

Halbert Frederick Gates
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Outline of Studies

Major subject: Physics
Minor subject: Mathematics

Biographical Items

Born, October 30, 1919, Milwaukee, Wisconsin

Undergraduate Studies, Milwaukee State Teachers College,
1936-1940

Graduate Studies, University of Wisconsin, 1940-1944,
continued 1947-1948, Michigan State College,
1950-1954

Experience: Graduate Assistant, University of Wisconsin,
1941-1944; Officer in Charge, Principles of Elec-
tricity Section, Theater Signal Corps School
(Germany), 1946; Assistant Professor of Physics,
Berea College, 1948-1950; Special Graduate Research
Assistant, Michigan State College, 1951-1954

Member of American Physical Society, American Association
of Physics Teachers, Society of Sigma Xi, Sigma Pi
Sigma, Pi Mu Epsilon, Kappa Delta Pi

DETERMINATION OF THE RATIO p/q OF THE PHOTOELASTIC
CONSTANTS OF OPTICAL GLASSES BY MEANS OF
TWO DIFFERENT ULTRASONIC METHODS

by

Halbert Frederick Gates

AN ABSTRACT

Submitted to the School of Graduate Studies of Michigan
State College of Agriculture and Applied Science
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Physics and Astronomy

1954

Approved E. A. Fiedemann

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ABSTRACT

Mueller (1) has described, in theory, two dynamic methods for the determination of the ratio p/q of the strain-optical constants of glass. The strains are produced by ultrasonic waves in the glass, while the optical effects are observed in the diffraction patterns produced by light which has passed through the sound field. The theory of Mueller is an extension of that of Raman and Nath (2) which explains the diffraction of light by a sound wave on the basis of variations of the index of refraction of the medium traversed by the sound wave.

Measurements were made on a series of thirteen American optical glasses and fused silica, following Mueller's methods "B" and "C". The ultrasonic frequency was in the neighborhood of ten megacycles per second.

Method "B" involves the methods of Hiedemann (3). Measurements are made of the polarizations in the diffraction pattern at several amplitudes. An extrapolation to zero sound amplitude leads to a value for p/q .

The experimental results are in excellent agreement with the theory. Values of p/q were determined for all of the samples, the ratios ranging from 1.11 to 2.34.

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Method "C" involves the methods of Bergmann and Fues (4). Measurements are made of intensity ratios in the diffraction patterns at several sound amplitudes.

The experimental values at various sound amplitudes, while of the correct order of magnitude, do not agree with the theory. Extrapolation cannot be performed as indicated in the theory, and no value of the ratio p/q can be determined. The behavior of the data is, however, consistent and systematic and it is hoped that some significance may be attached to it.

The values of the ratio p/q of the strain-optical constants, as obtained by method "B", are given, together with other data on the samples. The behavior of the data as obtained by method "C" is described.

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INTRODUCTION

Definitions of Photoelastic Constants

Pockels' strain-optical constants. Pockels (1) has given a set of thirty-six phenomenological constants which relate the general strain tensor to the optical effects which accompany it. In the case of an isotropic material these constants, p_{ij} (the subscripts run from one through six), are reduced by symmetry considerations to two, p_{11} and p_{12} .

Neumann's strain-optical constants. Neumann (2) describes the photoelastic behavior of an isotropic medium by means of two strain-optical constants, p and q . These constants relate a strain to the changes which it produces in the index of refraction n for light vibrating parallel and normal respectively to the direction of the strain. Equation (1) defines the Neumann constants in terms of a typical strain z_z .

$$\begin{aligned}dn_z &= n_z - n = -n^2 q z_z \\dn_x &= n_x - n = -n^2 p z_z\end{aligned}\tag{1}$$

Equation (2) shows the relation between the Neumann constants and the Pockels constants for an isotropic material.

$$p_{11} = \frac{2q}{n} \qquad p_{12} = \frac{2p}{n}\tag{2}$$

It is assumed that the fine-annealed optical glass used in this investigation is an isotropic material, and the discussion is in terms of the Neumann strain-optical constants, p and q .

Stress-optical constants. The optical effects may also be described in terms of the stress which causes them. Indeed, in measurements beyond the elastic limit, Filon and Jessop (3) find evidence that stress is the fundamental quantity related to the optical effects. It is assumed in this investigation that the deformations are small and that Hooke's Law applies and that stress and strain constants are equally significant.

Static Determination of Photoelastic Constants

Determinations of the photoelastic constants of glass by means of the application of steady stress have been made by Mach (4), Pockels (5), Filon (6), Twyman and Perry (7), and Schaefer and Nassenstein (8). These static methods often involve difficult optical measurements requiring large or specifically shaped samples, two "identical" samples, and/or precise temperature control. The samples are subject to relaxation or plastic flow as described by Coker and Filon (9).

Dynamic Determination of Photoelastic Properties of Glass by means of Ultrasonic Waves

The dynamic strains are produced in the glass by

ultrasonic waves. The optical effects are observed in the diffraction of a beam of light which has passed through the sound field.

Diffraction of light by ultrasonic waves. Early observations of the scattering or diffraction effect of an ultrasonic field on a beam of light were made by Debye and Sears (10) and by Lucas and Biquard (11). The use of a point source of light to show the diffraction of light by many sound waves simultaneously was developed by Schaefer and Bergmann (12) (13) (14). The use of a slit source of light to show the diffraction by a single wave was developed by Hiedemann and Hoesch (15) (16) (17). This is the method used in this investigation. The slit is made parallel to the sound wave front and the light beam is normal to the sound beam. A line diffraction pattern is then formed. This investigation also makes use of the polarization techniques of Hiedemann and Hoesch (18) for the separation of the effects of longitudinal and transverse sound waves.

Raman and Nath (19) have given a theory of the diffraction of light by a sound field. The theory is based on the phase changes in the light wave as it passes through the periodic structure of variations of index of refraction caused by the sound field. The effect on the light is the production of a "corrugated wave front" which leads to a diffraction pattern. The theory gives the directions, intensities, and frequencies of the light in the diffraction

orders. The theory has been verified on the basis of experiments by Bär (20) and Sanders (21).

Determination of the ratio p/q of the strain-optical constants. Mueller (23) has extended the Raman and Nath theory. He evaluates the index variations in terms of the strain-optical constants of the medium. This leads to expressions for the directions, polarizations, frequencies, and intensities of the elements of the diffraction pattern in terms of the strain-optical constants and the sound amplitude. Many of the conclusions have been experimentally verified by Hiedemann (24) (25).

Mueller (23) gives three methods for the experimental evaluation of the ratio p/q .

Method "B" involves measurements of the polarizations of the diffraction orders in the Hiedemann spectrum. Forms of this method have been used on glasses by Vedam (26) (27) and by Schaefer and Dransfeld (28).

Method "C" involves measurements of the intensity ratios in the diffraction orders. A partially similar method has been used on glasses by Bergmann and Fues (29).

Method "A" applies to certain crystals (but not also to glasses, as do "B" and "C"), yielding ratios of strain-optical constants.

Measurements have been made on crystals by Burstein, Smith, and Henvis (30) and by Galt (31). Galt reports agreement with the theory of photoelasticity in cubic crystals

given by Mueller (32), while Burstein and Smith (33) indicate the need of some modifications.

These dynamic determinations offer some characteristic advantages. The results are obtained directly in terms of the strain-optical constants, which are, according to Mueller (23), of greater theoretical importance than the stress-optical constants. Small samples are sufficient. There are no effects due to relaxations with time constants which are long with respect to the period of the sound.

Purposes of this Investigation

This investigation had two purposes. The first was to determine dynamic values of the ratio p/q for a series of American optical glasses and fused silica. The second was to compare methods "B" and "C" as a check on the values and on the theory.

These results are made more interesting by other work completed or projected in this laboratory. While this investigation was in progress Barnes (34) determined dynamic values of the elastic constants of the same set of samples. A projected problem involves static measurements leading to values of p and q for the same samples.

Theory of Photoelasticity in Glass

A theory of photoelasticity in amorphous solids has been given by Mueller (35). The photoelastic behavior is ascribed to two effects. The first involves elastic

alterations in the Lorentz-Lorenz interactions between dipoles. The second involves the production of artificial optical anisotropy of the atoms. Under the action of pressure the first produces positive birefringence while the second produces negative birefringence. The second is usually the larger, but when the index of refraction reaches large values the first may surpass it.

An effect due to the alignment (by deformation) of optically anisotropic molecules is discussed by Treloar (36) and Braybon (37), but is probably of significance only for long high-polymers.

THEORY

Basis of the Mueller Theory

The basic ideas in Mueller's extension of the Raman and Nath theory are outlined in the Introduction. Mueller's methods "B" and "C" for the determination of the ratio p/q are identified.

Special Conditions for this Investigation

Homogeneous and isotropic material. It is assumed that fine-annealed optical glass is homogeneous and isotropic. Each of the samples was good enough to permit almost complete extinction when placed between crossed polarizers.

Longitudinal sound waves. The observed effects were entirely due to longitudinal sound waves. The transducer used was an x-cut quartz crystal (a thickness vibrator) so that the primary wave in the glass was a longitudinal wave. Frequencies were chosen at which the blocks exhibited strong resonances for the longitudinal modes and weak resonances for transverse modes. In method "B" the polarizer and analyzer are set at forty-five degrees to the sound wave front. Hiedemann and Hoesch (18) have shown that this arrangement eliminates the effect of transverse waves. In method "C" the absence of significant transverse waves can be checked visually since their velocity differs from that of longitudinal waves and the corresponding diffraction patterns

have different spacing.

Standing sound waves. In a one inch cube of a low-absorption material like glass it would be very difficult to produce anything other than standing waves. The appearance of intense diffraction patterns due to resonances as the frequency was changed gave evidence of the standing wave condition. The theory of Raman and Nath notes that each line in the diffraction pattern due to standing waves is composed of sub-components having different frequencies and polarizations. The diffraction orders are identified by the index m . The value zero corresponds to the central order while the index one corresponds to the first order and so on. The sub-components are identified by the index r , which may be zero or have positive integral values.

Diffraction order and sub-component. Only the zero sub-component ($r=0$) of the first diffraction order ($m=1$) was considered in any of the measurements. Justification for this will appear below.

Applicability of Raman and Nath Theory. The sound frequencies used were in the neighborhood of ten megacycles per second. The optical path length in the sound field was about two centimeters. These values place this investigation in the region in which the Raman and Nath theory applies. Experimental verifications of the theory (in liquids) have been mentioned above. The wave lengths correspond to those involved in this work. A theoretical condition for

applicability has been given by Rytov (38). It is

$$\frac{L \lambda}{(\lambda^*)^2} < \frac{1}{2} \quad (3)$$

where L is the optical path length in the sound field, λ is the optical wave length, λ^* is the acoustic wave length. Willard (39) gives for a criterion

$$\frac{L \lambda v}{(\lambda^*)^2} < \frac{1}{2} \quad (4)$$

including the Raman and Nath argument v , which, in this work, is seldom greater than unity. Both of these criteria place these measurements in the province of the theory.

Conclusions of the Mueller Theory for Standing Longitudinal Sound Waves

Arrangement for method "B". The arrangement for method "B" requires the light beam to be perpendicular to the sound propagation direction. The slit is parallel to the sound wave fronts. The polarizer is set at forty-five degrees to the slit. This optical arrangement is diagramed in Figure 6.

