

CHANGE THE STORY, CHANGE THE CURRICULUM:
THE CURRICULUM-AS-STORY METAPHOR AS A FLEXIBLE LENS
FOR INTERPRETING CURRICULAR (IN)COHERENCE
FROM STUDENTS' PERSPECTIVES AND BEYOND

By

Brady Anthony Tyburski

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ABSTRACT

A common educational assumption is that coherence is a pre-requisite for a “good” curriculum. Indeed, in mathematics education this perspective has persisted both nationally and internationally as a foundational principle for curriculum design, reform, and evaluation. While curricular coherence is often unquestioningly accepted as desirable for student learning, some researchers have urged caution, arguing that “curricular coherence” is loosely defined with no widespread agreement over its meaning. Yet, disciplinary, logico-rational forms of coherence (i.e., retrospective expert perspectives) tend to dominate curricular discourses in mathematics education, often in ways that position these disciplinary forms of coherence as objective evaluations of curricula. Other perspectives on what it means for curricula to be “coherent”—particularly those of students—are rarely centered, which has epistemological as well as ethical consequences for who/what is positioned as coherent (i.e., “ideal”) and who/what is positioned as *incoherent* (i.e., abnormal, aberrant, incomplete). This binary imposes a distribution of “sensible” mathematics learning, thereby perpetuating a harmful culture of exclusion in mathematics education.

In this dissertation, I critically investigate curricular coherence in mathematics education by interrogating the notion of coherence itself and problematizing the dominance of a singular perspective on coherence. To do so, I conceptualize curriculum as a storied artform and view coherence as an individual’s holistic aesthetic judgement of curricular stories. These judgements are highly subjective and may vary from person to person as well as discipline to discipline, destabilizing the myth that curricular coherence is an objective evaluation with a singular definition. Rather, I contend that curricular coherence must be defined kaleidoscopically via a plurality of disciplinary and stakeholder perspectives. To this end, I investigate three interrelated questions: (1) Ontologically, what *is* coherence in its many forms? In other words, what does

“coherence” refer to in both mathematics and science education, as well as in other disciplines? Additionally, according to these ontologies, *who* is positioned with the authority to make judgements or evaluations of (in)coherence? (2) What are the aesthetic, ethical, and onto-epistemological foundations behind the common (and often implicit) assumption that coherence (in its many forms) is desirable? What are the consequences of these philosophical assumptions for curriculum? For learning? For how learners are positioned? In other words, I question *curricular coherence for what purpose*? (3) Finally, what are the flexible possibilities (and tensions) for conceptualizing curriculum using an aesthetic curriculum-as-story metaphor to investigate various forms of curricular (in)coherence from multiple stakeholder perspectives?

I inquire about these overarching questions through three interrelated studies—one theoretical and two empirical—situated within an arts-based research paradigm. These inquiries serve as a type of disciplinary-cultural analysis and artistic critique from both my own and students’ perspectives with the overriding goal of interrogating and shifting the normative value of (curricular) coherence in mathematics education. More broadly, this dissertation spotlights the aesthetic dimension of learning mathematics as well as the danger of divisive and dehumanizing politics of aesthetics inherent to uncritical conceptualizations of so-called “desirable” modes of teaching and learning, such as the privileged logico-rational definition of curricular coherence that is the current status quo.

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To Gideon and Hazel (Attorneys at Law), the most erudite feline duo I know.
From you, I learned that sometimes the answer is to just take a nap in a sunny spot.

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CHAPTER 1: INTRODUCTION

What I'm about to claim is that the narrative art of storytelling—written, oral, visual, or otherwise—and particularly the *forms, structures, and genres* of these narratives across cultural and disciplinary traditions convey more than just entertaining tales of heroics and adventure (i.e., “content”). They also convey powerful messages and cultural values around what it means to learn, to make sense, and the extent to which absolute certainty and coherence *should* coexist (or not) with ambiguity and incoherence (Freeman, 2010; McAdams, 2006). In other words, narrative conveys aesthetic, ethical, and onto-epistemological values about learning, living, and being (Stephens & McCallum, 1998). As Rancière (2000/2004) put it, artforms such as storytelling serve to “distribute the sensible” of a given culture, suggesting what/who is seen as aesthetically ideal and, in turn, what/who is seen as aesthetically aberrant (Hyvärinen et al., 2010; Strawson, 2004). For this reason, literary theorist Mieke Bal (2017) contends that the study of narrative ought to be one of culturally situated interpretation of stories as they are read by individuals, rather than an objective classification of their elements. What matters is not just *what* stories are being told (namely, the “content” of stories, e.g., which characters? Which settings? Which sequence of events?), but also *how* the broader narrative is experienced and interpreted by readers—aesthetically and otherwise—alongside the *cultural messages and values* the narrative might communicate.

With these questions in mind, I adopt the perspective that mathematics curriculum can be viewed as a storied artform (Dietiker, 2015a, 2015b) and employ a narrative framing to investigate the holistic *form* of mathematical stories as interpreted through multiple stakeholder viewpoints on curricular coherence. I view the investigations as a type of disciplinary-cultural analysis and critique with the overriding goal of interrogating and shifting the normative value of (curricular) coherence in (mathematics) education. I do so with the aim of offering up alternative

perspectives of curricular (in)coherence that might be valued if we were to interpret curricular stories through a multiplicity of storytelling traditions (with varying structures, genres, etc.).¹

Before proceeding further, it feels appropriate and only fair that I start by telling *my own* story and what has led me to carry out this research given the centrality of storytelling to this dissertation. It has been a long journey, and what I’m about to share herein is—or at least feels like—a dramatic departure from everything I’ve done before. Nevertheless, I find strength and courage in the adage that has been passed down to me through generations of scholars: “All research is autobiographical”.² This dissertation tells my biography so far, my *desire* to do research (Loveless, 2019).

After completing my initial research practicum a little over two years ago (Tyburski, 2022), I felt out of touch with what I was doing. The theories, philosophies, and approaches to research I was drawing on didn’t feel consistent with my ways of knowing and being as a person. They also felt loveless and detached from the very relational and human-centered approaches I try to bring into my own teaching practice. The analysis techniques I chose to use had served their stated purpose, but they had simultaneously failed to express the data that felt most important and moved me, or, as MacLure (2013) has called it, data that “glow” and “shimmer”. These data—students’ creative and aesthetic stories about different types of multivariable

¹ Throughout the dissertation, I use the phrase “(in)coherence” rather than just “coherence” to remain open to different perspectives and valuations of coherence wherein coherence is *not* assumed as a monolithic, universal good. This choice allows me to remain open to aesthetics of *incoherence* in curricular stories and the utility of such aesthetics for student learning, as I detail throughout this chapter and the subsequent one. While I will sometimes still use “coherence” on its own, this is to refer to literature that explicitly centers on *just* coherence or when I use a phrase such as “valuations of coherence” (see Chapter 3) where it is clear that I am considering possible value systems where the value of coherence may be rejected in favor of valuing, say, *incoherence*, for example.

² To embrace the relational nature of research and acknowledge those who have played a role in my own research journey, I share that these words of wisdom were first passed down to me by James Drimalla, who had himself learned them from Nico Gómez Marchant. It was only a few years later when I finally met Nico and learned he had originally heard them from James Hiebert. Of course, wisdom such as this transcends one genealogy (e.g., Glanfield et al., 2022; Leggo, 2008; Pinar et al., 1995), and I am grateful to the *many* other scholars who (through their work or in personal conversations) have continued to guide me as I embrace such wisdom in my scholarship.

functions—ultimately were not centered in my practicum analysis because my approaches to research served as a barrier to following the glow. Instead, I felt forced to reduce the worlds of these stories into words (in the sense of Dominguez and Abreu, 2022). Students’ idiosyncratic, relational knowledges and strengths were reduced to just a few codes thrown into the swirling chaos of the rest of the data. When averaged with the remaining data, those stories were drowned out by deficit messages repeated in much of the literature on students’ understandings of functions: “They don’t get it”, “They don’t get it”, “They don’t get it”. This was despite my explicit efforts to use asset-based, anti-deficit approaches (Peck, 2020) to conceptualize student learning. I knew I needed to change my approach if I wanted to allow students’ stories to shine. For me, this was not just a practical obligation; it was an ethical one.

Since then, I’ve taken time to articulate and critically reflect upon my philosophical worldview, in an effort to *live* these deeply held philosophies and allow them to guide all my theoretical and methodological considerations (Drimalla et al., 2024; Stinson, 2020). I’ve also taken the time to reflect on what is possible with our research methods, rejecting the pervasive myths of objectivity (Abreu et al., 2022; Bowers, 2022) that percolate throughout our field and lead us to predominantly follow scientific and post-positivistic approaches. By blurring the boundaries between feeling and knowing, I’ve re-attuned myself to the artful, creative, and aesthetic ways of learning and knowing (Eisner, 1985; Loveless, 2019; Sinclair, 2009) that have served me well in my life so far. The power of storytelling and of sharing and discussing stories with others is now centered rather than ignored in my arts-based approach to scholarship (Chilton & Leavy, 2020; Loveless, 2019). And, perhaps most importantly, following the healing wisdom of bell hooks (2001) and others (Abreu, 2022a, 2022b; Bowers et al., 2024; Yeh et al., 2021), I’ve been intentional about centering *loving* philosophies and *loving* approaches to

interacting with people.

In a reflection from fall 2022 that framed the introduction to my dissertation proposal, I wrote:

At this phase in my journey, I haven't yet made all the connections I hope to make. There are still some loose ends and some aspects that proudly read "TBD". Part of this is by design and consistent with the nature of the arts-based work I plan to do (As McNiff, 2018, reminds us, "trust the process"). But this is also because in some cases I genuinely don't have the words to describe what I someday hope to convey. Still, I believe what I offer in these pages is enough to continue this journey where I will hopefully learn the words—or drawings or poems or ...—from my participants and from doing.

Reading these words again, I'm struck by just how thematically relevant they are to a dissertation study which explores the role of curricular *coherence* and interrogates the largely unquestioned positive association with learning, among other descriptors (e.g., "wholeness", "goodness", "satisfaction", "unification"), while *incoherence* meanwhile is associated with mostly negative descriptors (e.g., "incompleteness", "badness", "confusion") often seen as barriers to learning (Buchmann & Floden, 1991; Hyvärinen et al., 2010). Yet, in my reflection, there is no such strict binary. I mention the "connections" I hope to make (i.e., a process of coherence seeking, Sikorski & Hammer, 2017) but also how these connections co-exist with "loose ends" and cases where I "genuinely don't have the words" (i.e., perceived incoherence). Yet, I embrace the value of such uncertainty with conviction by declaring how some aspects "proudly read 'TBD'" and that I intend to "trust the process". Shades of coherence and incoherence blend together harmoniously. After all, the complexity is not cause for despair but rather joy as I embrace an "aesthetic of unfolding" (Irwin, 2003) and its many ambiguities, uncertainties, and possibilities for sense-making (as well as *nonsense-making*, Appelbaum, 2010).

Indeed, now that I am nearing the end of this dissertation journey, I can look back on the

seemingly chaotic state of in/coherence I began from and smile.³ Not because I’ve “figured it out” or systematically excised all the incoherence. But because I’ve realized that living symbiotically with all this chaos that continues to accompany me *is* a form of learning. After all, I fully acknowledge that there are *still* loose ends and uncertainties. I *still* don’t know all the words to describe what I someday hope to convey. *However*, these are different from the ones I had in mind two years ago. Rather, they are *new*—uncertainties which unfolded and revealed themselves only after I pulled on the previous loose ends for long enough. I began with the goal of challenging my research practice by commencing a turn toward participatory, humanizing approaches to working alongside students and radically centering their perspectives (Osibodu et al., 2023) while embracing a “new aesthetics of mathematics education” (Bowers et al., 2022; Dubbs, 2021) in part through arts-based approaches to research. Through the guidance of my mentors and colleagues in the Program in Mathematics Education and the multidisciplinary communities of artist-scholars at MSU (especially the Arts-Based Research Learning Community and the STEAMpower fellowship program), and of course, the students I worked alongside, I can confidently say that I’ve made great strides toward these goals. Though, like before, I openly admit I still have a way to go. Such is life. Such is learning.

To the Gods and Goddesses of Research
Give us then the courage
To challenge the privileged paradigm
To break the illusion of objectivity
 To carry lightly the loud weight of words
 For we are longing for poetry
 Woven through with dance
 And drama performed with music

³ I use the phrase “in/coherence” when referring to a specific perspective on (in)coherence that embraces the complex (and sometimes contradictory) dialectic between coherence and incoherence where these notions coexist, mutually informing one another, rather than existing as distinct binaries or opposing forces. I use the slash (/) here in much the same way as Irwin (2003). For more on this complex dialectic, see Chapter 2, where I introduce and adopt the arts-based paradigm of research and its ontological stance of dialectical pluralism (Chilton et al., 2015). Later in the same chapter, I also unpack and dig further into literature which acknowledges this specific dialectic of in/coherence.

Let us look with both eyes open
At our unexamined subjectivities
Let us crack the categories of our thinking
And find an epistemology of the senses
Where wonder and passion interplay with reason

-Sally Atkins (2012)

What Led Me to this Dissertation Study?

In the previous section, I shared how my personal history and positionality as a *researcher* orients this dissertation study. Next, I take one step further back in time to explicate how my history as a *learner* of mathematics and subsequently a *teacher* of mathematics came to be entangled with my positionality as a researcher as I began to question implicit assumptions about mathematics (learning) and curriculum design through my experiences as a teacher. What follows is effectively the origin story of this dissertation research and what motivated me to study the aesthetics of curricular stories.

I come to this study as a university mathematics instructor with almost ten years of experience teaching mathematics courses ranging from college algebra to linear algebra and differential equations. I am also a trained mathematician who has typically thrived in the mathematics courses I have taken up through graduate school. This success is due in part to the many privileges I have enjoyed as a straight, white, cisgender man with an upper-middle class upbringing that has always afforded me access to well-resourced and high-quality educational institutions throughout my K-16 mathematics education. Even in the moments when mathematics did not feel fully coherent to me, I was consistently compelled to work out any unsolved logical puzzles with the goal of piecing together a sensible overarching “story” of mathematics. As an in-member of the demographic groups who have historically possessed the power to define the discipline of academic mathematics, including what “counts” as coherent mathematics and which mathematical stories are privileged over others, I have rarely

experienced instances where my definitions of “sensible” fundamentally deviated from those espoused by the mathematics curriculum. And even when I did, I felt included as a full member of the mathematics spaces I have navigated, meaning I almost always felt comfortable consulting with a peer or mentor as I worked to resolve such incoherence. Consequently, for much of my life, I did not recognize the valued-laden and socially constructed nature of logical forms of coherence which serve as a colonizing invisible hand that defines which (genres of) mathematical stories are privileged within and by our curriculum.

I carried this view of learning as forming coherent mathematical stories into my teaching practice. Tasked as the instructor of record for fast-paced, higher-level university mathematics courses with curricula that felt more like a collection of miscellaneous formulas than a coherent whole—i.e., multivariable calculus—I moved to craft stories which I could use to engage my students and support their learning. My hope was that students might then take up these stories and come to see the course as an interconnected series of skills, rather than a scattered mess. Building on my own learning experiences across the undergraduate mathematics curriculum, I gravitated toward stories based on function as a recurring and unifying mathematical “character” (Zandieh et al., 2017). And, to my delight, this story framing seemed to support and even engage several of my students in “making sense” of the courses I was teaching. But simultaneously I noticed that in relying on this function-based curricular story, other students seemed to be *more* confused or even bored by my constant references back to the same story. It was not until later that I started to realize that my perspective of seeing curriculum as a story—and specifically a “coherent” story—was not a perspective shared by all mathematics students, a reality I quickly learned from the many furrowed brows of pre-service mathematics teachers when I asked them about the “story” of their lesson plan. Indeed, as Dietiker (2015a, 2015b) detailed, such a storied

view of curriculum is not a default perspective in mathematics education.

Beyond this potential issue, I also learned in my research with multivariable calculus students (e.g., Tyburski, 2023) that not all undergraduate students viewed function as a unifying cross-curricular theme in the same way that mathematicians or curriculum designers might. It started to become much clearer why the story I had crafted for teaching multivariable calculus had not been as successful as I had hoped. It was around this time that I finally began to understand the potential harm I had inadvertently imposed on my past students by putting one curricular story on such a high pedestal. Namely, any students with different aesthetic and epistemological sensibilities about what constitutes a cross-curricular mathematical theme or what made a story—or a course—“mathematically coherent” were being repeatedly sent the implicit message that *their* stories, *their* ways of knowing, and *their* aesthetics were subordinate to or, in some cases, incompatible altogether, with learning multivariable calculus.

Consequently, I approach this study with the kind of epistemological humbleness (Barone, 2008) that is necessary to unlearn decades of assumptions regarding what “counts” as mathematical coherence, what “counts” as a “coherent” (curricular) story, and whether coherence—particularly in its logical, disciplinary forms—should hold the privileged status it boasts at present in mathematics education. By attending to more expansive views of (in)coherence, my hope across this dissertation is to learn how we might design engaging and impactful curricular stories. These are stories that are not one-size-fits-all but rather ones that are responsive to students’ varied aesthetic sensibilities and the cultural backgrounds and forms of storytelling that inform these sensibilities. In approaching this study, therefore, I purposefully eschew a singular view of story, such as the poetics of Aristotle (350 B.C.E./1995) which continue to be employed centuries later as a “model aesthetic structure” for a quality story,

particularly in popular Western culture. Such views impose a harmful politics of aesthetics (Rancière, 2000/2004) which serve to position those with different aesthetics as “tasteless” and even “objectively wrong” (as the derisive discourse on social media often goes). Instead, I remain open to axio-onto-epistemologies of story which are postmodern and purposefully incoherent; ones which are non-linear and do not feature a clear beginning, middle, and end; and even ones which are logically nonsensical while simultaneously aesthetically coherent.

Overview, Aims, and Guiding Questions

In mathematics curricular documents (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; Zorn, 2015), it is common practice to outline cognitive goals, habits of mind, and standards of practice that are intended to organize students’ curricular experiences. Correspondingly, there is extensive research on the degree to which students engage with and understand these pre-determined, cross-curricular themes, such as function, equivalence, and abstraction (e.g., Warren & Cooper, 2009; Zandieh et al., 2017). Indeed, cognitive research along these lines (often to develop learning progressions or trajectories) is viewed as the *sine qua non* for curricular design (Fortus & Krajcik, 2012). Yet, even when students are involved in the curricular design process or related research, they rarely have a say on which questions, research priorities, and standards of evidence are considered. In other words, we do not often ask students directly about the themes or stories *they* construct to make sense of their experiences across the mathematics curriculum. In mathematics and science courses, too, we do not tend to provide students with opportunities to engage in thematic reflection, or other forms of sense-making that are aligned with *their* views of what “sense” means or what “themes” they notice (Sikorski & Hammer, 2017). Some natural questions arise: How are mathematics students organizing their experiences across the mathematics curriculum (logically, aesthetically, and otherwise)? What forms of sense-making are they engaged in? What

stories do they tell themselves or others to explain the interconnections they see (or don't see) within and across courses?

A Brief Overview of Curricular Coherence in Mathematics Education

These questions led me to investigate how curricular coherence is conceptualized in mathematics education literature. In particular, I examined how such definitions left space for students' perspectives in determining the ways a lesson, course, or sequence of courses was seen as (in)coherent. After all, research reminds us that students do not always take up course ideas and themes in the ways instructors intend (Clift & Brady, 2005; Lew et al., 2016). I found that the word "coherence" can mean many things. Schmidt et al. (2002), for instance, argued that a coherent curriculum is one that consists of "a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives" (p. 9). They added further that this requires that the particulars should evolve to "deeper structures inherent to the discipline." Others, such as Cuoco & McCallum (2018), have proposed definitions that focus on coherence of *practices* in addition to coherence of content. A coherent curriculum, in this view, is one that enables students to leverage recurring mathematical practices, such as using structure and abstraction, to make connections and "take advantage of a coherent curriculum" (p. 252). Ultimately, *much* of the literature featured types of *disciplinary coherence* of this form (Fortus & Krajcik, 2012; Morony, 2023), where coherence is based on alignment with so-called disciplinary knowledge, logico-deductive sequencing, and engagement with pre-determined mathematics practices. Another type of coherence present in the literature was based on the metaphor of curriculum as a story (e.g., Stigler & Hiebert, 1999). Dietiker and her colleagues (Dietiker, 2015a; Richman et al., 2019) have developed this metaphor to attend to how story coherence is *interpretive* and based on the intertwined nature of logic *and* aesthetics as a reader (i.e., student) engages with a curricular

story. Unlike disciplinary coherence perspectives, Dietiker and colleagues' view of story coherence centers students' interpretations *and* attends to students' aesthetic reactions to the mathematical stories we weave in our curricula.

While there are shimmers of attending to students' perspectives and interpretations of curricula (e.g., Dietiker, 2015a), the dominant messages throughout the curricular coherence literature in mathematics education are that coherence is a property of the curriculum itself, rather than an interaction between students and the curriculum. If we are to design "coherent" curricular experiences from this view, then disciplinary experts must first design and sequence content and opportunities in ways that encourage recurring use of disciplinary practices. The role of the student is primarily to be influenced or guided to "coherent experiences" by the curriculum. In other words, the student is the consumer of a coherent curriculum, rather than the producer. Further, these approaches to curricular coherence tend to favor a particular way of consuming and coming to know: one that moves from particulars to the general and puts a premium on pre-established mathematical and logical practices as a way of building connections between experiences. I'm reminded of an observation by Doxiadis (2003):

At age five or six, a child lives in a storied internal environment, i.e. an environment cognitively organized by stories of all kinds, of family, of home, of daytime routine, of behavior, of neighborhood, of games, of friends, of animals, of dream. The main characteristics of the storied world are integration and emotional richness. With the introduction to mathematics, the child is de-storied, a neologism that sounds suspiciously close to "destroyed". (p. 20)

By defining curricular coherence so narrowly, what opportunities for sense making do we shut out? In what ways do these narrow definitions destroy students' natural inclination to integrate the everyday and the mathematical as well as the emotional and the logical?

To be clear, not all approaches to curriculum design in mathematics education downplay the role of the student and their approaches to sense-making to this degree. For example, the

recent undergraduate inquiry-oriented curricula that have been created in differential equations (Rasmussen et al., 2018), linear algebra (Wawro et al., 2013), and abstract algebra (Larsen et al., 2013) were all designed following the principles of realistic mathematics education (Gravemeijer & Terwel, 2000), which center students' guided reinvention of mathematical concepts. While this is a definite step toward viewing the student as playing an active role in sense-making throughout the curriculum, what students reinvent is typically pre-selected by curriculum designers who tend to be mathematicians and mathematics educators. Like most research used to discern curricular coherence, students are involved, but they do not necessarily have full agency to determine the forms of coherence that are relevant or the questions they would like to pursue. So, even in this case, the views of coherence that get centered are likely to be from the perspective of others, rather than of students.

That said, (mathematics) teacher education program design and evaluation, however, is one research area that shows some signs of defying this trend by explicitly centering students' perspectives into evaluations of program coherence (Grossman et al., 2008; Richmond et al., 2019). More recently, Canrinus et al. (2017) as well as Nguyen and Munter (2024) have conducted studies devoted to evaluating program coherence from the pre-service teacher perspective. Although teacher education program design and evaluation is just one area of research that exhibits this tendency, it provides some hope that the field's interpretation of curricular coherence may broaden to include additional stakeholder perspectives. In science education, such a broadening to center student perspectives on curricular coherence has already begun to occur (Reiser et al., 2021; Sikorski & Hammer, 2017). Sikorski and Hammer (2017) introduced a distinction between premeditated coherence—the kind common in mathematics education—and students' coherence seeking: the ongoing process of “trying to build meaningful

mutually consistent relationships between information” (Sikorski, 2012, p 153). Premeditated coherence, they argued, has the tendency to undermine and undervalue the active role students must play while seeking coherence. These authors work from the assumption that students are actively seeking some form of coherence, so the question becomes not *whether* students are seeking coherence but rather *what kinds* of coherences they are seeking (linguistic, narrative, conceptual, etc.), *how* they are seeking these coherences, and for *what reasons*. They advance a broader, three-pronged view of coherence seeking that includes developing: (1) Conceptual coherence—an integration of knowledges and experiences into coherent frameworks, (2) An affective sense of unity across curricular experiences, and (3) the inclination to actively seek coherences. Classic definitions of (premeditated) disciplinary coherence in mathematics and science education often attend to conceptual coherence, but they ignore the importance of these other two dimensions.

The Unquestioned, Value-Laden Nature of Curricular Coherence

The many forms of curricular coherence introduced so far point to the number of possible perspectives and dimensions that could be considered when making judgements about a curriculum’s coherence. Yet, one trend runs throughout almost all these definitions: coherence and coherence seeking as a universal “good” for a curriculum and, in turn, for students’ learning. Even Sikorski and Hammer (2017), who significantly broaden the notion of curricular coherence to include students’ views, assume that students (and people in general) will naturally seek coherence. This is also a widely held view in curriculum design research (Morony, 2023; Wan & Lee, 2022).

On the other hand, some have argued that coherence (of curriculum, a narrative, etc.) is a valued-laden notion (Buchmann & Floden, 1991; Freeman, 2010; Herbert, 2004) that ought to be questioned. Indeed, the subjective, value-laden nature of coherence is often concealed by the

predominance of traditional Western views of learning as well as of narrative which privilege coherence as an objective standard against which all experiences and texts can be evaluated (Herbert, 2004; Hyvärinen et al., 2010; Strawson, 2004). As Richmond et al. (2019) urge us to contemplate: “Coherence for whom? According to whom? To what end(s)?” (p. 188). In other words, *why* design curriculum to be coherent? What evidence is provided that curricular coherence supports students’ learning? *Which* students’ learning? And, fundamentally, what axiological, ethical, and onto-epistemological assumptions underlie these assumptions about coherence as a universal good in curriculum design and learning? Do these assumptions hold in all cases?

These questions culminate in another question: *Might there be other forms of intentional incoherence that could also benefit student learning?* Sinclair (2005), for example, speaks of destabilizing devices, like contradictions, ambiguity, and purposeful gaps as a design choice to further engage students. From the perspective of Dietiker’s (2015a) curriculum-as-story metaphor, too, scripting a story for moments of uncertainty, tension, and anticipation (i.e., incoherence) are a way to aesthetically hook and captivate students. Incoherence and ambiguity, in other words, can be engaging (Irwin, 2003). Indeed, there is emerging conversation in the narrative learning theory literature on how the widely held assumption that coherent narratives are ideal can result in deficit views of those who favor alternative ways of making sense of their life (Freeman, 2010; Hyvärinen et al., 2010). This is despite the fact that some genres of story and some cultural views of storytelling may embrace ambiguity, unknowability, and other forms of sense-making that have often been associated with “incoherence” in not only the disciplinary culture of mathematics but also popular Western culture (Appelbaum, 2010; Jones, 2021).

Overarching Aims of Inquiry

With this background and context in mind, I investigate three interrelated questions in

this dissertation.

(Q1) First, I take an additional step back and wonder “**What is coherence?**” with the goal of remaining open to several possible forms of (curricular) coherence. To answer this question, I explore multidisciplinary perspectives on coherence as presented in various literatures (including linguistics, logic, epistemology, communication, and, of course, curricular coherence in mathematics education) and then conduct a study of undergraduate STEM students’ definitions of (in)coherence as it relates to their mathematics curricular experiences. Across these explorations, I question *the ontology of coherence*—does “coherence” refer to *connections* in a network of ideas? Alignment between two structures? Something else? In these various fields, is coherence seen as a binary? A spectrum? A complex blend of some form? Additionally, I attend to *who/what decides what is coherent*. Is coherence positioned as an objective evaluation? Or a subjective interpretation? In either case, *who* is positioned with the authority to make such judgements?

(Q2) Second, I investigate and subsequently interrogate the aesthetic, ethical, and onto-epistemological foundations behind the common (and often implicit) assumption that coherence (in its many forms) is desirable (whether aesthetically, for learning, etc.). I wonder: *What are the consequences of these philosophical assumptions for curriculum? For learning? For how we position learners?* In other words, I ask *curricular coherence for what purpose?* To explore these questions, I attend to both theoretical and philosophical arguments as well as empirical studies which suggest curricular coherence is beneficial for student learning. In doing so, I question the commonly held view of coherence as an unassailable good and explore possibilities for the coexistence of coherence and incoherence, including as it relates to learning (e.g., the examples provided last section).

(Q3) Third, I explore flexible possibilities (as well as tensions) for using the curriculum-as-story metaphor to investigate various forms of curricular (in)coherence (and from multiple stakeholder perspectives). I will argue that this metaphorical framing allows for a complex, subjective view of (in)coherence that need not be conceived of as a strict binary and enables us to investigate not only disciplinary coherence but also aesthetic, emotional, and other forms of coherence as interpreted by students. I suggest that changing the *form* of story that is considered through this metaphor (e.g., its structure, genre, cultural/disciplinary storytelling tradition), enables us to flexibly interpret curriculum from *several* perspectives of (in)coherence. Finally, I note some tensions of using such an interpretive narrative framing for curricular analysis.

Structure of Dissertation

This dissertation consists of three thematically related studies that address these overarching aims of inquiry.

Chapter 2 serves the dual role of introducing the theoretical framework I use throughout the remainder of the dissertation in the first part—Dietiker’s (2015a) curriculum-as-story metaphor interpreted from an aesthetic perspective aligned with the arts-based research paradigm (Chilton et al., 2015; Conrad & Beck, 2015)—followed by a second part presenting a theoretical-philosophical investigation of the value-laden notion of (curricular) coherence (i.e., Q1 and Q2), as presented in various disciplinary literatures (both in and outside of curricular studies). I conclude by synthesizing lessons from both parts of the chapter to propose a viewpoint from which the curriculum-as-story metaphor might serve as a flexible analytic tool for investigating curricular (in)coherence in its various forms and from multiple stakeholder perspectives (Q3). This conclusion also serves to clarify my perspective on curricular (in)coherence I subsequently use throughout the dissertation. With an eye to introducing the theoretical-philosophical

orientations for all subsequent chapters as a connected and cohesive whole, I choose to keep both of these parts within one longer chapter, rather than breaking them up into two smaller ones.

Chapter 3 presents an empirical study of six undergraduate STEM students' interpretations and valuations of mathematical curricular (in)coherence. In this participatory, arts-based study, the metaphor of curriculum as story is used as a framing device to support “conspiratorial” conversations (Barone, 2008) with and between students about their varied interpretations and valuations of cross-curricular (in)coherence (i.e., Q1). In the discussion, I consider the relationship between students' perspectives of (in)coherence and the types of coherence mentioned in the mathematics and science education literature (as introduced in this chapter and explored further in Chapter 2). Additionally, I take up students' views of (in)coherence and discuss how they might be considered from the perspective of the curriculum-as-story metaphor to explore additional possibilities for forms and genres of stories that might be used to conceptualize curriculum and analyze curricular (in)coherence (i.e., Q3).

Having investigated *students'* curricular stories and perspectives on curricular (in)coherence, I next present a textbook analysis in Chapter 4 attending to *disciplinary* stories and the aesthetics of (in)coherence they privilege. By juxtaposing these stories, I consider how the textbook—as a didactic cultural artifact of the discipline of mathematics (Plut & Pesic, 2003)—might support or hinder the transmittal of privileged disciplinary-pedagogical meta-narratives (Stephens & McCallum, 1998) about the role of function. Specifically, I interrogate the so-called “disciplinary coherence” of stories told about three different types of multivariable functions in a commonly adopted calculus textbook (i.e., Q2), questioning a strict adherence to aesthetics of disciplinary coherence when crafting curricular stories. Leveraging the curriculum-as-story metaphor once again, I conduct an arts-based analysis of my own design where I *read*

textbook stories about these different types of functions as I would a literary novel. Ultimately, in the discussion, I take up the interpretations from my reading to consider the (in)coherence of possible stories that might be told with/through the multivariable calculus curriculum, attending to ethical concerns such as STEM students' socialization into multiple disciplinary cultures which privilege different disciplinary forms and aesthetics of storytelling (i.e., Q3).

Finally, Chapter 5 presents a summary of conclusions from across all prior chapters and aims of inquiry, followed by a discussion of overarching themes across studies to make the significance of the set of studies transparent. Alongside the discussion of these themes, I explore lingering questions that could motivate future research directions as well as personal lessons I learned while carrying out this dissertation.

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CHAPTER 2: CHANGE THE STORY, CHANGE THE CURRICULUM: THE CURRICULUM-AS-STORY METAPHOR AS A FLEXIBLE LENS FOR INTERPRETING CURRICULAR (IN)COHERENCE

The universe is made of stories,
Not of atoms.

Excerpt from Muriel Rukeyser's (1968/2006) *The Speed of Darkness*

At the heart of this dissertation is a focus on stories as a narrative artform in relation to curriculum materials and student experiences. I review work herein that uses either story, narrative, or both. So, I begin by clarifying that I use *story* to refer to how events unfold sequentially through a text (e.g., on page one *this* happens, followed by *that* on page two). Meanwhile, *narrative* is a broader interpretive framing used to refer to an individual reader's interpretation of a story. Throughout this dissertation, I adopt the perspective that mathematics curriculum can be conceptualized as a story (Dietiker, 2015a; Gadanidis & Hoogland, 2003) and interpreted using narrative frameworks (i.e., Bal, 2017; Dietiker, 2013). Like a story, curricula feature (mathematical) characters inhabiting settings and engaging in actions that constitute the plot. A curriculum could also be interpreted as featuring themes and morals in the way a classic story might. As mathematics curricula (and textbooks in particular) are used for enculturating students into the discipline of mathematics, the stories they tell—as well as the ways these stories are told—should not be an afterthought. After all, the stories we are told and then re-tell (ourselves privately or others publicly) influence how we organize our experiences and make sense of the world (Bruner, 1986; Clark & Rossiter, 2008; Stephens & McCallum, 1998). Indeed, the power of story and its unique potential to transform and re-humanize how we conceptualize the teaching and learning of mathematics has been acknowledged repeatedly by mathematicians (Doxiadis & Mazur, 2012; Thomas, 2007) as well as mathematics educators (Burton, 1999; Dietiker, 2015b; Healy & Sinclair, 2007). It is also a perspective espoused by mathematical storyteller Apostolos Doxiadis who has argued for narrative as a soulful force in mathematics

education that

sets the basis for a non-formalist, non-Platonist view, a view of mathematics not as something pinned like a dead moth for Euclidean purists to examine – and in this form taught to our [students]–, but mathematics as it is lived by human beings, as it is loved, as it is explored, feared, created, dreamed of... By human beings. (2003, p. 6)⁴

Despite the polyphony of voices calling for a shift to viewing the teaching and learning of mathematics through the lens of story, curricular analyses from this perspective tend not to treat curriculum as an artform with the potential to aesthetically and emotionally impact its audience (Dietiker, 2015b; Gadanidis & Cendros, 2023). Yet, aesthetic engagement with stories (mathematical or otherwise)—such as surprise experienced during a shocking plot twist that defies expectations (Dietiker, 2016; Gadanidis et al., 2016; Ryan & Dietiker, 2018), suspense felt as “the plot thickens” and a story’s mystery unfolds in unexpected ways (Irwin, 2003; Richman et al., 2019), or the accompanying pleasure (or displeasure) felt when a story resolves in a satisfying (or unsatisfying) way (Doxiadis, 2003; Miežys, 2023)—is not merely incidental or “for entertainment”. As McKee (1997) opined in his classic book on screenwriting:

The world now consumes films, novels, theatre, and television in such quantities and with such ravenous hunger that the story arts have become humanity’s prime source of inspiration, as it seeks to order chaos and gain insight into life. . . . Some see this craving for story as simple entertainment, an escape from life rather than an exploration of it. But what, after all, is entertainment? To be entertained is to be immersed in the ceremony of

⁴ Mathematical storyteller is an apt title for Doxiadis, given his authorship of multiple bestselling literary works based on the history of mathematics, including most recently the graphic novel *Logicomix* (Doxiadis et al., 2009); however, Doxiadis prefers to refer to himself as a “paramathematician”. This title originates from the same essay quoted in text, where Doxiadis argues for the necessity of a new, multidisciplinary branch of mathematics called “paramathematics” dedicated to artfully crafting mathematical stories that aesthetically engage students and other interested audiences in the history, content, context, and utility of mathematics. He suggests such a field would be at the intersection of mathematics (including the history and philosophy of mathematics), education theory, cognitive psychology and research on narrative modes of thinking (e.g., Bruner, 1986), sociology, and anthropology, among other disciplines. Though he uses the prefix “para-” meaning “on the side” in the name of this proposed sub-discipline, Doxiadis clarifies that this is not to *separate* storytelling from the work of professional mathematicians and mathematics educators but instead to acknowledge the additional expertise, artistry, and craft that goes into creating mathematical stories beyond the skills traditionally taught to professional mathematicians and educators. He invokes this etymology primarily to call for a “benign revolution” in mathematics, whereby mathematical storytelling would become recognized as a powerful force for advancing the teaching and learning of mathematics and, more broadly, re-humanizing how mathematics is seen as a discipline.

story to an intellectually and emotionally satisfying end. (p. 12)

In other words, participation in the “ceremony of story” is an active *process* of sense making that is fundamentally aesthetic, emotional, *and* epistemological in nature (Allen, 1995; Bruner, 1986). As Doxiadis (2003) cautions, “To derive from a book, a play, a film, a concert, a higher ‘message’ is highly commendable. But unless you are also entertained, there is no hope of instruction” (p. 2). While *no* hope of instruction is overly hyperbolic, I wholeheartedly endorse the sentiment that the aesthetic *form* of curriculum is at least as important as its content (Eisner, 2004). By adopting the theoretical perspective of curriculum as a storied artform (Dietiker, 2015b), I purposefully position myself to interpret curriculum holistically with attention to the logical, aesthetic, and emotional factors involved in interpreting stories. Specifically, I opt to attend to the holistic aesthetic dimension of *coherence*, which is used by both artists and laypeople to refer to their interpretations on the ways in which the individual parts of an object “stick together” to form a whole (Aschenbrenner, 1985).

Indeed, the holistic *form* and *structure* of a story interpreted as a form of narrative conveys powerful messages and cultural values around what it means to learn, to make sense, and the extent to which absolute certainty and coherence *should* coexist (or not) with ambiguity and incoherence (Freeman, 2010; McAdams, 2006). In other words, narrative conveys disciplinary-cultural aesthetic, ethical, and onto-epistemological values about learning, living, and being (Stephens & McCallum, 1998). As Rancière (2000/2004) put it, artforms such as storytelling serve to “distribute the sensible” of a given culture, suggesting who/what is seen as aesthetically ideal and, in turn, who/what is seen as aesthetically aberrant (Hyvärinen et al., 2010; Strawson, 2004). For this reason, literary theorist Mieke Bal (2017) contends that the study of narrative ought to be one of culturally situated interpretation of stories as they are read by individuals, rather than an objective classification of their elements. What matters is not just

what stories are being told (i.e., the characters, settings, sequencing of events), but also *how* the broader narrative is experienced and interpreted by readers (aesthetically and otherwise), alongside the *cultural messages* the narrative might communicate.

With these questions in mind, I employ a narrative framing (Bal, 2017; Dietiker, 2013) throughout this dissertation to investigate the holistic *form* of mathematical stories as interpreted through multiple stakeholder viewpoints on curricular coherence. I view this as a type of disciplinary-cultural analysis and critique with the overriding goal of interrogating and shifting the normative value of (curricular) coherence in (mathematics) education. I do so with the aim of offering up alternative perspectives of curricular (in)coherence that might be valued if we were to interpret curricular stories through a multiplicity of storytelling traditions (with varying structures, genres, etc.).

Preliminary Remarks on Definitions of Curriculum

Before expanding on this theoretical perspective in the subsequent sections, I first take a moment to specify what I mean by curriculum throughout this dissertation. Among both practitioners and curriculum designers at the undergraduate level, there are several views of what constitutes “the curriculum” (Fraser & Bosanquet, 2006). Dietiker (2015a) clarifies that the metaphor of mathematics curriculum as a story can be used to conceptualize any form of curriculum, ranging from the intended curriculum (i.e., curricular policy documents and standards, see e.g., Gadanidis and Cendros, 2023) to the enacted curriculum (i.e., mathematics lessons, see e.g., Andrà, 2013; Dietiker et al., 2023; Weinberg et al., 2016). My curricular focus shifts throughout the chapters of this dissertation. Within this chapter, I synthesize the literature on curricular coherence at several grain sizes, yet my discussion concerns the notion of curricular coherence more broadly as an interpretive holistic criterion and aesthetic value for evaluating multiple forms and grain sizes of curriculum. In Chapter 3, I turn my attention to the enacted

curriculum (Remillard, 2005) from the student perspective, sometimes referred to as the *learned curriculum*, when I ask students to reflect across their mathematics curricular experiences and share their personal interpretations and valuations of curricular (in)coherence across these course experiences. However, I do so not to study the learned curriculum, per se, but rather to learn about students' views on curricular (in)coherence so these can subsequently be used to perturb status quo definitions of coherence and inform curriculum design of the intended curriculum. Finally, in Chapter 4, I turn exclusively to the intended curriculum and the “curricular material” (Remillard, 2005) of the written “textbook curriculum” (Tarr et al., 2008), or as Valverde et al. (2002) called it, the *potentially* implemented curriculum, given the intermediary mediating role that textbooks play between the intentions of curriculum designers and the implementation of these intentions in the classroom by teachers. That said, throughout the entire dissertation, I focus on curriculum with an eye to students and learning, as opposed to forms of teaching or assessment.

Outline

This chapter is organized into three parts. In the first part, I detail the metaphor of mathematics curriculum as a storied, narrative art form and lay out my axio-onto-epistemological perspective that story is socio-cultural in nature, meaning that the intended curriculum (and textbooks in particular) are cultural (didactical) artifacts of the discipline of mathematics.⁵ In the second part, I synthesize literature on (curricular) coherence across several disciplines (from in

⁵ “Axio-onto-epistemology” is a portmanteau of *axiology* (the philosophical branch focused on the study of value, often subdivided into aesthetics and ethics), *ontology*, and *epistemology* that I adopt to acknowledge that these philosophical considerations are always inseparably related to one another (e.g., Eisner, 1985). “Onto-epistemology” is common parlance in education research, yet I have only seen *axio*-onto-epistemology used on rare occasions (e.g., Abreu, 2022). Notably, philosopher Karen Barad (2007) introduced *ethico*-onto-epistemology, but her focus was on *ethics*, rather than the wider category of axiology, including aesthetics. This state of affairs is emblematic of the disproportionate attention placed on ontology and epistemology compared to axiology—and particularly aesthetics—in (mathematics) education research (Bowers, 2022; Eisner, 2004; Tyburski, 2023).

and outside of education) to explore various forms and ontologies of coherence (Dissertation Q1). This discussion is followed by an interrogation of the axio-onto-epistemological foundations behind the often-implicit assumption that (curricular) coherence in its many forms is desirable (Dissertation Q2). Finally, in the third part, I conclude by bringing parts one and two together to propose the curriculum-as-story metaphor as a flexible lens for interpreting curricular (in)coherence in its many forms. Specifically, I suggest that such flexibility is only made possible by leaning into the socio-cultural view of story and attending to not just one cultural view, genre, or aesthetic form of story when utilizing this metaphor but instead considering many.

Part 1: The Curriculum-as-Story Metaphor

Personal Aesthetics and Aesthetic Experience

Despite the reductive view of aesthetic engagement alluded to previously as being merely peripheral or “just for entertainment” that often pervades educational and popular discourses⁶, there is plenty of evidence that one’s personal aesthetics are fundamentally intertwined with how they make sense of the world (Eisner, 1985; Jasien & Horn, 2022; Stott, 2018). Indeed, this alternative view of aesthetics aligns with the original etymological meaning of aesthetics as “knowledge acquired through sensory perception” (Marini, 2021, p. 40). From this perspective, aesthetic experience is an idiosyncratic response a person has to a particular object or event (Dewey, 1934/2005; Saito, 2007), which could be positive (joy, wonderment, curiosity), negative (disgust, boredom, disappointment), neutral, or an eclectic mix of all these. Such a view challenges classical Western perspectives on aesthetics where objective aesthetic evaluations are

⁶ Such a view likely originates from the stultifying and unnatural historical barricade that has been gradually erected between art and science, between feeling and thinking, in education and Western society at large (Eisner, 2004; Laird, 2013).

possible, instead treating such evaluations as highly subjective and contextual, varying from person to person and even moment to moment. Additionally, this view suggests that people's everyday aesthetic experiences and choices are impacted (often at the unconscious level) by their idiosyncratic aesthetic preferences, which are entangled with their embodied emotional and sensory responses (Dietiker, Riling, et al., 2023; Marini, 2021; Norman, 2007; Sinclair, 2008; Stott, 2018).⁷ For example, when going on a walk, the path you take is likely influenced by the possible sights, smells, and sounds you may wish to encounter (or avoid) on your stroll, whether you consciously acknowledge this or not.

Similarly, a person's engagement with a mathematics lesson or, more generally, curriculum is inescapably aesthetic in nature. "A mathematical experience can be a transformative, compelling enterprise of impulses and anticipation (see Hofstadter, 1992). Aesthetic is the motivating influence that can advance an individual (mathematician or student) through challenges and setbacks and dissuade the person from giving up" (Dietiker, 2015b, p. 3). The many multi-faceted roles that aesthetics play in both professional mathematicians' and students' mathematical activity have been well-documented (Burton, 2004; Jasien & Horn, 2022; Sinclair, 2004, 2006). Some roles are motivational in nature, as Dietiker (2015b) observed in the quote above. For example, aesthetic reactions like wonderment, surprise, and curiosity have been repeatedly linked to sustained engagement with mathematical inquiry (Dietiker, Singh, et al., 2023; Dietiker & Richman, 2021; Gadanidis et al., 2016). Yet, aesthetic considerations also influence the *nature* of one's mathematical inquiry and engagement, ranging from influencing what features in a situation one attends to or ignores (Andrà, 2013; Gadanidis & Hoogland,

⁷ Insight, for instance, is an excellent example of a phenomenon that is simultaneously aesthetic, emotional, and epistemological in nature (e.g., Borwein, 2007; Irwin, 2003). What someone considers an insightful moment is grounded in their personal aesthetics of learning and paired with a sudden unfolding moment (the "aha!") which tends to be pleasurable.

2003) to the questions one deems as interesting and “worthy” of further consideration (Fiori & Selling, 2016; Jasien & Horn, 2022).

The Intersubjective Nature of Aesthetics and Politics of Aesthetics

The *personal* aesthetic sensibilities alluded to in the previous section are not developed within a vacuum but instead in relation to the *intersubjective* aesthetic sensibilities of the socio-cultural contexts we inhabit. This philosophical perspective is consistent with an arts-based research (ABR) paradigm (Chilton et al., 2015; Conrad & Beck, 2015), which I embrace throughout this dissertation to conceptualize aesthetics (as well as art).⁸ A primary ontological assumption of ABR is that “we are all, at a fundamental level, creative and aesthetic beings in intersubjective relation with each other and our environment . . . art is fundamental to our capacity to make meaning and give value to life; human beings are fundamentally aesthetic beings” (pp. 7–8). Following Dissanayake (1992), we are “homo aestheticus” in the sense that art and aesthetics play a unique role across cultures in making sense of experience and communicating what we value (Cajete, 2012; Dewey, 1934/2005; Grosz, 2008). These stances are consistent with the original etymological meaning of aesthetics mentioned previously, further highlighting the inseparable relationship between aesthetics and epistemology. A second core tenet the ABR paradigm embraces is a stance of ontological pluralism (Chilton et al., 2015). This pluralism alludes to the worldview that “truth” is represented through multiple aesthetic and artistic realities. These forms of (aesthetic) reality are paradoxical in nature—simultaneously logical and illogical, real and surreal, in time and timeless, linear but spatial—yet, they co-exist and intermingle in a complex dialectic that ABR positions as harmonious and natural rather than

⁸ I call ABR a paradigm of research inquiry (Lather, 2006; Stinson & Walshaw, 2017), in the sense that it features a distinct philosophical worldview (Stinson, 2020) with corresponding methodological approaches. See Chapter 3 for further details on the nature of arts-based research methodology. In this section, I introduce the core philosophical tenets of the ABR worldview solely to clarify my perspective on aesthetics.

aberrant (see e.g., Irwin, 2003). Simply put, what one person considers to be an artistic masterpiece could be interpreted by someone else as an uninspiring dud. Both interpretations—though paradoxical—can coexist as plural aesthetic truths with something to offer the intersubjective conversation about the art piece under consideration. When subjective aesthetic interpretation is viewed as a primary way of coming to know, there is no objective, singular Truth.

The ABR paradigm acknowledges that the multiple realities stemming from particular aesthetic sensibilities are always co-informed by idiosyncratic, individual experience in the constructivist sense of Dewey's (1934/2005) *Art as Experience* (see e.g., the previous section) and intersubjective, meta-social context in the sense of Rancière's (2000/2004) *Politics of Aesthetics*. Rancière argued that which aesthetics become privileged within a community of practice (e.g., the discipline of mathematics) is political insofar as shared aesthetic practices result in distributions of what is considered sensible, often in ways that are covert. Consequently, aesthetic sensibilities that are dominant within a socio-cultural context (e.g., the mathematics classroom) have a powerful influence on not only an individual's personal aesthetics but also how one's personal aesthetics are perceived (by the individual *and* others from within the socio-cultural context, see e.g., de Freitas & Sinclair, 2014). Further, intersubjective aesthetics becomes a yardstick against which all other aesthetic sensibilities are measured and judged: privileged socio-cultural aesthetic sensibilities become a way of policing who/what is considered “aesthetic” and who/what is not.

Mathematics Curriculum as a Storied Artform

A corollary to the axiom that aesthetic engagement is inextricably bound up with learning is that curriculum design is best viewed as a kind of *art* whose *form* and content are inseparable and equally important. Eisner (2004) introduces this lesson as follows:

How something is said is part and parcel of what is said. . . . *How* one speaks to a child matters, *what* a classroom looks like matters, *how* one tells a story matters. . . . Change the cadence in a line of poetry and you change the poem's meaning. (pp. 6–7, emphasis in original)

This call to contemplate aesthetic form in addition to content when considering students' curricular engagement has been taken up by several educators—both in and outside of mathematics education (e.g., Fiori & Selling, 2016; Uhrmacher, 2009). Several suggested strategies have emerged from this collective contemplation, such as incorporating moments of surprise, suspense, tension, or wonder into curricula (Dietiker, Singh, et al., 2023; Gadanidis & Cendros, 2023; Parrish, 2009) while simultaneously allowing for moments of ambiguity and uncertainty that spotlight the “unfolding” of curricular experiences as emergent and aesthetic in nature rather than fixed, objective, and pre-determined (Appelbaum, 2010; Irwin, 2003). Yet, a natural question arises upon embracing the metaphor of mathematics curriculum as art: *Which artforms?* One proposal is to view mathematics teaching and learning as a theatrical production or, more generally, a kind of performance art (Gadanidis & Borba, 2008; Gerstberger, 2009; Rodd, 2003). Sinclair (2005), in likening mathematical text(book)s to forms of art, alludes to poetry as another possibility before broadening her focus to other forms of narrative. Along similar lines, both Appelbaum (2010) and Borasi and Brown (1985) have contemplated what we could learn from comparing mathematics curricula and textbooks, respectively, to literary novels. Though the possibilities abound, what many of these artforms have in common—from film to theatrical performance to novels—is a focus on mathematics curriculum as a *story* (Dietiker, 2015a; Gadanidis & Hoogland, 2003).

The prevalence of story across these conversations about curriculum as artform is apt given what is known about the crucial role that stories and storytelling play in how humans learn and make sense of the world. Stories are one way we organize our experiences, as “excuses,

myths, reasons for doing and not doing, and so on” (Bruner, 1991, p. 4). In fact, neuroscience research suggests that narrative comprehension may be one of the earliest mental capacities that babies develop (Nelson, 2006). These narrative ways of knowing complement and intertwine with logico-deductive ways of knowing, yet they cannot be reduced to one another (Bruner, 1986; Clark & Rossiter, 2008; Polkinghorne, 1988; Worth, 2008). As alluded to previously, stories are much more than a form of entertainment. Even in a discipline like mathematics that has historically privileged logic, narrative forms of knowing have repeatedly been shown to play a prominent role in how professional mathematicians, students, and teachers learn (e.g., Burton, 1999; Healy & Sinclair, 2007). The metaphor of curriculum as story fully acknowledges that narrative and logico-deductive forms of sense-making go hand-in-hand: after all, the aesthetic and emotional impact of *how* a story is told is equally (if not more) important than the logical construction of the story’s many components (Dietiker, 2015a, 2015b; Gadanidis & Hoogland, 2003).

In addition to the congruence of the metaphor of curriculum as story with perspectives on narrative sense-making, there are other analytic reasons such a perspective serves the stated goals of this dissertation. Namely, treating curriculum as a story directs our attention to (1) the order in which curricular events unfold, (2) possible rearrangements of how curricular events *could* unfold, and (3) the aesthetic impact these orderings might have on learners (Dietiker, 2015a). Though other approaches to curriculum design and analysis undoubtedly attend to sequencing, Dietiker (2015b) has argued that the power of viewing curriculum as a story is in how it reframes sequencing with an eye to planning for possible aesthetic impact:

Just as a fiction writer carefully chooses the moment in a story to introduce a character or to reveal 'who done it', so too might a [curriculum designer] deliberate on the point in a sequence to introduce mathematical objects or reveal important properties or

relationships.⁹ (p. 9)

In other words, sequencing from this perspective attends to the logical sequencing of content alongside the aesthetic impact of the narratives conveyed by different possible sequences. In analyzing curricular stories, attention is paid to a question such as *in what ways does the curricular plot unfold with a dynamic pace and rhythm that build tension and suspense, encouraging students to actively wonder about what might happen to the mathematical characters next, leading to further aesthetic and epistemological engagement?* Gadanidis and Hoogland (2003) suggest that the answer is often “not many” in the context of most mathematics classrooms:

In contrast [to good stories that trigger pleasurable reactions], the mathematics story that is experienced in school is often flat-lined. It does not have the peaks and valleys of a good story, it does not engage students' mathematical attention, and it offers minimal opportunities for experiencing mathematical insight. (p. 489)

Consequently, Gadanidis and his colleagues propose that a major criterion for curricular success is whether students leave the classroom eager to re-tell curricular mathematics stories to their friends and family, in the same way they might after watching a memorable movie (Gadanidis et al., 2022; Gadanidis & Borba, 2008). Accordingly, Gadanidis et al. (2022) have suggested that “good” mathematics stories are ones that travel with students across time and space. First, good stories travel in space outside the classroom, as they ideally captivate students to re-tell the story outside of the classroom to their friends and family. Second, good stories travel in time across the curriculum and are “*regenerative*, as they live fruitfully in future mathematics stories” (Gadanidis & Hoogland, 2003, p. 490, emphasis mine). In other words, “good” stories intertwine with future mathematical stories in a way that makes future stories more engaging or meaningful.

