}\left[c_i\tau_{ij}\right]^{-\theta}} c_i^{-\theta} \sum_{K=0}^{\infty} \sum_{p\in G_{ij}^{kl}(K)} \tilde{\tau}_{ijp}^{-\theta} \\
=& \frac{\gamma^{-\theta}}{\sum_{i\in I}\left[c_i\tau_{ij}\right]^{-\theta}} c_i^{-\theta} \sum_{K=0}^{\infty} \sum_{p\in G_{ij}^{kl}(K)} \prod_{k=1}^{K} t_k(p)^{-\theta} \\
=& \frac{c_i^{-\theta}\, b_{ik} a_{kl} b_{lj}}{\sum_{i'\in I} c_{i'}^{-\theta}\, b_{i'j}} \\
=& c_i^{-\theta}\left[b_{ik}a_{kl}b_{lj}\right]\Phi_j^{-1}
\end{aligned}
\tag{1.39}
$$

Dividing equation 1.39 by equation 1.38 we can calculate the probability of any good traveling through a kl edge, conditional on being sold from origin i to destination j. Given that X_{ij} is the total value of direct flows (trade) between i and j, we can express the total volume of traffic between k and l as:

$$
\Xi_{kl} = \sum_i \sum_j X_{ij} b_{ik} a_{kl} b_{lj} b_{ij}^{-1} \tag{1.40}
$$

This expression is the structural expression for indirect flows Ξ, as a function of trade costs and direct flows X only. Both flows are total flows, hence comprising both corporate and interbank market.

As in Ganapati et al. (2020), conditional on the observed trade values X_{ij}, the contribution of trade between i and j to the traffic between legs k and l is invariant to multilateral resistance or marginal costs, since they affect trade from that country and others proportionally on all routes. Therefore, following Allen and Arkolakis (2019), we can establish that there exists a unique set of banking *direct* flows X consistent with observed country characteristics, market clearing conditions, and the banks' optimization. This implies that there exists a unique *indirect* matrix Ξ of banking flows. We can now recover the costs a_{ij} and b_{ij} that rationalize observed bilateral direct and indirect flows. The theoretical model predicts that the indirect flows are defined by the following matrix formulation of equation 2.17:

$$
\Xi^{\text{expected}} = A \odot B'(X \oslash B)B' \tag{1.41}
$$

Recall that the overall goal of the estimation is to estimate the unobserved costs τ and t, which define the matrices A and B, that rationalized the total observed direct and indirect banking flows:

$$\min_{\beta} \sum_{ij} \left| \Xi_{kl} - \Xi_{kl}^{\text{expected}}(\tau, t|X) \right|$$

Let $\Pr(g_{ij} = 1) = a_{ij}$. This relates the extensive margin to the intensive margin: the net benefit of forming a link is inversely related to bilateral costs incurred in extending a loan (see equation 1.10)[24]:

$$a_{ij} = t_{ij}^{-\theta} = \frac{1}{1 + \exp(Z'_{ij}\beta)} \in [0, 1] \tag{1.42}$$

where

$$Z'_{ij}\beta = \alpha + \beta_1 \Xi_i + \beta_2 \Xi_j$$

The matrix of observables Z_{ij} can be extended to include further bilateral and country-specific variables, leaving the results fairly similar. The relationship between the extensive margin derived in the first part of the work serves a double purpose: it provides a microfunded interpretation for the edge costs, which are proportional to the costs incurred to set up or maintain the endogenous matrix structure. Moreover, it allows to naturally rely on a parametric assumption for the A matrix, thus avoiding imposing a sparsity condition on the network, which might be too strong in this context. To summarize the estimation procedure: given the set of parameters, we obtain a guess for the A matrix, hence the B matrix, and have a prediction for the indirect trade, following equation 2.22. Importantly, our goal is not to pursue inference, but to saturate the moments but minimizes the distance between the observed and the model-based traffic[25]:

$$\min_{\beta} \sum_{ij} \left| \Xi_{kl} - \Xi_{kl}^{\text{expected}} \left(A(\beta), B(A(\beta)), X_{ij} \right) \right| \tag{1.43}$$

[24] This relationship does not fully allow for a feedback in the other direction, since internalizing the amount of flows into the global intermediaries' problem introduces a dynamic aspect of the network formation which requires a different set of equilibrium solutions.

[25] Although not necessary for our purpose, it is possible to bootstrap the standard errors.

1.7.1 Data

Our model requires two sources of data: a trade, representing the direct trade, and a traffic matrix, indirect trade. More specifically, the direct matrix captures the flow from *origin* i to *destination* j, while in the indirect matrix the bilateral flow ij represents the amount of banking flows going *through* i and j, summing across all origin-destination pairs. The direct trade matrix is taken from the Consolidated Banking Statistics (CBS) of the Bank for International Settlements (BIS), while the Locational Banking Statistics (LBS) of the BIS provide data on the traffic matrix. Using the equivalence result that relates the decentralized path to the centralized path, we can use data on all sectors, bank to bank, and bank to nonbank[26]. The BIS data is the most comprehensive source of information on international banking, available at quarterly frequency. In details, the LBS are reported according to the residence principle and the reporting banks include subsidiaries and branches. Instead, CBS data consolidate positions across parent banks. Let's consider an example, a subsidiary of a US bank located in Italy providing a loan to a firm in Germany. This would be considered a USA to Germany transaction in the CBS data, while the LBS will report it as Italy to Germany. The object traded are mainly claims over deposits and loans[27]. The main difference between LBS and CBS is in terms of cross sectional availability. In fact, the reporting countries in the CBS amount of a subset of 30 reporting countries, which will be hence our sample for both datasets. It would be possible to overcome this issue with further development in the availability of data on the liability side, which could allow to expand the set of countries by using both sides of the balance sheet, as it is currently possible for the LBS (see Brei and von Peter (2018)). While several studies have relied on one of the two datasets in order to estimate gravity equations (e.g. Buch (2005), Brüggemann et al. (2011), Brei and von Peter (2018) use the LBS, Aviat and Coeurdacier (2007) use the CBS), this paper first uses the variation between the two datasets to recover the unobserved costs that are usually proxied but geographical distance or alternative observable measure.

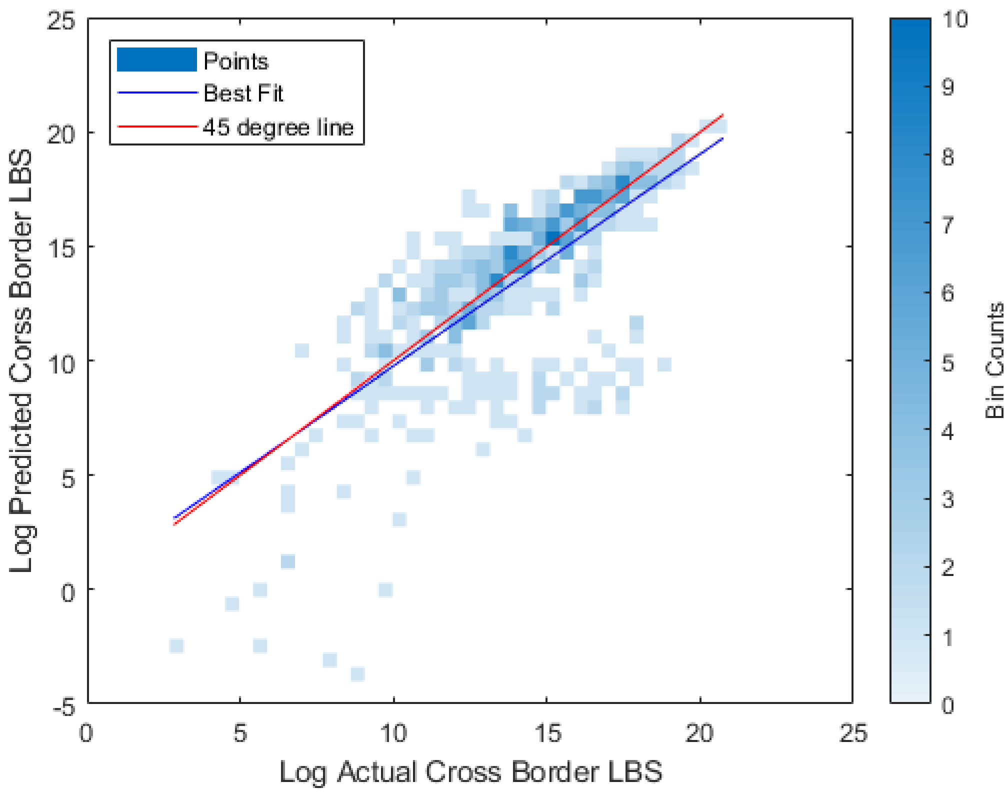
[26] The differential impact of a cost change through the corporate and the interbank market is left to future research.

[27] The LBS data provide a detailed breakdown on the type of instruments, which we do not pursue in this work.

1.7.2 Results

We show the results computed for the 2014q4 data[28]. We check our estimation results by comparing the LBS (traffic, indirect flows) predicted by the model against their observed counterparts in the data (Figure 2.3). We include both a best fit line and a 45 degree line, representing the perfect fit scenario. In general, we fit the data extremely well, with a correlation between the observed and predicted shares of 0.903.

Figure 1.4: Model Prediction



The estimated bilateral costs t, up to scale θ, are visualized in figure 1.5, while the scaled *inverse* of the expected costs τ, are in figure 1.6. Therefore a low value for the A matrix corresponds to a low cost, while the opposite is true for matrix B (see equation A.10). For example, the estimation shows that, as expected, the UK has low costs both as a lender (rows) and as a borrower (column), especially with respect to the US, Germany, France and Japan. Japan, instead, has low costs as a

[28]The results on different quarters and years are available upon request.

lender but not necessarily as a borrower. As a final note, both the A and B matrix are not symmetric, consistently with our assumptions.

Figure 1.5: Matrix A

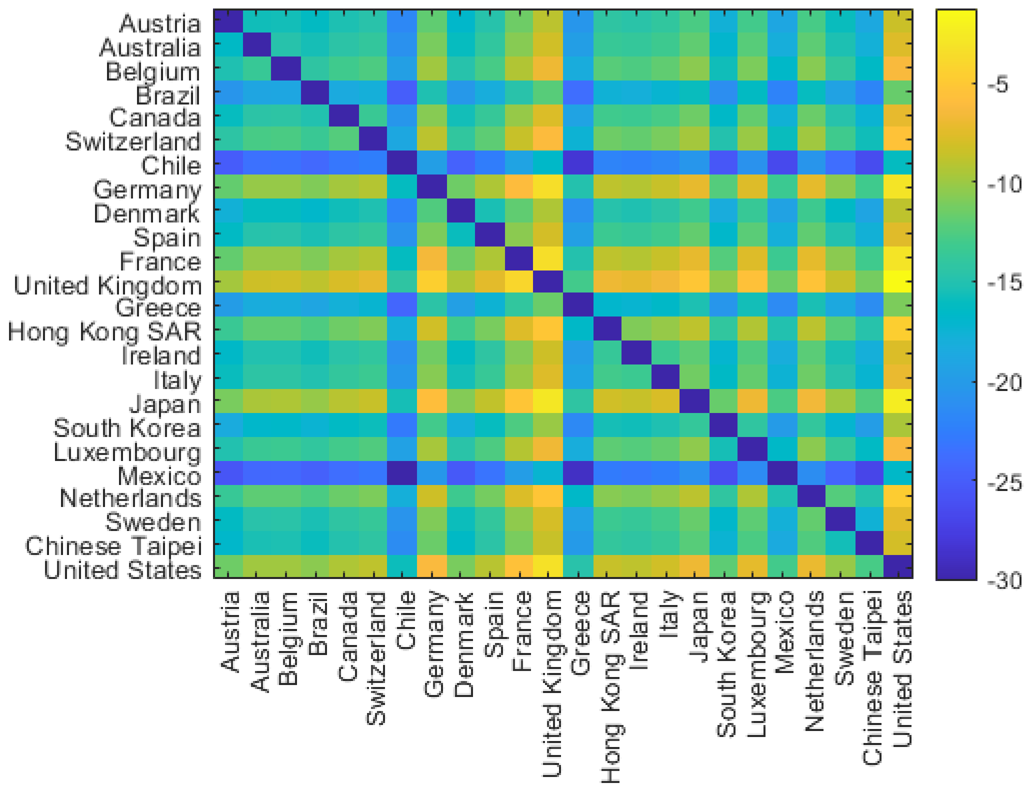
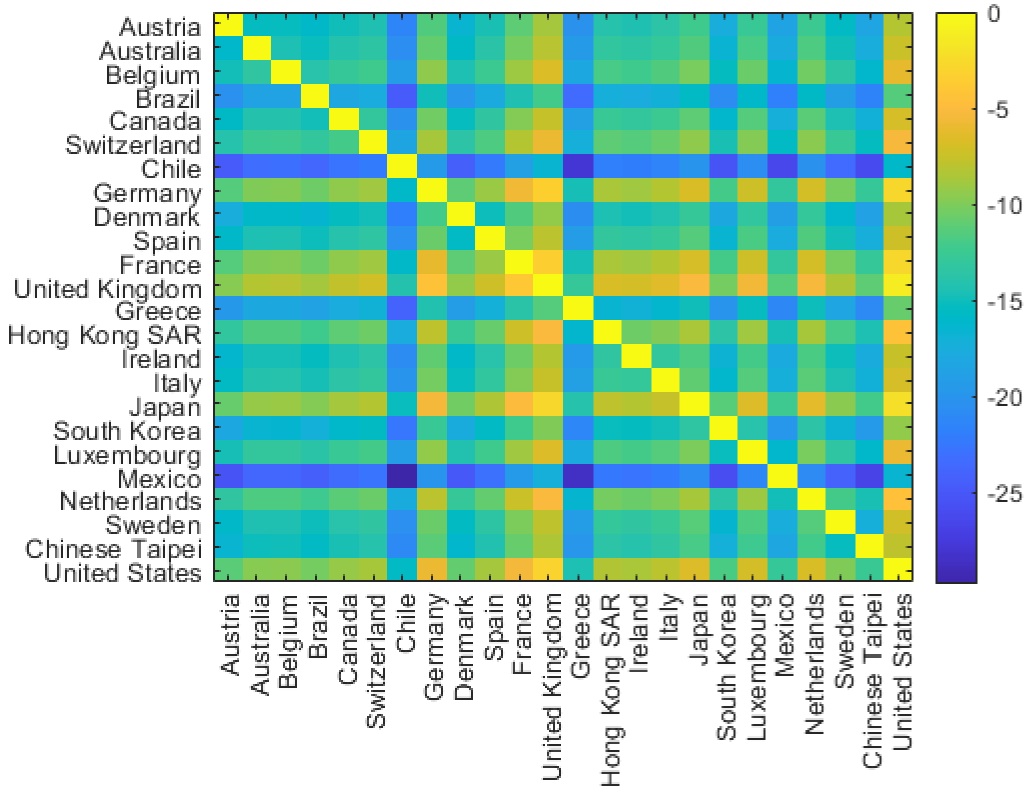


Figure 1.6: Matrix B



1.8 Conclusion

Trade in banking flows occurs both directly and indirectly, through a variety of intermediators. This paper has proposed a structural model to estimate unobserved trade cost in the context of international banking. We present an endogenous network formation process that models the existence of the network. The costs associated with setting the network are proportional to the bilateral cost faced when moving funds, e.g. loans and deposits, across borders. We embedded Allen and Arkolakis (2019) transportation model in a Ricardian structure, where heterogeneous banks provide different types of financial products $\omega \in \Omega$ to firms. We obtained the probability that a trade happens from country k to country l, conditional on origin i and destination j. The derivations rely on the Frechét distributional assumption of the idiosyncratic shocks that occur by type and by path. Moreover, in the derivations of the interbank market, we relax the assumption made for corporate lending, i.e. that lenders optimize over the entire path. A combination of a Pareto and Poisson results in a Fréchet distribution for the price of liquidity, which implies that the path-optimizing behavior can be related to a sequence of decentralized decisions in the network. We use the gravity structure to model indirect banking flows as function of direct flows and unobservable trade costs only. We exploit both CBS and LBS data from the Bank for International Settlements to obtain the necessary variation to recover the unobserved costs, which rely on a parametric assumption which helps saturate the moments and minimize the distance between the observed indirect flows (LBS) and the indirect flows predicted by the model. The results confirm the role of the United Kingdom as a core financial hub in 2014q4. On the policy front, the paper provides a measure to assess the centrality of a country in the banking network. However, the model cannot fully capture dynamic decisions that affect the evolution of the network, which might be crucial to perform a counterfactual exercise. We leave this and other issues to future research.

APPENDICES

APPENDIX A

OPTIMAL PATH DERIVATIONS

Firms receive bids for financing their capital investments. Banks are competitive and each bank from country i and industry ω makes firms face the same interest rate P_i^X. Hence, price is given by:

$$p_{ij}^X(\omega) = c_i^X \ \tau_{ij}(\omega) \tag{A.1}$$

The goal is derive the probability that a route r is the lowest-cost route from i to j for good ω and country i is the lowest-cost supplier of good ω to j. We want to know the probability that any given good ω is shipped from i to j on a specific route p. Firms choose the lowest-cost route r from i to j for ω from all routes $p \in G$ and consumers in j choose the lowest-cost supplier of good ω from all countries $i \in I$. We will observe ω being shipped on route r from i to j if the final price of ω including both the marginal cost of production and shipping cost on route r from i to j, $p_{ijnr}(\omega)$, is lower than all other prices of good ω from all other country-route combinations.

Therefore we will find i) the probability that a country i provides loans to country j at the lowest price; ii) the price of the loan that a country i actually pays to country j is independent of j's characteristics.

A.0.1 Lenders

The *unconditional* probability that taking a route p to lend from country i to j for a given product ω that costs less than a constant τ is:

$$\begin{aligned} H_{ijp\omega}(\tau) \equiv & \Pr\Big(\tau_{ij}(p,\omega) \leq \tau\Big) \\ = & \Pr\left(\frac{\tilde{\tau}_{ij}(p)}{\xi_{ij}(p,\omega)} \leq \tau\right) \\ = & 1 - \Pr\left(\xi_{ij}(p,\omega) \leq \frac{\tilde{\tau}_{ij}(p)}{\tau}\right) \\ = & 1 - \exp\left\{ - \left[\frac{\tilde{\tau}_{ij}(p)}{\tau}\right]^{-\theta} \right\} \qquad \Big[\xi \sim \text{ Fréchet}(1,\theta)\Big] \end{aligned} \tag{A.2}$$

Because the technology is i.i.d across types, this probability will be the same for all goods $\omega \in \Omega$.

So far we have considered the potential trade cost, however we do not observe bilateral ex-ante cost, but the cost that each country applies ex-post, after choosing the cheapest path. The probability that, *conditional* on banks choosing the least cost route, the cost in ω is less than some constant τ:

$$\begin{aligned}
H_{ij\omega}(\tau) \equiv & \Pr\left(\tau_{ij}(\omega) \leq \tau\right) \\
= & \Pr\left(\min_{p \in G} \tau_{ij}(p,\omega) \leq \tau\right) \\
= & 1 - \Pr\left(\min_{p \in G} \tau_{ij}(p,\omega) \geq \tau\right) \\
= & 1 - \Pr\left(\bigcap_{p \in G} \left[\tau_{ij}(p,\omega) \geq \tau\right]\right) \\
= & 1 - \prod_{p \in G} \left[\Pr\left(\tau_{ij}(p,\omega) \geq \tau\right)\right] \quad \text{by independence} \\
= & 1 - \prod_{p \in G} \left[1 - \Pr\left(\tau_{ij}(p,\omega) \leq \tau\right)\right] \\
= & 1 - \prod_{p \in G} \left[1 - H_{ijp\omega}(\tau)\right] \quad \text{by eq F.3} \\
= & 1 - \exp\left\{-\tau^{\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right\}
\end{aligned} \tag{A.3}$$

To summarize, this is the probability that, given that banks choose the lower cost route, the cost is below a certain value.

A.0.2 Borrowers

Similar to equation F.3, the probability that the price is below a certain constant is the following:

$$\begin{aligned}
G_{ijp\omega}(r) \equiv & \Pr\left(r_{ij}(p,\omega) \leq r\right) \\
= & 1 - \exp\left\{-\left[c_i \frac{\tilde{\tau}_{ij}(p)}{r}\right]^{-\theta}\right\}
\end{aligned} \tag{A.4}$$

Firms minimize the price they pay across countries and routes:

$$\begin{aligned}
G_{j\omega}(r) \equiv & \Pr\left(\min_{i\in I, p\in G} r_{ij}(p,\omega) \leq r \right) \\
= & 1 - \Pr\left(\min_{i\in I, p\in G} c_i \tau_{ij}(p,\omega) \geq r \right) \quad \text{by eq 1.15} \\
= & 1 - \Pr\left(\bigcap_{i\in I} \bigcap_{p\in G} \left[c_i \tau_{ij}(p,\omega) \geq r \right] \right) \\
= & 1 - \prod_{i\in I} \prod_{p\in G} \left[\Pr\big(c_i \tau_{ij}(p,\omega) \geq r \big) \right] \quad \text{by independence} \\
= & 1 - \prod_{i\in I} \prod_{p\in G} \left[1 - G_{ijp\omega}(r) \right] \quad \text{by def} \\
= & 1 - \prod_{i\in I} \prod_{p\in G} \exp\left\{ - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\} \quad \text{by eq A.4} \\
= & 1 - \exp\left\{ \sum_{i\in I} \sum_{p\in G} - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\} \\
= & 1 - \exp\left\{ - r^{\theta} \sum_{i\in I} c_i^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\}
\end{aligned} \tag{A.5}$$

A.0.3 Market Making

Finally, we can combine the two sides of the market, i.e. the probability that a firm in country j producing ω chooses to borrow from a bank country i, and that the route from country i to j is the minimal cost route. In other words, we compute the probability that, picking any other route-country pair, the price will be higher than the optimal one.

$$
\begin{aligned}
\pi_{ijp\omega} \equiv& \Pr\left(r_{ij}(p,\omega) \le \min_{k \neq i,\, s \neq p} r_{kj}(s,\omega) \right) \\
=& \Pr\left(\bigcap_{k \neq i} \bigcap_{s \neq p} \left[c_k \tau_{kj}(s,\omega) \ge r_{kj}(p,\omega) \right] \right) \\
=& \prod_{k \neq i} \prod_{s \neq p} \left[\Pr\left(c_k \tau_{kj}(s,\omega) \ge r_{kj}(p,\omega) \right) \right] \quad \text{by indep} \\
=& \prod_{k \neq i} \prod_{s \neq p} \left[1 - G_{kjs\omega}\left(r_{kj}(p,\omega) \right) \right] \\
=& \int_0^\infty \prod_{k \neq i} \prod_{s \neq p} \left[1 - G_{kjs\omega}(r) \right] dG_{ijp\omega}(r) \\
=& \int_0^\infty \prod_{k \neq i} \prod_{s \neq p} \left[1 - G_{kjs\omega}(r) \right] \frac{d}{dr} \left[1 - \exp\left\{ - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\} \right] dr \\
=& \int_0^\infty \prod_{k \neq i} \prod_{s \neq p} \left[1 - G_{kjs\omega}(r) \right] r^{\theta-1} \theta \left[c_i \tilde{\tau}_{ij} \right]^{-\theta} \exp\left\{ - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\} dr \\
=& \int_0^\infty \prod_{k \neq i} \prod_{s \neq p} \left[\exp\left\{ - \left[c_k \frac{\tilde{\tau}_{kj}(s)}{r} \right]^{-\theta} \right\} \right] \left(r^{\theta-1} \theta \left[c_i \tilde{\tau}_{ij} \right]^{-\theta} \right) \exp\left\{ - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\} dr \\
=& \int_0^\infty \prod_{i \in I} \prod_{p \in G} \left[\exp\left\{ - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\} \right] \left(r^{\theta-1} \theta \left[c_i \tilde{\tau}_{ij} \right]^{-\theta} \right) dr \\
=& \int_0^\infty \left[\exp\left\{ - r^\theta \sum_{i \in I} c_i^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\} \right] \left(r^{\theta-1} \theta \left[c_i \tilde{\tau}_{ij} \right]^{-\theta} \right) dr \\
=& \int_0^\infty \left[\exp\left\{ - r^\theta \sum_{i \in I} c_i^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\} \right] \left(r^{\theta-1} \theta \left[c_i \tilde{\tau}_{ij} \right]^{-\theta} \right) dr \, \frac{\sum_{i' \in I} c_i'^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{i'j}(p) \right]^{-\theta}}{\sum_{i' \in I} c_i'^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{i'j}(p) \right]^{-\theta}} \\
=& \int_0^\infty \left[\exp\left\{ -r^\theta \sum_{i \in I} c_i^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\} \right] \left(r^{\theta-1} \theta \right) \sum_{i \in I} c_i^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} dr \, \frac{\left[c_i \tilde{\tau}_{ij} \right]^{-\theta}}{\sum_{i' \in I} c_i'^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{i'j}(p) \right]^{-\theta}} \\
=& \int_0^\infty -\frac{d}{dr} \exp\left\{ - r^\theta \sum_{i \in I} c_i^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\} dr \, \frac{\left[c_i \tilde{\tau}_{ij} \right]^{-\theta}}{\sum_{i' \in I} c_i'^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{i'j}(p) \right]^{-\theta}} \\
=& - \exp\left\{ - r^\theta \sum_{i \in I} c_i^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\} \Bigg|_0^\infty \frac{\left[c_i \tilde{\tau}_{ij} \right]^{-\theta}}{\sum_{i' \in I} c_i'^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta}} \\
=& \left[- \frac{1}{\exp(\infty)} + \frac{1}{\exp(0)} \right] \frac{\left[c_i \tilde{\tau}_{ij} \right]^{-\theta}}{\sum_{i' \in I} c_i'^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{i'j}(p) \right]^{-\theta}} \\
=& \frac{\left[c_i \tilde{\tau}_{ij}(p) \right]^{-\theta}}{\sum_{i' \in I} c_i'^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{i'j}(p) \right]^{-\theta}} \qquad \text{(A.6)}
\end{aligned}
$$

By the law of large numbers, given the continuum of products, this is also the share of all loans sold from i to j in industry ω and take route p.

