ESSAYS ON INNOVATION, PRODUCT DEVELOPMENT AND STRATEGIC SUPPLY CHAIN MANAGEMENT - AN ANALYTICAL INVESTIGATION OF STRATEGIES FOR COMPETITION AND COLLABORATION

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By

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ABSTRACT

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In this dissertation, four essays are presented to investigate the strategic incentives of firms to make investments in innovation and quality. Differential game models are used to examine the impact of investments in innovation on competitive advantage and on supply chain channel power.

The first essay considers a simultaneous game between two competing firms. Decisions regarding the investments in innovation are made by the two firms at the same time. The second essay extends the competitive scenario in the first essay by considering a sequential game. These games are typically representative of leader-follower or incumbent-entrant type of competitive situation and involve information asymmetry.

In the third essay a competitive situation between a supplier and a buyer is modeled. The buyer is currently locked-in by the supplier and is assumed to be dependent on the supplier for carrying out product development activities successfully. To become independent, the buyer invests in creating a substitute technology. The supplier in turn knowing such a motive on the part of the buyer, strategically manages its production. The fourth essay considers a more general competitive scenario between a supplier and a buyer. The two firms make investments under uncertainty in innovation capabilities, which would allow them to increase channel power and consequently to lock-in the other.

Dedicated to my beloved wife *Anjali*
for her time, support, and encouragement.

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Chapter 1

Introduction

1.1 Introduction

In today's highly competitive environment, new challenges and opportunities are arising in the new product development arena. Driven by global markets, global competition, the global dispersion of engineering talent, and the advent of new information and communication technologies the new vision of product development is that of a highly disaggregated process with people and organizations spread throughout the world. At the same time products are becoming increasingly complex requiring close to a million engineering decisions to bring them to market (Eppinger et al. 1994, Eppinger 1998, Ulrich 2001).

Competitive pressures mean that time to market has become key to new product success. However, at the same time it is important to keep the innovation and quality dimensions of the new product at their optimal level. Moreover, firms operating in a collaborative network should be cognizant of the overall power structure and their own channel power. A real product advantage rooted in difficult-to-copy capabilities and a translation of that product advantage into a fundamental market franchise that reinforces its own momentum is a powerful combination (Wheelwright and Clark 1992). In this context, it is asserted that strategic investments in innovation and quality present unique competitive advantage.

This dissertation explores the area of product development, innovation and quality based competition and inter-organizational relationship in a supply chain. It is structured into four essays to investigate several interesting questions. The central question of this dissertation is, how should firms invest in innovation and what are the implications of such investments for competitive advantage? In a supply chain context, this research examines the relative motivation for supply chain partners to invest in innovation. The analysis provides insights regarding evolution of power structure in supply chain as an outcome of inter-organization dialectics (Zeitz 1980).

Cox et al.(2002) note that despite the undoubted significance of resource dependence perspectives in disciplines such as political science, social psychology, sociology and economics, it has played relatively minor role thus far in the supply chain management literature dealing with relationships between suppliers and buyers. The relative invisibility of power in this literature may be partly attributable to a focus by academics and practitioners on concepts such as lean supply, which implies that firms should be more open, trusting and collaborative. While such notions might seem ethically appealing it is contended that they are not based upon a sound understanding of what is actually possible, or desirable in buyer-supplier relationships.

Understanding why a firm benefits from investments in innovation and quality illuminates issues of competitive strategy and industrial organization. In the field of competitive strategy, much attention has been devoted to the concept of core capabilities (Teece et al. 1997). Understanding how firms make optimal investments in the face of competition reveals the nature of competition and provides theoretical and managerial implications for developing core competence and dynamic capabilities.

This chapter is divided into five sections. The introduction completes the first section. In the second section, the gaps found in existing literature are highlighted. The third section presents the conceptual framework for this study. The fourth section presents the research methodology used in this dissertation and finally in the fifth section an outline of the chapters is provided.

1.2 Gaps in existing literature

The literature pertaining to each essay is provided in the specific chapter in which it is discussed. This section highlights the specific gaps found in literature that is addressed by means of this thesis.

It has been noted that typically in competitive analysis involving R&D decisions, the focus is on breakthrough innovations. These studies consider patent race between competing firms. However, it is very common to observe firms competing by investing in incremental improvements in products. Recently some researchers have explored this form of competition (Cohen et al. 2000, Bayus et al. 1997) in the extant literature. These studies however do not consider active investment by firms after the launch of the product. This is an important aspect when innovation is considered to be manifested in the product quality, process improvements and in overall quality culture in an organization. Post launch incremental improvement of different aspects of product quality, improvements in various business processes and an incremental adoption of quality culture is quite evident in real-world. This thesis explicitly considers these long term innovation strategies that span the entire life cycle of a product.

Many firms operate in a simultaneous product launch situation while others compete by adopting the role of leader or follower. The strategic implications in these diverse circumstances are quite different. The investigation of these structures of competition in a single framework is presently lacking in the extant literature. In this dissertation a model is explored in which firms compete in a simultaneous entry situation. This model is further extended to extract implications for investments when this competitive game is of a sequential entry type.

In recent years with the emergence of e-business and a supply chain view for product development process, multiple firms with varying and at times conflicting objectives enter into collaborative arrangements. In such situations, the competitive strategy based on quality and innovation could potentially permeate into these collaborative setups. The existing literature doesn't explicitly consider the implications of innovation based competitive strategies in a collaborative supply chain context. It can be noted that when innovation and quality levels form the core of a firm's capabilities, each member in the supply chain would have an incentive to invest and improve their dynamic capabilities. This leads to tacit competition among collaborative product development partners by means of active investment in innovation and quality. This issue is treated in literature at a conceptual level and a formal theoretical grounding is lacking. This dissertation fills this gap by presenting models of innovation based competition between a buyer and a supplier.

1.3 Conceptual Framework

The objective of this dissertation is to address the gaps mentioned in the previous section. The issue of innovation in a product development context is addressed and the strategic importance of innovation and quality is highlighted. This thesis provides insights into the dynamics between competing firms as well as the competitive dynamics between collaborating firms when quality and innovation are considered to be the strategic levers.

The product development literature is quite broad encompassing areas in marketing, operations and engineering. It has been noted that while how products are developed differs not only across firms but also within the same firm over time, what decisions are being made seems to remain fairly consistent at a certain level of abstraction (Krishnan and Ulrich 2001). Different organizations will make different choices and may use different methods, but all of them make decisions about a collection of issues such as the product strategy and planning, product concept, innovation investments, configuration, total quality management, procurement and distribution arrangements and relationships. Among these decisions, this research explores investments in innovation and quality and strategic implications of these investments on buyer-supplier relationships.

It has been recognized that although the forces of innovation are central to competition in young, technically dynamic industries, they also affect mature industries where life cycles historically were relatively long, technologies mature, and demands stable (Wheelwright, Clark 1992). A strategy for technology must confront, in the first instance, what the focus of technical development will be. The question is what technologies are critical to the firm's competitive advantage. In this context, technology must include the know-how the firm needs to create, produce, and market its products and deliver them to customers. As a first step in creating a technology strategy, 'focus' defines those capabilities where the firm seeks to achieve a distinctive advantage relative to competitors. For most firms, there are a large number of important areas of technological know-how but only a handful where the firm will seek to create truly superior capability.

Establishing 'focus' defines targets for investment in technical capabilities, but leaves open the question of source. This is the second critical aspect of a technology strategy. Technological capability may be developed internally through investment in people, equipment, facilities, and methodologies, or through advanced development projects. But technology may also be acquired from outside the firm. Specifically in a supply chain context, a firm can enter into a partnership with suppliers in the development of advanced process and product technology. Thus the key questions the technology strategy must answer about sources are: (1) What roles will external and internal sources play, and (2) How will they be integrated? (Wheelwright, Clark 1992)

Having determined the focus of technical development and the source of capability, the firm must establish the timing and frequency for innovation efforts. Part of the timing issue involves developing technical capabilities, and the rest involves introducing technology into the market. The frequency of implementation and associated risks will depend in part on the nature of the technology and the markets involved (e.g. disk drive vs. automotive technology), but in part on strategic choice. At the extreme, a firm may adopt what has been called the rapid inch-up strategy - frequent, small changes in technology that cumulatively lead to continuous performance improvement. The polar opposite is what might be called the great leap forward strategy. In this approach, a firm chooses to make infrequent but large-scale changes in technology that substantially advance the state of the art. In essay 1 and 2, where innovation strategies between competing firms are examined, the "rapid inch-up" form of innovation strategy is considered. The third and fourth essay considers active investments in innovation to investigate power structures in supply chain.

As an example of the importance of innovation strategy in product development, it can be noticed that IBM created and continues to dominate the mainframe segment, but it missed by many years the emergence of the minicomputer architecture and market. The minicomputer was developed, and its market applications exploited by firms such as Digital Equipment and Data General. Many R&D programs involve the exploration of several possible alternatives to develop a single product. For example, an auto manufacturer may develop several prototypes for a new car design; or a communications company may investigate several techniques to develop a new microwave relay system. In 1994, Ford Motor Company announced a complete change that will allow it to build the vehicles it sells to North America and Europe off common platforms. Similarly, for the new Accord, Honda developed discrete new vehicles for America, Asia, Europe and Japan using flexible platforms and rationalized components.

It is asserted that innovation efforts are driven by investments, which in turn are manifested in terms of product and process quality and in the overall quality culture in the organization. Considering quality from a holistic perspective leads one to think of quality in terms of: performance, features, reliability, conformance, durability, serviceability, aesthetics and perceived quality (Garvin 1988). Moreover, when viewed from an organizational context it includes aspects of total quality management like quality leadership and quality culture. For analytical investigation in this dissertation, a composite measure of quality is considered to encompass these aspects.

In the field of industrial organization and technological innovation there is a rich literature on the make-buy decision and on vertical integration (Armour and Teece 1980, Fine and Whitney 1996, Langlois and Robertson 1989, Masten 1984). Much of the work since the 1970s has been grounded in transaction cost economics and in the concept of asset specificity. The central element of this theory is that asset specificity gives rise to vertical integration because of the threat of opportunistic behavior on the part of at least one of a pair of interdependent firms (Williamson 1985). One of the most significant specific asset in manufacturing is product-specific component designs and the associated production tooling. A better understanding of the conditions under which innovation investments are important sheds light on the question of why asset specificity may exist in a particular situation, thus revealing a driver of an industrial organization variable previously treated as exogenous (Ulrich and Ellison 1999). Moreover, it would aid in gaining insight regarding strategic behavior among the supply chain partners to achieve advantages of supplier and customer lock-ins.

There are many benefits of outsourcing the design (innovation) decisions to supply chain partners viz. minimizing investments, exploiting the benefits of economies of scale of the supplier and maintaining organizational focus. On the other hand, the motivation to invest in innovation and quality considerations by a firm is to diffuse the technology in the market by satisfying holistic customer requirements and to achieve total quality improvements. This in turn enables customer lock-ins. This gain translates into lock-ins of supplier who would deliver the components for the product because of associated financial benefits from collaboration. This inherent complexity in the underlying dynamics of a collaborative supply chain presently lacks a firm theoretical grounding. Specifically, in a supply chain context, the three important strategic questions (i) who among the supply chain partners would invest in innovation (ii) how would the collaborating partners invest and, (ii) what are the conditions under which a firm would invest for structural and competitive advantage, remains largely unattended.

Based on the understanding from relevant research literature, a conceptual framework is developed. This research is an analytical investigation. The development of conceptual framework however provides a framework for interpretation of the key constructs used in the study. To incorporate the aspects of product and process innovation we follow the conceptualization as given by Khanna and Iansiti (1997). The conceptualization based on empirical investigation of NPD(New Product Development) projects, suggests that the effort proceeded in two basic and distinct stages. These stages were typically demarcated by the creation of a detailed technical specification document. The first stage, which is defined as research was aimed at the exploration and integration of the new techniques necessary to produce a product with the targeted specifications. During this stage, researchers explored new possibilities, and experimented with stretching old capabilities. By the end of this stage, project members developed a firm approach to the delivery of the performance targets described at the beginning of the project. The second stage, termed as development, was aimed at the development of a reliable process. This stage involved considerable refinement of the techniques and capabilities chosen in the research stage. The objective here was to find a specific production process that would maximize production yield, and minimize product cost. This conceptualization provides motivation to consider specific functional forms for different parameters and variables in the model.

Following standard practice (Lancaster 1966 , Wilkie and Pessemier 1973 , Rosen 1974), a product and process are defined by a vector of attributes. As stated earlier, innovation is conceptualized in this thesis as being manifested in terms of quality. Moreover, today it is very important for firms to pursue continuous total quality improvement for the entire product life cycle. This is achieved by making investments in total quality management to propagate quality culture within the organization. The investments in quality and innovation efforts get translated into a firm's "know-how". It is asserted that this learning reinforces further investments in quality and innovation by means of increased revenue and profits. The conceptual framework is presented in figure 1.1.

1.4 Research Methodology

This dissertation presents four essays and each essay uses a specific form of differential game approach to address the issue of innovation based competition. In a differential game each player maximizes, his/her objective functional subject to a number of constraints, which include, in particular, a differential equation describing the evolution of the state of the game. Optimization problems of this type are known as optimal control problems and are widely used in economic theory and management science. The following paragraphs provide information regarding the methodology. The required analytical details are provided in appendix.

In the first essay a simultaneous game between competing firms is considered. Each firm's choice influences the evolution of the state of the game via a differential equation (the system dynamics) as well as the objective functional of the competing firms. The important assumptions for the first essays are that the players make their choices simultaneously and that they represent the solutions to their control problems by Nash equilibrium strategies. In this essay, open-loop Nash equilibrium conditions are derived.

In the second essay, a class of differential game in which some firms have priority of moves over others is considered. The firm that has the right to move first is called the leader and the other competing firm is called the follower. A well-known example of this type of hierarchical-moves game is the Stackelberg model of duopoly. The open-loop Nash equilibrium conditions in a sequential move game is derived and the results are analyzed. A comparison of the strategies of leader and the follower is provided.

In the third essay a stochastic differential game is considered. The class of stochastic differential game used are commonly referred to as piecewise deterministic differential games. A piecewise deterministic process is a system which evolves in a deterministic way, except at certain jump times, at which the deterministic law of motion switches from one mode to another. Both the jump times and the system modes which govern the motion between jump times are randomly selected.

Finally, the fourth essay uses stochastic differential game with white noise. In these games uncertainty enters, not in the form of a piecewise deterministic process, but in the form of a Wiener process. Wiener processes are also known as Brownian motion or white noise processes and play an important role in different fields.

1.5 Outline

The dissertation is divided into six chapters. The overall underlying theme of the thesis is innovation and quality investments and its strategic consequence on competition and collaboration. The research is structured in the form of four essays characterized by different problem contexts and research questions. The first and second essay use a differential game based approach to investigate the nature of strategy between two competing firms while, third and fourth essays are targeted to examine strategic approach adopted by collaborating firms.

The introduction completes the first chapter. In the second chapter, the simultaneous differential game of first essay is presented. This essay examines equilibrium results for investments in innovation for a competitive setup in which two firms are simultaneously launching the new product. The implications of firm asymmetries and various parameter values are illustrated and discussed. It is conjectured that the Nash equilibrium investment is a function of time and peaks at the date of launch.

The mathematical background pertaining to this essay and the proofs for theorems presented in the essay are provided in Appendix A.

The third chapter covers the sequential differential game used for second essay. The second essay extends the framework in essay 1 by considering a sequential game. These games are characterized by a leader and follower based on the time they initiate their innovation efforts and launch the product in market. The open-loop Nash equilibrium results for this sequential game are presented and implications of firm asymmetries are discussed. The mathematical background pertaining to this essay and the proofs for theorems presented in the essay are provided in Appendix B.

Chapter 4 presents the third essay in which a stochastic differential game model between competing supplier and buyer is considered. A model is formulated in which the buyer actively invests in substitute technology to come out of the lock-in created by supplier. Stationary Markov-perfect Nash equilibrium conditions are derived and several insights are detailed. The mathematical background pertaining to this essay and the proofs for theorems presented in the essay are provided in Appendix C.

The fifth chapter examines the fourth essay that uses stochastic differential game with white noise. In this essay both discounted payoff maximization and utility maximization objectives are examined. The discounted payoff game considers the competitive setting between supplier and buyer akin to those in patent race based models. In this case, one of the collaborative partners locks-in the other and gains the overall surplus. The utility maximization based game investigates the case in which the competition doesn't necessarily result in a zero-sum game. Instead, in this case, it is hypothesized that the two collaborating partners appropriate utility from the relationship. The buyer and supplier compete to maximize their respective utility in the supply chain relationship. The mathematical background pertaining to this essay and the proofs for theorems and propositions presented in the essay are provided in Appendix D.

Finally, the sixth chapter concludes this thesis. The theoretical and managerial implications of the results from the four essays are summarized. The chapter also presents directions for future research.

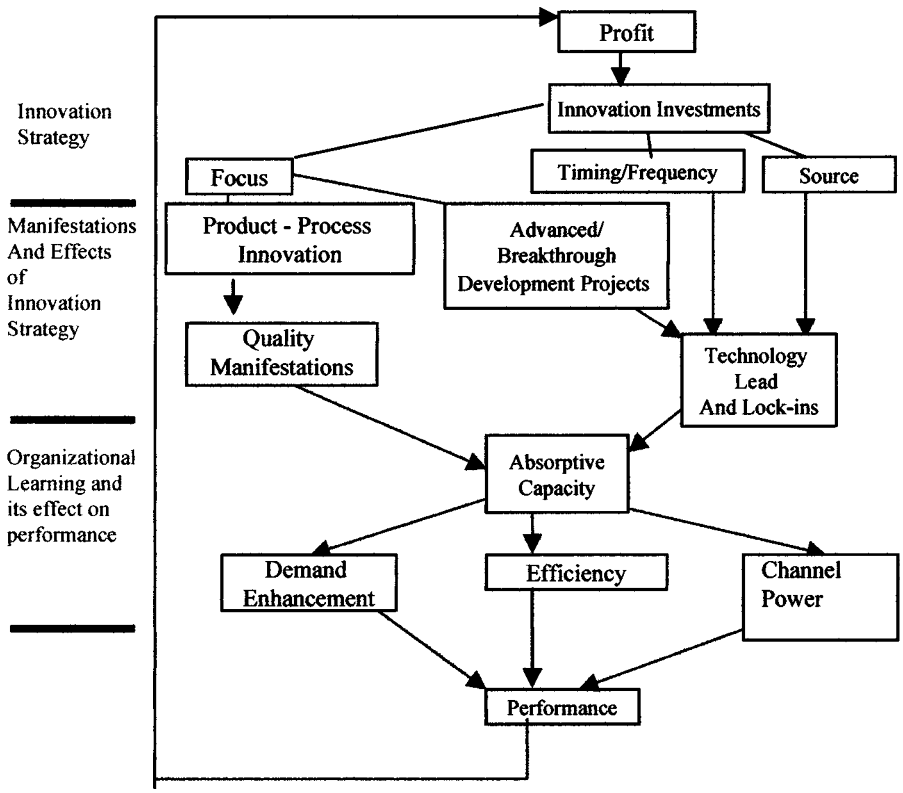

Figure 1.1: Conceptual Framework

Chapter 2

Simultaneous Differential Game Between Two Competing Firms

2.1 Introduction

Product-process innovation is increasingly becoming the core strategy of many leading firms. Firms compete by introducing new products in the market. The profitability depends on the initiation and launch of a new product and the quality levels attained through the development process. The emphasis on technology-driven product differentiation is a function of exogenously defined technological change. The decisions made by firms are to a great extent influenced by the internal and external environment. This essay explores the internal drivers for product-process innovation. Specifically, the investments made in innovation efforts are critically examined.

In line with the assertion for reduced time to market as proposed by Smith and Reinertsen (1991), many leading firms such as General Electric, Hewlett Packard adopt time-to-market as their principal product development metric. At the same time a competing line of thought by Zirger and Maidique (1990), Cooper and Kleinschmidt (1987) provide evidence for the importance of quality of a new product. Boeing, BMW, Mercedes Benz, for instance, use innovation and quality as their key metric for new product success. Fortuna (1990) has suggested that new product performance is often the decisive factor in the purchase of technologically advanced products. Underlying the need to reduce the speed to market and improve product quality, is the associated cost. It is evident that there exists an inherent tradeoff in the adoption of one perspective over the other.

Today product-based competition is not limited to investments in product design before launch. Firms compete by adopting a continuous improvement program. Post-launch, firms keep investing in product improvement for example improvements in feature, fit and finish; in process innovation and in development of quality culture to sustain competitive advantage for the entire product life cycle. It is asserted that an investigation of such a context of competition would indeed aid our understanding of the dynamics of investment and associated quality improvement.

There are persuasive reasons for firms to consider the evolution of quality in their product-process innovation decisions. During the 1960s and 1970s many domestic U.S. companies lost their quality leadership to new, aggressive competition. The most obvious consequence was loss of market share. Moreover, there was a gathering awareness by companies that they have been enduring excessive costs due to chronic quality-related wastes. In a context where competition is primarily dictated by quality levels, it is of utmost importance to consider aspects of quality planning and associated investments as an integral part of product management. Numerous specific quality crises and problems have been traced to the way in which quality was planned in the first place. Furthermore, competing with quality requires a clear understanding of how to manage investments in the long-run with a strong strategic mindset.

This essay explores competitive strategy of firms competing by means of quality. A differential game model is considered in which two firms compete by simultaneously introducing a new product into the market. The time of product launch is same. The two firms compete by dynamically managing the interaction between their relative product quality and cost. The game-theoretic model is solved for open-loop Nash equilibrium and the implications and insights from these equilibrium strategies is discussed.

The chapter is divided into five sections. The introduction completes the first section. In the second section, related literature is reviewed. In the third section the model is presented, the structure of open-loop Nash equilibrium strategies and their implications are analyzed in the fourth section. The fifth section concludes the essay with discussion of the results. The mathematical preliminaries and proofs of theorems are provided in Appendix A.

2.2 Literature Review

Many previous studies have investigated product and process innovation, time to market and the quality implications of product development. A detailed review of literature in this area can be found in Bayus (1995), Kamien and Schwartz (1982), and Reinganum(1989). The related literature most closely related to this essay is summarized below.

Research in the field of innovation has taken two approaches. The first one relates to breakthrough innovation. The analytical models in this approach usually consider a technology race for patents. The trade-offs between time to market , total resources spent and competition on the supply of new technology are defining characteristics of this line of enquiry. See for example, Reinganum (1982), Fudenberg and Tirole (1984), Tirole (1990), and Hendricks (1992). The model considered in this essay differs from this approach in that it considers how firms invest in product development once the technological breakthrough has already taken place.

Clark and Fujimoto (1991) cite many problems with this approach of product development: difficulty in designing for simplicity and reliability, failure to pay enough attention at the design stage to the likely quality of the manufactured product, excessive development times, weak design for producibility, inadequate attention to customers, weak links with suppliers, and neglect of continuous improvement. Technological progrss rests on a foundation of both incremental improvements and radical breakthroughs, and finding the right balance between them is a constant challenge. The traditional approach relied excessively on the radical breakthrough, with companies attempting to leverage the strength they have in creative research and advanced development, but balance is required. Frequently, developing several products with incremental improvements will be a better approach than trying to create one product that represent a breakthrough (Clausing, 1994).

Other studies have looked at the technology diffusion process. For example, Chatterjee and Eliashberg (1990) investigate the innovation diffusion among the customers and Reinganum (1981), McCardle (1985) investigate the diffusion of innovation within an organization. Adner and Levinthal (2001) consider an explicit interaction between technology change and demand. This essay differs from this stream of research in that the effect of innovation on demand is not explicitly modeled. However, the quality manifestations of investments in innovation influence the terminal revenue function. The influence on demand is therefore implicit in the conceptualization.

The models of technology breakthrough discussed above primarily consider a single innovation. However, it can be asserted that in reality the breakthrough innovation is accomplished by many incremental innovations. The repeated-innovation strand of the R&D literature focuses on this modification of the traditional breakthrough innovation models. Some examples of repeated innovation models can be found in Grossman and Helpman (1991a, 1991b), Aghion and Howitt (1992), and Sergerstrom et al. (1991). In line with these models of repeated innovation, the model in this chapter focuses on continuous product and process innovation over a period of time. The current state of technology is assumed to be exogenously given to all the competing firms. The focus of this chapter is to investigate how the innovation investments evolve over time and what are the implications on the dynamics of product quality.

Several product development models have extended this stream of research in innovation by presenting the argument that after the breakthrough innovation is achieved, this new technology must be further improved and consciously incorporated in the new product before it can be brought into the market. The central issues are therefore the speed to market, level of product quality and the cost associated with the product development endeavor. The tradeoffs associated with these objectives have been taken into account in many studies. The quality of a product can be improved by increasing the development time of the product. However, this might result in overly delaying the new product into the market and thereby losing the profits for that period or even worse being locked-out of the market. Examples of these models can be found in Dutta et al. (1995), Reinganum (1982), Cohen et al. (1996a, 1996b), Bayus et al. (1997), Bayus(1998) and Gjerde et al. (2002). The model in this chapter builds on these product development models. Specifically, this essay explicitly evaluates new product development based duopolistic competition.

Clausing(1994) integrates the product development aspects with quality improvements. The approach, aptly termed as - Total Quality Development - views specific product development in the context of the total corporate strategy, which determines when the development of a product will start and when, after the product has been produced, its life cycle will be terminated through withdrawal. Total quality development is conceptualized to be composed of three major elements: Basic concurrent engineering (basic improvements in clarity and unity), enhanced quality function deployment and quality engineering using robust design. *Basic concurrent engineering* was implemented by many American companies since 1980s. It consists of two elements: (1) improved process(better game plan), which provides greater clarity to the activities, and (2) closer cooperation (better teamwork), which creates greater unity within the team that does the work. The improved process that provides improved quality has four features - (1) concurrent process, (2) focus on quality, cost, and delivery, (3) emphasis on customer satisfaction, and (4) emphasis on competitive benchmarking. The closer cooperation that improves unity consists of (1) integrated organization, (2) employee involvement (participative management), and (3) strategic relations with suppliers. Beyond basic concurrent engineering, enhanced quality function deployment and quality engineering using robust design emphasizes the satisfaction of customer needs and the consistency of the product's performance. Together they help the teams to practice more vigilant information processing that provides strong responsiveness to the voice of the customer and helps ensure the viability of the core concepts, the robustness of functional quality, the economical precision of production, the success of integration, and effective reusability. Such a total quality development framework is considered such that firms invest in "quality culture" throughout the growth phase of the product. The competing firms influence the quality of their new product by investing in innovation and other quality-related activities such as employee training, six sigma, statistical process control and quality leadership. Firm asymmetries are considered by investigating different parameter values and their implications on the nature of equilibrium strategies.

2.3 The Model

2.3.1 Differential Game Formulation

The notations and symbols used in the model are as follows:

N: Number of firms

T: Finite time horizon for the strategies

i, j: Superscript to denote competing firms in a duopoly

t: An instant of time in the dynamic game setup

$u(t)$: Investments in innovation effort (expenditure per unit time)

$R(t)$: Net revenue rate for the firm at t

R_0: Product category net revenue rate for the existing product

R_1: Product category net revenue rate for the new product

x_0: Quality level of the existing product

Several examples illustrate the importance of innovation and quality as competitive weapons. IBM replaced Remington Rand as a market leader in the mid 1950s, and its subsequent growthoutran its competitors so rapidly that by 1963, its data processing revenues were four times larger than the combined revenues of its eight main rivals in the U.S. market. In part, its advanve was due to high R&D spendings and innovative products. Its early 704 computer was the fastest and largest general purpose computer yet produced during 1960s. It was one of the first computers to incorporate in its hardware floating-point arithmetic, and it carried advanced features, such as index register and magnetic-core memories (Flamm 1988 pp.84; Bashe et al. 1986). But high R&D spending and innovative success were only a limited part of the story. The links from R&D spending to innovative success and from innovative success to market share, were far from strong throughout this period (Hoffman 1976). Systems that were outstanding success as in terms of technical performance could fail to achieve more than a tiny market share (as was the case with Burroughs B5000 or Bull's Gamma 60), whereas some of the greatest successin terms of market share were no more than average in terms of technical performance (example include the Honeywell H200; see Hoffman 1976, pp.347-8). What consumers wanted was a "good" level of technical performance, a reasonable price relative to performance, and excellent backup in terms of servicing, software and general support. This essay considers such holistic approach taken by firms by considering investments made in "quality culture" as explained next.

A product is the output of any process. The quality aspects associated with a product have two perspectives. One is the "external" quality dimension which covers aspects that a customer can directly observe. Specifically, these includes performance, features, durability etc. The second dimension is "internal" quality dimension. This dimension ensures freedom from deficiency in a product. A customer evidently expects high levels on both of these dimensions of quality.

As in Juran(1992), this external dimension of quality increases customer satisfaction and makes a product more saleable. This dimension directly influences competition and increases market share, provide sales income and secure premium price for the product. The major effect of this quality is on sales and it is conjectured that usually, higher external dimension of quality costs more. The internal dimension on the other hand reduces error rates, reduces rework, waste, field failures and warranty charges. This results in reduction of customer dissatisfaction. The internal dimension of quality results in reduced inspection and enables shortened time to market. The yields and capacity is improved and so is the delivery performance. It is suggested in Juran (1992) that the major effect is on costs and usually, higher internal dimension of quality costs less.

In this essay quality connotes both of these dimensions. It is asserted that firms actively competing with quality invest continuously in product-process innovation for the entire product life cycle. The emphasis of quality dimension might be changing with different phases of product life cycle but investments are always targeted towards further improvements. Especially, the distinction between 'Big Q' and 'Little Q' provided in Juran (1992) (pp.11-12, Figure 1-5) illustrates the two views regarding quality. The figure is reproduced in figure 2.10 at the end of this chapter for explanatory purposes. As can be seen in figure 2.10, "Big Q" represents the broader quality culture within an organization; whereas "Little Q" has a restricted meaning confined to functional boundaries. In this essay quality covers aspects stated under Big Q.

The formulation in this essays extends the model in Cohen, Eliashberg and Ho(1996a,1996b, 2000). Specifically it is considered that firms compete by means of continuous improvement of quality for the entire product growth phase. This is achieved by means of increasing the level of development resources in product-process innovation and in total quality management. The level of development resources is measured in dollars. It is a strategic development decision. Enhancements in quality are achieved by climbing a "performance ladder." Let the quality of the product at time t be $x(t)$.

In addition to the Cobb-Douglas function used in Cohen et al.(2000), in this essay the performance ladder is also explicitly dependent on the present state of quality $x(t)$. In the context of total quality management, it is asserted that as quality levels increase it becomes even more difficult to climb the "performance ladder". The hypothesis behind the formulation is that a firm need to make higher innovation investments targeted towards improvement of quality. To capture this dynamics a negative feedback effect of present state of quality on the rate of change of product quality ($\dot{x}(t)$) is considered. The state dynamics is,

$$\dot{x}(t) = K[u(t)]^{\alpha} - Lx(t) \tag{2.1}$$

where, K is proportional to the level of capital investment in development technology and L is the proportionality constant for the influence of present quality on the speed of further quality improvements. α is the innovation resource productivity parameter.

This state dynamics is an extension of the model presented in Cohen et al.(2000). In the paper Cohen et al.(2000) investigate investments in innovation made by a firm till product launch. This essay extends the conceptualization by considering continuous investment in product, processes and quality culture. Notice that according to the formulation, there is a natural decay in the rate of quality improvement when a firm stops its continuous improvement program. In firms that are engaged in continuous quality improvement initiatives, quality is front and center throughout the company; i.e. the importance of quality is "visible" throughout the organization all the time. Quality improvement is a very important activity within the organization. In such a business philosophy, "investments" in improvement of quality sends a visible *signal* to the entire organization regarding its continuous quality improvement endeavor. These investments are primarily focused on product and process innovation but also extend to aspects like training, managing for quality and shop floor control of quality (internal failure costs, external failure costs, waste reduction, six sigma, statistical process control). Such an approach towards improvement propogates "quality culture". The state dynamics suggests that in the absence of investment in innovation and quality activities, the quality culture which is manifested in product quality starts deteriorating. Such decay effects have been considered in extant literature pertaining to investments in innovation (Grilliches 1979). In the words of Grilliches(1979; pp.101), "if one distinguishes between the firm-specific knowledge capital and the general state of knowledge in the industry as a whole, then at least as far as the first is concerned, it is quite clear that its earning capacity erodes over time, both because better products and processes become available and because its own knowledge begins to lose its specificity." Garvin(1988) observes that superior quality is associated with well-defined management practices and not simply a supportive corporate culture. Design, purchasing, and manufacturing activities all play a role, but they must be accompanied by the right policies and attitudes. In the problem context, the competing firms invest in management practices involving product, processes and quality culture for a planning horizon that spans beyond product launch. Based on (2.1), the quality of the product at time t is:

$$x(t) = x_0 + \int_0^t [K[u(s)]^\alpha - Lx(s)]ds \tag{2.2}$$

The two firms in the duopoly pre-commits the date of launch and simultaneously launch the new product. Post-launch, the product quality provides a means for evaluating the product's attractiveness in the market in the presence of other competing products. The firm's market share is a function of both its own product quality and the product quality of rivals. Attraction model is a reasonable market share function, frequently used in the marketing literature and has received empirical support (Bell, Keeney, and Little, 1975; Cooper and Nakanishi, 1988). The net revenue rate at time

t for the firm i that develops and introduces the new product is, the product of the new product category demand rate, the profit margin, and the firm's market share. The market share is approximated by the relative product quality. The objective functional is modeled similar to that in Cohen et al.(2000). The difference lies in the planning horizon. In this competitive setup, the planning horizon extends beyond the date of launch and the competing firms continue investing in product-process innovation till the end of growth phase of a product.

$$R(t) = \begin{cases} R_0 \cdot \frac{x_0}{x_0^i + x_0^j}, & 0 \leq t < T_P, \\ R_1 \cdot \frac{x^i(T)}{x^i(T) + x^j(T)}, & T_P \leq t < T, \end{cases} \tag{2.3}$$

The cumulative development cost of the new product at time t is given as:

$$TC(t) = \int_0^T \{u(s)\} ds \tag{2.4}$$

The firm's cumulative profit at time t is determined as follows,

$$T\Pi(t) = TR(t) - TC(t), \tag{2.5}$$

where $TR(t)$ and $TC(T)$ are total revenues and costs at time t, respectively. The total revenue function is given by:

$$TR(t) = \int_0^T R(s) ds \tag{2.6}$$

where $R(.)$ is given in (2.3). The firm's decision set is $\Delta = \{u(t)\}$. Notice that the firms pre-commit on the date of product launch T_p. The cumulative profit function, $T\Pi(\delta)$, is defined as the total profit by end of the window of opportunity with decision

$\delta \in \Delta$. The firm's decision problem can be stated as,

$$\max_{\delta \in \Delta} T\Pi(\delta) = TR(\delta^*) - TC(\delta^*) = T\Pi^*(\delta^*) \tag{2.7}$$

The combination of equations (2.1) through (2.6) generates an explicit representation of firm i's cumulative profit by the end of time horizon. This substitution yields:

$$\begin{aligned} T\Pi^*(\delta^*) = max_{\delta\in\Delta}\Big[& R_0 \cdot \frac{x_0}{x_0^i + x_0^j} \cdot T_P + \\ & + R_1 \cdot \frac{x_0 + \int_0^T K_1[u(s)]^\alpha - Lx(s)ds}{x_0 + \int_0^T K[u(s)]^\alpha - Lx(s)ds + x^j(T)} \cdot (T - T_P) - \\ & - \int_0^T \{u(s)\}ds\Big] \end{aligned} \tag{2.8}$$

where x_0^j and $x^j(T)$ respectively are the quality of existing and new product of the competitor. Considering a duopoly, the optimization problem written above can be reformulated as a differential game problem with state variable for the firm i (competing firm is represented by superscript j) given as $x^i(t)$; the control variable $u^i(t)$. In the terminology used in optimal control and differential games, the salvage term for firm i, $\Phi^i(T, x(T))$ is defined as follows:

$$\begin{aligned} \Phi^i(T, x(T)) \triangleq & R_0 \cdot \frac{x_0^i}{x_0^i + x_0^j} \cdot T_P^i + \\ & + R_1 \cdot \frac{\{x_0^i + \int_0^T K_1[u^i(s)]^{\alpha^i} - L_1 x^i(s)ds\}(T - T_p)}{x_0^i + \int_0^T K_1[u^i(s)]^{\alpha^i} - L_1 x^i(s)ds + x^j(T)}. \end{aligned} \tag{2.9}$$

where,

$$x^j(T) = x_0^j + \int_0^T [K_2[u^j(s)]^{\alpha^i} - L_2 x^j(s)]ds \tag{2.10}$$

Notice that since it is a simultaneous game the time of launch for the two players is the same (T_p). A continuous improvement in the product is considered and therefore the entire time interval $t \in [0, T]$ needs to be optimized. The salvage value is the value of the game at terminal time T. The differential game formulation for player i is:

$$\max T\Pi^i(u^i(t)) = -\int_0^T \{u^i(s)\}ds + \Phi^i(T, x^i(T)) \tag{2.11}$$

subject to,

$$\dot{x^i}(t) = K_1[u^i(t)]^\alpha - L_1 x^i(t) \tag{2.12}$$

$$x^i(0) = x_0^i, T \text{are fixed,} \tag{2.13}$$

$$x^i(T), \text{ is free.} \tag{2.14}$$

The game formulation is symmetric for the two firms in duopoly, therefore the differential game formulation for player j is:

$$\max T\Pi^j(u^j(t), T_p) = -\int_0^T \{u^j(s)\}ds + \Phi^j(T, x^j(T)) \tag{2.15}$$

subject to,

$$\dot{x^j}(t) = K_2[u^j(t)]^\alpha - L_2 x^j(t) \tag{2.16}$$

$$x^j(0) = x_0^j, T \text{are fixed,} \tag{2.17}$$

$$x^j(T), \text{ is free.} \tag{2.18}$$

2.4 Analysis of the Model

In this essay the open-loop Nash equilibrium conditions for the differential game stated in the previous section are investigated. The rationale for the evaluation of open-loop Nash equilibrium is the relative analytical tractability. However, more importantly, as reasoned in Reinganum (1981) and Bayus et al.(1997), it is assumed that each firm must precommit itself to an introduction date and product performance level.