⁹ Dietiker (2015b) uses “teacher” in this quote rather than “curriculum designer”, yet she specifies that the view of curriculum as story is applicable to both the enacted curriculum (and teachers) *as well as* the written curriculum (and curriculum designers).

That said, what constitutes a “good” mathematical story is idiosyncratic, contextual, and culturally bound. In this dissertation, I am not suggesting that we poke and prod at curriculum to discern the “ultimate story form” for the mathematics curricula but rather that we take the time to learn from the ancient human craft of storytelling in its *many* instantiations across cultures, allowing us to leverage the *art* of storytelling and consider aesthetic narrative engagement as we craft and contemplate curriculum design. Just as art is a process, rather than just a product (Eisner, 2004), I view these curricular efforts as a process of perpetual engagement not marked by certain or objective answers but rather a contingent “unfolding” that is ever evolving and never certain (Irwin, 2003).

Mathematical Discourse as a Form of Narrative

Perspective on Reading and Reader-Oriented Theory

I take a reader-oriented perspective on texts, which holds that meaning is not contained in the text itself but instead that meaning is developed through a transactional process as the reader engages hermeneutically with the text (Rosenblatt, 1978, 1988; Weinberg & Wiesner, 2011). In other words, there is no “objective” or “correct” reading of a narrative—no two readers will experience a text in quite the same way. In fact, it is not uncommon for different people to read the same narrative but come away with drastically different interpretations and aesthetic reactions. One reason for this is that different readers may attend to and analyze elements of a text in different ways. After all, one reader’s “riveting extended metaphor” is another reader’s “dull pages full of description and exposition”. Ultimately, a reader’s interpretation of a text is influenced by their goals and motivations for reading the text, as well as the cultural and socio-historical context in which the reading is taking place (Rosenblatt, 1978, 1988). As I detail later, the way one directs their attention while reading is highly dependent on their disciplinary and cultural upbringing. Disciplinary literacy studies, for instance, demonstrate how mathematicians’

reading practices vary from those in other disciplines (Shanahan et al., 2011).

A reader's aesthetic sensibilities—which may themselves be influenced by disciplinary norms (de Freitas & Sinclair, 2014; Rancière, 2000/2004)—further influence the elements of the text a reader directs their attention toward (Gadanidis & Hoogland, 2003). Rosenblatt (1986), however, distinguishes between an *aesthetic* stance toward reading and an *efferent* one, where the goal is primarily to glean information from the text. An aesthetic stance toward reading, on the other hand, is characterized by attention to the sounds of words, felt tensions while reading, and imagery evoked by actions, characters, or settings among other aesthetic reactions. In this study, I adopt such an aesthetic stance while reading curricular texts as stories (Dietiker, 2015a, 2015b).

An upshot of reader-oriented theory is that a text is open to *many* possible readings. A natural consequence of this is that I am *not* suggesting that everyone reads mathematical textbooks and curricula as stories from an aesthetic stance but instead that it is *possible* and beneficial to read them this way (see e.g., Dietiker & Richman, 2021). Indeed, some textbook reading studies suggest that undergraduate mathematics students tend to take a mostly efferent stance while reading textbooks, treating them primarily as a review source for practice problems and supplemental information, rather than as stories in their own right (Randahl, 2012; Weinberg et al., 2012).

Mathematics Texts as a Form of Narrative

To frame the curriculum-as-story metaphor, I follow Dietiker (2013, 2015a) and draw on literary theory (Bal, 2017) to first define the broader concept of narrative. To qualify as a narrative, a text must be perceived by the reader as consisting of events: “transition[s] from one state to another state” (Bal, 2017, p. 5). Additionally, the reader must perceive these events as being sequentially related to one another (temporally or otherwise). Though this does mean that

the order in which events are introduced to the reader tends to be linear in nature, it does not necessitate that events be introduced to the reader in the order they occur in time. After all, literary narratives frequently flirt with non-linearity by relaying events to the reader in a non-chronological order (e.g., flashbacks, foreshadowing) for aesthetic or thematic impact.¹⁰

Similarly, while juxtaposing mathematical proofs with narrative, Thomas (2007) observed how proofs also feature a form of “flashback” when they remind a reader of relevant hypotheses, lemmas, and other results before logically chaining these together to move the proof forward.

Indeed, many have proposed that mathematics and narrative share much in common. Specifically, mathematical proof (Doxiadis, 2012; Netz, 2005), mathematical texts and textbooks (Dietiker, 2013; Gowers, 2012; Sinclair, 2004), and, more generally, mathematical discourse and activity (Burton, 1999; Healy & Sinclair, 2007; Margolin, 2012) have each been investigated as forms of narrative. What is more, this view has been espoused by mathematicians, mathematics educators, historians of mathematics, writers of mathematical fiction and nonfiction, literary theorists, and other storytellers, including poets and playwrights. It has also been the subject of multiple interdisciplinary, international conferences on this topic (see e.g., Boyd, 2003; Doxiadis, 2004, 2005; Doxiadis & Mazur, 2012a).^{11,12}

Still, there are those who refute the notion of mathematical texts as narrative in nature. Solomon and O’Neill (1998), for instance, analyzed historical letters exchanged between mathematicians and concluded that they represented instances of mathematics embedded in everyday written narrative, rather than evidence that mathematical text itself could be seen as

¹⁰ For example, the classic novel *Dracula* (Stoker, 1897) is told through a series of journal entries, letters, and other documents which are not always ordered chronologically, thereby building further unease and narrative tension at key moments. In film, *Memento* (Nolan, 2000) and *Eternal Sunshine of the Spotless Mind* (Gondry, 2004) both contemplate themes related to (loss of) memory and are told in partially reverse chronological order to further immerse the audience in each main character’s perspective.

narrative. Ultimately, they argued that “Mathematics is constituted by a logical structure that is not reducible to a temporal sequence of events. . . . [Therefore,] mathematics cannot be narrative for it is structured around logical and not temporal relations” (pp. 216–217). Many others, however, have pushed back on this claim, challenging this Platonic onto-epistemology of mathematics as a collection of “timeless” truths while drawing attention to alternative forms of temporality and sequentially present in mathematical discourse. For example, Lloyd (2012) conducted a historical investigation on the competing Aristotelian perspective in the context of constructing geometric proofs. They concluded that from this onto-epistemological perspective:

[Mathematical] reasoning does involve a sequence of steps that are essential to reveal, or as Aristotle would say to actualize, the truths that are there in potentiality in the geometric figures or the quantities discussed. In the sense that the proof depends on a construction or procedures that are carried out at some point *after* the statement of what is to be shown, in the sense the mathematical reasoning shares the sequentially, if not the temporality, of narrative. (p. 403, emphasis in original)

Similarly, when de Freitas (2012) asked a mathematician to tell a story about a mathematical diagram, she observed how they sequentially animated (i.e., “actualized”) specific parts of the diagram, thereby creating a temporalized narrative sequence with a chronology of events for the seemingly static and “timeless” geometric diagram. Healy and Sinclair (2007) meanwhile

¹¹ These references point to relevant keynote presentations or publications from these conferences. Two such meetings were sponsored by the Canadian Fields Institute for Research in Mathematical Sciences: in 2003, *Mathematics as Story: A Symposium on Mathematics Through the Lens of Art and Technology* was organized by Gadanidis, Hoogland, and Kamran Sedig, then the following year in 2004 *Online Mathematical Investigation as a Narrative Experience* was organized by Gadanidis, William Higginson, and Sedig. While the websites for these conferences are no longer accessible, invited speakers included those in the arts and humanities (such as Ellen Dissanayake, cited previously) as well as those in mathematics education (e.g., Nathalie Sinclair, Andrea A. diSessa). Two additional conferences were sponsored by Thales + Friends—a non-profit organization that aspires to bridge the gap between mathematics and other forms of cultural activity—and held in Greece: *Mathematics and Narrative* took place in 2005 and was organized by Doxiadis and then in 2007 a second conference followed, organized by Doxiadis and Barry Mazur, which culminated in an edited collection of essays about mathematics and narrative, *Circles Disturbed* (Doxiadis & Mazur, 2012). Contributors included mathematicians, mathematics educators, narratologists, playwrights, and others. Further information about the presentations and contributors is still available as of writing at <https://thalesandfriends.org/>.

¹² While most of these conversations focus on narratives of *people* doing mathematics (e.g., literary accounts of historical mathematicians, pre-service mathematics teachers’ personal mathematical autobiographies, and the fundamentally narrative nature of mathematical learning), my focus in this study is primarily mathematical text(books) as narrative.

observed how the motion of dynamic mathematical visualizations supported students in constructing mathematical narratives to guide their learning. Using this evidence, they make an empirical and theoretical case that mathematical activity involves both narrative and logico-deductive ways of thinking (Bruner, 1986), leading them to suggest that while formal mathematical text might not be narrative in nature, that does not discount a student's (or perhaps a mathematician's) mathematical activity from being a form of narrative.

Ultimately, because I adopt a reader-oriented approach to narrative, Solomon and O'Neill's (1998) interpretation of mathematical text as non-narrative due to its "timelessness" can coexist with my own interpretation, where I choose to read texts with a broader sense of sequentially and temporality. Indeed, Bruner (1991) argues for multiple ways to conceive of temporality in the context of narrative, suggesting that "even nonverbal media have conventions of narrative diachronicity, as in the 'left-to-right' and 'top-to-bottom' conventions of cartoon strips and cathedral windows" (p. 6). In the end, though, rather than take a hardline stance on this issue (like Solomon and O'Neill), I adopt one similar to Bal (2017) who suggested that "not everything is narrative, but practically everything in culture has a narrative aspect to it, or at least can be perceived, interpreted, as narrative" (p. xix). Similarly, rather than wading into the fray of the mathematics as narrative debate, Dietiker (2013) instead proposed a third option by establishing a *metaphor* through which mathematical texts could be interpreted as a narrative.

Framework for Viewing Curriculum as Story

Three Layers of Mathematical Narrative

Dietiker's (2013) metaphor of curriculum as story is based on Russian formalist narrative tradition (Bal, 2017), which distinguishes between three interconnected layers of meaning: the text, story, and fabula. The *text layer* refers to the modalities through which the narrative is delivered (e.g., written text, spoken dialogue, imagery, animation). Meanwhile, the *story layer*

refers to the sequence of events and how they unfold sequentially through the text (e.g., on page one *this* happens, followed by *that* on page two), while the *fabula layer* represents a reader's reconstruction and sense-making around how these events are related—chronologically or otherwise—possibly deviating from the sequencing of and explicit connections between events as presented in the text.¹³ For example, in the case of a literary flashback, the story layer refers to the events in the order they were sequenced in the narrative—say, present event A, followed by the flashback event, then present event B. On the other hand, a reader's corresponding chronological fabula might begin with the flashback, followed by present event A, then present event B (assuming they recognize the flashback depicted a past event). The fabula, then, refers to an individual reader's idiosyncratic interpretation of the story layer, including any deeper truths, meanings, or morals they take away from engaging with the narrative.¹⁴

To establish the metaphor between literary and mathematical narrative, Dietiker (2013) defined *mathematical fabula* by focusing primarily on a reader's "logical re-construction of mathematical events", or, in other words, the "logical line of reasoning" (p. 16) a reader used to connect events. This kind of relationality and sequencing recalls the Aristotelian actualization (Lloyd, 2012) or sequential "animation" de Freitas (2012) observed while one mathematician told the story of a mathematical diagram. At the same time, mathematical fabulae need not be strictly linear in nature, as Dietiker (2013) demonstrated using a multi-linear concept map to depict one possible fabula. In this dissertation, I choose to adopt a broader view of mathematical fabula by acknowledging *any* types of connections a reader (i.e., students in Chapter 3; myself in Chapter 4) uses to interpret or relate mathematical events—logical or otherwise. Such a choice is

¹³ Note that following the definition of narrative introduced earlier, a text is *not* considered narrative unless the reader can discern some sort of sequentiality or connection between events.

¹⁴ These are lowercase *t* subjective truths from the perspective of the reader. Different readers may discern dramatically different morals from the same story.

not entirely unprecedented. After all, Dietiker (2013) clarified that a reader's logical re-construction of mathematical stories could occur in many different ways, including through "re-defining, noticing a pattern, connecting, and conjecturing" (p. 16), and mathematical sense-making processes such as these are fundamentally aesthetic and embodied (e.g., Fiori & Selling, 2016; Jasien & Horn, 2022), as well. Further, this choice to attend to other forms of relationality is also consistent with research suggesting that a reader's interpretation of multi-modal texts is impacted by factors that are not strictly logical (e.g., Stöckl & Bateman, 2022). As detailed previously, aesthetic forces and expectations may direct a reader's attention to certain (culturally constructed) narrative structures and textual modes of organization when making sense of how a narrative hangs together (e.g., Gadanidis & Hoogland, 2003; Shanahan et al., 2011). As a simple example, in the context of reading mathematical textbooks, whether a figure appears in text or in the margin may influence the degree to which readers attend to it, as well as their overall interpretation of the perceived quality of the mathematical exposition (Woollacott et al., 2023). Collectively, these considerations suggest the need to attend to not only logical but also aesthetic and emotional relations a reader might consider when interpreting and relating mathematical events in my definition of mathematical fabula.

To give a curricular example demonstrating the use of these three narrative layers, take the case of a mathematics textbook. The text layer would include the written text, ranging from paragraphs of prose to boxed equations to captions of figures. The text layer also includes the imagery of the figures themselves, such as pictures, graphs, diagrams, etc. Meanwhile, the story layer refers to the order that mathematical events, character introductions, etc. occur on the page, as perceived by the reader: perhaps an example is used to subsequently introduce a mathematical definition, followed by a formal statement of a related theorem alongside a visual depiction of

the theorem in the margin, concluding with several additional examples of the theorem in action. After engaging with the story, a particular reader's mathematical fabula might instead be arranged *starting* with the statement of the theorem, followed by the examples as instances of the theorem (including the first one) alongside the visual depiction. Another reader, though, might be more aesthetically and emotionally drawn to the visual depiction on the page as a way of making sense of the story. This reader's mathematical fabula might start with the visual, including the examples and theorem as several different possible paths for interpreting the visual. Note how in both these examples, the sequencing and relationality of the readers' fabulae differ from that of the story. Examples of this have been observed empirically, too. Andrà (2013), for instance, investigated students' interpretations of the story of a mathematical lecture via the notes they took (an instantiation of their mathematical fabulae), finding great differences in the relationships between events and concepts each student chose to foreground, background, or even omit entirely.

Ultimately, it is the dynamic interplay between the final two narrative layers—story and fabula—that allows for an analysis of mathematical narrative which considers the ways in which a reader might be emotionally and aesthetically engaged with how a mathematical story develops. By attending to the sequencing of mathematical events and the ways in which the unfolding of these events may result in a reader having to dynamically adjust and update their interpretations of a story (i.e., their mathematical fabula) as they (re-)read, this framework can be used to investigate how a story could aesthetically and emotionally impact readers. Yet, mathematical stories are often sequenced in a way that first gives away the punchline (answer, theorem, etc.) followed by a thorough explanation (through examples, proof, explanation, etc.), often foreclosing the opportunity for anticipation, tension, or surprise leading up to said

punchline (Gadanidis & Hoogland, 2003; Ryan & Dietiker, 2018). However, even simple re-arrangements in the sequencing of mathematical events could transform how a reader experiences a story and the ways in which they are aesthetically and emotionally pulled to engage with the story (Dietiker, 2015a). In other words, how a mathematical story is told matters! While the fabula layer is undoubtedly crucial to conceptualizing different student interpretations to mathematical stories, Dietiker (2013) contends that the mathematics education research community should redirect some of its attention to the story layer “with an aim of enhancing the potential dramatic effects of mathematical texts” (p. 19). In other words, this literary narrative framework enables us to attend to the artistry of crafting mathematical curricular narratives (Dietiker, 2015b) by attending to the structures of these stories and how events unfold within them, not to mention common genres or structures of story and other aesthetic considerations involved in crafting stories that are more likely to engage students.

Elements of Mathematical Story

Sequencing is a quintessential aspect of stories, but this is but one element to consider when interpreting a story. While there are a multitude of story elements one could attend to, I follow Dietiker (2015a) who extended her initial narrative metaphor (Dietiker, 2013) to include some core elements of literary narrative which have endured over time—characters, action, settings, and plot. I begin by briefly describing each of these elements, followed by a more detailed discussion of mathematical characters and plot, respectively, as these are the elements I attend to in particular detail throughout this dissertation. Keep in mind throughout the discussion of these story elements that a narrative does not a priori include characters, settings, actions, etc. Rather, a reader interprets a narrative as featuring characters performing actions in certain settings. Therefore, what “counts” as a character, a setting, or an action is subjective and up to

reader interpretation.¹⁵

Mathematical **characters** include mathematical objects, concepts, representations or anything else that a reader anthropomorphizes or treats as an entity which can act or be acted upon (Dietiker, 2015a; Weinberg et al., 2016). Possible examples could include variables (x , y , z , t , etc.), a function (e.g., $f(x,y,z)$) or the *concept* of a particular type of function, such as vector fields), or forms of representation (the general notion of an xyz -coordinate plane, a *particular* xyz -coordinate plane on which the function $f(x,y,z)$ is graphed, or perhaps a table of values that gets referred to multiple times in text). As Weinberg et al. (2016) note and as some of the prior examples demonstrate, mathematical characters can be either particular (the function $f(x,y,z) = x^2+y^2+z^2$) or general (multivariable, real-valued functions of the form $f(x,y,z) : \mathbb{R}^3 \rightarrow \mathbb{R}$). The mathematical **settings** which characters inhabit include different forms of representation (symbols, tables, graphs, etc.) as well as the physical contexts or courses a reader identifies mathematical characters to be inhabiting at a particular moment in the story. A reader might interpret, say, an electric (vector) field introduced visually as inhabiting both a graphical setting as well as the disciplinary setting of physics. Mathematical **action** refers to moments when a character or characters are manipulated mathematically (e.g., a vector field $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ is split into its component functions) or when a new character is created or introduced that results in some form of change (e.g., $\mathbf{r}(t)$ is plotted, leading to the introduction of the geometric *space curve* $\mathbf{r}(t)$).

Unlike anthropomorphic literary characters who are often interpreted as possessing agency to carry out actions themselves, many mathematical narratives (especially textbooks) are written in a third person omniscient point of view (Herbel-Eisenmann, 2007; Love & Pimm,

¹⁵ For example, a reader could interpret a mathematical function as a character (or object) in its own right, an action that acts on other characters (a process), or perhaps both at the same time (Breidenbach et al., 1992; Sfard, 1992).

1996) that positions mathematical characters as primarily being acted upon with little agency of their own.¹⁶ Mathematical actions, then, are carried out by mathematical actors, including the (omniscient) narrator, the reader themselves, or perhaps human characters within the mathematical narrative. Finally, the mathematical **plot** refers to a reader's aesthetic reactions to the unfolding of the story as they engage with and form their interpretations of the story (i.e., their *fabula*). Note that *plot* and *story* are sometimes viewed as synonyms in common language as well as other literary traditions, but within the Russian formalist tradition Bal (2017), they have distinct meanings.

Dietiker's (2013, 2015a) metaphor of curriculum as story is designed to describe several types of curricula at varying grain sizes. This narrative framework can be used to analyze any form of curriculum (Dietiker, 2015a), ranging from curricular policy documents constituting the intended curriculum (Gadanidis & Cendros, 2023) to the potentially implemented textbook curriculum (Dietiker & Richman, 2021; Huffman Hayes, 2024; Miežys, 2023) all the way to the enacted curriculum of mathematics lessons (Andrà, 2013; Dietiker et al., 2023; Weinberg et al., 2016). Additionally, within each type of curriculum, the tools of the framework can be leveraged to attend to various grain sizes of mathematical story, ranging from the individual lesson level, the unit level, course level, cross-course level, etc. (Dietiker, 2015a). As I have defined narrative as events arranged in some kind of sequence relative to one another, a narrative can be as simple as a single clause or as complex as an entire series of stories. That said, while the same general narrative constructs (character, action, setting, etc.) can be applied across narrative grain sizes, they may need to be reinterpreted accordingly. For example, an analysis of the story told across

¹⁶ de Freitas' (2012) account of mathematical characters is a notable exception to this point of view. Though she does not explicitly define the term, she assigns agency and perspective to the mathematical characters one mathematician introduced while telling a story about a geometric diagram.

an entire calculus textbook might consider only one or two major events from each chapter (e.g., the introduction of the character of the derivative or the first time the fundamental theorem of calculus is stated) and only the characters that play major roles across multiple chapters (e.g., limits, derivatives, and integrals). In such an analysis, the events that occurred in, say, example 2 of section 1.6 that led to evaluating the limit of a particular function would likely be omitted. On the other hand, in the case of Chapter 4, where I zoom in to analyze the stories of just three chapters in the textbook, I will include events at this scale.

Characters

In Chapter 4, I treat three different types of function found in multivariable calculus as characters and juxtapose the *stories* that are told about each of these characters in the chapter of a textbook in which they are introduced. Because I devote particular attention to the characters in that study, I take the time in this section to introduce additional nuances and details concerning this story element. While all elements of a story have idiosyncratic roles, compelling, memorable characters can be a crucial way to hook readers and keep them invested in the events of a story (Corbett, 2013). Mathematical characters likely play a similar role in mathematical stories. For instance, Andrà (2013) found that most students in her study gravitated toward depicting mathematical characters in their notes from a mathematics lecture, backgrounding mathematical actions in the process, even though the instructor had intended to highlight these actions in his storytelling. She concluded by suggesting that this attention to characters might be a natural human tendency: “Paying attention to the characters' names recalls a common practice: when we meet a new person, we record her name” (p. 22). Indeed, studies of narrative understanding provide evidence that characters—and particularly the main characters—often have a large influence on how readers make sense of a story (e.g., Morrow, 1985). Having established that characters are important, I provide a more thorough definition of mathematical character,

including how characters are introduced in mathematical narrative. Following this, I introduce additional aspects of character and some considerations for narrative character analysis.

What constitutes a mathematical character is up to reader interpretation. However, Dietiker (2015a) defines mathematical characters as objects—“‘figures’ brought into existence (objectified) through reference or inference in the text”—and clarifies that “what is objectified by a mathematical story is determined by its reader based on the positioning of content within the text” (p. 292). She offers the example of a polynomial “ x^2-6x+3 ”, which might be interpreted as a single character (object) or a collection of multiple characters (e.g., x^2 , $6x$, and 3) depending on the story context. Weinberg et al. (2016), following narrative learning theorists Czarniawska (2004) and Polkinghorne (1988), suggest that this process of determining mathematical characters is fundamentally iterative and abductive in nature. A reader initially identifies characters (and other story elements) based on their usefulness for interpreting the story and forming a fabula. Then, as they continue reading (or re-read a previous segment of the text), they revise these interpretations, adding or omitting characters and story elements that support the refinement of their interpretations in ways that are personally satisfying (e.g., to increase the coherence of any identified themes or morals of the story).

Mathematical characters can be introduced in a story in multiple ways, including explicitly as well as implicitly (Dietiker, 2015a). In math textbooks, for example, characters—particularly the *main* characters or *protagonists*—may be introduced through a term bolded in the text or a red box depicting an important definition of a particular character. Still, many other characters—particularly *secondary and supporting characters*—are introduced implicitly and must be discerned through a process of interpretive abduction described above. Often, this occurs via a mathematical action (for example, adding “2” and “3” could be interpreted as resulting in a

new character “5”, even if this is just an intermediary action in a larger event, Dietiker, 2015a). While even main characters might be introduced implicitly, *when* and *how* a character is introduced does impact a reader’s interpretation of that character (e.g., Chrysanthou, 2022). For instance, a reader may suspect that characters introduced in the opening of a story will return to play a role later (e.g., Dietiker, 2015a).

Like literary characters, a mathematical character can be interpreted as possessing *character traits* (e.g., a function could be *even*), and they may undergo mathematical *character development* as the reader learns more about these traits. The process by which a reader learns about a character and their development is fundamentally relational in nature. By juxtaposing the qualities of one character with other characters, a reader discerns the qualities that make the character unique, an interpretive process which Bal (2017) called “fleshing out” a character. A reader may not notice, for example, that a function is *even* until they are introduced to other functions which share this trait, as well as other functions which are *odd*, and perhaps even some that are *neither even nor odd*. Given the relational nature of this interpretive fleshing out process, characters—literary or mathematical—must be analyzed in relation to one another (Margolin, 1989; Thomas, 2007). Further, in the case of mathematical narrative, Andrà (2013) provided several examples of how mathematical character development is intertwined with the representational setting a character is presented within, meaning that characters are best analyzed in relation to not only other characters but also other story elements.

A primary reason I have spent this much time introducing characters is because Dietiker (2015a) has proposed that reading for character development can be a useful form of curriculum analysis for both teachers and researchers:

Rather than reading textbooks with a fully developed understanding of the complexity of mathematical objects, this framing enables a teacher to read how new properties are

subtly revealed throughout the sequence. Specifically, a teacher might recognize how different tasks assume radically different characterizations of mathematical objects that may render them unrecognizable to students. (p. 293)

Indeed, the textbook analysis in Chapter 4 follows such a model. By juxtaposing the character development of three different types of multivariable functions in the chapters they are first introduced, I interpret implicit cultural meta-narratives about function(s) that may go unnoticed were these characters' stories not considered side-by-side.

Plot

The construct of mathematical plot is the heart and soul of the curriculum-as-story metaphor. The plot—a reader's aesthetic reactions to the unfolding of the story as they engage with and form their interpretations of the story—is based on a reader's holistic interpretation of all aforementioned story elements coordinated with their interpretation of the sequencing of a story. In other words, the plot stretches *across* both the story and fabula layers of the narrative to attend to both the logical *and* aesthetic sense-making a reader engages in while reading a mathematical story. The plot accomplishes the stated goal of viewing curriculum as a form of art, which must be interpreted based on how its form and content intertwine (Eisner, 2004). Specifically, the plot allows for detailed analysis of how a reader

experiences a mathematical story, perceives its structure (and thus look for order, find patterns, sense rhythm, etc. [*sic*]), and anticipates what is ahead (by wondering, imagining, asking questions). As a tension between the pursuit of mathematical ideas through inquiry and the revelation of information, it is the potential temporal dynamics of the story, that which encourages (or discourages) a reader to continue to advance through the mathematical story. (pp. 298–299).

A plot analysis considers the tight feedback loop of aesthetic engagement, emotional reaction, and logical interpretation.

Richman et al. (2019) draw on Bal (2017) and introduce two additional characteristics of

mathematical plot to Dietiker's (2015a) curriculum-as-story framework—density and rhythm.¹⁷

The *density* of a plot is based on the number of questions that are open and unanswered within the plot. When the number of open questions increases, the plot becomes denser (i.e., “the plot thickens”), which can build narrative tension for the reader, possibly increasing student engagement as they continue reading in search of answers (e.g., Dietiker, Singh, et al., 2023; Dietiker & Richman, 2021). On the other hand, as questions are answered and the density lessens, narrative tension is often released. Meanwhile, *rhythm* refers to “the pattern created by the opening and closing of questions over the course of the story” (Richman et al., 2019, p. 4). While too much of a steady, regular rhythm may come off as boring for a reader, too many irregular rhythmic shifts may be interpreted as jarring. Purposeful changes in a plot’s rhythm can also lead a reader to feel more aesthetically engaged, as in the case when several questions are answered and new questions are immediately posed during the climax of a story.

Stories as Cultural and Textbooks as Cultural Artifacts

Onto-Epistemology of Narrative Interpretation

While those who claim that mathematical discourse can be considered narrative in nature may agree on that point, many of them express distinct onto-epistemologies of interpreting mathematics as narrative as well as the stories themselves (e.g., Thomas, 2007, positions stories as platonic in nature). Therefore, I take a moment to specify that my stance on (mathematical) story is socio-cultural in nature and most aligned with the following description:

Literary characters are contingently created, incompletely determined abstract objects or person-types, products, or artifices constructed by authors at specific space-time points and existing in interpersonal cultural space . . . mathematical objects and truths exist in . . . neither a physical nor a psychological realm but that of shared human cultural understanding, like cultural artifacts. Mathematics in this view is fallible and corrigible,

¹⁷ Richman et al. (2019) also introduce a third characteristic of the plot—coherence. I save an in-depth discussion of plot coherence for the next section, in which I give a detailed account of several interpretations of curricular coherence.

being humanly created, not discovered. (Margolin, 2012, p. 494)

Such a view is consistent with Bal's (2017) stance on narrative interpretation as culturally situated in nature, as opposed to a process of objective classification. It is also consistent with Burton's (1996) stance on narrative learning in mathematics:

I claim that a narrative approach to mathematics and its pedagogy is consistent with a view of mathematics as being socially derived and with the understanding of mathematics as being socially negotiable . . . By engaging with the narrative, we place the mathematics in its context and personalise it, making it come alive to the conditions of the time. Context provides meaning . . . By narrating, we make use of our power to employ language to speculate about, enquire into, or interrogate that information. (p. 32–33)

An implication of this socio-cultural view of (mathematical) narrative interpretation is that *genres of story*—including those told across the discipline of mathematics (e.g., proofs, Netz, 2005) or in the classroom (e.g., Schleppegrell, 2004)—are not objectively given but instead subjectively and socially constructed. The upshot of this is that common *forms* of a story and the values they convey about, say, coherence, are not absolute but rather cultural in nature. In other words, preferred cultural story forms are intersubjective and governed by disciplinary-cultural politics of aesthetics which have the function of directing the axio-onto-epistemological values that become normalized for those within that culture.

The Intended Curriculum as a Cultural Artifact and Cultural Meta-Narrative

Taking a socio-cultural perspective on the curriculum-as-story metaphor implies, more generally, that the intended curriculum, including the textbook curriculum, is a cultural artifact. In undergraduate education, many instructors use textbooks as a primary curricular guide (Fraser & Bosanquet, 2006). So, even though mathematics students do not read them cover to cover (Weinberg et al., 2012), the stories conveyed (or not) in textbooks play a powerful role in the reproduction of mathematical culture and specifically the meta-narratives that are valued by the discipline (Plut & Pesic, 2003). By a *meta-narrative*, I mean a “cultural narrative schema which

orders and explains knowledge and experience” (Stephens & McCallum, 1998, p. 6). In other words, meta-narratives are narratives that recur (implicitly or explicitly) across cultural artifacts which individuals subsequently leverage to frame, explain, *and story* their past and subsequent experiences. For example, the meta-narrative that “good always conquers evil” is common across children’s books. That said, as Brown (2022) demonstrated in the case of introduction to proof textbooks, the meta-narratives perpetuated by mathematics textbooks do not always align with those held by members of the discipline of mathematics. In these cases, textbooks might convey messages that serve a counterproductive role toward enculturating students into the discipline. These results are consistent with Bressoud’s (2011) observation that (retellings of) historical stories about mathematics do not always line up with how stories are told in textbooks. In general, there should be no expectation that textbooks (as didactic texts) align with “expert” mathematical texts (e.g., Love and Pimm, 1996). More generally, Bernstein (1999) suggested that the translation of disciplinary knowledge, practices, and texts into curriculum always involves a form of “recontextualization” which is never a direct translation. So, while disciplinary narratives may be culturally *related* to disciplinary didactic narratives, they are certainly not the same.

Part 2: A Deep Dive into Perspectives on (Curricular) Coherence

Having established curriculum as a storied form of art that must be considered aesthetically and logically as a holistic gestalt of both its form and content (Dietiker, 2015b; Eisner, 2004), I next turn my attention toward one such dimension of consideration, that of coherence. There are certainly other aspects of art (or curriculum) that could be investigated with an eye to a holistic analysis. Yet, I choose to attend specifically to coherence because (a) it is already a common principle used to evaluate both narrative (Brockmeier, 2004; Repp, 2017) and curriculum (Morony, 2023b; Nguyen & Munter, 2024) and (b) its broad, multidisciplinary usage

positions me to be open to a multiplicity of perspectives from which curricular stories could be conceptualized as (in)coherent, especially those not often considered from within mathematics education (including, perhaps, the perspectives of interdisciplinary STEM students in Chapter 3, none of whom were pursuing a mathematics major or minor). In other words, a focus on coherence allows me to investigate varied notions of holistic sense-making by which a student might organize their cross-curricular experiences—whether it be through some form of curricular themes, habits of mind (Cuoco et al., 1996) or other mathematical practices (Cuoco & McCallum, 2018), aesthetically and emotionally engaging stories (Dietiker, Singh, et al., 2023), or other forms of relationality.

Coherence has seen wide uptake and consideration across several disciplines including the arts and humanities in the realm of aesthetic critique (Aschenbrenner, 1985; Repp, 2017), philosophy in the study of epistemology (BonJour, 1985; Thagard, 2000), logic and mathematics in reference to consistent axiomatic systems free of contradiction (Daya, 1960), as well as education in studies of curriculum design and student learning (Jin et al., 2019; Morony, 2023b; Schmidt et al., 2005). But coherence is not just academic jargon. Since undertaking this study of coherence, I rarely go a day without seeing the word used in everyday life—from friends sharing their opinions on the latest TV show (“What’d you think about *Doctor Who* this week? To me, the plot felt incoherent and unsatisfying after that ambiguous ending.”) to learning about how a theoretical framework should provide coherence to a research study (Cai et al., 2019; Lerman, 2019).¹⁸

¹⁸ This tendency for many to attend to and value coherence in even ordinary contexts provided an additional impetus for focusing my attention on this notion. Indeed, in the study reported on in Chapter 3, the fact that I could begin by directly asking participants, “What does it mean for something to be coherent?” proved beneficial given that I sought to investigate students’ interpretations of cross-curricular coherence. It immediately allowed for a more conversational tone.

The multidisciplinary attention to coherence spanning both academic and everyday spheres is likely a consequence of the polysemous meaning of the word “coherence” itself, which derives from the Latin *coharere* meaning, “to stick together” (Kolln, 1999). In other words, a “coherent whole” is one in which its parts are perceived as sticking together—or being related—in a satisfactory way. A new kind of coherence is born each time a choice is made about the type(s) of “glue” or relation(s) to consider between parts of a whole—logical coherence, aesthetic coherence, emotional coherence, thematic coherence, conceptual coherence, or even some combination of these (Brockmeier, 2004; Merriam-Webster, n.d.; Thagard, 2000). The list goes on, and each term is by no means stringently defined, varying according to personal preferences as well as disciplinary and cultural norms and values around what is meant by “sticking together in a satisfactory way” (e.g., Hyvärinen et al., 2010; McAdams, 2006).

The multiplicity of types of coherence and their cultural relativity topples the implicit assumption that attention to coherence is a defined, *a priori* aspect of (narrative, curricular, artistic) critique: rather, coherence is best thought of as an axio-onto-epistemological *value* (Buchmann & Floden, 1991; Freeman, 2010; Herbert, 2004). Coherence, in other words, refers to the value that the parts of something (life, narrative, curricular experience, etc.) ought to come together into a unified, related whole. Given the intimate relationship between aesthetic and ethical values (Rancière, 2000/2004), coherence tends to become associated with goodness, completeness, and beauty, while incoherence becomes synonymous with badness, incompleteness, ugliness (Buchmann & Floden, 1991). Yet, the subjective, value-laden nature of coherence is often concealed by the predominance of “traditional” Western views of narrative and learning which privilege coherence as an objective standard against which all experiences and texts can be evaluated (Herbert, 2004; Hyvärinen et al., 2010). Indeed, conversations of

curricular coherence in STEM education—including those with the explicit goal of pushing the disciplinary boundaries of how curricular coherence is defined—tend to more or less start with the nearly unquestioned assumption that coherence of some form is desirable for all students’ learning (e.g., Morony, 2023; Sikorski & Hammer, 2017). Emblematic of this status quo, for instance, Wan and Lee (2022) begin their introduction to a study of textbook coherence with the contention that “Securing coherence, especially in curriculum and resource development, is something that few educators would ever oppose or find fault with” (p. 882).

So far, I have argued that (a) there are many different types of coherence and uses for coherence, in and outside of the context of curriculum, which helps demonstrate that (b) coherence is an axio-onto-epistemological value that could hypothetically be rejected (or considered differently). I now proceed to flesh out these points by introducing and discussing several forms of coherence originating from disciplines beyond education, which I use to frame a more detailed discussion of the forms of curricular coherence from the mathematics and science education literature that I briefly introduced in Chapter 1. Afterwards, I present arguments that have been given *for* and *against* the value of coherence, concluding with an acknowledgement of the complex, non-binary nature of this debate (Buchmann & Floden, 1991; Freeman, 2010).

Types of Coherence

As a way of further clarifying what I mean (and do not mean) by coherence, I introduce some relevant perspectives on coherence from text linguistics, logic, communication, and epistemology, with a focus on the different types of coherence proposed from each perspective. First, in the context of text linguistics, the related notion of *cohesion* is used to describe how sentences “stick together” on a sentence-by-sentence level, while coherence is a form of “global cohesion” which involves the reader considering the textual structures beyond the sentence level through the lens of their prior knowledge and narrative expectations (Kolln, 1999). Cohesion, in

other words, concerns properties internal to the text and coherence fundamentally involves consideration of external properties related to reader sensemaking (Stöckl & Bateman, 2022). For the sake of this dissertation, my attention is primarily on “global cohesion” or coherence.

A second type of coherence occurs in logic and mathematics when defining axiomatic systems and possible logical foundations of mathematics. A *theory* based on an *axiomatic system* (i.e., set of axioms) is defined as the set of axioms in the axiomatic system as well as any theorems that can be logically proven from a combination of one or more of the axioms. A theory is then considered to be coherent (or *consistent*) if for every proposition p in the theory, $\sim p$ is not in the theory. In other words, the law of contradiction—that p and $\sim p$ cannot be simultaneously true—must apply to the theory for it to be considered coherent. Daya (1960) adds, “The coherence of a set of syntactical rules . . . cannot be established by sheer inspection, intuition, or self-evidence” (p. 194). Rather, this form of logical coherence can only be definitively established through a rigorous consistency proof (which may or may not actually exist, depending on the axiomatic system). Notably, this form of logical coherence is strictly binary in nature: if a single contradiction is discovered, then

a change in the axiomatic set is sought in such a way as to preserve the [desirable] derivations within the system. From the logical point of view, however, *all* the derivations within a system which at any point give rise to p and $\sim p$ must be invalid. (Daya, 1960, p. 194)

Any individual contradictions found in a theory render the entire theory incoherent.

In contrast to the view of coherence from textual linguistics, in this context coherence is seen as residing primarily within the logical system (i.e., theory) itself (the “text” so to speak). It is up to the “reader” to deduce whether the theory is coherent or not; however, a reader’s interpretations (intuitions, prior knowledge) do not change the outcome. In fact, in some cases, when no proof of coherence exists, the reader may never know with certainty whether the theory

is coherent or not. Another contrast to the textual perspective is that in this form of logical coherence, local and global coherence collapse into one another because a single local incoherence (i.e., there exists a p such that p and $\sim p$ coexist in the theory) is equivalent to the whole system being incoherent. As alluded to in the previous Daya quote, even in the case where a reader deduces a sub-theory consisting of desirable deductions (i.e., local coherence), the discovery of a single instance of local *in*coherence may result in the local patch of coherence turning out to be incoherent in nature. That is, unless the reader can edit the axiomatic system to re-generate the local patch of coherence without the simultaneous presence of any additional incoherences. Many mathematical forms of curricular coherence often feature a strong logical component (e.g., Cuoco & McCallum, 2018; Schmidt et al., 2005), likely owing at least partially to the disciplinary value placed on this kind of logical coherence used to evaluate axiomatic systems. Though forms of curricular coherence proposed in mathematics rarely share such a strict and dichotomous view of such logical, structural coherence.

A third type of coherence was put forth by Fisher (1984, 1987) as a key part of his narrative paradigm view of rationality and human communication. Fisher argued that humans are natural storytellers, i.e., *homo narrens*, and consequently a good story does more to convince than a logically-sound argument. This narrative paradigm was a direct challenge to the classic Rational World Paradigm, which privileges the logical rationality of experts who possess disciplinary knowledge and training in how to construct and appropriately evaluate disciplinary forms of logic involved in formulating sound arguments. In contrast, from the perspective of Fisher's (1987) "anti-elite" paradigm

the lay audience can test [experts'] stories for coherence and fidelity. The lay audience is not perceived as a group of observers, but as active, irrepressible participants in the meaning-formation of the stories that any and all storytellers tell in discourses about nuclear weapons or any other issue that impinges on how people are to be conceived and

treated in their ordinary lives. (p 72)

Experts in this view are positioned as knowledgeable counselors to the people, but it is the people who ultimately judge whether the arguments and stories put forth by experts are probable (what he called coherent) and therefore convincing. Fisher suggested three forms of narrative coherence be considered: (1) *structural* coherence of a story and any arguments the storytelling is making, (2) *material* coherence, established by juxtaposing alternative stories with the current one to consider whether crucial elements or counterarguments were missing from an otherwise internally coherent story, and (3) *characterological* coherence, based on the reliability of the narrator and any other actors involved in the story. As he put it,

A character may be considered an organizational set of actional tendencies. If these tendencies contradict one another, change significantly, or alter in ‘strange’ ways, the result is a questioning of character. Coherence in life and in literature requires that characters behave characteristically. (Fisher, 1987, p. 47)

Of note, structural coherence is a consistent consideration across all forms of coherence introduced so far. Material coherence at least implicitly has been involved in both prior types of coherence, even if it is stated most explicitly in Fisher’s formulation of coherence. In the case of linguistic text coherence, one’s sense-making is governed by prior knowledge and expectations of the text, meaning that deviations or missing elements get considered in discerning coherence. Meanwhile, coherence of logical systems effectively requires a reader to attend to what is *not yet* present (i.e., all possible propositions *p* in the theory, including those not currently proven) when discerning coherence. Out of Fisher’s three types of coherence, however, characterological coherence is unique and implies that characters have a defining role to play in any convincing narrative, echoing Corbett (2013) and aligning with Andrà’s (2013) empirical results. I revisit this type of coherence in Chapter 4, when I consider coherence across three different function characters’ stories.

In a similar vein to Fisher (1984, 1987), philosophers and psychologists have also proposed coherentist theories of inference, justification, and truth as a rejection of classic, purely logico-rational epistemologies (e.g., BonJour, 1985; Thagard, 2000). From these perspectives,

Our knowledge is not like a house that sits on a foundation of bricks that have to be solid, but more like a raft that floats on the sea with all the pieces of the raft fitting together and supporting each other. A belief is justified not because it is indubitable or is derived from some other indubitable beliefs, but because it coheres with other beliefs that jointly support each other. (Thagard, 2000, p. 5)

Unlike the logical system view of coherence mentioned previously, justification of a decision, ethical value, or belief(s) does not emanate from a consistent axiomatic foundation from which the inference is logically derived. Rather, an individual engages in a (possibly unconscious) process of reflection during which they iteratively adjust their system of beliefs (adding new beliefs, discarding outdated beliefs, considering other connections between beliefs, etc.), fine-tuning how they interpret their system of beliefs until they again reach a coherent state. While a completely coherent system is rarely possible due to the complexity of such systems, coherentist epistemologies suggest that individuals reflect in an effort to construct mental representations of the world that maximize coherence (e.g., Thagard, 2000). From this perspective, coherence is on a spectrum, rather than a binary (as with the form of logico-mathematical coherence of axiomatic systems introduced previously): an individual can construct one mental representation or model of the world and can evaluate it as “more” or “less” coherent than an alternative mental representation. Thagard (2000) has maintained that there are at least six interrelated types of coherence, including logical, perceptual, and conceptual, among others. Further, he considers emotions (and therefore aesthetics) as being involved in an individual’s determination of coherence:

When a situation “makes sense” to us, we feel a general well-being, whereas a situation that we are unable to comprehend can cause anxiety. The usually pleasant feeling that something makes sense involves an overall assessment of coherence, in contrast to the

confusion and anxiety that often accompany incoherence. I call these *metacoherence* emotions, because they require an overall assessment of how much coherence is being achieved. (Thagard, 2000, p. 193, emphasis in original)

One example metacoherence emotion is surprise, in the case where one's system of beliefs and inferences previously interpreted as relatively coherent suddenly becomes less coherent in light of an unexpected new observation. This shift necessitates a substantially different interpretation of the system before it can be seen as coherent once again. Note that Thagard's coherentist epistemology embraces the value of coherence seeking as synonymous with goodness. Indeed, he explicitly associates positive emotions with maximizing and (re-)establishing coherent interpretations of belief systems.

Forms of Curricular Coherence

Against the broader backdrop of the interdisciplinary perspectives of coherence discussed in the prior section, I now synthesize several forms of *curricular* coherence that have been introduced in both mathematics and science education. First, I begin with a general overview of curricular coherence and specify the grain size I am most interested in to focus the subsequent discussion. Afterwards, I survey forms of curricular coherence in the education literature, beginning with those in which coherence is considered *internal* to the curriculum—namely, disciplinary or so-called “canonical” forms of coherence (Fortus & Krajcik, 2012; Morony, 2023b; Wan & Lee, 2022)—and gradually transitioning toward forms that consider coherence as primarily *external* to the curriculum and based on students' interpretations (Reiser et al., 2017; Sikorski & Hammer, 2017). Along the way, I continually revisit the question of how coherence is decided from each perspective and *who* makes these decisions (Richmond et al., 2019), as a way to build toward a critical discussion in the subsequent section on the axio-onto-epistemological foundations of the value-laden nature of coherence in its many forms.

“Curricular coherence” is a common phrase used in curriculum research and reform, yet

just like the uses of “coherence” in everyday life and across other disciplines, it can refer to several different aspects of curriculum. Many suggest that despite its common use, the phrase is still only loosely defined at best, given the lack of widespread agreement over its meaning (Muller, 2022; Thompson, 2008). To help put my subsequent discussion of curricular coherence on firmer ground, therefore, I take a moment to clarify the classes of curricular coherence I will be attending to throughout the remainder of this section. There are several possible grain sizes of curricular coherence—ranging from the broadest policy or standards level (Schmidt et al., 2005) to the program level of a school district or higher education degree program (Newmann et al., 2001; Nguyen & Munter, 2024) to the specific level of a single course (Foster et al., 2021) to a particular unit or lesson within a course (Han et al., 2020). Beyond grain size, curricular coherence has been used to refer to the coordination within and between various curricular components, such as the instruction, instructional materials (e.g., textbooks), (structure of the) content, forms of student engagement and participation, cross-cutting themes, and learning progressions (Fortus & Krajcik, 2012; Morony, 2023b). Finally, coherence might be considered over the span of an entire curriculum—between units—or within a particular unit (Fortus & Krajcik, 2012; Morony, 2023c). For the sake of this dissertation, I attend primarily to longer-term coherence within one mathematics course (i.e., Chapter 4, an analysis of a multivariable calculus textbook) or across multiple courses (i.e., Chapter 3, an analysis of undergraduate STEM students’ interpretations and valuations of mathematical cross-curricular (in)coherence). Across both studies, I focus primarily on curricular coherence concerning the (structure of) *content* and cross-cutting *themes* from the perspective of students (Chapter 3) and also as interpreted in the textbook curriculum (Tarr et al., 2008, Chapter 4). With this scope in mind, I next survey several forms of coherence across both mathematics and science education.

Disciplinary Coherence

Perhaps the most common class of coherence within the mathematics education literature is *disciplinary coherence*, a form of logico-structural coherence referring to the alignment between the organization and sequencing of curricular content and the so-called logico-hierarchical knowledge structures of the discipline (Cuoco & McCallum, 2018; Morony, 2023b; Schmidt et al., 2005). Disciplinary coherence has also been called canonical coherence (referring to coherent curriculum as reflecting the so-called “canon” of the parent discipline), as well as conceptual coherence (Muller, 2009) and logical coherence (Jin et al., 2022). These last two descriptors refer to how a disciplinary coherent curriculum privileges the logical arrangement of concepts. Schmidt and his colleagues (Schmidt et al., 2002; Schmidt et al., 2005; Schmidt & Houang, 2007, 2012) are attributed with popularizing this form of coherence in mathematics education, which they introduced to compare the school mathematics and science standards of top-performing countries from the 1995 TIMMS Study with the U.S. standards at the time (Schmidt et al., 2002; Schmidt et al., 2005). They argued that a coherent curriculum is one that consists of “a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives”, which requires that the particulars should evolve to “deeper structures inherent to the discipline” (Schmidt et al., 2002, p. 9). Schmidt et al. (2005) initially clarified that this definition did not imply that there is only one coherent curricular sequence, but later Schmidt and Houang (2012) added that “it would be very unlikely that alternative modes of coherence would be very different from each other given the logical organization of the discipline” (p. 298). These comments seem to convey the belief that disciplinary coherence (at least in Schmidt et al.’s formulation) mirrors a nearly binary logical form of coherence considered in the context of axiomatic systems which is germane to the discipline of mathematics (Daya, 1960).

Others, such as Cuoco and McCallum (2018) have proposed expanded definitions of disciplinary coherence which focus on coherence of *practices* in addition to coherence of content. A coherent curriculum, in this view, is one that enables students to leverage recurring mathematical practices, such as using structure and abstraction, to make connections and “take advantage of a coherent curriculum” (p. 252). Jin et al., (2022), taking a similar perspective on coherence, argue that mathematization is one such cross-cutting practice in the context of the science curriculum, for example. Taking a step back, such a practice view of coherence suggests that coherent curricula may have recurring *themes*, whether these are mathematical habits of mind as Cuoco and McCallum propose or some other form of recurrence.

From the perspective of disciplinary coherence, the anthropomorphized discipline of mathematics itself alongside those who can discern its logico-hierarchical organization—i.e., disciplinary experts—are those who are positioned with the authority to evaluate the ways in which a curriculum is or is not coherent. For instance, Cuoco and McCallum (2018) conclude their argument by suggesting that a powerful way curricular coherence can be achieved is when mathematicians form partnerships with mathematics educators: “[mathematicians] can communicate a sense of what the subject itself is all about, a sense of excitement and power and coherent view that makes [curriculum] make sense” (p. 255). Though well-intentioned, this call to action seems to signal a return to the assumption that disciplinary experts (mathematicians or those with sufficient mathematical training) are the primary purveyors of coherence. Namely, for students to have coherent curricular experiences, disciplinary experts must first design and sequence content and opportunities to encourage recurring use of disciplinary practices. The role of the student in this case is primarily to be influenced or guided to a coherent learning experience by a curriculum (Buchmann & Floden, 1991; Sikorski & Hammer, 2017) which has

been organized in terms of mathematicians' perspectives, knowledges. In other words, the student is positioned as the passive consumer of a coherent curriculum, rather than the active producer.

While student viewpoints are not often *directly* sought out during this process, most curricular coherence literature (e.g., Fortus & Krajcik, 2012) suggests that designers hoping to establish disciplinary coherence should also seek out (cognitive) theories of student learning or research-based learning progressions to use alongside their disciplinary knowledge and practices (e.g., Jin et al., 2022). Yet, even when students are involved in the curricular design process or related research, they rarely have a say on which questions, research priorities, and standards of evidence are considered. Instead, these are determined by researchers and other disciplinary experts. Thompson (2008), for instance, proposed the radical constructivist method of conceptual analysis as a tool to design for curricular coherence, whereby a researcher reflects on how a student might engage with a particular mathematical topic.

Story Coherence

Another view of coherence that has been presented involves appealing to metaphors of curriculum as story, such as the one outlined previously that Dietiker (2015a) proposed. Prior to Dietiker, however, Stigler and Hiebert (1999) made a similar contention:

Imagine the lesson as a story. Well-formed stories consist of a sequence of events that fit together to reach the final conclusion. Ill-formed stories are scattered sets of events that don't seem to connect. As readers know, well-formed stories are easier to comprehend than ill-formed stories. And well-formed stories are like coherent lessons. They offer the students greater opportunities to make sense of what is going on. (p. 61)

Well-formed, coherent lessons from this perspective are defined in terms of connectedness, including how certain components are sequenced relative to one another. While there are many possible forms of connection that might be considered (e.g., between the various story elements introduced earlier), Stigler and Hiebert defined lesson coherence as “the connectedness or

relatedness of the mathematics across the lesson” (p. 60). In other words, story primarily serves as a metaphorical coat of paint for disciplinary coherence, which evokes imagery around the importance and impact of careful structural sequencing. Indeed, from Stigler and Hiebert’s perspective, connections are inherent to the discipline of mathematics and “other judgements about coherence, such as the flow of mathematical connections, are quite subtle and require a good deal of mathematical sophistication” (p. 63). In other words, disciplinary experts (i.e., teachers) design the “well-formed stories” so that their students can consume these stories which offer “greater opportunities to make sense of what is going on”. Students are positioned not as active readers interpreting these stories (as with the reader-oriented perspective, Rosenblatt, 1978, 1988, of narrative adopted by Dietiker (2015a) and myself) but primarily as passive: “One way to help students notice how ideas are related is to explicitly point out the connections among them” (p. 62). Additionally, this framing does little to attend to differences between learners, suggesting that “as readers know” there is an almost universal way of constructing stories in a well-formed way (i.e., through logical connections).

