MATHEMATICS INTERVENTIONS FOR STUDENTS WITH AUTISM: APPLICATION TO REALISTIC CLASSROOM SETTINGS

By

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ABSTRACT

This dissertation presents three studies exploring mathematics interventions for students with autism spectrum disorder (ASD) including a review of literature and two intervention studies using manipulative based interventions. Specifically, the researcher conducted a systematic review of literature exploring the current state of the literature on mathematics interventions for students with ASD. The results of the systematic review indicated a need for more research in realistic classroom settings (i.e., teacher implemented, small group instruction), and confirmed virtual manipulative-based interventions as an evidence-based practice (EBP). The first single-case experimental design (i.e., multiple probe across participants) study, which examined the effectiveness of the virtual representational abstract (VRA) instructional sequence when implemented by the classroom teacher to teach three elementary students with ASD to solve addition with regrouping problems. The researcher found a functional relation between the teacher implemented VRA instructional sequence and student accuracy in solving single- or double-digit addition with regrouping problems. The teacher was able to implement the intervention with over 90% treatment fidelity across phases and students. Finally, in the second study the researcher conducted a single case adapted alternating treatments experimental design study comparing the efficacy of a virtual manipulative and a finger counting strategy to teach single digit addition and subtraction without regrouping via small group instruction. The researcher found a functional relation between the virtual manipulative and accuracy however, the finger counting strategy also resulted in increased accuracy compared to baseline. Overall, the dissertation shows that virtual manipulative based interventions are an evidence-based practice and can be successfully implemented by the teacher and in a small group setting for students with ASD.

For my Mom and Dad
Thank you for always believing in me.

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TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

Mathematics is an important content area for all students, including students with disabilities, as success in mathematics has implications for mathematics achievement, job skills, and daily living skills later in life (Bullen et al., 2022; Nelson et al., 2022). Given the role of mathematics, it is unsurprising that expectations for mathematics instruction for students with disabilities, have increased over the past decade (Spooner et al., 2018). Despite the importance of learning and maintaining mathematics skills as well as the increased attention to grade-aligned mathematics instruction, students with disabilities continue to struggle in mathematics throughout K-12 education and adulthood (Nelson et al., 2022).

While many students find mathematics to be a challenging content area, students with autism spectrum disorder (ASD) may find this area particularly challenging (Cox & Root, 2021; Oswald et al., 2016). Students with ASD often display difficulties in executive functioning, flexible thinking, working memory, reading, and language which are skills necessary to build conceptual understanding of mathematics concepts (Cragg & Gilmore, 2014; Kim & Cameron, 2016; Root et al., 2017). Yet, previous researchers have shown students with ASD can learn complex mathematics skills when taught using explicit and systematic interventions and instruction (Bowman et al., 2019). More research is needed, however, to explore targeted interventions to support learning and maintaining mathematics skills for students with ASD.

Mathematics Interventions for Students with ASD

To ensure students with ASD receive equitable mathematics instruction that allows them to make meaningful progress, students with ASD should receive instruction involving evidence-based practices (Cox & Jimenez, 2020; King et al., 2016). Three systematic literature reviews aiming to identify evidence-based practices for teaching mathematics to students with ASD were published in 2016, representing the most recent available (Barnett & Cleary, 2016; Gevarter et al., 2016; King et al., 2016). The three systematic reviews, which included different inclusion and exclusion criteria and applied different quality indicators, suggested different conclusions in terms of the existing research base on teaching mathematics to students with ASD. While two of the reviews concluded insufficient evidence to claim any practices evidence-based (e.g., Barnett & Cleary, 2016; Gevarter et al., 2016), King et al. (2016) made a different determination. King et al determined that 71% of cases reviewed showed a functional relation and interventions generally produced moderate to large effects. However, consistent across the reviews was a finding that the majority of studies examining mathematics intervention and students with ASD involved one-on-one, researcher implemented interventions in school settings. Another consistency across the three reviews one consistency involved the identification of visual representations as one of the most widely used interventions for mathematics instruction for students with ASD. Recommendations from the three groups of researchers included that future studies explore maintenance and generalization as well as explore interventions in settings that are more generalizable to common educational settings (i.e., general education classrooms, small group instruction, teacher led interventions).

Mathematics Manipulatives

One of the most common visual representations used in K-12 general and special education mathematics classrooms are manipulatives (Carbonneau et al., 2013; Spooner at al., 2019). When one thinks of manipulatives one generally thinks of concrete manipulatives, which can be defined as physical objects that can be manipulated to support solving and conceptual understanding of mathematics problems (Carbonneau et al., 2013; Uribe-Florez & Wilkins, 2010). With the increase in technology use for students with ASD, there has been an increased focus on virtual manipulatives, which similar to concrete manipulatives but offered in a digital format (i.e., computer or app-based; Bouck & Flanagan, 2010; Bouck et al., 2020).

In a review of the literature on mathematics manipulatives for students with ASD and intellectual and developmental disabilities (IDD), Long et al. (2022) found virtual manipulatives in and of themselves and virtual manipulative-based instructional sequences to be evidence-based practices. While the evidence for virtual manipulatives was sufficient for classifying virtual manipulatives as evidence-based, Long et al. discussed several limitations of the current research base. First, the majority of participants across the study were middle school students with IDD, with a particular emphasis on students with mild intellectual disability. More research is needed to expand the current research to elementary and high school, including students with ASD. Further, all of the current research in mathematics manipulatives for students with ASD have been implemented in one-on-one settings with researchers serving as the implementer.

In research comparing concrete and virtual manipulatives for students with ASD, researchers found both manipulative types to be effective in supporting acquisition of the target mathematics skill (Bassette et al., 2019; Bassette et al., 2020; Bouck et al., 2014; Shurr et al., 2021). Bouck et al. (2014) used an alternating treatments design study to compare the effectiveness of concrete and virtual manipulatives to teach single and double-digit subtraction to three elementary-aged students with ASD. Bouck et al. found both concrete and virtual manipulatives to be effective in increasing accuracy and independence in solving the subtraction problems, however, virtual manipulatives resulted in faster increases in both accuracy and independence. In addition, all three students indicated a preference for the virtual manipulatives, noting they enjoyed the graphic animation and that they were easy to use on the computer.

Bassette et al. (2019) also taught three elementary-aged students with ASD to solve subtraction problems using concrete and app-based virtual base ten blocks. Using an alternating treatments design, Bassette et al. compared the accuracy and independence with which students completed the target mathematics skills. They determined that both manipulative types were effective for increasing accuracy, however, all three students were more independent with the virtual manipulatives. Consistent with other research comparing manipulative types, all three participants reported a preference for virtual manipulatives and parents and teachers of the participants supported the use of the virtual manipulatives as students were motivated by technology to learn new skills.

Bassette et al. (2020) taught three elementary-aged students with ASD to solve a mathematics skill at their instructional level (i.e., single-digit addition with regrouping, equivalent fractions, or fraction addition) using concrete and virtual manipulatives (i.e., base ten blocks or fraction tiles). While both manipulative types were effective in increasing accuracy and independence in the alternating treatments design study, researchers found two of the three students were most successful (e.g., were more accurate and completed more steps of the task analysis independently) with the virtual manipulatives. In social validity interviews, the students and their parents reported a preference for virtual manipulatives as they thought the virtual manipulatives were easier for students to keep organized and that technology could keep them more engaged. Most recently, Shurr et al. (2021) conducted an alternating treatments design study to compare concrete and virtual manipulatives to teach three elementary aged students with ASD to teach double digit addition problems. The results of this study were consistent with previous studies in finding both manipulative types were effective in increasing accuracy and independence, however, the virtual manipulatives were found to be more supportive than concrete manipulatives for all three participants.

Manipulative-Based Instructional Sequences

In addition to the use of manipulatives on their own, researchers have also explored the use of manipulative based instructional sequences including the concrete-representational-abstract (CRA) and virtual-representational-abstract (VRA) instructional sequences (Agrawal & Morin, 2016; Bouck & Sprick, 2018). The CRA and VRA instructional sequences begin with teaching students to solve the target mathematics skill with manipulatives (concrete or virtual), then transitioning to using representations or drawings, and finally using numerical strategies (Agrawal & Morin, 2016; Bouck & Sprick, 2018). For students with ASD, Stroizer et al. (2015) taught three elementary-aged students with ASD to solve addition with regrouping, subtraction with regrouping, and single digit multiplication using the CRA instructional sequence. Stroizer et al. (2015) found a functional relation between the CRA instructional sequence and accuracy with which the students solved all three problem types. Yakubova et al. (2016) used a multiple baseline across participants replicated across behaviors design to explore the effects of the CRA delivered via point-of-view video modeling on accuracy with which four students with ASD solve addition, subtraction, and number comparison problems. They found the intervention resulted in improvements in accuracy across math skills as well as students maintained their accuracy at higher than baseline levels for up to three weeks following the end of intervention.

An emerging recent research base examines the VRA and students with ASD. Park et al. (2020) explored an intervention package consisting of the VRA instructional sequence with fading supports to teach double-digit subtraction with regrouping to four middle school students (three with ASD). Park et al. (2020) found a functional relation between the intervention package and students accuracy in solving the subtraction problems. The students in the Park et al. study maintained their accuracy for up to six weeks following the end of intervention. Long et al. (2023) also explored the effects of an intervention package including the VRA instructional sequence, explicit instruction, and the system of least prompts (SLP) to teach subtraction with regrouping to four students with ASD (two elementary, two secondary) in an online environment. Long et al. (2023) found a functional relation between the intervention package and student accuracy. Two of the four participants struggled in the representational phase; however, they were able to reach mastery criteria in the abstract phase and maintain their accuracy with no instruction provided. In a variation, Root et al. (2020) taught three middle school students with developmental disabilities (two with ASD) to solve multiplicative word problems using the VRA integrated (VRA-I) instructional sequence, in which students are introduced to multiple representations at one time (i.e., virtual and representational; representational and abstract). Researchers found a functional relation between the VRA-I framework and student accuracy in solving the multiplicative word problems, and students were able to maintain their accuracy for up to three weeks following the removal of intervention (Root et al., 2020).

While research is emerging to explore manipulative-based mathematics interventions for students with ASD, there are two major gaps in the current literature (Long et al., 2023; Park et al., 2020; Root et al., 2020). As previously mentioned, there are currently no studies exploring manipulative-based mathematics intervention for students with ASD in small group settings or with teachers or paraprofessionals as implementors (Bottge, et al., 2018; Long et al., 2022). One of the main goals of special education research is to identify evidence-based practices teachers can implement in their classrooms to effectively teach students with disabilities, however, if researchers are implementing interventions in one-on-one settings, the results have implications mostly in these arrangements (Rumrill et al., 2020). To make effective practice recommendations to teachers, researchers must begin expanding their research to explore the true effectiveness of mathematics manipulatives in settings more closely aligned with real world classroom instruction such as teacher implementation and small group instruction (Singer et al., 2017). As such, there is significant work to be done to bridge the research-to-practice gap for virtual manipulatives (Long et al., 2022).

Purpose of the Study

Students with ASD have been successful at acquiring and maintaining a variety of mathematics skills (e.g., addition, subtraction, multiplication, fractions) with the use of virtual manipulatives and virtual manipulative based instructional sequences (Bassette, et al., 2019; Bassette et al., 2020; Bouck et al., 2014; Long et al., 2023; Park et al., 2020; Root et al., 2020; Shurr et al., 2021). Further, students with ASD report and preference for virtual manipulatives over concrete manipulatives as they are easier to use, more organized, and students find technology to be motivating when learning mathematics (Bassette, et al., 2019; Bassette et al., 2020; Bouck et al., 2014; Shurr et al., 2021). While there is significant evidence supporting the use of virtual manipulatives and virtual manipulative based instructional sequences as mathematics interventions for students with ASD, however, all of the current research exploring virtual manipulative based instructional sequences to support mathematics skills for students with ASD are researcher implemented in one-on-one settings (Long et al., 2022; Park et al., 2020; Root et al., 2020). There is a need for further research to fill the current gaps in the literature: small group and teacher implemented manipulative-based interventions for students with ASD (Long et al., 2022).

Study 1—Systematic Review of the Literature. Study one was an evidence-based synthesis focused on research on mathematics interventions for students with. Previous systematic reviews (Barnett & Cleary, 2016; Gevarter et al., 2016; King et al., 2016) reviewed studies published before 2015, therefore, study one included peer reviewed research published between 2015 and 2022. The researchers located studies via database, journal, and ancestral searches and coded them based on research design, intervention, target math skills, outcomes, and whether they study met the quality indicators and practice standards set by the Council for Exceptional Children (CEC, 2014; Cook et al., 2014). Researchers determined if each study was methodologically sound then if there was enough research to categorize the intervention as evidence-based for students with ASD. The results were analyzed to inform the current state of literature on teaching mathematics to students with ASD.

Study 2—Teacher Implemented VRA. The purpose of study three was to explore the efficacy of the VRA instructional sequence when implemented by classroom teachers (or paraprofessionals). Although there has been an increase in the use of virtual manipulatives in research and practice, all of the research involves researcher-implemented interventions (e.g., Bouck et al., 2014; Bassette et al., 2019; Bassette et al., 2020; Shurr et al., 2021). As such, there is significant work to be done to bridge the research-to-practice gap for virtual manipulatives (Long et al., 2022). This study employed a multiple probe across participants single case design to explore the effects of the VRA instructional sequence on the student accuracy solving double-digit addition with regrouping problems. Accuracy was defined as the number of problems students answered correctly out of five. Researchers were also interested in teacher implementation fidelity, which was calculated based on items completed on a researcher created checklist.

Study 2—Small Group Comparison. The purpose of study two was to explore the efficacy of the VRA instructional sequence when implemented in a small group setting (dyads). While research exploring mathematics manipulatives for students with ASD have only been conducted in one-on-one settings, small group instruction remains an effective practice in K-12 special education classrooms (Bottge, et al., 2018; Long et al., 2022). This study employed a multiple probe across dyads single case design to explore the effects of the VRA instructional sequence on the accuracy and independence with which elementary students with ASD solved single digit multiplication problems. Accuracy was defined as the number of problems students answered correctly out of five.

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CHAPTER 2

AN UPDATED SYSTEMATIC REVIEW OF MATHEMATICS INTERVENTIONS FOR STUDENTS WITH AUTISM SPECTRUM DISORDER

Evidence-based practices are crucial to successfully teaching students with autism spectrum disorder (ASD) in academic and life skills related areas (Hume et al., 2021). Hume et al. (2021) conducted a systematic review exploring a range of behavioral, developmental, and educational practices for individuals with ASD. Applying the single case and quantitative group design research quality indicators (Gersten et al., 2005; Horner et al., 2005), Hume et al. (2021) identified 28 practices that met the evidence-based practice (EBP) criteria for students with ASD. While these general EBPs are important for teachers who educate students with ASD across a variety of academic and life skills, teachers need a more nuanced understanding of EBPs for students with ASD in all areas, including mathematics education.

Previous Literature Reviews

Researchers have published several systematic reviews of the literature targeting mathematics interventions for students with ASD over the past decade (Barnett & Cleary, 2016; Gevarter et al., 2016; King et al., 2016). King et al. (2016) conducted a best-evidence synthesis of research involving mathematics interventions for students with ASD regardless of age or grade by applying the What Works Clearinghouse (WWC, 2014) quality indicators. Their review explored experimental design studies evaluating the effects of mathematics interventions on a mathematics outcome (e.g., percent accuracy) involving at least one student in grades K-12 with ASD (e.g., ASD, Asperger's PDD-NOS) located in peer-reviewed special education journals published prior to May 2014 using set search terms. Studies were excluded from the King et al. review if they did not use an experimental design, disaggregate data for students with ASD, or were not considered high quality based on WWC (2014) quality indicators. King et al. determined 21 studies fit their inclusion criteria and found a functional relation between the intervention and the targeted mathematics outcome in 71% of the studies. The majority of studies involved one-on-one and researcher-implemented interventions in school settings. The interventions in the studies included in King et al. review involved contingent praise, prompting, constant time delay, representations, strategy instruction, and antecedent strategies to target a range of mathematics skills (e.g., computation, functional skills, fluency, and early numeracy skills packages). Despite positive results in the included studies, none of the interventions reached the WWC threshold to label the practice as evidence based.

Barnett and Cleary (2016) also conducted a review of mathematics interventions for students with ASD. Barnett and Cleary included studies with no year limits published in peer-reviewed journals; included at least one elementary to post-secondary student diagnosed with ASD, Asperger's, or PDD-NOS participating in an educational setting (i.e., school, clinic, home-based, tutorial); and explored the effects of an intervention on a National Council of Teachers of Mathematics content standard (i.e., number and operations, algebra, geometry, measurement, data analysis). They found 11 studies meeting their inclusion criteria, which they divided into two pre-determined intervention categories: visual representations (n=6) and strategy instruction (n=5). Across the 11 studies, eight different interventions were found, with the highest frequency of three for a touch point intervention. Other interventions included video modeling, manipulatives, schematic diagrams, cognitive or meta-cognitive strategies, response repetition, counting on and next-dollar strategies, and a high-preference instructional arrangement to increase completion of low-preference tasks. Five studies targeted basic operations, three problem solving, two purchasing, and one study reported they focused on "math tasks." All of the studies occurred outside of the general education setting and were implemented by the researcher. Barnett and Cleary reported the state of the literature to be preliminary and noted more research was needed to make effective practice recommendations for teachers.

Additionally, Gevarter et al. (2016) conducted a systematic review of mathematics interventions for students with ASD. Gevarter et al. conducted two database and journal searches to locate studies from 1980-2015 first using search terms for ASD (i.e., autism, Asperger's, PDD) and mathematics (i.e., mathematics, math, arithmetic, algebra), and second using search terms for autism and academic engagement. They included studies if they targeted a mathematics outcome, had at least one participant diagnosed with ASD with no age or grade limit, and used an experimental design. Gevarter et al. identified 26 studies meeting their inclusion criteria with 53 participants with ASD. Gevarter et al. reported studies included in the review used interventions including representations (i.e., touch points, number lines, and manipulatives), strategy instruction (i.e., counting on, Solve-It, mnemonics, graphic organizers), and scripted curriculum. Studies included in the review targeted accuracy measures in basic operations (i.e., addition, subtraction, multiplication, division), money or purchasing, word problem solving, numeracy, time, and fractions. Gevarter et al. evaluated the studies for quality using Reichow et al.'s (2008) quality indicators, which were developed specifically to evaluate studies targeting students with ASD. They reported 13 of the 26 identified studies were high quality and found positive results. Interventions which targeted accuracy of the mathematics skill and interventions with behavioral components all reported increased engagement with the mathematics tasks. The majority of the included studies focused on researcher-implemented, one-on-one interventions in self-contained classrooms. Similar to Barnett & Cleary (2016) and King et al. (2016), Gevarter et al. determined insufficient research existed to make recommendations of specific instructional practices or label interventions as evidence based. Gevarter et al. recommended future researchers explore interventions in settings and context more generalizable to common educational settings (i.e., general education classrooms, small group instruction, teacher-led interventions).

While the three previous literature reviews offered important results and implications for practice, the methods were inconsistent and contained several limitations. First, search terms and publication year were inconsistent across the three reviews. King et al (2016) used all possible truncations of 26 terms related to specific mathematics skills (e.g., addition, number sense, telling time) and autism and included research published before May 2014. Barnett & Cleary (2016) used search terms pertaining to math (e.g., math, math instruction, teaching strategies) and autism (e.g., ASD, Asperger's) with no information on publication year. Gevarter et al. (2016) included similar search terms to Barnett & Cleary (2016) for autism and mathematics instruction but added an additional search with the term academic engagement and reviewed research published between 1980 and 2015. These methodological inconsistencies resulted in each locating a different set of studies to include in their analysis. Additional limitations also existed with each review. King et al. did not report the age or grade of the target students, which makes it difficult to make practice recommendations based on the results. Barnett & Cleary failed to provide explicit or clear information regarding their data analysis procedures. With the inconsistencies in the studies located and data analysis procedures, each review's limitations, and the amount of new literature since each was published, a need exists for an updated systematic review to explore the current literature on mathematics instruction for students with ASD.

Current Study

The researchers of the current study sought to update the previous reviews exploring mathematics interventions for students with ASD, attending to their limitations and inconsistencies, while analyzing the state of the literature for EBPs. The study explored the following research questions: (a) What is the current state of the research on mathematics interventions to support students with ASD? (b) What are the participant characteristics, outcomes, and settings of mathematics intervention research involving students with ASD; (c) How many studies, according to Cook et al. (2014) quality indicators, exploring mathematics interventions for students with ASD are methodologically sound; and (d) Can any mathematics interventions for students with ASD be classified as EBPs?