This assumption is reasonable because of the long and involved nature of the new product development process. The implication is that once the development project is underway any changes or adjustments are prohibitively costly. Moreover, the improvement in product performance which encompasses the innovation and quality attributes is a complex phenomenon. An objective measure of such a product performance for decision making is very difficult, if not impossible. The open-loop Nash equilibrium allows for the equilibrium control path to be dependent only on time. The nature of state trajectory is embedded in the solution and therefore doesn't require to be directly observed.

To solve the differential game, Pontryagin's maximum principle is used. Interested readers can refer Sethi and Thomson (2000), Dockner et al.(2000) for details regarding the Pontryagin's approach to solving optimal control and differential game problems. The following theorems present the analytical results obtained by solving the model. The proofs of all the theorems are provided in Appendix A.

2.4.1 Equilibrium Results

Theorem 1 *The maximized costate variables for a firm is a function of time and are given by,*

$$\lambda_1^* = \lambda_{01}e^{L_1 t}; \quad \lambda_1(0) = \lambda_{01} \textit{is a known positive constant} \tag{2.19}$$

$$\lambda_2^* = \lambda_{02}e^{L_1 t}; \quad \lambda_2(0) = \lambda_{02} \textit{is a known negative constant} \tag{2.20}$$

where λ_1^* and λ_2^* are the costate variables reflecting the marginal price for a unit increase in firm's own state and that of the state of the competitor.

where λ_1^* is the costate variable reflecting the marginal price for a unit increase in firm's own state.

Theorem 2 *The Nash equilibrium investment in product development is given as:*

$$u^i(t)^* = [K_1 \alpha^i \lambda_{01} e^{L_1 t}]^{\frac{1}{1-\alpha^i}} \tag{2.21}$$

$$u^j(t)^* = [K_2 \alpha^j \psi_{01} e^{L_2 t}]^{\frac{1}{1-\alpha^j}} \tag{2.22}$$

where $\psi_1(0) = \psi_{01}$ *is a known constant.*

Theorem 3 *The equilibrium state trajectory of quality improvement is given as:*

$$x^i(t) = \frac{e^{-L_1 t}}{L_1}\left[L_1 x_0^i + \frac{(-1+\alpha^i)[(K_1\alpha^i\lambda_{01})^{\frac{1}{1-\alpha^i}} - (K_1\alpha^i\lambda_{01}e^{L_1 t})^{\frac{1}{1-\alpha^i}}}{}\right] \tag{2.23}$$

2.5 Discussion

In this section, the equilibrium result is discussed by plotting the trajectories for the costate, control and state variables. Hypothetical values have been chosen for illustration purposes. The values are: $L_1 = 1$, $K_1 = 10$, $\alpha^i = 0.2, \lambda_{01} = 10$, $R_1 = 10000$, $x_0^i = 1$, $T = 10$, $T_p = 1.5$. For the analysis the two competing firms are assumed to be symmetric and therefore the corresponding values for firm j are also assumed to be the same.

Observation 1 *The costate variable increases at a faster rate with time.*

The costate variable measures the marginal utility of performance improvements. In other words $\lambda_1(t)$ denotes the highest hypothetical price that the firm acting as a rational decision maker would be willing to pay for an infinitesimally small increment in unit of the product performance (quality) at time t. Evaluating the expression for the costate variable for firm i, it can be observed that the equilibrium costate of a firm is dependent on λ_{01}, parameter L_1, and the present instant of time t.

The costate variable for both competing firms increase for the entire planning horizon. This is especially the case since the competition is based on quality levels.

Firms increase their market shares based on their quality relative to that of the competitor's. It can be conjectured that post launch, the product faces its introduction and growth stage. In these phases maintaining higher quality becomes even more important since the product sales are directly influenced by product quality. This leads to a convex increasing trajectory for the costate variable. A plot drawn using hypothetical values for parameters and for terminal time $T = 10$ is shown in figure 2.1. As can be noted the rate of increase in marginal utility of quality improvement increases with time.

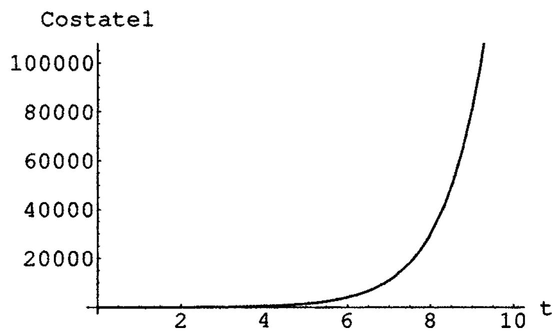

Figure 2.1: Costate trajectory

Observation 2 *The competing firms increase the rate of investment in innovation with time.*

Next, the open-loop Nash equilibrium strategy for investment in innovation is analyzed. As can be noted the expression for firm i is $u_i^* = [K_1\alpha^i\lambda_1^*]^{\frac{1}{1-\alpha^i}}$. The expression suggests that investments follow the pattern of costate dynamics. This relationship between u_1^* and λ_1^* suggest that a firm would invest in product innovation based on the hypothetical price associated with a marginal increase in product quality. This also motivates the need for continuous improvements for the entire planning horizon. Therefore, in a continuous improvement context, firms continue to increase their investment in innovation rapidly with time. Figure 2.2 presents a plot of the equilibrium investment trajectory.

It is conjectured that when viewed from the 'Big Q' perspective, even though there is no major investment towards design quality, other aspects of quality improvement become important. For example, aspects of reliability, and conformance to specifications become exceedingly important in the introduction and growth phase of a product. It is also quite common to observe firms improving the overall features, fit and finish, and aesthetic aspects of a product in multiple iterations post launch. There are investments associated with employee training, attaining quality leadership and in embracing quality culture. It is asserted that with an enlarged perspective of quality in the context of product development, the investment trajectory assumes a convex increasing shape.

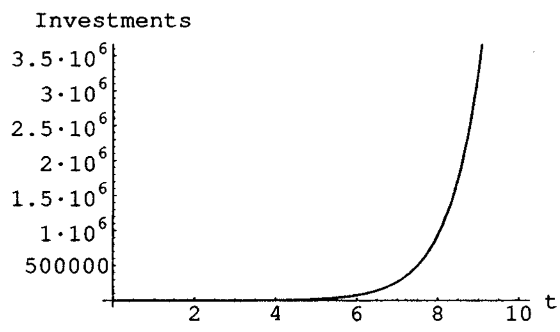

Figure 2.2: Control trajectory

In an empirical study it is very difficult to obtain a temporal investment patterns of firms in their R&D and quality activities. Thereby, evidence of longitudinal analysis of firm-level R&D and quality spending is almost absent in extant literature. However, longitudinal studies of industry level R&D spending can be observed in research literature. The convex shapes shown in figure 2.3 for investments relate with some of such studies. One such example can be found in Sutton(2001, pp.145) regarding the R&D spending in digital switch industry during 1980s. R&D spending escalated quite sharply during the 1980s. In a five year period beginning in 1984, industry R&D investments on digital switches doubled in nominal terms from an annual expensiture of 1.25 billion dollars to almost 2.5 billion dollars. The first generation of systems had involved outlays in the region of 50 million dollars to 100 million dollars. Northern Telecom for example, spent 76 million dollars on its DMS-1 system (Scherer 1992). Second generation systems were expected to involve outlays of hundreds of millions, and some industry exterts quoted estimates as high as 1 billion dollars for third generation systems. A natural wave of replacement of existing equipment was forecast for 1992-95, and the leading producers asimed to offer best-practice products by that date.

Observation 3 *The product quality exhibits a convex increasing trajectory.*

With the investment profile suggested earlier it is observed that the quality of a product increases for the entire time-horizon. The state dynamics suggest that present state exerts a negative feedback on the rate of improvement in quality. Moreover, in the absence of investments it was asserted that there is a natural decay akin to obsolescence. It can be argued that firms have multiple choice regarding their quality improvement strategy. Once can conjecture that firms invest in innovation such that quality increases linearly. Alternatively, it is also possible that firms invest such that quality grows in a convex increasing fashion and eventually stabilizes at certain level. The primary motivation for these two scenarios would be lower costs.

However, the state trajectory reveals a convex increasing trajectory. It is observed that in a continuous improvement context, the equilibrium strategy for the competing firms is to increase revenue by increasing quality at a faster rate with time. At equilibrium, firms invest at a much higher rate than what is required to ensure a mere positive growth in quality. In fact, the investments are geared towards achieving a convex increase in quality. Indeed this is a welcome scenario for customers while at the same time the competing firms attain a much higher level to compete for the next generation product.

When viewed from a holistic perspective for quality, the result questions the presumption that firms invest higher amount in innovation pre-launch and then reduce these investments post launch. It can be reasonably conjectured that when competition is directly affected by relative quality levels and the quality level of a product is influenced by a much broader scope of activities, the investments in innovation increase at a faster rate with time till the end of the growth phase of a product. The nature of state trajectory with hypothetical parameter values is plotted in figure 2.3.

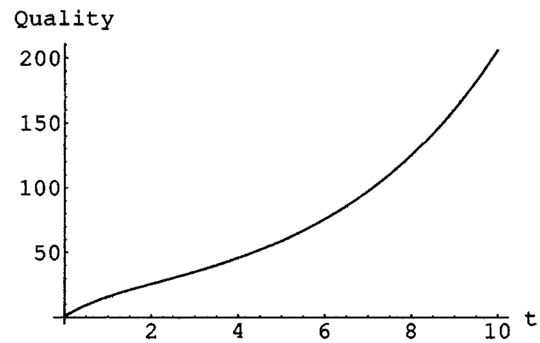

Figure 2.3: State trajectory

2.5.1 Firm Asymmetries

If the competing firms i and j are symmetric such that the parameters are identical then the trajectories of costate, control and state would also be identical. However, it is interesting to note the implications of firm asymmetries by considering different parameter values for the two firms (Bayus et al. 1997). Asymmetries may be addressed in at least three ways: First, it is possible that approaches and techniques such as total quality management, quality function deployment and cross-functional teams can be used to make the product development process more cost-efficient and effective. This would influence the value of the resource productivity parameter α. Second, it is possible to have an advantage in product development by making capital investment in development technology. This is akin to explicitly considering asymmetries in the value of K. Third, it is possible that the obsolescence parameter of firms denoted by L is different. The firm with higher value of L has a higher obsolescence or decay in quality. In this essay asymmetries are evaluated by considering different values for the parameters: (α^i and α^j), (K_1 and K_2), and (L_1 and L_2).

Parameter: α

Parameters α^i and α^j denote the innovation resource productivity parameter of the two firms. An asymmetry could result if the competing firms have differing capabilities in making a productive use of investments. The skill set of employees, training and development activities, high quality culture are some of the reasons for such an asymmetry. $(\alpha^i = 0.2) > (\alpha^j = 0.1)$ suggest that firm i has a higher innovation productivity over j. An investigation of different values of α suggest that the firm with higher innovation productivity also invests higher amount in product development. This is shown in figure 2.4.

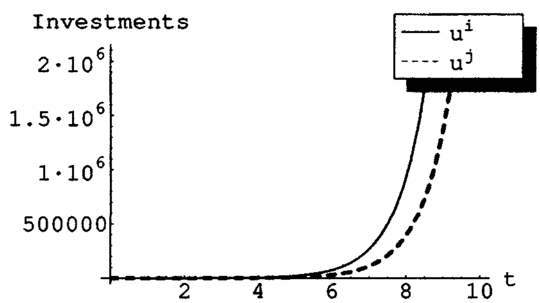

Figure 2.4: Control trajectory when parameter α is different for competing firms

As can be seen, with everything else considered equal between the competitors, the parameter α exerts a positive effect on innovation investments. Firms with an inherent advantage in innovation resource productivity make most use of it by means of investments. The resulting quality manifestation is such that the quality levels of firm i is higher than that of firm j. This results in higher revenues for firm i as compared to firm j. The result is presented in figure 2.5.

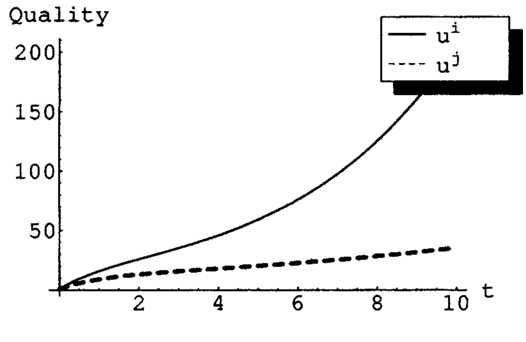

Figure 2.5: State trajectory when parameter α is different for competing firms

This result points towards the literature on resource based view (Wernerfelt(1984)) and on dynamic capabilities (Teece, Pisano and Shuen (1997)). It is reasoned in this paper that when competition do not have deep-seated competitive advantage, the moves and countermoves of competitors can often be usefully formulated in game-theoretic terms. However, analyzing how Chrysler should compete against Toyota and Honda or how United Airlines can best respond to Southwest Airlines using game-theory is quite difficult. In the above examples, for instance, Southwest's advantage is built on organizational attributes which United cannot readily replicate. Indeed, the entrepreneurial side of strategy-how significant new rent streams are created and protected-is largely ignored by the game-theoretic approach.The results suggest that a firm with relative competence intensifies investments to gain the maximum advantage in terms of product quality. As can be observed, the difference in quality and therefore revenue of the two firms increases with time. It can be conjectured that with relatively high value of R_1 as compared to costs u_i and u_j, firm i earns higher profit than firm j.

Parameter: K

K_1 and K_2 are proportional to the level of capital investment in development technology by the two competing firms. If $(K_1 = 10) > K_2 = 5)$ it suggests that firm i has higher levels of capital investment in development technology as compared to firm j. As in the case of investigation related to distinct α, an examination of different values of K also suggests that the firm with higher value of K also invests higher amount in product development. This is shown in figure 2.6.

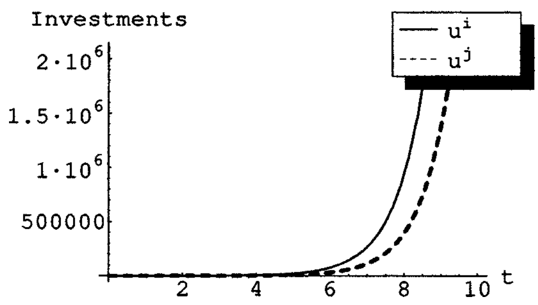

Figure 2.6: Control trajectory when parameter K is different for competing firms

The figure shows that parameter K exerts a positive effect on innovation investments. Similar to the conjecture regarding different values of α, the intuition behind this effect is that with higher level of capital investments in development technology, a firm targets the investments towards increasing the product quality. Figure 2.7 provides the pictorial representation.

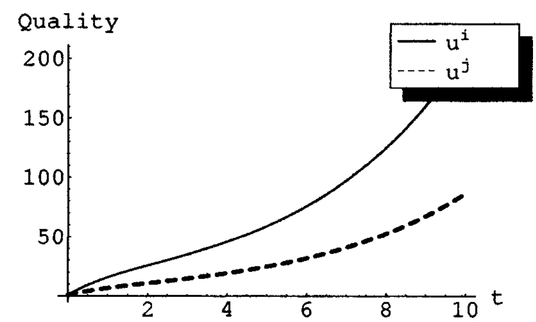

Figure 2.7: State trajectory when parameter K is different for competing firms

The resource-based approach sees firms with superior systems and structures being profitable not because they engage in strategic investments that may deter entry and raise prices above long-run costs, but because they have markedly lower costs, or offer markedly higher quality or product performance. The resource-based perspective puts both vertical integration and diversification into a new strategic light. Both can be viewed as ways of capturing rents on scarce, firm-specific assets whose services are difficult to sell in intermediate markets (Wernerfelt(1984)). The resource-based perspective also invites consideration of managerial strategies for developing new capabilities. In the model, the relative difference in resources due to strategic investments is captured in asymmetries in the value of K. From the figures it can be conjectured that with adequately high value of R_1, firm i would have relatively higher profits than firm j.

Parameter: L

In the formulation, L_1 and L_2 characterizes the decay or in other words obsolescence effect of quality. Different values of L_1 and L_2 helps evaluate the difference among the two firms regarding this effect. $(L_1 = 1) > (L_2 = 0.5)$ suggests that the obsolescence effect on firm i is higher than that on firm j. An investigation of the dynamics of innovation investments reveals that with higher value of the parameter L, a firm would make higher level of investments. Figure 2.8 illustrates the result.

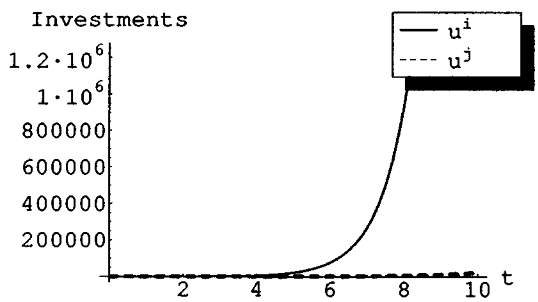

Figure 2.8: Control trajectory when parameter L is different for competing firms

Parameter L therefore exerts a positive effect on innovation investments. The growth in product quality for the two firms is shown in figure 2.9.

It is indeed interesting to note that initially the quality level is higher for the firm j that had the lower value for L^j. However this negative feedback lead to higher

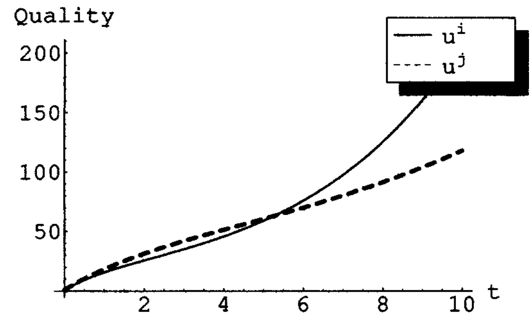

Figure 2.9: State trajectory when parameter L is different for competing firms

investments in innovation by firm i and therefore in the second half of the planning horizon, the quality of firm i is higher. The result suggests that with everything else being equal, the higher level of negative feedback makes the firm more aggressively pursue quality improvements. On the other hand, a lower negative feedback makes a firm complacent in innovation investments. Over time the firm with an inherent disadvantage achieves higher quality and therefore higher revenue. It can however be observed that although the relative difference in investments is very high, the difference in quality is not very much. Therefore, unless the value of R_1 is very high it is reasonable to expect that the profit of the firm with lower value of L would be higher. However, with very high values of R_1 the profits of firm i could be potentially higher than that of firm j.

Teece, Pisano and Shuen (1997) present a conceptual framework about dynamic capabilities. They suggest that the term 'dynamic' refers to the capacity to renew competencies so as to achieve congruence with the changing business environment. The term 'capabilities' emphasizes the key role of strategic management in appropriately adapting, integrating, and reconfiguring internal and external organizational skills, resources, and functional competencies to match the requirements of a changing environment. In the context of this essay, it can be asserted that the parameters allow an examination of dynamic capabilities. Along with the cases analyzed above, additional insights can be gained by considering different values for combination of parameters. This will allow an explicit evaluation of competition where one of the firms has an inherent advantage in one aspect for example, effectiveness of product development activity (α) whereas the rival firm has an advantage in the level of capital investment in development technology (K).

The approach of competing with strategic quality management through investments in innovation and quality culture has only recently entered the business plans of companies.Despite the uniqueness of specific industries and companies, certain subjects for strategic quality goals are widely applicable. According to Juran(1992) these include: product performance, competitive performance, quality improvement, cost of poor quality and performance of major processes. The product performance goal relates to major performance features which determine response to customer needs; promptness of service, fuel consumption, mean time between failures, courtesy. These features directly affect product salability. Competitive performance has always been a goal in market-based economies, but seldom a part of the business plan. The trend to make competitive quality performance a part of the business plan is recent but irreversible. Quality improvement goal may be aimed at improving product salability and/or reducing the cost of poor quality. Either way, the end result after deployment is a formal list of quality improvement projects with associated assignment of responsibilities. The goal of quality improvement usually includes a goal of reducing the costs due to poor quality. While these costs are not known with precision, they are known to be very high. Despite the lack of precise figures, it is feasible, through estimates, to bring this goal into the business plan and to deploy it successfully to lower levels. Finally, the goal of performance of major processes has only recently entered the strategic business plan. The goal relates to the performance of major processes that are multifunctional in nature, for example, new product launching, billing, bidding for business and purchasing.

Overall, in such an approach towards product development, management's role is to provide clarity, unity, and resources, and to lead improvement in "quality culture". The TQM practice of *hoshin kanri* (policy deployment, strategy deployment) and the various aspects of strategy (business, product, technology, manufacturing, field support, and marketing) ensure that the quality-enabled product development project is a high priority for corporate success. As in the case of production operations, product development programs also require on-line quality control. If the team never checks performance (progress) against the target (plan), then divergences tend to grow unchecked. The development control is a specific application of PDCA (plan, do, check, and act), the general management control approach advocated by Deming (1986). Reward systems enables reinforcing quality culture in an organization and top management should provides such sources of motivation. Adequate corporate resources must be allocated for competing with quality-driven product development process. As can be observed from the control trajectories, it is very important to have the resources in place at the right time. In return the rewards are shorter lead times, improved quality of product, higher market share, enhanced customer satisfaction and eventual reduction in variable costs due to learning effects. The ability to complete more product cycles than the competition ptovides greater product variety, increased flexibility, and increased corporate learning. The lead rapidly grows over the companies that are unable to foster a quality culture based product development environment.

Managers at leading companies have embraced such total quality development programs, which can be quite expensive, because they see a clear link between quality and profitability. Quality is often included explictly in the strategic planning process. Annual goals are set for quality improvement and made specific and actionable. Goals normally take into account the customer's perspective and are also matched against the expected performance of competitors. Garvin(1988) notes that GTE, as a part of its strategic planning process, requires that each business unit identify the place of quality in its business strategy, define the important quality elements in its strategic programs, establish long-range quality-related goals, and explain how it will develop the commitment and coordination of line and staff functions to meet those goals. Both internal costs of quality measures and external customer related measures are involved. Strategic quality management, is in essence, an extension of aspects like statistical process control and interfunctional teams. It is a comprehensive approach, is more closely linked to profitability and basic business objectives, is more sensitive to competitive needs and the consumer's point of view, and is more firmly tied to continuous improvement. Many companies mistakenly think that they have adopted the new approach when their programs merely include elements of quality assurance and quality control. For the most part, these companies are still thinking defensively about quality. However for achieving the competitive potential of quality, it ought to be deployed in a strategic manner.

As a concluding remark, an example from Garvin(1988) pp.30-33 is provided to emphasize the point. Between 1970 and 1980, Xerox's share of U.S. copier revenues fell from 96 percent to 46 percent, largely because of Japanese competition. These inroads led to a restructuring of the company. Several ambitious quality programs were initiated as a result. The first was competitive benchmarking. Comprehensive surveys were developed to monitor customer satisfaction and to compare customer's reactions to Xerox's products and to competitors'. Quality of products, services, and practices was then checked against the performance of world leaders. Targets for improvement were developed with all benchmarks incorporated into annual operating plans and five-year business plans. These steps marked a sharp change in company philosophy. Xerox historically had been self-contained and introspective. As a virtual monopolist, it measured progress by tracking its own performance over time, rather than by watching competitors. Design problems were addressed. The new product development process was completely overhauled; large investments were made in professional workstations and computer-aided design (CAD) capabilities; engineering and design teams were located closer together and given shared responsibility for design, ease of manufacture, and ease of service; designers were provided with additional training, including exposure to advanced techniques; and design teams began to work with suppliers much earlier in the development cycle. A new emphasis on employee involvement, including quality circles and problem-solving teams was introduced. Extensive training was provided and was coupled with systems of feedback, recognition, and reward. Top management involvement was recognized and a companywide quality control process, anchored in a clear statement of philosophy and goals was put forth. These steps soon produced impressive results. Assembly quality in the Reprographic Business Group improved 63 percent in two years. During the same period, the reliability of the group's products, as measured by customer reporting, increased 40 percent, and an index of customer satisfaction, compiled from monthly surveys of 50,000 customers increased 30 percent.

Topic	***Contents of Little Q***	***Contents of Big Q***
Products	Manufactured goods	All products, goods and services, whether for sale or not
Processes	Processes directly related to manufacture of goods	All processes; manufacturing support; business, etc.
Industries	Manufacturing	All industries; manufacturing; service; government, profit or not
Quality is viewed as:	A technological problem	A business problem
Customer	Clients who buy the products	All who are impacted, external or internal
How to think about quality	Based on culture of functional departments	Based on the universal Trilogy (planning, control and improvement)
Quality goals are included:	Among factory goals	In company business plan
Cost of poor quality	Costs associated with deficient manufactured goods	All costs which would disappear if everything were perfect
Improvement is directed at:	Departmental performance	Company performance
Evaluation of quality is based on mainly on:	Conformance to factory specifications, procedures, standards	Responsiveness to customer needs
Training in managing for quality is:	Concentrated in the Quality Department	Companywide
Coordination is by:	The quality manager	A quality council of upper managers

Figure 2.10: Contrast, Big Q and Little Q

Chapter 3

Sequential Differential Game Between Two Competing Firms

3.1 Introduction

A natural extension of the differential game considered in essay 1 is a game with sequential decision. Essay 2 extends the conceptualization of the competitive scenario of the first essay by considering a sequential game. These games are typically representative of leader-follower or incumbent-entrant competitive situations. The games are characterized by information asymmetry where the follower is aware of the innovation and quality levels of leader's products.

The motivation for considering this scenario is its close correspondence with many real life cases. In a market for jeans, for example, Levi has 56 percent of all market, whereas Lee, Arizona, Guess, Gap and Calvin Klein altogether constitute 32 percent of the market. Levi could therefore commit investments in innovation in new product development and achieve a first-mover advantage. At the same time, knowing the investment strategy of the leader, the rival firms can formulate their own strategies. Therefore, the firm acting as a leader chooses a decision path that maximizes the objective for all conceivable response that can be taken by the follower(s).

Another reason to explicitly consider this case is that the solution approach for a sequential game is quite distinct from that for a simultaneous game. In the case of sequential games, a hierarchical play differential game approach is used to model the competitive situation and to obtain the open-loop Stackelberg Nash equilibrium. The issue of subgame perfectness and commitment is extremely important in these solutions.

The essay is divided into five sections. The introduction completes the first section. In the second section, the model is presented. The structure of open-loop Stackelberg Nash equilibrium strategies and their implications are analyzed in the fourth section. This leads to the third section in which the model is presented and the differential game is formulated. The proposed differential game is solved in the third section. Finally, the fifth section concludes the essay with discussion of the results.

The mathematical preliminaries and proofs of theorems are provided in Appendix B.

3.2 The Model

3.2.1 Differential Game Formulation

The notations and symbols used in the model are as follows:

N: Number of firms

T: Finite time horizon for the strategies

i, j: Superscript to denote competing firms in a duopoly

t: An instant of time in the dynamic game setup

$u(t)$: Investments in innovation effort (expenditure per unit time)

$R(t)$: Net revenue rate at t for the firm

R_0: Product category net revenue rate for the existing product

R_1: Product category net revenue rate for the new product

x_0: Performance level of the existing product

T_p: Date of product launch by leader

$T_p + \tau$: Date of product launch by follower

The leader is represented by the superscript i and the follower is represented by the superscript j. Since it is a sequential game the time of product launch for the two players is such that the leader launches the product at time (T_p). Later, the follower launches the product after time τ at $T_p + \tau$. A continuous improvement in the product is considered and therefore the entire time interval $t \in [0, T]$ for the leader, and $t \in [T_p, T]$ needs to be optimized. As in essay 1, the state dynamics of the leader is,

$$\dot{x^i}(t) = K_1[u^i(t)]^{\alpha^i} - L_1 x^i(t) \tag{3.1}$$

The rate of quality improvement $\dot{x}^i(t)$ increases with investments $u^i(t)$. The factor $L_1 x^i(t)$ suggests a natural decay in quality in the absence of any investments. The differential game formulation for leader is:

$$\max T\Pi^i(u^i(t), T_p) = -\int_0^T \{u^i(s)\}ds + \Phi^i(T, x^i(T)) \tag{3.2}$$

where $\Phi^i(T, x(T))$ is defined as,

$$\begin{aligned}\Phi^i(T, x(T)) \triangleq R_0 \cdot \frac{x_0^i}{x_0^i + x_0^j} \cdot T_P^i + \\ + R_1 \cdot \frac{x^i(T)(\tau)}{x^i(T) + x_0^j} + R_1 \cdot \frac{x^i(T)(T - T_p - \tau)}{x^i(T) + x^j(T)}\end{aligned} \tag{3.3}$$

subject to,

$$\dot{x}^i(t) = K_1[u^i(t)]^{\alpha^i} - L_1 x^i(t) \tag{3.4}$$

$$x^i(0) = x_0^i, T \text{are fixed}, \tag{3.5}$$

$$x^i(T), \text{ is free.} \tag{3.6}$$

Next, the follower's problem formulation is discussed. Owing to a sequential nature of the game, the information about the leader's quality and about the leader's investments is known to the follower. Specifically, it is assumed that the knowledge gained by the leader's investments "spills over" to follower's quality improvement dynamics. It is asserted that in the context of a leader-follower competition, the level of quality improvements of follower indeed depends not only on its own innovation efforts but also on the knowledge pool available because of the leader's investments. The fact that the leader often cannot wholly conceal its efforts nor can it credibly announce the commitment it has made, make the situation quite complicated. To address these issues fully would require a subtle and rich analysis with the games of incomplete information. A formal treatment of "spillover effects" is also widely used in econometric settings. For example, Griliches (1979) present a knowledge-based Cobb-Douglas production function in which spillovers are considered. Using such a Cobb-Douglas function in the state dynamics of a differential game makes it analytically intractable. Instead, in this essay the issue is simplified by confining attention to a relatively simple framework. The spillover effect is modeled by considering a linear additive term $M_2u^i(t)$. Follower's state dynamics is:

$$\dot{x^j}(t) = K_2[u^j(t)]^\alpha - L_2x^j(t) + M_2u^i(t) \tag{3.7}$$

It is assumed that the spillover is a function of the investments made by the leader $u^i(t)$. In this term M_2 is a very small number which quantifies the amount of spillover from the leader to the follower. The term $M_2u^i(t)$ enables in restricting the analysis to the case where the follower emulates the innovation na dproduct quality of the leader, as opposed to setting its own technology and quality standards. This can be explained by means of an example from Sutton(2001) pp.119. In the global photographic film market the efforts put in by Kodak between 1957 and 1963 led to new quality standards in color film. Kodak had been involved in color film since 1921 and had spent over 60 million dollars on R&D upto 1957. It spent almost as much again during next six years. By 1963, the company had spent a total of 121 million dollars on color film research, and its C22 process, launched in 1953, became an industry standard. Kodak's efforts posed two problems for rivals. The first related to the pace of advance of color film technology. Few firms could finance the level of R&D spending now required from their current sales revenue. The second related to the process itself. Most users around the world had their film developed and printed by small local photo processors, and each type of film required a different treatment and possibly different equipment. Kodak's rivals were faced with sn invidious choice. They might choose to make their film products compatible with C22, in which case it could be processed in a manner identical to Kodak's. Otherwise, they would have to find some way of coping with the fact that, although all processes would have equipment to handle Kodak's films, few might wish to acquire equipment to handle less popular brand.

The main rival from Europe, Agfa-Gevaert invested heavily during this technology growth phase such that R&D spending running ahead of sales, the R&D/ Sales ratio rose steadily up to the end of 1970s, while the ratio of profits to sales declined. Throughout this period, the company faced a technical dilemma that would not be fully resolved until the late 1970s. Agfa's process was a water-based process, in contrast to Kodak's oil-based process. Through the early 1970s, Agfa's approach was to continue with improvements to its process rather than attempt to emulate Kodak. By the end of 1970s, however, the disadvantages of remaining with a water-based process were becoming increasingly clear, and this effort culminated in the introduction of Agfa's first compatible film in 1978: the CNS 400. An oil-based process offered better technical prospects in the long run (including better image stability, for example) but more importantly - it was the mainstream system. A film that could not be processed on the same equipment as Kodak's would be at a continuing disadvantage in the market. It became increasingly important to produce a film that was not only capable of being processed on the same equipment as Kodak's but was also fairly robust in processing. A film might sit in a processor's lab for some time before being dealt with and would then be processed as part of large batch with no adjustment for individual films. The need to produce films that would survive this unevenness in processing procedure placed ever-increasing demands on the manufacturers. In the mid-1970s Agfa redirected its R&D efforts towards the development of an oil-based process compatible with the Kodak C41 process. In the formulation of this essay such a leader-follower dynamics is considered.

The follower's differential game formulation is given as:

$$\max T\Pi^j(u^j(t),T_p) = -\int_{T_p}^{T}\{u^j(s)\}ds + \Phi^j(T,x^j(T)) =$$
$$= -\int_{0}^{T-T_p}\{u^j(s+T_p)\}ds + \Phi^j(T,x^j(T)) \qquad (3.8)$$

where $\Phi^j(T,x(T))$ is defined as,

$$\Phi^j(T,x(T)) \triangleq R_0 \cdot \frac{x_0^j}{x_0^i + x_0^j} \cdot T_P +$$
$$+ R_0 \cdot \frac{x_0^j}{x^i(T) + x_0^j} \cdot \tau + R_1 \cdot \frac{\{x^j(T)(T - T_p - \tau)\}}{x^i(T) + x^j(T)} \qquad (3.9)$$

subject to,

$$\dot{x^j}(t) = K_2[u^j(t)]^\alpha - L_2 x^j(t) + M_2 u^i(t) \qquad (3.10)$$

where, $M_2 u^i(t)$ represent the spillover of knowledge, assumed to be a function of investments by the leader, M_2 is a small constant such that $0 < M_2 << 1$.

$$x^j(0) = x_0^j, T \text{are fixed}, \qquad (3.11)$$

$$x^j(T), \text{ is free}. \qquad (3.12)$$

where,

$$x^i(T) = x_0^i + \int_0^t [K_1[u^i(s)]^{\alpha^i} - L_1 x^i(s)]ds \qquad (3.13)$$

$$x^j(T) = x_0^j + \int_{T_p}^t [K_2[u^j(s)]^{\alpha^j} - L_2 x^j(s) + M_2 u^i(s)]ds \qquad (3.14)$$

3.3 Analysis of the model

As in the first essay, Pontryagin's maximum principle is used to solve for open-loop Stackelberg equilibrium conditions. Interested readers can refer Dockner et al.(2000) for details regarding the Pontryagin's approach to solving sequential differential game problems. The equilibrium results are expresses in the form of theorems as given below.

