ESSAYS ON DIGITAL PLATFORMS

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ABSTRACT

This dissertation develops theoretical frameworks to examine how digital platforms shape and monetize interactions among consumers and producers. It analyzes the incentives that drive platforms' design and pricing decisions, as well as the regulatory implications of these choices.

Chapter 1: Price Coherence and Sponsored Search

Online marketplaces lose revenue when consumers discover products through the platforms but purchase directly from sellers at lower prices. To prevent this, platforms often enforce "price coherence," requiring sellers to offer their lowest prices through the platforms. Such practices face scrutiny for purportedly raising prices and reducing welfare. I explore the welfare implications of regulating price coherence when a platform may respond by adjusting its online search design. I develop a model in which a platform monetizes seller-consumer access through "transaction fees"—commission on intermediated transactions—and "referral fees"—proceeds from the sale of prominent search slots. With price coherence, the platform offers many prominent slots to help consumers find high value products but calibrates transaction fees to prevent any price reduction from increased competition. With a ban on price coherence, the platform loses its ability to influence prices through transaction fees and responds by offering fewer prominent slots, stifling competition to bolster its revenue from referral fees. My model thus reveals a novel tradeoff: prices are higher with price coherence, but consumers consider fewer products without it. I show that in face of this tradeoff, regulatory proscription of price coherence may unintentionally lower both total welfare and consumer surplus.

Chapter 2: Platform Coherence Policies with a Multiproduct Seller

I study a vertically differentiated product market intermediated by a monopoly platform. A monopoly seller offers a low- and a high-quality product to consumers with heterogenous

preferences to purchase through the platform rather than directly from the seller. Absent any restrictions imposed by the platform, the seller may draw consumers to purchase directly through differences in product prices and product availability between its direct and platform selling channels. I characterize the strategic pricing and assortment decisions made by the seller. Strategic assortment can substantially lessen the platform's ability to monetize the access it provides in buyer-seller interactions. The platform always finds it optimal to implement both cross-channel price and availability coherence policies if feasible. In contrast to general optimality of price coherence in similar markets supplied by a single-product seller, the platform may optimally allow for cross-channel price flexibility if it cannot enforce cross-channel availability coherence.

Chapter 3: Product Recommendations with Match Externalities

Platforms designing product recommender systems face a tradeoff between providing familiar "safe" recommendations likely to engage consumers or unfamiliar "discovery" recommendations with more engagement uncertainty but higher potential value. Committing to provide discovery recommendations when safe ones offer little value increases buyers' willingness to pay but may reduce successful consumer-producer matches. I show how match externalities to third-party advertisers can tip the scale of this tradeoff towards safe recommendations to prioritize consumer engagement over value. Although some consumers would prefer discovery, a platform provides them with safe recommendations to increase its revenue from third-party advertisers, compensating the consumers with lower participation fees. Despite reduced recommendation efficiency, consumer surplus increases with the presence of third-party advertisers. These match externalities typically cause a platform to prefer improving the "allure" (match likelihood) of discovery recommendations over their "suitability" (expected value from successful matches).

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INTRODUCTION

Digital platforms like Amazon, Facebook, Google, and Uber increasingly provide the infrastructure that underpins modern life and business. By shaping interactions among consumers and producers, their decisions have immediate and far-reaching consequences. This dissertation develops theoretical frameworks to study the incentives that drive digital platforms' design and pricing decisions, as well as the regulatory implications of these choices.

Fundamentally, a platform is an entity that facilitates interactions between distinct parties, each affiliated with the platform, either by providing *access* between parties or by offering *benefits* to one or more of them.[1] For example, Facebook Marketplace primarily provides access between buyers, sellers, and advertisers, while Apple Books primarily provides benefits to buyers and sellers by enabling books to be read on a digital device. In most cases, platforms provide both access and benefits to participants—for instance, Amazon offers market access as well as transaction benefits to both buyers and sellers.

Importantly, platforms do more than simply facilitate interactions; they shape them through design choices. By determining the conditions for access and the nature of the benefits they provide, platforms influence both the value generated in markets and how that value is allocated among consumers, producers, and platforms.

The chapters of this dissertation can be delineated by how they relate to platforms' design and provision of access and benefits: Chapter 1 examines how a platform monetizes access provision through its design and the implications this has for ongoing policy debates; Chapter 2 identifies and examines a tradeoff a platform faces between monetizing access and benefits; and Chapter 3

[1] This definition of a platform builds on that proposed by Hagiu & Wright (2015) by further characterizing the facilitation of interactions as the provision of access or benefits (or both).

examines how a platform's design of participation benefits depends on the number and nature of parties affiliated with the platform.

When considering access provision by platforms, Chapters 1 and 2 focus on "marketplace" platforms through which consumers discover and may transact with sellers. Marketplace platforms conventionally monetize consumer-seller access by levying a transaction fee on sellers for any sale made through the platform. However, faced with such a fee, a seller may offer a discount on its product through its direct sales channel to encourage consumers to leave the platform after discovering the product to purchase it directly, thereby cutting the platform out of the transaction and avoiding its fee. To prevent this behavior, platforms often require sellers to list their lowest prices through the marketplaces. These "price coherence" requirements face regulatory scrutiny for purportedly increasing prices and reducing welfare.

In Chapter 1, I demonstrate how a regulatory ban of price coherence requirements may lead to unintended consequences that ultimately reduce welfare. This is driven by the fact that platforms do not simply respond to regulation by adjusting their fees, but they also modify the design of their marketplaces. As a result, while a ban on price coherence may lead to lower prices, it also causes platforms to reduce the number of products easily accessible to consumers. The chapter introduces a novel tradeoff to the literature on price coherence that should be considered in ongoing and future regulatory discussions. More generally, the chapter highlights the importance of recognizing platforms' ability to shape market interactions through design choices, and of considering their incentives to do so when evaluating regulatory measures.

In Chapter 2, I examine a platform's attempt to monetize both access and benefit provision on a marketplace. I focus on the platform's use of transaction fees and policies that restrict seller behavior across sales channels. This includes price coherence requirements, as studied in Chapter

1, but also explores an additional policy, along with other features, that arise when a multiproduct seller participates on a marketplace.

While a single-product seller can only avoid transaction fees by strategically discounting its product through its direct sales channel, a multiproduct seller has a larger set of options. In addition to strategic pricing, a multiproduct seller can also employ strategic assortment—offering a larger variety of products through its direct channel—to avoid transaction fees. I identify this behavior among hotels and clothing sellers on marketplaces, where it is used as a tactic to encourage consumers to purchase certain products through direct websites, thereby avoiding platform transaction fees despite price coherence requirements. To prevent this, some platforms also enforce "availability coherence" across sales channels.

I study the interaction between a seller's strategic pricing and assortment tactics and a platform's use of price and availability coherence requirements. I demonstrate that a platform always prefers to enforce both price and availability coherence if possible, but if it cannot enforce availability coherence, then it may not be optimal to enforce price coherence. This is a contrast to much of the literature on price coherence that focuses on single-product sellers. The result arises from a tradeoff between monetizing access provision and benefit provision. Requiring price coherence aims to monetize access, but if a seller responds by delisting a product from the marketplace, the platform forfeits the opportunity to monetize transaction benefits in any sale of that product. This chapter demonstrates how multiproduct sellers introduce new strategic interactions and outcomes on marketplaces, offering insights relevant to both platforms and the sellers that they host.

In Chapter 3, I study a platform's decision to promote new product discovery through product recommendations when third-party advertisers value consumer engagement. In this environment, the platform provides both access between consumers, producers, and advertisers; and benefits to consumers in the form of tailored product recommendations that reduce search frictions. Although these recommendations are designed to help consumers find products more efficiently, I show that their quality is influenced by how much advertisers value access to consumers' attention. Specifically, a larger advertising market leads to less efficient product recommendations, such that consumers discover new products less often than they prefer. This result provides an explanation for repetitive recommendation "filter bubbles" that have been widely documented in practice.

Despite receiving worse recommendations, consumers are still better-off with the presence of advertisers. This reveals a tradeoff between the prevalent "premium" and "free" platform subscription structures for consumers. While consumers may receive better recommendations and greater overall value with ad-free services compared to free, ad-supported ones, the higher subscription fee associated with premium services completely offsets the value generated from the improved recommendations.

Finally, I show that with the presence of advertisers, platforms typically prefer to make recommendations more alluring—designed to increase the likelihood of consumer engagement—rather than more suitable. This is because advertisers derive value from consumer engagement, and that value does not depend on how much consumers like the products they engage with. Both the allure and suitability of recommendations increase consumers' value from participation, but allure additionally improves advertisers' payoffs by generating higher engagement.

Digital platforms play an increasingly central role in modern life and business, shaping interactions in ways that have broad economic and social consequences. This dissertation attempts to contribute to understanding these dynamics by identifying key questions in digital markets, developing theoretical frameworks to study them, and highlighting implications for consumers, producers, platforms, and policymakers.

CHAPTER 1

PRICE COHERENCE AND SPONSORED SEARCH

1.1 Introduction

Marketplace platforms provide access between sellers and consumers. Well-known examples include the physical goods marketplaces Amazon, eBay, and Etsy; the travel booking marketplaces Airbnb, Booking.com, and Expedia; and freelance labor platforms like TaskRabbit. Through these platforms, sellers gain access to consumers that they could not otherwise reach, and consumers gain access to a wider selection of products that they could not otherwise consider.

Platforms may monetize this access provision through two broad pricing mechanisms: "referral fees" and "transaction fees." Referral fees are charged for facilitating seller-consumer contact through online search results, e.g., by offering sponsored search slots that garner increased demand for sellers' products. In contrast, transaction fees are charged at the time of intermediated seller-consumer transaction. They are attractive to platforms for several reasons,[2] but they make access monetization susceptible to "disintermediation": if a platform charges a fee for access at the time of transaction, then, as access has already been granted, the seller and consumer have a joint incentive to cut the platform out of the relationship by transacting directly. For this reason, a platform often enforces sellers' "price coherence"—requiring each seller to offer its best price through the platform—to monetize access provision through transaction fees.

Such price coherence clauses have come under considerable regulatory scrutiny in recent times as they purportedly raise product prices and reduce welfare. Competition authorities both in the US and Europe have restricted their use across a wide range of platforms that offer products such

[2] For example, referral fees may eliminate participation by risk averse sellers (Hagiu & Wright, 2024), while transaction fees allow a platform to price discriminate under asymmetric information (Z. Wang & Wright, 2017) and influence prices to bolster other revenue streams (as shown in this chapter).

as physical goods, travel accommodations, eBooks, digital applications, and online games (see Baker & Morton (2018) for a review of regulation of price coherence clauses). In response to such regulatory attention, platforms often do not enact formal price coherence clauses directly but enforce them through platform design features. For example, Amazon eliminated formal price coherence clauses in Europe in August 2013 and in the US in March 2019, but it continues to punish sellers who break the terms of such clauses by steering consumers elsewhere through elimination of the "Buy Box" and demotion in search results.[3]

Beyond price coherence enforcement, platform design shapes the search environment and competition among sellers on a marketplace. It influences how effectively a platform can monetize access provision through its referral and transaction fees. Therefore, to assess a platform's response to regulation of price coherence clauses (or their informal enforcements), one must consider their interplay with both the platform's design and the fees charged by the platform. The goal of this chapter is to explore this interaction and draw out its regulatory implications.

A critical aspect of a platform's design is the management of sponsored search slots that grant sellers more prominent product placement in exchange for referral fees. Platforms actively design the search environment by choosing how many sponsored search slots to make available and by influencing the relative costs consumers incur to evaluate sponsored and non-sponsored products. For example, a platform may change the default number of products listed per-page to allow for more or fewer sponsored products to be easily inspected before moving to a subsequent results page. Alternatively, it may pad search results with repetitive or unrelated information to influence

[3] The US Federal Trade Commission sued Amazon for these and other practices in September 2023 (FTC, 2023b), and the EU bans such anti-discounting behaviors by Amazon and other large "gatekeeper" platforms under the Digital Markets Act (Regulation (EU) 2022/1925, 2022).

the number of actions (e.g., clicks; scrolls) needed to find a relevant non-sponsored product differently than those needed to find a relevant sponsored product.[4]

In this chapter, I focus on the design of the sponsored search environment as a key strategic lever that a platform may use in response to regulatory intervention. While a ban on price coherence may force a platform to reduce its transaction fee, which leads to lower prices, the platform may attempt to restore its loss of revenue by restricting the number of prominent slots. A reduction of prominent slots may reduce competition among sellers and increase the platform's revenue earned through referral fees. Thus, a ban on price coherence leads to a tradeoff between lower prices and lower product variety easily accessible to consumers. Indeed, in face of this tradeoff, I find that a ban on price coherence requirements may be counterproductive and reduce both total welfare and consumer surplus.

I develop a model of price coherence in a horizontally differentiated product market. Consumers search through a monopoly platform for the best product available from many single-product sellers, where product valuations are random across sellers. I model sponsored search by focusing on sponsored seller prominence and platform design of the search environment. The platform chooses the number of sponsored search slots to make available to sellers, and consumers face lower search costs to evaluate sponsored sellers relative to non-sponsored sellers. Each seller relies on the platform to become discoverable by consumers, but once a consumer discovers a seller, he may purchase either through the platform or directly from that seller. The platform monetizes access provision by charging sellers referral fees for placement in sponsored search slots and by charging transaction fees for intermediated transactions. I compare an unregulated environment, in which the platform may enforce price coherence, to one that proscribes this practice.

[4] In Appendix E, I provide further discussion of and empirical evidence for sponsored seller prominence and platform design as relevant features of online search.

Consumers may conduct their product search by several procedures. I begin by analyzing a stylized, baseline specification of the model that eschews any need to specify consumers' search procedure; then I demonstrate the relevance of these results with the baseline assumptions relaxed, considering the most common specifications of consumer search.

In the baseline model, consumers freely evaluate sponsored sellers, and they face prohibitively high search costs to evaluate non-sponsored sellers.[5] The first main result from the baseline analysis is that with price coherence, the platform provides maximal prominent product variety to consumers through sponsored search slots (Proposition 1.1). If the platform can enforce price coherence, then sellers cannot induce disintermediation, all transactions occur through the platform, and the platform earns its transaction fee on every purchase. The platform calibrates equilibrium prices through its choice of transaction fee, and in equilibrium, it earns the profit of a multi-product monopoly seller of all sponsored products. Increasing the number of sponsored search slots provides consumers with a wider selection of products, and on average, increases the match-value of their most preferred product. Aggregate demand therefore increases (at any given symmetric price among sellers). With price coherence, this "demand expansion effect" causes the platform to offer as many sponsored search slots as possible.

In contrast, with regulatory proscription of price coherence, the platform may reduce the observed product variety by keeping the number of sponsored search slots low (Proposition 1.2). If the platform cannot enforce price coherence, then sellers induce disintermediation of any sale facing a positive transaction fee from the platform. In equilibrium, the platform earns the sponsored seller industry profit. By increasing the number of sponsored search slots available to sellers, the platform puts more products into consumers' consideration. As a result, competition

[5] The baseline model builds on a more general version of the classic random-utility discrete choice model of Perloff & Salop (1985) that allows for a partially covered market, similarly to Rhodes & Zhou (2024).

among sponsored sellers increases, putting downward pressure on the equilibrium price. Because the platform cannot control this "price reduction effect" through its choice of transaction fee (as it can with price coherence), the platform has less incentive to offer sponsored search slots. Without price coherence, the platform's sponsored search design must therefore balance its countervailing effects on the industry profit: higher demand and lower prices.

I provide conditions under which the platform keeps the number of sponsored search slots low. One condition is that the market is covered regardless of the number of sponsored search slots. In that case, availing more sponsored search slots has no demand expansion effect, so the platform offers few slots to avoid any price reduction effect. In cases when this market coverage condition does not hold, the platform's design choice depends on the tail distribution of the random product valuations. If availing more sponsored search slots *eventually* makes prices competitive (i.e., the price reduction effect eventually overcomes the demand expansion effect), then the platform offers fewer sponsored search slots than feasible. In such cases, if it cannot enforce price coherence, the platform effectively reduces consumer choice in order to bolster the industry profit that it extracts through referral fees.

The literature has repeatedly shown that price coherence increases transaction fees and product prices (Boik & Corts, 2016; Calzada et al., 2022; Carlton & Winter, 2018; Edelman & Wright, 2015; Johnson, 2017). This is also true in my model; however, introducing sponsored search into the environment reveals a new tradeoff concerning price coherence. While prices are always higher with price coherence, product variety (that consumers can easily access) may be lower (Corollary 1.1). The proscription of price coherence may reduce both total welfare (Proposition 1.3) and consumer surplus (Proposition 1.4) due to this reduction of product variety.

The insights gained from the baseline model continue to hold in richer environments in which consumers incur positive search costs to evaluate sponsored sellers (Section 1.6) or when consumers may feasibly search among non-sponsored sellers (Section 1.7.2). To demonstrate this, I relax the baseline assumptions and adapt the model both to sequential search with observable prices—building on the "directed" search model of M. Choi et al. (2018)—and to search with unobservable prices—building on the "random" search model of Wolinsky (1986).

Next, I endogenize the search cost to sample sponsored sellers as a choice by the platform in the directed and random search environments. With price coherence, the platform always minimizes search costs. With the proscription of price coherence, however, the platform's decision depends on the consumer search procedure. Specifically, if consumer search is directed (by prices and their partial knowledge of product valuations), then the platform minimizes search costs (as it does with price coherence). In contrast, if consumer search is random, then the platform keeps search costs moderately high; a ban on price coherence unambiguously lowers both total welfare and consumer surplus (Proposition 1.5). This difference is driven by a result from the search literature: with directed consumer search, equilibrium prices decrease in search costs, but they increase in search costs with random search. Since without price coherence the platform cannot calibrate equilibrium prices through its transaction fee, it resorts to its choice of search costs to influence prices—in particular, it induces higher prices by choosing higher search costs if consumer search is random.

Last, I extend the baseline model to incorporate competition between two platforms through which consumers conduct their product search. With platform competition, the drawback of a ban on price coherence remains: consumers observe lower product variety without it. However, the usual motivation for banning price coherence is weakened or even negated, as prices might increase after such a ban. This is because when consumers search across multiple platforms, each platform has an incentive to encourage purchase through its own marketplace instead of its competitor's marketplace. With price coherence, a platform achieves this by lowering its transaction fee to induce lower prices on its marketplace. In equilibrium, the platforms enforce price coherence even if they would be better-off coordinating not to do so (to avoid the downward pressure on transaction fees and prices). Therefore, although price coherence *allows* the platforms to charge high transaction fees, in equilibrium, both transaction fees and prices remain low. Relative to the baseline model, platform competition thus expands the conditions under which banning price coherence harms consumers and total welfare; moreover, such a ban may act as a coordination device to the benefit of the platforms (Proposition 1.7).

The rest of the chapter is organized as follows. I place this work within the related literature in Section 1.2, and I introduce the formal model in Section 1.3. I then complete the baseline analysis: I derive equilibrium outcomes in the two regulatory environments in Section 1.4, and I study the welfare consequences of regulatory intervention in Section 1.5. In Sections 1.6 and 1.7, I adapt the baseline framework to allow for costly sequential search by consumers, endogenous search costs, transaction benefit provision by a platform, and platform competition. I conclude in Section 1.8. I provide all omitted proofs in Appendix A.

1.2 Related Literature

This chapter contributes to a growing literature on price coherence by considering a more complete picture of the strategies that platforms use to monetize seller-consumer access. It reveals a novel and contrasting set of results to the understanding of price coherence and its regulation.

Much of the literature finds detrimental welfare effects of price coherence (Boik & Corts, 2016; Calzada et al., 2022; Carlton & Winter, 2018; Edelman & Wright, 2015; Johnson, 2017). A common result is that price coherence increases intermediation fees, leading to higher product prices and reduced welfare. Expanding beyond this focus on prices, I consider how price coherence also shapes marketplace design and consumer search, revealing that it can have positive welfare effects. While other authors similarly identify positive welfare effects (Johansen & Vergé, 2017; Liu et al., 2021; Mariotto & Verdier, 2020), they abstract from a platform's role in providing consumer-seller access, which is the focus of this chapter and the usual motivation for enforcing price coherence. They find that price coherence may increase welfare by tightening seller participation constraints when sellers or consumers have heterogenous preferences across sales channels. In contrast, I find that price coherence may increase welfare through a mechanism that does not depend on any platform's provision of transaction benefits.

From the price coherence literature, C. Wang & Wright (2020) and C. Wang & Wright (2023) are most related to this chapter. They model consumer-seller access provision through consumer search, where consumers search for sellers directly at a high search cost or through a platform at a lower search cost. C. Wang & Wright (2020) show that price coherence reduces consumer surplus as long as the platform operates without it. I introduce an intra-platform difference in search costs between sponsored and non-sponsored products and find a contrasting welfare result. C. Wang & Wright (2023) study a platform's decision to reduce search costs and find that a monopoly platform has no incentive to do so without price coherence. In contrast, I show that a platform always has an incentive to change search costs; and it may even increase them without price coherence.

Another strand of the price coherence literature focuses on a platform's incentive to enforce price coherence (Calzada et al., 2022; Hagiu & Wright, 2024; Liu et al., 2021; Mariotto & Verdier, 2020; Chapter 3). I abstract from mechanisms that may make price coherence a suboptimal policy (such as multiproduct sellers or partial access provision).

Casner (2024), Hagiu & Wright (2024), and Xie & Zhu (2023) study access monetization by a platform and consider non-price design choices that may limit sellers' ability to induce intermediation (such as restricted communication and search steering), whereas I study access monetization when non-price design choices may not resolve any threat of disintermediation.

This chapter relates to work on (sponsored) consumer search. Foundational papers most relevant include M. Choi et al. (2018) on directed sequential search and Wolinsky (1986) on random sequential search.[6] Armstrong et al. (2009) studies the influence of an exogenously sponsored seller, and De Corniere (2016) studies endogenously sponsored search through a profit-maximizing platform. A platform with superior information about product-match quality may provide poor matches in search results (De Corniere, 2016; Hagiu & Jullien, 2011; Karle & Peitz, 2017). I abstract from such information asymmetry and show that a platform may still lower consumer search value. Work such as Athey & Ellison (2011) and Edelman & Schwarz (2010) focus on details of sponsored search slot auctions and take a more stylized approach to advertiser-consumer interactions, whereas I consider a more stylized sponsored search monetization scheme and focus on richer advertiser-consumer interactions.

I demonstrate how a ban of price coherence effectively shifts a platform's fee structure. This changes the platform's stake in a price-choice tradeoff that arises from increasing the number of sellers considered in the market. Several authors study a similar tradeoff as it relates to platform ownership, cross-network effects, buyer preferences, and product rankings (Belleflamme & Peitz, 2019; Dinerstein et al., 2018; Hagiu, 2009; Johnson et al., 2023; Nocke et al., 2007). Most related on this theme is Teh (2022), who studies how a platform's non-price design depends on the platform's fee structure. He shows that with two-part tariffs, a platform admits the socially optimal

[6] See Armstrong (2017) for a review covering both types of sequential search.

number of sellers and imposes socially optimal search costs; with lump-sum fees, it admits too few sellers and imposes too high search costs. Similar results play a role in this chapter, but I further study their welfare effects *across* fee structures, taking into account differences in equilibrium prices; I also consider platform competition. Further, the relationship between price coherence and a price-choice tradeoff is not obvious and is not identified in the literatures on either topic.

Several authors empirically study the effects of price coherence clauses following their ban in European countries (Ennis et al., 2023; Hunold et al., 2018; Ma et al., 2024; Mantovani et al., 2021a). Like the theoretical literature, they focus on how product prices change after a ban, while this chapter highlights the need to also study the impact on sponsored product variety, search costs, and overall marketplace design.

1.3 Baseline Model

A monopoly platform intermediates trade in a horizontally differentiated product market. The market consists of a countably infinite number of zero-cost single product sellers and a representative consumer with unit demand for the product. The sellers and their products are indexed by $i \in \mathbb{N}$. The consumer has valuation $v_i \in \mathbb{R}$ for product i. The v_i are drawn independently across i from the interval $[\underline{v}, \bar{v}] \subseteq \mathbb{R} \cup \{-\infty, +\infty\}$ according to the common distribution function H with a log-concave density h that is strictly positive and differentiable in $(\underline{v}, \bar{v})$.[7]

The platform provides essential access between sellers and the consumer; the consumer can only discover a seller through the platform. However, once the consumer has discovered a seller, the consumer can purchase the seller's product either through the platform or through the seller's

[7] I assume H is common knowledge. Log-concavity of h streamlines the exposition, but the qualitative nature of the results do not change if this condition is relaxed to $\phi_p(p; K) \leq 0$, where $\phi(p; K)$ is defined below Equation (1.2).

direct sales channel. Each seller i sets intermediated and direct prices that I denote as p_i and p_i^d, respectively. Both parties receive the same gross payoff whether they transact through the platform or directly, i.e., the platform does not provide any transaction benefits to the sellers or the consumer. If the consumer receives the same net payoff from purchasing through either sales channel, I assume he purchases through the platform (if he purchases at all).

To facilitate consumer search, the platform can determine and allocate "prominent" and "non-prominent" search slots among sellers. The consumer can sample a product in a prominent search slot more easily than one in a non-prominent search slot. In particular, the platform chooses the number of prominent search slots $K \leq \overline{K}$ available to the sellers, where $\overline{K}$ bounds K due to a technological constraint faced by the platform. Let $\mathcal{C} \subseteq \mathbb{N}$ denote the set of sellers in prominent search slots and let s_i be the consumer's search cost to observe $\left(v_i, p_i, p_i^d\right)$ and make a purchasing decision regarding product i. I assume that $s_i = s_{\mathcal{C}}$ for any prominent seller $i \in \mathcal{C}$ and $s_i = s_{\mathcal{C}'}$ for any non-prominent seller $i \in \mathcal{C}' \coloneqq \mathbb{N} \setminus \mathcal{C}$, where $s_{\mathcal{C}'} \geq s_{\mathcal{C}} \geq 0$. The search cost values $s_{\mathcal{C}}$ and $s_{\mathcal{C}'}$ are common knowledge.

For the baseline analysis, I assume that $s_{\mathcal{C}} = 0$ and $s_{\mathcal{C}'} > \int_{\underline{v}}^{\bar{v}} v dH(v)$. That is, it is costless for the consumer to sample all prominent sellers, but it is prohibitively costly to sample a non-prominent one. For simplicity, I also assume that the consumer samples all prominent sellers simultaneously.[8]

[8] The simultaneous sampling assumption avoids certain uninteresting technicalities that may arise from the extent of the (v_i, p_i)'s observability before the consumer conducts his search. For example, if each (v_i, p_i) is completely unobservable before search, there is an equilibrium in which the consumer does not sample any prominent seller, even though it is costless to do so. The consumer's prior information and beliefs play a more relevant role when the baseline assumption that $s_{\mathcal{C}} = 0$ and $s_{\mathcal{C}'} > \int_{\underline{v}}^{\bar{v}} v dH(v)$ is relaxed. I formalize the consumer's information and address that case in Sections 1.6 and 1.7.

The platform monetizes seller-consumer access provision through two pricing mechanisms: a per-transaction fee f and a referral fee r, both levied on the seller. The per-transaction fee f is charged for intermediated transactions, where f is paid by seller i if and only if product i is purchased through the platform. In contrast, the referral fee r is charged for placement in prominent search slots, where r is paid by seller i if and only if $i \in \mathcal{C}$. Whenever K prominent search slots are allocated among more than K eligible sellers (who offer to pay r), each eligible seller is randomly allocated a slot with equal probability. Because a seller $i \in \mathcal{C}$ pays r for prominence, I refer to such a seller interchangeably as a "prominent seller" and a "sponsored seller."

In addition to setting a transaction fee and a referral fee, the platform chooses whether to enforce sellers' "price coherence." If the platform enforces price coherence, then a seller's intermediated price cannot exceed its direct price. In other words, a seller i with $p_i^d < p_i$ is excluded from the consumer's search results. I refer to an environment in which the platform does not enforce price coherence as one with "price flexibility."

The timing of the baseline game is as follows:

1. The platform chooses whether to enforce price coherence. It chooses the number of prominent search slots K and sets the transaction fee f and referral fee r.
2. Each seller i observes all decisions made by the platform in stage 1. It decides whether to offer r to the platform for prominence. Up to K eligible sellers that offer r are selected for inclusion in the set $\mathcal{C}$ of prominent search slots.
3. Each seller i observes whether it has received a prominent search slot and sets its intermediated and direct prices p_i and p_i^d.
4. The consumer observes (v_i, p_i, p_i^d) for each seller $i \in \mathcal{C}$ and makes a purchasing decision.
5. All payoffs are realized.

I use weak perfect Bayesian equilibrium as a solution concept. I consider all equilibria in which sellers behave symmetrically within the set of prominent sellers $\mathcal{C}$ and within the set of non-prominent sellers $\mathcal{C}'$.

1.4 Baseline Analysis

In this section, I characterize the equilibrium of the game described above. In particular, I consider two regulatory environments: first in Section 1.4.1, I solve for the equilibrium with enforceable price coherence, and in Section 1.4.2, I consider regulatory proscription of price coherence. In what follows, I refer to the latter case as one of "mandated price flexibility."

1.4.1 Price Coherence

Suppose the platform enforces price coherence and let f, r, and K be given. Each seller i must set $p_i \leq p_i^d$ (otherwise it loses access to the consumer), the consumer always purchases through the platform (if it purchases at all), and the platform receives f for any completed transaction. I assume that $f \leq \bar{v}$ because $f > \bar{v}$ yields zero profit for the platform and is dominated. In what follows, I first solve for a symmetric equilibrium price $p(f, K)$ given a consideration set $\mathcal{C}$ with $|\mathcal{C}| = K$. I then study the platform's problem of choosing the optimal f, r, and K.

Let $\mathcal{C}$ with $|\mathcal{C}| = K$ be given. Suppose seller $i \in \mathcal{C}$ lists price p_i while each seller $j \in \mathcal{C} \setminus \{i\}$ lists price p. The consumer purchases i if and only if

$$v_i - p_i > \max_{j \in \mathcal{C} \setminus \{i\}} \{0, v_j - p\}. \tag{1.1}$$

Then seller i's profit is given by

$$(p_i - f) \Pr\left[v_i - p_i > \max_{j \in \mathcal{C} \setminus \{i\}} \{0, v_j - p\}\right] - r = (p_i - f) \int_{p_i}^{\bar{v}} H(v + p - p_i)^{K-1} dH(v) - r.$$

Note that seller i's profit gross of r is non-positive for $p_i \leq f$ and zero for $p_i \geq \bar{v}$ (or approaches zero as $p_i \to \bar{v}$ if $\bar{v} = \infty$), while it is strictly positive for all $p_i \in (f, \bar{v})$. Therefore, if there is a unique solution to the first order condition of seller i's profit maximization problem, it is also the unique global maximum of i's profit.

Applying Leibniz integral rule, the first order condition of seller i's profit maximization problem is

$$\int_{p_i}^{\bar{v}} H(v+p-p_i)^{K-1} dH(v)$$

$$- (p_i - f)\Bigg[H(p)^{K-1}h(p_i)$$

$$+ (K-1)\int_{p_i}^{\bar{v}} H(v+p-p_i)^{K-2} h(v+p-p_i) dH(v)\Bigg] = 0.$$

In equilibrium $p_i = p$, so a candidate equilibrium price solves

$$p = f + \phi(p; K), \tag{1.2}$$

where[9]

$$\phi(p;K) := \frac{[1 - H(p)^K]/K}{H(p)^{K-1}h(p) + \int_p^{\bar{v}} h(v) dH(v)^{K-1}}.$$

Lemma 1.1. *If $r = 0$, then the prominent sellers' equilibrium price $p(f, K)$ increases in f and decreases in K, where:*

(i) If $f \le \underline{v} - \phi(\underline{v}; K)$, then $p(f, K) \le \underline{v}$, such that the market is fully covered. Specifically, $p(f, K) = \underline{v}$ if $K = 1$ and $p(f, K) = f + \phi(\underline{v}; K)$ if $K \ge 2$.

(ii) Otherwise, $p(f, K)$ uniquely solves Equation (1.2) *and $p > \underline{v}$, such that the market is not fully covered.*

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[9] To see this, observe that imposing $p_i = p$ into the first order condition and rearranging terms obtains

$$p - f = \frac{\int_p^{\bar{v}} H(v)^{K-1} dH(v)}{H(p)^{K-1}h(p) + (K-1)\int_p^{\bar{v}} H(v)^{K-2} h(v) dH(v)} = \frac{\frac{1}{K}\int_p^{\bar{v}} [KH(v)^{K-1}h(v)]dv}{H(p)^{K-1}h(p) + \int_p^{\bar{v}} h(v)[(K-1)H(v)^{K-2}h(v)]dv} = \phi(p; K).$$

Given that h is log-concave, $\phi(p;K)$ decreases in p, so Equation (1.2) pins down the equilibrium price $p(f,K)$ as detailed in the lemma provided that r is not too large and the sellers' net profit at $p(f,K)$ remains positive. As Equation (1.2) indicates, the sellers' equilibrium price is equal to their effective unit cost f plus a markup $\phi(p;K)$. The numerator of the markup $\phi(p;K)$ in equilibrium is each seller's expected demand. The denominator is the absolute value of the slope of the demand at the equilibrium price. It measures the density with which the consumer purchases from a given seller while being indifferent between that seller and the outside option (this corresponds to the first term $H(p)^{K-1}h(p)$) or being indifferent between that seller and another seller (given by the second term $\int_p^{\bar{v}} h(v)dH(v)^{K-1}$).

The equilibrium price $p(f,K)$ increases in the transaction fee f, as sellers pass a part of the fee to the consumer. It decreases in the number of competing sellers K in the consideration set, as higher competition leads to lower prices.[10]

Now consider the platform's profit. A seller offers to pay the referral fee r if it can earn a nonnegative profit from placement in a prominent search slot, and the payment of r does not influence the equilibrium price $p(f,K)$. Therefore, for any f and K, the platform's optimal referral fee, $r(f,K)$, extracts all surplus from a prominent seller, i.e.,

$$r(f,K) = \frac{1}{K}[p(f,K) - f]\left[1 - H\big(p(f,K)\big)^K\right]. \qquad (1.3)$$

Additionally, the platform earns f whenever a transaction occurs. Its profit is thus given by

$$\Pi^{PC}(f,K) \coloneqq f\left[1 - H\big(p(f,K)\big)^K\right] + Kr(f,K) = p(f,K)\left[1 - H\big(p(f,K)\big)^K\right]. \qquad (1.4)$$

[10] That $p(f,K)$ decreases in K relies on the assumption of log-concave h. At a symmetric price p, as K increases, per-seller demand $[1 - H(p)^K]/K$ decreases. Log-concave h ensures that the density with which the preferred seller loses a purchase by marginally increasing its price, $H(p)^{K-1}h(p) + \int_p^{\bar{v}} h(v)dH(v)^{K-1}$, never decreases "too much" relative to per-seller demand; it ensures that the equilibrium markup term $\phi(p;K)$ decreases in K.

The platform chooses f and K to maximize $\Pi^{PC}(f,K)$. Notice from Equation (1.4) that

$$\Pi^{PC}(f,K) \leq \max_p p[1-H(p)^K] =: \Pi^M(K),$$

that is, the platform profit cannot exceed the monopoly profit of a seller who gets to set the price of all K products. In what follows, I will denote such a seller as a "K-product monopoly seller." Lemma 1.2 provides a summary of the monopoly price, which aids in showing that the platform can achieve the monopoly profit through its choice of f.

Lemma 1.2. *A K-product monopoly seller's optimal price $p^M(K) \coloneqq \arg\max_p p[1-H(p)^K]$ is unique and increases in K. Furthermore, $p(0,K) \leq p^M(K)$.*

Lemma 1.2 states that, for any given K, the platform may not achieve the upper bound on its profit of $\Pi^M(K)$ by setting $f=0$ because the induced equilibrium price would be too low. Recall that the equilibrium price $p(f,K)$ increases continuously in (and is larger than) the transaction fee f. Thus, for any K, the platform can uniquely achieve the profit of $\Pi^M(K)$ by setting f such that the induced equilibrium price satisfies $p(f,K)=p^M(K)$.

Next, consider the platform's choice of K. Observe that, by the envelope theorem, the platform's optimal profit $\Pi^M(K)$ increases in K. The platform thus optimally chooses $K=\overline{K}$, i.e., it avails to the consumer as many product options as possible.

Finally, it is easy to see that when price coherence is feasible, the platform always enforces it. If price coherence is not enforced and the platform sets any $f>0$, then in any equilibrium, each seller $i \in \mathcal{C}$ sets $p_i^d = p(0,K) < p_i$ and all transactions are direct, resulting in a profit of $\Pi^{PC}(0,K) \leq \Pi^M(K)$. Thus, the platform is always better-off enforcing price coherence.

Proposition 1.1 summarizes the equilibrium outcomes with enforceable price coherence.

Proposition 1.1. *The platform enforces price coherence if admissible. It sets $K^{PC}=\overline{K}$ to make the consumer consideration set as large as possible. It chooses transaction fee f^{PC} such that the* *equilibrium price satisfies* $p(f^{PC}, \overline{K}) = p^M(\overline{K})$. *It sets referral fee* $r^{PC} = r(f^{PC}, \overline{K})$ *given by Equation* (1.3). *The platform extracts the* $\overline{K}$*-seller industry revenue as profit, and its optimal profit is that of a* $\overline{K}$*-product monopoly seller.*

1.4.2 Mandated Price Flexibility

Now suppose that the platform cannot enforce price coherence due to regulatory constraints. Recall that if the platform sets any transaction fee $f > 0$ (with price flexibility), then in any equilibrium, each seller i sets $p_i^d = p(0, K) < p_i$ and all transactions are direct. If instead the platform sets $f = 0$, then in any equilibrium, the price paid by the consumer is $p(0, K)$, whether it is a direct or an intermediated transaction. Furthermore, players' equilibrium payoffs do not depend on f or the channel of transaction. Sellers always receive a price $p(0, K)$, and the platform optimally sets the referral fee $r(0, K)$ to extract all seller surplus.

The platform thus earns a profit of $\Pi^{PC}(0, K)$ with price flexibility, which I rewrite as

$$\Pi^{PF}(K) \coloneqq Kr(0, K) = p(0, K)\left[1 - H\big(p(0, K)\big)^K\right]. \tag{1.5}$$

The platform chooses K to maximize $\Pi^{PF}(K)$, which is the industry profit with K sellers.

Recall from Proposition 1.1 that with price coherence, the platform increases expected demand by increasing K while calibrating the equilibrium price through its choice of f. In contrast, with price flexibility, the platform cannot influence the equilibrium price through f. It fully realizes the effects of K on both demand and the equilibrium price. Holding price constant, K has an increasing effect on demand (because $1 - H(p)^K$ increases in K). However, K has a decreasing effect on the equilibrium price due to increased competition (as $p(0, K)$ decreases in K by Lemma 1.1). The platform must choose K to balance its countervailing effects on industry profit: higher demand and lower prices. Proposition 1.2 provides conditions under which the platform keeps the consumer's consideration set small by setting $K^{PF} < \overline{K}$.

Proposition 1.2. *With mandated price flexibility, equilibrium payoffs are invariant to the platform's transaction fee f^{PF}. The platform chooses the number of prominent search slots $K^{PF} \in \{1, \dots, \overline{K}\}$ to maximize the industry profit, and it sets a referral fee of $r^{PF} = r(0, K^{PF})$ given by Equation* (1.3). *Furthermore:*

(i) If $\underline{v} \geq \phi(\underline{v}; 1)$, then $K^{PF} = 1$.

(ii) If $\lim_{v \to \bar{v}} \frac{1-H(v)}{h(v)} = 0$, then $K^{PF} < \overline{K}$ for sufficiently large $\overline{K}$.

The conditions given in the proposition may be interpreted as conditions on market coverage and the thickness of the tail of the valuation distribution. By Lemma 1.1, the condition $\underline{v} \geq \phi(\underline{v}; 1)$ in Proposition 1.2(i) is equivalent to $p(0,1) = \underline{v}$ (or $p^M(1) = \underline{v}$). Since $p(0, K)$ decreases in K, it implies that for any $K \in \{1, \dots, \overline{K}\}$, the equilibrium price $p(0, K)$ is less than (or equal to) the consumer's minimum possible valuation $\underline{v}$. The market is fully covered in the sense that in equilibrium, the consumer always makes a purchase. Much of the literature on random utility oligopoly models focuses on this case (S. P. Anderson et al., 1995; Gabaix et al., 2016; Perloff & Salop, 1985). When the market is always covered, increasing K has no demand expansion effect (because $1 - H(p)^K = 1$ for all K) but always puts downward pressure on the equilibrium price. The platform thus opts to keep the equilibrium price as high as possible by setting $K^{PF} = 1$.

Proposition 1.2(ii) provides a condition concerning the tail distribution of the v_i: the inverse hazard ratio tends to zero at the upper bound of the consumer's product valuation. The condition ensures that the price reduction effect of increasing K *eventually* outweighs any demand expansion effect. It implies that the equilibrium price converges to the competitive price as K increases (i.e., $\lim_{K \to \infty} p(0, K) = 0$).

To see the role of the condition on the inverse hazard ratio, it is useful to rewrite the markup term of the equilibrium price as (see proof of Proposition 1.2 for derivation)

$$\phi(p;K) = \frac{1 - H(p)^K}{KH(p)^{K-1}h(p) + \int_p^{\bar{v}} \frac{h(v)}{1 - H(v)} dH_{(K-1:K)}(v)}, \tag{1.6}$$

where $H_{(K-1:K)}$ denotes the distribution function of the $(K-1)$-th order statistic of $\{v_i\}_{i=1}^K$, or the consumer's second most preferred product valuation. In expression (1.6), the numerator is total demand, and the denominator is the density with which the consumer is indifferent between his most preferred product and any other option. That $\lim_{v \to \bar{v}} \frac{1-H(v)}{h(v)} = 0$ ensures that the consumer's demand remains sufficiently elastic as the number of product options grows. When K is large, $H_{(K-1:K)}$ puts more weight on the higher values in $(\underline{v}, \bar{v})$ and the denominator grows unboundedly, driving the markup to zero. Thus, the platform has an incentive to keep K low to preserve the industry profit that it can extract upfront through its referral fee.

As an example, $\lim_{v \to \bar{v}} \frac{1-H(v)}{h(v)} = 0$ holds for the normal distribution but not for the heavier tailed exponential distribution. With a heavier tailed distribution, increasing the number of prominent sellers may expand demand too much relative to any increase in the likelihood that a preferred seller faces a close competitor. In such cases, the preferred product remains sufficiently un-substitutable as K gets large, and sellers maintain positive prices in the limiting equilibrium. Note that $\bar{v} < \infty$ is a sufficient condition for $\lim_{v \to \bar{v}} \frac{1-H(v)}{h(v)} = 0$, so $K^{PF} < \bar{K}$ for sufficiently large $\bar{K}$ always holds when the support of the product valuations is bounded above.[11]

[11] Assuming $\bar{v} < \infty$ and $\lim_{v \to \bar{v}} \frac{1-H(v)}{h(v)} = L^{-1} > 0$ implies $\frac{h(v)}{1-H(v)} \leq L$ and $-\log[1 - H(v)] = \int_{\max\{0,\underline{v}\}}^{v} \frac{h(x)}{1-H(x)} dx \leq vL$ for all v. This yields the contradiction $\infty = \lim_{v \to \bar{v}} \{-\log[1 - H(v)]\} \leq \bar{v}L \in \mathbb{R}$.

Neither of the conditions in Proposition 1.2 are necessary for $K^{PF} < \bar{K}$, but they encompass many commonly studied demand structures. The result clearly demonstrates that without price coherence, the platform has incentives to shrink the consumer's consideration set when competition reduces equilibrium prices.

1.5 Welfare Consequences of Price Coherence Regulation

As noted in the analysis above, when price coherence is enforceable, the platform effectively chooses the equilibrium price to maximize the industry revenue. However, with mandated price flexibility, it cannot control the equilibrium price through its transaction fee. Instead, the platform chooses the number of prominent search slots to maximize the industry profit that it extracts from the sellers through referral fees.

Hence, a welfare tradeoff arises. As the following Corollary (to Proposition 1.1 and Proposition 1.2) states, prices are higher with price coherence, but the consumer may have fewer options and receive worse product matches without it.

Corollary 1.1. *Prices are higher and the consideration set is larger with enforceable price coherence relative to mandated price flexibility, i.e.,* $p^{PC} \geq p^{PF}$ *and* $K^{PC} \geq K^{PF}$.

That $K^{PC} \geq K^{PF}$ is immediate because $K^{PC} = \bar{K}$. And employing Lemma 1.2,

$$p^{PC} = p^M(\bar{K}) \geq p^M(K^{PF}) \geq p(0, K^{PF}) = p^{PF}.$$

In the remainder of this section, I study the consequences of this price-choice tradeoff in terms of total welfare and consumer surplus.

Given an equilibrium price p and a number of prominent search slots K, total welfare and consumer surplus are respectively given by

$$W(p, K) \coloneqq \int_p^{\bar{v}} v dH(v)^K,$$

$$CS(p,K) := \int_p^{\bar{v}} (v-p)dH(v)^K.$$

Both $W(p,K)$ and $CS(p,K)$ decrease in p and increase in K. Thus, as Corollary 1.1 suggests, price coherence may either improve welfare (relative to price flexibility) as it ensures higher product variety or reduces welfare as it leads to higher prices. The following result ranks total welfare and consumer surplus under price coherence and price flexibility for large $\overline{K}$.

Proposition 1.3. *For $\overline{K}$ sufficiently large, the total welfare and consumer surplus may be ranked as follows.*

(i) Total welfare is higher with enforceable price coherence relative to mandated price flexibility, i.e., $W(p^{PC},K^{PC}) > W(p^{PF},K^{PF})$ if any one of the following three conditions hold: (a) $\bar{v} < \infty$, (b) $\underline{v} \geq \phi(\underline{v};1)$, or (c) $\lim_{v\to\bar{v}} \frac{1-H(v)}{h(v)} = 0$.

