THE DETERMINATION OF THE ELASTIC CONSTANTS OF OPTICAL GLASSES BY AN ULTRASONIC METHOD

By

JAMES MILTON BARNES

AN ABSTRACT

Submitted to the School of Graduate Studies of Michigan State College of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY

Department of Physics

1955

Approved: E. A. Hiedemann

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The Determination of the Elastic Constants of Optical Glasses by an Ultrasonic Method.

James M. Barnes

Abstract

The values of the elastic constants of thirteen samples of optical glass commercially produced in the United States and a sample of fused silica were calculated from the results of measurements made on these samples. The method used was a variation of the Schaefer-Bergmann ultrasonic method,[1] modified to use diffraction of higher orders than the first by using Hiedemann patterns.[2] It was found necessary to consider simultaneously the point source and slit source diffraction patterns in order to choose those of the latter type which were suitable for measurements. The accuracies of the experimentally determined values of the longitudinal and shear wave velocities were 0.13% and 0.25% respectively, while the accuracies in determining the elastic constants were 0.5% for the shear modulus, Lame's modulus, and the bulk modulus, 0.7% for Poisson's ratio, and 0.8% for Young's modulus.

The values of the surface tensions of the samples were calculated using the results of the measurements and Auerbach's equation.[3]

Abstract--References

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THE DETERMINATION OF THE ELASTIC CONSTANTS OF OPTICAL GLASSES BY AN ULTRASONIC METHOD

By

JAMES MILTON BARNES

A THESIS

Submitted to the School of Graduate Studies of Michigan State College of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

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ACKNOWLEDGMENTS

I wish to express my most sincere appreciation to Dr. E. A. Hiedemann for the inspiration and guidance which helped to bring this thesis to a successful conclusion.

I also wish to express my indebtedness to the Owens-Illinois Glass Company for providing funds and aiding in obtaining materials for the measurements included in this paper.

James M. Barnes

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TABLE OF SYMBOLS

θ = Diffraction Angle

m, n = Order of Diffraction

λ_o = Optical Wave Length in Air

Λ_L = Longitudinal Ultrasonic Wave Length

Λ_S = Shear Ultrasonic Wave Length

F = Focal Length of Objective Lens

r = Longitudinal Wave Diffraction Pattern Spacing

R = Shear Wave Diffraction Pattern Spacing

f = Frequency of the Ultrasonic Wave

V_l = Longitudinal Wave Velocity

V_S = Shear Wave Velocity

λ = Lame's Elastic Modulus

μ = Shear Elastic Modulus

k = Bulk Elastic Modulus

E = Young's Elastic Modulus

σ = Poisson's Ratio for the Elastic Medium

ρ = Density of the Medium

d = Comparison Grating Constant

x = Comparison Grating Diffraction Pattern Spacing

S = Surface Tension of the Medium

Introduction

The object of the study reported in this paper was the determination of the elastic constants of a series of optical glasses by a method using the diffraction of light produced by elastic waves of high ultrasonic frequency.

In 1932 P. Debye and F. W. Sears[1] in America and R. Lucas and P. Biquard[2] in France discovered simultaneously and independently that ultrasonic waves can act as optical diffraction gratings. If an ultrasonic wave is excited in a transparent medium and if light passes through it in a direction perpendicular to the direction of propagation of the acoustical wave, a diffraction pattern of the light is formed resembling that obtained by means of a simple line grating. The grating spacing corresponds to the length of the sound wave in the medium. The use of high ultrasonic frequencies allows one to obtain the condition for an "infinite" medium with rather small samples. It is only necessary that the dimensions of the sample are large compared with the wave length.

In the case of an "infinite" isotropic solid there exist only two types of elastic waves in the medium, namely the longitudinal and the shear waves. If the density of the medium is known, the two elastic constants of an ideal isotropic solid can be computed from the velocities

of propagation of the two types of waves. To determine the velocity of propagation of a wave one often measures both frequency and wave length. The frequency of ultrasonic waves can be obtained with ease from the measurement of the frequency of the electrical circuit driving the transducer. The main problem is therefore the measurement of the wave length.

The diffraction of light by ultrasonic waves offers various possibilities for the measurement of the ultrasonic wave length. Two different methods have been used for systematic studies of the elastic constants of optical glasses. Cl. Schaefer, L. Bergmann, and H. J. Goehlich[3] used the method of the Schaefer-Bergmann diffraction patterns.[4] K. H. Hoesch[5] used a method of E. Hiedemann[6] consisting of direct measurement of the wave length of the two different waves.

Schaefer, Bergmann, and Goehlich replaced the slit in the usual Debye-Sears arrangement by a small circular aperture. The light passing through the glass sample was slightly convergent, the image of the source being focused on a photographic film approximately one meter distant from the sample. The spacing of the orders on the film together with the constants of the optical system sufficed to determine the ultrasonic wave length in the glass. In isotropic media the Schaefer-Bergmann patterns are of the circular form. As their glass samples were cubes and the ultrasonic vibrations were excited by means of an X-cut quartz fastened to one surface, reflections of the

side lobes of the ultrasonic beam at the surfaces of the cube combined with internal coupling caused the waves to fill the block with the waves being propagated in many directions. Each wave propagation direction which was normal to the axis of the optical system produced a pair of diametrically opposite diffraction points. Due to the presence of both shear and longitudinal waves the loci of the diffraction points were two concentric circles around the central image point. Each circle corresponded to one of the two types of waves--the larger one to the shear waves. The spacing of the orders was found by measuring the diameters of the circles. Schaefer, Bergmann, and Goehlich calibrated their optical system by placing a plane grating just in front of the glass cube and correcting for the difference of position between it and the ultrasonic grating, which they proved to exist at essentially the center of the cube.

Hoesch took care to limit the presence of the stationary waves to one direction only. The different types of waves were observed separately by means of using the experimental procedures of photo-elastic analysis.[7] Hoesch measured the standing wave pattern directly through a microscope held on a traverse mount driven by a micrometer screw. In this manner a direct measurement of the ultrasonic grating was made over a large number of grating spaces. The method was very accurate, but it required some time.

It appeared possible that the accuracy of measuring the diffraction angles would be increased by using the sharp images of a slit

instead of those of a circular aperture. The diffraction pattern produced by the grating could have been measured by a traversing microscope. The author, however, decided that a measuring method involving a permanent record was more satisfactory and that the use of a line diffraction spectrum also called the Hiedemann pattern,[8] could produce greater precision by the use of higher orders.

Experimental Principle

The principle of this experiment follows closely the original arrangement of Debye and Sears or Lucas and Biquard respectively. A glass cube was excited into vibration by means of a piezo-electric quartz plate attached to one face.* The vibrations in the medium were in the form of standing waves of the same frequency as that of the voltage applied to the quartz. Parallel light passing through the block in a direction parallel to the face of the quartz plate was diffracted by the ultrasonic wave system and focused on a screen or photographic plate. Fig. 1 shows the relationship between the various directions at the glass block. Though the system of standing waves may become very complicated due to the reflections from the surfaces and internal coupling, it was generally possible to adjust the frequency such that most of the ultrasonic energy lay in those waves whose propagation direction was perpendicular to the face of the cube to which the driving quartz was attached. When this occurred, the interference pattern, hereinafter referred to as the diffraction pattern, had the maximum number of orders. The theory of the diffraction due to either the longitudinal or shear ultrasonic waves has been given by

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* The wave system produced in the glass depends also on the type of acoustical contact. Cementing the quartz to the sample generally favors the Schaefer-Bergmann patterns.