3.3.1 Equilibrium Results

Theorem 1 *The maximized costate variables for the follower are function of time and are given by,*

$$\lambda_1^* = \lambda_{01} e^{L_2 t} \tag{3.15}$$

$$\lambda_2^* = \lambda_{02} e^{L_1 t} \tag{3.16}$$

where λ_1^* and λ_2^* are the costate variables reflecting the marginal price for a unit increase in follower firm's own state and the state of the leader i; $\lambda_1(0) = \lambda_{01}$, $\lambda_2(0) = \lambda_{02}$ are known constants.

Theorem 2 *The Stackelberg equilibrium investment by the follower in product development is given as:*

$$u^j(t)^* = [K_2 \alpha^j \lambda_{01} e^{L_2 t}]^{\frac{1}{1-\alpha^j}} \tag{3.17}$$

Next the leader's problem is investigated. The leader knows the follower's best response to each control path $u^i(.)$.

Theorem 3 *The maximized costate variables for the leader are given by,*

$$\psi_1^* = \psi_{01} e^{L_1 t} \tag{3.18}$$

$$\psi_2^* = \psi_{02} e^{L_2 t} \tag{3.19}$$

$$\psi_3^* = -\psi_{03} e^{-L_2 t} \tag{3.20}$$

where ψ_1^* and ψ_2^* and ψ_3^* are the costate variables reflecting the marginal price for a *unit increase in leader's own state, the state of the follower and the costate of the follower; ψ_{01}, ψ_{02} and ψ_{03} are constants.*

Theorem 4 *The Stackelberg equilibrium investment by the leader in product development is given as:*

$$u^i(t)^* = \left[\frac{K_1\alpha^i\psi_{01}e^{L_1t}}{1 - M_2\psi_{02}e^{L_2t}}\right]^{\frac{1}{1-\alpha^i}} \tag{3.21}$$

Theorem 5 *The equilibrium state trajectory of performance improvement of the follower is given as:*

$$x^j(t) = \frac{e^{-L_2t}}{L_2}\left[L_2x_0^j + (-1+e^{L_2t})M_2\left[\frac{K_1\alpha^i\psi_{01}e^{L_1t}}{1 - M_2\psi_{02}e^{L_2t}}\right]^{\frac{1}{1-\alpha^i}} + \right.$$
$$\left. + (-1+e^{L_2t})K_2(K_2\alpha^j\lambda_{01}e^{L_2t})^{\frac{\alpha^j}{(1-\alpha^j)^2}}\right] \tag{3.22}$$

Theorem 6 *The equilibrium state trajectory of performance improvement of the leader is given as:*

$$x^i(t) = \frac{e^{-L_1t}}{L_1}\left[L_1x_0^i + (-1+e^{L_1t})K_1\left[\frac{K_1\alpha^i\psi_{01}e^{L_1t}}{1 - M_2\psi_{02}e^{L_2t}}\right]^{\frac{\alpha^i}{1-\alpha^i}}\right] \tag{3.23}$$

3.4 Discussion

In this section, the equilibrium result is discussed by plotting the trajectories for the costate, control and state variables. Hypothetical values have been chosen for illustration purposes. The values are: $L_1 = 1$, $K_1 = 10$, $\alpha^i = 0.2, \psi_{01} = 10$, $R_1 = 10000$, $x_0^i = 1$, $T = 10$, $T_p = 1.5$. For the analysis, the leader and the follower are assumed to be symmetric and therefore the corresponding values for the follower j are also assumed to be the same.

Observation 1 *The costate variable of both the leader and the follower increases more rapidly with time*

As in essay 1, the marginal utility of a unit increase in quality, exhibits a convex increasing trajectory. With similar values of parameters as in essay 1, the plot of costate trajectory of the follower is given in figure 3.1. The plot begins at time $t = T_p$ since the follower initiates product development activities only after the leader has already launched the product.

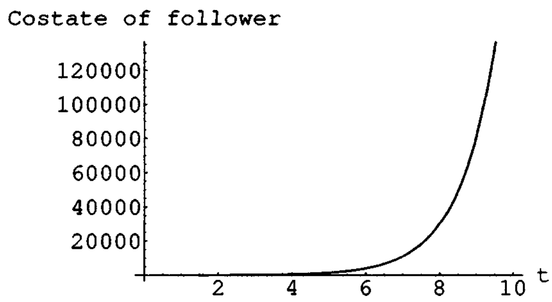

Figure 3.1: Costate trajectory of the follower

Next, the costate trajectory of the leader is analyzed. ψ_1 which represent marginal utility from a unit increases in quality of leader's product is of theoretical interest and is plotted in figure 3.2.

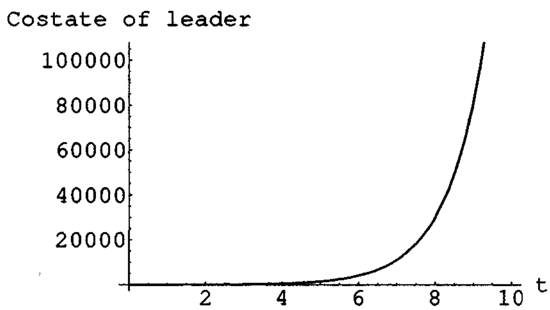

Figure 3.2: Costate trajectory of the leader

As in essay 1, the costate variable for both the leader and the follower firm increase for the entire planning horizon. This follows since the leader and follower are competing on the basis of their quality levels. Firms increase their market shares based on their relative quality to that of the competitor's. It can be conjectured that post launch, the product faces its introduction and growth stage. In these phases, maintaining higher quality becomes even more important since the product sales are directly influenced by product quality. This leads to a convex increasing trajectory for the costate variable.

Observation 2 *The investment in innovation made by the follower increases more rapidly with time.*

The investment strategies of the follower are purely a function of its own costate variable. As was illustrated, the follower's costate variable increases in a convex fashion. This in turn results in a convex increase in follower's control trajectory. With identical parameter values it can be conjectured that the follower compensates for a delayed entry into the market by increasing its investment intensity. Such an increase in investment results in increase quality and therefore high total revenue for the follower. The plot of investments by follower is shown in figure 3.3.

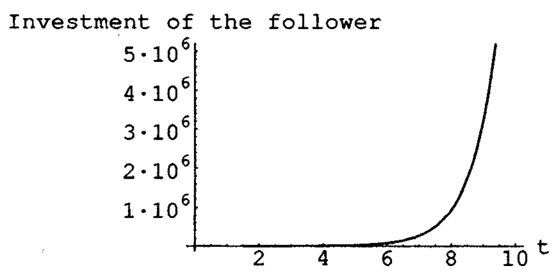

Figure 3.3: Control trajectory of the follower

Observation 3 *The investment in innovation made by the leader initially increases rapidly with time but later increases at a decreasing rate.*

Leader's investment trajectory is sigmoidal. Because of the nature of the game, the leader enters the market before the follower. While formulating its equilibrium investment strategy, the leader takes into account the evolution of its own costate and also the costate of the follower. It is asserted that being early in the market, the leader takes into account all possible courses of action that a follower may choose. The leader's investments in innovation are plotted in figure 3.4.

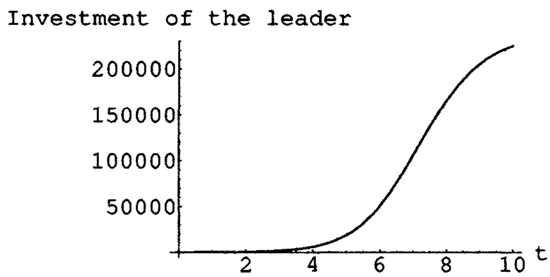

Figure 3.4: Control trajectory of the leader

It is interesting to note that the leader's rate of investment increases in the initial phase and then starts to decrease. It is conjectured that the leader capitalizes on the advantage of early market entry by increasing the intensity of investments and gaining a high market share. Subsequently, after the follower's entry, the leader reduces the intensity of investments and thereby reduces the amount of spillover that is potentially possible.

Observation 4 *When the parameter values of the leader and follower are identical, the follower invests higher than he leader.*

A comparative plot of the investment strategies of the leader and the follower are presented in figure 3.5. As can be observed from the plot in the initial time horizon, the follower maps its investment strategy to that of the leader. However subsequently the leader starts reducing the rate of investments, while the follower continues with the high investment rate based strategy.

Observation 5 *The rate of increase of follower's product quality increases with time.*

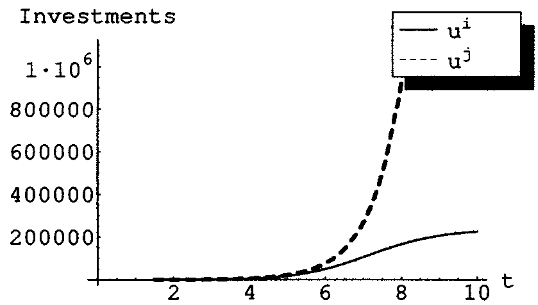

Figure 3.5: Comparative plot of investments by the leader and follower

The quality of follower's product increases for the entire time-horizon. Moreover, this increase is a convex function. As can be noted in the problem formulation, the quality level of the follower is a function of investments made by the leader and the follower. Specifically, the quality trajectory is influenced by both the convex profile of investments made by the follower and also by the spillover effect of the investments made by the leader. This results in a convex increase in quality improvements. The state trajectory of the follower is plotted in figure 3.6.

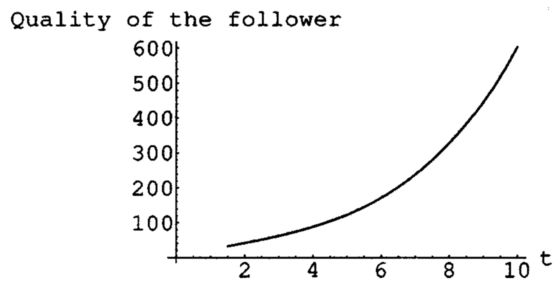

Figure 3.6: State trajectory of the follower

The follower starts accruing revenue only after $T_P + \tau$, where T_P is the date of launch of the leader. It can be asserted that since the game context is that of quality-based competition, the follower compensates for the delayed entry by increasing the quality levels at a fast rate.

Observation 6 *The rate of increase of leader's product quality decreases with time.*

The leader's investment trajectory follows a sigmoidal trajectory. In the problem formulation the leader's quality improvement is only a functions of its own investments. Moreover it is assumed that the leader doesn't obtain the advantages of spillover of knowledge gained by follower's investments. Furthermore, the leader needs to deliberately avoid maintaining very high investments to ensure that the follower doesn't achieve huge gains from spillovers. Under such a setup it was noted that the leader chooses a sigmoidal trajectory for investments. In such an investment profile the investment increases rapidly in the initial phase but then eventually tapers off. Corresponding to this investment profile, the rate of quality improvement is high in the initial phase but then eventually the rate of improvement starts decreasing. Thus, it is conjectured that the quality trajectory of the leader is concave. Figure 3.7. presents the plot of leader's state trajectory.

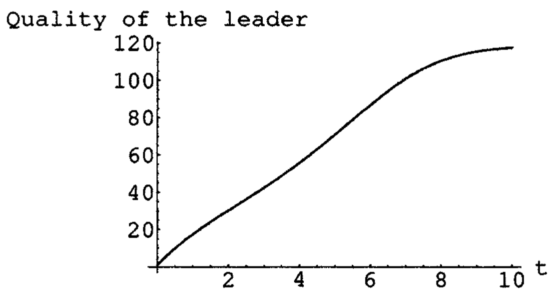

Figure 3.7: State trajectory of the leader

Observation 7 *When the parameter values of the leader and follower are identical, the quality of follower's product is higher than that of the leader's product.*

The combined plot of the leader and follower's state trajectory is presented next for comparison in figure 3.8. With identical parameter values the follower and leader are perfectly symmetric in their capabilities. Moreover, the leader has the advantage of earlier market entry, whereas the followers obtains the gains of information asymmetry and the associated knowledge spillovers from the leader. It is reasonable to assert that the follower employs its inherent competence and the benefit of spillover of knowledge from leader's investments to increase quality at a much faster rate. Under such circumstances the overall gain and loss in the competitive game is dictated by the relative revenues achieved and the costs incurred by the two players. With higher investments than the leader the follower incurs higher costs. At the same time these investments also lead to higher quality levels and therefore revenues for the follower. In the event of the value of R_1 being very high the follower wins the game while the results favor the leader if R_1 is low.

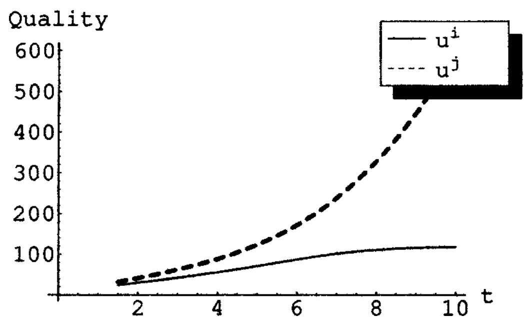

Figure 3.8: Comparison of state trajectories of the leader and follower

3.4.1 Firm Asymmetries

Similar to the analysis in essay 1, firm asymmetries and their implications are investigated in this essay. It can be conjectured that firms take up the role of leader and follower based on inherent strengths and weaknesses. Therefore, an explicit consideration of firm asymmetries is very important and could potentially lend more insights in a sequential game setup. Asymmetries may be addressed in at least four ways: First, it is possible that approaches and techniques such as total quality management, quality function deployment and cross-functional teams can be used to make the product development process more cost-efficient and effective. This would influence the value of the resource productivity parameter α. Second, it is possible to have an advantage in product development by making capital investment in development technology. This is akin to explicitly considering asymmetries in the value of K. Third, it is possible that the obsolescence parameter of firms denoted by L is different. Finally, different values of M suggest different levels of spillover of knowledge from the leader's investments to the follower. The firm with higher value of L has a higher obsolescence or decay in quality. In this essay asymmetries are evaluated by considering different values for the parameters: (α^i and α^j), (K_1 and K_2), (L_1 and L_2) and (M_1 and M_2).

Parameter: α

As in essay 1, parameters α^i and α^j denote the innovation resource productivity parameter of the two firms. An asymmetry could result if the competing firms have differing capabilities in making a productive use of investments. The skill set of employees, training and development activities, quality culture are some of the reasons for such an asymmetry. $(\alpha^i = 0.2) > (\alpha^j = 0.1)$ suggest that firm i has a higher innovation productivity over j. An investigation of different values of α suggest that the firm with higher innovation productivity also invests higher amount in product development.

From the results it can be observed that the follower has a relatively higher level of investments in innovation as compared to the leader. Therefore, an increase in the value of α of the follower will only result in making these investments still higher. Instead, it is asserted that an increase in leader's α would potentially provide some interesting insights. A plot of control trajectories of the leader and the follower's investment strategies in a sequential game with higher α for the leader (with the α of follower kept unchanged) is given in figure 3.9.

The figure clearly shows that the leader has a higher investment than when α^i was lower. Moreover, the investments are higher than that of the follower for

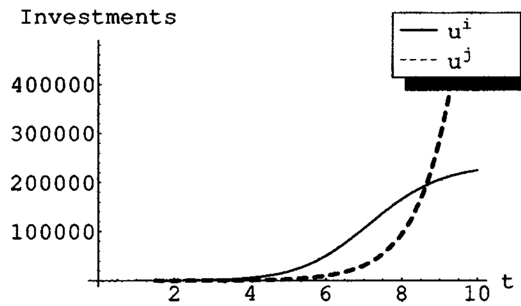

Figure 3.9: Control trajectory when parameter α of the leader is higher

almost the entire planning horizon. As in essay 1, this result clealry points towards the literature on resource based view (Wernerfelt(1984)) and on dynamic capabilities (Teece, Pisano and Shuen (1997)). In the presence of such a competence, a firm employs it to improve the competitive advantage. The resulting trajectory of quality is presented in figure 3.10.

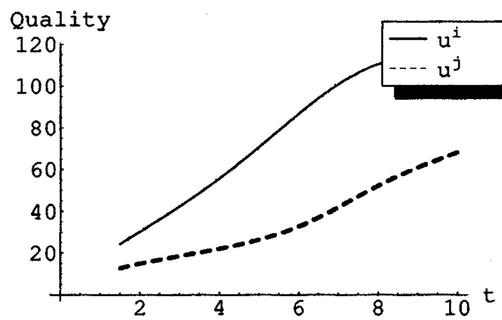

Figure 3.10: State trajectory when parameter α of the leader is higher

As can be noted, the quality of leader's product is always higher than that of the follower. Hence an asymmetry in terms of resource productivity parameter provides a totally different result than what was observed when the competing firms were symmetric. It suggests that the leader with an advantage in terms of early market entry as well as a higher level of resource productivity indeed continues having higher revenues. Under this situation the leader doesn't worry much about the spillover to the follower snce its own resource productivity enables attaining higher revenues by increasing quality levels relative to that of the follower. With a high value of R_1 it can be conjectured that the result favors the leader.

Parameter: K

Firm asymmetries can be analyzed by evaluating different values for K and the implications on control and state trajectories. As in essay 1, K_1 and K_2 are proportional to the level of capital investment in development technology by the two competing firms. If $(K_1 = 20) > (K_2 = 10)$, it suggests that the leader i has higher levels of capital investment in development technology as compared to the follower j. The implications of a higher value of K for the leader are analyzed. The plot is presented in figure 3.11.

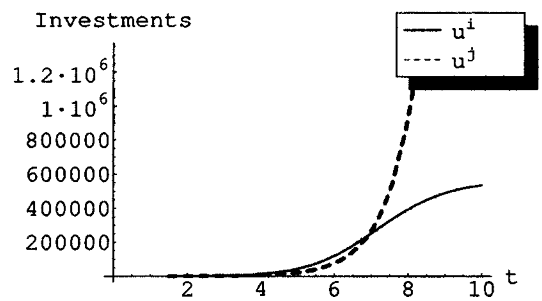

Figure 3.11: Control trajectory when parameter K of the leader is higher

The figure shows that parameter K exerts a positive effect on innovation investments. Similar to the conjecture regarding different values of α, the intuition behind this effect is that with higher level of capital investments in development technology, a firm targets the investments towards increasing the product quality. As can be seen, with a higher value of K the investment by the leader is relatively higher than that of the follower for most part of the planning horizon. However, it can also be observed that in the latter part of the planning horizon, the follower's investment infact shoots up while that of the leader tapers off. It can be conjectured that such an investment profile would have an impact on the state trajectory. The plot representing the evolution of product quality is provided in figure 3.12. It can be noted, that the product quality of the leader is higher than that of the follower for most part of the planning horizon. Eventually, at the end of the planning horizon, the high growth in follower's investment results in higher quality than that of the leader.

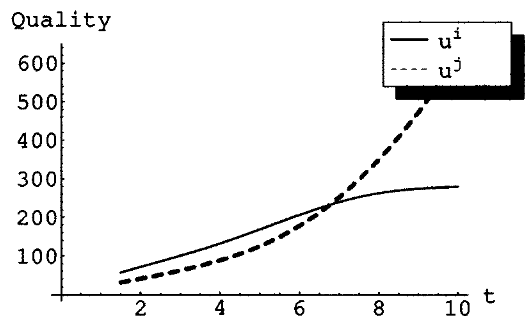

Figure 3.12: State trajectory when parameter K of the leader is higher

In the model, the relative difference in resources due to strategic investments is captured in asymmetries in the value of K. From the figures it can be conjectured that with adequately high value of R_1, the leader i would have relatively higher profits than the follower j.

Parameter: L

As in essay 1, L_1 and L_2 characterize the decay or in other words obsolescence effect of quality. Different values of L_1 and L_2 help evaluate the difference among the two firms regarding this effect. $(L_1 = 1) > (L_2 = 0.7)$ suggests that the obsolescence effect for leader i is higher than that for the follower j. An investigation into the dynamics of innovation investments reveals that with higher value of the parameter L, a firm would make higher level of investments. The control trajectory when the leader has a higher value of L is plotted in the figure 3.13.

Interestingly, with a difference in L_1 and L_2 the shape of investment trajectory of the leader is now convex. It is asserted that when the leader faces a high obsolescence effect the investments are increased at a faster rate to ensure an increasing state

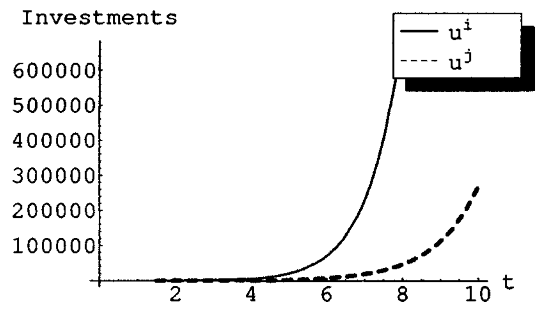

Figure 3.13: Control trajectory when parameter L of the follower is higher

trajectory. However, as opposed to parameter α and K such an increased control trajectory doesn't translate into higher quality. Infact, in this situation, the leader has lower product quality inspite of higher investments. It can be conjectured that a high decay effect puts the leader into a disadvantageous position. In such a situation, under equilibrium control the leader always loses the competitive game. The plot of the evolution of state trajectory is shown in figure 3.14.

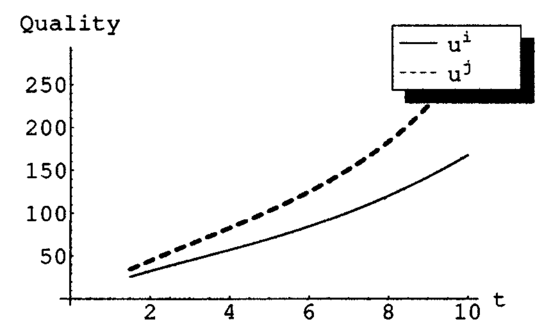

Figure 3.14: State trajectory when parameter L of the leader is higher

Spillover factor M_2

As was stated earlier, M_2 characterizes the amount of spillover from the leader to the follower. In this section the impact of reduction of the spillover factor M_2 is considered. Evidently in this case the leader would be less worried about the amount of advantage the follower obtains from the leader's own investments. The plot of the innovation investments with a reduced M_2 is provided in figure 3.15.

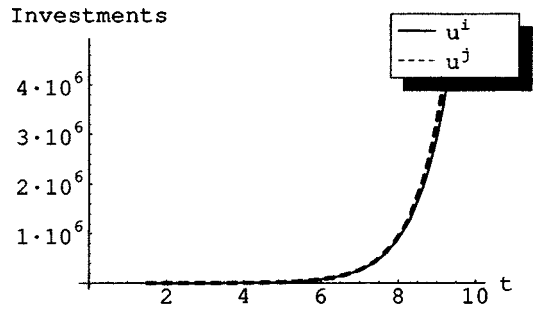

Figure 3.15: Control trajectory when parameter M_2 is reduced

As can be noted, if the spillover effect is low, the leader pursues investments more aggressively. Under such circumstances, the control trajectory of the leader is also convexly increasing and is only marginally lower than that of the follower. The impact of such investments on the state trajectory is presented in figure 3.16.

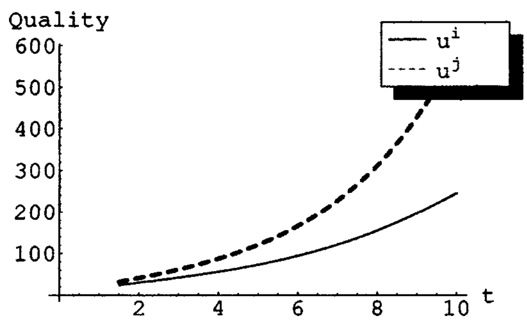

Figure 3.16: State trajectory when parameter M_2 is reduced

It can be noted, in such a situation, the quality of follower is higher than that of the leader and both obtain a convex increasing state trajectory. It can be argued that unlike α, K and L, a different value of M_2 doesn't correspond to any advantage related to resources, core competence or dynamic capabilities. This is purely an exogenously defined variable based on technology and industry types. Under a changed circumstances with respect to M_2 the leader doesn't really gain much in the competitive game. As can be observed from the plots, the results infact point to the follower winning the game due to much higher levels of quality and only marginally higher levels of investments.

It is interesting to note that there is apparently a paradox in trying to assess, both empirically and theoretically, the impact of competitive pressure on innovation and growth. On one hand, according to the tradition originating in Schumpeter (1942) the prospective reward provided by monopoly rent to a successful innovator is required to stimulate sufficient R&D investment and technological progress. On the other hand, the incentives to innovate are weaker for an incumbent monopolist than for a firm in a competitive industry (Arrow, 1962). When competition is intense in the product market, innovation may even be seen as the only way for a firm to survive. In a neo-Schumpeterian models of endogenous growth (Sergerstrom et al. 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992), innovation allows a firm in an industry to take the lead and gain profit. But the monopoly rent enjoyed by the winner is only temporary, and a new innovator, capitalizing on accumulated knowledge, is always able to "leapfrog" the leader unless the leader is endowed with advantages of firm asymmetries. In recent research literature, Aghion, Harris and Vickers (1997), and Aghion, Harris, Howitt and Vickers (2001), supposing a duopoly in each sector, both at the research and production levels, have introduced what they call "step-by-step innovation," according to which technological progress allows a firm to take the lead, but with the lagging firm remaining active and eventually capable of catching up. This model has been extended by Encaoua and Ulph (2000), allowing for the possibility that the lagging firm leapfrogs the leader, without driving it out of the market. The results in this essay support such a conjecture.

In this essay a simplified approach is adopted to consider spillover effects. Griliches(1979) lays out the conceptual framework and provides an early discussion of the importance of spillover effects of R&D. Later in 1992, Griliches reviewed the recent empirical evidence on spillovers, and tentatively concluded that spillover effects may be substantial.

The conventional approach used in innovation and R&D research viewed the process as one with constant returns, competitive output and factor markets and no externalities. However, such a framework doesn't offer a full explanation of productivity growth. For a better understanding it therefore becomes very important to consider increasing returns to scale, R&D spillovers and other externalities and disequilibria. This essay is an attempt to consider some of these essential features of innovation based competition.

Notwithstanding the nature of R&D process, however, two additional factors play an important role in setting a lower limit to the size of those firms engaged in developing and marketing new products. The first factor relates to the "unit project size" associated with the development and sales of a new product. For example, in the case of a new pharmaceutical drug, it is estimated to be of the order of 150 million dollars in 1990. This amounts to 0.1 percent of global sales revenue, which is equivalent to about 3 percent of the annual sales revenue of the industry's largest firms, and so it might seem at first glance to constitute a fairly modest entry fee. The second problem the small innovator faces relates to the size of the marketing effort required to support a new product launch. These factors should be considered to enrich our understanding of the competitive dynamics.

Chapter 4

Piecewise Deterministic Differential Game Between Two Collaborating Firms

4.1 Introduction

The study of innovation based competition has often considered aspects related to patent race and incremental product-process innovation to achieve distinctive advantage. However recently innovation based competition has has become an aspect of buyer-supplier relationships. There are many instances in manufacturing where we find situations of lock-ins created by innovative suppliers. For example, in the computer industry Intel and Microsoft as suppliers of microprocessor and operating system respectively to desktop manufacturers like IBM and Dell illustrate such innovation-based lock-ins. Indeed, there exists an evolving power structure in a supply chain driven by innovation competence of its members.

Cox et al.(2002) note that despite the undoubted significance of resource dependence perspective in disciplines such as political science, social psychology, sociology and economics, it has played relatively minor role thus far in supply chain management literature dealing with relationships between suppliers and buyers. The relative paucity of literature dealing with *power* in supply chain relationships might be partly attributed to a focus by academics and practitioners on concepts such as lean supply, which requires that firms be open, trusting and collaborative in their dealings with suppliers. While such notions might have appeal under certain circumstances it can be argued that other considerations such as the desirability of avoiding supplier lock-ins might induce technology and innovation based competition in buyer-supplier relationships.

Collaborative supply chain management is an active area of current research. The research theme ranges from operational coordination, for example, collaborative planning, forecasting and replenishment, to strategic aspects, for example, contracts and governance mechanism designs. In this essay, it is argued that collaborative supply chain management covers a broad spectrum of issues in which, aspects regarding cooperation and operational excellence fall on one end, and innovation-based "intrasupply chain competition" at the other end. Specifically, the competitive behavioral aspects associated with collaborative supply chains are investigated by examining the relative motivation for supply chain partners to invest in innovation. The theoretical analysis provides insights into evolution of power structure in supply chain as an outcome of inter-organization dialectics (Zeitz 1980).

This essay focuses on innovation-based buyer-supplier competition. A business context is considered in which, at any given point in time of the relationship, both the buyer and the supplier could be pursuing innovation simultaneously. It is recognized that the primary motivation for such investments in innovation by members of the supply chain is to increase their differential or relative channel power in the supply chain. Also in a situation where the buyer is locked-in by a supplier, the buyer may actively pursue the creation of a substitute technology by investing in innovation. The primary motivation for the buyer in this case would be to eliminate the technology lock-in and become independent.

The problem scenario considered in this research is one in which the supplier provides a product component to the buyer and it is assumed that the buyer is totally dependent on the supplier for developing the final product representing a situation of technology lock-in. The induced buyer-supplier competition is one in which the buyer actively pursues development of substitute technology to eliminate dependence on the supplier. A differential game formulation is used to examine the competition between the buyer and the supplier. Investigation of this type of buyer-supplier competition would lead to a better understanding of the dynamics of collaboration among supply chain partners.

An interesting aspect of the problem is the fact that the supplier must take into account the inherent incentives for the buyer to develop a "backstop" technology, which can be substituted for the supplied component. The supplier with the knowledge of this intent of the buyer acts such that the profits are maximized before the 'invention' of substitute technology by the buyer. The time when such innovation materializes is uncertain, but can be affected by R&D efforts. The competitive framework between the buyer and the supplier is presented in figure 4.1.

It can be observed in the framework that the buyer-supplier collaborative relationship is based on the notion of symbiosis. The reason why a buyer collaborates with a supplier is that the supplier's innovation capabilities enable the buyer to sell its products. This in turn results in an increased market share of the buyer and the supplier firm. However along with this synergetic inter-relationship, another form of relationship also evolves. This is represented in terms of the channel power. Supplier's innovation capability leads to an increase in supplier's relative channel power. This channel power of the supplier may be exploited by choosing (monopolistic) price and production quantities. In this situation locking in the buyer benefits the supplier and the buyer now faces a competitive situation within the supply chain. One of the means by which the buyer counters supplier's growth in channel power is by investing in innovation. As an example, Dell counters the lock-in created by innovative collaborating partners like Intel by investing in process innovation. With this process innovation the buyer can influence its own channel power. Yet another approach for the buyer is to invest in substitute technologies. A successful development of substitute technology would enable the buyer to set itself free from the lock-in created by the supplier. This new technology can now be used to improve its own market position.

The formulation is adapted from the research literature in exhaustible resources. Specifically the model formulation in this essay is similar to that in Harris and Vickers (1995), who analyze a dynamic game between a resource-exporting country and an importing country that is seeking to invent a substitute technology. The authors investigate the central question in the economics of exhaustible resources concerning incentives for the discovery of reproducible "backstop" technologies to substitute for finite natural resources that are being depleted. They assert that resource producers must take account of the possible invention of backstop technologies in deciding their extraction policies. Importing countries are motivated to discover backstop technologies not only to overcome the problem that resource stocks are finite, but also to reduce dependence upon resource producers, who often enjoy considerable degree of monopoly power over them.

The context of buyer-supplier competition that is considered in this essay is similar to the problem considered in Harris and Vickers (1995). In a supply chain environment, the supplier who is presently a monopoly supplier of components must take into account the possible creation of substitute technology on the part of the buyer in deciding on his pricing and production policies. By actively investing in innovation and consequently being successful in creating a substitute product, the buyer can come out of the lock-in created by a monopoly supplier.

The organization of this essay is as follows. Review of related literature is provided in the next section. The stochastic differential game model is presented in the third section. In the fourth section the differential game is analyzed and the Markov-perfect Nash equilibrium results are presented. Finally, the fifth section provides a discussion of the results. The proofs of theorems are provided in Appendix C.

4.2 Literature Review

Many recent research papers examine product development and supply chain management adopting an integrated approach. These research papers fall under two broad categories. One line of research investigates the design of physical supply chain for successful new product development. For example, Fisher (1997) argues that the optimal supply chain for innovative products is different from that of non-innovative products, because of the relative magnitude of direct production costs and the costs of a mismatch between supply and demand. Product design has also been found to contribute to leaner supply chains by postponing the point of differentiation in the order-fulfillment process (Lee 1996 , Lee and Tang 1997). Novak and Eppinger (2001) argue that complexity in product design and vertical integration of production are complements; that in-house production is more attractive when product complexity is high, as firms seek to capture the benefits of their investments in the skills needed to coordinate development of complex designs.

The other stream of research investigates strategic issues related to collaboration in a supply chain. It examines the evolution of relationship between collaborating firms while taking cognizance of the issues of power and lock-ins. The key issues involved with the innovation investment decisions of collaborating firms are explained in terms of transaction cost economics (Williamson, 1979) , property rights approach (Grossman and Hart, 1986), channel power and lock-ins (Cox et al. 2002).

Much of the work since 1970s has been grounded in transaction cost economics and in the concept of asset specificity. Asset specificity refers to the relative lack of transferability of assets intended for use in a given transaction to other users. Highly specific assets represent sunk costs that have relatively little value beyond their use in the context of specific transaction. High asset specificity requires strong contracts or internalization to combat threat of opportunism. The central element of this theory is that asset specificity gives rise to vertical integration because of the threat of opportunistic behavior on the part of at least one of a pair of interdependent firms (Williamson 1985) . In this paper asset specificity is captured by considering a one to one correspondence for supplier's component and the final product produced by the buyer. The model considers a case where the supplier and buyer are interdependent on each other and do not have any alternative firm to supply to or source to buy from.

Research on interorganizational relationships has attempted to identify fixed antecedents to cooperative relationships. For example, Aiken and Hage (1968) identify some of the internal organizational characteristics that lead to cooperation. Researchers have argued that resource dependency and uncertainty will affect both levels and types of collaboration (Pfeffer and Salancik 1978 ; Williamson 1975). Theoretical investigations have considered aspects of political economy (Berg and Zald, 1978) and institutional environments (Contractor and Lorange, 1988 ; Hall et al. 1977) as determinants of cooperation. Interactive theorists suggest that cooperation springs from the development of commitment between collaborating firms whereby firms come to care about their partners and cooperate out of altruism rather than specific exogenous requirements (Cook, 1977 ; Deutsch, 1962 ; Macneil, 1978).

In contrast to the above research themes, essays 3 and 4 investigate the formation of relationship as an outcome of an inherently competitive process. Firms enter into a relationship for gaining the advantage of partnership while at the same time managing the power dynamics in the channel. The research, commonly referred to as interorganizational dialectics, comes closest to this conceptualization of supply chain relationships. Within this theme, researchers have examined interorganizational cooperation arising in the context of a specific relationship and unfolding through an ongoing interaction. (Levinthal and Fichman, 1988 ; Van de Van and Walker, 1984 ; Zeitz, 1980 ; Heide and Miner, 1992).

A review of product development literature reveals that the new vision of product development is that of a highly disaggregated process. Products are becoming increasingly complex and it requires the collaboration of many firms to bring them to market (Eppinger et al. 1994 ; Eppinger, 1998 ; Ulrich, 2001). The literature on make-buy decisions and on vertical integration combines the inherent strategic inter-organizational issues with product development issues (Mahoney 1992 ; Armour and Teece, 1980 ; Fine and Whitney, 1996 ; Langlois and Robertson, 1989 ; Masten, 1984). Essays 3 and 4 consider such an integrated perspective on product development and examine some of the inherent strategic issues associated with it. In such an environment core competence and dynamic capabilities of a firm are of paramount importance. In a joint product development, the level of product-process innovation attained by each collaborating partner is one of the key indicators of core competence and dynamic capabilities (Wernerfelt, 1984 ; Prahalad and Hamel, 1990 ; Teece, Pisano and Shuen, 1997). Essay 3 and essay 4 focus on the possibility that anticipated future interactions may influence the development of sustainable competitive advantage by each collaborating partner.