In this case the polarization of the sub-component r of the diffraction order m of the Hiedemann spectrum is given by

$$\tan (\gamma_{m,r} + 45^\circ) = \frac{J_{s-r}(\frac{Rv}{2}) J_{s+r+1}(\frac{Rv}{2})}{J_{s-r}(\frac{v}{2}) J_{s+r+1}(\frac{v}{2})} \quad \text{if } m = 2s + 1$$

$$\frac{J_{s-r}(\frac{Rv}{2}) J_{s+r}(\frac{Rv}{2})}{J_{s-r}(\frac{v}{2}) J_{s+r}(\frac{v}{2})} \quad \text{if } m = 2s \quad (5)$$

where s is zero or any positive integer. $\gamma_{m,\nu}$ is the angle of rotation of the analyzer from the crossed position which causes extinction of the sub-component ν of the diffraction order m . J is the Bessel function of the first kind and of order indicated by the subscript. R is the ratio p/q of the strain-optical constants. v is the argument of the Bessel functions in the Raman and Nath theory, given by

$$v = \frac{2\pi\mu L}{\lambda} = \frac{4\pi^2 L n^2 q A}{\lambda \lambda^*} \quad (6)$$

where μ is the amplitude of variation of the index of refraction associated with the strain-optical constant q , L is the optical path length in the sound field, λ is the optical wave length in air, λ^* is the acoustical wave length, n is the index of refraction of the medium, and A is the acoustical amplitude. It is to be noted that the quantity v is proportional to the amplitude A of the sound wave.

In the case (used in this investigation) where $r=0$ and $m=1$, equation 5 becomes

$$\tan \theta = \tan(\gamma_{1,0} + 45^\circ) = \frac{J_0\left(\frac{Rv}{2}\right) J_1\left(\frac{Rv}{2}\right)}{J_0\left(\frac{v}{2}\right) J_1\left(\frac{v}{2}\right)} \quad (7)$$

where $\theta = (\gamma_{1,0} + 45^\circ)$.

θ is the experimental quantity measured in method "B".

Arrangement for method "C". The arrangement for method "C" requires the light beam to be perpendicular to the direction of propagation of the sound. The slit is parallel to

the sound wave fronts. This optical arrangement is diagrammed in Figure 7. Natural light passes through a Wollaston double image prism which splits it into two beams of equal intensity, the polarizations of which are normal to each other and parallel and normal respectively to the slit. The two beams diverge slightly (forty-two minutes in this work) and give rise to two separate diffraction patterns. The ratio $B_{r,m}$ of the intensities of corresponding lines in the two patterns is given by

$$\sqrt{B_{r,m}} = \frac{J_{s-r}(\frac{Rv}{2}) J_{s+r+1}(\frac{Rv}{2})}{J_{s-r}(\frac{v}{2}) J_{s+r+1}(\frac{v}{2})} \quad \text{if } m = 2s+1$$

$$\frac{J_{s-r}(\frac{Rv}{2}) J_{s+r}(\frac{Rv}{2})}{J_{s-r}(\frac{v}{2}) J_{s+r}(\frac{v}{2})} \quad \text{if } m = 2s \quad (8)$$

where the symbols have the same meaning as in equation 5.

In the case (used in this investigation) where $r=0$ and $m=1$, equation 8 becomes

$$\sqrt{B} = \sqrt{B_{0,1}} = \frac{J_0(\frac{Rv}{2}) J_1(\frac{Rv}{2})}{J_0(\frac{v}{2}) J_1(\frac{v}{2})} \quad (9)$$

B is the experimental quantity measured in method "C".

Analytical similarity of methods "B" and "C". A comparison of equations 7 and 9 indicates that

$$\tan \theta = \sqrt{B} = \frac{J_0(\frac{Rv}{2}) J_1(\frac{Rv}{2})}{J_0(\frac{v}{2}) J_1(\frac{v}{2})} \quad (10)$$

where θ is the extinction angle (from crossed position) plus forty-five degrees for the sub-component $r=0$ of the first diffraction order of method "B" and B is the intensity ratio

of the $r=0$ sub-components of the first diffraction orders of method "C". Thus two experimental quantities which may be independently measured are given by the same analytical expression.

B and θ as Functions of v
and of Sound Amplitude

B and θ as functions of v . Equation 10 cannot be solved to give values of R since v contains quantities which cannot be accurately measured. Neither sound amplitude nor sound field width, for example, can be precisely known.

However, as v approaches zero the right hand member of equation 10 approaches the value R . Thus, letting subscript zeros indicate limiting values, equation 10 becomes

$$\tan \theta_0 = \sqrt{B_0} = R \quad (11)$$

Figure 1, drawn from equation 10 shows the mode of approach of these quantities to their limit as v approaches zero. $\tan \theta$ and $\sqrt{B}$ are plotted against the square of v merely in order to give a relation which is very nearly linear. The straight lines are drawn to show the departure of the points from the linear relation as v increases. For example, when v is less than unity and R is less than two, a straight line is a good approximation. Thus the "best" straight line (criterion of least squares) (40) through the four points shown in Figure 1 for R equal to two, intersects the vertical axis at 1.989, giving an error of 0.55 percent. None of the

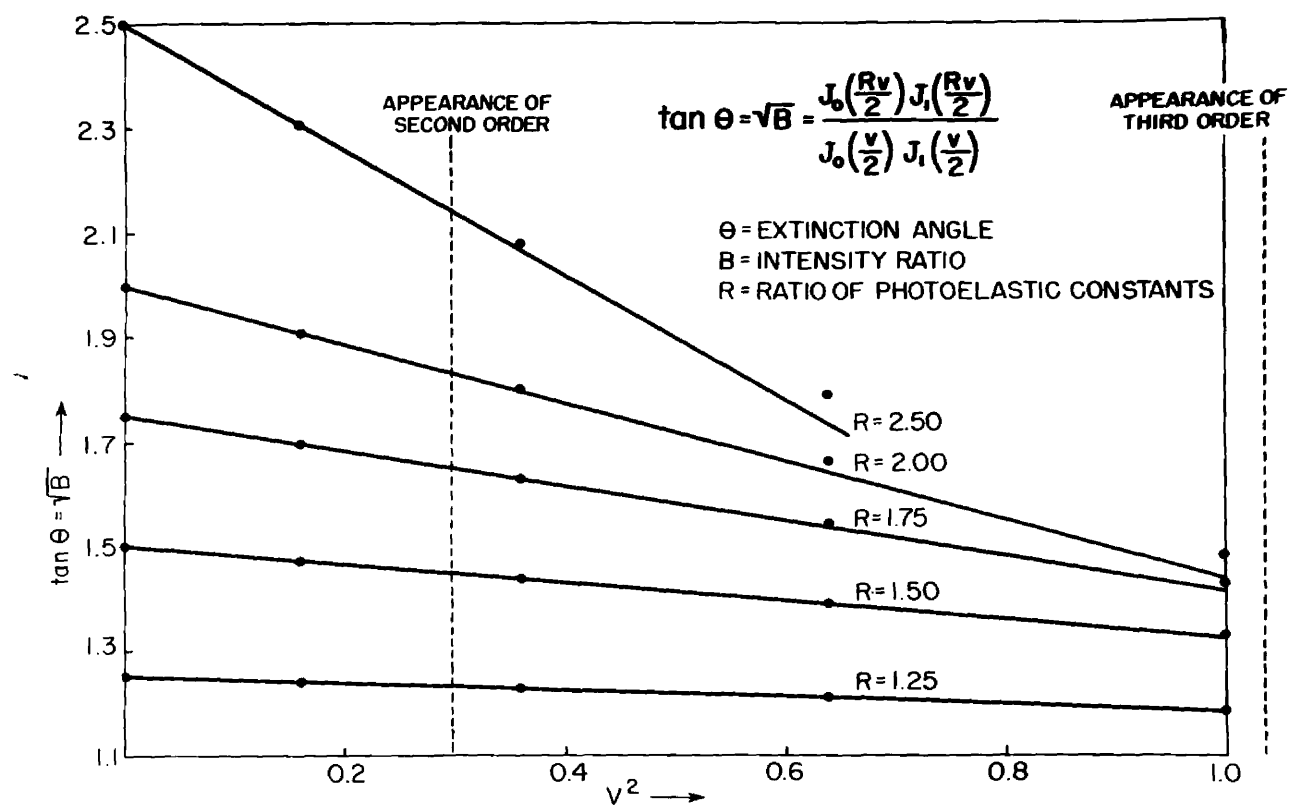


Fig. 1. Theoretical variation of $\tan \theta$ and $\sqrt{B}$ with v^2

glasses examined exhibited values of R greater than 1.83.

B and θ as functions of sound amplitude. Since v is proportional to sound amplitude, a similar linear relationship exists between $\tan \theta$ or $\sqrt{B}$ and the square of the sound amplitude, provided that the latter corresponds, in general, to values of v less than unity. In this case experimental points on a plot of $\tan \theta$ or $\sqrt{B}$ against the square of the sound amplitude can be extrapolated linearly to give a value for $\tan \theta_0$ or $\sqrt{B_0}$. Such an extrapolation was used in this investigation.

It is necessary to relate the experimental sound amplitudes to values of v in order to justify the linear extrapolation.

An approximate value of v corresponding to any given sound amplitude can be obtained from a comparison of the light intensities in the diffraction orders due to that sound. It is thus possible to consider only values of sound amplitude which correspond to acceptable values of v . The relationship between values of v and intensities in the diffraction orders is given, in the case of liquids, by the theory of Raman and Nath (19). Good agreement between experiment and theory has been obtained by Sanders (21). For solids the theory has been extended by Mueller (23) to include the effect of the photoelastic constants. Since the optics differ, the relationship is different for the arrangements for methods "B" and "C". Appendix I gives

Mueller's theoretical conclusions and lists numerical values in the range of Figure 1 and of this investigation.

For the special case of method "B" and a glass having R equal to 1.5, values of v corresponding to the appearance of second and third diffraction orders are indicated in Mueller's Figure 3 (23) (due to Hiedemann). These approximate values are indicated by the dotted vertical lines of Figure 1.

In general, the values of v at which the diffraction orders appear, decrease as R increases. The following overall conclusions may be drawn. For method "B", when the third order diffraction line does not appear, the sound amplitude corresponds to a value of v which lies on the essentially straight portion of the appropriate curve of Figure 1. Linear extrapolation is then in order. For method "C", when the second diffraction order does not appear, the sound amplitude corresponds to a value of v on the linear portion of the corresponding curve and extrapolation is justified.

An arbitrary criterion for the "appearance" of a diffraction order is an intensity in that order of approximately one percent of that in the central order. One criterion suggested by Sanders (21) is 0.86 percent.

Justification for Consideration of Only $r = 0$ Sub-components of First Diffraction Orders

The numerical values given in Appendix I also indicate the conditions under which it is permissible to consider only

the $r=0$ sub-components of the first diffraction orders. In method "B", when the third diffraction order does not appear, the ratio of the intensities in the $r=0$ and first sub-components of the first diffraction order is not more than a few percent. Thus the extinction of the $r=0$ sub-component causes a distinct minimum in the intensity of the line. In method "C", when the second order does not appear, the ratio of the intensities in the $r=0$ and $r=1$ sub-components of the first diffraction order is not more than about 0.1 percent. Thus an intensity measurement of the line is equivalent to an intensity measurement of the $r=0$ sub-component.