Method

Search Procedures

The researchers conducted a keyword search in the ProQuest, EBSCO, PsycINFO and ERIC databases from 2015-2022 using terms coming from three categories: disability (autis* OR Asperger OR PDD OR ASD), content (math* OR arithmetic), and intervention (intervention OR instruction). The researchers selected to begin their search with studies published in 2015 because all three previous reviews (Barnett & Cleary, 2015; Gevarter et al., 2016; King et al., 2016) evaluated studies prior to 2015. Studies were included in the review if they (a) were published in a peer reviewed journal between January 2015 and December 2022; (b) were written in English; (c) involved at least one dependent variable related to functional or academic mathematics outcome (e.g., subtraction, purchasing skills); (d) involved at least one participant with a reported identification of ASD; (e) used an experimental design (i.e., single case, group experimental); (f) included school-age students (i.e. K-12); (f) included sessions which occurred in a formal or informal K-12 educational or clinical setting; and (g) disaggregated mathematics data for participants with ASD from non-mathematics data and data from participants without ASD. Given that single case design studies allow for analysis at the individual student level, studies were still included in the review if there were participants without ASD (Ledford & Gast, 2018). Researchers applied the following exclusion criteria: the study (a) did not involve an empirical design (i.e., qualitative), (b) did not have outcomes related to mathematics, (c) did not explore the effects of an intervention, (d) were behavioral in nature (i.e., on task behavior in mathematics), or (e) could not disaggregate data for students with ASD from data for other students. Using these inclusion and exclusion criteria, the researchers reviewed the title and abstract and, if necessary, the whole article to determine inclusion (see Figure 1 for PRISMA diagram).

After searching the four databases using the keyword method, the researchers searched for articles published between 2015 and 2022 in eight peer-reviewed journals with the same search terms used for databases: *Education and Training in Autism and Developmental Disabilities, Exceptional Children, Exceptionality, Focus on Autism and Other Developmental Disabilities, Journal of Autism and Developmental Disabilities, Journal of Special Education, Journal of Special Education Technology,* and *Remedial and Special Education*. The journals were selected for the search because they frequently publish research involving students with ASD, are considered top tier journals publishing special education research in general or were searched in previous reviews (Barnett & Cleary, 2016; Gevarter et al., 2016; King et al., 2016). As with the keyword search, the researchers reviewed the title and abstract and, if necessary, the whole article to determine inclusion through the application of the inclusion and exclusion criteria.

After applying the inclusion and exclusion criteria to the studies found in the database and journal keyword searches, the researchers identified 24 studies. As a final step, the researchers conducted an ancestral search of the reference lists of the 24 included studies. Researchers examined the reference list of all included studies and identified any studies that may meet inclusion and exclusion criteria for the review. After reviewing the titles of articles in the references, the researchers located the studies, read the study abstracts, the whole article if necessary, and determined if the article was included in the review by applying the inclusion and exclusion criteria. The researchers located an additional five studies through the ancestral search, resulting in 29 total studies which met inclusion and exclusion criteria.

Article Coding

The researchers coded the 29 included studies for the total number of participants, their grade level, disability category, the setting where the study was conducted (i.e., school, home, virtual, summer camp, special education, general education), and the interventionist (i.e., researcher, teacher). The researchers then determined the study design, coding for single case design or group design study and specific design used (e.g., multiple probe across participants, ABAB design). The researchers also identified the dependent variable, the intervention, and the math focus for each included study. If the study used an intervention package, the researchers noted as such and coded what additional instructional practices were used (e.g., explicit instruction, the system of least prompts). Intervention package was defined as the use of additional instructional practices in addition to the main intervention.

Quality Indicator Coding

The researchers coded the 29 studies for quality using the quality indicators (QIs) established by Cook et al. (2014) and the Council for Exceptional Children (CEC). The researchers selected the QIs by Cook et al. (2014), as opposed to other options (e.g., Gersten et al., 2005; Horner et al., 2005), because the CEC QIs were developed based on a culmination of previously mentioned QIs and other researchers used these QIs in previous systematic reviews involving mathematics (e.g., Losinski et al., 2019; Park et al., 2020). The Cook et al. (2014) QIs can be used to evaluate the quality of studies as well as make EBP recommendations. The researchers used a researcher-created Google form to code each article for basic information such as the study title, researchers, and area of mathematics targeted. The form also included items for each QI, presented as a multiple-choice question. For each question the researcher indicated if the study met the QI, did not meet the QI, or if the QI was not applicable due to the design (i.e., if a QI only applied to group design studies).

The Cook et al. (2014) QIs included 28 items involving items for both single case and group design studies. Twenty-two of the items applied to single case research design and 24 to group design studies. The 28 QIs were categorized into: context and setting, participants, intervention agent, description of practice, implementation fidelity, internal validity, outcome measures or dependent variables, and data analysis (see Cook et al., 2014 for a complete list of QIs). After studies were coded for all QIs, they were classified as methodologically sound or not methodologically sound. For a study to be considered methodologically sound, the study must have met all of the QI's for its respective design (Cook et al., 2014). The study was considered not methodologically sound if the study did not meet one or more of the QI's.

Effect Sizes

For studies considered methodologically sound, the researchers calculated two different effect sizes—percent of non-overlapping data (PND) and Tau-U—to compare baseline and intervention phases for all cases that fit the inclusion criteria (e.g., students with ASD). If the original researchers did not provide PND and Tau-U in the results, the researchers of this systematic review used a computer software, WebPlotDigitizer, to extract graphed data from studies (Rohatgi, 2015). After determining the range of baseline data, the researchers calculated PND by using the following equation $\frac{number\ of\ non-ovelapping\ interveniton\ datapoints}{total\ number\ of\ intervention\ datapoints} \times 100 = PND$ (Ledford & Gast, 2018). Scruggs & Mastropieri (1988) suggested researchers interpret PND greater than 70% as an effective intervention, PND between 50% and 70% as neutral/mixed effects, and any PND below 50% as not effective. Researchers used an online calculator (http://singlecaseresearch.org/calculators/tau-u) to determine the Tau-U between baseline and intervention for each relevant case (Vannest et al., 2016). Researchers interpreted Tau-U effect sizes based on standard convention: 0.20 and 0.60 moderate effect, between 0.60 and 0.80 large effect, and any score greater than 0.80 a very large effect (Parker et al., 2011).

Inter-Observer Agreement

Two independent reviewers coded for interobserver agreement (IOA). The first researcher, a doctoral candidate in special education whose research involved mathematics interventions for students with disabilities, served as the primary data collector. A second data collector, a special education undergraduate student, was trained by the first researcher to code systematic review data. The second reviewer evaluated 578 of the 1925 articles (30%) located from database and journal searches and independently determined if the articles met inclusion criteria. After both reviewers independently determined if the studies would be included in the review, they compared their decisions and calculated IOA. The researcher calculated point-by point IOA by using the following equation $\frac{\#\ of\ agreements}{\#\ of\ agreements + \#\ of\ disagreements} \times 100$ (Ledford & Gast, 2018). IOA was calculated as 100% for inclusion.

The secondary data collector also independently coded for the study information (e.g., participants, setting, interventionist, dependent variables, intervention, mathematical focus, and intervention package details). The second data collector coded 9 of the 29 studies (31%) for everything except intervention package details; researchers calculated IOA at 98.6%. In terms of intervention packages, a second researcher coded seven of the 22 (31.8%) methodologically sound studies to the components of the intervention package. IOA for instructional practices was calculated as 91.7%. IOA for both was calculated by using the following equation $\frac{\#\ of\ agreements}{\#\ of\ agreements\ +\#\ of\ disagreements} \times 100$ (Ledford & Gast, 2018). The researchers discussed disagreements and decided if the article included an intervention package and what the intervention package components were. Finally, two reviewers used the researcher-created Google form outlining each QI detailed in Cook et al. (2014) indicating 'yes' if the study met the QI or 'no' if the study did not meet the QI. The first researchers compared coding for 34.5% of the articles and calculated IOA using $\frac{\#\ of\ agreements}{\#\ of\ agreements\ +\#\ of\ disagreements} \times 100$ (Ledford & Gast, 2018). The researchers discussed disagreements and decided if the article met the QIs in question and if the article was considered methodologically sound. IOA for QIs was 96.2%.

Results

Current State of the Literature

Researchers found 29 studies exploring mathematics interventions for students with ASD that fit the inclusion criteria. Twenty-eight studies were single case research design studies, and one was a group design study. The 29 studies included a total of 116 students with ASD, which represented 87.9% of the total number of participants. Of the 116 students with ASD, 32 were in elementary school, 78 in middle school, and six in high school. Across the 29 studies, five involved the teacher as the interventionist and 24 involved the researcher as the interventionist.

Two of the 29 studies involved small group instruction while the other 27 were conducted in a one-on-one setting. Of the 29 included studies, 12 targeted basic operations and 10 targeted word problem solving. Word problem types included compare, algebraic, proportion, percent of change, part-part whole, scaled pictograph, multiplicative, and fraction word problems. Other targeted skills included early numeracy skills (n=3), fractions (n=3), and price comparison (n=1). The most common interventions were manipulative-based interventions (n=14) and modified schema-based instruction (MSBI; n=8). Additional interventions included video-based interventions (n=4), early numeracy intervention (n=2), and cover-copy-compare (n=1).

Methodologically Sound Studies

Twenty-two of the 29 studies that met inclusion criteria, were identified as methodologically sound according to Cook et al.'s (2014) QIs (see Table 1). Across the 22 studies, there were 55 students with ASD, including 19 elementary-aged students, 35 middle school students, and one high school student (see Table 2). Across the 22 studies, five involved the teacher as the interventionist and 17 involved the researcher as the interventionist. Two of the 22 studies involved small group instruction while the other 20 were all conducted in a one-on-one setting. Of the 22 methodologically-sound studies, 10 targeted basic operations and eight targeted word problem solving. Other targeted skills included operational skills involving early numeracy skills (n=2) and fractions (n=2). Interventions included manipulative-based interventions (n=11), MSBI (n=7), video-based interventions (n=3), and an early numeracy intervention (n=1).

Of the 22 methodologically sound studies, 19 included instructional practices in addition to the main intervention, creating an intervention package. Eleven of the studies involving intervention packages included one additional instructional practice, four included two additional instructional practices, three included three additional instructional practices, and one included four additional instructional practices. The most common instructional practice implemented was explicit instruction (n=8), followed by the system of least prompts (n=5), manipulatives (n=4) and task analyses (n=4). Two studies each included overlearning, faded support, corrective feedback, technology (iPad), and visual supports; and one study involved each of the following: comprehension check, calculator, mnemonic, and visual activity schedules.

Of the 22 single-case studies that met all QIs, 15 used multiple probe across participants designs, two used multiple baseline across participants with an embedded alternating treatment design, two used multiple baseline across behaviors design, two used multiple baseline across student groups design, and one used ABAB reversal design. The Tau-U scores for 49 of the 55 indicated a very large effect (i.e., scores between 0.80 and 1.0). One participant’s Tau-U score was 0.68, indicating a large effect, and the other five participants’ scores were between 0.40 and 0.47, indicating a moderate effect. PND effect scores ranged from 40% to 100% across the 55 participants with ASD, with 100% being the mode. According to visual analysis, PND, and Tau-U effect sizes of all students with ASD (see Table 2), 20 of the 22 single case design studies showed a positive effect for all cases involving students with ASD, one showed positive results for two of the three cases, and one showed positive results for one of three cases (see Table 1).

Studies Found to be Not Methodologically Sound

According to the QIs, seven of the 29 studies were considered not methodologically sound (Cook et al., 2014; refer to Table 1). Six were single case design studies and one was a group design study. All seven did not meet QI 3.1 or QI 3.2, which both referred to the role of the intervention agent (e.g., describing the intervention agent/ their role, and the specific training required for the intervention agent to implement the study). In addition, two studies (i.e., Maras et al., 2019, Morton & Gadke, 2018) failed to also meet QI's 5.1, 5.2, and 5.3, which referred to assessing implementation fidelity throughout the study. All other QIs were met.

Evidence-Based Practice Classification

For an intervention to be considered evidence-based according to Cook et al. (2014) requires at least 20 participants across at least five studies considered methodologically-sound with no negative results. Researchers found 11 studies exploring manipulatives with 23 participants with ASD, seven studies focused on MSBI with 19 participants with ASD, three studies exploring video-based interventions (VBI) with nine participants with ASD, and one study exploring an early numeracy intervention with four participants with ASD. As such, two of the interventions had five of more studies published: manipulatives and MBSI. Of the 11 studies exploring manipulatives, six studies involved virtual manipulative based instructional sequences, three involved virtual manipulatives in and of themselves, one study compared virtual manipulatives to concrete manipulatives, and one study involved the CRA instructional sequence. There were a total of 23 participants with ASD across the 11 studies with no negative results reported. This suggests that, aggregated, manipulative-based interventions are an EBP for students with ASD; however, considering only one of the studies involved concrete manipulatives it is really virtual manipulatives that are an EBP. Of the seven studies exploring MBSI, only 19 students with ASD were included. While all 19 students had positive results, there were insufficient participants across the studies to classify MSBI as an EBP for supporting mathematics for students with ASD.

Discussion

It is important for teachers and researchers to have access to nuanced EBPs that target teaching specific content area skills, such as mathematics, to students with ASD (Hume et al., 2021). The purpose of the review was to update previous literature reviews exploring mathematics interventions for students with ASD (Barnett & Cleary, 2015; Gevarter et a., 2016; King et al., 2016), while both attending to their limitations and methodological inconsistencies and evaluating for EBPs. After locating and examining 29 studies exploring mathematics interventions for students with ASD published between 2015 and 2022, 22 met all the QIs according to Cook et al. (2014) and presented positive results for 55 students with ASD. The researchers found several main results. First, the researchers determined manipulative-based interventions to be evidence-based for students with ASD. The other interventions had an insufficient number of studies or students with ASD to quality them currently as evidence based. Second, the researchers found 19 of the 29 studies explored intervention packages, rather than stand-alone interventions. Third, while the quantity of mathematics research increased since the last set of systematic reviews, considering the narrow window of this review of the last eight years, the research is limited with regards to grade level, settings, and interventionist.

Evidence-Based Practice Classification

Researchers determined two interventions had a sufficient number of studies to be evaluated as EBPs: manipulative-based interventions and MSBI. The QIs by Cook et al. (2014) require at least five studies with 20 participants across the studies with no negative results to classify a practice as evidence based. While researchers concluded manipulative-based interventions an EBP for students with ASD given the seven studies and 19 participants, they were unable to do so for MSBI due to only 19 participants. With one more participant with ASD with positive results across the seven included studies, MSBI could have been classified as an EBP to support students with ASD in mathematics. Even if the EBP determined included studies from the years of Barnett & Cleary (2015), Gevarter et al. (2016), and King et al. (2016), there would not be additional participants with ASD due to the novelty of the intervention.

Although researchers deemed manipulative-based interventions as evidence-based, only one study existed which truly evaluated the effectiveness of concrete manipulatives. The ten studies involving 20 students with ASD that focused on virtual manipulatives suggest virtual manipulative-based interventions can stand on their own as an EBP. However, the researchers acknowledge the heterogenous nature of the virtual manipulative-based interventions (i.e., virtual manipulatives as stand-alone interventions and virtual manipulative based instructional sequences [e.g., VRA, VR, VA, VRA-I]). Of the 10 methodologically sound virtual manipulative-based intervention studies, three studies explored virtual manipulatives as a stand-alone intervention, six explored virtual manipulative-based instructional sequences (i.e., three VRA, one VR, one VA, and one VRA-I), and one study compared concrete to virtual manipulatives. Although combining all manipulative-based interventions into one category, as in the current study, is consistent with previous reviews (e.g., Long et al., 2022; Spooner et al., 2019), it fails to provide a nuanced understanding of what particular aspects of virtual manipulative-based interventions are evidence-based (i.e., virtual manipulatives as part of a graduated sequence of instruction as opposed to stand alone tools as well as if particular graduated sequences are more effective than others). As more studies are published exploring virtual manipulative-based interventions, future researchers should attempt to get a more refined understanding of the effects of each type of virtual manipulative-based intervention.

The results suggesting virtual manipulative-based interventions are an EBP and MSBI needing additional studies to confirm EBP determination are consistent with other researchers (e.g., Clausen et al., 2021; Long et al., 2022; Peltier et al., 2020). Researchers previously determined virtual manipulatives or virtual manipulative-based interventions to be an EBP for students with intellectual and developmental disability (Long et al., 2022), moderate and severe disabilities (manipulatives in general, inclusive of virtual manipulatives; Spooner et al., 2019), learning disabilities (i.e., Park et al., 2021), and disabilities in general (Peltier et al., 2020). Clausen et al. (2021), in applying the Horner et al. (2005) criteria for determining EBP, stated MSBI could not be classified as an EBP due to violations of the 5-3-20 criteria (i.e., there were not five high quality or acceptable studies conducted by three distinct research teams in three different geographical areas with at least 20 total participants).

Intervention Packages

Among the 22 included studies found to be methodologically sound, 19 explored the effects of an intervention package. Each of the 19 studies created an intervention package that paired the main intervention with one to four additional instructional practices. Given students with ASD have a variety of needs, incorporating additional instructional practices to support the main intervention presents a viable option to supporting students with ASD (Odom et al., 2021). All 19 studies that included intervention packages found the intervention package resulted in positive effects for the participants with ASD. From this review, the most common instructional practices involved in the intervention packages are all evidence-based for students with ASD: explicit instruction, the system of least prompts, manipulatives, and task analyses (Sam et al., 2019; Shepley et al., 2019; Spooner et al., 2019).

This review, however, did not evaluate the individual efficacy of each instructional practice, rather accepted interventions as packages. This review shows positive results for studies involving intervention packages, which is consistent with previous reviews that made EBP determinations based on intervention packages (e.g., Long et al., 2022; Spooner et al., 2019; Park et al., 2020). In recognizing the high rates of intervention packages, the researchers acknowledge that tools like virtual manipulatives themselves do not teach students with ASD. It is VM with an EBP used to teach students how to solve the mathematics with the VM that provides the support for students. Given the frequency with which intervention packages are used in mathematics research (Spooner et al., 2019), future research involving component analysis or studies exploring the effects of individual instructional practices may be beneficial for understanding how each instructional practice supports students unique needs (Spooner et al., 2019).

Context of Existing Research

The 22 studies determined to be methodologically sound were limited as far as the age range of the students, instructional arrangement, and the interventionist. The students with ASD who participated in the methodologically sound studies ranged from second to ninth grade, with the majority (63.6%) of the 55 students in middle school and only one (1.8%) in high school. For virtual manipulatives, there was an insufficient number of middle school students (n=12) to establish virtual manipulatives as an EBP for specifically middle school students with ASD. However, if researchers remove the one ninth grade student with ASD included in the virtual manipulative studies, the results confirm virtual manipulatives as an EBP for elementary and middle school students with ASD. As one of the primary goals of locating EBPs is to determine and communicate what works, for whom, under what conditions, understanding for whom virtual manipulatives-based interventions are actually an EBP is important (Cook & Cook, 2011; Cook et al., 2009; Spooner et al., 2019).

In addition to limited age range, the current review also found the majority of the interventions were implemented in one-on-one (91.0%) settings by a researcher (77.3%). In K-12 special education classrooms, small group instruction is a common practice when implementing interventions as teachers can deliver targeted instruction to a small group of students as opposed to focusing their attention on only one student (Ozen et al., 2017; Saadatzi, et al., 2018). Small group instruction is often preferred by teachers, as students are not only accessing academic learning opportunities but also social-emotional learning opportunities (Hoekstra et al., 2023). In addition, without exploring interventions in which the teacher is trained as the implementor, researchers cannot speak to the true effectiveness of the intervention in practice (Jones et al., 2022; William, 2019). Researchers are often highly trained and qualified to implement specific interventions in controlled environments with high treatment fidelity (Joyce et al., 2019). However, it remains unknown if teachers are able to implement the same interventions with high fidelity and what training and qualifications teachers need to ensure the interventions are implemented as intended. When interpreting the results of the current review, it is important to consider the insufficient research regarding the efficacy of mathematics interventions for students with ASD in realistic classroom settings. To make accurate EBP determinations and effective practice recommendations to teachers, researchers must explore the effectiveness of mathematics interventions in settings more closely aligned with real world classroom instruction, such as small group instruction delivered by a teacher (Singer et al., 2017).

