SOLAR RADIATION AVAILABILITY ON SURFACES IN THE UNITED STATES AS AFFECTED BY SEASON, ORIENTATION, LATITUDE, ALTITUDE AND CLOUDINESS

By

Clarence Frederick Becker

AN ABSTRACT

Submitted to the School for Advanced Graduate Studies of
Michigan State University of Agriculture and Applied
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Approved by James S. Boyd

Radiation from the sun is the primary source of all energy used by mankind. The life expectancy of fossil and nuclear fuel reserves are difficult to determine with accuracy; however, the earth's supply of fuels is limited and severe shortages of this form of fuel are possible within a century.

There is a need to learn how to use directly the daily supply of energy from the sun before stockpiles of other energies are exhausted. Some of the potential uses for solar radiation on the farm, such as final drying of grain, hay and other agricultural crops can utilize solar radiation with little disadvantage due to the intermittency of the radiation.

It is necessary to know the availability of solar radiation at the surface of the earth if it is to be used in engineering applications. The United States has a network of Weather Bureau stations which measure and record solar radiation incident upon a horizontal surface. Records of radiation incident on vertical surfaces are available for one United States station.

Methods for estimating the quantity of solar radiation available on surfaces with various orientations in the United States are presented in this thesis.

Calculations of hourly and daily total cloudless day radiation incident on surfaces with various orientations

and located at different north latitudes with elevations at or near sea level are presented. Seasonal curves showing the availability of the solar radiation on surfaces with various orientations have been constructed and polynomials have been determined for the curves representing the availability of solar radiation on a horizontal surface.

Optimum tilt angles of solar collectors for the various seasons and latitudes are presented and the ratios of solar radiation incident upon tilted surfaces with various tilt angles to that incident on horizontal surfaces are presented in the form of curves for which polynomials have been developed. The calculated ratios can be multiplied by the values given for a horizontal surface to give the corresponding value for a tilted surface at various tilt angles.

Curves and polynomials giving the percentage increase in solar radiation with altitude are shown which can be used to correct the values for conditions at or near sea level.

Comparisons between the calculated cloudless day radiation determined from the curves and polynomials developed above and recorded clear day solar radiation are presented.

The correlation between the ratio of observed to calculated cloudless day radiation and percentage of possible sunshine is developed. The resulting regression equation gives a method for correcting the cloudless day values for cloudiness by relating it to a parameter which is measured

at many places and for which long time averages are available.

The variability of the availability of solar radiation is discussed and the distribution of days with various amounts of solar radiation and the mean total solar radiation in various categories are shown for Madison, Wisconsin based on data beginning July of 1941. The sequence of dark days for the various months is determined for the same station during the same period.

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INTRODUCTION

Radiation from the sun is essentially the primary source of all energy used by mankind. Energy supplies that are most easily utilized, such as the fossil fuels, have undergone natural concentration. Energy from the sun lifts water vapor above the earth, and this energy is partially recoverable in the form of water power. Energy for moving masses of the atmosphere over the surface of the earth, or the production of wind, comes from the sun.

Statement of the Problem

The life expectancy of fossil fuel reserves is difficult to determine with accuracy, for it involves not only the uncertainty of estimates of the quantity of fuel present, but also the prediction of rates of production and of demand (2). In addition, there are coal, oil, and gas in the earth's crust that may never be used because it would not pay to dig to the depths necessary to recover them. The finding of large deposits of uranium in recent years will alleviate the energy situation, however, this type of fuel, like fossil fuels, cannot be considered inexhaustible.

It is evident that the earth's supply of fuels, wood, gas, oil and coal is limited and that severe shortages of this form of energy is possible within a century. There is a need to learn how to use the daily supply of energy from the sun directly before the stockpile of other energies is exhausted.

Solar radiation energy is immense in quantity but relatively low in intensity. Threlkeld (28) estimates that the solar radiation intercepted by the earth each day equals about 5600 million million BTU. Because of the vastness of the supply of solar energy, the direct conversion of solar radiation into useful forms of energy has great attractiveness. Solar radiation is currently being used on a limited basis for distillation of salt water, for manufacture of salt by solar evaporation, for water heating in Southern United States, and for generation of electricity by thermopiles, the photogalvanic effect, or by the solar battery utilizing a silicon p-n junction. Research is under way to determine other methods of transforming and storing solar energy. Work is being done in photosynthesis in an attempt to shorten the time necessary for completion of the process by which petroleum was naturally formed (22); this work is coupled with the culture of algae and the subsequent production of carbohydrates with the most desirable properties.

The utilization of solar energy by the above-mentioned methods is still very inefficient. However, the direct collection of solar energy as heat is much more efficient and the greatest progress is likely to take place in this direction in the near future. Experimental solar collectors for heating air have been built by Buelow (5), Telkes (26) and others (9), and collectors for heating water have been built by Hottel (12) and the University of Florida (9). These collectors show efficiencies as high as 50 to 80 percent.

The possibility of utilizing solar energy on the farm has been investigated only to a very limited extent, even though there are many potential uses. In many respects it seems to be the logical place to start, for one of the disadvantages of solar radiation, namely, its intermittency, is not critical for such uses as final drying of grain, hay and other agricultural crops. Where solar energy is to be used for space heating, some means of heat storage at relatively high temperatures is necessary. The efficiency of collection is thereby reduced, since threshold intensity of radiation must be reached before energy can be collected and stored. Solar radiation of relatively low intensity could be utilized for drying agricultural crops, where any amount of heating of air would be advantageous.

A solar energy collection and storage system could also supply supplemental heat to farm buildings.

There is a network of Weather Bureau Stations in the United States which measure and record solar radiation incident upon a horizontal surface. The first solar station in the United States was started at Madison, Wisconsin, in 1911. The number of stations has increased from ten in 1940 to twenty-five in 1949, and to seventy-five today. Daily total values of solar radiation incident upon a horizontal surface are published by the Weather Bureau (31). In addition to the data for radiation on horizontal surfaces, a station of the Weather Bureau at Blue Hill, Massachusetts, measures and publishes data for radiation incident upon vertical surfaces facing north, south, east and west.

Objective of Thesis

To utilize solar energy at the surface of the earth for engineering application, it is necessary to determine the availability of the supply.

In many cases it may be desirable to orient surfaces other than horizontally for more efficient collection of solar energy or for structural reasons. Frequently the collecting surface will be incorporated into the wall or roof of a house or farm building in which case the horizontal orientation may not be practical.

It is the purpose of this thesis to construct curves and develop equations useful for estimating quickly the quantity of solar radiation energy available on surfaces with various orientations anywhere in the United States.

History of the Use of Solar Radiation

Investigations on potential and actual use of solar radiation are not new since the sun was recognized as a source of energy hundreds of years ago. Green (9) made a brief summary of the history of solar utilization which is quoted as follows:

> In 1902 H. E. Willsie, American Engineer, built a binary-vapor pumping plant run by a collector. Heated water was stored and used to boil off sulphur dioxide which ran a small pumping engine. The heater and collector had an asphalt bottom, wooden sides, and was covered with glass. On later more efficient collectors he used a double glass cover and tilted the collector so that it was perpendicular to the sun's rays. Several such plants were built after 1909 by Frank Shuman, Philadelphia engineer.
>
> The oldest method of collecting solar energy at high temperature was by concentrating the rays of the sun upon a surface by the means of mirrors. Archimedes is said to have used this method as early as 214 B.C. to set fire to Roman ships while they were at considerable distance from the shores of the island of Sicily.
>
> In 1878 August Mouchot first generated power from the reflected and concentrated rays of the sun. He used a huge reflector shaped like a lamp shade and lined with burnished silver to reflect and concentrate rays of the sun upon a boiler located at the focal axis of the reflector. By using a large reflector area as compared to boiler area, he was able to get a steam pressure of 75 pounds per square inch (325 F approximately) and run a one-half horsepower engine. The diameter of the conoidal reflector was eight and one-half feet.
>
> The noted engineer, John Ericson, later experimented with this type of equipment, using a parabolic

> trough lined with small mirrors to reflect the rays of the sun upon a small tubular boiler located along the focal axis of the reflector. He obtained as high as 210 BTU per square foot of sunshine collector.
>
> About 1901 the English inventor, A. G. Eneas, built several reflectors of the truncated-cone type that absorbed as high as 223 BTU per square foot of sunlight collected at their Arizona location.
>
> In 1913 a successful steam plant using parabolic-trough reflectors which were geared to an engine so as to be kept facing the sun continually, was built by Frank Shuman and Professor Boys at Meadi, Egypt. This plant developed 50 horsepower and collects 13,269 square feet of sunshine. (...)
>
> The University of Florida built a parabolic-trough heater 3 x 4 feet in 1934 with one and one-quarter inch pipe placed inside a 2-inch pipe located at the focus. Water circulated between the two pipes by gravity, provisions were made to measure the quantity of water and temperature rise; results showed as high as 128 BTU per square foot per hour of sunshine collected was possible. Operating temperatures as high as 220 F. were readily obtained. When filled with water and with all outlets closed, the steam pressure rose to 50 pounds per square inch gage, indicating a temperature in the absorber of approximately 300 F. (...)

Solar Radiation Outside the Atmosphere

Solar Constant

The solar constant is the energy incident upon a unit area located at mean distance of the earth from the sun and oriented perpendicular to the sun's rays outside the atmosphere. The most recent data available on the value of the solar constant is presented by Johnson (18). His value is 2.00 ± 0.04 calories per minute per square centimeter or about 440 BTU per square foot per hour. The latest value given by the Smithsonian Institute is 1.95 calories per minute per square centimeter. The difference between these two values lies in a rediscussion of the corrections applied to the measured data for the ultra violet and infrared spectral regions lying outside the observed region.

Spectral Distribution

The spectral distribution of solar energy outside the atmosphere is such that for all practical purposes, the energy lies between the limits of 0.22 and 7 microns. About 0.14 percent of the energy has wave length greater than 7 microns and 0.02 percent of it has wave length less than 0.22 microns.

According to Fritz (7), the energy in the ultra violet below 0.4 microns comprises about 9 percent of the total incident energy; the energy in the visible range (which contains the peak at 0.46 microns) is 41 percent and that in the infrared beyond 0.72 microns contains about 50 percent.

In tracing the solar spectrum down through the atmosphere, the short wave constituents are absorbed high in the ionosphere, principally by ozone. The absorption coefficient is such that the spectrum of solar energy at the ground is cut off below 0.29 microns.

There appears to be irregular changes in solar activity which change the solar constant. However, the effect on the integrated energy available for non-selective power sources is very small (11).

Position of the Earth Relative to the Sun

The angle at which the sun's rays reach a surface on the earth changes from day to day and from hour to hour owing to the changing position of the earth relative to the sun and the rotation of the earth about its axis.

As the earth moves about the sun in its seasonal orbit, its axis maintains a nearly constant angle of 66.5 degrees with the plane of its orbit. On June 21, the summer solstice, the sun is vertically overhead at solar noon at the Tropic of Cancer and has the greatest noon elevation in all north latitudes greater than 23.5 degrees. On December 21, the winter solstice, the axis of the earth points away from the sun and the angle which the sun's rays make with a horizontal surface on the earth is least in northern latitudes. The sun is directly overhead at noon at the equator on March 21, the vernal equinox, and on September 21, the autumnal equinox. Days and nights are of equal length throughout the world at the time of the equinox.

The sun is slightly off the center of the elliptical path which the earth makes around the sun. As a result, at aphelion (July 1), the distance of the earth from the sun is 1.034 that at perihelion (December 1). Hence, the intensity of sunshine outside the atmosphere on December 1, other things being equal, is approximately 1.069 that on July 1.

INTENSITY OF SOLAR RADIATION UPON SURFACES LOCATED AT THE SURFACE OF THE EARTH DURING CLOUDLESS DAYS

Direct Radiation

Direct Radiation during Cloudless Days

The quantity of direct solar radiation incident upon a surface located on the earth's surface is given by the formula: (see Glossary, page 104, for definition of terms)

$$I_1 = (J_o/r^2)t^m \cos i \quad \ldots\ldots\ldots \quad \text{(a)}$$

where I_1 = the solar energy incident upon the surface

J_o = the solar constant

r = the radius vector of the earth

t = the transmission coefficient of the atmosphere

m = the air mass

i = the angle of incidence.

In tracing the solar spectrum down through the atmosphere during cloudless days, depletion of the direct beam takes place by scattering and absorption (7, 21). Scattering is caused primarily by air molecules, dust, and to a certain extent, by water vapor along the path of the sun's rays to the surface of the earth. The principal absorbing agents are water vapor, ozone and cloud particles.

Direct Solar Radiation Incident upon a Surface Perpendicular to the Sun's Rays

In a very important paper, Moon (23) has calculated the direct solar radiation incident upon a surface normal to the sun's rays during cloudless days. His calculations of the spectral distribution of the energy at sea level are based on an assumed atmospheric condition of 2 centimeters of precipitable water vapor, 300 dust particles per cubic centimeter and 0.28 centimeters of ozone at 760 millimeters of mercury pressure and 0 C. The calculations were made for air masses of 0, 1, 2, 3, 4 and 5. Curves constructed by Moon from the calculations show the spectrum of solar energy cut off below 0.29 microns wave length, and a large vertical separation of the curves for short wave lengths. This vertical separation shows the influence of ozone absorption as the sun passes through longer and longer paths. The absorption bands of water vapor are very evident in the infrared region; no energy reaches the surface of the earth in the stronger water vapor bands.

Moon's data have been taken as standard for cloudless day summer conditions by the American Society of Heating and Ventilation Engineers. The integrated direct solar radiation as a function of the solar altitude appears in the Heating, Ventilating and Air Conditioning Guide (1).

F. W. Hutchinson and W. P. Chapman (14) have applied Moon's calculations to a standard winter atmosphere with assumed atmospheric conditions of 33 F dew-point, 300 dust particles per cubic centimeter, and 0.28 centimeters of ozone. The winter curve in Figure 1 is a copy of Hutchinson's data which show direct solar radiation at normal incidence as a function of the solar altitude angle. The summer curve is the result of a plot of the information presented in the Heating, Ventilating and Air Conditioning Guide (*op. cit.*). It is noted that for a given solar altitude, the summer value is lower than the winter value. The summer value is smaller because of an assumed higher moisture content and due to a greater solar distance.

Solar Angles

The sun as seen by an observer on the surface of the earth follows a circular arc from horizon to horizon. This position can be defined by the solar altitude (b), and the solar azimuth (a), Figure 2. These angles vary continuously from sunrise to sunset and are different for various days of the year. The diurnal variation is symetrical with respect to a north-south line and therefore with respect to solar noon.