4.3 The Model

4.3.1 Differential Game Formulation

The notations and symbols used in the model are as follows:

S: subscript for the supplier firm

B: subscript for the buyer firm

T: Finite time horizon for the strategies

t: An instant of time in the dynamic game setup

$u_B(t)$: Buyer's innovation effort (R&D expenditure per unit time)

$u_S(t)$: Supplier's production effort (Expenditure in production processes and resources per unit time)

$p(u_S)$: Price/ Inverse demand curve

$c_S(u_S)$: unit production cost to be incurred by the supplying firm

c_B: unit production cost to be incurred by the buying firm (Assumption $c_S \geq c_B$)

$c_U(u_B)$: Effort cost incurred by the buying firm for achieving the level of innovation that would be a perfect substitute for the supplier S's innovation

τ: random instant of time at which the level of innovation (breakthrough innovation) is reached

$F(t)$: the probability that the buying firm B is successful in the innovation project by time t, that is $Pr(\tau \leq t) = F(t)$

k_{01}: a function representing the switching rate

$x(t)$: state dynamics represented by remaining demand for the product

Denote the supplier firm by S and the buyer firm by B. The context of buyer-supplier relationship that is based on innovation and quality parameters can be considered in the broad rubric of resource-based view of a firm. Specifically, the vendor S is providing a resource in the form of innovation capabilities to the buyer firm B. There is a fixed known demand for the product. A one to one correspondence is considered such that for every product sold by the buyer to the final consumer, one component is required from the supplier. The supplier is assumed to be a monopolist and the buyer faces perfect competition in the final consumer market.

It is asserted that a one to one correspondence assumption helps in the analysis in two distinct ways. First, this allows explicit examination of dependence between the two collaborating partners and the evolution of power structure based on innovation. Second, this lends analytical simplification for solving the problem. The supplier S is a monopoly producer of a product component for which the cost of production is $c_S(u_S) > 0$ per unit. Supplier S's rate of production is a function of production effort u_S and is assumed to be γu_S. For analytical simplicity γ is assumed to be equal to 1. The rate of production is defined as, $u_S : (0, \infty) \rightarrow [0, \bar{u_S}]$. The remaining market demand (measured as a percent of initial demand x_0) is the state variable for the problem. In the context of the model specification and the underlying assumptions, the remaining demand x is identical for the buyer and the supplier. This remaining demand x, is assumed to be a function of the supplier's production rate and the rate of change of remaining demand is,

$$\dot{x}(t) = -\gamma u_S(x(t)) = -u_S(x(t)); \qquad x(0) = x_0$$

As production rate increases, the remaining market demand falls more rapidly. The price function of the buyer firm B, $p(u_S)$ is a function of the production rate of the supplier. Buyer invests to create a substitute technology which is assumed to enable production of the product component at a unit cost $c_B \leq c_S$. In the event of a successful creation of such a technology there will be no demand for the supplier's component. The investment policy in innovation chosen by B is denoted by $u_B : (0, \infty) \to [0, \infty)$. This investment represents buyer's efforts in R&D and is a function of the remaining demand $x(t)$. By investing in innovation, buyer gains knowledge and the time path for the buyer's level of knowledge $k(t)$ is given by,

$$\dot{k}(t) = u_B(x(t)); \qquad k(0) = 0$$

The outcome of innovation investments is considered to be uncertain. Therefore, it is asserted that the R&D investments don't precisely determine the date of success of innovation. A natural and convenient way to represent uncertain innovation is to suppose that the instantaneous probability of the buyer B innovating, conditional on not having so far done so, is a function of its current R&D effort rate. This "hazard rate" approach, which derives from an exponential probability distribution, has been used by numerous authors, e.g. Dasgupta and Stiglitz (1981) , Harris and Vickers(1995) . The probability that the buyer B has innovated by t is assumed to take an exponential form, which is independent of the accumulated knowledge level, $k(t)$.

$$\frac{\dot{F}(t)}{1-F(t)} = u_B(x(t)) \tag{4.1}$$

where, $F(t)$ is the probability that the buyer B is successful in the innovation project by time t, that is $Pr(\tau \leq t) = F(t)$.

Let $\pi(u_S) = [p(u_S) - c_S(u_S)]\gamma u_S$ be supplier's instantaneous profit function. This instantaneous profit function acts as the objective functional for the supplier. In the problem context, the buyer faces perfect competition. The buyer invests in innovation to become independent of the lock-in created by the supplier. Since the buyer is facing perfect competition, it maximizes consumer surplus. A figure representing the consumer surplus is provided in figure 4.2. Let the consumer surplus be denoted as $\sigma(u_S) = \int_0^{u_S} p(y)dy - u_S p(u_S)$. After the successful invention of substitute technology, a new market is created for the buyer's product. Now, the buyer can gain discounted profits depending upon the price $p_B(u_B)$ and the variable cost of production $c_B(u_B)$. The payoff of the buyer firm B is $\int_0^{\tau} e^{-rt}[\sigma(u_S(x(t))) - c_U(u_B(x(t)))]dt + \int_{\tau}^{\infty} e^{-rt}\pi_B(u_B)dt$, where c_U is buyer's cost of innovation effort, τ is the random time at which innovation occurs, $r > 0$ is the discount rate, p_B is the price that the buyer can charge for the product once the innovation materializes and $\pi_B(u_B)$ is the profit earned by the buyer after time τ. It is a function of the price charged and the variable cost of production . The payoff of the supplier firm S is $\int_0^{\tau} e^{-rt}\pi(u_S(x(t)))dt$.

The strategies $u_S(\centerdot)$ and $u_B(\centerdot)$ constitute an equilibrium if and only if, for all $x_0 \geq 0$, taking $u_B(\centerdot)$ as given, the production effort $u_S(\centerdot)$ maximizes supplier's payoff among all remaining strategies; and similarly, taking $u_S(\centerdot)$ as given, the innovation effort $u_B(\centerdot)$ maximizes buyer's payoff among all possible strategies. The following assumptions are considered for the inverse demand function, the profit function and the cost of effort function.

(i) The inverse demand function $p(u_S)$ is twice continuously differentiable on $[0, \bar{u_S}]$;

$p' < 0$ on this interval; $p(0) > c_S \geq 0$; and $p(\bar{u_S}) = 0$. This assumption signifies that there is a finite price $p(0)$ to sustain a profitable production, the inverse demand function is downward sloping, and that the total demand for the supplier's output at any given time is bounded by the maximum possible production rate $\bar{u_S}$.

(ii) The profit function $\pi(u_S)$ is strictly concave on $[0, \bar{u_S}]$: $\pi'' < 0$, which is equivalent to $2p' + p''u_S < 0$.

(iii) The cost of production effort $c_S(u_S)$ is continuously differentiable on $[0, \bar{u_S})$; $c_S(0) = 0$ on $[0, \bar{u_S})$; and $c_S'(y)$ is a constant. The cost of innovation effort $c_U(u_B)$ is twice continuously differentiable on $[0, \infty)$; $c_U(0) = c_U'(0) = 0; c_U'' > 0$ on $[0, \infty)$; and $c_U'(y) \to \infty$ as $y \to \infty$. This assumption suggest that costs increase and are strictly convex in effort. It also suggests that zero R&D effort is sub optimal and for the strategy u_B we can confine our interest to interior solutions.

In this market setting, it is assumed that the product enjoys a very large fixed demand. Furthermore, whatever is produced by the supplier gets sold and hence the production rate of the supplier equals the quantity demanded per unit time in the market. The inverse demand function or in other words the price charged by the supplier is conceptualized as a linear decreasing function of units produced per unit time.

$$p(u_S(x(t))) = a - b\gamma u_S(t) = a - bu_S(t); \qquad a > c_S > 0,\ b > 0 \text{ are constants} \quad (4.2)$$

The variable cost of production is assumed to be linear and increasing function of production effort u_S.

$$c_S = c_1[u_S]; \qquad c_1 > 0 \text{ is a proportionality constant} \quad (4.3)$$

The cost of effort function for the buyer is assumed to be strictly convex and increasing in innovation effort u_B.

$$c_U = c_2[u_B]^2; \qquad c_2 > 0 \text{ is a proportionality constant} \tag{4.4}$$

Note that this problem has the same state dynamics for the buyer and the supplier. The objective functional of supplier S is the expected profit till the random instant of time τ,

$$\begin{aligned} J^S(x) &= E_{u_B(.)} \int_0^{\tau} e^{-rt}[p(u_S(x(t))) - c_S(u_S)]\gamma u_S(x(t))dt \\ &= E_{u_B(.)} \int_0^{\tau} e^{-rt}[a - bu_S(x(t))) - c_1 u_S(x(t)))]u_S(x(t)))dt \end{aligned} \tag{4.5}$$

Note that the supplier S receives zero profit for $t > \tau$ (since demand vanishes). The objective functional of firm B is the expected gain from innovation efforts, given by,

$$\begin{aligned} J^B(x, u_B) &= E_{u_B(.)} \int_0^{\tau} e^{-rt}[\sigma(u_S(x(t))) - c_U(u_B(x(t)))]dt + \int_{\tau}^{\infty} e^{-rt}\pi_B(u_B)dt \\ &= E_{u_B(.)} \int_0^{\tau} e^{-rt}[\sigma(u_S(x(t))) - c_2[u_B(x(t))]^2]dt + \int_{\tau}^{-rt\pi_B(u_B)dt} \end{aligned} \tag{4.6}$$

where,

$$\sigma(u_S(x(t))) = \int_0^{u_S(x(t))} p(y)dy - u_S(x(t))p(u_S) = \int_0^{u_S(x(t))} [a - by]dy - u_S(x(t))[a - bu_S(x(t))] \tag{4.7}$$

is the surplus gained by the customers if the buyer firm B is able to accomplish the required innovation level.

The problem formulation for the supplier is:

$$
\begin{aligned}
J^S(x) &= E_{u_B(.)}\int_0^{\tau} e^{-rt}[p(u_S(x(t))) - c_S(u_S)]u_S(x(t))dt \\
&= E_{u_B(.)}\int_0^{\tau} e^{-rt}[a - bu_S(x(t)) - c_1u_S(x(t))]u_S(x(t))dt
\end{aligned}
$$

subject to

$$
\dot{x}(t) = -u_S(x(t)); \qquad x(0) = x_0 \tag{4.8}
$$

The problem formulation for the buyer is,

$$
\begin{aligned}
J^B(x, u_B) &= E_{u_B(.)}\int_0^{\tau} e^{-rt}[\sigma(u_S(x(t))) - c_U(u_B(x(t)))]dt + \int_{\tau}^{\infty} e^{-rt}\pi_B(u_B)dt \\
&= E_{u_B(.)}\int_0^{\tau} e^{-rt}[\sigma(u_S(x(t))) - c_2[u_B(x(t))]^2]dt + \int_{\tau}^{\infty} e^{-rt}\pi_B(u_B)dt
\end{aligned}
$$

subject to,

$$
\dot{x}(t) = -u_S(x(t)); \qquad x(0) = x_0 \tag{4.9}
$$

The differential game defined by (4.8) and (4.9) belongs to the class of piecewise deterministic games. The game has two modes only: mode 0 is active before firm B has made the innovation and mode 1 becomes active if firm B succeeds in making the substitute technology. Thus, there can be at most one switch of mode. The switching time is the random variable τ and the probability distribution of τ is F, as given in equation (4.1).

4.4 Analysis of the model

The variables are written without the arguments for notational convenience. For example, $u_S(x(t)) = u_S$. In the game specified in equation (4.8) and equation (4.9), the nature of stationary Markov perfect Nash equilibrium conditions for the supplier and buyer are analyzed. Supplier S's stationary Markovian strategy for production effort $U_S(h,x)$ is a mapping $U_S : M \times (0,\infty) \mapsto [0, \bar{u_S}]$, where $M = 0,1$ is the set of modes. Buyer B's stationary Markovian R&D effort strategy $U_B(h,x)$ is a mapping $U_B : M \times (0,\infty) \mapsto [0,\infty)$. The solution approach in Harris and Vickers (1995) is adopted and is presented below.

Let the value functions of the supplier and buyer be defined as $V^S(h,s)$ and $V^B(h,s)$ respectively for $h \in M = \{0,1\}$ and $x \in (0,1)$. As required from the definition and theorem pertaining to piecewise deterministic differential game provided in Appendix C, these value functions are assumed to be bounded and continuously differentiable and for all $x \in [0,1]$ they satisfy the HJB equations,

$$\begin{aligned} rV^S(0,x) = max\Big\{&\pi(u_S) - u_S V_x^S(0,x) \\ &+ U_B(0,x)[V^S(1,x) - V^S(0,x)]\Big| u_S \in [0,\bar{u_S}]\Big\} \end{aligned} \tag{4.10}$$

$$rV^S(1,x) = 0 \tag{4.11}$$

$$\begin{aligned} rV^B(0,x) = max\Big\{&\sigma(U_S(0,x) - c_U(u_B) - U_S(0,x)V_x^B(0,x) \\ &+ u_B[V^B(1,x) - V^B(0,x)]\Big| u_B \in [0,\infty]\Big\} \end{aligned} \tag{4.12}$$

$$rV^B(1,x) = \pi_B(u_B) \tag{4.13}$$

Equations (4.11) and (4.13) present the value functions for the supplier and the buyer once the system switches to mode 1. As reasoned earlier, the supplier's profit stream becomes zero after the switch as there will be no demand for supplier's product component. On the other hand, in the event of being successful in making the innovation, the buyer earns the present value of the constant stream of consumer surplus over an infinite interval of time. Additionally, irrespective of the system mode, it can be observed that the supplier's profit will become zero if there is no remaining demand. In the absence of any production by the supplier, it is assumed that the buyer gets a constant expected payoff $\underline{V^B}$, which is the optimal value of a one player stochastic control problem with buyer B as the decision maker. Hence, the value functions are subjected to two constraints,

$$V^S(h,0) = 0 \quad \forall \quad h \in M$$

$$V^B(0,0) = \underline{V^B} \tag{4.14}$$

The maximizing sets of $u_S \in [0, \bar{u_S}]$ and $u_B \in [0, \infty)$, that maximize the respective value functions of the supplier and the buyer in equation (4.10) and (4.12) are denoted as $\Phi^S(x)$ and $\Phi^B(x)$ respectively. If $U_S(0,x) \in \Phi^S(x)$ and $U_B(0,x) \in \Phi^B(x)$ for all $x \in [0,1]$, and provided that the value functions can be found that satisfy HJB equations, then from the theorem in Appendix C, the strategy pair $(\mathrm{U}_S(h,x), U_B(h,x))$ is a stationary Markov perfect Nash equilibrium. To determine the value functions, the equations (4.10) and (4.12) can be rewritten as,

$$rV^S(0,x) = G^S(V^S_x(0,x)) - U_B(0,x)V^S(0,x) \tag{4.15}$$

$$rV^B(0,x) = \sigma(U_S(0,x)) - U_S(0,x)V^B_x(0,x) + G^B[\bar{V^B} - V^B(0,x)] \tag{4.16}$$

where,

$$G^S(z) = max\{\pi(u_S) - u_S z | 0 \leq u_S \leq \bar{u_S}\}; \quad G^B(z) = max\{z u_B - c_U(u_B) | 0 \leq u_B\}$$

$$U_S(0,x) = \text{argmax}\{\pi(u_S) - u_S V^S_x(0,x) | 0 \leq u_S \leq \bar{u_S}\} \tag{4.17}$$

$$U_B(0,x) = \text{argmax}\{[\bar{V^B} - V^B(0,x)]u_B - c_U(u_B) | 0 \leq u_B\} \tag{4.18}$$

Solution of the system of autonomous nonlinear differential equations (4.15) - (4.18) leads to the determination of the two value functions. Substituting the expressions for the demand function and the cost of effort function from equations (4.2) and equation (4.4) respectively,

$$G^S(z) = max\{[a - bu_S - c_1u_S]u_S - u_Sz|0 \leq u_S \leq \bar{u_S}\} \tag{4.19}$$

$$G^B(z) = max\{zu_B - c_2[u_B]^2|0 \leq u_B\} \tag{4.20}$$

$$U_S(0,x) = \text{argmax}\{[a - bu_S - c_1u_S]u_S - u_SV_x^S(0,x)|0 \leq u_S \leq \bar{u_S}\} \tag{4.21}$$

$$U_B(0,x) = \text{argmax}\{[\bar{V^B} - V^B(0,x)]u_B - c_2[u_B]^2|0 \leq u_B\} \tag{4.22}$$

where,

$$\bar{V^B} = \frac{\pi_B(u_B)}{r} \tag{4.23}$$

4.4.1 Equilibrium Results

Theorem 1 *If V^{S*} and V^{B*} denote the positive equilibrium value of the supplier and buyer firm, then,*

$$V^{S*} = \frac{a^2\sqrt{c_2}}{\sqrt{2\{8r(b+c_1)^2(\bar{V^B}+rc_2) - a^2b\}}} \tag{4.24}$$

$$V^{B*} = 2c_2r + \bar{V^B} - \frac{\sqrt{c_2}\sqrt{8r(b+c_1)^2(\bar{V^B}+rc_2) - a^2b}}{\sqrt{2}(b+c_1)} \tag{4.25}$$

constitute the equilibrium pair.

Theorem 2 *If U_S^* and U_B^* denote the positive equilibrium control functions of the supplier and buyer firm, then,*

$$U_S^*(0,x) = \frac{a - V_x^S(0,x)}{2(b+c_1)} = \frac{a}{2(b+c_1)} \tag{4.26}$$

$$U_B^*(0,x) = \frac{\bar{V^B} - V^B(0,x)}{2c_2} = \frac{\sqrt{8r(b+c_1)^2(\bar{V^B}+rc_2) - a^2b}}{2\sqrt{2}(b+c_1)\sqrt{c_2}} - r \tag{4.27}$$

Theorem 3 *At equilibrium the remaining demand at time t can be expressed as:*

$$x(t) = x_0 - \frac{at}{2(b + c_1)} \tag{4.28}$$

where $x(0) = x_0$ is the initial remaining market demand.

4.5 Discussion

Observation 1 *At equilibrium the supplier chooses a monopoly production rate.*

The observation can be explained by exploring the monopoly production rate of the supplier in the given setting. The monopoly profit of the supplier π_m is

$$\pi_m = max\{\pi(u_S)|u_S \in [0, \bar{u_S}]\} = max\{[a - bu_S - c_1[u_S]]u_S|u_S \in [0, \bar{u_S}]\}$$

The corresponding production effort required to attain a monopoly profit is therefore,

$$u_{Sm} = argmax\{\pi(u_S)|u_S \in [0, \bar{u_S}]\} = argmax\{[a - bu_S - c_1[u_S]]u_S|u_S \in [0, \bar{u_S}]\}$$

Taking the first order condition leads to the monopoly production rate given by:

$$u_{Sm} = \frac{a}{2(b + c_1)}$$

The supplier is assumed to be a monopoly. In the given setup the supplier employs monopoly production rate to gain as much of monopoly profits as possible before being locked out of the market. An illustration of such a monopoly production is provided in figure 4.3. At equilibrium the supplier exercises monopoly power and allows the buyer to pursue creation of substitute technology. The supplier sets the monopoly production rate at the start of the game and it remains time invariant for the entire planning horizon. Next, the observation regarding innovation effort of the buyer is presented.

Observation 2 *(i) The innovation effort of the buyer increases with the potential value of time discounted profit $\bar{V}^B$ that the buyer can receive in the event of successful creation of substitute technology.*
(ii) The monopoly production rate of the supplier exerts a negative effect on buyer's innovation rate.

The observation stated in 2(i) is intuitive and can be observed from the expression for buyer's equilibrium control given in equation (70). The motivation for the supplier to invest in substitute technology would indeed be dependent on the discounted profits that the buyer can obtain in case the innovation is successful. It can be noted that the time of successful 'invention' is uncertain but it can be influenced by buyer's innovation efforts. This is expressed in equation (4.1). For observation 2(ii) the expression for equilibrium innovation rate by the buyer U_B^* is revisited. With some algebraic manipulations the equilibrium innovation rate of the buyer can be expressed as:

$$U_B^* = \sqrt{\frac{\bar{V}^B + rc_2}{c_2} - \frac{(U_S^*)^2 b}{2c_2}} - r \qquad (4.29)$$

From the above expression it can be noted that the supplier's monopoly production rate exerts a negative effect on the buyer's innovation efforts. Observation 1 and 2 characterize the nature of equilibrium control of the buyer and the supplier. Indeed, in the given setting there exists no motivation for the two players to collaborate and each acts in its best interests. The supplier makes full use of the monopoly power and the buyer continues investments in innovation to create substitute technology.

However, as asserted by Schelling (1980) pp.4, if we confine our study to the theory of strategy, we seriously restrict ourselves by the assumption of rational behavior. It is important to note that the result obtained doesn't necessarily suggest "intelligent" behavior but the equilibrium results are suggestive of a behavior motivated by a conscious calculation of advantages, a calculation that in turn is based on explicit and internally consistent value systems. Still, the assumption of rational behavior is a productive one as it gives us a grip on the subject. In the problem context the availability of equilibrium solution based on the assumption of rationality permits us to identify our own analytical processes with those of the hypothetical participants in a conflict. By further investigation of certain kind of consistency in the behavior of our hypothetical participants, we can examine alternate courses of behavior according to whether or not they meet those standards of consistency.

The equilibrium control of the buyer and the supplier presents a dilemma in achieving a long-term collaborative supply chain partnership. The solution for this dilemma for both firms is to create close ties with one another so as to minimize the risk of opportunism. These ties should deter the supplier from using monopoly power and the buyer from investing in substitute technology. The solution is to build credible commitments into the relationship. The next observation presents insights for building such credible commitments.

Observation 3 *At equilibrium if the supplier's production rate is such that* $U_s^* = \sqrt{\frac{2}{b}[\frac{r\bar{V}^B+c_2}{c_2} - r^2]}$ *then the buyer sets its innovation effort to zero.*

Analytically, the above expression can be obtained by setting $U_B^* = 0$ in equation (4.29) and some algebraic manipulations. Note, that in equation (4.29) we require $\frac{\bar{V}^B+rc_2}{c_2} > \frac{(U_S^*)^2 b}{2c_2}$ to ensure a real value of the expression. Moreover, it is also intuitive to note that the only way supplier can make the buyer set its innovation efforts to zero is by increasing production rate. It can be observed from figure 3 that with a downward sloping demand curve, the price charged by the supplier decreases with an increase in its production. The inverse demand curve (price) of the supplier's product component is $(a - bu_S)$. The monopoly price charged by the supplier is therefore $p^* = a - b\left[\frac{a}{2(b+c_1)}\right]$. With the changed production rate, the price becomes

$\tilde{p} = a - b\Big[\sqrt{\frac{2}{b}\Big[\frac{r\bar{V}^B}{c_2} + c_2 - r^2\Big]}\Big].$

The observation suggests that there is a price bandwidth that the supplier can credibly commit to the buyer and achieve a long-term partnership. It can be reasonably asserted that based on the cost structures there is a $\underline{p}$ below which the supplier makes losses. Theoretically, at this price marginal cost equals average revenue. Therefore the price range that a supplier can charge and achieve a long-term relationship with the buyer can be given as $\underline{p} < p \leq \tilde{p}$. With this price range the supplier can create a disincentive for the buyer to invest in substitute technology. In the problem context considered in this paper, the motivation for the supplier to adopt such a policy depends on two considerations. First, the price should be atleast high enough to cover marginal costs. Second, if the supplier wishes to remove uncertainty in the relationship and intends to be the preferred partner for the entire planning horizon, the supplier will adopt this policy. Particularly the second consideration depends on the profits that can be earned for the time period $[0, \tau]$ using a monopoly pricing with the profits that can be earned for the entire planning horizon $[0, \infty)$ by judiciously choosing a price from the given range. The choice of price from the given range would be such that the supplier starts with a price $p = \tilde{p}$ and gradually reduces it over time till it reaches $\underline{p}$.

It is important to note that in the problem context, time τ is not deterministic and is characterized by the hazard rate. The time instant τ could approach ∞ or alternately it could be very close to 0, depending on the outcome of innovation investments by the buyer. The pricing scheme obtained from this analysis provides implications for the supplier to create conditions for trust-based governance. The formation of buyer-supplier trust emanates from certain proactive measures taken by the collaborating partners as a part of their contract. Cox et al.(2002), pp.14 suggest some other aspects for forming long-term relationships. To avoid potential conflict between a buyer and a supplier due to relative difference in innovation competence, one approach is to have either the buyer or the supplier make dedicated investments. The other partner could protect its interest by posting a bond that it would forfeit if the partner investing in innovation acts opportunistically. The authors asserts, "This bond, which is known in transaction cost parlance as a *hostage*, would be used to create a condition of bilateral dependency (more commonly referred to as interdependency)." Yet another approach to create conditions for interdependency is that both the collaborating partners invest in innovation. Such joint innovation activity strengthens the bond between collaborating firms and enables long-term relationship formation.

From the supplier's perspective an ideal contract is one that is large, relatively simple, and therefore cheap to service and that is likely to result in repeat business. The utility that a supplier attaches to a particular exchange relationship is typically highest in those instances that involve a key contract in a key segment. Along with the relative utility derived from the buyer's business, the supplier also considers the aspect of substitutability of buyer's business. Specifically, the supplier considers the likelihood that it will be able to replace the buyer's business with an exchange of equivalent utility, if it loses the business. The answer depends on the market structure in which the supplier sells and on its competitive position relative to other suppliers. A higher frequency of transactions offers the supplier stability and certainty. From the buyer's perspective, creating conditions for repeat business allows the supplier to cover fixed and semi-fixed costs and justifies supplier's investments in speculative investments.

The specific scenario considered in this essay is an instance of complex bargaining situation that prevails between the buyer and the supplier. Williamson (1987) pp.27-37, provide a detailed explanation of evolution of such situations in terms of market failures. This explanation is presented here for enhancing understanding of the underlying dynamics. The market failures referred here are only failures in the limited sense that they involve transaction costs that can be attenuated by substituting internal organization for market exchange. According to Williamson, the argument proceeds in five stages. The first three stages are concerned with characterizing a successively more complex bargaining environment in which small numbers relations obtain. The last two deals with the special structural advantages which, either naturally, or because of prevailing institutional rules, the firm enjoys in relation to the market. The first three stages are specifically of interest in the context of this essay and are explanined next.

In an industry that produces a multicomponent product, some of these components can be assumed to be specialized (industry specific), and that among these there are components for which the economies of scale in production are large in relation to the market. In such a situation, the market will support only a few efficient-sized producers for certain components. A monopolistic excess price over cost under market procurement is commonly anticipated in these circumstances. This is the specific context of this essay as well. However, Demsetz(1968) also noted that this need not be the case of there are large numbers of suppliers willing and able to bid at the initial contract award stage. According to Williamson, if we assume that large numbers bidding is not feasible, then the postulated condition afford an apparent incentive for assemblers (buyers) to integrate backward or suppliers to integrate forward.

One case that could exist in such a situation is that of bilateral monopoly. Bilateral monopoly requires that both price and quantity be negotiated. Both parties stand to benefit, naturally, by operating on rather than off the contract curve - which here corresponds to the joint profit-maximizing quantity (Fellner, 1947). This, however, just establishes the quantities to be exchanged. The price and other terms of exchange still need to be determined. Any price consistent with non-negative profits to both parties is feasible. Bargaining can be expected to take place. A potential adaptation under such circumstances is to internalize the transaction through vertical integration; but a once-for-all contract might also be negotiated.

The other case is one in which there is a competitive assembly with monopolistic supply. This is indeed the case considered in this essay. Such case leads to a more complicated stage of market failure referred to as "strategic misrepresentation risks" in Williamson (1987). To understand the characteristics of "strategic misrepresentation risks" stage it is useful to first understand the stage that precedes it and that succeeds the "static market" stage discussed in the previous paragraph. This stage is called "contractual incompleteness". Let us assume that the product is technically complex and that periodic redesign and/or volume changes are made in response to changing environmental conditions. Furthermore, the assumption of infeasibility of large numbers bidding at the initial contract award stage is relaxed. Three alternative supply arrangements can be considered in the stage of "contractual incompleteness": a once-for-all contract, a series of short-term contracts, and vertical integration.

According to Williamson, the dilemma posed by once-for-all contract is that contingent supply relations ought to be exhaustively stipulated because otherwise independent parties will interpret contractual ambiguities to their own advantage. In such situations, the differences can be resolved only by haggling, or ultimately, litigation. But, exhaustive stipulation is quite costly. "Thus, although, if production functions were known, appropriate responses to final demand or factor price changes might be deduced, the very costliness of specifying the functions and securing agreement discourages the effort. The problem is made even more severe where a changing technology poses product redesign issues. Here it is doubtful that, despite great effort and expense, contractual efforts reasonably to comprehend the range of possible outcomes will be successful." In such cases, short term contracts, which would facilitate adaptive, sequential decision making might therefore be preferred. However, if (a) efficient supply requires investment in special-purpose, long-life equipment or (b) the winner of the original contract acquires a cost advantage, say by reason of 'first mover' advantages (such as unique location or learning, including the acquisition of undisclosed or proprietory technical and managerial procedures and task specific labor skills) it poses problems. With condition (a), optimal investment considerations favor the award of a long-term contract so as to permit the supplier confidently to amortize the investments. But as was stated earlier, long-term contracts pose adaptive, sequential decision-making problem. Thus in this instance optimal investment and optimal sequential adaptation processes are in conflict. The "first mover" advantage also leads to potential problems. For example, unless the total supply requirements are stipulated, 'buying in' strategies are risky. An aggressive buyer may attempt to obtain a price at a level of current costs on each successive rounds and this could lead to haggling. Short-term contracts thus experience what may be serious limitations in circumstances where nontrivial first-mover advantages obtain.

In consideration, therefore, of the problems that both long and short-term contracts are subject to, vertical integration is one way out. The conflict between efficient investment and efficient sequential decision making is therefore avoided. It is relevant to note that the technological interdependency condition involving flow process economies between otherwise separate stages of production is really a special case of the contractual incompleteness argument. On the one hand, it may be prohibitively costly, if not infeasible, to specify contractually the full range of contingencies and stipulate appropriate responses between stages. On the other hand, if the contract is seriously incomplete in these respects but, once the original negotiations are settled, the contracting parties are lcoked into a bilateral exchange, the divergent interests between the parties will predictably lead to individually opportunistic behavior and joint losses. Williamson suggests, that advantages of vertical integration are not that technological (flow process) economies are unavailable to non-integrated firm, but that integration harmonizes interests (or reconciles differences often by fiat) and permits an efficient (adaptive, sequential) decision process to be utilized.

It can be noted that the "contractual incompleteness" stage develops where there is an *ex ante* but not necessarily *ex post* uncertainty. In contrast, the next stage, "strategic misrepresentation risks," are serious where there is uncertainty in both respects. Not only is the future uncertain but it may not be possible, except at great cost, for an outside agency to establish accurately what has transpired after the fact. The model considered in this essay presents such a situation. Under such circumstances, Williamson asserts that the advantages of internalization resides in the facts that the firm's *ex post* access to the relevant data is superior, it attenuates the incentives to exploit uncertainty opportunistically, and the control machinery that the firm is able to activate is more selective. In the model where the buyer faces a competitive market, and the component supplier is a monopoly, the monopolistic supply prices provide an occasion for vertical coordination. Such integration effort, however, is dependent on production technology and policing expense. Alternately, as illustrated in this essay, the monopolistic supply prices also allow exploration of price conditions that could lead to a long-term relationship between the supply chain partners.

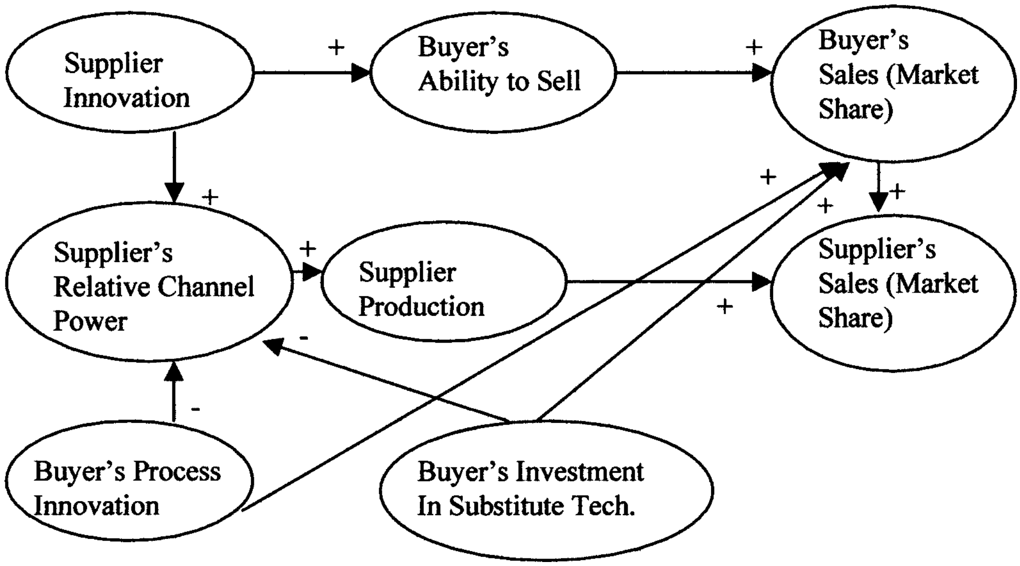

Figure 4.1: Buyer-Supplier Competitive Framework

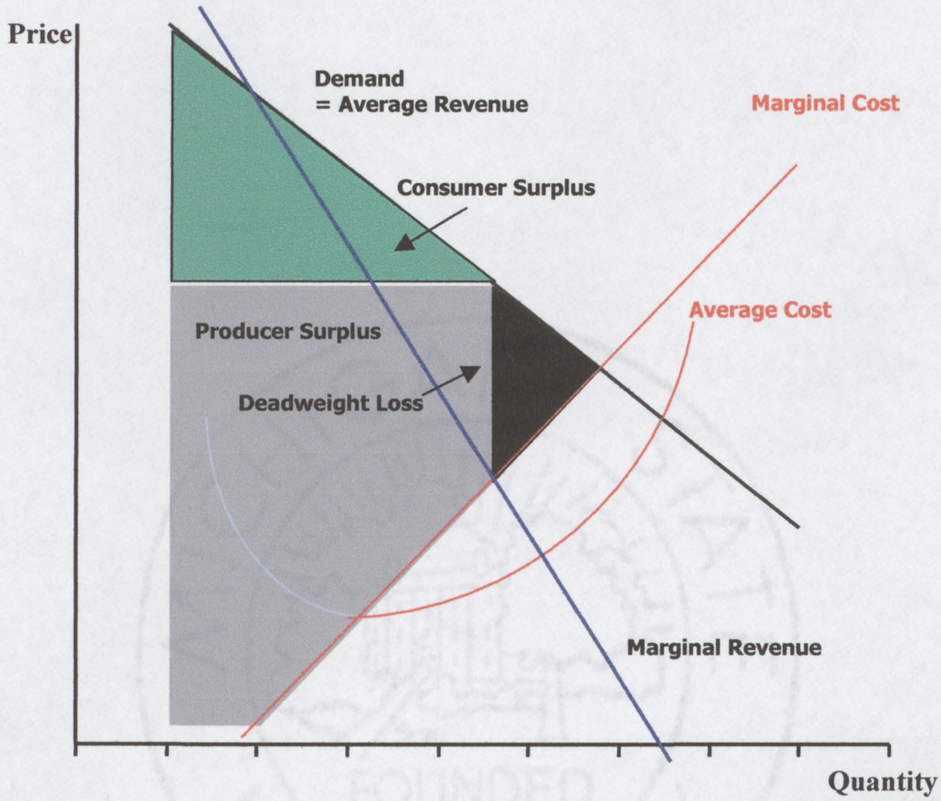

Figure 4.2: Consumer surplus

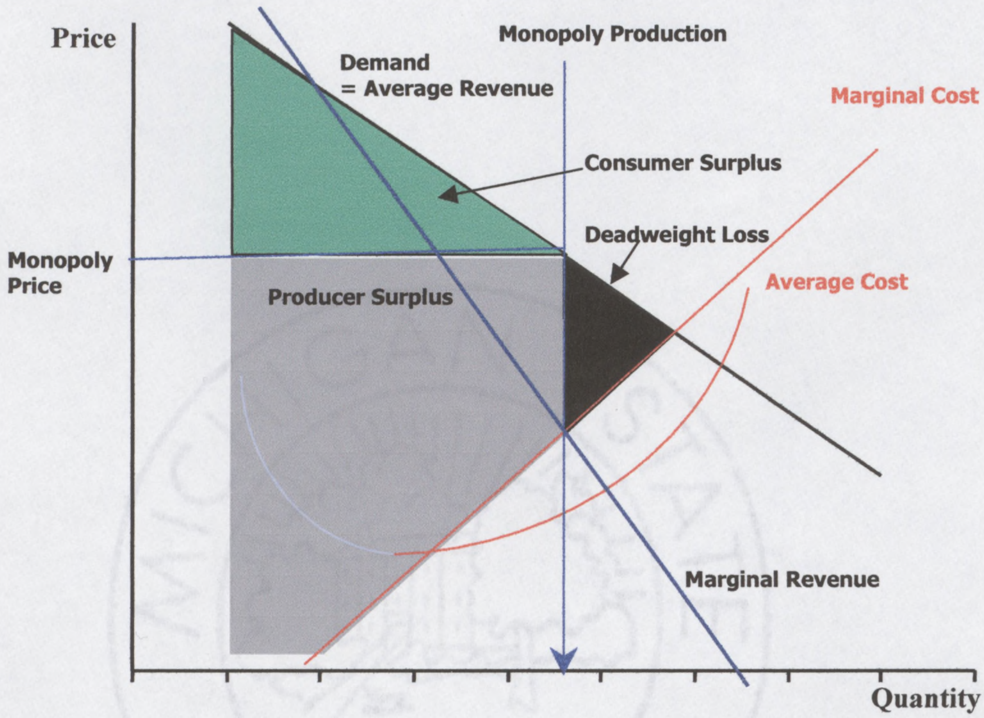

Figure 4.3: Monopoly production and price

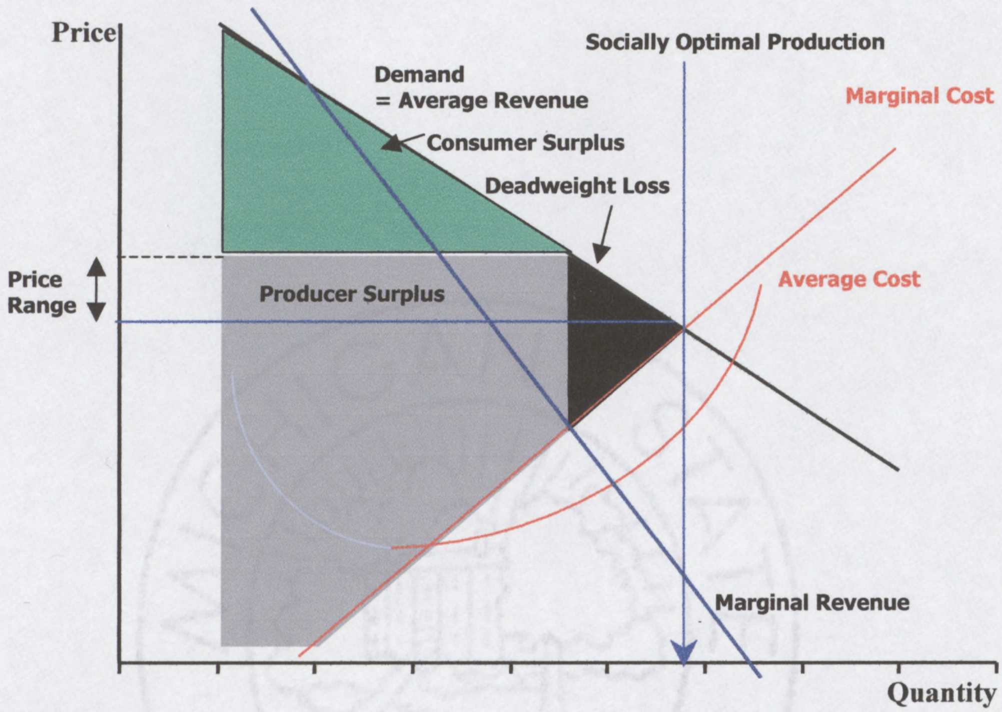

Figure 4.4: Price Range

Chapter 5

Stochastic Differential Game Between Two Collaborating Firms

5.1 Introduction

This chapter covers the fourth essay of this thesis. This essay characterizes the power struggle among supply chain partners. It is asserted that when firms are collaborating to deliver value in the market, the creation of critical asset requires more than just monopoly ownership of supply of a resource over competitors. A critical asset can only be truly owned and/or controlled effectively to leverage value if there is a dominance of one party in an exchange relationship over another. The implication is that to better understand the rent-earning capability of any supply chain resource, the relative power attributes of both buyer and supplier must be understood. Specifically these rents are earnings in excess of the firm's costs of production that are not eroded in the long run by new market entrants. In the language of economics, rents persist in long-run equilibrium while profits tend towards zero (Cox et al., 2002).