Implications for Practice

As a result of the repeated confirmation of the efficacy of virtual manipulative-based interventions in this review and prior reviews (e.g., Long et al., 2022; Park et al., 2021; Spooner et al., 2019), teachers should feel confident in using virtual manipulative-based interventions with their students with ASD, particularly middle school students. As such, a focus should be on teachers receiving training to implement manipulative-based interventions with fidelity, either through pre-service or in-service preparation, professional development, or coaching.

Additionally, given the efficacy of intervention packages from this review, teachers of students with ASD in mathematics should consider what EBP interventions (e.g., manipulatives, explicit instruction, task analysis, SLP) can be combined to support the individual mathematical and behavioral needs of their students (Odom et al., 2021).

Beyond VM-based interventions, which tended to focus more on computational skills for students with ASD in the literature, teachers should also consider using MBSI for problem solving. MSBI was not classified as an EBP but truly was only lacking one more participant with ASD with positive results. Clausen et al. (2021) report there is no effective alternative to MSBI for teaching problem solving skills to students with extensive support needs. Despite the lack of EBP identification in this and other reviews (i.e., Clausen et al., 2021), teachers should feel confident that MSBI is an effective (i.e., research-supported) intervention to implement to support word problem solving for students with ASD and implement it within the practice. Because of the continued confirmation that virtual manipulatives and MSBI are effective, teacher preparation programs and school sponsored professional development programs should be including instruction on how teachers can implement MSBI and virtual manipulative-based interventions in their mathematics instruction.

Limitations and Future Directions

While the results of the review provide important information regarding teaching mathematics to students with ASD, the review is not without limitations. While researchers searched databases and journals that frequently publish special education research focused on supporting students with ASD, the possibility remains that studies were missed. Specifically, dissertations or other grey literature were unable to be located through the keyword searches were unintentionally excluded from the study (Paez, 2017). The researcher did locate one dissertation; however, it was considered not methodologically sound according to the QI's. Second, the researchers chose to use Cook et al. (2014) QIs to determine if the studies located were methodologically sound. If researchers had chosen to use a different set of QIs such as What Works Clearinghouse (WWC; 2022), or Horner et al. (2005), the results may have been different. For example, WWC quality indicators (2022) require a minimum of six baseline sessions to meet standards without reservations, which would decrease the number of methodologically sound studies located in this review. Similarly, as noted by Clausen et al. (2021) in their review of MSBI, the 5-3-20 criteria of Horner et al. (2005) would likely have been violated by the virtual manipulative-based interventions, as there were not five independent teams of researchers. There is a need for additional research groups to begin exploring virtual manipulatives-based interventions.

Additionally, 86.36% of the studies deemed methodologically sound and used to make EBP decisions were intervention packages. While researchers chose to use studies involving intervention packages to make EBP classifications based on positive results and sufficient studies and participants, future researchers should examine intervention packages as EBPs. Given the high rate of intervention packages, it can be difficult to fully discern the efficacy of each element of the package. Future researchers should seek to validate packages as EBP and encourage replication of high-quality studies involving packages to aid obtaining either the 5-3-20 Horner (2005) rule or the Cook et al. (2014) standards for sufficient studies and participants for specific intervention packages. Finally, although the researchers concluded virtual manipulative-based interventions as an EBP for students with ASD, it was really these heterogenous group of interventions involving virtual manipulatives were evidence-based for elementary and middle school students. Researchers should continue to souse out the evidence-based of particular virtual manipulative interventions as well as at smaller age range intervals.

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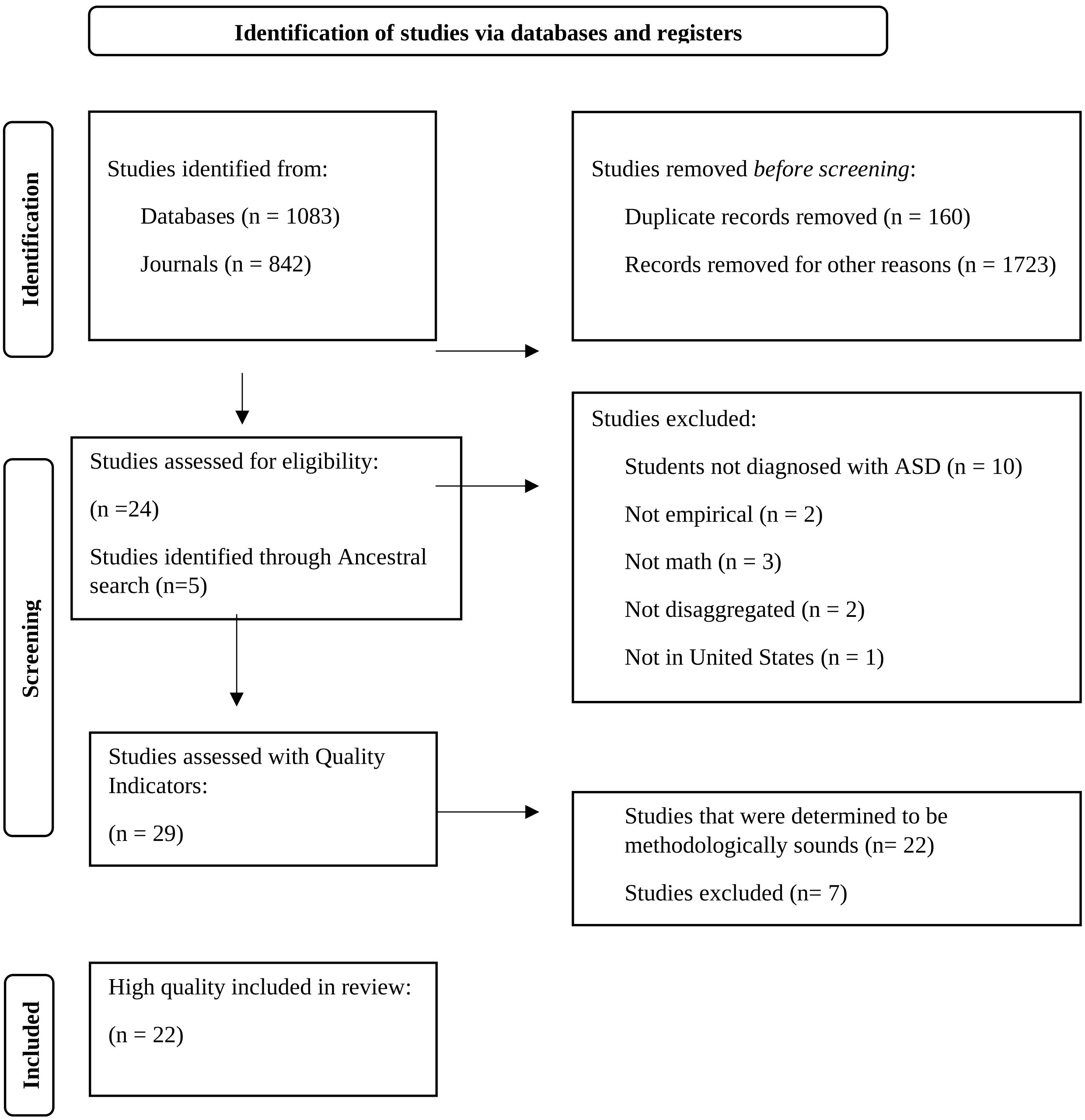
Figure 1

PRISMA Diagram



From: Page, M. J., McKenzie, J. E., Bossuyt, P. M., Boutron, I., Hoffmann, T. C., Mulrow, C. D., Shamseer, L., Tetzlaff, J. M., Akl, E. A., Brennan, S. E., Chou, R., Glanville, J., Grimshaw, J. M., Hróbjartsson, A., Lalu, M. M., Li, T., Loder, E. W., Mayo-Wilson, E., McDonald, S., … Moher, D. (2021). The PRISMA 2020 statement: An updated guideline for reporting systematic reviews. *BMJ*, *372*. https://doi.org/10.1136/bmj.n71

Table 1

Application of Cook et al. (2014) Quality Indicators

	1.1	2.1	2.2	3.1	3.2	4.1	4.2	5.1	5.2	5.3	6.1	6.2	6.3	6.5	6.6	6.7	7.1	7.2	7.3	7.4	7.5	8.2
Bassette et al., 2019	Y	Y	Y	N	N	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Bouck et al., 2021	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Bouck, Park, et al., 2020	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Bouck et al., 2019	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Bouck et al., 2020	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
Buncher, 2019	Y	Y	Y	N	N	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Cox & Root, 2020	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Jimenez & Besaw 2020	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Ledbetter-Cho, et al., 2020	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Long et al., 2022	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
Maras et al., 2019	Y	Y	Y	N	N	Y	Y	N	N	N	Y	Y	Y	Y	N/A	N/A	N/A	Y	Y	Y	Y	Y

Table 1 (cont'd)

Morton & Gadke, 2018	Y	N	N	N	N	Y	Y	N	N	N	Y	Y	Y	N/A	Y	N	N	N/A	N/A	Y	Y	Y
*Park et al. 2020	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Park et al., 2019	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Peltier et al., 2020	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Root & Browder, 2019	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Root, et al. 2019	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Root et al., 2017	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Root et al., 2022	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Root et al., 2021	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Root et al., 2018	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Shurr et al., 2021	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Stroizer et al., 2015	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y

Table 1 (cont'd)

Weng & Bouck, 2019	Y	Y	Y	N	N	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Wright et al., 2020	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Yakubova et al., 2022	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
*Yakubova et al. 2020	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
Yakubova et al., 2016	Y	Y	Y	N	N	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y
Yakubova et al. 2015	Y	Y	Y	N	N	Y	Y	Y	Y	Y	Y	Y	Y	N/A	Y	Y	Y	N/A	N/A	Y	Y	Y

Note: Y = Yes, quality indicator met; N = no, quality indicator not met; * indicates the study met all quality indicators. Quality indicators 6.4, 6.8, 6.9, 7.6, 8.1, and 8.3 were not included as they only applied to group design studies. See Cook et al. (2014) for the complete list of quality indicator (e.g., 1.1 – 8.2).

Table 2

Evidence-Based Classification Support for Mathematics Interventions for Students with ASD

Studies	Participants & Setting	Design	Int.	Intervention	Math Focus	Dependent Variable	Effect Size	Effects
Bouck et al., 2021	4 MS students (2 ID, 2 ASD); School (SE)	MP across participants	R	Virtual number line and corrective feedback	Addition of integers	Accuracy	PND: 100%, 85.7 Tau-U 1.0, 0.86	Positive effects
Bouck, Park, et al., 2020	3 MS students (1 LD, 1 ASD, 1 ID); School (SE)	MP across participants	R	App-based manipulatives; explicit instruction	Division	Accuracy	PND: 100% Tau-U: 1.0	Positive effects
Bouck et al., 2019	3 MS students (1 OHI, 1 IDD, 1 ASD); School (SE)	MP across participants	R	VA instructional sequence	Identifying fractions	Percent accuracy	Tau-U: 1.0 PND: 100%	Positive results
Bouck, Shurr, & Park (2020)	4 MS students (2 ID, 1 OHI, and 1ASD); School (SE)	MP across participants	R	VR Instructional Sequence	Multiplication, Division	Accuracy	PND: 100% Tau-U: 1.0	Positive effects
Cox & Root, 2020	2 MS students with ASD; After school, in home	ABAB reversal	R	Instruction using MSBI with visual supports, explicit instruction, and systematic prompting and feedback	Word problems, proportion	Mathematical word problems solving flexibility and communication measured by 4-point rubric	Tau-U: 1.0, 1.0 PND: 100%, 100%	Positive results

Table 2 (cont'd)

Jimenez & Besaw (2020)	2 Elem students (2 ASD); School (SE)	MP across behaviors across participants	T	Virtual manipulatives	Early Numeracy Skills	Accuracy and Engagement	PND: 80%, 90%, 75% Tau-U: 0.96, 0.99, 0.68 PND: 100%, 80%, 75% Tau-U: 0.96, 0.91, 0.83	Positive effects
Ledbetter-Cho, et al., 2020	5 Elem students (5 ASD); school (SE)	MP across participants	T	VBI	Addition	Percent of TA steps completed independently	PND: 100%, 100%, 100%, 100%, 100% Tau-U: 1.0, 1.0, 1.0, 1.0, 1.0	Positive results
Long et al., (2022)	2 Elem, 1 MS, 1 HS students (4 ASD); virtual Zoom meeting	MP across participants	R	VRA instructional sequence	Subtraction	Accuracy	PND: 100%, 100%, 100%, 100% Tau-U: 1.0, 0.75, 0.67, 0.43	Positive effects
Park, Bouck, & Fisher (2021)	3 MS students (1 ASD 2 LD); School (SE)	MP across participants	R	VRA instructional sequence	Multiplication	Accuracy	PND: 100% Tau-U: 1.0	Positive effects
Park, Bouck, Smith (2020)	4 MS students (3 ASD, 1 ID); School (SE)	MP across participants	R	VRA instructional sequence	Subtraction	Accuracy	PND: 100%, 100%, 100% Tau-U: 1.0, 1.0, 1.0	Positive effects
Peltier, et al., 2020	2 Elem students (2 ASD); school (SE)	MP across student groups	T	SBI	Word problems	Mathematical problem solving	PND: 100%, 87% Tau-U: 1.0, 0.96	Positive results

Table 2 (cont'd)

Root & Browder, 2019	3 MS students (3 ASD);	MP across participants	R	MSBI	Word problems, algebraic	Steps of problems solving completed independently	PND: 100%, 100%,75% Tau-U: 1.0, 1.0, 0.75	Positive results
Root et al., 2017	3 Elem students (3 ASD); School (SE)	MP across participants with embedded alternating treatment	R	MSBI	Word problems, comparison	Word problems solving via point system	PND: 100%, 100%, 100% Tau-U: 1.0, 1.0,1.0	Positive results
Root et al., 2019	3 Elem students with ASD Sumer camp for students with ASD	MP across participants	R	MSBI on an iPad	Word problems, scaled pictograph	Mathematical problem solving, measured by the number of critical steps of the task analysis completed independently correct	PND: 100%, 80%, 100% Tau-U: 1.0, 0.87, 1.0	Positive results
Root et al., 2022	6 MS students (3 ASD, 3 ASD/IDD); School (SE)	MP across dyads	T	MSBI	Word problems, multiplicative	Independence in solving word problems and cumulative correct problems solved	PND: 100%, 100%, 100% Tau-U: 1.0, 1.0, 1.0	Positive effects
Root et al., 2021	3 MS students (2 ASD 1 ID); School (SE)	MP across participants	R	VRA-I with visual support (poster) and graphic organizer	Word problems, multiplicative	Problem solving accuracy	PND: 91%, 91% Tau-U: 0.88, 0.99	Positive results
Root et al., 2018	3 MS students (3 ASD); School (SE)	MP across participants	R	MSBI	Word problems, algebraic	Accuracy and independence	PND: 100%, 100%, 100% Tau-U: 1.0, 1.0, 1.0	Positive results

Table 2 (cont'd)

Shurr et al., 2021	3 Elem students (3 ASD); School (SE)	MB across participants with embedded ATD	R	VM vs CM	Addition	Accuracy and independence	Virtual: PND: 100%, 100%, 100% Tau-U: 1.0, 1.0, 1.0 Concrete: Tau-U: 0.57, 0.40, 1.0 PND: 57%, 40%, 100%	Positive results for 1/3
Stroizer et al., 2015	3 Elem students (3 ASD); ESY program	MB across participants	T	CRA	Addition, Subtraction, Multiplication	Correct problems our of 10	PND: 97%, 96.7%, 96.7% Tau-U: 1.0, 0.91, 1.0	Positive results
Wright, et al., 2020	4 Elem students (3 ASD); School (SE)	MP across participants	R	Early Numeracy Intervention	Early Numeracy skills	Percentage correct of targeted math skills	PND: 50%, 94.4% Tau-U: 0.67, 0.97	Positive results
Yakubova et al., 2022	1 Elem student (1ASD); virtual Zoom meeting	MP across behaviors	R	VBI	Addition, Subtraction, Number Comparison	Accuracy	PND: 100%, 100%, 100% Tau-U: 1.0, 1.0, 1.0	Positive effects
Yakubova et al., 2020	3 MS students (3 ASD); Independent day school	MP across participants	R	VBI	Fractions	Accuracy	PND: 100%, 100%, 100% Tau-U: 1.0, 0.47, 1.0	Positive results for 2/3

Note: Elem = elementary; MS= middle school; HS = high school; SE= Special Education classroom; ASD= Autism Spectrum Disorder; ID= Intellectual disability, LD= learning disability; OHI=other health impairment; AATD= adapted alternating treatments design; ATD=alternating treatments design; MP=multiple probe; MB=multiple baseline; Int=interventionist; R=researcher; T=therapist; VRA=virtual-representational-abstract; VA=virtual-abstract; VR=virtual-representational; VM= virtual manipulative; CM = concrete manipulative; PND=percent of non-overlapping data.

CHAPTER 3

TEACHER IMPLEMENTATION OF THE VRA INSTRUCTIONAL SEQUENCE TO SUPPORT ELEMENTARY STUDENTS WITH ASD

Concrete manipulatives (CMs) are a common tool for all students, including students with disabilities, in mathematics classrooms (Carbonneau et al., 2013; Peltier et al., 2019). CMs are physical objects students can manipulate to support them in representing and solving mathematics problems (Peltier et al., 2019). For students with disabilities, CMs are commonly used to support students on their own or as part of a manipulative-based instructional sequences (Bouck & Park, 2018; Peltier et al., 2020). A concrete manipulative-based instructional sequence—referred to as the concrete-representational-abstract (CRA) instructional sequence—was deemed an evidence-based practice for students with learning disabilities (LD; Bouck et al., 2018). The CRA instructional sequence first teaches students to solve math problems with CMs, then transitions students to solving with representations or drawings, and finally to solving using just numerical strategies (Agrawal & Morin, 2016).

Although the majority of the research on the CRA instructional sequence focused on students with LD (Bouck et al., 2018), some researchers have explored this intervention for students with autism spectrum disorder (ASD). Stroizer et al. (2015) taught three elementary students with ASD to solve addition with regrouping, subtraction with regrouping, and multiplication with the CRA instructional sequence. They found a functional relation between the CRA instructional sequence and the three mathematical behaviors. Yakubova et al. (2020) taught three middle school students with ASD to solve fraction problems using an intervention package consisting of video modeling, the CRA instructional sequence, and a self-monitoring checklist. They also found a functional relation between the intervention package and student accuracy, and two of the three students generalized their skills to other fraction problems.

Virtual Manipulatives

With the increase of technology use in classrooms over the past two decades, researchers explored virtual manipulatives (VMs) as an alternative to CMs for students with disabilities (Satsangi & Miller, 2017). VMs are mathematics manipulatives offered in digital format (i.e., web or app based) students can manipulative on a computer or tablet (Bouck & Sprick, 2019; Satsangi & Miller, 2017). Within the literature, researchers found students with ASD to be just as successful with VMs as CMs in supporting mathematical learning (e.g., Bassette et al., 2019; Bouck et al., 2014, Shurr et al., 2021). Further, researchers suggested VMs and VM-based instructional sequences, such as the virtual-representational-abstract (VRA), to be an evidence-based practice for teaching mathematics to students with ASD and intellectual and developmental disabilities (IDD; Long et al., 2022; Long, ‘2023).

The VRA instructional sequence Is an adaptation of the CRA instructional sequence in which VMs replace CM within the first phase (Bouck & Sprick, 2019). As such, the VRA consists of first teaching the student to solve the target math problem using VMs then transitioning to solving using representations or drawings, and finally solving with numerical strategies (Bouck & Sprick, 2019). Park et al. (2020) explored the effectiveness of an intervention package consisting of the VRA instructional sequence and fading supports to teach subtraction with regrouping to four middle school students with ASD or IDD. Park et al. found a functional relation between the intervention package and students accuracy in solving the subtraction problems. Further, they found students maintained their accuracy for up to six weeks following the end of intervention. Long et al. (2023) explored the effects of a researcher-implemented intervention package including the VRA instructional sequence, explicit instruction, and the system of least prompts to teach subtraction with regrouping to two elementary with ASD and two secondary students with ASD in an online environment. Long et al. found a functional relation between the intervention package and student accuracy. Two of the four participants struggled in the representational phase, however, were able to reach mastery criteria in the abstract phase and maintain their accuracy with no instruction provided.