Not all story views of coherence are reducible to disciplinary coherence, however. Dietiker (2015a, 2015b) also focused on the sequencing and logical interconnections between story events (i.e., structural coherence) and when her research team extended the curriculum-as-story framework to story coherence, they even built on Stigler & Hiebert's (1999) work by focusing on mathematical connections: “we define story coherence as the extent to which the events and mathematical ideas of the mathematical story (i.e., a lesson) are connected to each other for a reader” (Richman et al. 2019, p. 4). Yet, their definition features a noticeable difference: (mathematical) connections are interpreted from a reader’s perspective. The *student* is positioned with agency to interpret the extent to which a mathematical story is coherent. The

introduction of this form of reader-oriented (Rosenblatt, 1978, 1988) story coherence marks a dramatic departure from Stigler and Hiebert's (1999) variant (a form of disciplinary coherence) which mirrors Fisher's (1987) rejection of traditional (logico-deductive) rationality in favor of a narrative rationality. The net effect is that the power of interpretation is (re)located into the hands of students, rather than disciplinary experts who espouse strict views of coherence governed by logico-rational notions of coherence. Specifically, as Richman et al. (2019) define story coherence as a characteristic of a mathematical plot, student interpretations of coherence are governed not only by notions of logical coherence but also the student's aesthetic engagement and emotional reactions to the unfolding revelations of the story. In other words, story coherence is a *mélange* of logical coherence as well as *aesthetic coherence* and *emotional coherence*. Such a view is not entirely unprecedented. Thagard's (2000) coherentist view holds that emotions (and in turn, aesthetics) were bound up in our judgements of coherence.

Putting all this together, the sequencing of content, questions, and ideas in a mathematical story has the power to engage a students' aesthetic and emotional sensibilities, which in turn plays a role in the students' engagement with and interpretations of the story. For example, Dietiker and Richman (2021) identified how students were motivated to engage in inquiry when they were driven by aesthetic reactions such as wonder and curiosity about sustained unanswered mathematical questions. An aesthetic pull to turn the next page toward a satisfactory resolution, then, can motivate students to learn. Richman et al. (2019) note that "coherence enables a reader to sense completeness and fitness if and when the threads of the story come together with clarity" (p. 4). But sensing completeness, fitness, and clarity are personally idiosyncratic and fundamentally governed by one's personal aesthetic (and emotional) preferences (e.g., Do they like to be left in suspense? Or do they prefer the suspense to dissipate?). Preferences such as

these have an indelible impact on one's interpretations of logical, aesthetic, and emotional coherence, and, in turn, their idiosyncratic interpretations of the extent to which they view a story as coherent. Indeed, an implication of Dietiker's curriculum-as-story framework is that there is no universal template for crafting a "well-formed" story.

In adopting the curriculum-as-story metaphor, there is a notable shift from seeing curricular coherence as being a strictly *internal*, objective property of a curriculum—as with Schmidt et al.'s (2005) brand of disciplinary coherence—to a view of coherence as a process of interaction between the student and curriculum featuring both internal (i.e., structural coherence) *and* external components (i.e., reader interpretation of the story, influenced in part by aesthetic and emotional dimensions of coherence). I call this shift notable because in nearly all the mathematics education literature I reviewed on curricular (content) coherence, the view of coherence as internal to the curriculum was largely entrenched. For example, in the recent ICMI study *Mathematics Curriculum Reforms Around the World*, coherence was defined as internal to the curriculum, while the separate notion of *relevance*, or, "the interaction between the curriculum and needs and aspirations (of students/young people, the workplace, university, society, etc.)" (Morony, 2023c, p. 120), was considered as external to the curriculum. The ontological separation of these two terms feels troublesome in that it seems to foreclose (at the very least) the possibility that coherence (in its many forms) might be relevant to some stakeholders. In another instance, McCallum (2023) introduced the "making-sense" stance, which he associated with coherence, as a dual to the "sense-making" stance, again drawing a semi-permeable separation between an individual's interpretation and sense-making and curricular coherence. Though McCallum is careful to acknowledge that these stances are dual and would ideally be coordinated, again the ontological separation of interpretive sense-making

and content structure feels like it may yet again foreclose certain forms of coherence. In a similar vein, Muller (2009) argued that adherence to primarily internal or primarily external forms of curricular coherence is not a binary but instead exists on a spectrum that often varies based on the nature of the privileged knowledge discourses of the parent discipline. Perhaps explaining my previous observation, he noted further that disciplines which privilege hierarchical knowledge discourses that are seen as building vertically¹⁹—such as mathematics, as evidenced by the prior discussion of disciplinary coherence—tend to favor a greater adherence to forms of internal coherence, given how careful sequencing is more likely to be of concern when ordering courses or introducing new concepts.

Speaking of this internal-external coherence spectrum, I next proceed to introduce two final perspectives on coherence that are almost entirely external in nature—that of students’ views of coherence. The fact that both these perspectives originated from science education—a discipline that is often viewed as being less strictly “vertical” than mathematics (Schmidt et al., 2005)—is perhaps further evidence of the feasibility of Muller’s (2009) claim.

Students’ Perspectives on Coherence

Designing for curricular coherence is often portrayed as a dynamic process that ideally involves several stakeholders (Bateman et al., 2009; Honig & Hatch, 2004; Morony, 2023a). Some even reframe curricular coherence entirely as not an end state but rather as a never-ending process which requires continual revision to be responsive to changing stakeholders’ needs and socio-cultural contexts (Richmond et al., 2019). Along these lines, in addition to valuing perspectives of disciplinary experts (from the disciplinary coherence perspective), recent

¹⁹ These terms are employed in the sense of sociologist of education Basil Bernstein (1999). Hierarchical knowledge discourses feature generalization and abstraction away from everyday particulars. Meanwhile, vertical knowledge structures are those in which “more mature” knowledge is seen as building on and subsequently replacing earlier forms of knowledge.

research has drawn attention to teacher and administrator perspectives and interpretations on curricular coherence (e.g., Baniahmadi, 2022; Doherty et al., 2023; Watanabe, 2007). Student perspectives, on the other hand, are much less frequently solicited, though there has recently been increased attention to valuing student perspectives on curricular coherence in teacher education program coherence research (Canrinus et al., 2017; Grossman et al., 2008; Nguyen & Munter, 2024). After all, students have been known to experience curricula in idiosyncratic ways that vary from the predictions of instructors, curriculum designers, and research on learning progressions (Clift & Brady, 2005; Lew et al., 2016; Sikorski & Hammer, 2017).

In science education, such a broadening to center students' perspectives of curricular coherence has already begun to occur. Sikorski & Hammer (2017) introduced a distinction between *premeditated coherence* and students' *coherence seeking*: the ongoing process of "trying to build meaningful mutually consistent relationships between information" (Sikorski, 2012, p. 153). Premeditated coherence, they argued, has the tendency to undermine and undervalue the active role students must play while seeking coherence (i.e., a view of coherence as external to the curriculum). Further, they provided examples of how premeditated definitions of coherence often lead to a conflation between coherence and correctness. Students' coherence seeking is often funneled to arrive at the connections curriculum designers intended (e.g., the logical connections historically privileged within the discipline of mathematics in the case of disciplinary coherence), rather than allowed to freely explore the potential connections and forms of coherence that resonate most. As Sikorski and Hammer put it, "Sequences that attempt to guide students through a series of investigations to arrive at the canonically accepted (and 'coherent') explanation may do so at the risk of students' own agency in deciding what coherence to seek next" (p. 935). These authors work from the assumption that students are

actively seeking some form of coherence, so the question becomes not *whether* students are seeking coherence but rather *what kinds* of coherences they are seeking (linguistic, narrative, conceptual, etc.), how they are seeking these coherences, and for what reasons. Alongside this anti-deficit perspective on students' coherence seeking, they advance a broader, three-pronged view of coherence seeking that includes developing: (1) *Conceptual* coherence—an integration of knowledges and experiences into coherent frameworks, (2) *An affective sense of unity* across curricular experiences, and (3) the *inclination to actively seek coherences*. Classic definitions of (premeditated) coherence in mathematics and science education often attend to conceptual coherence, while ignoring these other two dimensions. Sikorski and Hammer's (2017) call to redefine coherence from a student perspective is not an isolated one: Reiser et al. (2021) have also advocated for attention to students' perspectives on coherence that acknowledge students' epistemic agency to make sense of curricula on their own terms.

The Assumed Value Coherence

Across most literature on mathematics and science education on curricular coherence (and often curriculum more generally), it is assumed that coherence (of some form) is desirable and characteristic of a “good” curriculum. This is true both nationally and internationally—indeed, the recent ICMI study of international curricular reform featured curricular coherence (alongside relevance) as one of primary organizing themes for discussing curriculum reform and the author organizing the chapters on this theme suggested that “Coherence is likely to be a universal aspiration for mathematics curriculum reforms” (Morony, 2023b, p. 148). As demonstrated in the prior section, such a view is held both by authors who privilege internal coherence (e.g., Schmidt et al., 2005) and those who privilege external coherence (e.g., Sikorski & Hammer, 2017).

Curricular coherence is also a perennial value and design assumption across curricular

policy documents and standards. In the United States, current curricular standards for K-12 mathematics and science education including the Common Core State Standards for Mathematics and the Next Generation Science Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; NGSS Lead States, 2013), were designed with the goal of coherence in mind (McCallum, 2015; Reiser et al., 2021; Sikorski & Hammer, 2017). In higher education, the Mathematical Association of America's curriculum guide to majors in the mathematical sciences (Zorn, 2015) also emphasizes the importance of developing connections (i.e., structural and conceptual coherence) between courses across the postsecondary curricula. The case is similar internationally (Morony, 2023b; Muller, 2022; Office for Standards in Education, 2019).

However, curricular coherence is fundamentally a value-laden notion, which carries (implicit) assumptions about aesthetics, ethics, ontology, and epistemology. Buchmann and Floden (1991) noted further how coherence tends to be associated with other aesthetic notions which tend to be positioned as universally positive, such as “harmony” or “wholeness”:

Coherence is . . . a concept used with evaluative overtones that are usually positive. What is ‘coherent’ is supposed to have direction, systematic relations, and intelligible meaning, thus conveying a sense of purpose, order, and intellectual as well as practical control. (p. 65)

They went on to draw attention to the historical origins of curricular coherence with “efficient instruction” and “maximizing educational impact” based on strictly behaviorist views of learning which position learners as passive consumers acted on by the curriculum rather than active agents seeking coherence (e.g., Tyler, 1949). In light of this history, they suggested that curricular coherence and its axio-onto-epistemological entailments deserved critical examination before being taken for granted as an inherent good. Richmond et al. (2019) further suggest we must always ask, “Coherence for whom? According to whom? To what end(s)?” (p. 188). The

synthesis of various forms of curricular coherence in the previous section demonstrates how the answers to these questions—and therefore the axio-onto-epistemological entailments—vary by form of coherence.

On what grounds and according to what evidence has curricular coherence come to be associated almost unquestioningly with desirable learning outcomes in mathematics and science education? From the perspective of disciplinary coherence, mathematical practices that privilege the importance of seeking abstracted structures which serve to generalize particular notions and reframe the landscape of mathematics as an “affective unified whole” (Cuoco & McCallum, 2018; Sikorski & Hammer, 2017) are presented as evidence for the necessity of logical curricular structures. As famed mathematician Henri Poincaré put it,

Mathematics is the art of giving the same name to different things . . . when language is well chosen, we are astonished to learn that all the proofs made for a certain object apply immediately to many new objects; there is nothing to change, not even the words, since the names have become the same. (1908/2012, p. 375)

Effectively, the assumption is made that historical development of a discipline and the practices of experts within this discipline should inform how students might learn the discipline best. An association is drawn between historical development and practice and epistemology. Yet, many question this retroactive view of coherence, which dramatically simplifies if not outright ignores the nonlinear and messy processes of coherence seeking that historical mathematicians and scientists engaged in to arrive at conclusions which we can only look back on after the fact as coherent (Furinghetti, 2020; Sikorski & Hammer, 2017; Thompson & Carlson, 2017). Indeed, many have commented on the complex and often fraught nature of drawing direct conclusions about teaching and learning from historical accounts (Furinghetti, 2020; Thompson, 2008). One complicating factor is that historical figures lived and worked within different socio-cultural paradigms of knowledge and rationality than we do in the present day (Muller, 2009), making

one-to-one comparisons difficult.

I next turn to a brief overview of some other forms of empirical evidence that have been put forth to establish an association between curricular coherence and positive learning outcomes. Though I acknowledge that each study I mention below features a range of different axio-onto-epistemological commitments and assumptions (owing to the different forms of curricular coherence adopted in each one), I believe that there are still some important comparisons and similarities that can be noted. The initial evidence Schmidt and his colleagues presented to suggest disciplinary structural coherence might be linked to increased student learning was the observation that a compilation of curricular standards from top performing countries in the 1995 TIMMS study featured strong coherence of this form (Schmidt et al., 2002; Schmidt et al., 2005). While they did not immediately draw any correlational conclusions, William and Houang (2007) later noted a significant correlation between coherence and focus (considered collectively but not alone) of U.S. state standards and NAEP performance, controlling for other demographic factors. In a separate study of curricular reform in Chicago Public Schools in the 1990s, Newmann et al. (2001) found a significant correlation between increased instructional program coherence (of which curricular coherence is one dimension) and increased student achievement on state tests. These forms of evidence focus on long-term learning as measured by scores on politicized and value-laden standardized tests which tend to feature a narrow view of learning (Cobb & Russell, 2015; de Lange, 2007). Such evidence does more to suggest the political expediency of curricular coherence than it does to hint at its relationship with supporting student learning. At a smaller grain size, Fernandez et al. (1992) and Yoshida et al. (1993) used quantitative student recall studies to claim that structurally coherent lessons which feature clear connections and familiar structures (a) tended to support students in

forming coherent mental representations of the lesson, which, in turn, (b) better enabled students to construct knowledge. Newmann et al. (2001) further suggested that students may be more motivated to engage with lessons (and therefore learning) when it felt like a coherent whole, rather than a set of disconnected components.

As these last three studies illustrate, claims about the “natural” association between curricular coherence and learning often appeal to the axio-onto-epistemological foundations of positivist and cognitive (constructivist) theories of learning. Such theories tend to consider knowledge in terms of internal mental representations or structures (e.g., Putnam et al., 1990). For example, research in cognition suggests that when students’ experiences build on and feel connected to one another, they are more able to incorporate new understandings into their prior knowledge or augment their prior knowledge as needed (Greeno et al., 1996). Because these theories of learning presuppose a coherentist view of rationality (e.g., Thagard, 2000), they also (at least implicitly) adopt an accompanying normative principle of coherence (Shemmer, 2012) that suggests coherence is desirable and something that humans naturally seek out. As described above, when an individual cannot incorporate a new experience, belief, etc. into their existing mental representation made up of prior experiences, beliefs, etc., an assumption is that the learner will adjust their representation to cope with this local incoherence. In other words, the presence of any incoherence in one’s mental representations results in the experience of disequilibrium, which is resolved by seeking coherence (i.e., adjusting present mental structures or adding additional knowledges to existing mental structures with the goal of re-establishing coherence to overcoming disequilibrium, see e.g., Piaget, 1977/2001; von Glasersfeld, 1983). The prominent adoption of cognitive and constructivist theories of learning in mathematics education research (e.g., Lerman, 2010) explains, at least in part, the tendency for curricular

coherence to be seen as unquestionably beneficial for mathematics students' learning.²⁰ The tradition of narrative learning theory (Bruner, 1986; Polkinghorne, 1988), also based on largely constructivist foundations, is a different example of a learning theory that exhibits this tendency to associate coherence (seeking) with the process of learning: "We are all tellers of tales. We each seek to provide our scattered and often confusing experiences with a sense of coherence by arranging the episodes of our lives into stories" (McAdams, 1993, p. 11).²¹

While the tendency to associate (curricular) coherence as naturally tied to learning and sense-making is ubiquitous in (mathematics) education, paradigms of educational research built on different foundations could offer alternative axio-onto-epistemological appraisals of (in)coherence and learning. From a postmodern perspective, for example, the notion of structure and therefore *coherence* of structure is challenged. Writing from this perspective, Appelbaum, 2010) argues for the value of (*non*)sense-making:

In some cases it is impossible, speaking epistemologically, for mathematics as a discipline to 'make sense'—in others it might be more valuable pedagogically to treat mathematics 'as if' it does not make sense. To do so would celebrate the position of the learner, for whom much of the mathematics is new and possibly confusing anyway. Yet, so much of contemporary mathematics education practice is devoted to helping students make sense of mathematics! What if we stopped trying to make sense totally, and instead worked together with students to study the ways in which mathematics both does and does not make sense? (p. 10)

He continues to argue that several aspects of the discipline of mathematics could be seen as fundamentally *incoherent* in nature, from the use of *undefined* terms used in axioms to logical foundations that are fundamentally inconsistent (i.e., Gödel's Theorem). Proceeding in a similar way to those who suggest curriculum should reflect the structure of its parent discipline,

²⁰ Such an association is reflective of the frequency with which constructivism—a learning theory—has been uncritically mistranslated into a theory of instruction and instructional design (e.g., Davis & Sumara, 2002).

²¹ Notably, while Bruner adopts structuralist views of narrative in his accounts of narrative learning theory, he deviates from other narrative learning theorists by not assuming narratives are constructed solely for the aim of establishing coherence. For example, he suggests that narratives are "designed to contain the uncanniness rather than to resolve it" (Bruner, 1991, p. 16).

Appelbaum instead concludes that “Mathematics curriculum materials too often hide the messiness of mathematics where sense dissolves into paradox and perplexity; but what is more important is that they construct a false fantasy of coherence and consistency” (p. 10). Yet, Appelbaum is not alone in suggesting that an openness to curricular incoherence might be beneficial for learning. Irwin (2003), for one, considers how curriculum design might be guided by an aesthetic of unfolding, allowing for unplanned moments of uncertainty, ambiguity, and a lack of structure that engender opportunities for both teachers and learners to stare in awe at the chaos of the world and be left to wonder and make sense where it initially feels like there might be none.

Embracing the In-Betweens of In/Coherence

Taking up Richmond et al.'s (2019) advice, I conclude this investigation of curricular coherence by wondering: “Coherence for whom? According to whom?” (p. 188). If we ignore forms of coherence that do not conform to the status quo, what are the consequences? Who is impacted? Perhaps most importantly, is *anyone harmed*? Buchmann & Floden (1991) contend that the answer to this question could very well be yes, while simultaneously critiquing radical forms of internal coherence which position students as passive recipients of knowledge: “One must also question whether [students] are best served by having all connections laid out for them . . . A [curriculum] that is too coherent fits students with blinders, deceives them, and encourages complacency” (pp. 70-71). They raise an additional concern—which Appelbaum (2010) echoes—that if the value of “maximizing coherence” is taken too far, it may result in a rigidly-designed curriculum which positions learners as objects “that will be gradually shaped to one mold or equipped with one-sided views” (p. 69). And what about those who do not fit in this mold? Given the value-laden and axio-onto-epistemological nature of coherence, there is a danger that those who favor alternative forms of knowing, being, and making sense (or non-

sense!) of the world might be labeled as aberrant, positioned as outside of the normative aesthetic distribution of the sensible (Rancière, 2000/2004). Appelbaum (2010) expands on this point:

The ways in which mathematical narratives [i.e., of coherence] communicate a certain kind of arrogance in helping to constitute ‘others’ as ‘outside the domain’ through notions of ignorance. The curriculum can, in this sense, foster an interpretation that carries with it a socially constructed binary of a ‘knowing’ versus an ‘ignorant’ subject. (p. 17)

Given the political implications of such narratives, I purposefully embrace a non-binary view of (in)coherence and remain open to *several* forms and valuations of (in)coherence throughout this dissertation. Indeed, in the study reported in Chapter 3, I take a step back to also consider *students’* varied perspectives on cross-curricular (in)coherence alongside those I have detailed throughout this chapter that are present in the literature. My default axio-onto-epistemological orientation toward (in)coherence is one of *intersubjective aesthetic plurality* (Chilton et al., 2015; Conrad & Beck, 2015), consistent with the ABR paradigm. In other words, I acknowledge that several aesthetics of (in)coherence can reasonably coexist, while simultaneously remaining critically aware of the power relations that govern which aesthetics of (in)coherence are favored within a disciplinary culture and the harmful impacts this can have on those whose aesthetics do not align with the favored view.

So far, I have presented an abstract argument that a strict devotion to one form of coherence would be exclusionary. I now suggest this is not just an abstract ethical concern. The literature on narrative and narrative learning theory presents several examples of people for whom canonical narrative coherence is *not* a natural inclination (e.g., Hyvärinen et al., 2010; Strawson, 2004).²² Freeman (2010) concludes that “there are no doubt people whose lives and consequent ‘stories’ (should they even be called that) are dispersed, heterogeneous, even

²² At least from the structuralist tradition of narrative that is common in Western cultures.

fragmented” (Freeman, 2010, p. 167). Just as disciplinary coherence construes a vision of mathematics as a rigidly structured hierarchy, Western traditions of narrative dating back to Aristotle (350 B.C.E./1995) tend to favor particular story structures (beginning, middle, end) and forms of structural story analysis. These privileged Western views for story—like the privileged cognitive, constructive paradigm within mathematics education—often function to define what counts as coherent and sensible:

The question that has finally been posed in . . . feminist and postcolonial literatures is not simply why subjects deemed to be different . . . have not written ‘coherent’ narratives, but also how the imperative of coherence works to legitimize certain narratives while excluding or marginalizing others from the narrative canon. (Hyvärinen et al., 2010, p. 7)

The symmetry between the state of affairs in both the context of narrative as well as mathematics education leads me to embrace an open stance on (narrative) coherence through this dissertation that leans into complexity rather than reducing it (Sousanis, 2015). In doing so, I heed Freeman's (2010) wisdom surrounding the potential irony of rejecting coherence outright: “Now, it might be argued here that this ‘anti-coherence’—or even anti-narrativism—bespeaks a coherence of its own, that is in the inverted image of, and is thus parasitic upon, the very coherence it rejects and replaces” (pp. 167-168). Or, put simply, a switch to exclusively privileging incoherence over coherence would merely relocate the othering onto another group, redirecting rather than outright eliminating the epistemic harm. Therefore, I opt to consider the complex in-betweens of coherence and incoherence, remaining open to ways in which they might coexist (Dietiker, 2015b; Irwin, 2003). To accomplish this, I adopt a perspective similar to Richmond et al.'s (2019) conceptualization of coherence as a process which involves working alongside other stakeholders (i.e., students) toward a shared mission “that centers justice and continuously negotiates whether and how current notions of and efforts toward [curricular] coherence privilege the values and practices of dominant groups” (p. 188). My initial foray into living such

a praxis involved inviting STEM students into “conspiratorial conversations” (Barone, 2008) about the (in)coherence of the enacted mathematics curriculum wherein students acted as connoisseurs and critics (Eisner, 1991/2017) of the cross-curricular mathematical stories they had engaged with across their courses (see Chapter 3).

Part 3: The Curriculum-as-Story Metaphor as a Flexible Lens for Interpreting Curricular (In)Coherence

As established in the previous section, a strict and unquestioned devotion to a singular form of (in)coherence when crafting and critiquing (curricular) stories serves to perpetuate a culture of exclusion (Louie, 2017) in mathematics by implementing an implied politics of aesthetics (Rancière, 2000/2004) emanating from the aesthetic sensibilities associated with that view of coherence. Such a politics of aesthetics erects harmful boundaries between those who are positioned as “sensible” (i.e., whose readings of curricular stories are *coherent*) and those who are “insensible” (i.e., who express other aesthetic sensibilities regarding curricular stories). Yet, from the brief review of the narrative literature presented in the previous section read through the lens of a socio-cultural perspective of story, it becomes clear just *how* diverse the multitude of possible story forms can be, ranging from the postmodern and purposefully *incoherent* (Appelbaum, 2010) to the non-linear, the absurdist, and the illogical. Each of these story forms offers a *unique* and socio-culturally situated perspective of “narrative goodness” and “coherence”. In other words, each form of story presents an alternative aesthetic reality—they propose axio-onto-epistemological *possibilities*.

To avoid perpetuating a strict politics of aesthetics that serves to enforce compulsory assimilation and exclusion in mathematics education, our conceptualizations of curriculum *must* foster a stance of openness toward these alternate aesthetic realities. From the perspective of curriculum as a story, we work toward this goal by remaining open to a *multiplicity* of story

forms to metaphorically conceptualize curriculum while simultaneously interrogating the axio-onto-epistemological assumptions implicit to each story form taken up. Dietiker has proposed that this framing of curriculum allows us to ask questions about the *types* of curricular mathematical stories on offer to students (Dietiker, 2015a) and the variation (or lack thereof) in the *genres* of our curricular stories (Dietiker, 2015b), offering fertile ground for candidly subjective interpretation and critique of curricular aesthetics. However, we must be careful to move beyond mere literary or aesthetic cataloguing of these stories. I suggest that any interpretive critique of a curricular story ought to involve careful consideration of which aesthetic sensibilities and (mathematical) ways of knowing or being the story might privilege or ignore.

As I hope is apparent by now from the very topic of this dissertation, I am an unabashed proponent for the importance of viewing curriculum as an art form which has the potential to *aesthetically* impact students. We *should* study how aesthetically oriented and arts-based approaches to curriculum design enable us to consider alternative curricular futurities where dull, uneventful, and unengaging mathematical stories have become a relic of the past. Therefore, I offer the observations and recommendations in this section in the spirit of loving critique (Drimalla et al., 2024; Paris & Alim, 2014), with the goal of *growing* this aesthetic, storied approach to curricular analysis but in ways that are attentive to ethical concerns regarding how our advocacy for (or against) particular forms of curricular storytelling might *help* some students while simultaneously *harming* others.

I am particularly concerned about forms of literary cataloguing that espouse simplistic binaries, as they have the potential to enforce narrow views of story and therefore politics of mathematical aesthetics. As an example, recall how Stigler and Hiebert (1999) presented a

binary of “well-formed” and “ill-formed” stories based exclusively on the connectedness of story events. Such a binary left little room for aesthetic debate—a certain view of story was proposed as universally good, while another view of story was proposed as universally bad. Admittedly, I have yet to observe a binary this strict in the literature based on Dietiker’s curriculum-as-story metaphor. Indeed, most researchers who take up this metaphor explicitly acknowledge that what counts as a “good” story is subjective and may vary from reader to reader (owing to the built-in, reader-oriented assumption of narrative interpretation). But even *when* subjectivity of interpretation is acknowledged, I urge all researchers to avoid unnecessary aesthetic binaries given their overly simplistic nature and the potential harm they might do. Miežys (2023), for example, detailed several concrete examples of mathematical story sequences which students might interpret as *incoherent*, presenting a very thorough and engaging analysis that acknowledged how students’ aesthetic sensibilities toward these story sequences could vary. Yet, the framing and phraseology of the study does not convey this complexity, favoring instead definitive statements about “poor story design” as being the opposite of a “good” story. While I do not disagree that it is an admirable goal to craft aesthetically engaging mathematical stories that have wide appeal and therefore have the potential to impact many students (Dietiker, 2015a, 2015b; Dietiker, Singh, et al., 2023), I worry that such an aim and the simplistic binary language which can sometimes come with it (e.g., a “poor” vs. “good” story) might lead us to be stuck *cataloguing* rather than *critiquing*. Mirroring Richmond et al.’s, (2019) critical questions about curricular coherence: “Good” for *whom*? *Aesthetic* for whom? According to *whom*? To *which students* do these stories have wide appeal? Relatedly, are there students whose aesthetic sensibilities are consistently *ignored* if our goal is always to craft stories with wide appeal?

In other words, *what do we assume about stories a priori* when we aim to craft ones with

mass appeal? The association between the goodness of a story and its coherence, for instance, is one common assumption that *does* tend to be stated explicitly in research taking up the curriculum-as-story metaphor (Miežys, 2023; Richman et al., 2019). But in several cases, further assumptions are left implicit or unsaid. Presumably, the precise *form* of story being used as the template for the curriculum-as-story metaphor is guided by the research team’s personal or collective definition(s) of story, *as well as* the aesthetic sensibilities they associate with these views. To be clear, almost all researchers using the curriculum-as-story metaphor *do* specify they are using Bal’s (2017) narratological definition of story either directly or indirectly via reference to Dietiker’s research canon, sketching an initial outline of what they count (or not) as a narrative and therefore a story. However, Bal herself clarified that these definitions for narrative and story were purposefully open ended, so they could be used to analyze a wide range of stories from any genre (action, mystery, etc.), featuring various (socio-cultural) aesthetic sensibilities (in terms of story structure, purpose, etc.), and told through several narrative modalities (i.e., texts, ranging from film to visual art to comic books, see e.g., Bal, 2021). Therefore, alluding to this general definition of story is not enough to clarify the axio-onto-epistemological assumptions (explicit or implicit) bound up in a researcher’s use of the word “story”. Researchers ought to take the time to critically reflect on and make explicit the view(s) and/or form(s) of story that guide their thinking, as well as any aesthetic criteria they associate with “good” stories (e.g., (in)coherence, (non)linearity, particular narrative structure(s)). But reflection is not enough—this stance toward reflexivity should appear explicitly in any subsequent presentations of research to clarify and communicate the *idiosyncratic* source template(s) for “story” being used to conceptualize curriculum. To omit these reflections perpetuates the myth that there is an agreed-upon, singular notion of story and a corresponding “correct” politics of story aesthetics, thereby positioning

those whose aesthetic sensibilities that do not align with the privileged one as “other”.

A Flexible Re-Interpretation of the Curriculum-as-Story Metaphor: My Perspective on Curricular (In)Coherence

I next move to propose a *flexible* (re-)interpretation of the curriculum-as-story metaphor that is responsive to the critiques I outlined. To be clear, though, I see this as but one way of navigating the ethical dilemmas associated with investigating curricular aesthetics through the lens of story (i.e., one that is aligned with my axio-onto-epistemological worldview and view of story). I expect and eagerly anticipate additional proposals for navigating these tensions that are compatible with philosophical perspectives other than my own. The variant of the curriculum-as-story metaphor I propose in the remainder of this chapter also serves to clarify the stance of openness I take toward curricular (in)coherence throughout this dissertation to remain open to a plurality of interpretations and valuations of (in)coherence. In presenting my proposal, I focus on first delineating its theoretical and philosophical underpinnings to carefully situate this flexible orientation relative to other forms of story coherence. Later, in Chapters 3 and 4, I demonstrate the practical utility of this flexible perspective for studying cross-curricular stories and (in)coherence.

The idea for this flexible lens is rather simple. I begin with the same axio-onto-epistemological assumptions of narrative interpretation used to form Dietiker's (2015a) usual curriculum-as-story metaphor (with some additions, which I revisit in a moment). After all, this metaphorical correspondence favors a view of curricular coherence as subjective and based on an individual's *holistic* interpretation of a story which is formed from their personal aesthetic sensibilities and preferences for structural logics. This openness to multiple forces *and* personal aesthetic sensibilities positions this view of story coherence as an ideal candidate for a foundation of the flexible lens I construct. Next, I invoke the socio-cultural view of story I have

taken up following Bal (2017). As I have mentioned before, a chosen, socio-culturally situated *form* of story or narrative (e.g., genre, cultural form of storytelling, a particular narrative structure) features accompanying aesthetic sensibilities about what counts as a “good” or “coherent” story. Therefore, if we change the *form* of story that gets used as a template for the curriculum-as-story metaphor, this results in an idiosyncratic corresponding form of (curricular) story coherence. This is similar to how I observed dramatic differences between the views of story coherence espoused by Stigler and Hiebert (1999) compared to Richman et al. (2019), owing to their idiosyncratic axio-onto-epistemologies of story (and narrative interpretation). If I were to stop there and fix a particular form of story and consider (curricular) story coherence from this perspective, it would result in an accompanying set of (narrative) aesthetic sensibilities which would afford a new vantage on what curriculum *could* or *ought* to be. As I allude to in the title of chapter and the dissertation: *change the story, change the curriculum*.

But I cannot stop here. If I were to stop there and *fix* a particular form of story and consider the curriculum-as-story metaphor using this form, that would serve to enforce a singular politics of aesthetics emanating from the chosen form of story. So instead, I outright reject a *single* choice of story form and instead remain open to *flexibly* changing the story form used as the template for the curriculum-as-story metaphor however frequently as needed. For example, in Chapter 3, as I worked alongside students to critique curricular stories, I remained open to *their* preferred aesthetic sensibilities for story and used *these* in the moment as the template for the curriculum-as-story metaphor to define a notion of story (in)coherence until it was time to switch again to the preferred story form of another student. Effectively, this stance results in the flexible lens I suggested I would craft in that it consistently features a radical openness to a plurality of interpretations of valuations of curricular (in)coherence.

I conclude this chapter by outlining the axio-onto-epistemological assumptions about curricular (in)coherence that this flexible lens entails to further clarify my proposal and solidify the assumptions about (in)coherence that I adopt throughout this dissertation. Before doing so, I recall and summarize the core assumptions of the curriculum-as-story metaphor from the first part of this chapter, given that it serves as the foundation for the flexible lens I have proposed. I do so in a numbered list for ease of reference back to these assumptions and for brevity because all the details were presented earlier in this chapter.

Assumptions of the Curriculum-as-Story Metaphor

- (1) **Curriculum as a Form of Narrative.** Curriculum is an artform (Eisner, 2004)—and specifically a *storied* artform (Dietiker, 2015b)—which can be interpreted as a form of narrative (Dietiker, 2013).
 - a. **Definition of Narrative.** To be considered a narrative form, a reader (see Assumption 2, below) must interpret a text as containing events and relations or connections of *some* sort between these events. In deviating from others with an eye to ontological plurality of (in)coherence, I loosen the common requirement that these connections be *sequential* in nature.
 - b. **A Socio-Cultural Axio-Onto-Epistemology of Narrative.** Narrative forms (and therefore stories and storytelling) are socio-culturally bound and must be interpreted in this context (Bal, 2017, 2021). Genres, cultural forms of storytelling, preferences for narrative structures, and so on are therefore bound up in the aesthetic norms, values, and ensuing politics of aesthetics (Rancière, 2000/2004) of a given culture or discipline (i.e., mathematics).
- (2) **A Reader-Oriented, Interpretive Perspective on Curricular Stories.** Anyone who engages with the curriculum (including a student) is positioned as an active *reader* of curricular stories, via a process of narrative interpretation (Bal, 2017; Rosenblatt, 1978, 1988; Weinberg & Wiesner, 2011).
 - a. **Meaning is Subjective and Not “Contained in” a Narrative Text.** Rather, it develops through an interpretive transactional process between a reader and the text. There is no objective reading of narrative and no two readers will experience a story in quite the same way.
 - b. **Reading is Influenced by Personal Aesthetic Sensibilities.** A reader’s background, learning history, and aesthetic preferences govern their reading process. One’s goals and motivations for reading—e.g., aesthetic or efferent/informational (Rosenblatt, 1986)—also play a role.

- c. **A Socio-Cultural Axio-Onto-Epistemology of Narrative/Curricular Interpretation.** Reading is guided by personal aesthetic sensibilities (Assumption 2b) which are bound up in the aesthetic norms, values, and ensuing politics of aesthetics of the socio-cultural contexts that reader inhabits, including their disciplinary affiliations (Shanahan et al., 2011; Wiesner et al., 2020).
- (3) **The Precise View of Narrative Interpretation from the Curriculum-as-Story Perspective.** This theoretical conceptualization was proposed by Dietiker (2013, 2015a), when she crafted the curriculum-as-story metaphor building on Bal's (2017) theory of narrative interpretation. This conceptualization also adopts all prior assumptions (i.e., 1 and 2).
- a. **A Structuralist, 3-Layer View of (Mathematical) Narrative.**²³ Includes the text (medium of the narrative), story (a sequence of mathematical events), and fabula (a reader's logical re-construction of mathematical events).
 - b. **Epistemological Assumption of Constructivist Representationalism for Narrative Interpretation.** A reader's logical re-construction of mathematical events from the story are viewed as being "contained" within a cognitive representation (the fabula), which Dietiker (2013) has likened to a concept image. Consistent with Assumption 1a, I loosened the requirement that relations within the fabula be strictly logical with an eye to remaining open to ontological plurality of (story) coherence.
 - c. **Personal Aesthetic Sensibilities Influence Engagement with Curricular Stories and Therefore Learning.** A reader's aesthetic response while they engage with a curricular story (i.e., the plot) is based on the dynamic unfolding of the plot relative to that reader's expectations based on their interpretations within the current fabula they have constructed. In this way, aesthetics intertwine with one's motivation and engagement while learning (Wong, 2007).

Further Assumptions of Story (In)Coherence from the "Change the Story, Change the Curriculum" Perspective

- (1) **Stance of Ontological and Axiological Plurality Toward Interpretations of Curricular Coherence.** The willingness to consistently "change the story" forms(s) used as a template in the curriculum-as-story metaphor is emblematic of an openness toward varied forms or interpretations *as well as* valuations of (in)coherence (ontological and axiological plurality, respectively). This openness is consistent with the aesthetic, ontological plurality of the ABR paradigm (Chilton et al., 2015).
- (2) **In/Coherence as a Complex Dialectic.** Coherence and incoherence can be seen as more

²³ In Chapter 5, I reflect candidly about possible tensions and limitations that result from building my "flexible" perspective on curricular (in)coherence on the back of Bal's (2017) narratological theory based on a structuralist view of narrative which assumes a constructivist perspective on narrative interpretation.

than just a fixed binary. Rather, they form a complex dialectic with one another. This is consistent with the dialectic ontological plurality of the ABR paradigm (Chilton et al., 2015).

(3) Reader-Oriented Perspective on Interpreting Curricular (In)Coherence. Based on Curriculum-as-Story Assumption 2, anyone who engages with the curriculum (including students) is positioned as a *reader*, meaning that interpretations or evaluations of curricular (in)coherence are made by readers (e.g., students).

- a. **Coherence is Neither an Internal Trait of the Curriculum, nor an Exclusively External Trait.** Rather, following Curriculum-as-Story Assumption 2a, interpretations of the (in)coherence of curricular stories are judgements of the reader, who engages in a transactional process or interpretation which considers both idiosyncratic personal interpretative factors, as well as the nature of curricular stories themselves.
- b. **Judgements of Curricular (In)Coherence Involve Aesthetic and Logical Interpretations.** This is a direct consequence of several Curriculum-as-Story assumptions, including 2b/c and 3b/c which assume aesthetic sensibilities (both personal and in terms of socio-cultural values and norms) as well as logic governs interpretations of the (in)coherence of a curricular story.

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CHAPTER 3: UNDERGRADUATE STEM STUDENTS' INTERPRETATIONS AND VALUATIONS OF CURRICULAR COHERENCE ACROSS MATHEMATICS COURSES

The argument for openness to multiple forms of curricular (in)coherence I put forth in Chapter 2 destabilizes the notion that (curricular) coherence has a singular or objectively given meaning. This rhetorical move may seem counterproductive, given that in the same chapter I also acknowledged how critics of curricular coherence research have argued that “coherence” itself is ill-defined and requires clearer definition (Muller, 2022; Thompson, 2008). Some could interpret my argument as a philosophical sidestep, an abdication of researcher responsibility, or even an instance of needlessly overcomplicating an already tenuous theoretical issue. In response, I point to the repeated cautions offered by researchers both in *and* outside of education about the rash of implicit moral, aesthetic, and epistemological values that often accompany any given interpretation of what—and, by association, *who*—is considered (in)coherent (Appelbaum, 2010; Buchmann & Floden, 1991; Hyvärinen et al., 2010). Selecting definition(s) for curricular coherence, therefore, requires careful contemplation of the axio-onto-epistemological entailments of each possible definition and how adopting them (and their associated philosophical values) might shift the distribution of what are considered “sensible” forms of mathematical curricular storytelling. Adopting the flexible curriculum-as-story metaphor I have proposed, on the other hand, offers an *undefinition*—to channel Appelbaum (2010)—for curricular coherence by rejecting any one definition in favor of remaining open to *all* possible definitions. In other words, this metaphor offers a practical approach for critically contemplating *many* curricular aesthetics in a way that lays bare any philosophical underbrush, paving the way for candid discussion of possible definitions (or perhaps *undefinitions*) that would put subsequent curricular coherence research on firmer theoretical ground.

In this chapter, I showcase the utility of this proposed curriculum-as-story metaphor for

examining multiple curricular aesthetics and specifically interpretations of (in)coherence. With the aim of contributing to ongoing conversations concerning the multi-faceted *process* of designing coherent curricula (Bateman et al., 2009; Honig & Hatch, 2004; Morony, 2023a) which is responsive to multiple stakeholders’ evolving needs and socio-cultural contexts (Modeste et al., 2023; Richmond et al., 2019), I use this metaphor to center *students’* perspectives on curricular (in)coherence. As detailed in Chapter 2, the perspectives of education researchers, disciplinary experts, and to a lesser degree teachers and administrators have been investigated in extant studies of curricular (in)coherence (Baniahmadi, 2022; Cuoco & McCallum, 2018; Doherty et al., 2023; Fortus & Krajcik, 2012; Watanabe, 2007). However, students’ perspectives on curricular (in)coherence are most often considered only peripherally, in line with privileged forms of “cognitive” coherence (Fortus & Krajcik, 2012; Jin et al., 2022) which prioritize the interests, desires, and “retroactive” coherence seeking (Sikorski & Hammer, 2017) of the researchers and other disciplinary experts who design these investigations. That said, recent research—particularly in teacher education—has begun to buck this status quo by attending more directly to students’ perceptions of curricular (in)coherence (Canrinus et al., 2017; Nguyen & Munter, 2024; Richmond et al., 2019). This shift is timely given the recent critiques of cognitive coherence in science education that call into question the efficacy of privileging pre-meditated, retrospective forms of *expert* curricular coherence in curriculum design rather than students’ idiosyncratic, in-the-moment coherence seeking (Reiser et al., 2021; Sikorski & Hammer, 2017).²⁴

This marked shift to attending directly to students’ perspectives on curriculum naturally

²⁴ For a more detailed review of the literature on curricular coherence, including the contentions made in this paragraph, see Chapters 1 and 2. To avoid redundancy, I do not rehash these points further in this chapter. In lieu of a dedicated literature review section rehashing the conceptual framework for this chapter, I opt to provide a more detailed account of my theoretical orientation to this study in the next section.

raises broad questions about students' interpretations of curricular coherence and the nature of their cross-curricular coherence seeking. In particular, given that research on students' coherence seeking has been limited to science education contexts (Reiser et al., 2021; Sikorski, 2012; Sikorski & Hammer, 2017): What are the nature of students' perspectives on curricular coherence and their coherence seeking activities in the context of mathematics? Specifically, what are students' interpretations of cross-curricular mathematical coherence? And finally, in what ways do students' perspectives and forms of coherence seeking vary from the context of science? As a departure from these past studies in science education, however, I adopt a critical stance toward the value-laden assumption of coherence seeking by questioning the frequent axiological association of coherence with "goodness" (Buchmann & Floden, 1991) and opting to also investigate students' varied aesthetic sensibilities and differential *valuations* of (in)coherence. Inspired by Appelbaum's (2010) postmodern plea that *nonsense-making* ought to be considered alongside sensemaking, I remain open to not only students' coherence seeking but also their *incoherence* seeking.

In this exploratory study, I work alongside six undergraduate STEM students who had just completed an introductory multivariable calculus course to investigate their perspectives on curricular (in)coherence across their mathematics course taking experiences. To remain open to students' varied personal and disciplinary aesthetic sensibilities and perspectives on (in)coherence, I view curriculum as a storied *artform* (Dietiker, 2015a; Eisner, 2004) through the lens of the flexible curriculum-as-story metaphor (from Chapter 2). Following an ethical praxis of love (Bowers et al., 2024; hooks, 2001; Yeh et al., 2021), I aim to radically center students' perspectives by positioning them as *mathematical artists and critics* (Appelbaum, 2010; Eisner, 1991/2017) of cross-curricular stories in this participatory, arts-based study. Student-artists

participated in sustained individual and collective arts creation and reflection across their curricular experiences with the goal of fostering arts-based, “conspiratorial conversations” (Barone, 2000, 2008) on the topic of curricular (in)coherence. The aim of these critical conversations was to destabilize privileged politics of aesthetics underlying disciplinary coherence by drawing attention to a multiplicity of aesthetic sensibilities of (in)coherence through the generation of aesthetically and emotionally impactful artistic educational critique (Barone & Eisner, 2012). Across these conversations, I explore two research questions that encapsulate the broad questions introduced previously: (RQ1) What interpretations and valuations of (mathematical) (in)coherence do STEM students express as they engage in artistic reflection and critique of their curricular experiences across mathematics courses? How do these student perspectives on curricular (in)coherence relate (or not) to forms of (in)coherence discussed in the mathematics and science education literature? (RQ2) What implications do STEM students’ interpretations and valuations of (in)coherence have for curricular design and how we conceptualize learning in mathematics?

A Theoretical Orientation Toward Participatory, Arts-Based Curricular Analysis

Across the following sections, I detail the philosophical worldview (Stinson, 2020) and praxis that informed the design of this study, including relevant axio-onto-epistemological assumptions about learning, the mathematics curriculum, and how I position students as learners, artists, and co-conspirators in this participatory, arts-based study of curricular (in)coherence. I begin by re-introducing (from Chapter 2) the view of curriculum as a storied artform alongside the flexible stance of curricular coherence as story coherence I take up to remain open to a plurality of student perspectives on (in)coherence. I then proceed to justify and outline my positioning of students (and therefore study participants) as mathematical artists and critics of the curriculum (Appelbaum, 2010; Eisner, 1991/2017, 2004) whose views of (in)coherence ought to

be taken seriously in pursuing more just and inclusive curricular futurities. Next, I introduce the praxis of love (Bowers et al., 2024; hooks, 2001; Yeh et al., 2021) guiding my research efforts and lean into this praxis to situate this study as a form of participatory, arts-based research featuring “conspiratorial conversations” (Barone, 2000, 2008). These conversations act to destabilize the privileged politics of aesthetics underlying curricular (in)coherence through the generation of evocative, artistic educational critique. Finally, I conclude with some further comments on how the paradigm of arts-based research aligns with the stated goals of this study.

Arts-Based Research: Positioning Students as Mathematical Artists and Critics

As in Chapter 2, I view (mathematics) curriculum as art (Eisner, 2004)—and specifically a narrative art—meaning that its aesthetic *form* cannot be ignored in studies of curriculum. Specifically, I employ the metaphor of curriculum as a story (Dietiker, 2015a) alongside the flexible stance of curricular coherence as story coherence previously proposed. This stance is one of openness toward varying student interpretations and valuations of curricular (in)coherence, with such variance owing to the idiosyncratic aesthetic sensibilities and socio-cultural framings of story which a student leverages—implicitly or explicitly—at a particular moment in time. In other words, the choice of story form (genre, socio-cultural view of storytelling, preference for story structure, etc.) used to metaphorically conceptualize curriculum as a narrative art results in a particular axio-onto-epistemology of curricular (in)coherence. The openness to ontological plurality afforded by these theoretical framings situates and guides the forms of inquiry in this study, particularly in relation to RQ1.

The curriculum-as-story metaphor positions students (including the participants in this study) as active *readers* (Rosenblatt, 1978, 1988) and interpreters of curricular stories (Chapter 2, Assumption 2). Students’ aesthetic sensibilities influence their engagement with the plots of cross-curricular stories and therefore their learning experience (Dewey, 1934/2005; Wong, 2007)

as they construct subjective personal meanings and conclusions (e.g., Sinclair, 2005) and make *judgements* about the "goodness" or (in)coherence of said stories. In this way, students are "connoisseurs" (Eisner, 1991/2017) of the curriculum, privately interpreting and critically evaluating the gestalt (i.e., form *and* content) of curricular plots. Viewing students as *readers*, therefore, serves as a valuable reminder that aesthetic forces impact their curricular experiences. Yet, this metaphor is also limited: it suggests nothing about what students might subsequently *do* with their private curricular judgements, nor does it explicitly entertain the possibility of students as *artists*—i.e., *authors*—of their own (curricular) stories. Given that an aim of this study is to engage students in a participatory, collective process of curricular co-analysis and critique to radically center their perspectives on (in)coherence, (private) readerly connoisseurship will not suffice. Accordingly, I go further and position students (and the participants of this study) as not only *mathematical critics* but also *mathematical artists*.

Students as Mathematical Artists and Critics

As full mathematical artists, participants in this study engaged in the process of art making to holistically reflect on their experiences with the learned curriculum, thereby acting as mathematical critics of the very same curriculum. My use of the phrase "mathematical artist" is an intentional riff on (and a possible response to) Appelbaum's (2010) provocation: "What would [students] as mathematical artists do, then, if they would not primarily be practicing forms of mathematical representation?" (p. 3). As alluded to in Chapter 2, Appelbaum applied a postmodern lens to critique the dominant assumption that mathematics always "makes sense" by suggesting that sometimes it is natural for mathematics to be contradictory and fundamentally nonsensical. Applying these observations to the mathematics curriculum, he questioned the overreliance on sense-making in the curriculum as well as the dominance of learning goals that put student fluency with the *representation* of generalized mathematical ideas at the forefront.

Put simply, Appelbaum’s contention is that mathematics curriculum should put a premium not just on learning mathematical content, sense-making, or representational fluency but also on contemplating and appreciating the *aesthetics* of mathematics.

I position student participants as “mathematical artists” and “mathematical critics” in much the same spirit as Appelbaum. These are not just metaphors—students in this study created mathematical art, wrote their own artists statements, and then exhibited and discussed their artwork with fellow participants. At the same time, I do not necessarily claim to advance Appelbaum’s original aim of having students create “nonrepresentational mathematical art”.²⁵ My use of “artist” and “critic” are, however, are well aligned with the meanings put forth by Eisner (2004, 1991/2017). This view of aesthetic criticism is often associated with the academic genesis of arts-based educational research (Cahnmann-Taylor & Siegesmund, 2017; Chilton & Leavy, 2020) as a form of evocative, artistic educational critique (Barone & Eisner, 1997). Therefore, by positioning students as artists with agency to critique mathematics curriculum using their idiosyncratic aesthetic judgement, I consider them squarely as co-artists in this participatory, arts-based endeavor. Meanwhile, I see my role as primarily a facilitator of mathematical artistry—as one who expressly creates space for conversations of non-sensemaking (i.e., *incoherence*) as well as coherence and sense-making through the artistic prompts and activities I offer student-artists. Such facilitation is in the spirit of radical love, a praxis which has guided the design of this study from its inception. I detour briefly to share this ethical stance,

²⁵ Though I will point out that most of the artwork students created was *not* strictly representational and often embraced moments of incoherence (a possible form of nonsense-making). Additionally, from the perspective of the ABR paradigm within which I have situated this study (see Chapter 2), art is viewed complexly as more than just mere representation but rather as an entity whose purpose is bound up on the aesthetic impact it has on its creator and/or the audiences who engage with it. Impact is also not a strict binary—similar to Appelbaum’s postmodern framing, ambiguity and art that fosters *nonsense-making* is seen as valuable within the ABR paradigm. In this light, a reader might interpret this study as an advancement of Appelbaum’s goal. Other readers might not. In the spirit of celebrating uncertainty and *nonsense-making*, I will leave this point purposefully open to your interpretation, dear reader.

then subsequently conclude with an overview of how the theoretical-philosophical pieces introduced thus far form a foundation for this participatory, arts-based study which serves to advance my overarching research goals.

Participatory, Arts-Based Research Grounded in an Ethic of Love and “Conspiratorial Conversation”

In Lynette Guzmán's (2019) essay *Academia Will Not Save You*, she presents her guiding question for conducting scholarship: “How will I work to deconstruct and reconstruct mathematics (education) in ways that treat students, teachers, and prospective teachers with dignity and love?” (p. 328). In this study, my motivation to radically attend to and center *students’* perspectives on curricular coherence (and to be critical of how status quo perspectives of curricular coherence position students) is guided by this question and, more generally, an ethical stance of love in my research and teaching practice (Bowers et al., 2024; de Rijke et al., 2021). Love is not a topic that is often centered in mathematics education. Yet, I am reminded by Maturana and de Rezepka (1997) that

Only love expands our intelligent behavior, because it expands our vision. Love is visionary, not blind. Accordingly, for the educational space to be a relational space of expansion of the intelligent behavior of the students and teachers, it must be lived in the biology of love. (p. 19)

By explicitly adopting an ethic of love in the conceptualization and enactment of this study, my aim is to expand our field’s vision and subsequent intelligent behavior by learning from axio-onto-epistemologies of curricular (in)coherence valued by students. A responsiveness to various philosophies of (in)coherence, as I argue momentarily, has the potential to expand what our research and curriculum is capable of and diversify the populations that they serve.

Following bell hooks (2001), what I mean by love goes much further than just care. Cloninger (2008) echoes this sentiment: “love motivates one to care for another; without this faculty, care would be an empty vessel, a messageless bottle drifting at sea” (p. 209).

Specifically, hooks, following Peck (1978), defines love as “the will to extend one’s self for the purpose of nurturing one’s own or another’s spiritual growth” (p. 4). This is an active process—love is a choice we make, not just something we passively hope for. hooks further clarifies that “spiritual” growth need not be religious; rather, it refers to “that dimension of our core reality where mind, body, and spirit are one . . . an animating principle in the self—a life force (some of us call it soul)—that when nurtured enhances our capacity to be more fully self-actualized” (p. 13). Such spiritual growth and self-actualization cannot occur without first consulting the other party about their needs and *then* working to support them in achieving their goals.

In constructing the theoretical framing and methodological practice of this study, I was guided by a praxis of love (Bowers et al., 2024). I use arts-based research approaches purposefully to radically center multi-modal opportunities to learn *from* students in ways that empower them to choose *how* they will reflect on and share their experiences (through their choice of artistic practice). Simultaneously, I worked to ensure that students are positioned to catalyze their own becoming as they created art and told mathematical stories which normalized their experiences of “struggling” and reflected moments when they interpreted learning mathematics to be concurrently incoherent and coherent. Given the epistemological power of art to evoke and explore metacognitive, pre-verbal human experience in a way that is sensitive to embodied, aesthetic, and sensorial ways of knowing and being (Chilton & Leavy, 2020; Rolling, 2010), arts-based research is particularly suited to these aims. Additionally, this loving, arts-based approach to curricular analysis denotes a purposeful contrast from other studies of curricular coherence which are guided primarily by notions of “cognitive coherence” with the goal of developing a “coherent” learning trajectory to guide future curriculum design (Fortus & Krajcik, 2012). Rather, by positioning students as co-artists and critics of the curriculum, I aim to

position them as *guiding* our shared inquiry into and critique of the curriculum. I also allow students to decide not only the artistic forms of their individual inquiry but also to collectively direct the flow of our curricular conversations at critical junctures.