A.0.4 Expected costs

Skipping the steps (similar to AA2019), the cost between locations i and j is expected trade cost τ_{ij} from i to j across all lenders:

$$\begin{aligned}\tau_{ij} \equiv \mathbb{E}_{\omega}\left[\tau_{ij}(\omega)\right] = & \int_{p \in G_{ij}} \tau_{ijp}(\omega)\ dp \\ = & \int_{0}^{\infty} \tau\ dH_{ij\omega} \\ = & \Gamma\left(\frac{1+\theta}{\theta}\right)\left[\sum_{p \in G_{ij}} \tilde{\tau}_{ijp}^{-\theta}\right]^{-1/\theta}\end{aligned} \tag{A.7}$$

Define $\gamma \equiv \Gamma\left(\frac{1+\theta}{\theta}\right)$ and following AA2019:

$$\tau_{ij}^{-\theta} = \gamma^{-\theta} \sum_{p \in G_{ij}} \tilde{\tau}_{ijp}^{-\theta} \tag{A.8}$$

taking into account the length of the path, and all possible lengths:

$$\begin{aligned} = & \gamma^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}(K)} \tilde{\tau}_{ijp}^{-\theta} \\ = & \gamma^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}(K)} \prod_{k=1}^{K} t_k(p)^{-\theta} \qquad \text{by definition 1.13} \\ = & \gamma^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}(K)} \prod_{k=1}^{K} a_{ij} \qquad \text{defining } t_k^{-\theta} = t_{ij}^{-\theta} \equiv a_{ij} \\ = & \gamma^{-\theta} \sum_{K=0}^{\infty} A_{ij}^{K}\end{aligned}$$

Assuming that the spectral radius of A is less than one[1] then:

$$\sum_{K=0}^{\infty} A^K = (\mathbf{I} - A)^{-1} \equiv B \tag{A.9}$$

[1]AA19: "sufficient condition for the spectral radius being less than one is if $\sum_j t_{ij}^{-\theta} < 1$ for all i. This will necessarily be the case if either trade costs between connected locations are sufficiently large, the adjacency matrix is sufficiently sparse, or the heterogeneity across traders is sufficiently small.

Hence:

$$\tau_{ij} = \gamma^{-\theta} b_{ij}^{-1/\theta} \Leftrightarrow b_{ij} = \sum_{p \in G_{ij}} \tilde{\tau}_{ijp}^{-\theta} \tag{A.10}$$

As in Allen and Arkolakis (2019) this is the "an analytical relationship between any given infrastructure network and the resulting bilateral trade cost between all locations, accounting for traders choosing the least cost route".

A.0.5 Edge Probability

Substituting equation A.7 into equation A.6:

$$\begin{aligned}
\pi_{ijp} =& \frac{\left[c_i \tilde{\tau}_{ij}\right]^{-\theta}}{\sum_{i' \in I} c_i'^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{i'j}(p)\right]^{-\theta}} \\
=& \gamma^{-\theta} \frac{\left[c_i \tilde{\tau}_{ijp}\right]^{-\theta}}{\sum_{i' \in I} \left[c_i' \tau_{i'j}\right]^{-\theta}} \qquad \text{by equation B.8} \\
\equiv& \frac{\gamma^{-\theta}}{\Phi} \left[c_i \tilde{\tau}_{ijp}\right]^{-\theta} \qquad \text{by defining } \Phi \equiv \sum_{i' \in I} \left[c_i' \tau_{i'j}\right]^{-\theta} \\
=& \frac{\gamma^{-\theta}}{\Phi} c_i^{-\theta} \prod_{k=1}^{K} t_k(p)^{-\theta} \qquad \text{by definition 1.13}
\end{aligned} \tag{A.11}$$

where Φ is closely follows the price parameter as in Eaton and Kortum (2002).

For a pair of nodes linked by the edge e_{kl} the probability of going through an edge tk, summing across all the paths that lead from i to j involving e_{kl} is the following:

$$\begin{aligned}
\pi_{ij}^{kl} =& \frac{\gamma^{-\theta}}{\Phi} c_i^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}^{kl}(K)} \prod_{k=1}^{K} t_k(p)^{-\theta} \\
=& \frac{\gamma^{-\theta}}{\Phi} c_i^{-\theta} \left(b_{ik} a_{kl} b_{lj}\right) \\
=& \frac{\gamma^{-\theta}}{\sum_{i' \in I} \left[c_i' \tau_{i'j}\right]^{-\theta}} c_i^{-\theta} \left(b_{ik} a_{kl} b_{lj}\right) \\
=& \frac{\gamma^{-\theta}}{\sum_{i' \in I} \left[c_i' \tau_{i'j}\right]^{-\theta}} c_i^{-\theta} \left(b_{ik} a_{kl} b_{lj}\right) \\
=& \frac{c_i^{-\theta} \left(b_{ik} a_{kl} b_{lj}\right)}{\sum_{i' \in I} c_i'^{-\theta} b_{i'j}}
\end{aligned} \tag{A.12}$$

Summing across all products, origin and destination countries, we obtain the probability that trade happens in the edge between node k and node l:

$$\pi^{kl} = a_{kl} \sum_j b_{lj} \frac{\Phi_k}{\Phi_j} \tag{A.13}$$

which is also the fraction of the total trade, by the law of large numbers.

A.0.6 Price

Since borrowers in each industry observe all the bids from all countries and select the minimal offer, the lender country and the sector coincides in term of indexing. Therefore, the country level price index is the following:

$$\begin{aligned} P_j =& \left(\int_{\omega \in \Omega} p_j^{1-\sigma}(\omega)\, d\omega \right)^{\frac{1}{1-\sigma}} \\ =& a(\Phi)^{-1/\theta} = a \left(\sum_{i \in I} \left[c_i \tau_{ij} \right]^{-\theta} \right)^{-1/\theta} \end{aligned} \tag{A.14}$$

where $a = \Gamma\left(\frac{1-\sigma+\theta}{\theta}\right)^{\frac{1}{1-\sigma}} \gamma^{-\theta}$ is a constant.

A.0.7 Gravity

The probability that the banking sector of country i provides loans to the non-financial sector in country j at the lowest price is:

$$\begin{aligned}
\pi_{ij} &= \sum_p \pi_{ijp} \\
&= \sum_p \frac{\left[c_i \tilde{\tau}_{ij}\right]^{-\theta}}{\sum_{i' \in I} c_i'^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{i'j}(p)\right]^{-\theta}} && \text{from equation A.6} \\
&= \sum_p \frac{\gamma^{-\theta}}{\Phi}\left[c_i \tilde{\tau}_{ijp}\right]^{-\theta} && \text{by equation A.11} \\
&= \frac{\gamma^{-\theta}}{\Phi} c_i^{-\theta} \sum_p \tilde{\tau}_{ijp}^{-\theta} \\
&= \frac{\gamma^{-\theta}}{\Phi} c_i^{-\theta} \frac{\tau_{ij}^{-\theta}}{\gamma^{-\theta}} && \text{by equation A.8} \\
&= \frac{\left(c_i \tau_{ij}\right)^{-\theta}}{\Phi} && \text{analog of Eaton Kortum equation 8} \\
&= \frac{\left(c_i \tau_{ij}\right)^{-\theta}}{\gamma P_j^{-\theta} a^{\theta}} && \text{by equation A.14}
\end{aligned} \tag{A.15}$$

APPENDIX B

DECENTRALIZED EQUILIBRIUM DERIVATIONS

$$
\begin{aligned}
M\left[1-F(p)\right] =& M\left[1-\int_{\xi_0}^{\infty} G\left(p,\xi\right)\ dH(\xi)\right] \\
&= m\xi_0^{-\theta}\int_{\xi_0}^{\infty}\left[1-G\left(p/(\xi)^{1/\beta}\right)\right]\ dH(\xi) \\
&= m\xi_0^{-\theta}\int_{\xi_0}^{\infty}\left[1-G\left(p/(\xi)^{1/\beta}\right)\right]\theta\xi_0^{\theta}\xi^{-\theta-1}\ d\xi \\
&= m\int_{\xi_0}^{\infty}\left[1-G\left(x\right)\right]\theta\xi^{-\theta-1}\ d\xi \\
&= p^{-\theta}m\int_{(p/\xi_0)^{1/\beta}}^{\infty}\left[1-G\left(x\right)\right]\theta(-\beta)x^{\beta\theta-1}\ dx \\
&= p^{-\theta}m\int_{0}^{(p/\xi_0)^{1/\beta}}\left[1-G\left(x\right)\right]\theta\beta x^{\beta\theta-1}\ dx
\end{aligned}
\tag{B.1}
$$

where the third line uses a change of variable involving the CDF and the PDF of the Pareto distribution, and the fifth line uses a change of variable $x = (p/(\xi)^{1/\beta})$. Take the limit of the integral for $\xi_0 \to 0$, so that equation B.1 can be written as $G(p) = \exp\left\{-\kappa(x)p^{-\theta}\right\}$, where:

$$\begin{aligned}
\kappa =& m \int_0^\infty \left[1 - G\left(x\right)\right] \theta\eta x^{\eta\theta-1} \; dx \\
=& m \int_0^\infty \left[1 - e^{-\kappa x^{-\theta}}\right] \theta\eta x^{\eta\theta-1} \; dx \\
=& m \int_0^\infty \left[1 - e^{-y}\right] \eta\kappa^{\eta} y^{-\eta-1} \; dy \\
=& m\kappa^{\eta} \int_0^\infty e^{-y} y^{-\eta} \; dy \\
=& m\kappa^{\eta}\Gamma(1-\eta) \\
\kappa =& \left[m\Gamma(1-\eta)\right]^{1/(1-\eta)}
\end{aligned} \tag{B.2}$$

where in the third line we use a change of variable $y = \kappa x^{-\theta}$, in the fourth line we integrated by parts, and finally we use the definition of the gamma function $\Gamma(\cdot)$. η is thus the elasticity of the price to the upstream intermediate supplier's price. The final distribution is:

$$G_{ijp\omega}(p) = \exp\left\{-\kappa p^{-\theta}\right\} \tag{B.3}$$

APPENDIX C

SIMPLE GENERAL EQUILIBRIUM

A competitive equilibrium is a sequence of prices and allocations such that: i) taking prices as given, agents maximize their objective, ii) corporate and interbank loans are purchased from their lowest-cost provider, given lenders path minimization, subject to the trade costs iii) markets clear in all periods. The first order conditions are: Household block

$$[C_t]: \quad \frac{u'_{C_t}}{u'_{C_{t+1}}} = \beta R_t^D \qquad \text{Euler equation}$$

$$[H_t^S]: \quad -\frac{u'_{H_t}}{u'_{C_t}} = w_t \qquad \text{Real good labor supply}$$

$$[M_t]: \quad -\frac{u'_{M_t}}{u'_{C_t}} = w_t \qquad \text{Banking labor supply}$$

Firms block

$$[H_t^D]: \quad F'_{H_t} = w_t \qquad \text{Real good labor demand}$$

$$[I_t]: \quad \left[F'_{K_{t+1}} + (1-\delta) + P_{t+1}\right]/(P_t) = R_t^X \qquad \text{Aggregate demand for capital}$$

Banking block

$$[M_t^D]: \quad -\lambda_t \frac{X_t}{M_t} + \mathbb{E}_t \Lambda_{t,t+1} \lambda_{t+1} w_t = 0 \qquad \text{Banking labor demand}$$

$$[D_t]: \quad -\lambda_t \beta \frac{Z_t^S}{D_t} + \mathbb{E}_t \Lambda_{t,t+1} \lambda_{t+1} R_t^D = 0 \qquad \text{Deposit demand}$$

$$[Z_t^D]: \quad -\lambda_t (1-\beta) \frac{Z_t^S}{Z_t^D} + \mathbb{E}_t \Lambda_{t,t+1} \lambda_{t+1} P_t^Z = 0 \qquad \text{Liquidty demand}$$

Prices and Interest Rates:

$$P_j = a \left(\sum_{i \in I} \left[c_i \tau_{ij}\right]^{-\theta} \right)^{-1/\theta} \qquad c_i \in \{c_i^X, c_i^Z\}$$

$$\left.\begin{aligned} c_i^Z &= R_i^D p_i^Z \\ c_i^X &= w_i \end{aligned}\right\}$$

The remaining conditions are the following:

1. Bilateral price is $p_{ij} = c_i \tau_{ij}$, where τ_{ij} is the expected value

2. Aggregate demand for loans (gravity): $X_{ij} = a^{-\theta} \gamma^{-1} \tau_{ij}^{-\theta} w_i^{-\theta} P_j^{\theta} X_j$, where θ is the demand elasticity

3. Aggregate supply: $X_i = M_i (\frac{w_i}{P_i})^{\psi}$

4. Output market clearing: $X_i = \sum_j \pi_{ij} X_{ij}$

5. Exogenous deficits: $ED = \frac{\sum_i w_i X_i}{\sum_i \varsigma w_i X_i}$. If $\varsigma = 1$, flows are balanced, so that $ED = 1$

6. Normalized income: $\sum_i w_i X_i = 1$

Existence and uniqueness of the equilibrium relies on Allen et al. (2020).

APPENDIX D

NETWORK FORMATION DETAILS

This section borrows from Ballester et al. (2006) and Calvó-Armengol et al. (2009). Each agent simultaneously chooses the action z which consists of two components: idiosyncratic effort $x_i > 0$ i.e. does not depend on other agents' effort, and $z_i \geq 0$, which depends on the network. It obtains a payoff $u_i(x, z, \Sigma)$, where Σ represents the strength of the connections, measured as the inverse of the expected trade cost:

$$u_i(x, z; \Sigma) = f_i(X)x_i - \frac{1}{2}x_i^2 + \alpha_i z_i - \frac{1}{2}\sigma_{ii} z_i^2 + \sum_{j \neq i} \sigma_{ij} z_i z_j + \epsilon_i \tag{D.1}$$

where f_i represents exogenous heterogeneity, which consists of three components: individual heterogeneity X_i, peer heterogeneity $\sum_j \sigma_{ij} X_j$ and the homophily component $\sum_j \sigma_{ij} X_{ij}$. Hence, $f_i(X) = X_i' \delta_1 + \delta_2 \sum_j \sigma_{ij} X_j + \delta_3 \sum_j \sigma_{ij} X_{ij}$. X is a vector of country characteristics including, for example, the expected rate of return. We match equation 1.2 as follows: h_i comprises the observable expected return, and an unobservable component, given by direct and network effects. In a similar fashion, cost shifters are identified in an observable and unobservable component. The random error $\epsilon_i \overset{i.i.d}{\sim} \text{Gumbel}(0, 1)$

The solution of the idiosyncratic part does not involve other agents, hence the outcome of the game stems directly from the optimization. Hence, we focus on the network component of the game:

$$u_i(z; \Sigma) = \alpha_i z_i - \frac{1}{2}\sigma_{ii} z_i^2 + \sum_{j \neq i} \sigma_{ij} z_i z_j \tag{D.2}$$

We normalize $\sigma_{ii} = 1$. Let $\mathbb{I}$ be the identity matrix and $\mathbb{J}$ the matrix of ones. Moreover, define $\sigma_{ij} = \lambda g_{ij} - \gamma$ and $\sigma_{ii} = -\beta - \gamma$. Hence we can decompose the interaction matrix of the game

$\Gamma(\alpha, \Sigma)$ into own concavity, global substitutability and network complementarity effects:

$$\Sigma = -\beta\mathbb{I} - \gamma\mathbb{J} + \lambda\mathbb{G} \tag{D.3}$$

The model can hence be written as:

$$u_i(z, \mathbb{G}) = \alpha_i z_i - \frac{1}{2}(\beta - \gamma)z_i^2 - \gamma \sum_j z_i z_j + \lambda \sum_{j \neq i} \sigma_{ij} z_i z_j \tag{D.4}$$

We impose the same restrictions as in Calvó-Armengol et al. (2009): let $\alpha_i = \mu \sum_j g_{ij}$, $\beta = 1$, $\gamma = 0$:

$$u_i(z, \mathbb{G}) = \mu g_i z_i - \frac{1}{2}z_i^2 + \lambda \sum_{j \neq i} g_{ij} z_i z_j \tag{D.5}$$

If and only if $\lambda\rho(\mathbb{G}) < 1$, where $\rho(\cdot)$ is the spectral radius, then the matrix $[\mathbb{I} - \lambda\mathbb{G}]^{-1}$ is well defined and non-negative, such that there is a unique and interior Nash equilibrium[1]:

$$z^*(\mathbb{G}) = \frac{\mu}{\lambda} \mathsf{B}(\mathbb{G}, \lambda) \tag{D.6}$$

where $\mathsf{B}(\mathbb{G}, \lambda) = (\mathbb{I} - \lambda\mathbb{G})^{-1}(\lambda\mathbb{G}\mathbf{1})$ is the Katz-Bonacich centrality measure of node i. The condition $\lambda\rho(\mathbb{G}) < 1$ requires that the own-concavity is high enough to counter the payoff complementarity, in order to prevent the positive feed-back loops triggered by such complementarities to escalate without bound.

The individual best reply function is:

$$\begin{cases} x_i^* = f_i(X) = X_i'\delta_1 + \delta_2 \sum_j g_{ij} X_j + \delta_3 \sum_j g_{ij} X_{ij} & \text{(D.7a)} \\ z_i^* = \mu g_i + \lambda \sum_j g_{ij} z_j & \text{(D.7b)} \end{cases}$$

Hence, the individual equilibrium outcome is uniquely defined by[2]:

$$y_i^*(x, z\mathbb{G}) = f_i(X) + \frac{\mu}{\lambda} \mathsf{B}_i(\mathbb{G}, \lambda) \tag{D.8}$$

[1]Theorem 1 in Ballester et al. (2006)
[2]Proposition 1, Calvó-Armengol et al. (2009).

CHAPTER 2

THE NETWORK GRAVITY OF GLOBAL BANKING

with Raoul Minetti and Oren Ziv

2.1 Introduction

Firms acquire international funding in different ways. In the first, standard, case, a bank in country i and a firm in country j might arrange a contract where they provide the loan *directly*. Direct cross-border loans move from country i to country j, in a similar way as we might think of trade in goods in a standard framework, e.g. Eaton and Kortum (2002). The second option is related to the literature that focuses on *indirect* trade flows, which emerge either because of global value chains (e.g. Antràs and De Gortari (2020)), or because of transport hubs (Ganapati et al., 2020). Similarly, lending flows might be intermediated between source and destination, through the presence of affiliates and complex financial routes, as recently shown by Coppola et al. (2021), where there is a role for a third country k in the transaction originating at i with destination j[1].

While gravity models have worked well, both empirically and structurally, for trade in goods, their ability to capture financial bilateral flows is still an open question. For example, Buch et al. (2011) find that, in contrast with trade empirical regularities, almost all German banks hold at least some foreign assets. The authors call for "a model of international trade in financial services that accounts for additional sources of heterogeneity". Additionally, Delatte et al. (2017) show that an empirical gravity equation for banking is insufficient to fully rationalize the data, and they attribute to tax havens this "abnormal" activity, the puzzle between the data and the theory.

We propose one reconciliation to the banking gravity puzzle by adding an additional source of heterogeneity to the standard gravity framework. What is the role of intermediation networks in propagating shocks? How does a standard gravity compare with a gravity with network effects?

[1] An example of the latter situation is the presence of a subsidiary of a US bank in the UK, which finances projects in the EU. Hence, acquiring information on institutional details in EU, while staying in a country with similar institutions, language, etc.

What is the role for banking hubs? To address these questions and to to understand the geography of banking flows, we present a multicountry dynamic gravity model for international banking with intermediation network effects. We characterize multinational banks by taking a leaf from both the trade and the banking literature. Heterogeneous banks provide differentiated loans to firms, building on Eaton and Kortum (2002), and loans are produced with a loan-specific technology, as in Goodfriend and McCallum (2007).

The baseline dynamic gravity can already provide insights in the partial and general equilibrium forces that relate loans and physical capital investment in a three-country environment. However, as we saw, it cannot fully address the concerns raised by empirical works. To visualize the issue, consider the three counties in figure 2.1. Suppose that a loan is issued by a bank in country i for a firm in country j but that, because of the complexity of the financial system, the loan transits through country k. Direct gravity, from i to j, correctly captures the origin and destination demand and supply forces, but it fails to understand the true strengths in the relationships (distance) between countries. If the cost between the intermediary country k and the destination country j increases (e_{kl} goes up), it is possible that it is now cheaper to send it directly, via another intermediary country, or that the firm will find it too expensive, and look for a domestic bank.

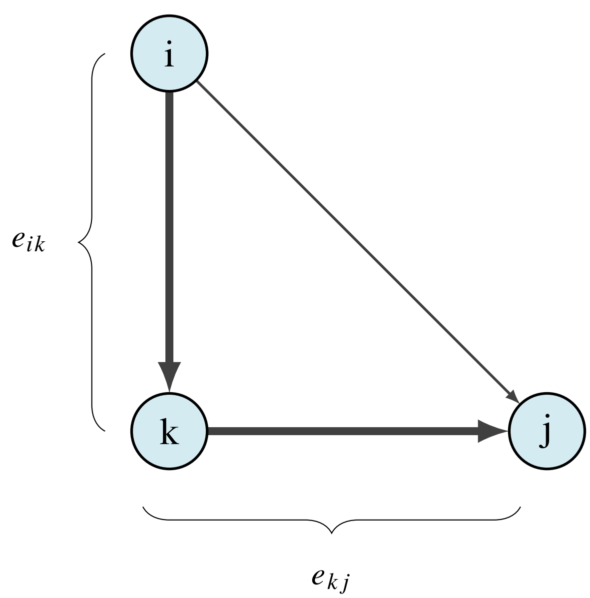


Figure 2.1: Direct and Indirect flows

We propose a relatively simple way to capture indirect flows and the richness of the banking network: building on the transportation model by Allen and Arkolakis (2019), in our economy banks can send loans to firms directly and indirectly, through paths across countries that are optimally chosen to minimize costs. We obtained a closed-form, microfunded measure of intermediation cost that embeds features of the betweenness network centrality. In other words, our approach models the international banking network via a network of bilateral, asymmetric costs, which fully capture the complexity of both direct and indirect flows. We show four applications of our model. First, we show that our model can be used to study how a shock to a *node* (TFP in one country, in our case) is propagated *through* the (banking) network. This exercise is in line with a vast literature in macroeconomics and spatial econometrics on shocks propagation. Second, we show how our model is the first DSGE that can incorporate shocks to the *edges*, the links of the network, which are endogenously determined. This is crucial in several settings, for example in supply chains disruptions, where the network does not simply propagate shocks, the links have been disrupted. Following an increase in the intermediation shock between two counties, the presence of a country with network centrality implies negative network effects on all bilateral flows, and positive effect on domestic loan issued in non-central countries. This has potential relevant implications for regulation policy, since network effects amplify the response of loan interest rates to intermediation shocks. Third, we show that how our model can be used to study a shock to multiple edges, for example Brexit, to understand whether the UK will maintain its central role as financial hub, or if other countries will challenge its leadership. Finally, we investigate the role of hubs, suggesting that the role of networks and indirect flows are crucial to differentiate between the cost minimization of banking hub, and potential agglomeration forces that exploit information-based scale economies. Some countries labeled as tax havens only serve the commonly thought purpose of lowering costs for the parties in the transaction. Other countries with similar lower fiscal or regulatory costs serve as information and expertise hubs: higher banking flows generate demand for professional workers in the sector. If the sector expands sufficiently enough, economies of scale can arise. This can be especially relevant when considering shocks such as Brexit: if costs between the UK and Europe increase but London manages to retain its banking and financial experts, the negative shock might be less severe. Overall, the analysis suggests that macroprudential policies targeting multinational banks and regulations can benefit from the quantification of network effects for global or local financial stability.

The paper unfolds as follows. The next section relates the analysis to prior literature. Section 2.2 lays out the model and solves for agents' decisions. Section 2.3 presents the general equilibrium and discusses the key features of the model. Section 2.5 presents the calibration results. Section 2.6 examines alternative model specifications and robustness. Section 2.7 concludes. The Appendix contains additional analysis.

Prior Literature

Reduced form gravity estimation was complemented by an effort in using heterogeneity to obtain a microfunded and closed-form expression for bilateral trade. The banking literature has followed a similar path. Restricting the scope of the review, the work of Martin and Rey (2004) remains an important starting point: in a two-county model for equity flows, they study how transaction costs generate scale effects, since smaller economies, relying on foreigners to finance their projects, are more heavily subject to the frictions.

On the empirical side, Buch (2005) uses BIS LBS data and reduced-form panel estimation to find that the role of distance is relevant and stable through the 80s and the 90s; Papaioannou (2009) confirms the relevance of geographical distance and adds the role of institutional quality as a key variable to predict cross border bank lending; Brüggemann et al. (2011) use a conditional equilibrium Eaton and Kortum (2002) framework to motivate a reduced-form banking gravity equation where frictions are interpreted as monitoring cost, and proxied with geographical distance.

The baseline gravity equation relies on a measure of distance between the country pairs. While this is a natural assumption in trade, it raises concerns for asset flows, which are not per se constrained by physical shipping and hence distance. A possible reconciling approach is to interpret geographical distance as a proxy for cultural, information, and transaction costs. While the link between distance can be captured by the use of data on time zones or presence of subsidiaries, the estimation still relies on finding a variable that can meaningfully proxy the relationship. Attempts to capture the bilateral costs include Portes and Rey (2005), Aviat and Coeurdacier (2007), Buch et al. (2013), Brei and von Peter (2018), with the inclusion of several other proxies for distance. Note that, while empirically using proxy variables is a feasible option to quantify the effect of frictions, it cannot capture the structural microeconomic foundations.

