A NARRATIVE INQUIRY OF POST-SECONDARY MULTILINGUAL INTERNATIONAL STUDENTS' LIVED EXPERIENCES RELATED TO MATHEMATICS

By

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ABSTRACT

The number of multilingual international students in post-secondary mathematics classrooms in the United States is increasing. As language is related to students' access to mathematics learning and their (mathematics) identities, it shapes their experiences as a doer of mathematics. However, as multilingual international students move to the United States, they routinely experience a change in the language of instruction, which influences their position as mathematics doers. Yet very little is known about their experiences in their current postsecondary mathematics classrooms and who they are. The purpose of this study is to understand their lived experiences that influenced their past, current, and future relationship to mathematics by using narrative inquiry. Through this study, I call for more attention to multilingual international students and further research about how to serve this growing population in postsecondary mathematics education.

In this narrative inquiry, I worked with four post-secondary multilingual international mathematics students, Jia, Maria, Helena, and Jihyun (all pseudonyms), to study their mathematics identities and see how they leverage their languages in different contexts. Each of the participants' essays about their relationship to mathematics was collected first, and then there were an hour to 1.5-hour synchronous interviews with each of the participants. Asynchronous interviews were then conducted in a written form based on their essay and synchronous interviews. All the verbal data were transcribed and analyzed to create a narrative for each participant using narrative analysis.

All four participants came from different countries. Jia shared her challenges in undergraduate mathematics courses, especially in a groupwork-based course. She also shared how she used her first language and English in different contexts. Although Maria shared some thoughts about being multilingual, she focused more on her experiences of being a woman in mathematics and the social challenges as an international student. Helena shared how her first language, Spanish, is intricate to how she sees her world. Her narrative raises questions about the perceived hierarchies of languages in mathematics. Lastly, Jihyun shared cultural and linguistic differences that she noticed as she moved to the United States. Her case shows how different stereotypes about gender and race intersectionally shape her actions.

Across the four participants' experiences, three main themes were found: 1) the participant's understanding of the roles of language in mathematics and how they use their languages in doing mathematics; 2) some participants' understanding of how race and nationality shape their actions, and 3) the participants' marginalized experiences because of their gender and their reaction to such a stereotype. Further, how intersections of these multiple social identities can influence participants' actions was explained using the case of Jihyun, a Korean woman.

This work contributes to the documentation of post-secondary multilingual international students' experiences in the United States and their identities from their past experiences, which were rarely focused on in existing literature on post-secondary mathematics education. Even though the mathematics education field as a research area has tried to escape the deficit perspective toward multilingual students, some participants were still attributing their challenges to some internal deficit they perceived. Changing of the narrative in the education field and providing a better learning environment for them where their languages are more valued would be necessary.

Copyright by JIHYE HWANG 2023 I dedicate this dissertation to all multilingual students who have valuable stories to be heard.

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CHAPTER 1: INTRODUCTION

As the world is globalizing, the number of international students in collegiate classrooms around the world, especially in the United States, has likewise been increasing. More than one million international students were enrolled in U.S. universities in the 2017-2018 academic year, and about half of them were studying STEM (Granovskiy & Wilson, 2019). In other words, more than 20% of STEM bachelor's degrees in the United States were awarded to international students (Granovskiy & Wilson, 2019). Similarly, more than 20% of Bachelor's degrees in mathematics and statistics were awarded to "non-resident aliens," which are international students (Data USA, n.d.). Therefore, the number of international students is substantial within the United States undergraduate mathematics education. Although statistical data is not the only reason that the mathematics education field needs to study international students, it adds one rationale for the importance of studying their learning and experiences around learning.

I believe that multilingual international students' experiences in proof-based mathematics classrooms should be investigated. The following is my main reasoning for the claim: 1) language is a part of one's identity and mathematics identity; and 2) multilingual international students experience a transition in their language of instruction (LOI) as they move to the U.S from their home country, which influences their experiences and their mathematics identities. In the rest of the introduction, I briefly expand upon my reasoning in the literature review and theoretical perspectives section.

The proportion of international students often becomes one way to measure whether an institution is diverse enough or not. This notion is because international students bring diversity to the U.S., such as racial, cultural, and linguistic diversity. I focus on the linguistic diversity that international students bring into mathematics classrooms. Among the about half a million international students in STEM who were enrolled in universities in 2017-2018, China, India,

Saudi Arabia, and South Korea were ranked as the four most common countries of their origins (Granovskiy & Wilson, 2019). There were 162,000 Chinese students, 154,000 Indian students, 20,000 Saudi Arabian, and 17,000 Korean students who came to the U.S. to study STEM areas in that academic year (Granovskiy & Wilson, 2019). Based on these four countries, it is very likely that these international students will bring at least one more language that they speak other than English. Assuming that this trend would have been similar in mathematics to STEM, the number of international students who speak more than one language in mathematics would be substantial.

I especially focus on linguistic diversity in a classroom because having a different first language than the LOI is directly related to communication, which can impact one's access to learning opportunities, one's ability to communicate, and one's mathematics identity. One of the common myths about mathematics is that it is independent of language because it uses its own symbols and equations (e.g., Schleppegrell, 2007; Takeuchi, 2015). However, once one needs to read aloud those symbols and equations, it requires one to use language. Multilingual international students in the classroom need to read those mathematical notations in the LOI, which might not be their first language. As a bilingual instructor in the United States, I had to learn how to read mathematical symbols and expressions in English, although I was already familiar with the mathematics content. For example, the order of reading fractions is opposite in Korean than in English. In Korean, people read aloud the denominator first and then the numerator, which is the opposite of the English order. Language plays an important role not only in reading symbols and expressions but also in-class activities and groupwork. When people in classrooms communicate, language is a primary and unavoidable medium, and hence it can

influence students' learning opportunities (Meyer et al., 2016). Therefore, mathematics is not language-independent but language-dependent (Schleppegrell, 2007).

In addition to the influences that language has on learning opportunities, language interacts with one's mathematics identities or one's relationship with mathematics. As "identity" is an important construct in this study, the following is the definition that I am employing for this study: one's identity is "how people understand their relationship to the world, how that relationship is constructed across time and space, and how people understand their possibilities for the future." (Norton, 1997, p. 410), which means that identity is a social construct that changes depending on space and time and is hence fluid. As identity is not static and shifts fluidly across social contexts where people interact with others, I view that people can have different relationships to the world, hence different identities, in different settings. Some relationships to the world are more present than others in different contexts. For example, my relationships to the world as Asian and Korean became more noticeable when I am in the United States than when I am in Korea. That is, one person has multiple identities, and one of the contexts can be mathematics. Therefore, in this study, I view mathematics identity as how one understands their relationships with mathematics, how those relationships are constructed across time and space, and how one understands their future possibilities in mathematics. On the other hand, there are some aspects that influence one's identity across different contexts, such as gender, race, nationality and ethnicity, although those aspects have different influences in different settings. For example, being an Asian in mathematics classrooms and history classrooms can have different influences on students' identities. I view language as one of such aspects of their identities because their language use is likely to be part of their identities in any setting, but language use takes different roles and meanings in different contexts.

With the concern of learning opportunities and their mathematics identities, I intended to study multilingual international students' learning and experiences in multilingual mathematics classrooms where teachers and/or students can speak multiple languages. However, some deficit perspectives on bilingual/multilingual students exist in our society. Often people focus on what multilingual international students do not do well, such as focusing on not speaking English as fluently as native English speaker students do. Some researchers have framed or assumed that multilinguals have linguistic deficiencies either implicitly or explicitly and then studied how that deficiency influences their learning and experiences (e.g., Campbell et al., 2007; Hwang et al., 2022; Neville-Barton & Barton, 2005). Even the manuscript that I authored has such a perspective (Hwang et al., 2022). In that article, we referred to one student, Jia, who uses Mandarin as a first language and uses English as a second language as a "non-native English speaker." The term "non-native English speaker" itself implicitly focuses on what a person does not have rather than what they have.

These perspectives are related to why I use "multilinguals" rather than other terms. One of the common understandings about bilinguals is that they speak both languages fluently, as if both are their native language (Planas & Setati, 2009). However, speaking multiple languages *equally* fluently is something that is neither possible nor measurable. Thus, in this study, I use the definition of multilingual as people who use one language in some situations and another in other situations (Planas & Setati, 2009). According to this, international students who are traditionally called "English as a Second Language (ESL)," "English as Another Language (EAL)," or "non-native English speakers" are "multilingual" as they use English during the official instruction of the course as well as some other contexts and often use their first language in other settings. Therefore, in this study, I try to be clear that I do not view multilingual students

in the proof-based mathematics classroom as a population with deficiencies but as a population who brings different resources (i.e., languages different than the LOI) into classrooms. As I delivered such students' experiences, there were situations where participants themselves attributed their challenge to their "lack of fluency" in the LOI. I delivered their stories and how they think about themselves without necessarily agreeing that their challenges are solely from them being multilingual.

When multilingual international students move to the U.S. from their home country, their pattern of language use changes: The language that they use in their home country loses its position as LOI and English, which is usually not a big portion of their language use, takes the position as LOI. The switch of the LOI is one of the struggles that international students report (Brenner, 1998; Macintyre et al., 1998). As such, their language usage changes regardless of various disciplines. Accordingly, mathematics becomes a context that will have specific language use in a setting because mathematics has its own discourse in which students need to learn to participate. When the LOI is not their first language, students can think their ability to contribute or participate in mathematical contexts becomes lower. For example, one bilingual international student, Jia, from Hwang et al. (2022) reported that she felt uncomfortable sharing what she knows and how she thinks with other English monolingual speaker peers. She felt that she was wasting their time because her explanation did not feel as efficient as others. At the same time, later in the semester, she reported that her work with other students who share the same first language as her was more efficient. This excerpt indicates that her mathematics identity shifted when she moved between different interactional contexts where the languages that her peers use are the same or different from what she uses, which shows the importance of language in one's mathematics identity.

During the transition, their past experiences around mathematics will likely shape their decisions and behaviors. Drawing from Ivanič (1998) and Dewey (2015), I view multilingual international students' experiences as influencing their identities that shape their current experiences and then, in turn, their past and current experiences influence their future experiences. More specifically, their experiences as a mathematics doer shape their mathematics identity that shapes their behaviors around mathematics learning. Because it is important for the field of mathematics education to help students have positive mathematics identities, in part so that they pursue further study and keep an interest in mathematics, I claim it is important to look at students' mathematical experiences including multilingual international students. However, even though a large number of international students exist in mathematics courses in United States universities, not a lot of studies, if any, about the experiences of those who are in or beyond the upper-level proof-based courses exist in the field. Hence, in this study, I aimed to understand multilingual international students' experiences, how they approach using their first or other languages, and how their past experiences shape their experiences and behaviors in their current context.

The original plan of the current study was to answer the following set of questions with a sole focus on the *multilingual* part of post-secondary multilingual international students' identities: What are multilingual international students' mathematics identities in various linguistic and mathematical contexts?

1. How do multilingual international students leverage their first and/or other languages when the LOI in proof-based mathematics courses is different from their first and/or languages?

2. What kinds of lived experiences do multilingual international students report having affected their mathematics identities?

As described in the original research question, I intended to study their changes in mathematics identities based on their changes in the language of instructions in doing mathematics and how they utilize their multiple languages. As I was working with my participants, however, their mathematics identities were shaped by other social markers, such as gender, race, and nationality, as well as the languages that they use in the United States. Thus, I revised my research questions as follows so that I could discuss their diverse experiences that shaped their relationships with mathematics and experiences: What are post-secondary multilingual international students' mathematics identities?

- 1. How do post-secondary multilingual students explain their relationships with mathematics?
- 2. What lived experiences do they share as having constructed their relationships with mathematics across time and space?
- 3. How do post-secondary multilingual students explain their understanding of their future possibilities in mathematics?

Through this study, I hope for our field to have a better understanding of multilingual collegiate students' experiences in mathematics classrooms, especially proof-based, so that the field can serve them in an equitable manner and help them develop positive mathematical identities.

CHAPTER 2: LITERATURE REVIEW

In this chapter, I review existing literature about multilingual students in mathematics education. Then, I explain how the roles of language in mathematics education and mathematics identity are framed in the current study based on existing literature.

Multilingual Students in Mathematics Classrooms

The mathematics education field has been moving toward more activity-andcommunication-based education from the past education that was more focused on procedures or how to find the right answers (CCSSI, 2010; National Council of Teachers of Mathematics, 2000). The field has been requiring students to communicate mathematically, both verbally and in writing. With such a movement, some researchers have focused on languages that are used in mathematics (Barwell, 2018; Halliday, 1978; Lew & Mejía-Ramos, 2019; Mejía-Ramos & Inglis, 2011; Moschkovich, 2007; Pimm, 1987). Across different researchers, Moschkovich (2002) categorized the field's work into three main perspectives about how researchers approached language in mathematics education: First, some researchers have focused on students' acquisition of words; second, others have focused on the building of multiple meanings in mathematics. The third and last perspective was approaching language as a resource and focusing on people's participation rather than people themselves. Moschkovich (2002) claimed that the first two perspectives, especially the first one, can lead researchers to focus on what multilingual students do not know or do not have, which can impose an implicit deficit view on the population. Hence, researchers have encouraged mathematics educators to focus on language in terms of their participation (Moschkovich, 2002) and how they use language during the participation (Takeuchi, 2016; Takeuchi et al., 2019).

Related to the recent emphasis on the importance of communication and participation, mathematics educators have claimed that the experiences of multilingual students should be

studied. Multilingual students switch between multiple languages that they use when they solve problems in mathematics, which can become another resource if the community values those languages. Existing K-12 mathematics education has shown that students and even teachers switching languages during class helped multilingual students' learning. Multilingual students performed better when they could use their native language, such as coming up with more advanced strategies or arriving at an answer more easily compared to when they are required to use the LOI (Domínguez, 2011). This outcome can be connected to the characteristics of mathematics language. Even though students share the same mathematical symbols across the world, they need to do some translation when they want to verbalize their interpretations because mathematical sentences with symbols are dense. Students need to have a conversation about their interpretations, which is related to Sfard's (2008) viewpoint. Sfard (2008) construed mathematics objects as discursive constructs and hence argued that doing mathematics is having a discourse of (mathematical) discourse, which requires intense linguistic capability. Therefore, mathematics needs significant linguistic ability to perform, which means students whose first language is not the LOI can struggle with mathematical languages.

Although there exists some understanding of multilingual students in K-12 mathematics education, the mathematics education field does not know a lot about the experiences of multilingual post-secondary students. Some differences exist between the K-12 multilingual population and those at the collegiate level. A significant portion of multilingual students at the post-secondary level are international students, and there are many different languages in one classroom. Although such a situation is possible for K-12 education as well, existing research usually has focused on classrooms with two languages, commonly Spanish and English, in the United States. Compared to K-12 education, where it is *more* likely that a teacher can be from

the same background as students or at least domestic, the compositions of students in postsecondary mathematics classrooms are more varied, and even professors are often from various countries. For those reasons, students might not be likely to be able to use their first language to help their learning during the class.

One relevant study at the post-secondary level was multilingual undergraduate students' experiences of active learning in intro-level mathematics classrooms by Rios (2022). Rios (2022) looked at multilingual students' experiences employing positioning theory and viewing language as political and as a resource. In the study, 26 multilingual students shared how they were positioned during the groupwork and how their narratives can challenge the dominant discourses about language in the classrooms that center on verbal participation and the nativeness of English. Even though there were some cases when multilingual students were positioned positively, there were many cases when they were deficit positioned, such as a "novice, subordinate, unintelligent, a burden, and non-legitimate speaker of English" [who are not valued as an English speaker] (Rios, 2022). At the end of the study, Rios (2022) suggested valuing other forms of communication and participation and raised concerns that the onus of being comfortable to say in the classroom seemed solely on multilingual students.

Language and Equity

Another important aspect of language is related to equity. I view language as having a critical role in equitable learning. As a perspective for looking at equity, I adapt Gutiérrez's (2009) work, which outlined a framework for equity using four dimensions: access, achievement, identity, and power. Access is related to resources that one can reach in order to participate in mathematical activities, including both material aspects (e.g., curriculum or technology) and social aspects (e.g., teachers and a positive atmosphere). Whereas the access dimension concerns

what students have as their resources, achievement is related to students' outcomes. In her framework, outcomes include not only test scores but also students' participation in class or which courses they are taking. In addition, Gutiérrez (2009) considered access and achievement as the *dominant axis*, which can explain how well students can do in mathematics, and in this framework, access is a precursor of achievement. As diversity is also an important factor, Gutiérrez (2009) introduced the *critical axis*, which influences how students can become critical citizens so that they can change society in the future. Identity and power are located on the critical axis, where identity includes the racial, gendered, linguistic, and economic background of students, and power means students can voice their opinions in the classroom and have mathematics as a tool to analyze society in critical ways.

With this framework by Gutiérrez (2009, Figure 1), language is relevant to all four dimensions and both axes. As already mentioned in the introduction, language can influence students' access to learning and their identities (Bishop, 2012). If students are not comfortable or confident using the LOI, they might not be able to participate fully in activities, even if they have a good curriculum or qualified teachers. As indicated by the dominant axis, this can impact their achievement as existing research showed that English language learners and those who seemed to be "fluent" in English had lower scores and lower pass rates in their high school exit exams than native speakers (Abedi & Lord, 2001). In addition, language is an inseparable aspect of one's identity (Civil & Hunter, 2015; Esmonde et al., 2009; Esmonde & Langer-Osuna, 2013; Planas, 2011) because language is also deeply connected to culture (Meyer et al., 2016). Then, this linguistic identity can influence students' power in the classroom because language can play a role when people decide whom to include or whom to address. Researchers have found that interacting with others while understanding new meanings is not easy (Halliday, 1978).

Moreover, some found that humor and social chat contribute to affording some power in classroom interactions, which also requires some level of fluency in the LOI (Civil & Hunter, 2015; Esmonde & Langer-Osuna, 2013).

Figure 2.1

Axes of equity by Gutierrez (2009)



Thus, in this study, I view the various languages that multilingual international students use as influential to their identity and argue that they should be viewed as a resource for their learning rather than as a deficiency. Hence, language is one of the important aspects that should be considered in terms of equity, and further it adds to the rationale that multilingual students' experiences should be focused on to see how mathematics educators can offer a more equitable learning environment to such students.

Mathematics Identity

Mathematics learning is more than learning mathematical content that is in textbooks; one of the primary goals of mathematics education should be to help students to have positive mathematics identities. Positive mathematics identities are important because they show how students view themselves as mathematical doers, which shapes what they can and cannot do, such as whether they participate in the discussion or ask questions. Although K-12 students are required to take mathematics courses, undergraduates are more likely to be in mathematics courses by their choice, and the mathematics identity issue remains a complicated one in postsecondary mathematics education as well (Solomon, 2007). For example, female students face a relatively lower ratio of female students in collegiate mathematics classrooms, which makes them struggle with their mathematics identities by making them think about whether mathematics has a place for them. Further, Solomon (2007) found that undergraduate students did not feel they belonged to the mathematics community and showed mathematics identities that hindered participation and feeling ownership of the work even though they chose to study mathematics as their major. Therefore, mathematics identities are important to study in postsecondary mathematics (Solomon, 2007).

Hence, to explore multilingual post-secondary students' experiences and their interpretations of their experiences, I employ the concept of identity as a construct. As mentioned in Chapter 1, in this study, I view, in general, one's identity as "how people understand their relationship to the world, how that relationship is constructed across time and space, and how people understand their possibilities for the future" (Norton, 1997, p. 410). One's identities are important to focus on in studies because one's current identities are a result of past educational experiences, and current experiences will change one's future identities, and those identities also shape one's attitude or behaviors. Researchers pointed out relationships between language and identity, which includes that people are not just exchanging information but also building and negotiating their identities during conversations, and hence, language is one important medium of such identity formation (Bustamante-López, 2008; Norton, 1997; Norton & Toohey, 2011). In

addition, languages that one uses are an important part of one's identity. Especially when multilingual students need to choose which languages they want to use in specific situations, they are negotiating and expressing their identities by selecting which part of themselves they want to show.

As interest in identity in education has grown, such interests have increased among mathematics educators as well, which is studied under the term "mathematics identity." Various researchers have employed different perspectives and drawn on different theoretical backgrounds. Fellus (2019) analyzed current identity-related studies in mathematics education into four aspects of identity using Ivanic's (1998) work. The four aspects were the autobiographical self, discoursal self, authorial self, and socioculturally available selfhoods. The autobiographical aspect of mathematics identity is focusing on one's past experiences related to mathematics and one's interpretations of those experiences with an assumption that past experiences impact who one is as a mathematics doer and hence in one's current and future experiences (e.g., Di Martino & Zan, 2010; Martin, 2006). The discoursal aspect focuses on which identities are addressed, attributed, and affiliated to a person by themselves and others during discourses. In this aspect, people have focused on how mathematics doers co-construct their mathematics identities throughout the way that they were talked about and to (e.g., Bishop, 2012; Sfard & Prusak, 2005). The authorial aspect focuses on one's voice and ownership during mathematical learning in terms of positions and power dynamics (e.g., Esmonde et al., 2009; Langer-Osuna, 2011; Takeuchi, 2016; Takeuchi et al., 2019). Lastly, socioculturally available selfhoods are prototypical identifications related to social groups to which a person is likely to belong, such as ethnicity, gender, or language. Regarding this aspect, people have studied how

mathematics doers accept and reject socioculturally available mathematics possibilities (e.g., Langer-Osuna, 2011; Marshall, 2010; Mendick, 2005).

All of these aspects of identities are not disconnected but closely related and interconnected. Burgess and Ivanič (2010) explained the relationships between the four aspects of identities, focusing on a writer's identity. Applying their theories in mathematics identity, socioculturally available possibilities of selfhood can shape the autobiographical self because one's sociocultural background can be closely related to one's access to certain ways of communication, acting, and being around mathematics. Because of what has been possible and allowed to a person, it can shape what they do and who they are in mathematics classrooms and around mathematics. Hence, socioculturally available possibilities of selfhood shape their relationship to mathematics, "values and beliefs, and a sense of relative authoritativeness and agency" (Burgess & Ivanič, 2010, p.244). However, one reflects on and interprets their experiences considering their contexts, and hence, everyone in the same sociocultural background takes their experiences differently and develops their unique mathematics identities based on all their experiences around mathematics up to the moment of being a mathematics doer.

Burgess and Ivanič (2010) also explained the relationship between the autobiographical self and the discoursal and authorial self. Burgess and Ivanič (2010) viewed that the discoursal and authorial self appears in a smaller timescale, such as within a single class session or solving one tasks, than the autobiographical self and socioculturally available selfhood that is developed over the one's life span or longer. One's so far developed autobiographical self, including their actions and beings around mathematics, shapes their experiences around mathematics at the moment. A person develops habitual actions over the years as part of their autobiographical self

and might continue to use the actions with which they are already comfortable. However, depending on the contexts, one might not be able to take their habitual actions and develop a different discoursal self than one would have intended. For example, when one international student who thinks of themselves as a leader role during groupwork in the mathematics classroom in their home country just moved to the United States and takes mathematics courses in a new setting, there can be a chance that they need to step away from the *leader* role and take different positions, such as listener, in the groupwork. In that case, their discoursal identity becomes a *listener* at the moment, and their authorial self as a mathematics doer can be different than what they usually felt in their home country.

The other direction of the influence between the autobiographical self and the authorial and discoursal self is possible. Using the same example above, if one repetitively experiences being positioned as a listener, that might change their autobiographical self in a long term. Thus, the four aspects of mathematics identity influence and interact with each other.

In this study, I mainly focused on the autobiographical aspect of post-secondary multilingual international students' mathematics identities because I intended to focus on their experiences over time and autobiographical identities that are guided by their socioculturally available selfhood based on the groups to which a person identifies as belonging (Di Martino & Zan, 2010; Ivanič, 2006; Martin, 2006). In this study, I could anticipate that their social markers about language, gender, race, and being an international student were likely to appear as part of their understanding of their socioculturally available selfhood. Of course, the discoursal self and authorial self in a shorter time span appeared in the data that I collected as a part of their experiences influenced their relationship with mathematics, but I mainly approached students'

mathematics identities through the autobiographic self guided by four socioculturally available selfhoods: language, gender, race, and being an international student, and possibly other aspects.

CHAPTER 3: METHOD

For this study, I use narrative inquiry as a research method. Narrative inquiry is a research method that focuses on understanding human's lived stories (Connelly & Clandinin, 1990). In this section, I review relevant literature about narrative inquiry in mathematics education and some from other education fields. Then, I explain the research participants and context and my research positionality. After that, I describe the pilot study–process and what I learn from the process. Lastly, data collection and analysis details will be followed by a summary of the pilot study findings and discussions.

Literature Review of Narrative Inquiry

Researchers have claimed that narratives have become an important tool for qualitative mathematics education research as they have been for other disciplines (Connelly & Clandinin, 1990; Foote & Bartell, 2011; Kim, 2011). Narrative inquiry is based on Dewey's two principles: the interaction principle and the continuity principle (Connelly & Clandinin, 1990). The interaction principle claims that experiences are shaped by one's internal condition and surrounding external conditions, and the continuity principle claims that one's past experiences influence their current experiences and their current experiences influence their future experiences (Dewey, 2015). Dewey claimed that these two principles are useful to understand whether one's experience is educative, which promotes learning and the continuous growth of a person, or mis-educative, which is the opposite of an educative experience. This theoretical background based on Dewey's theory of experience aligns with the intention of the current study.

Connelly and Clandinin (1990), who advanced narrative inquiry in the education field, claimed the following:

... humans are storytelling organisms who, individually and socially, lead storied lives ... education is the construction and reconstruction of personal and social stories; teachers and learners are storytellers and characters in their own and other's stories (Connelly and Clandinin, 1990, p.2)

People's experiences are in their stories as well as their beliefs, attitudes, and values, and their experiences are represented in a story as a whole rather than listed facts or figures (Foote & Bartell, 2011). Hence, according to this perspective, to understand students' experiences during their education, it is required to listen to people's stories of their education. By studying their stories, researchers can have a powerful tool to communicate, evoke emotions, empathize, and reflect on the experiences as well as provide insights into the factors of such experiences (Foote & Bartell, 2011; Martinie et al., 2016). Narrative inquiry is a useful tool to study anyone's experiences, but in the educational field, it has been used more often to study marginalized populations' experiences to invite readers to revisit their own views and experiences in the educational field (Kim, 2012; Martinie et al., 2016). For such a reason, narratives created from narrative inquiry become a beneficial tool for the professional development of teachers (Kim, 2011). In mathematics education, research subjects of the studies that used narrative inquiry were to understand teachers' experiences and identities (Kaasila, 2007; Martinie et al., 2016; Smith, 2006) and how researcher's positionalities are related to their experiences (e.g., Foote & Bartell, 2011). Yet, I could not find a peer-reviewed article about students' experiences using narrative inquiry as a research method in mathematics education.

