THE USE OF LIGHT REFRACTION FOR THE INVESTIGATION OF STATIONARY ULTRASONIC WAVES

by

Adolph Paul Loeber

AN ABSTRACT

Submitted to the School of Graduate Studies of Michigan State College of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Department of Physics and Astronomy

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Approved E. A. Hiedemann

Adolph P. Loeber

ABSTRACT

The refraction of light by an ultrasonic wave can be used as a tool for investigating the sound field, particularly in the region below one megacycle/second. An effect discovered by Lucas and Biquard (1), namely, the broadening of a narrow light beam as it passes through a sound field was explained by them as being due to refraction. Heuter and Pohlman (2) used a modified experimental arrangement to measure sound absorption.

A simple explanation has been made for the above phenomenon based on an experimental method due to Wiener (3). A simple mathematical theory is presented which describes the variation in intensity of the undeflected beam as the light beam is passed through various positions of a stationary sinusoidal sound wave. A similar but qualitative approach has been made to the case of a sinusoidal wave interfering with an oppositely directed sawtooth wave.

Experimentally, use has been made of variations of both the method of Lucas and Biquard as well as that of Hueter and Pohlman. These variations were used to obtain photographic records of the spatial pressure distribution in a stationary ultrasonic wave. A method is described for making rapid measurements of sound velocity in liquids which yields results accurate to within 1.5 percent. An approach to the problem of wave form determination in liquids has been made, and a method for measuring the pressure amplitude in a stationary ultrasonic wave is described.

Adolph P. Loeber

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Adolph Paul Loeber

candidate for the degree of

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Dissertation: The Use of Light Refraction for the Investigation of Stationary Ultrasonic Waves.

Outline of Studies

Major subject: Physics
Minor subject: Mathematics

Biographical Items

Born, February 1, 1920, Detroit, Michigan.

Undergraduate Studies, Wayne University, 1937-1941.

Graduate Studies, University of Chicago, Institute of Meteorology, 1942-1943; Wayne University, 1947-1948; Michigan State College, 1950-1954.

Experience: Technician, Chrysler Corporation Metallurgical Laboratory, 1941-1942; Meteorologist, Second Air Division, Eighth Air Force, 1943-1945; Engineer, Chrysler Truck Engineering, 1945-1947; Instructor in Physics, Wayne University, 1947-1950; Instructor in Physics, (part time) Marygrove College, 1948-1950; Graduate Assistant and Special Graduate Research Assistant, Michigan State College, 1951-1954.

Member of American Meteorological Society, The Acoustical Society of America, Society of the Sigma Xi, Sigma Pi Sigma, Pi Mu Epsilon.

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INTRODUCTION

Optical Methods in Ultrasonics

Early optical methods. Among the methods used for the qualitative and quantitative investigation of sonic and ultrasonic fields, the various optical methods are of considerable importance, especially at ultrasonic frequencies. Perhaps the earliest optical investigation of sound fields was made by Topler who in 1867 used a schlieren method to render sound waves from a spark source visible (1). Later, in 1899, Wood improved on this method (2), while still later Foley and Souder published their method of photographing shadowgraphs of waves from a spark source by using another spark as the source of light for casting the shadow and producing the picture (3).

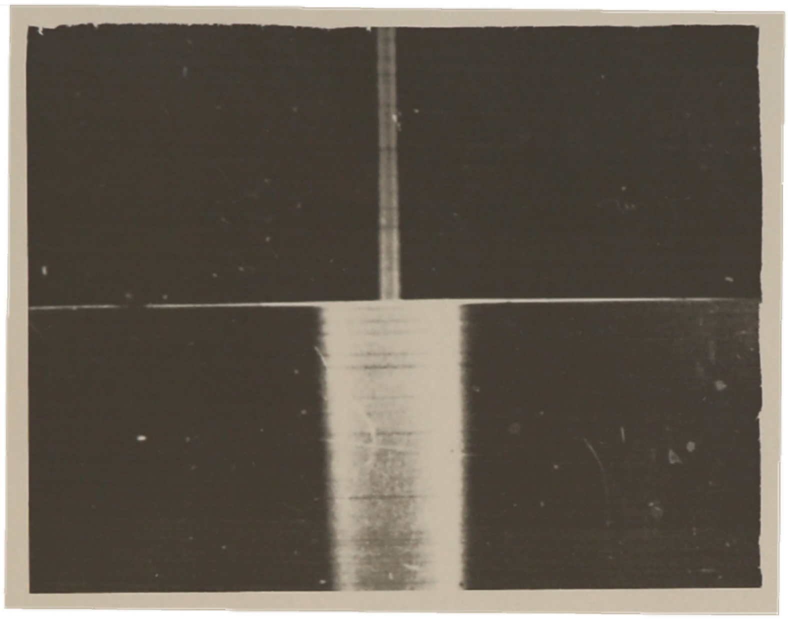
Later optical methods. More recently, since the discovery by Debye and Sears (4) and Lucas and Biquard (5) in 1932 that sound waves of short enough wave length can act as an optical diffraction grating for light which passes through the sound field, most of the optical methods which have been developed make use of this diffraction grating effect. For example, the adaptations of Bar (6), as well as those of Hiedemann and Hoesh (7) of Topler's method make use of this grating effect. As a result of the dependence of these newer methods on the existance of an acoustic grating, these methods generally find their most useful application in the ultrasonic frequency range above one megacycle/second. While it is true that the development of circuits for producing short duration spark light flashes (8) have permitted adaptations of the method of Foley and Souder (3) to higher frequencies, the applications in this case also have been made in the ultrasonic frequency range above one megacycle/second (9).

Light Refraction Method (Method I)

Besides the diffraction effect which ultrasonic waves can have on light traversing the sound field, there exists another effect, first discovered by Lucas and Biquard, and mentioned by them in their fundamental paper (5) which can be used as a tool for the investigation of sound fields when the wave length of the sound is greater than the width of a light beam which passes through the sound field. The effect referred to is the broadening of the image of a slit on a screen if the light which forms it is confined to a narrow beam which passes through the sound field. (See Figure 1). Lucas and Biquard explained this effect as due to the refraction of the light by the sound waves. In a later theoretical paper, Lucas pointed out that the refraction of light might be used for measuring sound absorption and reflection coefficients (10). The first use of the possibilities predicted by Lucas already in 1934 appears to have been made in much later work by Hueter and Pohlman who in 1949 used this effect to measure absorption of ultrasonic waves (11) and by Porreca, who in 1952 measured the distribution of light intensity across the slit image under various conditions (12).

Purpose and Scope of this Investigation

The purpose of this investigation is to examine the light refraction method with the purpose of developing its usefulness as a tool for



a)

b)

Figure 1. Broadening of a slit image due to light refraction by an ultrasonic wave. (Lucas-Biquard effect) a) sound off, b) sound on.

both the qualitative and quantitative investigation of ultrasonic waves, primarily stationary ultrasonic waves. Since the method is particularly adaptable to the longer ultrasonic wave lengths, it is hoped that this investigation might help in opening the door to optical methods in the frequency range below one megacycle/second.

THEORY

Theory of the Ray Deflection by Medium Having Refractive Index Gradient

<u>The method of Wiener</u>. The original explanation of the broadening of the slit image as given by Lucas and Biquard (5), is rather involved. However, this effect can be explained in a simple manner, based upon an experimental method due to Wiener (13). Wiener has shown that if a horizontally directed light beam impinges normally upon a transparent substance, in our case a liquid, having a continuously varying index of refraction whose gradient dn/dx is directed vertically, the beam will acquire a curvature in passing through the substance. The radius of curvature R of this beam will be given by

$$R = \frac{n}{dn/dx}, \tag{1}$$

where x is measured along the direction of the gradient, and n is the index of refraction at the point of incidence.

In passing through a layer of liquid of length δ, the beam of light will be deflected from its original direction so that the point where it impinges on a screen will be displaced from a position S to a position S_1. (See Figure 2). Let the distance (SS_1) be called d. Since, in practical cases, the value of dn/dx is very small, it is permissible to assume the same values of n at points A and E and to use the values of the incident and refraction angles at A in place of their sine and

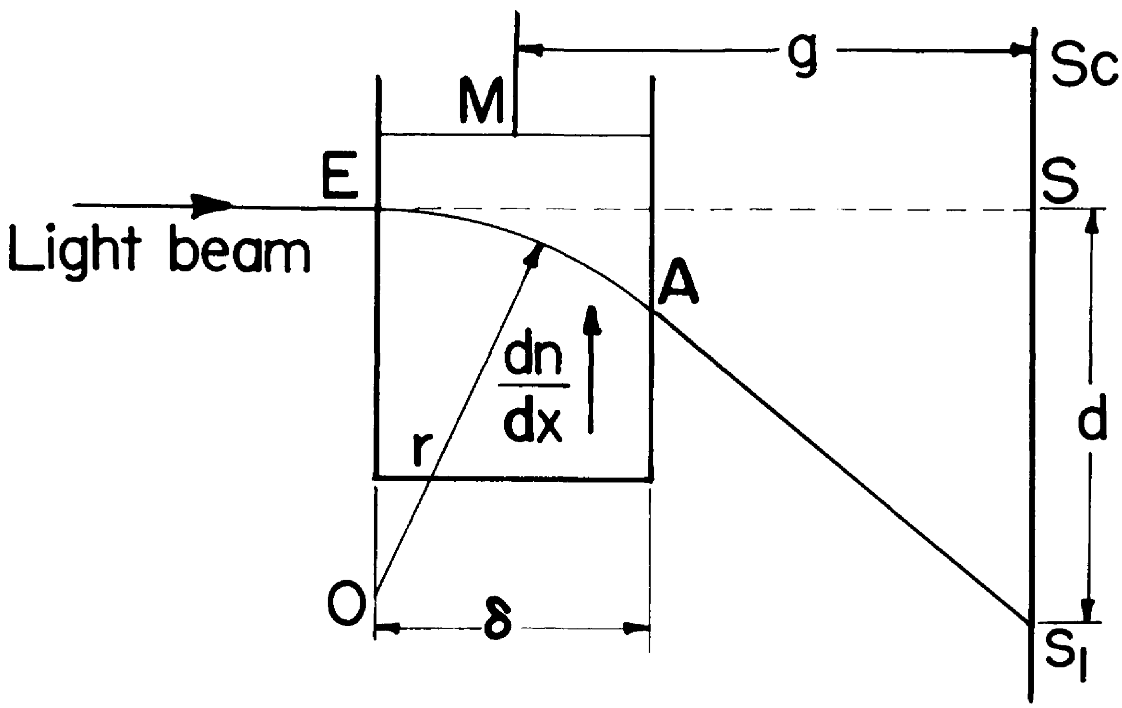


Figure 2. Refraction of a light beam in a liquid with continuously varying refractive index.

tangent functions. Under this assumption, it is possible to calculate the displacement d, with the aid of equation 2. The value of this displacement is given by

$$d = g \delta \frac{dn}{dx} \qquad (2)$$

where g is in reality the optical path length from the center M of the tank to the point S_1 on the screen. However, for practical purposes, the geometrical distance MS_1, or even the distance MS is a sufficiently accurate approximation.

The relation of equation 2 is also due to Wiener who used the above described effect for the measurement of diffusion rates of one liquid into another. More recently Wolin has attacked this problem from a somewhat different approach (14). The equation which he obtains however, reduces to equation 2 for the particular case considered here.

Application of Theory to Ultrasonic Waves

General. Although Wiener, as mentioned above, was primarily interested in the measurement of diffusion rates in liquids, and Wolin's interest in the problem was brought about by the necessity of knowing the degree of antenna pattern distortion caused by variation in the density in skin and core materials of radome sandwiches used in high precision tracking radars, the analysis is a general one which can be applied also to any material having a gradient of refractive index, whatever the mechanism which produces this gradient. This of course makes it immediately applicable to an ultrasonic wave, where variations in the pressure in various portions of the wave give rise to variations in the refractive index.

Application to progressive waves. In a plane progressive sinusoidal sound wave the sound pressure at a particular position x in space varies according to the relation

$$p = P\cos\left(\omega t - \frac{\omega x}{c}\right) \tag{3}$$

The derivative dp/dx then varies according to the relation

$$\frac{dp}{dx} = \frac{\omega}{c} P \sin\left(\omega t - \frac{\omega x}{c}\right) \tag{4}$$

so that at any particular position x in space as the sound wave passes the gradient of pressure will vary in a sinusoidal manner, returning to its original value after a time interval equal to one period. The variation in the pressure will produce a corresponding variation in the refractive index, so that along the wave there will exist a gradient of refractive index, which itself will vary sinusoidally.

If a beam of light which is small compared to the wave length of the sound is allowed to traverse the sound field in a direction normal to that of the sound wave propagation direction, it will be deflected, first to one side and then to the opposite side of its undeflected position as the sound wave passes. Since this to-and-fro motion of the light beam takes place at the frequency of the ultrasonic waves, the eye will not be able to follow the individual deflections, and the image produced by the light beam upon the screen will appear to be broadened. It should be noted that this broadening will be independent of the position x in the wave field provided we assume a plane wave, and no absorption. If absorption is considered, however, the broadening will decrease as the light beam is moved farther away from the sound source. It was essentially a modification of this method which was used by Huter and Pohlman to measure sound absorption in animal tissues (11).

Application to stationary waves. For a plane sinusoidal stationary sound wave, the sound pressure may be written in the following form:

$$p = P\cos\left(\omega t - \frac{\omega x}{c}\right) + P\cos\left(\omega t + \frac{\omega x}{c}\right) \tag{5}$$

or the equivalent form:

$$p = 2P\cos(\omega t)\cos\left(\frac{\omega x}{c}\right) \tag{6}$$

So the sound pressure at a pressure loop where $\cos(\omega x/c) = 1$ varies during one period between $+2P$ and $-2P$. At the pressure node the sound pressure is always zero. The gradient dp/dx, however, varies between it positive and negative maxima at the pressure nodes, and remains zero at the pressure loops. This is indicated in the expression

$$\frac{dp}{dx} = -\frac{2P\omega}{c}\cos(\omega t)\sin\left(\frac{\omega x}{c}\right) \tag{7}$$

At the pressure loops, $\sin(\omega x/c)$ is always zero while at the pressure nodes, where $\sin(\omega x/c) = 1$ the pressure gradient has a maximum in space. The value of this space maximum varies during one period between the values $-2P\omega/c$ to $+2P\omega/c$.

Thus in a stationary sound wave there will exist, as a result of the pressure gradient, a gradient of refractive index, the magnitude of which will vary between zero and a maximum within a quarter-wave length.

As indicated in equation 7, the periodicity in time in the stationary wave will produce a periodicity in the magnitude and direction of the pressure gradient, and therefore also in the refractive index gradient. If a beam of light of a width which is small compared to the wave length of the sound passes the sound field in a direction normal to that of the stationary wave, for example, at a node, then it will be deflected due to the periodicity in time of the refractive index gradient symmetrically to either side of the original path. This will once again produce a broadening of the image on the screen.