The sun's altitude and azimuth can be secured from the United States Hydrographic Office Tables 214 (29) if the latitude, local hour angle, and declination of the sun are known. The declination of the sun can be secured from any

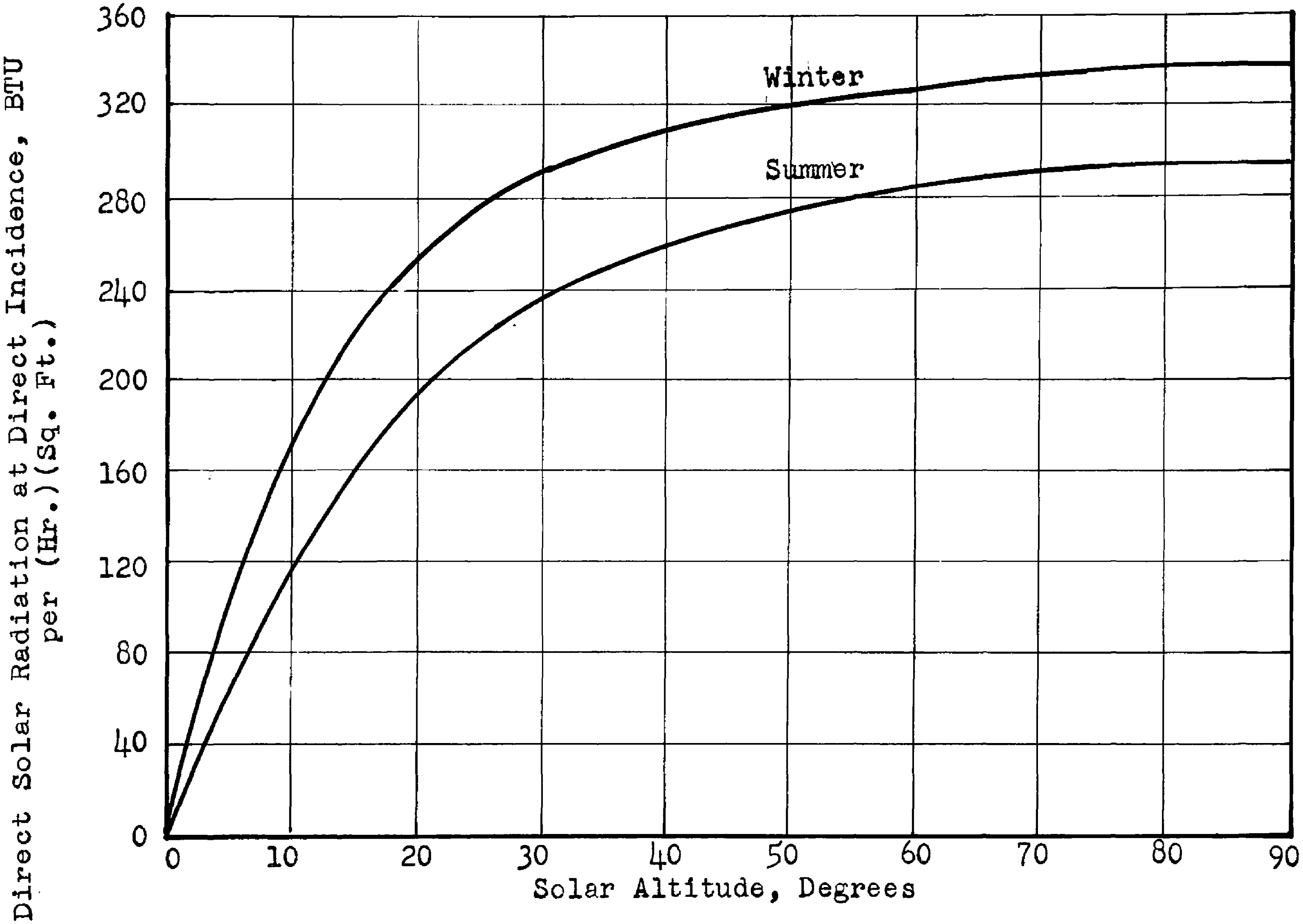
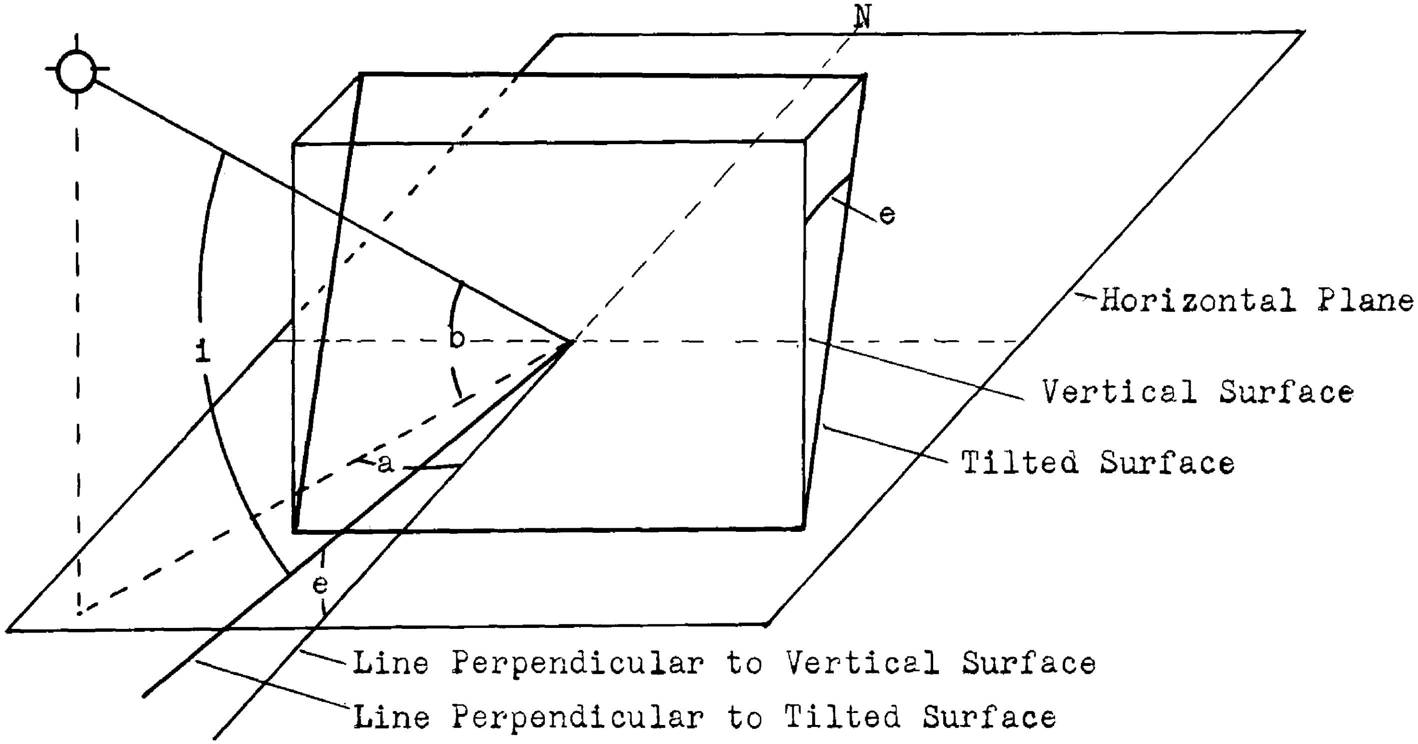


Fig. 1. Direct solar radiation incident upon a surface perpendicular to sun's rays at sea level on the earth during cloudless days. Winter curve from data by Hutchinson (14) and summer curve from data presented in 1954 Heating and Ventilating Guide (1). All data based on "Proposed Standard Radiation Curves for Engineering Use" by Moon (23).



a = Wall Solar Azimuth Angle
b = Solar Altitude Angle
e = Angle of Tilt from Vertical
i = Angle of Incidence

Fig. 2. Definition of solar angles

table of ephemeris. For all practical purposes, the declination is the same on January 21 and November 21, February 21 and October 21, March 21 and September 21, April 21 and August 21, and for May 21 and July 21; therefore, the solar geometry will be nearly the same on the dates with equal declinations. The local hour angle is expressed in degrees using 0 degrees for solar noon, 15 degrees for 11 a.m. and 1 p.m., 30 degrees for 10 a.m. and 2 p.m., etc.

Angle of Incidence for Direct Radiation

A formula for determining the angle of incidence of direct solar radiation upon a surface is given by Brown and Marco (4); their formula was changed to the following form by trigonometric substitution:

$$\cos i = \sin b \sin e + \cos b \cos a \cos e \quad \ldots. \quad (b)$$

where i = angle of incidence

a = wall solar azimuth

b = solar altitude

e = angular tilt of surface from vertical.

Figure 2 defines the solar angles used.

Horizontal surface. For a horizontal surface, e in equation (b) is 90 degrees and the equation becomes:

$$\cos i = \sin b \quad \ldots\ldots. \quad (c)$$

$$I_h = I_o \sin b \quad \ldots\ldots. \quad (d)$$

where I_h = direct solar radiation incident upon a horizontal surface,

I_o = direct solar radiation incident upon a surface normal to the sun's rays (Figure 1).

South-facing vertical surface. For a vertical surface, e in equation b is 0 degrees and the equation becomes:

$$\cos i = \cos b \cos a \quad \ldots\ldots\ldots \quad (e)$$

or

$$I_v = I_o \cos b \cos a \quad \ldots\ldots\ldots \quad (f)$$

where I_v = direct solar radiation incident upon a vertical surface.

South-facing tilted surface. For a south-facing tilted surface:

$$\cos i = \sin b \sin e + \cos b \cos a \cos e \ldots \quad (b)$$

$$I_t = I_o (\sin b \sin e + \cos b \cos a \cos e) \quad . \quad (g)$$

Sky Radiation

It was stated earlier that as the solar radiation passes through the atmosphere to the surface of the earth, the depletion of the direct solar radiation is due partially to scattering. A portion of the scattered radiation will return to space, but some of it will reach the surface of the earth as sky radiation.

According to **Fritz** (7), for an average clear sky, the diffuse sky radiation on a horizontal surface is about 16 percent of the total when the sun is high in the sky and about 37 percent of the total when the solar elevation is about 10 degrees.

In addition to diffuse radiation from the sky, some of the radiation reflected from the ground may reach a surface. The amount of the reflected radiation that reaches a surface will depend on the orientation of the surface.

The theory of radiation scattering is rather involved and at this time no theoretical method for calculating the quantity of sky radiation is known. Several investigators have measured sky radiation separately from the direct radiation. Klien (21) gives a method for computing sky radiation in terms of atmospheric conditions and Hand (10) has published information on the ratio of direct to sky radiation during typical winter cloudless days for horizontal surfaces.

Possibly the best information on sky radiation availability at this time has been presented by Parmelee (24). He has measured sky radiation on horizontal and vertical surfaces. In the paper cited, Parmelee plotted sky solar radiation (I_{dh}) versus I_h for a horizontal surface and sky solar radiation (I_{dv}) versus solar altitude (b) and wall solar azimuth (a) for the vertical surface; each was plotted for atmospheres with clearness ratios of 1.0, 0.8 and 0.6. He has defined the ratio of the observed intensity of direct radiation to that computed from Moon's data (Figure 1) as the clearness ratio because he noted that the direct radiation varied for a given solar altitude for cloudless skies and that as the intensity of direct radiation decreased, the intensity of the sky radiation increased. Figures 3 and 4 show Parmelee's data for a clearness ratio of one.

Total Solar Radiation

The total solar radiation during cloudless days incident upon a surface is found by adding the direct and sky radiation upon the surface.

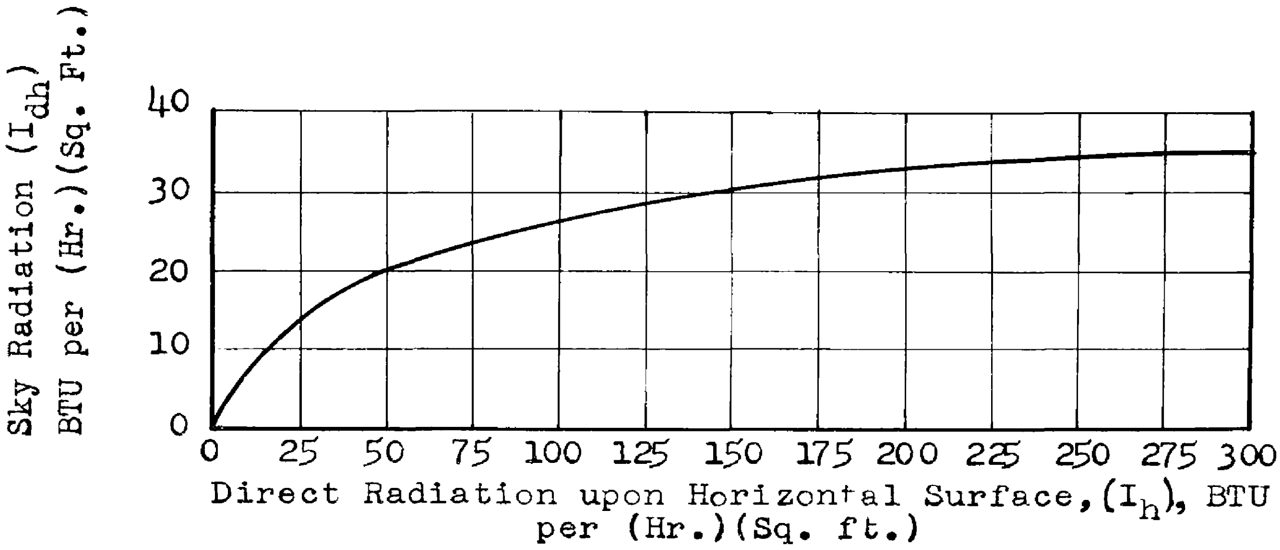


Fig. 3. Sky solar radiation (I_{dh}) for various values of direct radiation (I_h) upon a horizontal surface during cloudless days with clearness ratio equal to one. All values from Parmelee (24).

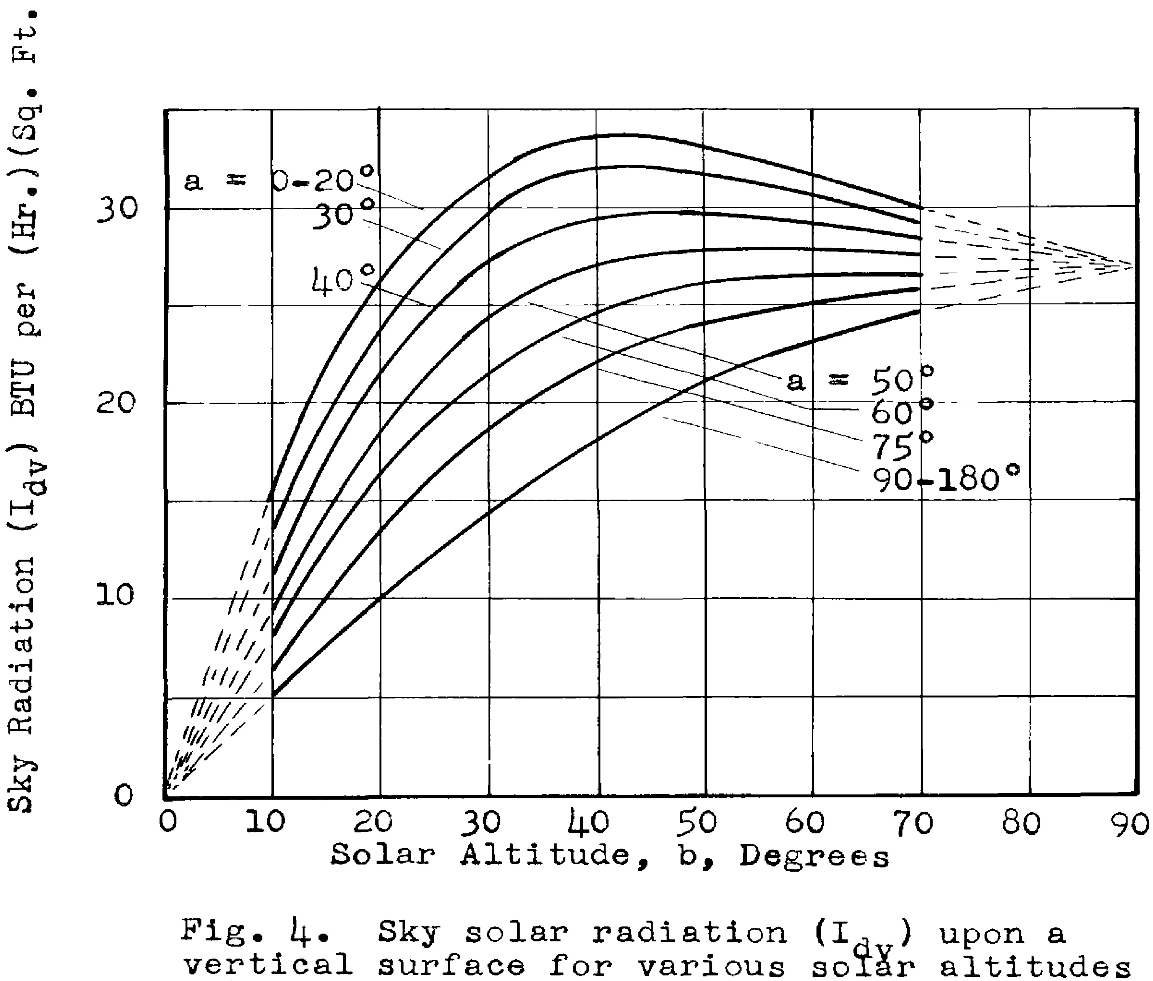


Fig. 4. Sky solar radiation (I_{dv}) upon a vertical surface for various solar altitudes (b) and wall solar azimuths (a) during cloudless days with clearness ratio equal to one.