(ii) Consumer surplus is lower with enforceable price coherence relative to mandated price flexibility, i.e., $CS(p^{PC},K^{PC}) < CS(p^{PF},K^{PF})$, if $\bar{v} < \infty$.

Proposition 1.3(i) provides conditions under which, for large $\overline{K}$, mandating price flexibility reduces total welfare. Total welfare in the first-best outcome is $\int_{\max\{0,\underline{v}\}}^{\bar{v}} v dH(v)^{\overline{K}}$, which approaches $\bar{v}$ as $\overline{K}$ gets large. Recalling that $K^{PC} = \overline{K}$, if price coherence is enforced, the platform can extract a larger share of the total welfare as $\overline{K}$ increases because dispersion in the distribution of the consumer's preferred product valuation decreases. In the limit, the platform can extract the entire first-best welfare (since $\lim_{\overline{K}\to\infty} \Pi^M(\overline{K}) = \bar{v}$).

In contrast, under the conditions of Proposition 1.3(i) and for large $\overline{K}$, the platform sets $K^{PF} < \overline{K}$. Under price flexibility, total welfare is therefore bounded below $\bar{v}$ because the platform restricts product variety, which keeps the consumer's preferred product valuation relatively low.

This observation reveals a severe drawback to the regulation of price coherence. Mandating price coherence unambiguously lowers prices, but a platform may reduce consumer choice to bolster the industry profit that it extracts through referral fees. This negative effect outweighs any price reduction effect on total welfare if the platform significantly reduces product variety without price coherence.

A different conclusion holds for consumer surplus: for large $\overline{K}$, mandating price flexibility increases consumer surplus (Proposition 1.3(ii)). This is because with price coherence, in the limiting case, the platform can extract all consumer (and seller) surplus.

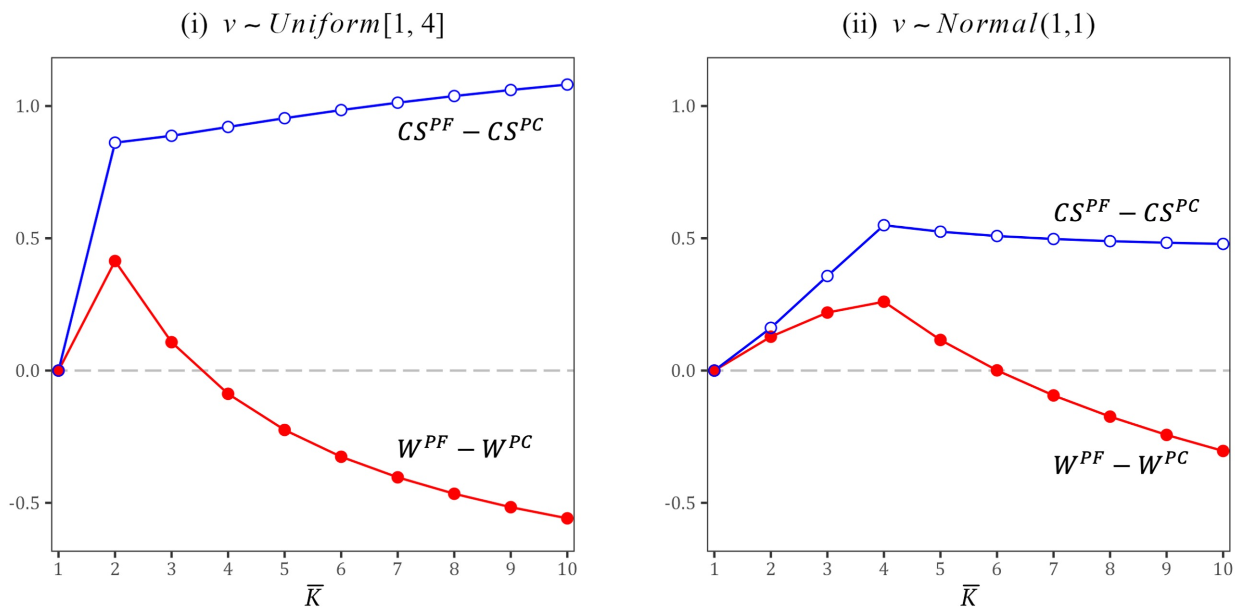


Figure 1.1: Welfare consequences of mandating price flexibility under uniform and normal distribution of product valuations

The welfare results of Proposition 1.3 are illustrated by Figure 1.1, which plots the impact of mandating price flexibility on total welfare ($W^{PF} - W^{PC}$) and consumer surplus ($CS^{PF} - CS^{PC}$)

as the maximum number of available sponsored slots ($\overline{K}$) varies. The figure considers two cases where at least one of the three conditions given in Proposition 1.3(i) are met. Under the uniform distribution over $[1,4]$, condition (a) (and hence (c)) is satisfied, and the mandate lowers total welfare when $\overline{K} \geq 4$. Under the normal distribution with mean 1 and variance 1, condition (c) is satisfied, and the mandate lowers total welfare when $\overline{K} \geq 6$.[12]

As Proposition 1.3 makes clear, for large $\overline{K}$ (and $\bar{v} < \infty$), whether a social planner should allow a platform to enforce price coherence depends entirely on whether the social planner is more concerned with total welfare or consumer surplus. However, as I discuss below, for small $\overline{K}$, the social planner's preferred regulation may not depend on the choice of welfare measure.

Under the conditions of Proposition 1.3, when might mandating price flexibility *improve* both total welfare and consumer surplus? If mandating price flexibility improves total welfare, then it also improves consumer surplus (because consumer surplus is total welfare less the platform's profit, which is always lower with price flexibility). There are two reasons why mandating price flexibility may improve total welfare (and hence consumer surplus). First, when $\overline{K}$ is small, with price flexibility, the platform may have incentive to make all products available to the consumer (i.e., set $K^{PF} = \overline{K}$) if the demand expansion effect of K outweighs any price reduction effect. In such a scenario, total welfare is higher with price flexibility as the prices are lower and the product variety remains the same. Second, total welfare may be higher with price flexibility if $\overline{K}$ is not large enough for the greater product variety with price coherence ($K^{PC} \geq K^{PF}$) to outweigh the

[12] In contrast, a price flexibility mandate never lowers total welfare under a heavier tailed distribution, for example, the exponential distribution, which does not satisfy any condition of Proposition 1.3(i) (as discussed in Section 1.4.2). If H is the exponential cdf with mean μ, then $p(0;K) = \mu$ for all K. Therefore increasing K has no price reduction effect, so $K^{PF} = \overline{K}$ due to the demand expansion effect of K. Hence $W^{PF} \geq W^{PC}$.

difference in prices ($p^{PC} \geq p^{PF}$). In both of these cases, a social planner should mandate price flexibility.

When might mandating price flexibility *reduce* both total welfare and consumer surplus? If mandating price flexibility reduces consumer surplus, then it is also reduces total welfare (as the contrapositive of this statement was argued above). Proposition 1.4 relates consumer surplus to the dispersion of the consumer's product valuations. Mandating price flexibility may reduce consumer surplus (and hence total welfare) if $\overline{K}$ is not too large and H is not too dispersed. Consider the environment of enforceable price coherence relative to that of mandated price flexibility. Depending on the dispersion of H, the fact that $K^{PC} = \overline{K} \geq K^{PF}$ has different effects on total welfare and the share of the total welfare received by the consumer (as determined by equilibrium prices). First, $K^{PC} \geq K^{PF}$ has an increasing effect on total welfare (since $\int v dH(v)^{K^{PC}} \geq \int v dH(v)^{K^{PF}}$), and this effect is "concave" in the dispersion of H (I clarify this notion below). Second, $K^{PC} \geq K^{PF}$ reduces the dispersion of the consumer's preferred product valuation such that the platform charges a higher price (i.e., $p^{PC} \geq p^{PF}$). Furthermore, this effect is "convex" in the dispersion of H. Therefore, if $\overline{K}$ is small and there is little dispersion in H, the welfare expanding effect of price coherence outweighs the price increasing effect (because $p^{PC} - p^{PF}$ is small relative to $\int v dH(v)^{K^{PC}} - \int v dH(v)^{K^{PF}}$). In such a case, a social planner should allow a platform to enforce price coherence.

To obtain specific results regarding the consumer surplus tradeoff, I focus on uniform H. Proposition 1.4 formalizes the above discussion.

Proposition 1.4. *Let* H *be the uniform distribution function over* $[\underline{v}, \bar{v}]$*. Given* $\overline{K} \geq 2$ *and* $\underline{v} > 0$*, there exists a cutoff for the upper valuation bound* $\bar{v}^{\dagger} > \underline{v}$ *such that consumer surplus is higher with enforceable price coherence relative to mandated price flexibility, i.e.,* $CS(p^{PC}, K^{PC}) >$

$CS(p^{PF}, K^{PF})$, if and only if $\bar{v} < \bar{v}^{\dagger}$. Furthermore, $\lim_{\underline{K} \to \infty} \bar{v}^{\dagger} = \underline{v}$, attaining Proposition 1.3*(ii) in the limiting case.*

Proposition 1.4 characterizes the consumer surplus tradeoff in terms of the upper valuation bound $\bar{v}$ with uniform H. It states that consumer surplus is higher under price coherence for sufficiently low values of $\bar{v}$. As $CS^{PC} > CS^{PF}$ implies $W^{PC} > W^{PF}$, the result demonstrates the existence of cases when a social planner should unambiguously allow a platform to enforce price coherence.

To illustrate the intuition behind this result, consider a sequence of environments in which $\bar{v}$ increases from an initial value of $\underline{v}$. As $\bar{v}$ increases, it differentially affects the level of total welfare and the share of the total welfare received by the consumer (which together make up consumer surplus).

First consider the consumer's expected value for his preferred product, which affects total welfare at a given price. For uniform H, the difference in the consumer's expected value from his preferred product across policies is

$$\int_{\underline{v}}^{\bar{v}} v dH(v)^{K^{PC}} - \int_{\underline{v}}^{\bar{v}} v dH(v)^{K^{PF}} = \left(\frac{K^{PC}}{K^{PC}+1} - \frac{K^{PF}}{K^{PF}+1} \right) (\bar{v} - \underline{v}),$$

which increases linearly in $\bar{v}$. As a result, as $\bar{v}$ increases, price coherence exhibits an increasing and linear effect on welfare at a given price.

Now consider equilibrium prices, which affect the share of welfare received by the consumer. If $\bar{v} = \underline{v}$, then there is no dispersion in the consumer's product valuations, and the platform extracts all surplus under both price flexibility and price coherence (by inducing $p^{PF} = p^{PC} = \bar{v}$). As $\bar{v}$ increases above $\underline{v}$, the dispersion in the consumer's product valuations increases, but the dispersion in the consumer's preferred product valuation increases *less* under price coherence compared to price flexibility (since $K^{PC} > K^{PF}$). As a result, as $\bar{v}$ increases, the platform induces a higher price under price coherence (i.e., $p^{PC} > p^{PF}$). Furthermore, under price coherence, the platform's ability to extract more consumer surplus increases with $\bar{v}$ (since $K^{PC} > K^{PF}$), and the difference in prices across policies increases convexly in $\bar{v}$. Therefore, as $\bar{v}$ increases, the platform extracts an increasing share of total welfare under price coherence.

Taken together, as $\bar{v}$ increases, total welfare always increases more under price coherence compared to price flexibility; however, the share of total welfare received by the consumer decreases with $\bar{v}$. For sufficiently low levels of $\bar{v}$, the consumer is better-off with price coherence because he earns a significantly higher product value with only marginally higher prices. For large values of $\bar{v}$, the platform extracts too much of any increased value through higher prices, and the consumer is better-off with price flexibility.

1.6 Sequential Search

Consumers often search sequentially. A consumer incurs a cost to evaluate a product and decides whether to end search or evaluate another at an additional cost. He repeats this process until all products have been sampled, a product has been purchased, or search has been aborted without a purchase. With this perspective, one may question whether the intuition derived from the baseline model holds in an environment with a more complex search procedure. In this section, I demonstrate robustness of the main results to sequential consumer search.

To consider sequential consumer search in the baseline model, I allow $s_C \geq 0$ and maintain $s_{C'} > \int_{\underline{v}}^{\bar{v}} v dH(v)$; i.e., the consumer may incur a positive cost to sample products in the prominent search slots, but it is still prohibitively costly to sample non-prominent products. In Stage 4 of the game, the consumer observes whether each seller i is prominent and searches sequentially.

As is standard in the literature, the consumer may conduct one of two types of sequential search: (i) "Directed" sequential search where a consumer has an informative signal concerning (p_i, v_i), usually full knowledge of p_i and a noisy signal of v_i, before incurring the search cost s_i to fully evaluate product i; and (ii) "Random" sequential search where the consumer has no information concerning (p_i, v_i) before evaluating product i.

While observability of the intermediated prices p_i before sampling depends on whether search is directed or random, I assume that the direct prices p_i^d are always unobservable before sampling. I adjust the baseline solution concept to account for unobservable prices. I continue to use weak perfect Bayesian equilibrium but refine it by assuming the consumer holds passive beliefs about the distribution of unobserved prices after observing any sequence of (intermediated or direct) prices.[13] This refinement is standard in the search literature. It is natural because all sellers set their prices before the consumer begins search.

Section 1.6.1 adapts the model to directed search as studied by M. Choi et al. (2018), and Section 1.6.2 adapts the model to random search as studied by Wolinsky (1986). The baseline model is the limiting case of both extensions as $s_C \to 0$.

1.6.1 Directed Search

Suppose the consumer freely observes all intermediated prices p_i before conducting any search.

To allow for a pure strategy pricing equilibrium, following the directed search literature, I assume $v_i = \hat{u}_i + u_i$ for each i, where the first component $\hat{u}_i$ is known to the consumer and reflects the consumer's prior valuation for product i, and the second component u_i is known to the consumer only after he incurs the search cost s_i to evaluate product i.

[13] Specifically, the consumer's passive beliefs imply that if the consumer observes an out of equilibrium price offered by one seller, he continues to believe that any unobserved prices remain as in equilibrium.

Assume $\hat{u}_i$ and u_i are respectively distributed according to distribution functions $\hat{G}$ and G with associated densities $\hat{g}$ and g. The utility components $\hat{u}_i$ and u_i are independent of each other and across i. The consumer searches sequentially with perfect recall.[14]

Now, consider the consumer's optimal search strategy. Let $u_i^* \coloneqq -\infty$ for all $i \in \mathcal{C}'$, and let $u_i^* \coloneqq u^*$ for all $i \in \mathcal{C}$, where u^* is implicitly defined by

$$s_{\mathcal{C}} = \int_{u^*}^{\infty} (u - u^*) dG(u).$$

Intuitively, u^* is a reservation value such that the net expected benefit from an additional search in $\mathcal{C}$ is equal to the search cost when the known valuation component $\hat{u}_i$ does not vary across i (i.e., when $\hat{G}$ is degenerate). As is well known due to Weitzman (1979), the consumer's optimal search strategy is as follows:

(i) The consumer visits the sellers in descending order of $\hat{u}_i + u_i^* - p_i$.

(ii) Let $\hat{\mathcal{C}}$ be the set of sellers that the consumer has so far evaluated. The consumer stops and takes the best option available if and only if

$$\max_{i \in \hat{\mathcal{C}}} \{0, \hat{u}_i + u_i - p_i\} > \max_{j \in \mathbb{N} \setminus \hat{\mathcal{C}}} \hat{u}_j + u_j^* - p_j.$$

Finally, let $w_i \coloneqq \hat{u}_i + \min\{u_i, u_i^*\}$ for each i. M. Choi et al. (2018) show that the consumer eventually purchases product i if and only if[15]

$$w_i - p_i > \max_{j \in \mathbb{N} \setminus \{i\}} \{0, w_j - p_j\}. \tag{1.7}$$

This expression is identical to Equation (1.1), except the w_i have replaced the *true valuations* v_i. The w_i can be interpreted as *effective valuations* whose values depend on the cost to realize the true valuations. Specifically, the consumer's purchasing decision depends more on the true

[14] Following M. Choi et al. (2018), I assume the consumer may not purchase product i without first incurring s_i.
[15] See M. Choi et al. (2018; Theorem 1) for proof.

valuations when search becomes less costly ($w_i \xrightarrow{d} v_i$ as $s_i \to 0$), and the consumer's purchasing decision depends more on the prior valuations when search becomes more costly ($w_i \xrightarrow{d} \hat{u}_i$ as $s_i \to \infty$).

Since the consumer's purchasing decision depends on the effective valuations w_i rather than the true valuations v_i, seller demand depends on the distribution of the w_i rather than that of the v_i. Let H^e and h^e respectively denote the distribution function and density of w_i for $i \in \mathcal{C}$; so,

$$H^e(w) = \int_{-\infty}^{u^*} \hat{G}(w-u)dG(u) + \int_{u^*}^{\infty} \hat{G}(w-u^*)\, dG(u),$$

$$h^e(w) = \int_{-\infty}^{u^*} \hat{g}(w-u)dG(u) + \int_{u^*}^{\infty} \hat{g}(w-u^*)\, dG(u).$$

Because of this equivalence of directed search outcomes to the discrete choice framework, the analysis and results of Sections 1.4 and 1.5 immediately follow with the obvious relabeling.[16]

1.6.2 Random Search

Suppose the consumer has no information (other than H) about (p_i, v_i) before sampling product i. Suppose the consumer expects each seller $i \in \mathcal{C}$ to charge $p_i = p$, and define v^* implicitly by

$$s_{\mathcal{C}} = \int_{v^*}^{\bar{v}} (v - v^*)dH(v).$$

Wolinsky (1986) shows that the consumer's optimal search strategy is as follows. Maintaining $\hat{\mathcal{C}}$ as the set of sellers that the consumer has so far evaluated, the consumer samples randomly in $\mathcal{C}$ and stops to take the best option available if and only if

[16] To be precise, define $\underline{w} \coloneqq \inf[\mathrm{supp}(\hat{g})] + \min\{\inf[\mathrm{supp}(g)], u^*\}$ and $\overline{w} \coloneqq \sup[\mathrm{supp}(\hat{g})] + u^*$. Then every result of Sections 1.4 and 1.5 follow with $(H, h, \underline{v}, \bar{v}) \mapsto (H^e, h^e, \underline{w}, \overline{w})$. The welfare measures remain valid due to M. Choi et al. (2018; Corollary 1). If $\mathcal{C}$ is interpreted strictly as a set of prominent sellers rather than a consideration set, then the development and interpretations in Sections 1.4 and 1.5 follow as well. M. Choi et al. (2018) discuss how (H^e, h^e) depends on the primitives $(\hat{G}, \hat{g})$ and (G, g) to ensure equilibrium existence.

$$\max_{i \in \hat{\mathcal{C}}}\{0, v_i - p_i\} \geq 1_{[\hat{\mathcal{C}} \subset \mathcal{C}]}(v^* - p),$$

where $1_{[\hat{\mathcal{C}} \subset \mathcal{C}]} = 1$ if $\hat{\mathcal{C}}$ is a strict subset of $\mathcal{C}$ and $1_{[\hat{\mathcal{C}} \subset \mathcal{C}]} = 0$ otherwise.

The above strategy is effectively identical to the optimal strategy under directed search except that the *expected* prices p replace the *actual* prices p_i (since the intermediated prices p_i are unobservable before sampling). Relative to the directed search environment, this difference relaxes competition among sellers in $\mathcal{C}$. With unobservable prices, a seller i has incentive to undercut other sellers to dissuade the consumer from searching any further after he samples product i. With observable prices, each seller i has this same incentive, but it is augmented by an additional incentive to reduce p_i to persuade the consumer to prioritize product i in his directed search sequence. Intuitively, this implies that the price reduction effect of increasing K is dampened with random search (relative to directed search). It suggests that mandating price flexibility would less frequently lower the number of search slots made prominent by the platform (with $K^{PF} < \overline{K} = K^{PC}$). This difference is not of primary significance, however, and every result from Sections 1.4 and 1.5 qualitatively holds with random consumer search. This analysis is relegated to Appendix B.

1.7 Extensions

1.7.1 Endogenous Search Cost

In this section, I extend the sequential search analysis from Section 1.6 by endogenizing the search cost $s_{\mathcal{C}}$ as a choice by the platform. I demonstrate that the main results continue to hold, and further, I show that the platform may maintain high search costs only with mandated price flexibility. This reveals an even stronger drawback to the regulatory proscription of price coherence.

Suppose the prominent seller search cost $s_{\mathcal{C}}$ is endogenously chosen by the platform, where any $s_{\mathcal{C}} \in \mathbb{R}_+$ is feasible.

It is easy to see that, under price coherence, the platform optimally sets $s_{\mathcal{C}}^{PC} = 0$ in either sequential search environment. With directed search, the platform's profit under price coherence is $\max_p p\left[1 - H^e(p)^{\bar{K}}\right]$, which decreases in $s_{\mathcal{C}}$ because H^e increases in $s_{\mathcal{C}}$ (since from its definition, H^e decreases in u^*, which decreases in $s_{\mathcal{C}}$). Similarly, with random search, the platform's profit under price coherence is $\max_{p \le v^*} p\left[1 - H(p)^{\bar{K}}\right]$, which decreases in $s_{\mathcal{C}}$ because v^* decreases in $s_{\mathcal{C}}$.

In contrast, with price flexibility, the platform's optimal choice of $s_{\mathcal{C}}^{PF}$ and consequent equilibrium and welfare outcomes differ significantly depending on the search environment.

Proposition 1.5. *Suppose g is log-concave.*

(i) (Directed search): Under enforceable price coherence and mandated price flexibility, the platform chooses search costs of $s_{\mathcal{C}}^{PC} = s_{\mathcal{C}}^{PF} = 0$, and all outcomes are equivalent to the non-sequential search environment.

(ii) (Random search): Under enforceable price coherence, the platform chooses a search cost of $s_{\mathcal{C}}^{PC} = 0$, and all outcomes are equivalent to the non-sequential search environment. Under mandated price flexibility, the platform sets $K^{PF} = \bar{K}$, and it chooses $s_{\mathcal{C}}^{PF} > 0$ to satisfy $v^* = \frac{1-H(v^*)}{h(v^*)}$. The equilibrium price is $p^{PF} = v^*$. Under price coherence, relative to mandated price flexibility, total welfare and consumer surplus are strictly higher.

Under price flexibility, the difference in outcomes between search environments is due to the competition intensifying effect of price observability. With unobservable intermediated prices, a higher search cost $s_{\mathcal{C}}$ lowers the benefit of searching, making the consumer less likely to leave for another seller and more likely to purchase from the current seller. Each seller faces more inelastic demand, and thus, charges a higher price. However, when prices are observable, these same effects of a higher search cost cause sellers to compete more intensely to attract the consumer earlier in his directed search sequence, lowering prices.

Under price flexibility, the platform leverages $s_{\mathcal{C}}$ to induce higher prices because it cannot do so through its choice of f (as it can under price coherence). With directed search, the platform thus lowers $s_{\mathcal{C}}$ as much as possible because prices decrease in $s_{\mathcal{C}}$.

In contrast, with random search, the platform maintains a high level of $s_{\mathcal{C}}$ because prices increase in $s_{\mathcal{C}}$. Specifically, the platform increases $s_{\mathcal{C}}$ up to the point that the consumer's participation constraint $p^{PF} \leq v^*$ is just satisfied. The platform eliminates any price reduction effect of increasing K, so it maximizes the demand expansion effect by setting $K^{PF} = \overline{K}$. Although $K^{PF} = K^{PC} = \overline{K}$, total welfare is unambiguously higher with price coherence due to lower equilibrium search costs and prices.

1.7.2 Non-Sponsored Search

In practice, while sponsored sellers enjoy better visibility over non-sponsored sellers, a consumer may viably consider both types of sellers in his purchasing decision. In this section, I demonstrate the robustness of the main welfare result to this possibility by allowing for an active non-sponsored search market.

Specifically, I relax the baseline assumption $s_{\mathcal{C}'} > \int_{\underline{v}}^{\bar{v}} v dH(v)$ so that the consumer may meaningfully consider sampling non-sponsored sellers. For clear insights and tractability, I maintain $s_{\mathcal{C}} = 0$, and I focus on the random sequential search environment. Therefore in Stage 4 of the game, the consumer freely observes (v_i, p_i, p_i^d) for each sponsored product $i \in \mathcal{C}$, and he may proceed to conduct random sequential search among the set of non-sponsored products $\mathcal{C}'$ at a search cost of $s_{\mathcal{C}'} \geq 0$. Proposition 1.6 demonstrates that the main welfare result, Proposition 1.3, also applies in this case.[17]

Proposition 1.6. *Suppose $s_{\mathcal{C}} = 0$ and the consumer conducts random sequential search among non-sponsored products at a search cost of $s_{\mathcal{C}'}$. Then prices and the number of sponsored search slots are higher with enforceable price coherence relative to mandated price flexibility, i.e., $p_{\mathcal{C}}^{PC} \geq p_{\mathcal{C}}^{PF}$, $p_{\mathcal{C}'}^{PC} \geq p_{\mathcal{C}'}^{PF}$, and $K^{PC} \geq K^{PF}$. Furthermore:*

(i) Given $\overline{K}$, there exist cutoffs $s_{\mathcal{C}'}^{PC}$ and $s_{\mathcal{C}'}^{PF}$ such that $s_{\mathcal{C}'}^{PC} < s_{\mathcal{C}'}^{PF}$ and:

(a) With enforceable price coherence, the consumer searches among non-sponsored products with positive probability if and only if $s_{\mathcal{C}'} \leq s_{\mathcal{C}'}^{PC}$.

(b) With mandated price flexibility, the consumer searches among non-sponsored products with positive probability if and only if $s_{\mathcal{C}'} \leq s_{\mathcal{C}'}^{PF}$.

(c) If $s_{\mathcal{C}'} \leq s_{\mathcal{C}'}^{PC}$, then total welfare and consumer surplus are lower with enforceable price coherence relative to mandated price flexibility, i.e., $W^{PC} < W^{PF}$ and $CS^{PC} < CS^{PF}$.

(ii) Given $s_{\mathcal{C}'}$, for $\overline{K}$ sufficiently large, total welfare and consumer surplus may be ranked as in Proposition 1.3.

A key distinction between Proposition 1.6 and Proposition 1.3 is that with a potentially active non-sponsored search market and any given $\overline{K}$, a welfare tradeoff exists only if it is sufficiently costly for the consumer to sample non-sponsored products relative to sponsored ones (i.e., $s_{\mathcal{C}'} > s_{\mathcal{C}'}^{PC}$).

[17] Assuming there are infinitely many sellers simplifies the proof of Proposition 1.6, but the insights may hold more generally with an arbitrary finite number of sellers. To demonstrate this, in Appendix C, I study the case with finitely many sellers under standard uniform H.

Without price coherence, the analysis and outcomes closely resemble those of the baseline model: the platform cannot set a positive transaction fee without sellers inducing disintermediation, so it maximizes the sponsored seller profit that it extracts through sponsored search referral fees.

With price coherence, the platform faces a choice between monetizing its access provision primarily through transaction fees or sponsored search. The consumer always makes a purchase if he searches among non-prominent sellers (since his optimal search strategy ends only with a purchase or failure to find another product to sample, which never occurs). Both sponsored and non-sponsored sellers face the transaction fee f, so a positive f induces higher prices from sponsored sellers (as in the baseline model) as well as from non-sponsored sellers. Further, increasing the transaction fee yields equivalent increases in prices from both types of sellers such that the consumer's (ex-ante) preference for a sponsored seller relative to a non-sponsored seller remains unchanged. Hence, provided that the non-sponsored seller price is not too high as to disqualify the consumer's search among non-sponsored products, increasing the transaction fee yields the platform a higher per-transaction payoff on the same number of transactions without changing its sponsored search revenue. There is a bound on the fee level, $\bar{f}$, beyond which the consumer does not search among non-sponsored products (because the prices are too high relative to the search cost). The platform thus faces a choice between monetizing access primarily through transaction fees by setting $f = \bar{f}$ or primarily through sponsored search by setting $f > \bar{f}$ to induce higher prominent seller prices (and earning $\Pi^M(\bar{K})$ as in the baseline model). The platform chooses to prioritize transaction fee revenue if sponsored search slots are not sufficiently prominent (i.e., if $s_{\mathcal{C}'} - s_{\mathcal{C}}$ is sufficiently small).

If with price coherence the platform monetizes access primarily through transaction fees (with $f^{PC} = \bar{f}$), the platform offers the same number of sponsored search slots as it does with mandated price flexibility (i.e., $K^{PC} = K^{PF}$). In such a case, mandating price flexibility lowers prices without changing the variety the consumer considers among the low search cost sponsored products, so total welfare and consumer welfare increase. If instead the platform monetizes access primarily through sponsored search (with $f^{PC} > \bar{f}$), which is true for sufficiently large $\bar{K}$, then the price-choice tradeoff of Proposition 1.3 persists in this environment. Mandating price coherence lowers prices but also reduces the variety considered among sponsored products. Welfare may suffer.

1.7.3 Transaction Benefit Provision

For a platform to profitably exist in a product market, it must either provide seller-consumer access or platform interaction benefits. I have focused so far on platform access provision, as it is the usual rationale for enforcing price coherence, but platforms often provide both of these services. For example, marketplace platforms provide access to markets by sellers and consumers while offering transaction benefits such as logistics services and secure transactions.

Inclusion of such benefit provision in the baseline model produces straightforward insights. In the baseline model, in response to a price flexibility mandate, the platform may shrink the consumer's consideration set to bolster its revenue from sponsored sellers. In certain cases, a platform's provision of transaction benefits may lessen this concern because it ties the platform's revenue closer to transaction volume. Specifically, a platform monetizes transaction benefits through its transaction fee, which produces an additional incentive for the platform to increase the number of transactions on which it collects the fee. This effect may counteract a platform's incentive to shrink the consumer's consideration set. If transaction benefits are large relative to the product market value, a regulatory authority may thus be less concerned about a platform limiting consumer choice in response to a price flexibility mandate. Otherwise, the insights from the baseline model persist if a platform offers transaction benefits in addition to market access.

I formalize this discussion by extending the baseline model such that the platform provides homogenous benefits to sellers and the consumer in intermediated transactions. This analysis is relegated to Appendix D.

1.7.4 Platform Competition

Consumers frequently search through more than one marketplace before making a purchasing decision. For instance, in a study of over 9.7 million mobile phone users in Shanghai and Shandong province, Huang et al. (2018) found that 43% of users visited more than one of the five most popular general merchandise e-commerce platforms in China within a week. In this section, I study such consumer multi-homing behavior across competing platforms.

I extend the baseline model to allow the consumer to search on two symmetric platforms, A and B. The consumer freely observes the prominent (sponsored) products on both platforms, and as in the baseline case, he does not view any non-prominent products. Each platform $P \in \{A, B\}$ chooses a number of prominent slots to offer, $K_P \leq \overline{K}$, and it chooses its referral and transaction fees, r_P and f_P. I assume that the platforms make their choices simultaneously in Stage 1 of the game. When possible, I consider equilibria in which the platforms make symmetric choices.[18]

Like in the baseline model, an infinite number of sellers are present on each platform. In a strict sense, it does not matter whether any (or how many) of the sellers are present on both platforms or on only one of them. If a seller is present on both platforms, then in any equilibrium, the seller cannot be (and cannot deviate to become) assigned a prominent search slot on both platforms with

[18] I consider all symmetric pure strategy equilibria if any such equilibrium exists. Otherwise, I do not apply any equilibrium selection and consider all (pure or mixed strategy) equilibria. The latter case applies only under mandated price flexibility, when integer constraints on the platforms' choices of K_A and K_B may not allow for any symmetric pure strategy equilibrium.

any probability because there are infinitely many sellers and only a finite number of slots.[19] Assuming the consumer only observes such a multi-homing seller's direct price and intermediated price on the platform through which he found the seller, the seller's non-prominent presence on the other platform makes no difference.

With platform competition, the implications of price coherence regulation are unambiguous. For large $\overline{K}$, mandating price flexibility reduces both total welfare and consumer surplus.

Proposition 1.7. *Let H be the uniform distribution function over* $[0,1]$*, and suppose the consumer conducts his search across two competing platforms. For* $\overline{K}$ *sufficiently large, both total welfare and consumer surplus are higher with enforceable price coherence relative to mandated price flexibility.*

The welfare result with platform competition contrasts with the baseline case with a monopoly platform, wherein mandating price flexibility reduces total welfare but increases consumer surplus for large $\overline{K}$ (Proposition 1.3). Platform competition thus introduces further drawbacks to a proscription of price coherence and allows for a more direct policy recommendation.

Proposition 1.7 derives from two key facts. First, with enforceable price coherence, the platforms set $K_A^{PC} = K_B^{PC} = \overline{K}$ such that the consumer observes more product variety compared to the case with mandated price flexibility (like in the baseline model). Each platform extracts its sponsored sellers' revenue as profit through its referral and transaction fees. By increasing K_P, a platform $P \in \{A, B\}$ increases the total likelihood that the consumer makes a purchase, while also increasing the likelihood that the consumer makes a purchase on P's marketplace relative to the other platform's marketplace. With price coherence, each platform can alleviate any equilibrium

[19] Aligning with this interpretation, in practice, the set of prominent products (which is small relative to the set of all products) often differs across competing platforms. For example, from October 2024 searches for three-star hotels in the 100 largest US cities, I find that an average of only 9.11 (or 36.4%) of the first 25 hotels on Booking.com also appear in the first 25 hotels on Hotels.com.

price reduction by increasing its transaction fee, so each platform always has an incentive to increase K_P, leading to $K_A^{PC} = K_B^{PC} = \overline{K}$ in equilibrium. As in the baseline model, without price coherence, the platforms cannot control prices through any positive transaction fees, so in equilibrium, each platform chooses K_P low to maintain higher sponsored seller profits that it can extract through referral fees.

The second key fact behind Proposition 1.7 is that with price coherence, platform competition drives down the platforms' transaction fees and hence drives down the equilibrium product prices. By lowering its transaction fee, a platform decreases the equilibrium price charged on its marketplace relative to the other marketplace and hence increases the probability that it intermediates a sale. As $\overline{K}$ increases, the consumer views more products (as $K_A^{PC} = K_B^{PC} = \overline{K}$) and becomes more particular about his choice of product (because products become more likely to be close substitutes). This increases each platform's incentive to lower the equilibrium price on its marketplace through a lower transaction fee. Indeed, in the limiting case, as $\overline{K} \to \infty$, the equilibrium transaction fees and prices approach zero. The platforms would both be better-off if they allowed for price flexibility (and set $K_A = K_B < \overline{K}$), but this cannot occur in an equilibrium because each platform has a unilateral incentive to increase K_P and alleviate any consequent price reduction through price coherence and a positive transaction fee. As a result, platform competition leads to lower transaction fees and prices, and for sufficiently large $\overline{K}$, prices are lower with enforceable price coherence relative to mandated price flexibility.

Relative to the baseline model, platform competition thus maintains the downside of a ban on price coherence but eliminates its upside: with mandated price flexibility, product variety is lower *and prices are higher* (for large $\overline{K}$). These facts directly lead to Proposition 1.7.

1.8 Conclusion

This chapter models price coherence regulation when a platform may monetize access both through transaction fees and referral fees, or sponsored search. The analysis reveals a new tradeoff that should caution future regulation of price coherence requirements. It is well understood that price coherence allows a platform to charge higher transaction fees, often resulting in higher prices. The usual argument is that proscribing price coherence makes sellers better-off because they regain a threat of disintermediation that disciplines a platform for charging excessively high transaction fees; consumers are better-off because they face lower prices, and total welfare increases. However, I demonstrate that when a platform may also monetize its access provision through sponsored search, its profit is closely tied to the sponsored sellers' industry profit. The platform can calibrate equilibrium prices through transaction fees only when it can enforce price coherence. Without price coherence, a large set of prominently sponsored product choices may drive down equilibrium prices and the industry profit. Because the platform loses its ability to influence the equilibrium price without price coherence, it reduces price competition by lowering the number of prominent sellers considered by consumers. This product variety reduction may outweigh any price reduction: proscribing price coherence may decrease both total welfare and consumer surplus. The result is robust in several consumer search and platform market environments and is exacerbated with platform competition. This price-choice tradeoff should be considered in ongoing and future price coherence regulation.

REFERENCES

Anderson, S. P., De Palma, A., & Nesterov, Y. (1995). Oligopolistic competition and the optimal provision of products. Econometrica: Journal of the Econometric Society, 1281–1301.

Anderson, S. P., & Renault, R. (1999). Pricing, product diversity, and search costs: A Bertrand-Chamberlin-Diamond model. The RAND Journal of Economics, 719–735.

Armstrong, M. (2017). Ordered consumer search. Journal of the European Economic Association, 15(5), 989–1024.

Armstrong, M., Vickers, J., & Zhou, J. (2009). Prominence and consumer search. The RAND Journal of Economics, 40(2), 209–233.

Athey, S., & Ellison, G. (2011). Position auctions with consumer search. The Quarterly Journal of Economics, 126(3), 1213–1270.

Bagnoli, M., & Bergstrom, T. (2006). Log-concave probability and its applications. Rationality and Equilibrium: A Symposium in Honor of Marcel K. Richter, 217–241.

Baker, J. B., & Morton, F. S. (2018). Antitrust Enforcement Against Platform MFNs. The Yale Law Journal, 2176–2202.

Belleflamme, P., & Peitz, M. (2019). Managing competition on a two-sided platform. Journal of Economics & Management Strategy, 28(1), 5–22.

Boik, A., & Corts, K. S. (2016). The effects of platform most-favored-nation clauses on competition and entry. The Journal of Law and Economics, 59(1), 105–134.

Boorsma, P. (2023). Platform Coherence Policies with a Multiproduct Seller. Available at SSRN 4531596.

Calzada, J., Manna, E., & Mantovani, A. (2022). Platform price parity clauses and market segmentation. Journal of Economics & Management Strategy, 31(3), 609–637.

Carlton, D. W., & Winter, R. A. (2018). Vertical most-favored-nation restraints and credit card no-surcharge rules. The Journal of Law and Economics, 61(2), 215–251.

Casner, B. (2024). Lowering the Garden Wall: Marketplace Leakage and Quality Curation. Available at SSRN 4969728.

Choi, M., Dai, A. Y., & Kim, K. (2018). Consumer search and price competition. Econometrica, 86(4), 1257–1281.

De Corniere, A. (2016). Search advertising. American Economic Journal: Microeconomics, 8(3), 156–188.

Dinerstein, M., Einav, L., Levin, J., & Sundaresan, N. (2018). Consumer price search and platform design in internet commerce. American Economic Review, 108(7), 1820–1859.

Edelman, B., & Schwarz, M. (2010). Optimal auction design and equilibrium selection in sponsored search auctions. American Economic Review, 100(2), 597–602.

Edelman, B., & Wright, J. (2015). Price coherence and excessive intermediation. The Quarterly Journal of Economics, 130(3), 1283–1328.

Ennis, S., Ivaldi, M., & Lagos, V. (2023). Price-Parity Clauses for Hotel Room Booking: Empirical Evidence from Regulatory Change. The Journal of Law and Economics, 66(2), 309–331.

FTC. (2023a, September). CASE NO.: 2:23-cv-01495-JHC. https://www.ftc.gov/system/files/ftc_gov/pdf/1910134amazonecommercecomplaintrevisedredactions.pdf

FTC. (2023b, September). FTC Sues Amazon for Illegally Maintaining Monopoly Power. FTC Press Release. https://www.ftc.gov/news-events/news/press-releases/2023/09/ftc-sues-amazon-illegally-maintaining-monopoly-power

Gabaix, X., Laibson, D., Li, D., Li, H., Resnick, S., & de Vries, C. G. (2016). The impact of competition on prices with numerous firms. Journal of Economic Theory, 165, 1–24.

Hagiu, A. (2009). Two-sided platforms: Product variety and pricing structures. Journal of Economics & Management Strategy, 18(4), 1011–1043.

Hagiu, A., & Jullien, B. (2011). Why do intermediaries divert search? The RAND Journal of Economics, 42(2), 337–362.

Hagiu, A., & Wright, J. (2024). Marketplace leakage. Management Science, 70(3), 1529–1553.

Huang, H., Zhao, B., Zhao, H., Zhuang, Z., Wang, Z., Yao, X., Wang, X., Jin, H., & Fu, X. (2018). A cross-platform consumer behavior analysis of large-scale mobile shopping data. Proceedings of the 2018 World Wide Web Conference, 1785–1794.

Hunold, M., Kesler, R., Laitenberger, U., & Schlütter, F. (2018). Evaluation of best price clauses in online hotel bookings. International Journal of Industrial Organization, 61, 542–571.

Johansen, B. O., & Vergé, T. (2017). Platform price parity clauses with direct sales. University of Bergen.

Johnson, J. P. (2017). The agency model and MFN clauses. The Review of Economic Studies, 84(3), 1151–1185.

Johnson, J. P., Rhodes, A., & Wildenbeest, M. (2023). Platform design when sellers use pricing algorithms. Econometrica, 91(5), 1841–1879.

Karle, H., & Peitz, M. (2017). De-targeting: Advertising an assortment of products to loss-averse consumers. European Economic Review, 95, 103–124.

Liu, C., Niu, F., & White, A. (2021). Optional intermediaries and pricing restraints. Available at SSRN 3825163.

Ma, P., Mantovani, A., Reggiani, C., Broocks, A., & Duch-Brown, N. (2024). The price effects of prohibiting price parity clauses: evidence from international hotel groups.

Mantovani, A., Piga, C. A., & Reggiani, C. (2021). Online platform price parity clauses: Evidence from the EU Booking. com case. European Economic Review, 131, 103625.

Mariotto, C., & Verdier, M. (2020). Platform–merchant competition for sales services. Journal of Economics & Management Strategy, 29(4), 834–853.

Nocke, V., Peitz, M., & Stahl, K. (2007). Platform ownership. Journal of the European Economic Association, 5(6), 1130–1160.

Perloff, J. M., & Salop, S. C. (1985). Equilibrium with product differentiation. The Review of Economic Studies, 52(1), 107–120.

Regulation (EU) 2022/1925 of the European Parliament and of the Council of 14 September 2022 on contestable and fair markets in the digital sector and amending Directives (EU) 2019/1937 and (EU) 2020/1828 (Digital Markets Act). (2022). In Official Journal of the European Union: Vol. L 265.

Rhodes, A., & Zhou, J. (2024). Personalized pricing and competition. American Economic Review, 114(7), 2141–2170.

Teh, T.-H. (2022). Platform governance. American Economic Journal: Microeconomics, 14(3), 213–254.

Wang, C., & Wright, J. (2020). Search platforms: Showrooming and price parity clauses. The RAND Journal of Economics, 51(1), 32–58.

Wang, C., & Wright, J. (2023). Platform investment and price parity clauses. The Journal of Industrial Economics, 71(2), 538–569.

Wang, Z., & Wright, J. (2017). Ad valorem platform fees, indirect taxes, and efficient price discrimination. The RAND Journal of Economics, 48(2), 467–484.

Weitzman, M. L. (1979). Optimal Search for the Best Alternative. Econometrica, 47(3), 641–654. https://doi.org/10.2307/1910412

Wolinsky, A. (1986). True monopolistic competition as a result of imperfect information. The Quarterly Journal of Economics, 101(3), 493–511.

Xie, Y., & Zhu, H. (2023). Platform leakage: Incentive conflicts in two-sided markets. Working Paper, https://questromworld. bu. edu/platformstrategy/wp-content ….

Zhou, J. (2017). Competitive bundling. Econometrica, 85(1), 145–172.

APPENDIX A: OMITTED PROOFS

Proof of Lemma 1.1: Applying integration by parts in the definition of $\phi(p;K)$, we can write

$$\phi(p;K) = \frac{[1-H(p)^K]/K}{h(\bar{v}) - \int_p^{\bar{v}} h'(v)H(v)^{K-1}dv} = \left\{\frac{Kh(\bar{v})}{1-H(p)^K} - \int_p^{\bar{v}} \frac{h'(v)}{h(v)} d\left[\frac{H(v)^K - H(p)^K}{1-H(p)^K}\right]\right\}^{-1} \quad (A1)$$

Denote $\phi_p(p;K) \coloneqq \frac{d}{dp}\phi(p;K)$ and $\phi_K(p;K) \coloneqq \frac{\partial}{\partial K}\phi(p;K)$. Equation $(A1)$ establishes $\phi_p(p;K) < 0$ and $\phi_K(p;K) < 0$. It is easy to see the first term increases in both p and K. In the second term, $\frac{H(v)^K - H(p)^K}{1-H(p)^K}$ is the distribution function of the highest order statistic $v_{(K:K)}$ of $\{v_i\}_{i=1}^K$ conditional on it being larger than p, which increases in both p and K in the sense of first order stochastic dominance. Further, $-\frac{h'(v)}{h(v)}$ increases in v using h log-concave, so the second term increases in both p and K. Therefore, $\phi_p(p;K) \le 0$ and $\phi_K(p;K) \le 0$.

Suppose $K \ge 2$. I will show that Equation (1.2), $p = f + \phi(p;K)$, has a unique solution. First notice that $p \le f + \phi(p;K)$ holds at $p = f$. Showing $\lim_{p\to\bar{v}}[p - f - \phi(p;K)] > 0$ completes the argument because $\phi_p(p;K) < 0$. Consider three cases. If $\bar{v} = \infty$, then this holds since $\phi(p;K)$ decreases in p and is finite for arbitrary p. If $\bar{v} < \infty$ and $h(\bar{v}) > 0$, then it holds since $\lim_{p\to\bar{v}} \phi(p;K) = 0$ from the definition of $\phi(p;K)$. Finally, if $\bar{v} < \infty$ and $h(\bar{v}) = 0$, then $\lim_{p\to\bar{v}} h'(p) < 0$ and $\lim_{p\to\bar{v}} \frac{d}{dv}[KH(v)^{K-1}h(v)] < 0$, and for p sufficiently close to $\bar{v}$ we have

$$\phi(p;K) \le \frac{\int_p^{\bar{v}} KH(v)^{K-1}h(v)dv}{KH(p)^{K-1}h(p)} < \frac{(\bar{v}-p)KH(p)^{K-1}h(p)}{KH(p)^{K-1}h(p)} = \bar{v} - p,$$

where the first inequality follows regardless of p; then $\lim_{p\to\bar{v}} \phi(p;K) = 0$ and $\lim_{p\to\bar{v}}[p - f - \phi(p;K)] = \bar{v} - f > 0$. Thus, Equation (1.2) has a unique solution $p^*(f,K) \in [f,\bar{v}]$. If $f \le \underline{v} - \phi(\underline{v};K)$, then $p^*(f,K) \le \underline{v}$, and $p^*(f,K) = f + \phi(p(f,K);K) = f + \phi(\underline{v};K)$ because

$\phi_p(p;K) = 0$ for $p \leq \underline{v}$. Along with the argument in the main text, this establishes (i) and (ii) for $K \geq 2$: the equilibrium price satisfies $p(f,K) = p^*(f,K)$.