Polkinghorne (1995) suggested two primary kinds of analysis in narrative inquiry: Analysis of narratives (pragmatic mode of analysis) and narrative analysis (narrative mode of analysis). Analysis of narratives is an analysis method that finds common themes and categories

from the collected stories—which is similar to other qualitative studies, and narrative analysis is building a narrative from the collected artifacts such as interviews from the participants or some physical artifacts. In narrative analysis, researchers find the essence of the story and reconstruct it as a coherent narrative or narrative based on the data. When results are presented from narrative analysis, there are different forms of presenting a narrative, such as an autobiography, biography, non-fiction, fiction, or poetry (See Figure 3.1) (Hannula, 2003; Kim, 2006, 2011, 2016; Martinie et al., 2016). For example, Kim (2006) presented a narrative of one student in an alternative school as an autobiographic journal, Kim (2011) presented multiple narratives from multiple people with different roles in the same alternative school from a first-person perspective of participants, and Martinie et al. (2016) presented their result as narratives combining multiple people's stories in several categories.

Figure 3.1

Different Types of Writing in Narrative Inquiry from Kim (2006)



Figure 6.3 Flirting With Polkinghorne, Mishler, and Labov

Researcher Positionality: As One of Multilingual International Students

I am one of the multilingual international students because I am from Korea and received my K-16 education mainly in Korean. As a student who has studied mathematics and mathematics education in the United States and an instructor who has taught numerous mathematics and mathematics courses at the post-secondary level, I had to face a challenge that I felt came from a linguistic difference. I have felt that my English is not fluent enough, which made me less confident and more hesitant in sharing my opinions or thoughts. Sometimes I could not express "easy" mathematics because I did not know how to say it in English, and I had to worry if my students or peers would think of me as not competent in mathematics because I use a lot of hedging words or do not know how to say words in English. The feeling that I could not communicate without any restriction hurt my confidence and attitude as a mathematics doer. Some of this might be related to my other social identities, such as Korean, Asian, and woman. I admit some of my experiences may have been influenced by other social markers, but I had attributed many of my painful experiences due to my fluency in English. Because of this experience, this research is personal to me and has additional importance to study. I attached a more detailed version of my researcher positionality as a narrative form in Appendix B. The researcher positionality was created using the narrative analysis method.

Narrative inquiry views a researcher as a co-participant of a study and co-constructor of a narrative because the researcher is questioning and listening to the participants' lived stories in inquiry. In those processes, because they knew that I was also one of "them," I hoped my participants would feel more comfortable sharing their stories. Also, as I could relatively easily relate to their experiences and share my experiences as well, I hoped it to be a chance as well that they could learn that they were not the only ones who had such experiences. For example, my

pilot study participant, Jia, was a Chinese female student. Although our first language is different, we share a lot of common social markers, especially being Asian female international students. I could easily relate to her experiences, and during the interview, she told me that it was good for her that she was not the only one who faced the challenges that she had. More reflections about the influences of my positionality in the current research are in Chapter 8.

In the meantime, I tried to be careful with my approach. I tend to attribute the reasons for my experiences to myself, especially to my lack of fluency in English. However, I firmly believe that should not be the case in how I view my participants because finding reasons solely from the person can imply that they have a deficiency. Participants attributed their experiences to "their deficiency," but I tried to find other implied reasons in their lived experiences by understanding the meanings of their experiences.

Participants and Context

Participants for the study are multilingual international post-secondary students whose K-12 education's LOI was primarily not in English. I chose participants who had K-12 education abroad in a different language than English because I wanted to understand their experiences under such a context that students need to deal with a different LOI than they had previously. In addition, as mentioned in the introduction, I wanted to know about their current experiences in upper-level proof-based mathematics courses and also in their previous mathematics education. I originally hoped they would be mathematics majors or minors so that they would be enrolled in or have taken proof-based mathematics courses such as Introduction-to-Proof (ITP), Analysis I or II, or Abstract I or II –the common proof-based mathematics courses. However, during the recruitment of participants, I did not get enough volunteers from the undergraduate students and hence broadened the participant pool to first-year graduate students as well. The reason that I expanded to first-year graduate students was that they were still adjusting to the new environments and taking qualifying courses with no teaching duties in the classrooms, so I believed that their experiences could be meaningful in understanding undergraduate students' experiences as well.

In the current study, including the pilot study participant Jia, I have four post-secondary multilingual international students, and details are in Table 3.1. Maria, Helena, and Jihyun are from the same cohort, as all of them were in the first year of their Ph.D. program at one Midwestern University in the United States. I sent out a recruiting email to all the first-year Ph.D. students in that program, and Jihyun responded first that she would be interested in participating. When I was sending the email, I told her that she did not need to feel pressured to participate because we were already friends at that time. Jihyun said she was still interested in participating and also advertised the study to her friends in the program. Also, as Jihyun shares the same first language as me, I told her that Jihyun could use any of the two languages when she was communicating about the study. Lastly, all the participants were monetarily compensated as a gratitude for their contribution.

Table 3.1

Pseudonym	Home Country	Languages	Graduate/Undergrad
Jia	People's Republic of China	Mandarin, English	Undergraduate
Maria	Mexico	Spanish, English	Graduate
Helena	Colombia	Spanish, English	Graduate
Jihyun	Republic of Korea	Korean, English	Graduate

Participants' Information

Pilot Study Method

One of the reasons that I became interested in this study was interviews with my pilot participant, Jia, who was a part of the Transitions-to-Proof (T2P) project that I was working on as a research assistant. Jia is a female Chinese student who majored in statistics and minored in Computational Mathematics, Science, and Engineering (CMSE), data science, and mathematics. She received her K-12 education in China, and her first language is Chinese, specifically Mandarin. I collected the data from her in two separate time frames, once in Fall 2019 and the other one in Fall 2021 and Spring 2022. In Fall 2019, the data sources for the T2P project were two in-person interviews, homework logs that recorded participants' experiences related to their course and homework, classroom observations, and class materials such as exams or homework. At that point, she was not minoring in mathematics and hence did not have any more proof-based mathematics courses. Therefore, we, the project team, stopped the data collection with her. As time went on, I found this topic important because her data presented to us how one multilingual international student's experiences could be in a proof-based mathematics classroom. Although her experiences were closely related to the course context that emphasized groupwork (Hwang et al., 2022), she shared other difficulties that she experienced in general in the classroom. Thus, I decided to reach out to her again to collect more data as a pilot of my dissertation.

In Fall 2021, I asked a few more questions based on her interviews and if, and so how, her thoughts or experiences had changed since Fall 2019. At that point, we were in the middle of a pandemic still, so I contacted her via email and asked whether she wanted to do in-person, Zoom, or email interviews. She wanted to do email interviews because she did not need to worry about time restrictions. Once I sent my questions, she responded with answers. Then, I added some of the comments and shared reflections of my experiences as well as some follow-up

questions. I took a day or two to respond to her responses and think deeply about which experiences that I have that might be similar to hers and what I want to ask next time. Jia also responded when she was available. Our email interactions included about 14 sets of exchanges, including the ones checking her consent on the interviews. As compensation, I offered her an Amazon gift card and some help for her application material for her master's degree. I analyzed her data and wrote her narrative from the data. I sent the narrative to her for member-checking purposes, and she responded with some feedback. After two cycles of member-checking, the narrative presented below was finalized.

From the pilot data collection, I learned that it would be beneficial to have some data to generate interview questions, as the T2P data collected in the first semester helped me understand Jia before I went more in-depth into her experience as a multilingual. Hence, I decided to ask participants to write their mathematics autobiographies. I also enjoyed the email interviews because of the benefit that they could give me and her, including but not limited to not worrying about meeting time or time restrictions for the interviews, being able to search how to express things in English, and being able to think deeply and react to what she told me in a thoughtful way. I especially appreciated that I could take enough time to think about what to ask next and how I could avoid imposing some deficit perspectives about multilingual international students. However, although I appreciated the email interviews, at the same time, sometimes I wished to have some opportunity to listen to and question their written experiences in the emails in the moment live with their facial expressions. I learned that there were some pros and cons to the email interview method. In addition, I felt that in-person or virtual synchronous interviews would be important to develop rapport and trust between my participants and me. As a result, I decided to have both synchronous interviews and asynchronous interviews for the data
collection. I discuss the findings from the pilot study after explaining the rest of the research method.

As working with Jia helped me in developing and improving data collection and analysis, I view her case as a pilot study for my dissertation. However, Jia's experiences share commonalities with other participants, Jia's narrative was included in the cross-analysis across participants.

Data Collection

Throughout the study, I collected different types of data: mathematics autobiography, synchronous interview(s), and follow-up interviews through emails. To approach the participants' experiences, I asked for their mathematics autobiography (Appendix C) titled "Me and Math: my relationship with mathematics up to now" (Di Martino & Zan, 2010). In addition to the title, I added some prompts indicating that I am interested in their memories of their experiences about and around mathematics. They might or might not have noticed which part of their experiences I was interested in because the recruiting email said that I was looking for multilingual international students, but I wanted to be open to the experiences that were important for them first. The autobiographies were resources for me to prepare for the interviews with them, as well as important data for the study. It was also a way that I could have an opportunity to listen to some of their experiences during the interview with added detail.

Synchronous Interviews

At the beginning of the synchronous interview (Appendix D), I asked for their background, such as their home country, languages that they use, when they moved to the United States, or which proof-based mathematics courses they have taken. I already had this information from the recruiting process, but I double-checked the information and asked for detail, such as

why they decided to come to the U.S. for their post-secondary education. After that, I asked more questions related to their autobiography, especially how those important events happened in their lives. Polkinghorne (1995) said, "The interviewer can solicit stories by simply asking the interviewee to tell how something happened" (p. 13) because people tend to say their answers as stories if an interviewer does not limit the type of answers. Also, if not mentioned in their mathematics autobiography, I asked questions about their experiences during the linguistic transition and how they felt and what they experienced when they moved to the United States.

Once I heard the stories about their past experiences, the interview focused on the current experiences, such as how they are using their first and/or other languages as a resource for their mathematics learning in the United States. During the synchronous interview, I sometimes shared my experiences as well so that they could feel "safe" to share their experiences after their responses and so that it did not force participants' answers in a similar direction to what I experienced. It was important to build a relationship and trust with my participants to share experiences, as mentioned previously. Developing a good relationship also helped me ask more questions to them, and I hope that it helped them share their thoughts during the follow-up interviews after the synchronous interview.

The synchronous interviews happened in Spring 2022 or Summer 2022. Helena wanted to have an in-person interview, so we had an in-person interview. With Maria and Jihyun, because either the participant or I was out of the country, each of their synchronous interviews happened on a video call. The lengths of the interviews were between 60 – 90 minutes. After the interviews, each of them was transcribed. Jihyun's interview was in Korean, so it was transcribed into Korean.

Asynchronous Interviews

After the synchronous interview, I planned to have follow-up interviews via email asynchronously. I decided to have asynchronous interviews in addition to synchronous interviews for several reasons. As an international student, I still feel pressured when I need to explain things, especially when I am asked to do so when I am not ready and need more time to think about it. I assumed it could be similar to other multilingual international students whose first language is other languages than what we were communicating with, English. By having an asynchronous method, I thought they would not need to worry about time and/or English use and could have time to think about their responses before they sent those to me. Also, as a researcher who uses English as a second language, I did not need to worry about English speaking or showing enough empathy in their responses. I also could try to connect the dots between what they wrote and said in the mathematics autobiography and the synchronous interview before I responded to them.

The asynchronous interview questions had two parts: 1) questions started from the participant's responses, and 2) narrative tasks. Narrative tasks were for them to provide some situations to provoke conversations about their understanding of themselves and others in proof-based mathematics classrooms. I asked what they noticed in the narrative, who they saw in the narrative, and whether they had a similar experience. This idea started from focus-group tasks in Leyva's dissertation (Leyva, 2016a), which focused on the power dynamics that Latin@ and/or women experience in STEM. I adapted the idea of providing a narrative so that they can think about whether they can see themselves taking such action in different settings, such as a lecture with the instructor or a recitation with a TA in Leyva (2016a), and if not, whom they see taking such an action. For example, in Leyva (2016a), the author provided different actions of students,

such as telling the TA about a mistake or answering different types of questions. In this study, the idea of using narrative to start the conversation was taken, but the specifics of the narratives were changed based on the goal of this study. The original task contexts were Calculus, which is different from the context that I focused on in this study. Therefore, I changed the narratives and questions accordingly. The narratives were written based on but modified from the observations collected as a part of the T2P data collection and Jia's case. I picked two settings: during groupwork to explore their experiences of interacting with peers and during the lecture to explore their interactions with the instructor and the whole class. The detailed tasks can be found in Appendix E. I decided to locate the interviews using narratives in the email interviews because reading and comprehending the narratives can take some time for multilingual international students, and reading those in front of other people, especially when people were waiting for them, could become a pressure. I thought it would be easier for me to ask follow-up questions in the synchronous interview, but them feeling comfortable was more important to me, and I could still ask follow-up questions via email.

With these goals and intentions, I sent an email to each participant with questions, and they responded in different ways. Helena stayed in the email (except once when she wanted to give feedback for the written narrative). Jihyun used email as well, but after one email exchange, she wanted to talk in person in Korean, so there was another in-person interview that corresponded to others' asynchronous interviews. Maria responded with a shared Word document to my first follow-up email. Starting from it, she wanted to have a conversation on the shared Word document, which became our primary way of communicating until I sent her a first draft of her narrative.

Data Analysis: Narrative Analysis

I mainly did *narrative analysis* for this study because I think the narratives can deliver who the participants have been in mathematics classrooms and what their experiences are as mathematics doers better than the patterns of their behaviors. I thought building a narrative from the data would be influential, especially in delivering how they felt during their experiences.

As mentioned above, narrative analysis is a way to synthesize an explanatory narrative from the data gathered using recursive movement between the data and an emerging thematic plot (Polkinghorne, 1995). Polkinghorne claimed:

The purpose of narrative analysis is not simply to produce a reproduction of observations; rather, it is to provide a dynamic framework in which the range of disconnected data elements are made to cohere in an interesting and explanatory way (du Preez, 1991 as written in Polkinghorne, 1995).

Researchers who use narrative analysis, hence, need to 1) interpret the gathered data to understand the experiences intended to study and 2) mediate understanding of the narrative for the readers (Kim, 2016). For the second, researchers can use five strategies: "focus, omission, addition, appropriation, and transposition," which are the strategies used for "narrative smoothing" (Kim, 2016; Polkinghorne, 1995). The purpose of narrative smoothing is to develop more engaging, coherent, and interesting narratives. However, it can introduce omissions through selective reporting or fails to offer enough context for the readers, and researchers have expressed some ethical research concerns (Kim, 2016). Researchers who employ narrative inquiry claimed that they often face this dilemma when they become faithful to what participants said and what makes it a good story (Kim, 2016).

Narrative Analysis Process

For this study, some of the analysis parts happened at the same time as I collected data so that I could generate questions to ask participants. Once the synchronous interview was finished, it was transcribed, and then, each experience that they mentioned was coded and written in the document. This stage happened repetitively as I collected more data from participants. By experience, I mean an event including their actions and emotions and others' behavior that influenced them in a certain amount of time (such as a class session or a day) in the same contexts. For example, in Jia's case, when she went to a Chinese female student on the first day of the class and shared each other's contact is an experience that Jia had in one class session. On the other hand, when Jia misheard one mathematical word that the professor said and could not understand until she found it after searching online at home, it is one experience that happened in a day.

After I coded for the experiences, I found some themes across the events as well as generated follow-up questions for the email interviews. For example, the following were a few themes for Jia's case: Challenges in communicating with groupmates during groupwork, Confidence in mathematics, Use of Chinese in learning mathematics, and Gender-related experience and motivations. After I found themes, I explored and came up with the plot to write their experiences as one coherent story. As making readers engaged in reading the story of participants' experiences is one important key in presenting their experiences as a story, this process required multiple trials to create a story, although it does not have one correct way of doing it. For this part, as the researcher who wrote the story, I had to decide which flow seemed to make sense the most. For example, in Jia's case (Appendix A), I did a thought experiment of writing in different orders, including chronological order, but the most engaging way of

presenting a story for me was the following: 1) introduce who she is, 2) her challenges, 3) her reactions to challenges/use of multiple languages in mathematics, 4) reasons/motivations of pursuing mathematics experiences that come from her past experiences. As mentioned above, every story is temporal, and there is no one way to the "right" or "valid" writing of the story (Foote & Bartell, 2011; Kim, 2016). My goal was to provide a story based on participants' experiences so that audiences can be engaged in their stories in order to understand, empathize, and reflect on their experiences as a teacher, a researcher, or fellow students (Foote & Bartell, 2011).

After I decided on the plot or the storyline, I looked at the data and started writing a story. In the process, I faced the dilemma of how much I would retain the voice of the participants. As a response to such an ethical question, I had to be sensitive about what they told me and its nuance (Kim, 2016). To decide how much I would keep their words and expressions and which part would be my explanations, the following were guidelines for my decisions for the narratives: 1) By reading the story, the audience should know and understand what the contexts for the story are. For example, in Jia's case, I added some description of how and how often groups were changed in a course in my words, although I based it on the collected data. If the part was more related to the context, then I filled in the gap so that readers could understand a better idea of the context. 2) However, when it comes to their behaviors and emotions, I tried my best to keep their words. For such a reason, I included some of their words directly. Especially related to adjectives, I tried to maintain their words unless their word choice could be misinterpreted. 3) For the flow of the story, I added some fillers, such as some signposting or connecting words to help transitions between the paragraphs or sections. These guidelines were based on the two goals of writing the narrative mentioned above: not only to interpret the

collected data to understand the causes that researchers were studying but also to help readers to understand the narrative. Again, I agree that this part of the analysis can be seen as subjective, and I think it is subjective. However, even when we present the result in more "traditional" ways, there are always interpretations and decisions made by the researcher.

The whole narrative that I wrote for Jia's case can be found in Appendix A, and Figures 3.2 and 3.3 are examples to show how I used the data to create the narrative using the guidelines mentioned in the previous paragraph. As can be seen in the writing, the story is written from the first-person perspective. According to researchers, stories written in the first-person perspective can give the impression that readers are hearing a spontaneous speech from the participants, which helps readers have a more empathetic understanding of the lived experiences of the participant (Kim, 2006). As mentioned previously, there are multiple forms of presenting the narratives (Figure 3.1). In this study, I decided to use the first-person perspective because I felt that the narrative from this perspective could be more powerful in provoking peoples' emotions and reflections.

Once I wrote a narrative, each narrative was sent to each participant for member checking. As researchers claimed, people's stories are not static and change as time passes, and people's horizon expands (Foote & Bartell, 2011; Kim, 2006). Therefore, the written narrative cannot be a direct copy of participants' experiences but one aspect of their experiences. Hence, I checked with each of the participants whether they agreed with my representation of their experiences from the perspective that they had at the point when they interacted with me so that they agreed with the narrative that I wrote. For example, I sent a previous draft of the narrative (Appendix A) to Jia, and she responded that she agreed with most of the parts and made two comments: One word that she thought could be misrepresentative and the other that she became

curious about whether I wanted her to provide more information. Following her response, I asked two more questions in an email and revised the narrative based on what she said. This process was similar to my work with the rest of the participants. In Helena's case, she said it was hard to express what she was thinking after reading the draft in writing, so she wanted to meet with me. We met and talked about how she felt, and I made some changes and resent the revised one to her.

After the participants agreed on the narrative, I asked them if they would be willing to record the narrative in their voices. It was completely voluntary, and so far, Jia and Helena have sent me their recording for full narratives, and Maria and Jihyun agreed to do some part of it. The collected audio recordings were and will be used to present the narratives to the public in addition to a written form.

Analysis of Narrative – Cross-case Analysis

After I present each participant's narrative, I share the findings that appeared across participants. A reason for including this cross-case analysis is to provide a chance to look at the participant's experiences together and find the themes across their experiences, which corresponds to "analysis of narrative" (Polkinghorne, 1995). This comparison across participants helps us understand the diversity within the group. Also, creating a narrative has limitations because of the need to be considerate of the flow of the story. Hence, including a cross-case analysis provides a place where I can bring the relevant data for the themes across participants.

The process is to find themes across their narratives and experiences, and hence it corresponds to "analysis of narrative." For this part of the analysis, the four social markers, language, race, nationality, and gender–which appeared in their narratives as socioculturally available selfhoods–were used as an analytical framework. As mentioned above, Jia's narrative

was included in the cross-case analysis because of the relevance and commonalities of her narrative with other participants' narratives.

I looked at the events and themes developed in the previous phase and found common themes under each category of language, race, nationality, and gender. For example, as the question about their opinions about the statement "language is mathematics free" was one of the interview questions, their responses were compared across the participants. *Valuing written communication, accents,* and *confidence in English* also appeared as common themes across participants in the category of language. Similarly, *stereotypes about Asians or Latinas* and *how they talk about their home countries* were discussed under the category of race and nationality. Lastly, *events of being discriminated against because of their gender* and *their thoughts and reactions to the gendered stereotypes in mathematics* were discussed under the category of gender.

While analyzing their experiences, I found that their behaviors and attitudes were not influenced by a single category of their socioculturally available selfhoods, and often their behaviors and attitudes appeared in the intersection of their multiple socioculturally available selfhoods. Hence, the last part of the cross-case analysis shares how their intersectional socioculturally available selfhoods influenced their interaction with other people through Jihyun's experiences, from whom I could find such intersectionality most.

Figure 3.2

Example of a narrative writing process

Collected Data

Jia :	I can give an example. When we do a problem, like something was asked and larger or
	smaller than a number. And I said, actually it's negative. So we need to change the
	signal. They didn't realize that. I just told them. But I was arranging my words and how
	to say that. I just write it down. I show it to them and a boy understand and the other
	one just don't understand and he explained to him. Yeah. Is it frustrating for you or?
Jihye:	
Jia:	Yes, a little bit, but that's common. (BIN 318-325)
Jia:	Sure. When I try to explain to them something but I cannot do it clearly, I just write it
	down. I show it to all the students. They just look at me and "Um, ah." They just don't
	give me the response I expected.
Jihye:	What is your expectation?
Jia:	Like they "Oh, I just realized. Oh, yes. That's the point." But sometimes they just do not
	do it, but usually they will do it. Yeah. (BIN 300-305)
Jia:	Yeah, because when your group need to speak English, and I want to tell somebody
	how to solve a problem, I cannot speak quickly to solve that. I may feel like I don't
	want to waste others' time.
Jihye:	You feel you waste others' time?
Jia :	Yeah, because it's just a small idea, but I spend so much time on explaining it, but
	others can just say it with a sentence, so I think that makes me a bit nervous. (TBI 489-
	494)

Written narratives

I always had ideas about how to solve the problems, but I could not write any steps clearly or I was not sure whether the words that I wrote were expressing my thinking in English. It happened several times in that semester. For example, something like this: Once, we were asked to compare a variable, x, with a number. In those processes, our group missed that x is a negative number and that led us to make a mistake by not changing the order of angle bracket (e.g., > or <). I said, "x is negative, so we need to change the signal" and they did not understand what I was telling them. I was expecting some positive reactions from them, like "Oh, yes, that's the point," but they did not get it and so could not give me such a reaction. Because I was not sure how to say it any other way, I just wrote it down on my note and showed it to them. Then, one of the groupmates understood and he explained that to the other people. It was a little bit frustrating, but this kind of experience was common at that time. After some of those experiences, I started staying quiet because I knew other people would usually say the same idea that I would have if I waited. Of course, if it took too long, I spoke about my idea, but sometimes I felt I was wasting others' time because other people just could explain the idea in one sentence.

Figure 3.3

Example of a narrative writing process II

Collected Data

I want to show that not only geniuses could learn math well. And I want refute the idea that many people think boys are smarter than girls. So far, I haven't achieved this goal for sure, but I won't give up. (Email interview 4)

Many people had said that boys are better in math than girls, which disappointed me when I failed to learn math well. I thought whether I could not get the knowledge because of my gender. At that time, I did not want to put much effort on math because I thought no matter how hard I study, I will still fail to be as good as other boys. I think those perspectives affected me unconsciously, which made me negative and inactive. However, I met a really good female math teacher in middle school. She encouraged me a lot and I enjoy math so much. I got full scores in that final exam and I was the only one. I asked her whether I should choose to learn social science in the future because that is "easier" for girls to learn. She said I have unlimited potential and I could do anything I want. I was greatly encouraged and decided to learn natural science in the future. Nowadays, there are more and more outstanding girls telling the world that many sayings are stereotypes. I am very happy to see that and hope I will be one of them in the future. (Email interview 7)

I planned to study abroad for mainly two reasons. The first reason is that my English was good on middle schools, so I was confident to concur the language problems at that time. The second reason is that my parents and I all want me to have a broader view of the world. (Email interview 6)

Written narratives

Also, there is another reason that I persevered in learning mathematics. I believe anyone can do mathematics. I want to show mathematics is not only for genius, and I want to refute the common claim that boys are smarter than girls or boys are better at mathematics than girls. Many people had said such a myth so many times, but that discouraged me when I could not do well on mathematics. At that time, I thought I could not do well in mathematics because of my gender, which made me think that I cannot do as well as other boys no matter how hard I study mathematics. That line of thought, in turn, hindered me from putting a lot of effort in studying mathematics. I do not think it was conscious, but it made me think negatively about my capability in mathematics and be passive and inactive about my learning of mathematics. Thankfully, I had a really good female mathematics teacher in my middle school. The teacher encouraged me a lot and helped me to enjoy mathematics. Once, I asked her whether I should choose social science as my area in the future because those subjects are "easier" for girls to learn. She told me "You have unlimited potential, and you can do anything you want to do." I appreciated that encouragement a lot and decided to pursue learning natural science rather than social science, then I came to the United States to see a larger world, world outside China as well.

Pilot Study Findings and Discussions

Summary of Findings

In this section, I summarize Jia's experiences that I learned from this pilot data, due to the length, but I have attached her narrative that I wrote after analyzing the pilot data at the end of this paper as Appendix A. Jia shared her challenges related to using English, such as being frustrated by not being able to precisely speak her thoughts and her efforts which led to those thoughts and efforts not being acknowledged by peers, and by mishearing what professors try to deliver or misunderstanding intentions of questions. However, she also found how to deal with her challenges using both English and Chinese. When she was stuck, Jia tried to explore other resources such as part of the textbook, lecture notes, or videos online, both in Chinese and English, or sometimes reached out to her Chinese friends. Also, Chinese was used to index her notes, write keywords and/or their definitions, and leave some memos to herself. Jia could use Chinese in different ways, but it was hard to use Chinese when she was working in groups, especially when she was with all native English speakers who did not speak Chinese.

