# FEEDBACK THAT MATTERS: MATHEMATICS TEACHER EDUCATORS' FEEDBACK OF STUDENT TEACHERS' LESSON PLANS

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## ABSTRACT

Although there is consensus on the relevance of the feedback given to student teachers, few instances in the literature have addressed this topic. Even less is said about feedback to mathematics lesson plans and how to characterize it. Lesson plans are artifacts that help establish a link between theory and practice, relate to a broader sense of assessment, beyond merely assigning grades, and keep the mathematics of the lesson as a central component. Focusing on the opportunities to think and reason that students will have on the lesson, the anticipatory work done when establishing a goal, selecting a task, and declaring teaching practices, makes the concept of cognitive demand to emerge. Teachers should be able to plan high cognitive demand lessons and implement them in ways that preserve this demand. Mathematics teacher educators, therefore, have a valuable opportunity when providing feedback to the cognitive demand of student teacher lesson plans. This research addresses the characterization of the feedback giving process of two Chilean Mathematics teacher educators, and eventual shifts that might have happened after a professional development workshop focused on cognitive demand of goals and tasks in a mathematics lesson plan, and the effects that certain teaching practices could have to this demand. Results suggest that the overall experience of reflecting about the cognitive demand of a mathematics lesson elicited different kinds of shifts at different stages of the research for both participants, who were able, at the end, to use the concepts of cognitive demand to articulate their feedback to student teacher mathematics lesson plans.

To my parents, for their unconditional support and love.
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## LIST OF ABBREVIATIONS

| | |
|---|---|
| TE | Teacher education |
| PST | Preservice teacher |
| CPEIP | Centro de Perfeccionamiento, Experimentación e Investigaciones Pedagógicas |
| MTE | Mathematics teacher educator |
| PD | Professional development |
| CLP | Chilean Peso |
| CD | Cognitive demand |

# CHAPTER 1: INTRODUCTION TO THE RESEARCH PROBLEM

*All teachers have the capacity to be stars – they just need access to opportunities to learn, reflect, and grow.*
Smith & Stein, 2012.

Reform efforts in Chile have called for an extensive improvement in Mathematics Education. There is a general call to make students engage mathematics in meaningful ways and, in general, to become mathematically proficient. This ambitious goal calls for soundly prepared mathematics teachers.

The publication of the Standards for Teacher Education by the Ministry of Education (MINEDUC, 2011) set a guideline in terms of what new teachers should know and be able to handle when graduated. In turn, this posed a challenge for Teacher Education (TE) programs, as the main responsible parties of providing adequate opportunities for their preservice teachers (PSTs) to acquire such knowledge and abilities. The new set of standards for high school mathematics (CPEIP, 2022) increased professional expectations for graduated teachers, and set the tone for the elementary mathematics standards, announced for late 2022.

The assumption that well-prepared Mathematics Teacher Educators (MTEs) positively impact the preparation of preservice mathematics teachers seems reasonable and widely accepted. However, there are few – if any – TE programs in Chile that provide a systematic professional preparation for their MTEs (Tatto et al., 2013). This is true not only in Chile, but also in other parts of the world (Zaslavsky and Leikin, 2003).

Learning to teach mathematics is no trivial matter. Just considering the vast amount of preparations and decisions made even before the lesson begins can become overwhelming. Therefore, planning a lesson in mathematics requires careful consideration. In a mathematics lesson plan, there are much more elements than it appears. Just for instance, when planning a mathematics lesson, establishing a goal that allows high cognitive demand work and selecting a task that properly aligns with said goal is a quintessential task for a mathematics teacher. It does not seem clear -however - whether Teacher Education programs are adequately preparing preservice teachers in this regard.

A crucial stage for PSTs when such skills are developed is during student teaching. Student teachers craft a lesson plan and bring it to a Mathematics Teacher Educator (MTE) before enacting it. A significant part of the 'fate' of the lesson is determined even before it starts. Low cognitive demand goals and tasks will not offer opportunities for students to engage with meaningful mathematics, regardless of the quality of the instruction (Smith and Stein, 2012). While high cognitive demand goals and tasks do not guarantee high-quality instruction, they do constitute an 'upper-bound' for the overall quality of the lesson. Thus, establishing goals and selecting tasks that allow for high cognitive demand work in the classroom are a necessary condition for the success of the lesson.

With so much happening before the lesson, MTEs get a valuable opportunity for formative assessment that can be often undervalued. Providing adequate feedback in order to overcome eventual issues with the lesson plan can effectively raise the potential ceiling of the lesson. But how (and whether) this opportunity is being taken advantage of by MTEs remains unclear. Delving deeper into the nature, content and focus of MTEs' feedback on a lesson plan might shed light on opportunities for improvement for TE programs.

This is precisely what this research builds on. The focus is on MTEs; how they provide feedback and if (and how) said feedback-giving process changes after experiencing a Professional Development workshop in which relevant elements of lesson planning are addressed. Thus, it becomes necessary to determine how to characterize MTEs feedback to lesson plans, what aspects are relevant when giving feedback to student teachers and which lesson-planning elements about feedback-giving are more important to focus on. The next section focuses on these ideas.

## 1.1 Overview of Chapters

This chapter serves as an introduction to the problem, so the reader gets a brief presentation to the major topics this work addresses.

In chapter two, a review of the pertinent literature is presented. The review is written in a way that serves multiple purposes: to articulate a theoretical framework for the research, to provide support for the relevance of the problem, and also to guide the reader about the decisions made when narrowing down the specific issues addressed by the research questions.

Chapter three presents all methodological aspects related to this work. This chapter is structured in a way that threads the research questions made with the design of the instruments used to answer them, and how the data was gathered and analyzed in the process.

Chapter four introduces the two main cases of this research. First, the reader is presented with a summary table for both participants, then details of the case of Mario Díaz are detailed; his background, his responses to the instruments, and a brief analysis. Next, the same structure is followed to introduce the case of Yasna Quezada.

Finally, chapter five addresses a discussion based on the results. Answers to research questions are presented and limitations are addressed. I also explicitly address my personal stance on the matter and how it might have influenced the research process. Then, possible next steps for the research on feedback on cognitive of student teachers' lesson plans are discussed. Finally, I present some final conclusions I have made throughout all of the research process.

# CHAPTER 2: LITERATURE REVIEW

There is consensus regarding the relevance of feedback for student teachers. For instance, Crichton & Valdera Gil (2015) identified the relevance of university tutors' feedback after a lesson as a valuable opportunity for student teachers to reflect on their own practice and identify ways to improve. Buhagiar (2013) synthesized student teachers' views on feedback from university tutors, stressing the need for TE programs to provide quality feedback to preservice teachers. Other authors (Hiebert et al., 2003; Pang, 2016) have proposed using lessons as experiments of shared knowledge, from which prospective teachers can reflect on or get external feedback from students, peers, school mentors and university tutors.

Much less is said, however, in relation to how this feedback should be characterized, and even less about feedback coming from MTEs. In general terms, student teachers tend to conceive feedback coming from university supervisors as highly related to theory (Crichton and Valdera Gil, 2015), or strongly attached to graded assessment (Borko and Mayfield, 1995, Snead and Freiberg, 2019). Furthermore, in some cases there is hardly any mathematics-specific feedback for a lesson (Borko and Mayfield, 1995). Kastberg, Lischka & Hillman (2020) made a first explicit characterization of MTEs' written feedback to prospective teachers' work; identifying lesson plans as instantiations of practice, reflecting on the effectiveness of different kinds of feedback, and openly calling MTEs to pursue further exploration on their feedback. Although this work explores MTEs' feedback to PSTs' work - which makes it relate the most with this study - it does not consider lesson plans.

Considering what has been addressed so far, if we had to list some desirable elements for feedback intended for student teachers, these might be:

- Establishing a link between theory and practice.

- Relating to a broader sense of assessment, beyond grades.
- Having mathematics as a central component.

I propose that lesson plans are a reasonable proxy for student teachers' practice and that focusing on how to provide adequate feedback to student teachers' mathematics lesson plans is a valuable opportunity for MTEs' professional development.

## 2.1 Why Lesson Plans?

There is wide agreement that well-prepared lesson planning is fundamental for effective teaching. From the perspective of a teacher educator fulfilling the role of university tutor for a student teacher, a lesson plan represents a readily available nexus between university and school. Indeed, lesson plans are artifacts that represent practice (Pang, 2016). As such, improvements on lesson planning will have an impact on teaching practices and, in turn, on the overall quality of the lesson.

Much of what has been studied in terms of feedback for student teachers happens after the lesson (Borko and Mayfield, 1995; Crichton and Valdera Gil, 2015) and often it is tied to a summative assessment (Snead and Freiberg, 2019). In reality, several crucial elements for an effective lesson happen before the lesson; and a considerable amount of mathematics is involved in this stage. Namely, Smith & Stein (2012) and Stein, Smith, Henningsen & Silver (2000) state that in establishing the goals for the lesson and selecting the tasks to be used, teachers make the decisions that will impact the most on the opportunities to learn that students will have during the lesson. The goal and the task greatly determine the 'upper bound' of the lesson in terms of cognitive demand. Low-level tasks and goals will yield low-level opportunities to learn, severely limiting the potential of the lesson.

Since planning happens before the lesson (but still with crucial elements for the lesson going on), providing feedback for lesson planning constitutes a perfect opportunity for formative assessment. Instead of providing directive feedback (just telling what was wrong, as it is often the case with post-lesson feedback), directing student teachers' attention to relevant aspects of lesson planning and all the mathematics in play during this process can lead to profound reflection and meaningful Student Teacher - MTE interactions.

In sum, since lesson plans are artifacts that:

- Directly link to practice
- Constitute a useful instance for formative assessment and
- Require careful consideration of the mathematics involved in the lesson

It makes sense to use them as a proxy for student teachers' practice, which in turn makes the study of MTEs' feedback to lesson plans become particularly relevant.

But a lesson plan has several elements. What to focus on in regards to feedback? To handle the tension between addressing relevant aspects of lesson planning with adequate depth while keeping the scope manageable, I propose to use the concept of cognitive demand as the articulating thread to analyze three salient elements of a lesson plan: goal, main task and declared teaching practices.

As it was stated before, establishing a goal that allows for high cognitive demand work and selecting a task that materializes such a work are necessary conditions for a successful mathematics lesson (Smith and Stein, 2012). Yet they are not sufficient; as teaching practices are crucial for either maintaining or causing the decline of planned, high cognitive demand work (Stein et al., 2000). However, as important as these practices are, in order to not broaden the focus any more than necessary, we will consider only those practices that can be observed from the lesson plan (e.g. paying attention whether the design of the lesson allows for mathematical discussion, or if there is any anticipation for how to elicit argumentation when common errors are detected).

## 2.2 Learning from Feedback

A strong case can be made for the idea that preservice teachers can significantly learn from feedback. Essentially, prospective teachers are demanded to learn to teach in a way that is “fundamentally different than how they were taught” (Borko and Mayfield, 1995, p. 502). In this process, reflecting on different instances of practice becomes particularly relevant. TE programs must ensure ample opportunities for preservice teachers to engage in this kind of reflection.

Schön (1983) distinguishes between ‘reflection in action’ and ‘reflection on action’ in terms of decisions practitioners make during and after a lesson, respectively. Since we have made the case for considering lesson plans as a valid instantiation of practice for preservice teachers, the term ‘reflection before action’ seems to fall in place naturally. Indeed, being able to anticipate some of the events that will happen on a lesson are foundational for the intended shift in the way preservice teachers will teach and, certainly, involves a considerable amount of mathematics (Pang, 2016; Smith and Stein, 2012).

## 2.3 Cognitive Demand

As we narrow down the focus of this work to feedback of lesson plans, the question arises: What aspect(s) of lesson plans are important to be given feedback?

I stated previously that there is consensus - not only in Chile - about the expectation of students engaging meaningfully with mathematics. Evidently, there are several ways of approaching this issue, but within a lesson plan, a case can be made that the selection of the task that will be presented to students, and the anticipated actions to implement this task, are the most important decisions a teacher needs to make before the lesson.

"Not all tasks are created equal, different tasks require different levels and kinds of student thinking" (Stein et al., 2000, p. 3). This quote not only serves as a motivation for stating the relevance of levels and kinds of student thinking and reasoning, but also as a means of defining the concept of cognitive demand. If the selection of a task carries so much importance, we can confidently claim that much of the fate of the lesson is decided even before it starts. In turn, if there are any issues with the cognitive demand of a task that can be detected before a lesson plan is enacted, it certainly is worth doing so. This creates a valuable opportunity for MTEs to provide effective feedback to their student teachers through their lesson plans.

Focusing on the cognitive demand of a planned lesson means paying dedicated attention to the opportunities students will have to think and reason when solving a mathematical task, which in turn will determine their learning gains (Stein et al., 2000). Therefore, a focus on cognitive demand also means a focus on the quality of the lesson, in the sense that the selection of high cognitive demand tasks is a necessary (albeit not sufficient) condition for the success of the lesson.

In the Chilean context, consistently with Snead & Freiberg's (2019) description, feedback on lesson plans at the student teaching stage is mostly used to assign grades and, without a systematic way of paying attention to student thinking, reasoning, or learning. This results in a significant missed opportunity to both increase the quality of the lesson and model better feedback practices for student teachers.

# CHAPTER 3: METHODOLOGY

Considering the relatively small size of the TE program in which the research is conducted (less than 10 MTEs), but also that there is some degree of variability among the participants in terms of background, preparation and experience, I decided that a Case Study would allow for a reasonable depth of analysis about the kind of feedback MTEs would give, shed light on which aspects of lesson planning they consider most relevant and, most importantly, what kind of things they could learn from the intervention about the process of providing feedback before a lesson. In this section, I will present the research questions, provide a description of the research context, as well as detailing the methods used for data collection and analysis.

## 3.1 Research Questions

The overarching question to be addressed through this research is: **How can MTEs' feedback on cognitive demand of student teachers' lesson plans can be characterized and how it shifts after a PD workshop?** In order to gather evidence to answer the question above, I stated the following sub-questions:

- *What are the characteristics of the feedback on cognitive demand MTEs usually provided before the workshop?*
- *How do MTEs respond to scenarios where they are asked to respond to cognitive demand elements of PSTs' lesson plans by providing feedback on learning goals, tasks and declared teaching practices, before and after the workshop?*

## 3.2 Research Context

The research was conducted in a TE program within a major, private Chilean University. Participants were MTEs of this program who have been in charge of overseeing Elementary PSTs at the student- teaching stage during their fifth semester, which has a strong focus on mathematics.

The researcher has been one of the MTEs in this program, and also was the supervisor of the Mathematics Education department at the time the study was performed.

Three main stages of data collection were considered: pretest (first interview), intervention (workshop) and posttest (second interview).

The pretest was conducted in a one-on-one interview form. Due to sanitary restrictions, a video-conference software was used for the interview, and audio/video data was recorded. Additionally, during the pretest, some participants' background information was collected (education background, mathematics background, years at the institution, etc.)

The workshop was designed so all participants could attend simultaneously. Due to scheduling issues derived from irregular working hours during the pandemic, two versions of the workshop were held so all participants could attend. Also, considering sanitary restrictions, a video-conference software was used for this workshop, and audio/video data was recorded.

The posttest was conducted in the same way as the pretest.

All data was stored digitally in a secure server.

## 3.3 Research Design

Answering the research questions required to properly depict how MTEs initially gave feedback to PSTs, design an intervention that addressed key elements of mathematical content and effectiveness of feedback (Kastberg et al., 2020) and gather evidence to determine whether and how the initial characteristics of MTEs' feedback on cognitive demand shifted as a result of the intervention.

Considering the relatively small number of MTEs in the program, pursuing depth in the gathering of data became a priority. To this end, the pretest and the posttest were designed as one-on-one interviews, and the intervention itself as a workshop that involved all participants.

Along with the information within the subsections below, appendix A could be useful to observe a general overview of the research questions and how the design was crafted to answer these questions.

## 3.4 Research Procedures and Instruments

### 3.4.1 Pretest

First, participants took the pretest. This was a one-on-one interview through a video conference software. This instrument gathered

- Background information (education background, mathematics background, years at the institution, etc.)
- Current (at the time of the interview) features of the feedback-giving process, through direct questions: how it worked, focus, assessment, the mathematics involved, etc.
- Participants' responses to situated scenarios, where PSTs provide certain goals and/or tasks or make claims about goals and tasks used in lesson plans, and MTEs were asked to react in terms of providing feedback on the cognitive demand of these goals, tasks and/or claims, making sure to discuss the mathematics involved in this process, as shown below:

1. One of your student teachers plans a lesson with the features below:

**Content**: Addition with whole numbers (Thousands)
**Goal**: Solving problems involving addition with whole numbers (Thousands)
For the main task, she poses the following problem
"Last weekend, Sarah went to the movies. She spent $2.942 (CLP) in the ticket and $1.432 (CLP) in popcorn. How much did she spend altogether?"
As to how to use this problem in the lesson, she proposes having the students follow the list of steps below:

1. Read and understand
2. Identify data
3. Identify the operation
4. Perform the operation
5. Answer the question and check the answer

Within the rationale section of the lesson plan, she claims that following these steps will help students understand the problem, organize the information and get the correct solution.

a. What do you think of this lesson plan?
b. What kind of Mathematics is involved in the task?
c. What opportunities would students have to think and reason with this task?
d. What can you say about the CD of the task?
e. What kind of feedback about the CD of the task would you provide and why? Provide examples

***Figure 3.1:*** *Example of a pretest question. For the full instrument see the Appendix*

The pretest was designed to gather relevant information to be used during the analysis stage. Background information was collected to enrich the description of each case and build on the existing variability of MTEs. The questions about the features of the feedback-giving process at the time of the interview were created to characterize the feedback each MTE was giving at the time. Since these answers were likely mediated by institutional requirements, participants were asked to respond to situated scenarios to help bring forth individual thoughts and beliefs about feedback that might have not come up through direct questions. These reactions enriched and complemented MTEs' self reports.