If $K = 1$, then $\phi(p;K) = \frac{1-H(p)}{h(p)}$. The prominent seller sets the usual monopoly price $p(f,K) = p^*(f,K)$ if $f > \underline{v} - \phi(\underline{v};K)$ and otherwise prices at $p(f,K) = \bar{v}$.

Finally, differentiating the first order condition, Equation (1.2), yields $\frac{\partial}{\partial f}p^*(f,K) = \frac{1}{1-\phi_p(p^*;K)} \geq 0$ and $\frac{\partial}{\partial K}p^*(f,K) = \frac{\phi_K(p^*;K)}{1-\phi_p(p^*;K)} \leq 0$. That $\phi_p(p;K) \leq 0$ and $\phi_K(p;K) \leq 0$ also implies $p(f,K)$ weakly increases in f and weakly decreases in K in the case that $K = 1$ and $f \leq \underline{v} - \phi(\underline{v};K)$. ■

Proof of Lemma 1.2: A log-concave density has log-concave associated cumulative and cumulative complementary distribution functions (Bagnoli & Bergstrom, 2006; Theorems 1 & 3). The density associated with H^K is $KH^{K-1}h$. Since h is log-concave, so is H. Then $KH^{K-1}h$ is log-concave because it is the product of log-concave functions. Then the associated cumulative complementary distribution function $1 - H^K$ is log-concave; the inverse hazard rate $\lambda(p;K) \coloneqq \frac{[1-H(p)^K]/K}{H(p)^{K-1}h(p)}$ decreases in p.

It is immediate that $p^M(K) = p(0,K)$ for $K = 1$, so assume $K \geq 2$. I will show that the first order condition for $\max_p p[1 - H(p)^K]$, $p = \lambda(p;K)$, has a unique solution. First notice that $p < \lambda(p;K)$ holds at $p = \max\{\underline{v}, 0\}$. Showing $\lim_{p\to\bar{v}}[p - \lambda(p;K)] > 0$ confirms that the first order condition has a unique solution since $\lambda(p;K)$ decreases in p. Consider three cases. If $\bar{v} = \infty$, then this holds since $\lambda(p;K)$ decreases in p and is finite for arbitrary p. If $\bar{v} < \infty$ and $h(\bar{v}) > 0$, then $\lim_{p\to\bar{v}}\lambda(p;K) = 0 < \bar{v}$. Finally, suppose $\bar{v} < \infty$ and $h(\bar{v}) = 0$. We must have $h'(v) < 0$ for v

sufficiently close to $\bar{v}$; hence, $\frac{d}{dv}KH(v)^{K-1}h(v) < 0$ for v sufficiently close to $\bar{v}$. Then for p sufficiently close to $\bar{v}$, we have

$$\lambda(p;K) = \frac{\int_p^{\bar{v}} KH(v)^{K-1}h(v)dv}{KH(p)^{K-1}h(p)} < \frac{KH(p)^{K-1}h(p)(\bar{v}-p)}{KH(p)^{K-1}h(p)} = \bar{v} - p.$$

Then $\lim_{p\to\bar{v}} \lambda(p;K) = 0 < \bar{v}$. Therefore, $p^M(K)$ uniquely solves $p = \lambda(p;K)$.

Now observe

$$\lambda(p;K) = \frac{[1-H(p)^K]/K}{H(p)^{K-1}h(p)} > \frac{[1-H(p)^K]/K}{H(p)^{K-1}h(p) + \int_p^{\bar{v}} h(v)dH(v)^{K-1}} = \phi(p;K).$$

Recall that $\lambda(p;K)$ and $\phi(p;K)$ both decrease in p. Then $p^M(K)$, which solves $p = \lambda(p;K)$, must be larger than $p(0,K)$, which solves $p = \phi(p;K)$.

To see that $p^M(K)$ increases in K, observe that

$$\lambda(p;K+1) - \lambda(p;K) = \frac{1}{H(p)^K}\frac{1-H(p)}{h(p)}\left[\frac{K - \sum_{k=1}^K H(p)^k}{K(K+1)}\right] > 0.$$

Thus, as K increases, the RHS of the first order condition $p = \lambda(p;K)$ increases; so $p^M(K)$ must also increase. ■

Proof of Proposition 1.2: I first derive a useful expression for $\phi(p;K)$. Let $H_{(K-1:K)}$ and $h_{(K-1:K)}$ respectively denote the distribution function and density of the second highest order statistic $v_{(K-1:K)}$ of $\{v_i\}_{i=1}^K$. Specifically,

$$H_{(K-1:K)}(v) = H(v)^K + K[1-H(v)]H(v)^{K-1},$$

$$h_{(K-1:K)}(v) = K(K-1)H(v)^{K-2}[1-H(v)]h(v).$$

Then we can write

$$\phi(p;K) = \frac{1-H(p)^K}{KH(p)^{K-1}h(p) + \int_p^{\bar{v}} \frac{h(v)}{1-H(v)} dH_{(K-1:K)}(v)}. \qquad (A2)$$

Consider statement (i). Suppose $\underline{v} \geq \phi(\underline{v}; 1)$. Then $p(0,1) = \underline{v}$ and $p(0,K) \leq \underline{v}$ for all $K \in \{1, \dots, \overline{K}\}$ since $p_K(0,K) < 0$ by Lemma 1.1. The market is fully covered for all $K \in \{1, \dots, \overline{K}\}$. The platform's profit reduces to $\Pi^{PF}(K) = p(0,K)$, which decreases in K. Hence $K^{PF} = 1$. (From Equation $(A2)$ one can see that, with a covered market, $p(0,K)$ decreases in K provided that $1 - H$ is log-concave, which is a weaker requirement than h log-concave, as assumed in Lemma 1.1. This observation is also made by Zhou (2017).)

Now consider statement (ii). Suppose $\lim_{v \to \bar{v}} \frac{1-H(v)}{h(v)} = 0$. Then $\lim_{K \to \infty} \phi(p; K) = 0$ from Equation $(A2)$, and $\lim_{K \to \infty} p(0,K) = 0 = \lim_{K \to \infty} \Pi^{PF}(K)$ by Equation (1.2) and Equation (1.5). Then $\arg\max_{K \in \mathbb{N}} \Pi^{PF}(K)$ is finite and non-empty, and $K^{PF} < \overline{K}$ for sufficiently large $\overline{K}$. ∎

Proof of Proposition 1.3: Recall that $\lim_{\overline{K} \to \infty} \Pi^{PC} = \bar{v}$, so $\lim_{\overline{K} \to \infty} W^{PC} = \bar{v}$. Under any of the conditions statement (i), $\lim_{\overline{K} \to \infty} K^{PF} < \infty$ by Proposition 1.2; hence $\lim_{\overline{K} \to \infty} W^{PF} = \int_{p^{PF}}^{\bar{v}} v dH(v)^{K^{PF}} < \bar{v}$. If $\bar{v} < \infty$, then $\lim_{\overline{K} \to \infty} CS^{PC} = \lim_{\overline{K} \to \infty} W^{PC} - \Pi^{PC} = 0$, while $\lim_{N \to \infty} CS^{PF} > 0$. ∎

Proof of Proposition 1.4: Write $\bar{v} = \underline{v} + \hat{v}$. We have $H(v) = \frac{v - \underline{v}}{\hat{v}}$ and $h(v) = \frac{1}{\hat{v}}$ on the support $[\underline{v}, \underline{v} + \hat{v}]$. Take $\overline{K}$ as given. I will first derive some facts in the game with price coherence. Proposition 1.1 yields $K^{PC} = \overline{K}$ and

$$\frac{p^{PC} - \underline{v}}{\hat{v}} = \frac{1 - \left(\frac{p^{PC} - \underline{v}}{\hat{v}}\right)^{\overline{K}}}{\overline{K}\left(\frac{p^{PC} - \underline{v}}{\hat{v}}\right)^{\overline{K}-1}} - \frac{\underline{v}}{\hat{v}}, \tag{$A3$}$$

which implies that $\frac{p^{PC} - \underline{v}}{\hat{v}} < \frac{1}{(\overline{K}+1)^{\frac{1}{\overline{K}}}}$. Differentiating Equation $(A3)$ with respect to $\hat{v}$ yields

$$\frac{\partial p^{PC}}{\partial \hat{v}} = \frac{\overline{K}\left(\frac{p^{PC}-\underline{v}}{\hat{v}}\right)}{(\overline{K}-1)+(\overline{K}+1)\left(\frac{p^{PC}-\underline{v}}{\hat{v}}\right)^{\overline{K}}} > \frac{p^{PC}-\underline{v}}{\hat{v}},$$

where the inequality follows using $\frac{p^{PC}-\underline{v}}{\hat{v}} < \frac{1}{(\overline{K}+1)^{\frac{1}{\overline{K}}}}$. Then

$$0 < \frac{1}{\hat{v}}\left[\frac{\partial p^{PC}}{\partial \hat{v}} - \frac{p^{PC}-\underline{v}}{\hat{v}}\right] = \frac{\partial}{\partial \hat{v}}\frac{p^{PC}-\underline{v}}{\hat{v}}.$$

Finally, compute

$$CS^{PC} = \int_{p^{PC}}^{\underline{v}+\hat{v}}\left[1-\left(\frac{v-\underline{v}}{\hat{v}}\right)^{\overline{K}}\right]dv = \hat{v}\left[\frac{1}{\overline{K}+1}\left(\frac{p^{PC}-\underline{v}}{\hat{v}}\right)^{\overline{K}+1} - \frac{p^{PC}-\underline{v}}{\hat{v}} + \frac{\overline{K}}{\overline{K}+1}\right].$$

Now consider the game without price coherence. First assume that $\underline{v} \geq \hat{v} = \phi(\underline{v};1)$, so the market is covered without price coherence. Proposition 1.2 yields $K^{PF} = 1$ and $p^{PF} = \underline{v}$. From this, we can compute $CS^{PF} = \frac{\hat{v}}{2}$. We have

$$CS^{PC} - CS^{PF} = \hat{v}\left[\frac{1}{\overline{K}+1}\left(\frac{p^{PC}-\underline{v}}{\hat{v}}\right)^{\overline{K}+1} - \frac{p^{PC}-\underline{v}}{\hat{v}} + \frac{1}{2}\frac{\overline{K}-1}{\overline{K}+1}\right].$$

The polynomial $\frac{1}{N+1}x^{N+1} - x + \frac{1}{2}\frac{N-1}{N+1}$ strictly decreases over $x \in \left[0, \frac{1}{(\overline{K}+1)^{\frac{1}{\overline{K}}}}\right]$ and is strictly positive at $x = 0$. From Equation $(A3)$ we have $\lim_{\hat{v}\to 0}\frac{p^{PC}-\underline{v}}{\hat{v}} = 0$, and $\frac{\partial}{\partial \hat{v}}\frac{p^{PC}-\underline{v}}{\hat{v}} > 0$ for all $\hat{v}$. Therefore, there exists $\hat{v}^{\dagger} > 0$ such that $CS^{PF} < CS^{PC}$ if and only if $\hat{v} < \hat{v}^{\dagger}$. To see that $\hat{v}^{\dagger} < \underline{v}$, note that if $\hat{v} = \underline{v}$, Equation $(A3)$ implies $\frac{p^{PC}-\underline{v}}{\underline{v}} > \frac{1}{(2\overline{K}+1)^{\frac{1}{\overline{K}-1}}}$, and

$$CS^{PC} - CS^{PF} < \hat{v}\left[\frac{1}{\overline{K}+1}\left(\frac{1}{(2\overline{K}+1)^{\frac{1}{\overline{K}-1}}}\right)^{\overline{K}+1} - \frac{1}{(2\overline{K}+1)^{\frac{1}{\overline{K}-1}}} + \frac{1}{2}\frac{\overline{K}-1}{\overline{K}+1}\right]$$

$$= \hat{v}\left[\frac{2-(2\bar{K}+1)^{\frac{\bar{K}}{\bar{K}-1}}\left[2(\bar{K}+1)+(\bar{K}-1)(2\bar{K}+1)^{\frac{1}{\bar{K}-1}}\right]}{2(\bar{K}+1)(2\bar{K}+1)^{\frac{\bar{K}+1}{\bar{K}-1}}}\right] < 0.$$

We have shown that there exists $\hat{v}^{\dagger} \in (0, \underline{v})$ such that, if $\underline{v} \geq \hat{v} = \phi(\underline{v}; 1)$, then $CS^{PF} < CS^{PC}$ if and only if $\hat{v} < \hat{v}^{\dagger}$.

To complete the proof, suppose $\underline{v} < \hat{v}$. If $K^{PF} = 1$, then

$$CS^{PF} = \frac{1}{2}\left[\underline{v} + \hat{v}\left(\frac{\hat{v}-\underline{v}}{2\hat{v}}\right)^2\right],$$

$$\frac{\partial CS^{PF}}{\partial \hat{v}} = \frac{1}{2}\left(\frac{\hat{v}^2 - \underline{v}^2}{4\hat{v}^2}\right) > 0.$$

So $CS^{PF}|_{\hat{v}>\underline{v}} > CS^{PF}|_{\hat{v}=\underline{v}} > CS^{PC}$ for $K^{PF} = 1$. Compared to this case, if $K^{PF} > 1$, then the consumer faces lower prices (since $p_K(0, K) < 0$) while enjoying a higher expected match value, so $CS^{PF} > CS^{PC}$ also holds for $\hat{v} > \underline{v}$ and $K^{PF} > 1$.

To see that $\hat{v}^{\dagger} \to 0$ as $\bar{K} \to \infty$, observe that $\hat{v}^{\dagger}$ solves $CS^{PC} = \frac{\hat{v}}{2}$ and

$$CS^{PC} - \frac{\hat{v}}{2} \leq \left[\int_{\underline{v}}^{\underline{v}+\hat{v}} v dH(v)^{\bar{K}} - \Pi^M(\bar{K})\right] - \frac{\hat{v}}{2},$$

Where, for any fixed $\hat{v}$, the last expression has a negative limit as $\bar{K} \to \infty$. ■

Proof of Proposition 1.5: Consider the game without price coherence. First consider the directed search environment of Section 1.6.1. M. Choi et al. (2018; Proposition 5) show that g log-concave implies the equilibrium price $p(0, K)$ decreases in s_C. Then for any K, the platform's profit $p(0,K)\left[1 - H\big(p(0,K)\big)^K\right]$ decreases in s_C because $\max_p p[1 - H(p)^K]$ increases in p over $[p(0,K), p^M(K)]$. Thus, $s_C^{PF} = 0$. This proves statement (i).

Now consider the random search environment of Section 1.6.2 and statement (ii). By the equilibrium pricing condition, Equation $(B1)$, $\tilde{p}(0,K)$ decreases in v^* if and only if $\tilde{\phi}(p,K)$ decreases in v^* (notation follows Appendix B). Observe

$$\frac{1}{\tilde{\phi}(p;K)} = \frac{1}{1-H(p)^K}\left\{\int_{v^*}^{\bar{v}}\left[\frac{h(v^*)}{1-H(v^*)}+\frac{h'(v)}{h(v)}\right]dH(v)^K - \int_p^{\bar{v}}\frac{h'(v)}{h(v)}dH(v)^K\right\},$$

$$\frac{\partial\left[\tilde{\phi}(p;K)\right]^{-1}}{\partial v^*} = \frac{h(v^*)}{1-H(p)^K}\left[\sum_{k=0}^{K-1}H(v^*)^k - KH(v^*)^{K-1}\right]\left[\frac{h(v^*)}{1-H(v^*)}+\frac{h'(v^*)}{h(v^*)}\right] > 0.$$

The first and second multiplicands of $\frac{\partial\left[\tilde{\phi}(p;K)\right]^{-1}}{\partial v^*}$ are positive. h log-concave implies $\frac{h(v^*)}{1-H(v^*)}$ increases in v^*, which, along with $\log(\cdot)$ increasing, implies $0 > \frac{d}{dv^*}\log\left[\frac{h(v^*)}{1-H(v^*)}\right] = \frac{h(v^*)}{1-H(v^*)} + \frac{h'(v^*)}{h(v^*)}$. Thus, $\tilde{\phi}(p,K)$ decreases in v^*. We have shown that $\tilde{p}(0,K)$ decreases in v^* and increases in $s_{\mathcal{C}}$.

For any K, the platform's profit is $\tilde{p}(0,K)\left[1-H\big(\tilde{p}(0,K)\big)^K\right]$, which increases in $\tilde{p}(0,K)$ as long as $\tilde{p}(0,K) \leq v^*$ (beyond which the consumer does not participate). From Equation $(B1)$, $\tilde{p}(0,K) = v^*$ if and only if $v^* = \frac{1-H(v^*)}{h(v^*)}$. Thus, the platform chooses $\tilde{s}_{\mathcal{C}}^{PF}$ such that $v^* = \frac{1-H(v^*)}{h(v^*)}$, and it earns $v^*[1-H(v^*)^K]$ for any K, which increase in K; it chooses $\tilde{K}^{PF} = \bar{K}$. Then $\widetilde{CS}^{PF} = 0 < \widetilde{CS}^{PC}$. Since $\tilde{p}(0,\bar{K})$ increases in $s_{\mathcal{C}}$ and $\tilde{s}_{\mathcal{C}}^{PF} > 0 = \tilde{s}_{\mathcal{C}}^{PC}$, we have $\tilde{p}^{PC} < v^* = \tilde{p}^{PF}$, and

$$\widetilde{W}^{PF} = v^*\left[1-H(v^*)^{\bar{K}}\right] = \int_{v^*}^{\bar{v}} v^* dH(v)^{\bar{K}} < \int_{\tilde{p}^{PC}}^{\bar{v}} v\,dH(v)^{\bar{K}} = \widetilde{W}^{PC}. \qquad \blacksquare$$

Proof of Proposition 1.6: Suppose all sponsored sellers charge $p_{\mathcal{C}}$, while all non-sponsored sellers charge $p_{\mathcal{C}'}$.

Suppose $s_{\mathcal{C}'} \leq \int_{\max\{0,\underline{v}\}}^{\bar{v}}\big(v-\max\{0,\underline{v}\}\big)dH(v)$ and define $v_{\mathcal{C}'}^* \in \left[\max\{0,\underline{v}\},\bar{v}\right]$ implicitly by

$$s_{\mathcal{C}'} = \int_{v_{\mathcal{C}'}^*}^{\bar{v}} \left(v - v_{\mathcal{C}'}^*\right) dH(v).$$

If $\max_{i \in \mathcal{C}}\{0, v_i - p_{\mathcal{C}}\} \geq v_{\mathcal{C}'}^* - p_{\mathcal{C}'}$, then the consumer does not search among $\mathcal{C}'$ and makes his purchasing decision considering only $\mathcal{C}$. Otherwise, the consumer conducts search among $\mathcal{C}'$ according to his optimal strategy described in Section 1.6.2 (with $v_{\mathcal{C}'}^*$ replacing v^*).

Suppose the consumer searches among $\mathcal{C}'$. If a non-prominent seller $i \in \mathcal{C}'$ charges price p_i instead of $p_{\mathcal{C}'}$, then upon sampling i, the consumer purchases i if and only if $v_i - p_i \geq v_{\mathcal{C}'} - p_{\mathcal{C}'}$ because he still expects other non-prominent sellers to charge $p_{\mathcal{C}'}$. Therefore the non-prominent seller's deviation profit is given by

$$(p_i - f)[1 - H(p_i + v_{\mathcal{C}'} - p_{\mathcal{C}'})].$$

The first order condition determines the optimal price (due to log-concave h), which is given by

$$p_{\mathcal{C}'}(f) = f + \frac{1 - H\left(v_{\mathcal{C}'}^*\right)}{h\left(v_{\mathcal{C}'}^*\right)}.$$

Note that if $v_{\mathcal{C}'}^* - p_{\mathcal{C}'}(0) < 0$, or equivalently $v_{\mathcal{C}'}^* < \frac{1 - H\left(v_{\mathcal{C}'}^*\right)}{h\left(v_{\mathcal{C}'}^*\right)}$, then the consumer never considers searching among $\mathcal{C}'$ and everything reduces to the baseline analysis.

Define $\bar{f}(s_{\mathcal{C}'}) \coloneqq v_{\mathcal{C}'}^* - \frac{1 - H\left(v_{\mathcal{C}'}^*\right)}{h\left(v_{\mathcal{C}'}^*\right)}$ as the highest transaction fee under which the consumer would consider sampling a non-sponsored product in the induced equilibrium. Observe that $\bar{f}'(s_{\mathcal{C}'}) < 0$; that is, the platform may charge a higher transaction fee in an active non-sponsored search market with lower search costs. Also, if $f \leq \bar{f}(s_{\mathcal{C}'})$, then the consumer purchases a product with probability one since there are infinitely many sellers.

Now assume $f \leq \bar{f}(s_{\mathcal{C}'})$ and consider the problem of a prominent seller $i \in \mathcal{C}$. If i charges p_i instead of $p_{\mathcal{C}}$, then the consumer purchases i if and only if

$$v_i - p_i > \max_{j \in \mathcal{C} \setminus \{i\}} \{v_{\mathcal{C}'}^* - p_{\mathcal{C}'}(f), v_j - p_{\mathcal{C}}\}.$$

Then seller i's profit is given by

$$(p_i - f) \Pr\left[v_i - p_i > \max_{j \in \mathcal{C} \setminus \{i\}} \{v_{\mathcal{C}'}^* - p_{\mathcal{C}'}, v_j - p_{\mathcal{C}}\}\right] - r$$

$$= (p_i - f) \int_{p_i + v_{\mathcal{C}'}^* - p_{\mathcal{C}'}}^{\bar{v}} H(v + p_{\mathcal{C}} - p_i)^{K-1} dH(v) - r.$$

Imposing $p_i = p_{\mathcal{C}}$ into the first order condition yields

$$p_{\mathcal{C}} = f + \phi\big(p_{\mathcal{C}} + v_{\mathcal{C}'}^* - p_{\mathcal{C}'}(f); K\big). \tag{$A4$}$$

The equilibrium sponsored seller price $p_{\mathcal{C}}(f, K)$ uniquely solves Equation ($A4$) in $(f, \bar{v})$. Recall $\phi_p, \phi_K \leq 0$. That ($A4$) has a unique solution for $K \geq 2$ follows by a similar argument used in Lemma 1.1 without an active non-sponsored search market; in this case ($A4$) has a unique solution even for $K = 1$. If $K = 1$ and $\bar{f} > \underline{v}$, then if $p_{\mathcal{C}} = f$ we have $p_{\mathcal{C}} < f + \phi\big(p_{\mathcal{C}} + v_{\mathcal{C}'}^* - p_{\mathcal{C}'}(f); K\big)$ since $\phi\big(p_{\mathcal{C}} + v_{\mathcal{C}'}^* - p_{\mathcal{C}'}(f); K\big) = \phi\big(\bar{f}; K\big) > 0$. If $K = 1$ and $\bar{f} \leq \underline{v}$, then $p_{\mathcal{C}} \geq \underline{v} - \big[v_{\mathcal{C}'}^* - p_{\mathcal{C}'}(f)\big]$ must hold optimally (otherwise the seller gets a lower price with less than all of the demand); if $p_{\mathcal{C}} = \underline{v} - \big[v_{\mathcal{C}'}^* - p_{\mathcal{C}'}(f)\big]$, we have

$$p_{\mathcal{C}} \leq f + \phi\big(p_{\mathcal{C}} + v_{\mathcal{C}'}^* - p_{\mathcal{C}'}(f); K\big) \Leftrightarrow \underline{v} - \phi\big(\underline{v}; K\big) \leq v_{\mathcal{C}'}^* - \phi\big(v_{\mathcal{C}'}^*; K\big)$$

which holds since $v - \phi(v; K)$ increases in v and $v_{\mathcal{C}'}^* \geq \underline{v}$. That $\lim_{p \to \bar{v}} [p_{\mathcal{C}} - f - \phi\big(p_{\mathcal{C}} + v_{\mathcal{C}'}^* - p_{\mathcal{C}'}(f); K\big)]$ follow as in the $K \geq 2$ case.

Having solved for the induced seller equilibrium prices, consider the platform's problem. Because each non-sponsored seller earns a negligible profit, the platform sets the referral fee to extract the sponsored seller industry profit. The platform's profit functions with and without price coherence are respectively given by

$$\Pi^{PC}(f,K) = \begin{cases} f + [p_{\mathcal{C}}(f,K) - f]\left[1 - H\left(p_{\mathcal{C}}(f,K) + v_{\mathcal{C}'}^* - p_{\mathcal{C}'}(f)\right)^K\right], & f \leq \bar{f} \\ p_{\mathcal{C}}(f,K)\left[1 - H\left(p_{\mathcal{C}}(f,K) + v_{\mathcal{C}'}^* - p_{\mathcal{C}'}(f)\right)^K\right], & f > \bar{f} \end{cases},$$

$$\Pi^{PF}(K) = \Pi^{PC}(0,K) = p_{\mathcal{C}}(0,K)\left[1 - H\left(p_{\mathcal{C}}(0,K) + v_{\mathcal{C}'}^* - p_{\mathcal{C}'}(0)\right)^K\right].$$

Suppose the platform enforces price coherence. For any K, suppose $f < \bar{f}$. The prominent seller price $p_{\mathcal{C}}(f,K)$ solves Equation $(A4)$ such that $\frac{\partial}{\partial f}p_{\mathcal{C}}(f,K) = \frac{d}{df}p_{\mathcal{C}'}(f) = 1$ and $\frac{\partial}{\partial f}\Pi^{PC}(f,K) = 1 > 0$. Hence $f^{PC} \leq \bar{f}$ implies $f^{PC} = \bar{f}$.

I next show that if $f^{PC} = \bar{f}$, then $K^{PC} = K^{PF}$. For any (f,K) with $f \leq \bar{f}$, we can compute

$$\frac{\partial}{\partial K}\Pi^{PC}(f,K) = \left[1 - H\big(p_{\mathcal{C}} + v_{\mathcal{C}'}^* - p_{\mathcal{C}'}\big)^K\right]\left[1 - \frac{\phi\big(p_{\mathcal{C}} + v_{\mathcal{C}'}^* - p_{\mathcal{C}'};K\big)}{\lambda\big(p_{\mathcal{C}} + v_{\mathcal{C}'}^* - p_{\mathcal{C}'};K\big)}\right]\frac{\partial p_{\mathcal{C}}}{\partial K}$$

$$- p_{\mathcal{C}}(0,K)H\big(p_{\mathcal{C}} + v_{\mathcal{C}'}^* - p_{\mathcal{C}'}\big)^K \ln\big[H\big(p_{\mathcal{C}} + v_{\mathcal{C}'}^* - p_{\mathcal{C}'}\big)\big],$$

$$\frac{\partial p_{\mathcal{C}}}{\partial K} = \frac{\phi_K\big(p_{\mathcal{C}} + v_{\mathcal{C}'}^* - p_{\mathcal{C}'};K\big)}{1 - \phi_p\big(p_{\mathcal{C}} + v_{\mathcal{C}'}^* - p_{\mathcal{C}'};K\big)}.$$

Now $\frac{\partial}{\partial f}\big[p_{\mathcal{C}} + v_{\mathcal{C}'}^* - p_{\mathcal{C}'}\big] = 0$, so $\frac{\partial}{\partial K}\Pi^{PC}(f,K)$ does not depend on f. In particular, $\frac{\partial}{\partial K}\Pi^{PC}\big(\bar{f},K\big) = \frac{\partial}{\partial K}\Pi^{PC}(0,K) = \frac{\partial}{\partial K}\Pi^{PF}(K)$. Hence $f^{PC} = \bar{f}$ implies $K^{PC} = K^{PF}$.

Suppose $f^{PC} > \bar{f}$. I will show that $K^{PC} = \bar{K}$ with $p_{\mathcal{C}}^{PC} = p^M(\bar{K})$. If $p_{\mathcal{C}}\big(\bar{f},\bar{K}\big) < p^M(\bar{K})$, then the platform can induce $p^M(\bar{K})$ with $f > \bar{f}$ to earn $\Pi^M(\bar{K})$, where $\Pi^M(K^{PF}) > \Pi^{PC}(f,K)$ for any (f,K) with $f > \bar{f}$ and $K < \bar{K}$; hence $K^{PC} = \bar{K}$ must hold, and $p_{\mathcal{C}}^{PC} = p^M(\bar{K})$. Now suppose $p_{\mathcal{C}}\big(\bar{f},\bar{K}\big) \geq p^M(\bar{K})$. Then $p_{\mathcal{C}}\big(\bar{f},K\big) \geq p^M(K)$ for all $K \leq \bar{K}$ (since $p_{\mathcal{C}}\big(\bar{f},K\big) - p^M(K)$ decreases in K). For any (f,K) with $f > \bar{f}$ and $K \leq \bar{K}$, the platform thus earns a profit no greater than

$$p_{\mathcal{C}}\big(\bar{f},K\big)\left[1 - H\left(p_{\mathcal{C}}\big(\bar{f},K\big)\right)^K\right] < p_{\mathcal{C}}\big(\bar{f},K\big)\left[1 - H\left(p_{\mathcal{C}}\big(\bar{f},K\big)\right)^K\right] + \bar{f}H\left(p_{\mathcal{C}}\big(\bar{f},K\big)\right)^K$$

$$= \Pi^{PC}\big(\bar{f},K\big).$$

That is, the platform can do better by setting $f = \bar{f}$; $p_{\mathcal{C}}(\bar{f}, \bar{K}) < p^M(\bar{K})$ is thus a necessary condition for $f^{PC} > \bar{f}$.

I have determined that $f^{PC} \le \bar{f}$ implies $f^{PC} = \bar{f}(> 0 = f^{PF})$ and $K^{PC} = K^{PF}$, from which it follows that $p_{\mathcal{C}}^{PC} > p_{\mathcal{C}}^{PF}$. Also, $f^{PC} > \bar{f}$ implies $K^{PC} = \bar{K} \ge K^{PF}$ and $p_{\mathcal{C}}^{PC} = p^M(\bar{K}) \ge p(0, K^{PF}) = p_{\mathcal{C}}^{PF}$. In summary, $K^{PC} \ge K^{PF}$ and $p_{\mathcal{C}}^{PC} \ge p_{\mathcal{C}}^{PF}$. That $p_{\mathcal{C}'}^{PC} = p_{\mathcal{C}'}(\bar{f}) \ge p_{\mathcal{C}'}(0) = p_{\mathcal{C}'}^{PF}$ also holds.

It remains to determine under what conditions $f^{PC} = \bar{f}$ or $f^{PC} > \bar{f}$. From the above results, we can write the platform's optimal profit with an active non-sponsored search market as

$$\Pi^{PC}(\bar{f}, K^{PF}) = \bar{f}(s_{\mathcal{C}'}) + \max_{K \le \bar{K}} \left\{ p_{\mathcal{C}}(0, K) \left[1 - H\left(p_{\mathcal{C}}(0, K) + \bar{f}(s_{\mathcal{C}'}) \right)^K \right] \right\}$$

and compute

$$\frac{\partial}{\partial s_{\mathcal{C}'}} \Pi^{PC}(\bar{f}, K^{PF}) = \bar{f}'(s_{\mathcal{C}'}) \left[1 - \frac{\phi\left(p_{\mathcal{C}}\left(\bar{f}(s_{\mathcal{C}'}), K^{PF}\right); K^{PF}\right)}{\lambda\left(p_{\mathcal{C}}\left(\bar{f}(s_{\mathcal{C}'}), K^{PF}\right); K^{PF}\right)} \right] < 0.$$

Let $(\hat{f}, \widehat{K})$ maximize the platform's profit without an active non-sponsored search market. Then

$$\Pi^{PC}(\hat{f}, \widehat{K}) = \max_{K \le \bar{K}, f \ge \bar{f}(s_{\mathcal{C}'})} p(f, K) \left[1 - H\left(p(f, K)\right)^K \right] \le \Pi^M(\bar{K}),$$

and $\frac{\partial}{\partial s_{\mathcal{C}'}} \Pi^{PC}(\hat{f}, \widehat{K}) \ge 0$ due to the loosening of the constraint on f (since $\bar{f}'(s_{\mathcal{C}'}) < 0$).

Now notice that if $s_{\mathcal{C}}' = 0$, then $\bar{f}(s_{\mathcal{C}}') = \bar{v}$, so

$$\Pi^{PC}(\bar{f}, K^{PF}) = \bar{v} > \Pi^M(\bar{K}) \ge \Pi^{PC}(\hat{f}, \bar{K}).$$

If $s_{\mathcal{C}'}$ is large such that $v_{\mathcal{C}'}^* = \frac{1 - H\left(v_{\mathcal{C}'}^*\right)}{h\left(v_{\mathcal{C}'}^*\right)}$, then $\bar{f} = 0$, so

$$\Pi^{PC}(\bar{f}, K^{PF}) = \Pi^{PF}(K^{PF}) \le \Pi^M(\bar{K}) = \Pi^{PC}(\hat{f}, \widehat{K}).$$

Therefore, there exists $s_{\mathcal{C}'}^{PC}$ such that $f^{PC} = \bar{f}$ (or equivalently $\Pi^{PC}(\bar{f}, K^{PF}) \ge \Pi^{PC}(\hat{f}, \widehat{K})$) if and only if $s_{\mathcal{C}'} \le s_{\mathcal{C}'}^{PC}$. Let $s_{\mathcal{C}'}^{PF}$ solve $\bar{f}(s_{\mathcal{C}}') = 0$. We have $s_{\mathcal{C}'}^{PF}$ and $s_{\mathcal{C}'}^{PC}$ that satisfy statement (i). If

$s_{\mathcal{C}'} \leq s_{\mathcal{C}'}^{PC}\left(< s_{\mathcal{C}'}^{PF}\right)$, then total welfare and consumer surplus are higher with mandated price flexibility since $K^{PF} = K^{PC}$, $p_{\mathcal{C}}^{PF} < p_{\mathcal{C}}^{PC}$, and $p_{\mathcal{C}'}^{PF} < p_{\mathcal{C}'}^{PC}$.

Last, consider statement (ii). Given any $s_{\mathcal{C}'}$, $\lim_{\hat{K}\to\infty} \Pi^{PC}\left(\hat{f}, \hat{K}\right) = \bar{v}$ and K^{PF} is bounded under the conditions of Proposition 1.3 by the same arguments regarding $\phi(p; K)$. The statement follows directly.

Now suppose $s_{\mathcal{C}'} > \int_{\max\{0,\underline{v}\}}^{\bar{v}}\left(v - \max\{0, \underline{v}\}\right)dH(v)$. If $\underline{v} < 0$, then $s_{\mathcal{C}'} > \int_{\underline{v}}^{\bar{v}} v dH(v)$ as in the baseline model. Suppose $\underline{v} \geq 0$. If $s_{\mathcal{C}'} > \int_{0}^{\bar{v}}(v - 0)dH(v) = \int_{\underline{v}}^{\bar{v}} v dH(v)$, then again the model reduces to the baseline model. Otherwise, $v_{\mathcal{C}'}^{*}$ is well-defined with $v_{\mathcal{C}'}^{*} \leq \underline{v}$. In such case, the only equilibrium exhibits $p_{\mathcal{C}'} = \infty$ (S. P. Anderson & Renault, 1999) so the consumer never searches among $\mathcal{C}'$ and the baseline analysis again applies. ■

Proof of Proposition 1.7: For each platform $P \in \{A, B\}$, suppose there are K_P prominent sellers on platform P who face a transaction fee f_P, and let $\mathcal{C}_P$ denote the set of those prominent sellers. Suppose that in equilibrium, prominent sellers on platform P set the price $p_P (\leq p_P^d)$.

Consider a deviation by a prominent seller $i \in \mathcal{C}_A$. The consumer purchases i iff

$$v_i - p_i > \max_{j_A \in \mathcal{C}_A \setminus \{i\}, j_B \in \mathcal{C}_B} \left\{0, v_{j_A} - p_A, v_{j_B} - p_B\right\},$$

Seller i therefore has a deviation profit of

$$(p_i - f_A) \int_{p_i}^{\bar{v}} H(v + p_A - p_i)^{K_A - 1} H(v + p_B - p_i)^{K_B} dH(v) - r_A.$$

The first order condition of i's profit maximization problem is

$$\int_{p_i}^{\bar{v}} H(v + p_A - p_i)^{K_A - 1} H(v + p_B - p_i)^{K_B} dH(v)$$

$$- (p_i - f_A) \Bigg[H(p_A)^{K_A - 1} H(p_B)^{K_B} h(p_i)$$

$$+ \int_{p_i}^{\bar{v}} h(v) dH(v + p_A - p_i)^{K_A - 1} H(v + p_B - p_i)^{K_B} \Bigg] = 0.$$

Let $\Delta \coloneqq p_A - p_B$ denote the difference in equilibrium prices. Imposing $p_i = p_A$ into the first order condition obtains the candidate equilibrium condition

$$p_A = f_A + \frac{\int_{p_A}^{\bar{v}} H(v)^{K_A - 1} H(v - \Delta)^{K_B} dH(v)}{H(p_A)^{K_A - 1} H(p_B)^{K_B} h(p_A) + \int_{p_A}^{\bar{v}} h(v) dH(v)^{K_A - 1} H(v - \Delta)^{K_B}}.$$

An analogous condition may be obtained for p_B. With uniform H over $[\underline{v} = 0, \bar{v} = 1]$, these two conditions simplify to

$$p_A = f_A + \int_{p_A}^{\bar{v}} H(v)^{K_A - 1} \left[\frac{H(v - \Delta)}{H(\bar{v} - \Delta)} \right]^{K_B} dv\,, \tag{A5}$$

$$p_B = f_B + \int_{p_B}^{\bar{v}} H(v)^{K_B - 1} \left[\frac{H(v + \Delta)}{H(\bar{v} + \Delta)} \right]^{K_A} dv\,. \tag{A6}$$

Assuming $f_A, f_B \leq \bar{v}$, we can show that the system of equations $(A5)$ and $(A6)$ has a unique solution in $(0, \bar{v})^2$. This is because the RHS of Equation $(A5)$ decreases in p_A, so it is easily verified that given any $p_B \in (0, \bar{v})$, Equation $(A5)$ has a unique solution $p_A^*(p_B) \in (0, \bar{v})$. Similarly, given any $p_A \in (0, \bar{v})$, Equation $(A6)$ has a unique solution $p_B^*(p_A) \in (0, \bar{v})$. Continuity of $p_A^*(p_B)$ and $p_B^*(p_A)$ follows by the implicit function theorem, so a solution to the system exists in $(0, \bar{v})^2$ due to Brouwer's fixed point theorem. This solution is unique as follows. First, from $(A5)$ we may compute

$$\frac{dp_A^*(p_B)}{dp_B} = \frac{-\int_{p_A}^{\bar{v}} H(v)^{K_A-1}\frac{\partial}{\partial\Delta}\left\{\left[\frac{H(v-\Delta)}{H(\bar{v}-\Delta)}\right]^{K_B}\right\}dv}{1+H(p_A)^{K_A-1}H(p_B)^{K_B} - \int_{p_A}^{\bar{v}} H(v)^{K_A-1}\frac{\partial}{\partial\Delta}\left\{\left[\frac{H(v-\Delta)}{H(\bar{v}-\Delta)}\right]^{K_B}\right\}dv} \in [0,1],$$

where I have used $\frac{\partial}{\partial\Delta}\left\{\left[\frac{H(v-\Delta)}{H(\bar{v}-\Delta)}\right]^{K_B}\right\} \leq 0$. Substituting $p_A^*(p_B)$ into Equation $(A6)$ fully characterizes the system of equations $(A5)$ and $(A6)$ in terms of p_B, and the resulting equation has a unique solution since its RHS decreases in p_B (and its LHS increases in p_B).

We may conclude that given any $f_A, f_B \leq \bar{v}$ and $K_A, K_B \leq \bar{K}$ (and provided that the referral fees are low enough to induce seller participation), the equilibrium product prices on each platform uniquely solve the system of equations $(A5)$ and $(A6)$.

Note that $p_A \geq p_B$ if and only if $f_A \geq f_B$, and hence $f_A = f_B$ implies $p_A = p_B$. To see this, observe that $p_A \geq p_B$ (i.e., $\Delta \geq 0$) and $f_A < f_B$ yield

$$f_B + \int_{p_B}^{\bar{v}} H(v)^{K_B-1}H(v+\Delta)^{K_A}dv = p_B < f_A + \int_{p_B}^{\bar{v}} H(v)^{K_A+K_B-1}dv,$$

where the equality follows by Equation $(A6)$, and the inequality follows by Equation $(A5)$ and $p_A \geq p_B$; but this is a contradiction, as $f_A < f_B$ and $\int_{p_B}^{\bar{v}} H(v)^{K_A+K_B-1}dv \leq \int_{p_B}^{\bar{v}} H(v)^{K_B-1}H(v+\Delta)^{K_A}dv$. Also note that if $f_A = f_B = f$, then $p_A = p_B = p(f, K_A+K_B)$, where $p(f, K_A+K_B)$ is the equilibrium price from the baseline model, which in this case uniquely solves

$$p = f + \frac{1-H(p)^{K_A+K_B}}{K_A+K_B}.$$

Now consider the platforms' choices with enforceable price coherence, which ensures that any transaction is intermediated in equilibrium. Given f_B and K_B, platform A sets its referral fee to extract all of its prominent sellers' profits, and adding its transaction fees, platform A earns as profit its prominent sellers' revenue of

$$\Pi_A(f_A, K_A) \coloneqq p_A \int_{p_A}^{\bar{v}} H(v - \Delta)^{K_B} dH(v)^{K_A}.$$

As $p_A \geq f_A$ always holds and implies that $\Pi_A(\bar{v}, K_A) = 0$, the platform optimally sets $f_A < \bar{v}$ given any choice of K_A.

I claim that in any symmetric equilibrium in which the platforms enforce price coherence, each platform offers $\bar{K}$ sponsored slots to sellers. Suppose (for contradiction) that there exist $f \in [0, \bar{v})$ and $K < \bar{K}$ such that $f_A = f_B = f$ and $K_A = K_B = K$ in equilibrium. The equilibrium price is thus given by $p_A = p_B = p(f, 2K)$.

Now for platform A, consider the deviation to $K_A = K + 1$ and $f_A = \hat{f}$, where $\hat{f}$ is chosen such that $p_A = p(f, 2K)$ remains the induced equilibrium price on A. Denoting the new induced price on B by $p_B = \hat{p}_B$, this means that

$$p(f, 2K) = \hat{f} + \int_{p(f,2K)}^{\bar{v}} H(v)^K \left[\frac{H(v - p(f, 2K) + \hat{p}_B)}{H(\bar{v} - p(f, 2K) + \hat{p}_B)}\right]^K dv\,,$$

$$\hat{p}_B = f + \int_{\hat{p}_B}^{\bar{v}} H(v)^{K-1} \left[\frac{H(v + p(f, 2K) - \hat{p}_B)}{H(\bar{v} + p(f, 2K) - \hat{p}_B)}\right]^{K+1} dv.$$

Recall that $p(f, 2K)$ decreases in its second argument, so $\hat{f} > f$ and $p(f, 2K) > \hat{p}_B$ must both hold.

Since p_A has not changed, the platform's deviation is profitable if it yields more demand on platform A than before, i.e., if

$$\int_{p(f,2K)}^{\bar{v}} H\big(v - (p(f, 2K) - \hat{p}_B)\big)^K dH(v)^{K+1} > \frac{1 - H\big(p(f, 2K)\big)^{2K}}{2}.$$

To see that this holds, first observe that Equation ($A6$) and $p(f, 2K) > \hat{p}_B$ imply

$$K(\hat{p}_B - f) = \int_{\hat{p}_B}^{\bar{v}} H(v + p(f, 2K) - \hat{p}_B)^{K+1} dH(v)^K,$$

where $\int_{\hat{p}_B}^{\bar{v}} H(v + p(f,2K) - \hat{p}_B)^{K+1} dH(v)^K$ is platform B's demand after the deviation by A. Since total demand after the deviation is $1 - H\big(p(f,2K)\big)^{K+1} H(\hat{p}_B)^K$, we have

$$\int_{p(f,2K)}^{\bar{v}} H\big(v - (p(f,2K) - \hat{p}_B)\big)^K dH(v)^{K+1}$$

$$= 1 - H\big(p(f,2K)\big)^{K+1} H(\hat{p}_B)^K - \int_{\hat{p}_B}^{\bar{v}} H(v + p(f,2K) - \hat{p}_B)^{K+1} dH(v)^K$$

$$= 1 - H\big(p(f,2K)\big)^{K+1} H(\hat{p}_B)^K - K(\hat{p}_B - f)$$

$$> 1 - H\big(p(f,2K)\big)^{2K} - K(p(f,2K) - f)$$

$$= 1 - H\big(p(f,2K)\big)^{2K} - K\frac{1 - H\big(p(f,2K)\big)^{2K}}{2K} = \frac{1 - H\big(p(f,2K)\big)^{2K}}{2},$$

as desired. We thus conclude that in any symmetric equilibrium in which the platforms enforce price coherence, $K_A = K_B = \bar{K}$ must hold.