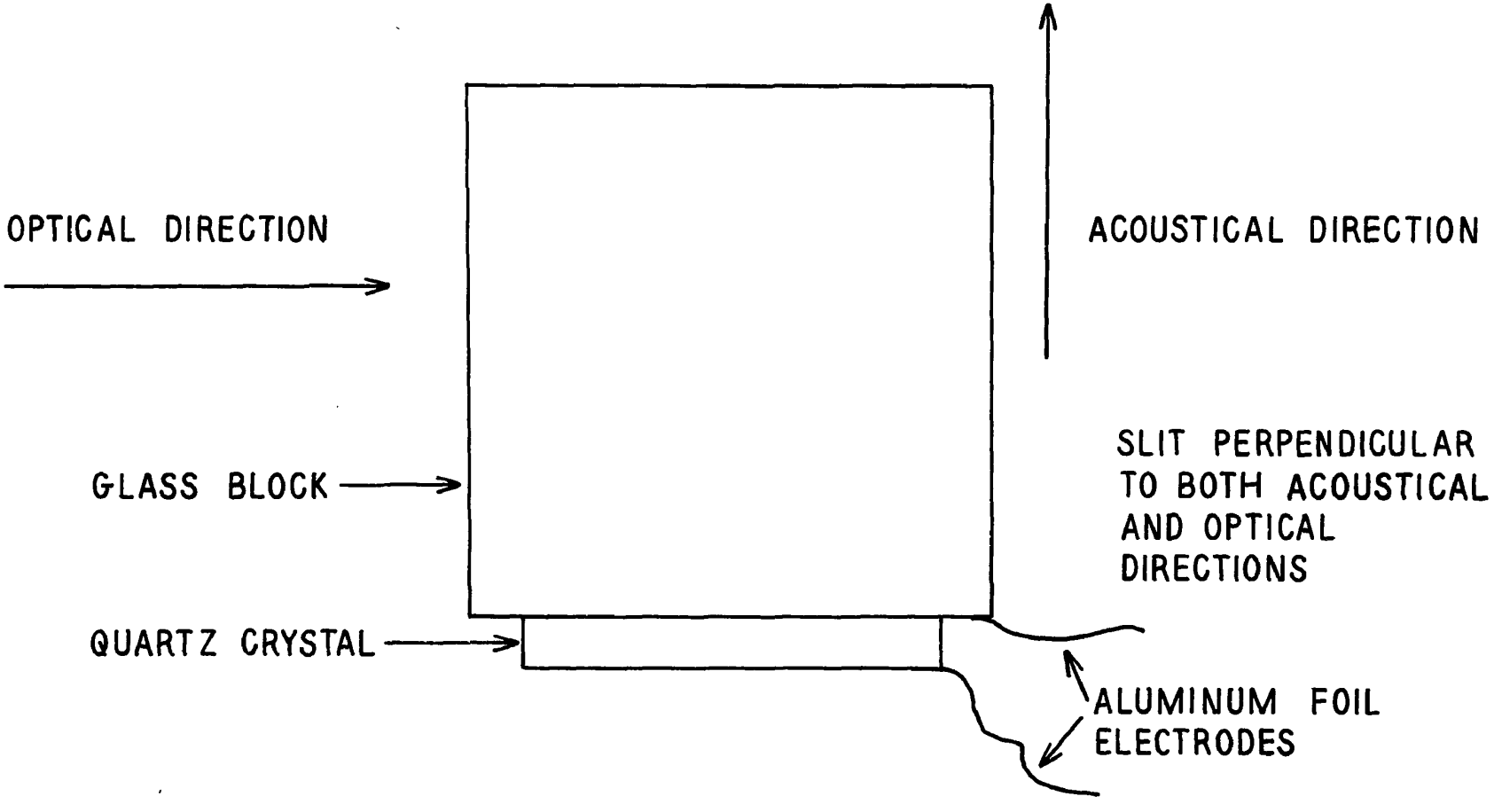

FIG. 1. DETAIL OF GLASS BLOCK ASSEMBLY

H. Mueller[9,10], and N. S. N. Nath and H. Mueller[11]. A very illuminating theoretical explanation was given by Schaefer, Bergmann, and Goehlich[3].

The type of source aperture used determines the type of diffraction pattern obtained. A point source consisting of a small round aperture will produce a diffraction pattern consisting of a set of points of light. A slit source produces a pattern which is a set of parallel lines. Each type of source has its advantage. The diffracted points of a point source lie along a straight line passing through the central order. The direction of this line is the direction of propagation of the particular wave which is causing the diffraction. If more than one standing wave of sufficient intensity to produce diffraction occurs within the glass cube, there will occur several lines of points of equal spacing but with different directions depending on the direction of the diffracting ultrasonic wave. With enough of these waves in different directions one obtains the characteristic Schaefer-Bergmann circular diffraction patterns.

A slit source may be considered as a large number of point sources set along in a row. Each diffracted point as described in the preceding paragraph becomes a diffracted line. Obviously a set of standing waves which produces a good Schaefer-Bergmann pattern will produce a hopeless confusion of lines with a slit source. Fig. 2, a photograph taken when a point source and a line source were used

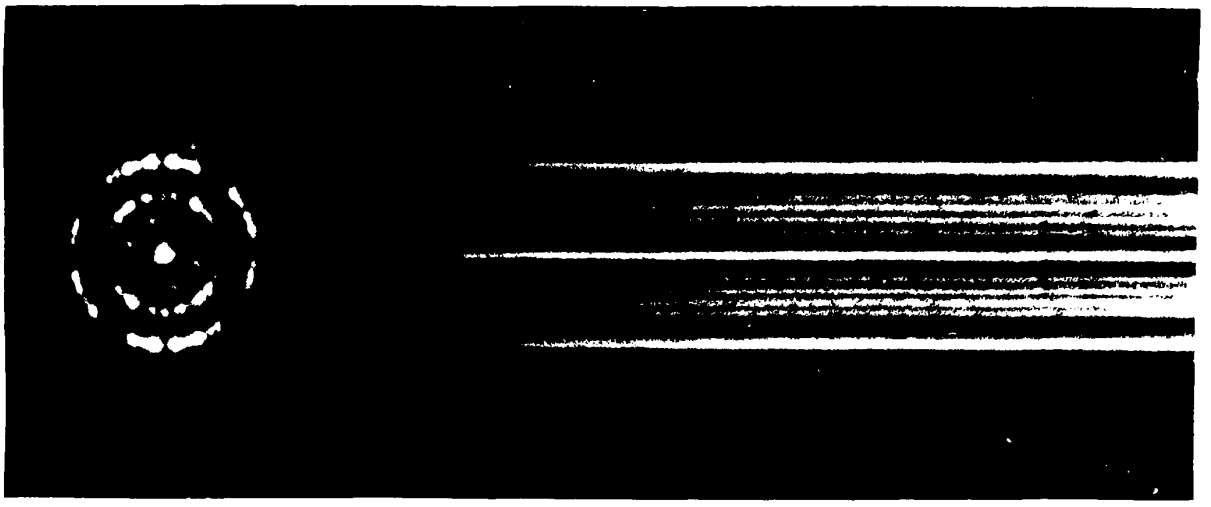

FIG. 2. PHOTOGRAPH OF A SIMULTANEOUS SCHAEFER-BERGMANN POINT SOURCE DIFFRACTION PATTERN AND A LINE DIFFRACTION PATTERN SHOWING HOW THE POINTS ON THE LONGITUDINAL WAVE (INNER) AND SHEAR WAVE (OUTER) CIRCLES PRODUCE POORLY DEFINED AND SPACED LINES WHICH ARE UNSUITABLE FOR MEASUREMENT

simultaneously, illustrates this case. (Fig. 2 was taken in the fused silica sample at an ultrasonic frequency of 11.748 mc/sec.) However, a slit source has the advantage that it may easily be adjusted for size, something which is not practical for small circular apertures. Thus for this investigation a slit source was used in order to obtain as fine a diffraction pattern as possible. The advantage of a point source was used to avoid mistakes due to "apparent" diffraction lines.

In evaluating a diffraction photograph to determine whether or not it is worth putting through the measuring process, one must know if the perpendicular distance between the lines is a true indication of the effect one wishes to measure. Referring again to Fig. 2, one can see that no measurement between lines could be indicative of the true longitudinal diffraction pattern spacing. In contrast, consider the two parts of the photograph shown in Fig. 4. Although this photograph is presented primarily to illustrate a different phenomenon, note that the pattern from the point source consists of a line of dots, and that at best only a few dim points remain of a circular pattern. Also note that as far as brightness of the pattern allowed, there is a line for every point and the spacing between lines corresponds to the spacing between points. In general in the measured photographs there were no more dots visible than there were lines, and the lines, by nature of the slit used, had varying widths so that a position could be found which allowed optimum accuracy of setting of the measuring instrument

used.

In all cases, then, the source used had a combination of a point opening and a slit opening of varying width. For all measurements diffraction patterns were searched for in which the diffraction pattern from the point part of the source consisted of a line of dots perpendicular to the quartz plate (and thus perpendicular to the slit). By varying the frequency one could watch the patterns of the standing waves change until a pattern of the desired configuration was found. With some glass samples it was impossible to find the desired patterns within certain of the frequency ranges used. With both the point pattern and the line pattern showing on the finished photograph, a permanent record was obtained of the pattern together with its generating point configuration. On each plate there also appeared a record of the pattern due to the calibration grating (explained farther on) so that possibilities of change due to emulsion shrinkage or other effects were minimized.

