SECONDARY MATHEMATICS TEACHER INTERNS' LEARNING THROUGH TEACHING: A CASE STUDY

By

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ABSTRACT

Many mathematics teachers, at one time or another, have exclaimed that "I really didn't understand mathematics, until I had to teach it." Secondary mathematics teachers' knowledge, both subject matter and pedagogical content, plays an important role in their teaching of the subject. Teacher education programs are, therefore, designed to give preservice teachers satisfactory preparation in both domains. However, a teachers' knowledge does not purely remain in the mind, it is also developed by the practices learned as a member of a community. Hence, practical experiences compliment academic preparation, which is why most teacher preparation programs culminate with the student teaching period. Ettiene Wenger once argued that learning is the same thing as one's identity development in a community. So, it is in this teaching internship period that mathematics teacher interns develop their professional identity through the interrelationship of their *Aspects of Self-in-Community* (i.e., their role as a teacher in a classroom) and their *Aspects of Self-in-Mind* (i.e., their pedagogical and subject matter knowledge). The purpose of this project is to better understand the role that teaching practice has on secondary mathematics teacher interns' perceptions of their evolving sense of knowing mathematics. In this dissertation research project, I relied on case study methodology to engage with each intern in an ongoing and open dialogue (through the use of digital reflection journals, interviews, and observations) as they worked to created reflective narratives throughout one school year in order to explore their notions of what they think means to understand a mathematical concept and to understand the ways in which they think that the work of teaching influenced their perception of their personal mathematical content knowledge. I analyzed all data using teacher noticing and teacher's mathematical knowledge for teaching frameworks. From each intern's narrative I developed claims about their mathematical knowledge by interpreting their experiences through the lens of Mathematical Knowledge for Teaching. From their experiences, I found that their curricular context, influence of their mentor teachers, and collective learning by teaching were the most influential aspects of their experiences.

Keywords: learning through teaching, secondary mathematics teaching, novice mathematics teacher learning, student teacher experiences

This work is dedicated to my parents, my sister, and steadfast wife without whose constant support this dissertation might not have been completed

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TABLE OF CONTENTS

CHAPTER 1: INTRODUCTION

A few years ago, while I was helping a colleague analyze interview data, I heard an experienced elementary school teacher remark: "I never really understood mathematics until I had to teach it." This phenomenon, I believe, is a common sentiment felt by many teachers, and I do not believe that it is limited to only elementary school teachers. When I initially imagined the trajectory for investigating this phenomenon, I thought about this statement as a teacher's reflection on their own readiness to teach mathematics. This initial line of inquiry led me to research related to mathematics teacher preparation, mathematics teacher identity, and teacher reflection. While the focus of my research has changed slightly, these domains still play a role in how I designed and carried out investigation into the aforementioned phenomenon.

While I now see this phenomenon as representing the intersection of and interaction between two domains, namely teacher mathematical knowledge and teacher practice, this study is not an attempt to identify the mechanisms for transforming preservice mathematics teachers' (PSMTs) subject matter knowledge into pedagogical knowledge by implementing established instruments for doing so (e.g., Ball & Hill, 2008). At the heart of this project, I still believe this phenomenon is an instantiation of how teachers view themselves. A novice teachers' identity varies as they transition out of the university setting into classroom settings where their roles also evolve from being a student to becoming an in-practice teacher. This stage of transition is unique because in one sense they are expected to know something, having completed, or nearly completed, an undergraduate mathematics program. In another sense, they are still novices and have very little practical teaching experience. Further, through practical experiences they come to interact with mathematics in a qualitatively different way than they did as a student. Mathematics teacher identity, therefore, played a key influence on the direction of this project;

specifically, this project explores novice mathematics teachers' identities by bringing together their perception of their growing mathematical knowledge because of their teaching experiences. It is unique that such research into PSMTs' knowledge takes into account their own perspectives of their experiences.

Before moving on to discuss the rationale and purpose for this project, one primary motivation for designing this project around the early professional experiences of secondary mathematics teacher interns was my desire to further my own professional development as a mathematics teacher educator. Personally, when I was in the first few years of my teaching career, I struggled, as many young teachers do, to teach in a way that I had envisioned as being ambitious. Some of my struggle, I think could be attributed to not having mentors around me that viewed teaching in quite the same way, but also because my perception of what it meant to know mathematics was in transition. I held the view that my students should learn mathematics in much the same way as I had, and my teaching reflected this (i.e., focused around demonstrating procedures and providing ample opportunities to practice). Even though I taught in a traditional way, I found myself coming away with a sense of knowing content more thoroughly, particularly in courses such as calculus that I struggled in as an undergraduate student. This project, therefore, was intended to provide the teacher interns I worked with opportunities to reflect on their initial teaching experiences and for me to further develop my own practice as a teacher educator.

Research Problem

In this section, I will broadly discuss how scholars have conceptualized and investigated the development of PSMTs' mathematical knowledge for teaching (MKT). Specifically, I add to these discussions by answering the question of what experiences contribute to the development of PSMTs' MKT by bringing in their perspective into the conversation. As a teacher, developing your professional identity is critical in learning how to participate and grow in becoming an effective teacher. Sachs (2005, p. 15) viewed a teacher's identity as a personal framework by which one can construct ideas of 'how to be', 'how to act', and 'how to understand' in their work. Additionally, Wenger (1998) viewed learning, in general, as developing modes of participating with communities. By reporting the experiences of novice mathematics teachers' learning to participate in educational spaces we can gain a unique perspective on how MKT develops by honing-in on how the work of teaching influences their perceptions of their own mathematics understanding.

Mathematics Teacher Knowledge

Most would agree that teachers need to possess, at minimum, sufficient content knowledge of the subject to be taught (Ball et al., 2008; Hill et al., 2005; Krzywacki, 2009; Shulman, 1986, 1987). In addition to their subject matter knowledge, teachers become more effective when they can make pedagogical decisions concerning the content they teach, when they are prepared to navigate the complexities of diverse learners, and when they attend to a given curriculum with a purposeful approach, amongst other responsibilities (Ball et al., 2008; Shulman, 1986, 1987). Researchers have argued that to prepare PSMTs for entrance into the teaching profession, they must develop an appropriate level of both mathematical content and pedagogical knowledge (Ball et al., 2008; Schoenfeld & Kilpatrick, 2008).

One of the more important ideas in relation to teacher knowledge to emerge in the last few decades is the idea that the knowledge required for teaching is a specialized content knowledge. Prior to the late 1980s, research into what teachers need to know for teaching well focused mainly on general pedagogical knowledge. Shulman (1986, 1987) argued that to teach well, teachers need pedagogical content knowledge. This type of knowledge goes beyond general know-how of subject matter and describes a content knowledge specifically for the work of teaching. Shulman (1986, 1987) named this profession-specific knowledge as *Pedagogical Content Knowledge* and described it as the kind of knowledge a teacher develops that uniquely blends content understanding with pedagogical considerations.

Ball et al. (2008) applied Shulman's description of pedagogical content knowledge to mathematics, a type of mathematical knowledge specific to the profession of teaching and different from the mathematical knowledge needed for other professions. To these researchers, in whatever way a teacher interacts with mathematics (lesson planning, grading, answering questions, etc.), it represents a unique mathematical problem. They identified the knowledge gained from these unique activities as Mathematical Knowledge for Teaching (MKT). The discussion around these types of mathematics problems rarely comes up in general undergraduate mathematics courses, so many preservice mathematics teachers believe that their readiness to teach mathematics correlates with their performance in their courses (Krzywacki, 2009). In Ball et al.'s view, MKT is comprised of two domains, namely, Subject Matter Knowledge and Pedagogical Content Knowledge, both of which are made up with three sub-domains. A natural question stemming from these frameworks for teacher knowledge is "How do novice mathematics teachers develop different kinds of mathematical knowledge?"

How do teachers grow in this form of professional mathematical knowledge? In more recent years, researchers have investigated how teachers learn, and one strand of research focuses on learning through teaching (Leikin, 2010; Leikin & Zazkis, 2010). In other words, teachers continue to learn while "on the job." In addition to learning pedagogical skills, curricula design, etc. while on the job, teachers continue to grow in their content knowledge because of their pedagogical work. Outside of teacher education, many psychologists have argued that a good way for children to learn a subject is for them to teach it to their peers. One would think that teachers who have already successfully demonstrated their capabilities as students (by way of successfully graduating secondary school and passing undergraduate coursework at minimum) would also continue to learn because of the act of teaching, but this knowledge would look different from the knowledge displayed by children in the psychology studies. Therefore, it became my purpose for this study to design opportunities for interns who would participate to provide personal accounts of their changing mathematical knowledge.

In addition to the kind of mathematical knowledge teachers may possess, the quality of their mathematical knowledge is decidedly important. Boaler (1998) and Skemp (1976) argued that the way someone knows mathematics is critical to their experience with the field. I theorize that the way novice teachers learned mathematics as students has implications for how they would later be inclined to explain concepts. They may come to the realization, after having assumed full teaching responsibilities, that the way they learned mathematics as students is different from what is needed as teachers (Krzywacki, 2009). I theorize that, in addition to pedagogical concerns they may soon encounter, they are also faced with learning mathematics in a qualitatively different way than they are accustomed to (Ball et al., 2008; Boaler, 1998; Skemp, 1976) and for a different purpose. Instead of learning mathematics for success as students, they are learning mathematics with the express purpose of explaining it to others.

Mathematics Teacher Preparation

Teacher preparation, and specifically mathematics teacher preparation, has been discussed at length over the last few decades and views on mathematics teacher preparation best practices have evolved. Many discussions have centered around: the breadth of content, the quality and focus of content, the types of preservice experiences to design methods courses around, and the length and requirements for student teaching experiences, amongst other things. Additionally, policy makers, researchers, and in-service teachers have agreed that to improve students' mathematical performance we need excellent teachers. The conversations about excellence have made use of phrases such as: *teaching mathematics with high expectations* (National Council of Teachers of Mathematics [NCTM], 2016), *well-prepared beginning teachers of mathematics* (AMTE, 2017), or *ambitious teaching practices* (Lampert et al., 2011). Surrounding these contributions about what constitutes an excellent mathematics teacher, many have spoken into how to adequately prepare future mathematics teachers.

Beginning any new career can be daunting, and this is certainly the case with teaching. Some people who have excelled in one craft or another have misjudged how difficult teaching can be and assumed that simply because they have the appropriate know-how, they can also teach, often citing the adage "those who can, do; those who can't, teach." People who hold this sentiment either may take for granted the work that teachers do or are ignorant of the specific professional knowledge required to teach (Shulman, 1986). In addition to navigating the interpersonal, cultural, and professional expectations placed upon teachers, becoming a teacher requires a person to think differently than he or she did as a student. The academic skills one uses to succeed throughout secondary school and college may not directly transfer to teaching students of one's own. Students can become adept at learning content, for example, merely for the purpose of progressing through their coursework. Making a certain grade often becomes the primary focus rather than gaining deeper and connected understanding of mathematical ideas (Ball et al., 2008; Boaler, 1998; Skemp, 1976). University teacher education programs have developed to a point where preservice teachers are given the opportunity to see many aspects of the teaching profession prior to taking lead of their own classrooms. While many studies have been designed around exploring specific aspects of the teacher preparation experience, research is needed to explore how preservice teachers view their teaching preparation experiences and how specifically these experiences attributed to their growth in content knowledge.

I view this project being placed at the intersection of mathematics teacher identity, mathematics teacher knowledge, and mathematics teaching practice. To better understand the phenomenon mentioned earlier, we need research that presents the stories of novice mathematics teachers' experiences learning mathematics for teaching.

Research Purpose

The broad goal for this project is to understand how PSMTs' early teaching experiences - specifically their student teaching experience – shapes their perceptions of their knowledge for teaching mathematics. Using case study methods, I designed this project to explore the stories that the interns told me about their own mathematics understanding throughout their internship by interpreting their reflections with the *MKT* framework. To these ends, the research questions that guided this project are:

1. *What aspects of mathematical knowledge for teaching are reflected in the narratives that PSMTs provide about their learning during student teaching?*
2. *What teaching experiences do the interns emphasize as being critical in helping them understand differently the content they teach?*

In this case study, I hope to contribute to the overall conversation about secondary teacher preparation by providing an in-depth look into the role that practical teaching experiences had on novice teachers' descriptions of their mathematics knowledge. Further, I hope that the insights gained from my study adds to our understanding of MKT by helping researchers and teacher educators understand the effect that field experiences have on particular aspects of novice teachers' Pedagogical Content Knowledge.

CHAPTER 2: LITERATURE REVIEW

The original idea for this study came about when I was helping a colleague analyze interview transcripts of elementary mathematics teachers. During one interview a teacher exclaimed, “I really didn’t understand mathematics until I had to teach it.” It is my belief that this phenomenon is not unique to elementary school teachers. I think that many novice secondary mathematics teachers also come away with a greater sense of understanding mathematical content when they teach. So, I set out to better understand secondary math learning through teaching and mathematics teacher identity provided insight into how they develop professional knowledge.

The teaching internship is a period of transition for novice teachers which is marked by a change of their identities (that of being a student to that of being a teacher) (van Zoest & Bohl, 2005a). Not only is this transitional period distinguished by gaining practical classroom experience, but their transition to becoming a teacher is also marked by a transition in their perception of what it means to know mathematics. Teacher professional identity is a construct that many researchers have used as a theoretical lens by which to understand prospective secondary mathematics teachers’ field experiences, the learning of mathematical topics, and the participation within communities (Losano & Cyrino, 2017; Sachs, 2005). Recent research into mathematics teacher’s professional identity has described teacher professional identity as “standing at the core of the teaching profession. It provides a framework for teachers to construct their own ideas of ‘how to be’, ‘how to act’, and ‘how to understand’ their work” (Sachs, 2005). For a novice mathematics teacher, therefore, learning who they are professionally influences how they think what it means to know mathematics for teaching. Looking at how novice mathematics teachers develop a sense of coming to know mathematics for teaching by utilizing their professional identity builds off this prior research.

Using mathematics teacher identity to investigate how they come to know mathematics identity provides insight into how MKT is developed not previously seen before. Like students' learning by teaching experiences, MKT develops through personal interactions. It is argued that a teacher develops MKT when they think about how to communicate key developmental understandings to others and reflect upon students' reception of these understandings (Silverman & Thompson, 2008). Further, reflecting upon how they understand mathematics, in terms of their professional mathematics identity can give us additional insights into the role that teaching practice has on PSMT's MKT development.

To provide a structure for and to situate this study within larger conversations, I made use of Van Zoest and Bohl's (2005) *Mathematics Teacher Identity* (MTI) framework. In the next chapter, I will go into more detail about this framework, but for now I will highlight a few features of the MTI framework that structurally support this literature review. Van Zoest and Bohl (2005) conceptualized a teacher's identity in relation to the communities they are members of as well as their own perceptions of themselves. They described one aspect of a teacher's identity through the communities they practiced in, a teacher's *Aspect of Self-in-Community*. The second aspect of a teacher's identity was described in terms of the knowledge, perceptions, and beliefs a teacher possessed, collectively termed *Aspect of Self-in-Mind.* These aspects-of-self, Van Zoest and Bohl (2005) argued, are not disjoint, rather they are overlapping and interrelated. In order to explore the relationship between novice teachers' *Aspects of Self-in-Community* and *Aspects of Self-in-*Mind, this study makes use of teacher reflection (Akinbode, 2013; Beauchamp, 2015; Caniglia et al., 2017; Dewey, 1938; Spalding & Wilson, 2002; Taggart &

Wilson, 2005) and noticing (van Es, 2011; van Es et al., 2017; van Es & Sherin, 2004) to gain insight into their personal experiences. This literature review is organized according to these two parts of the MTI framework, which I will flesh out in more detail in the next section.

With the above aspects of the MTI framework in mind, I have organized this literature review into three sections. First, I broadly discuss aspects of teacher knowledge that are applicable to this study. Since I am mainly interested in dimensions of content knowledge (as opposed to general pedagogical knowledge), I primarily focus on the work of Debora Ball and colleagues (2008), specifically their *Mathematics Knowledge for Teaching* framework. Second, I will explore the work of teaching by elaborating on the idea of *communities of practice*, drawing heavily from Wenger's (1998) book of the same name. Finally, I discuss aspects of learning through teaching, drawing from the work of Leiken and others (Leikin, 2005, 2010; Leikin & Zazkis, 2010), in an attempt to establish a relationship between a teacher's knowledge from community and their content knowledge.

As stated in the introductory chapter, I designed and carried out this case study with three secondary mathematics teacher interns with the goal of understanding how their teaching experiences influenced their sense of understanding the content. The research questions that guided the design and execution of this study are:

1. *What aspects of mathematical knowledge for teaching are reflected in the narratives that PSMTs provide about their learning during student teaching?*
2. *What teaching experiences do the interns emphasize as being critical in helping them understand differently the content they teach?*

In this study, I investigated secondary mathematics teacher interns' developing sense of what it means to understand a concept in relation to their teaching work. Specifically, I wanted to better understand the phenomenon that many novice teachers claim, “I really didn’t understand mathematics, until I had to teach it” through their own reflections on the relationship between their teaching experiences and its resulting influence on their content knowledge. When I initially thought about this phenomenon in terms of the broader conversations that it could speak into, I considered the phrase “I really didn’t understand mathematics” as reflection on one’s own mathematical understanding. Naturally, this brought me to research conversations concerning different kinds of teacher knowledge. When I thought about the phrase “until I had to teach it,” this led me to think about the work of teaching, teaching practice, and learning to teach in specific communities. This research project contributes to the broader conversation around teachers’ learning through teaching by considering the impact that their contexts and the practices that they learned in those contexts had on their own perception of understanding the content which they taught.

Aspects of Self-in-Mind

Van Zoest and Bohl (2005) conceived of a mathematics teacher’s aspect of self-in-brain as consisting of one’s knowledge, beliefs, commitments, and intentions. Teacher knowledge has been widely researched and developed over the last several decades, and in this section I will situate Van Zoest and Bohl’s (2005) work within the greater conversation of teacher knowledge. Leikin (2006) asserted that there were three dimensions to a teacher’s knowledge: the kinds of teachers’ knowledge, the sources of teachers’ knowledge, and the conditions and forms of knowledge. The first dimension, discussed in this section, draws from Shulman’s (1986, 1987) classification of teacher knowledge into subject matter knowledge, pedagogical knowledge, and curricular knowledge. In the context of this study, I will also add in the work of Ball et al. (2008) as they expand Shulman’s work and apply it to the kind of knowledge needed to teach mathematics. The second dimension, sources of teachers' knowledge, is based on the work of Kennedy (2002) and will be discussed in the next section. This aspect of teacher knowledge identifies the sources of teachers' knowledge as systematic knowledge, craft knowledge, and prescriptive knowledge. Systematic knowledge is knowledge gained from research and is taught in school settings. Craft knowledge is knowledge that is learned through experience in the classroom and interactions with students (i.e., 'on the job' knowledge). Prescriptive knowledge is knowledge that is learned from institutional policies and is manifested in tests, accountability systems, and texts. For this project, I focus primarily on secondary mathematics teacher interns' craft and prescriptive knowledge. The final dimension to a teacher's knowledge are the conditions and forms of knowledge, drawn mainly from Atkinson and Claxton (2000). This dimension distinguishes between intuitive knowledge and formal knowledge, or those teacher actions which cannot be planned in advance verses those that can be planned in advance.

Van Zoest and Bohl (2005) held the belief that learning was synonymous with developing one's identity and one part of a mathematics teacher's professional identity is based on their professional knowledge. For this dimension of a teacher's identity, they drew heavily on Shulman's (1986, 1987) framework for teacher knowledge and organized his seven categories of teacher knowledge into three domains: content and curriculum, pedagogy, and professional participation. For my study, because I wanted to study the phenomenon "I really didn't understand X, until I had to teach it," I need to unpack the idea of 'understanding X,' and this is in line with other researchers who have also investigated mathematics teachers' professional knowledge, particularly Shulman (1986, 1987) and Ball et al. (2008).

Kinds of Teacher Knowledge

Before one can teach an idea to someone else, they must first understand it well enough themselves. But is knowing content well enough to teach students and well enough to earn a degree the same type of knowledge? This is one question that Shulman (1986, 1987) expounded upon. For him, there was a qualitative difference between general knowledge of a subject and, what he calls, pedagogical content knowledge. Teacher knowledge, Shulman (1987, p. 8) argued, can be organized into seven categories: general pedagogical knowledge; knowledge of learners; knowledge of educational contexts; knowledge of educational ends, purposes, and values; content knowledge; curriculum knowledge; and pedagogical content knowledge. It was pedagogical content knowledge (PCK), specifically, that Shulman argued was particular to the teaching profession. In his view, pedagogical content knowledge was that "special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding."

Van Zoest and Bohl (2005) recognized that one's professional identity is tied, in part, to one's "Self-in-Mind." Specifically, part of a mathematics teacher's professional identity is tied to their knowledge and beliefs, commitments, and intentions about mathematics. To capture this aspect of identity, they leveraged Shulman's (1986, 1987) view of professional teacher knowledge and organized his categories into three domains, mainly: content/curriculum, pedagogy, and professional participation. Further, the MTI framework, I argue, would fit well with Ball et al.'s (2008) conception of *Mathematical Knowledge for Teaching (MKT)* because they both expounded upon Shulman's idea of PCK to further develop the particular kinds of knowledge needed for teaching mathematics. In the next section, I describe aspects of Ball et al.'s (2008) MKT framework and highlight the dimensions of mathematics teacher knowledge important to my study. I purposely focus on the influence that teaching practice has on novice mathematics teachers' perceptions of their own content knowledge using aspects of Ball et al.'s (2008) work to describe their experience.

Mathematics Knowledge for Teaching

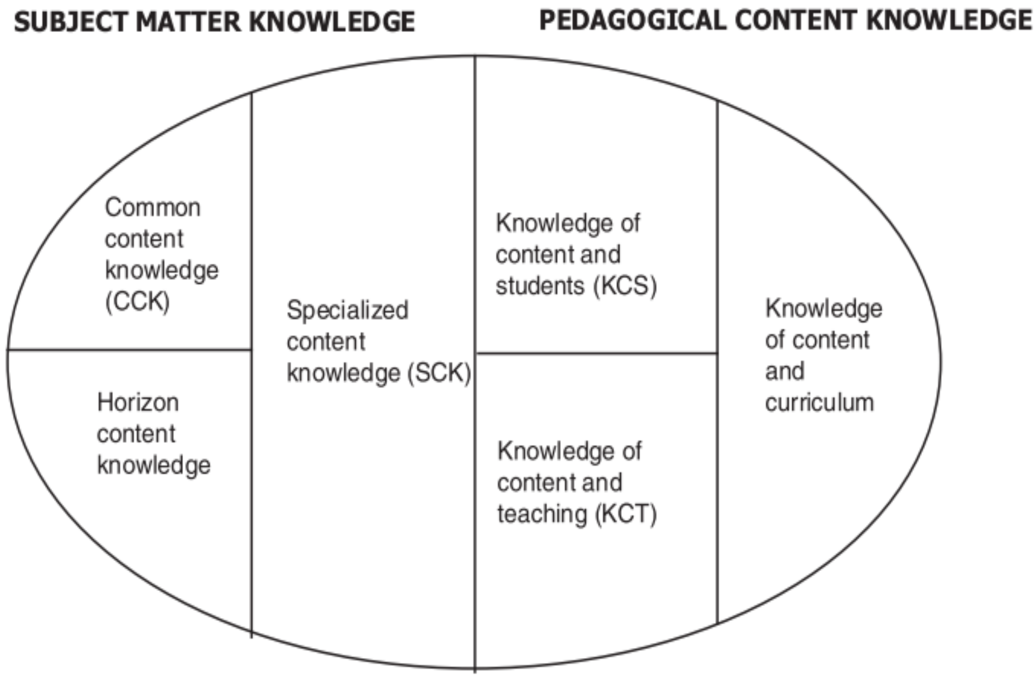
A mathematics teacher's knowledge, according to Ball et al. (2008), consists of subject matter knowledge and pedagogical content knowledge. In this section, I will explain how teaching ultimately requires a specific form of content knowledge, one that is able to be applied to everyday teaching tasks. Subject matter knowledge (SMK) is mathematical knowledge that is not mixed with pedagogical or student knowledge, and Ball et al. (2008) further delineated it into common content knowledge, horizon content knowledge, and specialized content knowledge. They reflected Shulman's (1986, 1987) description of PCK as a blend of content knowledge and pedagogical knowledge, but were more specific about their conception of PCK, organizing it into knowledge of content and curriculum (KCC), knowledge of content and students (KCS), and knowledge of content and teaching (KCT). All their domains of MKT are summarized in Figure 2-2 below. First, I will describe in further detail the different kinds of subject matter knowledge for mathematics; finally, I will discuss Ball et al.'s (2008) description of PCK.

Subject Matter Knowledge

In terms of mathematics, common content knowledge (CCK) is the mathematics that one may learn, in general, not for any particular profession (Ball et al., 2008; Shulman, 1986, 1987). This is the knowledge one would learn as a student or from a book. In relation to teaching, it is understood that this form of knowledge is the bare minimum that someone needs to teach. Without basic understanding of the topic, how would you be ready to teach it to someone else? Over time, as one learns more mathematics, the learner is then able to connect ideas across more difficult content. This ability to describe the development of mathematical ideas in terms of how they lay the foundation for other ideas is described as horizon content knowledge.

Figure 2 - 1

Domains of Mathematical Knowledge for Teaching Framework



General mathematics understanding, in terms of understanding procedures and concepts and the ability to describe the conceptual development of other ideas, is not specific to any profession. These parts of mathematical knowledge are necessary for a teacher to understand, but not sufficient on their own. Another way to understand mathematics is knowing it for use in a specific field. An engineer may need to use mathematics in specific ways to their profession, a statistician may employ other techniques specific to their profession, and teachers need to understand mathematics in ways specific to the teaching profession. This professional knowledge, each unique to specific professions, has been termed specialized content knowledge (SCK) (Ball et al., 2008), and MKT is the specialized content knowledge for teaching mathematics. Additionally, this form of mathematical knowledge would not be used outside of the teaching profession and is directly related to the work of teaching. Skills that characterize MKT would be knowing how to present mathematical ideas, responding to students' "why" questions, modifying tasks to be either easier or harder, amongst many other ideas. For example, being able to pose questions that may already have a set answer as a way to elicit someone else's viewpoint is not usually done in other professions outside of teaching. In summary, subject matter knowledge considers both general mathematics understanding, and the baseline knowledge and skills required for the teaching profession.

Pedagogical Content Knowledge

Shulman (1986, 1987) argued that knowledge for teaching needs to take into account content-specific ideas and be able to apply them to the everyday tasks of teaching. Ball et al. (2008) emphasized this as well when they developed their aspects of PCK within their MKT framework. As I mentioned above, in this framework, PCK consists of knowledge of content and curriculum (KCC), knowledge of content and student (KCS), and knowledge of content and teaching (KCT). Because both Van Zoest and Bohl (2005) and Ball et al. (2008) developed portions of their frameworks from Shulman's (1986, 1987) work, there are similarities between their work. In my discussion of Ball et al.'s (2008) aspects of PCK, I will bring in the corresponding portions of Van Zoest and Bohl's (2005) MTI framework. Although Ball et al. (2008) used similar categories as Shulman (1987), their conception of PCK is broader and often includes aspects of teacher knowledge that Shulman identified as distinct.

Knowledge of Content and Curriculum. The first dimension of Ball et al.'s (2008) conceptualization of PCK is knowledge of content and curriculum (KCC). Similarly, Van Zoest and Bohl's (2005) organized this aspect of Shulman's (1986, 1987) understanding of teacher knowledge into the content/curriculum portion of their *Aspects of Self-in-Mind*. KCC takes into account a teacher's knowledge of how mathematical concepts develop across courses, how different curricula may present a topic, and how to interact with the materials given to them. Included in this aspect of teacher knowledge would be skills such as: appraising and adapting the mathematical content of textbooks and connecting a topic being taught to topics from prior or future years. Curricular knowledge is similar to horizon knowledge in that with this kind of knowledge, teachers understand how content relates to other ideas both *vertically* and *laterally*. (Shulman, 1987, p. 10). By vertically, Shulman meant that a teacher is able to relate current curricular knowledge to other ideas within the same content area (i.e., how current ideas draw upon earlier mathematical ideas and also lay the foundation for future concepts). Since vertical curricular knowledge is concerned with relating ideas within the same content area, then lateral curricular knowledge is concerned with how content is related to other ideas in other subject areas.

Knowledge of Content and Students. In this aspect of PCK, a teacher considers their knowledge of mathematics content with respect to knowledge of their students. Ball et al. (2008, p. 401) compare this kind of knowledge to a teacher's subject matter knowledge by explaining that "recognizing a wrong answer is CCK…sizing up the nature of an error is SCK…and familiarity with common errors and deciding which of several errors students are likely to make are examples of KCS." Included in this aspect of teacher knowledge would be skills such as: responding to students' "why" questions, developing appropriate tasks in light of the current abilities of students, evaluating the plausibility of students' claims, and anticipating common conceptions and misconceptions students might hold about an idea. Van Zoest and Bohl (2005,

p. 335) included this aspect of a teacher's identity in their pedagogy domain and described it as "knowledge of one's own students and their preferences and their abilities."

Knowledge of Content and Teaching. In Ball et al.'s (2008, p. 401) PCK model, knowledge of content and teaching is a specific knowledge that makes use of both knowledge of teaching and knowledge of mathematics. In their view, tasks that demonstrate that this kind of PCK is being employed are choosing a sequence of tasks for students to work through, choosing specific activity structures to encourage deeper exploration of content, evaluating the usefulness of particular representations used to teach a specific idea, etc. KCT encompasses teacher knowledge about mathematics along with knowledge about how to teach it. For Van Zoest and Bohl (2005), KCT was essentially their pedagogy domain (although they also included KCS and PCK in this domain as we saw above).

Professional Participation Domain

In addition to the elements of PCK discussed above, Van Zoest and Bohl (2005) also included a third domain which reflected more than the content-specific aspects of a teacher's knowledge. Using Shulman's (1986, 1988) teacher knowledge framework, they grouped together knowledge of school practices, the goals and ends of education, and the influence of educational contexts, etc. My study, for the most part, was not designed to uncover the relationship between the teacher intern's teaching experience and this element of teacher knowledge. As we will see later on, however, learning to participate within their specific contexts was an important part of the interns' experience.

Teacher Knowledge vs Teacher Beliefs, Commitments, and Intentions

Generally, knowledge is referred to as what is gained from learning and, in terms of the MTI framework, is considered by Van Zoest and Bohl (2005) as generally accepted without debate and as the purely ‘in the brain’ portion of *Aspect of Self-in-Mind*. This simple definition, however, can be problematic because one person’s knowledge can be another person’s belief or non-belief. Van Zoest and Bohl (2005) make this distinction by further dividing up their domains of teacher knowledge into knowledge and beliefs, commitments, and intentions (represented in Figure 2-1 within the *Aspects of Self-in-Mind* in the framework). They define knowledge to be those ideas that are universally socially accepted, or nearly so, and not open to public debate. Further, this conception of knowledge has typically been the ‘constructs’ that cognitive psychologists study. Finally, this conception of knowledge is usually what teachers assess, as it is most easily identifiable.

A teacher’s beliefs, commitments, and intentions, Van Zoest and Bohl (2005) outlined, are those ideas that are not universally held and are open to debate. One’s beliefs may include personal values and are often the justification, consciously or unconsciously, relied upon for acting in a particular way. Additionally, one’s commitments and/or intentions are the desire one has to act or not act in response to a particular situation and the reasons for doing so. Therefore, this is an important framework I will use to analyze secondary mathematics teacher interns’ developing sense of professional identity, as it plays out with respect to their content knowledge.

The Quality of Mathematical Understandings

Closely related to kinds of mathematical knowledge is the quality of that knowledge. The quality of a particular mathematical understanding has been referred to in different ways. Usually in such discussions, mathematical knowledge has been characterized according to a duality of conceptual versus procedural knowledge. Skemp (1976) described the quality of mathematical knowledge in terms of *instrumental* versus *relational knowledge.* Similarly, Boaler (1998) used terms such as *open* and *closed* mathematics. What is common between both of these authors is that they do not discuss knowing the same mathematics in different ways; rather, they argue that they are, in fact, knowing a different kind of mathematics.

The way that a mathematics teacher understands the subject matter they teach makes a difference in how readily they are able to apply their mathematical knowledge to different teaching tasks. Preservice mathematics teachers are likely to have developed proficiency in learning mathematics in an instrumental way throughout their primary and undergraduate schooling (Boaler, 1998; Skemp, 1976). Many times, students become proficient at memorizing rules or procedures, but lack a fuller understanding of the subject. Skemp (1976, p. 2) described instrumental understanding as “rules without reason.” Further, he went on to say that students were not the only ones to understand mathematics in this way. Teachers, likely having learned mathematics as rules without reason, continue to teach mathematics in this way. Such knowledge can also be described as closed knowledge (Boaler, 1998), which is characterized by following rules, looking for cues as to which procedure to apply, etc. Preservice mathematics teachers, having grown accustomed to being successful mathematics students in settings that relied on closed mathematical knowledge, may find it difficult to apply such knowledge to new situations, specifically with the various teaching tasks described in the previous section. Opposed to learning mathematics in an instrumental (closed) way, understanding mathematics in a fuller sense–one that is able to be applied to new situations, make connections across concepts, and understand the “why” behind procedures–is considered relational understanding, or open (Boaler, 1998; Skemp, 1976).

Having an instrumental understanding of mathematics versus having a relational understanding of mathematics are so qualitatively different that Skemp (1976) claimed these are, in fact, two different mathematics. Boaler (1998) made a similar argument in her description of students from two different schools who had learned mathematics in dramatically different ways. According to Boaler, students from Amber school, who learned mathematics in a closed way, learned a qualitatively different form of mathematics than students from Phoenix school, who learned mathematics in an open way. The breadth of mathematical knowledge was not as important to her research as the quality of that knowledge. Students who understood mathematics in an open way were able, and more willing, to apply their knowledge to new situations or contexts, Boaler concluded; while students who understood mathematics in a closed way, understood mathematics to be a more rigid system and could not as easily apply their knowledge to new situations without having some cues as to what to do.

In terms of this research project, I suspect it is more likely that teachers who claim to have come away with a feeling of knowing a subject better after teaching it have likely come away with more relational understanding. Because teaching requires more than just Common Content Knowledge (CCK), one would assume that being required to engage in practices that make use of pedagogical content knowledge would naturally transform a teacher's prior CCK into a different kind of knowledge. Having previously discussed the kinds of teacher knowledge, in the next section I delineate the sources of teacher knowledge by looking at the research around learning through teaching, teacher noticing, and teacher reflection.

Sources of Teacher Knowledge

Kennedy (2002) explained that teachers develop knowledge through systematic and craft sources. By this she meant that a teacher's knowledge either originates out of formal settings (such as in university contexts) or from 'on-the-job' experiences. Because I am interested in understanding the role that the work of teaching has on three secondary mathematics teacher interns' perceptions of their own mathematics understanding, I rely on their personal accounts and reflection on their experiences. From a particular point of view, the purpose of this project is to gain insight into how novice mathematics teachers develop craft knowledge and to describe how that craft knowledge is different from their perceptions of their earlier systematic knowledge.

In this section, I will look at two approaches to teacher learning and professional development, particularly with an eye towards preservice teacher learning. In the first approach, I will review the research around learning through teaching that Leiken and Zazskis (2010) and others developed. In the second approach, I will look at some views that researchers have laid out concerning the role of noticing (van Es, 2011; van Es et al., 2016, 2017) and reflection (Caniglia et al., 2017; Taggart & Wilson, 2005) in preservice teacher development.

Learning Through Teaching

Outside the education discipline, learning by way of teaching is commonly accepted to be an effective way of learning a skill, concept, or trade. Psychologists attribute this phenomenon to particular practices inherent to teaching that change the nature of the teacher's knowledge, specifically retrieval, social interactions, interactivity, and self-reflection (Cohen et al., 1982; Cortese, 2005; Fiorella & Mayer, 2013; Galbraith & Winterbottom, 2011; Hoogerheide et al., 2016; Kobayashi, 2019; Koh et al., 2018; Roscoe & Chi, 2008). It seems that the very act of teaching a concept to others has been shown to transform a teacher's own knowledge. For example, Koh, Lee, and Kim (2018) theorize that it is through a *retrieval benefit* that a learner enhances their own knowledge when explaining an idea to others. Most of the aforementioned studies discuss learning through teaching in terms of as a benefit for students teaching peers and refer to the process of learning-by-teaching as the *teaching effect*. For student-teachers, it does not seem that simply preparing a lesson plan has the same effect as preparing the plan with the anticipation of actually teaching it to others, known as *interactivity* (Kobayashi, 2019). For example, a preservice teacher who goes through the actions of creating lesson plans to meet a requirement in a university course does not seem to receive the same impact on their learning of the content as when preparing a lesson to teach to actual students. Many university course assignments, such as lesson plans, often are limited in their scope (i.e., they might put together a sample lesson of any random topic or unit), and this might not lead to a robust, connected understanding of, say, algebra. During their teacher internship year, however, they have more opportunities to experience teaching concepts across units and for an entire course. The process of actually teaching the content provides even more of an opportunity for student-teachers to learn content more widely than simply preparing to teach content (Fiorella & Mayer, 2013). In view of the earlier discussion around teacher knowledge and teaching specific content knowledge, one might wonder how a professional teacher's content knowledge transforms through their work of teaching.

Coming into this project, I held the assumption that novice mathematics teachers are still students of mathematics, and we should not view them as finished products when it comes to their knowledge at the very beginning of their careers (Feiman-Nemser, 2003). By this I mean that coming into their initial teaching experiences, while they have demonstrated successful completion of their subject knowledge in university coursework, they need to learn how to apply that knowledge in teaching situations. In fact, as I will explain in this section, the initial years of teaching demonstrate the importance of the work of teaching on helping teachers further round out their own content knowledge.

Ball et al. (2008) argued that the mathematical knowledge needed for teaching was a specific type of mathematical knowledge, and Shulman (1986, 1987) pointed out that teachers needed to know more than just pedagogical skills to teach their content. While preservice teachers are introduced to useful knowledge during their teacher education programs, I would argue that learning on the job has a more lasting impact and plays a more important role in their transition from students to teachers. Social constructivist learning models (Vygotsky, 1978) contend that meaning is constructed in social settings through interactions. One would assume that this construction of knowledge happens not only for students but for teachers as well, particularly novice teachers. Through teaching interactions, teachers can experience knowing mathematics in the way that Ball et al. (2008) described. This phenomenon has been named learning through teaching (LTT) (Leikin & Zazkis, 2010).