Now let us derive the (an) equilibrium transaction fee level. First note that $\frac{\partial p_A}{\partial f_A} > \frac{\partial p_B}{\partial f_A} > 0$ holds as follows. Consider an increase in f_A. The RHS of Equation ($A5$) strictly increases in f_A and p_B and decreases in p_A. Hence, if p_B weakly decreased after the change, p_A must have strictly increased; but this produces the contradiction that p_B increased due to Equation ($A6$). Therefore $\frac{\partial p_B}{\partial f_A} > 0$. Now the RHS of Equation ($A6$) strictly increases in Δ and strictly decreases in p_B, so $\frac{\partial \Delta}{\partial f_A} \leq 0$ would wrongly imply $\frac{\partial p_B}{\partial f_A} \leq 0$. Hence $\frac{\partial \Delta}{\partial f_A} > 0$, or $\frac{\partial p_A}{\partial f_A} > \frac{\partial p_B}{\partial f_A}$. We thus have $\frac{\partial p_A}{\partial f_A} > \frac{\partial p_B}{\partial f_A} > 0$.

Next, I will show that $\arg\max_{f_A} \Pi_A(f_A, K_A) > 0$. Using $\frac{\partial p_B}{\partial f_A} > 0$, compute

$$\frac{\partial}{\partial f_A} \Pi_A(f_A, K_A) > \frac{\partial p_A}{\partial f_A} \frac{\partial}{\partial p_A}\left[p_A \int_{p_A}^{\bar{v}} H(v - \Delta)^{K_B} dH(v)^{K_A}\right]$$

$$= \frac{\partial p_A}{\partial f_A} K_A \left[\frac{\int_{p_A}^{\bar{v}} H(v)^{K_A-1} H(v-\Delta)^{K_B} dH(v)}{H(p_A)^{K_A-1} H(p_B)^{K_B} + \int_{p_A}^{\bar{v}} H(v)^{K_A-1} dH(v-\Delta)^{K_B}} - p_A \right]$$

$$> -\frac{\partial p_A}{\partial f_A} K_A f_A,$$

where the last inequality follows from the first order condition giving p_A. (Intuitively, it follows since if $f_A = 0$, the equilibrium price p_A is smaller than the price that maximizes the joint profit of all prominent sellers on A.) Therefore $\frac{\partial}{\partial f_A} \Pi_A(f_A, K_A) > 0$ at $f_A = 0$, and $\arg\max_{f_A} \Pi_A(f_A, K_A) > 0$. As allowing for price flexibility is equivalent to setting $f_A = 0$, this implies that the platforms both enforce price coherence in equilibrium if it is feasible to do so.

By the continuity of $\Pi_A(f_A, K_A)$ over $f_A \in [0, \bar{v}]$, the platform optimally sets $f_A \in (0, \bar{v})$. Since $\Pi_A(f_A, K_A)$ is differentiable in f_A over $(0, \bar{v})$, there exists a solution to the first order condition (for f_A) in platform A's profit maximization problem, and if it is unique, then it is also the unique global maximum of A's profit (given K_A).

The first order condition for f_A, i.e., $\frac{\partial}{\partial f_A} \Pi_A(f_A, K_A) = 0$, with the equilibrium conditions $f_A = f_B = f$ and $K_A = K_B = \bar{K}$ imposed, is given by

$$p^{2\bar{K}-1} + \frac{\bar{K}}{2\bar{K}-1}\left(1 - p^{2\bar{K}-1}\right) - \frac{1}{2\bar{K}} \frac{1 - p^{2\bar{K}}}{p} = \frac{\partial p_B}{\partial f_A} \left[\frac{\partial p_A}{\partial f_A}\right]^{-1} \frac{\bar{K}}{2\bar{K}-1}\left(1 - p^{2\bar{K}-1}\right), \qquad (A7)$$

where $p = p(f, 2\bar{K})$, and from differentiating $(A5)$ and $(A6)$,

$$\frac{\partial p_A}{\partial f_A} = 1 - \frac{\partial p_A}{\partial f_A} H(p)^{2\bar{K}-1} - \left[\frac{\partial p_A}{\partial f_A} - \frac{\partial p_B}{\partial f_A}\right] \frac{\bar{K}}{2\bar{K}-1} \int_p^{\bar{v}} [1 - H(v)] dH(v)^{2\bar{K}-1},$$

$$\frac{\partial p_B}{\partial f_A} = -\frac{\partial p_B}{\partial f_A} H(p)^{2\bar{K}-1} + \left[\frac{\partial p_A}{\partial f_A} - \frac{\partial p_B}{\partial f_A}\right] \frac{\bar{K}}{2\bar{K}-1} \int_p^{\bar{v}} [1 - H(v)] dH(v)^{2\bar{K}-1}.$$

From this, one can show that

$$\frac{\partial p_B}{\partial f_A}\left[\frac{\partial p_A}{\partial f_A}\right]^{-1} = \frac{\frac{\bar{K}}{2\bar{K}-1}\int_p^{\bar{v}}[1-H(v)]dH(v)^{2\bar{K}-1}}{1+H(p)^{2\bar{K}-1}+\frac{\bar{K}}{2\bar{K}-1}\int_p^{\bar{v}}[1-H(v)]dH(v)^{2\bar{K}-1}}.$$

Notice that $\frac{\partial p_B}{\partial f_A}\left[\frac{\partial p_A}{\partial f_A}\right]^{-1}$ decreases in p; hence, the RHS of $(A7)$ decreases in p. It is easily verified that the LHS of $(A7)$ increases in p. Recalling that $p = p(f, 2\bar{K})$ increases in f, $(A7)$ has a unique solution $f^{PC} \in (0, \bar{v})$ as follows. If $f = 0$, then the LHS of $(A7)$ is negative and hence smaller than the RHS. As $f \to \bar{v}$, the LHS of $(A7)$ is positive and increases, while the RHS goes to zero. Hence, there exists a unique equilibrium fee level $f^{PC} \in (0, \bar{v})$ that solves $(A7)$.

Finally, observe that $\lim_{\bar{K}\to\infty} \frac{\partial p_B}{\partial f_A}\left[\frac{\partial p_A}{\partial f_A}\right]^{-1} = 0$ regardless of the value of $\lim_{\bar{K}\to\infty} p(f^{PC}(\bar{K}), 2\bar{K})$. Then from $(A7)$, it must be that $\lim_{\bar{K}\to\infty} p(f^{PC}(\bar{K}), 2\bar{K}) = 0$, requiring $\lim_{\bar{K}\to\infty} f^{PC}(\bar{K}) = 0$.

Now if price flexibility is mandated, platform A earns a profit of

$$\Pi_A(0, K_A) = \frac{K_A}{K_A + K_B} p(0, K_A + K_B)[1 - p(0, K_A + K_B)^{K_A + K_B}] = K_A p(0, K_A + K_B)^2$$

As $\lim_{K_A\to\infty} p(0, K_A + K_B) = 0$, there exists a bound K_A^{max}, which does not depend on $\bar{K}$ or K_B, such that $\arg\max_{K_A \le \bar{K}} \Pi_A(0, K_A) < K_A^{max}$ (or this holds for all $K \in \arg\max_{K_A \le \bar{K}} \Pi_A(0, K_A)$ if the set is not a singleton). That is, for $\bar{K} \ge K^{max}$, $K_A \in \{K^{max}, \dots, \bar{K}\}$ is strictly dominated for A and not played in equilibrium. Since the platforms are symmetric, this same bound applies analogously to platform B.

To complete the proof, we must consider the limiting total welfare and consumer surplus. Given a symmetric price p and total number of sellers $K_A + K_B$ across platforms, total welfare and consumer surplus are given by $W(p, K_A + K_B)$ and $CS(p, K_A + K_B)$, as given in the baseline model. As $\lim_{\bar{K}\to\infty} p^{PC} = 0$ and $\lim_{\bar{K}\to\infty} (K_A^{PC} + K_B^{PC}) = \lim_{\bar{K}\to\infty} 2\bar{K} = \infty$, we have $\lim_{\bar{K}\to\infty} W^{PC} = \lim_{\bar{K}\to\infty} CS^{PC} = \bar{v}$.

On the other hand, $\lim_{\bar{K}\to\infty} p^{PF} > p(0, 2K^{max}) > 0$ and $\lim_{\bar{K}\to\infty} (K_A^{PF} + K_B^{PF}) < 2K^{max}$, so $\lim_{\bar{K}\to\infty} W^{PF} < \bar{v}$ and $\lim_{\bar{K}\to\infty} CS^{PF} < \bar{v}$. ■

APPENDIX B: RANDOM SEQUENTIAL SEARCH

I distinguish notation in this environment from the main model with a tilde (e.g., the equilibrium transaction fee with price coherence is denoted $\tilde{f}^{PC}$).

To allow for a non-trivial equilibrium, I assume $s_{\mathcal{C}} \leq \int_{\underline{v}}^{\bar{v}}(v-\underline{v})dH(v)$. This ensures that v^* is well-defined and $v^* \geq \underline{v}$. If $v^* < \underline{v}$, then each seller must set an infinite price in equilibrium (for details see S. P. Anderson & Renault, 1999).

A seller $i \in \mathcal{C}$'s profit from charging p_i while all other sellers in $\mathcal{C}$ charge p is given by

$$(p_i - f)\left\{\Pr[v_i - p_i > v^* - p]\frac{1}{K}\sum_{k=0}^{K-1}\Pr\left[\max_{j\in\hat{\mathcal{C}}} v_j < v^* \middle| |\hat{\mathcal{C}}| = k\right]\right.$$

$$\left. + \Pr\left[\max_{j\neq i}\{0, v_j - p\} < v_i - p_i < v^* - p\right]\right\}$$

$$= (p_i - f)\left\{[1 - H(p_i + v^* - p)]\frac{1}{K}\frac{1 - H(v^*)^K}{1 - H(v^*)} + \int_{p_i}^{p_i + v^* - p} H(p + v - p_i)^{K-1} dH(v)\right\}.$$

The first order condition with equilibrium condition $p_i = p$ imposed is

$$p = f + \tilde{\phi}(p; K), \qquad (B1)$$

where

$$\tilde{\phi}(p;K) := \frac{[1 - H(p)^K]/K}{H(p)^{K-1}h(p) + \left[\frac{1}{K}\frac{1 - H(v^*)^K}{1 - H(v^*)} - H(v^*)^{K-1}\right]h(v^*) + \int_p^{v^*} h(v)dH(v)^{K-1}}$$

$$= \frac{1 - H(p)^K}{\frac{1 - H(v^*)^K}{1 - H(v^*)}h(v^*) - K\int_p^{v^*} h'(v)H(v)^{K-1}dv}.$$

Let $\tilde{p}(f, K)$ denote the solution (a solution) to Equation $(B1)$ if it exists. Assumption B1 ensures the existence of a non-trivial equilibrium for all K when $f = 0$. It is necessary because the consumer will not participate if $p > v^*$, whereas the consumer would participate with $s_C = 0$ up to $p = \bar{v} > v^*$ in the discrete choice framework.

Assumption B1. $v^* \geq \frac{1-H(v^*)}{h(v^*)}$.

The following result is analogous to Lemma 1.1.

Lemma B1. *Suppose* $r = 0$, $f \leq v^*$, *and Assumption* B1 *holds. Then the prominent sellers' equilibrium price* $\tilde{p}(f, K)$ *increases in* f *and decreases in* K, *where:*

(i) If $f \leq \underline{v} - \tilde{\phi}(\underline{v}; K)$, *then* $\tilde{p}(f, K) \leq \underline{v}$, *such that the market is fully covered. Specifically,* $\tilde{p}(f, K) = \underline{v}$ *if* $K = 1$ *and* $\tilde{p}(f, K) = f + \tilde{\phi}(\underline{v}; K)$ *if* $K \geq 2$.

(ii) Otherwise, $\tilde{p}(f, K)$ *uniquely solves Equation* $(B1)$ *and* $p > \underline{v}$, *such that the market is not fully covered.*

Proof: We can write

$$\frac{1}{\tilde{\phi}(p; K)} = \frac{h(v^*)}{1 - H(v^*)} \frac{1 - H(v^*)^K}{1 - H(p)^K} - \int_p^{v^*} \frac{h'(v)}{h(v)} d\left[\frac{H(v)^K - H(p)^K}{1 - H(p)^K}\right].$$

One can show that the first term increases in K, and the second term does by the same argument from the Proof of Lemma 1.1 (v^* replacing $\bar{v}$ makes no difference). The first term also increases in p. Since v^* replaces $\bar{v}$ in the second term, we cannot write it as an expectation as before. We may nevertheless directly compute

$$\frac{\partial}{\partial p} \frac{1}{\tilde{\phi}(p; K)}$$

$$= \frac{KH(p)^{K-1} h(p)}{[1 - H(p)^K]^2} \left[\frac{h(v^*)}{1 - H(v^*)} [1 - H(v^*)^K] + \frac{h'(p)}{h(p)} [1 - H(p)^K] - \int_p^{v^*} \frac{h'(v)}{h(v)} dH(v)^K\right]$$

$$\geq \frac{KH(p)^{K-1} h(p)}{[1 - H(p)^K]^2} [1 - H(v^*)^K] \left[\frac{h(v^*)}{1 - H(v^*)} + \frac{h'(v^*)}{h(v^*)}\right],$$

where the inequality follows using the fact that $\frac{h'(v)}{h(v)}$ decreases in v for log-concave h. Finally, observe that $\frac{h(v^*)}{1-H(v^*)} + \frac{h'(v^*)}{h(v^*)} = \frac{d}{dv^*}\log\left[\frac{h(v^*)}{1-H(v^*)}\right] \geq 0$. The sign holds since h log-concave implies $1-H$ is log-concave, so $\log\left[\frac{h(v^*)}{1-H(v^*)}\right]$ is an increasing transformation of an increasing function of v^*. We thus have $\tilde{\phi}_K(p;K) \leq 0$ and $\tilde{\phi}_p(p;K) \leq 0$.

Suppose $K \geq 2$. I will show that Equation $(B1)$ has a unique solution. If $p = f$, then $p < f + \tilde{\phi}(p;K)$ holds. If $p = v^*$, then

$$f + \tilde{\phi}(p;K) = f + \frac{1-H(v^*)}{h(v^*)} \leq v^* = p,$$

where the inequality follows by Assumption B1. Then Equation $(B1)$ has a unique solution. The remaining results follow analogously to those in Lemma 1.1. ■

For any choice (f, K), the platform sets the referral fee $\tilde{r} = \tilde{r}(f,K)$ to extract any seller surplus, where

$$\tilde{r}(f,K) = \frac{1}{K}[\tilde{p}(f,K) - f]\left[1 - H\big(\tilde{p}(f,K)\big)^K\right]. \qquad (B2)$$

Again, $\tilde{p}(0,K) < p^M(K)$ since the second and third terms in the denominator of the definition of $\tilde{\phi}(p;K)$ are both positive. Then by previous arguments, $\tilde{K}^{PC} = K^{PC} = \overline{K}$ and $\tilde{f}^{PC}$ is chosen to satisfy $p(f,\overline{K}) = \min\{p^M(\overline{K}), v^*\}$.

We have the following result, which is very similar to Proposition 1.1.

Proposition B1. *Suppose Assumption* B1 *holds. The platform enforces price coherence if admissible. It sets* $\tilde{K}^{PC} = \overline{K}$ *to make the consumer consideration set as large as possible. It chooses transaction fee* $\tilde{f}^{PC}$ *such that the equilibrium price satisfies* $p\big(\tilde{f}^{PC},\overline{K}\big) = \min\{p^M(\overline{K}), v^*\}$. *It sets referral fee* $\tilde{r}^{PC} = \tilde{r}(f^{PC},\overline{K})$ *given by Equation* $(B2)$*. The platform extracts the* $\overline{K}$*-seller industry revenue as profit, and its optimal profit is that of a* $\overline{K}$*-product monopoly seller.*

Notably, $\widetilde{K}^{PC} = K^{PC} = \overline{K}$ and $\tilde{p}^{PC} = p^{PC}$ if $p^M(\overline{K}) \le v^*$. A similar exact equivalence result does not always hold without price coherence because the platform cannot calibrate prices through f without price coherence and $\tilde{p}(0, K) \neq p(0, K)$. There are, however, natural conditions under which $\widetilde{K}^{PF} < \overline{K}$, and these are qualitatively similar to the conditions in Proposition 1.2.

Proposition B2. *Suppose Assumption* B1 *holds. With mandated price flexibility, the platform chooses the number of prominent search slots* $\widetilde{K}^{PF} \in \{1, \dots, \overline{K}\}$ *to maximize the industry profit, and it sets a referral fee of* $\tilde{r}^{PF} = r\left(0, \widetilde{K}^{PF}\right)$ *given by Equation* $(B2)$. *Furthermore:*

(i) If $\underline{v} \ge \tilde{\phi}\left(\underline{v}; 1\right)$, *then* $\widetilde{K}^{PF} = 1$.

(ii) If $\lim_{v \to \bar{v}} \frac{1-H(v)}{h(v)} = 0$, *then there exists* $s_{\mathcal{C}}^{\dagger} > 0$ *such that, if* $s_{\mathcal{C}} < s_{\mathcal{C}}^{\dagger}$, *then* $\widetilde{K}^{PF} < \overline{K}$ *for sufficiently large* $\overline{K}$.

Proof: The main statement and statement (i) follow from previous reasoning. Taking the limit of Equation $(B1)$ with $f = 0$ yields

$$\lim_{K \to \infty} \tilde{p}(0, K) = \frac{1 - H(v^*)}{h(v^*)} = \lim_{K \to \infty} \widetilde{\Pi}^{PF}(K).$$

From the definition of v^*, we have $\lim_{s_{\mathcal{C}} \to 0} v^* = \bar{v}$, so $\lim_{s_{\mathcal{C}} \to 0} \frac{1-H(v^*)}{h(v^*)} = \lim_{v \to \bar{v}} \frac{1-H(v)}{h(v)}$. Suppose $\lim_{v \to \bar{v}} \frac{1-H(v)}{h(v)} = 0$ and choose an arbitrary $K \in \mathbb{N}$. Observe that

$$\lim_{s_{\mathcal{C}} \to 0} \widetilde{\Pi}^{PF}(K) = \Pi^{PF}(K) > 0 = \lim_{s_{\mathcal{C}} \to 0} \lim_{\overline{K} \to \infty} \widetilde{\Pi}^{PF}(\overline{K}).$$

Then there exists $s_{\mathcal{C}}^{\dagger}$ such that if $s_{\mathcal{C}} \le s_{\mathcal{C}}^{\dagger}(K)$ and $\overline{K}$ is sufficiently large with $\overline{K} > K$, then $\widetilde{\Pi}^{PF}(K) > \widetilde{\Pi}^{PF}(\overline{K})$; hence, $K^{PF} < \overline{K}$. ■

We have $\widetilde{K}^{PF} = K^{PF} = 1$ if the market is fully covered for all K (Proposition B2(i)). The condition of Proposition B2(ii) is the same as that of Proposition 1.2(ii). It implies $K^{PF} < \overline{K}$ for sufficiently large $\overline{K}$, but the same is not true for $\widetilde{K}^{PF}$. This is due to the unobservability of prices.

We have $\lim_{K\to\infty} \tilde{p}(0,K) = \frac{1-H(v^*)}{h(v^*)} > 0$ for $v^* < \bar{v}$, so each firm maintains significant market power with random search even as $K \to \infty$ because sellers cannot undercut each other to draw the consumer to them. Competition can only drive the equilibrium price low enough to ensure $\widetilde{K}^{PF} < \overline{K}$ if $s_{\mathcal{C}}$ is sufficiently low. If $s_{\mathcal{C}}$ is low, then the consumer is sufficiently demanding of a winning seller (i.e., v^* is high), making him more sensitive to prices and the seller demand more elastic.

The following analog to Corollary 1.1 holds.

Corollary B1. *Under Assumption* B1, *prices are higher and the consideration set is larger with enforceable price coherence relative to mandated price flexibility, i.e.,* $\tilde{p}^{PC} \geq \tilde{p}^{PF}$ *and* $\widetilde{K}^{PC} \geq \widetilde{K}^{PF}$.

Consider welfare. Given an equilibrium K and $p \leq v^*$ conforming to Proposition B1 or Proposition B2, the following observations hold. The probability that the consumer searches exactly k times is $H(v^*)^{k-1}[1-H(v^*)]$ for $k \leq K-1$ and $H(v^*)^{k-1}$ for $k = K$; the consumer's expected search cost is then

$$H(v^*)^K \cdot K s_{\mathcal{C}} + \sum_{k=0}^{K-1} H(v^*)^k[1-H(v^*)] \cdot (k+1) s_{\mathcal{C}} = s_{\mathcal{C}} \frac{1-H(v^*)^K}{1-H(v^*)}.$$

The consumer earns a surplus (gross of search costs) of $v_i - p$ if search ends because he finds a product i with $v_i \geq v^*$, which occurs with probability $1-H(v^*)^K$. The consumer earns a surplus (gross of search costs) of $v_{(K:K)} - p$ if search ends with recalled comparison of all products $n \in \mathcal{C}$, which occurs with probability $H(v^*)^K$. Trade occurs only if $v_{(K:K)} > p$, which occurs with probability $1-H(p)^K$. Sellers earn no surplus, and the platform's profit is $p[1-H(p)^K]$. Let $\widetilde{CS}(K,p)$ and $\widetilde{W}(K,p)$ respectively denote consumer and total welfare. Then

$$\widetilde{CS}(K,p) = [1-H(v^*)^K] \frac{\int_{v^*}^{\bar{v}} (v-p) dH(v)}{1-H(v^*)} + \int_p^{v^*} (v-p) dH(v)^K - s_{\mathcal{C}} \frac{1-H(v^*)^K}{1-H(v^*)}$$

$$= \int_p^{\bar{v}} [\min\{v, v^*\} - p] dH(v)^K ,$$

$$\widetilde{W}(K,p) = [1 - H(v^*)^K] \frac{\int_{v^*}^{\bar{v}} v dH(v)}{1 - H(v^*)} + \int_p^{v^*} v dH(v)^K - s_C \frac{1 - H(v^*)^K}{1 - H(v^*)}$$

$$= \int_p^{\bar{v}} \min\{v, v^*\} \, dH(v)^K.$$

Note that I have used the definition of v^* when rewriting each welfare measure. Proposition B2 provides conditions under which K^{PF} is bounded above. Specifically, it provides conditions under which there exists $\bar{K}^{PF}$, which does not depend on $\bar{K}$, such that $K^{PF} \leq \bar{K}^{PF}$.

Proposition B3. *Suppose there exists $\bar{K}^{PF}$ (that does not depend on $\bar{K}$) such that $K^{PF} < \bar{K}^{PF}$. For $\bar{K}$ sufficiently large, the total welfare and consumer surplus may be ranked as follows.*

(i) Total welfare is higher with enforceable price coherence relative to mandated price flexibility, i.e., $\widetilde{W}(p^{PC}, K^{PC}) > \widetilde{W}(p^{PF}, K^{PF})$.

(ii) Consumer surplus is weakly lower with enforceable price coherence relative to mandated price flexibility, i.e., $CS(p^{PC}, K^{PC}) < CS(p^{PF}, K^{PF})$.

Proof: For any (K,p), we have $\widetilde{W}(K,p) < v^* \leq \bar{v}$. Therefore

$$\lim_{\bar{K}\to\infty} \widetilde{W}\left(\widetilde{K}^{PF}, \tilde{p}\left(0, \widetilde{K}^{PF}\right)\right) < v^* = \lim_{\bar{K}\to\infty} \widetilde{W}\left(\widetilde{K}^{PC}, \tilde{p}\left(\tilde{f}^{PC}, \widetilde{K}^{PC}\right)\right),$$

where the equality follows since the platforms profit with price coherence, $\max_{p \leq v^*} p\left[1 - H(p)^{\bar{K}}\right]$, approaches v^* as $\bar{K} \to \infty$. Again using this fact, we have

$$\lim_{\bar{K}\to\infty} \widetilde{CS}\left(\widetilde{K}^{PF}, \tilde{p}\left(0, \widetilde{K}^{PF}\right)\right) > 0 = \lim_{\bar{K}\to\infty} \left[\widetilde{W}^{PC} - \max_{p\leq v^*} p\left[1 - H(p)^{\bar{K}}\right]\right] = \lim_{\bar{K}\to\infty} \widetilde{CS}\left(\widetilde{K}^{PC}, \tilde{p}\left(\tilde{f}^{PC}, \widetilde{K}^{PC}\right)\right). \quad \blacksquare$$

The following proposition provides a sufficient condition for consumer surplus to be higher with price coherence for uniform H. Like the non-sequential search case, this holds when there is little dispersion in match-values, but the search cost must also be sufficiently low so that the consumer searches enough to benefit from an enlarged search set $\mathcal{C}$: if $s_{\mathcal{C}}$ is too high, then $\widetilde{K}^{PC} > \widetilde{K}^{PF}$ does not materialize as a significant benefit to the consumer, while $\tilde{p}^{PC} > \tilde{p}^{PF}$ still holds. Recall the cutoff $\bar{v}^{\dagger}$ found in Proposition 1.4 such that $V^{PF} < V^{PC}$ if and only if $\bar{v} < \bar{v}^{\dagger}$.

Proposition B4. *Let H be the uniform distribution function over* $[\underline{v}, \bar{v}]$*, where* $\underline{v} > 0$*. For any* $\bar{v}^{\tilde{\dagger}} < \bar{v}^{\dagger}$*, there exists* $s_{\mathcal{C}}^{\tilde{\dagger}} > 0$ *such that* $\tilde{V}^{PF} < \tilde{V}^{PC}$ *for all* $\bar{v} < \bar{v}^{\tilde{\dagger}}$ *and* $s_{\mathcal{C}} < s_{\mathcal{C}}^{\tilde{\dagger}}$*.*

Proof: We can write

$$\tilde{V}^{PC} - \tilde{V}^{PF} = [V^{PC} - V^{PF}] - \left[\int_{v^*}^{\bar{v}} (v - v^*) dH(v)^N - \int_{v^*}^{\bar{v}} (v - v^*) dH(v)\right],$$

where the first term does not depend on $s_{\mathcal{C}}$ and is positive for $\bar{v} < \bar{v}^{\tilde{\dagger}}$ (Proposition 1.4) and the second term is positive and approaches zero as $s_{\mathcal{C}} \to 0$. ■

APPENDIX C: NON-SPONSORED SEARCH WITH FINITELY MANY SELLERS

In this Appendix, I extend the baseline model to allow feasible search among non-sponsored sellers when there are finitely many sellers. Specifically, I suppose $s_{\mathcal{C}} = 0$, $s_{\mathcal{C}'} \geq 0$, and assume there are $N \in \mathbb{N}$ sellers. After freely sampling all sponsored sellers, the consumer conducts random sequential search among non-sponsored sellers (if he so chooses to sample non-sponsored sellers).

Allowing for finitely many sellers introduces two differences compared to the case with infinitely many sellers (as studied in Section 1.7.2). First, with infinitely many sellers, each non-sponsored seller always earns zero profit (because it has negligible demand in equilibrium), while it may earn a positive profit if the sellers are finitely numbered. Consequently, with finitely many sellers, the platform cannot extract the entire sponsored seller industry profit and must consider the effect of its fee and sponsored search design on sellers' willingness to pay for sponsorship (beyond just a non-negative profit constraint). The second difference is mainly technical: with finitely many sellers, the consumer may sample all sellers without finding a product satisfying his reservation cutoff. This introduces "return" demand into each seller's demand function, which results from the consumer purchasing a product after he has once sampled that product and moved on to sample another product.

I focus on the case that valuations are uniformly distributed over $\underline{v} = 0$ and $\bar{v} = 1$. I solve for the pricing equilibrium given an arbitrary choice of f and K by the platform. I then numerically solve for the platform's equilibrium choices given various values of the exogenous parameters $s_{\mathcal{C}'}$, N, and $\overline{K}$. These simulations provide suggestive evidence that the insights from the analysis with infinitely many sellers, as summarized by Proposition 1.6, persist in this environment with finitely many sellers.

C.1 Analysis

Recalling the reservation value $v^*_{\mathcal{C}'}$, the consumer makes his purchasing decision as described in the proof of Proposition 1.6, taking the best option available if he eventually samples all products. That there are finitely many sellers makes no difference, as shown in Wolinsky (1986). Note that with standard uniform H, we have $v^*_{\mathcal{C}'} = 1 - \sqrt{2s_{\mathcal{C}'}}$.

Let f, K, and r be given, and suppose the platform enforces price coherence. If non-sponsored sellers set a price $p_{\mathcal{C}'}$ such that $p_{\mathcal{C}'} > v^*_{\mathcal{C}'}$, then the consumer never searches among non-sponsored sellers, and the baseline analysis applies. Supposing the consumer searches among non-sponsored sellers with positive probability in equilibrium, I will show that there is a unique symmetric subgame equilibrium.

Suppose each sponsored seller sets a price $p_{\mathcal{C}}$ and each non-sponsored seller sets a price $p_{\mathcal{C}'}$, where $p_{\mathcal{C}'} \leq v^*_{\mathcal{C}'}$. To clarify the derivation of sellers' demand functions, I will refer to a seller's "fresh" and "return" demand. Fresh demand is the probability that the consumer purchases a seller's product after first sampling it, and return demand is the probability that the consumer returns to purchase a seller's product that he has already sampled, after sampling all N products.[20]

A sponsored seller $i \in \mathcal{C}$ who deviates to price at p_i earns a profit of

$$(p_i - f)\left\{\underbrace{\Pr\left[v_i - p_i > \max_{j \in \mathcal{C}\setminus\{i\}}\{v^*_{\mathcal{C}'} - p_{\mathcal{C}'}, v_j - p_{\mathcal{C}}\}\right]}_{\text{fresh demand}}\right.$$

$$\left. + \underbrace{\Pr\left[\max_{j \in \mathcal{C}', l \in \mathcal{C}\setminus\{i\}}\{0, v_j - p_{\mathcal{C}'}, v_l - p_{\mathcal{C}}\} < v_i - p_i < v^*_{\mathcal{C}'} - p_{\mathcal{C}'}\right]}_{\text{return demand}}\right\} - r$$

[20] The sellers' demand functions are similar to those studied by Armstrong et al. (2009). Those authors' discussion may be straightforwardly adapted to clarify the equalities below, and I refer the reader to their paper for further details.

$$= (p_i - f)\left\{\int_{v_{\mathcal{C}'}^{*}-p_{\mathcal{C}'}+p_i}^{\bar{v}} (v - p_i + p_{\mathcal{C}})^{K-1} dv + \int_{p_{\mathcal{C}'}}^{v_{\mathcal{C}'}^{*}} v^{N-K}(v - p_{\mathcal{C}'} + p_{\mathcal{C}})^{K-1} dv\right\} - r.$$

A non-sponsored seller $j \in \mathcal{C}'$ who deviates to price at p_j earns a profit of

$$(p_j - f) \cdot$$

$$\left\{\underbrace{\Pr\left[v_j - p_j > v_{\mathcal{C}'}^{*} - p_{\mathcal{C}'}\right] \Pr\left[\max_{i\in\mathcal{C}} v_i - p_{\mathcal{C}} \le v_{\mathcal{C}'}^{*} - p_{\mathcal{C}'}\right] \frac{1}{N-K} \sum_{n=0}^{N-K-1} \Pr\left[\max_{l\in\hat{\mathcal{C}}\setminus\mathcal{C}} v_l < v_{\mathcal{C}'}^{*} \middle| \left|\hat{\mathcal{C}} \setminus \mathcal{C}\right| = n\right]}_{\text{fresh demand}}\right.$$

$$\left. + \underbrace{\Pr\left[\max_{l\in\mathcal{C}'\setminus\{j\}, i\in\mathcal{C}} \{0, v_l - p_{\mathcal{C}'}, v_i - p_{\mathcal{C}}\} < v_j - p_j < v_{\mathcal{C}'}^{*} - p_{\mathcal{C}'}\right]}_{\text{return demand}}\right\}$$

$$= (p_j - f)\left\{\left[1 - \left(v_{\mathcal{C}'}^{*} - p_{\mathcal{C}'} + p_j\right)\right] \frac{\left(v_{\mathcal{C}'}^{*} - p_{\mathcal{C}'} + p_{\mathcal{C}}\right)^K}{N-K} \frac{1 - {v_{\mathcal{C}'}^{*}}^{N-K}}{1 - v_{\mathcal{C}'}^{*}} + \int_{p_{\mathcal{C}'}}^{v_{\mathcal{C}'}^{*}} v^{N-K-1}(v - p_{\mathcal{C}'} + p_{\mathcal{C}})^K dv\right\}.$$

The first order conditions with equilibrium conditions $p_i = p_{\mathcal{C}}$ and $p_j = p_{\mathcal{C}'}$ imposed are given by

$$p_{\mathcal{C}} = f + \rho(p_{\mathcal{C}}; p_{\mathcal{C}'}), \qquad p_{\mathcal{C}'} = f + \eta(p_{\mathcal{C}'}; p_{\mathcal{C}}), \tag{$C1$}$$

where

$$\rho(p_{\mathcal{C}}; p_{\mathcal{C}'}) \coloneqq \frac{1 - \left(v_{\mathcal{C}'}^{*} - p_{\mathcal{C}'} + p_{\mathcal{C}}\right)^K}{K} + \int_{p_{\mathcal{C}'}}^{v_{\mathcal{C}'}^{*}} v^{N-K}(v - p_{\mathcal{C}'} + p_{\mathcal{C}})^{K-1} dv\,,$$

$$\eta(p_{\mathcal{C}'}; p_{\mathcal{C}}) \coloneqq \frac{\dfrac{\left(v_{\mathcal{C}'}^{*} - p_{\mathcal{C}'} + p_{\mathcal{C}}\right)^K}{N-K}\left(1 - {v_{\mathcal{C}'}^{*}}^{N-K}\right) + \int_{p_{\mathcal{C}'}}^{v_{\mathcal{C}'}^{*}} v^{N-K-1}(v - p_{\mathcal{C}'} + p_{\mathcal{C}})^K dv}{\dfrac{\left(v_{\mathcal{C}'}^{*} - p_{\mathcal{C}'} + p_{\mathcal{C}}\right)^K}{N-K} \dfrac{1 - {v_{\mathcal{C}'}^{*}}^{N-K}}{1 - v_{\mathcal{C}'}^{*}}}.$$

$$= \left(1 - v_{\mathcal{C}'}^{*}\right)\left\{1 + \frac{1}{1 - {v_{\mathcal{C}'}^{*}}^{N-K}} \int_{p_{\mathcal{C}'}}^{v_{\mathcal{C}'}^{*}} \left(\frac{v - p_{\mathcal{C}'} + p_{\mathcal{C}}}{v_{\mathcal{C}'}^{*} - p_{\mathcal{C}'} + p_{\mathcal{C}}}\right)^K dH(v)^{N-K}\right\}.$$

Recall that we assumed $p_{\mathcal{C}'} \le v_{\mathcal{C}'}^{*}$ to consider an active non-sponsored search market, and notice in an equilibrium we have $p_{\mathcal{C}'} \ge \eta(p_{\mathcal{C}'}; p_{\mathcal{C}}) \ge 1 - v_{\mathcal{C}'}^{*}$; therefore $v_{\mathcal{C}'}^{*} \ge \frac{1}{2}$ must hold (or equivalently $s_{\mathcal{C}'} \leq \frac{1}{8}$ must hold). Also, $\frac{d}{dp_{\mathcal{C}'}}\eta(p_{\mathcal{C}'};p_{\mathcal{C}}) \leq 0$ holds immediately by inspection of the second expression for $\eta(p_{\mathcal{C}'};p_{\mathcal{C}})$, and using $v_{\mathcal{C}'}^* \leq 1$ we have

$$\frac{d}{dp_{\mathcal{C}}}\rho(p_{\mathcal{C}};p_{\mathcal{C}'}) = -\left(v_{\mathcal{C}'}^* - p_{\mathcal{C}'} + p_{\mathcal{C}}\right)^{K-1} + \int_{p_{\mathcal{C}'}}^{v_{\mathcal{C}'}^*} v^{N-K}(K-1)(v - p_{\mathcal{C}'} + p_{\mathcal{C}})^{K-2} dv$$

$$\leq -\left(v_{\mathcal{C}'}^* - p_{\mathcal{C}'} + p_{\mathcal{C}}\right)^{K-1} + \int_{p_{\mathcal{C}}}^{v_{\mathcal{C}'}^* - p_{\mathcal{C}'} + p_{\mathcal{C}}} dv^{K-1} = -p_{\mathcal{C}}^{K-1} < 0,$$

from which we may also compute $\frac{\partial}{\partial p_{\mathcal{C}'}}\rho(p_{\mathcal{C}};p_{\mathcal{C}'}) = -\frac{d}{dp_{\mathcal{C}}}\rho(p_{\mathcal{C}};p_{\mathcal{C}'}) - p_{\mathcal{C}'}^{N-K}p_{\mathcal{C}}^{K-1} \in (0,1)$.

I now show that $(C1)$ has a unique solution that characterizes the equilibrium. First, given any $p_{\mathcal{C}}$, $p_{\mathcal{C}'} = f + \eta(p_{\mathcal{C}'};p_{\mathcal{C}})$ has a unique solution $p_{\mathcal{C}'}^*(p_{\mathcal{C}}) \in \left[f + 1 - v_{\mathcal{C}'}^*, f + \frac{1}{2}\right]$. If $p_{\mathcal{C}'} = f + 1 - v_{\mathcal{C}'}^*$, then we have $p_{\mathcal{C}'} = f + 1 - v_{\mathcal{C}'}^* < f + \eta(p_{\mathcal{C}'};p_{\mathcal{C}})$. If $p_{\mathcal{C}'} = f + \frac{1}{2}$, then we have $p_{\mathcal{C}'} > f + \eta(p_{\mathcal{C}'};p_{\mathcal{C}})$ if and only if $v_{\mathcal{C}'}^* \geq \frac{1}{2}$, which holds as argued above. The claim thus holds since $\eta(p_{\mathcal{C}'};p_{\mathcal{C}})$ decreases in $p_{\mathcal{C}'}$.

Now suppose $p_{\mathcal{C}'} \in [f, f+1]$. I claim that $p_{\mathcal{C}} = f + \rho(p_{\mathcal{C}};p_{\mathcal{C}'})$ has a unique solution $p_{\mathcal{C}}^*(p_{\mathcal{C}'}) \in [f, f+1]$. If $p_{\mathcal{C}} = f$, then $p_{\mathcal{C}} = f < f + \rho(p_{\mathcal{C}};p_{\mathcal{C}'})$. If $p_{\mathcal{C}} = f + 1$, then

$$f + \rho(p_{\mathcal{C}};p_{\mathcal{C}'}) = f + \frac{1 - \left(v_{\mathcal{C}'}^* + (p_{\mathcal{C}} - p_{\mathcal{C}'})\right)^K}{K} + \int_{p_{\mathcal{C}'}}^{v_{\mathcal{C}'}^*} v^{N-K}\left(v + (p_{\mathcal{C}} - p_{\mathcal{C}'})\right)^{K-1} dv$$

$$\leq f + \frac{1 - \left(v_{\mathcal{C}'}^*\right)^K}{K} + \int_{p_{\mathcal{C}'}}^{v_{\mathcal{C}'}^*} v^{N-K}v^{K-1} dv$$

$$= f + \frac{1 - \left(v_{\mathcal{C}'}^*\right)^K}{K} + \frac{\left(v_{\mathcal{C}'}^*\right)^N - (p_{\mathcal{C}'})^N}{N} < f + \frac{1}{K} \leq f + 1,$$

where the first inequality follows using $\frac{\partial}{\partial p_{\mathcal{C}}}\rho(p_{\mathcal{C}};p_{\mathcal{C}'}) < 0$.

As $p_{\mathcal{C}}^*(p_{\mathcal{C}'})$ and $p_{\mathcal{C}'}^*(p_{\mathcal{C}})$ are continuous, by the Brower fixed point theorem, the system of equations $(C1)$ has at least one solution $(p_{\mathcal{C}}, p_{\mathcal{C}'}) \in [f, f+1]^2$. The solution is unique as follows. Compute

$$\frac{d}{dp_{\mathcal{C}'}} p_{\mathcal{C}}^*(p_{\mathcal{C}'}) = \frac{\dfrac{\partial}{\partial p_{\mathcal{C}'}} \rho(p_{\mathcal{C}}; p_{\mathcal{C}'})}{1 - \dfrac{d}{dp_{\mathcal{C}'}} \rho(p_{\mathcal{C}}; p_{\mathcal{C}'})} \in (0,1).$$

Then upon substitution of $p_{\mathcal{C}} = p_{\mathcal{C}}^*(p_{\mathcal{C}'})$, the system $(C1)$ is characterized by

$$p_{\mathcal{C}'} = (1 - v_{\mathcal{C}'}^*) \left\{ 1 + \frac{1}{1 - {v_{\mathcal{C}'}^*}^{N-K}} \int_{p_{\mathcal{C}'}}^{v_{\mathcal{C}'}^*} \left(\frac{v - \left(p_{\mathcal{C}'} - p_{\mathcal{C}}^*(p_{\mathcal{C}'})\right)}{v_{\mathcal{C}'}^* - \left(p_{\mathcal{C}'} - p_{\mathcal{C}}^*(p_{\mathcal{C}'})\right)} \right)^K dH(v)^{N-K} \right\},$$

which has a unique solution since the RHS strictly decreases in $p_{\mathcal{C}'}$. Therefore, the equilibrium is unique.

From $(C1)$, one can show that non-sponsored sellers price higher than sponsored sellers ($p_{\mathcal{C}}' > p_{\mathcal{C}}$) and both equilibrium prices increase in f, where $p_{\mathcal{C}'} \leq v_{\mathcal{C}'}^*$ if and only if $f \leq \bar{f} \coloneqq 2v_{\mathcal{C}'}^* - 1$. The (subgame) equilibrium profits for each sponsored and non-sponsored seller are respectively given by

$$\pi_{\mathcal{C}} \coloneqq (p_{\mathcal{C}} - f)^2 - r,$$

$$\pi_{\mathcal{C}'} \coloneqq (p_{\mathcal{C}'} - f)^2 \frac{\left(v_{\mathcal{C}'}^* - p_{\mathcal{C}'} + p_{\mathcal{C}}\right)^K}{N-K} \frac{\left(1 - {v_{\mathcal{C}'}^*}^{N-K}\right)}{1 - v_{\mathcal{C}'}^*}.$$

Consider now the platform's problem. Given choices (f, K) that induce an active non-sponsored search market, the optimal referral fee is

$$r(f, K) = (p_{\mathcal{C}} - f)^2 - (p_{\mathcal{C}'} - f)^2 \frac{\left(v_{\mathcal{C}'}^* - p_{\mathcal{C}'} + p_{\mathcal{C}}\right)^K}{N-K} \frac{\left(1 - {v_{\mathcal{C}'}^*}^{N-K}\right)}{1 - v_{\mathcal{C}'}^*},$$

such that $\pi_{\mathcal{C}} = \pi_{\mathcal{C}'}$ in the induced equilibrium. With price coherence, the platform's profit is given by

$$\Pi^{PC} = \max\left\{\max_{f>\bar{f},K\leq\bar{K}} p(f,K)\left[1-H\big(p(f,K)\big)^K\right], \max_{f\leq\bar{f},K\leq\bar{K}} f\left[1-p_{C'}^{N-K}p_C^K\right]+Kr(f,K)\right\},$$

where $p(f,K)$ is the equilibrium price in the baseline analysis without sponsored search.

As in other cases, mandated price flexibility effectively constrains the platform's transaction fee to be zero, so the platform's profit is given by $\Pi^{PF} = \max_{K\leq\bar{K}} Kr(0,K)$.

The platform clearly prefers to enforce price coherence, but determining its optimal choices in either regulatory environment seems intractable. I therefore conclude this section by deriving the total welfare and consumer surplus functions, and in the next section I present numerical results to study the platform's equilibrium choices and consequent welfare effects of price coherence regulation.

Let K, p_C, and $p_{C'}$ be given. If $p_{C'} > v_{C'}$ such that the consumer never samples non-sponsored sellers, then total welfare and consumer surplus are just $W(K,p_C)$ and $CS(K,p_C)$ from the baseline analysis. Otherwise, total welfare is

$$\int_{v_{C'}^*-p_{C'}+p_C}^{\bar{v}} v\,dH(v)^K + H\big(v_{C'}^*-p_{C'}+p_C\big)^K\left[1-H\big(v_{C'}^*\big)^{N-K}\right]v_{C'}^*$$

$$+\left\{\int_{p_{C'}}^{v_{C'}^*} v\,dH(v-p_{C'}+p_C)^K H(v)^{N-K}\right.$$

$$\left.-(p_{C'}-p_C)K\int_{p_{C'}}^{v_{C'}^*} v^{N-K}(v-p_{C'}+p_C)^{K-1}dv\right\}.$$

The first term is the total surplus when the consumer makes a purchase among sponsored sellers, without sampling any non-sponsored sellers. The second term is total surplus when the consumer samples at least one non-sponsored seller (which occurs with probability $H\big(v_{C'}^*-p_{C'}+p_C\big)^K$) and purchases from a non-sponsored seller as a "fresh" consumer. The third term is total surplus that results from a "return" purchase. Note that $\int_{p_{C'}}^{v_{C'}^*} v\,dH(v-p_{C'}+p_C)^K H(v)^{N-K}$ augments the utility from any (return) purchased sponsored product by $p_{\mathcal{C}'} - p_{\mathcal{C}}$, so $p_{\mathcal{C}'} - p_{\mathcal{C}}$ must be subtracted whenever a sponsored seller wins return demand, which occurs with probability $K \int_{p_{\mathcal{C}'}}^{v^*_{\mathcal{C}'}} v^{N-K}(v - p_{\mathcal{C}'} + p_{\mathcal{C}})^{K-1} dv$.

Consumer surplus is given by the total welfare less the platform's and sellers' profits (of $\Pi + K\pi_{\mathcal{C}} + (N-K)\pi_{\mathcal{C}'}$).

C.2 Consequences of Price Coherence Regulation

Figure C.1 plots, for various values of the exogenous parameters $s_{\mathcal{C}'}$ and N, the impact of mandating price flexibility on total welfare as the maximum number of available sponsored slots $\overline{K}$ varies. The figure and its underlying simulations align with the predictions of Proposition 1.6 in this environment with finitely many sellers.

First, in the underlying simulations, prices and the number of sponsored search slots are always higher with enforceable price coherence relative to mandated price flexibility, i.e., $p_{\mathcal{C}}^{PC} \geq p_{\mathcal{C}}^{PF}$, $p_{\mathcal{C}'}^{PC} \geq p_{\mathcal{C}'}^{PF}$, and $K^{PC} \geq K^{PF}$. The main price-choice tradeoff of price coherence regulation therefore persists: mandating price flexibility lowers prices but also lowers prominent product variety.