The purpose of this essay is to provide theoretical framework and insights regarding power and competition in a collaborative supply chain setup. The essay builds on the resource-based school of strategic thinking specifically in the context of collaboration. Williamson's(1975, 1985, 1999) transaction cost approach provides yet another grounding for understanding the fundamental basis on which relationship between buyer and supplier takes place. With multiple firms constituting a supply chain, investments by supply chain partners have implications that transcend the traditional cost minimization or revenue/profit maximization objectives. In today's dynamic environment, firms are investing in unpredictable innovations and associated strategies to gain the first-mover advantage.

Today firms strategically decide to enter a collaborative relationship. In a joint product development context, many firms outsource the manufacturing process of components which would be used in the final product. At times, this outsourcing goes beyond just the manufacturing of a fully specified component to allowing and expecting the supplier to build resource competence through active innovation. An example can be found in the supply chain of cars as provided in Cox et al.(2002). Automobile production begins with design, which consists of three main elements. The first element is the concept itself. Thereafter, the design of the vehicle can be usefully divided into the macro-design (the development of the basic chasis, sub-assembly and component specification) and the micro-design (the development, to agreed specifications, of the vehicle's constituent components). The first two elements of design tend to be undertaken by the car assembler. In particular, the assembler takes charge of concept origination. In the face of intense competition, however, the costs associated with developing new vehicle prototypes have increasingly forced car assemblers to source the design of sub-assemblies and components (the micro-design) from external suppliers. The degree to which such outsourcing is undertaken by assemblers varies between different firms. In the European car industry, some assemblers, notably the German manufacturers of prestige/executive vehicles, tend to be quite conservative in their outsourcing strategies. Other companies, like Rover, tend to compete more on cost than quality, and have been much keener to rid themselves of the burden of micro-design overheads.

The assemblers who outsource the micro-design to external suppliers have a motivation to let these suppliers grow larger so that the supply bases can be brought up to global standards. These larger suppliers would then be required to take full responsibility for the design of sub-assemblies and for the coordination of the second- and third-tier component manufacturers that contribute to the product. Today, many of the big sub-assemblers in Europe for example, Bosch, Lucas-Valeo, Magnetti, ZE and GKN, are almost as large as some of the car assemblers that they supply (EIU 1997a,b). The strong market position of these sub-assemblers is further enhanced by product specialization. No single supplier produces all types of sub-assembly. Bosch, which is the world's largest automotive equipment manufacturer, targets its efforts on starter systems, spark plugs, braking systems, lighting and windscreen wipers.

Valeo dedicates its efforts towards starter equipment, heating and cooling systems, clutches and lighting. Tenneco, Bosal and Arvin are Europe's major suppliers of car exhaust systems; Delphi, Johnson, Lear, Recaro and Bertrand Daure are the major manufacturers of seating; and Fichtel Sachs, Valeo, AP and Quentin Hazell are the principal producers of clutches. The net result of market consolidation is that the supply of particular sub-assembly systems has become concentrated amongst just a handful of manufacturers.

In such circumstances, firms enter the crossroads of a very delicate strategic supply chain relationship. Specifically, a strategy ought to be in place to defend the ability to appropriate and accumulate value by ensuring that the suppliers of the resources that the firm chooses not to own are not able to put themselves in a position to leverage value from the firm. The PC industry provides an excellent example of power diffusion up the supply chain. In 1981 IBM designed product, process and supply chain such that it sources the microprocessors from Intel and the operating system and application software from Microsoft. The outcome was a phenomenally successful product design but a disastrous supply chain design for IBM. Today, the power of Intel in the supply chain for PCs is undisputed. The new innovations that occur in this industry are to a great extent defined by this upstream supplier of microprocessors. The lesson learnt is to beware of "Intel inside" syndrome (Fine, 1999).

Extending this argument to the upstream microprocessor industry also provides some interesting observations. During 1960s, the practice of second sourcing whereby innovative firms license production to one or more manufacturers that can act as a second source of any new product had already developed. It was alleged that some sole suppliers of semiconductors "exploited" their customer firms once they had locked in their product designs to that of supplier's product. This feature of the industry profoundly affected the evolution of market structure, for it opened up a new and attractive strategy for second sourced suppliers. A firm would enter as a second sourcer and learn to produce high volumes efficiently while offering a leading edge product identical to that of the innovating firm. Once this hurdle was surmounted, it could use its growing cash flow to support a larger R&D effort with a view to developing its own next generation products. For example, AMD operated as a second sourcer in its early years. The company's president, Jerry Saunder's described its strategy as one of "planting cash crops" with a payback period of a few months and moving to longer-term projects only as cash flows grew (Wilson, Ashton and Egan 1980, pp.58-9).AMD's annual R&D budget in the early 1970s stood at 6 million dollars compared to Intel's 193 million dollars (Dorfman 1987, pp.211). By 1975, however, AMD could launch its own (4-bit) microprocessor (Wilson, Ashton and Egan 1980, pp.96). By providing a family of compatible devices, AMD achieved considerable success, and by 1978 half a dozen companies, were now second sourcing AMD's product (Sutton 2001).

With increasing formation of collaborative supply chain networks, research regarding channel power and lock-in circumstances in a supply chain is of paramount importance. Cox et al. (2002) highlight the need to undertake rigorous analytical research in this sphere of supply and value chain networks to augment our understanding of power regimes. The results of an analytical research could potentially provide understanding of the types of countervailing strategy to shift the balance of power in a supply chain.

This essay analytically examines this issue using a differential games based approach. A model of competitive dynamics between a supplier and a buyer is presented using the theory of stochastic processes and differential games. The chapter analyzes the context of buyer-supplier competition, by adapting and building on Browne (2000), which is primarily targeted towards investigation of portfolio investment strategies in finance. In this research, the analysis and theoretical results in

Browne (2000) are extended to glean insights associated with strategic supply chain management.

The organization of this essay is as follows. The background literature for this essay is similar to that provided in essay 3 and is therefore skipped. The next section presents the model formulation. In the third section, the nature of the game is detailed. The model is analyzed in the fourth section and the results are presented in the form of theorems and propositions. Finally, the fifth section discusses the results. The mathematical preliminaries and proofs for the theorems and propositions is provided in Appendix D.

5.2 The Model

5.2.1 Differential Game Formulation

The notations and symbols used in the model are as follows:

S: subscript for the supplier firm

B: subscript for the buyer firm

T: Finite time horizon for the strategies

t: An instant of time in the dynamic game setup

$I(t)$: Risky innovation stock

$J(t)$: Risk less "ordinary" stock

$u(t)$: Investment in breakthrough innovation efforts (risky investments)

$g(t)$: Investment in quality, labor and capital (risk-free investments)

W: Wiener process or Brownian motion

$(\Omega, \mathcal{F}, \{\mathcal{F}(t)\}, P)$: Filtered probability space

$\mathcal{F}(t)$: P-augmentation of the natural filtration $F^W(t) := \sigma(W_S^u, W_B^u; 0 \leq u \leq t)$

θ: risk-adjusted return on investments in innovation

ρ: Correlation coefficient between the Wiener processes $W_B(T)$ and $W_S(T)$ for the buyer's and supplier's overall gain processes

r: Rate of return on risk-free investments

σ: A function defined on Ω

X_t^u: Overall gain at time t. It represents a diffusion process controlled by supplier $X_t^{u_S}$ and buyer $X_t^{u_B}$

$Z_t^{u_S,u_B} := \frac{X_t^{u_S}}{X_t^{u_B}}$, Jointly controlled diffusion process which is a function of the investment policies u_S and u_B

$\tau_y^{u_S,u_B} := \inf\{t > 0 : Z_t^{u_S,u_B} = y\}$: The first hitting time to the point y under the specific investment policies u_S and u_B

$v^{u_S,u_B}(z)$: expected payoff function under the policy pair (u_S, u_B)

$p(z)$: "price" that a supplier can demand based on her overall wealth accumulation $h(z)$: a known function for $z = a, z = b$, with $h(b) < \infty$

Following Myerson (1991) , we assume that the partnership formation game is such that each firm simultaneously announces the set of firms it wishes to ally with. Upon entering an alliance firms bargain over profit shares, and they write the results of the bargaining in a verifiable and enforceable contract. We can then assume that the compatibility costs are split evenly since bargaining can provide any other split through a transfer of profits.

The model considers two investment opportunities for the buyer and the supplier firm: investment in innovation stock $I(t)$ and investment in risk free "ordinary" stock $J(t)$. The growth process of innovation stock for the supplier and the buyer firm is assumed to follow a geometric Brownian motion and $I(t)$ satisfies the stochastic differential equation for supplier S and buyer B.

$$dI_S(t) = \mu_S I_S(t)dt + \sigma_S I_S(t)dW_S(t)$$

$$dI_B(t) = \mu_B I_B(t)dt + \sigma_B I_B(t)dW_B(t) \tag{5.1}$$

where $\mu_i, i = B, S$ are positive constants. The risk-free ordinary stock is assumed to evolve according to:

$$dJ_S(t) = rJ_S(t)dt$$

$$dJ_B(t) = rJ_B(t)dt \tag{5.2}$$

where $r \geq 0$. To avoid a trivial solution, we assume $\mu_i > r$, for $i = S, B$ since if $\mu_i < r$ then the firms would invest in risk-free investments, which yield higher returns. Let the parameter θ_i denote the risk-adjusted excess return of innovation stock $I_i(t)$ over the risk-free rate of return, for $i = S, B$. Specifically,

$$\theta_i = \frac{\mu_i - r}{\sigma_i}, \text{for } i = S, B \tag{5.3}$$

Let $u_S(t)$ denote supplier's overall investments in innovation at time t under an investment policy $u_S = u_S(t), t \geq 0$, and similarly, let $u_B(t)$ denote buyer's investments in innovation at time t under an investment policy $u_B = u_B(t), t \geq 0$. It is assumed that both $u_S(t), t \geq 0$ and $u_B(t), t \geq 0$ are suitable admissible $\mathcal{F}(t)$- adapted control processes. In other words, $u_S(t)$ and $u_B(t)$ are nonanticipative functions that satisfy $\int_0^T {u_S}^2 dt < \infty$ and $\int_0^T {u_B}^2 dt < \infty$ for every $T < \infty$. The values of $u_S(t)$ and $u_B(t)$ are restricted to non-negative values.

Let $X_t^{u_S}$ denote the overall gain of the supplier firm at time t, if the firm follows policy $u_S = u_S(t), t \geq 0$ with $x(0) = x_0$. This overall payoff includes monetary benefits associated with these investments, for example, increased revenue, profits and non-monetary benefits, for example, channel power owing to innovation. It is assumed that the proportion of investment not invested in innovation is put into risk free investment options. The evolution of the process can thus be obtained from (5.1) and (5.2) and using the definition in (5.3). The equation can be written as:

$$
\begin{aligned}
dX_t^{u_S} &= u_S(t)X_t^{u_S}\frac{dI_S(t)}{I_S(t)} + X_t^{u_S}[1-u_S(t)]\frac{dJ_S(t)}{J_S(t)} \\
&= X_t^{u_S}[(r+u_S(t)\sigma_S\theta_S)dt + u_S(t)\sigma_S dW_S(t)] \\
&\qquad 0 \le u_S(t) \le 1 \;\forall\, t
\end{aligned} \tag{5.4}
$$

Similarly the equation for the buyer can be obtained as follows:

$$
\begin{aligned}
dX_t^{u_B} &= u_B(t)X_t^{u_B}\frac{dI_B(t)}{I_B(t)} + X_t^{u_B}[1-u_B(t)]\frac{dJ_B(t)}{J_B(t)} \\
&= X_t^{u_B}[(r+u_B(t)\sigma_B\theta_B)dt + u_B(t)\sigma_B dW_B(t)] \\
&\qquad 0 \le u_B(t) \le 1 \;\forall\, t
\end{aligned} \tag{5.5}
$$

These equations representing supplier's and buyer's individually controlled overall gain processes are similar to the wealth equation first studied by Merton (1971).

In a supply chain environment the investments in innovation by buyer and supplier firm are expected to be correlated. To allow for this requirement, $W_S(t)$ is considered to be correlated with $W_B(t)$, with the correlation coefficient ρ , that is, $E(W_S(t), W_B(t)) = \rho t$. In this illustration it can be observed that $X_t^{u_S}$ is the diffusion process controlled by supplier and $X_t^{u_B}$ is the diffusion process controlled by the buyer firm. The jointly controlled diffusion process can thus be defined as $Z_t^{u_S,u_B} := \frac{X_t^{u_S}}{X_t^{u_B}}$, where, supplier maximizes $Z_t^{u_S,u_B}$ and buyer minimizes $Z_t^{u_S,u_B}$ by appropriate investments. It is a function of the investment policies u_S and u_B. Applying Ito's formula and utilizing the results from (5.4) and (5.5) gives

$$
dZ_t^{u_S,u_B} = Z_t^{u_S,u_B}[m(u_S(t),u_B(t))dt + u_S(t)\sigma_S dW_S(t) - u_B(t)\sigma_B dW_B(t)] \tag{5.6}
$$

where the function $m(u_S(t), u_B(t))$ is defined as:

$$
\begin{aligned}
m(u_S(t), u_B(t)) &\equiv m(u_S(t), u_B(t) : \sigma_S, \sigma_B, \theta_S, \theta_B, \rho) \\
&= u_S(t)\sigma_S\theta_S - u_B(t)\sigma_B\theta_B + u_B(t)^2\sigma_B^2 - \rho\sigma_S\sigma_B u_S(t)u_B(t)
\end{aligned} \tag{5.7}
$$

The state dynamics of the stochastic differential game is given by equation (5.6). For the process $z(t)$ in equation (5.6), let $\tau_y^{u_S,u_B} := inf\{t > 0 : Z_t^{u_S,u_B} = y\}$ be the first hitting time to the point y under the specific investment policies u_S and u_B. For given number a, b where $a < Z_0 < b$, let $\tau := min\{\tau_a^{u_S,u_B}, \tau_b^{u_S,u_B}\}$ denote the first escape time for the interval (a, b) under the policies $u_S(t)$ and $u_B(t)$. The escape time signifies the point at which either the supplier or the buyer gains channel power owing to innovation capabilities.

The objective functional under the policies $u_S(t)$ and $u_B(t)$ are defined as:

$$
\begin{aligned}
v^{u_S,u_B}(z) = E_z\Bigg(\int_0^{\tau^{u_S,u_B}} & p(Z_t^{u_S,u_B}) exp\{\int_0^t \lambda(Z_r^{u_S,u_B})dr\}dt + \\
& + h(Z_t^{u_S,u_B}) exp\{-\int_0^t \lambda(Z_r^{u_S,u_B})dr\}\Bigg)
\end{aligned} \tag{5.8}
$$

where $\lambda(z)$ is a given nonnegative function representing the discounting function. This function represents a risk-adjusted discounting of the pay-off function. For ease of mathematical analysis we only consider the cases when the discounting function is a constant λ. The function $p(z)$ is the "price" that a supplier can demand based on her overall wealth accumulation. It is assumed to be a given real bounded continuous function and $h(z)$ is known function for $z = a, z = b$, with $h(b) < \infty$. The function allows an investigation of discounted payoff and utility maximization games within the context of the above problem formulation, as will be explained later. The supplier would choose a control function $u_S(t)$ in order to maximize $v(Z_t^{u_S,u_B})$ while simultaneously the buyer would choose a control function $u_B(t)$ to minimize $v(Z_t^{u_S,u_B})$.

Perfect revelation of choices made by the buyer and the supplier firms is assumed. For the supplier firm the objective functional and state dynamics are:

$$v^{u_S,u_B}(Z_t^{u_S,u_B}) = \max \ E_z\left(\int_0^{\tau^{u_S,u_B}} p(Z_t^{u_S,u_B}) exp\{\int_0^t \lambda(Z_r^{u_S,u_B})dr\}dt + \right.$$
$$\left. + h(Z_t^{u_S,u_B}) exp\{-\int_0^t \lambda(Z_r^{u_S,u_B})dr\} \right)$$

subject to

$$dZ_t^{u_S,u_B} = Z_t^{u_S,u_B}[m(u_S(t),u_B(t))dt + u_S(t)\sigma_S dW_S(t) - u_B(t)\sigma_B dW_B(t)] \quad (5.9)$$

For the buyer, the objective functional and the state dynamics can be written as:

$$v^{u_S,u_B}(Z_t^{u_S,u_B}) = \min \ E_z\left(\int_0^{\tau^{u_S,u_B}} p(Z_t^{u_S,u_B}) exp\{\int_0^t \lambda(Z_r^{u_S,u_B})dr\}dt + \right.$$
$$\left. + h(Z_t^{u_S,u_B}) exp\{-\int_0^t \lambda(Z_r^{u_S,u_B})dr\} \right)$$

subject to

$$dZ_t^{u_S,u_B} = Z_t^{u_S,u_B}[m(u_S(t),u_B(t))dt + u_S(t)\sigma_S dW_S(t) - u_B(t)\sigma_B dW_B(t)] \quad (5.10)$$

5.3 Nature of Games

In this essay, games related to the achievement of relative performance goals and shortfalls are considered. Specifically, for numbers a, b with the associated condition, $aY_0 < X_0 < bY_0$, if the performance in terms of the objective of supplier is considered, upper performance goal b is said to have been reached if $X_t^{u_S} = bY_t^{u_B}$, for some $t > 0$ and that the lower performance shortfall level a occurs if $X_t^{u_S} = aY_t^{u_B}$, for some $t > 0$. In general the supplier wins if performance goal b is reached before performance shortfall level a is reached, while the buyer firm wins if the converse happens.

Two classes of games involving investments by supplier and buyer firms are considered. The first class of game considers the discounted payoff maximization (minimization) objectives. In this class, first a stochastic differential game is considered in which the supplier tries to maximize the expected discounted gain that is achieved upon outperforming the buyer in innovation capability. The buyer at the same time tries to minimize this expected discounted gain to be accrued by the supplier. Since the structure of the differential game is symmetric for both the buyer and the supplier, a straightforward inference can be made for the case when the buyer is investing in innovation to minimize the potential loss if the supplier outperforms. In this class of games, the ratio of the two gains processes $Z_t^{u_S,u_B} := \frac{X_t^{u_S}}{X_t^{u_B}}$ is a sufficient statistic to evaluate the investment strategies for the supplier and the buyer firms. This feature makes the objective functional to be dependent only on the gain process and renders the game a structure similar to that in Dirichlet problem.

As an alternative formulation, the other game examined in this essay has time-dependence and is similar in structure to a nonlinear Cauchy problem. It can be noted that the previous structure of the game allowed only one winner. The results suggest the optimal strategies to be adopted by supplier and buyer firm such that both firms try to outperform each other and thereby obtain overall surplus. In light of this argument, it can be asserted that by making the strategies to be time-dependent, the relative utility gained by a supplier and buyer can be obtained at any time instant. In these type of games both buyer and supplier receive utility (or disutility) from the ratio of the gain processes (i.e., from the relative performance of their respective gains achieved by investment in innovation stock and in ordinary stock.) where the game is played for a fixed duration of time . As in the previous class of game, in these games also the ratio $Z_t^{u_S,u_B} := \frac{X_t^{u_S}}{X_t^{u_B}}$ acts as the pertinent state variable.

5.3.1 Class I: Discounted Payoffs

In this game, supplier makes investments in innovation to maximize the expected discounted gains of reaching the upper goal b, while the buyer firm invests in innovation to minimize the potential gains by the supplier. For this game to have a solution we require the following condition to hold,

$$F^*(z) = \sup_{u_S} \inf_{u_B} E_z(e^{-\lambda \tau_b^{u_S,u_B}}) = \inf_{u_B} \sup_{u_S} E_z(e^{-\lambda \tau_b^{u_S,u_B}}), \text{for } \ z < u \tag{5.11}$$

Since the objective is to reach the upper goal b, we can set $h(b) = 1$. This value models the situation where the supplier attains control of the channel power. This game has the structure of the nonlinear Dirichlet problem and therefore we can apply Theorem 1. Specifically, in equation (5.9) and equation (5.10) we can substitute a known discount factor $\lambda(z) = \lambda > 0$. The following three cases are considered regarding the value of $p(z)$:

(a) Case A: $p(z) = 0$

In this case the supplier invests in innovation to maximize the terminal value of the objective functional by maximizing the gain process at the terminal time. The buyer on the other hand invests in innovation to minimize this terminal value. When the upper goal is reached the function $h(z)$ takes the value $h(b) = 1$. This is manifested in the form of lock-ins and externalities that can be gained by the dominant partner.

(b) Case B: $p(z) = p$, a constant price

Case A looks at a situation where the investment in innovation is aimed at maximizing the terminal value of the objective functional. The value of $p(z)$ was set to zero to make the required analytical simplification to focus only on the terminal value. In case B, a positive non-zero value of "price"

$p(z)$ is considered. In this case, the role of "price" in the nature of optimal control function is evaluated.

(c) Case C: $p(z)$ = a concave increasing function of z

In this case, the evolution of the "price" as a function of z is considered. The gains are manifested in the value of p which is a function of the joint gain process z at an instant of time in the game, and also on the terminal value of the objective functional.

5.3.2 Class II: Utility-based games

In this class of game there is an explicit time dependence of the strategies by supplier and buyer firms. The objective functional is of the form given in equation (86), and now the utility derived by the buyer and supplier firm is a function of the innovation level at an instant of time t. For this class of game, the following cases are investigated.

(a) Case A': $\beta(z) = 0, \lambda(z) = \lambda$ and $U(z) = z^{\alpha}$, for $0 < \alpha < 1$

In this case, the investments in innovation are aimed towards the terminal value of the game. In this game it can be noted that the function $U(z)$ denotes the concave increasing utility a firm attains by investments in innovation whereas $h(z)$ denotes a function that takes value at $z = a$ and $z = b$. A concave increasing utility function is quite widely used to represent the utility derived by investments in such strategic aspects as, innovation, quality, and advertising.

(b) Case B': $\beta(z)$ = a concave increasing function of z, $\lambda(z) = \lambda$ and $U(z) = z^{\alpha}$, for $0 < \alpha < 1$

In case B', a positive discounting factor and a concave increasing payoff function $\beta(z)$ and utility function $U(z)$ are considered.

5.4 Analysis of the Model

5.4.1 Discounted Payoffs - Case A

Theorem 1 *If the degree of advantage of a supplier over buyer denoted by κ satisfies the conditions,*

$\kappa < \tilde{\kappa}^-$ *and* $\kappa > \tilde{\kappa}^+$ (5.12)

where $\tilde{\kappa}^-$ and $\tilde{\kappa}^+$ are as defined in equation (96), then the value of the discounted game (5.11) is given by,

$$F^*(z) = \left(\frac{z}{u}\right)^{\eta^+} \quad \textit{for} \quad z < u \tag{5.13}$$

η^+ is defined in equation (94) and the associated saddle point is given by:

$$u_S^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\frac{\rho}{\kappa}-1)\eta^+ - 1}{(1-\rho^2)(\eta^+)^2 - 1}\right), \quad u_B^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1-\rho\kappa)\eta^+ - 1}{(1-\rho^2)(\eta^+)^2 - 1}\right), \tag{5.14}$$

5.4.2 Discounted Payoffs - Case B

Theorem 2 *If the degree of advantage of a supplier over buyer denoted by κ satisfies the conditions,*

$\kappa < \tilde{\kappa}^-$ *and* $\kappa > \tilde{\kappa}^+$ (5.15) *where $\kappa < \tilde{\kappa}^-$ and $\kappa < \tilde{\kappa}^+$ are as defined in equation (96), then the value of the discounted game (5.11) is given by,*

$$F^*(z) = \left(\frac{z}{u}\right)^{\eta^+}(1-\frac{p}{\lambda}) + \frac{p}{\lambda} for \;\; z < u \tag{5.16}$$

η^+ is defined in equation (94) and the associated saddle point is given by:

$$u_S^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\frac{\varrho}{\kappa}-1)(1+\eta^+)-1}{(1-\rho^2)(1+\eta^+)^2-1}\right), and \;\; u_B^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1-\rho\kappa)(1+\eta^+)-1}{(1-\rho^2)(1+\eta^+)^2-1}\right), \tag{5.17}$$

It can be noted that the value of competitively optimal control would accordingly be higher than that in expression (91).

5.4.3 Discounted Payoffs - Case C

Theorem 3 *If the degree of advantage of a supplier over buyer denoted by κ satisfies the conditions,*

$\kappa < \tilde{\kappa}^-$ *and* $\kappa > \tilde{\kappa}^+$ (5.18)

where $\kappa < \tilde{\kappa}^-$ and $\kappa < \tilde{\kappa}^+$ are as defined in equation (111), and θ_B and ρ satisfies the condition $\theta_B^2\rho^2 \geq 2$, then the value of the discounted game (5.11) is given by,

$$F^*(z) = (\frac{z}{u})^{\eta^+} for \;\; z < u \tag{5.19}$$

η^+ is defined in equation (109) and the associated saddle point is given by:

$$u_S^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\frac{\varrho}{\kappa}-1)\eta^+-1}{(1-\rho^2)(\eta^+)^2-1}\right), \;\; and \;\; u_B^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1-\rho\kappa)\eta^+-1}{(1-\rho^2)(\eta^+)^2-1}\right), \tag{5.20}$$

Corollary: If $\theta_B^2 \rho^2$ is equal to 2, the results are identical to that in case A. However, more generally the relationship between θ_B, the risk-adjusted return on investments in innovation for the buyer and ρ, the correlation coefficient between the Wiener processes $W_B(t)$ and $W_S(t)$ for the buyer's and seller's overall gain processes, can be written as,

$$\theta_B \geq \frac{\sqrt{2}}{\rho} \quad \blacksquare \tag{5.21}$$

5.4.4 Utility Maximization - Case A'

Theorem 4 *If the buyer and supplier are competing in utility maximization objective by maximizing the terminal value with investment in innovation and the discounting factor is λ, then the competitively optimal strategies are given as:*

$$u_{SJ}^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\rho/\kappa - 1)\alpha - 1}{(1-\rho^2)\alpha^2 - 1}\right) \text{ and } u_{BJ}^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1-\rho\kappa)\alpha - 1}{(1-\rho^2)\alpha^2 - 1}\right) \tag{5.22}$$

and the value of the game $F^*(t,z)$ is given by, $F(t,z) = e^{q(\alpha)(T-t)} z^\alpha$ where $q(\alpha)$ is,

$$q(\alpha) := \alpha\theta_B^2 \frac{(1-\kappa^2) - \alpha(1+\kappa-2\rho\kappa)}{2[(1-\rho^2)\alpha^2 - 1]} - \lambda \tag{5.23}$$

5.4.5 Utility Maximization - Case B'

Theorem 5 *If the buyer and supplier are competing in payoff maximization and terminal utility maximization objectives then the form of competitively optimal strategies are similar to equation (115).*

$$u_{SJ}^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\rho/\kappa - 1)\alpha - 1}{(1-\rho^2)\alpha^2 - 1}\right) \text{and } u_{BJ}^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1-\rho\kappa)\alpha - 1}{(1-\rho^2)\alpha^2 - 1}\right) \tag{5.24}$$

and the value of the game $F^*(t,z)$ is given by, $F(t,z) = e^{q(\alpha)(T-t)}z^{\alpha}$ *where* $q(\alpha)$ *is,*

$$q(\alpha) := \alpha\theta_B^2 \frac{(1-\kappa^2) - \alpha(1+\kappa-2\rho\kappa)}{2[(1-\rho^2)\alpha^2 - 1]} - \lambda + 1 \tag{5.25}$$

5.4.6 Propositions

Proposition 1 *The solution to the differential game involving discounted payoffs yield highest value when,*

$$\kappa \geq -\rho + \sqrt{\rho^2+1} \tag{5.26}$$

Proposition 2 *In a discounted payoff game, the complementarities and synergies in the investments in innovation are a function of who among the supplier and buyer exerts innovation based channel power.*

Proposition 3 *In a discounted payoff game, for negatively correlated gain processes, and for maximum value of the game (evaluated at* $\lim_{\eta^+ \mapsto 0}$*), investments in innovation by supplier firm would be strictly higher in case B than in the case A and case C, whereas the buyer will not make any investment in innovation in case B but would make investments* $\lim_{\eta^+ \mapsto 0} u_B^* = \frac{\theta_B}{\sigma_B}$ *in case A and case C.*

Proposition 4 *For discounted payoff game, in case B,*

(a) If the ratio, $\frac{\kappa}{\rho} > 1$*, both supplier and buyer firms would invest higher than in case A and case C;*

(b) If the ratio is such that, $0 < \frac{\kappa}{\rho} \leq \frac{1}{2}$*, the buyer would make lesser investments (would make equal investment to case A and case C if* $\frac{\kappa}{\rho} = 1$*) than those made in case A and case C, and the supplier would make no investments,*

(c) If the ratio is such that $\frac{1}{2} < \frac{\kappa}{\rho} \leq 1$*, the buyer would make lesser investments than those made in case A and case C. The supplier would make lower investments*

if the correlation coefficient $\rho = 1$ (would make equal investment to case A and case C if $\frac{\kappa}{\rho} = 1$) and would make higher investments otherwise.

Proposition 5 *In a discounted payoff game, investments in case A and case C would be identical when,*

$$\kappa = -\rho + \sqrt{\rho^2 + [2\lambda(1-\rho^2)+1]} \tag{5.27}$$

In contrast to the discounted payoff maximization game, in utility maximization games, the competitively optimal controls for the supplier and buyer firm are identical in case A' and case B'. Analyzing the competitively optimal control functions for the supplier and buyer firm by taking $\lim_{\alpha \mapsto 0}$ and $\lim_{\alpha \mapsto 1}$, it can be observed that the resulting expressions are exactly similar to those in the analysis of the discounted payoff game. Hence, the propositions follow in the case of utility maximization game as well. The only difference lies in the value of the two games considered in case A' and case B'. Comparing the value of the games in case A' and case B' of the utility maximization game, it can be observed that,

$$F^*_{CaseB'}(t,z) = e^1 F^*_{CaseA'}(t,z) = 2.718 F^*_{CaseA'}(t,z)$$

$$\therefore \quad \frac{F^*_{CaseB'}(t,z)}{F^*_{CaseA'}(t,z)} = e = 2.718$$

5.5 Discussion

In this section, the results in the previous section are discussed. From the theorems it can be observed that in all the cases pertaining to discounted payoff game as well as the utility maximization game, the investments made by the supplier and buyer in innovation are directly proportional to the risk adjusted returns of these investments. This result is quite intuitive because, if the firms can extract better returns from risk-free investments they would indeed do so. Moreover, these investments are also inversely proportional to the variability associated with the returns. Further exploration can be carried out by evaluating,

$$Investments \propto \frac{\theta_i}{\sigma_i}; \qquad i = B, S \tag{5.28}$$

Substituting $\theta_i = \frac{\mu_i - r}{\sigma_i}$ in the above equation leads to,

$$Investments \propto \frac{\mu_i - r}{\sigma_i^2}; \qquad i = B, S \tag{5.29}$$

The expression suggests that a firm would consider investing in risky innovations if the risk-free rate of returns is high and if the variance in the evolution of innovation stock is low. The stationary Markov perfect Nash equilibrium results are time invariant and are characterized completely by the parameters of the model.

It can be noticed that the impact of correlation between the growth of buyer's and supplier's innovation stock ρ and the power advantage κ cannot be straightforwardly conjectured from the equilibrium expressions. The propositions obtained from further analysis provide some implications of ρ and κ on the nature of relationship and innovation investment strategies.

By evaluating expression (130) in appendix D with the equality sign, it can be noted when $\rho = 0$ we have $\kappa \geq 1$. This implies that the degree of advantage of the supplier is either equal to or higher than that of the buyer. Therefore, when the investment plans are absolutely independent, the supplier gains the advantage in the relationship. This could be explained from the conjecture that the supplier may potentially invest into creating resources that could be supplied to other buying firms. The buyer's innovation investments on the other hand are geared towards creating value proposition for the final customers. In the long run therefore, the supplier channel power grows while the buyer's power depends on exogenous demand for resources developed through innovation investments.

For a perfect (positive) correlation ($\rho = 1$)between the gain processes of the buyer and supplier we obtain $\kappa \geq 0.414$. For such a situation, if $0.414 \leq \kappa < 1$, the degree of advantage rests with the buyer and for $\kappa \geq 1$, the supplier attains the degree of advantage. It can asserted that this perfect correlation is akin to the synergistic wealth formation by the supplier and the buyer. This is representative of the situations when the supply chain partners are collaborating in the "basic research" for creating a new product. The results intuitively suggest that with strong positive correlation between the wealth processes, both buyer and supplier firms could potentially attain the degree of advantage.

Finally, when the stochastic processes are negatively correlated, the supplier will always have higher degree of advantage than the buyer firm because now the value of κ is strictly greater than 1. An explanation for this is that with a negative correlation the two collaborating partners are creating substitute technologies. In such circumstance the result suggests that the gain that a supplier could achieve by creating technology which is potentially used by competitors of firm B, is higher than what the buyer could achieve by locking out the existing supplier.