In the case of a progressive wave, it was pointed out that the magnitude of this broadening should be independent of the position in the wave, assuming a plane wave and no absorption. In the case of a standing wave however, the degree of broadening will depend upon the position in the wave at which the light beam passes, being greatest at the pressure nodes and least (theoretically zero) at the pressure loops.

If the container of liquid is moved parallel to the sound propagation direction, while the position of the light beam remains fixed, a periodic broadening and narrowing of the image on the screen can be observed. By measuring the displacement of the container and the number of corresponding maxima (or minima) of the image width on the screen, one can determine the ultrasonic wave length in the liquid. If the frequency of the source is also measured, one can easily obtain the velocity.

Modification of the Method Using Decrease in Intensity of Undeflected Beam (Method II)

The light refraction method which makes use of the broadening of the slit image may be modified somewhat to yield a method which seems to be more useful for certain purposes. In place of merely observing the slit image broadening on a screen, the screen is removed and a narrow slit is substituted for it. The slit is placed so that the center of the undeviated beam is allowed to pass through. When the sound is turned on, the beam will become broadened, and therefore less light will pass through the slit. In general the broader the beam, the smaller the intensity of the light which passes through. In a progressive wave, for example, the more intense the sound, the broader would be the beam, and the less the intensity of the light passed by the slit (11). In a stationary sound wave, on the other hand, the slit would transmit the greatest amount of light when the beam passes through a pressure antinode (where the broadening is least) and would transmit the least at the pressure node (where broadening is greatest). A measure of the relative intensity of the light transmitted by the slit as the beam is allowed to traverse various portions of a standing wave should yield information about the wave form of the standing wave. It may therefore be well to investigate analytically the manner in which the intensity of light transmitted by the slit can be expected to vary as the light beam is allowed to pass through various portions of the standing wave.

Theory of Intensity Decrease of Undeflected Beam Applied to a Sinusoidal Stationary Ultrasonic Wave

Consider an arrangement as shown in Figure 3. Parallel light is

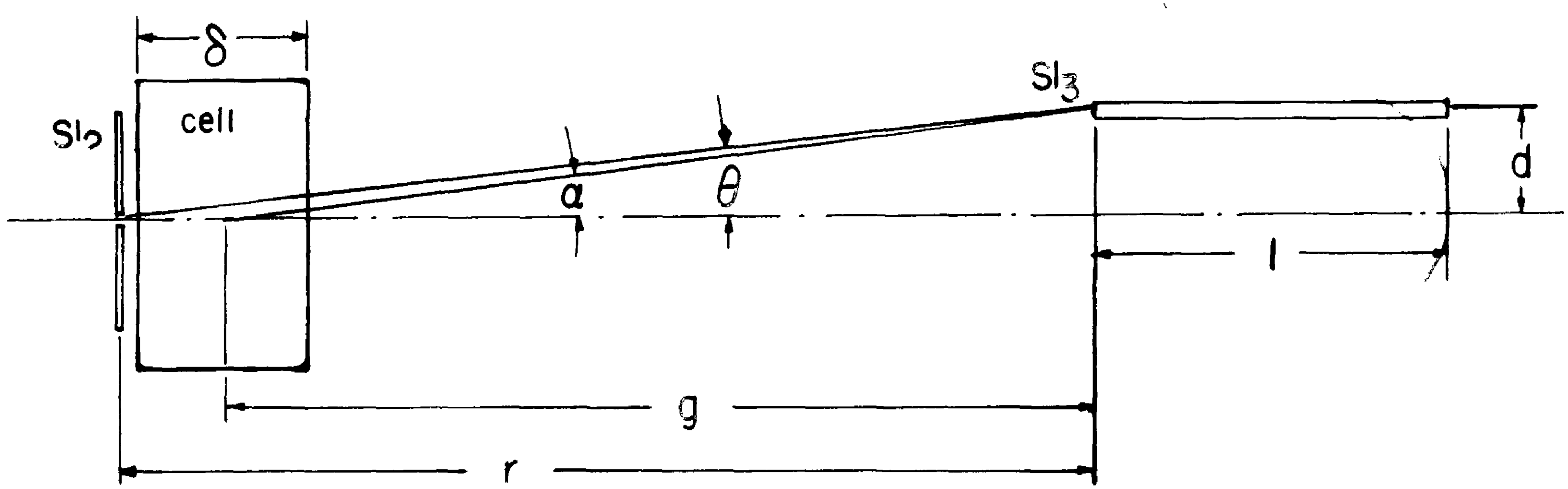


Figure 3. Schematic arrangement used in calculating intensity decrease of undeflected beam passing through sinusoidal stationary sound wave.

incident upon the slit Sl_2, which limits the width of the light beam passing through the cell containing the medium in which the stationary ultrasonic wave is to be set up. The light beam passing through the cell continues on to the plane of slit Sl_3. Here a slit diffraction pattern will be observed. That portion of the light in the diffraction pattern which is incident directly on the slit Sl_3 will pass through, while the remainder will be interrupted. The purpose of this analysis is to determine the ratio of light passed by slit Sl_3 when there is no sound in the cell, to that passed when the sound is turned on. Consider a point on the diffraction pattern the distance of which from the optic axis is designated by d. Let the angle subtended by this point and the optic axis at the slit Sl_2 be called θ, while the corresponding angle subtended at the center of the sound cell be designated α. Let r be the distance along the optic axis between the two slits, and g the distance from the center of the cell to slit Sl_3. Let the length of the final slit Sl_3 be ℓ, its width be s and the width of slit Sl_2 be a.

Light intensity distribution in the slit diffraction pattern. The intensity of light I at any given position in the diffraction pattern can be written as follows:

$$I = I_o \left[\frac{\sin \pi a \phi}{\pi a \phi} \right]^2 , \qquad (8)$$

where I_o is the maximum intensity (at the center of the diffraction pattern, and ϕ is given by the expression

$$\phi = \frac{\sin \theta_1 + \sin \theta}{\lambda_\ell} , \qquad (9)$$

where θ_1 is the incidence angle of the light at slit Sl_2, and λ_ℓ is the wave length of the light.

For the particular case under consideration the angle of incidence θ_1 is zero. Therefore equation 9 may be written:

$$\phi = \frac{\sin\theta}{\lambda_\ell}, \tag{10}$$

which substituted into equation 8 yields for the intensity I in the diffraction pattern yields

$$I = I_0 \left[\frac{\sin \pi a \frac{\sin\theta}{\lambda_\ell}}{\pi a \frac{\sin\theta}{\lambda_\ell}} \right]^2. \tag{11}$$

Since θ is a very small angle, the approximation

$$\sin\theta \backsimeq \theta \backsimeq \tan\theta \tag{12}$$

may be used, and since

$$\tan\theta = \frac{d}{r}, \tag{13}$$

equation (11) finally may be written:

$$I = I_0 \left[\frac{\sin \frac{\pi a d}{\lambda_\ell r}}{\frac{\pi a d}{\lambda_\ell r}} \right]^2 \tag{14}$$

which yields the intensity I in the diffraction pattern at any point d in terms of the maximum intensity and constants of the system.

<u>Displacement of the diffraction pattern due to sound wave</u>. When the sound in the cell is turned on, the beam of light passing through

the cell will be refracted to and fro at the frequency of the sound. Let the assumption be made that the entire diffraction pattern moves across the plane of the slit as a unit.

Although in reality it is the diffraction pattern which moves to and fro, for simplicity, consider that the slit Sl_3 is moving to and fro across the diffraction pattern. The relative motion between slit and pattern, which is the important consideration, will be unchanged by this assumption.

According to the equation of Wiener (equation 2), the displacement d of the diffraction pattern in the plane of Sl_3 due to the refractive index gradient dn/dx in the sound wave is given by

$$d = \gamma \delta \frac{dn}{dx}, \tag{15}$$

where δ in this case is the width of the sound beam through which the light passes. For the gradient of pressure in the standing wave, we have the relation of equation 7

$$\frac{dp}{dx} = -\frac{2P\omega}{c} \cos(\omega t) \sin\left(\frac{\omega x}{c}\right) \tag{16}$$

where 2P is the pressure amplitude in the standing wave.

It is possible to obtain a relation between the pressure gradient dp/dx and the refractive index gradient dn/dx by use of the Lorentz-Lorenz relation

$$\left(\frac{n^2-1}{n^2+2}\right)\frac{1}{\rho} = K \tag{17}$$

in which n is the refractive index of the medium, p its density, and K is a constant. Equation 17 may be re-written in the form

$$\left(\frac{n^2-1}{n^2+2}\right)V = K \, , \tag{18}$$

where V is the specific volume of the liquid being considered. By differentiating equation 18 we obtain

$$V\left[\frac{(n^2+2)(2n\,dn) - (n^2-1)(2n\,dn)}{(n^2+2)^2}\right] + \left[\frac{n^2-1}{n^2+2}\right]dV = 0 \tag{19}$$

or, rearranging

$$V\left[(n^2+2)(2n\,dn) - (n^2-1)(2n\,dn)\right] + \left[(n^2-1)(n^2+2)\right]dV = 0 \tag{20}$$

which yields

$$6n\,dn = -(n^2-1)(n^2+2)\frac{dV}{V} \, . \tag{21}$$

Now, from the definition of compressibility, the expression

$$-\frac{dV}{V} = \mathcal{K}\,dp \tag{22}$$

may be obtained, where K, the compressibility, will be considered to be a constant in this analysis. So, combining equations 21 and 22 yields

$$dn = \frac{\mathcal{K}}{6n}(n^2-1)(n^2+2)\,dp \, , \tag{23}$$

or we may write

$$\frac{dn}{dx} = \frac{\mathcal{K}}{6n}(n^2-1)(n^2+2)\frac{dp}{dx} \, . \tag{24}$$

Since dn will be very small with respect to n, the n in the right side of equation 24 may be considered constant and equation 24 may be written

$$\frac{dn}{dx} = k\,\frac{dp}{dx}\,, \tag{25}$$

where k is a constant given by

$$k = \frac{K}{6n}\,(n^2-1)(n^2+2)\,. \tag{26}$$

Equation 25 shows a direct proportionality between the pressure gradient and the refractive index gradient. Combining the equation of Wiener (equation 15), the equation for the intensity in a slit diffraction pattern (equation 14), the equation for the pressure gradient in a stationary wave (equation 16), and the equation relating the pressure gradient with the refractive index gradient (equation 24) will yield

$$I = I_0\left[\frac{\sin \dfrac{-\pi a g \delta K(n^2-1)(n^2+2)\,2P\omega}{\lambda \varrho r\, 6n\, c}\cos(\omega t)\sin\left(\frac{\omega x}{c}\right)}{\dfrac{-\pi a g \delta K(n^2-1)(n^2+2)\,2P\omega}{\lambda \varrho r\, 6n\, c}\cos(\omega t)\sin\left(\frac{\omega x}{c}\right)}\right]^2 \tag{27}$$

For simplicity in notation let

$$A = \frac{-\pi a g \delta K(n^2-1)(n^2+2)\,2P\omega}{\lambda \varrho r\, 6n\, C}\,, \tag{28}$$

and let

$$B = A\,\sin\left(\frac{\omega x}{c}\right). \tag{29}$$

then finally equation 27 becomes

$$I = I_0 \left\{ \frac{\sin[B\cos(\omega t)]}{B\cos(\omega t)} \right\}^2 \tag{30}$$

Equation 30 is essentially an expression for the intensity of light on the optic axis at any time t for a fixed value of B. Fixing the value of B fixes the position in the standing wave. Moreover, B contains A as a factor. A is made up entirely of constants of the system except for the factor P, which is the half amplitude of the pressure fluctuations in the stationary wave.

Light energy received through final slit Sl_3. Let the assumption be made that the intensity of light at the center of the final slit is equal to the average intensity over the width of the slit. This assumption would make for the greatest error where the diffraction pattern has either a maximum or minimum, but should not be excessive if the width s, of the final slit is small compared to the size of the diffraction pattern. Now consider the light energy which is transmitted through slit Sl_3. In a small increment of time, dt, the increment of energy ds passed by Sl_3 is given by

$$dS = I \ell s \, dt \tag{31}$$

I being the light intensity at the slit, and ℓ and s the dimensions of the slit. The total light energy S, passed by the slit during one-half period will then be given by:

$$S = \ell s \int_0^{T/2} I \, dt, \tag{32}$$

T being the period of oscillation of the sound wave. Using the value of I from equation 30, equation 32 may be written:

$$S = \ell s I_o \int_0^{\frac{T}{2}} \left\{ \frac{\sin\left[B\cos(\omega t)\right]}{B\cos(\omega t)} \right\}^2 dt. \tag{33}$$

Now, for convenience, the following change in variable is introduced. Let

$$z = \cos(\omega t). \tag{34}$$

Then

$$dz = -\omega \sin(\omega t)\, dt, \tag{35}$$

and

$$dt = -\frac{dz}{\omega \sin(\omega t)} = \frac{dz}{\omega\sqrt{1-\cos^2(\omega t)}} = \frac{-dz}{\omega\sqrt{1-z^2}}. \tag{36}$$

Making use of these relations (equations 34, 35, 36) equation 33 becomes

$$S = \ell s I_o \int_{+1}^{-1} -\left[\frac{\sin Bz}{Bz}\right]^2 \frac{dz}{\omega\sqrt{1-z^2}}. \tag{37}$$

Using the trigonometric identity

$$\sin^2 Bz = \frac{1}{2} - \frac{1}{2}\cos 2Bz \tag{38}$$

equation 37 may be written

$$S = \frac{-\ell s I_o}{\omega} \int_{+1}^{-1} \left\{ \frac{\frac{1}{2} - \frac{1}{2}\cos 2Bz}{B^2 z^2} \right\} \frac{dz}{\sqrt{1-z^2}}, \tag{39}$$

which, making use of the cosine series may be written

$$S = \frac{-\ell s I_0}{\omega B^2}\left[\int_{+1}^{-1}\frac{-dz}{2z^2\sqrt{1-z^2}} + \int_{+1}^{-1}\frac{1-\frac{2^2B^2z^2}{2!}+\frac{2^4B^4z^4}{4!}-\frac{2^6B^6z^6}{6!}+\cdots}{2z^2\sqrt{1-z^2}}\,dz\right] \tag{40}$$

or

$$S = \frac{\ell s I_0}{\omega B^2}\left[\cancel{\int_{+1}^{-1}\frac{-dz}{2z^2\sqrt{1-z^2}}} + \cancel{\int_{+1}^{-1}\frac{dz}{2z^2\sqrt{1-z^2}}} + \int_{+1}^{-1}\frac{-\frac{2^2B^2z^2}{2!}+\frac{2^4B^4z^4}{4!}-\frac{2^6B^6z^6}{6!}+\cdots}{2z^2\sqrt{1-z^2}}\,dz\right] \tag{41}$$

which simplifies to

$$S = \frac{2\ell s I_0}{\omega}\int_{+1}^{-1}\left(-\frac{1}{2!\sqrt{1-z^2}} + \frac{2^2B^2z^2}{4!\sqrt{1-z^2}} - \frac{2^4B^4z^4}{6!\sqrt{1-z^2}}+\cdots\right)dz. \tag{42}$$