Our characterization of multinational banks is a simpler version of more complex banking structure, in order to preserve tractability and focus on the network innovation of the model. Among many, the double-decker banking model in Bruno and Shin (2015) presents a setting where regional banks face loan demand by firms, and diversify away regional shocks by borrowing from global banks; Niepmann (2015) develops a theoretical framework to further refine this category, such that global, international and foreign banking emerge from the different way banks acquire deposits; (Cao et al., 2021) show the tradeoff that arises from the presence of global banks, facing TFP or banking shocks.

The paper also relates to the literature on dynamic gravity: Ghironi and Melitz (2005), Ravikumar et al. (2019), Caliendo et al. (2019), Park (2021). In particular, Olivero and Yotov (2012) find instead a significant role played by geographical distance in a dynamic framework for investment gravity. Finally, our work finds its static trade counterpart in Ganapati et al. (2020), who use the path structure developed in Allen and Arkolakis (2019) to understand the role of transportation hubs in containerized trade.

2.2 A Dynamic Gravity Model for Banking

Our model is a multi-country, dynamic, discrete time gravity model for banking flows. Bilateral banking loans flow across a finite number of countries through network paths, building on Allen and Arkolakis (2019) transportation model, which accounts for direct and indirect flows. In each country $i \in \mathcal{N}$ a representative, forward-looking household supplies labor in the goods market and to the banking sector, and consumes a single non-tradable final good, produced by competitive domestic firms. Capital and labor are immobile across countries. Consumption good firms pay labor, own the physical capital stock, and demand a diversified aggregate loan to finance their capital investment. Because of the complexity of the diversification choice, they delegate consulting firms to acquire the individual loans[2]. Consulting firms have the ability to collect loans internationally and produce the aggregate loan demanded by the firms via CES bundling. Individual loan varieties, indexed by $\omega \in \Omega$, are offered by monopolistic banks. Bank, risk-neutral profit maximizers, offer debt contracts to consulting firms internationally. Countries differ in their geography, as captured by a square matrix of deterministic intermediation coefficients, $\tilde{\tau}_{ij}$, and idiosyncratic shocks $\xi_{ij}(p, \omega)$, which affect the intermediation efficiency of a bank ω in country i which supplies a loan to to consulting firms in country j through a path p[3]. Banks choose the cheapest path to reach final demand, giving rise to a network of direct and indirect lending, according to the cost structure between country pairs. Finally, capital goods producers are introduced so to derive market price for capital. All agents are owned by households.

It is useful to introduce here the network terminology[4]. A graph G is an ordered pair of disjoint sets (V, E) such that E is a subset of the set $V^{(2)}$ of unorderd pairs of V, the set of vertices, or nodes, and E is the set of edges, where an edge $\{x_1, x_2\}$ joins the nodes x_1 and x_2. We assume V and E finite. A path, or route, is a graph P which consists of an ordered sequence of nodes and a set of edges that connects the nodes, following the sequence, i.e. $V(P) = \{x_0, x_1, \dots, x_N\}$, and

[2]The framework captures the continuum of financial products that firms need from banks, e.g. FX swaps, credit lines, term financing. Throughout the paper, we use the word loans for simplicity.

[3]Path and route are used interchangeably in the route choice modeling literature.

[4]See Bollobás (2013) for a general reference on graph theory.

$E(P) = \{x_0x_1, x_1x_2, \ldots, x_{N-1}x_N\}$.

In this section, we specify the agents' problems and we determine the aggregate variables of the heterogenous banking sector of the model.

2.2.1 Households

Households earn the wage rate w_t^H on labor supplied in the goods sector (H_t) and the wage rate w^M on labor supplied in the banking sector as loan monitoring activity (M_t). They also earn a gross rate of return $(1 + R_t^D)$ from deposit holding D_t, as well as profits Π from owning banks, firms, and capital producers. They use their funds for consumption C_t and to hold bonds:

$$\max_{\{C_{i,t}^M, M_{i,t}\}_{t\geq 0}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_{i,t}^M - k_H \frac{H_{i,t}^{1+\epsilon}}{1+\epsilon} - k_M \frac{M_{i,t}^{1+\varphi}}{1+\varphi} \right)$$
$$s.t. \quad C_{i,t} + D_t = (1 + R_{i,t-1}^D) D_{i,t-1} + w_{i,t}^H H_{i,t} + w_{i,t}^M M_{i,t} + \Pi_{i,t} \tag{2.1}$$

where ϵ is the inverse of Frisch elasticity for labor supplied to the production of goods and φ is the inverse of Frisch elasticity for labor supplied to banking activities. The parameters k_H and k^M govern the disutility from labor in the two sectors. Moreover, $\Pi_{i,t} \equiv \Pi_{i,t}^F + \Pi_{i,t}^B + \Pi_{i,t}^K$ denotes net transfers received from firms, banks and capital producers. Define the stochastic discount factor $\Lambda_{t,t+j} = \beta^t \frac{u'_{C_{t+j}}}{u'_{C_t}}$. Households maximize their lifetime utility by choosing consumption $C_{i,t}$, deposits $D_{i,t}$ holdings, labor supply $H_{i,t}$ to the entrepreneurs, and labor supply $M_{i,t}$ to the banking sector:

$$[C_{i,t}]: \quad 1 = \mathbb{E}_t \Lambda_{t,t+1}(1 + R_{i,t}^D) \tag{2.2}$$

$$[H_{i,t}]: \quad k_H H_{i,t}^{\epsilon} = \frac{w_{i,t}^H}{C_{i,t}} \tag{2.3}$$

$$[M_{i,t}]: \quad k_M M_{i,t}^{\varphi} = \frac{w_{i,t}^M}{C_{i,t}} \tag{2.4}$$

In the remaining of the paper, we will drop the subscript on monitoring wage, $w^M = w$.

2.2.2 Capital producers

Competitive capital producing firms can invest in $I_{i,t}$ units of capital goods at the cost of $I_t \left[1 + f(I_{i,t}/I_{i,t-1})\right]$ units of consumption goods. We assume that there are adjustment costs associated with producing new capital. We assume households own capital producers and are the recipients of any profits $\Pi^K_{i,t}$. The capital-good firms chooses the amount new capital $I_{i,t}$ to maximize the present discounted value of lifetime profits:

$$\max_{I_{i,t}} \qquad \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{i,0,t} \left\{ P^K_{i,t} I_{i,t} - \left[1 + f\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right] I_{i,t} \right\}$$

such that the price of capital goods $P^K_{i,t}$ is equal to the marginal cost of producing capital goods. $\Lambda_{t,t+j} = \beta^t \frac{u'_{C_{t+j}}}{u'_{C_t}}$ is the stochastic discount factor, since households own the firms. The function $f(\cdot)$ captures the adjustment cost in the capital-producing technology, and satisfies $f(1) = 0$, $f'(1) = 0$, and $f''(\cdot) > 0$. The first order condition determines the price of the investment good in terms of consumption:

$$P^K_{i,t} = \left\{ 1 + f\left(\frac{I_{i,t}}{I_{i,t-1}}\right) + \frac{I_{i,t}}{I_{i,t-1}} f'\left(\frac{I_{i,t}}{I_{i,t-1}}\right) \right\} - \mathbb{E}_t \Lambda_{i,t,t+1} f'\left(\frac{I_{i,t+1}}{I_{i,t}}\right) \left(\frac{I_{i,t+1}}{I_{i,t}}\right)^2 \tag{2.5}$$

2.2.3 Firms

The representative firm uses labor $H_{i,t}$ and capital $K_{i,t-1}$ to produce final good $Y_{i,t}$, via an increasing, concave, and constant returns to scale technology. To finance purchases of capital investment, firms can take loans from local or foreign banks[5]. We posit that firms face liquidity, trade credit, macroeconomic or other forms of risk that might hinder capital investment, hence entrepreneurs want to hold a diversified funding portfolio. We assume firms do not have the internal skill set to take the portfolio decision, so they delegate it to a consulting firm[6]. Representative firms only choose an aggregate loan bundle $X_{i,t}$ with net aggregate interest rate $R^X_{i,t}$, leaving the consulting

[5]The usual budget constraint is $\Pi^F_{i,t} + P^K_{i,t} I_{i,t} + (1 + R^X_{i,t-1}) X_{i,t-1} = Y_{i,t} + X_{i,t} - w^H_{i,t} H_{i,t}$. However, because of static nature of loan repayments and the investment assumption in equation 2.9, it reduces to equation 2.6.

[6]See Craig and Ma (2020): " [..] borrowing banks delegate their borrowing from lending banks to a small subset of large and well diversified intermediary banks".

firm to determine its composition. Representative firms in country i solves the following:

$$\max_{\{H_{i,t},K_{i,t},X_{i,t}I_{i,t}\}} \mathbb{E}_0 \sum_{j=0}^{\infty} \Lambda_{i,t,t+j+1}\Pi^F_{i,t}$$

$$s.t. \quad \Pi^F_{i,t} + (1+R^X_{i,t})X_{i,t} = Y_{i,t} - w^H_{i,t}H_{i,t} \tag{2.6}$$

$$Y_{i,t}(K_{i,t-1}, H_{i,t}) = K^{\alpha}_{i,t-1}H^{1-\alpha}_{i,t} \tag{2.7}$$

$$K_{i,t} = (1-\delta)K_{i,t-1} + I_{i,t} \tag{2.8}$$

$$X_{i,t} = P^K_{i,t}I_{i,t} \tag{2.9}$$

The first order conditions are:

$$[H^D_{i,t}]: \quad \frac{(1-\alpha)Y_{i,t}}{H_{i,t}} = w^H_{i,t} \tag{2.10}$$

$$[K_{i,t}]: \quad -P^K_{i,t}(1+R^X_{i,t}) + \mathbb{E}_t\left[\Lambda_{i,t,t+1}\left((1-\delta)P^K_{i,t+1}(1+R^X_{i,t+1}) + \frac{\alpha Y_{i,t+1}}{K_{i,t}}\right)\right] = 0 \tag{2.11}$$

2.2.4 Consulting firms

A representative, perfectly competitive agent costlessly assembles intermediate loans to produce an aggregate nontraded loan[7]. Consulting firms use a standard Dixit-Stiglitz aggregator, so that all banks essentially serve all firms with slightly differentiated loan contracts $x(\omega)$, providing diversified access to international finance by combining the $\omega \in \Omega$ loans with constant elasticity to construct a composite loan, according to the technology:

$$Y^X_{j,t} = \left(\sum_i \int_{\omega\in\Omega} x_{ij,t}(\omega)^{\frac{\sigma-1}{\sigma}}\, d\omega\right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the constant and symmetric elasticity of substitution across imperfectly substitutable intermediate loans. We now proceed to specify a static loan production structure of the economy.

[7]We abstract from possible microfundation on the portfolio decision of consulting firms. This could arise because the projects that they finance are imperfectly correlated (Martin and Rey, 2004), but we posit, as in Brei and von Peter (2018), that capital producers have a diversification motive over the types of financial product.

2.2.5 Banks

In each country there is a continuum of financial products, called loans for simplicity, indexed by $\omega \in \Omega$[8], and each loan market is perfectly competitive[9]. Every bank is infinitesimal and takes aggregate price and quantities as given. Banks extend loans to consulting firms, and the representative household divides its deposit holdings across the entire continuum of banks. Building on Goodfriend and McCallum (2007), banks issue new loans by a combination of liquidity and monitoring effort via Leontief technology[10].

Moreover, in order to send loans abroad, banks face bilateral intermediation cost $\tau_{ij} > 1$, interpreted as intermediation asymmetries that prevent a frictionless allocation of resources. Part of this intermediation cost is idiosyncratic and bank specific, banks draw a ξ intermediation productivity from a continuous distribution $F(\xi)$. In order to maintain tractability, we posit that banks return dividends to the households at the end of every period[11]. Given the static nature of the problem, we drop the time subscript to simplify notation. The problem of the ω bank consists of two stages: first, it chooses monitoring $M_i(\omega)$ and liquidity $D_i(\omega)$ to maximize cash flows, given monitoring wage w and liquidity net interest rate R^D. Second, it optimizes the path along which they send the loans abroad, choosing the cheapest path. We first describe the input maximization problem.

[8] Alternatively, see Niepmann (2015) for a model where monitoring quality is heterogeneous.

[9] The banking sector could be alternatively modeled via monopolistic competition: as in Gerali et al. (2010), switching costs, market structure, regulatory restrictions are potential sources of market power, which will put our model into a Melitz (2003) framework. We opt for an Eaton and Kortum (2002) setup to simplify the analysis. We provide the alternative Melitz model in appendix E.0.2.

[10] Our model can be related to the two tier Gerali et al. (2010) banking structure, since the Leontief production function makes the two inputs complements, hence making both wholesale and loan office necessary for the issuance of a loan.

[11] This assumption is isomorphic to a setting with banks exiting in every period. See Boissay et al. (2016) for a model with this structure.

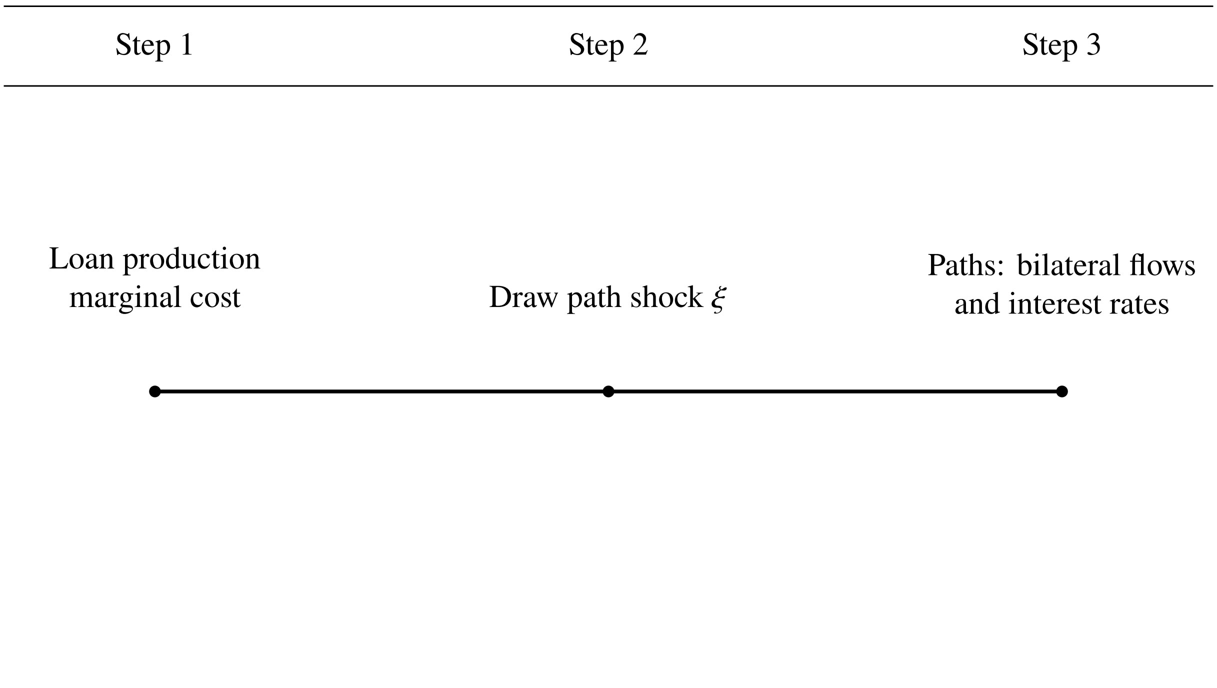


Figure 2.2: Within period decisions and shocks for bank ω.

A bank ω maximizes cash flows $[1 + r_i(\omega)]\; x_i(\omega) - w_i M_i(\omega) - \left[1 + R_i^D\right] D_i(\omega) - \pi^B(\omega)$, subject to a balance-sheet constraint $x_i(\omega) = D_i(\omega) + \pi^B(\omega)$ and production function $y_i^X(\omega)$. Substituting the balance-sheet constraint, the problem reduces to:

$$\max_{M_{i,t}(\omega), D_i(\omega)} r_i(\omega) y_i^X(\omega) - w_i M_i(\omega) - R_i^D D_i(\omega)$$
$$s.t. \quad y_i^X(\omega) = \min\left\{\Phi_i \frac{M_i(\omega)}{\zeta}, \frac{D_i(\omega)}{1-\zeta}\right\}$$

where Φ_i is a country-specific aggregate monitoring productivity, taken as exogenous. It represents a reduced-form approach to capture the role of scale economies in the loan management, which helps understand how financial hubs exploit both lower regulatory and fiscal costs *and* informational scale economies (e.g. Okawa and Van Wincoop (2012)). Competitive banks in i selling to j price their loans at marginal cost. The observed net interest rates for the loans at j are

$$r_{ij}(\omega) = c_i \tau_{ij}(\omega)$$

with

$$c_i = \frac{\zeta w_i}{\Phi_i} + (1-\zeta) R_i^D \tag{2.12}$$

where purchasers of loan ω at j source the lowest cost supplier globally. We now describe the path optimization problem that gives rise to aggregate direct and indirect bilateral banking flows.

2.2.5.1 Trade Paths

A bank ω in country i can reach a borrower in country j directly. In this case, they face a bilateral deterministic intermediation friction is $e_{ij} > 1$: it takes into account bilateral determinants such as distance, language, regulatory frictions at a country level, i.e. they are not bank ω specific. Hence, e_{ij} may represent the standard friction in the trade literature. Building on the transportation model by Allen and Arkolakis (2019), banks in country i can also reach country j indirectly, i.e. through another country k. In this case, the bank in i faces an intermediation cost equivalent to $\tilde{\tau}_{ij} = e_{ik}e_{kj}$, as in figure 2.1. Hence, the general expression for the deterministic component of the intermediation friction along a specific path p, $\tilde{\tau}_{ij}(p)$, is the cost of going from i to j along path $p \in G$:

$$\tilde{\tau}_{ij}(p) = \prod_{k=1}^{K_p} e_{k-1,k}$$

where $e_{k-1,k}$ is the edge specific cost, i.e. the cost between the countries $k-1$ and k along the path p. A path p is comprised of a series of K_r legs of a journey with $K_r - 1$ stops along the way between the origin, i such that $k = 1$, and destination j such that $k = K_r$. The length of the path, K_r, is path-specific. Each path belongs to a G graph[12].

Other that deterministic frictions, loans are subject to bank and route-specific idiosyncratic shocks[13], $\xi_{ij}(p, \omega)$. The following distributional assumption follows a vast literature.

[12] We abstract from the network formation. In appendix 1.3, we provide a potential *centralized* microfundation of the network formation, in a similar spirit as in Fajgelbaum and Schaal (2020).

[13] Antràs and De Gortari (2020): "two chains flowing across the same countries in the exact same order may not achieve the same overall productivity due to (unmodeled) idiosyncratic factors, such as compatibility problems, production delays, or simple mistakes."

Assumption 1 *Intermediation shocks $\xi_{ij}(p,\omega)$ are assumed to be realizations of draws from a Fréchet distribution:*

$$F(\xi_{ij}(r,\omega)) = \exp\left\{-\xi^{-\theta}\right\}$$

where the location parameter $\theta > \max\{1, \sigma - 1\}$ is the shape parameter which governs comparative advantage through the degree of heterogeneity in across loans. When θ is large, banks have similar productivities while when θ is small banks are highly heterogeneous.[14]. Intermediation cost τ depends on the potential paths across the network available to the lender, for each product ω. Hence, the overall intermediation friction $\tau_{ij}(\omega)$ consists of the two components: the deterministic friction $\tilde{\tau}_{ij}(p)$ along the path, and the path and bank-specific idiosyncratic element $\xi_{ij}(p,\omega)$, such that:

$$\tau_{ij}(p,\phi) = \frac{\tilde{\tau}_{ij}(p)}{\xi_{ij}(p,\omega)}$$

Lenders choose the least cost path, or route, to send their type ω to capital producers around the world, such that:

$$\tau_{ij}(\omega) = \min_{p\in G} \tau_{ij}(p,\omega) \tag{2.13}$$

We assume that $\tau_{ij} < \infty$ even in the case of no trade: it is possible that the friction from i to j is finite, but the benefits of connecting to j are less than the cost paid to form a link[15].

Given the stochastic nature of the ex-ante price, we want to know the probability that any given good ω is shipped from i to j on a specific path p. Lenders choose the lowest-cost path p from i to j for ω from all routes $p \in G$ and borrowers in j choose the lowest-cost supplier of good ω from all countries $i \in I$. In other words, the probability that a country i provides loans to country j at the lowest price[16].

[14]The interpretation of the latter, as pointed out by Allen and Arkolakis (2019), is that "θ can be considered as capturing the possibility of mistakes and randomness in the choice of routes, with higher values indicating greater agreement across traders. In the limit case of no heterogeneity, $\theta \to \infty$, all traders choose the path with the minimum aggregate trade cos".

[15]See section 1.3 for a potential microfundation.

[16]A priori, each borrower could receive funds from multiple lenders. However, given the stochastic nature of interest rate determination, the probability that any borrower faces two lenders with the same price is zero.

The probability that, given that lenders choose the lower cost route, the cost is below τ is given by[17]:

$$H_{ij\omega}(\tau) = 1 - \exp\left\{ - \tau^{\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \right\}$$

Moreover, we can derive that the product-level prices with which the bank will serve market j are distributed according to:

$$G_{j\omega}(r) = 1 - \exp\left\{ - r^{\theta} \sum_{i' \in I} c_i^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \right\}$$

We now can combine the two sides of the market.

Proposition 6 *The probability that a bank in country j with liquidity need υ chooses to borrow from a bank in country i, and that the route from country i to j is the minimal cost route, is:*

$$\pi_{ijp\omega} = \frac{\left[c_i \tilde{\tau}_{ij}(p)\right]^{-\theta}}{\sum_l c_l^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{lj}(p)\right]^{-\theta}}$$

Proof. See appendix B.1.

By the law of large numbers, this is also the share of all productions sold in j in industry that come from i and take path p, λ_{ijp}. Intuitively, as the bilateral trade costs rise, the origin country is the least cost provider in fewer goods. The equations shows that countries with lower marginal costs and countries with relatively lower bilateral trade costs will account for a larger fraction of the loans given to country j.

We now derive a crucial concept for our framework: the expected intermediation cost $\mathbb{E}_{\omega}\left[\tau_{ij}(\omega)\right]$, which takes into account all the possible paths in the network, and the consequent distributional assumption. The expected cost τ_{ij} is the network-equivalent, cost-based distance measure usually found in gravity models. In the rest of the paper we prefer working with the cost measure b_{ij}, which is proportional to the expected cost τ_{ij}, as the derivation of the following proposition show:

[17] See appendix B.1.0.1 for the derivations.

Proposition 7 *Assume banks choose the route that minimizes the cost of sending a loan from country i to country j. Define the matrix A, where each element is a function of the deterministic edge friction, such that each element $a_{ij} \equiv e_{ij}^{-\theta}$. Then, each element b_{ij} of the matrix $B \equiv (\mathbf{I} - A)^{-1}$ represents the expected cost, summing over the possible routes p:*

$$\begin{aligned} b_{ij} &= \sum_{p \in G_{ij}} \tilde{\tau}_{ij}(p)^{-\theta} \\ &= \sum_{p \in G_{ij}} \prod_{k=1}^{K_p} e_{k-1,k}(p)^{-\theta} \end{aligned}$$

Proof. See appendix B.2.

Note that the advantage of our model is that we can capture, with a microfunded and relatively simple way, the richness of the network that usually justifies third country fixed effects. Our probabilities ψ, and hence our (expected) costs τ, are a microfunded measure of network betweenness centrality. Moreover, bilateral intermediation frictions are spatially dependent: they are a combination of bilateral frictions and frictions on the other pairs of the network.

2.2.6 Direct and Indirect Gravity

This section provides aggregate variables for both composite interest rate and bilateral loan, the latter via commonly defined as gravity in the trade literature. We first derive a closed-form expression for the composite loan interest rate $R_{j,t}^X$ [18]:

$$R_j^X = \vartheta \left(\sum_i c_i^{-\theta} b_{ij} \right)^{-\frac{1}{\theta}} \tag{2.14}$$

where $\vartheta = \Gamma \left(\frac{\theta + 1 - \sigma}{\theta} \right)^{\frac{1}{1-\sigma}}$ is a constant.

Note that in a one-country setup, the aggregate interest would be equal to[19] $R_i^X = w_i + R_i^D$, and we would obtain a simple expression similar to the interest rate spreads in Goodfriend and McCallum (2007) or Gerali et al. (2010).

[18] See derivation at appendix C.0.1.

[19] Simplifying the weights on the two factors in the Leontief technology.

2.2.6.1 Direct Gravity

Summing across routes and using proposition 6, we obtain the share:

$$\lambda_{ij,t} = \frac{c_{i,t}^{-\theta} b_{ij,t}}{\sum_l c_{l,t}^{-\theta} b_{lj,t}} \tag{2.15}$$

Moreover, substituting the expression for the composite loan interest rate

$$\lambda_{ij} = \frac{c_{i,t}^{-\theta} b_{ij,t}}{\vartheta^{\theta} R_{j,t}^{-\theta}}$$

This expression is equivalent to the so-called structural gravity equation, which can be obtained substituting the share equation into the identity relating bilateral flows and (banking) expenditure:

$$X_{ij} = \lambda_{ij} X_j \tag{2.16}$$

Doing so, and substituting back the definition of b_{ij} helps better understand how the model captures the network complexity:

$$X_{ij,t} = \vartheta^{-\theta} R_{j,t}^{\sigma} X_{j,t} c_i^{-\theta} \left(\sum_{p \in G_{ij}} \prod_{k=1}^{K_p} e_{k-1,k}(p)^{-\theta} \right)$$

Our distance measure takes into account the entire network, including the possibility of different path lengths. Accounting for the complexity of the banking network in a standard gravity equation will need the use of a large number of third and higher level country effects. Instead, our model suggests a relatively simple way to incorporate network effects via a betweenness centrality approach. The network structure is fully captured in the expected costs, proportional to b_{ij}.