In this work, third order lines never appeared during measurements by method "B" and second order lines never appeared during measurements by method "C".

Indication of Sound Amplitude

Since the curve of $\tan \theta$ or $\sqrt{B}$ plotted against the square of sound amplitude is extrapolated to zero sound, it is, fortunately, not necessary to determine absolute values of sound amplitude. Values of some quantity which is proportional to sound amplitude are sufficient. Such a quantity is quartz (transducer) current or the current at any point in the series transducer circuit, as long as the geometry and frequency remain constant. A relationship between acoustic intensity J and piezoelectric transducer current I is given

by Cady (41).

$$J = I^2 R_s / 2 \quad (12)$$

R_s is the (constant) series resistance of the transducer equivalent network and involves frequency, piezoelectric constants, dimensions, acoustic impedances, and wave velocity. Thus for a given transducer, coupled to a constant load and driven at a constant frequency, sound amplitude is proportional to transducer current. The independent variable in this investigation has been transducer current. The linear extrapolations to $\sqrt{B_0}$ and $\tan \Theta_0$ were made on that basis.

APPARATUS AND METHODOLOGY

Ultrasonic Generator

The ultrasonic generator designed and constructed as a part of this investigation consists of six main parts, connected to operate as a unit and mounted in a relay rack.

Oscillator and power amplifier. Figure 2 shows the circuits of the oscillator and power amplifier which are constructed on the same chassis. Table I identifies the components.

The oscillator is an electron-coupled Hartley circuit, modified to give maximum stability. Its frequency range is continuous from about 5.2 megacycles per second to about 13.6 megacycles per second.

The radio-frequency power amplifier can deliver from one hundred and fifty to two hundred electrical watts. Plug-in output coils cover the ranges from 5.7 to 12 megacycles per second and from 12 to 24 megacycles per second, with the output stage doubling in the upper range.

When the circuit is well warmed up it is able to maintain a frequency in the neighborhood of ten megacycles per second constant within one kilocycle per second for several hours.

Stability was checked by means of a General Radio Type 620-A Heterodyne Frequency Meter while the approximate

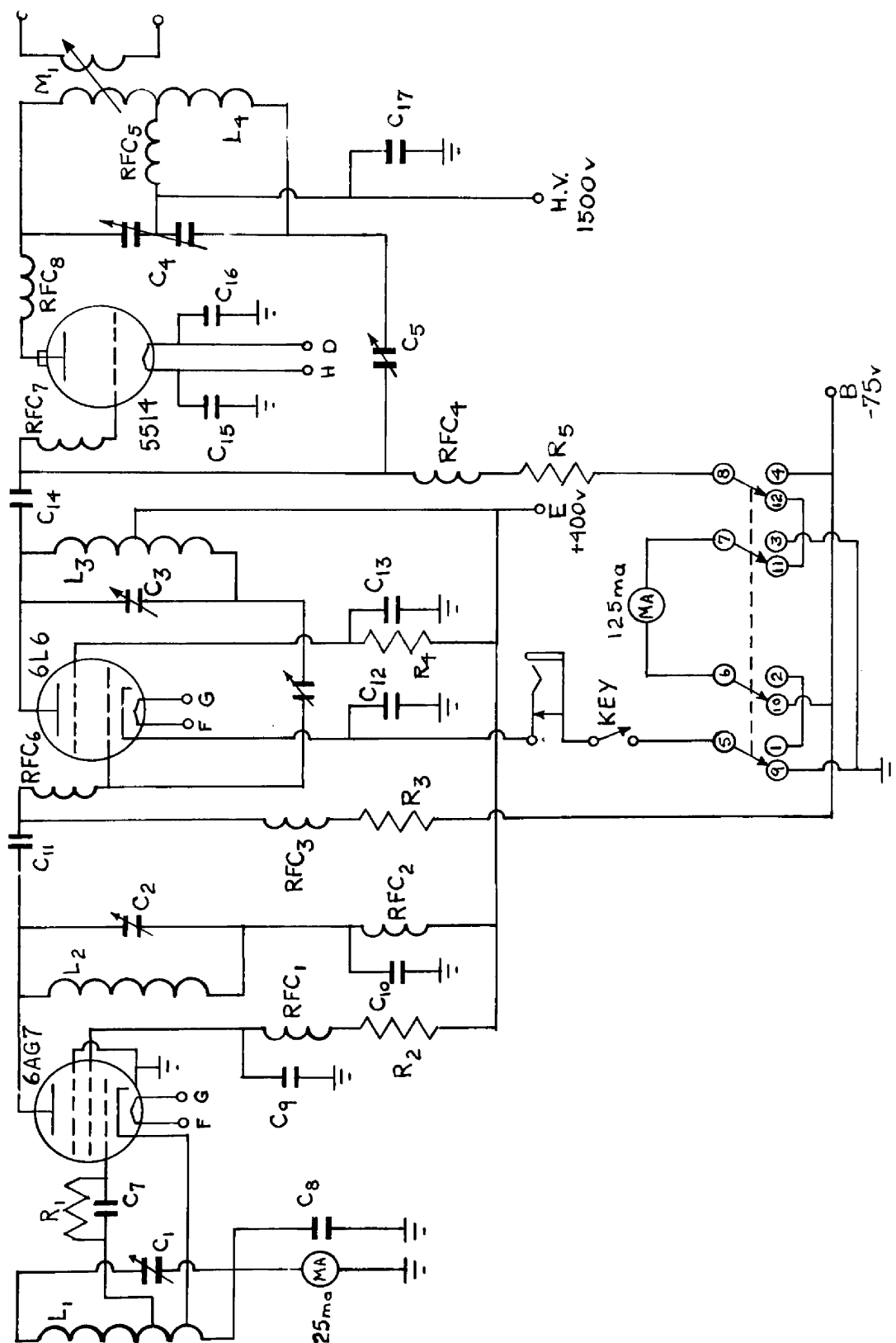


Fig. 2. Oscillator and power amplifier

TABLE I
ELECTRICAL COMPONENTS

Components for Oscillator and Amplifier (Figure 2)			
$L_{1,2,3}$	3.5 microhenrys	C_{14}	130 mmfd mica 600 v
L_4 a)	18 microhenrys	C_{17}	0.001 mfd mica 3000 v
b)	2.6 microhenrys	R_1	50,000 ohm zero temp
M_1	swinging link	R_2	68,000 ohm 1 w
$C_{1,2,3}$	13-300 mmfd	R_3	10,000 ohm 1 w
C_4	11-150 mmfd dual	R_4	25,000 ohm 1 w
C_5	10 mmfd variable	R_5	500 ohm 5 w
C_6	0.5 mmfd variable	$RFC_{1,2}$	2.5 mh 100 ma
C_7	75 mmfd zero temp	$RFC_{3,4}$	2.5 mh 100 ma
C_8	0.01 mfd	RFC_5	1 mh 200 ma
$C_{9,10,13}$	0.01 mfd 500 v	$RFC_{6,7}$	14 turns $\frac{1}{4}$ "
$C_{15,16}$	0.01 mfd 500 v	RFC_8	8 turns $\frac{1}{2}$ "
C_{11}	50 mmfd mica 600 v		
Components for Voltage-regulated Power Supplies (Figure 3)			
T_1	5 v 3 a	R_3	15,000 ohm 5 w
T_2	550 v CT 250 ma	R_4	90 ohm
T_3	600 v CT 120 ma; 5 v 3 a; 6.3 v 5 a	$R_{5,6}$	1000 ohm 1w
T_4	7.5 v 3 a	R_7	470,000 ohm 2w

TABLE I CONTINUED

Components for Voltage-regulated Power Supplies (Figure 3)

T ₅	6.3 v 3 a	R _{8,9}	100 ohm $\frac{1}{2}$ w
T ₆	6.3 v 1 a	R ₁₀	12,000 ohm 25 w
L ₁	24 h 200 ma	R _{11,12}	1 meg 1 w
L ₂	25 h 75 ma	R ₁₃	5,000 ohm 10 w
C _{1,2}	4 mfd 1000 v oil	R ₁₄	250,000 ohm 1 w
C _{3,4}	16 mfd 600 v	R ₁₅	100,000 ohm $\frac{1}{2}$ w
C _{5,6}	0.1 mfd 400 v	R ₁₆	390,000 ohm 2 w
C ₇	0.2 mfd 600 v	R ₁₇	50,000 ohm 2 w
C ₈	4 mfd 600 v oil	R ₁₈	120,000 ohm 1 w
R ₁	72 ohm 5 w	R ₁₉	500,000 ohm 1 w
R ₂	5,000 ohm 50 w	S ₁	VR tube firing relay

Components for High Voltage Power Supply (Figure 4)

T ₁	2.5 v 10 a 10,000 v insulation	L ₁	5/25 h 225 ma swinging choke
T ₂	1500 v (dc) 200 ma CT	L _{2,3}	8 h 225 ma filter choke
T ₃	0.4 kva autotrans- former	C _{1,3}	4 mfd 2,000 v oil
R ₁	50,000 ohm 50 w	C ₂	2 mfd 2,000 v oil
S ₁	thermal delay relay		

TABLE I CONTINUED

Components for Matching Circuit (Figure 5)			
L_1	22 turns	2" diameter	2" length
C_1	20-200 mmfd		

frequency given for each run depends upon measurements made with a General Radio Type 566-A Wavemeter.

Voltage-Regulated Power Supplies. Figure 3 shows the circuits of the voltage-regulated power supplies which are constructed on the same chassis. Table I identifies the components.

The minus seventy-five volt bias supply is regulated by two OA3 gas discharge tubes operating in parallel. The load through these tubes must be carefully balanced in order to avoid excessive current in either. A delay relay applies the firing voltage to this pair simultaneously. As the firing process is photosensitive, it is sometimes necessary to illuminate one or both tubes in order to fire both.

The four hundred volt plate supply is adapted from one described by Elmore and Sands (42). It uses the series reactance tube principle and maintains a regulation of about 0.1 percent with a ripple of about five millivolts.

High voltage power supply. Figure 4 shows the circuit of the high voltage power supply. Table I identifies the components. The filtering is good and the circuit provides voltages from zero to fifteen hundred volts at currents from zero to two hundred milliamperes.

