LEVERAGING MATHEMATICAL JUSTIFICATION OPPORTUNITIES TO SUPPORT STUDENTS' AGENCY IN THE LEARNING PROCESS: A CASE STUDY OF ONE TEACHER'S PRACTICE AND PERSPECTIVES

By

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ABSTRACT

Mathematical reasoning is essential to knowing and doing mathematics, and mathematical reasoning practices can be leveraged by teachers to support student voice and choice in the learning process. This dissertation explored how one elementary teacher provided opportunities for students to engage in the mathematical reasoning practice of justification and how these opportunities supported her goals for equity in a classroom of diverse students. Given that agency emerged as a prominent theme in the teacher's equity goals, this dissertation focused analysis of findings on the relationship between mathematical justification and agency, with connections to interrelated equity constructs (such as mathematical identity) expressed by the teacher, as well. Mathematical justification can be defined as, "the process of supporting your mathematical claims and choices when solving problems or explaining why your claim or answer makes sense" (Bieda & Staples, 2020, p. 103). Case study methodology was used to examine the teacher's curriculum materials, classroom enactment strategies, equity goals, and perceptions on the relationship between justification and equity. This study was situated in a self-contained, racially/ethnically diverse, fourth-grade classroom in an urban district in the Midwest, taught by a female Teacher of Color. Findings are reported in the form of three articles, each discussing connections between justification and equity (with a focus on agency). Each article was written for different primary audiences: the first for mathematics education researchers, the second for mathematics teacher educators, and the third for practitioners.

The first article presents the broadest overview of how and why the teacher provided justification opportunities, with the second two articles focusing more in-depth on specific types of justification opportunities. This first article, which is Chapter 2 of the dissertation, was written for researchers as the primary audience and will be submitted for publication consideration to the *Journal of Mathematics Teacher Education* (JMTE). This article was guided by two research questions: (1) *How does a teacher facilitate opportunities for students to engage in the mathematical practice of justification?* (2) *How does the teacher perceive mathematical justification as supporting her equity goals?* Findings showed that the teacher was committed to regularly incorporating justification opportunities in her mathematics lessons. She facilitated justification opportunities through purposeful task design and enactment strategies, situated within a conducive classroom environment. These opportunities were driven by her equity goals, which included a focus on supporting student agency in the learning process and cultivating the

development of positive mathematical identity in students. Given how infrequent justification is implemented in classrooms, and the limited research on the relationship between justification and equity in the literature, these findings help illuminate how justification can be leveraged by a teacher in ways relevant to their classroom context and equity goals.

The second manuscript (Chapter 3) was written for mathematics teacher educators as the primary audience and is intended to be submitted for publication consideration to the *Mathematics Teacher Educator* (MTE). This article was guided by two research questions: (1) *What are one teacher's goals for promoting equity in the classroom?* (2) *How does the teacher leverage the practice of mathematical justification to meet her equity goals?* Findings showed that facilitating students' agency and supporting students in developing positive mathematical identity were the primary equity goals for the teacher. Advancing these goals involved engaging students in problem-based learning experiences that focused on discovering and justifying mathematical patterns and relationships in ways that centered the students' ideas and reasonings. The article elaborates how and why the nature of problem-based learning experiences opened up spaces for leveraging justification to meet these equity goals.

The third manuscript (Chapter 4) is a practitioner-focused piece written for the journal, *Mathematics Teacher: Learning and Teaching PK-12* (MTLT). This article discusses how the beginning of curricular units can be conducive places to facilitate justification opportunities in ways that support students' engagement with new content and agency in the learning process. The article describes how and why the teacher in this study facilitated justification opportunities at the beginning of units and offers guidance for other teachers hoping to modify their practice to provide their students with more mathematical justification opportunities.

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CHAPTER 1: INTRODUCTION

Teacher practice plays a key role in advancing equitable experiences and outcomes for students. Specifically, teacher practice that facilitates mathematical reasoning opportunities can advance many aspects of equity. For example, engaging students in mathematical reasoning can strengthen students' problem-solving skills, promote conceptual understanding of content, and contribute to productive dispositions towards mathematics (Boaler, 2022; Staples et al., 2012). Justification is a particularly powerful reasoning practice because it can be leveraged to engage students in the process of making their own mathematical discoveries, defend claims and conjectures they make during that discovery process, develop sensemaking to understand why their ideas or their peers' ideas might make sense, and build knowledge together as a classroom community. Justification can provide opportunities for students to take ownership of the mathematics, in ways that center their voices, perspectives and ideas, sharing in authority of knowledge with the teacher (Ball & Bass, 2008; Bieda & Staples, 2020). Bieda and Staples (2020) define mathematical justification as, "the process of supporting your mathematical claims and choices when solving problems or explaining why your claim or answer makes sense (p. 103)." Justification can be considered a "learning practice" because it supports students in learning and doing many aspects of mathematics (Cohen & Ball, 2001; Staples et al., 2012). This practice is further supported in the *Standards for Mathematical Practice*, particularly the practice, *construct viable arguments and critique the reasoning of others*, which specifies students, "justify their conclusions, communicate them to others, and respond to the arguments of others" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 6-7). Bieda and Staples (2020) describe justification as an "equity practice" because it supports students' access to content and agency in the learning process. With regard to agency, Bieda and Staples (2020) state that providing students' opportunities to justify their reasoning builds their agency since students are generating mathematical ideas themselves in ways that make sense to them rather than practicing doing someone else's mathematics. This dissertation focuses primarily on the relationship between justification and agency, because agency emerged as a prominent equity goal for the teacher in the study. By *agency*, I refer to when students are empowered to have influence in their learning environment, by which they play a role in their own mathematical development, and when they learn from/with their peers

through the decentering of the teacher as a mathematical authority (Grootenboer & Jorgensen, 2009).

The mathematical reasoning practice of justification has benefits for both students' learning and teachers' instruction. Similarly, as described above, for students, the justification process can support students in talking about mathematics and learning from one another's contributions, facilitate their agency in the learning process, increase engagement, strengthen reasoning skills, support conceptual understanding of content, and foster positive mathematical identities (Bieda & Staples, 2020; Boaler, 2022; Staples et al., 2012). For teachers, eliciting students' justifications can help with assessing students' working understanding of content at a given time (Staples et al., 2012). This can help teachers be responsive to or build from what students know in ways that facilitate students' agency to guide them(selves) to the new content to be learned.

While there are many benefits to incorporating justification into mathematics instruction, it is a practice that is lacking in classrooms (Jacobs et al., 2006; Melhuish et al., 2020). One reason could be that some curriculum materials do not offer guidance to teachers on how to support justification (Staples et al., 2012). However, Bieda et al. (2010) found that even with curriculum materials that did provide some guidance, many teachers still did not engage students in justification opportunities. One reason for this could be that many teachers have limited conceptions and experiences with justification (Knuth, 2002; Staples et al., 2012). For instance, many teachers did not experience learning how to develop justifications as mathematics students themselves, or justification may not have been a prominent focus in their methods courses during teacher preparation. Additionally, standardized tests, which predominantly include multiple-choice questions to be able to quickly assess many students' answers to questions at scale, tend to focus on procedural fluency over argumentation practices like justification. Since various federal or district policies hold teachers accountable to their students' standardized tests scores in some capacity (NCLB Act of 2001, 2002), it makes sense that teachers' instruction would continue to be influenced by a focus on procedures with less attention to reasoning practices like justification. While policies can influence teacher practice, so can a teacher's goals and beliefs (Aguirre & Speer, 1999). Given the various reasons why justification may not be a prominent practice despite its many benefits, this dissertation sought to examine why one teacher perceived justification to be an important practice for advancing her equity goals and how she facilitated

justification opportunities to meet those goals. Furthermore, I argue that understanding how the justification process is facilitated can be better understood when analyzed through the lens or context of the teacher's equity goals. Given there is limited research in the field on justification as a process (Karunakaran & Levin, 2022) and a large gap in the literature on research that examines the relationship between justification and equity (Bieda et al., 2023), this dissertation sought to better understand how justification opportunities can be leveraged to meet a teacher's equity goals.

Conceptualizing Equity

There are many conceptualizations of the meaning of equity in the field (Aguirre et al., 2013; Cochran-Smith & Keefe, 2022; Gutiérrez, 2009; Horn, 2012). Cochran-Smith and Keefe (2022) conducted a literature review on the word "equity" and identified three themes in the discourse on equity in teacher education literature: (1) *Equity as a by-product of accountability*, (2) *Equity as differentiated from equality*, and (3) *Equity as a result of "best practice."* Regarding the meaning of the first, this refers to the many policies and initiatives that were designed to make teacher education more accountable to the public, such as rigorous teacher licensure, accreditation standards, and teacher evaluation measures. The assumption was that high teacher quality results in high student achievement, suggesting that all students succeeding or achieving at higher levels means more equitable student outcomes. The second theme refers to the notion that *equity* does not mean *equal*, *fair*, or *same*, which recognizes that students are unique with different learning styles, circumstances, and needs and can thus require different resources to learn the same content. Teacher education studies explain that learning differences among students are not a result of deficits in students, but rather, all students have strengths and bring cultural funds of knowledge which teachers can be responsive to and leverage to support their access to content and thus achieve equitable outcomes. The third theme centers around equity being a result of "best practice," which says that there are "high-leverage practices" or "core practices" (e.g., facilitating discussion, eliciting and interpreting student thinking, coordinating and adjusting instruction) that teachers can do to promote equitable experiences and outcomes for students (Ball & Forzani, 2009). This dissertation study focused on equity as related to the third theme, given the focus on understanding mathematical justification as a key practice for advancing equity for students.

Considering the vast use of the word equity in the literature, Cochran-Smith and Keefe (2022) distinguish between what they call *thin equity* and *strong equity*. In defining thin equity, Cochran-Smith and Keefe (2022) describe, "When we talk about *thin equity* in teacher education policy and practice… we refer to initiatives that assume equity has to do with an individual's access to education resources" (p. 19; emphasis in original). In describing strong equity, Cochran-Smith and Keefe (2022) explain, "We refer to *strong equity* to acknowledge the complex and intersecting historical, economic, and social systems that create inequalities in access to teacher quality in the first place" (p. 19, emphasis in original). Essentially, thin equity describes policies and practices that center around individualism and neoliberalism, whereas strong equity describes the complexities of learning as being socially, historically, and economically interrelated. A focus on strong equity can lead to positive social change and better learning experiences and outcomes for students, particularly students from historically and currently marginalized groups, such as Black and Latinx students.

Gutiérrez (2009) also notes different uses of equity in the field, specifically within mathematics education. Gutiérrez (2009) explains there has been a "dominant" discourse on equity that has centered around access to mathematics content and achievement in the subject. *Access* refers to resources and opportunities (e.g., high quality teachers, curriculum materials, tools) that allow each individual student to fully participate in mathematics, and *achievement* refers to student outcomes on institutionally acknowledged performance and participation measures (e.g., standardized tests). These dominant dimensions of equity have similar characterizations to the individual and neoliberal notions of thin equity described by Cochran-Smith and Keefe (2022). Gutiérrez (2009) states that to attain "true equity" (p. 6) for students, the field must also center constructs of power and identity, which are described as *critical dimensions of equity*, to also support students' access to and achievement in mathematics. Gutiérrez describes the *identity* dimension of equity as students seeing themselves reflected in and as doers of mathematics. *Power* is characterized by student empowerment, voice, and influence in the mathematics classroom and beyond. Gutiérrez's frames these four equity dimensions by situating them on intersecting axes to illustrate that they are interrelated and work together to fully realize equity. Access and achievement comprise the "dominant axis," and identity and power comprise the "critical axis." Gutiérrez (2009) calls for an increased focus on the identity and power dimensions, given the dominant focus in the field (historically and

currently) on the constructs of access and achievement. I argue that mathematical justification, as driven by a teacher's disposition that includes a focus on critical dimensions of equity, can be a powerful practice for advancing equity for students, particularly in a racially/ethnically diverse classroom of students.

I, as supported by others in the literature, view agency to be a critical equity construct. Bieda and Staples (2020) and Berry (2020) both view agency as an equity construct, interrelated with other equity dimensions. Specifically, Bieda and Staples (2020) view agency and access as interrelated equity constructs, and Berry (2020) views agency and identity as interrelated equity constructs. Bieda and Staples (2020) define *agency* as "when students develop a sense that they can do mathematics and generate mathematical ideas; each student sees mathematics as a tool to use," and *access* as referring to, "each student in a class having opportunities to engage in rigorous mathematics and to learn mathematics that is meaningful" (p. 104). Bieda and Staples (2020) describe agency and access as "intertwined," stating that students' agency is developed in relation to the opportunities they are provided (access), and students' access to mathematics is also shaped by how they engage with content and challenges, participate in class, and perceive themselves as capable (agency). Berry (2020) uses NCTM's (2014) *Principles to Actions* framing of equity to describe how agency and identity are interrelated constructs that can advance equity. In defining agency, Berry (2020) cites Aguirre et al. (2013) and situates identity in the definition of agency, "Agency is our identity in action and the presentation of our identity to the world" (slide 15). In other words, when a student identifies as being good at mathematics, they adopt behaviors and actions associated with smartness in mathematics; and when students have the opportunities to make choices and decisions associated with agency in the learning process, this is cultivating and reflecting their mathematical identities. Chapter 3 provides further elaboration of Berry's (2020) framing of agency and identity as interrelated constructs that can advance equity.

Given that agency and identity are interrelated equity constructs (Berry, 2020) and identity is part of the critical axis (Gutiérrez, 2009), I view agency to be an equity construct, situated on the critical axis. The construct of agency takes into account the learning environment as a social (not individual) system, where students feel empowered to use their voice, make their own decisions, have their ideas centered, and learn from one another, sharing in authority of knowledge with the teacher. Having agency provides opportunities for students' identities to be

reflected in the learning process, and in turn, they are building positive mathematical identities by seeing themselves as capable knowers and doers of mathematics. Because having agency means a more active and student-centered learning process (e.g., versus passively receiving information via lecture to be memorized and regurgitated, whereby conceptual understanding becomes less certain), this can result in more active student engagement, increased access to content, deeper understanding of mathematics, and thus higher achievement outcomes. Webb et al. (2014) found that teacher practice that supported student agency through centering their ideas and reasonings led to increased participation, engagement, and achievement in mathematics. For these reasons, I view agency as a critical equity construct. Chapter 3 elaborates the relationship between agency and identity (with connections to justification), from the literature and from the perspective of the teacher participant in the study.

Bieda and Staples (2020) define justification as an "equity practice" that can advance agency. Justifying one's reasoning supports agency because students are tasked with generating and defending mathematical ideas themselves in ways that make sense to them, rather than practicing doing the teacher's mathematics. Bieda, Staples, and colleagues (2023) situate their conceptualization of *justification* from their earlier work (i.e., Bieda & Staples, 2020) between identity and access on Gutiérrez's axes. They view identity as part of agency, and access and agency as intertwined equity constructs that can be advanced through justification. Figure 1 shows the image they created in Bieda et al. (2023) to illustrate where they situate their conceptualization of justification on Gutiérrez's axes.

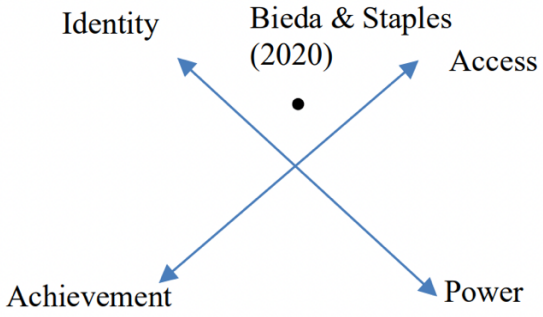

Figure 1. Bieda et al.'s (2023) Conceptualization of *Justification* (Bieda & Staples, 2020) on Gutiérrez's Dimensions of Equity Axes

Gutiérrez (2009) states that all four dimensions are necessary for attaining "true equity" (p. 6) and explains that the critical axis and the dominant axis have an inverse relationship which

can support one another. “Learning dominant mathematics may be necessary for students to be able to critically analyze the world, while being able to critically analyze the world may provide entrance into dominant mathematics” (p. 6). Thus, a focus on identity and power is essential for supporting students’ access to and achievement in mathematics. I argue that justification opportunities (which can center students’ ideas, perspectives, claims, and decisions) can advance all aspects of equity, as supporting students’ identity and power in mathematics can, in turn, promote access and achievement. Given Gutiérrez’s (2009) claim that identity and power do not frequent the dominant discourse of equity in mathematics (yet are necessary for advancing equity), I argue that a focus on the critical dimensions will be important for justification opportunities to truly advance equity for students, particularly in a classroom of racially/ethnically diverse students.

Leveraging justification to support critical aspects of equity, such as agency, requires the teacher to share authority of knowledge with students. I draw on Zavala and Aguirre’s (2024) conceptualization of *distributing intellectual authority* to frame how justification opportunities can be leveraged to support critical dimensions of equity. Zavala and Aguirre (2024) define distributing intellectual authority as involving, “positioning students as mathematical authorities in the classroom, using participation structures that expand student mathematical contribution opportunities, and embracing the role of teacher as facilitator and co-constructor of knowledge in the classroom” (p. 99). This definition encompasses the constructs of agency, identity, and justification, as it mentions providing students the autonomy to have their ideas and identities reflected in the learning process. Zavala and Aguirre (2024) describe three “instructional approaches” involved in distributing intellectual authority: (1) *position students as holders of math knowledge*, (2) *expand use of collaborative participation structures*, and (3) *embrace the teacher-as-facilitator role*. Zavala and Aguirre’s (2024) conceptualization of distributing intellectual authority is a helpful framework for explaining ways in which justification opportunities can support critical dimensions of equity, such as advancing students’ agency and cultivating positive mathematical identities, which will be elaborated in Chapter 3. Figure 2 shows where I situate justification (when leveraged with an intentional focus on agency) on Gutiérrez’s axes. Chapter 3 provides the most in-depth elaboration of the relationship among justification, agency, and identity.

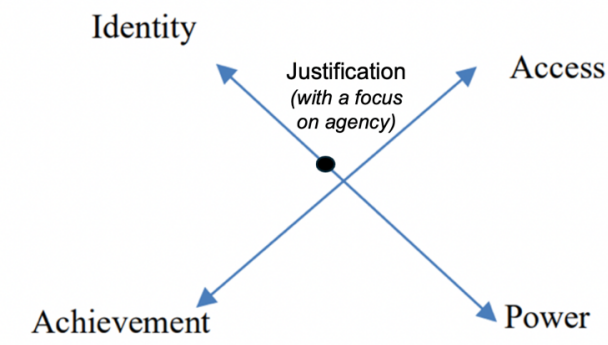

Figure 2. Conceptualization of Justification on Gutiérrez's Dimensions of Equity Axes

In this dissertation, I conceptualize equity as *strong equity* (Cochran-Smith & Keefe, 2022) and considered all four *dimensions of equity* (Gutiérrez, 2009) in the selection of the teacher participant and analysis of her equity goals. Because my views on equity align with strong equity (given its focus on learning being influenced by multiple social systems), and because I sought to understand the relationship between critical dimensions of equity and mathematical justification, the participant I selected was a teacher who I knew also held strong equity views, based on the type of school she worked at and my experiences with her as a former colleague. More about the depth of her equity goals that were uncovered through the study will be elaborated throughout the manuscripts of the dissertation.

Methodology of the Dissertation

Research Focus. The research questions guiding this dissertation centered around the relationship between justification and equity, with analysis focusing on the teacher's specific equity goals (in which agency emerged as a central theme). The dissertation explored questions pertaining to the justification opportunities the teacher provided and her perceptions of the impact of these opportunities on students. The overarching research questions guiding the study were (1) How does a teacher facilitate opportunities for students to engage in the mathematical practice of justification? (2) How does the teacher perceive mathematical justification as supporting her equity goals? The specific research questions that guided each manuscript will be outlined in the next section, which summarizes each article.

Case Study Methods. This dissertation was conducted using case study methodology, which refers to a "case" bounding the parameters of the study (Merriam & Tisdell, 2015). The case that bounded this study was one teacher's practice and perspectives, regarding justification opportunities and her equity goals. The teacher's practices and perspectives were examined

through interviews, classroom observations, and implemented curriculum materials. The study is situated in Merriam's (1998) constructivist pragmatic epistemological assumptions of case study, which describes using findings from research to inform practice. One goal of this dissertation was to provide insight into how one teacher incorporated justification opportunities to meet her equity goals, with the overarching aim to support other teachers in leveraging justification within their own contexts and to meet their equity goals.

School Context and Participant Background. The context of this study was one fourth-grade classroom in an urban district in the Midwest, with students predominantly from marginalized groups (including Black and Latinx students). The classroom was situated in a school with Title I status. Because the majority of students qualified for free or reduced lunch, the school provided all students free breakfast, lunch, and snacks during the school day. The school, Midwest Academy (pseudonym chosen by the participant), was known for its commitment to place-based education and social justice. The teacher in the study defined place-based learning in the following way, and relative to the context of her school:

> *[Midwest Academy] defines it as creating critical thinkers who contribute to the wellbeing of their community. I guess that place based is like putting the learning in the context of your place. Sometimes you can solve problems in that place that you know can come from your learning, or sometimes it's just you can learn about your place to develop and foster love of it and stewardship of it.*

At the grain size of a curricular unit, the teacher described place-based learning as,

> *Usually, a place-based unit will begin with a question, I guess, that you're trying to answer. It could be about the environment, it could be about history, it could be about perception, it could be about identities. You're using your place to answer, I think they call it, an essential question.*

The participant in this study was the teacher of this fourth-grade classroom. The teacher, Josie (her chosen pseudonym) was selected from a combination of *convenience sampling methodology* due to proximity and a pre-existing relationship I had with the teacher and *purposeful sampling methodology* based on practices and dispositions held by the teacher as someone hired to teach at a place-based school (Merriam & Tisdell, 2015). From having worked together at the same school for one year, I recalled that Josie was passionate about creating community-oriented and culturally relevant lessons, and that all teachers at Midwest Academy received yearly professional development on place-based learning. Place-based learning is situated in strong equity perspectives, given how place-based learning recognizes socio-cultural

and community-based knowledge. I knew incorporating student voice and ideas into mathematics instruction was important to Josie, and I was curious to learn more about how she conceptualized the relationship between justification and equity. Josie identifies as a Filipina woman and immigrated to the United States at age 13, which could have influence on her commitment to connecting culture, community, and school content because experiencing schooling in two countries informs her perspectives on education, and I know staying connected to her culture is important to her. At the time of the study, Josie had nine years of teaching experience, with six of those years being at Midwest Academy.

Researcher Positionality. Given that I taught at Midwest Academy at one point when Josie was a teacher there as well, I brought a somewhat insider lens to the research context. For example, I knew that Josie was passionate about place-based education, social justice, and student-centered learning. Given the potential for mathematical justification to advance critical aspects of equity, I was curious about how Josie thought about this connection and how she facilitated student-centered justification opportunities in her classroom. Since I never collaborated with Josie in lesson planning or observed her instruction when I worked at the school, I did not have a full understanding of her practice and perspectives and thus had more of an outsider lens with regard to her own classroom. This insider/outsider perspective I brought suggested to me that Josie's practice and perspectives could provide an interesting context to explore how justification opportunities are facilitated in ways that might relate to or advance critical dimensions of equity, given that these dimensions are left out of dominant discourse but crucial for realizing, what Gutiérrez (2009) calls, "true equity."