Teacher Implementation of Mathematics Interventions

In the aforementioned research involving the VRA instructional sequence for students with ASD (e.g., Long et al., 2023; Park et al., 2020), researchers acted as the interventionist. This is consistent for mathematics research for students with ASD and other extensive supports needs (ESN; King et al., 2016; Long et al., 2022). In fact, limited research exists exploring teacher implemented mathematical interventions for students with ASD and ESN. Browder et al. (2018) evaluated a teacher-led modified schema-based instruction (MSBI) intervention to support 10- to 13-year-old students with moderate intellectual disability to solve addition and subtraction word problems. Researchers found a functional relation between MSBI and the number of task steps students performed correctly, the number of problems students solved, and students' ability to discriminate problem type. The researchers used a coaching model to train teachers to implement the intervention, which involved the researcher modeling how to implement the intervention on the first session. This was then followed by the teacher taking over the role of implementor with prompts and feedback from the researcher for the reminder of the intervention. Teachers reported they liked this model and felt confident teaching the MSBI with this model. Researchers reported the teachers were able to implement the intervention in the classroom with high levels of fidelity.

More recently, Root et al. (2022) explored the effects of teacher implemented MSBI on problem solving for three small groups of high school students with ESN. Researchers found a functional relation between the intervention and word problems solving across participants. In terms of teacher implementation, Root et al. used similar training procedures to Browder et al. (2018), including ongoing coaching and feedback throughout the study. The teachers implemented the intervention with high levels of fidelity. While teachers reported they found the ongoing coaching and feedback throughout the study contributed to their high levels of fidelity, Browder et al. (2018) and Root et al. (2022) both reported this as a limitation as it is not feasible to offer this level of intensive support outside of the context of the research.

Current Study

There is strong evidence for VMs, as well as the VRA instructional sequence specifically, to support the teaching and learning of mathematics for students with ASD (Long et al., 2022; Long et al., 2023). However, there is a lack of research examining teachers as interventionists. As such, more research is needed to ensure teachers can implement interventions with their students with high levels of fidelity. The current study explored a teacher-implemented VRA instructional sequence intervention to support elementary-aged students with ASD in a targeted mathematics area of need within a classroom setting to answer the following research questions (a) What are the effects of a teacher implemented VRA instructional sequence on the accuracy with which students with ASD solve addition with regrouping problems?; (b) To what extent do participants maintain their accuracy of responding with no instruction preceding?; (c) Are teachers able to implement the intervention with high treatment fidelity?; and (d) What are student and teacher perceptions of the VRA instructional sequence as an intervention?

Method

Participants

Participants were three students with ASD educated in the same special education classroom. All three students attended the same self-contained special education classroom that supported students with autism in academic and functional curriculum. The special education teacher held a bachelor's degree in psychology and elementary education with endorsements in early childhood education, cognitive impairment, and ASD. Participants were nominated by their teacher to benefit from additional support in mathematics and were included in the study if they: (a) had a diagnosis of ASD indicated on the Individualized Education Program (IEP); (b) demonstrated at least one year below grade level mathematics skills in at least one area as measured by the KeyMath-3 diagnostic assessment (Connolly, 2017); (c) answered addition with regrouping baseline probes with less than 40% accuracy; (d) provided signed parental consent to participate; and (e) student assented to participate.

Zach

Zach was a 11-year-old, white, male student in fourth grade. Zach received special education services under the category of ASD. Zach's current IEP goals involved double and triple-digit addition and subtraction with and without regrouping, as well as solving one and two step word problems. Zach was in the special education classroom for the majority of his day but spent on average two hours per day with his general education peers for specials. Zach exhibited challenging behavior such as swearing, throwing materials, and aggression towards teachers and peers. He had a behavior intervention plan that addressed antecedent strategies to prevent challenging behavior, a plan for how to handle challenging behavior when it occurred, and behavior goals focused on decreasing aggression. Zach was able to vocalize in full sentences to communicate with the teacher and researcher. According to the KeyMath-3 assessment (Connolly, 2007), Zach's numeration raw score was 12 (grade equivalency 1.8) and mental computation and estimation was a raw score of 6 (grade equivalency 1.8). For the addition and subtraction subtests, Zach's raw score was 9 (grade equivalency 2.1).

Kevin

Kevin was an eight-year-old, white, male student in third grade. Kevin received special education services under the category of ASD. Kevin's current IEP goals involved telling time on an analog clock to the half and quarter hour, as well as single-digit addition and subtraction with regrouping. Kevin spent about two hours in the general education class per day during literacy instruction and specials (e.g., gym, art). Kevin had a limited vocal repertoire. He could count and read math word problems, but his speech was frequently echolalic. He used a speech generating device to communicate with the teacher and researcher. According to the KeyMath-3 assessment (Connolly, 2007), Kevin's numeration raw score was 6 (grade equivalency K.5) and mental computation and estimation was a raw score of 0 (grade equivalency ≤ K.0).

Ian

Ian was a nine-year-old, white, male student in third grade. Ian was receiving special education services under the eligibility of ASD. Ian survived a significant medical event when he was in preschool. This resulted in Ian missing a significant amount of in school instruction during preschool, kindergarten, and first grade. He has been back in school full time for two years, however, still has a significant number of absences due to his medical needs. Ian's current IEP goals involved single-digit addition and subtraction with regrouping and solving addition and subtraction word problems. Ian spent about two hours in the general education classroom during specials (e.g., gym, art). Like Kevin, Ian had a limited vocal repertoire and used a speech generating device or non-vocal communication such as pointing or pulling the adult to an area to communicate his wants and needs with the teacher and researcher. He could count and read the math problems and could answer yes/no questions. According to the KeyMath-3 assessment (Connolly, 2007), Ian's numeration raw score was 1 (grade equivalency ≤ K.0) and mental computation and estimation was a raw score of 1 (grade equivalency ≤ K.0).

Setting

All three students attended an elementary school in a Midwest state. The district enrolled 3,659 students across eight schools, including one high school, one middle school, and six elementary schools. About one-third of the student body received free or reduced lunch. The school itself enrolled 289 students in grades PK-5 with 55% of the student body identified as white, 14.2% identified as Black, 9.7 identified as Hispanic, 8.3% identified as Asian, and 11.1% identified as two or more races. In the school, about two-fifths of the student body received free or reduced-price lunch. Sessions occurred in the self-contained special education classroom either at the student's desk or in the small group room, which was a small room in the classroom consisting of a table and two chairs. Intervention sessions lasted no more 20 minutes per day per student and study sessions occurred twice per week.

Materials

Materials for the study included an iPad, Brainingcamp Virtual Manipulative Apps, probe sheets, learning sheets, a pencil, task analysis data collection sheets, and "I am working for ___" sheets (see Appendix A & B for examples of the data collection sheets, and 'I am working for ___" sheet). The Brainingcamp Virtual Manipulative Apps (2021) is an app-based library of virtual manipulatives. For this study, students used the base ten blocks app. The base ten blocks app contained a blank whiteboard, tens-rod and ones blocks on the left side, and a tool bar at the bottom. The tool bar consisted of options that allowed students to clear their screen, access a virtual marker, highlighter, and eraser that appeared in a pop-up window on the right side when clicked (see Figure 2 for examples of the app).

Probe sheets, used in baseline and maintenance sessions, consisted of five single- or double-digit addition with regrouping problems and were printed in black ink on white printer paper. Learning sheets were used in intervention and consisted of one problem used to model, one problem used for guided, and five problems used for independent (i.e., the probe). To create probe and learning sheets, the researcher listed all possible double-digit addition with regrouping problems with a sum under 100 and single-digit addition without regrouping problems randomized them, and assigned the problems to probes and learning sheets. Probe and learning sheets were numbered by session and did not contain repeated problems. Problems could be repeated on different probes or learning sheets but never occurred twice on the same sheet. “I am working for__” sheets were a sheet of white printer paper with “I am working for ____” printed at the top, and circles for the number of problems they would be completing at the bottom (refer to Appendix B). To collect data, researchers created a task analysis-based data collection sheet, which included the steps required to solve the problems and a place for the teacher and researcher to indicate the level of prompting required at each step and if the student answered the problem correctly or incorrectly (refer to Appendix A).

Independent and Dependent Variables

The independent variable was the VRA instructional sequence, taught via explicit instruction. Researchers measured two dependent variables in this study: teacher implementation fidelity and student accuracy. Implementation fidelity was defined as how closely the teacher implemented the intervention as intended (Ledford & Gast, 2018). Researchers calculated teacher implementation fidelity by calculating the percent of steps implemented as intended as measured on the fidelity checklist (see Appendix A). For virtual and representational phases, there were 20 total steps for double-digit addition with regrouping and 16 for single-digit addition with regrouping. For the abstract phase, there were 16 total steps for double-digit addition with regrouping and 13 steps for single-digit addition with regrouping. Student accuracy was calculated by indicating if the answer provided correctly corresponded to an answer key. The teacher recorded accuracy data during each session and the researcher divided the number of correct answers by the total number of opportunities (i.e., 4/5 [80%]) to get a percentage.

Experimental Design

To test the effects of VRA instructional sequence implemented by the classroom teacher, researchers used a concurrent multiple probe across participants design. Researchers chose this design as they were interested in exploring the effects of an intervention on a non-reversible behavior (i.e., a skill acquisition task) across the four participants (Ledford & Gast, 2018). Consistent with multiple probe across participants single case design, participants entered baseline simultaneously (Ledford & Gast, 2018). After completing three baseline sessions with a decelerating or zero-celerating trend, the first participant, Zach, entered the first phase of intervention: virtual. The remaining two participants continued to complete baseline probes until it was their turn to enter intervention systematically. When the first student achieved at least 100% accuracy for two consecutive sessions with virtual manipulatives, they entered the representational phase, and the second student entered the virtual phase pending a stable and zero-celerating or decelerating baseline trend (Ledford & Gast, 2018). Participants continued to enter intervention in this way until all three participants completed intervention. Intervention was completed when each student reached 100% accuracy for two consecutive session in each phase:

virtual, representational, and abstract. After intervention, students entered maintenance for two sessions across two weeks.

Procedure

All sessions were implemented by the classroom special education teacher; the researcher was there to collect data and provide feedback to the teacher after the session. The teacher had over 30 years of teaching experience. The researcher was the first author of the study and was a fourth-year doctoral candidate in special education whose research focused on math interventions for students with ASD. The researcher was present during all sessions of the study. All sessions were conducted in person and all data were collected in real time.

Teacher training

Before beginning the study, the researcher trained the teacher on how to implement the VRA intervention. Consistent with Root et al. (2022), the researcher used a two-step teacher training modeled after the principles of behavior skills training (BST). First, the researcher discussed the timeline of the study, the VRA instructional sequence, explicit instruction, and the goals of the study. Next, researcher trained the teacher on explicit instruction and the VRA instructional sequence for one hour. The researcher modeled sessions for each phase of the VRA and provided the teacher with opportunities to role play to practice what was modeled. This continued until the teacher was 100% independent in implementing the VRA intervention via explicit instruction in a role play situation. The teacher implemented the intervention with over 90% fidelity for the first session and thus needed a second training session. The teacher's fidelity was 100% fidelity for the second session. Throughout intervention, the researcher provided feedback to the teacher after each session. This feedback included discussing any items on the treatment fidelity checklist the teacher did not implement with fidelity as well as answering any questions asked by the teacher. In this study, the feedback mainly involved the inclusion of an advanced organizer.

Consistent with the structure of work sessions occurring in the classroom, at the beginning of each session, the teacher presented the students with a choice between two preferred items: one edible and one tangible item. The teacher asked each student “what do you want to work for?” with the items present. When the student chose for what they wanted to work, the teacher wrote at the top of the “I am working for ___” paper. As the students completed problems, they crossed out the circles to keep track of how many they completed and how many problems they had left (refer to Appendix B).

Baseline

For baseline sessions, the teacher presented the student with a probe sheet consisting of five addition with regrouping problems and instructed the student to complete the problems to the best of their ability. Zach completed double-digit addition with regrouping and Kevin and Ian completed single-digit addition with regrouping problems. While students had access to the Brainingcamp base ten blocks manipulative during baseline sessions, the teacher did not instruct them on how to use the manipulative prior to completing baseline probes. The iPad with the app open was available on the table while students solved the problems. Data were collected on accuracy and teacher treatment fidelity. If participants failed to initiate solving the problems within five minutes or if they indicated they did not know how to complete the problems, the session was terminated, and accuracy was scored as zero. The first participant completed at least three baseline sessions, the second at least four, and the third at least five.

Intervention

The teacher used explicit instruction to teach each phase of the VRA instructional sequence. Consistent with explicit instruction, the teacher began each session throughout the three phases of intervention (i.e., virtual, representational, abstract) with an advanced organizer in which they provided context for the math skill the student was learning and connected it to real life. The teacher stated people use addition every day like when we are budgeting to buy things at a store, cooking, or determining how many of something they need. She then modeled how to solve one problem. Next, each student solved one problem with prompts and cues as needed (i.e., guided phase). If passed, the student completed five problems independently. During guided practice, the student solved the problem independently. However, if the student made a mistake, did not know the next step, or did not initiate the next step within 10s, the teacher implemented the system of least prompts (SLP). The teacher implemented a four-level SLP, starting with a gesture prompt (e.g., point to the problem), followed by an indirect verbal prompt (e.g., "do you have enough ones?"). If students continued to make an error or not initiate, the teacher delivered a direct verbal prompt (e.g., "you need to regroup"), and, if the student still did not complete the next step, the teacher modeled (e.g., showed the student what to do). If participants did not achieve 100% accuracy and at least 85% independence during guided practice, they did not progress to the independent practice phase of explicit instruction (i.e., the probe), the session was terminated, and the learning sheet was repeated the next session.

Virtual phase. During the virtual phase, the teacher modeled for the student how to solve the problem using the VM. The teacher began with the advanced organizer to connect the math content to real life. The teacher then modeled how to solve one problem paired with a think aloud based on the task analysis (see Figure 2 for example of task analysis steps). To solve double-digit addition with regrouping problems with VMs, the teacher explained the student would be adding the numbers together and could write the problem on the left side of the app (e.g., $38 + 14 =$). Next, the teacher explained they would be using a tens and ones chart, or a T-Chart, to keep themselves organized and they would be using the blue rods to represent tens and the yellow cubes to represent the ones. The teacher instructed the student to set up the problem, by representing the first addend 38, with three tens and eight ones. After the first addend was represented, the teacher modeled how to represent the second addend underneath (14) with one ten and four ones. After the problem was set up, the teacher explained they were ready to add. The teacher described how they would add the ones and see if they were able to make a ten block with ten ones. The teacher stated that eight ones plus four ones is 12 ones. This meant they could make one tens block with two ones blocks left in the ones column. The teacher then explained they needed to highlight ten ones blocks and click the group button to group the ten ones into one ten block (i.e., regrouping). The teacher then moved the new tens block to the tens column and indicated they were ready to find the answer. In the ones column, there were two ones, so she wrote a two in the ones place. Then, the teacher counted five tens-rods and wrote a five in the tens place. Thus, 38+14 equals 52. Once the teacher modeled one problem, the participant solved one guided practice problem and, if independent in guided, five independent practice problems.

For single-digit addition with regrouping, the teacher also first presented the advanced organizer connecting the math skill to real life. The teacher then modeled how to solve one problem paired with a think aloud based on the task analysis (see Figure 2 for example of task analysis steps). She explained the student would be adding the numbers together and they would write the problem on the left side of the app (e.g., $4 + 8 =$). The teacher modeled drawing two circles to represent each of their addends (e.g., 4 & 8). Once the teacher drew the two circles on the screen, the teacher modeled that they would put ones blocks in each of the circles to represent the addends, four blocks in the first circle and eight blocks in the second circle. The teacher expressed to researchers she thought the students would need more support and structure in where to represent each of the addends on the screen. So, in collaboration with the teacher the researcher decided to add drawing circles to provide additional visual support. Once the teacher was done adding all of the blocks, she explained to the students the counting on strategy. The teacher told the students they would hold 4 in their head and count on from four to get the answer. Once the teacher completed modeling how to solve one problem, the student solved one guided practice problems and, if independent, five independent practice problems.

Representational. During the representational phase, the teacher modeled for the student how to solve problems using representations or drawings. Each session, she presented the advanced organizer to connect the math skill to real life. Then, the teacher modeled setting up the problem on the paper first by writing the problem (e.g., $38 + 14 =$). Next, the teacher explained they would be using a tens and ones chart or a T-Chart, like with VMs. However, instead of the blocks, students would be drawing lines to represent the tens and dots to represent ones. To set up the problem, they needed to represent the first addend 38 with three lines for three tens and eight dots for the ones. The teacher then modeled representing the second addend underneath with one line for one ten and four dots or squares for four ones. After they set up the problem, the teacher indicated they needed to add the ones. The teacher explained eight ones plus four ones was 12, and the student could determine this by starting with eight and counting on (i.e., 9, 10, 11, 12). She then modeled regrouping by circling ten ones and drawing one ten in the tens column and pointed out two ones left in the ones column. The teacher explained they needed to cross out ten ones, draw one ten in the tens place, and then add. The teacher stated there were two ones so they would write a two in the answer ones place. Then, they counted five tens and wrote a five in the answer tens place: thus, 38+14 equaled 52.

For single digit addition with regrouping, the teacher first explained they were going to draw a picture to help solve the problems. The teacher began by introducing the advanced organizer connecting the math skill to real life. Then, the teacher wrote the problem (e.g., $4 + 8 = \_$) at the top of the page. The teacher explained they were going to draw two circles to represent each of the addends (e.g., 4 and 8). Once the teacher drew the two circles, the teacher stated that they would draw dots or squares in each of the circles to represent the addends, four dots or squares in the first circle and eight dots or squares in the second circle. Once the teacher was done drawing the dots, she indicated the student could count on to find the total for the problem starting with four (i.e., 5, 6, 7, 8, 9, 10, 11, 12). Once the teacher completed modeling how to solve one problem, the participant solved one guided practice problem and, if independent in the guided portion, five independent practice problems.

Abstract. In the abstract phase, the teacher taught participants numerical strategies to solve the problems, such as counting on and partial sums. As in previous phases, the teacher presented the advanced organizer, then modeled how to solve one problem paired with a think aloud using the numerical strategies. For double-digit addition with regrouping, the teacher explained the student was going to solve the problem with the numerical strategy of partial sums. First, the teacher wrote the T-chart and the problem (e.g., 38+14) within of the T-Chart (see Figure 2). The teacher modeled starting in the ones place using the counting on strategy to add the digits in the ones place. The teacher indicated they are going to start by holding eight in their head and count up to add four (e.g., 9, 10, 11, 12). Twelve is one ten and two ones and the teacher explained they can write down 12. The teacher then stated they needed to add the digits the tens place. The teacher noted they have 3 tens, plus 1 ten, which equals 4 tens or 40. The teacher wrote down 40, with 4 in the tens place and 0 in the ones place. Finally, the teacher explained they have their partial sums and were ready to find their final answer by adding 40+12 to get the final answer of 52.

For single-digit addition with regrouping, the teacher began with the advanced organizer connecting the math skill to real life scenarios and then explained students were solving the problem with the numerical strategy of counting on. For the example problem $8 + 4 = \_$, the teacher indicated they hold eight in their head and count up to add four on their fingers (e.g., 9, 10, 11, 12), resulting in 12. Once the teacher modeled one problem, the participant engaged in one guided practice and, if independent in guided, five independent problems.

Maintenance

Students completed two maintenance probes, one per week for two weeks, following the end of intervention. Students were not provided any instruction from the researcher or teacher prior to completing the maintenance probes. The teacher provided the student with a probe sheet consisting of five problems (i.e., double- or single-digit addition with regrouping), access to the iPad with the base ten blocks app, and a pencil. The teacher presented the iPad, an example of work completed in the representational phase, and example of the work presented in the abstract phase and delivered the directive to solve the problems the best they could using any of the three strategies they learned. Data were collected on accuracy and teacher treatment fidelity.

Social Validity

After the completion of the maintenance phase, the researcher engaged the students and teacher in a social validity interview regarding their perceptions of the intervention. Because two of the three students had a limited vocal repertoire, the researcher created visuals to accompany the social validity interview including pictures of the strategies as well as thumbs up and thumbs down visuals, in which the students circled their choice. The researcher asked the students the following questions: (a) Which strategy did you like the best, why is this your favorite?; (b) Do you think learning these different strategies helped you to learn to solve the problems, why or why not?; (c) Which did you like best to help you solve the problems: the VMs, drawings, or numerical strategies, and why?; (d) Do you want you teacher to teach you more math using this method?; I (e) Is there anything else you would like to tell me? Researchers asked teacher the following questions: (a) Do you think the intervention helped your students to learn to solve the problems?; (b) Did you feel you were prepared to deliver this intervention with your students?; (d) Do you think this is an intervention you would use again with your students?; (e) is there anything else you would like to tell me?