One thing that I am interested in, which has been addressed in part by Leikin and Zazkis (2005; 2010), are the teaching experiences that preservice teachers identify as being important for deepening their mathematical knowledge. Instructional interactions, according to Leikin and Zazkis (2005; 2010), are central mechanisms by which teachers may learn through their teaching practice. They describe the teacher-learning process mainly in terms of six themes of instructional interactions (Leikin, 2005; Leikin & Zazkis, 2010): purpose, initiation, motives, reflection, actions, and focus. Without going into further detail on the aspects of their model yet drawing on an important point, they claim that experience in and of itself does not guarantee learning through teaching, rather it is through reflection on one's practice and on the other instructional interactions one has. "Every interaction," Leikin (2005, p. 238) explains, "in a way, relates to people's previous experiences." This is related to Wenger's (1998) description of learning within communities in that learning happens when there is an imbalance between a person's experience and the community's regime of competence. Further, this idea that learning through reflection happens when there is an imbalance is similar to the idea of *dissonance* that

Dewey (1910) posited. Further, Caniglia et al. (2017) built upon Dewey's notion of dissonance when they sought to identify 'aha' moments in preservice mathematics teachers' experiences through reflection. Like Caniglia et al. (2017), I designed my study around mathematics teacher interns' noticing and reflecting on their teaching experiences. In the next two sections, I will review how others have described teacher noticing and reflection.

Teacher Noticing

Ambitious teaching practices, as Lampet et al. (2011) describe, aims to support all students in exploring authentic contexts through application of their subject matter knowledge. Student thinking, I believe, is an important part of this vision, and teachers must be able to notice, unpack, and make use of students' ideas. Being attuned to student thinking, planning exploration of mathematical ideas around student thinking, and using student thinking as the focus of classroom discussions is a skill that takes time to develop. Teacher noticing is central to these skills, and, in this section, I briefly discuss a few perspectives on what teacher noticing is, how it is learned, and how it applies to this study.

In general, teacher noticing refers to what teachers see during classroom interactions and how they make use of it in the classroom environment (Lam & Chan, 2020; Russ et al., 2011). More specifically, teacher noticing, according to Van Es and Sherin (2002), can be described as encompassing the following: (1) identifying what is important or noteworthy about a classroom situation; (2) making connections between specifics of classroom interactions and the broader principles of teaching and learning they represent; and (3) using what one knows about the context to reason about classroom interactions. Similarly, Lam and Chan (2020, p. 578) describe teacher noticing as an "interrelated set of skills that includes teachers' ability to attend to noteworthy events, make sense of those noticed events, and decide how to respond." Both of these descriptions of teacher noticing convey a developed presence of mind. Being able to balance the multiple responsibilities simultaneously all-the-while focusing on student thinking is a skill that does not come naturally for novice teachers. As such, noticing to the extent discussed above is a skill that takes time to develop (Berliner et al., 1988; Star & Strickland, 2008; van Es & Sherin, 2002).

Lam and Chan (2020) distinguish two kinds of teacher noticing: in-the-moment noticing and delayed noticing. They describe in-the-moment noticing as happening during instruction which requires more immediate improvisation on the part of the teacher. Further, because this kind of noticing happens in real time, making sense of the data and reflecting on it often happens after instruction. This level of noticing develops with experience and requires the teacher to monitor multiple avenues of thought concurrently. A few key important takeaways about teacher noticing: (1) it doesn't come naturally for novice teachers; (2) noticing skills can be developed and honed; (3) novice teachers benefit from direction of a knowledgeable guide.

Because attaching meaning to in-the-moment noticing is challenging for novice teachers, recent approaches to developing this skill have relied on the use of analyzing self-capture video (Star & Strickland, 2008; van Es et al., 2016, 2017). Through video segments, preservice teachers can collectively analyze teaching episodes, and more importantly, they are given an opportunity to learn from more knowledgeable practitioners. Sherin and Han (2004) & van Es et al. (2016, 2017) extensively reported on the use of self-capture video in teacher education coursework. Through this mode of inquiry, they argued preservice teachers can collaboratively learn how to pay attention to aspects of teaching and learning. Most of the research around teacher noticing focuses on teachers noticing students' thinking. While this formed the basis of many of the instruments used in my project, I also made use of teachers noticing their own thinking through reflection on classroom interactions.

Reflection in Teaching Practice

Reflection has long been understood to be an integral aspect of learning and teaching; further, it has been actively researched over the last several decades. This overview of some basic concepts related to reflection in teaching is not meant to be exhaustive, but I will point out some aspects of this practice that played a role in the design of my project. Dewey (1938, p. 110) described reflection as being similar to inquiry and that “it is defined to look back over what has been done so as to extract the net meanings, which are capital stock for intelligent dealing with future experiences.” In other words, learning occurs through reflection when one carefully thinks back to what has happened from prior experiences in order to make necessary changes to future events. As described in the last section, Dewey referred to the idea of *dissonance*, or perplexity, as playing an important part of reflection, and this, as we will see later, will play an important role in preservice teachers’ learning through reflection. Caniglia et al. (2017) rely on these ideas of dissonance and reflection in the design of their study around ‘aha’ moments during student teaching experiences for preservice secondary science and mathematics teachers. In their study, they made some astute observations: (1) preservice teachers were able to identify moments of dissonance through explicit interventions; (2) they struggled to move beyond descriptive responses during their reflections; (3) and they benefited from guidance from a mentor. These three findings, as we will see, are important to the design of my project.

There are both positives and negatives associated with the implementing reflection into preservice teacher education. On the positive side, reflection is essential for “promoting growth in critical analysis of teaching, systematically reflecting on self-development and on actions within classroom and work contexts, and linking understanding with classroom practice (Taggart & Wilson, 2005, p. 98)." Negatively, although reflection can be beneficial for helping preservice teachers learn from their own teaching experiences, researchers have noted that reflection in an analytical way is not natural for young practitioners and they must be taught how to do it (Beauchamp, 2015; Spalding & Wilson, 2002; Taggart & Wilson, 2005). Additionally, there is evidence that explicit instruction is not as effective as working with mentor teachers in encouraging young teachers to reflect on what they have learned from their teaching (Beauchamp, 2015; Caniglia et al., 2017; Spalding & Wilson, 2002; Taggart & Wilson, 2005).

Now that I have briefly discussed the aspects of teacher noticing and reflection that have influenced this dissertation project, I will highlight the main portions of Wenger's (1998) *communities of practice* theory which Van Zoest and Bohl (2005) use as the foundation for their *Aspects of Self-in-Community*. Throughout the results chapter I will add more details of Wenger's theory that are at play in my data.

Aspects of Self-in-Community

As mentioned earlier, Van Zoest and Bohl (2005) ground their framework in Wenger's (1998) assumption that learning is taken to be the same thing as identity development in a community. Further, when learning happens in a community, it takes place because of an imbalance between a person's experience and the community's regime of competence. A person's interactions with the process of negotiating experience/competence imbalances are connected to his or her continued identity development. In this section, I will address the aspects of Wenger's (1998) work that influenced the development of the *Aspects of Self-in-Community* portion of the MTI framework (Van Zoest & Bohl, 2005), how communities of practice are defined, how identities influence a teacher's participation and meaning making, as well as point out the communities of practice central to this project.

A community of practice, according to Wenger (1998), is a specific type of community that constitutes the arenas in which individuals learn. He described that communities of practice are defined and cohered by three characteristics, namely: joint enterprise, mutual engagement, and shared repertoire. A *joint enterprise* is defined as 'something to do.' *Mutual engagement* is described as 'others to do it with.' Finally, a *shared repertoire* are the common resources (these can be physical, communicative, etc.) in which to act in carrying out the joint enterprise. Taken together, Van Zoest and Bohl (2005), included these domains underneath the umbrella of 'dimensions of competence.'

Communities of practice are naturally limited by their size and scope and made up of different types of members. Wenger (1998) introduced the term *boundaries* to describe these limitations. Further, he explained, communities of practice have members that are central to its function and those that are peripheral. In a classroom community, for example, the central figures are the teacher and students. The peripheral members could be other teachers in the same department, school administration, and parents. The communities of practice central to this project are each intern's specific classroom community and their internship professional learning community; additionally, their university class community could play a tangential influence on the data collected. For their classroom community, the central participants are the interns as the teacher, their mentor teachers, and their specific students. For their internship PLC, the central participants are me, as the facilitator, and each intern, as members.

A core aspect of Wenger's (1998) work was the idea of developing one's professional identity. His description of professional identity is larger in scope than what one would normally associate as identity. He included both how one viewed themselves and how others viewed them (so both an internalized view and an externalized view). Identities, he argued, are the vehicle by which we participate in communities and are transferable from one community to another. By this he means, your identity provides you with the funds of knowledge and experience necessary for you to be considered a participant in that community. Identities are continually being developed and negotiated between members. Further, identities allow members to create meaning within the community.

Central to the idea of meaning making within a community for Wenger (1998) are his ideas of *participation* and *reification*. He explains these as three ways one can participate within a community: (1) engaging mutually with others in activity; (2) understanding and fine-tuning the tasks at hand; and (3) developing a shared repertoire of resources. In the context of this project, for example, each intern participates within their classroom community by engaging with their mentor teacher to collaboratively teach their students mathematics. Over the course of time, as the interns gain more experience, they begin to fine-tune the tasks that their mentor teacher taught them and create their own teaching habits. Finally, the interns may make use of or collaborate to create shared resources (such as shared activities, assessments, etc.). Wenger (1998) also described reifications as an important aspect of meaning making in community. By a reification he meant the products (both conceptual and physical) of the participation. For example, when collaborating with their mentor teacher, they may collectively create activities that help meet the outcomes for their students by the end of a unit. Further, over the course of several units, teachers assess students' work and assign end of term grades to reflect students' collective understanding. Both of these are examples of reifications in the teaching community of practice.

Finally, as new participants within their classroom and school communities, novice teachers learn to participate by coming to understand what it means to be competent members of these communities. Wenger (1998) described this competence in terms of collective accountability, rather than discussing it in terms of a trait that an individual has. He described a *regime of competence* as the collection of ways of participating within a community that are relevant to the shared enterprise. He further described *regime of mutual accountability* in terms of a collectively held set of values that forms the framework of participation within the community.

Research Purpose Summary

It is in the context of learning to teach within specific classroom communities and a specific university context that the interns in this study came to reflect on the role that teaching practice had on their perceptions of what it meant for them to understand mathematics. The themes covered in this literature review are the foundation by which I designed the research questions, data collections methods, and data analysis. As I mentioned earlier, this study focuses on learning through teaching by way of professional noticing and reflection to explore the role that teaching practices have on three secondary, mathematics teacher interns' perceptions about what it means for them to understand the content which they teach. The research questions guiding this study are:

1. *What aspects of mathematical knowledge for teaching are reflected in the narratives that PSMTs provide about their learning during student teaching?*
2. *What teaching experiences do the interns emphasize as being critical in helping them understand differently the content they teach?*

It is my hope that in answering these questions, this project adds to our overall understanding about the influence of teaching practice on novice teachers' pedagogical content knowledge by hearing their own accounts of their experiences. Through these experiences we might also gain insight into how novice teachers' perceptions of what it means to understand a concept evolve after having taught mathematics for the first time. To aid in my analysis of the interns' written reflections, I made use of van Es's (2011) framework for learning to notice student mathematical thinking to first highlight what they were noticing and describe the importance that they attributed to their observations; further I employed Ball et al.'s (2008) Mathematics Knowledge for Teaching framework to view the interns' experiences through the lens of the pedagogical content knowledge they were developing.

CHAPTER 3: CONCEPTUAL FRAMEWORK

The purpose of this project is to explore how aspects of PSMT's evolving identities inform their self-evaluation of their knowledge. Specifically, the research questions that guided this project are

1. *What aspects of mathematical knowledge for teaching are reflected in the narratives that PSMTs provide about their learning during student teaching?*
2. *What teaching experiences do the interns emphasize as being critical in helping them understand differently the content they teach?*

In order to guide the design and implementation of my data collection and analysis, I made use of several frameworks that supported these research questions. Overall, because I was specifically interested in how PSMT's mathematics teacher identity influenced their perception of their evolving mathematical knowledge, I made use of a framework that incorporated both mathematics teacher identity and teacher knowledge, namely Van Zoest and Bohl's Mathematics Teacher Identity (MTI) framework (2005). This framework was particularly helpful because it allowed me to view the connection between a mathematics teacher's identity and how it influenced their perception of their own mathematical knowledge specifically because of how they viewed teacher identity as the intersection between teacher practice and teacher knowledge. Additionally, a large part of their framework was based around Shulman's theory of teacher knowledge; I, however, made use of Ball et al.'s (2008) Mathematics Knowledge for Teaching (MKT) framework to inform how I interpreted the PSMT's reflections. The MKT framework allowed me to discuss the PSMT's teaching experiences through the lens of their mathematical knowledge and is discussed in detail in my literature review.

This study focuses on learning through teaching (Cohen et al., 1982; Galbraith & Winterbottom, 2011; Hoogerheide et al., 2016; Kobayashi, 2019; Koh et al., 2018; Leikin, 2005, 2010; Leikin & Zazkis, 2010; Roscoe & Chi, 2008) by way of professional noticing and reflection (Akinbode, 2013; Beauchamp, 2015; Caniglia et al., 2017; Dewey, 1938; Spalding & Wilson, 2002; Taggart & Wilson, 2005; van Es, 2011; van Es et al., 2017; van Es & Sherin, 2004, 2002) as a way to explore the role that *Aspects of Self-in-Community* (Wenger, 1998) have on three secondary, mathematics teacher interns' *Aspects of Self-in-Mind* (Ball & Hill, 2008; Shulman, 1986, 1987), specifically their beliefs about what it means to understand a concept. In this section, I will explain how the phenomenon I am interested in fits into the *Mathematics Teacher Identity* (MTI) framework (Van Zoest & Bohl, 2005).

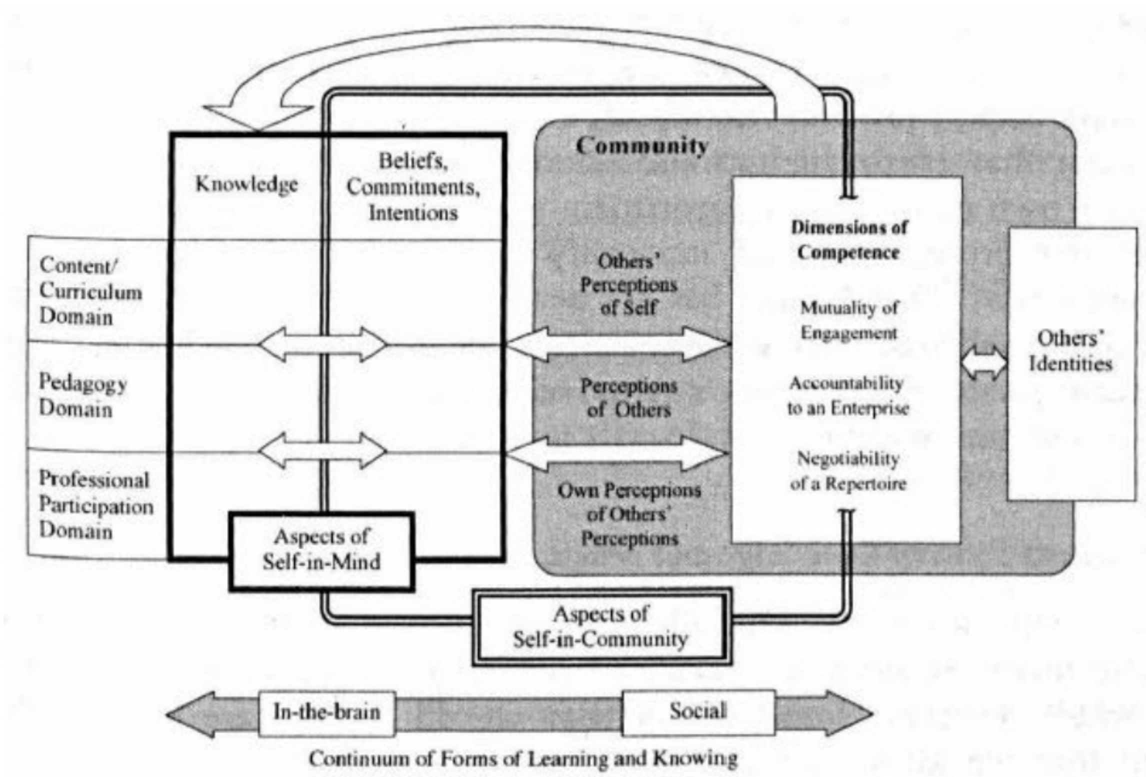
Mathematics Teacher Identity Framework

I made use of the MTI framework (Van Zoest & Bohl, 2005) to guide my research design, data collection, and data analysis. As mentioned previously, Van Zoest and Bohl (2005) described a mathematics teacher's identity as being composed of two interrelated components, namely *Aspects of Self-in-Mind* and *Aspects of Self-in-Community,* seen in Figure 2-1 below. Broadly, Van Zoest and Bohl (2005) draw from Wenger's (1998) theory of communities of practice to develop their *Aspects of Self-in-Community* and Shulman's (1986, 1987) work on knowledge for teaching to form their *Aspects of Self-in-Mind.*

In this framework, learning plays a central role and they view teacher learning along a continuum (i.e., either learning 'in-the-brain' or learning within a community). Learning, according to Wenger (1998), is synonymous with developing one's identity within a specified community of practice. A mathematics teacher's identity, from Van Zoest and Bohl's (2005) view, consists of their collection of capacity and understanding. These assumptions form the basis for the MTI framework (2005); further, Van Zoest and Bohl (2005) designed their framework in a way that described a teacher's professional identity by merging this socio-cultural view of learning with prior theories of teacher learning that were more cognitive in nature, namely Shulman's (1986, 1987) theory.

Figure 2 - 2

Mathematics Teacher Identity Framework



Because learning, according to Wenger (1998), can be conceived as developing one's identity within a community, I hypothesized that learning to teach within specific communities of practice would influence a teacher's perception of self-in-mind, specifically in the way they view their own content understanding. As such, my study sought to unpack and explore this relationship between *Aspects of-Self-in-Community* and *Aspects of Self-in-Mind* by analyzing three secondary mathematics teacher interns' reflections on their individual experiences. In terms of the MTI framework, my study was designed to explore the arrow that loops back from *Aspects of Self-in-Community* to *Aspects of Self-in-Mind.*

In order to explain the details of the above framework and explain how it influenced the design and implementation of this study, it was necessary to identify aspects of Shulman's (1986, 1987) and Wenger's (1998) work that most influenced the way that Van Zoest and Bohl (2005) understood teacher learning. In my literature review, I elaborated more on Van Zoest and Bohl'*Aspect of Self-in-Mind* by connecting their conception of teacher knowledge to that of mathematics knowledge for teaching.

CHAPTER 4: METHODS

The very act of teaching and its effect on enhancing a teacher's own learning of content material has been documented primarily in peer-to-peer tutoring settings (Kobayashi, 2019; Koh et al., 2018). My study provides narrative evidence of this *teaching effect* by investigating the role that *teaching practices* have on novice teachers' perceptions of their own *content knowledge.* This study will not undertake an investigation of the underlying mechanisms of the teaching effect; rather I have attempted to provide narrative evidence of three novice teachers' lived experiences as a way to explain the role that various *communities of practice* have on their individual mathematics teacher identities, specifically their *Aspects of Self-in-Mind* with regards to their content knowledge (Shulman, 1986, 1987; van Zoest & Bohl, 2005a; Wenger, 1998). It is my hypothesis that when first participating in extended periods of direct teaching and experiencing the interactivity of teaching, the interns will begin to develop Pedagogical Content Knowledge and through this professional knowledge will feel like they are gaining a broader understanding of the math that they teach. To guide my study, I put forward the following research questions:

1. *What aspects of mathematical knowledge for teaching are reflected in the narratives that PSMTs provide about their learning during student teaching?*
2. *What teaching experiences do the interns emphasize as being critical in helping them understand differently the content they teach?*

Because this study is focused on understanding novice teachers' lived experiences, it was natural to adopt qualitative research methods to guide data collection and data analysis, particularly case study methods (Stake, 1995; Yin, 2018). In order to create a layered account for each intern's experience, I collected varied data sources (i.e., interviews, observations, written reflections, teaching artifacts, written and video-recorded correspondence, focus-group discussions, and a pre-internship survey). In this chapter, I explain in detail each step of my research process, from the design and data collection to the process by which I make claims from the data analysis, as well as provide a rationale for the choices I made throughout this project.

The rest of this chapter is divided into four primary sections, namely: research design (where I will describe case study research and provide a rationale for this choice), a description of the context and participants who agreed to be a part of this study, my data collection methods (where I describe the instruments used and other forms of data collected), and my data analysis approach. Included in each of these sections are explicit descriptions about how these research method choices were made with the research questions in mind.

Research Design

Brené Brown wrote that “If research is going to serve people, it has to reflect their experiences” (2021). I agree with her research perspective and so have designed this project to serve educators by providing in-depth narratives for three secondary mathematics teacher interns’ experience teaching mathematics for the first time. Specifically, these narratives will provide readers with personal accounts of each intern's experiences from teaching and learning mathematics over the course of an academic year with an intent to explain the phenomenon of coming to know mathematics more robustly because of having participated in teaching practice. As I formulated the details of this dissertation, I decided to use case study research methods (Stake, 1995; Yin, 2018) which ultimately helped me gain an in-depth perspective on the relationship between learning how to teach and the role that teaching practice has on novice teachers’ sense of understanding mathematics concepts.

Researchers have gone back and forth on defining exactly what a case study is, but Yin (2018, p. 15) summarized the common features that are shared in many definitions. Case studies are empirical studies that: "investigate a contemporary phenomenon (the "case") in depth and within its real-world context, especially when the boundaries between phenomenon and context may not be clearly evident." Further, case studies cope with the technically distinctive situation in which there will be many more variables of interest than data points, benefit from the prior development of theoretical propositions to guide design, data collection, and analysis, and rely on multiple sources of evidence, with data needing to converge in a triangulating fashion.
With these criteria in mind, I will lay out the scope and features of this case study by: (a) describing the cases that were studied and the real-world context in which they were situated; (b) describing the secondary mathematics interns who participated in this project; (c) and introducing my data sources and describing how I analyzed the data.

In an instrumental case study (Stake, 1995; Yin, 2018), the focus of investigation is on issues more than the particular case, as opposed to intrinsic case studies, which are focused on the uniqueness of a particular case. Because I was interested in understanding how mathematics teacher interns reflected on their own content knowledge after having experienced teaching over an extended period, my research questions were crafted in a way that would help me better explore the nuances of this case. For case studies, why and how questions are commonly used (but not limited to) because they lend themselves to understanding how issues develop over time (Stake, 1995; Yin, 2018).

Recall that the motivation for this study was to better understand the phenomenon that many novice teachers have verbalized: "I really didn't understand mathematics, until I had to teach it." This phenomenon, in my thinking, indicates reflection over time and, as such, influenced how I crafted my research questions for this case study. Further, this phenomenon has two components, which are reflected in the research questions. For Research Question 1, I simply wanted to narrate how the interns' descriptions of their mathematical knowledge changed throughout their transition from student to teacher. From their descriptions, I would then make connections to the kinds of mathematical knowledge they described as well as the quality of their knowledge. Before implementing this study and as the interns gained more experience with varied teaching tasks, I expected that their descriptions of their mathematical knowledge would likely change. Tracking their descriptions of their evolving mathematical understanding, I hoped to make connections from their mathematical knowledge to their teaching experiences that they described. As for Research Question 2, my goal was to specifically highlight the kinds of teaching tasks that the interns indicated were critical to their developing mathematical knowledge. By answering this question, I hoped to identify the specific mathematical knowledge for teaching (Ball et al., 2008) they would likely be gaining through varied teaching tasks.

Further, case study research is designed around understanding propositions which help to direct our attention to something that should be studied within the scope of the case (Yin, 2018). In the introduction of this writing, I stated that, overall, I hope to observe and better understand the claim that many early-career teachers make when they say: "I didn't really understand X, until I had to teach it," where X could be *mathematics* or any particular concept in mathematics, such as *trigonometric identities.* This phenomenon, in my view, is influenced by experiencing mathematics in a qualitatively different way than they had as students. Ball et al. (2008)argued that mathematical knowledge for teaching (MKT) is a specific type of content knowledge. Thus, teachers who claim "I didn't really understand X, until I had to teach it" may actually be experiencing more to knowing mathematics than just "how to" procedures and that knowing mathematics for the purpose of teaching gives them the sense of having a more well-rounded grasp of the subject. In the next two subsections, I will describe the research context, the interns who agreed to participate in my project, the variety of data I collected, and the approach I used to analyze the data.

Research Context

The goal of this study was to explore secondary mathematics teacher interns' evolving sense of understanding mathematics content as they reflected on their own teaching practice over the course of one academic school year. In the following sections, I provide a full description of the research setting (i.e., the internship year experience), describe the participating teacher intern's and the selection process, as well as describe my positionality as teacher educator/researcher.

Research Setting

Before moving on it is important to clarify a few terms that will help give a fuller picture to this study. For many teacher-education programs the capstone professional experience, called the student teaching semester, happens during the last semester of the student teacher's undergraduate teacher education program (Greenberg et al., 2011). The participants in my study, however, all are recent graduates of a large, Midwestern research university that has a unique approach to teacher education program.

Prior to graduation, the participants of this study progressed through a rigorous teacher education program, one that is highlighted by extensive field experiences. Like many secondary teacher preparation programs in the United States, preservice teachers from this university study both content specific coursework and education specific coursework. This program is characterized by an increased emphasis on field work experience prior to graduation. These field experiences are founded on the belief that preservice teachers need opportunities to construct their own knowledge based upon concrete experiences and have an opportunity to reflect how these experiences interact with their Pedagogical Content Knowledge. In lieu of a semester-long student teaching experience within their undergraduate studies, preservice teachers complete a year-long experience, referred to from now on as their *teaching internship year*. Additionally, this internship takes place following graduation (usually the school year antecedent to graduation, but that's not always the case) and because all participants have completed their undergraduate major are referred to as *teaching interns* by the university. For this study, when referring to the participants prior to graduation, I will use the term preservice mathematics teacher; because most of the data collection period took place during the internship year they will be referred to as *secondary mathematics teacher interns*. Most states within the United States require a minimum length student teaching experience; however less than half of the states in the U.S. have a requirement that student teaching should be longer than ten weeks (Greenberg et al., 2011). As mentioned above, the teacher education program at the university in this case study requires teacher interns to complete a year-long teaching internship, highlighted by extensive lead teaching periods, ongoing graduate coursework, and action-research projects focused on improving their own practice. I believe that this extended internship year afforded the opportunity for the interns to gain experience in more varied and more extensive teaching tasks, which added to their development of mathematics knowledge for teaching.

Participants and Positionality

The goal of this study was to provide narrative evidence to explore secondary mathematics teacher interns' evolving sense of understanding mathematics content as they experience the work of teaching over the course of one academic school year. In the year prior to the 2021-2022 academic school year (during which a portion of this project work took place), I co-taught a sequence of teaching methods courses to a cohort of preservice secondary mathematics teachers in the final year of their undergraduate coursework. As I developed the initial design for this project, it was my intention to recruit participants from these courses, as they were slated to begin their internship the following year. Further, having been their instructor the year prior, I began to develop a rapport with them which helped to establish our ability to work collaboratively.

Throughout the 2021-2022 school year, I acted as a participant and a researcher, which is one common way for case study data to be collected (Stake, 1995; Yin, 2018). I participated in the internship by serving as the interns' field instructor (i.e., the university representative who works alongside interns and their mentor teachers to guide and assess them). This role allowed me to be immersed in the interns' teaching work. This opportunity to be involved in and aware of their everyday experiences is a quality that many case studies are characterized by and was required for collecting data spanning a significant length of time, specifically an academic school year (Stake, 1995; Yin, 2018). Further, because lived experiences influence one's beliefs and perceptions over time, choosing to use case study methods over the course of a school year enabled me to get a fuller sense of the interns' teaching experiences than if I had restricted my study to shorter or disjoint amounts of time. With these things in mind, as the interns' field instructor I was ideally situated to interact with them regularly. Additionally, because there was an assessment component attached to my role as a field instructor, I had decided to forego analysis of the data until the completion of each semester. Further, participation in and completion of the data collection was completely voluntary and was not attached in any way to their grades for completing the internship. Since I had already gained substantial trust and mutual respect with the interns, I could expect those who participated in my case study to respond in a forthcoming manner and without reservation.

My role as researcher never superseded my responsibilities as a field instructor. That is, as the field instructor, I was primarily concerned with the interns' professional growth and supporting them in a way that they could fulfill their classroom responsibilities without too much undue stress. During the 2021-2022 academic school year, I was the field instructor for five interns, all who started the year participating in this study. I regularly emphasized that the work done for this study was not a part of their internship requirements. During the Fall semester, I allowed two of the interns to back out of my data collection work so that they could focus more intentionally on their teaching responsibilities.

I initially planned on selecting participants by using a series of surveys and interviews and would, essentially, select some to be their field instructor for the school year. As I discovered along the way, the College of Education at the university had their own procedure in place for pairing up teacher interns with field instructors and had their own research approval process. While I did follow through with the planned surveys and interviews with PSMTs who agreed to take it, I was not able to select the participants on my own. Rather, over the course of the summer before the internship, I was placed with five interns in the geographic region surrounding the university. Logistically this turned out to be beneficial because it reduced the amount of travel time on my part.

The Internship Year

I explained earlier that I wanted to gain a clearer understanding of how teaching experience influenced the way three secondary mathematics teacher interns described their sense of understanding mathematics content. By choosing a case study methodology, I hope this research will add to the ongoing conversation around teacher knowledge and teacher preparation by presenting in-depth perspectives from three teacher interns as they navigate the complexities of their teaching internship year. In case study research, the focus of inquiry is usually on an individual but also considers the complexities of the context in which the case takes place; thus, this work focuses on the experiences of each intern while exploring the relationship between their teaching work and their developing mathematical knowledge for teaching.

This study took place in the Midwest United States through various contexts, the primary being local public high school mathematics classrooms. Throughout their internship, teacher interns worked daily with a mentor teacher at their school placement and weekly with their field instructor, namely me, who represented the university teacher education program. The mentor teacher is an experienced high school teacher, and, as the name implies, mentors the intern throughout the school year by overseeing the intern's progress and collaborates on their teaching responsibilities. Prior to the school year beginning, the interns met with their mentors to discuss the details of the internship, particularly by selecting their focus class. The focus class helps an intern to transition from an observer or classroom aid into a lead teacher, and typically the intern assumes responsibility as the lead teacher much sooner in the year. My role as the field instructor was to evaluate the intern according to program standards, support the intern's trajectory of learning, and assist the mentor teacher as needed. I was required to observe each intern a minimum of three times per semester, but because of this research project, the actual number of observations was much greater than three.

The teaching internship year through this university is separated into five stages across the standard two-semester, school-year calendar. Teacher interns experience an interval-like training schedule as they take on more instructional responsibility, periods of exertion punctuated by periods of recovery with an overall path of slowly taking on more responsibility (see Table 3 – 1 for a visual representation of the internship year). The five stages of the internship are the introductory period (where interns take on responsibility for their focus class), two Guided Lead Teach (GLT) periods in the Fall semester, an extended Lead Teach period in the Spring, and the end-of- year transition period. In the Fall semester, interns assume responsibility of their focus class, almost immediately, and then slowly take on more teaching responsibility during the two GLT periods, separated by a period where they are given room to reflect and learn from these initial experiences involving more teaching responsibility. The first semester's GLT periods prepare interns for the more intensive ten-week lead-teach period in the Spring, where they take on teaching responsibility for four courses. Throughout the entire school year, interns are given two prep hours, one of which is intended for teaching prep and the other to work on their coursework. I bring up their schedule because it directly influenced how I planned data collection periods which I will discuss in more detail in the next section.

Table 3 - 1

Rhythm of the Internship Year

6 Prep Hour								
5 2nd Prep Hour								
4								
3								
2								
1 Focus Class								
Estimated Duration	*4-5 weeks*	*GLT1 2 weeks*	*3 weeks*	*GLT2 2 weeks*	*4-5 weeks*	*2 – 3 weeks*	*Lead Teaching 10 weeks*	*4 weeks*
Semester	*Fall*					*Spring*		

Data Collection

In this case study, I collected data both prior to and throughout the teaching internship year using surveys, interviews, and written artifacts. In this section I will provide further detail on how these data sources were chosen to support the research questions and how the data was collected. The data sources are organized into two general categories: pre-internship data and internship data. The pre-internship data is made up of a pre-internship survey and pre-internship interviews. During the internship, I collected data through written artifacts (message board conversations, teaching reflection journals, lesson and unit plans, and assessments), interviews, and records of follow-up correspondence. A general timeline for my data collection is listed in Table 3 – 2. This project received IRB approval on October 6, 2020 and is considered exempt (STUDY00005142).

Table 3 - 2

Data Collection Schedule

Semester	Month	Data Source	Research Question
Spring 2021	March	Pre-Internship Survey	1
Spring 2021	April	Pre-Internship Interviews	1
Fall 2021	September – October	Learning to Notice & Reflect PLC	1 & 2
Fall 2021	October – November	GLT 2 Written Reflections, Interviews, and Artifacts	1 & 2
Spring 2022	January – March	LT Written Reflections Interviews, and Artifacts	1 & 2
Spring 2022	March	Post Data Collection Group Interview	1 & 2

Research Question 1 was supported from pre-internship data and from written items from interns throughout the school year. To answer Research Question 2, I collected written reflections, supplemental data that usually stemmed from follow-up conversations, transcripts from group interviews, and artifacts from their teaching experiences. Further, it should be noted that the focus for this project was modified after having adopted the *Mathematics Teacher Identity* framework (van Zoest & Bohl, 2005a) and this influenced how I used the data from early on in the collection process, as it was designed to meet another purpose, namely gauging the interns' sense of their own readiness to teach secondary mathematics.

Pre-Internship Data

As mentioned above, the data collected the semester preceding the 2021-2022 internship school year was originally intended as both a way to select the interns I would work with the following academic year and to begin to understand their self-perceived readiness to teach high school mathematics. As a part of the recruiting process, I intended this data to give me a glimpse into how willing the preservice teachers would engage in later discussions and how thoroughly they might respond to written prompts. Before administering the pre-internship survey and conducting interviews, I solicited volunteers for the first part of this study by announcing to the preservice teachers enrolled in TE 408 what I would be doing for my dissertation project and how I hoped to serve as a field instructor for a few of them.

Because I intended to serve as the field instructor for each of these interns so as to give me the best possible knowledge of their day-to-day teaching experiences, I concurrently sought out the appropriate teaching assistantship from the Department of Teacher Education and requested permission to carry out research with my interns, which I received over the summer. Through my communication with the internship coordinators, I was informed that it was not logistically possible for me to select my own interns, as I had originally intended. As such I was assigned the field instructor for five interns from the TE 408 cohort all in the same geographic area immediately in the vicinity of the university.

Pre-Internship Survey

During the last half of March 2021, I recruited 8 preservice teachers (out of 16 total enrolled in TE 408) to participate in the initial part of the study, which consisted of the *Pre-Internship Survey* and the *Pre-Internship Interview*. Prior to conducting the interview, the preservice teachers responded to the Pre-Internship *Survey* (Appendix A), using the online tool, Google Forms. The purpose of this survey was originally designed to help me begin to get a sense of their self-assessed readiness to teach high school mathematics content and students and was made up of both free-response and multiple-choice items. In the following paragraphs, I will explain why I originally decided to use this survey and how I ultimately made use of the data to help answer Research Question 1, even after having refocused the project.

To begin to answer my original research question about perceived readiness to teach, I needed an instrument that would allow me to: (a) understand how prepared the preservice teachers thought they were to teach high school mathematics, and (b) to begin to see what this belief was founded upon. While designing this study, I came across Hine's (2015a, 2015b) surveys which I adapted to fit my own context to better understand how preservice secondary teachers perceive their readiness to teach mathematics before and after going through a semester-long teaching practicum. Both surveys consisted of items related to PSMTs self-perceived readiness to teach mathematics in terms of their Mathematics Content Knowledge (MCK) and their Pedagogical Content Knowledge (PCK). The first survey consists of ten questions (Hine, 2015a, 2015b). The first four questions collect background information from the participant (biographical and academic profile data), and the next six questions help researchers better understand how preservice teachers describe the extent to which they perceive their readiness to teach mathematics in terms of their MCK and their PCK. While these surveys were originally chosen because of their connection with readiness-to-teach, the surveys added to the overall narrative for each intern by giving me a glimpse into their backgrounds as students and by hearing about any early teaching experiences. Further, it specifically helped me to answer part of Research Question 1 because in some instances the interns indicated how they viewed their mathematical knowledge for specific subjects as well as helping me to understand their approach to learning mathematics up through their undergraduate years.

Pre-Internship Interviews

As mentioned in the prior section, I followed up the *Pre-Internship Survey* with a short interview for each preservice teacher. Originally, the interview was conceived as helping to narrow down my participant pool, but because I was logistically unsure who I would be a field instructor for, I decided to carry out the survey and interview with anyone willing to do both (thus I had data for many interns by the time the internship began). The interview primarily served as an opportunity for me to follow up with each preservice teacher's responses on the survey and gave me a chance to add more detail to their prior experiences as a mathematics student and to collect further insight into how they perceived what it meant to learn and teach mathematics.

For the interview, I generally followed the protocol found in Appendix B, but also created individual follow-up questions based on each preservice teachers' responses on the survey. This protocol is less influenced by Hine's work but does draw some questions from his interviews found in Hine and Thai (2018, 2019), which helped me gain a more in-depth picture of each preservice teacher's personal story when it came to learning mathematics as a student and to hear about their decision to become a teacher. Further, I structured the interview questions around Jacob and Furgerson's (2012) principles for designing and carrying out qualitative interviews.

Internship Data

The goal for my data collection going into the 2021-2022 school year was to prepare the interns to craft reflective narratives during two periods in their internship, the first period being towards the end of the Fall semester during the interns' second Guided Lead Teach period (roughly in early November 2021) and the second during their Spring semester Lead Teach period (roughly early February 2022). Throughout each of these time frames, I collected data primarily using daily reflection logs, various modes of follow-up conversations, and some supporting artifacts (lesson plans, worksheets, observation notes) as necessary. I will discuss the structure of these data collection periods in more depth in the following sections, but the goal was to prepare them to reflect upon their teaching experience through a unit of instruction and how that teaching experience influenced their perception of their own mathematical knowledge. Prior to these two periods of focused data collection, I designed a "warm-up" period, in the form of an asynchronous professional learning community, throughout September and October to support the interns' development of their noticing and reflection skills. In the rest of this section, I describe each phase of data collection and how I implemented the data collection instruments, as well as provide the rationale for how I used each instrument.