Although it was possible to take diffraction pictures without them, the addition of a polarizer and an analyzer, one on either side of the glass block, allowed increased selectivity of the type of pattern to be recorded. The nature of the strains due to the longitudinal or the shear wave in the block was such that plane-polarized light passing through the glass specimen emerged with a change in polarization depending on whether the diffraction orders were due to the one or the other type of wave.[6] For a specific case, consider a longitudinal and

a shear wave in the glass, each with the same, say vertical, direction of propagation. With the polarizer and analyzer parallel and perpendicular to the propagation direction (crossed with one another), only the shear wave diffraction pattern is observed. With the polarizer and analyzer still crossed, but each making an angle of 45° with the propagation direction, only the longitudinal orders are observed. Inasmuch as the longitudinal orders were generally much brighter than those due to the shear wave, the parallel and perpendicular arrangement was almost a necessity for observing and photographing the latter. In addition the brightness of the central order was greatly diminished, and scattered light arising from the various elements of the optical system was eliminated.

The spacing of the diffracted lines in the case where the slit was perpendicular to the ultrasonic wave propagation direction was a function of the length of the standing wave in the glass as well as of the constants of the optical system. These waves acted as a grating to diffract the light into one or more orders. Since the spacing of the ultrasonic grating was one wave length, the angle θ of the n^{th} order of diffraction is given by the well-known relation

$$\sin\theta = \frac{n\,\lambda_o}{\Lambda} \tag{1}$$

where λ_o is the optical wave length in air and Λ is the ultrasonic wave length in the medium. If a lens of focal length F is used to focus the

diffraction pattern on the film plane, the spacing r between the zero or central order and the n^{th} order is given by

(2) $$r = \frac{F n \lambda_o}{\Lambda}, \qquad F \gg r.$$

Recognizing that there were two waves of different wave lengths produced in the glass, this last equation takes the following forms

(3a) $$R = \frac{F n \lambda_o}{\Lambda_s}$$

(3b) $$r = \frac{F n \lambda_o}{\Lambda_l}$$

where R is the shear wave diffraction spacing, r is the longitudinal wave diffraction spacing, Λ_s is the shear wave length, and Λ_l is the longitudinal wave length of the ultrasonic wave in the glass. Since the ultrasonic wave lengths are the unknown quantities, the equations are more usable in the following form:

(4a) $$\Lambda_s = \frac{F n \lambda_o}{R}$$

(4b) $$\Lambda_l = \frac{F n \lambda_o}{r}$$

With the use of a comparison grating the actual calculation of the velocity was much simplified. If x is the spacing between the central and the m^{th} order of the comparison grating diffraction pattern, taken with exactly the same optical bench arrangement except with the grating at the glass block position, and if d is the constant of this grating, then

(5) $$x = \frac{F\, m\, \lambda_o}{d}$$

Combining this last equation with the two preceding equations, one eliminates the product $F\lambda_o$ and has

(6) $$\frac{d\, x}{m} = \frac{r\, \Lambda_l}{n} = \frac{R\, \Lambda_s}{n}$$

or separately

(7a) $$\Lambda_l = \frac{d\, x\, n}{r\, m}$$

(7b) $$\Lambda_s = \frac{d\, x\, n}{R\, m}$$

Knowing the frequency f of the ultrasonic waves, the velocities of the waves in the glass sample are given by

(8a) $$V_s = f\, \Lambda_s$$

(8b) $$V_l = f\, \Lambda_l$$

where V_s is the shear wave velocity and V_l is the longitudinal wave velocity.

From elastic theory the dependence of the shear and longitudinal velocities on the elastic constants of the medium is given by several simple equations. The values of the elastic constants for this paper were calculated from the following:

(9) $$\mu = \rho V_S^2$$

(10) $$\lambda = \rho V_L^2 - 2\mu$$

(11) $$k = \rho V_L^2 - 4/3\mu$$

(12) $$\sigma = \frac{\lambda}{2(\lambda + \mu)}$$

(13) $$E = 2\mu(1 + \sigma)$$

where μ is the shear modulus, λ is Lame's modulus, k is the bulk modulus, E is Young's modulus, σ is Poisson's ratio, and ρ is the density of the glass.

Surface Tension

In 1948 R. Auerbach[13] published a paper in which an empirical relation was given, good for gases, liquids, or solids, for the surface tension in terms of the density and the sound velocity. Inasmuch as the densities and sound velocities of the various glasses were measured for this paper, it is appropriate that Auerbach's relation be given and the surface tension of each glass sample calculated. If S is the surface tension in dynes per centimeter, this relation is

$$S = 6.30 \times 10^{-4} V^{3/2} \rho \tag{14}$$

where the velocity used would be the longitudinal velocity in meters per second and the density would be in grams per cubic centimeter.

Multiple Diffraction

The phenomenon of multiple diffraction by ultrasonic waves was already considered in the theory of C. V. Raman and N. S. N. Nath[14]. Due to the finite thickness of an ultrasonic grating, light diffracted after passing through part of the ultrasonic grating will be diffracted again by the succeeding parts of the grating. Multiple diffraction increases with the length of the optical path in the ultrasonic grating and with the intensity of the waves. Raman and Nath explained in this way the frequency distribution over the subcomponents of a diffraction line; each successive diffraction produces a Doppler effect to the amount f. The effects of multiple diffraction produced by two ultrasonic waves of different wave lengths were first observed by L. Bergmann and E. Fues[15]. A rigorous theory of the multiple diffraction by ultrasonic waves in solids was given by N. S. N. Nath[16]. Multiple diffraction by means of ultrasonic waves of different wave lengths gives rise to the appearance of additional diffraction lines. These additional or "combination" lines and their production will now be described in detail for the special case of glass.

In the glass block two simultaneous gratings are produced, one by each the shear and the longitudinal wave. These gratings are thick, that is, they may occupy up to an inch of space along the optical path, and they are intermingled. Thus there is ample opportunity for multiple diffraction to occur. As an example, light diffracted into the

first longitudinal order may be rediffracted by the shear wave grating into a plus or minus first shear order with the longitudinal order as central order. One may just as easily consider the longitudinal and shear grating roles to be interchanged in the last statement. Inasmuch as the shear wave under normal conditions never produces an order higher than the first**, it is more convenient to consider the shear diffraction as the first diffraction, followed by the longitudinal diffraction.

In order to understand how the multiple diffraction pattern develops, Fig. 3 has been prepared. The parts of this figure are diffraction pattern photographs taken at the same frequency (f = 11.973 mc/sec.) in the same material (Bausch and Lomb glass type LF-1) with only the intensity of the ultrasonic standing wave pattern

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** This was explained by H. Mueller[10] in correlation with the results of E. Hiedemann and K. H. Hoesch[17]. Rötger[18] has shown that it is possible to increase the intensity of the shear waves enough to obtain the second diffraction order. He used a glass block of triangular cross-section; ultrasonic waves entering through one face of the block were reflected at the next face under an angle favoring optimum production of shear waves.

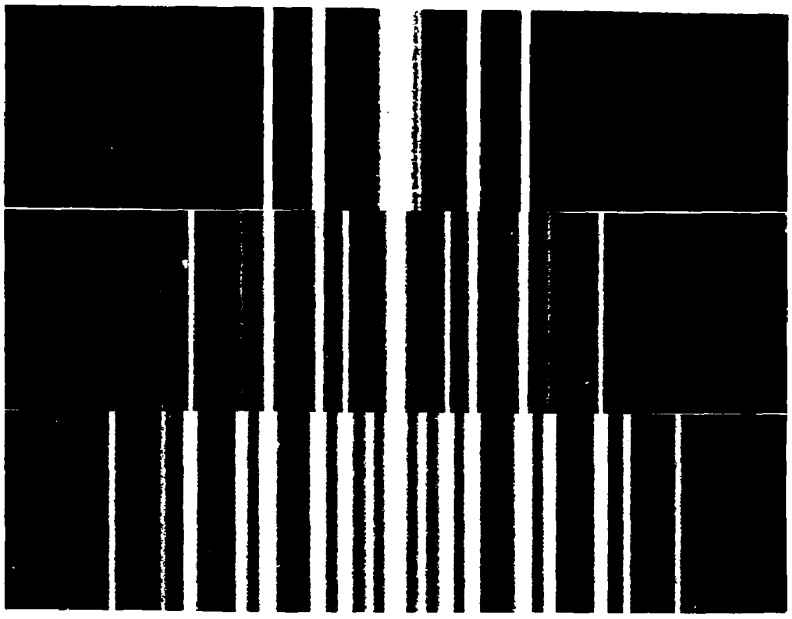

FIG. 3. A SET OF PHOTOGRAPHS OF ULTRASONIC DIFFRACTION PATTERNS TAKEN AT THE SAME FREQUENCY IN THE SAME GLASS SAMPLE, SHOWING THE EFFECT OF INCREASING THE ENERGY IN THE ULTRASONIC STANDING WAVE IN THE BLOCK. THE CHANGES IN LINE WIDTH ARE DUE TO DIFFERENT EXPOSURE TIMES.