DAILY TOTAL SOLAR RADIATION AVAILABLE AT THE SURFACE OF THE EARTH DURING CLOUDLESS DAYS FOR LOW ELEVATION AREAS

Procedure for Calculating Hourly Rates

The calculation of hourly rates of direct and sky radiation was carried out for the twenty-first of each month using the following procedure:

(1) The solar declination was secured for the twenty-first of each month from the Ephemeris (30).

(2) With the solar declination known, the solar altitude and the solar azimuth were secured for each daylight hour of the day at 30°, 35°, 40° and 45° north latitude from the United States Hydrographic Office Tables Number 214 (29).

(3) With the solar altitude for each hour known, the direct solar radiation at normal incidence (I_o) was secured from Figure 1, using the winter curve for the months of September through March and the summer curve for the remainder of the months.

(4) The hourly rate of direct solar radiation incident upon a horizontal surface, south-facing vertical surface, and south-facing surfaces tilted 30° and 60° from vertical, was calculated by using formulas d, f, and g, respectively.

(5) With hourly values for the solar altitude, the wall solar azimuth, and the direct radiation incident upon a horizontal surface known, Figures 2 and 3 were used to determine the hourly rate of sky radiation incident upon a horizontal surface (I_{dh}) and vertical surface (I_{dv}).

Since there is no information on sky radiation incident upon a tilted surface (I_{dt}), this value was estimated by linear interpolation utilizing the equation:

$$I_{dt} = I_{dh} + (I_{dv} - I_{dh}) \frac{(90° - e)}{90°} \quad ..... \quad (h)$$

(6) The total hourly rate of solar radiation was determined by adding the direct and the sky components.

Procedure for Calculating Daily Total Radiation

To determine the daily total radiation incident on the various surfaces, it is necessary to integrate the hourly values over the hours of the day. This integration was accomplished by using Simpson's Rule.

Results

Tables IV through VII show the results of the computations for 30°, 35°, 40° and 45° north latitude. The hourly rates of direct solar radiation agree fairly well with those presented by Hutchinson (15, 16, 17) except for the tilted surfaces, where he did not allow for any incident energy when the solar azimuth angle was greater than 90 degrees.

Horizontal Surface

Curves showing the daily total solar radiation incident upon a horizontal surface at 30°, 35°, 40° and 45° north latitude during cloudless days are shown in Figure 5.

A comparison of the curves in Figure 5 shows that the horizontal surface intercepts a maximum of radiation in summer, with the amount being nearly the same for all four latitudes in late June and early July at about 2650 BTU per day per square foot. This near equality is explained by the fact that the effect of greater solar altitudes near the middle of the day for the southern latitudes is counterbalanced by more daylight hours and by greater solar altitudes for the northern latitudes during the morning and later afternoon hours (see Tables IV through VII). The cloudless day radiation decreases from June to December to a value of about 50 percent of the summer value at 30° latitude and 25 percent of the summer value at 45° latitude. The decrease from summer to winter is explained by the greater angle of incidence owing to lower solar altitudes

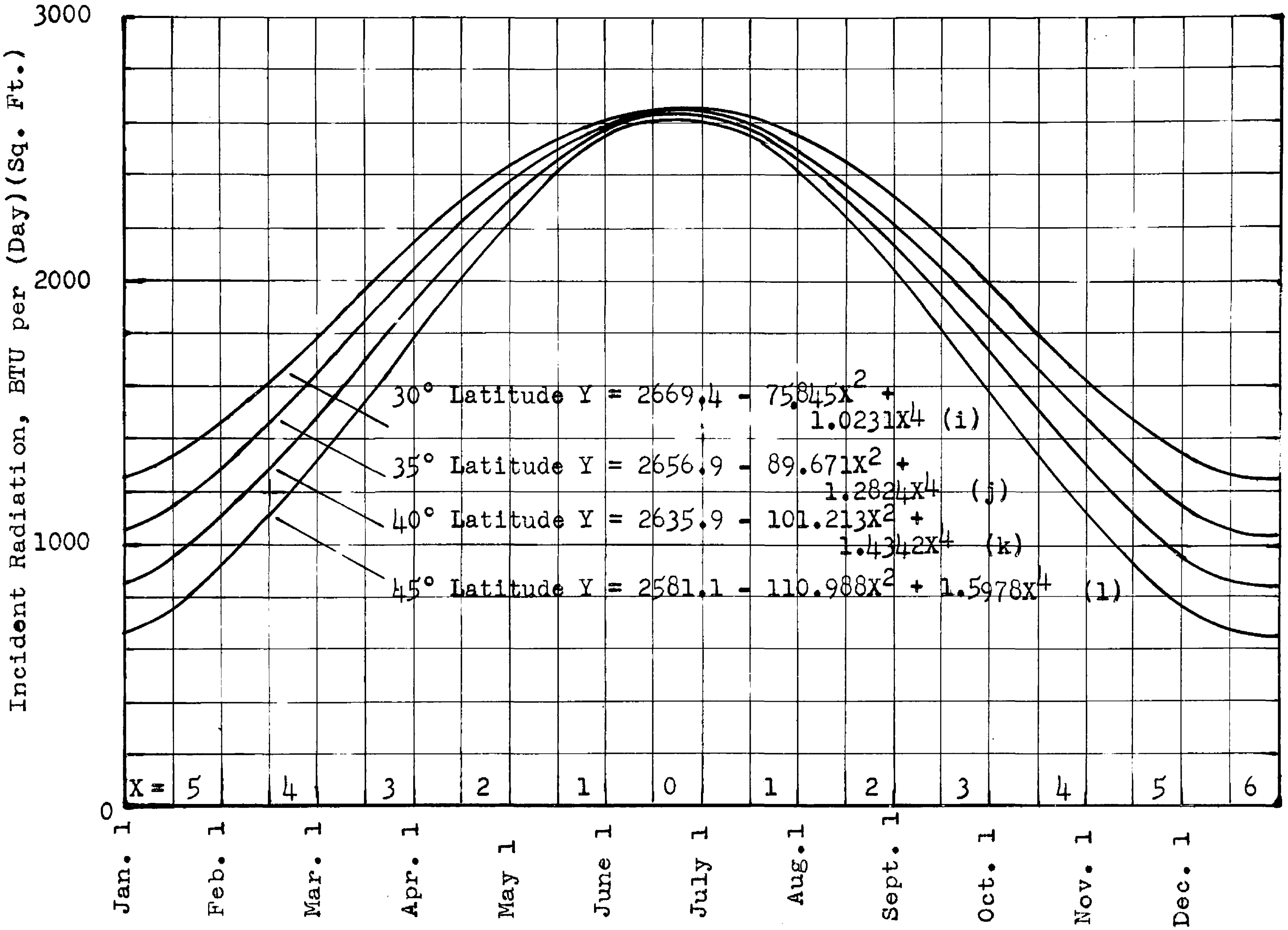


Fig. 5. Daily total direct and sky radiation incident upon a horizontal surface at various north latitudes during cloudless days.

and the smaller number of daylight hours for a given latitude in winter. The more pronounced change for northern latitudes in the number of daylight hours from summer to winter explains the larger decrease from summer to winter for 45° latitude as compared with 30° north latitude.

The shape of the curves was such that it suggested the possibility of being able to write equations for them. A method for determining the coefficients for a fourth degree polynomial suggested by Baten (3) was used in obtaining equations i, j, k, and l in Figure 5.

In computing these polynomials, the origin ($X = 0$) was located at June 21. X is the number of months, or factions thereof, from June 21. By using this origin and using $X = 6$ twice in the computation, fourth degree polynomials with exponents of the X and X^3 terms equal to zero were secured.

A sample computation using the computed data for 35° latitude is shown in Appendix II. The overall average deviation between the values determined from the polynomials and the corresponding calculated value for the 21st of each month is 15 BTU per square foot per day. The deviation amounts to less than 1 percent.

South-Facing Vertical Surface

Figure 6 shows the daily total solar radiation incident upon a south-facing vertical surface during cloudless days.

The curves of Figure 6 show that the solar radiation incident upon a south-facing vertical surface is at a maximum during the winter months. It is a maximum then because the lower solar altitudes have a more favorable angle of incidence on a vertical surface as shown by equation e . The normally higher values for northern latitudes are also explained by consistently lower solar altitudes than for southern latitudes. The dip in the curve for the more northern latitudes in December, which occurs even though the angles of incidence are more favorable then, is due to the increased depletion of the atmosphere and fewer daylight hours. It is noted that the curves for the four latitudes peak at between 1800 and 1900 BTU per day per square foot. The peaks are very nearly equal because the more favorable angle of incidence for the northern latitude is offset by more daylight hours and less depletion by the atmosphere for the southern latitudes.

South-Facing Tilted Surfaces

Curves for daily total solar radiation incident upon a south-facing surface tilted 30 degrees and 60 degrees from vertical are shown in Figures 7 and 8, respectively.

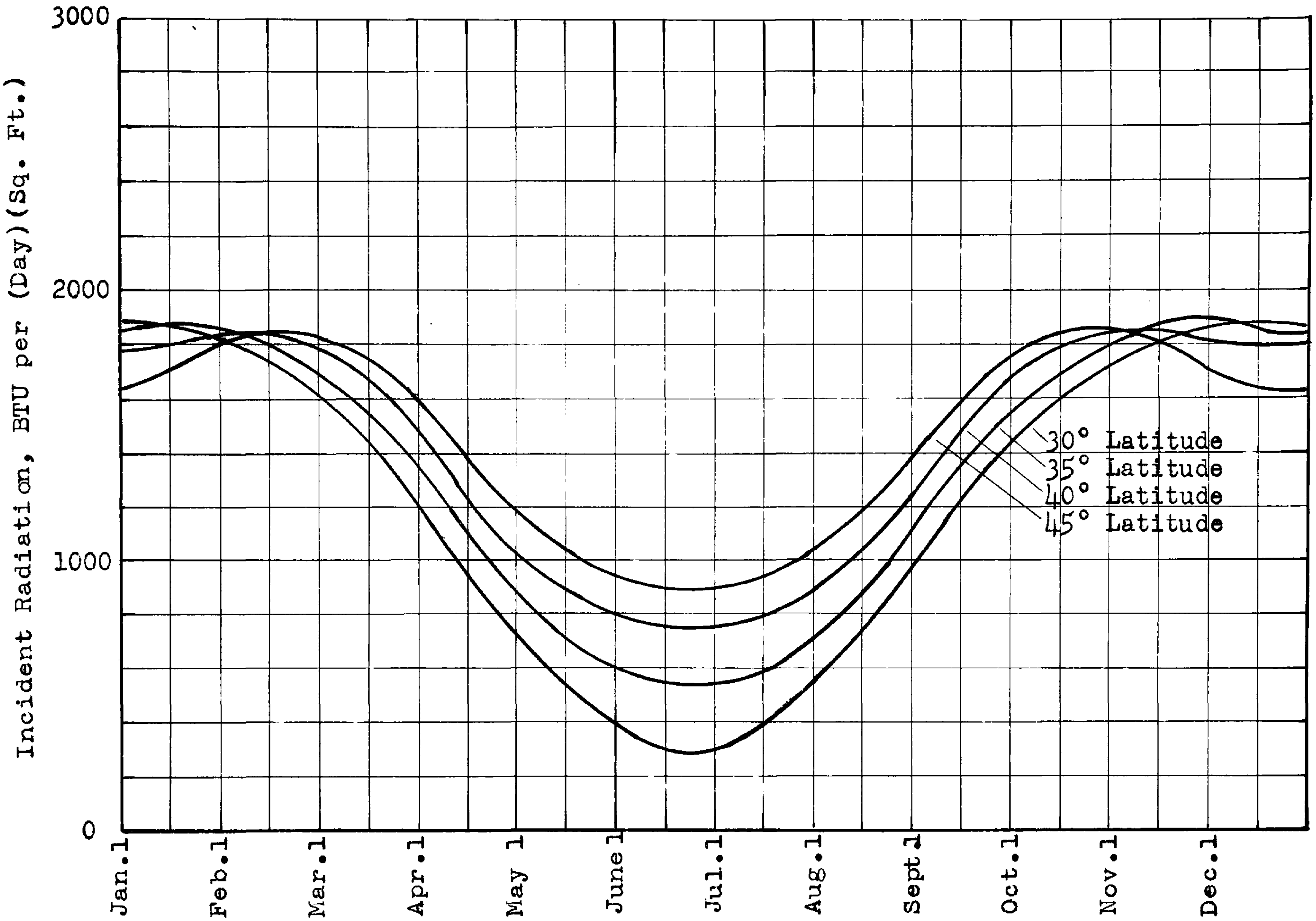


Fig. 6. Daily total direct and sky radiation incident upon a vertical south-facing surface at various north latitudes during cloudless days.