2.2.6.2 Indirect Gravity

As we illustrated using figure 2.1, bank loans can take a direct path from country i to country j, as well as indirect path, through a series of countries $k, l, \ldots, K$. In this section, we derive an *indirect* gravity equation, i.e. the amount of loans that go through an edge e_{kl}, without necessarily originating at k nor stopping at l. Let $\psi_{kl|ij}$ the conditional probability of a bank ω serving country

j from county i, and going through countries k and l, and recall the following matrix definitions from Proposition 7:

$$[A]_{ij} \equiv e_{ij}^{-\theta}$$

$$B \equiv (\mathbb{I} - A)^{-1}$$

where e_{ij} is the deterministic edge friction between i and j. The following proposition shows the key result of the network aspect of our model.

Proposition 8 *The probability of any financial product ω traveling through a kl edge, conditional on being sold from origin i to destination j:*

$$\psi_{kl|ij} = \frac{b_{ik}a_{kl}b_{lj}}{b_{ij}} = \left(\frac{\tau_{ij}}{\tau_{ik}e_{kl}\tau_{lj}}\right)^{\theta}$$

Proof: see appendix B.3.

The interpretation is intuitive: proposition 8 is a microfunded, probabilistic version of the betweenness centrality formula[20], such that the importance of a node depends on the amount of shortest paths that go through that node. In our case, in expectation.

In the above equation, we expressed the conditional probability in quadruplets because it provides a clear expression for bilateral k to l indirect gravity, and we will use it to estimate the network-based costs. Before showing the quadruplet, cost estimating gravity, we present a triangular ikj version of the same network gravity, which is more intuitive: Ξ_{ikj} is the probability of a flow going from country origin i (e.g. the US) to destination j (e.g. Italy), through a third

[20]Allen and Arkolakis (2019): "the numerator on the right hand side is the expected trade cost on the least cost route from i to j, whereas the denominator is the expected trade cost on the least cost route from i to j through the transportation link e_{kl}. Hence, the more costly it is to travel through the link e_{kl} relative to the unconstrained least cost route, the less likely a trader is to use the link e_{kl}."

country k (e.g. the UK):

$$
\begin{aligned}
\Xi_{ikj} &= \sum_l \psi_{kl|ij} X_{ij} \\
&= \sum_l \psi_{kl|ij} \lambda_{ij} X_j \\
&= \psi_{k|ij} \lambda_{ij} X_j
\end{aligned}
$$

where the second line uses the definition of the bilateral flows, as the product of expenditure and its share λ. Note that this triangular gravity relates our model to equation 7 in Arkolakis et al. (2018) model of multinational production.

Back to our quadruplet $iklj$ setting, we can obtain the total volume of indirect flows between k and l, Ξ_{kl}[21], summing over all the origin i and destination j countries:

$$
\begin{aligned}
\Xi_{kl} &= \sum_i \sum_j X_{ij} \psi_{kl|ij} \\
&= \sum_i \sum_j X_{ij} \frac{b_{ik} a_{kl} b_{lj}}{b_{ij}} \qquad (2.17) \\
\Xi &= A \odot B'(X \oslash B)B'
\end{aligned}
$$

where the last line is the equation in matrix form, $\odot$ and $\oslash$ being element-wise product and division, respectively.

As in Ganapati et al. (2020), the structural expression for indirect flows Ξ is a function of trade costs and direct flows X only: conditional on the observed trade values X_{ij}, the contribution of trade between i and j to the traffic between legs k and l is invariant to multilateral resistance or marginal costs, since they affect trade from that country and others proportionally on all routes. Therefore, following Allen and Arkolakis (2019), we can establish that there exists a unique set of banking direct flows X consistent with observed country characteristics, market clearing conditions, and the banks' optimization. This implies that there exists a unique indirect matrix Ξ of banking flows.

Note that the advantage of our model is that we can capture, with a microfunded and relatively simple

[21] It includes flows bound for l and those continuing onward to other destinations.

way, the richness of the network that might justify the introduction of third country fixed effects in gravity equations. Our probabilities ψ, and hence our (expected) costs τ, are a microfunded measure of network betweenness centrality, as discussed before. We now present the equilibrium concepts of our economy.

2.3 Aggregation and General Equilibrium

We define aggregate loan supply and aggregate monitoring and deposit demand in the banking sector by aggregating across the individual loan varieties:

$$Y_{i,t}^X = \int_\Omega y_{i,t}^X(\omega)d\omega\,, \qquad M_{i,t} = \int_\Omega M_{i,t}(\omega)d\omega\,, \qquad D_{i,t} = \int_\Omega D_{i,t}(\omega)d\omega$$

The aggregate demand for monitoring hours reads:

$$M_{i,t} = \vartheta\zeta Y_{i,t}^X \tag{2.18}$$

where ϑ is a constant and ζ is the monitoring share of the loan production. In equilibrium demand equals supply and it pins down the wage in the monitoring labor market.
The aggregate deposit demand reads:

$$D_{i,t} = \vartheta(1-\zeta)Y_{i,t}^X \tag{2.19}$$

In equilibrium demand equals supply and it pins down the wage in the interest rate in the deposit market. The individual loan markets are already assumed to clear since we replaced the consumer demand directly in the sales of the bank for each loan.
Aggregate loans clear:

$$R_{i,t}^X Y_{i,t}^X = \sum_j \lambda_{ij,t} R_{j,t}^X X_{j,t} \tag{2.20}$$

The social resource constraint,

$$\sum_i C_{i,t} + \left[1 + f\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right] I_{i,t} = \sum_i Y_{i,t}$$

requiring that world goods markets clear, can be omitted by Walras law.

Profits

Aggregate transfers to households equal the sum of final good firms, capital good producer and banks profits:

$$\Pi_{i,t} = Y_{i,t} - w_{i,t}^H H_{i,t} - (1 + R_{i,t}^X)X_{i,t} + P_{i,t}^K I_{i,t} - \left[1 + f\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right] I_{i,t} \tag{2.21}$$

2.3.1 Comparative Statics

The network structure of our model allows us to tackle the complexity of this economy. On one hand, we face an “edge problem”: a change in the cost on one edge of the network has an effect on the entire edge set (E), since flows are indirect, such that trade occurs over multiple edges, i.e. paths. On the other hand, there is a “node problem”: the change in one edge will have complex general equilibrium effects on non-banking sectors of the economies of the countries, which will in turn feed back into the edges of the banking network. This paper addresses both concerns by developing a model with a rich network structure which leads to a structural estimation of the intermediation frictions. The intuition is the following: a country with a high level of outstanding direct and indirect flows must be appealing to both. Even more so, if the amount of direct flows is significantly less then the indirect flows, it must be that the country presents cheaper bilateral intermediation costs. Recall that the gravity equation for loans has the following form:

$$X_{ij} = \lambda_{ij} X_j$$

Then the static effect of a change of the deterministic edge costs is the following:

$$\frac{\partial X_{ij}}{\partial e_{kl}} = \underbrace{\frac{\partial X_j}{\partial e_{kl}}\lambda_{ij} + X_j \frac{\partial \Phi_j^{-\theta}}{\partial e_{kl}} \frac{\lambda_{ij}}{\Phi_j^{-\theta}}}_{\text{gravity effect}} + \underbrace{X_j \frac{\partial \tau_{ij}^{-\theta}}{\partial e_{kl}} \frac{\lambda_{ij}}{\tau_{ij}^{-\theta}}}_{\text{network effect}}$$

where Φ is the denominator of equation 2.15, here with the standard symbol indicating multilateral resistance. The first term is the effect of a change in trade frictions on total demand X_j. The second and third terms is the effect on the probability/fraction of the loans, which operates thought two channels: the effect through the multilateral resistance, and the change on the expected path cost. While the first channel is common in standard trade models, the latter captures the effect that the edge shock has on the network cost.

2.4 Intermediation Cost Estimation

This section takes the model and to the data, in order to estimate the unobserved bilateral costs e_{ij} and the expected bilateral cost τ_{ij}. We use the estimated trade costs to calibrate our general equilibrium. Recall from proposition 8 that we obtained an expression for indirect trade, conditional on observing direct flows:

$$\Xi^{\text{expected}} = A \odot B'(X \oslash B)B' \tag{2.22}$$

The goal of this section is to estimate the unobserved costs e, which define the matrices A and B that rationalize the total observed direct and indirect banking flows[22]:

$$\min_{\beta} \sum_{ij} \left| \Xi_{kl} - \Xi_{kl}^{\text{expected}}(e|X) \right|$$

To estimate the matrix, we opt for a parametric assumption for the A matrix, thus avoiding imposing a sparsity condition on the network, which might be too strong in this context[23]:

$$a_{ij} = e_{ij}^{-\theta} = \frac{1}{1 + \exp(Z_{ij}'\beta)} \in [0, 1] \tag{2.23}$$

where

$$Z_{ij}'\beta \equiv \alpha + \beta_1 \Xi_i + \beta_2 \Xi_j$$

In our baseline, the matrix of observables Z_{ij} consists of total indirect inflows (Ξ_j) and outflows (Ξ_i)[24]. To summarize the estimation procedure: given the set of parameters, we obtain a guess for the A matrix, hence the B matrix, and have a prediction for the indirect trade, following equation 2.22:

$$\min_{\beta} \sum_{ij} \left| \Xi_{kl} - \Xi_{kl}^{\text{expected}} \left(A(\beta), B(A(\beta)), X_{ij} \right) \right| \tag{2.24}$$

Data. Our model suggests that, to estimate betweeness centrality costs, we only need two sources of data: a direct and an indirect flow matrix. The direct trade matrix is taken from the Consolidated Banking Statistics (CBS) of the Bank for International Settlements (BIS), while the Locational Banking Statistics (LBS) of the BIS provide data on the traffic matrix. The BIS database is the most

[22] Note that our goal is not to pursue inference, but to saturate the moments.

[23] See appendix 1.3 for a potential network formation microfundation for this logistic parametric structure. According to this interpretation, the edge costs are proportional to the costs incurred to set up or maintain the endogenous matrix structure.

[24] It can be extended to include further bilateral and country-specific variables, such as distance, language, legal, currency, colonial relationship (see CEPII gravity dataset), leading to similar results.

comprehensive source of publicly available data on international banking, available at quarterly frequency. In details, the LBS are reported according to the residence principle and the reporting banks include subsidiaries and branches. Instead, CBS data consolidate positions across parent banks. Let's consider an example, a subsidiary of a US bank located in Italy providing a loan to a firm in Germany. This would be considered a USA to Germany transaction in the CBS data, while the LBS will report it as Italy to Germany. The object traded are mainly claims over deposits and loans. While several studies have relied on one of the two datasets in order to estimate gravity equations[25], this paper first uses the variation between the two datasets to recover the unobserved costs that are usually proxied but geographical distance or alternative observable distance measure[26]

Results. We show the results computed for the 2014q4 data[27]. We check our estimation results by comparing the LBS (traffic, indirect flows) predicted by the model against their observed counterparts in the data (Figure 2.3). We include both a best fit line and a 45 degree line, representing the perfect fit scenario. In general, we fit the data extremely well, with a correlation between the observed and predicted shares of 0.903.

The estimated bilateral costs e, up to scale θ, are visualized in the A matrix (figure 1.5 in chapter 1), while the scaled inverse of the expected costs τ, are in the B matrix (figure 1.6). The estimation suggests, for example, that the UK has low costs both as a lender (rows) and as a borrower (column), especially with respect to the US, Germany, France and Japan. Japan, instead, has low costs as a lender but not necessarily as a borrower, confirming our modelling choice of asymmetric costs.

[25] See, e.g. Buch (2005), Brüggemann et al. (2011), Brei and von Peter (2018) for the LBS, Aviat and Coeurdacier (2007), among others, for the CBS.

[26] The main difference between LBS and CBS is in terms of cross sectional availability. We restrict to 24 countries reporting to the CBS, the list is available in appendix G. It would be possible to overcome this issue with further development in the availability of data on the liability side, which could allow to expand the set of countries by using both sides of the balance sheet, as it is currently possible for the LBS (see Brei and von Peter (2018)).

[27] The results on different quarters and years are available upon request.

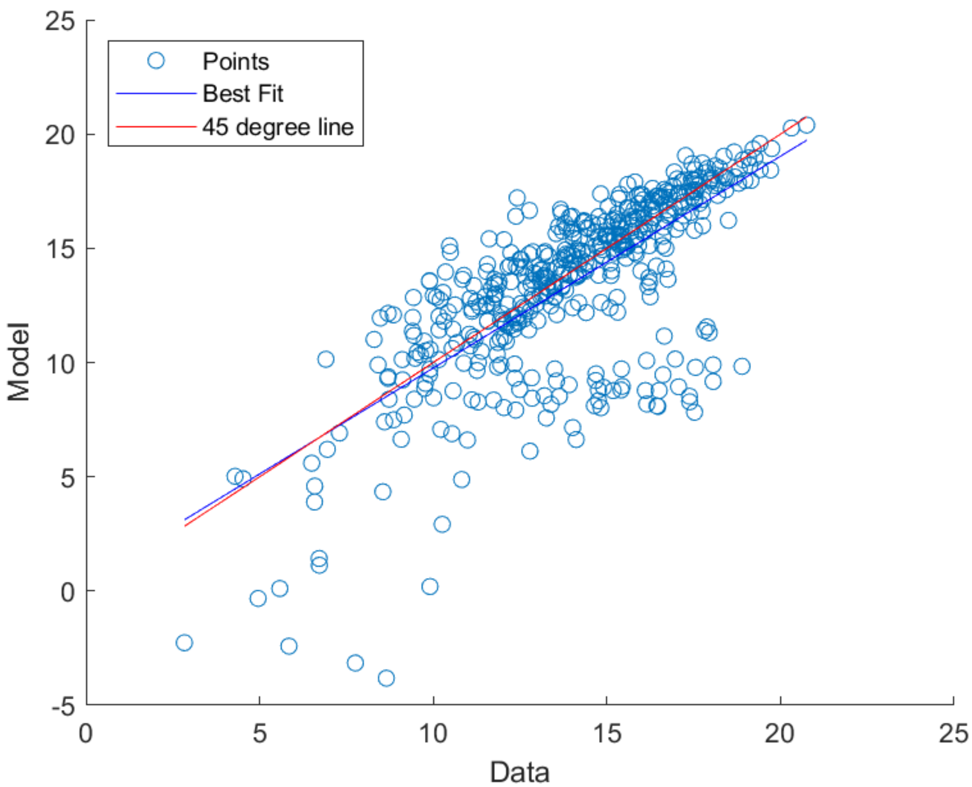


Figure 2.3: Indirect banking flows 2014q4: Model vs Data, in logs

2.5 Applications

This section presents different applications of the model. The first two exercises are illustrative to the potential of the intersection between the macroeconomics, trade and network literature. To avoid computational issues and best focus on the qualitative properties of the model, we limit these first dynamic exercises to a 3-country economy. We first study the impulse responses to node shocks, in line with the literature on shock propagation *through* a network. Second, we study the impulse responses to an intermediation shock, i.e. a shock to the matrix, allowing for a new equilibrium. Next, we present two static exercises in a N-country economy. In the third section, we study the effect of an increase of a cost increase from the United Kingdom to the European Union. Fourth, we investigate the long-debated question on the role of geographical distance in gravity models. Finally, we study the role of monitoring hubs in amplifying the network effects of intermediation shocks.

2.5.1 Node disruption

We first study the impulse responses to shocks to a node (country). There is a large literature that explores the transmission of shocks that arise in a country or in a sector, *through* selected channels, in the spirit of the spatial literature (see LeSage and Pace (2009)). For example, international transmission of the Lehman Brothers collapse via syndicated loans (De Haas and Van Horen, 2012) monetary policy transmission via production networks, business cycle synchronization via trade linkages (Juvenal and Monteiro, 2017), or the role of trade credit in amplifying financial shocks (Altinoglu, 2021). Most theoretical works assume an exogenous network. For example, the literature on production networks, where input–output linkages can function as a shock propagation mechanism, building on Acemoglu et al. (2012) seminal work, take the network as given. However, there has been a recent push to relax the network exogeneity assumption, being too strong in several settings. Theoretically, (Oberfield, 2018) provides a partial equilibrium where producers probabilistically match upstream and downstream firms, giving rise to an endogenous input-output structure; while Acemoglu and Azar (2020) build on the previous exogenous work. Econometrically, estimators that allow for endogenous and dynamic weights of the matrix can be both frequentist (Kuersteiner and Prucha, 2020) or bayesian (Han et al., 2021).

In our model, the network (or spatial matrix) is endogenous. Cross-country transmission of shocks occurs via direct and indirect banking flows, which are determined by costs. In turn, the costs depend on a set of bilateral edge costs e and on the marginal costs of the loan production inputs. This allows to fully capture the role of indirect effects.

Calibration. For calibration, we use fairly standard parameters for preferences and technology. We calibrate all the three country in the same way, except for the intermediation costs, to best isolate the effect of direct and indirect banking. The model is calibrated to quarterly frequency and solved numerically by locally approximating around the non-stochastic steady state. We use fairly standard parameters for preferences, technologies and the government sector (see Table E.1). Frisch elasticity of labor supply is set to 4 in both the final goods and the banking sectors, in line with the suggestion by Chetty et al. (2011) for macro models. The discount factor is calibrated to 0.9979, implying a yearly steady state deposit rate (R^D) of around 1%, as in Goodfriend and McCallum (2007). In the final goods sector, the depreciation rate of capital is set to $\delta = 0.025$. In the investment sector, following Gertler et al. (2012), we set f"(1) = 1, so that the steady-state elasticity of capital price to investment is 1. In the banking sector, we set the loan elasticity of substitution $\sigma = 1.471$ as in Gerali et al. (2010). We set $\theta = 4$, following Ganapati et al. (2020); we show alternative parametrizations in section 2.6. We set the share of monitoring in the loan production $\zeta = 0.350$, as in Goodfriend and McCallum (2007).

Table 2.1: Calibration of common parameters

Description	Symbol	Value
Preferences		
Household discount factor	β	0.960
Inverse Frisch elasticity H	ϵ	4.000
Inverse Frisch elasticity M	φ	4.000
Technology		
Capital share of output	α	0.330
Capital depreciation	δ	0.025
Inverse elasticity of I to P^K	f"(1)	1.000
Loans		
Loans elasticity of substitution	σ	1.471
Fréchet shape parameter	θ	4.000
Monitoring share of loan production	ζ	0.350

Intermediation costs. The model requires the calibration of the 3×3 matrix of intermediation frictions. We set the parameters in order to show an economy with sufficient centrality in one node, so that the three-country model does not collapse into trivial exercises, e.g. two-country or single country economies. The baseline calibration implies greater distance between country 1 and country 3, measured by the edge intermediation cost e_{ij}. The edge cost matrix is the following:

$$EC = \begin{bmatrix} 1.2 & 1.4 & 5.3 \\ 1.4 & 1.2 & 1.3 \\ 6.4 & 1.8 & 1.2 \end{bmatrix}$$

Hence, the parameters are chosen to let country 2 be a more central node, as graphically illustrated in figure 2.4, where thicker lines represent lower frictions.

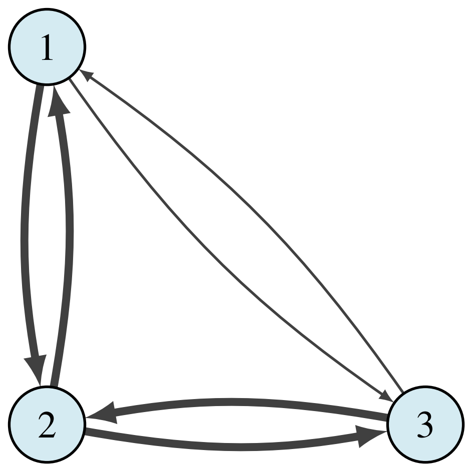


Figure 2.4: Country 2 as a central node

As a technical requirement, the spectral radius of the matrix A (defined in proposition 7), must be less than one in order to apply the geometric sum that leads to matrix B. A sufficient condition for the spectral radius being less than one is $\sum_j e_{ij}^{-\theta} < 1$ for all i, which is satisfied in our calibration.

TFP shock. We show the response of our economy to a one-standard deviation TFP shock in country 1. A TFP shock in country 1 has a standard effect on the domestic economy, with consumption, capital, wages and investment increasing . However, instead of a standard business cycle synchronization, country 2 and 3 experience a decline in output (see figure A.2). Country 1 increased need to finance its expanding economy, including investment, diverts resources away from real good production to the banking sector, see figure A.3. The transmission is non-standard, since our baseline model abstract from trade in goods, as common in the international RBC literature, which can be included by making non trivial assumptions on the interaction between trade in goods and trade in loans.

2.5.2 Single edge disruption

To the best of our knowledge, this is the first exercise of a shock to a network edge within a DSGE model. Hence, the shock does not hit one node (country, sector) and it gets transmitted to other parts of the economy via a network. Instead, the shock hits the the edges, the links between countries or sectors.

We experiment with an unexpected one-standard-deviation positive shock, an increase in the intermediation cost from country 1 to country 2, that is, a shock to e_{ij} as defined in section 2.2.5.1. The impulse responses for the economy describing banking gravity are in figures 2.5 and 2.6: dashed lines represent the responses to the economy without network, while the solid line include the network effect, as presented in section 2.3.1. In the first we show the responses to the bilateral flows in loans, i.e. banking gravity, for both direct (dotted and solid lines) and indirect flows (dash-dotted lines). In the second (Fig. 2.6) we show the main variables of the banking sector: the aggregate interest rate R^X, monitoring hours M and monitoring wage w.

An increase in the intermediation cost from country 1 to country 2 increases the overall costs sourcing from country 1 (Fig. 2.5): hence the decrease in bilateral flows for all country pairs with country 1 as an origin. Without network effects (dashed lines), country 2 substitutes the now-expensive loans from 1 with domestic loans (X_{22}), and loans from country 3 (X_{32}). Country 2 decreases, via multilateral resistance, the loans sent to country 3 X_{23}. The multilateral resistance determines a substitution effect for flows where 1 is the destination, with an increase in bilateral loans X_{21} and X_{31}.

Network effects are captured by the difference between the solid and the dotted lines, and refer to the last term in the equation 2.3.1. Network effects are negative on all bilateral flows X_{ij}, while they have a positive effect, relative to the baseline without paths, on the amount of domestic loans in both 1 (X_{11}) and 3 (X_{33}). Given than going from 1 to 3 is costly, and 2 being a central node to send both *direct* and *indirect* loans, firms in counties 1 and 3 find optimal to shift their portfolio closer to home, since substituting country 2 with the respective 1 or 3 is still costly, given our calibration. Indirect flows decrease everywhere, since in our three-country model there is limited role for alternative paths, given the central role of country 2.

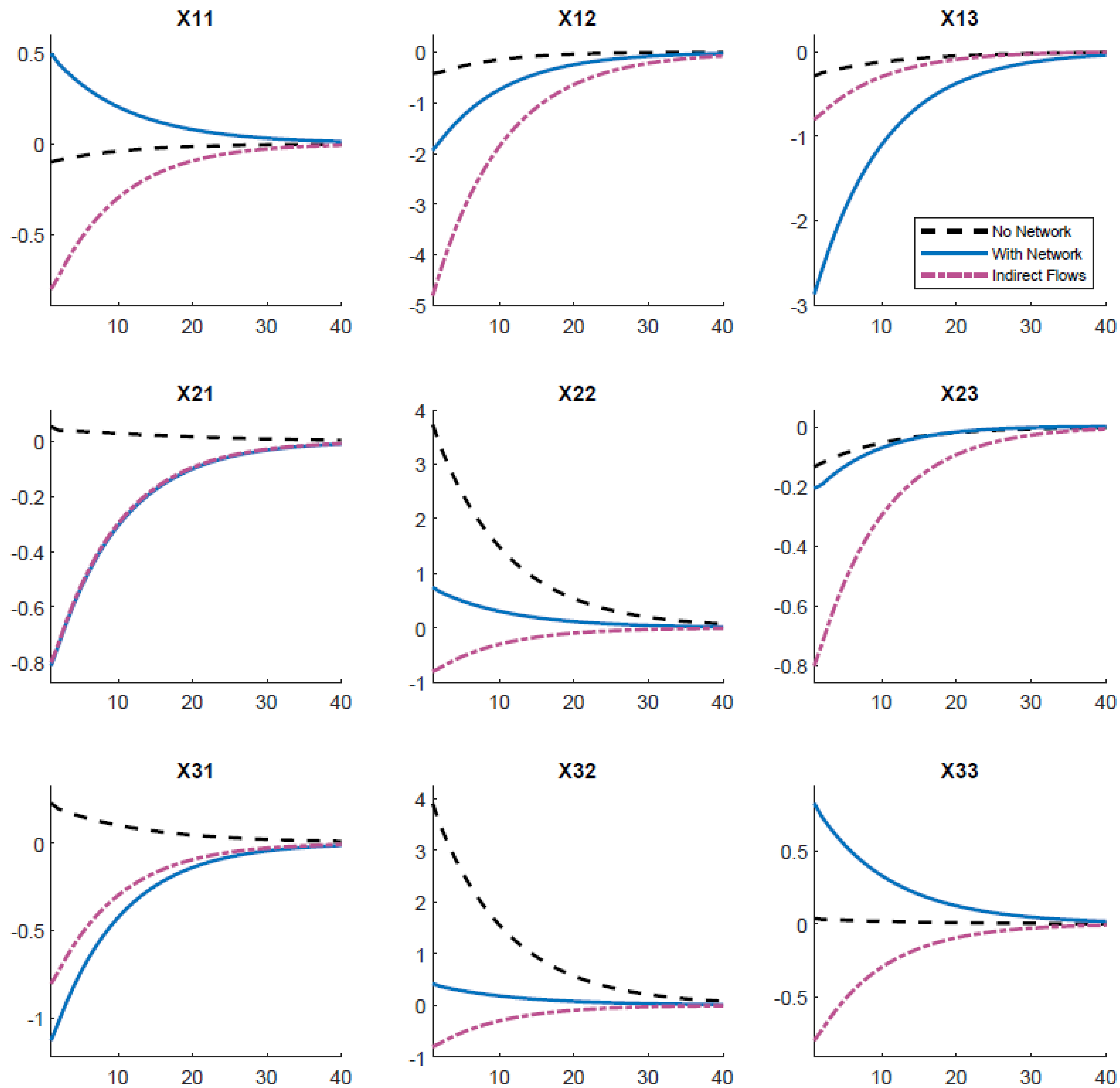


Figure 2.5: Bilateral flows (gravity) X from country i to country j, shock to intermediation costs, with and without network effects, percentage deviations.