Case B investigates a situation, in which there is a constant "price" charged by the supplier. With a negatively correlated gain process it can be argued that the supplier would channel the payoffs into creating substitute products and innovation. By doing so, the supplier could potentially increase the market through other competing buyer firms. On the other hand, with no incentive to influence the "price", the buyer would set her innovation efforts to zero. In contrast, in case A the "price" is set to 0 and therefore both supplier and buyer firms target their innovation investments to gain channel power. In such a hypothetical situation the buyer and supplier would invest a positive amount to ensure that they attain the degree of advantage in the channel at terminal time. In case C, since the "price" is now influenced by the relative gain Z^{u_S,u_B}, the buyer has incentive to invest and compete in the collaborative setup.

From these cases, some inference can be made. First, the onus lies on the buyer to create conditions for the supplier so that the investments are synergistic. As could be observed, owing to the certainty of attaining the degree of advantage, the supplier firm would prefer an uncorrelated or negatively correlated gain process. This suggests that for any buying firm that wishes to be successful in a collaborative relationship, an understanding must be developed about how to own and control critical assets that provide opportunities to create customer dependency and supplier 'lock-ins'. Such understanding normally requires a competence in effective demand, supply, procurement and strategy management.

The results and the discussion connect well with the literature on core capabilities (Prahalad and Hamel 1990) . An uncorrelated or negatively correlated gain process associated with investments in risky innovation by the supplier and buyer firm implies that the responsibilities of the two firms in a partnership are set a priori. This is particularly relevant to cases when the supplier is chosen based on certain unique capabilities. These capabilities could be patented or possibly protected owing to high costs associated with replication. Similarly, a negatively correlated Brownian motion could be conceptualized as a situation in which one of the two supply chain partners is investing to create substitute technology. In light of the above argument, a supplier would gain by making investments in innovation that are independent of the investments made by the buyer. Furthermore, a supplier would also gain by making investments in innovations that are possible substitutes of the buyers product-process portfolio. On the other hand, the buyer firm would never gain the degree of advantage if the investments in innovation are such that either they are uncorrelated with the investments made by the supplier or alternately, if the investments are potentially aimed at creating a substitute to the suppliers offering.

This can be interpreted in yet another way. It can be inferred from the results that a synergistic supply chain is dependent on the fact as to who among the supply chain partners actually commands the degree of advantage. In a supply chain where the supplier has the channel power owing to innovation, the innovation investment based gain processes are expected to be negatively correlated and the supplier and the buyer firms would be expected to invest in innovation to create substitutes. Whereas, in a supply chain where the channel power associated with innovation rests with a buyer, it is expected that the investments by the buyer and the supplier firm would be directed towards complementary gains. It can be asserted that although the gain process has been modeled as Brownian motion, the correlation between the gain processes of the buyer and supplier firm can be influenced by managerial actions. Specifically, when the buyer firm has the degree of advantage, the incentive for innovation for the supplier can be structured in such a way that the supplier would prefer considering synergistic and complementary innovation instead of substitutes.

If buyer firms intend to become effective in their ability to appropriate and accumulate value they must understand how to effectively manage the supply relationships (Cox et al. 2002). Therefore, in a collaborative context a competence in procurement management is at least as important for business success as effective demand and strategy management. To achieve sustainable business success, therefore, it is not enough to understand how to innovate with supply so that conditions are created for market closure to competitors. It is also essential to understand the power struggle over value appropriation and accumulation that occurs between buyers and suppliers at all stages in the supply chain networks that are created to produce products and services for final consumers.

There are several examples of strategies adopted by collaborating buyers to achieve sustained competitive advantage. For example, if a component or sub-assembly is considered commercially critical to a vehicle then, as a matter of routine, a Japanese assembler will take an equity stake in the supplier. Where an assembler does not own an equity stake in a supplier, however, it rarely opts to single source. Instead, it will choose to source from two suppliers simultaneously, so that, if one supplier underperforms, then it knows that the business will go instead to a second supplier. With such arrangements many of the opportunism found in western context is reduced, because the Japanese assemblers insist upon cost transparency on the part of the supplier as a precondition of signing a contract. Thus, because Japanese assemblers have fewer suppliers to manage, the costs associated with monitoring them for opportunism are correspondingly lower.

A plausible explanation for the failure of lean production in a Europena context might be the form of implementation (Cox et al. 2002). Japanese practice has been about creating complex networks of interlocking supplier dependencies as illustrated in the previous paragraph. The assembler dominates the sub-assembler, who in turn dominates the component manufacturer. This allows the assembler to drive its operational improvement program throughout the supply network. Many European assemblers, however, have simply outsourced the design, production and supply chain coordination of key sub-assemblies without bothering to ensure that the structures of power that will support such an enterprise are in place.

The economics literature makes an important distinction between information that is *public* and information that is *private* (Molho 1997). The idea of public information broadly equates to the notion of common knowledge, which simply means something that is widely known. Private knowledge, by contrast, is information that is tightly controlled. In the buyer-supplier exchange, this refers to information that is only known by one of the contracting parties. The existence of such an information asymmetry is the prerequisite of concealment and manipulation. Without this asymmetry, the objective interests of both parties are revealed and, power resources permitting, they can be aggressively pursued.

According to Williamson (1975, 1985) the risk of a buyer becoming dependent on a supplier was not a function of power, but occured simply as a result of bad management. Ex post dependency, for example, was a function of inadequate contracting that had left room for opportunism. A farsighted buyer should from past experience if nothing else, have been appraised of the circumstances that might place it in a position of dependency. By learning from the experience, therefore, it should have been able to construct appropriate governance structures to avoid a repeat of the problem. If the buyer failed to display such iterative learning, at least behavior of its suppliers would reach such a level that its competitiveness and, ultimately, its survival would be threatened.

Specifically for earning rents the buying firm ought to develop critical supply chain assets. In pp.6-8 Cox et al.(2002) suggest that critical assets are based on supply chain resources that can be made relatively scarce, and that allow their owners both to close the market for this particular supply chain resource to other potential competitors, and to effectively leverage value from their downstream customers and upstream suppliers. The relative scarcity of the resources on which such assets are based implies that only a very small number of firms are likely to have them within any particular supply chain or market at any given time. Rents are earned through the continuous actualization of potential supply chain and market power. In other words, a firm earning rents will recognize that it has to focus on both supply chain and market power and will employ that power effectively. Essentially, there are three mechanisms through which firms without critical assets might seek to reconfigure the existing structure of power in any particular market, or supply and value chain. These are product innovation, process innovation and supply chain innovation.

Another aspect to be considered by buyers is to create opportunities for reducing the search costs. Buyers with high costs of search, and unable to get easily at the true costs of production of the supplier, should expect, to pay high prices relative to those buyers that have somewhat lower search costs. Furthermore, the supplier will have an incentive to raise the search costs for each of its customers across the board. This can be achieved by increasing the level of uncertainty surrounding the product offering through innovation. Such a move would make information more difficult to acquire and process, and would leave buyers less well informed.

Supply chain power is based upon owning or controlling a supply chain resource that combines high degrees of utility and scarcity for a buyer or a supplier in the context of a particular transaction. It is always this combination of the two variables that provide the basis for supply chain power. If, for example, a buyer is facing an extensive pool of interchangeable and openly competitive suppliers, it makes no difference how high the utility is of the resource that it is attempting to buy. In these circumstances none of the suppliers can exert supply chain power because the relative scarcity of the resource in question is low. None of the supplier in such situation would possess a critical asset because there is high imitability leading to a highly contested market. The possibility of supply innovation means that both dominant and dependent firms should be constantly aware that most power relations are unlikely to be permanent. There is a strong probability, particularly over the longer term, that potential competitors, as well as dependent buyers or suppliers, will attempt to reduce their vulnerability by searching for supply innovation, or by seeking out new suppliers or customers through product, process or supply chain innovation (Cox et al. 2002).

Finally, the size and scale of operations of a firm is a determining factor in the power dynamics. In a car supply chain for example, the scale is the prerequisite to an assembler's ability to maintain the R&D spending. Maxton and Wormald(1994) explain the importance of size with the following example: If a prestige assembler, like Mercedes-Benz has sales of around 450,000 vehicles a year and incurs R&D costs of 5 per cent, it may only have a little over 1 billion dollars a year to re-invest. Conversely, although a volume assembler is unable to charge as much for its products, if it achieves a sales of 4.7 million vehicles a year, a similar research burden would still leave it with much deeper pockets.The money available for innovation process could well be five times that available to the prestige assembler. In an industry that is so competitive and where particular models often fail to find a market, the commercial vulnerability of the smaller player is obvious. Scale matters, and over time this fact is bound to lead to further concentration of the market and further consolidation of the powerful player's position.

Chapter 6

Conclusions

6.1 Introduction

This dissertation is a theory-building endeavor and has resulted in many interesting insights. This chapter concludes the dissertation by emphasizing the theoretical and practical implications and providing directions for future research. This chapter is structured into three sections. The first section discusses the theoretical contributions of this research. This is followed by a discussion of managerial implications in second section. Finally, the third section presents directions for future research.

6.2 Theoretical contributions

This dissertation contributes to theory development in operations management in several ways. The conceptual framework, model formulation and insightful analytical results provide a strong groundwork to integrate the role of operations management with overall business of an enterprise. Specifically, the context of product development and collaboration allows an examination of important aspects from marketing, strategy, organizational theory and the broad field of operations management.

The first contribution of this dissertation is that it extends the theory of R&D based competition beyond the models found in economics and marketing literature. Specifically, an investigation of continuous incremental improvement of product quality by means of investments in innovation provides a setting for evaluating R&D policies which remain unaddressed in models examining patent race. Although, organizational learning is not explicitly modeled, it is implicitly shown to be an important constituent of the overall dynamics. This conceptualization aids future theory-building activities in product development to consider an integrated framework incorporating innovation, continuous improvement of quality and learning.

The second contribution is that this research uncovers the importance of asymmetries by investigating the difference between firms in terms of key parameters. Moreover, for the examination of inter-firm competition through product development, the context of simultaneous entry and sequential entry are treated separately. This allows a deeper understanding of the implications of information asymmetry and commitment which have been regarded as important determinants in many game-theoretic studies.

The third contribution of this dissertation is that it presents the competitive role of innovation among collaborating firms. The research provides reasoning for inter-firm incentives in forming collaborative arrangements for product development. The dynamics of relationship among supply chain partners is viewed in terms of their respective innovation competence. It is emphasized that varying power arrangements in a supply chain leads to different implications for investments in innovation by buyer and supplier. The incentive for buyer and supplier to strategically maneuver their overall innovation levels by appropriate investments is highlighted.

This thesis asserts that the competitive strategy based on innovation could potentially permeate into collaborative setups aimed towards dispersed product development. The existing literature in operations management doesn't explicitly consider the implications of innovation and quality based competitive strategies in a collaborative supply chain context. Two theoretically insightful models have been presented that provide motivation and starting point for the literature in operations management to explicitly consider competition among buyer and supplier in supply chains. The notion of lock-ins and power shifts is implicit in the model development and has implications for the future theory-building activities in this area.

Finally, from a methodological standpoint, this dissertation provides motivation to use game-theoretic analytical study to investigate some of the interesting questions in operations management. It extends the research on innovation and quality-enabled competition by considering a dynamic model of competition utilizing a differential game approach. The theory of differential games originated as an extension of optimal control theory (which is concerned with one-person dynamic optimization problems in continuous time). The analytical tools of differential games are therefore influenced by those of optimal control (e.g., maximum principles, Hamilton-Jacobi-Bellman equations of dynamic programming, state space analysis). However, differential game theory has long since transcended its origins in one-person dynamic optimization and has moved on to become a subclass in its own right of the broader field of dynamic game theory.

The use of differential games is very common among researchers in economics, marketing and management science. The use of such an approach to examine issues related to strategic supply chain management is quite sparse. By explicitly considering strategic issues in operations and supply chain management, this research enhances the existing methodological foundations in operations management. It is emphasized that the competitiveness achieved with operations is deeply rooted in core capabilities and relates with the dynamic capabilities view of the firm (Teece et.al. 1997). It is asserted that the key operations constructs and variables ought to be viewed from a strategic framework and differential games provide one of the tools for a rigorous treatment of the problem context.

6.3 Managerial contributions

The analytical results of this dissertation provide insights into managerial implications regarding strategic issues concerned with product development and supply chain planning. The conceptual framework provides a foundation for managers to view competitive advantage emanating from product development in a more integrated manner.

The results of the first and second essay point towards a time variant innovation investment strategy. It suggests that firms competing by means of new product development must choose instantaneous investment in innovation such that it increases with time. The quality manifestations exhibit an increasing trajectory. In a symmetric competition, the investment profile of the leader is sigmoidal while that of the follower is convex increasing. The impact of such an investment profile is aptly reflected in the quality improvement trajectory.

The results provide implications for a firm for innovation investments by considering their relative strengths and weaknesses. The exploration of firm asymmetries provide some interesting implication that a firm should consider when competing via new product development. The results can be translated into executable decisions for investments. These results provide insights to evaluate different scenarios for pursuing strategies which would lead to best market outcome.

The third and fourth essays provide a way of thinking about business strategy and operational alignment in a collaborative network. The results provide directions for strategies to be adopted in a supply chain context. These are becoming increasingly important in the present business environment, where many firms are joined together in a collaborative network. In industries driven by innovation e.g. semiconductors, biotechnology, having the control on the overall innovation levels that drive the technology landscape could mean a strong strategic advantage for one partner over other contributing firms.

Innovation competence plays an important role in this approach on supply chain relationship. In essay 3 the specific scenario modeled is representative of the practical situation. As an example, it can be observed that IBM is actively involved in creating quantum computers. One reasoning for investments in quantum computers could be an extension of technology landscape. However, equally important is the realization that a success of such an endeavor could lead to lock-out of Intel from this newly formed market for quantum computers. The simplistic model used in essay 3 allows for an exploration of such a situation. The stationary Markov perfect Nash equilibrium investment strategy of the buyer and supplier is found to be time invariant and is characterized by the parameters in the model. An interesting insight obtained as a result of analysis of the model is that the supplier can influence the motivation of the buyer to invest in substitute technology. The underlying mechanism can be translated into pricing strategy that results in a long-term buyer-supplier relationship.

The analysis of a model of buyer-supplier relationship in the fourth essay allows to investigate a more generalized setup of buyer-supplier innovation-based competition. The aspect of channel power is tightly integrated in the analysis to understand the complex behavioral aspects using a simple framework. These results indicate that risk plays an important role in innovation investment decisions. Moreover, the variance in wealth formation is an important indicator of how much to invest. These results are quite intuitive and enable managers to objectively resolve some of these strategic decisions.

The propositions provide insights into the dynamics of relationship between a buyer and a supplier firm. These propositions present the role and responsibility of buyer in creating motivation for the supplier to collaborate. The importance of critical assets is aptly amplified in the results. The possession of critical assets, often observed with the upstream partner gives the supplier a potential to achieve relative market closure through a position of dominance over competitors. It is likely that a firm in possession of such a critical asset also has the potential to achieve effective leverage over collaborating partners and suppliers. The responsibility of the buyer is to create a delicate balance between managing supplier and customers. Similar to the conceptual development in Cox et al. 2002, the results suggest that for a buyer that wishes to be successful an understanding must be developed about how to own and control critical assets that provide opportunities to create customer dependency and 'lock-in.' A competence in procurement management forms the cornerstone for success of long-term business strategy.

6.4 Directions for future research

As discussed in the previous sections, this dissertation addresses an important issue facing many enterprises. The thesis adopts a theoretical approach to focus on some key research issues. This indeed aids in clear characterization of these issues and generation of important insights. Many extensions can be considered for this research theme and some of them are discussed below as potential research directions.

First, within the framework of first and second essays, the model can be extended by evaluating the improvement in product quality with learning effect. The learning effect is very well documented in research literature and the extension of this research to incorporate learning effect is straightforward. Specifically, learning can be used to characterize the dynamics of evolution of product quality and in the characterization of costs associated with innovation investments.

For analytical simplicity the revenue function is treated as a salvage value. As an extension the incorporation of the dynamics of evolution of revenue in the analysis of the model can be explored. In the sequential play game, the state dynamics of the follower can be considered to be dependent on the state of the leader at previous time instant. This modification would allow for analysis of a model in which the leader uses open-loop Nash equilibrium strategy for innovation investments whereas the follower would adopt a Markovian Nash equilibrium strategy. Additional insights can be gained by this modification for leader-follower competitive dynamics in new product development.

Firm asymmetries can be explored by considering multiple parameters at the same time. This would enable a richer understanding of the strategies that a leader and a follower should adopt based on their strengths and weaknesses.

By means of the third and fourth essay, the competitive dynamics of a buyer and supplier are introduced. The essays present the competitive role of innovation among collaborating firms. The research asserts that the buyer and the supplier could be in a competitive relationship due to efforts in innovation. Existing literature in operations management doesn't explicitly consider the implications of innovation based competitive strategies in a collaborative supply chain context. Models are presented that provide motivation to explicitly consider such competitive situations. The notion of lock-in and channel power is implicit in the model and presents an approach for theory-building activities in this area.

The specific scenario modeled is representative of the practical situation. As an example, it can be observed that IBM is actively involved in creating quantum computers. One reasoning for investments in quantum computers could be an extension of technology landscape. However equally important is the realization that a success of such an endeavor could potentially lead to independence from the microprocessor technology of Intel, a dominant supplier in the personal computer industry. The simplistic model used in this paper allows for an exploration of such a situation. The stationary Markov-perfect Nash equilibrium results present optimal strategies of the competing players and also provide some guidelines for long-term relationships.

I acknowledge that the model presented in this paper is a simplified version of the dynamics between a buyer and a supplier. The constraints placed on different parameters and the context of a monopoly supplier and a buyer facing perfect competition is an approximation for analytical convenience. However, this enables a theoretical investigation of some important aspects underlying buyer-supplier relationships. Specifically, the research provides a way of thinking about business strategy in collaborative networks. These are becoming increasingly important in the present business environment, where many firms are part of a collaborative network. In industries driven by innovation for example, semiconductors and biotechnology, having control over the overall innovation levels could mean a strong strategic advantage for one partner over other contributing firms. At the same time, the firm already having the channel power can take come actions that would enable a long-term relationship built on trust based governance.

Several extensions could be considered to better represent such 'co-opetitive' relationships between collaborating firms. Given the objective interests, it is necessary for collaborating partners to know how far the other side is prepared to concede before it is no longer profitable to be in a relationship. Information about the cost structures of the collaborating partners and their relative utility from the exchange relationship is critical to an understanding of power in exchange relationship. Hence, one of the extension of the problem is to consider aspects of information asymmetry. The two key problems generated by private information and imperfect observability are - adverse selection and moral hazard. The adverse selection is a condition of supplier opportunism that occurs prior to a signing of a contract. Moral hazard on the other hand refers to supplier opportunism that occurs once the buyer has signed a contract (Cox et al.(2002). A rational buyer tries to possess information that counters the above motives on the part of supplier. Future research could investigate the means by which a buyer can gain such important information. Aspects related to detailed price comparison between the supplier, its competitors and the range of substitute provides means to investigate buyer strategies. An investigation of the implications of threat by buyer of placing the contract elsewhere to get additional price concessions from its preferred supplier can be carried out.

The assumptions made in the essays and the constraints put on variables can be relaxed to obtain a better representation. A scenario with multiple suppliers/buyers enables enrichment of the findings in the dissertation. The result regarding guidelines for price bandwidth to create long-term relationship can be explored in further detail. The implications on profits by adopting a strategy for long-term relationship can be compared and contrasted with those of adopting the Markov-perfect Nash equilibrium strategies. The insights obtained from the research also provide a background for future studies in supply chain contracts. The problem context and results clearly demonstrate the competitive role of innovation between the buyer and the supplier. It provides a rationale for investigation of supply chain contracts by explicitly considering the evolution of innovation competence and critical assets of collaborating partners.

In the fourth essay, the stochastic differential game model considers Brownian motion for the evolution of wealth for supplier and buyer. These Brownian motions can be treated to be autocorrelated to investigate the associated implications of investment strategy. The model in essay 4 can be extended to examine a sequential game and the associated investment strategy for buyer and supplier. Another extension to the model is consideration of spill-over effects. The spill-over of knowledge created by investments in innovation is presented in literature pertaining to economics and management science. Such a spill-over of knowledge may provide some interesting results regarding buyer-supplier power structure and investment strategy.

The results obtained in these essays provide grounding for hypothesis for future empirical studies. The availability of secondary data could aid in validating the results and thereby fulfilling the theory testing requirement. Primary data can be collected based on the insights generated from this study to explore some of the psychological aspects involved with inter-firm competition in product development. The aspect of buyer-supplier relationship as reflected by the psychological school of thought can be integrated with the results in this study by using survey data from buyer and supplier firms involved in collaborative product development.

Finally, from methodological standpoint, the analysis can adopt classical game-theory and evolutionary game theory to broaden the scope of investigation. Complex adaptive systems based approach can be used to simulate the models in this study. The results obtained from this analytical study may aid in defining key relationships among agents, and can enhance our understanding of supply chain dynamics.

APPENDIX A

A.1 Mathematical Preliminaries, Definitions and Theorems:

In a differential game each player maximizes his objective functional subject to a number of constraints which include, in particular, a differential equation describing the evolution of the state of the game. Optimization problems of this type are known as optimal control problems and are widely used in economic theory and management science. In a differential game involving N players, each of these players seeks to maximize his objective functional - the present value of utility derived over a finite or infinite time horizon - by designing a strategy for those variables which are under his control. His choice influences the evolution of the state of the game via a differential equation (the system dynamics) as well as the objective functionals of his opponents. In the scope of this essay a game is considered with the assumption: (i)that players make their choices simultaneously and (ii) that they represent the solutions to their control problems by Markovian strategies. In this section, the conditions which can be used to verify that a given N-tuple of Markovian strategies constitutes a Nash equilibrium is presented.[1]

A.1.1 Nash equilibrium

Consider a differential game which extends over the bounded time interval $[0, T]$ or the unbounded time interval $[0, \infty)$. For ease of mathematical presentation, let $[0, T\rangle = [0, T]$, if $T < \infty$, and $[0, T\rangle = [0, \infty)$, if $T = \infty$. The state of the game at each instant $t \in [0, T\rangle$ is described by a vector $x(t) \in X$ where $X \subseteq \Re^n$ is the state space of the game. The initial state of the game is a fixed constant $x_0 \in X$. There are N players $i = 1, 2, ..., N$. Player-specific variables, functions, and parameters are denoted by upper indices. At each instant of time $t \in [0, T\rangle$, each player $i \in \{1, 2, \ldots, N\}$ chooses a control variable $u^i(t)$ from his set of feasible controls $U^i(x(t), u^{-i}(t), t) \subseteq \Re^{m^i}$. In general, this set depends on time t, the current state $x(t)$, and the vector $u^{-i}(t)$

[1]The proofs of all the definitions and theorems in this section can be found in Dockner et al.(2000)

consisting of all other players' controls at time t, that is,

$$u^{-i}(t) = (u^1(t), u^2(t), \dots, u^{i-1}(t), u^{i+1}(t), \dots, u^N(t))$$

The state of the game evolves according to the differential equation

$$\dot{x}(t) = f(x(t), u^1(t), u^2(t), \dots, u^N(t), t), \quad x(0) = x_0$$

where the system dynamics f are defined on the set

$$\Omega = \{(x, u^1, u^2, \dots, u^N, t) | x \in X, t \in [0, T\rangle, u^i \in U^i(x, u^{-i}, t), i = 1, 2, \dots, N\}$$

and have values in $\Re^n$. Each player $i \in 1, 2, , \dots, N$ seeks to maximize his objective functional,

$$J^i(u^i(.)) = \int_0^T e^{-r^i t} F^i(x(t), u^1(t), u^2(t), \dots, u^N(t), t)dt + e^{-r^i T} S^i(x(T))$$

Here, $F^i : \Omega \mapsto \Re$ is player i's utility function, r^i his individual rate of time preference, and $S^i : X \mapsto \Re$ his scrap value function. In the case $T = \infty$ we assume that $S(x) = 0$ for all $x \in X$. A Nash equilibrium is an N-tuple of strategies $(\phi^1, \phi^2, \dots, \phi^N)$ such that, given the opponents' equilibrium strategies, no player has an incentive to change his own strategy. Player i's decision problem can be rewritten as

$$\text{Maximize} J^i_{\phi-i}(u^i(.)) = \int_0^T e^{-r^i t} F^i_{\phi-i}(x(t), u^i(t), t)dt + e^{-r^i T} S^i(x(T))$$

$$\text{subject to } \dot{x}(t) = f^i_{\phi-i}(x(t), u^i(t), t), \quad x(0) = x_0, \quad u^i(t) \in U^i_{\phi-i}(x(t), t) \qquad (1)$$

where,

$$F^i_{\phi-i}(x, u^i, t) = F^i(x, \phi^1(x,t), \dots, \phi^{i-1}(x,t), u^i, \phi^{i+1}(x,t), \dots, \phi^N(x,t), t),$$

$$f^i_{\phi-i}(x,u^i,t) = f(x,\phi^1(x,t),\ldots,\phi^{i-1}(x,t),u^i,\phi^{i+1}(x,t),\ldots,\phi^N(x,t),t),$$

$$U^i_{\phi-i}(x,u^i,t) = U^i(x,\phi^1(x,t),\ldots,\phi^{i-1}(x,t),\phi^{i+1}(x,t),\ldots,\phi^N(x,t),t),$$

A Markovian Nash equilibrium for the differential game can thus be defined as follows,

Definition 1 *The N-tuple $(\phi^1,\phi^2,\ldots,\phi^N)$ of functions $\phi^i : X \times [0,T\rangle \mapsto \Re^{m^i}, i \in 1,2,\ldots,N$, is called a Markovian Nash equilibrium if, for each $i \in 1,2,\ldots,N$, an optimal control path $u^i(.)$ of the problem (1) exists and is given by the Markovian strategy $u^i(t) = \phi^i(x(t),t)$.*

The definition suggests that finding a Markovian Nash equilibrium of an N-player differential game amounts to finding Markovian strategies for the solutions of a system of N interdependent optimal control models. If we replace the assumption that optimal paths are defined by Markovian strategies by the assumption that the optimal paths are given by open-loop strategies, then we obtain the following definition.

Definition 2 *The N-tuple $(\phi^1,\phi^2,\ldots,\phi^N)$ of functions $\phi^i : X \times [0,T\rangle \mapsto \Re^{m^i}, i \in 1,2,\ldots,N$, is called an open-loop Nash equilibrium if, for each $i \in 1,2,\ldots,N$, an optimal control path $u^i(.)$ of the problem (1) exists and is given by the open-loop strategy $u^i(t) = \phi^i(t)$.*

The conditions for general Markovian Nash Equilibrium of the N-player game considered above is given by theorem 4.1 of Dockner et al.(2000). The theorem is presented below:

Theorem 1 *Let $(\phi^1,\phi^2,\ldots,\phi^N)$ be a given N-tuple of functions $\phi^i : X \times [0,T\rangle \mapsto \Re^{m^i}$ and make the following assumptions:*

(i) there exists a uniquely absolutely continuous solution $x : [0,T\rangle \mapsto X$ of the initial value problem,

$$\dot{x}(t) = f(x(t),\phi^1(x(t),t),\phi^2(x(t),t),\ldots,\phi^N(x(t),t),t), x(0) = x_0,$$

(ii) for all $i \in 1,2,\ldots,N$ there exists a continuously differentiable function $V^i : X \times [0,T\rangle \mapsto \Re$ such that the HJB equations

$$r^iV^i(x,t) - V^i_t(x,t) = max\{F^i_{\phi^{-i}}(x,u^i,t) + V^i_x(x,t)f^i_{\phi^{-i}}(x,u^i,t)|u^i_{\phi^{-i}}{}^i(x,t)\} \quad (2)$$

are satisfied for all $(x,t) \in X \times [0,T\rangle$,

(iii) if $T < \infty$ then $V^i(x,T) = S^i(x)$ for all $i \in 1,2,\ldots,N$ and all $x \in X$,

(iv) if $T = \infty$ then for all $i \in 1,2,\ldots,N$ either V^i is a bounded function and $r^i > 0$ or V^i is bounded below, $r^i > 0$, and $lim\ sup_{t\to\infty} e^{-r^it}V^i(x(t),t) \leq 0$.

Denote by $\Phi^i(x,t)$, the set of all $u^i \in U^i_{\phi^{-i}}(x,t)$ which maximize the right-hand side of equation(2). If $\phi^i(x(t),t) \in \Phi^i(x(t),t)$ holds for all $i \in 1,2,\ldots,N$ and almost all $t \in [0,T\rangle$ then $(\phi^1,\phi^2,\ldots,\phi^N)$ is a Markovian Nash equilibrium. (If $T = \infty$, optimality is understood in the sense of catching up optimality.)

Alternatively, the theorem can be restated in terms of Hamiltonian.

Theorem 2 *Let an N-tuple $(\phi^1,\phi^2,\ldots,\phi^N)$ of functions $\phi^i : X \times [0,T\rangle \mapsto \Re^{m^i}$ be given and let assumption (i) of theorem 1 be satisfied. Define for all $i \in 1,2,\ldots,N$ the Hamiltonians $H^i_{\phi^{-i}} : X \times \Re^{m^i} \times \Re^n \times [0,T\rangle \mapsto \Re$ by*

$$H^i_{\phi^{-i}}(x,u^i,\lambda^i,t) = F^i_{\phi^{-i}}(x,u^i,t) + \lambda^i f^i_{\phi^{-i}}(x,u^i,t)$$

and the maximized Hamiltonians $H^i_{\phi^{-i}}* : X \times \Re^n \times [0,T\rangle \mapsto \Re$ by

$$H^{i*}_{\phi^{-i}}(x,\lambda^i,t) = max\{H^i_{\phi^{-i}}(x,u^i,\lambda^i,t)|u^i \in U^i_{\phi^{-i}}(x,t)\}$$

Assume that the state space X is convex, the scrap value function S^i are continuously *differentiable and concave, and that there exist N absolutely continuous functions $\lambda^i : [0, T\rangle \mapsto \Re^n$ such that*

(i) the maximum condition $H^i_{\phi^{-i}}(x(t), \phi^i(x(t), t), \lambda^i(t), t) = H^{i*}_{\phi^{-i}}(x(t), \lambda^i(t), t)$ holds for all $i \in 1, 2, \ldots, N$ and almost all $t \in [0, T\rangle$,

(ii) the adjoint equation $\dot{\lambda}^i(t) = r^i\lambda^i(t) - (\partial/\partial x)H^{i*}_{\phi^{-i}}(x(t), \lambda^i(t), t)$ holds for all $i \in 1, 2, \ldots, N$ and for almost all $t \in [0, T\rangle$,

(iii)if $T < \infty$ then $\lambda^i(T) = S^i_x(x(T))$ holds for all $i \in 1, 2, \ldots, N$,

(iv) if $T = \infty$ then either $lim_{t\to\infty}e^{-r^it}\lambda^i(t)\tilde{x}(t) = 0$ holds for all $i \in 1, 2, \ldots, N$ and all feasible state trajectories $\tilde{x}(.)$, or there exists a vector $a \in \Re^n$ such that $x \geq a$ for all $x \in X$, $\lambda^i(t) \geq 0$ for all $i \in 1, 2, \ldots, N$ and all sufficiently large t, and $lim\ sup_{t\to\infty}e^{-r^it}\lambda^i(t)[x(t) - a] \leq 0$,

(v) the function $x \mapsto H^{i*}_{\phi^{-i}}(x, \lambda^i(t), t)$ is continuously differentiable and concave for all $i \in 1, 2, \ldots, N$ and all $t \in [0, T\rangle$.

Then $(\phi^1, \phi^2, \ldots, \phi^N)$ is a Markovian Nash equilibrium. (If $T = \infty$, optimality is understood in the sense of catching up optimality.)

A.1.2 Time Consistency and Subgame Perfectness

Definition 3 *Let $(\phi^1, \phi^2, \ldots, \phi^N)$ be a Markovian Nash equilibrium for the game $\Gamma(x_0, 0)$ and denote by $x(.)$ the unique state trajectory generated in this equilibrium. The equilibrium is called time consistent if, for each $t \in [0, T\rangle$, the subgame $\Gamma(x(t), t)$ admits a Markovian Nash equilibrium $(\psi^1, \psi^2, \ldots, \psi^N)$ such that $\psi^i(y, s) = \phi^i(y, s)$ holds for all $i \in \{1, 2, \ldots, N\}$ and all $(y, s) \in X \times [t, T\rangle$.*

Theorem 3 *Every Markovian Nash equilibrium of a differential game is time consistent.*

Definition 4 *Let $(\phi^1, \phi^2, \ldots, \phi^N)$ be a Markovian Nash equilibrium for the game $\Gamma(x_0, 0)$. The equilibrium is called subgame perfect if, for each $(x, t) \in X \times [0, T\rangle$,* *the subgame $\Gamma(x(t), t)$ admits a Markovian Nash equilibrium $(\psi^1, \psi^2, \ldots, \psi^N)$ such that $\psi^i(y, s) = \phi^i(y, s)$ holds for all $i \in \{1, 2, \ldots, N\}$ and all $(y, s) \in X \times [t, T\rangle$.A Markovian Nash equilibrium which is subgame perfect is also called a Markov perfect nash equilibrium.*

Theorem 4 *Let $(\phi^1, \phi^2, \ldots, \phi^N)$ be a given N-tuple of functions $\phi^i : X \times [0, T\rangle \mapsto \Re^{m^i}$ and make the following assumptions:*

(i) for every pair $(y, s) \in X \times [0, T\rangle$ there exists a unique absolutely continuous solution $x_{y,s} : [s, T\rangle \mapsto X$ of the initial value problem

$$\dot{x}(t) = f(x(t), \phi^1(x(t), t), \phi^2(x(t), t), \ldots, \phi^N(x(t), t), t), \; x(s) = y,$$

(ii) condition (ii)-(iv) of theorem 1 are satisfied with the additional requirement that in (iv), when V^i is not bounded above, lim $sup_{t\to\infty} e^{-r^i t} V^i(x_{y,s}(t), t) \leq 0$ must hold for all $(y, s) \in X \times [0, T\rangle$ (here $x_{y,s}(.)$ is defined in condition (i) above).

Denote by $\Phi^i(x, t)$ the set of all $u^i \in U^i_{\phi^{-i}}(x, t)$ which maximize the right-hand side of (2). If $\phi^i(x, t) \in \Phi^i(x, t)$ holds for all $i \in 1, 2, \ldots, N$ and all $(x, t) \in X \times [0, T\rangle$ then $(\phi^1, \phi^2, \ldots, \phi^N)$ is a Markov perfect Nash equilibrium. (If $T = \infty$, optimality is understood int he sense of catching up optimality.)

Definition 5 *Let $(\phi^1, \phi^2, \ldots, \phi^N)$ be a Markovian Nash equilibrium for the game $\Gamma(x_0, 0)$. The equilibrium is said to be weakly subgame perfect if, for each pair $(x, t) \in X \times [0, T\rangle \cap A$, the subgame $\Gamma(x, t)$ admits a Markovian Nash equilibrium $(\psi^1, \psi^2, \ldots, \psi^N)$ such that $\psi^i(y, s) = \phi^i(y, s)$ holds for all $i \in \{1, 2, \ldots, N\}$ and all $(y, s) \in X \times [t, T\rangle$.*

A.2 Theorems and Proofs

Theorem 5 *The maximized costate variables for player i is a function of time and are given by,*

$$\lambda_1^* = \lambda_{01} e^{L_1 t}; \quad \lambda_1(0) = \lambda_{01} \textit{is a known constant} \tag{3}$$

$$\lambda_2^* = \lambda_{02} e^{L_2 t}; \quad \lambda_2(0) = \lambda_{02} \textit{is a known constant} \tag{4}$$

where λ_1^* and λ_2^* are the costate variables reflecting the marginal price for a unit increase in firm i's own state and the state of the competing firm j.