Impedance matching circuit. Figure 5 shows the circuit of the impedance matching unit which is connected to the transducer. Table I identifies the components. This circuit would probably require modifications for operation above fifteen

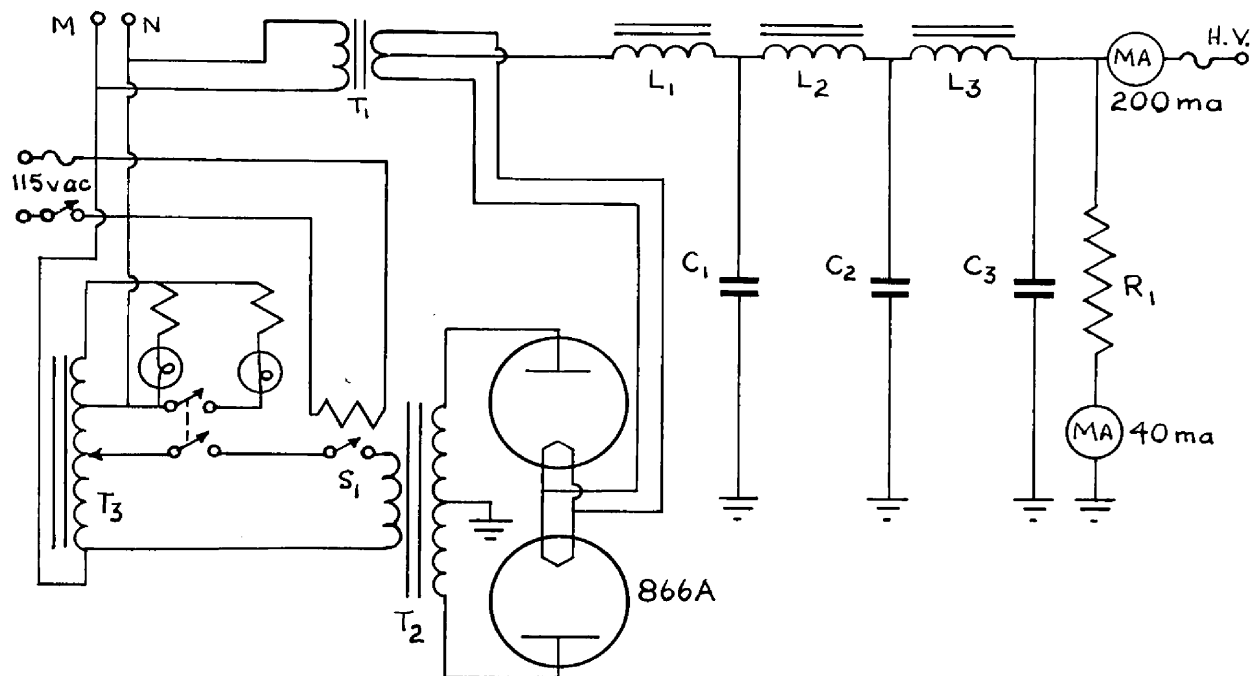


Fig. 4. High voltage power supply

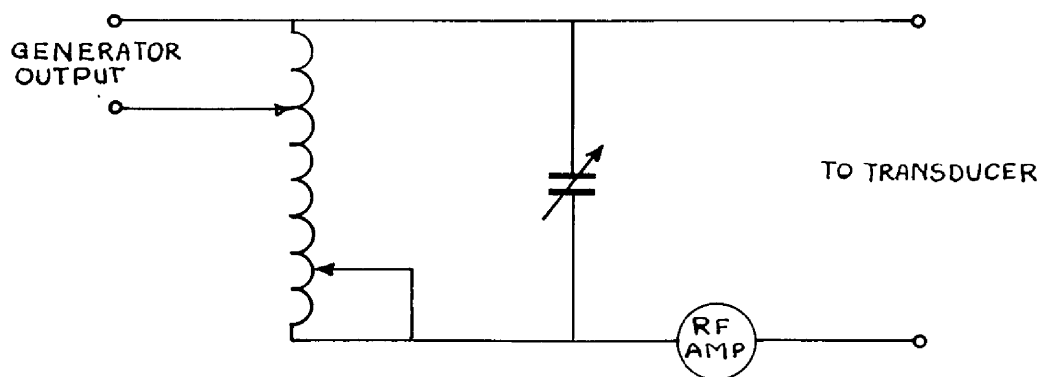


Fig. 5. Transducer matching circuit

megacycles per second. The radio-frequency ammeter is the one used to indicate sound amplitudes by means of their proportionality to transducer current.

Transducer

The transducer used for all measurements was an x-cut quartz crystal (thickness vibrator), driven near its fundamental frequency of ten megacycles per second. The crystal was one inch square. Aluminum foil electrodes, about three-quarters of an inch square provided the electrical connections. Silicone grease provided acoustic coupling between the crystal and the glass. The sandwich was held in place against the glass block by means of light spring pressure.

Optical Arrangements and Measurements

Method "B". Figure 6 shows the optical system for method "B". The light source is a General Electric Type A-H4 high intensity mercury lamp. Condenser lens L_1 illuminates a slit of aperture about 0.05 millimeters. The Gaertner Type L 541 E monochromatic filter combination isolates the Hg5461 line. Lens L_2 collimates the light. The polarizing prism is set at forty-five degrees to the slit. The light beam passes through the glass block normal to the direction of propagation of the sound waves. The sound wave fronts are parallel to the slit. The analyzer is initially set in the "crossed" position for extinction. When the sound field is present, a diffraction pattern is formed in the focal

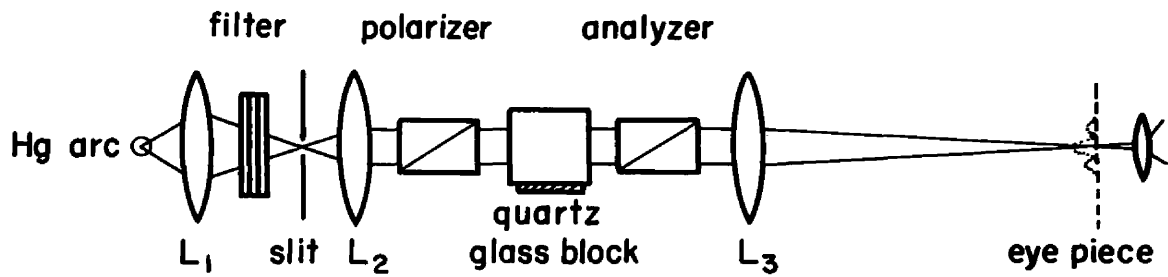


Fig. 6. Optical system for method "B"

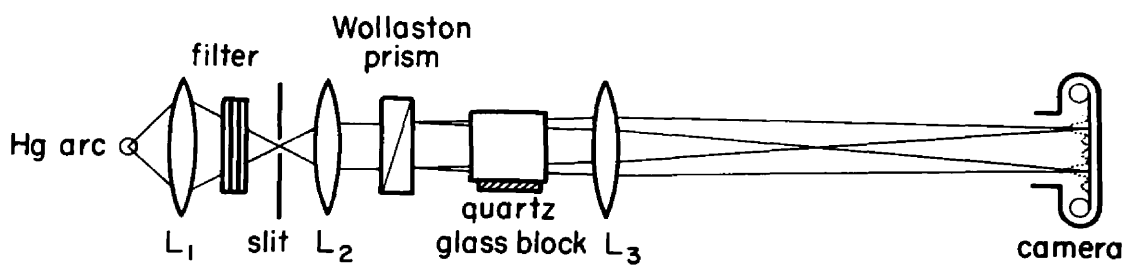


Fig. 7. Optical system for method "C"

plane of lens L_3 (focal length, twenty inches). The diffraction pattern is viewed by means of an eyepiece. The analyzer is then rotated through an angle γ (from the crossed position) to produce extinction (or a minimum) in the first diffraction order. Ten minimum settings were made and averaged to determine the angle.

This is the condition of extinction of the $r=0$ sub-component of the first diffraction order. $\tan \theta$ can then be computed as in equation 7. This measurement was repeated for at least four different sound amplitudes corresponding to four values of transducer current. The maximum amplitude was made at least twice the minimum, but not great enough to produce third order diffraction lines.

Typical plots of $\tan \theta$ versus the square of the transducer current I are shown in Figure 8.

A linear extrapolation can then be made, as shown, to give values of $\tan \theta_0$, which are values of R as indicated by equation 11. It is more convenient and accurate, however to perform the extrapolation analytically (40), using the least squares criterion of best fit. Appendix II gives a sample calculation for the borosilicate crown glass of Figure 8. The data of this investigation was obtained by analytical extrapolation.

Method "C". Figure 7 shows the optical system for method "C". The production of the collimated beam is the same as for method "B". In this case a Wollaston double

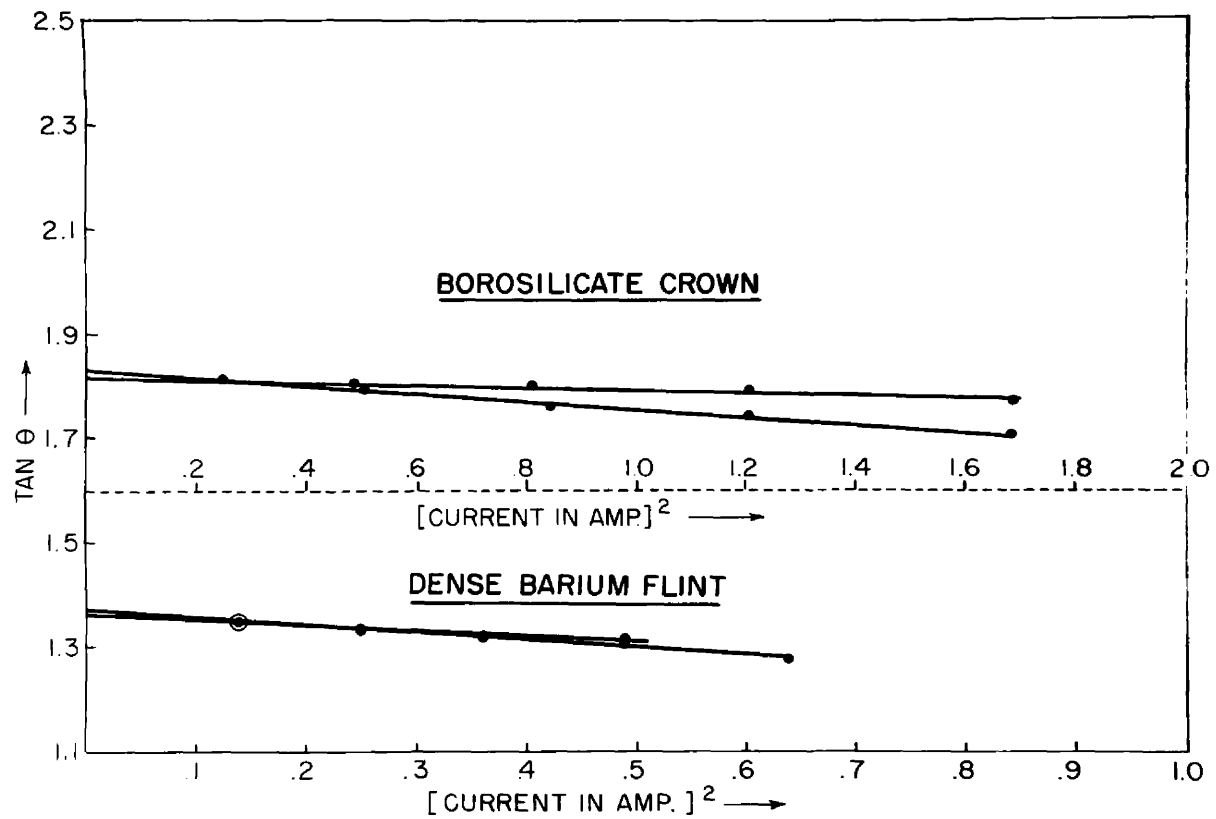


Fig. 8. Experimental variation of $\tan \theta$ with I^2

image prism is inserted before the glass block. Two beams are thus formed, having polarizations normal and parallel respectively to the slit and to the sound wave fronts. The beams diverge at an angle of forty-two minutes. When a sound field is present in the glass, two diffraction patterns are formed in the focal plane of lens L_3 . These patterns are recorded photographically, as are also, for a comparison of intensities, the images of the slit when no sound is present. Exposures were made for four or more sound amplitudes, corresponding to as many values of transducer current. The maximum amplitude was made at least twice the minimum, but not great enough to produce second order diffraction lines.