It is important to acknowledge that I brought to this research a particular status, as a White researcher affiliated with a prominent university who entered a classroom space of predominantly Students of Color and a Teacher of Color. Because I knew my presence could have some influence on the space, it was important to me that I learned from a teacher with whom I had an established and trusting relationship with and who remembered me as a teacher at one point. Josie knew that my role was to learn from her. Mindful of this status I brought, I tried to decenter myself by quietly observing at the edge of the classroom, and by centering Josie's voice during our meetings by asking questions that positioned her as expert and never offering or answering questions about my opinions. The insider/outsider lens I brought allowed for the

selection of, and deep learning from, an exceptionally equity-minded teacher and a trusting researcher-participant relationship.

Summary of Manuscripts

This dissertation contains three manuscripts, each intended to be submitted to a different journal. All have an overarching focus on mathematical justification. Given that supporting student agency emerged as a prominent equity goal for the teacher in the study, all three manuscripts discuss connections between justification and agency. Each manuscript also discusses other interrelated constructs within Josie's equity goals. For example, Manuscripts One and Two elaborate Josie's connections to mathematical identity. The intention of writing three articles with the same overarching focus (on justification and agency) but each for a different primary audience (researchers, teacher educators, and practitioners, respectively), is to support the ongoing challenge in the field of bridging research and practice, aligning with constructivist pragmatic case study methodology.

Manuscript 1: Researcher-focused Audience. The intended journal for the submission of this manuscript was the Journal of Mathematics Teacher Education (JMTE). The primary audience of JMTE is teacher education researchers. Two research questions guided this paper: (1) How does a teacher facilitate opportunities for students to engage in the mathematical practice of justification? (2) How does the teacher perceive mathematical justification as supporting her equity goals?

Findings indicated that Josie was committed to regularly implementing justification opportunities in her mathematics lessons. She did this through purposeful task design and enactment strategies, and by establishing a conducive classroom environment. These opportunities were all driven by her personal equity goals, which focused on increasing student agency in the learning process. Given the intertwinement of agency and identity, Josie's perspectives on the relationship between agency, identity, and justification are also elaborated. This study helps fill gaps in the literature that suggest a limited understanding on the relationship between mathematical justification and critical dimensions of equity (Bieda et al., 2023). A better understanding of mathematical practices (such as justification) that can advance students' agency and positive mathematical identity development is important for supporting equitable outcomes for students in mathematics.

Manuscript 2: Teacher Educator-focused Audience. This manuscript was written with the intention to submit to the Mathematics Teacher Educator (MTE). The primary audience of MTE is teacher educators. This paper was guided by the following two research questions: (1) What are one teacher's goals for promoting equity in the classroom? (2) How does the teacher leverage the practice of mathematical justification to meet her equity goals?

For Josie, advancing equity in mathematics meant a focus on facilitating students' agency and supporting students in building positive mathematical identity, which she saw as intertwined. This involved engaging students in problem-based learning experiences that centered on noticing and justifying patterns, rather than imparting these herself. This meant students were generating and justifying their own mathematical concepts, rather than solely receiving and practicing Josie's mathematics. When examining the justification opportunities that Josie provided, problem-based learning emerged as a prominent theme. This article discusses Josie's perspectives on student agency and identity in mathematics. The paper presents two cases of how she facilitated justification opportunities through problem-based learning experiences to support her equity goals. These cases are important for mathematics teacher educators because they provide examples, as described within one classroom context, of how and why problem-based learning can open spaces for leveraging justification opportunities to advance equity for students.

Manuscript 3: Practitioner-focused Audience. This manuscript will be submitted to the journal, Mathematics Teacher: Learning and Teaching PK-12 (MTLT). MTLT is a practitioner journal, with in-service and pre-service K12 teachers as the primary audiences. This article discusses how to incorporate opportunities for mathematical justification at the beginning of units to support students' engagement with new content and their development of agency in the learning process. The beginning of Josie's fractions unit is provided as an example. Shared are the curricular tasks she implemented at the start of the unit, excerpts of dialogue from classroom discussion, Josie's thoughts after debriefing the lesson, and guidance for other teachers.

The three manuscripts in this dissertation illustrate a powerful connection between justification and agency (with some attention to other interrelated equity dimensions) from the perspective of one Teacher of Color and her students in a racially/ethnically diverse classroom of students. Findings are written in the form of three articles, intended for publication consideration in three journals, with the goal of reaching three major audiences in the field of education: researchers, teacher educators, and practitioners. The next chapter, Chapter 2, presents the

broadest overview of how and why Josie provided justification opportunities, (with Chapters 3 and 4 focusing more in-depth on specific types of justification opportunities).

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CHAPTER 2: EXPLORING HOW AND WHY ONE TEACHER FACILITATED MATHEMATICAL JUSTIFICATION OPPORTUNITIES TO SUPPORT HER GOALS FOR EQUITY

Abstract

Teacher practice plays an important role in advancing equitable experiences and outcomes for students. Teacher practice that encourages students to engage in mathematical reasoning that promotes inquiry with justification can support students' agency and engagement with content in the learning process. This study examined how one fourth-grade Teacher of Color leveraged the mathematical practice of justification to advance her goals for equity in a classroom of racially/ethnically diverse students. Findings indicated that the teacher was committed to regularly implementing justification opportunities in her mathematics lessons. She did this through purposeful task design and enactment strategies, and by establishing a conducive classroom environment. These justification opportunities were driven by her personal equity goals, which were focused on advancing students' agency in the learning process and cultivating students' development of positive mathematical identities.

Background and Objectives

Mathematical reasoning has been described as an inherent component of what it means to know and do mathematics (Ball & Bass, 2003). Scholars have examined the role that mathematical reasoning practices can serve in advancing equitable experiences and outcomes for students (Bartell et al., 2017; Boaler & Staples, 2008). Inquiry-oriented lessons that leverage mathematical reasoning practices can improve students' problem-solving skills, promote conceptual understanding of content, and contribute to productive dispositions towards mathematics (Boaler, 2022). Discussion-based classrooms that center students' mathematical reasonings as a basis for forming collective understanding also serve to build and uphold the skills and dispositions of democracy (Ball & Bass, 2008). Justification is a particularly powerful reasoning practice because it can be leveraged to support students in making their own mathematical discoveries, defend claims and conjectures they make during that discovery process, and build mathematical knowledge together through sensemaking as a classroom community. Justification opportunities can support students in taking ownership of the mathematics, as it opens spaces for teachers to center their voices, perspectives, and ideas rather than regurgitating and practicing the teacher's mathematics (Ball & Bass, 2008; Bieda & Staples, 2020).

Mathematical reasoning is characterized by providing both *explanations* and *justifications* for mathematical claims and choices that are made (Cioe et al., 2015). While students are used to providing explanations that describe *how* they solved a problem or *what* mathematical pattern/relationship they notice (i.e., explanation), they typically will not offer a justification for *why* their choice of solution strategy or mathematical pattern/relationship they notice makes sense (i.e., justification) unless further prompted (Cioe et al., 2015). Students' predisposition to offer explanations without justifications can be influenced by their past experiences learning mathematics, which may have traditionally involved responding to prompts from the teacher that elicited explanation of what they did to solve a problem (i.e., "the how") without being pressed enough for the rationale behind it (i.e., "the why"). Cioe and colleagues (2015) distinguish between explanation and justification to explain how engaging students in mathematical reasoning (an essential component of doing and learning mathematics) involves supporting students in moving from "the how" to "the why." Cioe et al. (2015) defines an *explanation* as articulating the methods for solving a problem (without communicating why the method is correct or appropriate) or sharing evidence that some mathematical pattern or relationship holds (without communicating why the relationship must hold). In contrast, a *justification* provides reasoning for why a mathematical relationship holds or why a solution strategy is appropriate or correctly applied. Essentially, supporting students in moving from explaining to justifying involves eliciting rationales from students. Eliciting rationales supports students' full and authentic engagement in mathematical reasoning, because mathematical reasoning is characterized by both explanations and justifications.

Given how essential justification is to the mathematical reasoning process, Staples et al. (2012) describes justification as a "core mathematics practice" (Staples et al., 2012, p. 447). More specifically, Staples and colleagues (2012) describe justification as a "teaching and learning practice," (p. 447) which can support both teachers' instruction and students' learning. For example, justification is a practice that teachers can leverage to engage students with the content (e.g., in conceptually oriented ways) and with one another (e.g., supporting collaboration and collective knowledge building), and it can be used as a tool to assess what students know. For students, engaging in justification can support them in learning content in ways that are conceptually oriented and argumentation-based, building valued mathematical skills and dispositions. Bieda and Staples (2020) describe justification as an "equity practice" (p. 102),

explaining that justification is a practice that can be leveraged to support students' access to mathematics content and agency in the learning process.

Given the value that engaging in reasoning practices can provide for students' learning of mathematics, yet the ongoing challenge of bridging the theory of justification into practice in mathematics classrooms (Melhuish et al., 2020), and the limited research in the field that examines the relationship between justification and equity specifically (Bieda et al., 2023), this study sought to better understand how justification opportunities can be leveraged to meet a teacher's equity goals. It would make sense that a teacher who views justification as a practice than can be leveraged to meet their equity goals would be more likely to implement it in their classroom. Understanding how and why a teacher facilitates justification through the lens of their context and equity goals could provide insight into how other teachers might be supported in incorporating the practice in ways they view as relevant to their contexts and goals. This study examined two questions (1) How does a teacher facilitate opportunities for students to engage in the mathematical practice of justification? (2) How does the teacher perceive mathematical justification as supporting her equity goals?

Conceptual Framework

This study examined how one teacher facilitated opportunities for students to engage in the practice of mathematical justification, as defined by Bieda and Staples (2020), to understand connections between justification and equity goals, given its value as a student-centered reasoning practice, yet limited understanding of its relationship to equity in the existing literature. Bieda and Staples (2020) define justification as, "the process of supporting your mathematical claims and choices when solving problems or explaining why your claim or answer makes sense" (p. 103). Staples et al. (2012) is one study that provides a helpful understanding of the relationship between mathematical justification and a teacher's goals. Staples et al. (2012) explored how twelve middle school teachers conceptualized justification and why they found it to be a beneficial practice for their students. There were six "purposes" for using justification that surfaced with the teachers. The six purposes were: *promoting conceptual understanding*, *fostering valued math skills and dispositions*, *assessing* [what students know], *fostering valued life-long skills and dispositions* (e.g., taking a stand and supporting it with evidence), *managing diversity* (e.g., promoting access for a wider range of students), and *influencing social relationships* (e.g., shifting authority of knowledge from the teacher to be

shared with the students). While these purposes can be situated within various equity constructs (e.g., influencing social relationships relates to agency), this study builds on the work of Staples and colleagues (2012) to more specifically elaborate how justification as a practice might be leveraged to support a teacher's goals for equity more broadly.

Melhuish and colleagues (2020) also situate justification as a teaching and learning practice, similar to Staples and colleagues' (2012) conceptualization, explaining that justification is a practice teachers can leverage to support students in doing mathematics and enhance their understanding of mathematics. Melhuish et al.'s (2020) definition of *justifying* integrates mathematical goals for using the practice and the role of the classroom community. Melhuish and colleagues (2020, p. 37) define justifying as,

> *Reason[ing] with meaning of ideas, definitions, mathematical properties, established generalizations to*
>
> - *Show why an idea/solution is true;*
> - *Refute the validity of an idea;*
> - *Give mathematical defense of an idea that was challenged.*

Melhuish and colleagues (2020) elaborate to explain that their definition of justifying closely aligns with Stylianides' (2007, p. 291) definition of proof,

- *It uses statements accepted by the classroom community (set of accepted statements) that are true and available without further justification;*
- *It employs forms of reasoning (modes of argumentation) that are valid and known to, or within the conceptual reach of, the classroom community; and*
- *It is communicated with forms of expression (modes of argument representation) that are appropriate known to, or within the conceptual reach of, the classroom community.*

Melhuish et al. (2020) explain that the defining feature of justifying is the process of building on claims, meanings, and arguments that students accept as true, in order to build collective understanding of mathematics as a classroom community. Stylianides' (2007) definition encompasses this with a focus on the justification product (i.e., statement of the argument), whereas Melhuish and colleagues (2020) shift their focus to the process or the "act of justifying" (p. 38). This shift in focus supported Melhuish and colleagues (2020) in examining teachers' understanding, noticing, and use of the practice. After a two-year professional development in 25 elementary schools (with all third- through fifth-grade teachers as participants), Melhuish et al. (2020) found that few teachers attended to justification. Melhuish

and colleagues (2020) attributed this to teachers holding varying, nonstandard conceptualizations of reasoning (which were deeply rooted, and for many teachers, did not change even after being exposed to more precise or authentic definitions in professional development), and these differences in conceptions affected teachers' noticings and implementations of justification. Given the complexity of justification, it would make sense that teachers can face challenges in learning, noticing, and implementing the practice.

Ball and Bass (2003) found that facilitating justification, (or what they call "the reasoning of justification"), is complex work that involves many factors. Facilitating justification opportunities specifically with the goal to cultivate skills and dispositions characteristic of a democracy in a classroom, involves purposeful task design and pedagogy that supports student inquiry and opportunities to learn from one another in ways that build collective understanding of mathematics as a classroom community. They define the reasoning of justification as resting on two foundations, and describe what is required in building these foundations,

> *We characterized two foundations of the mathematical reasoning of justification: a base of public knowledge and mathematical language. The reasoning of justification requires pedagogy, and design, just as the teaching of any other aspect of mathematics. Students must learn to use publicly-established ideas, methods, and language to make, inspect, validate, improve, and extend the mathematical knowledge of the class. And teachers must provide the resources and create an environment that encourages and makes possible complex student work (p. 43).*

Both Ball and Bass (2003) and Melhuish et al. (2020) describe justification facilitation as involving centering students' ideas and reasonings to build collective understanding of mathematical concepts as a classroom community. Centering students is an essential component of both justification and many equity frameworks, including the equity frameworks presented in Aguirre et al. (2013), Cochran-Smith and Keefe (2022), and Seda and Brown (2021). These frameworks describe centering students as eliciting and building from their ideas, perspectives, and funds of knowledge rather than regurgitating and practicing the teacher's mathematics. Ball and Bass (2003) explain that facilitating the reasoning of justification is complex work given the attention to multiple factors (e.g., task design, pedagogy, context). Similarly, Remillard and Heck (2014) describe the curriculum enactment process as also involving the interplay of various factors related to task design, pedagogy, values, and context. Remillard and Heck's (2014) framework for conceptualizing the mathematics curriculum enactment process further illustrates this complexity, explaining that tasks (and practices) become enacted as influenced by the

following factors: curricular (or task) design, district policy around curriculum use, teachers (including their pedagogy, beliefs, and goals) and students (including their identities, knowledge, and beliefs). Remillard and Heck describe how these are multiple systems that are interacting in complex ways to influence classroom activity, which suggests that understanding the enactment of any practice (e.g., mathematical justification) involves examining multiple systems. Looking just at the design of the justification task or just the teacher's pedagogy, for instance, may not provide insight into the broader picture of how and why justification opportunities get implemented in a classroom.

In Figure 3, I conceptualize justification opportunities as influenced by multiple interacting systems situated within one another. The framework starts at the level of the curricular task. The impact of the implementation of a written curricular task selected by a teacher for a lesson is situated *within* purposeful enactment strategies (e.g., pedagogical strategies such as questioning techniques or discourse moves that promote student inquiry and elicit reasoning), and situated within a particular context (e.g., a conducive classroom environment), and also situated within a teacher's equity goals.

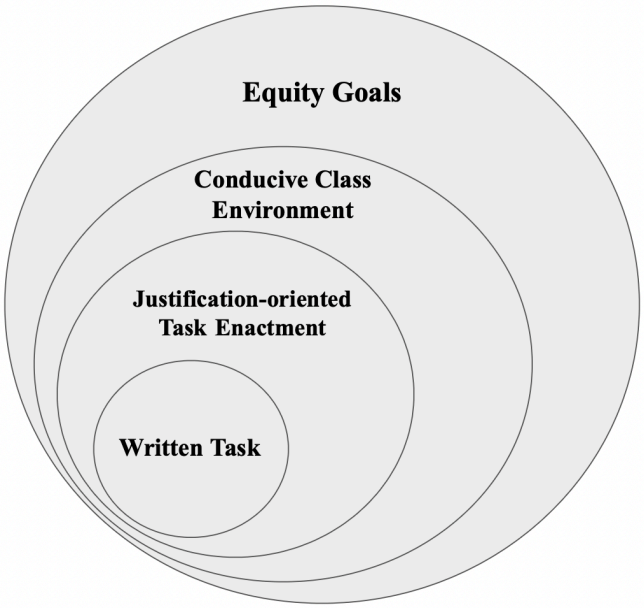

Figure 3. Framework for Conceptualizing a Teacher's Implementation of Justification Opportunities

A written task that is intended to elicit justification might include key words such as *prove*, *conjecture*, *justify*, *verify*, *show*, *convince* (Bieda et al., 2014). These verbs become

situated within *tasks of high-cognitive demand*, which Stein and Smith (1998) describe as tasks that are conceptually (versus procedurally) oriented, multi-step, and open-ended (e.g., does not prescribe the use of a particular solution strategy). Considering this definition, a task might say, "Calculate the area of the irregular shape. Justify your solution strategy and answer." Another example might be, "Jordan claims that if you double the perimeter of a rectangle, then the area of the new rectangle will be double the area of the original rectangle. Prove whether Jordan's claim is true or not." As mentioned above, Remillard and Heck (2014) explain that a given task can be enacted in various ways as influenced by various external factors, including a teacher's pedagogical moves. How a teacher enacts questioning strategies and discourse moves can be helpful in supporting students in communicating their thinking about a problem, orienting their ideas to one another, and ultimately justifying their ideas. Some examples of discourse moves, adapted from Chapin et al. (2009) and Cioe et al. (2015), that could support students during the mathematical justification process include, "Why would it make sense that…?" "Who agrees or disagrees with X, and why?" and "How could you convince someone that doesn't believe you?"

It is important for curricular tasks and their enactment to be situated within a conducive classroom environment, which is carefully cultivated by the teacher. This includes a classroom culture where students feel safe sharing their ideas and taking risks (Klosterman, 2016). Because being good at mathematics has traditionally been associated with quickness and correctness, producing high levels of math anxiety across students in mathematics classrooms, norms that value more than this as constituting mathematical "smartness" (Boaler, 2015; Cohen et al., 1999) could support students in feeling more comfortable talking through their in-progress or "rough draft math" ideas (Jansen, 2020) during the justification process. Norms shape the type of mathematical activity that occurs in a classroom (Cobb et al., 1992). They shape the development of mathematical routines and practices, and they shape interaction among students and between teacher and students (Yackel & Cobb, 1996). As a result, norms influence what counts as valued mathematical contributions (Franke et al., 2007; Goos, 2004). Norms that support students in feeling comfortable sharing their ideas and taking risks might include regular opportunities for open discussion, upholding the notion that learning is more important than performing, and valuing perseverance and revision of thinking more than immediate correctness (Boaler, 2015; Jansen, 2020). Part of a conducive classroom environment can include a *pedagogy of care*, which is characterized by pedagogy that shows students concern for both their

academics and well-being, and this is built through strong relationships and trust with and between students (Goralnik et al., 2012).

The outer circle in the framework presented in Figure 3 is *Equity Goals*, which everything the teacher does with the task, how it gets enacted, and how the classroom culture is established, becomes situated within or is driven by. Because "equity" is a complex and multifaceted word, I draw on Gutiérrez's (2009) framework for dimensions of equity (1) to communicate what I mean by equity and (2) to frame ways in which justification opportunities might advance equity. Gutiérrez describes four dimensions of equity: access, achievement, identity, and power. *Access* involves having resources and opportunities that allow students to fully participate in mathematics, such as high-quality curricular tasks designed to elicit justification. *Achievement* refers to institutionally acknowledged performance and participation, such as students' participation in rigorous mathematics classes that involve high order thinking such as justification. *Identity* means students seeing themselves reflected in and as doers of mathematics. *Power* is characterized by student empowerment, voice, and influence in the mathematics classroom and beyond. Gutiérrez describes access and achievement as part of a "dominant axis" and identity and power as comprising the "critical axis." Depending on a teacher's goals for using justification, this practice may be implemented in different ways and potentially with a focus on different dimensions of equity.

Methodology

Research Questions. The research questions guiding this study focused on understanding the opportunities for mathematical justification that one teacher provided students in her classroom and the impact she perceived these had on equity for her students. The first research question was, *How does a teacher facilitate opportunities for students to engage in the mathematical practice of justification?* The second research question was, *How does the teacher perceive mathematical justification as supporting her equity goals?*

Study Design. This research applied case study methodology, which can be described as a detailed description and analysis of a bounded system, where the parameters of the study are bounded by the "case" (Merriam & Tisdell, 2015). In this study, the teacher's practice, dispositions, and perspectives were the "case," and these were explored through interviews, classroom observations, and analysis of implemented curriculum materials. This study was situated in Merriam's (1998) constructivist pragmatic epistemological assumptions of case study,

meaning findings might be applied to offer insight into ways other teachers may consider providing similar opportunities, as adapted within their own contexts. The goal of this study methodology was to understand and amplify the practices and perspectives of one teacher and how she provides opportunities for mathematical justification in ways that centered students' ideas and voices.

Sampling and Participant Background. The participant in this study was one teacher, Josie (her chosen pseudonym). The selection of this teacher drew from a combination of *convenience sampling methodology* due to proximity and my pre-established relationship with the teacher and *purposeful sampling methodology* based on the teacher's existing practices that align with the study (Merriam & Tisdell, 2015). For instance, when considering participants for the study and speaking with Josie about it, she mentioned that she starts every lesson with a Number Talk (Parrish, 2014) and that she often incorporates justification opportunities in her lessons. Additionally, it was ideal that Josie and I already had an existing relationship, having both taught at the same school at one point. Because the teacher and I already established a relationship from formerly working together, this provided a strong foundation for a trusting research-participant relationship.

At the time of the study, Josie had nine years of teaching experience, primarily in self-contained, upper elementary classrooms, teaching students of racially/ethnically diverse backgrounds and in low socioeconomic regions. She also had some experience teaching middle school mathematics. During the study, Josie taught fourth grade. Josie identifies as a Filipina woman and immigrated to the United States from the Philippines at age 13. From once working in the same school with Josie, I knew she had a strong commitment to addressing issues of social justice, reflecting students' identities in the content, and incorporating student voice and agency into the classroom. This was Josie's sixth year at the school.