In doing so, I channel Barone's (2000) conceptualization of ABR as a “conspiratorial conversation” (p. 150) by viewing this participatory study as an opportunity to work with students to challenge a strict adherence to the dominant narratives that (1) curricular coherence is solely disciplinary coherence and that (2) curricular coherence is a universally desirable goal for student learning and therefore curricular design. As Barone (2008) puts it, “a conspiracy suggests a communion of agents engaged in exploratory discussions about possible and desirable worlds” (p. 39). In this study, I worked alongside students as co-artists to draw attention to a multiplicity of perspectives on curricular coherence with a goal of destabilizing the dominant politics of aesthetics in mathematics education which places logical forms of (in)coherence on a pedestal. Indeed, such views of curricular (in)coherence are based almost exclusively on forms of disciplinary coherence (Cuoco & McCallum, 2018; Schmidt et al., 2002, see Chapter 2) which privilege logico-rational forms of knowledge and favor assimilation to fixed mathematical practices (e.g., generalization, abstraction) that have historically been grounded in colonizing, predominantly White, cis-heteropatriarchal, and ableist discourses that often foreclose complexity, favor objectivity, and reduce axio-onto-epistemological plurality (e.g., McNeill & Jefferson, 2024). An unquestioned adherence to such singular views of coherence serve to enforce a narrow politics of mathematical aesthetics, thereby erecting a culture of exclusion (Louie, 2017) which positions students with different aesthetic sensibilities or ways of knowing (e.g., those who place positive value on *incoherence*) as deficit, “abnormal”, or aberrant (e.g., Appelbaum, 2010; Hyvärinen et al., 2010, recall Chapter 2). Toppling this harmful culture of

exclusion requires that we re-imagine possible curricular futurities by working alongside students to craft cross-curricular stories which are responsive to their *varied* philosophies of (in)coherence, aesthetic sensibilities, and ways of knowing/being, thereby embracing a politics of mathematical *plurality* and axio-onto-epistemological inclusion. If curriculum truly is art, then alternative interpretations and critiques ought to be celebrated and taken seriously in curriculum design, not castigated. This study aims to explore what taking a step in this direction might look like.

Brief Comments on Arts-Based Research

Arts-based research (ABR) refers to processes of inquiry whereby the researcher, alone or with others, engages the making of art as a primary mode of inquiry” (McNiff, 2014, p. 259).²⁶ ABR can stretch across any or all phases of the research process beginning from problem generation all the way through to data collection, analysis, and research communication (Leavy, 2020; Scotti & Chilton, 2018). A consistent feature across ABR is the high value placed on the *process* of art making as a form of aesthetic knowing (Eisner, 1985). Indeed, Norris (2011) has proposed that artwork which transforms those who make it satisfies a *pedagogical* criteria for quality ABR, meaning that the impact the art-making process has on the involved *artists* (i.e., in this study, the students who reflected on their curricular experiences by making art) can be just as important as the impact the art has on other intended audiences.

²⁶ The names used to refer to arts-based forms of inquiry are diverse, ranging from research-creation (Loveless, 2019) and scholartistry (e.g., Shanks & Svabo, 2018) to practice-based research (Candy et al., 2021; Vaughan, 2005) and a/r/tography (a phrase alluding to the intersection of **art**/research/teaching practices, see Springgay et al., 2005). Chilton and Leavy (2020) present a lexicology of related terms and detail some of the subtle differences between these terminologies, owing to their idiosyncratic disciplinary origins and purposes. That said, I use the term “arts-based research” to collectively refer to this broad body of arts-based inquiry practices given its wide-reaching, global usage (Leavy, 2020) and in following the lead of several recent handbooks and handbook chapters on this topic (e.g., Cahnmann-Taylor & Siegesmund, 2017; Chilton & Leavy, 2020; Leavy, 2018). This choice is not to downplay differences between these forms of inquiry but instead to emphasize their philosophical and methodological similarity, particularly vis-à-vis more common paradigms of (math) education research.

While process is central to ABR, I view ABR as more than just a methodology. Following several arts-based researchers (Chilton et al., 2015; Conrad & Beck, 2015; Leavy, 2020; Rolling, 2010), I adopt the perspective that ABR is a newly emergent paradigm of research inquiry (Lather, 2006; Stinson & Walshaw, 2017), in the sense that it features a distinct philosophical worldview (See Chapter 2) with corresponding methodological approaches. In other words, ABR is *more* than just using “art on the side” to collect data or share research results—it features careful attention to and integration of aesthetic, artistic ways of knowing and being in ways that are not central to existing research paradigms (see e.g., Viega, 2016).

I now briefly mention the practical intentions of and interpretive nature of ABR, with an eye to contextualizing this study and foreshadowing future choices in this chapter. When it comes to judging quality in ABR, there is no rigid “gold standard” (Leavy, 2020). As McNiff (2018) quipped, “when we talk and write with fixed jargon, we have left art” (p. 30). Research quality in the paradigm of ABR, therefore, is based more on the flexible criteria by which one might judge art or other humanities-oriented research (American Educational Research Association, 2009). For example, *usefulness* and *impact* are central criteria for judging the quality of ABR (Barone & Eisner, 2012; Chilton & Leavy, 2020; McNiff, 2014). Artist-researchers are encouraged to continuously question the “so what?” and purpose of their work—ABR is not so much concerned with whether a piece of art is “good” in a solely artistic sense but instead what it is “good for” (Leavy, 2020). Specifically, does the art make a difference? For whom? How and why? (Cahnmann-Taylor, 2017; Finley, 2011). In the case of this study, the goal of artmaking was to foster conspiratorial conversation with and among students to learn more about their perspectives on curricular (in)coherence. Specifically, this was with an eye to re-imagining what future curricula (and cross-curricular stories) might look like if we were open

to incorporating these varied aesthetic sensibilities. Later, I will invite you to “walk” through an “exhibit” of students’ art pieces so that *you* can also join in these re-imagining efforts.

That said, due to the aesthetic, intersubjective nature of ABR, artfulness of an ABR study is interpreted largely by the *aesthetic impact on its intended audience* (e.g., *you*, as a reader of this paper). Impactful ABR inspires a *generativity* of thought in its audience members that can lead them to consider new worldviews, possibly even transforming their own (Barone & Eisner, 2012; Chilton & Leavy, 2020). But impactful art rarely has a single, straightforward message—it features ambiguity and is open to multiple interpretations, affording multiple meanings and realities (Barone & Eisner, 2012; Bochner & Riggs, 2014). Art—and therefore ABR—that generates “puzzlements” (Eisner, 2008, p. 22) and interesting questions rather than straightforward answers may very well have succeeded in its purpose (e.g., Barney & Kalin, 2014). Art is always open to interpretation and therefore encourages constant *re*-interpretation of social situations (educational or otherwise). As mentioned before, this is the precise aim of the “conspiratorial conversations” I report on in this chapter. In the aforementioned exhibit of participant artwork, I purposefully do *not* reduce all the ambiguity or provide you with the full context. So, as you walk through the exhibit, I invite you to *embrace* the possibly generative nature of any uncertainty or *not* knowing that you encounter as the art elicits *questions* rather than solely answers.

Participants and Methodology

Participants

The mathematical artists and critics in this study were six undergraduate mathematics students from various disciplines who were enrolled in multivariable calculus at Michigan State University (MSU) in one of the two summer 2023 semesters. By recruiting students with diverse majors, my aim was to include students with a variety of (disciplinary) aesthetic sensibilities,

which I suspected would promote the sharing of a multiplicity of perspectives on curricular (in)coherence across their mathematics courses. At the same time, I recruited students taking the *same* course to ensure participants would have some shared mathematical content experiences to foster collective conversation and minimize instances when a participant would be unfamiliar with the mathematical terminology needed to join the conversation. Multivariable calculus was chosen as the course not for its specific content but rather to increase the chance that participants had taken at least two university mathematics courses they could reflect across throughout the study.^{27,28} I chose to focus on undergraduate mathematics students because they have several years of mathematical experiences—both secondary and post-secondary—over which to reflect. Additionally, whether these multivariable calculus students self-identify as struggling in their current classes or not, they are *successful mathematics students* in the sense that they have persevered through several courses to reach the conclusion of the calculus sequence. In this way, their perspectives can teach us much about not only the undergraduate mathematics curriculum but also (in a retrospective sense) the K-12 mathematics curriculum.

Participants were recruited by email using a list of students enrolled in multivariable calculus at the start of each of the two summer semesters obtained from the university registrar's office with IRB consent. In my recruiting, I made it clear that the intent of the study was to hear students' perspectives and stories of taking undergraduate mathematics courses. I also stated we

²⁷ Based on current curricular trends in the United States, I assumed many prospective participants would have taken a course equivalent to the first semester of single-variable calculus in high school, in which case they would have subsequently taken at least the second semester of single-variable calculus and multivariable calculus at MSU, resulting in two university mathematics courses to reflect across.

²⁸ Another reason multivariable calculus was selected is based on my experience teaching and single- and multi-variable calculus students in formal and informal settings paired with expertise as a researcher of multivariable calculus. This background afforded me flexible knowledge of the form and content of the calculus curriculum as well as a degree of comfort in interacting with students at this mathematical level. This knowledge and comfort allowed me to be flexible with my ABR methodology, particularly when I facilitated artistic activities and related conversations.

would be creating art as a form of reflection; however, it was made very clear that the process was more important than the product and that students did not need to be artists to participate. Prospective participants were promised a gift card as compensation for each session they attended. All students who signed up to participate were invited to participate in this study—the six students who responded to my emails are the mathematical artists and critics featured in this study, though not everyone completed all activities due to scheduling restrictions. Table 3.1 shares self-reported demographic information for each participant. In this table and throughout this chapter, participants are referred to by their “artist name”, which they crafted as part of an artist statement they wrote early in the study (details to follow).

All six mathematical artists were studying STEM disciplines, but none of them were pursuing a mathematics major or minor. Many of them were computer scientists, though their second majors and minors varied, and they each had idiosyncratic STEM course backgrounds. All students had successfully passed two semesters of single-variable calculus and a semester of multivariable calculus, aside from Fred who chose to drop out of multivariable calculus part-way through the summer semester and take it again later. Fred was also unique—alongside GUo3—in the sense that they had both successfully completed the equivalent of two semesters’ worth of single-variable calculus in their secondary education. The four other participants reported completing both semesters of single-variable calculus (i.e., Calculus I and Calculus II) as well as multivariable calculus at MSU. None of them reported taking any mathematics courses beyond multivariable calculus. In terms of academic standing (by credit hours) and the number of years they had been attending MSU, there was great diversity—some had just started at MSU, some were in the middle of their studies, and others were in their final year. All participants identified as Asian, but they each traced their heritage back to distinct geographic locations in Asia from

Table 3.1*Demographic Information of the Mathematical Artists*

Name	Disciplines (<u>M</u> ajor, <u>m</u> inor)	Academic Standing / Year at MSU	STEM Courses Taken	Further Demographics ²⁹
CinematicHue	M: Computer Science m: Business, Entrepreneurship, & Innovation	Junior / 3	Computer Sci. Statistics	Asian Man Maybe autistic Superhero and comics fan
Fred (Test Subject 838462904)	M: Computer Science m: Linguistics	Freshman / 1	Biology Chemistry Computer Science Engineering Physics	South Asian Man
GUo3	M: Computer Engineering m: Undecided	Sophomore / 2	Computer Science	Chinese Man International Student Has registered learning accommodations but does not use them in math courses
Joy	M: Data Science, Psychology (Previously Computer Science) m: Leaning toward Computer Science	Senior / 5	Chemistry Computer Science Computational Math, Science, & Engineering Physics Statistics	South Asian Woman (she/her) Pakistani Woman Daughter of Immigrants Heterosexual Aspiring Data Scientist
VKN	M: History, Philosophy and Sociology of Science; Psychology (Previously Pre-Med, now Pre-Law) m: Bioethics; Science, Technology, and Public Policy	Senior / 4	Biology Chemistry	Indian / Asian Woman Bisexual Diagnosed with ADHD Has registered learning accommodations & uses them in math courses
wuyen	M: Computer Science (Previously Pre-Med) m: Business	Junior / 2	Chemistry Computer Science Engineering Physics	Asian Woman Straight

²⁹ Participants were asked to demographic identities they felt were relevant to their experiences learning mathematics and/or that they wanted me to share when introducing them and their artwork in subsequent presentations of this research. I suggested some identities students might use to describe themselves (gender identity; racial/ethnic identities; whether they identified as having a disability, neurodivergence, or chronic illness; and registered learning accommodations). Following an ethic of love, participants were told to disclose whatever identities they wished. These logistics explain why some participants have more demographics listed than others as well as the varied language choices.

China to Pakistan to India. Additionally, while most of these artists were Asian Americans who had completed their secondary education in the United States, one of them (GUo3) is a Chinese international student who had only recently moved to the U.S. to pursue his post-secondary studies. Throughout the study, these demographic similarities fostered some natural comradery among artists which allowed for deeper conversation based on their shared identities and curricular experiences. Simultaneously, their demographic differences afforded a multiplicity of curricular perspectives with respect to (in)coherence.

Arts Creation and Reflection

The mathematical artists came together for a series of four approximately one-hour sessions of individual and collective arts creation and reflection on their curricular experiences, with a particular emphasis on their interpretations and valuations of (in)coherence across mathematical stories (see Table 3.2 for a summary of activities across all sessions). Following an arts-based approach, participants created and then discussed art in the form of their choosing to allow for a holistic, multimodal, and multisensory investigation of the learned mathematics curriculum (Bagnoli, 2009; Chilton & Leavy, 2020; Papoi, 2017; Rolling, 2010). This reflection centered opportunities for introspection on the roles that *aesthetic* as well as logical forces played in making curricular stories connect and cohere (Clark & Rossiter, 2008; Sinclair, 2009) across their everyday mathematical experiences. To allow for cross-pollination of perspectives (Leavy, 2020) on curricular (in)coherence across artists and their artwork, private exhibitions of their art were used to elicit richer conversation and subsequent mathematical storytelling. Interactions were also spread out over a period spanning three months to allow sustained space for artists to consider alternative perspectives and (co-)develop more complex points of view and targeted critiques of their curricular experiences. All sessions were video recorded using technology that captured all artists' faces and their art as they made it. In the remainder of this section, I provide

Table 3.2*Summary of Arts Creation and Reflection Activities*

Activity	Description (See Appendices A-C for further details)	Duration	Participants
Session 1			
Initial (In)Coherence Conversation	• Group / Individual • A framing conversation on (in)coherence	~20 mins	All
Arts Creation: Reflecting on Curricular (In)Coherence	• Individual • Created visual art to reflect on a curricular (in)coherence prompt	~40 mins	All
Session 2			
Written Artist Statement	• Individual • Participants created artist statement for visual art from Session 1	~35 mins	All
Individual Art Conversation	• Individual • Semi-structured discussion about art and artist statement	~25 mins	All
Session 3			
Group Discussion 1 (GD1): Art Exhibition & Group (In)Coherence Conversation	• Group • Snacks provided • Exhibition of student artwork • Gallery walk (individual) • Student-led discussion about (in)coherence • Decided goals for Session 4	50 mins	GUo3 VKN wuyen
Session 4			
Group Discussion 2 (GD2): Mathematical Story Creation & Sharing	• Group • Dinner • Discussed curriculum-as-story metaphor and various (visual) story structures • Each student artistically crafted their mathematical story • Share out: mathematical stories • Brief wrap-up discussion	100 mins	Joy VKN wuyen

an overview of the activities across these sessions.

Initial (In)Coherence Conversation.³⁰ We began with a framing conversation on (in)coherence to break the ice, ensure participants had a small amount of time to reflect on what “coherence” meant to them before they responded to the first art prompt, and to inform them of the intended foci of this study. I began with the disclaimer that there were no right or wrong answers but rather different perspectives and experiences that were all equally valid (in line with the goals of this study and the ABR paradigm). Drawing inspiration from the curriculum-as-story metaphor and a flexible view of story coherence (see Chapter 2), I first asked about participants’ views of (in)coherence in familiar, everyday, and/or narrative contexts: “What does it mean for something to be coherent? For instance, a story, a TV show, or movie, a song, a conversation...” (followed by an analogous follow-up question for incoherence). Afterwards, I transitioned toward questions that ask about participants’ views on *mathematical* (in)coherence across the curriculum.³¹ This sequencing of questions resulted in participants sharing a multiplicity of idiosyncratic perspectives on (in)coherence—both in and outside of mathematics contexts—which served as a rich framing that encouraged creative approaches to reflecting on curricular (in)coherence in the next activity.

Arts Creation: Reflecting on Curricular (In)Coherence. With the remaining time from the first session (~40 minutes), participants then responded to the following arts creation prompt:³²

³⁰ I originally planned to bring together all participants to meet each other and create art in community during this activity so they could get to know each other before being asked to participate in sustained group conversation and curricular critique in later sessions. However, due to scheduling constraints, only the male artists (Fred, GUo3, and CinematicHue) could meet at the same time. The remaining female artists, therefore, completed this session individually.

³¹ See Appendix A for a precise list of prompts for the (in)coherence conversation, as well as most other arts creation and reflection activities from this study.

³² Most participants finished their artwork before leaving the session. The only exception is wuyen, who opted to take some of art supplies home to finish her piece.

Consider your journey as an undergraduate mathematics student so far, and, in particular, the mathematical content and skills you've learned about across your courses. **What are some patterns, themes, coherences, or incoherences you've noticed or felt across your journey?** While I'd like you to focus your reflection on the mathematical content/skills you've learned, I recognize that your experience cannot be reduced to just a list of topics. Please do not shy away from also incorporating your own emotions, feelings, opinions, personal aesthetics, identities, backgrounds, relationships with other people, etc. when responding to this question.

Feel free to use any of the artistic resources provided to you to organize and make sense of your experiences. What product you create is up to you: Create a 3D model, write a story, a poem, or perform a song, etc. Feel free to choose the artistic medium that you feel is best suited for reflecting on your experiences.

The goal of this prompt was to allow students space to individually reflect in further detail on their mathematics experiences across courses using whichever definition(s) of (in)coherence felt most appropriate. To further encourage creativity and choice of artistic medium, I provided a range of arts supplies, including a variety of paper, cardboard, old magazines, colored pencils and markers, painting supplies, scissors, string, rubber bands, tape, and more (See Figure 3.1). In the sign-up form for the study, students could optionally request supplies in advance—CinematicHue and VKN both requested paint and a canvas, which they did end up using; meanwhile, Fred requested various mathematical tools—a calculator, 3D online graphing calculator, ruler, and compass—which I provided but he ended up not using. During the session, I also encouraged participants to ask for additional supplies they might want. Given the unconventional nature of this prompt in a mathematics context, let alone a research study, participants often asked if there were any requirements or limits for their artistic response. I responded by pointing them to the bold part of the prompt and reassuring them that they could be as creative and unconventional as they wanted because there are no right or wrong answers when sharing idiosyncratic experiences.

Written Artist Statement. About a week after the first session, participants were individually invited back to further reflect on the artwork they had created. First, they were

Figure 3.1

Arts Supplies Provided to Participants Throughout Study



tasked with reflecting privately on what their artwork meant to them by writing an artist statement. As the prompt I provided to participants explains:

An artist statement is a not-too-long series of sentences (a few paragraphs) that describe what you made and why you made it. It's a stand-in for you, the artist, talking to someone about your work in a way that adds to their experience of viewing that work. (Hotchkiss, 2018)³³

Such statements are widely used in the arts and arts education (Damrongmanee, 2016) and often accompany a piece of art when it is exhibited serving to “explain, justify, or contextualise an artist’s work to the viewer” (Hocking, 2021, p. 104). In the act of contextualizing a piece of art, these statements tend to express an artist’s identities relative to their motivations for creating the art and the creative processes and materials which were used in creating the art (Hocking, 2021). In the context of this study, I prompted participants to write artist statements to not only further position them as *artists* but also to provide an additional opportunity for them to reflect more—in their own words—on how their art related to who they are as people, learners of mathematics, and artists. To introduce the task of writing an artist statement, participants read through two

³³ This description was adapted from Hotchkiss' (2018) *How to Write an Artist Statement* with only minor changes to the first sentence to make the artist statement prompt more specific to the context of this study. I specified “a not-too-long” series of sentences as a few paragraphs and changed the tense of “make” to “made” to reflect that not all participants considered themselves to be artists who regularly make art.

example artist statements from professional artists, were given the definition quoted above, and presented with some common components of an artist statement they were encouraged to include alongside anything else they wished to say about their art. See Appendix B for the full prompt participants were given, as well as the example artist statements students were shown.

I included this activity to further position students as artists but also to encourage them to situate their artwork in terms of their broader worldview of learning (mathematics) and how they viewed themselves as mathematics learners, given the interconnections between (in)coherence, epistemology, and identity. In a particularly poignant example, Joy lamented in her artist statement:

I love the color blue, and it saddens me to use it as a representation of how low mathematics has made me feel in the past; however, I can think of no better colors to highlight how isolating it felt in some of my math courses.

But not everyone addressed these points *explicitly*—wuyen, for instance, wrote an evocative reflection about her “memories of girlhood” and playing dress up without direct reference to mathematics. Still, personal narratives such as wuyen’s proved to be fertile ground for further conversation about mathematics in subsequent activities, so I saw this as a feature rather than a bug of the artist statement prompt. On that note, while many participants expressed enthusiasm at the rare prospect of doing creative writing in a mathematics-coded context, others shared that they found this form of writing to be daunting as STEM students who rarely engaged in non-technical writing. In those moments, I reminded participants that even professional artists are not always comfortable writing such statements (Damrongmanee, 2016; Hocking, 2021) and assured them that they had full creative license to take the statement in whatever direction they felt most comfortable.

Individual Art Conversation. After students finished their artist statement, I read their artist statement myself and then we had a one-on-one conversation about their artwork and specifically how it connected to their perspectives of mathematics and curricular (in)coherence. I asked each artist, in particular, about which meanings of (in)coherence they considered while creating their art and later for specific examples of changes they would propose to make their experiences across mathematics courses more coherent. This conversation also served the dual role as a prime opportunity for me to get a sense of the diversity of participant views so that I could effectively prepare for and facilitate the upcoming group discussions.

Group Discussion 1 (GD1): Art Exhibition and Group (In)Coherence Conversation. About a month after everyone had crafted their artist statement, each participant was invited back for an approximately hour-long, private exhibition of each other's artwork followed by a group discussion motivated by students' engagement with the exhibit. Ultimately, GUo3, VKN, and wuyen attended.³⁴ This group discussion is where I imagined that the conspiratorial conversations proper would begin, as participants engaged in collective reflection as well as curricular analysis and critique based on their experiences taking mathematics courses. To kick things off, participants were first told to do a gallery walk of the six pieces of art they had collectively made, which were exhibited alongside their printed artist statements (See Figure 3.2). I purposefully gave no specific prompt for the gallery walk to allow the art to stand alone and give participants the chance to make their own initial observations which could frame subsequent conversation.

I facilitated the beginning of this subsequent semi-structured conversation by first reminding students of the goals I had for the study and ensuring everyone had a chance to speak

³⁴ Joy had also planned to join us but had to cancel last minute due to an unexpected emergency.

Figure 3.2

Art Exhibition of Students' Artwork Displayed During Both Group Discussions



Note. The paintings and paper art pieces were exhibited in stands on a table (left), while the colored pencil art was hung on a nearby whiteboard (right) to ensure sufficient space for participants to move around and view each other's art. More detailed images of the artwork appear later in the chapter.

throughout. Then, I gave each artist space to share their initial feelings or reactions to the exhibit and *also* one theme, pattern, coherence, or *incoherence* that resonated from the exhibit. After asking these questions, I tried to step back and allow the conversation to flow in the direction participants took it. One memorable example was when VKN asked GUo3 what it was like to feel like mathematics was mostly coherent and if he had ever experienced any incoherence. Given that the backdrop of this discussion was the participants' own artwork, this led them to consider *similar* interpretations and valuations of (in)coherence across each other's art and experiences, *while also* remaining open to the singular, idiosyncratic realities their peers had shared.

Group Discussion 2 (GD2): Mathematical Story Creation and Sharing. At the conclusion of the third session, I suggested some possibilities for our final group session and left it up to the remaining artists to decide what they wanted to do to build off our first group discussion. Ultimately, there was unanimous agreement to further explore the metaphor of the curriculum as a story (which I had just revealed had been my guiding framing of this study from

the beginning³⁵). While participants were openly blunt about adding that they did not consider most mathematical stories as “good” or aesthetically appealing, they did express appreciation for this unexpected humanities-oriented metaphor as a way of further reflecting on the ((in)coherence of) cross-curricular, mathematics stories that span across courses.

When Joy, VKN, wuyen, and I finally came together, it had been about a month and a half since our previous group discussion.³⁶ Once dinner had arrived, I began with a brief recap of the curriculum as story metaphor, giving examples of mathematical characters, settings, morals, etc. I followed this by sharing several visual examples of how stories and story structures have been depicted across cultures, with the hope of demonstrating to participants just how many ways a story could be depicted before they artistically crafted their own mathematics stories. Some of these story structures were linear, like the three-act structure (Field, 2005) with a clear beginning, middle, and end (Aristotle, 350 B.C.E./1995) or Freytag's (1990) pyramid (i.e., exposition, rising action, climax, falling action, resolution). Some were cyclic, such as the “Hero’s Journey” (Campbell, 2009). I also shared some non-linear story structures based on forms of cultural storytelling of particular First Nations tribes in Australia and Canada (Judge, 2024; Stimson, 2020).³⁷

Finally, artists were given the challenge to artistically craft a cross-curricular mathematical story: “If you thought of the **mathematical content and skills** you’ve learned

³⁵ This methodological approach of openly sharing mathematics education theory with students and then asking them to comment on, critique, and build on this theory is something that Stinson (2009) has also done with students.

³⁶ All participants who attending Group Discussion 1 were invited to attend to build on their prior conversations. Joy was also invited because she had planned to attend the prior discussion. I asked her to arrive early, so she could view everyone’s artwork, and I could give her a rough overview of what we had discussed previously. Out of the four participants invited, only GUo3 declined the invitation.

³⁷ The first example was of a floorplan from a First Nations museum in Australia designed so that attendees can experience the exhibits in any order (Judge, 2024). The second was Stimson's (2020) *Stampede Story Map*, a work of art on bison robe inspired by traditional First Nations forms of storytelling where the story “spirals outwards” from any one point, rather than beginning in one distinct spot and ending in another.

across your math courses as a story, what kind of story would it be?” Participants were encouraged to draw explicit connections between the curriculum-as-story metaphor (e.g., characters, morals, etc.), but the prompt also clarified that, “*your story could be coherent, incoherent or somewhere in between—this is YOUR interpretation and mapping of a story told across your math courses*” (see Appendix C for the full prompt and the related handout artists were provided). I then pointed them in the direction of the art supplies they had used previously (recall Figure 3.1) and allowed them to reflect and create for approximately one hour.³⁸ To wrap up, each participant took 5-10 minutes to share their mathematical story, how they had artistically depicted it, and the ways they interpreted these stories as being (in)coherent. Afterwards, with the limited time we had left, we went around the room and each participant was given a moment to share their final takeaways from either this session or from across *all* the sessions.

Analysis

In line with the participatory design of the study and overall research aim to explore students’ perspectives on curricular (in)coherence, I spotlight individual and collective participant analyses (i.e., artistic curricular *critique*) to the extent possible. Analysis through our conspiratorial conversation—in line with most ABR—went beyond reducing artwork, curricular critique, and complex discussion into a list of collectively agreed upon themes (i.e., a *similarity* analysis) (Dominguez & Abreu, 2022; MacLure, 2006, 2013). As McNiff (2018) puts it, “Of course, themes exist in life and art, but there are many other things too—characteristics, features, aspects, principles, ideas, patterns, structures, designs, compositions, similarities and differences” (pp. 29–30). With an eye to multiplicity, our arts-based analyses were open to

³⁸ I had initially planned for 30 minutes; however, after the first half hour, participants unanimously agreed they were finding the artmaking therapeutic and agreed to stay longer than planned to finish up their stories.

exploring complex boundaries between similarity and difference across participant experiences by remaining open to difference, disagreement, and singular cases. The goal of these analyses—our conspiratorial conversations—were to generate aesthetic, evocative insights through artwork which holds the power to change minds via the raising of additional possibilities and puzzlements about curricular (in)coherence. In the following sections, I detail the two flanks of this analysis: (1) the *participatory analyses* conducted by student artists across both group discussions based on each other's artwork and (2) a subsequent *collage analysis* which supported me in re-living the conspiratorial conversations and taking on students' varied perspectives of and metaphors for curricular (in)coherence. This individual analysis allowed me to wade through the complexity of our conversations with an eye to developing a format and structure through which to exhibit students' art and participatory analyses. Finally, I conclude by discussing some complexities around flexible criteria for *what counted* as I curated this exhibit of students' interpretations and valuations of curricular (in)coherence.

Situating this Study Within the Participatory Paradigm

While this study is primarily situated within an ABR paradigm, it simultaneously represents the beginnings of a turn toward embracing a complementary participatory research paradigm (Heron & Reason, 1997; Osibodu et al., 2023). Indeed, as noted by Conrad and Beck (2015), an ABR paradigm shares an inherently relational axiological orientation (e.g., Finley, 2011) with a participatory paradigm in that intersubjective human encounters (with art) which embrace multiple aesthetic sensibilities and ways of knowing are seen as prime sites for working collectively toward societal transformation and pathways to fostering human flourishing. Consistent with the participatory paradigm, students in this study are positioned as co-researchers (i.e., co-artists) with sway to guide the path of research inquiry as they join in conspiratorial conversations with the aim of disrupting singular, dominant, and potentially

harmful forms of curricular (in)coherence to envision more inclusive and loving curricular futurities that are responsive to student viewpoints (Commitments 1 and 5 from Osibodu et al., 2023). Simultaneously, the aesthetic intersubjective paradigm of ABR (Chilton et al., 2015) catalyzed by conversations *across* several art pieces depicting students' varied curricular realities afforded a space for conspiratorial conversation wherein disparate forms of knowing were continually brought into contact with one another and tensions between curricular realities were seen as generative instead of obstructive (Commitments 2 and 4 from Osibodu et al., 2023).³⁹

Consistent with a *beginning* turn toward the participatory paradigm, however, I acknowledge that not all aspects of this study were fully participatory in nature. On one hand, participants' relationship with each other, myself, and the research space tended to be more participatory and could be best described as "moving toward partnership", with some "paternalistic" reliance on myself as the lead researcher (Osibodu et al., 2023, Table 1, p. 227). Though students did not choose the research questions or methods (a nearly undisputed requirement for a fully participatory research space), this study was designed to foster open conversation and student choice: students chose the forms of art they used to express themselves, their interests guided both group discussions, and they selected the goals and modes of inquiry for the second group discussion. By our final interaction, students expressed comfort with artistically riffing off my initial prompts in ways that were meaningful to them. At this point, they also regularly referenced each other's art and perspectives alongside their own, positioning

³⁹ The remaining third commitment of participatory researchers that Osibodu et al. (2023) share is that "people, institutions, and practices are historicized" (p. 228). In the discussion section of this chapter, I engage in some brief historicization, which I hope to expand upon in future discussions of curricular (in)coherence. However, this commitment was not a pre-planned aim, limiting the extent to which this study might be considered as fully critical and participatory. Still, as Osibodu et al. stress, these participatory commitments are not meant to be mere checkboxes but rather commitments toward "emancipatory sensibilities" (p. 226). I characterize this study as a *beginning* turn toward the participatory paradigm in the sense that my goals are consistent with these sensibilities and oriented by *many* of the core philosophical tenets of this paradigm.

their peers as fellow co-artists and equally knowledgeable colleagues. Before we parted ways, all three participants expressed excitement about being provided the space to critique their curricular experiences and propose alternative views of curricular (in)coherence with a goal of envisioning more inclusive, loving curricular futurities.⁴⁰ On the other hand, I recognize that my relationship to the larger community space—the MSU (undergraduate) mathematics community—was much less developed. While I *do* have a genuine interest in improving undergraduate students’ experiences, I have not yet made concerted efforts outside of this study to connect with others in the *local* community context.⁴¹ Such relationship building is undoubtedly crucial in the long-term to create impactful and sustainable local change and to avoid harmful research ethics of extraction (Osibodu et al., 2023). In this study, I strive to honor the relationships I *did* build and the trust students put in me by curating and exhibiting their artwork carefully and ethically, with an eye to catalyzing further conversation between mathematics educators and students about curricular (in)coherence.

Participatory Analysis

The participatory analyses I highlight in this study occurred throughout Group Discussions 1 and 2 as student-artists shared their reactions to each other’s artwork (GD1) and cross-curricular stories (GD2). From GD1, I focus on (1) the initial discussion in which individual participants introduced their interpretations of the exhibited artwork—including their initial reactions to the exhibit alongside one theme, pattern, or (in)coherence that resonated with

⁴⁰ While not initially planned, multiple student artists volunteered to co-write their curricular stories with me at the conclusion of Group Discussion 2. Students’ interest in and comfort with proposing this option is demonstrative of the agency they felt in our research *partnership*. Unfortunately, due to scheduling limitations in the semesters I wrote this chapter, this co-writing has not yet materialized, but I fully intend to work alongside interested students as I revise and write subsequent iterations of this chapter.

⁴¹ At least beyond my relationships with students in the few undergraduate mathematics courses I have taught at MSU. That said, I have laid groundwork by building some relationships with faculty and staff in the MSU Mathematics Department which might enable me to exhibit students’ artwork sometime in the future, thereby forging a stronger relationship with this local community.

them—and (2) the subsequent participant-led discussion in which they talked across each other’s individual interpretations to note both shared and idiosyncratic perspectives on curricular (in)coherence. From GD2, I focus on (1) the interpretations for curricular (in)coherence that participants expressed metaphorically in the artistic representation of their cross-curricular story *or* their oral re-telling of the story, particularly (2) moments when participants referenced each other’s interpretations of curricular (in)coherence when sharing their own story *or* when they discussed final takeaways across stories (i.e., when they referenced each other’s art, from GD1, *or* stories, from GD2).

Aligned with both the arts-based and participatory research paradigms, these focal moments of participatory analysis spotlighted instances where disparate forms of knowing were brought into contact with one another to generate rich discussion and push participants’ points of view (Osibodu et al., 2023). In other words, these moments allowed for a rich *cross-pollination* (Leavy, 2020) of artists’ perspectives on curricular (in)coherence across several modalities (orally *and* in various artistic media of their choice), forms of art, and sessions (i.e., time). Participants commented on, learned from, and then riffed off each other’s artwork as the study progressed, which led to the germination of hybrid perspectives and increasingly complex curricular contemplation and critique over time. Indeed, when participants shared their cross-curricular stories at the end of the study, *all* of them (1) referenced at least one other participant’s artwork or perspectives on (in)coherence as artistic influences for their story *and* (2) expressed a non-binary view of curricular (in)coherence, where *coherence* naturally co-existed with *incoherence* (see *The In-Betweens of In/Coherence* for more).

To document and report out on the participatory analyses, I wrote field notes and memos during or shortly after each discussion reflecting on the various interpretations and valuations of

curricular (in)coherence that student-artists introduced verbally, in writing, or through their art. I also noted which art piece(s) participants associated with these perspectives *and* other relevant experiences they shared from their course-taking experiences to explain their perspective. To complement these initial observations, I re-watched these discussions multiple times and continued to write reflective memos as I edited the initial auto-generated transcripts of these discussions. During this time, I noted the resonances multiple student-artists acknowledged as they contemplated their interpretations of each other's artwork. Most often, this occurred when one or more participants explicitly affirmed or built on another participant's interpretation or curricular experience immediately after it was shared; however, I also noted moments when a participant called back to someone else's earlier point. Additionally, given the aims of this study, I did not shy away from noting any *differences* in perspective or moments when a student-artist mentioned an idiosyncratic point of view that was not affirmed or built upon by another.

Collage Analysis

To further reflect on the participatory analyses with an eye to deepening my understandings of how students' perspectives on curricular (in)coherence were situated relative to each other and to the literature on curricular coherence (i.e., Research Question 1), I crafted a collage as a form of secondary, arts-based analysis. As an artform, collage traditionally involves selecting images, text, or other objects, cutting them out or otherwise altering them, arranging them thoughtfully, and then attaching (usually by gluing) these ephemera onto paper or cardboard (Butler-Kisber, 2008; Chilton & Scotti, 2014). As a space where the visual and the written are combined, collage is a dynamic "intertextual surface" on which "texts and pictures . . . form a mutual, living dialogue, a unified story" (Sava & Nuutinen, 2003, p. 532). In other words, collage is a space of artistic possibility where visual imagery and words meet to create something unique that would not be expressible with just words or images alone (e.g., McCloud,

1993; Sousanis, 2015).

Similar to Larsen (2010), I use collage not to evaluate perspectives, nor to institute typologies or hierarchies but instead to identify relationships between perspectives. Moreover, in line with my aim of attending to both difference and similarity of perspective, “in collage a single, coherent notion ‘gives way to relations of juxtaposition and difference’ (Rainey, 1998, p. 124), and these fragments ‘work against one another so hard, the mind is sparked’ (Steinberg, 1972, p. 14) into new ways of knowing” (Butler-Kisber, 2008, p. 268). This framing of a “coherent” collage as being formed from several, possibly incoherent or paradoxical “fragments” further echoes my axiological stance of valuing the dialectic of in/coherence as a possible catalyst for learning.⁴² Collaging allowed me to reflect across participants’ perspectives with an eye to how similarity and difference, coherence and incoherence, blended in complex ways across the boundaries and intersections of these perspectives. As collage is always situated and bound to a moment in time (Vaughan, 2005), this artform further encouraged me to remain close to the idiosyncratic context of our conspiratorial conversations to better attend to the nuances in participant perspectives on curricular (in)coherence.

Effectively, I used collage to catalyze what Hunter et al. (2002) refer to as *incubation*: “the process of living and breathing the data . . . the intellectual chaos phase” (p. 389). The embodied nature of collage making—the tactile snipping, arranging, and gluing of ephemera—allowed me to *live* in this intellectual chaos as I artistically explored how students’ perspectives sat in relation to theoretical perspectives in the literature (Chilton & Scotti, 2014).⁴³ Far from

⁴² For more on how the theory and philosophy of collage synergizes with the ABR paradigm and framing of this study, see Appendix D.

⁴³ Culshaw (2019) similarly sought to enrich the theorization of an educational phenomenon (*struggling as a teacher*) using a collage methodology when she asked stakeholders whose perspectives had not been centered in the extant literature on the topic (i.e., teachers) to create collages that depicted their various experiences with struggle.

being an objective or a pre-determined method, I embraced collage as a subjective, open-ended (Vaughan, 2005), and spontaneous (Freeman, 2020) process whereby one “moves from intuitions and feelings to thoughts and ideas. Image fragments are chosen and placed to give a ‘sense’ of something rather than a literal expression of an idea” (Butler-Kisber, 2008, p. 269). Collage is “symbolism-laden artwork” (Scotti & Chilton, 2018, p. 361) where objects are meant to be interpreted metaphorically rather than literally (Butler-Kisber, 2008; Chilton & Scotti, 2014). Essentially, collaging became a way to *live and breathe* students’ metaphors, allowing me to further empathize with their perspectives (Margolin, 2014) as I explored the nuances of these metaphors in order to artistically depict and arrange them relative to one another in the physical collage space.⁴⁴

Collage was particularly well suited for this analysis because it allowed for the juxtaposition of perspectives in the natural forms they were expressed across the study—in texts, visuals, or *both simultaneously* (e.g., 2D and 3D visual art pieces, written artist statements, transcripts from group discussions)—without the need for further translation or reduction.⁴⁵ The stockpile of ephemera I used to craft this collage consisted primarily of photocopies or imitations of every piece of student artwork and any accompanying artist statements alongside a curated subset of student quotes and phrases from across the study which I typed up, stylized, and printed. To curate these textual artifacts, I reviewed all transcripts, field notes, and reflective memos one final time and selected any memorable student words, phrases, and quotes which I

⁴⁴ In many ways, this felt like a multi-modal form of in vivo coding (e.g., Saldaña, 2015), where I re-lived our conspiratorial conversations almost exclusively through students’ metaphorical language and imagery. Similar to in vivo coding, my aim was to spotlight student perspectives, language, and images as much as possible.

⁴⁵ This collage analysis was akin to creating *found poetry* from not only students’ quotes (i.e., text) but also their visual art. Collaging, therefore, shares many similarities with the arts-based method of *poetic transcription* (Faulkner, 2018; Shenfield & Prendergast, 2022), where researchers or participants create found poetry from interview transcripts. Forms of poetic transcription have seen emerging use in mathematics education as a critical methodology used to spotlight the metaphorical and poetic complexity of participant’s words and perspectives (Helme, 2021; Staats & Helme, 2023; Tremaine, 2022) in much the same way I intend to with my collage analysis.

felt were evocative of key metaphors or moments from our conspiratorial conversations. To complement these primary student artifacts, I located a small amount of secondary imagery online to visually depict any evocative student metaphors or allusions to pop culture which were not already explicitly depicted in their art (e.g., Joy spoke of coherence seeking in terms of a moth being drawn to a flame, so I printed out a moth to complement her artistic depiction of fire, though I also allowed myself to be creative when choosing).⁴⁶ Finally, to bring the collage together, I used any arts supplies that had been provided to students, including some magazines which I clipped imagery from (recall Figure 3.1).

The collage I created (see Figure 3.3, Image A) is organized into roughly six zones which intermingle across their porous boundaries. Each of these zones—or “sub-exhibits” as I will now call them given my aim of curating an *exhibit* featuring students’ artwork and analyses for the subsequent section—draws attention to a unique perspective or message about (in)coherence that I learned from our conspiratorial conversations. For example, the top-right sub-exhibit (Image C in Figure 3.3) depicts student reflections on the *purposes and relevance* of cross-curricular mathematical coherence to their personal lives, career aspirations, and scientific innovation across STEM fields. Yet, decisions about where to display each artifact were not as simple as choosing one sub-exhibit: many belonged best at the boundaries or even stretched across multiple locations. As the curator of this collage and exhibit, I chose not only which sub-exhibits to spotlight but also how to arrange in space and therefore which boundaries would be visible *and* invisible—i.e., under the black wheel.

I use this spinning wheel purposefully to riff on the visual metaphor wuyen used (see

⁴⁶ All these images were copyright-free and obtained via Free Range Stock (<https://freerangestock.com/>) save for two copyrighted images depicting specific allusions students made to visual media. Copyrighted images were used in ways consistent with U.S. copyright law related to fair use—i.e., for transformative purposes as only small pieces within the larger collage (see e.g., Butler-Kisber, 2008; Scotti & Chilton, 2018).

Figure 3.3

Analytic Collage



Figure 3.12) to explain the “tunnel vision” she experienced whenever she had to focus on learning the concepts from *one* course at the expense of “seeing” the underlying stories connecting *all* her other courses. At the same time, she knew that under the surface these were all connected, which she demonstrated by lifting the black, monochromatic wheel to reveal the colorful interrelationships between mathematical concepts and courses. In this collage, the spinning wheel functions analogously, this time drawing attention to the dangers of embracing a

singular perspective on curricular (in)coherence amidst a plurality of other possible perspectives, which often intertwine with one another (at the boundaries as well as underneath the wheel if it were to be removed). Given the figurative and literal centrality of the wheel to the collage, I affixed some student quotes and imagery that resonated with this openness to plurality that guided the study and my collage making (see Figure 3.3B). In particular, the quote that wraps around the perimeter of the wheel—as well as the *spinning* of the wheel itself—serves as a reminder of the cyclic dialectic between (“levels” or different forms of) knowing and *not* knowing, between coherence and *incoherence*. To avoid spoiling the exhibit for you or stealing the thunder of the student-artists, I have purposefully presented only a *brief* preview of this exhibit and how I curated it via collage. Very soon, you will experience the exhibit itself, where the artwork and words of the student-artists will guide you through the complexity of our conspiratorial conversations on curricular (in)coherence.

What Counted as (In)Coherence?

Before proceeding to the art exhibition, I briefly discuss three complexities that had to be navigated while determining what counted as a student perspective on cross-curricular, mathematical (in)coherence for the sake of these analyses. The first difficulty was establishing a taken-as-shared perspective for “curriculum” as well as a focal grain size of curriculum (i.e., *across* courses rather than *within* a course) during our conspiratorial conversations. Initially, many participants associated “the mathematics curriculum” with either the syllabus, textbook, homework assignments, or policies of a single instructor or course (i.e., multivariable calculus). Given the lack of consensus about what counts as “the curriculum” among even university instructors (Fraser & Bosanquet, 2006), it was unsurprising that we did not share the same perspective on curriculum at the outset. Therefore, to help get everyone on the same page as the study progressed, I repeatedly shared the goal of the study and highlighted any student examples

which concerned curricular (in)coherence *across* their mathematics courses as they arose. By the final group discussion, our definitions had more or less converged, as demonstrated by students' mathematical stories which all referred to experiences *across* several mathematics courses they had taken.

A second complexity was that students often went in different directions with how they chose to artistically explore their perspectives on curricular (in)coherence. Some student-artists were *very* explicit about how their artwork related to (in)coherence. Others interpreted the artmaking prompts more abstractly, treating the *making itself* as a metaphor for (in)coherence they used to fuel their reflection on the topic. For example, Joy started with the color of how her mathematics courses often made her feel—blue—while wuyen meanwhile explored how the coherence of the whole depends on its constituent parts.⁴⁷ Even in these cases where connections to curricular (in)coherence may not have been clear (to me) from the art alone, student-artists regularly clarified their intentions and the connections they saw to (in)coherence in their artist statement and subsequent discussions about their art. Therefore, in these analyses, I did not restrict myself to “counting” just the art that *explicitly* or *immediately* appeared related to curricular (in)coherence. Rather, I chose to honor the *many* artistic approaches students took to reflect on (in)coherence, trusting that they shared their art in response to the prompt for a reason.

The final complexity I draw attention to is the *entangled* nature of students' interpretations and valuations for cross-curricular mathematical (in)coherence with (1) their perspectives on (in)coherence in other contexts (i.e., ones they do not consider mathematical) and (2) their views of learning mathematics (i.e., epistemology). This study was deliberately framed using prompts that consistently encouraged student-artists to reflect on the abstract notion

⁴⁷ You will have the chance to further engage with Joy and wuyen's artwork later as you walk through the exhibit (Figures 3.10 and 3.9, respectively).

of cross-curricular (mathematical) (in)coherence through familiar, everyday contexts and examples, such as *stories* (via the curriculum-as-story metaphor). Inevitably, this led students to introduce several points of view on (in)coherence they might not have if they had exclusively been asked about (in)coherence in mathematical contexts. I see this as beneficial and chose not to shy away from including these interpretations in the analyses given the exploratory nature of this study. Attending to these varied interpretations, after all, offers a promising opportunity to re-envision how we might craft cross-curricular mathematical stories that appeal to students' broader aesthetic sensibilities. To keep the analysis focused, though, I attended primarily to the perspectives on (in)coherence that were introduced as more than just a passing comment, including any that were brought up repeatedly by one or more students across sessions *as well as* any that a student expanded upon, even if the perspective is not repeated.

The observation that students' perspectives on cross-curricular (in)coherence were entangled with their personal philosophies of learning mathematics (including their views on what it means to be a "successful" mathematics student) is consistent with a previous conclusion from Chapter 2 that curricular coherence is a value-laden notion which carries with it axio-onto-*epistemological* assumptions. In light of these findings, I did not attempt the impossible analytic task of disentangling student perspectives on curricular (in)coherence from their epistemological views. Instead, I aimed to *contextualize* students' perspectives by simultaneously considering any epistemological, aesthetic, or moral stances they expressed that might help explain their perspectives. The full investigation from Chapter 2 was completed only *after* I conceptualized this study, however, so my contextualization of student perspectives is admittedly limited. Still, consistent with the exploratory nature of this study and the ABR paradigm, I chose to embrace these complexities of entanglement rather than reduce them, engaging in philosophical

deconstruction of student perspectives when possible with an eye to laying foundations for subsequent theorizations of curricular (in)coherence.

Learning from Student Artwork

To radically center students' perspectives, I introduce each of the following lessons I (we) learned about curricular (in)coherence using participant artwork paired with an accompanying artist statement and/or some additional words each mathematical artist used to discuss their art. Each art piece has been carefully curated to engage you artistically in one or more facets of the lesson and “hook” you before you proceed to any subsequent written analysis. This aesthetic structural choice is consistent with the paradigm of ABR—I am viewing art not just as mere data but rather as a standalone product with the power to aesthetically and emotionally impact and transform. However, I am *not* implying that each individual piece of art “represents” an entire lesson or that the art will make complete sense to you before additional context is added—after all, participants created their art primarily for individual and collective conversation among themselves and not with the immediate purpose to exhibit it to anyone else. With this backdrop in mind, please treat each piece as an *invitation* for further exploration and aesthetic contemplation about what each mathematical artist *might* have had in mind, as well as how this sits alongside or challenges your existing worldviews, lived experiences, and perspectives related to mathematical (in)coherence. Think of this as an immersive exhibit—a unique chance for you to be transported back in time to our group discussions in fall 2023, as if you were in the room alongside the mathematical artists as they first learned from each other's art.⁴⁸

⁴⁸ This section has two titular purposes: I share some of what *I* learned from these mathematical artists in the past while also offering *you* the chance to enter a space of learning by engaging with student artwork in your present.

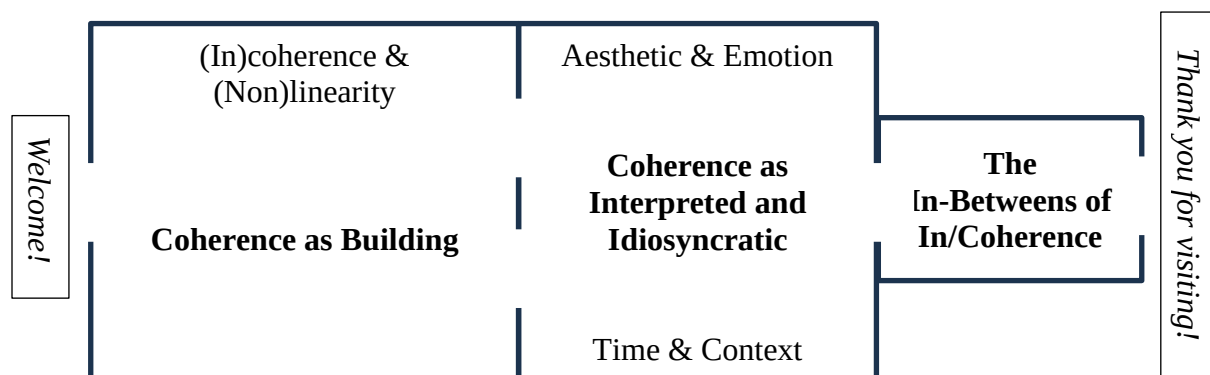
Welcome to the (In)Coherence Exhibit!

Welcome, welcome! I see you have a ticket for our illustrious (in)coherence exhibit. An excellent choice, if I do say so myself! (Though I may be biased given my role in curating this particular wing of the museum alongside our dynamic featured artists...) At any rate! Please deposit your ticket in the box to my right and grab a map of the exhibit—it will be my absolute pleasure to serve as your guide.

As you can see on the map (See Figure 3.4), the (in)coherence exhibit we are about to enter features three rooms that are themed based on recurring topics of discussion between our featured artists. In the first room, I invite you to learn more about the ways these student-artists expressed coherence as being about *building*, attending to differing views regarding the (non)linearity of such building followed by an introduction to how these artists viewed (in)coherence as deeply entangled with what mathematics *built to* (i.e., the purpose of building in the first place). In the next room of the exhibit, we will explore the ways artists positioned (in)coherence as an idiosyncratic, interpretive judgement (or not), with attention to the factors that the student-artists identified as impacting these judgements, including aesthetics and emotions as well as other contextual factors and the notion of time). Finally, I will lead you to the final room of this exhibit, featuring art that highlights our featured artists' conceptualizations of the relationship between coherence and incoherence (often in ways that moved beyond treating these notions as a strict binary). Having said all this, I would like to remind you, as the curator of this exhibit and your humble docent, that any sense of strict division between the parts of this exhibit is often illusory and primarily a limitation of how the exhibit has been arranged—linearly, in a more or less traditional academic format featuring hierarchically arranged sections—rather than an indication of strict boundaries between these views of coherence in the artists' artwork or perspectives on (in)coherence. As an attempt to mimic the overlaps between

Figure 3.4

Map of the (In)Coherence Art Exhibit



the artificial boundaries present in my initial floorplans⁴⁹ and draw attention to these porous boundaries, I do my utmost to point out any works of art which could have been placed in multiple parts of the exhibit.

Finally, I remind you once again that unlike other art museums, here you are *enthusiastically encouraged* to step over the red cordon and stand less than five feet away from the art so you can get up close and personal with it. Please *do* touch the artwork, make sure to turn *on* the flash of your camera whenever you want to take a picture for later, and, of course, *you have my permission* to run through the halls or *not keep up* with the group if you prefer to experience the exhibit in a different way. Right, I think that's all! Any questions before we start the tour?