Professional Learning Community

In response to the literature that novice teachers need time to learn to reflect and to notice (Hatton & Smith, 1995; Spalding & Wilson, 2002; van Es et al., 2017; van Es & Sherin, 2004), and because their reflections serve as my main source of data in answering the research questions, I used the first two months of the teaching internship as an opportunity to support my interns in establishing reflective habits and to hone their reflections in a way that was more analytical in nature, through implementation of an asynchronous professional development committee (PLC). This PLC served as a training period for my teacher interns to collectively learn these skills together. Further, a PLC made sense for a couple of reasons. First, instead of working individually with each intern, it saved time to work as a group. Second, professional development models for learning to reflect and notice existed, and these were designed in context for collective learning. Finally, the PLC served as a micro-community of practice between myself and the interns, and the use of self-capture videos served as the reification on which we participated. The primary purpose for this PLC was to provide a low-stakes space for the interns to collectively practice being attentive and reflective mathematics teachers. Being able to notice student thinking as well as being able to reflect in a way that gave me a glimpse into the experiences that helped to transform their own thinking was crucial for answering my research questions. Further, another purpose of the PLC was to provide the intern opportunities to reflect on their notions of what it means to understand a concept and to notice how core classroom interactions might influence their perceptions of what it means to know mathematics.

I went about designing the PLC around the principles laid out by Brodie (2014). In her discussion about the characteristics and principles for PLCs in mathematics education contexts, she pointed out that PLCs should fundamentally be about learning and should focus on supporting teachers to develop their own mathematical knowledge and their mathematical knowledge for teaching, particularly in relation to student thinking, and be supported by a well-established knowledge base as well as by data. So, with these characteristics in mind, I set out to design our PLC around teacher noticing (Mason, 2002, 2011; van Es, 2011; van Es et al., 2017; van Es & Sherin, 2004) and teacher reflection (Akinbode, 2013; Rodgers, 2002).

Specifically, I structured the agenda for this PLC around van Es et al.'s (2017) *Learning to Learn Through Teaching* course for mathematics teachers. Their course was divided into three phases and was built around preservice teachers analyzing self-captured teaching episodes. Phase 1 introduced preservice teachers to research-based frameworks in mathematics teaching focused on the five core dimensions of classroom interactions (student thinking, task quality, teacher questioning, classroom discourse, and formative assessment) (Black et al., 2004; Boaler & Humphreys, 2005; Carptenter & Lehrer, 1999). In this phase, preservice teachers viewed self-captured videos and identified their observations with regards to the specific type of classroom interactions they were focusing on. Phase 2 emphasized attending to the details of interactions, to what occurred in the classroom interaction and creating accurate depictions of those events. Phase 3 was based on research on lesson analysis to promote interns' noticing of classrooms as interactional spaces that link students, teaching, and content.

Van Es et al. (2017) designed the *Learning to Learn from Teaching* course to fit within an entire semester's schedule, but I had to condense their schedule to meet the goals of my project. In their description of Phase 1, four weeks were dedicated to the five dimensions of classroom interactions. For my setting and because many of these topics had been covered in their coursework the year prior, I provided the interns these frameworks as resources. I designed the PLC with the intent that it would support teacher noticing skills in the interns by relying on the teacher noticing framework described by van Es et al. (2017): attending to features, elaborating on what is observed, and integrating observations to make sense of complex phenomena. Collective and individual learning is an important feature of this PLC. The main function of this PLC was to create a space that formed a small community of secondary mathematics teacher interns who mutually supported each other in learning how to notice and reflect on mathematics teaching and learning. Each intern took turns collecting data (by way of video-recorded lessons, student work, and lesson plans), made observations of that data, responded to each other's ideas, and reflected on the process of noticing their own and their students' thinking.

To further condense a semester into an eight-week period, I designed each week's agenda to work on Phase 2 and Phase 3 concurrently. Each week had a content focus (one of the five dimensions of classroom interactions), activities, and assignments. One concern that I had prior to starting the PLC was that it might generate even more stress on the interns as they were adjusting to their teaching responsibilities and university course load. I initially planned for this to be a weekly, online face-to-face meeting, but had difficulty finding a common time in each intern's schedule; as such I adjusted the mode of interaction to allow for most of the work to be done asynchronously (I designed a Google Classroom as our platform). Weekly sessions were guided by Borko et al.'s (2008) Problem-Solving Cycle (PSC) professional development workshop framework: solve the problem & develop lesson plans, video the implementation of the lesson, observe the teacher's role, observe students' thinking. I chose this framework for weekly sessions because Borko et al. (2008) designed it to support PLC coordinators to orchestrate productive discussion around video cases, which seemed to be in line with the goals for my PLC. Each week two interns volunteered to provide a video-recorded teaching segment. At this point all five of my interns were participating in the PLC, and I wanted each intern to have the opportunity to record and analyze their own teaching. These self-capture videos formed the basis for discussions within the PLC (Borko et al., 2008, 2014; van Es et al., 2016, 2017).

The prompts and activities for weeks 1-4 and 6-8 can be viewed in Appendix C. During week 5 we met over zoom to synthesize the work we had done up until that time and explicitly practiced phase 2 (describing and interpreting observed events) and phase 3 noticing (integrating and elaborating on the observations). Prior to our meeting, I selected shorter clips from each intern's video recording from the weeks prior that I thought could provide rich discussion opportunities. The discussion protocol for this meeting can be viewed in Appendix D.

The work during the PLC set the stage for each intern's individual noticing and reflection work for the bulk of data collection periods of the second Guided Lead Teach and Lead Teach periods. Although the intent for this group was to prepare them for later reflection, it also provided keen insights into their experiences early in the semester. First, it helped me to continue answering Research Question 1 by allowing them space to discuss how teaching influenced their perception about what it meant to understand a concept. Second, by interacting with each intern about specific teaching instances observed through their video recorded lessons, I became privy to the elements of their teaching that they valued. This PLC experience provided invaluable information to me not only as a researcher but also as a field instructor.

Reflective Narratives

To gain insight into the lived-experiences of my interns, especially in connection with how they viewed what it meant to "understand X" and understand the relationship they saw between "aspects of self-in-community" and "aspects of self-in-mind" (i.e., the relationship they saw between the work of teaching and their own mathematics knowledge), we made use of a strategy called reflective narrative, and the interns engaged in this strategy during their second Guided Lead Teach period in the Fall semester and during their Lead Teach period in the Spring.

By this point in the project, only Matthew, Claire, and Scott continued on for this portion of the data collection, as the other two interns opted not to continue.

The end result of this portion of data collection was for each intern to create a lived-experience narrative for the instructional unit that they taught through in this phase of the internship. Their reflective narrative would ultimately make connections between the classroom interactions they viewed as transforming their own mathematical knowledge. To help them construct this narrative, I planned for them to engage in several reflecting activities adapted from Johns' (2010, p. 28) six dialogical movements, similar to the approach taken by Akinbode (2013).

Johns' (2010) six dialogs are: dialogue with self, dialogue with the story, dialogue with self and the story, dialogue with guides, dialogue with emerging texts, and dialogue with others. For this phase of data collection, the interns primarily worked through movements 1, 2, 4, and 5. Each data collection period was structured into three phases: maintaining a daily synopses log, thinking back over each week's written logs, and then constructing the reflective narrative at the end of the unit. In addition to these activities, the interns provided me with artifacts from their instructional units (lesson plans, daily activities, and assessments) to supplement the overall context of their stories.

The first phase of the process served as the interns' first dialogical movement. By maintaining a daily journal/account of the day's events, I intended these synopses to help them remember big events throughout the unit and for them to use these interactions as the grist for their reflections. In order to guide them along the process, I provided prompts to help focus thinking (these prompts are found in Appendix E and Appendix F – I updated them after the first semester). These prompts were designed with the aim of guiding the interns into making a record of their planning process, keeping as many “real time” monitoring/field notes as possible (writing down ‘aha’ moments, interesting questions students ask, etc.), and then immediately (or as close to as possible) writing down a recounting of what happened in the particular class.

The interns kept all written documents in a shared Google folder that I provided. This gave me access to their work as they were writing and gave me the ability to have digital conversations through the comment function. Such an intervention (i.e., personalized feedback and follow up questions) has previously been shown to help young teachers develop into reflective practitioners (Hatton & Smith, 1995; Spalding & Wilson, 2002) and this made-up the 4th dialogical movement. Further, at the end of each week of their instructional unit, each intern looked back over their daily logs to begin to identify themes in their classroom interactions and to reflect on those themes. This served as the second dialogical movement, and by responding to prompts (Appendix G), it aided the reflection into brainstorming for the end-of-unit Reflective Narrative.

After concluding their instructional units and all the response elements, the interns began to craft their reflective narratives. To help the interns get started with this, I not only provided more prompts for them to think about (Appendix H), but also had ongoing conversations with them (by an electronic chat function as well as other electronic correspondence) about the direction of their narratives. To help them identify the big ideas of their narratives, I referred them back to instructional patterns that they wrote about which seemed to play an important role in their experiences.

Supplemental Data

Throughout each phase of data collection, interesting moments arose out of the natural course of events. When these instances happened, I wanted to find out more about what the intern meant by a specific comment, so I asked them to provide more detail to me. This data proved, the few times it occurred, valuable to my project because it provided specific insight into some of the most influential aspects of their teaching work. I did not give any prompts for this or require that they write up their explanations to my follow up questions (one intern, for example, made a video explaining how he wrote up a grading key based on the curriculum's intended goals). Other examples of supplemental data were alluded to in earlier sections, specifically: artifacts from their teaching, observation notes, and conversations. For the most part this data served to fill out the stories of their teaching experiences by providing concrete examples and, in some cases, verifying their narratives.

Data Analysis

In the previous section I described how I collected data from various sources: (a) a pre-internship survey and interview, (b) written responses as a way of participating in the PLC, (c) written synopses and reflections over two instructional units, and (d) supporting data (artifacts, follow-up written or verbal explanations). Further, I transcribed all video recorded data (i.e., interviews, PLC discussions, supplemental data, etc.), and, once having collected the data and reformatted it into an easily readable form, I organized all data chronologically according to each intern. It made sense to organize the data this way because it was in line with my goal of understanding each interns' teaching experience as a case study over the course of their internship year.

I designed and viewed this data as an instantiation of their entire internship teaching experience, as best as I could replicate. In this section, I describe how I analyzed the data so that I could make claims about each intern's experience and how I used these claims to answer the research questions. For ease of reference a table showing the connection between research questions, data sources, and data analysis can be found in Table 3 – 3 below. Note that claims are discussed in detail in Chapter 4. In this chapter, they are only described briefly and are included to show the connection between how the data was analyzed and how research questions were answered.

Table 3 - 3

Connection Between Each Data Source, Claims, and Research Questions

Data Source	Goal of Analysis	Claim Argument Use	RQ Supported
Pre-Internship Survey and Interview	To tell the background story for each intern and to begin to get a sense of their current perception of their own mathematical understanding	Claims for Matthew, Scott, and Claire's	RQ 1
Learning to Reflect & Notice PLC	To add in initial influence of teaching mathematics content	Claims for Matthew, Scott, and Claire's	RQ 1 RQ 2
GLT 2 Reflective Narrative Process	To identify, describe, and interpret the teaching experiences that appeared to be the most influential to their thinking	Claims for Matthew, Scott, and Claire's	RQ 1 RQ 2
LT Reflective Narrative Process	To identify, describe, and interpret the teaching experiences that appeared to be the most influential to their thinking	Claims for Matthew, Scott, and Claire's	RQ 1 RQ 2
Supplemental Data	To become aware of and understand context specific teaching experiences that were not anticipated as being influential for the interns' thinking	Claims for Matthew, Scott, and Claire's	RQ 2

I worked cyclically through the data, applying a different lens during each iteration, so that I could identify patterns or insights within each intern's story (Stake, 1995; Yin, 2018). This process, in turn, helped me to establish claims about those teaching experiences that influenced a change in the interns' perceptions about what it meant for them to know mathematics. To guide my analysis, I relied on my research proposition (Yin, 2018), stated in the opening section of this proposal, that when a novice teacher claims, "I really didn't understand X, until I had to teach it," they are, in fact, claiming that they noticed a change in the quality of their mathematical knowledge, specifically they would have developed Pedagogical Content Knowledge, and this would give them the sensation of knowing the mathematics in a new way. This proposition, based on my understanding about the role of learning through teaching, and the role that noticing and reflection has on a teacher's practice, influenced the design of this project and was instrumental in how I looked at the data collected.

Analytic Method

My analytic approach was informed by the frameworks that influenced the design and implementation of this study. Because I am interested in the perceived influence that teaching practice had on the interns' perception of their own mathematics content knowledge, I developed a descriptive and concept-based coding system based upon the *Mathematical Knowledge for Teaching* (Ball et al., 2008) and *Learning to Notice Student Mathematical Thinking* (van Es, 2011) frameworks. Because I viewed all the data as representative of each intern's teaching internship experience, I used the same analytic approach for each segment of their data when it applied. In this section I will present my analytic method (see Table 3 – 4) by describing and demonstrating my coding process. I worked through the data in three cycles, beginning very broadly and becoming finer grained with each iteration. All analysis cycles made use of a grounded theory approach to organizing and scrutinizing the story.

Table 3 - 4

Analytic Method and Coding Scheme Overview

First Cycle
• Read through all data and made general observations about the data with a lens about making claims about each interns' experience in view of the research aim

Second Cycle
Learning to Notice Students' Mathematical Thinking Framework (van Es, 2011)
What Teacher's Notice
• Noticing Student Mathematical Thinking • Noticing Teacher Pedagogy • Noticing Behavior • Noticing Whole Class Environment

Third Cycle		
Mathematics Knowledge for Teaching (Ball et al., 2008)		
Subject Matter Knowledge		
Common Content Knowledge	*Specialized Content Knowledge*	*Horizon Content Knowledge*
- Simply calculating an answer - Knowledge not specific to teaching	- Presenting mathematical ideas - Recognizing what is involved in a particular representation	- Connecting a topic being taught to topics from prior or future years
Pedagogical Content Knowledge		
Knowledge of Content & Students	*Knowledge of Content & Curriculum*	*Knowledge of Content & Teaching*
- Leading a discussion of students' solutions - Evaluating students' claims - Modifying tasks to meet students' needs	- Appraising and adapting the mathematical content of textbooks - Modifying tasks to meet students' needs - Writing a grading key based on curricular goals	- Planning Classroom activities - Leading a discussion

First Cycle Analysis

To begin my analysis, I simply read through each intern's data and created analytic memos along the way. These memos served as an opportunity for me to make general observations about the intern's experience, pull out interesting moments that I thought spoke to my research aims and made connections to existing literature. This cycle of analysis served two purposes: first, it simply reminded me of what happened earlier in the school year (I began analysis in December at the end of the first semester but took a break and picked it back up in March after the Spring semester data collection concluded); second, it helped become familiar with the details and key moments throughout each intern's story.

Second Cycle Coding

Because this project relied on the intern's own observations, it made sense for me to pull an aspect of teacher noticing into my analysis. For this stage of my analysis, I used part of van Es's (2011) *Learning to Notice Student's Mathematical Thinking* framework. Her framework is organized into two categories, namely: What Teachers Notice, and How Teachers Notice. For my study, however, I only used the "what teachers notice," because I was interested in better understanding how their specific teaching experiences influenced their perceptions of their mathematical knowledge. Van Es's (2011) category of "What Teachers Notice" is further comprised of four elements: noticing student mathematical thinking, noticing teacher pedagogy, noticing student behavior, and noticing the whole class environment. These elements served as descriptive codes as I worked through each intern's data. This coding method allowed me to get a sense of what the interns paid attention to while they were teaching and what teaching experiences they highlighted in their writing.

This framework allowed me to parse and organize each passage into smaller chunks. So, for each written transcript, I first read through and identified larger chunks of moments that the intern highlighted and coded these as "what teachers notice." Within each "what teachers notice" coded passage, there were likely several types of coded segments that made up the entire noticing. The following excerpt from Scott's GLT2 Reflective Narrative demonstrates this coding process. Scott summarized the instructional unit from a student thinking perspective and a teaching pedagogy perspective.

> As students became further versed with the multiple strategies presented, we noticed that some methods had become fan favorites while others, not so much. Such personal preference validated our reason for teaching all methods as some students quickly found comfortability with certain strategies that others may not have. As a result, as we neared the end of my GLT2 period, we were able to evaluate how students would approach solving quadratics from multiple angles. And while a preference of method is allowed, some students elected to switch between multiple methods as some problems promoted one over another. From a teacher's perspective, this was an absolute success because at the end of the day, all strategies shall work, but some situations may already be prepared for specific methods.

In this paragraph, the following codes were generated:

- "As students became further versed with the multiple strategies presented, we noticed that some methods became fan favorites while others, not so much" [Noticing Student Mathematical Thinking]
- "As a result, as we neared the end of my GLT2 period, we were able to evaluate how students would approach solving quadratics from multiple angles." [Noticing Teacher Pedagogy]
- "And while a preference of method is allowed, some students elected to switch between multiple methods as some problems promoted one over another." [Noticing Student Mathematical Thinking]
- "From a teacher's perspective, this was an absolute success because at the end of the day, all strategies shall work, but some situations may already be prepared for specific methods." [Noticing Teacher Pedagogy]

All coding done throughout this analysis was through the qualitative data analysis computer program, MAXQDA2022. After I had coded all data, I was then able to view all coded segments organized by specific codes. It was necessary for me to break up the data in this way because it later helped me provide context to key passages by describing the specific kind of teaching task the interns were engaged in during pivotal moments in their reflections.

Third Cycle Coding

In addition to using the teacher noticing coding cycle, I analyzed the data for "mathematical tasks for teaching" through the lens of "mathematical knowledge for teaching." One assumption that I held to going into this study is that with the *Mathematical Knowledge for Teaching* (MKT) framework (Ball et al., 2008) one can attribute meaning to specific tasks a teacher does by interpreting their actions as demonstration of the kind of mathematical knowledge that are being developed. So, during this third coding cycle, I built off the work done during the second analysis phase by organizing their mathematics teaching tasks that the interns wrote about and subsequently interpreted these tasks as demonstrations of their developing MKT.

Recall that within the MKT framework, Ball et al. (2008) conceptualized a teacher's mathematical knowledge as comprised of Subject Matter Knowledge and Pedagogical Content Knowledge. For this phase of analysis, I organized my codes using the subcategories from Ball et al.'s (2008) (see Table 3 – 4 at the beginning of this section). Working off my assumption that the teaching work one does reflects the knowledge that they have, I organized and added to Ball et al.'s (2008, p. 400) description of mathematical tasks of teaching and assigned them to the subcategories of their MKT framework. The challenging part of this coding approach was identifying what exactly which kind of mathematical knowledge for teaching was at play during a particular task. For example, when engaged in a task, such as leading a discussion of students' solutions, multiple kinds of PCK are at play. The teacher must be aware of how to effectively lead classroom discussions in a way that gets at what students are trying to communicate and encourages other students to actively listen to and respond to their peer's ideas (KCT). At the same time, the teacher should be aware of what aspects of the student's idea to focus the conversation around (KCS).

The following excerpt from one of Claire's daily synopses from her second Guided Lead Teach Period demonstrates this coding process. In this passage, she responded to an earlier question I had about how her lesson planning process had developed up until this point in the semester.

> You commented previously asking what my lesson planning process has looked like for GLT2 - my mentor and I have resources from the other teacher who teaches precalc and one who previously taught precalc. We'll see what they have for the section we're covering, and then edit it to fit our classes' needs and what we think we need to emphasize. I'll then work through the notes myself, and then I pick out a warm-up to go with the lesson - usually either something from yesterday's work to reinforce, or something to connect to and start off the new lesson. Sometimes, after grading a test/quiz, it looks like a problem we had some issues with. I'll also pick out homework problems that match what we're doing/emphasizing in class. When I go to upload my notes to the Google Doc and type up my sequence of events, I normally make note of which parts I'd like to do whole class vs. small group, and any questions I have that I can think of in that moment.

In this paragraph, the following codes were generated:

- "My mentor teacher and I have resources from the other teacher who teaches precalc and one who previously taught precalc. We'll see what they have for the section we're covering, and then edit it to fit our classes' needs and what we think we need to emphasize." [PCK/Knowledge of Content and Curriculum]
- "I'll then work through the notes myself" [SMK/Common Content Knowledge]
- "Then I pick out a warm-up to go with the lesson - usually either something from yesterday's work to reinforce, or something to connect to and start off the new lesson." [PCK/Knowledge of Content and Teaching/Knowledge of Content and Curriculum]
- "Sometimes, after grading a test/quiz, it looks like a problem we had some issues with. I'll also pick out homework problems that match what we're doing/emphasizing in class." [PCK/Knowledge of Content and Student/Knowledge of Content and Teaching]
- "I normally make note of which parts I'd like to do whole class vs. small group, and any questions I have that I can think of in that moment." [PCK/Knowledge of Content and Teaching]

After having dissected the data using multiple lenses, I asked myself “what can I say happened in their experience?” and “what was the big takeaway from their story?”. In responding to these questions, I noticed patterns, recurring themes, and big moments in each intern’s experience. This formed the basis for making claims about their teaching experience with respect to how they viewed their mathematical knowledge. To provide further details to their claims, I pulled together the codes generated from each cycle of analysis to provide the language for answering my research questions with the goal of connecting their teaching work to their growing Pedagogical Knowledge.

CHAPTER 5: RESULTS

The goal of this study is to explore secondary mathematics teacher interns' evolving sense of understanding mathematics content as they reflect on their own teaching practice over the course of one academic school year. My hypothesis going into this study is that as novice teachers consistently participate in teaching practices and communities that help to develop their mathematics pedagogical content knowledge, this coincides with a sense of knowing the mathematics in a deeper way. More specifically, the research questions that guided the design and execution of this study are:

1. *What aspects of mathematical knowledge for teaching are reflected in the narratives that PSMTs provide about their learning during student teaching?*
2. *What teaching experiences do the interns emphasize as being critical in helping them understand differently the content they teach?*

In order to answer the above research questions and to explore the way secondary mathematics teaching contexts, understood as their own communities of practice, relate to novice teachers' developing pedagogical content knowledge (van Zoest & Bohl, 2005a), I present the results of this study in the form of narratives with the goal that these narratives serve as *arguments by example* (Weston, 2017). For each intern's narrative, I first begin by describing the intern's background, internship placement, and teaching experiences throughout the academic school year. Following this description, I then set forth to describe their overarching story, in the form of a claim. For the claims, I provide evidence by describing key moments through their internship that bring the moral of their story into clearer view. Finally, following this line of presenting evidence, I will seek to unpack the evidence and discuss the importance of the claim within the context of better understanding communities of practice (van Zoest & Bohl, 2005a;

Wenger, 1998) and their developing mathematics pedagogical content knowledge (Ball et al., 2008). Finally, after laying out each intern's narrative, I circle back around to answer the research questions using the claims made during the narratives.

The Cases

In earlier sections, I described my role as the field instructor for these three teacher interns and how the schedule of their teaching internship dictated, to an extent, my data collection schedule. In this section, I communicate the results from the data organized by each intern's case and roughly follow the flow of the internship year, according to my data collection schedule (i.e., Pre-Internship Data, Preparation Phase Data, Guided Lead Teach 2 Data collection period, and the Lead Teach data collection period). While I designed my project with the intention of getting the fullest sense of the interns' experiences through their Reflective Narratives, moments of rich thinking happened throughout the internship at unexpected times. Since case study research is grounded in lived experiences over time, I often bring in unplanned data sources such as when an intern formally explains their experiences to me (e.g., written or video-recorded explanations) as a follow up to an earlier conversation we had. I begin by telling both Matthew and Scott's narrative, mainly because they were both placed at the same school for their internship and there were similarities in their experiences, and then I will move on to discuss Claire's experience. The names of interns and institutions have been replaced with pseudonyms to protect the anonymity of the participants and host schools. As mentioned in the previous chapter, all interns graduated from the same local university, and, at times, they refer to its name in their data which I will refer to as Midwest State University (not the same institution as Midwestern State University in Texas).

The Case of Matthew

Matthew graduated from Midwest State's secondary mathematics teacher program at the end of the 2020-2021 school year like the rest of the interns in this study. From this program, he earned a bachelor's degree in mathematics, a minor in chemistry, and enrolled in specific teacher education coursework. Prior to the internship, I sat down with him to get his perspective on the upcoming school year. Matthew felt confident about being in the classroom, primarily because of his experience as a tutor and from the coursework he had in his teacher preparation program, as can be seen in the passages below:

> I feel very prepared to teach math. I currently tutor high school students and actually have been substitute teaching for a high school class while the teacher was on maternity leave since mid-March.
> (Matthew, Pre-Internship Survey, Pos. 1)

At this moment, Matthew highlights his extensive experience tutoring core high school mathematics content. His assumption, coming into the school year, was that this tutoring experience gave him a thorough content understanding; in other words, he was confident about his ability to teach high school content because of his robust common content knowledge (CCK) (Ball et al., 2008). Further, it seemed that there was some separation in his view of content understanding and pedagogical understanding, as he theorized that his knowledge about teaching would deepen through lived teaching experiences, in other words his "aspect of self-in-mind" teacher knowledge could deepen with experiential "aspects of self-in-community" teacher knowledge (van Zoest & Bohl, 2005a).

> I think Midwest State did a good job educating me about what it means to be a teacher, how to interact with students and about students from many different backgrounds, experiences and perspectives. *A lot of "head knowledge" about teaching. I think I would like more practical experience actually using that knowledge*
> (Matthew, Pre-Internship Survey, Pos. 1)

Prior to internship, it did not seem that he was thinking about the possible effect teaching practice would have on his own content knowledge. For Matthew, he felt confident with his content understanding, particularly the algebra courses, and his ability to explain concepts clearly, but perhaps not as confident with pedagogical knowledge.

Matthew's Internship Placement

For his teaching internship, Matthew was placed at Big Horn High School, a large suburban high school (approximately 1700 students, ~35% minority) in the Midwest United States. Matthew's mentor teacher, Greg, has been teaching at Big Horn High School for about 10 years, with close to 20 years total teaching experience, and had mentored a total of five student teachers prior to this school year. Mentor teachers play an important role in an intern's professional development and experience (Izadinia, 2015), especially at the beginning of the school year; often times, interns will observe and shadow their mentor teachers for the first part of the school year. So, it is common to see interns mimicking their mentors' instructional style early on (Izadinia, 2015; Weasmer & Woods, 2003).

The layout of Matthew's classroom context supported a traditional high school mathematics approach. The room was organized in a way that one would expect to see in a typical mathematics classroom, a fronted classroom where most of the board work was done by the teacher and students sat at typical high school desks organized into rows. The rows were organized into three general sections (two sections perpendicular with side walls, one section perpendicular with back wall) all, in general, facing the board at the front. In addition to a white board spanning the entire front wall, the classroom was equipped with a smartboard and projector system, which Matthew used extensively throughout the year.

Matthew and Greg regularly experimented with different desk arrangements, pre-organized groups, and seating charts. From post-observation discussions with them, it seemed many of these decisions were made with classroom management strategies in mind. While they admitted that group work was desirable, they described it as challenging finding ideal groups because of their perception that some students would not work well together. In fact, when students sat individually in rows, their seating assignment was pre-assigned and normally changed up every few weeks as an attempt to find optimal seating placement to support student engagement.

Matthew's Internship Teaching Experience

Matthew and Greg were assigned two sections of Algebra A/B (i.e., essentially what is commonly called Algebra 1) and two sections of Algebra C/D (i.e., Algebra 2). At the beginning of August, Matthew took on one of the Algebra A/B courses as his focus class. As the year progressed, both Matthew and Greg explained to me their major instructional challenges of teaching in their specific contexts. Due to the COVID-19 pandemic, many of the students lacked much content preparation and were consistently unengaged in class, much of this being attributed to their lack of accountability with online learning experiences during the previous year.

In general, Matthew's teaching style mimicked his mentor teacher's style and, in general, was dictated by the specific worksheet they were using that day (these worksheets would prove to be very influential for Matthew's thinking). Usually, class would begin with a warm-up task, usually reviewing previous days' work or reviewing a skill necessary for that day's lesson objective, followed by an introductory example and whole class discussion around new content, after which students either worked in groups or individually through a worksheet. It did seem, however, that teachers had some sense of autonomy about how much time to spend on certain topics, depending on how well they observed that their students were doing, how to directly teach ideas and choose examples by which to demonstrate these ideas.

Matthew was adept at clearly explaining ideas and paced his instruction well. Throughout individual or group work he consistently circulated the classroom to help students through problems. From my personal observations of his lessons, it seemed that he had to reteach much of what he already demonstrated during the whole group discussion while students worked their way through the worksheets. Students would often make comments such as "I don't know how to do it" or asked clarifying questions. He seemed to take the view that students, in fact, could not reason through these tasks on their own or with their group members because he nearly always showed them how to do the problem.

Big Horn's mathematics department had done away with assigning textbooks in their courses. Instead, they had designed courses around carefully sequenced worksheets. At times, these worksheets served as an introduction to a new concept in lieu of a classroom lecture; other times they served as practice of a concept they had seen their teacher demonstrate. Here is Matthew's description of the curricular context.

> Lastly, a final thing that has grown my ability to and understanding of teaching content is that Big Horn does not use a textbook, but rather a curriculum of crafted worksheets that give students to explore new ideas and practice old ideas. Some days, this functions very much like a textbook: here is a warmup where we do an example problem together, then we'll discuss and make sure everybody is on the same page, and then here is this worksheet for you to practice what you learned as well as try to extend your thinking. Other times though, it's much different. Sometimes the worksheet has very clear directions and pointed questions so that I don't teach anything up front at the board, but instead the majority of class time is working and struggling through the worksheet with their peers. This has really challenged my idea of what a math classroom is "supposed" to look like and it continues to challenge my ideas of what content is essential to understanding the overall course content AND sets you up for your future classes. (Matthew, GLT2 Reflective Narrative, Lines 88-99)

This description creates a good picture of what you might expect to see when walking into any of Matthew's classroom. Behind the scenes, Matthew elaborated more on the structure of this in-house curriculum. Algebra A/B was designed to explore ideas systematically, according to function type; that is, every unit had a similar goal of finding properties of the particular function, writing rules (more on this later) with that function, inverting and solving with that type of function, and mathematizing situations which are like story problems. This curricular context, as we will see, proved be a key influence not only on his teaching experiences but how his pedagogical content knowledge developed over this school year.

A Curricular Community of Practice Frame on Matthew's Case

> *Exposure to and participation in Big Horn's self-developed mathematics curriculum was a central influence on Matthew's first teaching experience. From this curriculum and by collaborating with experienced colleagues, he was given a consistent structure to support students' exploration of ideas, introduction to new ways to explain these ideas to students that were consistent with overall goals of the course and learned how to use his curricular knowledge to assess their understanding. Through these experiences, he described coming away with a sense of knowing the "whole story" of high school algebra and adding "new layers" to his previous understanding.*

In order to present evidence for this claim, I will first highlight four events throughout Matthew's internship experience that helped to develop his pedagogical content knowledge as a result of the curricular context he found himself in. Generally, these events take place prior to his internship, in the first two months of the Fall semester, and during his second Guided Lead Teach period. This sequence of events will show Matthew's trajectory of maturing Knowledge of Content and Curriculum (KCC) (Ball et al., 2008) and its influence on other aspects of his Pedagogical Content Knowledge (PCK). Following the description of these events, I will then unpack the evidence and expound upon why these moments are important to highlight and identify the transferable knowledge from Matthew's story.

In the semester leading up to his internship, Matthew met with his initial mentor teacher (who later stepped away from mentor teacher to the role of department head) and through this initial meeting learned about Big Horn's self-developed mathematics curriculum, which he communicated to me during my initial interview with him.

> One of the things I'm excited about is how he kind of explained how the whole math system department curriculum, whatever we're doing it for their works. Because, as I did, and found out from my Your Choice Project they don't use textbooks, but they do, he was saying it's, it's very like adaptive. So, it's a lot of, like, it's very fluid, you know, depending on how the class is doing what they're struggling with, like, it moves a lot, which he really likes because it allows him to kind of meet his students where they're at. He said it might be a little more challenging for me because it's not going to be like, you know, plan a month in advance where it's like, you know, chapter 1.1, 1.2, 1.3. It's a lot different you know it's not going to be that structured, but I'm excited for that and I think, too, *it'll help me really understand like what's important and how to order topics and concepts* and not just like rely on the textbook, because that's kind of what I've always done it's like "okay this is the first chapters, this is the first thing we're going to learn."
> (Matthew, Pre-Internship Interview, Pos. 61)

As understood in this passage, Matthew anticipated what his internship would be like by highlighting his initial sense of how teachers adapt the lessons and units for the needs of their own students as well as how it might give him a broader sense of what is ultimately important in high school mathematics courses. He expounds more on how he might be able to use the curriculum by responding to his individual students' needs when I had followed up with a question.

> CJF: So, do you think you'll have the opportunity to try some things that you've been hoping to try in person?
> Matthew: I think so, um, he did make it sound like I would kind of have some freedom on that. He said that, *especially at the beginning I wouldn't have to worry a ton about like trying to like create content and craft lessons and stuff*. It would be like he would help me a lot with like, you know, "This is what we'll talk about today, but then kind of approach it how you'd like." …. But it did sound like there's a lot of freedom, and especially talking with him and like what he likes to do. *He seems like he likes to do a lot of discovery stuff with the students where they can you know kind of work through things, discover it on their own, kind of*

like, almost stumble across information, which I'm kind of excited about too. And it sounds like there is a lot of freedom and like how to go about that.
(Matthew, Pre-Internship Interview, Pos. 79-80)

Going into his internship year, Matthew was excited to experience something new, when it comes to math instruction. He was excited for the opportunity to learn from a department how to identify the big picture ideas in his courses, structure lessons that helped students explore those ideas, and adapt curriculum to respond to their needs.

As I mentioned in the general description of Matthew's internship year, he began the year by assuming responsibility for a section of Algebra A/B, his focus class. The next episode of his story occurs during his first Guided Lead Teach period, roughly a month and a half into his internship in early October. This moment in his story comes from a share out that he wrote up for the other interns during our asynchronous professional learning community (PLC) and demonstrates the curricular context's influence on his own thinking up until this point. In the previous week's PLC responses, he had made an off-handed comment that lesson planning required intentional curricular choices which required him to talk with other teachers unless he have the "wrong idea;" this was interesting to me, and I had asked him to put together a more in-depth write up to share with his peers. While I pull out a portion of this write up in this section, the full share out is included in Appendix A.

The content and the way it is structured at Big Horn has been new and enlightening to me because *it is so different from how I was taught*.
(Matthew, Weekly PLC, Pos.4)

The goal of this lesson was to analyze their thinking process as a whole class and generalize this into the point-slope form of a line. Getting to this lesson was not as simple. My mentor teacher and I talked and talked about how to best introduce the point-slope form, but we seemed to get stuck. *The only way I knew how to do it was in the following way*:
$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} \rightarrow (y_2 - y_1) = m(x_2 - x_1) \rightarrow$
$y_2 = m(x_2 - x_1) + y_1 \rightarrow y = m(x - x_1) + y_1$

> However, we knew this wasn't how we wanted to teach it. So, we talked to another teacher, and they gave us the suggestion of completing missing values in a table. They said, "most students will know what to do to find the next value when they see a table, and then you can generalize what they are doing and get the rule that way.". So, we did, and it worked well…. (Full worked out example and commentary) ....
>
> Teaching the lesson/concept this way was definitely longer and more involved than simply manipulating the slope equation. However, *it really is a deeper understanding of the material when taught this way. It takes the learning away from moving around numbers in an equation to thinking about your thinking process*, which is a very important skill to have. This lesson, in particular, made me realize that *just because I know how to teach something one way, doesn't mean it is the best way, but it also means that I might not have a full or deep understanding of the concept if I can only teach it in one way*. I always knew that knowing how to apply or use a concept (such as this one) is a different skill and a different level of understanding than being able to teach somebody that same concept. But now I know that being able to teach a concept in multiple different ways is an even deeper understanding of the content. It's like an extra layer of understanding.
> (Matthew, Weekly PLC, Lines 3-15, 39-49)

This passage is revealing for several reasons, and it provides an important look into what Matthew had taken away from the curriculum up until that point. First, this lesson helped him see the limits of his previous content knowledge and helped him see how certain approaches to explaining ideas may not support student sense-making. Second, it gives us an example of how Big Horn's mathematics department functioned and how they made instructional choices based on students' prior knowledge and with the overall curriculum in mind.

As the semester progressed, Matthew became more aware of just how much of an impact Big Horn's mathematics curriculum influenced his own thinking. By November, during the second Guided Lead Teach period, Matthew reflected and reported on a unit of instruction within his Algebra A/B course, specifically a unit about quadratic functions. During this time, there are two specific instances that demonstrate just how important Big Horn's curricular context is for his teaching practice and his thinking about the content. Like the description of his experience during the first Guided Lead Teach Period, I will include portions of larger texts, namely a video transcript of his description of the assessment decisions that guided his grading of a Learning Check (Appendix B), and his Reflective Narrative (Appendix C).

The first instance in the second Guided Lead Teach period where Matthew demonstrated the role that Big Horn's curricular context had on his teaching practice happened at the end of the first week of teaching through quadratics. Up until that point in the data collection period, I required him to write up a daily synopsis of what was happening in class that day by responding to prompts. At the end of this first week, instead of covering new content, Matthew and Greg had planned for students to complete a Learning Check. Instead of writing up a synopsis about this day, he decided to take the opportunity to explain to me both the purpose of that specific Learning Check within the unit and, at the same time, explain Big Horn's greater assessment practice. To do this, Matthew recorded himself with Zoom and essentially walked me through the Learning Check answer key in One Note while also added commentary and notes within the program. To best capture this moment, Appendix B includes both the transcript of his verbal description of the grading process as well as screenshots of his notes that he was referring to.

In this first instance, Matthew demonstrated mastery of being able to communicate the instructional goals of the unit and how they assess students on those goals. He began this video by explaining how they design and assess learning checks (Lines 4-16).