However, Jia did not think her challenges related to language meant that she was not good at mathematics. The title of this narrative "I love math, hope math loves me too" is a direct quote from Jia's email that I think represents Jia's relationship with mathematics well. The quote suggests, at least to me, that Jia loves mathematics, but she is not sure whether mathematics is loving her, which makes us question where that feeling started. However, at the same time, I could infer that these different social markers might have influenced Jia's sense of belonging in the mathematical community or group of people who can do mathematics. Especially with gender-biases, Jia thinks that she needs to prove that girls can do mathematics as well. I suggest

this is one of the reasons that made her hope that mathematics also loves her as she maybe felt less welcomed by mathematics or the community around it.

As Jia still loved mathematics, she sees mathematics will stay as an important part of her life. Jia did not see herself using proof-based mathematics course contents such as Analysis or Abstract Algebra directly in her career, but she expected the logics and ways to structure an argument will help her academic career. The following is a quote from her email showing how she saw herself using mathematical logic:

I think mathematics and proofs give me the basic foundation of all other areas and actually bring me to another level of thinking. They taught me to think logically and ask why. For instance, there are lots of algorithms useful in building models. Also, when I am doing code problems, I always think of mathematical proof structures, especially induction proof. I need every step to be clear and logical.

Jia said that she will use this kind of mathematical knowledge (i.e., logical thinking or how to construct arguments logically) frequently in her future. Further, although her career is not directly related to mathematics but relevant to mathematics, Jia wanted to become a professor who has a power that can help her future students not feel frustrated and helpless as she felt, which is another way that she saw mathematics in her career.

Reflections and Discussions

Jia's experience is one possible shape of many different experiences that multilingual international students have. There are multiple messages that her case can deliver. Jia's experiences show connections between her past experiences, her current experiences, and her relationship with mathematics, especially in how she reacted to the challenges. For example, Jia reported being discouraged because of the gendered mathematics myth that she had been hearing

from the people around her. However, having an important teacher who encouraged her gave her confidence that she could do and learn mathematics well and gave her motivation to show the world that the myth is just a myth. Having this experience in middle school helped her to withstand the challenges that she had in undergraduate mathematics education.

Her narrative shows that multilingual international students' experiences are not simple to understand and can be connected to their other sociocultural identities, such as home country culture, gender, and race, to name a few. As mentioned previously, Jia's past experience was an example of how detrimental the gendered myth about mathematics can be (Esmonde et al., 2009; Leyva, 2017; Mendick, 2005). She also talked about some pressure that comes from racial myths regarding Asians (McGee, 2018; McGee et al., 2017a; Yadavalli et al., 2022). Considering everything, it leads to the importance of doing intersectional analysis for this population (Hsieh et al., 2021; Leyva, 2016b). Her case supports other researchers' claims that it is risky to focus on only one aspect of social identity and overlook other parts of social identity (Leyva, 2016b). Considering that intersectionality studies in mathematics education have been focused on race and gender, this study suggests another important aspect–languages–that can be part of intersectionality.

In terms of multilingual students' experiences, this study is a documentation of how one multilingual student leverages multiple languages in different mathematical contexts. The result shows that it can be especially difficult for multilingual international students to participate in groupwork because it is not easy to use multiple languages to take advantage of different languages. Jia could use Chinese in various ways in other contexts, such as listening to lectures, taking notes, or doing homework, but it was hard to use Chinese when she was working in groups, especially when she was with all native English speakers who could not speak Chinese.

In addition, different languages can easily create some power dynamics between students (Rios, 2022). Jia was positioned as one who has less power to speak English during groupwork, which is one kind of deficit positioning that appeared in students' experiences in Rios (2022).

Although Jia attributed the reasons for her challenging experiences during groupwork to herself, there are external reasons that were found across her interviews. Jia said when she was in a group with at least one other familiar student, who was usually Chinese, she felt more comfortable sharing her thoughts. While she felt connected to having a peer from the same culture, she mentioned that she felt excluded when other groupmates were having conversations about their lives outside the classroom. Jia said that she could not understand or follow the conversation. Given that sometimes conversations that are not themselves relevant to course material create changes in power dynamics between students (Esmonde & Langer-Osuna, 2013), not being able to join small talk because of cultural and linguistic differences can influence the power dynamics in the classroom. Also, what Jia reported about feeling comfortable when she had one person with whom she felt familiar aligns with existing K-12 research that multilingual students' roles were changed in a positive direction when they had a friend in the same group (Takeuchi, 2016).

Therefore, the result presents the need to consider how instructors can help multilingual international students not to be positioned as people with deficiencies and instead to be able to utilize the multiple languages that they have. For example, this finding implies that mathematics instructors need to think about how to form a group when they compose the groups. Across the T2P project data set, different instructors employed different methods in composing groups, such as randomizing, assigning by proximity, or letting students choose their groups (Smith III et al., 2020). Each way had affordances and constraints. The process of forming groups in a classroom

requires that mathematics educators think about how to mitigate the power dynamic between students and reflect on whether they are indirectly implying the notion that students should use English in the classroom and are not encouraged to use their other languages. In her interview, for example, Jia noted that she asked a question to a Chinese teaching assistant, but he responded in English, which Jia thought might be his job. However, I question if and why the teaching assistant felt that he needed to answer in English even though both he and Jia probably knew that their conversation would be clearer and more efficient when they spoke in Chinese. This decision might have come from the notion that English is the official LOI. However, I believe it is worth thinking about if this notion would be beneficial for students and, if not, what mathematics educators need to provide a better environment for multilingual students to leverage their first and other languages in their mathematics learning.

In this pilot study, I introduced only Jia's perspective as one of the numerous experiences of multilingual international students' experiences in mathematics classrooms. However, the mathematics education field still needs to know more and be more aware about how multilingual international students are positioned or viewed in the classroom. I believe there are a lot of students who have both similar and different experiences than what Jia had.

In the T2P data, our research team has seen that several domestic students describe that they experienced the existing "linguistic barriers" with international students, which might indirectly imply how general students consider international students. Also, by saying there is a barrier, some people consider it as inevitable. In addition, often, people imply that such a "barrier" should be solely overcome by multilingual students, and monolingual English speakers do not share the responsibility. One example is when Barton and colleagues (2005) suggested,

"If the most effective way to improve mathematics learning for these students is to address their language requirements, then that is what should be done" (p.728).

As the classrooms in the U.S. have been becoming more diverse, how to deal with the perpetuated notion that multilingual international students have "linguistic barriers" in society is an important aspect to think about when mathematics educators teach students. Also, I hope the narratives that I provide in the current study, starting from the next chapter, contribute to challenging such a notion.

CHAPTER 4: MARIA'S STORY

"I know the answer, but why I don't have the courage to raise my hand and say it?"

I am Maria. I am from a small city in Mexico, and so my first language is Spanish. I am also very fluent in English and now studying mathematics for my Ph.D. degree in the United States.

How has my relationship with mathematics been? I think my relationship with mathematics started since I was little. I was very curious about all the things around me, and so I used to go to the forest and collect weird plants and mushrooms, I always wanted to know how and why things worked the way they did. Then, in high school, there was the event that made me choose math. One time the math professor-we call high school teachers and middle school teachers as professors too-assigned some homework. It seemed hard for me and that was weird because at the time the math class was not challenging for me at all. I remember that I struggled to complete that homework and I even went to bed late that night realizing that it had been hours since I started working on it. The time flew by! and I was happy to have completed it. The next day I handed the homework to my professor, and he said I did the wrong sheet of exercises, that those exercises were too hard and were not homework. He did grade the work and most of it was wrong. But, I did not regret doing the homework because I enjoyed it. Since then, I realized that I wanted to do something with mathematics when finishing high school. The math professor told me there was a science school offered by the department of science of the University in the hometown. This science school offered a weekly class for high school students who want to get involved in math and physics. I started attending and that is where I met Professor Rodrigo who is an important person in my relationship with mathematics.

Prof. Rodrigo was extremely passionate about teaching mathematics. He is the person that made me go into mathematics because he always expressed mathematical ideas in a way that makes it look like art. He always told me about the relevance of mathematicians in the world. On the one hand we have the applied math that is done to solve or understand a problem in any field of research, such as encoding information or modeling an equation that solves for the velocity on some system. On the other hand, we have math that is not solving something that we can use, at least not explicitly or that we know, but is also relevant because math is part of the human culture, so is relevant because it enriches our culture. While talking about those and in general, he seemed so happy doing his job that he made me think that I want to do whatever he is doing no matter what. Also, it was eye-opening for me that I could get paid by doing mathematics. When he told me that he was paid to do only mathematics, I thought "Wow, it is for me" and I hadn't doubt on it. So, I told my mom and dad that I wanted to study mathematics and wanted to become a mathematician. They didn't know what the job of a mathematician was. So, they were scared. Like, okay, like what if you don't find a job? And I remember telling them, "No, I'm going to study my PhD in somewhere else and get paid for it", something like that. So I convinced them, and I remember I risked. When I accepted the risk, what if I didn't like it? Or what if I wasn't good enough? That was also a question because that seemed hard. I went into math, not because that was easy but it's because I like the challenge. But, also the other question I gave to myself at that time was, "what if I find out I don't like it?" However, in the end, I just said, "No, I'm going to do that." And so, I decided to study for a bachelor's in mathematics at the University in my hometown, and it turned out I am good at mathematics, and studying mathematics was the best decision that I've ever made.

What It Means to Me to Study Mathematics

Since I started to study math, I encountered several barriers. I am going to classify them in two types: the mathematical difficulties, and the social barriers. When I went into my undergraduate, it turned out that I did not know what's mathematics, like how to solve abstract problems. At that time, high school mathematics was based on concrete things and calculations. I thought that mathematics was just solving geometric problems like intersections of circles and lines and applying that to very simple mechanics like when you throw a ball in the air and you get a parabola. I was still curious about calculus as I was reading the proofs in the calculus textbook in my high school. I remember thinking, "oh, here's like the general case." I didn't know that it was a proof, but I was rather thinking like "it's talking about the general case, and this works for anything," which was all about in my undergraduate mathematics education.

Now, I feel doing mathematics is following rules that we have to solve some abstract problems. But, sometimes we don't know when I am getting into something difficult or that has a deeper meaning than I can understand. I feel doing mathematics is like me swimming in the ocean. I am on the surface where the sun hits, which means there are definitions and tools that I know how to use and the result. As I am learning more mathematics, I am enlightening the water under me. But there are a lot of waters below me. I don't know how deep the water is or what is deep down there. Sometimes it will be shallower than me so that I can easily make it brighter. I am curious to enlighten everything. What will be under the water? This feels like a challenge, which I like and makes me continue to study mathematics.

Yes, mathematics is challenging because I still have so many things that I don't know, and the time that I spend studying mathematics has increased. However, I still enjoy doing mathematics. I think for me the biggest challenge was the social aspect of it. I am a very social

person and I love sharing my thoughts and experiences with other people like my aunts or my middle school and high school friends. Since my middle school and high school, I have been changed with my experiences. I like the current version of me and want my people to know about it and talk about such changes. But I do not know how to explain what I am doing. What I was thinking for my life like research and a job after the degree is very different than what my friends in Mexico are thinking. My friends around my age were talking about getting married and having babies. I don't get along with them as I did before. And at the beginning, that was hard to get, like you're different. So that's hard. And also, my friends are like all very proud of me and whatever, but the friends, also they don't understand what I do. And some old people even do not know what a doctoral degree means. When I talk about "doctor," they think about the medical doctor. For example, my grandpa is like, "Oh, I heard you're going to do a doctor. You're going to study to be a doctor. You're going to heal me, then?" And I'm like, "No, grandpa. I'm going to do this." I try to explain it to him in the simplest way. But then, inevitably, I said the word mathematics. And he was like, "I don't know what is mathematics." And so I was like, again, trying to explain to him. So, it takes me time to make people sort of getting an idea. It is frustrating that I couldn't talk about what I am doing with my people.

Despite this difficulty in sharing what I am doing for my career, I still like working on mathematics because I like defeating mathematical challenges. But then I realized that is not very common for someone to like mathematics, and then I found that it was less common for a woman to be a mathematician. During my undergraduate, I felt alone because there weren't enough women with me. My undergraduate mathematics cohort was small; we were only five. All of the others were men, and we did not have any female professors, so I was the only woman in the classes. Sometimes, one of the classmates disvalued my work by saying, "Maria got a

good grade because the professor just gave it to her," or "Maria did well on the exam because her exam was easier than ours." He always tried to bring me down. The worst part? None of the other classmates said anything to him. They did not stand up for me even though some of them were my friends. They didn't say anything. I don't know if they didn't want to get into trouble or whatever.

But I think it's something of the culture here in Mexico, like how we grow up and how they teach us, I don't know, in our families, because in Mexico, the people that are heard the most is the man of the house, like the dad or the husband. I think men in Mexican society have more freedom to speak their minds and not be judged, so if I think about the classrooms and who participated in them: Men, they participated and said like the dumbest things. On the other hand, I did not participate as much as my classmates even though I wanted or knew the answer. And I remember thinking, "I know the answer, but why I don't have the courage to raise my hand and say it?" At that moment, I didn't think much about it. Thinking back now, I was afraid to give wrong answers in front of others. Also, in my high school, my professors never picked me up even though I raised my hand, which was very frustrating, but I gave up participating in the classroom since then. Overall, I had a hard time feeling comfortable in the group.

I love doing math and the challenges that come with it, but I knew this situation that I experienced was unacceptable, and I think that from that moment on, my motivation changed; I was pursuing a career in math not only because I like it but because it is necessary for a woman to be represented in this profession. Like we need someone to represent us everywhere. Like, we need people from different contexts and also women doing that. I realized my role in this. I think for a long time, that was the main reason why I do mathematics.

To Graduate School

Although the department of science was small, it had professors from all over the world. All of them were particularly good professors, and Rodrigo was outstanding. I think one of the things that make him a good professor is that he is very friendly, but at the same time very hard on students. His classes were always the hardest ones because the homework was extremely hard, but they were also the most enjoyable ones. Also, he liked to get involved with students to help them in any situation besides mathematics that he could help with. He was enthusiastic about working with us and helping us to study mathematics. Indeed, many of my cohort members in the undergraduate said, "Rodrigo convinced me to get in here," as he contributed to my decision to be in that university as well. He also told us that the department is to prepare students to do a doctorate or master's degree somewhere else by teaching mandatory basic courses in analysis, algebra, and topology. He encouraged us to study more by saying, "If you don't get another degree, like a master's or doctorate, you have limited opportunities to work in Mexico, or even in maybe another part of the world." Rodrigo and other professors also recommended getting the degree abroad, which is why I am in the U.S. now.

It's been a year since I have been here, and I just passed all of my qualifying exams which I am very proud of. It's the proudest accomplishment that I have made in mathematics so far. Although it was a different environment than what I was used to, I successfully finished my first year in the U.S. I noticed some differences between where I was used to and in the U.S.; here are some differences.

English and Mathematics

The first is related to English. My English is fluent. I passed the English requirement and stopped taking English classes after middle school. Since then, I have watched YouTube Videos

in English and so on. I practiced my English with YouTube videos because there's a wider variety of videos I could watch in English than in Spanish. So, I forced myself to watch the videos in English and became pretty good at English. Also, in my undergraduate, the first four semesters were in Spanish, but the later four semesters were taught in English, which helped me when I moved to the U.S. because I was already used to hearing all the mathematical terms in English. I think the professors wanted to help us so that we were ready to go out of the university and study abroad. And English is everywhere.

At the beginning of the transition from Spanish to English in my undergraduate, I was confused a bit. I started doing the homework using English and then changed to Spanish and then English. I didn't realize that I was doing that until I read the complete homework to review it. Then I was thinking, "what the..." I was using Spanish sometimes. What happened was: In mathematics, you do like an explanation and then an equation, and "therefore," or something like that. So, I stopped at each paragraph before writing an equation, like, and thinking something, and then writing in Spanish, then thinking something, and then in English. Those pauses caused me to mix Spanish and English, I guess. Maybe if it were a continuous paragraph, that wouldn't happen. I think it was because of my brief breaks in writing it. Also, there were times when "Oh, I don't know how to write in English" when I had to do homework in English. But as the textbooks were in English already, I could look at them and copy their languages. Then, I remembered and could use it later. But, I have been doing homework in English for a long time, so I am used to doing it in English now. I think in English when I am doing homework, and I don't have any issues with it.

The biggest difference here was the speed of English. The professors in the U.S. speak much quicker than the professors in my hometown. I think the professors in my hometown tell

slowly, even if they know English, and try to do it as naturally as possible to help us. Because... okay, I was like advanced in English compared to my classmates. So, professors knew they couldn't speak fast because not everyone was getting it. However, it was different here. For example, I remember taking the Real Analysis class in the first semester, and the professor was a native speaker. So, he did the lecture very, very fast. That was fast. In addition to the speed of verbal communication, sometimes he used contractions that were different from what we used in my undergraduate in Mexico. For example, to aggregate "continuous," we use C-N-T-S, and here the professors did it with cont, like C-O-N-T. So, when reading it that fast, I was confused, like, "what is this?" And then, from the context, I figured out what he meant, but he was very fast. He was explaining and speaking very fast.

Another difficulty was the accent. Like I remember I took a numerical analysis class in the first semester, and the professor was Chinese and had an accent. I remember not getting anything in the first class, but that was not my problem only because I talked with other classmates–including domestic students, and they were like, "I don't get it. I don't," too. But the good thing is that he always used slides, so everything was there, and he just read the slides, so I didn't need his voice.

Although I think language has a very important role in mathematics, I am glad that mathematics also has mathematical symbols and expressions to communicate with others. For example, Jihyun and I share the same language, English, but sometimes because English is not our first language for both of us, we cannot find the right word to express ourselves. In that case, one of us grabbed the chalk, went to the whiteboard, and wrote what we wanted to communicate. Then, we get the idea with symbols. So, maybe symbols are another language, but I think it's the simplest one. For example, when we see the less or equal symbol, we both understand. But if

someone says that A is bounded with B, I don't get it in my head right away. And then when someone starts writing it, I become, "oh, I see what you mean this." Of course, language is important in the communication of mathematics. In mathematics, you have to be very specific, which I like. Also, discussing and sharing how we think and perceive mathematics is fascinating, and I like it. Everyone is different. I like listening to others and sharing mine. I think languages like English and Spanish are not the only way to communicate mathematics.

I appreciate that we can communicate using mathematical symbols, but sometimes only listening to abstract things like mathematics is not easy. For example, when I have to understand some abstract concept, maybe I need to write down some examples or rewrite the definition in other ways, such as expanding something or using another definition that is equivalent to the original definition so that I can understand it. Or, even when some people tell me a complicated mathematical expression such as the definition of liminf of sequence, "liminf of x_n when n goes to infinity is supremum of infimum of x_m when m is greater than equal to n, for n is greater than equal to 0 " then I need to write down everything like this " $\lim_{n\to\infty} \inf x_n := \sup_{n\geq 0} \inf_{m\geq n} x_m =$ $\sup\{\inf \{x_m: m \geq n\}: n \geq 0\}$." Written things are important in communication.

But overall, I did not struggle a lot in doing mathematics with English. The more challenging thing for me was knowing cultural norms to interact with people in the U.S. I like joking with people a lot. I was confident with the friends in my hometown and always joked with them, and we had a good time together. Maybe it's because we've shared a lot of time together already. But now, at this current university, I am unsure if people will get offended by what I say. I need to know the limit first, and that has been slow. I now know whom I can joke with as I was in my undergraduate. It was similar when I interacted with professors, too. I think the mathematics department in my hometown was more casual. Like, the interaction with the professors was more relaxed. Maybe we-both students and professors-knew that we were very respectful to the professors already, so there's no need to put boundaries for the professors. But, now that I'm studying at my current university, I don't know how to interact with professors. It's awkward because, in my undergraduate, everything was very easygoing. I didn't have to think much when sending an email or whatever. But at graduate school, it's harder for me. I think twice about sending an email, maybe also because of English. I don't know the right words to express myself, but I'm a little shyer with professors here.

Another one is a hug. In the teaching orientation that I had in the U.S., the session's leaders told us that if we wanted to hug someone, we ask first. I get it because some people are not comfortable with physical contact. But in my hometown, you don't ask anyone that you hug someone. I feel the United States way is more professional in some way, like putting a distance or understanding that everyone has a different background and not everyone is comfortable with the same things. But now I know with whom I can do that. But that was tough because I'm very social. I want to interact with people, so at the beginning, I didn't feel free to express myself in the way I wanted to because I didn't have anyone that understood my jokes or my feelings to hug someone, right? So that was a hard time in the beginning.

But I am enjoying my time at my current graduate school. One thing that I really like is having fellow women in my cohort. We have six female Ph.D. students in my year, and I feel more comfortable being in a mixed group. I realized that I was not comfortable with my undergraduate cohorts compared to my experience at the graduate school. I don't see (at least among my classmates) misogynies behaviors. So here I feel more normal, which is good. I sometimes wonder if my role in representing women in mathematics became less clear, but it is still an important purpose of doing mathematics. In the future, I think I am going to still do

mathematics and like it. Mathematics has given me so much. It gave me a purpose and the opportunity to be more independent.

I still love mathematics, and I like that there is always something to do in mathematics. As I said earlier, I want to see what is under the deep water. It's been only a few days with nothing, but I already want to go back to something interesting, which is mathematics for me!

Reflection of Maria's Narrative

Maria's narrative shows her experiences around mathematics in chronological order, from her K-12 education, undergraduate school in Mexico, and graduate school in the United States. Her relationship with mathematics has changed depending on the stages where she was. Overall, it seemed like she enjoyed defeating mathematical challenges. As shown in her metaphor in the narrative, she was curious about what would be below the water she was floating around and was interested in illuminating the dark side, which became one of her motivations. The two main points from Maria's narrative are her gendered experiences as a Latina woman and her changes and challenges in social interactions around mathematics and/or being in the United States.

Being a Latina Woman in Mathematics

Winning in mathematical challenges was also related to Maria's other motivation; Maria was pursuing this career because she believed it was important to have women representations in mathematics, a belief that came from her challenging experiences as a woman in mathematics. There were no woman professors in her undergraduate university, and there were no women in her cohort year. Further, her undergraduate cohorts sometimes minimized her work as if she could not contribute to mathematics, while none of her men friends in her cohorts fought against these misogynies, which made her uncomfortable being in her learning environment.

Maria was also uncomfortable asking questions in front of others. There were experiences that influenced Maria's fear of giving wrong answers in front of others. Maria first speculated that it might be related to her memory of her high school professors never picking her to ask questions and her being frustrated about this. Her story echoes other cases where teachers call on female students fewer times than on male students (Sadker et al., 2009). Also, the aforementioned experience can negatively impact a sense of belonging in the mathematics classroom (Reinholz et al., 2022), suggesting that, in a similar way, Maria might have become more afraid to ask questions.

Together with gender discrimination, the authority of the teachers and her cultural norms of how to react to authorities might have hindered her from asking questions in front of others. As an undergraduate, she had one particular professor who yelled at students, so she wanted to avoid asking questions. This showed that the professor was authoritarian and reminded me of the experience of Isabelle, a Latina student from Leyva et al. (2022). Isabelle shared that it was not easy to argue with people with authority, and she had to accept what the authority said in her culture. She further explained that such a cultural atmosphere might influence Latin@ students' participation even in tasks where they were asked to find and criticize mistakes. Maria's cultural norms could have been different. However, Maria also mentioned experiencing patriarchal dynamics in Mexican culture. Considering all the factors mentioned in this paragraph, Maria might also have been worried about asking questions because she might have felt that she was challenging the authority by not accepting what they were telling her.

When Maria came to graduate school, she mentioned that she was asking more questions in graduate school than before. There were six female students in her year, which made her realize how uncomfortable she was in her undergraduate program when she was the only

woman. Maria noted that she felt "more normal" in graduate school, which could imply that she felt she was "less normal" or "abnormal" before. Maria's story aligns with other female post-secondary international students' reports that they felt the United States is more gender-neutral than their home countries in their earlier years (Lim et al., 2021).

Social Aspects of Maria's Experience

As well as the patriarchal dynamics and gender bias in mathematics, she might have also felt "abnormality" in relation to how her friends' lives were unfolding at the same age. She said that her friends focused on getting married and raising children, which was different from her life. She chose a "different life" than what her friends were doing, a choice that goes beyond a one-way relationship–which is about how she feels about mathematics–because mathematics has also influenced Maria's social life. As mentioned previously, she was living a different life from that of the "majority" of women at a similar age in her hometown. Besides that, choosing mathematics made it difficult to share her life with her people, including family and old friends, even though sharing was something she had valued in her life. In turn, it restricted the number of people she could talk to. For example, she could talk about mathematics to fellow graduate students but not to others who did not study mathematics which had been tough for her.

Such a social aspect appeared as one possible challenge for international students as well, particularly as a part of acculturation into a new environment (Yeh& Inrose, 2003). In her culture, she was used to joking around and hugging people. However, as she moved to the new country, she had to learn how to interact with people by following the cultural norm in the United States. Maria indicated that she felt more restricted and shier in the current situation regarding social interactions. Given that she is such a social person, it is again striking how the environment and past experiences in the classroom influenced her behaviors to the degree that

she did not feel comfortable participating in her past mathematics classrooms. According to the literature, Latin American countries are more collectivistic than North American (excluding Mexico) or European countries that presumably value individualism. Within collectivistic countries, one's identity tends to be influenced by close others and one's connection with them (Yeh & Inose, 2003). This cultural difference could lead international students to feel lonely and is related to their social connectedness, and less social connectedness becomes an indicator of acculturative stresses (Yeh & Inose, 2003). Especially for Maria, who already felt distance from her friends by continuing mathematics and graduate school, moving to the United States added a physical distance on top of the existing stress.

Being a Multilingual from Maria's Experiences

Regarding being multilingual, Maria did not feel particularly challenged in using English as she felt prepared enough to use it. Maria stopped taking English classes after middle school and watched YouTube videos in English, which helped her learn the language. Also, during the last two years of her undergraduate education, her courses were in English, and thus, she started using English in mathematics. When she first started doing mathematics in English, however, there was a transition period between Spanish and English even though she had prepared herself during high school and the first two years of her undergraduate education. Until she became used to using English for her mathematics, she switched between languages (i.e., English to Spanish to English). To learn how to write mathematical explanations in English, she used the English textbook as a reference to learn expressions. However, thanks to the fluency that she had developed in English over the years, she felt her transition was smooth. Her challenges in English moving to the United States were more about different accents and the speed of English

or the choice of contractions (e.g., CONT or CNTS for continuous), which she resolved by reading the notes and slides and guessing from contexts.

