THREE ESSAYS ON MATCHING PROBLEMS

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ABSTRACT

This dissertation studies opportunity and equity in college admissions, as well as how higher education affects labor market outcomes. The first two chapters explore how state policies affect enrollment at public institutions of higher education. The third chapter characterizes outcomes in a labor market in which agents with different skill levels (levels of education) choose roles and match with each other. In each chapter, I build a theoretical two-sided matching framework to explore how limited opportunities are allocated, and motivate theoretical predictions with empirical evidence.

Chapter 1: Performance Funding and Equity of Access to Public Universities

While state appropriations to public universities have historically been determined by student headcount (enrollment funding), an alternative is to fund based on completion metrics such as number of degrees conferred (performance funding). Advocates argue that performance funding incentivizes universities to decrease inefficient over-enrollment, while critics argue that performance funding incentivizes universities to admit fewer under-represented minority applicants, as they are less likely to graduate. I develop a framework in which a social planner and universities systematically differ in their expected returns to enrolling students and show that there exist many enrollment and performance funding rules that realign the university's enrollment problem with the social planner's problem. Ultimately, level of funding affects enrollment, not structure of the funding rule. I also identify conditions such that funding changes disproportionately affect under-represented minority enrollment. To assess theoretical predictions, I estimate changes in selectivity and demographic composition of incoming first-time, full-time cohorts at public four-year universities in Ohio and Tennessee, states that switched to performance funding in 2009 and 2010 respectively. Ohio decreased funding in the long-run, while Tennessee did not; as predicted, I find evidence of increased long-run selectivity in Ohio but not in Tennessee. I also find the proportion of Black enrollment in Ohio decreased by 1.13 percentage points in the long-run, while it increased by 2.93 percentage points in Tennessee.

Chapter 2: The Enrollment Effects of Regional Campuses

The Ohio public university system has several institutions, including its flagship Ohio State University (OSU), that are split between "main" and "regional" campuses. While OSU's regional campuses are independently accredited institutions, they also have a strong transfer function: if a regional campus student has a minimum 2.0 GPA and 30 credit hours, they are guaranteed the option to transfer into the main campus. I build a general theoretical framework of first-time and transfer admissions with multiple institutions; the theory predicts that opening a regional campus causes community colleges to enroll a less academically prepared first-time student body, and may cause community college students to be crowded out by less prepared regional campus students in transfer admissions. As such, there are always both students who are strictly better and worse off after a regional campus opens, and opening a regional campus is not always welfare increasing. I show that a social planner prefers to modestly expand enrollment at a main campus over opening a larger regional campus if the regional campus is insufficiently differentiated from a community college.

Chapter 3: Choosing Sides in a Two-Sided Matching Market

I model a competitive labor market in which agents of different skill levels (e.g., college educated or not) decide whether to enter the market as a manager or as a worker. After roles are chosen, a two-sided matching market is realized and a cooperative assignment game occurs. There exists a unique rational expectations equilibrium that induces a stable many-to-one matching and wage structure. Positive assortative matching occurs if and only if the production function exhibits a condition that I call *role supermodularity*, which is stronger than the strict supermodularity condition commonly used in the matching literature because a high skilled agent with a role choice is only willing to enter the market as a worker if she expects that it is more profitable to cluster with only other high skilled agents than to exclusively manage. The wage structure in equilibrium is consistent with empirical evidence that the wage gap is driven both by increased within-firm positive sorting as well as between-firm segregation.

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INTRODUCTION

The choice to pursue higher education is one of the most significant decisions that many make in their lifetime. Those with a bachelor's degree have higher average lifetime earnings than those whose highest educational attainment is a high school diploma or equivalent, and both have a different earnings profile than those who exit with an associate's degree (*Education pays* 2023). Even within the group of people with a bachelor's degree, those who graduate from highly selective institutions may be advantaged in the labor market (Chetty, Deming, and Friedman 2023), especially under-represented minority graduates (Dale and Krueger 2014).[1] Therefore, it is important to understand the match between a student and the institution she enrolls at, as well as the labor market outcomes that people with and without a bachelor's degree can expect to face.

This dissertation characterizes opportunity and equity in college enrollment, as well as how this carries over to future labor market outcomes. My goal is to better understand how limited enrollment slots at institutions of higher education are allocated among prospective students, and how allocations change in response to external shocks. Looking into the long-term consequences of enrollment changes, I also explore how educational attainment affects occupational choice and wage determination in the labor market. I approach this research agenda using a combination of theory and empirics. I theoretically model college admissions and a competitive labor market as large, decentralized two-sided matching markets to generate predictions on how changes in these environments affect outcomes. I then empirically explore how actual changes in public policy and technological innovation affect outcomes in these markets to establish the internal validity of theoretical predictions and provide guidance on how these frameworks could be applied more broadly.

To explain why I model these environments as two-sided matching markets, first consider what a traditional supply and demand framework says about college enrollment. Demand for limited enrollment slots exceeds supply at many colleges—especially at selective institutions. In a supply

[1] Under-represented minorities in higher education include students who identify as Black or African American, Hispanic or Latino/a, Native Americans or Alaskan Native, and/or Native Hawaiian or other Pacific Islander. Unless otherwise specified, though, this dissertation concentrates on Black and Hispanic students among the overarching group of under-represented minorities.

and demand framework, "price" of college (tuition) increases with the level of scarcity, "producers" of higher education (colleges) expand supply of enrollment slots in the long-run, and "consumers" (students) who have lower willingness-to-pay will stop demanding higher education. Yet these predictions fall short of reality. While less selective institutions have expanded enrollment over the last decade, more selective institutions tend to hold enrollment fixed (Blair and Smetters 2021). We are also far from the cynical prediction that a scarce supply of seats should result in tuition rising until only high income students can afford to pursue a post-secondary degree: while there is significant disparity in college enrollment by income level, around half of high school graduates in the lowest income quintile pursue higher education (Reber and Smith 2023). So the supply and demand framework inadequately explains what is happening in the "market" for college admissions.

College admissions and the labor market setting I consider in this dissertation are more appropriately modeled as decentralized two-sided matching markets, in which there are two mutually exclusive sets of economic agents who would like to match with an agent on the other side of the market. In the context of higher education, the two sides of the market are "colleges" and "students". In the labor market I consider, they are "managers" and "workers". Crucially, the match itself is an important outcome in these models: students care about getting higher education in general, but they also care about matching with an institution that they prefer. Similarly, workers in the labor market care about who their coworkers are. In the U.S., these markets are decentralized because agents independently make decisions on how to match with others after observing market conditions, and the markets are large enough that individuals cannot affect outcomes on their own.[2]

Moreover, both college enrollment and the labor market are affected by external conditions such as public policy and technological innovation. For example, federal Pell Grants are a major source of financial support for low-income students pursuing a bachelor's degree. Another example are state and local "Promise Programs", which make community college free for local high school graduates and are intended to encourage students who otherwise would not pursue higher education

[2]In a *centralized* two-sided matching markets, a social planner could select a mechanism to assign matches after observing agent characteristics and/or preferences (e.g., algorithmic matching of deceased donor organs to waitlisted recipients). That is, individual agents do not seek out their matches independently.

to attend a community college. Technological growth also impacts the labor market that students face upon leaving higher education: it affects what types of jobs are in demand, wages upon employment, and productivity within industries. Changes in external conditions naturally affect outcomes in enrollment and in the labor market; an advantage of theoretically modeling these markets is that I can offer predictions on how shocks like the introduction of a new state policy will affect downstream outcomes.

Chapters 1 and 2 focus on how state-level public policies affect enrollment at public universities, using Ohio as the primary case study. Chapter 1 explores how performance-based state appropriations affect first-time, full-time enrollment at public four-year universities. State appropriations subsidize tuition for in-state students, and act like a public investment into future productivity; changes in appropriations should therefore affect enrollment. Historically, state funds are appropriated to public universities based on headcount ("enrollment funding"), but an alternative funding rule is to appropriate funds based on completion metrics such as number of degrees conferred ("performance funding"). Enrollment funding may introduce a perverse incentive to admit as many applicants as possible, disregarding whether an applicant is likely to graduate. This is undesirable due to the large financial and opportunity costs of attending college, especially if the student doesn't graduate. Some states used performance funding as early as the 1990s, in response to the fact that college enrollment was increasing but completion rates were decreasing during the 1980s (Bound, Lovenheim, and Turner 2010), which could indicate that universities indeed over-enrolled students. By linking funding to completions, performance funding intentionally incentivizes increased selectivity—but this could indirectly target under-represented minority students, who are systematically less likely to graduate. So performance funding may also unintentionally introduce an incentive to admit fewer under-represented minority students.

I study the implementation of performance funding in Ohio and Tennessee, focusing more on the case of Ohio. These are the only two states with long histories of consistently using performance funding to determine the majority of state appropriations. I build a theoretical framework of first-time college admissions to predict how funding changes affect enrollment, apply this model to each state's implementation of performance funding to predict its enrollment effects, and assess the theoretical predictions using a synthetic control approach to estimate changes in enrollment level and composition post-implementation. A key modeling assumption is that a state policymaker's valuation of enrolling students systematically differs from a representative university's valuation, such that a social planner would always like the university to enroll more students than the university would in the absence of external funding. As such, state funding rules can be used as a policy tool to realign the university's enrollment problem with the social planner's problem. I justify this assumption with the inherent existence of state appropriations: if state policymakers did not want universities to enroll more in-state students, then they wouldn't subsidize in-state tuition.

I find that pinning down the correct level of funding, rather than the structure of the funding rule, is the crucial component to realigning the state policymaker's enrollment goals with the university's admissions problem. The intuition is similar to identifying when marginal cost equals marginal benefit in a traditional supply and demand framework. In practice, a naive implementation of performance funding is equivalent to a decrease in funding, and as intuitively expected, the theory predicts that a decrease in funding should decrease enrollment levels.[3] I also identify conditions under which changes in funding level will disproportionately affect enrollment of under-represented minority students.

I then apply the framework to Ohio's implementation of performance funding. Using synthetic controls, I show that theoretical predictions are consistent with long-run enrollment trends in Ohio after switching to performance funding. While state appropriations increased immediately after the switch to performance funding, they decreased in the long-run. Accordingly, I find a temporary short-run increase in enrollment, but that overall enrollment levels decreased by approximately 1 percentage point in the long-run. The effect is larger for enrollment level of Black students, which decreased by 2.8 percentage points. Looking at the demographic composition of incoming cohorts, I also find that the proportion of Black students decreased by 1.13 percentage points. This provides

[3]To see that a naive switch to performance funding is equivalent to a decrease in funding, suppose that the state government is willing to allocate up to $100 per student, and this doesn't change after switching from enrollment to performance funding. If payment becomes contingent upon completion, then because some students do not graduate, universities receive strictly less funding under performance funding.

internal validity to the theoretical framework applied to the case study of Ohio, and suggests that the framework can be informative to policymakers in other states as well. The model assumptions are not restrictive, and policymakers can apply additional structure to the model based on their state's idiosyncratic features to predict how different funding rules would affect enrollment.

In Chapter 2, I explore sorting of students between different institutions of higher education in Ohio in the first-time and transfer admissions processes. I concentrate on a unique feature of Ohio's public education system: including the flagship Ohio State University (OSU), several public universities have a main campus and one or several regional campuses. Regional campuses blend the traditional institutional missions of two-year colleges and four-year universities; for example, OSU regional campuses are independently accredited to confer both associate's and bachelor's degrees. Regional campuses also facilitate transfer into the main campus. Students who start at a regional campus are guaranteed the option to transfer into OSU's main campus if they have a minimum 2.0 GPA and 30 credits.

Ex-ante, it is unclear if adding a regional campus is beneficial to the overall landscape of higher education. Regional campuses are good for the OSU system as they allow OSU to expand enrollment while maintaining selectivity at the main campus. However, it isn't immediately obvious how regional campuses affect local community colleges, or if they make students better off. To explore how regional campuses impact the overall landscape of public higher education, I model first-time and transfer admissions in an environment with multiple institutions that vary in cost and value of attendance, and many students who vary in academic preparedness and preferences over institutions. The theory predicts that regional campuses make community colleges worse off in first-time admissions, and that community college transfer applicants may be crowded out by less academically prepared regional campus transfer applicants.

In first-time admissions, regional campuses attract some highly academically well-prepared students away from community colleges, which causes community colleges to enroll less prepared student bodies. This could be especially problematic because Ohio is a performance funding state; less prepared students are also less likely to graduate, which could cause funding to community colleges to decrease, further exacerbating a community college's ability to serve its student body.

Transfer applicants from community colleges may be worse off because the main campus prioritizes all regional campus transfers above a minimum threshold. If the minimum threshold for regional campus students is too low, then the main campus could become so selective over community college transfers that it would reject a community college transfer applicant in favor of a less academically prepared regional campus transfer applicant. This is consistent with the fact that the average external transfer-in student at OSU has a stronger academic profile than the minimum regional campus transfer requirements.

If a social planner considers students and institutions complements—and therefore prefers enrolling the most academically well-prepared students at the main campus over a regional campus over a community college—then the welfare effects of adding a regional campus are always mixed. Regional campuses attract students away from community colleges in first-time enrollment, which is welfare increasing. However, they also attract students away from the main campus, which is welfare decreasing. It is also welfare decreasing for community college transfer applicants to be crowded out by less prepared regional campus transfers. Although opening or expanding regional campuses can be overall welfare increasing, the theory predicts that there will always exist students who are worse off afterwards. While policymakers in other states may consider regional campuses an appealing investment, especially if their public universities are capacity constrained in enrollment, they should take into account the fact that there may be unintended consequences on students and for their local community colleges in order to minimize welfare losses.

Finally, in Chapter 3, I model a competitive labor market in which individuals vary in skill level and there are two employment roles available, managers and workers. We can think of the two skill levels as having or not having a college degree. It is costly to become a manager, but managers and workers must match together for production to occur—so some agents must become managers. The main novelty in this model is that people choose their roles before matching; in the standard two-sided labor market matching framework, roles are exogenously predetermined. Role choice reflects the fact that people with similar skills may choose different occupational tracks. For example, some professors take on administrative titles, while others avoid them.

After roles are chosen, a large, decentralized two-sided matching market is realized; agents simultaneously try to match with somebody of the other role *and* maximize individual wages. In the unique rational expectations equilibrium outcome, the matching and wage structure are uniquely determined by a condition that I call "role supermodularity". If role supermodularity is satisfied, then the equilibrium matching is positive assortative: college graduates who become managers only match with college graduates who become workers. I call this a "clustering" equilibrium, since people cluster with those who share their skill level. If role supermodularity is not satisfied, then the unique equilibrium matching is negative assortative: college graduates specialize in management and oversee non-college graduate workers. I call this a "specialization" equilibrium.

To contextualize what role supermodularity means, first note that strict supermodularity is a common condition in models of two-sided matching with transfers and implies that the production technology treats the manager and worker roles as complements. In the standard matching model without role choice, strict supermodularity implies that the equilibrium matching is positive assortative; it is negative assortative otherwise. However, strict supermodularity is insufficient to generate a unique wage structure in equilibrium. Role supermodularity is a stronger condition than strict supermodularity that results from the pre-matching role choice and determines both a unique matching *and* wage structure in equilibrium. Role supermodularity is a stronger condition because it requires that roles are "strong" complements to each other. The wage structure in equilibrium is unique because role choice introduces more structure to the labor market given that agents rationally optimize their choices.

Equilibrium wage differentials can be decomposed into two components: wage differences between individuals in the same role but different levels of educational attainment, and wage differences between individuals with the same level of educational attainment but different roles. These differentials behave differently in the clustering vs. specialization equilibria. Trends in U.S. wage dispersion are more consistent with the clustering equilibrium. This has a notable policy implication that could potentially improve income equity: it may be possible to simultaneously increase productivity and decrease wage differentials between people with and without a college degree by making it less costly for individuals without a college degree to become managers.

Methodologically, what unites this dissertation is the use of two-sided matching frameworks to characterize the choice to pursue higher education alongside supporting empirical evidence to validate theoretical predictions. Thematically, all three chapters characterize opportunity and equity in higher education by following the decision to enroll in college from admissions through the eventual labor market outcomes that those with or without a degree will face. My hope is that this work will inform thoughtful, nuanced policy on how to improve the lives of those who choose to pursue higher education as well as those who do not or cannot.

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CHAPTER 1

PERFORMANCE FUNDING AND EQUITY OF ACCESS TO PUBLIC UNIVERSITIES

1.1 Introduction

U.S. state governments have historically appropriated funds to public universities based on headcount (enrollment funding, "EF"), but an alternate rule is to reward universities based on performance metrics such as degree completions (performance funding, "PF"). PF initially gained traction during the 1980s, when the proportion of high school graduates enrolling in post-secondary institutions was increasing while the college completion rate was decreasing (Bound, Lovenheim, and Turner 2010). Because this trend could indicate that universities over-admit students who are not likely to graduate—which is undesirable due to the large financial and opportunity costs of attending university—PF advocates argue that using outcomes-based metrics correctly incentivizes universities to decrease over-enrollment. However, critics argue that PF unintentionally incentivizes universities to admit fewer applicants whose observable characteristics correlate with a lower probability of degree completion.[1] If so, then switching from EF to PF may disproportionately restrict college accessibility to under-represented minority (URM) students, who are less likely to graduate than non-URM students.[2]

Although 33 states used some form of PF during 2020 (Ortagus, Rosinger, and Kelchen 2021), the majority of states do not consistently use PF and/or only use PF to determine small amounts of funding. Two important exceptions are Ohio and Tennessee (the "PF states"), which have used PF to determine almost all state appropriations for over a decade. As such, the enrollment effects of completely switching from EF to PF are limited and not well understood. Characterizing the effects of funding changes on enrollment is important because all higher education outcomes are downstream of a student being admitted in the first place—therefore, understanding the overarching

[1]This chapter centers around *enrollment effects* only, but PF may also affect enrolled students. Similar to the traditional moral hazard setting, we can think of a policymaker as a principal who would like to induce universities (agents) to invest in programs that make students more likely to graduate. However, there may be a perverse incentive to lower completion requirements.

[2]Causey, Lee, Ryu, Scheetz, and Shapiro (2022) report that the six-year completion rate at four-year public universities for the Fall 2016 cohort is 68% overall, but only 50.2% for Black students and 57.1% for Latino/a students.

effects of funding rules begins by understanding its impact on college accessibility.

After giving a literature review and background on PF implementation in Section 1.2, I develop a theoretical framework of state funding in Section 1.3. A social planner (or state policymaker, SP) and universities systematically differ in how they evaluate the expected returns to enrolling a given student. I show that under complete information, the SP has a high degree of flexibility in picking a funding rule that realigns the university's enrollment problem with the SP's enrollment problem; in particular, there are many EF and PF rules that work (Theorem 1). Ultimately, level of funding determines the university's optimal enrollment cut-offs, not the timing of the funding rule. Taking into consideration that Ohio and Tennessee impose additional restrictions on their PF rules, I show in Proposition 2 that there exists a restricted funding rule that realigns the university's enrollment problem in Ohio, but not in Tennessee. I then show in Section 1.3.2 there exist gaps between theory and implementation and argue that the wedge between the SP's and the university's underlying valuation of enrolling students may cause Ohio's premium on URM degree completions to be incorrectly specified.

I show comparative statics results in Proposition 3; under general conditions, changes in funding rules have intuitive results on enrollment. When the level of funding increases, level of enrollment is also expected to increase; on the other hand, when the emphasis on degree completions increases, level of enrollment is expected to decrease. Finally, I identify conditions that hold if and only if a change in the level of funding has a larger effect on URM enrollment than on non-URM enrollment in Proposition 4.

Given that the switch to PF caused funding per student to decrease in the long-run in Ohio but funding levels were volatile in Tennessee, the main testable hypothesis follows from Proposition 3: the theory predicts that there was a long-run decrease in enrollment level at Ohio public four-year universities following PF implementation, but the opposite or no effect in Tennessee. Using synthetic controls, I find evidence that trends in enrollment after PF implementation are consistent with these predictions: in Ohio, overall enrollment level decreased by 1 percentage point, while there is no evidence of statistically significant changes in enrollment levels in Tennessee. The effect is more pronounced for Black enrollment in Ohio; I find enrollment level for Black students decreased by 2.8 percentage points. Looking at the proportion of Black students within first-time, full-time cohorts, the proportion of Black enrollment decreased by 1.13 percentage points in Ohio after switching to PF. This provides evidence that conditions identified in Proposition 4 hold.

1.2 Literature Review and Background

This chapter contributes to three overarching strands of literature. First, it fits into research on pay for performance. For further reading, Lazear (2000) reviews the effect of performance pay in the private sector, van Thiel and Leeuw (2002) review performance pay in public sector, and Podgursky and Springer (2007) review the effects of performance pay in the K-12 educational sector. A recent paper that similarly examines the interplay between incentivizing improved outcomes while potentially introducing unintentional incentives to also increase selectivity in the context of health care is Gupta (2021), who finds that a program aimed at reduced hospital readmissions was successful, but half of the reduction is explained by increased quality while the other half is explained by increased selectivity over returning patients. Second, this chapter is adjacent to a growing literature on how university financing affects institutional operations that started from work by Brown, Dimmock, Kang, and Weisbenner (2014) on the importance of university endowments. My comparative statics results are similar to how Bulman (2022) finds that an increase in a university's endowment correlate with universities enrolling proportionately fewer under-represented minority students.

Third, this chapter primarily complements existing studies that focus on studying PF's effects on its intended goal, increasing degree completion. Overall, the literature finds PF has no impact on graduations. Like this chapter, Hillman, Fryar, and Crespin-Trujillo (2018) and Ward and Ost (2021) study Ohio and Tennessee. They find that while degree completion increased after PF, the increase was not caused by PF. Hillman, Tandberg, and Gross (2014), Kelchen (2018), Tandberg and Hillman (2014), and Rutherford and Rabovsky (2014) study states that have implemented smaller degrees of PF and find similar results. Other papers show that effects depend on institutional characteristics and policy-specific premiums. Boland (2020) finds that PF had no effect on degree completion at historically Black colleges and universities. Favero and Rutherford (2019) and Hagood (2019) find that PF may disproportionately increase funding at universities that already perform well and are less resource constrained. Similarly, Birdsall (2018) finds institutions that are less dependent on state funding increase graduation rates after switching to PF. Finally, Li (2020) finds that adding a premium on STEM degrees increases the number of degrees conferred in STEM fields.

It is less clear how PF affects college admissions. Kelchen (2018) finds no effect on enrollment. Gandara and Rutherford (2020) find that universities admit cohorts with higher standardized test scores after PF, and find some negative short-run effects on URM enrollment. Umbricht, Fernandez, and Ortagus (2017) study PF in Indiana and find weak evidence that URM enrollment decreased, but these results are confounded by the fact that Indiana simultaneously implemented other higher education initiatives. Gandara and Rutherford (2018) show that adding premiums to degrees completed by URM students mitigate unintended negative enrollment effects. Ward and Ost (2021) find evidence that Hispanic enrollment decreased in the PF states, but enrollment is not the focus of their study. To my knowledge, I am the first to look at long-term enrollment dynamics in the PF states. This may explain why my empirical results differ from other studies: I find a decline in enrollment of Black students begins 3 years after implementation in Ohio only.

For a full history of PF in the U.S., see Dougherty, Jones, et al. (2016) and Dougherty and Natow (2015); I describe specific implementation details for Ohio and Tennessee in the following two sub-sections.

1.2.1 Ohio

Ohio has 14 public four-year universities with considerable heterogeneity across institutional characteristics. I exclude Northeast Ohio Medical University from analysis due to being a medical school. Most public universities primarily enroll in-state students (>70%), with the exceptions of Central State University, Ohio State University, and Miami University. Central State University is a historically Black university, which are relatively rare in the Midwest, and likely attracts out-of-state students who do not have a historically Black university or college in their home state. Ohio State

Table 1.1: Information on Ohio Public Universities

University	Enrollment	% In-State	6-Year Graduation %	Other Info
Bowling Green State University	13,853	88%	61%	
Central State University	5,406	55%	25%	Historically Black university
Cleveland State University	9,776	85%	49%	
Kent State University	20,418	80%	65%	
Miami University	16,864	62%	81%	
Ohio State University	46,123	66%	88%	Public flagship, has hospital
Ohio University	18,113	88%	65%	
Shawnee State University	3,091	84%	35%	Open admissions
University of Akron	11,323	92%	48%	Test optional admissions[a]
University of Cincinnati	29,663	78%	72%	Has hospital
University of Toledo	11,965	74%	55%	Open admissions,[b] has hospital
Wright State University	6,938	95%	44%	Test optional admissions
Youngstown State University	8,822	76%	49%	Open and test optional admissions

Data are from the NCES College Navigator. Enrollment is the total undergraduate population as of Fall 2022. 6-year graduation rate is for the Fall 2016 cohort.

[a] Test optional universities do not require applicants to disclose their standardized testing scores, but generally still require a high school transcript.

[b] Open admissions universities automatically admit any students who meet a publicly posted minimum standardized testing scores and/or high school GPA. Most also accept some students with scores below the threshold after a holistic review.

University and Miami University are highly ranked public universities and attract more out-of-state applicants due to their prestige.

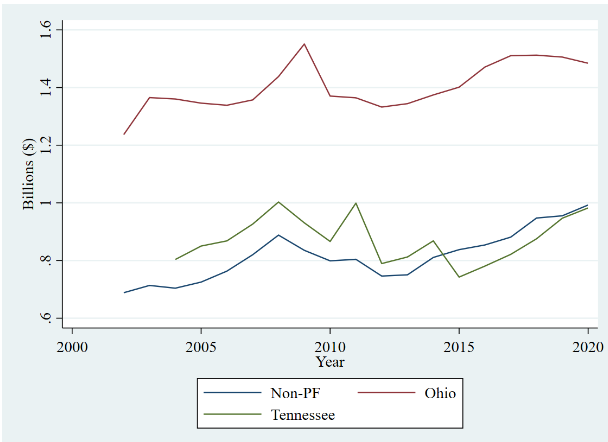
Ohio switched from EF to PF in 2009, with a four year transition period during which EF was phased out incrementally. According to the Ohio *State Share of Instruction Handbook* (2022), the PF metrics are course completions (30%) and degree completions (50%), with an additional 20% set aside for discretionary funding (e.g., a university could receive more money if it has a medical center). Out-of-state graduates not employed in Ohio do not count towards degree completion, while out-of-state graduates employed in-state count for half a completion. There are two sets of risk premiums for course and degree completions respectively. The only risk factors considered for course completions are financial and academic status. Risk factors for degree completions additionally include age, race, and first generation status. Premiums are explicitly intended to incentivize enrollment of at-risk students given that they are less likely to complete courses and degrees, and they are calculated based on observed differences in historical graduation rates (Carey 2014).

As seen in Figure 1.1, total state appropriations in Ohio have fluctuated between $1.24 billion and $1.55 billion over the last two decades. There was a one-time spike the year after PF implementation followed by a return to pre-implementation levels the year after, and has since been trending upwards. Although Ohio has higher levels of total funding than other states on average, this is driven by the fact that Ohio universities also enroll more students on average. Looking at state appropriations per full-time equivalent (FTE) student, the overall trend in funding is similar to total appropriations, but funding per FTE in Ohio is lower than in the non-PF states.

Figure 1.2 shows that funding per FTE at most institutions mirrors the overall trend around the time of implementation, but institutions show different trends 3 years after implementation. There are two major patterns. Some institutions, including the three top-ranked universities (Ohio State University, Miami University, and University of Cincinnati) have faced fairly constant funding levels after the one-time drop post-implementation. Other institutions, including Ohio University and Cleveland State University, experienced funding increases after the one-time drop and have returned to pre-implementation levels of funding by 2019. Two outliers are Bowling Green State University, where funding per FTE has been constant outside of the one-time increase during 2009, and University of Toledo, which saw a one-time increase in funding in 2008 that has persisted across time. This suggests considerable heterogeneity in responses to the change in funding rule.

Figure 1.1: State Appropriations Trends

(a) Total State Appropriations



(b) State Appropriations per FTE

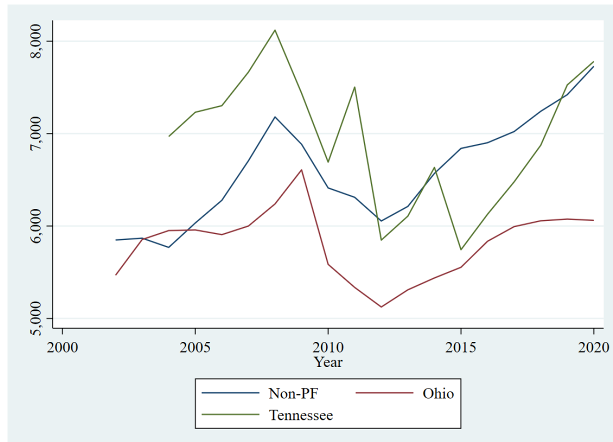


Figure 1.2: Ohio State Appropriations per FTE - Institution Level

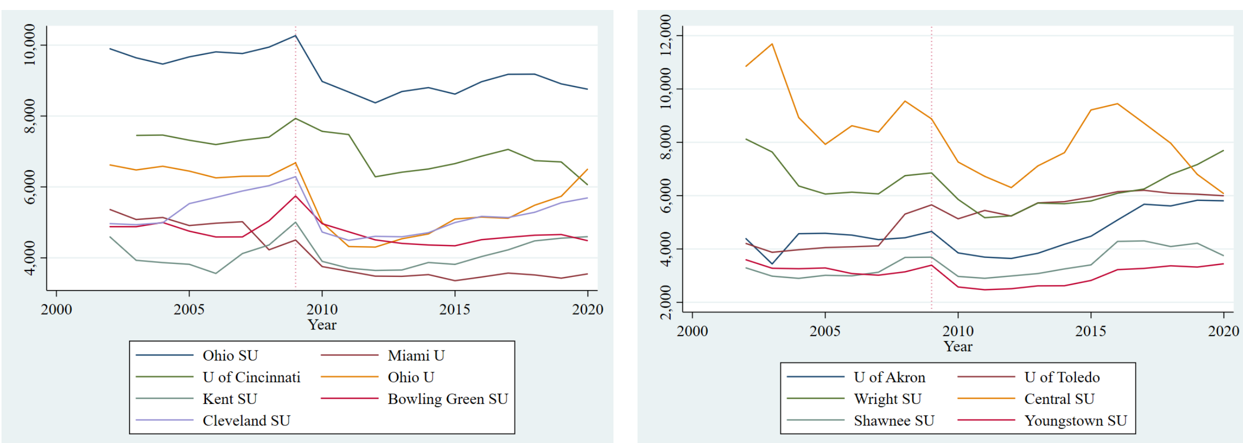


Table 1.2: Information on Tennessee Public Universities

University	Enrollment	% In-State	6-Year Graduation %	Other Info
Austin Peay State University	8,120	88%	43%	Open admissions
East Tennessee State University	10,554	75%	55%	
Middle Tennessee State University	17,438	89%	55%	
Tennessee State University	7,678	38%	32%	Historically Black university, open admissions
Tennessee Technological University	8,537	94%	60%	
University of Memphis	16,708	81%	48%	
University of Tennessee-Chattanooga	9,884	89%	52%	Open admissions
University of Tennessee-Knoxville	27,039	54%	73%	Public flagship, has hospital
University of Tennessee-Martin	6,165	90%	53%	Open admissions

Data are from the NCES College Navigator. Enrollment is the total undergraduate population as of Fall 2021. 6-year graduation rate is for the Fall 2016 cohort.

Table 1.3: Tennessee Performance Funding Weights

	24 Hours	48 Hours	72 Hours	Bachelors	Masters	Doctorates	Degrees/100 FTE	6-Year Grad %	Research
APSU	1.3%	2.5%	5.8%	44.3%	10.6%	23.4%	11.2%	0.0%	0.9%
ETSU	2.3%	3.7%	7.4%	29.0%	3.4%	20.0%	12.7%	16.4%	5.1%
MTSU	1.4%	2.9%	8.2%	42.9%	3.5%	21.7%	15.4%	2.4%	1.7%
TSU	1.5%	2.6%	6.2%	24.8%	6.7%	20.2%	10.5%	6.6%	21.1%
TTU	1.3%	2.9%	8.0%	32.9%	5.1%	37.9%	7.5%	0.7%	3.7%
UM	1.0%	2.1%	5.6%	29.9%	3.0%	22.5%	8.2%	20.4%	7.2%
UT-C	1.5%	2.9%	7.7%	31.2%	6.9%	39.0%	5.3%	2.9%	2.8%
UT-K	0.6%	1.7%	3.9%	19.2%	3.0%	18.2%	7.6%	21.9%	23.9%
UT-M	1.1%	2.1%	5.5%	27.5%	5.4%	55.2%	2.5%	0.0%	0.8%

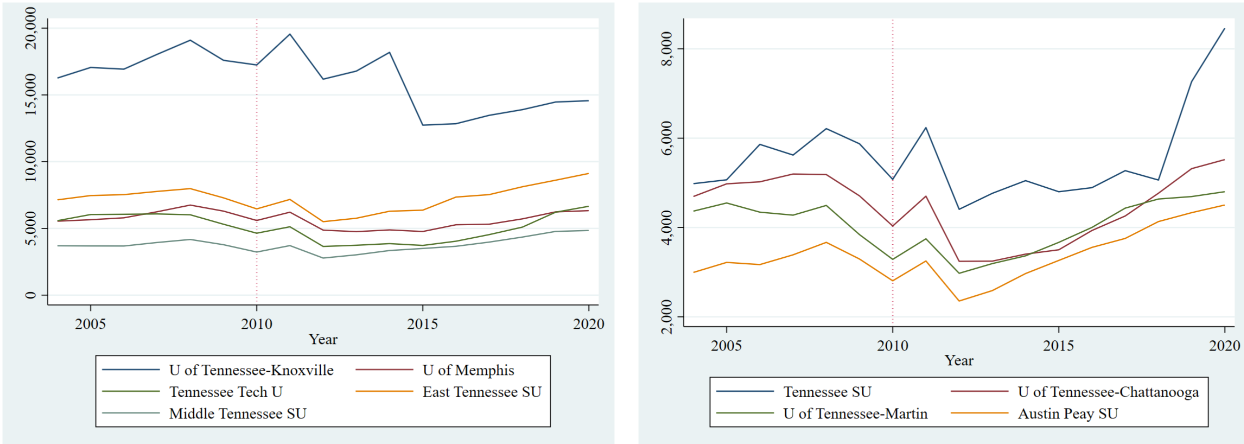
1.2.2 Tennessee

Tennessee has 10 public four-year universities, four of which are affiliated with each other through the University of Tennessee system. I exclude University of Tennessee-Southern, as it was a private institution until 2021. Tennessee public universities have less variance in size, proportion of in-state enrollment, and graduation rates compared to Ohio public universities. Overall, Tennessee universities are smaller and enroll proportionately more in-state students than Ohio universities, with the exceptions of Tennessee State University and University of Tennessee-Knoxville due to being a historically Black university and the state flagship respectively.

Tennessee switched to PF in 2010 with a three year transition period similar that of Ohio. According to the *2015-2020 Outcomes-Based Funding Formula Overview* (2016), PF metrics differ across universities depending on institutional mission and Carnegie classification. State policymakers assign weights with guidance from universities; see Table 1.3. PF incentives may be weaker in Tennessee than Ohio, as institutions may advocate for weights that align with pre-existing institutional characteristics. As suggestive evidence of this, note that actual 6-year graduation rates do not strongly correlate with the size of the PF weight on 6-year graduation rate. For example, East Tennessee State University and Middle Tennessee State University have similar characteristics, including 6-year graduation rates, but assign weights of 16.4% and 2.4% to 6-year graduation rates respectively. Almost all institutions place small weights on course completions and large weights on post-graduate degree completions, with Tennessee State University and the public flagship University of Tennessee-Knoxville additionally placing strong weights on research. There are only two risk categories considered across all metrics: age and financial status. In-state and out-of-state students are treated the same.

As seen in Figure 1.1, total state appropriations in Tennessee have increased from $804 million in 2004 to $947 billion in 2019. Funding amounts were highly volatile until 2015, after which appropriations have consistently increased. As can be seen in Figure 1.3, this is generally also true at the institution level. The trend in overall funding is similar to state appropriations per FTE. Funding per FTE in Tennessee is more comparable to average funding per FTE in the non-PF

Figure 1.3: Tennessee State Appropriations per FTE - Institution Level



states than Ohio. Unlike Ohio, state appropriations (total and per FTE) decreased at the time of PF implementation, followed by a return to pre-implementation levels the following year.

1.3 Theoretical Framework

There is one social planner (or state policymaker, SP), a finite set of universities indexed by k, and a continuum of in-state students. The SP and universities do not discount the future. In-state students differ across two dimensions: caliber $x \in [0, 1]$ and group $r \in \{\text{M}, \text{URM}\}$ (majority and under-represented minority), so that the full type space is $\Theta = [0, 1] \times \{\text{M}, \text{URM}\}$. An arbitrary student i is described by her type $\theta_i = (x_i, r_i) \in \Theta$. Caliber is a composite score that describes a student's academic strength, and $x|r \sim Uniform[0, 1]$.[3]

There are three time periods, $t \in \{0, 1, 2\}$. At $t = 0$, the SP sets and credibly commits to a publicly observable funding rule. After that, applications, admissions, and enrollment occur. Simultaneously, students send applications to all universities they find acceptable, and universities admit all students they find acceptable.[4] All accepted applicants then enroll at their favorite offer.

After enrollment, a student realizes one of three outcomes: she leaves early during $t = 1$, leaves late during $t = 2$, or graduates at the end of $t = 2$. This captures the spirit of existing PF rules: universities can receive funding for students who complete some education but fail to

[3] Assuming a uniform distribution simplifies notation, but I can let caliber follow any arbitrary probability distribution over $[0, 1]$, in which case I must take into account the measure of students of every caliber level.

[4] The admissions game can be either sequential (with students moving first, then universities) or simultaneous. Outcomes are equivalent.

graduate. We can think of "early" exiters as students who don't complete general requirements, while "late" exiters have completed general requirements but not major-specific requirements. This simplification captures the broad strokes of college attrition: historically, about half of exits occur during the first year of enrollment.[5]

Denote the expected probability that i has exited before $t = 1$ ends as $Early(x_i|r_i) : [0, 1] \rightarrow [0, 1]$, and the expected probability that she has exited before $t = 2$ ends as $Late(x_i|r_i) : [0, 1] \rightarrow [0, 1]$. By definition, $Late(x|r) \geq Early(x|r)$ for all (x, r), as a student who has exited before $t = 1$ ends has necessarily also exited before $t = 2$ ends, while a student can exit late but complete her general requirements at $t = 1$. Let both exit functions be continuously differentiable and weakly decreasing in x_i. Additionally, $Early(x_i|URM) \geq Early(x_i|M)$ for all x_i, and the equivalent inequality holds for $Late()$. The functions $Early()$ and $Late()$ are known to the SP and university through prior student bodies.