School Context. Josie's classroom was situated within an urban, public charter school. The school, Midwest Academy (pseudonym chosen by Josie), was known for its commitment to social justice and place-based education. The school hired teachers who shared this mission and required teachers to incorporate place-based projects into their curriculum. In describing what place-based learning means, in the context of her school, Josie explained,

> *[Midwest Academy] defines it as creating critical thinkers who contribute to the wellbeing of their community. I guess that place based is like putting the learning in the context of your place.*

Sometimes you can solve problems in that place that you know can come from your learning, or sometimes it's just you can learn about your place to develop and foster love of it and stewardship of it.

The majority of the students at Midwest Academy were from groups that are currently and historically marginalized, including Black and Latinx students. The school is designated a Title I status, and because the majority of students qualified for free or reduced lunch, the school provided all students free breakfast, lunch, and snacks during the school day.

Researcher Positionality. Given that I formerly taught at this school and had a pre-established relationship with Josie, I brought a somewhat "insider" lens to this context. For instance, I knew that this school and Josie had a strong commitment to place-based education and social justice, and that integrating these within their existing mathematics curriculum was sometimes more challenging than other subject areas. I also knew that new teachers at the school were typically sent to a professional development on *Responsive Classroom* (Responsive Classroom, 2020) prior to the start of the school year, and that a major focus of the beginning of the school year in Josie's classroom was on responsive classroom routines. While I was at the school, I did not collaborate in lesson planning with Josie nor observe her instruction, which means the lens I brought toward her specific classroom verged toward the "outsider" lens. This insider/outsider perspective I brought suggested to me that Josie's classroom would provide an interesting context to explore how justification opportunities could be provided, and particularly, in ways that might advance critical dimensions of equity. I recognize I brought to this research a particular status, as a White researcher from a prominent research university entering the space of a classroom of predominantly Students of Color and a Teacher of Color. Because I knew my presence could have some influence on the space, it was important to me to work with a teacher with whom I had an established and trusting relationship, who knew me as a teacher at one point as well, and where my role was to learn from her. Mindful of my status, I was conscientious about decentering myself by quietly observing while in the classroom and centering Josie's voice during our meetings, by asking questions to position her as expert and by not offering or answering questions about my opinions.

Data Collection and Analysis. Data sources included recordings and transcripts of interviews and lesson planning meetings, field notes of classroom observations, and implemented curriculum materials. The first classroom observation was conducted in November 2023 to

gather a general idea of Josie's practice and routines established that school year, and to inform the development of pre-interview questions. A two-part pre-interview (totaling 130 minutes) was conducted in January 2024 to gather background information (e.g., about Josie's identity and teaching experience) and Josie's initial thoughts on justification and equity. For example, I asked Josie, "What relationship, if any, do you see between equity and the practice of justifying?" Regular classroom observations occurred two days per week (typically consecutive) from mid-February through April 2024, totaling 16 observations, in which I took field notes. Field notes were taken on types of student justifications (e.g., justifications of their ideas, claims, choices, or mathematical relationships) and on Josie's justification implementation (including pedagogical strategies and norms for eliciting or promoting justification, such as discourse moves or statements that decentered herself as the mathematical authority). Classroom observations began approximately halfway into the school year in order to examine how justification opportunities would be facilitated well-after routines and norms were established by Josie in the classroom, and prior to any influence that me entering the space as an observer could have.

Dispersed throughout the classroom observations were six (30-60 minute) lesson planning meetings, where Josie debriefed past lessons and shared plans for upcoming lessons. In these meetings, Josie debriefed the past week's justification opportunities, reflected on questions I asked about observed lessons in relation to justification and her equity goals, and shared her curriculum and instruction plans for the upcoming week. To facilitate lesson debrief conversations, I asked Josie to reflect on the justification opportunities that played out during the previous week, including what she thought went well and what challenges she experienced. I asked Josie what she noticed about students in relation to her equity goals across those lessons and how she was considering those equity goals as she planned for the upcoming week. Sometimes I asked questions to understand more about something I observed or about occurrences earlier in the school year. For instance, in order to better understand the types of and reasons for the Number Talks Josie created, I asked, "You also did a Number Talk related to something that you were doing in class, which was your Read Harder Challenge. Have you done other Number Talks outside of these two– the Loyalist-Patriot Debate Number Talk and Read Harder Challenge Number Talk– that have also related to other things happening in your classroom?" This was followed up with the question, "Why are these types of number talks important to you?"

Josie uploaded all implemented curriculum materials during the February through April observation window (which included the materials from lessons I observed and those I did not observe) into a shared folder. Viewing the curricular tasks implemented before and after lessons I observed helped to provide a better understanding of curricular coherence and how Josie's lessons built toward supporting her equity goals during different stages of a curricular unit. Keyword analysis and thematic analysis were conducted on the design of all curricular tasks she implemented during the observation window. Keyword analysis was used across these curriculum materials to identify words and phrases related to justification, such as identifying problems in worksheets that included words such as *why*, *prove*, *true, explain, notice,* and *pattern*. Thematic analysis (Braun & Clarke, 2006) was used to group tasks by themes (such as tasks that were designed by Josie for cultural relevance, tasks involving patterns, etc.) to look for potential connections for supporting the equity goals she expressed. For example, cultural relevance was one theme that emerged. A task that showed cultural relevance included mention to students' experiences, interests, or community (e.g., a task that referenced the amount of rainfall in the students' city) because of the connection to their community. Mathematical patterns was another theme that emerged during thematic analysis of the curricular tasks. One example of a task grouped into this theme included the direction, "Describe a pattern that you notice is true about multiplication for numbers that end in zeros." Within this task about noticing patterns was a connection to agency because students were being tasked with developing a conjecture about what happens when numbers are multiplied by powers of ten, whereby students are assuming ownership of the mathematics since the teacher is not imparting the mathematical rules or procedures.

In May, an exit interview (60 minutes) was conducted to gather Josie's final reflections and more deeply examine questions around justification and equity that emerged during the study. For instance, since Josie reported the beginning of units to be important places to leverage the practice of justification for supporting her equity goals, I asked, "In what ways do the beginning of units or new topics lend themselves well to incorporating opportunities to justify?" To help me summarize some of her strategies and perspectives, I also asked Josie what advice she would give to other teachers looking to provide more justification opportunities. I probed to ask about how another teacher should go about designing tasks as well as what strategies or

questions the teacher should ask to support students' own inquiry, claims, and reasonings along the way.

Transcripts (of interviews and lesson planning sessions) and field notes (of classroom observations) were also analyzed using a thematic analysis approach to understand Josie's practice and perspectives around justification and equity concepts. This analysis involved reading all field notes and transcripts to identify quotes that could help explain or elaborate each circle of Figure 3, and organizing these quotes into themes. The same process used to identify themes from quotes as described earlier for curricular tasks (which is circle 1) was applied for each circle of the framework in Figure 3. This involved creating a separate data table for each circle of Figure 3. For the first circle (the innermost circle), *Written Task*, keyword analysis and thematic analysis (as described earlier) were conducted to look for patterns in the types of tasks Josie designed and selected to implement. For the other circles, thematic analysis was conducted on field notes and transcripts (of interviews and lesson planning meetings) separately for each circle, resulting in multiple rounds of analysis of the same data, but with attention to each circle separately. For the second circle, thematic analysis was conducted to look for themes in Josie's justification-oriented task enactment. Themes included activating background knowledge, pressing for reasoning, involving peers, and sharing in authority of knowledge. For example, in one lesson, Josie pressed for one students' reasoning to elaborate on why they thought the answer was wrong. Her statement, "What makes this wrong? Why does it all need to be single digits?" is eliciting a justification for an answer a student is disagreeing with. For the third circle, analysis involved looking for themes around how Josie provided a conducive classroom environment. References to math norms and statements around supporting students in feeling comfortable or confident to participate were themes coded for under the conducive classroom environment category. For the fourth circle (the outermost circle), thematic analysis was used to look for themes around explicit connections to the equity goals expressed by Josie. The goal of examining themes within each circle of the framework in Figure 3 was to examine the multi-system nature of justification opportunities facilitated by Josie and her intended impact on equity for students.

Results

In this section, findings are shared through the lens of the framework in Figure 3. First, examples from Josie's classroom and perspectives that elaborate each circle of the framework

are shared, and then an example that cuts across the circles is discussed. Any quotes where the word, *like*, was interjected as a filler and not needed for context, was sometimes removed for readability. All names used are pseudonyms.

Task Design. Josie designed most of her curriculum materials herself, reporting that her primary curriculum "doesn't suit the kids at our school." She described the primary curriculum material as heavily procedural, overly scaffolded, not inquiry-oriented, and not culturally relevant, which is especially important to her working in a school focused on place-based learning. According to Josie, creating materials herself allowed her to connect content to the students and to their community, and opened spaces for justification opportunities. Keyword analysis revealed that tasks included references to their city (e.g., examining the amount of local rainfall), names of students in the class, interests held by students in the class (e.g., calculating the number of bottles of glue needed to make slime for a given amount of friends), and references to other events happening in their classroom or school (e.g., calculating the percentage of their class who have completed their reading challenge thus far).

Key terms that surfaced in tasks for eliciting justifications included words such as *pattern*, *noticing*, *true*, *prove*, *why, explanation*. Phrases included: *Using what you know about…*, *Find the patterns or make up a rule for…*, *Describe a pattern that you notice is true about…*, *Use at least X strategies.* None of the tasks included the word "justification" or "justify." When asked why she used the word "explanation" instead, Josie said that she thought it was a more accessible word to her students. However, Josie intentionally incorporated correlated key words in tasks to signal to students that she was looking for a justification. For example, "Write (in words) your explanation for why this is a true equation. Is your explanation true for other examples? Give two equations that are similar to the example that follow your explanation." Here, the word "why" signals justification, and the request for multiple examples associated with the proving of something to be "true" could denote a way to support or enhance the justification.

Justification-oriented Task Enactment. Some of the tasks Josie created did not include directions for justification, but justifications were elicited through her enactment strategies. These included pressing for reasoning, involving peers to build from one another's ideas, activating background knowledge to elicit a conjecture to explore, and sharing authority of knowledge with students.

In describing how to elicit a justification, Josie explained that it required a lot of prompting to get at students' reasoning. She elaborated, "But definitely *the why* requires more prompting. 'Cause it is, you know, like deeper thinking. Like you really have to notice whatever it is, the pattern that you're trying to explain, and that sometimes takes time for most people." When asked what types of prompts she used, Josie said, "Maybe I'll ask them something specific like, *Why is there always this number here?* or *Why did you know you could do that specific 'whatever' operation?* Or it's just, I guess it depends on the ask. But just very specific questions about what they're working on." Some questions that I observed Josie ask to elicit justification during math lessons included, "Why did you care about making it equal?"; Do you want to talk about why you added the values?"; and "How does that make sense?"

Another enactment strategy used by Josie to facilitate justification was involving peers. In describing why she did this (and referencing a student from class whom I frequently observed ask questions), Josie explained, "Maybe some kids who are too afraid to ask are then learning from that other kid who's just gonna ask why. Anyways, she's great to have in class, because she's like, *I don't understand. Why is it?*" Sometimes Josie relied on other students' words to justify concepts to their peers in a way she was not able to or asked them to rephrase in their own words what they understood their peers did. She shared,

> *Honestly, she helped me. She was like, she said it in a way that I never would have said it. And I was just trying to repeat it in my own words… Sometimes I ask other kids to explain what they understood about what somebody said.*

Josie explained that sometimes another student's justification is what can click for someone and "if there's twenty-four different people sharing a perspective, there's more chances that that might happen." Persuading a peer is another strategy she used to elicit justification. For example, in a lesson I observed involving equivalent fractions, Josie asked, "Did you persuade the other person that it is or isn't equivalent?"

Part of Josie's pedagogical approach was supporting students in noticing patterns or relationships and coming up with "rules" themselves based on their inquiry, which involved activating students' background knowledge to get them to generate a conjecture to be explored and justified. At the beginning of a lesson I observed on equivalent fractions, Josie began by telling the class what they would do that day and reminding them where they left off the previous day, "We're going to come up with a rule for equivalent fractions today. [Yesterday], you used

the fraction bars to create equivalent fractions. The last question asked you to notice a pattern and write a rule, but we ran out of time." Josie summarized what three students did yesterday, "Aniah found 1/8 is the same as 2/16," and then Josie drew the corresponding fraction bars beside it. Josie continued, "Jared noticed 1 times 2 is 2, and 8 times 2 is 16. Jess saw 9/12 is the same as ¾. She said you multiply the top and bottom by the same number. Is this always true? Does it work for every fraction? Talk about it with your partner." After taking time to talk this through with a partner, Josie gathered the class back together. The discussion led to the following dialogue, which discussed multiplying by odd and even numbers:

> ***Josie:*** *Could I get from 1/8 to 4/32 without knowing 2/16? How?*
>
> ***Student:*** *Knowing we're doing it by 2s, we can also do it by 4s, 6s, and 8s. 1 times 4 is 4, and 8 times 4 is 32.*
>
> ***Josie:*** *Does it also work for times 10, 12, 14? Does this only work for even numbers? You said 3/6 is one-half, one time. You said 5/10 is one-half. You were very certain of that. What about odd numbers?*
>
> ***Student:*** *You can still multiply by odd numbers to get another equivalent fraction.*
>
> ***Josie:*** *What do I multiply ½ by to get 5/10?*
>
> ***Student:*** *Multiply the numerator and denominator by 5.*
>
> ***Josie:*** *Is that true?*
>
> ***Student:*** *Yes, because 1x5 is 5 and 2x5 is 10.*

Now, Josie felt ready to ask the class, "Who can make up a rule for equivalent fractions knowing what we know?" One student answered, "If you multiply the numerator and the denominator by the same number, it will stay equivalent." Josie recorded this on the board and then said to the class, "I'm going to check with these problems on our worksheet and check if that's true" and proceeded to talk through that process with the class, to conclude the exploration of this math rule for equivalent fractions that the class generated and tested together. Given that time was running short at the end of class, it makes sense that Josie facilitated this last step in a more teacher-centered way quickly, but in a way that wrapped up the ideas that originated from the students.

An overarching theme of Josie's pedagogy and justification enactment strategies was sharing mathematical authority with students. While this surfaced in the enactment strategies previously described, I elaborate some more of her thoughts and examples here. It was important to Josie that she was "not the knower of everything," as she told me. She tried not to give

answers to students, stating, “I very rarely try and give the answers to them, ‘cause, you know, I think I've said this before, but I don't want to be like the keeper of math. I want them to be the thinkers.” Through her pedagogy, Josie endeavored to send the message to students that they were doers of mathematics and should feel empowered to have the authority and agency to make discoveries and claims, and to justify those themselves. Similarly, Josie purposefully did not answer questions sometimes, explaining, “I do it pretty frequently, so they know if I'm not answering a question, they're gonna stop asking because I want them to work through it.” Incorporating daily discussion and inquiry into instruction served to shift authority of knowledge to the students. Josie shared,

> *I love, you know, doing math discussions, and I think that engages the kids, too, to feel like they can share ideas, you know, and not just like, “Hey Miss. [Josie] told me to do this specific thing for this problem.” But they could be like, “I noticed this pattern. What if we tried this pattern across these other things, will we see the same thing? And then they ask, “Well, why didn't we- ? Or why did we- ?” So we always start with discussion, and that's built into like every day for us, too.*

Building discussion into everyday practice normalized for students that mathematics is about exploring, sharing, and justifying one’s own ideas and learning from peers doing the same.

Conducive Classroom Environment. Justification opportunities were importantly situated within a *conducive classroom environment*, where students felt safe sharing and taking risks (Klosterman, 2016). For Josie, establishing a conducive classroom environment involved co-creating classroom agreements, building students’ confidence in mathematics, promoting their curiosity, and humanizing the mathematics learning experience so that students could feel comfortable sharing ideas (including uncertain or in-progress ideas) and taking risks (such as disagreeing with peers who may often get correct answers early on).

When Josie was asked why she thought her students were so comfortable sharing their ideas even when they might be wrong or disagreeing with their peers, she referenced co-creating math agreements together at the beginning of the school year and establishing these as norms throughout the year,

> *I think setting the norms and making the agreements together first… So we were like, everyone is going to feel like they contribute to this classroom and you're gonna be a person that helps in that. You know what I mean? Not prevents people from their own learning. And I think they*

understood that. So, I think part of it was the kids buying in and willing to be those kind of people for each other.

Part of the math agreements were that each student had a responsibility to contribute their ideas and also a responsibility to support others in their efforts to contribute, which meant uplifting the knowledge that students brought to class and being intentional about building their confidence and sharing their mathematical thinking with others. Josie explained, "Everyone came to the classroom with a tool belt of knowledge. And they can share whatever is in there, and we can add to it and everyone can contribute. So I think that is part of agency." Emphasizing this from the start meant boosting the confidence of more reserved students early on. Josie mentioned, "For some kids, especially kids that aren't as comfortable, you have to really find those moments… like taking their notebook and showing the class, like look at what this person did. And I think that takes a lot of deliberate teacher action at the beginning." These agreements developed into norms, as they were not just written at the start of the school year but, as I observed, carried deep into the school year. Josie elaborated, "It has to be a culture. It has to be within this space, a space where I feel safe to share an idea and be okay with being wrong or being celebrated for being." Sometimes Josie modeled for students when she made mistakes and how to respectfully disagree with a peer. She shared, "I practice also, a lot like, telling the kids to disagree with me... I think it helps them understand that they're all capable, not just that an adult can tell them something, but that they can see sometimes that, you know, adults slip up, or anybody, and that there is a kind way to engage with somebody and help them understand something, and don't agree with them." This also contributed to Josie's efforts to position students as capable and share in authority of knowledge. Josie iterated,

I think a student that sees themselves as capable they're more willing to share their thinking… I think that that is more important than me being an authority of knowledge in a classroom, and as part of learning, because I cannot give my knowledge to somebody. I can't. That's never– you know what I mean? If I could, my job would be so easy.

Supporting students' ease and comfort in the mathematics classroom involved inspiring students' curiosity to want to explore and share their noticings as well as to develop an overall joy for and comfort in doing mathematics. Josie shared, "I just think if a child or person is more curious than afraid, they're more willing to try. So I think that helps them access it, if they're willing to try." Multiple times, she shared that one of her top goals as a mathematics teacher is "promoting a curious mind" and "engaging a curious mind." For Josie, a lot of joy can come

from being curious. She explained, “There's so much joy coming from being curious, you know, and to want the desire to learn, the desire to find out about something you just decided to be interested in because of whatever reason. I don't know what would happen to somebody that lost that.” For Josie, promoting curiosity was more important than speed and immediate correctness, which contributed to her conducive classroom environment. She believed, “If you understood that you could be curious and ask questions and explore something instead of having to know it right away. But then you, there's a lot of self-confidence that has to be built with a kid that sees math and problems that way.” For Josie, she promoted curiosity and reinforced for students it is okay to not always know the answer, (disrupting the math anxiety associated with traditional mathematics learning), in order to build students’ confidence and comfort to participate in the justification process.

Josie facilitated a justification-oriented conducive classroom environment so well that students felt comfortable disagreeing with their peers who were perceived to be the “math whiz” of the class, and even these “math whiz” students felt comfortable sharing when they were not certain of their correctness. After observing a lesson where this was very apparent, I asked Josie about it and she explained,

> *I know that the kids perceive Jonah especially, even since second, third grade, because he's been in Faith’s (pseudonym for Josie’s daughter) classes like the math whiz or whatever. He's just really natural with it. He understands patterns really well. And so I know the kids kind of see that. But even sometimes, and I'm like, don't want them to always go with what they say, you know. And sometimes they don't, which I'm like, okay so they're still thinking for themselves, and they perceive not Jonah as the keeper of math, but all of them as able to do math, and they can disagree with even somebody that, like Jonah, who is often right.*

Describing another example with a different “math whiz” in the class, Josie shared, “‘Cause sometimes in math what's intuitive isn't always, you know, the answer or something. And sometimes you didn't– Evan will get something that is, like seems intuitive, like it should be this, and it's not. And so there's been instances where that's happened.” Contributing to Josie’s conducive classroom environment was students seeing instances in which the “math whizzes” were also offering answers with justifications that ended up being incorrect.

Both Jonah and Evan were White boys in a class of predominantly Students of Color, so I asked Josie how she thought other students thought about Jonah and Evan both being White boys and seemingly intuitive with math. Josie stated that the “culture around race and gender is also

really different than other schools that I've experienced." I asked her to say more about that and she explained that "identity is so celebrated at our school," sharing how they make identity wheels at the beginning of the year and that kids are open about their pronouns, including kids who identify as "they." Josie also shared about a female Student of Color who presented to others as being great at mathematics and how tracking who shared during Number Talks provided opportunities for students to see their peers of all genders and races (particularly marginalized identities) showcase their mathematical smartness.

> *I guess we talk about race pretty openly and gender pretty openly… I've had students that are nonbinary, that are openly that, and openly are like, I prefer 'they'... I think identity is so celebrated at our school. We make identity wheels at the beginning of the year. We talk about inclusivity all the time, that even though it happens to be that the two, two of the kids in the class that are really vocal about their method are White boys. There are also a lot of examples of like Amina. She's Mexican or no, she's Colombian and Arabic. Half and half. And she's really great at math. So there's like other examples of identities that they see that are super capable at math as well, but maybe do it more quietly, or do it less boisterously or something. But they still have good shares. And because I think Numbers Talks is set up the way that it is, we're like, you can't share every day, like a different person, different three people are highlighted every day. They still see those identities as people doing that. So I think it's just culturally at our school, we just always talk about identities and how there's value in every identity. And maybe a lot of them have been going to that, just that school, since kindergarten. Maybe they've never lived in a world where the perception is like, you know, men are good at math and women aren't, or something.*

Josie believed that the reason students in her classroom (which were predominantly Students of Color) seemed comfortable disagreeing with math whizzes, Jonah and Evan (who were White boys), was attributed to (1) there being a culture of inclusivity and discussion of identity school-wide and (2) seeing regular examples of girls, non-binary students, and Students of Color showcasing mathematical smartness during lessons in her classroom.

Disposition for Leveraging Justification Opportunities to Support Equity Goals. Josie perceived it possible for justification opportunities to advance all four dimensions of equity (*access*, *achievement*, *identity*, and *power*) (Gutiérrez, 2009). Gutiérrez (2009) states that addressing all four dimensions is required for fully realizing equity. Because Josie believed justification opportunities can advance each dimension, and because she saw each dimension as intertwined with the others, this suggests a correlation for Josie between justification

opportunities and equity advancement. Shared below are Josie's thoughts on the relationship between justification opportunities and each of these dimensions of equity. Furthermore, for Josie, a focus on facilitating students' agency and cultivating students' development of positive mathematical identities emerged as prominent themes in her practice and perspectives, suggesting these to be critical constructs for her in influencing the other equity dimensions. I am using the equity construct of *agency* to mean students being empowered to have influence in their learning environment, by which they play a role in their own mathematical development and when they learn from/with their peers through sharing authority of knowledge with the teacher (Grootenboer & Jorgensen, 2009). By *positive mathematical identity,* I refer to students seeing themselves as doers of mathematics and perceiving confidence in their capabilities to engage in rigorous mathematical practices (Darragh, 2013).