> So, every Learning Check and test that we do is broken down into goals.
> So, on this Learning Check, you can see that it's broken down into goals. So, on this one, we are checking goal number six and goal number seven.
> And then you see these: Basic, Proficiency, and Expert. So, this is the part that I'm going to talk about right now. Um, as well as kind of the long-term grading purpose of these learning checks.
> So, switching back here, so like I said, everything is broken down into goals and we grade each of those goals on a four-point scale.

From this introduction, he provided a thorough explanation of the kind of student responses one might expect to see for each element of the 4.0 scale (see Lines 14-67), description of their modified grading scale (Lines 69-83), how they ultimately assign grades to students by unit learning goals (Lines 84-138), and how he might write up an answer key for this specific Learning Check and how he might assess to a partially incorrect student response (Lines 165-223). While Matthew intended this video as a way for him to explain what I might see on the Learning Check, it communicated much more to me about his content knowledge for teaching, namely it indicated to me that he was able to make use of the department's curricular goals as a way to assess student understanding efficiently and helpfully. By the end of the video, he concluded with a statement about how the grading system present at Big Horn helped him get a better understanding of each student's thinking as well as how to use the data from assessments in a way to reflect on his own teaching.

> So, I hope this helps, as far as the grading system and what the learning checks look like. Again, if you have any questions about this stuff, I'd love to talk about it. I love grading. *It's really good for me because it helps me see really where the students are at what they need to work on*, but also helps me understand like, "oh, I see a lot of the students messed up this part. *So that means maybe as the teacher, I need to focus on this aspect of the task a little bit better*."
> (Matthew, GLT 2 Learning Check Video Transcript, Lines 225-229)

The second instance that exemplified Matthew's curricular knowledge was within his Reflective Narrative at the end of this Guided Lead Teach period. Not only did Matthew reflect upon the unit but he also extended it to express how the curriculum influenced his entire internship. The crux of Matthew's story came at the beginning of his Reflective Narrative (italics added for emphasis):

> Yes, this is my first time teaching in a classroom full time, but it is not my first time teaching. For starters, in my TE 407 class, we had an opportunity to work with Midwest State students in a college algebra class where we co-taught two lessons with a partner. On top of that, this is my 5th year working as a math tutor

> for students from several different, local high schools and middle schools. I have extensive experience in Algebra 1, 2 and Pre-Calculus and those are the classes that I am teaching right now, with my focus class in Algebra 1. *Because of this experience, I thought I knew how to teach and explain the content and what it looked like for my students to understand the material*. However, working with Big Horn's math department has completely turned this theory on its head. *It's not that before I didn't know the content or that I couldn't explain it, but I realized that up until this year I didn't really know what it meant to teach the content*. Big Horn teaches students the content in ways that I have never even experienced before, and I think that it does an amazing job while doing so. They do a great job emphasizing the "why" of the math involved, as well as allowing space for students to discover and make connections on their own instead of simply telling them things found in the chapter of the textbook.
> (Matthew, GLT2 Reflective Narrative, Lines 8-22)

For Matthew, the singular catalyst in his maturing sense of what it means to teach mathematics content was the curricular context that he is in. He mentioned that while he had instances of teaching experience that gave him confidence with regards to his subject matter knowledge and his ability to explain how he understood the material, it is precisely learning to teach within Big Horn's mathematics department that introduced him to an approach to teaching mathematics that changed his view on what math teaching and learning should look like.

As we saw in his video explanation of the department's assessment practices, Matthew's day-to-day teaching skills were directly influenced through interactions with the more veteran teachers in the department. This can also be seen in the following passage (italics added for emphasis):

> On the first day of GLT2, we discussed each of the coefficients in vertex form and talked about what their meaning is. This is one example where Big Horn's curriculum differs with my experience. When I learned about the coefficients, I learned about them in association with transformations and talked about how each letter changed the graph. However, in our department meeting before this unit, one of the veteran teachers and our department chair both said that it would be a mistake to interpret this lesson as transformations. Instead, this is paraphrased what they said:
>
> "It would be a mistake to interpret this lesson in the unit as simply a lesson on transformations. Instead, *we need to think about our rule as a series of steps that*

take an input and transform it into an output, and each of the coefficients are a part of that series. For example, changing h shouldn't be thought of as 'moving the graph left and right'. It should be thought of as 'when I change my h value, I am now subtracting h from my previous input. That means to get the previous output, I need to add h to my input'. This is the same for the k value. I do all of my steps to the input and the very last step is to add k. Therefore, when changing k, I am changing the very last thing added to my output, so as a result my output would increase or decrease based on k…. Also, *when thought of as a series of steps, this creates a necessity for the mirroring/symmetry that we see in parabolas*. $(x-h)^2$ can be seen as finding the distance between our input x and the value of h, and squaring that distance. Therefore, if 2 input values are equidistant from h, then the $(x-h)^2$ part in the rule would have the same value, and lastly adding k to that value will still ensure that both inputs equidistant from h will have the same output, thus creating symmetry about the line $x = h$"

This was all a challenge to my previous thinking! For starters, *I needed to start thinking of an equation as a series of steps that changes an input to an output*. In fact, this is why they started calling it a rule instead! The word "equation" is implying that the left and right side of the equal sign are indeed equivalent, and that is not the emphasis we want to have. So, by changing to the word "rule", we more strongly imply a course of action to be followed. On top of that, *I was really challenged by my department chair about this idea of "mathematical necessity."* He said that when we create scenarios or showcase situations in which there is a mathematical necessity, it gives meaning to our work and eliminates the ideas of "we're just doing what we've been told" or "math has all these made-up things that don't make sense and I have no idea where they come from or why it's important".
(Matthew, GLT2 Reflective Narrative, Lines 41-73)

This passage is particularly interesting because we see Matthew being introduced to important curricular design principles that a teacher may not understand if teaching directly out of a textbook. Specifically, he picks up on the rationale for supporting sense-making using the word "rule" instead of "equation." This choice of vocabulary evidently has been a purposeful, department-wide decision that Matthew gradually became aware of throughout the semester. Additionally, by way of veteran teachers, Matthew is introduced to how tasks are selected and sequenced by the idea of "mathematical necessity." This is a curricular feature that I became aware of through observations, both in his classroom and in the two other interns' classrooms at Big Horn; often, the worksheets built towards introducing a new idea by first connecting back to an earlier concept and then "throwing a wrench" into the students' problem solving by giving them a task that was an extension of what they had done before but with a slight twist. This is what I think Matthew meant by "creating scenarios or showcasing situations in which there was mathematical necessity."

Finally, Matthew reflected on the curricular context that influenced his understanding of what it meant to teach quadratic functions by making an astute observation. In the following passage, he goes back to a similar unit he had designed in one of his teacher prep courses, early in his undergraduate coursework.

> Overall, my student teaching experience and especially these past 2 weeks of our quadratic unit has challenged me and deepened my understanding of quadratics and what it looks like to teach a unit on quadratics. Going back to TE 150, I wrote a quadratics unit for Algebra 1 students, very similar to the students I am teaching now! It was very interesting and enlightening to read back through that unit I created and compare it to the unit I am teaching now. *What I am teaching now seems to be much more simplified and 'why' focused, and much less focused on procedures*. I am teaching students skills that help them reason through solving problems and in teaching them the why and showing them the why, I believe this is leading to a deeper understanding of the content compared to the unit I made in TE 150. On top of that, *I think I have a much wider range of ways to teach the content than I did before*. I used to have a huge emphasis on factoring or the quadratic formula to solve quadratic equations, but now I understand how useful completing the square can be and especially how naturally it can be used to build off of students ' understanding of linear rules and inversing linear rules. Most of all*, I've learned how important it is to think of the class as a whole story and thinking about how each unit connects to the next*, not just how content connects within a unit. Big Horn does a very good job with this storyline idea and certain things such as using the word "rule" or thinking about an inverse as traveling a path on a map backwards really creates coherent understandings that transfer from unit to unit and also from class to class.
> (Matthew, GLT2 Reflective Narrative, Lines 163-181)

There are numerous important features about Matthew's experience present in this passage. First, as mentioned previously, Matthew had been teaching at Big Horn for about 2.5 months as he prepared for teaching the quadratic functions unit, and he had participated in department meetings and worked as part of an instructional team with an experienced mentor teacher. Also, the way the curriculum was designed to structure content in a consistent way, he had come away with having a greater sense of the “storyline” of high school algebra. As he mentioned earlier in the Reflective Narrative, he began to take notice of the sequencing and selection of tasks as a way to move the storyline along in a way that provided students the opportunity to see the mathematical necessity of ideas and procedures. Because of these things, as Matthew stated, he came away with a “much wider range of ways to teach the content than before.” Having gained a wider repertoire for teaching ideas in a way that supported the department’s goal of supporting student sense-making, he then looked back at a similar unit he had planned as a course project. Without Big Horn’s curricular influence, Matthew recognized his inclination to teach procedurally.

Key Takeaway from Matthew’s Case

Key Takeaway: When novice teachers learn to teach within well-defined and coherent curricular communities of practice, they can develop mature Knowledge of Content and Curriculum (KCC) within their subject area. In turn, using their KCC, they can apply this knowledge to other aspects of their teaching and develop other aspects of their Pedagogical Content Knowledge, particularly Knowledge of Content and Teaching and Knowledge of Content and Students. All things considered, this practical knowledge for teaching complements their Subject Matter Knowledge.

It is important to think of Matthew’s overall experience in view of the curricular community of practice he learned to teach in (van Zoest & Bohl, 2005a; Wenger, 1998) and its influence on his sense of understanding the algebra content he taught this semester. Considering this influence, I will unpack how the Big Horn curricular community of practice played a central role in developing Matthew’s knowledge of content and curriculum (KCC) and this, in turn, influenced other aspects of his Mathematics Knowledge for Teaching (MKT), particularly his Knowledge of Content and Teaching (KCT) and Horizon Content Knowledge (HCK) (Ball et al., 2008).

As Wenger (1998) explained, a community of practice is defined and cohered by the following three characteristics: a joint enterprise (something to do), a mutuality of engagement (others to do it with), and a shared repertoire (physical or social resources by which to act). Big Horn's Mathematics Department, as observed through Matthew's narrative, meets these criteria. First, they were involved in a *joint enterprise* of teaching their students mathematics; further, they were dedicated to teaching their students in a way that supported their sense-making and connection-making of mathematical ideas across units. Second, the department had *mutual engagement* amongst central and more peripheral participants – that is, the teachers who spearheaded this curricular design for the department, other teachers who went along with the curricular changes, and novice teacher interns all worked within the curricular framework and worked collaboratively to apply this framework to individual classroom. Finally, because of the department's decision to forego textbooks, they created a physical *shared repertoire* (course worksheets, activities, assessments, and access to unit plans on a department share-drive), and a social *shared repertoire* in settings such as department meetings where they would discuss upcoming units, lessons, etc. in view of their overall curricular goals and in more informal discussions with colleagues.

This was the setting in which Matthew learned to teach mathematics and came away with curricular knowledge that framed the rest of his work. Van Zoest and Bohl (2005a) summarized Wenger's (1998) view of learning within communities of practice below, and with this in mind, I will describe Matthew's learning to teach within Big Horn's mathematics department.

As one learns (either within or outside of a community), one develops new ways to participate with, and within, communities. The whole of a person's history of participation and knowledge is called his or her experience. When learning happens in a community, it takes place because of an imbalance between a person's experience and the community's regime of competence (van Zoest & Bohl, 2005b, p. 322).

Viewing learning to teach with this lens helps us to better understand Matthew's experience. First, prior to his internship year, Matthew had many teaching experiences (i.e., extensive one-on-one tutoring, substitute teaching, and non-academic teaching positions) and because of these experiences had confidence coming into the classroom, mostly because this gave him recent experience explaining subject matter to students. While these experiences are helpful, potentially helping him gain a sense of the overall picture of a course (especially if tutoring the same student consistently over the year), they were limited in scope and application. When he started his internship at Big Horn high school and became more of a central participant, through taking on more lead teach responsibility and participating in department meetings, he noticed an imbalance between his perceptions of what it meant to teach mathematics as a tutor and what it meant to teach mathematics as a teacher at Big Horn High School. Specifically, during the department meeting that he referenced in his Reflective Narrative (lines 41-73), he communicated his awareness of a community *regime of competence*. Again, Wenger (1998), as interpreted through Van Zoest and Bohl (2005b), described this concept in the following excerpt:

Within any one given community of practice there are certain types of knowledge and ability, specific modes of participating, and particular ways of experiencing that participation that are communally held and valued… A *regime of competence* is the collection of modes of participation in the community that are held to be relevant to its enterprise. Competence, then, is not something that someone has as an individual. It is, rather, only defined in relation to some undertaking and the various forms of participating in that undertaking (2005b, p. 321).

In Matthew's reflection, he summarized the department's application of curricular goals on teaching specific content, and he used this to make comparisons with his initial tendency to explain algebra content and concluded that the department's approach was better because it was more consistent with their joint enterprise of supporting students' sense-making.

Prior to and early on in the internship year, Matthew was a *peripheral participant* (van Zoest & Bohl, 2005a; Wenger, 1998) in the Big Horn Mathematics Department. As his internship progressed, his role took on a more inward trajectory, and, as he took on more lead teaching responsibility and participated in department meetings, he became a more *central participant*. Because of this experience, he developed KCC (Ball et al., 2008) specific to his teaching context. He became aware of the curricular approach almost immediately, first because of the shared curricular materials and worked to implement them as faithfully as he could. As he got further into his internship year, he began to appreciate the consistent way the curriculum structured exploration of content. Every unit in the curriculum is structured similarly by setting out to achieve the following goals: finding the properties of a given function, writing rules in different forms for that function, solving problems using inverse operations with that type of function, and mathematizing (which are like story problems) situations. This consistent structure was an important feature of Matthew's experience, and he communicates how teaching in this context helped to give him a more connected perspective of the content:

> Most of all, I've learned how important it is to think of the class as a whole story and thinking about how each unit connects to the next, not just how content connects within a unit. Big Horn does a very good job with this storyline idea and certain things such as using the word "rule" or thinking about an inverse as traveling a path on a map backwards really creates coherent understandings that transfer from unit to unit and also from class to class.
> (Matthew, GLT2 Reflective Narrative, Lines 175-180)

In addition to working within the consistent curricular structure, Matthew became competent within Big Horn's curricular community by demonstrating evidence of KCC through communicating the rationale behind curricular decisions to focus on emphasizing, such as the choice of emphasizing the word "rule" instead of equation. Further, he noticed the idea of "mathematical necessity" for the way tasks were selected and sequenced. With both of these curricular understandings, Matthew was able to make instructional decisions that were both consistent with the overall curricular enterprise and supportive of student understanding.

Matthew's story is important because it gives an example of how a novice teacher's instructional practices are influenced by their curricular knowledge and the community within which they learned to teach. In addition to developing KCC, Matthew developed a form of KCT that was consistent with the curricular goals set out by the department. He described how the impact of working within the Big Horn mathematics department gave him a wider repertoire of teaching strategies and how teaching through the unit gave him a deeper understanding.

> I am teaching students skills that help them reason through solving problems and in teaching them the why and showing them the why. I believe this is leading to a deeper understanding of the content compared to the Unit I made in TE 150. On top of that, *I think I have a much wider range of ways to teach the content than I did before*. I used to have a huge emphasis on factoring or the quadratic formula to solve quadratic equations, but now I understand how useful completing the square can be and especially how naturally it can be used to build off of students ' understanding of linear rules and inversing linear rules.
> (Matthew, GLT2 Reflective Narrative, Lines 169-165)

Matthew notes that because of the curriculum's emphasis on justification for procedures, it helped to deepen his own content understanding and helped him see a connection between concepts. This emphasis on justification of procedures seems not to have only shown a different way to present ideas but also to have helped deepen his own understanding of the procedures of quadratic functions. So, when he taught through the unit and developed a KCT consistent with the curricular context, this, in turn, also deepened his own Common Content Knowledge (CCK) by helping him see the connections between procedures (i.e., he specifically mentions how he used to solve quadratic equations compared with the procedure he learned through teaching).

Well-developed curricular knowledge also has an influence on a teacher's assessment practices which makes use of both KCT and KCS. As we saw in Matthew's description of his assessment practice on the Learning Check, he was able to apply his curricular knowledge in a way that was instructive for students and made use of his experience with student thinking to create a rubric of possible student responses. Then, as the semester progressed, Matthew's responsiveness to student thinking in support of learning proves to be helpful for his own thinking (KCS) as well. By assessing students' interactions with other students and reflecting on ways to be responsive to students seems to further Matthew's thinking by helping him gain a repertoire of conceptual skills. Prior to this teaching experience, Matthew's idea about how to teach the content stemmed from the way he best understood the idea. Now he views understanding an idea by the number of different ways he can explain the same concept (using different representations and using those ideas to make sense of procedures) (KCT). In summary, Matthew's role as a teacher encouraged him to think about mathematics in a qualitatively different way than he had previously.

To conclude this section, I string together many of the comments we have already observed Matthew make throughout the Fall semester to reinforce the transferable knowledge from his story, in his own words:

> The content and the way it is structured at Big Horn has been new and enlightening to me because it is so different from how I was taught. It's not that before I didn't know the content or that I couldn't explain it, but I realized that up until this year I didn't really know what it meant to teach the content. Most of all, I've learned how important it is to think of the class as a whole story and thinking about how each unit connects to the next, not just how content connects within a

> unit. I think I have a much wider range of ways to teach the content than I did before. What I am teaching now seems to be much more simplified and 'why' focused, and much less focused on procedures. Now I know that being able to teach a concept in multiple different ways is an even deeper understanding of the content. It's like an extra layer of understanding.

The Case of Scott

Scott, also a graduate from the regional university's secondary mathematics teacher program at the end of the 2020-2021 school year, did not always know that he wanted to be a high school mathematics teacher. For the first two years of his undergraduate studies, he pursued a business degree. It was during these first two years, however, that he experienced being a teaching assistant in an introductory computing class. Further, after he made the switch from studying business to mathematics, he again served as a teaching assistant in the department's College Algebra course. He described himself as a very "hands on TA," and these initial teaching experiences were, in part, influential on his decision to pursue a secondary mathematics teaching licensure from the university's teacher preparation program. Through this program, he earned a bachelor's degree in mathematics, and enrolled in specific teacher education coursework.

Scott's Internship Placement

Like Matthew, Scott was also placed at Big Horn High School. His mentor, Drew, had been teaching at Big Horn High School for about twenty years and was widely recognized as an expert mathematics teacher in his school and in the district. For Scott, who was excited to begin his internship and make as much progress as he could, having Drew as his mentor was ideal for him to better learning mathematics teaching. Drew had a close working relationship with the university's teacher education program, often serving as a host for undergraduate preservice teachers' field placements, a commentator in mathematics education publications, and had served as a mentor teacher for about a dozen interns over his many years in Big Horn's mathematics department. As we will see later, Scott's opportunity to develop specific aspects of his Pedagogical Content Knowledge (PCK) was due, in part, to the *overlap* between his classroom community of practice and the university community of practice (van Zoest & Bohl, 2005b; Wenger, 1998).

The layout of Scott and Drew's classroom context supported a collaborative style of teaching and learning. Throughout the room, desks were clustered into groups of three to four students each and were spaced enough so that Scott could easily circulate amongst groups throughout their activities. While the room had a front whiteboard (most board work was done by the teacher) there was also a whiteboard on the back of the room, which students regularly used to display their work or for instructors to elaborate ideas to small groups of students. In general, Scott followed Drew's lead on how to conduct direct instruction and practice activities, and lead student discussions. Lessons usually followed a following format: an initial task to pique students' interest, followed by direct instruction, then group work to explore presented ideas. Sometimes students, at the beginning of a session, worked exclusively on a given worksheet that reviewed previously learned concepts until they reached a task they could not solve, at which time Scott would then use these tasks to extend earlier ideas. Under Drew's influence, Scott's experience using features of curricular tasks to help create a classroom culture based on exploration and struggle was an important aspect of his professional learning.

Scott's Internship Teaching Experience

Scott and Drew were assigned three sections of Honors Algebra A/B and two sections of Geometry. At the beginning of August, Scott took on one of the Algebra A/B courses as his focus class. As described in Matthew's case, Big Horn's mathematics department had a self-designed curriculum, and this is the context within which Scott also learned to teach. Because Big Horn's mathematics department had structured courses around carefully sequenced worksheets, these worksheets, at times, served as an introduction to a new concept in lieu of a classroom lecture. Other times they served as practice of a concept they had seen their teacher demonstrate. While the curriculum influenced his teaching experience, in some sense, by dictating the way students experienced mathematics on any given day, Drew spent more time summarizing and reflecting upon the instructional choices he made in response to student thinking. While Matthew spent much of his time reflecting on how the curriculum was important for his own learning, Scott, however, spent more time summarizing what happened in his classroom and used classroom interactions and instructional decisions to demonstrate how the curriculum was implemented in his specific context. As we will see, he wrote about specific curricular features – such as the use of productive struggle – as decisions made by himself and his mentor teacher.

From my observations of his teaching throughout the year, I was impressed with his ability to lead student discussions and create interesting ways to launch lessons that encouraged students to engage with the task. Throughout individual or group work he consistently circulated the classroom to help students tackle problems. Scott was adept at clearly explaining ideas and paced his instruction well.

From my perspective, Scott generally did not spend much time planning each day's lessons on his own. His approach to planning was to take the given worksheet for the day and work through the problems ahead of time so that he was prepared to help students as needed. Instead of spending time thinking about his own instructional choices or about the rationale of task sequencing, he initially relied more on observation of his mentor teacher's lesson prior to teaching it on his own. In the passage below, Scott reflected on his Fall semester's planning practices and how this, in turn, impacted his understanding.

> Before, when I would watch what my mentor teacher would do and attempt to mimic his teaching, I was more so focused on what to do rather than why to do it. I would become so hyper focused on the presentation that at times, I wouldn't fully comprehend the web of concepts that he was teaching and would have a tunnel vision about the structure of the lesson.
> (Scott, LT Reflective Narrative, Lines 11-14)

This bare minimum planning and his posture to mimic his mentor's presentation limited his potential for gaining deeper curricular knowledge, in my opinion. As we will see, this began to change when he was required, as a part of his university coursework, to extensively plan an instructional unit using backward design principles (Wiggins & McTighe, 2005) and ultimately teach through the unit during his Lead Teach period. As we will look at in more detail later, by studying the curriculum materials on his own ahead of time, explicitly identifying how big picture goals are developed on a daily basis and planning ways for his students to achieve these goals, he began to move beyond a "hyper focused" view of the content to a more connected "web of concepts" understanding.

In the next section, I will make a claim about Scott's teaching internship experience in relation to how his content knowledge for teaching grew as a result of the practices he engaged in. To support this claim, I pull together excerpts from his daily synopses, reflections, and participation within the professional learning community.

The Influence of an Experienced Community Broker in Scott's Case

> *Scott's opportunity to develop Pedagogical Content Knowledge (PCK) would have been more limited if it had not been for the influence of two specific communities of practice, namely the classroom community and the university community. In the classroom, Scott learned to teach under the guidance of Drew, a veteran teacher, and a leader in the school's mathematics department. Scott relied heavily on Drew's teaching style to implement lessons throughout the Fall semester of his internship. From Drew, Scott assimilated a specific way to implement Big Horn's curriculum by reflecting on the*

relationship between instructional decisions and student understanding. In the university community, course projects, such as the Unit Plan project, encouraged Scott to take a closer look at the Big Horn curriculum. As he planned and carried out a unit of instruction in the Spring semester (work for this began in the Fall), he began to show evidence of developing KCC. So, through close shadowing of an experienced teacher who was focused on creating a classroom designed around exploration and collaboration and because of university required "big picture" planning, Scott came away with knowledge of content and teaching (KCT), knowledge of content and students (KCS), and to a limited extent knowledge of content and curriculum (KCC).

Coming into this study, one of the assumed outcomes was that novice teachers, especially those without extensive tutoring or substitute teaching experiences, would have forgotten some specifics of secondary school mathematics, particularly if they had not used those strategies or concepts in recent coursework. Consequently, simply by teaching through material they were reminded of concepts or strategies they had previously known or had not been introduced to. This was particularly true for Scott early in the semester. While he did have some experience as a TA for college algebra, he still admitted to having gaps in his understanding. Early in the internship year he identified one concept that he had previously forgotten (italics added for emphasis):

> Learning math for the first time was always a struggle for me. When I first decided to become a teacher, I had believed that I had a deep understanding of the material and was confident that I could help others reach this level. But being at my placement, *I forgot many of the niche details that come with algebra*. An example that comes to mind is that domain and range is not only an interval, but also includes the set of numbers within the domain and range. *While monitoring students working on a task* that highlights this detail, it became a common occurrence to see students working in rational numbers consistently. But given the context of the situations I was provided with the opportunity to retell these situations in a way that highlighted the fact that the domain lied in whole numbers. Part of this mistake was a misunderstanding of the story problems which I had forgotten had many hints within them. Commonly, I would look out for keywords that would highlight rate, signs, y-intercept, etc. But it was apparent that my mathematical literacy had overlooked the set of numbers previously. *After teaching the lesson I find that it's important to look for more than numbers in a story problem and be careful to re-read situations in case there are details to be missed*. In this case, looking out for small details that can have calculation altering effects, and being diligent in reading to improve mathematical literacy to help

> gain a better understanding of the situation is how I found a "new and deeper way" to understand the problems (and concepts) in front of me.
> (Scott, Weekly PLC Prompts & Responses, Pos. 26)

He recounted this illustration of learning through teaching during our weekly PLC and in this instance, he was responding to a similar share-out from Matthew in which the interns were responding to a prompt in which I had asked in what ways teaching had influenced the way they thought about a concept. In this excerpt, Scott identified a concept (i.e., domain and range) that had developed because of monitoring student progress through context-based problems. In this example, it is inferred that by noticing students' misconceptions he realized that he had forgotten to communicate to his students how the context of a task could influence the way the domain and range of the problem was represented.

Towards the end of our PLC (middle of October), Scott again identified a concept that he had learned more about because of teaching. This passage, however, took on a slightly different tone and hinted at the effect of teaching over a longer length of time, as opposed to how one specific lesson influenced his thinking.

> One example I can think of where I have noticed my own understanding changing is factoring. When I was in high school, my teacher taught me one method to factor, and in my mind, that was what factoring was. However, at this point, *I know multiple different methods to factor*. I understand better now that factoring isn't putting numbers into little boxes and getting two smaller polynomials out - factoring is undoing the distributive property. That method that I was originally taught (as well as the other ones I have learned) isn't what factoring is - rather, it's a way to help organize my thinking as I factor.
> (Scott, Weekly PLC Prompts & Responses, Pos. 38)

From the last two passages, there is evidence for my original suspicion about the outcomes of this study in relation to novice teachers' subject matter knowledge because of teaching. During the first two months of the internships, Scott identified both short term growth in common content knowledge (i.e. how a task's context influences its domain/range) and more extended horizon knowledge (Ball et al., 2008) after having taught through a unit on factoring. While these examples are evidence of the effect that teaching may have on novice teachers' subject matter knowledge, Scott's teaching experience also demonstrates how being within communities of practice helped to develop his PCK. To show this, I pull excerpts from his reflective narratives in his Guided Lead Teach period at the end of the Fall semester and during his Lead Teach period in the Spring semester.

During Scott's second Guided Lead Teach period in November, he taught through and reflected on a unit focused on solving quadratic functions. During this portion of the internship, the influence of his mentor teacher's classroom community is the most evident in how he describes both instructional decisions and observations of students' thinking and responses to his teaching. As I mentioned above, while Big Horn's curricular framework is present in the background of his writing, he talks about the curriculum in terms of instructional decisions that both he and his mentor teacher make. From my point of view, these decisions were features of Drew's established classroom routine and, as such, Scott picked up on teaching strategies and reflected on them in terms of how his students responded to their tasks.

Throughout the narrative, Scott highlighted a particular instructional strategy that formed the backbone of this unit, namely allowing students to struggle as they explored different strategies for solving quadratic functions, and this strategy seems to have formed an important impression on his view of what teaching mathematics could look like. For the purpose of providing evidence for my claim about Scott's experience, I include excerpts from his reflective narratives during the Fall and Spring semesters but have included them in their entirety in Appendix D and Appendix F.

> *A calculated risk that was taken throughout the launch of this unit was to intentionally allow for students to struggle*. By doing so, we wanted for students

> to be able to identify what caused them trouble. From a teacher's perspective, the hope was that by doing so, students would gain a deeper understanding and further be able to identify the strategy that they felt best suited to find the solution. When students first approached the unit, they assumed that they could solve the same way that we would with linears. As a result, this became their obstacle. They quickly took note that they could not do so. This irritated them. They ran into a wall that they did not know how to climb. When first presenting such issues, it was instinctual to help the students. But this time around, *we resisted the urge to and allowed for the students to struggle as a result. By doing so, we hoped that students would rise to the challenge and find the result of their struggles*. Normally, students would seek to find a solution and finish the task. In this case, that would not be a success. Rather the ability to simply identify one's own struggle was. This was not a single mathematical conversation, but rather one that would continue for days on end. We challenged these students over and over again with the hope that they would keep on persevering through their hurdles. (Scott, GLT2 Reflective Narrative, Lines 13-27)

It may be helpful to see an example of what Scott is talking about here. In Appendix E, we have one of the earlier worksheets from this unit and an example of where the sequencing of tasks was designed in a way to draw on students' prior knowledge as a way to help them identify the need for new knowledge. In the first five tasks, students work through solving increasingly more complex linear functions (which they had previously learned in an earlier unit). By the time groups arrive at the last two tasks on the worksheet (the reverse side included gradually more complex quadratic functions), Scott hoped that students would realize that they could not solve the quadratic function using their earlier strategies solving linear functions. As he mentioned above, all this work came without direct instruction about how to solve quadratics on their own. Instead, Scott learned to implement a specific instructional strategy present in the curriculum, namely allow students to come up with their own ideas for resolving this new kind of problem.

From prior interactions with Matthew, it didn't appear to me that teachers were required to teach the content a specific way. Presumably a teacher could have taken a different approach and walked their students through the worksheets using an "I show, now you practice" approach. In the above situation as described by Scott, he has come away with a particular knowledge for teaching quadratic functions (and mathematics more generally). He mentioned in lines 13-15 they hoped students would come away with the ability to analyze situations and reflect on the challenge of analyzing earlier strategies they had learned. While he described this in a way that one might assume he wanted students to do this on their own without any outside assistance, he viewed his role as a facilitator and consolidator. From his notes on the worksheet, we see specific questions he planned to help focus students' attention on analyzing their work by comparing strategies between tasks. When finished, he consolidated their thinking to discuss strategies for solving quadratic functions. He also mentioned in lines 25-27 that this was an ongoing conversation and teaching strategy not limited to one day.

In addition to gaining practical knowledge teaching content using a particular framework, Scott also learned that by monitoring student thinking throughout the unit they, as instructors, were able to reflect meaningfully on the impact of their instructional choices. As he explained, simply being able to find a solution to any given quadratic equation was not their primary goal for the student; we saw above that they intentionally wanted students to come away with the ability to analyze their own thinking and make reasoned choices about which strategies to use. This necessitated that he paid careful attention to students' ideas and responded to their work. We see this first in the passage given above in lines 17-20 when he mentioned students' frustration with their inability to apply strategies for solving linear equations to quadratic situations. This frustration indicated to him that students noticed that something was different about the new tasks. He followed up on this thought in the following excerpt:

> As we continued our unit, eventually the students would be able to work around such struggles. We had created moments of uncertainty for the students, and it helped to connect students to mathematics. *They were eager for the solution*. Like a good mystery novel, we slowly unveiled the strategies for doing so. By week's end, we had finally reached the big reveal. We introduced our first strategy for doing so by instructing how to convert quadratics in simplified form to vertex

> form. After students had previously identified their struggles and how it related to dealing with multiple independent variables, we had come to a possible workaround that would satisfy. *Students were thrilled to finally be able to solve quadratics.* Their connection to mathematics proved deeper than before as they were thrilled to be able to solve such equations and no longer had to be uncertain about the tasks that they received. Little did they know that we would continue to peel back the layers that enveloped solving quadratic equations.
> (Scott, GLT2 Reflective Narrative, Lines 28-38)

From this passage, we get a sense of what it is like to be a student in Scott's classroom at this time. While they were initially frustrated as they struggled to find a solution to solving equations with multiple independent variables, he mentioned that they were now excited to have one strategy for solving quadratic problems. In these two passages, Scott highlighted their reasoning behind allowing students to struggle.

> As students became further versed with the multiple strategies presented, *we noticed that some methods had become fan favorites* while others, not so much. Such personal preference validated our reason for teaching all methods as some students quickly found comfortability with certain strategies that others may not have. *As a result, as we neared the end of my GLT2 period, we were able to evaluate how students would approach solving quadratics from multiple angles. And while a preference of method is allowed, some students elected to switch between multiple methods as some problems promoted one over another*. From a teacher's perspective, this was an absolute success because at the end of the day, all strategies shall work, but some situations may already be prepared for specific methods.
> (Scott, GLT2 Reflective Narrative, Lines 49-57)

From this unit, Scott came to understand the content better by reflecting on his learned approach to teaching the content by putting himself in his students' shoes, so to speak. Because this was the first time that he had taught through a unit on solving quadratic functions, it makes sense that he was reintroduced to strategies that he had previously forgotten and added to his overall understanding, which he alludes to in the following passage. Scott, however, emphasized that it was more than just being introduced to new strategies for solving quadratics that helped him come away with deeper knowledge. Scott's ability to empathize with his students as they struggled through new tasks while distilling new strategies seems to have helped him come away with a deeper sense of knowing the material.

> I found myself gaining a deeper understanding of solving quadratics as I taught the unit. When I was introduced to the concept, the quadratic formula was one of the concepts I left high school being able to repeat at any moment. It was my default and I treated it as such. But understanding now that given a quadratic already in vertex form, it is much simpler for me to use reverse orders of operations. *I did not anticipate learning as much as I had from this unit when I first launched it. But seeing my students try new strategies helped me gain this understanding that what is most comfortable may not be what is most efficient*. I think that throughout the process, one of the most rewarding findings that I had was that when faced in the presence of such uncertainty, students rose to the occasion. It was good to see that through such challenging tasks, students would not be discouraged. As a student, I knew that I would struggle often at this age. And that left an effect on me where sometimes I may be tentative to have such high expectations. But seeing students who continued to try out different strategies through tasks which were not necessarily possible for them at the time helped put away my hesitance. I am hoping to build off such strides and continue to hold my students to these high expectations.
> (Scott, GLT2 Reflective Narrative, Lines 58-72)

From this teaching experience, Scott had a changed perspective on what it meant to struggle through new tasks. As he mentioned, the resulting confidence that students appeared to have after this unit contrasted to his own experience as a secondary student and this was due, in part, to how they experienced mathematics in his classroom. This seemed to be a powerful learning experience for Scott, one that has helped change his own perspective of what it means to learn mathematics.

Throughout the Fall semester, the primary influence on his learning to teach seems to have been from his in-class interactions with his mentor teacher, as they deliberated on upcoming lessons and reflected on students' progress. As such, he came away with a view of presenting content to students by way of allowing for productive struggle and using their challenges to highlight important ideas. Towards the end of this semester, however, another community began to influence his view of teaching mathematics, namely his university course community. As mentioned in an earlier chapter, the interns were concurrently enrolled in two sequences of graduate coursework. In their mathematics specific course sequence, they were assigned an extensive unit plan project, which began at the end of the Fall semester and continued into Spring. Guided by project requirements, Scott began to study Big Horn's curriculum in more detail to form the framework of his unit plan.

As he explained below, Scott not only planned this unit but also was required to teach through the unit. For the Spring semester, he selected a unit on investigating general polynomials. Having known that for this unit he had spent more time thinking about the big picture as part of his unit plan project, the conversations that I had with him when helping him think through his teaching experience were focused more on his planning than in-class interactions. So, the tone of this reflective narrative is more general.:

> As a part of TE 802/804, we were tasked with creating a unit plan for one of our units of Lead Teach. This unit just so happened to be my first unit of Lead Teach. When teaching the unit, each lesson felt much more interconnected than units that I had taught previously. Each day seemed to transition into each other and the conversations that we were creating at the beginning and end of each lesson felt much more organic yet organized than the prior semester. *Teaching after having done the unit plan was like watching a movie and seeing the trailer beforehand. I had an idea of what was going to happen and had more access to anticipating students. But when you don't watch the trailer, you may still have an idea of what it is about, but you really have no expectations*. Because of this, I was able to better anticipate the mathematical conversations being held and facilitate the discussion so that it'd be easier to transition into further concepts.
> (Scott, LT Reflective Narrative, Lines 1-10)

Not only did Scott describe his unit plan experience as helping him to come away with a more connected feeling which, in turn, influenced his ability to orchestrate productive mathematics discussions, but he also used an interesting simile to describe the impact of his experience. A well-designed unit plan, he described, influenced his teaching like going to see a movie and having watched the trailer ahead of time. As such, he was now empowered to anticipate students' conversations and use their conversations to lead their discussions. He also admitted that having spent a significant amount of time studying the curriculum materials in order to design this unit plan freed him up to think more about how concepts connected to each other, rather than focusing on the instructional decisions that had to be made daily, as seen in the passage below.

> Before, when I would watch what my mentor teacher would do and attempt to mimic his teaching, I was more so focused on what to do rather than why to do it. I would become so hyper focused on the presentation that at times, I wouldn't fully comprehend the web of concepts that he was teaching and would have a tunnel vision about the structure of the lesson.
> (Scott, LT Reflective Narrative, Lines 11-14)

Further, having a fully fleshed-out unit plan helped Scott choose how to make better use of each day's launch and debrief time, as evidenced below:

> With access to my own unit plan, it made it easier for me to see how I could start lessons based on the ideas we would unravel as a class the day prior. And using the end of the class to delve deeper into the day's concept or continue this transition gave me a sense of fluidity in my lessons. Knowing which concepts would be harder for students to understand made dedicating a debrief versus continuing to push forward student thinking a much easier decision to make.
> (Scott, LT Reflective Narrative, Lines 15-19)

In essence, the unit plan also served as a playbook for Scott to make use of to guide his in-class teaching decisions.