First, consider the social planer's problem. I assume for now that the SP has complete information on students and university characteristics. This is an unrealistically strong assumption, but I impose it to show a naively constructed funding rule might fail to achieve its intended goals even in ideal circumstances.

The SP is willing to enroll a student i to university k depending on the expected social returns of her enrollment. I assume there are two components to social returns: receiving education and degree completion. A student who completes only her general requirements generates social returns to her enrollment due to receiving education, but always less than if she had successfully completed the degree.[6]

[5] The *Yearly Success and Progress Rates: Fall 2015 Beginning Postsecondary Student Cohort* (2022) report finds that 78.9% of students in the Fall 2015 starting cohort at public four-year universities returned after their first year and 61.8% completed a degree at their starting institution, so 55% of attrition occurred during the first year.

[6] This is consistent with the fact that the median weekly earnings for people with some college education but no degree is higher than those with a high school diploma and lower than those with a bachelor's degree, according to *Education pays* (2023). That said, the returns to post-secondary education when the student does not graduate are not as well understood as returns to education conditional on graduation (Lovenheim 2023). However, Maurin and McNally (2008) find evidence that additional years of college education increase lifetime earnings given that the student graduates.

Denote the net social returns to education if i enrolls at k and persists through $t = 1$ as

$$Educ_k^{SP}(x_i|r_i) : [0,1] \to [0,1]$$

and similarly the net social returns to a degree completed by i at k as

$$Degree_k^{SP}(x_i|r_i) : [0,1] \to [0,1],$$

such that both functions are weakly increasing in x_i and continuously differentiable.[7]

I assume that $Educ_k^{SP}(x_i|URM) \geq Educ_k^{SP}(x_i|M)$ and $Degree_k^{SP}(x_i|URM) \geq Degree_k^{SP}(x_i|M)$ for all x_i. This rules out *only* the possibility that the SP systematically values the social returns to enrolling a URM student less than non-URM enrollment.

Finally, let the expected operating cost of education per student at $t \in \{1, 2\}$ be $c_t \in [0, 1]$, which is sunk by university k at the beginning of each time period.[8] The expected social return to admitting i at k is

$$\begin{aligned} EV_k^{SP}(x_i|r_i) = &- c_1 + (1 - Early(x_i|r_i))[Educ_k^{SP}(x_i|r_i) - c_2] \\ &+ (1 - Late(x_i|r_i))Degree_k^{SP}(x_i|r_i). \end{aligned} \tag{1.1}$$

I assume that (c_1, c_2) is such that for both r, there exists some $\hat{x}_r \in (0, 1)$ such that $EV_k^{SP}(x_r|r) < 0$ for all $x_i < \hat{x}_i$ and $EV_k^{SP}(x_r|r) \geq 0$ for all $x_i \geq \hat{x}_i$.

Because $EV_k^{SP}()$ is weakly increasing in caliber, following Azevedo and Leshno (2016), I can characterize a stable matching between students and a university as a group-specific cut-off x_r: if the SP sets a cut-off x_r for group r such that a student of type (x_r, r) is admissible, then the SP must be willing to also admit all students with caliber above the cut-off.[9] The socially efficient

[7]To simplify modeling, I assume that $Degree_k^{SP}(x_i|r_i)$ captures both the returns from education and graduation at $t = 2$.

[8]By operating cost, I specifically mean the expected cost of running classes, which can be calculated/forecasted based off of past years. (This is part of Ohio's funding rule calculation.) There may be variance in how much an arbitrary student costs outside of classes, but I allow this to be absorbed by the net valuation functions $Educ_k^{SP}()$ and $Degree_k^{SP}()$ respectively. This reduces the number of components that depend on student type, which is more tractable for later analysis.

[9]This is in contrast to papers like Chade, Lewis, and Smith (2014), where the caliber of an application is mapped to a probability of being admitted—in this model, students are either assigned a probability 0 or 1. These papers focus on highly selective schools that receive many applications from high-performing students and must consider other dimensions of the student to prevent admitting students over the school's capacity. At public universities, the cut-off rule is more realistic.

enrollment policy maximizes the total expected social returns to education. With some minor abuse of notation, the socially efficient cut-offs when capacity constraints do not bind are (x_M^{SP}, x_{URM}^{SP}) that solve $EV_k^{SP}(x_r^{SP}|r) = 0$; that is, the SP is willing to admit every student whose expected social returns to enrolling at k are net (weakly) positive.[10]

That said, it is likely that capacity constraints *do* bind for many public universities. In that case, the socially efficient cut-offs can be derived as follows:[11]

1. Take some arbitrarily small $\varepsilon > 0$. Pick the group with the higher expected social returns at the caliber level $1 - \varepsilon$, and admit all students in that group with caliber at least as high as $1 - \varepsilon$.

2. Compare the expected social returns to admitting URM vs. M students at the top ε of caliber who have not yet been admitted and admit students in that group.

3. Repeat until capacity constraints are met.

Call the SP "race blind" if $Educ_k^{SP}(x|M) = Educ_k^{SP}(x|URM)$ for all x and analogously for $Degree_k^{SP}()$. If the SP is race blind, then the efficient URM cut-off is always weakly higher than the M cut-off.

Proposition 1. *Let the SP be race-blind. Then* $x_{URM}^{SP} \geq x_M^{SP}$.

Proof. Suppose capacity constraints do not bind. The two socially efficient cut-offs solve $EV_k^{SP}(x|r) = 0$, so

$$(1 - Early(x_M^{SP}|M))[Educ_k^{SP}(x_M^{SP}) - c_2] + (1 - Late(x_M^{SP}|M))Degree_k^{SP}(x_M^{SP}|M)$$
$$= (1 - Early(x_{URM}^{SP}|URM))[Educ_k^{SP}(x_{URM}^{SP}) - c_2] + (1 - Late(x_{URM}^{SP}|URM))Degree_k^{SP}(x_{URM}^{SP}).$$

[10]The abuse of notation is because efficient cut-offs should be specific to k. However, I avoid comparing between cut-offs at different institutions, so I drop any indicator for k when describing the cut-offs in terms of caliber level x for ease of notation.

[11]This procedure assumes no competition over applicants, which is unrealistic. However, it can be modified to take into account universities' expectations over students' accept rates if there is competition, and is structurally very similar.

Now suppose for a contradiction that $x^{SP}_{URM} < x^{SP}_{M}$. It follows that $Early(x^{SP}_{URM}|URM) > Early(x_M|URM) > Early(x_M|M)$, and similarly for the late exit functions. For equality to hold it must be the case that one of the two conditions hold:

1. $Educ^{SP}_k(x^{SP}_M|M) = Educ^{SP}_k(x^{SP}_M|URM) < Educ^{SP}_k(x^{SP}_{URM}|URM)$ or
2. $Degree^{SP}_k(x^{SP}_M|M) = Degree^{SP}_k(x^{SP}_M|URM) < Degree^{SP}_k(x^{SP}_{URM}|URM)$.

But neither condition can hold because $Educ^{SP}_k()$ and $Degree^{SP}_k()$ are increasing in caliber and $x^{SP}_{URM} < x^{SP}_M$. Hence the contradiction.

Now suppose capacity constraints bind. Again suppose for a contradiction that $x^{SP}_{URM} < x^{SP}_M$. But then it must be the case that the social returns *all* the URM students with caliber $x \in [x^{SP}_{URM}, x^{SP}_M]$ yield lower social returns to enrollment than the M student at the M-type caliber, contradicting the admissions procedure with capacity constraints. □

Next, consider the university's admissions problem. Since c_1, c_2 are operating costs, the university k and the SP "agree" on these parameters, as well as the exit functions. Denote k's net private returns to educating i at $t = 1$ as $Educ_k(x_i|r_i) : [0,1] \to [0,1]$ and denote k's net private returns to graduating i as $Degree_k(x_i|r_i) : [0,1] \to [0,1]$. I assume:

1. $Educ_k(x_i|r_i)$ and $Degree_k(x_i|r_i)$ are weakly increasing in x_i: Universities expect higher net returns from higher quality students, because these students are more likely to bring prestige to the institution and positively affects the university's ranking, external funding options, investment into research, etc.

2. $Educ_k(x_i|URM) \geq Educ_k(x_i|M)$ and $Degree_k(x_i|URM) \geq Degree_k(x_i|M)$ for all x_i: Universities do not get strictly lower returns from educating and graduating URM students. It is possible for universities to systematically value URM education/graduation higher, and it is also possible that universities are race-blind.

3. $Educ^{SP}_k(x_i|r_i) > Educ_k(x_i|r_i)$ and $Degree^{SP}_k(x_i|r_i) > Degree_k(x_i|r_i)$. This is because the university k takes into account additional non-operating costs that the SP may not explicitly

consider (e.g., outreach/retention programs, health services, community centers, or adding resources to admissions offices to increase the quality of the admissions process indirectly).

The last assumption also captures the fact that state appropriations are intended to subsidize the enrollment of students whom universities could not afford to enroll without additional aid. If it were not the case that the SP wants more students enrolled at universities than universities are willing to enroll in the absence of funding, then the SP would not appropriate any funds.

Put altogether, the expected private return to k for admitting i in the absence of additional funding is

$$\begin{aligned} EV_k(x_i|r_i) = & -c_1 + (1 - Early(x_i|r_i))[Educ_k(x_i|r_i) - c_2] \\ & + (1 - Late(x_i|r_i))Degree_k(x_i|r_i). \end{aligned} \tag{1.2}$$

As a benchmark, consider the SP's problem when the SP can mandate cut-offs and is willing to pay university k so that the mandated cut-offs are not strictly unprofitable to k. An SP who is not budget constrained can use a continuum of compensation schemes to ensure that university k finds the cut-offs acceptable.[12] On one end of the continuum, the SP can extract all "rents" from the university and give the university $EV^{SP}(x_r^{SP}|r) - EV_k(x_r^{SP}|r)$ for every admitted student in group r. This funding rule is the least expensive for the SP to implement. On the opposite end of the spectrum, the SP can exactly break even and give the university $EV^{SP}(x_i|r_i) - EV_k(x_i|r_i)$ for every admitted student. This funding rule is the most expensive for the SP to implement.

1.3.1 The Admissions Problem and Efficient Funding Rules

In reality, U.S. state policymakers cannot directly influence admissions, but the SP can pick and commit to a funding rule that enters into the university's optimization problem. Call an arbitrary funding rule $\rho = (p_{1,M}, p_{1,URM}, p_{2,M}, p_{2,URM})$ where $p_{t,r} \in \mathbb{R}_+$ is funding given at time $t \in \{1, 2\}$ for a student in group r provided she meets the rule's criterion. I define two classes of funding rules:

[12] If the SP *is* budget constrained, the statement still holds and she would use a procedure analogous to the one for admitting students under capacity constraints.

1. Enrollment funding (EF): Funds are awarded for every student at the beginning of a time period, before outcomes are realized.

2. Performance funding (PF): Funds are awarded conditional on course completion at the end of $t = 1$ and/or degrees conferred at the end of $t = 2$.

Call a funding rule "socially efficient/optimal" if implementing the funding rule causes the university's optimal cut-offs to coincide with the SP's optimal cut-offs. Socially efficient EF and PF rules exist.

Theorem 1. *The following rules implement the socially efficient cut-offs.*

1. *Pure EF:*

$$p_{1,r}^{EF} = (1 - Early(x_r^{SP}|r))[Educ_k^{SP}(x_r^{SP}|r) - Educ_k(x_r^{SP}|r)],$$
$$p_{2,r}^{EF} = \frac{(1 - Late(x_r^{SP}|r))}{(1 - Early(x_r^{SP}|r))}[Degree_k^{SP}(x_r^{SP}|r) - Degree_k(x_r^{SP}|r)].$$

2. *Pure PF:*

$$p_{1,r}^{PF} = Educ_k^{SP}(x_r^{SP}|r) - Educ_k(x_r^{SP}|r),$$
$$p_{2,r}^{PF} = Degree_k^{SP}(x_r^{SP}|r) - Degree_k(x_r^{SP}|r).$$

Proof. The proposed funding rules exactly align k's enrollment problem with the social planner's problem at the socially efficient cut-offs for each group and time period. Because $EV_k(x|r)$ is weakly increasing in caliber, any student in group r with caliber $x \geq x_r^{SP}$ has weakly larger expected value to k than the student at the group cut-off given the proposed funding rules, so is admissible. □

The proposed pure EF and PF rules are not the only funding rules that implement the socially efficient cut-offs. As long as the funding rule promises the university the efficient amount of funding in expectation, the SP can spread payments across the first and second time periods, and could even completely frontload funding into $t = 1$ or backload funding into $t = 2$.

Corollary 1. *Socially efficient EF and PF rules are not unique.*

Proof. Take the socially optimal PF rule from Theorem 1. Let $a, b \in [0, 1]$. Any funding rule that pays upon completion and satisfies

$$p_{1,r} = a \cdot p_{1,r}^{EF} + \frac{(1 - Late(x_r^{SP}|r))}{(1 - Early(x_r^{SP}|r))} \cdot b \cdot p_{2,r}^{EF},$$

$$p_{2,r} = \frac{(1 - Early(x_r^{SP}|r))}{(1 - Late(x_r^{SP}|r))} \cdot (1 - a) \cdot p_{1,r}^{EF} + (1 - b) \cdot p_{2,r}^{EF}$$

also implements the socially optimal admissions cut-offs. The same is true for an appropriately constructed linear combination of the pure PF rule. □

The key takeaway is that switching from EF to PF *does not* inherently move universities closer to socially efficient cut-offs. If switching to PF is intended to decrease over-enrollment, then it only does so if it reduces overall funding. This is indeed what would happen if the SP switches from EF to PF in the most naive way possible: pay the same amounts as in any arbitrary EF rule, but only after outcomes are realized.

Next, I discuss gaps between theory and actual implementation in the PF states.

1.3.2 Constrained Performance Funding

An immediate implication from Theorem 1 is that optimal funding rules must be implemented at the *institution* level. While Tennessee implements PF at the institution level, Ohio implements PF at the state level. Given the heterogeneity between Ohio public universities, a statewide funding rule is therefore efficient for at most one institution.

Putting aside the level of implementation, there are additional constraints on how both PF states design their rules which may further prevent efficiency. I constructed pure EF and PF rules that realign the university's problem with the social planner's problem such that each component $p_{t,r}$ is independent of the other components. However, both PF states have policy aspects that relate funding components across time or between groups:

1. In both PF states, funding for course completions is the same between groups: $p_{1,URM} = p_{1,M}$.

2. Ohio assigns a premium for URM degree completions: for some $\beta \in \mathbb{R}_+$, $p_{2,URM} = (1 + \beta)p_{2,M}$.

   In Tennessee, funding for degree completions are the same between groups: $p_{2,URM} = p_{2,M}$.

3. Both states assign weights to course and degree completions respectively. We can think of funding as follows: the SP is willing to fund up to $p \in \mathbb{R}_{++}$ per enrolled student. A fraction $\alpha \in [0, 1]$ of p is paid at the end of $t = 1$ conditional on course completion and the rest is paid conditional on degree completion. That is,

$$p_{1,M} = (1 - \alpha)p,$$

$$p_{2,M} = \alpha p.$$

Define the class of "Ohio PF rules" as $\rho^{OH} = (p, \alpha, \beta)$ such that:

1. $(1 - \alpha)p$ is paid for every student still enrolled at the end of $t = 1$,

2. αp is paid for every M student who graduates at the end of $t = 2$, and

3. $\alpha(1 + \beta)p$ is paid for every URM student who graduates at the end of $t = 2$.

Tennessee PF rules are a subset of this class that set $\beta = 0$.

There exists an Ohio PF rule that induces the socially optimal cut-offs. The intuition is similar to Corollary 1: the SP can frontload or backload funding so long as the funding rule gives the optimal total amount of funding in expectation. The SP should first choose (p, α) to induce the university k to choose the socially efficient cut-off for M students, then use β as a backloaded "lever" to compensate for differences between URM and M students at their respective cut-offs.

Proposition 2. *A unique socially optimal Ohio PF rule exists.*

Proof. By construction. First, for notational convenience, denote

$$\Delta Educ_k(x_r|r) = Educ_k^{SP}(x_r|r) - Educ_k(x_r|r),$$

$$\Delta Degree_k(x_r|r) = Degree_k^{SP}(x_r|r) - Degree_k(x_r|r).$$

First, set (α, p) to induce the optimal M cut-offs,

$$(1-\alpha)p = \Delta Educ_k(x_M^{SP}|M),$$
$$\alpha p = \Delta Degree_k(x_M^{SP}|M).$$

Then separate β into two components, $(\beta_1, \beta_2) \in \mathbb{R}^2$ such that $\beta = \beta_1 + \beta_2$, and set them to satisfy

$$\alpha\beta_1 p = \frac{1 - Early(x_{URM}^{SP}|URM)}{1 - Late(x_{URM}^{SP}|URM)}(\Delta Educ_k(x_{URM}^{SP}|URM) - (1-\alpha)p),$$
$$\alpha\beta_2 p = \Delta Degree_k(x_{URM}^{SP}|URM) - \alpha p.$$

The β_1 component realigns k's problem at $t = 1$ with the SP's problem, and analogously for β_2. Solving for each component,

$$\beta_1 = \frac{1 - Early(x_{URM}^{SP}|URM)}{1 - Late(x_{URM}^{SP}|URM)}[\frac{\Delta Educ_k(x_{URM}^{SP}|URM)}{\Delta Degree_k(x_M^{SP}|M)} - \frac{1-\alpha}{\alpha}],$$
$$\beta_2 = \frac{\Delta Degree_k(x_{URM}^{SP}|URM)}{\Delta Degree_k(x_M^{SP}|M)} - 1,$$

and implementing $(p, \alpha, \beta_1 + \beta_2)$ aligns k's problem with the SP's problem at the efficient cut-offs. □

In practice, though, Ohio's URM premium is based *only* on "the decreased likelihood of students graduating based on their risk category" (*State Share of Instruction Handbook* 2022). This implies that Ohio's URM premium is

$$\beta^{OH} = Late(x_{URM}^{SP}|URM) - Late(x_M^{SP}|M) > 0, \tag{1.3}$$

where the inequality is imposed because the URM premium used in practice is strictly positive.[13] This is a restrictive condition that almost always prevents social efficiency, except in a knife-edge case.

[13] According to *State Share of Instruction Handbook* (2022), this is the closest to how Ohio state policymakers calculate β^{OH} in practice. However, analysis would not change much if β^{OH} were instead an arbitrary function of the difference in graduation rates $f(Late(x_{URM}^{SP}|URM) - Late(x_M^{SP}|M)) : [0,1] \to \mathbb{R}_+$ (e.g., taking a proportion of the difference); simply substitute $Late(x_{URM}^{SP}|URM) - Late(x_M^{SP}|M)$ with $f()$ in Corollary 2.

Corollary 2. *Let the proposed Ohio PF rule be efficient over M students. k's optimal URM cut-off is efficient if and only if*

$$Late(x_{URM}^{SP}|URM) - Late(x_{M}^{SP}|M) = \frac{1 - Early(x_{URM}^{SP}|URM)}{1 - Late(x_{URM}^{SP}|URM)} \left[\frac{\Delta Educ_k(x_{URM}^{SP}|URM)}{\Delta Degree_k(x_{M}^{SP}|M)} - \frac{1-\alpha}{\alpha} \right] + \frac{\Delta Degree_k(x_{URM}^{SP}|URM)}{\Delta Degree_k(x_{M}^{SP}|M)} - 1.$$

For the sake of discussing whether or not the knife-edge case is likely (or in general, if the URM premium β is likely to be close to efficiency), note that two of the components in Corollary 2 can be approximated.

First, consider the component $\frac{1-Early(x_{URM}^{SP}|URM)}{1-Late(x_{URM}^{SP}|URM)}$.[14] *Snapshot Report: First-Year Persistence and Retention* (2017) and Causey, Pevitz, Ryu, Scheetz, and Shapiro (2022) show that for the Fall 2015 cohort, the first year persistence rate for Black students at four-year public universities was 64.8%, while the six-year graduation rate was 51.3%. So this component is approximately 1.26.

Second, consider $\frac{1-\alpha}{\alpha}$. In Ohio, the weight on course completion is 30% and degree completion is 50%. Disregarding set-aside funds, that implies $\alpha = 0.625$ ($\frac{1-\alpha}{\alpha} = 0.6$).

Although the components

$$\frac{\Delta Educ_k(x_{URM}^{SP}|URM)}{\Delta Degree_k(x_{M}^{SP}|M)}, \frac{\Delta Educ_k(x_{URM}^{SP}|URM)}{\Delta Degree_k(x_{M}^{SP}|M)}$$

can't be approximated without imposing further assumptions, such as a precise functional form, understanding how they affect the size of the optimal URM premium is important for characterizing why β^{OH} may be incorrectly specified. Recalling that $EV_k^{SP}(x_i|r_i) > EV_k(x_i|r_i)$ for all (x_i, r_i) by assumption, note that the size of the right-hand side of Corollary 2 depends on how large $\Delta Educ_k(x_{URM}^{SP}|URM) > 0$ and $\Delta Degree_k(x_{URM}^{SP}|URM) > 0$ are relative to $\Delta Degree_k(x_{M}^{SP}|M) > 0$. Fixing $\Delta Degree_k(x_{M}^{SP}|M)$, as the difference in how the SP and the university k value URM enrollment becomes larger, the optimal premium becomes larger. On the other hand, fixing the numerators, as the difference in how the SP and university k value M enrollment becomes larger, the optimal premium becomes smaller.

[14] Also note that by construction, this component is always larger than 1.

Therefore, the key issue with β^{OH} is that by considering only the difference in graduation rates, it fails to comprehensively compensate for all the differences between how the SP and the university k value the returns to enrolling and graduating students. Note that this gap is driven purely by the fact that the SP and the university k are different—it has nothing to do with differences in how the SP *or* how the university k evaluate students differently based on group. In fact, even if the SP and k are *both* completely race-blind, a wedge between their valuations still exists because I assume that the SP's net returns functions are strictly larger than k's.

1.3.3 Comparative Statics

Suppose that k faces an arbitrary Ohio PF rule $\rho^{OH} = (p, \alpha, \beta)$ that is not necessarily socially efficient, nor does it impose $\beta = \beta^{OH}$. Throughout, I use primes to denote derivatives when the notation is clear and ∂ notation when it could be ambiguous or difficult to read.

First, define $\bar{\beta}$ as:

$$\bar{\beta} = \frac{Late(x^*_{URM}|URM) - Early(x^*_{URM}|URM)}{1 - Late(x^*_{URM}|URM)}. \tag{1.4}$$

By construction, $\bar{\beta} > 0$.

The main comparative static results follow.

Proposition 3. *Let $\frac{\partial}{\partial x^*_M} EV_k(x^*_M|M) > 0$. k's optimal M cut-off is decreasing in the level of funding p and increasing in the proportion of funding determined by degree completions α.*

Let $\frac{\partial}{\partial x^*_{URM}} EV_k(x^*_{URM}|URM) > 0$. k's optimal URM cut-off is decreasing in p and β. Moreover, let $\beta \in [0, \bar{\beta}]$ ($\beta > \bar{\beta}$), where $\bar{\beta}$ is defined in Equation 1.4. k's optimal URM cut-off increasing (decreasing) in α.

Proof. First, by construction, $EV_k()$ is weakly increasing in caliber for both groups. For the M cut-off,

$$\frac{\partial}{\partial p}EV_k(x_M^*|M) = (1-\alpha)(1-Early(x_M^*|M)) + \alpha(1-Late(x_M^*|M))$$

$$> 0,$$

$$\frac{\partial}{\partial \alpha}EV_k(x_M^*|M) = p[Early(x_M^*|M) - Late(x_M^*|M)]$$

$$< 0,$$

$$\frac{\partial}{\partial x_M^*}EV_k(x_M^*|M) \geq 0,$$

so the Implicit Function Theorem gives that $x'_M(p)$ has the opposite sign of $\frac{\partial}{\partial x_M^*}EV_k(x_M^*|M)$, and $x'_M(\alpha)$ has the same sign as $\frac{\partial}{\partial x_M^*}EV_k(x_M^*|M)$. Letting $\frac{\partial}{\partial x_M^*}EV_k(x_M^*|M) > 0$ hold, the results for the M cut-off comparative statics hold.

For the URM cut-off,

$$\frac{\partial}{\partial p}EV_k(x_{URM}^*|URM) = (1-\alpha)(1-Early(x_{URM}^*|URM)) + \alpha(1+\beta)(1-Late(x_{URM}^*|URM))$$

$$> 0,$$

$$\frac{\partial}{\partial \alpha}EV_k(x_{URM}^*|URM) = p[(1+\beta)(1-Late(x_r^*|r)) - (1-Early(x_{URM}^*|URM))],$$

$$\frac{\partial}{\partial \beta}EV_k(x_{URM}^*|URM) = (1-Late(x_{URM}^*|URM))\alpha p$$

$$> 0,$$

$$\frac{\partial}{\partial x_{URM}^*}EV_k(x_{URM}^*|URM) \geq 0,$$

so the Implicit Function Theorem gives that $x'_{URM}(p)$ and $x'_{URM}(\beta)$ have the opposite sign of $\frac{\partial EV_k(x_M^*|M)}{\partial x_M^*}$. Letting $\frac{\partial}{\partial x_{URM}^*}EV_k(x_{URM}^*|URM) > 0$, the results for the URM cut-off comparative statics in p and β hold.

The sign of $\frac{\partial}{\partial \alpha}EV_k(x_{URM}^*|URM)$ depends on how large β is, and is negative so long as $\beta \leq \bar{\beta}$. So if the premium is sufficiently small, then $x'_{URM}(\alpha)$ has the same sign as $\frac{\partial}{\partial x_M^*}EV_k(x_M^*|M)$. If the premium is too large, the opposite is true. □

First, note that the conditions $\frac{\partial}{\partial x_M^*}EV_k(x_M^*|M) > 0$ and $\frac{\partial}{\partial x_{URM}^*}EV_k(x_{URM}^*|URM) > 0$ are not strongly restrictive and are satisfied so long as $EV_k()$ is not constant in caliber around (x_r^*, r) for both r. A stronger assumption on $EV_k()$ that guarantees the conditions hold is to let $EV_k()$ be strictly increasing in caliber.

However, if the strictly increasing marginal caliber conditions do not hold, then it is unclear how an increase in p affects enrollment levels. For example, if capacity constraints bind, then an increase in p may actually decrease enrollment because students both at and just above the group-optimal cut-off have an expected return of 0, so k is no worse or better off by slightly increasing cut-offs. If capacity constraints are not binding, though, then the increase in p may still increase enrollment, because students just below the optimal cut-off before the funding increase have the same, strictly positive expected return as students at the cut-off.

Next, the size of the URM premium β determines how an increase in the fraction of funding determined by degree completions α affects level of enrollment. If β is large and α increases under the URM cut-off condition, then level of enrollment for URM students may increase in spite of the increased emphasis on degree completion. This is because a large β may distort the university's incentives if the "reward" for graduating URM students is so high that universities are willing to increase their exposure to risk and increases enrollment of URM students to potentially yield a high "reward" later on.

Finally, Proposition 3 can be used to predict the *dynamics* of PF implementation. Recall from Figure 1.1 that PF caused different trends in funding post-implementation between the two PF states.

In Ohio, most universities experienced an initial increase in funding per FTE followed by a long-run decrease; in Tennessee, funding was volatile following the implementation of PF, but has trended towards being higher than the pre-implementation level from 2018 onward. As such, Proposition 3 predicts that in Ohio, there was an initial increase in enrollment followed by a long-term decrease. It predicts the opposite or no effect in Tennessee.

I conclude by assessing under what conditions a change in funding affects URM enrollment more.

Proposition 4. *Let* $\frac{\partial}{\partial x^*_M} EV_k(x^*_M|M) > 0$ *and* $\frac{\partial}{\partial x^*_{URM}} EV_k(x^*_{URM}|URM) > 0$. *A change in* p *induces* *a larger change in URM enrollment than M enrollment if and only if*

$$\frac{\frac{\partial}{\partial x}EV_k(x^*_M|M)}{\frac{\partial}{\partial x}EV_k(x^*_{URM}|URM)} < \frac{(1-\alpha)(1-Early(x^*_{URM}|URM))+\alpha(1+\beta)(1-Late(x^*_{URM}|URM))}{(1-\alpha)(1-Early(x^*_M|M))+\alpha(1-Late(x^*_M|M))}. \tag{1.5}$$

Proof. By the Implicit Function theorem, the magnitude of change in cut-offs caused by an increase in p is larger for the URM cut-off if and only if

$$\begin{aligned}
|x'_M(p)| &< |x'_{URM}(p)| \\
\frac{\frac{\partial}{\partial p}EV_k(x^*_M|M)}{\frac{\partial}{\partial p}EV_k(x^*_M|M)} &< \frac{\frac{\partial}{\partial p}EV_k(x^*_{URM}|URM)}{\frac{\partial}{\partial p}EV_k(x^*_{URM}|URM)} \\
\frac{\frac{\partial}{\partial x}EV_k(x^*_M|M)}{\frac{\partial}{\partial x}EV_k(x^*_{URM}|URM)} &< \frac{\frac{\partial}{\partial p}EV_k(x^*_{URM}|URM)}{\frac{\partial}{\partial p}EV_k(x^*_M|M)} \\
\frac{\frac{\partial}{\partial x}EV_k(x^*_M|M)}{\frac{\partial}{\partial x}EV_k(x^*_{URM}|URM)} &< \frac{(1-\alpha)(1-Early(x^*_{URM}|URM))+\alpha(1+\beta)(1-Late(x^*_{URM}|URM))}{(1-\alpha)(1-Early(x^*_M|M))+\alpha(1-Late(x^*_M|M))}.
\end{aligned}$$

□

Equation 1.5 is more likely to hold if (i) the marginal private returns to enrolling the student (x^*_{URM}, URM) is large relative to the marginal private returns to enrolling the student (x^*_M, M), (ii) the probability of URM persistence and retention at the URM cut-off is large relative to the probability of M persistence and retention at the M cut-off, or (iii) the URM premium β is large. Condition (ii) never holds if assuming that $x^*_{URM} < x^*_M$. This follows from Arcidiacono, Kinsler, and Ransom (2023), who show that URM enrollment at Harvard and University of North Carolina-Chapel Hill would become more selective if admissions were based on test scores alone. It is unclear if condition (iii) holds in Ohio, but it does not hold in Tennessee, where the URM premium is 0.

Continuing to assume that $x^*_{URM} < x^*_M$, so that the right-hand side of Equation 1.5 is strictly less than 1, whether the inequality holds first depends on if the expected private returns to k is convex or concave. If $EV_k()$ is convex, then the marginal return to the cut-off URM student is always smaller than that of the cut-off M student, so the left-hand side of Equation 1.5 is greater than 1 and the inequality never holds. If $EV_k()$ is instead concave, then the left-hand side of Equation 1.5 is strictly less than 1. Concavity alone is not enough, though; the inequality might or might not hold depending both on *how* concave $EV_k()$ is as well as the difference in cut-offs $x_M^* - x_{URM}^*$.

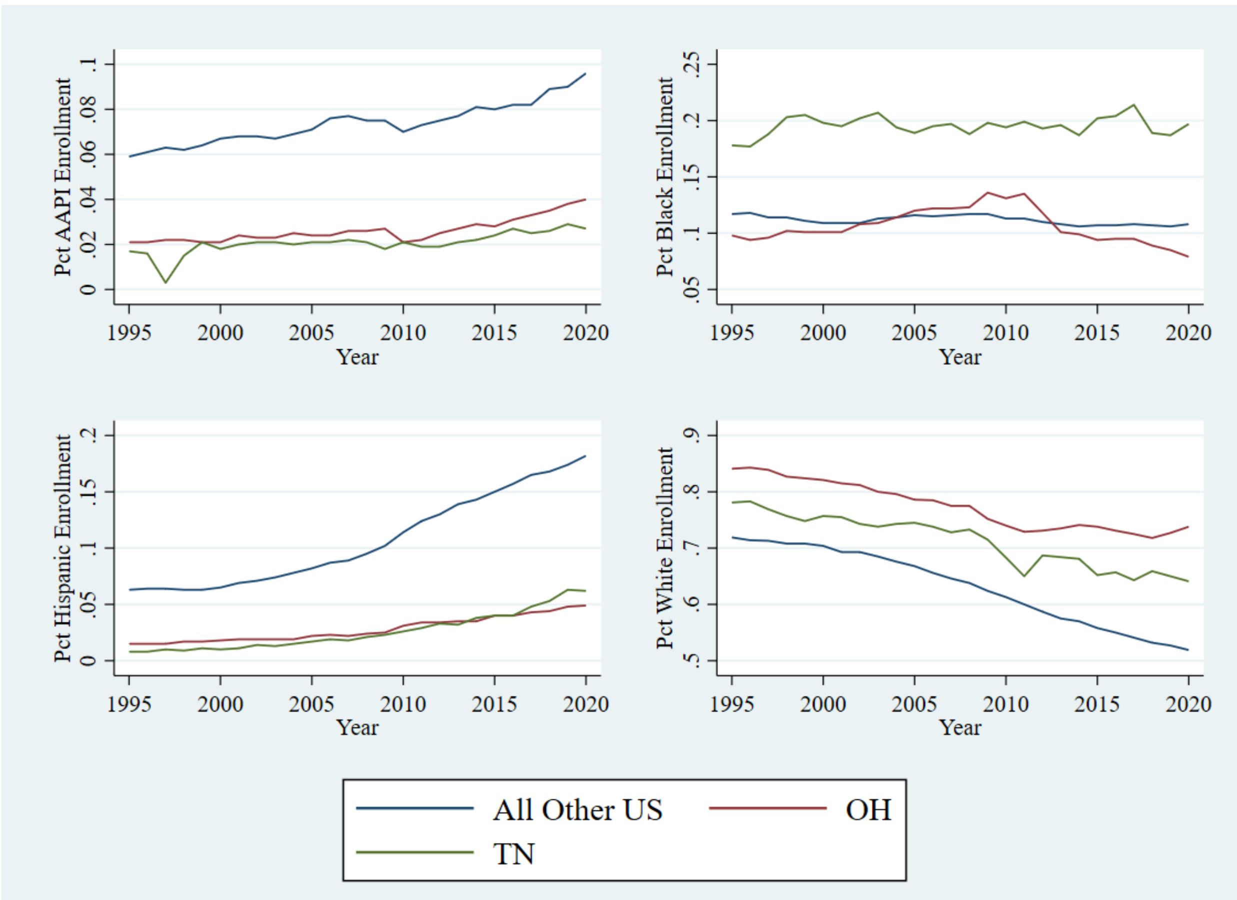
Proposition 4 provides guidance for future work on estimating the returns to education, especially for researchers interested in the equity effects of policy changes on enrollment. There are many obstacles to this estimation problem, especially if the goal is to completely characterize $EV_K()$, because universities are all selective to some degree. It may be difficult, for example, to estimate the shape of the returns to education for students at a given university whose caliber is much lower than the average student at the university—because it is unlikely that such a student would ever be accepted at the university to begin with. However, an advantage of Proposition 4 is that it only requires that the researcher estimate the returns to education around the existing cut-offs at a given university. Therefore, the researcher may be able to take advantage of policy changes at both the state and institution level that cause small changes in level of enrollment—this alone is enough to make predictions on the equity effects of policy changes, allowing the researcher to side-step the much more onerous task of fully estimating $EV_k()$.

1.4 Data and Approach

The main testable hypothesis from the theoretical framework follows from Proposition 3: if switching to PF causes a decrease (increase) in funding, then level of enrollment also decreases (increases). Based on the trends in funding per FTE shown in Figure 1.1, the theory therefore predicts: (1) a short-run increase in level of enrollment followed by a long-term decrease in Ohio, and (2) the opposite or no overall effect in Tennessee. Tennessee institutions are able to self-select their PF weights to some extent (as discussed in Section 1.2.2), which likely mutes the effects of changes in funding. Also, if I find that enrollment changes disproportionately affect URM student enrollment, this is suggestive evidence that the condition identified in Equation 1.5 from Proposition 4 holds.

I use data from the Integrated Postsecondary Education Data System (IPEDS) and Common Core of Data (CCD). IPEDS contains annual institution-level data on U.S. universities and colleges that receive Title IV funding. I use enrollment data from 1995-2020 on public four-year post-secondary institutions that primarily award baccalaureate degrees. The CCD contains annual data on the U.S. public elementary and secondary educational systems. I use education agency membership data from 2000-2020. I supplement with data on state unemployment from the U.S. Bureau of Labor Statistics.

Figure 1.4: Enrollment Trends



The main approach is synthetic control.[15] The outcomes of interest are: (1) the number of students enrolled at any public university divided by the number of 12th grade students in a given demographic group (measure of enrollment level) and (2) the number of first-time Black students divided by the number of all first-time students enrolled (proportion of Black enrollment). All

[15]The difference-in-differences parallel trends assumption is not likely to hold by visual comparison of enrollment trends in the PF and non-PF states in Figure 1.4.

Table 1.4: Summary Statistics: Enrollment

	Candidate Pool	Ohio	Tennessee
Total	16,326	38,411	17,884
	(10,610)	(3,540)	(2,410)
Asian	820	1,014	377
	(1,104)	(263)	(121)
Black	2,060	4,102	3,495
	(2,072)	(790)	(515)
Hispanic	1,204	1,098	495
	(1,722)	(503)	(364)
White	11,070	29,618	12,643
	(6,927)	(1,326)	(1,082)

outcomes are measured at the state level.[16] I use percentages to scale outcomes relative to the size of potential/overall enrollment, as Table 1.4 shows that the PF states have higher enrollment numbers than non-PF states on average.

I call the first group of outcomes measures of "enrollment level". An increase in enrollment level is equivalent to a decrease in selectivity. Ideally, I would be able to track the proportion of *in-state applicants* who apply to at least one public university in-state and eventually enroll by demographic group. The way that I calculate the measure of enrollment level in practice has issues both in the numerator and denominator.

The numerator and the theoretical model do not account for the fact that universities may substitute in-state students for high caliber out-of-state and international students. Bound, Braga, Khanna, and Turner (2020) show universities enroll more international students when facing funding cuts, while Arcidiacono, Kinsler, and Ransom (2023) show that universities are more selective over out-of-state students. Combining these insights, it is likely that in-state and out-of-state enrollment will tend to move in opposite directions, as public universities may consider non-local students as imperfect substitutes for local students. However, as seen in Tables 1.1 and 1.2, public universities in the PF states predominantly enroll in-state students, so any effects of performance funding on

[16]Outcomes are more volatile at the institution level, leading to institution level synthetic controls that are not well-matched on pre-treatment outcomes. I aggregate to the state level to smooth out this volatility.

in-state enrollment should still dominate the overall effect on enrollment, although this attenuates estimates towards 0.