Because "equity" is a word that is multifaceted and conceptualized in nuanced ways across the literature and school contexts, I provided Josie a brief description of each dimension of equity as defined by Gutiérrez (2009), to help guide our conversation in a way that provided for shared language around the word. After sharing Gutiérrez's conceptualization of equity, I asked Josie, "Where do you see the practice of justifying within each of these dimensions?" She believed, "Justifying supports all of those things." Josie discussed each dimension as intertwined with another dimension, interestingly aligned with how Gutiérrez frames them. Regarding the relationship between justification and *identity*, Josie mentioned when you're justifying, "the reasoning is your own" and "you're able to voice your own critical thinking of whatever it is." With *power*, her perception was, "You did the math yourself, so that part is like identity. And it's part of power because that's, again, centered around yourself… You do the task without, I guess, being trapped in an algorithm. You don't have to follow the step-by-step process right now." Josie explained that attending to dimensions of identity and power (which have connections to agency), in turn, promoted access and achievement. Referencing the achievement dimension, Josie stated,

> *I think all of that probably leads to the achievement part where kids are more open to participating, because nobody, they don't feel maybe the pressure of the correct strategy or the correct thing that it should look like. They can talk it through. And that they're told it's okay to have an unfinished thought, even to share that out because there's still value in that.*

Similarly, Josie shared that focusing on mathematics as a justification process can make mathematics more accessible to students because it removes the pressure of having to produce a correct final product (e.g., focus on correct answer) and students can be creative in their problem-solving (reflecting their mathematical identities and agency to do so),

> *I think that thinking of math as a process makes it more accessible to a lot of kids. Makes it friendlier, makes it feel maybe less pressure. Math isn't just you show up, you do the thing, and you get the answer. You're like allowed to be more creative with it. And I think a lot of kids do well with creative tasks, because they can kind of put themselves into it.*

When justification was a norm situated within a conducive class environment where students did not feel pressured to do mathematics in a constrained way, students' agency and identity were cultivated. Talking about both, Josie explained,

> *Sometimes they come, and they are so hung up on the correct answer, and I think for them to discover that they can explain their thinking, and that they can logic through stuff... I think just as part of their identity, perceive math to be like, 'Oh, it's something I can think about and actually do. And there's, I'm not stuck in one way, and if I don't know the way I'm just instantly wrong or dumb, or whatever.' I always tell them, 'Look at it first. Use what you know to figure out what you don't. You can see things. You came into this classroom already knowing things.'*

Josie saw that all students brought a wealth of knowledge, experiences, and perspectives with them that they could apply to their solving of problems, and Josie saw justification as opening opportunities to reflect students' identities in the learning process. It was important to Josie that students did not get fixated on mathematics just being about correct answers, but that doing mathematics meant discovering, problem-solving, and explaining your thinking using what you know. When I asked Josie directly about the relationship between agency, identity, and justification, she rhetorically asked,

> *Don't they kind of go hand-in-hand? Or to me, it feels that way. Like, isn't the opportunities to have their thinking be heard and valued and uplifted, is kind of the agency part? I think that justification opportunities and conversations inherently provide kids with agency.*

Josie saw cultivating students' agency and identity as intertwined. These were the primary equity goals Josie had for her students inside and outside of the math classroom, and she leveraged mathematical justification to support these goals. By doing so, Josie believed she was influencing other dimensions of equity, including access and achievement. Being a facilitator of students' agency in the learning process and their positive mathematical identity development

was important to Josie's goal of cultivating free-thinking students inside and outside of the mathematics classroom. Josie believed, "Because really, that's where we're leading them to is like to be outside of our walls and be free thinking humans out in the world."

For Josie, sharing authority of knowledge with students was particularly supportive for facilitating their agency and development of positive mathematical identity. This was evident through her practice and perspectives. As previously shared, Josie expressed, "I very rarely try and give the answers to them, 'cause, you know, I think I've said this before, but I don't want to be like the keeper of math. I want them to be the thinkers." This is why it was important to Josie to provide students opportunities to discover and justify mathematical patterns/relationships themselves in ways that centered their ideas and working understanding of content. According to Josie, supporting students' agency and positive mathematical identity development ultimately can support access to content and achievement in mathematics, and justification is a practice that can be leveraged to promote these goals.

Summarizing Back the Framework. For Josie, leveraging justification opportunities to support her equity goals involved purposefully designed curricular tasks, such as those that involved students in discovering and justifying mathematical patterns or relationships themselves whereby they took ownership of the mathematics. Engaging students in justification (in general, and particularly in ways that met her equity goals) involved more than just a purposefully designed curricular task. As illustrated in Figure 3, what drove how and why a given justification task was enacted were Josie's dispositions for critical equity constructs. For Josie, her primary equity goals of supporting students' agency and positive mathematical identity development meant implementing justification in ways that shared authority of knowledge with students. This more active student role required a conducive classroom environment where students felt comfortable sharing their ideas and taking risks.

Example that Moves through the Framework. Here, I use the framework shared earlier (i.e., Figure 3) to help illustrate how Josie's tasks were embedded within particular enactment strategies, which were embedded within a conducive classroom environment, and driven by her equity disposition to support students' agency and development of positive mathematical identities, in order to facilitate equity in her classroom. In this section, I elaborate one of Josie's tasks to illustrate how that task and the multiple systems represented by the framework were

embedded within one another in order to support equity. As described earlier, one task Josie gave students was to develop a rule for finding equivalent fractions.

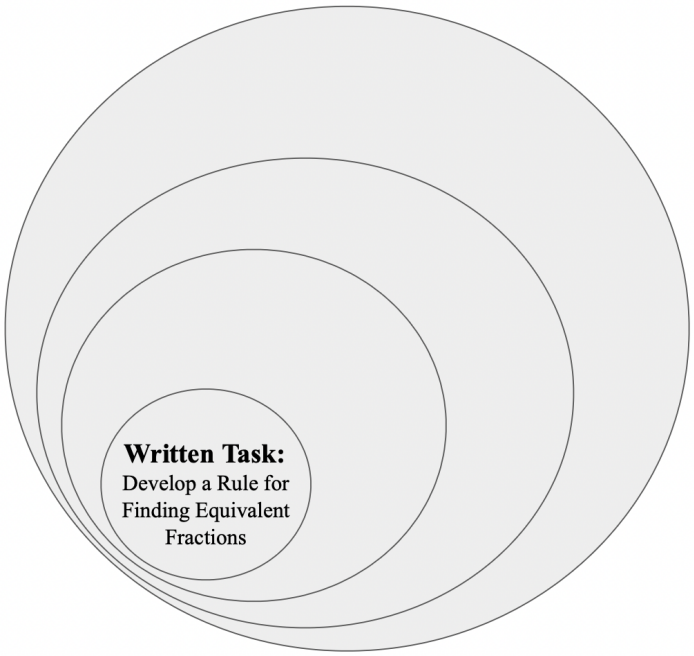

Figure 4. Written Task

For Josie, enacting this task in ways that met her equity goals meant sharing authority of mathematical knowledge with students. This involved first activating students' background knowledge, which Josie did when she reminded the class about the equivalent fractions that Aniah, Jared, and Jess had noticed. Josie also asked prompting questions and pressed for reasoning. She asked students, "Could I get from 1/8 to 4/32 without knowing 2/16?"; "Does this also work for even numbers?…What about odd numbers?"; and "Is that true?" These types of questions supported students in thinking for themselves and needing to justify the mathematics behind their claims and ideas.

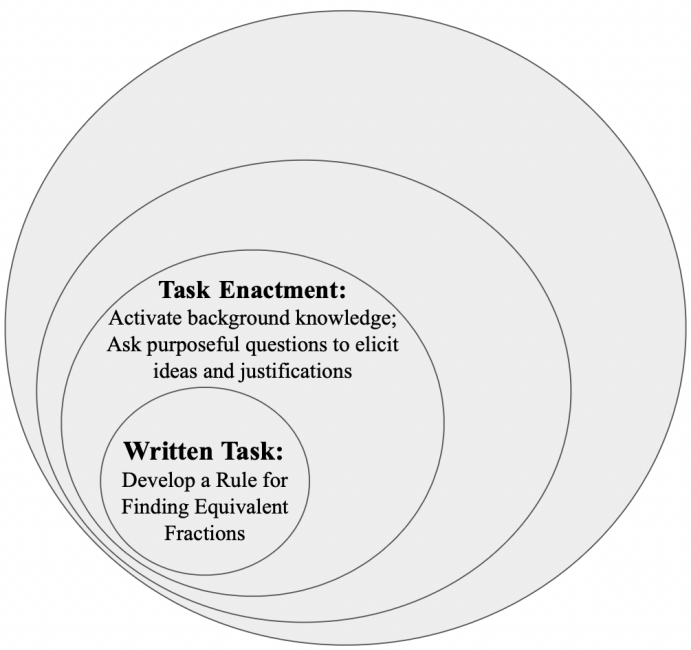

Figure 5. Written Task, Situated in Task Enactment

This task and its enactment were embedded within a conducive classroom environment. Josie provided students time to explore over the course of multiple days. She positioned students with authority of knowledge, to build their confidence so that they could feel comfortable sharing their in-progress ideas. This lesson took place in April, so the math agreements had solidified into norms, where students learned that part of the learning process was exploring and justifying claims, which meant not having answers right away and raising potentially incorrect ideas was okay.

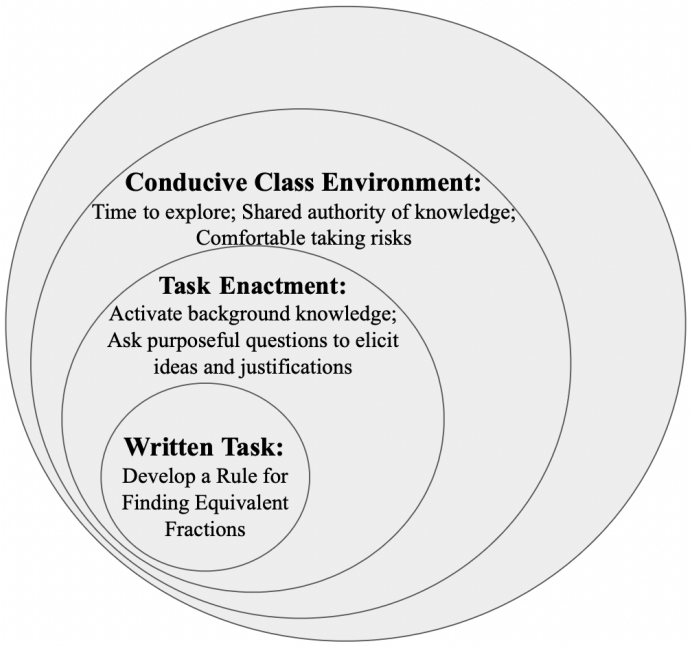

Figure 6. Written Task, Situated in Task Enactment, Situated in Conducive Class Environment

The written task, *as embedded within* Josie's enactment, *as situated within* the conducive class environment she fostered, *were all driven by* Josie's equity disposition to support students' agency and positive mathematical identity development. This is why it was important to Josie to let students come up with a mathematical rule themselves rather than her imparting the mathematical relationship. This involved giving students the agency to explore, discover, and justify mathematical ideas in ways that made sense to them and reflected themselves in the learning process. For Josie, this built toward her goal of wanting students to be free thinking humans inside and outside of the mathematics classroom.

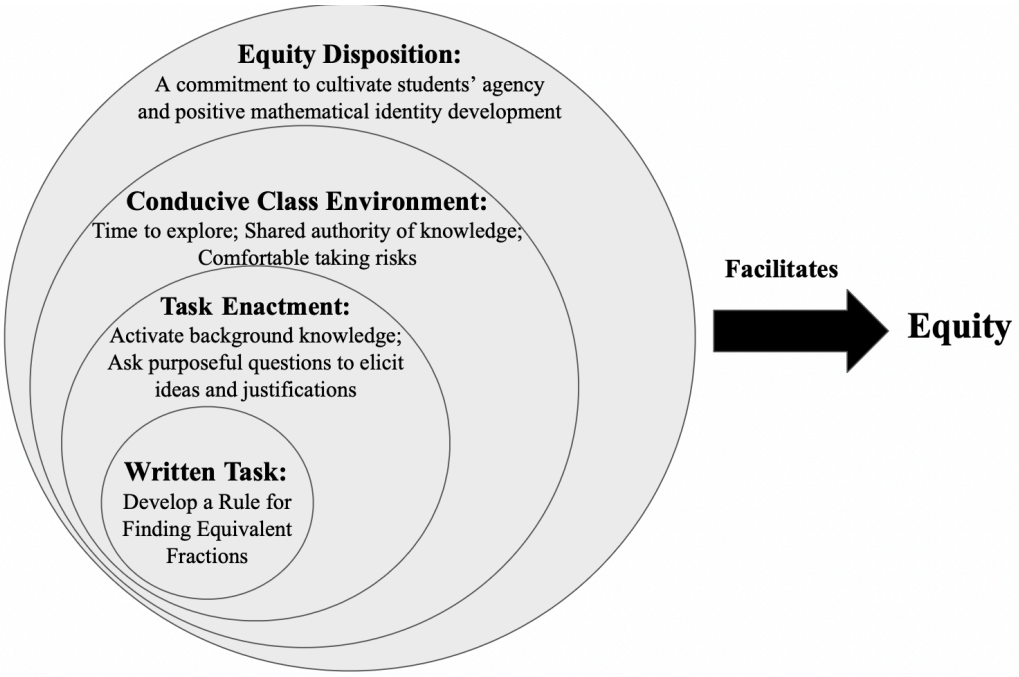

Figure 7. Written Task, Situated in Task Enactment, Situated in Conducive Class Environment, Situated in Equity Disposition for Supporting Agency and Positive Mathematical Identity

Discussion and Conclusion

Given how infrequent justification is incorporated into classrooms and how challenging it is for teachers to notice and implement the practice (Melhuish et al., 2020), it can be said that incorporating justification would be complex work and that teachers would need to view the practice to be relevant to their context and equity goals in order for them to implement it. To better understand how and why one teacher who was committed to regularly incorporating justification opportunities perceived justification as a practice that can be leveraged to support her goals for advancing equity, this study explored the questions, *How does a teacher facilitate opportunities for students to engage in the mathematical practice of justification?* and *How does the teacher perceive mathematical justification as supporting her equity goals?* Examining Josie's facilitation of justification opportunities illustrated the complexity of this work because it involved attention to multiple factors situated within one another (i.e., purposeful task design, situated within justification-oriented enactment, situated within a conducive classroom environment). Furthermore, Josie's attention to these factors were driven by her goals for facilitating students' agency and development of positive mathematical identity in order to support equity. According to Josie, justification can advance all four dimensions of equity

(access, achievement, identity, and power), which Gutiérrez (2009) describes all as intertwined and required for realizing equity. It makes sense that focusing on facilitating students' agency and positive mathematical identity development was possible through justification, given that the practice opens spaces for centering students' voices and perspectives through their mathematical claims and reasonings rather than using and practicing the teacher's mathematics. It also makes sense that supporting students' choice and voice in the learning process (agency), and supporting students in seeing their ideas and perspectives reflected in the learning process (identity) can support access to mathematical content and achievement in the subject. When students feel connected to the content and are actively engaged with the content, these can facilitate opportunities for access and increased participation patterns and achievement. However, more research is needed to understand the relationship between students' agency in the learning process, positive mathematical identity, and mathematical achievement. Fernandez et al. (2024) is one study that shows a beginning understanding of this relationship, finding a statistically significant relationship between a student's positive mathematical identity and achievement, with pedagogy that centers students' reasonings as a contributing factor. In describing this relationship, Fernandez and colleagues (2024) explained, "students who see themselves as individuals who can do and practice mathematics tend to perform better on mathematics tasks, even net of multiple other covariates of both identity and achievement" (p. 2018). Though more research is still needed to examine the role of justification in advancing equity (including developing a better understanding of the relationship between mathematical identity and achievement), it does make sense that pedagogy which leverages mathematical reasoning practices where students can see themselves as the "authors of mathematics" (Fernandez et al., 2024, p. 2017) can play a key role in supporting students' mathematical identity and mathematical achievement.

Leveraging justification opportunities could be particularly conducive to supporting equity in classrooms of racially/ethnically diverse students, whose voices and perspectives have and continue to be marginalized. Not all students perceive themselves as doers of mathematics, or see or know people who look or identify similar to themselves as doers of mathematics. Advancing equity means being intentional about supporting students of marginalized backgrounds to see themselves as capable mathematics doers, who have choice and voice in the learning process, and see themselves reflected in the learning process. This study suggests that

justification could be a powerful practice that can be leveraged to facilitate equity, though more research is needed to understand the impact on students' perceptions and experiences since the unit of analysis in this study was the teacher. Examining the student experience could involve administering surveys and conducting focus groups or interviews with students, to look for potential alignment between the teacher's equity goals and impact on students.

Furthermore, because justification facilitation is complex work and suggests potential for advancing equity for students, practice-based methods could be supportive for teachers to learn the practice and in ways relevant to their contexts and goals. During Josie's exit interview, I asked, "Is there anything else you think would be helpful for researchers and/or teachers to know related to the role of justifying in the math class or how to provide students these opportunities?" Josie said watching examples of how some teachers facilitate justification opportunities can be helpful. She referenced a two-week long professional development that she attended two summers prior to the study that was particularly helpful for her, where she observed a teacher facilitating mathematical reasoning practices with students in relational ways. Josie recommended, "Just watch somebody because a lot of the things, a lot of these like micro opportunities really happen so fast, it would be impossible to like write it in a textbook, you know." According to Josie, part of providing justification opportunities involves micro-level enactment moves that may not be included in all curriculum materials. She also mentioned that getting better at facilitating justification opportunities takes practice. Josie explained,

> *I think it's like anything, it's like part of it is seeing it in practice, and part of it is practicing. And the more you do it, the more of those small moments you'll recognize and know how to inject a justification opportunity into. So you have to do it a lot and you have to see somebody effectively do it to recognize it. You know, it's exposure, I guess.*

Learning to provide justification opportunities involves seeing examples of implementation in practice and practicing implementing it oneself. It takes time and practice to learn any new practice, particularly as it needs to be adapted to be culturally relevant and responsive to one's own students, classroom context, and goals. Given the complexity of justification facilitation, this suggests providing practice-based opportunities may be helpful in supporting teachers in learning to facilitate justification opportunities in ways that are responsive to students and situated in critical dispositions for equity. Practice-based opportunities might involve seeing examples of how other teachers facilitate justification opportunities in similar and

different ways within their unique contexts (i.e., *representations of practice*) and engaging in *approximations of practice* to practice implementing some justification opportunities in mediated settings before trying out on real students to minimize students being "practiced on" (Grossman et al., 2009). Mediated practice opportunities, such as trying out the delivery of parts of justification opportunities, can give the professional development facilitator opportunities to offer feedback to a teacher and for the teacher to retry and refine justification-oriented implementation strategies (such as discourse moves for eliciting justification). While there may be concern that practicing in a removed setting could lead to prescriptive instructional methods across contexts, research has found that practice-based opportunities can support teachers in feeling more prepared later on when they go to implement new teaching practices with their students, and research has shown teachers are able to apply those practices they learned in ways responsive to their students (Kavanagh et al., 2020; Schiera, 2021).

In the final question of the exit interview, I asked Josie, "Anything else you want researchers or teachers to know?" and she responded, "Everyone should do it! Kids really like it! They're really good at explaining themselves." According to Josie, justification opportunities can be leveraged to intrigue students' curiosity, enhance their engagement with and enjoyment of mathematics, contribute to the development of a positive mathematical identity, and support their agency in the learning process. This study illustrated that justification has the potential to be a powerful, equity-oriented mathematical practice a teacher can leverage to provide students the agency to make/defend choices and to generate mathematical meaning/discoveries, whereby students are taking ownership of the mathematics, reflecting and cultivating positive mathematical identities in the learning process regarding what it means to know, do, and be good at mathematics.

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CHAPTER 3: THE ROLE OF JUSTIFICATION-ORIENTED PROBLEM-BASED LEARNING EXPERIENCES IN SUPPORTING STUDENTS' AGENCY AND MATHEMATICAL IDENTITY

Abstract

Mathematical reasoning practices, such as justification, can advance equity for students by facilitating their agency in the learning process and supporting positive mathematical identity development in students. Problem-based learning lessons can provide a helpful context for teachers to leverage justification opportunities in ways that can advance these aspects of equity. This study examined how one Teacher of Color in a racially/ethnically diverse fourth-grade classroom of students in an urban, Title I school facilitated problem-based learning experiences that leveraged justification opportunities to support equity for her students. This article discusses how and why justification-oriented problem-based learning opportunities can be conducive for supporting a teacher's equity goals. Curricular tasks, instructional strategies, and classroom vignettes from one teacher's classroom are shared.

Background

The Standards for Mathematical Practice (SMP) specify the importance of students engaging in mathematical reasoning practices, such as *reason abstractly and quantitatively* (SMP 1) and *construct viable arguments and critique the reasoning of others* (SMP 3) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 6). Mathematical reasoning practices engage students in sensemaking, promoting critical thinking skills and conceptual understanding of content (Bieda et al., 2022). When embedded in inquiry-oriented learning experiences (such as problem-based learning lessons), mathematical reasoning practices can advance students' agency by providing them opportunities to explore and make mathematical discoveries themselves (Hansen, 2022; Schettino, 2016). In the *problem-based approach,* "students are expected to solve problems or make sense of mathematical situations for which no well-defined routines or procedures exist. In introductory activities, as well as in application settings, students are expected to explore problems, make conjectures, and draw generalizations about mathematics concepts and processes" (Erickson, 1999, p. 516). In problem-based learning experiences, students are making mathematical discoveries, rather than having the mathematics directly imparted to them. Thus, the role of the teacher is to use students' curiosity and ideas to drive learning. In problem-based learning, because students are making

mathematical discoveries, this creates opportunities for students to generate and defend claims, or engage in the process of mathematical justification. *Justification* refers to "the process of supporting your mathematical claims and choices when solving problems or explaining why your claim or answer makes sense" (Bieda & Staples, 2020, p. 103). By design, problem-based learning experiences can provide opportunities for students to engage in mathematical justification. Justification is described by Bieda and Staples (2020) as an "equity practice" that can develop students' agency. Specifically, Bieda and Staples (2020) state that opportunities to justify one's reasoning supports students' agency because students are generating mathematical ideas themselves in ways that make sense to them rather than practicing doing the teacher's mathematics. There is limited research on both the justification facilitation process (Karunakaran & Levin, 2022) and on the relationship between justification and equity (Bieda et al., 2023). I argue that examining the process of how justification opportunities are facilitated by a teacher can help highlight ways in which justification opportunities are provided and how those opportunities relate to a teacher's equity goals. Given how infrequent justification occurs in classrooms (Melhuish et al., 2020), it would make sense that a teacher who views justification as supporting their equity goals would incorporate the practice as part of their pedagogy. This study explored two questions: (1) *What are one teacher's goals for promoting equity in the classroom?* (2) *How does the teacher leverage the practice of mathematical justification to meet her equity goals?* Understanding how justification opportunities are facilitated in the context of a teacher's equity goals may help teacher educators better be able to support teachers in incorporating justification opportunities into instruction in ways that those teachers view responsive to their contexts, students, and goals.