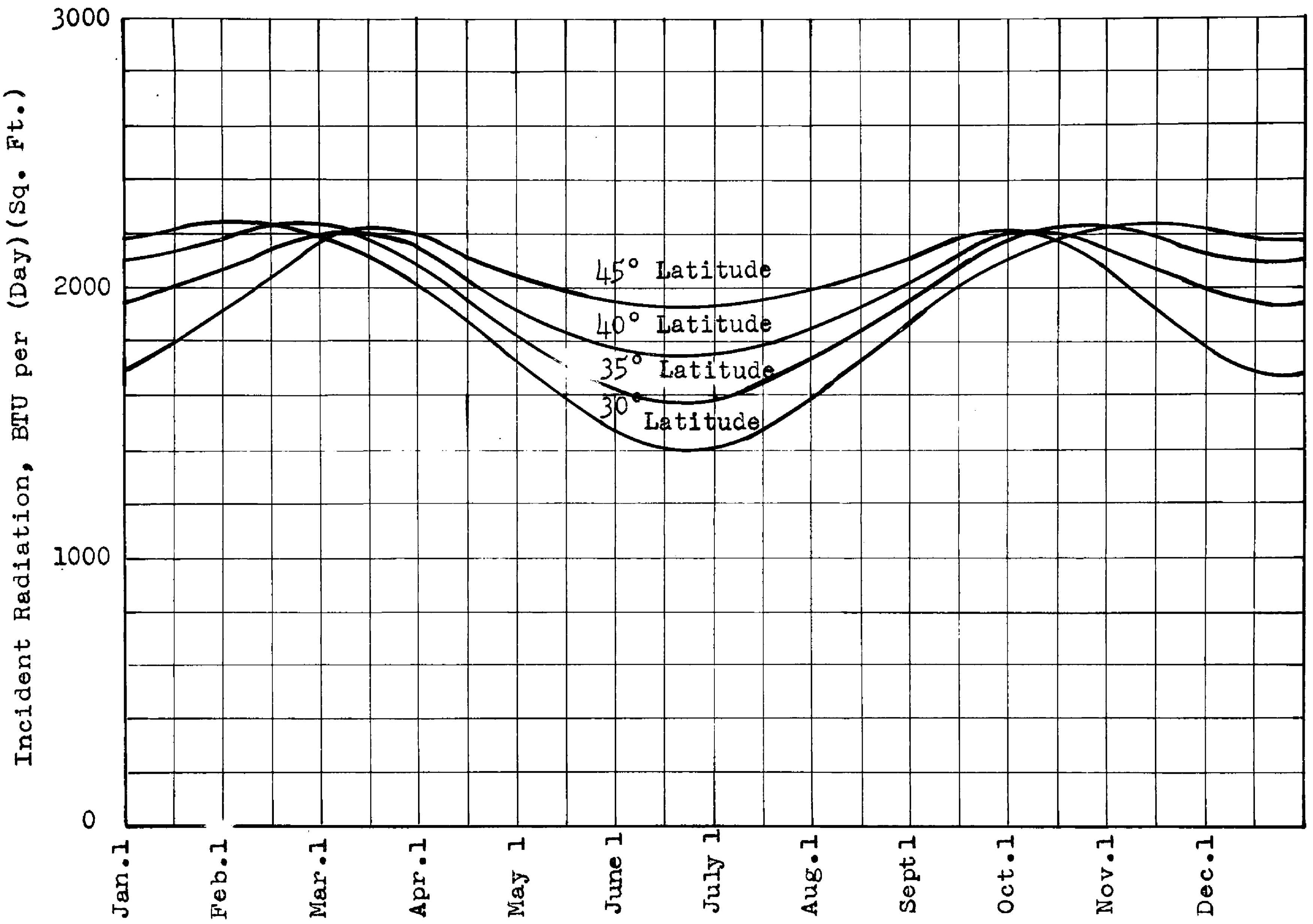


Fig. 7. Daily total direct and sky radiation incident upon a south-facing surface tilted 30 degrees from vertical at various north latitudes during cloudless days.

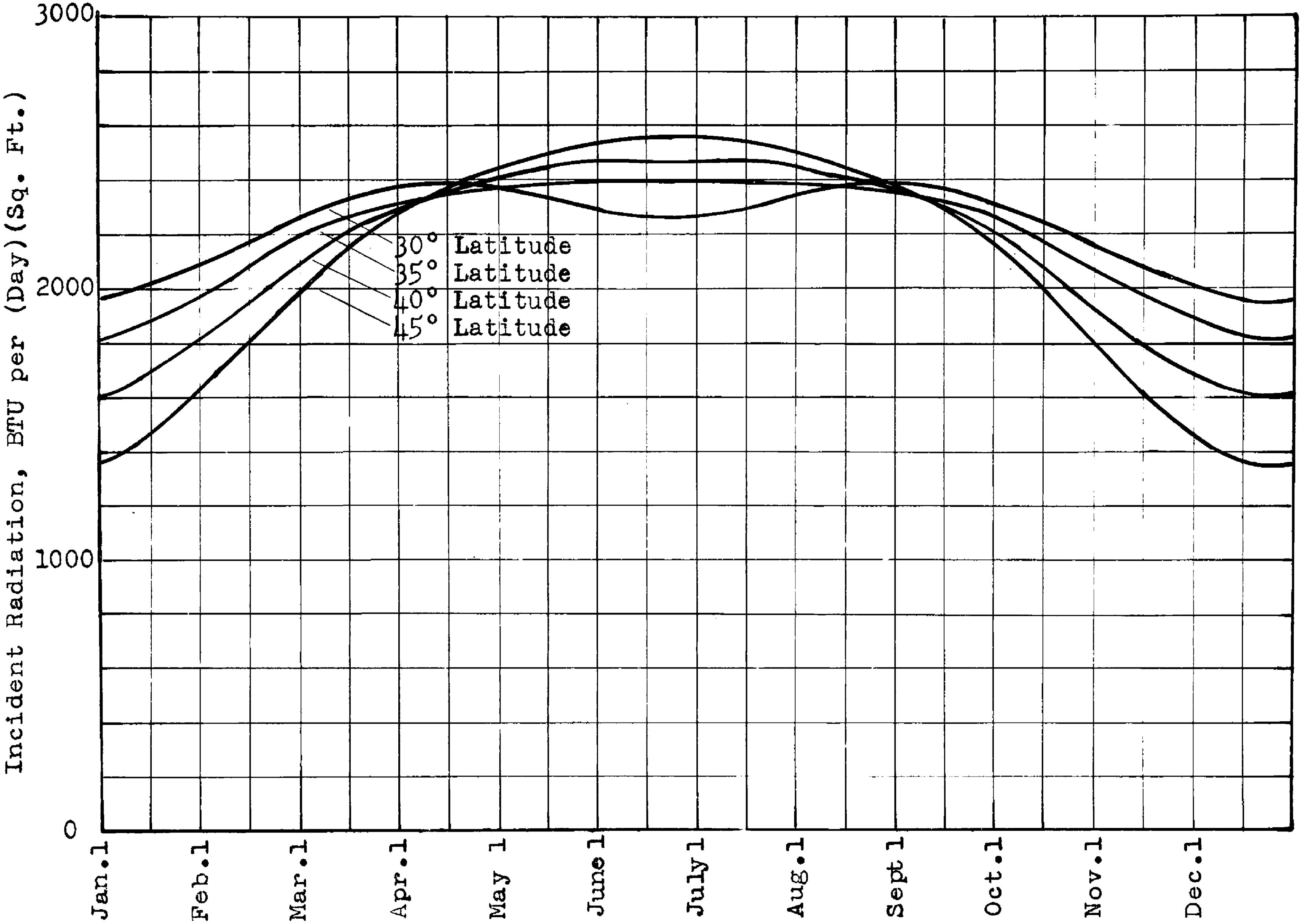


Fig. 8. Daily total direct and sky radiation incident upon a south-facing surface tilted 60 degrees from vertical at various north latitudes during cloudless days.

These curves reveal the importance of proper orientation of solar collector surfaces for the most efficient collection of solar energy. It is to be noted, for example, that for a surface tilted 30 degrees from the vertical, as compared with the vertical surface, the tilted surface shows a slight increase of incident cloudless day energy during mid-winter, and a substantial increase to a maximum of slightly more than 2200 BTU per day per square foot during the spring and fall months for north latitudes of 30°, 35°, and 40°. For 45° latitude, the tilted surface shows increased incident energy during spring, summer and fall, and a slight decrease during mid-winter.

A comparison of the curves for a south-facing surface tilted 60 degrees from vertical with curves for a horizontal surface shows that a slight attenuation occurs during the mid-summer period with small increases during the other summer months and much higher values for the remainder of the year.

Optimum Tilt Angles for Solar Collectors

It is apparent that the optimum tilt angle for a collector of solar radiation will depend on the latitude and on seasonal demand based on the uses to which the collected energy is to be put. In some cases it may be necessary to determine the optimum tilt angle for a period of time rather than for a particular time because a solar collector on a farm, for example, will usually be incorporated into, or made a part of, a farm structure.

The daily total radiation incident upon a surface, for all practical purposes, will be a maximum if a south-facing surface is tilted so that the sun's rays are perpendicular to it at solar noon. A south-facing surface will be perpendicular to the sun's rays at solar noon if the tilt angle from vertical is equal to the solar altitude for that time. The solar altitudes for the 21st of each month at 30°, 35°, 40° and 45° north latitude are given in Tables IV through VII.

Figure 9 shows curves which give the number of degrees to tilt a south-facing surface from vertical to make it perpendicular to the sun's rays at solar noon and thus, for all practical purposes, the optimum tilt angle from vertical for maximum incident solar radiation for various times of the year. However, there is a slight difference to be noted. Calculations using formula g for June 21 showed a slight increase in daily total incident energy with small increases in the tilt angle from the apparent optimum angle. The small increase occurs because the increase in incident energy during early morning and late afternoon hours due to additional tilting of the surface more than counterbalances the decrease during mid-day. The above situation

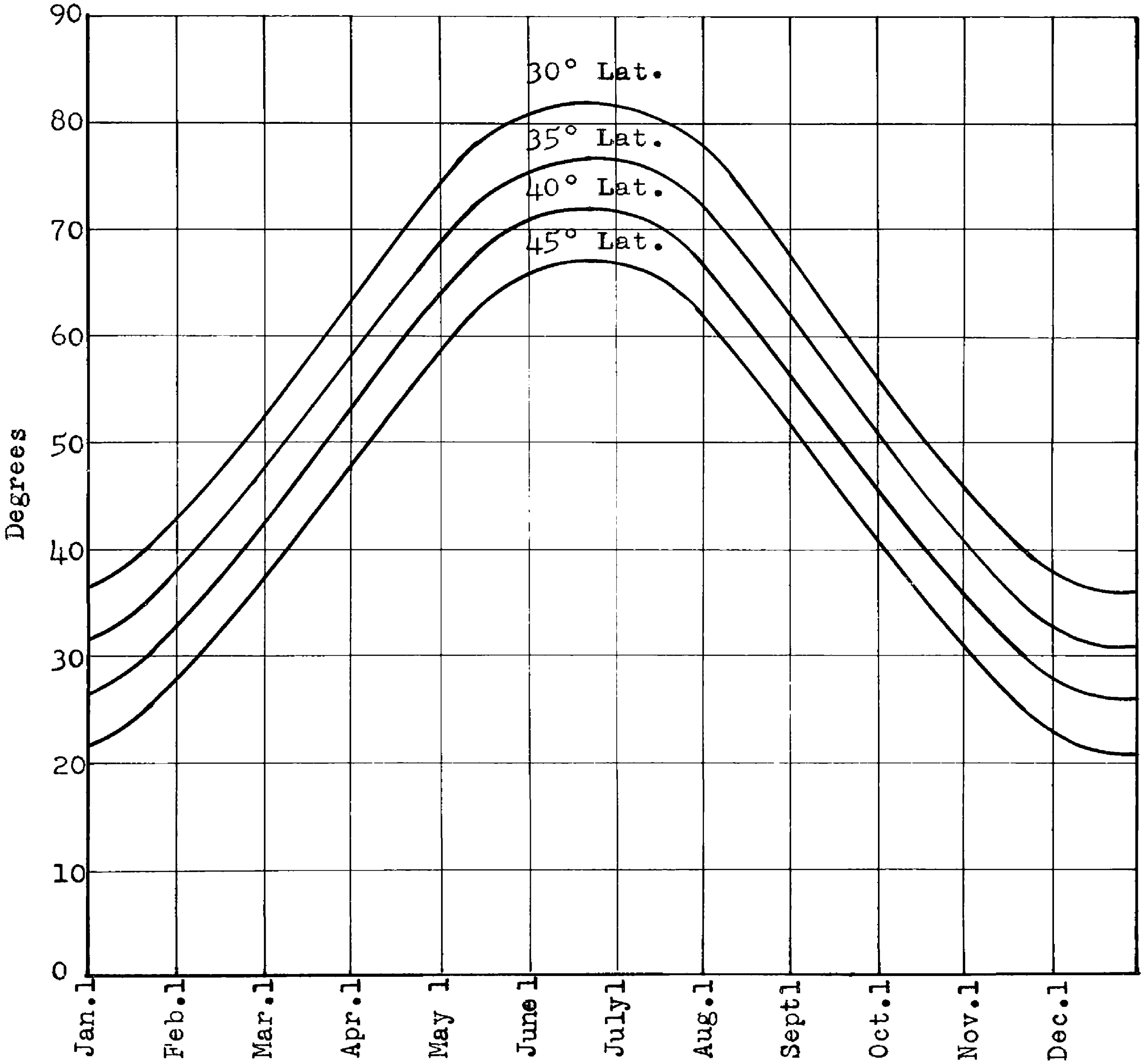


Fig. 9. Number of degrees to tilt a south-facing surface from vertical to make it perpendicular to the sun's rays at solar noon.

is true only during mid-summer when the solar azimuth angles are relatively small during the early morning and late afternoon hours. It should be noted that the above situation would be even more pronounced at more northerly latitudes.

Ratio of Tilted to Horizontal Surface Incident Radiation

The ratios of the daily total solar radiation incident upon a south-facing vertical surface and upon south-facing surfaces tilted 30 degrees and 60 degrees from vertical to that incident upon a horizontal surface for the latitudes mentioned before, were calculated for 21st of each month. The calculated ratios are shown in Table VIII.

Figures 10 and 11 show a plot of the ratio of tilted to horizontal surface incident radiation. A polynomial, of the form shown in equation m below, was fitted by the method of least squares to each of the curves.

$$Y = a + b X + c X^2 \quad \ldots\ldots \qquad (m)$$

where X = the slope of the south-facing surface in degrees from vertical

Y = the ratio of tilted to horizontal surface incident radiation.

Table I shows a tabulation of the coefficients of equation m for various latitudes and seasons. Equation m can be used to secure factors that can be multiplied by

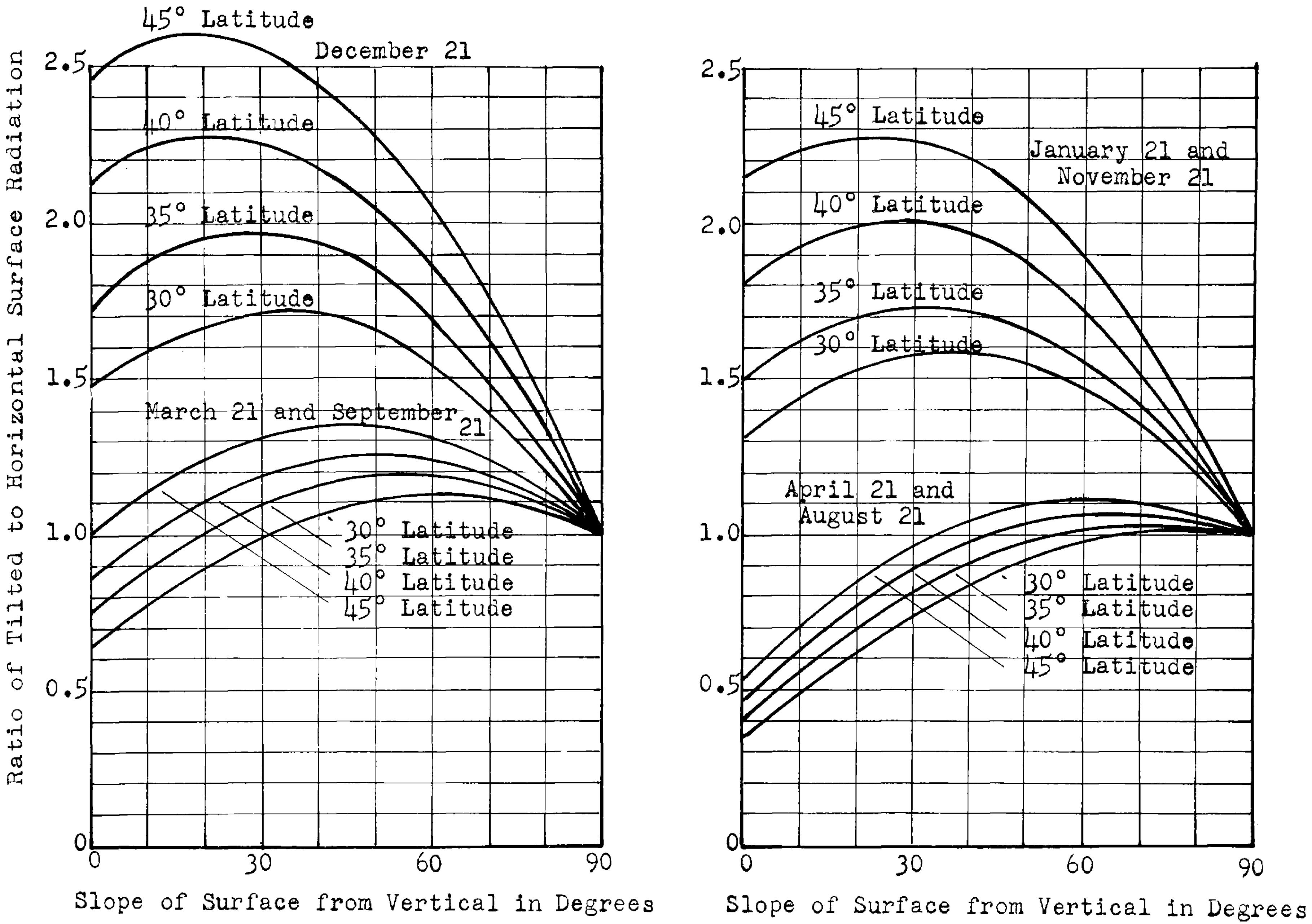


Fig. 10. Ratio of direct and sky radiation incident upon south-facing surfaces with various tilt angles to radiation incident upon a horizontal surface.