The choice of numerator is additionally problematic for Tennessee because concurrent policy changes may also affect it. The non-profit organization KnoxAchieves was started in 2008 with backing from local politicians to provide scholarship funds for low-income college freshmen in Knox County, and later expanded to the state level. The scholarship became state funded in 2014 when Tennessee legislators created the "Tennessee Promise" program. In its current iteration, the Tennessee Promise guarantees funding for all in-state high school graduates who enroll at a two-year program. Nguyen (2020) shows this caused short-run substitution between two-year and four-year institutions and indirectly increased enrollment at four-year institutions. The numerator may therefore increase due to changes in student preferences and bias estimates upwards. As there is no strong reason to think this effect dominates the out-of-state student substitution effect (or vice versa), I suggest interpreting *all* Tennessee outcomes with caution.

The denominator uses the population of 12th grade students as the pool of potential applicants, but not all high school students will apply to public four-year universities.[17] The denominator therefore overestimates the pool of potential college students and attenuates estimates towards zero.

An additional problem arises for Ohio, which experienced a drop in 12th grade enrollment that exactly coincides with the time of PF implementation. 12th grade enrollment decreased from 134,522 in 2008 to 118,872 in 2009—a difference of 15,650 students (or an 11.6% drop). As can be seen in Figure 1.5, this is driven by a one-time decrease in white 12th graders.[18] Figures 1.5 and 1.6 show that this decrease does not appear in other states. After discussion with the Ohio Department of Education, I was able to confirm that this is likely due to a change in accounting, as there was a business rule change on how to count students that were previously counted more than

[17]The CCD has limited public data on the number of high school completions, and IPEDS does not have detailed applicant data. Also, note that using the total number of applications received at the state level is a worse candidate to use as the denominator—potential students may send multiple applications. Also, note that modest one advantage of using the population of 12th grade students is that the concurrent Tennessee policy changes don't cause problems with the denominator, only the numerator.

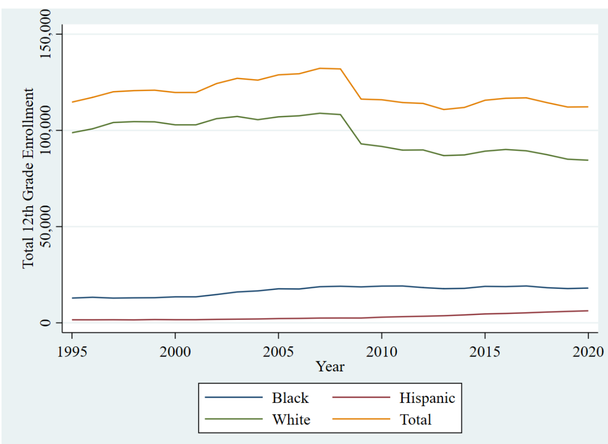
[18]This decrease also affects 11th grade enrollment between 2008-2009 to roughly the same magnitude. However, lower grades are not affected.

once in the data submission, and does not indicate a “true” drop in the population.

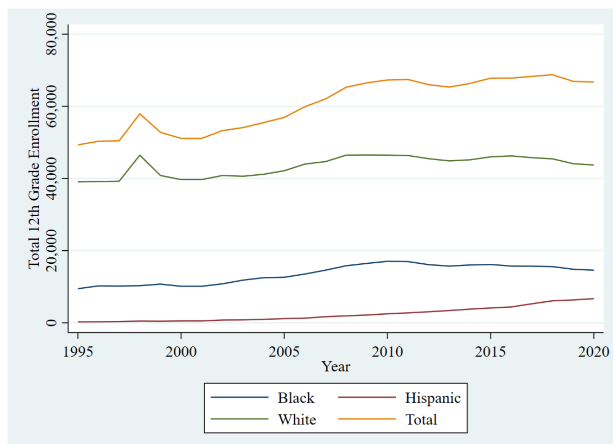
This accounting decrease in 12th grade enrollment will mechanically cause the measure of enrollment level in Ohio to increase at the time of PF implementation, but it does not indicate a “true” change in enrollment. I therefore interpret the enrollment level outcomes for overall enrollment and enrollment of white students in Ohio with caution. Additionally, I run all of the enrollment level outcomes using a modified version of the 12th grade population that removes the one-time drop and present those results as well. Because the populations of Black and Hispanic 12th graders is steady across time, I argue that the enrollment levels for Black and Hispanic students can be meaningfully interpreted without modification.

Figure 1.5: 12th Grade Enrollment Trends

(a) Ohio

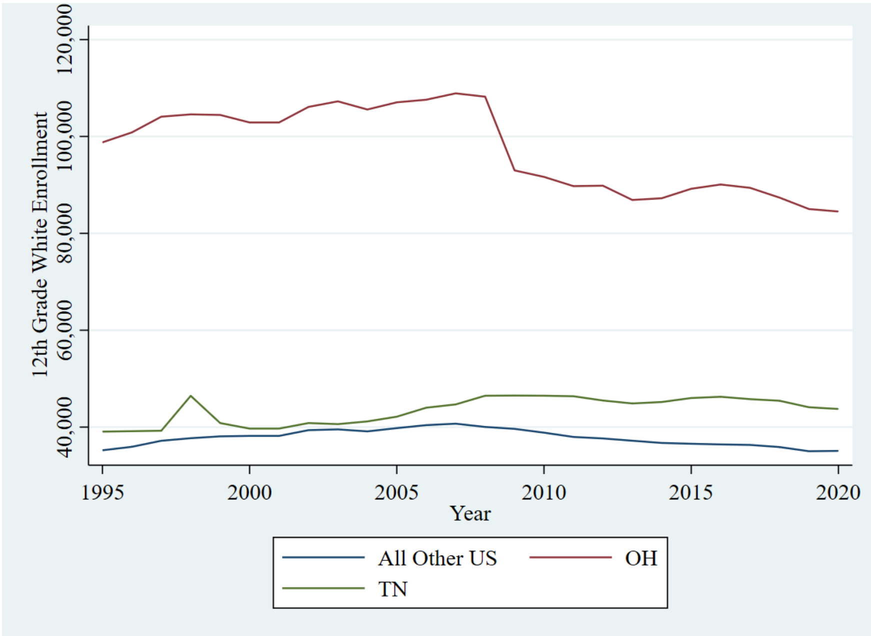


(b) Tennessee



The other outcome of interest, proportion of Black enrollment, captures changes in enrollment composition and is used to assess if the condition in Proposition 4 is likely to hold. A decrease in proportion of Black enrollment indicates that this demographic was disproportionately affected by changes in enrollment. Although Hispanic students are also under-represented minorities, I estimate proportions of Black and Hispanic enrollment separately, as Black, Cortes, and Lincove (2020) show that there are differences in application behavior and applicant readiness across Black and Hispanic students. Hispanic students have higher average college readiness than comparable Black students, but have a lower propensity to apply. As such, universities may perceive the academic readiness of Hispanic applicants differently from Black applicants of similar caliber.

Figure 1.6: 12th Grade White Student Enrollment Trends



My preferred candidate pool for the synthetic controls includes 36 states for enrollment level and 37 states for proportion of Black enrollment. I drop Alaska, Hawaii, California, Texas, the Far West states that have not already been dropped, and the New England states.[19] For enrollment level, I additionally drop Idaho due to missing data on 12th grade students. I consider other candidate pools in Appendix 1D. I drop Alaska and Hawaii due to being non-continental states. I drop California and Texas are dropped due to their top percentage admissions policies. I drop the Far West and Southwest states because of these states' differences in demographics and public university characteristics compared to the PF states. For example, states in these regions (other than California, which is already dropped) tend to have a small number of public four-year universities, which I believe makes them poor comparisons to Tennessee and especially Ohio, where in-state students face a variety of options over public post-secondary education, with significant heterogeneity over things like institutional mission, location, and other important traits that affect students' preferences over attending university. I drop the New England states again because of the small number of in-state options and because geographic proximity in that region may cause enrollment in the New

[19]I use the IPEDS classifications. Far West states include Alaska, California, Hawaii, Nevada, Oregon, and Washington. New England states include Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island, and Vermont

England states to be more regionally interlinked than in other states.

The researcher has leeway in selecting the matching variables to create the synthetic control. To avoid specification searching, I follow recommendations from Ferman, Pinto, and Possebom (2020) and match on all pre-treatment outcomes in the main specification. For enrollment level, the pre-treatment period begins in 2000; for proportion of Black enrollment, it begins in 1995. I present results using additional matching variables in Appendix 1E. The main results hold under alternate specifications, though inference generally differs.

Finally, I also perform the same analyses for private, not-for-profit four-year universities in Appendix 1C as a placebo test: private universities do not receive state funding and should not be affected by state funding changes, but are otherwise exposed to the same conditions as nearby public universities. I do not find any evidence that private universities respond to changes in state funding, which supports that the main results indeed capture the causal effects of switching to PF.

1.5 Results

1.5.1 Enrollment Level

Figures 1.7 and Table 1.5 summarize the synthetic control results for enrollment level overall and by demographic groups (Black, Hispanic, and white students). Information on weights are in Appendix 1A.

Table 1.5: Enrollment Level - Ohio: Summarized Treatment Effect and Inference

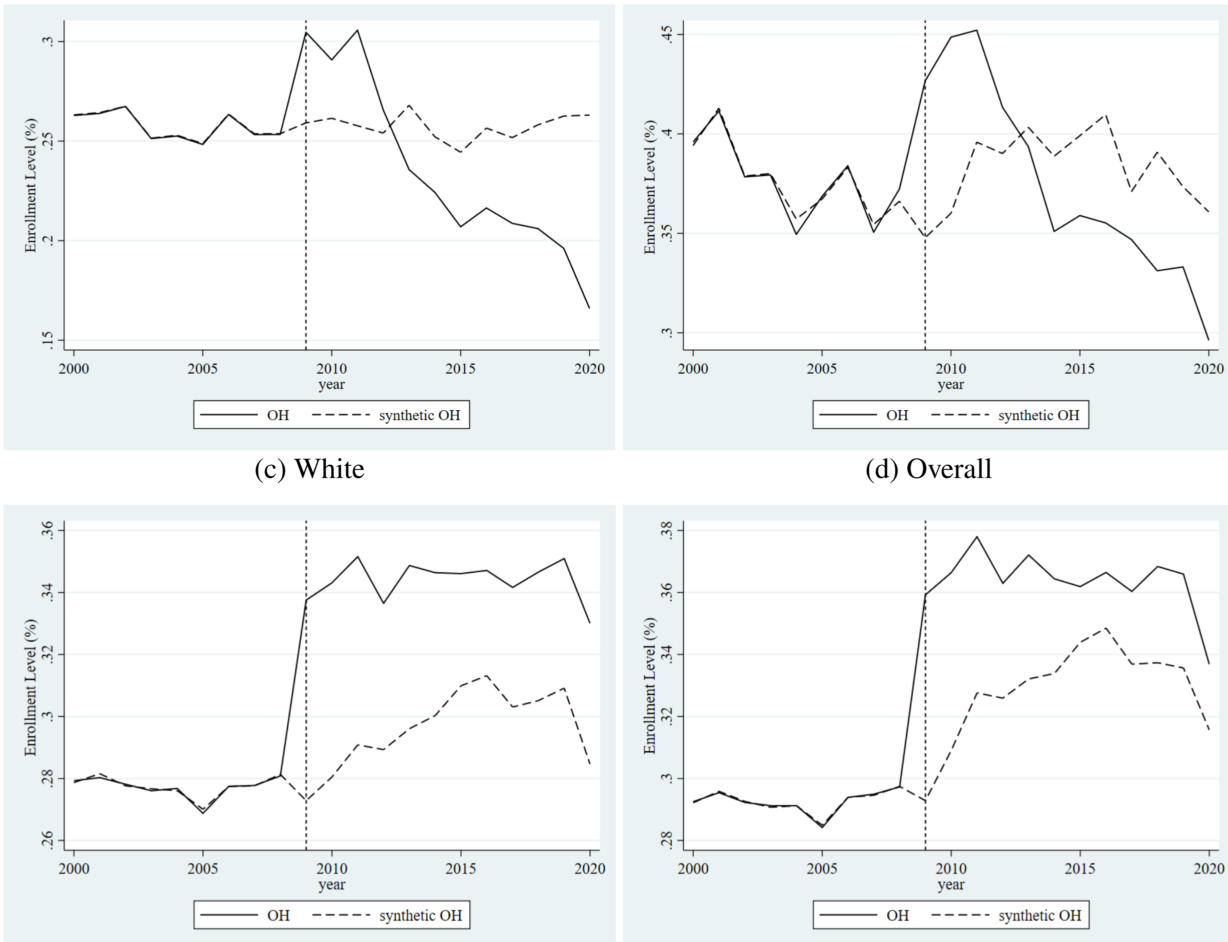
	Treatment Effect	p-value[20]
Overall	0.032	0.028
Black	-0.028	0.028
Hispanic	-0.015	0.111
White	0.046	0.056

While these results initially suggest there was a permanent increase in overall enrollment as well as enrollment of white students that started in 2009 (p-values=0.028 for overall enrollment, 0.056 for white enrollment), this is likely driven by the accounting change that caused 12th grade

[20]The p-value is calculated per-period, but does not change across time except for Hispanic enrollment. I show the highest p-value over the post-treatment period.

Figure 1.7: Enrollment Level - Ohio

(a) Black (b) Hispanic (c) White (d) Overall



students to be counted differently before and after 2009. Synthetic Ohio also experiences an increase in overall enrollment level, but in 2011 and at a smaller magnitude, while enrollment of white students does not display any noticeable one-time changes at any point post-implementation. Also, in Appendix 1C, I find that there are similar results for private universities. Put altogether, the changes in overall and white enrollment levels as presented in panels (c) and (d) of Figure 1.7 likely do not pick up on the true effect of changes in funding.

The population of Black and Hispanic 12th graders is steady across the time period, so enrollment levels do not suffer from measurement issues for these groups. The trend in Black student enrollment matches well with predictions from Proposition 3 (p-value = 0.028). There is an initial increase in enrollment level, followed by a sharp and persistent decrease that begins 3 years after

Table 1.6: Enrollment Level - Ohio: Black and Hispanic Enrollment Treatment Effects

	Black Enrollment	Hispanic Enrollment
2010	0.029	0.089
2011	0.048	0.056
2012	0.012	0.023
2013	-0.032	-0.010
2014	-0.028	-0.038
2015	-0.037	-0.040
2016	-0.040	-0.054
2017	-0.043	-0.024
2018	-0.052	-0.060
2019	-0.066	-0.040
2020	-0.097	-0.065
Average	-0.028	-0.015

implementation. While the long-run effect is a 2.8 percentage point decrease in enrollment level, as can be seen in Table 1.6, this masks the fact that the gap becomes larger at the tail end of the time period—the estimated treatment effect for 2020 is -0.097. The trend for Hispanic student enrollment is very similar (p-value=0.11), but the effect is muted compared to enrollment of Black students.

In Figure 1.8 and Table 1.7, I show the results of artificially eliminating the one-time drop in the population of white 12th grade students in 2009 by adding 15,000 (the approximate size of the drop) for 2009 and after.[21] The trends match with what Proposition 3 predicts: enrollment increases more in Ohio than synthetic Ohio for 2 years after implementation, but then decreases relative to synthetic Ohio in the following years. The trend for enrollment level of white students follows a similar pattern but is not statistically significant at the 15% level or lower.

Panels (a) and (b) from Figure 1.8 along with panels (a) and (b) from Figure 1.7 are consistent with what Proposition 3 predicts and also provides suggestive evidence that the condition in Proposition 4 holds. Enrollment level initially increases, but decreases in the long-run, and this effect is stronger for Black students.

I do not find any results in Tennessee with a p-value lower than 0.194, which is also consistent with Proposition 3. However, recall that this could potentially also be driven by some combination

[21] Results don't change much if I instead subtracted 15,000 for 2008 and earlier.

Figure 1.8: Modified Enrollment Level - Ohio: White and Overall Enrollment

(a) White (b) Overall

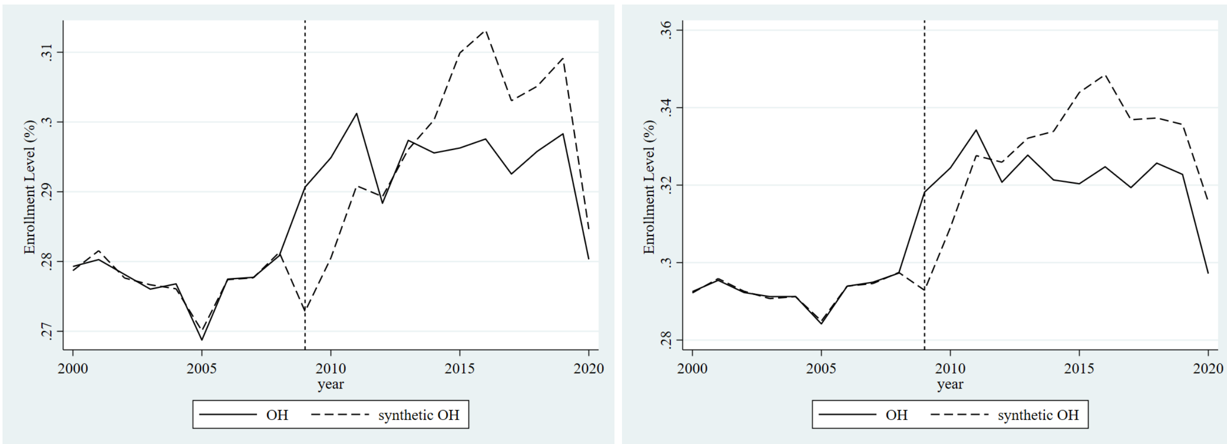


Table 1.7: Modified Enrollment Level - Ohio: White and Overall Enrollment Treatment Effects

	White Enrollment	p-value	Overall Enrollment	p-value
2010	0.014	0.056	0.015	0.028
2011	0.010	0.111	0.007	0.056
2012	-0.001	0.139	-0.005	0.083
2013	0.001	0.167	-0.004	0.083
2014	-0.005	0.167	-0.013	0.083
2015	-0.014	0.167	-0.024	0.083
2016	-0.016	0.167	-0.024	0.083
2017	-0.011	0.167	-0.018	0.056
2018	-0.009	0.194	-0.012	0.056
2019	-0.011	0.194	-0.013	0.083
2020	-0.004	0.194	-0.019	0.056
Average	-0.004	0.157	-0.010	0.068

of self-selection of PF weights, the effect of the Tennessee Promise, and volatility in funding post-PF implementation. The synthetic control figures are in Appendix 1A. Although not statistically significant, note that enrollment level is fairly constant across the post-treatment period with a one-time drop in 2015, but there is considerably more volatility when looking at enrollment of white students and Black students respectively. In particular, enrollment level for Black students increases from 2017-2020—which is consistent with several Tennessee public universities experiencing funding per FTE increases towards the end of the post-treatment period according to Proposition 3.

1.5.2 Proportion of Black Enrollment

I show the synthetic control results in Figure 1.9 and summarize results in Table 1.8. Information on the synthetic control weights are in Appendix 1A. I also show results for other demographic groups in Appendix 1B.

Table 1.8: Proportion of Black Enrollment - Ohio: Summarized Treatment Effect and Inference

	Treatment Effect	p-value
Ohio	-0.0133	0.059
Tennessee	0.0293	0.059

In Ohio, the overall treatment effect is a decrease of 1.13 percentage points that is concentrated in the last 7 years of the post-treatment period. The proportion of Black enrollment increased for the first 3 years after implementation before sharply dropping. While synthetic Ohio also shows a long-term decrease in the proportion of Black enrollment, the magnitude of the drop is much larger in Ohio, and much of it is concentrated in the one-time decrease 3 years after implementation. Interestingly, synthetic Ohio and Ohio appear to roughly move in parallel even in the post-period treatment except for the large in proportion of Black enrollment in Ohio from 2012-2014, which could possibly suggest that universities were highly responsive either to changes in funding or learning from experience after being exposed to the new funding rules for a few years.

Figure 1.9: Proportion of Black Enrollment - Ohio

(a) Ohio Synthetic Control (b) Tennessee Synthetic Control

Also, this result mirrors enrollment levels for Black students as shown in panel (a) of Figure 1.7 closely, but the pre-treatment trends are different. While enrollment level for Black students is fairly constant for both Ohio and synthetic Ohio during the pre-treatment period—and remains constant for synthetic Ohio post-treatment—the *proportion* of Black enrollment was increasing throughout the pre-treatment period. Since the number of Black 12th graders is relatively constant, this implies that the number of Black 12th graders who applied to public universities has been increasing over time, and public universities may have been expanding enrollment of Black students to respond to increased demand for higher education from Black students. However, after PF implementation, the number of potential Black applicants became much less predictive of how many Black students would go on to eventually enroll (although it remained predictive in synthetic Ohio, as seen by the fact that synthetic Ohio is fairly steady across the post-period treatment in panel (b) of Figure 1.7). This is further suggestive evidence that Black enrollment was indeed disproportionately affected by changes in funding.

On the other hand, Tennessee shows markedly different results, as expected: the overall treatment effect is an increase of 2.93 percentage points. While proportion of Black enrollment in synthetic Tennessee trends downward steadily, it fluctuates at between 19%-20% in Tennessee for most of the post-implementation time period with the exception of a spike in 2016-2017. Because synthetic Tennessee is trending downward during this time, the treatment effect is positive for all post-treatment time periods.

1.6 Policy Discussion and Conclusion

In this chapter, I present a theoretical framework of state funding under general assumptions on how state policymakers and universities evaluate the returns to enrolling students to assess the two main enrollment arguments for and against PF: (1) PF increases efficiency in enrollment, and (2) PF disproportionately affects enrollment of URM students.

I show in Theorem 1 that neither PF nor EF is inherently more efficient than the other, debunking the main argument (1) for PF. Rather, level of funding determines whether a funding rule is able to realign the university k's enrollment problem with the SP's enrollment problem. If state policymakers are concerned with over-enrollment, then they should decrease overall funding. That said, switching rules may be more politically viable than announcing a cut in funding, as it is possible to use the switch to PF as an opportunity to decrease funding without directly calling it a funding cut. Indeed, Figure 1.2 shows this is what happened in Ohio in practice: after PF was implemented, funding per FTE decreased. I then turn to assessing the gaps in efficiency that result from differences between theory and implementation. While I show in Proposition 2 that an efficient Ohio PF rule exists, I show in Corollary 2 that in general, Ohio's URM premium is likely incorrectly specified. I show that this is driven by the fact that choosing a premium based only on differences in graduation rates fails to comprehensively account for all differences between how the SP and the university k evaluate the returns to educating and graduating students.

In Proposition 3, I show how universities respond to changes in in funding rules. This generates the main testable hypothesis of this chapter: if switching to PF causes a decrease in funding, then level of enrollment should decrease. Finally, I give conditions in Proposition 4 such that changes in funding will disproportionately affect URM cut-offs, which would indicate that argument (2) against PF holds. Using synthetic controls, I find evidence to support the main testable hypothesis and suggestive evidence that the condition I identify in Proposition 4 holds, as the proportion of Black students enrolled was affected more strongly than other changes in student composition after both PF states switched to PF.

This chapter shows that there can be unintended and inequitable side effects to state policies that do not intend to affect equity of opportunity. Ohio policymakers explicitly recognize that URM students are less likely to graduate and included a premium for URM completions based on the difference in graduation rates. In spite of this, Black students were disproportionately affected by the change in funding rule; a back-of-the-envelope calculation suggests that this resulted in approximately 375 fewer Black students enrolled every year as first-time students at public four-year universities in Ohio from 2010-2019, or a total of around 3,780 fewer Black students enrolled over the entire time-span.[22] A policymaker who cares for equity in college accessibility must be

[22] I drop 2020 from this back-of-the-envelope calculation as it is unclear how the COVID-19 pandemic affected Ohio enrollment relative to other states.

aware of *all* systematic differences between URM and non-URM students, not just differences in graduation rates, to correctly incentivize universities to change their enrollment decisions.

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APPENDIX 1A

ENROLLMENT LEVEL - SUPPLEMENTARY FIGURES AND TABLES

Table 1A.1: Enrollment Level - Ohio: Synthetic Control Weights

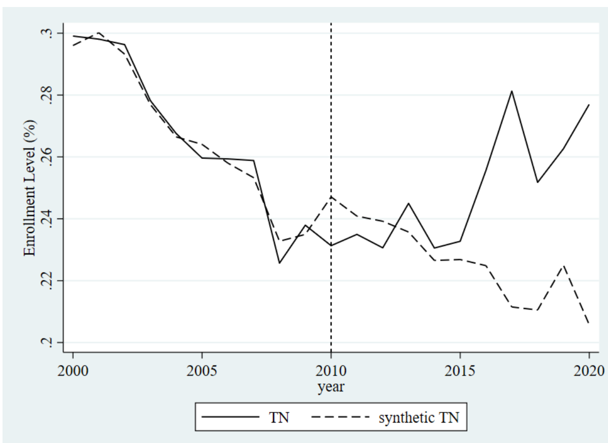
	Overall	White	Black	Hispanic
AL				0.07
AZ	0.152			
CO			0.068	
DC				0.29
DE			0.109	
GA			0.186	
IA	0.097	0.393		0.288
IL	0.1	0.243		
IN		0.152		
KY			0.202	0.03
MD		0.022		
MN		0.046	0.131	
MS	0.404	0.085		
ND	0.016			
NE	0.008			
NJ			0.225	
NM			0.04	
OK		0.034		
SC	0.083	0.017		
SD				0.119
UT	0.071	0.009		
WV	0.069		0.02	0.204
WY			0.02	

Table 1A.2: Proportion of Black Enrollment - Ohio: Synthetic Control Weights

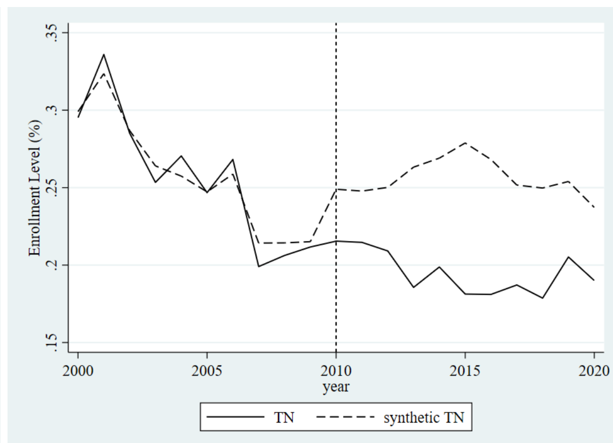
Ohio		Tennessee	
AL	0.113	AL	0.592
DE	0.005	DC	0.003
GA	0.02	FL	0.116
KY	0.112	MN	0.15
MD	0.052	MO	0.03
MN	0.175	WV	0.109
MO	0.523		

Figure 1A.1: Enrollment Level - Tennessee

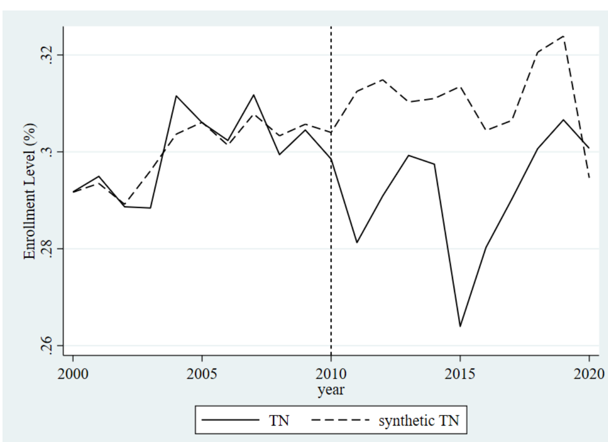
(a) Black



(b) Hispanic



(c) White



(d) Overall

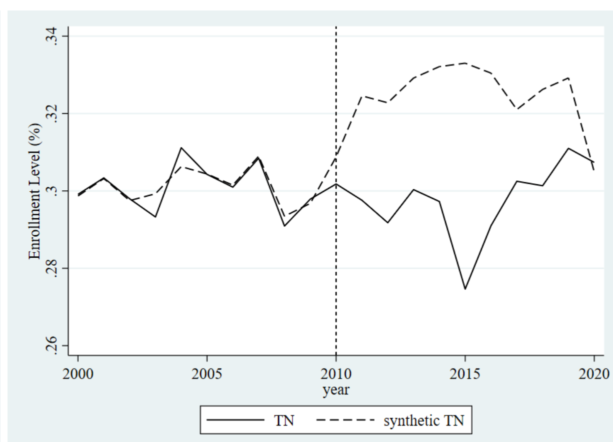


Table 1A.3: Enrollment Level - Tennessee: Synthetic Control Weights

	Overall	White	Black	Hispanic
AL				0.071
DE	0.015	0.053	0.257	
FL	0.16		0.415	
IA			0.149	0.133
IL	0.078	0.296		
KY			0.088	0.12
MI				0.293
MN		0.154		
MO	0.128			
MS	0.278			
MT			0.007	
NE			0.037	
NM			0.022	
OK		0.029		
SC	0.344	0.105		0.108
NE				
NM				
UT				0.276
WI		0.253		
WV			0.026	
WY		0.11		

APPENDIX 1B

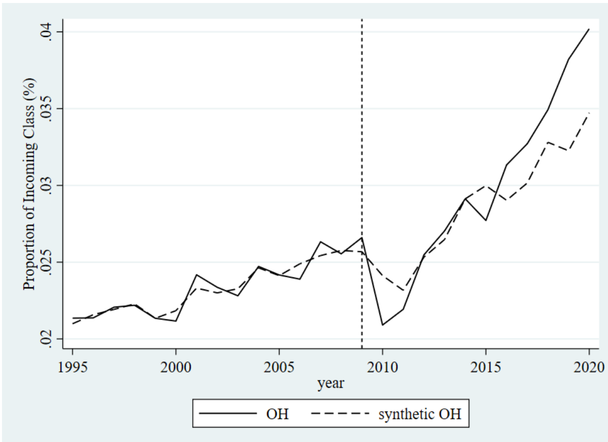
PROPORTION OF ENROLLMENT - OTHER DEMOGRAPHIC RESULTS

Using the same candidate pool and matching variables as in the main analysis, I do not find there were any practically or statistically significant (at the 10% level or lower) changes in proportion of enrollment for other demographic groups.

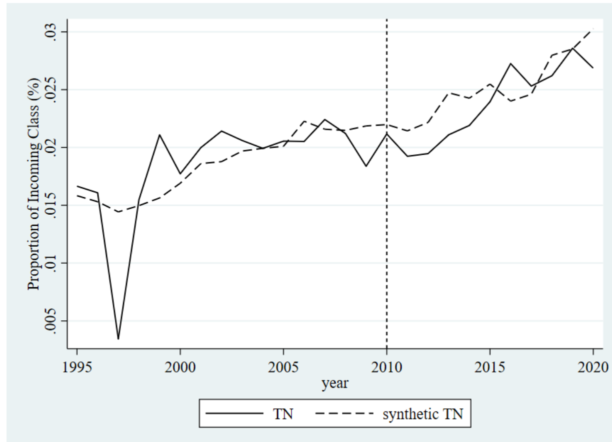
While there is some evidence that Ohio universities also admitted fewer Hispanic students following funding changes, the effect on proportion of enrollment is not statistically significant. This may reflect differences between Hispanic and Black applicants as discussed in Section 1.4. Also, note that the changes in funding did not generate any statistically significant positive preferential effects—that is, no specific demographic appears to have experienced a systematic increase enrollment level in response to funding changes.

Figure 1B.1: Proportion of Enrollment: Other Demographic Groups

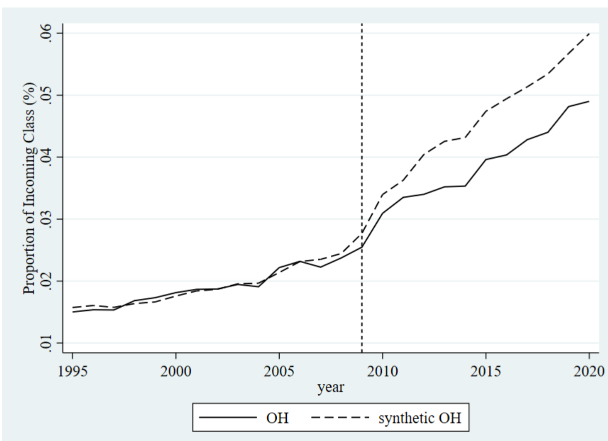
(a) Ohio: Asian Enrollment



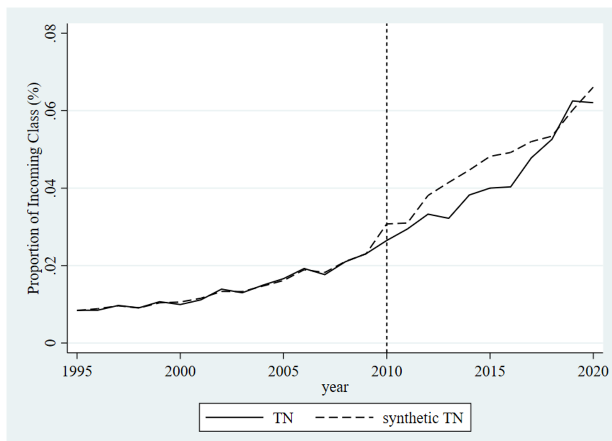
(b) Tennessee: Asian Enrollment



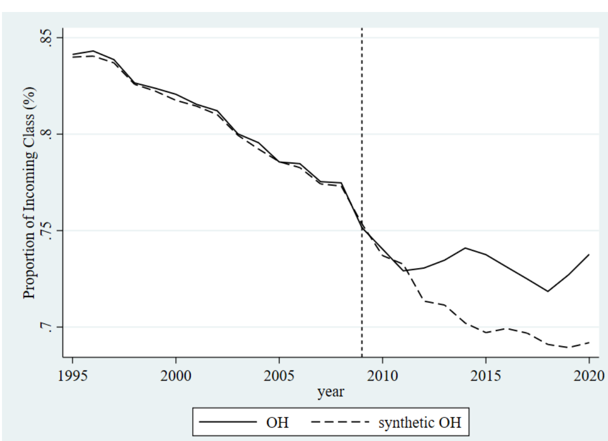
(c) Ohio: Hispanic Enrollment



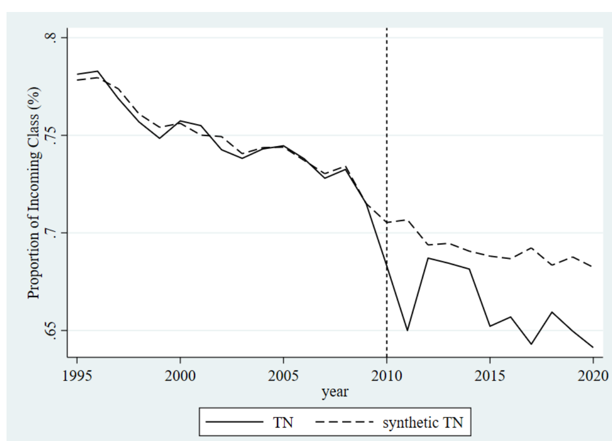
(d) Tennessee: Hispanic Enrollment



(e) Ohio: White Enrollment



(f) Tennessee: White Enrollment



APPENDIX 1C

PRIVATE UNIVERSITIES

I perform the empirical analyses as outlined in Section 1.4, but I instead use private four-year not-for-profit universities. Private universities are exposed to the same conditions as public universities in the same state *except* for public funding policies. If results in Section 1.5 are purely driven by statewide trends, then we should see similar results for public and private university. However, I find no evidence of similar changes in enrollment after PF implementation, which supports that the main results correctly capture the effects of PF.

Throughout, I use the same candidate pool as in the main analysis, but I additionally drop Wyoming due to not having candidate institutions in all years. I match on all pre-implementation incomes, as in the main estimations.

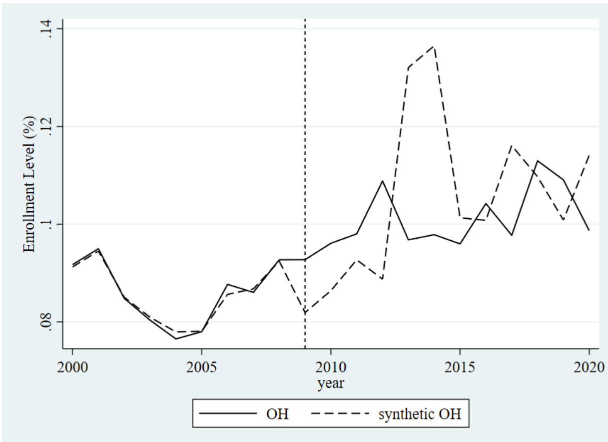
Enrollment Level

Figure 1C.1 shows overall level of enrollment and enrollment level by demographic group. Again, results for overall enrollment and enrollment level of white students are affected by the accounting change in how 12th grade students were counted that went into place in 2009; I therefore also show the modified enrollment levels in panels (e) and (f). Overall, taking into consideration the modified enrollment levels for white students and overall enrollment, there is no evidence that private schools responded to changes in state funding, and Ohio generally behaves very similarly to synthetic Ohio in the post-treatment period. One possible concern is that for level of Black student enrollment, synthetic Ohio exhibits a sudden spike in 2013-2014, but this likely reflects idiosyncratic behavior in the synthetic control rather than a true divergence between Ohio and its synthetic control.

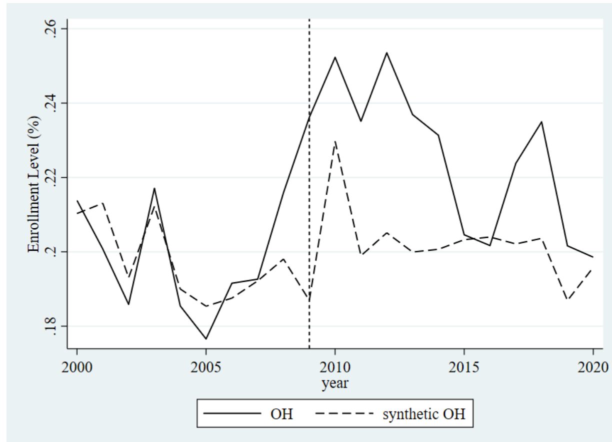
Figure 1C.2 shows that in Tennessee, there are no changes in level of enrollment after 2010 looking at overall enrollment and enrollment of Black students. While there is a statistically significant (p-value=0.029) decrease in enrollment of white students, this moves in the opposite direction as public universities.