### 3.4.2 PD Intervention (workshop)

The intervention is designed as an online workshop, attended by all participants. This workshop considered three main sections:

1. Working with a framework for cognitive demand of a task, sorting tasks according to their cognitive demand, discuss implications for MTEs' feedback to lesson plans in terms of cognitive demand of a task.
2. Analyzing lesson goals, discerning whether they allow high cognitive demand work, discuss implications for MTEs' feedback to lesson plans in terms of the (potential) cognitive demand of a goal.
3. Evaluating where declared practices can be found on a lesson plan and whether they could maintain/decrease the cognitive demand of the work, discuss implications for MTEs' feedback to lesson plans in terms of the declared teaching practices. Each section took approximately one hour, so the overall time for the workshop was about three hours. The main topics addressed in each section are detailed below:

**Section 1:** Working with a framework for cognitive demand of a task, sorting tasks according to their cognitive demand, discuss implications for MTEs' feedback to lesson plans in terms of cognitive demand of a task.

Example (for the full section, check the Appendix):

Small group discussion (some lesson plans as handouts)

- Where's the math in the lesson plan?
- What about the Math do I usually provide feedback to and how?
- Specific about Math in goals/task/practices Whole group discussion: Sharing what happened in small groups (Hopefully, some ideas about CD emerged from last discussion)

Whole group

- A framework for CD
- CD framework (Smith & Stein)
- One round together with a few tasks (procedures without and with connections)
- Sorting tasks according to CD (small groups)

Examples from our students

a) Some tasks and claim about the tasks

b) What do we think about these?

c) What opportunities would students have to think and reason in each of the cases? How does this relate to CD?

d) What kind of feedback would we provide to these student teachers?

e) Do we think there would be any change if we consistently started giving this kind of feedback?

**Section 2:** Analyzing lesson goals, discerning whether they allow high cognitive demand work, discuss implications for MTEs' feedback to lesson plans in terms of the (potential) cognitive demand of a goal.

Example

Small groups: Re-examine goals A, B and C from pretest:

a) What do we think about these?

b) How would you lead the discussion from here onwards? Provide examples

c) What would you do to make the mathematics explicit in the discussion? Provide examples

d) What opportunities would students have to think and reason in each of the cases? How does this relate to CD?

e) What can we say about the CD of possible tasks for each goal?

f) What is the expected takeaway of possible tasks for PSTs with the discussion you proposed?

g) What kind of feedback would we provide?

- Private (Provide examples)
- Public (Provide examples)

Whole group discussion: Sharing what happened in small groups

**Section 3:** Evaluate where declared practices can be found on a lesson plan and whether they could maintain/decrease the cognitive demand of the work, discuss implications for MTEs' feedback to lesson plans in terms of the declared teaching practices.

Example

Small groups:

- Re-examine lesson plans 1 and 2 from pretest

Whole group:

- QUASAR project
- The fate of tasks set up as doing mathematics
- The fate of tasks set up as procedures with connections
- Factors associated with the decline of high-level CD
- Factors associated with the maintenance of high-level CD

SO:

a) How does this relate to lesson planning?

b) What kind of elements of the lesson plan we need to pay attention to 'detect' potentially some of these teaching practices?

c) What kind of feedback could we provide to:

- Encourage potential practices that would help maintain a high CD? Provide examples
- Call for a revision on potential practices that would cause a decline of CD? Provide examples

This design helped bring out what happened during the pretest interview and use it as a starting point for discussing and reflecting as a community of practice (Zaslavsky and Leikin, 2003). The information gathered during this process was helpful to shed light on issues and opportunities for improvement, directly built on ideas of actual MTEs.

#### 3.4.3 Posttest

The posttest had a similar design to the pretest, except that it did not start by gathering participants' background information and current state of the feedback-giving process. It included some questions at the end where MTEs were asked to reflect on the whole process, and state whether any of their initial ideas or beliefs changed as a result of it.

### 3.5 Data Sources and Analysis

#### 3.5.1 Sources

The main research question addressed two main aspects; characterizing MTEs' feedback and determining whether said feedback changes after a professional development workshop. For the former, the main data sources were pretest interviews. For the latter, in addition to pretest interviews, posttest interviews were also considered.

#### 3.5.2 Analysis

In this research, each one of the participant MTEs constituted a case. Properly answering the research question required the analysis of both pretest and posttest. Since these instruments were designed as interviews, it was natural to have the unit of analysis being each MTE's answer to each question. For the first part of the research question (characterizing MTEs' feedback), a categorical analysis was performed, looking for emerging categories throughout the data. These categories were compared with the kinds of feedback existing in the literature for similar instances (mainly lesson feedback). Given that there was no existing research addressing this particular topic, this contrast seems reasonable. There was no guarantee a priori that feedback given to lesson plans would be similar to other types of feedback. For the second part of the research question (determining whether the MTEs' feedback shifts after the PD workshop), the same categorical analysis was performed to the posttest interviews. There was not nearly enough statistical power to perform a quantitative comparison, but there is arguably enough depth in each one of the cases for a qualitative contrast between what MTEs declared before and after the workshop in relation to what they considered most relevant when giving feedback to student teacher lesson plans.

### 3.5.3 Validity and Reliability

To ensure results were trustworthy (valid, qualitatively); pretest questions were previously piloted. Also, despite the small size of the sample, efforts were made to provide a degree of variation in terms of participant backgrounds. Additionally, members of the dissertation committee and the researcher provided systematic reflexive examination of both the instruments and the methodology of this research.

Triangulation of researchers (researcher, members of the committee and at least one external researcher) was put in place to ensure that the category analysis was reliable.

# CHAPTER 4: RESULTS

## 4.1 Introduction

Tables 4.1 and 4.2 present a brief summary of the responses Mario Díaz and Yasna Quezada gave during pre and post interviews. It should serve to the reader to get a first comparison at a glance of any eventual shifts that might have happened in between interviews. A deeper presentation of these results follows.

| **From (Pre Interview)** | **To (Post Interview)** |
|---|---|
| **PREFERENCES ON FEEDBACK**<br>• Formal aspects: goal, task, coherence.<br>• Mathematics: Emphasis on methods, but focused on curriculum and alignment with standards.<br>• No spontaneous mention of cognitive demand. | **PREFERENCES ON FEEDBACK**<br>• Sustains what he said in the first interview<br>• Mathematics: Still emphasis on curriculum and standards<br>• Spontaneous mention of cognitive demand. |
| **FEEDBACK ON CD OF TASKS**<br>• Mainly through questions: most calling for reflection, some directive.<br>• Mathematics of tasks seen through content<br>• Opportunities to think and reason related to secondary aspects, such as the link of problems with real life context.<br>• Mention of cognitive demand only when specifically asked.<br>• Informal use of cognitive demand terms. | **FEEDBACK ON CD OF TASKS´**<br>• Mainly through questions: mostly centered around the role of student teachers in the lesson.<br>• Mathematics of tasks seen through content.<br>• Noticed mathematical issues with tasks.<br>• Opportunities to think and reason related to chances of discussion.<br>• Formal use of cognitive demand terms. |
| **FEEDBACK ON CD OF GOALS**<br>• Opportunities to think and reason related to secondary aspects such as difficulties for students.<br>• Mention of cognitive demand only when specifically asked.<br>• Informal use of cognitive demand terms. | **FEEDBACK ON CD OF GOALS**<br>• Goals offer different opportunities to think, reason and discuss mathematics.<br>• No explicit mention of cognitive demand, but it appears implicit within his discourse. |
| **FEEDBACK ON PRACTICES**<br>• Opportunities to think and reason related to aspects such as the structure of anticipated mediation of student difficulties. | **FEEDBACK ON PRACTICES**<br>• Teacher interventions seem to take away opportunities to think and reason because of giving too much information. |

***Table 4.1:*** *Mario Díaz, pre and post interviews responses summary*

*Table 4.1 (cont'd)*

| | |
|---|---|
| • Effects of teaching practices on cognitive demand focused on opportunities for students to discover mathematical ideas. | • Cognitive demand is not properly categorized, but he identified that "declared" practices can have an effect on cognitive demand. |

| From (Pre Interview) | To (Post Interview) |
|---|---|
| **PREFERENCES ON FEEDBACK**<br>• Broader elements, such as student learning, anticipating possible difficulties, etc.<br>• Mathematics: the kind of mathematical knowledge put into play in the plan, problem solving.<br>• Indirect mention of cognitive demand through problem solving. | **PREFERENCES ON FEEDBACK**<br>• Sustains what she said in the first interview, in terms of going beyond rigid structures and focusing on student learning.<br>• Explicit mention of cognitive demand and, specifically about the relevance of the framework for CD.<br>• Mathematics from a problem solving perspective. |
| **FEEDBACK ON CD OF TASKS**<br>• Mainly through questions: most calling for reflection, some directive.<br>• Attention to the amount of decisions the task allows students to make as a proxy for cognitive demand<br>• Opportunities to think and reason linked with space for making decisions.<br>• Use of formal cognitive demand terms. | **FEEDBACK ON CD OF TASKS**<br>• Evident connection between "opportunities to think and reason" and "cognitive demand".<br>• Questions almost exclusively calling for reflection.<br>• Use of formal cognitive demand terms.<br>• CD as a means of articulating feedback.<br>• Epiphany: Link between her own idea of decision-making with opportunities to think and reason (which in turn link to CD. |
| **FEEDBACK ON CD OF GOALS**<br>• Ability to extend the framework of cognitive demand of tasks to lesson goals.<br>• Give feedback through questions beyond content: what do you intend to achieve? Emphasis? What would a student think? | **FEEDBACK ON CD OF GOALS**<br>• Through questions that call for reflection.<br>• Centered on student learning and mathematical production.<br>• CD as a means to articulate feedback. |
| **FEEDBACK ON PRACTICES**<br>• Some teaching practices, such as giving away too much information, can take away opportunities to think and reason from students.<br>• A task that has been planned as high cognitive demand can end up being implemented with lower cognitive demand. | **FEEDBACK ON PRACTICES**<br>• Even if a task has a high CD, some teaching practices can lower it significantly.<br>• Careful feedback on monitoring is needed so teachers do not take away students' mathematical authority.<br>• Insists on the link between decision-making and opportunities to think and reason. |

***Table 4.2**: Yasna Quezada, pre and post interviews responses summary*

## 4.2 Case 1 – Mario Díaz: The journey of acquiring technical language proficiency

In this section, I will present and discuss the case of Mario Díaz. Specifically, I will describe the shifts that occurred from our first interview to the second one, related to the two salient aspects of this research: cognitive demand and feedback of lesson plans. For clarity purposes, instead of following a chronological description (pre-interview, workshop, post-interview), I decided to construct a narrative within each section of the interviews, starting by describing what happened in the first one, immediately followed by the corresponding part of the second one. The workshop is addressed afterwards. This setting should help the reader make the contrast of what happened before and after the workshop, and get a better feel of the shifts that are taking place.

### 4.2.1 Background: A school teacher discovering a framework for cognitive demand

Mario is an elementary teacher with a specialization in Mathematics and graduate studies in Mathematics Education. He has been an instructor at the institution the research was conducted in for at least three years. He has prior experience as an elementary teacher at public schools, as well. He has supervised student teachers at least twice before participating in this study.

Mario's journey is one of evolution. During the first interview, his answers seemed standard, mostly focused on elements than processes. For instance, when asked about the kind of mathematics involved in a certain task, he would mention the specific content (e.g. whole numbers, addition and the like) rather than the way of doing mathematics, or the type of teaching-learning practices in place. Similarly, when asked open questions about important aspects about planning a lesson, establishing goals, selecting tasks, or even analyzing latent teaching practices, he would often mention standard procedures; and, even when talking about pedagogical or didactic elements of a mathematics lesson, there would be no mention of cognitive demand. It was only when directly asked about cognitive demand that he would speak of it, but only in very general terms (low, mid, high). In terms of feedback, he always attempted to provide questions, but these questions were not always reflective; sometimes, he was more directive than intended.

But that would change. And the beginning of the change can be traced to Mario's participation in the workshop. Sharing with colleagues was particularly important for him. At first, he did not participate much, but as the workshop kept going on, he "tuned in" and felt more comfortable sharing and participating. By the time the participants at the workshop were sorting tasks by cognitive demand, he had already picked up the technical terms.

His performance during the second interview would confirm what happened in the workshop. Just asking "what mathematical elements would you consider when giving feedback to mathematics lesson plans?" was enough for him to mention cognitive demand, among other important elements. And, whenever asked about the cognitive demand of a task, he no longer talked only in terms of low/mid/high cognitive demand, but he rather analyzed the specific category in Smith & Stein's framework. (e.g. "I think this is a high cognitive demand task, but I am still debating myself whether it is procedures with connections or doing mathematics... it seems it is mostly oriented toward processes rather than construction of meaning").

Such was Mario's shift. Of course, he still offered some standard, rigid answers by the second interview. In terms of feedback, he still offered lots of questions, although some of them were still directive. But, in terms of cognitive demand, there was a clear evolution. He acquired technical language proficiency to a significant degree.

### 4.2.2 Cognitive Demand and Feedback - Pre and post interviews: Breaking the limits of a rigid feedback that does not consider cognitive demand

This section's purpose is to build a narrative where Mario's participation in the interviews is described through the lenses of cognitive demand and feedback. As described in the introduction, the main parts of the interview will be presented and, starting from what happened in the first interview, followed by the description of the same part of the second one.

#### 4.2.2.1 Characteristics of the feedback

***Describing the way of giving feedback so far: Sticking to a customary feedback***

In terms of the feedback-giving process at the time of the first interview, Mario follows the standard procedures; student teachers send an email with the lesson plan attached a few weeks before the implementation, and he provides written feedback in the document and sends it back. The number of iterations for this process highly depends on the student teacher he is interacting with, varying from one iteration for student teachers that write adequate lesson plans from the beginning, to several (up to five) iterations for those student teachers who require more attention and assistance:

> *There is a limited time where student teachers can turn the lesson plan in, some suggestions and adjustments are given to them through email. If needed, we meet in person right after that (...) There were times when certain student teachers wrote adequate lesson plans right away, so they did not need much feedback (...) There were student teachers that needed much more mediation in that regard. Then maybe they got up to five feedback instances before they taught the lesson.*

Since the former question addressed elements that would not change at least until next academic year, it was only asked in the pre-workshop interview.

***Personal preferences on giving feedback: Alignment to standards, content and structure***

The first question about personal preferences on giving feedback is open, and it directly asks about the elements of a lesson plan that he considers most important to give feedback to. The openness of the question is intentional, since we want to know what aspects of a lesson plan the MTE brings forth naturally, without further prompts or directions. Mario replied that he pays attention to the goal of the lesson, how the tasks correlate with the goal, and how the three "moments" of the lesson (launch, implementation, closure) are aligned to the content:

> *The lesson goal, if it adequately matched the tasks. . . or, let see, how can I say it? Maybe if the tasks matched the lesson goal so the main objective could be reached. What else? The three moments of a lesson, asking questions back to students - which are important in terms of the content that is being taught. . . the closing of the lesson, too.*

When asked again this question in the post-workshop interview, Mario said he sustains what he said in the first interview.

***Prioritizing elements of a lesson plan: Goal-Task coherence from a chronological perspective***

When asked to order the list of lesson plan elements shown below, according to his personal level of importance:

A. Formal elements of the lesson plan (completeness, adequate writing of goals, task description, resources, etc.)
B. Coherence between established goal and selected task
C. Connection with relevant mathematical ideas

D. Establishing a learning goal

E. Selecting a main task

Mario answered consistently with the previous question. The three most important elements for him were (in order): establishing a goal for the lesson (D), coherence between the goal and the main task (B), and selection of the main task (E). This order seems to follow chronological considerations rather than mathematical ones and, in any case, excludes any mention of cognitive demand. When asked again this question in the post-workshop interview, Mario said he would just make a small change, switching (B) and (E). Therefore, the three most important elements for him this time were (in order): establishing a learning goal (D), selecting a main task (E) and coherence between the goal and the task (B). Despite the slight change, Mario still seems to be thinking chronologically, rather than assigning importance to the listed elements.

***Mathematics involved in feedback: Departing from structure-only feedback. The first spontaneous mention of cognitive demand.***

Next, Mario was asked what mathematical elements he takes into account when giving feedback to a mathematics lesson plan. He mentioned that he pays attention to the methods involved in the planning; also, he checks that all mathematics in general stays strongly connected to the established goal and to the national standards for content.

> *(...) All mathematical elements, therefore, must have a strong connection with the goal of the lesson, with the learning goal obtained from the standards; then, after attending this first requirement, they can analyze how this element develops, that mathematical task, that mathematical object. From the ideas that are involved, maybe the methods that are being used, students' interactions to achieve the development of the mathematical object, the kind of question to check whether the mathematical ideas in*

> *play are learnt, the establishment of relationships. . . instead of having just one exercise to work on, maybe imbuing it with a real-life situation. Depending on what is being worked within the lesson plan.*

During the second interview, he was asked the same question. He kept some of the elements from the first interview. Namely, paying attention to the goal, to national standards and - overall - checking that there is coherence between the goal and the task. But, there were some relevant new elements that he brought forth. Most relevant among them, an explicit and unprompted mention to cognitive demand.