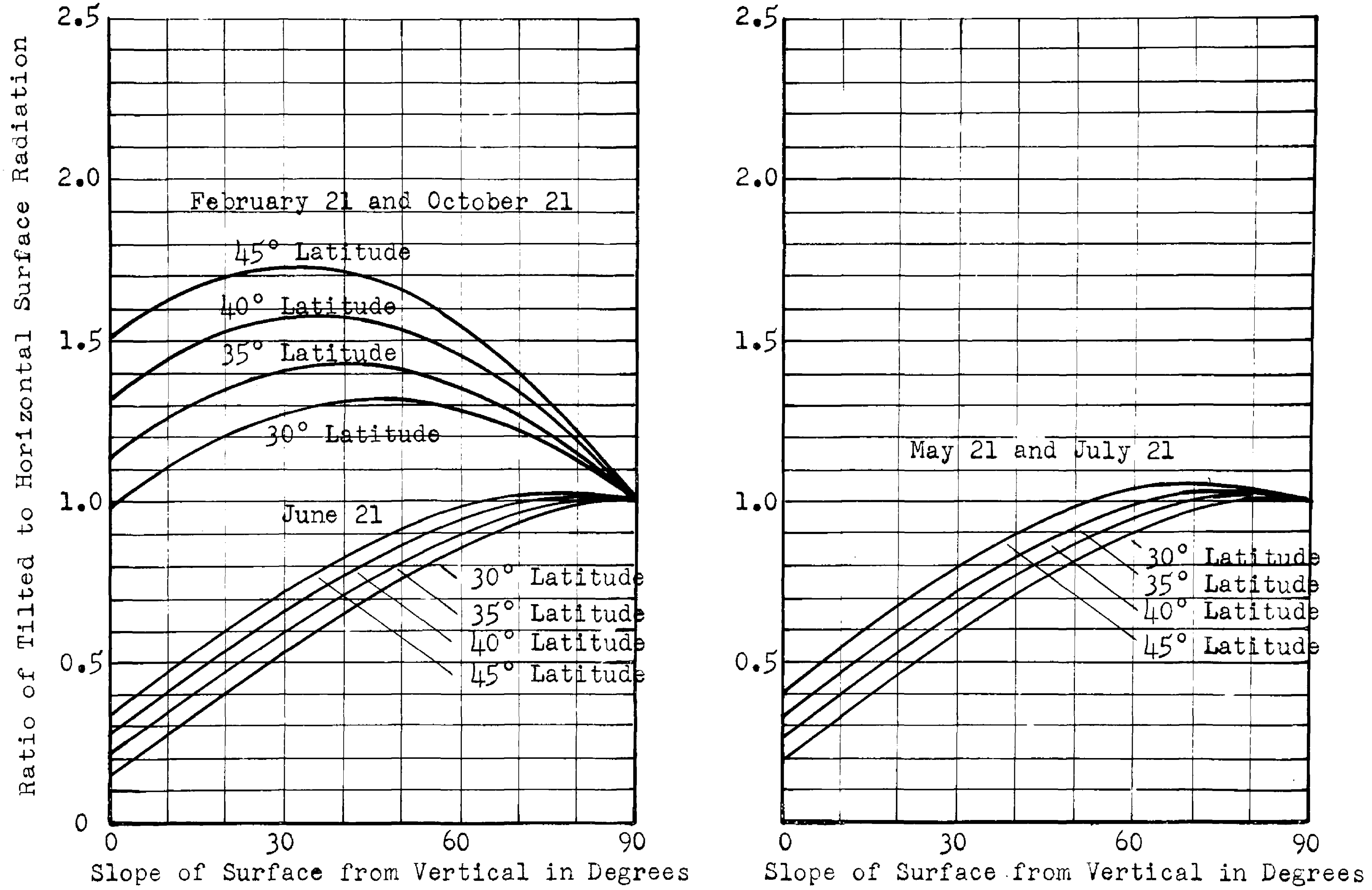


Fig. 11. Ratio of direct and sky radiation incident upon south-facing surfaces with various tilt angles to radiation incident upon a horizontal surface.

TABLE I

COEFFICIENTS OF THE POLYNOMIAL, $Y = a + bX + cX^2$, (m), FOR THE VARIOUS CURVES OF FIGURES 10 AND 11

Date	Latitude	Coefficient		
		a	b	c
Dec. 21	30°	1.496	0.012657	-0.0002025
	35°	1.734	.014313	.0002502
	40°	2.132	0.12366	.0002774
	45°	2.499	.010621	.0003033
Jan. 21 and Nov. 21	30°	1.322	0.014395	-0.0001999
	35°	1.510	.013339	.0002112
	40°	1.827	.013236	.0002500
	45°	2.154	.012117	.0002776
Feb. 21 and Oct. 21	30°	0.997	0.014531	-0.0001613
	35°	1.141	.015173	.0001860
	40°	1.312	.014250	.0001972
	45°	1.500	.013619	.0002139
Mar. 21 and Sept. 21	30°	0.638	0.016279	-0.0001361
	35°	.768	.015605	.0001445
	40°	.872	.015237	.0001532
	45°	1.011	.015365	.0001722
Apr. 21 and Aug. 21	30°	0.314	0.017447	-0.0001083
	35°	.400	.017659	.0001221
	40°	.492	.016878	.0001249
	45°	.571	.018014	.0001472
May 21 and July 21	30°	0.186	0.016350	-0.0000806
	35°	.244	.016964	.0000944
	40°	.345	.015827	.0000944
	45°	.399	.016884	.0001132
June 21	30°	0.105	0.016696	-0.0000744
	35°	.205	.016145	.0000805
	40°	.277	.016064	.0000889
	45°	.326	.017028	.0001055

values of Figure 5, or by the interpolated values of equations i, j, k and l, to secure the daily total cloudless day solar radiation incident upon a south-facing surface at any tilt angle between horizontal and vertical. Ratios for any day of the year can be secured by interpolation between the values secured for the 21st of the months between which the desired day comes.

Figures 10 and 11 show the importance of seasonal demand on proper orientation of a collector surface. It is noted that little can be gained by an orientation other than horizontal during the mid-summer months. The curves show the distinct advantage of a surface that is vertical, or very nearly vertical, during mid-winter months. The tilt angles for maximum incident radiation for the various latitudes agree with those given in Figure 9.

Comparison between Calculated Cloudless Day and Recorded Clear Day Radiation

Procedure

The information published in Climatogical Data, National Summary (31) on recorded total solar radiation was utilized to check the calculations previously presented. The information available is restricted to horizontal surfaces, except for the measurements made at Blue Hill, Massachusetts on vertical surfaces oriented at the cardinal compass points.

Supplements of Local Climatological Data (33) were secured from the United States Weather Bureau for Lincoln, Nebraska; Madison, Wisconsin; Boston, Massachusetts; Lander, Wyoming; Albuquerque, New Mexico and East Lansing, Michigan for the five-year period, 1950 through 1954. The average sky cover from sunrise to sunset in tenths was recorded for each day in the supplements mentioned. When the average sky cover from sunrise to sunset is three-tenths or less, the day was recorded as clear; four-tenths to seven-tenths, partly cloudy; and eight-tenths or more, cloudy. Technical Paper No. 12 of the United States Weather Bureau (32) summarizes the average number of clear, partly cloudy, and cloudy days from sunrise to sunset for over 200 United States Weather Bureau Stations; the Summaries included data for periods up to 77 years. Table IX shows a copy of the data for the stations used in connection with this study.

Dates when the average sky cover from sunrise to sunset was three-tenths or less were recorded for each of the stations mentioned for the five-year period, 1950 through 1954. Daily total incident radiation was then secured for the recorded clear days from Climatological Data, National Summary (op. cit.) for a horizontal surface at each of the stations and for a south-facing vertical surface at Blue Hill, Massachusetts. Average sky cover data was not available at the same location that solar radiation data was taken for Blue Hill, Massachusetts, so the sky cover data were taken from records made at the Logan International Airport, Boston, Massachusetts.

The solar radiation data was recorded in units of gram-calories per square centimeter per minute; the units were converted to BTU per square foot per hour by multiplying by 3.68.

Calculated solar radiation during cloudless days incident upon a horizontal surface for the 21st of each month was determined for each of the stations by interpolating between values secured from the equations in Figure 5 for the latitude of each station. Calculated cloudless day radiation was determined for a south-facing vertical surface at Blue Hill, Massachusetts by interpolation of values given in Figure 6.

Curves showing the calculated incident cloudless day radiation were constructed for each of the stations. Plots of points representing the recorded radiation during clear days were made in order to make a comparison between the recorded and the calculated cloudless day values for the various days of the year.

Results

Figures 12, 13, 14 and 15 show the comparison between calculated cloudless day and recorded clear day solar radiation incident upon a horizontal surface at Madison, Wisconsin;

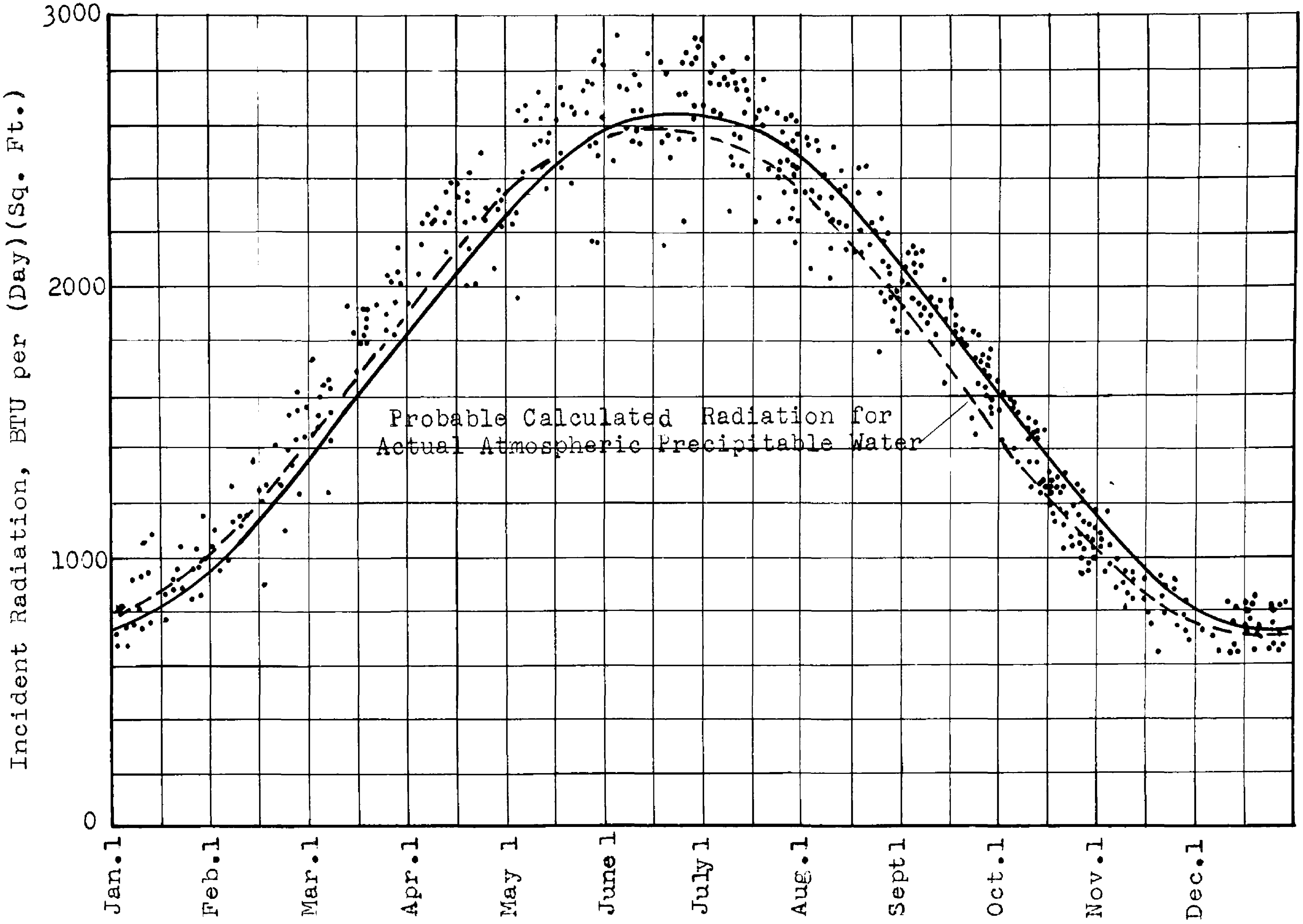


Fig. 12. Comparison between recorded and calculated direct and sky radiation incident upon a horizontal surface at Madison, Wisconsin during days with 0 - 3 tenths cloud cover, 1950 through 1954.