A. LOW ULTRASONIC INTENSITY
B. MEDIUM ULTRASONIC INTENSITY
C. HIGH ULTRASONIC INTENSITY

changed. Part A was taken at low excitation intensity***. It shows the simplest diffraction pattern obtainable with the two waves present in the sample. The inner pair of lines is the first order of the longitudinal diffraction pattern, and the outer pair is the first order of the shear diffraction pattern.

Part B is a photograph taken at medium ultrasonic intensity and shows, besides the orders of part A, the appearance of the second orders of the longitudinal wave and the four lines which are the first multiple diffraction lines. These lines are the first and last lines out from the central order. They may be grouped in symmetrical pairs using the first orders of either the longitudinal or shear wave primary patterns as their central orders.

On further increasing the ultrasonic intensity, the orders added to the above in part C are the third orders of the longitudinal wave pattern and four more lines due to multiple diffraction. Considering the primary shear orders as central orders for multiple diffraction, each has a pattern of two orders of longitudinal spacing about it. In general, if the longitudinal wave diffracts the light into n orders and the shear order is relatively strong, there should be visible n orders

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*** For comparison, the voltage applied to the plate of the final power amplifier for this part was 150 volts, for part B about 300 volts, and for part C about 800 volts.

of primary longitudinal diffraction, 1 order of primary shear diffraction, and 2n orders of multiple diffraction, plus the central order. Thus for $n = 5$, 33 lines should be visible. Fig. 4, at least on the original negative, had this many lines. The multiple processes of photographic reproduction necessary for preparing the figures may reduce the definition of the picture.

Fig. 4 has been prepared to further clarify the grouping arrangement of a multiple diffraction pattern. Both the line and the point source pattern have been included to show the relationship of one to the other. The photograph was taken in the Bausch and Lomb type DBF-1 glass sample at a frequency of 10.019 mc/sec. Thin lines have been extended down from the photograph, and superimposed on these lines are the grouping patterns in heavy lines for both primary sets of orders and the two multiply diffracted sets of orders. If the orders are not visible in the line spectrum, they are certainly visible in the point diffraction spectrum, where the author counted enough points of light on the original negative to have generated 45 lines.

Observation of numerous diffraction patterns through the optical system indicated that the best multiple diffraction patterns occurred when both the longitudinal and shear standing waves were simultaneously strong and normal to the vibrating quartz. In the case of Fig. 4, this condition was exceedingly well fulfilled. It is not

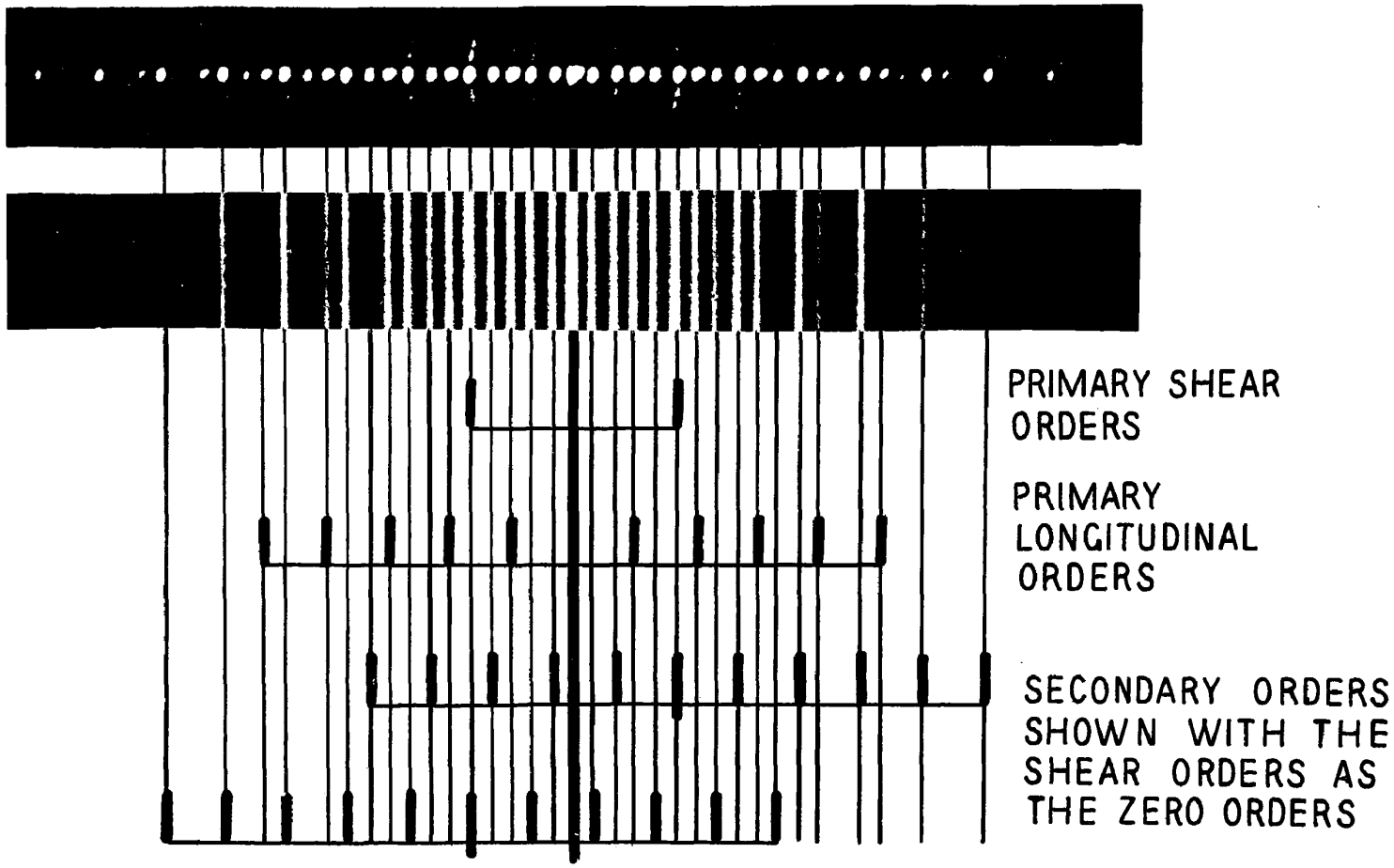


FIG. 4. MULTIPLE DIFFRACTION SPECTRUM SHOWING LINE AND POINT SPECTRA FROM THE SAME NEGATIVE. F = 10.0 MC.

easy to find a frequency for which a particular glass block thickness is so very closely identical to an integral multiple of both longitudinal and shear wave lengths that such a perfect pattern can be observed.

As an example of another type of multiple diffraction picture, Fig. 5 is included. The primary diffraction orders of this picture, taken in the sample of fused silica with unpolarized light at an ultrasonic frequency of 11.858 mc/sec. are the Schaefer-Bergmann circular diffraction pattern and a strong linear point pattern normal to the surface of the vibrating quartz. These are indicated in Part B of the figure. Part C of the figure illustrates the additional circular patterns which are due to multiple diffraction. The first and second orders of the linear point pattern act as centers, or central orders, for additional circular patterns, the multiply diffracted patterns. In addition two new lines of points occur. The spacing of these points vertically is equal to the spacing of the longitudinal diffraction pattern, or the radius of the longitudinal diffraction circle. Horizontally, however, the spacing is equal to neither the radius of the longitudinal nor of the shear diffraction circles. One notes that these points lie on the shear diffraction circles, and on closer examination it can be seen that they lie on the intersection of two shear diffraction circles. Not every intersection of shear circles produces a point. From observation and measurement of a number of different photographs, some showing a good many more of these extra points, it was found