Substituting now for z and dz their equivalents as given in equations 34 and 35, equation 42 becomes

$$S = \frac{2\ell s I_0}{\omega}\left[\int_0^{\frac{T}{2}}\frac{\omega}{2!}\,dt - \int_0^{\frac{T}{2}}\frac{2^2B^2\cos^2(\omega t)\,\omega}{4!}\,dt + \int_0^{\frac{T}{2}}\frac{2^4B^4\cos^4(\omega t)\,\omega}{6!}\,dt - \cdots\right], \tag{43}$$

which can be written in the following form:

$$S = \frac{2\ell s I_0}{\omega}\left[\int_0^{\frac{T}{2}}\frac{\omega}{2!}\,dt + \sum_{n=1}^{\infty}\int_0^{\frac{T}{2}}(-1)^n\frac{2^{2n}B^{2n}\cos^{2n}(\omega t)\,\omega\,dt}{(2n+2)!}\right]. \tag{44}$$

Letting

$$\left[(-1)^n \frac{2^{2n} B^{2n} \cos^{2n}(\omega t)\,\omega\, dt}{(2n+2)!}\right] = F_n \,, \tag{45}$$

and integrating the first term we have for equation 44

$$S = \frac{2 l_s I_o}{\omega}\left[\frac{\pi}{2} + \sum_{n=1}^{\infty} \int_0^{\frac{T}{2}} F_n \right. . \tag{46}$$

<u>Integration of the F_n terms.</u> The problem now becomes that of evaluating the integrals of the F_n terms which form the series making up equation 46. For n = 1, the F_n integral becomes:

$$\int_0^{\frac{T}{2}} F_1 = \int_0^{\frac{T}{2}} - \frac{2^2 B^2 \cos^2(\omega t)\omega\, dt}{4!} \,, \tag{47}$$

which upon integration becomes

$$\int_0^{\frac{T}{2}} F_1 = -\frac{2^2 B^2}{4!}\left[\frac{\omega t}{2} + \frac{\sin 2\omega t}{4}\right]_0^{\frac{T}{2}}$$

$$= -\frac{2^2 B^2}{4!}\left[\frac{\cancel{2}\pi \cancel{T}}{\cancel{2T}2} + \underbrace{\frac{1}{4}\sin\frac{\cancel{2}\cdot 2\pi \cancel{T}}{\cancel{T}\ \cancel{2}}}_{=0}\right]$$

$$\int_0^{\frac{T}{2}} F_1 = -\frac{2^2 B^2}{4!}\left[\frac{\pi}{2}\right] . \tag{48}$$

For n = 2 we have

$$\int_0^{\frac{T}{2}} F_2 = \int_0^{\frac{T}{2}} \frac{2^4 B^4 \cos^4(\omega t)\,\omega\, dt}{6!}\,, \tag{49}$$

which upon integration becomes

$$\int_0^{\frac{T}{2}} F_2 = \frac{2^4 B^4}{6!}\left[\frac{1}{4}\cos^3(\omega t)\sin(\omega t)\right]_0^{\frac{T}{2}} + \frac{3}{4}\int_0^{\frac{T}{2}} \cos^2(\omega t)\omega\, dt. \tag{50}$$

The first term (in the brackets in equation 50) will always be zero for the limits involved while the second term, except for the constant multipliers, is identical with the integral for the case for n = 1. Therefore equation 50 becomes

$$\int_0^{\frac{T}{2}} F_2 = \frac{2^4 B^4}{6!}\left[\frac{3}{4}\right]\left[\frac{\pi}{2}\right]. \tag{51}$$

For n = 3 we have

$$\int_0^{\frac{T}{2}} F_3 = \int_0^{\frac{T}{2}} \frac{2^6 B^6 \cos^6(\omega t)\,\omega\, dt}{8!}\,, \tag{52}$$

which upon integration becomes

$$\int_0^{\frac{T}{2}} F_3 = -\frac{2^6 B^6}{8!}\left\{\left[\frac{1}{6}\cos^5(\omega t)\sin(\omega t)\right]_0^{\frac{T}{2}} + \frac{5}{6}\int_0^{\frac{T}{2}} \cos^4(\omega t)\omega\, dt. \tag{53}$$

In equation 53 the first term of the right side will always be zero for the limits involved while the second term is again identical with the n = 2 term except for the constant multipliers. So therefore equation 53 yields

$$\int_0^{\frac{T}{2}} F_3 = -\,\frac{2^6 B^6}{8!}\left[\frac{3}{4}\right]\left[\frac{5}{6}\right]\frac{\pi}{2}\ . \tag{54}$$

and similarily for higher values of n. So that equation 46 which gives the energy passing the final slit Sl_3 becomes

$$S = \frac{2\ell_s I_o}{\omega}\left\{\frac{\pi}{2} - \frac{2^2 B^2}{4!}\left[\frac{\pi}{2}\right] + \frac{2^4 B^4}{6!}\left[\frac{3}{4}\right]\left[\frac{5}{6}\right]\frac{\pi}{2} - \frac{2^6 B^6}{8!}\left[\frac{3}{4}\right]\left[\frac{5}{6}\right]\frac{\pi}{2} + \cdots\right\} . \tag{55}$$

Taking out the factor $\pi/2$ in the series, and writing ω as $2\pi/T$ results in equation 55 taking the form

$$S = \frac{\cancel{2}\,\ell_s I_o T \cancel{\pi}}{\cancel{2}\cancel{\pi}\quad 2}\left\{1 - \frac{2^2 B^2}{4!} + \frac{2^4 B^4}{6!}\left[\frac{3}{4}\right] - \frac{2^6 B^6}{8!}\left[\frac{3}{4}\right]\left[\frac{5}{6}\right] + \cdots + (-1)^n \frac{2^{2n} B^{2n}}{(2n+2)!}\,\frac{3.5\ldots(2n+1)}{4.6\ldots(2n)}\right\} , \tag{56}$$

which may also be written in the general form

$$S = \frac{l_s I_o T}{2} \sum_{n=0}^{\infty} \frac{(-1)^n B^{2n}}{(2n+1)\, n!\, (n+1)!} \quad . \qquad (57)$$

Equation 57 is then the expression for the light energy transmitted through slit Sl_3 in a time interval of one-half period for a given value of B. The sound pressure amplitude, the position in the stationary wave, and constants of the system are all contained in B (see equations 28 and 29).

Now, in the same length of time, i.e. one-half period, the light energy S_o, which would pass through the slit Sl_3 if no sound were in the cell can be expressed by the relation

$$S_o = \frac{l_s I_o T}{2} \qquad (58)$$

So that the ratio of the energy passed with the sound on to that passed with the sound off S/S_o, or the relative energy passed S_R, is given by the expression

$$S_R = \frac{S}{S_o} = \sum_{n=0}^{\infty} \frac{(-1)^n B^{2n}}{(2n+1)\, n!\, (n+1)!} \quad . \qquad (59)$$

If the first few terms of equation 59 are written out and if for B its equivalent from equation 29 is substituted, the expression for S_R appears in the following form

$$S_R = \left\{ 1 - \frac{A^2}{6} \sin^2\left(\frac{\omega x}{c}\right) + \frac{A^4}{60} \sin^4\left(\frac{\omega x}{c}\right) - \frac{A^6}{1008} \sin^6\left(\frac{\omega x}{c}\right) + \cdots \right\} . \qquad (60)$$

From this expression it is possible to calculate the values of S_R for different positions x along the wave, assuming certain values for the parameter A. Several theoretical curves corresponding to various values of A are shown in Figure 4. The minimum points on the curves of Figure 4 correspond to the nodes of pressure in the stationary wave, i.e., the positions where $\sin(\omega x/c) = 1$, or where the series of equation 60 becomes

$$S_{R_{min}} = \left\{ 1 - \frac{A^2}{6} + \frac{A^4}{60} - \frac{A^6}{1008} + \cdots \right\} \qquad (61)$$

Therefore, in theory at least if it were possible to obtain experimentally, the value of S_R for various positions along the wave, it should be possible not only to obtain some idea about the form of the sound wave, but, since the A could be determined, and since it contains the pressure amplitude as a factor, this method should provide an approach to the determination of the pressure amplitude in a stationary sound wave.

Theory of Sawtooth Wave Formation in Waves of Finite Amplitude

In the discussion above the assumption has been made that the waves are sinusoidal. This is a usual assumption, but it should be worth while to recall under what conditions such an assumption is valid, assuming the original source to be sinusoidal. The problem of what happens to a sinusoidal wave form as it is propagated is one which has been considered for many years, Stokes having mentioned the problem as early as 1848 (15). Investigations were subsequently made by Riemann (16) and Earnshaw (17) and mentioned by Raleigh (18) in his book.

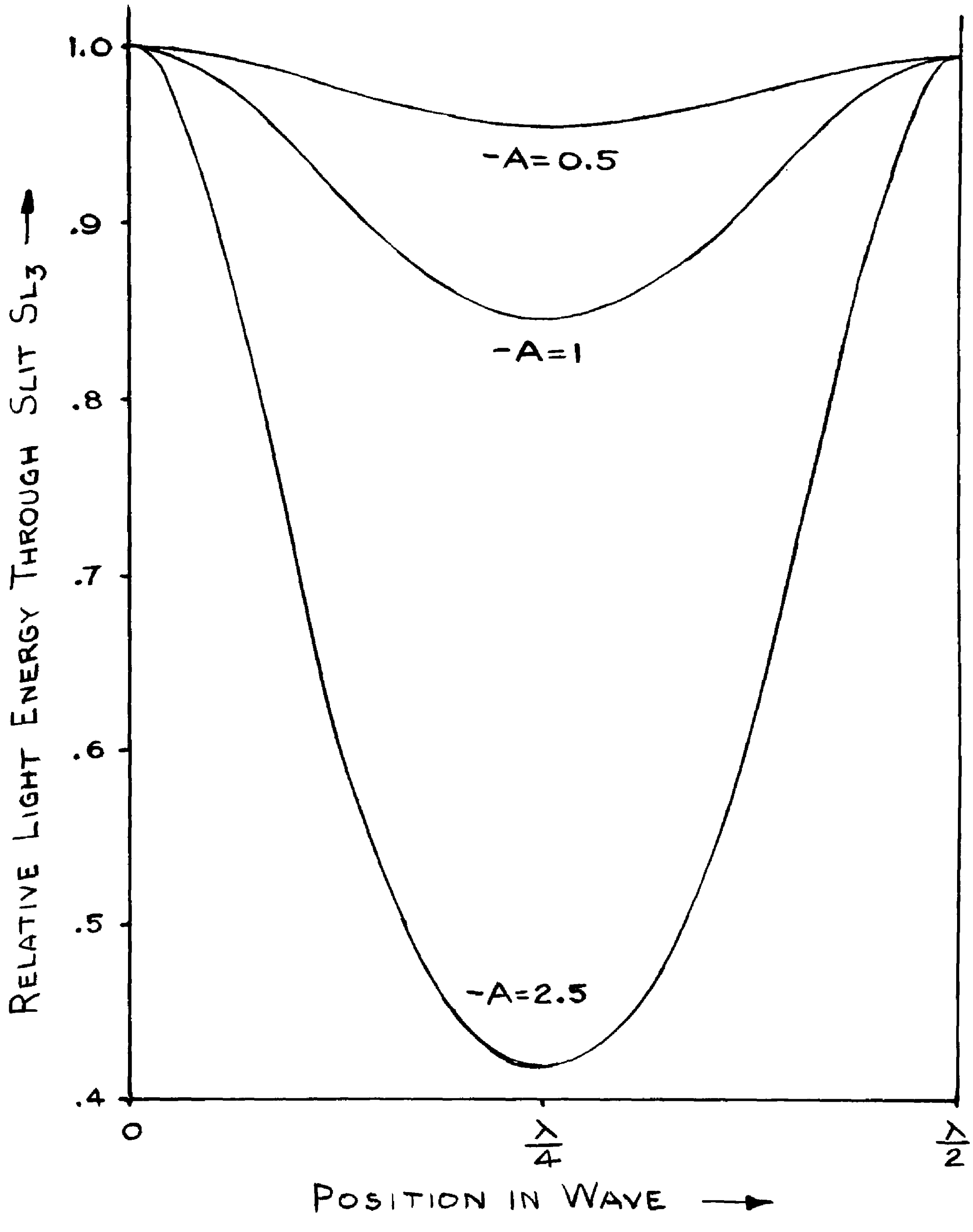


Figure 4. Theoretical curves of relative light energy through final slit Sl_3 vs. position in wave.

The problem arises from the fact that in the development of the usual equations for a plane sound wave an approximation is made which limits the application of the equations to cases of infinitely small amplitudes. This amounts to neglecting any change in compressibility of the medium under consideration with change in pressure. The result is that the waves behave as the theory predicts if the compressibility is a constant, or if it is not, the equations are still applicable provided that the amplitude is small enough. If these conditions are not fulfilled, i.e., if we have a wave of finite amplitude in a medium having a non-constant compressibility in the pressure range under consideration the velocity of propagation of the wave is no longer a constant, but is different for different portions of the wave, the result being that the condensations gain continually on the rarefactions in the wave with the result that there is a tendency for the condensations to overtake the rarefactions, tending to form a sawtooth shaped wave. As Raleigh has pointed out (18) this process can not go on indefinitely.

Fay (19) and Biquard (20) have investigated this problem mathematically. Fay obtained an exact solution to the equation of motion in the form of a Fourier series. When there is a non-linear relationship between pressure change and specific volume change, i.e., a non-constant compressibility, there is found to be a gradual transfer of energy from the low frequency components to those of higher frequency, again tending to form a sawtooth wave. Since the high frequencies tend to be absorbed more rapidly than do the low frequencies, one might expect the wave form to become stabilized after a time. However, the conditions for stability vary with intensity, so that no permanent stable form results, but only a so-called "most stable form" which changes its shape more gradually than any other form of the same wave length and intensity. The most stable form for very low intensities becomes a sine form.

The distortion of waves into a sawtooth form has been observed experimentally in air by Hubbard, et. al. (21), while Mikhailov (22) claims to have observed a non-linear effect of several liquids on sound waves traversing the liquids. By subjecting a liquid simultaneously to sounds of two different frequencies and then detecting not only the original frequencies, but also sum and difference frequencies, he concluded that the liquids had non-linear characteristics. If this is true, one might also, in accordance with the theory outlined above, expect possible sawtooth wave formation in liquids.

Theory of Intensity Decrease of Undeflected Beam Applied to a Sinusoidal Wave Interfering with a Sawtooth Wave Traveling in Opposite Direction

In the mathematical theory developed earlier, sinusoidal waves have been assumed throughout. If, however, a sawtooth wave form does develop from an original sine wave, the curve of light intensity on the optic axis as a function of the position in the wave through which the light has passed would be expected to vary from the type of curve plotted in Figure 4. Just how the final curve should look would depend upon the degree to which a sawtooth form had developed, and whether the wave with which it was interfering was essentially sinusoidal or sawtooth in nature. However, if a sawtooth form does develop, it would be more likely to be found at some distance from the sinusoidal source than very near the source. By the same token one might expect to find sinusoidal waves near the source. Thus, for oppositely directed waves interfering with one another, there is some possibility that near the source a sinusoidal wave traveling in one direction might be interfering with a sawtooth wave traveling in the opposite direction. It might be of interest to investigate what type of a final curve this situation might produce, and in particular, to see if such a situation produces a non-symmetrical curve.