Proof: Nash equilibrium conditions are derived by constructing the Hamiltonians for the two firms. Since the game formulation is symmetric in terms of objective functional and state dynamics, a symmetric equilibrium is sought such that: $u^{i*}(t) = u^{j*}(t) = u^*(t)$. Analytical solution is derived for firm i by writing the Hamiltonian as:

$$\begin{aligned} H = &-[u^i(t)] + \lambda_1[K_1 u^i(t)^{\alpha^i} - L_1 x^i(t)] + \\ &+ \lambda_2[K_2 u^j(t)^{\alpha^j} - L_2 x^j(t)] \end{aligned} \tag{5}$$

The necessary conditions for optimality are:

$$H_{u^i} = 0; \tag{6}$$

$$\dot{\lambda_1}^* = -H_{x^i}; \quad \dot{\lambda_2}^* = -H_{x^j} \tag{7}$$

$$H(T^*) = -\Phi^{i*}_{T}, \text{since } T \text{ is free} \tag{8}$$

From (23),

$$\dot{\lambda_1^*} = L_1 \lambda_1; \quad \lambda_1^* = \lambda_{01} e^{L_1 t} \tag{9}$$

where $\lambda_1(0) = \lambda_{01}$ is the known positive constant denoting the initial value of the costate λ_1.

$$\lambda_2^* = \lambda_{02} e^{L_2 t} \tag{10}$$

where $\lambda_2(0) = \lambda_{02}$ is a known negative constant denoting the initial value of the costate λ_2. ■

Theorem 6 *The Nash equilibrium investment in product development is given as:*

$$u^i(t)^* = [K_1 \alpha^i \lambda_{01} e^{L_1 t}]^{\frac{1}{1-\alpha^i}} \tag{11}$$

$$u^j(t)^* = [K_2 \alpha^j \psi_{01} e^{L_2 t}]^{\frac{1}{1-\alpha^j}} \tag{12}$$

where $\psi_1(0) = \psi_{01}$ is a known positive constant denoting the initial value of the costate ψ_1.

Proof: Differentiating (21) with respect to $u^i(s)$, we obtain:

$$H_{u^i} = -1 + \lambda_1 \alpha^i K_1 [u^i(t)]^{\alpha^i - 1} \tag{13}$$

By equating(42)to zero and some algebraic manipulations, the following expression is obtained for optimal effort in product development:

$$u^i(t)^* = [K_1 \alpha^i \lambda_1^*]^{\frac{1}{1-\alpha^i}} \tag{14}$$

Substituting the expression for λ_1^* in (14):

$$u^i(t)^* = [K_1 \alpha^i \lambda_{01} e^{L_1 t}]^{\frac{1}{1-\alpha^i}} \tag{15}$$

Because of the underlying symmetry in problem formulation, the corresponding value for $u^j(t)^*$ can be expressed as in equation (12). ■

Theorem 7 *The equilibrium state trajectory of performance improvement is given* *as:*

$$x^i(t) = \frac{e^{-L_1 t}}{L_1}\left[L_1 x_0^i + \frac{(-1+\alpha^i)[(K_1\alpha^i\lambda_{01})^{\frac{1}{1-\alpha^i}} - (K_1\alpha^i\lambda_{01}e^{L_1 t})^{\frac{1}{1-\alpha^i}}}{}\right] \tag{16}$$

Proof: The expression for the optimal state trajectory can be obtained by considering the state dynamics given in equation (2.16). Substituting $u^i(t) = u^i(t)^*$ in (2.16) the following first-order differential equation can be obtained:

$$\dot{x}^i(t) = K_1[K_1\alpha^i\lambda_1^*]^{\frac{\alpha^i}{1-\alpha^i}} - L_1 x^i(t) \tag{17}$$

The first order differential equation can be solved with the initial condition $x^i(0) = x_0^i$. The resulting expression for $x^i(t)^*$ is:

$$x^i(t) = \frac{e^{-L_1 t}}{L_1}\left[L_1 x_0^i + \frac{(-1+\alpha^i)[(K_1\alpha^i\lambda_{01})^{\frac{1}{1-\alpha^i}} - (K_1\alpha^i\lambda_{01}e^{L_1 t})^{\frac{1}{1-\alpha^i}}}{}\right] \tag{18}$$

■

APPENDIX B

B.1 Mathematical Preliminaries, Definitions and Theorems

The theoretical background presented in essay 1 forms the core for solving a hierarchical play game as well. This section covers some of the salient analytical aspects specific to a hierarchical play game.

For a finite horizon T let L and F denote the leader and follower respectively. Let x denote the vector of state variables, u^L the vector of control variables of the leader, and u^F the vector of control variables of the follower. Assume $x \in \Re^n$, $u^L \in \Re^{m^L}$, *and* $u^F \in \Re^{m^F}$.

Definition 1 *The initial value of the follower's costate variable λ_i is said to be non-controllable if $\lambda_i^*(0)$ is independent of the leader's control path $u^L(t)$. Otherwise, it is said to be controllable.*

The definition suggests that if the costate variable is controllable, the follower's control variable $u^F(t)$ at time t depends also on future values of $u^L(.)$, that is, on values $u^L(s)$ with $s > t$.

B.2 Theorems and Proofs

Theorem 1 *The maximized costate variables for the follower are function of time and is given by,*

$$\lambda_1^* = \lambda_{01} e^{L_2 t} \tag{19}$$

$$\lambda_2^* = \lambda_{02} e^{L_1 t} \tag{20}$$

where λ_1^* and λ_2^* are the costate variables reflecting the marginal price for a unit increase in follower firm's own state and the state of the leader i; $\lambda_1(0) = \lambda_{01}$, $\lambda_2(0) = \lambda_{02}$ are constants.

Proof: Stackelberg equilibrium conditions are derived by constructing the Hamiltonians. Analytical solution is derived for the follower firm j by writing the Hamiltonian as:

$$\begin{aligned} H^j &= -[u^j(t)] + \lambda_1[K_2 u^j(t)^{\alpha^j} - L_2 x^j(t) + M_2 u^i(t)] + \\ &+ \lambda_2[K_1 u^i(t)^{\alpha^i} - L_1 x^i(t)] \end{aligned} \tag{21}$$

The necessary conditions for optimality are:

$$H^j_{u^j} = 0; \tag{22}$$

$$\dot{\lambda_1}^* = -H^j_{x^j}; \quad \dot{\lambda_2}^* = -H^j_{x^i} \tag{23}$$

$$x^j(0) = x^j_0 \tag{24}$$

From (23),

$$\dot{\lambda_1^*} = L_2\lambda_1; \quad \lambda_1^* = \lambda_{01}e^{L_2 t} \tag{25}$$

$$\dot{\lambda_2^*} = L_1\lambda_2; \quad \lambda_2^* = \lambda_{02}e^{L_1 t} \tag{26}$$

where $\lambda_1(0) = \lambda_{01}$ is the known positive constant denoting the initial value of the costate λ_1 and $\lambda_2(0) = \lambda_{02}$ is a known negative constant denoting the initial value of the costate λ_2. ■

Theorem 2 *The Stackelberg equilibrium investment by the follower in product development is given as:*

$$u^j(t)^* = [K_2\alpha^j\lambda_{01}e^{L_2 t}]^{\frac{1}{1-\alpha^j}} \tag{27}$$

Proof: Differentiating (21) with respect to $u^j(t)$, we obtain:

$$H_{u^j} = -1 + \lambda_1\alpha^j K_2[u^j(t)]^{\alpha^j - 1} \tag{28}$$

By equating(28)to zero and some algebraic manipulations, the following expression is obtained for optimal effort in product development:

$$u^j(t)^* = [K_2\alpha^j\lambda_1^*]^{\frac{1}{1-\alpha^j}} \tag{29}$$

Substituting the expression for λ_1^* in (29):

$$u^j(t)^* = [K_2\alpha^j\lambda_{01}e^{L_2t}]^{\frac{1}{1-\alpha^j}} \quad \blacksquare \tag{30}$$

Theorem 3 *The maximized costate variables for the leader are given by,*

$$\psi_1^* = \psi_{01}e^{L_1t} \tag{31}$$

$$\psi_2^* = \psi_{02}e^{L_2t} \tag{32}$$

$$\psi_3^* = -\psi_{03}e^{-L_2t} \tag{33}$$

where ψ_1^* and ψ_2^* and ψ_3^* are the costate variables reflecting the marginal price for a unit increase in leader's own state, the state of the follower and the costate of the follower; ψ_{01}, ψ_{02} and ψ_{03} are constants.

Proof: Stackelberg equilibrium conditions are derived by constructing the Hamiltonian. Analytical solution is derived for the leader firm i by writing the Hamiltonian as:

$$\begin{aligned} H^i &= -[u^i(t)] + \psi_1[K_1u^i(t)^{\alpha^i} - L_1x^i(t)] + \\ &+ \psi_2[K_2u^j(t)^{\alpha^j} - L_2x^j(t) + M_2u^i(t)] + \psi_3[\lambda_1L_2] \end{aligned} \tag{34}$$

The necessary conditions for optimality are:

$$H^i_{u^i} = 0; \tag{35}$$

$$\dot{\psi_1}^* = -H^i_{x^i}; \quad \dot{\psi_2}^* = -H^i_{x^j}; \quad \dot{\psi_3}^* = -H^i_{\lambda_1} \tag{36}$$

$$x^i(0) = x^i_0 \tag{37}$$

From (36),

$$\dot{\psi}_1^* = L_1\psi_1; \quad \psi_1^* = \psi_{01}e^{L_1 t} \tag{38}$$

$$\dot{\psi}_2^* = L_2\psi_2; \quad \psi_2^* = \psi_{02}e^{L_2 t} \tag{39}$$

$$\dot{\psi}_3^* = -\psi_3 L_2; \quad \psi_3^* = \psi_{03}e^{-L_2 t} \tag{40}$$

where ψ_1^* and ψ_2^* and ψ_3^* are the costate variables reflecting the marginal price for a unit increase in leader's own state, the state of the follower and the costate of the follower; $\psi_{01}, \psi_{02}$and ψ_{03} are constants.

The Stackelberg equilibrium investment by the leader in product development is given as:

$$u^i(t)^* = \left[\frac{K_1\alpha^i\psi_{01}e^{L_1 t}}{1 - M_2\psi_{02}e^{L_2 t}}\right]^{\frac{1}{1-\alpha^i}} \tag{41}$$

Proof: Differentiating (21) with respect to $u^i(t)$, we obtain:

$$H_{u^i} = -1 + \psi_1\alpha^i K_1[u^i(t)]^{\alpha^i - 1} + \psi_2 M_2 \tag{42}$$

By equating(42)to zero and some algebraic manipulations, the following expression is obtained for optimal effort in product development:

$$u^i(t)^* = \left[\frac{K_1\alpha^i\psi_1^*}{1 - M_2\psi 2^*}\right]^{\frac{1}{1-\alpha^i}} \tag{43}$$

Substituting the expression for ψ_1^* and ψ_2^* in (43):

$$u^i(t)^* = \left[\frac{K_1\alpha^i\psi_{01}e^{L_1 t}}{1 - M_2\psi_{02}e^{L_2 t}}\right]^{\frac{1}{1-\alpha^i}} \tag{44}$$

■

Theorem 4 *The equilibrium state trajectory of performance improvement of the follower is given as:*

$$x^j(t) = \frac{e^{-L_2 t}}{L_2}\left[L_2 x_0^j + (-1+e^{L_2 t})M_2\left[\frac{K_1\alpha^i\psi_{01}e^{L_1 t}}{1-M_2\psi_{02}e^{L_2 t}}\right]^{\frac{1}{1-\alpha^i}} + \right.$$
$$\left. + (-1+e^{L_2 t})K_2(K_2\alpha^j\lambda_{01}e^{L_2 t})^{\frac{\alpha^j}{(1-\alpha^j)^2}}\right] \tag{45}$$

Proof: The expression for the optimal state trajectory can be obtained by considering the state dynamics given in equation (2.16). Substituting $u^i(t) = u^i(t)^*$ and $u^j(t) = u^j(t)^*$ in (2.16) the following first-order differential equation can be obtained:

$$\dot{x}^j(t) = K_2[K_2\alpha^j\lambda_1^*]^{\frac{\alpha^j}{1-\alpha^j}} - L_2x^j(t)(T_p+\tau-t) + M_2\left[\frac{K_1\alpha^i\psi_1^*}{1-M_2\psi 2^*}\right]^{\frac{1}{1-\alpha^i}} \tag{46}$$

The first order differential equation can be solved with the initial condition $x^j(0) = x_0^j$. The resulting expression for $x^j(t)^*$ is:

$$x^j(t) = \frac{e^{-L_2 t}}{L_2}\left[L_2 x_0^j + (-1+e^{L_2 t})M_2\left[\frac{K_1\alpha^i\psi_{01}e^{L_1 t}}{1-M_2\psi_{02}e^{L_2 t}}\right]^{\frac{1}{1-\alpha^i}} + \right.$$
$$\left. + (-1+e^{L_2 t})K_2(K_2\alpha^j\lambda_{01}e^{L_2 t})^{\frac{\alpha^j}{(1-\alpha^j)^2}}\right] \tag{47}$$

■

Theorem 5 *The equilibrium state trajectory of performance improvement of the leader is given as:*

$$x^i(t) = \frac{e^{-L_1 t}}{L_1}\left[L_1 x_0^i + (-1+e^{L_1 t})K_1\left[\frac{K_1\alpha^i\psi_{01}e^{L_1 t}}{1-M_2\psi_{02}e^{L_2 t}}\right]^{\frac{\alpha^i}{1-\alpha^i}}\right] \tag{48}$$

Proof: The expression for the optimal state trajectory can be obtained by considering the state dynamics given in equation (3.4). Substituting $u^i(t) = u^i(t)^*$ in (3.4) the following first-order differential equation can be obtained:

$$\dot{x}^i(t) = K_1 \left[\frac{K_1 \alpha^i \psi_1^*}{1 - M_2 \psi 2^*} \right]^{\frac{\alpha^i}{1-\alpha^i}} - L_1 x^i(t) \tag{49}$$

The first order differential equation can be solved with the initial condition $x^i(0) = x_0^i$. The resulting expression for $x^i(t)^*$ is:

$$x^i(t) = \frac{e^{-L_1 t}}{L_1} \left[L_1 x_0^i + (-1 + e^{L_1 t}) K_1 \left[\frac{K_1 \alpha^i \psi_{01} e^{L_1 t}}{1 - M_2 \psi_{02} e^{L_2 t}} \right]^{\frac{\alpha^i}{1-\alpha^i}} \right] \tag{50}$$

■

APPENDIX C

C.1 Mathematical Preliminaries

A piecewise deterministic process is a system which evolves in a deterministic way, except at certain jump times $T_1, T_2, \ldots$ at which the deterministic law of motion switches from one model to another. Both the jump times T_1 and the system modes, which govern the motion between jump times, are randomly selected. In the purview of piecewise deterministic differential games, in this essay, analysis is restricted to models where there are finitely many different modes and where the evolution of the process between the jump times is described by a deterministic differential equation, which may depend on the current mode. Furthermore, attention is focused on the case of an autonomous problem defined over the unbounded time interval $[0, \infty)$. The discount rate is assumed to be strictly positive, and the utility function is assumed to be bounded so that the present value of utility is finite for all feasible control paths. The analytical aspects of these games are mentioned below.

Let $X \subseteq \Re^n$ denote the state space of the model and let M be a finite set of modes. One may think of the mode as an additional state variable with discrete values. Let $x(t)$ denote the state at time t and $u(t)$ the control chosen at time t. The set of controls, which are feasible at time t when the current mode is $h \in M$ and the state is equal to $x(t) \in X$, is given by $U(h, x(t)) \subseteq \Re^m$. The evolution of the state during an interval in which the mode of the system equals h is described by the differential equation $\dot{x}(t) = f(h, x(t), u(t))$, where $f(h, \cdot, \cdot)$ maps the set $\Omega(h) = \{(x, u) | x \in X, u \in U(h, x)\}$ into $\Re^n$. The instantaneous payoff rate at time t, when the current mode is equal to h, is given by $F(h, x(t), u(t))$, where $F(h, \cdot, \cdot)$ is a real-valued function defined on $\Omega(h)$. In addition, the decision maker receives the lump sum payoff $S_{hk}(\cdot)$ if the jump from mode h to mode $k \neq h$ occurs at time t. The function $S_{hk}(\cdot)$ is defined on X and has values in $\Re$. All payoffs are discounted at the constant rate $r > 0$ and the initial state and mode are deterministic constants, $x_0 \in M$ and $h_0 \in M$, respectively.

In these differential game models, the dynamic system can switch between modes in a nondeterministic way. The mathematical tool to describe the evolution of the system mode as a function of time is a continuous-time stochastic process $h : [0, \infty) \times \Xi \mapsto M$. Here Ξ is a set of points ξ representing possible realizations of some random phenomenon (like a coin toss). Subsets of Ξ are called events and each event has a certain probability. If the event A is characterized as the set of all those $\xi \in \Xi$ which satisfy a certain condition a, then we denote the probability of A simply by $Pr\{a\}$. For example, the event that the mode of the system at time t is equal to h is $\{\xi \in \Xi | h(t, \xi) = h\}$ and its probability is denoted by $Pr\{h(t, \xi) = h\}$. Quite often the argument ξ is suppressed for notational simplicity so that the probability is written as $Pr\{h(t) = h\}$.

Conditional probabilities play a major role in probability theory and are also important in the description of the stochastic process. If A and B are two events characterized by conditions a and b, respectively, and if $Pr\{b\} > 0$, then the probability of A conditional on the occurrence of B is given by $Pr\{a|b\} = \frac{Pr\{a \text{ and } b\}}{Pr\{b\}}$. To describe the probability law that governs the switches between modes in a stochastic control model, assume that for every pair of modes $(h, k) \in M \times M$with $h \neq k$ there exists a function $q_{hk} : \Omega(h) \mapsto \Re_+$,such that the following relation holds

$$\lim_{\Delta \to 0} \frac{1}{\Delta} Pr\{h(t + \Delta) = k | h(t) = h\} = q_{hk}(x(t), u(t)) \quad k \neq h \tag{51}$$

This means that the probability that the system switches from mode h to another mode k in the short time interval $(t, t + \Delta]$, given that the mode was h at time t, is, to a first approximation, proportional to the length of the interval Δ. The factor of proportionality is equal to $q_{hk}(x(t), u(t))$. In addition to (51) we assume that the probability that two or more switches occur during the interval $(t, t + \Delta]$, divided by Δ, converges to 0 as Δ approaches 0.The functions $f(h, .)$, $F(h, ., S_{hk}(.)$, and $q_{hk}(.$

are sufficiently smooth and that $F(h, .), S_{hk}(.)$, and $q_{hk}(.)$ are bounded. Given the initial state $x_0 \in X$, the initial mode $h_0 \in M$, and the control path $u(.)$, the system dynamics

$$\dot{x} = f(h(t), x(t), u(t))$$

and equation (51) determine a unique stochastic process $h : [0, \infty) \times \Xi \mapsto M$. This process is called a piecewise deterministic process and it is known to be continuous from the right. This means that $lim_{k \to \infty} h(S_k, \xi) = h(t, \xi)$ holds for almost all $\xi \in \Xi$, for all $t \in [0, \infty)$, and for every sequence of real numbers $s_k > t$ with $lim_{k \to \infty} s_k = t$.

For simplicity, the dependence of the process on the initial conditions is not made explicit in the notation presented here. However it is important to note that the probability law governing the process depends also on the chosen control path $u(.)$ and therefore the probabilities or expectations with respect to these laws can be written as $Pr_{u(.)}$ or $E_{u(.)}$. Furthermore, it should be realized that not only is $h(.)$ a stochastic process but so is $x(.)$. This follows from the fact that the system dynamics f depend on the realization of $h(.)$. Therefore $x(.)$ is in fact $x(t, \xi)$but, for notational simplicity it is written as $x(t)$.

Different realizations of the stochastic process (i.e., different realizations $\xi \in \Xi$) result in different payoff stream for the agent even if the control path is fixed. Therefore, it is assumed that the decision maker maximizes the expectation of the discounted payoff stream, conditional on the given initial state and mode.

C.2 Definitions and Theorems

The following definition and theorem (see Definition 8.2 and Theorem 8.2 in Dockner et al. (2000)) pertaining to the theory of piecewise deterministic games are utilized in this essay.

Definition 1 *An N-tuple* $(\phi^1, \phi^2, \ldots, \phi^N)$ *of functions* $\phi^i : M \times X \mapsto \Re^{m^i}, i \in \{1, 2, \ldots, N\}$ *, is a stationary Nash equilibrium of the game* $\Gamma(h_0, x_0)$ *if, for each* *player $i \in \{1, 2, \ldots, N\}$, an optimal control path $u^i(.)$ of the problem (12)-(14) exists and is given by the stationary Markovian strategy $u^i(t) = \phi^i(h(t), x(t))$. If $(\phi^1, \phi^2, \ldots, \phi^N)$ is a stationary Markovian Nash equilibrium for all games $\Gamma(h, x)$ with $(h, x) \in M \times X$ then it is called subgame perfect.*

$$J^i_{\phi^{-i}}(u^i(.)) = E_{u(.)}\Big\{ \int_0^\infty e^{-r^i t} F_{\phi^{-i}}(h(t), x(t), u^i(t))dt + \\ + \sum_{l \in \mathbb{N}} e^{-r^i T_l} S_{h(T_l-)h(T_l)}(x(T_l)) | x(0) = x_0, h(0) = h_0 \Big\} \tag{52}$$

subject to the constraints

$$\dot{x}(t) = f^i_{\phi^{-i}}(h(t), x(t), u^i(t)); \quad x(0) = x_0; \quad u^i(t) \in U^i_{\phi^{-i}}(h(t), x(t)) \tag{53}$$

where the piecewise deterministic process $h(.)$ is determined by the initial condition $h(0) = h_0$ and the switching rates

$$q^i_{\phi^{-i},hk}(x(t), u^i(t)) \tag{54}$$

The functions $F^i_{\phi^{-i}}, f^i_{\phi^{-i}}, U^i_{\phi^{-i}}$ and $q^i_{\phi^{-i},hk}$ are defined by,

$$F^i_{\phi^{-i}} = F^i(h, x, \phi^1(h, x), \ldots, \phi^{i-1}(h, x), u^i, \phi^{i+1}(h, x), \ldots, \phi^N(h, x))$$

$$f^i_{\phi^{-i}} = f(h, x, \phi^1(h, x), \ldots, \phi^{i-1}(h, x), u^i, \phi^{i+1}(h, x), \ldots, \phi^N(h, x))$$

$$U^i_{\phi^{-i}} = U^i(h, x, \phi^1(h, x), \ldots, \phi^{i-1}(h, x), u^i, \phi^{i+1}(h, x), \ldots, \phi^N(h, x))$$

$$q^i_{\phi^{-i},hk}(x, u^i) = q_{hk}(x, \phi^1(h, x), \ldots, \phi^{i-1}(h, x), u^i, \phi^{i+1}(h, x), \ldots, \phi^N(h, x)) \tag{55}$$

All players j use the strategies $u^j(t) = \phi^j(h(t), x(t))$

Theorem 1 *Let $(\phi^1, \phi^2, \ldots, \phi^N)$ be a given N-tuple of functions $\phi^i : M \times X \mapsto \Re^{m^i}, i \in 1, 2, \ldots, N$, and assume that the piecewise deterministic process defined by* *the state dynamics,*

$$\dot{x}(t) = f(h(t), x(t), \phi^1(h(t), x(t)), \phi^2(h(t), x(t)), \ldots, \phi^N(h(t), x(t)) \tag{56}$$

and the switching rates,

$$q_{hk}(x(t), \phi^1(h(t), x(t)), \ldots, \phi^{i-1}(h(t), x(t)), u^i, \phi^{i+1}(h(t), x(t)), \ldots, \phi^N(h(t), x(t))) \tag{57}$$

is well defined for all initial conditions $(h(0), x(0)) = (h, x) \in M \times X$*. Let there exist* N *bounded functions* $V^i : M \times X \mapsto \Re, i = 1, 2, \ldots, N$*, such that* $V^i(h, x)$ *is continuously differentiable in* x *and such that the HJB equations,*

$$\begin{aligned} r^i V^i(h, x) = max\Big\{ & F^i_{\phi^{-i}}(h, x, u^i) + V^i_x(h, x) f^i_{\phi^{-i}}(h, x, u^i) + \\ & + \sum_{k \neq h} q^i_{\phi^{-i},hk}(x, u^i)[S^i_{hk}(x) + V^i(k, x) - V^i(h, x)] | u^i \in U_{\phi^{-i}}(h, x) \Big\} \end{aligned} \tag{58}$$

are satisfied for all $i \in \{1, 2, \ldots, N\}$ *and all* $(h, x) \in M \times X$*. Denote by* $\Phi^i(h, x)$ *the set of all* $u^i \in U^i_{\phi^{-i}}(h, x)$ *, which maximize the right hand side of equation (58). If* $\phi^i(h, x) \in \Phi^i(h, x)$ *holds for all* $i \in \{1, 2, \ldots, N\}$ *and all* $(h, x) \in M \times X$ *then* $(\phi^1, \phi^2, \ldots, \phi^N)$ *is a stationary Markovian Nash equilibrium. Moreover, the equilibrium is subgame perfect. As per the above theorem, the piecewise deterministic dynamics for this essay are given by* $\dot{x}_S(t) = f(h(t), x_S(t))$ *and the switching rate is given by* $k_{01} = K(h(t), x_S(t))$*. Initial conditions are* $h(t) = 0$ *and* $x_S(0) = x_{S0} > 0$*. .*

C.3 Theorems and Proofs

Theorem 2 *If* V^{S*} *and* V^{B*} *denote the positive equilibrium value of the supplier and buyer firm, then,*

$$V^{S*} = \frac{a^2\sqrt{c_2}}{\sqrt{2\{8r(b + c_1)^2(\bar{V^B} + rc_2) - a^2 b\}}} \tag{59}$$

$$V^{B*} = 2c_2r + \bar{V^B} - \frac{\sqrt{c_2}\sqrt{8r(b+c_1)^2(\bar{V^B}+rc_2) - a^2b}}{\sqrt{2}(b+c_1)} \tag{60}$$

constitute the equilibrium pair.

Proof: From equations (4.19), (4.20), (4.21) and (4.22), the maximization of the expression in the right hand side leads to the expressions,

$$G^S(z) = \frac{(a-z)^2}{4(b+c_1)} \tag{61}$$

$$G^B(z) = \frac{z^2}{4c_2} \tag{62}$$

$$U_S(0,x) = \frac{a - V_x^S(0,x)}{2(b+c_1)} = \frac{a - V_x^S(0,x)}{2(b+c_1)} \tag{63}$$

$$U_B(0,x) = \frac{\bar{V^B} - V^B(0,x)}{2c_2} \tag{64}$$

The system of autonomous nonlinear differential equations as specified in equations (4.15) and (4.16) can be analyzed by exploring the corresponding equilibrium points or critical points of the equations. These points are obtained by setting $V_x^S = 0$ and $V_x^B = 0$ respectively. Equations (4.15) and (4.16) thus reduces to,

$$rV^S(0,x) = G^S(0) - U_B(0,x)V^S(0,x) \tag{65}$$

and,

$$rV^B(0,x) = \sigma(U_S(0,x)) + G^B[\bar{V^B} - V^B(0,x)] \tag{66}$$

Substituting for G^S, G^B, $U_S(0,x)$ and $U_B(0,x)$ from equations (61), (62), (63) and (64), and solving for $V^S(0,x)$ and $V^B(0,x)$ leads to:

$$V^{S*} = \frac{a^2\sqrt{c_2}}{\sqrt{2\{8r(b+c_1)^2(\bar{V^B}+rc_2) - a^2b\}}} \tag{67}$$

and,

$$V^{B*} = 2c_2r + \bar{V^B} - \frac{\sqrt{c_2}\sqrt{8r(b+c_1)^2(\bar{V^B}+rc_2) - a^2b}}{\sqrt{2}(b+c_1)} \quad \blacksquare \tag{68}$$

Theorem 3 *If U_S^* and U_B^* denote the positive equilibrium control functions of the supplier and buyer firm, and then,*

$$U_S^*(0,x) = \frac{a - V_x^S(0,x)}{2(b+c_1)} = \frac{a}{2(b+c_1)} \tag{69}$$

$$U_B^*(0,x) = \frac{\bar{V^B} - V^B(0,x)}{2c_2} = \frac{\sqrt{8r(b+c_1)^2(\bar{V^B}+rc_2) - a^2b}}{2\sqrt{2}(b+c_1)\sqrt{c_2}} - r \tag{70}$$

Proof: The equilibrium value of $U_S^*(0,x)$ is obtained by substituting $V_x^S(0,x) = 0$ and finding the argument u_S that maximizes the right hand side of equation (4.21). Similarly, the equilibrium value $U_B^*(0,x)$ is obtained by finding the argument u_B that maximizes the right hand side of equation (4.22) and substituting the equilibrium point for the value of buyer $V^{B*}(0,x)$ from equation (60). Therefore,

$$U_S^*(0,x) = \frac{a - V_x^S(0,x)}{2(b+c_1)} = \frac{a}{2(b+c_1)}$$

$$U_B^*(0,x) = \frac{\bar{V^B} - V^B(0,x)}{2c_2} = \frac{\sqrt{8r(b+c_1)^2(\bar{V^B}+rc_2) - a^2b}}{2\sqrt{2}(b+c_1)\sqrt{c_2}} - r \quad \blacksquare$$

Theorem 4 *At equilibrium the remaining demand at time t can be expressed as:*

$$x(t) = x_0 - \frac{at}{2(b+c_1)} \tag{71}$$

where $x(0) = x_0$ is the initial remaining market demand.

Proof: The theorem follows by substitution of $u_S = U_S^*$ in the equation for state dynamics and solving the resulting first order differential equation.

$$\dot{x}(t) = -U_S^* = -\frac{a}{2(b+c_1)} \quad \blacksquare$$

which leads to,

$$x(t) = x_0 - \frac{at}{2(b + c_1)}$$

where $x(0) = x_0$ is the initial remaining market demand.

APPENDIX D

D.1 Mathematical Preliminaries

D.1.1 Probability Spaces

Underlying the mathematical descriptions of random variables and events is the notion of a probability space $(\Omega, \mathcal{F}, P)$. A sample space Ω is a non empty set that represents the collection of all possible outcomes of an experiment. The elements of Ω are called sample points. The sigmafield $\mathcal{F}$ is a collection of subsets of Ω that includes the empty set ϕ (the "impossible event") as well as the set Ω (the "sure event") and is closed under the set operations of complements and finite or denumerable unions and intersections. The elements of $\mathcal{F}$ are called measurable events or simply events. The probability measure P is an assignment of probabilities to events (sets) in $\mathcal{F}$ and is subject to the conditions : (i) $0 \leq P(F) \leq 1$ for each $F \in \mathcal{F}$; (ii) $P(\phi) = 0, P(\Omega) = 1$; *(iii)* $P(\bigcup_i F_i) = \sum_i P(F_i)$, for any finite or denumerable sequence of mutually exclusive (pairwise disjoint) events $F_i, i = 1, 2, \ldots$ belonging to $\mathcal{F}$.

The closure property of $\mathcal{F}$ ensures that the usual applications of set operations in representing events do not lead to nonmeasurable events for which no (consistent) assignment of probability is possible. The required countable additivity property (iii) gives probabilities a sufficiently rich structure for doing calculations and approximations involving limits. Two immediate consequences of (iii) are the following so-called continuity properties: if $A_1 \subset A_2 \subset \ldots$ is a nondecreasing sequence of events in $\mathcal{F}$ then, equation (1) below can be thought of as the limiting sequence of $\bigcup_{n=1}^{\infty} A_n$.

$$\bigcup_{n=1}^{\infty} A_n = \lim_n P(A_n) \tag{72}$$

By considering complements, one gets for decreasing measurable events $A_1 \supset A_2 \supset$

. . . that,

$$\bigcap_{n=1}^{\infty} A_n = \lim_n P(A_n) \tag{73}$$

While (1) holds for all countably additive set functions μ (in place of P) on $\mathcal{F}$, finite or otherwise, (2) does not hold if $\mu(A_n)$ is not finite for at least some n (onwards). If Ω is a finite or denumerable set, then probabilities are defined for all subsets F of Ω once they are specified for singletons, so $\mathcal{F}$ is a collection of all subsets of Ω. Thus if f is a probability mass function (p.m.f.) for singletons. i.e., $f(\omega) \geq 0$ for all $\omega \in \Omega$ and $\sum_\omega f(\omega) = 1$, then one may define $P(F) = \sum_{\omega \in F} f(\omega)$. The function P so defined on the class of all subsets of Ω is countably additive, i.e. P satisfies (iii). So, $(\Omega, \mathcal{F}, P)$ is easily seen to be a probability space. In this case the probability measure P is determined by the probabilities of singletons ω.

In the case when Ω is not finite or denumerable, e.g. if Ω is the real line or the space of all infinite sequences of 0's and 1's, then the above formulation is no longer possible in general. Instead, for example, in the case $\Omega = \Re^1$, one is often given a piecewise continuous probability density function (p.d.f.) f, i.e. f is nonnegative, integrable, and $\int_{-\infty}^{\infty} f(x)dx = 1$. For an interval $I = (a, b)$ or $(b, \infty), -\infty \leq a < b \leq \infty$ one then assigns the probability $P(I) = \int_a^b f(x)dx$ by a Riemann integral. This set function P may be extended to the class $\mathcal{C}$ comprising all finite unions $F = \bigcup_j P(I_j)$. The class $\mathcal{C}$ is a field, i.e., $\emptyset$ and Ω belongs to $\mathcal{C}$ and it is closed under complements and finite intersection (and therefore finite unions). But, since $\mathcal{C}$ is not a sigmafield, usual sequentially applied operations on events may lead to events outside of $\mathcal{C}$ for which probabilities have not been defined. But a theorem from measure theory, the *Carathéodory Extension Theorem*, asserts that there is a unique countably additive extension of P from a field $\mathcal{C}$ to the smallest sigmafield that contains $\mathcal{C}$. In the case of $\mathcal{C}$ above, this sigmafield is called the Borel sigmafield $\mathcal{B}^1$ on $\Re^1$ and its sets are called Borel sets of $\Re^1$.

In general, such an extension of P to the power set sigmafield, that is the collection of all subsets of $\Re^1$, is not possible. The same considerations apply to all measures (i.e. countably additive nonnegative set functions μ defined on a sigmafield with $\mu(\emptyset) = 0$), whether the measure of Ω is 1 or not. The measure $\mu = m$, which is defined first for each interval I as the length of the interval, and then extended uniquely to $\mathcal{B}^1$ is called the Lebesque measure on $\Re^1$. Similarly, one defines the Lebesgue measure on $\Re^k (k \geq 2)$ whose Borel sigmafield $\mathcal{B}^k$ is the smallest sigmafield that contains all k-dimensional rectangles $I = I_1 \times I_2 \times \cdots \times I_k$, with I_j a one-dimensional rectangle (interval) of the previous type. The Lebesgue measure of a rectangle is a product of the lengths of its sides, i.e., its volume. Lebesgue measure on $\Re^k$ has the property that the space can be decomposed into a countable union of measurable sets of finite Lebesgue measure; such measures are said to be sigma-finite (the most commonly used measure).

D.1.2 Differential games with white noise

In stochastic differential games with white noise, uncertainty enters in the form of a Wiener process. Wiener processes are also known as Brownian motion or white noise processes. A standardized k-dimensional Wiener process w with time domain $[0, T\rangle$, where $[0, T\rangle = [0, T]$ if T is a finite number and $[0, T\rangle = [0, \infty)$ if $T = \infty$, is a continuous-time stochastic process with values in $\Re^k$, that is, $w : [0, T\rangle \times \Xi \mapsto \Re^k$, where Ξ is a set of points ξ representing possible realizations of some random phenomenon (like a coin toss). Subsets of Ξ are called events and each event has a certain probability. The defining properties of a Wiener process are:

1. $w(0, \xi) = w_0$ for all ξ in a set of probability 1 where $w_0 \in \Re^k$ is an arbitrary initial value;

2. for any finite sequence of real numbers $(t_1, t_2, \ldots, t_l)$ with $0 \leq t_1 < t_2 < \cdots < t_l \leq T$ it holds that the random variables $w(t_1, .)$ and $w(t_{i+1}, .) - w(t_i, .), i \in 1, 2, \ldots, l-1$ are stochastically independent.

3. for all pairs (s,t) of real numbers such that $0 \leq s < t \leq T$, the random variable $w(t,.) - w(s,.)$ has a normal distribution with mean vector $0 \in \Re^k$ and covariance matrix $(t-s)I$, where $I \in \Re^{k \times k}$ denotes the $k \times k$ unit matrix.