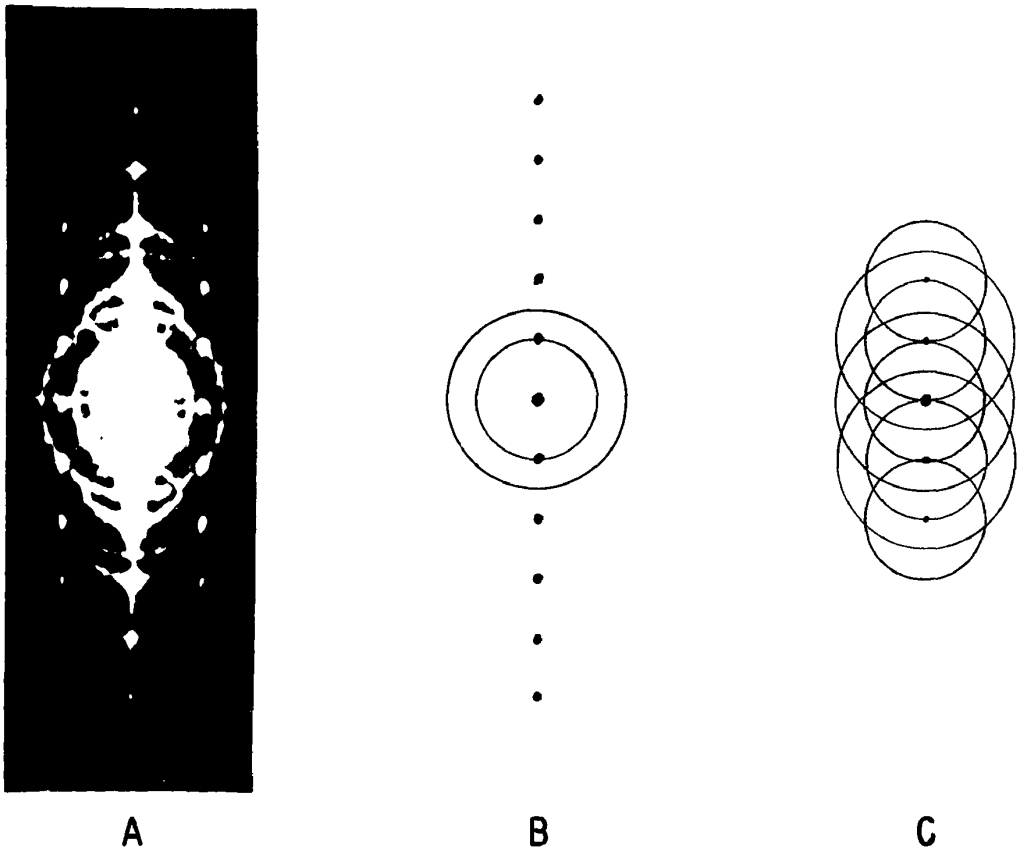


FIG. 5. A. PHOTOGRAPH OF MULTIPLE DIFFRACTION OF A SCHAEFER-BERGMANN CIRCULAR PATTERN TAKEN WITH UNPOLARIZED LIGHT IN THE FUSED SILICA SAMPLE. B. APPROXIMATE SCALE DRAWING SHOWING THE PRIMARY ORDERS OF DIFFRACTION. C. DRAWING SHOWING HOW THE EXTRA CIRCLES DUE TO MULTIPLE DIFFRACTION HAVE AS THEIR CENTRAL ORDERS THE FIRST AND SECOND ORDERS OF THE LONGITUDINAL WAVE POINT DIFFRACTION PATTERN.

that the loci of these points are the intersections of any shear wave diffraction circle with one centered about a longitudinal diffraction point two orders away in either direction. Schaefer, Bergmann, and Goehlich point out that the light appearing in the shear orders is elliptically polarized and that the sense of rotation of adjacent orders is opposite. Thus it would be that light arriving at the same point from alternate diffraction orders would tend to reinforce, while light arriving at the same point from adjacent orders would tend to cancel. The fact that some of these points appear where the shear wave circles no longer have enough intensity to be seen may be due to either this reinforcement by invisible but present circles of diffraction or by diffraction of the light in the few bright orders near the horizontal axis of the photograph by the strong longitudinal wave which produced the central vertical line of diffracted points. In this latter case there would indeed be multiple diffraction at the indicated point, e. g., rediffraction of light appearing in orders whose origin was multiple diffraction.

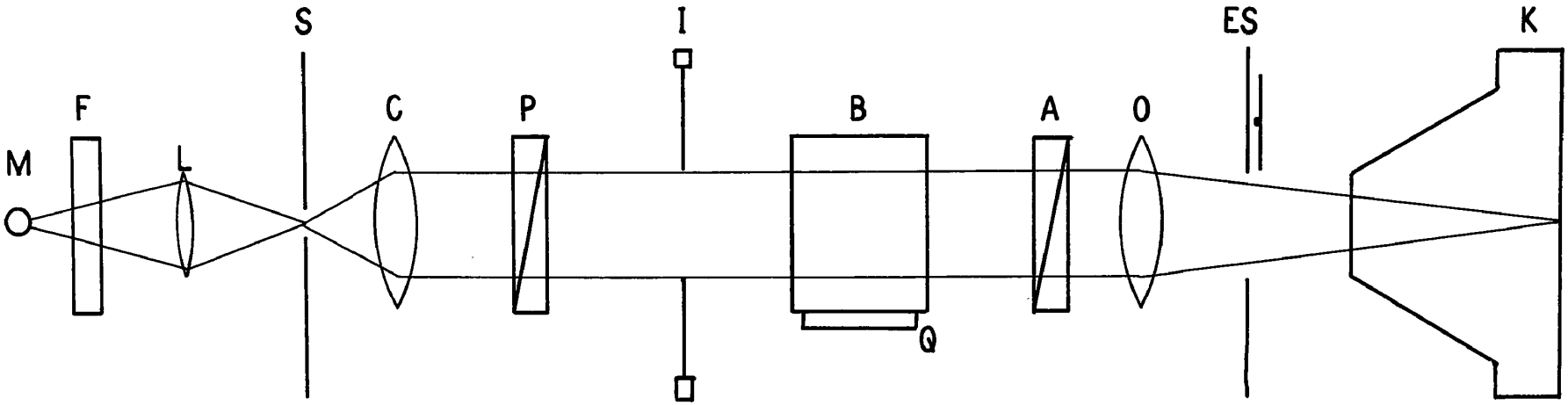
Description of the Experimental Apparatus

The Optical System

The arrangement of apparatus on the optical bench is shown schematically in Fig. 6. The light source M was a mercury arc lamp. The light from this lamp passed through the filter F and was focused on the slit S by a short focal length lens L. The light from the slit was collimated by the lens C and passed through the polarizer P, the glass block B, and the analyzer A, to the lens O. This lens focused the image of the slit on the film plane of the camera K. An iris diaphragm I placed just before the glass block limited the light beam to keep light from passing around the block. An electric shutter ES was placed between the lens O and the camera. Some notes about these optical elements:

1. The mercury arc serving as a light source was fitted with the filter F for eliminating all but the λ5461 green line. It was found to be feasible to remove the blue section of this filter for most purposes. By doing so the intensity of the green line was appreciably increased, allowing shorter exposure times, while the effect of the faint red line passed by the remainder of the filter was negligible. In some cases the red line did produce a diffracted first order, but it was weak, out of focus, and easily ignored.

2. The slit was a fully adjustable commercial unit with the jaws replaced by razor blades. Early attempts to adjust the blades to be



M - MERCURY ARC
F - FILTER
L - LENS
S - SLIT

C - COLLIMATING LENS
P - POLARIZER
I - IRIS
B - GLASS BLOCK
Q - QUARTZ PLATE

A - ANALYZER
O - OBJECTIVE LENS
ES - ELECTRIC SHUTTER
K - CAMERA

FIG. 6. COMPONENTS OF THE OPTICAL SYSTEM

parallel were fortunately not entirely successful, and as soon as the convenience of non-parallel jaws was discovered, they were allowed to remain as they were. Due to the changing brightness of the diffraction patterns from block to block, the total exposure time was difficult to judge. The converging slit allowed more latitude in this judgment. During the early adjustment of the slit a nick was accidentally put in the blades at the narrower end. This nick, nearly round in shape, was used as the aperture for the point source pattern of each diffraction picture; it proved to be an important part of the optical apparatus.

3. The polarizer and analyzer were two-inch disks of H-Polaroid mounted in glass. Complete extinction was not possible with these elements, which fact was helpful in that the crossed polarizer-analyzer position which passed the most light when viewing the weak shear orders still allowed just enough of the strong longitudinal orders to pass to photograph the two simultaneously.