> *(...) I could keep some elements; for example, consulting the national standards document, learning objectives and such, given that they normalize and center us in terms of practice itself. It is also important to pay attention to coherence between the learning goals that were established and the selected task. (...) Also, the analysis, what are the opportunities that students have to reflect about what they are doing; the task that is being presented, the opportunities they have to interact with said task, and also whether its cognitive demand is low or high, and the purpose of why the teacher selected a task with a certain cognitive demand level in relation to the group of students.*

#### 4.2.2.2 Cognitive demand and feedback of tasks

**First scenario for tasks - Pre interview: Feedback through questions. But what kind of questions?**

The first lesson plan excerpt presents the following scenario:

One of your student teachers plans a lesson with the features below:

- Content: Addition with whole numbers (Thousands).
- Goal: Solving problems involving addition with whole numbers (Thousands).

For the main task, she poses the following problem: *"Last weekend, Sarah went to the movies. She spent* \$2.942 *(CLP) on the ticket and* \$1.432 *(CLP) on popcorn. How much did she spend altogether?"*

As to how to use this problem in the lesson, she proposes having the students follow the list of steps below:

1. Read and understand
2. Identify data
3. Identify the operation
4. Perform the operation
5. Answer the question and check the answer

Within the rationale section of the lesson plan, she claims that following these steps will help students understand the problem, organize the information and arrive at the correct solution.

***Figure 4.1:*** *Excerpt from lesson plan used on first scenario of pre interview*

It is worth mentioning that, in every scenario from here onward, before asking about the feedback on cognitive demand, the approach to inquire about cognitive demand itself will follow three increasingly specific questions: First, a question related to the general thoughts that the MTE had about the scenario, which answer might or might not address cognitive demand. Next, there is a question about the kind of mathematics involved in the situation (again, the answer might or might not be related with cognitive demand). Then, a question asking indirectly about cognitive demand, stated in terms of the opportunities to think and reason that students might have in the scenario. Finally, a direct question about the cognitive demand of the situation (task, goal, or other) described in the scenario.

When asked about general thoughts, Mario immediately started suggesting feedback for the student teacher in the form of questions. Although the first questions were of the calling-forreflection kind (e.g. how to achieve that students can understand the problem? Why are these data relevant?), towards the end, these suggestions became more of the directive kind:

*This makes me wonder a bit about. . . how can we achieve this, because, of course, there are five steps to solve this problem, but how can we make sure this is going to help the student understand the problem? How? The "how" ... I mean, we have these steps, but how do we do it? At the time of, maybe, identifying the data, why are these data relevant? Maybe I would ask something back there. Identifying the operation... maybe asking something back here, again.*

*(...) And maybe I would ask another question within this fourth step, why did you choose this operation?, sorry, within the third step, identify the operation, why did you choose it? And for the fourth step. . . what would happen if we used the opposite operation? Does it make sense? (...) If we change the operation, or if we use the opposite operation than the one selected, does the total money spent make sense? I think I would add more questions in between these steps, in order to... I mean, we are not sure that even with my subjective questions that the student will understand the problem.*

In terms of the kind of mathematics involved in the task, Mario took it in the direction of the content (e.g. whole numbers, arithmetic), instead of talking about cognitive demand or other similar concepts. He also showed some uncertainty while answering this question:

*Well... whole numbers, right? Addition with whole numbers. . . Well, there is proper arithmetic involved here (...) The kind of mathematics as a concept. I don't know. But the only thing I can infer is that this is a kind of mathematics task related to arithmetic, number range within whole numbers. Sorry, numbers up to the thousands. That's all (...) but the kind of mathematics, thinking about it from a technical standpoint, I am not sure if I answered the question.*

When asked about the opportunities to think and reason that students would have with this task, Mario mainly talked about the relation that the task had with everyday life:

> *Well, first of all, it resembles a situation within a real-life context. Maybe, the opportunities to think, I imagine. . . I think they are elementary school students, so it allows them to think that mathematics is useful to buy bread (laughs), to purchase something, a product. To reason. . . that there are certain difficulties that everyday life presents, and that we need to use our mathematics background in order to solve these difficulties.*

When directly asked about the cognitive demand of the task, Mario first discussed how it depends on the students' context, to then ascertain that the task had a "low to mid cognitive demand":

> *What is these students' social context? I think that first. . . because if it is a group of students that, maybe, I don't know. . . has certain characteristics that might be different from a school where parents' cultural capital allows them to support their children's learning process, I would dare to say that the cognitive demand, I don't know if it's high level, high demand, with a high impact. I think it would all depend on what context we are dealing with to be able to, at least from my point of view, claim something about the cognitive demand of this task. I know nothing about the students' context.*
>
> *(...) Oh, as a teacher. I would think this task is somewhere between low and mid cognitive demand.*

Finally, in terms of the feedback that he would provide, Mario insisted that he would pose questions for the student teacher. Again, some of the questions called for reflection and some suggestions were directive:

*I would add the questions I mentioned at the beginning, such as: when she chooses the operation, when she identifies the mathematical operation, asking: why do you think that is the right operation? Maybe verbally. . . if she is presenting the problem to the whole group and she somehow selects a single student, asking: but why? And also asking some other questions like: Okay, but what happens? I don't know. . . for example, with division, does it allow to solve the problem?*

*(...) Suggesting, of course, to generate questions within each step in order to understand whether the student is effectively understanding said step.*

*(...) What I would add for students that, for instance, solve the exercise quickly, we could have another task as an expansion. Those groups that finish early, present a new task; maybe with a similar context, but adding additional elements, in a way that increases the complexity of the task.*

**First scenario for tasks - Post interview: Using technical terms for cognitive demand and questions calling for reflection**

For the post-workshop interview, the first scenario is shown below:

One of your student teachers plans a lesson with the features below:

- **Content:** Operation with whole numbers (Hundreds).
- **Goal:** Solving problems involving different operations with whole numbers (Hundreds).

For the main task, she poses the following problem "Beatrice has $953 (CLP). Her older sister borrowed $125 from her and her little brother borrowed $150 from her? How much money did they borrow altogether?"

As to how to use this problem in the lesson, she proposes having the students strongly focus on keywords. For this purpose, she proposes having the students read the problem and highlight the important numbers and words. Then, they must determine which operation to use and solve the problem.

Within the rationale section of the lesson plan, she claims that following these steps will help students understand the problem, organize the information and get the correct solution.

***Figure 4.2:*** *Excerpt from lesson plan used on first scenario of post interview*

Just as in the first interview, Mario was asked to give his general thoughts about this lesson plan. Similarly as he did in the first interview, Mario had the tendency to quickly go into questions, mainly rooted into the the role that the amount Beatrice had at the beginning plays:

> *What are the questions that would be asked in return for an eventual difficulty, presuming that students use the total amount that Beatrice has, right? And also, how can this learning situation with these students be faced and solved. And also asking about the purpose, maybe an intentioned purpose that this teacher has at the time of including the total amount of money Beatrice has.*

One notable difference from the first interview relates to noticing issues with the task. It is worth mentioning that, in both interviews, the task corresponding to the first scenario intentionally had questionable aspects. Namely, in the first interview, the task rigidly divided the problem solving process into a series of steps instead of acknowledging the dynamic process of solving a problem. In the second interview, the task is a good example of why we should not only rely on keywords. The term "lend" seems to call for subtraction, but the problem actually requires addition. In the first interview, Mario did not talk about this issue during the first interview, but he noticed the issue in the second interview:

> *And it might happen, for example, that a student (...) uses a subtraction. And what the questions would be to face this situation.*

Then, Mario was asked about the kind of mathematics involved in the task. Again, he mainly addressed the question content wise.

> *The number range, as I understand, is up to the hundreds. What have been the previous exercises, the previous problems that have been presented to the class? Why does she*

*consider that this situation is actually a problem? Maybe she addressed different registers of representation.*

Next, Mario was asked about the opportunities to think and reason that students would have with this task. He mentioned that there are several opportunities at hand, and he was quick to mention several aspects of the task that could be taken advantage of for rich discussion. He did not focus on the limitations that the issues of the task imposed on the cognitive demand, but rather paid attention to the potential that the task offered as is.

> *Wow. I think that there are several, because now, what happens if we consider the total amount Beatrice has? (...) For instance, that students might create new questions for this problem. I think the previous might be used as an extension question, and then these new questions can be solved by other groups. (...) Then, generate a plenary discussion, that from one single problem could stem multiple questions in regards to the presented information, and also transforming the problem. I think they have several opportunities for reflection and analysis with their classmates.*

When directly asked about the cognitive demand of the task, Mario answered that, depending on some conditions, it could vary from memorization to procedures with connections. The emphasis that the student teacher proposed on keywords would bring down the cognitive demand to memorization if this is a routine to be done without questioning, or to procedures without connections at most. If the inclusion of the total amount that Beatrice had is properly used, then it could bring the cognitive demand up to procedures with connections.

> *If students use keywords (...) it is going to be low cognitive demand, considering that she justifies that following these steps will help students understand the problem, and if students have to underline certain words or key phrases, I think it will be a low*

*cognitive demand task (...) I think it can move between 'without connections' and 'memorization', because if students have already worked with similar problems, they will replicate their resolution (...) using the strategy. But, since it is mentioned that Beatrice has $953, we might jump, perhaps, to a higher level of cognitive demand. How to discriminate how to use that amount in relation to what is being asked to me (...) I think that we could say in this case that it could be procedures with connections, having the need to discern that the total amount is irrelevant to how much was borrowed from Beatrice. Then, it could vary within these three levels, depending on the circumstances.*

There is a significant shift here. In the first interview, it was hard for Mario to discuss cognitive demand if not fully considering the students' context. In the second interview, however, he was able to detach a bit from the context and address the cognitive demand of the task itself. Furthermore, he went from using general terms like 'low cognitive demand' to fully formal terms of Smith & Stein's framework, such as 'memorization', 'procedures without connections' and 'procedures with connections'. It seems reasonable to think that Mario picked up these terms during the workshop.

Finally, in terms of the feedback that he would provide, Mario replied in a similar way than in the first interview: he would ask questions for the student teacher. This time, however, questions were almost exclusively calling for reflection. He did not mention any significant directive questions:

*I am going to suppose a level of procedures without connections (...) I will focus on achievement level: What will happen with students that were not able to solve the problem? (...) I would also ask some other questions: Which is the keyword that hindered the solving process? Which keyword or key phrase helped? (...) And I would* *ask about simplification and extension questions in order to attend to the diversity of the group (...) Also, using the idea of cognitive demand, these four levels: How would the student teacher sort this problem according to its cognitive demand level? How to increase the cognitive demand level of this problem?*

**Second scenario for tasks - Pre interview: Questions, high regard for PST work, but feedback is still not fully articulated**

The second lesson plan excerpt posed the following scenario:

1. Two of the student teachers you oversee are planning lessons about area of rectangles. Each one of them designs a main task as shown below:

| | |
|---|---|
| **PST 1: Martha's Carpeting Task** Martha was recarpeting her bedroom which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?<br>**Rationale:** In order to have the students reinforce the area formula for rectangles, I decided to use this task. I wanted to make sure that the problem could be finished within one lesson. Since the problem is framed within a real-world context, it will be meaningful for students. | **PST 2: The Fencing Task** Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen in which to keep the rabbits.<br>a. If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?<br>b. How long would each of the sides of the pen be if they had only 16 feet of fencing?<br>c. How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.<br>**Rationale:** I chose this problem because it will allow to delve into two intertwined concepts: area and perimeter of rectangles. I expect the students to offer multiple ways of solving the problem. |

***Figure 4.3:*** *Second scenario for tasks. Pre interview*

Since this is the second scenario about tasks, the first question Mario was asked is about the opportunities to think and reason that each task provided to elementary students. Mario naturally pointed towards specific instances of opportunities to learn for each of the tasks. He treated all opportunities as the same in nature, focusing on listing the opportunities he identified rather than differentiating them in terms of depth, demand or any other categorization:

> *I would start with the second PST. To think and reason. . . I think that there are some of these opportunities for this class, as they establish the tight relationship between perimeter and area. That they get to decide the moment when the concepts of perimeter and area emerge.*
>
> *On the other hand, I think that in Martha's carpeting task, the opportunities to think. . . Well, to some extent this could happen even at home, right? Looking at it with the opportunities lens. I mean, how many times have faced a situation where our parents want to change a wallpaper or something like that? Well, there is actual math here, here we can see that the computation of an area is present.*

When asked about the cognitive demand of each of the tasks, Mario concluded that the second task had a high cognitive demand, stressing that his classification was made under the assumption that students were actually able to solve the problem. In contrast, he mentioned that the first task had a low to mid cognitive demand, considering that the area formula for rectangles might pose an access issue for solving the task:

> *Okay, I will presume the student can solve the task. I think the second task can actually be considered as high cognitive demand for a common classroom, I will work with this presumption. And for the first task, I would sort it somewhere between mid and low cognitive demand. But there might be students that do not understand how to do it, because they forget the formula.*

The final question for this scenario was to provide hypothetical feedback to each one of the student teachers. Although he was emphatic that there were some issues with task one, he also explicitly mentioned that he would be careful of not just listing the negative aspects of the task, but rather stating questions that call for reflection: "what happens if the student doesn't know the formula?" "How can we be sure that all students actually know the area formula?" From these questions, he intended to start a dialogue about what would be a proper way to present the task to students.

In terms of the second task, he would also provide feedback through questions: "how is the task going to be presented to students?" "Are they going to work in small groups?" "Are students supposed to finish part a before they can move on to the next, and so on? Mario did not elaborate on how to proceed after providing feedback in this case.

Besides these instances of feedback, designed to be private, Mario was also asked to think of a public way of giving feedback in a situation like this. He would choose to start with student teacher 2, asking why she decided not to directly use the formula, and also whether the task had an adequate level of cognitive demand for the class. If not, it might be reasonable to use part of the task from student teacher 1 as a starting point:

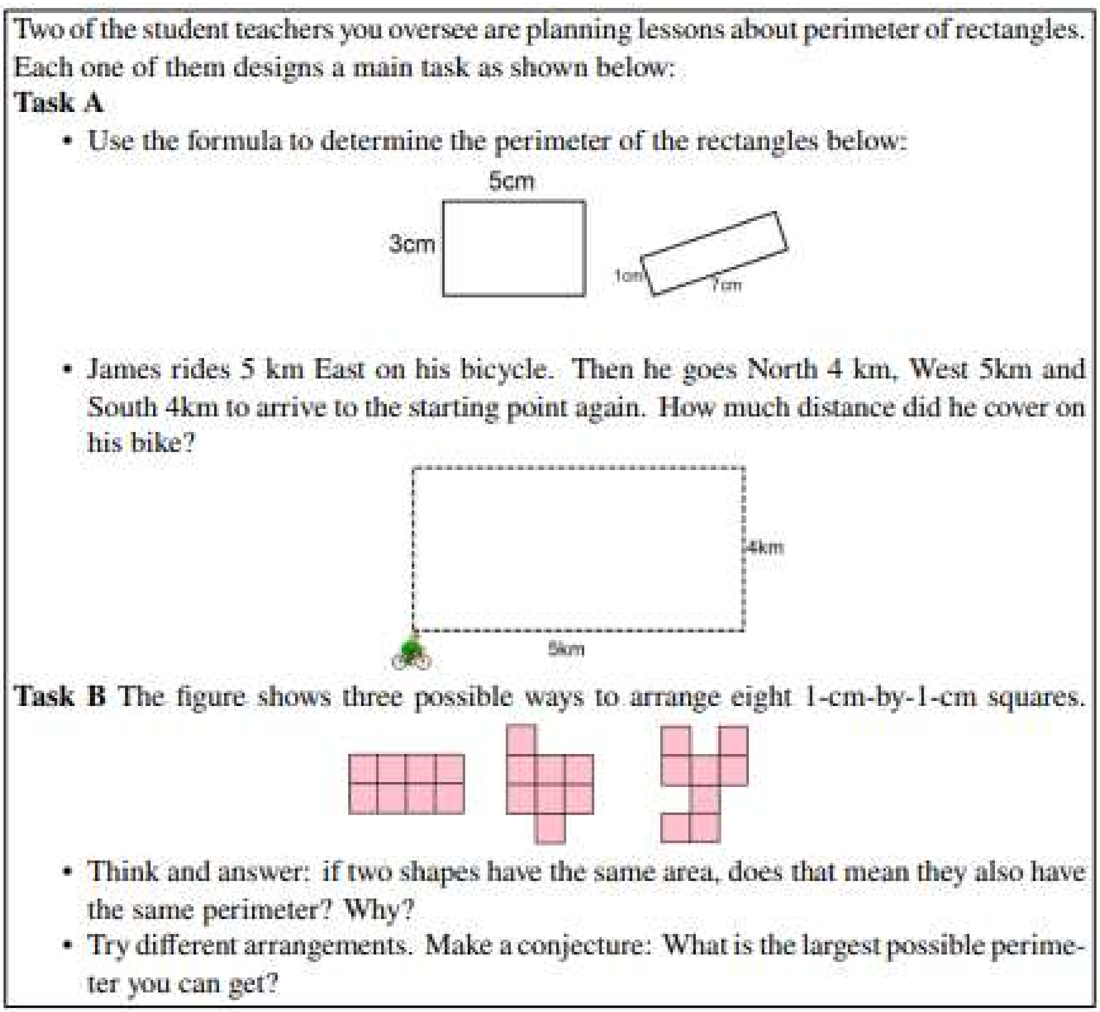
> *Well, what we could do, then, is consider Manuela's [sic] task, task number 1. And if we consider that task number 2 has a high cognitive demand level, then we could present this task first, but eliminating a bit that they could tell how many square meters, maybe we could give students a sheet of grid paper.*

Mario did not elaborate specifically on public feedback for student teacher 1. He rather mentioned he would like to establish some sort of link between both tasks, not disregarding any of them:

*Grid paper. To students. . . but maybe taking away the questions, and asking students to create the question for this problem, related to these sheets of grid paper. . . that they make up a question, but through practice. And once they have solved this task, present task number two, but going little by little, depending on time. And, by doing it like this, generating a connection between both tasks, without belittling any task regarding the other one.*

**Second scenario for tasks - Post interview: Opportunities to think and reason, and the use of cognitive demand as a means for giving feedback**

For the post-workshop interview, Mario was presented the following scenario:


Two of the student teachers you oversee are planning lessons about perimeter of rectangles. Each one of them designs a main task as shown below:

**Task A**

- Use the formula to determine the perimeter of the rectangles below:



- James rides 5 km East on his bicycle. Then he goes North 4 km, West 5km and South 4km to arrive to the starting point again. How much distance did he cover on his bike?