Figure 1C.1: Enrollment Level - Ohio Private Universities

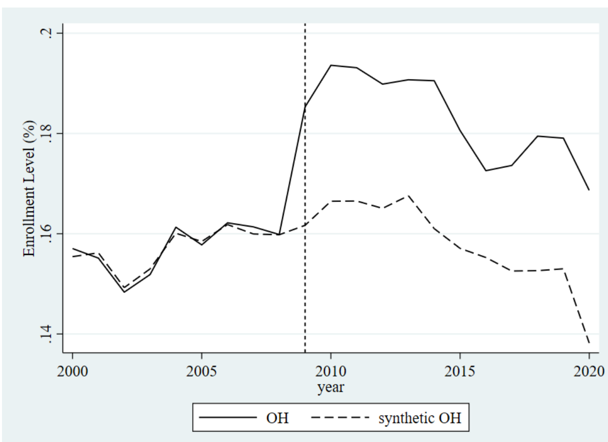
(a) Black



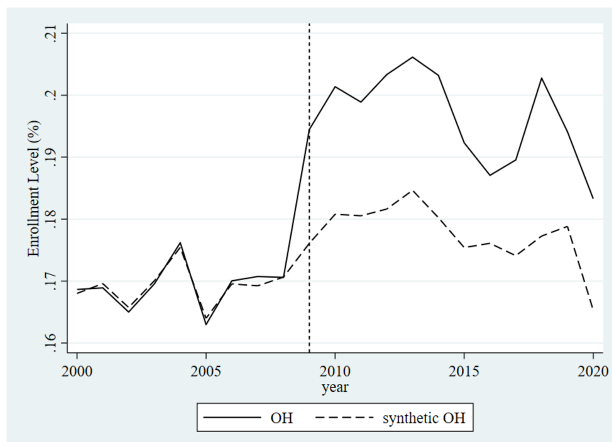
(b) Hispanic



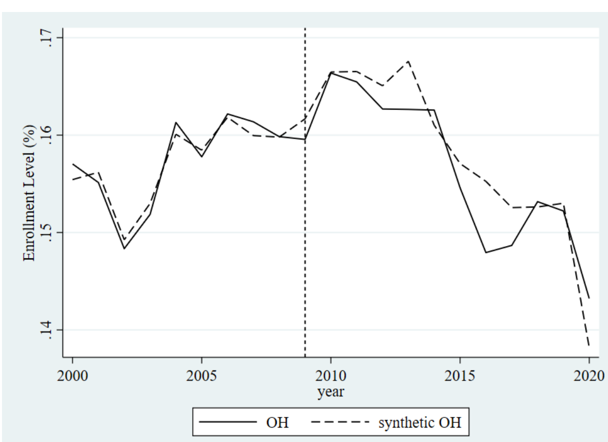
(c) White



(d) Overall



(e) White (Modified)



(f) Overall (Modified)

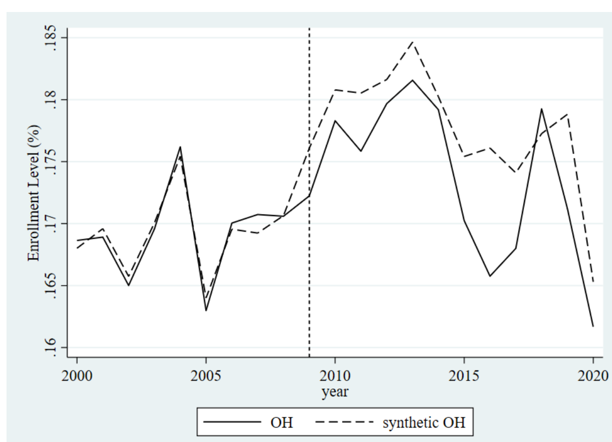
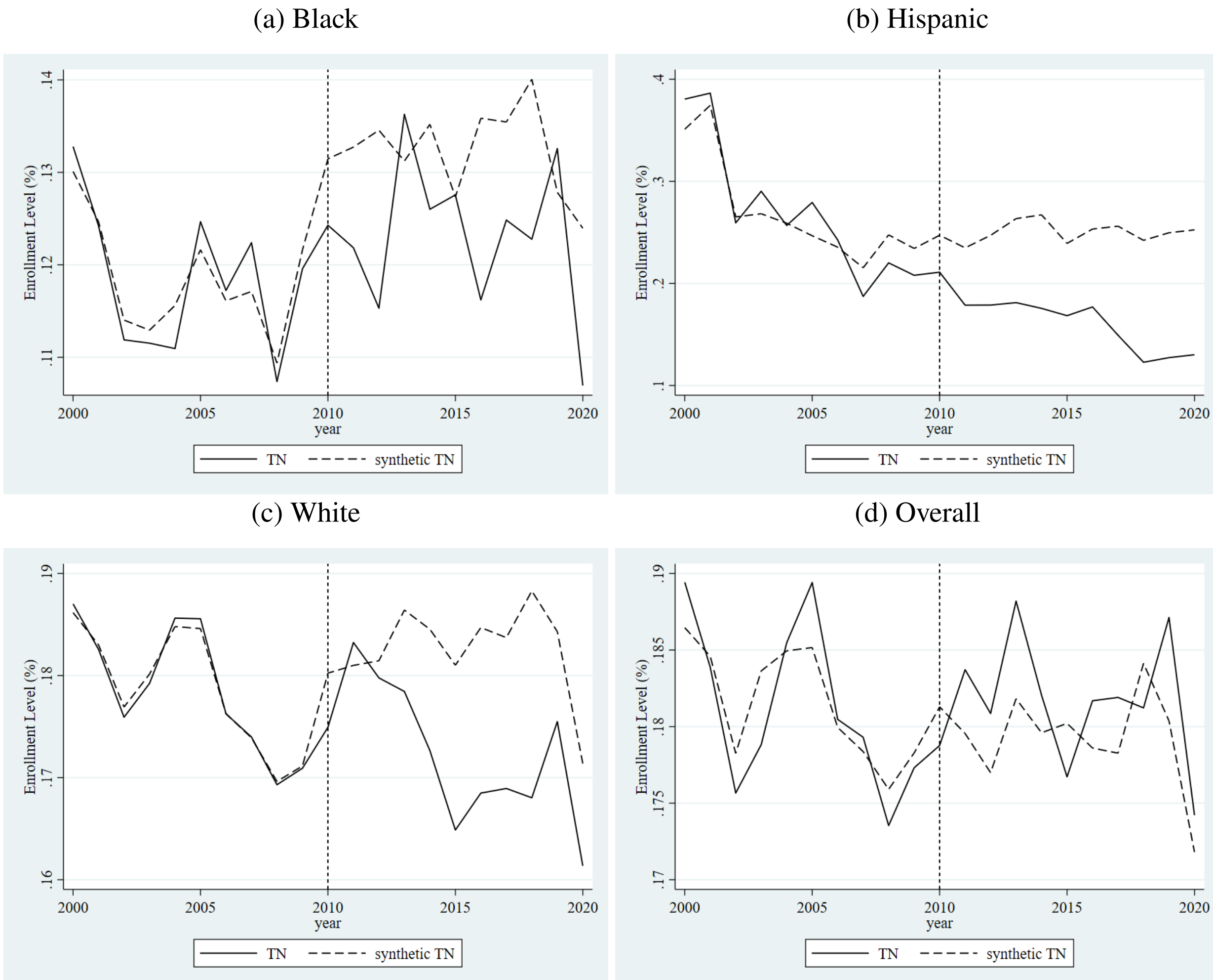


Figure 1C.2: Enrollment Level - Tennessee Private Universities

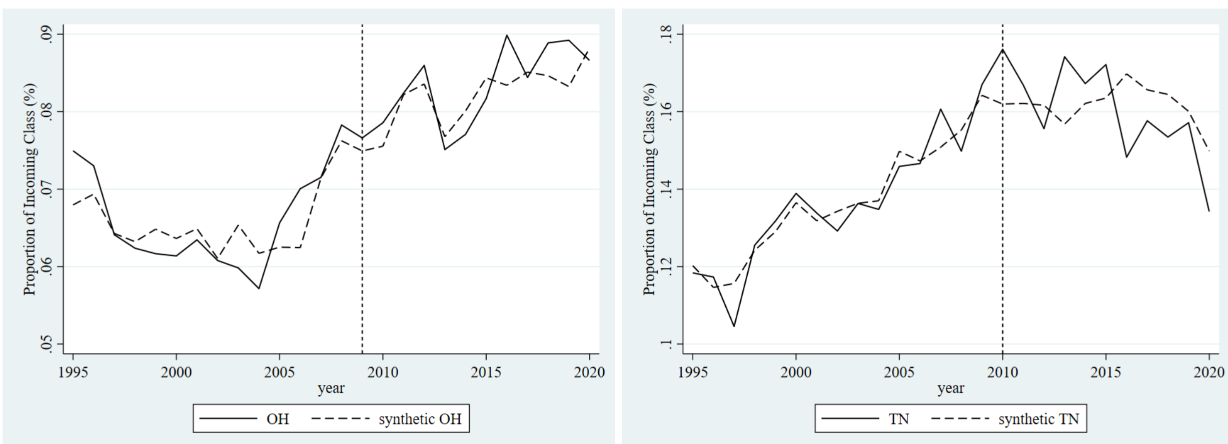


Proportion of Black Enrollment

As seen in Figure 1C.3, neither Ohio nor Tennessee private universities appear to change proportion of Black enrollment after PF implementation.

Figure 1C.3: Proportion of Black Enrollment - Private Universities

(a) Ohio (b) Tennessee



APPENDIX 1D

ALTERNATE CANDIDATE POOLS

I additionally considered several different candidate pools for the synthetic control as described in Table 1D.1.

Table 1D.1: Alternate Candidate Pools

1	Drop only the other treatment state, AK, and HI
2	Additionally drop CA, TX
3	Additionally drop Far West and New England states
4	Additionally drop Southwest states[1]

Recall that the candidate pool I used for the main analyses is group 3. Candidate pools 1 and 2 result in synthetic controls that assign more weights than there are pre-treatment outcomes to match on for most of the enrollment level outcomes, hence I only present candidate pool 4 for those outcomes.

Using candidate pool 4 does not substantially change results on enrollment level outcomes for Tennessee; outcomes remain statistically insignificant (lowest p-value=0.151) and the synthetic controls generated are virtually the same as in the main estimation. I therefore discuss only Ohio results for the rest of this subsection.

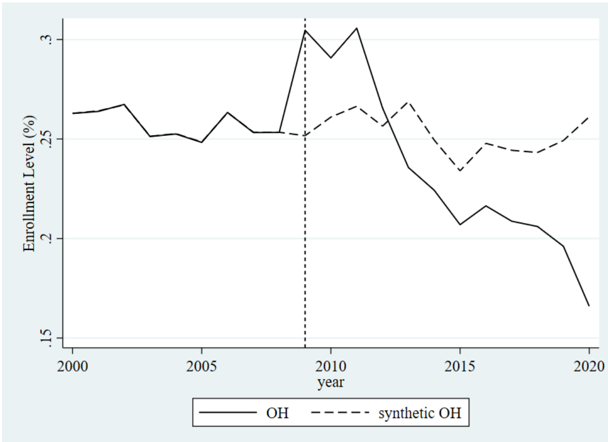
While the synthetic control weights change for virtually all enrollment level outcomes, I still find a statistically significant divergence in selectivity over Black enrollment (p-value = 0.03) in Ohio that follows the same trend as when I use candidate pool 3, though per-period treatment effects are slightly muted when compared to the main estimation. Also, the per-period post-implementation treatment effects on enrollment for Hispanic students in Ohio is similar to the main estimation, but the p-value is lower (p=0.061).

Including inference, using candidate pool 4 both with and without altering the population of white 12th grade students gives similar results to using candidate pool 3 for overall enrollment and enrollment of white students in Ohio.

[1]Using the IPEDS classification, Southwest states include Arizona, New Mexico, Oklahoma, and Texas.

Figure 1D.1: Enrollment Level - Ohio: Candidate Pool 4, Part 1

(a) Black



(b) Hispanic

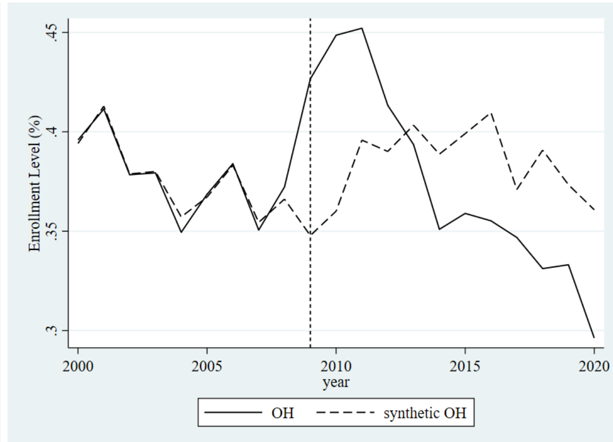
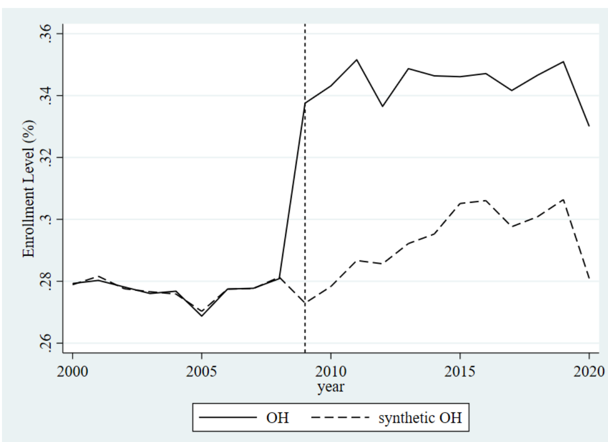
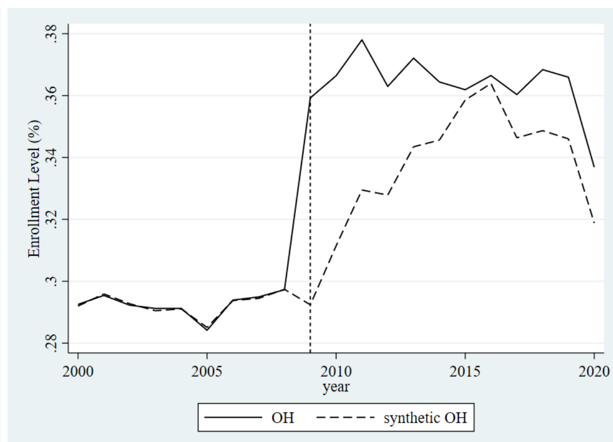


Figure 1D.2: Enrollment Level - Ohio: Candidate Pool 4, Part 2

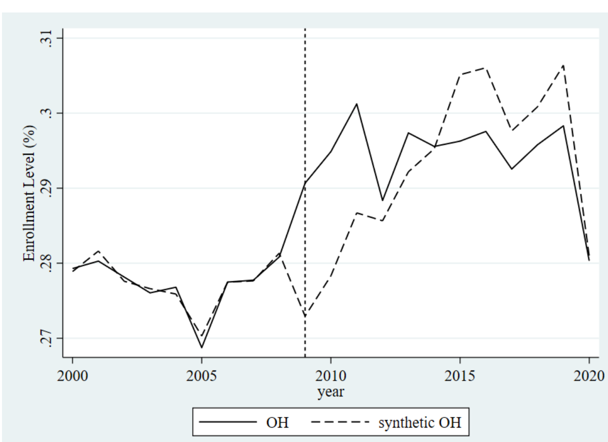
(a) White



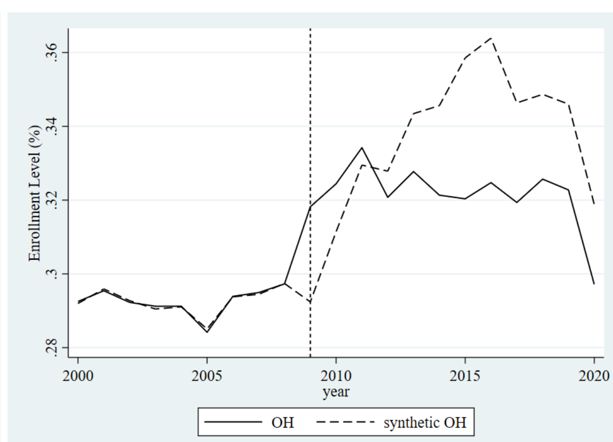
(b) Overall



(c) White - Modified



(d) Overall - Modified



Finally, for proportion of Black enrollment in both states, I find that the candidate pool affects only inference—all 4 candidate pools generate the same synthetic controls. For Ohio, the p-values are 0.125 for candidate pool 1, 0.087 for candidate pool 3, and 0.059 for candidate pool 4. For Tennessee, the p-values are 0.146 for candidate pool 1, 0.152 for candidate pool 2, and 0.059 for candidate pool 4.

APPENDIX 1E

ALTERNATE MATCHING SPECIFICATIONS

I follow recommendations from Ferman, Pinto, and Possebom (2020) and run alternate specifications, using candidate pool 3 throughout. I consider up to 4 alternate specifications: even and odd years only, both with and without additional controls (five year average of state unemployment rate and mean in-state tuition before implementation).[1]

I do not show Tennessee results in this document, as for all alternate specifications I check, results remain statistically insignificant.[2]

Concentrating on the alternate matching specifications for synthetic Ohio, I show here the results from using either even or odd years with additional controls for enrollment level. I do not show results from using even/odd years only because they result in synthetic controls that assign more weights than there are pre-treatment outcomes. I also do not show overall enrollment level and enrollment level of white students in Ohio due to the accounting change that affected how 12th grade white students were counted. For proportion of Black enrollment, I show specifications using even/odd years only as well as even/odd years with controls.

In general, pre-treatment match quality is worse when discarding some of the pre-treatment outcomes as matching variables. While overall patterns in outcomes still hold, inference substantially degrades, likely because of the worse pre-treatment match quality. The p-values for Black enrollment level vary between 0.056 to 0.083 (evens and controls) but goes as high as 0.194 for odds and controls. The p-values for enrollment level of Hispanic enrollment vary between 0.028 to 0.083 (evens and controls) but goes as high as 0.222 for odds and controls.

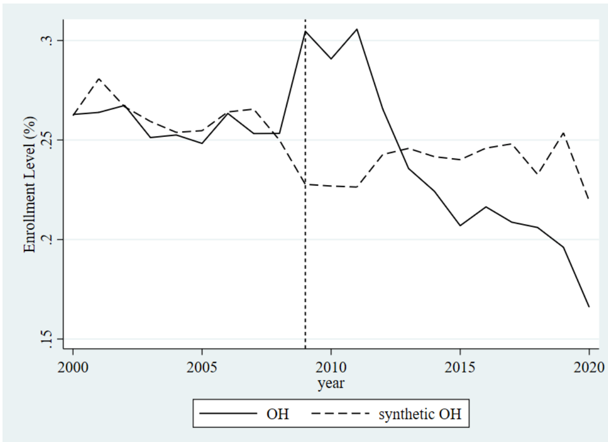
For proportion of Black enrollment in Ohio, the specification using only even years is fairly close to the specification using all pre-treatment years to match on; the p-value for both even year specifications is 0.054. Match quality for specifications using only odd years is visibly worse,

[1]Ferman, Pinto, and Possebom (2020) additionally suggest other specifications that are not well-suited for this analysis; for example, they recommend the first half of pre-treatment outcome variables—but this leads to synthetic controls that do not match with the treatment units very well in the second half of the pre-treatment period, and I believe that capturing the trend right before treatment occurs is important.

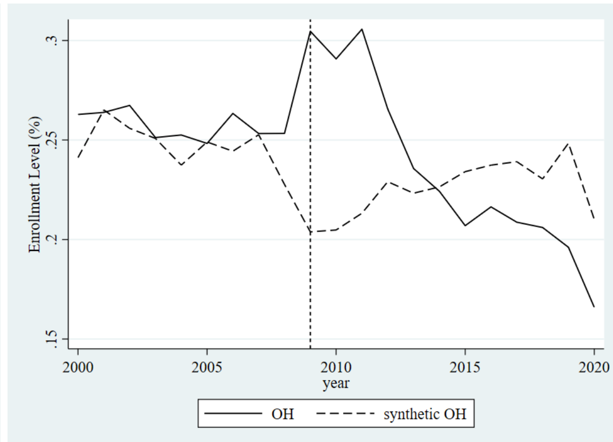
[2]The lowest p-value I ever observe is 0.25.

Figure 1E.1: Enrollment Level - Ohio: Alternate Matching Variables

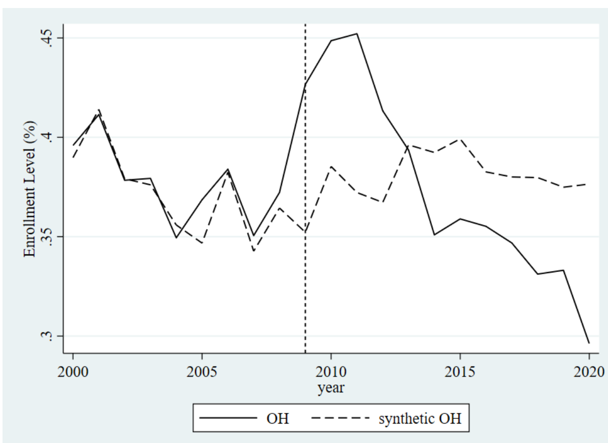
(a) Black Enrollment - Evens + Controls



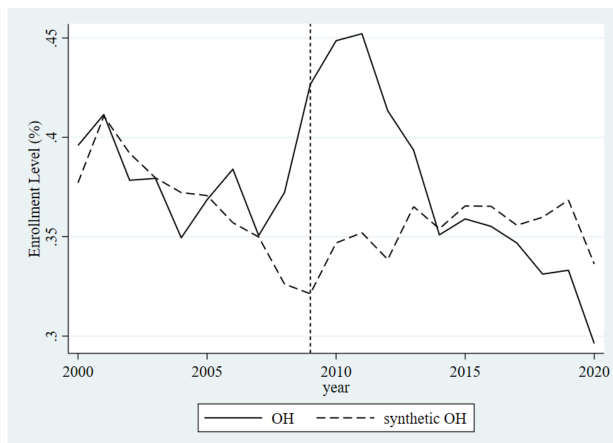
(b) Black Enrollment - Odds + Controls



(c) Hispanic Enrollment - Evens + Controls



(d) Hispanic Enrollment - Odds + Controls



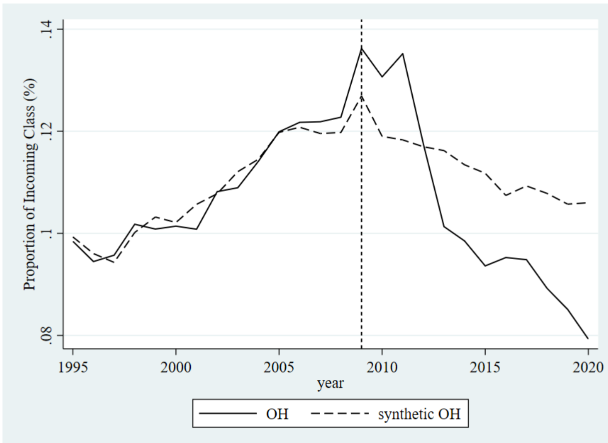
and the poorer match quality causes Ohio to perform worse in the standard placebo tests used for inference with synthetic controls (the lowest p-value for odd year specifications is 0.162).

The same is true for proportion of Black enrollment in Tennessee. The specifications using only even years are fairly close to the specification using all pre-treatment years to match on, especially from 2016 onward (p-value=0.054, same as in the main estimation), while the match quality for specifications using only odd years performs worse. P-values vary between 0.027 to 0.081 for the odd years only specification, and between 0.054 to 0.108 for the odd years plus controls specification. Also, note that using even years generates the same synthetic control as using even years plus additional controls.

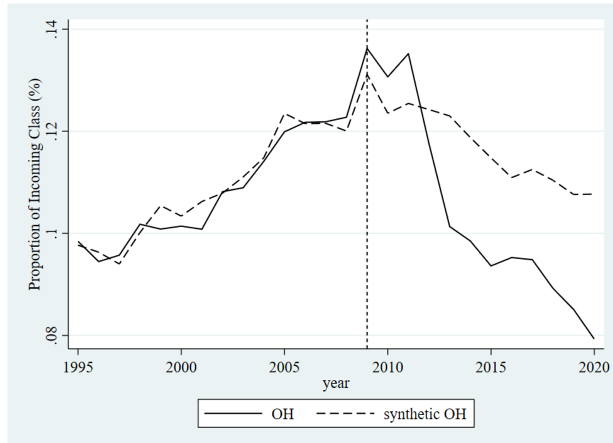
However, for all outcomes checked, changing the matching variables does not strongly affect

Figure 1E.2: Proportion of Black Enrollment - Ohio: Alternate Matching Variables

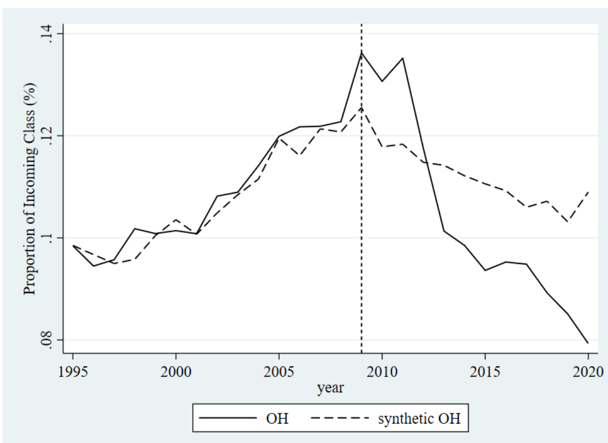
(a) Evens



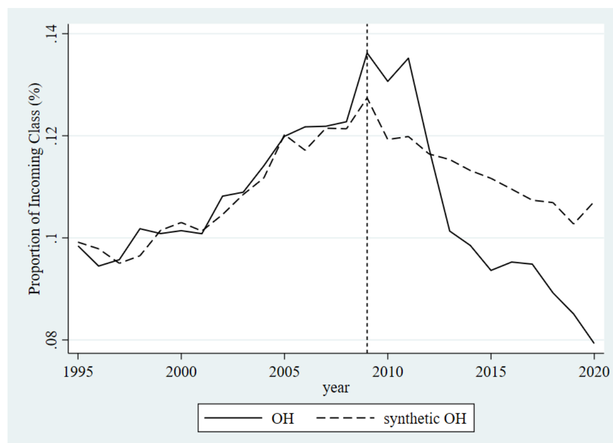
(b) Evens + Controls



(c) Odds

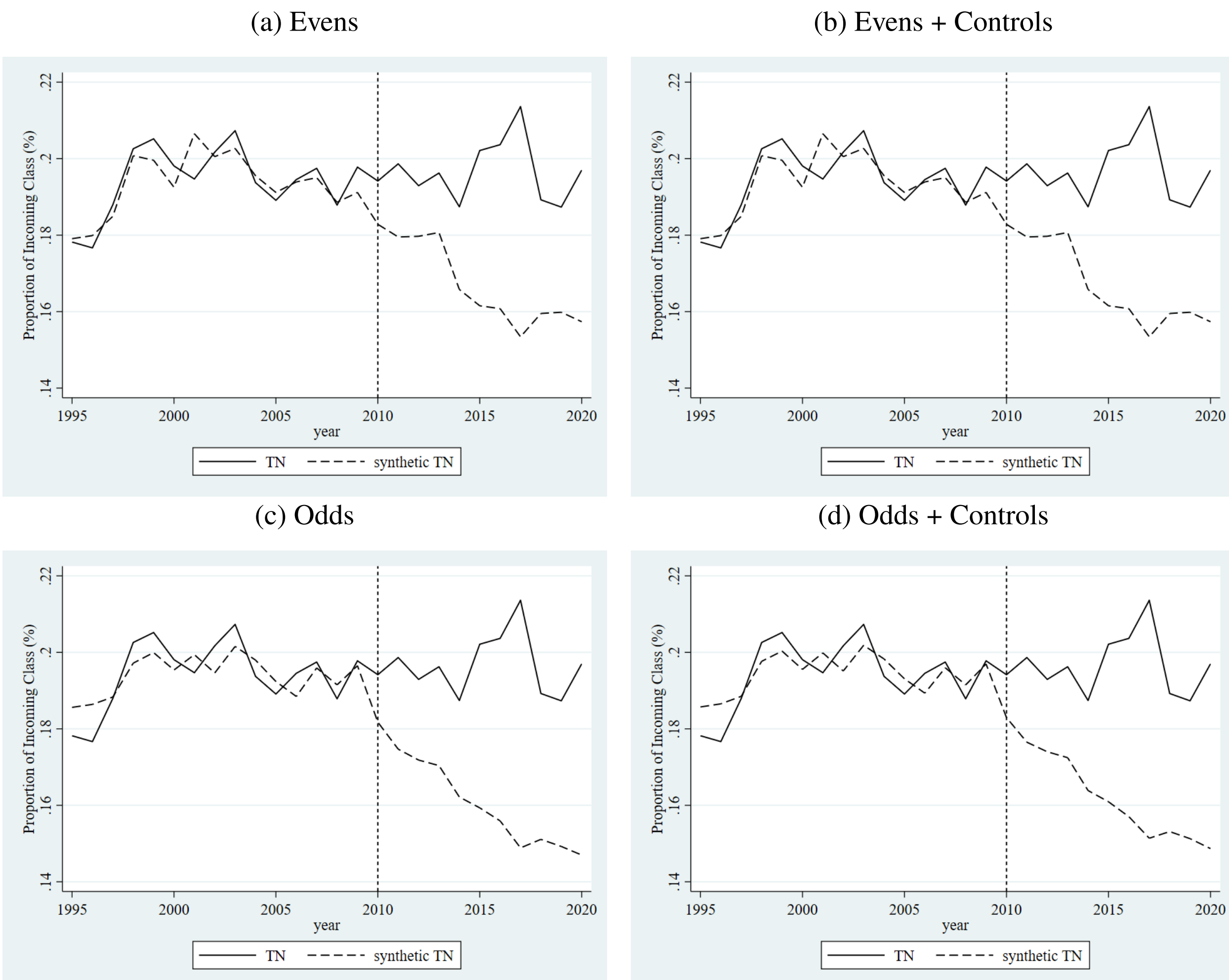


(d) Odds + Controls



the trend in how the synthetic controls diverge from outcomes in the PF states post-implementation.

Figure 1E.3: Proportion of Black Enrollment - Tennessee: Alternate Matching Variables



CHAPTER 2

THE ENROLLMENT EFFECTS OF REGIONAL CAMPUSES

2.1 Introduction

Several Ohio public universities, including the state flagship (Ohio State University, OSU), consist of a main campus along with one or several independently accredited regional campuses. This structure differs from public university systems in which members are independent peer institutions (e.g., the University of California or State University of New York systems), because Ohio regional campuses also share characteristics with community colleges: a core part of their institutional mission is to facilitate transfer into the main campus. For example, at OSU, all students enrolled at a regional campus are guaranteed the option to transfer into the main campus if they have a minimum 2.0 GPA and 30 credit hours.

Little is currently known about the academic pathways of students who attend Ohio regional campuses. As a first step to understanding the eventual outcomes of students who start at regional campuses, this chapter looks into the enrollment effects of adding a regional campus. It is clear that OSU benefits from opening or expanding regional campuses in two ways. First, regional campuses increase overall enrollment level. Second, students can be sorted between the main and regional campuses. The university can admit students with stronger applications to the main campus and students with weaker application strength to the regional campuses. That is, regional campuses allow institutions to simultaneously keep overall enrollment high while being selective at the main campus, which is important for prestige purposes (i.e., college rankings).

However, local community colleges could be made worse off by regional campuses. If students who would have enrolled at the community college prefer the regional campus after it becomes an alternative, this may lead to decreased enrollment and/or a less academically prepared student body at community colleges. Also, because students at the regional campus receive priority in transfer admissions, it may become more difficult for the community college to successfully transfer students into the main campus. Put altogether, regional campuses may make it more difficult for community colleges to fulfill both aspects of their institutional missions (conferring two-year degrees and facilitating transfer).

Finally, it is not immediately clear whether regional campuses make students better off. On one hand, students benefit from an increase in the options for higher education. Intuitively, this is especially beneficial for students who prefer the main campus, but are unable to enroll as a first-time student: regional campuses offer a streamlined, priority pipeline into their top choice. Regional campuses are also less expensive to attend, both in financial cost and opportunity cost. On the other hand, students who start at the community college and later try to transfer to the main campus may be crowded out by transfer applicants from the regional campus.

In this chapter, I consider two questions. First, how does introducing a regional campus affect first-time and transfer enrollment? Second, is it socially optimal for a university to expand its main campus modestly, or to open a new regional campus? To answer these questions, I build a framework in which students differ in wealth and academic preparedness, while institutions of higher education differ in price and perceived value of attendance. Under some general assumptions, introducing a regional campus causes local community colleges to enroll academically less prepared first-time student bodies, and the main university campus becomes more selective over transfer students from community colleges. This is because regional campuses attract some well-prepared students away from community colleges in first-time admissions, and regional campus students may "crowd out" better prepared community college students in transfer admissions.

I then consider a social welfare exercise: should a social planner subsidize tuition to modestly grow its main campus or open a larger regional campus? I suppose that the social planner is positive assortative in matching students to institutions; that is, she always prefers to enroll the most academically prepared student who is not yet enrolled somewhere to the main campus over the regional campus over the community college. I find that subsidizing tuition at the main university is always welfare-increasing, whereas the welfare effects of opening a regional campus are ambiguous. Tuition subsidization expands overall enrollment in a way that preserves positive sorting of students to institutions: decreased tuition induces some low wealth, highly academically prepared students to enroll at the main university instead of the community college. This then opens up capacity at the community college and in transfer enrollment, allowing students who "just barely" were inadmissible before to gain enrollment. Opening a regional campus has mixed welfare effects in first-time admissions because the regional campus draws students away from both the main university and the community college. It is welfare reducing for academically well-prepared students who would be accepted by the main university to convert their preferences away from the main university to the regional campus, but potentially welfare increasing for students to convert their preferences away from the community college to the regional campus. There are also mixed effects on welfare over transfer admissions if community college students are crowded out by less prepared regional campus students.

This chapter rests within the literature on student pathways over higher education. For a review, see Lovenheim and J. Smith (2023). Students' pathways over higher education are varied; a significant proportion of students enroll at more than one post-secondary institution. Andrews, J. Li, and Lovenheim (2014) study in-state students who were enrolled for the first time at public Texas institutions between 1992-2002 and find that 31.4% of students transferred at least once (most from a community college). Moreover, they find that transfer among students who eventually received a bachelor's degree is high: around half of eventual degree recipients transfer at least once, and 16% of degree recipients transferred more than once. Similarly, the National Student Clearinghouse Research Center reports in *Two-Year Contributions to Four-Year Completions* (2017) that almost half (49%) of students who completed a bachelor's degree in the 2015-16 academic year had enrolled at a two-year public institution in the last ten years.

However, this masks significant heterogeneity in attainment based on starting institution. More students are enrolled at community colleges than at public four-year institutions (9.6 million vs. 7.3 million in 2019), and more first-time students started at community colleges than at four-year universities in pre-COVID years (Fink 2024). That only half of bachelor's degree recipients therefore reflects systematic differences in completion. In spite of the fact that the majority of students who start at a community college report their eventual goal is a bachelor's degree (*What We Know About Transfer* 2015), most fall short of their original goal. Only 31% of community college students in the Fall 2010 cohort eventually transferred to a four-year institution, and less than half of transfers (14% of the cohort) went on to complete a bachelor's degree (Shapiro et al. 2017). As a point of comparison, the 6-year graduation rate for the Fall 2010 cohort of full-time, first-time students at four-year institutions was 60% (de Brey et al. 2019).

Understanding how first-time students sort between different institutions of higher education is also important due to existing gaps in the populations that enter two-year vs. four-year institutions. Public two-year institutions proportionally serve more under-represented minority students than public four-year institutions. According to the National Center for Education Statistics (NCES), 32.7% of students enrolled at a public four-year institution in Fall 2022 identified as Black or Hispanic, compared to 41.3% of students at public two-year institutions (*Total fall enrollment in degree-granting postsecondary institutions, by level and control of institution and race/ethnicity or nonresident status of student: Selected years, 1976 through 2022* 2023). Because community colleges are more affordable than four-year institutions and more accommodating to non-traditional and part-time students, they serve an important role in promoting upwards mobility (*An Introduction to Community Colleges and Their Students* 2021).

This chapter also relates to research on how community colleges affect degree completion. Within the field of economics, this strand of literature started with Rouse (1995), who shows that the enrollment effect of community colleges on eventual bachelor's degree completion is not clear ex-ante. There are two conflicting effects: some students might only pursue a four-year degree after being exposed to higher education through community college ("democratization"), while other students who would have otherwise attended a four-year institution may be attracted to the community college ("diversion"); I partially build off of the framework when considering transfer admissions in this work. To contextualize these effects, Mountjoy (2022) studies tenth grade students who enrolled at a Texas public high school between 1998-2002 and finds that increased access to two year institutions overall increases educational attainment and later earnings, but as theoretically predicted, there are two opposing effects. Around 1/3 of community college students were diverted from a four-year institution, while the other 2/3 of students were democratized. See also research papers on the effects of “Promise” programs that make community college free (or highly cost subsidized) to local high school graduates, including Nguyen (2020), Bell (2021), Acton (2021), and Billings, Gándara, and A. Li (2021). Finally, Kane and Rouse (1999) provides an overview of community college’s role in the larger U.S. higher education environment.

This work complements research on how students choose to apply to universities (e.g., “demand side” changes as opposed to institutional “supply side” changes). See Bound, Hershbein, and Long (2009) for a review on how students change their admissions behavior in response to increased competition, which is especially relevant to the environment I study: as I discuss in the next section, one reason OSU expanded its regional campuses is in direct response to increasing competitiveness in applications. See also Chade, Lewis, and L. Smith (2014), Pallais (2015), and Knight and Schiff (2022) for research on how changes in student application behavior affect enrollment.

Finally, I add to research that build theoretical frameworks on how either or both student application behavior and institutional preferences shape college enrollment. I follow the matching framework from Azevedo and Leshno (2016), who show that for a large, decentralized two-sided matching environment with a continuum of agents, a stable matching can be equivalently characterized in a supply and demand framework. In the context of college admissions, prices are analogous to level of institutional selectivity; in equilibrium, a university picks a cut-off level of selectivity. Blair and Smetters (2021) uses the same framework and imposes assumptions on positive sorting that are similar to those I use to characterize the social planer’s problem in Section 2.4. See also Che and Koh (2016), as well as Epple, Romano, and Sieg (2006) and Fu (2014) for equilibrium models of college admissions.

To my knowledge, I am the first to research regional campuses. I concentrate on characterizing enrollment, as this is necessarily the entry point for every student’s post-secondary pathway; understanding enrollment is therefore crucial to contextualizing all academic outcomes. Since regional campuses have similarities with both two-year and four-year institutions, it is not clear a priori whether regional campuses are more comparable two-year or four-year institutions. In the next section, I characterize OSU’s regional campuses and show how they compare to OSU’s main campus and comparable community colleges. In Section 2.3, I construct a theoretical framework of first-time and transfer admissions with multiple institutions, and show the theory predicts that adding a regional campus has negative effects on first-time enrollment for community colleges and for students applying to transfer into the main campus from the community college. I also present a social welfare analysis comparing the predicted welfare effects of tuition subsidization vs. opening a regional campus in Section 2.4. I show that from the social planner's perspective, tuition subsidization is always welfare-increasing. However, there are always both students who generate strictly more and strictly less social welfare after a regional campus opens. I conclude in Section 2.5.