The ratios of the intensities in the first order lines of the two patterns were determined photographically. Before development, an emulsion calibrating exposure was made with Hg5461 light and a rotating step-wedge to give a pattern of densities corresponding to ten or twelve exposures. Each step represents an exposure of one and one half times the previous value. These densities were compared to the densities of the diffraction pattern lines by means of a Jarrel-Ash Type JA-200 Microphotometer. Assuming the reciprocity relation and no intermittency effect, the ratio of intensities B of the diffraction pattern lines was calculated from the ratio of exposures in the step-wedge pattern.

Two or three exposures were made for each sound amplitude and two or three photometer readings were made on each first order line on each exposure. A correction was applied

for the (small) difference in intensities of the no-sound beams due to any slight misalignment of the Wollaston prism.

Plots of $\sqrt{B}$ versus the square of the transducer current I are shown in Figures 9, 10, 11, and 12. The data was obtained by this method. No correspondence with the theoretical curves of Figure 1 is evident. No value of $\sqrt{B_0}$ or of R is indicated.

Appendix III gives a sample calculation by this method for the crown flint glass of Figure 12.

Temperature

Importance of temperature. Since a change of temperature varies both the dimensions of the glass block and the velocity of sound, it may disturb the standing wave resonance condition and so alter the proportionality between transducer current and sound amplitude. Temperature gradients in the block might produce inhomogeneities or anisotropies which would disturb the optical measurements. Harris (43) and Filon (44) report small changes in photoelastic constants with changes in temperature.

Room temperature. Since the glass block was cooled by a current of air, the temperature of this air ("room temperature") was the nominal temperature of the block. Room temperature was measured by means of an ordinary mercury thermometer placed in the neighborhood of the block. In general, a run was not satisfactory when the room temperature varied much more than one half of a degree centigrade.

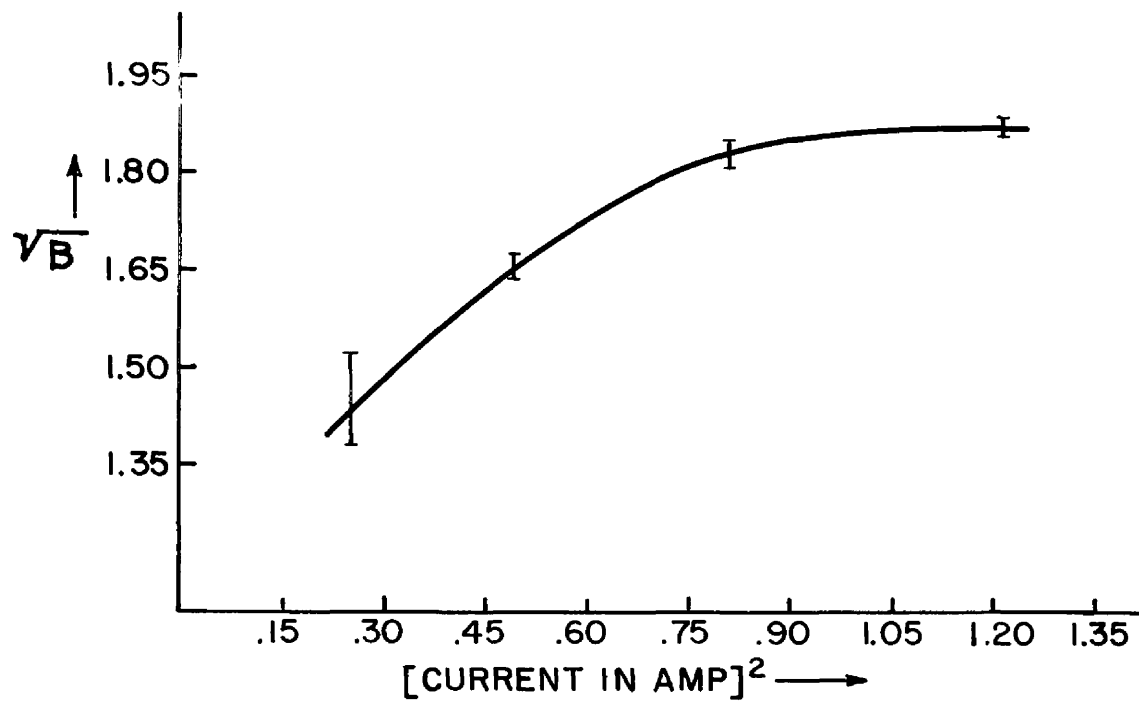


Fig. 9. Experimental variation of $\sqrt{B}$ with I^2 in BSC-1.

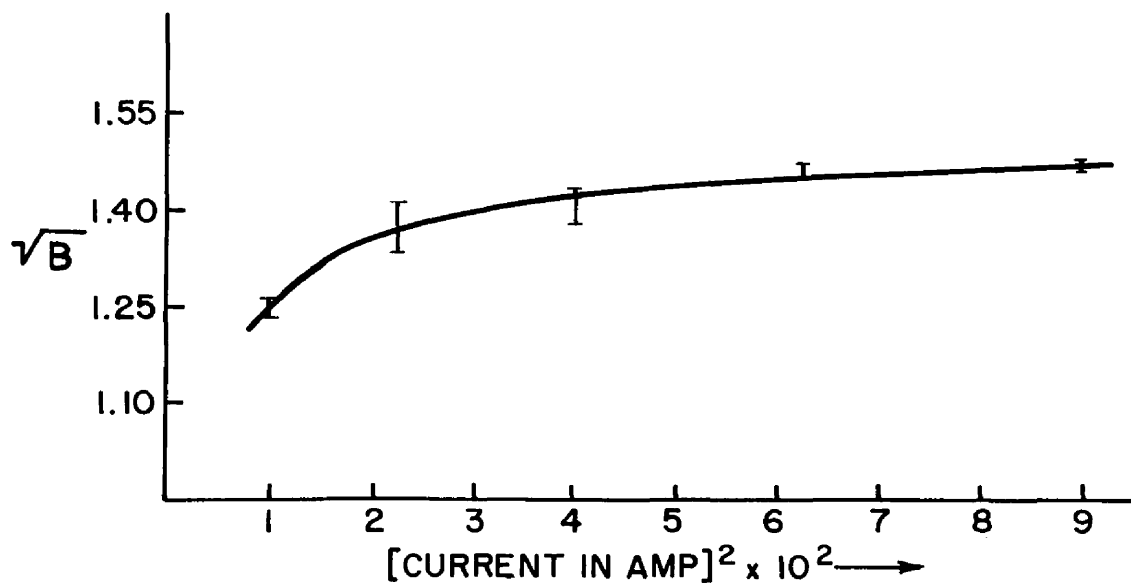


Fig. 10. Experimental variation of $\sqrt{B}$ with I^2 in DBF-1.

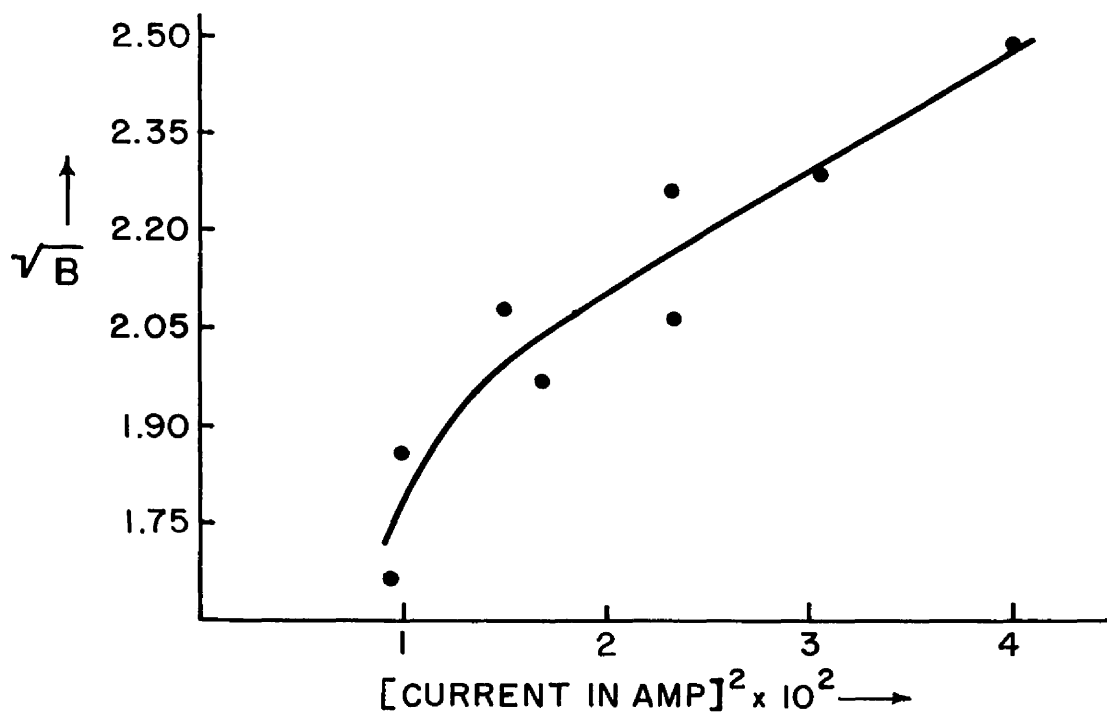


Fig. 11. Experimental variation of $\sqrt{B}$ with I^2 in fused silica.