**Task B** The figure shows three possible ways to arrange eight 1-cm-by-1-cm squares.

- Think and answer: if two shapes have the same area, does that mean they also have the same perimeter? Why?
- Try different arrangements. Make a conjecture: What is the largest possible perimeter you can get?

***Figure 4.4:*** *Second scenario for tasks. Post interview*

Just as in the pre-workshop interview, the first question for the second scenario is about opportunities to think and reason that each task would provide to elementary students. For task A, instead of focusing on opportunities to think and reason, Mario started to think of questions to ask the student teacher, which seems to imply he did not see many of such opportunities. For task B, he mentioned that experimenting with the squares opened several opportunities to reflect and analyze the situation, and that it would be beneficial to give actual squares to students to allow for an open exploration.

> *In the first task, I would ask the same question: What opportunities to think and reason do students have when only using the formula? (...) For the second part of the same task, I would ask: What is the reason you decided to create both situations without mentioning the concept of perimeter? (...) They seem two different foci, is the first part of task A really necessary? (...) In terms of task B (...) they have several opportunities for experimentation (...) Maybe giving students some manipulative squares would provide possibilities for analysis and reflection.*

Then, Mario was asked about the cognitive demand of each task. In contrast with the preworkshop interview, Mario used several technical terms this time. He asserted that the first part of task A was a memorization task, whereas the second part of the task was procedures with connections and, if the image were to be removed, it could even be considered as construction of mathematics. As for task B, he briefly mentioned that making and testing a conjecture about maximum perimeter made it a construction of mathematics task.

> *Task A, exercise 2, might be classified as high cognitive demand, procedures with connections. Maybe the image tells you something, there is an orientation towards the perimeter. (...) I would think about whether it is worth it to eliminate the image. Without*

*the image, it could even be construction of mathematics, because they would be generating a conjecture.*

The final question for this scenario was to provide hypothetical feedback to each one of the student teachers. This time, Mario followed a different rationale than in the first interview. He said that he would use the same strategy with both student teachers: start asking them to sort their task according to Smith & Stein's cognitive demand framework and justify their chosen category. Then, he would question whether the task had a high cognitive demand, and in the event it would not, ask what changes could be made to the task to make it a high cognitive demand one.

*I believe what I mentioned before. To reflect if these two tasks (...) what is the cognitive demand that each of these exercises would imply and why, so later - during public 36 feedback - she could warrant her answer (...) And with PST number two, I would do the same; ask her to sort her task according to the cognitive demand she wants students to work with. And, based on what I mentioned, to analyze (...) if it corresponds to a high cognitive demand or a low cognitive demand task, and what is the corresponding subdomain in each case.*

**4.2.2.3 Goal feedback: From cognitive demand as an obstacle to cognitive demand as a source of opportunities to think and reason**

Then, it was time to ask Mario about feedback of cognitive demand for lesson goals. Just as it was made with tasks, he was presented a scenario, and then progressively asked about cognitive demand. The scenario is shown below:

During your weekly (whole group) meeting with the student teachers you oversee, you start talking about the lesson they need to teach in a few weeks. The content is area of rectangles. You ask them to draft possible goals for the lesson. They propose three possible goals. As they talk, you write them on the board:

**Goal A:** Students will examine several examples of rectangles, identify the length and width and learn the area formula for rectangles (A = L x W)

**Goal B:** Students will be able to identity length and width of rectangles in pictures and use the area formula for rectangles (A = L x W) to solve a series of missing value problems

**Goal C:** Students will recognize that a rectangle with length L and width W can be covered with L groups of W 1-unit-by-1-unit squares without gaps or overlaps. Thus, there are (L x W) square units within the rectangle

Adapted from Smith & Stein, 2012

***Figure 4.5:*** *Scenario for goals. Pre interview*

For this scenario, the first question that relates (indirectly) to cognitive demand is: *what opportunities to think and reason would students have with each of these goals?*

For Goal A, Mario said that, from experience, he doubted students could distinguish between length and width in rectangles. It is unclear the effect this confusion would have on the concept of area according to Mario, because he did not elaborate further on the topic. In terms of Goal B, Mario briefly translated the goal into his own words, without further analysis. Regarding Goal C, he stated that its intention was to understand how the area formula emerges for a rectangle, as well as showing why length times width results the same as width times length. This last idea is particularly interesting, since for Mario the confusion between length and width was problematic:

> *I think, for goal A, I don't think they can identify what is length and what is width, because there is always a confusion, at least what I have observed in students; which*

*one is the length? Which one is the width? (...) Well, I think what I value from goal A is that they can identify length and width in different rectangles. For goal B (...) Here, what is the goal? If we know length and width and I am missing - for example - the length [sic], we can make the computation for that rectangle. And for goal C (...) here I think it means a deeper understanding of the formula, how it emerges in a rectangle. Oh, that's why length times width is the same as width times length and it represents the area. I don't know, I imagine students in a situation like I just described. I think goal C contributes to that; students' understanding of how the area formula for rectangles emerges.*

The next question was: What can you say about the cognitive demand of possible tasks that could emerge from each one of these goals? At this point, it seems Mario felt the need to address the idea of cognitive demand from a more technical approach, but still lacking 'official' terms to do so, since he attempted to sort the tasks with a low-mid-high cognitive demand scale:

*I think that goal A, I don't know, might have a low impact. Goal B. . . mid, because of the missing value. And goal C, high. Low, mid and high.*

When asked to provide feedback for each goal, Mario again stated questions; for Goal A he would ask why the formula is so relevant for the lesson, while being cautious of also providing positive feedback. He would also ask how the rectangles would be presented to students; would all be horizontal, vertical or diagonal? He seemed focused on the need that students were first able to distinguish between length and width.

For Goal B, the main questions would be: why is it important to consider missing value problems? If we take away the phrase 'and use the area formula for rectangles', is the goal still attainable? Again, Mario seemed focused on the importance of students being able to distinguish between length and width in a rectangle.

Goal C produced different questions; while he still focused somewhat on the distinction between length and width, he delved deeper into some concepts: why would you work with a grid? Why 1-unit-by-1-unit squares? Why mention 'without gaps or overlaps'? This time, it seems the goal helped direct the discussion in a slightly more significant direction:

> *[F]irst goal, why does she think that the formula is relevant in this lesson? But also a positive feedback. I think that I would value a lot that students could start by identifying length and width in each of the rectangles. Also, asking how these rectangles will be presented, all horizontal, with the base as width? Or are we going to present them diversely, right?*
>
> *Goal B (...) Well, I think we are going to start with opportunities, why did she decide to present a series of missing value problems? Why did she conceive the goal in this way? I also want to value that they can identify length and width in these rectangles; but, on the other hand, I am going to ask: but why the formula? If we take away the phrase "they will use the area formula for rectangles" is goal B achievable? I mean, will students always be able to identify length and width of a series of rectangles to solve missing value problems?*
>
> *And for goal C (...) the same as goal B, value the fact that students will identify length and width, (...) and why does this PST want to work with 1-unit-by-1-unit squares, without gaps and overlaps? Why did she conceive it that way?*

For the post-workshop interview, the scenario about goals was the following one:

During your weekly (whole group) meeting with the student teachers you oversee, you start talking about the lesson they need to teach in a few weeks. The content is perimeter of rectangles. You ask them to draft possible goals for the lesson. They propose three possible goals. As they talk, you write them on the board:

| |
|---|
| **Goal A:** Students will examine several examples of rectangles, identify all the sides, realize that opposites sides are congruent and learn the perimeter formula for rectangles (P = 2L + 2W) |
| **Goal B:** Students will be able to identify all sides of the rectangle, noting that opposite sides are congruent and use the perimeter formula for rectangles (P = 2L + 2W) to solve a series of missing value problems |
| **Goal C:** Students will recognize that a rectangle with length L and width W can be covered with L groups of W 1-unit-by-1-unit squares without gaps or overlaps. Thus, L square sides cover the length and W square sides cover the width without gaps or overlaps. The total distance around the rectangle is 2L + 2W. |

Adapted from Smith & Stein, 2012

***Figure 4.6:*** *Scenario for goals. Post interview*

The first question relates (indirectly) to cognitive demand: what opportunities to think and reason would students have with each of these goals? Mario decided to start with goal C. He stated that it offers several, diverse opportunities to think and reason, especially if students can experiment with manipulatives such as square tiles. He sees opportunities for an active, dynamic lesson where there is plenty of discussion and overall, with a high cognitive demand.

> *I would start with goal C. I think there are several, diverse opportunities to reason. Especially if students have the opportunity to experiment. I imagine this goal to be very dynamic and active, working in groups, with manipulatives, with the squares that might be brought from home, right?, so they can experiment with the goal itself, and so they can realize that something happens, and from here we can infer this idea of the perimeter being 2L + 2W. So goal C offers a high cognitive demand.*

He then moves on to goal B. Overall he thinks that, although students will be able to identify all side lengths, giving them the formula seems to take away opportunities to think and reason. Inspired by goal C, he asked what would happen if the formula was not provided from the beginning, and whether it could help to increase cognitive demand.

> *Goal B, although students will be able to identify all the sides of a rectangle, and that opposite sides are congruent, but what happens at the moment of using the perimeter formula, right? (...) to solve a series of missing value problems, so the formula does not emerge, it is being given to students. Inspired by goal C, I would discuss what happens if we take the perimeter formula away when they solve missing value problems. So, generate this kind of dialogue with students. If we take away the perimeter formula, can it have a high cognitive demand?*

Finally, he addressed goal A. Mario suggested several modifications to the task, showing that - as is - there were not many apparent opportunities to think and reason. Some of the suggestions he made was to change the orientation of the rectangles, make them measure with a ruler and check if opposite sides are congruent, and ask students to create their own problems. He did not mention explicitly that goal A would result in low cognitive demand work, but it seems implicit, given the amount of changes he suggested.

> *Well, the same. That they get to identify and observe different kinds of rectangles, maybe with different orientations, identify the sides, and also. . . if they have a ruler, that they can measure and verify that opposite sides are congruent. And then again, what happens with learning the formula? Why not design problems with different rectangles? Maybe students themselves can create a problem. And maybe bring problem 2 from task A, that we just analyzed. I think that - with some modifications - these three goals can result into high cognitive demand problems, removing, of course, that part of 'learning the formula'.*

#### 4.2.2.4 Practice feedback: Discovering the impact of teaching practices on cognitive demand

The final part of the interview addressed elements of latent teaching practices that could be seen from the lesson plan. For the purpose of this interview, they were called "declared practices". The scenario for this section, described as excerpts from a lesson plan, is shown below:

**Lesson plan**
**Goal:** Students will examine different rectangular configurations for a given perimeter, conjecture about what is the maximum possible area for that perimeter and find a way to test/support their conjecture
**Content:** Area and Perimeter of Rectangles
**Task:** The Fencing task (see above)
(...)

- Have them work in groups from the beginning.
- Monitor groups
- If they are struggling, mention that they must find the area of all rectangles with a perimeter of 24
- If more help needed, suggest they start with a 1x11 rectangle
- If needed, remind them to use the A=LxW formula
- In whole group discussion, if no one has the right answer, suggest to divide by 4 and test

***Figure 4.6:*** *Scenario for practices. Pre interview*

The first question was: (based on what can be seen in the excerpts) *What opportunities to think and reason would students have in this lesson?* Mario started with the idea that working in groups and interacting with each other is crucial. He also claimed that the questions this teacher anticipated would be helpful for understanding the topic. Even if further help is needed, there are questions prepared for the situation. However, the last suggestion (divide by 4 and test), was borderline excessive for Mario; he wondered whether it was too much scaffolding. In any case, by the end, he decided that the teacher was not giving the answer away. In his answers, it seems Mario focused more on the structure the teacher used to anticipate questions (some help needed, more help needed, even more help needed) rather than the effects these questions might have on the cognitive demand of the lesson. At one point, he questioned whether there was too much help, but he ended up deciding there was not:

*Well, I think that working in groups. . . they can interact with each other, I think this is crucial. Besides, these extension questions that the teacher is asking if students are having complications, mentioning that they must find the area of all rectangles with a perimeter of 24 units; I mean, she is helping so they can understand what is behind the problem. Also, I understand that - if students require further assistance, she is also asking some other questions; suggesting that they start with a 1-by-11 rectangle, because it fits the 24 units, right?, as perimeter, in that sense. . . (reads) 'if necessary, remind them to use the formula area = length times width'... Yes, I also think that we cannot belittle what students are thinking; I mean, we all grew up with the formula, so I think we cannot take value away from it. Additionally, she is using it as a last. . . I mean, second to last resort. And lastly, whole group discussion. If nobody has the right answer, suggesting that they divide by four and see what happens. Eh, she has already provided a lot of scaffolding, but I don't think she is giving away the answer. . . so, I think she planned all her interventions so she can keep these interventions as a last resort. . . divide by four and test, right? I think students have every opportunity to think and reason.*

Last, Mario was asked: What effects can teaching practices have on cognitive demand? He declared that it was important to involve relevant mathematical concepts when addressing mathematical content, so students are able to discover formulas (for example), rather than the teacher presenting it to them. He did not elaborate on how some teaching practices can help maintain or diminish the initial cognitive demand of a task:

*Yes, I think it provides footing so they can later formalize the formula autonomously. I think this is the effect it has. I mean, not presenting the formula immediately, but rather* *let students themselves say 'wow! That's why!', 'ooh!'... these things teachers hear when students say 'ooh!, this is why you multiply length times width!', right? And not base times height [sic], for instance. Then, I think some mathematical concepts are being involved (...) I think I have grown conscious lately (...) Not presenting the formula right away, and instead let it emerge from students, let them realize 'oh, that's why'*

A similar scenario was posed during the second interview:

> A preservice teacher plans to use the problem below in a lesson: 'Create a real-life problem that is solved with the division $1\frac{3}{4} : \frac{1}{2}$'
> Within the lesson plan, the following notes are included:
> (...)
> - Make them work in groups from the beginning
> - Monitor groups constantly
> - If they struggle, remind them different roles of dividend and divisor in a division
> - If they are still struggling, ask them how many times a half 'fits' into one whole and three fourths.

***Figure 4.6:*** *Scenario for practices. Post interview*

The first question for this scenario was about the opportunities to think and reason school students would have with this setting. Mario stated that he found few opportunities to think and reason within the design. Furthermore, he mentioned that this was not only due to the excessive help eventually offered by the teacher, but also by the task itself. At this point of the interview, Mario felt comfortable using technical terms for cognitive demand, and he claimed that the task fell into the memorization category. Maybe, if the teacher asked for different ways of solving the problem, it might just reach procedures without connections. He notices that planned interventions are directive, taking away opportunities to make decisions from students, while also pointing out that, in his opinion, the task is centered on procedures, not reasoning.

*They will create a problem, yes. . . They might even go through different scenarios. However, I think. . . from what we have been talking about cognitive demand, I would* *think this is a memorization task (...) Maybe if the teacher asked the students to solve the problem in two different ways, maybe. . . it would reach procedures without connections (...) But I think students would have very few opportunities to think and reason with this task (...) Because the feedback she gives, her interventions are highly directive, then there are not many opportunities for students to reflect by themselves. Maybe if students are given manipulatives. . . it might be different. (...) It seems her plan is centered around computational strategies rather than delving deeper in the meaning of division. Well, I think we would still be in memorization. . . we would stay in that state. Maybe changing something, using different registers of representation so the situation is not as obvious, not as evident. (...) But, effectively, it has an impact, I think it maintains a low cognitive demand, basically through memorization.*

### 4.2.3 Workshop

A workshop was conducted in between the first and second interview. During this workshop, some elements about the interview were discussed in small groups and with the whole group (for more details, the reader can refer to the methods section and/or the appendix). Theoretical elements about cognitive demand were also addressed, as well as some practical implications to the work of providing feedback to student teachers' lesson plans. Smith & Stein's (2012) framework for cognitive demand categorization was used as a guideline.

As the researcher, my main role was to guide every section of the workshop. Presenting slides, designating groups, managing discussion time and executing plenary discussions. Farther along, I formally introduced Smith & Stein's (2012) theoretical framework and oversaw some practical assignments. Finally, I led the summary discussion, tying everything up to make conclusions.

***Mario's workshop experience: The shift's source***

Mario's participation in the workshop started in a similar manner than his first interview. While he was permanently open to participate and give his opinion, as well as to receive comments about what he said, at the beginning, he showed very little mention of cognitive demand and, when he did, it was using general terms, such as high or low. As the workshop progressed, however, more mentions of cognitive demand started to appear. Furthermore, after a group assignment, where they had to sort tasks according to Smith & Stein's framework, Mario's mentions of cognitive demand started to pick up these technical terms, which suggests he felt they were useful for his discourse and he felt comfortable with them. Naturally, the general categorization of high or low cognitive demand came easier to him than deciding which specific level better applies for a task, although discussing and coming to an agreement with his group was achieved without any major difficulty. Mario showed an evolution process inside the workshop itself, which came accompanied by his body language: starting somewhat tight and tense, and becoming more comfortable and loose as the workshop progressed.