Coherence as Building

Before we step over the threshold of the exhibit, I direct your attention overhead to the top of the doorframe that leads into the (in)coherence exhibit where we have hung up a quote from VKN, one of our featured mathematical artists: "A lot of people don't experience math the same way I do . . . there are different lenses and different levels of coherence" (Final reflection

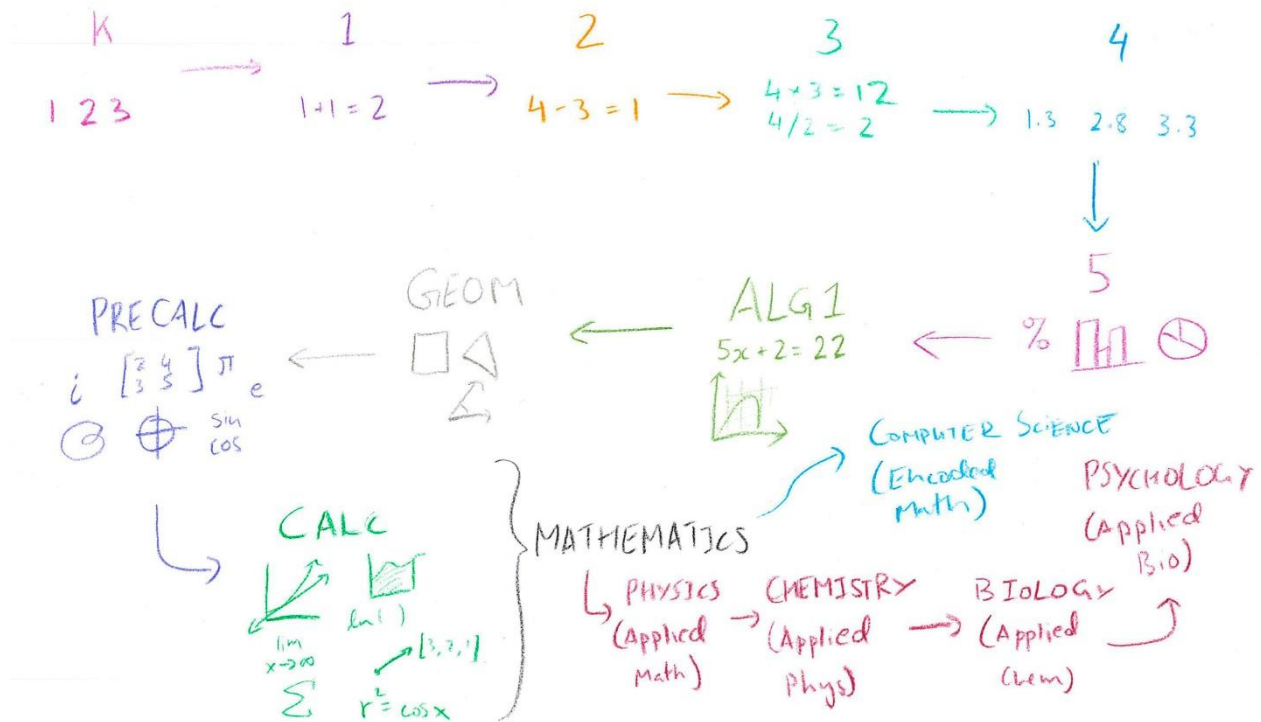
⁴⁹ I.e., my collage analysis

from Group Discussion 2). Like VKN, the mathematical artists featured across this exhibit express varying perspectives and valuations of curricular (in)coherence in their artwork based on their idiosyncratic experiences with learning mathematics and aesthetic sensibilities. As you follow me into the first room of the exhibit, take some time to engage with the three featured art pieces scattered in the entryway. Almost immediately, you may begin to take note of some differing perspectives on (in)coherence.

[Please walk toward the first piece of art on the next page.]

Figure 3.5

All Adding Up by Fred



"As a person, I'm a very conceptual learner. I love mathematics and the fact that each mathematical concept always builds on top of one another. People usually struggle with calculus but I actually really love calculus because it's the first math class where I get to conceptualize all the algebra and trigonometry that I was taught. I also like how it has a lot of practical applications in the real world. That is why I made the art the way it is, because I want to express how all the mathematics concepts we have learned over the years adds up and all makes sense the way it is."

-Fred's artist statement.

[If you turn around, you'll see the next painting behind you. Flip the page.]

Figure 3.6

Connections by VKN



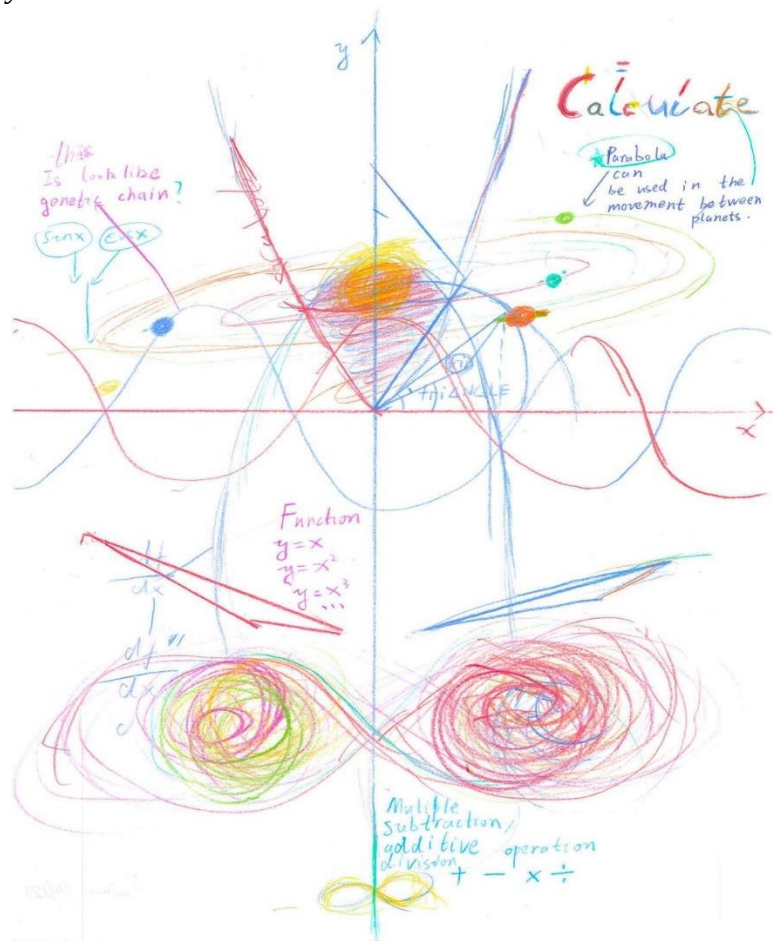
“The initial gray paint on the canvas represents fundamental mathematics, simplified and palatable. From these theories, comes complex connections, higher level thinking that compounds on itself. The pink and yellow and green inviting the learner to test the limits of their imaginations, to venture into theoretics and what-ifs. From this comes industrial applications and scientific advancements - from this comes atomic bombs and quantum mechanics. And yet, the gray is ever-present, a foundational cornerstone.”

-VKN's artist statement

[When you are ready, the final piece of art is just to your right. Please continue there.]

Figure 3.7

Math Universe by GUo3



“The drawing that I drew is based on the cartesian coordinate system with colored pencil. As we know, math can’t leave the coordinate, if you want to describe a function or equation, the best way is to sketch the coordinate to understand. . . . As for me, math put some magic on me, and let me think a lot. Everything about engineering is based on math, and even financial problems.”

-Excerpt from GUo3’s artist statement^{50,51}

“I wanted to do that like a tree. . . . The y is a vertical. For the basement, you have to learn how [to] plus, subtract, divide, or multiply—how they calculate the number and group, group, group. . . . [Trees] need the root to absorb nutrition, right? So it is like math. Math also needs your brain to absorb how to extract your basic math skills to apply [to] the harder, more complicated, math problems. That’s what I have.”

-GUo3’s explanation of his art from Group Discussion 1

⁵⁰ In the artist statements hung throughout this exhibit, I do my utmost to preserve the original spelling and grammar of the artist without the use of corrections in brackets or [sic] notes so as not to editorialize their voice. I only editorialize when necessary for clarity.

⁵¹ Any artist statement not presented in its entirety within the exhibit can be read in full by stopping at our giftshop (“Appendix E”) before you leave the museum.

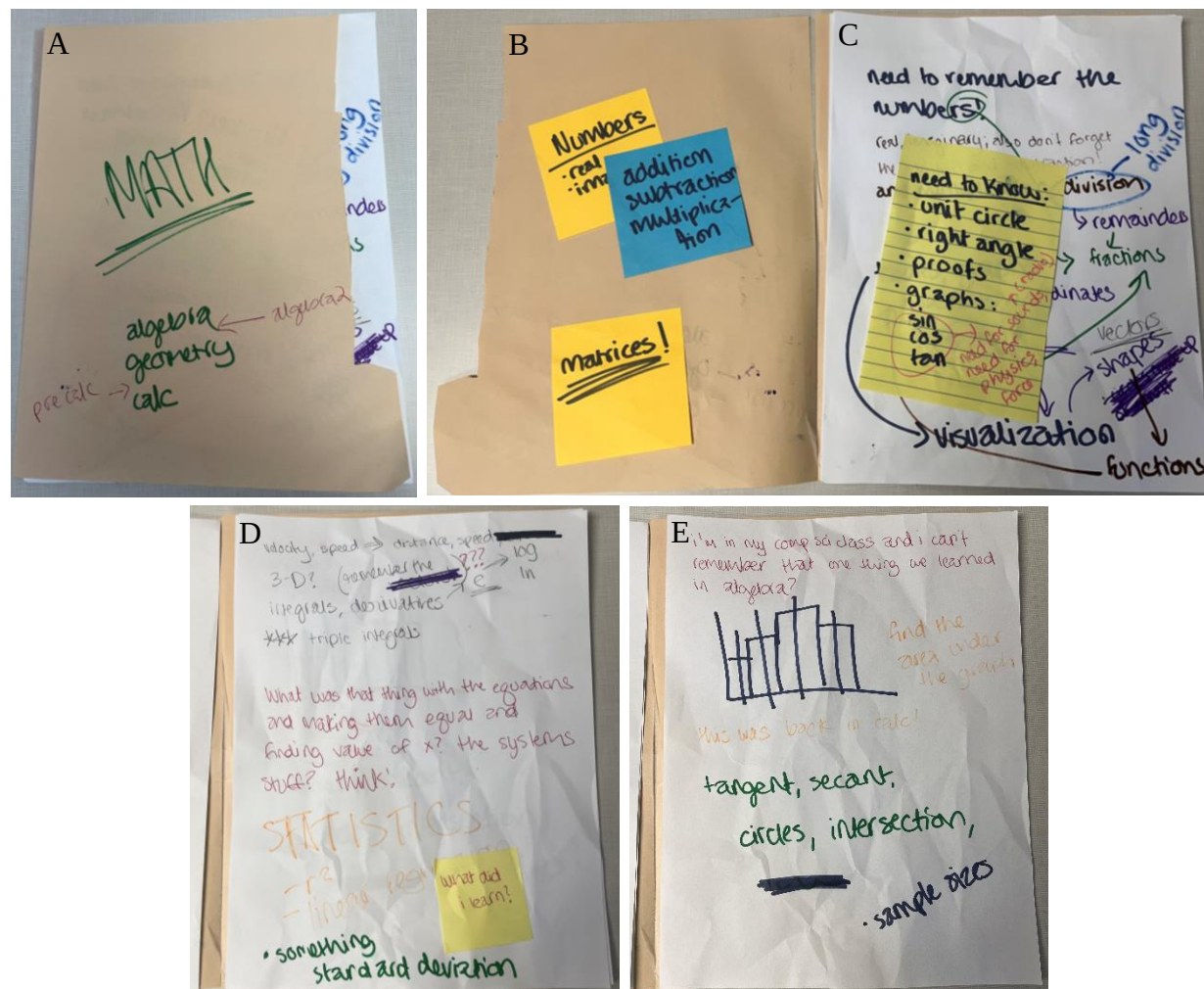
Coherence as a form of “building” was one of the unanimously agreed upon themes identified by student-artists when they came together for their first group discussion. However, each artist expressed unique interpretations of “building” including (but not limited to) outgrowth from basic foundations; crafting a sequence or ordering elements so that they “flow” together and “make sense”; or creating an organized structure. Within this larger room, there are opportunities to engage with these different forms of building split across two interrelated conversations happening in both parts of this room. The first is about the relationships between (in)coherence and (non)linearity, and the second focuses on questioning the *purpose* of coherence—in other words, *what are we building to and why?*

[Please walk toward the spotlighted piece of art up ahead once you are ready to engage with the first part of the exhibit.]

Coherence and (Non)Linearity

Figure 3.8

VKN's Mathematical Story



VKN crafted a file folder art piece using paper, colored markers, and a variety of sticky notes to share her cross-curricular mathematical story. The images above depict the front cover (A), inside cover (B), and the pages of content (C, D, and E). She introduced her story in Group Discussion 2 as follows: "My idea was to make an analogy between a computer file and thinking of your brain as data storage and files. . . . So the first thing is even the folder title, I'm like, 'Wait, what else do I need to know? There was algebra and geometry and calc and what?' The little arrows and comments and things are supposed to be—there's not a lot of linearity in this, but it's like, 'Oh yeah, when I remember it, put it in the file.' It's supposed to be a lot more cluttered. I also meant to crumple a couple of papers to remember, 'Oh, this is super old. I totally discarded it, but I need it now'⁵² It's meant to represent a bunch of haphazard notes put together.

⁵² As depicted in the images of VKN's art piece, she later crumpled all three pages in the folder.

A common view of coherence as a form of *building* expressed by student-artists was that coherence involves sequencing or arranging the elements of a larger whole to create an overarching sense of *flow*. This definition was proposed by CinematicHue when asked about what it meant for something to be coherent in the initial (in)coherence conversation (which also included Fred and GUo3). He responded by claiming that “In a [coherent] story, everything flows smoothly, *so it makes sense*. It is all in order . . . sequential”. He continued a moment later, “In order [for a story] to flow, you need a script (what forms the story). Everything is put together, so it makes sense from every little detail, the costumes to the character and everything”. Fred also took up this definition for coherence within this conversation and beyond. In his art piece *All Adding Up*, Fred created a literal *flow* diagram depicting courses in the K-16 mathematics curriculum as building linearly on top of one another, sharing how it was “about making sense and flowing” while simultaneously expressing how “all the mathematical concepts we have learned over the years adds [sic] up and all make sense the way it is” (Artist Statement). GUo3’s *Math Universe* depicts a similar logico-rational stance toward mathematics concepts building upon one another; however, he placed more emphasis on the advanced mathematical concepts which grow out of the “roots” of what VKN called “fundamental mathematics” in her artist statement (i.e., basic operations, functions, etc.).

In *Connections* (Figure 3.6), VKN takes up a similar stance of building from the so-called “simplified and palatable” (Artist Statement) fundamentals of mathematics; however, her interpretation of *building* is in terms of a “tangled string” (Individual Art Interview) that piles up on top of itself. While technically linear (i.e., one string), the tangled nature of the string and how it piles on top of itself in three dimensions is representative of how VKN is honest about *not* seeing her cross-curricular mathematical experiences as strictly “coherent”. When she described

her art, VKN clarified: “This is the jumble of foundational mathematics that I still haven’t grasped, and yet we’re continuously moving forward and adding more comprehensive topics that still draw back on these topics that I still don’t understand” (Individual Art Conversation). While the curriculum of her mathematics classes *moves forward* (seemingly linearly), her *organization* of the story of these courses continues to become more non-linear, tangled, and confusing to her as time goes on, a reality that she captured in her mathematical story art (Figure 3.8) in which she tangibly depicts the mathematical incoherence she is currently experiencing in the form of a messy folder.

[Exhibit continues on the next page]

Building to What? Coherence and Purpose

Figure 3.9

Doll Up, Dress Up, Beautify by wuyen



“As a young child, I often drew on my imagination to create entire worlds where nothing was out of my reach. Whether it be a box of colorful crayons or crafting with fancy paper, I embarked on exploring the limitless possibilities that come with the act of creating. In the realm of my imagination, I discovered the joys of self-expression. It was a world where I could be anyone and do anything, building the foundation for my journey of self-discovery and growth.

Girlhood is unique to each individual’s narrative which is what I hoped to convey through my origami inspired piece. Girlhood is both diverse and beautiful, with many stories yearning to be heard. This was the reasoning for the varying dresses created to represent the many unique experiences of being a girl. The piece invites viewers to reflect on how this shared human experience shapes our lives.”

-Excerpt from wuyen’s artist statement

“I wanted to connect math with dress up in a way to be the skills that you learn—well, this is tying to a bigger picture—but it is the skills that you learn in your mathematics courses or material that you learn can apply in multiple aspects of your life. And then that’s where dress up comes in, where you can be whoever you want to be. . . . Wherever you want to go for dress up ties back to math because it’s like math will go wherever you want to go.”

-wuyen, Individual Art Conversation

“I hope to convey that math applications can be applied in different, um, all aspects of your life. And I feel like I represented that in the different dresses that I made representing the different careers that a person might be able to apply mathematics to.”

-wuyen, Group Discussion 1

[Exhibit continues on the next page]

Another unanimously agreed upon theme by the artists in Group Discussion 1 as they discussed each other's artwork was the importance of considering what coherence built to—in other words, what the *purpose* of coherence was in the first place for the student going along on the curricular “journey”. In wuyen's *Doll Up, Dress Up, Beautify*, she constructed a closet of three origami dresses which represented future roles she wished to assume that would require learning particular mathematical skills. The white dress, for example, was representative of her occupational goal to be either a nurse or a doctor. The blue dress, meanwhile, represented her desire to one day be a mother, which she also saw as requiring mathematical know-how:

When you're taking care of the household, there's different things that you have to do, lots of jobs around the house. But let's say, for example, cooking. You have to measure your ingredients. You have to do perfect timing and other things and weighing your ingredients as well. There's math in that, so math in all aspects of life. Oh, and gardening too, I guess... (wuyen, Individual Art Conversation)

For wuyen, the *purpose* of cross-curricular mathematical coherence was acquiring skills that could have a larger impact on various aspects of her everyday life. A coherent mathematics experience for her was one that allowed for “limitless possibilities”, as dress-up had when she was a young girl. Indeed, wuyen spoke frequently about how everyday life and activities, no matter how mundane—from playing dress up to brushing her teeth each morning, could be seen as “coherent” because they had a bigger purpose or meaning to play in her day or life.

Ontologically, wuyen often gravitated toward considering the purpose of coherence in relation to how the *parts* of something related to the whole. This view of coherence was even what led her to reflect on curricular (in)coherence *using* origami, which she described as, “a series of folds and patterns to create something great as a whole” (Individual Artist Conversation).

A second interpretation of the *purpose* of mathematical coherence that was mentioned by nearly every artist at one point was applications of mathematics to areas of STEM.

CinematicHue, for instance, argued that a coherent mathematics course would discard mindless

and repetitive tasks or homework in favor of projects with realistic applications and opportunities to learn about modern technologies, like AI. Oftentimes, artists positioned these “extra-mathematical” applications as end goals for mathematical coherence that were “more advanced” than mathematical foundations such as addition and subtraction, integrals, or functions. This point of view is expressed in Fred’s, GUo3’s, and VKN’s artwork (see Figures 3.5–3.7). For instance, in Fred’s *All Adding Up*, the scientific disciplines appeared at the end of his flow diagram after all mathematics courses. Meanwhile, VKN referred to “industrial applications and scientific advancements” like “atomic bombs and quantum mechanics” as the pink or green string, the highest possible level of mathematical thinking in her artist statement.

While almost every artist positioned these types of applications as the most advanced and occurring toward the end of their curricular journeys, Joy reflected at the end of Group Discussion 2 that “it would’ve been important to know right then and there while you were doing it. *Why* is this important? *What* are we trying to achieve here? Because it doesn’t do you much good two years later.” From Joy’s perspective—similar to wuyen’s view of mathematics as a form of dress up—there ought to be a clear sense of purpose for each part of the mathematics curriculum. A curriculum that treated mathematical applications as mysterious and supposedly important end goals that would be learned “eventually” might as well be *incoherent* from the perspective of a student trying to make sense of how one part of a curriculum they are experiencing in the moment relates to grander end goals. For example, VKN questioned, “Where does it all—where does it connect? What does it result in?” (Group Discussion 1). She frequently reflected on how *others* (e.g., Fred or GUo3) seemed to see the connections, but she could only understand them in the abstract as she viewed their artwork. Further, she noted how she did not feel like this in courses outside of mathematics:

We study history to learn international relations, to understand public policy, to understand all of these different topics that connect to the real world and the function of things. Whereas math always seems a little bit—I guess I can also work personally on familiarizing myself with how things happen, but I just don't really understand, I guess on a more fundamental level of, "Okay, in what process or where do triple integral come in in terms of building a spaceship?" (Individual Art Conversation)

Whereas the cross-curricular stories in other courses were coherent and aesthetically appealing for VKN, she frequently positioned the cross-curricular stories of mathematics as an incoherent enigma, almost like a summer blockbuster that everyone she knew loved but that she could never fully understand the aesthetic appeal of.

Coherence as Interpreted and Idiosyncratic

The focus of the next room of the (in)coherence exhibit concerns perspectives involving idiosyncratic, interpretive *judgement*. From some perspectives on mathematical coherence—particularly disciplinary ones—coherence is seen as an “objective” assessment. The featured artists, on the contrary, regularly noted how judgements of coherence are highly subjective. Fred, for example, explained how his experiences across mathematics courses felt “flowy” and coherent, but then added a caveat: “Some people kind of struggle with math and, yeah, we all have our own strengths and weaknesses. I’m not really a good essay writer, so that is probably coherent to someone and that’s not coherent to me” (Individual Art Conversation). Throughout this room of the exhibit, you will find several examples of our featured artists reflecting on the ways in which curricular coherence involves subjective judgement. In the first part of the room, we feature art that highlights how one’s aesthetic sensibilities and emotions influence their judgements of (in)coherence. Meanwhile, in the second part of this room, you will find instances of our artists suggesting that judgements of coherence are contextual and time bounded.

Emotional and Aesthetic (In)Coherence

Figure 3.10

The Smoke Trail by Joy



The smell of ink on paper. The emotion tied with flipping a page. The worlds brought to life. Growing up, I have always loved literature. Reading works of fiction brought a warmth to my life much like sitting by a fire on a cool night. Subjects like mathematics, on the other hand, have often left me feeling blue, as though I could see the smoke trail leading to a warm fire, and I had no clue how to actually get there. It should be as simple as following the smoke or path laid out for me. Yet I felt like when I would take a step in that direction, the smoke would come into my lungs and choke me out as opposed to gently guiding me along. . . . On days that mathematics made sense, I loved it. There was satisfaction in being able to successfully follow the smoke trail or build the fire. On the days that it didn't, however, I felt lost in an abyss of blue. When it got bad enough, I'd feel as though I had been left under water and the top surface had frozen under me. Those were the times I thought I could never catch up or make it to the fire, and it was difficult not to give up in those times."

-Excerpt from Joy's artist statement

"One thing just about the piece itself is that I can't say it was fully coherent to me. Even in the beginning, I didn't know what I was really making. I started with the color blue because that's how it made me feel. . . . isolated. Sad."

-Joy, Individual Art Conversation

[Exhibit continues on the next page]

Joy's *The Smoke Trail* is perhaps one of most evocative pieces we have on display in this exhibit, particularly with regards to emphasizing the role that emotion can play in some students' judgements of mathematical (in)coherence. Much like VKN alluded to earlier, in her art, Joy reflected on how she frequently saw the seemingly coherent "fire" of the mathematics curriculum but would often struggle to reach this fire in practice (i.e., to interpret her mathematics experiences as fully coherent), even when she had a smoke trail leading her to it. The deceptiveness of the smoke trail and how it "would come into [Joy's] lungs and choke [her] out as opposed to gently guiding [her] along" (Artist Statement) were part of why her entire painting takes place next to a frozen lake. As she shared in her Individual Art Conversation, "I started with the color blue because that's how [mathematics] made me feel. . . . isolated. Sad." The frozen-over blue lake that dominates the foreground of Joy's art emphasizes that judgements of mathematical coherence for her are governed by a looming sense of sadness, as well as other emotionally painful memories that she likened to being choked, drowned, or burned alive. In particular, she likened her emotional struggle with coherence seeking in mathematics as akin to an episode of the animated TV series *SpongeBob SquarePants* (Lender et al., 2002) where

There's lots of filing cabinets on fire and there's just a lot of SpongeBobs running around. Whenever I don't understand something, my brain just imagines that what's going on inside my head is that everything's on fire and there's little "me's" running around everywhere trying to fix everything. (Individual Art Conversation)

For Joy, mathematical coherence seeking can be a panic-inducing experience that is inherently emotional.

For wuyen, emotion also played a leading role in interpreting whether a story (mathematical or otherwise) was coherent to her. In the Initial (In)Coherence Conversation, for example, she shared that for a story to make sense to her, it must also make her feel: "I think

emotional is a big component for me to make things make sense because I'm a very emotionally driven person, more than logic for sure" In particular, she found Taylor Swift's discography to be coherent because of the emotional stories told in each of her songs and albums. She also expressed a similar view of mathematical coherence when I introduced the curriculum-as-story metaphor in Group Discussion 1:

The different applications of math as a story was [*sic*] very interesting because to me during our first conversation, we were kind of talking about how stories make sense to me if it was emotionally touching in some way to me. So, applying that to the bigger idea now of the whole math experience being a story, I feel like emotions still should play a part. Well, not *should*. For me, it *does* play a part in the math journey to make it make sense.

wuyen expressed a clear aesthetic preference for stories that involved emotion and made her feel. For her, *these* stories were coherent. However, she was quick to clarify that she did not like to feel *afraid* and therefore avoided horror movies, which she did not consider to be coherent. Aesthetic and emotional sensibilities, in this way, can go hand-in-hand with judgements of (in)coherence. Similarly, Fred explained his love for math as similar to his love of Disney animated movies, concluding, "I'd say math is kind of similar, *if* you care about math. I kind of care a little too much about math. Other people don't give a shit about it" (Initial (In)Coherence Conversation). In both these cases, wuyen and Fred allude to the inseparable relationship between aesthetic sensibilities, emotion, and preferences for particular kinds of (curricular) stories.

[Exhibit continues on the next page]

Coherence, Time, and Context

Figure 3.11

Joy's Mathematical Story



Joy crafted her art piece with multicolored sticky notes and pencil drawings to share her cross-curricular mathematical story. The full story (A) as well as two sequences of sticky notes taken from the beginning (B) and near the end (C) of the full story are displayed above. The sticky note color designates whether it represents a moment in the future (orange), the past (yellow), or the present (blue). Joy explained her story further in Group Discussion 2:

“One thing I was trying to get across both in the sticky notes as well as the content of the sticky notes is repetition. . . . I also wanted to iterate that these repeating concepts come across in math courses that may come across in circumstances that happen in your life. And I guess this way of storytelling is also very nonlinear, just like that is I do jump around a lot, so that's what my mind feels like.

So, my first [sticky note, top left of image B] is the future, and it's just amazing how I've learned how everything can all come together. I actually got that inspiration from a couple of [the pieces of student art] I saw today. [In painting my art prior piece, see Figure 3.10], I was focusing on [how] I would get really confused a lot of the time. But there's also a lot of people that took away all the things they can build from their math concepts or things that they can create from learning [math]. So that's something I also tried to get across. So, in 2015, I'm in algebra class learning logarithms, and I'm just sitting there. I don't get it and I'm like, ‘When will I really need it?’ Then next year in chemistry class we're supposed to know logarithms, and I'm like, ‘I don't understand them,’ and I'm telling my teacher that, so he tries explaining them to me: ‘Does that make sense?’ And I'm like, ‘A little bit more but not really. I still just don't get it.’ Then we come to the present and before coming here I was just taking a chem exam, and I actually had the thought while I was plugging [it into] my calculator, ‘I'm glad I understand the logarithm enough now to be able to do a half-life problem—that came in handy now. So, then I jumped back to 2019 after a calc class in college and I'm like, ‘it took a few years, but I finally understand logs now’, and it's back to the present and chem class: ‘I still can't say I understand chemistry even after three years. . . . Coming back to present, I got into soap making very recently, and I just had this thought last week. I was like ‘Soap making makes chemistry fun because I never thought there was anything I could find fun with chemistry. And so maybe some parts of chem do make sense.’ You have to take measurements of the oils, and you also have to get the right measurement of lye and water and quantify the oil or you won't have a safe soap bar. So, then I had a future with the homemade soap that ‘oh wow, it's great how everything can come together and it feels really nice when it all makes sense.’ Then I jumped back to 2014 . . .”

-Joy, Group Discussion 2

[Exhibit continues on next page]

Several artists expressed how judgements of coherence are contextual and influenced by time. For instance, the first definition that VKN proposed for coherence in the Initial (In)Coherence Conversation was *accessibility*. Shortly afterward, she suggested that inequities in schooling based on the area code one lived in were a prime example of *incoherence* from her perspective. She explained:

You still have to teach [the same] topic to people that might not be coming in with the same background So, I guess [coherence is] not necessarily even about the topic itself or the work or whatever it is, but there's outside socioeconomic factors that make things inaccessible.

Similarly, when Joy added further context to her painting, she claimed that

Environment also plays a role in how well a fire can be built. A person who is asked to build a fire on a dry, heated day might have better success than a person trying to build one in the rain. When my life met difficult times, I could not understand how I could possibly be asked to do certain mathematical tasks successfully

Both VKN's and Joy's quotes stand alongside several other examples of artwork in this exhibit to help clarify that for many coherence is *contextually bound* and involves much more than just "the content" of a story or curriculum. Aesthetics, emotion, and even *time* can impact judgements of coherence. On the topic of *time*, multiple artists interpreted coherence (seeking) as a reflective process that occurs in time. wuyen, for example, suggested that mathematics students ought to have several opportunities for sustained reflection *over time* that would allow them to contemplate cross-curricular stories that span multiple mathematics courses. Meanwhile, several other artists hinted that coherence seeking was something which required the passage of time and occurred iteratively. Joy, for instance, proposed that "coherence is something that comes together in the end" (Individual Art Conversation).

Joy's sticky note art (Figure 3.11) serves to further highlight how interpretations of coherence are not fixed and may change with time. The story she told in her sticky notes frequently involves moments from the past, present, and future intermingling in a highly non-

linear order to demonstrate the complex relationship Joy saw existing between time and mathematical coherence. In particular, she drew a comparison between the TV series *The Haunting of Hill House* (Averill et al., 2018) and her sticky note story in order to share her perspective on curricular (in)coherence:

[The Haunting of Hill House] jumps between time a lot. And so I think originally when I think of coherent, I think of something that flows well together, but I don't think it necessarily always has to be linear, and there's moments where it's incoherent. You don't know what's happening yet, but as the story progresses and all the pieces come together, it forms an entire story and you're like, okay, at the end it's coherent, but all those steps along the way, you just didn't know what was happening. (Group Discussion 2)

According to Joy, sometimes the iterative nature of coherence seeking can act in unexpected ways. She added that sometimes the flow of time can even feel like it has halted entirely, resulting in a moment where “nothing is coherent” and “nothing makes sense” (Individual Coherence Conversation).

[Exhibit continues on the next page]

The In-Betweens of In/Coherence

Figure 3.12

Fire in the Firmament by CinematicHue



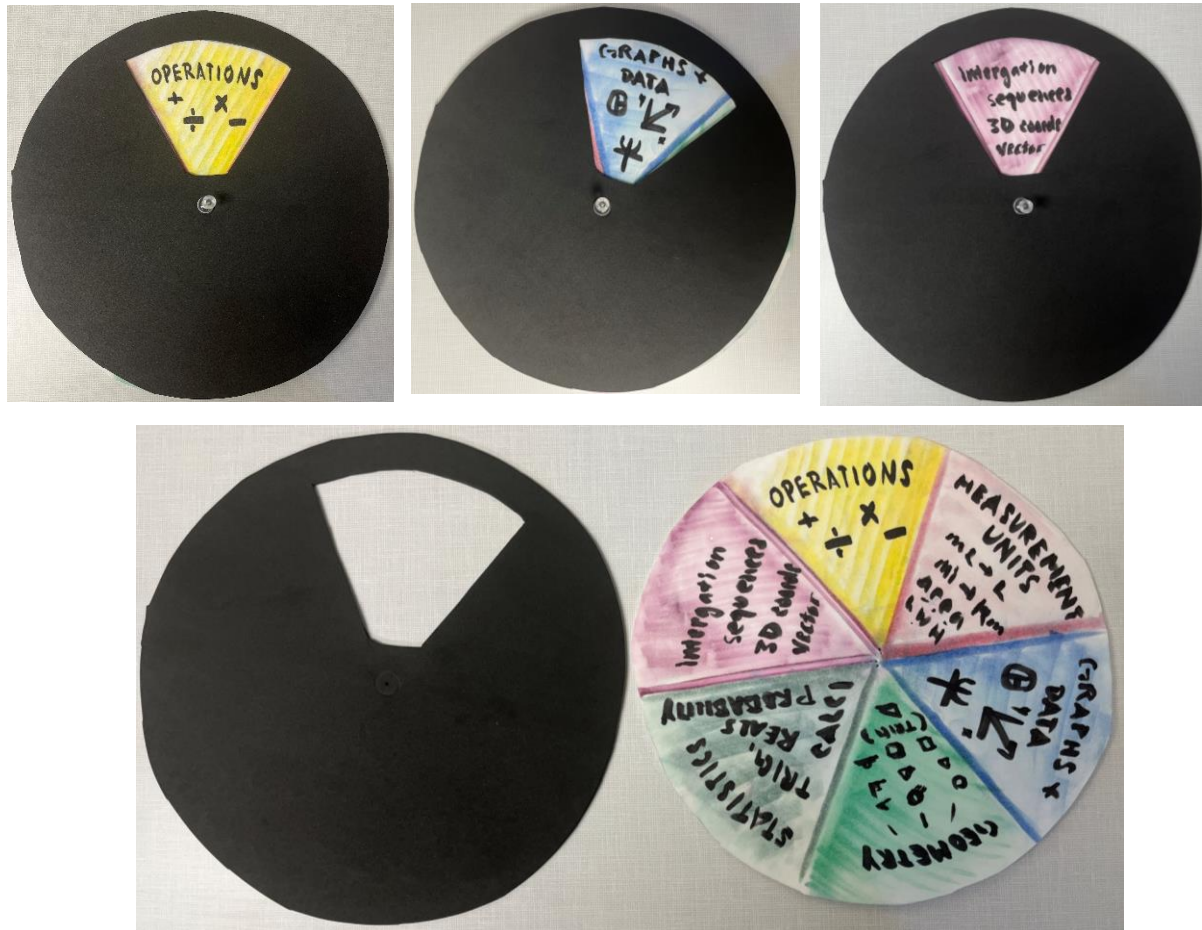
“Inspired by the myth of Prometheus, this artwork delves into the dualistic nature of knowledge. Much like Prometheus stole fire from the gods to give to humanity, knowledge too can be a gift or a curse, depending on its application. The painting aims to convey this nuanced understanding. The elements of lightning and fire serve as powerful symbols in the artwork: they are simple yet evoke a range of interpretations, capturing the essence of what I aim to communicate— that knowledge is both enlightening and dangerous, capable of creation and destruction.”

-Excerpt from CinematicHue’s artist statement

[Exhibit continues on the next page]

Figure 3.13

wuyen's Mathematical Story



wuyen crafted a spinning wheel using foam sheets and colored markers to share her cross-curricular mathematical story. The top images show the wheel at its start (“operations”), third phase (“graphs + data”), and end (calculus, “where we are today”). The bottom image depicts the top and bottom layers of the wheel deconstructed. She told her story in Group Discussion 2 as follows: “Based on everybody’s experiences from [Group Discussion 1], I feel like for some people math makes sense to them, but, to me, not really so much anymore. Some people, they look at this circle and it all makes sense to them, but for me, sometimes not. Sometimes for me in order to—not even just grasp the concept—just to try my best to stay afloat, I have to look at some things through a lens and hyperfixate on them in order to understand them. But, at the end of the day, they are a part of one circle, and they all tie back into each other. And then the wheel was to kind of represent how it’s a never-ending cycle I feel like. So it just keeps going and going and then everything ties together like that.”

[Exhibit continues on the next page]

When the featured artists first came together to create the artwork for this exhibit, many of them seemed to see mathematics as inherently *coherent*, with the onus being exclusively on them to “make sense of” this mathematical coherence. However, by the time they reached the second group discussion, all of them appeared to have arrived at the realization that *incoherence* regularly permeates the mathematics curriculum and may even be a natural part of any learning experience. This *final* room of the exhibit dives into the gray area in-between coherence and incoherence, the non-binary space where many artists found themselves contemplating at the conclusion of this reflective experience.

One possible “in-between” is expressed in CinematicHue’s *Fire in the Firmament*, featuring a watercolor depiction of the myth of the Greek god Prometheus giving fire to humanity. CinematicHue explained, “Fire can destroy a forest . . . but then at the same time there’s some situations where it can be used for good” (Individual Art Conversation). He continued to explain that *mathematical knowledge* is similar to fire in that it can be used in ways that benefit or harm society. In his painting, CinematicHue frames this ethical duality against the backdrop of a cliffside—a purposeful gray area. As recounted earlier in the exhibit, Joy expressed a similar duality in her discussion of how fire (i.e., mathematical coherence) can either provide warmth or burn down the forest. Despite this potentially dangerous duality, Joy expressed how she felt like a moth drawn to the flame, compelled to seek mathematical coherence, despite its potential to harm her.

Another “in-between” and the final one of today’s exhibit is expressed metaphorically in wuyen’s wheel art (see Figure 3.13). Using her wheel, wuyen shared that she had a difficult time seeing a “coherent” cross-curricular mathematical story because she found herself hyperfixating on topics from within one course or unit, a form of what she called “tunnel vision” where all

other aspects of her mathematics experience necessarily became shrouded in darkness as she focused on learning new ideas. Next, she removed the black layer, acknowledging that “at the end of the day, [the sectors] are a part of one circle, and they all tie back into each other”. Despite her clear struggles with navigating this mathematical tunnel vision, wuyen concluded with a message of acceptance of the never-ending *cycle* of coherence seeking that is necessarily punctuated with painful moments of navigating incoherence and darkness: “The wheel was to kind of represent how it’s a never-ending cycle I feel like. So, it just keeps going and going and then everything ties together like that.” Both VKN and Joy’s reactions to wuyen’s mathematical story demonstrated their own stances toward embracing a complex, non-binary dialectic of curricular in/coherence they had been experiencing across their mathematics courses. Joy, on one hand, came to embrace the messiness of living with both coherence and incoherence: “I think life is always the ups and downs and feeling like you figured one thing out and now ten things are just opened up and you’re like, okay.” VKN, on the other hand, appeared to come to terms with her complicated relationship to mathematical coherence:

You know what? Just because math is not necessarily my strong suit and it is someone else's, that doesn't necessarily mean that I'm any less valid or smart. Everybody has their own talents, and it's not the end of the world if you don't understand [mathematics].

Discussion

In the prior section, I exhibited six STEM students’ interpretations and valuations of (mathematical) (in)coherence, answering the first half of RQ1. Though each of these student artists *valued* coherence and continually agreed that mathematics curricula ought to be coherent, not everyone valued the same *forms* of coherence. I next take a moment to synthesize these various perspectives on (in)coherence and situate them in the context of the existing mathematics and science education literature. To answer the second half of RQ1, I begin by noting various student perspectives that were similar to forms of (in)coherence present in the literature and

conclude by highlighting perspectives that deviated from what is present in the literature.

First and foremost, in line with disciplinary perspectives on coherence that espouse logico-hierarchical views of cross-curricular coherence (Cuoco & McCallum, 2018; Schmidt et al., 2002), almost *every* participant treated curricular coherence as a form of *building*, with many focusing on the construction of “logical” sequences or organizations of mathematical ideas (Fred, GUo3) that “flowed” and “made sense” (CinematicHue). Additionally, most students attended carefully to the sense of coherence they perceived between the mathematical skills they were learning across the curriculum and the *purposes* they hoped to use these skills for in their eventual careers (e.g., wuyen). Though much less present in the mathematics education literature on curricular coherence, Muller (2009) speaks briefly about “occupational coherence” and its importance when education is meant to prepare future professionals. A focus on this form of coherence also runs across the literature on mathematics teacher education program coherence (e.g., Nguyen & Munter, 2024). While logic and practicality were certainly involved in most students’ perspectives on (in)coherence, several students also regularly alluded to *aesthetic* and *emotional* factors as key to their idiosyncratic judgements of coherence (e.g., Joy, wuyen). Though much less present in the mathematics and science education literature on curricular coherence, Dietiker’s (2015a) curriculum-as-story metaphor holds that holistic interpretations of curriculum—including about its (in)coherence—fundamentally involve an interplay between both logical *and* aesthetic factors. Beyond mathematics education, recall from Chapter 2 that Thagard (2000) proposed *emotional coherence* as a consideration individuals taken into account when considering the feasibility of their belief systems. Finally, students regularly expressed that establishing coherence for themselves was a dynamic, ongoing process that continued over time (e.g., Joy, VKN, wuyen), which aligns with perspectives in the literature suggesting that

designing coherent curricula is an ongoing process that must be regularly revisited (Bateman et al., 2009; Honig & Hatch, 2004).

At the same time, many other students' perspectives deviated from forms of (in)coherence reported on in the mathematics and science education literature. For instance, though all students saw mathematical coherence as a form of *building*, not all participants expressed the strictly linear and hierarchical perspectives consistent with forms of disciplinary coherence. For instance, each of the mathematical stories that Joy, VKN, and wuyen crafted during Group Discussion 2 expressed curricular aesthetics that *eschewed* linear building and hierarchical relationality between mathematical concepts and experiences, embracing instead different story structures consistent with non-linear philosophies of time and amorphous conceptualizations of relationality. Simultaneously, these student-artists regularly reflected on how the cross-curricular mathematical stories they had experienced (that privileged disciplinary forms of coherence) were not as coherent *to them* as they were to other students. Though, consistent with Han et al.'s (2020) notion of *retroactive coherence*, some of them admitted that their courses did begin to feel more coherent with the benefit of hindsight.

Moving along, another unexpected perspective on coherence was introduced by CinematicHue, who primarily contemplated the coherence of his past mathematical curricular experiences by reflecting on the dualistic nature of mathematics—as a tool which can simultaneously do great harm *or* provide immense benefit to humanity. But he was not the only student to bring up the *ethical* ramifications about what is considered “coherent” or “sensible” mathematics for students to learn—Joy, in particular, discussed this topic at length. Though the “goodness” of mathematics was an important consideration for multiple students when considering curricular (in)coherence, conversations about the ethics of curricular coherence

remain relatively scarce in the mathematics education literature. One final perspective on (in)coherence which deviated from those germane to the literature was wuyen's *holistic* interpretation of mathematical coherence. Unlike most other participants, wuyen would often contemplate how even mundane (and seemingly *incoherent*) parts of learning mathematics might actually be considered coherent if they had even a small role to play in the bigger picture. She talked, for instance, about brushing her teeth as being a "coherent" activity because it is how she remains healthy long term. In this sense, wuyen considered a *much* wider context than just her experiences in mathematics class when making judgements about mathematical (in)coherence. This perspective shares a striking resemblance to East Asian conceptualizations of "everyday" aesthetics (Saito, 2007) but deviates notably from the mathematics education literature on curricular coherence that very rarely takes into consideration students' wider, everyday contexts. Instead, most of the literature tends to favor a focus on the *larger discipline of mathematics* and/or close attention to the *text* of a given curriculum.

Some Immediate Implications for Curricular Coherence Research

In light of these diverse student perspectives on curricular (in)coherence and particularly those that deviate *and even clash* with those espoused in the literature, the potential harms of privileging just one strict view of (in)coherence become apparent. For starters, adherence to just *one* curricular aesthetics of (in)coherence (such as disciplinary coherence) can *never* hope to capture the idiosyncrasies that individuals attend to as they engage in their unique (in)coherence seeking processes. Worse, as detailed in Chapter 2, a devotion to just one set of aesthetic values can enforce an exclusionary politics of aesthetics (Rancière, 2000/2004) that convey harmful messages about who/what is (in)coherent (Appelbaum, 2010; Buchmann & Floden, 1991). This phenomenon was on full display across Joy, wuyen's, and VKN's curricular reflections and critiques, where they often positioned themselves as struggling or incapable as they strove to

interpret the cross-curricular mathematical stories they had engaged with as coherent. Recall, for example, Joy's hauntingly evocative artist statement for *The Smoke Trail* (Figure 3.10) in which she recounted how seeking disciplinary coherence across her mathematics courses has led to her repeatedly feel as if she was being choked or even burned alive by the *supposedly* ideal "fire" of (disciplinary) coherence. As a reminder, *all* three of these participants were highly successful mathematics students in the grand scheme of the K-16 mathematics curriculum given that they had successfully completed most of the undergraduate calculus sequence and all preceding prerequisite courses. Still, I listened repeatedly as they expressed perpetual frustration about "not seeing" the (disciplinary) curricular coherence across mathematical stories expressed in Fred's and GUo3's artwork. Though each of these students eloquently expressed highly idiosyncratic perspectives on curricular (in)coherence that deviated from mathematical disciplinary coherence, they appeared to have internalized that disciplinary forms of coherence were "more desirable" and "more correct" than their own. It was not until the final group discussion that each of these student-artists started to challenge these deficit views they held about their own aesthetic judgements, as noted toward the end of the exhibition. This appeared to be a positive and perhaps even empowering outcome for these participants. But what about the other students navigating these exclusionary politics of mathematical aesthetics without sufficient space for critical reflection?

While the culture of exclusion (Louie, 2017) in mathematics classrooms is not something that will be solved overnight or with just one change, there are some small steps that can be taken to mitigate these harms in future curricular coherence research. As I noted in Chapter 2, we can remain open to *several* perspectives on curricular (in)coherence, rather than strictly espousing one perspective. I argue this can be accomplished, in part, by explicitly

acknowledging the fundamentally *aesthetic* nature of making judgements about holistic qualities of curricula and curricular stories. This messaging makes it clear that coherence seeking is *not* an objective evaluation but an idiosyncratic judgement based on one's personal aesthetic sensibilities. In other words, not everyone will find the same curricular story to be "coherent". While the definition of "coherence seeking" offered by Sikorski and Hammer (2017) does much to shift power into the hands of students to make their own judgements about what is coherent, it still positions this activity as a mostly *cognitive* act. Almost nothing is said about the *emotional* and *aesthetic* forces that influence coherence seeking (as demonstrated across students' perspectives in this study). Future research, therefore, ought to more explicitly embrace (as well as further investigate) how affective forces and other value judgements (including *aesthetics*) influence and *drive* coherence seeking. As past research in mathematics education repeatedly demonstrates, such aesthetic forces play a highly non-trivial but often overlooked role in students' mathematical activities (e.g., Fiori & Selling, 2016; Jasien & Horn, 2022). It is high time that we fully acknowledge how these personal subjectivities impact judgements of mathematical coherence and, in doing so, begin to rid our discipline of the myth that mathematics is objective, pure, or immune to harmful politics of aesthetics.

A second clear takeaway from this study is an empirical affirmation of the theoretical observations noted in Chapter 2 about the fundamentally complex, non-binary, and dialectic nature of in/coherence. As the final three participants expressed while they reflected across the mathematical stories they had crafted, *incoherence* need not be a dirty word and *many* times it can be a powerful force for learning and aesthetic curricular engagement (Appelbaum, 2010; Irwin, 2003; Richman et al., 2019). Though coherence seeking research primarily privileges *coherence*, these perspectives espoused by students suggest research in this area ought to

simultaneously consider students' *incoherence* seeking to avoid missing crucial forces that impact their coherence seeking activities. In other words, research in this area ought to remain critical of the all-too-common implicit assumption that coherence and goodness go hand in hand to avoid reifying a reductive binary aesthetic of (in)coherence which feeds the exclusionary politics of aesthetics discussed earlier.

A final lesson researchers focused on coherence can learn from these student perspectives concerns the complex relationship between coherence (seeking) and time. Students repeatedly expressed how coherence seeking was intertwined with time, often in ways that eschewed strictly linear notions of temporality that most extant literature on curricular coherence (implicitly) assumes.⁵³ Joy, for instance, shared her sticky note story (recall Figure 3.11), where she reflected on her coherence seeking experiences across years of her life, switching back and forth between the past, present, and future to explore the non-linear, twisty temporal path along which she finally came to see certain aspects of mathematics as coherence (e.g., logarithms). Meanwhile, wuyen's mathematical story (recall Figure 3.13) featured a cyclic conception of temporality, whereby forces of coherence and incoherence seeking remained in perpetual balance. These students did not see time as an isolated force, however, and they often spoke about how *emotions* and other factors in their lives impact their perception of the flow of time. At one point, Joy remarked how "sometimes nothing is coherent", such as during the years of the COVID-19 pandemic, when she felt like "nothing made sense" which contributed to a sense of time being frozen.

Though uncommon in the curricular coherence research I reviewed, there is an emerging

⁵³ For example, even the flexible curriculum-as-story metaphor I use throughout this dissertation to conceptualize (in)coherence assumes a certain linear sequentially and therefore passage of experiential time! See Chapter 5 for further reflections on possible tensions that arise from adopting theories of narrative interpretation that feature these assumptions about temporality.

body of education research featuring critical contemplation regarding how assumptions about temporality influence our conceptualizations of curriculum and education research more broadly (Cole et al., 2024; Mikulan & Sinclair, 2023). Within this body of literature, Gerth van den Berg (2024) has argued that “research in curriculum and education studies might instead respond to a time that is *felt* and *sensed* instead of a time that inevitably ticks on” (p. 249, emphasis in original), mirroring the “intuitive” sense of time Joy alluded to where her emotional state impacted her perception of the flow of time (see also A. S. King, 2021). Similar to students’ varied perspectives about the relationship between (in)coherence and time, this body of research on time often destabilizing and deconstructs ever-present (implicit) assumptions about time’s arrow as always proceeding forward linearly, exploring instead alternative conceptualizations of temporality and how these might lend themselves to alternative curricular and educational futurities (Blumenfeld-Jones, 2024). The nuanced perspectives students expressed concerning the relationship between (in)coherence seeking and time suggests that future curricular coherence research should draw on this time research to present more fleshed out accounts of possible views of (in)coherence, including (in)coherence seeking.

I end this section with a clarification: I am *not* suggesting that any of the above facets should entirely usurp dominant views of coherence seeking as a cognitive activity governed by primarily logico-rational considerations. Rather, I am arguing that *multiple* factors and perspectives, such as aesthetics, must be included in our research on curricular (in)coherence if we hope to diffuse the exclusionary politics of curricular aesthetics that is currently in place in mathematics education. Consideration of many philosophical factors—axiological, ontological, and epistemological—also serves to re-iterate how (in)coherence seeking is not a solely epistemological endeavor. As Eisner (1985) remarked:

Alfred North Whitehead once commented, ‘Most people believe that scientists inquire in order to know. Just the opposite is the case. Scientists know in order to inquire.’ Scientists, Whitehead believed, are drawn to their work *not by epistemological motives but by aesthetic ones*. The joy of inquiry is the driving motive for their work. Scientists, like artists, formulate new and puzzling questions in order to enjoy the experience of creating answers to them” (p. 27, emphasis my own)

Implications for Representations of Student Learning

In the next two sections, I turn to my second research question and reflect on the possible implications of these students’ interpretations and valuations of (in)coherence have for how we conceive of curricular design and student learning in mathematics education. As noted in the previous section, student-artists repeatedly expressed *nonlinear* conceptualizations of coherence seeking (and time) and also acknowledged in/coherence as a complex *dialectic*, noting how *incoherence* can also be a productive force for learning. Collectively, these students’ perspectives on (in)coherence suggest an image of “the process” of learning that is *far* from a linear affair that need not proceed strictly forward in time. Recall, for instance, how wuyen depicted her course taking journey as a circle, depicting learning as an inescapable and never-ending cycle of in/coherence. VKN, meanwhile, depicted her coherence seeking using a messy, highly non-linear folder with any new mathematical sense-making depicted as sticky notes that are constantly in motion, being moved around the folder, added, discarded, and then possibly added back once again.

Representations of learning present in much of the mathematics and science education literature, however, are largely linear, featuring an “arrow of time” conceptualization of temporality. The dominant such representation in mathematics education research is *learning trajectories* (Ellis et al., 2016; Fortus & Krajcik, 2012; Myers et al., 2015), which are typically depicted as linear (or multi-linear), metaphorically implying that learning is always progressing forward. Meanwhile, curriculum is often mapped out in either a linear or perhaps a spiral

fashion, conveying (at least implicitly) a rather linear view of how students might interact with it. Though simplified representations have their use, there is a danger that such simplified imagery might inadvertently convey reductive views of learning that no longer model the highly complex and non-linear process of learning (as depicted in students' artwork on (in)coherence from across this study). Indeed, Davis (2008) observed how these linear representations of learning have come to influence the ways that some expect learning ought to occur, often with problematic effects: "in the desire to pull learners along a smooth path of concept development, we've planed off the bumpy parts that were once the precise locations of meaning and elaboration. That is, we've created obstacles in the effort to avoid them" (p. 84). Students' perspectives on learning and (in)coherence presented in this study, therefore, serve as a potent reminder that researchers must be critical of any simplistic depictions of learning. Further, they encourage (and perhaps even support us with) crafting learning theories and alternative representations that display the innate complexities of the learning process (see e.g., Pirie & Kieren, 1994).

Future Possibilities for Reframing Curricular (In)Coherence Using the Curriculum-as-Story Metaphor

Moving on to the issue of curriculum, in this section I consider different possibilities for how we might conceptualize curricular mathematical stories (using the flexible curriculum-as-story metaphor) based on the aesthetic sensibilities students expressed in this study. In other words, building on my contention at the end of the last chapter that we ought to conceptualize curriculum through a multiplicity of story forms to remain open to various perspectives on (in)coherence, I wonder: *which story forms might we consider as templates for contemplating possible curricular futurities?* Specifically, I consider different story forms that might be compatible with the varied aesthetic sensibilities students expressed across this study and then speculate on what these story forms might teach us about new possibilities for curricular

(in)coherence that transcend those consistent with status quo perspectives of disciplinary coherence. As expressed previously in the exhibition, student-artists unanimously valued using the familiar context of *stories* to reflect on their cross-curricular experiences. VKN, for instance, affirmed the value of considering the mathematics curriculum through the lens of a story, while simultaneously noting wryly how the mathematical stories she had engaged with had not always been easy to follow:

My experience with the story [of mathematics] is not as laid out as [Fred's *All Adding Up*, recall Figure 3.6]. It's comparing reading *Percy Jackson* and keeping the characters straight versus reading *Game of Thrones* and trying to keep the characters straight, y'know? But, I definitely can totally see [mathematics like] seeing a story. (Group Discussion 1)

In this section, my goal is to use the power of story that even students acknowledged to reflect further on story structures—different cultural traditions of storytelling, genres of story, and other possible presentations of stories—that might be used to productively conceptualize curriculum. Specifically, I contemplate what we might learn from contemplating story forms that admit aesthetics of (in)coherence compatible with those expressed in student-artists' mathematical stories from Group Discussion 2, such as non-linearity, or a cyclic, never-ending nature.