With this in mind, a graphical analysis has been made of such a situation. An arbitrary sinusoidal pressure wave traveling to the right has been assumed to interfere with an arbitrary sawtooth pressure wave of the same amplitude traveling to the left. (See Figure 5). In the particular case chosen an extreme type of sawtooth has been chosen for simplicity, though this extreme type could not occur in reality.

The analysis was carried out as follows: The sine pressure curve and the sawtooth pressure curve were drawn as in Figure 5, where their positions correspond to the time $t = 0$. The sine curve was assumed to move to the right with a definite velocity, while the sawtooth wave moved to the left with a velocity of equal magnitude. The relative positions of the waves were then drawn for times of $t = T/18$, $t = 2T/18$, etc., where T is the period of vibration. The resultant pressure within a wave length was then obtained for each time ($T = 0$, $T/18$, $2T/18$, etc.), by algebraically adding the pressures due to the sine and sawtooth component waves. This resulted in eighteen different curves, each representing the pressure distribution along a wave length for a given instant

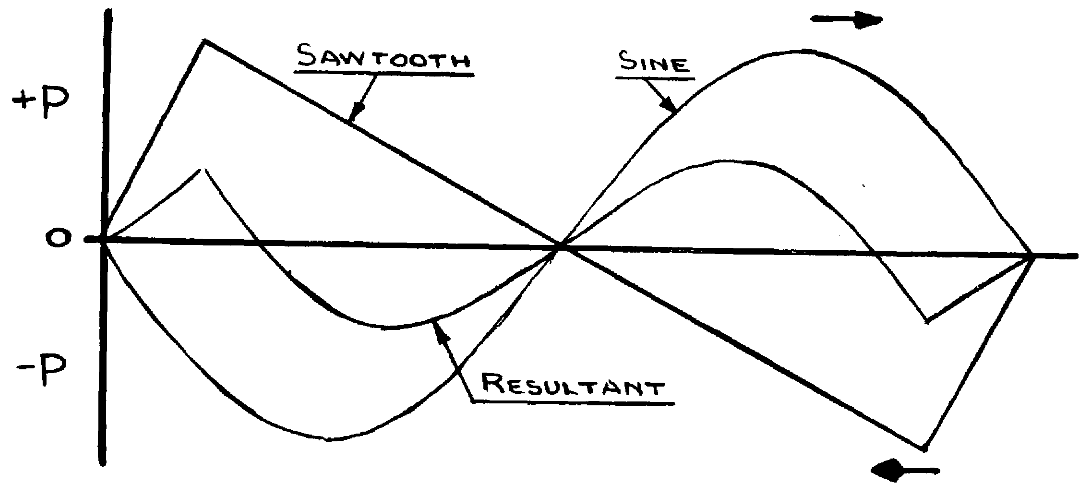


Figure 5. Resultant of sinusoidal wave traveling to right and sawtooth wave traveling to left (time t = 0).

in time corresponding to one of the times t = 0, T/18, 2T/18, etc. Now, the deflection of the light beam through a point in the sound field at a given time is a direct function, not of the pressure but of the pressure gradient. Therefore, it was necessary to determine the slope of the pressure curves for various positions in the wave. An approximation to the average slope over a small portion of a wave length was obtained by measuring graphically the slope of the chord connecting two points on the pressure curve separated by a distance of $\lambda/18$, λ being the wave length. Thus, for each pressure curve, a value for its slope was obtained for eighteen equally spaced positions within one wave length. The deflection d of the light beam through a given position in the wave at any instant is directly proportional to this slope of the pressure curve at the point. The deflection d_{max} caused by the maximum slope obtained in the above analysis was arbitrarily set equal to one-half the width of the central maximum of the diffraction pattern of slit Sl_2 in the plane of slit Sl_3. This caused the light intensity on the optical axis to become zero for this case. Because of the proportionality existing between d and the slope of the pressure curve, this procedure automatically tied any given value for the slope to a definite value of d. The values for d corresponding to the measured values of the slopes were then computed. The amount of light passing the final slit Sl_3 for any given d value could then be determined if the light distribution in the diffraction pattern were known. A recorder trace of this light distribution was obtained as follows: A photomultiplier tube of a microphotometer was placed behind slit Sl_3 and connected to a recorder. Slit Sl_3 together with the photomultiplier tube were mounted on a micrometer screw which was driven by a synchronous motor across the slit diffraction pattern at a rate of one millimeter per minute. Meanwhile the recorder paper was moving at a rate of two inches per minute. Thus, a trace was obtained on the recorder paper which showed light intensity versus position in the diffraction pattern. (See Figure 6). Since the recorder scale was a linear one, while that of the microphotometer was not quite linear, the recorder did not read a true intensity, and it shall hereafter be referred to as "recorder intensity."

Now, since the deflections d for a light beam passing through a given portion of the wave were known for time t = 0, T/18, 2T/18, etc. It was a simple matter to obtain the "recorder intensity" on the optical axis at these times with the aid of Figure 6. From the eighteen values obtained of this "recorder intensity" for a given position in the wave a

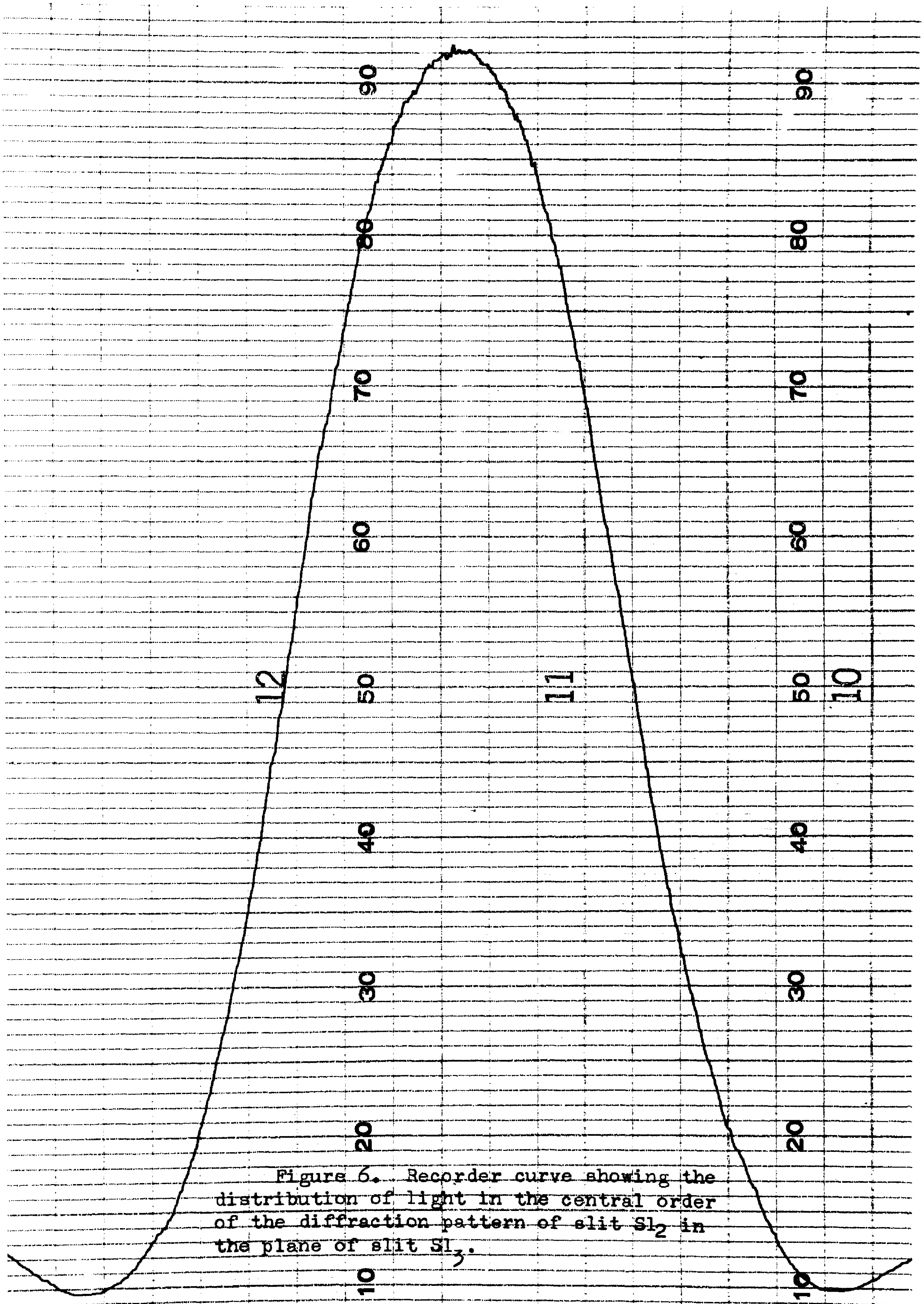


Figure 6. Recorder curve showing the distribution of light in the central order of the diffraction pattern of slit Sl_2 in the plane of slit Sl_3.

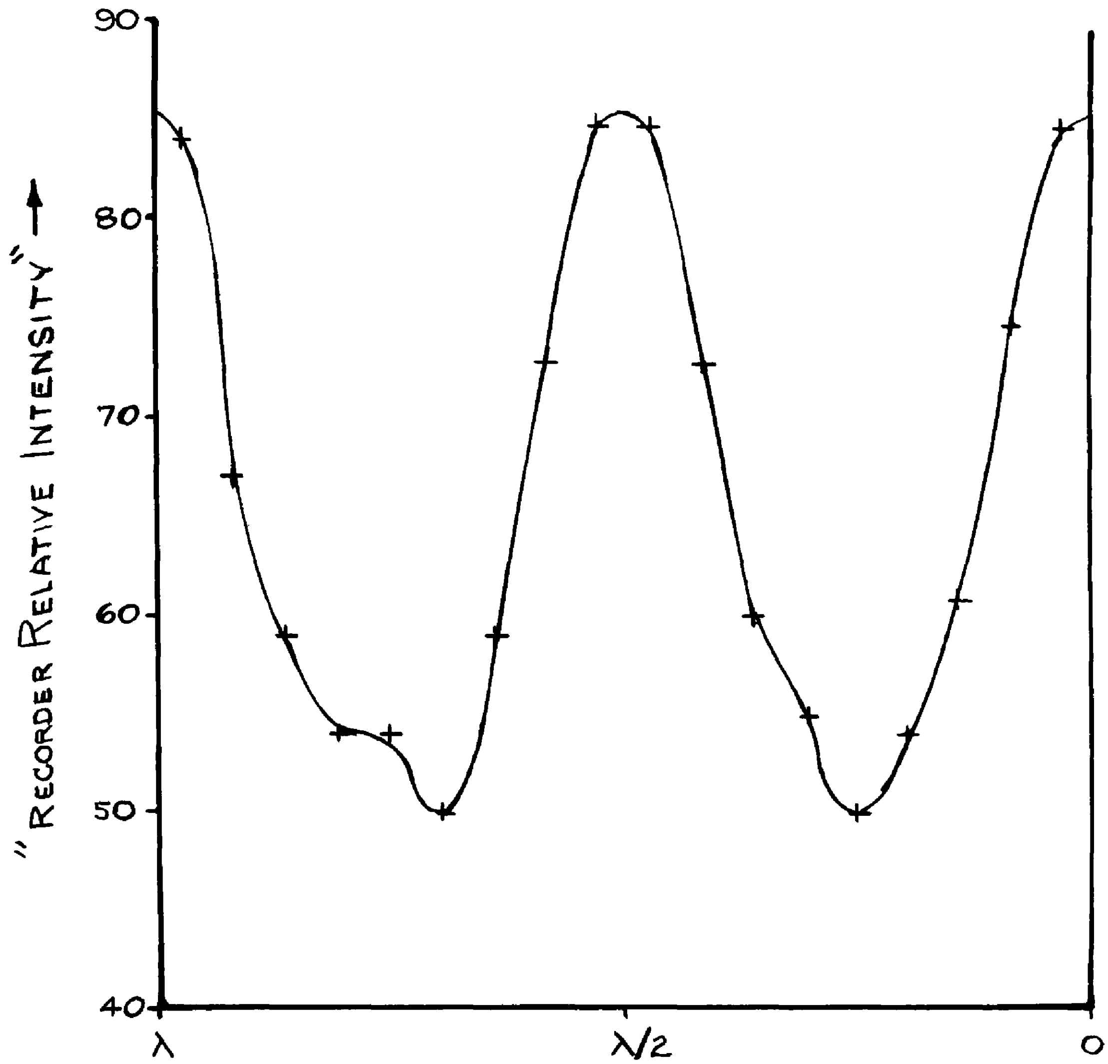


Figure 7. Curve of theoretical "recorder relative intensity" vs. position in wave due to sinusoidal wave interfering with oppositely directed sawtooth wave.

mean value over the period T was computed. This represented the "mean recorder intensity" for one position in the wave. Similarly values of "mean recorder intensity" were obtained for each of the eighteen equally spaced positions along a wave length. Dividing these "mean recorder intensity" values by the maximum "recorder intensity" at the central maximum of the curve of Figure 6, produced what might be called "relative recorder intensity" readings for each of the eighteen points along the wave length. These readings were plotted against position in the wave and resulted in the curve shown in Figure 7. Although this curve is a theoretical curve, yet it shows "recorder intensities" rather than actual intensities since all its intensities were obtained from Figure 6, which is itself a recorder record, and therefore all its intensities are "recorder intensities."

Although the manner of approach in this analysis has been rather arbitrary, and although it is realized that the particular shape of the final curve will depend upon some factors which have been arbitrarily assumed, as for example, the amplitude of the component waves, and the particular deflection corresponding to a given pressure gradient, it is nevertheless hoped that some significance can be attached to the asymmetry and the general shape of the curve.

EXPERIMENTAL APPARATUS

Oscillator

The oscillator used for driving the transducers used in this investigation was a commercial type manufactured by Brush Electronics Company under the trade name "Hypersonic Generator, model BU-204." This model can be equipped with different tuning units for covering different frequency ranges. In this investigation the tuning unit used was that designated by the Brush Company as "Tuning Drawer, model BU-404-B", and covered the frequency range 0.3 megacycle/second to one megacycle/second.

This generator is essentially a self-excited Hartley oscillator using a type 810 tube as the oscillator and using two type 866-A tubes as rectifiers. Its nominal radio frequency output is rated by the manufactjrers as 250 watts.

Transducers

The transducers used in this investigation were pre-polarized barium titanate ceramic elements manufactured by Brush Electronics Company.