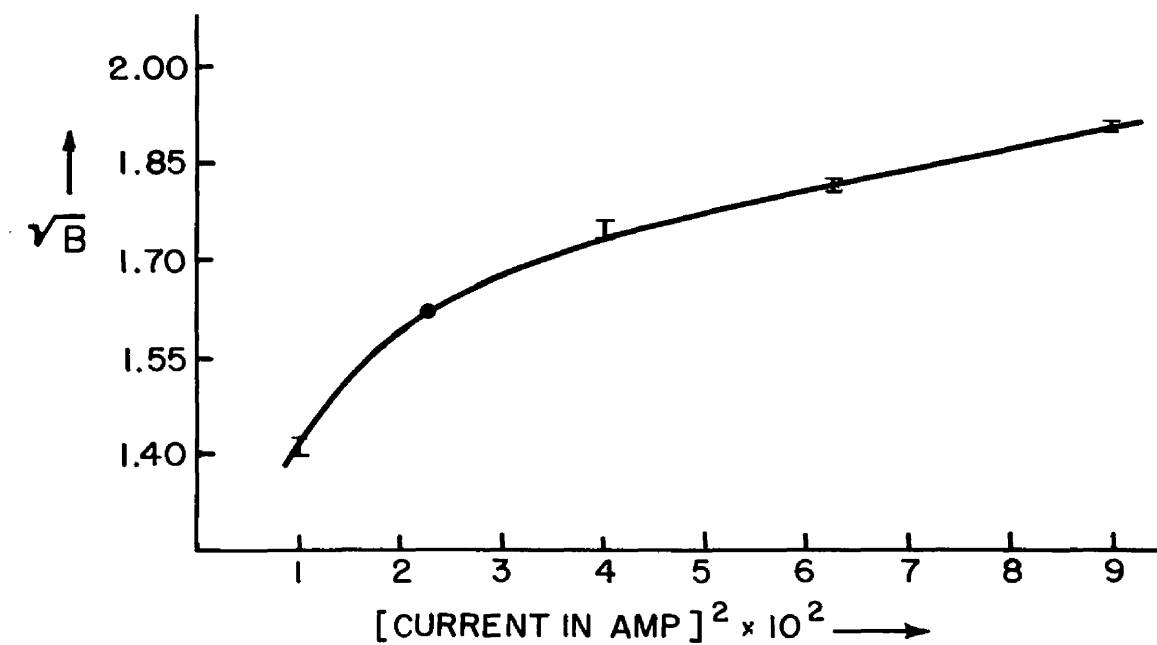


Fig. 12. Experimental variation of $\sqrt{B}$ with I^2 in CF-1.

Block temperature. The temperature difference between the surface of the block and the surrounding air was measured by means of a thermocouple in contact with the surface and insulated from the air. This measurement gives some indication of the internal temperature of the block. Data was taken only when this temperature difference was less than one quarter of a degree centigrade.

Glass and Fused Silica Samples

Measurements were carried out on thirteen samples of fine-annealed optical glass and one sample of fused silica. The ordinary glasses were supplied by the Bausch and Lomb Optical Company, the rare earth glasses by the Eastman Kodak Company, and the fused silica block was fabricated in Germany and secured by the Owens-Illinois Glass Company through the Hanovia Chemical Company. The Bausch and Lomb samples and the fused silica block were one inch cubes. The Eastman samples were three quarter inch cubes or near cubes.

Table II gives descriptions of the samples. B&L represents Bausch and Lomb, E represents Eastman, H represents Hanovia. The densities are those determined in this laboratory by Barnes (34) with the exception of the value for block number nine which is the manufacturer's nominal density. n_D is the nominal index of refraction for Na5893 light. V is the nominal dispersion, being given by the formula $V = (n_D - 1)/(n_F - n_C)$ in which n_F refers to H4861

TABLE II
DESCRIPTION OF GLASS SAMPLES

Block No.	Mfgr.	Type	Density gm/cm ³	n _D	V	Young's Modulus <u>dynes</u> <u>cm²</u> <u>x10¹¹</u>	Bulk Modulus <u>dynes</u> <u>cm²</u> <u>x10¹¹</u>
1	B&L	BSC-1	2.48	1.51100	63.5	7.64	4.39
2	B&L	C-1	2.53	1.52300	58.6	7.57	4.30
3	B&L	CF-1	2.69	1.52860	51.6	6.20	3.54
4	B&L	LF-1	3.18	1.57250	42.5	5.91	3.50
5	B&L	LBC-2	3.14	1.57250	57.4	7.55	4.85
6	B&L	DBF-1	3.60	1.61700	38.5	5.98	3.64
7	B&L	DF-2	3.64	1.61700	36.6	7.81	5.58
8	B&L	DBC-2	3.78	1.61700	54.9	5.64	3.41
9	B&L	EDF-1	3.91	1.64900	33.8	7.98	4.40
10	B&L	EDF-4	4.72	1.75060	27.7	5.45	3.57
11	E	EK-110	4.13	1.69680	56.15	10.24	8.04
12	E	EK-330	4.57	1.75510	47.19	10.88	8.58
13	E	EK-450	4.63	1.80370	41.80	11.80	9.52
14	H	Fused silica	2.20			7.30	3.67

and n_D to H6563. The elastic constants are preliminary values of dynamic determinations made in this laboratory by Barnes (34) using ultrasonic waves of the same order of frequency as those used in this work.

Table III gives the compositions of the samples as supplied by the manufacturers.

The letters in the Bausch and Lomb designations for the glass types have the following significance: BSC, borosilicate crown; C, crown; CF, crown flint; LF, light flint, LBC, light barium crown; DF, dense flint; DEF, dense barium flint; DBC, dense barium crown; EDF, extra dense flint.

TABLE III
COMPOSITIONS OF GLASS SAMPLES IN PERCENT BY WEIGHT

Block No. Type Melt No.	1 BSC-1 0-9560	2 C-1 0-7763	3 CF-1 0-9841	4 LF-1 0-9447	5 LBC-2 0-8762
SiO ₂	70.8	71.7	67.8	54.3	49.1
K ₂ O	12.1	2.0	11.2	8.0	7.8
Na ₂ O	7.5	13.7	2.0	3.0	0.5
PbO			9.0	32.5	
BaO				2.1	31.0
SrO					
CaO	1.3	9.0			
B ₂ O ₃	8.0	1.1			3.4
ZnO		1.5	4.0		7.5
ZrO ₂					
TiO ₂					
Al ₂ O ₃					
Sb ₂ O ₃		1.0	6.0		0.5
As ₂ O ₃	0.3			0.1	0.2
La ₂ O ₃					
ThO ₂					
Ta ₂ O ₅					
WO ₃					
Al+Si+Zr+Ti (oxides)					

TABLE III CONTINUED

Block No. Type Melt No.	6 DBF-1 0-9271	7 DF-2 0-9001	8 DBC-2 0-8472	9 EDF-1 0-4432	10 EDF-4 0-8030
SiO ₂	45.0	46.6	39.0	42.1	31.5
K ₂ O	6.3	6.4		7.1	1.6
Na ₂ O	2.0				
PbO	38.2	46.3	1.6	48.8	64.5
BaO	7.6		42.7		
SrO					
CaO					
B ₂ O ₃			4.7		
ZnO	0.8	0.4	5.4		
ZrO ₂			0.5	0.2	0.1
TiO ₂				1.5	
Al ₂ O ₃			5.1		
Sb ₂ O ₃		0.3	0.8		2.0
As ₂ O ₃	0.1		0.2	0.3	0.3
La ₂ O ₃					
ThO ₂					
Ta ₂ O ₅					
WO ₃					
Al+Si+Zr+Ti (oxides)					

TABLE III CONTINUED

Block No. Type	11 EK-110	12 EK-330	13 EK-450	14 Fused silica
SiO ₂				100
K ₂ O				
Na ₂ O				
PbO				
BaO	14	12	} 6	
SrO	6			
CaO				
B ₂ O ₃	40	30	22	
ZnO				
ZrO ₂				
TiO ₂				
Al ₂ O ₃				
Sb ₂ O ₃				
As ₂ O ₃				
La ₂ O ₃	20	28	40	
ThO ₂	20	12	6	
Ta ₂ O ₅		18	11	
WO ₃			4	
Al+Si+Zr+Ti (oxides)			11	

PRESENTATION OF DATA

Method "B"

Only method "B" led to data which agreed with the theory and permitted extrapolation. Therefore values of R were obtained only by this method.

Accuracy of data. Figure 1 shows the departure of the theoretical points from a straight line. As is mentioned above, an analytical straight line extrapolation for the case in which R equals two and v is not greater than unity leads to an error of 0.55 percent in the value of R . Since all glasses tested exhibited values of R less than 1.83, the "theoretical" error introduced by linear extrapolation is certainly less than 0.5 percent.

The linearity of typical experimental data is shown in Figure 8.

Since the result is obtained by extrapolation to zero transducer current, absolute accuracy of current determination is unnecessary. Linearity is sufficient for the meter readings. In one case (block number eight) successive determinations were made with different ammeters. The resulting values for R differed by 0.31 percent, less than the average percent of difference between runs using the same meter. Although the analytical extrapolation method (40) assumes deviations only in the values of the ordinates, errors

in abscissa (current) values must also reflect in the "probable error" of the y-intercept. Although only four or five points were involved in each determination, a formal value of the probable error p_R in the value of R due to the deviation of the data points from the straight line was calculated (as in Appendix II) for each determination. In every determination of R for a glass sample p_R was less than 0.3 percent. This gives some indication of the effect of any non-linearity in the ammeter and of errors in the scale readings.

Each value of extinction angle used in the calculations was an average of ten settings and readings of the analyzer circle. Each value of the "crossed" position of the analyzer was an average of ten settings and readings. Angle determinations are most difficult for the faint lines corresponding to low sound amplitudes and for materials of large R which require a greater analyzer rotation and consequently permit more background illumination. Since p_R was less than 0.3 percent for all glass determinations, it appears that the contributions of errors in angle measurements to non-linearity must be smaller than that.

Two independent determinations of R were made for each glass sample. They agree, in every case, to within about one percent.

Fused silica requires some special consideration. Because of the large value of R it is especially necessary to

use small values of current (low sound amplitude, small v) in this case. Thus the extrapolation was made from only three points, not including a doubling of sound amplitude. Because of this narrow base for extrapolation the uncertainty in R is greater. Therefore four determinations were made for fused silica. Each differs from the average by less than 0.8 percent.

It seems safe to conclude that the average values for R are correct to within two percent.

Numerical results. Table IV lists the individual determinations with corresponding temperatures and approximate frequencies, together with the final average values for R , the ratio p/q of the Neumann strain-optical constants.

Method "C"

Accuracy of data. The remarks about the measurement of transducer current which were made in regard to method "B" apply also in this case.

Some idea of the dependability of the photographic method can be gained from an examination of the spread of experimentally determined values. Figures 9, 10, 11, and 12 show this experimental spread. This check is especially significant for the case of block number one (borosilicate crown) for which the experimental spread at each current includes four determinations. The spreads for block number six include two values while those for block number three include two or three values.

TABLE IV
NUMERICAL RESULTS

Block No.	Type	Freq. mc/sec	Room Temp. °C	Points for Extrapo- lation	R	p/q	p _R	Aver. R
1	BSC-1	9.8	24.0	5	1.827	.0025		1.82
1	BSC-1	9.8	25.0	4	1.816	.0011		
2	C-1	10.2	25.0	4	1.823	.0004		1.82
2	C-1	10.0	25.0	4	1.819	.0004		
3	CF-1	9.4	26.5	5	1.672	.0014		1.68
3	CF-1	10.0	26.0	5	1.689	.0010		
4	LF-1	9.9	26.5	4	1.440	.0038		1.45
4	LF-1	10.0	26.0	4	1.452	.0004		
5	LBC-2	10.1	25.5	4	1.625	.0003		1.62
5	LBC-2	9.8	25.0	4	1.618	.0009		
6	DBF-1	10.5	26.0	5	1.366	.0009		1.36
6	DBF-1	10.2	24.5	4	1.361	.0004		
7	DF-2	10.2	28.0	4	1.456	.0010		1.46
7	DF-2	9.9	27.0	4	1.459	.0003		
8	DBC-2	9.8	26.0	5	1.281	.0010		1.28
8	DBC-2	10.2	22.5	4	1.285	.0006		
9	EDF-1	10.1	25.0	4	1.459	.0010		1.46
9	EDF-1	9.4	25.0	5	1.464	.0007		

TABLE IV CONTINUED

Block No.	Type	Freq. mc/sec	Room Temp. °C	Points for Extrapo- lation	R	p/q	p _R	Aver. R
10	EDF-4	9.6	25.0	5	1.109		.0045	1.11
10	EDF-4	10.5	26.0	4	1.113		.0013	
11	EK-110	9.9	24.0	4	1.530		.0017	1.53
11	EK-110	10.2	26.5	4	1.536		.0017	
12	EK-450	10.2	27.5	4	1.548		.0016	1.55
12	EK-450	10.0	27.0	4	1.554		.0016	
13	EK-330	10.1	30.0	4	1.595		.0007	1.60
13	EK-330	9.8	27.5	4	1.596		.0007	
14	Fused silica	10.1	27.0	3	2.343			2.34
14	Fused silica	10.4	27.0	3	2.349			
14	Fused silica	10.1	26.5	3	2.318			
14	Fused silica	10.0	25.5	3	2.334			

Multiple diffraction cannot have caused trouble, for only the first order lines appeared.