It is no secret that literary stories frequently flirt with non-linearity by relaying events to the reader in a non-chronological order, often with the goal of increasing aesthetic engagement (e.g., building narrative tension or immersion, misleading the reader) or forwarding narrative themes.⁵⁴ Similarly, though many (Western) stories follow a linear or cyclic structure (Aristotle, 350 B.C.E./1995; Campbell, 2009; Freytag, 1990), often featuring a clear beginning, middle, and

⁵⁴ One example of a narrative that leverages non-linearity to great effect to forward its themes is the TV series *Pachinko* (Hugh et al., 2022–Present) adapted from Min Jin Lee's best-selling novel of the same name. This historical epic tells the tale of four generations of a Korean family who immigrate to Japan in the early 1900s during Japan's colonization of Korea. While the novel follows a strictly linear chronology, the TV adaptation deftly blends the origin story of the matriarch of the family beginning in 1915 with the struggles of her grandson in the 1980s in ways that spotlight the deleterious and often covert impact that forces including systemic racism, colonialism, and generational trauma exert on their family tree.

end, this is not a requirement. Many indigenous cultural forms of storytelling, for instance “may begin with the ending or seem to start in the middle, then move to the beginning of the story. Oftentimes, [American] Indian stories don’t really end, but continue for a lifetime” (Jones, 2021, para 8). Such story forms are highly non-linear, amorphous, and constantly changing, rather than adhering to a strict story structure or template. Because stories are often seen as ongoing, those engaging with indigenous stories are expected to “start where they are” (Judge, 2024) and “live” the story. Even if they are not yet familiar with the ongoing characters or plotlines, readers are expected to remain open to listening and learning as they slowly fill in the blanks. As an example, consider Stimson's (2020) *Stampede Story Map*, a sequence of artistic pictographs on bison robe that depicts the stories of the historic Calgary Stampede from the perspective of First Nations peoples.⁵⁵ This artistically depicted narrative purposefully has no defined start or end; however, the pictographs are designed in spiral, snake-like, and linear patterns, so once a viewer chooses where to start, they can follow the story along multiple trajectories. Could we also consider complex possibilities such as these when crafting cross-curricular mathematical stories that allow for students to “choose their own adventure”, rather than following a strictly linear path? Less rigid story forms such as these have the potential to broaden what is meant by curricular coherence, as they would allow a priori for *many* possible routes toward “making sense” of cross-curricular mathematical stories, in line with a student-centered, “coherence seeking” perspective on curricular coherence (Sikorski & Hammer, 2017). At the same time, I believe great care and ethical consideration must be undertaken if we were to seek inspiration from Indigenous story forms. Many Indigenous traditions, after all, feature ethical traditions around the sharing and (re)telling of stories (e.g., King, 2003; Wilbur & Keene, 2023).

⁵⁵ See the University of Calgary’s Institute for the Humanities webpage (<https://tinyurl.com/StampedeStoryMap>) for an image of the art, alongside a video of the author introducing the art and the process behind making it.

In addition to different story structures, another question that ought to be considered is how different *genres* of curricular stories might offer new possible curricular aesthetics and answers to what (in)coherence *could* mean. Gadanidis and Hoogland (2003), for example, proposed that *romance* may be an apt genre template for mathematical cross-curricular stories, as these stories tend to be about overcoming adversity in search of happiness and wholeness. At the same time, the association of happiness with wholeness seemingly evokes many of the same problematic associations between coherence (wholeness) and goodness (happiness), suggesting that coherence seeking ought to be the default, without considering the value of *incoherence*, as the student-artists in this study often did. Additionally, many stories within the romance genre feature a similar ending (the “Happily Ever After” trope), suggesting that there may be a single destination or goal (i.e., coherent end state), rather than several possible ways to seek coherence. This view does not appear to remain open to the cyclic and never-ending portrayal wuyen gave of (in)coherence, for instance. On the other hand, the mystery genre leaves room for ambiguity, uncertainty, and even misdirection and red herrings to engage readers with an unfolding plot (Appelbaum, 2010; Dietiker, 2015b). This genre explicitly features exciting blends of coherence *and* incoherence that are always evolving, suggesting that coherence seeking is an iterative, non-linear process that evolves over time, more in line with the perspectives expressed by student-artists in this study. Meanwhile, the science fiction and fantasy genres often feature extensive worldbuilding and exploration in addition to other plot points. Coherence seeking in these types of stories is perhaps more akin to an ongoing process of a reader “living” in varied mathematical curricular landscapes, as opposed to ever establishing an “end state” of absolute coherence. Could these genres, therefore, be considered as possible candidates for exploring new aesthetics of curricular (in)coherence more in line with those expressed by the student-artists? Further,

what would it even *look like* to try to craft a mathematics story in the science fiction or fantasy genre? I *have* seen a number of mathematical stories explicitly presented as a romance or a mystery, but I cannot say I recall ever experiencing a mathematical sci fi or fantasy!

I conclude with one final question concerning narrative form and the curriculum-as-story metaphor. In much of the literature I have reviewed, the literature is treated as a *literary* story, evoking certain imagery about the nature of curriculum. Though there are exceptions, there is very often a sense of *linearity* in how literary stories are presented physically. Such stories are most often read left to right across a single line, then top to bottom down all lines of text, then they turn to the next page. There is a clear intended directionality, a temporal arrow governing how one *ought* to read a story.⁵⁶ *Might there be value in considering different narrative presentations as templates for the curriculum-as-story metaphor?* After all, Bal claims that her narrative interpretive theory can be applied to most any form of narrative. How might other narrative presentations be useful for contemplating curricular futurities and alternative aesthetics of (in)coherence in mathematics curriculum?

For example, what if we considered curriculum through the lens of *comics*? McCloud (1993) defines comics as “juxtaposed pictorial and other images in deliberate sequence, intended to convey information and/or produce an aesthetic response in the reader” (p. 20).⁵⁷ I could contemplate many facets of comics as a narrative form and how they might enable us to reconceptualize curricular aesthetics, ranging from how comics are almost unanimously seen as a form of entertainment to how comics combine pictures *and* text in ways that transcend what

⁵⁶ Whether individual readers adhere strictly to this arrow or engage with a narrative text more non-linearly by re-reading, skipping chapters, etc. is another issue entirely.

⁵⁷ While Bal (2021) does not discuss comics, she acknowledges this is largely due to her own lack of knowledge on the topic, instead referring readers to McCloud’s (1993) *Understanding Comics: The Invisible Art*, which she dubs as “a narratology of the comic” (p. 196).

might be conveyed narratively with just either of those alone (e.g., Sousanis, 2015). But, for brevity, I stick to considerations of (non-)linearity, as I have focused on while considering other forms of story throughout this section. Like Bal's definition of a *story*, McCloud's definition of a comic requires "a deliberate sequence". In other words, a comic is not just a miscellaneous set of images. Rather, there is a purposeful sequentiality of how comic panels line up and flow together. Yet, as McCloud details, pages of comic books can be arranged in ways that are purposefully *non-linear* or *multi-linear*, providing the reader with options for how to proceed. The purposeful ambiguity of such panel arrangements makes it clear that the readers' choices and interpretations are valued. There is also a sense that the author has taken a step back and left the reader to their own devices to be pulled by the lines and trajectories drawn on the page (Ingold, 2013).

In addition to stretching the notion of sequentiality, comics also afford complex conceptualizations of temporality. Sometimes, a panel offers just a frozen snapshot in time. Other times, as readers scan across a panel, speech bubbles or other visual cues depict the occurrence of events across the panel (e.g., dialogue spoken across multiple speech bubbles). Going further, the transition from one panel to another may represent a difference of just one second, several minutes, or an even longer stretch of time. And the change is rarely fixed from panel to panel. As McCloud explains, in some sense, it is as if time is contained on the page itself in the "gutters" between panels. Temporality and the arrangement of space (i.e., panels) swirl together, allowing for complex, non-linear conceptualizations of narrative not as frequently on offer through the modality of literary storytelling confined to the pages of a book. What might *curriculum* look like if it were modeled off these complex notions of temporality and spatiality? What aesthetics of (in)coherence would such a curriculum convey? I do not claim to have

answers to this question at this moment, but I pose this question as but *one* example of the generative possibilities of reconsidering which story forms are used as templates for analyzing curriculum through the curriculum-as-story metaphor.

A Final Note as You Leave the Exhibit

Take these students' stories and artwork expressing their views of curricular (in)coherence. They have been gratefully offered to you by the mathematical artists and critics I worked alongside. They're yours. Do what you will with them. Use them to critically reflect on your teaching practice. Get inspired to host your own conspiratorial conversations based on them! Forget them. But don't say in the years to come that you would have lived your life as a researcher, teacher, or *person* differently if only you had heard these stories. You've heard them now.⁵⁸

⁵⁸ The structure of this concluding message is a paraphrased and in some cases directly quoted riff off the concluding structure Thomas King (2003) used to end each chapter in his collection of essays *The Truth about Stories* (pp. 29, 60, 89, 119, 151). If you have not already, I encourage you to engage with King's original essays, which read more like stories, to experience the original power of this narrative framing structure firsthand. For more on how King uses this narrative structure alongside a lovely philosophical discussion about how the stories we implicitly or explicitly invoke in our research play a defining ethical and discursive role in (arts-based) research-creation, see Loveless' (2019) *Manifesto for Research-Creation* Chapter 1.

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APPENDIX A: LIST OF ARTS CREATION AND REFLECTION PROMPTS

Initial (In)Coherence Conversation Prompts

1. What does it mean for something to be coherent? For instance, a story, a TV show, or movie, a song, a conversation...
 - a. Follow-up: What does it mean for something to be *incoherent*?
2. What does it mean for an experience in your life to be coherent or incoherent?
3. What does it mean to you to have a coherent or incoherent learning experience across your courses at MSU?
 - a. Follow-up: [If they do not mention] Is coherence influenced by the specific course content?
 - b. Follow-up: What about in math classes in particular?

Arts Creation Prompts for Reflection on Curricular (In)Coherence

Consider your journey as an undergraduate mathematics student so far, and, in particular, the mathematical content and skills you've learned about across your courses. **What are some patterns, themes, coherences, or incoherences you've noticed or felt across your journey?** While I'd like you to focus your reflection on the mathematical content/skills you've learned, I recognize that your experience cannot be reduced to just a list of topics. Please do not shy away from also incorporating your own emotions, feelings, opinions, personal aesthetics, identities, backgrounds, relationships with other people, etc. when responding to this question.

Feel free to use any of the artistic resources provided to you to organize and make sense of your experiences. What product you create is up to you: Create a 3D model, write a story, a poem, or perform a song, etc. Feel free to choose the artistic medium that you feel is best suited for reflecting on your experiences.

Individual Art Conversation Prompts

Immediately before this conversation, participants were asked to create an artist statement (See Appendix B for the full prompt and associated handout given to students). After I read through their artist statement, we had a conversation about their artwork using the following questions as a rough guide.

1. [Brief clarifying questions to clear up any aspects of their artist statement I did not immediately follow or understand after reading]
2. What were some of the meanings of coherence/incoherence that you considered while creating your art piece?
3. What were some of the patterns, themes, coherences, and/or (in)coherences that you observed when reflecting on your experiences across (undergraduate) math courses?

4. [Any remaining questions I had for each artist after viewing their art and artist statement. Some of these were prepared in advance prior to this session; others were prepared on the spot based on the conversation up until this point]
5. In your next math class, what kind of components would be helpful (in a perfect world!) to help make that new class cohere with the previous math classes you've taken?
 - a. Follow-up: Say the person who organizes the undergraduate math curriculum at a large university was in the room with us right now. What specific suggestions would you give them to help ensure the undergraduate math curriculum is coherent *across courses* for students?

Initial Prompts to Generate Conversation from Group Discussion 1

Participants viewed an exhibit of everyone's artwork immediately before responding to these prompts.

1. What are your initial feelings and reactions after viewing this exhibit?
2. Please share one theme, pattern, coherence, or incoherence that resonates with you and connect it back to the art in the exhibit.

APPENDIX B: ARTIST STATEMENT HANDOUT WITH PROMPT

Figure 3.14

Artist Statement Handout with Prompt

~ ~ ~

To start today's conversation, I'd like you to create an artist's statement for the piece of art you created last time.

What is that?

An artist statement is a not-too-long series of sentences (a few paragraphs) that describe what you made and why you made it. It's a stand-in for you, the artist, talking to someone about your work in a way that adds to their experience of viewing that work.

Components to Include (Not necessarily in this order)

Title. Decide on a title for your work.

Who? Talk about who you are – as a person, learner of mathematics, & artist. Treat this like you are introducing yourself to someone. I welcome you to include your identities, cultural backgrounds, and *anything else* about you that influences who you are as a learner of mathematics.

What? Make sure to state the medium of your art (drawing with colored pencils, water color painting, sculpture, etc.).

Why? What is the meaning or story behind the art you created? What were you hoping to convey? Why did you create it in the way you did? What did you think about while creating your artwork?

Anything else you'd like to say about your art!

Figure 3.14 (cont'd)

A Few Examples

Madhukanta Sen — Flow (2018)

Even though the world seems full of darkness, I can bring the light of hope through the various forms of art I practice: painting, poetry and vocal music. All of my work draws upon my childhood in India and its centuries old cultural traditions. Inspired by my memories of swirling colors, lush flowers, sweet fragrances, lively festivals and the joie de vivre of my people, I combine them with the energy and directness of my adopted home in America, particularly the work of the great mid-century Abstract Expressionist painters.

Image of the painting has been removed for copyright compliance but can be found on Sen's personal webpage linked below

I love colors and use them to put impressions of my life to canvas, and I use words to instill hope in the human heart.

Overall, my art reflects my deep belief in the power of human goodness. This belief has been inspired by living in two great nations and participating in their cultures. It is my goal to create work which celebrates and honors this experience.

This is just an excerpt of Sen's artist statement. The full version can be found on her website:
<https://www.artbymadhu.com/>

Edvard Munch — The Scream (1893)



I was walking along a path with two friends – the sun was setting – suddenly the sky turned blood red – I paused, feeling exhausted, and leaned on the fence – there were blood and tongues of fire above the blue-black fjord and the city – my friends walked on, and I stood there trembling with anxiety – and I sensed an infinite scream passing through nature.

APPENDIX C: MATHEMATICAL STORY CREATION HANDOUT FROM GROUP DISCUSSION TWO

Mapping Mathematical Stories

Like a story, the math that you learn across your courses features....

Mathematical characters. Which can have character arcs and character growth. Or not...they might be static, background characters rather than main characters.

Ex: Numbers!

Plot points. Things that happen to the characters.

Ex: $5 - 7 = -2$, a new character: a negative number!

Settings. The context in which the mathematical plot occurs.

- The course you learn something in
- A particular field of study (e.g., a math application to physics, finance, etc.)
- The setting of a story problem
- How the math itself is represented in the story (through symbols, graphs, tables, pictures, etc.)

Themes, Morals, or Takeaways. The recurring and overarching themes *across* mathematics courses. What was the point of the story? What were your takeaways?

Stories come in *many* different forms and structures. See the separate handout for several visual examples...

Quick Reflect!

What are your OWN preferred languages or mediums to represent and/or share stories? Come up with 2-3 and write them below.

Some possibilities: Poetry, pictures, diagrams, photography, collage, a mood board, a video, drawing(s), written text, flow diagram, ...

Prompt

If you thought of the **mathematical content and skills** you've learned across your math courses as a story, what kind of story would it be? For example...

- What would be some of the overarching themes or morals?
- What are some of the main characters?
- What would the primary plotlines be across your math courses?

In a medium of your choosing, share one of the main storylines you've noticed across your mathematics courses. Try to make explicit connections to particular mathematical concepts and skills you've learned and the courses you've learned them in.

Feel free to use the story structures we looked at before for inspiration, but you don't need to follow these. Be creative and express the story of mathematics in a way that makes sense to you. *Your story could be coherent, incoherent, or somewhere in between—this is YOUR interpretation and mapping of a story told across your math courses.*

APPENDIX D: THEORY OF COLLAGE AS A PHILOSOPHICAL STANCE AND METHOD

The power of collage stems from the mixing and matching of selected ephemera outside of their original context in ways they were not originally intended, destabilizing the structuralist notion that objects have one purpose, one definition, or one use; instead, collage is open to multiplicity of meaning and paradoxical dialectics of juxtaposition (Butler-Kisber, 2008; Scotti & Chilton, 2018; Vaughan, 2005). In this way, collage is a (postmodern) philosophical stance and methodological approach (Brockelman, 2001; Scotti & Chilton, 2018; Vaughan, 2005) that is consistent with the ABR paradigm. Indeed, feminist philosopher Sandra Harding (1996) proposes collage as a model for a “borderlands epistemology” (p. 22) which blends distinctive cultural forms of knowing—including dominant *and* non-dominant knowledges—to create *new* forms of knowledge, *new* insights. However, she clarifies that because there is no perfect representation of the world, the goal of this blending is *not* to create a “maximally ideal knowledge system” (p. 22). Instead, the aim is to craft a collage of local, contextual knowledges that “simultaneously emphasize[s] personal meanings, history, culture, and tradition in such a way as to bring disparate voices of the internal-personal and external-contextual to a common place” (Finley, 2001, p. 17). Collage embraces this integration of multiple experiences, exploring the coexistence of these (perhaps paradoxical) perspectives through artful juxtapositions and overlappings in physical space (Scotti & Chilton, 2018; Vaughan, 2005). Such a “piecing together creates resonances and connections that form the basis of discussion and learning” (Vaughan, 2005, p. 40), serving as a form of cultural critique with the aim of aesthetic and/or political transformation. In light of this framing, collage is a well-suited methodology for my purposes of situating participants’ multiple perspectives on and critiques of curricular (in)coherence relative to one another and the literature.

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APPENDIX E: FULL ARTIST STATEMENTS

All artist statements that were not reproduced in their entirety previously can be found within this appendix.

CinematicHue – *Fire in the Forge*

As an artist deeply influenced by philosophy, abstract art, cinema, and literature—ranging from poetry to comics from both mainstream and independent publishers—I find that art serves as a universal language. Art has been an essential part of human culture since prehistoric times, and its influence is undiminished in the modern age. I am also fascinated by mathematics and its abstract principles, which inspire me to explore the concept of knowledge in my work.

The medium of choice for this particular piece is color painting, with stylistic cues taken from 19th-century abstract art. It's not merely an aesthetic choice; the historical context adds a layer of depth to the work's exploration of knowledge dissemination.

Inspired by the myth of Prometheus, this artwork delves into the dualistic nature of knowledge. Much like Prometheus stole fire from the gods to give to humanity, knowledge too can be a gift or a curse, depending on its application. The painting aims to convey this nuanced understanding. The elements of lightning and fire serve as powerful symbols in the artwork: they are simple yet evoke a range of interpretations, capturing the essence of what I aim to communicate—that knowledge is both enlightening and dangerous, capable of creation and destruction.

In creating this piece, my goal is to prompt the viewer to contemplate the ambivalent power of knowledge, as encapsulated in the timeless myth of Prometheus.

GUo3 – *Math Universe*

The drawing that I drew is based on the cartesian coordinate system with colored pencil. As we know, math can't leave the coordinate, if you want to describe a function or equation, the best way is to sketch the coordinate to understand. Also, I put the red and blue color to represent the negative and positive. But the other element in drawing is colorful to show the diversity of math, function, geometry and so on. I try to make this drawing abstract, because you know, math always tries to make people hard to understand.

As for me, math put some magic on me, and let me think a lot. Everything about engineering is based on math, and even financial problems. Calculating is wonderful, makes people think, imagine, associate...

Joy – *The Smoke Trail*

The smell of ink on paper. The emotion tied with flipping a page. The worlds brought to life. Growing up, I have always loved literature. Reading works of fiction brought a warmth to my life much like sitting by a fire on a cool night. Subjects like mathematics, on the other hand, have often left me feeling blue, as though I could see the smoke trail leading to a warm fire, and I had

no clue how to actually get there. It should be as simple as following the smoke or path laid out for me. Yet I felt like when I would take a step in that direction, the smoke would come into my lungs and choke me out as opposed to gently guiding me along.

I love the color blue, and it saddens me to use it as a representation of how low mathematics has made me feel in the past; however, I can think of no better colors to highlight how isolating it felt in some of my math courses.

A fire can bring both warmth and destruction to a person that tries to yield it, and it is not always expected which action fire will take. It can bring a form of shelter to a person just as easily as it could burn down a forest.

There are also multiple ways to build a fire. If asked to build a fire using only what nature provides, a person that learned to build one only with lighter fluid would be at an impasse. Mathematics has often left me feeling stuck, especially when I learned past topics in a certain way or only to a certain extent, and it would feel like a sucker punch to the stomach to be told I should already know something that I did not necessarily feel I did.

Environment also plays a role in how well a fire can be built. A person who is asked to build a fire on a dry, heated day might have better success than a person trying to build one in the rain. When my life met difficult times, I could not understand how I could possibly be asked to do certain mathematical tasks successfully.

On days that mathematics made sense, I loved it. There was satisfaction in being able to successfully follow the smoke trail or build the fire. On the days that it didn't, however, I felt lost in an abyss of blue. When it got bad enough, I'd feel as though I had been left under water and the top surface had frozen under me. Those were the times I thought I could never catch up or make it to the fire, and it was difficult not to give up in those times.

wuyen – Doll Up, Dress Up, Beautify

Untamed spirit and boundless wonder remain present in all aspects of my life. Inspired by the memories of girlhood, *Doll Up, Dress Up, Beautify*, highlights how memories of this phase in life constantly dance with the reality of the present. This piece is a testament to how creativity and adaptivity is weaved into our everyday lives.

As a young child, I often drew on my imagination to create entire worlds where nothing was out of my reach. Whether it be a box of colorful crayons or crafting with fancy paper, I embarked on exploring the limitless possibilities that come with the act of creating. In the realm of my imagination, I discovered the joys of self-expression. It was a world where I could be anyone and do anything, building the foundation for my journey of self-discovery and growth.

Girlhood is unique to each individual's narrative which is what I hoped to convey through my origami inspired piece. Girlhood is both diverse and beautiful, with many stories yearning to be heard. This was the reasoning for the varying dresses created to represent the many unique experiences of being a girl. The piece invites viewers to reflect on how this shared human experience shapes our lives.

CHAPTER 4: DISCIPLINARY INCOHERENCE? META-NARRATIVES ABOUT FUNCTION(S) CONVEYED BY THE STORY OF A COMMONLY ADOPTED MULTIVARIABLE CALCULUS TEXTBOOK

Many consider the concept of mathematical function—a univalent mapping (or transformation) from one set of elements to another—to be a crucial recurring theme across the story of the K-16 mathematics curriculum (e.g., Graf et al., 2019; CCSS-M, 2010; Zorn, 2015). Some mathematics education researchers even go as far as to contend that the concept of function is “the single most important mathematical concept studied from kindergarten to graduate school” (Harel & Dubinsky, 1992, p. vii). Mathematicians have also written about the centrality of the function concept. For instance, Gowers et al. (2008) stated how “One of the most basic activities of mathematics is to take a mathematical object and transform it into another one” (p. 10). In this sense, function is positioned as a unifying, cross-curricular theme that occurs across nearly every subdiscipline of mathematics.

This disciplinary *meta-narrative* about the role of function has endured for decades. Recall from Chapter 2 that by a meta-narrative, I mean “a cultural narrative schema which orders and explains knowledge and experience” (Stephens & McCallum, 1998). Yet, this meta-narrative about the role of the function concept is not a narrative that many students readily take up, according to a large body of research (Martínez-Planell & Trigueros, 2021; Melhuish et al., 2020; Zandieh et al., 2017). Although there are likely several reasons for this, in this chapter I examine the role that the stories told about functions in a commonly adopted calculus textbook (Stewart et al., 2021)—a didactic cultural artifact of the discipline of mathematics (Plut & Plesic, 2003)—might play in transmitting or failing to transmit this cultural meta-narrative to students. In this arts-based textbook analysis, I take up the metaphor of curriculum as a story (Dietiker, 2015a) and treat different types of functions as mathematical characters in this curricular story. Specifically, I examine the stories told about the three different types of *multivariable* functions

featured in an introductory multivariable calculus course—parametric, vector-valued functions; multivariable, real-valued functions; and vector fields. I consider two research questions while reading these stories as I would a literary novel: *(RQ1) How are these characters portrayed in the chapters they are introduced and how do these portrayals compare with one another? (RQ2) What meta-narratives about function(s) are conveyed by these character introductions when they are considered collectively?*

I inquire into these research questions with the broader goal of investigating the nature of cross-curricular mathematical stories as told in the intended, textbook curriculum (Remillard, 2005; Tarr et al., 2008). Owing to the dominance of disciplinary coherence (Cuoco & McCallum, 2018; Schmidt et al., 2002) as a guiding axio-onto-epistemological principle for “good” curricular stories in mathematics education (see Chapter 2), mathematical stories in the intended curriculum are often constructed with primarily *logical connections* between story components in mind, as evaluated from a (retrospective) expert perspective. Research on student learning is also considered when constructing cross-curricular stories (i.e., methods of ensuring cognitive coherence, Fortus & Krajcik, 2012; Jin et al., 2022) which *indirectly* takes into account students’ perspectives on what constitutes a “good” (mathematical) story (see Chapters 2 and 3). Yet, the aesthetic principle of disciplinary coherence puts disproportionate weight on preserving privileged disciplinary structures and sequences when experts recontextualize (Muller, 2009) disciplinary stories to craft didactical ones (even if, in practice, such a process is acknowledged as being complex rather than a straightforward, one-to-one transposition from one context to the other, e.g., Love & Pimm, 1996). Sikorski and Hammer (2017), however, called into question the epistemological utility of such pre-meditated forms of coherence, arguing instead that active readers of these curricular stories—students *as well as* disciplinary experts—engage in

idiosyncratic forms of coherence seeking that may deviate from or even conflict with disciplinary forms of coherence. In other words, a (didactical) mathematical story considered “good” or “coherent” by one person (i.e., an expert) may be aesthetically judged otherwise by someone else given their differing axio-onto-epistemological sensibilities.

In this exploratory study, I take up Sikorski and Hammer's (2017) call to be suspicious of narrow, pre-meditated aesthetics of coherence in the context of mathematics education via cross-curricular stories from the textbook curriculum. I do so by *reading* these stories from a literary, *aesthetic* stance rather than a strictly efferent one (Dietiker, 2015a; Rosenblatt, 1986) to reflect on the ways that stories considered “coherent” from a disciplinary perspective might simultaneously be considered *incoherent*, even from the perspective of someone socialized into the discipline of mathematics like myself. In particular, I attend to story themes as a form of coherence, noting any meta-narratives that are salient across my readings of curricular stories (i.e., RQ2), and subsequently interrogating the ways in which these are consistent (or not) with privileged disciplinary meta-narratives (i.e., function as a unifying, cross-curricular theme). By challenging a strict devotion to disciplinary coherence as the privileged paradigm for crafting curricular stories, I spotlight how a multiplicity of other forms of (in)coherence—logical, aesthetic, and otherwise—might be useful in contemplating *whose* stories get told and which stories *ought to be* told when designing curricula that is attentive to a variety of (interdisciplinary) aesthetic sensibilities (Hyvärinen et al., 2010; Irwin, 2003; Modeste et al., 2023; Richmond et al., 2019).

To investigate how the cross-curricular theme of function is developed *across* the curriculum, I zoom into a critical transition point in the development of this theme in the undergraduate curriculum—introductory multivariable calculus. Not only does this course

feature three different types of functions in one disciplinary context and therefore *three* different stories that can be juxtaposed, but it also represents one of the first points in the mathematics curriculum where functions other than single-variable ones begin to play a substantial role. Additionally, despite the importance of multivariable calculus as a key transition point between “introductory” and “advanced” mathematics courses and its nature as an interdisciplinary service course for multiple STEM disciplines, there has been limited research dialogue in mathematics education questioning what the curricular content, goals, and *stories* ought to be in this consequential intermediary course. For example, despite the crucial role of vector-valued functions (i.e., parametric functions and vector fields) in introductory multivariable calculus *as well as* in subsequent STEM courses (Bollen et al., 2017; Dray & Manogue, 2023; Gire & Price, 2014) and mathematics courses such as linear algebra (Slye, 2019; Zandieh et al., 2017), multivariable calculus function research attends almost exclusively to real-valued functions. As such, by reading and interrogating the curricular stories from the intended multivariable calculus curriculum—particularly those involving *different* types of multivariable functions (real- *and* vector-valued)—this study contributes not only to the general literature on cross-curricular coherence and crafting engaging curricular stories but also serves to catalyze further conversation about the curriculum of multivariable calculus itself. Curricular deliberation along these lines is well overdue and has clear ethical implications for STEM students’ socialization into their respective disciplines, especially given the varied and sometimes even *conflicting* disciplinary meta-narratives and aesthetic sensibilities of mathematics compared to other STEM fields (e.g., Dray & Manogue, 2004).

Theoretical Background and Past Research

Textbook Curriculum as Story

I adopt the curriculum-as-story metaphor (Dietiker, 2013, 2015a) in the form introduced

in Chapter 2 to conceptualize the intended, textbook curriculum as a *story* which could be read like a literary novel. This framing emphasizes that *logical* connections between curricular story elements (i.e., disciplinary forms of coherence) are equally as important as the *aesthetic* impact that the story has on its reader as it unfolds (i.e., the mathematical *plot*). Further, I view *reading* from a reader-oriented perspective (Rosenblatt, 1978, 1988), in the sense that reading is an active, socio-culturally informed process whereby meaning is developed through a hermeneutic process between the reader and the text. From this perspective, what is considered a “logical connection” between story elements and what “counts” as an *engaging* or “good” story is deeply subjective and based on a reader’s axio-onto-epistemological sensibilities.

In addition to the process of reading being socio-culturally bound, I take the perspective that (mathematical) *stories* and *storytelling* are socio-cultural in nature. Therefore, based on the theoretical framing of curriculum developed so far, mathematics textbooks are disciplinary didactical artifacts from the *culture* of mathematics (Plut & Pesic, 2003). The relationship between the stories in these didactical artifacts and those in other disciplinary artifacts, however, is complex and may not completely align (because crafting didactical stories always requires a process of translating—or recontextualizing—disciplinary knowledges, practices, and stories into another form (Bernstein, 1999; Bosch et al., 2021; Love & Pimm, 1996). Consequently, the stories told in textbooks are of a unique, socially constructed (and discipline-specific) genre which features language unique to the didactic norms of that discipline and also schooling more generally (Schleppegrell, 2004). As a result, curricular stories in textbooks may *or may not* convey messages or themes consistent with disciplinary meta-narratives (Stephens & McCallum, 1998), meaning that curricular stories may be interpreted by students in ways that could hinder their enculturation into the discipline of mathematics (e.g., Brown, 2022; see also Zarkh, in

press, for an example of this phenomena in the context of enacted curricular stories).

Analysis of Textbook Stories

Textbook analyses help reveal the idiosyncrasies of textbook stories—including crucial differences between stories which might at first appear similar, unintended messages stories could send to readers, as well as whether these stories achieve the stated aims of their authors when read by students (Dietiker & Richman, 2021; Herbel-Eisenmann, 2007; Wagner, 2012). The utility of such analyses has led to a growing interest in the careful conceptualization of textbook analysis (Fan et al., 2013) to study the pedagogical and mathematical intentions of textbooks as well as the (disciplinary) cultural traditions represented in textbooks (Pepin et al., 2001). Charalambous et al. (2010) further delineate between the “vertical” study of a concept or cross-cutting practice throughout a textbook from a more general “horizontal” study of a textbook for its overall properties. A vertical textbook analysis (such as this one) primarily attends to topic-specific matters, including definitions, practices, conventions, and connections between a chosen concept (i.e., function) and other relevant constructs that are advocated for within a textbook (or textbooks). Analyses of this form often involve attention to the different contexts or representational settings through which a given concept is explored over the course of a textbook story (e.g., Lankeit & Biehler, in press). Given the importance of *all* representational settings and how they relate to one another, non-textual elements—including illustrations or other visual mathematical representations—should not be ignored in a textbook analysis (Love & Pimm, 1996; Otte, 1983). In order to read textbook stories *holistically* in terms of their logical connections and aesthetic impact (Dietiker, 2015a), it is necessary to pay attention to any non-textual elements in terms of how they connect to other mathematical ideas and representations but *also* the aesthetic impact these visual elements might have on readers (see e.g., Kim, 2009)

Prior textbook analyses suggest several other facets which ought to be considered when reading the stories of textbooks beyond the pacing and sequencing (Dietiker, 2015a). One such facet is the organizational structure and sequencing of the story. The most prevalent such story structure in mathematics textbooks begins with some form of exposition, transitions to several examples, then concludes by leaving the reader to work on exercises to check their understanding (Love & Pimm, 1996). According to Harel (2021), this trend applies to multivariable calculus textbooks as well:

A conspicuous common feature to these textbooks is that their content presentation is organized around a sequence of lessons, each comprised of three phases: a pre-formal statement phase, formal statement phase, and post-formal statement phase. The first two phases are typically brief, while the third phase, consisting mostly of examples illustrating the concept and procedures for its application, occupies the lion's share of the lesson. (pp. 718–719)

More specifically, in a separate study of linear algebra textbooks, Harel (1987) noted four storytelling devices through which mathematical concepts (i.e., mathematical characters) were introduced. One common strategy was to make a connection between the new, mathematical concept and an everyday, non-mathematical concept (i.e., an analogy) or a previously introduced mathematical concept (via an isomorphism). Another was to introduce examples which were then generalized to justify and introduce the new concept. He also noted that in several cases, the author of the story began by simply postponing further discussion about a core concept (e.g., “as you will soon see, multivariable functions are essential to multivariable calculus”).

It is also important to attend to factors related to *the reader* of a story, particularly regarding the “voice” of a textbook (Herbel-Eisenmann, 2007; Love & Pimm, 1996) and how the language used throughout a textbook impacts not only a reader's aesthetic engagement with the story but also how a reader is positioned relative to the discipline of mathematics (Herbel-Eisenmann & Wagner, 2007). The language used in mathematics textbooks often highlights an

aesthetics of *depersonalization* whereby personal pronouns are omitted, readers are told *what* to do and *how* they should be thinking as they read (e.g., “As seen previously...”), and abstract objects are positioned with more agency to move the story forward than the reader (e.g., “the graph tells you that...”) (Herbel-Eisenmann, 2007). Textbooks that are not open to multiple interpretations and instead favor funneling all readers down a single interpretive path are what Eco (1979) called *closed* texts (see also Weinberg and Wiesner, 2011). Mathematical texts—including textbooks—tend to be closed (Herbel-Eisenmann & Wagner, 2007; Wagner, 2012), imposing certain constraints on *who* can read these textbook stories and *how* these readers can engage with a story (Weinberg & Wiesner, 2011). While these texts may serve to socialize some into the disciplinary language norms and aesthetics of mathematical discourse, they may also serve a destructive socialization function that significantly chokes creativity and diversity among students of mathematics (Brown, 2022; Wagner, 2012). *Open* texts, on the other hand, can be read in multiple ways by readers with differing aesthetic sensibilities.

Function as a Disciplinary Meta-Narrative and Privileged Cross-Curricular Theme

Textbooks have been shown to characterize function(s) in many different ways across a variety of representational settings (e.g., Mesa, 2010). Indeed, function is consistently seen as a multi-faceted notion (Ayalon & Wilkie, 2019; Dreyfus & Eisenberg, 1982; Niss, 2020) and, as Thompson & Carlson (2017) caution, there is no monolithic concept of function. Even modern mathematicians use at least two non-equivalent set-theoretic definitions of function which sometimes give opposite answers to basic questions, such as “Is *this* a function?” (Mirin et al., 2021). With the goal of being attentive to myriad possible characterizations in my own textbook analysis, I begin by synthesizing literature on what “function” *could* mean from a variety of perspectives.

From the formal, set-theoretic perspective, a *function* is defined as a univalent (Even &

Bruckheimer, 1998) mapping (or *transformation*) from one set of elements— X , “the domain”—to another— Y , “the codomain”—i.e., $f: X \rightarrow Y$. For example, the most common type of functions introduced in high school algebra including linear, quadratic, and exponential families of functions (Cooney et al., 2010; Graf et al., 2019) are all single-variable, real-valued functions of the form $f: \mathbb{R} \rightarrow \mathbb{R}$. But functions need not (and often do not) have the same domain and codomain and may involve multiple variables in either their domain or codomain. For example, a binary operation on two elements, such as addition, multiplication, or the operation from any algebraic group could also be considered a function, $f: X^2 \rightarrow X$ (Wasserman, 2023). According to APOS theory (Asiala et al., 1996; Breidenbach et al., 1992), one of the most commonly adopted theoretical frameworks for conceptualizing mathematicians’ and students’ perspectives about function(s), a function could be conceptualized as either an *action* which can be evaluated at one input in the domain, a *process* which could be carried out on *any* input in the domain set to output an element of the codomain set, or an *object* in its own right that could itself be acted on by another mathematical operation (e.g., taking the derivative or integral of a function). In line with the set theoretic perspective, this object perspective treats function as a mathematical *structure*. But not all perspectives on function focus on *structural* character traits. Another common perspective on the character of function involves viewing a function as a covarying relationship between two continuous quantities (Jones, 2022; Thompson & Carlson, 2017). This is a perspective most often used to model or conceptualize dynamic, realistic situations (Beckmann & Michelsen, 2022; Bettersworth, 2023; Carlson et al., 2002).

Functions appear across a variety of mathematical settings and the same functional relationship can be represented in several ways (Cooney et al., 2010; Gagatsis & Shiakalli, 2010; Hitt, 1998). Mesa (2004), for example, in a textbook analysis of problems involving function

across a sample of middle school mathematics textbooks, noted how functions were represented symbolically as either a rule (e.g., $y = f(x)$) or an ordered pair (e.g., $(x, f(x))$), as well as in visual contexts (depicted graphically or geometrically), and verbally in contexts involving physical phenomena and modeling. In a paired analysis of multivariable calculus textbooks and lessons, Harel (2021) similarly observed three different modes of presentation—algebraic, graphical, and physical—and noted how the physical context often took a back seat to the other two, providing a likely explanation for why he noted that covariational perspectives for function were rarely taken up explicitly.

The flexible nature of the function concept to be viewed from many perspectives and in many representational contexts helps explain why mathematicians and mathematics educators have consistently positioned function as a privileged theme and even a meta-narrative of the discipline. In an entry of the *Encyclopedia of Mathematics Education* on the teaching and learning of functions, Niss (2020) contends:

The concept of function is one of the most important in all of mathematics. Functions form the basis of many mathematical fields, and they are the key means of mathematization for a whole host of mathematical and non-mathematical applications. They help us establish, understand, and describe relationships, making them an important part of everyday life. However, function is also one of the most complex mathematical concepts. This makes it one of the most fundamental and significant concepts not only in mathematics, but also in mathematics education. (p. 303)

Several historians suggest that function has played a crucial role in the development of (continental) mathematics over the last several centuries (Boyer, 1946; Monna, 1972; Youschkevitch, 1976), though others temper this claim by arguing that it is the result of modern sensibilities being used to retroactively conceptualize the history of the discipline (Kjeldsen & Petersen, 2014; Thompson & Carlson, 2017). Still, the centrality of function to at least *modern* mathematics is demonstrated by the idiosyncratic nature of the different types of functions that

have been introduced to study the various sub-disciplines of mathematics. For instance, linear algebra features *linear transformations* of vector spaces which map vectors following certain linearity properties (Slye, 2019; Zandieh et al., 2017). Meanwhile, abstract algebra introduces *homomorphisms* and *isomorphisms* between algebraic structures as structure-preserving maps between algebraic structures, such as groups or rings (Hausberger, 2017; Melhuish et al., 2020). From a structural perspective, several mathematicians view each of these different types of functions through a unified and unifying lens (Gowers et al., 2008). Specifically, mathematicians have introduced *morphisms* as generalizations of homomorphisms which each preserve a particular algebraic structure pertinent to a given sub-discipline of mathematics (Baker et al., 1971; Hausberger, 2017). For example, *homeomorphisms* between topologies preserve topological structure. This type of generalization has been taken further still in *category theory*—a mathematical meta-language which uses this generalized notion of morphism to abstractly conceptualize the discipline of mathematics itself (Piaget et al., 1990/1992; Riehl, 2017). Yet, mathematicians—both historical and modern—do not unanimously agree on the precise nature of this unifying “structural view” of mathematics and the degree to which it ought to be adopted (Hausberger, 2017).

Mathematics education researchers have also taken up a “unified notion of function” characterization to conceptualize students’ learning about different types of functions across course contexts (Hausberger, 2017; Melhuish et al., 2020; Zandieh et al., 2017). However, these researchers adopt a broader perspective on “unification”, attending to the various ways students see different types of functions as connected (or not), rather than adhering to a strict category theory perspective. Zandieh et al., (2017) define a “unified notion” like so: “By expressing a *unified notion* of a mathematical construct, we mean understanding various constructs as

examples of the same phenomenon, regardless of differences in the specific contexts” (p. 24, emphasis added). In particular, they operationalized this idea for function by investigating the connections students expressed between their concept images (Tall & Vinner, 1981) for function and linear transformations. Similarly, Melhuish et al. (2020) adapted this unified notion of function perspective to study the coherence of abstract algebra students’ concept images for function and homomorphism. Although this perspective on function has only been explicitly used in these two course contexts at present, Zandieh et al. (2017) propose that this “unified notion” construct could be applied more widely to understand the conversations occurring about function understanding across other undergraduate courses.

As in mathematics, function has been a focal consideration in research and curriculum design for several decades (Niss, 2020). In curricular documents, function is often viewed as a cross-curricular theme, leaning into a *unified* perspective on function. The Common Core State Standards for high school mathematics (2010), for example, identify function as one of the six “conceptual categories” used to organize and provide coherence to the standards, alongside other central themes like “number and quantity”, “algebra”, and “modeling.” Function is also mentioned explicitly as a concept that “crosses a number of traditional course boundaries” (p. 57). The CCSSM further specify that students should be able to recognize two different function types as instances of a function with a particular domain and codomain *as well as* recognize sequences and geometric transformations as functions (Standards F-IF.3, G-CO.2). There is emerging empirical evidence suggesting students may benefit from reasoning about new types of functions in terms of old types of functions they have previously learned about, as it may support metaphorical thinking across different course contexts, at least in the undergraduate mathematics education context (Melhuish et al., 2020; Zandieh et al., 2017).

Another recurring recommendation in mathematics education research is that students should be exposed to functions across several representational settings (e.g., verbal, symbolic, tables, graphical), so they can learn to recognize functions across these settings (Carlson et al., 2002; Cooney et al., 2010; Gagatsis & Shiakalli, 2010). From an APOS theory perspective, it is recommended that students assimilate function actions and processes across these representational contexts with the aim of crafting a schematic *structure* for function as an object in its own right that merely inhabits multiple contexts (Asiala et al., 1996; Breidenbach et al., 1992). Meanwhile, Eames et al. (2021) and Graf et al. (2019) propose a learning progression for function that echoes many previously stated recommendations, adding that students ought to eventually attend to the domain of a function as a major characteristic of a function's identity from a structural perspective.

A final perspective commonly presented in the literature proposes that students ought to first learn about function from a covariational reasoning perspective before engaging with set-theoretic perspectives which emphasize function as a mathematical object which relates a domain and codomain set via a univalent mapping (Oehrtman et al., 2008; Thompson, 1994). Paoletti & Moore (2017), for instance, argue that introducing this structural definition prematurely in mathematics curriculum is akin to putting the cart before the horse. This covariational reasoning hypothesis proposes that a covariational perspective on function to model dynamic, real-life scenarios serves as “the horse” to motivate subsequent learning about function from a structural perspective. While the arguments for this hypothesis are theoretically sound, those who advocate for this hypothesis have said much less about how a covariational perspective for function might support students as they transition from reasoning about functions on continuous domains (where covariational reasoning is an applicable process) to those on

discrete, abstract domains (where such reasoning is not immediately applicable). There has also been limited empirical evidence to date for the claims of this hypothesis.

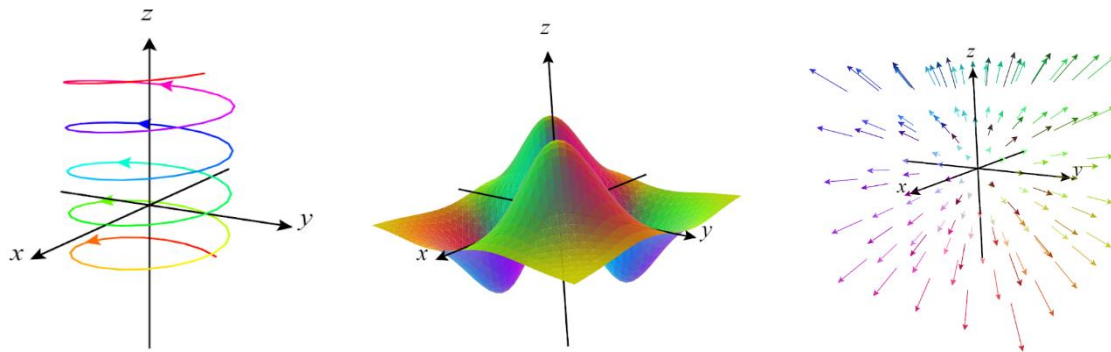
The Role of Function(s) in the Multivariable Calculus Curriculum

Multivariable calculus (MVC) features can be conceptualized as featuring three different types of multivariable functions (see e.g., Bettersworth, 2023, Table 1, pp. 82–83) defined by the dimensionality of their domains and/or codomains: 1) parametric functions, of the form $f: \mathbb{R} \rightarrow \mathbb{R}^n$; 2) multivariable, real-valued functions, of the form $f: \mathbb{R}^m \rightarrow \mathbb{R}$; and 3) vector fields, of the form $f: \mathbb{R}^m \rightarrow \mathbb{R}^n$. From a structural perspective, the dimensionality of each domain and codomain distinguishes these function types from one another and the single-variable, real-valued functions students have primarily encountered prior to MVC. Perhaps most importantly, these defining properties result in different possible representations for each function type (e.g., a graph of a surface plotted in $\mathbb{R}^{m+1}$ or a heat map plotted in $\mathbb{R}^m$ for multivariable, real-valued functions as opposed to a curve plotted in $\mathbb{R}^n$ for a parametric function, see Figure 4.1) and qualitatively distinct kinds of mathematical techniques that are used to analyze each function type. For example, the calculus of each function type outlined above is distinct based on how “rate of change” and “accumulation” could be interpreted with respect to each function’s inputs and outputs.

The traditional introductory MVC curriculum, as depicted in commonly adopted textbooks such as Stewart et al. (2021), consists of roughly four parts. Following Martínez-Planell et al. (in press), the first part is an introduction to vector basics and graphing in 3D (i.e., introductory notions), the second is the calculus of curves (i.e., parametric functions), the third is differential and integral calculus (on real-valued, multivariable functions), and the fourth is vector calculus (on vector fields). As I have in my past teaching (see Chapter 1), the bulk of the

Figure 4.1

Visual Representations of Three Different Types of Multivariable Functions



Note. (From left to right) A parametric, vector-valued function (i.e., a curve); a real-valued function (a surface); and a vector field

introductory MVC curriculum could be seen as the story of “How can we do calculus on *this* type of multivariable function?” In essence, the structure of the MVC curriculum corresponds to the different types of multivariable functions that are introduced.

Though curricular research in the context of MVC remains limited (Martínez-Planell & Trigueros, 2021), the extant literature features one dominant curricular point of view that relates to the teaching and learning of (multivariable) function(s)—MVC “builds on” single variable calculus and is primarily about generalizing single variable concepts to construct their multivariable analogies. For example, much of the research on (students’ learning of) multivariable functions focuses on multivariable, real-valued functions as a generalization of single variable, real-valued functions (Dorko, 2023; Kabaël, 2011; Martínez-Planell & Gaisman, 2012). Similarly, MVC is viewed as being a context in which derivatives and integrals of single-variable functions are generalized into multivariable analogs, such as the total derivative (Harel, 2021; Lankeit & Biehler, in press) or line integrals of vector-valued functions (Jones, 2020). In the context of function research, however, the objects of study are almost always real-valued functions, rather than vector-valued ones (Hahn & Klein, in press; Tyburski, 2023), suggesting

that certain *kinds* of generalization are valued over others, at least in mathematics education. On the other hand, physics education researchers have conducted plenty of research on how students create and translate between vector field representations of physical phenomena, such as electric and magnetic fields (e.g., Baily & Astolfi, 2014; Bollen et al., 2017; Gire & Price, 2014). In these studies, the attention is primarily on interpreting these vector fields as representing physics phenomena, rather than their structural, set-theoretic properties, perhaps partially owing to Dray and Manogue's (2004) observation that physicists discuss functions in different ways than mathematicians.

Students' Interpretations of *Our* Curricular Stories about Function(s)

Time and time again, students' perspectives on function have been shown to vary from those of instructors and other mathematicians (e.g., Sinclair et al., 2011; C. G. Williams, 1998). As Thompson (1994) remarked, "Tables, graphs, and expressions might be multiple representations of functions to us, but I have seen no evidence that they are multiple representations of anything to students" (p. 23). In this section, I reject the view that tables, graphs, and expressions mean *nothing* to students and instead take up an anti-deficit (Peck, 2020) perspective by attending to literature on students' *coherence seeking* (Sikorski & Hammer, 2017) across situations that might be seen by a disciplinary expert as involving function(s). I focus not on the ways students "struggle" to adopt disciplinary meta-narratives pertaining to function but rather on normalizing and explaining their varied interpretations of our curricular stories. To do so, I highlight several entrenched features in our curricular stories (about function) which could lead students to interpret these stories in ways other than intended.

In Tyburski (2022, 2023), I asked three undergraduate MVC students how they viewed functions as being used across their STEM courses. Each student had rich and idiosyncratic extended metaphors to share. Robin, for example, told me the following:

I would say functions are kind of like canvases, and we're being trained as painters. And so, all right, here's this blank canvas and paint this operation on this canvas. And we're learning how to paint really well. It doesn't really matter what kind of canvas you have, but you know, whatever kind of strokes you use or whatever kind of brush you use or whatnot is what's important. . . . The function is just the backdrop.

Meanwhile, Amelia suggested that she saw functions everywhere, going as far as to call them “the ABCs of math”. Both students used evocative language to describe the meta-role they saw function(s) playing across several of their courses, yet when I asked them to tell me about the different types of functions they had encountered in their MVC courses, they were notably confused. “What do you mean by different types of function?” Amelia asked after a long pause. While students’ general stories about function revealed developed views concerning the role function plays as a cross-curricular theme compatible with disciplinary meta-narratives about function(s), all three students did not seem to use this same extended metaphor to conceptualize the stories they had constructed to make sense of experiences in MVC. In the end, however, all students engaged in some form of coherence seeking and listed instances of what they considered as functions in the story of MVC they were constructing. This tendency across participants to start forming salient connections between newly introduced mathematical characters and the mathematical character(s) they associated with “function” has also been observed in several other course contexts, including in linear algebra with linear transformations (Slye, 2019; Zandieh et al., 2017) and abstract algebra with homomorphisms and isomorphisms (Hausberger, 2017; Melhuish & Lew, 2019). A similar phenomenon has been observed with pre-service mathematics teachers when it comes to drawing connections between secondary and post-secondary mathematical functions (e.g., Wasserman, 2023).

In the lists of functions students eventually provided, *different* functions were organized roughly by either (a) the dimension of the *representation* they could be visualized within (e.g., $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ could be graphed in $\mathbb{R}^3$, whereas $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ could not) or (b) the coordinate systems in

which a function was written (rectangular, polar, cylindrical, or spherical). As students explained, what mattered the most about the function is what they were asked to do with it and, to them, different dimensions of representations or coordinate systems felt like they required qualitatively different strategies in most circumstances. A similar coherence seeking phenomenon was observed by Montiel et al. (2008, 2009) when they asked MVC students to identify (single and multivariable) functions and non-functions expressed in various coordinate systems. When distinguishing between functions and non-functions, several students applied the vertical line test—a valid univalence check for a single-variable, real-valued function *in rectangular coordinates*—across all representations without further translation. Zandieh et al. (2017) observed a similar tendency in some linear algebra students when asked about whether a linear transformation could represent a function. Montiel et al. (2009) concluded that students’ general definition(s) for function “seems to often be lost among the different representations students are exposed to, without recognizing any implicit hierarchy” (p. 152).

Yet, research into students’ reasoning with linear transformations and homomorphisms in abstract algebra refutes the claim that students see *no* connections across such scenarios (Melhuish et al., 2020; Slye, 2019; Villabona et al., 2020). While students have been shown to largely ignore the domain and range (Graf et al., 2019; Markovits et al., 1986) even when a formal, set-theoretical definition is provided for a kind of function, they *do* attend to other salient (and characteristic) properties, such as the homomorphism or linearity properties (Melhuish et al., 2020; Slye, 2019; Villabona et al., 2020). This finding led Slye (2019) to conclude that a definition alone is not enough support for students to conceptualize different function types from a formal, set-theoretical perspective.

This body of research suggests that students *do* engage in coherence seeking across

situations that are considered (from a disciplinary perspective) as involving multiple kinds of function(s). So what causes students to engage in *these* forms of coherence seeking, rather than disciplinary forms of coherence seeking that would more readily foster a unified meta-narrative about function? One factor that has been identified in MVC is the (implicit) assumption by instructors that generalizing functions from two dimensions to three dimensions is relatively straightforward and spontaneous (Harel, 2021), despite a sizeable body of research evidence suggesting this is not the case (Dorko, 2023; Martínez-Planell & Gaisman, 2012; Montiel et al., 2009). Harel further observed that MVC textbooks tended to introduce key concepts without proper motivation and introduce conceptual shortcuts before providing students with sufficient context or motivation for them. Indeed, McGee et al. (2015) noted that in commonly used calculus textbooks, explicit conversations linking representations of single-variable and multivariable functions are virtually nonexistent.

Another institutional barrier presented in curricular stories that has been proposed is the compartmentalized nature of the post-secondary mathematics curriculum into separate courses, which often silo related core concepts across course boundaries (e.g., Hausberger, 2017). Going further, the *language* used for different types of functions *across* these course boundaries can vary in a dramatic way. In fact, in the latter half of the postsecondary curriculum, students encounter several function types that do not go by the name of “function”, including vector fields in MVC, transformations in linear algebra, homomorphisms in abstract algebra, and homeomorphisms in topology among others. In their comparative curricular research Ayalon and Wilkie (2019) concluded that:

[The] word ‘function’ might act as a connecting link or middle term . . . which then calls for all the work of the teacher and teaching resources to connect explicitly the word ‘function’ to students’ existing concept images and to multiple meanings through functional activities, to support an increasingly coherent and rich system of interrelated

meanings. (p. 16)

Although I am aware of no analogous study at the undergraduate level, this conclusion suggests students might benefit when consistent vocabulary is used to draw their attention—even at the surface level—to possible connections between new and old mathematical characters, as this might help trigger processes of coherence seeking. Fortus and Krajcik (2012) call this *language coherence*, arguing that

In coherent curricula it is important either to use terms in a consistent manner across all contexts or to explicitly clarify the different meanings the terms have in different places, why they are used in one place in one way and a different way in another place. (p. 51)

In other words, drawing students' attention to surface-level differences (i.e., vocabulary) might provide a “flag” for them to engage in further reflective coherence seeking across two seemingly unrelated notions.