Conceptual Framework

Framing this study is Schettino's (2016) conceptualization of relational problem-based learning, which considers the interaction between concepts of justification, curriculum, ownership of knowledge, and shared authority. I also draw on Berry's (2020) conceptualization of agency and identity to frame how these constructs are dimensions of equity and how they are interrelated. Then, I discuss the role of *distributing intellectual authority* (Zavala & Aguirre, 2024) in facilitating justification opportunities in ways that can empower students' agency and build positive mathematical identities.

Relational Problem-based Learning with a Justification Focus. Problem-based learning is a student-centered pedagogy and relational learning process that involves mathematical reasoning practices, including justification (Erickson, 1999; Schettino, 2016). In describing the "essential elements" of the problem-based approach, Erickson (1999) states, "students can explore and make conjectures, listen to one another's explanations and justifications, and reach consensus about the concept or about how particular ideas are related" (p. 518). Because students are tasked with making discoveries themselves rather than being told by the teacher, this process involves students defending their claims and learning from ideas their peers have raised. Erickson explains, "In this teaching approach, students make reasoned conjectures about a problem-solving task, justify their thinking, and listen to and consider other students' ideas" (Erickson, 1999, p. 518)." In addition to the problem-based approach affording opportunities for justification, I argue that justification is an integral component of problem-based learning.

While there are many definitions and conceptualizations of problem-based learning (Merritt et al., 2017), I draw from Schettino's (2016) framework of teaching mathematics with *relational problem-based learning*. Schettino describes problem-based learning as a relational and collaborative, not individual, process. In this framework, the themes that characterize relational problem-based learning are (1) *ownership of knowledge*, (2) *justification- not prescription*, (3) *the connected curriculum*, and (4) *shared authority*. Ownership of knowledge refers to centering students' solution strategies, ideas, perspectives, and discoveries in the learning process. This requires that the students justify their claims and decisions, as the teacher is not prescribing methods for solving the problem. Scaffolding and supporting students in not having to rely on the teacher for solving problems with mathematical concepts that are very new to them requires a curriculum that coherently connects mathematical concepts to one another and also connects to students' background knowledge. Sharing authority of knowledge means all members of the class are contributing to the knowledge building of the classroom, requiring teachers to decenter themselves. Schettino found that relational problem-based learning allows students to have more agency in the learning process, supports students in viewing mathematics more positively, and thus contributes to students seeing themselves as more confident doers of mathematics.

Erickson (1999) says that facilitating problem-based learning requires the teacher to establish a classroom environment where students feel safe taking risks, such as disagreeing with

a peer, and comfortable sharing ideas that are still uncertain or in-progress, without fear of embarrassment or ridicule. Schettino (2016) also echoes the importance of a comfortable classroom environment for facilitating problem-based learning, particularly because it is such a relational process. Schettino (2016) states, "In the context of creating a relational learning environment, empowering student voice and agency is facilitated by creating a safe environment, further demonstrating the interdependence of the relational framework on each of its parts" (Voice and Agency section, para. 3). According to Schettino, a safe environment is an essential part of supporting students' agency and voice through problem-based learning.

Agency and Identity as Equity Constructs. Advancing equity for students involves supporting their development of a positive mathematical identity and facilitating their agency in the learning process. Berry (2020) elaborates the National Council of Teachers of Mathematics' (NCTM's) *Principles to Actions* and frames equity using the interrelated constructs of identity and agency. According to Berry, teaching mathematics in ways that advance equity for students involves more than just teaching mathematical skills but also empowering students to see themselves as capable, confident doers of mathematics (identity). This involves integrating students' cultural experiences and contexts into the mathematics as well as expanding students' perceptions of what it means to do and be good at mathematics (which extends beyond quick calculations and correct answers, to also include engaging in mathematical discourse, collaboration, and productive struggle). According to Berry, when students develop a positive mathematical identity, this in turn can empower their agency to make decisions that increase their participation and engagement in mathematics class. Berry (2020) states, "If one identifies themselves as being smart and good at mathematics, then they present themselves and adopt behaviors and actions of smartness and being good at mathematics" (slide 16). This may include more frequently asking questions, participating in mathematical discourse, and internalizing increased motivation to persevere. Citing Aguirre and colleagues (2013), Berry states, "Agency is our identity in action and the presentation of our identity to the world" (slide 15). Having agency within the mathematics classroom and beyond is a reflection of having a positive mathematical identity. Echoing the importance of cultivating students' identity and agency in mathematics, in the November 2024 NCTM president message, Knighten (2024) cites the NCTM *Catalyzing Change* series which states, "Mathematics instruction should be consistent with research-informed and equitable teaching practices that nurture children's positive mathematical identities and strong

sense of agency." Knighten explains that connecting to students' cultural experiences, building from what they know, and providing opportunities for students to learn from and with their peers allows students to "take ownership of the learning process" and "put themselves in the driver's seat."

Figure 8 shows the relationship I conceptualize between relational problem-based learning and equity. Problem-based learning (with a focus on relational components that include justification) can advance equity for students through supporting their development of a positive mathematical identity. Having a positive math identity can, in turn, facilitate students' agency. As students have more agency in the learning process, they are also building a more positive mathematical identity. As a result, the constructs of agency and identity can positively influence one another.

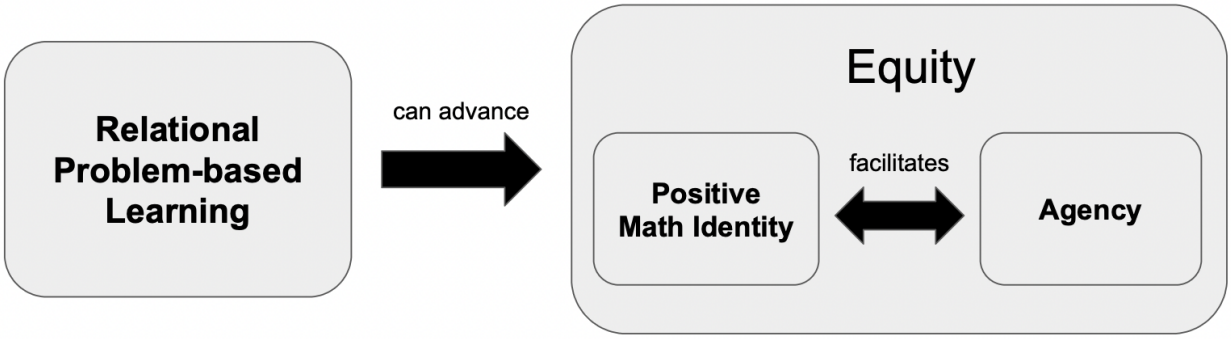

Figure 8. Framework for Relational Problem-based Learning Facilitating Students' Agency and Positive Mathematical Identity Development

Distributing Intellectual Authority to Support Students' Agency and Identity Development. Supporting students in building positive mathematics identities and empowering their agency in the learning process involves the teacher sharing authority of knowledge with students. Zavala and Aguirre (2024) state that distributing intellectual authority can "make space for multiple forms of knowledge and communication" (p. 98). They explain that moving from a teacher-centered to student-centered mathematics classroom is one way to advance students' mathematical authority. Zavala and Aguirre (2024) define *distributing intellectual authority* as involving, "positioning students as mathematical authorities in the classroom, using participation structures that expand student mathematical contribution opportunities, and embracing the role of teacher as facilitator and co-constructor of knowledge in the classroom" (p. 99). They explain that this involves providing multiple opportunities for students to communicate what they know and have choice in how they do so, and these opportunities must be frequent enough and

facilitated in ways that allow students to see themselves and each other as mathematical authorities. Supporting students in seeing themselves as mathematical authorities is part of building positive mathematical identities, and having mathematical authority inherently is advancing agency in the learning process. Zavala and Aguirre (2024) describe three "instructional approaches" involved in distributing intellectual authority: (1) *position students as holders of math knowledge*, (2) *expand use of collaborative participation structures*, and (3) *embrace the teacher-as-facilitator role*. Regarding the first, positioning students as holders of math knowledge means recognizing the multiple forms of background knowledge, experiences, and perspectives students bring, and eliciting and leveraging these to provide space and support for students to apply their ideas and strategies to analyze new mathematical situations in which they can generate their own ideas and justify them using the mathematics. The second instructional approach involves providing students frequent and varied opportunities for collaboration. Collaborative participation structures Zavala and Aguirre (2024) name as conducive to distributing intellectual authority include "turn and talk," "think-pair-share," "collaborative group work," and "gallery walks." These all provide varied ways for students to voice their ideas, learn from other's thinking, expand their curiosity by asking questions, and build upon others' ideas. The third instructional approach involves the teacher taking on a role as facilitator and co-constructor of knowledge, rather than the traditional math teacher role of imparting all knowledge. This means providing space for students to engage in productive struggle and utilize questioning strategies that engage students in generating and justifying mathematical claims and ideas they notice, discover, and wonder about. Zavala and Aguirre (2024) affirm, "When intellectual authority is distributed, more voices, ideas, and actions can be leveraged for individual and collective mathematics learning" (p. 97). Distributing intellectual authority is a key component involved in facilitating justification opportunities, particularly in ways that aim to advance students' agency and build positive mathematical identities.

Sampling and Participant Background

The participant in this study is one teacher, Josie (her chosen pseudonym). Josie was chosen for the study through a combination of *convenience sampling methodology* due to proximity and my pre-existing relationship with her as a former colleague and *purposeful sampling methodology* based on existing teaching practices and equity-oriented dispositions (Merriam & Tisdell, 2015). When considering participants for the study and speaking with Josie

about it, she mentioned that she starts every math lesson with a Number Talk (Parrish, 2014) and that she incorporates justification opportunities within and outside of Number Talks. Because Josie knew me as a teacher too at one point, this aided in providing a foundation for a trusting, researcher-participant relationship during the study.

Josie had nine years of teaching experience at the time of the study, with the most recent six years being at the school in which the study took place. The school is a Title I, K8 public charter school, located in an urban district in the Midwest with students predominantly from historically and currently marginalized backgrounds, including Black and Latinx students. During the study, Josie taught in a self-contained, fourth-grade classroom. Most of her classroom experience was in elementary classrooms, situated in urban communities with students of racially/ethnically diverse backgrounds and in low socioeconomic regions. She also had some experience as a middle school mathematics teacher. Josie identifies as a Filipina woman, and she immigrated to the United States from the Philippines when she was 13. From once working together with Josie at the same school, I knew that she had (and the school's mission was about) strong commitments to social justice and place-based learning.

Data Collection and Analysis

Study Design. The research questions guiding this study focused on how Josie provided justification opportunities in her classroom to meet her equity goals. The first question this study explored was, *What are one teacher's goals for promoting equity in the classroom?* The second was, *How does the teacher leverage the practice of mathematical justification to meet her equity goals?* These questions were examined through case study methodology (Merriam & Tisdell, 2015). In this study, Josie's practice and perspectives (about the relationship between equity and justification) were the "case" in which the study was bound.

Data Collection and Analysis. To examine Josie's practice and perspectives around equity and justification, data included recordings and transcripts of interviews and lesson planning meetings, field notes of classroom observations, and curriculum materials. Data collection began with a pre-observation in November 2023 to get a general idea of Josie's practice and inform some pre-interview questions, such as clarifying questions about types of justification opportunities observed. In January 2024, a two-part pre-interview (totaling 130 minutes) was conducted to gather background information (e.g., Josie's identity and teaching experience),

understand Josie's equity goals, and gather her general perspectives on the relationship between justification and equity.

Classroom observations were conducted two days per week (typically consecutive) from mid-February through April 2024, totaling 16 observations. Observing lessons during the second half of the year (and before standardized testing) provided a helpful timeframe for examining lessons with classroom routines well-solidified into the school year by Josie, and well-established prior to my entry into the classroom space. These observations spanned beginning, middle, and end of units, and across topics including multiplying and dividing by powers of ten, division with remainders, foundational fraction concepts (e.g., meaning), creating equivalent fractions, adding/subtracting fractions, and early decimals concepts (e.g., naming, meaning). During classroom observations, field notes were taken on Josie's justification-oriented, problem-based enactment strategies and on student justifications and agency. These field notes included types of student justifications (e.g., justifications of their ideas, claims, choices, or mathematical relationships) and Josie's justification implementation (including norms and pedagogical strategies for eliciting justification, such as discourse moves). Notes were also taken on instances in which Josie provided opportunities for supporting students' mathematical identity development or agency (such as what she would say to decenter herself as the mathematical authority) and instances of or statements made by students exhibiting agency, such as questioning a peer.

Spread throughout these observations were six (30-60 minute) lesson planning meetings, where Josie debriefed the previous week's lessons and shared plans for the upcoming week. Specifically, Josie debriefed the past week's justification opportunities, which included reflecting on questions I asked about observed lessons in relation to justification and her equity goals. For instance, I asked Josie what successes and challenges she experienced in supporting justification in the past week. I also asked Josie to reflect on what she noticed about students' agency and identity in the past week, and how she was considering these noticings in her plans for the upcoming week. Sometimes I asked questions to clarify or better understand something I observed in a lesson, about what happened during a day I could not observe that week, or about lessons earlier in the school year. For instance, after observing a Number Talk that connected to their class' reading challenge, I asked Josie if she had done any Number Talks earlier in the

school year that related to something happening in class or in the school, and to tell me about those types of Number Talks and why they were important to her.

During the observation window, Josie shared all curriculum materials she implemented into a folder in Google Drive. These included tasks I observed and did not observe so that I could get (1) a sense of curricular coherence and (2) a better understanding of how her lessons built upon one another, potentially in ways that supported students' agency during different stages of a unit. Analysis was conducted on all tasks in this folder. Keyword analysis was used to identify words and phrases within the curriculum materials related to justification, such as *why*, *prove*, *true, explain, pattern*. Thematic analysis (Braun & Clarke, 2006) was used to look for themes in tasks with potential connections for supporting students' agency or identity. One example of a theme that emerged was task design for cultural relevance. A task was grouped into the theme of cultural relevance when it related to students' experiences, interests, or community (e.g., a task that referenced the amount of rainfall in the students' city), because of the connection to their community. Another theme centered around patterns. An example of a task that was grouped into the theme of patterns included the direction, "Describe a pattern that you notice is true about multiplication for numbers that end in zeros." This theme connected to agency because students were being tasked with ownership of knowledge by developing a conjecture about what happens when numbers are multiplied by powers of ten, rather than the teacher telling them.

In May, a 60-minute exit interview was conducted to gather the teacher's final reflections about justification and equity concepts, ask questions that spoke across observations or emerged from the study, and gather the teacher's advice for other teachers looking to advance students' agency and identity through justification practices.

Analysis of pre- and exit interviews, lesson planning meetings, and field notes of classroom observations were also conducted using a thematic analysis approach, to look for connections between Josie's equity goals and her justification-oriented curricular enactment. One example of a theme that emerged was the construct of curiosity. Josie talked about inspiring students' curiosity in interviews and in lesson planning meetings. Grouped into this theme included transcript quotes that explicitly used the word, curiosity. For example, the quote, "I just think if a child or person is more curious than afraid, they're more willing to try," illustrates the teacher's desire to support students in developing positive mathematical identity. Additionally, being curious is connected to facilitating students' agency to engage with the mathematics as it

provides a starting point to build from, which is to uncover or better understand something stemming from what one is curious about. Another theme centered around background knowledge. Grouped into this theme included references to background knowledge that the teacher mentioned in interviews or lesson debrief meetings, such as this quote from the exit interview, "All kids need the background knowledge to enter into the problem-based space, like the thinking of the problem, I guess. They need to be able to use their background knowledge and their reasoning as they justify their thinking." Field notes were also analyzed to look for statements or questions the teacher asked to connect to students' background knowledge. For example, in observation 14, the teacher directed students to think about their answer in a previous problem and consider what they know about money and fractions to answer the next question, "What fraction of a dollar is 5 dimes?" The exact statement said by the teacher that was grouped into the theme of activating background knowledge was, "Then I asked what fraction of a dollar is your answer to question 5… You know what money is, you know what a fraction looks like." Activating students' background knowledge was an important theme because of the connection to the knowledge students bring (their identities) and how drawing on background knowledge can support students' agency in the learning process by using what they know to help themselves get started without the teacher's help. Themes that emerged from thematic analysis showed connections to Zavala and Aguirre's (2024) framing of distributing intellectual authority. In the findings, I describe Josie's equity goals as situated within the three instructional approaches for distributing intellectual authority. Then, I share classroom examples that showed how Josie facilitated relational problem-based learning with a justification focus, as connected to her equity goals.

Findings

For Josie, advancing equity in mathematics required intentional focus on facilitating students' agency and supporting students in building positive mathematical identity, which she saw as intertwined. This involved engaging students in problem-based learning experiences that focused on noticing and justifying patterns, rather than imparting these herself, and decentering herself as the mathematical authority. When examining the justification opportunities that Josie provided, problem-based learning emerged as a prominent theme. Discussed below are Josie's perspectives on student agency and identity in mathematics instruction, which show connections to Zavala and Aguirre's (2024) framing of distributing intellectual authority. Then, two cases are

shared of how Josie facilitated relational problem-based learning experiences (focused on justification) to support her equity goals. All names presented in the findings are pseudonyms. In quotations where the word "like" was interjected as a filler and not needed for context, it was sometimes removed for readability.

Josie's Equity Goals: Supporting Agency and Identity. Josie saw agency and identity as intertwined, and relational problem-based learning experiences with a justification focus as inherently cultivating both of these. How Josie thought about the relationship between justification, agency, and identity has connections to Zavala and Aguirre's (2024) conceptualization of *distributing intellectual authority*. According to Zavala and Aguirre (2024), distributing intellectual authority provides opportunities for collaboration and for students' voices, ideas, and reasonings to be heard, and doing this requires the teacher to share authority of knowledge with and among students. Zavala and Aguirre (2024) describe three instructional approaches for distributing intellectual authority: (1) *position students as holders of math knowledge*, (2) *expand use of collaborative participation structures*, and (3) *embrace the teacher-as-facilitator role*. This section describes Josie's perspective on the relationship between justification, agency, and identity as situated within Zavala and Aguirre's (2024) framework of three instructional approaches for distributing intellectual authority.

Position Students as Holders of Math Knowledge: Josie provided justification opportunities for students because it was important to her that her students saw themselves as holders of math knowledge, and she leveraged justification opportunities to center students' reasoning as the mathematical authorities. Supporting students in generating and justifying ideas for themselves involved not answering many questions students asked that would stop them from thinking or would give away answers. Josie shared, "I always try and promote that I'm not the knower of everything… I just don't answer questions… So they know if I'm not answering a question, they're gonna stop asking because I want them to work through it." Not answering or imparting the knowledge sought by students' questions (and instead, supporting students in producing the knowledge and answers themselves) was Josie's way of showing that she is not the "knower of everything." Josie used similar language when she talked to me about not wanting to be the "keeper of math." She shared, "I very rarely try and give the answers to them, 'cause, you know, I think I've said this before, but I don't want to be the keeper of math. I want them to be the thinkers."

Josie supported students in being the thinkers by providing justification opportunities that provided students entry points into the problem, activated their background knowledge, and inspired their curiosity. She explained,

> *Justifying allows everyone to have, to contribute, to have an entry point. So even if you can't do long division, you can explain it with a diagram by cutting, you know, tick marks into whatever. It allows everyone to share 'cause everyone came to the classroom with a tool belt of knowledge. And they can share whatever is in there, and we can add to it and everyone can contribute, so I think that is part of agency.*

According to Josie, all students are holders of math knowledge because they all "came to the classroom with a tool belt of knowledge," and they can learn from and build off of one another's ideas to work toward collective understanding. In another conversation, Josie added that sometimes students need to be reminded about or supported in accessing the knowledge they bring, and inspiring their curiosity is one way that can activate that. Josie stated, "When I teach math, I want kids to be curious and know that they, if they really looked at it, they know more than they're realizing."

Expand Use of Collaborative Participation Structures: Because Josie viewed all students as holders of math knowledge, she created frequent opportunities for students to collaborate so they could learn from one another. Josie explained that with twenty-four students in the room, there are so many perspectives and ideas to be learned from, outside of her own. She shared,

> *It's common that I'll allow them to talk to each other, too, which is really important, like give them time to either be with a partner or a small group. And be very clear that everybody should contribute, you know. I think that's important, too, especially for those tasks that are looking at a lot of numbers and really trying to notice as much as they can. The more brains you have, obviously the more you might notice, and more conclusions might come to something.*

In another conversation, Josie echoed these ideas, sharing that all students come to the classroom as different people, who think and learn in different ways, and one students' way of unique thinking could be what helps something click for another student, especially with there being twenty-four students in the class. She explained, "They come to your classroom as different people… You never know what's really going to help them click in their head. So if there's twenty-four different people sharing a perspective, there's more chances that that might happen."

Embrace the Teacher-as-Facilitator Role: Because supporting students' agency and building positive mathematical identities were prominent equity goals for Josie, she saw her role as that of a facilitator. For Josie, being a facilitator involved leveraging justification opportunities, situated within relational problem-based learning experiences, that centered students' ideas and reasonings. When asked about the relationship she saw between justification and agency, Josie rhetorically asked,

> *Don't they kind of go hand-in-hand? Or to me, it feels that way… The opportunities to have their thinking be heard and valued and uplifted is kind of the agency part…I think that justification opportunities and conversations inherently provide kids with agency.*

According to Josie, the mathematical practice of justification and agency "go hand-in-hand." In particular, Josie described justification as affording opportunities for students' ideas and perspectives to be "heard and valued and uplifted," (an expression and reflection of students' identity in the learning process), which inherently advanced agency. Further describing how justification opportunities provided spaces for students' voices to be heard, reflecting their identities and facilitating their agency as they access they mathematics, Josie explained,

> *The reasoning is your own. You and your voice is, you know what I mean? You're able to voice your own critical thinking of whatever it is. So nobody told you this is a solution or this is a strategy. You did the math yourself, so that part is like identity… You do the task without I guess being trapped in an algorithm. You don't have to follow the step-by-step process right now. You can, you know, think of it in your own way. I think that allows for a lot of access.*

Being a facilitator meant not telling students a solution or strategy to use and letting them apply their own way of solving the problem. Facilitating students' motivation and confidence to do so involved inspiring their curiosity. For Josie, reasoning and curiosity were directly related. She believed, "How could a child be curious if they're not allowed to ask questions and explore those questions? That's, you know, reasoning in a classroom." When asked about how justification supported her goals as a teacher, Josie shared, "Math isn't just like you show up, you do the thing, and you get the answer. You're allowed to be more creative with it. And I think a lot of kids do well with creative tasks, because they can kind of put themselves into it." Facilitating students' creativity and joy for learning was important for Josie, because to her, math was more than calculating answers. Josie mentioned, "I love learning. I love engaging a curious mind, whether it's a kid or an adult, and I couldn't imagine doing any other job that would be fulfilling." Josie went on to add, "There's so much joy coming from being curious, you know–

the desire to learn, the desire to find out about something you just decided to be interested in because of whatever reason. I don't know what would happen to somebody that lost that." For Josie, being curious was more conducive to students' mathematical identity development and agency building than having to know answers right away, and helping them see this sometime involved boosting their confidence. Josie mentioned,

> *If you understood that you could be curious and ask questions and explore something instead of having to know it right away. But then you, there's a lot of like self-confidence that has to be built with a kid that sees math and problems that way.*

Providing students opportunities to be curious, creative, and discover and justify mathematical ideas themselves through problem-based learning meant taking on a role as a facilitator, which meant leveraging justification opportunities. For Josie, justification opportunities provided the space for her students' ideas to be heard, backed up by the mathematics to support their claims and ideas. Being able to approach problems using their own knowledge, in turn, facilitated their agency in the learning process. The teacher-as-a-facilitator role was important to Josie so that they could be the thinkers. Josie believed, "Because really, that's where we're leading them to is like to be outside of our walls and be free thinking humans out in the world." For Josie, building students' positive math identity (through creative opportunities to engage in reasoning) can support students in being agentic, free thinking humans within and beyond the mathematics classroom.