> *(Excerpt from the beginning of the workshop)*
>
> *I think a mathematical task has to be.. 'reachable' by students. (...) Students should be able to. . . use what they have learnt previously and use it to give an answer to the situation. More than an exercise, try to make it challenging, yet feasible.*
>
> *(Excerpt from the beginning of the workshop)*
>
> *Will this question be of a high cognitive level? Squares have four sides, so. . . I don't know (...) Are we presuming they have some basic knowledge? (...)*
>
> *(Excerpt from the beginning of the workshop)*

*(...) About the kind of cognitive demand, about the goal. . . and the implications that the cognitive demand level set at the moment of establishing the goal might have for the task.*

As it was mentioned in the methodology section, the workshop was divided into three main sections. To better understand Mario's journey, now that we have already examined in detail his participation in both the first and second interviews, we will delve further into his performance in these three sections. This way of organizing the results should shed light not only on what shifts occurred during the research, but also whether any of these shifts might have stemmed from the workshop.

**Section 1: Framework for cognitive demand and feedback of cognitive demand of tasks - Laying foundations for technical proficiency**

Before addressing Smith & Stein's framework for cognitive demand, we spent some time laying some necessary groundwork. This was done through questions to be worked in small groups. At the moment of the design, the intention for including these questions before addressing the aforementioned framework was to determine whether there were any spontaneous mentions of cognitive demand by any of the participants, and if these eventual mentions triggered any mentions by other participants. Mario's performance in this setting is summarized below.

***Where is the Math in a lesson plan?***

Mario mentioned that - in his view - there should be mathematics throughout the lesson, but student teachers tend to concentrate mathematical considerations mostly on the implementation and closing phases.

*Well... Where I usually see it is in the implementation phase of the lesson, where the main task is addressed. But I can also see it during the closure phase, where* *information is processed and organized, depending on whether there is interaction among students. But I believe that it should be visible throughout the lesson plan, all the way from what is done in the beginning. . . motivation and the like. It should be present through all three phases. But where I mostly see it present is during implementation and closure.*

***Where is the Math in the goals and the tasks?***

At this point, it seems that Mario is still getting used to the workshop, so there is still some uncertainty in his answers. For this question, he states that, within a task, it is important to focus on the kind of questions that are anticipated when implemented with students. Questions designed to stimulate the exchange of ideas and opinions can become an inflection point leading to mathematical discussion and a good closing phase. Mario's ideas are certainly powerful, but do not completely answer the given question. When asked what a good task looks like, Mario took the initiative and stated that it should be challenging, but doable. It should be more than an exercise, and it should 'mobilize' students, while remaining connected to both the curriculum and the standards.

*In summary, the kind of questions asked by the teacher, in order to generate mathematical discussion within the class room. I also agree with Ursula, we can observe the lesson plan as a whole (...) but I would tend to emphasize on the kind of questions the teacher is asking, and whether those questions can generate student discussion. (...) A task should be doable by students, when they face the situation - bringing all their previous knowledge - they have the tools to solve the situation. More than an exercise. (...) while also being truthful to the goals and the learning indicators found in the standards.*

As it can be seen, when asked about the Math that a lesson plan has/should have, Mario's considerations tend to focus on curricular aspects rather than the cognitive demand of the task. To some extent, it can be said that the original intention for including these questions worked, in the sense that it could be determined that Mario, at the time, did not really consider cognitive demand when conceiving a mathematics lesson. At least not spontaneously.

***A framework for cognitive demand***

Next, all participants were formally presented Smith & Stein's framework for cognitive demand, dividing tasks into two main categories: low cognitive demand and high cognitive demand. In turn, each of these categories was divided into two subcategories, as it is shown below:

Low cognitive demand

- Memorization
- Procedures without connections

High cognitive demand

- Procedures with connections
- Doing mathematics

This framework was used throughout the study as the main resource to refer to cognitive demand. It was even extended when discussing cognitive demand of goals for the lesson.

The decision of including a formal theoretical framework for cognitive demand was motivated by the participants' diversity of backgrounds. At the moment of the design, it was unclear whether all participants knew or used a formal conceptualization for the subject. Providing one common framework would secure an established theoretical minimum, which in turn would help when making comparisons and drawing conclusions.

After the presentation, participants were asked to engage in a practical activity, where they had to sort a series of tasks according to the framework of cognitive demand they had just seen. The first two tasks were seen as examples, with the participation of the researcher.

Mario was hesitant at the beginning. Most times, he could distinguish between low and high cognitive demand, but still had some troubles when deciding the specific category:

> *[The task] Has a low cognitive demand (...) I can't discriminate between memorization and procedures without connections, uhm. . . can I see the table again?*

But there were some occasions when he had trouble distinguishing between low and high cognitive demand, as it happened when sorting the task below:

**Fractions and pencils** Sarah and Brandon have 20 pencils altogether. Sarah says that ¼ of the pencils are hers. Bradon states that 15 pencils are his. Explain how it is possible for both of them to be right. Use words or drawings.

***Figure 4.7:*** *Example of task used in the workshop to sort by cognitive demand*

> *There might be some students who solve it right away and they only have to explain it (...) Within the premise that the required basic knowledge is acquired, I really don't know if it has a high cognitive demand.*

Despite the fact that he occasionally struggled, providing Mario with a theoretical framework proved to be foundational for his journey, as the first instances of narratives of cognitive demand of his own can be traced back to this point of the study.

Then, participants were asked to apply what they had learned by analyzing tasks drawn from lesson plans written by actual student teachers from previous cohorts. This section was included in the design to establish a link between the notion of "opportunities to think and reason" with cognitive demand, through the practical exercise of analyzing excerpts of actual lesson plans from student teachers.

The first set of tasks is shown below:

| Task A | Task B |
|---|---|
| Test if the addition is correct:<br>13 + 24 = 37<br>**Step 1:** Identify the addends<br>**Step 2:** Subtract one addend from the sum (e.g. 37 – 24 =)<br>**Step 3:** If the result is the same than the other addend, then the original sum is correct.<br>**Justification** Giving the students a set of steps to test whether a sum is correct helps them to organize their thinking and to learn in a better way. | **Task B** Find different subtractions which result is 180.<br>e.g.: 360 – 180<br>**Justification** A question of this kind allows for flexible reasoning, while it also allows looking for strategies that can be later applied in more complex problems. |

***Figure 4.8:*** *Example of tasks drawn from real lesson plans to be used in the workshop*

To analyze these tasks, participants were asked a series of questions. A summary of Mario's answer to each of these questions can be found next:

***What opportunities to think and reason would students have with each task?***

For task A, Mario states that the teacher is providing part of the reasoning, but he also claims that this is something positive, since students are given a way to test if their additions are correct. At the beginning, Mario did not talk about task B.

> *I think that the teacher in task A is providing a reasoning model, a guide. . . she is modeling a way of reasoning to check whether a sum is correct. I think that - at the moment of including the steps - she is reinforcing this idea of how to check the sum. (...) I think this is positive.*

Mario's opinion about task B came only after another participant in the small group talked about this task. Mario valued that it was presented as a problem and that it had multiple possible answers. Students need mental organization to solve this problem.

> *I hadn't seen it that way, Ursula, thanks. Actually, task B is also presented as a problem, because it has multiple answers. But we are thinking in this kind of context, where students need some sort of mental organization.*

***How does our previous analysis relate to cognitive demand?***

Mario's groupmate gave an extensive answer to this question, talking about how learning by exploration and discovery (related to high cognitive demand tasks) meant a long-lasting learning experience, while the opposite, just following steps would be easily forgotten. Mario agreed with his partner, not offering further answer to this question.

Although the intention was to offer participants the opportunity to further develop their ideas about cognitive demand and to explore the relationship this concept has with thinking and reasoning, it did not come up quite as intended, given that it was Mario's groupmate who talked the most, relegating him to a secondary spot. This is not to say that the exercise was futile. Mario would draw from this interaction in the second interview to build his own notion of cognitive demand.

***How would you provide feedback to each task?***

Mario said he would ask the student teacher why she chose these two specific tasks, and how they compare to each other. He also agreed with his groupmate, who said that exploration in task B might be worth the time and effort, because students tend to better remember what they explore, rather than what they do following a set list of steps.

At the moment of the design, this one probably the main question to be included, since it directly asked participants to provide examples of feedback. As such, it was expected that participants would delve deeper into examples of feedback. In the end, Mario was giving bits and pieces of feedback examples throughout this section, rather than concentrating them on this last question.

***Section 2: Cognitive demand of goals and feedback - Ambitious goals should not be obstacles, but opportunities.***

In this section, participants were asked to start re-analyzing goals from the pretest interview:

**Goal A:** Students will examine several examples of rectangles, identify the length and width and learn the area formula for rectangles (A = L x W)

**Goal B:** Students will be able to identity length and width of rectangles in pictures and use the area formula for rectangles (A = L x W) to solve a series of missing value problems

**Goal C:** Students will recognize that a rectangle with length L and width W can be covered with L groups of W 1-unit-by-1-unit squares without gaps or overlaps. Thus, there are (L x W) square units within the rectangle

Adapted from Smith & Stein, 2012

***Figure 4.9:*** *Goals revisited at the workshop*

Some questions from the pretest were revisited. Namely, participants were asked the following:

***What can we say about the cognitive demand of possible tasks built from each of these goals?***

Mario directly states that goal A will likely result in a low cognitive demand task, because the actions that this goal describes greatly constrain what students will do; examine, identify and learn a formula.

For goal B, Mario claims it is similar to goal A, but it adds an application for the formula, and not just learning it. He implies that goal B also would yield low cognitive demand tasks.

It is on goal C that Mario makes a clear distinction, since it implies discovering what ‘without gaps and overlaps’, ‘L’ and ‘W’ mean for a rectangle. It appears like an invitation for a teacher to explore and discover in a lesson. Hence, goal C would most likely result in high cognitive demand tasks.

Mario's groupmate gave a similar answer, but she used technical terms (e.g. procedures without connections). Although Mario initially did not use such terms, he engaged effortlessly with what his partner said. It appears that he was almost ready to take this step from a non-technical language to a technical one. It seems a logical conclusion that both the content of the workshop and engaging with his groupmate discourse might have been the trigger for Mario to effectively start using these technical terms, since during the last interview he continued doing so, to some extent.

At the moment of the design, the intention for this question was to explicitly bring the discussion to a focus on cognitive demand. In this regard, it seems that the objective was fulfilled, since Mario and his groupmate directly referred to cognitive demand of possible tasks that emerged from the listed goals.

***What kind of feedback would we give to each goal? Provide examples***

Consistently to what he answered during the first interview, Mario proposed feedback mostly based on questions. For goals A and B, he would ask: 'why is the formula important?' From there, he mentioned that it could be even possible to use this as an instance for public feedback, starting a discussion with a group of student teachers, talking about where is it more convenient to deal with the area formula for rectangles; at the beginning as a definition?, in the middle, as a means of application?, or towards the end, as a consequence of exploration?

Mario and his groupmate strongly centered the feedback on (school) student learning. They mentioned that it would always be convenient to ask questions such as: '¿what do you think your students will do when facing this task?', 'what opportunities to think and reason they will have when engaging with this task?', and the like.

The intention of asking again this question (it was also asked during the first interview) was twofold; on the one hand, it would allow to determine whether there were any changes that might have been caused by the workshop and, on the other hand, it would also provide the opportunity for participants to talk about this topic with their group mates. In the case of Mario, both intentions were fulfilled; since it was possible to verify that his discourse was mostly consistent with what he answered during the first interview, but it also got refined through the interaction with his groupmate.

**Section 3: 'Declared' practices, effect on cognitive demand and feedback of practices - Learning about cognitive demand marked a significant shift when thinking about teaching practices and cognitive demand**

The final section of the workshop addressed some latent teaching practices that can be anticipated in a lesson plan. We labeled these as 'declared' practices, since they had not happened yet, but they eventually would, should the lesson happen as planned.

Participants were shown the lesson plan for the fencing task (see above, in the first interview), and were asked for their opinion, this time as a small group discussion.

Without any major prompt, Mario immediately claimed that the interventions listed in the plan would lower the cognitive demand if implemented, since they would take away opportunities to think and reason from students. Then - again without any prompt - he talked about how he would give feedback to this lesson plan; some of his feedback was of the reflective kind (e.g. 'how would you rephrase your third note so it is not that explicit?'), but some of it was directive (e.g. 'You should use your fourth note only as a simplification of the problem, if and only if all the other instances were not enough', 'in your fifth note, I would suggest using manipulatives'), just as it happened during the first interview with him.

At the moment of designing the workshop, this question was intentionally open, since the topic was already discussed during the first interview. The main intention was to check whether participants would answer in terms of cognitive demand and feedback and to provide the opportunity for a small group discussion. In the case of Mario, this worked just as planned. He was asked about his opinion, and he immediately answered in terms of cognitive demand (increasingly using technical terms) and then talking about the feedback he would give in this scenario. His intervention was decisive enough that his groupmate fully agreed with him and did not add any other major elements to the discussion.

### 4.2.4 Conclusions: Professional growing of a caring, thoughtful teacher

Mario's journey during this research was, undoubtedly, a productive one. There are evident shifts both in terms of the role that cognitive demand plays in the planning of a mathematics lesson, and in the ways of providing feedback to these plans. At first, Mario focused diligently on the coherence between goals and tasks, but mostly based on contents and standards considerations, rather than setting a high cognitive demand that could lead to rich mathematical discussion. He did not mention cognitive demand at all, unless explicitly asked to talk about it and, although his main way of providing feedback was through questions, these questions were sometimes directive and somewhat shallow, given the lack of consideration to deep, conceptual mathematical ideas.

During the workshop, Mario started picking up the formal aspects of the framework for cognitive demand used throughout this instance. This experience seemed to have made an impact for him; not only because he increased the frequency and technicality of use of such terms, but also because he looked comfortable using them, as shown by the fluency of his discourse and his body language.

Mario's participation in the second interview only comes to confirm the previous statement. When asked about the mathematical elements he considers most important to include in a feedback to a lesson plan, among other concepts, he mentioned cognitive demand of the task, and the establishment of a goal that allows for such a task. There was no prompt to do so. He included these ideas on his own volition. This kept occurring during the rest of the interview. Different aspects of cognitive demand - including spontaneous sorting of tasks and goals - emerged continuously throughout our interaction. Mario's statements were certainly perfectible, as he still made occasional mistakes, but there is definitely a major shift in the way he conceives a mathematics lesson plan and, accordingly, the way that a mathematics educator should provide feedback to it.

## 4.3 Case 2 – Yasna Quezada: From cognitive demand as a theoretical construct to a personal link with student decision-making

This section is for presenting and analyzing the case of Yasna Quezada. As with all other cases, the final goal is to document the shifts that occurred from the first interview, through the workshop, up until the final interview. The main aspects we are observing are cognitive demand and feedback of lesson plans. Just as with the previous case, the narrative will not be presented in a chronological way. Instead, I will first dissect Yasna's participation in both interviews into their main parts. Within each part, I will first discuss the first interview, followed by the second one. The workshop will be addressed afterwards.

### 4.3.1 Background: A special education teacher finding grounds for relevant use of cognitive demand

Yasna is also a teacher, with specializations in Mathematics and graduate studies in Mathematics Education. She has been an instructor at the institution where the research was conducted for at least three years. She has supervised student teachers at least twice before participating in this study.

Yasna showed technical proficiency from the first interview. When she was asked about cognitive demand, she would use formal terms from Smith & Stein's framework comfortably and with precision. She situated student learning as a top priority, permanently using it as a guideline to answer the questions I asked her and to build the arguments she gave during our interactions. Yasna would sustain this proficiency up until the end, but there are two aspects that would shift to some degree: The priority given to cognitive demand when providing feedback and, more interestingly, how she linked her personal idea of students making decisions with opportunities to think and reason and, in turn, with cognitive demand.

As I just mentioned, when asked about cognitive demand, Yasna would proficiently speak about it, bringing up theoretical frameworks and technical terms. But at the time of the first interview, Yasna would only talk about cognitive demand if explicitly asked to do so. She would not bring it up to the conversation if there was no prompt. By the second interview, however, Yasna would often speak of cognitive demand when asked about general thoughts about a scenario and always speak of it when asked about opportunities to think and reason. This appears to show (and what happened during the second interview seems to confirm it) that cognitive demand became a top priority when conceiving a mathematics lesson and, therefore, when giving feedback to a mathematics lesson plan.

Furthermore, Yasna seems to have made some interesting connections. She brought with her an idea from the very beginning: the space that students have to make decisions greatly determines if a task can be considered a routine exercise, or a non-routine problem. As students get to decide what to do, they can generate their own heuristics, develop strategies, test their ideas and, eventually, solve a problem. In contrast, the less space they get to make decisions will constrain their ability to think on their own, and most likely they will end up reproducing whatever the teacher wanted them to reproduce. But, of course, not every decision will have this impact. There are trivial decisions and more relevant ones. When Yasna was repeatedly asked about opportunities to think and reason, she noticed that those decisions that relate with thinking and reasoning are the ones she referred to from the beginning. She later also made the connection between opportunities to think and reason and cognitive demand, and it was at this point that the major connection was made: her idea of students' space for decision making is strongly related to cognitive demand, via thinking and reasoning. From this point onward, the second interview started to gravitate around this one major concept: What opportunities to think and reason do students have? whatever their decision-making space allows. What is the cognitive demand of this product? high, if they get space to make decisions, think and reason. Low, otherwise. How high or how low? directly correlated to the amount of such space students get.