For the photographing of the stationary waves as well as some preliminary work on the other aspects of this investigation a barium titanate (Brush ceramic A) disc, used as a thickness vibrator, was employed. The nominal diameter of the disc was one inch and its nominal resonance frequency was 0.4 megacycle/second. For the remaining portion of the investigation a barium titanite plate (Brush ceramic B), also a thickness vibrator, was used. Its nominal resonance frequency was likewise 0.4 megacycle/second and its nominal size originally was 2-3/16 inches square. This was cut into two pieces so that the piece actually used in the work was 2-3/16 inches by approximately 1-1/16 inches. Both transducers when used were mounted in holders with an air backing for radiating to one side.

Optical Arrangements

Several slightly different optical arrangements were used. For the photographing of the sound pressure distribution in the sound wave by the light refraction method (Method I), the basic arrangement used was that shown in Figure 8. A source of intense light L illuminated a slit Sl_1 through a condenser lens C. The lens L_1 was used to illuminate the slit Sl_2 with a fairly narrow beam, while lens L_2 was used to focus a sharp image of the edges of slit Sl_2 on the screen Sc, which was replaced in some cases by a photographic film. The light from Sl_2 was allowed to pass through the cell containing the sound wave on its way to the screen.

The need for this fairly complicated optical arrangement arose from the type of light source used. It was a General Electric type 32C 6-8 volt single filament headlight bulb. With this source, it was very difficult to obtain even illumination over a large enough area by illuminating slit Sl_1 directly with the condenser in the ordinary manner, i.e., by forming an image of the filament at Sl_1. Therefore a housing with a small hole was placed around the lamp L and the image of the hole was focused on Sl_1. The second lens L_1 then became necessary in order to

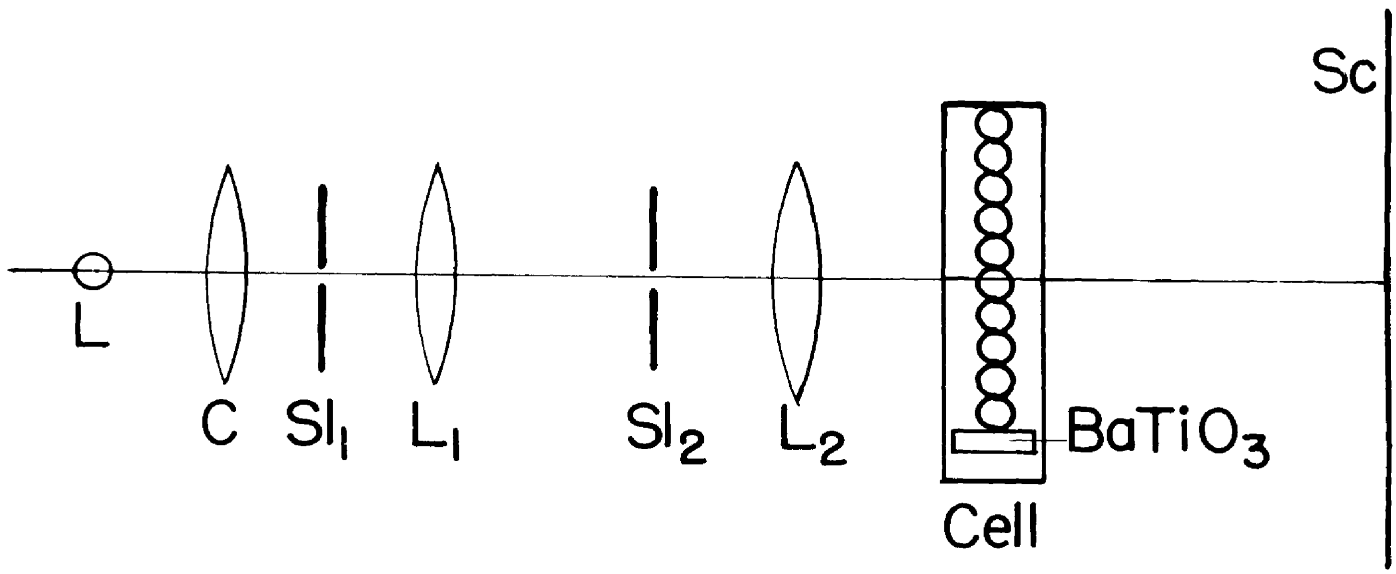


Figure 8. Basic optical arrangement used for light refraction method (Method I).

produce a sharp, narrow, yet well-defined beam both through the cell, as well as on the screen. The adjustment of the width of slit Sl_2 was very critical, since it was necessary to keep the beam width through the cell small compared to a wave length of sound, and yet if Sl_2 were made too narrow, the spreading out due to the diffraction effects would make the beam too wide in the cell. Variations of this basic arrangement were used for photographing the sound pressure distribution in the sound wave.

For later work (Method II) a somewhat different optical arrangement was used. This arrangement is shown in Figure 9. The light source L in this case was a General Electric type AH-4 100-watt mercury vapor lamp. An image of the source was focused on slit Sl_1 by means of the condenser C. The slit Sl_1 then became a secondary source, and the light from it was rendered parallel by lens L_1 and passed through the sound cell after first being limited in width by the slit Sl_2. The final slit Sl_3 was a Pohl precision type slit manufactured by Spindler and Hoyer in Germany. A Gaertner type L-541-E filter was employed behind the slit Sl_3 to isolate the mercury 5461 green line. The light passing through the final slit was measured by an American Instrument Company No. 10-210 Aminco Photomultiplier Microphotometer. The output of this microphotometer was fed directly into a Minneapolis-Honeywell Brown Electronik recorder, model No. 153X11V-X-28. Since the meter on the microphotometer, which read relative light intensity was not a linear scale, it was necessary to calibrate the recorder scale against the scale on the microphotometer. A calibration curve showing the relation between the two scales is shown in Figure 10.

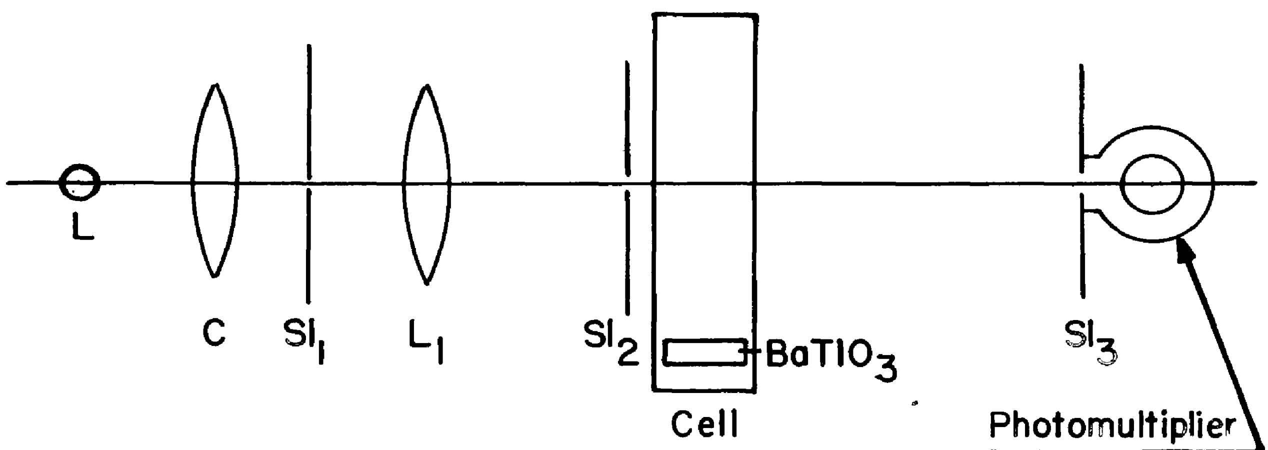


Figure 9. Optical arrangement for obtaining recorder curves of intensity change in undeflected beam. (Method II).

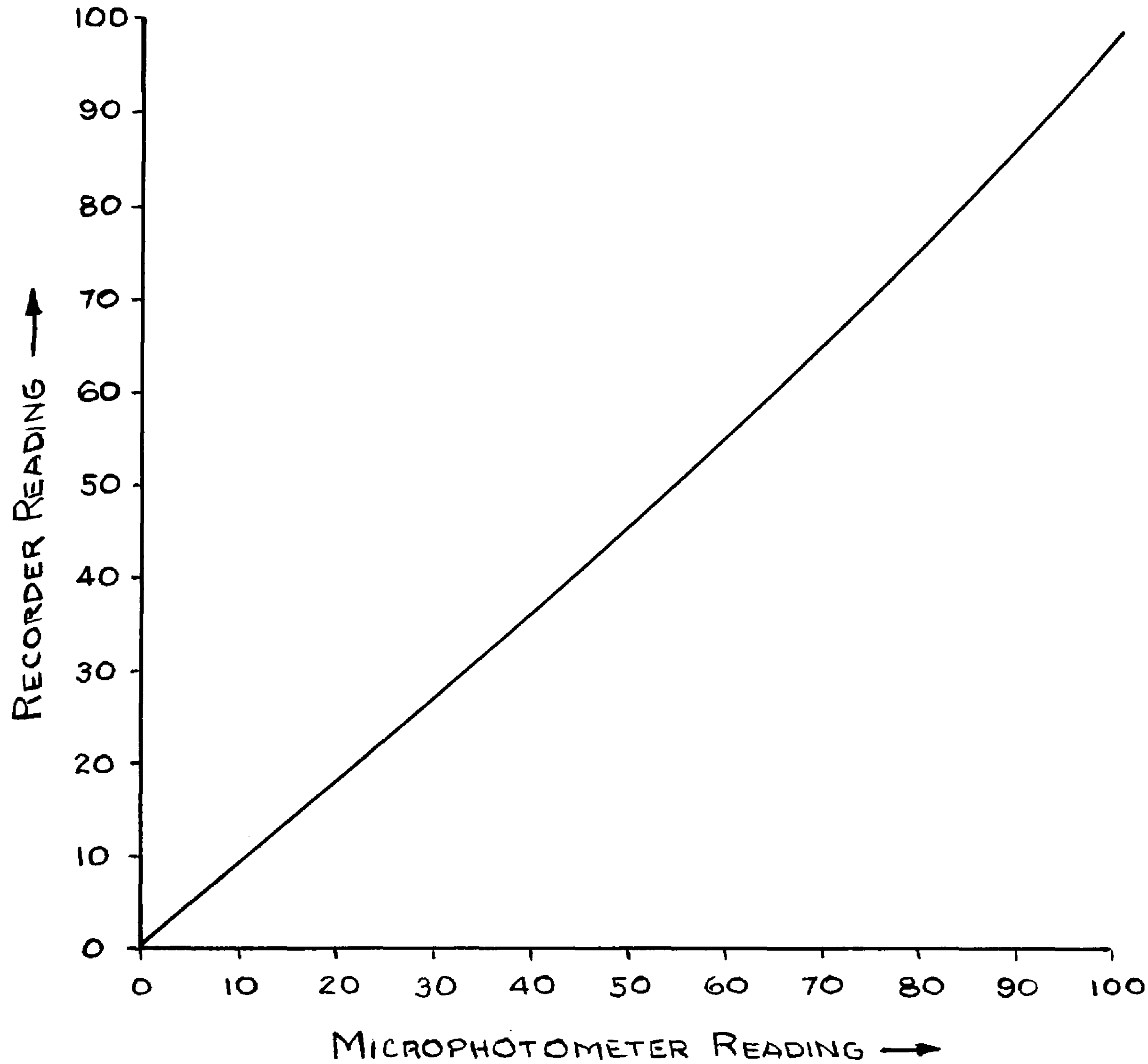


Figure 10. Calibration curve. Microphotometer reading vs. recorder reading.

Frequency and Current Meters

Measurements of frequency for use in calculating sound velocity were made with a U.S. Army Signal Corps type BC-221-J frequency meter for use in the range 0.125 to 20 megacycles/second. Nominal values of frequency were made with a similar but uncalibrated instrument for the measurements where frequency did not enter directly into the calculations.

The value of the R.F. current into the barium titanate transducer was measured by a Simpson 0-500 ma. thermocouple type R.F. milliammeter.

METHODOLOGY AND RESULTS

Photographic Record of Spatial Pressure Distribution in a Stationary Ultrasonic Wave

Photographic record by light refraction method (Method I). Photographing the spatial pressure distribution in a stationary sound wave by the light refraction method (Method I) may be accomplished as follows (23): The basic optical arrangement has been described in a previous section and is shown in Figure 8, page 37. A somewhat simplified schematic diagram of the arrangement showing the modifications used in the basic set-up is shown in Figure 11.

Light, after passing through the slit Sl_2 was confined to a very narrow beam. It is then passed through lens L_2 which forms a sharp image of the edges of Sl_2 upon the rotating cylinder which has replaced the screen. This rotating cylinder had a photographic film mounted on it. The cell containing the stationary sound wave was then moved by means of a micrometer screw driven by a variable speed motor, and simultaneously, the cylinder was rotated slowly by means of a second similar motor. The light reaching the photographic film was limited in the vertical direction by a fairly wide horizontal slit placed immediately in front of the cylinder. The alternate increase and decrease in the broadening of the beam as it traversed the various portions of the stationary sound wave resulted in the type of photographic record shown in Figure 12.

As has been pointed out earlier, the broadening of the beam is

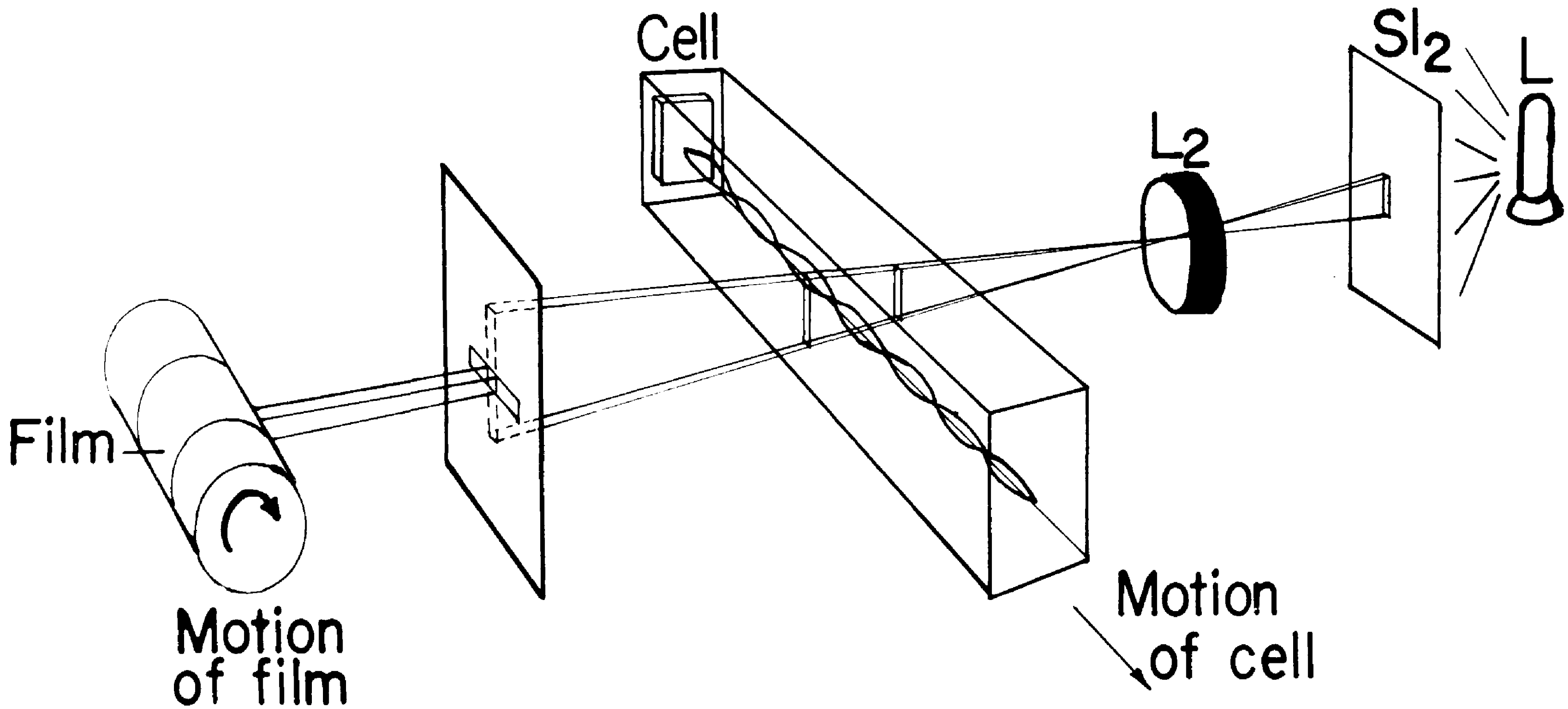


Figure 11. Simplified schematic diagram of optical arrangement for the photographic recording of the spatial pressure distribution in a stationary ultrasonic wave by the light refraction method (Method I).