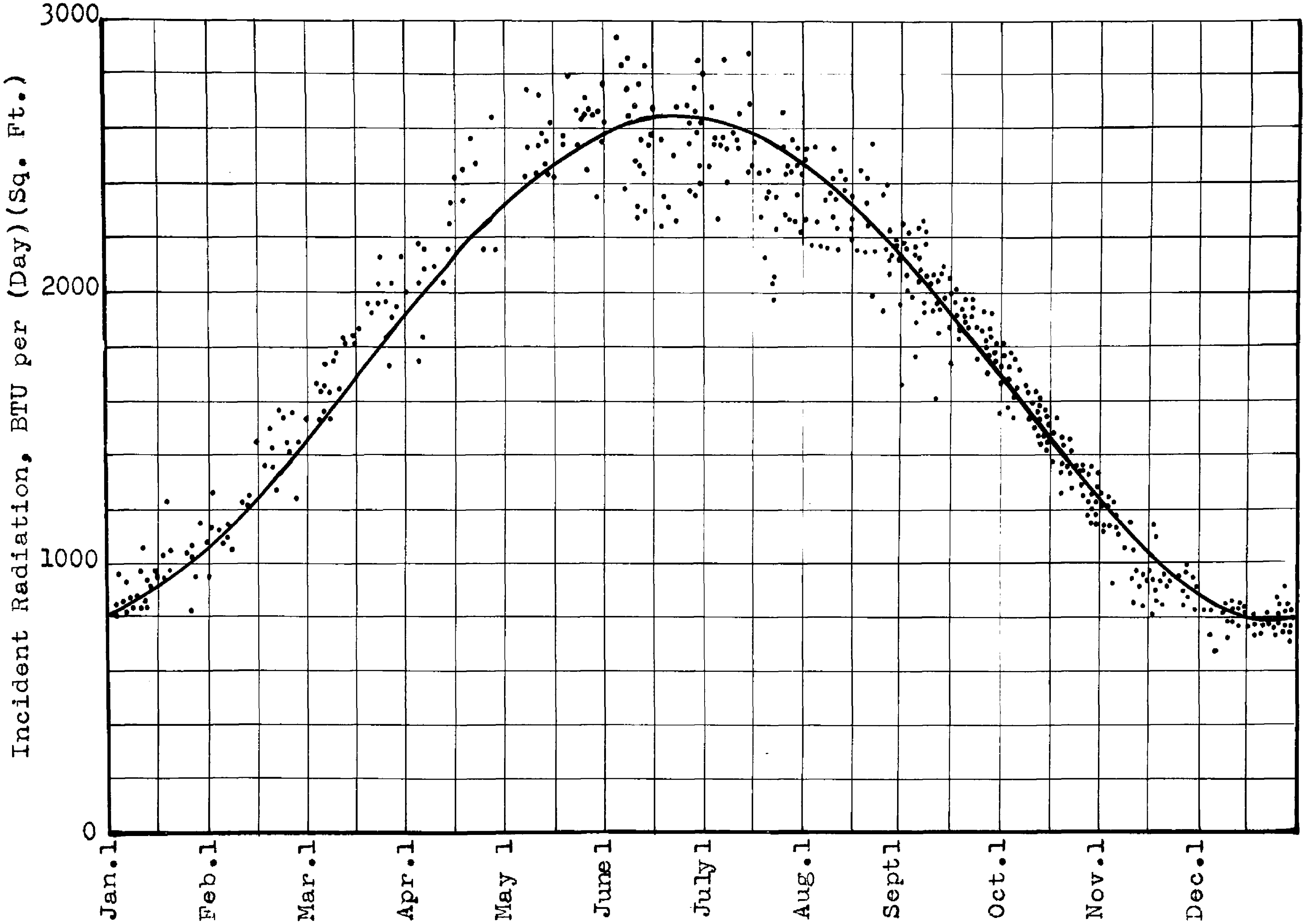


Fig. 13. Comparison between recorded and calculated direct and sky radiation incident upon a horizontal surface at Lincoln, Nebraska, during days with 0 - 3 tenths cloud cover, 1950 through 1954.

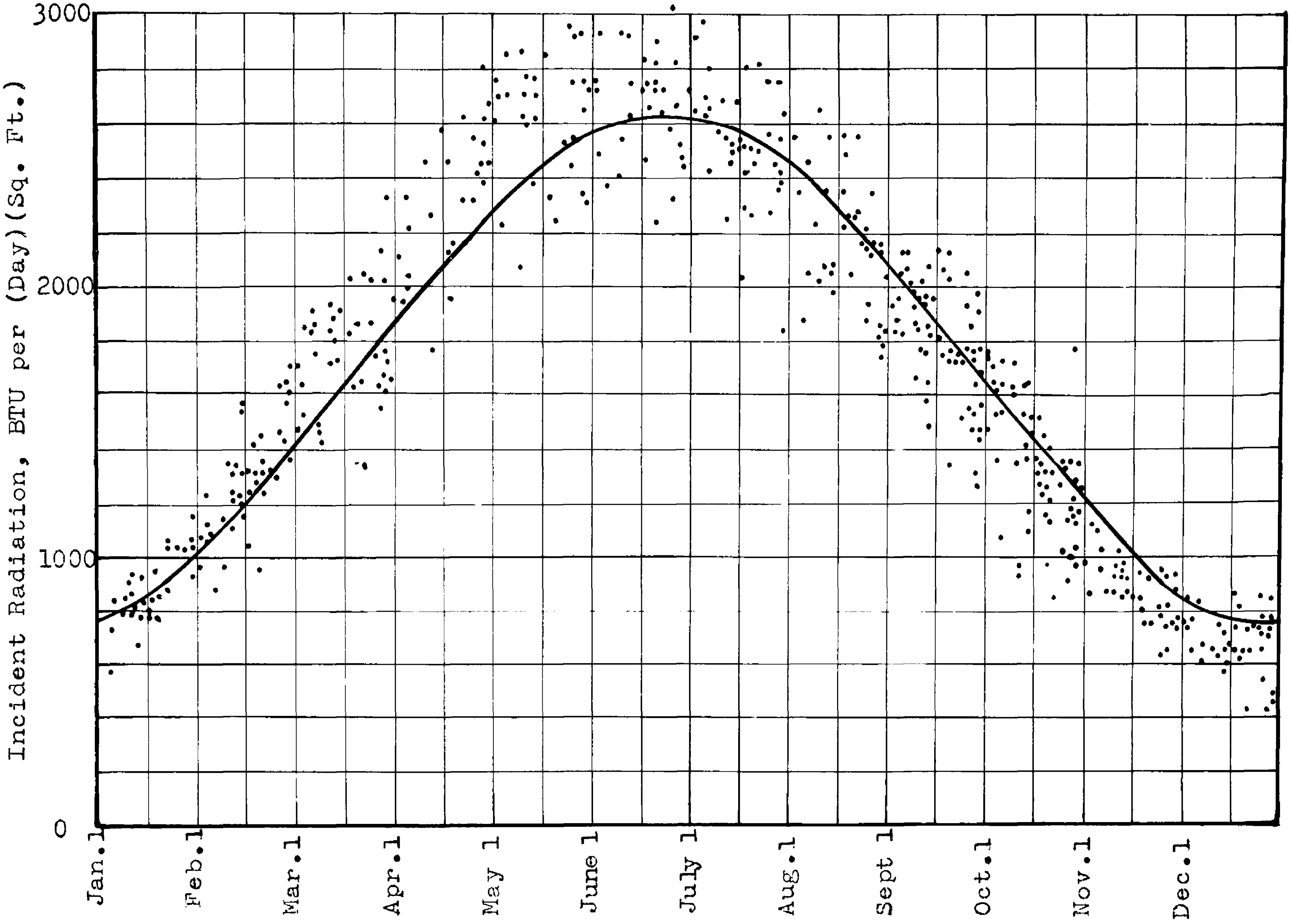


Fig. 14. Comparison between recorded and calculated direct and sky radiation incident upon a horizontal surface at Blue Hill, Massachusetts, during days with 0 - 3 tenths cloud cover, 1950 through 1954.

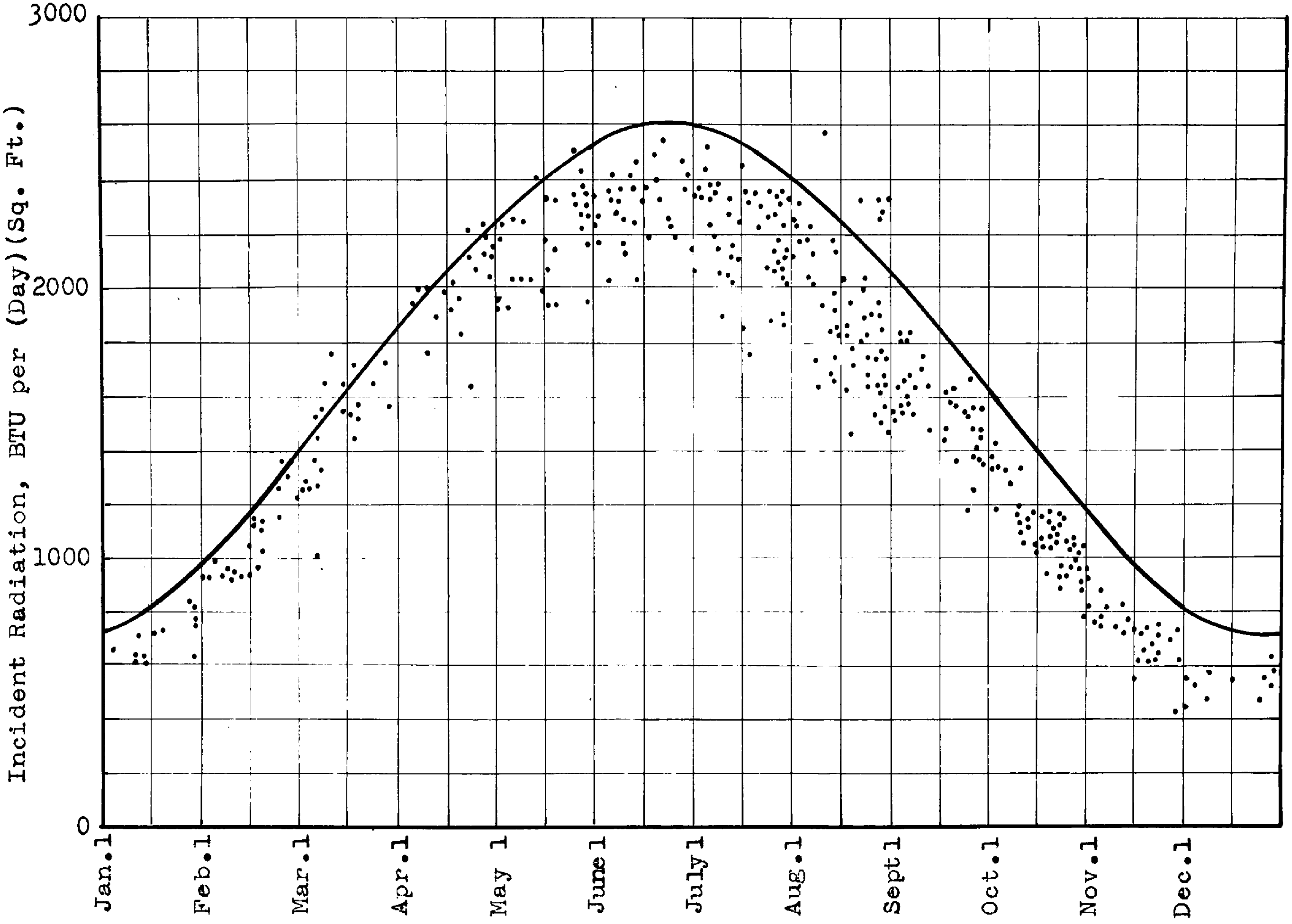


Fig. 15. Comparison between recorded and calculated direct and sky radiation incident upon a horizontal surface at East Lansing, Michigan during days with 0 - 3 tenths cloud cover, 1950 through 1954.

Lincoln, Nebraska; Blue Hill, Massachusetts and East Lansing, Michigan, respectively. It is evident that the calculated curves, based on assumed standard atmospheric and sea level conditions, gave reasonably good correlation with the observed radiation at Madison, Lincoln, and Blue Hill which have elevations of 672, 938, and 1184 feet above sea level, respectively. It is noted, in each case, that the calculated values are low during the spring and early summer period and high during the last part of October and first part of November. This difference is possibly due to higher atmospheric moisture content than assumed during the October-November period and a lower atmospheric moisture content than assumed during the spring months. Table XII shows values of monthly mean precipitable water for all days in the United States published by the Weather Bureau (34) and values of precipitable water for cloudless days used for the computation of direct radiation at direct incidence (Figure 1). It is to be expected that cloudless day values will be less than the mean recorded values. However, the data indicates a tendency for low values of precipitable water during the spring months as compared with the fall months. The dashed curve of Figure 12 is a plausible calculated curve for actual precipitable water during cloudless days.

Figure 14, for Blue Hill, shows a great deal more scatter of the recorded radiation than do Figures 12 and 13 for Madison and Lincoln, respectively. One possible explanation of the greater scatter could be the location of the measuring pyrheliometer. Hand (10), in a report on solar radiation measuring stations in the United States, indicated that the Blue Hill pyrheliometer is located 10 miles south of Boston and that there is almost no smoke or dust except with winds of a northerly component which carry smoke from Boston. He also indicated that the pyrheliometer at Madison is located on top of a building located at the University of Wisconsin and that little smoke interference is noted. The Lincoln pyrheliometer is located in downtown Lincoln (since 1940).

Figure 15, which shows the comparison for East Lansing, Michigan, indicates that the computed values are consistently too high, varying from about 25 percent to 10 percent too high for winter and summer, respectively. The tendency for comparatively higher recorded values during spring is also noted for East Lansing. A possible explanation for the consistently low recorded values at East Lansing may be due to the fact that Michigan is surrounded, to a great extent, by large bodies of water over which the masses of air must move in their predominant easterly movement. The bodies of water and the large industrial activity in the whole area undoubtedly tend to increase the haziness and smoke content of the atmosphere above the assumed amount for a normal atmosphere. Results of work by Fritz (8), who has constructed isolines of cloudless day solar radiation for horizontal surfaces in the United States for each month of the year, based on close analysis of recorded data supplemented by computed values where there were no recording stations, also showed consistently lower values for the Great Lakes Region, and in particular for Michigan, than for other parts of the country at the same latitude. Crabb (6) also noted the relative low amount of solar radiation at East Lansing as compared with other stations in the United States.

The variation noted for all stations can be attributed, in part, to varying cloud amounts because the recorded values used were for days with average cloud cover between zero and three-tenths. Additional variation may be caused by occasional presence of dust or moisture on the glass cover of, or by improper leveling of, the measuring pyrheliometer.