The effect on monitoring wage w and hours M is standard (Fig. 2.6): an increase in the intermediation cost increases the demand for domestic loans, pushing monitoring value upwards (dashed lines). However, with network effects (solid lines), country 1 will be less relevant for both *direct* and *indirect* flows, as show in the decrease in bilateral flows both inwards $X_{i,1}$ and outwards $X_{1,j}$ (Fig. 2.5). Importantly, taking networks into account reveals the higher transmission of the shocks in the aggregate interest rates R^X. Country 2 suffers from the increased cost from 1, but its central position in the network seems to compensate the overall effect. However, both country 1 and country 3 experience higher interest rates taking network effects into account, which raises questions for the role of regulation policy.

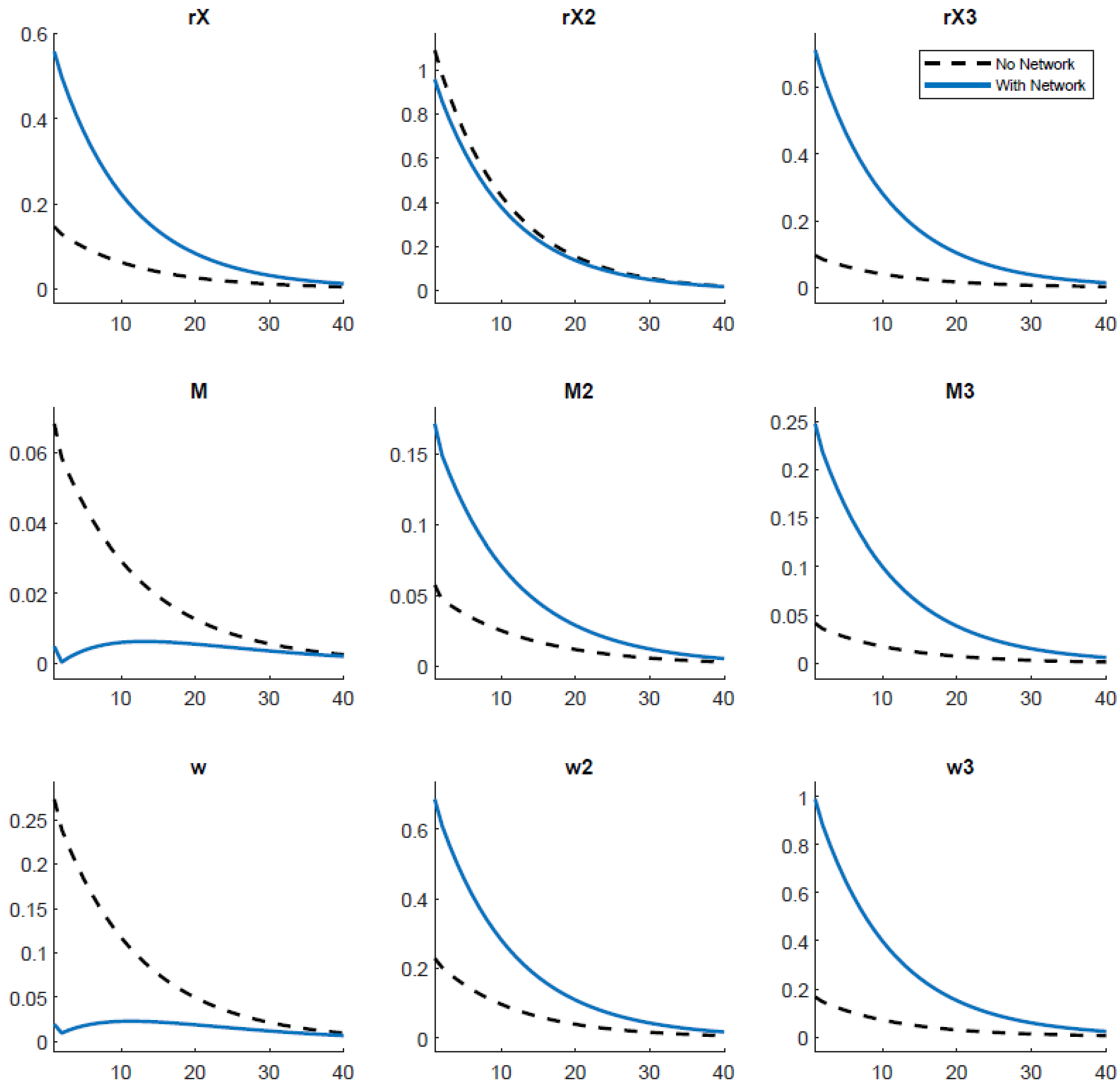


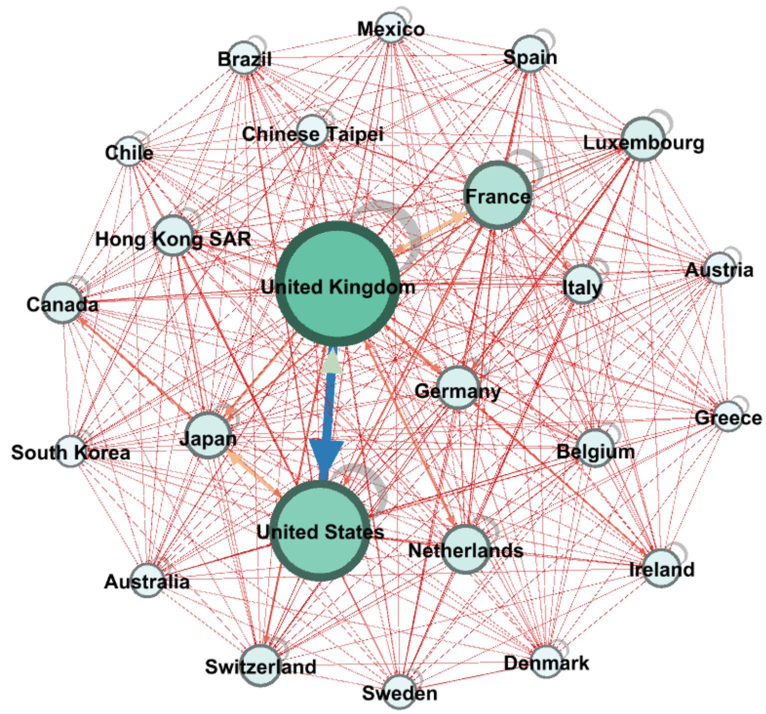
Figure 2.6: Banking variables, shock to intermediation costs, with and without network effects, percentage deviations. First row: aggregate interest rate R^X. Second row: monitoring hours M; third row: monitoring wage w. Columns indicate country.

2.5.3 Multiple edges disruption

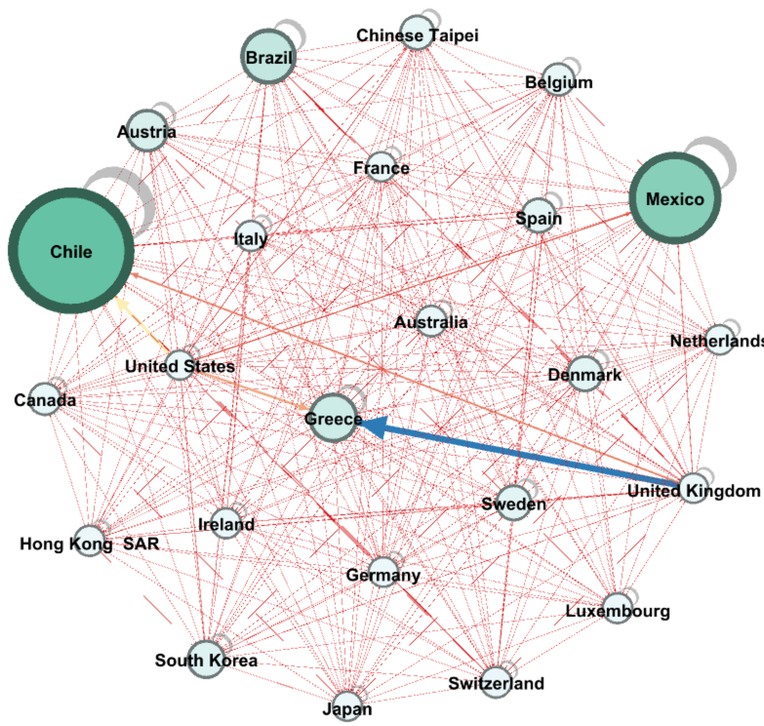
Shocks can hit several links at once. Examples include regional trade or exchange rate agreements (e.g. the European Union or Bretton Woods), the 2021 Suez Canal obstruction, the 2022 Russian war sanctions. In our model, we explore a simplified version of Brexit. Changes to intermediation costs are implemented as changes to link costs e_{kl}, an increase in intermediation costs for banking flows between the UK and each of the EU countries in our sample[28]. To maintain computational tractability, we perform this exercise in the steady-state of the economy.

First, we investigate how the network structure changes after a multiple edge shock. Figure 2.7a plots the indirect network of banking flows in the 4th quarter of 2014. Node sizes (and color) are weighted by their "pagerank" centrality, an adjusted measure of betweeness centrality. Not surprisingly, the United Kingdom and the United States are central nodes, with strong outflows from the UK to the US. Figure 2.7b plots the network after a 50% increase in the "Brexit" costs. The UK looses most of its centrality, and outside-EU countries are the ones the mostly benefit from the cost increase. While the qualitative mechanism is consistent with our intuition, quantitative implications can only be derived by considering a larger sample.

[28] Austria, Belgium, Denmark, France, Germany, Greece, Italy, Ireland, Luxembourg, Netherlands, Spain, Sweden



(a) Indirect flows, baseline.



(b) Indirect flows after a 50% increase in multiple bilateral e costs.

Figure 2.7: Indirect flows, node sizes and color reflect centrality.

As part of the previous exercise, it is possible to investigate the severity of nonlinearities in the network centrality: in the following table we report the centrality measures depending on the shock intensity (1%, 5%, 10%, 50%) to the edges. In our example, the network centrality seems to converge rather quickly, while be of great interest in financial stability exercises.

Table 2.2: Node centrality given 1%, 5%, 10% and 50% multiple edge shocks

Country	Baseline	1%	5%	10%	50%
1	*0.013*	*0.039*	*0.04*	*0.042*	*0.047*
2	0.021	0.011	0.011	0.011	0.011
3	*0.021*	*0.015*	*0.015*	*0.015*	*0.016*
4	0.024	0.094	0.093	0.093	0.089
5	0.032	0.015	0.015	0.015	0.015
6	0.033	0.012	0.012	0.012	0.012
7	0.013	0.338	0.335	0.332	0.319
8	*0.063*	*0.008*	*0.008*	*0.008*	*0.008*
9	*0.021*	*0.022*	*0.023*	*0.024*	*0.025*
10	*0.031*	*0.021*	*0.021*	*0.022*	*0.023*
11	*0.071*	*0.007*	*0.007*	*0.007*	*0.007*
12	0.173	0.007	0.007	0.007	0.008
13	*0.013*	*0.053*	*0.055*	*0.058*	*0.069*
14	0.038	0.010	0.010	0.010	0.010
15	*0.027*	*0.009*	*0.009*	*0.009*	*0.010*
16	*0.030*	*0.009*	*0.010*	*0.010*	*0.010*
17	0.070	0.007	0.007	0.007	0.007
18	0.017	0.034	0.034	0.034	0.033
19	*0.034*	*0.009*	*0.009*	*0.009*	*0.009*
20	0.017	0.221	0.218	0.216	0.208
21	0.041	0.008	0.008	0.008	0.008
22	*0.025*	*0.022*	*0.023*	*0.024*	*0.025*
23	0.018	0.023	0.023	0.023	0.022
24	0.156	0.007	0.007	0.007	0.007

Finally, we investigate the role of network effects. We compare two alternative counterfactual economies, one with paths and another without paths (indirect links). How much direct flows are more or less than in the case without paths in the Brexit counterfactual economy? For example, after the increase in 1% in the "Brexit" costs, the US keeps receiving funds, while outflows reduce, with an average negative network effect for the US as a sender of -15.15%.

2.5.4 Learning Banking Hubs

As discussed in section 2.2.5, monitoring productivity helps understand how banking hubs exploit both lower regulatory and fiscal costs *and* informational scale economies. The role of scales is particularly relevant in the presence of networks and indirect flows through paths. Countries might be central in the banking network because of lower regulatory, fiscal, cultural costs, captured by the intermediation cost in our model. However, banking hubs could also reflect the role of informational expertise in processing loans. This is relevant for domestic or international regulation that addresses tax havens. Some countries labeled as tax havens only serve the commonly thought purpose of lowering costs for the parties in the transaction. Other countries with similar lower fiscal or regulatory costs serve as information and expertise hubs: higher banking flows generate demand for professional workers in the sector. If the sector expands sufficiently enough, economies of scale can arise. This is especially relevant when considering shocks like Brexit: if costs between the UK and Europe increase but London retains its banking and financial experts, the negative shock might be less severe. In the main section of the paper we have assumed that the monitoring scale effect is exogenous. However, it is natural in our setting to endogenize it. We explore two alternatives. In the first, the scale effect is linked to the amount of *indirect* flows that go through that country. More specifically, it increases or decreases monitoring productivity (above or below 1), if the total of indirect flows (in and out) are more or less than $1/N$, i.e. the case where all flows are evenly distributed:

$$\Phi_{i,t} = 1 + \left(\frac{\sum_j \Xi_{ij,t} + \Xi_{ji,t}}{\sum_{i,j} \Xi_{ij,t}} - 1/3 \right)$$

The second approach uses the centrality measure showed in Proposition 8. Similar to the first, we define the scale effect as deviation of the average centrality:

$$\Phi_{k,t} = 1 + \left(\sum_{i,j} \frac{b_{ik,t} a_{kk,t} b_{kj,t}}{b_{ij,t}} - \sum_{i,j,k} \frac{b_{ik,t} a_{kk,t} b_{kj,t}}{b_{ij,t}} \right)$$

We perform the exercise in a three-country economy: while this setup has partial ability in capturing the richness of the full N-country network, it provides tractable dynamic insights of the mechanisms that drive bilateral flows, both direct and indirect, in the presence of agglomeration forces. The mechanism is shown in figure 2.8, where we compare the baseline model with networks (dashed lines) and the same economy augmented with a monitoring scale effect (increase in Φ). As expected, the exercise suggest the presence of nonlinearities in the role of agglomeration forces. In our context of international banking flows, this is relevant not only for a Brexit exercise, as previously discussed, but also in the tax havens debate. In one case, countries can gain network centrality in the banking network because of fiscal and/or regulatory advantages. In the other case, the same centrality can be achieved because of learning or information hub effects. For some countries, the line between the two cases can be far from sharp, posing an extra challenge to regulators tackling tax havens, who might need to consider general equilibrium effects of the presence of knowledge and information scale effects.

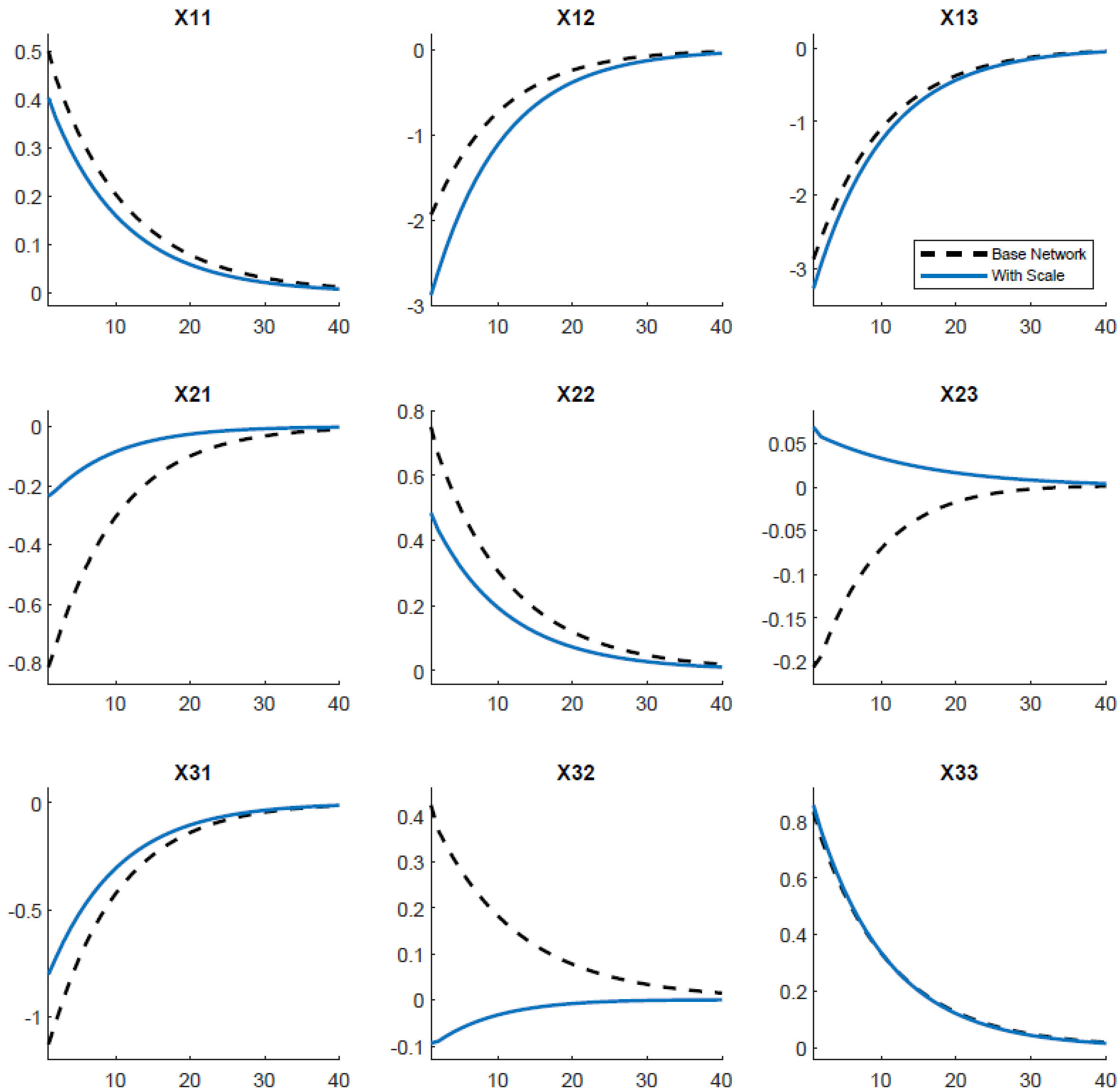


Figure 2.8: Bilateral flows (gravity), shock to intermediation costs, with network effect, with and without monitoring scale effects on country 2, percentage deviations.

2.6 Robustness and alternative specifications

In this section, we consider alternative specifications of the banking sector.

The role of θ. Recall that θ is the shape parameter of the Fréchet distribution, governing the heterogeneity among banks and paths, the possibility of mistakes and randomness in the choice of the optimal route within the network. We present the results for both direct flows (Fig. A.4) and indirect flows (Fig. A.5) in the appendix. As expected, it has a qualitative minor role on direct flows, while being relevant for indirect flows: since higher values indicate greater agreement across loan traders, it amplifies the centrality of the nodes, given the origin and destination of the shock.

Cobb-Douglas production function. In the model, we specified a Leontief loan production technology. In appendix E.0.1, we show the model where banks use a Cobb-Douglas technology. In the calibration, we set the share of monitoring in the loan production $\zeta = 0.350$, as in Goodfriend and McCallum (2007).

2.7 Conclusion

Building on the well-documented features of multinational banking, this paper has studied the role of network effects in the propagation of intermediation cost shocks. In our economy, banks can send loans to firms either directly or indirectly, through paths across countries that are optimally chosen to minimize costs. We obtained a closed-form, microfunded measure of intermediation cost that embeds features of the betweenness centrality, allowing to capture in a relatively simple way the richness of the banking network. We have found that network effects are important to understand how bilateral shocks are transmitted globally. Following an increase in the intermediation shock between two counties, the presence of a country with network centrality implies negative network effects on all bilateral flows, and positive effect on domestic loan issued in non-central countries. This has potential relevant implications for regulation policy, since network effects amplify the response of loan interest rates to intermediation shocks. Finally, we have investigated the role of hubs, suggesting that the role of networks and indirect flows are crucial to differentiate between the cost minimization of banking hub, and potential agglomeration forces that exploit information-based scale economies.

The analysis suggests that macroprudential policies targeting multinational banks and regulations can benefit from the quantification of network effects for global or local financial stability, e.g. in stress tests. However, multinational banks can optimally select the nodes, for example by opening new branches or engaging in brownfield investments (e.g., acquire local banks), giving rise to an endogenous network, where paths can be complements, substitutes, or both. We leave these and other issues to future research.

APPENDICES

APPENDIX A

FIGURES AND TABLES

Figure A.1 summarizes the structure of the economy.

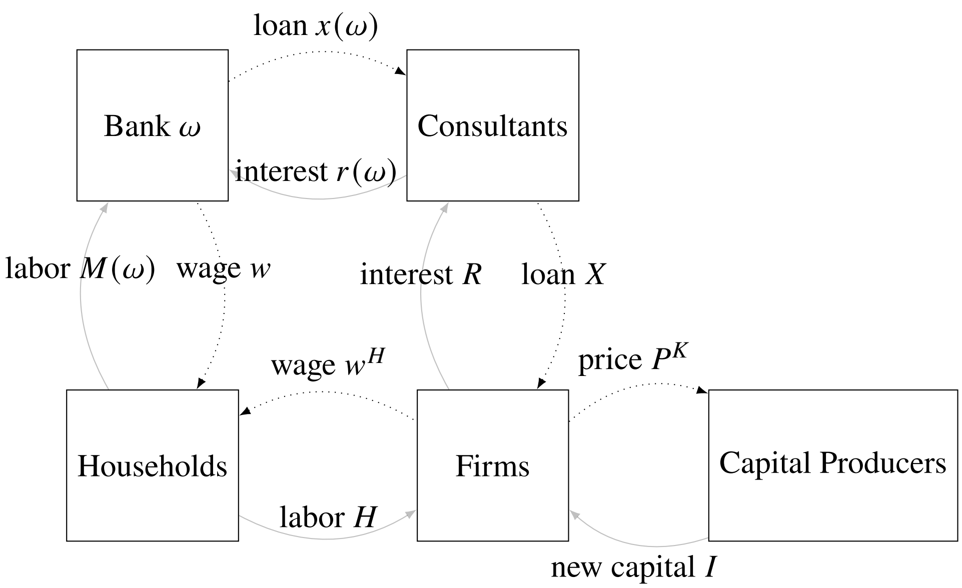


Figure A.1: Static flows in the model economy.

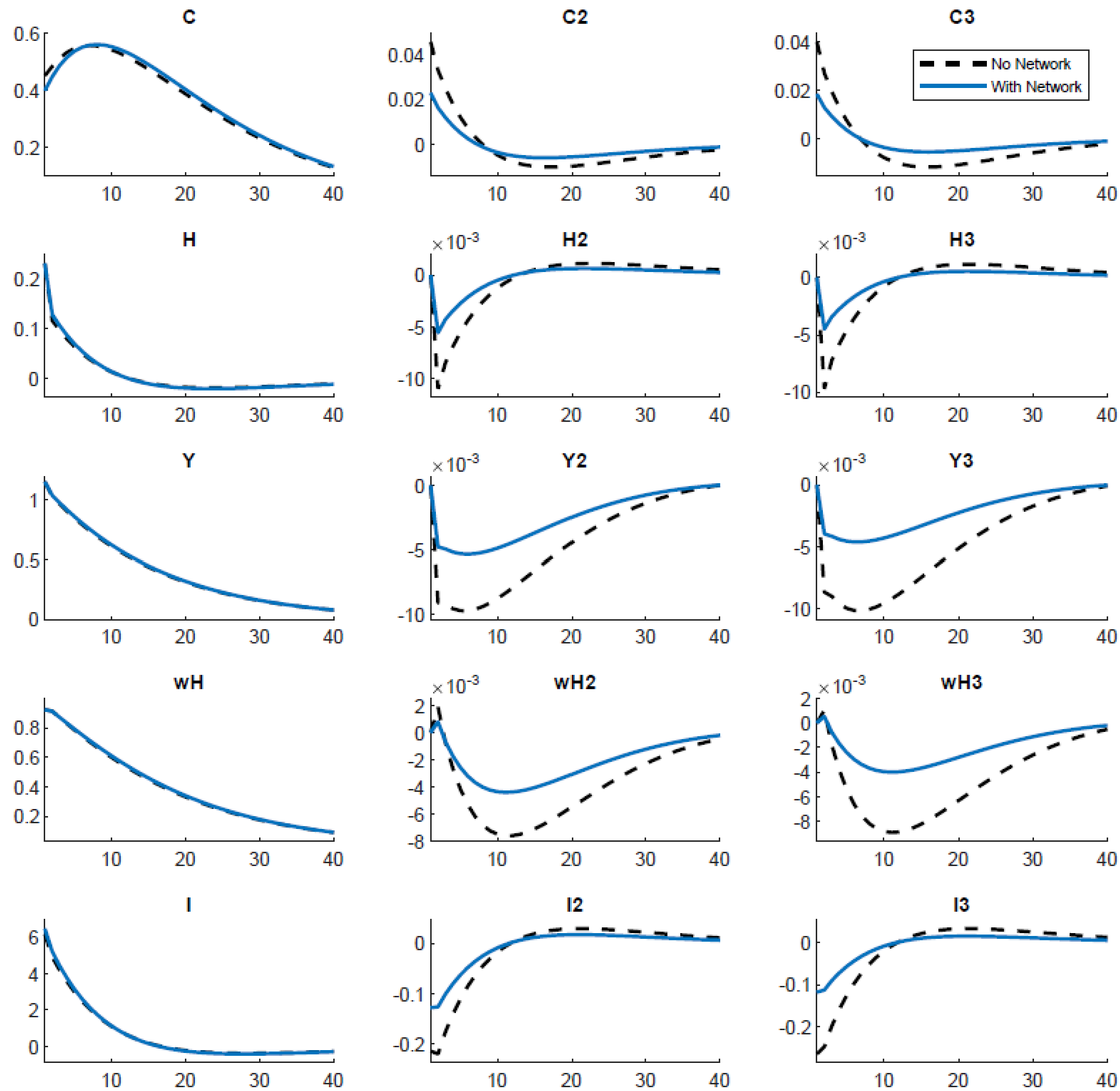


Figure A.2: TFP shock to country 1, IRFs to the real side of the economy, percentage deviations. First line: consumption C; second line: final good hours H; third line: final good output Y; fourth row: final good wage w^H, fifth row: investment I.