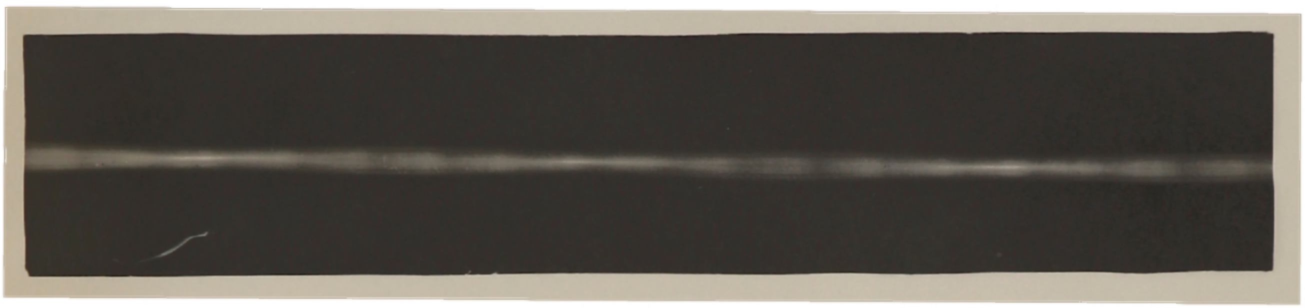


Figure 12. Photographic record of spatial pressure distribution in a stationary ultrasonic wave obtained by light refraction method (Method I).

greatest at the pressure nodes and least at the pressure antinodes. Therefore in the photograph of Figure 12 the broad portions represent the pressure nodes, while the narrow portions represent the pressure loops.

Photographic record by intensity variation in undeflected beam (Method II). If instead of photographing the broadening of the beam directly as described above, use is made of the variation in intensity of the undeflected beam (Method II) (11), a different type of photograph is obtained, which also shows the spatial pressure distribution in a stationary ultrasonic wave (24). The optical arrangement for this case is still basically that shown in Figure 8, page 37, modified as shown in the schematic diagram of Figure 13. In this case, a narrow slit Sl_3 is placed into the path of the light beam between the sound wave and the film. As the cell containing the sound wave is moved in the direction indicated, the light reading the film will be varied; more light passing through the slit Sl_3 when the beam is narrow and less when the beam is broad. Thus, the greater the variation in refractive index in the thin layer which the beam traverses through the sound field, the greater will be the broadening, and the smaller will be the exposure on the film.

Therefore, if the film is moved in the same direction as the cell containing the stationary sound wave, an image of the sound wave is produced in which pressure gradient variations in the sound field are converted into density variations on the photogram. Figure 14 shows a photograph obtained in this manner. Since on the photographic film (negative) the greater exposures will occur where broadening is least,

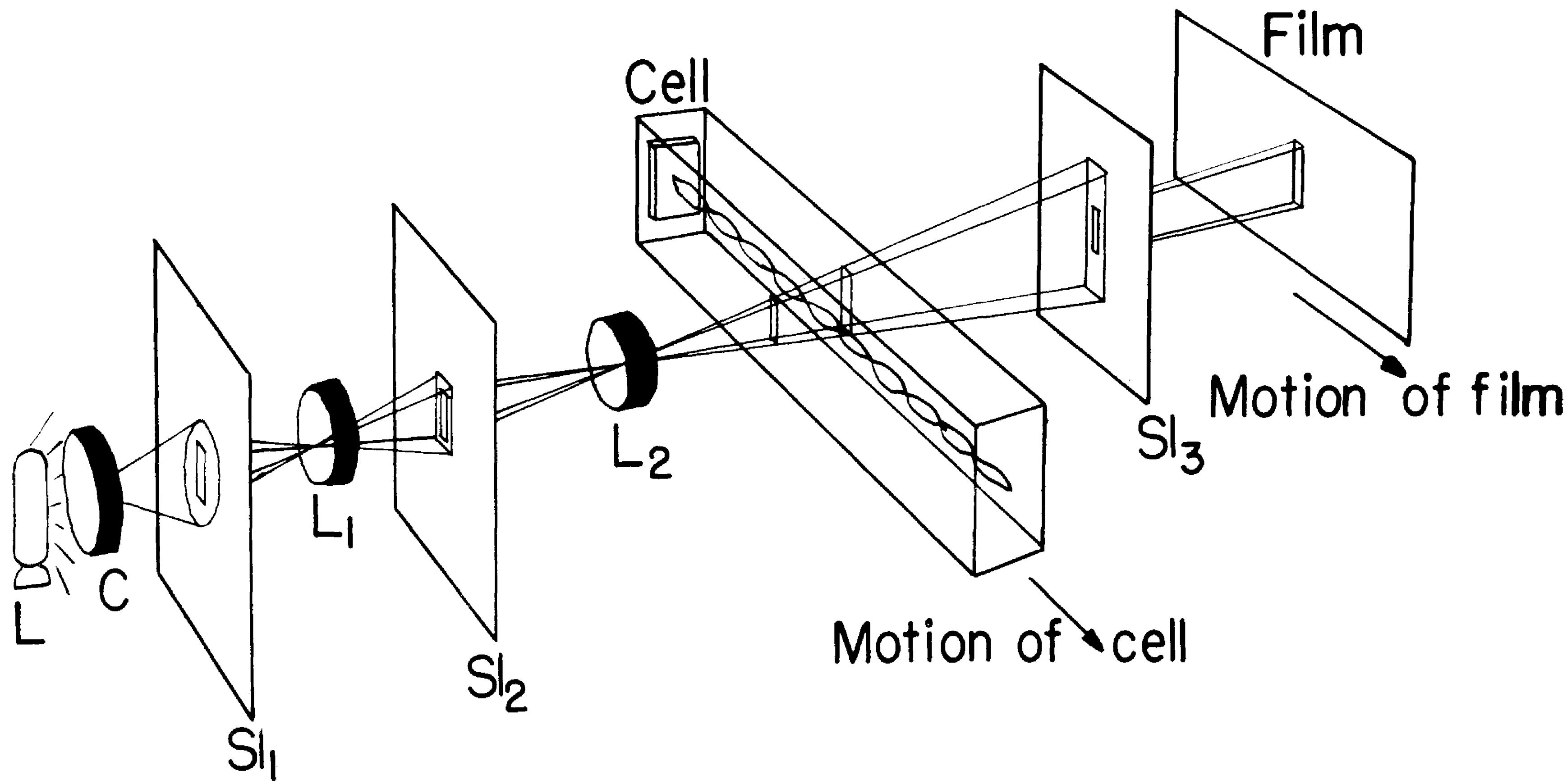


Figure 13. Optical arrangement for photographically recording the spatial pressure distribution in a stationary ultrasonic wave by the method of intensity variation of the undeflected beam (Method II).

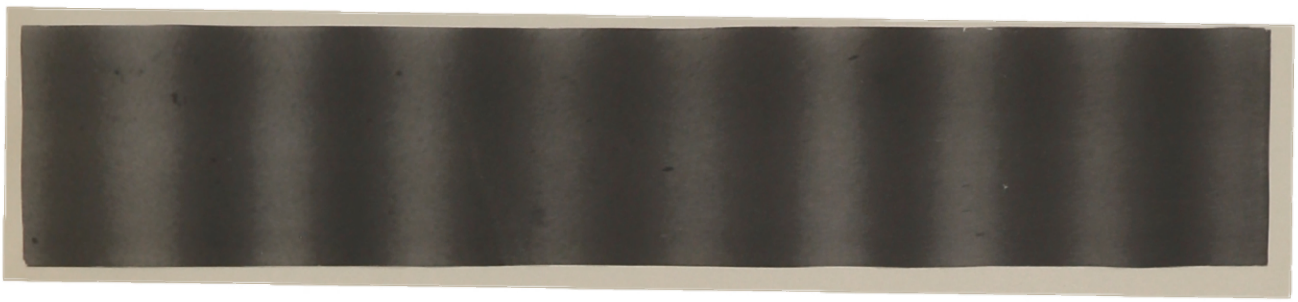


Figure 14. Photographic record of spatial pressure distribution in a stationary ultrasonic wave obtained by the method of intensity variation of the undeflected beam (Method II).

i.e., at pressure loops, the dark portions of the positive reproduction of Figure 14 represent the pressure nodes, and the lighter portions pressure loops, or the dark portions may be considered to represent presssure gradient loops and the lighter portions nodes of pressure gradient.

It may be of interest to point out that this method can be explained by considering the stationary ultrasonic field in a transparent liquid to be optically equivalent to a series of cylindrical lenses; alternately divergent and convergent. The location of these lenses is fixed in relation to the sound field but the lenses vary continuously from a maximum positive power to a maximum negative power. At the nodes of the sound pressure gradient there is nothing to cause a divergence or convergence of the light. A light beam passing through these positions will therefore show no deviation and pass through slit Sl_3. Between these positions in the sound wave, light will be deflected away from the slit. The greater the variation in pressure gradient, the more light will be deflected and the smaller the exposure of the film.

Measurement of Sound Velocity

The method of decrease in intensity of the undeflected beam was found to be rather easily adaptable to the fairly rapid measurement of sound velocity in transparent liquids. The set-up used was basically that shown in Figure 8, page 37. The method of velocity measurement was as follows: The cell in which the position of the transducer had been adjusted for optimum standing waves was mounted on a micrometer screw traverse mount which could be operated manually, causing the cell to move at right angles to the light beam. The sound was turned on and the micrometer screw adjusted to give a minimum deflection on the recorder. The frequency meter was adjusted for zero beat. A position reading for the cell was then taken from the micrometer screw. The cell was then moved by means of the micrometer screw at right angles to the light beam. The distance traveled in most of the measurements was slightly less than five centimeters. As the cell moved across the light beam a series of maxima and minima were drawn on the recorder chart. As the screw neared the end of its travel the photometer needle was watched closely and the effort was made to stop the motion as closely as possible at an exact minimum reading, so that insofar as was possible a distance equal to an integral number of half-wave lengths would be traversed. The motion of the screw was then stopped and a second position reading of the cell was taken. The difference between initial and final position readings gave the total distance traversed by the cell. The frequency was read from the frequency meter dial, and the number of half-wave lengths traversed could easily be counted from the recorder chart. From the number of half-wave lengths covered and the distance traversed the wave length, λ, could be determined. By use of the frequency reading, f, the velocity c was determined from the equation

$$c = f\lambda. \qquad (62)$$

A temperature reading of the medium was taken before and after a run with a mercury in glass thermometer, and the mean value taken as the temperature.

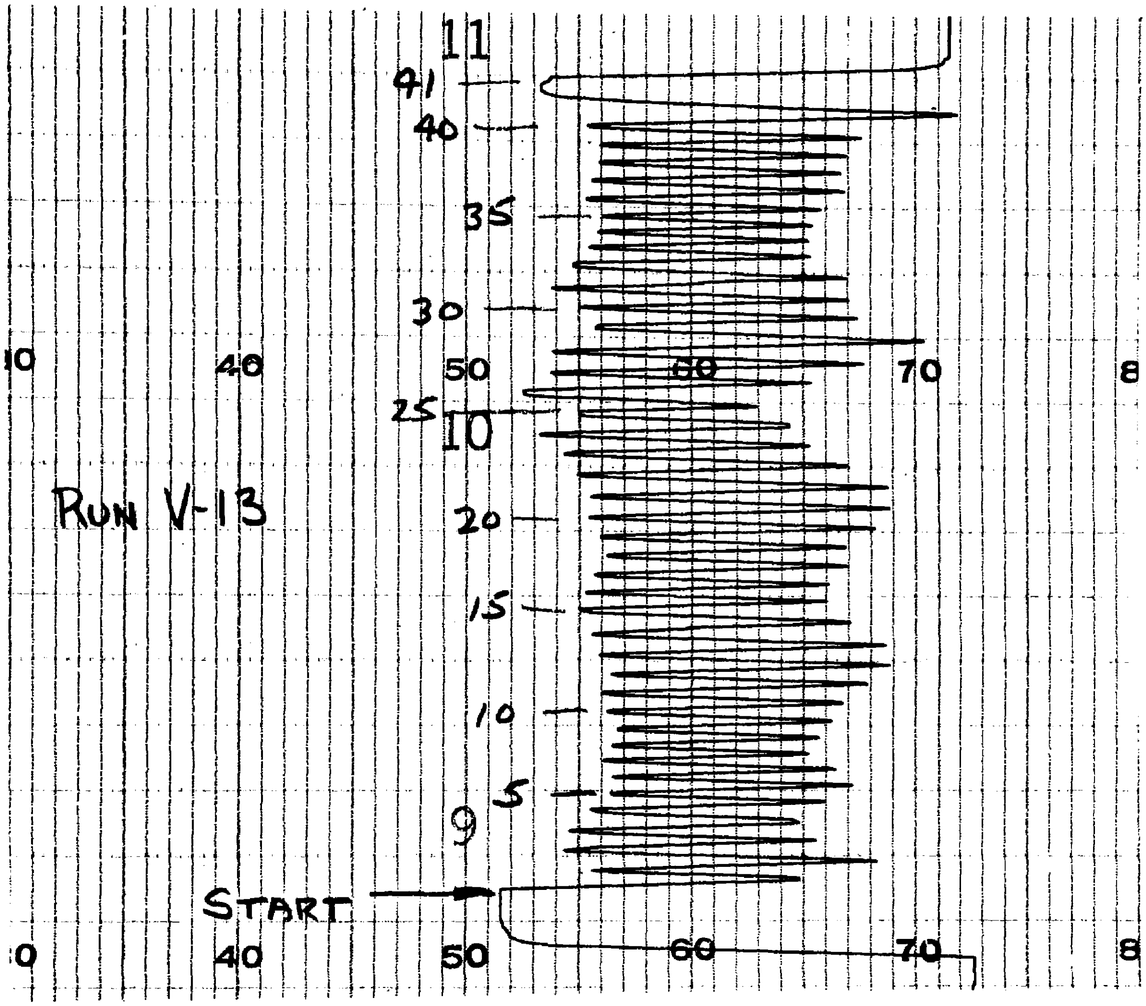


Figure 15. Typical recorder trace obtained in making sound velocity measurements by the method of intensity variation of the undeflected beam.