Inter-Observer Agreement and Treatment Fidelity

Accuracy inter-observer agreement (IOA) data were collected on 33.33-50% of baseline and intervention sessions and 50% of maintenance sessions. During sessions in which IOA data were collected, the researcher also collected accuracy data. After the session, the researcher compared their accuracy data to the teacher's accuracy data for agreement. The researchers calculated IOA using the following equation $\frac{\#\ of\ agreements}{\#\ of\ agreements\ +\#\ of\ disagreements} \times 100$ (Ledford & Gast, 2018). IOA was calculated as 100% for each student for each phase and condition.

The researcher also collected treatment fidelity data during each session to ensure the teacher was implementing the intervention with fidelity. The checklist included items involving each step of implementing the intervention (see Appendix C and D). Treatment fidelity was calculated at 98.8% for sessions with Zach (98% virtual, 100% representational, 99% abstract), 99.1% for sessions with Kevin (98% for virtual, 98.4% for representational, and 100% for abstract), and 99.4% for sessions with Ian (100% for virtual, 98.4% for representational, and 100% for abstract). The only item in the treatment fidelity checklist the teacher did not complete across all sessions was the advanced organizer. She missed the advanced organizer in six of the 36 total intervention sessions across the three participants (three times in virtual, two times in representational, and once in abstract).

Data Analysis

To analyze data, researchers graphed accuracy data in Excel and conducted visual analysis as well as calculations. Researchers compared immediacy of effect using visual analysis which required researchers to compare the last baseline session to the first intervention session. (Gast & Spriggs, 2014). To calculate trend within each phase researchers utilized the split middle method. After finding the middle point of the data, researchers drew a line connecting the mid-date and mid-rate for each phase and determined if the line was zero-celerating (straight slope), accelerating (increasing slope), or decelerating (decreasing slope; White & Haring, 1980). The researchers used the 80-25 rule to calculate level across each phase by calculating if 80% of the data fell within 25% of the mean in each phase. The researchers determined the Tau-U between baseline and intervention sessions using an online calculator. Vannest et al. (2016) indicated the effect size moderate if it fell within the range of 0.20 to 0.60, large if it fell within 0.60 to 0.80, and very large if it was above 0.80.

Results

Overall, researchers found a functional relation between the teacher implemented VRA instructional sequence taught via explicit instruction and the dependent variable of accuracy (see Figure 3). All three students acquired each phase (i.e., virtual, representational, and abstract) and maintained their accuracy in solving their target addition skills (single- or double-digit addition with regrouping). The teacher was successful in implementing the intervention with over 97% fidelity across all three students. She thought the intervention was easy to implement and helped her students to acquire the math skill. Students reported a preference for the VMs.

Zach

During baseline, Zach was 0% accurate for all three baseline probes resulting in a stable, zero-celerating trend (see Figure 3). Zach answered with 40%, 60%, and 80% accuracy for his first three virtual sessions. In session four, he was unable to complete a session due to challenging behavior. He completed virtual sessions five and six with 100% accuracy. His virtual intervention data were variable and accelerating. Zach completed both of his representational sessions with 100% accuracy; his data were stable and zero-celerating. In the abstract phase, Zach did not pass guided his first session. He did not want to complete the problems using the partial sums strategy the teacher was modeling. He continued to draw to solve the problems and, if interrupted, engaged in challenging behavior such as aggression and elopement. In his second abstract session he was 0% accurate using the abstract strategy but solved all five problems correctly with drawings. His third abstract session did not occur due to challenging behavior. This was followed by a session at 60% accuracy, in which he completed three of five problems using the partial sums abstract numerical strategy. He still completed two problems with drawings. In abstract session five, challenging behavior prevented the session. In sessions six and seven, Zach completed the problems using the abstract numerical strategy with 100% accuracy. The data in abstract were variable and accelerating. The Tau-U between baseline and intervention data was 0.91, indicating a very large effect. Zach was 100% accurate in both of his maintenance sessions. He chose to solve all problems in both maintenance sessions using the

VM. Between maintenance sessions, there were two planned sessions that no data collection occurred due to challenging behavior. Maintenance data were stable and zero-celerating.

Kevin

During baseline, Kevin was 0% accurate for each of his four baseline probes, resulting in a stable and zero-celerating trend (see Figure 3). Kevin did not pass guided on his first session in virtual. This was followed by 40%, 100%, and 100% accuracy on sessions two through four respectively. The data were variable and accelerating. Kevin completed his first representational session at 40% accuracy, his second at 80% accuracy, and his third and fourth at 100% accuracy. Data were variable and accelerating. He was unable to pass guided in his first abstract session and completed his second through fourth session with 20%, 40%, and 80% accuracy. This was followed by two sessions at 100% accuracy. The Tau-U between baseline and intervention data was 1.0, indicating a very large effect. Kevin was 100% accurate in both of his maintenance sessions. He chose to use the VM during both maintenance sessions. Maintenance data were stable and zero-celerating.

Ian

During baseline, Ian was 0% accurate for all five of his baseline probes, resulting in stable data with a zero-celerating trend (see Figure 3). Ian was 100% accurate on both of his first two virtual sessions. The data were stable and zero-celerating. Ian did not pass guided on his first representational session. He was 80% accurate in his second representational session, followed by 100% accurate in his third and fourth representational sessions. His representational intervention data were stable and accelerating. Ian was 20%, 40%, and 80% accurate in his first three abstract sessions, respectively. In his fourth and fifth abstract sessions, he was 100% accurate. The Tau-U between baseline and intervention data was 1.0, indicating a very large effect. Ian was 100% accurate in both of his maintenance sessions; his data were stable and zero-celerating. He first solved the problem with numerical strategies but chose to use the VM to check his answers during both maintenance sessions.

Social Validity

When asked what his favorite strategy was during the post-intervention social validity interviews, Zach indicated he liked the VM best because he liked using the iPad and it was easy for him to use. He expressed that he did not like using the numerical strategy as he did not think it was quick enough. Zach told the teacher that he liked working with her and the researcher and wanted to use the iPad for more math in the classroom. Kevin did not vocally engage in the interview but was able to indicate preference for the VM. He also indicated the strategies helped him to solve the problems, and he would like the use these strategies to learn math in the future. Like Kevin, Ian also did not vocally engage in the interview, but was able to indicate that he preferred the VM and circled drawing second, he thought the strategies helped him to solve the problems and would like the use these strategies to learn math in the future.

The teacher reported the intervention was very effective in helping her students learn to solve the target math problems and she would like to incorporate this intervention into her daily mathematics instruction. She also indicated the training provided by the researcher was sufficient to make her feel confident implementing the intervention. Since ending the study, the teacher reported she has started to teach math content with explicit instruction and the VRA instructional sequence to solve math problems and is seeing increased accuracy.

Discussion

Manipulatives, including VMs as part of a manipulative-based instructional sequence (e.g., VRA), are an evidence-based practice for students with IDD as well as ASD (Long, 2023;

Long et al., 2022). However, the existing research predominately examines middle school students and almost exclusively focuses on one-on-one researcher implemented instruction (Long, 2023; Long et al., 2022). This study explored a teacher implemented VM-based instructional sequence (i.e., VRA) taught via explicit instruction on the accuracy of three elementary students with ASD in solving single- or double-digit addition problems. Researchers found two main results. First, a functional relation existed between the intervention and student accuracy for all three elementary students, and the three students maintained their targeted skill for a brief time. Second, the teacher successfully implemented the intervention with her students, following two training sessions, with consistently high treatment fidelity.

Virtual Manipulative Based Interventions

The results of this study align with prior research exploring the VRA instructional sequence for students with ASD. Prior researchers found students were successful in acquiring computational skills via the VRA instructional sequence (e.g., Long et al. 2023; Park et al., 2020; Root et al., 2020). However, much of the prior research involving the VRA instructional sequence for students with ASD is focused on middle school students (Long, 2023). Of the eight participants with ASD in previous VRA studies, six were middle school students, one was in high school, and one was in elementary school (Long et al., 2023; Park et al., 2020; Park et al., 2021). This study extends the literature to suggest the efficacy of the VRA instructional sequence for elementary school students with ASD.

Beyond the efficacy of the VRA instructional sequence, the results of the study also suggest a continued preference for virtual manipulatives by students with ASD. During maintenance sessions, students were allowed to select the specific VRA strategy of their choosing and all consistently chose to use the virtual manipulatives. A preference for VMs among elementary students with ASD is consistent with prior studies in which VMs were compared to CMs (Bassette, et al., 2019; Bassette et al., 2020; Bouck et al., 2014; Shurr et al., 2021). A preference towards technology-based interventions for students with ASD exists outside of the manipulative and mathematical literature (Kim et al., 2022; Spriggs et al., 2015). Further, Wong et al. (2015) determined technology to be an EBP for students with ASD. While teachers may be hesitant to make the shift from using CMs, especially for younger students with disabilities, elementary students with ASD often embrace and prefer technology as well as experience success with VMs.

Teacher Implementation

Prior to this study, all studies examining the VRA instructional sequence for students with ASD included the researcher as the interventionist (Long, 2023). This study extends the literature to suggest teachers can effectively, efficiently, and with limited training implement the VRA instructional sequence, similar to other mathematical interventions (e.g., MSBI; Browder; Root). Previous studies involving a researcher implemented VRA instructional sequence with students with ASD involved three to eight sessions per phase and 10-19 total intervention sessions (Long et al., 2023; Park et al., 2020; Park et al., 2021). The current study involved two to six session per phase and 11-14 total intervention sessions. Although different mastery criteria was in place across the different studies, teacher implementation did not extend the length to achieve student mastery and maintenance. Further, teacher fidelity in the current study remained over 95% throughout intervention. Given the limited initial professional development time (i.e., two hours), it suggest the ease of intervention implementation. While Root et al. (2022) found teachers needed continued feedback and coaching throughout the intervention and the researcher continually provided feedback throughout this study, the feedback was relatively minimal and could probably have been successfully phased out. Further researchers should seek to examine teachers implementing the VRA instructional sequence without continual coaching to determine fidelity without ongoing training.

Implications for Practice

This study supports teacher use of the VRA instructional sequence to teach addition to elementary age students with ASD. The VRA instructional sequence can help students to build a conceptual understanding of the math concept with manipulatives and drawings before moving to abstract numerical strategies (Bouck & Sprick, 2019). In addition, this instructional sequence provides students with multiple strategies to solving these problems allowing students to independently choose the strategy they prefer to use. Another implication is that the VRA instructional sequence can be implemented successfully with high degrees of fidelity by a classroom teacher with as few as two hours of training. When treatment fidelity was less than 100%, it was the exclusion of the advanced organizer. While advanced organizers are an important element of explicit instruction, it does not necessarily impact students ability to learn the content being modeled when the focus is on procedural knowledge (Archer & Hughes, 2011; Doabler & Fien, 2013). With the continued research confirming the value of virtual manipulatives for students with disabilities, more teacher preparation and school based professional development programs should provide teachers with instruction on how to incorporate virtual manipulative based interventions in their classrooms.

Limitations and Future Directions

While the results of the current study were positive, the study is not without limitations. First, Zach exhibited challenging behavior throughout the duration of the study. While this is not abnormal for students with ASD, and Zach has a behavior intervention plan in place to manage his behavior, the aggression and property destruction sometimes prevented sessions from occurring. Future researchers should attempt to incorporate additional behavioral interventions to support students who exhibit challenging behavior to continue accessing the academic content. Second, the researchers in the current study used explicit instruction to teach students double- and single-digit addition with regrouping via the VRA instructional sequence. Traditionally, explicit instruction involves modeling at least two problems and students engaging in two guided practice problems (Doabler & Fien, 2013). For the current study, the researcher and teacher made the decision to model only one problem and have students engage in one guided practice problem. The researcher and teacher made this adjustment to support the students specific needs as it would align with students current work sessions which lasted around 15 minutes. Future researchers should seek to evaluate explicit instruction and the efficacy of the number of problems modeled and guided to further understand how to support students unique needs.

Third, there was no generalization phase in the current study. Generalization is important and should always be a part of programming for students with ASD (Shurr et al., 2019). Future researchers should identify ways to plan for and explore students' ability to generalize the skills taught in the study. For example, future researchers could consider generalization to other implementors (e.g., general education teacher, paraprofessional), in other environments (i.e., the general education classroom), or with other materials (i.e., problems presented horizontally, different base ten block manipulative). Fourth, researchers did not evaluate for long-term maintenance of the mathematical skills. Long-term maintenance is crucial for students to be able to apply their skills to real-life situations (Spooner et al., 2017; Szekely, 2014). Park et al. (2020; 2021) suggested the use of additional instructional practices such as fading support and overlearning to create effective intervention packages to support extended maintenance of mathematics skills for students with disabilities. Future researchers should continue to explore the use of intervention packages to meet the unique needs of students with ASD across learning stages other than acquisition (i.e., fluency, maintenance, generalization). Finally, while the results of the current study suggest a functional relation between the VRA instructional sequence and students accuracy, the data show consistent decreases in accuracy when transitioning between phases (i.e., virtual to representational, representational to abstract). Decreases in performance when transitioning between phases is a noted limitation of the VRA instructional sequence. This decrease in performance may suggest that students do not see the phases as connected, but separate skills (Root et al., 2021). Root et al. (2021) recently explored the VRA-Integrated (VRA-I) instructional sequence to teach problem solving to three students with IDD. This integrated approach involves teaching students all three representations simultaneously as opposed to sequentially during each phase of instruction (Root et al., 2021; Strickland, 2022). The VRA-I could be a viable alternative to the VRA instructional sequence to prevent these dips in the data as students have already been exposed to instruction on the subsequent phases over the course of the intervention.

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APPENDIX A: DATA COLLECTION SHEETS

SLP	I: *Independent* (no prompts)	G: *Gesture*	IV: *Indirect verbal* – Now what?	DV: *Direct verbal* – state what to do	M: *Model* – Show what to do

Student Name: Session:	Guided	
	1	2
1. Write the problem (i.e., 38+14=)		
2. Draw T Chart		
3. Represent tens in the first addend (i.e., 3)		
4. Represent ones in the first addend (i.e., 8)		
5. Represent tens in the second addend (i.e., 1)		
6. Represent ones in the second addend (i.e., 4)		
7. Add ones		
8. Regroup ones		
9. Add tens		
10. Write/State the answer		
Total independence		
Total number of prompts		
Answered correctly?		

Accuracy in Independent Portion				
1	2	3	4	5

+ correct
- incorrect

For Abstract only use steps 1, 2, 7, 8, 9, 10

SLP	I: *Independent* (no prompts)	G: *Gesture*	IV: *Indirect verbal* – Now what?	DV: *Direct verbal* – state what to do	M: *Model* – Show what to do

Student Name: Session:	Guided
	1
Write the problem (e.g., 3 + 8)	
Draw two circles	
Represent the first addend	
Represent the second addend	
Count on	
Write/state the answer.	
Total independence	
Total number of prompts	
Answered correctly?	

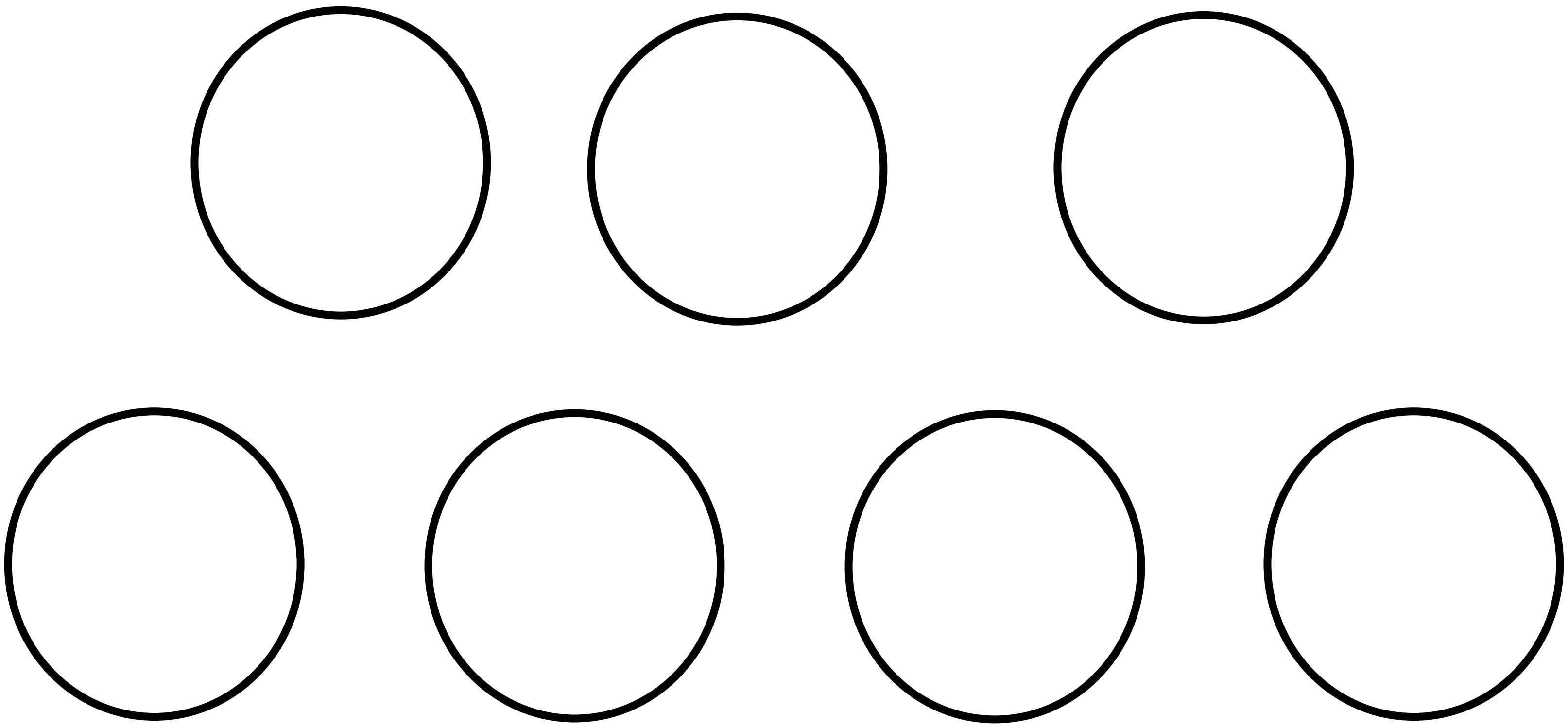
Accuracy in Independent Portion				
1	2	3	4	5

+ correct
- incorrect

For abstract only use steps 1,5, & 6

APPENDIX B: EXAMPLE OF “I AM WORKING FOR___” SHEET.