As a result of his collaborative approach to teaching in the Fall semester and his more in-depth planning experience during the unit plan project, Scott thought explicitly about how the way he introduced concepts to students connected to their earlier experiences and set the course by which they would come to think more deeply about key concepts. We see in the following excerpt how his planning for such experiences had developed:

> When planning, I am considering how I want to structure my lessons from finding interesting launches that connect to what we will be working on, all the way to what big ideas I want for students to reach by the end of the lesson. Working on

the task and envisioning how I would do it versus how my students might, gives me room to anticipate potential strategies that they may employ and how I can further guide their thinking. Seeing their work when grading allows for me to see which connections to concepts are being made and what material may need further reinforcement.
(Scott, LT Reflective Narrative, Lines 49-55)

While Scott was prompted to think deeply about the entirety of a unit of instruction ahead of time, which wasn't his normal practice on his own initiative, we see him making connections between this big picture approach to planning and his own on-the-job teacher learning. While his earlier approach to planning was focused on mimicking his mentor teacher's approach and doing a quick review of the content ahead of time, by this point his planning took on a more all-encompassing approach. It appears that he then saw the direct impact of looking at the entire story ahead of time and its benefit for being able to reflect on students' thinking in a way that influenced his future instruction.

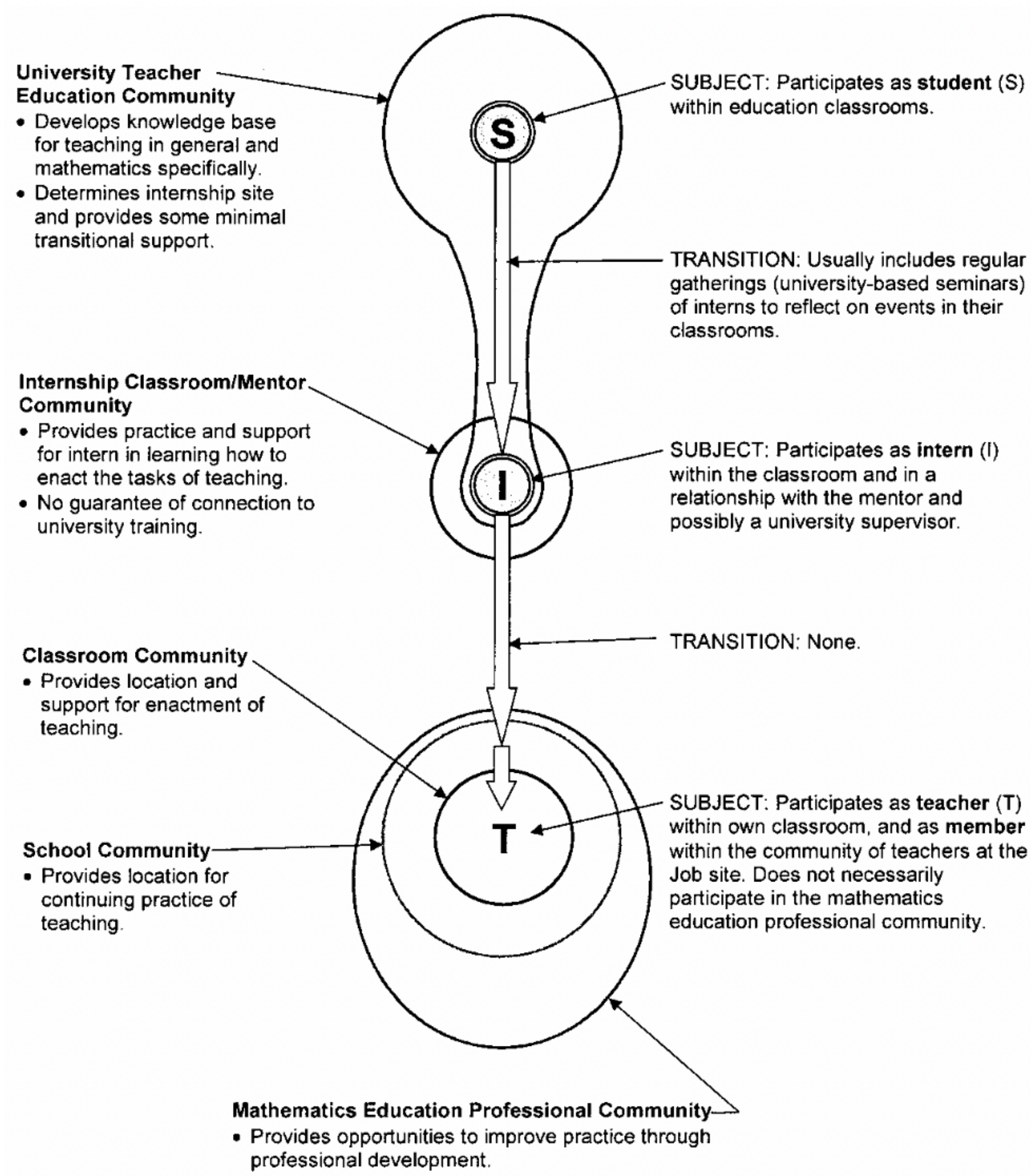
Key Takeaway from Scott's Case

Key Takeaway: When novice teachers learn to teach in an environment led by an expert mentor teacher, particularly one that is experienced in implementing an ambitious curriculum, they can come away with developed Knowledge of Content and Teaching (KCT) as well as Knowledge of Content and Students (KCS). As a result, when they proceeded to study curriculum materials, they have concrete teaching experiences on which to reflect which, in turn, helps them connect curricular goals with visible student responses.

Scott's internship year demonstrated the influence of overlapping and interacting communities of practice on a novice teacher's experience. As we see in the figure below, Van Zoest and Bohl (2005b) describe a teacher's development in terms of key transitions within the context of *constellations of communities* (i.e., related communities of practice). In this figure, they describe a teacher's development in relation to the communities of practice that have the most influence on them throughout their preparation. In this section, I will unpack Scott's claim about learning through teaching and how two communities of practice influenced his experiences and by highlighting aspects of Wenger's (1998) theory of learning in communities not discussed in Matthew's case. More generally, Scott's experience demonstrates how various communities influence novice teachers' learning differently.

Figure 4 - 1

Van Zoest & Bohl's (2005) Current Configuration of Communities



From Scott's internship, we see the influence of two main communities: his internship Classroom/Mentor Community and the university teacher education community. Within his classroom community, the central participants are Drew, his mentor teacher, and his students. Within the university teacher education community, the central participants are his instructors (namely me, as his field instructor, and his course instructors) and other teacher interns enrolled in the course. While Van Zoest & Boh's (2005b) description of the transition from student to teacher is helpful in our discussion, Scott's case wasn't a purely linear progression. Throughout his internship we see different communities playing a more prominent role at various intervals.

Learning happens when there is a contrast between a person's prior experience and the community of practice's regime of competence (van Zoest & Bohl, 2005b; Wenger, 1998). We see early in Scott's internship during the Fall semester that his classroom/mentor community seems to have had the most influence on his development of Pedagogical Content Knowledge (PCK). As we saw in an earlier section, Scott's reflections focused mostly on the instructional decisions made by both himself and Drew. From my point of view, many of these decisions were primarily made by Drew and replicated by Scott. First, throughout the Fall semester, I worked with Scott to provide more detailed lesson plans for official observations. Initially the lesson plans that he provided were not acceptable (essentially just providing an unfilled worksheet that they were working on that day). As we saw in Lines 11-14 of his Lead Teach Reflective Narrative, he relied upon observing his mentor teacher first teaching through the lesson to make instructional decisions. His reflections during the Fall semester, however, drew heavily from the structure of the curriculum (i.e., using struggle as an instructional tool) and talked about these ideas as instructional decisions. Because I had access to his written reflections, I knew that there was more going on between him and his mentor teacher by the way he described the instructional decisions made on a day-to-day basis. Towards the end of the first semester, I made it clear that he needed to be explicit about these instructional decisions in his plans, and he began to demonstrate the ability to plan in detail.

Scott's case demonstrates the role that a veteran mentor teacher has on effectively enacting lessons that are in line with the spirit of the curriculum. In Scott's experience, Drew acted as a *broker* (van Zoest & Bohl, 2005b; Wenger, 1998) between Big Horn's math department community of practice and the classroom community of practice. Having been a central participant in helping to develop the department's curriculum, Drew was an ideal mentor teacher to observe the enactment of the curriculum in the classroom. As mentioned in an earlier section, Drew's classroom learning environment was designed around collaboration. As such, through observation and consistent discussion around instructional decisions with Drew, Scott came away with an experiential knowledge of how the content could be implemented. Further, instrumental in their classroom community of practice was to reflect on the instructional decisions through observing student thinking.

The second community of practice that influenced Scott's learning was the university teacher education community, primarily through the unit plan project. For this, interns were introduced to unit planning using backward design principles, as laid out by Wiggins and McTighe (2005). Generally, this approach is broken down into three stages: identify desired results, determine acceptable evidence, and plan learning experiences and instruction. For Scott's internship, this course planning framework served as a *boundary object* (van Zoest & Bohl, 2005b; Wenger, 1998) by which he participated in both communities. As mentioned earlier, the Big Horn mathematics department used the same *Understanding by Design* (Wiggins & McTighe, 2005) resource to build course materials. As such, all teachers in the department had access to these unit plans on a shared computer drive. So, as Scott was introduced to the elements of this framework in his university course, he gained deeper insight into the ideas behind how the units and lessons at Big Horn were designed.

Scott's case is important because through it we see how the interaction and influence of communities of practice can initiate experiences for novice teachers that give them the opportunity to develop pedagogical content knowledge. Although Matthew and Scott both completed the unit plan assignment at the same time, in some sense Scott was primed to understand it differently than Matthew. As we saw earlier in this section, Scott had been experiencing the "spirit" of the curriculum throughout the Fall semester before he dug into the unit design framework. Scott's classroom and Matthew's classroom looked very different in the way the teachers and students interacted with each other. While Matthew understood the "letter" of the curriculum earlier in his internship, Scott experienced the "spirit" of the curriculum by being in an environment structured around student exploration and collaboration.

In summary, Scott's case demonstrates the result of communities of practice pulling novice teachers in as full participants and how this inclusion, in turn, impacts their sense of knowing the content that they teach. When Scott began his internship, he initially relied on the procedures and strategies he remembered from his school days for teaching his lessons. Instructional decisions, however, did not come naturally to him, and through observation and conversations with his mentor teacher, Scott began to learn essential teaching practices and how to make instructional decisions to guide student thinking in a way that supported their collaboration and exploration. Further, Scott learned what it meant to plan for effective learning experiences by accessing department resources, fulfilling university course requirements, and by keeping up with the professional expectations of the internship. While this level of detailed planning did not come naturally to him at first, from his teaching experiences in the Fall semester, instructional decisions began to make more sense to him as a result of deeper planning, and he began to see the overall storyline of the algebra course.

The Case of Claire

Claire graduated from Midwest State's secondary mathematics teacher program at the end of the 2020-2021 school year. From this program, she earned a bachelor's degree in mathematics, a minor in physics, and enrolled in specific teacher education coursework. Prior to the internship, I sat down with her to get her perspective on the upcoming school year and get a picture of why she decided to become a teacher.

Like Scott, Claire's journey to become a mathematics teacher was not a direct path out of high school. Initially, when she started her undergraduate program, she enrolled in the engineering program, primarily because math and physics were her favorite subjects in high school:

> I had always considered teaching in the back of my mind ever since I was a kid, (but when) I got to high school, and I got to AP Physics, and I was one of two girls, and everybody was going into engineering. And I was like "I'm not sure that I want to teach. My dad teaches and I think I want to do something different." But I knew I liked math and science and I knew I liked problem solving
> (Claire, Pre-Internship Interview Transcript, Pos. 21)

After a few semesters in engineering, however, she realized that, while she enjoyed studying engineering, she wanted, ultimately, to work with kids. In the passage below we see that choosing to enter the secondary mathematics teacher preparation program helped her to continue studying mathematics and physics, but also begin working with students.

> I liked working with other people so it's like, well look at engineering. I can work on a team and solve problems. And I got there, and it was fine, and I liked it, but I realized that at the end of the day what I really wanted was to work with kids and young people. But as much as I loved math and science, I still wanted to study it. I knew I wouldn't be content in my career, if that was what I was doing was just working with numbers rather than people
> (Claire, Pre-Internship Interview Transcript, Pos. 21)

Prior to this school year, Claire had limited in-class teaching experiences. While the teacher preparation in which she was enrolled in includes field experiences throughout the program, many of these experiences were adapted to fit safety protocols and often were online because of the COVID-19 pandemic. She looked forward with some nervousness about her upcoming teaching internship, and, after she initially met with her mentor teacher, Katrina, she began to realize what exactly to expect. She explained: “I am looking forward to the internship and I'm excited to start and to engage with the mathematics and the students and to start to learn how to manage my own classroom” (Claire, Pre-Internship Interview Transcript, Pos. 49).

Claire’s Internship Placement

For her internship year, Claire was placed at Longship High School, a medium-sized, suburban school (approximately 900 students; 23% minority) in the same geographic region as the university in which she was enrolled. Claire’s mentor teacher, Katrina, was also a graduate of Midwest State’s secondary mathematics teacher preparation program and had been teaching in the school district for approximately a dozen years. During Katrina’s career, she had mentored several teacher interns, but Claire was the first intern while Katrina was employed at Longship.

> I'm excited to learn from Katrina. I mean I've only talked to her once so far, but she was very, I really like her vibe. I’m her first intern at Longship, but not her first intern overall. She's been at Longship for four years, but she hasn't had an intern hasn't yet because she's been on maternity leave, multiple times. But she has taken interns before and so she was talking about how one of her favorite parts of teaching is like passing on that knowledge to the next teachers. So, I'm excited to learn from her and see what she has to offer.
> (Claire, Pre-Internship Interview Transcript)

Longship’s mathematics department was organized like many typical high school mathematics departments in the United States. Unlike Big Horn’s mathematics department where they eschewed textbooks and developed their own curriculum resources, Longship’s mathematics department assigned textbooks to each of their courses and teachers were generally free to teach through the curriculum how they saw fit. Further, teachers worked collaboratively with others that were teaching the same subjects, often sharing resources, and creating activities together, as is common within many schools.

Claire’s Internship Teaching Experience

Claire and Katrina were assigned two sections of Pre-Calculus, two sections of AP Statistics, and one section of Pre-Algebra, and, for this school year, Claire chose one of the Pre-Calculus sections as her focus class. Katrina supported Claire’s development as a novice teacher by the way she viewed her role as a mentor teacher. Of the five interns I worked with this year, Claire and Katrina's relationship was more like that of co-teachers rather than mentor/intern. This working relationship, I believe, was an important part of Claire's identity development and had an impact on the kinds of experiences she was exposed to early on in her internship. Claire described this relationship in her Lead Teach Reflective Narrative (Appendix H).

> My mentor and I have been strong collaborators throughout the course, and this unit was no exception. She has taught AP Stats for several years now and has a really good sense of what works and what students need to know for the exam. She and I picked activities together, used note sets from prior years, and planned out the sequencing together. I did more day-to-day specific planning, by working through the notes and activities, reviewing material, anticipating what students might ask, etc.
> (Claire, Lead Teach Reflective Narrative, Pos. 6)

From this passage we see how Claire and Katrina worked together throughout the school year. By doing big-picture planning together (sequencing lessons/activities, designing assessments, etc.), Claire was then able to use their meetings to do her day-to-day lesson planning. As her field instructor, I thought this kind of working relationship was helpful for Claire, and it was evident by how detailed and thorough her lesson plans were. Further, Katrina, being a graduate of the same math teacher education program, knew many of the assignments and experiences required of Claire; as such, when doing big-picture planning, Katrina offered guidance and suggestions on the kinds of activities to plan, thus preparing Claire for her university course assignments.

Claire and Katrina's room was organized around student collaboration and exploration. This was evident by how desks were organized into groups of three to four students each. Katrina, and by proxy Claire, wanted her students to experience mathematics first through concrete activities, then use these experiences to build towards abstraction. These experiences usually happened at the beginning of a unit and lasted a full day or two. Other days, discussions were connected back to these experiences but took on a more traditional approach. In the passage below, an excerpt from her Reflective Narrative from her GLT2 period (the full write up is in Appendix G) Claire described a typical day in her class.

> For a typical daily lesson, we'd start with a warm-up, either continuing content covered the previous day or scaffolding skills for the new content. We then move into notes, balancing between introducing content to the whole class and working together in small groups on example problems. Once notes are done, students are assigned some kind of practice (usually either book problems or a worksheet) for homework and have the rest of class to work on it together and ask questions.
> (Claire, GLT2 Reflective Narrative, Pos. 12)

While these typical days did have some direct instruction, again we see the emphasis on students working together in groups. Often Claire circulated the classroom in order to listen to students' ideas and select these contributions for later use in whole-class discussion.

Throughout her internship, Claire gained valuable experience noticing students' thinking and using their ideas to reflect upon her own teaching. Being attentive to student thinking, particularly in relation to the overall story of the unit, was critical for her professional learning this school year. In the following excerpts we can see the level of detail that she remembers about students' ideas and her ability to dig deeper to attach meaning to these ideas. In the first instance Claire reflects on a common issue and uses it to make a claim about a greater misunderstanding she believed to be present.

> The most commonly asked question was about finding the solution of the equation $2^{2-x} = 7^{2x}$. Most students were able to apply the logarithm to the equation, but then had trouble figuring out how to get x by itself. Even when they used the law to bring the $2x$ down in front of the log (getting $2x\log_2 7$) some struggle still occurred. *I think some of them were having trouble seeing* $\log_2 7$*as being an actual number. They had more of a mindset of "needing to get rid of it" rather than just dividing by it and putting it in their calculator.* To try to get around this, we put it in a calculator (getting about 2.81) to try and make the connection between treating it like a constant. The ones who were still struggling after that are the ones who have had consistent algebra issues since the beginning of the year.
> (Claire, GLT2 Synopsis 11-17, Pos. 11)

This excerpt told me a lot about Claire's level of sophisticated noticing. Not only was she able to notice the aspect of a task that caused her students' the most difficulty, she demonstrated that she also was able to unpack their struggle and interpret what the deeper issue really was.

In another instance, during a unit review day, she spotted an answer that she instinctively knew was incorrect but had a moment of uncertainty because of her students' confidence in their responses.

> (An) interesting moment I can think of - at the very end of the review, my final question (for ten points and the chance to win it all) was "What is ln (1)?" All three (groups) got the answer at just about the same time, so we gave a tiebreaker, which was ln(0). I picked the question because I knew it was undefined and thought that would be interesting, but then all three students wrote "1" as the answer. They all were so confident in their answers, and my mentor teacher said, "I think Ian had it first" that I was like "Maybe I'm wrong???" and we awarded the points. We had a moment of realization afterwards that ln(0) is in fact undefined - the whole sequence is in the video I took, and it's honestly a little bit funny.
> (Claire, GLT2 Synopsis 11-19, Pos. 10-11)

In this passage we see how a classroom interaction injected some uncertainty into her confidence about her own content understanding. She used this instance to also reflect on her own teaching and student thinking after the lesson. One interesting aspect of her practice that demonstrated to me how well she was paying attention to student thinking and allowed her to write up these instances of detailed noticing was her use of monitoring notes. This usually looked as simple as writing down a few observations to later highlight during the whole class discussion onto a large sticky note. It indicated to me, however, that she viewed these notes as an important tool for her instructional practice and one that she planned to do ahead of time.

Not only did Claire pay attention to student thinking in the classroom, but she also reflected on their affective responses to learning opportunities and how those responses influenced the way students positioned themselves within the classroom.

> This lesson had me thinking not just about student thinking, but also about student competence - particularly about how students assign themselves/their peers' competence. I have one student in particular who seems to have been assigned the status of "most competent" - through my observation, it appears to be both by self and peer assignment. The student in question picks up on things very quickly and is very bright, often asking questions that are beyond the scope of this course - and his friends really push the fact that he's quite bright. This status came out particularly during today's lesson - there were multiple, multiple times in which immediately after his team member went up to the board, he went to stand next to them and started giving them step by step directions. A lot of these students are students who are doing well in the course - who are putting in the work and getting the material - but who may not be as fast as he is. I had to remind him multiple times to sit down and let his team member actually try, but it got to the point where he started ignoring me when I asked. My mentor teacher had to go up to him and tap him on the shoulder and remind him that he had been asked multiple times to sit down and needed to follow the rules of the game if he wanted to continue to play. It was interesting to me that this happened because we had nothing like this in Second Hour when we played the game. We have a lot of competitive people in that hour, and a lot of students who are really excelling in the course (at least as much as the student in question), however, no one person stood out as holding that perceived status.
> (Claire, GLT2 Synopsis 11-19, Pos. 10-11)

I mention all these instances throughout her internship to highlight how paying attention to student thinking and positioning was an important part of her practice. In the next section we will see this practice of reflecting on and analyzing student thinking proved to be a catalyst for her own learning.

The Impact of Noticing and Reflecting on Student Thinking in Claire's Case

> *While Claire demonstrated evidence of re-learning content because of the teaching effect, it was through co-teaching and collaboration with her mentor and actively implementing lessons across units that she came away with a fuller understanding of the content. This curricular knowledge was further developed by her continual practice of paying attention to and analyzing student thinking. By applying her curricular knowledge in assessment situations, she learned to communicate big ideas in a succinct and clear manner. Further, these experiences helped her to reflect on future instructional decisions.*

Like Scott's experience, Claire also had moments of learning or being reminded of new content knowledge because of teaching through concepts. Prior to her internship she admitted to being hesitant to teach any statistics course, primarily due to her limited exposure to the ideas, as we see in the passage below.

> Yeah, so I'll probably be teaching stats next year. That's one of her subjects (although) she doesn't know for sure what she's teaching yet because they haven't finalized the schedule but when she was talking to me, she said "I'll probably be teaching AP Stats because I'm the person who does that." I was like, "okay… I've only taken one statistics course in all of my math career", it was Statistics 431.
> (Claire, Pre-Internship Interview Transcript, Pos. 51)

She unpacked her prior experience with statistics in more detail during Lead Teach reflective narrative and discussed coming away with a sense that she now understands the topic better as a result of teaching through it (in this case the unit was focused on exploring Chi-Squared Tests).

> If we are being 100% honest, this was one of the units I felt most the shaky on in Statistics going into this year. My own Statistics professor covered it, but *the class I took was very calculations focused, with less emphasis on the conceptual*. I spent a lot of time reviewing the material myself for this unit and *making sure that I understood not just what we were doing, but why we were doing it*. Now, having taught the unit, I feel really great about the material. I have definitely noticed my own understanding deepening through the process - I can clearly articulate why things happen, and I can't think of a question that a student asked that I didn't know the answer to. I have learned this year that I actually know more statistics than I originally gave myself credit for, and I pick up on the details very quickly with a little bit of review
> (Claire, Lead Teach Reflective Narrative, Pos. 5)

Again, like Scott, we see one way that novice teachers gain more competence in their subject matter understanding is first by reviewing the material on their own. In Claire's case, she indicates that she worked through the activities on her own, but with more of an eye for understanding the conceptual reasons behind the procedures rather than just to get the problems done. She concluded that she came away with being fully prepared for students' queries, was able to explain the ideas in more depth, and realized that she knew more than she initially believed. At surface level, Claire's experience supports the argument that simply by teaching through content she thought she was unfamiliar with, she came away from teaching with more confidence in her content knowledge, known as the *teaching effect* (Kobayashi, 2019; Koh et al., 2018). In her case, one might argue that it was the cumulative effect of having taught through the entire unit that gave her a deeper sense of understanding the concept of Chi-Squared Tests.

Cumulative Teaching Effect

During Claire's Fall semester second Guided Lead Teach period, she reflected on an instructional unit in her focus class, specifically one organized around exploring exponential and logarithmic functions. As was common prior to any unit they taught, Claire sat down with Katrina and co-planned the unit. This co-planning helped Claire to begin to see the unit as a piece in a larger story by identifying the big ideas to be covered, the assessments, and the daily schedule.

> My mentor and I did a lot of co-planning for what content we wanted to focus on during this unit. We started off by mapping out approximately what the unit was going to look like - which sections of the book were covered on what days, when we had a test/quiz, number of review days & when. We have resources from a teacher that formerly taught the course, that the other Pre-Calc teacher also uses. We sat down together and edited her guided note sheets to fit what we wanted the class to emphasize & what we wanted to give them vs. what we wanted to just say. I typically picked out warm-ups and homework problems myself when I worked through the notes and planned for myself what I wanted to say. On our two whole class review days, I planned the review myself by picking out problems that fit with what we'd done and putting together the reviews.
> (Claire, GLT2 Reflective Narrative, Pos. 10)

It is noteworthy how Claire relied on other teachers' resources to get a sense of their approach to teaching this unit. She clarified how this habit of building off others' work influenced how she interacted with the content daily in view of telling the overall story.

> My mentor and I have resources from the other teacher who currently teaches Pre-Calc and one who previously taught Pre-Calc. We'll see what they have for the section we're covering, and then edit it to fit our classes' needs and what we think we need to emphasize. I'll then work through the notes myself, and then I pick out a warm-up to go with the lesson - usually either something from yesterday's work

> to reinforce, or something to connect to and start off the new lesson. Sometimes, after grading a test/quiz, it looks like a problem we had some issues with. I'll also pick out homework problems that match what we're doing/emphasizing in class. When I go to upload my notes to the Google Doc and type up my sequence of events, I normally make note of which parts I'd like to do whole class vs. small group, and any questions I have that I can think of in that moment.
> (Claire, GLT2 Synopsis 11_10, Pos. 12)

In this passage, Claire clarified one aspect of her daily routine of preparing for lessons by first looking at others' lesson plans, editing these plans to fit her specific students' needs, and then choosing tasks that help students practice ideas they continue to struggle with.

Also, it is informative to see how she then independently prepared for the lesson. She mentioned, "I'll then work through the notes myself, and then I pick out a warm-up to go with the lesson." It appears that her working through the notes served two purposes: first, it reminded her of the specifics of the examples and the details of the concepts to be taught. Second, it served as a way for her to identify warm-up and homework problems that would best serve her students' learning. This kind of work was different from how she described learning mathematics as a student: "In high school, I just memorized these and took them as fact….my Algebra 2 teacher definitely approached the subject as being a list of things to memorize" (Claire, GLT2 Reflective Narrative, Pos. 10). For Claire, while individual teaching-effect moments happened, it appears that the cumulative result of teaching day-in-day-out, especially in a supportive co-teaching environment, gave her the sense that she understood the story of course content. This began, as we noted above, in her co-teaching meetings with Katrina, where they did macro-planning together.

Claire and Katrina's habit of working through the unit ahead of time as a team not only helped remind Claire of concepts that she had previously forgotten but helped her see the big picture. This, as we will see, also impacted the way she assessed students.

> I genuinely feel like my content knowledge has greatly increased and deepened by teaching this course. In addition to understanding the material better*, approaching it from the perspective of a teacher allows me to see broader connections across units.* Stats felt a little bit disjointed to me when I was a student, but *as a teacher, I can clearly see the sequencing and how things we have already done have helped prepare for the new content*.
> (Claire, Lead Teach Reflective Narrative, Pos. 10)

Although Claire's focus class was Pre-Calculus throughout the year, she was very involved with teaching statistics during the Fall semester. Because Claire and Katrina worked together more like co-teachers, Claire was very much involved in every course by monitoring groups working, helping to grade assignments, and even taking lead occasionally at the end of the day when Katrina had to attend district meetings. Additionally, during her second Guided Lead Teach period, she taught through a unit on binomial random variables. These experiences, in addition to her lead teaching experience in the Spring, collectively added to her sense of knowing the subject across units. What's more, being included in the big-picture decision making about courses she was not directly taking lead in seemed to have prepared her to better understand the content she taught in the Spring. As she indicated above: "…approaching it from the perspective of a teacher allows me to see broader connections across units…how things we have already done have helped prepare for the new content." This passage is also key to getting a glimpse of Claire's perception about what it means to "understand" mathematics. There is understanding the content in an instrumental way that students are expected to come away with. And there is knowing the content "from the perspective of a teacher" that needs to see the overall structure of a course and see connections between units. It is evident from her passage that she views her growth in content knowledge as having both better subject matter understanding and pedagogical content knowledge.

There is more to Claire's story, however, than just coming away with further developed content knowledge. During her internship, as we noted earlier, student thinking was central to her practice. She interacted with students through selecting and assessing their thinking daily but also demonstrated the ability to use her curricular understanding that she gained by working closely with Katrina to grade students' written responses on exam questions. Throughout the rest of this section, I will describe her interactions with students and content from both her Fall second Guided Lead Teach and Spring Lead Teach periods as well as use an instance from her statistics course in the Spring where she was able to apply her big picture understanding of the topics to assess student thinking.

Noticing, Reflecting on, and Assessing Students' Thinking

One practice that Claire consistently incorporated into her teaching was noticing students' thinking and being responsive to their ideas. As noted previously, Claire and Katrina's co-planning prior to units served as the foundation for Claire's daily planning. Her daily planning also focused on being responsive to student thinking gathered from recent assessments or from her observations of their progress the day prior. This usually warranted either adjusting the pacing of the unit to provide more time for practice, selecting appropriate warm-up activities to bridge concepts across lessons, and clarifying students' needs. I use the three passages below to again point out Claire's detailed attention to student thinking but also to highlight the ways in which she responded to their thinking as she made instructional decisions.

In this first instance, during her reflection from the lesson on 11/8/21 she highlighted several questions raised by students throughout the hour. At one point she mentions that "there was another question about 3^{-1}, which I was not surprised by, however, I was surprised by the guess that $3^{-1} = -3$" (Claire, GLT2 Synopsis 11_08, Pos. 10). This, evidently, suggested a common misconception that many of her students consistently demonstrated (i.e., misunderstandings about the properties of exponents). Her response was to plan a whole-class discussion around the idea by implementing the concept again a few days later as a warm-up activity.

> I felt like the first one was a good reminder of a couple of our exponent rules - what to do with a negative exponent, and moving between an exponent and a radical, which from my noticings have been the two we most consistently struggle with
> (Claire_GLT2_Synopsis 11_10, Pos. 10)

Here, Claire made an instructional choice (i.e., including a task to help review exponent rules) in response to one student's misunderstanding of exponent rules and created another opportunity to address this misconception for the benefit of all students.

In another instance, a student asked a question that Claire had not thought to address in their introduction to exponential and logarithmic functions where the number e came from:

> One more question stands out in my brain as being noteworthy - towards the end of class, a girl (our foreign exchange student) asked me where we got the value e from. It occurred to me that no one actually told me that until I got to college, and I have no idea if any of my students have ever seen where e comes from. I'd like to look at the lessons ahead and try to take a moment to talk about how we get e and address that with the class - I think that can be especially helpful with understanding the connection between the two compounding equations.
> (Claire_GLT2_Synopsis 11_09, Pos. 14)

In this section, Claire used a student's question to reflect on her own experience and use that as a lens in which to look at her planning. In this instance it does not appear that she actually built his exploration into future content, or at least she did not mention or include it in her lesson plans, but it would not be surprising the next time she teaches Pre-Calculus that she would include some exploration around the origin of the number e.

Finally, in this last instance, Claire often mentioned how pacing through the lesson played out in her daily write-ups. In this passage, she used this type of noticing to highlight potential student struggle and to build more time for practice into subsequent lessons.

> It still took them a lot longer than I was anticipating. When we went over log properties, I showed them how we got the properties from the exponential form, and that these were shortcuts instead of moving between forms - I wrote little notes by the properties about this. *Later, I noticed a few students trying to use those little notes as properties rather than the properties themselves - I'm worried that might have confused them a bit*. We spent a lot of time on this and didn't get through everything planned, so we adjusted our plans tomorrow to spend another day working with these. First, we'll finish up the notes, and then we'll have a lot more practice time.
> (Claire. GLT2_Synopsis 11_10, Pos. 11)

In this passage, Claire demonstrated several teaching practices which indicates to me that she made use of a different posture towards the content. First, she noticed students' thinking by mentioning the pacing "took a lot longer than I was anticipating" and "I noticed a few students trying to use those little notes as properties… I'm worried that might have confused them a bit." Second, she reflected on her students' apparent struggles by responding with how she planned to address this by "adjusting our plans tomorrow to spend another day working with these…we'll finish up the notes, and then we'll have a lot more practice time." In both cases, Claire was in-tune with the kinds of misconceptions that were present in her class and reflected on how to best address them. These kinds of teaching tasks (i.e., noticing and evaluating student thinking and deciding how to respond) required her to apply her mathematical knowledge in ways that are unique to teaching.

In the Spring semester, as described in Scott's case, each intern had to plan and implement an instructional unit as part of their university course requirements. For this assignment, Claire selected a topic within her AP Statistics course. Through the co-teaching experiences described earlier and the in-depth planning she had done individually for this unit, Claire had developed a mature curricular knowledge and was able to apply this knowledge to analyze student work. One goal of their statistics course, she described, was to prepare students for the AP Stats exam at the end of the school year. For this exam, students have several "free response" questions. To help prepare their students for this kind of writing, Claire and Katrina included similar kinds of questions on their end of unit exams. In the following excerpt (her full write-up is included in Appendix I), Claire explained how this kind of grading was different from her earlier experiences and how she made use of her curricular knowledge.

> I think this grading format is really interesting because it forces you to look at what the most important parts of the concepts are. As the teacher, you have to consider what the key takeaways for each concept are, and ask yourself the question - are they displaying that knowledge through the work they've put on this paper? It may not match exactly what the sample "correct" answer is, but are they getting what they need? One basic example I can think of for this is a correctly applied formula with an arithmetic error. Typically, in a class such as Algebra, Geometry, etc., an arithmetic error will result in the loss of a point - the student will likely get either partial or most of the credit, but they'll still see a reduction in points. In Statistics, typically as long as the student has recognized what formula to use, and makes a correct conclusion based on the value they got, they'll get an "E" for the section. The value itself may not be exactly correct, but they've still understood the essentials of the concept. It would have to be a fairly egregious calculation error to be marked down from an E to a P.
> (Claire, Lead Teach Grading Write-Up, Pos. 3)

This experience proved to be valuable for Claire because it required her to identify the key curricular ideas present in the task and assess students through the lens of "are they displaying that knowledge through the work they've put on this paper?" Being able to decide which ideas are present in and critical for a solution requires deeper content knowledge about the concept that goes beyond an instrumental understanding of that concept and is evidence that the knower has more of a relational view of the idea (Skemp, 1976).

Further, Claire added that this grading process pushed her mathematics understanding even further than simply teaching through the content. Her in-depth and extensive experience grading students' free response questions pushed her to synthesize and apply her knowledge in a way that she had not done as much prior to this unit.

> I also think that by grading this way, I see an increase in my own understanding of the concepts. I have felt pretty good about my own understanding after teaching it, however, after reading and evaluating 80 FRQ's (2 per student), I see an increase in my own understanding not only with the 'what we're doing', but also 'why we're doing'. I find myself being able to articulate important concepts more succinctly, and really get at the core of what's important. If I were ever in the position to be teaching Stats after my internship year, I think this grading process would help improve my practice especially with knowing what to focus on
> (Claire, Lead Teach Grading Write-Up, Pos. 4)

For Claire, teaching the content was helpful for increasing her subject matter knowledge, but she now describes her understanding in terms of what she is able to do because of this teaching task. We see from this excerpt that this experience seemed to have helped give her the confidence to transfer her curricular and student knowledge into other aspects of her pedagogical content knowledge, namely being able to articulate important ideas succinctly and make instructional decisions about the kinds of tasks to focus on.

Claire's internship experience pushed her mathematics understanding beyond well-versed subject matter knowledge. Through her working relationship with her mentor, Claire gained a big picture view of the subjects taught, she approached practicing content for review with a relational understanding lens, and she learned how to apply her conceptual understanding to assess student thinking in a clear and concise manner. In the next section, I will describe the main lesson we can learn from her story.

Key Takeaway from Claire's Case

Key Takeaway: Having a mentor teacher who collaborates with their intern in a way that invites the intern into full participation in the classroom community by viewing them as a co-teacher helps support the intern's confidence and provides a clear mode of participation. Further, full engagement in every aspect of teaching helps novice teachers gain broad experience applying their content to different situations (i.e., answering students' *questions, grading students' work, making instructional decisions in response to misconceptions, etc.). Because of these experiences, they quickly come to the realization that knowledge-for-teaching is different from subject matter knowledge.*

Similar to both Matthew and Scott, Claire's learning-to-teach experience can be interpreted through analyzing her classroom community of practice. In Claire's case, Katrina played a pivotal influence on how Claire participated within their classroom community. We know that in Claire's case, Katrina established their roles within the classroom even before the internship began. Her mentoring philosophy was to include Claire as a central participant in the classroom from day one by establishing their relationship as co-teachers. Although Claire had a different lead-teach load than Katrina, both worked together to plan units, shared in the workload to grade student work, and interacted with groups regularly.

From my perspective as her field instructor, Claire moved through the intern stage of a novice mathematics teacher's identity development (van Zoest & Bohl, 2005a) – along the same path as Figure 4-1 mentioned in Scott's case – and was capable and confident in making instructional decisions similar to any full-time in-service teacher fairly early in her internship. This was partially because of her work ethic, in my opinion, but also because of the unique working relationship she had with her mentor teacher. Katrina viewed Claire as a capable mathematics teacher and had full confidence in leaving any of her courses for Claire to teach (and often did when she had other meetings). In this case, Katrina acted as a broker (van Zoest & Bohl, 2005b; Wenger, 1998) within their classroom community by the way she invited Claire's full participation in the classroom community; more broadly, this case gives us an example by which we can see the role that a broker can play in the trajectory of novice community members.

Katrina's role in Claire's classroom community adds to our understanding of the concept of a broker. Wenger described brokering as the process of "introducing elements from one practice to another," (Wenger, 1998, p. 236) thus a broker is the person that serves in completing this function. Further, within communities of practice, some members serve as central participants, while others are more periphery. As participants gain more experience and develop more competence within the community, they have an inward trajectory towards being a more central participant. Through Katrina, Claire became a central participant within their mathematics classroom community. Veteran community members introduce novice members to the roles, practices, and routines of the community and can serve as a key influence in inclusion into community regime of competence. For mathematics teacher interns, in general, the mentor teacher is a central influence for initial teaching experiences. In Claire's case, as we discussed, she was invited into full participation within the classroom community and was treated as an equal; this positioned her for taking on teaching tasks and making instructional decisions early in the mentorship.

Because of Claire's full inclusion in the classroom community early in her internship, she gained much experience in all aspects of teaching mathematics. In her work with Katrina, she learned how to sequence units, adjust the pacing of lessons, create activities designed around collaboration and exploration, build assessments, and grade in a way that distills key concepts into concise feedback for students. While there was not one single experience that transformed her view of her own understanding, it was the cumulative result of applying her content knowledge to different teaching situations over the course of the year. She described her transformation in terms of the way she positioned herself to the content. Specifically, she described that in order to teach the way that was expected she needed to have a more connected understanding of the content, not dissimilar to the way that Skemp (1976) described a relational understanding.