Transverse waves cannot have caused trouble since the frequencies used were non-resonant for them, and, since their diffraction pattern has a different spacing than that due to the longitudinal waves, their presence would have been obvious.

Due to the action of the Wollaston prism, the beams of light are not quite normal to the sound beam but have an obliquity of twenty-one minutes. Parthasarathy (45) (46) has investigated the **asymmetrical** intensity distribution in the diffraction pattern due to oblique incidence of light on progressive waves in liquids. He reports an effect only at wave lengths shorter than those involved in this work. In addition, since the two beams from the Wollaston prism are symmetrical in incidence and since the sound field consists of standing waves, any effect should disappear when the two first order intensities in each pattern are averaged in the calculation. Further, an examination of the data indicates no significant intensity **asymmetry**.

There is thus no apparent reason to doubt the experimental values.

Numerical results. Since the experimental points do not follow the linear pattern indicated by the theory they do not lead to values of R . The values of the intensity ratios for various currents are the only numerical results of method "C". These values are shown on the curves of

Figures 9, 10, 11, and 12.

Values of R from Other Investigations

It is well known that glasses of similar composition or even "identical" samples may differ significantly in their properties. See, for example, Coker and Filon (9) and Mueller (35). In addition, effects of aging on photoelastic properties and temperature dependence are reported by Harris (43) and Filon (44). So it is with grave reservations as to its significance that the data in Table V is given. The glass types are merely similar, identified on the basis of incomplete data on density, index of refraction, elastic constants, or composition. The parenthesis in the Table indicate that the similarity of types is quite doubtful. The measurements due to Pockels (5), to Twyman and Perry (7), and to Schaefer and Nassenstein (8) are by static methods. Those due to Vedam (26) (27) are by Mueller's method "B" with sound intensities inferred from input voltage to the power amplifier. Those due to Schaefer and Dransfeld (28) are by Mueller's method "B" with sound amplitudes inferred from intensities in the diffraction pattern and extrapolation from two points giving an announced accuracy of five percent. Bergmann and Fues (29) used the intensity ratio method without extrapolation, being content with sound amplitudes low enough to avoid second order lines. Their results are, therefore, of doubtful significance. Bergmann has since remarked on the necessity of extrapolation (47). It is to be noted that Schaefer and

TABLE V
VALUES OF p/q GIVEN BY OTHER INVESTIGATORS

(Glass types compared in this table are merely similar, with parenthesis indicating especially doubtful similarity)

This Investigation			Other Investigations		
Block No.	Type	p/q	Type	Reference	p/q
1	BSC-1	1.82	BK-1	(8)	1.76
			BK-1	(28)	1.73
			BK-12	(8)	1.82
			BK-12	(28)	1.89
3	CF-1	1.68	KF-7	(8)	1.64
			KF-7	(28)	1.64
4	LF-1	1.45	(LF-1)	(8)	(1.45)
			(LF-1)	(28)	(1.50)
			LF-4	(8)	1.32
			LF-4	(28)	1.39
			0.2154	(5)	1.44
			(x)	(7)	(1.51)
			(10)	(27)	(1.70)
5	LBC-2	1.62	BaK-4	(8)	1.73
			BaK-4	(28)	1.75
			(9)	(27)	(1.84)
6	DBF-1	1.36	BaSF-1	(8)	1.29
			BaSF-1	(28)	1.29
			(12)	(27)	(1.50)
7	DF-2	1.46	(0.1571)	(5)	(1.27)
			(14)	(27)	(1.41)
9	EDF-1	1.46	(SF-2)	(8)	(1.18)
			(SF-2)	(28)	(1.26)
			(SF-9)	(8)	(1.20)
			(SF-9)	(28)	(1.22)
10	EDF-4	1.11	(SF-4)	(8)	(1.07)
			(SF-4)	(28)	(1.07)
			0.500	(5)	1.11
			(16)	(27)	1.05
			(17)	(27)	(1.18)
14	Fused Silica	2.34	Fused Silica	(26)	2.85

Nassenstein used the same set of glasses as Schaefer and Dransfeld.

SUMMARY AND CONCLUSIONS

Mueller has described, in theory, two methods for the determination of the ratio p/q of the strain-optical constants of glass. Both methods involve the diffraction of light by means of ultrasonic waves. Method "B" involves measurements of the polarization of the diffraction orders. Method "C" involves measurements of ratios of intensities in the diffraction orders.

The appropriate experimental arrangement was provided for each method. Data was taken, by both methods, on three glass samples and on a fused silica sample.

The data given by method "B" was in excellent agreement with the theory. This method was then applied to a set of thirteen American optical glasses, including three rare earth glasses, and to fused silica. The ratio of the strain-optical constants was calculated for each sample for dynamic harmonic strains having a frequency in the neighborhood of ten megacycles per second.

The data given by method "C" did not agree with the theory and did not lead to values for the ratio of the strain-optical constants. The data followed a definite pattern, however, and the pattern was consistent among the four samples tested.

It may be concluded that method "B" is applicable as described and leads to dynamic values of the ratio of the strain-

optical constants. It may also be concluded that method "C" does not apply as described. However, on the basis of the pattern and consistency of the data obtained by method "C" it is hoped that it may be shown to have significance.

APPENDIXES

APPENDIX I

Theoretical Intensity Ratios in the
Diffraction Orders

Arrangement of Mueller's Method "B". The optical system for Mueller's method "B" (23) is shown in Figure 6. For the case of standing longitudinal waves in glass, incident light polarized at forty-five degrees to the slit, crossed analyzer, and unit incident intensity, the intensity $I_{m,r}$ in the r sub-component of the m diffraction order is given by the Mueller theory as

$$I_{m,r} = \begin{cases} \frac{1}{4} \left[J_{s-r} \left(\frac{v}{2} \right) J_{s+r} \left(\frac{v}{2} \right) - J_{s-r} \left(\frac{Rv}{2} \right) J_{s+r} \left(\frac{Rv}{2} \right) \right]^2 & \text{if } m = 2s \\ \frac{1}{4} \left[J_{s-r} \left(\frac{v}{2} \right) J_{s+r+1} \left(\frac{v}{2} \right) - J_{s-r} \left(\frac{Rv}{2} \right) J_{s+r+1} \left(\frac{Rv}{2} \right) \right]^2 & \text{if } m = 2s+1 \end{cases} \quad (\text{I-1})$$

where the quantities are the same as those in equation 5.

The total intensity I_m in the order of index m is given by

$$I_m = \begin{cases} 2 \sum_{r=1}^{\infty} I_{m,r} + I_{0,m} & \text{if } m = 2s \\ 2 \sum_{r=0}^{\infty} I_{m,r} & \text{if } m = 2s+1 \end{cases} \quad (\text{I-2})$$

Numerical substitutions for R and v give the following ratios of the total intensity in the third order to the total intensity in the zero or central order.

TABLE VI
TOTAL INTENSITY RATIOS: THIRD ORDER
TO ZERO ORDER, METHOD "B"

v	R 1.5	R 2.0	R 2.5
0.8	.0136	.0206	.0292
1.0	.0221	.0327	.0464

Numerical substitutions for R and v give the following ratios of the intensity in the first sub-component to the intensity in the zero sub-component for the case of the first diffraction order.

TABLE VII
SUB-COMPONENT INTENSITY RATIOS: FIRST TO ZERO
SUB-COMPONENTS, FIRST ORDER, METHOD "B"

v	R 1.5	R 2.0	R 2.5
0.8	.0223	.0591	.156
1.0	.0460	.155	.610

Arrangement of Mueller's Method "C". The optical system for Mueller's method "C" (23) is shown in Figure 7. For the case of standing longitudinal waves in glass, incident light polarized parallel to the slit, and unit incident intensity, the intensity $I_{m,r}$ in the r sub-component of the m diffraction order is given by the Mueller theory as

$$I_{m,v} = \begin{cases} \left[J_{s-v}\left(\frac{Rv}{2}\right) J_{s+v}\left(\frac{Rv}{2}\right) \right]^2 & \text{if } m = 2s \\ \left[J_{s-v}\left(\frac{Rv}{2}\right) J_{s+v+1}\left(\frac{Rv}{2}\right) \right]^2 & \text{if } m = 2s+1 \end{cases} \quad (\text{I-3})$$

where the quantities are the same as those in equation 5.

The total intensity in the m order is given by equation I-2.

Numerical substitutions for R and v give the following ratios of the total intensity in the second order to the total intensity in the zero or central order.

TABLE VIII

TOTAL INTENSITY RATIOS: SECOND ORDER
TO ZERO ORDER, METHOD "C"

v	$R \ 1.5$	$R \ 2.0$	$R \ 2.5$
0.4	.000788	.00256	.00650
0.6	.00420	.0141	.0368
0.8	.0155	.0487	.127
1.0	.0368	.127	.299

Numerical substitutions for R and v give the following ratios of the intensity in the first to that in the zero sub-component of the first diffraction order.

TABLE IX

SUB-COMPONENT INTENSITY RATIOS: FIRST TO ZERO
SUB-COMPONENTS, FIRST ORDER, METHOD "C"

v	R 1.5	R 2.0	R 2.5
0.4	.000122	.000429	.00106
0.6	.000697	.00229	.00602
0.8	.00229	.00805	.0226
1.0	.00602	.0226	.0702

APPENDIX II

Sample Data and Calculation for Method "B"

Glass: Block number 1, type BSC-1

Frequency: 9.8 megacycles (oscillator dial 20.36)

Room temperature: 24° centigrade

Thermocouple thermometer scale reading maintained less than
2.5 (temperature difference: less than one quarter
degree centigrade)

Maximum permissible transducer current: no third order at
1.5 amperes

Analyzer angle readings in degrees: (for extinction or a
minimum)

<u>No sound</u> <u>("crossed")</u>	<u>.50 amp</u>	<u>.71 amp</u>	<u>.92 amp</u>	<u>1.10 amp</u>	<u>1.30 amp</u>
44.75	60.8	60.2	60.2	60.3	60.1
44.80	61.6	60.8	60.6	59.1	59.0
44.85	61.3	60.0	60.5	59.8	60.0
44.85	60.5	61.0	60.3	60.2	59.5
44.80	61.3	60.3	60.3	60.7	59.1
44.75	61.2	(59.4)	59.6	60.1	59.0
44.80	61.0	60.9	60.3	59.5	59.4
44.80	60.8	60.9	60.1	59.5	59.2
44.85	61.0	61.0	59.5	59.7	59.2
44.80	60.0	60.6	60.5	60.1	59.4

Analyzer angle averages:

44.80	60.90	60.63	60.19	59.90	59.39
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Values of θ : (θ = minimum setting minus "crossed" setting plus forty-five degrees. See equation 7)

	<u>.50 amp</u>	<u>.71 amp</u>	<u>.92 amp</u>	<u>1.10 amp</u>	<u>1.30 amp</u>
	61.10	60.83	60.39	60.10	59.59
Tan θ :	1.811	1.791	1.759	1.739	1.704

For linear extrapolation assume a curve of form $y = a + bx$, where $x = I^2$ and $y = 10(\tan \theta - 1.6)$, which gives best fit in sense of least squares.