I am working for _____



APPENDIX C: TREATMENT FIDELITY: DOUBLE-DIGIT ADDITION

<table>
<tr><td colspan="5">Before Intervention</td></tr>
<tr><td></td><td colspan="3">Teacher has all appropriate materials</td><td>Y / N</td></tr>
<tr><td></td><td colspan="3">Teacher presents student with two choices (1 edible and 1 tangible) and asks “What do you want to work for”</td><td>Y / N</td></tr>
<tr><td></td><td colspan="3">Teacher writes what the student is working for on top of I am working for paper.</td><td>Y / N</td></tr>
<tr><td colspan="5">During Intervention</td></tr>
<tr><td rowspan="14">Modeling</td><td colspan="3">Teacher models one problem</td><td>Y / N</td></tr>
<tr><td colspan="3">Begins with advanced organizer (i.e., orient the students to the problems and connect to real life)</td><td>Y / N</td></tr>
<tr><td colspan="3">Models each of the following steps: (Circle the phase)</td><td></td></tr>
<tr><td>Virtual</td><td>Representational</td><td>Abstract</td><td></td></tr>
<tr><td>Write the problem (i.e., 38+14=)</td><td>Write the problem (i.e., 38+14=)</td><td>Write the problem (i.e., 38+14=)</td><td>Y / N</td></tr>
<tr><td>Draw T Chart</td><td>Draw T Chart</td><td>Draw T Chart</td><td>Y / N</td></tr>
<tr><td>Represent tens in the first addend (i.e., 3)</td><td>Draw tens in the first addend (i.e., 3)</td><td>Add ones</td><td>Y / N</td></tr>
<tr><td>Represent ones in the first addend (i.e., 8)</td><td>Draw ones in the first addend (i.e., 8)</td><td>Regroup ones</td><td>Y / N</td></tr>
<tr><td>Represent tens in the second addend (i.e., 1)</td><td>Draw tens in the second addend (i.e., 1)</td><td>Add tens</td><td>Y / N</td></tr>
<tr><td>Represent ones in the second addend (i.e., 4)</td><td>Draw ones in the second addend (i.e., 4)</td><td>Write/State the answer</td><td>Y / N</td></tr>
<tr><td>Add ones</td><td>Add ones</td><td></td><td>Y / N</td></tr>
<tr><td>Regroup ones</td><td>Regroup ones</td><td></td><td>Y / N</td></tr>
<tr><td>Add tens</td><td>Add tens</td><td></td><td>Y / N</td></tr>
<tr><td>Write/State the answer</td><td>Write/State the answer</td><td></td><td>Y / N</td></tr>
<tr><td rowspan="4">Guided</td><td colspan="3">Student completes one guided problem</td><td>Y / N</td></tr>
<tr><td colspan="3">If student makes mistake, indicates they do not know what to do, or 10s without response teacher initiates SLP</td><td>Y / N</td></tr>
<tr><td colspan="3">Delivers prompts in correct order: gesture, indirect verbal, direct verbal, model</td><td>Y / N</td></tr>
<tr><td colspan="3">Moves to intervention if at least 100% accurate and 85% independent.
OR
If less than 100% accurate and 85% independent teacher repeats model and guided</td><td>Y / N</td></tr>
<tr><td></td><td colspan="3">Student completes five problems independently (no prompting)</td><td>Y / N</td></tr>
</table>

APPENDIX D: TREATMENT FIDELITY: SINGLE-DIGIT ADDITION

	Before Intervention			
	Teacher has all appropriate materials			Y / N
	Teacher presents student with two choices (1 edible and 1 tangible) and asks "What do you want to work for"			Y / N
	Teacher writes what the student is working for on top of I am working for paper.			Y / N
	During Intervention			
Modeling	Teacher models one problem			Y / N
	Begins with advanced organizer (i.e., orient the students to the problems and connect to real life)			Y / N
	Models each of the following steps: (Circle the phase)			
	Virtual	Representational	Abstract	
	Write the problem (i.e., 8+4=)	Write the problem (i.e., 8+4=)	Write the problem (i.e., 8+4=)	Y / N
	Draw two circles	Draw two circles	Count on	Y / N
	Represent first addend (i.e., 8)	Represent first addend (i.e., 8)	Write answer	Y / N
	Represent second addend (i.e., 4)	Represent second addend (i.e., 4)		Y / N
	Count on	Count on		Y / N
	Write/State the answer	Write/State the answer		Y / N
Guided	Student completes one guided problem			Y / N
	If student makes mistake, indicates they do not know what to do, or 10s without response teacher initiates SLP			Y / N
	Delivers prompts in correct order: gesture, indirect verbal, direct verbal, model			Y / N
	Moves to intervention if at least 100% accurate and 85% independent. OR If less than 100% accurate and 85% independent teacher repeats model and guided			Y / N
	Student completes five problems independently (no prompting)			Y / N

Figure 2

Screenshot of Digital Tool in each Phase and Example of Task Analyses

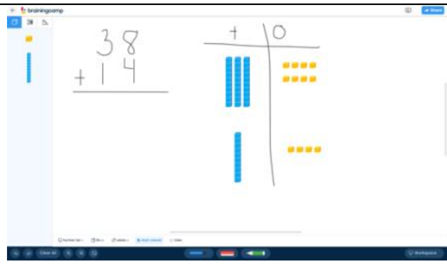
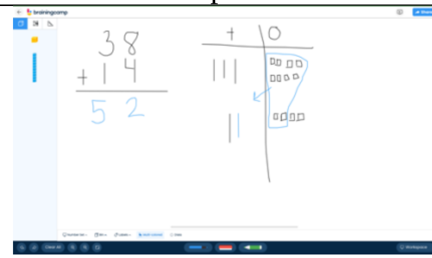
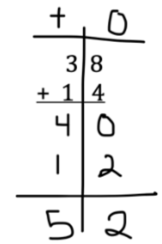
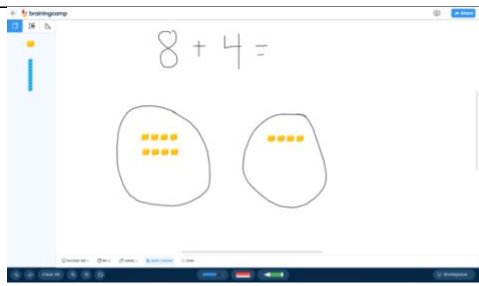
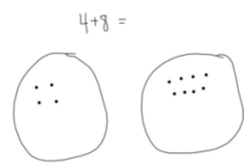
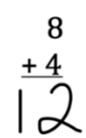
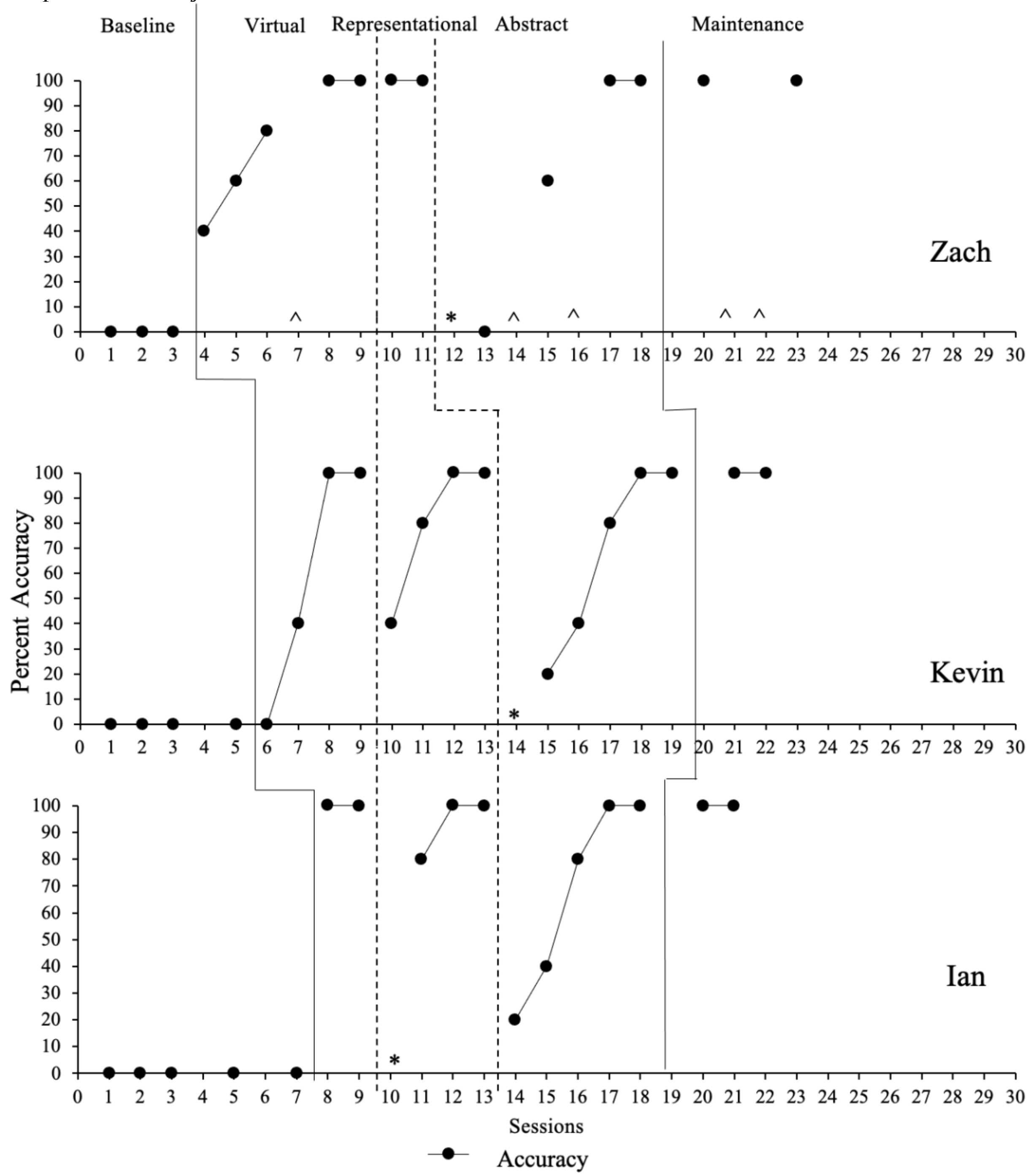
	Virtual	Representational	Abstract
Double digit Addition with Regrouping			
	Write the problem (i.e., 38+14=). Draw T Chart. Represent tens in the first addend (i.e., 3). Represent ones in the first addend (i.e., 8). Represent tens in the second addend (i.e., 1). Represent ones in the second addend (i.e., 4). Add ones. Regroup ones. Add tens. Write/State the answer.	Write the problem (i.e., 38+14=). Draw T Chart. Draw tens in the first addend (i.e., 3). Draw ones in the first addend (i.e., 8). Draw tens in the second addend (i.e., 1). Draw ones in the second addend (i.e., 4). Add ones. Regroup ones. Add tens. Write/State the answer.	Write the problem (i.e., 38+14=). Draw T Chart. Add ones. Regroup ones. Add tens. Write/State the answer.
Single Digit Addition with Regrouping			

Figure 2 (cont'd)

Single Digit Addition with Regrouping	Write the problem (i.e., 8+4=). Draw two circles. Represent first addend. Represent second addend. Highlight all ones blocks. Regroup. Write/State the answer.	Write the problem (i.e., 8+4=). Draw two circles. Draw lines or dots to represent first addend. Draw lines or dots to represent second addend. Count all lines or dots. Write/State the answer.	Hold first addend in head. Count on. Write/State the answer.

Figure 3

Graphed Accuracy Data



Note: * denotes sessions in which the participant did not pass the guided portion of explicit instruction, ^ denotes sessions in which challenging behavior prevented the session from occurring.

CHAPTER 4

COMPARISON OF A VIRTUAL MANIPULATIVE TO A FINGER COUNTING STRATEGY TO TEACH ADDITION AND SUBTRACTION TO STUDENTS WITH ASD

One of the primary goals of special education research is to identify evidence-based practices (EBPs) teachers can implement in their classrooms to effectively teach students with disabilities (Rumrill et al., 2020). For students with autism spectrum disorder (ASD) specifically, researchers have worked to identify EBPs to support academic and behavioral skills (Odom et al., 2010; Simpson, 2005; Wong et al., 2015). While researchers have identified some EBPs, they are still not being implemented nor being implemented with fidelity (i.e., as intended) by teachers in general and special education settings (Brock et al., 2020; Dynia et al., 2020; Odom et al., 2013). In other words, a research-to-practice gap exists with regards to educating students with ASD and the implementation of EBPs in the classroom (Greenwood & Abbott, 2001).

The research of interventions is more controlled environments than actual classroom environments operates as a challenge in closing the research-to-practice gap (Rumrill, 2020). While there are several evaluation methods to assess the quality of the research base before deeming instructional practices as evidence-based, many do not consider the implementation of these interventions in settings that closely resemble a realistic classroom environment (Rumrill et al., 2020). Three of the most widely used sets of quality indicators to evaluate a research base for special education single case design research are from the Council for Exceptional Children (CEC, 2014; Cook et al., 2014), Horner et al. (2005), and What Works Clearinghouse (WWC; WWC, 2020). While all three require high-quality articles to report details of the setting in which the study is conducted, none require any studies be implemented in a variety of settings, such as in small or whole group settings (Cook et al., 2014; Horner et al., 2005; WWC, 2020). This is concerning because to make accurate practice recommendations to teachers, intervention research should reflect realistic classroom settings (Rumrill, 2020; Singer et al., 2017). Specifically for mathematics interventions, there has been limited attention to small group implementation of many evidence-based interventions (Long, 2023; Long et al., 2022)

Virtual Manipulatives for Students with ASD

Within the domain of mathematics, one intervention recently emerged as an EBP for students with ASD: manipulative-based interventions (Long, 2023; Long et al., 2022). For students with ASD Long et al. (2023) found virtual manipulative-based interventions, inclusive of using virtual manipulatives (VMs) on their own or as part of a graduated sequence of instruction, an EBP. Over the past few decades, VMs emerged as an alternative mathematical tool for concrete manipulatives (CMs; Bouck & Flanagan, 2010; Moyer et al., 2002). Given the commonality of CMs in mathematics teaching and learning for students (Carbonneau et al., 2013; Maccini & Gagnon, 2000; Peltier et al., 2019), early researchers compared the efficacy of VMs to CMs including for elementary students with ASD (Bassette et al., 2019; Bassette et al., 2020; Bouck et al., 2014; Shurr et al., 2021). In a single-subject alternating treatments design study, Bouck et al. (2014) conducted a single-subject alternating treatments design study to compare the effectiveness of concrete and virtual base ten blocks. All three elementary-aged students with ASD increased their accuracy using both manipulatives. However, students were more accurate and independent with the VM. Bassette et al. (2019) also compared CMs and VMs to teach elementary students with ASD to solve subtraction problems, with a focus on efficiency (i.e., independent completion of task analysis steps per minute). Students completed more steps of the task analysis independently with the VM.

Bassette et al. (2020) used an alternating treatment design to compare the efficiency (i.e., steps completed independently per minute) and accuracy with which three students with ASD solved single-digit addition, equivalent fraction, and adding fraction problems using CMs and VMs. Two of the three students were more accurate and efficient with the VM across all three behaviors. Most recently, Shurr et al. (2021) compared CMs and VMs to teach three elementary-aged students with ASD to solve double-digit addition and word problem solving. While student accuracy increased with both manipulative types, Shurr et al. found the use of VMs resulted in a greater impact on student accuracy and independence. Across the existing research base comparing VMs and CMs for students with ASD, all participants expressed a preference for the VM, regardless of which manipulative was found to be more efficient or beneficial. In addition, the teachers expressed interest in VMs for use in their classrooms (Bassette et al., 2019; Bassette et al., 2020; Shurr et al., 2021). However, no research has directly assessed the efficacy of VM-based interventions in non-one-on-one instructional arrangements, such as small group settings (Long, 2023; Long et al., 2022).

Small Group Mathematics Instruction

In K-12 special education classrooms, small group instruction is a common practice as small group instruction allows teachers to deliver targeted intervention to multiple struggling students at once (Ozen et al., 2017; Saadatzi, et al., 2018). However, research on mathematics manipulatives for students with ASD has only been conducted during one-on-one instruction (Long, 2023). To make effective practice recommendations to teachers, researchers must explore the effectiveness of mathematics interventions, like manipulatives, in settings more closely aligned with real world classroom instruction, such as small group instruction (Singer et al., 2017). Browder et al. (2018) examined modified schema-based instruction (MSBI) with embedded pictorial task analysis, graphic organizers, systematic prompting, and feedback to teach mathematics problem solving skills to dyads of middle school students with intellectual disability. All eight participants were able to correctly follow the task analysis and independently solve the problems after the small group intervention (Browder et al., 2018). More recently, Root et al. (2022) used teacher implemented MSBI to teach small groups of fifth and sixth grade students with extensive support needs mathematical problem solving. A functional relation was determined to exist between the intervention and mathematical problem solving. All six students increased their accuracy and independence in word problem solving in a small group setting.

The Current Study

Although VMs are an EBP for students with ASD (Long, 2023), a need exists for research exploring their efficacy in realistic settings as well as with younger students. Further, all comparison studies involving VM compared the intervention to CM; no studies exist comparing VMs to non-manipulative based interventions. To examine the efficacy of VMs in small group settings for students with disabilities, researchers compared the effects of VMs to a numerical strategy (i.e., finger counting strategy) to teach young children at risk for ASD to solve single-digit addition and subtraction problems without regrouping. Researchers explored the following research questions: (a) Is the VM or finger counting strategy more effective in teaching young children at risk for ASD to solve single-digit addition and subtraction without regrouping problems; (b) Are students able to maintain their accuracy using their best treatment?; and (c) What are student and teacher perceptions of the different strategies used in the study.

Method

Participants

The current study included three 1st grade students currently receiving special education services and being evaluated for ASD during the school year in which the study occurred. The resource room teacher was asked to nominate students who: (a) received special education services for mathematics; (b) had precursor skills such as rote counting to 10, counting with 1:1 correspondence, and identify numerals; (c) struggled with addition and subtraction; and (d) had parental consent to participate. Students were excluded from the study if they: (a) could perform single-digit addition and subtraction without regrouping problems and (b) exhibited challenging behavior that would prevent them from participating in a small group setting (i.e., aggression towards other students). All three students were identified by their resource room teacher as having mathematics Individualized Education Program (IEP) goals related to single-digit addition and as students who would benefit from additional support in math. All of the participating students were educated in separate general education first grade classrooms yet received pull-out special education services from the same special education teacher. The researcher worked with all three students at the same time in a small group, consistent with their typical experiences when receiving special education mathematics services.

Oscar

Oscar was a six-year-old, male student in first grade. His family were refugees from Iran and his parents and siblings only spoke Arabic in the home. Oscar's high school brother served as a translator for the family. Oscar was receiving special education services under the eligibility of early childhood developmental delay (ECDD) but shortly after the study ended the school was going to re-evaluate Oscar for ASD eligibility. When Oscar first began school, he did not speak

English or Arabic reliably and would only answer questions in head nods, so professionals were unable to determine an IQ score. Oscar's current IEP goals involved counting to 100, single-digit addition within ten, and rote counting and writing numbers within 20. According to the KeyMath-3 assessment (Connolly, 2007), Oscar's numeration raw score was 6 (grade equivalency ≤ K.0) and mental computation and estimation was a raw score of 1 (grade equivalency ≤ K.0). For the addition and subtraction subtests, Oscar's raw score was 0 (grade equivalency ≤ K.0).

Ivan

Ivan was a six-year-old, male student in first grade. Ivan was Oscar's twin brother. Like Oscar, Ivan was receiving special education services under the eligibility of ECDD and was also going to be revaluated for ASD eligibility. Similarly, when Ivan began school, he did not speak English or Arabic reliably and would only answer questions in head nods, so professionals were unable to obtain an IQ score. Ivan's current IEP goals involved counting to 100, single-digit addition within ten, and rote counting and writing numbers within 20. According to the KeyMath-3 assessment (Connolly, 2007), Ivan's numeration raw score was 3 (grade equivalency ≤ K.0) and mental computation and estimation was a raw score of 1 (grade equivalency ≤ K.0). For the addition and subtraction subtests, Ivan's raw score was 1 (grade equivalency K.2).

Tina

Tina was a six-year-old, Black, female student in first grade. Tina was receiving special education services under the eligibility of speech and language impairment. Tina had a full-scale IQ of 35 according to the Wechsler Intelligence Scale for Children-5th edition (WISC-5th edition). Shortly after the study ended, the school was going to re-evaluate Tina to determine ASD eligibility. Tina was also diagnosed with a seizure disorder and a cleft pallet. Her file indicated that for kindergarten her parents opted for an online education option offered by the district. However, her file also indicated that she did not attend online sessions. She returned to in person school in first grade. Tina's current IEP goals involved understanding the concept of zero, counting from 1-10 independently, and single-digit addition within ten. According to the KeyMath-3 assessment (Connolly, 2007), Tina's numeration raw score was 4 (grade equivalency ≤ K.0) and mental computation and estimation was a raw score of 1 (grade equivalency ≤ K.0). For the addition and subtraction subtests, Tina's raw score was 1 (grade equivalency K.2).

Setting

All three students attended an urban elementary school in a Midwest state. The district enrolled 9,989 students across 29 schools and about 73% of the student body received free or reduced lunch. The school itself enrolled 264 students in grades PK-3 with 30.3% of the student body identified as white, 18.6% identified as Black, 28% identified as Hispanic, and 22% identified as two or more races. Over 90% of the school's student body received free or reduced-price lunch. Sessions occurred in an unused classroom on the same hallway as the special education classroom. All sessions occurred at a kidney-shaped table with students sitting around the table facing the researcher. Intervention sessions lasted no more 30 minutes per day and occurred twice per week.