Even though I could not find a place that flowed within her narrative for the following part, it felt important to note that she mentioned that being able to use English made more resources (e.g., textbooks and papers in English) available to her, given that she did not have a lot of Spanish mathematics textbooks in her hometown university library. In addition to her note that she appreciated being able to communicate and interact with her cohort members from across the world in her graduate school, this note also shows that her possible resources were expanded as she added one more language to her linguistic capacities.

CHAPTER 5: HELENA'S STORY

"Don't get me wrong. I am glad that I speak English, but Spanish is my language."

Hello, my name is Helena. I am a Latina from Colombia, currently studying mathematics in the United States for my Ph.D. I did my bachelor's degree and master's degree in Colombia before coming here. I've visited the United States before, but it's my first time living here. I just finished my first year in my Ph.D. program, and overall, I am grateful and feel blessed that I can study in the United States.

What I really appreciate about this Ph.D. is that I am in a really good environment. The campus is beautiful, and the buildings are well-maintained. I am especially thankful that I met good people from my program, my religious group, and life outside school. For example, from my program, everyone in the department is friendly and says, "whatever you need, you can talk with us," They also offer food at every seminar. Everything is awesome. I also have bonded with my cohorts pretty well from the beginning. There were many events in the first year, including preparing for the qualifying exam. We have been there for each other throughout the year and studied together for the courses. Not only study with them, but I also hang out and do parties with them very often. I love those guys. As well as my friends from the Ph.D. program, I found a really good Catholic community that welcomes me with their arms open. I am a Catholic and my religion and faith are important to me. So, I am happy to have a strong bond with the girls that I found in my Catholic community.

Languages–Spanish and English–to Me

I have other amazing people around here, who are my roommies. They are also Latinas, and it has been very helpful. Because, maybe I seem fluent in English, but I still struggle with English. Like some classes, I miss what a professor said. I'm like, "I don't know what he said." Or, if a lot of people are talking simultaneously, I lose track of the conversation or information.

These kinds of struggles sometimes give me some frustration, so having Latin@s around me and speaking Spanish with them is important to me. Don't get me wrong, I am glad that I speak English, but *Spanish* is *my language*.

I think a language is not just a language, but it is connected to how my world is defined. For example, what is this (Figure 5.2)? What is the first thing that came to your mind when I said, "what is this?" For me, this is "*botella*," and the English word is a bottle, but this is a *botella* to me. It's Spanish for me, and it will be maybe English, Korean, Chinese, or something else, for you. The word is not just a word. It's the object itself. When we learn an object, its name comes in a form of language. For that reason, many of the words that I've grown up with are in Spanish. However, there were certain words that I learned in English, too. For example, when I learned "basement," I learned it in English because in Colombia, "basements" are not common. So, when I need to speak in Spanish, "basement" would come first to my mind to talk about it even though there's the corresponding word "sótano" in Spanish. "basements" is the word that I learned to define that object. Like this, before coming to the United States, my world was defined by Spanish words. Spanish is how I define my world. I can express myself fully and completely when I am doing it in Spanish. So, I do love talking in Spanish. I love when I have the opportunity to speak in Spanish.

Figure 5.1 Botella



On the other hand, I don't think my mathematics world is solely defined by Spanish. This is maybe because I have spent a lot of time doing mathematics in English as well. At least, since my undergraduate study, I have used both English and Spanish. When I reached my second or third year in my university as a mathematics major, most of the materials were in English, such as textbooks and articles. My professors still used Spanish when they were writing on the board and speaking to us. As I moved further to specific mathematics such as TDA (Topological Data Analysis), there was no translation to Spanish. But for the majority of the undergraduate mathematics courses, we had Spanish translation. So, we had English references from our textbook, and we communicated with Spanish words. For such reason, I feel my mathematics world is built using both languages, especially undergraduate-level mathematics.

Also, contrary to how strongly I feel a connection between a word and an object in daily life, I see less connection between the word and the object in mathematics. Mathematical objects
are abstract, like love, minds, and God. I feel it's more like a name for a bigger concept than a concrete one. I do not have a concrete object for "1" because it's an abstract concept, so it is hard to say that "1" is "uno" itself. It's hard to put into a word, but it feels different than daily terms. However, I don't know why, but when I think about the very basic mathematical things, I go to Spanish. For example, if I need to count something, I go to Spanish without thinking. You know you count how you learned. "*Uno, dos, tres,* …" Even though I said that I do not think my mathematics world is that tightly connected to Spanish like my daily life, if I think about one of the abstract concepts, it is inevitably going to take me to the Spanish word that I can decouple faster. I don't know why. It's weird.

Although I don't know why I go to Spanish, I know this: English is a *tool* for me. Speaking about my experiences around English, I remembered that one of my friends in Colombia said "I love English, I like to speak in English." But I disagreed with her and still disagree with her. English is a way that I can express myself, and I can understand people in the United States or vice versa. For example, when I study mathematics, I read and write mathematics in English because I need to communicate with others. Especially in proof-based mathematics courses, I feel you need more English than in computation-based ones because you are "telling a story." Although I verbally communicate with others using English here, I think in Spanish when I speak to myself. I process the idea in Spanish and then translate it when I get it. (Maybe it's not all the time, and I am probably doing it in English as well when I am not aware. That's a possibility.)

I also need to use English as a mathematician if I want to succeed in the world of mathematics. English is the language of the world, which has some upsides but also a downside. All the research is in English; All the textbooks are in English. So, you are kind of compelled to

learn English if you want to publish in a good journal. The United States itself is a ... *potencia* (translated to "power" in English). They have great universities and top universities. (But a lot of their professors are from China, which I found kind of funny). But either way, everything is in English. It's a good thing because everyone can understand each other. So, even if you are from Saudi Arabia, I am going to be able to read your research because you are publishing it in English. But at the same time, it's like the people living in your country may not read what you are doing because they don't know English. That's bad. I hope Latin countries invest more in STEM education so that there's more resources and articles of STEM in Spanish.

So far, I've talked about how language matters to my relationship with the world and mathematics, but language also matters in how I interact with people. For example, although it depends on the kind of groupwork, in Colombia, I usually take the leader position in a lab or in writing an article, leading the flow of work, delegating tasks, and putting all the ideas together. But, I tend to be quieter in English than in Spanish. When I went to a summer school in Canada a few years ago, I didn't take the leadership position because I felt I was not an expert on the subject, and I felt insecure with my English. I still miss some information in English, and it takes a few more milliseconds when I am thinking and doing mathematics in English. So, sometimes it's challenging when I also have to understand other people in English while I am doing mathematics.

I also experience such challenges when I am working in the mathematics help room, sometimes I get stuck with a student. I know how to do it, but sometimes I cannot find good words to explain myself. As I need to teach a class next semester, I am really worried about that. What if I am talking, and my students don't understand me? I will try to perfect my English as

soon as I start teaching classes, but I think that's the difficult part. I am scared that I am not explaining myself well.

Even though I need to use English to communicate about mathematics with others, I still think mathematics itself is a language, which is interesting. For me, mathematics is a collection of words that we created to describe our mathematical elements. It's a new language that you learn in doing mathematics. It looks complicated, but you need to learn new terms and how to write them. It's a universal one because even if a textbook is in Chinese, the equations are going to look exactly the same, like $x^2 + 6x + 7$. No matter your language and how you would read it verbally, it will look exactly the same if it's in French or whatever. I think that's pretty cool. Because of these universal notations, I can follow classes even if it is sometimes challenging. For instance, I didn't understand anything from my numerical analysis professor, and it was not just me. All the international students had the same feeling, and the native English speakers agreed, saying "yeah, it's hard to follow." I think one of the main reasons was, I am still struggling with the Chinese accent and the professor was Chinese. I think it is related to that I do not know how their accent works. For example, when I am speaking with Latin@ people in English, we have the same accent in pronunciation, so we know what we're talking about. But the Chinese have their accent and Indians have their accent, too. So, I think both of them are the most difficult for me to understand, and there are a lot of people from those two in our department. I got three Chinese professors so far during my first year. Sometimes, it's been hard. Also, professors who use English as their first language speak fast, which is also challenging. The speed makes things difficult too.

So, anyway, when I was in the numerical class, I didn't follow the classes but just did read. I think it's lucky that it is mathematics and people write what they say. I can follow what

they were writing with a little bit of what they are saying. But, if they were saying something without writing it, it was like maybe I got it, or maybe I didn't. I am like "let's hope it was not important. I will ask my friends later." But the worse thing was that he used a PDF presentation and just read it. It was hard to follow what he said, but if I had read the presentation, then I would have about 80% of the information. So, there was no point in attending class and I stopped attending the course.

For me, the more challenging part of using English is using English in real life. For example, it happened when I was in Toronto for summer school. And there was a day I went to ask for a salad, and the people... the server asked me about the dressing. And he said something like, "Da, da, da dressing. Da, da, da." Like he presented me the three or four dressings. I had no idea what he said. And I was like, "... the first one." No idea what I asked for. So, the most challenging part here was talking with people. Another example is: During spring break, I went down to Kentucky to repair houses as a volunteer, and all the terms around repairing houses were so hard in English terms. Oh, my gosh. They were talking so fast. They were using slang. Five people were talking at the same time. We were in a car, so I just couldn't handle the noise and everything. So much information is blasting you, and you get exhausted.

Anyway, studying mathematics in English has been fine and not that challenging. I love mathematics.

Love for Mathematics

From what I remember since childhood, I have always been a good student, particularly a skillful student in doing mathematics. The first decisive moment in my relationship with mathematics was in high school; this is the point where you decide what degree you want to pursue. At first, I wanted to study something related to mathematics, but not mathematics itself

since I thought it was too nerdy and engineering was not appealing to me, so I decided on statistics. However, my mathematics teacher told me that I should study mathematics. Then, if I didn't like it, I could easily change to statistics, and I thought it was a good plan. I knew it was not a bad career choice. I thought someone who is a mathematician would always get a job since everything needs mathematics. Nevertheless, it was a pretty uninformed or naïve decision. In my mind, mathematics was about doing calculations, and I really had no idea what it was about.

Then, when I entered college, I realized how mistaken I was. Mathematics is not calculations; it is about abstract thinking. The first college class was proof-based and called "Mathematics Foundations," similar to set theory. The way to think proofs was something that I didn't encounter before. The closest thing we did in the K-12 education was geometry, which was really easy geometry for school... So, this kind of logic, like start with a statement, add other, add other. Then, you have other elements and do the proof. It's not only, "Take this matrix and find the determinant." It's more, "Let me show you why the determinant zero implies it's not invertible, and it's the same as being linearly dependent. And if you have like ten different statements, these following are equivalent, and the eigenvalues and everything is related, and so on." It was fascinating to me.

One professor said this in my first year: Being a mathematician is "living in the boundary of the knowledge," always pushing further, trying to discover properties and relationships of objects and structures that sometimes only live in our minds, and I *loved* it. It was completely different from what I was expecting, but it was for the best. He also added, "What are you going to learn here is far from the boundaries. You're going to reach those boundaries when you are far in there. Not even a bachelor, like far in your Ph.D. or something." And that moment expanded my horizon, and I remember I was thinking, "Wow, maybe I can go there."

Over the years in college, my love for mathematics just increased exponentially. On the one hand, I learned how beautiful and powerful they are: from being a tool to discover and describe phenomena from chemistry, biology, physics, medicine, and so on to creating equations, manifolds, and algebraic structures that only live in the realm of human imagination. On the other hand, I got deep satisfaction and enjoyment in doing mathematics. Even though sometimes they were not easy, I managed to understand them and achieved high grades in my college years. I think my learning had an exponential growth, learning itself is not linear. It's just that because, in the beginning, you don't know a lot of things. Well, you're learning calculus. Then, you're doing an algebra course. So, in the beginning, it's like, "Wow, this is interesting," but it's not the core yet. But then, it's when you're starting to do the deeper stuff that you start like, "Wow, this is cool!" So, not because it's growing really fast, but because it started really slow. I think currently it's not going to be the same slope as I've been in. I might have an inflection point in my growth, so I don't know where I will go from now. However, with my exponential love for mathematics, I decided to pursue a Ph.D. degree in mathematics.

One of the reasons that I am doing my Ph.D. it's because I want to become a professor. My mom is a teacher, and I really liked that she had the same vacation as I did. We could spend a lot of time together, which I'd like to have with my family in the future too. Also, I can meet people from various backgrounds if I work at a university. When I went to college, I really loved having the opportunity to know people from any field. As I played soccer, I met people from engineering and veterinary medicine, friends I wouldn't have met otherwise. It's a pretty integral environment. In an enterprise, you are with people that do exactly like you, exactly what you do, or even like maybe in the same field, such as people from human resources or engineers. But here, in principle, you can meet people from any field. Also, students are young people, so if I

get old, what I'm going to do? Everyone's going to get old. Even though students are tired or whatever, they are young, so they have a different energy than older people. Other reason is education. I think education is the basis for everything. So, being part of it, like passing knowledge to new people, I really like the idea and I want to do it. I know people don't like academia in part because it doesn't pay enough. Maybe it's biased, but it doesn't pay well in Colombia. And here, whatever university, it's going to be better than in Colombia, I am already saying like, "If I work here, for example, I'm going to be paid better than in Colombia. So, anything you're going to offer me is perfect." Like, I don't need a lot of money. I don't want a lot of money. I don't care about money. So academia, it's a good place for me.

Nonetheless, being a professor does not mean everything to me. Back in my bachelor's degree, I put a huge part of my value as a person in being a mathematician, but after different experiences, I realized my career is an important part of my life but not my priority. There are things beyond mathematics and my profession: God, family, friends, and life itself. At this point, I do love doing mathematics, and I'm passionate about my future research, yet, they are my job, not my life. No matter what I do (professionally), no matter my mistakes, and no matter my relations, God loves me, and I'm worthy for just being me.

Why am I sharing my experiences? I think everyone has rights and duties. I think this is a duty as an international student. Jihye needed me and my experiences for her research, and I have time, so why not? Also, I think it's cool that we can share our experiences. I don't know, but maybe what I am doing now is going to help someone in the future. It's one of my philosophical models. That is, everything that you do in your life is transcendent, no matter what. Like even if I decided to pick up trash today from the ground. It's going to be transcendent. How? You don't know. How much? You don't know. But it's better to live that way because

everything has importance. For example, after you graduate, what is transcendent is not just your grade, but is going to be the knowledge and experiences that it's going to be useful for others in the future. So, I think that's the reason.

Reflection of Helena's Narrative

Helena's narrative shows her interesting perspective on language, as shown in the title of her narrative. Helena talked about her perspective toward languages, Spanish and English, and Mathematics as languages. Then, the narrative continued with her journey as a mathematician, her love for mathematics, and her future career.

Helena's lens to look at the world was closely tied to languages, especially Spanish. She said, "Spanish is *my* language," which contrasts with her saying that English is just a tool for her. She noted that the word is the object itself, especially when the objects are concrete. I felt it was related to who she is and how she sees her world. Her world was defined mainly through Spanish. Helena defined her world using Spanish, and Spanish is the language in which she feels most confident about expressing herself. According to the definition of identity in the current study–"how people understand their relationship to the world, how that relationship is constructed across time and space, and how people understand their possibilities for the future" (Norton, 1997, p. 410), Spanish is closely related to her identity. Spanish is the main sources of meaning with which she has built her relationship with the world and hence part of her identity as well. Helena's narrative is an important example to reflect and imagine how multilingual students, especially those whose first language is not LOI, could feel in their classrooms. They are losing their primary source of meaning in the classrooms.

In Helena's case, as her undergraduate mathematics education had some English components, such as textbooks, she did not think her mathematics world was solely defined by

Spanish. However, when she was thinking about mathematics, she admitted that she inevitably went to Spanish first. Helena noted that it was weird, but from my perspective, maybe it is natural, given that her world is defined in Spanish. She added that she still thinks in Spanish first and then translates it to English when she tries to make sense of some mathematics. For Helena, the reason she translated to English was to communicate with others; thus, she did not have the need to force herself to think in English when she had a more comfortable language, Spanish.

English as a Tool of Power

It was clear that English was a tool for her to do mathematics and to live her daily life in the United States, which contrasts how she thought of Spanish. If mathematics educators are not aware of different languages and pretend there is only one single language in the classroom, then they are neglecting a part of students' identities in mathematics classrooms. Engaging with different languages can impact one's actions in their learning environment. Helena said that she became quieter in English, and she took different roles in groupwork in the United States because she still felt insecure about her English. In particular, she was worried that she would miss some information and was stressed about needing to understand others' English while thinking about mathematics for herself.

Helena also added that she needed more time to think about mathematics in English. Related to this, Helena mentioned that she might be using English when thinking as well, even if she might not have been aware of it. The specific process of how she thinks and why she felt she needed more time is not within the scope of the current study. However, how multilingual students think and do mathematics using multiple languages is still understudied, even though it is necessary to have a better understanding of this issue to serve them better. Understanding how

students use their languages in doing mathematics at the moment, in addition to self-report reflections of their actions, would be beneficial to understand multilingual students better.

Another interesting point from Helena's narrative is that English is the language of the world. In the United States, English is the LOI, so often, it often has more power and is valued more in the classrooms. However, Helena raises this is true outside of the classrooms as well. Because the political status of the United States and the institutions in it, English has power. To write in a good journal and to read recent research, Helena said she needed to use English. It is related to Barwell's (2018) perspective on language. He criticized that viewing languages as resources could imply that the role of language is neutral. However, language is related to various ideologies, social norms, and people's identities, and thus, it is not neutral. Barwell viewed language as "stratified" and "stratifying," which aligns with Helena's perspective about English. Languages are already stratified. Depending on the context, one language is preferred to other languages, such as English is preferred socially in the United States classrooms. In turn, this stratification influences social stratification (Barwell, 2018). People tends to unitize form of their languages, including certain terms or different languages, during the conversation. In that unitization, already existing hierarchy between languages plays a role and often it strengthens the existing hierarchy (Barwell, 2018). Through this process, languages other than English are less valued than English in the classrooms, this can cause issues and can negatively impact multilingual students' identities. For example, some multilingual students pretended that they are monolingual English speaker hiding their other languages (Rios, 2022). In addition to this, multilingual students shared deficit positioning of themselves, such as a novice, unintellectual, or as a burden (Rios, 2022). How to value various languages in linguistically diverse classrooms is a task that mathematics educators have to solve together.

Mathematics as Language

While Helena thinks of English as a powerful tool for communication, she thinks of mathematics itself as a language. She described that "mathematics is a collection of words that we created to describe our mathematical elements. It's a new language that you learn in doing mathematics. It looks complicated, but you need to learn new terms and how to write them." This description resonates with Sfard's (2008) perspective that mathematical objects are discursive constructs. Sfard viewed mathematics as a discourse about discursive (mathematical) objects. However, Sfard did not agree with the understanding of that mathematics as a language or a register, which seems to contrast Helena's understanding of mathematics itself as a language. Sfard did not think of that mathematics itself as a language because 1) she viewed discourse as a human activity and language as a symbolic system, and 2) for her, discourse includes more forms of communication than just verbal languages. In her narrative, Helena focused on the mathematical symbols that she considered universal and on the specific meanings of the words that were defined in mathematics. This understanding seemed more related to the mathematics register (Halliday, 1978), a set of specific meanings of language, including words, grammar, and modes of writing, in mathematics. However, Helena's quote frames mathematics as created by humans and as something that she uses to communicate with others in different modes, including nonverbal. Moreover, it seemed like Helena was focusing on "language" as meanings rather than just a series of alphabets. Thus, Helena's understanding sounded similar to Sfard's definition of mathematics.

Even though I interpret that her meaning behind "mathematics itself is language" goes beyond the symbols, the more or less universal symbols became important resources for her

learning. As noted in the narrative, when she missed some information because of the accent or speed, she could use the written symbols as a resource for her mathematics learning.

Mathematics for Helena

Helena's view of mathematics before she decided to study mathematics was "it is too nerdy", which is also related to the common portrayals of mathematicians (Wilson & Latterell, 2001). She noted that her mathematics teachers were role models at that time since she did not know big names like Gauss. The teachers were diligent and smart, not lazy, and she equated good mathematicians with good students. But she did not feel that mathematicians were cool. Wilson and Latterell reviewed popular culture in which characters are mathematicians and found that the focus was often on mathematicians' mental illnesses and that mathematicians were depicted as lonely figures. Moreover, when they asked elementary students to draw mathematicians and caption their drawings, students drew "bald men with glasses and beards, who were working at a blackboard or a computer," with some captions mentioning mathematician's lack of social life, who only thinks hard about mathematics (Wilson & Latterell, 2001, p.177). Even though I do not know Helena's childhood exposure extent to popular culture, her image was not surprising given the floating-around narrative about mathematicians.

When she was learning about proofs, however, Helena changed her mind: mathematics is not about calculations but about abstract thinking. The chain of logical thinking and figuring out how to expand the area of mathematics using those types of abstract thinking was fascinating to her. Throughout her essay, many positive adjectives about mathematics appeared, such as fascinating, beautiful, powerful, and satisfied, which really showed me how much Helena loves mathematics. She noted that her love for mathematics increased exponentially as she learned

more mathematics. Helena thought that she might be at the "inflection point" with her growth, but she still loved mathematics and was pursuing a mathematics Ph.D. degree.

Helena's Religion

Helena is a sincerely religious person, which is a big part of her identity. The ideas related to her religion came up several times in her essay and during the conversations. It seemed like she spent time thinking about abstract concepts, such as God, love, and minds. Maybe somewhat related, Helena said that mathematical concepts existed only in the human imagination and our minds when she was describing why she loves mathematics. I wonder if existing in the abstract was a common aspect of religion and mathematics. The last example of her relationship with mathematics and religion is her sharing that her mathematical career was not her priority, even though mathematics is important to her. Her faith takes priority, as well as having a good relationship with her people. Helena believed that she was worthy of being herself and that God loved her no matter what. I wonder if that helped Helena stay balanced and be less swayed by the external environment changes, such as moving to the United States. At the same time, she believes what she does in her life will help someone in a transcendental way, which might also be related to her Catholic faith. The belief that her actions would be transcendental led her to participate in this study as well. She viewed participating in this study and helping me as a duty as an international student, and she would be rewarded in the future in some way somehow.

CHAPTER 6: JIHYUN'S STORY

"Does Paradise Exist Nowhere?"

Hello, my name is Jihyun. I am a Korean woman studying mathematics for my Ph.D. at a large public university in the United States. I will share my experiences studying mathematics in the United States as an international student and what I realized through those experiences.

Compared to My Experiences in Korea

The obvious difference in my transition from Korean to U.S. education was the change in the language of instruction from Korean to English. In my undergraduate in Korea, there were mathematics courses taught in "English," and many of the mathematics courses used English textbooks. But, it was taught by Korean professors, so it was "Korean-style English." In the class, professors used English words for the mathematical terms, but the prepositional particles and the connecting words were in Korean. The numbers were also in Korean as well. So, even though I had some mathematics classes in English, I was nervous about taking courses in English in the U.S. when I first came. I worried, "What if I cannot follow what the professor says?" I really had to focus fully on the class. I did not want to miss a single sentence. It was more for English, not for mathematics. After 50 minutes of class, I was drained. I did not have the energy to study after taking classes.

One example that was more for English than mathematics was: I was taking an online PDE (Partial Differential Equation) class in my second semester. What was really tough for me was that the professor was reading while scrolling through a PDF. He was not a native English speaker either, and he was speaking really, really fast. I knew I should've focused on the contents, but I was more focused on English. The contents were Hilbert space or Hölder's inequality that I already knew, but I still couldn't understand and was like, "what is he saying?" In the end, I did not watch the video but just read his notes more than 100 times.

These are some other examples of small things that take my energy: I can't visualize mathematical expressions in my head when someone is telling me mathematical expressions verbally, which is different than when I am using Korean to study mathematics. When I hear mathematical expressions in Korean, the image of the expressions appears in my head without effort. Not being able to have a visual image in my head is not a big challenge, or it is maybe tedious, and I need to write it down to visualize it, so it takes up my energy. Especially when it has fractions of multiple complicated expressions, it takes some time for me to process it.

Another example comes from similar sounds but different meanings. For example, when someone says "[ɛks] plus [i]," which means "x+e," but I sometimes think of "x+2" because we read "2" as "[i]" in Korean. Now, after two semesters, it has been getting better. Say, if I were using 100 of my energy at the beginning, I might use 70 now. I still miss some things, but now I kind of know what I am confused about and what to do.

Because I want to continue my mathematics career in the United States, I try to use English to work on mathematical things even when I am thinking in my head. If the sentence is simple or the problem is easy enough, then I think in English. Or, when some questions come up while studying English, I write in English rather than Korean, which is a change from when I was in Korea. Meanwhile, when the sentence is complicated and long or if I am stuck understanding some mathematical things, I think in Korean and return to English. Although it's not a big change, I think it is a benefit that I speak two languages. Korean and English have different structures and orders in sentences, so I can think of one mathematical thing in two ways. When I read out verbally, processing a mathematical definition in Korean and doing it in English have very different orders, which is sometimes helpful. This leads to another interesting point about mathematics. Even though I ran into some people who use different languages, such

as Russian or Chinese, we might still be able to communicate with mathematical symbols even though we process their meanings in different languages. I heard this from somewhere, and I really do agree with what that person said after I thought about it. Since then, I have thought mathematics is language free.

If there's another thing, I feel my learning experience is different from my domestic peers, it is how we understand mathematical terms. For example, when there are "locally compact" or "locally convex set," for me, "locally" in English does not carry a specific meaning or context to it because it is not the word that I have used in my daily life. Rather than guessing meanings from "locally," I think "locally convex set" is one mathematical term. Another similar example is "continuous." Maybe monolingual English speakers would imagine some kind of image or something, but for me, the mathematical definition comes first, something like "there exists an open ball and blah blah...." And I think sometimes it takes a longer time for the meanings of such terms to come across to me.

Different Culture?