The y-intercept y_0 is given by the value of the coefficient a which is, in turn given (40) by

$$y_0 = a = \frac{\sum x^2 \sum y - \sum x \sum xy}{n \sum x^2 - (\sum x)^2} \quad (\text{II-1})$$

Evaluating a :

<u>x</u>	<u>y</u>	<u>xy</u>	<u>x²</u>
.250	2.11	.5275	.0625
.504	1.91	.9625	.2540
.846	1.59	1.345	.7140
1.21	1.39	1.6825	1.464
<u>1.69</u>	<u>1.04</u>	<u>1.755</u>	<u>2.856</u>
4.50	8.04	6.2725	5.3505

Substituting in equation II-1,

$$y_0 = a = 2.2729$$

Evaluating R :

$$R = \tan \theta_0 = 1.6 + y_0/10$$

$$R = 1.827$$

Evaluating p_R :

If the data points are assumed to have a random distribution about a theoretical straight line, the probable error p_R in the y-intercept is given formally by (40)

$$P_R = \frac{1}{10} P_a = 0.675 \sqrt{\frac{\sum[(y-a-bx)^2]}{n-2}} \quad (\text{II-2})$$

where

$$b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} \quad (\text{II-3})$$

In this work the data points do not follow a relation which is, theoretically, perfectly linear. (See Figure 1). Nevertheless, this calculation is carried through formally to give some indication of the reliability of the data.

For this sample,

$$b = -0.7394$$

$$p_a = 0.0254$$

$$p_R = 0.0025$$

APPENDIX III

Sample Data and Calculation for Method "C"

Glass: Block number 3, type CF-1

Frequency: 9.4 megacycles (oscillator dial 20.06)

Room temperature: 23⁰ centigrade

Thermocouple thermometer scale reading maintained less than 2,
(temperature difference: less than one quarter degree
centigrade)

Maximum permissible transducer current: no second order at
300 milliamperes

Emulsion sensitivity exposure: rotating step-wedge with
step to step exposure ratio of 1.5

Diffraction pattern exposure:

<u>Exposure number</u>	<u>Exposure time (sec)</u>	<u>Transducer current (ma)</u>
1	1/2	no sound
2	1/2	100
3	1	100
4	1/2	150
5	1/2	150
6	1/2	200
7	1/2	200
8	1/2	250
9	1/5	250
10	1/5	300
11	1/5	300
12	1/2	100
13	1/5	250

Photometer readings on successive sector steps:

1)	2.0	1.5	1.5	Average:	1.67
2)	4.5	4.7	4.8		4.67
3)	10.3	10.3	10.5		10.37
4)	19.9	20.3	19.5		19.90
5)	35.3	35.5	36.0		35.60
6)	50.2	48.8	50.0		50.00
7)	64.5	63.0	62.8		63.43
8)	75.5	75.4	75.6		75.50
9)	80.7	80.3	81.2		80.73
10)	86.7	86.5	86.6		86.60
11)	90.9	91.0	90.8		90.90
12)	93.8	93.8	93.4		93.67
13)	95.6	95.8	95.4		95.60

The average photometer readings are plotted against the logarithm (to the base 1.5) of the rotating sector exposures. A standard s-shaped emulsion sensitivity curve results.

Pairs of photometer readings on the two lines of exposure

1 (no sound):

63.4	65.0	65.8	67.0	65.8	66.9
64.7	66.0	64.6	65.6		

Average difference: 1.22

Difference in log exposure (read from sensitivity curve): 0.084

This difference, due to the inequality of the two beams from the Wollaston prism, will be applied below as a correction.

Photometer readings on diffraction lines and calculations:

L_1 and L_2 ; photometer readings on the diffraction lines of one beam. (Light polarized normal to slit).

L_3 and L_4 ; photometer readings on the diffraction lines of the other beam.

A_{12}	; average of L_1 and L_2 .
A_{34}	; average of L_3 and L_4 .
Bkg	; photometer reading on background near lines.
Bkg Cor	; Correction to A due to presence of background. $Bkg\ Cor = Bkg (100 - A)/100$.
A_c	; Corrected average photometer readings on lines.
G_1	; Logarithm of average exposure of diffraction lines of one beam. G is determined from A_c by means of sensitivity curve.
G_2	; Logarithm of average exposure of diffraction lines of the other beam.
$D = G_2 - G_1$	; Logarithm of ratio of exposures of diffraction lines of the two beams. Since the exposures are made simultaneously, D is the logarithm of the ratio of the intensities of the diffraction lines of the two beams.
D_a	; Average value of three determinations of D.
$D_{ac} = D_a - .08$	; D_a corrected for beam inequality.
$B = 1.5^{(D_{ac})}$	; Intensity ratio of the diffraction lines.

Exp	L ₁	L ₂	A ₁₂	Bkg	Bkg Cor	A _c	G ₁	L ₃	L ₄
2	15.4	15.9	15.65	2	1.7	13.95	3.43	41.2	40.2
2	13.9	14.0	13.95	2	1.7	12.25	3.24	36.4	35.4
2	13.1	12.5	12.80	2	1.7	11.10	3.11	32.2	31.2
4	28.8	28.9	28.85	1	0.7	28.15	4.53	67.6	66.4
4	25.3	25.2	25.25	2	1.5	23.75	4.23	60.2	59.4
4	21.3	22.0	21.65	3	2.3	19.35	4.00	54.0	54.3
5	28.1	29.2	28.65	2	1.4	27.25	4.47	65.1	65.0
5	25.5	25.2	25.35	2	1.5	23.85	4.25	59.4	58.8
5	22.1	22.2	22.15	3	2.3	19.85	3.93	54.4	53.5
6	47.2	49.0	48.10	2	1.0	47.10	5.84	81.6	81.9
6	42.8	43.4	43.10	2	1.1	42.00	5.49	77.0	77.4
6	36.0	37.5	36.74	2	1.3	35.45	5.04	72.2	73.0
7	47.0	48.0	47.50	1	0.5	47.00	5.83	81.0	81.4
7	42.0	43.3	42.65	1	0.6	42.05	5.50	76.4	77.4
7	34.9	35.8	35.35	2	1.3	34.05	4.94	70.7	71.2
8	61.2	61.6	61.40	3	1.3	60.10	6.75	87.0	87.0
8	56.4	56.8	56.60	2	0.9	55.70	6.44	84.4	84.4
8	51.0	51.7	51.35	2	1.0	50.35	6.07	81.4	81.4
9	6.1	6.4	6.25	0	0.0	6.25	2.34	39.7	39.8
9	4.7	4.3	4.50	0	0.0	4.50	1.95	35.0	34.7
9	5.8	6.0	5.90	0	0.0	5.90	2.28	38.0	39.0
10	13.7	14.6	14.15	0	0.0	14.15	3.47	59.2	59.9
10	11.6	12.2	11.90	0	0.0	11.90	3.20	56.3	56.0
10	14.5	15.2	14.85	0	0.0	14.85	3.52	59.7	60.3
11	11.6	12.5	12.05	0	0.0	12.05	3.21	58.0	57.6
11	9.9	11.2	10.55	0	0.0	10.55	3.02	52.3	52.9
11	9.7	9.4	9.55	0	0.0	9.55	2.88	50.0	50.6
12	13.4	13.1	13.25	1	0.9	12.35	3.25	37.3	36.8
12	12.9	13.0	12.95	2	1.7	11.25	3.13	35.7	35.6
12	12.3	12.3	12.30	2	1.8	10.50	3.02	33.1	31.3
13	6.1	6.2	6.15	0	0.0	6.15	2.31	41.5	42.0
13	4.7	5.4	5.05	0	0.0	5.05	2.08	35.6	36.7
13	6.3	7.6	6.95	0	0.0	6.95	2.45	43.8	43.3

Exp	A ₃₄	Bkg	Bkg Cor	A _c	G ₂	D	D _a	D _{ac}	$\sqrt{B}$
2	40.70	1	0.6	40.10	5.36	1.93			
2	35.90	2	1.3	34.60	4.98	1.74	1.75	1.67	1.40
2	31.70	2	1.4	30.30	4.68	1.57			
4	67.00	1	0.6	66.40	7.18	2.65			
4	59.80	2	0.8	59.00	6.67	2.44	2.45	2.37	1.62
4	54.15	2	0.9	53.25	6.27	2.27			
5	65.05	1	0.3	64.75	7.06	2.59			
5	59.10	1	0.4	58.70	6.65	2.41	2.45	2.37	1.62
5	53.95	1	0.5	53.45	6.29	2.36			
6	81.75	1	0.2	81.55	9.00	3.16			
6	77.20	1	0.2	77.00	8.21	2.72	2.85	2.77	1.75
6	72.60	1	0.3	72.30	7.70	2.66			
7	81.20	1	0.2	81.00	8.92	3.09			
7	76.90	1	0.2	76.70	8.24	2.74	2.80	2.72	1.74
7	70.95	2	0.6	70.35	7.51	2.57			
8	87.00	2	0.3	86.70	10.00	3.25			
8	84.40	2	0.3	84.10	9.47	3.03	3.04	2.96	1.82
8	81.40	2	0.4	81.00	8.91	2.84			
9	39.75	0	0.0	39.75	5.34	3.00			
9	34.85	0	0.0	34.85	4.99	3.04	3.00	2.92	1.81
9	38.50	0	0.0	38.50	5.25	2.97			
10	59.55	0	0.0	59.55	6.71	3.24			
10	56.15	0	0.0	56.15	6.48	3.28	3.25	3.17	1.90
10	60.00	0	0.0	60.00	6.74	3.22			
11	57.80	0	0.0	57.80	6.59	3.38			
11	52.60	0	0.0	52.60	6.22	3.20	3.28	3.20	1.91
11	50.30	0	0.0	50.30	6.06	3.18			
12	37.05	1	0.6	36.45	5.10	1.85			
12	35.65	1	0.6	35.05	5.00	1.87	1.80	1.72	1.42
12	32.20	2	1.4	30.80	4.71	1.69			
13	41.75	0	0.0	41.75	5.48	3.17			
13	36.15	0	0.0	36.15	5.08	3.00	3.11	3.03	1.85
13	43.55	0	0.0	43.55	5.60	3.15			

The values of $\sqrt{B}$ from the last column are plotted against the square of the transducer current in Figure 12.

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