Materials

Materials for the study included three iPads, Brainingcamp (2022) virtual manipulative app, probe sheets, learning sheets, and a pencil. Researchers downloaded Brainingcamp (2022)—a library of app-based virtual manipulatives—onto each iPad. The current study used the linking cubes virtual manipulative within the Brainingcamp library of apps (see Table 3 for a screenshot). The linking cubes app represents a mathematical tool similar to concrete manipulative unifix cubes. The linking cube app displayed a blank whiteboard, 10 different colored cubes represented both horizontally and vertically on the left side of the screen, a virtual pen, marker, and eraser at the bottom of the screen. Students were able to write with the virtual marker on the whiteboard background and used their finger to drag cubes from the left side of the screen onto the blank background.

During baseline and maintenance sessions, researchers used probe sheets consisting of five problems. For intervention sessions, researchers used learning sheets including two problems for modeling, two problems for guided practice, and five problems used for independent practice. The five-problem independent practice during intervention served as intervention session probes. The probe sheets and learning sheets were printed in black ink on standard white printer paper. To create probe and learning sheets, the researcher listed all possible single-digit addition without regrouping or single-digit subtraction without regrouping problems including relevant reversals (e.g., $3 + 4; 4 + 3;\ 6 - 2$), randomized them, and assigned the problems to addition-only and subtraction-only probes and learning sheets (see Appendix A for an example of a probe sheet and a learning sheet). Problems could be repeated on different probe or learning sheets but never occurred twice on the same sheet.

Independent and Dependent Variables

The dependent variable in the study was student accuracy in solving the addition or subtraction problems. Researchers calculated accuracy by dividing the number of correct answers as compared to an answer key, by the total number of opportunities to obtain a percentage (i.e., 3/5 [60%]). The independent variable was the strategy used to complete the problem during each session: finger counting or the virtual linking cubes (i.e., VM), both taught via explicit instruction. Explicit instruction involved the researcher first modeling how to solve two problems with the appropriate intervention (VM or finger counting strategy), the students solving two problems with prompts as needed with the assigned intervention, and the students solving five problems independently with the assigned intervention.

Experimental Design

Researchers evaluated the effectiveness of both strategies on student's accuracy in solving single-digit addition without regrouping and single-digit subtraction without regrouping problems using an adapted alternating treatments design (AATD). Researchers used an AATD as this design allows for the comparison of the effects of instructional practices or interventions on non-reversible behaviors like skill acquisition tasks, such as solving addition and subtraction problems (Ledford & Gast, 2018). In an AATD, researchers apply independent variables to separate behavior chains which are of comparable difficulty, independent of one another, and are not currently within the students' skillset (Ledford & Gast, 2018). Researchers randomly assigned single-digit addition without regrouping problems with a finger counting strategy, and single-digit subtraction without regrouping problems with the virtual linking cubes. After completing intervention, students entered into a best treatment phase.

Procedures

All sessions were implemented by the first author, who served as the interventionist. The interventionist was a doctoral candidate in special education with three years of experience implementing mathematics interventions with students with disabilities. A second data collector was present in sessions in which inter-observer agreement (IOA) data were collected. The second data collector was a special education doctoral student trained by the first author to compare student answers to the answer key to determine the percent accuracy for each session. All sessions were conducted in person and all data collected in real time.

Baseline

In baseline sessions, the researcher presented each student in the small group with the same probe sheet consisting of five single-digit subtraction without regrouping or single-digit addition without regrouping problems, depending on the session, and instructed the students to complete the problems the best that they could. The researcher ensured students had access to the Brainingcamp VM during baseline subtraction sessions by sitting the iPad next to the student with the app open on the screen. However, the student received no prior instruction on how to use the app or how to solve the problems before beginning baseline probes. The students did not have access to the VM during addition sessions in baseline, nor was instruction provided for finger counting. Researchers collected accuracy data during all baseline sessions. If students failed to initiate solving the problems within five minutes or if they indicated they did not know how to complete the problems, the session was terminated, and accuracy was scored as zero. All three students complete six baseline sessions: three addition and three subtraction sessions.

Intervention

The researchers taught the students to solve single-digit subtraction without regrouping problems with a linking cubes virtual manipulative, and single-digit addition without regrouping problems with a finger counting strategy. Researchers randomized sessions assigning a strategy-behavior pair to each session, with each strategy-behavior pair not occurring more than twice consecutively. After completing at least five sessions with each strategy-behavior pair, completing an equal number of strategy-behavior pairs, and all students completing at least two sessions with 100% accuracy for one of the two strategy-behavior pairs, the intervention phase ended, and students entered the best treatment phase. In the current study, students completed 14 intervention sessions total, with seven sessions per strategy-behavior pair.

Virtual manipulatives and single-digit subtraction without regrouping. During the VM sessions, the researcher provided each student with an iPad with the virtual linking cubes app open and guided access enabled so students were unable to leave the linking cubes app without a password. During these sessions, researchers also provided each student with learning sheets including two problems for modeling on the first page, two problems for guided practice on the second page, and five problems for independent practice (i.e., the probe) on the third page. The researcher began with an advanced organizer, which oriented the students to the problems they would be solving and connected the mathematics to real life scenarios. The researcher explained the students would be learning single-digit subtraction without regrouping, and connected subtraction to real-life scenarios, such as using subtraction when buying things at a store to make sure you have enough money or when you are cooking or baking. Then the researcher continued by explaining that they were going to solve a subtraction problem using linking cubes (e.g., $6 - 2 = $ ___; see Table 3). The researcher explained subtraction means they are taking away or finding the difference. To set up the problem, the researcher modeled setting up the minuend (e.g., 6) by dragging six cubes onto the screen. Then, the researcher stated they were ready to subtract. The researcher showed the students how to pick up the pen or marker from the bottom of the screen and explained to take away two cubes they were going to cross them out with the pen. The researcher modeled crossing out two blocks, counting aloud as they crossed out the blocks. Finally, the researcher explained the number of blocks left, or not crossed out, was the difference, which was the answer to the problem. The researcher counted each remaining block and wrote the answer (i.e., 4) on the learning sheet. After modeling, students participated in guided practice, where each solved two problems. If the researcher noticed a student was not completing the next step, engaged in an incorrect step, or told the researcher they did not know how to solve the problem, the researcher delivered prompts and cues to the individual student who required the prompting. Students then completed the five-problem subtraction probe sheet independently without feedback or prompts from the researcher.

Finger counting and single-digit subtraction without regrouping. During the finger counting sessions, the researcher provided each student with learning sheets including two single-digit addition without regrouping problems for modeling on the first page, two problems for guided practice on the second page, and five problems for independent practice (i.e., the probe) on the third page. The researcher began with an advanced organizer, which oriented the students to the problems they would be solving and connected the mathematics to real life scenarios. The researcher stated they would be solving addition problems and addition is important in many areas of their life including when they are trying to determine how many of something they need, determining how much money they may need to buy items at a store, or when cooking or building. The researcher continued by telling the students they would be using a finger counting strategy (e.g., $3 + 4 =$; see Table 3). After reading the problem, the researcher explained they were going to represent the two single-digit numbers with their fingers and count them. To start adding, the researcher modeled representing the first addend (i.e., 3) with three fingers. The researcher then explained they needed to add the second addend by adding the additional number of fingers (i.e., 4). Finally, the researcher stated they would count all of the fingers to get the answer (e.g., 7). After modeling, students participated in guided practice where each solved two problems. If the researcher noticed a student was not completing the next step, was engaging in an incorrect step, or told the researcher they did not know how to solve the problem, the researcher delivered prompts and cues to the individual student who required the prompting. Students then completed the five-problem addition probe sheet independently without feedback or prompts from the researcher.

Best Treatment

Following intervention, students entered the best treatment phase. The best treatment phase consisted of three sessions that measured students accuracy maintenance using their best treatment with its paired mathematical behavior when no instruction was provided. To determine which intervention was the students best treatment, researchers used Percent of Nonoverlapping Data (PND) to compare accuracy in intervention to baseline accuracy. To calculate PND, researchers first determined the range of data point values for baseline for both VMs and finger counting. Next, researchers counted the number of data points that fell outside of that range, divided the number of intervention data points that fell outside of that range by the total number of intervention datapoints, and multiplied by 100 to get a percentage (Ledford & Gast, 2018). If the PND was the same for both finger counting and VMs, researchers compared the average accuracy to determine which intervention was more effective. Oscar's PND for finger counting and VMs were 100%, his average accuracy across the seven sessions for finger counting was 48.6% and for VMs was 88.6%. Ivan's PND for finger counting was 85.7% and VMs was 100%. Tina's PND for finger counting and VMs was 85.7%, her average accuracy across the seven sessions for finger counting was 40.0% and for VMs was 80.0%. Based on the PND and average accuracy, all three students used virtual linking cubes for their best treatment sessions.

Inter-Observer Agreement and Procedural Fidelity

IOA data were collected on 33.3% of baseline sessions, 42.9% of intervention sessions, and 33.3% of best treatment sessions. During sessions in which IOA data were collected, a secondary data collector independently scored each problem for accuracy according to an answer key and calculated the total percent accuracy and compared this to the researchers initial calculation. After the session, researchers compared their data for agreement. The researchers calculated IOA using the following equation $\frac{\#\ of\ agreements}{\#\ of\ agreements + \#\ of\ disagreements} \times 100$ (Ledford & Gast, 2018). IOA was calculated as 100% for accuracy for all three students for all phases.

The researcher also collected procedural fidelity data during each session. The checklist included (a) modeled how to solve two problems with think aloud; (b) students completed two guided practice problems and the researcher delivered prompts and cues as needed; (c) students completed five problems independently; (d) students had access to all appropriate materials during each condition (i.e., virtual during subtraction sessions, nothing during addition sessions); (e) the researcher did not provide prompts during independent practice; and (f) at least two students were present for each session. Procedural fidelity was calculated at 100%.

Social Validity

Following the best treatment phase, the researcher asked each student a series of social validity questions in the small group setting. The researcher asked students the following questions: (a) do you think it is important for you to learn math problems like these, why or why not?; (b) do you think learning these different strategies helped you to learn to solve the problems, why or why not?; (c) which did you like best to help you solve the problems, the VMs, or finger counting strategies, and why?; (d) what did you think about learning with your peers in a group?; (e) is there anything else you would like to tell me? The researchers also conducted a brief interview with the teacher and asked the following questions: (a) do you think the strategies (VMs and finger counting) helped your students to learn the content?; (b) what are your perceptions of small group instruction versus one-on-one mathematics instruction?; (c) do you think this intervention is something you could implement with your students?; and (d) is there anything else you would like to share?

Data Analysis

Researchers conducted visual analysis as well as a series of calculations consistent with single case design research to analyze data and interpret the results of the study (Ledford & Gast, 2018). First, researchers graphed accuracy data and conducted a visual analysis of the graphed data to determine immediacy of effect across the three students. Second, researchers used the split-middle method to calculate trend. To accomplish this, researchers found the middle point of the data, located the mid-rate, and mid-date for each phase, the interventionist then drew a line and determined if the line was zero-celerating (straight slope), accelerating (increasing slope), or decelerating (decreasing slope; White & Haring, 1980). Then, researchers used the 80-25 rule to calculate level across each phase by calculating if 80% of the data fell within 25% of the mean in each phase (Ledford & Gast, 2018). The researchers determined the Tau-U between baseline and intervention sessions using an online calculator (http://singlecaseresearch.org/calculators/tau-u). Tau-U effect sizes over .80 were considered very large effect, any effect sizes between 0.60 and 0.80 were considered a large effect, and effect sizes between 0.20 and 0.60 were considered moderate effect (Vannest et al., 2016).

Results

All three students increased their accuracy in solving single-digit addition and subtraction without regrouping problems using both VMs and a finger counting strategy (see Figure 4). However, after comparing PND and average accuracy, all three students were more accurate with the VM than the finger counting strategy. Researchers also established a complete separation in the data between VMs and the finger counting strategy after a four-week break involving student absences due to illness and a two-week winter break. Researchers were able to establish a functional relation between the VM and the accuracy with which students solve single digit subtraction without regrouping problems.

Oscar

Oscar answered all three baseline single-digit addition without grouping sessions with 0% accuracy, resulting in a zero-celerating trend and stable data. During the finger counting sessions paired with single-digit addition without regrouping, he was 60%, 20%, 80%, and 60% accurate for his first four sessions, respectively. After the four-week break Oscar was 20% accurate on session five, 60% accurate on session six, and 40% accurate on session seven. Oscar’s intervention data for finger counting were variable and decelerating. The Tau-U between baseline and intervention for finger counting was 1.0, indicative of a very large effect.

Oscar also answered with 0% accuracy for all three baseline single-digit subtraction without regrouping, resulting in a zero-celerating trend and stable data (see Figure 4). For the virtual linking cubes intervention paired with single-digit subtraction without regrouping, he was 100% accurate for his first two sessions, followed by 40% accuracy in his third session. A four-week break occurred after session three and Oscar was 80% accurate on session four after his return. For sessions five, six, and seven, Oscar was 100% accurate. Oscar’s intervention data for virtual linking cubes were stable and accelerating. The Tau-U between baseline and intervention for virtual linking cubes was 1.0, indicative of a very large effect. During best treatment involving the VMs and single-digit subtraction without regrouping, Oscar maintained his accuracy at 100% for all three sessions. The best treatment data were stable and zero-celerating.

Ivan

Ivan was 20% accurate on his first baseline single-digit addition without regrouping session, and 0% accurate for baseline sessions two and three, resulting in a decelerating trend and stable data. During the first four finger counting sessions paired with single-digit addition without regrouping, he was 80%, 20%, 80%, and 60% accurate, respectively. After the four-week break, Ivan was 40% accurate on session five and 60% accurate on his final two sessions. Ivan's intervention data for finger counting were stable and zero-celerating. The Tau-U between baseline and intervention for finger counting was 0.95, indicative of a very large effect.

During baseline, Ivan answered with 0% accuracy for three baseline single-digit subtraction without regrouping sessions resulting in a zero-celerating trend and stable data. For the virtual linking cubes paired with single-digit subtraction without regrouping condition, he was 100% accurate on first session, followed by 60% and 80% accurate on his second and third sessions. There was a four-week break after subtraction session three and Ivan was 80% accurate on session four after his return. For sessions five, six, and seven, Ivan was 100% accurate. Ivan's intervention data for virtual linking cubes were stable and accelerating. The Tau-U between baseline and intervention for virtual linking cubes was 1.0, indicative of a very large effect. During best treatment involving VMs and subtraction without regrouping, Ivan maintained his accuracy at 100% for all three sessions. The best treatment data were stable and zero-celerating.

Tina

Tina answered all three baseline single-digit addition without grouping sessions with 0% accuracy, resulting in a zero-celerating trend and stable data. During the finger counting sessions paired with single-digit addition without regrouping, Tina was 0% accurate for her first two sessions followed by 80% and 60% accurate for sessions two and three, respectively. After the four-week winter break, Tina was 40% accurate on session five, 60% accurate on session six, and 40% accurate on session seven. Tina's intervention data for finger counting were variable and accelerating. The Tau-U between baseline and intervention for finger counting was 0.71, indicative of a large effect.

During baseline, Tina also answered with 0% accuracy for all three baseline single-digit subtraction without regrouping, resulting in a zero-celerating trend and stable data. For the virtual linking cubes (paired with single-digit subtraction without regrouping), she was 100% accurate for her first session, followed by 80% accuracy in her second session and 0% accurate in her third session. A four-week break occurred after VM session three, and Tina was 80% accurate on session four after her return. For sessions five, six, and seven, Tina was 100% accurate. Tina's intervention data for virtual linking cubes were stable and accelerating. The Tau-U between baseline and intervention for virtual linking cubes was 0.86, indicative of a very large effect. During best treatment, Tina maintained her accuracy at 100% for the first and third session but made a counting error during best treatment session two and completed that session with 80% accuracy. The best treatment data were stable and zero-celerating.

Social Validity

During the social validity interviews, all three students told the researcher they thought these math problems were important to learn and the strategies were helpful in learning to solve the problems during the study. When students were asked to choose which of the strategies was their favorite, Oscar and Tina both indicated they liked using the VM, while Ivan indicated his preference for the finger counting strategy. Oscar and Ivan both told the researcher they enjoyed working in a group setting and would like to learn in groups more. However, Tina expressed that she did not like working with the other two students and would like to work with the researcher by herself. When asked to explain their answers, none of the students elaborated on their reasoning for their answers. When conducting the social validity interview with the teacher, she expressed that the strategies helped her students to solve the problems and was interested in learning more about the strategies to use with her other students. She was happy the students were able to make progress in the small groups as she has to use a lot of small group instruction in her classroom. She also thought this would be something she could implement in her classroom if she had some additional time to explore different VM options.

Discussion

For researchers to make effective practice recommendations and close the research-to-practice gap, it is important to evaluate EBPs in realistic classroom settings (Beahm & Cook, 2021). Teachers have reported they are interested in implementing VMs in their classrooms (Bassette et al., 2019; Shurr et al., 2021), however, the majority of research exploring VMs for students with ASD is conducted in one-on-one settings (Long, 2023; Long et al., 2022). The current study compared the accuracy of first grade students with disabilities using VMs and a finger counting strategy to solve single-digit addition and subtraction without regrouping problems in a small group setting. There were two main results of this study. First, although the young students were able to increase their accuracy to higher than baseline levels with both the VM and finger counting strategy, they were more accurate with the VM. Second, the students were successful with both interventions implemented in a small group setting.

Virtual Manipulatives

Student accuracy increased when using both finger counting and VMs in solving single-digit addition and subtraction problems without regrouping, respectively, as compared to baseline levels. However, researchers identified a clear separation in the data, with students more accurate with the VMs. While the results are consistent with previous research suggesting VMs to be more effective than another intervention, all previous research compares VMs to CMs (Bouck et al., 2014; Bassette et al., 2019; Shurr et al., 2021). Yet, the prior research comparing VMs to CMs found VMs only marginally more effective than CMs and failed to capture consistent separation in the data (Bouck et al., 2014; Bassette et al., 2019; Shurr et al., 2021).

Based on this research and prior research suggesting VMs to be more effective with regards to independence than CMs, there may be something inherent in VMs that assist in supporting students with or at-risk for ASD mathematically. One hypotheses involves the draw to technology researchers suggest exists for students with ASD (Wong et al., 2015). Another hypothesis is the embedded scaffolds or features present in VMs but not CMs or numerical strategies provide students additional support (Bouck et al., 2020). For example, in this study, the finger counting strategy required students to represent the accurate addends with their fingers and continue holding their fingers accurately until the students could count. The researcher observed several times in which the students would change the number of fingers after representing the problem. When using the VM, once students represented the minuend with the virtual linking cubes, the cubes were locked into the screen. The embedded features of the VM in this study removed the fine motor demand (i.e., holding fingers accurately) as well as potential for error making, and resulted in higher accuracy.

Small Group Instruction

Although researchers determined VM-based interventions an EBP for students with ASD, existing research only explored VM-based interventions delivered via one-on-one instructional arrangements (Long, 2023; Long et al., 2022). In the current study, young students at-risk for ASD were successful when VMs were implemented in a small group setting. The finger counting strategy was also successfully implemented in a small group setting, although students were less accurate. While research involving controlled environments with one-on-one researcher implemented interventions provide important insights into the effectiveness of mathematics interventions (Beahm & Cook, 2021; Rumrill et al., 2020), it is critical to confirm the efficacy of these interventions in settings more closely aligned to a classroom (Beahm & Cook, 2021; Brock et al., 2020; Rumrill, et al., 2020). For students with ASD specifically, who often experience social and academic struggles, small group instruction allows for targeted academic instruction with additional opportunities for social interaction and incidental learning (Ledford & Wehby, 2015). By examining mathematics interventions researchers previously found effective in more realistic settings, researchers can make more informed practice recommendations (Gersten et al., 2017). This approach not only ensures the efficacy of interventions in realistic environments but also helps to close the research to practice gap by providing teachers with accurate practice recommendations. (Behmn & Cook, 2021; Rumrill et al., 2020).