> I genuinely feel like my content knowledge has greatly increased and deepened by teaching this course. In addition to understanding the material better, a*pproaching it from the perspective of a teacher allows me to see broader connections across units*. Stats felt a little bit disjointed to me when I was a student, but as a teacher, I can clearly see the sequencing and how things we have already done have helped prepare for the new content.
> (Claire, Lead Teach Reflective Narrative, Pos. 10)

Contrasted with the way she learned and experienced mathematics as a student, with a focus on memorization procedures, teaching required her to adopt a new posture when learning the content, as we see in the excerpt below.

> The class I took was very calculations focused, with less emphasis on the conceptual. I spent a lot of time reviewing the material myself for this unit and making sure that I understood not just what we were doing, but why we were doing it. Now, having taught the unit, I feel really great about the material. I have definitely noticed my own understanding deepening through the process - I can clearly articulate why things happen.
> (Claire, Lead Teach Reflective Narrative, Pos. 5)

We can see from these passages that she had two modes of preparing for lessons (and units) that she was unfamiliar with. First, the act of preparing for the traditional view of teaching (i.e., introducing new ideas through direct instruction, leading classroom discussions, answering students' questions, etc.) required Claire to assess her own subject matter understanding and review the ideas she was less confident with ahead of time. Second, her detailed and thorough planning, which she adopted a shared format from her university course, helped her begin to think about the material in a qualitatively different way than simply review the material to make sure she knew how to do it. As she worked through problems she not only practiced for procedural competence, but she also made certain that she could justify the reasons behind the procedures.

Educational psychologists have theorized that through teaching experiences, students can come away with better content understanding (Galbraith & Winterbottom, 2011; Hoogerheide et al., 2016; Kobayashi, 2019; Koh et al., 2018; Roscoe & Chi, 2008) which is referred to as the *teaching effect* (these studies primarily investigate phenomenon in peer-to-peer settings). I theorize that novice mathematics teachers also experience the teaching effect, though it looks different for teachers than it does for students. Because knowledge for teaching requires a unique application of content understanding that is distinctive to the teaching profession (Ball et al., 2008; Shulman, 1986, 1987), novice teachers also experience a realization of having learned something new as the result of teaching. Claire's case demonstrates that through sustained teaching experiences she was given the opportunity to apply her content knowledge in different situations. Through these experiences, she came away with a deeper sense of understanding the content. In the student-focused studies mentioned previously participants were not professional teachers, and the view of teaching was normally limited to explaining concepts to others. For actual teachers, they do more than just explain content and respond to students' questions. When broadened to include all teaching tasks that a professional teacher does, we observed a similar teaching effect for a novice mathematics teacher but distinctly different because she came away with more than subject matter knowledge.

Answering the Research Questions

Having presented each intern's case and having made claims about their teaching experience, I now present an organized summary of the results based on the claims. This summary is organized according to the three research questions of this study. Each research question is listed and an answer to the question is presented based on the claims explained beforehand.

Answering Research Question 1

First research question:

What aspects of mathematical knowledge for teaching are reflected in the narratives that PSMTs provide about their learning during student teaching?

Recall that Ball et al. (2008) describe Mathematics Knowledge for Teaching (MKT) in terms of both Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK). In the following answer to Research Question 1, I will interpret each intern's description of their own mathematical understanding through the lens of MKT. From Matthew's Claim, we see that by the end of the school year, he came to view his mathematics understanding, after having taught using Big Horn's mathematics curriculum, as "knowing the whole story" and having "new layers."

Coming into the internship, Matthew came into the year with confidence about his own content knowledge and his ability to apply that knowledge in teaching situations. This confidence came from his extensive tutoring experiences over the preceding years. After having taught, he began to view his understanding differently. From Matthew's point of view, the quality of his mathematical knowledge had changed. Using Skemp's (1976, p. 2) description, Matthew described his understanding as instrumental (i.e., 'rules without reason') prior to his experience at Big Horn High School. We see this especially clear when he compared an instructional unit that he taught in the Fall to a similar unit that he had designed early in his teacher education program.

> Going back to TE 150, I wrote a Quadratics Unit for Algebra 1 students, very similar to the students I am teaching now! It was very interesting and enlightening to read back through that unit I created and compare it to the unit I am teaching now. *What I am teaching now seems to be much more simplified and 'why' focused, and much less focused on procedures*.
> (Matthew, GLT2 Reflective Narrative, Lines 163-181)

This was a common theme throughout much of his written data. At times, when trying to make a greater point about the way he was learning to teach the content at Big Horn, he explained the way that he had previously learned an idea in order to contrast it with how he now viewed teaching the content. In the passage below, we see how he compared the way he learned to describe the vertex-form of a quadratic form in terms of the number of transformations one does to a parent function.

> My mentor teacher and I talked and talked about how to best introduce the point-slope form, but we seemed to get stuck. *The only way I knew how to do it was in the following way*:
> $m = \frac{(y_2 - y_1)}{(x_2 - x_1)} \rightarrow (y_2 - y_1) = m(x_2 - x_1) \rightarrow$
> $y_2 = m(x_2 - x_1) + y_1 \rightarrow y = m(x - x_1) + y_1$
> However, we knew this wasn't how we wanted to teach it…. Teaching the lesson/concept this way was definitely longer and more involved than simply manipulating the slope equation.
> (Matthew, Weekly PLC, Lines 3-15, 39-49)

From this example, and others like it, Matthew's primary instructional decision centered around explaining the concept, drawing from how he knew to do the procedure.

After having taught at Big Horn for an extended period of time, his narrative around his own mathematics knowledge took on a new tone, which proved to be a central theme to his experience. We see in the following passage how he came away with a knowledge of content and curriculum (KCC).

> I've learned *how important it is to think of the class as a whole story and thinking about how each unit connects to the next*, not just how content connects within a unit. Big Horn does a very good job with this storyline idea and certain things such as using the word "rule" or thinking about an inverse as traveling a path on a map backwards really creates coherent understandings that transfer from unit to unit and also from class to class.

From his experience at Big Horn, Matthew now began to characterize his understanding as "knowing the whole story." In that story there were themes that spanned the entirety of the course, tied together subplots, and formed the structure for each unit. It is evident that this curricular structure helped him to think about teaching mathematics content through a particular lens which influenced all aspects of his teaching practice.

Finally, Matthew summarized the feeling he came away with about his own understanding by contrasting it to his earlier understanding. First, the confidence he gained from his tutoring experiences helped him come into the internship with confidence in his ability to explain his Subject Matter Knowledge. He explains, “It’s not that before I didn’t know the content or that I couldn’t explain it, but I realized that up until this year I didn’t really know what it meant to teach the content” (Matthew, GLT2 Reflective Narrative, Lines 8-22). Being in Big Horn’s mathematics department gave him a new vision about what it meant to teach mathematics in a way that went beyond explaining the procedure. Through Matthew’s reflections, we see his teaching emphasis shift to the ‘why’ behind mathematical procedures.

From Scott’s Case, he described his mathematics understanding, after having taught through the year, as now knowing the whole story of algebra (using the image of having seen a movie trailer ahead of time). As mentioned earlier in this chapter, Scott’s experience demonstrated the *teaching effect* (Galbraith & Winterbottom, 2011; Hoogerheide et al., 2016; Kobayashi, 2019; Koh et al., 2018; Roscoe & Chi, 2008) throughout much of the internship. I think the reason why we did not see this as much with Matthew was because he came into the internship with extensive tutoring experience. “I believed that I had a deep understanding of the material and was confident that I could help others reach this level,” he explained. “But being at my placement, I forgot many of the niche details that come with algebra” (Scott, Weekly PLC Prompts & Responses, Pos. 26). From this early experience, it seems that Scott initially experienced learning through teaching by realizing the details that he had forgotten about algebra. Further, as we see below, the cumulative teaching effect of having implemented an entire instructional unit, exposed him to a wider repertoire of strategies that he was previously not as familiar with.

> I found myself gaining a deeper understanding of solving quadratics as I taught the unit. When I was introduced to the concept, the quadratic formula was one of the concepts I left high school being able to repeat at any moment. It was my default and I treated it as such. But understanding now that given a quadratic already in vertex form, it is much simpler for me to use reverse orders of operations. *I did not anticipate learning as much as I had from this unit when I first launched it. But seeing my students try new strategies helped me gain this understanding that what is most comfortable may not be what is most efficient*.
> (Scott, GLT 2 Reflective Narrative, Lines 58-64)

Prior to teaching this unit, we see that Scott relied on one strategy for solving quadratic functions, but, as one might expect, teaching through a unit was, in a way, a thorough way to review prior content.

In addition to a cumulative teaching effect, Scott came away with viewing the content that he taught as if understanding a story, similar to Matthew.

> When teaching the unit, each lesson felt much more interconnected than units that I had taught previously. Each day seemed to transition into each other and the conversations that we were creating at the beginning and end of each lesson felt much more organic yet organized than the prior semester. *Teaching after having done the unit plan was like watching a movie and seeing the trailer beforehand. I had an idea of what was going to happen and had more access to anticipating students. But when you don't watch the trailer, you may still have an idea of what it is about, but you really have no expectations*. Because of this, I was able to better anticipate the mathematical conversations being held and facilitate the discussion so that it'd be easier to transition into further concepts.
> (Scott, LT Reflective Narrative, Lines 1-10)

This description was more than a cumulative teaching effect that he came away with. Through focused and structured planning, Scott came away with a deeper and more connected sense of understanding the content. From this Knowledge of Content and Curriculum, he was able to transfer it into his teaching practice and orchestrate classroom discussions because he was able to anticipate student thinking beforehand.

Like both Matthew and Scott, Claire described her prior mathematics understanding, both in high school and college, as having more of an emphasis on memorized procedures to solve problems. "In high school," she explained, "I just memorized these and took them as fact….my Algebra 2 teacher definitely approached the subject as being a list of things to memorize" (Claire, GLT2 Reflective Narrative, Pos. 10). Later on, we hear a similar description of a course she took in college: "This was one of the units I felt most shaky on in Statistics going into this year. My own Statistics professor covered it, but the class I took was very calculations focused, with less emphasis on the conceptual" (Claire, Lead Teach Reflective Narrative, Pos. 5). While Claire may have felt uncertain about her content knowledge coming into this semester, teaching not only solidified her procedural understanding, but seemed to have transformed it by the way she had to approach reviewing ahead of time. She mentioned this in the passages below:

> I have definitely noticed my own understanding deepening through the process - I can clearly articulate why things happen, and I can't think of a question that a student asked that I didn't know the answer to. I have learned this year that I actually know more statistics than I originally gave myself credit for, and I pick up on the details very quickly with a little bit of review.
> (Claire, Lead Teach Reflective Narrative, Pos. 5)
>
> I genuinely feel like my content knowledge has greatly increased and deepened by teaching this course. In addition to understanding the material better, approaching it from the perspective of a teacher allows me to see broader connections across units. Stats felt a little bit disjointed to me when I was a student, but as a teacher, I can clearly see the sequencing and how things we have already done have helped prepare for the new content.
> (Claire, Lead Teach Reflective Narrative, Pos. 10)

Before teaching, the content felt "disjointed". After teaching, she could now see "broader connections" and the "sequencing" of ideas. From being involved with all aspects of teaching the content consistently over the course of a school year, her Subject Matter Knowledge (SMK) developed horizon content knowledge (HCK). This is evident when she mentioned "I can clearly see…how things we already done helped prepare for the new content."

Answering Research Question 2

Second research question:

What teaching experiences do the interns emphasize as being critical in helping them understand differently the content they teach?

Recall that, according to Wenger(1998), a person's experience is the whole of their history of participation within a community. Further, when they learn within a community, it happens because of an imbalance between their prior experience and the community's regime of competence. From Research Question 1, we observed each intern describing a change in the substance of their mathematics understanding. What teaching experiences, or practices, did they identify as being critical to this transformation?

For much of Matthew's write-ups, he focused on how exposure to and participation in Big Horn's self-developed mathematics curriculum influenced how he viewed his evolving sense of understanding mathematics. First, he realized that there was an imbalance between his personal experience learning mathematics and the way courses were structured in Big Horn's mathematics department. "The content and the way it is structured at Big Horn has been new and enlightening to me because *it is so different from how I was taught*" (Matthew, Weekly PLC, Pos.4). In the passage below he summarized how Big Horn's curricular design pushed his perception of what it means to learn mathematics:

> A final thing that has grown my ability to and understanding of teaching content is that Big Horn does not use a textbook, but rather a curriculum of crafted worksheets that give students to explore new ideas and practice old ideas. Some days, this functions very much like a textbook…Other times though, it's much different. Sometimes the worksheet has very clear directions and pointed questions so that I don't teach anything up front at the board, but instead the majority of class time is working and struggling through the worksheet with their peers. *This has really challenged my idea of what a math classroom is "supposed" to look like and it continues to challenge my ideas of what content is*

> *essential to understanding the overall course content AND sets you up for your future classes.*
> (Matthew, GLT2 Reflective Narrative, Lines 88-99)

As he planned for upcoming lessons and units, through collaboration with more experienced teachers he came to see not only the structure, but the rationale behind the structure. We see this influence of working with more experienced peers reflected in Matthew's writing as he recognized the way content is taught which might emphasize ideas differently than the curriculum intended. In the instance during the PLC where he described a moment from a department meeting, he listened to a discussion about how to best introduce the point-slope form of a line. In this example, Matthew came away from the discussion realizing that:

> Teaching the lesson/concept this way was definitely longer and more involved than simply manipulating the slope equation. However, *it really is a deeper understanding of the material when taught this way. It takes the learning away from moving around numbers in an equation to thinking about your thinking process.*
> (Matthew, Weekly PLC, Lines 3-15, 39-49)

From Scott's Claim, through close shadowing of an experienced teacher, Drew, who was focused on creating a classroom designed around exploration and collaboration and because of university-required "big picture" planning, Scott came away with knowledge of content and teaching (KCT), knowledge of content and students (KCS), and to a limited extent knowledge of content and curriculum (KCC). From Drew, Scott assimilated a specific way to implement Big Horn's curriculum by reflecting on the relationship between instructional decisions and student understanding. In the university community, course projects, such as the Unit Plan project, encouraged Scott to take a closer look at the Big Horn curriculum.

From Claire's Claim, through co-teaching and collaboration with her mentor and actively implementing lessons across units, she came away with a fuller understanding of the content. While preparing to teach content she was unfamiliar with, she realized that she needed to do more than simply practice problems. Rather, she explained that she needed to "make sure that I understood not just what we were doing, but why we were doing it" (Claire, Lead Teach Reflective Narrative, Pos. 5).

This curricular knowledge was further developed by her continual practice of paying attention to and analyzing student thinking. Further she came away from one particular assessment experience (i.e., grading AP Stats free-response questions on an exam) with a fuller sense of the content.

> By grading this way, I see an increase in my own understanding of the concepts. I have felt pretty good about my own understanding after teaching it, *however, after reading and evaluating 80 FRQ's (2 per student), I see an increase in my own understanding not only with the 'what we're doing', but also 'why we're doing'.* I find myself being able to articulate important concepts more succinctly, and really get at the core of what's important. If I were ever in the position to be teaching Stats after my internship year, I think this grading process would help improve my practice especially with knowing what to focus on
> (Claire, Lead Teach Grading Write-Up, Pos. 4)

By applying her curricular knowledge in assessment situations, she learned to communicate big ideas in a succinct and clear manner. Further, these experiences helped her to reflect on future instructional decisions. Collectively these experiences required her to apply her content knowledge in a way that was especially different than learning content as a student.

CHAPTER 6: DISCUSSION

The goal of this study was to explore secondary mathematics teacher interns' evolving sense of understanding mathematics content as they reflect on their own teaching practice over the course of one academic school year. More specifically, the research questions that guided the design and execution of this study were:

1. *What aspects of mathematical knowledge for teaching are reflected in the narratives that PSMTs provide about their learning during student teaching?*
2. *What teaching experiences do the interns emphasize as being critical in helping them understand differently the content they teach?*

These questions, when answered together, were designed to help explain and unpack the interns' teaching experiences in a way that helped to understand which aspects of their mathematics knowledge for teaching were being developed. My hypothesis going into this study was that when novice teachers describe their mathematics knowledge as having changed because of teaching consistently over several months that they were, in-fact, beginning to develop pedagogical content knowledge. In other words, developing mathematics knowledge for teaching would coincide with a sense that their mathematical understanding took on new dimensions.

To answer the above research questions and to explore the way secondary mathematics teachers experienced novel teaching contexts, I collected data from (a) various written sources (reflections, synopses, asynchronous message board responses, etc.), (b) transcripts and fieldnotes from personal observations and conversations with the interns, (c) and artifacts provided to me from their classrooms (example tests/quizzes, rubrics, etc.). After analyzing the data, I made claims about each intern's experience, and answered the research questions using those claims. In this chapter, I will discuss how the data, claims, takeaways, and answers to the research questions from the present study connect with current and past research on learning through teaching and teacher professional development. This chapter will wrap up with what I ascertain to be the study's limitations, suggestions for future research, and concluding remarks.

Discussion

To begin this discussion, I will first recount the claims and takeaways for each intern. Following this, I will discuss how this project helps to add to the overall conversation around learning through teaching and teacher professional development. From Matthew's case, I made a claim that argues for the importance of curricular coherence in a novice teacher's understanding of the mathematics content and how this influenced his other teaching decisions. Through learning to teach in Big Horn's mathematics department, where the curricular structure was common across courses, Matthew came away with a fuller sense of knowing the mathematics content. His claim directly answered Research Question 2 by identifying curricular coherence as the most influential experience that gave him a fuller understanding of high school algebra. From Scott and Claire's cases, we came away with a view of the importance of productive mentorship. In Scott's case, we saw the impact that learning to teach from Drew, who made instructional decisions in-line with the spirit of the Big Horn curriculum, had on Scott's understanding of the curriculum. From this mentorship, Scott picked up on instructional practices that helped to support student learning by way of structuring lessons around their exploration and discovery of new ideas. In Claire's case, she gained broad teaching experience early on in her internship because of the way that her mentor teacher involved her as a co-teacher. As such, Claire came away with a thorough understanding of the content that she taught because of the many authentic teaching experiences she gained from the start. Their cases helped me to answer Research Question 2 by discussing the importance of community brokers (van Zoest & Bohl, 2005a;

Wenger, 1998) in the inclusion of novice teachers into ambitious teaching practices (Lampert et al., 2011); through participation in these practices the interns came away with a sense of knowing procedures in more depth, as well as and more importantly, knowing how procedures connect to and build upon each other. Having summarized the main takeaways from each case, I will connect my research to three broad research areas, namely: mathematics knowledge for teaching, novice teacher professional development, and learning through teaching.

Mathematics Knowledge for Teaching and Teacher Professional Development

As noted numerous times throughout this writing, the phenomenon that inspired the design, implementation, and analysis of this project is the statement that many novice teachers articulate: "I really didn't understand math, until I had to teach it." My hypothesis going into the analysis of the data was that when novice teachers claim this, they are, in fact, saying that their mathematical knowledge has transformed because of their teaching experience. As such, I made use of the *Mathematics Knowledge for Teaching* (Ball et al., 2008) framework to describe three specific interns' developing sense of knowing mathematics. A natural question that arises from the MKT framework is "How do mathematics teachers develop such knowledge?" Ball et al. (2008, p. 405) posed a similar line of inquiry into "how different approaches to teacher development have different effects on particular aspects of teachers' pedagogical content knowledge." Put another way: "What teaching experiences help novice mathematics teachers develop pedagogical content knowledge?" The narratives described in this study help add to the conversation around this question by providing the perspective of novice teachers. Recall the quote from Brené Brown (2021) in Chapter 3: "If research is going to serve people, it has to reflect their experiences." If we want to understand more about the experiences that help novice teachers develop their PCK, then we need to hear their perspectives about what they find helpful.

In this section I will discuss the role of curricular knowledge in the interns' overall experience, the impact of developing a teaching practice around student thinking, and the role that collaborating communities had on their professional development.

The Role of Curricular Knowledge

To some extent curricular knowledge, or knowledge of content and curriculum (KCC) (Ball et al., 2008), was influential for each intern's growth of other aspects of their PCK. The curricular context formed the structure by which they learned how to participate in their placements and what it meant to teach mathematics. For Matthew and Scott, exposure to and participation in Big Horn's coherent curricular context influenced the way they understood students' thinking and other instructional tasks. Big Horn's curricular structure and philosophy was a novel experience for Matthew, in particular. By preparing for and teaching through the worksheets, interacting with colleagues to understand the rationale for how tasks were sequenced, and understanding how units were designed with specific learning goals, he was then able to apply that knowledge to other teaching tasks. In his explanation of the grading system, for example, he skillfully applied curricular knowledge to describe how students' ideas were assessed in relation to the established goals for the unit. From Matthew's perspective, learning to teach in the Big Horn curricular context helped him develop an evolved knowledge of content and curriculum (KCC), and this, in turn, influenced the growth of other teaching practices, such as analyzing student work in view of the curricular goals.

Curricular knowledge was influential for Scott, as well, although he described his experience differently. From Scott's perspective, learning to teach through Big Horn's curricular context from a mentor who was an experienced practitioner helped him develop important instructional strategies that supported student exploration and collaboration. Through Drew's influence, Scott experienced what it was like to teach in line with the philosophy of the curriculum by using instructional choices such as productive struggle and uncertainty to encourage student exploration and collaboration. By the time he came explicitly to prepare his own unit plan, he was primed to understand the nuances of Big Horn's curricular design because he had already experienced what they set out to achieve through learning to teach under Drew's guidance. Teaching decisions now made more sense to Scott because they were seen through the lens of the curricular structure.

Although Claire did not write extensively about the curricular structures that were in place, she also gained curricular knowledge and was then able to use that knowledge to help develop other aspects of her PCK. By co-teaching with Katrina, Claire learned how to sequence lessons within a unit, design activities to help students explore concrete representations of mathematical ideas and use these activities as the foundation by which to investigate unit learning goals. Further, she demonstrated how the core ideas of a unit formed the framework by which she assessed student thinking, as she explained in the AP Statistics free response questions. Through all these experiences, curricular knowledge was the framework around which she structured assessments and instruction, as Matthew also did. This knowledge helped Claire to focus on ideas that were present in students' work and helped her streamline her feedback to them.

The Influence of Student Thinking on Novice Mathematics Teacher's Practice

In addition to curricular knowledge, interacting with students seemed to be particularly influential for Matthew, Scott, and Claire given the greater context of this project. Before beginning the internships, numerous interns (not just the ones followed in this study) indicated that much of their field experience had been atypical due to the COVID-19 pandemic. In each of their experiences, the interns developed skills for interpreting, evaluating, and making sense of students' ideas in varied instructional interactions. As such, Scott and Claire focused much of their writing around the interactions they had with their students. Some of this may have been due to the structured prompts they were required to write for their in-class experiences. Since many of the instruments used in this study were designed around noticing and reflecting on student understanding (van Es et al., 2017), moving away solely from theory to interacting with students had a considerable impact on their developing PCK. Scott's writing focused heavily on students' affective responses as a way to reflect on his instructional decisions. In particular, through his unit planning he was better able to interpret students' ideas and respond to their thinking. That is, through detailed planning he came away with a curricular knowledge and applied it to the way he listened to and used students' ideas to move classroom discussion forward similar to the idea of *generative listening* (Johnson & Larsen, 2012). This kind of listening refers to the ability to respond to students' ideas and make in-the-moment decisions concerning the lesson trajectory. Claire also focused many of her reflections on unpacking student thinking, such as when she noticed students misinterpreting negative exponents or when she had to make sense of their free-response questions on the end of unit AP Statistics exam. Through these experiences, Claire demonstrated the ability to listen *interpretively* and *evaluatively* (Johnson & Larsen, 2012), or the ability to make sense of students' thinking in the moment and to make judgements about their thinking. From their teaching internships, Scott and Claire came away with lived knowledge of how students learn and think about mathematics. Johnson and Larsen (2012) made the argument that given opportunities to interact with students' ideas, that mathematicians could have the opportunity to develop MKT in the courses that they taught; I found that this is also true for novice secondary mathematics teachers. In future teaching experiences, they will likely be able to make curricular and instructional choices based upon their KCS from this particular school year, being familiar with students' common misconceptions, the activities that encourage student collaboration and hold their attention, as well as how much time to allot to cover specific topics.

According to Wenger (1998), communities of practice that relate to each other are called *constellations of communities* and the interactions between communities are called *overlaps.* Within the context of the teaching internship, multiple communities are collectively engaged in the professional development of interns. In Claire, Matthew, and Scott's internship, collaboration between the university and school provided opportunities for the application of pedagogical theory to their everyday teaching experiences. For example, by way of a course assignment, the interns were introduced to the elements of backward design (Wiggins & McTighe, 2005) to plan an extensive unit, and to design and carry out an action-research project. This work provided them the opportunity to make sense of their school's curricular approaches and to understand the rationale for curricular decisions as well as investigate an aspect of teaching or learning that they found interesting. As such, they were able to apply curricular knowledge to classroom interactions and view their teaching not simply as an opportunity to develop skills but also to view their classrooms as contexts for learning more about how their students think about mathematical concepts. This overlapping relationship between their classroom and university communities was also an important aspect of providing professional oversight into their progression to becoming an in-service teacher. Cooperation between a school's mentor teacher and a university's field instructor allows for direct intervention should an intern not be making satisfactory progress in becoming a competent teacher. This mutual collaboration is an important feature of a university's teacher education program in that it provides an intern with the support they need to successfully progress through the internship.

Connections to Learning Through Teaching Research

Recall that in the literature around learning by teaching (Cohen et al., 1982; Cortese, 2005; Fiorella & Mayer, 2013; Galbraith & Winterbottom, 2011; Hoogerheide et al., 2016; Kobayashi, 2019; Roscoe & Chi, 2008), the very act of teaching educational materials to others has been observed to enhance the teacher's own learning of these materials. This phenomenon is also known as the *teaching effect*. Notably, the title "teacher" used in most of these studies simply refers to the person explaining concepts to others, usually in peer-to-peer contexts, such as in tutoring experiences, and is not generally referring to professional teachers.

Matthew is an interesting case because his career trajectory began with extensive tutoring experiences prior to completing a year-long teaching internship. During the school year, he often referenced in his write-ups how this tutoring experience gave him confidence in his ability to explain ideas. Specifically, he discussed feeling very prepared to teach specific content which indicated to me that tutoring over an extended period of time provided an opportunity for him to relearn and retain high school mathematics subject matter knowledge, which is similar to what Fiorelli and Mayer (2013) concluded. Additionally, he described that he not only knew the material but could also explain it clearly. Recall Ball et al.(2008) described specialized content knowledge as the knowledge required to carry out the common tasks of teaching (i.e., explaining ideas, answering students' questions, etc.) and is specific to the teaching profession. Through tutoring, Matthew gained experience explaining ideas to students and answering their questions in one-on-one settings, so he came away with experience using specialized content knowledge for much of the content one would normally have in high school mathematics courses.

Tutoring often serves as a springboard for many preservice teachers' careers, and I believe it prepares them for the application of subject matter knowledge for some of the tasks unique to teachers. Tutors have the opportunity to deepen their own understanding through discussion and explanation, making links between concepts, testing and clarifying their understanding, and working through ideas repeatedly to secure their understanding (Galbraith & Winterbottom, 2011). It makes sense then that Matthew, having extended tutoring experiences over the years leading up to the internship, felt like he had a solid grasp of high school content. I would argue, however, that he likely did not feel that he knew the content differently than when he was a student; rather he would have claimed that he knew the content thoroughly and in-depth. In fact, he stated later that the knowledge he gained from tutoring was not the same knowledge he gained from his internship experience (Matthew, GLT 2 Reflective Narrative, Lines 8-22). His teaching internship not only helped him gain more specialized content knowledge, but he came away with a more mature content knowledge and had a sense that he now understood the content differently.

As we saw from Matthew's Case, exposure to and experience from teaching in Big Horn's curricular context gave him a sense of the overall story of algebra and a consistent structure around which he could plan learning experiences. From this curriculum, for example, he saw the importance of choosing terms carefully so as not to unintentionally communicate an idea that would go counter to the curricular goals (he used the example of using the term 'rule' instead of equation). Because learning through professional teaching is distinctly different from learning by tutoring, I argue that early professional teaching experiences may afford novice teachers many of the same benefits as tutoring does, but they now have the potential to develop mathematics content knowledge which can only be applied in professional teaching situations.

Scott and Claire also experienced learning through teaching but without having experienced tutoring prior to the internship. Early in their internships, they described what they thought it meant to learn through teaching as remembering material they had forgotten. In Scott's case, he described having forgotten many details of the domain and range concept early in his experience with teaching algebra, particularly with how the context of a problem could affect the solution of a task. Through interactions with students, he came to realize that his lesson preparation had not been as thorough as it should have been (Scott, Weekly PLC Prompts & Responses, Pos. 26). We saw this happen again for Scott after he taught through a unit on factoring. By preparing for and implementing activities in this unit, he learned more strategies for factoring trinomials than he had previously known; more importantly, he came to view factoring trinomials as more than a procedure. Instead, he came to view the procedure in relation to another concept, namely the distributive property (Scott, Weekly PLC Prompts & Responses, Pos. 38). For Claire, she described how she prepared to teach a concept in AP Statistics that she felt shaky about going into the unit. In planning for these lessons, she practiced concepts until she was confident with the procedures, but also practiced in a way so that she could justify her procedures. In both cases they experienced the *teaching effect* at the lesson and unit level numerous times. Their experiences were consistent with the literature in that preparing to teach a lesson with the anticipation of actually teaching it to students (Kobayashi, 2019), the act of preparing to teach through practicing procedures (Galbraith & Winterbottom, 2011), and actually teaching to students (Fiorella & Mayer, 2013) helped them understand the concepts better.

Yet they also experienced what I described as the *collective teaching effect*; by this I mean that through being involved in every aspect of professional teaching tasks (sustained planning and implementation of units, grading, collaborating with colleagues, noticing student thinking, using student ideas to orchestrate classroom discussions, etc.), they came away with a transformed sense of understanding the content. In Scott's case, through extended participation within a classroom focused on student exploration and collaboration and completing an extensive unit plan project, he described his overall perception of knowing the content as if he had seen the movie trailer ahead of time and was better able to anticipate the classroom interactions as a result (Scott, LT Reflective Narrative, Lines 1-10). For Claire, through co-teaching she quickly gained a sense of how to structure and sequence learning experiences across units; and by the time she began grading free-response questions in the AP Stats class, she was able to use that curricular knowledge to create a detailed grading key that helped her identify the key components of students' responses and enabled her to respond to their ideas in concise ways. The collective teaching effect, as demonstrated by Scott and Claire, is similar to the relationship between mathematics and pedagogy described by Leikin (2010). By this I mean that through teaching in specific teaching communities, they gained flexibility and improvisation in their ability to explain ideas and respond to students' queries; additionally, they learned how to create situations in which students' autonomy and creativity were supported in their exploration of new mathematical ideas. Interacting with mathematics content in these varied ways gave Scott and Claire the opportunity to re-learn content they had forgotten, to extend their procedural knowledge through learning multiple solutions paths to solving mathematical concepts, and to transform their procedural knowledge into relational knowledge (Skemp, 1976).

Another possibility for learning from teaching is that the interns approached preparing for lessons differently because they felt a sense of responsibility towards their students. As Claire indicated, when she worked on problems as a student, she usually just worked through them to understand how to complete the procedures well enough to pass the course. As a teacher, however, she wanted to make sure that she could justify her work and made sure she understood the connections between procedures. One part of a mathematics teacher's *Aspects of Self-in-Community* as Wenger (1998) described is the notion of regime of mutual accountability. It is likely that due to a sense of what it meant to be the teacher in the classroom that the interns felt the need to thoroughly prepare to teach the content. This notion would agree with Poling's (2020) assertion that mathematics teachers may make instructional decisions, in part, because of a sense of being responsible for their students' opportunities to learn. In the case of novice mathematics teachers, such as Matthew, Claire, and Scott, one part of being responsible to their students was being thoroughly prepared to explain the content they taught.

Further, an important aspect of each intern's experience was the role that reflection played in helping them realize there was a dissonance between previous learning experiences and their internship experience (Caniglia et al., 2017; Dewey, 1938; Wenger, 1998). As Dewey (1938, p. 110) described reflection as a way of thinking, similar to inquiry. He defined it as the ability to "look back over what has been done so as to extract the net meanings, which are the capital stock for intelligent dealing with future experiences." Further, the interns' learning experiences were each defined by a dissonance between their previous way of learning or understanding mathematics and the way they now viewed their understanding. Matthew, for example, described the dissonance he experienced when he noticed the uniqueness of Big Horn's curricular approach; he claimed that "it is so different than how I was taught (mathematics)"

(Matthew, Weekly PLC, Pos.4) and then consistently reflected on how the features of this curricular experience helped him understand mathematics in a new way. Claire also realized a dissonance between her experience learning statistics as a student (i.e., focused on memorizing procedures) when she compared her prior experience of learning statistics to the experience of teaching statistics, pointing out that teaching required making connections between units. From reflecting on her knowledge after having taught statistics, she had realized how disjointed her previous understanding was and attributed the change in her understanding to how she approached learning to teach versus how she approached learning as a student.

For Matthew and Claire, they described their mathematical knowledge mostly in terms of procedures to be memorized. Further, Matthew and Scott described their previous understanding as inflexible (i.e., 'the only way I knew how to explain it was by…"). Through teaching, each intern realized this dissonance from their previous way of knowing mathematics content and realized that they needed a more conceptual understanding or more ways to explain ideas. According to Matthew, when participating in Big Horn's mathematics department meeting, he came to the realization that his way of understanding how to introduce the point-slope form of a line was not in line with the department's curricular goals and supported student memorization rather than sense-making. Similarly, Claire described a dissonance between the way she worked exercises as a student to how she worked on them as a teacher. As a student, she purely practiced for the purpose memorizing procedures. As a teacher, not only did she practice procedures to review them but also made an effort to understand the conceptual justifications for the procedures.

From a socio-cultural point of view (Vygotsky, 1978; Wenger, 1998), learning is an inherently social activity. We saw in an earlier chapter that from Wenger's (1998) theory individual learning is the development of modes of participation with others in society. He went on to expound upon this when he described that learning happens in a community, and it happens because of an imbalance between a person's experience and the community's regime of competence (van Zoest & Bohl, 2005a; Wenger, 1998). Viewing Matthew, Scott, and Claire's experience through this lens helps us to understand how they learned through teaching.

Recommendations

This study was designed as a way to understand novice mathematics teachers' experiences of learning mathematics content through their teaching practice. As a result of participating in their internship as a field instructor, a researcher, and a teaching assistant in their university graduate course, I gained thorough and valuable insight into the varied influences of mathematics teacher preparation. From their stories, we can make recommendations for the different stakeholders that will support the future preparation of mathematics teachers.

Teacher Education Program Coordinators and Course Designers

As we saw for Matthew, Claire, and Scott, school placements played a central influence in their experiences. While it may not be feasible to provide this experience for every teacher intern, being placed in a mathematics department that had created a community of practice around curricular coherence provided an opportunity for Scott and Matthew to observe ambitious teaching practices that were in line with greater curricular goals. As such, both had the opportunity to gain a curricular vision that guided the rest of their pedagogical experiences. For teacher education program coordinators, it would be important to become aware of such schools in the geographic regions that they serve and develop relationships with such schools.

Another factor, and one that is not surprising, is the importance of continued involvement in the university teacher education program throughout their first year of teaching. Most teaching internships (or student teaching experiences) in the United States usually last for a semester with most states setting a minimum of ten weeks (Greenberg et al., 2011). Recommendations for teacher prep program designers – have a course component as part of the intern experience. For the course instructor – continue to have or be sure to include unit planning. I think it is critical that this type of planning happens while interns are teaching, rather than just planning a unit prior to internship teaching experiences. This makes sense in terms of the *learning by teaching* literature (there is more of a teaching effect when learners are planning with the anticipation of actually having to teach the unit to their own students).

Further, for teacher education course designers, assignments should include actual implementation in the interns' classrooms. For example, in Matthew, Scott, and Claire's graduate coursework, they each designed a unit of instruction with the expectation that they would implement the unit, reflect on their teaching, and revise the unit plan after having taught it. I believe that this assignment element that required the interns to teach their units to students motivated a sense of urgency in them because of their anticipated interactions with students. These anticipated interactions, or as Kobayashi (2019) described as interactivity, seems to have had some impact on their own learning. In Claire's case, she selected a unit from a class that she had previously identified as one that she was not as confident with.

Recommendations for Mentor Teachers

The role of the mentor teacher in a novice teacher's internship is crucial to what an intern's experience will be like and as such plays an important role in helping an intern develop pedagogical content knowledge. Depending on how the mentor views and establishes their own role in relation to the intern, it could have a positive or detrimental effect on the intern's internship experience (Weasmer & Woods, 2003). From my experience working with interns for two academic years, I have observed many mentor-intern relationships. In my view, mentors that include their interns in every aspect of teaching, like Katrina did with Claire, give their interns valuable experience early in the teaching experience, but it also requires mentors to be cognizant of interns' capability to handle such responsibility. There have been instances (not with the three interns in this study) of mentors wanting their interns to mimic essentially everything they do, thereby giving them very little opportunity to develop their own teaching styles and, as a result, give them less authentic experience throughout the year.

Recommendations for Field Instructors

The role of the field instructor is also an important influence on interns' opportunities to develop their pedagogical content knowledge because of teaching. A question that came up routinely from my interns was "how detailed should my lesson plans be?" In my opinion, field instructors should work closely with their interns to establish norms for lesson and unit plans that go beyond what they might expect to see their mentor teacher doing. Their mentor teachers likely have taught their courses many times and often do not plan lessons with explicit background thinking (such as anticipating students' thinking, implementing plans in a way that supports exploration, etc.). This kind of thinking comes with experience, but novice teachers need explicit guidance for doing the necessary behind-the-scenes thinking that goes beyond being able to procedurally complete the tasks for that day. Early on in Scott's experience, for example, I noticed that he was simply working through the worksheets on his own and observing his mentor teach before having to do so on his own. As time went on, his plans became more detailed (after having required him to make use of the format supplied to him in his university coursework). Further, by the time he was required to design a detailed unit plan using backward design principles, he benefited from unit planning practice in that he was able to apply his knowledge in the classroom by anticipating student thinking and leading students in whole class discussions by using their ideas.