While studying in the United States, I was sometimes surprised by how freely people asked questions. This is one example from today's colloquium. The colloquium speaker drew a diagram on the board during the talk, and they noted the domain as omega and the boundary as J. As the domain had a hole in it, the boundary had two pieces, including the inside one, and the speaker said, "J is not connected." Then, one professor questioned, "You mean simply connected?" Then the speaker was like, "huh..?" So, the professor thought "J" was the domain, not the boundary. I was thinking, "if you read the board, then you could figure it out," and that was really amusing to me that they could ask such questions. Something that you could know if you think about it for one minute, but just asking questions to the speaker in public. It is similar

in classes too. The guys in the classrooms just say things regardless of whether their answer is valid. I sometimes wonder how they could ask such silly questions. Other times I think they look smart and I become jealous while questioning, "am I not talented as a mathematician?" When professors ask us questions, it is hard for me to answer right away because it takes me time to think and speak. I need to think at least three times before I verbally say something. I feel native English speakers think faster than I do. Combined with my personality, I also need to think about English. When my classmates discuss, I am busy following what they say. I do not have enough time to also think about the topic. But it is hard to tell this to my friends, and honestly, it would be annoying if they said something like, "Oh, Jihyun is not good at speaking English, so we need to wait for her."

I think maybe it's coming partly from Korean education. For example, in elementary school, none of my teachers said "that's a great question" to students' questions. Even to speak up and present in the classrooms, the norm was to wait to raise our hands until the teacher gave us a chance to speak up. Compared to that, here, even when professors are explaining, we could raise our hands. Also, there were not many discussions during my education in Korea. I would say my education was more "cramming education," where you are asked to accept what teachers tell you and memorize and use those. I was rushed to learn the contents and was not used to asking the question "why." For example, in proof-based mathematics courses, there are certain ways to prove things like contrapositive, mathematical induction, contradiction, etc. I have never questioned these methods, and it has felt natural to do it that way. I do not know how I would behave if I didn't behave as I did. Maybe asking more questions? But about what? I think this is maybe a consequence of cramming education. I kept using things and applying things before I understood, and then I forgot about the "why" part.

Another example that shows different styles of education (or language-related difference) is how to write proofs. When I am writing proofs, I feel I lack vocabulary. I usually use pretty similar sentences in writing proofs "Assume not," "Suppose," "therefore," or "hence." These are almost it. Then I saw one monolingual English friend's proof in his assignment. Even though I was aware that he is a lot more talkative than I am, I was surprised to realize that he used a variety of forms of sentences. The basic structure of the proofs is the same, but there are more filler words in the proofs. It's more like I am using templates that I always use, but the native speakers were really writing an essay and having a conversation with others. That felt different to me. Sometimes I am jealous, and other times I wonder if it's just not my style. I sometimes skip some details of the proofs, but at the same time, I wonder if I skip the details because "I really do know well" or because "I don't know how to express that in English or do not want to be bothered to think about possible expressions."

Perfectionism: Gender, Race, and Ethnicity

Experiences in Korea: Gender

As I said earlier, I think three times before I say something. It is related to my perfectionism which developed over the years. I believe that every adult's personality is formed based on their past experiences, and I am not an exception to it. So, to explain where my perfectionism might have come from, I need to tell you about my childhood.

I am the oldest daughter in our family. My dad and grandmother really wanted to have a boy. My grandmother never bought me snacks or good toys and pressured my mom, asking why she couldn't have a boy. My dad always bought me blue things and never bought me a doll. We were rich enough to buy more than 100 dolls, but they never did that for me. I remember when my younger cousin was born, my grandmother really adored him. It was different than how she

treated me. As you can see, my grandmother is conservative, but she complimented me only when I did well in mathematics, saying, "Oh, this girl is good at mathematics," and "it's interesting that she likes mathematics." I liked that my grandmother showed her interest in me. Even though I was good at all the other subjects, doing mathematics well made me feel special. People say STEM is for men, but I was doing better than these nerd boys.

I also had several experiences related to mathematics and being a girl outside the home. I also started learning mathematical content beyond my school grade when I was nine, and I was pretty good at it. It was not part of my public K-12 education in Korea. I went to some private academies called "*hagwons*," which are for-profit private institutions or academies where students can take remedial classes to catch up or take advanced classes to learn beyond their grade levels. My first teacher in my first mathematics hagwon was stubborn and strict, but I couldn't be happier when she complimented my work. She thought I was really good at understanding and solving problems. There was a boy who was jealous of me and wanted to have all the teacher's attention. Because he was not ready for the content I was learning, I had to have lessons before the official time or run to the restroom to avoid him when he appeared. I felt that I was exclusive and special, and I learned the content ahead of others. That teacher made me a little bit more interested in mathematics.

Later, I moved to another hagwon which was preparing students to enter "science high schools," which are supposed to be high schools for talented STEM students. There were 13 students in the classroom, and I was the only girl. I pre-learned mathematics content, but not the sciences. I had always been the first in any classes before this, but I was not in that class. And I was treated badly. When I asked questions, teachers dismissed my questions as if my questions were not important. Also, a chemistry teacher in that class said something like, "여자 카가

들어와서 물 흐린다(A girl makes the water cloudy)," which is similar to "A rotten apple (= a girl = me) spoils the whole barrel" in English. I decided to study science harder and show that I could do better than these boys.

These experiences have influenced my tendency to be perfect. I always wanted to get everything correct because I wanted to meet the expectation that comes from compliments. I also had to prove that I was better than boys and hide what I was not good at. Even though I did well on a hundred things, if I miss one thing, I become a person who is not good at it. People would tell me, "I knew you were like this because you are a girl."

Experience in the U.S.: Race and Nationality

When I moved to the United States, there were two additional aspects that I came to think of: I am Korean and Asian. Sometimes I feel I am representing Koreans, since I might be the only Korean they know or talk to. That's why I always have to be nice to people and humble. If I do something wrong or say something stupid, I am afraid they would think every Korean is neither good at mathematics nor nice, etc.

I know it is weird to say it because I am studying mathematics, but it is a prevalent myth that Asians are good at mathematics. To be honest, I know I have that bias too. When someone comes into the mathematics help room, I automatically assume they would be good at mathematics. I mean, it can have pros and cons. It might be good for me because other people would think I would be good at mathematics. But, on the other hand, it puts me under pressure. Again, I need to meet that expectation. If I do one thing wrong, then people would think I am an outlier as an Asian, or I can give people a chance to talk badly about Asians not being good at mathematics. I am trying to leverage this bias to be a good thing for me. One day, when I am

seeking a job, I hope being an Asian woman can be a benefit as the U.S. always talks about diversity and diversity.

In contrast to people expecting Asians to be good at mathematics, they anticipate Asians not being good at English. I really try to speak better English, with a better accent and pronunciation. None has told me my English is bad, but I can feel it. When I walk downtown with my White guy friends, people never ask me for directions. I don't know if it is because I am Asian, a woman, or maybe both. I feel that people assume I will not understand what they are saying. It's not explicit discrimination, but I feel that way. I really wish I could fully express myself in English.

I have really worked hard on improving my English. As mentioned earlier, I try to think in English while doing mathematics and taking notes during lectures. I go to my office to talk to my cohort members even though I enjoy studying alone more. Even in my daily life, I try to think in English. For example, a few days ago, I fell down the stairs. Rather than standing up, I was sitting there for more than a minute, thinking, 'how should I say that *I fell down the stairs* in English?' Or, while I was talking to my brother in Korea, he said he saw "*on the news, a mother who neglected* (岂大(하다) a 6-year-old child with a disability was sentenced to 20 years in

prison." Then I was thinking, 'how should I say this sentence in English? What is 방치하다 in English?'

I know I could have just said that 'left him alone,' but it does not have the nuance for '방치하다' in Korean. There was a book that I liked and read about half in Korean. I started reading the book in English, and I couldn't move on when I did not know some words. I opened Korean and English books and compared the same parts to see how people translated them. Also, when communicating with others, I focus on people's expressions. When someone uses a certain expression that I did not know of, I think, 'Oh, that person expresses this in that way." Then I forgot what I was thinking and what I was about to tell them. When I return home, I try to remember what that person said and get upset when I cannot remember.

Like this, I tried to think in English. Overall, I feel pretty comfortable using English now in day-to-day life. But, I cannot fully express myself in English as my English vocabulary is limited compared to my Korean vocabulary, and I feel dumb in English. During summer camp, I was hiking with my professor. When we did about 1/3 of that hike, my professor asked me, "how far are we?" I wanted to tell him that we were at "1/3" of our hike, but I couldn't think of "onethird" at that moment. I know I could've said "one over three," but I felt that was not the right occasion for "one over three." I was frustrated. Not just mathematics, but there are four-character idioms and sayings in Korean that I do not know in English. Maybe I can make sense of it if I listen to it, but I cannot think of those off the top of my head.

Another part that I am developing is my pronunciation. In Korean, "L" and "R" correspond to the same character, " \equiv ." So, it is not easy for me to pronounce "r." For example, it is hard for me to say "real analysis." Once, I said "real," and someone heard it as "weird." Because it can cause miscommunication, I try to pronounce it correctly and try to erase my Korean accent as much as I can. I really wish for English to become like my first language. That's why I think in English, and speak in English even if I am alone. Now, I sometimes dream in English. Maybe it's a good sign?

Imposter Syndrome

Having said all of this, I have imposter syndrome. I even told this to my advisor when I asked him to be my advisor:

These are some things that you need to know about me. First, I can communicate in English, but sometimes it is difficult for me. I am not good at English; I don't think I speak English really well. Second, I think I am not good at mathematics, so I wonder if I have potential in this career and am curious about what you think about it. Overall, I have imposter syndrome.

Thankfully, my advisor said that he had not thought that I was not good at English. I told him, "it's been less than a year since I started speaking in English. I cannot be good at it." I added further that when I was in Korea, I had read passages and books and knew what English sounded like, but I had not been in a conversation while I was thinking at the same time. It is hard to follow if someone just spits mathematical things verbally without writing. Not just for mathematical expressions but statements too. So, I told him asking to be patient with me. He assured me that I could always tell him if I had some concerns and asked if I had any concerns at that time. Soon, I will do a reading course with my advisor and another student who is a male and a senior to me. I told my advisor was worried about 'what if he goes too fast and I cannot follow the contents but stay quiet and not be able to ask him to go slower?' my advisor said if that happens, he will have a separate meeting with us or tell the senior student to go slower. We will see how it goes.

One more difference that I noticed in the United States was that my peers were referring to themselves as "mathematicians." Because I do not have a Ph.D. degree and am still a student, I hadn't considered myself a mathematician. As I said earlier, I still sometimes question myself if I don't have talent as a mathematician when I need much time to think before I verbalize my thoughts. So, it was a surprise to me when I first heard my peers think they are mathematicians, even though now I am trying to remind myself that I am a mathematician as well.

I started studying math because I thought I was good at it. Then I decided to become a mathematician since I felt exclusive when I did mathematics, when I came up with a brilliant idea to solve one problem, or when I realized some concepts through my heart. Nevertheless, I sometimes feel that I have come too far with mathematics. I had chances and resources that I could go to other routes than mathematics to make more money and more "honor," like being a medical doctor or lawyer. My dad wanted me to be a lawyer, but I insisted on studying mathematics. I sometimes feel that maybe I am doomed to be a mathematician. While preparing qualifying exam, there were times when I became gloomy. I thought if I gave up this qualifying exam and went to Korea, there would be no place for me. I felt I would be stigmatized as a "loser" because it felt to me that I was losing and failing. I was thinking, 'who would hire me if I even couldn't endure this one year?" I felt there were no other paths for me anymore.

I am continuing to study mathematics, but I do not think I am in love with mathematics. When I think about what love is, I feel there is an end to love. There are some ups and downs in love, and I feel there is also hate in love. But I do not have such a feeling for mathematics, so I cannot say I am in love with it. I had ups and downs, but I am rather calm now. I do not have many emotional interactions with mathematics anymore, and I do not think about why I am doing this. It's natural, without question. As I thought maybe I liked mathematics just because it was the only thing I had studied intensively, I explored other subjects during my undergraduate. I hated sitting in calculus classes passively, so instead, I explored my interest, read many books, and tried Chemistry, Physics, Philosophy, etc. But nothing caught my interest. Since then, I felt "I just need to do this. This is my path." Mathematics is my life. Mathematics is who I am. I am doing mathematics because it is all in my life. I am addicted to mathematics. Even if my life is doomed to be a mathematician, I like how my life goes with mathematics.

Reflection of Jihyun's Narrative

Jihyun's narrative starts with the differences and challenges she noticed as she moved to the United States after her master's degree in mathematics in Korea. It leads to parts about different cultures and education systems and then her personality of perfectionism. She explains possible experiences that influenced her perfectionism, many of which are connected to being a Korean woman. In the end, she finishes her narrative with her imposter syndrome and her current relationship with mathematics. Jihyun's narrative highlights language differences between Korean and English.

Languages: Korean and English

Throughout her narrative, Jia's experiences were shaped by four different themes: Language, Nationality, Race, and Gender. The change of LOI was apparent and had a big impact on her experiences in the United States. Her language-related experiences share some characteristics with what other participants mentioned, such as accent, speed, and the importance of written communication, which are discussed in Chapter 8. In addition, Jihyun provided detailed examples of her challenges and confusion using both languages, which may be easier for her to share because I speak the same first language and have a similar background to hers. For example, she mentioned different structures and grammar between English and Korean. Figure 6.1 is an example of the different order of sentences in Korean and English regarding the definition of convergence of a sequence. As shown in Figure 6.1, the order of the sentence structure is the opposite.

Figure 6.1

Comparison of Korean and English Versions of the Definition of Convergence



The different order does not appear only in full sentences but also in a part of the sentence or specific terms. For example, when she read a fraction, there was a difference depending on which language she was using. In English, people read 1/3 as one-third or one over three, which is always read numerator first. However, in Korean, people read 1/3 as "3 분의 1" (a total of three parts and one of them, if I had to translate in order), which reads the denominator first. Jihyun probably would not translate some of the words for more advanced mathematics content and keep them in English. Jihyun said that she uses English words for mathematical terms and uses Korean prepositional particles, which means the sentence's general structure and

These different orders can mean that there is a different way to think about and understand the same mathematical sentences and indicate there is a possibility that people would make sense of the definition in a different order depending on their language. Using mathematical symbols–even though the structure of mathematical symbols follows the English order–might be a way that students can make sense of the meaning of the symbol in the language

order were still following those of Korean.

and the way that they are familiar and comfortable with. Jihyun thought mathematics is language-free because even though they have different orders of reading the fraction or mathematical definitions, she can communicate with other people who speak different languages using the symbol. These different ways of sense-making arising from language differences might also be possibly related to Jihyun's challenges of not being able to have the images of mathematical expressions in her head when she heard. The parts that one would say at the end in Korean will be at the beginning in English and vice versa.

Another notable aspect related to the language of Jihyun's narrative is that she connected daily language and mathematical terms. She noted that she keeps mathematical terms in English even though she uses Korean structure, but those terms do not carry daily meanings and contexts in her mathematics. As she mentioned in the narrative, "continuous" connects to its mathematical definition first, rather than the meanings such as unbroken and uninterrupted. Even though she knows what the word "continuous" means in daily life English, the more familiar meaning to her is its mathematical definitions. This can have pros and cons. When one does not have a connection to the word's usage in daily life, it is hard to have an instinct about its meaning but follows formal mathematical definitions. On the other hand, this focus first on mathematical definitions prevents possible semantic contamination and confusion caused by daily life words (Mejía-Ramos & Inglis, 2011; Monaghan, 1991; Tall & Vinner, 1981). For example, students' understanding of limits as a boundary in daily life influences students' understanding of the *limit* in mathematical contexts (Monaghan, 1991). Tall and Vinner (1981) also found that students' understanding of *continuity* in mathematics was influenced by their daily life usage of *continuity*. These confusions are unlikely to happen to multilingual students whose daily language is not the LOI.

Compared to the semantic contamination within one language, multilingual students can experience confusion between different languages with similar sounds. Jihyun mentioned in her narrative that you could have the same or similar sounds but different meanings. She gave the [i] sound as an example, which means 2 in Korean when it is "e" in English. Another similar example is [0] in Korean. The sound [0] means 5 in Korean, which can be confused when English speakers read 0 as [0]. In these cases, the written symbols, 2 and 5, will be helpful for communication. However, sometimes different languages have similar sounds but have very or subtly different meanings. For example, "*nombre décimal*" *a*nd "decimal number" look similar, but n*ombre décimal me*ans finitely many non-zero digits in French, and a decimal number in English can include infinitely many non-zero digits ⁽⁽⁾(urand-Guerrier et al., 2016)⁽⁾⁾.

Further Durand-Guerrier et al. (2016) pointed out that seemingly universal mathematical symbols are not universal. For example, some people write C_k^n for the binomial coefficient, and others write C_n^k , or for a half-open interval, the U.S. uses [a, b) and French or German use [a, b] (Durand-Guerrier et al., 2016). Hence, being aware of such possibilities and clarifying the meanings of symbols would benefit both multilingual and monolingual students.

Mathematics Identity: Intersection of Different Social Identities

Jihyun's mathematics identity, which includes her relationship to mathematics, experiences in mathematics, and future self in mathematics, were greatly influenced by her gender, race, nationality, and language. Also, she explains how these aspects influence her behavior in mathematics.

Since she was young, she has experienced gendered discrimination. Especially in Korea, her mathematics identity was closely related to being a woman. Jihyun became special when she did well in mathematics and was discriminated against because she was a girl. In her narrative,

she referred to herself as the oldest daughter, which is interesting because she did not note herself as the oldest child. In Korea, there are often some expectations toward the oldest daughter which are different from that toward the oldest son. Jihyun has one younger sister and one younger brother-the youngest. This chain of siblings usually implies that the parents wanted to have a son-at least from my family and my extended family-and that might have been true for her family as well, given her descriptions of her grandmother and father's expectations. Also, being the oldest daughter in Korean culture often means they are expected to become a caregiver for the family (김희진, 1995; 이은지, 2017). According to 김희진 (1995), Korean mothers with an oldest daughter tended to expect more obedience from their oldest daughter than Korean mothers with an oldest son or U.S. mothers with any gender of oldest child. Although this was about 30 years ago now, considering Jihyun's age, it was the atmosphere of Korean culture then. In Korea, there is an old saying, " 큰 딸은 살림 밑천이다(first daughter is a foundation for family finances-if I had to translate into English)," which has different interpretations about the origin. Some interpreted this as a reminiscence of the past when the groom and their family provided some financial support to the bride's family. A more recent common understanding of the saying is that the first daughters take care of their siblings and do housekeeping while their parents work to earn money. Either way, the interpretations imply the sacrifice of the first daughter, which was expected to those in Korea. I do not think Jihyun was free from the old saying. Even though it did not make it into the narrative, Jihyun noted that she enjoyed her first year in the United States because she could spend her time by only herself. In Korea, she had to help her grandmother and care for her siblings. There were always some events in the family, and she could not spend as much time as she wanted to study mathematics. In contrast, as she

lived by herself in the United States, she did not need to care about other family members and could use her time for herself. I interpret this as that she was sacrificing her time to meet the expectations.

While Jihyun was living up to the expectation as a caregiver and experiencing some discrimination, she found that doing mathematics well put her in a different position. She became a special girl who could do well in mathematics, which is a "field for men." Her grandmother acknowledged Jihyun after all the years of discrimination. It probably was a different kind of role and expectation of her from the family than what she was given before. Imagine how it might have felt to her. First, she probably did not want to lose that attention from the family. It came from something she did rather than something she could not do anything about it—born as the first daughter. Jihyun said, "Of course, I was happy [to have the attention and compliments]. They are saying that I am better than those nerd boys." Second, I wonder if she internalized the family's expectations, which were accustomed to her. If so, she probably had to meet the expectations to do well in mathematics, which probably led her to become afraid to make mistakes and try to be perfect.

This perfectionism tendency continued in the United States, and two other aspects of her identity were added: Korean and Asian. The myth that "Asians are good at mathematics" pressured her to be perfect again. She did not want to be the one who disappoints people, which is a stereotype threat and appears in other existing literature (McGee et al., 2017b; Shah, 2017; Yadavalli et al., 2022). Jihyun was worried that others would think that she was not "Asian" or that Asians are not good at mathematics if she did not do well in mathematics. Even though Jihyun knew that it was a stereotype, she wanted others to see that she was good at mathematics, probably because it was the way that she became special to her important people, family.

However, this stereotype again strengthened her high self-expectation in mathematics, and Jihyun felt she should be good at it.

This perfectionism in mathematics could likely be connected to her perfectionism in English. Jihyun noted another stereotype about Asians: "Asians do not speak English well." Several times, she expressed her frustration and annoyance about this stereotype during the interview. Jihyun did not want to hear this and practiced hard to avoid this kind of comment. As she repeatedly appeared in the narrative, she really tried hard to think in English. She stayed for a minute after she fell down the stairs to think about English; She went to the office even though she thought of herself as a person who does better when she works alone; and she compares the pages of the Korean and English versions of the same book to see the translation. It is hard work and represents the stress and pressure she was dealing with.

Jihyun was pressured to be perfect in both mathematics and English, which restricted her behaviors. She had to check herself three times before she verbalized her thoughts to avoid mistakes as much as possible. As she noted, it is also related to the "cramming education" in Korea that does not give her enough space for her to ask questions. In Korea, private education is a huge business; most students are likely to have a separate private education, and mathematics is the biggest business. Depending on their "achievement level," students get supplementary material for their year or get to learn the contents beyond their current grade level at their hagwons.

Consequently, people tend to think the farther their hagwon mathematics content grade level from their current grade, the better the students are at mathematics. Hence, many hagwon systems and teachers try to move on as fast as possible, which in turn causes "cramming education" that does not provide enough time for students to make sense of and ask deep

questions about the contents. In addition, the KSAT mathematics exam-probably the ultimate goal of studying mathematics for many students-asks complicated questions that students need to apply the knowledge but does not ask why some piece of knowledge is true. I do think there is value in KSAT mathematics questions, but it probably unintentionally restricted students from asking "why" questions.

In the last section, Jihyun confessed her imposter syndrome, which is not rare to find in woman students' experiences in STEM education (Collins et al., 2020; McGee et al., 2017b; Shapiro & Williams, 2012). I view her imposter syndrome as also closely related to her goal to prove that she can do mathematics, which in turn leads to higher expectations of herself and becoming harsh on herself than on others. She feared she would become a "loser" if she gave up studying mathematics. In her deep mind, she might have been worried that she would not be "exclusive and special" anymore if she stopped doing it. She noted that she thought people would say they knew she would not do well in mathematics because she is a woman even if it is a single mistake she made after one hundred good performances. It shows the level of her pressure. Jihyun thought she needed to be perfect. During the interview, Jihyun mentioned that her grandmother now adores her the most because Jihyun is doing a Ph.D. degree in mathematics in the U.S. She was happy with it but, at the same time, it felt like pressure, which is maybe connected to why she said she does not have emotional interaction with mathematics. Maybe she did not have the capacity to think about the emotional aspect as there already has been enough emotional consumption around mathematics, and she just needed to be good at it.

These pressures and imposter syndrome might have influenced Jihyun to start thinking that she just needed to do mathematics because she was "doomed" to be a mathematician. She explored other subjects because she was bored in high school and the first year of university, but

nothing interested her other than mathematics. How she described to me the reason for doing mathematics felt like she had no other choices anymore. However, I felt her claim about being fated was insecure as she noted that she still questioned whether she was qualified to do mathematics well compared to other cohort members. She thought about possibly returning to Korea when she was preparing for the qualifying exams. If she really felt that she was fated, then she would not need to worry about doing mathematics. Maybe that is why I felt throughout the interview and writing this narrative that she was trying to convince herself to do mathematics because she was fated to do so more than because she really liked it.

When I looked at Jihyun's experiences, her relationship with mathematics is complicated and influenced by various aspects of her social identity and her experiences throughout her life. As Jihyun said that she believes that every adult's personality has formed from their previous experiences, her experiences show how her current mathematics identity, including her future self in mathematics, has been shaped. Many of her experiences happened during the interaction with other people or external environments, which makes me hope that students' mathematics identity can move toward a more positive direction when their learning environments are changed. How mathematics educators can provide those experiences and environments is still a question for further investigation.

CHAPTER 7: CROSS-CASE ANALYSIS

In this chapter, I present cross-case comparisons between my participants' mathematics identities, including Jia. Each participant is from a different country, but it is important to note that they do not represent or speak for every person from their home country. Each narrative is a story of a multilingual international student's experience. I compare their experiences, highlighting the commonalities and differences between their experiences in order to develop an understanding of multilingual students' challenges and backgrounds. Further, doing a cross-case analysis allows for additional data to be examined, analyzed, and shared that could not be included in the narrative because of the flow of the narratives.

Linguistic Aspects

Is Mathematics Language-Free?

All participants acknowledged the importance of language in mathematics; however, not all agreed with the statement "Mathematics is Language-free." Jia and Helena disagreed while Jihyun and Maria agreed. Jia and Maria specifically mentioned that they think language is necessary to communicate their mathematics ideas with other people. Maria mentioned that mathematics requires a specific and precise language, which she liked. She thought discussing mathematics with other people is an important part of mathematics because everyone is different, and communication is a way that they can share the way they think about mathematics and how they perceive mathematics. Similarly, Jia noted that mathematics could be applied to other areas only with the help of some languages.

Another interesting perspective about mathematics and language is that two of the participants (Helena and Jia) considered mathematics itself as a language. They both focused on specific words, grammar, and symbols that mathematics use, which connects to the idea of mathematics register.

<u>Jia:</u> I think math is a language because it has its grammar and signals [symbols], such as $\because \therefore \rightarrow \Sigma$, to name just a few. Just like any other language, math is special and mysterious, if you don't learn the language, you will not understand what all the things represent. If I am not learning Real Analysis II, I cannot tell what " $\forall \epsilon > 0$, $\exists P$ partition, such that U(f,P)-L(f,P)< ϵ " is. (Even now, I feel hard when I suddenly look at all these things without my notes.)

Helena: I think one thing is mathematics itself is a language, so with our friends, it's like, "What is this? What are you talking about?" And part of this is just jargon. Like it's just words that we create to describe our mathematics elements, so that's why it looks so complicated, but it's a new language because you need to learn. You need to learn new terms. You need to learn how to write it. But it's a universal one, because even if I assume the textbook is in Chinese, the equations are going to be exactly the same, X squared plus six X plus seven. No matter what you are, it's going to look exactly the same if it's in French or whatever. So, I think that's pretty cool. Like more about notations, sometimes people use different notations. You can understand an integral, no matter how you talk.

In the email interview, Helena added the following:

I think I disagree with the sentence *[mathematics is language free]*. As I said before, mathematicians have developed a particular language to communicate mathematical concepts, and all the notation and symbols are part of the mathematical language. Sure, if you are thinking in languages like Spanish, English, or Finnish, we can argue mathematical language is independent of them as the notation is more or less universal, and yet, as any other human activity, we need language to convey ideas.