The consequence of this price-choice tradeoff on total welfare is depicted in Figure C.1. Aligning with Proposition 1.6(i), given $\overline{K}$, mandating price flexibility increases total welfare for sufficiently low levels of the search cost. To see this in the figure, observe that for any $\overline{K}$ such that $W^{PF} - W^{PC} < 0$ (and for any N), the difference in welfare $W^{PF} - W^{PC}$ increases in $s_{\mathcal{C}'}$, and this difference eventually becomes positive for $s_{\mathcal{C}'}$ sufficiently small.[21]

[21] For $N = 30$, Figure C.1 does not consider $s_{\mathcal{C}'}$ small enough for $W^{PF} - W^{PC}$ to be positive for $\overline{K} \in \{27, 28, 29\}$, but this holds for example if $s_{\mathcal{C}'} = \frac{1}{2^m} \cdot \frac{1}{9}$ with $m \in \{5, 6, 7\}$.

Next, aligning with Proposition 1.6(ii), given $s_{C'}$, the sign of $W^{PF} - W^{PC}$ decreases in $\bar{K}$. A difference from Proposition 1.6(ii) is that given N, $W^{PF} - W^{PC}$ need not be negative for sufficiently large $\bar{K}(\leq N)$. The reason for this is analogous to the reasoning that total welfare may be higher with mandated price flexibility for small $\bar{K}$ in the baseline model: if N is small, then $\bar{K}$ is also small such that the greater product variety with price coherence ($K^{PC} \geq K^{PF}$) may not outweigh the difference in prices in the welfare calculation. In Figure C.1, given $s_{C'}$, $W^{PF} - W^{PC}$ is negative for sufficiently large $\bar{K}$ provided that N is not too small. It suggests that Proposition 1.6(ii) applies in this environment for sufficiently large N.

Taken together, these simulations suggest that the intuitions derived from the baseline model (with only sponsored search) persist with viable non-sponsored search with an arbitrary number of sellers.[22]

[22] It can be verified numerically that $CS^{PC} < CS^{PF}$ holds for all underlying simulations of Figure C.1, which also aligns with the predictions of Proposition 6. In view of Proposition 1.4, this is not surprising with standard uniform H as with that valuation distribution it also holds in the baseline model.

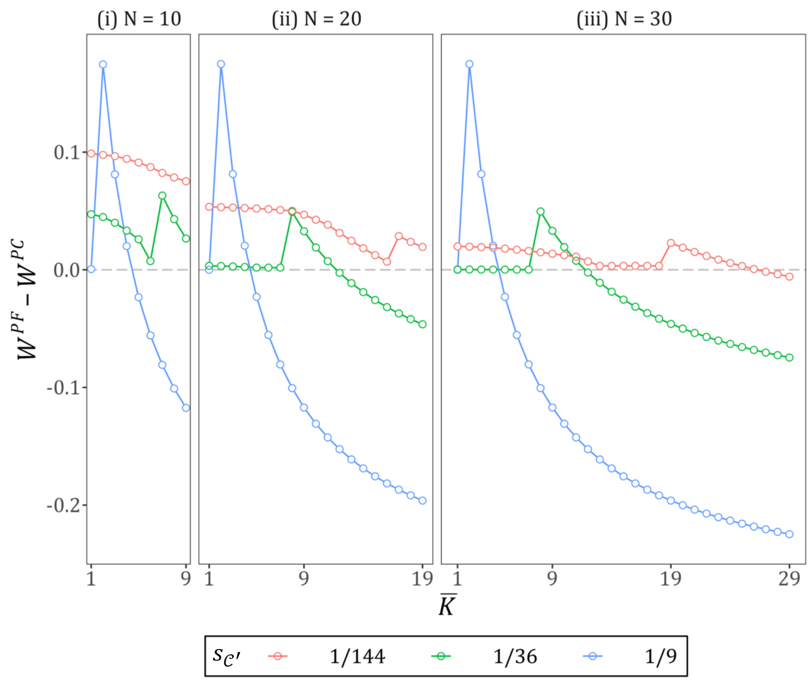


Figure C.1: Welfare consequences of mandating price flexibility with finitely many sellers under the standard uniform distribution of product valuations

APPENDIX D: TRANSACTION BENEFIT PROVISION

Consider the baseline model, and suppose that in any intermediated transaction, each seller i receives a homogenous benefit b_S and the consumer receives a homogenous benefit b_C. Specifically, a seller's payoff (net of any referral fee) is $b_S + p_i - f$ and the consumer's payoff is $b_C + v_i - p_i$. I will distinguish notation in this environment from the baseline model with a check (e.g., the equilibrium transaction fee with price coherence is denoted $\check{f}^{PC}$).

Suppose seller $i \in \mathcal{C}$ lists price p_i while each seller $j \in \mathcal{C} \setminus \{i\}$ lists price p. The consumer purchases i if and only if

$$b_C + v_i - p_i > \max_{j \in \mathcal{C} \setminus \{i\}} \{0, b_C + v_j - p\}.$$

Then seller i's profit is given by

$$(b_S + p_i - f) \Pr\left[b_C + v_i - p_i > \max_{j \in \mathcal{C} \setminus \{i\}} \{0, b_C + v_j - p\}\right] - r$$

$$= (b_S + p_i - f) \int_{p_i - b_C}^{\bar{v}} H(v + p - p_i)^{K-1} dH(v) - r.$$

Imposing $p_i = p$ into the first order condition obtains

$$p = f - b_S + \phi(p - b_C; K).$$

Let $\check{f} := f - b_S - b_C$ be the amount of transaction fee that exceeds any intermediated transaction benefit. We can rewrite the candidate equilibrium price condition as

$$(p - b_C) = \check{f} + \phi\big((p - b_C); K\big).$$

Hence the equilibrium price is given by $\check{p}(f, K) = b_C + p\big(\check{f}, K\big)$, where $p\big(\check{f}, K\big)$ is the equilibrium price in the baseline model (as summarized in Lemma 1.1).

Consider the platform's problem. As before, the platform's optimal referral fee, $\check{r}\big(\check{f}, K\big)$, extracts all surplus from a prominent seller, i.e., if transactions are intermediated,

$$\check{r}(\check{f},K) = \frac{1}{K}[(b_S + \check{p}(f,K) - f)][1 - H(\check{p}(f,K) - b_C)^K]$$

$$= \frac{1}{K}\left[\left(p(\check{f},K) - \check{f}\right)\right]\left[1 - H\left(p(\check{f},K)\right)^K\right].$$

Without price coherence, transactions occur directly if and only if $\check{f} > 0$. Then the platform's profit with and without price coherence are respectively given by

$$\breve{\Pi}^{PC}(\check{f},K) = f[1 - H(\check{p}(f,K) - b_C)^K] + K\check{r}$$

$$= (b_S + b_C)\left[1 - H\left(p(\check{f},K)\right)^K\right] + p(\check{f},K)\left[1 - H\left(p(\check{f},K)\right)^K\right],$$

$$\breve{\Pi}^{PF}(K) = \breve{\Pi}^{PC}(0,K).$$

With price coherence, for any K the platform calibrates the equilibrium price through its choice of $\check{f}$ to maximize $\breve{\Pi}^{PC}(\check{f},K)$, and $\breve{K}^{PF} = \overline{K}$ by the envelope theorem. Without price coherence, the platform chooses $\breve{K}^{PF}$ to maximize $\breve{\Pi}^{PF}(K)$.

We must add one condition to Proposition 1.3 to obtain an analogous welfare result. If $b_S + b_C > \max_{K\in\mathbb{N}} \breve{\Pi}^{PF}(K)$, then for sufficiently high $\overline{K}$, the platform sets $\breve{K}^{PF} = \overline{K}$: it is willing to sacrifice access monetization through sponsored search in order to maximize the number of transactions on which it can monetize transaction benefits. If $b_S + b_C \leq \max_{K\in\mathbb{N}} \breve{\Pi}^{PF}(K)$, then $\breve{K}^{PF}$ is bounded and the welfare results follow as in the baseline model. I conclude with a summary of this result.

Proposition D1. *For $\overline{K}$ sufficiently large, the total welfare and consumer surplus may be ranked as follows.*

(i) *If $b_S + b_C > \max_{K\in\mathbb{N}} \breve{\Pi}^{PF}(K)$, then total welfare and consumer surplus are lower with enforceable price coherence relative to mandated price flexibility, i.e., $\breve{W}^{PC} < \breve{W}^{PF}$ and $\breve{CS}^{PC} < \breve{CS}^{PF}$.*

(ii) *Otherwise, total welfare and consumer surplus may be ranked as in Proposition* 1.3.

Note that a sufficient condition for $b_S + b_C \leq \max_{K \in \mathbb{N}} \breve{\Pi}^{PF}(K)$ is $b_S + b_C \leq \max_{K \in \mathbb{N}} \Pi^{PF}(K)$ ($\geq \Pi^M(1) \geq \underline{v}$). Intuitively, the baseline welfare result (Proposition 1.3) holds if the transaction benefits are not large relative to the value of the product market.

APPENDIX E: SEARCH ON AMAZON

In this Appendix, I further discuss the existence and relevance of sponsored seller prominence and platform design on online search marketplaces. These two features of online search motivate the model and play significant roles in the results. To provide empirical evidence of these features, I use consumer search on Amazon as an illustrative case study.

In its recent complaint against Amazon, the FTC states that 70% of Amazon shoppers do not click past the first search results page when searching for a product (FTC, 2023a). Given the importance of the first page of search results to the typical Amazon shopper's experience, I collected all product listings from the first page of search results for 111 randomly generated search queries on Amazon. Each query comprises a product category from a random object generator and one or more adjectives to make the category more specific. For example, "Chef's Knife" is the query used for the object "Knife." For each listing on the first search results page, I gather all available product information, including the name, brand, seller, price, review information, search ranking, and whether the product is in a sponsored search slot.

E.1 Sponsored Seller Prominence

Regarding sponsored seller prominence on Amazon, the FTC states, "advertisements typically occupy the most desirable space on the search results page," and that "Amazon typically buries organic search results beneath advertisements, making them harder to find and less likely to be clicked." The web-scraped search results align with these claims. Table E.1 provides summary statistics for the first page of search results over all queries. Sponsored products disproportionately appear in the highest-ranking search results: while on average sponsored products make up 37% of all products, they make up 65% of the first quarter of results, which are most immediately visible to consumers. Sponsored sellers (i.e., sellers that have at least one product sponsored) tend to have products in the majority of all search slots and have products in nearly 80% of the first quarter of search slots. These results demonstrate that sponsored sellers enjoy prominent visibility to consumers. In terms of 1-5 star ratings from customers, there is no significant difference in quality between sponsored and unsponsored products or sellers.

Table E.1: Summary of Amazon First Search Results Page from 111 Random Search Queries

Average over Queries	Sponsored Products	Unsponsored Products	Sponsored Seller Products	Unsponsored Seller Products
Mean Star Rating (1-5)	4.49	4.51	4.51	4.50
Share of Slots	0.37	0.63	0.59	0.41
Share of First Quarter of Slots	0.65	0.35	0.77	0.23

Notes—Sponsored products are products that appear in search slots labeled "sponsored," and sponsored seller products are products sold by the seller of any sponsored product. Sponsored (seller) products are prominently placed, disproportionately making up the top-ranking positions. There is no significant difference in quality between sponsored and unsponsored (seller) products.

E.2 Platform Design

A platform may make many choices when designing consumers' search experiences. I next provide several examples of these choices that influence the variety of products observed by consumers and their costs to evaluate a given number of products.

E.2.1 Number of products per page

From the web-scraped data, the first page of search results showed as few as 22 products and as many as 80 products (both including repeated listings of products). Figure E.1 provides evidence that such differences in the number of products per page need not be due to exogenous factors such as the supply of products. In Figure E.1(i), the search for an "Electric Toothbrush" yields a total of 511 products spread over 7 pages of search results. From the same browser and screen size, in Figure E.1(ii), the search for a "Smart TV" yields a total of 540 products spread over 20 pages.

The difference in the number of products per page (73 compared to 27) is a deliberate design choice. Five electric toothbrush products are displayed per row, whereas only one smart tv product is displayed per row—a consumer must take a different number of costly actions to see the same number of products across the two search queries.

E.2.2 Repetition of Products

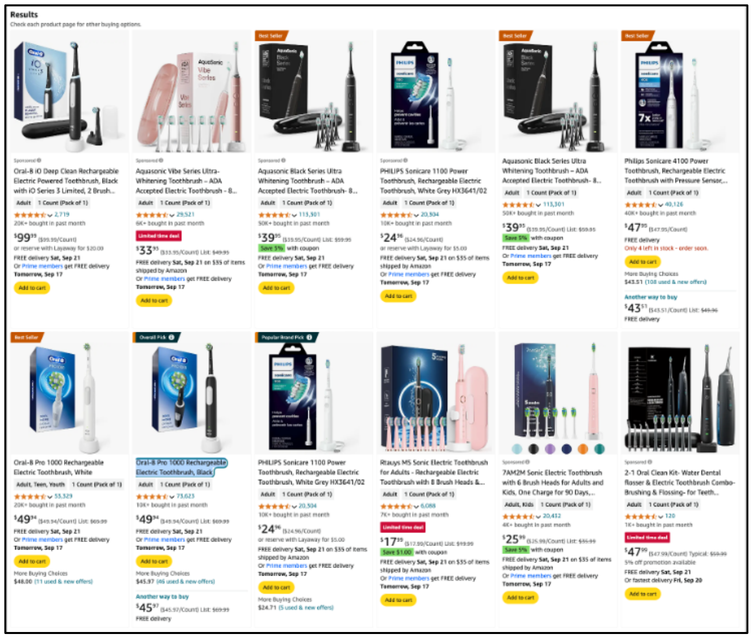
Distinct products in the Amazon search queries appear between one and eight times within the first page of results. Such repetition of the same products within a search page decreases the variety of products observed by consumers and increases the costs incurred to observe a given number of distinct products.

E.2.3 Number of Sponsored Search Slots

A platform may vary the number of sponsored search slots made available to sellers. The number of search slots labeled “sponsored” in the first page of Amazon search results varies between six and 29. The number of search slots with a product that appears anywhere on the page in a sponsored slot varies between seven and 68. Consumers’ ability to easily see prominently placed products varies with the number of sponsored products.

A platform eventually faces some constraints to the number of sponsored search slots that it can make available. These include limited screen space and legibility of product information. For example, by making product slots smaller, a platform can add more prominent slots near the top of a search page, but if a platform makes too many sponsored slots available, the slots eventually lose their prominence, a defining feature of sponsored slots.

(i) “Electric Toothbrush”



(ii) “Smart TV”

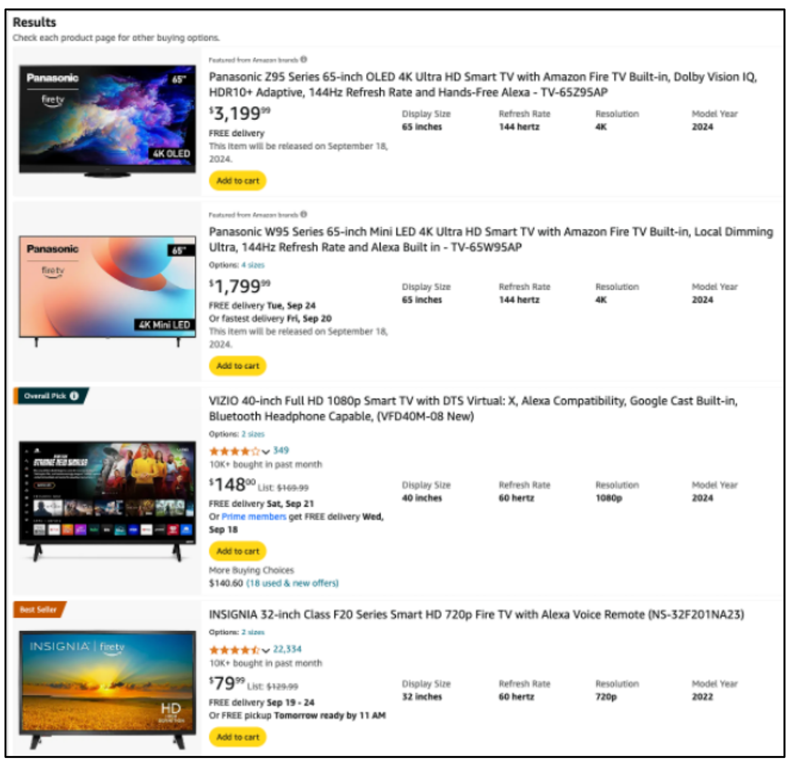


Figure E.1: Two Example Search Queries

Notes—The number of products displayed per page is an example of a platform design lever. The Panel(i) query yields 73 products per page over 7 pages, while the Panel(ii) query yields 27 products per page over 20 pages. Such choices influence consumers’ search experience by altering the number of prominent products and the costs incurred to view a given number of products.

CHAPTER 2

PLATFORM COHERENCE POLICIES WITH A MULTIPRODUCT SELLER

2.1 Introduction

Digital platforms like Amazon and Booking.com contribute value to seller-consumer interactions in many industries. They do so either by providing *benefits* to either party or *access* between parties (or both). For example, online marketplaces provide benefits to consumers in the form of quick and secure transactions; they provide benefits to sellers in the form of lower logistical costs; and they provide sellers with wide market access because consumers begin product search through marketplaces. A platform can easily monetize any benefits that it provides in seller-consumer interactions by charging a fee to intermediate any such transaction. It is more difficult, however, for a platform to monetize access provision. This is because benefits are provided during or after a transaction occurs, while access is provided *before* any transaction occurs. To illustrate, suppose a seller lists a product through a platform. Once a consumer finds the seller through the platform, seller-consumer access has been granted by the platform without any transaction having occurred. The consumer knows about the seller and may then purchase a product either through the platform or through the seller's direct sales channel. Transaction through the direct channel circumvents any transaction fee that the platform may charge in attempt to monetize access provision. In this chapter, I study the strategic interactions between a platform and a multiproduct seller who compete over the surplus generated by a platform's access provision.

Suppose a platform implements a selling fee that charges for seller-consumer access at the time of intermediated transaction. Conditional on any other costs and benefits across sales channels, a seller prefers a direct seller-consumer transaction, while the platform prefers an intermediated transaction. A multiproduct seller has two tactics available to induce direct transactions: cross-channel *strategic pricing* and cross-channel *strategic assortment*. A platform has a natural policy response available for each of these tactics: a cross-channel *price coherence clause* and a cross-channel *availability coherence clause*, respectively. These are the main strategic seller and platform behaviors studied in this chapter. I define these concepts and motivate their importance below.

First, for a product listed both through a platform and directly, a seller may offer a lower direct price compared to its platform price to induce a consumer to purchase directly. This is strategic pricing, and it is the only tactic available to a single-product seller to induce a direct sale. A price coherence clause prohibits this behavior by requiring a seller to make its platform-listed price for any given product the lowest available price for that product, and in particular, the direct price must be at least as high as the platform-listed price. As discussed in Chapter 1, platforms have controversially imposed price coherence clauses in attempt to monetize the market access they provide to sellers. Competition authorities and many authors have investigated platforms' use of these clauses, and antitrust policies have been implemented to restrict this practice in numerous markets. Baker & Morton (2018) provide a review of antitrust approaches taken to regulate price coherence clauses. More recently, platforms do not always explicitly enact price coherence clauses, but they enforce the policies through other platform design mechanisms. For example, Amazon eliminated formal price coherences clauses on its marketplace in Europe in August 2013 and in the US in March 2019, but it continues to punish sellers who break the terms of such policies by steering consumers elsewhere through elimination of the "Buy Box" and demotion in search results. Amazon effectively enforces price coherence due to the importance of these features in generating sales (FTC, 2023a).

While much of the discussion of platform coherence policies has centered around platform and seller pricing behaviors (see Chapter 1 for further discussion of this), most sellers offer multiple products and may thus strategically vary the menu of products that they list across selling channels. This strategic assortment is the second tactic available to a multiproduct seller to induce direct sales. Sellers on Amazon indeed participate in this behavior, as documented in the Harvard Business Review:

> Selling on Amazon does not have to be an all-or-nothing decision. Some brands sell a few products on Amazon while also encouraging customers to buy directly from their own website. This hybrid strategy allows them to use Amazon to build awareness and acquire customers but also drives purchasers to their own website.[23] (Israeli et al., 2022)

Even in face of an effective price coherence requirement, a multiproduct seller may list a subset of its products exclusively through its direct channel to induce a consumer who prefers those products to purchase directly.

Considering sellers' strategic assortment decisions, a platform may impose another type of cross-channel coherence policy, in addition to a price coherence clause. An availability coherence clause requires a seller to offer any product it lists directly also on the platform.

Some platforms explicitly enact availability coherence clauses in practice. In its early years the online ticketing platform Ticketmaster often required that event managers supply Ticketmaster with their full inventory of publicly available tickets in order to sell any tickets through the

[23] Basic online product searches yield specific examples of strategic assortment in vertically differentiated product markets. The following come from February 22, 2023 search results. The "Gap Store" on Amazon.com offers 29 men's jeans options, each listed for no more than $69.99, but the "Gap" direct website offers 176 men's jeans options, with 60 options listed for at least $70. For a March 24-26, 2023 stay, "Quality Inn University" in Lansing, Michigan lists a "King Room" and a "Double Room" through Booking.com, but their direct booking website offers these rooms in addition to a "Poolside King Suite."

platform (Budnick & Baron, 2012; pg. 62). As another example, consumers download mobile applications (apps) from Apple App Store and Google Play Store for a fee, or for free, and they may subsequently purchase various features or services for the downloaded apps (e.g., characters and avatars; content subscription services; cloud software and services). Both of these platforms require that any app downloads and any such unlockable "in-app" features or functionality be paid for through the platforms' billing services. With these requirements, Apple App Store and Google Play Store effectively enact both price and availability coherence clauses for developers. By doing so, they eliminate any viable direct sales channel within apps: if a developer lists an app through a platform, then all monetization of that downloaded app must occur through intermediated transactions.[24] [25] As in the case of price coherence, platforms may enforce availability coherence through platform design mechanisms other than explicit coherence clauses. For example, marketplaces may steer consumers away from sellers who do not implement availability coherence.

In this chapter, I develop a model to study a seller's cross-channel strategic pricing and assortment decisions in relation to a platform's coherence policy choices. I consider a monopoly platform that intermediates a vertically differentiated product market. A monopoly seller offers a low- and a high-quality product to consumers, where the seller obtains a higher margin on sales of the high-quality product gross of any selling fees. It can offer each product through the platform, directly, or through both selling channels. Consumers are unaware of the seller if it has no presence on the platform, but in the absence of any restrictions imposed by the platform, the seller can draw

[24] See 6 June 2023 Apple Store Review Guidelines: Item 3.1.1 and 29 August 2023 Google Play Developer Program Policies: Payments Section 2.

[25] Online travel agencies like Booking.com implement limited forms of availability coherence, requiring hotels to list a minimum share of available rooms in order to list any rooms through the platforms (Hunold et al., 2018).

consumers to purchase directly through cross-channel differences in product prices and availability. The seller provides an inferior purchasing experience to consumers through direct sales relative to platform-intermediated sales. Consumers incur heterogenous costs to directly purchase a product listed both directly and through the platform, and they incur these and additional costs to locate and directly purchase a product not listed through the platform. The platform may restrict the seller's cross-channel pricing and assortment decisions by enforcing price coherence and (or) availability coherence clauses. The platform charges an ad valorem fee to the seller for each intermediated transaction.

I first characterize the seller's pricing and assortment decisions when faced with various fees and platform policies. The seller's assortment choice of which products to sell through each channel depends critically on the relative cost-intensity of production of each product, where the cost-intensity of production is defined as the ratio of the unit cost of a product to consumers' valuation for that product. When two products yield the same gross margin for the seller, an ad valorem fee reduces the seller's realized margin more sharply for the more cost-intensive product. Thus, even though the seller earns a higher gross margin on the high-quality product, a sufficiently high fee may lead the seller to prefer selling the low-quality product through the platform (depending on the relative product margins and cost structures). Under price coherence, the seller loses its ability to make direct sales of any product it lists through the platform. If platform-selling fees are too high, the seller gains consumer awareness by only listing one product through the platform and induces some buyers to purchase the other product directly.

I next consider the platform's choice of coherence policies and selling fee. The platform always prefers to enforce both price and availability coherence. The number of intermediated transactions at any fee level is decreasing in the viability of the seller's direct sales option. If availability coherence is not enforced, the seller retains the ability to draw consumers to purchase directly through either cross-channel price or availability differences. Enforcing both price and availability coherence eliminates both mechanisms for the seller to induce sales outside of the platform. This leaves the platform only with a seller participation constraint on its fee level, and the platform maximizes the revenue it earns per transaction, crucially, on *all* consumer transactions. Without either a price or availability coherence policy in place, the platform must either lower its fee to tax all consumer transactions or tax fewer transactions to maintain its optimal price and availability coherence fee level.

While the platform always prefers price and availability coherence, this strong policy imposition may face regulatory limitations in some industries since it effectively eliminates any off-platform selling option. A platform may also find it infeasible to implement in some cases due to monitoring or commitment failures. In contrast to numerous results that demonstrate general optimality of price coherence in a single-product setting, I show that if availability coherence cannot be implemented, the platform may optimally allow for cross-channel price flexibility.

Some basic intuition for this result is as follows. While the platform is concerned with both the number of intermediated sales and the per-sale revenue it earns from each intermediated sale, the seller is mainly concerned with the transaction-weighted average margin it earns between intermediated and direct sales. Price flexibility only changes the margin the seller earns through intermediated sales, but price coherence restricts the margin the seller earns through both intermediated and direct sales. The platform must compensate the seller for this difference with a lower selling fee in order to tax the high-surplus product in intermediated sales under price coherence. Once the platform induces intermediated sale of the high-surplus product, however, the seller induces less marketplace leakage to direct sales under price coherence because its outside sales option becomes less profitable. When implementing price coherence, the platform thus faces a tradeoff between lower per-intermediated-transaction revenue and a larger number of intermediated transactions. Price coherence may be suboptimal if the seller retains a sufficiently profitable outside sales option from direct-only product listings to induce a significant number of direct sales.

The rest of the chapter proceeds as follows. Section 2.2 reviews the relevant literature. Section 2.3 outlines the model. Section 2.4 analyzes the model, beginning by deriving the seller's optimal assortment and pricing strategies given arbitrary platform coherence policies and transaction fee levels, and proceeding to characterize equilibrium platform decisions under various implementable policy sets. Propositions are proven in the main text, and Appendix A contains proofs of all lemmas. Section 2.5 concludes.

2.2 Related Literature

This chapter relates to a large literature that studies the economic effects of price coherence policies in platform markets. Edelman & Wright (2015) consider a market in which firms can make both intermediated and direct sales to consumers who have heterogenous costs to join a platform yet receive positive convenience benefits from intermediated transactions. Price coherence allows platforms to raise selling fees that are passed through to all consumers whether they join the platform or not. C. Wang & Wright (2020) and C. Wang & Wright (2023) model price coherence when platforms act as search engines for products. While a price coherence clause allows a platform to fully monetize the search cost reduction it provides to consumers, it harms consumers through higher prices and is not necessary to prevent showrooming. Hagiu & Wright (2024) study a variety of strategies platforms may use to prevent marketplace transaction leakage to direct sales. Their baseline model is closely related to mine in that a monopoly platform intermediates sales between a monopoly seller and consumers who incur heterogenous costs to purchase directly. A common result in these models so far discussed is that monopoly platforms always find it optimal to enforce price coherence.[26]

In contrast to general optimality of price coherence found by the above authors, I find that a monopoly platform need not prefer to implement price coherence. Other authors find similar results when platforms compete instead of monopolize intermediated product markets. Boik & Corts (2016) and Carlton & Winter (2018) find that price coherence policies tend to raise platform imposed selling fees and equilibrium product prices, and they may raise prices so high that price coherence hurts platform profits depending on the elasticity of aggregate demand. Johansen & Vergé (2017) and Calzada et al. (2022) come to a similar conclusion but focus on the constraint that a seller's options to delist from a platform in response to a price coherence clause imposes on transaction fees. Aside from a driving role of competing platforms, some authors identify separate reasons why price coherence may lower a monopoly platform's profits. Liu et al. (2021) show that price coherence may be suboptimal when the platform provides a convenience benefit to some consumer transactions but a share of consumers only ever consider purchasing directly. Price coherence increases the number of intermediated transactions but lowers the fee the platform charges for intermediated transactions because all consumers must realize this fee through their purchase price, not only those who transact through the platform. The authors show that the latter effect dominates under certain demand conditions. Mariotto & Verdier (2020) demonstrate how price coherence may be suboptimal for a monopoly platform when the platform provides heterogenous *seller*-side benefits and, like in Liu et al. (2021), the platform is not necessary for seller profitability. Finally, Hagiu & Wright (2024) show that price coherence may be suboptimal

[26] Hagiu & Wright (2024) do show price coherence may be suboptimal in an extension of their baseline model, as discussed further below.

when buyers do not depend on the platform to discover the seller and the platform is uncertain about the nature of consumer preferences for intermediated transactions. I identify a new mechanism for why price coherence may be suboptimal for a monopoly platform, related to the presence of a multiproduct seller rather than a single-product seller on the platform.

None of the above-mentioned papers consider multiproduct sellers, and no papers that I am aware of study the relationship between multiproduct firms and price coherence. Miao (2022) studies seller strategic pricing in a related setting in which multiproduct firms sell a basic good and an ancillary good directly or through a platform. This could apply, for example, to airlines who sell tickets through online travel agencies and collect baggage fees directly on-site. He shows that with ad valorem fees sellers have incentives to shift revenue to the less taxed good. Miao (2022) takes the platform fee levels and the seller assortment decisions as exogenously given, whereas I endogenize and focus on these elements. He also considers platform fees that vary between the basic and ancillary good, whereas I consider a single platform fee that applies to both goods sold. I consider a single fee because I study vertically differentiated goods rather than complementary goods, and platforms tend to set a single fee that applies to all goods in a given product category.

Z. Wang & Wright (2017) study platform fee choice when a platform taxes with a single fee many independent products of various consumer valuations. Each product is sold by a single-product seller. They show that with a fixed per-transaction fee, the price elasticity of demand is too high for low-value goods and too low for high-value goods since the fee takes up a larger percentage of costs for low-value goods. An ad valorem fee alleviates this problem and can implement price discrimination. Z. Wang & Wright (2017) assume that firms may only sell through the platform and do not have their own direct sales channels. My results suggest that the efficiency of an ad valorem fee may be dampened when sellers have a direct sales channel and market power.

With the product structure of Z. Wang & Wright (2017) and a direct sales channel, sellers have more incentive to sell high-value goods directly than they do for low-value goods. An ad valorem fee implements price discrimination when high value goods are sold through the platform, but it introduces per-product participation constraints on the platform's fee that tighten for higher value products.

Several authors have empirically studied the (non-assortment) effects of price coherence clauses following their bans in several European countries. Hunold et al. (2018) use travel metasearch data from the price comparison website Kayak to study effects of price coherence clauses on hotel pricing behaviors and their use of online travel agencies. They show that in Germany more hotels listed any rooms through online travel agencies, hotels more frequently undercut their intermediated prices with direct prices, and hotels listed rooms more frequently through online travel agencies after the ban on price coherence clauses relative to countries without any restriction of price coherence clauses. Mantovani et al. (2021b) use a natural experiment in France to investigate hotel pricing effects of Booking.com's price coherence clause. They show that removal of the clause brought about a decrease in the average intermediated hotel listing price in the short run but had a less significant effect in the medium run. Ennis et al. (2023) use hotel-level transaction data to analyze changes in pricing behavior in European countries in response to the ban of price coherence clauses on Booking.com and Expedia. They find that hotel chains directly undercut the online travel agency listed prices without a price coherence clause in mid-level and luxury hotels but somewhat surprisingly not in budget hotels. In their analysis, Ennis et al. (2023) study the average price of each room booking through each channel in their main outcome variables but do not account for or consider different room types that make up that average. Undercutting direct prices with intermediated prices for a given room does not match economic theory, but considering how price coherence changes the menu of rooms offered through direct and intermediated selling channels as well as the prices of those rooms, as this chapter calls attention to, may clarify their unexpected result.

2.3 Model

A monopoly platform M intermediates trade in a vertically differentiated product market. A monopoly seller S sells a low-quality product L at unit cost c_L and a high-quality product H at unit cost $c_H \geq c_L$. A unit mass of buyers has homogenous valuations v_L and $v_H \geq v_L$ for goods L and H, respectively, where $v_H - c_H > v_L - c_L$.

M provides essential access between consumers and S; consumers are unaware of S unless they discover S through M. However, once a consumer discovers S, the consumer can purchase a product either through M or through S's direct sales channel. More specifically, S makes cross-channel product assortment and pricing decisions. It sells both products directly, and it makes an assortment decision consisting of a set $\mathcal{M} \in 2^{\{L,H\}}$ of *intermediated products* listed for sale through M. S sets an *intermediated price* p_K for each intermediated product $K \in \mathcal{M}$, and it sets a *direct price* $\dot{p}_K$ for each product $K \in \{L, H\}$. If $\mathcal{M}$ is empty, then S has no presence on M, and no consumers discover S. If $\mathcal{M}$ is non-empty, then all consumers discover S through the products $\mathcal{M}$ listed on M. Once consumers discover S, they freely observe S's intermediated and direct products and prices, and they may purchase either product through either available sales channel.

Consumers have heterogenous preferences to purchase a product through M rather than directly. Each consumer incurs a cost s to directly purchase any product listed both through M and listed directly, where s is distributed over the interval $[0, \bar{s}]$ according to a log-concave distribution function G with associated probability density function g that is differentiable and strictly positive in $(0, \bar{s})$. If S has presence on M, then each consumer incurs a search and switching cost $s(1+\psi)$ to directly purchase any product listed only directly.

M monetizes its transaction benefit provision and seller-consumer access provision through a transaction intermediation fee and policies governing S's pricing and assortment behaviors across sales channels. It sets an ad valorem fee $\tau \in [0,1]$ to be paid by S for each intermediated transaction (where S pays M a fee of τp_K each time $K \in \mathcal{M}$ is purchased through M). M also enacts price and availability restriction policies $(\mathcal{P}, \mathcal{A}) \in \{PF, PC\} \times \{AF, AC\}$. A *price flexibility policy* $\mathcal{P} = PF$ places no cross-channel restrictions on product prices, whereas a *price coherence policy* $\mathcal{P} = PC$ requires that any intermediated price is lower than the associated product's direct price (i.e., $p_K \leq \dot{p}_K$ if $K \in \mathcal{M}$). An *availability flexibility policy* $\mathcal{A} = AF$ places no cross-channel restriction on product availability, whereas an *availability coherence policy* $\mathcal{A} = AC$ requires that any product listed directly must also be listed through M (i.e., $\mathcal{M} = \{L, H\}$).

The timing of the game is as follows:

1. M enacts price and availability policies $(\mathcal{P}, \mathcal{A})$ and sets its fee τ.
2. S observes M's choices. It makes its assortment choice $\mathcal{M}$ and sets its intermediated prices $\{p_K\}_{K \in \mathcal{M}}$ and direct prices $\{\dot{p}_K\}_{K \in \{L,H\}}$.
3. Consumers draw their switching costs s. If $\mathcal{M}$ is non-empty, they discover S and observe all intermediated and direct products and prices. They choose whether to purchase, which product to purchase, and through which channel to purchase.
4. All payoffs are realized.

Costs, valuations, and the distribution of the s are common knowledge. The solution concept is wPBE.

2.3.1 Discussion of Assumptions

Homogenous valuations—The main mechanisms and results identified from the analysis of the model are robust to relaxing the assumption of homogenous consumer tastes for product quality. As written, the model allows for two interpretations due to its assumption of homogenous consumer tastes for quality. From a strict interpretation, production of L has mainly strategic assortment motivations because S would only make sales of the high-margin product H without any need for M. An alternative and preferred interpretation is that of a stylized model of more general buyer-platform-seller relationships which allow for heterogenous consumer valuations. From this latter interpretation in mind, $v_H - c_H \geq v_L - c_L$ is a very natural assumption because H would otherwise not be sold without any need for M. I present all results with the latter interpretation in mind. For example, S lists both L and H through the direct or intermediated selling channels in certain cases of equilibrium characterizations, even though it only makes sales of one product through each channel. This is because S would (depending on τ) make sales of both products through any channel that it lists multiple products through with heterogenous consumer tastes for quality. I maintain the assumption of homogenous valuations to keep the analysis tractable. Heterogenous valuations complicate formal analysis by introducing more feasible allocation and pricing strategies by S, but in Appendix C I show that the main mechanisms and results identified from analysis of the stylized model persist in a model that allows for heterogenous quality preferences.

Disutility from direct transactions—I follow Hagiu & Wright (2024) in assuming that consumers have heterogenous costs to purchase directly from S. These costs may be interpreted in many ways. In the model, consumers are unaware of S, and they begin their product search process through M. Thus, the disutility s may be interpreted as costs to switch channels and purchase directly. It may also encompass conveniences that M provides to reduce costs of transactions, such as shipping services that reduce waiting time to receive a product, a sense of security from exposing oneself to financial privacy risks, and efficient checkout processes that reduce time spent inputting shipping and payment data. Consumers realize an extra cost ψs to directly purchase a product that is only listed directly. This may represent costs incurred to locate and evaluate products listed only through the direct channel once a consumer discovers the seller.

Ad valorem fees—I assume that M sets an ad valorem fee rather than a fixed fee to be paid by S in order to match the fee structure typically used in practice by platforms such as Amazon and Booking.com. These platforms set a single ad valorem fee that applies to all products sold from a given product category. Since I study a vertically differentiated product market, I thus consider a single ad valorem fee that applies to both product types L and H. Many authors assume that platforms set a fixed per-transaction fee to be paid by sellers to simplify analysis and argue that their results are robust to this choice of fee structure. The reason that ad valorem and fixed fees yield similar results in such analyses is because they consider (ex ante) homogenous single product sellers. Ad valorem fees differentially affect margins of products with different cost structures, and the results of this chapter are closely tied to the underlying cost-structure of the two goods provided by S because of this.

Non-optional intermediary—M is necessary to facilitate discovery between buyers and sellers such that S cannot operate without at least partial presence on M. The strategic mechanisms identified in this chapter would persist if M was "optional" to S's operation, but this would introduce another set of economic forces already studied by other authors (Johansen & Vergé (2017) and Calzada et al. (2022) in settings with competing platforms and Liu et al. (2021) and Mariotto & Verdier (2020) in settings with monopoly platforms). These authors study economic outcomes of price coherence and show that it may be a suboptimal pricing policy due to fee constraints imposed by sellers' abilities to delist and operate without presence on a platform. I assume that presence on M is necessary for buyer-seller discovery to isolate the role of strategic assortment by a multiproduct seller separately from the role of an optional intermediary.

2.4 Analysis

I proceed by backward induction to solve for a wPBE. Section 2.4.1 solves for S's optimal assortment and pricing strategies for given fee level τ and relevant policy choices $(\mathcal{P}, \mathcal{A})$. Section 2.4.2 characterizes M's optimal fee level and policy choices under various implementable policy sets.

2.4.1 The Seller's Problem

2.4.1.1 Price and Availability Coherence

Let τ be given. I consider first the most stringent platform policy demands. Suppose $(\mathcal{P}, \mathcal{A}) = (PC, AC)$ so that M enforces both price and availability coherence (PAC). PAC effectively eliminates any direct selling option for S because S must gain consumer awareness through M. Availability coherence makes S's assortment decision an all-or-nothing decision, and price coherence ensures that no consumer purchases directly (because they always prefer to purchase through M). Therefore, S lists both products through M (if it is profitable to do so), and it makes only intermediated sales if it makes any sales. It makes intermediated sales only of the product for which it earns a higher intermediated margin after adjustment for the ad valorem fee rate τ.

To make S's pricing decision and sales outcomes precise, I compare outcomes from S making intermediated sales of L, S making intermediated sales of H, and S making no sales at all. If S makes intermediated sales of L, then it does so at a price of $p_L = v_L$ to extract all consumer surplus; it earns a gross margin of $v_L - c_L$ and a net margin of $(1-\tau)v_L - c_L$. S can increase its gross margin by $(v_H - c_H) - (v_L - c_L) > 0$ by instead making intermediated sales of H at a price of $p_H = v_H$, while the transaction fee would increase by $\tau(v_H - v_L)$. Therefore, S prefers to make intermediated sales of H rather than of L if and only if $\tau \leq \tilde{\tau} := \frac{(v_H - c_H)-(v_L - c_L)}{v_H - v_L}$.

For whichever product $K \in \{L, H\}$ that S prefers to sell through M, intermediated sales of K are profitable to S if and only if $(1-\tau)v_K - c_K \geq 0$, or $\tau \leq \bar{\tau}_K := \frac{v_K - c_K}{v_K}$. The threshold $\bar{\tau}_K$ is the maximal fee rate under which S finds intermediated sales of product K profitable. Define $\bar{\tau}$ as the maximal fee rate that S finds intermediated sales of either L or H profitable, i.e., $\bar{\tau} := \max\{\bar{\tau}_L, \bar{\tau}_H\}$.

To summarize, S makes intermediated sales of H if both $\tau \leq \tilde{\tau}$ (so that intermediated sales of H are more profitable than those of L) and $\tau \leq \bar{\tau}_H$ (so that intermediated sales of H are profitable); it makes intermediated sales of L if $\tau > \tilde{\tau}$ (so that intermediated sale of L are more profitable than those of H) and $\tau \leq \bar{\tau}_L$ (so that intermediated sales of L are profitable). Whether S's decision to make intermediated sales of L or H differs over $\tau \in [0, \bar{\tau}]$ (i.e., whether $\tilde{\tau} \leq \bar{\tau}_H$ and $\bar{\tau}_H \leq \bar{\tau}$) depends on the relative *cost-intensity* of each product, defined as follows.

Definition. The cost-intensity of product $K \in \{L, H\}$ is the ratio of its unit cost to consumers' valuation for the product: $\frac{c_K}{v_K}$. A product is more cost-intensive than another if it has a weakly higher cost-intensity.

It is easily shown that H is more cost-intensive than L (i.e., $\frac{c_H}{v_H} \geq \frac{c_L}{v_L}$) if and only if S can profitably sell L for higher values of τ compared to H (i.e., $\bar{\tau} = \bar{\tau}_L \geq \bar{\tau}_H$) and S prefers to make intermediated sales of L compared to H for relevant fee levels (i.e., $\tilde{\tau} \leq \bar{\tau}$).

Proposition 2.1 summarizes S's assortment and pricing decisions and consumers' purchasing decisions under PAC.

Proposition 2.1 (Assortment, Pricing, and Sales Under PAC). *Suppose* $(\mathcal{P}, \mathcal{A}) = (PC, AC)$ *so that* M *enforces both price and availability coherence. Then* S *makes no sales if* $\tau > \bar{\tau}$*. If* $\tau \leq \bar{\tau}$*, then* S *lists both products through* M *with* $\mathcal{M} = \{L, H\}$*, and all consumers purchase through* M*. Moreover:*

(i) *Suppose* H *is more cost-intensive than* L*. If* $\tau \in [0, \tilde{\tau}]$*, then all consumers purchase* H *at* $p_H = v_H$*. If* $\tau \in (\tilde{\tau}, \bar{\tau}]$*, then all consumers purchase* L *at* $p_L = v_L$*.*

(ii) *Suppose* L *is more cost-intensive than* H*. If* $\tau \in [0, \bar{\tau}]$*, then all consumers purchase* H *at* $p_H = v_H$*.*

2.4.1.2 Price or Availability Flexibility

Before proceeding to characterize S's optimal assortment and pricing decisions given a specific platform policy other than PAC, it is useful to consider S's problem given an arbitrary platform policy $(\mathcal{P}, \mathcal{A}) \neq (AC, PC)$. Under any such platform policy, either $\mathcal{P} = PF$ or $\mathcal{A} = AC$, and S can induce direct sales in one of two ways. First, if $\mathcal{P} = PF$, then, for an intermediated product $I \in \mathcal{M}$, S can set $\dot{p}_I < p_I$ so that a consumer with sufficiently low switching costs prefers to purchase I directly. I refer to this first disintermediation tactic as strategic pricing. Second, if $\mathcal{A} = AC$, then S can list a product $D \notin \mathcal{M}$ only directly so that a consumer who would like to purchase D must do so directly. I refer to this second disintermediation tactic as strategic assortment.

The following result characterizes S's optimal assortment choice when it has a feasible direct sales option through either strategic pricing or strategic assortment.

Lemma 2.1 (Assortment with a Direct Sales Option). *Suppose* $\mathcal{P} = PF$ *or* $\mathcal{A} = AF$ *so that* M *allows for flexible cross-channel prices or availability. Then:*

(i) If $\tau \neq \tilde{\tau}$, then S optimally chooses one intermediated product $I \in \mathcal{M}$ and one direct product $D \in \{L, H\}$ such that it makes all intermediated sales (if any) of I and all direct sales (if any) of D.

(ii) If $\tau = \tilde{\tau}$, then S may make intermediated sales of both L and H, but it optimally chooses one direct product $D \in \{L, H\}$ such that it makes all direct sales (if any) of D. If S optimally makes any intermediated sales, then it can achieve its optimal profit by choosing one intermediated product $I \in \mathcal{M}$ such that it makes all intermediated sales of I.

Lemma 2.1 states that S can always achieve its optimal profit by only making sales of at most one product through each sales channel.[27] Note that I and D may be the same or different products. If $I = D$, then S engages in strategic pricing to induce any direct sales. If $I \neq D$, then S engages in strategic assortment to induce any direct sales.

Given this result, consider S's pricing decision when it makes sales of at most one product through each sales channel. Specifically, suppose S makes any intermediated sales of an arbitrary product $I \in \mathcal{M}$ at an intermediated price of p_I, and suppose it makes any direct sales of an arbitrary product $D \in \{L, H\}$ at a direct price of $\dot{p}_D$.