Selected Curricular Stories and my Approach to Reading Them

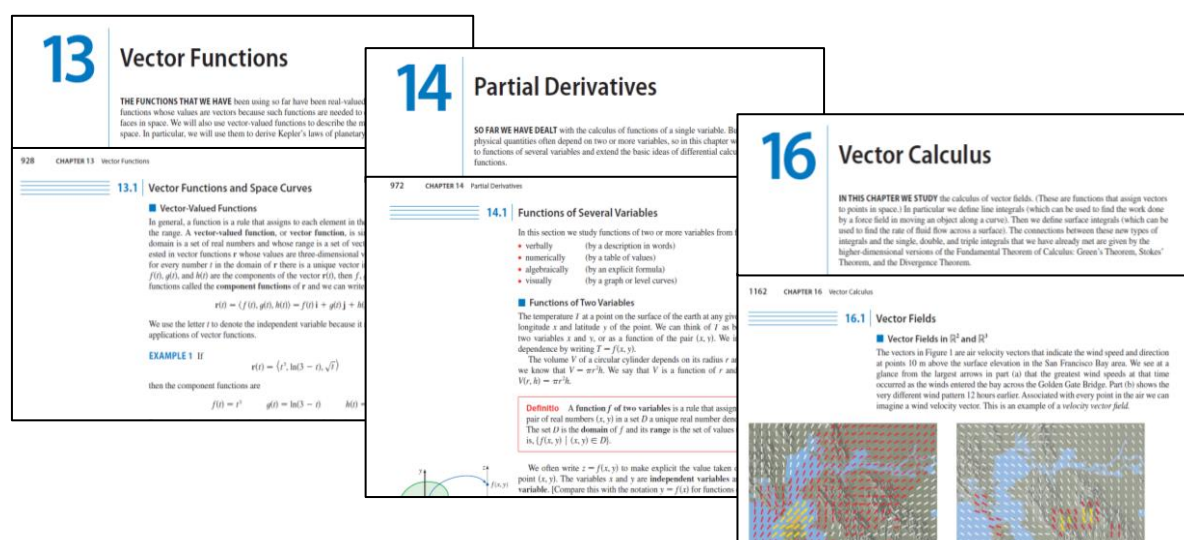
Textbook Stories of Interest

I analyzed the character introductions of three different multivariable function types in Stewart et al.'s (2021) *Calculus*, a commonly adopted textbook in U.S. undergraduate classrooms for teaching the calculus series (Mesa, 2010; Mkhathshwa, 2022). For brevity, I will refer to *Calculus* as “the textbook” from now on. By “character introduction”, I mean the first section within the unit in which each function type is introduced (e.g., 13.1). Hereafter, I refer to these sections as “chapters” as a reminder that I am conceptualizing the textbook as a literary story. The three chapters analyzed were: (1) 13.1: Vector Functions and Space Curves, within the Vector Functions unit, in which parametric, vector-valued functions are introduced; (2) 14.1: Functions of Several Variables, within the Partial Derivatives unit, in which multivariable, real-valued functions are introduced; and (3) 16.1: Vector Fields, within the Vector Calculus unit, in which vector fields are introduced. I also included the introductory page for each unit because

they immediately precede each chapter and briefly introduce each function type alongside the goals for the unit. Each of these pages included a large image with a caption followed by a few written sentences. See Figure 4.2 for a visual depiction of the text of the three textbook stories analyzed. MVC is covered in units 12–16, so these selected chapters are taken from various points in the overarching story of the intended curriculum for the course.

Figure 4.2

Visual Depiction of the Beginning of the Three Textbook Stories



Note. Excerpts from Stewart et al. (2021) pp. 927–928, 971–972, 1161–1162. The large images from each unit introduction have been omitted to save space and comply with copyright law.

Both Huffman Hayes (2024) and Lanius (in press) conducted analyses of curricular textbook stories of the definite integral (i.e., a mathematical concept) from the perspective of the curriculum-as-story metaphor. My analysis largely follows suit at a similar curricular grain size (i.e., chapter/unit vs. entire textbook). However, unlike both Huffman Hayes and Lanius, my character of interest is introduced *immediately* in the first chapter. Therefore, unlike how they determined their story (unit) of analysis by reading multiple chapters within a textbook unit until their character had been formally introduced, I reached this point in just one chapter per function type. While I am sensitive to Dietiker and Richman's (2021) critique that “analyses focused on

narrow parts of curricula can miss important large-scale features of a lesson that can substantially affect that part” (p. 326), I was careful to review subsequent chapters within each unit and noted that they *did not* feature subsequent attention to the three multivariable functions as primary characters to the degree the first chapter did. Regardless, for this exploratory study, my intent was to attend to the characters in the important moment when they are *first* introduced, for reasons I detail next.

Characters Do More than Just Tell a Story

In my reading of these textbook stories, I focus my attention on function *characters*. I do so, in part, because research on the psychology of reading has suggested that characters play an essential role in creating a vivid story landscape and, in turn, an immersive, aesthetically engaging reading experience (Alderson-Day et al., 2017; Gowers, 2012). In particular, characters serve as a primary conduit by which many readers connect with and learn from fictional stories (Gabriel & Young, 2011; Webber et al., 2022). Some have even proposed that readers’ judgements about the overall reliability and *coherence* of a story are strongly impacted by the believability of its characters. Fisher (1987), for example, called this *characterological coherence*:

A character may be considered an organizational set of actional tendencies. If these tendencies contradict one another, change significantly, or alter in ‘strange’ ways, the result is a questioning of character. Coherence in life and in literature requires that characters behave characteristically. (p. 47)

Similarly, Bal (2017) has suggested that established characters painted with a “detailed portrait” over the course of the story are more likely to be interpreted as *realistic* with “a certain measure of coherence” to them (p. 106). Yet, characters are more than just fictional, abstract constructions emanating from the pen of their author. They have agency in the sense that they act on and influence how their authors and any readers experience not only the story the character

originated from but also any subsequent stories (de Freitas, 2012; Pemberton et al., 2022). In other words, mathematical characters—and especially those that recur, such as function—have the potential to influence how students *read*, engage with, and judge the aesthetics of subsequent curricular stories. Hence, mathematical characters cannot be ignored when crafting cross-curricular mathematical stories.

Because characters are fleshed out based on their *relationship to* other story elements, such as the settings they inhabit or their interactions with other characters (Andrà, 2013; Bal, 2017; Dietiker, 2015a), I read stories *holistically*, considering not just characters but also any other story elements that stand out to me as a reader at a given moment. Specifically, I view different types of multivariable functions as distinct mathematical characters and read the stories of their character introductions *in relation* to one another (i.e., RQ1) with an eye to discerning the (in)coherence of messages or themes conveyed across these stories and how these themes align with the privileged meta-narrative of function as a unifying concept (i.e., RQ2).

I focus on character introductions rather than other parts of the textbook stories because, as with any story, the initial “meeting” a reader has with a character influences how they view this character and their role in the story going forward. Chrysanthou (2022), remarks, for example, that “the kind of information which readers receive when they meet a [character] for the first time in a narrative has a considerable impact on the way in which they look upon that character and consider the entire following story” (p. 29). Of course, some characters—in this case, some types of multivariable functions—develop and transcend their initial characterizations as the story progresses beyond their introduction; however, it is also the case that people’s first impressions have an oversized impact on how they view someone or something, owing in part to the well-studied human primacy bias (Demarais & White, 2007; Denrell, 2005). Additionally,

research has suggested that initial aesthetic and affective hooks play an important role in curricular sequencing and drawing students into curricular stories (Dietiker, 2016; Ryan & Dietiker, 2018). In other words, how stories are framed and how characters are introduced can have a large impact on the subsequent organization and structure of a mathematical story (e.g., Weinberg et al., 2016).

Methodology

I follow an arts-based research (ABR) paradigm (Chilton et al., 2015; Conrad & Beck, 2015, see Chapter 2). Specifically, I *read* curricular stories as a form of analytic interpretation and aesthetic critique of these stories while simultaneously creating artistic marginal notes in a variety of modalities (i.e., poetry, pictures, written notes, etc.). This arts-based practice supports holistic reflection in ways that allowed me to attend to not only my logico-rational judgements about a story but also my personal aesthetic sensibilities, how my emotions were stirred while reading, and any other extra-rational dimensions that factor into my experience as a reader. This framing is also consistent with Dietiker's (2012) initial characterization of her curriculum-as-story endeavors as humanities-oriented research (AERA, 2009) and fully enables me to investigate curricular artifacts as a form of *storied art* that ought to be read holistically and subjectively with attention to both its aesthetic *form* and *content* (Dietiker, 2015b; Eisner, 2004).

Reading as a Form of Analysis

For this textbook analysis, I intentionally set out to read mathematical stories in the textbook in much the same way I read literary novels, acknowledging critical and close *reading* as a powerful form of analysis (e.g., Gallop, 2000). In doing so, I position myself—*the reader*—as the primary instrument of analysis (Merriam & Tisdell, 2015). For this particular reading, I allowed myself to react, predict, wonder, and engage aesthetically with the developing plot. I also scribbled notes in the margins in much the same way I might while making sense of a dense

classic novel. Following other researchers who have used Dietiker's (2015a) curriculum-as-story framework for textbook analysis, I purposefully allowed myself to engage my didactical disciplinary literacies (Weinberg et al., 2023; Wiesner et al., 2020) and wonder from the perspective of students “reading” this story for the first time (e.g., Dietiker & Richman, 2021; Huffman Hayes, 2024; Miežys, 2023). For instance, I considered the prior events, characters, and other elements students would be acquainted with from engaging with past chapters in the textbook’s story while reflecting on how students might react aesthetically to the unfolding, sequencing, and structuring of the story, as well as the coherences or incoherences they might observe (through e.g., cheap plot tricks or plot holes, see Miežys, 2023).

At the same time, I also deviate from past users of Dietiker's (2015a) curriculum-as-story framework by fully embracing my own subjectivity, in line with the ABR paradigm. In other words, I do not apologize for or explain away the fact that I am reading mathematical stories as myself—a trained mathematician, teacher-researcher, and avid reader of stories. When I read stories, I am constantly making judgements about the characters as well as how the story has been constructed. When I’m invested, I’ll exclaim, “You fool! Don’t go into that house! It’s obviously haunted!” Or, on the opposite end of the spectrum when I’m reading a story that doesn’t engage me, I may wonder snarkily: “Why didn’t the author write the story like *this* and not *that*?” If I am frustrated or bored by a story, feeling like the author has led me down a particularly confusing or frustrating path that I do not want to go down (i.e., the textbook story is functioning as a *closed* text), I openly acknowledged this in my marginal notes. I am not alone in reading stories like this: Rosenblatt (1986) detailed an aesthetic stance toward reading where the readers “focus attention on what is being lived through in relation to the text *during* the reading event” (p. 124, italics in original), including the sounds of words, felt tensions while reading, and

imagery evoked by actions, characters, or particular settings, among other aesthetic reactions. Therefore, in reading mathematical stories, I purposefully choose *not* to shy away from recording aesthetic and emotional comments stemming from my engagement with a story or lack thereof via my marginal notes. My reason for this is not to insult the authors of the textbook but rather to ensure I am reading authentically, rather than holding back, as I investigate the aesthetics of (in)coherence within these particular textbook stories. This stance toward reading better allowed me to follow the elements of the text that “shone” (MacLure, 2013) and resonated with my experience, intuition, and sense as a disciplinary expert.

For each character introduction, I read the story of the chapter three times while making marginal notes. Given my familiarity with reading mathematics textbooks, including Stewart et al.'s (2021) *Calculus*, I would sometimes find myself drifting back into old habits of reading *effervently*, for content and logical coherence, rather than attending to coherence more holistically (and aesthetically) via the interplay between various literary elements. Whenever I found myself drifting to reading the textbook more like a technical manual (usually every hour or so), I purposefully read a literary short story to “refresh” my point of view and reorient myself to reading the textbook as a story.

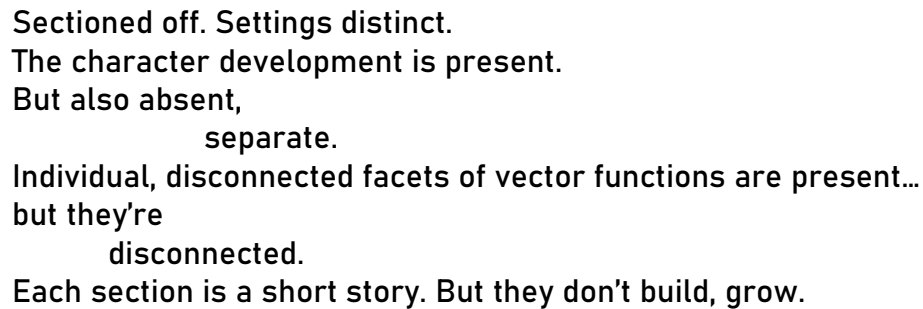
Arts-Based Reflective Analysis While Reading—Crafting Marginal Notes

Following an arts-based approach (Leavy, 2018; McNiff, 2018), I did not restrict the structure or modality of my marginal notes while reading. Instead, I focused on recording my thoughts, feelings, and observations in whichever visual arts modality I felt best captured my experience in the moment, leveraging various forms of visual art (Holm et al., 2018; Papoi, 2017) and the powerful interrelationship between text and visuals (McCloud, 1993) as needed to express or reflect on my reading experience. Sometimes, I would record a written comment or pose a question; other times, I would draw a sketch to represent my interpretations or write a

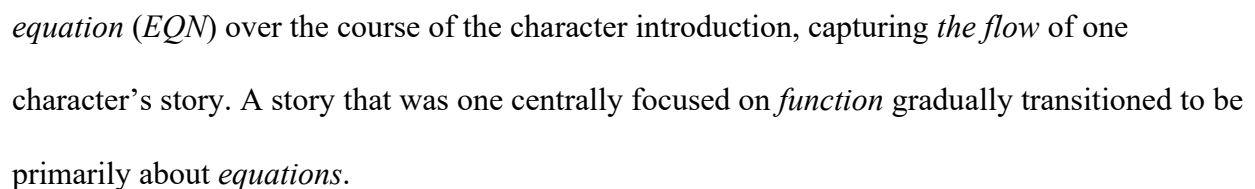
quick poem to artistically convey my reactions as a reader (see e.g., Figure 4.2). Each of these art forms and modalities has unique axiological and epistemological entailments and can therefore be seen as idiosyncratic ways of knowing and methods of inquiry (see e.g., Abreu, 2022a; Sousanis, 2012, 2015; Tremaine, 2022) that I chose to collage together. To add focus to my analysis, I attended primarily to the story elements outlined in Dietiker’s (2015a) framework—character, action, setting, plot—in choosing what to reflect and create notes about. However, I did not shy away from attending to other dimensions that were salient in my reading of each story, especially those related to my aesthetic interpretations of how the story was told or the overall moral of each story. This choice was in line with my intent to lean into my subjectivity and unique positionality and to take an “aesthetic stance” toward reading these mathematical stories. To give a sense of the nature of these marginal notes, I next discuss a few examples to further flesh out my arts-based approach to reading.

In Figure 4.3—*Failure to Reach Escape Velocity*—I reflected on the stop-start *rhythm* of the sections in Chapter 13.1 which introduced parametric, vector-valued functions. I used poetry to depict my frustration at what I perceived as incoherence between sections which did not seem to build to a greater purpose. To complement the poetry, I visually represented the persistent building tension that I experienced throughout each section, as well as the dissipation of this tension without payoff as each section ended and another began with seemingly no sense of forward trajectory or moral to the story. In Figure 4.4, *The EQN-FXN Flow*—I used line art to visually depict the story of Chapter 13.1 in another way in terms of the *nicknames* used for parametric, vector-valued functions. The art portrays the gradual—but very noticeable—transition from positioning this function character as a *function* (*FXN*) to positioning it as an

Failure to Reach Escape Velocity



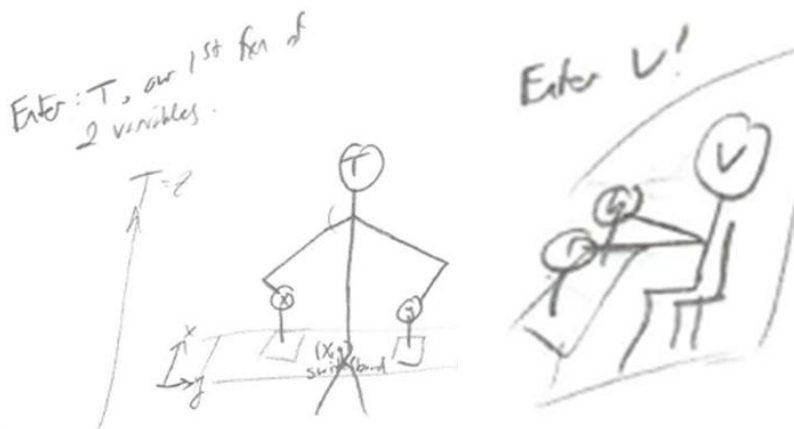
The FXN-EQN Flow



Finally, in Figure 4.5—*What’s a Functional Relationship Look Like Anyway?*—I share a drawing I used to visually reflect on the *relationships* between multivariable, real-valued functions (the functions T and V) and their character traits (domain, range, input, output, etc.). Specifically, my goal was to investigate whether the language of the textbook story was positioning these character traits as *characters* in their own right with relationships to the “main” function character, *or* merely ways to further characterize this function character.

Figure 4.5

What’s a Functional Relationship Look Like Anyway?



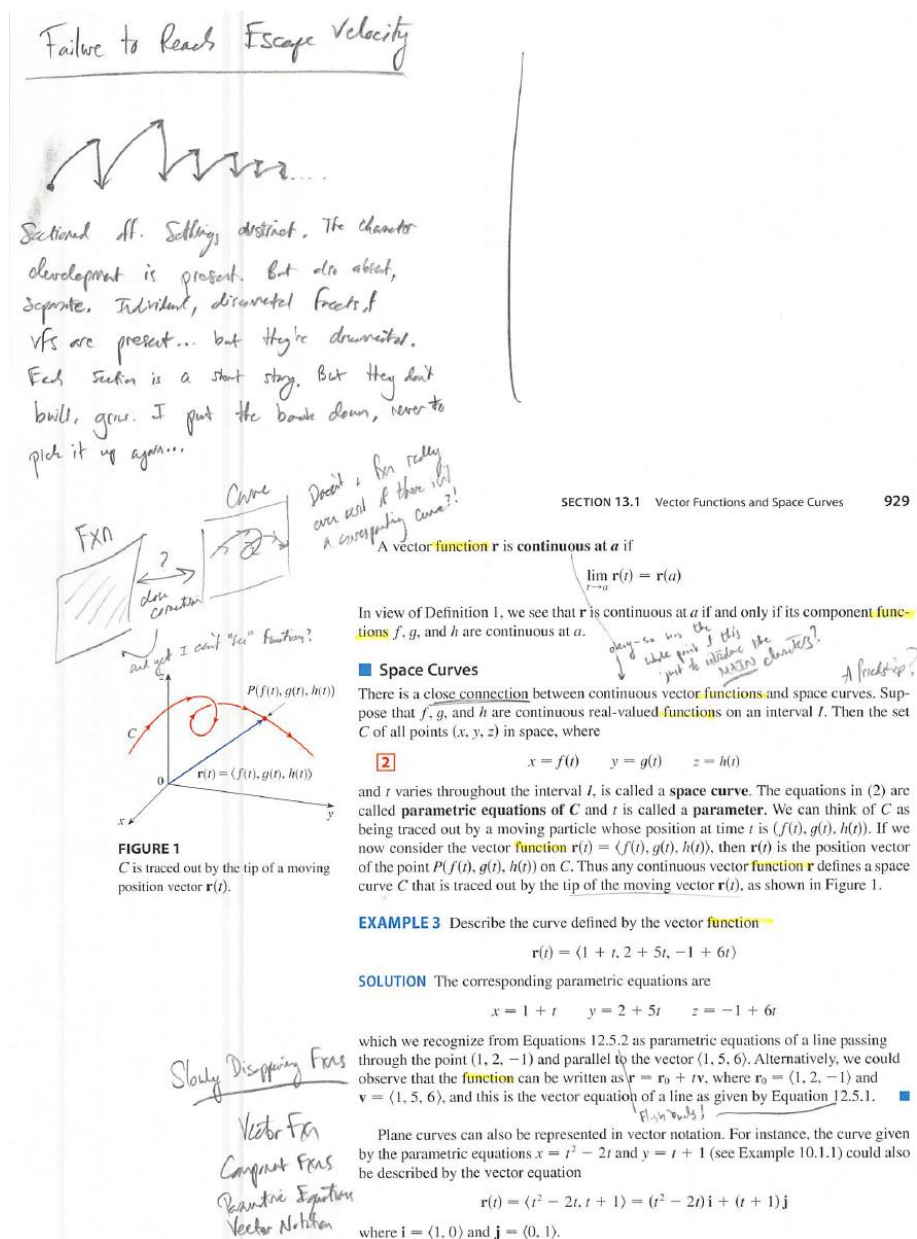
While the number of marginal notes on each page varied, most had at least two or three marginal notes in various locations on the page—sometimes in the actual margins and other times closer to the written words themselves. To ensure I had a sufficiently large artistic canvas while creating these reflective marginal notes, I printed the pages of each textbook story on 13 inch by 9 inch paper with extra blank space at the top. See Figure 4.6 for an example of one page of analysis from Chapter 13.1 which, in particular, shows the original context in which I created the *Failure to Reach Escape Velocity* marginal note.

Secondary Analyses and Sharing my Interpretive Readings

Following this open-ended analysis, I re-read and reflected across my marginal notes and

Figure 4.6

A Sample Page of Analysis with Several Marginal Notes



conducted secondary analyses to follow up on recurring themes and questions that had appeared across all the character introductions. In this paper, I share results from two such analyses: (1) a “nickname analysis” of the changing names and phrases used to refer to each character (e.g., function, equation, curve), how this evolved throughout each chapter, and what this conveyed

about each character's identity paired with (2) a plot structure analysis of each story. To share my reading, I first provide my plot analysis concerning the characterization of each function type (answering RQ1). I follow this with an overall analysis wherein I reflect on the meta-narratives about function(s) conveyed *across* these stories (answering RQ2).

Plot Analyses

Parametric, Vector-Valued Functions

Within the first few words of the unit introduction, this new type of character is introduced as something different from the existing cast of functions:

The functions that we have been using so far have been real-valued functions. We now study functions whose values are vectors because such functions are needed to describe curves and surfaces in space. We will also use vector-valued functions to describe the motions of objects through space. (p. 927)

From the first few words of this story, the purpose of these new function characters is clearly signposted: they are used to describe *geometric* objects, like curves and surfaces, which can be used to *model physical phenomena* including the motion of objects. Little else is revealed about these function characters in this brief introduction, aside from the characterization that their values (i.e., outputs) are vectors, distinguishing them from previously introduced single-variable functions whose values were real numbers that have featured so far in the overarching story of (multivariable) calculus. Flipping to the next page, the chapter title, “Vector Functions and Space Curves” continues to foreshadow that these new types of functions likely have an intimate relationship with geometric curves. However, it is not until a little over one page and two short sections later that this relationship is laid bare.

Before this foreshadowing gets paid off, the chapter opens with an approximately half-page section entitled “Vector-Valued Functions” in which vector(-valued) functions are briefly introduced and fleshed out. See Figure 4.7 for the full depiction of this character introduction,

Figure 4.7

Formal Character Introduction to Parametric, Vector-Valued Functions

In general, a function is a rule that assigns to each element in the domain an element in the range. A **vector-valued function**, or **vector function**, is simply a function whose domain is a set of real numbers and whose range is a set of vectors. We are most interested in vector functions $\mathbf{r}$ whose values are three-dimensional vectors. This means that for every number t in the domain of $\mathbf{r}$ there is a unique vector in V_3 denoted by $\mathbf{r}(t)$. If $f(t)$, $g(t)$, and $h(t)$ are the components of the vector $\mathbf{r}(t)$, then f , g , and h are real-valued functions called the **component functions** of $\mathbf{r}$ and we can write

$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t) \mathbf{i} + g(t) \mathbf{j} + h(t) \mathbf{k}$$

We use the letter t to denote the independent variable because it represents time in most applications of vector functions.

Note. Excerpt from Stewart et al. (2021) p. 928

which is notably unboxed. Without further context or transition, readers are immediately met with a quick sentence reminder about the nature of a “general” function using a *correspondence* characterization which highlights how a function is a rule relating the elements of the domain and range. Afterwards, vector(-valued) functions are introduced formally in terms of this correspondence characterization: “A **vector-valued function** or **vector function**, simply a function whose domain is a set of real numbers and whose range is a set of vectors” (p. 928).⁵⁹ My experience suggests that this new character is *far* from “simple” for many student readers to become acquainted with, and I suspect it does not help matters that it is interchangeably referred to using two names from the beginning of the story without any rationale for why.⁶⁰ Fortunately,

⁵⁹ I use the phrase “parametric, vector-valued function” to distinguish between the type of vector-valued functions introduced in this textbook chapter and vector *fields*, a distinct type of vector-valued function introduced later in the textbook’s story of MVC. Each of these function types have vector outputs, but parametric, vector-valued functions have only scalar (real-valued) inputs while vector *fields* can also take vector inputs. Notably, the definition Stewart et al. use here to characterize vector(-valued) functions clashes with mine and *does not* include vector fields as examples of vector-valued functions (even though they have vector outputs). I unpack the consequences of this choice later in my plot analysis of the story in which these authors introduce vector fields.

⁶⁰ The unit introduction unveils these as “vector-valued functions”, then a few lines later at the top of the next page, the title of Chapter 13.1 drops the compound adjective, calling them “vector functions”, and just a line later in the

the story adds further detail as it transitions to verbally unpack what the previously introduced definition means in terms of inputs, outputs, the domain, and an implicit reference to the codomain (i.e., V_3 , even though “range” is not explicitly mentioned). Shortly thereafter, the symbolic terminology $\mathbf{r}(t)$ is introduced to refer to the vector output of such vector functions, and readers are introduced to the *action* of breaking a vector function into the single-variable, real-valued components functions which “make up” said vector function— $f(t)$, $g(t)$, and $h(t)$. The importance of this action is made clear with how “**component functions of $\mathbf{r}$** ” is bolded, suggesting it is on the same level of importance as the *name(s)* of the function character itself.

Indeed, after component functions are introduced, the primary action carried out with vector functions through the remaining story involves quickly reducing them to their component functions for further computation. This treatment of vector-valued functions as effectively a collection of several single-variable, real-valued functions sends a strong message about the defining traits of such functions: they decompose into multiple other single-variable functions and *these* functions are the most important. The example immediately following the definition of vector functions, for instance, features such a decomposition into component functions to determine the domain of a particular vector function. However, this is the first and final instance in the chapter where the component functions are used to discern the character traits of a vector

title of the first section of the chapter, the compound adjective is restored. The formal definition used in the text clarifies soon after that these are both names for the same character after which “vector function” is used exclusively throughout the remainder of the chapter (with the adjective “vector-valued” only being used two more times in the remainder of textbook). Even as a seasoned reader of mathematics stories, I felt whiplash because of this back-and-forth use of different names. Then, when the adjective “vector-valued” disappeared entirely I was confused, wondering if I had missed something. After re-reading the previous few sentences, I realized I had missed nothing—at least nothing written explicitly into the story being told. The authors do little to guide readers through why the compound adjective is dropped here. For readers familiar with this terminology, this omission may have less of an impact, but this is also the *first* time that “real-valued” is used to refer to all prior function types that had been introduced as characters in the story of (multivariable) calculus so far. Given the newness of this terminology to refer to the nature of elements in the range, I suspect this sleight of hand obscures rather than simplifies many readers’ introduction to this character. At any rate, from now on in this plot analysis, I follow the authors’ lead and use “vector function” instead of “vector(-valued) function”.

function. As the “Vector-Valued Functions” section ends, the story takes a brief detour to a section on the limits and continuity of vector functions, beginning with another instance of breaking a vector function into its real-valued component functions (see Figure 4.8). Unlike the definition provided for a vector function, *the limit of a vector function* involving this action of decomposition is boxed, suggesting (at least implicitly) that the character of vector functions play primarily a passing or secondary role in the story of calculus. After all, calculating such a limit barely requires any acknowledgement that vector functions are new characters at all, as long as one remembers that they can be re-written as three single-variable functions and dealt with using almost exclusively knowledge of single-variable functions.

Figure 4.8

Example of Decomposing Vector Functions into their Component Functions

The **limit** of a vector function $\mathbf{r}$ is defined by taking the limits of its component functions as follows.

1 If $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$, then

$$\lim_{t \rightarrow a} \mathbf{r}(t) = \left\langle \lim_{t \rightarrow a} f(t), \lim_{t \rightarrow a} g(t), \lim_{t \rightarrow a} h(t) \right\rangle$$

provided the limits of the component functions exist.

Note. Excerpt from Stewart et al. (2021) p. 928

The subsequent section further relegates vector functions to their role as supporting characters which are used mostly to introduce and subsequently support the *true* main character of this chapter: space curves. As foreshadowed by the unit introduction, this section begins by introducing space curves in relation to *continuous* vector functions, (see Figure 4.9) as C , the set of points $(x, y, z) = (f(t), g(t), h(t))$, where t varies over the interval, I , on which f , g , and h are continuous. This definition presents another example of how the single-variable component functions of the vector function $\mathbf{r}(t)$ are emphasized rather than the multivariable function itself.

Figure 4.9

Space Curves are Introduced in Relation to Continuous Vector Functions

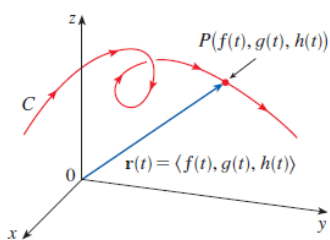


FIGURE 1

C is traced out by the tip of a moving position vector $\mathbf{r}(t)$.

■ Space Curves

There is a close connection between continuous vector functions and space curves. Suppose that f , g , and h are continuous real-valued functions on an interval I . Then the set C of all points (x, y, z) in space, where

$$\boxed{2} \quad x = f(t) \quad y = g(t) \quad z = h(t)$$

and t varies throughout the interval I , is called a **space curve**. The equations in (2) are called **parametric equations of C** and t is called a **parameter**. We can think of C as being traced out by a moving particle whose position at time t is $(f(t), g(t), h(t))$. If we now consider the vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$, then $\mathbf{r}(t)$ is the position vector of the point $P(f(t), g(t), h(t))$ on C . Thus any continuous vector function $\mathbf{r}$ defines a space curve C that is traced out by the tip of the moving vector $\mathbf{r}(t)$, as shown in Figure 1.

Note. Excerpt from Stewart et al. (2021) p. 929

For one, the interval, I , is presented primarily as a subset of the domain(s) of f , g , and h without any explicit reference to how it is *simultaneously* a subset of the domain of $\mathbf{r}(t)$. The variable t is introduced as a *parameter* in relation not to its status as the independent variable of the multivariable vector function $\mathbf{r}(t)$ but as the independent variable for each component function. Of course, I am referring to mathematical equivalent statements; however, the story of this chapter (at least up until the *formal* definition given here for space curves) has almost entirely dropped any explicit references back to $\mathbf{r}(t)$ as initially characterized as either a function with a distinct domain and range *or* a function whose outputs are vectors. And this occurred only one page after readers are first introduced to these new types of functions, where the story explicitly acknowledges that these new multivariable functions deviate from all other (real-valued) function characters readers would be familiar with from engaging with the story of calculus thus far.

As depicted in the second half of the text as well as the graphical curve shown in Figure 4.9, however, the characterization of $\mathbf{r}(t)$ as having vector outputs does make a brief comeback to suggest a visual *intuition* and meaning for a space curve if $\mathbf{r}$ is interpreted as representing a position vector. Namely, as indicated in the provided diagram, “ C is traced out by the tip of a

moving position vector $\mathbf{r}(t)$ ”, suggesting a more *covariational* characterization of this vector function, where the output (i.e., *tips* of position vectors in $\mathbb{R}^3$) covary with the input (i.e., the parameter t) as t spans the domain, I . The visual characterization of space curves persists throughout the remaining five pages of the chapter detailing several largely computational examples involving describing and/or sketching space curves given by a particular vector function, however, this covariational characterization largely does not.⁶¹ While space curves take center stage for the remaining 80% of the pages in the chapter, the moment in the story just outlined is effectively the last gasp for vector functions to play a notable role. After the relationship between these two characters has been established, “vector function” only appears two more times, both in examples concerning space curves where the multivariable function is quickly reduced once again to its single-variable component functions to consider the parametric equations needed to sketch a corresponding space curve. Across the entire story, “function” is mentioned only about half as frequently as “curves” and about the same number of times as (parametric) “equation”.

Multivariable, Real-Valued Functions

The character of multivariable, real-valued functions, on the other hand, takes center stage throughout its chapter—from beginning to end. The unit introduction begins by situating the story of this chapter relative to past calculus stories involving single variable functions:

So far we have dealt with the calculus of functions of a single variable. But, in the real world, physical quantities often depend on two or more variables, so in this chapter, we turn our attention to functions of several variables and extend the basic ideas of differential calculus to such functions. (p. 971)

In addition to motivating this turn toward multivariable functions based on how functions can

⁶¹ Aside from a few passing mentions, that is. At the conclusion of Example 6 (p. 931), for example, the text notes how “The arrows . . . indicate the direction in which C is traced as the parameter t increases”.

model physical (“real world”) *phenomenon*, the unit image also depicts sweeping sand dunes with a caption suggesting that “a function of two variables can describe the shape of a surface like the one formed by these sand dunes” (p. 971). Unlike both other chapters analyzed, this one starts *immediately* with a detailed statement about the goals and structure of the chapter: “In this section, we study functions of two or more variables from four points of view: verbally, numerically, algebraically, and visually” (See Figure 4.10 for how this is formatted in text). Immediately, this phrasing positions the *settings* to appear throughout this chapter (e.g., words, tables, formulas, graphs, and level curves) as being stages on which multivariable, real-valued functions get to be in the spotlight (in marked contrast to the relationship between parametric, vector-valued functions and geometric space curves, where the vector functions played only a supporting role). Notably, this is also the same framing that is used in the very first (single variable calculus) chapter of the textbook (i.e., Chapter 1.1 titled “Four Ways to Represent a Function”, p. 8) to introduce the many different *settings* in which single-variable functions could be represented. Consistent with the framing from the unit introduction which suggests that this new chapter will extend past ideas to a new type of function, this parallel framing device draws a clear connection between prior characterizations of (real-valued) functions as readers have encountered them previously in the story of calculus and the new, multivariable types of function they are about to become acquainted with. The implication is clear: multivariable, real-valued functions will be similar in some ways to their predecessors—real-valued, single-variable functions—at least insofar as they can be viewed simultaneously from several different representations, each with their own benefits and drawbacks.

The differences between this story and the previous one continue as the chapter begins its

Figure 4.10

Framing Device for Chapter Introducing Multivariable, Real-Valued Functions

14.1 | Functions of Several Variables

In this section we study functions of two or more variables from four points of view:

- verbally (by a description in words)
- numerically (by a table of values)
- algebraically (by an explicit formula)
- visually (by a graph or level curves)

Note. Excerpt from Stewart et al. (2021) p. 972

first proper section (“Functions of Two Variables”) with a couple of brief but concrete examples of multivariable, real-valued functions introduced both *verbally* and *symbolically* using this as an opportunity to present $z = f(x,y)$ -type notation while simultaneously drawing further attention to how these new types of functions can *model* realistic *physical phenomena* (e.g., how temperature on the surface of the earth varies with the variables latitude and longitude). These two examples additionally demonstrate a commitment to depicting multiple representations of these new function types simultaneously, cohering with the previously introduced narrative frame. After both examples conclude, the story continues with a definition box offering a formal characterization of “a function f of two variables” (See Figure 14.11). Unlike in the story introducing parametric, vector-valued functions, the formal characterization of multivariable, real-valued functions of two variables here is boxed, further emphasizing the importance of this new function character. Similar to the characterization of vector-valued functions, though, these multivariable functions are also introduced using a *correspondence* definition as “a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$ ” (p. 972). In the next line of the definition, both domain *and* range are explicitly bolded, with range being explicated in not only words but also using formal set-builder notation, suggesting specifically that these functions are *set-theoretic* in nature. Just outside of the box, the meaning

Figure 14.11

*Formal Characterization of a Function of Two Variables*⁶²

Definitio A function f of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is, $\{f(x, y) \mid (x, y) \in D\}$.

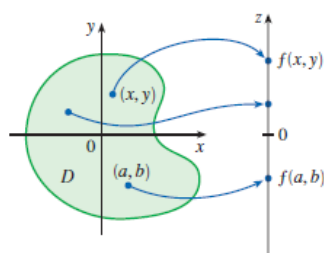


FIGURE 1

We often write $z = f(x, y)$ to make explicit the value taken on by f at the general point (x, y) . The variables x and y are **independent variables** and z is the **dependent variable**. [Compare this with the notation $y = f(x)$ for functions of a single variable.]

A function of two variables is just a function whose domain is a subset of $\mathbb{R}^2$ and whose range is a subset of $\mathbb{R}$. One way of visualizing such a function is by means of an arrow diagram (see Figure 1), where the domain D is represented as a subset of the xy -plane and the range is a set of numbers on a real line, shown as a z -axis. For instance, if $f(x, y)$ represents the temperature at a point (x, y) in a flat metal plate with the shape of D , we can think of the z -axis as a thermometer displaying the recorded temperatures.

If a function f is given by a formula and no domain is specified, then the domain of f is understood to be the set of all pairs (x, y) for which the given expression defines a real number.

Note. Excerpt from Stewart et al. (2021) p. 972

of the symbolic “ $z = f(x, y)$ ” is explained and unpacked and the independent (x and y) and dependent (z) variables are introduced (in bold), followed immediately by a bracketed aside that encourages a reader to draw yet another comparison between multivariable, real-valued functions and the previously introduced single variable, real-valued functions: “Compare this with the notation $y = f(x)$ for functions of a single variable” (p. 927).

The following paragraph moves to immediately supplement this initial characterization of multivariable, real-valued functions introduced formally and symbolically using a *visual* arrow diagram representation to further highlight these functions as a *correspondence* between their domain (a subset of $\mathbb{R}^2$, depicted on the xy -plane) and range (a subset of $\mathbb{R}$, depicted on the z -axis). This visual characterization serves to accentuate the importance of the domain and range of these functions by depicting these character traits *visually* to complement and expand on their initial symbolic and set-theoretic definitions. Further, this characterization foreshadows the

⁶² Note that in the official electronic version of the textbook, the “n” is missing from “Definition” in *all* definition boxes, including this one. This typo is a quirk unique to this edition of the textbook.

graphical, visual characterization of these types of multivariable functions in three-dimensional space spanned by the x -, y -, and z -axes (i.e., $\mathbb{R}^3$), which is soon introduced in the next section two pages later. Before that section, though, the story proceeds through four examples that further flesh out the character of multivariable, real-valued functions with recurring, explicit references to their set-theoretic nature (i.e., the domain and range) across all four representational settings foreshadowed by the framing narrative. For instance, the prompt of Example 2 is to “find the domain and range” of a particular multivariable function, $g(x,y)$, and in this example the domain is depicted symbolically using set-builder notation *as well as* visually in $\mathbb{R}^2$ while the range is depicted symbolically in set-builder notation *and* interval notation. Subsequent examples across the story of the chapter continue this trend, including Example 8: “Find the domain and range and sketch the graph of $h(x,y)$ ” (p. 976). Even examples that are not explicitly framed in terms of *finding* the domain and range continue to feature comments about the domain (and less so the range), such as Example 4, which concludes with a note that the domain of the function of interest is “ $\{(L, K) \mid L \geq 0, K \geq 0\}$ because L and K represent labor and capital and are therefore never negative” (p. 975).

Across the remainder of this twelve-page story (which is not only double the length of the other two stories introducing new types of vector-valued functions *but also* one of the longest chapters in the textbook’s story of MVC), clear transitions are employed to move the story along between sets of examples that help make this story feel like a much smoother read compared to the more fragmented and unmotivated sections of the chapter introducing parametric, vector-valued functions (which, as I noted in my marginal note shown in Figure 4.3, felt like it failed to reach to ever reach “escape velocity”). In contrast to the confusing whiplash I alluded to in the prior plot analysis, this chapter is an almost pleasant read with a steady rhythm, thanks to these

repeated transitions between major events that continually refer back to the framing of the chapter as an investigation of multivariable, vector-valued functions across several representational settings. After examples 1 and 2 which depicted these multivariable functions in primarily *visual* and *symbolic* settings, examples 3 and 4 feature them in *tabular* and *verbal* settings in the context of *modeling physical phenomena*. The text signals these changing settings with two quick sentences: “Not all functions can be represented by explicit formulas. The function in the next example is described verbally and by numerical estimates of its values” (p. 973). Another such transition appears at the beginning of the next section immediately after Example 4 to demarcate the shift to the *visual* setting as *graphs* are formally introduced (See Figure 4.12): “Another way of visualizing the behavior of a function of two variables is to consider its graph” (p. 975). Immediately afterwards, the formal definition for a graph of a multivariable, real-valued function is introduced, boxed, building explicitly on the *set-theoretic* characteristic of the *domain* of each of these functions. This verbal and symbolic definition is accompanied soon after by a visual depiction in the margin of such a graph as a geometric *surface* “lying directly above or below its domain D in the xy -plane” (p. 975). Notably, the focus on the function’s domain is present throughout each of these descriptions, establishing a continuity of characterization of multivariable, real-valued functions (as *set-theoretic*) across representational settings. Several graphical examples follow spanning the next three pages at which point the story transitions to the penultimate section concerning one final *visual* setting by taking stock of all previous visual settings that had been explored: “So far we have two methods for visualizing functions: arrow diagrams and graphs. A third method, borrowed from mapmakers is a *contour map* on which points of constant elevation are joined to form *contour curves*, or *level curves*” (p. 977, emphasis in original).

Figure 4.12

The Story Formally Introduces the Graph of a Multivariable, Real-Valued Function

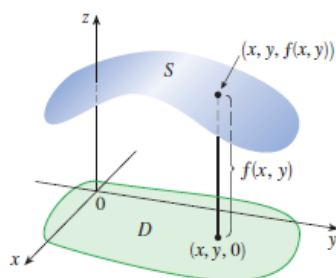


FIGURE 5

■ Graphs

Another way of visualizing the behavior of a function of two variables is to consider its graph.

Definitio If f is a function of two variables with domain D , then the **graph** of f is the set of all points (x, y, z) in $\mathbb{R}^3$ such that $z = f(x, y)$ and (x, y) is in D .

The graph of a function f of two variables is a surface S with equation $z = f(x, y)$. We can visualize the graph S of f as lying directly above or below its domain D in the xy -plane (see Figure 5).

Note. Excerpt from Stewart et al. (2021) p. 975

Finally, after several examples depicting multivariable, real-valued functions of *two* variables in this final visual setting, the chapter concludes with a short (two-page) section which formally characterizes two related (but new) function characters. The story first introduces a definition for real-valued, multivariable functions of *three* variables and shortly afterwards introduces a definition for such functions of *any number* of variables. The definition for a real-valued, multivariable function of three variables mirrors the formal definition used to introduce its two-variable kin using the exact *correspondence* phrasing employed before (recall Figure 4.11) with only minor changes to the dimensionality of the domain made to account for the additional variable (i.e., elements of this function’s domain are *ordered triples* in $\mathbb{R}^3$). The definition characterizing this new function is unboxed, however, hinting that perhaps this character is not as important as the multivariable function of two variables that has featured up until now.

After just a few very brief examples across one page fleshing out these function types across a few representational settings—symbolic and then in terms of level surfaces, a three-dimensional analog of level curves—the story of the chapter wraps up by first noting that “functions of any numbers of variables can be considered. A **function of n variables** is a rule

that assigns a number $z = f(x_1, x_2, \dots, x_n)$ to an n -tuple $(x_1, x_2, \dots, x_n)$ of real numbers. We denote by $\mathbb{R}^n$ the set of all such n -tuples. . . . The function f is a real-valued function whose domain is a subset of $\mathbb{R}^n$ ” (pp. 983–984, emphasis in original). This definition and characterization appear on the last half page, unboxed, and the story seems to leave it up to reader interpretation whether this characterization is meant to suggest that *each* choice for n (i.e., $n = 2, 3, \dots$) generates a *distinct* character or if this definition is being introduced to clarify that *all* functions of this form—i.e., *all* multivariable, real-valued functions of two, three, four, or *any* number of variables—are actually instances of the same character (or at least *family* of characters). What is left unambiguous, however, is that each of these characters are, indeed, functions and have ties back to prior functions encountered in the story of mathematics and specifically single-variable calculus. Indeed, across the chapter, “function” is far and away the most common name used to refer to multivariable, real-valued functions (87 instances). The use of this name does not subside, even as “level surfaces” (51 instances) and geometric names (such as “graph” or “surface”, 44 instances) are introduced as competing alternatives.

Vector Fields

The caption for the image featured in the vector calculus unit introduction immediately states how “vector fields can be used to model such diverse phenomena as gravity, electricity and magnetism, and fluid flow” (p. 1161). Nearby, the first sentence of the main text reads, “In this chapter we study the calculus of vector fields (These are functions that assign vectors to points in space)” (p. 1161). The use of parentheses to convey the technical details persists throughout this paragraph and feels almost like the narrator is whispering to the reader, implicitly conveying that perhaps the function definition of vector fields is a sidenote. The emphasis on introducing vector fields for the purpose of *modeling physical phenomena* from engineering and physics contexts carries into the beginning of the chapter introducing vector fields, which features nearly a full

page of four example velocity fields depicted *visually* alongside written interpretations of what meaning the plotted vectors convey (See Figure 4.13). Notably, these visual depictions nor the accompanying text refer to any of these vector fields as functions. Rather, the emphasis is on the nature of the vectors themselves and interpreting what they mean in context. For example, the chapter starts by stating that

The vectors in [the first velocity field depicted in Figure 4.13] are air velocity vectors that indicate the wind speed and direction at points 10 m above the surface elevation in the San Francisco Bay area. We see at a glance from the largest arrows in part (a) that the greatest wind speeds at that time occurred as the winds entered the bay area across the Golden Gate Bridge” (p. 1162)

However, this example (as well as one of the four other examples given on this page) conclude by briefly interpreting the vector field as a *correspondence* between points and vectors:

“associated with every point in the air we can imagine a wind velocity vector” (p. 1162). Even though the word “function” is not explicitly used, this phrasing might remind some readers of a common prior characterization of functions.

On the following page, the story formally characterizes vector fields as functions in a way that leans further into this *correspondence* characterization: “In general, a vector field is a function whose domain is a set of points in $\mathbb{R}^2$ (or $\mathbb{R}^3$) and whose range is set of vectors in V_2 (or V_3)” (p. 1163). Though unboxed, this statement serves to characterize vector fields as *set-theoretic* in nature as a relation between a domain (i.e., a set of points) and a range (i.e., a set of vectors).⁶³ Immediately afterwards, the story proceeds to formally define and characterize vector fields on $\mathbb{R}^2$ and on $\mathbb{R}^3$; however, these introductions are done separately and even feature separate boxed definitions. This initial (unboxed) statement about a *general* vector field suggests

⁶³ As alluded to in the plot analysis of the prior textbook story that introduced parametric, vector-valued functions (recall Footnote 52), this story *does not* refer to vector fields explicitly as “vector-valued” even though vector fields are characterized as having vector outputs. This serves to position vector fields as “outside” of the classification of function types as either real- or vector-valued that was introduced in the prior story.

Figure 4.13

Vector Fields are Introduced Visually as Depicting Physical Phenomena

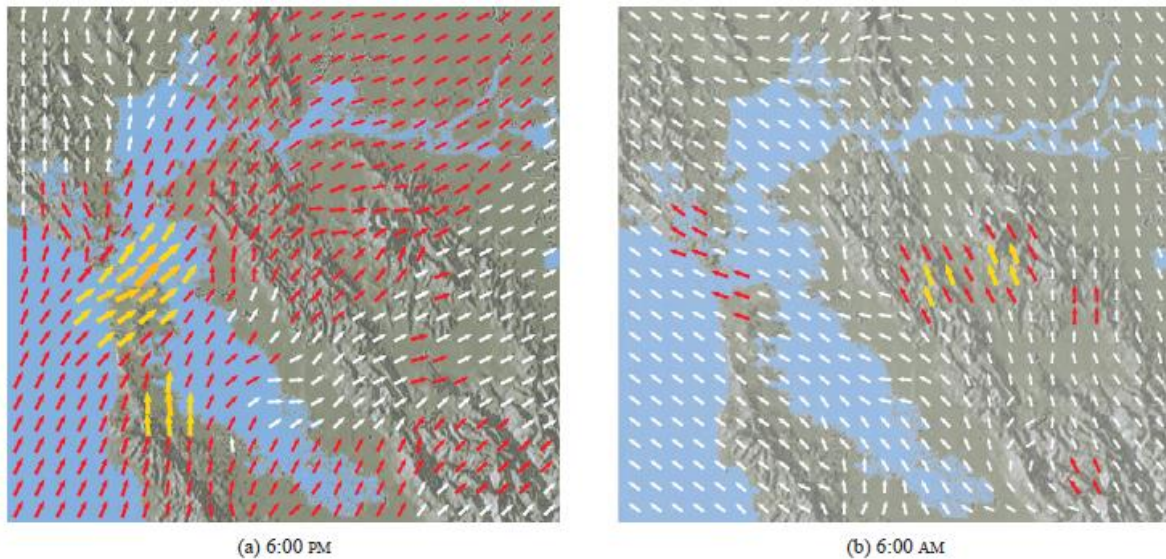


FIGURE 1 Velocity vector fields showing San Francisco Bay wind patterns on a particular spring day

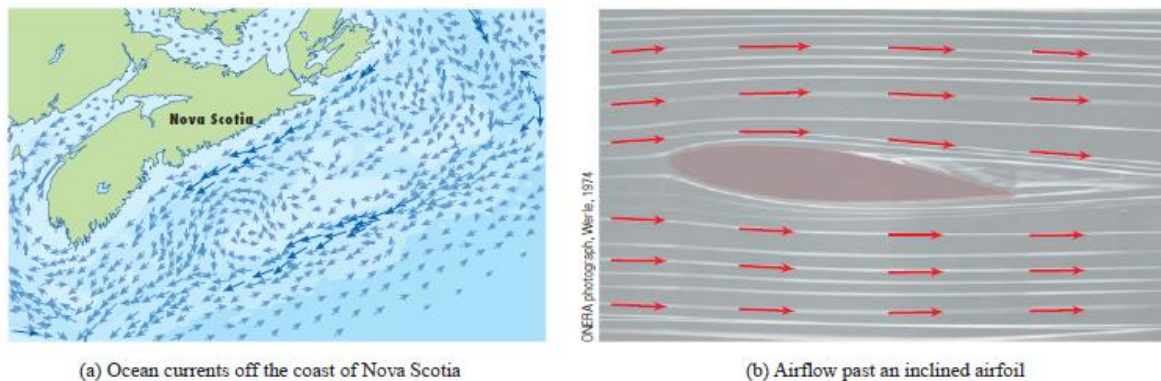


FIGURE 2
Velocity vector fields

Note. Excerpt from Stewart et al. (2021) p. 1162

that these characterizations refer to the same or at least closely related characters (deviating from the characterizations of multivariable, real-valued functions of two, three, and more variables from the previously analyzed chapter). At the same time, the separately boxed definitions seem to imply the opposite, insinuating that these types of functions are different enough to require distinct characterizations.

At any rate, the story first proceeds to characterize vector fields on $\mathbb{R}^2$ (see Figure 4.14).

This begins with a formal boxed definition introducing these as a “function $\mathbf{F}$ that assigns to each point (x, y) in D a two-dimensional vector $\mathbf{F}(x, y)$ ” (p. 1163) leaning into a *correspondence* characterization of these functions with a less explicit focus on any sets, such as the domain or range. Next, the story takes a moment to unpack this definition and detail how these vector fields can be pictured by drawing the vector output $\mathbf{F}(x, y)$ starting at every such (x, y) , using an accompanying *visual* representation on $\mathbb{R}^2$ in the margin to depict this process of assigning a vector output to each ordered pair input (see Figure 4.14). Much like with parametric, vector-valued functions, the action of decomposing vector fields into component functions is depicted symbolically to wrap up this characterization. The importance of this action is conveyed by not only bolding “component functions” (as in the prior chapter that introduced parametric, vector-valued functions) *but also* by specifying the nature of these component functions, $P(x, y)$ and $Q(x, y)$, as “scalar” functions (or fields) of two variables, to clarify that these functions output scalars and are not themselves vector fields. While the phrase “scalar fields” is bolded, minimal fanfare or further explanation is provided for the need for this new term except to demarcate that the component functions of a vector fields are not themselves vector fields. While these component functions are always multivariable, real-valued functions in this textbook’s story of MVC, no explicit connection is made between vector fields and the fact that their component functions are multivariable, real-valued functions, which had been the focus of the two units immediately preceding this one on vector calculus. As a reader at this point, I was left wondering “Why?” Did all these different and seemingly disconnected adjectives describing types of functions need to be introduced? In this moment, it felt like they were serving to clog the story with alternative nicknames for pre-established characters which served to muddy the introductions to the *new* character of vector fields. Consequently, I was left uncertain about the

Figure 4.14

Formal Characterization of Vector Fields on $\mathbb{R}^2$

1 Definitio Let D be a set in $\mathbb{R}^2$ (a plane region). A **vector field on $\mathbb{R}^2$** is a function $\mathbf{F}$ that assigns to each point (x, y) in D a two-dimensional vector $\mathbf{F}(x, y)$.

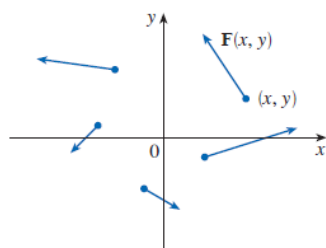


FIGURE 3
Vector field on $\mathbb{R}^2$

The best way to picture a vector field is to draw the arrow representing the vector $\mathbf{F}(x, y)$ starting at the point (x, y) . Of course, it's impossible to do this for all points (x, y) , but we can form a reasonable impression of $\mathbf{F}$ by drawing vectors for a few representative points in D as in Figure 3. Since $\mathbf{F}(x, y)$ is a two-dimensional vector, we can write it in terms of its **component functions** P and Q as follows:

$$\mathbf{F}(x, y) = P(x, y) \mathbf{i} + Q(x, y) \mathbf{j} = \langle P(x, y), Q(x, y) \rangle$$

or, for short,

$$\mathbf{F} = P \mathbf{i} + Q \mathbf{j}$$

Notice that P and Q are scalar functions of two variables and are sometimes called **scalar fields** to distinguish them from vector fields.