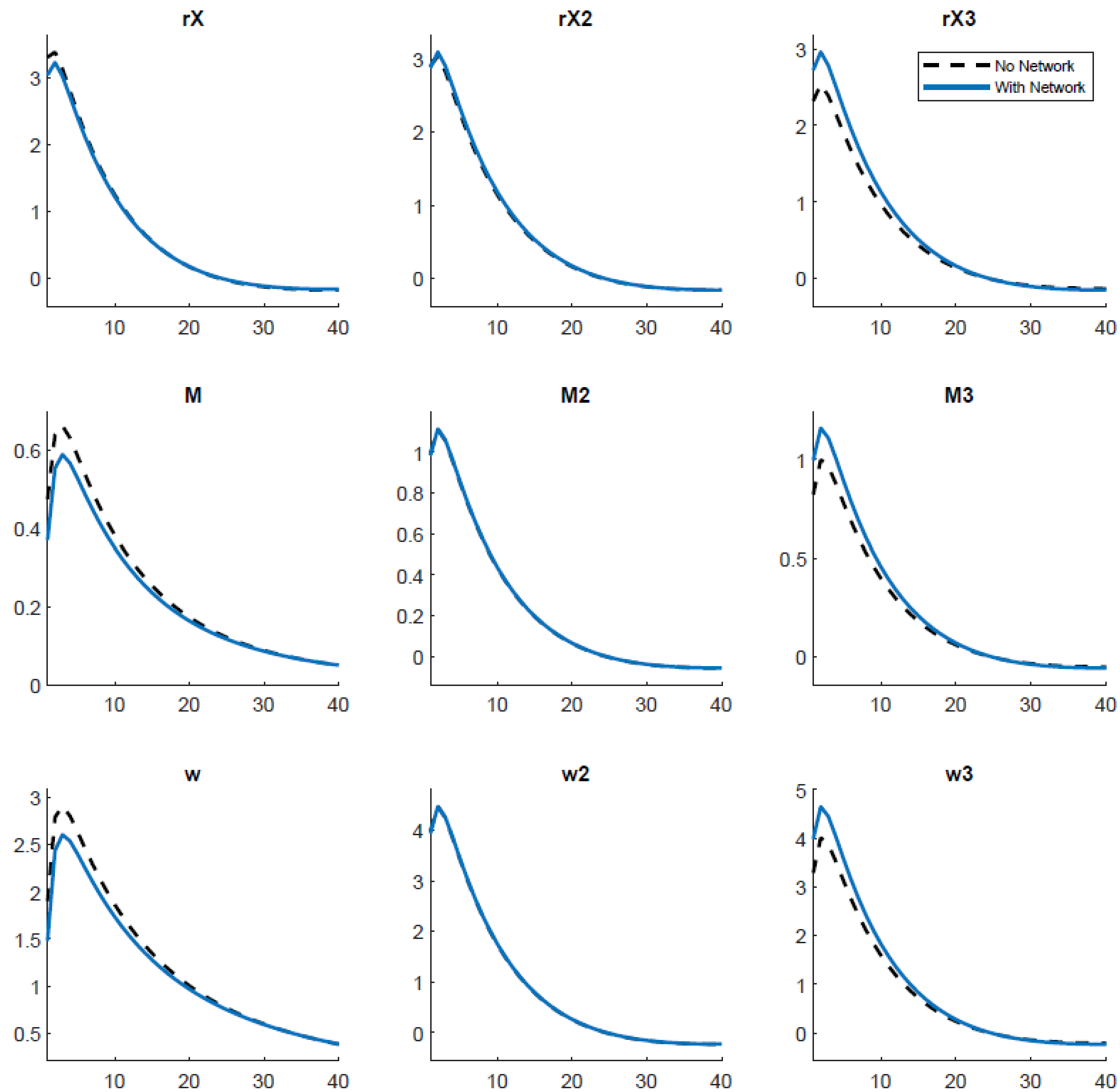


Figure A.3: TFP shock to country 1, IRFs to the banking side of the economy, percentage deviations. First line: aggregate interest rate R^X; second line: monitoring hours M; third line: monitoring wage w^M.

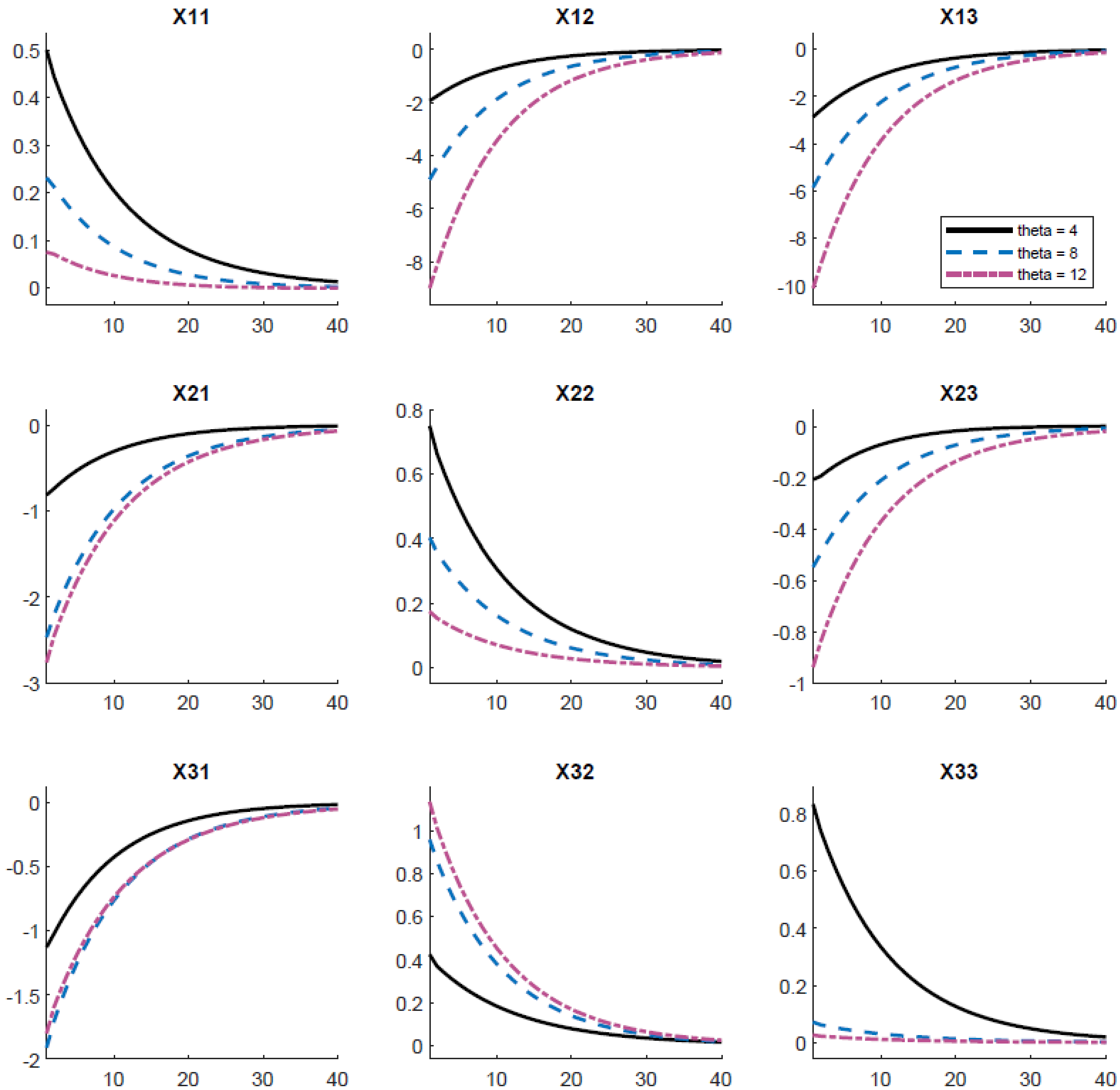


Figure A.4: Direct flows, sensitivity to the shape distribution parameter θ

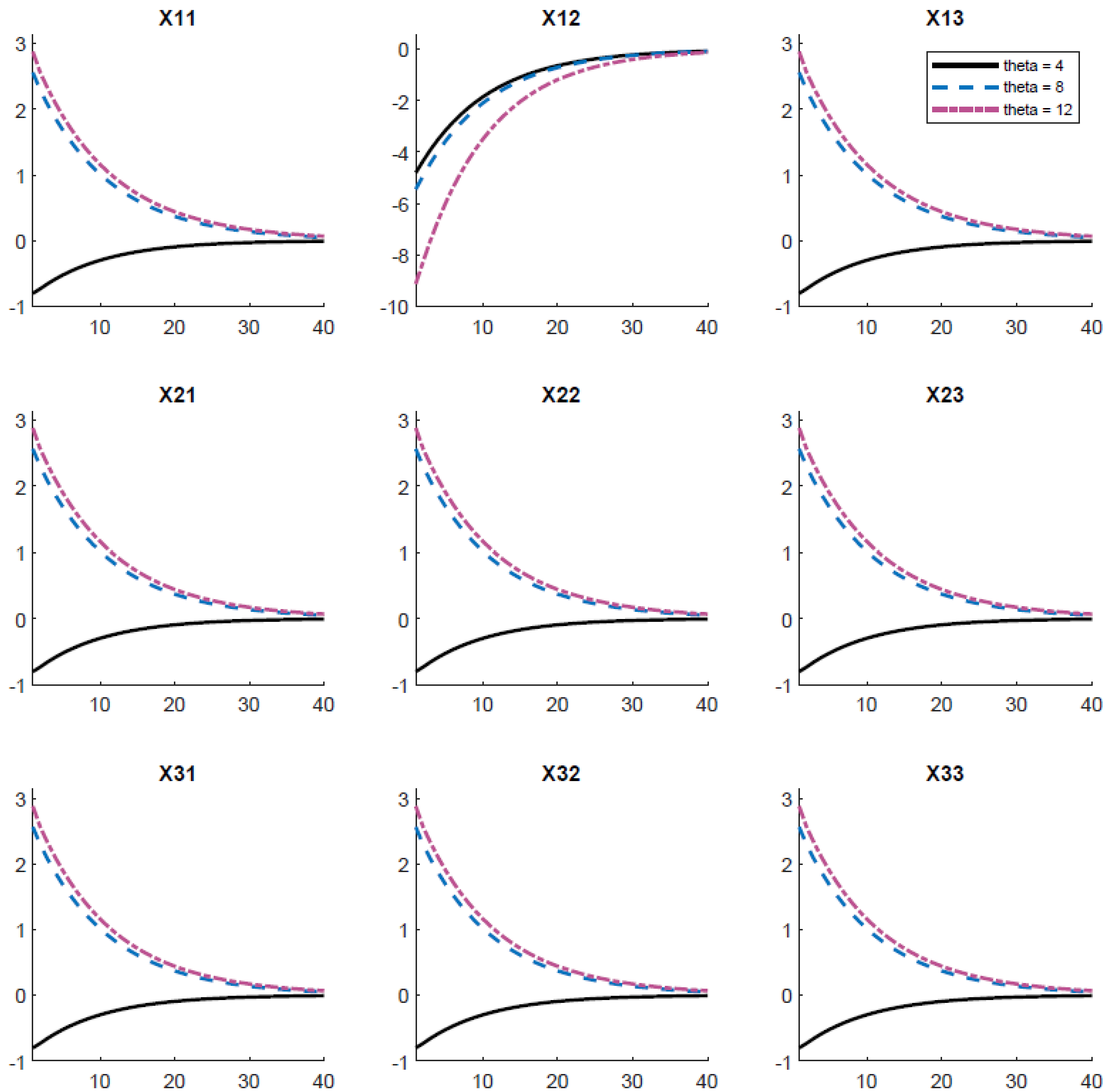


Figure A.5: Indirect flows, sensitivity to the shape distribution parameter θ

APPENDIX B

PROOFS OF PROPOSITIONS

B.1 Proof of Proposition 6 - Gravity Path Probability

Firms receive bids for financing their capital investments. Banks are competitive and each bank from country i and industry ω makes firms face the same interest rate P_i^X. Hence, price is given by:

$$p_{ij}^X(\omega) = c_i^X \ \tau_{ij}(\omega) \tag{B.1}$$

The goal is derive the probability that a route r is the lowest-cost route from i to j for good ω and country i is the lowest-cost supplier of good ω to j. We want to know the probability that any given good ω is shipped from i to j on a specific route p. Firms choose the lowest-cost route r from i to j for ω from all routes $p \in G$ and consumers in j choose the lowest-cost supplier of good ω from all countries $i \in I$. We will observe ω being shipped on route r from i to j if the final price of ω including both the marginal cost of production and shipping cost on route r from i to j, $p_{ijnr}(\omega)$, is lower than all other prices of good ω from all other country-route combinations.

Therefore we will find i) the probability that a country i provides loans to country j at the lowest price; ii) the price of the loan that a country i actually pays to country j is independent of j's characteristics.

B.1.0.1 Lenders

The *unconditional* probability that taking a route p to lend from country i to j for a given product ω that costs less than a constant τ is:

$$
\begin{aligned}
H_{ijp\omega}(\tau) \equiv & \Pr\Big(\tau_{ij}(p,\omega) \leq \tau\Big) \\
= & \Pr\left(\frac{\tilde{\tau}_{ij}(p)}{\xi_{ij}(p,\omega)} \leq \tau\right) \\
= & 1 - \Pr\left(\xi_{ij}(p,\omega) \leq \frac{\tilde{\tau}_{ij}(p)}{\tau}\right) \\
= & 1 - \exp\left\{-\left[\frac{\tilde{\tau}_{ij}(p)}{\tau}\right]^{-\theta}\right\} \qquad \Big[\xi \sim \text{ Fréchet}(1,\theta)\Big] \qquad \text{(B.2)}
\end{aligned}
$$

Because the technology is i.i.d across types, this probability will be the same for all goods $\omega \in \Omega$.

So far we have considered the potential trade cost, however we do not observe bilateral ex-ante cost, but the cost that each country applies ex-post, after choosing the cheapest path. The probability that, *conditional* on banks choosing the least cost route, the cost in ω is less than some constant τ:

$$
\begin{aligned}
H_{ij\omega}(\tau) \equiv & \Pr\Big(\tau_{ij}(\omega) \leq \tau\Big) \\
= & \Pr\Big(\min_{p\in G} \tau_{ij}(p,\omega) \leq \tau\Big) \\
= & 1 - \Pr\Big(\min_{p\in G} \tau_{ij}(p,\omega) \geq \tau\Big) \\
= & 1 - \Pr\Big(\bigcap_{p\in G} \big[\tau_{ij}(p,\omega) \geq \tau\big]\Big) \\
= & 1 - \prod_{p\in G}\Big[\Pr\Big(\tau_{ij}(p,\omega) \geq \tau\Big)\Big] \quad \text{by independence} \\
= & 1 - \prod_{p\in G}\Big[1 - \Pr\Big(\tau_{ij}(p,\omega) \leq \tau\Big)\Big] \\
= & 1 - \prod_{p\in G}\Big[1 - H_{ijp\omega}(\tau)\Big] \quad \text{by eq F.3} \\
= & 1 - \exp\left\{-\tau^{\theta}\sum_{p\in G}\Big[\tilde{\tau}_{ij}(p)\Big]^{-\theta}\right\} \qquad \text{(B.3)}
\end{aligned}
$$

To summarize, this is the probability that, given that banks choose the lower cost route, the cost is below a certain value.

B.1.0.2 Borrowers

Similar to equation F.3, the probability that the price is below a certain constant is the following:

$$
\begin{aligned}
G_{ijp\omega}(r) \equiv & \Pr\left(r_{ij}(p,\omega) \leq r\right) \\
= & 1 - \exp\left\{ - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\}
\end{aligned} \tag{B.4}
$$

Firms minimize the price they pay across countries and routes:

$$
\begin{aligned}
G_{j\omega}(r) \equiv & \Pr\left(\min_{i\in I, p\in G} r_{ij}(p,\omega) \leq r \right) \\
= & 1 - \Pr\left(\min_{i\in I, p\in G} c_i \tau_{ij}(p,\omega) \geq r \right) \quad \text{by eq 2.13} \\
= & 1 - \Pr\left(\bigcap_{i\in I} \bigcap_{p\in G} \left[c_i \tau_{ij}(p,\omega) \geq r \right] \right) \\
= & 1 - \prod_{i\in I} \prod_{p\in G} \left[\Pr\left(c_i \tau_{ij}(p,\omega) \geq r \right) \right] \quad \text{by independence} \\
= & 1 - \prod_{i\in I} \prod_{p\in G} \left[1 - G_{ijp\omega}(r) \right] \quad \text{by def} \\
= & 1 - \prod_{i\in I} \prod_{p\in G} \exp\left\{ - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\} \quad \text{by eq F.1} \\
= & 1 - \exp\left\{ \sum_{i\in I} \sum_{p\in G} - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\} \\
= & 1 - \exp\left\{ - r^{\theta} \sum_{i\in I} c_i^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\}
\end{aligned} \tag{B.5}
$$

B.1.0.3 Market Making

Finally, we can combine the two sides of the market, i.e. the probability that a firm in country j producing ω chooses to borrow from a bank country i, and that the route from country i to j is the minimal cost route. In other words, we compute the probability that, picking any other route-country pair, the price will be higher than the optimal one.

$$
\begin{aligned}
\pi_{ijp\omega} \equiv& \Pr\left(r_{ij}(p,\omega) \le \min_{k\neq i,\, s\neq p} r_{kj}(s,\omega) \right) \\
=& \Pr\left(\bigcap_{k\neq i} \bigcap_{s\neq p} \left[c_k \tau_{kj}(s,\omega) \ge r_{kj}(p,\omega) \right] \right) \\
=& \prod_{k\neq i} \prod_{s\neq p} \left[\Pr\left(c_k \tau_{kj}(s,\omega) \ge r_{kj}(p,\omega) \right) \right] \quad \text{by indep} \\
=& \prod_{k\neq i} \prod_{s\neq p} \left[1 - G_{kjs\omega}\left(r_{kj}(p,\omega) \right) \right] \\
=& \int_0^\infty \prod_{k\neq i} \prod_{s\neq p} \left[1 - G_{kjs\omega}(r) \right] dG_{ijp\omega}(r) \\
=& \int_0^\infty \prod_{k\neq i} \prod_{s\neq p} \left[1 - G_{kjs\omega}(r) \right] \frac{d}{dr} \left[1 - \exp\left\{ - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\} \right] dr \\
=& \int_0^\infty \prod_{k\neq i} \prod_{s\neq p} \left[1 - G_{kjs\omega}(r) \right] r^{\theta-1} \theta \left[c_i \tilde{\tau}_{ij} \right]^{-\theta} \exp\left\{ - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\} dr \\
=& \int_0^\infty \prod_{k\neq i} \prod_{s\neq p} \left[\exp\left\{ - \left[c_k \frac{\tilde{\tau}_{kj}(s)}{r} \right]^{-\theta} \right\} \right] \left(r^{\theta-1} \theta \left[c_i \tilde{\tau}_{ij} \right]^{-\theta} \right) \exp\left\{ - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\} dr \\
=& \int_0^\infty \prod_{i\in I} \prod_{p\in G} \left[\exp\left\{ - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\} \right] \left(r^{\theta-1} \theta \left[c_i \tilde{\tau}_{ij} \right]^{-\theta} \right) dr \\
=& \int_0^\infty \left[\exp\left\{ - r^\theta \sum_{i\in I} c_i^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\} \right] \left(r^{\theta-1} \theta \left[c_i \tilde{\tau}_{ij} \right]^{-\theta} \right) dr \\
=& \int_0^\infty \left[\exp\left\{ - r^\theta \sum_{i\in I} c_i^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\} \right] \left(r^{\theta-1} \theta \left[c_i \tilde{\tau}_{ij} \right]^{-\theta} \right) dr \, \frac{\sum_{i'\in I} c_i'^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{i'j}(p) \right]^{-\theta}}{\sum_{i'\in I} c_i'^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{i'j}(p) \right]^{-\theta}} \\
=& \int_0^\infty \left[\exp\left\{ -r^\theta \sum_{i\in I} c_i^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\} \right] \left(r^{\theta-1} \theta \right) \sum_{i\in I} c_i^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} dr \, \frac{\left[c_i \tilde{\tau}_{ij} \right]^{-\theta}}{\sum_{i'\in I} c_i'^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{i'j}(p) \right]^{-\theta}} \\
=& \int_0^\infty -\frac{d}{dr} \exp\left\{ - r^\theta \sum_{i\in I} c_i^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\} dr \, \frac{\left[c_i \tilde{\tau}_{ij} \right]^{-\theta}}{\sum_{i'\in I} c_i'^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{i'j}(p) \right]^{-\theta}} \\
=& - \exp\left\{ - r^\theta \sum_{i\in I} c_i^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\} \Bigg|_0^\infty \frac{\left[c_i \tilde{\tau}_{ij} \right]^{-\theta}}{\sum_{i'\in I} c_i'^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta}} \\
=& \left[- \frac{1}{\exp(\infty)} + \frac{1}{\exp(0)} \right] \frac{\left[c_i \tilde{\tau}_{ij} \right]^{-\theta}}{\sum_{i'\in I} c_i'^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{i'j}(p) \right]^{-\theta}} \\
=& \frac{\left[c_i \tilde{\tau}_{ij}(p) \right]^{-\theta}}{\sum_{i'\in I} c_i'^{-\theta} \sum_{p\in G} \left[\tilde{\tau}_{i'j}(p) \right]^{-\theta}}
\end{aligned}
\tag{B.6}
$$

By the law of large numbers, given the continuum of products, this is also the share of all loans sold from i to j in industry ω and take route p, $\lambda_{ijp\omega}$.

B.2 Proof of Proposition 7 - Expected Cost

Assume banks choose the route that minimizes the cost of sending a loan from country i to country j. Define the matrix A, where each element is a function of the deterministic edge friction, such that: $a_{ij} \equiv e_{ij}^{-\theta}$. Then, each element b_{ij} of the matrix $B \equiv (\mathbf{I} - A)^{-1}$ represents the expected cost, summing over the possible routes p:

$$b_{ij} = \sum_{p \in G_{ij}} \tilde{\tau}_{ij}(p)^{-\theta} \tag{B.7}$$

Proof.

B.2.0.1 Expected Cost

The cost between locations i and j is expected trade cost τ_{ij} from i to j across all lenders:

$$\begin{aligned}
\tau_{ij} \equiv \mathbb{E}_{\omega}\left[\tau_{ij}(\omega)\right] &= \int_{p \in G_{ij}} \tau_{ijp}(\omega)\, dp \\
&= \int_0^{\infty} \tau \; dH_{ij\omega} \\
&= \int_0^{\infty} \tau \frac{d\, H_{ij}(\tau)}{d\, \tau}\, d\tau \\
&= \int_0^{\infty} \tau \frac{d}{d\, \tau}\left(1 - \exp\left\{ - \tau^{\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right\}\right) d\tau \\
&= \theta \Phi_j \int_0^{\infty} \tau_j^{\theta} \exp\left\{-\tau^{\theta}\Phi_j\right\}\; d\tau \\
&= \int_0^{\infty} \left(\frac{x}{\Phi_j}\right)^{\frac{1}{\theta}} \exp\{-x\}\; dx \\
&= \Phi_j^{-\frac{1}{\theta}} \int_0^{\infty} (x)^{\frac{1}{\theta}} \exp\{-x\}\; dx \\
&= \Phi_j^{-\frac{1}{\theta}} \Gamma\left(\frac{\theta+1}{\theta}\right) \\
&= \Gamma\left(\frac{1+\theta}{\theta}\right)\left[\sum_{p \in G_{ij}} \tilde{\tau}_{ijp}^{-\theta}\right]^{-1/\theta}
\end{aligned} \tag{B.8}$$

B.2.0.2 Expected cost with Paths

Let $\gamma \equiv \Gamma\left(\frac{1+\theta}{\theta}\right)$. Following Allen and Arkolakis (2019):

$$\tau_{ij}^{-\theta} = \gamma^{-\theta} \sum_{p \in G_{ij}} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}$$

taking into account the length of the path, and all possible lengths:

$$= \gamma^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}(K)} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}$$

$$= \gamma^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}(K)} \prod_{k=1}^{K} e_{k-1,k}(p)^{-\theta} \qquad \text{by definition 2.2.5.1}$$

$$= \gamma^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}(K)} \prod_{k=1}^{K} a_{ij} \qquad \text{defining } e_{k-1,k}^{-\theta} \equiv a_{ij}$$

$$= \gamma^{-\theta} \sum_{K=0}^{\infty} A_{ij}^{K}$$

Assuming that the spectral radius of A is less than one[1] then:

$$\sum_{K=0}^{\infty} A^{K} = (\mathbf{I} - A)^{-1} \equiv B \tag{B.9}$$

Hence[2]:

$$\tau_{ij} = \gamma^{-\theta} b_{ij}^{-1/\theta} \Leftrightarrow b_{ij} = \sum_{p \in G_{ij}} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \tag{B.10}$$

Hence the expected bilateral price is:

$$r_{ij} \equiv \mathbb{E}\left[r_{ij}(\omega)\right] = c_i \mathbb{E}\left[\tau_{ij}(\omega)\right] = c_i \gamma^{-\theta} b_{ij}^{-1/\theta}$$

[1] AA19: "sufficient condition for the spectral radius being less than one is if $\sum_j e_{ij}^{-\theta} < 1$ for all i. This will necessarily be the case if either trade costs between connected locations are sufficiently large, the adjacency matrix is sufficiently sparse, or the heterogeneity across traders is sufficiently small (i.e. θ is sufficiently large"

[2] As in Allen and Arkolakis (2019), this is the "analytical relationship between any given infrastructure network and the resulting bilateral trade cost between all locations, accounting for traders choosing the least cost route".