A consumer can purchase I through M to obtain a surplus of $v_I - p_I$, or he can purchase D directly from S to obtain a surplus of $v_D - \dot{p}_D - s\left(1 + 1_{[D \in \mathcal{M}]}\psi\right)$, where $1_{[D \in \mathcal{M}]} = 1$ if $D \in \mathcal{M}$ and $1_{[D \in \mathcal{M}]} = 0$ otherwise. Therefore, if a consumer purchases a product at all, he purchases D if and only if $s < \frac{v_D - \dot{p}_D - (v_I - p_I)}{1 + 1_{[D \in \mathcal{M}]}\psi}$. Provided that a consumer considers purchasing through either sales channel (i.e., $p_I \leq v_I$ and $\dot{p}_D \leq v_D$), S's profit is thus given by

[27] For ease of exposition, in any case that $\tau = \tilde{\tau}$ and S may achieve its optimal profit by making intermediated sales of H, L, or both H and L, I assume that S only makes intermediated sales of H.

$$[(1-\tau)p_I - c_I]\left[1 - G\left(\frac{v_D - \dot{p}_D - (v_I - p_I)}{1 + 1_{[D\in\mathcal{M}]}\psi}\right)\right] + (\dot{p}_D - c_D)G\left(\frac{v_D - \dot{p}_D - (v_I - p_I)}{1 + 1_{[D\in\mathcal{M}]}\psi}\right). \quad (2.1)$$

First notice that S optimally sets $p_I = v_I$ if intermediated sales of I at $p_I = v_I$ are profitable (i.e., $\tau \leq \bar{\tau}_I$). This is because giving consumers a positive surplus in intermediated sales makes it more costly for S to induce direct sales and lowers the margin earned in intermediated sales. Specifically, if $\tau \leq \bar{\tau}_I$ and $p_I < v_I$, then, as is clear from Equation (2.1), S could increase both p_I and $\dot{p}_D$ to increase its intermediated and direct profit margins while keeping the intermediated and direct product demand unchanged. If instead $\tau > \bar{\tau}_I$, then S should not make any intermediated sales of I because it is not profitable to do so.

S's optimal profit is thus given by

$$\max_{\dot{p}_D} 1_{[\tau\leq\bar{\tau}_I]}[(1-\tau)v_I - c_I]\left[1 - G\left(\frac{v_D - \dot{p}_D}{1 + 1_{[D\in\mathcal{M}]}\psi}\right)\right] + (\dot{p}_D - c_D)G\left(\frac{v_D - \dot{p}_D}{1 + 1_{[D\in\mathcal{M}]}\psi}\right),$$

where $1_{[\tau\leq\bar{\tau}_I]} = 1$ if $\tau \leq \bar{\tau}_I$ and $1_{[\tau\leq\bar{\tau}_I]} = 0$ otherwise. The first order condition of S's profit maximization problem is

$$\dot{p}_D = c_D + 1_{[\tau\leq\bar{\tau}_I]}[(1-\tau)v_I - c_I] + \left(1 + 1_{[D\in\mathcal{M}]}\psi\right)\frac{G\left(\frac{v_D - \dot{p}_D}{1 + 1_{[D\in\mathcal{M}]}\psi}\right)}{g\left(\frac{v_D - \dot{p}_D}{1 + 1_{[D\in\mathcal{M}]}\psi}\right)}. \quad (2.2)$$

Equation (2.2) sets the direct price of D equal to S's opportunity cost of a direct sale of D plus a markup. The opportunity cost of a direct sale of D includes the unit cost of a direct sale and the forgone profit from an intermediated sale of I. The markup is the ratio of direct demand to the loss in direct demand from a marginal increase in the direct price.

Denote a solution to Equation (2.2) by $\dot{p}_D^{\{I\}}(\tau)$. (The set notation in the superscript $\{I\}$ is meant to associate the intermediated product I with the intermediated assortment set $\mathcal{M} \supseteq \{I\}$.) The following lemma summarizes S's optimal prices and sales outcomes.

Lemma 2.2 (Pricing and Sales with a Direct Sales Option). *Suppose $\mathcal{P} = PF$ or $\mathcal{A} = AF$ so that M allows for flexible cross-channel prices or availability. Suppose S optimally makes any intermediated sales of product $I \in \mathcal{M}$ and any direct sales of product $D \in \{L, H\}$. Then:*

(i) S makes no intermediated sales if $\tau > \bar{\tau}_I$; otherwise, it makes any intermediated sales of I at a price of $p_I = v_I$.

(ii) S makes no direct sales if $\tau < \frac{v_I - c_I - (v_D - c_D)}{v_I}$; otherwise, it makes any direct sales of D at a price of $\dot{p}_D(\tau) = \max\left\{\dot{p}_D^{\{I\}}(\tau), v_D - \left(1 + 1_{[D \in \mathcal{M}]}\psi\right)\bar{s}\right\}$, where $\dot{p}_D^{\{I\}}(\tau)$ uniquely solves Equation (2.2).

(iii) For all $\tau \in [0, \bar{\tau}]$, the mass of direct sales induced by S increases in the difference between S's maximal direct and intermediated profit margins; that is, $G\left(\frac{v_D - \dot{p}_D}{1 + 1_{[D \in \mathcal{M}]}\psi}\right)$ increases in $v_D - c_D - [(1 - \tau)v_I - c_I]$.

Lemma 2.2(i) and Lemma 2.2(ii) characterize S's pricing decision with a viable direct sales option. Lemma 2.2(iii) states that S induces more direct sales when it can earn a higher margin from its direct product compared to its intermediated product. It implies that, given an assortment choice, as τ increases, S prices more aggressively in its direct sales channel to draw more consumers away from the platform after discovery.

In the remainder of this section, I apply Lemma 2.1 and Lemma 2.2 in S's assortment and pricing problems given relevant specific platform policies $(\mathcal{P}, \mathcal{A})$ under which S can viably induce direct sales.

2.4.1.3 Price Flexibility

Suppose $\mathcal{P} = PF$ so that M does not regulate cross-channel prices. Whether $\mathcal{A} \in \{AF, AC\}$ is inconsequential. To see this, suppose $\mathcal{A} = AF$, so S can choose to list a product D only directly.

By doing so, S induces a direct sale for any sale of D at a direct price of $\dot{p}_D$. If instead $\mathcal{A} = AC$, then S must list D through M, but it can achieve the previous outcome by setting a high intermediated price $p_D > v_D$ so that no consumer purchases D through M. Due to PF, S can maintain its direct price $\dot{p}_D$ to continue to induce any direct sales of D, so (PF, AF) is payoff-equivalent to (PF, AC).

Proposition 2.2 fully characterize S's assortment and pricing decisions under PF.

Proposition 2.2 (Assortment, Pricing, and Sales under PF). *Suppose $\mathcal{P} = PF$ so that M does not regulate cross-channel prices. Then $H \in \mathcal{M}$. S makes no intermediated sales if $\tau > \bar{\tau}$. If $\tau \leq \bar{\tau}$, then S optimally chooses an intermediated product $I \in \mathcal{M}$ such that it makes all intermediated sales (if any) of I at a price of $p_I = v_I$, and it optimally makes direct sales of product H at a price of $\dot{p}_H(\tau) = \max\left\{\dot{p}_H^{\{I\}}(\tau), v_H - \bar{s}\right\}$. Furthermore:*

(i) Suppose H is more cost-intensive than L. If $\tau \in [0, \tilde{\tau}]$, then $I = H$. If $\tau \in (\tilde{\tau}, \bar{\tau}]$, then $I = L$.

(ii) Suppose L is more cost-intensive than H. If $\tau \in [0, \bar{\tau}]$, then $I = H$.

Note that S's choice of intermediated product follows a cutoff rule, under which it makes any intermediated sales of the product that provides the highest net margin in intermediated sales. That is, under price flexibility, S chooses the intermediated product $I = H$ if and only if $\tau \leq \min\{\tilde{\tau}, \bar{\tau}\}$. It will be useful to contrast this result with S's choice of intermediated product under price coherence.

2.4.1.4 Price Coherence

I conclude the section with a characterization of S's assortment and pricing decisions under $(\mathcal{P}, \mathcal{A}) = (PC, AF)$. Note that, under price flexibility, S's assortment choice $\mathcal{M}$ could not be completely pinned down in Proposition 2.2 (because, as long as $H, I \in \mathcal{M}$, whether $D \in \mathcal{M}$ has no effect on outcomes). In contrast, under price coherence, S's assortment choice is unambiguous.

Again, by Lemma 2.1, S optimally chooses one product $I \in \mathcal{M}$ for any intermediated sales and one product $D \in \{L, H\}$ for any direct sales. With price coherence, it must be that I is a different product than D (otherwise S cannot induce any direct sales of D). This implies that S's assortment choice is either $\mathcal{M} = \{H\}$ or $\mathcal{M} = \{L\}$.

Proposition 2.3 fully characterize S's assortment and pricing decisions under price coherence.

Proposition 2.3 (Assortment, Pricing, and Sales under PC and AF). *Suppose* $(\mathcal{P}, \mathcal{A}) = (PC, AF)$ *so that* M *enforces price coherence but does not regulate cross-channel product availability. Then* S *makes no intermediated sales if* $\tau > \bar{\tau}$. *If* $\tau \leq \bar{\tau}$, *then there exists a cutoff* $\tau^* < \min\{\tilde{\tau}, \bar{\tau}\}$ *such that:*

(i) *If* $\tau \in [0, \tau^*]$, *then* S *optimally sets* $\mathcal{M} = \{H\}$ *and makes intermediated sales of* H *at a price of* $p_H = v_H$. *It makes no direct sales if* $\tau < \frac{v_H - c_H - (v_L - c_L)}{v_H}$; *otherwise, it makes direct sales of* L *at a price of* $\dot{p}_L(\tau) = \max\left\{\dot{p}_L^{\{H\}}(\tau), v_L - (1+\psi)\bar{s}\right\}$.

(ii) *If* $\tau \in (\tau^*, \bar{\tau}]$, *then* S *optimally chooses* $\mathcal{M} = \{L\}$ *and makes any intermediated sales of* L *at* $p_L = v_L$. *It makes direct sales of* H *at a price of* $\dot{p}_H(\tau) = \max\left\{\dot{p}_H^{\{L\}}(\tau), v_H - (1+\psi)\bar{s}\right\}$.

Under price coherence, S's choice of intermediated product follows a cutoff rule. It chooses $\mathcal{M} = \{H\}$ if and only if $\tau \leq \tau^*$. This is similar to S's choice of an intermediated product under price flexibility. In both cases, S makes intermediated sales of the high margin product H if and only if τ is sufficiently low, below some cutoff value.

However, S employs a lower cutoff value (of τ^*) under price coherence compared to the cutoff (of $\min\{\tilde{\tau}, \bar{\tau}\}$) that it employs under price flexibility. Under price flexibility, S makes intermediated sales of whichever product yields a higher intermediated margin. In contrast, under price coherence, S makes intermediated sales of L rather than of H even in cases that it earns a lower intermediated margin by doing so (i.e., whenever $\tau \in [\tau^*, \tilde{\tau}]$). The reason for this is that, under price coherence, if S lists H through M, then it cannot induce any direct sales of H through strategic pricing; it must sell L in any direct transactions. This is costly for S to replace H with L in direct transactions because L yields a lower direct margin than H. Therefore, under price flexibility, S maintains L as its intermediated product for higher fee levels.

2.4.2 Platform Policy and Fee Choice

From Section 2.4.1, we have seen that S only ever makes sales of a single product type through either sales channel. After observing M's fee and policy choice, S chooses an intermediated product $I \in \mathcal{M} \subseteq \{L, H\}$ such that I is purchased in any intermediated sales at a price of $p_I = v_I$ provided that such sales are profitable ($\tau \leq \bar{\tau}_I$); it chooses a direct product $D \in \{L, H\}$ such that D is purchased in any direct sales at a price of $\dot{p}_D$. The intermediated product, the direct product, and the direct price each depends on the fee level τ and platform policies $(\mathcal{P}, \mathcal{A})$.

Now consider M's problem. For given choices of τ and $(\mathcal{P}, \mathcal{A})$, M earns a profit of $\tau v_I \left[1 - G\left(\frac{v_D - \dot{p}_D(\tau)}{1 + 1_{[D \in \mathcal{M}]}\psi}\right)\right]$ (provided that $\tau \leq \bar{\tau}_I$). Importantly, M's profit depends on two components: its per-intermediated-transaction revenue τv_I, and the volume of intermediated transactions $1 - G\left(\frac{v_D - \dot{p}_D(\tau)}{1 + 1_{[D \in \mathcal{M}]}\psi}\right)$. It will be useful to discuss the latter component in terms of transaction *leakage*, where leakage is the number of direct sales $G\left(\frac{v_D - \dot{p}_D(\tau)}{1 + 1_{[D \in \mathcal{M}]}\psi}\right)$ on which M does not earn a transaction fee. All else equal, M prefers higher per-intermediated-transaction revenue and a higher volume of intermediated transactions, or equivalently higher per-intermediated-transaction revenue and lower transaction leakage.

From this formulation, it is easy that the platform optimally chooses $(\mathcal{P}, \mathcal{A}) = (PC, AC)$. Recall that PAC eliminates any direct sales option for S, so there is no transaction leakage for any fee level τ. All consumers purchase through M provided that the participation constraint $\tau \leq \bar{\tau}_I$ is satisfied. Therefore, M chooses the fee level $\tau = \tau^{PAC}$ that maximizes its per-intermediated-transaction revenue τv_I subject to $\tau \leq \bar{\tau}_I$. Proposition 2.4 provides the optimal fee level, accounting for S's specific choice of the intermediated product I as a function of τ, as characterized by Proposition 2.1.

Proposition 2.4 (Optimality of PAC). *M enforces $(\mathcal{P}, \mathcal{A}) = (PC, AC)$. Moreover:*

(i) If H is more cost-intensive than L, then M's fee is given by

$$\tau^{PAC} = \begin{cases} \tilde{\tau}, & \tilde{\tau} v_H \geq \bar{\tau} v_L, \\ \bar{\tau}, & \tilde{\tau} v_H < \bar{\tau} v_L. \end{cases}$$

(ii) If L is more cost-intensive than H, then M's fee is given by $\tau^{PAC} = \bar{\tau}$.

2.4.2.1 Mandated Availability Flexibility

With some exceptions, availability coherence may not be prevalently observed in practice due to enforcement or regulatory limitations. In this section, I study M's optimal policy and fee rate when it must allow for availability flexibility by setting $\mathcal{A} = AF$.

In choosing a price restriction policy $\mathcal{P} \in \{PF, PC\}$, M faces a tradeoff between its per-intermediated-transaction revenue and transaction leakage. To highlight this tradeoff, I first consider how an exogenously given fee rate performs across price restriction policies. This exercise also aids in studying M's optimal policy choice with an endogenous fee. The following result follows from application of Proposition 2.2, Proposition 2.3, and Lemma 2.2(iii).

Lemma 2.3 (Platform Profit and Leakage Across PF and PC). *Suppose $\mathcal{A} = AF$ so that M does not regulate cross-channel product availability and consider an exogenously given fee rate τ.*

(i) If $\tau \in [0, \tau^*]$, then under $\mathcal{P} = PF$, relative to $\mathcal{P} = PC$, M's per-intermediated-transaction revenue is the same, and transaction leakage is higher. M prefers $\mathcal{P} = PC$.

(ii) If $\tau \in (\tau^*, \min\{\tilde{\tau}, \bar{\tau}\}]$, then there exists a cutoff value $\psi^*(\tau)$, increasing in τ, such that under $\mathcal{P} = PF$, relative to $\mathcal{P} = PC$, transaction leakage is higher if and only if $\psi > \psi^*(\tau)$. M's per-intermediated transaction revenue is higher under $\mathcal{P} = PF$, so it prefers $\mathcal{P} = PF$ if and only if ψ is sufficiently small.

(iii) If $\tau \in (\min\{\tilde{\tau}, \bar{\tau}\}, \bar{\tau}]$, then under $\mathcal{P} = PF$, relative to $\mathcal{P} = PC$, M's per-intermediated-transaction revenue is the same, and transaction leakage higher. M prefers $\mathcal{P} = PC$.

Now consider M's choice of a price restriction policy $\mathcal{P} \in \{PF, PC\}$ when it may vary its transaction fee across the two policies.

If $\mathcal{P} = PF$, then using Proposition 2.2, M earns a profit of

$$\max_{\tau \in [0,\bar{\tau}]} 1_{[0 \le \tau \le \min\{\tilde{\tau},\bar{\tau}\}]} \tau v_H \left[1 - G\left(v_H - \dot{p}_H^{\{H\}}(\tau)\right)\right] + 1_{[\min\{\tilde{\tau},\bar{\tau}\} < \tau \le \bar{\tau}]} \tau v_L \left[1 - G\left(v_H - \dot{p}_H^{\{L\}}(\tau)\right)\right],$$

where the first term is the profit from inducing S's intermediated sale of H and the second term is the profit from inducing S's intermediated sale of L.

If $\mathcal{P} = PC$, then using Proposition 2.3, M earns a profit of

$$\max_{\tau \in [0,\bar{\tau}]} 1_{[0 \le \tau \le \tau^*]} \tau v_H \left[1 - G\left(v_L - \dot{p}_L^{\{H\}}(\tau)\right)\right] + 1_{[\tau^* < \tau \le \bar{\tau}]} \tau v_L \left[1 - G\left(v_H - \dot{p}_H^{\{L\}}(\tau)\right)\right],$$

where the first term is the profit from inducing S's intermediated sale of H and the second term is the profit from inducing S's intermediated sale of L.

Let τ^{PF} and τ^{PC} denote M's optimal fee choices under $\mathcal{P} = PF$ and $\mathcal{P} = PC$, respectively. Unlike many results in the literature on general optimality of price coherence, M may optimally allow for price flexibility in certain cases.

Proposition 2.5 (Optimal Pricing Restriction Policy under AF). *Suppose $\mathcal{A} = AF$ is mandated so that M cannot regulate cross-channel product availability.*

(i) If $\tau^{PF} \le \tau^*$, then M optimally implements $\mathcal{P} = PF$.

(ii) If $\tau^{PF} > \tau^*$, then $\mathcal{P} = PC$ need not be M's optimal price restriction policy.

To see Proposition 2.5(i), suppose $\tau^{PF} \leq \tau^*$. Then S sells H through M under price flexibility and continues to do so under price coherence at $\tau = \tau^{PF}$. By Lemma 2.3(i), S induces less leakage under price coherence, so switching from price flexibility to price coherence at $\tau = \tau^{PF}$ maintains M's per-transaction revenue and increases the number of intermediated transactions. Since M's profit can be improved by price coherence at $\tau = \tau^{PF}$, the fee that maximizes M's profit under price flexibility, M's optimal profit under price coherence must be greater than that under price flexibility.

I verify Proposition 2.5(ii) with the numerical example in the following section. Some basic intuition for why the optimal pricing restriction policy may be ambiguous is as follows. While M is concerned with both the number of intermediated sales and the revenue it earns from each intermediated sale, S is mainly concerned with the transaction-weighted average margin it earns between intermediated and direct sales because it always makes sales to every consumer. Price flexibility only changes the margin S earns through intermediated sales, but price coherence restricts the margin S earns through both intermediated and direct sales when S makes intermediated sales of H. M must compensate S for this difference in order to tax H in intermediated sales. This compensation may be suboptimal if S retains a sufficiently profitable outside sales option and induces a significant number of direct sales.

2.4.2.2 Numerical Example

To illustrate M's tradeoff between price flexibility and price coherence and to verify Proposition 3.5(ii), I numerically solve the model with uniform switching costs. I keep specific choices of v_H, v_L, c_H, $\bar{s}$, and ψ constant, and I vary c_L over the interval $[v_L - (v_H - c_H), c_H]$, which includes all permissible values for c_L such that $c_L \leq c_H$. As c_L varies over this interval, the difference in cost intensities $\frac{c_L}{v_L} - \frac{c_H}{v_H}$ varies symmetrically about zero. Figure 2.1 shows the results from this exercise. Appendix B demonstrates robustness of these results to the fixed parameter choices.

A main observation from Figure 2.1 is that M prefers price coherence for low and high levels of c_L, but it prefers price flexibility for intermediate levels of c_L. This is driven by a tradeoff M faces between per-intermediated-transaction revenue and number of intermediated transactions. Notice that M sets $\tau^{PC} \leq \tau^{PF}$ whenever it induces sale of the same product under both pricing restriction policies. In such situations, it earns lower per-intermediated-transaction revenue under price coherence; however, it taxes more intermediated sales under price coherence because S is taxed by a weakly lower fee, it must account for extra costs ψs experienced by consumers to induce direct sales, and it may sell the relatively lower margin product L in direct sales. Below I explain in detail these and all other aspects of Figure 2.1.

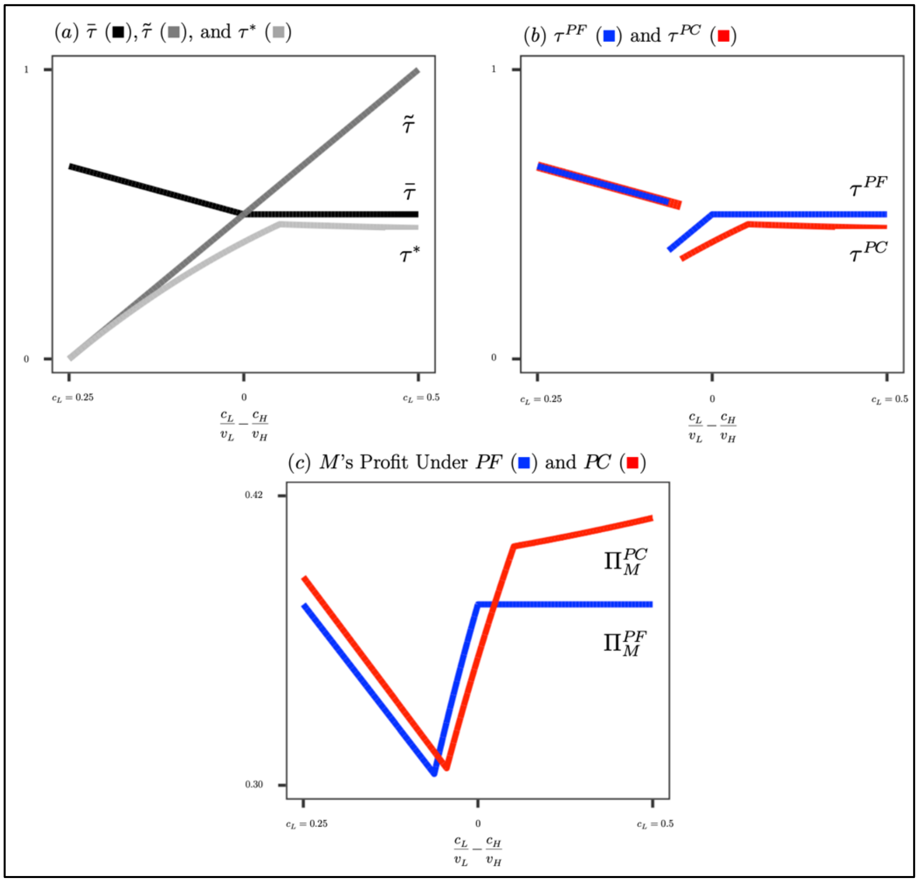


Figure 2.1: Numerical Example

Notes—The model is specified by $s \sim U[0, \bar{s} = 1]$, $v_H = 1$, $v_L = 0.75$, $c_H = 0.5$, $\psi = 0.1$, and $c_L \in [0.25, 0.5]$.

In the example illustrated by Figure 2.1, $\bar{s}$ is relatively high such that consumers with the maximal switching cost $s = \bar{s}$ would not purchase either product directly at any positive price. A high level of $\bar{s}$ is not necessary for price flexibility to dominate, but it simplifies exposition in the following way.[28] S never optimally covers the market only through direct sales, even if it earns zero margin in intermediated sales. Further, as shown in Panels (a) and (b) of Figure 2.1, M never finds it optimal to lower its fee below what is needed to make S indifferent about selling M's targeted intermediated product through M. Under price flexibility, M either chooses a high fee

[28] For instance, price flexibility also dominates price coherence for some c_L in the example with each $\bar{s} \in \{0.25, 0.50, 0.75\}$ rather than $\bar{s} = 1$, as shown in Appendix B.

level $\bar{\tau}$ to extract all surplus in intermediated sales or the lower fee level $\tilde{\tau}$ to just induce S to make intermediated sales of H. Under price coherence, M either chooses a high feel level $\bar{\tau}$ to extract all surplus from intermediated sales of L or the lower fee level τ^* to just induce S to make intermediated sales of H.

Recall that M must set $\tau \leq \tilde{\tau}$ to induce S to sell H through M under PF, and M must set $\tau \leq \tau^*$ to induce S to sell H through M under PC. When $\frac{c_H}{v_H} > \frac{c_L}{v_L}$, ad valorem fees are relatively more costly to S's sale of H than they are to its sale of L. In addition to this, for low levels of c_L, S's margin is similar for direct sales of L and H in this example. Thus for low levels of c_L, M must set τ very low to induce S to sell H through M under either pricing policy ($\tilde{\tau}$ and τ^* are low for low c_L). Instead, M optimally sets $\tau^{PF} = \tau^{PC} = \bar{\tau} = \frac{v_L - c_L}{v_L}$ for low levels of c_L to extract all surplus from intermediated sales of L rather than a small share of surplus from intermediated sales of H. M prefers PC over PF for small c_L because PC increases the cost for M to induce leakage through direct sales of H due to additional search costs ψs.

As $\frac{c_L}{v_L}$ increases to $\frac{c_H}{v_H}$, $\bar{\tau} = \frac{v_L - c_L}{v_L}$ decreases so that M earns less per-transaction revenue by inducing intermediated sale of L, meanwhile $\tilde{\tau}$ and τ^* increase so that more per-transaction revenue may be earned by inducing intermediated sale of H. Eventually, as $\frac{c_L}{v_L}$ increases but remains below $\frac{c_H}{v_H}$, S switches to the lower fees $\tau^{PF} = \tilde{\tau}$ and $\tau^{PC} = \tau^*$ in order to induce sales of H rather than L through M. This switch occurs sooner under PF because it is more costly for M to induce this switch under PC ($\tau^* < \tilde{\tau}$) for any given $c_L > 0$. This is because S must forgo direct sales of H when it sells H through M under PC but not under PF. M thus achieves increasing profits by switching to a lower fee level under PF before it does so under PC, and PF dominates PC for some values of c_L for which M sets $\tau^{PF} = \tilde{\tau}$ (increasing in c_L) and $\tau^{PC} = \bar{\tau}$ (decreasing in c_L).

Once M switches to the lower fee level under both pricing policies, it faces the following tradeoff. On one hand, $\tau^{PC} < \tau^{PF}$ so that M earns a higher per-transaction revenue under PF. On the other hand, more transactions occur through M under PC because $\tau^{PC} < \tau^{PF}$ and S retains a less attractive direct sales option of L rather than H under PC. The former effect dominates the latter for intermediate values of c_L because S's direct sales option remains sufficiently profitable to induce significant leakage under PC. PF dominates PC for intermediate levels of c_L. Once c_L is high enough so that $\frac{c_L}{v_L} = \frac{c_H}{v_H}$, M sets $\tau^{PF} = \bar{\tau} = \frac{v_H - c_H}{v_H}$ to extract all surplus from intermediated transactions of H under PF. Any further increase in c_L has no effect on M's problem under PF because S sells H both through M and directly. However, further increase in c_L continues to decrease S's direct sales margin under PC and increases the volume of intermediated transactions of H under PC. Although τ^{PC} remains below τ^{PF} for high c_L, the loss in per-transaction revenue relative to PF is offset by the increase in the number of intermediated transactions. M prefers PC over PF for high c_L.

A mismatch in M's and S's incentives plays a central role in why PF may dominate PC. While M is concerned with both the number of intermediated sales and the per-intermediated-sale revenue it earns from each intermediated sale, S is mainly concerned with the transaction-weighted average margin it earns between intermediated and direct sales because it always makes sales to every consumer. PF only changes the margin S earns through intermediated sales, but PC restricts the margin S earns through both intermediated and direct sales when S makes intermediated sales of H. M must compensate S for this difference in order to tax H in intermediated sales. This compensation may be suboptimal if S retains a sufficiently profitable outside sales option and induces a significant number of direct sales.

2.5 Conclusion

In this chapter, I argue that existence of a multiproduct seller on a platform introduces two strategic mechanisms that a firm and a platform can use to interact in a typical marketplace: strategic assortment and availability coherence clauses. A seller may make strategic assortment decisions by varying its menu of available products across selling channels, and a platform may restrict this behavior by enforcing an availability coherence clause. I show that strategic assortment plays a significant role in seller-platform relationships that lessens the power of platforms in many situations. While a price coherence clause allows a platform to fully monetize the access it provides between a seller and consumers in a single-product firm setting, a multiproduct firm's ability to gain market access by partial presence on a platform weakens and, in some cases, reverses this result. A platform can recover its ability to fully monetize access by pairing a price coherence clause with an availability coherence clause, but this may not be feasible from an implementation or regulation standpoint, as it effectively eliminates any off-platform sales option available to a participating seller. As platforms continue to carry out fundamental capacities in many buyer-seller relationships across industries, the strategic roles introduced by varying product scope provided by sellers should be considered in study and further regulation evaluations.

In this chapter, I focus on the relationship between the presence of a multiproduct seller on a platform and platform coherence policies. Considerable attention has recently been drawn to algorithmic steering on platforms, where platforms may punish certain seller behaviors by hiding their products from consumer view. Even without price and availability coherence clauses, if platforms steer based on product availability, then strategic assortment may be less effective for sellers than it is in this setting. Platforms have also drawn attention for acting dually as intermediaries and sellers, using data collected by third party sellers to inform their own market entry and product design decisions. This may present another important component of strategic assortment. Optimal assortment strategies may change if a platform is fundamental to seller discovery but also increases competition in product markets present on the platform. This chapter is one of the first to consider the role that multiproduct sellers play in platform markets, and it would be interesting to investigate their role in these and other aspects of platform markets in future work.

REFERENCES

Baker, J. B., & Morton, F. S. (2018). Antitrust Enforcement Against Platform MFNs. The Yale Law Journal, 2176–2202.

Boik, A., & Corts, K. S. (2016). The effects of platform most-favored-nation clauses on competition and entry. The Journal of Law and Economics, 59(1), 105–134.

Budnick, D., & Baron, J. (2012). Ticket masters: The rise of the concert industry and how the public got scalped. Penguin.

Calzada, J., Manna, E., & Mantovani, A. (2022). Platform price parity clauses and market segmentation. Journal of Economics & Management Strategy, 31(3), 609–637.

Carlton, D. W., & Winter, R. A. (2018). Vertical most-favored-nation restraints and credit card no-surcharge rules. The Journal of Law and Economics, 61(2), 215–251.

Edelman, B., & Wright, J. (2015). Price coherence and excessive intermediation. The Quarterly Journal of Economics, 130(3), 1283–1328.

Ennis, S., Ivaldi, M., & Lagos, V. (2023). Price-Parity Clauses for Hotel Room Booking: Empirical Evidence from Regulatory Change. The Journal of Law and Economics, 66(2), 309–331.

FTC. (2023, September). CASE NO.: 2:23-cv-01495-JHC. https://www.ftc.gov/system/files/ftc_gov/pdf/1910134amazonecommercecomplaintrevisedredactions.pdf

Hagiu, A., & Wright, J. (2024). Marketplace leakage. Management Science, 70(3), 1529–1553.

Hunold, M., Kesler, R., Laitenberger, U., & Schlütter, F. (2018). Evaluation of best price clauses in online hotel bookings. International Journal of Industrial Organization, 61, 542–571.

Israeli, A., Schlesinger, L. A., Higgins, M., & Semerkant, S. (2022). Should your company sell on Amazon? Reach comes at a price. Harvard Business Review, 100(9–10), 38–46.

Johansen, B. O., & Vergé, T. (2017). Platform price parity clauses with direct sales. University of Bergen.

Liu, C., Niu, F., & White, A. (2021). Optional intermediaries and pricing restraints. Available at SSRN 3825163.

Mantovani, A., Piga, C. A., & Reggiani, C. (2021). Online platform price parity clauses: Evidence from the EU Booking. com case. European Economic Review, 131, 103625.

Mariotto, C., & Verdier, M. (2020). Platform–merchant competition for sales services. Journal of Economics & Management Strategy, 29(4), 834–853.

Miao, C.-H. (2022). The pricing of ancillary goods when selling on a platform. International Journal of Industrial Organization, 83, 102847.

Wang, C., & Wright, J. (2020). Search platforms: Showrooming and price parity clauses. The RAND Journal of Economics, 51(1), 32–58.

Wang, C., & Wright, J. (2023). Platform investment and price parity clauses. The Journal of Industrial Economics, 71(2), 538–569.

Wang, Z., & Wright, J. (2017). Ad valorem platform fees, indirect taxes, and efficient price discrimination. The RAND Journal of Economics, 48(2), 467–484.

APPENDIX A: OMITTED PROOFS

Proof of Lemma 2.1: Without loss of generality, assume $\psi = 0$. Suppose $\tau \neq \tilde{\tau}$ and consider statement (i).

Suppose for the sake of contradiction that S makes a positive mass of intermediated sales of both L and H. Then consumers must be indifferent between purchasing L and H through M, so $v_L - p_L = v_H - p_H$. S earns a higher profit from intermediated sales of H at p_H compared to L at p_L if and only if $(1-\tau)p_H - c_H \geq (1-\tau)p_L - c_L$, or $\tau \leq \frac{p_H - c_H - (p_L - c_L)}{p_H - p_L}$. If S makes direct sales of a product $D \in \{L, H\}$, then the total direct demand (of all products sold directly) is given by $G\big(v_D - \dot{p}_D - (v_D - p_D)\big)$.

If $\tau < \frac{p_H - c_H - (p_L - c_L)}{p_H - p_L}$, then intermediated sale of H is more profitable than the that of L; S can increase its profit by increasing p_L (and $\dot{p}_L$ by the same amount if S makes direct sales of L) so that all intermediated sales are of H and demand in either channel remains unchanged. Similarly, S can induce all intermediated sales of L as a profitable deviation if $\tau > \frac{p_H - c_H - (p_L - c_L)}{p_H - p_L}$. Therefore, $\tau = \tau = \frac{p_H - c_H - (p_L - c_L)}{p_H - p_L}$ must hold.

Given $\tau = \frac{p_H - c_H - (p_L - c_L)}{p_H - p_L}$, consider the following four cases. If $p_H < v_H$ and $p_L < v_L$, then S can raise p_H, p_L, and $\dot{p}_D$ for each $D \in \{L, H\}$ with a positive mass of direct sales by the same small amount; demand for each product in each channel would be unchanged, but S would earn a higher profit margin on each sale. If $p_H = v_H$ and $p_L < v_L$, then S could raise p_H, p_L, and $\dot{p}_D$ for each $D \in \{L, H\}$ with a positive mass of direct sales by the same small amount; demand in each sales channel would be unchanged, but S would sell only L in intermediated sales with a higher profit margin. Similarly, if $p_H < v_H$ and $p_L = v_L$, then S can induce intermediated sales only of H as a profitable deviation. Finally, if $p_H = v_H$ and $p_L = v_L$, then $\tau = \tilde{\tau}$, which contradicts the original assumption. Therefore, S makes intermediated sales of at most one product.

Now suppose for the sake of contradiction that S makes a positive mass of direct sales of both L and H. Then consumers must be indifferent between purchasing L and H directly, so $v_H - \dot{p}_H - s = v_L - \dot{p}_L - s$ for all s. But this implies $\dot{p}_H - c_H - (\dot{p}_L - c_L) = v_H - c_H - (v_L - c_L) > 0$, so S would be better-off increasing its direct price for L so that all direct sales are of H. Therefore, S makes direct sales of at most one product.

The first part of statement (ii) follows from the proof of statement (i). The second part, that S can achieve its optimal profit by listing only one product through M is obvious. If it makes intermediated sales of both products, consumers must earn zero surplus from intermediated purchase. It can increase the intermediated price of either product so that consumers only purchase the other through M. Demand and profit in both channels is unchanged. ■

Proof of Lemma 2.2: Statement (i) follows from the main text. Consider statement (ii). If $\tau < \frac{v_I - c_I - (v_D - c_D)}{v_I}$, then $(1-\tau)v_I - c_I > v_D - c_D$; this implies that S would lower its profit by setting $\dot{p}_D < v_D$ to induce any direct sales because it earns a higher margin on intermediated sales of I at $p_I = v_I$. Suppose $\tau \geq \frac{v_I - c_I - (v_D - c_D)}{v_I}$ and repeat Equation (3.2) as

$$LHS \coloneqq \dot{p}_D = c_D + 1_{[\tau \leq \bar{\tau}_I]}[(1-\tau)v_I - c_I] + \left(1 + 1_{[D \in \mathcal{M}]}\psi\right) \frac{G\left(\frac{v_D - \dot{p}_D}{1 + 1_{[D \in \mathcal{M}]}\psi}\right)}{g\left(\frac{v_D - \dot{p}_D}{1 + 1_{[D \in \mathcal{M}]}\psi}\right)} \coloneqq RHS.$$

LHS increases in $\dot{p}_D$, and RHS decreases in $\dot{p}_D$ using G log-concave. If $\dot{p}_D = c_D$, then $LHS \leq RHS$ because $\tau \leq \bar{\tau}_I$ implies that $(1-\tau)v_I - c_I \geq 0$ (and $LHS < RHS$ clearly holds if $\tau > \bar{\tau}_I$). Now consider $\dot{p}_D = v_D$. If $g(0) > 0$, then $LHS \geq RHS$ because $\tau \geq \frac{v_I - c_I - (v_D - c_D)}{v_I}$ implies $v_D - c_D \geq (1-\tau)v_I - c_I$. Suppose $g(0) = 0$. Then $\lim_{x \to 0} g'(x) > 0$; hence, for $\dot{p}_D$ sufficiently close to v_D,

$$RHS = c_D + 1_{[\tau \leq \bar{\tau}_I]}[(1-\tau)v_I - c_I] + \left(1 + 1_{[D \in \mathcal{M}]}\psi\right) \frac{\int_0^{v_D - \dot{p}_D} g\left(\frac{x}{1 + 1_{[D \in \mathcal{M}]}\psi}\right) dx}{g\left(\frac{v_D - \dot{p}_D}{1 + 1_{[D \in \mathcal{M}]}\psi}\right)}$$

$$< c_D + 1_{[\tau \leq \bar{\tau}_I]}[(1-\tau)v_I - c_I] + \left(1 + 1_{[D \in \mathcal{M}]}\psi\right) \frac{(v_D - \dot{p}_D) g\left(\frac{v_D - \dot{p}_D}{1 + 1_{[D \in \mathcal{M}]}\psi}\right)}{g\left(\frac{v_D - \dot{p}_D}{1 + 1_{[D \in \mathcal{M}]}\psi}\right)}$$

$$= c_D + 1_{[\tau \leq \bar{\tau}_I]}[(1-\tau)v_I - c_I] + \left(1 + 1_{[D \in \mathcal{M}]}\psi\right)(v_D - \dot{p}_D).$$

Then $\lim_{\dot{p}_D \to v_D} LHS - RHS > v_D - c_D - 1_{[\tau \leq \bar{\tau}_I]}[(1-\tau)v_I - c_I] \geq 0$. Therefore, Equation (3.2) has a unique solution $\dot{p}_D^{\{I\}}(\tau) \in [c_D, v_D]$, where $\dot{p}_D = \dot{p}_D^{\{I\}}(\tau)$ maximizes S's profit if $\dot{p}_D^{\{I\}}(\tau) \geq v_D - \left(1 + 1_{[D \in \mathcal{M}]}\psi\right)\bar{s}$. Otherwise, $\dot{p}_D^{\{I\}}(\tau)$ induces all sales to be direct, which S can achieve with the higher price $\dot{p}_D = v_D - \left(1 + 1_{[D \in \mathcal{M}]}\psi\right)\bar{s}$.

Finally, consider statement (iii). To simplify the notation, without loss of generality, assume $\psi = 0$. I will compare the case that S makes any intermediated sales of $I \in \{L, H\}$ and any direct sales of $D \in \{L, H\}$ to a case in which S makes any intermediated sales of $i \in \{L, H\}$ and any direct sales of $d \in \{L, H\}$. Without loss of generality, suppose

$$OC(D, I) \coloneqq v_D - c_D - [(1-\tau)v_I - c_I] \geq v_d - c_d - [(1-\tau)v_i - c_i] =: OC(d, i).$$

I must show that $G(v_D - \dot{p}_D) \geq G(v_d - \dot{p}_d)$. I do so in each of the following four cases, one of which must apply.

<u>Case 1</u>: $\tau \in \left[0, \max\left\{\frac{v_I - c_I - (v_D - c_D)}{v_I}, \frac{v_i - c_i - (v_d - c_d)}{v_i}\right\}\right)$.

Then $G(v_D - \dot{p}_D) = 0 = G(v_d - \dot{p}_d)$ by statement (ii).

<u>Case 2</u>: $\frac{v_I - c_I - (v_D - c_D)}{v_I} < \frac{v_i - c_i - (v_d - c_d)}{v_i}$ and $\tau \in [0, \bar{\tau}] \cap \left[\frac{v_I - c_I - (v_D - c_D)}{v_I}, \frac{v_i - c_i - (v_d - c_d)}{v_i}\right)$.

Then $G(v_D - \dot{p}_D) \geq 0 = G(v_d - \dot{p}_d)$ by statement (ii).

<u>Case 3</u>: $\frac{v_I - c_I - (v_D - c_D)}{v_I} > \frac{v_i - c_i - (v_d - c_d)}{v_i}$ and $\tau \in [0, \bar{\tau}] \cap \left[\frac{v_i - c_i - (v_d - c_d)}{v_i}, \frac{v_I - c_I - (v_D - c_D)}{v_I}\right)$.

Then $G(v_D - \dot{p}_D) = 0$ by statement (ii), and I must show that $G(v_d - \dot{p}_d) = 0$ as well. First note that $I = H$ and $D = L$ must hold; otherwise, $\frac{v_I - c_I - (v_D - c_D)}{v_I} \leq 0$, which yields the contradiction $\tau < 0$.

That $I = H$ and $D = L$ implies $\tau \leq \tilde{\tau}$ because $\frac{v_I - c_I - (v_D - c_D)}{v_I} \leq \frac{v_I - c_I - (v_D - c_D)}{v_I - v_D} = \tilde{\tau}$. Then $i = L$ and $d = H$ cannot hold because it yields the contradiction $OC(D, I) < OC(d, i)$ using $I = H, D = L$, and $\tau \leq \tilde{\tau}$. Similarly, $i = H$ and $d = L$ cannot hold since it yields the contradiction $\frac{v_I - c_I - (v_D - c_D)}{v_I} = \frac{v_i - c_i - (v_d - c_d)}{v_i}$. Therefore, $i = d$ must hold.

If $\mathcal{P} = PC$, then S cannot induce any direct sales with $i = d$, and $G(v_d - \dot{p}_d) = 0$, as desired.

If $\mathcal{P} = PF$ and $\tau = 0$, then no direct sales are optimal, and $G(v_d - \dot{p}_d) = 0$, as desired.

Finally, suppose $\mathcal{P} = \mathcal{P}F$ and $\tau > 0$. I will show $I = H$ and $D = L$ is not optimal and conclude that this case need not be considered. To see this, suppose that S makes any direct sales of L at $\dot{p}_L$. Consider a deviation in which S sells H at $\dot{p}_H = \dot{p}_L + v_H - v_L$ and increases the direct price of L from $\dot{p}_L$. A consumer earns a surplus of $v_H - \dot{p}_H - s = v_L - \dot{p}_L - s$ from purchasing H directly, so intermediated and direct demand remain unchanged. But S earns a higher direct profit because its new margin is $\dot{p}_H - c_H = \dot{p}_L - v_L + v_H - c_H > \dot{p}_L - c_L$ (using $v_H - c_H > v_L - c_L$). This is a profitable deviation if S induces any direct sales at $\dot{p}_L$. Otherwise, its profit is equivalent to that earned by selling H through M at $p_H = v_H$ and directly at $\dot{p}_H = v_H$ (i.e., making no sales of H directly). However, by making a small decrease to the direct price of H from v_H increases S's margin earned from consumers with low switching costs who purchase directly after the price change. Specifically, this margin increases by $v_H - c_H - [(1-\tau)v_H - c_H] = \tau v_H > 0$. Thus, it is a profitable deviation to make direct sales of H.

<u>Case 4</u>: $\tau \in \left[\max\left\{\frac{v_I - c_I - (v_D - c_D)}{v_I}, \frac{v_i - c_i - (v_d - c_d)}{v_i}\right\}, \bar{\tau}\right]$.

Then $\dot{p}_D = \max\left\{\dot{p}_D^{\{I\}}(\tau), v_D - \bar{s}\right\}$ and $\dot{p}_d = \max\left\{\dot{p}_d^{\{i\}}(\tau), v_d - \bar{s}\right\}$ by statement (ii). Therefore, $G(v_D - \dot{p}_D) = G\left(\min\left\{v_D - \dot{p}_D^{\{I\}}(\tau), \bar{s}\right\}\right)$ and $G(v_d - \dot{p}_d) = G\left(\min\left\{v_d - \dot{p}_d^{\{i\}}(\tau), \bar{s}\right\}\right)$.