Discovering and Justifying Patterns. To support these goals of students being free-thinking humans, Josie provided students opportunities to discover and justify mathematical patterns and relationships themselves through problem-based learning experiences (in ways that reflected their identity), rather than her imparting these ideas to students (facilitating their agency). These opportunities were leveraged at the beginning of new topics and units, by connecting to students' background knowledge. When directly asked about the relationship she sees between problem-based learning, justification, and background knowledge, Josie said, "All kids need the background knowledge to enter into the problem-based space, like the thinking of the problem, I guess. They need to be able to use their background knowledge and their reasoning as they justify their thinking." Problem-based learning involves engaging students in noticing patterns, activating background knowledge to apply in the investigation of those patterns, and justifying a rule or reason that can explain the mathematics behind those patterns. During the justification

process, those patterns become understood and then ultimately proven or disproven. For Josie, math was all about noticing patterns, and this was why she created opportunities for students to discover and justify patterns at the beginning of new units. Connecting again to her theme of "gamifying" math, Josie explained,

> *So much of math is patterns, looking at patterns, thinking about patterns, or understanding why those patterns happen. And I think that is a way to kind of 'gamify' the experience for kids. And they're good at that… And especially at the beginning of units, if you didn't tell them yet what something is, they can just look at the raw numbers, or whatever it is, and see those things and be like, 'Hey, actually, I came up with this.'*

For Josie, math discussions were fundamental parts of the pattern-noticing and justification process, and all of this was important for cultivating their agency and building positive math identity. Josie shared,

> *I love, you know, doing math discussions, and I think that engages the kids, too, to feel like they can share ideas, you know, and not just like, "Hey Miss. [Josie] told me to do this specific thing for this problem." But they could be like, "I noticed this pattern. What if we tried this pattern across these other things, will we see the same thing?" And then they ask like, "Well, why didn't we…? Or why did we…?" So we always start with discussion, and that's built into every day for us, too.*

Described below are two cases from Josie's class where she facilitated students' agency by engaging them in discovering and justifying patterns in their own ways that supported building their mathematical identities. The curricular tasks, Josie's enactment strategies, and the students' ideas are highlighted. In both of these cases, students noticed patterns, developed a rule, and engaged in a justification process aiming to prove or disprove it.

Figure 9. Continuum of Noticing to Justifying Patterns for Proving/Disproving Math Rules

<u>*Case 1: Investigating Divisibility Rules:*</u> During two consecutive lessons, the class was investigating divisibility patterns of single digit numbers. Although Josie could have provided students with a list of "divisibility rules," supporting her equity goals of advancing students' agency and building positive mathematical identity meant providing students a problem-based learning experience for discovering and justifying these divisibility rules or patterns themselves. Prior to this lesson, Josie gave students a pre-assessment on reasoning with divisibility to gather

some of their background knowledge before implementing this task. For Josie, building from and connecting to background knowledge is a key part of facilitating problem-based lessons. Josie created her own worksheet for students so that they could have an organized space where they could record their ideas as they explored patterns in numbers divisible by particular digits. The directions on the worksheet said, "Find the pattern or make up a rule for divisibility by single digit numbers." The worksheet had two columns. Running down the left column was a row for each digit 1-10 (with 7 omitted). There was a box with each of these digits that left space for students to show their work for the patterns they noticed with numbers divisible by each of these digits. The specific worksheet direction in that column said, "What patterns do you notice? Show your work." Running along the right-hand column was space to record, "Our Group's Rule." This was a two-day lesson, where students spent the majority of class investigating divisibility patterns/rules with a partner, and then gathering as a whole class at the end of the lesson to discuss what they noticed that day. Students investigated these patterns by choosing their own numbers to divide and see what they noticed, and skip counting to see what they noticed. Josie recorded the rules students noticed each day on an anchor chart (see Figure 10). When discussing a rule for divisibility by 1, the conversation was short, as was the justification, but it appeared sufficient for Josie as she did not spend much time on it:

Student: *Every number is divisible by 1.*

Ms. Josie: *Can you prove that?*

Student: *With division you cut into groups. I can cut any number into groups of 1.*

The discussion that came about during numbers divisible by 3 was more complex. Below is a segment from one pattern students noticed, highlighting Josie's questioning strategies for supporting their justifications. (The chart in Figure 10 shows other patterns students also noticed and justified with numbers divisible by 3).

Ms. Josie: *What patterns and rules did you notice for numbers I can divide by 3?*

Student 1: *It goes odd, even, odd, even.*

Ms. Josie: *Is that only true for 3s or is it also true for other numbers?*

Student 1: *It also works for... (and other students jumped in to add more examples).*

Ms. Josie: *So, because it's an odd number, it alternates. Why does it do that?*

Student 2: *I think it's because 3 is made up of an odd number and an even number.*

Ms. Josie: *Tell me how.*

Student 2: *3 is a 2 and a 1.*

Ms. Josie: *Is every odd number made up of an odd and an even part? Think about it for a minute. 63, 77.*

Here, Josie asked students if this pattern is only specific to 3s or if dividing by other digits presents this alternating pattern too, to help students think about the nature of 3 versus other numbers. Josie also asked *why* the numbers alternate in the way that they do. When a student said that 3 is made up of an odd and even part, Josie pressed for reasoning by asking *how,* and she presented some examples to consider.

While Josie could have presented all the divisibility rules for students upfront to "save" instructional time, this would not have advanced her equity goals of facilitating students' agency and allowing students' identities to be reflected in the learning process. Allowing students to discover divisibility patterns and justify the rules they came up with themselves in ways that made sense to them, promoted agency and afforded students the opportunity to build positive mathematical identity, and also supported students in a deeper exploration and understanding of numbers and division.

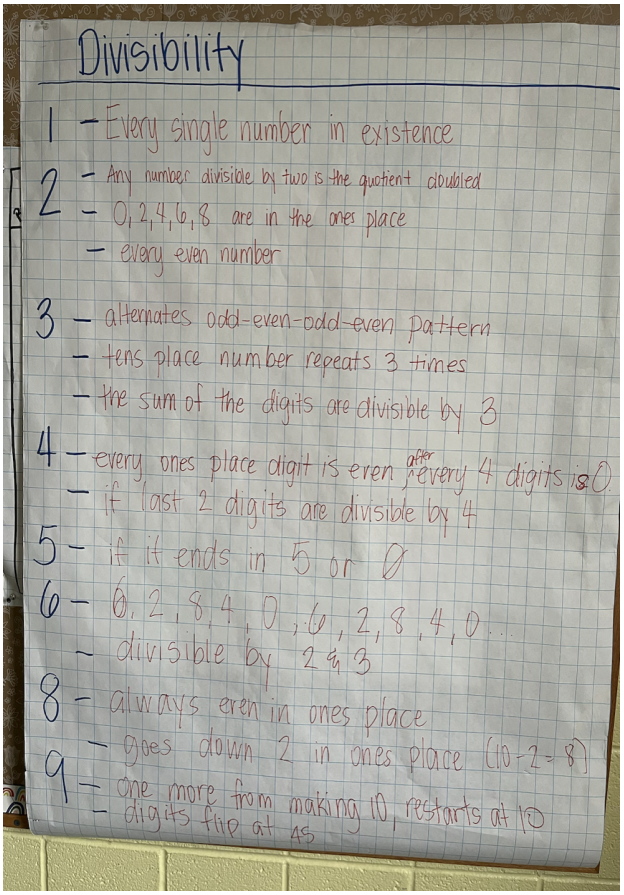

Figure 10. Chart of Students' Divisibility Noticings

When reflecting on this lesson in a debrief meeting later, Josie said she felt, that across all the lessons they did in the past week, these two days provided students the most opportunities to engage most deeply in the process of mathematical justification. For her, that meant, "time to actually sit and play with numbers." When I asked how Josie thought this lesson supported students' agency or mathematical identities, she talked about the task being accessible because anyone can notice a pattern and share their ideas, whether correct or incorrect. Linking how access supports agency, Josie explained,

> *Anybody can notice a pattern. And everybody feels just like they can participate, you know. Like, 'I can look at this and share a thought, and it's not gonna be wrong. I'm just looking for a pattern to share out loud, and whatever it is fine, as long as we have proof that it's true, based on, you know, the numbers that we see. So I guess, I don't know, like agency, just anyone can start, at least start it, even if they don't get to as much depth as like– there's varying degrees of, you know, math that the kids get to. But everyone can start with something.*

Elaborating on students' agency, Josie talked about the importance of providing students time to notice, explore, and explain.

> *I think, you know, I often feel rushed for time. But I think it's important to give them the time to even just sit, even if they're not productive, you know, in that 15 minutes the whole entire time, or for that 20 minutes. Just to be able to sit there and look at it and putz around a little bit and do something else. And then come back to and be like, "Oh, actually, now that I'm sitting here, I notice this." I often feel like, you know, impatient, like okay, let's just get to work. But I think sometimes I have to remember just let them look at it. Because I mean, most of these kids are curious learners anyway. So they want to, they want to be able to share something. They love sharing their thinking, so I think just giving them time to sit and really get to find something out on their own.*

<u>*Case 2: "1 is equal to 10 in the decimal world":*</u> In a later lesson that was connecting the end of a unit focused on fractions to a new unit focused on decimals (and the relationship between the two), Josie gave students the task, "Write the fraction ½ as a decimal." At this point in time, students had not yet learned about writing fractions as decimals in school. In order to engage students in this task, Josie activated students' background knowledge and supported them in noticing patterns. This exploration led to an interesting discussion around place value and Zoe's (all names pseudonyms) metaphor of "the decimal world."

As a bridging question leading up to this task, the prior question students were asked to answer and justify was what fraction of a dollar is 50 cents. Students answered with 50/100, 5/10, and ½. The class agreed that one dollar represents 100 cents. One student's justification was, "Because 1 is half of 2, and 50 is half of 100. If the top number is half of the bottom number, it's equivalent to ½." Another student added, "So we all know 5/10 is ½. Let's say we multiply both those by 10. That's 50/100." Noticing that the numerator was half of the denominator, and checking by multiplying a numerator and denominator by the same number, convinced the class that 50 cents can be represented as fractions that include 50/100, 5/10, and ½.

This background knowledge that was activated engaged students in noticing a particular pattern that could be extended to the next question of considering how ½ would be written as a decimal. During this task, Josie pulled out a blank number line as a tool to support one student's process of justifying her claim that ½ is written as 0.5. Josie guided the class in filling out the number line to support them in noticing any place-value related patterns in how decimals are written as they relate to fractions. As Josie guided students in filling in the number line with their noticings, she scaffolded by asking students to look at the patterns. Investigating these patterns was pivotal to the justification process of showing ½ is 0.5, and ultimately extending students' understanding of place value, fractions, and decimals (and how they relate). Figure 11 shows how the students thought through the place value relationship of whole numbers, fractions, and decimals to create the decimal number line, as recorded on the projector by Josie.

Zoe: *Zero point five.*

Ms. Josie: *Why?*

Zoe: *One whole is like a ten, in the decimal world. It's kind of hard to explain.*

Ms. Josie: *It's okay, you're doing a good job.*

Zoe: *On the number line, if you went–*

Ms. Josie: *I have a number line right here. I knew you were going to say something about the number line, so I had one handy.*

Zoe: *1 is equal to 10 in the decimal world. If you cut the 1 in half, it's not going to be five.*

Ms. Josie: *Like this?*

Zoe: *Kind of.*

Ms. Josie: *These represent tenths. Zoe is kind of getting there. 1 is 10, in this example of the decimal world.*

Ms. Josie: *What is 2?*

Zoe: *Two tenths.*

Ms. Josie: *How would I write two tenths?*

Zoe: *2/10*

Ms. Josie: *If each one, if the unit fraction is the 10, what would I call this?*

Zoe: *Four tenths.*

Ms. Josie: *Could I fill out the rest with this pattern?*

Multiple Students: *Yes.*

Ms. Josie: *If I was here at the dot, what would this fraction be called? Look at the pattern.*

Multiple Students: *Six tenths.*

Ms. Josie: *What about this one?*

Multiple Students: *Seven tenths.*

Ms. Josie: *I'm looking for the fraction, three tenths, as a decimal. Who could come up to the board and find it? Even though they're written as a fraction or decimal, their word form is the same. This [points to 4/10] is four tenths and this [points to 0.4] is four tenths. What is this fraction?*

Multiple Students: *One tenth.*

Ms. Josie: *What's this fraction?*

Multiple Students: *Zero tenths.*

Ms. Josie: *What's this decimal's name? What fraction is it equivalent or equal to?*

Multiple Students: *One half.*

Ms. Josie: *5/10 or ½*

Ms. Josie: *[And] this one?*

Multiple Students: *One whole.*

Ms. Josie: *Ten tenths or one whole.*

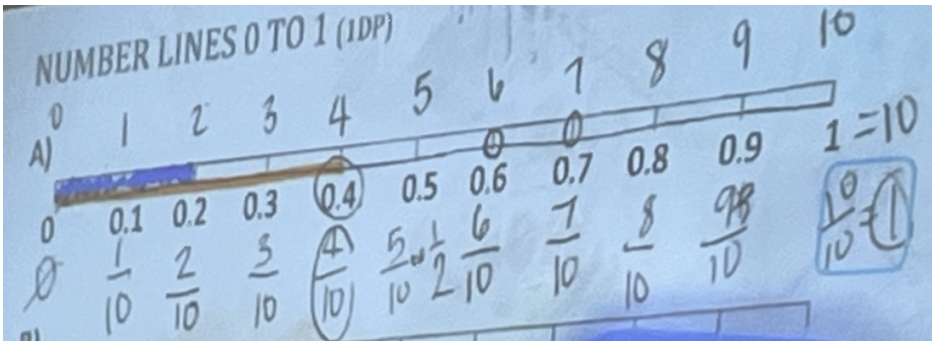

Figure 11. Number Line

Josie took out a decimal number line to help support the class in understanding Zoe's claim that ½ is written as 0.5 as a decimal. When Zoe presented this claim, she did not yet have a justification developed and said it was "kind of hard to explain." Though Zoe did have a partial justification, which was "one is equal to ten in the decimal world," she was still trying to figure out how to describe this with mathematical clarity and evidence. Because Zoe mentioned a number line, Josie took out a number line to help elaborate Zoe's claim in a way that started with the ideas Zoe initiated. Though Josie seemed to lead much of the justification development, she used Zoe's claim (i.e., ½ is written as 0.5 as a decimal) and Zoe's starting justification for this (i.e., "one is equal to ten in the decimal world") as building blocks to scaffold the class in connecting the relationship between whole numbers, fractions, and decimals together on a number line, which helped illustrate Zoe's idea that "one is equal to ten in the decimal world," and ultimately produce justification for the claim, "½ is written as 0.5 as a decimal."

In debriefing this lesson later, Josie talked about why math discussions are important to have in class and how a student's idea can sometimes convey a concept to other students in a way she would not have thought of. Referring to Zoe, Josie explained, "Honestly, she helped me. She was like, she said it in a way that I never would have said it. And I was like just trying to repeat it in my own words, to see if like, if I said it again, who would see it again?" The design and enactment of this problem-based lesson afforded Zoe the opportunity to explain her own working understanding of decimals and fractions through a metaphor that made sense to her, reflecting a positive mathematical identity. Josie supported Zoe's agency in the learning process by working to understand her metaphor, helping elaborate Zoe's idea through a process of explanation and justification, and supporting the class in understanding Zoe's metaphor. Josie explained that having math discussions is important because every class is different and "they come to your classroom as different people." Cultivating students' mathematical identity means recognizing that each student brings to class different perspectives and providing opportunities that give students the agency to apply their perspectives to the content in ways that make sense to them. Sometimes one student's idea can help other students. Josie affirmed,

> *I think that's why I think these conversations are so important and discussion is so important. 'Cause you really don't know. Every group is different. Their context is different. Their perspective is different. They come to your classroom as different people. And you can plan all you want, but you never know what's really going to help them click in their head. So if there's*

twenty-four different people sharing a perspective, there's more chances that that might happen. I don't know what, how that helped the kids that nodded, but it did. It helped them click something.

The design and enactment of Josie's problem-based lessons, in which she leveraged the mathematical practice of justification, supported her goals of facilitating students' agency and providing opportunities for students' identities to be reflected in the learning process.

Foundation of Conducive Classroom Environment. For Josie, supporting students' agency and positive mathematical identity development in problem-based lessons required a *conducive classroom environment*. Cultivating a conducive classroom environment was a culture built over time through development of norms, trust, and relationship-building with students over the course of that school year. How her students viewed what it meant to do and be good at mathematics came from the class culture established from the start of the year and how that culture became reinforced each day into the year. Klosterman (2016) characterizes a *conducive classroom environment* as a classroom culture where students feel safe taking risks and sharing their in-progress ideas. As Josie's students were working to justify their claims, they took risks by sharing ideas they were not yet certain of but tried to back-up with mathematics. For Josie, setting the foundation of a conducive classroom environment meant talking through "math agreements" with students at the beginning of the school year and creating them over the course of several days. Josie shared,

We spent a whole week making the agreements. So we were like, 'Everyone is going to feel like they contribute to this classroom and you're gonna be a person that helps in that.' You know what I mean? Not prevents people from their own learning. And I think they understood that. So, I think part of it was the kids buying in and willing to be those kind of people for each other... I think part of it is a culture thing from the school, but also, we– I just, I really pushed really hard to make it so that they knew I was serious about holding them to these agreements, you know? And yeah, we just talked a lot about everybody deserves a chance to learn, everybody can contribute. Everybody knows something, and you know what I mean? I think because we also had those discussions and they helped make the norms and they were the ones that dictated what my job was as a teacher.

Figure 12 shows the math agreements the class came up with. The student agreements started with ideas that Josie presented to the students that they built from. The teacher agreements were made by the students. The green posters show the teacher agreements, and the yellow posters show the student agreements. Josie reinforced these throughout the year, evolving

the agreements into norms. Students learned that in their mathematics classroom, everyone's ideas were valued, mistakes were part of the learning process, authority of knowledge was shared amongst, and that they had a responsibility to understand one another and hold one another accountable.

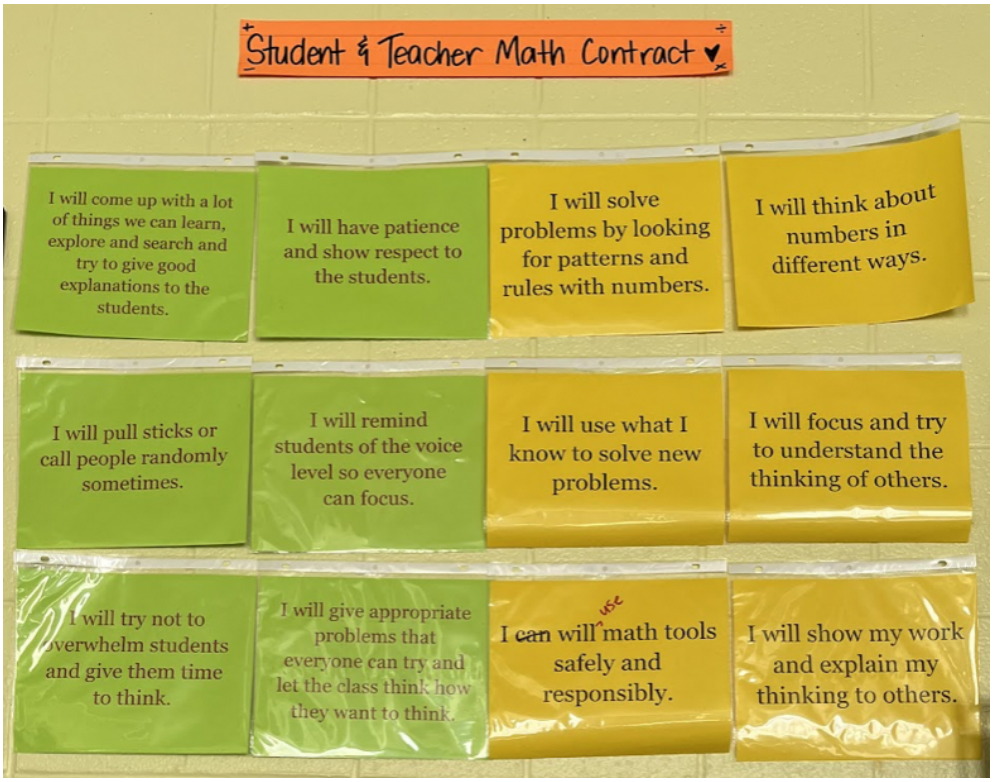

Figure 12. Math Agreements Developed by Ms. Josie's Class

When I asked Josie which of the agreements she thought might be most supporting students' agency, she said that they all do. Josie explained,

> *They all in a way ask the students to be critical thinkers and be able to share that critical thinking. So not just be students that can, say, memorize what I said and then do it. But that they have the agency to solve things, to think about things, to focus on what somebody else is saying which holds the, you know, the speaker accountable to their peers, not just to me.*

For Josie, it was important that students saw themselves as doers of mathematics and that they had opportunities to construct knowledge themselves, which is why she incorporated problem-based approaches to mathematics pedagogy. Josie shared,

> *I think a student that sees themselves as capable they're more willing to share their thinking… I think that that is more important than me being an authority of knowledge in a classroom, and as*

part of learning, because I cannot give my knowledge to somebody. I can't. That's never– you know what I mean? If I could, my job would be so easy.