Figure 16 shows the comparison between calculated and recorded values of incident radiation on a south-facing vertical surface at Blue Hill, Massachusetts. Reasonably good average correlation is again shown. A great deal of variation in the recorded radiation is noted, particularly during the winter months. In addition to the explanation given for the variation of solar radiation on a horizontal surface at Blue Hill, the vertical surface will be subjected

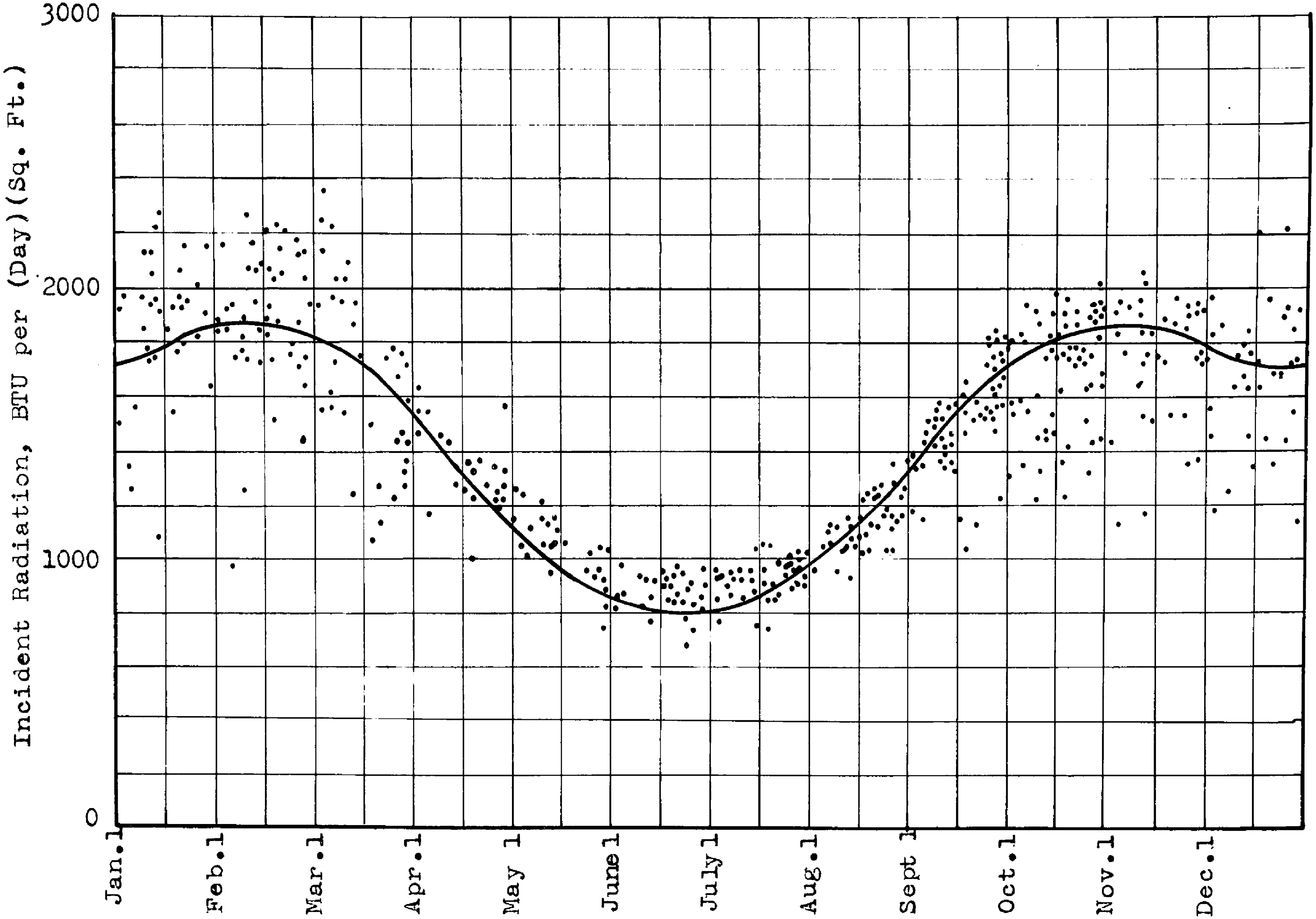


Fig. 16. Comparison between recorded and calculated direct and sky radiation incident upon a vertical surface at Blue Hill, Massachusetts during days with 0 - 3 tenths cloud cover, 1950 through 1954.

to varying amounts of reflected radiation from the ground. The high recorded values can reasonably be attributed to increased ground reflection when the ground is covered with snow.

Comparison between Calculated and Observed Ratio of Vertical to Horizontal Surface Incident Radiation

Procedure

The only possible check of the calculated ratios of solar energy incident upon south-facing surfaces to that incident upon a horizontal surface, as presented in Figures 10 and 11, is to check the ratio of south-facing vertical to horizontal surface incident radiation for Blue Hill, Massachusetts.

The calculated ratio of south-facing vertical to horizontal incident radiation for Blue Hill was determined for the 21st of each month by interpolation between the values given for 45° and 40° north latitude in Table VIII for 42°-13', which is the latitude of Blue Hill. The curve in Figure 17 was constructed from the points determined by the method mentioned above.

Supplements of Local Climatological Data for Boston, Massachusetts (33) were used to determine the dates during the five-year period, 1950 through 1954, when the average sky cover from sunrise to sunset was classified as clear or

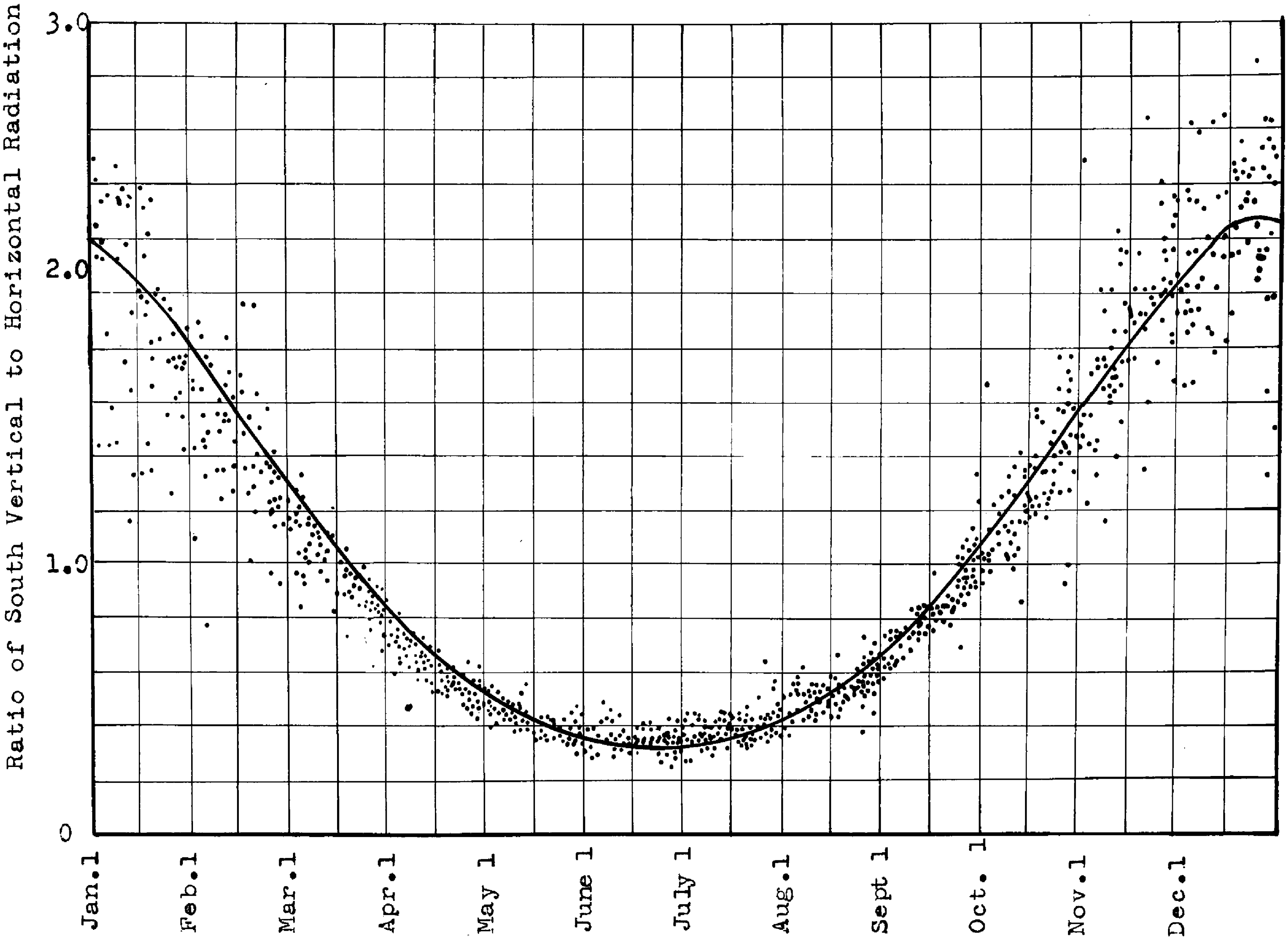


Fig. 17. Comparison between actual and calculated ratio of total radiation incident upon a south-facing vertical surface to that incident upon a horizontal surface at Blue Hill, Massachusetts, during days with 0 - 7 tenths cloud cover, 1950 through 1954.

partly cloudy (0-7 tenths cloud cover, inclusively). Daily total solar radiation measured and recorded during the clear and partly cloudy days determined above, was secured from Climatological Data, National Summary (31) for the horizontal and south-facing vertical surface at Blue Hill. The ratio of the vertical to horizontal surface incident radiation was calculated and the values for the various days were plotted on Figure 17.

Results

Figure 17 shows good average correlation between calculated and observed ratio of daily total solar radiation incident upon a south-facing vertical surface to that incident upon a horizontal surface at Blue Hill, Massachusetts for clear and partly cloudy days. Only clear and partly cloudy days were used for this comparison because only a very small portion of the total radiation is available during cloudy days.

It is noted that there is considerably more scatter of points during winter months than during summer months. The very high ratios during the winter can be attributed to the increased radiation incident upon the south-facing vertical surface owing to increased reflection from the ground caused by snow cover. This increased reflection will affect the vertical surface more than the horizontal surface and consequently, increase the observed ratio. The observed ratios which fall considerably below the calculated ratios occurred on the relatively cloudy days because with increased cloudiness, the radiation tends to become equal in all directions, or the ratio tends toward unity.

VARIATION IN SOLAR RADIATION INTENSITY WITH ALTITUDE

Review of Literature

Preliminary comparisons between calculated cloudless day and recorded clear day solar radiation for Lander, Wyoming (5,563 ft.) and Albuquerque, New Mexico (5,310 ft.) indicated that the calculated results, based on the atmospheric and sea level conditions which were assumed, gave values that were much too low. The higher recorded values for Lander and Albuquerque are to be expected because the solar radiation has a shorter path through the atmosphere and less chance of depletion in reaching a surface at high altitude.

The earliest known information on the variation in solar radiation intensity with altitude was published in 1919 by Kimball (19). Prior to that time, in cooperation with the Weather Bureau and the Smithsonian Institute, he made studies on the increase in solar radiation intensity with altitude westward from the Atlantic Coast of the United States. Records of solar radiation intensity were taken at various places near sea level, in the Great Plains, and at various places at high altitude; such as Hump Mountain, North Carolina; Mount Wilson, California; Cheyenne, Wyoming; Flagstaff, Arizona; and Santa Fe, New Mexico. From these studies, he arrived at a monthly mean increase in solar radiation with altitude as shown by the yearly curve of Figure 19. This work was continued and in 1948, Klein (21) summarized work which by that time had included results from 56 plateau and mountain stations supplemented by information from balloon ascents. He noted that the variation of the transmission of solar radiation with altitude depended on the season of the year and the length of the path of the sun's rays (air mass). His work indicated that the variation with altitude was logarithmic for the various air masses. (See Glossary for definition of Air Mass.) Figure 18 shows the results of his findings for summer conditions with air masses of one and two and for winter conditions with air mass of two. It is noted that the winter and summer lines, for air masses of two, come very nearly to being equal at four kilometers. Below this level, atmospheric transmission of solar radiation is lower during summer, which is as expected due to higher moisture content of the atmosphere.

Both Klein's (ibid.) and Kimball's (loc. cit.) data show the increase in atmospheric transmission of solar radiation with altitude as compared with that at three-tenths of a kilometer (984.2 ft.) elevation because at lower elevations no definite relationship between transmission and

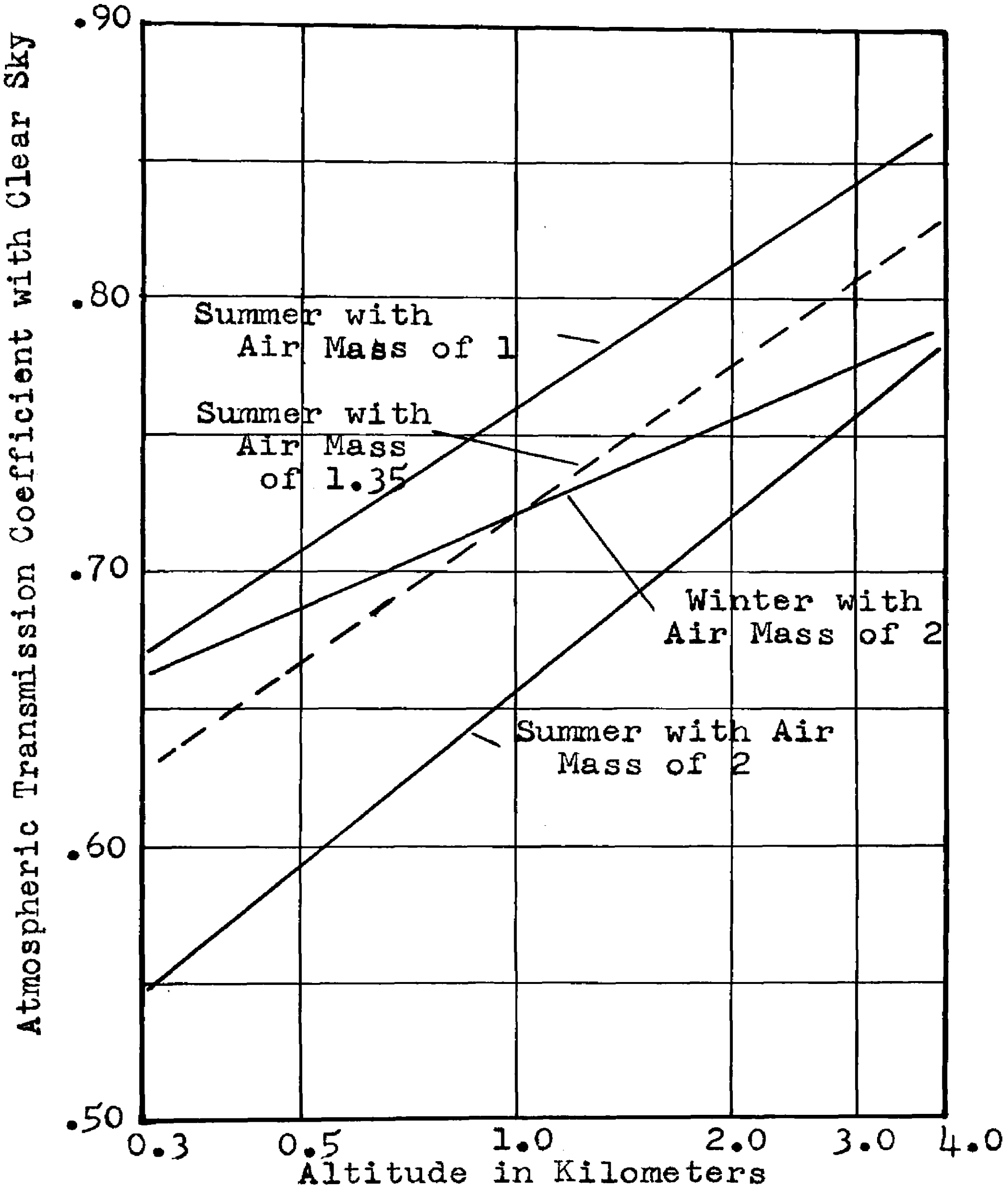


Fig. 18. Mean summer and winter transmission coefficients with cloudless sky at high level stations as a function of altitude, from data by Klein (21).