2.2 Background

I build the model in Section 2.3 based on the Ohio State University (OSU) campus system.[1] The OSU main campus is in Columbus (the OSU-Main Campus henceforth), and OSU regional campuses are located in Lima, Mansfield, Marion, Newark, and Wooster. The Wooster campus houses OSU's Agricultural Technical Institute; because of its specialized focus, I do not consider the Wooster campus in this analysis. When I refer to "regional campuses", I mean the Lima, Mansfield, Marion, and Newark campuses only.

I identify four key differences between the main and regional campuses: (1) institutional missions, (2) affordability, (3) profile of the student bodies, and (4) student outcomes. I conclude the section by discussing (5) differences between regional campuses and comparable community colleges.

Institutional missions. The OSU regional campuses are independently accredited and treated as separate institutions from the OSU-Main Campus and each other by the National Center for Education Statistics (NCES). While the OSU-Main Campus primarily confers baccalaureate degrees to its undergraduate students, the OSU regional campuses have Carnegie classifications of either

[1] OSU is not the only public university in Ohio with regional campuses: Bowling Green State University, Kent State University, Miami University, Ohio University, University of Akron, University of Cincinnati, and Wright State University have at least one regional campus as well; however, to my knowledge, some of these universities treat regional campuses as commuter campuses and do not prioritize transfers from regional campuses. Central State University, Cleveland State University, Shawnee State University, University of Toledo, and Youngstown State University do not have any regional campuses.

"Baccalaureate College: Diverse Fields" (Lima) or "Baccalaureate/Associate's College: Mixed Baccalaureate/Associate's" (all others). As such, OSU advertises two academic pathways at its regional campuses: (1) completing either an associate or baccalaureate degree at the regional campus, or (2) starting a degree before transferring to the OSU-Main Campus (*Undergraduate Admissions: Regional Campuses* 2023).

A key feature of regional campuses is that their students may request a "campus change" to the OSU-Main Campus after completing 30 credit hours with at least a 2.0 GPA (*Campus change: FAQs* 2023).[2] Campus changes are considered an internal movement within the OSU system, but because regional campuses are independently accredited, the NCES counts them as transfers. Campus changes students are not required to send a separate application to the OSU-Main Campus, and there are no frictions in transferring credits.

However, there still exist other potential frictions in the transfer process. Remaining at the student's home campus is the default; a student must proactively initiate a campus change. Also, the minimum requirements to enter specific programs offered at the OSU-Main Campus may be higher than the minimum requirements to enter the OSU-Main Campus. For example, to apply for the Fisher College of Business at the OSU-Main Campus, a student must have a "[m]inimum Ohio State GPA of 3.10 or better" (*Admission to Major Program and Specialization Criteria* 2024). Because Business Management is offered as a bachelor's degree at all regional campuses, a student who marginally qualifies for a campus change but has a strong preference to pursue a business-related major may choose not to change campuses.

As seen in Table 2.1, around 90% of OSU undergraduate students are enrolled at the OSU-Main Campus, but the regional campuses have a much stronger emphasis on enrolling local students (*Regional Campus Vision and Goals* 2017). Regional campuses were founded in the 1950s to serve students living in each campus's home county and adjacent counties, but regional campus enrollment began expanding in the early 2000s in response to increased admissions competition at

[2]The federal government considers 12 credit hours the minimum to be considered a full-time student; OSU recommends taking at least 15 credit hours if a student's goal is to graduate in four years (*Finish in Four* 2024). A full-time regional campus student would therefore qualify to apply for a campus change and start attending the OSU-Main Campus at the start of her second year of enrollment at the earliest.

the OSU-Main Campus. Virtually all students at regional campuses are from in-state, while around 1/4 of undergraduate students at the OSU-Main Campus are from out-of-state.

Table 2.1: Undergraduate Enrollment at Ohio State University

	Main Campus	Lima	Mansfield	Marion	Newark
Total Enrollment	45,728	739	843	884	2,422
In-State Enrollment	33,993	733	824	878	2,412
In-State Pct.	74.3%	99.2%	97.7%	99.3%	99.6%
First-time, Full-time enrollment					
Fall 2023	7,983	260	843	884	2,422
Fall 2022	7,960	273	827	900	2,263
Fall 2021	8,350	279	940	1,047	2,727
Fall 2020	8,602	345	1,011	1,157	2,870
Fall 2019	7,630	360	1,075	1,262	2,939

Data from the Ohio State University Analysis and Reporting;
https://web.archive.org/web/20240221051625/http://oesar.osu.edu/student_enrollment.aspx

Differences in enrollment characteristics reflect the fact that OSU has different strategic goals for the main and regional campuses. According to the *Accelerating Excellence, Access and Service: Strategic Enrollment Plan for The Ohio State University, 2022-2024* (2021), OSU has dual goals of expanding enrollment while improving the quality of incoming classes at the OSU-Main Campus. Admissions are selective; the Fall 2022 admissions rate was 53%. In contrast, the administration is focused only on expanding enrollment at regional campuses, especially to local and under-represented students (*Autumn 2023: Enrollment Report* 2023). There are no specific goals to increase the number of campus change students. Also, regional campuses are not intended to be selective: students who have never attended a university but have completed a high school degree or equivalent can be admitted to a regional campus (*Undergraduate Admissions: Regional Campuses* 2023).

Finally, a consequence of differing institutional missions is a perceived difference in prestige of attending the OSU-Main Campus vs. an OSU regional campus. OSU was ranked no. 43 among national universities by the *U.S. News Best Colleges* (2023), and is sometimes cited as a “Public Ivy”. On the other hand, the OSU regional campuses are unranked.

Affordability. I summarize information on costs and financial aid across OSU campuses in

Table 2.2. In-state tuition for the 2022-2023 academic year was $12,485 at the OSU-Main Campus and $8,944 at the regional campuses, according to the NCES College Navigator. That is, attending a regional campus costs around 30% less relative to the OSU-Main Campus. Moreover, the average net price of the OSU-Main Campus is between $4,000-$6,000 more than at the regional campuses.[3] Variance in net price at regional campuses appears to be driven by the fact that residential housing is only available at the Mansfield and Newark campuses.

The net price for the OSU-Main Campus is *lower* at the regional campuses for the two lowest income brackets ($0-$48,000), but higher at all other income levels. In spite of this, the OSU-Main Campus has the lowest proportion of students receiving federal Pell Grants, suggesting that in-state students at very low income brackets make enrollment choices do not fully base their enrollment decisions on net price. This likely reflects that there are opportunity costs associated with enrolling at the OSU-Main Campus not captured by sticker or not price, which I discuss more later.

The amount and composition of financial aid also differs between campuses. Roughly the same proportion of students receive some form of financial aid across all campuses, but the average amount received is lower at the regional campuses (between $5,147-$7,083) than at the main campus ($10,993). Much of this difference is driven by the fact that students at the main campus receive around $7,000 more in institutional aid compared to students at the regional campuses. However, regional campus students are more likely to receive any federal grant or scholarship, including Pell Grants.

The opportunity cost of enrolling at the OSU-Main Campus also differs from that of regional campuses. Regional campuses do not require students to live on campus, and students who start at a regional campus but transfer to the main campus are not required to live on-campus if they graduated from high school more than two years ago (*Campus change: FAQs* 2023). Since most students wouldn't qualify for a campus change until completing 2-3 semesters, some campus change students may *never* be required to live on-campus. On the other hand, the OSU-Main Campus requires most students to live on campus for at least two years (*Undergraduate Admissions: FAQs*

[3]Net price can be thought of as the "actual" price a student should expect to pay to attend a given institution, inclusive of sources of funding such as financial aid and costs such as room and board.

Table 2.2: First-Time, Full-Time Undergraduate Financial Aid and Tuition (2021-2022) at Ohio State University

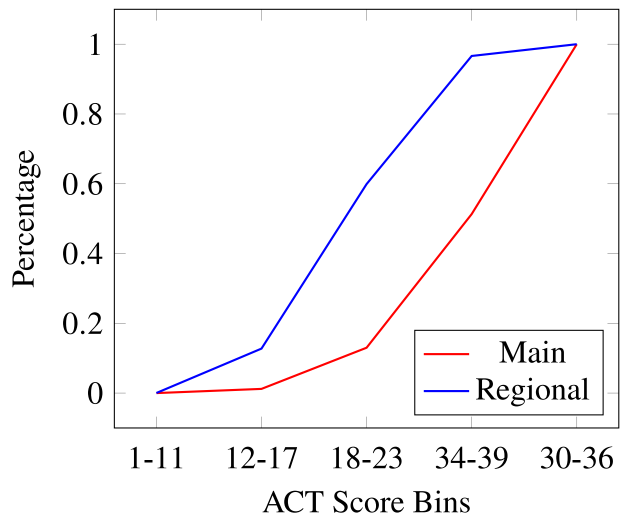
	Main	Lima	Mansfield	Marion	Newark
Avg. Financial Aid					
Num. Awarded	7,007	231	302	304	1,042
Pct. Awarded	83%	86%	84%	82%	80%
Avg. Amt Awarded	$10,993	$5,766	$7,083	$5,147	$6,718
Grants & Scholarships					
Federal, Any					
Num. Awarded	3,372	153	230	158	789
Pct. Awarded	40%	57%	64%	42%	61%
Avg. Amt Awarded	$4,094	$4,474	$5,004	$4,346	$4,852
Federal, Pell Grants					
Num. Awarded	1,527	88	140	83	475
Pct. Awarded	18%	33%	39%	22%	37%
Avg. Amt Awarded	$5,012	$4,474	$5,084	$4,871	$4,852
State					
Num. Awarded	1,013	67	111	67	371
Pct. Awarded	12%	25%	31%	18%	29%
Avg. Amt Awarded	$2,706	$724	$1,232	$704	$371
Institutional					
Num. Awarded	5,576	185	240	274	817
Pct. Awarded	66%	69%	67%	74%	63%
Avg. Amt Awarded	$9,313	$2,528	$2,486	$2,675	$2,755
Loans					
Federal					
Num. Awarded	29,666	105	202	110	540
Pct. Awarded	35%	39%	56%	30%	42%
Avg. Amt Awarded	$5,250	$4,910	$5,343	$4,825	$4,755
Other					
Num. Awarded	445	8	24	6	37
Pct. Awarded	5%	3%	7%	2%	3%
Avg. Amt Awarded	$18,192	$7,173	$8,652	$11,358	$10,194
In-State Tuition	$12,485	$8,944	$8,944	$8,944	$8,944
Net Price					
Average	$19,582	$13,086	$15,908	$13,377	$15,024
By Income Level					
$0-$30,000	$6,956	$9,465	$12,615	$9,109	$11,852
$30,001-$48,000	$8,402	$9,790	$13,529	$9,738	$12,608
$48,001-$75,000	$13,620	$11,823	$16,328	$11,158	$14,455
$75,001-$110,000	$22,528	$15,421	$19,664	$15,511	$18,098
$110,001+	$26,186	$16,878	$20,588	$16,323	$19,005

Data from NCES - College Navigator.

for parents and families 2023). Being untied to a specific residence may also free up students at the regional campus to continue living at home and/or more easily pursue part-time employment.

Profile of student bodies. First-time students who start at the OSU-Main Campus are more academically prepared based on observable characteristics. The average ACT score for the incoming class of 2021 at the OSU-Main Campus was 28.6, and between 20.6-22.8 at the regional campuses. Moreover, as seen in Figure 2.1, the cumulative distribution of students in each ACT score bin at the OSU-Main Campus stochastically dominates the ACT distribution at the regional campuses.

Figure 2.1: Cumulative Distribution of ACT Scores at Main vs. Regional Campuses



All OSU students appear to value the prestige of attending the OSU-Main Campus. The percentage of first-time students who enrolled at a regional campus but reported that the OSU-Main Campus was their first choice has risen from 38% in 2017 to 52% as of 2021 (*New First Year Students by Campus Choice* 2022).

Student outcomes. For the Fall 2016 cohort, the six-year graduation rate for full-time, first-time students who started at the OSU-Main Campus is 88%. The transfer-out rate is 7%. In contrast, the six-year graduation rate is between 16%-34% at regional campuses, and the transfer-out rates are between 45%-67%. See Table 2.3 for institution-level details. Note that graduation rates shown only count students who were awarded a degree at their home institution.

Student outcomes are consistent with the fact that the OSU-Main Campus is selective and that

Table 2.3: Full-Time Undergraduate Student Outcomes (Fall 2016 Cohort) at Ohio State University

Campus	Graduation Rate	Transfer-Out Rate
Main Campus (Columbus)	88%	7%
Lima	21%	56%
Mansfield	16%	67%
Marion	17%	62%
Newark	34%	45%

Data from the NCES College Navigator. Note that in the NCES data, changing from a regional campus to the main campus is counted as a transfer-out, although OSU considers this an internal campus change.

regional campuses are meant to facilitate transfer into the OSU-Main Campus. It also implies that students who start at the regional campus have a strong preference for transfer. As a point of reference, the average transfer-out rate at U.S. community colleges was 31.5% for the Fall 2010 cohort.

Regional campuses vs. community colleges. To illustrate the difference between regional campuses and community colleges, I show information on four community colleges that OSU has articulation agreements with in Table 2.4.[4] These community colleges are the closest external comparison to regional campuses because they have reduced transfer frictions into the OSU-Main Campus compared to other community colleges. Moreover, all of the colleges except Columbus State Community College share buildings and resources with the regional campus in the same locale (*Regional Campus Vision and Goals* 2017).

That said, there are important differences between the comparison community colleges and regional campuses. First, the comparison community colleges confer only associate's degrees. Second, the community colleges enroll proportionately more part-time students. Third, the OSU regional campuses are more expensive than the community colleges. Fourth, regional campuses enroll fewer students. Finally, transferring into the OSU-Main Campus is a higher burden for students at the community college, who must send an application and ensure that classes taken at their original institution will count for credit at the OSU-Main Campus.

While the regional campuses and comparison community colleges have similar graduation

[4]Articulation agreements are intended to reduce frictions in transferring credits between institutions (*Pathway Agreements* 2023).

Table 2.4: Information on Comparison Community Colleges

	Central Ohio Technical University	Columbus State Community College	North Central State College	Rhodes State College
Location	Newark	Columbus	Mansfield	Lima
Total Enrollment (Fall 2022)	2,614	25,129	2,439	3,533
In-State Tuition, 2022-23	$5,016	$5,188	$4,624	$4,532
Avg. Net Price, 2021-22	$10,103	$6,964	$3,885	$8,991
Outcomes, Fall 2019 Cohort				
4-Year Graduation Rate	30%	26%	27%	27%
Transfer-Out Rate	17%	16%	13%	18%

Data from the NCES College Navigator.

rates, the regional campuses have much higher transfer-out rates and the community colleges have higher rates of non-completion. Also, compared to the average community college, the transfer-out rate at the comparison group of community colleges is about half of the historical national average (Shapiro et al. 2017). This suggests that the comparison community colleges are not only systematically different from regional campuses, but they are also different from other community colleges. This is suggestive evidence that regional campuses have spillover effects to similarly located community colleges.

Table 2.5: Number of Campus Change and External Transfer Students

Year	Campus Change	External Transfers
Fall 2023	1,124	1,827
Fall 2022	1,161	1,857
Fall 2021	1,286	2,070
Fall 2020	1,412	2,158
Fall 2019	1,372	2,415
Fall 2018	1,422	2,388

Data from *Autumn 2023: Enrollment Report* (2023).

Finally, I show the profile of *all* incoming external transfer students at the OSU-Main Campus. This includes students transferring from *any* community college as well as those transferring from other four-year institutions. Looking at Table 2.5, the number of campus change students (who transfer from a regional campus) is consistently smaller than the number of students transferring in from other institutions.

Table 2.6: Academic Profile of External Transfer Students

Semester	Avg. Credits Transferred	Avg. GPA
Fall 2022	43	3.23
Fall 2021	41	3.22
Fall 2020	45	3.18
Fall 2019	46	3.17
Fall 2018	47	3.14
Fall 2017	48	3.19
Fall 2016	54	3.18
Fall 2015	54	3.18

Data retrieved from http://undergrad.osu.edu/apply/transfer/admission-criteria using the Internet Archive.

OSU recommends that external students only apply to transfer into the OSU-Main Campus if they have a GPA of at least 2.5 (*Transfer applicants: Admission criteria* 2024). In practice, the average external transfer student has a GPA of at least 3.14 and enters with 41-54 credit hours. Assuming that the majority of campus change students enter the OSU-Main Campus the semester after they complete 30 credit hours at their home campus, external transfer students come into the OSU-Main Campus with 1-2 more semesters of classes than campus change students.

As OSU does not, to my knowledge, have publicly posted information on the profile of the average campus change student, I cannot directly compare between the two. However, it is reasonably likely that the marginally admitted campus change student is almost certainly academically weaker than the marginally admitted external transfer student, since OSU recommends a minimum GPA of 2.5 for external transfer students, but guarantees regional campus students can transfer in with a minimum GPA of 2.0.

Summary. I briefly recap the institutional features that a theoretical framework of college admissions should capture. First, both prices and the intrinsic prestige of attending a given institution should be decreasing from the main campus to the regional campus to the community college. Second, a student's wealth endowment should be a major determinant of her preferences over institutions. Students who are income constrained may prefer a more affordable institution, regardless of how academically well-prepared they are. Third, students at the regional campus who apply with an intent to transfer into the main campus should tend to maintain their preference. Finally, transfer admissions should prioritize regional campus students over the community college students whenever students from both institutions apply.

2.3 Theoretical Model

2.3.1 Unaffiliated Environment

I first consider an environment with 2 institutions, a main university M and a community college C. Call this the "unaffiliated" environment (as opposed to the second environment that I will consider, in which M is affiliated with a regional campus). In the unaffiliated environment, the set of institutions is therefore $J_U = \{M, C\}$. Denote an arbitrary institution $j \in J_U$. Each institution has capacity for first-time, full-time students denoted $\kappa_j \in (0, 1)$ for all $j \in J_U$. Furthermore, let $\kappa_M + \kappa_C < 1$. The main university M also has capacity for transfer students $\kappa_T \in (0, \kappa_C)$, to be discussed in more detail later.

There is a unit mass of in-state students who demand higher education and differ across two dimensions, academic preparedness $a \in [0, 1]$ and wealth $w \in [0, 1]$. Both a and w are independently drawn from the distribution $Uniform[0, 1]$. That is, an arbitrary student i is described by her type $(a_i, w_i) \in [0, 1]^2$. The distribution over type is common knowledge. If student i applies to a given institution, then she directly reveals her academic preparedness a_i to that institution. Also, since total capacity $\kappa_M + \kappa_C < 1$, there are always some students who are unable to enroll at any institution due to capacity constraints.

Institutions differ in price and value, which together determine a student's *initial* preference ranking. Before applying, students do not know their personal value of attending a given institution, so all students assign each institution the same expected value (say, based on the prestige of the institution or average outcome of its graduates). They also face the same expected net price at each institution.[5] Fix the price and value of attending the community college C to 0 and denote the price and value of attending the main university M as $p_M \in (0, 1]$ and $v_M \in (0, 1]$ respectively.[6]

Students get utility from both wealth and the value of the institution attended; the outside option of getting no higher education is strictly negative. The utility a student i yields from attending C is

$$u(w_i, C) = ln(1 + w_i), \tag{2.1}$$

and the utility from attending M is

$$u(w_i, M) = ln(1 + w_i - p_M) + v_M. \tag{2.2}$$

[5]In practice, lower income students generally receive more financial aid at universities than at community colleges. However, Table 2.4 shows that the average net price at community colleges is lower than that of the OSU-Main Campus. Also, many low income, highly academically well-prepared students do not apply to selective universities in spite of the fact that those who do apply have similar enrollment behavior and academic performance as their higher income peers (Hoxby and Avery 2013). This is likely at least partially due to information gaps, whether on the difference between sticker and net prices, or a lack of knowledge on admissions probability (Cortes and Lincove 2018).

[6]Later, when considering the social planner's problem, I will assume that the social planner prefers for more academically prepared students to attend the main university M; that is, the value of attending M is not constant, but increasing in a_i. But here, I assume that a student does not know how valuable attending M is for herself personally. She knows only that attending M is more prestigious than her other options.

Under two conditions on (p_M, v_M), students' preferences can be characterized by a cut-off rule on their wealth endowments. The first condition is $ln(1 - p_M) + v_M < 0$, which implies a student with the lowest possible endowment $w_i = 0$ prefers the community college. The second condition is $ln(2 - p_M) + v_M > ln(2)$, which implies a student with the highest possible endowment $w_i = 1$ prefers the main university. Given that $u(w_i, C)$ and $u(w_i, M)$ are strictly increasing in w_i, under these two conditions, Equation 2.2 crosses Equation 2.1 exactly once from below. So wealth endowment determines students' initial preferences over higher education: students with sufficiently large enough wealth prefer the main university. I summarize the two conditions on the unaffiliated wealth cut-off as follows:

Condition 1. *(p_M, v_M) together satisfy $ln(1 - p_M) + v_M < 0$ and $ln(2 - p_M) + v_M > ln(2)$.*

I now characterize student preferences in the unaffiliated environment.

Lemma 1. *Let Condition 1 on the unaffiliated wealth cut-off hold. There exists $\hat{w}$ such that i initially prefers the main university M if and only if $w_i \geq \hat{w}$; furthermore,*

$$\hat{w} = \frac{1 - e^{v_M}(1 - p_M)}{e^{v_M} - 1} \in (0, 1). \tag{2.3}$$

Proof. $\hat{w}$ is determined by setting $u(\hat{w}, C) = u(\hat{w}, M)$ from Equations 2.1 and 2.2, then solving for $\hat{w}$:

$$ln(1 + \hat{w}) = ln(1 + \hat{w} - p_M) + v_M$$
$$\Rightarrow \hat{w} = \frac{1 - e^{v_M}(1 - p_M)}{e^{v_M} - 1}.$$

To see that $\hat{w} \in (0, 1)$, first suppose not and that $\hat{w} \leq 0$. Since $e^{v_M} - 1 > 0$ for $v_M \in (0, 1]$, it must be the case that

$$1 - e^{v_M}(1 - p_M) \leq 0$$
$$\Rightarrow ln(\frac{1}{1 - p_M}) \leq v_M$$
$$\Rightarrow 0 \leq ln(1 - p_M) + v_M,$$

contradicting that $ln(1 - p_M) + v_M < 0$ from Condition 1 on the unaffiliated wealth conditions. Now suppose that $\hat{w} \geq 1$. Then,

$$1 - e^{v_M}(1 - p) \geq e^{v_M} - 1$$
$$\Rightarrow 2 \geq e^{v_M}(2 - p_M)$$
$$\Rightarrow ln(2) \geq ln(2 - p_M) + v_M,$$

contradicting that $ln(2 - p_M) + v_M > ln(2)$ from Condition 1 on the unaffiliated wealth conditions. So $\hat{w} \in (0, 1)$. □

As expected, an increase in p_M increases the unaffiliated wealth cut-off $\hat{w}$ (fewer prefer the main university M), while an increase in v_M decreases it (more prefer M); see Corollary 6 in Appendix 2A for details.

There are two time periods, $t \in \{1, 2\}$. Each time period covers two years of higher education; this is to capture the fact that transfer often occurs around two years after first-time enrollment. At the beginning of $t = 1$, the first-time admissions process occurs, and then general education happens. At the beginning of $t = 2$, transfer admissions occurs.[7]

At the beginning of $t = 1$, first-time admissions happens over two steps:

1. Students who prefer the main university M apply there. M is selective, and the best applicants based on academic preparedness are accepted up to M's chosen cut-off level of academic preparedness, $a_M \in (0, 1)$. Following Azevedo and Leshno (2016), this cut-off rule constitutes a stable matching.

2. Students who prefer the community college C or were previously rejected at the main university M apply to C. As C is not selective, the best applicants based on academic preparedness are accepted up to capacity.

The main university M cares both about enrollment and selectivity; these incentives naturally compete with each other. Denote the quantity of students above M's cut-off level of academic

[7] I focus on enrollment effects, so I do not include a third period during which final academic outcomes are realized.

preparedness a_M as $q_M = 1 - a_M$.[8] Let $\pi_M \in \mathbb{R}_{++}$ be the expected and constant "profit" of enrolling a student. Let $\rho_M(q_i) : [0,1] \to \mathbb{R}_+$ be a continuous and differentiable function that captures M's prestige concern, which satisfies $\rho_M(0) = 0, \rho_M(1) > \pi$, $\frac{\partial}{\partial q_M}\rho_M(q_M) > 0$, and $\frac{\partial^2}{\partial q_M^2}\rho_M(q_M) > 0$. Finally, denote the inverse of the prestige concern function as $\rho_M^{-1}()$.

The main university M's enrollment decision is:

$$\max_{q_M} \quad (1-\hat{w}) \int_0^{q_M} (\pi_M - \rho_M(q_M)) q_M dq_M \tag{2.4}$$

subject to the capacity constraint $(1-\hat{w})q_M \leq \kappa_M$. For an institution $j \in J_U$, I denote a_J^* the optimal cut-off level of academic preparedness for j in the unaffiliated environment. Also, denote q_J^* as the quantity of students with academic preparedness at least a_J^* in the unaffiliated environment.

A few points about the prestige concern function $\rho_M()$ merit discussion. At the highest level of selectivity, there is no prestige cost induced ($\rho_M(0) = 0$), so the main university is always willing to enroll at least some students. However, the marginal prestige cost of expanding enrollment is always increasing in level of enrollment (e.g., dropping the cut-off from 0.5 to 0.45 incurs a larger prestige cost than dropping it from 0.9 to 0.85). Because of the conditions I impose on the extreme values of $\rho_M()$, if capacity constraints don't bind, then the optimal level of enrollment has a straightforward marginal cost vs. marginal benefit interpretation: enroll until $\pi_M = \rho_M(q_M^*)$.

If the capacity constraint binds, then the main university M would like to enroll more students, but cannot. M therefore enrolls the best applicants up to capacity. However, going forth, I restrict attention to the case where first-time capacity constraints at M do not bind. This ensures that M is *equally selective* over in-state students in both environments I will consider, so any differences in outcomes between the unaffiliated and affiliated environments cannot be attributed to institutional changes at M.[9] This condition also aligns with OSU's strategic enrollment plan as described in *Accelerating Excellence, Access and Service: Strategic Enrollment Plan for The Ohio State*

[8] Note that q_M isn't the quantity of students that the main university M ultimately enrolls, since only students with wealth $w_i \geq \hat{w}$ will apply to M.

[9] Recall that I look at in-state students. The university may simultaneously be expanding enrollment of out-of-state students to fill seats. Bound, Braga, Khanna, and Turner (2020) show universities enroll more international students when facing funding cuts, while Arcidiacono, Kinsler, and Ransom (2023) show that universities are more selective over out-of-state students. That is, in-state and out-of-state enrollment should tend to move in opposite directions, as public universities treat them as imperfect substitutes.

University, 2022-2024 (2021): the university system prioritizes selectivity in first-time enrollment at the main campus. I summarize the condition on selectivity at M as follows:

Condition 2. *The main university M's first-time admissions problem is not bound by its capacity constraint at the optimum.*

Under Conditions 1 and 2 as well as an additional condition on capacity at the community college κ_C, I now characterize first-time enrollment in the unaffiliated environment.

Proposition 5. *Let Condition 1 on the unaffiliated wealth cut-off and Condition 2 on selectivity at the main university M hold. Also, let $\kappa_C > \hat{w} \cdot \rho_M^{-1}(\pi_M)$. Then the first-time cut-offs are:*

$$a_M^* = 1 - \rho_M^{-1}(\pi_M)$$
$$a_C^* = \hat{w} + (1 - \hat{w})a_M^* - \kappa_C.$$

Proof. If capacity does not bind, then the optimal cut-off at the main university M is determined by the optimization problem defined in Equation 2.4, which by the Fundamental Theorem of Calculus and the assumption that $\rho_M(0) = 0$, implies that

$$\pi_M - \rho_M(q_M^*) = 0 \iff q_M^* = \rho_M^{-1}(\pi_M) \iff a_M^* = 1 - \rho_M^{-1}(\pi_M).$$

Then, since the community college C fills up enrollment to capacity from the strongest applicants,

$$\kappa_C = \hat{w}(1 - a_M^*) + (a_M^* - a_C^*),$$

and solving for a_C^*, the cut-off at C is $a_C^* = \hat{w} + (1 - \hat{w})a_M^* - \kappa_C$. □

I impose a condition on capacity at the community college C, $\kappa_C > \hat{w} \cdot \rho_M^{-1}(\pi_M) = \hat{w}(1 - a_M^*)$, to ensure that admissions is always strictly less selective at C than at the main university M, as this more accurately reflects the actual landscape of college admissions. Students with wealth $w \in [0, \hat{w}]$ and $a \in [a_M^*, 1]$ who are admissible at M always enroll at C as a first-time student due to personal preference. The capacity condition ensures that C serves not only these students, but also some students who were rejected from M. So, the cut-off at C is lower than at M: $a_C^* < a_M^*$.

As intuitively expected, an increase in the unaffiliated wealth-cut-off $\hat{w}$ decreases the measure of students who most prefer and will apply to the main university M and increases selectivity at the community college C. This is because more students with high levels of academic preparedness choose not to apply to M and enroll at C instead. Also, if C increases capacity κ_C, then selectivity decreases, because C will admit the students who were previously just barely inadmissible.

After admissions, general education occurs. For expositional purposes, this is the first two years of higher education, during which students at the community college C work on an associate's degree and/or completing courses with the intention to transfer, and students at the university main campus work on general requirements. At the end of $t = 1$, a student at C realizes one of three outcomes: she either (1) exits without a degree, (2) exits with a degree, (3) applies to transfer to the main university M.

I allow for students at the community college C to potentially change their initial preferences over institutions during $t = 1$, due to being "exposed" to higher education. Following Rouse (1995), students with wealth $w < \hat{w}$ and initially preferred C may become "democratized" and apply to transfer into the main university M. Of the students who initially preferred the community college C, denote $\delta \in (0, 1)$ the fraction of students who complete their first two years of education, are democratized, and apply to transfer to M.

All students who initially preferred the main university M but could not enroll as a first-time student maintain their preference. This diverges from Rouse (1995)'s framework; she allows for students with wealth $w \geq \hat{w}$ and initially preferred the main university M to potentially become "diverted" away from applying to transfer to M after being exposed to the community college. However, I include in this framework *only* the democratization effect, to concentrate on how students are able to gain access to the main university M through transfer admissions. Of course, this is a strong assumption, but I interpret this as a scenario in which all failures to transfer are institutionally driven (e.g., lacking capacity and/or M being selective). Also, this will make some comparisons cleaner when comparing the unaffiliated and affiliated environments.

At the beginning of $t = 2$, students at the community college C may apply to transfer into the main university M. I assume that M does not have a prestige concern over transfer admissions, because four-year universities' incentive structures almost exclusively focus on the performance of their first-time, full-time students. External measures of prestige such as the *U.S. News Best Colleges* (2023) ranking only take into account the outcomes of students who *started* at a given institution. Also, four-year institutions are not required to submit data on graduation rates of transfer-in students to the federal Integrated Postsecondary Education Data System, which is used to generate publicly available information on institutional performance for the NCES College Navigator.[10] As such, the incentive to admit transfer students shifts away from selectivity and towards enrollment. To capture this, I assume that M admits the best transfer students based off of academic preparedness up to transfer capacity $\kappa_T \in (0, \kappa_C)$.

If transfer capacity κ_T is too small, then students who initially preferred the main university M cannot transfer. They are completely crowded out by highly academically prepared, democratized students.

Lemma 2. *If $\kappa_T \leq \hat{w} \cdot \delta(1 - a_M^*)$, then first-time community college students who prefer the main university M cannot enroll at M as a transfer student.*

Proof. If $\kappa_T \leq \hat{w} \cdot \delta(1 - a_M^*)$, then the main university M's optimal cut-off for transfer admissions satisfies $a_T^* > a_M^*$. Because all students who were previously rejected from M have an academic preparedness level strictly less than a_M, they are not admitted. All capacity is filled by democratized students who initially preferred the community college and have high levels of academic preparedness. □

However, comparing the number of external transfers from Table 2.5 to first-time enrollment in Table 2.1, transfer capacity seems reasonably high; the ratio of transfers to first-time students in Fall 2022 was 0.233. In practice, the OSU-Main Campus is unlikely to be in the case such that

[10] To be more specific, institutions are only required to submit information on graduation rates of students who were first-time, full-time students at that institution. Transfer students who complete a degree are included in data on the number of degrees conferred, and these data do not distinguish between degrees completed by a student who started at that institution or elsewhere.

transfer capacity is so scarce that it completely restricts entry from students who initially preferred the main university M.

I summarize the two conditions on the unaffiliated capacity constraints as follows:

Condition 3. *$\kappa_C > \hat{w}(1 - a_M^*)$ and $\kappa_T > \hat{w} \cdot \delta(1 - a_M^*)$.*

Denote a_T^* as the optimal cut-off for transfer applicants in the unaffiliated environment: community college student i who applies to transfer is admitted if and only if $a_i \geq a_T^*$. Transfer admissions in the unaffiliated environment is characterized as follows:

Proposition 6. *Let Condition 1 on the unaffiliated wealth cut-off, Condition 2 on selectivity at the main university M, and Condition 3 on the unaffiliated capacity constraints hold. Then the transfer cut-off is*

$$a_T^* = \frac{\hat{w} \cdot \delta + (1 - \hat{w})a_M^* - \kappa_T}{\hat{w} \cdot \delta + (1 - \hat{w})}.$$

Proof. Under $\kappa_T > \hat{w} \cdot \delta(1 - a_M^*)$, both democratized students who initially preferred the community college C and non-diverted students who continue to prefer the main university M will successfully transfer. Hence,

$$\kappa_T = (1 - \hat{w})(a_M^* - a_T^*) + \hat{w} \cdot \delta(1 - a_T^*),$$

and rearranging to solve for a_T^* gives the result. □

For comparative statics, see Corollary 7 in Appendix 2A. Under Condition 3, transfer admissions are less selective than first-time admissions ($a_T^* < a_M^*$). A positive measure of students who initially preferred the main university M can gain access as a transfer student. Therefore, in the unaffiliated environment, there are three academic pathways for students who successfully enroll somewhere as a first-time student: (1) start at the main university and stay there, (2) start at the community college and stay there, or (3) start at the community college and transfer to the main university.

2.3.2 Affiliated Environment

Now consider an environment with 3 institutions. In addition to the main university M and community college C, there is now a non-selective regional campus R with capacity $\kappa_R \in (0,1)$ that is affiliated with M. I discuss first-time enrollment at R and changes in transfer admissions in more detail after showing how adding R affects student preferences. Call this the "affiliated" environment, and the set of institutions is now $J_A = J_U \cup \{R\} \equiv \{M, R, C\}$. With some light abuse of notation, I continue to use j to refer to an arbitrary institution. Capacity constraints at M and C are the same as before, and $\kappa_M + \kappa_R + \kappa_C < 1$.

Denote the price and value of attending the regional campus as $p_R \in (0, p_M)$ and $v_R \in (0, v_M)$ respectively. The utility a student i gets from attending R is

$$u(w_i, R) = ln(1 + w_i - p_R) + v_R. \tag{2.5}$$

Similar to the unaffiliated environment, under some conditions on prices and values at the main university (p_M, v_M) and the regional campus (p_R, v_R), students' preferences can be characterized by a pair of cut-off rules on their wealth endowments. I first derive the "upper" affiliated wealth cut-off that determines if a student prefers the main university M over the regional campus R or vice versa.

Lemma 3. *Let $ln(1 - p_M) + v_M < ln(1 - p_R) + v_R$ and $ln(2 - p_M) + v_M > ln(2 - p_R) + v_R$. There exists $\bar{w}$ such that i initially prefers the main university M over the regional campus R if and only if $w_i \geq \bar{w}$; furthermore,*

$$\bar{w} = \frac{e^{v_R}(1 - p_R) - e^{v_M}(1 - p_M)}{e^{v_M} - e^{v_R}} \in (0, 1). \tag{2.6}$$

Proof. The proof is similar to that of Lemma 1, so I omit some set-up. The upper affiliated wealth cut-off $\bar{w}$ is determined by setting $u(\bar{w}, R) = u(\bar{w}, M)$ and solving for $\bar{w}$,

$$\begin{aligned} ln(1 + w_i - p_R) + v_R &= ln(1 + w_i - p_M) + v_M \\ \Rightarrow \bar{w} &= \frac{e^{v_R}(1 - p_R) - e^{v_M}(1 - p_M)}{e^{v_M} - e^{v_R}}. \end{aligned}$$

To see that $\bar{w} \in (0,1)$, first suppose not and that $\bar{w} \leq 0$. Then there will be a contradiction on $ln(1-p_M)+v_M < ln(1-p_R)+v_R$. Next, suppose not and $\bar{w} \geq 1$, then there will be a contradiction on $ln(2-p_M)+v_M > ln(2-p_R)+v_R$. □

Similar to Lemma 1, the two conditions $ln(1-p_M)+v_M < ln(1-p_R)+v_R$ and $ln(2-p_M)+v_M > ln(2-p_R)+v_R$ imply that Equation 2.2 (the utility of attending the main university M) crosses Equation 2.5 (the utility of attending the regional campus R) from below exactly once. An increase in p_M or v_R increases the upper affiliated wealth cut-off $\bar{w}$ (fewer prefer M), while increase in p_R or v_M decreases $\bar{w}$ (more prefer M); see Corollary 8 in Appendix 2A for details.

Next, I derive the "lower" affiliated wealth cut-off that determines if a student prefers the the regional campus R over the community college C or vice versa.

Lemma 4. *Let $ln(1-p_R)+v_R < 0$ and $ln(2-p_R)+v_R > ln(2)$. There exists $\underline{w}$ such that i initially prefers the regional campus R over the community college C if and only if $w_i \geq \underline{w}$; furthermore,*

$$\underline{w} = \frac{1-e^{v_R}(1-p_R)}{e^{v_R}-1} \in (0,1). \tag{2.7}$$

Proof. Structurally, the proof is identical to Lemma 1, so I omit the details. □

Comparative statics are analogous to Corollary 6 in Appendix 2A, so are omitted. An increase in the price of the regional campus p_R increases the lower affiliated wealth cut-off $\underline{w}$ (fewer prefer the regional campus R), while an increase in v_R decreases it (more prefer R).

Finally, an additional condition is needed to guarantee that the cut-offs upper and lower affiliated wealth-cut-offs, $\bar{w}$ and $\underline{w}$, are intuitively ordered,

$$(e^{v_M+v_R}-e^{v_M})p_M > (e^{v_M+v_R}-e^{v_R})p_R.$$

This condition is similar to a supermodularity or convexity condition. It must be the case that the increased price of attending the main university p_M relative to that of attending the regional campus p_R is "justifiable" in that the value of attending the main university v_M is also "significantly" higher than the value of attending the regional campus v_R. For example, if the values of attending each institution v_R and v_M are very close, but the regional campus is significantly cheaper than the main university, then the wealth cut-offs will not be well-behaved. When this condition is satisfied, along with the previous restrictions, students with the highest wealth endowments prefer M, those with moderate endowments prefer R, and those with low endowments prefer C.