Josie cultivated a conducive classroom environment so strong that students even felt comfortable challenging their peers whom they consider "math whizzes." Josie's definition of "math whiz" was a student who can see patterns very naturally and who often produces correct answers. In describing this dynamic in her classroom, Josie elaborated,

I know that the kids perceive Jonah especially, even since second, third grade, because he's been in Faith's (Josie's daughter) classes like the math whiz or whatever. He's just really natural with it. He understands patterns really well. And so I know the kids kind of see that. But even sometimes, and I don't want them to always go with what they say, you know. And sometimes they don't, which I'm like, okay so they're still thinking for themselves, and they perceive not Jonah as the keeper of math, but all of them as able to do math, and they can disagree with even somebody that, like Jonah, who is often right.

For Josie, it was important that students did not see her or the "math whizzes" as the "keepers of math," but rather, all see themselves as knowers and doers of mathematics. Josie mentioned that often the math whizzes will still share ideas that are wrong.

'Cause sometimes in math, what's intuitive isn't always, you know, the answer or something, and sometimes you didn't– Evan will get something that is– seems intuitive– like it should be this, and it's not. And so there's been instances where that's happened.

Jonah and Evan were students that Josie specifically referred to as math whizzes. A major testament to the conducive classroom environment Josie created can be seen relative to the identities of the class' math whizzes. For one, the students in her class felt comfortable countering or disagreeing with Jonah and Evan because they could also see themselves as knowers and doers of mathematics. Second, the identities of Jonah and Evan were not centered around having to always be right (which can sometimes happen with kids used to excelling in mathematics class), as shown by their willingness to present ideas that were in-progress and potentially be mathematically incorrect.

Discussion and Conclusion

This study examined how one teacher provided justification opportunities to meet her equity goals through mathematics instruction. Specifically, the study explored the questions, *What are one teacher's goals for promoting equity in the classroom?* and *How does the teacher leverage the practice of mathematical justification to meet her equity goals?* Josie's goals for

advancing equity for students meant facilitating their agency in the mathematics learning process and supporting students' development of positive mathematics identities. This involved relational problem-based learning experiences that included inspiring students' curiosity, sharing authority of knowledge with and among students, supporting students in discovering and justifying mathematical patterns and relationships themselves, and establishing a conducive classroom environment by which they felt comfortable and empowered to engage in these opportunities.

Given the limited research in the field on how the process of justification opportunities are facilitated (Karunakaran & Levin, 2022), how infrequent justification occurs in classrooms (Melhuish et al., 2020), and the gaps in the literature on the role of justification in advancing equity (Bieda et al., 2023), this study helped highlight examples of how justification could be leveraged to support a teacher's equity goals. Because Josie's equity goals included supporting students' agency and positive mathematical identity development, this suggests that justification may be a powerful practice in advancing critical equity dimensions. Importantly, understanding how justification is facilitated in the context of a teacher's equity goals could better help teacher educators support other teachers in incorporating justification into their instruction in ways that are relevant to each teacher's context and goals. Because every classroom context is different, and practices are contextually situated, teacher educators need to be able to work within a teacher's equity goals in order for teachers to view the practice as valuable and to be leveraged in ways conducive to their students.

It can be easy to wonder whether Josie's practice and perspectives are about equity or if her approach to mathematics instruction is simply "just good teaching." Often, *equity* and *good teaching* are terms used interchangeably. If good teaching, by definition, means instruction that is supporting positive learning experiences and producing positive student outcomes for all students (particularly marginalized students), it would seem that good teaching would inherently advance aspects of equity. Equity is a broad word and multidimensional construct that signifies positive learning experiences and an impact of positive outcomes for all students. If equity means positive student outcomes are realized (especially for marginalized students), then good teaching practice is a means by which those can become achieved. This paper defined Josie's primary equity goals as a desire to empower students' agency and advance their mathematics identity development, and the study explored a teaching practice by which Josie strove to

achieve those equity goals. For Josie, teaching practice that incorporated relational problem-based learning approaches (that leveraged justification opportunities) was the type of "good teaching" she perceived to advance equity for her students. In the article, "But That's Just Good Teaching! The Case for Culturally Relevant Pedagogy," Ladson-Billings (1995a) affirms that culturally relevant pedagogy is indeed a description of good teaching, stating,

> *One usual response to what I share is the comment around which I have based this article, "But, that's just good teaching!" Instead of some "magic bullet" or intricate formula and steps for instruction, some members of my audience are shocked to hear what seems to them like some rather routine teaching strategies that are a part of good teaching. My response is to affirm that, indeed, I am describing good teaching, and to question why so little of it seems to be occurring in the classrooms populated by African American students" (p. 159).*

In following work, Ladson-Billings (1995b) studied the pedagogical practices of eight teachers in classrooms with Black students and found that the "good teaching," which was achieved through culturally relevant pedagogy, rest on three criteria: (1) students achieve academic success (2) students develop cultural competence (3) students develop a critical consciousness through which they challenge inequities. It makes sense that pedagogy that is relevant to students' lives, where students see content connected to their culture and community and where they have a voice to analyze and critique that content, can support access (to a better understanding of the content) and engagement (through finding meaning in the content). For Josie, who taught in a classroom of predominantly Students of Color, it was important to her that she connected mathematics content to students' lives, that students saw themselves as doers of mathematics and as a community (a mathematical community and community more broadly as affiliated with their shared place), and that students had choice and voice in the learning process.

Equity is a complex and multifaceted construct. The broad use of terms such as *equity* and *culturally relevant pedagogy* in the literature has implications for the field to be clearer about the ways they are defining and using these constructs. Gutiérrez's (2009) framework for *dimensions of equity* is one example of a helpful framework that names and describes the multiple facets of equity, within the context of mathematics teacher education. Gutiérrez defines equity as comprising four interrelated dimensions (*access*, *achievement*, *identity*, and *power*), all of which are described as needed to realize equity. Josie believed that justification can promote all of these equity dimensions. When I asked, "Where do you see the practice of justifying within each of these dimensions?" Josie stated, "Justifying supports all of those things," and went on to

elaborate how. Although Josie saw justification as a practice that can promote equity (because it can promote all four dimensions), the focus on this paper was on her attention to the critical dimensions in order to illuminate the relationship between justification, agency, and identity. When the field discusses equity goals, it is important to be intentional about attending to aspects of critical dimensions of equity to broaden our understanding beyond the dominant dimensions of equity, as access and achievement have dominated the discourse, according to Gutiérrez. Since Gutiérrez also states that all dimensions must work together to realize equity, more attention to the critical dimensions can lead the field closer to supporting equitable outcomes and experiences for all students.

More research is needed to understand the relationship between justification opportunities embedded in relational-problem-based learning experiences and students' agency and identity. In particular, how and why teachers provide these opportunities across various contexts needs further exploration. Because justification opens spaces to center students' voices and ideas and requires the teacher to be responsive to the perspective brought by students, the field could benefit from research that explores the potential of culturally relevant and culturally responsive justification practices. In particular, what would pedagogy that facilitates opportunities for culturally responsive justification look like, and what impact could it have on equity for students? How might culturally responsive justification further advance students' agency and positive mathematical identity development? This has implications for the development of frameworks or tools that seek to understand what culturally relevant and responsive justification could look like and how to support educators in facilitating this type of pedagogy. Additionally, while this study only measured the teacher's perceptions of equitable experiences, future studies could benefit from a deeper analysis of the student experience. For example, understanding students' own perspectives on the effects that culturally relevant or responsive justification opportunities have on their agency and identity would be important for understanding the alignment between the teacher's intentions and the impact on students. This could involve surveying students about their perspectives on how they feel about their own level of understanding and how they view what it means to do and be good at mathematics (including how they situate themselves within that description), throughout the year. A better understanding of how and why teachers can facilitate justification opportunities to meet their equity goals (and

the impact on students) could better help teacher educators support teachers in leveraging justification in ways that are responsive to teachers' contexts.

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CHAPTER 4: INCORPORATING MATHEMATICAL JUSTIFICATION AT THE BEGINNING OF UNITS TO SUPPORT STUDENTS' ENGAGEMENT WITH NEW CONTENT AND AGENCY IN THE LEARNING PROCESS

A fourth-grade class just began their first unit on fractions, having not yet had any lessons on fractions this school year. The teacher introduced the task shown in Figure 13. As expected, the most common student answers shared were ⅓ and ½. One student's justification for the answer of ⅓ was that 1 out of 3 pieces are shaded in. One student's justification for the answer of ½ was that the full bottom half of the shape is shaded in. Class that day ended on a cliffhanger, having not come to a consensus on the answer to this problem.

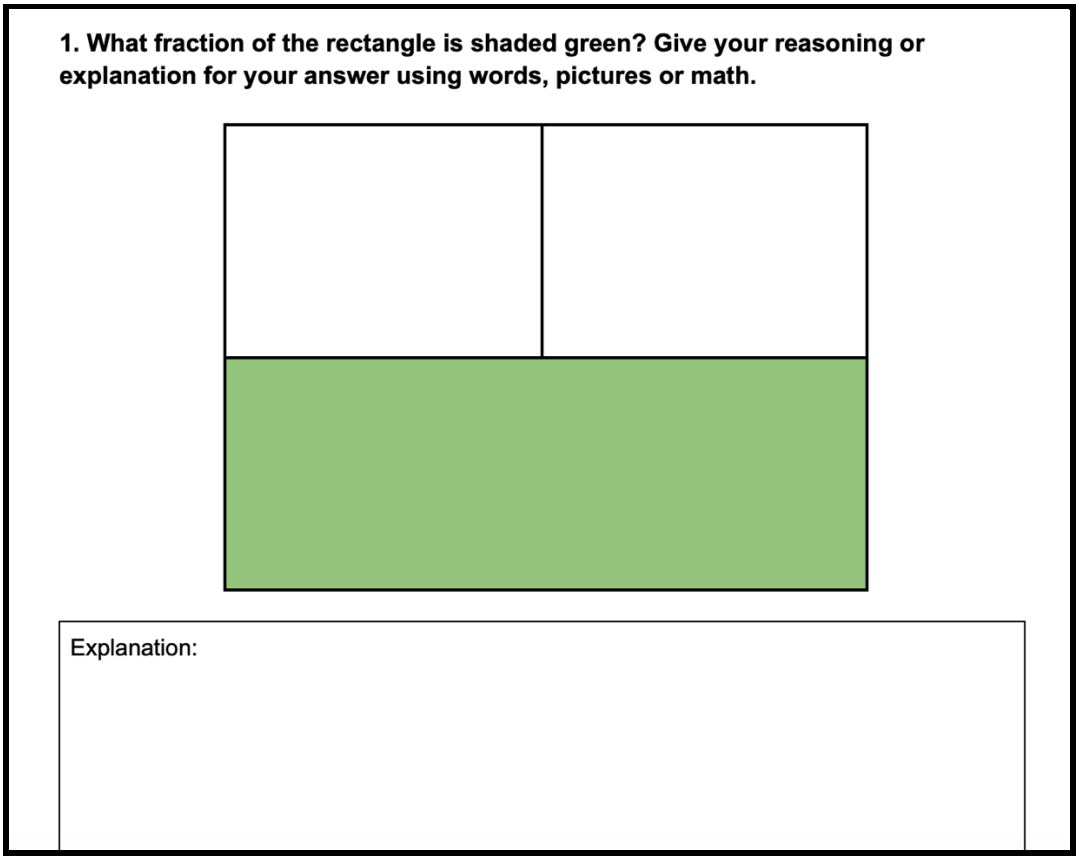

Figure 13. First Fractions Problem

In response to students' answers and justifications for their thinking, the teacher prepared a hands-on lesson for engaging students in an exploration and discussion of thirds, halves, and the meaning of fractions the following day. The teacher gave students the direction, "Use this Play Doh to make a pastry, and cut your pastry into thirds." In groups of two or three, students used their Play Doh to form giant cookies, brownies, marshmallows, and other desserts. Then

they cut their pastries into three pieces, some of equal sizes and some of different sizes. The teacher asked students to justify their reasoning for cutting their pastries in the ways that they did. She asked one group who made a cookie that they cut into three equal-sized pieces, "Why did your group choose to cut your cookie this way?" After eliciting their reasoning, she asked the class about one group's marshmallow cut into three pieces of different sizes, "What do you think about this cut?" The students' justifications paved the way for a productive discussion on the meaning of fractions, which will be elaborated in this paper. Using this context, this article discusses how the mathematical practice of justification can be leveraged at the beginning of a new unit in ways that support students' engagement with new content and agency in the learning process.

Establishing a Foundation for Fraction Concepts

Having a strong conceptual understanding of fractions is important for setting a foundation for the building blocks of higher level math concepts later, such as proportional reasoning and algebra. In fourth grade particularly, where students will learn to create equivalent fractions and add and subtract fractions with like denominators (that are more complex than the fractions they worked with in third grade), it is foundational to first understand the meaning of fractions as being equal-sized parts of the same whole. A teacher may find it first helpful to activate students' background knowledge and check for their understanding about the meaning of fractions. Activating students' background knowledge, in ways that invite opportunities to consider notions of justice related to equal shares, alongside their recollection of fraction concepts, can be generative for supporting engagement. Children often become invested in topics around justice, fairness, and equal shares, which can hook them into the content and potentially support them in recalling what they may have learned about fractions in the past. Keeping in mind that fairness does not mean sameness, the word "fair" as used in the classroom dialogue shown later (and as used thereon out), will be used to refer to "equal shares" (as opposed to sameness). Having opportunities to explore and discuss fraction concepts encourages students to really think about fractions, understand that fractions have meanings, and notice that the pieces (of unit fractions which share the same whole) should be equal. Leveraging the mathematical practice of justification can help provide these meaning-making opportunities for students.

The Role of Mathematical Justification

Bieda and Staples (2020) define mathematical *justification* as, "the process of supporting your mathematical claims and choices when solving problems or explaining why your claim or answer makes sense" (Bieda & Staples, 2020, p. 103). Justification is a mathematical practice that can benefit both students' learning and teachers' instruction. For students, the justification process can support students in talking about mathematics and promote knowledge mobility (Liljedahl, 2020), increase engagement, strengthen reasoning skills, support conceptual understanding of content, and foster positive mathematical identities and productive dispositions towards mathematics (Boaler, 2022; Staples et al., 2012). For teachers, eliciting students' justifications can help with assessing students' working understanding of content at a given time (Staples et al., 2012). Eliciting students' ideas about fractions, scaffolding their understanding about fraction concepts, and supporting them in the communication of and reasonings behind their ideas involves the teacher decentering themself as the mathematical authority. Sharing in authority of knowledge with students opens up the space for students to have the agency to be able to pose ideas, generate their own claims, and justify the claims or ideas they come up with and decisions they make as they are exploring new content at the beginning of a unit. Bieda and Staples (2020) describe justification as an "equity practice" because it can support both students' access to content and their agency in the learning process. This article builds from Bieda and Staples' conceptualization to consider how the beginning of a unit can be particularly conducive for leveraging mathematical justification to facilitate students' agency and engagement with new content.

Classroom Discussion of Fractions

The dialogue below features snippets of the discussion facilitated by Ms. Josie (all names pseudonyms) in which she engaged students in discussion about their reasonings and justifications for the ways they cut their Play Doh pastries. Ms. Josie leveraged the justification process in ways that supported students' engagement with the content in the new unit and their agency during the learning process. This involved activating students' background knowledge, eliciting justifications of students' ideas, and connecting to the theme of "fairness" (i.e., equal shares in this case) to scaffold students' understanding of fractions as equal-sized pieces. Ms. Josie's students felt comfortable sharing their uncertain or in-progress ideas because of the

comfortable class environment Ms. Josie cultivated over time and the norms around math discussion participation established from the start of the school year.

Discussion of Cookie Cut into Three Equal-sized Pieces.

> ***Ms. Josie:*** *Why did your group choose to cut your cookie this way? (See Figure 14).*
>
> ***Malik:*** *The reason my group cut it that way is because it was the easiest way to cut it instead of finding the most complex way.*
>
> ***Ms. Josie:*** *Why is it the easiest?*
>
> ***Malik:*** *It's the easiest way, than trying to find a harder way. It was easier to make them equal.*
>
> ***Ms. Josie:*** *Why did you care about making it equal?*
>
> ***Malik:*** *We didn't want to find a more complex way.*

Figure 14. Play Doh Cookie Attempted to Be Cut into Three Equal-sized Pieces

In the dialogue above, Ms. Josie pressed for students' reasoning in their justification for cutting the cookie in the way that they did. She did this by asking Malik *why* his group chose to cut his cookie in that way, *why* it was the easiest way for them, and *why* they cared about making it equal. From this conversation, all we know is that the shape in which the group partitioned the cookie was the easiest way for them to cut it into thirds, but the concept of fractions having equal-sized parts not did not clearly surface.

Next, Ms. Josie moves on to elicit justifications from a group that cut their Play Doh pastry into pieces of unequal sizes. In the dialogue below, Ms. Josie decides to engage the class in discussion about the cut of one group's marshmallow (not pictured) into different sized pieces to see what discussion that would generate regarding the size of parts. She begins by asking the class what they notice about the cut of the marshmallow, then asks students how they would feel about receiving different pieces of a dessert, and finally transitions to asking the class about the naming of the pieces. Ms. Josie uses the justification process to tie together students' noticings of

and feelings about portion sizes of dessert shares, with naming conventions and meaning of fractions.

Discussion of Marshmallow Cut into Three Different-sized Pieces.

Ms. Josie: *What do you think about this cut?*

Shana: *It looks uneven.*

Ms. Josie: *How would you feel if you had this piece (points to the smallest piece) and the other two people had these pieces?*

Taylor: *It would be unfair because that size piece is way smaller.*

Ms. Josie: *But the other day, we said this is still one-third because it's 1 of 3 pieces. So if I were sharing this cookie with you and I said I'm getting one-third and you two are also getting one-third, even though these pieces are different sizes, can I say that?*

Multiple Students: *It's unfair.*

Jordan: *You can take a piece off the bigger piece to give to the person with the smaller piece.*

Ms. Josie: *Let's say I went to the coffee shop with Amir. And I said, let's split this cookie in half, (reusing the cookie from the previous group as a manipulative). What would you say if I said this piece (points to the bigger piece) is my half and the other is his half?*

Multiple Students: *Unfair!*

Ms. Josie: *Half is a fraction right? How would I write one-half? Write it with your finger in the air.*

Adriana: *There's a 1 on the top, then a line, then a 2 on the bottom.*

Ms. Josie: *You're saying these both are still written as ½?*

Jonah: *No! That's not a half!*

Destiny: *That's like one-quarter.*

Ms. Josie: *So if I really did want to cut this cookie in half, where would I cut it?*

Destiny: *Down the middle. (Comes to the projector and cuts the cookie down the middle).*

Destiny: *Now that's one-half.*

Ms. Josie: *Why is this one-half now?*

Destiny: *Because it went down the middle.*

Ms. Josie: *This piece and that piece are the same...*

Multiple Students: *Size!*

In the dialogue above, Ms. Josie appealed to the idea of fairness in equal shares of dessert portions and referenced back to a more familiar fraction (one-half) to activate students' background knowledge and scaffold their understanding towards a justification of fractions

having equal-sized pieces. After Shana described the three marshmallow pieces as "uneven," Ms. Josie asked the class how they would feel if they received the smaller piece and others received the larger pieces, all while referring to each piece as "one-third." Multiple students expressed it would be unfair, and Taylor's justification for the reason it was unfair (and why multiple other students likely felt it to be unfair) was because the piece was smaller. Ms. Josie reminded the class, though, that on the previous day they had said that one-third is one of three pieces. Next, Ms. Josie proceeded to cut the first group's cookie into two (distinctly different-sized) pieces (see Figure 15) and asked if that represented half, because it is two pieces. Students were familiar with the fraction ½ meaning two equal-sized pieces since the partitioning is "down the middle," which was a justification that seemed to help activate background knowledge and scaffold students' understanding that the fraction of thirds should also signify equal-sized pieces.

Figure 15. Play Doh Cookie Cut into Two Pieces

Summarizing the Discussion.

> ***Ms. Josie:*** *What's an important thing we need to know about fractions based on what we just did?*
>
> ***Aaron:*** *They have to be equal!*
>
> ***Ms. Josie:*** *What does?*
>
> ***Mya:*** *The pieces.*
>
> ***Ms. Josie:*** *The pieces that I cut have to be equal.*
>
> ***Mya:*** *That doesn't look equal.*
>
> ***Ms. Josie:*** *You're right. It's hard for me to cut this exactly equal.*
>
> ***Ms. Josie:*** *If I cut something into fourths, all of the pieces have to be?*
>
> ***Multiple Students:*** *Equal!*

After engaging in discussion about halves meaning two equal-sized pieces and thirds meaning three equal-sized pieces, Ms. Josie did a quick check with students to ask about fourths and what would be important to remember about the size of fourths. In third grade, the students likely learned that fractions signify equal-sized pieces of a whole, but the hands-on activity Ms. Josie facilitated that elicited justifications (e.g., for their decisions on the cuts they made and their ideas about fractions and sharing) helped her elicit what students remembered and activate background knowledge about fractions in ways that the students cared about and made sense to them (e.g., fairness of having equal-sized shares of desserts). These justification opportunities supported students' engagement with the content and their agency in the learning process.

Debriefing on Students' Agency and Engagement with New Content in the Lesson

When debriefing this lesson, I asked Ms. Josie, "How do you think students engaging in that process of justifying supported their mathematical identities and agency?" Ms. Josie explained,

> *I guess, maybe they just like, force them to really think about fractions, especially really common ones, like halves and thirds. And that those fractions have a real meaning, like not just 1 out of however many, 3 pieces, but that the pieces should be equal. Or, you know, you're going to get unfairness, or maybe a wrong recipe, or that the idea that the pieces of a fraction are equal was important 'cause they felt it. They were mad about it.*

Curious to understand more about her pedagogical approach, I inquired, "If you just told them that fractions need equal size pieces, and then just moved on without letting them explore, what effect do you think that would have had?" Ms. Josie reflected on having done that in the past and shared,

> *I've definitely done that. I think once we move on to equivalent fractions– because this has happened before– where, when the pieces aren't equal exactly. For example, if you have, you know, one-third and two-sixths or something. I think it's harder for kids to see that equivalence then if they don't really understand about, like, "Here's the whole, two of the same whole, and that here are the equal pieces that they are cut into." So that's happened before. I think, just, I don't know, maybe [students] not like [having] as deep of an understanding, and just like [thinking], "These are fractions and I know there's a top number and a bottom number," but never have to think about– Like, how in the green problem [students justifying], "Oh, it's 1 out of 3 pieces, so that must be one third." So having that still same misconception.*

From past experience, Ms. Josie learned that quickly reminding or telling students that fractions need to have equal-sized pieces does not provide a strong enough foundation for understanding the meaning of fractions needed for solidifying equivalent fraction concepts in ways that set them up for success later in the unit. Rather, through her years of teaching experience, Ms. Josie found that providing students justification opportunities actively engaged them in learning new content and supported their agency in the learning process. In the case of the beginning of this unit, justifications that connected to halves being cut down the middle and the notion of fairness in equal shares of dessert portions, supported students in understanding that thirds (and other unit fractions that share the same whole) have equal-sized pieces. Reflecting on how sharing authority of knowledge with students and providing students opportunities to explore and justify their ideas is important for both supporting students' agency and engagement with new content, Ms. Josie shared, "I cannot like give my knowledge to somebody. Like I can't. That's never– you know what I mean? If I could, my job would be so easy." Although it takes more time than direct instruction to provide students opportunities to explore, pose and justify ideas in ways that build from their background knowledge, students are developing their agency in the learning process and engaging more actively with the content.