B.3 Proof of Proposition 8 - Indirect Gravity

The probability of going through an edge tk, conditional on origin i and destination j, is:

$$
\begin{aligned}
\psi_{kl|ij} =& \sum_{K=0}^{\infty} \sum_{p \in G_{ij}^{kl}(K)} \frac{\tilde{\tau}_{ij}(p)^{-\theta}}{\sum_{K=0}^{\infty} \sum_{p \in G_{ij}(K)} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}} \\
=& \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}^{kl}(K)} \tilde{\tau}_{ij}(p)^{-\theta} \\
=& \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}^{kl}(K)} \prod_{k=1}^{K} e_{k-1,k}(p)^{-\theta} \\
=& \frac{1}{b_{ij}} \left(b_{ik} a_{kl} b_{lj}\right)
\end{aligned} \tag{B.11}
$$

where in the last step we isolate the kl step and follow the matrix algebra in Allen and Arkolakis (2019), such that $\sum_{K=0} \sum_{L=0}^{K-1} A^L A A^{K-L-1} = (I-A)^{-1} A (I-A)^{-1}$.

The conditional probability is:

$$
\psi_{kl|ij} = \frac{b_{ik} a_{kl} b_{lj}}{b_{ij}} = \left(\frac{\tau_{ij}}{\tau_{ik} e_{kl} \tau_{lj}}\right)^{\theta} \tag{B.12}
$$

where the last step was obtained by plugging the expected cost definition in B.10.

APPENDIX C

DERIVATIONS

C.0.1 Aggregate Interest Rate

Let $G_{ij}(\phi)$ be the Pareto (equilibrium) probability density function of the productivities of banks from country i that sell to country j such that the measure of banks from country i with productivity ϕ is $N_i\, dG_i(\phi)$. Then we can write the aggregate interest rate in j as:

$$R_j = \vartheta \left(\sum_i c_i^{-\theta} b_{ij} \right)^{-\frac{1}{\theta}}$$

where $\vartheta = \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}}$.

Proof.

$$\begin{aligned}
R_j^{1-\sigma} &= \int_{\Omega} r_{ij}(\omega)^{1-\sigma}\, d\omega \\
&= \int_0^{\infty} p^{1-\sigma}\, dG_j(p) \\
&= \int_0^{\infty} p^{1-\sigma} \frac{d}{dp} \left(1 - \exp\{-p\Phi\}\right)\, dp \\
&= \theta\Phi \int_0^{\infty} p^{\theta-\sigma} \exp\{-p\Phi\}\, dp \\
&= \int_0^{\infty} \frac{x}{\Phi}^{\frac{1-\sigma}{\theta}} \exp\{-x\}\, dp \\
&= \Phi^{-\frac{1-\sigma}{\theta}} \int_0^{\infty} x^{\frac{1-\sigma}{\theta}} \exp\{-x\}\, dp \\
&= \Phi^{-\frac{1-\sigma}{\theta}} \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right) \\
R_j &= \vartheta \left(\sum_i c_i^{-\theta} b_{ij} \right)^{-\frac{1}{\theta}}
\end{aligned}$$

where $\vartheta = \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}}$.

APPENDIX D

EQUILIBRIUM CONDITIONS

Quantities: $C, H, D, M, \Pi, I, Y, K, X, Y^X$

Prices: w^H, R^D, w, P^K, R^X, c

	Equilibrium equations
Households	
1	$C_{i,t} + D_{i,t} = (1 + R^D_{i,t-1})D_{i,t-1} + w^H_{i,t}H_{i,t} + w_{i,t}M_{i,t} + \Pi_{i,t}$
2	$\mathbb{E}_t\Lambda_{i,t,t+1}(1 + R^D_{i,t}) = 1$
3	$k_H H^{\epsilon}_{i,t} = \frac{w^H_{i,t}}{C_{i,t}}$
4	$k_M M^{\varphi}_{i,t} = \frac{w_{i,t}}{C_{i,t}}$
Capital Producers	
5	$P^K_{i,t} = \left\{1 + f\left(\frac{I_{i,t}}{I_{i,t-1}}\right) + \frac{I_{i,t}}{I_{i,t-1}} f'\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right\} - \mathbb{E}_t\Lambda_{i,t,t+1} f'\left(\frac{I_{i,t+1}}{I_{i,t}}\right)\left(\frac{I_{i,t+1}}{I_{i,t}}\right)^2$
Firms	
6	$Y_{i,t} = K^{\alpha}_{i,t-1}H^{1-\alpha}_{i,t}$
7	$K_{i,t} = (1-\delta)K_{i,t-1} + I_{i,t}$
8	$X_{i,t} = P^K_{i,t}I_{i,t}$
9	$\frac{(1-\alpha)Y_{i,t}}{H_{i,t}} = w^H_{i,t}$
10	$-P^K_{i,t}(1 + R^X_{i,t}) + \mathbb{E}_t\left[\Lambda_{i,t,t+1}\left((1-\delta)P^K_{i,t+1}(1 + R^X_{i,t+1}) + \frac{\alpha Y_{i,t+1}}{K_{i,t}}\right)\right] = 0$
Banks	
11	$c_{i,t} = \zeta w_{i,t} + (1-\zeta)R^D_{i,t}$
12	$R^X_{j,t} = \vartheta\left(\sum_i c^{-\theta}_{i,t}b_{ij,t}\right)^{-\frac{1}{\theta}}$
13	$M_{i,t} = \vartheta\zeta Y^X_{i,t}$
14	$D_{i,t} = \vartheta(1-\zeta)Y^X_{i,t}$
Other conditions	
15	$\Pi_{i,t} = Y_{i,t} - w^H_{i,t}H_{i,t} - (1 + R^X_{i,t})I_{i,t}$
Market clearing	
16	$R^X_{i,t}Y^X_{i,t} = \sum_j \lambda_{ij,t}R^X_{j,t}X_{j,t}$
17	$\sum_i C_{i,t} + \left[1 + f\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right] I_{i,t} = \sum_i Y_{i,t}$

where the last equation is redundant by Walras law.

D.0.1 Steady State Equations

	Steady-state equations
Households	
1	$C_i = Y_i + (1 + R_i^X)I_i + R_i^X Y_i^X$
2	$\beta(1 + R_i^D) = 1$
3	$k_H H_i^{\epsilon} = \frac{w_i^H}{C_i}$
4	$k_M M_i^{\varphi} = \frac{w_{i,t}}{C_i}$
Capital Producers	
5	$P_i^K = 1$
Firms	
6	$Y_i = K_i^{\alpha} H_i^{1-\alpha}$
7	$I_i = \delta K_i$
8	$X_i = P_i^K I_i$
9	$\frac{(1-\alpha)Y_i}{H_i} = w_i^H$
10	$\frac{\alpha Y_i}{K_i} = P_i^K (1 + R_i^X) \left(\frac{1}{\beta} - (1 - \delta)\right)$
Banks	
11	$c_i = \zeta w_i + (1 - \zeta) R_i^D$
12	$R_j^X = \vartheta \left(\sum_i c_i^{-\theta} b_{ij}\right)^{-\frac{1}{\theta}}$
13	$M_i = \vartheta \zeta Y_i^X$
14	$D_i = \vartheta (1 - \zeta) Y_i^X$
15	$R_i^X Y_i^X = \sum_j \lambda_{ij} R_j^X X_j$

APPENDIX E

ALTERNATIVE SPECIFICATIONS

E.0.1 Cobb Douglas Banking Technology

$$c_i = a\,(w_i)^{\zeta}\left(R_i^D\right)^{1-\zeta} \tag{E.1}$$

and $a = \left[\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} + \frac{1}{\Phi}\left(\frac{\alpha}{1-\alpha}\right)\right]$ is a constant. Since $R^D = 1$ by timing assumption, $c_i = a\,w_i^{\zeta}$.

The labor market clearing condition for monitoring hours reads:

$$w_{i,t}M_{i,t} = \zeta R_{i,t}^X Y_{i,t}^X \tag{E.2}$$

where ϑ is a constant and ζ is the monitoring share of the loan production.

The aggregate deposit clearing condition reads:

$$R_{i,t}^D D_{i,t} = (1-\zeta) R_{j,t}^X Y_{i,t}^X \tag{E.3}$$

Table E.1: Calibration of common parameters

Description	Symbol	Value
Preferences		
Household discount factor	β	0.960
Inverse Frisch elasticity H	ϵ	4.000
Inverse Frisch elasticity M	φ	4.000
Technology		
Capital share of output	α	0.330
Capital depreciation	δ	0.025
Inverse elasticity of I to P^K	f''(1)	1.000
Loans		
Loans elasticity of substitution	σ	1.471
Monitoring share of loan production	ζ	0.350
Fréchet shape parameter	θ	4.000

E.0.1.1 Equilibrium Conditions

Quantities: C, H, B, M, Π, I, Y, K, X, D, Y^X

Prices: w^H, R^B, w, P^K, R^X, c

Equilibrium equations	
Households	
1	$C_{i,t} + B_{i,t} = R^B_{i,t-1} B_{i,t-1} + w^H_{i,t} H_{i,t} + w_{i,t} M_{i,t} + \Pi_{i,t}$
2	$\mathbb{E}_t \Lambda_{i,t,t+1} R^B_{i,t} = 1$
3	$k_H H^{\epsilon}_{i,t} = \frac{w^H_{i,t}}{C_{i,t}}$
4	$k_M M^{\varphi}_{i,t} = \frac{w_{i,t}}{C_{i,t}}$
Capital Producers	
5	$P^K_{i,t} = \left\{1 + f\left(\frac{I_{i,t}}{I_{i,t-1}}\right) + \frac{I_{i,t}}{I_{i,t-1}} f'\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right\} - \mathbb{E}_t \Lambda_{i,t,t+1} f'\left(\frac{I_{i,t+1}}{I_{i,t}}\right)\left(\frac{I_{i,t+1}}{I_{i,t}}\right)^2$
Firms	
6	$Y_{i,t} = K^{\alpha}_{i,t-1} H^{1-\alpha}_{i,t}$
7	$K_{i,t} = (1-\delta) K_{i,t-1} + I_{i,t}$
8	$X_{i,t} = P^K_{i,t} I_{i,t}$
9	$\frac{(1-\alpha) Y_{i,t}}{H_{i,t}} = w^H_{i,t}$
10	$-P^K_{i,t}(1 + R^X_{i,t}) + \mathbb{E}_t\left[\Lambda_{i,t,t+1}\left((1-\delta) P^K_{i,t+1}(1 + R^X_{i,t+1}) + \frac{\alpha Y_{i,t+1}}{K_{i,t}}\right)\right] = 0$
Banks	
11	$c_{i,t} = a\,(w_{i,t})^{\zeta}\left(R^D_{i,t}\right)^{1-\zeta}$
12	$R^X_{j,t} = \vartheta\left(\sum_i c^{-\theta}_{i,t} b_{ij,t}\right)^{-\frac{1}{\theta}}$
13	$w_{i,t} M_{i,t} = \zeta R^X_{i,t} Y^X_{i,t}$
14	$R^D_{i,t} D_{i,t} = (1-\zeta) R^X_{j,t} Y^X_{i,t}$
Other conditions	
15	$\Pi_{i,t} = Y_{i,t} - w^H_{i,t} H_{i,t} - (1 + R^X_{i,t}) X_{i,t} + P^K_{i,t} I_{i,t} - \left[1 + f\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right] I_{i,t}$
Market clearing	
16	$R^X_{i,t} Y^X_{i,t} = \sum_j \lambda_{ij,t} R^X_{j,t} X_{j,t}$
17	$B_{i,t} = 0$
18	$\sum_i C_{i,t} + \left[1 + f\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right] I_{i,t} = \sum_i Y_{i,t}$

where the last equation is redundant by Walras law.

E.0.1.2 Steady State

	Steady-state equations
Households	
1	$C_i = w_i^H H_i + w_i M_{i,t} + \Pi_i$
2	$\beta R_{i,t}^B = 1$
3	$k_H H_i^{\epsilon} = \frac{w_i^H}{C_i}$
4	$k_M M_i^{\varphi} = \frac{w_{i,t}}{C_i}$
Capital Producers	
5	$P_i^K = 1$
Firms	
6	$Y_i = K_i^{\alpha} H_i^{1-\alpha}$
7	$I_i = \delta K_i$
8	$X_i = P_i^K I_i$
9	$\frac{(1-\alpha)Y_i}{H_i} = w_i^H$
10	$\frac{\alpha Y_i}{K_i} = P_i^K (1 + R_i^X) \left(\frac{1}{\beta} - (1-\delta)\right)$
Banks	
11	$a\,(w_i)^{\zeta} \left(R_i^D\right)^{1-\zeta}$
12	$R_j^X = \vartheta \left(\sum_i c_i^{-\theta} b_{ij}\right)^{-\frac{1}{\theta}}$
13	$w_{i,t} M_{i,t} = \zeta R_{i,t}^X Y_{i,t}^X$
14	$R_i^D D_{i,t} = (1-\zeta) R_j^X Y_i^X$
Other conditions	
15	$\Pi_i = Y_i - w_i^H H_i - (1 + R_i^X) X_i$
Market clearing	
16	$R_{i,t}^X Y_{i,t}^X = \sum_j \lambda_{ij,t} R_{j,t}^X X_{j,t}$
17	$B_i = 0$

E.0.2 Banking Market: Monopolistic Competition

Most equations are the same, except for:

1. marginal cost

$$r_{ij}(\omega) = \frac{\sigma}{\sigma - 1}\frac{c_i}{\phi}\tau_{ij}(\omega)$$

2. gravity

$$X_{ij,t} = \vartheta R_{j,t}^{\sigma} X_{j,t} N_{ii,t} c_i^{-\sigma} b_{ij,t}^{\frac{\sigma}{\theta}}$$

3. shares

$$\begin{aligned} \lambda_{ij,t}^{E} &\equiv \frac{W_{ij}}{W_j} \equiv \frac{W_{ij}}{\sum_l W_{lj}} \\ &= \frac{N_{ii} c_i^{1-\sigma} b_{ij}^{\frac{\sigma-1}{\theta}}}{\sum_l N_{ll} c_l^{1-\sigma} b_{lj}^{\frac{\sigma-1}{\theta}}} \end{aligned} \tag{E.4}$$

APPENDIX F

PROOFS OF PROPOSITIONS WITH MELITZ

In the derivations of the propositions we simplify notation by writing the double sum, over the path p and the length of the path K: $\sum_{K=0}^{\infty}\sum_{p\in G_{ij}(K)}$, simply as $\sum_{p\in G_{ij}}$, if not noted otherwise.

Some Lemmas

Interest rate distribution conditional on minimum route

We first derive a price distribution which serves in the propositions.

Let us consider the probability that country i is able to offer country j loan $\omega \in \Omega$ for a price less than p. Because the technology is i.i.d across goods, this probability will be the same for all goods $\omega \in \Omega$. The *unconditional* probability that taking a path p to lend from country i to j for a given product ω whose price is less than a constant p is:

$$\begin{aligned} G_{ijp\omega}(r) \equiv& \Pr\left(r_{ij}(p,\omega) \le r\right) \\ =& \Pr\left(\frac{\sigma}{\sigma-1}\frac{c_i}{\phi}\frac{\tilde{\tau}_{ij}(p)}{\xi_{ij}(p,\omega)} \le p\right) \\ =& 1-\Pr\left(\xi_{ij}(p,\omega) \le \frac{\sigma}{\sigma-1}\frac{c_i}{\phi}\frac{\tilde{\tau}_{ij}(p)}{r}\right) \\ =& 1-\exp\left\{-\left[\frac{\sigma}{\sigma-1}\frac{c_i}{\phi}\frac{\tilde{\tau}_{ij}(p)}{r}\right]^{-\theta}\right\} \qquad \left[\xi \sim \text{ Fréchet}(1,\theta)\right] \end{aligned} \tag{F.1}$$

Because the technology is i.i.d across types, this probability will be the same for all goods $\omega \in \Omega$.

So far we have considered the potential price, however we do not observe bilateral ex-ante price, but the price that each country applies ex-post, after choosing the cheapest path. The probability that, *conditional* on banks choosing the least cost route, the price in ω is less than some constant p:

$$
\begin{aligned}
G_{ij\phi}(r) \equiv & \Pr\left(\min_{p\in G} r_{ij}(p,\omega) \leq r\right) \\
= & 1 - \Pr\left(\min_{p\in G} \frac{\sigma}{\sigma-1}\frac{c_i}{\phi}\tau_{ij}(p,\omega) \geq r\right) \quad \text{by eq 2.13} \\
= & 1 - \Pr\left(\bigcap_{p\in G}\left[\frac{\sigma}{\sigma-1}\frac{c_i}{\phi}\tau_{ij}(p,\omega) \geq r\right]\right) \\
= & 1 - \prod_{p\in G}\left[\Pr\left(\frac{\sigma}{\sigma-1}\frac{c_i}{\phi}\tau_{ij}(p,\omega) \geq r\right)\right] \quad \text{by independence} \\
= & 1 - \prod_{p\in G}\left[1 - G_{ijp\omega}(r)\right] \quad \text{by def} \\
= & 1 - \prod_{p\in G}\exp\left\{-\left[\frac{\sigma}{\sigma-1}\frac{c_i}{\phi}\frac{\tilde{\tau}_{ij}(p)}{r}\right]^{-\theta}\right\} \quad \text{by eq F.1} \\
= & 1 - \exp\left\{\sum_{p\in G} -\left[\frac{\sigma}{\sigma-1}\frac{c_i}{\phi}\frac{\tilde{\tau}_{ij}(p)}{r}\right]^{-\theta}\right\} \\
= & 1 - \exp\left\{-(r\phi)^{\theta}\left(\frac{\sigma}{\sigma-1}\right)^{-\theta} c_i^{-\theta}\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right\} \qquad \text{(F.2)}
\end{aligned}
$$

Cost distribution conditional on minimum route

First, let us consider the probability that bank ω in i is able to offer country j loan $\omega \in \Omega$ for a cost less than τ. Because the technology is i.i.d across goods, this probability will be the same for all goods $\omega \in \Omega$:

$$
\begin{aligned}
H_{ijp\omega}(\tau,p) \equiv & \Pr\left(\tau_{ij}(p,\phi) \leq \tau\right) \\
= & \Pr\left(\frac{\tilde{\tau}_{ij}(p)}{\xi_{ij}(p,\omega)} \leq \tau\right) \\
= & 1 - \Pr\left(\xi_{ij}(p,\omega) \leq \frac{\tilde{\tau}_{ij}(p)}{\tau}\right) \\
= & 1 - \exp\left\{-\left[\frac{\tilde{\tau}_{ij}(p)}{\tau}\right]^{-\theta}\right\} \qquad \left[\xi \sim \text{ Fréchet}(1,\theta)\right] \qquad \text{(F.3)}
\end{aligned}
$$

So far we have considered the potential trade cost, however we do not observe bilateral ex-ante cost, but the cost that each country applies ex-post, after choosing the cheapest path. The probability that, *conditional* on banks choosing the least cost route, the cost in ω is less than some constant τ:

$$
\begin{aligned}
H_{ij\omega}(\tau) \equiv & \Pr\left(\min_{p \in G} \tau_{ij}(p,\omega) \leq \tau\right) \\
= & 1 - \Pr\left(\min_{p \in G} \tau_{ij}(p,\omega) \geq \tau\right) \\
= & 1 - \Pr\left(\bigcap_{p \in G} \left[\tau_{ij}(p,\omega) \geq \tau\right]\right) \\
= & 1 - \prod_{p \in G} \left[\Pr\left(\tau_{ij}(p,\omega) \geq \tau\right)\right] \quad \text{by independence} \\
= & 1 - \prod_{p \in G} \left[1 - \Pr\left(\tau_{ij}(p,\omega) \leq \tau\right)\right] \\
= & 1 - \prod_{p \in G} \left[1 - H_{ijp}(\tau)\right] \quad \text{by eq F.3} \\
= & 1 - \exp\left\{-\tau^{\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right\}
\end{aligned}
\tag{F.4}
$$

To summarize, this is the probability that, given that banks choose the lower cost route, the cost is below a certain value.

Harmonic Average Interest Rate, only Paths

$$
\begin{aligned}
\mathbb{E}_\xi\left[r_{ij}(\phi,\xi)^{1-\sigma}\right] &= \left(\int_0^\infty r_{ij}^{1-\sigma}(\phi,\xi)\ dG_{ij\phi}(p)\right) \\
&= \int_0^\infty p^{1-\sigma}\,\frac{d\,G_{ij\phi}(p)}{d\,p}\,dp \\
&= \int_0^\infty p^{1-\sigma}\,\frac{d}{d\,p}\left(1-\exp\left\{-(\phi p)^\theta\left(\frac{\sigma}{\sigma-1}\right)^{-\theta} c_i^{-\theta}\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right\}\right)dp \\
&= \theta\Phi_j\int_0^\infty p^{\theta-\sigma}\exp\left\{-p^\theta\Phi_j\right\}\ dp \\
&= \int_0^\infty\left(\frac{x}{\Phi_j}\right)^{\frac{1-\sigma}{\theta}}\exp\{-x\}\ dx \\
&= \Phi_j^{-\frac{1-\sigma}{\theta}}\int_0^\infty (x)^{\frac{1-\sigma}{\theta}}\exp\{-x\}\ dx \\
&= \Phi_j^{-\frac{1-\sigma}{\theta}}\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right) \\
&= \varsigma\phi^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma}\left(\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma-1}{\theta}} \qquad \text{(F.5)}
\end{aligned}
$$

where $\varsigma = \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)$.

Harmonic Average Interest Rate, only Paths, second version

$$\begin{aligned}
\mathbb{E}_{\xi}\left[r_{ij}(\phi,\xi)^{-\sigma}\right] &= \left(\int_0^{\infty} r_{ij}^{1-\sigma}(\phi,\xi)\ dG_{ij\phi}(p)\right) \\
&= \int_0^{\infty} p^{-\sigma}\,\frac{d\,G_{ij\phi}(p)}{d\,p}\,dp \\
&= \int_0^{\infty} p^{-\sigma}\,\frac{d}{d\,p}\left(1-\exp\left\{-(\phi p)^{\theta}\left(\frac{\sigma}{\sigma-1}\right)^{-\theta} c_i^{-\theta}\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right\}\right) dp \\
&= \theta\Phi_j \int_0^{\infty} p^{\theta-1-\sigma}\exp\left\{-p^{\theta}\Phi_j\right\}\ dp \\
&= \int_0^{\infty}\left(\frac{x}{\Phi_j}\right)^{-\frac{\sigma}{\theta}}\exp\{-x\}\ dx \\
&= \Phi_j^{\frac{\sigma}{\theta}}\int_0^{\infty}(x)^{-\frac{\sigma}{\theta}}\exp\{-x\}\ dx \\
&= \Phi_j^{\frac{\sigma}{\theta}}\Gamma\left(\frac{\theta-\sigma}{\theta}\right) \\
&= \dot{\varsigma}\phi^{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} c_i^{-\sigma}\left(\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma}{\theta}} \qquad \text{(F.6)}
\end{aligned}$$

where $\dot{\varsigma} = \Gamma\left(\frac{\theta-\sigma}{\theta}\right)$.

Harmonic Average Productivity

We first derive a measure proportional to average productivity:

$$\begin{aligned}
\int_{\phi_{ij}^*}^{\infty} \phi^{\sigma-1}\, dG_{ij}(\phi) =& \int_{\phi_{ij}^*}^{\infty} \phi^{\sigma-1}\, \frac{dG_{ij}(\phi)}{d\phi} d\phi \\
=& \int_{\phi_{ij}^*}^{\infty} \phi^{\sigma-1}\, \frac{d\left(1-\phi^{-\kappa}\right)}{d\phi} d\phi \\
=& \int_{\phi_{ij}^*}^{\infty} \phi^{\sigma-1} \kappa \phi^{-\kappa-1}\, d\phi \\
=& \kappa \int_{\phi_{ij}^*}^{\infty} \phi^{\sigma-\kappa-2} d\phi \\
=& \frac{\kappa}{\kappa+1-\sigma} \left(\phi_{ij}^*\right)^{\sigma-\kappa-1} \\
=& \frac{\kappa}{\kappa+1-\sigma} \left(\left[\frac{\sigma}{\varsigma} \frac{c_i f_j}{X_j R_j^{\sigma}} \right]^{\frac{1}{\sigma-1}} \frac{\sigma}{\sigma-1} c_i \left(\sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \right)^{-\frac{1}{\theta}} \right)^{\sigma-\kappa-1} \\
=& \varpi \left(\left[\frac{c_i f_j}{X_j R_j^{\sigma}} \right]^{\frac{1}{\sigma-1}} c_i \left(\sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \right)^{-\frac{1}{\theta}} \right)^{\sigma-\kappa-1} \qquad \text{(F.7)}
\end{aligned}$$

where $\varpi = \frac{\kappa}{\kappa+1-\sigma} \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-\kappa-1} \left(\frac{\sigma}{\varsigma}\right)^{\frac{\sigma-\kappa-1}{\sigma-1}}$.