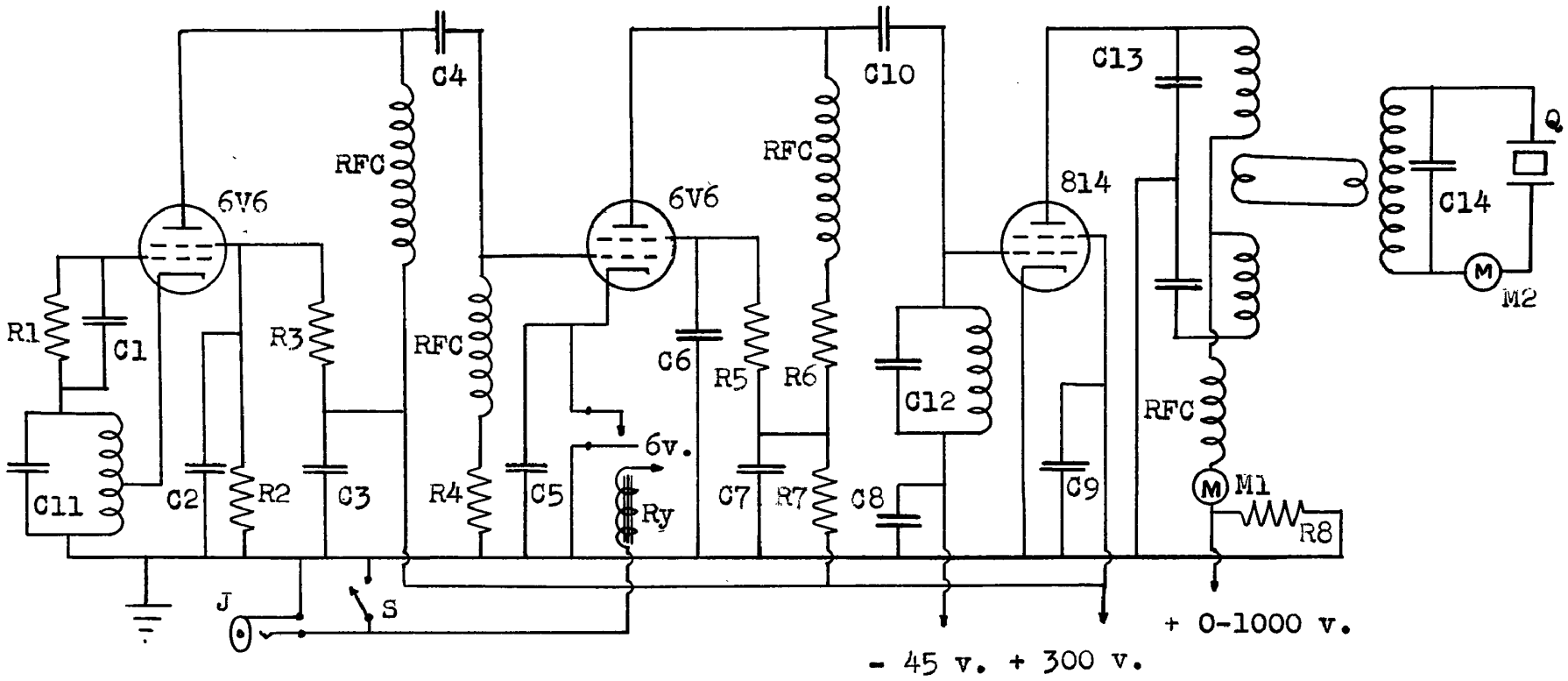
4. The films or plates were held in a lensless plate-back camera. The shutter was retained for convenience in preparing for a photograph. For photographic plates, several cut-film holders were modified to hold two-inch squares cut from Eastman Kodak 103-F spectroscopic plates. In general fast photographic emulsions were used to decrease exposure time, thus lessening the chance of frequency and temperature drift.

5. The electrically-operated shutter was built by the author

to be as simple a mechanism as possible. Its sole purpose was to interrupt the light beam during the part of the exposure when the driving voltage applied to the quartz was cut off. It consisted of a two-inch disk of light-weight blackened metal mounted on a thin counterweighted rod pivoted in the middle. A 28 volt DC relay was arranged to open the shutter when voltage was applied. The shutter disk was arranged to cover a 1 1/2 inch hole in the aluminum plate on which it was mounted, and the plate was attached to a heavy rod to allow mounting on the optical bench in such a way that the hole was centered on the axis of the optical system.

The Electrical System

The electrical system consisted of three parts, the radio-frequency oscillator which excited the quartz transducer, the exposure timing circuit, and the thermo-couple circuit. The circuit diagram for the oscillator unit is shown in Fig. 7. The circuit itself is a modified version of a short-wave transmitter. It was constructed by the author. It consisted of a variable frequency oscillator, a buffer-doubler stage, and a power amplifier. The output of the amplifier was link-coupled to a tuned tank circuit for matching impedance to the high impedance quartz. The variable frequency oscillator allowed continuous tuning over small ranges while hunting for diffraction patterns, and its control was placed in such a position as to be convenient to the operator while peering through the rear of the camera. A radio-frequency



R1=68K ohms	R7=5K ohms	C5=.1 mf.	C11, C12, C13,	RFC's-- R.F. Chokes
R2=100K ohms	R8=1.6M ohms	C6=.1 mf.	C14--variable	Q-- X-cut quartz
R3=5K ohms	C1=.0008 mf.	C7=.00022 mf.	tank capacitors	J--phone-type jack
R4=47K ohms	C2=.1 mf.	C8=.00025 mf.	M1--300 ma.	Ry-- keying relay
R5=5K ohms	C3=.00022 mf.	C9=.00047 mf.	meter	S-- keying switch
R6=5K ohms	C4=.0001 mf.	C10=.0001 mf.	M2-- 0-5 a. meter	

FIG. 7 . Radio-frequency Oscillator Circuit

ammeter was installed in series with the quartz crystal for the purpose of tuning controls other than the oscillator for maximum output. The voltage applied to the power amplifier was adjustable by means of a continuously variable transformer. The keying circuit of the original transmitter was retained to allow intermittent operation of the oscillator during the exposure as controlled by the timing circuit.

The timing circuit, shown in Fig. 8, served the dual purpose of recording total exposure time and breaking up the exposure into a number of short intervals. It consisted of a one RPM motor which operated a micro switch by means of various cams. The cams turned the instruments on for a short time, then off for a longer time, continuously. Three seconds on and twelve off, three seconds on and seventeen off, five seconds on and ten off, were various useful intervals built into the cams. The cam-operated microswitch controlled the relay, which in turn operated three pieces of equipment. When the relay closed, the electric shutter opened, the oscillator was keyed on, and a timing clock started. When the microswitch opened, the relay opened and turned off the three instruments. An extra switch allowed the timer to be used independently of the timing circuit for photographic developing work. The choice of cams depended on the heating effect of the ultrasonic wave in the glass sample, as measured by the thermocouple circuit. This circuit saved the operator much effort and allowed him to undertake other necessary duties in the dark-