Note. Excerpt from Stewart et al. (2021) p. 1163

overall importance of these new characters to the overall story of MVC.⁶⁴

Next, the textbook story proceeds (for the next one-and-a-half pages) to step through one example of sketching a vector field on $\mathbb{R}^2$, followed by a second example of sketching a vector field on $\mathbb{R}^3$, both of which are provided initially in *symbolic* notation. In the first example, a *tabular* representation is used to list ordered pairs and their corresponding function outputs for $\mathbf{F}(x, y) = \langle -y, x \rangle$ and a side-by-side *visual* sketch of the vector field is offered to depict the *process* of sketching a vector field (see Figure 4.15). In the second example, the tabular characterization is skipped, and the story jumps straight into a *visually* sketched representation. Throughout these examples, the story devotes minimal attention to correspondence between inputs and outputs. Instead, the story proceeds to center the *interpretation* of the vectors making up each of these vector fields.

⁶⁴ Indeed, similar to the terminologies “vector-valued functions” and “real-valued functions” which are used sparingly after their introduction, the names “scalar function” and “scalar field” are only used seven additional times in the textbook’s story of MVC. Despite “scalar field” being bolded at this point in the story, though, the phrase “scalar function” is the only one to appear later in this chapter, as I detail momentarily. As in the previous chapter introducing vector-valued, parametric functions, there is a sense of nickname whiplash present here, too.

Figure 4.15

Tabular and Visual Characterizations of an Example Vector Field on $\mathbb{R}^2$

(x, y)	$\mathbf{F}(x, y)$	(x, y)	$\mathbf{F}(x, y)$
$(1, 0)$	$\langle 0, 1 \rangle$	$(-1, 0)$	$\langle 0, -1 \rangle$
$(2, 2)$	$\langle -2, 2 \rangle$	$(-2, -2)$	$\langle 2, -2 \rangle$
$(3, 0)$	$\langle 0, 3 \rangle$	$(-3, 0)$	$\langle 0, -3 \rangle$
$(0, 1)$	$\langle -1, 0 \rangle$	$(0, -1)$	$\langle 1, 0 \rangle$
$(-2, 2)$	$\langle -2, -2 \rangle$	$(2, -2)$	$\langle 2, 2 \rangle$
$(0, 3)$	$\langle -3, 0 \rangle$	$(0, -3)$	$\langle 3, 0 \rangle$

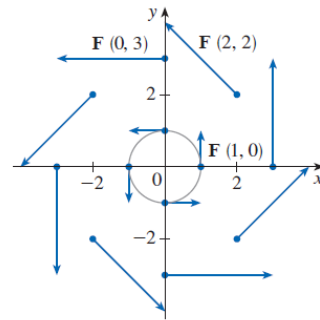


FIGURE 5
 $\mathbf{F}(x, y) = -y \mathbf{i} + x \mathbf{j}$

Note. Excerpt from Stewart et al. (2021) p. 1163

For instance, in the first example, after the cursory visual sketch is created, the text notes that “It appears from [the visual representation of this vector field] that each arrow is tangent to a circle with center the origin” (p. 1164). The remaining story covering this example shifts to confirming this claim mathematically and concludes with another comment about the nature of the vectors: “Notice also that . . . the magnitude of the vector $\mathbf{F}(x, y)$ is equal to the radius of the circle” (p. 1164). In the second example, a visual sketch is provided immediately, and the remainder of the example text is interpretation: “Notice that all vectors are vertical and point upward above the xy -plane or downward below it. The magnitude increases with distance from the xy -plane” (p. 1164). Throughout these two examples, vector fields are characterized primarily as *fields* of multiple vectors to be interpreted, more consistent with their initial introduction in the chapter than the formal correspondence definitions which had been used to characterize them just moments ago (e.g., Figure 4.14).

After these two examples, the story shifts primarily to what can best be described as extended vignettes of notable examples of vector fields depicting *physical phenomena*—a velocity field (as in the beginning of the chapter, see Figure 4.13), a gravitational field, followed

by an electric field, and concluding with gradient fields. In other words, there is a return to the initial characterization and purpose of vector fields in the beginning of the story. These four examples span the remaining half of the story (three pages) and complete a transition toward characterizing vector fields as *modeling physical phenomena*. Aside from using function and vector notation as a symbolic necessity, once technology is introduced as a way of plotting vector fields after the initial two vector field examples, these characters are portrayed primarily as a tool for modeling realistic physics and engineering phenomena, with their *structural* characterization (i.e., domain and range) never being mentioned again. Indeed, there are only five explicit references to vector fields as “functions” across this entire chapter, most of which occur near the formal definitions. The word “function” returns briefly when the gradient of a function, $\nabla f(x, y) = \langle f_x(x, y), f_y(x, y) \rangle$, introduced previously in the textbook’s story of MVC is reintroduced as an example of a vector field.⁶⁵ However, this word is only used to clarify that f is a *scalar function* in this context, as opposed to a vector-valued function (or scalar field)—the gradient is referred to as a “gradient vector field” (in bold) and *not* explicitly as a function in its own right.

Analysis Across Stories: Real-Valued Functions are Characterized as Functions, Vector-Valued Functions are Characterized Primarily in Other Ways

These story analyses emphasize key differences between the character introductions of these three types of multivariable functions in one commonly used MVC textbook. While multivariable, real-valued functions are clearly positioned as a main character and repeatedly characterized as a function akin to those previously encountered in the story of calculus (i.e., single-variable, real-valued functions), the same cannot be said about both types of vector-valued

⁶⁵ Note that f_x and f_y are the symbols used in this textbook story for partial derivatives of a multivariable, real-valued function f .

functions.

The difference is most extreme in the case of parametric, vector-valued functions, which play the role of a supporting character meant to introduce geometric space curves. These functions are introduced formally using a structural, correspondence-based characterization; however, their characterization almost immediately shifts to being primarily about the action of decomposing these multivariable, vector-valued functions into their single-variable, real-valued component functions for the sake of various computations in service of sketching and discerning properties of space curves. These parametric, vector-valued characters are introduced mostly as *symbolic* entities, and it is not until space curves are introduced that the story shifts to the *visual* setting at all. It would not be a stretch to say that parametric, vector-valued functions are essentially characterized as the set of its single-variable component functions dressed up in a trench coat. Yet, unlike the textbook's introduction to multivariable, real-valued functions which are clearly tied to previously introduced function characters using a multiple representations framing device (recall Figure 4.10), a similar connection is not established for these single-variable functions.

Vector fields, on the other hand, remain the main character of their story; however, their *structural* and *correspondence* character traits emphasized in their formal definition are mostly de-emphasized throughout the rest of their story. Rather, vector fields are consistently characterized as *modeling physical phenomena*, from the unit introduction and beginning of the chapter proper featuring a one-page spread introducing applied vector fields both *visually and verbally* (recall Figure 4.13) to the end where three pages of “vector field vignettes” introduce and interpret the physical meanings of several vector fields used in physics and engineering contexts. As with parametric, vector-valued functions, implicit connections are made between

vector fields and other types of functions. For example, the action of decomposing vector fields into their component functions—multivariable, real-valued functions—is highlighted similarly to parametric, vector-valued functions; however, little is done to *explicitly* connect these characters to the textbook’s prior story arcs concerning functions. In fact, the connection is *obscured* when the textbook story introduces a new name—scalar fields—to describe these component functions with minimal explanation for why or reference to how this is but a *nickname* for a previously-introduced type of function.

Considered collectively, the story about multivariable, real-valued functions is positioned as a clear sequel to the story of (single-variable) functions; meanwhile, the stories for both types of vector-valued functions are introduced as largely disconnected from the previous stories about single-variable functions. *Both* types of vector-valued functions are introduced using a plethora of alternative nicknames and other descriptors (e.g., real-valued vs. vector-valued functions, scalar vs. vector functions) that not only induce confusion and whiplash while reading their individual stories but also serve to create a rift between their characterizations and prior characterizations of functions across the textbook’s story of (multivariable) calculus. Although both types of vector-valued functions are *formally* introduced using analogous *set-theoretic* or *correspondence*-based definitions (see Figures 4.7, 4.14) which mirror multivariable, real-valued functions (Figure 4.11), this characterization is *spotlighted* in the story of multivariable, real-valued functions while being downplayed in favor of alternative characterizations for both vector-valued functions. The consequence is that these stories may perpetuate the meta-narrative that the most important type of multivariable functions in MVC are the real-valued ones. According to this textbook’s stories, vector-valued ones are also functions—at least formally—but this is portrayed as almost a technicality or sidenote that is not a fact used frequently in

practice. Readers (our students) could be more likely to come away with alternative messages about vector-valued functions that are foregrounded in these stories, such as how readily they can be decomposed into their component functions for further computation or, in the case of vector fields, the importance of physically interpreting the constituent vector outputs. The disparate messages about the nature of these characters does little to build a sense of “characterological coherence” (to use Fisher’s term) across these stories, giving readers little reason to see them as instances of a singular, *unified* function character.

Discussion

In this study, I read *one* textbook’s stories for introducing three different types of multivariable function to investigate the messages about function(s) that are portrayed at one critical junction in the curriculum, as students are exposed to different types of functions beyond single-variable ones. On one hand, the textbook’s story about real-valued, multivariable functions was positioned as a natural *sequel* to the stories about single-variable functions from high school and, more recently, single-variable calculus. The textbook’s story about this type of multivariable function is clearly couched within the dominant perspective in the MVC literature of positioning MVC as primarily about generalizing single-variable notions to derive analogous multivariable notions. Collectively, these points suggest that multivariable, real-valued functions are positioned as a *main* character within the context of MVC.

Meanwhile, both types of vector-valued functions were portrayed differently, as *side* characters which serve specific purposes that are *not* as clearly aligned with the single-variable to multivariable generalization perspective. Rather than being an “extension” of single-variable functions, these vector-valued functions are often *reduced back* into them when they are broken into single-variable component functions for further algebraic-symbolic analysis. Both these function types are introduced with a formal set-theoretic definition with attention drawn to

domains, codomains, etc. However, shortly afterwards they are treated, respectively, as having distinct character traits and tendencies. At the same time, these observations are not surprising given that the majority of MVC research in mathematics education focuses on real-valued rather than vector-valued functions (Hahn & Klein, in press; Tyburski, 2023).

Given the dramatically different characterizations of these three different types of functions across this textbook's stories, the characterological coherence for "function" as a unifying, thematic character might be interpreted as rather limited. Consequently, readers may be led to believe that function is an unnecessary boondoggle, "extra", or definitional formalism that only plays a passing role in the overarching story of MVC and, more generally, mathematics. Already, research suggests that MVC students may benefit from additional guidance as they make sense of multivariable, real-valued functions as a generalization of single-variable real-valued functions (Martínez-Planell & Trigueros, 2021; McGee et al., 2015; Yerushalmy, 1997). The same may well be true for making sense of vector-valued functions as multivariable functions, *especially* if there are even fewer textbook supports for making this generalization.

A Cautious Pitch for Function as an Organizing Story for Multivariable Calculus

"One possible benefit of the interpretation of mathematics curriculum as a mathematical story is the empowerment it offers [curriculum designers] to recognize how mathematical characters are positioned within a development (Is it a central character? Supporting?). When a [curriculum designer] sees a potential benefit for making a character more central, he or she could purposefully adjust the mathematical story to draw more attention to it" (Dietiker, 2012, p. 160).

In the spirit of this quote and given the story of what brings me to this dissertation work in the first place (see Chapter 1), I propose that the MVC curriculum could be organized based around the three types of multivariable function spotlighted in this study. After all, calculus is done on *functions*, so MVC could reasonably be recast as the study of making the appropriate changes to calculus to define multivariable analogues for derivatives and integrals of each of these three multivariable function types. This has at least three benefits. First, from a narrative

perspective, students have often suggested to me that MVC feels like a hodge-podge of loosely connected ideas, calculations, and visualizations. And who can blame them? The visual representations of these functions (recall Figure 4.1) look perhaps even less qualitatively similar than various representations of single-variable functions. And they are called vector *fields*—since when is a field of vectors a *function*? A “framing narrative” (Weinberg et al., 2016) based around multivariable function types could help students organize their experience within MVC. Second, in addition to this *narrative* benefit, emerging empirical evidence suggests that students who view functions from across course contexts from a unified perspective are better positioned to succeed in both linear and abstract algebra. Might this also be the case in MVC? I hypothesize the answer could be yes and that students may similarly benefit from recognizing vector-valued *as well as* real-valued multivariable functions as types of functions, as it would better position them to recognize structural similarity across several different types of calculus required for each function type (e.g., the Chop, Multiply, Add conceptual pattern for integration, Dray & Manogue, 2023). Finally, mathematics students might benefit from being exposed early in the undergraduate curriculum to this type of *regenerative story* (Gadanidis et al., 2016) and be better prepared as they transition to future courses (*like* linear and abstract algebra). In essence, this organizing function story could provide what Savinainen and Viiri (2008) called “conceptual framework coherence” across different courses with respect to different types of function(s).

Simultaneously, I recognize that while an overarching story for MVC centered on function may support students’ enculturation into the discipline of mathematics, it may have the opposite effect on students’ enculturation into other STEM disciplines, given the differing ways that scientists and mathematicians conceptualize functions (e.g., Dray & Manogue, 2004) and, more generally, the differing aesthetic sensibilities (i.e., perspectives on *curricular coherence*,

Modeste et al., 2023) and curricular stories (Lanius, in press) across scientific disciplines.

Several of the above arguments were based on an implicit premise that students would take more mathematics courses, but MVC is service course for *several* STEM disciplines (e.g., Dray & Manogue, 2023; O’Leary et al., 2021; Page et al., 2024) and meta-narratives are fundamentally cultural in nature (Stephens & McCallum, 1998). Therefore, the unified and unifying meta-narrative of function in MVC may *not* be a regenerative story for students whose final mathematics course is MVC—in fact, it may have a deleterious effect depending on the discipline(s) that student is hoping to join.

Even *for* mathematics students, stories of the form I am proposing favor a disciplinary brand of coherence which privileges the use of mathematical practices including *formalizing, unifying, generalizing, and simplifying* (i.e., FUGS, see Hausberger, 2017, p. 418). Indeed, *almost all* dominant curricular perspectives authored by mathematicians and mathematics educators on function learning—ranging from APOS theory to covariational reasoning to a unified notion for function—favor a narrow aesthetics of “FUGS” inherent to *structural* forms of thinking common to the discipline of mathematics. These aesthetics of unification and generalization are best epitomized using some quotes. Benis-Sinaceur (2014), for example, defined a particular form of generalization called idealization as “leaving aside or discarding all other aspects, especially specific substantial space-time aspects. This operation has been called idealization because it comes down to extracting a *form* from sundry situations” (p. 94, emphasis in original). Similarly, recall how Poincaré claimed that

Mathematics is the art of giving the same name to different things . . . when language is well chosen, we are astonished to learn that all the proofs made for a certain object apply immediately to many new objects; there is nothing to change, not even the words, since the names have become the same. (1908/2012, p. 375)

The so-called “art” of FUGS involves *discarding* all sense of context in favor of *extraction* in

order to categorize *and* name everything the same. Greeno (1992) has called this a “thinking-with-basic” philosophy in the sense that all concepts are seen as building hierarchically from the “basics”. Yet, what is “ideal” for some is not “ideal” for all—not even mathematicians universally subscribe to such a limited form of aesthetics, at least not exclusively (Hausberger, 2017). Still, Harel (2021) has observed that MVC textbook stories often favor this aesthetics of storytelling and disciplinary coherence. Wagner (2012) urged caution toward such overly narrow aesthetics of storytelling which can lead to the creation of *closed* texts that privilege and normalize particular epistemological and aesthetic points of view at the expense of others, even if these points of view can have value. I offer my pitch for *a* narrative framing of MVC in terms of function with a similar degree of caution.

Multivariable Calculus as an Anthology Featuring *Many Stories*

In this spirit, I pose the question: *What other overarching curricular stories could we tell across the MVC curriculum?* Dietiker (2012) has suggested, for instance, that we might consider how *changing* our mathematical characters might change the overarching curricular story. What other characters might we consider? Given the emerging state of research in MVC, there has been minimal deliberation in the mathematics education literature concerning what stories we *ought* to tell. Harel (2021) proposed that *linearization* ought to take center stage. Meanwhile, Dray and Manogue (2023) advocated for a MVC curriculum that focuses on differentials and writing equations describing relationships between differentials. In effect, they argue that MVC is *not* about the functions themselves, but the “tiny changes” between quantities being *related* by functions, in line with a covariational reasoning perspective. *What other stories might we consider? Whose stories might we consider?*

Regardless of the chosen story, however, each story choice privileges certain forms of (in)coherence or other (disciplinary) aesthetics. For example, both alternative stories still lean

into disciplinary forms of coherence that favor “FUGS”-like aesthetics. So, for each story that is proposed, we ought to consider several ethical questions. For example, what forms of (in)coherence are privileged in these stories and therefore which forms of *aesthetics* do these stories forward? Which students from which (disciplinary) cultures *benefit* from each story? Which students are *harmed* by each story? No single story is *aesthetically* resonant for everyone, and choices of curricular story have clear ethical implications, particularly in a course context like MVC that is quintessential for several students across disciplines but still taught primarily by those from the discipline of mathematics.

In following my “Change the story, change the curriculum” framing of the curriculum-as-story metaphor (See Chapter 2), alongside the unique, interdisciplinary crossroads that MVC represents, I argue that we should *not* choose just one story. Rather, I propose that it may be productive to conceptualize the MVC curriculum as an *anthology or collection of several* stories. In this framing, mathematics curricular stories are not merely *one-off* tales but rather *interconnected* sagas which often have at least some loose recurring characters, thematic similarity, or connective tissue of some sort across selected stories. All stories mentioned across this chapter so far could, for example, reasonably co-exist (and in many cases synergize) with each other. But then the question becomes: *Whose* stories are included in this anthology? And how can we approach the ethical conundrum of deciding *how many stories* become a part of this anthology?

First, this curricular effort requires collegial interdisciplinary conversation, as we build curricula in terms of interdisciplinary forms of coherence (Modeste et al., 2023). As mathematicians and mathematics educators, we must exhibit humbleness and a willingness to be critical of the disproportionate *power* our disciplinary stories have (and have had) over students

enrolled in MVC given that departments of mathematics are often designated to teach MVC rather than other science departments. Part of being modest involves listening to and learning from the stories proposed by those in other STEM disciplines, as we kindle a complex interdisciplinary conversation (J. Williams et al., 2016). Modeste et al. propose that “a central aspect [of this dialogue] will be to articulate the rationalities/epistemologies of other disciplines with the rationalities/epistemologies of mathematics” (p. 169). I suggest it will also involve articulating our disciplinary *aesthetics*, as they are fundamentally intertwined with these rationalities and epistemologies (Corfield, 2012; Laudan, 1984). Finally, in line with the aims of Chapter 3, I argue that these conversations *must also* involve students (both current and past) to ensure our stories remain open to various forms of coherence seeking, rather than being guided solely by (retroactive) (inter)disciplinary coherence(s).

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CHAPTER 5: CONCLUSION

Having reached the end of one research journey and the beginning of presumably many more, I begin this final chapter by recapping my overarching aims of inquiry followed by a synthesis of how each of the three studies of this dissertation advanced these inquiry goals. Across these summaries, I detail some notable themes which emerged across this dissertation—some expected and others spontaneous. Finally, I conclude in the explorative and speculative spirit I began by outlining several lingering questions and *incoherences* I am left contemplating. I frame these discussions by reflecting on my recent personal growth, noting where I came from when I first embarked on this journey, the strides I have made, and the areas in which I hope to continue growing in the future.

Overarching Inquiry Goals and Progress Made

In Chapter 1, I outlined three overarching and interconnected aims of inquiry for this dissertation related to the aesthetics of cross-curricular mathematical stories, with a particular focus on interpretations of the *(in)coherence* of these stories from multiple stakeholder perspectives. Namely, I wanted to take a step back and investigate the ontological question *What is coherence?* (Q1) not with the goal of arriving at a singular answer but rather to contemplate the *many* ways someone might answer this question and how their answers might be related to their philosophical assumptions about mathematics, (mathematics) learning, curriculum, stories and storytelling, and aesthetics more generally. In other words, my intent was not to solely *list* several ontological possibilities for defining curricular coherence. Rather, I aimed to interrogate the axio-onto-epistemological assumptions that undergird the widely held aesthetic value that (curricular) coherence is universally desirable by considering: *What are the consequences of these philosophical assumptions for curriculum? For learning? For how we position learners?* (Q2). In doing so, I was careful to adopt the terminology of “(in)coherence” as a reminder that

incoherence as well as the dialectic between coherence and incoherence (i.e., in/coherence) has a powerful role to play in an individual's aesthetic interpretation of (cross-curricular) stories and therefore in curricular design and student learning (Appelbaum, 2010; Dietiker, 2015; Irwin, 2003). In other words, I purposefully remained open to the possibility that the assumption of coherence (Shemmer, 2012) could be rejected for the sake of considering different story aesthetics and therefore different curricular aesthetics. My overall inquiry into these questions about curricular (in)coherence was framed in terms of treating curriculum as a story to investigate the flexible possibilities (and tensions) for using a curriculum-as-story metaphor to investigate various perspectives on curricular (in)coherence (Q3).

Across the chapters of this dissertation, I characterized several possible answers to the first overarching inquiry question: *What is coherence?* In Chapter 2, I presented a review of the multi-disciplinary literature on “coherence” shedding light on several conceptualizations of what coherence could mean, ranging from logical coherence of an axiomatic system (Daya, 1960) to characterological coherence of stories told to the public (Fisher, 1987) to the emotional coherence of an individual's system of beliefs (Thagard, 2000). Then, I refined my focus to *curricular* coherence in the context of mathematics and science education and conducted an in-depth investigation of various forms of curricular coherence introduced in the literature, ranging from dominant disciplinary forms of coherence which privilege expert disciplinary perspectives and alignment with disciplinary logics and pre-established hierarchies (Cuoco & McCallum, 2018; Schmidt et al., 2002) to perspectives from science education that spotlight *students'* idiosyncratic, in-the-moment coherence seeking activities (Sikorski & Hammer, 2017). Inspired by this perspective from science education, I continued this line of inquiry into Chapter 3 by characterizing and exhibiting six undergraduate STEM students' multiplicity of perspectives on

cross-curricular mathematical (in)coherence. While some students' ontologies of coherence aligned with those presented in the mathematics and science education literature, other students' views repeatedly pushed beyond those that have been put forth by disciplinary experts in the literature. Namely, students repeatedly shared interpretations of cross-curricular (in)coherence that positioned (in)coherence not as an objective evaluation but rather a subjective and highly contextual judgement, based on one's aesthetic sensibilities, emotions, and learning history. Further, multiple students philosophized about the complex, non-linear relationship between time and coherence, challenging strict logico-rational views of coherence which associate "making sense" of cross-curricular stories as a strictly linear affair.

By detailing these various perspectives on coherence, this dissertation destabilizes the myth conveyed by much of the existing mathematics literature that the notion of curricular coherence is a foregone conclusion with just a single ontology or definition. As a concrete instance, in Chapter 4 I analyzed one curricular (textbook) story crafted primarily using logics of disciplinary coherence, highlighting the ways in which it might be interpreted as *incoherent* by even readers who *are* disciplinary experts. I concluded by arguing that *mathematical* disciplinary coherence ought to be considered alongside other forms of *interdisciplinary* coherence (Modeste et al., 2023), particularly when contemplating which curricular story (or stories) to privilege in service courses that serve students with divergent career goals, such as MVC. Indeed, some researchers have suggested that the notion of "curricular coherence" itself is *not* well-defined and requires a more careful definition before productive conversations can continue in this branch of curriculum research (Muller, 2022; Thompson, 2008). Though I argued against a singular definition for (in)coherence in this dissertation, I simultaneously addressed several ontological facets that ought to be considered in future research conversations about (un)definitions for

“curricular coherence”. First, I argued that *aesthetic* and *emotional* forms of (in)coherence cannot be ignored as a complementary force and necessary counterbalance to the overreliance on strictly logical views of coherence in mathematics education. Second, I demonstrated how coherence should be considered as more than just a binary with incoherence, suggesting less reductive alternatives such as a *spectrum* view or a view of coherence and incoherence as a complex dialectic (in line with the philosophical tenet of ontological dialectical pluralism of the ABR paradigm, Chilton et al., 2015). Third, I contested a view of (curricular) (in)coherence as an *objective evaluation* of the curriculum, forwarding instead the view that (in)coherence is a *subjective (aesthetic) judgement*. Specifically, I expounded on how such judgements often involve an ongoing iterative process of engagement of a student (or other individual) with the “text” of the curriculum, meaning that curricular coherence is not merely an internal property or feature of a curriculum itself. Rather, these idiosyncratic judgements form as one engages in processes of (in)coherence seeking that proceed along trajectories that need not be strictly linear, often adhering to alternative notions of temporality, as detailed in students’ perspectives on (in)coherence in Chapter 3. Fourth and finally, I called for further attention to *students’* (in-the-moment) perspectives of (in)coherence and (in)coherence seeking to complement expert, pre-meditated views of (curricular) coherence that dominate the mathematics education literature. Such student perspectives have the potential to expand which axio-onto-epistemological facets are considered in the name of (in)coherence, ensuring that we are not systematically ignoring the perspectives of those whom curriculum is meant to impact. Considering my exploration of each of these ontological facets of (curricular) (in)coherence collectively, this dissertation cautions that *any* subsequent mathematics education research which focuses on (in)coherence of any form—curricular or otherwise—ought to be critical of the definition of (in)coherence itself,

including what past researchers have meant by this term *and* what the researcher means by the term. Such criticality should be part and parcel of any definition of (curricular) (in)coherence, given the *many* possible ontologies of (in)coherence and their differential implications for how students are positioned as learners.

On that note, in pursuing the second inquiry question for this dissertation, I documented how any definition of (in)coherence is value-laden and therefore how (in)coherence should not be treated as a foregone conclusion or something that is a universal good (Buchmann & Floden, 1991; Hyvärinen et al., 2010), as it has been in past curriculum research (e.g., Jin et al., 2022; Morony, 2023c). Specifically, in Chapter 2 I concluded my literature review of (curricular) coherence with a critical investigation of the philosophical foundations for various forms of (curricular) (in)coherence intertwined with a critical interrogation of empirical evidence that has been offered up to argue that curriculum ought to be coherent. Throughout this investigation, I noted that past research frequently *assumes* the goodness of something being coherent (whether it be curriculum or a narrative) with minimal philosophical or empirical justification of the purpose such coherence serves *or* consideration of alternative dimensions of holistic aesthetic judgement.

Hyvärinen et al. (2010) argued the assumption that “good” narratives must be coherent is an implicit value stemming from Western ways of knowing and structures of storytelling. Indeed, in the subsequent chapters of their edited volume, they featured several compelling personal narratives from real people that might be considered “incoherent” according to traditional narrative learning theory. In response, Freeman (2010) concluded with a call to reconsider how narratives are judged so as to remain open to various aesthetics of storytelling when considering the ways that individuals make sense of their lives and personal experiences.

As I argued in Chapter 2, these assumptions of coherence are not limited to narrative learning theory. Rather, such assumptions are foundational to *most* Western learning theory grounded in constructivism (Shemmer, 2012). Additionally, these assumptions are present in axio-onto-epistemologies of story and storytelling originating from Western cultures (Herbert, 2004). More broadly, my inquiry into this second dissertation question suggests that assumptions of coherence and associations between coherence and aesthetic goodness are entrenched in many Western values, art forms, and ways of knowing, explaining why these assumptions are not unpacked explicitly in much of the research I reviewed. I return to this theme of the goodness of coherence as a Western aesthetic value in the next section; however, I remark briefly here that I hope this dissertation serves to reveal this implicit assumption and acts as a possible stepping stone for more sustained critical and philosophical interrogation of coherence assumptions underlying mathematics education research. By “research”, I do not just mean “curriculum research” and literature on curricular coherence as has been my focus in this dissertation but *all* areas of mathematics education research that feature this coherence assumption (either implicitly or explicitly), such as the philosophy of mathematics (education) and the development of learning theories, conceptual analysis and learning trajectories research for particular mathematical concepts (e.g., Izsák & Beckmann, 2019; Thompson, 2008), and studies of student learning and the assessment of student learning (e.g., Savinainen & Viiri, 2008). Similar to how Mikulan and Sinclair (2023, 2024) have noted that entrenched assumptions of temporality grounded in Western ways of knowing limit the possibilities in our research and enforce certain ethics of education, I argue that assumptions of coherence play a similar role in education (research) and ought to be reconsidered and deconstructed in a critical light.

This is more than just a theoretical concern. As I detailed in Chapter 2 (and as students’

stories in Chapter 3 demonstrated), the taken-for-granted association of coherence with “goodness” has ethical implications for who or what is considered “good”, “aesthetic”, and/or “smart” (Appelbaum, 2010; Buchmann & Floden, 1991), similar to what Hyvärinen et al. (2010) noted with “coherent” narratives in the context of narrative learning theory. Such an association imposes a politics of aesthetics (Rancière, 2000/2004) thereby imposing a hierarchy that privileges certain aesthetic preferences over others, positioning some mathematics students as “usual” and others as “aberrant”. It is for this reason that Richmond et al. (2019) contend that developing coherent curricular requires continually revisiting the questions *coherence for whom?* and *coherence for what purpose?* Buchmann and Floden (1991), for instance, noted the problematic historical association between behaviorism and curricular coherence which positioned students as passive receptacles of “coherent” knowledge, spotlighting the imminent danger of research that fails to question the implicit assumption that coherence is an inherent aesthetic and epistemological good. Throughout the dissertation, I repeatedly argued against such a strict binary of “coherence good, incoherence bad” is harmful not only because it results in the enforcement of a rigid politics of mathematical aesthetics but because it ignores the many ways that *incoherence* can motivate and catalyze learning (Appelbaum, 2010; Dietiker, 2015; Irwin, 2003). By embracing the conclusions of my second aim of inquiry, I considered different possible valuations of (in)coherence across Chapters 3 and 4, positioning (in)coherence as a holistic but idiosyncratic *aesthetic judgement* of an object (be it a curriculum, story, or something else). In doing so, I remained open to considering many possible views of (in)coherence and therefore many possible axio-onto-epistemologies of (in)coherence.

Finally, in line with my third aim of inquiry, I attended to various perspectives on (in)coherence by leveraging the curriculum-as-story metaphor to explore curricular coherence as

a refractive diamond (Buchmann & Floden, 1991, p. 69) or “disco ball” (Canrinus, 2024, p. 258) defined as an amalgamation of several varying perspectives on the topic. In part one of Chapter 2, I first outlined the curriculum-as-story metaphor as introduced by Dietiker (2013, 2015) based on the narrative interpretive theory of Bal (2017). Later in this chapter, I introduced my re-interpretation of this metaphor as a flexible lens for interpreting various perspectives on curricular coherence. I presented this re-interpretation as one possible response to my argument that research using the curriculum-as-story metaphor *must* be critical of the story aesthetics being invoked either implicitly or explicitly by the definitions or forms of story that a researcher employs (e.g., privileged story structures, cultural forms of storytelling, genres). As a response to my own loving critique of past research and past claims that “curricular coherence” is ill-defined, I also outlined the philosophical entailments of the flexible curriculum-as-story perspective I proposed to use throughout the dissertation, including those related to curricular coherence.

In the remaining chapters, I subsequently demonstrated the pragmatic nature of this flexible re-interpretation of the curriculum-as-story metaphor as a deliberate tool to remain open to varying stakeholders’ perspectives on curricular (in)coherence, including students (Chapter 3) *and* disciplinary experts (Chapter 4). Further, in the Chapter 3 discussion, I built off students’ perspectives on (in)coherence to contemplate forms of stories which could espouse compatible aesthetics of (in)coherence, including Indigenous forms of storytelling, comic books, and various possible story genres. In the Chapter 4 discussion, I used the generative, flexible potential of the curriculum-as-story metaphor to propose that the MVC curriculum might be productively viewed as an *anthology* of curricular stories from various STEM disciplines—including mathematics—with some shared elements (such as characters and settings) but also some unique elements and the possibility of different and possibly even contradictory disciplinary aesthetics of

(in)coherence across stories. In doing so, I urged caution for adopting just one overarching curricular story for MVC, given the multiple interdisciplinary stakeholders and their differing (and possibly contradictory) interdisciplinary views of (in)coherence.

The Tensions and Limitations of the Curriculum-as-Story Metaphor

Despite the many possibilities of the flexible curriculum-as-story metaphor I illustrated throughout this dissertation, I often found myself reflecting on possible tensions and limitations of its use. Having synthesized the contributions of this dissertation, I now turn my attention to some further reflections on the origins of the curriculum-as-story metaphor and possible tensions that arise from these origins, as promised in Chapter 2. Throughout this section, I maintain the speculative nature of the dissertation so far, offering few definitive answers in favor of carefully posed and in-depth lines of questioning that I believe ought to be addressed in future research.

In this dissertation, I proposed the curriculum-as-story metaphor as being adaptable to analyzing many possible forms of story. In doing so, I proposed that this metaphor could be used to investigate a panoply of story aesthetics and the corresponding perspectives on curricular (in)coherence espoused by such story aesthetics. Personally, I did not run into any significant bumps while applying this flexible metaphor in Chapters 3 and 4. However, careful attention to Dietiker's original formulation of the metaphor and Bal's underlying theory of narrative interpretation on which this metaphor is based has led me to doubt whether this metaphor—even the re-interpreted version I proposed—could truly be used to interpret *all* forms of narrative and therefore *all* forms of curricular (in)coherence. I specifically began to contemplate one theoretical question. Bal's theory assumes a *structuralist* view of narrative (interpretation), so is it possible to analyze story aesthetics and story forms that purposefully *eschew* structuralist assumptions and traditions via the curriculum-as-story metaphor?

As stated previously in Chapters 1 and 2, Bal (2017) acknowledged quite explicitly that

narrative interpretation occurs in a socio-cultural context and milieu, even taking time to position her narrative interpretive theory as a form of contextual cultural critique rather than an attempt to classify all possible story types (as is the classical structuralist aim). In other words, Bal purposefully crafted her theory so that it could be used to analyze *several* different forms of narrative—literary, artistic, and otherwise (Bal, 2021). In doing so, she took inspiration from a number of other theories of narrative (interpretation) beyond structuralism, resulting in what she claims (in Bal, 2017) to be a modern theory of narrative applicable across paradigmatic boundaries. As she claimed in the preface to the first edition of her 1985 book introducing this theory:

One need not adhere to structuralism as a philosophy in order to be able to use the concepts and views presented in this book. Neither does one need to feel that adherence to, for example, a deconstructionist, Marxist, or feminist view of literature hinders the use of this book. I happen to use it myself for feminist criticism, and feel that it helps to make that approach more convincing, because of the features a systematic account entails. (Bal, 2017, p. viii)

For Bal, therefore, the utility of structuralism is its systematic nature that allows for a certain kind of narrative interpretive bookkeeping. Namely, Bal’s theory features a structural, three-layer perspective on narrative interpretation by distinguishing between the text, story, and fabula. However, as she clarifies immediately in Bal (2021), she adopts this convention primarily to *challenge* reductive views espoused by early structuralist theories of narrative:

For me, it has primarily been an attempt to overcome the binary connotations of the older division in two – text and fabula, or story and plot. This older division is bound to a distinction, at risk of becoming a separation, of form and content. (p. 3)

Therefore, this separation of layers is primarily a practical consideration—rather than a theoretical boundary—allowing for subsequent analysis of the effects a narrative text has on its readers. Indeed, Bal regularly refers to these layers as fundamentally interconnected. She clarifies, “distinction does not entail separation. On the contrary: the connections between the

three layers are the point of an analysis based on the distinction, which is by definition provisional” (Bal, 2021, p. 3). Still, despite Bal’s contention that such a distinction is not akin to separation and is merely provisional, the structuralist axio-onto-epistemological assumption that these layers *can* be separated—even just theoretically—likely reduces the range of story forms amenable to analysis via Bal’s theory.

Perhaps more concerning, however, are Bal’s rather platonic structural assumptions about the nature of the fabula and story layers relative to one another. Specifically, Bal often seems to imply that the fabula pre-exists the story and is “out there” to be re-arranged in certain ways which generate different possible stories for a similar “underlying” narrative. For example, she writes about “a fabula that has been ordered into a story” (2017, p. 8), as if a fabula is a pre-existing special clay from which stories are molded. Simultaneously, she speaks of the fabula as “the result of the mental activity of reading” (p. 9), implying that the fabula is constructed *after* a reader engages with a story. Bal resolves this apparent tension by seemingly treating “the” fabula of a narrative as an approximate consensus about the core characters, events, and other elements which *most* readers construct as events, characters, or other elements within their idiosyncratic fabulae. It is in this sense that Bal suggests the notion of a fabula exists in both the past and the present, both preceding the story *and* originating from a reader’s ex post facto interpretation of said story. Yet, this was not an oversight on Bal’s part: she fully acknowledges the paradoxical nature of her assumptions about the fabula and does not dispute the cautions that have been offered in the narratology literature concerning the platonic view that the fabula pre-exists the story (e.g., Smith, 1980). She admits (without any further discussion), “although I maintain this distinction, I fully agree with these analyses of the problem inherent to it” (Bal, 2017, p. 152).

This structuralist assumption that the fabula layer exists before the story layer would

seemingly reduce the variety of story forms that could be analyzed using Bal's theory to those that do not explicitly contradict this Platonic perspective. At the same time, perhaps this issue is sidestepped entirely with the curriculum-as-story metaphor, thanks to Dietiker's (2013) formulation of the *mathematical* fabula as merely *one* reader's idiosyncratic "logical re-construction of the mathematics events beyond the text and story" (p. 16). Though I believe Dietiker avoided opening the bulk of the Platonic can of worms present in Bal's theory, I am not so optimistic that we can conclude definitively that this foundational assumption does not permeate *other* features of Bal's theory that Dietiker may have admitted into the curriculum-as-story metaphor she crafted. Given the centrality of this assumption to Bal's theory, such a conclusion would require a more careful theoretical analysis of the list of tenets from Bal's theory that were adopted wholesale alongside those that were augmented in some way in the metaphor Dietiker crafted.

Another ever-present assumption of Bal's narrative theory (that Dietiker *does* adopt) is the inherent *linearity* of narrative interpretation. A story, for instance, is assumed to be a *sequential* ordering of events and narrative interpretation (event A comes first, then B, then C, ...). This is used to great effect by both Bal and Dietiker to explain readers' differential aesthetic reactions based on the (mis)alignment between a reader's expectations about how events might happen (i.e., how elements are organized within one's fabula) and how they end up unfolding in the story. Yet, as detailed in Chapter 3, there are some forms of stories and storytelling that eschew this linearity assumption in favor of multilinear or non-linear perspectives on the temporality of experience (e.g., Indigenous forms of storytelling, comic books). As Mikulan and Sinclair (2023) and others note, there are *many* different philosophies of temporality that transcend the assumption that the experience of time (and therefore of narrative experience) must

proceed like a straight arrow). As such, I have begun to wonder whether a curriculum-as-story metaphor built on Bal's theory of narrative interpretation will truly be flexible enough to consider the aesthetics of (in)coherence of story forms that embrace radically different philosophies of time and therefore "narrative unfolding". I do not believe, however, this musing can be answered with a simple "yes" or "no". Just as I began to lean toward suspecting the answer is likely "no", I recalled how Bal firmly contended that her theory could be used to analyze *most* forms of literature as well as various other forms of narrative. The evidence for this contention appears to be rooted in her complex, paradoxical conceptualization of the fabula introduced previously, which distorts the flow of experiential time to entertain philosophies of temporality that are not strictly linear. Bal (2021) expounds:

We can grasp how what I will call 'multitemporarily' and what has been called 'multidirectionality' join forces in complicating the sense of history as a chronological series of events: in turning (factual) history into (subjective) memory. Memory militates against a binary view of narrative as a text telling a story, or a story telling a plot. For the sake of this . . . , I propose we consider the fabula a kind of history – but then, due to the story level, frequently imagined rather than necessarily having occurred in the past. Or both. The string of events we call history now turns from a line into a constellation from which rays go out in all directions. Futurity itself, then, is multidirectional, encompassing the past as well as the times of others. (p. 7)

This passage serves as a counterargument to my suspicion that Bal's theory admits a strictly linear sense of time and sequentially. However, the flexibility of the theory also appears to rely on the adoption of paradoxical assumptions about the fabula as simultaneously preceding the story *and* originating from the interpretation of the story, as both history and a memory all at once. But recall that Dietiker (2013) adopted modified conceptualizations of the fabula in crafting the curriculum-as-story metaphor. Does this mean that *this* metaphor is not as open to alternative conceptualizations of temporality of narrative interpretation and therefore less able to analyze (curricular) story forms that do not adhere to traditional linear sequencing? I do not have a certain answer at this moment, but I *do* believe further attention to this issue is warranted to

accurately assess any notable limitations of this current metaphor for analyzing a range of possible curricular story forms and aesthetics.

While Bal's theory has proven to be a useful template for the current curriculum-as-story metaphor, the possible limitations introduced throughout this section leave me to suggest that it may be worth investigating *other* possible theories of narrative interpretation as alternative templates. Undoubtedly, each of these will also introduce its own limitations, but remaining open to different perspectives on narrative interpretation—Indigenous, poststructuralist, or any number of other story forms—would likely enable us to contemplate other curricular aesthetics and therefore different curricular futurities. By remaining open to different privileged lenses of narrative interpretation, we remain critical and introspective about what a “story” might be in ways that subvert Bal's structuralist assumptions. Fludernik (1996), for instance, problematized the very notion of a “natural” narratology, deriving a different narrative theory from an analysis of colloquial forms of narrative (like dialogue) that were considered “non-canonical” at the time she crafted her theory. I suggest that we ought to do the same in mathematics education by entertaining perspectives other than Bal's for analyzing curriculum as a storied artform.

Though I have tried to remain open to many aesthetic possibilities, I am very much including myself in this “we”. As I wrote this dissertation, it became increasingly apparent the degree to which my personal views of story, narrative, and aesthetics were entrenched in Western (often structuralist) perspectives. I, of course, alluded briefly to some other perspectives at select points. I even purposefully read several sources that *eschewed* perspectives on narrative that were most familiar to me. However, looking back across the lists of references that made it into this dissertation with the critical lens I honed throughout the process of writing, I can see there is a clear bias toward authors who adopt perspectives similar to my own. This is not strictly

a bad thing. After all, everyone (including me) has their own aesthetic sensibilities and preferences, and these *are* largely influenced by cultural upbringing and socialization. However, given that the goal in this dissertation was to remain open to various curricular aesthetics and perspectives on (in)coherence as expressed across various story forms, I certainly have some learning (and reading) to do about different (cultural) story forms and perspectives on narrative interpretation if I *truly* hope to critically interrogate my own biases and live up to the initial aspirations of this dissertation. To not do so would likely lead me to (inadvertently) enforce the very same strict politics of mathematical aesthetics that I aimed to dismantle at the outset.

Directions for Future Research and Personal Growth

In this final section, I reflect on possible directions for future research as well as how I hope to grow as a researcher going forward using what I learned from the experience of completing this dissertation.

Further Interrogation of the Politics of Curricular Aesthetics

As mentioned toward the beginning of this chapter, a major revelation of this dissertation, at least with respect to my journey as a researcher, was that coherence is not just a catch phrase or area of research I used to frame my work (which is admittedly how I arrived at it originally with a desire to focus on how students *holistically* organized their curricular experiences across courses). Rather, the assumption of coherence as an unquestionable good is an *aesthetic* value that is baked into many Western learning theories and philosophies. It took me some time to locate literature that explicitly dove into and questioned the axiomatic nature of this value (e.g., Hyvärinen et al., 2010; Shemmer, 2012). But now that I'm familiar with language like "the coherence principle" or "the consistency principle", I have *names* for this assumption. Having a name for these slippery abstract philosophical values is exciting. I feel like I have gained the power and confidence to discuss and critique these ideas and to read about and imagine possible

philosophies that augment or even outright reject such principles.

Now that I have unearthed this privileged aesthetic value in mathematics education and begun to deconstruct and critique it by cataloguing several differing curricular aesthetics (of (in)coherence), I hope to next work on adopting a much sharper critical stance toward my research inquiry. On one hand, I am proud and excited at how I have fundamentally transformed my research approaches and philosophies in just under two years while working on this dissertation. On the other, there is so much more I wish I could do in my research and particularly the way I *write up* my research to avoid inadvertently reifying existing politics of aesthetics. The *true* criticality I hope to work toward requires going further than I have at present. Rather than merely remaining “open” to different aesthetics of (in)coherence, I plan to devote more attention to systematic analyses of systems of power and privilege that serve to perpetuate certain politics of mathematical aesthetics while marginalizing others. After all, chosen curricular story aesthetics have non-neutral implications for who/what is deemed (in)coherent, so they are *not* neutral choices and should not be treated as such.

Throughout this dissertation, I have merely gestured toward Rancière’s politics of aesthetics as a relatively simple framing device to affirm the political nature of a given community’s aesthetic sensibilities. A concrete step I hope to take that will advance my goal of greater criticality is to learn more about the underlying philosophy that accompany these ideas so I can employ them to further interrogate and disrupt systems of power in ways that are more direct, specific, and pointed than I felt capable of doing as I wrote this dissertation. Adopting such a perspective is a natural extension of the various postmodern sensibilities and approaches present across this work (e.g., Appelbaum, 2010; the ABR paradigm; collage as a method from Chapter 3). Moving in this direction is also consistent with my commitment to further embracing

a participatory research paradigm (Osibodu et al., 2023), which requires further deconstruction of the historicized power dynamics and any corresponding axio-onto-epistemological assumptions with an eye to the (aesthetic) emancipation of those who feel constrained and boxed in by the curricular stories that are told (or not) in the mathematics classroom.

Loving Approaches to Research

Throughout this dissertation, I explored arts-based approaches to research as a way to question dominant axio-onto-epistemologies of (mathematics) education research and explore paradigms of research better aligned with my worldviews. This was a resounding personal success—not only because ABR aligned well with a dissertation about stories, storytelling, and aesthetics but also because I *finally* felt like the research approaches I crafted and employed across this dissertation were consistent with my worldviews and the ways I inquire about the world outside of academia. One goal, in particular, was to consider how ABR could serve as a participatory, loving approach to working *with* students as co-researchers. This was, as far as I can tell from my own experience and students' reactions, a success. As demonstrated in the art exhibit presented in Chapter 3, all participants were overwhelmingly creative as they reflected on their curricular experiences through art in whatever modality felt most appropriate to them. Though there were some initial growing pains (e.g., students took some time to acclimate to the unexpectedly open-ended prompts I gave them), the result was well worth it. Several student-artists *thanked me* for providing a friendly space where they could be creative and reflect without judgement across their mathematics curricular experiences. These participants repeatedly expressed how they did not see this as something they were often able to do in their STEM, let alone mathematics, courses. The final three student-artists even chose to stay later into the evening days before the week of final exams because they were *enjoying* crafting and sharing their curricular stories. To them, creating art was therapeutic, unlike many of their other

mathematical experiences they brought up, many of which they called outright traumatic. I suspect students' enjoyment was due, in part, to the open-ended possibilities afforded by the arts-based approaches to research I employed. However, I strongly believe that it was also how I positioned students as co-conspirators, philosophers, and artists that differentiated this experience well beyond past research experiences for me. Unlike past projects, from the beginning, I constantly aimed to create a participatory space that could support students' reflection and spiritual growth in ways that were supportive of their goals, guided by an ethics and praxis of radical love (Bowers et al., 2024; hooks, 2001; Yeh et al., 2021). Love was at the heart of why this was an overwhelming success.

This first participatory, arts-based study was meant to be a trial run, but it was already a considerable success as far as I could tell. I cannot imagine ever going back to the way I used to do research, which now feels sterile and inauthentic in comparison. Given that this was just my first foray into research informed by ethics of love, I plan to continue reflecting on what it means to cultivate research spaces grounded in an ethics of love and particularly how I can work to ensure that participants benefit from these spaces. As Laura (2013) contended, "taking love seriously in social research means that the process and product of scholarship has real consequences for the lives of three-dimensional human beings . . . not for imagined 'others' somewhere out there" (p. 291). When I came across this quote recently, it gave me pause. Though participants shared with me that they benefitted from this research experience, I simultaneously felt a twinge of guilt because I had not taken much explicit time to think about the immediate implications of this study given its speculative nature. In particular, as I discussed in Chapter 3, I did not consider the possible implications of this work for the local community of undergraduate mathematics students at MSU. Yes, I had been busy completing this dissertation

and, yes, this was only the first time I have carried out a participatory study of this nature. I had *purposefully* tried to keep the scope reasonable. And yet, this quote led me to wonder how I could do better next time to avoid perpetuating an ethics of *extraction* which largely contradicts an ethics of love. Going forward, I will not allow this to be afterthought again, choosing instead to ask these questions at the same time as I generate any research questions or goals (and ideally *alongside* participants next time).

Concrete Research Questions and the Aesthetic Dimension of Theorizing

In addition to the more theoretical future research questions and goals I relayed about the curriculum-as-story metaphor earlier and the more general personal goals I outlined in the last couple of sections, there are a couple of concrete directions for future work I hope to pursue in the future. First, I would like to conduct an updated version of the participatory, arts-based study in Chapter 3, where I use what I learned from this first iteration to redesign the study protocols to center students' *philosophical worldviews* from the outset. Given how Chapter 2 revealed just how intertwined an individual's axio-onto-epistemological assumptions are with their perspectives on curricular (in)coherence, I now believe that a more philosophical focus throughout the study would be helpful to disentangle *why* certain students hold certain perspectives on (in)coherence. Further, by making philosophy the focus of conversations from the outset, it might help catalyze the types of conversations that happened in the final group discussion where participants began to reflect critically on dialectics of in/coherence and the non-binary relationship between coherence and incoherence. Careful reconsideration would be required to reflect on how to best incorporate art prompts and/or further discussion questions that more explicitly broach relevant philosophical dimensions, but it would likely be worth it given that this would allow for a deeper dive into students' ontologies and valuations for mathematical (in)coherence.

Second, the discussion and conclusion of Chapter 4 served as a clarion call for further interdisciplinary conversations about what curricular stories ought to be included in the “anthology” of MVC. At the conclusion of this dissertation, I am left realizing that I have no answers to this question and that there are also very few answers in the MVC literature itself. This suggests there is need for sustained research and dialogue in this area from various stakeholder perspectives. I proposed some possibilities in Chapter 4 itself, but building on what I learned across this dissertation, I am particularly interested in subsequent studies *with students* (like the one in Chapter 3) concerning the curricular stories *they* constructed across MVC and particularly those that they found useful in subsequent STEM courses. Specifically, I would be interested in developing a methodology for participatory storytelling, storycrafting, and curricular reflection similar to what I had originally hoped to do in Chapter 3, using some of the successful prompts (like the final one from group discussion two). I believe such a methodology could prove invaluable to future interdisciplinary conversations that ought to involve all stakeholders.

Though these previous two research directions are natural next steps after my dissertation that I very well may undertake in the future, there is another, more indirectly connected, line of inquiry that I am interested in pursuing immediately upon completion of this dissertation. The time I spent on this dissertation served as a meditative space that allowed me to carefully network theorizing and other approaches to research in ways that resonated with my philosophical worldviews (Stinson, 2020). For the first time, I allowed my axio-onto-epistemological assumptions about mathematics education and the world around me to *fuel* the work I was doing. Theory was the lifeblood of this endeavor, not just a required section in each chapter. For the first time in my career as a researcher, I treated theory less as a “tool”—a mere

means to an end—and more like a *living force* (de Freitas & Walshaw, 2016) guiding the praxis of not only my research *but also* my teaching endeavors. Along this journey, I came to realize the importance of the *aesthetic* dimension of theorizing in mathematics education research, as I recognized the many ways in which my choices about which theories to use (or not) were governed by my personal aesthetic sensibilities (e.g., Tyburski, 2023). As an arts-based researcher, this was a breakthrough for me that I had not previously considered: theorizing is not only informed by an individual’s epistemological, ontological, and ethical values but also their *aesthetic* ones. Yet, the literature introducing the purpose and roles of “theory” and “theoretical frameworks” to emerging mathematics education scholars tends to present theory as primarily utilitarian rather than personal in nature (e.g., Cai et al., 2019; Cobb, 2007). As such, philosophical issues including epistemology, ontology, and to a lesser extent ethics are often discussed in relation to theory (e.g., Bikner-Ahsbabs et al., 2024) but *aesthetics* are notably absent in education research and specifically *mathematics* education research. The consequence is that many in our field—including emerging scholars—likely come to see theory as divorced from their personal subjectivities and aesthetic values.

This myth about theorizing perpetuated across the literature is dangerous because it indirectly suggests that decisions about theory are clinical decisions, rather than matters of personal taste. Particularly when combined with the myths of objectivity that pervade mathematics education research (Abreu et al., 2022), these views about theorizing can lead to the perpetuation of a politics of aesthetics that favors mere reproduction of past theories over personally resonant efforts to theorize in ways that align with one’s aesthetic sensibilities of research alongside their philosophical worldviews. As soon as I recognized this issue, it has been hard for me to unsee it. Going forward, therefore, I plan to conduct a literature review exploring

how entrenched this view of theorizing is in mathematics education literature. In writing up the results of this study, my goal would be to draw attention to this omission in the literature, paired with *several* concrete examples of how personal aesthetics are involved in choices related to theory, using examples of arts-based reflection examples I used across my dissertation studies to demonstrate my point. This effort naturally dovetails off past research where my colleagues and I have argued that more attention ought to be devoted to how emerging scholars are supported in learning to theorize (Drimalla et al., 2024) as well as how arts-based approaches to research can support such theoretical reflection (Lockett & Tyburski, under review). Ultimately, this new project idea offers an exciting opportunity to apply what I learned about aesthetics across this dissertation study to my past research in the field of graduate education that I plan to pursue going forward.

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