Guidance for Your Classroom

This section elaborates what Ms. Josie did in her classroom to leverage justification opportunities for supporting students' agency and engagement with new content, and includes guidance for your classroom. Discussed are the affordances of the beginning of a unit, the role of sharing authority of knowledge with students, the design of exploratory tasks, and connections to formative assessment that supports a responsive approach to instruction.

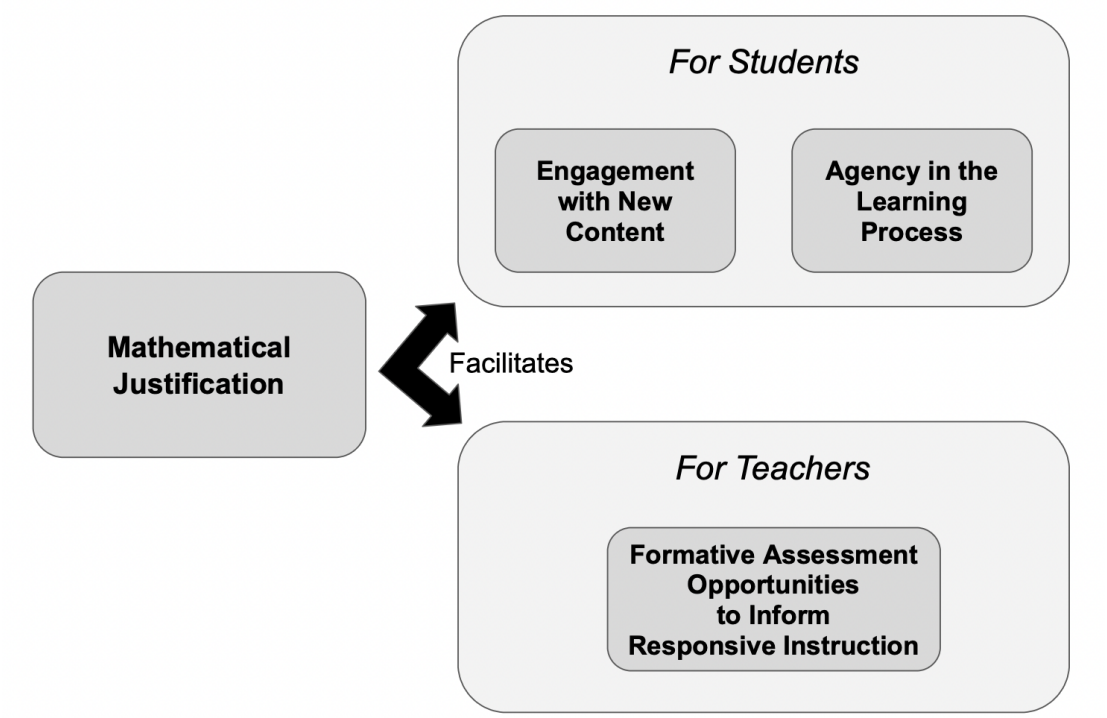

Figure 16. Leveraging Mathematical Justification to Facilitate Students' Agency and Engagement with New Content and to Facilitate Teachers' Formative Assessment Opportunities to Inform Responsive Instruction

Affordances of the Beginning of a New Unit. The beginning of a new unit is an especially conducive place in the curriculum for leveraging the mathematical practice of justification in ways that can support students' agency and engagement with new content. This is because justification can serve as a bridge that connects students' background knowledge to the new content to be learned.

Figure 17. Justification Process to Bridge Students' Background Knowledge and New Content

At the beginning of a new unit, a teacher has an important choice: tell students the new mathematics that they can memorize passively, or give students the agency to explore, discover, and justify concepts themselves as active doers of mathematics. The beginning of new units or

topics are particularly conducive places for supporting students' agency and engagement with the mathematics exactly because the content is so new to students. Leveraging the mathematical practice of justification (in ways that support students in building on background knowledge from previous units or experiences) can be a helpful tool in this process of discovery and exploration of new content. Given that teachers have a loaded curriculum with extensive content to cover, it can be tempting to rely on direct instruction instead, to perhaps quickly impart to students new mathematical procedures, relationships, or meanings. However, this is not as engaging or effective long-term, which could result in taking time to re-teach later. Ms. Josie explained,

> *You're still gonna have to go back and reteach the rest that didn't memorize it. 'Cause really, that's what that is. If there's not like– the most, majority of your class is not gonna be a bunch of memorizers. So you're gonna have to spend that time anyway.*

Spending extra time at the beginning of a new unit in intentional ways sets a strong foundation that can support the solidifying of key concepts for students early-on, ultimately setting up students for success through the progression of the unit. Providing students exploratory opportunities to engage in justification, with an eye to where in the unit you are leading them to, can help you make decisions about what to spend more/less time on throughout a unit. Ms. Josie affirmed,

> *But as much as you can give those opportunities, you know. Because, the reality of teaching is that there's not a lot of time and that curriculum, they jam pack it with all kinds of stuff. You feel like you can't get through all of it. But I think that's also why I say look at the big picture, look at your end goal for that unit, and pick the most relevant things that you want to cover. Sometimes it all also helps to know where you are leading kids to.*

Share Authority of Knowledge with Students. Sharing authority of knowledge with students is important for facilitating students' agency, supporting students in activating their background knowledge, and ultimately for setting the stage for an active (versus passive) learning process that engages students. Decentering oneself as the mathematical authority means not readily giving away reasonings, definitions, or answers and tasking students to develop their own justifications instead. For example, instead of telling her fourth-grade students that one-third (or any unit fraction) means equal-sized parts, Josie engaged the students in discussion about fairness in ways that activated their background knowledge about a familiar fraction, one-half. After talking through a real-world scenario with students where a cookie could be cut into two

pieces (of equal then unequal sizes), instead of telling students which represented one-half, Josie pressed for students to offer the justification. Destiny reminded the class that the cookie cut into two equal-sized pieces represents one half, with the justification, "because it went down the middle," (referencing the cut going down the middle). Speaking to her pedagogy of sharing authority of mathematical knowledge with students, Ms. Josie shared, "I don't want to be like the keeper of math. I want them to be the thinkers." In order for students to generate answers and procedures or to discover mathematical relationships and concepts themselves, they need time to explore and to be supported in drawing on their background knowledge to help them make connections to the new content, and to develop justifications of the claims they make through this discovery process. Because the teacher is not prescribing procedures or imparting definitions, answers, or reasonings, leveraging the mathematical practice of justification can support students in using mathematics to back up the claims and ideas they are exploring. In explaining her role as the teacher in this process, Ms. Josie explained, "I think it's my job to listen and understand students' thinking, and then like, decide what my next move will be to help them get where I need them to be." Ms. Josie finds that eliciting students' justifications supports her in assessing what students know at a given time, which can help her build from students' working understanding as a bridge to guide them toward the new content in ways that make sense to them.

Sharing authority of knowledge with students means intentionally providing students opportunities to justify their ideas. To support students in communicating their ideas and claims, Ms. Josie suggests providing sentence stems, particularly at the beginning of the year if students have minimal experience engaging in mathematical discussion in previous grade levels. Though Josie did not provide sentence stems in this lesson (which occurred in April), she found that overtime students did not need them anymore after enough practice. One sentence stem suggested by Ms. Josie is, "Here is a pattern I noticed and I think this pattern happens because…" Others might include:

- I think… because I remember (from last year, from earlier in the year)...
- My claim is… My warrant/reasoning is…
- I conjecture… because I know…
- I used this solution strategy because…
- I tried or tested… because…

Exploratory Task. Exploratory tasks, like the task Ms. Josie designed, can be conducive for supporting students' agency through justification opportunities. I define an exploratory task as one that is open-ended, accessible, and intriguing. *Open-ended* means there are multiple ways to approach the problem and students are not prescribed a method to use. For instance, Ms. Josie did not tell students how to cut their pastries. As students were forming their pastries and cuts, some asked her if the pieces needed to be the same size and all she told students was, "What do you think?" and "We will talk about it as a group." *Accessible* means all students have background knowledge or experiences that they are supported in drawing from to enter into the problem space. Ms. Josie's students knew that "thirds" relates to the number 3 and they were familiar with halves meaning two pieces that are equal-sized because the whole is cut down the middle. *Intriguing* means the task hooks students in, piques their curiosity, is relevant to students, students care about it, and/or students are interested or invested in finding the answer or solving the task at hand. In the case of this lesson, students cared a lot about fairness (i.e., sharing desserts where everyone gets the same size portion) which hooked them into the task, particularly when they were presented with a scenario that they perceived to be unjust or unfair. Ms. Josie mentioned, "The idea that the pieces of a fraction are equal was important 'cause they felt it. They were mad about it."

Formative Assessment. Eliciting students' mathematical justifications provides teachers a window into students' current thinking, which can support the responsive instruction required for facilitating students' agency in a lesson. The only way to elicit students' working understanding of any mathematical concept is to provide them opportunities to share their thoughts and reasonings about it. Asking students to explain and defend their claims, ideas, and decisions (whether written, pictorially, or verbally) helps teachers understand students' conceptions, and potentially, misconceptions. Ms. Josie picked one group that cut their pastry into three equal-sized pieces and one group that cut their pastry into three different-sized pieces to elicit their current understanding and thus inform her next instructional decision. After Malik stated his reasoning for cutting the cookie into equal-sized pieces, Ms. Josie pressed for justification by asking, "Why did you care about making it equal?" It could be easy to make the assumption that the reason Malik and his partner cut their cookie into three equal-sized pieces was because they understood the definition of "thirds" as three equal-sized pieces. However, eliciting Malik's justification showed that was not clear. A circle is not easy to cut into three pieces, which makes

sense that the reasoning for his type of cut could have had to do more with ease of cutting than equal shares. This prompted Ms. Josie to ask the class their thoughts on the cut of the marshmallow. After Shana said, "It looks uneven," Ms. Josie asked the class, "How would you feel…?" about receiving the smaller piece, alongside reminding them that the previous day they had said one-third is 1 of 3 pieces. In response to continued unclarity on the meaning of thirds (but also engagement around unfairness or injustice regarding unequal shares of desserts), Ms. Josie activated students' background knowledge on halves as a fraction which means two equal-sized parts, not two parts of any size. This supported scaffolding students' understanding of all unit fractions of the same whole having equal-sized pieces, as supported by engagement in a justification process about a scenario that interested students.

Conclusion

The beginning of a unit can be an especially conducive place to leverage the mathematical practice of justification in ways that help teachers elicit and build from students' background knowledge to support their engagement with new content and agency in the learning process. This involves the teacher sharing authority of knowledge with students, tasking students with generating and justifying claims they make through an exploratory and discussion-based learning process. While the beginnings of units are particularly conducive places for leveraging justification to support students' understanding of new content, justification is a mathematical practice that can be incorporated into any lesson at any stage of a unit to facilitate students' reasoning, agency, and engagement with content. According to Ms. Josie, "You can insert opportunities to justify anywhere… It could be the smallest moment. It could be, like, Why do you think that?…Do you agree? Why do you think that?" The mathematical practice of justification is more than just asking students *why*. Justification is a powerful practice aimed at facilitating students' agency, strengthening students' reasoning skills, centering their voices, and increasing their engagement with content.

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CHAPTER 5: CONCLUSION

This dissertation examined how one teacher, Josie, provided opportunities for her fourth-grade students to engage in the mathematical reasoning practice of justification and how these opportunities supported her goals for equity in a racially/ethnically diverse classroom of students. Because agency emerged as a prominent equity goal for Josie, all three articles analyzed the role that mathematical justification opportunities can play in facilitating students' agency in the learning process. In particular, justification opportunities allowed students to share authority of knowledge during lessons, opening spaces for them to discover mathematical relationships themselves, generate claims, and justify their ideas. All of this supported their engagement with content and development of positive mathematical identities. Sharing the findings of this dissertation study for three major audiences in the field of education (researchers, teacher educators, and practitioners) could be helpful in the ongoing challenge of bridging research and practice. Article One (Chapter 2) provided an account of the multifaceted ways in which Josie facilitated justification opportunities, to provide a more comprehensive examination of the various factors involved so that researchers could build from the framework in future studies, given the complexity of justification facilitation. The factors that this article discussed as influencing Josie's facilitation of justification opportunities were purposefully designed curricular tasks, justification-oriented task enactment strategies, a conducive classroom environment, and a disposition for critical dimensions of equity (with a focus on agency and related equity constructs such as identity). Article Two (Chapter 3) and Article Three (Chapter 4) focused in on selected examples from Josie's classroom in ways that could help support teacher educators and practitioners (respectively) in translating Josie's strategies for facilitating justification to be applied in and adapted to the contexts of their own classrooms and goals. Article Two described the affordances of relational problem-based learning in leveraging justification, and Article Three discussed how and why the beginning of new units could be conducive places in the curriculum for facilitating justification. Relational problem-based learning experiences and the beginning of units can both offer conducive opportunities for supporting students' agency via a justification process. This is because relational problem-based learning lessons, and particularly lessons at the beginning of units, are supportive for engaging students in activating background knowledge in ways that they can apply to their noticings of

new content; from there, students can develop and reason through claims and conjectures about the content through the justification process.

This study sought to address gaps that exist in the literature on both the justification process (Karunakaran & Levin, 2022), the lack of justification opportunities in classrooms (Melhuish et al., 2020), and on the role of justification in promoting a teacher's equity goals (Bieda et al., 2023). Given that this study only examined one teacher's practice and perspectives, more research is needed to understand how justification may be leveraged across a diverse set of classroom contexts and teachers' equity goals for their own students. Researching how a practice gets implemented across contexts, and how practice-based experiences might support teachers in learning new practices, can be helpful for addressing the concern about practices being implemented in uniform or prescriptive ways across contexts (Philip et al., 2018). Aligned with situative perspectives of learning (Peressini et al., 2004), I view practices to be taken up and implemented by teachers in relation to their contexts and goals. I also argue that practice-based methods can support teachers in learning new practices in ways that can be relevant and responsive to their classrooms and equity goals for their students, especially a practice as complex as justification. Below, I offer suggestions for future research to continue to examine how justification opportunities can be leveraged to meet teachers' equity goals, and how teachers may be supported in learning the practice of justification.

Implications

This study demonstrated a powerful connection between mathematical justification and equity. Specifically, it illuminated how the process of justification can be leveraged to provide opportunities to support students' agency in the learning process, which is a critical equity construct. Supporting students' agency in the learning process can increase students' engagement with content and help cultivate students' positive mathematical identities (which are additional, and intertwined, equity constructs associated with agency). Supporting equitable experiences and outcomes for all students involves attending to multiple dimensions of equity, because as Gutiérrez (2009) states, all dimensions are needed to realize equity. This may involve focused attention to the non-dominant dimensions (e.g., dimensions of identity and power) which have historically gotten less attention, such as considering how to build students' agency and identity development through mathematics learning in order to also support their access to mathematics and achievement in the subject.

This study has implications for more research to better understand how justification opportunities can be leveraged to advance teachers' equity goals across diverse contexts. This research might begin with additional case studies across contexts unique from one another geographically, demographically, economically, and in terms of curriculum used, teaching approach, and district leadership. Understanding nuances by context can illuminate similarities and differences in how and why justification opportunities are facilitated, as well as help explain challenges and successes that are experienced alongside teachers' respective equity goals. Because every classroom context is different, with varying student needs and teacher goals, understanding how justification opportunities can be adapted across contexts will be helpful in supporting teachers in learning to facilitate justification opportunities in ways they view relevant to their equity goals and responsive to their students. Findings from additional case studies could inform participatory action research (PAR) studies, where researchers apply what they learned from these case studies to work alongside teachers to build their capacity for facilitating justification opportunities. Findings from the case studies and PAR studies could inform the design and delivery of professional development geared towards (1) teachers to learn strategies for incorporating justification opportunities into their own classrooms, where facilitators could work with them to examine how to apply those strategies in ways responsive to their contexts and equity goals and (2) teacher educators looking to work alongside pre-service and in-service teachers as they learn to facilitate justification opportunities in culturally responsive ways.

Additional Case Studies. Additional case studies that seek to understand how more teachers across various contexts (e.g., diverse student cultures, varying geographic and economic regions, different grade levels, students with disabilities, teaching approach, curriculum used, etc.) incorporate regular justification opportunities can help the field better understand ways that teachers use this practice to be responsive to their students and advance their equity goals. This could involve examining the design and adaptation of their curriculum materials, their lesson planning processes, their enactment strategies, their classroom environment, and their personal equity goals for students, to look for similarities and nuances in the facilitation of justification opportunities across contexts and goals. Because justification offers opportunities to center students' perspectives and ideas (and for the teacher to be responsive to these), case studies that seek to understand what culturally responsive justification practices look like would be helpful

for better understanding the role of justification in building students' mathematical identities in ways where they can see themselves reflected in the mathematics.

Exploring the student learning perspective in mathematics classes with regular justification opportunities is also important for understanding the impact of these opportunities (e.g., the alignment between the teacher's intentions or goals and the student experience). For example, if a teacher incorporates justification opportunities with the equity goal of increasing students' agency in the learning process, it would be helpful to know whether students report experiencing agency in their mathematics class. Surveying and interviewing students throughout the school year about what their mathematics classes have taught them regarding what it means to do and be good at mathematics (and where they situate themselves along those views) could provide insight into their perception of their own agency as mathematics learners.

Participatory Action Research. As the field learns more about how teachers across contexts leverage justification to meet equity goals (e.g., through more case studies), participatory action research (PAR) studies could be beneficial for teachers who may not be incorporating justification regularly but want to learn more about how to implement these opportunities in ways that support their visions for equity and are culturally responsive to their contexts. PAR studies could allow teachers and researchers to learn from/with one another over the course of a school year. Teachers and researchers could partner together to analyze, create, and adapt the design of curricular tasks to be more conducive to justification opportunities in ways that are intended to meet their equity goals and in ways that are culturally responsive. The teachers and researchers might debrief on the successes and challenges the teachers experience with their curricular enactment strategies for facilitating justification opportunities. For example, if facilitating student agency is a goal, this could involve examining questioning strategies or discourse moves that may have advanced or hindered students' agency during the justification process. Analyzing power dynamics among students, and between the teacher and students, could also provide insight into who is considered a mathematical authority and how that influences students' individual and collective contributions during the justification process. As discussed above, studying students' experiences alongside the teacher's would also be conducive for understanding the alignment between the teacher's goals and impact on students. This could involve conducting classroom observations, collecting student work, administering surveys to students, and facilitating interviews and focus groups with students.

Professional Development. Findings from these case studies and PAR studies could inform professional development for teachers and teacher educators. For teachers, professional development could be designed around supporting teachers in learning strategies for incorporating justification opportunities into their own classroom. These strategies would be informed by the successes and challenges experienced by teachers as they facilitated justification opportunities during the case studies and PAR studies. Distinguishing between macro level strategies and micro level strategies for facilitating justification could also be helpful. Macro level strategies might include those for establishing a classroom environment conducive to justification, and micro level strategies could include particular discourse moves for eliciting students' reasoning and orienting their ideas to one another. Additionally, professional development facilitators could work with teachers to examine together how to apply the macro and micro level strategies other teachers have found success with, in ways that are responsive to the classroom culture and equity goals of the teachers in the professional development. Findings from the case studies and PAR studies could also inform professional development for teacher educators, including mathematics coaches and teacher preparation methods course instructors. This professional development could include a similar design, sharing strategies for facilitating justification opportunities and thinking alongside the teacher educators about how to make sense of those strategies within their institutional contexts and goals.

Professional development geared toward both audiences (teachers and teacher educators) could include *practice-based* methods for learning about justification opportunities and to practice facilitating them (Grossman et al., 2009). Ms. Josie found that watching examples of justification opportunities playing out in practice in other teachers' classrooms, as well as practicing them herself (in a professional development and overtime in the classroom), was supportive in her own learning of facilitating justification opportunities. As referenced in Chapter 2, Josie mentioned,

> *I think it's like anything, it's like part of it is seeing it in practice, and part of it is practicing. And the more you do it, the more of those small moments you'll recognize and know how to inject a justification opportunity into. So you have to do it a lot and you have to see somebody effectively do it to recognize it. You know, it's exposure, I guess.*

Other research has also found practice-based methods to be supportive for teachers in translating new or complex teaching practices that they hear about in theory (e.g., in courses or

professional development) in a way that they can find both learnable to do themselves later and applicable in their own classroom contexts (Ball & Forzani, 2009; Kavanagh et al., 2020; Schiera, 2019 & 2021). Types of practice-based methods include *representations*, *decompositions*, and *approximations of practice*. *Representations* of the practice of justification could include video clips of teachers (e.g., from the case studies and PAR studies) facilitating justification opportunities. An important part of learning a new practice is seeing it play out in classrooms, particularly in similar and different ways by context, so that teachers can see what the practice looks like then debrief about how to make sense of the practice within their own classrooms. *Decomposing* justification opportunities could look like breaking down macro and micro level strategies for facilitating justification into more learnable parts, and discussing how the distinct parts make sense together as a whole. For example, the professional development could work with teachers to think of norms that could contribute to a classroom environment conducive to justification in their classrooms (a macro level strategy). Additionally, the professional development could decompose discussion facilitation strategies, such as pedagogy and discourse moves that decenters the teacher and positions students with authority of knowledge in ways that elicits and centers their ideas, discoveries, and reasonings. After seeing justification opportunities represented in practice and decomposing parts or types of justification opportunities, *approximating* the practice of justification can be supportive in helping teachers apply what they learned to practice facilitating justification opportunities themselves. Practicing in a mediated setting with the professional development facilitator could provide opportunities for teachers to try out strategies and get feedback, engage in do-overs, and debrief. Because justification opportunities are not prescriptive but look different across contexts and is a practice that is inherently responsive to student ideas, approximations of practice can support teachers in being more responsive later when trying out the practice with their actual students. Kavanagh and colleagues (2020) found that engaging in approximations of practice in mediated settings can support teachers in implementing the practice in ways that are more responsive to their students because of the time they had to practice beforehand.

Closing

In conclusion, this dissertation sought to understand how Josie provided opportunities for mathematical justification in a diverse, fourth-grade classroom to meet her equity goals for her students. Given that agency emerged as a prominent equity goal for Josie, analysis focused on

the relationship between justification opportunities and her perceptions of the impact these opportunities had on students' agency. To conclude my exit interview with Josie, I asked if there was anything else she would like the field to know. She enthusiastically offered her encouragement for more teachers to incorporate justification opportunities into their mathematics instruction, too, because of how engaged students become in the process. She shared, "Everyone should do it! Kids really like it! They're really good at explaining themselves." Josie considers justification a "human skill," as it can empower students' agency within and beyond the mathematics classroom. She affirmed, "Because really, that's where we're leading them to is to be outside of our walls, and be free thinking humans out in the world."

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