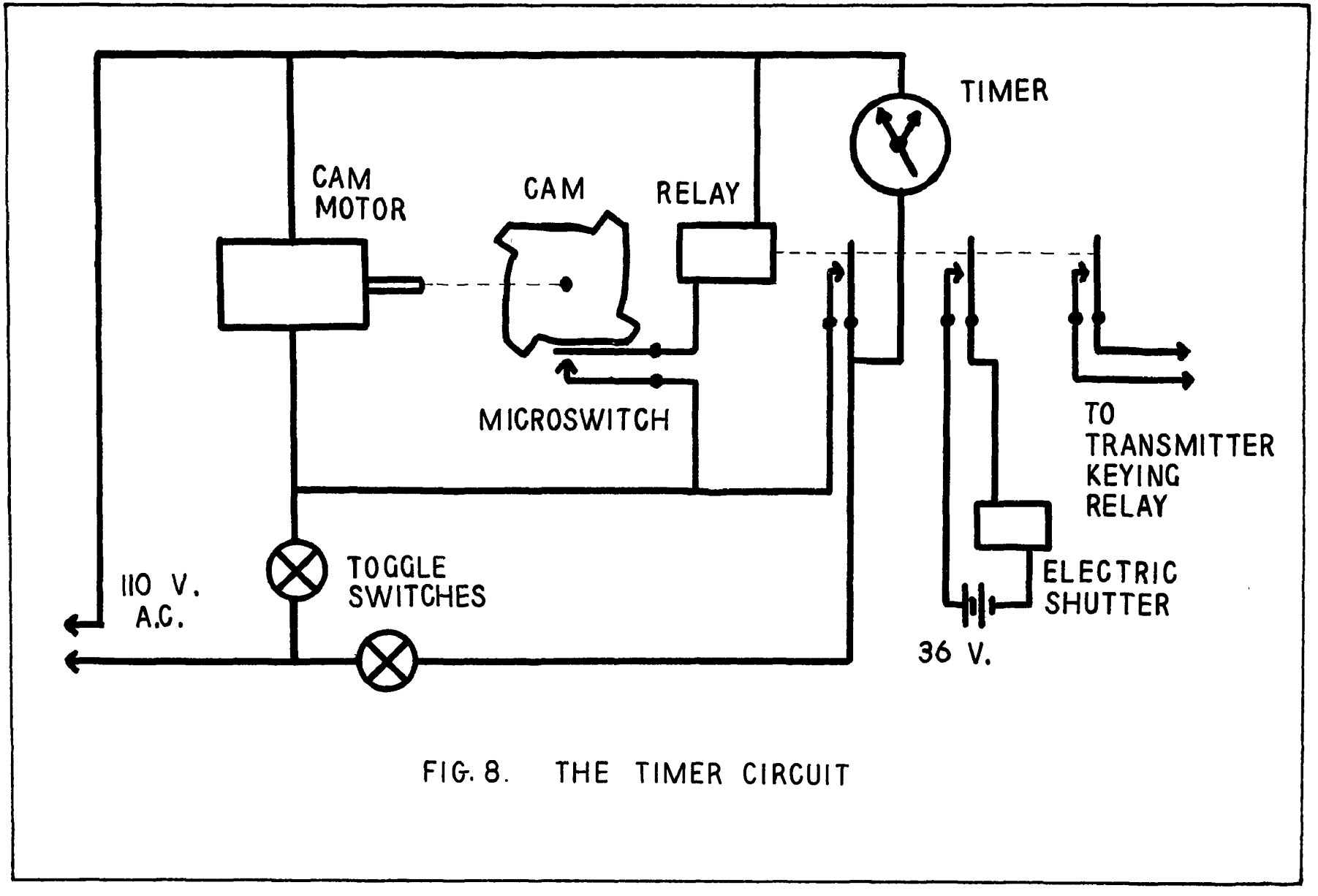


FIG. 8. THE TIMER CIRCUIT

ened room when exposure times of long duration were required.

The thermocouple circuit consisted of a fine copper-constantan junction soldered to a 1/4 inch diameter disk of thin copper, an ice bath, and a continuously operating, self-balancing, recording potentiometer. The electrical connections were conventional. The small copper disk was pressed against the upper surface of the glass block under a small pad of sponge rubber by means of a flat spring. The thermocouple circuit was not intended to give a true temperature of the glass block, but only to give the temperature of a portion of the surface in order to prevent overheating and indicate when a steady state had been reached. Temperature readings were taken to the nearest 1/2 degree centigrade.

Mounting of the Glass Block

The glass block itself was mounted on a small glass-covered table on the optical bench. The method of mounting the quartz crystal to the block was as follows:

After polishing all surfaces of the glass cube, it was placed upside down on the workbench. A small dot of silicone stopcock grease the size of a match head was placed in the center of the upper surface. Then a strip of aluminum foil 1 1/2 inches wide and 4 inches long was lightly placed over the grease spot. A plate glass block one by two by two inches, not otherwise used in the investigation, was placed on the aluminum foil and pressed down with as much force as possible to

spread the grease under the foil into a thin film. This method generally eliminated all air between the foil and the glass surface. The heavy block was then removed. The foil was folded down on two surfaces adjacent to the greased surface so that one could hold the block and foil as a unit without one slipping with respect to the other during the next step.

An equal amount of grease was then put on the upward surface of the foil on the block. The quartz crystal, one inch square, was cleaned and placed on this dot of grease. The plate glass block was used again for pressing the quartz down and creating another thin grease layer while the block and foil on it were held firmly. When as much of the grease had been squeezed out as possible, the quartz was centered on the glass block by sliding it while still under pressure. The heavy piece of plate glass was laid aside. The assembly of quartz, glass sample, and aluminum foil was inverted and the aluminum foil trimmed so that one piece about two inches long extended from one side. This piece of foil served as the upper electrode connection on the quartz, while the thin grease layer served as the acoustical conductor from quartz to glass. For some glass samples which seemed difficult to drive acoustically, a plastic cement was used in place of the grease.

The glass on the small table on the optical bench was covered with a piece of flat aluminum foil to serve as the lower quartz

electrode. Mounting the quartz and block on the optical system simply consisted of placing them on the foil-covered table and connecting the electrodes to the oscillator with small clips. Since no grease was used under the quartz, the thin film of air served as a mismatch of acoustical impedance, and little energy was radiated downward. This method of mounting allowed one to change glass samples in the shortest possible time.

Measurement Procedure

The procedure for taking a measurement photograph began with the adjustment of the optical system. With the ground glass viewing plate in the camera and a magnifying lens focused on this plate, the camera was moved until the slit image was in sharp focus. Then with the removal of the ground glass, the slit image was viewed directly through the magnifying lens. Next the frequency of the oscillator was adjusted until a diffraction configuration was found which one wished to record. The timer circuit was started as well as the temperature recorder, and the system was allowed to run several minutes. A small blower placed two feet or so above the block was turned on to keep the air stirred up in the vicinity of the glass block. A rise in temperature was always expected due to dissipation of the ultrasonic energy within the block in the form of heat. As the temperature changed, the frequency of the oscillator had to be readjusted to keep the desired diffraction pattern. This was due to the dependence on temperature of the elastic constants and volume expansion of the block with increase of temperature which caused a slight change in the standing ultrasonic wave system within the block. If the temperature kept increasing, either the cam operating the timer was replaced with one which gave less quartz excitation time per minute, or the voltage applied to the radio-frequency power amplifier was lowered to reduce the energy being dissipated in the glass. When it appeared that the

system had stabilized, the room was darkened, the photo plate was put in place, the electric shutter was positioned in the optical system, the timing clock was reset to zero, and the exposure was begun. A mark was put on the temperature record at the beginning and end of the exposure by shorting the thermocouple leads momentarily with a tapping key. During the exposure the frequency meter was read several times. For each exposure the following quantities were recorded in the date record book: date, photo plate number, type of emulsion, glass sample number, exposure time, surface temperature from the recorder, R. F. quartz current, R. F. amplifier plate voltage, polarizer-analyzer position, timer cam type, and frequency.

The length of actual exposure depended mainly on the photographic emulsion speed and the brightness of the diffraction image. The latter soon became something the operator could judge readily, and with the converging slit a wider latitude of error in this judgment was acceptable. During the last several hundred exposures made by the author on Eastman Kodak 103-F Spectrographic plates the exposure time varied between 1/2 to 1 minute. The actual time elapsed during the exposure was generally of the order of 3 to 10 minutes, however.

Calibration of the optical system was done with a comparison grating. This grating was obtained by the author by photographically reducing a large drawing onto a glass photographic plate. An optical bench accessory consisting of a cross slide which held two posts was

used to hold the comparison grating and the glass block table. Immediately after each exposure for data the calibration grating was moved into the position formerly occupied by the glass sample, the photographic plate holder raised in the camera about 3/4 of an inch, the polarizer rotated 90^{o}, and an exposure of about 6 seconds made. In this manner every ultrasonic diffraction pattern could be compared with the pattern due to the line grating.

A comparator having a least count of 0.001 mm. was used for all linear measurements. In order to obtain the grating constant for the comparison grating, it was measured over a range of 40 to 60 lines about 20 times and an average value calculated. For measurement of the data recorded on the photographic plates, a setting was made and read on each line ten times.

The frequency of the voltage applied to the piezo-electric quartz was measured to the nearest kilocycle per second on a reliable frequency meter. This meter was checked against radio station WWV and found to have a deviation much less than one kilocycle per second. The maximum error in determining the frequency was estimated to be less than one kilocycle per second on all readings.

The density of the glass samples was determined to five significant figures by weighing the blocks in water and in air on an analytical balance, taking every precaution. These measurements were repeated twice with good agreement.

Experimental Materials and Results

The velocities and densities of thirteen different types of optical glass were measured by the methods given in this paper. Nine samples of Bausch and Lomb optical glass were chosen from their catalog to cover the range from the lightest to the most dense types. One inch cubes were cut, annealed, and polished for us by that company. Three samples of rare earth optical glass were obtained from the Eastman Kodak Company. These samples were slightly smaller than one inch cubes. The fused silica sample was obtained for us by the Owens-Illinois Glass Company. It was understood that this sample was of the highest quality in respect to its homogeneity. Tables A and B give the percentage composition by weight of these samples as given by the respective glass manufacturer. The fused silica sample was assumed to be 100% silicon dioxide.