I summarize the conditions on the affiliated wealth cut-offs as follows:

Condition 4. *(p_R, v_R) and (p_M, v_M) together satisfy:*

1. $ln(1 - p_M) + v_M < ln(1 - p_R) + v_R$
2. $ln(2 - p_M) + v_M > ln(2 - p_R) + v_R$
3. $ln(1 - p_R) + v_R < 0$
4. $ln(2 - p_R) + v_R > ln(2)$
5. $(e^{v_M + v_R} - e^{v_M})p_M > (e^{v_M + v_R} - e^{v_R})p_R$

When I refer to Condition 1 on the unaffiliated wealth cut-off and Condition 4 on the affiliated wealth cut-offs together, I call them conditions on "wealth cut-offs" without specifying the environment.

Next, I characterize student preferences in the affiliated environment and compare them to preferences in the unaffiliated environment. Because the regional campus R is introduced as a new alternative, fewer students rank both the community college C and the main university M as their top choice.

Proposition 7. *Let Conditions 1 and 4 on the wealth cut-offs hold. Then $\bar{w} > \hat{w} > \underline{w}$.*

Proof. First, for a contradiction, suppose that $\bar{w} \leq \hat{w}$. Then

$$\frac{e^{v_R}(1 - p_R) - e^{v_M}(1 - p_M)}{e^{v_M} - e^{v_R}} \leq \frac{1 - e^{v_M}(1 - p_M)}{e^{v_M} - 1}$$

$$\Rightarrow e^{v_M + v_R}(1 - p_R) + e^{v_R}p_R - e^{v_M}p_M \leq e^{v_M + v_R}(1 - p_M)$$

$$\Rightarrow e^{v_M + v_R}(p_M - p_R) \leq e^{v_M}p_M - e^{v_R}p_R$$

contradicting the condition $(e^{v_M+v_R} - e^{v_M})p_M > (e^{v_M+v_R} - e^{v_R})p_R$.

Then for another contradiction, suppose that $\hat{w} \leq \underline{w}$. Then

$$\frac{1 - e^{v_M}(1 - p_M)}{e^{v_M} - 1} \leq \frac{1 - e^{v_R}(1 - p_R)}{e^{v_R} - 1}$$

$$\Rightarrow -e^{v_M+v_R}(1 - p_M) - e^{v_M}p_M \leq -e^{v_M+v_R}(1 - p_R) - e^{v_R}p_R$$

$$\Rightarrow e^{v_M+v_R}(p_M - p_R) \leq e^{v_M}p_M - e^{v_R}p_R$$

again contradicting the condition $(e^{v_M+v_R} - e^{v_M})p_M > (e^{v_M+v_R} - e^{v_R})p_R$.

Finally, to close the loop, suppose for a contradiction that $\bar{w} \leq \underline{w}$. Then

$$\frac{e^{v_R}(1 - p_R) - e^{v_M}(1 - p_M)}{e^{v_M} - e^{v_R}} \leq \frac{1 - e^{v_R}(1 - p_R)}{e^{v_R} - 1}$$

$$\Rightarrow e^{v_M+v_R}(p_M - p_R) \leq e^{v_M}p_M - e^{v_R}p_R,$$

once again contradicting the condition $(e^{v_M+v_R} - e^{v_M})p_M > (e^{v_M+v_R} - e^{v_R})p_R$. □

The next corollary immediately follows because fewer students initially prefer M.

Corollary 3. *Let the conditions from Proposition 7 hold. The main university M enrolls weakly fewer students and becomes weakly less selective after a regional campus is added.*

Under Condition 2 on selectivity at the main university M, the cut-off at M is the same in both the unaffiliated and affiliated environments, but M enrolls strictly fewer first-time students in the affiliated environment because of Proposition 7. If Condition 2 does not hold—which is not the case that I concentrate on—then M becomes strictly less selective and enrolls weakly fewer students. The capacity constraint either continues to bind (and M enrolls the same amount of students in both environments), or begins to bind (and M enrolls strictly fewer students).

Now to derive the first-time cut-offs in the affiliated environment. The main university M and community college C have the same enrollment behavior as before. Condition 2 holds and M is selective in admissions, while C enrolls up to its capacity κ_C. The regional campus R is also non-selective and enrolls students up to its capacity κ_R. This aligns with *Accelerating Excellence, Access and Service: Strategic Enrollment Plan for The Ohio State University, 2022-2024* (2021), which states that the main enrollment goal at regional campuses is expansion. Also recall that any first-time students with a high school degree or equivalent are admissible at regional campuses (*Undergraduate Admissions: Regional Campuses* 2023). To summarize institutional differences in enrollment, only M is selective and sets enrollment above capacity, while R and C are non-selective.

There are still two time periods $t \in \{1, 2\}$, but at the beginning of $t = 1$, first-time admissions now happens over three steps:

1. Students who most prefer the main university M apply there.

2. Students who most prefer the regional campus R or who were previously rejected at M apply to R, and the best applicants are accepted up to capacity.[11]

3. Students who most prefer the community college C or who were previously rejected at both M and R apply to C, and the best applicants are accepted up to capacity.

For an institution $j \in J_A$, I denote a_J^{**} the optimal cut-off of academic preparedness for j in the affiliated environment. Under Conditions 1, 2, and 4 as well as additional conditions on capacities at the regional campus κ_R and community college κ_C, I now characterize first-time enrollment in the affiliated environment.

Proposition 8. *Let Conditions 1 and 4 on the wealth cut-offs and Condition 2 on selectivity at M hold. Also, let* $\kappa_R > (\bar{w} - \underline{w})(1 - a_M^*)$ *and* $\kappa_C > \underline{w}(1 - \frac{(\bar{w}-\underline{w})+(1-\bar{w})a_M^* - \kappa_R}{1-\underline{w}})$. *Then the first-time cut-offs are:*

$$a_M^{**} = a_M^* = 1 - \rho_M^{-1}(\pi_M)$$
$$a_R^{**} = \frac{(\bar{w} - \underline{w}) + (1 - \bar{w})a_M^{**} - \kappa_R}{1 - \underline{w}}$$
$$a_C^{**} = \bar{w} + (1 - \bar{w})a_M^{**} - \kappa_R - \kappa_C$$

[11] In practice, students use the Common Application to apply to the OSU-Main Campus and regional campuses simultaneously. It is equivalent to assume that students apply to both the main university M and regional campus R simultaneously, and the university system sorts applicants by academic preparedness by simultaneously choosing cut-offs for both campuses.

Proof. a_M^{**} follows from the Condition 2 on selectivity at M. Then,

$$\kappa_R = (\bar{w} - \underline{w})(1 - a_R^{**}) + (1 - \bar{w})(a_M^{**} - a_R^{**})$$
$$\kappa_C = \underline{w}(1 - a_C^{**}) + (1 - \underline{w})(a_R^{**} - a_C^{**}),$$

and rearranging to solve for a_R^{**}, a_C^{**} gives the result. □

Similar to the unaffiliated environment, I impose some conditions on capacity constraints which simplify to $\kappa_R > (\bar{w} - \underline{w})(1 - a_M^{**})$ and $\kappa_C > \underline{w}(1 - a_R^{**})$. Together, they ensure that admissions is most selective at the main university M and least selective at the community college C. It also implies that the regional campus R enrolls not only students who most prefer R but also some students who were rejected from M, and that the community college C enrolls a mix of students who most prefer M, R, or C.

For comparative statics on a_R^{**} and a_C^{**}, see Corollary 9 and further discussion in Appendix 2A. I concentrate here on the outcomes at the community college C, which must lower its first-time cut-off after a regional campus R is introduced.

Proposition 9. *Let Conditions 1 and 4 on the wealth cut-offs and Condition 2 on selectivity at M hold. Also, let $\kappa_R > (\bar{w} - \underline{w})(1 - a_M^{**})$ and $\kappa_C > \underline{w}(1 - a_R^{**})$. Then the cut-off at C is lower in the affiliated environment than in the affiliated environment: $a_C^{*} - a_C^{**} > 0$.*

Proof. Suppose not, and $a_C^{**} \geq a_C^{*}$. Then

$$\bar{w} + (1 - \bar{w})a_M^{**} - \kappa_R - \kappa_C \geq \hat{w} + (1 - \hat{w})a_M^{*} - \kappa_C$$
$$\Rightarrow (\bar{w} - \hat{w})(1 - a_M^{*}) \geq \kappa_R$$
$$\Rightarrow (\bar{w} - \underline{w})(1 - a_M^{**}) > (\bar{w} - \hat{w})(1 - a_M^{*}) \geq \kappa_R,$$

but $\kappa_R > (\bar{w} - \underline{w})(1 - a_M^{**})$, hence the contradiction. □

It is also of interest to characterize how different the cut-offs at the community college C are in the two environments. Since the comparative statics are straightforward, I present the result without further proof.

Corollary 4. *Let the conditions from Proposition 9 hold. Then the gap between cut-offs at the community college C,*

$$a_C^* - a_C^{**} = (\hat{w} - \bar{w})(1 - a_M^{**}) + \kappa_R > 0,$$

is decreasing in $\bar{w}$, and increasing in both a_M^{**} and κ_R.

As the upper affiliated wealth cut-off $\bar{w}$ increases, more students initially prefer the regional campus R over the main university M, which increases competition for limited first-time enrollment slots at R and causes the students who were marginally admissible at R to "trickle down" to the community college C. Since R has a higher cut-off than C, this increases the cut-off at C.

As the cut-off at the main university a_M^{**} increases, the community college C benefits in both settings, but it benefits C more in the unaffiliated setting than the affiliated setting. In the unaffiliated setting, C directly admits the students who "trickle down" from the main university M. In the affiliated setting, those students first "trickle down" to the regional campus R, and the weakest students at R who lose enrollment seats are then enrolled at C. This dilutes the effect on enrollment in C in the affiliated environment.

Finally, as capacity at the regional campus κ_R increases, the regional campus R is able to enroll more students, which draws some well-prepared students away from the community college C.

These insights generate the following hypotheses:

Hypothesis 1. *As more students prefer the regional campus over the main university, average and minimum levels of academic preparedness for first-time community college students increase.*

Hypothesis 2. *Average and minimum levels of academic preparedness for first-time community college students increase as the main university increases selectivity.*

Hypothesis 3. *Average and minimum levels of academic preparedness for first-time community college students decrease after a regional campus is introduced. Moreover, both are decreasing in the enrollment level at regional campuses.*

Now to consider transfer admissions. To simplify analysis, I assume that students at the regional campus who initially preferred the main university do not change their preferences, due to being part of the same university system. Of course, this is a strong assumption, but is not too unrealistic for students at OSU regional campuses. Recall that about half of students entering regional campuses state that the OSU-Main Campus was their first choice, and about half of students eventually transfer out. This suggests students at the regional campus either have very consistent preferences, or that the inflow of democratized students is about the same as the outflow of diverted students.

While the main university M remains non-selective over transfer enrollment, M treats transfers from the regional campus R and the community college C differently. M exogenously chooses an R-specific minimum cut-off $a_{Min} \in (a_R^{**}, a_M^{**})$ and guarantees that any transfers from the regional campus R with academic preparedness at least as high as a_{Min} will be accepted, and prioritizes filling transfer capacity with students from R before filling remaining capacity with applicants from C. Because C is not affiliated with M, I also call transfer students from C "external transfers". I assume that a_{Min} is exogenously chosen because the choice at the OSU-Main Campus was likely picked due to being exactly average (a 2.0 GPA), not strategically, as I was able to verify using the Internet Archive that the cut-off has been unchanged since at least May 2014. Also, the bounds on a_{Min} reflect that while campus change is not selective, not all regional campus students can transfer to the main university.

As was the case in the unaffiliated environment, if transfer capacity is too low, then no community college students transfer.

Lemma 5. *If $\kappa_T \leq (1 - \bar{w})(a_M^{**} - a_{Min})$, then first-time community college students who prefer the main university M cannot enroll at M as a transfer student.*

Proof. If $\kappa_T \leq (1 - \bar{w})(a_M^{**} - a_{Min})$, then because regional campus students are prioritized, capacity is completely filled by these students. □

Clearly, though, the OSU-Main Campus admits both campus change and external transfer students every year. Going forth, I only consider the case in which $\kappa_T > (1 - \bar{w})(a_M^{**} - a_{Min})$.

I summarize the three conditions on the affiliated capacity constraints as follows:

Condition 5. $\kappa_R > (\bar{w} - \underline{w})(1 - a_M^{**})$, $\kappa_C > \underline{w}(1 - a_R^{**})$, *and* $\kappa_T > (1 - \bar{w})(a_M^{**} - a_{Min})$.

When I refer to Condition 3 on the unaffiliated capacity constraints and Condition 5 on the affiliated capacity constraints together, I call them conditions on "capacity constraints" without specifying the environment.

Denote a_T^{**} as the optimal cut-off for external transfer applicants in the affiliated environment: community college student i who applies to transfer is admitted if and only if $a_i \geq a_T^{**}$. Transfer admissions in the affiliated environment is characterized as follows:

Proposition 10. *Let Conditions 1 and 4 on the wealth cut-offs, Condition 2 on selectivity at M, and Conditions 3 and 5 on capacity constraints hold. Let M prioritize admitting transfer applicants from the regional campus with academic preparedness of at least a_{Min}. Then the optimal transfer cut-off for community college students is*

$$a_T^{**} = \frac{\underline{w} \cdot \delta + (1 - \bar{w})(a_M^{**} - a_{Min}) - \kappa_T}{\underline{w} \cdot \delta}.$$

Proof. Since $a_T^{**} > a_{Min}$, there are two kinds of students who transfer. First, students from the regional campus R, who receive priority. Second, democratized community college students. Hence,

$$\kappa_T = (1 - \bar{w})(a_M^{**} - a_{Min}) + \underline{w} \cdot \delta(1 - a_T^{**}),$$

and rearranging gives the result. □

Proposition 10 holds in general, and says nothing about the sign of $a_{Min} - a_T^{**}$. Empirically, though, it is natural to test this to see whether regional campus students benefit from facing lower transfer-in requirements:

Hypothesis 4. $a_{Min} < a_T^{**}$.

Comparative statics on the transfer cut-off a_T^{**} that community college students face follow.

Corollary 5. *Let the conditions from Proposition 10 hold. The optimal transfer cut-off for community college students is increasing in $\underline{w}$ and a_M^{**}, and decreasing in $\bar{w}$ and a_{Min}.*

Proof.

$$\frac{\partial}{\partial \bar{w}} a_T^{**} = -\frac{a_M^{**} - a_{Min}}{\underline{w} \cdot \delta} < 0,$$
$$\frac{\partial}{\partial \underline{w}} a_T^{**} = \frac{\kappa_T - (1-\bar{w})(a_M^{**} - a_{Min})}{\underline{w}^2 \cdot \delta} > 0,$$
$$\frac{\partial}{\partial a_M^{**}} a_T^{**} = \frac{1-\bar{w}}{\underline{w} \cdot \delta} > 0,$$
$$\frac{\partial}{\partial a_{Min}} a_T^{**} = -\frac{1-\bar{w}}{\underline{w} \cdot \delta} < 0.$$

The first inequality follows from Condition 5; specifically, that $\kappa_T >= (1-\bar{w})(a_M^{**} - a_{Min})$. The second inequality follows from the assumption that $a_{Min} \in (a_R^{**}, a_M^{**})$. □

As the lower affiliated wealth cut-off $\underline{w}$ increases, more people prefer the community college C over the regional campus R. Transfer admissions from C is more selective, since more highly prepared students will start at C and increase competition for external transfers. As the first-time cut-off at the main campus a_M^{**} increases, some students lose access to the main university M and both R and C are able to enroll a more academically prepared first-time student body, which also increases competition for external transfers.

As the upper affiliated wealth cut-off $\bar{w}$ increases, more people prefer R most and will not transfer. Similarly, as the regional campus transfer cut-off a_{Min} increases, fewer students from R transfer. Both of these cause external transfer to become less selective, since regional campus transfers take up less transfer capacity.

These comparative statics generate four more hypotheses:

Hypothesis 5. *As more students prefer the community college over the regional campus, the main university becomes more selective over external transfers.*

Hypothesis 6. *The main university becomes more selective over external transfers as selectivity at the main university increases.*

Hypothesis 7. *As more students prefer the regional campus over the main university, the main university becomes less selective over external transfers.*

Hypothesis 8. *The main university becomes less selective over external transfers as the regional campus transfer cut-off increases.*

Finally, I identify conditions such that community college transfer applicants lose access to the main university M after the regional campus is added.

Theorem 2. *Let Conditions 1 and 4 on the wealth cut-offs, Condition 2 on selectivity at M, and Conditions 3 and 5 on capacity constraints hold. Let M prioritize admitting transfer applicants from the regional campus with academic preparedness of at least a_{Min}. The transfer cut-off for community college students is strictly higher in the affiliated environment if and only if*

$$a_{Min} < a_M^* \left(1 - \frac{(1-\hat{w})\underline{w} \cdot \delta}{(1-\bar{w})(\hat{w} \cdot \delta + (1-\hat{w}))}\right) + \frac{(1-\hat{w})\underline{w} \cdot \delta}{(1-\bar{w})(\hat{w} \cdot \delta + (1-\hat{w}))} - \frac{(\hat{w} - \underline{w})\delta + (1-\hat{w})}{(1-\bar{w})(\hat{w} \cdot \delta + (1-\hat{w}))}\kappa_T.$$

Proof. $a_T^{**} > a_T^*$ if and only if

$$\frac{\underline{w} \cdot \delta + (1-\bar{w})(a_M^{**} - a_{Min}) - \kappa_T}{\underline{w} \cdot \delta} > \frac{\hat{w} \cdot \delta + (1-\hat{w})a_M^* - \kappa_T}{\hat{w} \cdot \delta + (1-\hat{w})},$$

and rearranging to solve for a_{Min} gives the result. □

Theorem 2 says that if the regional campus transfer cut-off a_{Min} is too low, then students at the regional campus R crowd out community college students in the transfer process. The weakest regional campus transfer students (those just barely acceptable) are prioritized even above the most academically prepared community transfer applicants. However, if a_{Min} is high enough, then there is no crowd-out effect because the regional campus students who have priority would still have been able to transfer even without priority. This generates the following hypothesis:

Hypothesis 9. *The main university M is more selective over external transfers after the regional campus is introduced.*

If the hypothesis holds, then the main university M would be able to enroll an overall stronger transfer student body by increasing a_{Min}, which reduces crowd-out of highly academically prepared community college applicants. This is especially important to note because in this model, there is an equity implication to crowd-out: students with low wealth endowments ($w \in [0, \underline{w}]$) are exactly the students who initially most prefer C. Highly academically prepared, low wealth students who are democratized are also the students who are at most risk of being pushed out by regional campus transfers.

2.4 A Social Planner's Problem

Now consider the following situation: a state policymaker (or interchangeably, social planner, SP) decides how to invest in higher education in an unaffiliated environment such that Conditions 1 and 4 on wealth cut-offs, Condition 2 on selectivity at the main university M, and Conditions 3 and 5 on capacity constraints hold. The SP can either expand M modestly, or open a regional campus R. If the SP does the latter, then M will prioritize admitting transfer applicants from R with academic preparedness of at least a_{Min}.

Of course, under Condition 2 on selectivity at the main university M, directly expanding capacity at M has no effect on first-time enrollment. Therefore, I suppose that the SP would partially subsidize the price of enrollment p_M (and Condition 2 continues to hold) to expand enrollment at M. Under subsidization, students at the main university M pay $p^0 = p_M - \psi$ for some $\psi \in \mathbb{R}_{++}$ small. Since we are in the unaffiliated environment, this lowers the wealth cut-off to

$$\hat{w}^0 = \frac{1 - e^{v_M}(1 - p^0)}{e^{v_M} - 1}, \text{ and}$$
$$\hat{w} - \hat{w}^0 = \frac{e^{v_M} \cdot \psi}{e^{v_M} - 1}.$$

Since all students at the main university M benefit from subsidization, the social cost of subsidizing the price of enrollment is

$$\text{Social Cost} = (1 - \hat{w}^0)(1 - a_M^*)\psi.$$

To make the comparison as clean as possible, suppose that if the SP instead spent the same amount on opening a regional campus R, then R would have capacity $\kappa_R^0 \in ((\hat{w} - \hat{w}^0)(1 - a_M^*), 1)$ such that $\kappa_M + \kappa_R^0 + \kappa_C < 1$. The lower bound on κ_R^0 reflects that opening the regional campus would increase total enrollment more than subsidizing tuition at the main campus.

The SP is *positive assortative* in matching students to institutions, but cannot directly make matches.[12] Social benefit is linearly increasing in a student's academic preparedness a_i, such that for $b_M \in \mathbb{R}_{++} \setminus (0,1]$ and $b_R \in (1, b_M]$, a student i generates social benefit $b_M \cdot a_i$ for enrolling at the main university M, social benefit $b_R \cdot a_i$ for enrolling at the regional campus R, and social benefit a_i for enrolling at the community college C. I assume that this social benefit is accrued at the *last* institution that a student attends. So if student i attends M as a first-time student, then the social benefit is $b_M \cdot a_i$. If student i' starts at C but successfully transfers to M, then the social benefit is $b_M \cdot a_{i'}$.

I limit discussion to the case where $a_M^{**} = a_M^* > a_T^{**} > a_T^* > a_{Min} > a_R^{**} > a_C^* > a_C^{**}$ and subsidies are not enough to change the ordering of these cut-offs. The first equality $a_M^{**} = a_M^*$ comes from Condition 2 on selectivity at the main university M. $a_M^* > a_T^{**}$ and $a_{Min} > a_R^{**}$ come from Condition 5 on affiliated capacity constraints. $a_T^{**} > a_T^*$ assumes that the condition in Theorem 2 holds, and $a_T^* > a_{Min}$ comes from Hypothesis 4. That is, the regional campus is beneficial to some students who can only transfer to the main university M because of prioritized transfer admissions, because a_{Min} is strictly less than the transfer cut-offs. $a_R^{**} > a_C^*$ does not come from any particular result, but is true whenever $\kappa_R^0 < \kappa_C$. Finally, $a_C^* > a_C^{**}$ comes from Proposition 9.

Social benefit of subsidization. Social welfare increases in three ways. First, students with academic preparedness $a \in [a_M^*, 1]$ and wealth $w \in [\hat{w}^0, \hat{w})$ now enroll as first-time students at the main university M, whereas they would have previously preferred and enrolled at the community college C and only a fraction δ of them would become democratized and successfully transferred. The welfare gain is

$$B_{Main}^S = \frac{1}{2}(\hat{w} - \hat{w}^0)(1-\delta)(1-(a_M^*)^2)(b_M - 1) > 0.$$

Second, the first-time enrollment cut-off at the community college C cut-off is lower. This is because some highly-prepared students initially start at the main university M instead of C after subsidization, which allows C to admit students who were previously marginally inadmissible.

[12] If admissions were centralized and run by the SP, then the SP would first enroll the best students based on academic preparedness who are not already admitted elsewhere to the main university until its capacity is full, then to the regional campus (if available) until its capacity is full, and finally to the community college until its capacity is full.

Denote the new cut-off $a_C^0 < a_C^*$. Since these students are never admissible at M in either first-time or transfer admissions, the welfare gain is

$$B^S_{College} = \frac{1}{2}((a_C^*)^2 - (a_C^0)^2) > 0.$$

Finally, the transfer cut-off is lower. Previously, democratized community college students with academic preparedness $a \in [a_M^*, 1]$ and wealth $w \in [\hat{w}^0, \hat{w})$ would have transferred to the main university M. After subsidization, these students now start at M, which frees up some transfer capacity for students who were previously marginally inadmissible. Denote the new cut-off $a_T^0 < a_T^*$. The welfare gain is

$$B^S_{Transfer} = \frac{1}{2}(\hat{w}^0 \cdot \delta + (1 - \hat{w}^0))((a_T^*)^2 - (a_T^0)^2)(b_M - 1) > 0.$$

Under subsidization, welfare increases because more students start at the main university M in the first-time admissions process, and this further "trickles down" to first-time admissions at C and to transfer admissions. Moreover, these enrollment effects maintain positive sorting by academic preparedness. That is, some students gain access to C in first-time admissions, and these are the most qualified students who were only just unacceptable before subsidization, and similarly for accessibility to M in transfer admissions.

Social benefit of the regional campus. I fully back out the welfare changes from opening a regional campus in Appendix 2B and summarize the results here. Unlike the case of subsidization, there are both gains *and* losses to opening a regional campus.

The certain welfare gains from opening a regional campus are

$$B^R_{Gain} = \frac{1}{2}[(1 - \bar{w})((a_T^*)^2 - (a_{Min})^2)(b_M - 1) + (\bar{w} - \underline{w})((a_T^*)^2 - (a_{Min})^2)(b_R - 1) + (1 - \underline{w})((a_{Min})^2 - (a_R^{**})^2)(b_R - 1) + ((a_C^*)^2 - (a_C^{**})^2)].$$

Welfare gains are accrued by two kinds of students: (1) those who would have enrolled at the community college C and would certainly fail to transfer to the main university M in the unaffiliated environment, but enroll at the regional campus R in the affiliated environment (first three components of the expression), and (2) those who are not enrolled anywhere in the unaffiliated environment, but enroll at C in the affiliated environment (last component of the expression). In particular, students of the first kind especially benefit if they most prefer M and are only are able to transfer to M because they start at R and enjoy preferential treatment in transfer enrollment (first component of the expression). These are precisely the students who crowd out highly academically prepared community college students in transfer admissions.

The certain welfare losses are

$$B^R_{Loss} = \frac{1}{2}[(\bar{w} - \hat{w})(1 - (a^*_T)^2)(b_R - b_M) + \underline{w}((a^{**}_T)^2 - (a^*_T)^2)(1 - b_M)\delta].$$

Welfare losses are accrued by two kinds of students: (1) those who enroll at the main university M in either first-time or transfer admissions in the unaffiliated environment, but choose to enroll and stay at the regional campus R in the affiliated environment (first component of the expression), and (2) those who always initially enroll at the community college C, and can only transfer in the unaffiliated environment due to being crowded out in the affiliated environment (second component of the expression). Of course, the latter are the students who are crowded out by preferential treatment for regional campus students in transfer admissions.

Finally, ambiguous welfare effects are

$$B^R_{Ambig} = \frac{1}{2}(\hat{w} - \underline{w})(1 - (a^*_T)^2)(\delta \cdot b_M + (1 - \delta) - b_R)).$$

These welfare changes are accrued by highly academically prepared students who most prefer the community college C in the unaffiliated environment, but most prefer the regional campus R in the affiliated environment. In the unaffiliated environment, a fraction δ of these students would become democratized and successfully transfer to the main university M—while the rest stay at C. But in the affiliated environment, all of these students most prefer and start at R, and will not try to transfer. If the social benefit of enrolling at M is high (b_M is very large relative to b_R, or b_R is not that much larger than 1, or both) and/or democratization is high (δ is large), then this is more likely to be a welfare loss.

Without adding more assumptions on the model parameters, it is unclear whether adding a regional campus would increase or decrease social welfare. It depends on the distance between social benefit parameters as well as the distance between wealth cut-offs. First, as the social benefit of enrolling at the regional campus R decreases and approaches the social benefit of enrolling at the community college C ($b_R \to 1$), welfare gains decrease, welfare losses increase, and ambiguous welfare effects are more likely to be welfare decreasing. That is, if R doesn't deliver a substantially better educational experience than a community college, then the institutional characteristic that primarily distinguishes R is that it facilitates transfer. But crowd-out of highly prepared community college students is welfare reducing as well. Conversely, as the social benefit of enrolling at the main university M decreases and approaches the social benefit of enrolling at R ($b_M \to b_R$), welfare gains increase, welfare losses decrease, and ambiguous welfare effects are more likely to be welfare increasing.

Second, the distances between the wealth cut-offs in the two environments, $\bar{w} - \hat{w}$ and $\hat{w} - \underline{w}$, also affect welfare. If the difference between the upper affiliated wealth cut-off and unaffiliated wealth cut-off $\bar{w} - \hat{w}$ is small, then welfare losses decrease because not many students switch their preference from the main university to the regional campus after it opens. Also, for a small democratization level δ, if the difference between the unaffiliated wealth cut-off and the lower affiliated wealth cut-off $\hat{w} - \underline{w}$ is large, then welfare gains increase, because students who initially preferred the community college switch to preferring the regional campus once it is introduced. Put altogether, if $\bar{w} - \hat{w}$ is small relative to $\hat{w} - \underline{w}$, then adding the regional campus is more likely to be welfare increasing. To achieve that, the regional campus needs to have similar value as the main university ($v_M - v_R$ is small) and/or similar affordability as the community college (p_R is small).

Finally, some welfare loss always occurs because of incorrect sorting. Unlike with subsidization, some well-qualified community college students are crowded out by less prepared regional campus students in transfer admissions; this is welfare-reducing because social welfare is positive assortative.

2.5 Conclusion

The primary contribution of this chapter is the construction of a general theoretical framework that describes first-time and transfer admissions with multiple institutions. Using the model, I find that opening a regional campus may adversely affect community colleges in both first-time and transfer admissions. I provide some suggestions here for an empirical strategy to test the predictions generated by the theoretical framework. To summarize, there are nine hypotheses. I summarize them, reordered to group similar hypotheses, below:

1. As more students prefer the regional campus over the main university, academic preparedness at community colleges increases.

2. As more students prefer the community college over the regional campus, selectivity over external transfers increases.

3. As more students prefer the regional campus over the main university, selectivity over external transfers decreases.

4. Increasing selectivity at the main university increases academic preparedness at community colleges.

5. Increasing selectivity at the main university increases selectivity over external transfers.

6. Increasing the regional campus transfer cut-off decreases selectivity over external transfers.

7. Opening and/or expanding a regional campus decreases academic preparedness at community colleges.

8. Opening and/or expanding a regional campus increases selectivity over external transfers.

9. The main university sets a higher "minimum requirement" for external transfers than for regional campus transfers.

However, not all of these hypotheses are testable. Hypotheses (1), (2), and (3) require truthfully eliciting student preferences. Hypotheses (4) and (5) cannot be empirically tested using Ohio as a case study because OSU did not become a selective institution until *after* the regional campuses were founded.[13] Enrollment changes would partially reflect that the value of attending the OSU-Main Campus was changing and attracting a different pool of applicants, and it is not clear how to untangle changes in student application behavior from institutional changes in selectivity. Finally, Hypothesis (7) cannot be tested because OSU has not had any variation in the regional campus cut-off; as discussed earlier, the minimum GPA requirement has been set at 2.0 for at least a decade.

Combining the insights from the hypotheses generated, though, there are still two things that can potentially be tested with internal validity. First, is the OSU-Main Campus more selective over external transfer students? Second, does enrollment at regional campuses indirectly affect enrollment at community colleges? I propose several empirical strategies that could be employed with data on OSU transfer applicants to test the first question. Ideally, data on external transfers would have information on *all* applicants, including those who are rejected. It is also preferred to have data linking applicants to student outcomes, in order to get a more standardized measure of "academic preparedness".[14] Identifying variation is in the type of students' home institutions: regional campus or community college.

First, I suggest using a regression discontinuity approach to verify that the minimum GPA cut-off for regional campus student transfer-ins works as expected. I propose a modified version of the approach in Hoekstra (2009), using application GPA as the running variable. Second, it would be illustrative to graphically show the distributions in different measures of student preparedness for external vs. regional campus transfer-ins, and test for a difference in sample means (e.g., a t-test) and/or a difference in probability distributions (e.g., a Kolmogorov-Smirnov test).

Next, I suggest an empirical test based on Arcidiacono, Kinsler, and Ransom (2023), who model admissions at selective institutions. The underlying model for external admissions is

$$x_i = z_i\beta + \varepsilon_i,$$

[13]The regional campuses were founded 1957-1960 and the OSU-Main Campus was an open admissions university through the 1980s.

[14]That is, students at different community colleges could have the same entering GPA, but different levels of "true" preparedness, due to idiosyncrasies in grading at the institution level. By tracking students by their performance at the OSU-Main Campus after enrollment, one could measure student performance when measured across the same benchmark.

where for applicant i, x_i is a measure of applicant quality, z_i is a vector of observable characteristics, and ε_i are unobservable characteristics following a logistic distribution. In the presence of a regional campus, applicants from the community college are admitted if:

$$z_i\beta + \varepsilon_i + \geq a_T^{**}.$$

The empirical specification is therefore a logit model on the probability of admission as the outcome of interest, using the student's application characteristics as controls, and the coefficient on the type of home institution is a measure of the average marginal effect of being an external transfer (relative to a regional campus student) on transfer admissions.

Finally, I suggest using a quantile regression approach to test for differences in the distribution of academic preparedness at different percentiles between the two transfer-in groups, especially if there is evidence that the cut-off (as tested in the regression discontinuity approach) is strongly predictive of regional campus transfers. Because regional campus students potentially benefit from facing less selective transfer-in criteria, the distribution of academic preparedness in this group may be positively skewed; conversely, the distribution of academic preparedness for external transfer-ins may be negatively skewed. It would therefore be instructive for measuring the potential size of incorrect sorting (which is welfare-reducing) to understand if there is a pile-up of regional campus students at the minimum 2.0 GPA cut-off for a campus change.

Another contribution of this work is a policy recommendation on how to expand higher education enrollment. If a state policymaker can choose between subsidizing tuition at a flagship institution or opening a regional campus, I characterize when each policy is more welfare-improving. I find that subsidization is socially preferred if enrollment at the main university can be substantially expanded and/or the social returns and the perceived value to students of attending the main university are relatively high compared to the alternatives. On the other hand, opening a regional campus is socially preferred when the previous conditions don't hold, but additionally if the regional campus's value is similar to the main university's value, and/or the regional campus's price is close to the community college's price. Finally, the stronger the preferential treatment in transferring students from the regional campus, the less preferred opening a regional campus becomes, because transfers from the regional campus will crowd out highly academically prepared transfer applicants from the community college.

I conclude with some additional remarks on how this work may additionally be useful for policymakers. Enrollment at selective institutions of higher education has not kept pace with increased student demand for it (Bound, Hershbein, and Long 2009). Whether or not a state policymaker can improve upon this situation depends on if institutions *choose* not to enroll to capacity, or if they *cannot* increase capacity (e.g., the campus cannot physically expand). In the latter case, opening a regional campus may be the only feasible way to increase enrollment. However, policymakers should be aware that this may have spillover effects onto community colleges and readjust to prevent these institutions from unintentionally losing state appropriations—especially if states use or are considering switching to performance funding—or other forms of governmental support.

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APPENDIX 2A

SUPPLEMENTAL PROOFS

Unaffiliated Environment

Recall that the unaffiliated wealth cut-off in the environment with a main university M and community college C is

$$\hat{w} = \frac{1 - e^{v_M}(1 - p_M)}{e^{v_M} - 1} \in (0, 1),$$

where $p_M, v_M \in (0, 1)$ are the price and value of attending M respectively. The unaffiliated wealth cut-off $\hat{w}$ determines a student's initial preferences over institutions: a student i prefers M if and only if $w_i \geq \hat{w}$. Comparative statics from Lemma 1 follow:

Corollary 6. *Let Condition 1 on the unaffiliated wealth cut-off hold. $\hat{w}$ is increasing in p_M and decreasing in v_M.*

Proof.

$$\frac{\partial}{\partial p_M}\hat{w} = \frac{e^{v_M}}{e^{v_M} - 1} > 0,$$
$$\frac{\partial}{\partial v_M}\hat{w} = \frac{-e^{v_M} p_M}{(e^{v_M} - 1)^2} < 0,$$

since for $v_M \in (0, 1)$, $e^{v_M} > 1$. □

Next, recall that the transfer cut-off in the unaffiliated environment is

$$a_T^* = \frac{\hat{w} \cdot \delta + (1 - \hat{w})a_M^* - \kappa_T}{\hat{w} \cdot \delta + (1 - \hat{w})},$$

where $a_M^* \in (0, 1)$ is the first-time enrollment cut-off at the main university M, $\delta \in (0, 1)$ is the fraction of community college students who are democratized and apply to transfer to M, and $\kappa_T \in (0, 1)$ is capacity at M for transfer students. Students at the community college who apply to transfer are admitted if and only if $a_i \geq a_T^*$. Comparative statics from Proposition 6 follow:

Corollary 7. *Let Condition 1 on the unaffiliated wealth cut-off, Condition 2 on selectivity at M, and Condition 3 on the unaffiliated capacity constraints hold. a_T^* is decreasing in transfer capacity κ_T*

and increasing in the democratization fraction δ. Also, a_T^* is increasing in the unaffiliated wealth cut-off $\hat{w}$ if and only if $a_M^* < (1-\delta)a_T^* + \delta$.

Proof. First,

$$\frac{\partial}{\partial k_T} a_T^* = -\frac{1}{\hat{w}\cdot\delta + (1-\hat{w})} < 0,$$
$$\frac{\partial}{\partial \delta} a_T^* = \frac{\hat{w}[(1-\hat{w})(1-a_M^*) + \kappa_T]}{(\hat{w}\cdot\delta + (1-\hat{w}))^2} > 0,$$

since $\hat{w}, a_M^*, \delta \in (0,1)$. Next,

$$\frac{\partial}{\partial \hat{w}} a_T^* = \frac{\delta(1-a_M^*) - (1-\delta)\kappa_T}{(\hat{w}\cdot\delta + (1-\hat{w}))^2},$$

so $\frac{\partial}{\partial \hat{w}} a_T^* > 0$ if and only if $\delta(1-a_M^*) > (1-\delta)\kappa_T$. Substitute in $\kappa_T = (1-\hat{w})(a_M^* - a_T^*) + \hat{w}\cdot\delta(1-a_T^*)$ and suppose not:

$$\delta(1-a_M^*) \leq (1-\delta)((1-\hat{w})(a_M^* - a_T^*) + \hat{w}\cdot\delta(1-a_T^*))$$
$$\Rightarrow a_M^* \geq (1-\delta)a_T^* + \delta,$$

contradicting that $a_M^* < (1-\delta)a_T^* + \delta$ by assumption. □

Increasing transfer capacity always reduces transfer selectivity, and increasing the fraction of democratized community college students increases transfer selectivity. However, an increase in the unaffiliated wealth cut-off $\hat{w}$, which increases the measure of students who initially prefer the community college C, has two effects that work in opposite directions. First, more students will most prefer the C over the main university M. This also means the level of academic preparedness at the community college increases. Second, it increases competition for transfer slots among democratized community college students. So an increase in $\hat{w}$ decreases transfer competition from the group of students who switched their preferences from M to C (recall that students who start at C but prefer M will always want to transfer), but the democratized community college students who apply to transfer have higher academic preparedness than before. The expression $a_M^* < (1-\delta)a_T^* + \delta$ could be interpreted as a check on which of those effects is stronger.