TABLE I

Results of Sound Velocity Measurements

Substance	Sound Velocity (m./sec.)		Temperature (Deg. C)
Acetone* (3 runs)	1155	at	27.0
	1165	at	27.8
	1169	at	27.6
mean value	1163	at	27.5
Carbon Tetrachloride** (4 runs)	916.1	at	28.1
	918.9	at	28.2
	915.3	at	28.3
	917.9	at	28.4
mean value	917.1	at	28.3
O-Xylene (4 runs)	1353	at	26.3
	1342	at	26.4
	1351	at	26.5
	1346	at	26.6
mean value	1348	at	26.5
Dow-Corning 200 fluid (1 centistoke) (3 runs)	981.6	at	26.1
	987.1	at	26.4
	985.1	at	26.5
	984.6	at	26.3

* Freyer (26) gives 1190 m./sec. at 20° C for acetone with $\Delta v/\Delta T$ = -5.6 m./sec. deg. C. This yields 1148 m./sec. at 27.5° C.

** Freyer (26) gives 904.0 m./sec. at 30° C for carbon tetrachloride with $\Delta v/\Delta T$ = 3.1 m./sec. deg. C. This yields 909.3 m./sec. at 28.3° C.

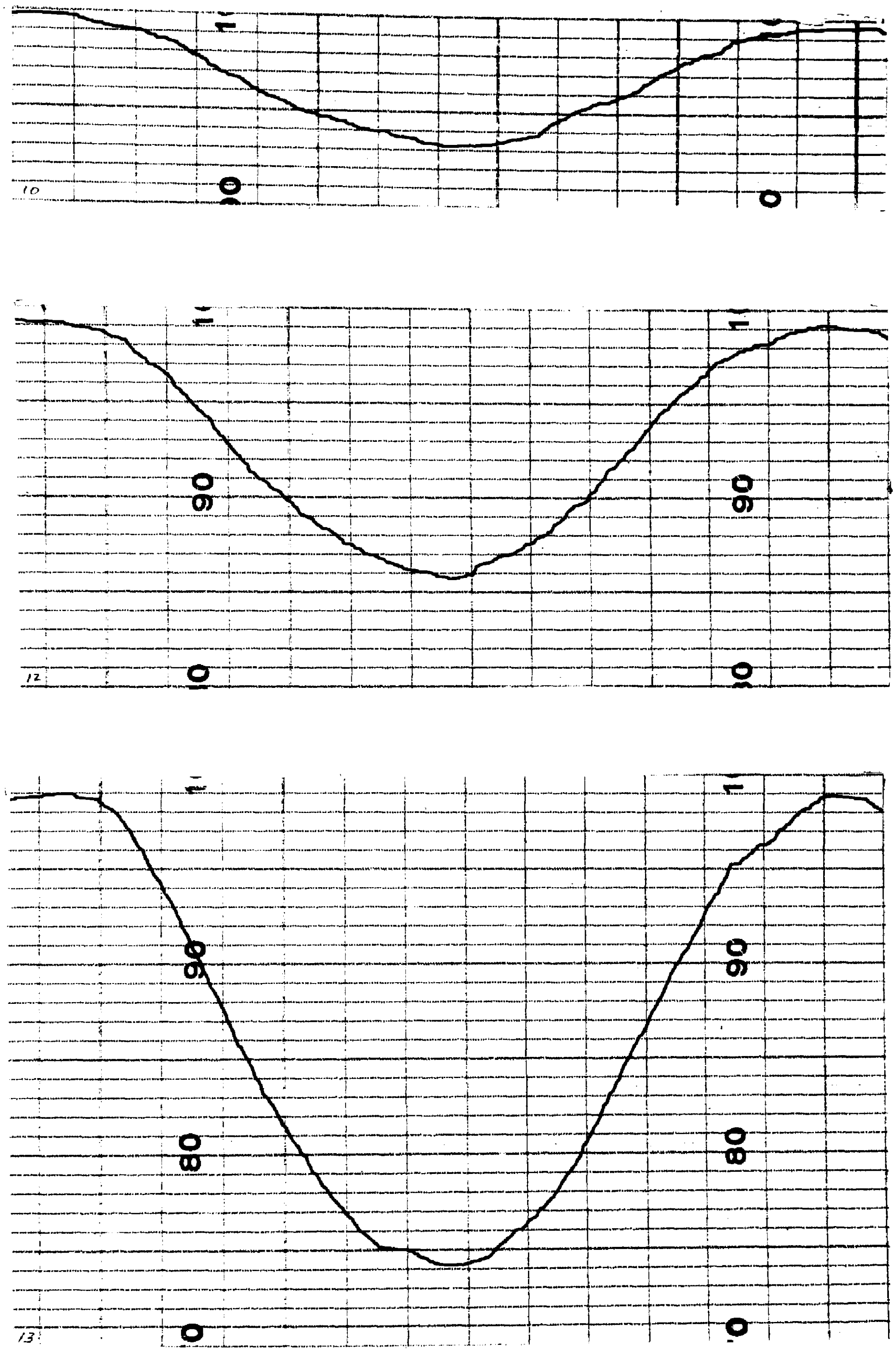
A typical recorder record of a typical run is shown in Figure 15.

Individual runs in a given substance were found to agree generally within less than 1.5% between extreme values, and also seemed to agree well with values given in the literature. A listing of some of the measured values is shown in Table I.

Although other methods exist for the measurement of sound velocity in liquids which are certainly more precise than this method, even at these frequencies (25), nevertheless it is felt that the rapidity with which a run can be made (10 minutes or less including calculations) should make this method of some use, even with the relatively low precision. The method could be made almost fully automatic and probably faster if this should be thought desirable.

Approach to Wave Form Determination

An attempt was made to apply the method of decrease in intensity of the undeflected beam to the study of wave form of the standing wave. The method was first applied to glycerine. The experimental curves obtained (Figure 16) in this liquid seemed to follow the general shape of the theoretical curves predicted from the theory (Figure 4, page 26) and thus indicated the existance of waves in this liquid at least approximately sinusoidal in nature. Figure 16 shows the experimental recorder trace of the "recorder relative intensity" for three differing sound intensities, indicated by transducer currents of 100, 200, and 300 milliamperes respectively. These curves were obtained for the light beam passing approximately 2.4 cm. from the transducer. This placed it approximately



a)

b)

c)

Figure 16. Recorder trace of "recorder relative intensity" of light through final slit versus position in the wave for a stationary wave in glycerine. Transducer current a) 100 ma., b) 200 ma., c) 300 ma.

14 cm. from the reflecting end of the sound cell.

Similar curves, however, were not obtained for other liquids investigated, except perhaps at very low intensities, where it was difficult to determine the shapes of the curve. Rather curves of an unsymmetrical nature were obtained and the curves were not necessarily the same shape from one wave to the next. As an example of this asymmetry Figure 17 of a curve for benzene taken approximately 2.3 cm. from the transducer, and Figure 18 of a curve for acetone, taken at about the same position relative to the sound source are shown. It was the consistancy with which these asymmetrical curves appeared which led to the qualitative investigation of the type of curve one would expect to obtain if a sinusoidal wave interfered with a sawtooth type wave. In this connection the curve of Figure 19 is included because of the interesting similarity in shape between it and the curve obtained from the analysis referred to above, and shown in Figure 7, page 33. The curve in Figure 19 was obtained for carbon tetrachloride at a distance approximately 8.5 cm. from the transducer. The comparison between the curves is mentioned mainly because of the interesting similarity, and no claim is made that the asymmetry is necessarily proof of the existance of a sawtooth wave form in the liquid. There are other possibilities, such as reflections from the boundaries of the cell which might possibly give rise to such asymmetrical curves and further investigation, both theoretical and experimental, in this matter is probably called for before a definite statement can be made about the wave shape in these liquids. However, it is hoped that this method provides an approach to this problem which can

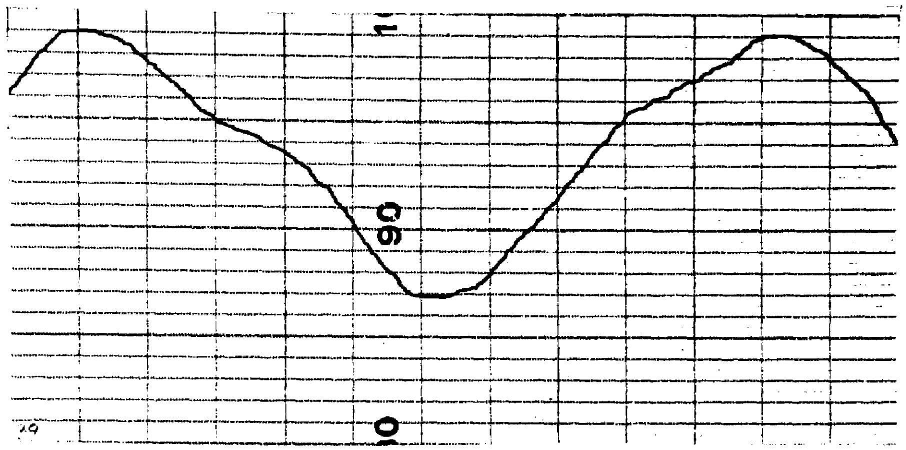


Figure 17. Recorder trace for standing wave in benzene.

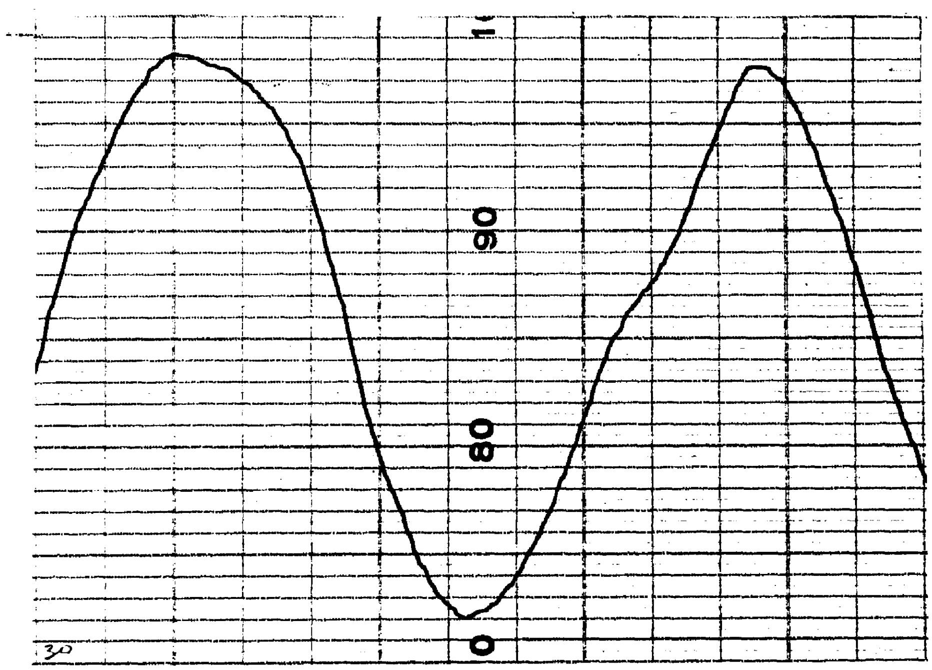


Figure 18. Recorder trace for standing wave in acetone.

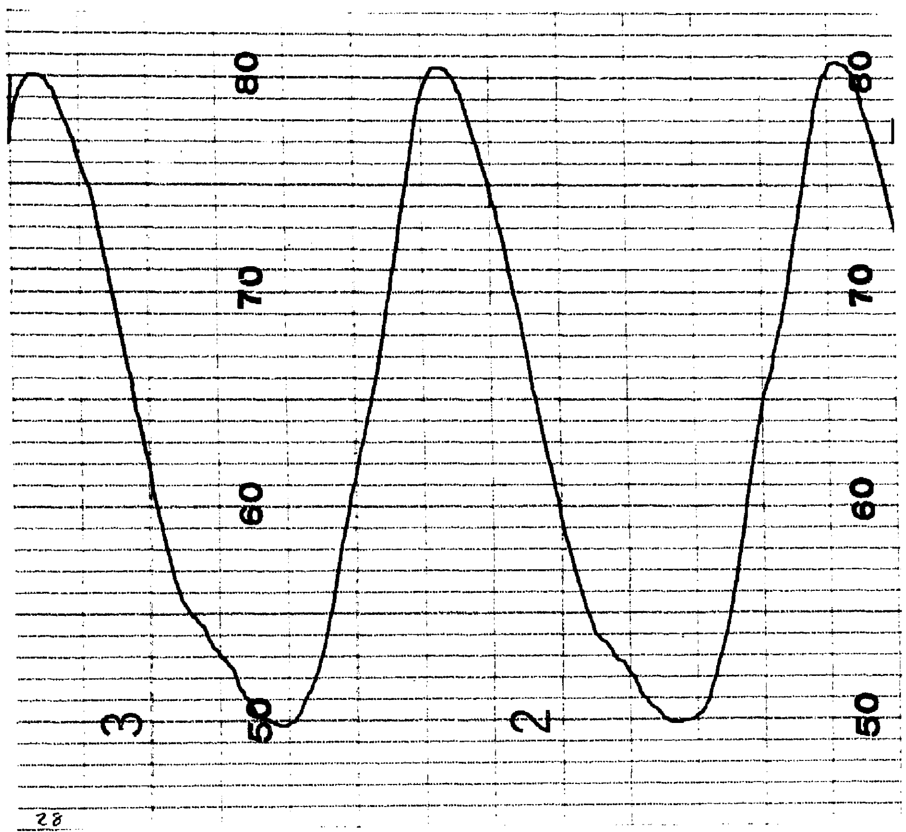


Figure 19. Recorder trace for standing wave in carbontetrachloride.

be improved and investigated further in the future.