TABLE A

Per Cent Composition by Weight, Dispersion, and Index of Refraction of Bausch and Lomb Glass Samples

Glass Type Melt No.	BSC-1 0-9560	C-1 0-7763	CF-1 0-9841	LF-1 0-9447	LBC-2 0-8762	DF-2 0-9001	DBF-1 0-9271	DBC-2 0-8472	EDF-4 0-8030
SiO_2	70.8	71.7	67.8	54.3	49.1	46.6	45.0	39.0	31.5
K_2O	12.1	2.0	11.2	8.0	7.8	6.4	6.3	--	1.6
Na_2O	7.5	13.7	2.0	3.0	0.5	--	2.0	--	--
PbO	--	--	9.0	32.5	--	46.3	38.2	1.6	64.5
BaO	--	--	--	2.1	31.0	--	7.6	42.7	--
CaO	1.3	9.0	--	--	--	--	--	--	--
B_2O_3	8.0	1.1	--	--	3.4	--	--	4.7	--
ZnO	--	1.5	4.0	--	7.5	0.4	0.8	5.4	--
ZrO_2	--	--	--	--	--	--	--	0.5	0.1
Al_2O_3	--	--	--	--	--	--	--	5.1	--
Sb_2O_3	--	1.0	6.0	--	0.5	0.3	--	0.8	2.0
As_2O_3	0.3	--	--	0.1	0.2	--	0.1	0.2	0.3
Nominal n_D	1.511	1.523	1.539	1.573	1.573	1.617	1.617	1.617	1.751
Nominal Dispersion	63.5	58.6	51.6	42.5	57.4	36.6	38.5	54.9	27.8

TABLE B

Approximate Per Cent Composition by Weight
of Eastman Kodak Rare Earth Glass Samples

Glass Type→	EK-110	EK-330	EK-450
La_2O_3	20	28	40
ThO_2	20	12	6
BaO	14	12	--
SrO	6	--	--
B_2O_3	40	30	22
Ta_2O_5	--	18	11
BaO + SrO	--	--	6
WO_3	--	--	4
Al + Si + Zr + Ti (oxides)	--	--	11

Diffraction patterns were obtained from each of the samples as far as possible at frequencies of 10, 10.5, 12, 13.5, 14, 15, and 16.5 megacycles per second, using X-cut quartz plates, 1 inch square, of fundamental frequencies 1.5, 2, 3, 4, and 5 megacycles per second. These patterns were measured with a reliable comparator, along with the 3 or 4 order diffraction pattern produced by the comparison grating on the same glass plate. Knowing the comparison grating constant ($d = 0.2319$ mm. $\pm 0.04\%$), the wavelengths were calculated by using equations (7a) and (7b). The velocities were calculated from the values of the frequency and the wavelengths using equations (8a) and (8b).

Table C is a sample tabulation of all the velocity measurements for a particular sample of glass, in this case the Bausch and Lomb type DBF-1. The frequency column shows that a full spread from 10 to 16.5 megacycles per second was possible for this type of glass. In certain of the samples, notably the Eastman Kodak glasses, it was not possible to find suitable diffraction patterns at some of the higher frequencies. The temperatures in the next column were those at a point on the upper surface of the sample and, as said before, were not truly indicative of the internal temperature of the glass sample during its period of excitation. They are for comparison purposes only. No trend of change of velocity with this temperature was noted.

TABLE C

Sample Tabulation of all Measurements on the Sample of Bausch and Lomb Type DBF-1 Glass

Trial	Frequency	Temperature	Longitudinal Velocity	Shear Velocity	Orders
	mc/sec	°C.	m/sec	m/sec	
1	11.749	26.5		2589.	
2	11.815	26.5	4361.		4
3	10.021	27.	4370.	2585.	4
4	13.980	29.		2589.	
5	14.042	29.	4367.		8
6	14.731	28.	4366.	2590.	4
7	16.605	25.5	4369.		2
8	13.532	29.	4362.		2
9	10.535	25.5	4358.	2597.	4
10	13.534	27.5	4366.		6
11	13.465	28.		2591.	
Mean Velocities			4365.	2590.	
Probable Numerical Error			3	3	
Probable Error (%)			.07%	.10%	

The next two columns are the ultrasonic velocities as calculated from the measurements of the diffraction patterns. The number of orders of the longitudinal wave diffraction pattern for the particular photograph is given in the last column. The shear wave patterns had, of course, only first order patterns. Note that it is only occasionally that one gets a useful diffraction pattern resulting from both the longitudinal and shear waves. The patterns where simultaneous diffraction due to both waves occurred often showed multiple diffraction.

The mean values of the velocities are shown underneath the velocity columns. The numerical and percentage probable errors are given next below. Among all the samples the maximum probable per cent errors were 0.13% for the longitudinal velocities and 0.25% for the shear velocities. Averages values were respectively 0.10% and 0.17%. A weighted mean of the longitudinal velocities of each of the samples, weight being assigned to each measurement according to the number of orders measured, was considered, but in general the change of the value of the mean was less than the probable numerical error.

The results of the measurements of the velocities and densities of all the glass samples are listed in Table D. The probable error claimed for this set of measurements is the maximum value noted in the last paragraph, 0.13%, for longitudinal velocities and

TABLE D

Velocities and Densities of the Glass Samples

Glass Type	Longitudinal Velocity (m/sec)	Shear Velocity (m/sec)	Density (g/cc)
BSC-1	5893.	3572.	2.4766
C-1	5797.	3473.	2.5268
CF-1	5086.	3088.	2.6924
LF-1	4597.	2762.	3.1742
LBC-2	5333.	3109.	3.1424
DBF-1	4365.	2590.	3.6008
DF-2	5156.	2908.	3.6441
EDF-4	3704.	2185.	4.7189
DBC-2	4140.	2469.	3.7813
EK-110	5679.	3102.	4.1317
EK-330	5578.	3039	4.5720
EK-450	5804.	3143.	4.6293
Fused Silica	5963.	3766.	2.2027

0.25% for shear velocities. The error in the measurement of the densities was less than 0.05%.

The elastic moduli of the samples are given in Table E, as calculated from the values in Table D and equations (9) to (13). Calculated with the mathematics of propagation of precision indices,[18] the probable errors in μ , λ , and k are 0.5%, while those of σ and E respectively are 0.7% and 0.8%.

In Table F are the results of the calculations of the surface tensions of the various samples according to Auerbach's method, using equation (14). Inasmuch as the precision of the velocity and density measurements gives these quantities to four significant figures while the empirical constant 6.70 is of fewer significant figures, no accuracy can be estimated.

TABLE E

Elastic Moduli of the Glass Samples

Glass Type	Shear Modulus ($dyne\ cm^{-2}$)	Lame's Modulus ($dyne\ cm^{-2}$)	Poisson's Ratio	Young's Modulus ($dyne\ cm^{-2}$)	Bulk Modulus ($dyne\ cm^{-2}$)
	$x10^{11}$	$x10^{11}$		$x10^{11}$	$x10^{11}$
BSC-1	3.160	2.289	0.2100	7.638	4.393
C-1	3.048	2.396	0.2201	7.437	4.428
CF-1	2.567	1.830	0.2081	6.203	3.541
LF-1	2.422	1.865	0.2175	5.897	3.479
LBC-2	3.037	2.862	0.2426	7.548	4.887
DBF-1	2.416	2.030	0.2283	5.934	3.640
DF-2	3.082	3.524	0.2668	7.807	5.579
EDF-4	2.253	1.968	0.2332	5.556	3.470
DBC-2	2.305	1.871	0.2240	5.643	3.408
EK-110	3.976	5.374	0.2874	10.237	8.024
EK-330	4.222	5.080	0.2730	10.751	7.895
EK-450	4.573	6.448	0.2925	11.822	9.497
Fused Silica	3.124	1.584	0.1682	7.299	3.667

TABLE F

Surface Tensions of the Glass Samples

Glass Type	Surface Tension dynes/cm.
BSC-1	706.
C-1	703.
CF-1	615.
LF-1	623.
LBC-1	771.
DBF-1	663.
DF-2	850.
EDF-4	670.
DBC-2	635.
EK-110	1110.
EK-330	1200.
EK-450	1290.
Fused Silica	639.

Summary

Using a system which involved the diffraction of light by ultrasonic longitudinal and shear waves, the lengths of these waves were measured in thirteen samples of optical glasses at frequencies ranging between 10 and 16.5 megacycles per second. The probable errors in the determination of these wavelengths were 0.13% for the longitudinal waves and 0.25% for the shear waves. From these values together with the densities of the materials, the elastic constants were calculated. The calculated probable errors for these values were 0.5% for the bulk modulus, shear modulus, and Lame's modulus, 0.7% for Poisson's ratio, and 0.8% for Young's modulus.

It was pointed out that the true spacing of the diffraction lines requires that the wave producing the diffraction have its direction of propagation perpendicular to the slit and that this condition is best investigated by observing the diffraction patterns due to a point source and a slit source photographed simultaneously.

The surface tensions of the samples were calculated from the measurements of sound velocity and density using Auerbach's relation.

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