I'd do a comparison with music, musical notation is more or less universal and an orchestra with people around the world can play together without knowing the same language or even actually talking between them. But how much better would their music be if they can communicate between them and the conductor?

They both noted that 1) mathematics itself is a language and 2) you need to learn the language to interpret and engage in mathematics.

Even though Jihyun and Maria did not mention that mathematics itself is a language, they both agreed that the symbols used in mathematics are universal so that they can use these symbols to communicate with people who speak other languages. The existence of symbols made some participants (Maria and Jihyun) believe that mathematics is language-free (even though Maria questioned this later). Maria, Jihyun, and Helena are in the same cohort and took similar courses in their first year. Maria provided an example:

Even though Jihyun and I share the same language–English, but sometimes we cannot find the right word to express ourselves. Then, one grabs the chalk, goes to the whiteboard, writes it on the board, and says, "this." Then, the other one says, "Ah, yeah, I know what you mean," and gets the idea with symbols.

After thinking for seconds, Maria continued:

So, well, maybe symbols are another language, but I think it's like the simplest one. I don't know, the less or equal symbol, for example, it's like, we are both thinking of something, but no, but this is bounded with this, and but I don't get it in my head. And then someone starts writing it, and, oh, I see what you mean this. So, I think it's language free.
All the participants appreciated mathematical symbols that make communication easier even though depending on how they interpreted the role of symbols, their opinion about mathematics being language-free differed as described previously. Although Maria initially agreed with the notion of mathematics being language free, she made a similar point to Helena and Jia. All three agreed that despite mathematics having "universal" mathematical symbols, verbal communication about mathematics should be meditated by some language, often English, in the United States. However, as I mentioned in Jihyun's narrative reflection, not all symbols have a universal meaning. The grammar of mathematics writing with symbols follows an English order (maybe other languages as well, but at least the order is quite different from Korean). Therefore, she differs in her view of mathematics having "universal" symbols when compared to Maria, Helena, and Jia.

Valuing Written Communication

All of the participants' narratives about the relationship between language and mathematics show the importance of written communication for them to do mathematics. In all four participants' experiences, there were cases that they benefitted or could have benefitted by using written communication. When Jia's group mates in the ITP course at one point did not understand what she was trying to explain, she wrote what she wanted to convey on the paper and showed it to them. Maria and Jihyun both mentioned that it is helpful for them to have written expressions on the board as it is not easy to picture mathematical expressions in their head when they listen to them in English. As mentioned earlier, the three graduate student participants mentioned that their communication with each other benefitted by using mathematical symbols and a whiteboard.

The advantages of written communication were clear during the lecture too. As the data was collected during the pandemic, some participants took online courses. Jia and Jihyun mentioned that they were frustrated when the instructors were reading the PDF while scrolling down it rather than explaining it. However, as they had written documents from the instructors, they could study and review them by themselves later. Also, Maria and Helena also mentioned that when they were struggling with their professors' accents, they used the written material as their resources. Helena said:

I'm lucky this is mathematics, and people write, so they write what they're saying. So, I just needed to follow what they were writing a little with what they are saying, and I could follow. But if, for example, they were saying something without writing it, it was like, 'maybe I got it. Maybe I didn't. Let's hope it was not important. I'm going to ask [to my friends] later.'

Helena's quote echoes what other participants shared about written communication. Even though she missed something, she had something to point out to her friends to ask for help.

Accents

Helena and Maria's comments about accents were also a common theme across participants. Helena and Maria's first language are Spanish, and they both said it was harder for them to understand the Chinese accent in comparison to other accents. Helena explained this way:

I didn't understand anything of the professor. And it was not me. All the international students had the same feeling, and the native speakers were like, "Yeah, it's hard to follow."... I'm still struggling with Chinese [accent]. Like if I'm speaking with Latino people in English, we have like the same errors in pronunciation, so we know how we're

talking. But the Chinese have their own errors. Not errors, like- Their accent. Yeah, their accent, as well as Indian. So, I think both of them are the most difficult for me to understand, and there's a lot [in mathematics].

As the participants noted, it was not rare to find multilingual mathematics instructors in postsecondary mathematics education, and it can be challenging for multilingual students whose first language is not the language of instruction.

Although Jihyun did not mention specific instances of challenges with her instructors' accents, she still brought up the role of her accent. She said she "tried not to speak like Asians," which meant that she wanted to erase her accent as much as possible and improve her pronunciation. Jihyun was aware of how the Chinese accent was considered, and she did not like Chinese accents either, feeling that they were not trying hard enough to "fix" their accents. Even though she thought having different accents was natural and unavoidable, Jihyun still wanted to change her accent. This was due to her frustrating experiences when people misunderstood her. It is worth noting that throughout their narratives, both Helena and Maria focused on others' accents, but Jihyun focused on her accent, which raises questions surrounding participants' confidence in their English.

Confidence in English

The levels of confidence and comfort between participants that I felt during the conversation with them varied. Maria was clearly very confident about her English as well as Helena, who was confident with English but still shared some concerns. Jihyun and Jia shared more concerns about English with me. Maria stopped taking English classes after middle school because she already had high enough TOEFL scores at that time, and then she honed her English by watching English YouTube videos. She said the transitions from Spanish to English were not

difficult for her, and the only two aspects that she felt challenging were speed and various accents of English in the United States. Helena was not very concerned because she thought English for her was a tool to communicate with others, which is quite a different stance than what she said about Spanish, "Spanish is my Language." She also pointed out that the United States has *potencia* (power) and everything is in English, which is problematic but also means that she needs to learn English to succeed in the world of mathematics. Even though she was fairly confident about her English in general, when doing mathematics she noted that she tended to take on a different role and be quieter in English than in Spanish. Helena was still worried as she sometimes missed some information and needed a few more milliseconds in English. This might be related to her reflections on her preference to work alone, and she noted that she thinks in Spanish and translates to English later when she is doing mathematics. Helena also said living daily life is more challenging than doing mathematics in English, as she felt she could not express herself fully in English.

On the other hand, Jia and Jihyun were concerned about their English. When Jia and I were doing the first interview, she made comments about her English several times, such as "what's wrong with my English." After more than two years of mathematics education in English, Jia said she was still concerned about her English as sometimes she misses what her instructor says. However, she said she was more confident than before as she became more fluent. Also, as shown in the narrative, she reflected on her behavior, that she was not annoyed but patient when she met a foreigner whose first language is neither English nor Chinese. Jia, as a senior, thought that people in the United States should be the same, understanding, willing to help, and patient.

Jihyun made it clear to her possible advisor before asking him to be her advisor that she did not feel confident in her English even though she had improved her English over the first year. There were some differences and challenges that she noticed, such as semantic contamination or different structure of sentences. She put effort into improving her English, which allowed for the energy that she needed to put into taking a course to decrease. However, her eagerness to improve her English was different from others, and it especially contrasts with Helena's. Contrary to Helena, who made it clear that English is a tool and Spanish is her language, Jihyun tried to think in English even though she was alone, whether it was about mathematics or daily life. She also mentioned that she wanted English to be like her first language. It was not just about vocabulary and fluency, but she wanted to erase her Asian/Korean accent. One of the reasons that Jihyun wanted to improve her English was that she disliked the stereotype that Asians are not good at English. This emphasis on race and nationality was not isolated to Jihyun's perspective on accents but was broader and throughout the participants' narratives in relation to their mathematical identity.

Race and Nationality

Jihyun and Jia, both of whom are Asian, talked about their race during the data collection. As previously mentioned, Jihyun really hated the notion that "Asians are not good at English" and felt that it is an automatic anticipation for Asians. She was considering living the rest of her life in the United States, so she thought she needed to be able to express herself fully in English. On the contrary, Jia did not mention the expectations toward Asians in terms of English but brought up race in other ways.

Both Jia and Jihyun shared their thoughts about the myth that "Asians are good at mathematics." Jia said that she heard this myth all the time. She thought it was not because

"Asians" are good at mathematics but more of a difference in their education system. Jia noted that she did a lot of practice in her K-12 education, which helped her do well in mathematics. She also credited the different curriculums between China and U.S. for this thought, as people typically think that Asians are good at mathematics when Asians demonstrate a particular skill or knowledge not taught in the U.S. curriculum. Jia thought this was just because some students learned that content earlier than others. Jia noted that her good performance in mathematics was because of her efforts and diligence.

Jihyun was also aware of the myth and admitted that she had this racial bias toward the students as a tutor in the mathematics helping room. She saw both pros and cons of the myth. The pro was that she was a mathematician and people would expect that she would be good at mathematics. The con was that she needed to meet the expectation of being good at mathematics. This was also a pressure for Jia even though she disagreed with the myth. Jia and Jihyun both were students who had been "successful" in mathematics, but both still felt some level of pressure to do well in mathematics because of the racial myth. They were concerned that people would think they were not Asians or people would say "Asians are not good at mathematics" if they made mistakes. Even though Jia did not agree with the myth, she still was afraid and did not want to be the one who did not meet the expectation and hear something like "how could you perform so badly in math as you are Chinese" or "you are a shame of Asians." As they both did not want to disappoint people, this pressure was ironically some motivation as well for them.

On the other hand, Helena and Maria did not explicitly talk about what it means to be a Latin@ in mathematics. When I asked Helena this question, she said her personality came from being a Latina and more specifically being a Colombian, such as being "more open and warmer." However, she noted that she had not thought about what it meant to be a Latina mathematician,

and so far, she had not noticed any differences. Maria said that she has been "breaking the stereotype and made it possible for another woman in [her] hometown to become a mathematician." It seemed like she was talking about Latina as a combined racial and gender identity, but when I asked what the stereotype that she mentioned in the above quote is, she said gendered stereotype.

Both Helena and Maria discussed more structural issues within their countries related to mathematics and science. Helena said she appreciated nice buildings and installations in comparison to people who do not care about the public property and university facilities in Colombia. Maria talked about not having enough budgets in universities to hire faculty members, which led to small departments. Both Helena and Maria mentioned they hope each of their governments invests more money in science.

Gender

All of the participants were aware of the gendered myth that "men are better at mathematics than women" or "women can't do mathematics." Jia, Jihyun, and Maria had clear experiences and had stronger opinions about this gender issue than Helena. In each of these three women's narratives, they were discriminated against or discouraged because they are women. Jia did not share specific experiences, but Jihyun and Maria shared their lived experiences in their family, hagwons, and schools. Both Jia and Maria had the experience of being the only woman in the mathematics classroom, which in turn led to unpleasant and discriminatory experiences. In Maria's case, her undergraduate cohort members tried to devalue her achievement because she was a woman. In Jihyun's case, her hagwon teacher implied that she did not belong in the classroom by saying that "여자애가 들어와서 물 흐린다 (A girl makes the water cloudy)."

well and wanted to represent women in the field. Jia wanted to become a professor to have more power to help students avoid the same experiences as she did, who were as helpless or frustrated. Maria also had a similar goal to represent women in mathematics. The number of female students in her current graduate program made Maria question her role in this goal, but it was still an important motivation for her to do mathematics.

On the other hand, Helena viewed this as more of a natural tendency even though she was aware of less presence of women in mathematics. In one of the interview questions, I shared a narrative of one student asking questions publicly during the lecture and then asked whom she imagined while reading the narrative. Helena answered that she imagined a young boy with a uniform. I prompted further explanation, and the following is her response in her email:

I'm not sure why I imagined it that way, I found it weird too. Maybe it was common that in my classroom, the guys were asking questions. To be fair, since 10th grade, when we could choose between mathematics, arts, and natural sciences, and I chose mathematics, there were more guys than girls in all my classroom. And that hasn't changed in all my years in mathematics. So, statistically speaking, there's a greater probability that questions were asked by guys.

To extend my answer, I know there should be more girls in mathematics, but at the same time, women have a natural tendency to choose careers related to people and helping them, that's why there are more female nurses or female teachers. So, even if I would love to have more female classmates, and we have to work to stimulate more girls to be interested in math, I don't feel discriminated by the fact that most of my partners are men, moreover, sometimes I feel/felt special being part of the few girls but having better grades than the guys (a little arrogant, I know, but it felt good).

As shown in the response, she was aware of the disproportion in the classroom, but she viewed it as a natural tendency. In the follow-up email, she said that this was probably because of the possibility of motherhood and added, "I honestly think our biology does influence who we are and what we choose in life. I think there are, in fact, psychology studies showing this."

Another interesting point from the quote in Helena's previous paragraph is that she felt special being part of the few women in mathematics and doing better than the guys. This is similar to the other three participants in that they all want to prove their presence, but especially with Jihyun's narrative. Jihyun also had situations where other people treated her as special for doing mathematics well, which led her to feel special about herself too. Even though both Jihyun and Helena felt special by doing well in mathematics, how they viewed mathematics in their life was contrasting.

Jihyun viewed herself as "doomed" to be a mathematician or "addicted" to mathematics. It did not sound like she actively chose mathematics with her will. Rather, it was more like she did not have any other choices than mathematics. One of the pieces of evidence Jihyun provided was that she tried to explore other areas, but nothing interested her. Also, she noted that "mathematics is my life," so she could not fail in mathematics and was inevitably desperate for success in mathematics. In the narrative, she was afraid that she would be a "loser" and could not find any other jobs if she failed the qualifying exam and went back to Korea. It sounded like her life was failing when she imagined failing qualifying exams for the mathematics Ph.D. program. On the other hand, Helena agreed that mathematics is a big part of her identity, but she was clear that it was not the entirety of her life. She still had other important things, such as religion and people around her, and believed that she was worthy of just being her. (Although it is not about mathematics, this pattern appears in how they deal with English as well. Jihyun wanted English

to be like her first language, and Helena was clear that English is just a tool to live in the United States.)

Intersectionality in Interactions

Although previous results have centered sole characteristics of identity, throughout the participant narratives, there were cases that showed the complexities of their experiences, with multiple layers of marginalization. This aspect was clearer in Jia's and Jihyun's cases for me as they shared how their experiences were shaped by their gender and language more than Helena and Maria did. Jihyun explicitly described how her actions were influenced by her gender, race, and languages, whereas Jia's narrative was more nuanced and inferred.

Jihyun noted that she needs to think three times before she verbalizes her thoughts and that she does not lead the discussion in groupwork but rather is quiet or works alone. She also noted that she "never can be" a person who asks a question in front of the whole class. She "just accept[s] things without questioning," and she blamed herself for not understanding the material even when she had struggles in understanding some parts of the lecture. Hence, she does not ask questions right away but takes her time to digest the material and ask questions privately after the class.

When I asked if her careful personality and actions would be related to her previous experiences, she answered, "Of course," adding that she believes everyone's personality is determined by their past experiences. Her experiences of hearing gendered discourse from family led her to be careful before she gave her opinions. Jihyun reflected on her feelings:

"I always had to prove that I'm better than boys, so I can't get caught doing what I'm not good at. Even if I did well on hundred things, if I cannot do a single thing, I would become a person who is bad at every other thing for the reason that I am a girl."

This quote shows that she had to hide what she was not good at and tried to avoid such situations on account of her gender, thereby influencing her actions and experiences in mathematics.

In addition to pressure from her gender, she also had pressure that came from being Asian and Korean. Such pressure shaped her actions:

Sometimes I feel I am representing Koreans since I might be the only Korean that they know or talk to. That's why I always have to be nice to people and humble. If I do something wrong or say something stupid, I am afraid that they would think every Korean is not good at math or not nice, etc. On the other hand, people have a bias that Asians are good at math and bad at English. So, sometimes I feel a little pressure to meet their expectation in math and to show that I can do better at English.

In addition to the pressure that she already had from Korea regarding gender, she now has a different type of pressure that came from being in the United States. Both of these pressures were at work within the classroom and knowing these pressures is helpful in understanding her personality and actions.

Other experiences might have influenced her personality and actions, such as her family culture, the personalities of family members, and her past interaction with peers. But even without those aspects, if one needs to simultaneously be good at mathematics to represent women in mathematics and be good at English to disprove that Asians are bad at English, it would not be an easy setting to share thoughts without a lot of inter dialogues. Even though Jia did not share the pressure of speaking English as an Asian, Jia also shared similar pressures about mathematics.

Their Mathematics Identities: Answering the Research Questions

Before moving to the discussion, I summarize the mathematics identities of the participants, Jia, Maria, Helena, and Jihyun, at the time when they shared their lived experiences. As a reminder, the definition of mathematics identity in the current study is the following: how one understands their relationships with mathematics, how those relationships are constructed across time and space, and how one understands their future possibilities in mathematics. In this section, I describe each of the three parts of their mathematics identities based on the following primary research question and sub-questions: What are post-secondary multilingual international students' mathematics identities?

- 1. How do post-secondary multilingual students explain their relationships with mathematics?
- 2. What lived experiences do they share as having constructed their relationships with mathematics across time and space?
- 3. How do post-secondary multilingual students explain their understanding of their future possibilities in mathematics?

Participants' Understanding of Their Relationships with Mathematics

In this subsection, I summarize each participant's relationships with mathematics.

Jia's Relationships with Mathematics

Because Jia was a pilot study participant, the interview questions were different, and hence, it was not easy to see how her relationships with mathematics have changed over the years. Although she did question if she would be a part of the mathematics community and more broadly STEM community in her middle school because of the gendered-stereotype, she was doing an undergraduate degree in a mathematics-related STEM field, including a mathematics minor. As the title of her narrative shows, Jia loved mathematics and was hoping to be loved by mathematics. Even though she struggled with how she was positioned during the groupwork and adjusting to changes in the LOI to English, she believed that she could do well in mathematics no matter what languages she used to study mathematics and no matter where she studied mathematics.

Maria's Relationships with Mathematics

Maria viewed doing mathematics as "following rules that we have to solve some abstract problems" and swimming on the surface of the ocean where she does not know the depth and what is in it. She viewed mathematics as enriching our culture and fascinating to work on. She analogized defeating challenges in mathematics to illuminating the water below while she was swimming on the surface of the ocean.

As well as mathematics challenges that she liked to defeat, doing mathematics gave her other types of challenges. Doing mathematics created barriers between her and her people, such as her family and friends. She noticed that it was not common to like mathematics and even less so for women. It changed and restricted her interaction with her friends who do not study mathematics because she could not share what she does. Further, she experienced some misogynies regarding gender, which hurt her. Since then, representing women became a motivation for her, as mathematics was a gendered space for her.

Thus, for Maria, at the time of our conversations, mathematics was what gave her purpose–representing women–and the opportunity to be independent–to live outside her home country, Mexico pursuing her degree with funding.

Helena's Relationships with Mathematics

For Helena, mathematics is "a collection of words that we created to describe our mathematical elements," and "being a mathematician is 'living in the boundary of the knowledge,' always pushing further, trying to discover properties and relationships of objects and structures that sometimes only live in our minds." The former led to the conversation about why she thought language has an important role in mathematics and how language can shape her action as a mathematics doer. She became a less active member of groupwork in English than in Spanish, and sometimes she had to try to convince herself that what she missed in the lecture was not important. It further relates to her perspective on the role of English in the field of mathematics. Helena viewed that to succeed in mathematics as a mathematician, she needs to use English.

Both quotes mentioned at the beginning of the previous paragraph also describe Helena's understanding of mathematics as a field: Helena thought mathematics is a human creation and only lives in humans' minds. She loved the idea of pushing the boundary, and since then, her love for mathematics increased exponentially. Before she decided to study mathematics, she thought of mathematics as nerdy, but at the time of our conversation, she loved mathematics and described it as fascinating, satisfying, and bringing enjoyment.

Despite her exponentially grown love for mathematics, she does not think that is all for her life. It is not the most priority in her life but just a part of her life. One of the reasons that she is studying mathematics for her Ph.D. degree is to become a professor so she can spend time with her future family, which is more important than doing mathematics. She believed doing mathematics well does not determine how worthy she is as a person. She had faith that she is worth herself.

Jihyun's Relationships with Mathematics

On the contrary to Helena, for Jihyun, doing well in mathematics is closely related to her worth. In her family, her grandmother, who had a gender preference toward boys, recognized Jihyun when she did well in mathematics after years of indifference. Mathematics made her special. It was not just in her family, but those experiences were repeated outside as well. As well as being recognized, she experienced some discriminative words toward her as she was a woman. Those experiences made Jihyun study mathematics harder than boys and want to be perfect in mathematics. She thought a single mistake would hide a hundred things that she did well and make her as incompetent.

Since she came to the United States, representing Asians and Koreans became another pressure on her. In the United States, doing well in mathematics had more connotations about her based on her socioculturally available selfhood, including anticipations about Asian English. Doing mathematics for Jihyun was dealing with multiple layers of sociocultural expectations.

Contrary to the other three participants who expressed that they love mathematics, Jihyun said she does not love mathematics. She decided to do mathematics because it made her special and exclusive. Jihyun now feels that she does not have other choices anymore but to do mathematics. She said mathematics is her life and who she is. Jihyun said that she would have felt like a "loser" if she failed in qualifying exams and if she went back to Korea. She felt that she could not do something else and no one would hire her. Even though she feels that she is "doomed" to be a mathematician, she said she likes how her life is flowing with mathematics.

For all the participants, their relationships with mathematics evolved throughout their years in mathematics. As they faced different aspects of mathematics, their understanding of mathematics changed. As they interacted with various people and as they moved to different

contexts, they were positioned and positioned themselves differently. In those changes, sociocultural categories where they belong influenced their relationships, which will be discussed in the next subsection.

Participants' Experiences Having Constructed Their Relationships With Mathematics

The narratives were constructed to present the experiences that the participants shared as having influenced their relationships across time and space. As reviewed in the current chapter, their experiences were shaped by their socioculturally available selfhoods, such as their languages, races, nationalities, and gender. As well as these socioculturally available selfhoods, their personalities (such as Maria claiming herself as a social person or Jia claiming herself as not a social person) and their satisfying moments in doing mathematics influenced their experiences around mathematics.

The most noticeable experiences influencing their relationships to mathematics were the participants' gendered experiences. Their gendered experiences started before they moved to the United States. For Jia, it was when she was in middle school; For Jihyun, it started when she was a child outside mathematics and continued in STEM education during her middle school ages; For Maria, it was during the undergraduates. Representing women in mathematics became a goal for the three of the four participants.

Compared to gendered experiences that shaped their relationships with mathematics before they moved to the United States, racialized experiences and experiences as multilingual have been shaping their relationships with mathematics currently in their lives in the United States. For Jia and Jihyun, being Asians (as well as being Chinese and Korean, respectively) meant that they needed to do well in mathematics to meet external expectations, which was not the pressure that they had to worry about when they were in their home countries.

Regarding being multilingual, more specifically using their second language–English–to study mathematics, Maria did not show noticeable changes in her relationships with mathematics. One possible influence might be that she might have become more appreciative of written communication in mathematics as she communicated with people from different countries and has benefitted from that mode of communication, which is a change that appears in other participants. For Helena and Jia, they were positioned and positioned themselves differently when they needed to communicate in English. They both shared that they took less active roles in interaction either because they felt their opinions were not communicated well or because they were worried about the chance that they might miss some parts of the discussion.

Participants' Understanding of Their Future Possibilities in Mathematics

All of the participants are pursuing graduate degrees in mathematics or mathematicsrelated fields, which might be an indication of how they see their future possibilities in mathematics. Jia, Maria, and Helena said they like and love mathematics. Jia was confident in her capabilities in doing mathematics. Even though she is pursuing a degree not exactly in mathematics but in a relevant field, she viewed that mathematics provided a foundation for herself as it helped her learn different ways of thinking. Also, she wants to become a professor to help students so that they would not experience her helpless and frustrating experiences.

Maria also wanted to represent women in mathematics. As she met many female colleagues in her current institution, she was questioning her role in representing women in the field. However, she still thought that her success in mathematics would help other Latina students in her hometown. She was still curious about mathematics and wanted to illuminate what would be "under the water" that she could not access.

Helena thought she might be at the inflection point for her exponential love for mathematics. She was not sure where she would go during her journey in the Ph.D. program. Becoming a professor was a big part of herself during her undergraduate, which is related to her other values, such as being able to have the same break as her future children as her mother did or having a chance to meet diverse people. However, she now thinks there are other more important things in her life than being a professor, and doing mathematics is only a part of who she is.

Lastly, Jihyun was seeing herself as "doomed to be a mathematician." As of the time of data collection, she was not trying to or could not have any emotional interactions with mathematics, which probably is related to what she went through with mathematics. She wanted to stay calm in that relationship. However, she still questioned her future possibilities in mathematics and told her advisor that she thought she was not good at mathematics. As she noted, Jihyun was experiencing imposter syndrome, so it seemed her future possibilities in mathematics were insecure. She was questioning her capacities and her talent in mathematics at times during our conversations. However, Jihyun was trying to convince herself to view herself as a mathematician, and at least she was happy with how her life was going with mathematics.

Throughout this Chapter, four participants had some similarities and some contrasting stances in their understanding of their relationships with mathematics, how they interpreted and reflected on the experiences that they felt have influenced their relationships, and how they see their future possibilities in mathematics. In the next Chapter, I describe the contributions of the current study by connecting them to existing literature.

CHAPTER 8: DISCUSSIONS

This research contributes to understanding post-secondary multilingual international students and their experiences around mathematics. Reflections for each case were already shared after each case; therefore, in this chapter, I discuss implications from cross-case analysis, followed by reflections about the current study and future directions.

Reflections from Cross-Case Analysis

Reflections on the Relationship between Language and Mathematics

In the first section of the cross-case analysis, I presented how participants view the relationship between mathematics and language and how language plays a role in mathematics. Even though Maria and Jihyun agreed that "mathematics is language-free," and Helena and Jia disagreed, they all noted the usefulness of written symbols that they thought were universal to communicate with people who do not speak the same language. Participants also mentioned that mathematics has a specific set of words and grammar that have different meanings in mathematics compared to daily life, which is related to the concept of the mathematics register (Halliday, 1978; Pimm, 1987). *Mathematics register* refers to "the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes" (Halliday, 1978, p. 195). I interpret the mathematics register as a set of meanings distinct in mathematics compared to other contexts, which was what the participants in the study were discussing.

Because of this characteristic of the mathematics register, Helena and Jia saw mathematics itself as a language, and Maria later also seemed to agree with the notion. Given that Jihyun expressed that she did not have semantic contamination in English, there is potential that this could explain the others' view of mathematics as a language even though they did not mention semantic contamination (Mejía-Ramos & Inglis, 2011; Monaghan, 1991; Tall & Vinner,

1981). As mentioned in her reflection section, Jihyun noticed that she did not borrow meanings or nuances from daily language when she was learning new mathematical terms in English because she was not familiar with the contexts where those English words are used in daily life. As researchers have reported that students sometimes have semantic contamination in the existing literature (Mejía-Ramos & Inglis, 2011; Monaghan, 1991; Tall & Vinner, 1981), these multilingual students might have had that phenomenon in learning mathematical terms in their first languages, which in turn, leads to the next point.