Rewriting Equation (2), $\dot{p}_D^{\{I\}}(\tau)$ and $\dot{p}_d^{\{i\}}(\tau)$ solve

$$v_D - \dot{p}_D^{\{I\}}(\tau) = OC(D, I) - \frac{G\left(v_D - \dot{p}_D^{\{I\}}(\tau)\right)}{g\left(v_D - \dot{p}_D^{\{I\}}(\tau)\right)},$$

$$v_d - \dot{p}_d^{\{i\}}(\tau) = OC(d, i) - \frac{G\left(v_d - \dot{p}_d^{\{i\}}(\tau)\right)}{g\left(v_d - \dot{p}_d^{\{i\}}(\tau)\right)}.$$

Since G/g is an increasing function, we have $v_D - \dot{p}_D^{\{I\}}(\tau) \geq v_d - \dot{p}_d^{\{i\}}(\tau)$ if and only if $OC(D, I) \geq OC(d, i)$. Hence, $G(v_D - \dot{p}_D) \geq G(v_d - \dot{p}_d)$, as desired. ■

Proof of Proposition 2.3: Denote S's respective profits from $\mathcal{M} = \{H\}$ and $\mathcal{M} = \{L\}$ as

$$\Pi_S^{\{H\}}(\tau) := \max_{\dot{p}_L} 1_{[\tau \leq \bar{\tau}_H]}[(1-\tau)v_H - c_H]\left[1 - G\left(\frac{v_L - \dot{p}_L}{1+\psi}\right)\right] + (\dot{p}_L - c_L)G\left(\frac{v_L - \dot{p}_L}{1+\psi}\right),$$

$$\Pi_S^{\{L\}}(\tau) := \max_{\dot{p}_H} 1_{[\tau \leq \bar{\tau}_L]}[(1-\tau)v_L - c_L]\left[1 - G\left(\frac{v_H - \dot{p}_H}{1+\psi}\right)\right] + (\dot{p}_H - c_H)G\left(\frac{v_H - \dot{p}_H}{1+\psi}\right).$$

I will show that there exists $\tau^* \in (0, \min\{\tilde{\tau}, \bar{\tau}\})$ such that $\Pi_S^{\{H\}}(\tau) \geq \Pi_S^{\{L\}}(\tau)$ if and only if $\tau \leq \tau^*$.

Clearly $\Pi_S^{\{H\}}(0) > \Pi_S^{\{L\}}(0)$ because $\Pi_S^{\{H\}}(0) = v_H - c_H$ is the maximal total welfare that cannot be improved upon.

Also, $\Pi_S^{\{H\}}(\tau) < \Pi_S^{\{L\}}(\tau)$ for all $\tau \geq \min\{\tilde{\tau}, \bar{\tau}\}$. S can always earn a higher direct profit from selling H rather than L directly at a fixed level of direct demand, and its intermediated profit is higher from selling L through M for $\tau \geq \tilde{\tau}$.

Now since $\Pi_S^{\{H\}}(\tau)$ and $\Pi_S^{\{L\}}(\tau)$ each decrease continuously over $[0, \min\{\tilde{\tau}, \bar{\tau}\}]$, it suffices to show that $\Pi_S^{\{H\}}(\tau)$ decreases at a faster rate than $\Pi_S^{\{L\}}(\tau)$ at all fee levels $\tau \in [0, \min\{\tilde{\tau}, \bar{\tau}\}]$; that is, if $(0 \geq) \frac{d}{d\tau}\Pi_S^{\{L\}}(\tau) \geq \frac{d}{d\tau}\Pi_S^{\{H\}}(\tau)$ for all $\tau \in [0, \min\{\tilde{\tau}, \bar{\tau}\}]$, then $\Pi_S^{\{H\}}(\tau)$ and $\Pi_S^{\{L\}}(\tau)$ have a single crossing that defines τ^*.

First consider $\Pi_S^{\{L\}}(\tau)$. By Lemma 2.2, $p_H^{\{L\}}(\tau)$ is well-defined and unique, and $\Pi_S^{\{L\}}(\tau)$ is achieved by $\dot{p}_H = \max\left\{\dot{p}_H^{\{L\}}, v_H - (1+\psi)\bar{s}\right\}$. Then $\frac{d}{d\tau}\Pi_S^{\{L\}}(\tau) = -v_L\left[1 - G\left(\frac{v_H - \dot{p}_H^{\{L\}}}{1+\psi}\right)\right] \cdot 1_{[\tau \leq \bar{\tau}_L]}$.

Now consider $\Pi_S^{\{H\}}(\tau)$ in two cases: $\tau < \frac{v_H - c_H - (v_L - c_L)}{v_H}$ and $\tau \geq \frac{v_H - c_H - (v_L - c_L)}{v_H}$.

If $\tau < \frac{v_H - c_H - (v_L - c_L)}{v_H}$, then by Lemma 2.2, S makes no direct sales to achieve $\Pi_S^{\{H\}}(\tau)$. Then $\frac{d}{d\tau}\Pi_S^{\{H\}}(\tau) = -v_H \leq -v_L \leq \frac{d}{d\tau}\Pi_S^{\{L\}}(\tau)$, as desired.

Now suppose $\tau \geq \frac{v_H - c_H - (v_L - c_L)}{v_H}$. By Lemma 2.2, $p_L^{\{H\}}(\tau)$ is well-defined and unique, and $\Pi_S^{\{H\}}(\tau)$ is achieved by $\dot{p}_L = \max\left\{\dot{p}_L^{\{H\}}, v_L - (1+\psi)\bar{s}\right\}$. Then $\frac{d}{d\tau}\Pi_S^{\{H\}}(\tau) = -v_H\left[1 - G\left(\frac{v_L - \dot{p}_L^{\{H\}}}{1+\psi}\right)\right]$. Then $\frac{d}{d\tau}\Pi_S^{\{L\}}(\tau) \geq \frac{d}{d\tau}\Pi_S^{\{H\}}(\tau)$ holds if and only if

$$v_H\left[1 - G\left(\frac{v_L - \dot{p}_L^{\{H\}}}{1+\psi}\right)\right] \geq v_L\left[1 - G\left(\frac{v_H - \dot{p}_H^{\{L\}}}{1+\psi}\right)\right] \cdot 1_{[\tau \leq \bar{\tau}_L]},$$

which holds by Lemma 2.2(iii) (and because $v_H \geq v_L$). ■

APPENDIX B: ROBUSTNESS OF THE NUMERICAL EXAMPLE

Section 2.4.2.2 demonstrates how PC need not dominate PF. Here I demonstrate the robustness of this result to differing values of the fixed parameters in that example. As in Section 2.4.2.2, I numerically solve the model with uniform switching costs. I normalize $v_H = 1$ and consider various values for ψ and $\bar{s}$. For each choice of ψ and $\bar{s}$, I allow (v_L, c_H) to uniformly vary over a discretized set $[0, 1] \times [0, 1]$. For each specification of $(\psi, \bar{s}, v_L, c_H)$, I solve for M's optimal profits under PF and PC for all c_L over a discretized interval $\left[c_L^{\min}, c_L^{\max}\right]$, where $c_L^{\min} \coloneqq \min\{0, v_L - (v_H - c_H)\}$ and $c_L^{\max} \coloneqq \min\{v_L, c_H\}$. Thus, for each ψ and $\bar{s}$, I consider all permissible values of (v_L, c_H, c_L) in the model such that $c_L \leq c_H$. For each specification $(\psi, \bar{s}, v_L, c_H)$, I document the fraction of costs c_L over $\left[c_L^{\min}, c_L^{\max}\right]$ for which PF dominates PC.[29] Figure B.1 records the results of this exercise.

It is clear from Figure B.1 that for all choice of ψ and $\bar{s}$ considered, PC is more likely to dominate PF for randomly chosen values of (v_L, c_H, c_L). PF is relatively more likely to dominate PC for moderately high values of v_L. This is because direct sale of L must be sufficiently profitable relative to the sale of H in order to induce significant leakage for PF to dominate PC. Next, PF is relatively more likely to dominate PC for lower values of ψ because high values of ψ make it more costly for S to induce leakage under PC. Finally, PF is relatively more likely to dominate PC for higher values of $\bar{s}$ considered. Recall that M's main tradeoff between PC and PF is between per-intermediated-transaction revenue and leakage. If $\bar{s}$ is low, then leakage is easily induced by S and may be more costly than a loss in per-transaction revenue for M. Overall, Figure B.1 verifies that

[29] To account for any small numerical errors, I only record PF as dominating PC if $\Pi_M^{PF} > \Pi_M^{PC} + 0.001$.

while PC is more likely to dominate PF overall, Proposition 3.5(ii) is relevant in many circumstances in which PF may dominate PC.

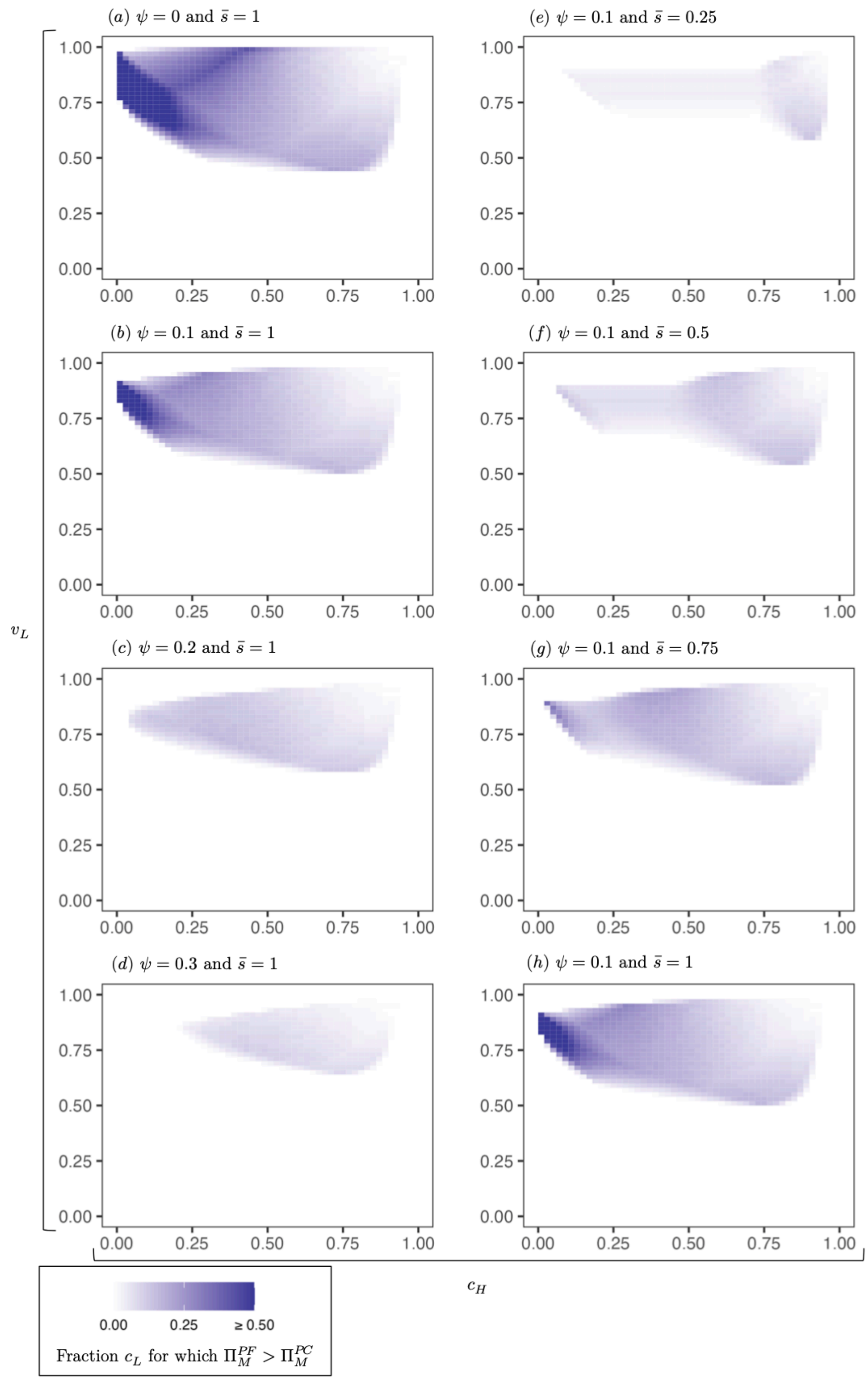


Figure B.1: Robustness of the Numerical Example

Notes—The figure shows the fraction of costs $c_L \in [c_L^{\min}, c_L^{\max}]$ for which PF dominates PC at various other parameter levels. Switching costs are distributed uniformly over $[0, \bar{s}]$. The high valuation v_H is normalized to one.

APPENDIX C: HETEROGENOUS CONSUMER VALUATIONS

Assume that a fraction α of consumers (called type 1) have valuation v_L for both L and H, and the remaining fraction $1-\alpha$ of consumers (called type 2) have valuation v_L for L and v_H for H. Type 1 consumers have no taste for quality beyond that provided by L, and type 2 consumers are sensitive to improvement in quality beyond that provided by L. As S acts after M acts in the game, assume that consumers choose to purchase the product that maximizes S's profit when indifferent between purchasing L and H. Finally, assume that consumers' quality preference types and their switching costs are independently realized in stage three of the game. Note that the main model is the special case of this extension when $\alpha = 0$. Below I establish S's assortment options and pricing problems under PAC, PF, and PC given an arbitrary fee $\tau \leq \bar{\tau}$. A second source of consumer heterogeneity introduces significant complexities in the analysis, so upon solving for S's assortment and pricing problems under each policy, I proceed to demonstrate the robustness of the main results by numerically solving this extended model with uniform switching costs.

PAC—Under PAC, S faces an all-or-nothing listing decision and all transactions are facilitated through M. S will list both products through M provided $\tau \leq \bar{\tau}$. S will sell L to type 1 consumers if $\tau \leq \frac{v_L}{v_L - c_L}$; otherwise, it will not sell to type 1 consumers. S will sell H to type 2 consumers if $\tau \leq \min\{\tilde{\tau}, \bar{\tau}\}$; it will sell L to type 2 consumers if $\tau \in [\tilde{\tau}, \bar{\tau}]$. In all cases, S prices at valuation for each product it makes sales of so that all consumers earn zero surplus.

PF—By arguments analogous to those in the baseline analysis, S will minimize the surplus it offers to consumers in sales through M. S will only make sales of a product through M if it is profitable to do so, and it will make sales of the more profitable product through M whenever possible.

Suppose H is more cost-intensive than L and $\tau \le \tilde{\tau}$. S can make direct sales only of L, only of H, or of both L and H. These strategies earn respective profits of

$$\max_{p_L^d} \alpha[(1-\tau)v_L - c_L]\big[1 - G\big(v_L - p_L^d\big)\big] + (1-\alpha)[(1-\tau)v_H - c_H]\big[1 - G\big(v_L - p_L^d\big)\big]$$

$$+ \big(p_L^d - c_L\big)G\big(v_L - p_L^d\big),$$

$$\max_{p_H^d} \alpha[(1-\tau)v_L - c_L]\big[1 - G\big(v_L - p_H^d\big)\big] + (1-\alpha)[(1-\tau)v_H - c_H]\big[1 - G\big(v_H - p_H^d\big)\big]$$

$$+ \alpha\big(p_H^d - c_H\big)G\big(v_L - p_H^d\big) + (1-\alpha)\big(p_H^d - c_H\big)G\big(v_H - p_H^d\big),$$

$$\max_{p_L^d, p_H^d} \alpha[(1-\tau)v_L - c_L]\big[1 - G\big(v_L - p_L^d\big)\big] + (1-\alpha)[(1-\tau)v_H - c_H]\big[1 - G\big(v_H - p_H^d\big)\big]$$

$$+ \alpha\big(p_L^d - c_L\big)G\big(v_L - p_L^d\big) + (1-\alpha)\big(p_H^d - c_H\big)G\big(v_H - p_H^d\big),$$

where the third problem is solved subject to the type 2 incentive compatibility constraint $p_H^d - p_L^d \le v_H - v_L$. If H is more cost-intensive than L and $\tau \in (\tilde{\tau}, \bar{\tau}]$, then S's three strategies yield respective profits

$$\max_{p_L^d}[(1-\tau)v_L - c_L]\big[1 - G\big(v_L - p_L^d\big)\big] + \big(p_L^d - c_L\big)G\big(v_L - p_L^d\big),$$

$$\max_{p_H^d} \alpha[(1-\tau)v_L - c_L]\big[1 - G\big(v_L - p_H^d\big)\big] + (1-\alpha)[(1-\tau)v_L - c_L]\big[1 - G\big(v_H - p_H^d\big)\big]$$

$$+ \alpha\big(p_H^d - c_H\big)G\big(v_L - p_H^d\big) + (1-\alpha)\big(p_H^d - c_H\big)G\big(v_H - p_H^d\big),$$

$$\max_{p_L^d, p_H^d} \alpha[(1-\tau)v_L - c_L]\big[1 - G\big(v_L - p_L^d\big)\big] + (1-\alpha)[(1-\tau)v_L - c_L]\big[1 - G\big(v_H - p_H^d\big)\big]$$

$$+ \alpha\big(p_L^d - c_L\big)G\big(v_L - p_L^d\big) + (1-\alpha)\big(p_H^d - c_H\big)G\big(v_H - p_H^d\big),$$

where again the third problem is solved subject to the type 2 incentive compatibility constraint $p_H^d - p_L^d \le v_H - v_L$.

Now suppose L is more cost-intensive than H. S can again directly sell only L, only H, or both L and H. If $\tau \le \frac{v_L - c_L}{v_L}$, then these strategies yield respective profits

$$\max_{p_L^d} \alpha[(1-\tau)v_L - c_L]\left[1 - G\left(v_L - p_L^d\right)\right] + (1-\alpha)[(1-\tau)v_H - c_H]\left[1 - G\left(v_L - p_L^d\right)\right]$$

$$+ \left(p_L^d - c_L\right)G\left(v_L - p_L^d\right),$$

$$\max_{p_H^d} \alpha[(1-\tau)v_L - c_L]\left[1 - G\left(v_L - p_H^d\right)\right] + (1-\alpha)[(1-\tau)v_H - c_H]\left[1 - G\left(v_H - p_H^d\right)\right]$$

$$+ \alpha\left(p_H^d - c_H\right)G\left(v_L - p_H^d\right) + (1-\alpha)\left(p_H^d - c_H\right)G(v_H - p_H^d),$$

$$\max_{p_L^d, p_H^d} \alpha[(1-\tau)v_L - c_L]\left[1 - G\left(v_L - p_L^d\right)\right] + (1-\alpha)[(1-\tau)v_H - c_H]\left[1 - G\left(v_H - p_H^d\right)\right]$$

$$+ \alpha\left(p_L^d - c_L\right)G\left(v_L - p_L^d\right) + (1-\alpha)\left(p_H^d - c_H\right)G\left(v_H - p_H^d\right),$$

subject to $p_H^d - p_L^d \leq v_H - v_L$ in the third problem. If $\tau \in \left(\frac{v_L - c_L}{v_L}, \bar{\tau}\right]$, then S's three strategies yield respective profits

$$\max_{p_L^d}(1-\alpha)[(1-\tau)v_H - c_H]\left[1 - G\left(v_L - p_L^d\right)\right] + \left(p_L^d - c_L\right)G\left(v_L - p_L^d\right),$$

$$\max_{p_H^d}(1-\alpha)[(1-\tau)v_H - c_H]\left[1 - G\left(v_H - p_H^d\right)\right] + \alpha\left(p_H^d - c_H\right)G\left(v_L - p_H^d\right)$$

$$+ (1-\alpha)\left(p_H^d - c_H\right)G\left(v_H - p_H^d\right),$$

$$\max_{p_L^d, p_H^d}(1-\alpha)[(1-\tau)v_H - c_H]\left[1 - G\left(v_H - p_H^d\right)\right] + \alpha\left(p_L^d - c_L\right)G\left(v_L - p_L^d\right)$$

$$+ (1-\alpha)\left(p_H^d - c_H\right)G\left(v_H - p_H^d\right),$$

subject to $p_H^d - p_L^d \leq v_H - v_L$ in the third problem.

PC—S can list only L through M, only H through M, or both L and H through M. Since sales of both products may be made through the same channel, listing both products through M is no longer immediately strictly dominated. If only one product is listed through M, then leakage may be induced only by the other product due to PC, and it is expensive due to the extra location costs ψs incurred by consumers. If S lists only L through M, then it sets $p_L = v_L$ and earns

$$\max_{p_H^d} 1_{\tau\in\left[0,\frac{v_L-c_L}{v_L}\right]} \cdot [(1-\tau)v_L - c_L]\left\{\alpha\left[1-G\left(\frac{v_L-p_H^d}{1+\psi}\right)\right]+(1-\alpha)\left[1-G\left(\frac{v_H-p_H^d}{1+\psi}\right)\right]\right\}$$

$$+\left(p_H^d-c_H\right)\left[\alpha G\left(\frac{v_L-p_H^d}{1+\psi}\right)+(1-\alpha)G\left(\frac{v_H-p_H^d}{1+\psi}\right)\right]$$

If S lists only H through M, then it sets either $p_H = v_L$ or $p_H = v_H$. These two strategies yield respective profits

$$\max_{p_L^d} 1_{\tau\in\left[0,\frac{v_L-c_H}{v_L}\right]}$$

$$\cdot [(1-\tau)v_L - c_H]\left\{\alpha\left[1-G\left(\frac{v_L-p_L^d}{1+\psi}\right)\right]\right.$$

$$\left.+(1-\alpha)\left[1-G\left(\frac{v_L-p_L^d-v_H+v_L}{1+\psi}\right)\right]\right\}$$

$$+\left(p_L^d-c_L\right)\left[\alpha G\left(\frac{v_L-p_L^d}{1+\psi}\right)+(1-\alpha)G\left(\frac{v_L-p_L^d-v_H+v_L}{1+\psi}\right)\right],$$

$$\max_{p_L^d} 1_{\tau\in\left[0,\frac{v_H-c_H}{v_H}\right]} \cdot [(1-\tau)v_H - c_H](1-\alpha)\left[1-G\left(\frac{v_L-p_L^d}{1+\psi}\right)\right]+\left(p_L^d-c_L\right)G\left(\frac{v_L-p_L^d}{1+\psi}\right).$$

Finally, suppose S lists both products through M. Suppose H is more cost-intensive than L. If $\tau \in [0,\tilde{\tau}]$, then S earns

$$\alpha[(1-\tau)v_L - c_L] + (1-\alpha)[(1-\tau)v_H - c_H].$$

If $\tau \in (\tilde{\tau},\bar{\tau}]$, then S earns $(1-\tau)v_L - c_L$. Now suppose H is less cost-intensive than L. If $\tau \in \left[0,\frac{v_L-c_L}{v_L}\right]$, then S earns

$$\alpha[(1-\tau)v_L - c_L] + (1-\alpha)[(1-\tau)v_H - c_H].$$

If $\tau \in \left(\frac{v_L-c_L}{v_L},\bar{\tau}\right]$, then S earns $(1-\alpha)[(1-\tau)v_H - c_H]$.

Given S's assortment options and pricing problems, M's profit equation can easily be written as a function of $(\mathcal{P},\mathcal{A},\tau)$. It is immediate that Proposition 1 holds in this extended model with heterogenous consumer valuations in that PAC remains M's optimal policy. A viable direct sales channel constrains M's per-intermediated-transaction revenue at a fixed number of intermediated transactions beyond profitability of S. PAC eliminates any viable direct sales channel such that M can maximize its per-intermediated-transaction revenue subject to S's profitability and without any effect on the number of intermediated transactions.

To demonstrate the robustness of the tradeoff between PF and PC to heterogenous consumer preferences for quality, I numerically solve the model with uniform switching costs as in Figure 2.1. I consider the same fixed parameters used in Figure 2.1 ($s \sim U[0, \bar{s} = 1], v_H = 1, v_L = 0.75, c_H = 0.5, \psi = 0.1, c_L \in [0.25, 0.5]$). I allow the heterogenous taste parameter α to vary uniformly over $[0, 1]$ and find that PF dominates PC for a positive fraction of costs $c_L \in [0.25, 0.5]$ for all $\alpha \in [0, 1)$. This fraction decreases to zero as $\alpha \to 1$. Over all possible $\alpha \in [0, 1]$, PF dominates PC for an average of 12.5 percent of the cost levels $c_L \in [0.25, 0.5]$.

Figure C.1 demonstrates that the underlying mechanism for these results is the same with both homogenous and heterogenous valuations by illustrating the case of maximal consumer heterogeneity ($\alpha = 0.5$). Again, it is more costly under PC for M to induce S to sell its targeted product set through M because S loses its ability to sell those products directly if they are sold through M. As a consequence of this, M lowers τ^{PF} below $\frac{v_H - c_H}{v_H}$ to induce intermediated sales of H and to achieve increasing profits for lower values of c_L than when it lowers τ^{PC} below $\frac{v_H - c_H}{v_H}$ to induce intermediated sales of H and to achieve increasing profits. Once M induces sale of both products through M, all leakage is eliminated under PC but some leakage remains under PF. The tradeoff between lower per-intermediated-transaction revenue and lower levels of leakage identified in the main analysis persists. Note that the right end behavior in Figure C.1 differs from that in Figure 2.1 because M's "targeted" intermediated product set differs between environments for high values of c_L. When $\alpha = 0$ (Figure 2.1), M targets only H under both PF and PC, and there is no intermediated sale of L for high c_L. Thus further increases in c_L have nondecreasing effects on Π_M^{PC} and Π_M^{PF}. When $\alpha = 0.5$ (Figure C.1), however, M targets both L and H under both PF and PC for high c_L. Thus, further increases in c_L have decreasing effects on Π_M^{PC} and Π_M^{PF}. Overall, for different values of $\alpha \notin \{0, 0.5\}$, M's targeted intermediated product set may change for fixed levels of c_L, but the underlying mechanism remains in that PC still makes it costly to induce the targeted product set's intermediated sale compared to PF because of the restriction PC places on direct sales and consequently direct sales margins.

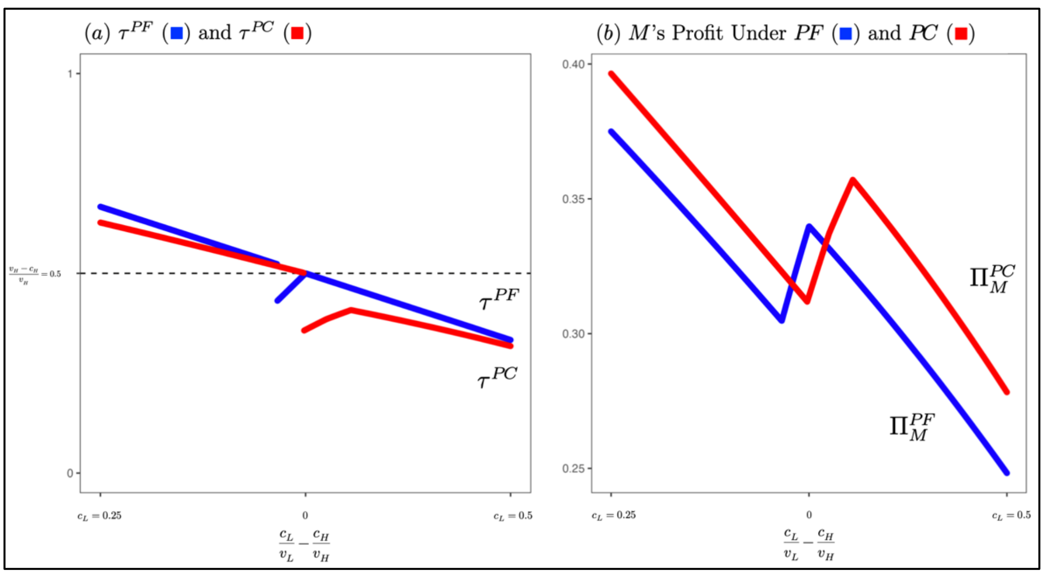


Figure C.1: Numerical Example with Heterogenous Valuations

Notes—The model is specified by $\alpha = 0.5$, $s \sim U[0, \bar{s} = 1]$, $v_H = 1$, $v_L = 0.75$, $c_H = 0.5$, $\psi = 0.1$, and $c_L \in [0,25, 0.5]$. When $\frac{c_L}{v_L} \leq \frac{c_H}{v_H}$, S lists only L through M under PC. When $\frac{c_L}{v_L} > \frac{c_H}{v_H}$, S lists both L and H through M.

CHAPTER 3

PRODUCT RECOMMENDATIONS WITH MATCH EXTERNALITIES

3.1 Introduction

Online platforms like streaming service providers (e.g., YouTube, Netflix, Spotify) and marketplaces (e.g., Amazon.com, Booking.com) provide consumers access to effectively limitless product options. However, due to search frictions, these options would offer consumers little incremental value without product recommendations to guide their search. Platforms' product recommender systems are constrained by accessible data, introducing a tradeoff in their design. On one hand, platforms can leverage their observations of individual consumer-producer interactions to offer familiar product recommendations. If consumer preferences remain relatively stable, then these "safe" recommendations are likely to be engaged with again. On the other hand, platforms can provide unfamiliar recommendations (e.g., through collaborative filtering) to promote new product discovery. While these "discovery" recommendations may provide more value to consumers compared to their safe recommendations, they also carry greater uncertainty regarding consumer engagement. This chapter formally studies this safe-discovery recommendation tradeoff faced by platforms, as well as its implications for consumers.

A recent study by Chen et al. (2024) clearly demonstrates this tradeoff on a music streaming platform. The authors experimentally implemented a relatively more discovery-oriented recommender system and found that it increased some users' consumption diversity but decreased overall consumption on the platform. Different platforms implement systems on opposite sides of the safe-discovery recommendation tradeoff. For example, Ricks & McCrosky (2022) show that although YouTube provides tools for consumers to give feedback on videos, user feedback often does not impact future video recommendations. From complementary survey data, the authors document that consumers on YouTube must resort to other tactics, like strategically adjusting viewing activities, to influence their video recommendations. This suggests that YouTube's recommender system may prioritize volume of engagement over value from engagement. In contrast, Netflix places high emphasis on providing recommendations that maximize consumer value—it has even released data and hosted a public prize contest in attempt to improve its ability to predict individual customer content preferences (Thompson, 2008). A key difference between these two platforms is their revenue structures. YouTube is mainly ad-funded, while Netflix is subscription-funded.[30] Due to the presence of third-party advertisers on YouTube, a video view creates a positive externality from the consumer-producer interaction by granting advertisers access to the viewer's attention. The value of this view-externality depends not on how much the consumer likes the video but only on whether the consumer watches the video. This could explain why YouTube prioritizes engagement over consumer value relative to Netflix. I formalize this intuition and other results by modeling platforms' recommendation efficiency tradeoff under varying levels of match-externalities.

To study this issue, I build a model in which a platform provides consumers with access to many products for a participation fee. Due to search frictions, consumers rely on the platform to provide them with product recommendations. For each consumer, the platform considers two products to recommend: a "safe" product and a "discovery" product. Due to the data each recommendation derives from, consumers are more likely to engage with safe recommendations compared to discovery recommendations. Consumers know their valuations for safe recommendations due to their familiarity, while they only know distributional features of the value that discovery

[30] Ricks and McCrosky (2022) do not gather the share of participants in their study who have premium (ad-free) and free (ad-present) YouTube accounts. However, when much of their data was collected in 2021, less than 3.5% of all YouTube users were estimated to be premium subscribers, suggesting that their results are largely driven by the ad-present version of YouTube (Statista, 2023, 2024a).

recommendations provide. Third-party advertisers value access to successful consumer-producer matches, and the platform sells this access for a fee. The key friction here is that third-party advertiser preferences only depend on volume of engagement by consumers, while consumers with low safe recommendation values prefer a low match-likelihood discovery recommendation over their safe recommendation.

The model encompasses many platform service structures. For example, media streaming and social media platforms provide advertisement space for which demand increases in the volume of engagement by consumers but does not depend on how much consumers value their interactions. Similarly, online marketplaces may sell advertisements in complementary product spaces once transactions are completed.

When analyzing the model, I first characterize the platform's choice of recommendation strategy and consumer- and advertiser-participation fees. If the value of the advertising market increases, the platform provides discovery recommendations to consumers who prefer them less often in order to increase overall match volume and increase the advertiser participation fee. This result provides an explanation and conditions for repetitive or consumption-narrowing product recommendation "filter bubbles" often observed in practice (Pariser, 2011). The platform charges consumers lower participation fees to sustain their participation with deteriorated recommendations. To induce participation from consumers with low safe recommendation valuations, the platform cannot charge any consumers who prefer discovery for the heterogenous values they receive from safe recommendations. Thus, even though consumers experience lower quality recommendations, consumer surplus generally increases with advertiser value. This reveals a tradeoff between the prevalent "premium" and "free" platform subscription structures for consumers. While they may receive better recommendations and greater aggregate value with ad-free services compared to ad-funded services, the increase in participation fee incurred may significantly offset any value generated from better recommendations.

Next, I study the platform's incentives to improve discovery recommendations. The quality of discovery recommendations comprises two components: match-likelihood or "allure" and match-conditional expected value or "suitability." An increase in allure or suitability increases consumer value from discovery recommendations in similar ways, but increasing allure also produces a positive advertiser demand effect by increasing the match-likelihood for all discovery recommendations provided. I demonstrate that when match externalities are sufficiently large, the platform typically prefers to improve allure over suitability when improving its recommendation quality. This is true even in cases when an improvement in suitability has a larger effect than allure on recommendation quality. Further, at any fixed recommendation quality level, the platform always prefers to tradeoff recommendation suitability in favor of recommendation allure.

The rest of the chapter proceeds as follows. Section 3.2 reviews the relevant literature on recommender systems. Section 3.3 builds the model. Section 3.4 discusses the modeling assumptions and features in the context of a motivating example. Section 3.5 analyzes the model and develops the main results, with all proofs provided in an Appendix. Section 3.6 concludes.

3.2 Related Literature

Much of the literature on product recommender systems studies the internal mechanics of specific recommender algorithms and how they produce varying degrees of value through social learning (Che & Hörner, 2018; Feng et al., 2022; Kremer et al., 2014; Lee & Wright, 2021). I abstract from such details concerning how a recommender system learns consumer preferences to focus on how a platform uses such information once it is obtained.

The empirical literature demonstrates that platforms' recommender systems have varying effects on consumption patterns. Personalized product recommendations have been shown to lower consumption diversity in the audio streaming industry (A. Anderson et al., 2020; Holtz et al., 2020) but increase it in the online retail industry (Brynjolfsson et al., 2011; Oestreicher-Singer & Sundararajan, 2012). Both the theoretical and empirical literatures attribute these differences to differences in the design of platforms' recommender systems (Chen et al., 2024; Fleder & Hosanagar, 2009; Holtz et al., 2020). Chen et al. (2024) reveal a possible explanation for why such differences in design exist in practice. In an audio streaming field experiment, they demonstrate that a platform's promotion of higher consumption diversity through new product recommendations comes at a cost of lower engagement from users who end up with bad recommendations. The focus of this chapter is to study this empirically documented consumption diversity tradeoff faced in recommender system design within a theoretical framework, and I show that the tradeoff may lead to biased product recommendations.

Other authors study product recommendation bias in environments in which a platform has incentives to bias search to increase sponsored search revenue (Bourreau & Gaudin, 2022; De Corniere & Taylor, 2014; Hagiu & Jullien, 2011) or to preference a platform's own products over its competitors' products (Aridor & Gonçalves, 2022; Hagiu et al., 2022; Zou & Zhou, 2024). I present an additional form of recommendation bias that arises when third-party advertisers benefit from successful recommendations.

3.3 Model

3.3.1 Players

There are four types of players: producers, consumers, third-party advertisers, and a monopoly platform that facilitates discovery between consumers and producers and provides advertisement space through successful consumer-producer matches.

Producers—An infinite number of zero cost producers produce a differentiated product. I index producers and their products by $j \in J$. Each producer has zero marginal cost and zero outside option. Each producer is thus willing to accept a zero lump-sum transfer from the platform to supply the market. These simplifying assumptions allow us to ignore the supply side of the market to focus on recommendations.

Consumers—A unit mass of consumers indexed by $i \in I = [0, 1]$ have unit demand for the product. Consumers may only sample one product and must decide whether to consume a product after sampling it. Each consumer i has valuation $m_{ij}u_{ij}$ for product j. The first component $m_{ij} \in \{0, 1\}$ is a binary "match" indicator that is observable at the time of sampling and determines whether a consumer receives any value from a product. The second component u_{ij} is a consumer's match-conditional valuation for a product, which is learned through experience and realized only after consumption. Consumers have a homogenous outside option value of zero, and the valuations $m_{ij}u_{ij}$ follow some known distribution with expectation less than zero.[31] Because of this, consumers do not sample a product randomly and depend on the platform for a recommendation. Following Lee & Wright (2021), consumers may only sample one product. I omit the consumer index i in most notation.

[31] I discuss the case of a non-zero consumer outside option value in Section 3.5.4.

Advertisers—A unit mass of advertisers indexed by $k \in K = [0, 1]$ benefit from visibility in consumer-producer matches. Let n_m denote the mass of successfully matched interactions between producers and consumers. An advertiser k has valuation $\beta_k v(n_m)$ for advertisement through the platform, where β_k is drawn independently across advertisers from the common distribution function G_β with associated density g_β over support $[0, \bar{\beta}]$. I assume that v is increasing and concave and that G_β is twice continuously differentiable with a strictly increasing hazard rate $\frac{g_\beta(x)}{1-G_\beta(x)}$. I omit the advertiser index k in most notation.

Platform—A monopoly platform facilitates discovery between consumers and producers by making product recommendations to consumers. The platform can provide one of two types of recommendations to consumers: "safe" and "discovery." From the collection of consumer data (e.g., unmodelled previous consumption data), the platform has a "safe" recommendation $S_i \in J$ for each consumer i, where $m_{S_i} = 1$ with certainty.[32] The (match-conditional) safe recommendation valuations u_{S_i} are drawn independently across consumers from the common distribution function G_S with weakly decreasing density g_S over support $[0, \bar{u}]$. The platform may also provide a "discovery" recommendation $D_i \in J$, where $m_{D_i} = 1$ with a fixed probability ρ. Conditional on a successful match (i.e., $m_{D_i} = 1$), the discovery recommendation valuations u_{D_i} follow a distribution with the commonly known expectation μ. I refer to ρ as the "allure" of discovery recommendations, to μ as their match-conditional "suitability," and to the expected value $\rho\mu$ as their overall "quality." Finally, the platform has two revenue sources. It may charge a participation fee f_c to consumers, and it may charge a participation fee f_a to advertisers.

[32] The safe recommendation need not have unit match-likelihood. All results qualitatively hold as long as a safe recommendation has a higher match-likelihood than a discovery recommendation.

3.3.2 Information

Consumers and the platform only know the distributional features ρ and μ of the discovery valuations $m_{D_i}u_{D_i}$. In contrast, as the safe recommendations are familiar to consumers, and this is known by the platform, consumers and the platform have better information about the safe valuations $m_{S_i}u_{S_i}$. Each consumer i and the platform know that sampling S_i yields a successful match $m_{S_i} = 1$. The consumer also knows the eventual consumption value u_{S_i} of the familiar recommendation, while the platform only knows the distribution of u_{S_i}.

With a consumer-invariant participation fee, the most the platform can learn from consumer-revelation of the u_{S_i} is equivalent to allowing each consumer to send a signal $s \in \{S, D\}$ to the platform specifying his desired recommendation type—safe or discovery.[33] This matches commonly observed tools on platforms that allow consumers to provide input for product recommendations. A recommendation strategy for the platform is a function $\sigma: \{S, D\} \to [0, 1]$, where $\sigma(s)$ specifies the probability with which the platform recommends D_i when it receives signal s from consumer i.

3.3.3 Timing

The timing of the game is as follows. In Stage 1, the platform sets its fees and publicly announces its recommendation strategy. In Stage 2, consumers decide whether to participate (knowing their eventual safe valuations) and send signals to the platform. Advertisers decide whether to participate. In Stage 3, consumers receive recommendations according to the platform's recommendation strategy, and all payoffs are realized.

[33] A consumer's expected value is maximized with recommendation D if and only if $u_S < \rho\mu$. Consider a platform strategy that recommends D with probability $\sigma(u_S)$ upon consumer revelation of u_S. Without a negative consequence due to the invariance of f_c, any participating consumer with $u_S < \rho\mu$ would reveal $\arg\max \sigma(u_S)$, and any participating consumer with $u_S \geq \rho\mu$ would reveal $\arg\min \sigma(u_S)$. Thus, truthful revelation requires that $\sigma(\cdot)$ has a range made up of no more than two values.

I use subgame perfect Nash equilibrium as a solution concept. I focus on an equilibrium satisfying the natural condition $\sigma(D) \geq \sigma(S)$ under which consumers truthfully signal their preferred recommendation types.

3.4 Illustrative Example and Discussion of Assumptions

The model encompasses many different platform services, but it is helpful to illustrate its main features through a motivating example. In this section, I use YouTube as an example and provide further discussion of the modeling assumptions throughout.

Players—YouTube is an online video sharing service, and all three players in addition to the platform are present in the market. Video creators (producers) upload content to the platform and consumers watch videos through the platform. Third-party advertisers benefit from access to consumer attention when consumers watch creators' videos. This benefit depends on whether consumers watch creators' videos, and it does not depend on how much consumers value watching creators' videos.

A need for recommendations—Consumers face inordinate video choices—over 500 hours of videos were freely uploaded to YouTube every minute as of June 2022 (Statista, 2024b). Due to search frictions, this growing variety is virtually meaningless to consumers without a recommendation system to orient their search. By providing video recommendations, YouTube makes video consumption manageable and enables video uploads to increase consumer value or engagement, even with such a vast amount of content already on the platform.

Safe and discovery recommendations—Consumers generally watch many videos through YouTube over time, so the platform accumulates historical consumer viewing behavior on each consumer. As long as consumer preferences for given types of content remain relatively stable, past viewing behavior is predictive of future viewing of repetitive or similar content—safe recommendations have higher match-likelihoods than experimental recommendations ($\rho < 1$). However, because of the vast and continually increasing availability of content on YouTube, as well as differing amounts of data the platform has on individual consumers, product discovery through collaborative filtering or other experimental recommendation algorithms may yield higher expected value for some consumers depending on their viewing history and consequent safe recommendations ($\mu > 0$). YouTube thus faces a choice between offering safe and discovery recommendations to consumers.

Single-product sampling—In the model, consumers only consider one product recommendation (as in Lee & Wright, 2021). This is a simplifying assumption, but it may still be justified. First, if evaluating the recommended video is costless (Armstrong et al., 2009; Zou & Zhou, 2024) or less costly than evaluating another video, and if subsequent evaluation costs are high relative to the value from consumption, then a consumer may only be willing to consider one recommendation. Alternatively, consumers may experience a lack of attention beyond the first recommendation. For example, if YouTube videos are considered a "distraction" for consumers, then their outside option may increase with the number of products they evaluate. Finally, online videos may be considered an experience good. Consumers may not have time to sample another video once they have started evaluating a recommended video.[34]

By considering single-product sampling, I assume that consumers know their safe recommendation valuations but cannot watch the safe recommendation if they receive their discovery recommendations, even if the discovery recommendations do not produce successful matches. The lack of attention or experience good justifications for single-product sampling can

[34] In the case of experience goods, we must assume that consumers can evaluate products for their match success $m_j \in \{0, 1\}$ before giving any attention to third-party advertisers, i.e., if a producer is not a successful match, then a consumer stops watching the video before seeing any advertisements.

also rationalize this assumption. It may also be the case that consumers do not know which videos are similar to those they have watched in the past, but they recall the value they received from their past experiences. Without consequence to any results, we can also think of the realization of the u_{S_i} and message-sending interactions taking place in the first stage of the game. This view more closely resembles what happens on YouTube, where consumers can provide feedback as future recommendation input while they are watching a video—users can press an "I like this" button, signaling that they want to see similar content in the future (i.e., signaling $s = S$), or they can press a "not interested" button, signaling they want to see something new in the future (i.e., signaling $s = D$). In that case, consumers may not be able to find a repeat or similar video when it is not recommended to them if they have poor recall of the previous learning and feedback stage. Regardless of any direct mapping from the simplified single-product sampling setting to reality, the key element introduced by single-product sampling that drives results in the model is that consumers generate less advertisement revenue for the platform when they receive more recommendations that are not successful matches.

Recommendation quality—Quality of discovery recommendations are characterized in the model by their match-likelihood or "allure" ρ and their match-conditional expected value or "suitability" μ. On YouTube, a video description must be alluring to entice consumers to click on it. If a platform knows more about what catches the interest of consumers, then it can supply more alluring recommendations. However, an alluring video may be either suitable or unsuitable. While many niches of video may catch the attention of a given consumer, some video niches may provide the consumer with greater satisfaction from actually watching the videos compared to others.

Participation fees—On YouTube, video creators may freely upload content to the platform. Some producers are paid through advertisement-revenue sharing with the platform, but I abstract away from this feature in the model. Consumers may freely watch videos on the platform ($f_c = 0$), potentially due to a binding non-negative consumer price constraint (considered below). Third-party advertisers pay for visibility when consumers watch videos ($f_a > 0$).

As shown below, the platform's optimal recommendation strategy depends on the participation fees it charges. It is interesting to note that there is evidence that YouTube recommendations are inefficient from consumers' point of view, in line with the predictions of the model. With no consumer participation fees and positive advertiser participation fees, the platform does not always have an incentive to provide consumers with their desired recommendation types, and Ricks & McCrosky (2022) show that YouTube often does not provide recommendations in line with consumer-generated input.

No "freemium" service menu—YouTube and other streaming and media services offer a "freemium" consumer service menu in practice, through which consumers may choose between a high-cost (premium) ad-free and a free ad-present service option. The model here captures recommendation tradeoff incentives within either service option. I do not allow for multiple service options because platforms likely do not price discriminate along consumers' preferences for familiar content (i.e., the u_{S_i}), but they instead likely adopt a menu of service options to price discriminate along time-invariant heterogenous preferences for certain service features like consumers' nuisance costs from exposure to advertisements or their overall service quality preferences (Jeon et al., 2022; Sato, 2019).

3.5 Analysis

3.5.1 Equilibrium

In this section, I solve for a unique subgame perfect equilibrium. Given $\sigma(D) \geq \sigma(S)$, a participating consumer signals $s = D$ if and only if $u_S < \rho\mu$. I refer to such a consumer as a "D-

signalling consumer." Similarly, I refer to a consumer with $u_S \geq \rho\mu$ as an "S-signalling consumer." I denote equilibrium choices and outcomes with an asterisk (e.g., $\sigma^*(D)$ and f_a^* denotes the equilibrium $\sigma(D)$ and f_a).