This is the tone that my last interaction with Yasna had. She ended up building this major idea, which she used as a constant guideline to think of a mathematics lesson. Situating student learning at the very center, and permanently considering how much space they get to make decisions, think and reason. This should evidently reflect on the lesson plan, so she also used this guideline to provide feedback in forms of questions: What do you expect students to learn? How can this task offer opportunities for your students to decide, think and/or reason? How does this goal allow for a high cognitive demand task? What are students doing at point X? What are you doing at point Y? How are your interactions not restraining their thinking? These were the kinds of questions Yasna ended up building for feedback. Powerful questions.

### 4.3.2 Cognitive Demand and Feedback - Pre and post interviews: Departing from a customary, institutionalized feedback to a meaningful use of cognitive demand

In this section, I will describe and analyze Yasna's participation in the interviews through the lenses of cognitive demand and feedback. As described in the introduction, to help the reader better see the shifts that are taking place, the process will not be presented chronologically; first, each domain of the interviews will be addressed, within which the interaction in the first interview will be described, followed by the corresponding interaction in the second interview.

#### 4.3.2.1 Characteristics of the feedback

***Describing the way of giving feedback so far: Standardized feedback constrained by institutional requirements***

At the time of our first interview, Yasna described a standard feedback-giving process: student teachers would send her through email a proposed lesson plan, and she would reply with some feedback within the text document before the lesson could be approved to be enacted. This process would take place at least once per lesson (student teachers had to teach five lessons throughout the semester).

***Personal preferences on giving feedback: Centered on student learning, but not necessarily on cognitive demand***

When asked what aspects of the lesson plan she considers most important to provide feedback to, Yasna said that, instead of starting from specific elements, such as the writing of the goal or other formal elements, she likes watching broader elements of the lesson, such as how the student teacher is thinking about student learning, what kind of questions she poses, how she anticipates possible difficulties or misconceptions, error management, methods, assessment, and the like.

> *All that has to do with the making of the objective, the goal, I care more about the general feel of the class; I mean, how I see it in the paper or the screen, I can tell how that PST, that teacher candidate, is thinking in turn about how her students learn. Then, the kind of questions that she poses, how she anticipates mistakes, what goes in the bulk of the plan, I would say. How she thinks the launch phase, how she thinks the closure phase, what are the details that shed light about her didactic choices, in sum. And how these choices impact on the plan, definitively. That is the part I try to pay. . . I mean, I do it for the whole plan, but what I care the most about really is that. . . and, on the other hand, what that PST intends to have for assessment.*

When asked again this question during the second interview, Yasna withheld what she originally said. She insisted that her main intention usually is attempting to go beyond the rigid structures that usually take place when planning and that are required by design; namely, those related to standards and curriculum, and make student teachers reflect about the planning of a lesson.

***Prioritizing elements of a lesson plan: Task fidelity is important***

When asked to order the list of lesson plan elements shown below, according to her personal level of importance:

A. Formal elements of the lesson plan (completeness, adequate writing of goals, task description, resources, etc.).
B. Coherence between established goal and selected task.
C. Connection with relevant mathematical ideas.
D. Establishing a learning goal.
E. Selecting a main task.

Yasna chose the following elements as her top three: Selecting a task (E), then Coherence between goal and task (B), and Connection with relevant mathematical ideas (C).

During the second interview, Yasna was again asked this question. This time, she decided on the following order: E, D, B, C, A. While she still decided on the selection of a main task as the most relevant element, this time the establishment of a goal jumped up to second place, relegating goal-task coherence to third place. Yasna argued that the task is still extremely relevant, as it determines the outcome in terms of student learning, but she changed her mind from the previous interview in terms of the importance that the fidelity to the lesson goal represents.

**Mathematics involved in feedback: Mathematical knowledge, problem solving. . . emphasis on cognitive demand appears after the workshop**

The next question was related to the mathematical elements Yasna considers when giving feedback to a mathematics lesson plan. She said she pays attention to the kind of mathematical knowledge that is put in play into the plan. How this knowledge threads the standards, the curriculum, to a goal and then made into a problem solving task, and how these kinds of problems and concepts are given priority over a series of exercises or procedures. Overall, the student teacher should show she is thinking about problems that would allow to develop this kind of mathematics.

> *What is the mathematical knowledge included in the plan, which the PST intends to develop, and that is usually shaped after the standards; and how that mathematical knowledge is posed from a problem solving perspective. What time is assigned and what priority is given to that problem, and not to a series of exercises, or to a procedure written in a plan, but how all of these are a part of a whole, in which the PST is thinking about the problems that allow the development of the intended mathematics.*

When faced again with the same question in the second interview, Yasna immediately spoke about cognitive demand, and how our previous conversation and the experience of the workshop made her think about the increasingly relevant role that the framework for cognitive demand had been taking place throughout the content and methods courses in the institution, and how this should also be implemented more and more into the student teaching stage, as well.

> *And I think this goes beyond the workshop, because it has to do with the way we have been incorporating this framework about cognitive demand levels throughout the regular coursework. One question that I think should be always asked is: What is the cognitive demand level of your lesson plan proposal? They tend to think, especially when they are planning their first lessons, that their tasks have a high cognitive demand level even if they don't, so I think we should ask: Why? Where are you coming from to assert this? What are your arguments? I would add the need of an intentional focus on the levels of cognitive demand. I think our feedback is still lacking in this regard.*

Yasna made a relevant connection when answering this question; while she was asked - generally - about the mathematical elements that should be present in a lesson plan feedback, she spontaneously mentioned cognitive demand, and how the framework that was already being used in methods and content courses, should also be utilized in the student teaching courses, specifically when giving feedback to lesson plans.

#### 4.3.2.2 Cognitive demand and feedback of tasks

**First scenario for tasks - Pre interview: Feedback through meaningful questions, uncertainty about what 'kind of mathematics' refers to**

For an overview of the scenario (and any scenario used in this case), the reader can refer to the first case. Alternatively, the full interview can be found in the appendix section.

When asked for her general thoughts about the first scenario about tasks, Yasna questioned the choice of providing rigid steps, arguing that - when it is taken that far - it restrains students' ability to make decisions and over-structures their actions, causing that the overall problem solving process cannot be seen as a whole anymore, rather than providing guidance for adequately solving the problem. Yasna also mentions that she would ask the student teacher about the intended implementation of the task, and also when and how students are supposed to mathematically produce a relevant result.

> *[To] me, this 'steps' thing always comes to mind, because it is used way too often; it's a very 'Polya-like' thing, a frequently found idea, (...) in the end, these steps often transform into a straitjacket (...) because what usually happens is that worksheets state things like: identify the data, select an operation, execute the operation and finally (...) they become overly-structured scripts, where students unpack the problem solving process to the extent that, later, when they have to define the solution, they don't remember what they had to do anymore. Then, that would be my first feedback to this PST, after watching this. I am also missing what she thinks or considers about the representation of the problem. I think this is a missing step, so I would ask: What do you think about representations here? At what point would students represent the problem? In order to identify the best possible path for a solution (...) at what point do you think that students will produce mathematically? Unregarding the path that was followed.*

Regarding the kind of mathematics involved in the task, Yasna briefly mentioned that it is 'just mathematics', while also pointing out that there is a chain of mathematical knowledge about addition and subtraction put into place, which establishes a relation between how students learn and what the teacher does so this learning can actually occur. Similarly to other cases, Yasna showed some uncertainty while answering this question. Apparently, the term ‘kind of mathematics’ is too vague for her.

> *I acknowledge my ignorance in this question, because I don’t really know what we mean with ‘what kind of mathematics’... just math. It is a kind of mathematical knowledge somewhere along the chain of knowledge about addition and subtraction, where it is somehow involved how children learn, and what the efforts that the teacher makes so that learning happens. But I don’t have an answer. I don’t know what kind of mathematics is involved.*

When asked about the opportunities to think and reason that students would have with this task, Yasna argued that she found no evidence of opportunities for students to think and reason and, therefore, she thinks there would be few of them:

> *I have no evidence to say that this is happening. I mean, I would say that few opportunities, fundamentally because I don’t have any evidence for the opposite. And because within the planning, in the end, in the execution of that planning I should have evidence. . . maybe with more information, it is expected that with these steps each child gets the possibility to face the task and then there would be a discussion (...) What are the different types of reasoning that could be seen? With what I am seeing, I don’t see opportunities.*

Next, Yasna was explicitly asked about the cognitive demand of the task. She stated that it was hard to make a judgment without further context but, as she saw it, it was somewhere in between procedures without connections and procedures with connections. It should be highlighted that Yasna explicitly used these technical terms without being prompted to do so any more than asking her about cognitive demand.

> *I'd say it would be like. . . I actually don't know if I have all the information, but I would dare to say this is procedures with connections, I wouldn't say it is memorization nor procedures without connections. . . maybe, because there is a series of steps to follow. . . I would say it is between procedures without or with connections.*

Finally, about the feedback that she would provide, Yasna focused on asking for further information because, as it was standing, the cognitive demand of the task seemed low, and it directly related to the amount of information provided in terms of how the task would be presented to the students. This way of providing feedback appears to have a mixture of asking questions (additional information) and somewhat directive instructions (direct assessment of low cognitive demand).

> *First, adding more information, I think I would give this as a feedback, give me more information, where I can observe how things will unfold (...) because this written performance is way different if the teacher is in front of the class and says (...) 'okay, step 1. . . identify the data', and the children look for numbers, don't read the problem, look for numbers and write them, 'step 2. . . and all of them do the same at the same time'. This is all very different than saying 'children, you are going to solve this problem with your partner. . . you can follow these steps, but you will have to explain how you solved the problem later', you see? Deep down, both scripts work in a fundamentally different way.*

**First scenario for tasks - Post interview: Understanding and making decisions rather than following procedures**

When asked about her general opinion, Yasna said she did not like the task, since emphasizing keywords hindered the students' overall ability to solve problems, having them focus on rigid structures and procedures rather than thinking and reasoning about mathematics. This kind of overly-procedural approach often leads to mistakes, when the wording does not exactly correlate with the operation needed to find the answer. Yasna gives the example of comparing two numbers, where the words "more than" might be used, but a subtraction would be needed to find said difference.

> *What do I think? I don't like it. Because something that I hold as a foundation of life is that we hinder student learning when we make them focus on keywords, because keywords are not always a rigid measurement; for example, in problems that deal with comparison by difference, children get confused since they see the word "more than", but they must subtract. . . or sometimes, when you say "she lent", it might mean add or subtract, same with "eating", or "giving away" (...) If they focus on these elements, on keywords, they will start looking only at these things, instead of observing the problem as a whole.*

Next, Yasna was asked about the kind of mathematics involved in the task. She still was unsure about what the expression "kind of mathematics" meant, but this time she asked right away if I meant cognitive demand. I told her that she could decide on any characteristic that she thought might characterize the mathematics involved in the task.

> *I don't know if I fully understand what 'kind of mathematics' means. But there is a need for solving a problem. For me, a series of rigid, controlled steps does not proceed.*

*There are several times when students are only asked what they have just been taught. I taught addition. . . I ask about addition. And I ask with the kind of numbers and under the exact circumstances I taught you. I don't know if this problem reaches the 'doing mathematics' category, but it certainly is along the lines of procedures with connections. There are connections, because the students must make decisions. And that is the kind of mathematics that I see. It is very difficult for me to separate the problem in parts and categorize it.*

From this excerpt we can see that Yasna still struggles - at least to some extent - when asked a general question about the 'kind of mathematics' that is involved in the task. It seems that she is looking for a theoretical framework under which she can categorize the mathematics of the task, although she was given total freedom to describe it. Nevertheless, there is a clear shift from her first interview, where she described the kind of mathematics as 'just mathematics'. This time, even though she still struggled with the term 'kind of mathematics', she directed her attention to the cognitive demand of the problem. Consistently with what we have seen from her in the post interview, she has brought the importance of cognitive demand to the highest priority, to the extent of using the concept of cognitive demand - and its framework - as a means for articulating her answers around them.

Following the script, next Yasna was asked about the opportunities to think and reason offered by the task. At this point, she made an interesting distinction between the task itself and the task as it appears in the lesson plan. She claimed that the task offered several opportunities to think and reason, as the situation is an inverse problem, where the described action (lending) does not directly correlate with the operation needed to reach the answer to the problem (addition). But, if we consider how the task was planned to be implemented, the opportunities basically disappear, given the extremely rigid structure of following exact steps and focus on specific keywords.

> *I think the task offers several opportunities, if I depart from the intention stated in the lesson plan and, instead of focusing on keywords, we think of developing a plan to solve the problem. Especially if we add a component of collaborative work, I'm thinking about working in groups. The problem looks simple, but there are several possible paths to a solution, and they might be negotiated within each group. Therefore, there are also collective opportunities to think and reason, also. (...) Without departing from the intended plan, the cognitive demand dies. If we look at the problem, I can see at least two different ways of solving it, then getting into this keywords thing it means a dead end, no return path.*

At this stage, it is clear that Yasna has explicitly made the connection between 'opportunities to think and reason' and cognitive demand. She has gotten even further, if we consider how throughout our conversation during the post interview she added this idea of also incorporating the idea of 'making decisions' as intrinsically related to 'opportunities to think and reason' and, therefore, also to cognitive demand. The way she expresses her ideas also helped lay a bridge between this topic and the subsequent section of how teaching practices affect the cognitive demand of a task. She explicitly stated that a high cognitive demand task might actually 'die' (citing her words) if the lesson plan does not adequately implement it.

Next, Yasna was directly asked about the cognitive demand of the task. Unsurprisingly, Yasna said that the task itself represented a high cognitive demand problem, within the procedures with connections category. She also mentioned that, taking into account the way the implementation of the problem was planned, the level of cognitive demand would be extremely procedural and rather algorithmic, so it would fall under the category of procedures without connections.

> *The problem opens a perspective of autonomous thinking, that I could perfectly place in a high cognitive demand category; I don't know if doing mathematics, but at least procedures with connections. If I look at the implementation proposal of this student teacher, that implies a series of steps, I think in this case it would be rather algorithmic, the one of procedures without connections, because it is a list of steps to follow (...) and students are not making any decisions. If we look at the way the teacher directs these steps, I think it would fall to a lower cognitive demand.*

Yasna continues the development of her newfound relationship between decision making and cognitive demand. She thinks of the decision making space that a certain task generates, and directly correlates this space with the amount of opportunities to think and reasons students would have and, in turn, with the cognitive demand of the task. Finally, Yasna was asked about the kind of feedback she would provide this student teacher, in terms of the cognitive demand of the task. She said she would rely on questions, trying to cause the student teacher to reflect about what overly relying on keywords implies in terms of the students' ability to reason, think autonomously, and solve problems. Moreover, she would ask the student teacher to bring elements from their coursework (content and methods courses) to anticipate what students would do when solving the task, what they might struggle with, and to think of ways of representing the situation described in the task, and what these representations (or lack thereof caused by limitations on the way of implementing the task) means in terms of cognitive demand.

> *I would try, through questions, that the student teacher might reflect and realize what focusing too much on keywords might cause. I have seen in schools how teachers tend*

*to rely on using keywords to solve word problems as their first - and sometimes only - tool. It also appears in textbooks. Arguably, it has become a part of the belief system of some teachers. I would try to make student teachers enter in a dialogic dimension, which I think can be achieved through this question: What would students do? Describe what you think they would do. Would they think? Or would they rather blindly follow the steps? (...) Do we want students that follow instructions? Or do we want students that can think autonomously? (...) One of the things we want is for student teachers to bring elements from their other courses to answer these questions. And, personally, I emphasize representation. I think asking for representations comes to replace Polya and the misrepresentations of his steps. When we advance on representing a problem, we also advance our understanding of the problem and the mathematics underlying it.*

Consistently with our first interview, Yasna gave examples of feedback in the form of questions. Also similarly, the questions would revolve around what students would do, and what that implies. One notable difference is that questions this time were not directive at all, but rather intended to cause reflection and dialogue.

**Second scenario for tasks - Pre interview: Student decision-making and problem solving; foundations for an upcoming epiphany**

After examining the scenario, Yasna was asked about the opportunities to think and reason that school students would have with each of the tasks. She made a clear distinction between the first and second task; the first one appears as limited and 'encapsulated' (in her own words), as students get very little space to make decisions, whereas the second one allows for mathematical knowledge to emerge as a tool to solve problems, offering significantly more opportunities to think and reason.

Yasna continues to construct a consistent narrative where concepts such as cognitive demand, thinking, reasoning and making decisions are tied together and intertwined with each other.

> *Well, clearly there are two very different ways of teaching here. One, for me, is totally questionable, because - just as student teacher 1 says - she is expecting a formula to be reinforced, within an encapsulated problem (...) we have asked ourselves in other scenarios when a student is actually solving a problem and (...) we have defined that it is in the moment when the student has to decide something, and that decision-making possibility, I think, is borderline non-existent in the first case; (...) this is just a way of exercising what was previously taught, because if this student teacher is saying that she wants to reinforce the formula is because the formula was already taught, versus - evidently - what we observe in the second case, where students go into a mathematical activity, where knowledge finally emerges as the tool that allows them to solve the stated problem. Then, evidently, this possibility is significantly more present in the second case than in the first one.*

When asked about the cognitive demand of each of the tasks, Yasna started with the second one, stating that it reached the highest level of doing mathematics, since there is a challenge for which there is no preset strategy, whereas in the first one there is not, placing it in the category of procedures without connections.