Multilingual students have mathematics registers in their first languages, and different languages have different mathematics registers. Thus, it might be the case that multilingual students add and connect some words and phrases to their existing register as they experience their transitions from one language to another. I believe it is a strategy and process for surviving in a new environment. Moschkovich (2002) raised a concern that focusing on acquiring words and grammar can lead to a deficit perspective toward bilingual students. However, the problem here is that people have approached multilingual students thinking that *they have "deficiencies" in their language skills, so they must learn the language (often LOI)*. Instead, we need to look at students like Maria as they already have their mathematics registers in their own language – like monolingual English speakers, and they are expanding their resources so that they can communicate with people who cannot speak their first language. Mathematics educators, hence, need to look for ways to help their expansion of linguistic skills.

While they were expanding their mathematic registers to other languages, participants used symbols as a tool for communication. For example, when Jihyun and Maria had trouble communicating in English, they could use mathematical symbols on the board. Maria wrote down the mathematical expressions when they were complicated to understand in verbal English.

Each person already had meanings of those mathematical symbols in each of their mathematics registers, so they could understand the mathematical expressions in the way that each was more comfortable with.

Combining a lessened possibility of semantic contamination in their second language and the experiences that they were able to communicate with English, it is possible that it might have been easier for these international multilingual students to feel mathematics as a separate language than monolingual English speakers, as their main meaning of English mathematical terms are mathematical ones rather than daily meanings.

Importance of Written Communication

As all the participants emphasized the usefulness and convenience of symbols in communication, participants' narratives showed the importance of written communication. Compared to verbal communication, which disappears physically as soon as it is spoken, written communication stays until someone removes it. In their narratives, they showed different ways in which they used it. The written communication becomes a source that they can review after the class and ask questions to other people. Even though this method of teaching was not pedagogically advantageous, participants could study using slides or PDFs that their instructors uploaded when they were not learning much from the lecture. Written communication was also important when working with others. When participants could not find a good verbal expression to communicate mathematical ideas, they could write symbols on the board or paper to communicate with others. This implies that creating a groupwork environment that values written communication as well as verbal communication would be important. Further, it would be helpful to have a shared writing place so that everyone can work together, as Liljedahl (2016) suggested.

Accents

Another common theme that appeared in the study was the participants' comments about accents. Except for Jia, all three mentioned understanding people who have accents from their other languages as their challenge. I honestly think it is a difficult aspect to discuss. As a multilingual student who studied English using certain "standard" accents—probably White middle class from certain areas, it was challenging for me as well to understand English with different accents. While I understand their frustration regarding the accents, these various accents are the product of the diversity that we need to value. Also, it is evidence that they can speak languages other than English or the LOI, which should not be something that they feel they need to hide but should be something valued. In Rios (2022), some multilingual students hid that they could speak other languages and pretended that they were monolingual English speakers, which means that their multilingual identity was not valued in their environment.

Participants mentioned that certain accents are more difficult to understand than others; usually Asians' accents were the toughest ones to understand. Although it is understandable to me that accents similar to someone's first language are easier to understand, I still have some concerns with the notion. The comments about Asian instructors' English in mathematics are not rare (Subtirelu, 2015), and this is related to a language ideology that values "nativeness" and thereby values certain accents. People sometimes give compliments something like "you don't have accents," or claim that they experience "language barriers" with multilingual students. According to Subtirelu (2015), it is a way to position monolingual English speakers as having no responsibilities and "normal" and place all the responsibilities of "incomprehensible" communication on multilingual students whose first language is not English. As the participants here are also multilingual, it shows that they are eager to be like "nativeness," as Jihyun noted in

the narrative, which could be seen as they attempt to protect themselves from possible deficit perspective toward them. However, Shuck (2006) raises the point that race, nationalities, and languages are closely related, and commenting on their accent is beyond just about language but also related to their race and nationalities. Hence, the discussion about accents can be on the subtle and thin line between racism–if it becomes worse–and natural frustration.

Accepting that dealing with accents can be challenging for multilingual students, it would be great if there is a way that can improve their learning environment. First, the perspectives and narratives surrounding accents need to be changed. As mentioned previously, accents are a product of the diversity in society, and they show that individuals can speak multiple languages. This changes the narrative from solely focusing on English to emphasizing multilingual diversity. As for some practical changes, there were not many practices that I found and came up with. Written communication would be helpful again in this discussion about accents, although it will still be limited help, as sometimes instructors will give verbal explanations and instructions without writing. For online courses, closed captioning can be an option (Tisdell & Loch, 2017). Tisdell and Loch (2017) provided manually edited English captions and auto-translated captions for other languages to university students. In their study, one of the reasons students found captions helpful was that it helps clarify accents and explanations that students could miss just by watching videos. One caution of this approach is that someone needs to edit manually because auto-generated captions probably miss mathematical expressions. In their study, students asked if editing captions could be public so that everyone could edit them (Tisdell & Loch, 2017), which would reduce the amount of work that the instructor needs to put in.

Reflections on Race, Nationality, and Gender: Stereotype Threat and Stereotype Lift

In the participants' narratives and cross-case analysis, several mathematical stereotypes were discussed, such as: "mathematics is language-free," "Asians are good at mathematics," and "women are not good at mathematics." I have already discussed the implications of "Mathematics is language free,"– so this section centers on race and gender within participants' experiences in this section.

Race in Mathematics

Jia and Jihyun shared their experiences as Asians and their thoughts about the stereotypes regarding race. They shared how stereotypes about Asians– "Asians are good at mathematics" and "Asians are bad at English"–are influencing them. Both Jia and Jihyun shared that they do not want to be an outlier of the stereotype "Asians are good at mathematics," so their goal is to meet the expectation, which meant that they internalized racialism (Gupta et al., 2011). Jihyun admitted that she has such a stereotype in her mind. When she saw an Asian student coming into the help room, she assumed that the student would be better at mathematics.

This notion, in turn, could lead to a higher level of stress and pressure to meet the excessive expectation (McGee, 2018; McGee et al., 2017; Yadavalli et al., 2022). Existing literature pointed out that sometimes Asian students' experiences were overlooked because of stereotypes and the myth of the "model minority," focusing only on achievement. Researchers also criticized that Asian students' academic success has been normalized without focusing on what they were enduring in their experiences (McGee et al., 2017). The current study echoes existing literature that Asian students feel pressured to meet expectations, which requires them to do extra labor. This is similar but different to students from Black, Latin, or other minoritized groups, where they also have to put in extra labor, but to challenge the stereotypes (McGee,

2018). Jia's narrative implied that she wanted to get credit for the amount of practice and work that she invested rather than attributing it to her race. This pressure also could lead Asian students to seek help less than others and hide or minimize their successes to avoid possible unfriendly responses (Gupta et al., 2011; McGee, 2018), which could be found in Jia and Jihyun's narratives. Even though Jia was very successful in the ITP course, she did not show off or talk about her achievement highly when we first met. Jihyun also told her advisor that "she does not think she is good at mathematics" even though she already had a bachelor's degree and a master's degree in mathematics and was pursuing a Ph.D. degree in mathematics.

Compared to the discussions about Asians that arose naturally in conversation with Jia and Jihyun, there had not been explicit conversations about being Latin@ in the conversations with Helena and Maria. I asked them what it meant for them to be a Latina in mathematics, but both of them did not mention any of the stereotypes regarding their race. However, they talked about what it meant to be a woman and Latin"a" in mathematics, emphasizing the gendered aspects over the racial aspects.

Gender in Mathematics

All four participants shared their thoughts about being a woman in mathematics. They all were aware that women are a minority and marginalized in mathematics. This was especially clear from Maria's and Jihyun's experiences when they were the sole woman in their community in mathematics or STEM. In those experiences, Jia felt discouraged, Jihyun was humiliated, and Maria's achievement was devalued solely for being a woman, which adds to the existing literature about students' gendered experiences in STEM and mathematics (e.g., Lim et al., 2021; Mendick, 2005). Especially, Maria's male cohort members minimized her achievements and capacity by claiming those as a product of "female advantage" (Lim et al., 2021). Maria viewed

such "female advantage" as a very unpleasant perspective, but Jihyun shared that she could take advantage of such a perspective, for example, to find a future job. This points to the potential that Jihyun internalized the gendered stereotype as something that she could take advantage of. Further, Jihyun and Helena noted that they felt special in that they could do mathematics well, which also can be interpreted as a product of internalizations of the stereotype. Even though I felt that Jihyun internalized the stereotype on some level, she still wanted to challenge it. Jihyun, Jia, and Maria want to represent women in mathematics and/or be role models for future students, which became a motivation for their study in mathematics (Lim et al., 2021; Mendick, 2005).

The above experiences were in their home countries, China, Korea, and Mexico, and they claimed patriarchy is more apparent in their home countries than in the United States. In addition, they also considered their experiences in the United States to be more gender-blinded, which echoes what Asian female international students in STEM shared in Lim et al. (2021). The researchers interpreted earlier students' perspective that the United States is more gender-blind as their coping mechanism to adapt to the new environment and empower themselves to depart from their traumatic experiences in their home country (Lim et al., 2021). However, Lim and colleagues also found that international students who are early in their career seemed to find the United States more gender-blinded and merit-based, but students who are later in their career again reported micro-aggressive discrimination toward them regarding their gender and race.

Much of the existing research about gender in mathematics focused on achievement and student engagement (e.g., Hsieh et al., 2021; Johnson et al., 2020; Laursen et al., 2014). Even though there is some research soliciting students' experiences and narratives in STEM (e.g., Collins et al., 2020; Lim et al., 2021), there were very few in mathematics, and even fewer at the

post-secondary level (e.g., Mendick, 2005). Thus, the current study contributes to documenting female post-secondary students' experiences in mathematics in the existing literature.

Intersectionality

Within existing literature on gender stereotypes in mathematics, Leyva (2017) called for an intersectional analysis of gendered experiences. Intersectionality theory (Crenshaw, 1991) is a framework that provides a way to explore how multiple layers of marginalization, such as one's race, gender, and ethnicity, interplay with each other and (re)produce their experiences. It is also a way to highlight varied experiences between different subgroups within an in-group (Leyva, 2016b, 2017).

In mathematics education, intersectionality theory is mainly used between race and gender (Hsieh et al., 2021; Leyva, 2016b, 2017), but I would like to call attention to language as one of the aspects of intersectionality (Lim et al., 2021). "Nativeness" in English is one of the ideologies in education that marginalizes multilingual students whose first language is not English (Subtirelu, 2015). Further, the linguistic aspect of multilinguals' lives is not easy to separate from ethnicity, nationality, and race (Lim et al., 2021; Subtirelu, 2015). Languages have origins or are associated with some culture, and students' mathematics identity varies depending on their other social identities, as shown in the current study. As mentioned in the previous chapter about cross-case analysis, the participants' experiences show some evidence of intersectionality in their experiences. Jihyun's experience showed how her mathematics identity was shaped by multiple layers of marginalization and discrimination. Also, Maria and Helena talked about being "Latina," which always combined their race and gender. Maria especially did not separate what it means to her to come from a Latin country, but always talked about being a woman from the country. In addition, the language differences mentioned previously might have

been mingled with the participants' experiences differently. The language aspect of one's identity might become more challenging to some multilingual students and less so to others.

Also, language is perhaps more fluid than other social aspects of social identities. Although gender can be a fluid social element of identity, one's perspective about language changes rather quickly and more often than the other identities. Jihyun and Jia mentioned that they felt more comfortable with English after one or more years of experience in the United States. I focused on those who are relatively early in their career in the United States in this study to explore how they manage their first few years, but it would also be meaningful to see how multilingual people's perspectives toward the languages changed over the years and how that interacts with their other social identities.

Reflections on Research Methods

In this section, I would like to share some reflections, including limitations and notes about the current study.

Reflections on (Possible) Influences of My Positionality

First, I noticed that I could empathize with certain populations more than others and noticed that participants might have felt similarly. For example, Jihyun is a Korean woman who has a lot of similarities with me. We both are Korean women who speak Korean as our first language, our educational backgrounds are similar, and we both are the oldest daughters who have two siblings (younger sister and youngest brother). The interview happened in Korean and English, and I could understand the contexts, such as hagwon and cramming education, and what she was saying without having further explanations. Also, as Helena and Jihyun both mentioned, it is easier to understand English from people who speak the same language as there is a pattern in each language, and one can just translate those sentences to their first language.

Even though Jia did not share the same first language as me, I felt she was very open to sharing her experiences with me. I wonder if it was easier for her to assume that she could share about her experiences in the United States because both of us are East Asian women. The other way might be true as well. I felt it easier to ask Jia how it is to be an Asian in the United States as I have lived my version of such life and spoken with many of my friends in the United States. In contrast, I was cautious when I asked Helena and Maria about being Latinas because I felt I did not have good background knowledge about their previous lives and experiences in the United States. For example, when Helena and Maria shared some issues in their countries that they wanted to be improved, I did not know what to say. I did not want to sound judgmental or pretend that I knew their previous and current contexts. Hence, it was relatively harder to form a rapport with Helena and Maria because I was censoring myself much more.

At the same time, I wonder how Helena and Maria really felt during the interview and data collection. They both mentioned less about being Latin@ compared to how Jia and Jihyun talked about their experiences as Asians and the stereotype "Asians are good at mathematics." They might have never faced one of the narratives related to their race, or they might not have felt safe talking about that aspect to me. Even though I do not like Asians to be positioned as a "model minority," I was worried about how they would feel and react when being asked about their experiences as a Latina by a person who is from the "model minority," which is one of the groups stereotyped to perform better than their group. I was wondering if they would have been trying to protect their races because I am an outsider to their Latin@ community. It was noticeable to me that they did not share a lot about races, where we belong in different groups, compared to what they shared about gender, where we belong in the same group. Existing research claimed that people are more likely to protect their group when they were outperformed

by the other group and enhance their group when their group outperformed the other group (Chen et al., 2002), which aligns with my experiences in data collection.

Nevertheless, I still firmly believe both of them shared their valuable voices and important experiences with me, but I wonder whether and how different it would be if they were interviewed by a Latina researcher.

Anti-deficit Perspectives Toward the Population

When I was writing and interpreting the data, one of the concerns that I had was how I should deal with the deficit perspectives that participants have for themselves. I did not want people to view multilingual students as people with deficiencies and wanted to challenge the notion. However, it was also important to have their voices and represent how they view themselves to the audience. In the end, I kept their perspectives in their narratives even though it felt like a deficit perspective, but I tried to avoid deficit interpretations of their experiences when I wrote reflections and discussions. For example, Jihyun's narrative has some parts that can be interpreted as deficit perspectives. Jihyun talked about the amount of energy that she had to invest in taking classes and questioned her qualification to be a mathematician because she felt English speakers would think faster than her. She also shared with her advisor that she felt she was still not good at communicating in English and was restricted by the number of expressions and vocabulary.

I decided to keep these parts in narratives for the following reasons. They are related to the challenges that they were facing in the classrooms, and some of the participants attribute the challenges to themselves. However, these deficit perspectives toward themselves probably would not have just come from internally. That might be because there are not many things that they

can do about the external aspects, but they can try to "improve" themselves, which is supported by the efforts that some of them shared what they did to improve their English.

I also hoped that these narratives would provide the contexts where these deficit perspectives came from. For example, the stereotype that "Asians are not good at English" probably would not have been helpful to Jia and Jihyun. Also, it probably would not have been rare to hear complaints about the accents, such as "I cannot understand Professor X because of their accent," as appeared in these participants' narratives. Even though accents are valuable as it shows that a person can speak at least one more language, it has been viewed as a deficiency. Then, it is not surprising that multilingual students who have accents also think they have a deficiency in their English. Jihyun is an example of those. She wanted to erase her accents as much as possible.

I think it is important to have relatively holistic portraits of their experiences, including how students view themselves and where they stand to challenge their deficit perspectives toward themselves. Further, I hope other parts of the narratives help readers not just to take away their "deficiencies," and I suggested an anti-deficit perspective focusing on their experiences rather than their internal features through the reflections and this chapter. As Adiredja (2019) suggested, there is a self-sustaining cyclic system of deficit narratives. Deficit master/research narratives influence deficit perspectives for marginalized students' mathematical work, which leads to deficit interpretations of their work, followed by deficit stories about those populations. In turn, such deficit stories enhance the deficit narratives. The current study shows this cycle as well, and I wanted to disturb the cycle by presenting anti-deficit interpretations of my participants.

Future Directions

Implication for Teaching

The goal of the current study was to explore and share post-secondary multilingual international students' experiences related to mathematics. Hence, providing practical suggestions for teaching was not the main purpose of the study. However, the followings are some possible suggestions for instructors in collegiate mathematics education based on this study and situated within the literature.

First, I would recommend valuing written communication as well as verbal communication. All the participants shared some way of benefitting from written communication. Rather than uploading the lecture note after the class, which students could use only after the class, lecture notes can be uploaded in advance so that students can download and write some additional memos and notes on the provided one. This will help multilingual students, as well as other students, focus on what is being talked about in the classroom rather than doing multiple jobs of listening, understanding, and writing. Even though closed caption service was mentioned previously because of the accents, having closed caption can be a good tool for multilingual students if they can download and review the transcripts later (Tisdell & Loch, 2017). Instructors can also value written conversations between students. As mentioned previously, providing a shared place to work during groupwork will be helpful (Liljedahl, 2016) as it provides a place where multilingual students can contribute and communicate using writing more easily rather than working on a separate sheet of paper.

Secondly, I would suggest being considerate when composing groups. Based on the experiences of Jia, who felt better when she had at least one more student who spoke Chinese in the same group, my colleagues and I suggested having at least one more multilingual student

within a group (Hwang et al., 2022). However, there should be more. Some of the participants shared their frustrations with other accents that they were not familiar with, especially Chinese accents, which made me think it could be unproductive if they have an additional "hierarchy" of languages in the group. As Takeuchi (2016) noted, friendship might be an important aspect to consider for multilingual students to take various roles in groupwork rather than passive roles. On the other hand, Jihyun said that she would never have shared her concerns about imposter syndrome with her advisor if she was still in Korea. Also, Jihyun and Maria shared their discriminated experiences as women in their home countries. Based on these observations, I question if it would be helpful for them to be in the same group with other men from their home countries. For all of the reasons above, I still do not think it is a good strategy to have randomly generated groups, but I could not find a single suggestion that will work for every class. However, I still believe that instructors need to be considerate about their group composition method, and one way to deal with the situation might be to ask students about their preferences after some weeks later in the semester once they have worked with different people to know whom they feel more comfortable with-as Jia's instructor did (Hwang et al., 2022).

Lastly, I urge instructors to view multilingual students as someone who has more linguistic resources than as someone who has a "language barrier." My hope is that the narratives of this study to help others understand multilingual students in classrooms. If one has a perspective that multilinguals have deficiencies and a language barrier, then everything becomes the multilingual students' fault and responsibility to be like "native English speakers." However, there exists more tension and power dynamics that come from the ideologies of our society, such as racism, as mentioned previously. With the perspective that multilingual students have more

linguistic resources, I hope instructors can find more equitable teaching practices to improve their learning environment.

Implication for Research

As a next step of the current study, there are three different kinds of studies that I am currently visioning.

First, related to the previous section about teaching implications, research about teaching practices that can help multilingual students' learning environments is necessary. From the current study, there were some possible teaching practices that can provide better learning environments, such as considerate group composition (Hwang et al., 2022) or valuing written communication (e.g., providing shared space for writing (Liljedahl, 2016)). However, a further discussion needs to happen. Also, it is necessary to change the discourse within mathematics education about multilingual students from people who have deficits to people who have multiple resources in mathematics education practice. For this purpose, a potential option might be to have a participatory study to find teaching practices and/or principles that could be useful to practitioners. In the process, I hope the narratives from the study could become a tool to understand multilingual students and start the conversation.

Second, directly related to the current study, it would be interesting to see multilingual graduate students' mathematics identity both as a student and as a teacher. Many mathematics graduate students become teaching assistants where they need to teach using their second language. While they manage both roles as a student and as a teacher, it would be interesting to see how their perspective from one role influences the other. Also, in post-secondary mathematics education, a significant portion of the instructors are multilingual as well (Subtirelu, 2015). Hence, studying multilingual graduate teaching assistants will also provide insights into

the challenges of being multilingual teachers and their strategies to manage the situations. Multilingual instructors would also have linguistic challenges, but it could be even harder when it is combined with their other identities, such as being a woman (Lim et al., 2021).

Lastly, there could be a more detailed study about how multilingual students use their multiple languages. For example, I am interested in looking at how they would use their multiple languages when they were understanding new definitions or facing some challenging problems. I believe it would be helpful to devise a teaching practice when the field has a better understanding of how they use their languages.

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APPENDIX A JIA'S STORY

"I Love Math, Hope Math Loves me too"

My name is Jia. I am a Chinese woman, was born in China, and lived there until I came to the U.S. for my undergraduate education. So, Chinese, or Mandarin, is my first language and English is my second language. As I am studying in the United States, the language "officially" used in the classroom is English, which I had some concerns with. As my major is statistics and minors are Computational Mathematics Science and Engineering (CMSE), data science, and mathematics, I have taken a lot of mathematics courses. Of course, languages matter in other disciplines as well, but today I am telling you about my experiences in mathematics courses.

Intro-to-Proof

One of the mathematics courses that I remember is the Introductions-to-Proof course. I took the course in the fall semester of my sophomore year, and, as its name indicates, it was a transition course between Calculus courses and more proof-based mathematics courses. Actually, before the course started, I did not have a lot of ideas about the course. I asked some of my Chinese friends and they said that the course is not difficult, but hard to do perfectly. They said that sometimes it asks you to explain your thoughts, but it can be quite difficult for international students, like us, to express thoughts clearly. And, I started to agree with them as the course was going on. There were several points that I was worried about my English while taking the course although I did well in this course. Sometimes I could not understand what the problem was asking to me or had difficulties understanding the definition, and other times were difficult because of the interactions with other people, like with the instructor or groupmates. I think these challenges were both from my language and my education, which are different here content-wise and teaching method-wise.

One of my biggest challenges was that the ITP course was based on groupwork. We had a lot of groupwork in the course, about two-thirds of the class each week was set aside for us to solve problems in groups, which I enjoyed and found helpful in general because groupwork helps me come up with some ideas or understand the material that I could not understand from my instructor's explanation. But, I think we need some "scientific way" of assigning groups. I think it will be good to have at least one person who can lead the discussion and some others that can benefit from the group discussion. If you ask me what an ideal form of a group would be, first of all, I want people who are active and passionate about what we are learning. Then, I want someone who is super smart who can help the group when we fall behind for some reason, another one who is humorous who can make the atmosphere of the group nice, and a prudential one who can notice the tiny mistakes that we make. Lastly, I want a person from different culture, I mean not Chinese, so that they can provide different perspectives or ideas. In addition, for me, personally, it is important to have a person who I am already familiar with as well because I am a person who likes to talk to the people that I know.

Anyway, as the ITP course changed groups somewhat regularly, my experiences were diverse. The first week, we formed a group based on where we were sitting. As I told you just before, I wanted to have some familiar people in the class, so when I stepped into the classroom, I went to a Chinese female student. And then, there was another Chinese female student, so we became a group. We added each other on WeChat (a SNS that mainly Chinese use) and became good friends, and we sometimes asked questions about the class. (Actually, I became a good friend with one of them and we still hang out sometimes.) As we sat close to each other, they became my groupmates for the first two weeks in the class, which was a good experience for me. I liked it because I could use Chinese with them, which I think is efficient.

After that, my instructor and TA switched groups roughly every two weeks during the first half of the semester. The TA somehow generated the groups and wrote them on the board. And then, I started realizing that who I am assigned to work with in a group influences my groupwork experience. None of my second groupmates were Chinese and all of them looked like their first language was English. I think communication between groupmates is important. But, in this group, I felt isolated. No, let me rephrase that: I was not very engaged. The other people in the group were discussing enthusiastically, but I was like in an, "Ahh, yeah- (indifferent)" kind of mode. I felt my language was the biggest reason. I felt like I did not speak that good.

I always had ideas about how to solve the problems, but I could not write any steps clearly or I was not sure whether the words that I wrote were expressing my thinking in English. It happened several times in that semester. For example, something like this: Once, we were asked to compare a variable, x, with a number. In those processes, our group missed that x is a negative number and that led us to make a mistake by not changing the order of angle bracket (e.g., > or <). I said, "x is negative, so we need to change the signal" and they did not understand what I was telling them. I was expecting some positive reactions from them, like "Oh, yes, that's the point," but they did not get it and so could not give me such a reaction. Because I was not sure how to say it any other way, I just wrote it down on my note and showed it to them. Then, one of the groupmates understood and he explained that to the other people. It was a little bit frustrating, but this kind of experience was common at that time. After some of those experiences, I started staying quiet because I knew other people would usually say the same idea that I would have if I waited. Of course, if it took too long, I spoke about my idea, but sometimes I felt I was wasting others' time because other people just could explain the idea in one sentence.

I did not think my experience was challenging only because I did not know how to explain mathematics in English. One other reason that I remember was that, sometimes, I could not be sure which ways or methods they learned to approach the same type of questions in their mathematics in their K-12 education. I thought we all would have learned similar mathematics, but it seemed not. I think it made the situation harder because I think that it would have been easier if we all had similar background knowledge about mathematics. Another part of my experience that I think added more difficulty was that I could not easily jump in on their small talk. This could also be related to my personality; I don't think I am a very social person. But sometimes they speak fast or say things about things from outside the classroom that I did not understand, and then I felt excluded. But I still thought that it was better than doing all of the work alone.

After another round of group rotations, I again was in a group with my friends from the first week as well as another Chinese female student. About middle of the way through the semester, our instructor sent us a survey asking if we had a preference on whom to work with and if so, who we wanted to work with. I put the three Chinese female students on my list, and so did the other three. So, we became a permanent group for the rest of the semester. I was very happy with the group because I felt our communication was so efficient; being able to speak Chinese reduced the time we needed to explain what we were not understanding in English. Compared to when I was in groups where there was the pressure that I needed to explain things precisely, which made me stutter and feel even more nervous, it was so easy to communicate. I became a lot more talkative. We could do small talk as well, and I enjoyed talking with them about my life outside the classroom.