Since the initial value of the Wiener process w_0 has no importance in the equilibrium results, this value is typically taken to be equal to 0. The state variable x is described by a stochastic differential equation of the form,

$$dx(t) = f(x(t), u(t), t)dt + \sigma(x(t), u(t), t)dw(t), x(0) = x_0 \tag{74}$$

Here f is a function defined on $\Omega = \{(x, u, t) | x \in X, u \in U(x(t), t), t \in [0, T\rangle\}$ with values in $\Re^n$. The function σ is also defined on Ω and takes values in $\Re^{n \times k}$. In other words, $\sigma(x, u, t)$ is an $n \times k$ matrix for every $(x, u, t) \in \Omega$. A component $\sigma_{ij}(x, u, t)$ of this matrix measures the direct influence of the jth component of the k-dimensional Wiener process on the evolution of the ith component of the n-dimensional state vector. Note that in case both k and n are equal to 1 then the function σ is a real-valued function. Furthermore, if σ is identically equal to 0 then the state dynamics is similar to that in the deterministic differential game. $x(.)$ is a solution to the stochastic differential equation (74) if $x(.)$ satisfies the integral equation

$$x(t) = x_0 + \int_0^t f(x(s), u(s), s)ds + \int_0^t \sigma(x(s), u(s), s)dw(s) \tag{75}$$

for all ξ in a set of probability 1. The first integral on the right hand side is the usual Riemann integral while the second integral has to be interpreted as the limit

$$\lim_{\delta \to 0} \sum_{l=1}^{L-1} \sigma(x(t_l), u(t_l), t_l)[w(t_{l+1}) - w(t_l)] \tag{76}$$

where $0 = t_1 < t_2 < \ldots t_L = t$ and $\delta = max\{|t_{l+1} - t_l| : 1 \leq l \leq L-1\}$. Ito's lemma, a basic result in stochastic calculus is used for solving stochastic differential games with white noise.

Ito's Lemma 1 *Suppose that $x(.)$ solves the stochastic differential equation (74). Let $G : X \times [0, T\rangle \mapsto \Re$ be a (nonrandom) function with continuous partial derivatives G_t, G_x, G_{xx}. Then the function $g(t) = G(x(t), t)$ satisfies the stochastic differential equation,*

$$\begin{aligned} dg(t) = \{ & G_t(x(t),t) + G_x(x(t),t)f(x(t),u(t),t) + \frac{1}{2}tr[G_{xx}(x(t),t)\sigma(x(t),u(t),t) + \\ & + \frac{1}{2}tr[G_{xx}(x(t),t)\sigma(x(t),u(t),t)\sigma'(x(t),u(t),t)]\}dt + \\ & + G_x(x(t),t)\sigma(x(t),u(t),t)dw(t) \end{aligned} \tag{77}$$

An important point to be noted is that any solution to the stochastic differential equation (74) is a stochastic process depending on the realization of $\xi \in \Xi$. This implies that solutions $x(t)$ and $u(t)$ are in fact $x(t, \xi)$ and $u(t, \xi)$ respectively.

D.2 Definitions and Theorems

In this section, some definitions and theorems are provided that are necessary for solving the differential game problem.

D.2.1 Competitively Optimal Strategies

The lower and upper values of the game are given by, $\underline{v}(z) = \sup_{u_S \in U} \inf_{u_B \in U} v^{u_S, u_B}(z)$, and $\bar{v}(z) = \inf_{u_B \in U} \sup_{u_S \in U} v^{u_S, u_B}(z)$ respectively, where U is the set of all admissible controls. If $\underline{v}(z) = \bar{v}(z)$ for every z, then the value of the game is given by $v(z) := \underline{v}(z) = \bar{v}(z)$. This value can be attained if a Nash equilibrium or equivalently a saddle point for the payoffs exist. This implies that for all other admissible strategies for investments in innovation by the supplier and the buyer u_S and u_B, for all $z \in (a, b)$ there exists strategies $u_S^* = \{u_S^*(t), t \geq 0\}$ and $u_B^* = \{u_B^*(t), t \geq 0\}$ such that,

$$v^{u_S, u_B^*}(z) \leq v^{u_S^*, u_B^*}(z) \leq v^{u_S^*, u_B}(z) \tag{78}$$

These saddle point strategies u_S^* and u_B^* are referred to as the equilibrium, or competitively optimal strategies.

D.2.2 Differential Operator

For an arbitrary function $\psi(z) \in \mathcal{C}^2$ let Γ denote the differential operator defined by

$$\Gamma\psi(z) := (1-\rho^2)[\psi_z(z) + z\psi_{zz}(z)]^2 - \psi_z(z)^2 \tag{79}$$

D.2.3 Conditions for sufficiently fast increasing function

An increasing concave function $\psi(z) \in \mathcal{C}^2$ (and therefore $\psi_{zz} < 0$ is suffiently fast increasing on an interval (a, b) if the following condition holds:

$$2\psi_z(z) + z\psi_{zz}(z) > 0, \text{for all } \ a < z < b \tag{80}$$

D.2.4 Conditions for sufficiently fast increasing function

The parameter κ, which represents the degree of advantage, is the ratio of the market prices of risk for the investments in innovation made by the supplier and buyer firm. Specifically, for θ_i defined in (5.3), for $i = S, B$, the parameter κ is defined as:

$$\kappa := \kappa(\theta_S, \theta_B) = \frac{\theta_S}{\theta_B} \tag{81}$$

Supplier is said to have the advantage if $\kappa > 1$ and the buyer firm has the advantage if $\kappa < 1$. In the case when the two are symmetrically positioned in a supply chain $\kappa = 1$.

Next. two theorems are presented that provide the analytical background for the problem formulation of this essay. Specifically, these theorems relates to the solution of nonlinear Dirichlet problem and the solution of nonlinear Cauchy problem.

D.2.5 Solution to a Nonlinear Dirichlet Problem

Theorem 1 [2] *Suppose that $\psi(z) : (a,b) \mapsto \Re$ is a C^2 concave, sufficiently fast increasing solution to the nonlinear Dirichlet problem for $a \leq z \leq b$:*

$$\frac{z\psi_z(z)^2}{2\Gamma\psi(z)}\theta_B^2[(1-\kappa^2)\psi_z(z)-(1+\kappa^2-2\rho\kappa)(\psi_z(z)+z\psi_{zz}(z))]+p(z)-\lambda(z)\psi(z)=0 \quad (82)$$

where $\psi(a) = h(a)$, and $\psi(b) = h(b)$. Also suppose that $\psi(z)$ satisfies the following conditions:

(i) For all admissible policies u_S and u_B, and for all $t \geq 0$ either,

(a) there exists integrable random variables X_1, X_2, such that $X_1 \leq \psi(Z_t^{u_S,u_B}) \leq X_2$ or ,

(b) the following moment condition holds

$$\int E[(Z_r^{u_S,u_B}\psi_z(Z_r^{u_S,u_B}))^2(u_{Sr}^2 + u_{Br}^2)]dr < \infty$$

(ii) The function $zH(z)$ is bounded on (a,b), where,

$$H(z) := \frac{\psi_z(z)[\psi_z(z) + z|\psi_{zz}(z)|]}{|\Gamma\psi(z)|}$$

(iii) The function $zH(z)[1 + H(z)]$ is Lipschitz continuous [3] on (a,b)

[2] The proof of this theorem can be found in Browne (2000)

[3] In mathematics, a function $f : M \mapsto N$ between metric spaces M and N is called Lipschitz continuous (or is said to satisfy a Lipschitz condition) if there exists a constant $K > 0$ such that $d(f(x), f(y)) \leq Kd(x,y)$ for all x and y in M. In this case, K is called the Lipschitz constant of the map.

Then $\psi(z)$ is the value of the game,

$$v^{u_S,u_B}(Z_t^{u_S,u_B}) = E_z\left(\int_0^{\tau^{u_S,u_B}} p(Z_t^{u_S,u_B})exp\{\int_0^t \lambda(Z_r^{u_S,u_B})dr\}dt + \right.$$
$$\left. + h(Z_t^{u_S,u_B})exp\{-\int_0^t \lambda(Z_r^{u_S,u_B})dr\}\right)$$

subject to

$$dZ_t^{u_S,u_B} = Z_t^{u_S,u_B}[m(u_S(t), u_B(t))dt + u_S(t)\sigma_S dW_S(t) - u_B(t)\sigma_B dW_B(t)]$$

that is, $\psi(z) = v(z) \equiv v^{u_S^*,u_B^*}(z)$ and moreover this value is achieved at the saddle point control functions, or competitively, optimal portfolio strategies, u_{Sv}^* and u_{Bv}^* given by,

$$u_{Sv}^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{\psi_z(z)}{\Gamma\psi(z)}\right)\left((\frac{\rho}{\kappa} - 1)(\psi_z(z) + z\psi_{zz}(z)) - \psi_z(z)\right) \tag{83}$$

$$u_{Bv}^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{\psi_z(z)}{\Gamma\psi(z)}\right)\left((1 - \rho\kappa)(\psi_z(z) + z\psi_{zz}(z)) - \psi_z(z)\right) \tag{84}$$

D.2.6 Solution to a Nonlinear Cauchy Problem

Theorem 2 [4] *Suppose that $\Upsilon(t,z) : [0,T]\times(0,\infty) \mapsto \Re$ is a $\mathcal{C}^{1,2}$ concave, sufficiently fast increasing solution(in z) to the nonlinear Cauchy problem:*

$$\Upsilon_t + \frac{z\Upsilon_z^2}{2\Gamma\Upsilon}\theta_B^2[(1-\kappa^2)\Upsilon_z - (1+\kappa^2-2\rho\kappa)(\Upsilon_z + z\Upsilon_{zz})] + \beta - \lambda\Upsilon = 0, with \Upsilon(T,z) = U(z) \tag{85}$$

Also suppose that $\Upsilon(z)$ satisfies the conditions (i), (ii), (iii) of Theorem 1, *then*

[4]The theorem is taken from Browne (2000)

$\Upsilon(t,z)$ *is the competitively optimal value function of the game,*

$$J^{u_S,u_B}(t,z) = E_{t,z}\left(\int_t^T \beta(Z_s^{u_S,u_B}) exp\{\int_t^r \lambda(Z_v^{u_S,u_B})dv\}dr + \right.$$
$$\left. + U(Z_T^{u_S,u_B}) exp\{-\int_t^T \lambda(Z_r^{u_S,u_B})dr\} \right) \tag{86}$$

where $\beta(z)$ and $U(z)$, are concave increasing utility functions, $J^{u_S,u_B}(t,z)$ is the expected payoff function under the policy u_S and u_B for a given fixed terminal time T. The notation $E_{t,z}(.)$ denotes $E(.|Z_t = z)$. $J(t,z)$ denotes the value of the above specified game, if it exists such that,

$$J(t,z) = \inf_{u_B}\sup_{u_S} J^{u_S,u_B}(t,Z) = \sup_{u_S}\inf_{u_B} J^{u_S,u_B}(t,z)$$

This implies $\Upsilon(t,z) = J(t,z)$ and the competitively optimal control functions are given by

$$u_{SJ}^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{\Upsilon_z(t,z)}{\Gamma\Upsilon(t,z)}\right)((\frac{\rho}{\kappa} - 1)(\Upsilon_z(t,z) + z\Upsilon_{zz}(t,z)) - \Upsilon_z(t,z)) \tag{87}$$

$$u_{BJ}^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{\Upsilon_z(t,z)}{\Gamma\Upsilon(t,z)}\right)((1 - \rho\kappa)(\Upsilon_z(t,z) + z\Upsilon_{zz}(t,z)) - \Upsilon_z(t,z)) \tag{88}$$

D.3 Theorems and Proofs

D.3.1 Discounted Payoffs - Case A

Theorem 3 *If the degree of advantage of a supplier over buyer denoted by κ satisfies the conditions,*

$\kappa < \tilde{\kappa}^-$ *and* $\kappa > \tilde{\kappa}^+$ (89)

where $\tilde{\kappa}^-$ *and* $\tilde{\kappa}^+$ *are as defined in equation (96), then the value of the discounted game (5.11) is given by,*

$$F^*(z) = \left(\frac{z}{u}\right)^{\eta^+} \textit{for} \;\; z < u \tag{90}$$

η^+ *is defined in equation (94) and the associated saddle point is given by:*

$$u_S^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\frac{\rho}{\kappa}-1)\eta^+ - 1}{(1-\rho^2)(\eta^+)^2 - 1}\right), \; u_B^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1-\rho\kappa)\eta^+ - 1}{(1-\rho^2)(\eta^+)^2 - 1}\right), \tag{91}$$

Proof: In this case Theorem 1 is applied with $\lambda(z) = \lambda > 0, p(z) = 0$ and setting $h(b) = 1$. Specifically, according to Theorem 1, $F^*(z)$ must be a fast increasing concave solution to,

$$\frac{zF_z(z)^2}{2\Gamma F(z)}\theta_B^2[(1-\kappa^2)F_z(z)-(1+\kappa^2-2\rho\kappa)(F_z(z)+zF_{zz}(z))]-\lambda F(z) = 0, \text{for} z < u \tag{92}$$

with $F^*(b) = 1$. Solution of the nonlinear Dirichlet problem of (92) are of the form $(\frac{z}{b})^\eta$, where η is a root to the quadratic,

$$\eta^2[\theta_B^2(1+\kappa^2-2\rho\kappa)+2\lambda(1-\rho^2)] - \eta\theta_B^2(1-\kappa^2) - 2\lambda = 0 \tag{93}$$

The discriminant of this quadratic is:

$$D = [\theta_B^2(1-\kappa^2)]^2 + 8\lambda[\theta_B^2(1+\kappa^2-2\rho\kappa)+2\lambda(1-\rho^2)]$$

which is positive. It can be observed that equation (93) admits two real roots

$\eta^+(\lambda;\kappa,\rho)$ and $\eta^-(\lambda;\kappa,\rho)$ where

$$\eta^{+,-} = \frac{\theta_B^2(1-\kappa^2) \pm \sqrt{D}}{2[\theta_B^2(1+\kappa^2-2\rho\kappa)+2\lambda(1-\rho^2)]} \tag{94}$$

Since $\lambda > 0$ and $\theta_B^2(1+\kappa^2-2\rho\kappa)+2\lambda(1-\rho^2) > 0$ we have $\eta^- < 0 < \eta^+$. However, since we require $F_z > 0$, and $2F_z + zF_{zz} > 0$, only the positive root η^+ is relevant. It is important to note that for concavity we require an additional condition that $\eta^+ < 1$. Therefore, we must have the following condition,

$$Q_2(\kappa) := \kappa^2\theta_B^2 - \kappa\rho\theta_B^2 - \lambda\rho^2 > 0 \tag{95}$$

This equivalence condition follows from the elementary fact that for the quadratic equation ax^2+bx+c, with $a>0$, the requirement that the larger root be less than 1, that is $\frac{[-b+\sqrt{b^2-4ac}]}{2a} < 1$, is equivalent to the requirement that $a+b+c>0$. Applying this fact to equation(93) the condition $Q_2(\kappa) > 0$ is obtained. The equation $Q_2(\kappa) = 0$ admits two roots,

$$\tilde{\kappa}^-(\lambda) = \frac{\rho}{2}\left(1-\sqrt{1+\frac{4\lambda}{\theta_B^2}}\right) \text{ and } \tilde{\kappa}^+(\lambda) = \frac{\rho}{2}\left(1+\sqrt{1+\frac{4\lambda}{\theta_B^2}}\right) \tag{96}$$

Equation $Q_2(\kappa) > 0$ holds only for $\kappa < \tilde{\kappa}^-(\lambda)$ and $\kappa > \tilde{\kappa}^+(\lambda)$. It can be noted that with no discounting, equation (96) yields $\tilde{\kappa}^-(\lambda) = 0$ and $\tilde{\kappa}^+(\lambda) = \rho$. Applying equation (83) and equation (84) the competitively optimal control functions for innovation strategies for the supplier firm and the buyer firm can be obtained as:

$$u_S^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\frac{\rho}{\kappa}-1)\eta^+-1}{(1-\rho^2)(\eta^+)^2-1}\right),\ u_B^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1-\rho\kappa)\eta^+-1}{(1-\rho^2)(\eta^+)^2-1}\right), \blacksquare \tag{97}$$

D.3.2 Discounted Payoffs - Case B

Theorem 4 *If the degree of advantage of a supplier over buyer denoted by κ satisfies the conditions,*

$\kappa < \tilde{\kappa}^-$ *and* $\kappa > \tilde{\kappa}^+$ (98)

where $\kappa < \tilde{\kappa}^-$ and $\kappa < \tilde{\kappa}^+$ are as defined in equation (96), then the value of the discounted game (5.11) is given by,

$$F^*(z) = \left(\frac{z}{u}\right)^{\eta^+}(1-\frac{p}{\lambda}) + \frac{p}{\lambda} for \quad z < u \tag{99}$$

η^+ is defined in equation (94) and the associated saddle point is given by:

$$u_S^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\frac{\rho}{\kappa}-1)(1+\eta^+)-1}{(1-\rho^2)(1+\eta^+)^2-1}\right), and \quad u_B^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1-\rho\kappa)(1+\eta^+)-1}{(1-\rho^2)(1+\eta^+)^2-1}\right), \tag{100}$$

Proof: In this case $\lambda(z) = \lambda > 0$, and $p(z) = p$. For this case, $F(z)$ must be a fast increasing concave solution to

$$\frac{zF_z(z)^2}{2\Gamma F(z)}\theta_B^2[(1-\kappa^2)F_z(z) - (1+\kappa^2-2\rho\kappa)(F_z(z)+zF_{zz}(z))] + p - \lambda F(z) = 0, \text{for} z < u \tag{101}$$

with $F^*(b) = 1$. This problem has a solution which is of the form,

$$\left(\left(\frac{z}{b}\right)^{\eta^+} + \frac{p}{\lambda}(1-(z/u)^{\eta^+})\right)$$

where η^+ is defined in equation(94). Substituting this expression in equation (83) and equation (84) of Theorem 1, the competitively optimal investment strategies for the supplier and buyer firm can be obtained as:

$$u_S^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\frac{\rho}{\kappa}-1)(1+\eta^+)-1}{(1-\rho^2)(1+\eta^+)^2-1}\right), \text{and} \ \ u_B^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1-\rho\kappa)(1+\eta^+)-1}{(1-\rho^2)(1+\eta^+)^2-1}\right)\blacksquare \tag{102}$$

D.3.3 Discounted Payoffs - Case C

Theorem 5 *If the degree of advantage of a supplier over buyer denoted by κ satisfies the conditions,*

$\kappa < \tilde{\kappa}^-$ *and* $\kappa > \tilde{\kappa}^+$ (103)

where $\kappa < \tilde{\kappa}^-$ and $\kappa < \tilde{\kappa}^+$ are as defined in equation (111), and θ_B and ρ satisfies the condition $\theta_B^2\rho^2 \geq 2$, then the value of the discounted game (5.11) is given by,

$$F^*(z) = (\frac{z}{u})^{\eta^+} for \ \ z < u \tag{104}$$

η^+ is defined in equation (109) and the associated saddle point is given by:

$$u_S^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\frac{\rho}{\kappa}-1)\eta^+-1}{(1-\rho^2)(\eta^+)^2-1}\right), \ \ and \ \ u_B^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1-\rho\kappa)\eta^+-1}{(1-\rho^2)(\eta^+)^2-1}\right), \tag{105}$$

Proof: In this case, Theorem 1 is applied with $\lambda(z) = \lambda > 0$, $p(z)$=a concave increasing function of z, and setting $h(b) = 1$. Specifically, according to Theorem 1, $F^*(z)$ must be a fast increasing concave solution to,

$$\frac{zF_z(z)^2}{2\Gamma F(z)}\theta_B^2[(1-\kappa^2)F_z(z)-(1+\kappa^2-2\rho\kappa)(F_z(z)+zF_{zz}(z))]+p(z)-\lambda F(z) = 0, \text{for} z < u \tag{106}$$

with $F^*(b) = 1$. To make the problem stated in equation (106) easy to solve we consider a particular functional form of p. Specifically we consider,

$$p(z) = \left(\frac{z}{b}\right)^{\eta^+} \tag{107}$$

where η^+ is the positive root of the quadratic

$$\eta^2[\theta_B^2(1 + \kappa^2 - 2\rho\kappa) + 2\lambda(1 - \rho^2)] - \eta\theta_B^2(1 - \kappa^2) - (2\lambda - 1) = 0 \tag{108}$$

The solution of the Dirichlet problem (106) with $p(z)$ defined in (107) is also of the form $(\frac{z}{b})^{\eta^+}$. The discriminant of equation (108) is:

$$D = [\theta_B^2(1 - \kappa^2)]^2 + 4(2\lambda - 1)[\theta_B^2(1 - \kappa^2 - 2\rho\kappa) + 2\lambda(1 - \rho^2)]$$

The positive root of equation (108) is:

$$\eta^+ = \frac{\theta_B^2(1 - \kappa^2) + \sqrt{D}}{2[\theta_B^2(1 - \kappa^2 - 2\rho\kappa) + 2\lambda(1 - \rho^2)]} \tag{109}$$

For concavity, we require an additional condition $\eta^+ < 1$. This also ensures that the expression for $p(z)$ in equation (107) is concave as required for the analysis. Therefore we must have the following condition,

$$Q_2(\kappa) := \kappa^2\theta_B^2 - \kappa\rho\theta_B^2 - \lambda\rho^2 + \frac{1}{2} > 0 \tag{110}$$

The equation $Q_2(\kappa)$=0 admits two roots,

$$\tilde{\kappa}^-(\lambda) = \frac{\rho}{2}\left(1 - \sqrt{1 + \frac{4}{\theta_B^2}(\lambda - 1/2\rho^2)}\right) \text{ and } \tilde{\kappa}^+(\lambda) = \frac{\rho}{2}\left(1 + \sqrt{1 + \frac{4}{\theta_B^2}(\lambda - 1/2\rho^2)}\right) \tag{111}$$

Equation $_2(\kappa) > 0$ holds only for $\kappa < \tilde{\kappa}^-(\lambda)$ and for $\kappa > \tilde{\kappa}^+(\lambda)$. With no discounting, it can be noted from equation (111) that $\tilde{\kappa}^{-}(0) = \frac{\rho}{2}\left(1 - \sqrt{1 - \frac{2}{\theta_B^2 \rho^2}}\right)$ and $\tilde{\kappa}^{+}(0) = \frac{\rho}{2}\left(1 + \sqrt{1 - \frac{2}{\theta_B^2 \rho^2}}\right)$. To ensure a real value for $\tilde{\kappa}^{-}$ and $\tilde{\kappa}^{+}$, the following condition is imposed on the possible values of θ_B and ρ.

$$\theta_B^2 \rho^2 \geq 2 \tag{112}$$

Applying equation (83) and equation (84) of Theorem 1, the competitively optimal control function for innovation strategies for the supplier firm and the buyer firm can be obtained as:

$$u_S^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\frac{\rho}{\kappa} - 1)\eta^+ - 1}{(1-\rho^2)(\eta^+)^2 - 1}\right), \text{ and } u_B^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1 - \rho\kappa)\eta^+ - 1}{(1-\rho^2)(\eta^+)^2 - 1}\right) \blacksquare \tag{113}$$

Corollary: If $\theta_B^2 \rho^2$ is equal to 2, the results are identical to that in case A. However, more generally the relationship between θ_B, the risk-adjusted return on investments in innovation for the buyer and ρ, the correlation coefficient between the Wiener processes $W_B(t)$ and $W_S(t)$ for the buyer's and seller's overall gain processes, can be written as,

$$\theta_B \geq \frac{\sqrt{2}}{\rho} \quad \blacksquare \tag{114}$$

D.3.4 Utility Maximization - Case A'

Theorem 6 *If the buyer and supplier are competing in utility maximization objective by maximizing the terminal value with investment in innovation and when the discounting factor is λ, then the competitively optimal strategies are given as:*

$$u_{SJ}^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\rho/\kappa - 1)\alpha - 1}{(1-\rho^2)\alpha^2 - 1}\right) \text{ and } u_{BJ}^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1 - \rho\kappa)\alpha - 1}{(1-\rho^2)\alpha^2 - 1}\right) \tag{115}$$

and the value of the game $F^*(t,z)$ is given by, $F(t,z) = e^{q(\alpha)(T-t)}z^\alpha$ where $q(\alpha)$ is,

$$q(\alpha) := \alpha\theta_B^2 \frac{(1-\kappa^2) - \alpha(1+\kappa-2\rho\kappa)}{2[(1-\rho^2)\alpha^2 - 1]} - \lambda \tag{116}$$

Proof: Let $F^*(t,z)$ denote the value of this game - should it exist. Therefore we have

$$F^*(t,z) = \inf_{u_B} \sup_{u_S} F^{u_S,u_B}(t,z) = \sup_{u_S} \inf_{u_B} F^{u_S,u_B}(t,z) \tag{117}$$

In this case, by Theorem 2, $F^*(t,z)$ must be a fast increasing concave solution to

$$F_t(t,z) + \frac{zF_z^2(t,z)}{2\Gamma F(t,z)}\theta_B^2[(1-\kappa^2)F_z(t,z) - (1+\kappa^2-2\rho\kappa)(F_z(t,z)+zF_{zz}(t,z))] - \lambda F(t,z) = 0, \tag{118}$$

The value function $F(t,z)$ is obtained as,

$$F(t,z) = e^{q(\alpha)(T-t)}z^\alpha \tag{119}$$

where $q(\alpha)$ is defined by

$$q(\alpha) := \alpha\theta_B^2 \frac{(1-\kappa^2) - \alpha(1+\kappa-2\rho\kappa)}{2[(1-\rho^2)\alpha^2 - 1]} - \lambda \tag{120}$$

Substituting the values in equation (87) and equation (88) the competitively optimal strategies can be obtained as:

$$u_{SJ}^*(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\rho/\kappa - 1)\alpha - 1}{(1-\rho^2)\alpha^2 - 1}\right) \text{ and } u_{BJ}^*(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1-\rho\kappa)\alpha - 1}{(1-\rho^2)\alpha^2 - 1}\right) \blacksquare \tag{121}$$

D.3.5 Utility Maximization - Case B'

Theorem 7 *If the buyer and supplier are competing in payoff maximization and ter-*

minal utility maximization objectives then the form of competitively optimal strategies are similar to equation (115).

$$u^*_{SJ}(z) = \frac{\theta_S}{\sigma_S}(\frac{(\rho/\kappa - 1)\alpha - 1}{(1-\rho^2)\alpha^2 - 1}) \textit{and } \; u^*_{BJ}(z) = \frac{\theta_B}{\sigma_B}(\frac{(1-\rho\kappa)\alpha - 1}{(1-\rho^2)\alpha^2 - 1}) \tag{122}$$

and the value of the game $F^*(t,z)$ is given by, $F(t,z) = e^{q(\alpha)(T-t)}z^\alpha$ where $q(\alpha)$ is,

$$q(\alpha) := \alpha\theta_B^2 \frac{(1-\kappa^2) - \alpha(1+\kappa-2\rho\kappa)}{2[(1-\rho^2)\alpha^2 - 1]} - \lambda + 1 \tag{123}$$

Proof: Let $F^*(t,z)$ denote the value of this game - should it exist. Therefore we have

$$F^*(t,z) = \inf_{u_B}\sup_{u_S} F^{u_S,u_B}(t,z) = \sup_{u_S}\inf_{u_B} F^{u_S,u_B}(t,z) \tag{124}$$

In this case, by Theorem 2, $F^*(t,z)$ must be a fast increasing concave solution to

$$F_t(t,z) + \frac{zF_z^2(t,z)}{2\Gamma F(t,z)}\theta_B^2[(1-\kappa^2)F_z(t,z) - (1+\kappa^2-2\rho\kappa)(F_z(t,z) + zF_{zz}(t,z))] + \beta(z) - \lambda F(t,z) = 0, \tag{125}$$

The value function $F(t,z)$ is obtained as,

$$F(t,z) = e^{q(\alpha)(T-t)}z^\alpha \tag{126}$$

where $q(\alpha)$ is defined by

$$q(\alpha) := \alpha\theta_B^2 \frac{(1-\kappa^2) - \alpha(1+\kappa-2\rho\kappa)}{2[(1-\rho^2)\alpha^2 - 1]} - \lambda + 1 \tag{127}$$

For ease of mathematical analysis, the functional form of $\beta(z)$ is considered identical to $F(t,z)$. The functional form of $\beta(z)$ is therefore assumed to be $\beta(z) = e^{q(\alpha)(T-t)}z^\alpha$. It can be noted that, since the value of α is assumed to be strictly between 0 and 1, the concavity requirement for $\beta(z)$ is satisfied. Substituting the values in equation

(87) and equation (88) the competitively optimal strategies can be obtained as:

$$u^*_{SJ}(z) = \frac{\theta_S}{\sigma_S}\left(\frac{(\rho/\kappa - 1)\alpha - 1}{(1-\rho^2)\alpha^2 - 1}\right) \text{and } u^*_{BJ}(z) = \frac{\theta_B}{\sigma_B}\left(\frac{(1-\rho\kappa)\alpha - 1}{(1-\rho^2)\alpha^2 - 1}\right) \blacksquare \tag{128}$$

D.4 Propositions and Proofs

Proposition 1 *The solution to the differential game involving discounted payoffs yield highest value when,*

$$\kappa \geq -\rho + \sqrt{\rho^2 + 1} \tag{129}$$

Proof: Evaluating the implications of the different cases in discounted payoff objectives, it can be observed that the solutions in Case A and Case C have equivalent representation, $F^*(z) = \left(\frac{z}{u}\right)^{\eta^+}$ for $z < u$. The corresponding solution for Case B is, $F^*(z) = \left(\frac{z}{u}\right)^{\eta^+}\left(1 - \frac{p}{\lambda}\right) + \frac{p}{\lambda}$ for $z < u$. It can be observed that both of these expressions yield the highest value of 1 when $\lim_{\eta^+ \mapsto 0}$. This implies,

$$\theta_B^2(1 - \kappa^2) + \sqrt{D} = 0$$

After some algebraic manipulations the following expression is obtained,

$$\theta_B^2(1 - \kappa^2 - 2\rho\kappa) = 2\lambda(\rho^2 - 1)$$

Since the right hand side of the above expression is less than or equal to 0, the following relation need to hold,

$$\kappa^2 + 2\rho\kappa - 1 \geq 0 \Rightarrow \kappa \geq -\rho \pm \sqrt{\rho^2 + 1}$$

Since, the value of κ should be positive, only the positive root is relevant, that is,

$$\kappa \geq -\rho + \sqrt{\rho^2 + 1} \quad \blacksquare \tag{130}$$

Proposition 2 *In a discounted payoff game, the complementarities and synergies in the investments in innovation is a function of whom among the supplier and buyer exerts innovation based channel power.*

Proof: Equation (130) can be rewritten as,

$$\rho \geq \frac{1-\kappa^2}{2\kappa} \tag{131}$$

Analyzing equation (131) with equality sign, it can be observed that if the supplier has a higher degree of advantage (and therefore, $\kappa > 1$) then the Brownian motion leading to gain process from innovation for the buyer and supplier is expected to be negatively correlated. On the other hand, if the degree of advantage is in favor of the buyer firm (and $\kappa < 1$) then the stochastic processes is expected to be positively correlated. $\blacksquare$

Proposition 3 *In a discounted payoff game, for negatively correlated gain processes, and for maximum value of the game (evaluated at* $\lim_{\eta^+ \mapsto 0}$*), investments in innovation by supplier firm would be strictly higher in case B than in the case A and case C, whereas the buyer will not make any investment in innovation in case B but would make investments* $\lim_{\eta^+ \mapsto 0} u_B^* = \frac{\theta_B}{\sigma_B}$ *in case A and case C.*

Proof: The value of η^+ is such that $0 < \eta^+ < 1$. For u_S^* and u_B^* in Theorem 1 , Theorem 2 , Theorem 3 we consider $\lim_{\eta^+ \mapsto 0}$ and $\lim_{\eta^+ \mapsto 1}$. For case A and case C, this leads to, $\lim_{\eta^+ \mapsto 0} u_S^* = \frac{\theta_S}{\sigma_S}$ and $\lim_{\eta^+ \mapsto 0} u_B^* = \frac{\theta_B}{\sigma_B}$. For case B, the expression reduces to $\lim_{\eta^+ \mapsto 0} u_S^* = \frac{\theta_S}{\sigma_S}\left(\frac{2-\rho/\kappa}{\rho^2}\right)$ and $\lim_{\eta^+ \mapsto 0} u_B^* = \frac{\theta_B}{\sigma_B}\left(\frac{\kappa}{\rho}\right)$. It can be noted that in all three cases, the investments in innovation by supplier and buyer firm is directly proportional to the risk adjusted return on these investments. However, in case B the optimal investments also depend on correlation between the gain processes and on the degree of advantage between the buyer and supplier firm. Specifically, it can be observed that if the gain processes are negatively correlated, then in case B, the investments in innovation by supplier is strictly greater than that in case A and case C. However, with negative correlation, the investment in innovation by the buyer would be equal to zero, since the value of control is lower bound by zero and negative values are not permitted. ■

Proposition 4 *For discounted payoff game, in case B,*

(a) If the ratio, $\frac{\kappa}{\rho} > 1$, both supplier and buyer firm would invest higher than in case A and case C;

(b) If the ratio is such that,$0 < \frac{\kappa}{\rho} \leq \frac{1}{2}$, the buyer would make lesser investments (would make equal investment to case A and case C if $\frac{\kappa}{\rho} = 1$) than that made in case A and case C, and the supplier would make no investments,

(c) If the ratio is such that $\frac{1}{2} < \frac{\kappa}{\rho} \leq 1$, the buyer would make lesser investments than that made in case A and case C. The supplier would make lower investments if the correlation coefficient $\rho = 1$ (would make equal investment to case A and case C if $\frac{\kappa}{\rho} = 1$) and would make higher investments otherwise.

Proof: For case A and case C, we have, $\lim_{\eta^+ \mapsto 0} u_S^* = \frac{\theta_S}{\sigma_S}$ and $\lim_{\eta^+ \mapsto 0} u_B^* = \frac{\theta_B}{\sigma_B}$. These expressions do not depend on ρ. However, for case B, the expression reduces to $\lim_{\eta^+ \mapsto 0} u_S^* = \frac{\theta_S}{\sigma_S}\left(\frac{2-\rho/\kappa}{\rho^2}\right)$ and $\lim_{\eta^+ \mapsto 0} u_B^* = \frac{\theta_B}{\sigma_B}\left(\frac{\kappa}{\rho}\right)$. For $\frac{\kappa}{\rho} > 1$, $\left(\frac{2-\rho/\kappa}{\rho^2}\right) > 1$ and hence the supplier would invest more than the investments made in case A and case C. Once again the result for buyer is a straightforward substitution of $\frac{\kappa}{\rho} > 1$ in $\lim_{\eta^+ \mapsto 0} u_B^* = \frac{\theta_B}{\sigma_B}\left(\frac{\kappa}{\rho}\right)$. For $0 < \frac{\kappa}{\rho} \leq \frac{1}{2}$, the numerator of the expression for

$\lim_{\eta^+ \mapsto 0} u_S^* \leq 0$ hence the supplier should set the control to zero. The corresponding proposition for the buyer firm is a straightforward result of substituting $\frac{0<\kappa}{\rho} \leq \frac{1}{2}$ in the expression for $\lim_{\eta^+ \mapsto 0} u_B^*$. Finally, for $\frac{1}{2} < \frac{\kappa}{\rho} < 1$, with perfect correlation ($\rho = 1$) the value of $\kappa < 1$. In such a case $2 - \rho/\kappa < \rho^2$ and therefore the investment made by the supplier in case B would be lower than that in case A and case C. Alternatively, when $(0 < \rho < 1)$, we will have $2 - \rho/\kappa > \rho^2$ and accordingly the investments made by the supplier in this case would be higher.■

Proposition 5 *In a discounted payoff game, investment in case A and case C would be identical when,*

$$\kappa = -\rho + \sqrt{\rho^2 + [2\lambda(1-\rho^2) + 1]} \tag{132}$$

Proof: It can be noticed that qualitatively, case A and case C are equivalent. The only difference in the two cases is the value of η^+, as a result of different values of the discriminant D. Therefore the investments to be equal we require the discriminant for Case A,

$$[\theta_B^2(1-\kappa^2)]^2 + 8\lambda[\theta_B^2(1-\kappa^2-2\rho\kappa) + 2\lambda(1-\rho^2)]$$

to be equal to that in case C,

$$D = [\theta_B^2(1-\kappa^2)]^2 + 4(2\lambda-1)[\theta_B^2(1-\kappa^2-2\rho\kappa) + 2\lambda(1-\rho^2)]$$

With some algebraic manipulations we can obtain the desired result, that is,

$$\kappa = -\rho + \sqrt{\rho^2 + [2\lambda(1-\rho^2) + 1]} \quad ■ \tag{133}$$

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