I begin with a preliminary result that simplifies the platform's recommendation strategy. Compared to any positive value of $\sigma(S)$, the platform strictly prefers $\sigma(S) = 0$. On the consumer-side, setting $\sigma(S) = 0$ does not change D-signalling consumer payoffs but increases S-signalling payoffs, allowing for a higher consumer participation fee holding all else equal. On the advertiser-side, setting $\sigma(S) = 0$ increases the total number of successful matches created because discovery recommendations have lower match-likelihoods than safe recommendations (i.e., $\rho < 1$). This allows for a higher advertiser participation fee holding all else equal. The platform thus sets $\sigma^*(S) = 0$, and, from an S-signalling consumer's perspective, platform recommendations are efficient in equilibrium.

Lemma 3.1. *The platform always recommends safe recommendations to consumers who prefer it, i.e., $\sigma^*(S) = 0$.*

Taking $\sigma^*(S) = 0$ as given, to simplify notation let $\sigma \coloneqq \sigma(D)$ denote the platform's discovery recommendation probability for D-signalling consumers. I now solve for equilibrium demand given platform choices (σ, f_c, f_a).

Employing Lemma 3.1, an S-signalling consumer (with $u_S \geq \rho\mu$) participates if and only if $u_S - f_c \geq 0$. A D-signalling consumer (with $u_S < \rho\mu$) participates if and only if $(1-\sigma)u_S + \sigma\rho\mu - f_c \geq 0$. The mass of consumers who participate at the fee level f_c and discovery probability σ is then given by

$$n_c(f_c, \sigma) \coloneqq \begin{cases} 1 - G_S\left(\dfrac{f_c - \sigma\rho\mu}{1-\sigma}\right), & f_c \leq \rho\mu, \\ 1 - G_S(f_c), & f_c > \rho\mu. \end{cases}$$

Note that $f_c \leq \rho\mu$ is the condition necessary for any D-signalling consumer to participate. If $f_c > \rho\mu$, then participation is not profitable for D-signalling consumers even if they always get their preferred recommendation (i.e., if $\sigma = 1$).

Because every safe recommendation results in a successful match while every discovery recommendation results in a successful match with probability ρ, the mass of consumers with successful matches at the fee level f_c and discovery probability σ is given by

$$n_m(f_c, \sigma) := \begin{cases} [1 - \sigma(1-\rho)] \left[G_S(\rho\mu) - G_S\left(\frac{f_c - \sigma\rho\mu}{1-\sigma}\right)\right] + 1 - G_S(\rho\mu), & f_c \leq \rho\mu, \\ 1 - G_S(f_c), & f_c > \rho\mu. \end{cases}$$

Now, an advertiser participates if and only if $f_a \leq \beta v(n_m)$. Given any value of n_m, the mass of advertisers who participate at the fee level f_a is then given by

$$n_a(f_a, n_m) := 1 - G_\beta\left(\frac{f_a}{v(n_m)}\right).$$

Having derived consumer and advertiser demand, we can write the platform's profit as

$$\Pi(f_c, f_a, \sigma) := f_c n_c(f_c, \sigma) + f_a n_a\big(f_a, n_m(f_c, \sigma)\big).$$

If $f_c > \rho\mu$, then D-signalling consumers' expected benefit from discovery recommendations do not cover their fee to participate, only S-signalling consumers may be willing to participate, and the platform's profit does not depend on σ. In such a case, the platform sets the standard monopoly platform market participation fee f_c^* to be the opportunity cost of an additional increase in the consumer participation fee (i.e., the loss in advertiser revenue through the indirect network effect) adjusted upwards by a factor related to the elasticity of consumer participation (see, e.g., Armstrong, 2006). It employs standard monopoly pricing for f_a^* given the endogenously determined n_m due to a lack of any indirect network effect from advertisers to consumers. The tradeoff between safe and discovery recommendations is the focus of this chapter, but in this case discovery recommendations are of too poor quality to warrant any induced participation from those who would benefit from them. In what remains, I assume that the discovery recommendation quality $\rho\mu$ is sufficiently large so that at least some consumers who prefer discovery recommendations participate in equilibrium, i.e., $f_c^* \leq \rho\mu$. A sufficient condition is

$$\rho\mu \geq \max_{f_c} f_c[1 - G_S(f_c)],$$

which ensures that $f_c^* \leq \rho\mu$ holds with no match externalities (i.e., if $\bar{\beta} = 0$). For example, if G_S is the standard uniform distribution function, then this condition is given by $\rho\mu \geq \frac{1}{4}$.

Now consider any platform choice (σ, f_c), where $f_c \leq \rho\mu$. All consumers participate if and only if $f_c \leq \sigma\rho\mu$. If $f_c < \sigma\rho\mu$, then all consumers earn a strictly positive surplus from participating, and the platform could increase f_c to improve consumer-side profits without any effect on advertiser-side participation or profits. If $f_c > \sigma\rho\mu$, then an increase in σ has two opposing effects on the number of successful matches n_m. First, an increase in σ improves D-signalling consumers' participation payoffs because they more often get their desired recommendation. Thus, an increase in σ draws more D-signalling consumers into the market. A $(1 - \sigma(1 - \rho))$-fraction of these new consumer participants end up with successful matches. Second, an increase in σ decreases the number of successful matches from D-signalling consumers who participate before the change in σ because those consumers get the lower match-likelihood discovery recommendation more often. Lemma 3.2 verifies that the positive drawing-in effect always dominates the negative effect on n_m, which implies that if $f_c > \sigma\rho\mu$, then the platform can profitably increase σ. Therefore, all consumers must participate in equilibrium.

Lemma 3.2. *The equilibrium consumer participation fee satisfies $f_c^* = \sigma^*\rho\mu$, and all consumers participate.*

With Lemma 3.1 and Lemma 3.2 and the demand equations derived above, the platform's profit simplifies to

$$\Pi(f_a, \sigma) = \sigma\rho\mu + f_a\left[1 - G_\beta\left(\frac{f_a}{v\big(n_m(\sigma)\big)}\right)\right],$$

where

$$n_m(\sigma) = 1 - \sigma(1-\rho)G_S(\rho\mu).$$

Note that $n_m(\sigma)$ decreases from one to $\underline{n_m} \coloneqq 1 - (1-\rho)G_S(\rho\mu)$ as σ increases over $[0, 1]$. If $\sigma = 1$, then recommendations are efficient from the consumers' perspective and yield $\underline{n_m} < 1$ successful matches.

Now consider the platform's advertising-side profit. The platform optimally chooses $f_a \in \left[0, \bar{\beta}v\big(n_m(\sigma)\big)\right]$ for any value of $n_m(\sigma)$, where $f_a \geq 0$ because there is no positive indirect network effect from advertiser participation. The first order condition for the platform's optimal advertiser participation fee is

$$\frac{\partial\Pi(f_a, \sigma)}{\partial f_a} = 1 - G_\beta\left[\frac{f_a}{v\big(n_m(\sigma)\big)}\right] - \frac{f_a}{v\big(n_m(\sigma)\big)}g_\beta\left[\frac{f_a}{v\big(n_m(\sigma)\big)}\right] = 0. \tag{3.1}$$

The strictly increasing hazard rate assumption implies the first order condition has a unique solution $f_a\big(n_m(\sigma)\big) \in \Big(0, \bar{\beta}v\big(n_m(\sigma)\big)\Big)$, which is given implicitly by

$$f_a\big(n_m(\sigma)\big) = v\big(n_m(\sigma)\big)\frac{1 - G_\beta\left[\frac{f_a\big(n_m(\sigma)\big)}{v\big(n_m(\sigma)\big)}\right]}{g_\beta\left[\frac{f_a\big(n_m(\sigma)\big)}{v\big(n_m(\sigma)\big)}\right]}. \tag{3.2}$$

Furthermore, given any value of $n_m(\sigma)$, $f_a\big(n_m(\sigma)\big)$ uniquely maximizes the platform's advertising-side profit $\Pi_a\big(f_a, n_m(\sigma)\big) \coloneqq f_a\left[1 - G_\beta\left(\frac{f_a}{v(n_m(\sigma))}\right)\right]$ because this function is nonnegative over all $f_a \in \left[0, \bar{\beta}v\big(n_m(\sigma)\big)\right]$ and zero at the endpoints. I show in the Appendix that

$f_a(n_m(\sigma))$ is increasing in $n_m(\sigma)$ and $\Pi_a\left(f_a(n_m(\sigma)), n_m(\sigma)\right)$ is increasing and concave in $n_m(\sigma)$ (see the proof of Proposition 3.1). Intuitively, as the number of successful matches increases, advertisers earn more participation value and are charged a higher fee, and the platform's advertising-side profit increases; advertisers earn diminishing marginal returns to successful matches, so the platform earns diminishing marginal returns to successful matches. We thus have that the equilibrium advertisement participation fee satisfies $f_a^* = f_a(n_m(\sigma^*))$, where $n_m(\sigma^*)$ is the equilibrium number of successful matches.

It remains to derive σ^*, the platform's optimal probability of promoting discovery for D-signalling consumers. At any fixed fee levels, these consumers always prefer higher levels of discovery, while advertisers always prefer lower levels of discovery due to its decreasing effect on n_m. An increase in σ thus allows the platform to maintain full consumer participation at higher consumer participation fees (i.e., it increases profit from consumers), but it decreases the profit from advertisers. Specifically, a marginal increase in σ increases profit from consumers by $\rho\mu$ because every consumer's fee is increased by this much, while it decreases profit from advertisers by

$$-\frac{\partial \Pi_a\left(f_a(n_m(\sigma)), n_m(\sigma)\right)}{\partial n_m(\sigma)} n_m'(\sigma) = \Pi_a\left(f_a(n_m(\sigma)), n_m(\sigma)\right)\frac{v'(n_m(\sigma))}{v(n_m(\sigma))}(1-\rho)G_S(\rho\mu),$$

an amount that is positive and increases in σ (see the proof of Proposition 3.1). The platform attempts to equalizes these two opposing marginal effects in choosing σ, giving the first order condition

$$\Pi_a\left(f_a(n_m(\sigma)), n_m(\sigma)\right)\frac{v'(n_m(\sigma))}{v(n_m(\sigma))} = \frac{\rho\mu}{(1-\rho)G_S(\rho\mu)}. \tag{3.3}$$

If it exists, let $\hat{\sigma}$ be the unique solution to Equation (3.3). The platform's equilibrium recommendation strategy is then characterized by

$$\sigma^* = \begin{cases} 1, & \dfrac{\rho\mu}{(1-\rho)G_S(\rho\mu)} > \Pi_a(f_a(1),1)\dfrac{v'(1)}{v(1)}, \\ \hat{\sigma}, & \dfrac{\rho\mu}{(1-\rho)G_S(\rho\mu)} \in \left[\Pi_a\left(f_a\left(\underline{n_m}\right),\underline{n_m}\right)\dfrac{v'\left(\underline{n_m}\right)}{v\left(\underline{n_m}\right)}, \Pi_a(f_a(1),1)\dfrac{v'(1)}{v(1)}\right], \\ 0, & \dfrac{\rho\mu}{(1-\rho)G_S(\rho\mu)} < \Pi_a\left(f_a\left(\underline{n_m}\right),\underline{n_m}\right)\dfrac{v'\left(\underline{n_m}\right)}{v\left(\underline{n_m}\right)}. \end{cases} \quad (3.4)$$

Equation (3.4) can be interpreted as follows. If the marginal potential advertising market value beyond the consumer-efficient number of matches $\underline{n_m}$ is sufficiently small, then the platform fully prioritizes consumers as its main revenue source and provides consumers with efficient recommendations (i.e., $\sigma^* = 1$). On the other hand, if this marginal potential advertising market value is sufficiently large, even as $n_m \to 1$, then the platform fully prioritizes advertisers as its main revenue source and never provides D-signalling consumers their preferred recommendations (i.e., $\sigma^* = 0$). Otherwise, the platform balances both consumers and advertisers as significant revenue sources and mixes between giving D-signalling consumers the discovery recommendation that the consumers prefer and the safe recommendation that the advertisers prefer (i.e., $\sigma^* \in (0,1)$).

Proposition 3.1 summarizes the platform's equilibrium choices and the equilibrium outcomes.

Proposition 3.1. *The platform always recommends the safe recommendation S to consumers who signal $s = S$ by setting $\sigma^*(S) = 0$. The platform recommends the discovery recommendation to consumers who signal $s = D$ with probability $\sigma^*(D) = \sigma^*$ given by Equation* (3.4). *The platform charges the participation fee $f_c^* = \sigma^*\rho\mu$ to consumers, and all consumers participate and signal their preferred recommendation types. The platform charges the participation fee $f_a^* =$*

$f_a(n_m(\sigma^*))$ *to advertisers, where* $n_m(\sigma^*) = 1 - \sigma^*(1-\rho)G_S(\rho\mu)$ *is the equilibrium number of successful matches and* $f_a(\cdot)$ *is given by Equation* (3.2).

The proposition fully characterizes the platform's recommendation decisions and fee levels. Below I further explore the roles of match externalities through advertiser value and the components of discovery recommendation quality $\rho\mu$ in the equilibrium outcomes, using Proposition 3.1 as a starting point.

3.5.2 Advertiser Value and Equilibrium Outcomes

The following result demonstrates that match externalities through advertiser value tend to decrease consumer recommendation efficiency as well as consumer fees, and they tend to increase advertiser fees.

Proposition 3.2. *An increase in match externalities in the sense of first order stochastic dominance of the distribution* G_β *decreases the discovery recommendation probability* σ^**, decreases the consumer participation fee* f_c^**, and has an ambiguous effect on the advertiser participation fee* f_a^**. An increase in match externalities in the stronger sense of hazard rate dominance of the distribution* G_β *unambiguously increases the advertiser participation fee* f_a^**.*

Proposition 3.2 provides an explanation for consumption-narrowing "filter bubbles" observed in practice, in which consumers tend to receive repetitive or similar content from recommendation algorithms. The proposition suggests that filter bubbles should be more prevalent in markets with match externalities and that they should reduce consumption diversity more when match externalities are larger.

Figure 3.1 illustrates the platform's equilibrium choices as a function of $\bar{\beta}$ when G_S and G_β are both uniform distribution functions and $v(n_m) = \sqrt{n_m}$. In this setting, an increase in $\bar{\beta}$

corresponds to an increase in match externalities in the sense of first order stochastic dominance and of hazard rate dominance.

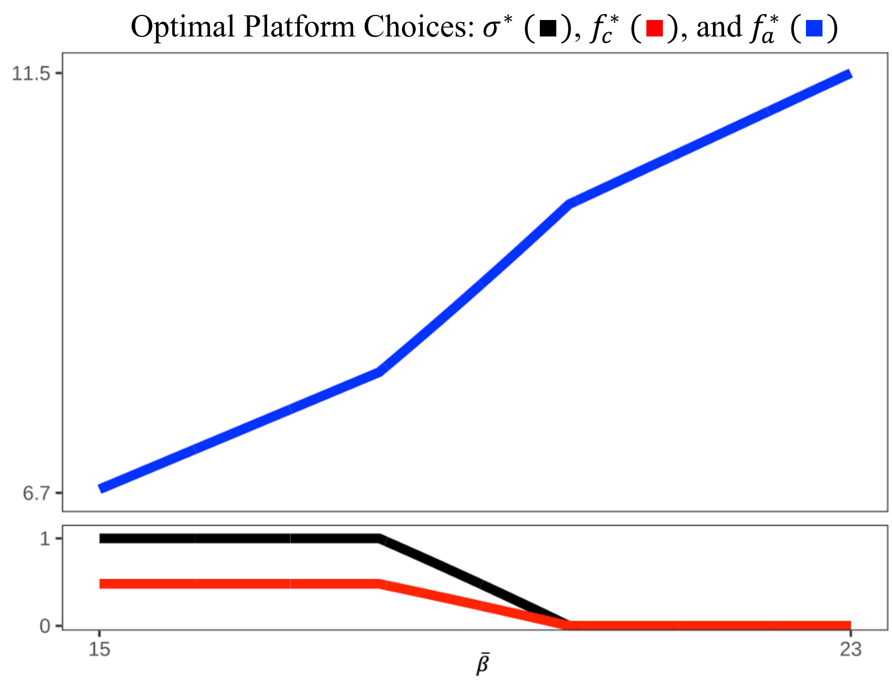


Figure 3.1: Optimal Platform Choices as a Function of Match Externalities

Notes—G_S and G_β are both uniform distribution functions and $v(n_m) = \sqrt{n_m}$. Other exogenous parameters are fixed at $\bar{u} = 1$, $\rho = 0.6$, and $\mu = 0.8$.

Intuitively, advertiser participation fees increase with advertisers' value from match access, $\bar{\beta}$. Consumer participation fees decrease with $\bar{\beta}$, and they become zero when advertisers value match-access sufficiently highly. Decreasing consumer participation fees are driven by deteriorated consumer recommendation efficiency. As $\bar{\beta}$ increases, recommendation efficiency for D-signalling consumers decreases through a decrease in σ^*; that is, as match externalities increase, consumer value generation decreases to preserve high match-likelihoods. More consumers do not receive their preferred recommendations because the platform's main revenue source shifts from consumers towards advertisers. Interestingly, however, if match externalities increase, consumer surplus increases even though consumer value decreases.

Proposition 3.3. *Consider an increase in match externalities in the sense of first order stochastic dominance of the distribution G_β. While consumer recommendation efficiency decreases, consumer surplus increases. Platform profit increases.*

Consumers are better-off with higher match externalities despite lowered consumer value generation because the platform compensates consumers with lower fees to sustain full participation.

S-signalling consumers receive the same recommendations no matter the existence or magnitude of match externalities, but they face a lower participation fee for higher levels of match externalities by Proposition 3.2. S-signalling consumers are thus better-off with larger match externalities.

D-signalling consumers receive their desired recommendations less often with larger match externalities, but they also face a lower participation fee. A D-signalling consumer earns a surplus of $(1-\sigma^*)u_S$, which decreases in σ^* and consequently increases with match externalities by Proposition 3.2. D-signalling consumers are fully charged for the discovery recommendations they expect to receive, but they do not pay for the safe recommendations they expect to receive. They are better-off with higher match externalities.

To further explore this result, it is useful to compare the extreme case of no match externalities with high levels of match externalities. When there are no match externalities (i.e., $\bar{\beta} = 0$), the platform sets $\sigma^* = 1$, and D-signalling consumers all receive the homogenous ex ante value from participation of $\rho\mu$. The platform can fully extract this value while sustaining full consumer participation. In contrast, when match externalities are sufficiently large, the platform sets $\sigma^* < 1$, and D-signalling consumers receive heterogenous ex ante values from participation of $\sigma^*\rho\mu + (1-\sigma^*)u_S$, which depend on u_S. The platform cannot extract any of the heterogenous participation value $(1-\sigma^*)u_S$ without losing participation from consumers with the lowest values of u_S. The platform thus forfeits some surplus to D-signalling consumers to sustain full participation.

Proposition 3.3 has significant practical relevance. It reveals a tradeoff between the prevalent "premium" and "free" platform subscription structures for consumers. While consumers may receive better recommendations and greater aggregate value with ad-free services compared to ad-funded services, the higher participation fee incurred completely offsets the consumer value generated from improved recommendation efficiency.

3.5.3 Incentives to Improve Recommendation Quality

I now further consider the effects of discovery recommendation allure ρ and suitability μ on platform's profit and study the platform's incentives to improve each of these aspects of recommendation quality.

An important observation for the following results is that ρ and μ have similar effects on the recommendation quality $\rho\mu$ and thus on consumer value from participation. However, they affect advertiser value from participation differently. I illustrate this point through the following thought experiment.

Consider an initial platform configuration with $\sigma > 0$, $f_a > 0$, and $f_c = \sigma\rho\mu$, where $\rho = \mu$ initially holds. Keeping f_a fixed, I consider how σ and f_c adjust in response to either an increase in ρ or an equivalent increase in μ, while ensuring that the platform's initial advertising-side profit remains unchanged. I then evaluate the net effect on the platform's consumer-side profit.

As $\rho = \mu$ initially holds, increasing either ρ or μ improves the recommendation quality $\rho\mu$ by an equivalent amount. Denote the improvement in recommendation quality by Δ and the updated discovery probability and consumer fee from an increase in ρ or μ by $(\sigma^{(\rho)}, f_c^{(\rho)} = \sigma^{(\rho)}(\rho\mu + \Delta))$ and $(\sigma^{(\mu)}, f_c^{(\mu)} = \sigma^{(\mu)}(\rho\mu + \Delta))$, respectively.

First consider the effect of increasing μ. At the initial discovery probability σ, the recommendation quality improvement (of Δ) allows the platform to charge a higher consumer fee while maintaining full participation, which increases consumer-side profit. However, it also draws some S-signalling consumers to instead signal D (as $G_S(\rho\mu)$ increases). This has no effect on consumer-side profits, but it reduces advertiser-side demand at f_a and σ because more consumers receive the low-match-likelihood D recommendation, resulting in fewer successful matches. This characterizes all of the effects of an increase in μ, and it implies that $\sigma^{(\mu)} < \sigma$ must hold in order to maintain the same number of successful matches before and after the change in μ.

An increase in ρ yields these same effects as an increase in μ through the equivalent improvement in recommendation quality; however, an increase in ρ has the addition effect of increasing the match-likelihood of all D recommendations. This has a positive effect on the number of successful matches and advertiser demand at f_a and σ. This implies that $\sigma^{(\rho)} > \sigma^{(\mu)}$ maintains the same number of successful matches before and after the change in ρ. Therefore, $f_c^{(\rho)} = \sigma^{(\rho)}(\rho\mu + \Delta) > \sigma^{(\mu)}(\rho\mu + \Delta) = f_c^{(\mu)}$, and the platform is able to maintain its initial advertising-side profit with a larger consumer-side profit after the increase in ρ, compared to an equivalent increase in μ.

What drives the preceding observation is the fact that either an increase in ρ or μ increases consumer participation value, but an increase in ρ additionally increases the match-likelihood of D. The analysis suggests that the platform tends to have more incentive to improve discovery recommendation allure compared to suitability, as the following proposition confirms. In contrast to the example above, however, an increase in ρ and μ have a differential effect on recommendation quality $\rho\mu$ when $\rho \neq \mu$. This makes it possible for the platform to prefer to improve μ when μ is sufficiently small and consumer-side revenue is a relatively significant source of the platform's profit.

Proposition 3.4. *The platform's profit increases in both discovery recommendation allure ρ and suitability μ, but with match externalities, the platform has more incentive to improve allure relative to suitability in the following two senses:*

(i) Fixing discovery recommendation quality $\rho\mu$, the platform always weakly prefers to tradeoff suitability μ in favor of allure ρ. The preference is strict if $\sigma^* > 0$ and $\bar{\beta} > 0$.

(ii) If $\sigma^* \in (0, 1)$, then $\frac{\partial \Pi^*}{\partial \rho} > \frac{\partial \Pi^*}{\partial \mu}$ if and only if $\rho < \mu + \frac{\rho\mu}{1-\rho} \frac{G_S(\rho\mu)}{G_S(\rho\mu) - \rho\mu g_S(\rho\mu)}$.

Proposition 3.4 implies that in the presence of match externalities, a platform tends to prefer discovery recommendations that are more alluring than they are suitable. In the context of ad-funded video streaming, for example, this suggests that a platform has higher incentives to make recommendations that resemble "click-bait" compared to improving the suitability of recommendations conditional on attracting engagement. If $\rho > \mu$, then increasing ρ has a smaller effect on recommendation quality $\rho\mu$ compared to increasing μ. Proposition 3.4(ii) states that even in such cases, the platform may strictly prefer to improve ρ over μ.

Figure 3.2 illustrates Proposition 3.4 when G_S and G_β are both uniform distribution functions and $v(n_m) = \sqrt{n_m}$. Equilibrium platform profit levels are represented by color on iso-recommendation-quality curves $RQ(q) := \left\{(\rho, \mu) \in [0, 1]^2 : \rho\mu = q \geq \frac{1}{4}\right\}$, where recommendation quality $\rho\mu$ is the same and equal to q for all points on each curve $RQ(q)$.

Observe that the platform's profit weakly increases in both discovery recommendation allure and suitability—profit increases (color gets darker) when travelling north or east from any point in the (ρ, μ)-plane. However, the platform tends to prefer higher levels of allure ρ compared to suitability μ, as profit is higher (colors are darker) to the right of the 45-degree line, where $\rho > \mu$. To see Proposition 3.4(i) in Figure 3.2, observe that on any iso-recommendation-quality curve $RQ(q)$, the platform always prefers to tradeoff suitability for allure—profit increases (color gets darker) when travelling counterclockwise on any iso-recommendation-quality curve. Proposition 3.4(ii) is particularly stark with G_S uniform because $\frac{\partial \Pi^*}{\partial \mu} = 0$ and $\frac{\partial \Pi^*}{\partial \rho} > 0$ for all (ρ, μ) when $\sigma^* \in (0, 1)$. That $\frac{\partial \Pi^*}{\partial \mu} = 0$ means that any consumer-value generation from an increase in suitability is completely offset by a loss in advertising profits. It is clear in the figure that the platform gains more from improved allure than from improved suitability—profit increases more (color gets darker) when travelling east compared to north in the (ρ, μ)-plane.

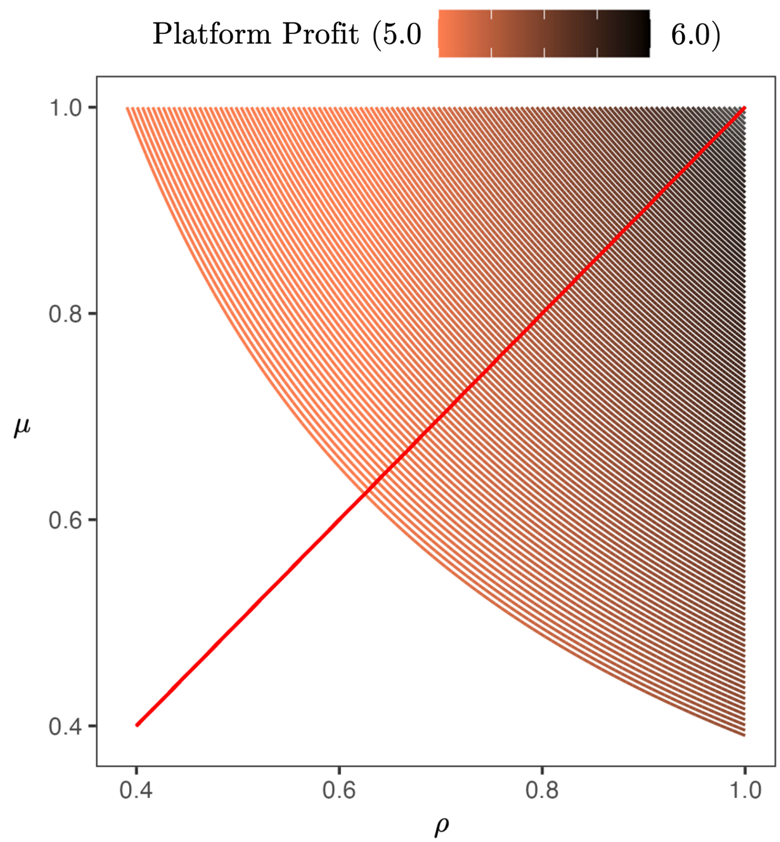


Figure 3.2: Platform Profit as a Function of Allure and Suitability

Notes—Profit is represented by color on iso-recommendation-quality curves $RQ(q) = \left\{(\rho, \mu) \in [0,1]^2 : \rho\mu = q \geq \frac{1}{4}\right\}$. G_S and G_β are both uniform distribution functions and $v(n_m) = \sqrt{n_m}$. Other exogenous parameters are fixed at $\bar{u} = 1$ and $\bar{\beta} = 20$.

3.5.4 Non-Zero Outside Option and Non-Negative Consumer Price Constraint

In some platform environments, a reasonable modeling prediction may require the platform to promote new product discovery with a positive probability in equilibrium (i.e., $\sigma^* > 0$). However, according to the previous analysis, for sufficiently large match externalities, $\sigma^* = 0$ always holds. This result relies on two important assumptions that streamlined the previous analysis and exposition but are likely not practically relevant. First, I assumed that consumers have no outside option value, although in practice they may have some attractive option that offers them positive utility from not participating on the platform. Second, I assumed that the platform faced no constraints on its choice of participation fees. In practice, however, platforms often face non-negative consumer price constraints due to adverse selection or opportunistic behaviors by consumers (Amelio & Jullien, 2012; J. P. Choi & Jeon, 2021). For example, in the video streaming context, if YouTube paid consumers to join and watch videos, then a consumer could programmatically create many user accounts to "watch" videos and earn revenue without supplying any actual consumer attention. Engagement in this case would not result in match externalities.

Allowing for consumers to have a homogenous outside option $u_0 > 0$ and considering a non-negative consumer price constraint ensures that $\sigma^* > 0$ always holds.[35] This is because if the platform sets $f_c \geq 0$ and $\sigma\rho\mu < u_0$, then some D-signalling consumers do not participate, and the platform can profitably increase σ.[36] Therefore, $\sigma^* \geq \min\left\{\frac{u_0}{\rho\mu}, 1\right\}$ must hold, and the platform always recommends D with positive probability in this case. That $\sigma^* > 0$ holds also strengthens the relevance of Proposition 3.4, which states that in the presence of match externalities, the platform tends to have strict preferences to improve allure over suitability only when $\sigma^* > 0$.

3.6 Conclusion

This chapter identifies and analyzes a key tradeoff in the development and implementation of product recommender systems. The quality of product recommendations is limited by historical data. On one hand, historical interactions may be used to provide familiar recommendations to consumers that are "safe" in the sense that consumers are likely to interact with them because they

[35] Allowing for $u_0 > 0$ with no constraint on f_c yields similar results to the analysis with $u_0 = 0$. Specifically, if match-externalities are sufficiently small, such that $f_c^* \geq 0$ holds, or if they are sufficiently large, then Lemma 3.2 holds, and the same analysis applies in both cases. For intermediate levels of match-externalities, however, the platform may set $f_c^* < 0$ and keep σ^* small enough to keep some D-signalling consumers from participating (i.e., Lemma 3.2 does not hold).

[36] This follows by the same argument used to prove Lemma 3.2 after adjusting n_c, n_a, and n_m to account for $u_0 > 0$.

have interacted with them or similar products in the past. On the other hand, a vast supply of producers makes it unlikely that "safe" recommendations provide the best expected match value for all consumers. Platforms can thus provide "discovery" recommendations that may provide more value to consumers at the risk of a lower successful match-likelihood. I show how consumer-producer match externalities to third party advertisers may incentivize a platform to deteriorate recommendation quality—providing consumers with their undesired recommendation types more often—to increase profits from advertisers. Despite lower recommendation quality, consumers are better-off with higher match-externalities because they face lower fees from the platform. To maintain high consumer participation despite lower value from participation, the platform does not charge for poor quality recommendations and forfeits some surplus to consumers. Due to the fact that consumer-producer match externalities depend on whether matches occur and not the value produced in those matches, platforms often have higher incentives to produce more "alluring" recommender systems that exhibit higher match-likelihoods compared to more "suitable" recommender systems that provide higher match-conditional expected values to consumers.

The chapter generates several practical insights. It explains why and under what conditions product recommender systems create "filter bubbles" that are inefficient for consumers. It reveals that these filter bubbles should be eliminated in ad-free service options, but consumers must pay for the increased value they receive such that their surplus ultimately decreases compared to ad-supported service options. Finally, it suggests that in environments with match externalities, platforms have strong incentives to prioritize "click-bait" recommendations over maximizing overall recommendation quality.

REFERENCES

Amelio, A., & Jullien, B. (2012). Tying and freebies in two-sided markets. International Journal of Industrial Organization, 30(5), 436–446.

Anderson, A., Maystre, L., Anderson, I., Mehrotra, R., & Lalmas, M. (2020). Algorithmic effects on the diversity of consumption on spotify. Proceedings of the Web Conference 2020, 2155–2165.

Aridor, G., & Gonçalves, D. (2022). Recommenders' originals: The welfare effects of the dual role of platforms as producers and recommender systems. International Journal of Industrial Organization, 83, 102845.

Armstrong, M. (2006). Competition in two-sided markets. The RAND Journal of Economics, 37(3), 668–691.

Armstrong, M., Vickers, J., & Zhou, J. (2009). Prominence and consumer search. The RAND Journal of Economics, 40(2), 209–233.

Bourreau, M., & Gaudin, G. (2022). Streaming platform and strategic recommendation bias. Journal of Economics & Management Strategy, 31(1), 25–47.

Brynjolfsson, E., Hu, Y., & Simester, D. (2011). Goodbye pareto principle, hello long tail: The effect of search costs on the concentration of product sales. Management Science, 57(8), 1373–1386.

Che, Y.-K., & Hörner, J. (2018). Recommender systems as mechanisms for social learning. The Quarterly Journal of Economics, 133(2), 871–925.

Chen, G., Chan, T., Zhang, D., Liu, S., & Wu, Y. (2024). The effects of diversity in algorithmic recommendations on digital content consumption: A field experiment. Available at SSRN 4365121.

Choi, J. P., & Jeon, D.-S. (2021). A leverage theory of tying in two-sided markets with nonnegative price constraints. American Economic Journal: Microeconomics, 13(1), 283–337.

De Corniere, A., & Taylor, G. (2014). Integration and search engine bias. The RAND Journal of Economics, 45(3), 576–597.

Feng, Y., Caldentey, R., & Ryan, C. T. (2022). Robust learning of consumer preferences. Operations Research, 70(2), 918–962.

Fleder, D., & Hosanagar, K. (2009). Blockbuster culture's next rise or fall: The impact of recommender systems on sales diversity. Management Science, 55(5), 697–712.

Hagiu, A., & Jullien, B. (2011). Why do intermediaries divert search? The RAND Journal of Economics, 42(2), 337–362.

Hagiu, A., Teh, T., & Wright, J. (2022). Should platforms be allowed to sell on their own marketplaces? The RAND Journal of Economics, 53(2), 297–327.

Holtz, D., Carterette, B., Chandar, P., Nazari, Z., Cramer, H., & Aral, S. (2020). The engagement-diversity connection: Evidence from a field experiment on spotify. Proceedings of the 21st ACM Conference on Economics and Computation, 75–76.

Jeon, D.-S., Kim, B.-C., & Menicucci, D. (2022). Second-degree price discrimination by a two-sided monopoly platform. American Economic Journal: Microeconomics, 14(2), 322–369.

Kremer, I., Mansour, Y., & Perry, M. (2014). Implementing the "wisdom of the crowd." Journal of Political Economy, 122(5), 988–1012.

Lee, G., & Wright, J. (2021). Recommender systems and the Value of User Data. National University of Singapore Working Paper, Https://836d83c2-C629-40a4-9ca7-B1bf324d720d. Filesusr. Com/Ugd/C6ffe6e391febde31845a08bc1ef6969182398. Pdf (June2021).

Oestreicher-Singer, G., & Sundararajan, A. (2012). Recommendation networks and the long tail of electronic commerce. Mis Quarterly, 65–83.

Pariser, E. (2011). The filter bubble: How the new personalized web is changing what we read and how we think. Penguin.

Ricks, B., & McCrosky, J. (2022). Does This Button Work? Investigating YouTube's Ineffective User Controls. Mozilla Foundation. Online: Https://Foundation. Mozilla. Org/En/Research/Library/User-Controls/Report.

Sato, S. (2019). Freemium as optimal menu pricing. International Journal of Industrial Organization, 63, 480–510.

Statista. (2023, December 8). U.S. YouTube Premium subscribers 2024 | Statista. https://www.statista.com/statistics/1261865/youtube-premium-subscribers/

Statista. (2024a, March 18). Global: YouTube users 2020-2029 | Statista. https://www.statista.com/forecasts/1144088/youtube-users-in-the-world

Statista. (2024b, April 11). YouTube: hours of video uploaded every minute 2022 | Statista. https://www.statista.com/statistics/259477/hours-of-video-uploaded-to-youtube-every-minute/

Thompson, C. (2008). If you liked this, you're sure to love that. The New York Times, 21.

Zou, T., & Zhou, B. (2023). Self-preferencing and search neutrality in online retail platforms. Available at SSRN 3987361.

APPENDIX: OMITTED PROOFS

Proof of Lemma 3.2: If $f_c < \sigma\rho\mu$, then all consumers earn a strictly positive surplus from participating and the platform can increase f_c without affecting n_c or n_a to increase its profit. Suppose $f_c > \sigma\rho\mu$ and compute

$$\frac{\partial n_m}{\partial \sigma} = [1-\sigma(1-\rho)]g_S\left(\frac{f_c - \sigma\rho\mu}{1-\sigma}\right)\frac{\rho\mu - f_c}{(1-\sigma)^2} - (1-\rho)\left[G_S(\rho\mu) - G_S\left(\frac{f_c-\sigma\rho\mu}{1-\sigma}\right)\right].$$

Because g_S is weakly decreasing, we have

$$\begin{aligned} G_S(\rho\mu) - G_S\left(\frac{f_c-\sigma\rho\mu}{1-\sigma}\right) &= \int_{\frac{f_c-\sigma\rho\mu}{1-\sigma}}^{\rho\mu} g_S(u)du \\ &\leq \frac{\rho\mu - f_c}{1-\sigma} g_S\left(\frac{f_c-\sigma\rho\mu}{1-\sigma}\right), \end{aligned}$$

which implies

$$\begin{aligned} \frac{\partial n_m}{\partial \sigma} &\geq [1-\sigma(1-\rho)]g_S\left(\frac{f_c-\sigma\rho\mu}{1-\sigma}\right)\frac{\rho\mu-f_c}{(1-\sigma)^2} - (1-\rho)\frac{\rho\mu - f_c}{1-\sigma} g_S\left(\frac{f_c-\sigma\rho\mu}{1-\sigma}\right) \\ &= \rho g_S\left(\frac{f_c-\sigma\rho\mu}{1-\sigma}\right)\frac{\rho\mu-f_c}{(1-\sigma)^2} > 0. \end{aligned}$$

That is, if $f_c > \sigma\rho\mu$, then the platform can increase σ with a positive effect on advertiser-side profit due to a positive effect on n_m. An increase in σ also increases consumer-side profit since $f_c > 0$ and n_c increases in σ. Therefore, increasing σ strictly improves the platform's overall profit.

We conclude that unless $f_c = \sigma\rho\mu$, the platform has a profitable deviation; $f_c^* = \sigma^*\rho\mu$ must hold. The result implies that $n_c(f_c^*, \sigma^*) = 1$. ∎

Proof of Proposition 3.1: The equilibrium platform choices follow from the analysis in the main text provided that $\Pi_a(f_a(n_m), n_m)$ is increasing and concave in n_m. Using the envelope theorem and first order condition (3.1), we may compute

$$\frac{\partial \Pi_a(f_a(n_m), n_m)}{\partial n_m} = \Pi_a(f_a(n_m), n_m)\frac{v'(n_m)}{v(n_m)} \geq 0.$$

Differentiating this expression with respect to n_m yields

$$\frac{\partial^2 \Pi_a(f_a(n_m), n_m)}{\partial n_m^2} = \frac{\partial \Pi_a(f_a(n_m), n_m)}{\partial n_m} \frac{v'(n_m)}{v(n_m)} + \Pi_a(f_a(n_m), n_m) \frac{v''(n_m)v(n_m) - v'(n_m)^2}{v(n_m)^2}$$

$$= \Pi_a(f_a(n_m), n_m) \frac{v''(n_m)}{v(n_m)} \leq 0.$$

Therefore, $\Pi_a(f_a(n_m), n_m)$ is increasing and concave in n_m, and the proposition follows from the main text.

Note that the first order condition (3.1) implies that $f_a'(n_m) \geq 0$. Condition (3.1) can be rewritten as

$$\frac{f_a(n_m)}{v(n_m)} = \frac{1 - G_\beta \left[\frac{f_a(n_m)}{v(n_m)}\right]}{g_\beta \left[\frac{f_a(n_m)}{v(n_m)}\right]},$$

so $\frac{\partial}{\partial n_m} \frac{f_a(n_m)}{v(n_m)} = 0$ must hold. Since $v'(n_m) \geq 0$, which has a decreasing effect on $\frac{\partial}{\partial n_m} \frac{f_a(n_m)}{v(n_m)}$, it must be that $f_a'(n_m) \geq 0$. ■

Proof of Proposition 3.2: Suppose the distribution function of the β_k increases in the sense of first-order stochastic dominance and denote the new equilibrium platform choices by $\left(\dot{f}_a^*, \dot{f}_c^*, \dot{\sigma}^*\right)$. Advertising profit $\Pi_a(f_a, n_m) = f_a \left[1 - G_\beta \left(\frac{f_a}{v(n_m)}\right)\right]$ increases at any choice (f_a, n_m), so the optimal advertising profit $\Pi_a(f_a(n_m), n_m)$ must increase at any n_m. From direct observation of Equation (3.4), this allows only $(\sigma^*, \dot{\sigma}^*) \in \left\{(1,1), \left(1, \dot{\hat{\sigma}}\right), \left(\hat{\sigma}, \dot{\hat{\sigma}}\right), (\hat{\sigma}, 0), (0,0)\right\}$ where $\dot{\hat{\sigma}}$ denotes a new solution to Equation (3.3): $\Pi_a(f_a(n_m), n_m) \frac{v'(n_m)}{v(n_m)} = \frac{\rho\mu}{(1-\rho)G_S(\rho\mu)}$. If $(\sigma^*, \dot{\sigma}^*) \neq \left(\hat{\sigma}, \dot{\hat{\sigma}}\right)$, then $\sigma^* \geq \dot{\sigma}^*$ obviously holds. It also holds if $(\sigma^*, \dot{\sigma}^*) = \left(\hat{\sigma}, \dot{\hat{\sigma}}\right)$. This is because $\Pi_a(f_a(n_m), n_m) \frac{v'(n_m)}{v(n_m)}$, which decreases in n_m, increases at each n_m after the change in advertising market value, while $\frac{\rho\mu}{(1-\rho)G_S(\rho\mu)}$ remains constant. Then n_m must increase after the change in match externalities, implying $\hat{\sigma} \geq \dot{\hat{\sigma}}$. Hence $\sigma^* \geq \dot{\sigma}^*$, and it is immediate that $f_c^* = \sigma^* \rho\mu - u_0 \geq \dot{\sigma}^* \rho\mu - u_0 = \dot{f}_c^*$.

There is an indeterminate effect on f_a^* unless we consider an increase in the advertisement market value in the stronger sense of hazard rate dominance (i.e., if $\frac{g_\beta(x)}{1-G_\beta(x)}$ decreases for all x after a change in the distribution of β_k). Hazard rate dominance implies first order stochastic dominance, so $\dot{n}_m^* \geq n_m^*$ because $\sigma^* \geq \dot{\sigma}^*$ from the previous result. Because $f_a'(n_m) > 0$, this increase in n_m has a positive effect on f_a^*. Further, at any fixed n_m, the RHS of Equation (3.2) increases under hazard rate dominance so that $\dot{f}_a(n_m) \geq f_a(n_m)$ for all n_m—another positive effect on f_a^*. Taken together, we have $\dot{f}_a^* = \dot{f}_a(\dot{n}_m^*) \geq f_a(n_m^*) = f_a^*$ under the stronger hazard rate dominance notion of increased match externalities. ■

Proof of Proposition 3.3: Consumer recommendation efficiency is decreasing in match externalities in the sense of first order stochastic dominance of G_β because σ^* decreases in match externalities in this sense from Proposition 3.2. An S-signalling consumer's equilibrium payoff is $u_S - f_c^* = u_S - \sigma^* \rho\mu$, and a D-signalling consumer's equilibrium payoff is $\sigma^* \rho\mu + (1 - \sigma^*) u_S - f_c^* = (1 - \sigma^*) u_S$. Both consumer types' payoffs decrease in σ^* and thus increase in match externalities. It is immediate that platform profit increases in match externalities because such a change increases advertiser demand n_a holding all platform choice variables constant, hence optimal platform profit must increase. ■

Proof of Proposition 3.4: The platform's optimal profit is

$$\Pi^* = \sigma^* \rho\mu - u_0 + \Pi_a\left(f_a\big(n_m(\sigma^*)\big), n_m(\sigma^*)\right),$$

where $n_m(\sigma^*) = 1 - \sigma^*(1 - \rho) G_S(\rho\mu)$. By the envelope theorem,

$$\frac{\partial \Pi^*}{\partial \rho} = \sigma^* \mu + \Pi_a\left(f_a\left(n_m(\sigma^*)\right), n_m(\sigma^*)\right)\frac{v'\left(n_m(\sigma^*)\right)}{v\left(n_m(\sigma^*)\right)}\sigma^*[G_S(\rho\mu) - (1-\rho)\mu g_S(\rho\mu)]$$

$$\geq \sigma^* \min\left\{\mu, \mu\left(1 - \frac{\rho\mu g_S(\rho\mu)}{G_S(\rho\mu)}\right) + \frac{\rho\mu}{1-\rho}\right\} \geq 0,$$

$$\frac{\partial \Pi^*}{\partial \mu} = \sigma^* \rho - \Pi_a\left(f_a\left(n_m(\sigma^*)\right), n_m(\sigma^*)\right)\frac{v'\left(n_m(\sigma^*)\right)}{v\left(n_m(\sigma^*)\right)}\sigma^*(1-\rho)\rho g_S(\rho\mu)$$

$$\geq \sigma^* \rho\left(1 - \frac{\rho\mu g_S(\rho\mu)}{G_S(\rho\mu)}\right) \geq 0,$$

where the second line in each computation follows by Equation (3.3) when $\hat{\sigma}$ is well-defined and otherwise because the RHS of Equation (3.3) is larger than the LHS iff $\hat{\sigma}$ is not well-defined and $\sigma^* = 1$.

Consider statement (i). When $\sigma^*, \bar{\beta} > 0$, at a fixed recommendation quality $\rho\mu$, the platform strictly benefits from increased advertiser demand without any change to consumer demand from trading off μ for ρ when keeping its recommendation strategy and fees unchanged. If either σ^* or $\bar{\beta}$ is zero, then such a change has no effect on the platform's profit. Statement (ii) follows by simple algebra. ■