F.0.0.1 Lemma

$$
\begin{aligned}
\int_{\phi_{ij}^*}^{\infty} \phi^{-1}\ dG_{ij}(\phi) &= \int_{\phi_{ij}^*}^{\infty} \phi^{-1}\ \frac{dG_{ij}(\phi)}{d\phi} d\phi \\
&= \int_{\phi_{ij}^*}^{\infty} \phi^{-1}\ \frac{d\left(1-\phi^{-\kappa}\right)}{d\phi} d\phi \\
&= \int_{\phi_{ij}^*}^{\infty} \phi^{-1} \kappa \phi^{-\kappa-1}\ d\phi \\
&= \kappa \int_{\phi_{ij}^*}^{\infty} \phi^{-\kappa-2} d\phi \\
&= \frac{\kappa}{\kappa+1} \left(\phi_{ij}^*\right)^{-\kappa-1} \\
&= \frac{\kappa}{\kappa+1} \left(\left[\frac{\sigma}{\varsigma} \frac{c_i f_{ij}}{X_j R_j^{\sigma}} \right]^{\frac{1}{\sigma-1}} \frac{\sigma}{\sigma-1} c_i \left(\sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \right)^{-\frac{1}{\theta}} \right)^{-\kappa-1} \\
&= C \left(\left[\frac{c_i f_{ij}}{X_j R_j^{\sigma}} \right]^{\frac{1}{\sigma-1}} c_i \left(\sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \right)^{-\frac{1}{\theta}} \right)^{-\kappa-1} \qquad \text{(F.8)}
\end{aligned}
$$

where $C = \frac{\kappa}{\kappa+1} \left(\frac{\sigma}{\sigma-1}\right)^{-\kappa-1} \left(\frac{\sigma}{\varsigma}\right)^{-\frac{\kappa+1}{\sigma-1}}$

Corollary

$$
\int_{1}^{\infty} \phi^{-1}\ dG_{ij}(\phi) = \frac{\kappa}{\kappa+1}
$$

Harmonic Average Productivity, second version

We first derive a measure proportional to average productivity:

$$\begin{aligned}\int_{\phi_{ij}^*}^{\infty} \phi^{\sigma} \, dG_{ij}(\phi) &= \int_{\phi_{ij}^*}^{\infty} \phi^{\sigma} \, \frac{dG_{ij}(\phi)}{d\phi} d\phi \\ &= \int_{\phi_{ij}^*}^{\infty} \phi^{\sigma} \, \frac{d\left(1-\phi^{-\kappa}\right)}{d\phi} d\phi \\ &= \int_{\phi_{ij}^*}^{\infty} \phi^{\sigma} \kappa \phi^{-\kappa-1} \, d\phi \\ &= \kappa \int_{\phi_{ij}^*}^{\infty} \phi^{\sigma-\kappa-1} d\phi \\ &= \frac{\kappa}{\kappa-\sigma} \left(\phi_{ij}^*\right)^{\sigma-\kappa}\end{aligned}$$

Harmonic Average Interest Rate, both Paths and Productivity

Let N_{ij} the mass of firms with $\phi \geq \phi^*$. We can derive the harmonic average interest rate:

$$\begin{aligned}\mathbb{E}_{\phi,\xi}\left[r_{ij}(\omega)^{1-\sigma}\right] &= \int_{\Omega} r_{ij}(\omega)^{1-\sigma} \, d\omega \\ &= \int_{\Omega} \varsigma \phi^{\sigma-1} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma} \left(\sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma-1}{\theta}} \, dG_{ij}(\phi) \\ &= \varsigma \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma} \left(\sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma-1}{\theta}} \frac{1}{1-G_i(\phi_{ij}^*)} \int_{\phi_{ij}^*}^{\infty} \phi^{\sigma-1} \, dG_{ij}(\phi) \\ &= \varsigma \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma} \left(\sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma-1}{\theta}} \frac{N_{ii}}{N_{ij}} \frac{\kappa}{\kappa+1-\sigma} \left(\phi_{ij}^*\right)^{\sigma-\kappa-1} \\ &= \vartheta c_i^{1-\sigma} \left(\sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma-1}{\theta}} \frac{N_{ii}}{N_{ij}} \left(\left[\frac{c_i f_j}{X_j R_j^{\sigma}}\right]^{\frac{1}{\sigma-1}} c_i \left(\sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{-\frac{1}{\theta}}\right)^{\sigma-\kappa-1} \\ &= \vartheta \frac{N_{ii}}{N_{ij}} \left(c_i \left(\sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{-\frac{1}{\theta}}\right)^{\kappa} \left[\frac{c_i f_j}{X_j R_j^{\sigma}}\right]^{\frac{\sigma-\kappa-1}{\sigma-1}} \qquad \text{(F.9)}\end{aligned}$$

where $\vartheta = \varsigma \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \varpi = \varsigma^{-\frac{\kappa}{1-\sigma}} \left(\frac{\kappa}{\kappa+1-\sigma}\right) \left(\frac{\sigma}{\sigma-1}\right)^{-\kappa} \sigma^{\frac{\sigma-\kappa-1}{1-\sigma}}$.

In the third line we used the density of banks exporting from i to j:

$$N_{ij} = \left(1 - G_i(\phi_{ij}^*)\right) N_{ii} \tag{F.10}$$

Corollary

With zero export fixed costs, the threshold coincides with the lower bound of the support of the Pareto distribution, assumed here to be 1, hence:

$$\int_\Omega r_{ij}(\omega)^{1-\sigma}\, d\omega = \varsigma \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma} \left(\sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma-1}{\theta}} N_{ii} \frac{\kappa}{\kappa+1-\sigma} \tag{F.11}$$

Harmonic Average Interest Rate, both Paths and Productivity, second version

Let N_{ij} the mass of firms with $\phi \geq \phi^*$. We can derive the harmonic average interest rate:

$$\begin{aligned}
\mathbb{E}_{\phi,\xi}\left[r_{ij}(\omega)^{-\sigma}\right] &= \int_\Omega r_{ij}(\omega)^{-\sigma}\, d\omega \\
&= \int_0^\infty \dot{\varsigma}\phi^\sigma \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} c_i^{-\sigma} \left(\sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma}{\theta}} dG_{ij}(\phi) \\
&= \dot{\varsigma} \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} c_i^{-\sigma} \left(\sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma}{\theta}} \frac{1}{1-G_i(\phi_{ij}^*)} \int_{\phi_{ij}^*}^\infty \phi^\sigma\, dG_{ij}(\phi) \\
&= \dot{\varsigma} \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} c_i^{-\sigma} \left(\sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma}{\theta}} \frac{N_{ii}}{N_{ij}} \left(\left[\frac{c_i f_j}{X_j R_j^\sigma}\right]^{\frac{1}{\sigma-1}} c_i \left(\sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{-\frac{1}{\theta}}\right)^{\sigma-\kappa} \\
&= \dot{\vartheta} \frac{N_{ii}}{N_{ij}} c_i^{-\kappa} \left(\sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\kappa}{\theta}} \left[\frac{c_i f_j}{X_j R_j^\sigma}\right]^{\frac{\sigma-\kappa}{\sigma-1}}
\end{aligned} \tag{F.12}$$

where $\dot{\vartheta} \equiv \dot{\varsigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \dot{\varpi} = \dot{\varsigma}\left(\frac{\kappa}{\kappa-\sigma}\right)\left(\frac{\sigma}{\sigma-1}\right)^{-\kappa}\left(\frac{\sigma}{\varsigma}\right)^{\frac{\sigma-\kappa}{\sigma-1}}$.
where $\dot{\varpi} \equiv \frac{\kappa}{\kappa-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-\kappa}\left(\frac{\sigma}{\varsigma}\right)^{\frac{\sigma-\kappa}{\sigma-1}}$.

Corollary

With zero export fixed costs, the threshold coincides with the lower bound of the support of the

Pareto distribution, assumed here to be 1, hence:

$$\int_{\Omega} r_{ij}(\omega)^{-\sigma}\ d\omega = \varsigma \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} c_i^{-\sigma} \left(\sum_{p\in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma}{\theta}} N_{ii}\frac{\kappa}{\kappa-\sigma} \tag{F.13}$$

where here $\varsigma \equiv \Gamma\left(\frac{\theta-\sigma}{\theta}\right)$

Harmonic Average Interest Rate, both Paths and Productivity, third version

We derive the harmonic interest rate needed to relate ω interest rate and quantity to aggregate quantity X.

$$
\begin{aligned}
\mathbb{E}_{\phi,\xi}\left[r_{ij}(\omega)^{2-\sigma}\right] &= \int_{\Omega} r_{ij}(\omega)^{2-\sigma}\ d\omega \\
&= \int_0^\infty \int_0^\infty p^{2-\sigma}\,\frac{d\,G_{ij\phi}(p)}{d\,p}\ dp\ dG_{ij}(\phi) \\
&= \int_0^\infty \int_0^\infty p^{2-\sigma}\,\frac{d}{d\,p}\left(1-\exp\left\{-(\phi p)^{\theta}\left(\frac{\sigma}{\sigma-1}\right)^{-\theta} c_i^{-\theta}\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right\}\right) dp\ dG_{ij}(\phi) \\
&= \int_0^\infty \theta\Phi_j \int_0^\infty p^{1+\theta-\sigma}\exp\left\{-p^{\theta}\Phi_j\right\}\ dp\ dG_{ij}(\phi) \\
&= \int_0^\infty \int_0^\infty \left(\frac{x}{\Phi_j}\right)^{\frac{2-\sigma}{\theta}} \exp\{-x\}\ dx\ dG_{ij}(\phi) \\
&= \int_0^\infty \Phi_j^{-\frac{2-\sigma}{\theta}} \int_0^\infty (x)^{\frac{2-\sigma}{\theta}}\exp\{-x\}\ dx\ dG_{ij}(\phi) \\
&= \int_0^\infty \Phi_j^{-\frac{2-\sigma}{\theta}}\,\Gamma\left(\frac{\theta+2-\sigma}{\theta}\right)\ dG_{ij}(\phi) \\
&= \int_0^\infty \ddot{\varsigma}\phi^{\sigma-2}\left(\frac{\sigma}{\sigma-1}\right)^{2-\sigma} c_i^{2-\sigma}\left(\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma-2}{\theta}}\ dG_{ij}(\phi) \\
&= \ddot{\varsigma}\left(\frac{\sigma}{\sigma-1}\right)^{2-\sigma} c_i^{2-\sigma}\left(\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma-2}{\theta}}\ \frac{1}{1-G_i(\phi_{ij}^*)}\int_{\phi_{ij}^*}^\infty \phi^{\sigma-2}\ dG_{ij}(\phi) \\
&= \ddot{\varsigma}\left(\frac{\sigma}{\sigma-1}\right)^{2-\sigma} c_i^{2-\sigma}\left(\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma-2}{\theta}}\ N_{ii}\frac{\kappa}{\kappa+2-\sigma}\left(\phi_{ij}^*\right)^{\sigma-\kappa-2} \\
&= \ddot{\varsigma}\left(\frac{\sigma}{\sigma-1}\right)^{2-\sigma} c_i^{2-\sigma}\left(\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right)^{\frac{\sigma-2}{\theta}}\ N_{ii}\frac{\kappa}{\kappa+2-\sigma} \\
&= \ddot{\varsigma}\left(\frac{\sigma}{\sigma-1}\right)^{2-\sigma} c_i^{2-\sigma} b_{ij}^{\frac{\sigma-2}{\theta}} N_{ii}\frac{\kappa}{\kappa+2-\sigma}
\end{aligned}
\tag{F.14}
$$

where $\ddot{\varsigma}\equiv\Gamma\left(\frac{\theta+2-\sigma}{\theta}\right)$.

F.0.1 Proof of Proposition 8 - Indirect Probability

Proposition 5

The probability of any good traveling through a kl edge, conditional on being sold from origin i to destination j:

$$\begin{aligned}\psi_{kl|ij} &= \frac{b_{ik}a_{kl}b_{lj}}{b_{ij}} \\ &= \left(\frac{\tau_{ij}}{\tau_{ik}e_{kl}\tau_{lj}}\right)^{\theta}\end{aligned} \tag{F.15}$$

Proof: We derive $\psi_{ijp\omega}$, the probability of a bank ω from i selling to j using a given path p:

$$
\begin{aligned}
\psi_{ijp\omega} \equiv & \Pr\left(\underset{s}{\operatorname{argmin}}\, \tau_{ij}(s,\omega) = p \cap \min_{s} \tau_{ij}(s,\omega) \leq \tau\right) \\
= & \Pr\left(\tau_{ij}(p,\omega) \leq \min_{s\neq p} \tau_{ij}(s,\omega)\right) \\
= & 1 - H_{ij\omega}\big(\tau_{ij}(p,\omega)\big) \qquad \text{by lemma F.4} \\
= & \int_0^\infty \left[1 - H_{ij\omega}(\tau)\right] dH_{ijp\omega}(\tau) \\
= & \int_0^\infty \left[1 - H_{ij\omega}(\tau)\right] \frac{d}{d\tau}\left[-\exp\left\{-\left[\frac{\tilde{\tau}_{ij}(p)}{\tau}\right]^{-\theta}\right\}\right] d\tau \qquad \text{by lemma F.3} \\
= & \int_0^\infty \left[1 - H_{ij\omega}(\tau)\right] \tau^{\theta-1}\theta \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \exp\left\{-\left[\frac{\tilde{\tau}_{ij}(p)}{\tau}\right]^{-\theta}\right\} d\tau \\
= & \int_0^\infty \exp\left\{-\tau^\theta \sum_{s\neq p, s\in G} \left[\tilde{\tau}_{ij}(s)\right]^{-\theta}\right\} \tau^{\theta-1}\theta \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \exp\left\{-\tau^\theta \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right\} d\tau \\
= & \int_0^\infty \exp\left\{-\tau^\theta \sum_{p\in G} \left[\tilde{\tau}_{ij}(s)\right]^{-\theta}\right\} \tau^{\theta-1}\theta \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} d\tau \\
= & \int_0^\infty \left[\exp\{\Theta\}\right] \tau^{\theta-1}\theta \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} d\tau \frac{\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}}{\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}} \\
= & \int_0^\infty \left[\exp\{\Theta\}\right] \tau^{\theta-1}\theta \sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta} d\tau \frac{\tilde{\tau}_{ij}(p)^{-\theta}}{\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}} \\
= & \int_0^\infty -\frac{d}{d\tau}\exp\left\{-\tau^\theta \sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right\} d\tau \frac{\tilde{\tau}_{ij}(p)^{-\theta}}{\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}} \\
= & -\exp\left\{-\tau^\theta \sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}\right\}\Bigg|_0^\infty \frac{\tilde{\tau}_{ij}(p)^{-\theta}}{\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}} \\
= & \left[-\frac{1}{\exp(\infty)} + \frac{1}{\exp(0)}\right] \frac{\tilde{\tau}_{ij}(p)^{-\theta}}{\sum_{p\in G}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}} \\
= & \frac{\tilde{\tau}_{ij}(p)^{-\theta}}{\sum_{K=0}^\infty \sum_{p\in G_{ij}(K)}\left[\tilde{\tau}_{ij}(p)\right]^{-\theta}}
\end{aligned}
$$

Note this expression is closely related to ψ_{iln} in equation 5 in Arkolakis et al. (2018), with $\rho = 0$.

All banks export, hence $\psi_{ij} \equiv (1 - G(\phi_{\min})) = 1$. We can show this summing across all routes:

$$\psi_{ij} = \sum_p \psi_{ijp}$$
$$= \sum_{p \in G} \frac{\tilde{\tau}_{ij}(p)^{-\theta}}{\sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}} = 1$$

The probability of going through an edge tk, conditional on origin i and destination j, is:

$$\begin{aligned}
\psi_{kl|ij} =& \sum_{K=0}^{\infty} \sum_{p \in G_{ij}^{kl}(K)} \frac{\tilde{\tau}_{ij}(p)^{-\theta}}{\sum_{K=0}^{\infty} \sum_{p \in G_{ij}(K)} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}} \\
=& \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}^{kl}(K)} \tilde{\tau}_{ij}(p)^{-\theta} \\
=& \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}^{kl}(K)} \prod_{k=1}^{K} e_{k-1,k}(p)^{-\theta} \\
=& \frac{1}{b_{ij}} \left(b_{ik} a_{kl} b_{lj}\right)
\end{aligned} \tag{F.16}$$

where in the last step we isolate the kl step and follow the matrix algebra in Allen and Arkolakis (2019), such that $\sum\limits_{K=0} \sum\limits_{L=0}^{K-1} A^L A A^{K-L-1} = (I - A)^{-1} A (I - A)^{-1}$.

The conditional probability is:

$$\psi_{kl|ij} = \frac{b_{ik} a_{kl} b_{lj}}{b_{ij}} = \left(\frac{\tau_{ij}}{\tau_{ik} e_{kl} \tau_{lj}}\right)^{\theta} \tag{F.17}$$

where the last step was obtained by plugging the expected cost definition in B.10.

F.0.2 Aggregate Interest Rate

Let $G_{ij}(\phi)$ be the Pareto (equilibrium) probability density function of the productivities of banks from country i that sell to country j such that the measure of banks from country i with productivity ϕ is $N_i\, dG_i(\phi)$. Then we can write the aggregate interest rate in j as:

$$R_j = \vartheta \left(\sum_i N_{ii} c_i^{1-\sigma} b_{ij}^{\frac{\sigma-1}{\theta}} \right)^{\frac{1}{1-\sigma}}$$

where $\vartheta = \frac{\kappa}{\kappa+1-\sigma}^{\frac{1}{1-\sigma}} \varsigma^{\frac{1}{1-\sigma}} \left(\frac{\sigma}{\sigma-1}\right)$.

Proof.

$$\begin{aligned}
R_j^{1-\sigma} &= \sum_i N_{ii} \int_\Omega r_{ij}(\omega)^{1-\sigma}\, d\omega \\
&= \sum_i N_{ii} \varsigma \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma} \left(\sum_{p \in G} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} \right)^{\frac{\sigma-1}{\theta}} \frac{\kappa}{\kappa+1-\sigma} \\
&= \frac{\kappa}{\kappa+1-\sigma} \varsigma \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \sum_i N_{ii} c_i^{1-\sigma} b_{ij}^{\frac{\sigma-1}{\theta}} \\
R_j &= \vartheta \left(\sum_i N_{ii} c_i^{1-\sigma} b_{ij}^{\frac{\sigma-1}{\theta}} \right)^{\frac{1}{1-\sigma}}
\end{aligned}$$

where $\vartheta = \frac{\kappa}{\kappa+1-\sigma}^{\frac{1}{1-\sigma}} \varsigma^{\frac{1}{1-\sigma}} \left(\frac{\sigma}{\sigma-1}\right)$.

APPENDIX G

COUNTRY LIST

Table G.1: Country List

	ISO2	Country
1	AT	Austria
2	AU	Australia
3	BE	Belgium
4	BR	Brazil
5	CA	Canada
6	CH	Switzerland
7	CL	Chile
8	DE	Germany
9	DK	Denmark
10	ES	Spain
11	FR	France
12	GB	United Kingdom
13	GR	Greece
14	HK	Hong Kong SAR
15	IE	Ireland
16	IT	Italy
17	JP	Japan
18	KR	South Korea
19	LU	Luxembourg
20	MX	Mexico
21	NL	Netherlands
22	SE	Sweden
23	TW	Chinese Taipei
24	US	United States

CHAPTER 3

RECESSIONS AND RECOVERIES. MULTINATIONAL BANKS IN THE BUSINESS CYCLE

with Qingqing Cao, Raoul Minetti and María Pía Olivero.

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3.1 Introduction

The dynamics of business cycles in advanced and emerging economies have been the object of an intense debate in recent years (Cecchetti et al., 2009, Reinhart and Rogoff, 2014). Concerns have grown about the length of recessions, especially following shocks to the banking sector (Cerra and Saxena, 2008). While in the past a popular view was that deep recessions would be accompanied by quick recoveries (Friedman and Schwartz, 2008), evidence on recent recessionary episodes suggests that banking disruptions trigger deep recessions and persistent output losses. Reinhart and Rogoff (2014) document that in 23 out of 30 episodes of banking disruptions occurred after 1990 the length of the recession from peak to recovery was no lower than 5 years and its depth (output drop, peak to trough) exceeded 5%. Cerra and Saxena (2008) estimate an output impact of 7% of banking crises, with the output drop remaining above 6% at a 10-year horizon. This debate calls for a deeper understanding of the forces that shape the dynamics of business cycles (Claessens et al., 2011).

The structure of a country's financial sector is often considered to be a driver of the dynamics of business cycles (Bordo and Haubrich, 2010, Claessens et al., 2011). This paper studies qualitatively and quantitatively to what extent a key feature of the financial sector, its openness to multinational banks, can affect the depth and duration of recessions, possibly helping explain cross-country differences, and the evolution over time, of business cycle dynamics. In recent decades, following the relaxation of foreign bank entry restrictions, multinational banks have significantly expanded their presence in advanced and emerging countries (Claessens and van Horen, 2014). The international claims of BIS reporting banks rose from $6 trillion in 1990 to $37 trillion in 2007, over 70% of world GDP (BIS, 2008). In Latin America and in Central and Eastern Europe, large European and U.S. banks have broadened their networks of affiliates, reaching a credit market share of over 25% in several countries, and above 50% in some countries (Allen et al., 2013). And the expansion of multinational banks is accelerating in transition countries, such as China and Russia.

How does the expansion of multinational banks influence the dynamics of business cycles of host countries? Do multinational banks attenuate or exacerbate the depth of recessions following banking disruptions and real shocks? Do they have the same impact on contractions and recoveries? To address these questions, we first present motivational evidence on the impact of multinational banks on business cycle dynamics. Using information from a broad panel of economies over the last three decades, we carry out a case study analysis of recessionary episodes as well as a regression analysis based on prior studies on business cycle dynamics. The evidence points to a different impact of multinational banks on the severity of contractions and recoveries in a country. We then embed multinational banks into a dynamic stochastic general equilibrium model with two countries (the host or domestic country and the foreign country). In both countries firms can borrow from local banks, which operate within the country's borders, and multinational banks, which have parent offices in the foreign country and affiliates in the host country.

In the model, we characterize multinational banks by taking a leaf from the banking literature. First, we posit that multinational banks have internal capital markets which allow them to transfer liquidity, subject to costs, as well as (partly) consolidated balance sheets between parents and affiliates. The literature has documented multinational banks' reliance on internal capital markets, which allow them to transfer liquidity across borders in a timely manner without the need to resort to costly local deposits (Cetorelli and Goldberg, 2012). Second, we posit that multinational banks experience disadvantages in allocating their liquidity to firms in the host country due to lower ability than local banks at extracting value from (monitoring, managing and liquidating) firms' collateralizable assets. Several studies document multinational banks' limited experience and information about assets and activities of local borrowers (Giannetti and Ongena, 2012, Mian, 2006), especially small and medium-sized informationally opaque firms or firms with limited international engagement.[1]

We analyze and quantify the impact of multinational banks on the depth and duration of recessions by calibrating parameter values and shock processes of the host country to data on Poland, a country featuring a significant presence of multinational banks. We ask our model: how do multinational banks shape the dynamics of business cycles in the host country? The analysis delivers a nuanced answer: depending on the nature of the shock, multinational banks can act as a stabilizer or an amplifier in the short-run, contractionary phase. Most interestingly, their presence can induce a trade-off between the short-run response of the economy (depth of the contraction) and its medium-run response (speed of the recovery). Consider first a negative shock to the capitalization of the domestic banking sector. Multinational banks can supplant the liquidity shortage in the host country through cross-border transfers to their affiliates via internal capital markets, playing a stabilizing role in the contractionary phase. However, over the medium run, this entails a progressive reallocation of local firms' borrowing and collateral assets from domestic banks, more expert about local firms' assets, to the less expert multinational banks. This reallocation in the credit market slows down the recovery of collateral asset values, credit, and output. Consider next a domestic TFP shock, which reduces multinational banks' return from lending to firms in the host country. On impact multinational banks amplify the shock by repatriating liquidity to their parents in the foreign country. However, in the medium run the reallocation of borrowing in the credit market (towards local banks this time) makes the economy rebound more quickly. Thus, depending on the nature of the shock, multinational banks can act as a stabilizer or an amplifier in the short-run, contractionary phase but their presence can be the source of a trade-off between the short-run response of the economy (depth of a recession) and its medium-run response (speed of the recovery).

[1] Foreign banks can have limited knowledge of local markets, assets, and legal procedures, especially when assets are inherently local and non-tradable, and when markets are informationally opaque (Dell'Ariccia et al., 1999, Mian, 2006). Domestic banks may possess private information not available to multinational banks. Further, foreign banks often have a shorter history in lending to local firms than domestic banks and, hence, may have to resort to expensive local experts (Giannetti and Ongena, 2012).

When we assess the effects quantitatively, we obtain that in the simulated economies the presence of multinational banks reduces the average depth of recessions (output drop, peak to trough) by up to 5% but it lengthens their average duration (years from peak to recovery) by about 10%. Aggregating these effects through the Reinhart and Rogoff (2014) composite measure of severity of recessions, multinational banks raise the severity index by about 8%.

In the last part of the paper, we examine whether banking regulations and macroprudential policies ameliorate the trade-offs induced by multinational banks. Regulations that increase the costs of multinational banks' transfers accelerate the recovery after a banking shock but reduce the stabilizing role of multinational banks in the contractionary phase. By contrast, higher consolidation of multinational banks' balance sheets (due, e.g., to regulations that incentivize entry through branches rather than subsidiaries) accelerates the recovery after a banking shock without diluting the short-run stabilizing role of multinational banks. The analysis also reveals that macroprudential policies that set countercyclical loan-to-value ratios ameliorate the trade-offs if they target multinational banks' loans.

The paper unfolds as follows. The next section relates the analysis to prior literature. Section 3 provides motivating evidence. Section 4 lays out the model and solves for agents' decisions. Section 5 presents the calibration and the simulation results. Section 6 studies regulations and policies. Section 7 examines alternative model specifications and robustness. Section 8 concludes. The online Appendix contains additional analysis.

The entire paper can be found on the publisher's website, including the online appendix and the CRediT authorship contribution statement.

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