Affiliated Environment

Recall that the upper affiliated wealth cut-off in the environment with a main university M, regional campus R, and community college C is

$$\bar{w} = \frac{e^{v_R}(1-p_R) - e^{v_M}(1-p_M)}{e^{v_M} - e^{v_R}} \in (0,1),$$

where $p_R, v_R \in (0,1)$ are the price and value of attending R respectively. The upper affiliated wealth cut-off determines if a student initially prefers the main university M over the regional campus R: a student i prefers M over R if and only if $w_i \geq \bar{w}$. Comparative statics from Lemma 3 follow.

Corollary 8. *Let* $ln(1-p_M) + v_M < ln(1-p_R) + v_R$ *and* $ln(2-p_M) + v_M > ln(2-p_R) + v_R$. $\bar{w}$ *is increasing in* p_M *and* v_R *and decreasing in* p_R *and* v_M.

Proof. First,

$$\frac{\partial}{\partial p_M}\bar{w} = \frac{e^{v_M}}{e^{v_M} - e^{v_R}} > 0,$$
$$\frac{\partial}{\partial p_R}\bar{w} = -\frac{e^{v_R}}{e^{v_M} - e^{v_R}} < 0,$$

since $v_M, v_R \in (0,1)$ and $v_M > v_R$. That is, e^{v_M}, e^{v_R}, and $e^{v_M} - e^{v_R}$ are strictly positive. Next,

$$\frac{\partial}{\partial v_M}\bar{w} = -\frac{e^{v_M}(1-p_M)}{e^{v_M} - e^{v_R}} - \frac{e^{v_M}[e^{v_R}(1-p_R) - e^{v_M}(1-p_M)]}{(e^{v_M} - e^{v_R})^2} < 0.$$

The first component is strictly negative, since $v_M, p_M \in (0,1)$ and $v_M > v_R$. The second component is strictly positive, since e^{v_M} is strictly positive and from Lemma 3, the numerator is also strictly positive. So $\frac{\partial}{\partial v_M}\bar{w} < 0$. Finally for similar reasoning,

$$\frac{\partial}{\partial v_R}\bar{w} = \frac{e^{v_R}(1-p_R)}{e^{v_M} - e^{v_R}} + \frac{e^{v_R}[e^{v_R}(1-p_R) - e^{v_M}(1-p_M)]}{(e^{v_M} - e^{v_R})^2} > 0.$$

□

The intuition is straightforward. As the price of the main university p_M or the value of the regional campus v_R increases, the relative utility of the main university M relative to the regional campus R decreases, so more students will prefer R. Conversely, as the value of the main university v_M of price of the regional campus p_R increases, the opposite occurs.

Next, recall that from Proposition 8, the first-time cut-off for the regional campus R in the affiliated environment is

$$a_R^{**} = \frac{(\bar{w} - \underline{w}) + (1 - \bar{w})a_M^{**} - \kappa_R}{1 - \underline{w}},$$

where κ_R is capacity at the regional campus R and $\underline{w}$ is the lower affiliated wealth cut-off which determines if a student initially prefers the regional campus R over the community college C: a student i prefers R over C if and only if $w_i \geq \underline{w}$. A student i who applies to R is accepted if $a_i \geq a_R^{**}$. Comparative statics follow:

Corollary 9. *Let Conditions 1 and 4 on the wealth cut-offs and Condition 2 on selectivity at M hold. Also, let $\kappa_R > (\bar{w} - \underline{w})(1 - a_M^{**})$ and $\kappa_C > \underline{w}(1 - a_R^{**})$. a_R^{**} is increasing in $\bar{w}$ and a_M^{**}, and decreasing in $\underline{w}$ and κ_R.*

Proof.

$$\begin{aligned}
\frac{\partial}{\partial \bar{w}} a_R^{**} &= \frac{1 - a_M^{**}}{1 - \underline{w}} > 0, \\
\frac{\partial}{\partial \underline{w}} a_R^{**} &= \frac{(a_M^{**} - 1)(1 - \bar{w}) - \kappa_R}{(1 - \underline{w})^2} < 0 \\
\frac{\partial}{\partial a_M^{**}} a_R^{**} &= \frac{1 - \bar{w}}{1 - \underline{w}} > 0, \\
\frac{\partial}{\partial \kappa_R} a_R^{**} &= -\frac{1}{1 - \underline{w}} < 0.
\end{aligned}$$

since $\bar{w}, \underline{w}, a_M^{**}, \kappa_R \in (0, 1)$. □

As the upper affiliated wealth cut-off $\bar{w}$ increases, more students prefer the regional campus R over the main campus M; this increases selectivity at R due to increased competition for enrollment at R. An increase in the first-time cut-off at the main university a_M^{**} has a similar effect. Conversely, as the lower affiliated wealth cut-off $\underline{w}$ increases, more students prefer the community college C

over R; this decreases selectivity at R. An increase in capacity at the regional campus κ_R has a similar effect.

Also from Proposition 8, the first-time cut-off for the community college C in the affiliated environment is

$$a_C^{**} = \bar{w} + (1 - \bar{w})a_M^{**} - \kappa_R - \kappa_C,$$

where κ_C is capacity at C. A student i who applies to C is accepted if $a_i \geq a_C^{**}$. Comparative statics follow:

Corollary 10. *Let Conditions 1 and 4 on the wealth cut-offs and Condition 2 on selectivity at M hold. Also, let* $\kappa_R > (\bar{w} - \underline{w})(1 - a_M^{**})$ *and* $\kappa_C > \underline{w}(1 - a_R^{**})$. a_C^{**} *is increasing in* $\bar{w}$ *and* a_M^{**}*, and decreasing in* κ_R *and* κ_C.

The math is straightforward so is omitted. The first-time affiliated cut-off at the community college a_C^{**} is increasing in the upper affiliated wealth cut-off $\bar{w}$. As $\bar{w}$ increases, more students initially prefer the regional campus R over the main university M, which has an indirect enrollment effect on the community college C. R is able to enroll a more academically prepared student body (due to more students initially preferring R), and this implies students who were previously marginally accepted at R must instead enroll at C, which causes academic preparedness to also increase at C. An increase in the first-time cut-off at the main university a_M^{**} has a similar effect. Finally, a_C^{**} is decreasing in capacities at the regional campus and community college, κ_R and κ_C, which is straightforward: as R increases capacity, we know from Corollary 9 that R becomes less selective and will enroll some students who previously had to enroll at C. This indirectly causes C to also become less selective. If κ_C increases, then C enrolls students who were previously marginally inadmissible and lowers selectivity.

APPENDIX 2B

FULL WELFARE ANALYSIS

As a brief reminder, I consider in Section 2.4 whether a state policymaker who is positive assortative in matching students to institution prefers to expand the main university M or open a regional campus R. Here, I show the details of welfare changes in the latter case. I consider the welfare analysis in subgroups, first based on the student's level of academic preparedness (which determines where she can enroll), and then by wealth endowment (which determines her preference over institutions). Recall that under Conditions 1 and 4 on the wealth cut-offs, $\bar{w} > \hat{w} > \underline{w}$, and students with wealth $w \in [\bar{w}, 1]$ always most prefer the main university M, students with wealth $w \in [\hat{w}, \bar{w})$ most prefer M in the unaffiliated environment but most prefer the regional campus R in the affiliated environment, students with $w \in [\underline{w}, \hat{w})$ most prefer the community college C in the unaffiliated environment but most prefer R in the affiliated environment, and students with $w \in [0, \underline{w})$ always most prefer C. Also recall that for this welfare analysis, I assume that $a_M^{**} = a_M^* > a_T^{**} > a_T^* > a_{Min} > a_R^{**} > a_C^* > a_C^{**}$.

<u>Group 1</u>. Students with $a \in [a_M^{**}, 1]$. These students are accepted at all institutions as both a first-time and transfer applicant.

1. $w \in [\bar{w}, 1]$: Students always most prefer and enroll at M as a first-time student. There are no changes in welfare.

2. $w \in [\hat{w}, \bar{w})$: In the unaffiliated environment, these students most prefer and enroll at M as a first-time student. In the affiliated environment, they most prefer and enroll at R as a first-time student and will not apply to transfer. The resulting loss in welfare is:

$$B_1^R = \frac{1}{2}(\bar{w} - \hat{w})(1 - (a_M^*)^2)(b_R - b_M) < 0.$$

3. $w \in [\underline{w}, \hat{w})$: In the unaffiliated environment, these students most prefer and enroll at C as a first-time student, and a fraction δ transfer to M. In the affiliated environment, they most prefer and enroll at R as a first-time student and will not apply to transfer. The resulting

change in welfare is:

$$B_2^R = \frac{1}{2}(\hat{w} - \underline{w})(1 - (a_M^*)^2) \underbrace{(b_M \cdot \delta + (1-\delta) - b_R)}_{\text{Ambiguous sign}}.$$

4. $w \in [0, \underline{w})$: Students always enroll at C as a first-time student, and a fraction δ always transfer to M. There are no changes in welfare.

Group 2. Students with $a \in [a_T^{**}, a_M^*)$. These students are accepted at both R and C as first-time students, and can always transfer to M.

1. $w \in [\bar{w}, 1]$: Students always most prefer M, but cannot enroll there as a first-time student. Regardless of environment, they always successfully transfer to M. There are no changes in welfare.

2. $w \in [\hat{w}, \bar{w})$: In the unaffiliated environment, these students most prefer M but must enroll at C as a first-time student, and always successfully transfer. In the affiliated environment, they most prefer and enroll at R as a first-time student and will not apply to transfer. The resulting loss in welfare is:

$$B_3^R = \frac{1}{2}(\bar{w} - \hat{w})((a_M^*)^2 - (a_T^{**})^2)(b_R - b_M) < 0.$$

3. $w \in [\underline{w}, \hat{w})$: In the unaffiliated environment, these students most prefer and enroll at C as a first-time student, and a fraction δ transfer to M. In the affiliated environment, they most prefer and enroll at R as a first-time student and will not apply to transfer. The resulting change in welfare is:

$$B_4^R = \frac{1}{2}(\hat{w} - \underline{w})((a_M^*)^2 - (a_T^{**})^2) \underbrace{(b_M \cdot \delta + (1-\delta) - b_R)}_{\text{Ambiguous sign}}.$$

4. $w \in [0, \underline{w})$: Students always enroll at C as a first-time student, and a fraction δ always transfer to M. There are no changes in welfare.

Group 3. Students with $a \in [a_T^*, a_T^{**})$. These students are accepted at both R and C as first-time students. In the unaffiliated environment, students at C can transfer. In the unaffiliated environment, students at R can transfer, but students at C cannot.

1. $w \in [\bar{w}, 1]$: Students always most prefer M, but cannot enroll there as a first-time student. Regardless of environment, they always successfully transfer to M. There are no changes in welfare.

2. $w \in [\hat{w}, \bar{w})$: In the unaffiliated environment, these students most prefer M but must enroll at C as a first-time student, and always successfully transfer. In the affiliated environment, they most prefer and enroll at R as a first-time student and will not apply to transfer. The resulting loss in welfare is:

$$B_5^R = \frac{1}{2}(\bar{w} - \hat{w})((a_T^{**})^2 - (a_T^*)^2)(b_R - b_M) < 0.$$

3. $w \in [\underline{w}, \hat{w})$: In the unaffiliated environment, these students most prefer and enroll at C as a first-time student, and a fraction δ transfer to M. In the affiliated environment, they most prefer and enroll at R as a first-time student and will not apply to transfer. The resulting change in welfare is:

$$B_6^R = \frac{1}{2}(\hat{w} - \underline{w})((a_T^{**})^2 - (a_T^*)^2)\underbrace{(b_M \cdot \delta + (1 - \delta) - b_R)}_{\text{Ambiguous sign}}.$$

4. $w \in [0, \underline{w})$: Students always enroll at C as a first-time student. A fraction δ transfer to M in the unaffiliated environment, but cannot transfer in the affiliated environment. The resulting loss in welfare is:

$$B_7^R = \frac{1}{2}\underline{w}((a_T^{**})^2 - (a_T^*)^2)(1 - b_M)\delta < 0.$$

Group 4. Students with $a \in [a_{Min}, a_T^*)$. These students are accepted at both R and C as first-time students. Students at R can transfer to M, while students at C cannot.

1. $w \in [\bar{w}, 1]$: Students always most prefer M, but cannot enroll there as a first-time student. In the unaffiliated environment, they enroll at C and cannot transfer. In the affiliated environment, they enroll at R and successfully transfer to M. The resulting increase in welfare is:

$$B_8^R = \frac{1}{2}(1 - \bar{w})((a_T^*)^2 - (a_{Min})^2)(b_M - 1) > 0.$$

2. $w \in [\hat{w}, \bar{w})$: In the unaffiliated environment, these students most prefer M but must enroll in C as a first-time student, and cannot transfer. In the affiliated environment, they most prefer and enroll at R as a first-time student and will not apply to transfer. The resulting increase in welfare is:

$$B_9^R = \frac{1}{2}(\bar{w} - \hat{w})((a_T^*)^2 - (a_{Min})^2)(b_R - 1) > 0.$$

3. $w \in [\underline{w}, \hat{w})$: In the unaffiliated environment, these students most prefer and enroll at C as a first-time student, and cannot transfer. In the affiliated environment, they most prefer and enroll at R as a first-time student and will not apply to transfer. The resulting increase in welfare is:

$$B_{10}^R = \frac{1}{2}(\hat{w} - \underline{w})((a_T^*)^2 - (a_{Min})^2)(b_R - 1) > 0.$$

4. $w \in [0, \underline{w})$: Students always enroll at C as a first-time student and cannot transfer. There are no changes in welfare.

Group 5. Students with $a \in [a_R^{**}, a_{Min})$. These students are accepted at both R and C as first-time students, and can never transfer.

1. $w \in [\bar{w}, 1]$: Students always most prefer M, but cannot enroll there as a first-time student. In the unaffiliated environment, they enroll at C and cannot transfer. In the affiliated environment, they enroll at R and cannot transfer. The resulting increase in welfare is:

$$B_{11}^R = \frac{1}{2}(1 - \bar{w})((a_{Min})^2 - (a_R^{**})^2)(b_R - 1) > 0.$$

2. $w \in [\hat{w}, \bar{w})$: In the unaffiliated environment, these students most prefer M but must enroll in C as a first-time student, and cannot transfer. In the affiliated environment, they most prefer

and enroll at R as a first-time student and will not apply to transfer. The resulting increase in welfare is:

$$B^R_{12} = \frac{1}{2}(\bar{w} - \hat{w})((a_{Min})^2 - (a^{**}_R)^2)(b_R - 1) > 0.$$

3. $w \in [\underline{w}, \hat{w})$: In the unaffiliated environment, these students most prefer and enroll at C as a first-time student, and cannot transfer. In the affiliated environment, they most prefer and enroll at R as a first-time student and will not apply to transfer. The resulting increase in welfare is:

$$B^R_{13} = \frac{1}{2}(\hat{w} - \underline{w})((a_{Min})^2 - (a^{**}_R)^2)(b_R - 1) > 0.$$

4. $w \in [0, \underline{w})$: Students always enroll at C as a first-time student and cannot transfer. There are no changes in welfare.

Group 6. Students with $a \in [a^*_C, a^{**}_R)$. These students are only accepted at C as first-time students in both environments, and can never transfer. There are no changes in welfare for this group.

Group 7. Students with $a \in [a^{**}_C, a^*_C)$. These students are only accepted at C as first-time students in the affiliated environment, and can never transfer. The resulting gain in welfare is:

$$B^R_{14} = \frac{1}{2}((a^*_C)^2 - (a^{**}_C)^2) > 0.$$

Group 8. Students with $a \in [0, a^{**}_C)$. These students are never enrolled. There are no changes in welfare for this group.

Putting it altogether, the certain welfare gains from opening a regional campus are

$$\begin{aligned} B^R_{Gain} = \frac{1}{2}[&(1 - \bar{w})((a^*_T)^2 - (a_{Min})^2)(b_M - 1)+ \\ &(\bar{w} - \underline{w})((a^*_T)^2 - (a_{Min})^2)(b_R - 1)+ \\ &(1 - \underline{w})((a_{Min})^2 - (a^{**}_R)^2)(b_R - 1) + ((a^*_C)^2 - (a^{**}_C)^2)], \end{aligned}$$

the certain welfare losses are

$$B^R_{Loss} = \frac{1}{2}[(\bar{w} - \hat{w})(1 - (a^*_T)^2)(b_R - b_M) + \underline{w}((a^{**}_T)^2 - (a^*_T)^2)(1 - b_M)\delta],$$

and ambiguous welfare effects are

$$B^R_{Ambig} = \frac{1}{2}(\hat{w} - \underline{w})(1 - (a^*_T)^2)(b_M \cdot delta + (1 - \delta) - b_R)).$$

CHAPTER 3

CHOOSING SIDES IN A TWO-SIDED MARKET

3.1 Introduction

A common assumption in the two-sided matching literature is that if the surplus generated by matching satisfies strict supermodularity, then the resulting stable matching will be positive assortative. This condition is often used in marriage market models as well as several labor matching models, including Kremer (1993)'s O-Ring theory. However, these models do not accurately capture labor markets with two distinct roles in which agents may be able to choose which role they prefer. For example, some doctors open their own practices, some financial analysts will enter their firm's management track, some professors will chair their department, and some entrepreneurs will start their own small business—but all such people likely have similarly qualified peers in their field who prefer to follow the lead of others. In this chapter, I model many-to-one labor markets that have a "lead" role and "support" role (which are filled by a manager and worker(s) respectively) that together generate output, in which agents have a pre-matching strategic choice over role.

The main contribution this chapter makes to the literature is adding a role choice to a many-to-one labor market with two sides—agents decide if they prefer to lead or support before the matching market is realized. I find that there exists a unique rational expectations equilibrium that induces a stable matching, and that the matching pattern is socially efficient. In equilibrium, both the matching pattern *and* the wage structure are unique. The latter is generally not the case when the solution concept requires only stability; the unique wage structure is driven by the pre-matching role choice. A condition stronger than strict supermodularity that I call *role supermodularity* determines the equilibrium matching pattern and wage structure, as positive assortative matching occurs if and only if the production function satisfies role supermodularity. A stronger condition is necessary because a high skilled agent is only willing to enter the market as a worker if she expects that she can profitably cluster with other high skilled agents. The theory predicts two kinds of wage differentials—differences in wages between agents of the same type in different roles, and differences in wages between agents of different types.

After the literature review in Section 3.2, I set up the model in Section 3.3 and solve for equilibrium outcomes. In Section 3.4, I provide comparative statics and discuss how wage differentials change in response to changes in underlying productivity. I show how the wage structure relates to observed trends in U.S. wage inequality, and discuss possible policy implications. I conclude in Section 3.5 by discussing directions for future work.

3.2 Literature Review

This chapter combines elements from the two-sided assignment model and the role assignment model.

In a two-sided assignment problem as in Shapley and Shubik (1971), agents are divided into two disjoint sets and match surplus is generated if agents belonging to different sets match with each other (e.g., men and women in the marriage market, firms and employees in the labor market, managers and workers in this chapter). I assume that match surplus is transferable via wages. See Chiappori (2020) for a full review of matching models with transfers, and Chapter 6 of Roth and Sotomayor (1992) for an overview of the literature on many-to-one matching. The chapter reassesses the assumption that strict supermodularity in inputs induces positive assortative matching, as introduced by Becker (1973) and often used in the two-sided matching literature.

The pre-matching role choice parallels the decisions agents face in investment and matching models with transferable utility (Chiappori, Iyigun, and Weiss (2009), Nöldeke and Samuelson (2015), and Zhang (2021)). Structurally, they are similar: a decision is made before matching (choice of role vs. an investment choice), interim outcomes are realized (each agent's role vs. changes in skill type), and then matching occurs. The first stage is non-cooperative, as agents strategize in anticipation of the matching market that they will face, while the second stage is a cooperative assignment problem. In the investment and matching framework, though, sides of the matching market are fixed and investment decisions change agents' skill levels; in contrast, I allow strategic choices to change the supply and demand of agents on both sides of the market while skill levels are fixed.

The model is also similar to social games (Jackson and Watts (2008), Jackson and Watts (2010)), which generalize the discrete marriage market problem. In a social game, players of different roles choose strategies and partners simultaneously. Unlike this paper, in a social game, a player's role cannot change (e.g., "men" and "women" in the marriage market", "firms" and "employees" in the labor market), but role in the social game is similar to type in this model in that it is fixed and under loose conditions, players in the scarcer role (in the social game) or type (in this paper) are able to coordinate on an advantageous outcome for the entire group. Both games also feature cooperative and non-cooperative elements that interact—individuals are strategic over who they are willing to match with to maximize their payoffs, but matching itself is cooperative.

Role assignment models (Kremer and Maskin (1997), Li and Suen (2001), McCann and Trokhimtchouk (2008), Anderson (2022)) are one-sided matching frameworks that analyze changes in worker sorting and their downstream effects on wage dispersion.[1] In role assignment models, the firm (acting as a social planner) first determines who is matched with whom, and then assigns roles within each match; in this chapter, the timing is reversed. Kremer and Maskin (1997) show that in the role assignment model, matching patterns become positive assortative as skill levels become dispersed, and mean skill level correlates positively with wage inequality. Li and Suen (2001) show that for sufficiently dispersed skill distributions, segregation by type and wage inequality depends on how the social planner chooses to sort the agents who are indifferent between managing a lower-skilled worker or working for a higher-skilled manager. My model captures a similar tension without requiring a social planner; the equilibrium matching pattern depends on whether high-skilled agents can profitably become a worker, or whether high-skilled agents always prefer to become a manager.

In role assignment, there are two standard assumptions that pull the matching pattern in opposite directions: (1) managers and workers are complements (i.e., a highly-qualified accountant should work in a junior position at a top firm), and (2) output is more sensitive to managerial skill (i.e., a highly-qualified accountant should work in a senior position at a lower-tier firm). I similarly impose

[1] I call it the role assignment model, following Anderson (2022). It has also been described as a two-step assignment problem (Gavilan 2012).

an assumption that the manager role is more sensitive to skill type, but unlike the standard role assignment environment, I do *not* restrict attention to supermodular production functions. Empirical justification for these assumptions is unclear. Due to data limitations and difficulty in quantifying productivity, empirical research on managerial impact is modest. There are multiple data issues: it is not clear how to choose the best measure of productivity, some types of productivity may be unobservable, and one would need rich data across many firms. However, existing papers validate the main assumptions. Lazear, Shaw, and Stanton (2015) show that in a technology-based service workplace, the average manager contributes more to output than the average worker. Bertrand and Schoar (2003) find that differences in corporate managerial practices are systematically and significantly related to differences in performance. Finally, Bloom and Van Reenen (2007) find that better managerial practices are significantly and positively related to higher productivity in manufacturing firms.

That said, assumptions on the production function are not innocuous and I do not claim results generalize to all labor markets. Unlike Eekhout and Kircher (2018), present a model of assortative matching in large firms, here I consider a setting that more closely aligns with small business ownership (e.g., a single owner employs a small number of workers and all agents perform a variety of tasks). However, results may also apply to larger firms in which internal distribution of human capital is important, such as technology-based service and/or innovation sectors, fields in which a high level of qualification is necessary to enter the market (e.g., law or academia), start-up companies, or sectors with a significant freelance presence. Finally, while a different application of matching models, Reynoso (2021) show conditions such that positive assortative matching among wives may emerge in marriage markets with polygamy which are comparable to the results I show in this chapter on labor market matching.

3.3 Model

The labor market is competitive with two employment roles, $r \in \{m, w\}$ such that a manager m must match with exactly $n \in \mathbb{N}$ workers for production to occur, where n is relatively small.[2]

[2] I consider a more specific model with endogenized $n \in \{1, 2\}$ in Appendix 3D.

Worker skill is additive. There is a unit mass of agents, all risk neutral, who are of a skill type $\theta \in \{H, L\}$ such that $H, L \in \mathbb{R}_{++}$ and $H > L$.[3] All agents have an outside option of 0. The measure of H-type agents is $M_H \in (0, \frac{1}{n+1})$. Denote θ_m the type of an arbitrary manager, and denote $\sum_{i=1}^{n} \theta_w^i$ an arbitrary worker composition.

The production technology is $f(\theta_m, \sum_{i=1}^{n} \theta_w^i) : \mathbb{R}_+^2 \to \mathbb{R}_+$ that satisfies three assumptions: monotonicity, inefficiency of mixed worker compositions, and managerial impact. The first assumption, monotonicity, imposes that increasing the number of H-type agents in the group increases productivity.

Assumption 1. ***Monotonicity.*** *f is monotone in both arguments:*

1. *$f(H, \sum_{i=1}^{n} \theta_w^i) > f(L, \sum_{i=1}^{n} \theta_w^i)$ for all $\sum_{i=1}^{n} \theta_w^i$, and*
2. *$f(\theta_m, H + \sum_{i=1}^{n-1} \theta_w^i) > f(\theta_m, L + \sum_{i=1}^{n-1} \theta_w^i)$ for all $\theta_m, \sum_{i=1}^{n-1} \theta_w^i$.*

The second assumption rules out the possibility that strictly mixed worker compositions are efficient and allows me to narrow down the possible worker compositions under consideration. For any manager, it is always more productive to either hire all H-type or all L-type workers rather than take a strictly mixed worker composition—if a manager is willing to hire one H-type worker, then she must also be willing to hire a second H-type worker, and so on. This is because marginal productivity is increasing in the number of H-type workers hired. In practice, therefore, I only need to consider four matching patterns: H-type manager with all H-type workers, H-type manager with all L-type workers, L-type manager with all H-type workers, and L-type manager with all L-type workers.

Assumption 2. ***Inefficiency of mixed worker compositions.*** *The marginal productivity of H-type workers is weakly increasing in the number of H-type workers:*

$$f(\theta_m, (\ell+1)H+(n-\ell-1)L) - f(\theta_m, \ell H+(n-\ell)L) \geq f(\theta_m, \ell H+(n-\ell)L) - f(\theta_m, (\ell-1)H+(n-(\ell-1))L)$$

[3] Working with a continuum of agents instead of discrete agents allows me to avoid markets such that an agent cannot match regardless of what role she chooses, such as if there are $n+1$ agents in the market. But even in the discrete agent setting, note that the aforementioned scenario can always be avoided after duplicating the market a sufficient number of times.

for all θ_m and $\ell \in \{1, 2, ..., n-1\}$.

Finally, the manager's role is more important to overall productivity than the worker composition. This implies that the "L-type manager with all H-type workers" matching pattern is always inefficient compared to the "H-type manager with all L-type workers" matching pattern.

Assumption 3. ***Managerial impact.*** *The marginal productivity from changing an L-type manager to an H-type manager is greater than changing the entire worker composition from L-type to H-type:*

$$f(H, \sum_{i=1}^{n} \theta_w^i) - f(L, \sum_{i=1}^{n} \theta_w^i) > f(\theta_m, nH) - f(\theta_m, nL)$$

for all $\sum_{i=1}^{n} \theta_w^i$.

Assumptions 2 and 3 are strong, especially as n becomes large. I discuss how to loosen these assumptions in Appendix 3B and show that that dropping them does not affect the structure of the main results.

Table 3.1: Summary of Commonly Used Notation

Notation	Explanation
M_H	Measure of H-type agents
$c(\theta)$	Cost to a θ-type agent of becoming a manager
P	1-to-n matching market
$P_{r\theta}$	Measure of θ-type agents in role r in P
$\mu(\theta, \ell)$	Measure of θ-type managers matched with ℓH and $(n-\ell)L$ workers
V	Wage vector
$v_{r\theta}$	Wage of a θ-type agent in role r given V

The game takes place over two stages. In the first stage, agents make strategic role choices. In the second stage, these choices resolve into a matching market P and a standard assignment game occurs.

1. **Strategic Stage:** Agents simultaneously make pre-matching strategic decisions over what role to enter into the market as. Becoming a manager has a known, type-dependent cost $c(\theta) \in \mathbb{R}_+$, and costs are relatively small compared to productivity.

   Let σ_θ be a type symmetric strategy, such that σ_H gives the fraction of H-type agents who chose to become a manager (and analogously for σ_L). Denote $\sigma = (\sigma_H, \sigma_L)$ an arbitrary strategy profile.

2. **Outcome Stage:** σ induces a matching market $P = (P_{mH}, P_{wH}, P_{mL}, P_{wL}) \in \mathbb{R}^4_+$ with transfers; in this setting, transfers take the form of wage determination. Denote $P_{r\theta}$ the measure of θ-type agents in role r (e.g., the measure of L-type workers in the matching market induced by σ is $P_{wL} = (1 - M_H)(1 - \sigma_L)$).

   Once P has formed, a cooperative, non-strategic assignment game occurs. A *market outcome* in the assignment game is a matching μ along with a wage vector V. Since worker composition can be expressed as a linear combination of skill types, I denote a matching μ as a function $\mu : \{L, H\} \times \{0, 1, ..., n\} \to [0, 1]$ such that $\mu(\theta, \ell)$ is the measure of θ-type managers matched with ℓ H-type workers and $(n - \ell)$ L-type workers. Denote V as a wage vector of up to four components, such that $v_{r\theta} \in \mathbb{R}_+$ is the wage of a θ-type agent in the role r whenever $\mu_{r\theta} > 0$.

Consider the assignment game that happens in the second stage. Given an arbitrary P, solutions to the assignment game are *stable market outcomes*, which is a pair (μ, V) that satisfies feasibility of the matching μ, consistency between μ and the corresponding wage vector V, and does not allow for blocking coalitions to form. First, I define a feasible matching.

Definition 1. *A* ***feasible*** *matching μ for a matching market P satisfies:*

1. *μ is such that all unmatched agents share the same role,*

2. *$\mu(\theta, \ell) \geq 0$ for all θ,*

3. *$\sum_{\ell=0}^{n} \mu(\theta, \ell) \leq P_{m\theta}$ for all θ,*

4. $\sum_{\ell=0}^{n} \ell(\mu(H,\ell) + \mu(L,\ell)) \leq P_{wH}$, *and*

5. $\sum_{\ell=0}^{n} (n-\ell)(\mu(H,\ell) + \mu(L,\ell)) \leq P_{wL}$.

Feasibility guarantees that (1) there are no unmatched agents who could find another unmatched agent on the other side of the market, (2) the measure of any manager-worker composition is strictly non-negative, (3) the total measure of θ-type managers matched not exceed the measure of θ-type managers available in the market, (4) the total measure of H-type workers matched does not exceed the measure of H-type workers available in the market, and (5) the total measure of L-type workers matched does not exceed the measure of L-type workers available in the market.

In addition to feasibility, stability additionally imposes structure on the relationship between μ and V.

Definition 2. *A* ***stable market outcome*** (μ, V) *for a matching market* P *is a feasible matching* μ *alongside a payoff vector* V *that satisfies:*

1. *Individual rationality:* $v_{r\theta} \geq 0$ *for all* r *and* θ.

2. *Pairwise efficiency: If* $\mu(\theta,\ell) > 0$, *then* $f(\theta, \ell H + (n-\ell)L) = v_{m\theta} + \ell v_{wH} + (n-\ell)v_{wL}$.

3. *Market efficiency:* $v_{m\theta} + \ell v_{wH} + (n-\ell)v_{wL} \geq f(\theta, \ell H + (n-\ell)L)$ *for all* θ, ℓ.

Individual rationality guarantees that all agents are willing to participate in the labor market. Pairwise efficiency ensures that wages are split so no productivity is wasted. Pareto efficiency gives that no blocking coalitions exist, as any coalitions are either no better off or are unsustainable given the wage demands that members of the coalition have. Put altogether, stability imposes two features: V is feasible and compatible with μ, and there do not exist any managers and groups of workers who all prefer to be matched with each other over their current assignment.

It has already been shown that for an arbitrary matching market P with wages (transfers), stable outcomes to the assignment game exist; while the stable matching is generally unique, it can be supported by a continuum of wage vectors (see Chiappori, Pass, and McCann (2016) and Chiappori (2020)). If f satisfies strict supermodularity, then the unique stable matching is positive assortative; otherwise, it is negative assortative. Because wage determination isn't unique, stability alone cannot generate unique predictions on what stable outcome(s) will occur in a competitive market setting. However, in this setting, P isn't fixed until agents have made their role choice; I show later in this section that this pre-matching role choice gives more structure to the potential wage vectors that can emerge.

The solution concept for the full game follows. I use a rational expectations equilibrium, as agents must have correct expectations about how their first stage role choices affect the stable market outcomes that occur in the second stage.

Definition 3. *A **rational expectations equilibrium** is a list $(\sigma^*, (\mu^*, V^*))$ that satisfies the following:*

1. *(μ^*, V^*) is a stable outcome in the matching market induced by σ^*.*

2. *σ^*_θ maximizes θ-type agents' expected wages minus costs incurred for all θ. If $P_{r\theta} > 0$ in the matching market induced by σ^*, then V^* explicitly defines the expected wage in the labor market, $v^*_{r\theta}$.*

   If $P_{m\theta} = 0$, then the wage that the θ-type agent expects to receive when individually deviating to becoming a manager is

$$v_{m\theta} = \max_{\sum_{i=1}^{n} \theta_w^i : P_{r\theta_w} > 0 \forall \theta_w^i} [f(\theta, \sum_{i=1}^{n} \theta_w^i) - \sum_{i=1}^{n} v^i_{w\theta_i}]. \tag{3.1}$$

   If $P_{w\theta} = 0$, then all workers are of the other type $\theta' \neq \theta$, so wage that the θ-type agent expects to receive when individually deviating to becoming a worker is

$$v_{w\theta} = \max_{\theta_m : P_{r\theta_m} > 0} [f(\theta_m, \theta + (n-1)\theta') - v_{m\theta_m} - (n-1)v_{w\theta'}]. \tag{3.2}$$

The first part of the definition is a consistency condition. If agents optimally play σ^* because they expect to face the stable market outcome (μ^*, V^*), then σ^* induces a matching market that not only can, but actually *does* sustain (μ^*, V^*) as a stable market outcome. This prevents cases where, for example, $\sigma_H = \sigma_L = 1$, but agents incorrectly expect to be matched in the second period. The second part is a utility maximizing condition. V^* along with Equations 3.1 and 3.2 together allow agents to have rational expectations on all wages they could possibly face, even if some type-role combinations are not in the market induced by σ^*. If $P_{r\theta} > 0$, then V^* explicitly defines $v_{r\theta}$; if not, then Equations 3.1 and 3.2 impose that θ-type agents determine what their wages from deviating to r would be by using pairwise efficiency.

Before solving for the equilibrium, consider the social planner's problem of assigning first roles and then matches.[4] $f(H, nH)$ is the most productive arrangement (f satisfies monotonicity and inefficiency of mixed worker compositions), but the opportunity cost of grouping H-type agents together is that the n H-type workers could have instead been managers to groups of L-type workers (f satisfies managerial impact). Hence, after taking into account the cost of becoming a manager, the social planner groups agents of the same type together if and only if

$$f(H, nH) + n \cdot f(L, nL) + n[c(H) - c(L)] > (n+1)f(H, nL). \tag{3.3}$$

I call this matching pattern *clustering*. If Equation 3.3 is not satisfied, then the social planner has all H-type managers become managers and matches them with n L-type workers, and assigns both roles to L-type agents such that the L-type workers who are unable to match with the relatively scarce H-type managers form clusters with an L-type manager. I call this matching pattern *specialization*.

The same condition determines the equilibrium outcome. To provide intuition for why this is the case before I formally state the result, consider a discrete example with $n = 1$, two H-type agents, four L-type agents, and no cost of entering as a manager. Consider the perspective of the H-type agents, who have three strategies available: $\sigma_H = \{0, \frac{1}{2}, 1\}$. Table 3.2 shows the possible matching patterns that each strategy can induce, excluding cases such that agents end up unmatched. I show formally when proving the main result that rational agents indeed prevent "unbalanced" markets from emerging in equilibrium. For each pair, the first is the manager type and second is the worker type (e.g., "HL" means a H-type manager matches with an L-type worker").

[4]Note that the social planner need not assign a stable market outcome, but she would pick one regardless.

Table 3.2: Example: σ_H and Possible Matchings

H-type strategy	Matching Patterns
$\sigma_H = 0$	LH LH LL
$\sigma_H = \frac{1}{2}$	HH LL LL
	HL LH LL
$\sigma_H = 1$	HL HL LL

L-type agents always mix strategies, implying that they are equally well off on both sides of the market, so by pairwise efficiency $v_{mL} = v_{wL} = \frac{1}{2}f(L,L)$ for all matching patterns. Given L-type wages, (1) $\sigma_H = 0$ is strictly dominated by $\sigma_H = 1$ since $f(H,L) > f(L,H)$, and (2) the HL LH LL matching is unstable as pairwise efficiency, Pareto efficiency, and the L-type wage condition cannot be satisfied simultaneously.

Hence the H-type agents face two possible outcomes: play $\sigma_H = \frac{1}{2}$ to induce clustering or $\sigma_H = 1$ to induce specialization. Their wages under clustering are, by the same logic as the L-type agent wage determination, $v_{mH} = v_{wH} = \frac{1}{2}f(H,H)$. By pairwise efficiency and the L-type wage determination, their wages under specialization are $v_{mH} = f(H,L) - \frac{1}{2}f(L,L)$. So H-type agents prefer to induce clustering if and only if

$$\frac{1}{2}f(H,H) \geq f(H,L) - \frac{1}{2}f(L,L),$$

which is equivalent to the SPP's cutoff condition in Equation 3.3 with $n = 2, c(H) = c(L) = 0$.

This intuition extrapolates into the general model: H-type agents' incentives align with the social planner's problem, so the same condition determines both the efficient and competitive, decentralized market outcomes.

Outside of knife-edge cases, a unique equilibrium always exists.[5] Equilibrium matching patterns are socially efficient.

1. If Equation 3.3 holds, then there exists a unique **clustering equilibrium**. The equilibrium strategies are $\sigma^*_{CE} = (\frac{1}{n+1}, \frac{1}{n+1})$. In the unique stable market outcome, all agents match with

[5] See Appendix 3C for details on the knife-edge cases.

their own type, and wages are

$$v^*_{m\theta} = \frac{1}{n+1}(f(\theta, n\theta) + n \cdot c(\theta)),$$
$$v^*_{w\theta} = \frac{1}{n+1}(f(\theta, n\theta) - c(\theta)).$$

2. If not, then there exists a unique **specialization equilibrium**. The equilibrium strategies are $\sigma^*_{SE} = (1, \frac{1-(n+1)M_H}{(n+1)(1+M_H)})$. In the unique stable market outcome, all H-type agents become managers, L-type agents mix such that the market clears, and the wages are

$$v^*_{mH} = f(H, nL) - \frac{n}{n+1}(f(L, nL) - c(L)),$$
$$v^*_{mL} = \frac{1}{n+1}(f(L, nL) + n \cdot c(L)),$$
$$v^*_{wL} = \frac{1}{n+1}(f(L, nL) - c(L)).$$

Appendix 3A has the full proof; I outline the argument here. Existence can be proven by construction. I first show the intuitive result that an equilibrium shouldn't induce an unbalanced market in which there are excess agents in one of the roles, as those agents can profitably deviate to the other side of the market. Given that the market is balanced, θ-type agents mix strategies if and only if expected wages net of costs on both sides of the market are the same. At least some L-type agents must always cluster for the market to clear, so $v_{mL} - c(L) = v_{wL}$ regardless of the matching pattern. Given that, utility-maximizing H-type agents consider whether to manage L-type agents (who demand lower wages) or to cluster with other H-type agents (a more productive arrangement). This trade-off makes the H-type agents' optimization problem equivalent to the social planner's problem, hence Equation 3.3 determines both the unique, stable matching pattern as well as the accompanying wage vector in equilibrium. Uniqueness follows by ruling out all other possibilities: H-type agents prevent inefficient matching patterns from emerging because they can always do better by disregarding what L-type agents do and clustering together.