> *I think, I would dare to say that (...) I think it would fall in the 'doing mathematics' category. There is a challenge to be solved, the answer is unknown, there are some tools to work with (...) there is intention of collaborative work, whereas the first one I would say it is. . . I wouldn't dare to say it is memorization, but I would say it is procedures without connections. Problem 2 is a good one!*

The final question for this scenario was to provide hypothetical feedback to each one of the student teachers. For the first task, Yasna thought of a series of questions: at what time during the lesson will students produce mathematics? When are they going to discuss? How are you going to deal with mistakes, are you going to intervene?, to name a few. The intention behind these questions was to call for reflection and subtly start bringing up the issue of the low cognitive demand of the task. For the second task, Yasna also thought of some questions: How can we make sure this work is actually owned by the students? How will you monitor the work?, among others. This time, the intention behind the questions was to make sure that teaching practices would not diminish the high cognitive demand set with the task. She also made the suggestion to give back questions instead of answers while monitoring student work. This suggestion is not necessarily directive, in the sense that also calls for a particular kind of reflection: what are good questions to give back to students when they are struggling to solve the task? Yasna also made some suggestions to use common mistakes in lesson plans for public feedback.

> *In the document, I think I would fill it with questions, I'm thinking. . . like, let's think first about case 1. I would say, write. . . maybe something like you have stated, at what time students. . . ? I would put a dialogue balloon, I think it has to be as concrete as that, at what time are students going to produce mathematically?, at what time are they going to discuss?, at what time do you expect them, for instance, to be a lesson closure where they can show and argue about their findings?, how will you deal with mistakes?, if you notice someone doing something wrong: will you intervene right away? Or will you wait until there is some product? I mean, I would really fill it with questions, to return something to the student teacher that can help her reflect about how to deal with these things. And in relation to the second case, there is also a big risk, in terms of*

*having this great idea planned that crumbles down when implemented, namely: how do we do so this is really a student work?, how will you monitor the work? If they are lost, if you see the groups are not making progress. . . give back questions that lead to reflection. And what I tried to do with public feedback, it wasn't so hard, was to take common elements that I saw. For example, I don't know, if I notice in two instances that error management is not addressed, what I would suggest when talking with the whole group is "well, I have seen several lesson plans and I feel we have not addressed this yet, let's see an example, what do you see", or. . . another thing that has been very useful, is to look for a video of an external person where this issue happened, have the student teachers watch this video so they could say "this is me. . . the same is happening to me, I am doing the same thing", that in terms of public feedback.*

**Second scenario for tasks - Post interview: Decision-making and opportunities to think and reason, it all has to do with cognitive demand!**

Consistently with the first task, for Yasna, Task A offered no significant opportunities to think and reason, since students were asked to reproduce a series of steps within a series of boundaries that are completely under control, steps that were already taught. Whereas Task B offered plenty of opportunities, especially if students work in groups, making and testing their own conjectures.

In terms of cognitive demand, Task A would follow under the 'procedures without connections' category, given the reasons she exposed when giving her general opinion for the task, and Task B would reach the 'doing mathematics' category.

When asked about the kind of feedback Yasna would give for these tasks, just like she did in our first interview, she mentioned she would ask several questions about expected learning outcomes, and anticipated strategies and complications for students. In this interview, however, Yasna explicitly mentioned cognitive demand (during our first interview she did it indirectly), and she even used the ideas and categorization of cognitive demand to articulate the feedback process.

#### 4.3.2.3 Goal feedback: Further strengthening the link between decision-making and cognitive demand

At this point in the interview, it was time to ask Yasna about feedback of cognitive demand for lesson goals. Just as it was made with tasks, she was presented a scenario, and then progressively asked about cognitive demand.

For the task scenario, the first question inquires indirectly about cognitive demand, by asking for opportunities to think and reason offered by each one of the three goals. Yasna briefly mentioned that opportunities are in crescendo with each goal. With very few in the first goal.

> *I'd say it goes in crescendo, more opportunities to think in the third goal, and very few opportunities to think and reason in the first one, especially in the first one, which is about learning the formula.*

Next question directly asked about cognitive demand of tasks that could emerge from each of the goals. Yasna seemed to be sure of her thoughts in this matter, and quickly answered similarly to the previous question. Roughly, the first goal should produce tasks in the category of procedures without connections, the second one in procedures with connections, and the third one in doing mathematics.

> *I'd say that, also, we are transitioning from a first goal, a cognitive demand of procedures without connections. And one. . . this is merely hypothetical, really, the second goal I think it is closer to procedures with connections, and the third one probably is doing mathematics.*

Then, Yasna was asked to provide feedback for each goal. She said that she mostly would give back questions to clarify goals beyond content: What do you intend to achieve? What sense and emphasis can we observe? What would a student reading this goal think about the lesson?, among other questions.

> *I mean, I would rather asking back some questions. Of course, I would ask questions directed towards the end I want to achieve, which is. . . with the goal you have made, because there is also much about - especially when they have pressure from the standards and the curriculum on them - of also using an algorithm, cutting the objective, separating somehow the lesson objective producing a goal, but it is. . . what do you expect to achieve with this? This goal you have proposed, if we look a different one, what means and emphases can we observe? In the first feedback, of course, I would say: what do we have here? What do you expect to achieve? What do you think a school student that sees this goal written on the board would think about the development of the lesson? I would ask these kind of questions, I think.*

#### 4.3.2.4 Practice feedback: Teaching practices as an upper bound for the cognitive demand of a lesson

The final part of the interview addressed elements of latent teaching practices that could be seen from the lesson plan. As it was declared in the first case, these were called "declared practices".

Based on what could be seen for the scenario for practices (the reader might want to refer to the first case to read the scenario), the first question of this part of the interview was about the opportunities to think and reason students would have in this lesson. Yasna agreed with most of the lesson settings: working in groups and monitoring groups as they work. But she found several issues with the interventions the student teacher planned for when students would struggle.

Namely, she thought that the student teacher might be giving away too much information and, in turn, taking away many opportunities for students to think, reason and make decisions. Instead, she proposed to give back questions so students could reflect and find their own answers.

> *Very few, I think. Because it is similar to what happens with mistakes, right? Make them work in groups from the beginning, ok. Monitor the groups, ok. If they are struggling, mention they must find the area of all rectangles with a perimeter of 24 units. . . I'm asking myself if this could be done in a different, non-directive way, maybe like a question (...) what do we have to do? Does anyone have an idea? I have this option, can anyone think of another one? But I think that, in what I see here, this is more like following instructions. If they require further help, suggest they start with a 1x11 rectangle. Remind them what the formula is, use the formula. . . okay, I'm not sure they have many opportunities. I think. . . toward other options, like letting the groups work (...) you know? This goes around my head all the time, lately, the time of actual mathematical production (...). So, the reflection I would give this student teacher back is. . . what do you do if - despite you telling them all this information - they don't solve it? "Ah, I would do it for them, or I would ask another student that does know", maybe the best option is to let it happen (...) I would say there might be an opportunity or two, but this is way too directive for me.*

Last, Yasna was asked: What effects can teaching practices have on cognitive demand? While she previously acknowledged the high cognitive demand of the task itself, she lucidly stated that the way the task is presented and managed will determine the kind of knowledge that emerges. Although working in the task would allow students opportunities for rich interaction, student teacher's interventions could terminate these opportunities if not executed properly.

*But of course, I feel the problem itself, like a problem that is. . . at this point I think this idea of "the way I implement the problem determines the knowledge that emerges" is so well put, there is a problem with several pauses about how. . . different stages, and to allow students to get into the mathematical activity, but - undoubtedly - the performance of the teacher could either kill it, or take advantage of it and make progress from it, and I think that this performance of the teacher absolutely (...) has to do with how much the teacher knows, how much she recognizes and acknowledges, and the importance of saying "I don't know, what do you think?" How is it possible that I'm not giving away the solution?, they say, How can I not tell the school students what to do?. So this, beyond the cognitive demand of the problem itself, might actually die due to the performance of a teacher that does not know clearly where she wants to get to in the end.*

### 4.3.3 Workshop

As it was stated in the first case, a workshop was conducted in between interviews. A brief description of what was addressed during the workshop can be found within that case. For a more detailed description, the reader can refer to the appendix.

**Yasna's workshop experience**

Yasna's participation in the workshop was fairly consistent with her performance during the first interview. She was able to offer her insights from the very beginning, given her knowledge about cognitive demand in general, and Smith and Stein's framework in particular. During small group work, she often was the first one to speak and get the discussion started. When sorting tasks according to cognitive demand, she was somewhat hesitant at the beginning, but she quickly got up to speed and showed proficiency up until the end. One notable feature of her discourse was that, whenever discussing mathematical tasks, she would mention how she considered that good mathematical tasks allow for students to make decisions. This would be the starting point of a revealing link of concepts that Yasna would make during the second interview.

### 4.3.4 Conclusions

Undoubtedly, the main point to be discussed from Yasna's participation in the workshop has to do with the concept of 'space for decision making'. She intuitively has the notion that the opportunities to learn that a given task can offer is strongly correlated with the amount of decisions students are allowed to make when solving the task. If few or none are left for students to make, then the task would offer severely limited opportunities to learn. Alternatively, if plenty of decisions are left for students to make, then there is space to think and reason and, therefore, for opportunities to learn. Furthermore, if too many decisions are left for students, then very likely the task would end up drafting away from productive mathematical work. Then, properly managing the amount of decisions left for students to make has an effect on the opportunities to learn the task has to offer for students. Yasna would later link her thoughts about this space for decision making explicitly with the framework of cognitive demand used in this study.

# CHAPTER 5: DISCUSSION AND IMPLICATIONS

## 5.1 Introduction

The topic of feedback for student teachers' mathematics lesson plans is not only under-researched, but also there seems to be no consensus on what the best protocol to enact this process is. This study was able to identify that standard procedures tend to cover mostly formal aspects of a lesson and, although content and methods are given attention at the time of conceiving and planning for a mathematics lesson, there are several missed opportunities in terms of paying enough attention to tasks, goals and practices in relation to their cognitive demand. There is consensus in the literature that selecting a task is arguably the most important decision that a mathematics teacher has to make before the lesson takes place (Smith and Stein, 2012). Then, the question arises: If we agree that this decision is so important, then why are we not paying enough attention when student teachers are learning to make it?

In this section, I take on the task of integrating all previous elements of the research to learn more about how feedback is given to student teachers' lesson plans, what MTEs consider important about lesson plans and how to provide feedback to them and how the practice of providing feedback to these plans can be improved.

## 5.2 Research Questions

The first research question of this study is related to characterizing MTE's feedback giving process to student teachers' mathematics lesson plans at the time of the first interview. As anticipated, when directly asking about the features of the feedback they gave, answers were strongly mediated by institutional requirements; this is, feedback used to be given mostly in a very standard way. Student teachers would email their MTE with a plan proposal in advance of each intervention (normally, each student teacher taught five lessons per semester). Each MTE would check the plan, provide written feedback within the same text document, and then send it back to the corresponding student teacher. This process would repeat a few times until the MTE considered the plan was good enough to be implemented. Exceptionally, some MTEs would ask for one-on-one meetings (either in-person or virtual, as circumstances allowed) if they considered there were specific issues that could not be addressed just with standard procedures. These instances, however, rarely occurred and strongly depended on individual willingness to make them happen, as they were not part of any official protocol.

To delve further than these institutionally mediated answers, asking about personal preferences for giving feedback was particularly useful. Questions such as "what aspects of a lesson plan do you consider most important to give feedback to?" or "which mathematical elements do you consider when giving feedback to a mathematics lesson plan?" allowed MTEs to analyze richer elements of planning a lesson, and to start the approach toward discussing cognitive demand in a lesson plan.

Generally speaking, both MTEs intend to give feedback for lesson plans mainly through asking questions centered around two main aspects: school students' learning and student teachers' preparation. However, the way they enact these principles into actual feedback varies from MTE to MTE. Some of them (e.g. Mario Díaz) asked questions that called for reflection, some that carried curricular considerations, but also some that - more than questions - seemed like directive instructions. Other MTEs (e.g. Yasna Quezada) consistently put in the center student learning, and every feedback related to this main aspect to different degrees by calling to question planned elements according to how these allowed students to think, reason and make decisions.

As both MTEs started from a different point in terms of their knowledge and use of cognitive demand elements in a lesson plan, it is only natural that their shifts also occurred in different ways and at different times. There were, however, two main ‘types’ of shifts, depending on the degree of preparation in Mathematics Education and experience in Teacher Education:

On the one hand, MTEs with less Mathematics Education background and experience in Teacher Education, such as Mario Díaz, got the most from the overall experience by increasing their knowledge about theoretical elements of cognitive demand and how to use them, first in conceiving a mathematics lesson, and - in turn - in how to provide feedback to lesson plans. This type of shift was characterized by a first interview with few or no spontaneous mentions of cognitive demand, unless explicitly asked about them and, even in this case, cognitive demand was referred to in vague terms, such as ‘low’ or ‘high’. During the workshop, MTEs that experienced this type of shift would progressively pick up formal terms for cognitive demand from the theoretical framework used in this instance and become proficient in sorting tasks using this framework. During the second interview, the shift appeared evident, as it was not necessary to explicitly ask about cognitive demand for it to emerge in the discussion and, when it did, it was often referred to by using formal theoretical terms, at least to some degree. One important aspect of this type of shift is that cognitive demand was also considered at relevant stages of the feedback process during the second interview, strongly contrasting with the absence of these considerations during the first interview.

On the other hand, MTEs with more Mathematics Education background and experience, such as Yasna Quezada, were not strangers to Smith & Stein’s theoretical framework for cognitive demand. Their shifts, consequently, occurred in a different way than the first group. These MTEs were able, at different points, to make a conscious link between the theory they already knew and the personal process of providing feedback. For instance, Yasna was able to establish a relation between the notion of ‘opportunities to think and reason’, related to the framework for cognitive demand, with a longtime personal view on how the space for decision making given by a task allowed for 'better' work in a mathematics lesson. By building the construct of 'opportunities to think, reason and make decisions' as a proxy for cognitive demand, Yasna made conscious connections that were not there before, marking a clear shift in her case.

## 5.3 Limitations

This work is a modest, yet important contribution to the topic of feedback on lesson plans, specifically in terms of their cognitive demand. It has brought forth the relevance of focusing on the opportunities students will have to think, reason and make decisions and how these opportunities, in turn, impact their learning gains. However, it is important to note that, although choosing Smith & Stein's (2012) framework for cognitive demand proved particularly useful in providing a lens for analyzing the tasks, this decision also implies certain limitations. Namely, it leaves out relevant aspects of equity, cultural background and social justice.

For instance, Aguirre & Zavala (2013) developed a lesson analysis tool that focuses on culturally responsive mathematics teaching, delving deeper in multiple dimensions that go beyond mathematical thinking (including language, culture and social justice), paying special attention to students' funds of knowledge. All of these provided opportunities for strategic lesson planning and purposeful discussions aimed at improving the quality of mathematics teaching.

Also, Bartell, Turner, Aguirre, Drake, Foote & Mcduffie (2017) make a call to teaching mathematics in ways that are responsive to students' backgrounds, knowledge and experiences, as well as focused on meaningful mathematics learning. They propose to better know students, to engage with the community and then create mathematics lessons "aimed at deepening children's mathematical understanding of a particular concept and connecting mathematics to community contexts" (p.327).

Finally, Berry III, Conway IV, Lawler & Staley (2020) propose an analysis and a series of lessons to work with mathematics from a social justice perspective. Among many contributions, the authors elaborate on the relevance of thoughtfully planning a lesson:

> When the teacher has a strong sense of the learning goals, has considered at least one pathway to those goals, and predicted how students may engage in that pathway, the teacher can make sound decisions based on unpredicted student interests, experiences, or questions. Establishing goals and strategies to assess ensures the teacher maintains a focus as the pathway changes. (p.68)

The authors emphasize the freedom students have to go in different pathways and insist on the relevance that teachers anticipate enough fundamental elements of the lesson so they respond accordingly, always providing space for meaningful mathematical discussion.

Undoubtedly, all these perspectives provide relevant, if not essential, elements to be considered within a lesson plan and, arguably, trace a necessary path for improvement of the present work.

## 5.4 Personal Stance

Researchers should face their field work with open minds, especially when they interact and research about peers. Nevertheless, and although every measure was taken to keep as much impartiality and as few biases as possible, it is inevitable that our personal perspectives mediate to some extent our investigative work.

The present work started because I detected what I considered a weakness in the way we as MTEs provided feedback to student teachers. I felt we were missing a valuable opportunity to engage in meaningful, dialogic ways of assessment through lesson plans. It was not clear to me at this point how to characterize this weakness, and even less how to overcome it. Yet, I thought it was worth to invest time and resources to find out the answer to these questions.

Through interactions with colleagues from different Teacher Education programs I noticed these issues seemed to happen throughout Chile, and studying the literature showed me that this is, in fact, an under-researched topic.

All contemporary reform attempts in Chile call for teachers to make their students engage meaningfully with mathematics. MTEs have the same goal with student teachers. In order to better understand where the roots for those meanings lie, we need to pay careful attention to what happens in the lesson, and especially what kind of mathematical activities school students are presented with. Thoughtful anticipation of the lesson might result in significant learning gains if adequately implemented.

This personal perspective influenced several components of this research. It was the search of a framework that strongly considered mathematical thinking and reasoning that led me to choose Smith & Stein's (2012) work. In turn, this strongly influenced the design of the interviews, and the decision of using situated scenarios with a focus on cognitive demand rather than direct questions.