Implications for Practice

Researchers identified several important implications for practice as a result of the current study. First, teachers should feel confident implementing VM interventions in their classrooms. The results of this study—VMs were a more effective intervention for young students with ASD than a numerical strategy—combined with validation of VMs and VM-based instructional sequences as an EBP for students with ASD (Long et al., 2023) support teachers using VMs with students with ASD. Second, while the finger counting strategy was not as effective as the VMs, teaching students a variety of strategies so they can independently make choices about the strategies they prefer to use is a best practice (Flores et al., 2014; Whitby et al., 2009). Students still increased their accuracy with single-digit addition without regrouping using the finger counting strategy over baseline levels. Thus, teachers should not feel as though they have to prevent students from using a finger counting strategy. The finger counting strategy is a viable strategy students can use to solve problems without additional tools or supports, such as manipulatives or an iPad. This study also supports the delivery of VMs in a small group setting. Students and teachers reported positive feedback regarding learning and teaching mathematics in small group settings, and students were successful in increasing their accuracy. Teachers should feel confident implementing virtual manipulative based interventions in small group settings with students who have similar mathematical needs, including early elementary. Previous researchers suggested benefits of peer-to-peer interaction and incidental teaching opportunities as a result of small group instruction (Winstead et al., 2019).

Limitations and Future Directions

One limitation of the study was the special education eligibility of the three students. At the time of the study, the teacher was preparing for upcoming re-evaluations to determine if the students would continue to receive special education services under the category of ASD. While the students were all struggling in math and had goals related to single-digit addition and subtraction within ten, their special education eligibility may shift. Future researchers should seek to replicate the current study with students who have an identification of ASD to increase the generalizability of the results. Additionally, the researchers did not apply the students' best treatment condition to the other behavior (i.e., the VM intervention to single-digit addition without regrouping). However, authors of other AATDs exploring manipulative-based interventions for students with ASD did not apply the best treatment to the other behavior being explored in the study (Bassette et al., 2019; Shurr et al., 2021). Future researchers should consider applying the best treatment to the other behavior to see if the student can acquire and maintain the skill that was not acquired to 100% accuracy.

Another limitation of the study involved all intervention sessions implemented by a researcher and occurring in a small empty classroom down the hall from the students general education classroom at the request of the resource room teacher. Future researchers should seek to implement interventions in the classroom to explore how other classroom factors could impact the results of the study. Future researchers should also consider training the teacher as the interventionist to explore the feasibility and effects of the intervention when implemented by a classroom teacher. Finally, because of scheduling, researchers were unable to work with the students for four weeks during the middle of the study. The four weeks included two weeks where students were absent due to illness, followed by two weeks of winter break. While this did not seem to impact the outcomes in this study and long breaks in winter and summer are common in K-12 education, future researchers should seek to implement interventions without large breaks in the data collection.

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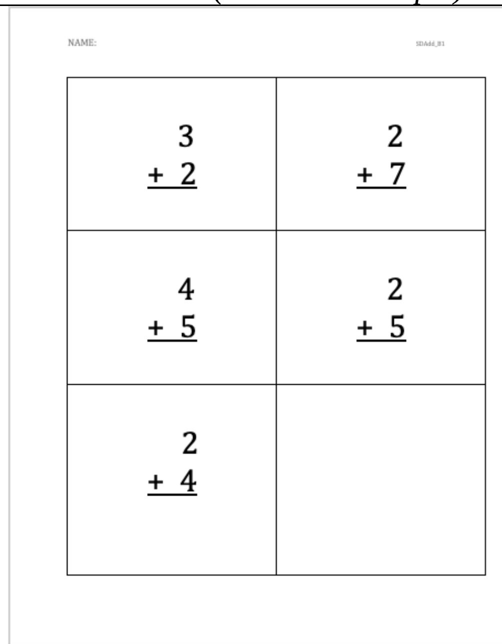
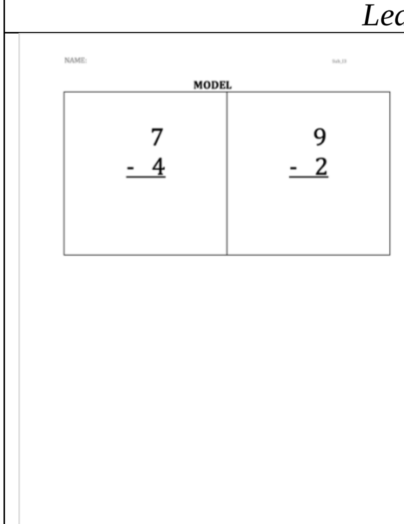
APPENDIX E: EXAMPLE PROBE AND LEARNING SHEETS

Probe Sheet (Addition Example)


NAME:

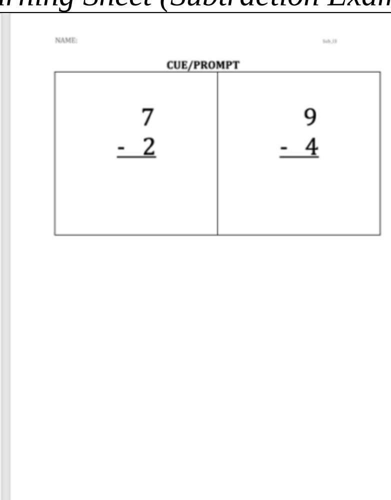
3 + 2	2 + 7
4 + 5	2 + 5
2 + 4	

Learning Sheet (Subtraction Example)


NAME:

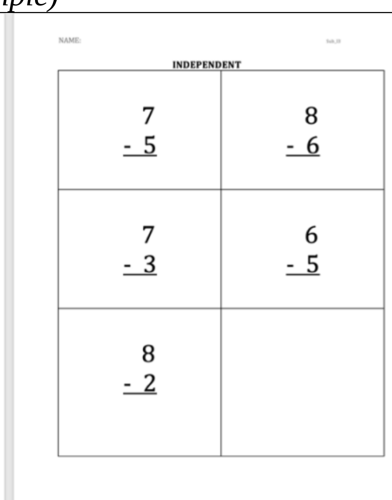
MODEL

7 - 4	9 - 2


NAME:

CUE/PROMPT

7 - 2	9 - 4


NAME:

INDEPENDENT

7 - 5	8 - 6
7 - 3	6 - 5
8 - 2	

Table 3

Screenshot of Example Problem and Strategies

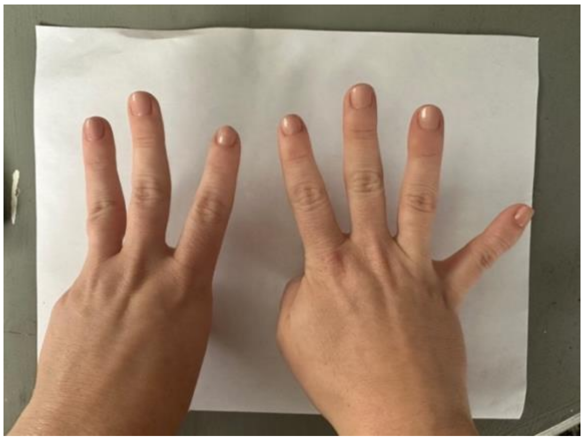
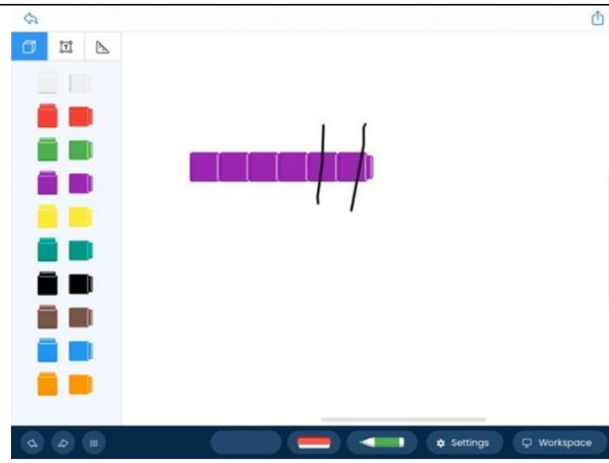
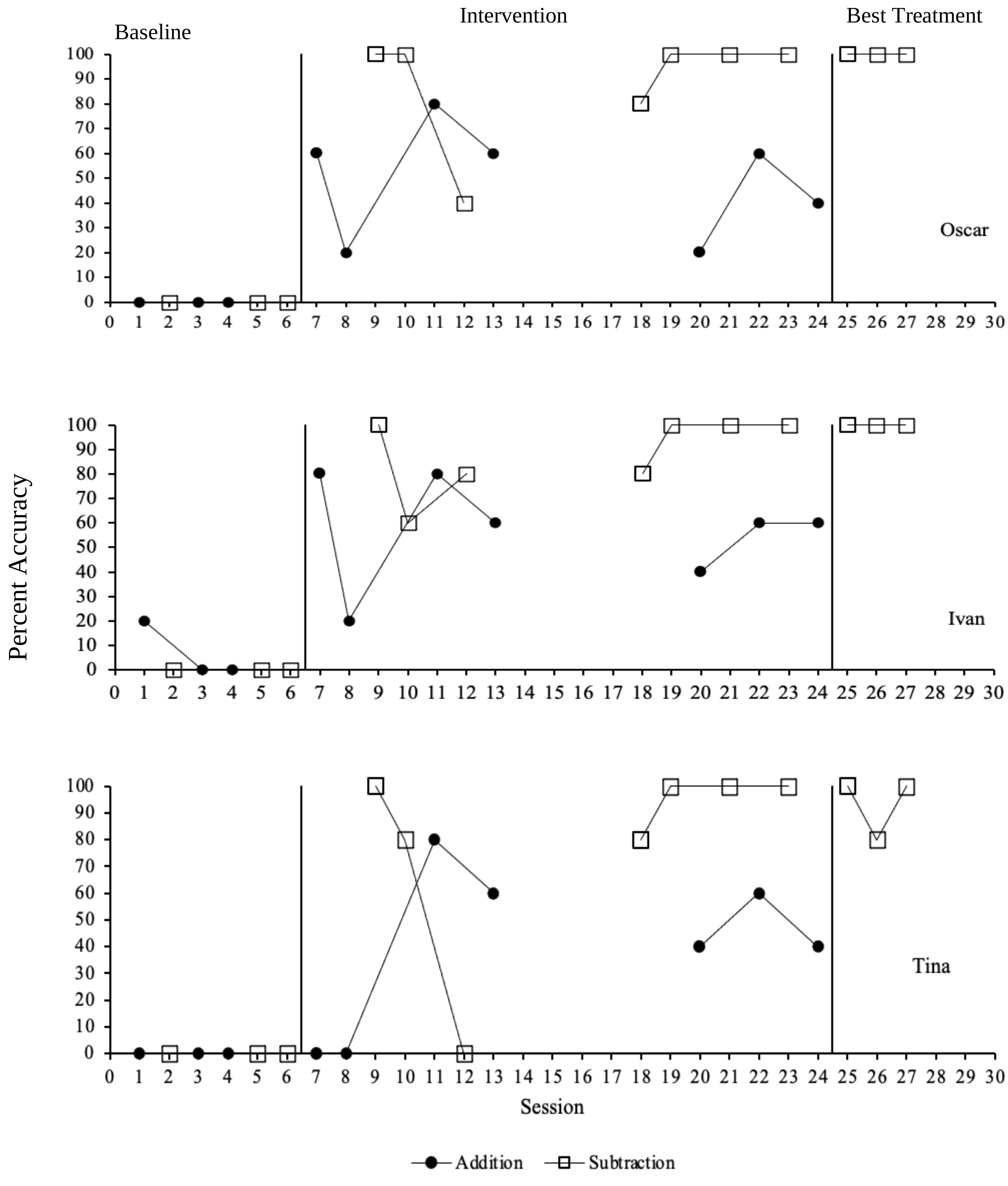
Addition	Subtraction
Finger Counting	Virtual Manipulatives
$3 + 4 =$	$6 - 2 =$
	

Figure 4
Graphed Student Accuracy Data



Note: Sessions 14-17 were intentionally left without data to depict the four-week break involving winter break, snow days, and student illness.

CHAPTER 5

OVERALL DISCUSSION

This dissertation presents three studies exploring mathematics interventions for students with autism spectrum disorder (ASD): a review of the literature on mathematics interventions for students with ASD and two intervention studies using virtual manipulative based interventions to support elementary students with ASD. One of the studies examined a teacher implemented virtual-manipulative-based intervention (i.e., VRA) to teach addition with regrouping to three elementary students with ASD and the other compared virtual manipulatives to an abstract strategy implemented in a small group setting with three elementary students at-risk for ASD. The results of the systematic review indicated a need for more research in realistic classroom settings (i.e., teacher implemented, small group instruction) and confirmed virtual manipulative-based interventions as an evidence-based practice (EBP) for students with ASD. In the study exploring the effectiveness of the VRA instructional sequence implemented by the classroom teacher, the researcher found a functional relation between the intervention and student accuracy in solving single- or double-digit addition with regrouping problems. The teacher implemented the intervention with over 95% procedural fidelity across phases and students. The researcher also found a functional relation between the virtual manipulative and student accuracy in the study comparing the efficacy of a virtual manipulative and a finger counting strategy to teach single digit addition and subtraction without regrouping via small group instruction.

Two main results derived from the overall dissertation. First, virtual manipulative-based interventions are an evidence-based practice for students with ASD in mathematics. Second, students were successful learning target mathematics skills taught via virtual manipulative-based interventions in realistic contexts, such as teacher implementation or small group settings.

Mathematics Evidence-based Practices for Students with ASD

The results of the systematic review found two interventions had sufficient number of studies to be evaluated as evidence-based practices (EBPs): virtual manipulative-based interventions and modified schema-based instruction (MSBI). Applying the Cook et al. (2014) quality indicators (QIs), an EBP determination requires at least five studies with 20 participants across the studies with no negative results. Based on the current research base, virtual manipulative-based interventions are an EBP for students with ASD. The current literature for students with ASD and virtual manipulatives consists of 10 studies and 20 students with ASD. However, the researcher was unable to conclude MSBI as an EBP as there were only 19 students across the seven studies. With one more participant with ASD with positive results, MSBI could have been classified as an EBP to support students with ASD in mathematics.

Although the researcher concluded virtual manipulative-based interventions as an EBP for students with ASD in mathematics, this determination was for the heterogenous grouping of virtual manipulative based interventions. For the purposes of the systematic review, virtual manipulative based interventions included virtual manipulatives as stand-alone interventions and virtual manipulative based instructional sequences [e.g., VRA, VR, VA, VRA-I]). Alone, insufficient number of studies existed to evaluate each as an EBP. Despite the need for a more nuanced exploration of the effects of manipulative-based interventions, this aggregation was consistent with previous evidence-based practice determinations (Long et al., 2022; Spooner et al., 2019). Each of the manipulative-based instructional sequences offer unique advantages and disadvantages, such as removal of the drawing phase (e.g., VA instructional sequence) for students who struggle to draw or for when the mathematics skill does not lend itself to drawing (i.e., fractions; Bouck et al., 2017). More research is needed across each variation to be able to determine the individual effectiveness of each, and, with a greater research base, researchers could examine as EBPs as well as the contributions of each instructional sequence to student understanding and outcomes. With more nuanced research in this area, researchers can make more accurate practice recommendations based on individual virtual manipulative based interventions as opposed to general recommendations.

Application to Realistic Classroom Settings

Despite the continued confirmation that virtual manipulative-based interventions are effective for students with disabilities in general and students with ASD in particular, studies are still predominantly researcher-implemented in one-on-one settings (Long et al., 2022; Long, 2023). The field lacks the nuance to claim these interventions are effective in realistic classroom contexts. The results of this dissertation, specifically the teacher implemented and small group studies, show virtual manipulative based interventions can be effective for students with ASD in realistic classroom settings. Prior to this dissertation no virtual manipulative intervention research involved the teacher as the implementor. The current study used similar teacher training protocols as Browder et al. (2018) and Root et all (2022), who both explored teacher implementation of MSBI for students with students with extensive support needs. The current study found similar results with regards to teacher implementation fidelity (i.e., over 95%) and student accuracy. All three students who participated in the current study were successful at acquiring and maintaining their accuracy in 11-14 sessions with the teacher implemented intervention. The results of this and previous studies (Browder et al., 2018; Root et al., 2022) confirm teachers can implement effective interventions for students with ASD, and the students can acquire the skills in a relatively similar number of sessions as researcher implemented interventions.

In terms of small group instruction, young students at-risk for ASD were also successful when virtual manipulatives were implemented in a small group setting. Small group instruction offers targeted academic instruction as well as opportunities for social interaction and incidental learning, which is particularly beneficial for students with ASD who often struggle socially and academically (Ledford & Wheby, 2015). This approach has the potential to positively impact the academic and social outcomes of students with ASD and improve their social and academic outcomes (Behmn & Cook, 2021; Rumrill et al., 2020). By applying evidence-based mathematics interventions to small group settings for students at-risk for ASD, educators can promote skill acquisition and generalization of mathematical concepts beyond one-on-one instruction (Gersten et al., 2017).

While we know evidence-based practices are essential for improving mathematics outcomes for students with disabilities, there are currently no quality indicators that require the application to realistic classroom settings—including teacher implementation or small group implementation (Cook et al., 2014; Horner et al., 2005, WWC, 2022). This is problematic as researchers can classify a practice as evidence-based before it is known if the intervention can be implemented successfully in settings that resemble a realistic classroom, which is the ultimate goal.

Implications for Practice

This dissertation offers several implications for practice. First, as a result of the repeated confirmation of the efficacy of manipulative-based interventions, teachers should feel confident in using virtual manipulative-based interventions with their students with ASD. Second, even though MSBI was not determined to be an EBP yet, Clausen et al. (2021) suggest there are currently no quality alternatives to teaching problem solving to students with ASD. Despite the lack of EBP identification in this and other reviews (i.e., Clausen et al., 2021), teachers should feel confident that MSBI is an effective (i.e., research-supported) intervention to implement to support word problem solving for students with ASD and implement it within the practice. Due to the continued support of virtual manipulative based interventions and MSBI, teacher preparation programs and professional development programs should consider training pre- and in-service teachers to implement these interventions in their classrooms.

The two single case design studies in this dissertation confirmed the use of virtual manipulative based interventions taught via explicit instruction in realistic classroom settings such as when implemented by the teacher and when implemented in a small group. Provided teachers receive the proper training to ensure the interventions are implemented as intended, teachers should feel confident implementing virtual manipulative interventions with students in a small group setting. Teachers should seek out opportunities for professional development to support them in implementing virtual manipulative based interventions as well as explicit instruction in their classrooms. Teacher preparation programs should provide instruction on virtual manipulatives as an EBP and how pre-services can implement these interventions in a variety of settings for students with a variety of needs.

Finally, given students with ASD have a variety of needs, incorporating additional instructional practices to support the main intervention presents a viable option to supporting students with ASD (Hume et al., 2021; Odom et al., 2021). Intervention packages appear to be increasing in mathematics research for students with ASD, as the majority (86.36%) of high-quality studies in the systematic review included intervention packages consisting of a main intervention (e.g., virtual manipulatives) and another instructional practice (e.g., explicit instruction, SLP). As such, teachers should assess their students skills and develop intervention packages using EBPs to support their unique needs.

Limitations and Future Directions

While this dissertation offers several important results and implications for practice, it is not without limitations. One limitation involved the lack of generalization phase in either of the single case studies. Generalization is important and should always be a part of programming for students with ASD (Shurr et al., 2019). For the teacher-implementation study, the teacher trained to implement the intervention was responsible for the majority of students' instruction on a daily basis. Future researchers could explore generalization to other teachers or settings, such as the students general education teacher or in the general education setting. In the small group study, the intervention was implemented by the researcher in a contrived environment (i.e., empty classroom). Future researchers should seek to assess generalization inside of a classroom as well as train the teacher to implement to a small group of students.

Additionally, this dissertation aggregates virtual manipulative interventions into a group, when in fact, there are many variations of virtual manipulative based interventions—even as evident in the two explored in the dissertation single case design studies. Although combining virtual manipulative based interventions into one group is consistent with previous reviews (e.g., Long et al., 2022; Spooner et al., 2019), it fails to provide a nuanced understanding of what particular aspects of virtual manipulative-based interventions are evidence-based (i.e., virtual manipulatives as part of a graduated sequence of instruction as opposed to stand alone tools as well as if particular graduated sequences are more effective than others). As more studies are published exploring virtual manipulative-based interventions, future researchers should attempt to get a more refined understanding of the effects of each type of virtual manipulative-based intervention. Finally, the researcher was not fully aware of the extent of students' challenging behavior prior to beginning the interventions. While challenging behavior only interrupted intervention sessions for one of the six students (i.e., Zach), the researcher acknowledges the importance of supporting academic interventions with additional behavioral components. Future researchers should attempt to incorporate additional behavioral interventions to support students who exhibit challenging behavior to continue accessing the academic content.

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