Approach to Sound Pressure Measurement

General. As has been mentioned earlier, the mathematical theory developed previously should make it possible to use the method of decrease in intensity of the undeflected beam (Method II) for the estimation of the sound pressure amplitude in the standing wave. Recall that S represents the light energy passing through the final slit Sl_3. If the light beam passes through a pressure node S will have a minimum value. S_o represents the light energy passed by slit Sl_3 for the case of no sound. The relative energy passed, S_R is the ratio S/S_o. The relation between $S_{R_{min}}$ and the parameter A is given by equation 61, page 25, which is written again here.

$$S_{R_{min}} = \left\{ 1 - \frac{A^2}{6} + \frac{A^4}{60} - \frac{A^6}{1008} + \cdots \right\} . \qquad (63)$$

It will be recalled that the parameter A is given by equation 28, page 17, which can be written in the form

$$A = \frac{2\pi^2 a g \delta K (n^2-1)(n^2+2)\, 2P}{6 n \lambda_l \lambda_s r} \qquad (64)$$

where

a = the width of the slit Sl_2,

g = the distance from the center of the sound cell to the slit Sl_3,

δ = the path length of the light beam in the sound field,

K = the compressibility of the medium,

n = the refractive index of the medium,

λ_ℓ = the wave length of light in the light beam,

λ_S = the wave length of the sound in the medium used,

r = the distance from slit Sl_2 to slit Sl_3 and

$2P$ = the pressure amplitude of the stationary sound wave.

Not considering the pressure amplitude 2P for the moment, all of the above quantities are either known or measurable, with the possible exception of the quantity Σ, the path length of the light in the sound field, which however, can be estimated. Assuming then that these quantities are known, the pressure amplitude can be obtained in terms of the parameter A. The relationship between this parameter and $S_{R_{min}}$ (equation 63) can be plotted as a curve. Such a curve is shown in Figure 20. Since $S_{R_{min}}$ can be obtained from the minimum value of the curve on recorder trace (Figure 16, page 53) and the recorder calibration curve (Figure 10, page 40), the curve of Figure 20 enables one to determine the value of A for a given measured value of S_R. Then by the use of equation 64, the pressure amplitude can be calculated.

Relation between transducer current and the parameter A. The values of A obtained from Figure 20 for the three wave-form curves for glycerine (see Figure 16, page 53), are shown plotted against transducer current in Figure 21. The relationship appears to be fairly linear in this case, although the amount of data on which this statement is based is rather small.

Although there is some doubt as to whether the theory can be applied

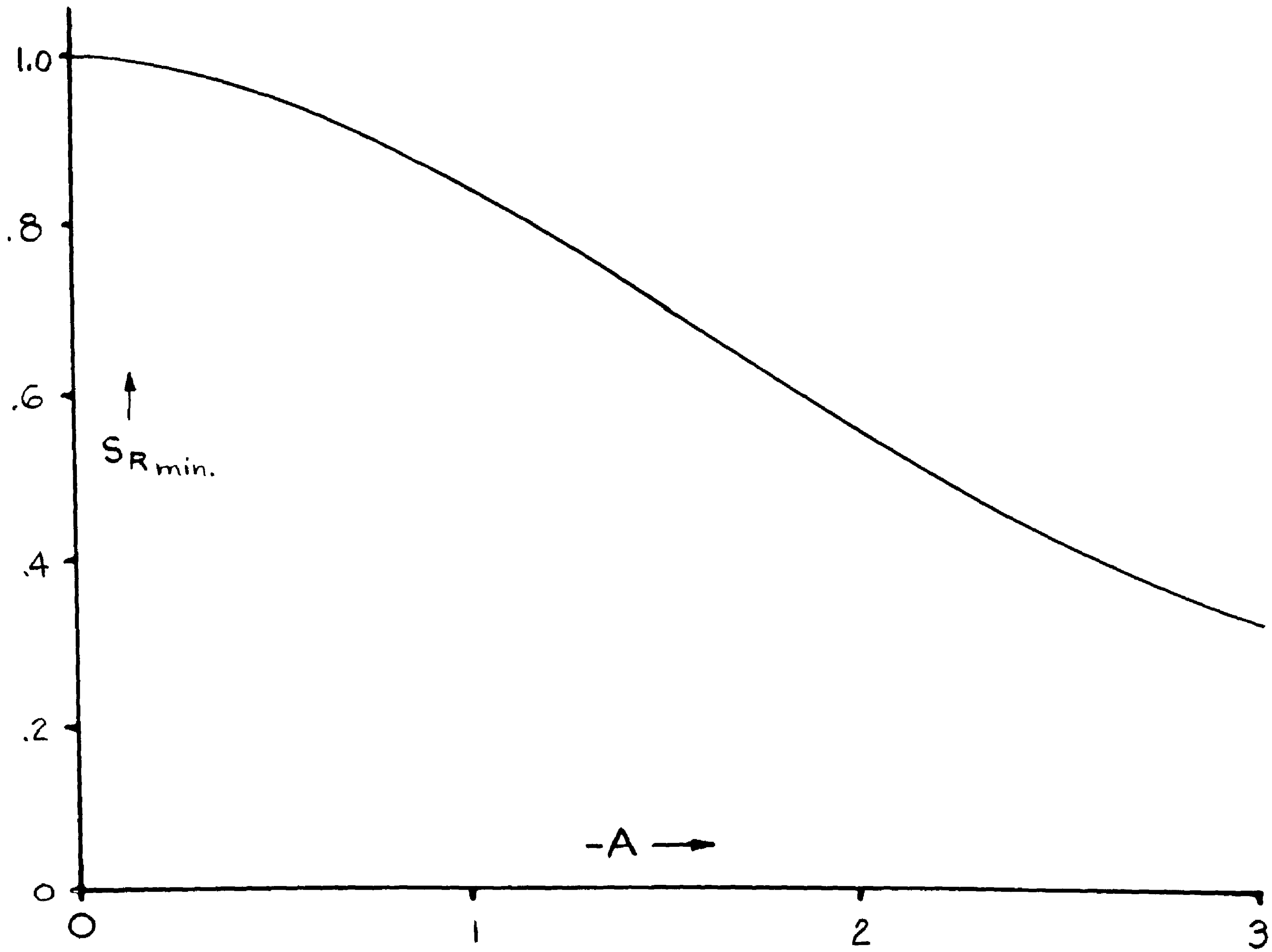


Figure 20. Variation of $S_{R_{min}}$ with parameter A.

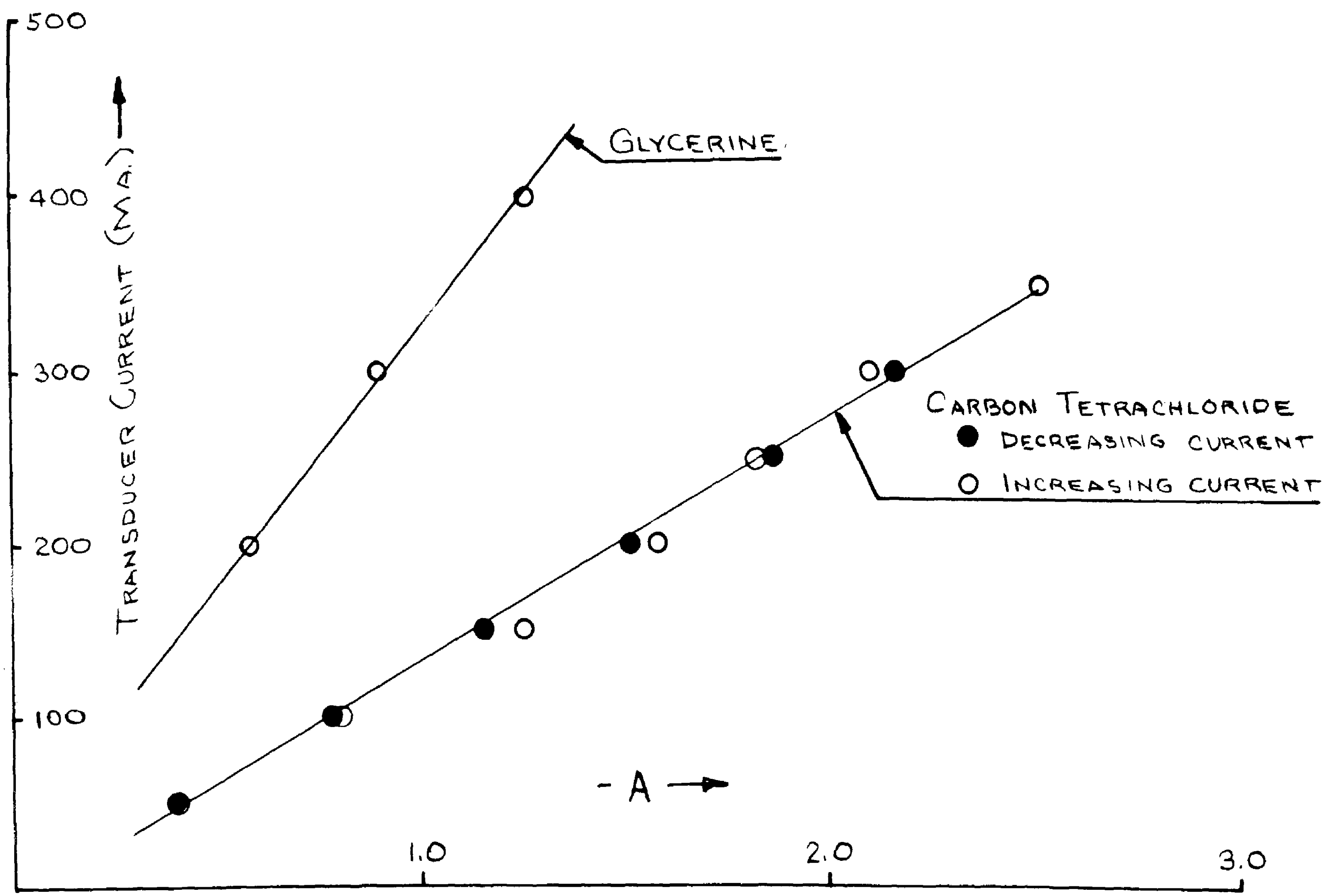


Figure 21. Transducer current vs. parameter A for runs in glycerine and carbon tetrachloride.

to curves which deviate markedly from the theoretical curves (see Figure 4, page 26), yet an attempt to plot curves similar to the one obtained for glycerine in Figure 21 was made. However, to obtain data which could be used it was necessary that nearly identical conditions except for the sound intensity be maintained, so that the resonance condition for setting up the standing wave would not be disturbed. It was usually found that it was not possible to maintain these conditions to a sufficient degree to obtain more than three points for any one curve. The data from these curves seemed to indicate that there was a linear region, but that perhaps at low sound intensities there was a non-linear relation between transducer current and A. However, the data for any one curve was never enough to establish this as a definite fact and must therefore be considered inconclusive.

In an effort to avoid this difficulty, another method was tried for obtaining experimental values of $S_{R_{min}}$. With carbon tetrachloride in the cell, the sound on and the transducer current adjusted to 300 ma., the cell was moved until the recorder indicated a minimum. The movement of the cell was then stopped, and the recorder allowed to run. The recorder then traced a relatively constant value while the transducer current was kept constant. The transducer current was then varied by steps of 50 ma. from the 300 ma. value down to a final value of 50 ma. At each new value of transducer current, the recorder pen would come to rest at a new reading. The process was immediately repeated in the reverse direction starting with the transducer current set at 50 ma. and finishing with it at 350 ma. This process had the advantage of enabling data for several points to be taken in a relatively short time and without turning the generator on and off several times. This helped to avoid changes in the experimental conditions while data for a single curve was being gathered. From the $S_{R_{min}}$ values obtained in this way, a curve was plotted of transducer current versus A for the carbon tetrachloride (Figure 21). These data also seem to show a fairly linear relationship between transducer current and A, though it should be noted that the points obtained during the first part of the run seem to be much more nearly on a straight line than do those obtained during the latter part of the run.

Calculation of pressure amplitude. Having determined the values of A, the application of equation 64 makes possible the determination of a value for 2P the pressure amplitude of the standing wave. Using values for the constants of the system shown in Table II, the pressure amplitude corresponding to the 200 ma. transducer current in glycerine turns out to be approximately 1.0 atmospheres, while that for the 400 ma. case is approximately 2.2 atmospheres. Making the assumption that the theory can be applied to carbon tetrachloride also, the pressure amplitudes turn out to be 0.1 and 0.6 atmospheres for the 50 ma. and the 350 ma. case respectively.

It should be borne in mind that the values obtained in this manner are probably not highly accurate; perhaps not much better than an order of magnitude approximation. The main difficulties probably lie in the uncertainty in the path length of the light in the sound field δ, and the fact that what is being measured is sort of an average effect over the entire width of the sound beam. However, the values obtained for the

TABLE II

Data Used for Sound Pressure Determination by Use of Equation 64

	Glycerine	CCl_4
a	.246 mm.	.246 mm.
g	53.3 cm.	53.3 cm.
δ	2.5 cm.	2.5 cm.
K	22×10^{-6}/Atm.	90.7×10^{-6}/Atm.
n	1.47	1.46
λ_ℓ	5461 Å	5461 Å
λ_s	$\sim$ 4.3 mm.	
r	58.4 cm.	60.4 cm.
f		.397 Mc/Sec.
c		919 m./Sec.
Temperature		28.2° C

pressure amplitudes do not seem unreasonable, and the method can undoubtedly be improved to the point where useful pressure measurements can be made.

The measurements of quantities which appear in equation 64 were made by usual methods. The values of K and n were obtained from the Handbook of Physics and Chemistry, Thirty-second Edition. The sound wave length λ_s was estimated from the trace on the recorder curve for the case of glycine and the frequency was measured in the case of CCl_4 and used together with the previously measured velocity to obtain the wave length. The width of the slit Sl_2 was calculated from the width of the central order of the diffraction pattern of slit Sl_2 in the plane of Sl_3 (see Figure 6, page 32).

SUGGESTIONS FOR FURTHER WORK

Further work, both theoretical and experimental, lying beyond the scope of this investigation has been suggested by the present work.

Relative to the theory, it is felt that the simple theory presented here could easily be modified to apply to progressive waves and that pressure amplitude could be measured in this case quite readily. Furthermore for the case of stationary waves, the theory ought to be extended next to the case of oppositely directed sinusoidal waves of different amplitudes interfering with one another. Application of such a theory might result in obtaining a method of determing reflection coefficients for various reflectors. The qualitative analysis of the interfering sinusoidal and sawtooth waves could perhaps be approached in a more rigorous and meaningful fashion.

Experimentally it might be desirable to improve the method of sound velocity measurement to make it faster, more automatic, and perhaps more precise. For both pressure and wave form determinations, it might be desirable to improve experimental techniques to eliminate confusion due to possible reflections. This could be done either by using much larger tanks or by using previously calculated wave guides where the modes to be set up were known in advance. The wave form could also be studied in progressive waves by use of an ultrasonic strobescope and measuring the deflection of the beam directly. Such a strobescope is under development in this laboratory.

SUMMARY

The refraction of light by stationary ultrasonic waves has been investigated, with the aim of using this effect to study sound fields. A simple theory has been presented for the variation in the intensity of the undeflected beam after it passes through a stationary wave. Use was made of this variation in intensity to measure sound velocity, to study wave form, and an attempt to make pressure measurements. Suggestions for further work have been made.

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