Since not all the faculty at the institution the research took place in were proficient with the chosen framework, I made the decision that one section of the professional development workshop should be dedicated to an introduction to this framework, perhaps missing on the opportunity of allocating time for deeper reflections, should this introduction had not been necessary.

Data interpretation was performed carefully. Still, some elements of my personal standpoint surely were present at this stage. Hence the decision of backing up every claim with as much evidence as possible and of providing the reader with elements of contrast - such as quotes, tables, or other similar - so they can also make conclusions of their own.

In sum, as it is impossible to completely detach ourselves and our perspectives from the research process, making these perspectives explicit and taking adequate measures to control for partiality and/or bias during the research constitutes an exercise of intellectual honesty that I considered necessary to openly address.

## 5.5 Next Steps

This research is only the first step in an under-researched topic. Although there was not nearly enough statistical power to make causal claims of any kind, the qualitative description of the cases involved in the study certainly contributed to establishing foundations for future research in this topic, not only for the specific context of Chile, but also for the interest of the international Mathematics Education community.

Major decisions are made before the lesson starts. Lesson plans are artifacts of practice containing relevant information related to the quality of the lesson. Giving feedback to these lesson plans, therefore, constitutes a valuable resource for MTEs, not only in terms of providing student teachers with a means for reflecting about the cognitive demand of the lesson - which was the main focus of this research - but also to anticipate teaching decisions related to topics left out in this work, such as culture, equity and social justice.

For instance, elements of Aguirre & Zavala's (2013) analysis tool might be used to link the aspects of mathematical thinking discussed in this research with students' funds of knowledge, language and culture in order to develop a way of providing feedback that articulates mathematical thinking not only from a cognitive perspective, but also including culturally responsive ways of thinking about mathematics lessons.

Furthermore, Berry III et al. (2020) provide complementary ways of thinking about Smith & Stein's (2012) five practices to orchestrate productive mathematical discussion, incorporating strategies that support teachers' discourse when planning a lesson with a social justice perspective. To Smith & Stein's "Anticipating likely student responses to challenging mathematical tasks and questions to ask students who produce them" (Berry III et al., 2020, p. 50), they propose "Anticipating likely student points of view and asking questions that help students identify specific points of agreement and disagreement") (p. 50).

In sum, culture, equity and social justice elements are fully compatible with cognitive demand. There is no reason to limit the feedback of lesson plans only to cognitive demand, and researchers should take note of this, as it appears as a natural way to make the present work grow.

## 5.6 Conclusion

Considering how important decisions made at the moment of planning a mathematics lesson are, it makes absolute sense to take time and resources to allow for a strong anticipatory work. The cognitive ceiling of a lesson is determined long before the lesson even starts, and some of the practices that might determine whether that ceiling is maintained are also decided at this time. Additionally, there are salient elements about culture, equity and social justice that should be anticipated at the moment of planning. Therefore, when student teachers are starting to take up the challenge of learning to plan a mathematics lesson, MTEs should be thoughtful and meticulous to help them realize how much is at stake; How will my students face and solve the task? What opportunities to think and reason will they have with this task? How does it relate to the goals? What strategies might they use? What difficulties might they have? What kind of errors do I anticipate? What will I do - as a teacher - to help guide them to the learning goals, holding them accountable for their learning and not taking away their mathematical authority over their work?

These and other questions are particularly relevant, and we should never take our minds away from them, because they matter. As MTEs, we are responsible for incorporating these questions when giving feedback to student teachers' lesson plans, because we fulfill a double role when doing so; on the one hand, we are preparing them to make better lesson plans but, on the other hand, we are also modeling how to give feedback, which will, in turn, mediate the kind of mathematical discourse our student teachers will have in the future. The feedback we give them, then, also matters.

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# APPENDIX A: INSTRUMENTS

**Overall organization of activities**

| Pretest | Workshop | Posttest |
|---|---|---|
| Questions about current state of feedback-giving process | | |
| Scenarios involving tasks | | Scenarios involving tasks |
| Scenarios involving goals | | Scenarios involving goals |
| Scenarios involving declared practices | | Scenarios involving declared practices |
| | | Looking back: did anything change? |

***Table A.1:*** *Research questions and overall organization of activities. Color coded for the reader's convenience*

**Organization of questions about feedback**

| Pretest | Workshop | Posttest |
|---|---|---|
| 1. In general terms, how does the process of submission and feedback of lesson plans work in this stage?<br>2. How many feedback instances are there for each lesson plan? | | N/A |
| 3. a. What are the elements of a lesson plan that you consider most important to give feedback to?<br>b. Order the following elements of a lesson plan according to the importance you give to each one of them (all elements are important, but the objective of this question is to prioritize among them):<br>A. Formal aspects of the lesson plan (completion, adequate writing of goals, description of tasks, resources, etc.).<br>B. Coherence between the goal of the lesson and the selected task.<br>C. Connection with relevant mathematical ideas.<br>D. Establishing a learning goal.<br>E. Selecting a main mathematical task. | Small group discussion:<br>• What are the main aspects that I consider most important to provide feedback on? (open question).<br>• How would I order these elements of lesson planning according to importance? Why?<br>A. Formal aspects of the lesson plan (completion, adequate writing of goals, description of tasks, resources, etc.).<br>B. Coherence between the goal of the lesson and the selected task.<br>C. Connection with relevant mathematical ideas.<br>D. Establishing a learning goal.<br>E. Selecting a main mathematical task.<br>Whole group discussion: Sharing what happened in small groups | • In our first interview, you stated that the main aspects on a lesson plan that you consider most important to give feedback about are: (summarize)<br>– ¿Is this still accurate? ¿Would you like to add or remove any of these?<br>• Last time we talked, you provided this order of importance for these elements of lesson planning: (summarize)<br>– Is there anything you would change in this order? |

***Table A.2:*** *Organization of questions about feedback*

*Table A.2 (cont'd)*

| Pretest | Workshop | Posttest |
|---|---|---|
| 4. What mathematical elements you take into account when giving feedback to student teachers' lesson plans? | Small group discussion:<br>• Where's the math in the lesson plan?<br>• Elaborate on what the participants said (if goal/tasks do not appear, then ask).<br>• What about the Math do I usually provide feedback to and how?<br>Specific about Math in goals/task | Last time, when I asked about the mathematical elements that you consider when giving feedback to your student teachers, you said this: (summarize) Based on all the PD experience, would you like to revise something? |

**Organization of scenarios and questions about tasks**

| Pretest | Workshop | Posttest |
|---|---|---|
| One of your student teachers plans a lesson with the features below:<br><br>Content: Addition with whole numbers (Thousands)<br>Goal: Solving problems involving addition with whole numbers (Thousands)<br><br>For the main task, she poses the following problem "Last weekend, Sarah went to the movies. She spent $2.942 (CLP) in the ticket and $1.432 (CLP) in popcorn. How much did she spend altogether?" As to how to use this problem in the lesson, she proposes having the students follow the list of steps below:<br><br>1. Read and understand<br>2. Identify data<br>3. Identify the operation<br>4. Perform the operation<br>5. Answer the question and check the answer<br><br>Within the rationale section of the lesson plan, she claims that following these steps will help students understand the problem, organize the information and get the correct solution. | (Hopefully some ideas about CD emerged from last discussion)<br><br>Whole group:<br>• A framework for CD<br>• CD framework (Smith & Stein)<br>• One round together with a few tasks (procedures without and with connections)<br>• Sorting tasks according to CD (small groups)<br><br>a. Examples from our students<br>b. Some tasks and claim about the tasks<br>c. What do we think about these?<br>d. What opportunities would students have to think and reason in each of the cases? How does this relate to CD?<br>e. What kind of feedback would we provide to these student teachers? Give an example<br>f. Do we think there would be any change if we consistently started giving this kind of feedback? | One of your student teachers plans a lesson with the features below:<br><br>Content: Operation with whole numbers (Hundreds)<br>Goal: Solving problems involving different operations with whole numbers (Hundreds)<br><br>For the main task, she poses the following problem "Beatrice has $953 (CLP). Her older sister borrowed $125 from her and her little brother borrowed $150 from her? How much money did they borrow altogether?"<br><br>As to how to use this problem in the lesson, she proposes having the students strongly focus on keywords. For this purpose, she proposes having the students read the problem and highlight the important numbers and words. Then, they must determine which operation to use and solve the problem.<br><br>Within the rationale section of the lesson plan, she claims that following these steps will help students understand the problem, organize the information and get the correct solution. |

***Table A.3:*** *Organization of scenarios and questions about tasks*

*Table A.3 (cont'd)*

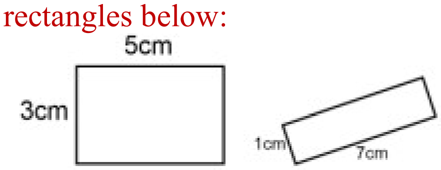
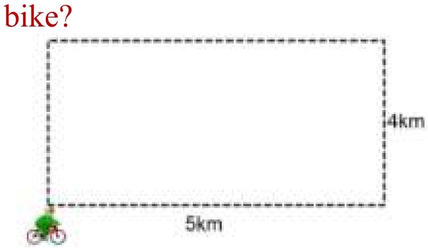
| **Pretest** | **Workshop** | **Posttest** |
|---|---|---|
| a. What do you think of this lesson plan?<br>b. What kind of Mathematics<br>is involved in the task?<br>c. What opportunities would students have to think and reason with this task?<br>d. What can you say about the<br>CD of the task?<br>e. What kind of feedback about the CD of the task would you provide and why? Provide examples | | a. What do you think of this lesson plan?<br>b. What kind of Mathematics is involved in the task?<br>c. What opportunities would students have to think and reason with this task?<br>d. What can you say about the CD of the task?<br>e. What kind of feedback about the CD of the task would you provide and why? Provide examples |
| Two of the student teachers you oversee are planning lessons about area of rectangles. Each one of them designs a main task as shown below:<br><br>**PST 1: Martha's Carpeting Task**<br>Martha was recarpeting her bedroom which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?<br><br>Rationale: In order to have the students reinforce the area formula for rectangles, I decided to use this task. I wanted to make sure that the problem could be finished within one lesson. Since the problem is framed within a real-world context, it will be meaningful for students. | Whole group:<br>What's the 'fate' of the lesson according to the CD of the task? QUASAR project<br>•Examples from our PSTs<br>•Comparison of two tasks, including rationale<br>•What do we think about these?<br>•What opportunities would students have to think and reason with different kinds of task?<br>• How does this relate to CD?<br>• What kind of feedback would we provide to these PSTs? Give examples<br>• Think about private feedback<br>• Think about public feedback.<br>• What is public feedback?<br>• How do we use it?<br>• How can we use it better? | Two of the student teachers you oversee are planning lessons about perimeter of rectangles. Each one of them designs a main task as shown below:<br>Task A<br>• Use the formula to deter-<br>mine the perimeter of the<br>rectangles below:<br><br>• James rides 5 km East on his bicycle. Then he goes North 4 km, West 5km and South 4km to arrive to the starting point again. How much distance did he cover on his bike?<br> |

*Table A.3 (cont'd)*

| Pretest | Workshop | Posttest |
|---|---|---|
| **PST 2: The Fencing Task** Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen in which to keep the rabbits.<br>a. If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?<br>b. How long would each of the sides of the pen be if they had only 16 feet of fencing?<br>c. How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.<br><br>Rationale: I chose this problem because it will allow to delve into two intertwined concepts: area and perimeter of rectangles. I expect the students to offer multiple ways of solving the problem. You must provide feedback before they enact the lesson plan.<br>• What opportunities would students have to think and reason with each task?<br>• What can we say about the CD of each of the tasks above?<br>• What kind of feedback would you provide to each one of the student teachers? What would you focus on?<br>Give examples | | You must provide feedback before they enact the lesson plan.<br>• What opportunities would students have to think and reason with each task?<br>• What can we say about the CD of each of the tasks above?<br>• What kind of feedback would you provide to each one of the student teachers? What would you focus on? Provide examples |

**Organization of scenarios and questions about goals**

| Pretest | Workshop | Posttest |
|---|---|---|
| During your weekly (whole group) meeting with the student teachers you oversee, you start talking about the lesson they need to teach in a few weeks. The content is area of rectangles. You ask them to draft possible goals for the lesson. They propose three possible goals. As they talk, you write them on the board:<br><br>**Goal A**: Students will examine several examples of rectangles, identify the length and width and learn the area formula for rectangles (A = L x W).<br>**Goal B:** Students will be able to identity length and width of rectangles in pictures and use the area formula for rectangles (A = L x W) to solve a series of missing value problems<br>**Goal C:** Students will recognize that a rectangle with length L and width W can be covered with L groups of W 1-unit-by-1-unit squares without gaps or overlaps. Thus, there are (L x W) square units within the rectangle<br>(Adapted from Smith & Stein, 2012) | Small groups:<br>Re-examine goals A, B and C from pretest:<br>a. What do we think about these?<br>b. How would you lead the discussion from here onwards? Provide examples<br>c. What would you do to make the mathematics explicit in the discussion?<br>d. What opportunities would students have to think and reason in each of the cases? How does this relate to CD?<br>e. What can we say about the CD of possible tasks for each goal?<br>f. What is the expected takeaway of possible tasks for PSTs with the discussion you proposed?<br>g. What kind of feedback would we provide? Provide examples<br><br>Whole group discussion:<br>Sharing what happened in small groups | During your weekly (whole group) meeting with the student teachers you oversee, you start talking about the lesson they need to teach in a few weeks. The content is perimeter of rectangles. You ask them to draft possible goals for the lesson. They propose three possible goals. As they talk, you write them on the board:<br><br>**Goal A:** Students will examine several examples of rectangles, identify all the sides, realize that opposites sides are congruent and learn the perimeter formula for rectangles (P = 2L + 2W)<br>**Goal B:** Students will be able to identify all sides of the rectangle, noting that opposite sides are congruent and use the perimeter formula for rectangles (P = 2L + 2W) to solve a series of missing value problems.<br>**Goal C:** Students will recognize that a rectangle with length L and width W can be covered with L groups of W 1-unit-by-1-unit squares without gaps or overlaps. Thus, L square sides cover the length and W square sides cover the width without gaps or overlaps. The total distance around the rectangle is 2L + 2W.<br>(Adapted from Smith & Stein, 2012) |

***Table A.4:** Organization of scenarios and questions about goals*

*Table A.4 (cont'd)*

| Pretest | Workshop | Posttest |
|---|---|---|
| a. How would you lead the discussion from here onwards? (main ideas to focus on)<br>b. What would you do to make the Math involved explicit in the discussion?<br>c. What opportunities would students have to think and reason with each goal?<br>d. What can we say about the CD of possible tasks for each goal?<br>e. What is the expected take-away for PSTs with the discussion you proposed?<br>f. What kind of feedback would you provide? Give an example | | a. How would you lead the discussion from here onwards? (main ideas to focus on)<br>b. What would you do to make the Math involved explicit in the discussion?<br>c. What opportunities would students have to think and reason with each goal?<br>d. What can we say about the CD of possible tasks for each goal?<br>e. What is the expected take-away for PSTs with the discussion you proposed?<br>f. What kind of feedback would you provide? Give an example |

**Organization of scenarios and questions about practices**

| Pretest | Workshop | Posttest |
|---|---|---|
| **Lesson plan**<br>**Goal:** Students will examine different rectangular configurations for a given perimeter, conjecture about what is the maximum possible area for that perimeter and find a way to test/support their conjecture<br>**Task:** The Fencing task (see above)<br>(...)<br>•Have them work in groups from the beginning.<br>• Monitor groups<br>• If they are struggling, mention that they must find the area of all rectangles with a perimeter of 24<br>• If more help needed, suggest they start with a 1x11 rectangle<br>• If needed, remind them to use the A=LxW formula<br>• In whole group discussion, if no one has the right answer, suggest to divide by 4 and test.<br><br>a. What opportunities did students have to think and rea-son?<br>b. What effects can teaching practices have on CD?<br>c. Where can we see declared teaching practices in a lesson plan?<br>d. How can we provide feedback on these declared practices to help (as much as possible) maintaining a high CD? Give examples. | Small groups:<br>• Re-examine plan<br><br>Whole group:<br>• QUASAR project<br>• The fate of tasks set up as doing mathematics<br>• The fate of tasks set up as procedures with connections<br>• Factors associated with the decline of high-level CD<br>• Factors associated with the maintenance of high-level CD<br><br>SO:<br>a. How does this relate to lesson planning?<br>b. What kind of elements of the lesson plan we need to pay attention to 'detect' potentially some of these teaching practices?<br>c. What kind of feedback could we provide to<br>a) Encourage potential practices that would help maintain a high CD? Give an example.<br>b) Call for a revision on potential practices that would cause a decline of CD? Give an example | **Lesson plan**<br>**Goal:** A preservice teacher plans to use the problem below in a lesson:<br><br>'Create a real-life problem that is solved with the division $1\frac{3}{4} \div \frac{1}{2}$.<br><br>Within the lesson plan, the following notes are included:<br>(…)<br>•Make them work in groups from the beginning.<br>• Monitor groups constantly<br>• If they struggle, remind them different toles of dividend and divisor in a division.<br>• If they are still struggling, ask them how many times a half 'fits' into one whole and three fourths.<br><br>a. What opportunities did students have to think and rea-son?<br>b. What effects can teaching practices have on CD?<br>c. Where can we see declared teaching practices in a lesson plan?<br>d. How can we provide feedback on these declared practices to help (as much as possible) maintaining a high CD? Give examples. |

***Table A.5:*** *Organization of scenarios and questions about practices*