To compare the equilibrium to the setting without a role choice, again set costs to 0 and let $n = 1$ (but continue to assume a continuum of agents). Then Equation 3.3 simplifies to

$$f(H, H) + f(L, L) > 2f(H, L) \qquad \text{(Role supermodularity)}$$

and positive assortative matching occurs in equilibrium if and only if role supermodularity is satisfied. I call the condition role supermodularity because the roles that an H-type agent is willing to take determines the equilibrium matching pattern. Role supermodularity is a stronger necessary condition for positive assortative matching than the standard strict supermodularity condition,

$$f(H,H) + f(L,L) > f(H,L) + f(L,H),$$

because of the role choice—H-type agents are never willing to become workers if her only choice is to match with an L-type manager. This is in contrast to the standard assignment model, where $f(L,H)$ matters because the matching market is pre-determined and H-type agents may end up a worker because she has no role choice.[6] Here, though, H-type agents are only willing to become workers if the high productivity of the H-type cluster outweighs the fact that L-type workers would demand a lower wage from her if she manages.

This theoretical prediction aligns with Adhvaryu, Bassi, Nyshadham, and Tamayo (2020)'s empirical study of a garment facturing firm in India. They find that negative assortative matching occurs even though the underlying production function displays complementarities between managers and workers, which is consistent with the hypothesis that a stronger condition than strict supermodularity (which can be interpreted informally as "inputs behaving more like complements than substitutes") such as role supermodularity (which informally requires that inputs behave *strongly* as complements) is needed to induce positive assortative matching in labor markets. In general, the model predicts that in labor markets in which agents must preemptively decide to enter as the lead role (e.g., entering the management track at a company may require extra training or external credentials), more complementarity between agents is needed to sustain positive clustering.

3.4 Wage Differentials and Productivity

Theorem 3.3 implies that two separate features describe wage inequality: (1) wage differentials between agents of different types in the same role (which are driven by differences in productivity and costs), and (2) wage differentials between agents of the same type in different roles (which are

[6]Consider, for a constrasting example, a marriage market. A high-skilled woman may prefer to marry a high-skilled man, but she is unable to if the fixed supply of such men is scarce, so she marries a low-skilled man or is unmatched, depending on her preferences. Hence $f(L,H)$ affects the matching pattern in the traditional marriage market setting.

driven by the ability to pass on the cost of entering as a manager to workers). The latter feature is straightforward to pin down: in both equilibria, whenever θ-type agents are on both sides of the market, $v_{m\theta} - v_{w\theta} = c(\theta)$.

In a clustering equilibrium, the wage differentials between agents of different types in the same role are

$$v^*_{mH} - v^*_{mL} = \frac{1}{n+1}[(f(H,nH) - f(L,nL)) + n(c(H) - c(L))] \text{ and}$$
$$v^*_{wH} - v^*_{wL} = \frac{1}{n+1}[(f(H,nH) - f(L,nL)) - (c(H) - c(L))].$$

Changes to $f(\theta, n\theta)$ and/or $c(\theta)$ affect only wages of θ-type agents, but affect both sides of the market. Given that, the main productivity-related reason that clustering may switch to specialization is growth in $f(H, nL)$, which shrinks marginal productivity of H-type clusters.

In a specialization equilibrium, the wage differential between H-type and L-type managers is

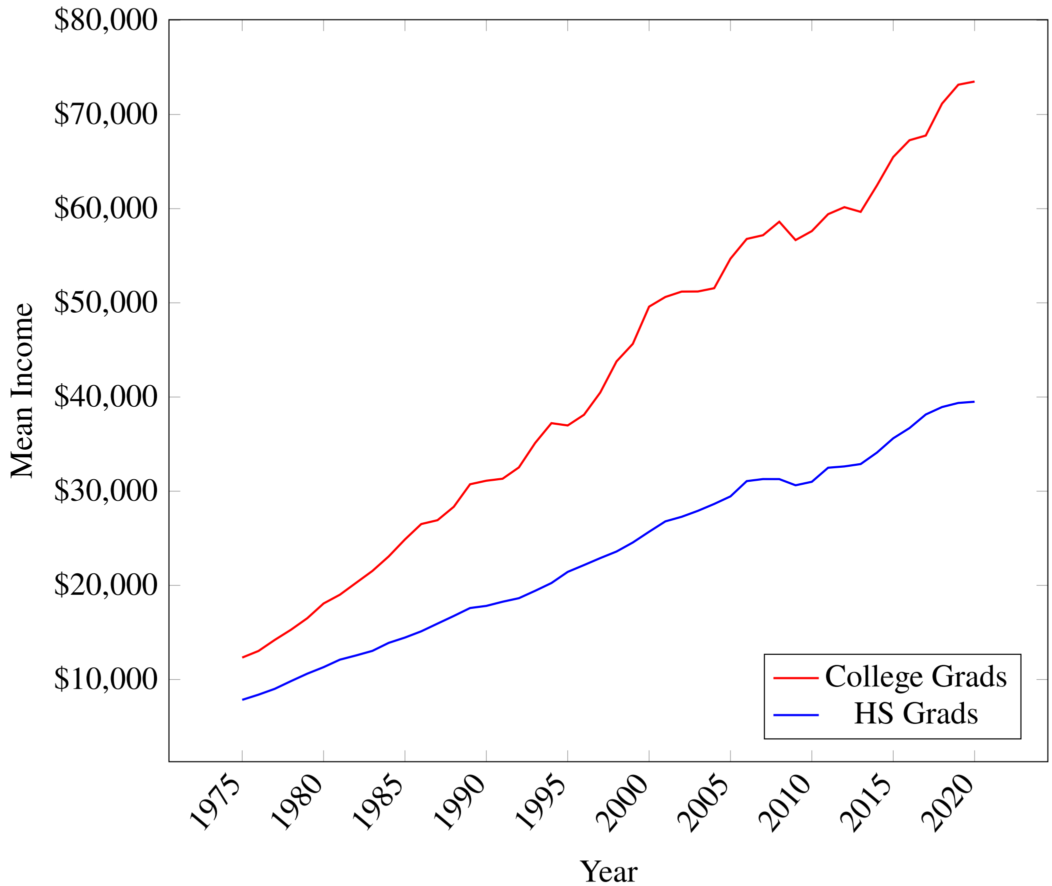
$$v^*_{mH} - v^*_{wH} = f(H,nL) - f(L,nL),$$

and $c(H)$ does not enter into the expression because H-type managers are unable to directly pass on $c(H)$ to workers, while managers of both types continue to pass a fraction of $c(L)$ onto L-type workers. Changes to $f(H, nL)$ and $c(H)$ affect only H-type managers' wages, while changes to $f(L, nL)$ or $c(L)$ affect all agents in the market: if L-type clusters become more productive, all L-type agents' outcomes improve while H-type managers are worse off, while increasing L-type cost works in the opposite direction. This suggests there are two productivity-related reasons that specialization may switch to clustering: (1) growth in $f(H, nH)$ or (2) growth in $f(L, nL)$.

How do these comparative statics match up with observed wage patterns in the U.S.? Data from U.S. Census Bureau (2022) as shown in Figure 3.1 shows that the mean income of college graduates has increased faster than that of high school graduates over the last four decades.[7] I also show the time trend in standard errors of mean income in Figure 3.2. In my model, agents of the same type have variance in wages due to $c(\theta)$. Interestingly, the pattern of dispersion in

[7]Individuals in the dataset are 18+.

Figure 3.1: Mean Income by Level of Education

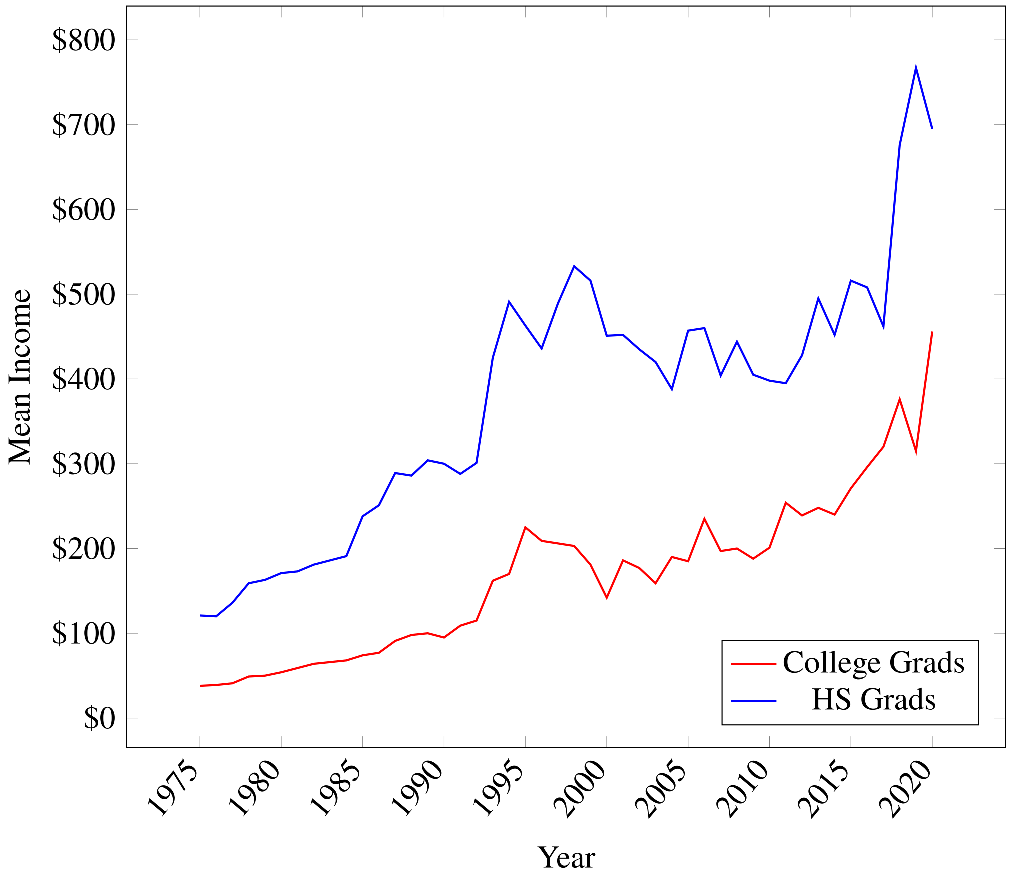


wages across agents of the same education level is similar—suggesting that the cost of becoming a manager is increasing in general—but has consistently been higher for high school graduates. This implies that the cost of becoming a manager is larger for L-type agents, perhaps because they face a larger opportunity cost or higher risk in entering into the market as a manager.

That college educated adults are experiencing faster income growth and variance in both groups' incomes is increasing over time are together consistent with the hypothesis that the overall landscape of the U.S. labor market has moved towards clustering equilibria over time. Specialization is ruled out because variance in college graduates' incomes would be steady in this case. This may be driven by fields like technology-based start-ups or finance, in which productivity has increased the fastest among high-skilled matches.[8] This is also consistent with Song, Price, Guvenen, Bloom, and von Wachter (2019), who find that between 1978 and 2013, increased within-firm positive

[8] I borrow the term "clustering" equilibrium from Silicon Valley tech clusters.

Figure 3.2: Standard Error of Mean Income by Level of Education



sorting correlates with increases in between-firm wage disparity.[9]

The model has a notable policy implication: in some cases, making it less costly for low-skilled agents to enter in the lead role can simultaneously increase productivity and decrease wage differentials. As previously noted, H-type clustering is always the most productive assignment disregarding costs; however, it may not occur in equilibrium because H-type agents prefer to manage L-type workers who demand lower wages. Suppose that firms and/or policymakers want to push the matching pattern towards clustering—for instance, if the most technically demanding projects are expected to generate positive externalities. If so, then they should subsidize L-type agents who wish to enter in the lead role: by decreasing $c(L)$, role supermodularity is easier to attain because v^*_{wL} is always decreasing in $c(L)$ and the H-type manager's trade-off between higher productivity and paying higher wages to H-type workers shifts towards the former. This simultaneously pushes

[9]Card, Heining, and Kline (2013) find similar patterns in Western Germany from 1985 to 2009.

the matching pattern towards the most "high powered" arrangement, clustering, and decreases wage differentials.

As an example, consider web and/or mobile app development, a sector that has multiple entry points depending on experience and training. Also, cost to receive credentials is increasing in type—aspiring developers without experience may attend short-term coding bootcamps to get their foot in the door of an entry-level job, but high profile jobs may require years of university education.[10] For example, senior mobile developers at Google require a bachelor's degree at minimum and consider an advanced degree substitutable with professional experience.[11] This model advises policymakers to subsidize short-term programs like coding bootcamps rather than providing scholarships for advanced degrees in computer science. By making entry-level coders better off, higher-level coders will prefer to group together.

3.5 Conclusion

I present a two-sided matching model of a labor market in which agents can choose their role. The pre-matching strategic decision causes positive assortative matching to become more difficult to attain in equilibrium compared to the standard assignment model; this is primarily driven by H-type agents' incentives. The production technology must satisfy role supermodularity, a stronger condition than the standard strict supermodularity condition, for positive assortative matching to occur.

However, these results follow from stylized assumptions on how matching works. For example, I impose that managers must match with *exactly* n workers. In Appendix 3D, I show that n can be endogenized when considering specific functional forms. I also impose two strong properties on the production function, inefficiency of mixed worker compositions, and the importance of managerial impact. However, these properties can be dropped while maintaining the overall structure of results, as I discuss in Section 3B.

A natural extension of the chapter is to add skill types. To conclude, I give some informal

[10]See https://web.archive.org/web/20240328130626/https://brainstation.io/ for examples of coding bootcamps. Most are several weeks long.

[11]See https://web.archive.org/web/20221101032812/https://careers.google.com/jobs/results/104727998504018630-senior-software-engineer-mobile-android-geo/?distance=50&q=Senior%20Software%20Engineer.

discussion about how I anticipate results would extrapolate to a model with more skill types. To simplify discussion, I assume $n = 1$ and only discuss matching patterns. Suppose there are three skill types, $H > M > L$, and f satisfies the following: (i) if $\theta_m > \theta'_m$, then $f(\theta_m, \bar{\theta}_w) > f(\theta'_m, \bar{\theta}_w)$, (ii) if $\theta_w > \theta'_w$, then $f(\bar{\theta}_m, \theta_w) > f(\bar{\theta}_m, \theta'_w)$, and (iii) if $\theta > \theta'$, then $f(\theta, \theta') > f(\theta', \theta)$.[12] I anticipate five possible equilibrium matching patterns:

1. Full clustering: All types match with their own type.
2. H-clustering: H-type agents match with their own type. M-type managers match with L-type workers.
3. M-clustering: H-type managers match with L-type workers. M-type agents match with their own type.
4. L-clustering: H-type managers match with M-type workers. L-type agents match with their own type.
5. Impure specialization: H-type managers match with M-type workers. M-type managers match with L-type workers.

By adding the assumption $f(H, M) - f(M, M) > f(H, L) - f(M, L)$, (i.e., M-type clustering is inefficient; this rules out M-clustering), the production assumptions are analogous to the setting in Anderson (2022), which generalizes Kremer and Maskin (1997). Moreover, the proposed equilibria matching patterns align with the matching pattern that Anderson (2022) derives: skill types in a connected interval form groups, and within those groups, specialization occurs.[13] This suggests that as more and more skill types are added, equilibria matching patterns in a two-sided framework may remain efficient.

[12]Unlike the case with two types, these assumptions are not enough for a complete ordering. For example, $f(L, H)$ cannot be compared to $f(M, M)$.

[13]Anderson (2022) terms this matching pattern "positive clustering". Unfortunately, our shared usage of the term "clustering" refer to opposite scenarios. In Anderson (2022)'s model, a cluster is a group of (potentially multiple) skill types. In my model, a cluster is a single skill type that prefers to match with own type if possible.

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APPENDIX 3A

MAIN THEOREM - PROOF

I first establish two basic facts about any potential equilibrium, then prove the main result.

Lemma 6. *If σ induces an unbalanced market, then σ cannot be part of an equilibrium.*

Proof. If the market is unbalanced, then some agents will be unmatched and receive 0. All unmatched agents must share the same role r. Suppose some of the unmatched agents are θ-type. Since θ-type agents are perfect substitutes for each other and the market is competitive, $v_{r\theta} = 0$ in any stable market outcome. Any θ-type agent in role r can do strictly better by deviating to the other role and matching with another θ-type agent who was previously unmatched with certainty, as by Equations 3.1 and 3.2, her wage from deviating is $f(\theta, \theta) > 0$. □

Lemma 7. *Let σ induce a balanced matching market. If $\sigma_\theta \in (0, 1)$, then a necessary condition for σ to be part of an equilibrium is that the expected payoff vector it induces must satisfy $v_{m\theta} - c(\theta) = v_{w\theta}$.*

Proof. As the market is balanced, agents are matched with certainty. A θ-type agent is indifferent between pure strategies if and only if $v_{m\theta} - c(\theta) = v_{w\theta}$. □

Outside of knife-edge cases, a unique equilibrium always exists.[1] Equilibrium matching patterns are socially efficient.

1. If Equation 3.3 holds, then there exists a unique **clustering equilibrium**. The equilibrium strategies are $\sigma^*_{CE} = (\frac{1}{n+1}, \frac{1}{n+1})$. In the unique stable market outcome, all agents match with their own type, and wages are

$$v^*_{m\theta} = \frac{1}{n+1}(f(\theta, n\theta) + n \cdot c(\theta)),$$
$$v^*_{w\theta} = \frac{1}{n+1}(f(\theta, n\theta) - c(\theta)).$$

[1] See Appendix 3C for details on the knife-edge cases.

2. If not, then there exists a unique **specialization equilibrium**. The equilibrium strategies are $\sigma^*_{SE} = (1, \frac{1-(n+1)M_H}{(n+1)(1+M_H)})$. In the unique stable market outcome, all H-type agents become managers, L-type agents mix such that the market clears, and the wages are

$$\begin{aligned}
v^*_{mH} &= f(H, nL) - \frac{n}{n+1}(f(L, nL) - c(L)),\\
v^*_{mL} &= \frac{1}{n+1}(f(L, nL) + n \cdot c(L)),\\
v^*_{wL} &= \frac{1}{n+1}(f(L, nL) - c(L)).
\end{aligned}$$

Proof. Denote (μ_{CE}, V_{CE}) the stable market outcome in the clustering equilibrium, and (μ_{SE}, V_{SE}) analogously for the specialization equilibrium.

Step 1. I show existence of the proposed equilibria. Suppose that f satisfies Equation 3.3. I claim that (μ_{CE}, V_{CE}) satisfies the consistency condition of the rational expectations equilibrium (REE). σ^*_{CE} induces a matching market P_{CE} such that every 1 in $n+1$ agents becomes a manager and all agents match with their own type. By definition of stability, the wage structure must satisfy pairwise efficiency,

$$f(\theta, n\theta) = v_{m\theta} + n \cdot v_{w\theta}.$$

Since θ-type agents are on both sides of the market in P_{CE}, Lemma 7 additionally imposes that

$$v_{m\theta} = v_{w\theta} - c(\theta).$$

Solving the system, V_{CE} has four components,

$$\begin{aligned}
v^*_{m\theta} &= \frac{1}{n+1}(f(\theta, n\theta) + n \cdot c(\theta)),\\
v^*_{w\theta} &= \frac{1}{n+1}(f(\theta, n\theta) - c(\theta)).
\end{aligned}$$

The proposed wages immediately satisfy individual rationality. To see that Pareto efficiency along with wage maximization is satisfied, recall that by MP1, I need only compare between clusters and specialized matches. Suppose for a contradiction that an H-type agent of either role along with n L-type workers have an incentive to rematch with each other. Since the market is large and competitive, L-type workers continue to make a wage $\frac{1}{n+1}(f(L, nL) - c(L))$, so it must be the case that the rematched H-type manager is better off. [2] Since her payoff is $f(H, nL)$ after subtracting off wages paid to her n L-type coworkers, for the rematch to be profitable, it must be the case that

$$f(H, nL) - \frac{n}{n+1}(f(L, nL) - c(L)) - c(H) > \frac{1}{n+1}(f(H, nH) + n \cdot c(H)) - c(H)$$
$$\Rightarrow (n+1)f(H, nL) > f(H, nH) + n \cdot f(L, nL) + n[c(H) - c(L)]$$

but this contradicts that f satisfies Equation 3.3. Hence (μ_{CE}, V_{CE}) is the unique stable, wage-maximizing outcome of the market P_{CE} induced by σ^*_{CE} in rational expectations.

Now suppose that f does not satisfy Equation 3.3. σ^*_{SE} induces a matching market P_{SE} such that all H-type agents become managers, and L-type agents are mixed between managers and workers such that the market is balanced. By definition of stability, wage structure V_{SE} must satisfy pairwise efficiency,

$$f(L, nL) = v_{mL} + n \cdot v_{wL}$$
$$f(H, nL) = v_{mH} + n \cdot v_{wL}.$$

Since L-type agents are on both sides of the market in P_{SE}, Lemma 7 additionally imposes that

$$v_{mL} = v_{wL} - c(L).$$

Solving the system,

$$v^*_{mH} = f(H, nL) - \frac{n}{n+1}(f(L, nL) - c(L)),$$
$$v^*_{mL} = \frac{1}{n+1}(f(L, nL) + n \cdot c(L)),$$
$$v^*_{wL} = \frac{1}{n+1}(f(L, nL) - c(L)).$$

The proposed wages immediately satisfies individual rationality. To see that Pareto efficiency is satisfied, recall that by MP1, I need only compare between clusters and specialized matches. Since H-type agents are only managers, there are no alternate matching configurations to consider, hence

[2] Note that I compare the H-type manager who deviates to the H-type manager who doesn't, but it would be equivalent to also compare her to the H-type worker who doesn't deviate, after readjusting for when $c(H)$ is incurred.

Pareto efficiency is satisfied. (μ_{SE}, V_{SE}) is the unique stable, wage-maximizing outcome of the market P_{SE} induced by σ^*_{SE} in rational expectations.

Step 2. I show the proposed equilibria is unique in the next two steps. In Step 2, I show that $\sigma_H = 1$ cannot be an equilibrium if f satisfies Equation 3.3. The matching pattern is as in (3) above, but pairwise efficiency and Lemma 7 together can hold, and we arrive at the wage structure as in the specialization case. This strategy is strictly dominated by $\sigma^*_H = \frac{1}{n+1}$ and matching with each other (leaving some L-type agents unmatched). To see this, suppose not. Then

$$\begin{aligned} f(H, nL) - \frac{n}{n+1}(f(L, nL) - c(L)) &> \frac{1}{n+1}(f(H, nH) + n \cdot c(H)) \\ \Rightarrow (n+1) f(H, nL) &> f(H, nH) + n \cdot f(L, nL) + n[c(H) - c(L)], \end{aligned}$$

so the assumption that Equation 3.3 holds is contradicted.

It can similarly be shown that $\sigma_H = \frac{1}{n+1}$ is not an equilibrium when f does not satisfy Equation 3.3.

Step 3. Finally, I rule out all other possible equilibria. Suppose for a contradiction that for some f, there exists $\sigma_H \neq \{\frac{1}{n+1}, 1\}$ that is part of an REE. Note that in all cases that follow, by Lemma 6 and the scarcity of H-type agents, σ_L is fully mixed (due to the scarcity of H-type agents) and chosen such that the the matching induced by (σ_H, σ_L) is balanced. Three cases follow:

Case 1. Suppose $\sigma_H = 0$ is part of an REE. L-type managers either match with n H-type workers or n L-type workers. Then by pairwise efficiency and Lemma 7, L-type agents' wages are $v_{mL} = \frac{1}{n+1}(f(L, nL) + n \cdot c(L))$ and $v_{wL} = \frac{1}{n+1}(f(L, nL) - \cdot c(L))$. Also by pairwise efficiency, $v_{wH} = \frac{1}{n}[f(L, nH) - v_{mL}]$. Given the stable market outcome in this matching market, I claim that this strategy is strictly dominated $\sigma_H = \frac{1}{n+1}$ and clustering.[3] To see this, suppose that H-types' wages under $\sigma_H = 0$ are larger than in a cluster:

$$\begin{aligned} \frac{1}{n}[f(L, nH) - \frac{1}{n+1}(f(L, nL) + n \cdot c(L))] &\geq \frac{1}{n+1}(f(H, nH) - c(H)) \\ \Rightarrow f(L, nH) - f(L, nL) &\geq n[f(H, nH) - f(L, nH) - n[c(H) - c(L)], \end{aligned}$$

[3]Note that $\sigma_H = \frac{1}{n+1}$ along clustering is only stable if f satisfies strict supermodularity, but it dominates $\sigma_H = 0$ nonetheless, and when f does not satisfy strict supermodularity that strategy may in turn be dominated by $\sigma_H = 1$.

which contradicts the managerial impact property of f for relatively small costs.

Case 2. Suppose $\sigma_H \in (0, \frac{1}{n+1})$ is part of an REE. H-type managers match with groups of n H-type workers, and L-type managers match with groups of n H-type workers or groups of n L-type workers. But then pairwise efficiency and Lemma 7 cannot simultaneously hold, so there is no stable (μ, V) that is consistent with σ_H.

Case 3. Suppose $\sigma_H \in (\frac{1}{n+1}, 1)$ is part of an REE. H-type managers match with groups of n H-type workers or groups of n L-type workers, and L-type managers work with groups of n L-type workers. But then pairwise efficiency and Lemma 7 cannot simultaneously hold, so there is no stable (μ, V) that is consistent with σ_H.

□

APPENDIX 3B

LOOSENING ASSUMPTIONS ON f

Recall that I impose three properties on f: monotonicity, inefficiency of mixed worker compositions, and managerial impact. While monotonicity is a reasonable assumption in general, the other two may not be, especially as n gets large (recall that I also assume in this chapter that n is relatively small). For example, for n sufficiently large, one might expect to eventually see diminishing marginal returns to adding more H-type workers to a group. Similarly, it is likely more realistic to assume that having enough H-type workers matched together can "outweigh" the lower managerial impact of an L-type manager when n is large enough. I show in this section that these two assumptions can individually be dropped without losing the overall structure of results. A unique equilibrium still exists, but the form of the "clustering" equilibria will be different.

First, suppose that I drop inefficiency of mixed worker compositions. Then for all f, there exists some $\bar{n} \in \{0, 1, 2, ..., n\}$ such that the marginal productivity of adding one more H-type worker is strictly larger than adding an H-type manager to an L-type worker cluster below $\bar{n}$ but smaller above $\bar{n}$. Denote this mixed worker composition as "impure clustering". Then there are two types of matches to compare between, "impure clustering" and "pure specialization", and the "impure role supermodularity" condition is

$$f(H, \bar{\theta}_w) + \frac{\bar{n}}{n+1} f(L, nL) + \bar{n}[c(H) - c(L)] > (\bar{n} + 1) f(H, nL). \tag{3B.1}$$

All results from the main theorem follow, adjusting for the new wage structure under "impure clustering".

Proposition 11. *Let f satisfy monotonicity and managerial impact. Outside of knife-edge cases, a unique equilibrium always exists. Equilibrium matching patterns are socially efficient.*

1. If Equation 3B.1 holds, then there exists a unique **impure clustering equilibrium**. The equilibrium strategies are $\sigma^*_{CE} = (\frac{1}{\bar{n}+1}, \frac{1-(n-\bar{n})}{n+1})$. In the unique stable market outcome, "impure clustering" occurs, in which H-type managers match with $\bar{n}$ H-type workers and

$n - \bar{n}$ L-type workers, and L-type managers match with their own types. The wages are

$$\begin{aligned}
v^*_{mH} &= \frac{1}{\bar{n}+1}[f(H, \bar{\theta}_w) + \bar{n}c(H) - \frac{n-\bar{n}}{n+1}[f(L, nL) - c(L)],\\
v^*_{wL} &= \frac{1}{\bar{n}+1}[f(H, \bar{\theta}_w) - c(H) - \frac{n-\bar{n}}{n+1}[f(L, nL) - c(L)],\\
v^*_{mL} &= \frac{1}{n+1}(f(L, nL) + n \cdot c(L)),\\
v^*_{wL} &= \frac{1}{n+1}(f(L, nL) - c(L)).
\end{aligned}$$

2. If not, then there exists a unique **pure specialization equilibrium**. The equilibrium strategies are $\sigma^*_{SE} = (1, \frac{1-(n+1)M_H}{(n+1)(1+M_H)})$. In the unique stable market outcome, all H-type agents become managers, L-type agents mix such that the market clears, and the wages are

$$\begin{aligned}
v^*_{mH} &= f(H, nL) - \frac{n}{n+1}(f(L, nL) - c(L)),\\
v^*_{mL} &= \frac{1}{n+1}(f(L, nL) + n \cdot c(L)),\\
v^*_{wL} &= \frac{1}{n+1}(f(L, nL) - c(L)).
\end{aligned}$$

The proof is exactly as in the main theorem, substituting the impure clustering wages and impure role supermodularity condition for the pure forms as in the main text.

Second, suppose that I drop the managerial impact property. Recall that in the main theorem, I show uniqueness by showing no other σ_H other than the proposed ones can be part of an equilibrium. Now, though, Step 3 - Case 1 may no longer follow. The "clustering" equilibrium will always be unique, but its form will additionally depend on whether

$$f(L, nH) - f(L, nL) \geq n[f(H, nH) - f(L, nH)] \tag{3B.2}$$

holds or not. If not, then the main theorem still holds. But if Equation 3B.2 holds, then $\sigma_H = 0$ now dominates $\sigma_H = \frac{1}{n+1}$ and H-type agents are better off becoming workers and match with an L-type manager. The equilibrium will still be unique, but in the main theorem, the clustering equilibrium is broken into two sub-cases, H-type *worker-only* clustering and **pure role clustering**.

Proposition 12. *Let f satisfy monotonicity and inefficiency of mixed worker compositions. Outside of knife-edge cases, a unique equilibrium always exists. Equilibrium matching patterns are socially efficient.*

1. *If Equation 3.3 holds, then there exists a unique* ***clustering equilibrium****.*

   Case 1. *If Equation 3B.2 additionally holds, then the equilibrium strategies are $\sigma^*_{CWE} = (0, M_H + \frac{1}{n+1}(1-2M_H))$. In the unique stable market outcome, all H-type agents become workers, L-type agents mix such that the market clears, and the wages are*

$$v^*_{wH} = \frac{1}{n}(f(L,nH) - \frac{1}{n+1}(f(L,nL) + n \cdot c(L)))$$
$$v^*_{mL} = \frac{1}{n+1}(f(L,nL) + n \cdot c(L)),$$
$$v^*_{wL} = \frac{1}{n+1}(f(L,nL) - c(L)).$$

   Call this the ***worker-only*** *clustering equilibrium.* ***Case 2.*** *If not, then the equilibrium strategies are $\sigma^*_{CE} = (\frac{1}{n+1}, \frac{1}{n+1})$. In the unique stable market outcome, all agents match with their own type, and wages are*

$$v^*_{m\theta} = \frac{1}{n+1}(f(\theta,n\theta) + n \cdot c(\theta)),$$
$$v^*_{w\theta} = \frac{1}{n+1}(f(\theta,n\theta) - c(\theta)).$$

   Call this the ***pure role clustering equilibrium****.*

2. *If not, then there exists a unique* ***specialization equilibrium****. The equilibrium strategies are $\sigma^*_{SE} = (1, \frac{1-(n+1)M_H}{(n+1)(1+M_H)})$. In the unique stable market outcome, all H-type agents become managers, L-type agents mix such that the market clears, and the wages are*

$$v^*_{mH} = f(H,nL) - \frac{n}{n+1}(f(L,nL) - c(L)),$$
$$v^*_{mL} = \frac{1}{n+1}(f(L,nL) + n \cdot c(L)),$$
$$v^*_{wL} = \frac{1}{n+1}(f(L,nL) - c(L)).$$

The proof is the exactly as in the main theorem, except that Step 1 and Step 3 - Case 1 are reversed in the case of the H-type worker-only clustering equilibrium.

APPENDIX 3C

EQUILIBRIA IN THE KNIFE-EDGE CASE

Suppose $f(H, nH) + n \cdot f(L, nL) + n[c(H) - c(L)] = (n+1) f(H, nL)$. Both the clustering and specialization equilibria can occur, and so can equilibria where the matching pattern is mixed between the clustering and specialization cases. To see this, set $n = 1$ and costs to 0. Agents expect to face the same wage regardless of role, $v_\theta = \frac{1}{2} f(\theta, \theta)$. Since f satisfies $f(H, H) + f(L, L) = 2 f(H, L)$, $v_H = \frac{1}{2} f(H, H) = f(H, L) - \frac{1}{2}$. Then any $\sigma_H \in [\frac{1}{2}, 1]$ can be sustained in equilibrium. To give an example, let $\sigma_H = \frac{3}{4}$ and σ_L be such that the market is balanced. Then a measure $\frac{1}{4} M_H$ of H-type managers match with H-type workers, a measure $\frac{1}{2} M_H$ of H-type managers match with L-type workers, and any remaining L-type workers match with L-type managers. No agents have an incentive to deviate, and the payoff vector is sustainable in rational expectations, as the matching and payoff vector together are a stable market outcome in the matching market induced by the agents' strategies.

Because of the multiplicity in equilibria that can arise, I rule out the knife-edge case altogether as a matter of convenience.

APPENDIX 3D

ENDOGENIZING n

In the main model, one manager must match with exactly n workers. Now let $n = \{1, 2\}$ be endogenous. The intuition behind Theorem 3.3 is that H-type agents compare whether they are better off under clustering or specialization and choose their strategy accordingly. However, if a manager can choose up to n workers to match with, then the manager now compares up to four different possible matching patterns: one worker clustering, one worker specialization, two worker clustering, and two worker specialization.

Fix $L = 1$, $M_H \in (0, \frac{1}{3})$, relax the assumption that mixed coworker groups are inefficient, and let the production technology to be constant returns to scale Cobb-Douglass: $f(\theta_m, \theta_W) = \theta_m^{\alpha}\theta_W^{1-\alpha}$ for $\alpha \in (\frac{1}{2}, 1)$. Recall that constant returns to scale Cobb-Douglas production functions have the property that if inputs are scaled up by a given factor, then productivity is scaled up by the same factor; although the comparison is not quite one-to-one as only additional workers can be added, a key question of interest is whether it is always preferable to add as many workers as possible in this setting. I am most interested in comparing one worker clustering with two worker specialization and the implications on wage inequality.

Comparing the clustering matching patterns, one worker clustering is always more efficient than two worker clustering for both types, since

$$Surplus(1cl) = \frac{1}{2}(M_H H + (1 - M_H))$$
$$Surplus(2cl) = \frac{2^{1-\alpha}}{3}(M_H H + (1 - M_H)).$$

Also, all agents prefer one worker clustering over two worker clustering, since $v_{\theta}^{1cl} = \frac{1}{2}\theta > \frac{2^{1-\alpha}}{3}\theta = v_L^{2cl}$ for all $\alpha \in (\frac{1}{2}, 1)$.

Next, I compare the specialization matching patterns. By the above, it is efficient and better off for excess L-type agents to form one worker clusters. Comparing the surpluses under one and two worker specialization,

$$Surplus(1sp) = M_H H^\alpha + \frac{1}{2}(1 - 2M_H)$$
$$Surplus(2sp) = 2^{1-\alpha} M_H H^\alpha + \frac{1}{2}(1 - 3M_H)$$
$$= M_H H^\alpha + \frac{1}{2}(1 - 2M_H) + M_H(H^\alpha(2^{1-\alpha} - 1) - \frac{1}{2}),$$

so two-worker specialization is more efficient if and only if $H^\alpha(2^{1-\alpha} - 1) > \frac{1}{2}$. As for H-types, $v_H^{2sp} = 2^{1-\alpha}H^\alpha - 1$ and $v_H^{1sp} = H^\alpha - \frac{1}{2}$, so H-type agents prefer two worker specialization over one worker specialization if it is also more efficient. Since I am interested in comparing two worker specialization with one worker clustering, impose the condition and set $H^\alpha(2^{1-\alpha} - 1) > \frac{1}{2}$.

Finally, I compare one worker clustering and two worker specialization. Since

$$Surplus(2sp) = 2^{1-\alpha} M_H H^\alpha + \frac{1}{2}(1 - 3M_H) = 2^{1-\alpha} M_H H^\alpha + \frac{1}{2}(1 - M_H) - M_H$$
$$Surplus(1cl) = \frac{1}{2}(M_H H + (1 - M_H)),$$

one worker clustering is more efficient if and only if $2^{1-\alpha} M_H H^\alpha - M_H < \frac{1}{2} M_H H \Rightarrow H + 2 > 2^{2-\alpha} H^\alpha$.

The wage rate for L-type agents is the same in both cases, $v_L^{1cl} = v_L^{2sp} = \frac{1}{2}$. As for H-types, $v_H^{1cl} = \frac{1}{2}H$ and $v_H^{2sp} = 2^{1-\alpha}H^\alpha - 1$, so the market outcome is one worker clustering if and only if $H + 2 > 2^{2-\alpha}H^\alpha$, hence the market outcome is efficient. Since $2^{2-\alpha}$ is bounded from above by $2^{1.5} < 3$, the condition holds whenever role supermodularity with $n = 2$ holds.

To summarize, there are two possible matching patterns in equilibrium. If $H + 2 > 2^{2-\alpha}H^\alpha$, then the market outcome is that θ-type managers match with one θ-type worker. If not, then the market outcome is that one H-type manager matches with two L-type workers, and excess L-type agents form one-worker clusters.

Example 1. *Let $f(\theta_m, \theta_W) = \theta_m^{\frac{3}{4}}\theta_W^{\frac{1}{4}}$. To guarantee that mixed worker compositions remain inefficient, fix $H \in (0, 30]$.*

Let $H = 1.5$. If the manager must hire two workers, then two worker specialization is the efficient market outcome. However, if the manager can hire either one or two workers, then two worker clustering is the efficient market outcome. Furthermore, the same is true for any $H \in (23, 30]$.

As α changes, the intervals of H that guarantee one worker clustering occurs instead of two worker specialization remains similar: if H is sufficiently large and if $H \in (1, 2)$. That is, if H-types are not that much more productive or significantly more productive than L-types, it is better to have H-type works together and take advantage of the fact that the H-type manager is most productive with an H-type worker. However, if H is in a "middle ground", then specialization becomes more productive because worker skill is additive.