A second recommendation for field instructors, and one that came directly from this study, was the use of self-capture video to guide discussions with interns. This experience was invaluable to me as their field instructor, because it proffered insight into what the interns were paying attention to during classroom discussions and served as a platform for helping them focus on student thinking, rather than their presentation of concepts. For example, simple discussions with them about where to place the camera and where to focus the recording helped them begin to see that the emphasis in teaching mathematics should be upon student interactions and thinking and not on their presentations. Also, using self-capture video gives interns the opportunity to reflect on classroom interactions after the fact and provides specific instances on which to focus discussions with the field instructor and their peers. A common feature of any teaching internship is official observation of an intern's teaching by the field instructor. Commonly after such observations, there is a written report and/or a debrief meeting with the intern. I did not find these kinds of interactions as impactful for the intern because they relied more on one-way discussion. Using self-capture video gives the intern the opportunity to observe their own teaching and the video can serve as a concrete point of focus for the conversation between the field instructor and intern.

Recommendations for Teacher Interns

A generally accepted assumption in teacher learning is that through interactions with students, teachers can come away with craft knowledge (Kennedy, 2002). A preservice teacher does not have to wait until their teaching internship or university field experiences to begin gaining experience teaching students. As we saw in Matthew's case, he came into the school year as an experienced tutor. As discussed earlier in this chapter, tutoring helps to develop some aspects of a teacher's MKT (specifically horizon content knowledge, specialized content knowledge, and common content knowledge) (Galbraith & Winterbottom, 2011; Roscoe & Chi, 2008). I recommend that preservice teachers gain as much experience prior to their initial full-time teaching placements mainly because it gives them opportunity to apply their subject matter knowledge in ways one normally would not find in a typical mathematics course.

Limitations and Future Research

This study had several limitations which constrained both the generalizability of the data but also the breadth of data. This section will explore some of these limitations and identify ways that this project might inform future research.

First, because of the longevity of this project and the number of responsibilities each intern juggled, participant fatigue played into the data becoming fragmented at times. Completing a year-long internship, while helpful for developing robust teaching skills prior to becoming a full in-service teacher, is tiring for a teacher intern. Besides the day-to-day responsibilities my interns had, they were also enrolled in two graduate-level courses, as noted earlier. The demands of keeping up with their weekly course requirements, teaching responsibilities, and added participation in this research project was a lot for them to handle. While I attempted to piggyback on the requirements for one of their courses (being a TA for the course and having access to its syllabus), not every aspect of my research data coincided with a course assignment. As such, the interns occasionally opted out or did not provide some detailed or thorough responses. I reported previously that I had five interns when I launched the project, but found that for two, the workload was too much and opted them out because of the voluntary nature of this project.

Second, because of the choice to use case study research methods, this data is, by nature, not intended to prove any existing teacher learning theories. Yet, I do not think that this is a negative; rather, research is supported and added to not only with quantitative demonstration but through the stories we hear. Each intern's narrative was designed with the intent to gain insight into their teaching experiences as a way to get clarity on the effect that teaching practice has on a novice teacher's mathematics understanding.

As discussed in the prior section, Matthew's prior teaching experience as a tutor made me think more about the opportunities for developing MKT in different kinds of teaching experience. My hunch is that tutoring helps the tutor begin to develop some aspects of MKT, but in a more limited capacity. Future research could explore the "MKT-building opportunities" present in early teaching situations (one-on-one tutoring, substitute teaching, student teaching, initial years of in-service teaching).

Concluding Remarks

One of the motivations for designing this project around mathematics teacher interns' experience learning to teach was to better inform my own practice in supporting intern development of pedagogical content knowledge. By gathering the interns' perspectives on what sort of experiences they thought helped push their understanding further, I was able to explore the relationship between teaching experience and how novice mathematics teachers transform their knowledge into ways that can be applied to teaching situations. Through this research experience, I am better prepared to lead novice mathematics teachers in their early teaching experiences by suggesting ways for them to reflect upon their students' thinking, to plan in ways that go beyond reviewing procedural competency, to seek to identify the key overarching ideas that create opportunities for their students to explore these ideas, and to organize opportunities for them to collaborate with peers and colleagues to analyze teaching situations. In my future teacher education endeavors, I will draw upon the experiences from this project to help support novice teachers toward becoming ambitious mathematics teachers.

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APPENDIX A: Pre-Internship Survey

Consent
This is the first part of the research study you agreed to be a part of. You are being asked to participate in research about how your content knowledge of mathematics evolves throughout your teaching internship. Your participation in this survey is voluntary. You can skip any question you do not wish to answer or withdraw at any time. You must be 18 or older to participate. If you have any questions, please contact myself (fessler7@msu.edu) or my advisor, Dr. Shiv Karunakaran (karunak3@msu.edu). You indicate that you voluntarily agree to participate in this research study by submitting the survey.

Biographical Information
1. Name
2. Age
3. Gender: How do you identify?
4. Related Information: Any other pertinent background information that you might think would be helpful for me to know.

Internship Information
1. What school is your internship taking place at (if known)?
2. If unknown, what area of the state do you hope to be placed in?
3. Do you know what courses you will be teaching yet?
4. If so, what courses?

Content Preparation and Background: The information asked for in this section will help me to begin developing your personal biography (I will use a pseudonym) and how your academic preparation helps to add to who you are, which will make up part of the context of my case study.
1. What is your major?
2. Do you have minor? If so, what is it and do you plan on incorporating it into your teaching internship?
3. Why did you decide to choose this major/minor?
4. How would you describe your content area academic preparedness?
5. How would you describe your overall content area experience (in terms of opportunities to learn)?
6. Do you think you need more content area preparation? If so, please be specific.

Pedagogical Preparation and Background: The information asked for in this section will help me to begin developing your personal biography (I will use a pseudonym) and how your pedagogical preparation and background helps to add to who you are, which will make up part of the context of my case study.
1. Describe any previous teaching experience(s) you may have had.
2. Why did you decide to pursue becoming a teacher?
3. How would you describe your pedagogical academic preparedness?
4. What topics do you feel the most confident to teach?
5. What topics do you feel less confident to teach?

6. What makes you confident about beginning your teaching internship?
7. Are there any other experiences you wished you had had prior to your internship?

APPENDIX B: Pre-internship Interview Protocol

Thank you for agreeing to meet with me today for this interview. The idea for this project originated from reflecting on my own experience as a young teacher and remembering the feeling that math made more sense to me after going through student teaching and my first year of teaching. For my dissertation I want to focus on the teaching internship year and am setting out to understand the ways in which teaching influences how you, as the teacher intern, perceive what it means to 'understand' a mathematics concept.

I would like this interview to serve as a glimpse into your background as a math student, the reasons why you want to become a math teacher, and how you envision what your teaching will look like next year.

I will be the only person who reviews this interview, and I will describe your answers in my research by referring to you with an alias. Before we begin, I would like to record this zoom session and use the auto transcribe feature, so that I can go back and have an accurate account of our conversation. Is this ok with you?

START RECORDING AND AUTO TRANSCRIP

Do you have any questions for me about this project before we begin?

Background and Experience as being a Mathematics Student

1. Tell me about your 6-12 mathematics experience.
    1. Where did you go to high school?
    2. Would you characterize it as a non-traditional or traditional approach to the subject?
    3. What did you like about it? What did you not like?
2. How did you go about learning math that made you understand it as a K-12 student?
3. What did you think (as a high schooler) it meant to understand math well?
4. Tell me about any moments in K-12 mathematics that were particularly challenging. What made them challenging? What did you do to overcome them?
5. Tell me about your experience learning math as a mathematics major at MSU.
6. How did learning math look different to you in college than it did in high school?

Choosing to become a mathematics teacher

1. What initially made you want to become a teacher? Why a math teacher?
2. What kind of math teacher do you want to be?
3. Were there any teachers from your K-12 or college experience that served as an inspiration for you to choose this career? What about their teaching approach do you want to emulate?
4. What do you want a student to understand about mathematics after having taken a course from you?

Looking forward to the internship year

1. What are you excited about for the internship year?
2. What are you worried or unsure about for the internship year?

3. How much influence do you think that teaching math content that you are less familiar with will play in the way you view what it means to understand it well?
4. How do you envision working with the mentor teacher? Do you prefer to figure things out on your own or rely on the advice of others?

Closing Script

My initial design for the pre-internship data was to also use it as a way to select my interns for next school year. However, coordinating with TE has been challenging, thus I'm doing all portions of the pre-internship data with anyone who will complete it. So, sometime over the summer we will both be notified about who your field instructor and mentor teacher will be. Prior to the school year starting, if we are paired up together, we will have a meet and greet with your mentor teacher (probably in person, but possibly via zoom), we will go over the syllabus for the internship, etc. I usually wait a few weeks before I come to do your first observation, so that you get some time to form a rapport with your students.

What you might expect if we are paired up is that in the first month or two you will begin the practice of doing reflections based off of prompts that I give you and that we will have a back/forth conversation about your teaching within the document and in-person.

APPENDIX C: PLC Weekly Prompts and Reflection Questions

Welcome to our Learning from Teaching PLC! Like I said in our initial email – this is meant to support your learning about your teaching practice, support your learning in TE 802, and provide practice in noticing both your students and your own thinking as well as reflecting on your practice. I'm planning on doing this asynchronously as much as possible – with us meeting in person once every few weeks to discuss video analysis, noticings, and reflections. Each week we will have one video of a teaching episode from two different interns (supporting your Inquiry Cycle 1 assignment). I'd like to give each of you the opportunity to have a "test run" video before doing the actual recording of the task for Inquiry Cycle 1. After you upload your video, write up a brief synopsis of the episode, what you were trying to accomplish with the task, how students responded, and how you might change or adapt the task. If you are not uploading a video recording, your to-do for the week would be to watch your colleagues' video, write up things that you notice (by commenting directly on their write up), and write up a reflection for that week – based on prompts that I will provide to you. Finally, I hope that these analyses, reflections, and noticings will serve as an opportunity for us to have ongoing conversations about teaching mathematics and that you collectively develop skills for noticing student thinking and reflecting on your own thinking because of this work.

Week 1 (September 5-11)

Reflection Prompt: For this first week, what overall impressions do you have about teaching secondary mathematics thus far? Was it like you were anticipating? Were you surprised by anything? What challenges do you anticipate with the rest of the semester? Are there any stories that you can tell us that illustrate any responses to the preceding prompts?

Week 1 (September 12 – 18)

Video Analysis Activity: Here is the space to provide write ups and conversations about teaching episodes for interns 1 & 2. If you are uploading a video, please provide us with a synopsis, what you were trying to accomplish with the task, how students responded, and how you might change or adapt the task; additionally, participate in the Jamboard. Analysis activity. If you are not the person to upload the video, your task is to: 1) read the synopsis; 2) watch the video on Go React; 3) participate in the analysis activity within the Jamboard.

Noticing & Analysis Prompts: For each video, respond to the following: What evidence of student understanding do you notice? What was their idea (be specific). What misunderstandings do you notice? Where did these understandings and misunderstandings come from? How do these ideas compare with others' in the classroom? How might this student reply to another problem?

Reflection Prompts: For this week, as Claire & Salima have completed the first set of videos, think about and respond to the following prompt: What sort of class structures do you notice helps to elicit student discourse, explanations, and visible evidence of student thinking? In order to observe student thinking in a video format (thinking about the requirements for the Inquiry Cycle 1 assignment), what kind of choices can you make as the instructor?

<u>Week 3 (September 19 – 25)</u>

Video Analysis Activity: Here is the space to provide write ups and conversations about teaching episodes for Adam & Matthew's video. If you are uploading a video, please provide us with a synopsis in this space, what you were trying to accomplish with the task, how students responded, and how you might change or adapt the task; additionally, participate in this conversation space with your noticings. If you are not the person to upload the video, your task is to: 1) read the synopsis; 2) watch the video on GoReact; 3) participate in the conversation about noticing in this space.

Noticing & Analysis Prompts: This week, let's focus on noticing the cognitive demand of the task in addition to how students demonstrated their thinking. What do you notice about the relationship between student engagement and the cognitive demand of tasks? What do you notice about the complexity (or lack thereof) of student thinking and the cognitive demand of this task? Any other general noticing about these teaching episodes?

Reflection Prompts: Take a look at Matthew's share out from last week. I'd like to hear some about how teaching has influenced the way you think about what it means to understand a concept. What aspect of teaching has helped you think about a concept in a "new and deeper way". Come up with an example such as when Brandon was teaching how to find the rate of change of linear functions from equations and tables in relation to point-slope form.

Week 4 (September 26 – October 2)

Video Analysis Activity: Here is the space to provide write ups and conversations about teaching episodes for Scott's video. If you are uploading a video, please provide us with a synopsis in this space, what you were trying to accomplish with the task, how students responded, and how you might change or adapt the task; additionally, participate in this conversation space with your noticings. If you are not the person to upload the video, your task is to: 1) read the synopsis; 2) watch the video on GoReact; 3) participate in the conversation about noticing in this space.

Noticing & Analysis Prompts: This week, let's focus on noticing the cognitive demand of the task in addition to how students demonstrated their thinking. What do you notice about how the teacher uses questions to promote student understanding? It may be helpful to review the types of questions to elicit student thinking from the Fall semester (i.e. funeling, focusing, advancing, & assessing). I'll attach the reading we did last year to this. Any other general noticing about these teaching episodes?

Reflection Prompts: Over these last few weeks, as you have begun to think about noticing student thinking and noticing different classroom interactions (levels of cognitive demand, teacher questioning, etc.) that help to elicit student thinking, what have been your main take-aways? What have you noticed your students really responding to? What does their high-levels of engagement look like? Conversely, what seems to lower student engagement? Why do you think students engage in the mathematics in different ways at different times?

Week 5 (October 3 – 9)

This week we met virtually on Zoom to collectively think back over specific segments for each interns' video recordings and to review what we've learned about noticing and reflection.

Week 6 (October 10 – 16)

Video Analysis Activity: We are now going to individually practicing what we did in the face-to-face meeting last week. Thanks again for Claire & Salima being the "guinea pigs" in this new stage of noticing & reflecting. For this stage of our PLC, if you are uploading the video then you will need to provide a 'play by play' of events that took place during your video segment. This "play by play" can be in outlined format, paragraph format, or whichever format you prefer. Some important features of this write up: 1) time ranges for events, 2) important sub-segments, 3) rationale of why you chose this segment. Identify a few important sub-segments during your video. This portion is very important in that it is a way for you demonstrate that you are noticing something important going on with relation to student thinking AND providing a rationale of it's importance. If you are not uploading a video this week, your assignment is to 1) read the synopsis/write up that your peers wrote up, 2) what else do you notice/wonder about the teaching segment in relation to student thinking in relationship with instruction?

Reflection Prompts: From this past week, I'd like you to reflect on an event that you thought students had an opportunity to demonstrate their thinking in class. How would you describe the quality of their thinking? Did anything surprise you by what they mentioned or demonstrated in class? Does the content of their understanding communicate the same goals that you were hoping they would understand?

Week 7 (October 17 – 23)

Video Analysis Activity: This week Steven & Brandon will be uploading their video tasks in this new stage of noticing & reflecting. For this stage of our PLC, if you are uploading the video then you will need to provide a 'play by play' of events that took place during your video segment. This "play by play" can be in outlined formate, paragraph format, or which ever format you prefer. Some important features of this write up: 1) time ranges for events, 2) important sub-segments, 3) rationale of why you chose this segment. Identify a few important sub-segments during your video. This portion is very important in that it is a way for you demonstrate that you are noticing something important going on with relation to student thinking AND providing a rationale of it's importance. If you are not uploading a video this week, your assignment is to 1) read the synopsis/write up that your peers wrote up, 2) what else do you notice/wonder about the teaching segment in relation to student thinking in relationship with instruction?

Reflection Prompts: As we are near the halfway point of your first semester of teaching, let's take a moment to reflect some more on knowing mathematics as a student vs knowing mathematics as a teacher. As a student, what do you see as the best way to move your understanding forward/deeper? As a teacher, how do you see your understanding of concepts? Is it stagnant or changing? If it's changing, what would you attribute that change to? For this reflection it is very helpful to be specific. To the best of your abilities identify examples to support your claims.

Week 8 (October 24 – October 31)

Video Analysis: This week Johnny will be uploading his video tasks in this new stage of noticing & reflecting. For this stage of our PLC, if you are uploading the video then you will need to provide a 'play by play' of events that took place during your video segment. This "play by play" can be in outlined format, paragraph format, or whichever format you prefer. Some important features of this write up: 1) time ranges for events, 2) important sub-segments, 3) rationale of why you chose this segment. Identify a few important sub-segments during your video. This portion is very important in that it is a way for you demonstrate that you are noticing something important going on with relation to student thinking AND providing a rationale of its importance. If you are not uploading a video this week, your assignment is to 1) read the synopsis/write up that your peer wrote up, 2) what else do you notice/wonder about the teaching segment in relation to student thinking in relationship with instruction?

APPENDIX D: Week 5 PLC Discussion Protocol

Don't forget to record the meeting!
Warm Up:
- High/low from your teaching this week? What made your response a "high" or "Low"?

Introduction:
- Over the last few weeks, we've all had a chance to video some of our teaching and to think about noticing & reflecting, specifically in relation to student thinking & understanding.
- Today, we'll begin to establish some norms of writing about the things we noticed about ourselves and student thinking during our teaching practices.
- It's important to be able to accurately communicate the instance that we want to highlight (both in the moment AND after the fact). Describing in-the-moment noticing accurately is more efficient when we use monitoring charts. Describing after-the-fact noticing accurately is easier when you have artifacts of the teaching session (i.e., a video or audio recording, student work, etc.). It is also important to begin to go beyond describing events to thinking about the meanings of events, their importance, and why these events happened.
- Instructions:
  - We'll analyze each video by noticing the instructional behavior & student mathematical reasoning present in each
  - Collectively, we will create an accurate description of each video clip that not only describes the events, but will attempt to analyze student responses
  - As we discuss each video segment, I need a volunteer to document group thoughts in the google document (taking turns)

Throughout the discussion, seek to build on the interns' ideas, ask them to clarify questions about their statements, and be inclusive and supportive.

Eliciting the Interns' Thinking about the Lesson Segment
- In each of the video clips…
  - What do you notice about the teaching moves present in the video? D
  - What do you notice about how students' ideas are used to move the mathematical conversation forward? D
- What do you notice about how students' ideas are used to move the mathematical conversation forward? D
- What do you notice about students' overall posture towards the concepts? D
- What might a student's explanation (or response) might reveal about her understanding? A
  - What else might a student's response might reveal about their understanding? A

Probing for Evidence of Their Claims
- What evidence of student reasoning is there?

Helping the Group to Connect Their Analysis to Key Mathematical & Pedagogical Ideas
- What connections do you see between evidence of student reasoning and instructional behaviors and design?
  - Teachers' Use of Questions/Level of cognitive demand

APPENDIX E: Second Guided Lead Teach Daily Synopsis Prompts

Overarching goal that we are working towards with the reflective narrative: *How does "understanding ____" play out in your classroom? In other words, how does "understanding ____" develop over the course of the lesson or unit for the student, for you, as the teacher, etc. What classroom interactions help to move "understanding _____" forward?*

Date:

Lesson Title:

Lesson Goals:

Synopsis:

Take 15-20 mins to write down everything you remember from today's lesson. Think of this as a "play-by-play" activity. When you were preparing for the lesson, were there any concepts or tasks that you felt unsure about? What were the "big" moments of today's lesson? What did you notice students did well with? What did they ask questions about?

APPENDIX F: Lead Teach Daily Synopsis Guide

Overarching goal that we are working towards with the reflective narrative: *How does "understanding ____" play out in your classroom in terms of different classroom interactions? In other words, how does "understanding ____" develop over the course of the lesson or unit for the student, for you, as the teacher, etc. What classroom interactions help to move "understanding _____" forward?*

Date:

Lesson Title:

Lesson Goals:

Synopsis:

Take 15-20 mins to write down everything you remember from today's lesson. Primarily, focus on thinking about the different ways you and your students interacted with the content.

Classroom interactions: Student thinking & understanding; Cognitively demanding tasks; Teacher Questioning to Promote Understanding; Promoting Classroom Discourse through student-student talk; Using formative Assessment to Gain Insight Into Student Thinking

APPENDIX G: Dialogue-with-Synopses Reflection Prompts

Prompts (based on Model for Structured Reflection from Johns, 2009, 20010):

1. Take a few mins to review your daily synopses, lesson plans, and monitoring sheets from this week.
2. What teaching practices did you engage in this week?
3. What classroom interactions did you engage in this week?
4. Which of these practices were key, in your mind, to helping students understand the unit and lesson's goals?
5. Which classroom interactions influenced the way you made instructional decisions, the way you perceived how students were understanding the content, etc.
6. What themes, big ideas, related to "understanding the content" do you notice? When it plays out from a teachers' point of view? When it plays out from a students' point of view?
7. What are your overall takeaways from this week?

APPENDIX H: Reflective Narrative Prompts

Name:

Course:

Unit Title:

1. **Review your pre-unit narrative, daily synopses, lesson plans, and monitoring sheets and weekly reflections.**
2. **Thinking back over the Guided Lead Teach period, particularly in your focus class, what story do you want to tell about your experience as the teacher? A few prompts that may help you form this story:**
    1. How did you go about picking this unit to be the focus for this study?
    2. What was your planning process like? To what extent did you collaborate with your mentor teacher for this unit?
    3. Explain what your daily lesson structure was like.
    4. What ways did you plan for students to demonstrate understanding of the content?
    5. What takeaways do you have about students' understanding of the content? What claims can you make about how they approach understanding mathematics?
    6. What goals do you have for your students in your course? In what ways did you see your students working towards that goal over the last two weeks?
    7. What did the students struggle with? Why do you think they struggled?
    8. What evidence do you have that the students achieved the goals for this unit?
    9. In what ways have you seen your content knowledge being used through different teaching tasks?
    10. As a teacher, what examples can you think of that demonstrates that you are developing a different kind of understanding of the content. In other words, thinking over the last

several months (with examples from this unit as demonstration) how have you seen your posture towards understanding a concept change as a result of your teaching experience.

APPENDIX I: Matthew's PLC Share Out (GLT1 Period)

So, the lesson itself was very simple. We had been working with students on how to find the rate of change of a linear function from tables and graphs, so we gave the students a partially filled out table and asked them to fill in the remaining missing values. The goal of this lesson was to analyze their thinking process as a whole class and generalize this into the point-slope form of a line. Getting to this lesson was not as simple. My mentor teacher and I talked and talked about how to best introduce point-slope form, but we seemed to get stuck. The only way I knew how to do it was in the following way:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} \rightarrow (y_2 - y_1) = m(x_2 - x_1) \rightarrow$$
$$y_2 = m(x_2 - x_1) + y_1 \rightarrow y = m(x - x_1) + y_1$$

However, we knew this wasn't how we wanted to teach it. So, we talked to another teacher and they gave us the suggestion of completing missing values in a table. They said, "most students will know what to do to find the next value when they see a table, and then you can generalize what they are doing and get the rule that way". So, we did, and it worked really well.

Just looking at the first table, most of the students were able to figure out that the rate of change was 5: for every 1 increase in x, there was an increase of 5 in y. From there, a lot of students were able to figure out that going to 8 is a change of 3, so y would change by 15, since we would go up by 5, 3 times or 5*3. Then to get the next point, you would have to add 15 to 59 and would get 74. From here, I would start to generalize it:

"So how did you figure out you need to add 15? … Oh, so you saw that from 5 to 8 was a change of 3, so we would have to go up by a change of 5*3 = 15. Okay so from there, what did we add the 15 to? … That's right, the previous y value of 59."

That gave us the equation of: 74 = 5(3) + 59. From there, we can say 5 was the rate of change or "slope" so let's replace it with the letter "m". Next, we got 3 from subtracting two x values, 5 and 8. To generalize this process, let's call the two x values x2 and x1, and then the two y values we had, let's call those y2 and y1. With these substitutions, our formula would look something like this:

$$y_2 = m(x_2 - x_1) + y_1$$

Now we have arrived at point-slope form. A few notes though: why was y1= 59 and y2= 74? Why not the other way around? Well, when we subtracted the x values to find a difference of 3, we did 8 - 5 = 3, which makes x2= 8 and x1= 5. Looking at the table, 59 was the y value that went with the x value of 5, so both of those need to be "ones", or simply, 59 = y1.

Teaching the lesson/concept this way was definitely longer and more involved than simply manipulating the slope equation. However, it really is a deeper understanding of the material when taught this way. It takes the learning away from moving around numbers in an equation to actually thinking about your thinking process, which is a very important skill to have. This lesson in particular made me realize that just because I know how to teach something one way, doesn't mean it is the best way, but it also means that I might not have a full or deep understanding of the concept if I can only teach it in one way. I always knew that knowing how to apply or use a concept (such as this one) is a different skill and a different level of understanding than being able to teach somebody that same concept. But now I know that being able to teach a concept in multiple different ways is an even deeper understanding of the content. It’s like an extra layer of understanding

APPENDIX J: Learning Check Assessment Explanation Video Transcript

Hey Chuck. So, just hanging out here on Friday night, figured I'd make this video for you while I'm kind of doing some reflection work.

So, I just wanted to explain to you our grading system and how these learning checks work. So let me start with our grading system.
So, every learning check and test that we do is broken down into goals.
So, on this learning check, you can see that it's broken down into goals. So, on this one, we are checking goal number six and goal number seven.
And then you see these: Basic, Proficiency, and Expert. So, this is the part that I'm going to talk about right now. Um, as well as kind of the long-term grading purpose of these learning checks.

Figure J-1
Learning Check Frontmatter Example

Learning Check #4

_____Goal 6: I can write a rule for a quadratic function

$y = x^2$

_____Goal 7: I can find properties for a quadratic function

$a = 1$

Basic: Do two problems on the front and one on the back

$2^{nd} = 2$

Proficiency: Do #1-6
Expert: Do #3-7

For the rules in 1-4, find the following properties and explain why this is the case (you only need to explain each property once):

a) Coordinates of the vertex
b) Direction the parabola opens
c) The 2nd difference of the function

So, switching back here, so like I said, everything is broken down into goals and we grade each of those goals on a four-point scale.

Figure J-2
Breakdown of Assessment Scale

0 - Didn't take the test or learning check

1- Basically just left the sheet blank

1.5 - Did some math, but still have a ways towards understanding

2- On the right track, but still not quite there.

2.5 - BASIC. You completed a basic problem correctly.

3 - PROFICIENCY. You completed proficient problems correctly.

3.5 - EXPERT. Completing the expert problems mostly correctly

4 - EXPERT. 100%

So, if you score a zero, that means that you didn't take the test or learning check. So, we do that just because you have to have some sort of holder in there. So, if you didn't take it, you get as 0, but that's quickly replaced whenever you take it.

If you get a 1 that means that you basically just left it blank. And so that's just us saying like, "Hey, you took it. We don't need to worry about you taking this, but you really didn't do any work."

If you get a 1.5, that means that you did some math, but not really on the right track or maybe I should say, but like still have a way to go. So it's like, you know, if I'm asking you to inverse this function, say I give you a linear equation and it's like, Y =-5(x+7)+6, you just kind of switch the signs. So that would be something that could be an example of a 1.5 where like you did some math, but you still kind of have a long ways to go of understanding what the inverse is.

Figure J-3
Matthew's Demonstrating Student Misunderstanding

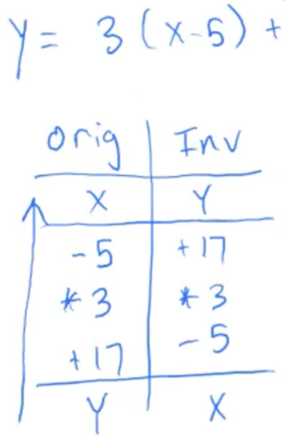
$$y = 5(x+7) - 6$$

$$-5(x-7)+6$$

So, after that, uh, would be a 2. So, if you score a two, that means that you were on the right track, but still Not quite there, I guess that's how I would describe it. So, it's like, again, same example here. If you're trying y=3(x-5)+17. And so, you understand that in our class, we've talked about doing this original and inverse table where you start with X and you write an order here. So, you have X. The first thing we're doing is subtracting five, multiplying by three and then adding 17. And that gives me Y . So, to write the inverse, I go backwards, but then maybe here you forget to switch the operations. So, it's still plus 17 times three minus five, but you understand that you are supposed to work back from Y to X, but you just miss the understanding point of switching the operations. So that would be something that's like an example of 2 (See Figure J-4 below).

Now a 2.5, this is what we call basic. And so, this is where you need to be to pass. Now, technically you can be like have combinations of 2.5's and 2's and still pass. But like we say, 2.5 is our cutoff to give them something to shoot for. So, this is Basic. So, this means that you completed a Basic problem correctly. So basically, like on the work, on the learning check that I showed, we had some labeled Basic. If you got all of those, correct. Awesome. You're had a Basic understanding, you know, like the minimum that you need to know to pass sometimes too, you can get the score if you complete the Proficiency problems, but have some minor mistakes. The next one would be Proficiency. And this is our 3 level. And this would mean that you completed the proficient problems correctly. Or it could mean that if you're trying the expert problems, you made good progress on the expert problems, but you made mistakes where you only solved at the Proficiency level. So again, I can kinda show you an example of that when I switch over to the learning chart.

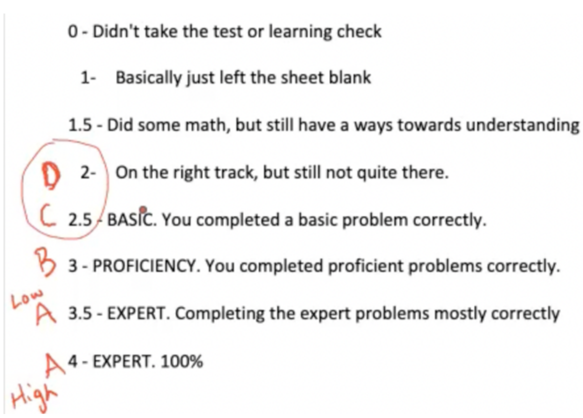
Figure J-4
Matthew Demonstrating Another Students' Misunderstanding



And then a 3.5 that is Expert. And so that is again just like the others, this would be completing the expert problems correctly. And now there is a small caveat here. So the 4 level is also expert, but this is like our traditional 100% in math classes. So if you get a hundred percent, 4, that's awesome. Okay. But this 3.5 expert, you can complete the expert problems correctly, or I should say mostly correctly.

So we have some expert problems and it's like, you know, if you get all of the Proficient stuff and you get some of the expert stuff, correct, you can still get a 3.5, but you know, you're just not at a 4 because you didn't get everything completely correct.

Figure J-5
Matthew's Further Explanation of Assessment Scale


0 - Didn't take the test or learning check

1- Basically just left the sheet blank

1.5 - Did some math, but still have a ways towards understanding

D 2- On the right track, but still not quite there.

C 2.5 - BASIC. You completed a basic problem correctly.

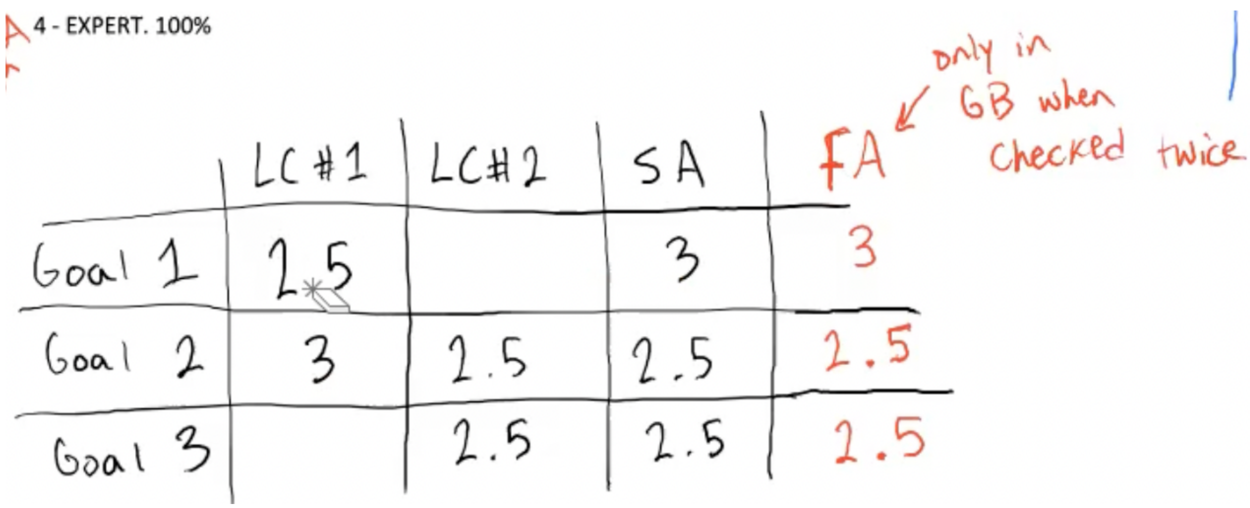
B 3 - PROFICIENCY. You completed proficient problems correctly.

Low A 3.5 - EXPERT. Completing the expert problems mostly correctly

A High 4 - EXPERT. 100%

So, when we do this, we modify our grading scale a little bit. And now I don't know it a hundred percent, but I know that 3.5 is an A, but it's kind of like a low A. A 4 would be a high A obviously because it's like our 100%. Proficiency would be a B. Basic would be somewhere in the C range. And then this would kind of be a D range. It's not like a hundred percent correct, um, on these two here, but it's enough to kind of be in the neighborhood. Because like you can't get all 2's and pass, you need to get like a little bit of 2.5's to pass. So, there’s each of the numbers. And here's kind of how it works as we go throughout the class, we have learning checks.

Figure J-6
Matthew’s Explanation of Final Grade Determination

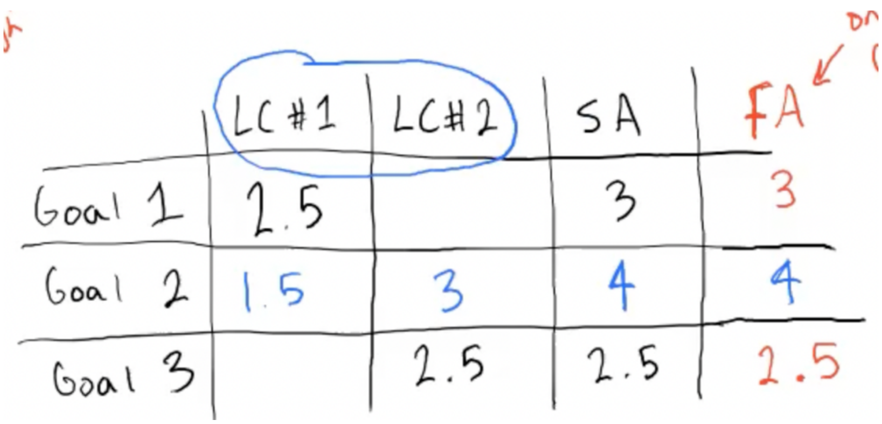


So, we have goals that we test that we check on and test on. And so, we have a formative assessment category and a summative assessment category.

So, say I'm working with goals so far. We just have goal 1, goal 2, and goal 3. So, say my first learning check, I cover goals 1 and 2 and I get a 2.5 in goal 1 and a 3 in goal 2. Then maybe I'm learning check number two, say it covers just the last two. So maybe I get a 2.5 here, a little bit worse. And then another 2.5 here on Goal 3. And then say I have my test. So, our tests, we call these summative assessments. So, I'm going to call it SA. And so, this, that would be over all the goals for the unit. So, say I do this again and I get a 3, a 2.5, and a 2.5 here. The last category that we have is this formative assessment category. So, each of these learning checks individually count for zero in the grade book. So, if I get a 2.5 on this learning check, it's not going to affect my grade. If I get a three, you know, a 2, 5, 2, 5, whatever on these learning checks on the goals, it's not going to affect my grade. But what we do is overall, we look at them as a cumulative (a collective) and that's how we establish this formative assessment grade. So, for example, in Goal 1, I have a 2.5 on learning check number one, and I have a 3 in summative assessment. I've shown that I was at a 2.5 level, but now I understand the material at a 3 level. And so, my formative assessment would be a 3. If I'm looking at goal two, I scored a 3 on the first one, a 2.5, and then a 2.5 on the test. So that shows me that I'm probably at a 2.5 level. Yes, I did score higher on this first one, but that was in the beginning. And then I scored two 2.5s in a row. So, it's saying, you know, maybe I just kind of had a better day that day or maybe it was just, the questions were a little bit easier, but overall, I'm probably at this 2.5 level and then here for goal number three, I have two, 2.5s. So, my formative assessment would be a 2.5. Now we only do this formative assessment category when we have checked on a goal twice. So, it can either be a learning check in a summative assessment to learning checks. We only put this in. So, it's only in the grade book, the GB when it's been checked twice. So that just allows for it to be more of a fair assessment of how you are doing now. This is kind of an easy example because we see some progress or some consistency, but let me change these numbers, uh, to show you just kind of how the system can be a little bit different.

So, say on this first one, let me change this right here. So, say I got a 3 on the first one and then I did not do as well. And I got a 2 here. Okay. That's a 2. And then I got a 2.5 on the summative assessment. So, this would still kind of make sense to have a 2.5, because I started at a 3, then I kind of dipped down to a two and I kind of leveled out here on the most recent score being at 2.5.