Thinking back, I should have encouraged myself to step out of my comfort zone more. Now, I just try to express my thoughts as clearly as I can. And I will encourage others, as well, if they are in a similar situation. I thought about what I would have done if I were in China and some of my classmates were international students who were not speaking Chinese perfectly. Then, I realized that I would have been patient and willing to listen to them. I think that it should be the same in the U.S. What do you think?

Other Concerns Related to Language Use

Although another two years have passed since the ITP course, I still do have some concerns about my English in mathematics-related courses. Sometimes I misunderstand the questions on the homework or exams or mishear some words or phrases during class, which makes it hard for me to connect concepts back to what I know in Chinese. For example, in Analysis II, my instructor said "partition," but I heard it as "portion," and I did not know what that meant in mathematics. I was confused throughout the whole class and had difficulties following his proof in the class. I noticed that I misheard what he was saying after I searched online when I came back home. However, now I know how to deal with this kind of situation. When I was stuck, as I said just before, I search online or ask my Chinese friends. For instance, I struggled a lot in my Abstract Algebra I course. As the course was so challenging, I needed to relearn the material from resources I found online. When I searched for the course materials online, I could find some parts of some textbooks as well as lecture notes about the course in Chinese. I read those and tried to learn the course material by myself. Although there was no perfect substitute for the course textbook in Chinese, it was still helpful. Sometimes I used videos on YouTube or Bilibili (video sharing platform like YouTube but in Chinese usually) as additional resources. When I search online, I use both of my languages. I usually start with

English, but when it seems it will take a lot of time for me to understand, then I switch to Chinese.

In addition to searching the course material to help my understanding of the material, I sometimes search for the terms that appear on the question. If I was stuck on some problem, then the main problem was usually that I misunderstood what the question was asking. Although I struggle sometimes because I need to communicate mainly with my second language in the classroom, I see some benefits of being able to use both languages and have figured out how to deal with the challenge. In addition to what I already told you, I know I can ask some friends who speak English and other friends who speak Chinese when I have questions. Also, I use Chinese in my note taking as well. I use it as an index so that I can quickly check where I need to look because I can notice Chinese easily and read Chinese a lot faster. Another time I use Chinese is for the definitions. I write some definitions or some keywords in Chinese so that I can remember them better. I switch between the two languages. I can show you one example (Figure X). I also write some memos or how I feel in Chinese, such as "don't fall asleep, you can do it!" "why is it so hard?" or "Confused, review after the class," so that I can focus more on the class and encourage myself.

Some Additional Challenges Because of COVID-19

Although I have been figuring out how to survive in mathematics classes, COVID-19 added another challenge. Because of COVID, all of my mathematics courses were changed to online, which led professors to explain mathematics lectures on zoom and write them on a sheet of paper. Compared to when they are writing on the board and explaining in front of physical students, they wrote a lot faster as they did not need to write it as big as they did on the board. Also, for some reason, they spoke faster and somehow it felt to me that they were reading the

book rather than explaining things to us. I sometimes could not write everything in my notes and could not completely understand what they were explaining. Also, they did not have a lot of groupwork during the class time, so I could not even ask questions to peers during the groupwork. So, I tended to review the videos instead of going to the synchronous class because I could rewind and stop whenever I wanted to. Then, when we came back to "in-person" mode, wearing masks became another issue. I personally can hear others clearly by looking at their mouths. For example, if I do not wear my glasses, I cannot hear what others are saying. Wearing masks had the same effect on me as I could not see their mouths. And further, I wanted to keep the social distance!

How I Think about Mathematics and My Future

I think so far I told you a lot of the challenges that I experienced. But, all of these challenges neither discouraged me from studying mathematics nor led me to lose my confidence. I think my challenges in mathematics courses related to language mean that I just need more time than native English speakers, but they do not mean that I am not good at mathematics. This is how I think about mathematics: I think mathematics itself is a language because of different symbols and grammars. Like any other language, if you have not learned it, mathematics is mysterious and special because you do not know how to interpret and use the language. For instance, if I had not taken Analysis II, then I would not know what " $\forall \varepsilon > 0$, $\exists P$ partition, such that U(f,P)-L(f,P)< ε " means. Because I view mathematics as another language, I think I just need to take more steps moving between the three languages, Mathematics, English, and Chinese. I think mathematics would have been hard even if I studied it in Chinese, but mathematics is charming so I would work hard anyway to understand the material. I am

confident that I will definitely learn well if I have enough time (although time just becomes an issue).

Also, there is another reason that I persevered in learning mathematics. I believe anyone can do mathematics. I want to show mathematics is not only for genius, and I want to refute the common claim that boys are smarter than girls or boys are better at mathematics than girls. Many people had said such a myth so many times, but that discouraged me when I could not do well on mathematics. At that time, I thought I could not do well in mathematics because of my gender, which made me think that I cannot do as well as other boys no matter how hard I study mathematics. That line of thought, in turn, hindered me from putting a lot of effort in studying mathematics. I do not think it was conscious, but it made me think negatively about my capability in mathematics and be passive and inactive about my learning of mathematics. Thankfully, I had a really good female mathematics teacher in my middle school. The teacher encouraged me a lot and helped me to enjoy mathematics. Once, I asked her whether I should choose social science as my area in the future because those subjects are "easier" for girls to learn. She told me "You have unlimited potential, and you can do anything you want to do." I appreciated that encouragement a lot and decided to pursue learning natural science rather than social science, then I came to the United States to see a larger world, world outside China as well.

I still but less encountered to such a myth, "*boys are better at mathematics than girls*" in the U.S. compared to when I was in China. None explicitly say such a statement to me, or I have not experienced any explicit gendered-discrimination, I think, but I still have been feeling a reminiscent of such a myth in the mathematics classroom. I felt that students tend to ask questions to men, and men are usually very willing to answer questions. Even when they do not

know the exact answers, they could help analyzing the process or share their ideas. I am aware that there can be other factors as well such as they knew each other before the class, or I might have been just too sensitive.

Nowadays, I see more outstanding girls who prove that those statements are just stereotypes and myths, which makes me happy. I want to be one of them. As a first step, I just graduated from my undergraduate university with a bachelor's degree in statistics and as well as having three minors, which are data science, mathematics, and CMSE. I wanted to study statistics because I think big data is quite useful and if I can make good use of statistics, I can benefit the whole society. Since I was young, I have been wanting to help all people's lives if I can become a powerful person. I still remember telling this goal to my parents at that time. For the goal, statistics was a good start, but I wanted to become more practical. I figured out that data science is more practical than statistics, and that is why I decided to apply for the master's degree in data science which I am now waiting for some offers. And after that, I might do some work, but I want to have a PhD degree as well so that I can become a professor. My memories about my education were not always happy-as I told you some of them today, I sometimes felt frustrated or helpless. In the future, if I become a professor, then I would like to let my future students know that they can do anything they want to. I do not want my students to encounter the similar discouragement and powerless experiences as I did, and if they experienced already, I want to help them get out of those thoughts.

APPENDIX B RESEARCHER POSITIONALITY: JIHYE'S STORY

Start of My Relationship with Mathematics

In my earlier memory, I did not like "math." I think, at that time, what is called mathematics was arithmetic, which I was not enjoying doing. I think that was my elementary school memory about mathematics. I did not think that I was bad at it because my mother sent me to the mathematics *hagwon* when I got 88% on one of my school's mathematics contests. Later, one of my friends was surprised that I wanted to become a mathematics teacher because she remembered I was bad at mathematics. So, I don't know what was right, but it is not important. I was pretty indifferent about mathematics. At that time, I wrote my plans for a future job as a pianist, artist/painter, or stylist–totally irrelevant to mathematics or mathematics education.

One of the reasons that I gave up the artistic side of my career at that age was because of finance. My parents were worried as that route would require more financial needs, and I had two younger siblings for whom my parents also needed their support. I grew up hearing the sentence "큰 딸은 살림 밑천이다" a lot as well as my father's side grandmother (and probably my dad as well) really wanted to have a son. I remember my mom told me that my grandmother did not seem to be happy or congratulate her when I was born. My family and extended family were pretty patriarchal.

Secondary Education – Mathematics Teacher

Then, when I became a middle school student, I met some mathematics teachers who made me want to become a mathematics teacher. Based on our first mid-term exam and probably IQ test, the school recruited some "gifted" students into a mathematics club. I wanted to join a club for other things, such as beads craft or drawing, but I was kind of forced to join the

mathematics club. Although it did not happen based on my preference, it changed my perspective on mathematics a lot. The teachers worked really hard to provide us with some experience outside of textbook mathematics. They brought some board games to find some mathematical component into it, asked us to read some mathematics novels and essays, and they brought us to some mathematics festivals where you could explore and experience the mathematical things that can be found in real life, something like prime numbers in flower petals or the fastest curve for a slide. In addition, we could run a booth at such a mathematics festival and also our school festival. This experience really gave me some interest in mathematics and then made me dream that I want to become a teacher who can introduce this kind of experience to my future students. This became a motivating force for me to study mathematics through high school.

For my high school, I went to a private school, one where you need to get admitted after an interview, because the chairman of the school was the author of the famous high school mathematics reference/workbook. (The set of his books was the second most sold in Korea when the most sold book was the Bible.) In school, there were some peers who were really fast in mathematics, not only in the calculation but also coming up with some brilliant ideas to solve problems. That was a real challenge, and I felt that I could not get into the top 20% of the school in mathematics, score-wise. Although it was challenging, I still liked doing mathematics and enjoyed it. I just enjoyed solving complicated problems and thinking about them for a few hours and days. It was very rewarding when I could solve those.

During high school, through some of the teachers, I learned that knowing mathematics and teaching mathematics are different. In our school, there were at least three teachers who had mathematics Ph.D. degrees from the most prestigious university in Korea. I felt that the teachers

knew how to do mathematics, but some of them did not know how to explain the logic and their reasoning because things were so natural to them. At this point, I became more and more leaning toward that I want to teach mathematics–arrogantly, I thought I would be good at it.

However, around that time, one of the teachers that I really admired said that *I was not that smart to do mathematics* compared to other peers in the school, so probably it would be better to choose mathematics education than mathematics when I was deciding between those two as my major. It was pretty bad and hurtful. Even though I was getting at least in the top 4% of national exams, he still said it to me, and that hurt my confidence and planted the seed of doubt in my mind regarding my possibility in mathematics.

And this also reminds me of another experience with one of the middle school teachers that I mentioned previously. The teacher–who I really liked and respected and who was the mathematics club teacher and also my mathematics subject teacher–told me that *girls do not do mathematics as well as boys*, even though the middle school was a girl's middle school and he was teaching in the school. Both interactions were unpleasant to me, but I had to take it, and it bothered me when I was doing mathematics. I thought that what they said was not true, but I have doubted myself throughout my life.

Undergraduate: Perfectionism

After these experiences, I felt that I had to prove that I could do mathematics well. Eventually, I went to my undergraduate university as a mathematics education major. During that time, I think I felt that I should do well. No other choices. I tried so hard to keep first place in the cohort throughout all four years of my undergraduate education. I could not get the highest score for every single mathematics exam, but at least in terms of GPA for each semester, I felt that I needed to be the highest. I did not want to give that spot to other people. It was related to

my scholarship as well because I thought I needed to get it because my parents have two other kids that they need to support and the full tuition scholarship was only given to the only one with the highest GPA in the entire department. But more than anything, I did not want to lose to other people, especially men.

The notion that men are better at mathematics than women was still very common in my undergraduate. One of my professors preferred male students in general during the class and implied that they were better at it, although she was a woman as well. I liked her as she was good at teaching (lecturing) the material, but I was annoyed when I figured out that she talked to a class a year after mine and told them that there was one female student who really worried about her scores to the point of reviewing her exam even though she did get good scores, which was me. Her nuance was that I was too sensitive to the grades and made a big deal of it. I was not happy that my effort to review what I missed was regarded in that way. I was not there to complain about her grading, even though I did care about my grades.

Anyway, the obsession that I could not lose first place made me more reluctant to work with others. I did not want to show that I also struggled and I was a person who needed time to think for a while to solve problems. I also wanted to hide that I sometimes searched online. I want to be seen as perfect in some ways. Many of my friends formed a study group, but I never joined any of those. I sometimes talked with other classmates about homework, but it was not common that I was sitting and talking with someone else. In the end, I graduated a summa cum laude for the cohort in our year anyway.

Even through these discriminations and biases faced in mathematics, I still wanted to study more for a higher degree. I really enjoyed studying mathematics and solving problems. I enjoyed thinking about different hypotheses based on the given information and using all of the

information to find the answer logically. But as my undergraduate degree in mathematics education was still heavily focused on mathematics, I was not sure whether I wanted to learn more content in mathematics or to study mathematics education more. I ended up decided to try mathematics first in the U.S. because of English. I was not good at English, and the fact that I needed to read a lot in English when I studied mathematics education sounded like a big hurdle for me.

At that time, I knew that I was bad at English. English was always the one that trapped my ankle when I was preparing for the university entrance exam, KSAT. My English education since when I was seven was heavily focused on reading and listening, very little about speaking and writing. There were cases that I could practice. My high school had a native Englishspeaking teacher, but we had one teacher for 38 students in a classroom. Later in high school, I participated in one of the physics competitions where my groupmates and I were solving a physics problem and presenting and defending our solution in English. During that time, I remember one of my groupmates pointed out that I always start speaking with "I am …" even though sometimes I need to say "I + verb" rather than "I am -ing" form. It was not that I did not know other forms grammatically, but speaking was just a whole different world for me. I think part of the reason was "I am …" was one of the first forms of English sentences that I learned, and I had not had a lot of chances to speak in English. Anyway, I thought that I was not good at English in my undergraduate (and still do).

However, I thought I would be fine studying mathematics because I was already learning mathematics with English textbooks in my undergraduate. Even though the professors explained in Korean, they used all the mathematical terms in English, so I thought I would be fine. For example, when they were explaining "homomorphism," even though other parts of the

explanations were in Korean, we still used "homomorphism" in English without translating. I guess I did not know how important the Korean explanation part was. The way that I studied in my undergraduate was that I attended class and took notes of the lecture while understanding those, and that was it. I had not sat down to review and understand the lecture part. To do that, Korean was very important, although I did not know then how big a difference it would make.

Graduate School in the United States

Unfortunately, even when I came to the U.S., those biases about gender and mathematics were still prevalent. I was the only female student out of 17 cohorts in the mathematics Ph.D. program, and the year above mine was similar. And by the time I arrived on the campus, the female student who was a year ahead of me decided to leave the program. One out of seventeen. The number itself meant a lot to me. I did not feel safe hanging out with friends being only one female, and that did not help me either when I was studying mathematics. I did not want to stay in the office place after sunset when all the others were men. It did not matter whether the students in my cohort were nice or not. It was just not a safe place for me. Moreover, the first teatime that all the first-year Ph.D. students and the graduate director met, I felt that the graduate director at that time ignored and did not respect me. He was asking questions to all the other students except me. I could see why the second-year female student wanted to leave the program. I was not valued, and I remember thinking 'was it because I am a woman? Or as I am Asian? Or both?', which did not help me at all with my confidence.

In the graduate mathematics classrooms, where, of course, all the classmates were men, I witnessed that they asked questions and answered professors' questions, which led me to think that they knew and understood the contents a lot faster than I did. I think it was not very helpful that English is my second language, and I was not fluent enough. Think about how much I

wanted to be perfect in my undergraduate. I had to prove that I could do mathematics, and now I do not speak or listen to the language of instruction that well even. Even though many of my professors were writing on the board, I felt I was missing so much information that they delivered verbally. Maybe the piece of information that they were saying verbally was not very helpful, but I could not know if that was crucial in understanding or not. As I had to translate so much information, I felt I did not have enough space to spend on understanding in my brain. No matter how well I did score-wise, those experiences crashed my confidence.

Later, a professor told me that the members of my cohort just asked questions regardless if they made sense or not, and I was doing well in the course and did well on the qualifying exams. I was happy to hear that, but it did not help me improve my confidence. I felt that I did not have such a huge amount of enthusiasm that my other cohorts had in mathematics. I knew (or tried to convince myself) that I could do a mathematics Ph.D. degree if I wanted to do it, but I did not see a reason to stay in the field in the end. Thinking back, I wonder whether I was worn and tired by that time of the biases that I experienced in mathematics and how much I was undervalued, feeling that I needed to prove myself as a person who could do mathematics. I felt that by that time, I had lost my interest in learning new mathematical content, and I still feel the same now.

Again, Mathematics Education

The reason that I could leave the mathematics Ph.D. program was that I found that my interest in mathematics education is still there. I hesitated so much as the professor in mathematics wanted me to stay in the program, using my score as a piece of evidence. Many people told me that people would think that I moved to mathematics education because I could not do mathematics. Remember what my high school math teacher told me? After 10 years, I

again faced the same words. But, at that point, I was indifferent to my learning in mathematics, and I did not see the point of doing it. (This is why I think positive mathematics identity is so important in addition to achievement, and I would like to listen to students' experiences more than focus on their content learning.) There were a lot of discussions, and I tried several different things, and then finally, I started taking courses in mathematics education.

As I guessed before, mathematics education courses require more English skills. There were a lot more readings, and the courses were discussion-based. I was comparing myself with other people. They were reading so fast, and they were able to do skim reading, but I could not. I also do not know if it is because English is my second language or if it is just me, but I do not remember what I read or even something that I wrote. People told me sometimes they were just bluffing even though they did not read the whole thing, but because of the insecurity that I felt, I couldn't do it.

The notion that I needed to be perfect was still there and did not help me in taking courses. I was just busy translating what other people said in the discussion, and I could not think about my opinions. Even if I was thinking of responding to others' comments, they slipped out of my mind when I had to translate another one. Translating, thinking, and speaking was too much for me. Also, I did not like talking about what I had not thought about enough. At the same time, I knew if I did not say anything, people would think of me as dumb. But I did not want to or could not say something. So, it was just a bad cycle. And I still do not like a discussion. If it's a presentation, it's better because I can prepare. Q&A? It is always the most challenging one because I am afraid about whether I can understand it and I cannot prepare in advance. My hope to be perfect and how I view I speak English cannot get along well.

Regarding The Current Research

All of my experiences led me to do the current study. I learned and grew up so much, and there were good memories as well. But it was painful for me to study mathematics and mathematics education in English. When I was struggling and frustrated with my English, people told me that my English was ok/good and that they could not imagine how difficult it would be to study a Ph.D. program in one's second language. I appreciated those, but sadly, it did not help me think better of myself. I still feel the glass ceiling every day that I would never be like a native English speaker. I am still afraid that I look dumb when I speak English poorly.

I feel hypocritical that I do this study using the anti-deficit framework when I have so much deficit perspective toward myself. I say that we need to view multilingual students as people with more resources for learning, but I view myself as someone with a deficiency. It's ironic. I cannot ever view myself as a person who can speak English in addition to Korean in the United States, but I feel that I am a person who can speak English as well as Korean in Korea. When I compare myself to how I speak in Korean, I become such a trifling person when I speak English. I think part of the reason is that English is valued in Korea, but Korean is, at least I feel, not valued in the United States. It does not help how I see myself that I can speak Korean in the United States. I just became a person who speaks "broken" English.

For that reason, I questioned and am still questioning myself so much if I am a person who can do this research. Am I qualified to do it? When I have such a deficit perspective toward myself, can I advocate for other multilingual students while avoiding such a perspective to my participants? I still do not know if I will succeed in the goal. And I do not have a clear vision of how we can provide a better environment for multilingual students because I did not find what is helpful to me. However, I still would have liked to deliver multilingual student voices to get

some mathematics educators' attention to these participants' experiences and find a way to help provide a better learning environment for these people. When I was chatting with a fellow Korean graduate student in another program, she also mentioned that she was helpless in taking graduate courses and she found herself attributing the reasons for her challenges to herself in the end as she could not do anything about it. I truly hope she and other multilingual students have a better experience and better understanding of their value than I did and that this study is one of the starting points of those changes.

APPENDIX C ESSAY PROMPT

Essay Prompt for Multilingual international students' mathematics identities in proofbased mathematics courses.

Shiv Smith Karunakaran, Jihye Hwang Michigan State University

Introduction and Purpose

Good morning/afternoon. Thank you again for participating in this study. As we discussed earlier, this interview explores your experiences of mathematics throughout your life, but especially focusing on undergraduate mathematics, as a multilingual international student.

Confidentiality

It is important for you to understand that your confidentiality is of the utmost importance to me. The complete details of the confidentiality agreement between you and I are listed in the Informed Consent Form that you have already signed. At this time, I want to ask you once again if you have any questions or concerns about your participation in this study. If so, please send me an email (<u>hwangji7@msu.edu</u>) or Dr. Shiv Karunakaran (<u>karunak3@msu.edu</u>)

"Me and Math: my relationship with mathematics up to now"

Please write an essay about your experiences around mathematics throughout your life. I am curious about your past/current/future experiences, especially

- What your thoughts about and/or relationship with mathematics were and are;
- Whether those thoughts have been changed and if so how and;
- Any events or people that you remember or that was important for you in terms of your relationship with mathematics.
 - \circ What were your emotions at that time?
 - Why do you think they became an important event or person?

APPENDIX D INTERVIEW SCHEDULE

Interview schedule for Multilingual international students' mathematics identities in proofbased mathematics courses.

Shiv Smith Karunakaran, Jihye Hwang Michigan State University

Subject: General

Introduction and Purpose

Good morning/afternoon. Thank you again for participating in this study. As we discussed earlier, this interview explores your experiences of mathematics throughout your life, but especially focusing on undergraduate mathematics, as a multilingual international student.

Interviewer and Participant Roles

It is important for me to try and understand how you think about the prompts in reasoning and proving tasks. The best thing you could do is to share your thinking and correct answer does not matter. At times, I might say, "Tell me more" or "What do you mean by ..." or "What led you to ..." When I ask these questions, it is often just to make sure I understand how you are thinking. I might ask you to tell me what you're thinking if you are working quietly. My questions and any notes that I might write are signs of my interest in what you are doing and my need to describe your thought process.

Confidentiality

It is important for you to understand that your confidentiality is of the utmost importance to me. The complete details of the confidentiality agreement between you and I are listed in the Informed Consent Form that you have already signed. At this time, I want to ask you once again if you have any questions or concerns about your participation in this study.

Also, at this time, have them sign the Amazon gift card list and consent form.

This Interview

I am going to ask about your experiences starting with what you wrote in your essay "me and math: my relationship with mathematics," to explore your experiences more, and to know more about you. If you could explain and give to me as much as detail you can give me, that will be very helpful and appreciated.

Do you have any questions before we begin?

If not, continue ...

Phase 1) Background

This section is designed to ask their background such as year, previous mathematics course, and experience in mathematics.

Ask:

- Which year are you in? Sophomore, Junior, or Senior?
- What is your major and/or minors?
- When will you graduate?
- What is your future plan after graduation?
- Where's your home country?
 - Although I am not sure how much I can infer about the culture from their home country, I have to know the context for their K-12 education.
- When did you come to the US?
- What are languages that you speak?
 - To have some background knowledge about them.
- How do you describe your relationship with English?
 - To know how they feel about English in general.
- What mathematics courses and when have you taken here?
- How was your K-12 mathematics education?
 - *Maybe already have this information from their essay, but to check.*
- What was your use of English in your K-12 education?
 - To have a bit more idea about how they felt about English before they came here.

<u>Phase 2) Questions related to their experiences written in "Me and Math: my relationship with</u> <u>mathematics up to now."</u>

The participant will answer to questions related to their essay. This part will be written once I read their experiences in the essay. So, I cannot write specific questions to ask in this phase of the interview, but here are few things that I want to focus:

If there're event/people that they mentioned in the essay, then ask more details about the event/people, how they felt, and what were the reasons for those emotions.
Especially focusing on a turning point of their relationships if available.

Then, in addition to what is written in the essay, ask more questions about the current university experience.

- How was studying mathematics in English for you?
 - Want to be open as much as I can to listen their stories.
- Were there any advantages and disadvantages of studying mathematics in English for you?
 - *Probably will have some follow-up questions.*
- How do you think the role of language use in mathematics? Is it important?

- Is your experience different from other subjects, such as chemistry, history, or any subjects that you have taken in English as the language of instruction?
 - To know how they think about role of language in mathematics and see whether there is some special character that they notice comparing to other subjects.
- How were your interactions with other people about mathematics in English? Other people can be professors, TA, MLC tutors, classmates, or any people.

Lastly, about their vision about use of mathematics in their life.

• How do you see your future relationship with mathematics?

This is the end of the interview. End it by saying/asking the following:

- Thank you for participating. Do you have any final thoughts that you would like to share?
 - To ask one's final thoughts, or whether one did not have chance to mention what they wanted to say.

APPENDIX E NARRATIVE TASKS FOR EMAIL INTERVIEWS

Narrative #1: Taking Up Spaces in the College Mathematics Classroom

You are taking [Math XXX] and the professor was trying to prove " $\sqrt{2}$ is irrational." As class is learning about contradiction, the professor assumed that $\sqrt{2} = \frac{p}{a}$ where p and q are integers.

From there, the professor and students together found that p should be divisible by 2 and so is q. The class seemed to be agreeing that they can assume p and q are relatively prime. At that point, one student raised their hand and waited. The professor noticed and asked what their question is. The student said, "I don't understand why we can assume there's no common factor." The professor said, "That's a reasonable question." The student proposed their conjecture: "because p and q is simplified...?" The professor agreed with the student and added some more detail.

Probing Questions:

1. When you were reading this narrative, who were you imagining as the student? Can you describe them?

2. Can you see yourself as the student in the narrative? In any of your proof-based mathematics courses? Why or why not?

3. If you behave differently in proof-based mathematics lectures, how do you behave? Why do you behave this way?

4. Are there any individuals from your college mathematics lectures or recitations sessions who engage like this? Who are they? In what ways do they behave similarly and differently?

Narrative #2: Groupwork

The mathematics course that you were taking now has some groupwork. In your group, there were four different people: One student was leading the discussion and checked with others if they were on the same page: "Are we good to move on to the next problem?" Another student did not hesitate to ask questions to the group when they were not sure what to do: "No, not yet. I am not sure how you made a move from here to here [pointing fingers on the notebook]" This person sometimes tried to do some joking so that they could feel comfortable and closer. The third person was staying quiet and listening to other's explanations. Sometimes it looks like this person has something to add but stays quiet. The last person did not seem to care about the groupwork and worked on their own.

1. Do you relate to this one of these four than others? Or, do you relate to some combination of these students?

1-1. If it is based on your experience, who were the other people and how did they actually behave in the groupwork in your memory?

1-2. If it is based on your imagination, who might be the other people? How would you describe them?

3. Would you have had a different role in groupwork if you do in your first language?