Figure J-7
Matthew's Further Explanation of Final Grade Determination

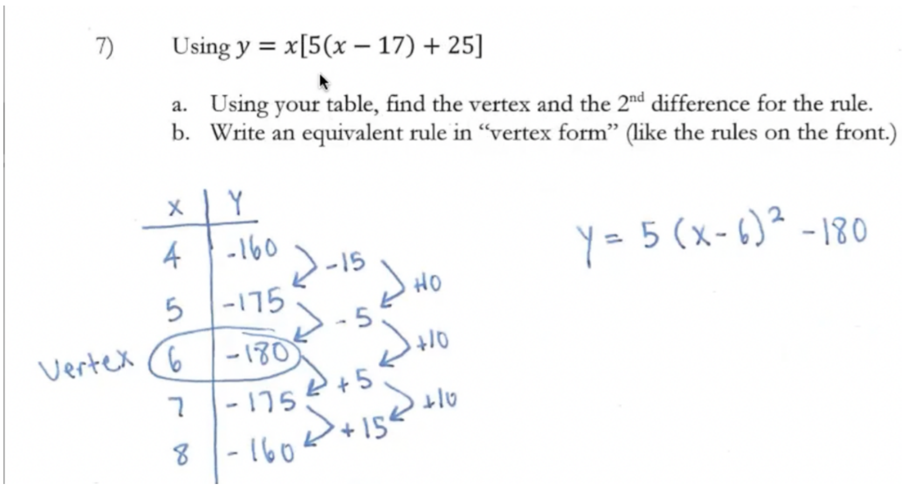


	LC #1	LC #2	SA	FA
Goal 1	2.5		3	3
Goal 2	1.5	3	4	4
Goal 3		2.5	2.5	2.5

Now if this one changed as well, I have seen it. Let me change all three of these. I've seen it before where we kind of have a jump of one. So, say it was a 2 and then a 3 and then a 2. This formative assessment, it's not, not like the highest score possible. So yes, my highest score is a 3, but this is showing me, "hey, I was at a 2 and then I scored at a 3 and then a 2. So maybe I put this right in the middle at a 2.5", or maybe if this 3 looks kind of a little bit fluky, I could bump this down to a 2, but I wouldn't normally do that. I would keep this at a 2.5. So, there is some subjectivity here, but most of the time it evens out pretty well. Most of the time we don't see scores where someone goes like a 1.5 and then jumps up something big, like a 3 and then a 4. Most of the time we don't see that. However, we do most of the time we do look left to right. Like if you're showing progress, this is a big jump. So, I might not say that you would end at a four, but maybe something like a three, five, but maybe looking at the test if you're like, man, you not, I knocked those two tests out of the park or these last two learning checks some of the assessment then maybe I would say a 4. That's just kind of where we have to look at the work and, and make some notes.

Now, last thing before I switch over to the learning check onto this learning check. We don't always have Expert questions. Some of that is because maybe we just have not learned the material enough to be at an Expert level. So, for example, it could be on the learning check that I'm just talking about right now, I'll switch over there. This one Expert was Question 3-7, but really the main Expert question was number 7. Let me, uh, get to the next page here. There we go. We're looking at this equation outside of vertex form, try and use your table. Find the vertex, second difference, write a rule in vertex form. This was something that we talked about in class, but maybe we hadn't got to this yet in class, we've only done this writing rules, finding properties stuff. Then maybe I don't have an Expert level on the learning check. So that could be why someone only scores a 3.

Figure J-8
Matthew's Example of an Expert Question



So hopefully that kind of makes sense. I know that was a little bit confusing. Um, but if you have questions about this grading system, please talk to me and maybe we can zoom and talk about it some more. Um, but you wanted to see the learning check. So, I just, I wanted to talk about that a little bit.

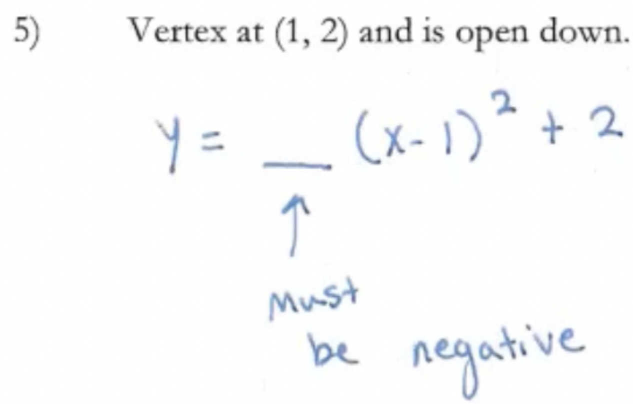
So, we break this down into goals and that is because we want them to see what they need to work on. So, we have goal 6 and goal 7 and we score them individually. So, you could get like a 2.5 on goal 6 and a 3 on goal 7, just for example. And that shows me okay if I got a 2.5 or something on goal 6 and that's not where I wanted to be, but I did score like a 3 or higher, and that's where I want for goal 7. Then I need to focus my attention on goal number six. It's one of the things that I like most about this grading system.

So, as I said, we break this down, telling them what they need to do for each level. So, for a basic level, a 2.5, you need to be able to do any two problems on the front. So, two of these four and then one problem on the back. So, there's this Expert question number 7 and then there's 5 and 6. So if you can do two on the front and one on the back then perfect, you're at a basic level, but then Proficiency, it's a little bit different. So, Proficiency, we want you to be able to do all of the ones on the front and 5 and 6, excuse me. On the back again, keeping 7 is like this Expert level.

So, if you can do everything, but number 7, you're at a pro you know, everything at like a B grade level and you're doing very well. Then lastly, for Expert, I could have said all problems (1 through 7). Most students will try all problems who are Expert level, but we do kind of limit it just to say “Hey, you know, if you can do 3 and 4, 5 and 6, and then 7, you'd be at an Expert level.” Cause most likely if you know how to do seven, it kind of points us that, “Hey, you probably know if you can do 7, 3 and 4, 5 and 6.” It's probably saying “Hey, you can probably do 1 and 2 as well".

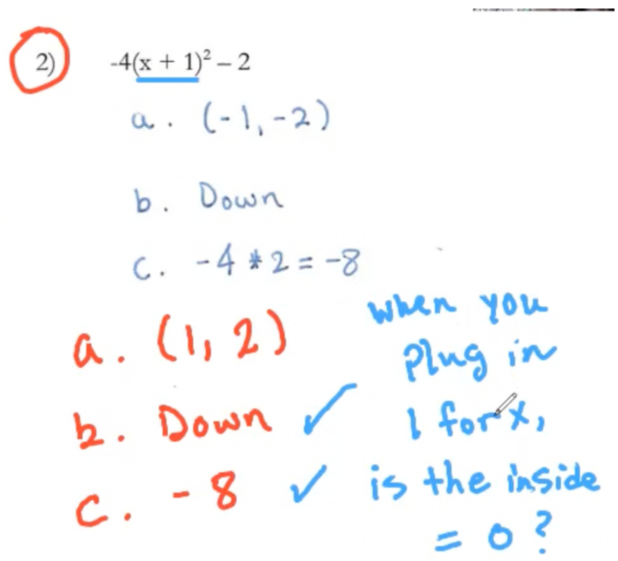
So as far as that goes, you know, on my answer key here, I just kind of write up some notes, kind of some things I'm looking for. It doesn't have to be exactly what I said, but this is just kind of a good idea of what I'd be looking for each question. I do have some ambiguity, I guess, here. So, this, I just left a blank and saying, “Hey, the vertex is at this point. I know I need to have $(x-1)^2+2$. And since it's opened down, it can be any negative number here”. So, I just made a note to myself, “Hey, we need to have a number here.” It's got to be negative, but it doesn't matter which one it is. And so, I would go through each of these and take, um, write down their scores. I would give them comments kind of just, um, sometimes questions, sometimes comments. I don't like to correct everything and write down the right answers, but I'll just kind of make some comments, give them some questions to help them down the right track.

Figure J-9
Matthew’s Example of Assessment Key



So, I guess, um, just to give you more of an example on that, if I was a student and, and uh, this question number 2, say my answers were (1,2), down, and -8. Now down is correct, -8 is correct. But this part is not correct. So, I might give him a question like "When you plug in 1 for x, Is the inside zero?" And I use this terminology because this is something that we've talked about a lot in class. We talked about this idea of the line of symmetry in that when this inside is equal to zero, we are on the line of symmetry. Our distance away from it is zero. We are on the line of symmetry. And so, our vertex happens when this inside of this x of, um, this inside X plus one, when that is equal to zero, we're at our vertex. So, giving them, this clue here, when you plug in one for X is the inside zero. I'm hoping that they go back and look and say, "Hey, well actually if I plug in 1, 1+1 is 2, it's not 0". So that's me saying like, "Hey, this answer is incorrect", but here's kind of some guidance to get the right one that way. It makes them think about it a little bit more without just giving them the right answer.

Figure J-10
Matthew's Further Example of Assessment Key



So, I hope this helps, as far as the grading system and what the learning checks look like. Again, if you have any questions about this stuff, I'd love to talk about it. I love grading. It's really good for me because it helps me see really where the students are at what they need to work on, but also helps me understand like, "oh, I see a lot of the students messed up this part. So that means maybe as the teacher, I need to focus on this aspect of the task a little bit better

APPENDIX K: Matthew's Reflective Narrative (GLT 2 Period)

So far, my first semester of the internship has been absolutely wonderful. I work with so many great people every day and I could not be happier with my placement. One of the reasons is that many of the mathematics department teachers have graduated from Midwest State or have had interns from Midwest State. This means that they really understand the internship process and know how to support me as I am growing and learning this year. However, the main reason that Big Horn has been so wonderful is that it has challenged my understanding of what it means to teach math. I think at this point, a little bit of history would be helpful.

Yes, this is my first time teaching in a classroom full time, but it is not my first time teaching. For starters, in my TE 407 class, we had an opportunity to work with Midwest State students in a college algebra class where we co-taught two lessons with a partner. On top of that, this is my 5th year working as a math tutor for Lansing area students from several different high schools and middle schools. I have extensive experience in Algebra 1, 2 and Pre-Calculus and those are the classes that I am teaching right now, with my focus class in Algebra 1. Because of this experience, I thought I knew how to teach and explain the content and what it looked like for my students to understand the material. However, working with Big Horn's math department has completely turned this theory on it's head. It's not that before I didn't know the content or that I couldn't explain it, but I realized that up until this year I didn't really know what it meant to *teach* the content. Big Horn teaches students the content in ways that I have never even experienced before, and I think that it does an amazing job while doing so. They do a great job emphasizing the "why" of the math involved, as well as allowing space for students to discover and make connections on their own instead of simply telling them things found in the chapter of the textbook. To illustrate my point, I am going to focus in on my GLT2 experience in my focus class. During those 2 weeks, students started their Quadratics unit and the main goals during the two weeks were the following:

1. Students can find properties of a quadratic function.
    0. This includes things such as the vertex, the direction it opens, the second difference and the line of symmetry.
2. Students can write a rule for a quadratic function.
    0. This could be writing a rule when given a table of values, when given certain properties, or when given a graph. Sometimes we asked for specific rules and other times we simply asked for *possible* rules.
3. Students can change forms of a quadratic rule in order to find properties.
    0. This involves changing a quadratic rule in vertex form (y=a[x-h]2+k) into simplified form (ax2+bx+c) and vice versa.

Before I began my two weeks of GLT2, the students did a <u>Desmos Activity</u> along with some discussion that helped introduce them to the basic idea of quadratic functions. Through the activity, they were able to see the basic shape of the graph (parabola) and see how changing the a, h and k values affected the graph. We started in vertex form, because this is the form we will use most often in the class. On the first day of GLT2, we discussed each of the coefficients in vertex form and talked about what their meaning is. This is one example where Big Horn's curriculum differs with my experience. When I learned about the coefficients, I learned about them in association with transformations and talked about how each letter changed the graph. However, in our department meeting before this unit, one of the veteran teachers and our department chair both said that it would be a *mistake* to interpret this lesson as transformations. Instead, this is (paraphrased) what they said:

"It would be a mistake to interpret this lesson in the unit as simply a lesson on transformations. Instead, we need to think about our rule as a series of steps that take an input and transform it into an output, and each of the coefficients are a part of that series. For example, changing h shouldn't be thought of as 'moving the graph left and right'. It should be thought of as 'when I change my h value, I am now subtracting h from my previous input. That means to get the previous output, I need to add h to my input'. This is the same for the k value. I do all of my steps to the input and the very last step is to add k. Therefore, when changing k, I am changing the very last thing added to my output, so as a result my output would increase or decrease based on k…. Also, when thought of as a series of steps, this creates a necessity for the mirroring/symmetry that we see in parabolas. (x-h)2 can be seen as finding the distance between our input x and the value of h, and squaring that distance. Therefore, if 2 input values are equidistant from h, then the (x-h)2 part in the rule would have the same value, and lastly adding k to that value will still ensure that both inputs equidistant from h will have the same output, thus creating symmetry about the line x=h"

This was all a challenge to my previous thinking! For starters, I needed to start thinking of an equation as a series of steps that changes an input to an output. In fact, this is why they started calling it a rule instead! The word "equation" is implying that the left and right side of the equal sign are indeed equivalent, and that is not the emphasis we want to have. So by changing to the word "rule", we more strongly imply a course of action to be followed. On top of that, I was really challenged by my department chair about this idea of "mathematical necessity". He said that when we create scenarios or showcase situations in which there is a mathematical necessity, it gives meaning to our work and eliminates the ideas of "we're just doing what we've been told" or "math has all these made up things that don't make sense and I have no idea where they come from or why it's important".

As a result of this mindset shift, how I talked with my class about finding the vertex (to begin with) was different as well. If (x-h) tells me how far away I am from the line of symmetry, I need an input such that x-h=0. So, in my class I repeated the phrase "what makes the inside 0?" over and over when talking about finding the input of the vertex. One way in which this grew me was that it showed me another way (and I think a better way) to explain how to find the vertex. I had always learned, and therefore always taught others that the vertex is at h and since the equation has x-h, if it is x+h then I make that h negative. This was really just a memorized and repeated process that I have internalized, but not a good way to teach. Teaching the idea of "our input (h) is what makes the inside 0 and our output (k) is what y equals when the inside is 0 is much better because it helps eliminate those common positive/negative errors.

Another cool thing that grew my understanding of teaching is that this rule idea can be consistently applied when talking about switching forms from simplified form to vertex form, as well as when finding inverses. For example, no longer is an inverse thought of as "switching x and y" or a "reflection across the line y=x". Now, an inverse is starting at my output and following my series of steps in reverse to get my input. Lastly, a final thing that has grown my ability to and understanding of teaching content is that Big Horn does not use a textbook, but rather a curriculum of crafted worksheets that give students to explore new ideas and practice old ideas. Some days, this functions very much like a textbook: here is a warmup where we do an example problem together, then we'll discuss and make sure everybody is on the same page, and then here is this worksheet for you to practice what you learned as well as try to extend your thinking. Other times though, it's much different. Sometimes the worksheet has very clear directions and pointed questions so that I don't teach anything up front at the board, but instead the majority of class time is working and struggling through the worksheet with their peers. This has really challenged my idea of what a math classroom is "supposed" to look like and it continues to challenge my ideas of what content is essential to understanding the overall course content AND sets you up for your future classes.

All this being said, this breakthrough and growth in my teaching knowledge is only good or important if it can make a difference in students' ability to learn and understand the material. To check their understanding, we used a few different methods. The first of these is the classic "quiz" concept, which we call "learning checks". As explained in my video, learning checks are assessments that do not count towards their grade, but rather simply serve as an opportunity for the students to show me and my mentor teacher what they know, and these give us feedback on what we need to adjust in our instruction. We do grade these and give feedback, but it does not count as a grade in the gradebook. Another assessment form that we use is we consistently give students ample amounts of work time on a worksheet and during this time, I walk around and look at student responses as well as working one-on-one with many students. This allows me to check in with them immediately after a lesson and see how well they understood the concept I introduced at the board, or it allows me to see how well they can use previous topics to extend their thinking when given a new worksheet with little instruction.

In regards to the first goal of finding properties, because of my emphasis on "what makes the inside 0", I had a handful of students on a learning check only write the x-coordinate when they were asked to give the coordinates of the vertex. This is an understandable mistake and since then, I have corrected my wording and now say a phrase more along the lines of "remember, to find the x-coordinate of our vertex we need to see what input makes the inside 0." In addition to this, I made sure to always stress that the vertex is a point, so it needs an input and output pair. In regards to using properties to write a rule, they did well when given specific properties (2nd difference and vertex), but still struggled when trying to write a rule from a table. When we discussed the idea of symmetry, we showed them that we can look for symmetry in the y-values of a table and the vertex is the "middle" of that symmetry. However, many students tried to write a linear rule for the table (since we had spent all semester up until this point on linear functions). To help fix this misunderstanding, as well as help to strengthen their ability to write quadratic rules, we had two follow-up worksheets for them. One was titled "Linear or Quadratic" where students were given multiple situations and asked if it could be modeled by a relationship that was linear, quadratic or neither. A second worksheet was called "Exploring Vertex Form" where we used calculators (which are always allowed on learning checks and tests) to help strengthen our understanding of the properties of a quadratic rule in vertex form, as well as writing rules in vertex form from a table. Both of these worksheets went very well and when walking around and working with students, many seemed to understand this content better by the end of class. I know they understood it better because when I asked them why a rule was linear or quadratic, or if I asked them how they found the vertex, they were able to explain their reasoning much clearer than previously. They were comfortable saying things like "5 is the input that makes the inside 0, and it gives an output of -3", or "this rule is quadratic because it has a constant 2nd difference" or "the vertex is at (-2, 6) because this the middle row that doesn't have a repeating y-value". I think that my emphasis on the "why" of a problem as a teacher helped convey to my students that the why was important, but also help them develop an understanding of the content because seeking to understand why results in learning content much better than seeking to memorize a pattern. Lastly, with regards to the last goal, we definitely need more work on learning how to switch forms of quadratic rules. Talking with a veteran Algebra A teacher, it sounds like Covid might be a little bit to blame for this. He said that typically, students understand the distributive property with one variable or number such as being able to distribute 4(x-3) or x(x+7), and so the switch to distributing twice when multiplying binomials isn't too big of a stretch. However, I was telling him that with my Algebra A classes, many students were struggling with the basic distribution. He said that this might be due to content that was cut out of earlier curriculum when Covid struck and moved things online for a year and a half. It could also be due to many students not learning as well during online school and the resulting knowledge gap or deficiency could include this distribution skill. Regardless of the reason, after Thanksgiving Break, we will be working on strengthening this skill because they need it for when we start to inverse quadratic rules. When it comes to switching from simplified form to vertex form, the "standard" algebraic method is completing the square. In all honesty, this is a confusing process for many *college* students. To help students understand this process, Big Horn is using a modified version of completing the square where you simply follow the steps in reverse that you do when multiplying two binomials. (I will upload a graphic organizer that we made this week after GLT2 to help students understand the process better as well as come up with the shortcut of h= -ba2) students need more help with this as well and it is something we plan to work on when we come back after Thanksgiving. We know they still need help on this because we had a learning check before we left for Thanksgiving break and many students could not switch forms of a quadratic rule in either direction.

Overall, my student teaching experience and especially these past 2 weeks of our quadratic unit has challenged me and deepened my understanding of quadratics and what it looks like to teach a unit on quadratics. Going back to TE 150, I wrote a Quadratics Unit for Algebra 1 students, very similar to the students I am teaching now! It was very interesting and enlightening to read back through that unit I created and compare it to the unit I am teaching now. What I am teaching now seems to be much more simplified and why focused, and much less focused on procedures. I am teaching students skills that help them reason through solving problems and in teaching them the why and showing them the why, I believe this is leading to a deeper understanding of the content compared to the Unit I made in TE 150. On top of that, I think I have a much wider range of ways to teach the content than I did before. I used to have a huge emphasis on factoring or the quadratic formula to solve quadratic equations, but now I understand how useful completing the square can be and especially how naturally it can be used to build off of students ' understanding of linear rules and inversing linear rules. Most of all, I've learned how important it is to think of the class as a whole story and thinking about how each unit connects to the next, not just how content connects within a unit. Big Horn does a very good job with this storyline idea and certain things such as using the word "rule" or thinking about an inverse as traveling a path on a map backwards really creates coherent understandings that transfer from unit to unit and also from class to class. I am super thankful for this experience, and I am excited for what next semester will bring.

APPENDIX L: Scott's Reflective Narrative (GLT2 Period)

When preparing for GLT2, I had the opportunity to choose between 2 different units to focus on. The unit that I chose, Quadratic Inverses, and our new unit in Geometry, Circles. A primary factor when deciding which unit to choose was my comfortability with the class that I would be focusing on. For GLT2, I would be taking my 5th hour Geometry class for the first time. Whereas with my 6th hour Honors Algebra A class, I had already had the chance to lead this class previously. By gauging my comfort level with each class, I was able to determine which class I would be able to better focus on the content for rather than balance the needs of students adjusting to myself as the lead rather than my mentor.

The goal for this unit was for students to be able to solve quadratics using a variety of strategies. While solving is not a concept that they are all too unfamiliar with, quadratics present a specific hurdle unlike linears. Using the strategies presented, students would be able to receive a question and identify which approach would work best for them based on the identifying features presented to them. A calculated risk that was taken throughout the launch of this unit was to intentionally allow for students to struggle. By doing so, we wanted for students to be able to identify what caused them trouble. From a teacher's perspective, the hope was that by doing so, students would gain a deeper understanding and further be able to identify the strategy that they felt best suited to find the solution. When students first approached the unit, they assumed that they could solve the same way that we would with linears. As a result, this became their obstacle. They quickly took note that they could not do so. This irritated them. They ran into a wall that they did not know how to climb. When first presenting such issues, it was instinctual to help the students. But this time around, we resisted the urge to and allowed for the students to struggle as a result. By doing so, we hoped that students would rise to the challenge and find the result of their struggles. Normally, students would seek to find a solution and finish the task. In this case, that would not be a success. Rather the ability to simply identify one's own struggle was. This was not a single mathematical conversation, but rather one that would continue for days on end. We challenged these students over and over again with the hope that they would keep on persevering through their hurdles.

As we continued our unit, eventually the students would be able to work around such struggles. We had created moments of uncertainty for the students and it helped to connect students to mathematics. They were eager for the solution. Like a good mystery novel, we slowly unveiled the strategies for doing so. By week's end, we had finally reached the big reveal. We introduced our first strategy for doing so by instructing how to convert quadratics in simplified form to vertex form. After students had previously identified their struggles and how it related to dealing with multiple independent variables, we had come to a possible workaround that would satisfy. Students were thrilled to finally be able to solve quadratics. Their connection to mathematics proved deeper than before as they were thrilled to be able to solve such equations and no longer had to be uncertain about the tasks that they received. Little did they know that we would continue to peel back the layers that enveloped solving quadratic equations.

Through continued practice, students began to become comfortable with the new method that they had learned. And as this new strategy became their new normal, it was time to throw another twist at them. By highlighting the students' previous findings that multiple independent variables were the cause of all problems related to solving quadratics, we introduced the quadratic formula. Students recalled their struggles and were less than happy to hear that simplified form quadratics could be solved all along. The primary reason for delaying the existence of the quadratic formula was that in my district, this is not a tested concept. As we enter the home stretch before finals, it is imperative that we do our best to prepare them for the exam. Yet it is a core concept for standard mathematical practice, so we find it to be an important strategy for students to be aware of.

As students became further versed with the multiple strategies presented, we noticed that some methods had become fan favorites while others, not so much. Such personal preference validated our reason for teaching all methods as some students quickly found comfortability with certain strategies that others may not have. As a result, as we neared the end of my GLT2 period, we were able to evaluate how students would approach solving quadratics from multiple angles. And while a preference of method is allowed, some students elected to switch between multiple methods as some problems promoted one over another. From a teacher's perspective, this was an absolute success because at the end of the day, all strategies shall work, but some situations may already be prepared for specific methods.

Additionally, I found myself gaining a deeper understanding of solving quadratics as I taught the unit. When I was introduced to the concept, the quadratic formula was one of the concepts I left high school being able to repeat at any moment. It was my default and I treated it as such. But understanding now that given a quadratic already in vertex form, it is much simpler for me to use reverse orders of operations. I did not anticipate learning as much as I had from this unit when I first launched it. But seeing my students try new strategies helped me gain this understanding that what is most comfortable may not be what is most efficient.

I think that throughout the process, one of the most rewarding findings that I had was that when faced in the presence of such uncertainty, students rose to the occasion. It was good to see that through such challenging tasks, students would not be discouraged. As a student, I knew that I would struggle often at this age. And that left an effect on me where sometimes I may be tentative to have such high expectations. But seeing students who continued to try out different strategies through tasks which were not necessarily possible for them at the time helped put away my hesitance. I am hoping to build off such strides and continue to hold my students to these high expectations.

While this unit was about solving quadratics, I think that the deeper narrative was that in the face of a challenge, that we must try multiple approaches. Through the struggle, students would try using their prior knowledge from linear functions to attack such problems. They would continue to take on such hurdles with curiosity to find such a solution. And when they have exceeded their known strategies, this is when we introduce new methods of solving for them. We introduced taking simplified form and converting it into vertex form through our coefficient relationships or box and tail method. Once we reached this form then we could use our reverse order of operations in order to find the solution. And if students were not as keen on that train of thought, the quadratic formula is now readily available for their disposal. They can use the coefficients in simplified form in order to solve for their solution as a result. Not only was this the story of the unit for solving quadratics, but it was my students and mine. This story had us brave the storm to continue trying new things and seeing what works and what doesn't. More so, why certain strategies don't work. Identifying such hurdles and keeping our emotions in check in order to persist on. By enduring such blows to our mathematical arsenal, we came out stronger by advancing the tools that we knew. That was how we came to solve quadratic equations. That was our story.

APPENDIX M: Big Horn Quadratics Worksheet

Figure M-1
Scott's Quadratics Lesson

Honors Algebra A
[redacted]

Name_______________________
Date______________
TC#: ___

SOLVING

In each of the problems below, solve for x, if you are able to. Show all work.

1) $15 = 2x + 7$

SADMEP
$-7 \quad -7$
$8 = 2x$
$\div 2 \quad \div 2$
$4 = x$

2) $15 = -2x + 7$

SADMEP
$-7 \quad -7$
$8 = -2x$
$\div -2 \quad \div -2$
$-4 = x$

3) $15 = 2(x + 7)$

Distribution
$15 = 2x + 14$
$-14 \quad -14$
$1 = 2x$
$\div 2 \quad \div 2$
$0.5 = x$

4) $15 = 2(x + 7) + 7$

SADMEP/ Distribution
$-7 \quad -7$
$8 = 2(x + 7)$
$\div 2 \quad \div 2$
$4 = x + 7$
$-7 \quad -7$
$-3 = x$

Is this the only way we know how to solve this?
What other strategies do we have?

5) $15 = 2(x + 7) + x$

Distribution
Primes students
$15 = 2x + 14 + x$
$15 = 3x + 14$
$-14 \quad -14$
$1 = 3x$
$0.\overline{3} = x$

Why can't we solve this like #4?
What differences with #4 do we see?

6) $15 = 2x^2 + 7$

SADMEP
$-7 \quad -7$
$8 = 2x^2$
$\div 2 \quad \div 2$
$4 = x^2$

$\sqrt{4} = x$
$2 = x$

What changed between this question & the one's before?
Can we have more than one answer?

7) $15 = 2(x + 7)^2$

SADMEP
$15 = 2(x^2 + 14x + 48)$
$15 = 2x^2 + 28x + 96$
$-96 \quad -96$
$-81 = 2x^2 + 28x$
$= 4x + 28x$

$-81 = 32x$
$\div 32 \quad \div 32$
$-2.53 = x$

Do we notice like-terms?
Where in the process do we begin to struggle?

APPENDIX N: Scott's Reflective Narrative (Lead Teach Period)

As a part of TE 802/804, we were tasked with creating a unit plan for one of our units of lead teach. This unit just so happened to be my first unit of lead teach. When teaching the unit, each lesson felt much more interconnected than units that I had taught previously. Each day seemed to transition into each other and the conversations that we were creating at the beginning and end of each lesson felt much more organic yet organized than the prior semester. Teaching after having done the unit plan was like watching a movie and seeing the trailer beforehand. I had an idea of what was going to happen and had more access to anticipating students. But when you don't watch the trailer, you may still have an idea of what it is about, but you really have no expectations. Because of this, I was able to better anticipate the mathematical conversations being held and facilitate the discussion so that it'd be easier to transition into further concepts.

Before, when I would watch what my mentor teacher would do and attempt to mimic his teaching, I was more so focused on what to do rather than why to do it. I would become so hyper focused on the presentation that at times, I wouldn't fully comprehend the web of concepts that he was teaching and would have a tunnel vision about the structure of the lesson. With access to my own unit plan, it made it easier for me to see how I could start lessons based on the ideas we would unravel as a class the day prior. And using the end of the class to delve deeper into the day's concept or continue this transition gave me a sense of fluidity in my lessons. Knowing which concepts would be harder for students to understand made dedicating a debrief versus continuing to push forward student thinking a much easier decision to make.

For this same reason, making any adjustments to the lesson became easier. If we hadn't reached the discussion that I had hoped to that day, I knew that things would be okay as I could adjust how I would start my lesson the next. An example of this would be when we first introduced the idea of polynomials. That day, the hope was that we would debrief concepts that were meant to be uncovered by students throughout the task, but it ran longer than initially anticipated due to the quality of productive conversations that we were hearing when monitoring. As a result, I discussed with my mentor teacher about doing a recap the following day rather than doing a debrief in order to allow students to continue this self-discovery process.

Coming up with interesting launches has become a part of my teaching practice. It is something that I watched my mentor teacher do a countless number of times throughout the first semester, and a skill that came to me as time went on. Putting myself in students' shoes, it can become stale to go to the same class everyday with sole focus on academic content for 60 minutes at a time. But I know that if I can liven up the lesson, and provide a hook, that students might not feel the way about my teaching. Bringing excitement and enthusiasm to my teaching is something that I feel I always want to work on, and I think the students benefit from this.

These launches that I have come up with help with engagement. It frames the concepts that I am teaching in a unique and interesting way. And it's an added bonus if I can create real-life connections for students with my launches. Connecting launches to outside of the classroom helps revitalize relevancy for content from my own perspective as well as my students. My favorite launch this year has been framing class as an introductory course where the students are in college. Part of this launch is connecting mathematics to real-life experiences, but the main component I am finding is that honor students enjoy being treated older than they really are. For example, I launched our class about creating the box by introducing everybody to Engineering 101. The launch was intended to be immersive by creating a real-world example for how mathematics could connect to outside of the classroom. And while not every student was looking to become an engineer, many of the students did have their sights set on college. If I had introduced to the class an engineering problem, that may have deterred students who had no intention to become an engineer.

Teaching practices that I would engage with outside of the classroom primarily consist of planning and grading. When planning, I am considering how I want to structure my lessons from finding interesting launches that connect to what we will be working on, all the way to what big ideas I want for students to reach by the end of the lesson. Working on the task and envisioning how I would do it versus how my students might, gives me room to anticipate potential strategies that they may employ and how I can further guide their thinking. Seeing their work when grading allows for me to see which connections to concepts are being made and what material may need further reinforcement. On top of that, I've tried to become a part of the community by showing up to events after school. While it doesn't quite feel like a teaching practice, inserting myself outside of the classroom helps mold the relationship I have with my students whether I think of it so formally or not.

Inside of the classroom, I am partaking in using launches or HETTs to begin my lesson, and monitoring the class to see student work, including conversations being held. Throughout the year, I have found that letting students talk and listening to their ideas has become one of the most effective ways to cultivate a learning environment. As students would run into roadblocks, I would answer questions, but would always try to regain momentum rather than push students entirely. This of course doesn't always happen but is something that I am actively working on. In an ideal lesson, my job is to only present concepts and provide the resources students need to reach an understanding of them. By our debriefs or recaps, I am hoping that I am pushing these ideas less, and rather facilitating discussion amongst students.

By connecting my practices inside and outside of the classroom, I am enhancing my own understanding of the content by seeing how I think of the material versus how my students do. Finding any discrepancies and building a bridge between our understandings not only helps the students but provides me with a refreshing perspective about the subject matter. While I believe that I have a deeper understanding of the content, at times I find myself learning from my students just as much as they learn from me. When teaching, I need to constantly reassess the "why" of the material and think of content less procedurally as I do sometimes before teaching. When doing so, it makes the material not only more interesting, but also much more natural in a sense when considering the questions that may be held.

APPENDIX O: Claire's Reflective Narrative (GLT2 Period)

Course: Pre-Calculus
Unit Title: Exponential and Logarithmic Functions

Review your pre-unit narrative, daily synopses, lesson plans, and monitoring sheets and weekly reflections.

Thinking back over the Guided Lead Teach period, particularly in your focus class, what story do you want to tell about your experience as the teacher? A few prompts that may help you form this story:

How did you go about picking this unit to be the focus for this study?

- Honestly, it lined up with GLT2. We knew we were going to be started Chapter 3 around the time that I was started my second guided lead teach, so we lined up our schedule for the unit to start on the same day my GLT2 started, and last approximately the length of that time-period.

What was your planning process like? To what extent did you collaborate with your mentor teacher for this unit?

- My mentor and I did a lot of co-planning for what content we wanted to focus on during this unit. We started off by mapping out approximately what the unit was going to look like - which sections of the book were covered on what days, when we had a test/quiz, number of review days & when. We have resources from a teacher that formerly taught the course, that the other Pre-Calc teacher also uses. We sat down together and edited her guided note sheets to fit what we wanted the class to emphasize & what we wanted to give them vs. what we wanted to just say. I typically picked out warm-ups and homework problems myself when I worked through the notes and planned for myself what I wanted to say. On our two whole class review days, I planned the review myself by picking out problems that fit with what we'd done and putting together the reviews.

Explain what your daily lesson structure was like.

- For a typical daily lesson, we'd start with a warm-up, either continuing content covered the previous day or scaffolding skills for the new content. We then move into notes, balancing between introducing content to the whole class and working together in small groups on example problems. Once notes are done, students are assigned some kind of practice (usually either book problems or a worksheet) for homework and have the rest of class to work on it together and ask questions.

What ways did you plan for students to demonstrate understanding of the content?

- We had both a test and a quiz (pop quiz on logarithmic properties). We assign homework, but homework is checked for completeness, not correctness. Three review days before the test - one was working on a review packet, but two were whole class reviews. On day 1, students could display understanding on individual whiteboards. On day 2, we played a game in which students could demonstrate understanding in front of the whole class on the back whiteboard. We also often plan for students to come up to the whiteboard during lessons to demonstrate example problems.

What takeaways do you have about students' understanding of the content? What claims can you make about how they approach understanding mathematics?

What goals do you have for your students in your course? In what ways did you see your students working towards that goal over the last two weeks?

What did the students struggle with? Why do you think they struggled?

- They struggled at first with logarithmic properties - we had to push things back a day and include an extra work day just to practice with these. I did anticipate this ahead of time - many of them are typically very process oriented. They like having set steps to follow. Having to pick out themselves what to do (i.e., multiple solution pathways) can be difficult. We saw the same thing happen in a previous unit when we covered exponent rules, and there were multiple different ways to start.

What evidence do you have that the students achieved the goals for this unit?

- Test scores
- Review - students did well with problems. No problem stood out as students getting it less than the others. Overall, very solid especially on second day

As a teacher, what examples can you think of that demonstrates that you are developing a different kind of understanding of the content. In other words, thinking over the last several months (with examples from this unit as demonstration) how have you seen your posture towards understanding a concept change as a result of your teaching experience.

- I definitely believe that my understanding of math has changed - a lot of the tools we use are just that, tools, to get to an end, rather than being the actual mathematics themselves. For an example from this unit - logarithm properties. In high school, I just memorized these and took them as fact. Now, I know that they are fact, but I understand the why better. Having taught log properties now, and looking for ways to help my students understand, I think of them more as "shortcuts" that have been derived from exponential/logarithmic inverses. They are tools that can help me, and I can always fall back on the definition of the log function. I think I was aware of this beforehand, but my algebra 2 teacher definitely approached the subject as being a list of things to memorize. Teaching it has given me a new frame of reference for the concept

APPENDIX P: Claire's Reflective Narrative (Lead Teach Period)

If we are being 100% honest, this was one of the units I felt most shaky on in Statistics going into this year. My own Statistics professor covered it, but the class I took was very calculations focused, with less emphasis on the conceptual. I spent a lot of time reviewing the material myself for this unit and making sure that I understood not just what we were doing, but why we were doing it. Now, having taught the unit, I feel really great about the material. I have definitely noticed my own understanding deepening through the process - I can clearly articulate why things happen, and I can't think of a question that a student asked that I didn't know the answer to. I have learned this year that I actually know more statistics than I originally gave myself credit for, and I pick up on the details very quickly with a little bit of review.

My mentor and I have been strong collaborators throughout the course, and this unit was no exception. She has taught AP Stats for several years now and has a really good sense of what works and what students need to know for the exam. She and I picked activities together, used note sets from prior years, and planned out the sequencing together. I did more day-to-day specific planning, by working through the notes and activities, reviewing material, anticipating what students might ask, etc.

On a typical day, we start with a multiple-choice question of the day warm-up from an old AP exam. We've been working through a practice exam in order, so the questions may or may not relate. We take notes and do some examples with significance testing. Typically, we'll take notes on the details, and then students will work through the calculations with their groups before coming back as a whole class to discuss and summarize. At the end of class, time permitting, students have time to work on homework and ask questions.

I think the thing they struggled with the most was recognizing the difference between the test for homogeneity and the test for association - beyond the wording of the hypotheses. I understand why, as this is a very specific detail dealing with the difference between number of samples and numbers of variables. I feel like I didn't truly have it down until I taught it and was responsible for explaining it to other people.

After grading their tests, and looking over things, I feel like they have the big things down. On the multiple-choice questions, the most missed questions were generally very nitpicky, with one right answer but one or two that were almost right. We'll continue to work on those details as they prep for the AP exam, and the curve (we curve every test to be in line with the score distribution of the actual AP exam) will balance out those questions. I thought that the free response questions went very well also. Almost all students were able to recognize the second question as being a chi-square test for independence, although a few did forget how to find a p-value based on a test statistic, rather than a set of data. On the first question, there were a few who selected the wrong test, but generally received follow through credit for correct calculations and conclusions based on what they started with. I would say the most missed part of this question was having hypotheses that were not specific enough - which again, is a nitpicky detail that will wash out in the curve.

I genuinely feel like my content knowledge has greatly increased and deepened by teaching this course. In addition to understanding the material better, approaching it from the perspective of a teacher allows me to see broader connections across units. Stats felt a little bit disjointed to me when I was a student, but as a teacher, I can clearly see the sequencing and how things we have already done have helped prepare for the new content

APPENDIX Q: Claire's Reflective Narrative (Lead Teach Period)

If we are being 100% honest, this was one of the units I felt most shaky on in Statistics going into this year. My own Statistics professor covered it, but the class I took was very calculations focused, with less emphasis on the conceptual. I spent a lot of time reviewing the material myself for this unit and making sure that I understood not just what we were doing, but why we were doing it. Now, having taught the unit, I feel really great about the material. I have definitely noticed my own understanding deepening through the process - I can clearly articulate why things happen, and I can't think of a question that a student asked that I didn't know the answer to. I have learned this year that I actually know more statistics than I originally gave myself credit for, and I pick up on the details very quickly with a little bit of review.

My mentor and I have been strong collaborators throughout the course, and this unit was no exception. She has taught AP Stats for several years now and has a really good sense of what works and what students need to know for the exam. She and I picked activities together, used note sets from prior years, and planned out the sequencing together. I did more day-to-day specific planning, by working through the notes and activities, reviewing material, anticipating what students might ask, etc.

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