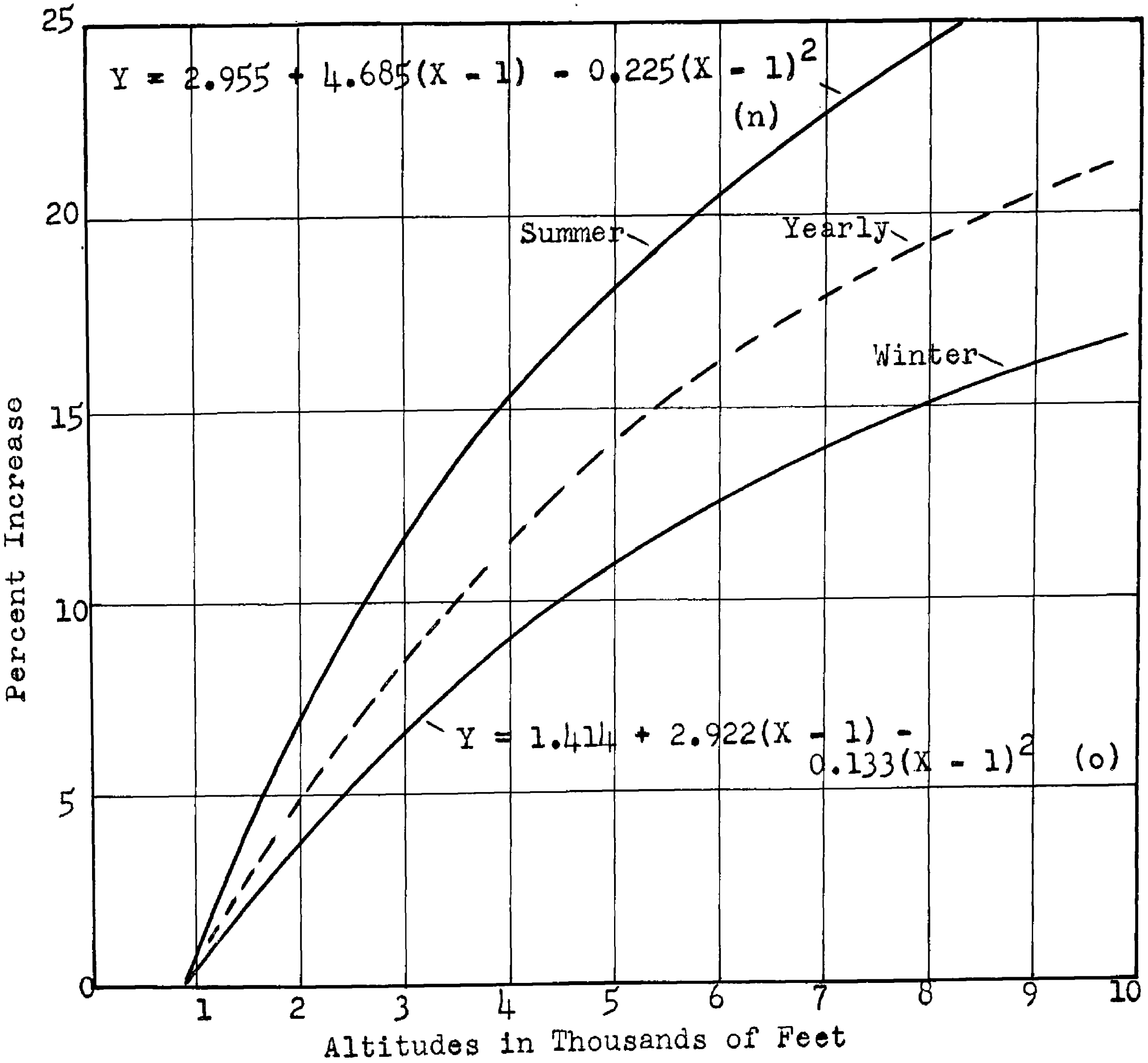


Fig. 19. Percentage increase of solar radiation intensity with altitude. Winter and summer curves calculated from data in Figure 18. Curve for yearly mean from data by Kimball (19).

elevation could be established and differences appeared to be more a function of local conditions.

Procedure for Calculating Corrections Due to Altitude

Solar altitudes for June 21 at 40° north latitude for the various daylight hours (Table VI) were integrated by using Simpson's Rule; this integration resulted in an integrated mean solar altitude of 47.7 degrees. A mean solar altitude of 47.7 degrees corresponds to an integrated mean air mass of 1.35 as the secant of the zenith angle of the sun is a good approximation of the air mass. Interpolation for an air mass of 1.35 was then made between values of atmospheric transmission for air masses of one and two for summer conditions as shown in Figure 18.

Using the same method used for June 21, the integrated air mass for December 21 turned out to be about three; but since data were not available for air masses greater than two, the data shown in Figure 18 for a winter air mass of two were used to determine approximate corrections for altitude in winter.

Percentage Increase of Solar Radiation Intensity with Altitude

The percentage increase of solar radiation intensity with altitude was determined for winter and summer conditions according to the increase in the atmospheric transmission coefficients shown in Figure 18 for winter air mass of two and summer air mass of 1.35, respectively. The percentage increase for winter and summer is shown in Figure 19. The yearly mean increase with altitude, determined by Kimball (<u>loc</u>. <u>cit</u>.) is also shown in Figure 19. It is of interest to note how closely the curve for yearly mean increase approximates being an average between the summer and winter increases.

The curves were drawn to show no increase in solar radiation intensity with elevation up to 1000 feet above sea level because no definite relationship between atmospheric transmission of radiation and elevations to 1000 feet could be established from Klein's (<u>loc</u>. <u>cit</u>.) and Kimball's (<u>loc</u>. <u>cit</u>.) data, and also due to the good correlation between the calculated and recorded values previously discussed for Blue Hill, Massachusetts; Lincoln, Nebraska; and Madison, Wisconsin which have altitudes above sea level of 672, 1184, and 938 feet, respectively.

The method of least squares was used to determine equations n and o of Figure 19 for the summer and winter curves, respectively. These equations can be used between altitudes of 2000 and 10,000 feet above sea level. Points calculated by the equations for 1000 feet intervals between 2000 and 10,000 feet fit the points used in plotting the curve with average deviations of 0.25 and 0.10 percent for summer and winter, respectively. The equations can be used to approximate the percentage increase of solar radiation with altitude for December 21 and June 21. Percentage increases for any other time of the year can be approximated by interpolation between the values given by the two equations.

Comparison between Calculated Cloudless Day and Recorded Clear Day Radiation for Two High Altitude Stations

Figures 20 and 21 show the comparison between calculated cloudless day and recorded clear day radiation incident upon a horizontal surface for Lander, Wyoming and Albuquerque, New Mexico, respectively. The period covered for the comparison was from April of 1950 through 1954 for Albuquerque because April 1 was the beginning date for publication of recorded solar radiation data in Climatological Data, National Summary (31) for Albuquerque. Publication of recorded data for Lander began July of 1950 and was interrupted during the first eleven months of 1951 and in January of 1952, which explains the relatively few points for the first months of the year.

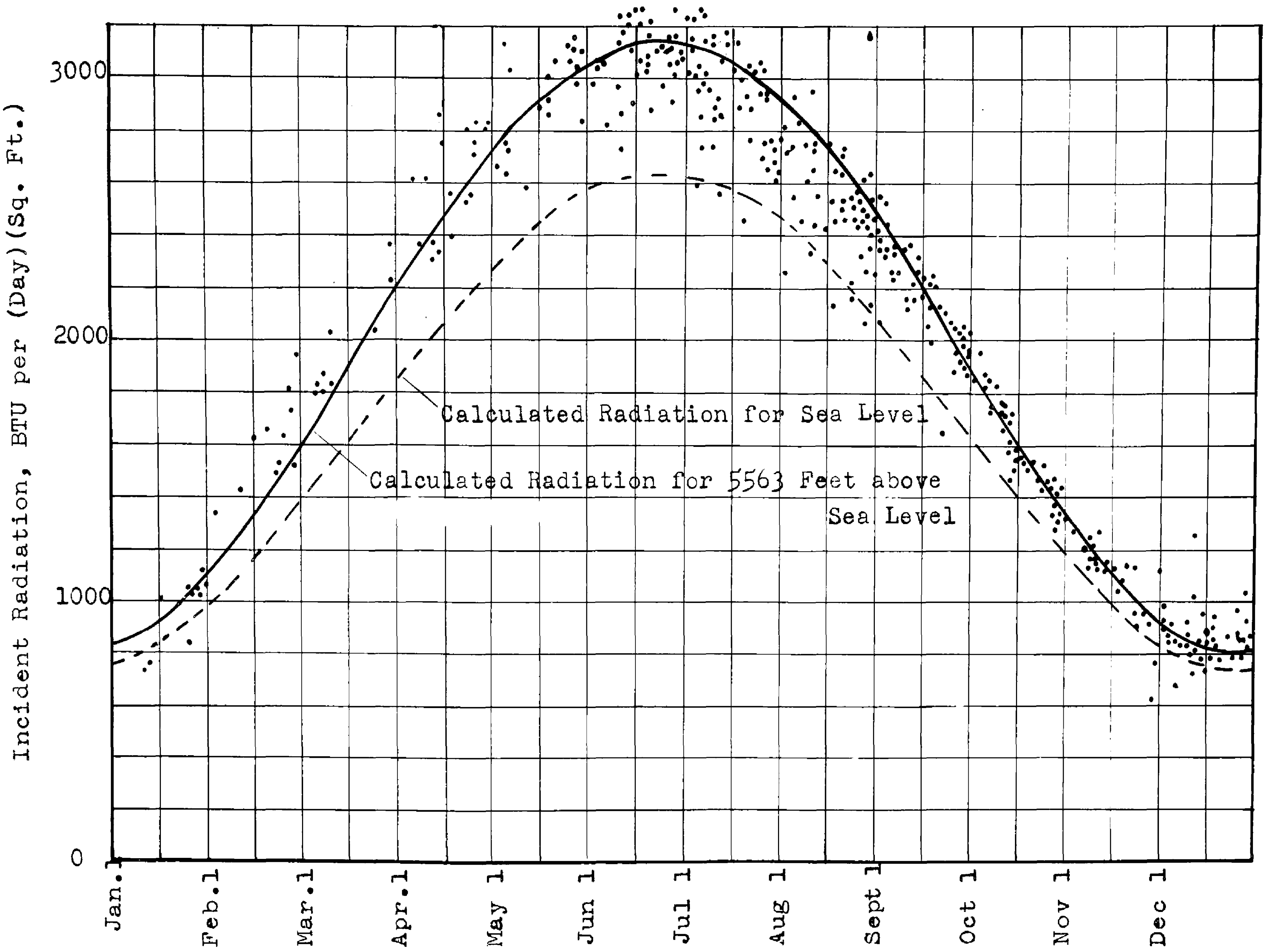


Fig. 20. Comparison between recorded and calculated direct and sky radiation incident upon a horizontal surface at Lander, Wyoming during days with 0 - 3 tenths cloud cover, 1950 through 1954.

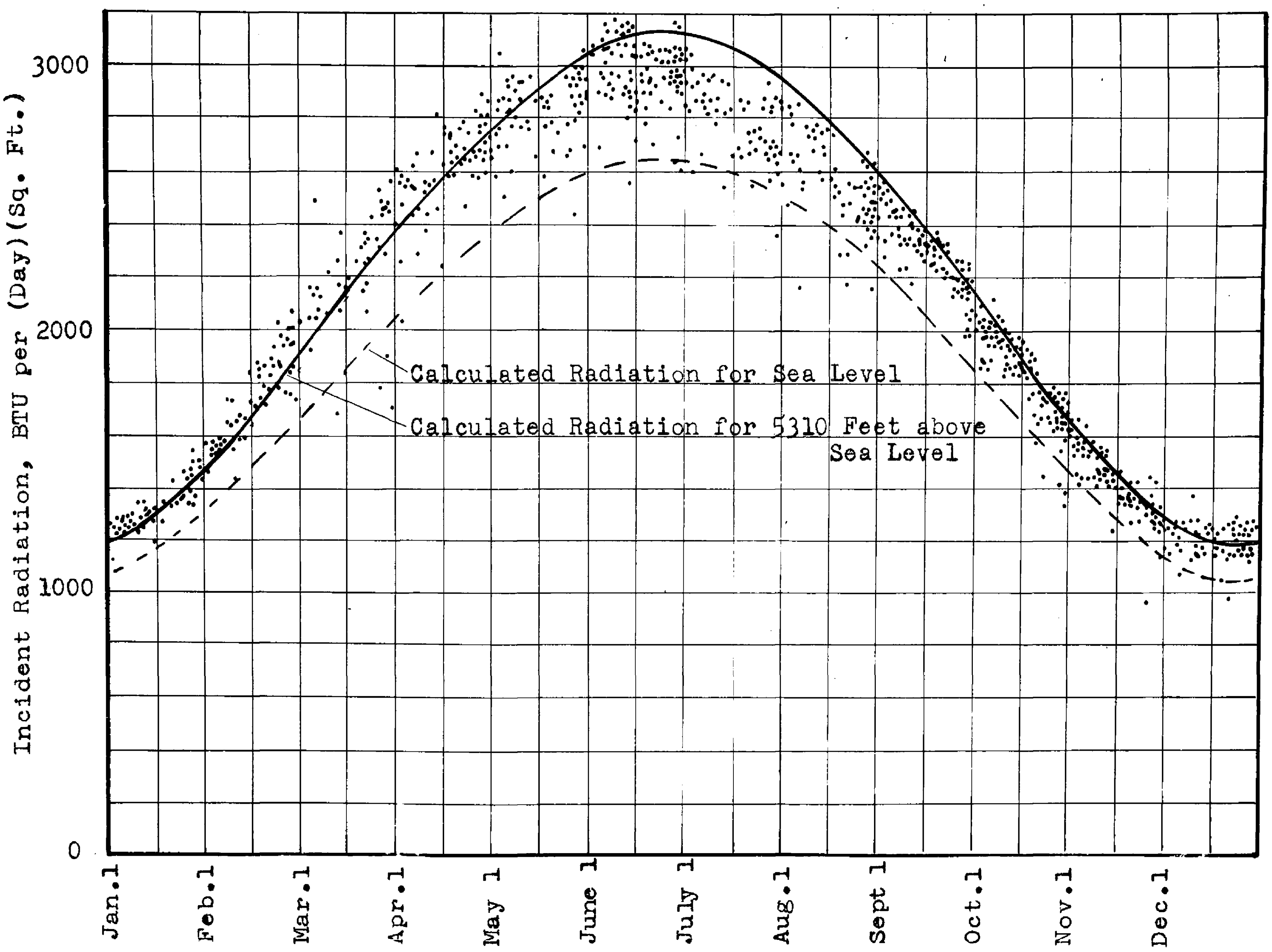


Fig. 21. Comparison between recorded and calculated direct and sky radiation incident upon a horizontal surface at Albuquerque, New Mexico, during days with 0 - 3 tenths cloud cover, 1950 through 1954.

The procedure used in making this comparison was the same as the one used for Blue Hill, Madison, Lincoln and East Lansing except that corrections for altitudes of 5563 and 5310 feet above sea level were made for Lander and Albuquerque, respectively. The dashed curves in each case represent the calculated daily total solar radiation for sea level conditions for the latitudes of the respective stations.

Again, reasonably good average correlation between recorded and calculated values is noted, except, during July and August for Albuquerque when the calculated values are high. A good explanation for the latter is now known, except that it could possibly be due to the numerous thunderstorms at Albuquerque during those months.

EFFECT OF CLOUDS ON SOLAR RADIATION INTENSITY

Review of Literature

So far the discussion presented for calculated solar radiation intensity at the surface of the earth has been for cloudless skies, and the observed values referred to have been for clear days, or days when the average percentage of cloud cover was 30 percent or less. The sky, however, is not always cloudless; in fact, in some places during some seasons, cloudy conditions are very prevalent. Because of the great effect clouds have on solar radiation, it is one of the most important considerations in determining the availability of solar energy. The effect of clouds is also one of the most difficult to determine.

Fritz (7) states that if we look at the earth as a whole, the planet reflects about 35 percent of the solar radiation incident upon it back to space and that clouds are the major cause for this reflection. Haurwitz (11) has reported that the ratio of total radiation with complete overcast to total radiation with cloudless skies varies from about 0.83 for cirroform type clouds to 0.18 for fog. This ratio depends not only on cloud type, but also on the mean free path of the sun's rays through the cloud, drop size and distribution, and liquid water content of the clouds. Because of the difficulty involved in approximating the parameters mentioned, which are needed to determine the effect of clouds, and owing to the non-homogeneous nature of clouds, it is more practical to correlate the measurements of solar radiation at a few stations with some other parameter, such as average cloudiness or percentage of possible sunshine, which are observed in many places.

Fritz (*op. cit.*), Kimball (20) and others, suggest correlating the ratio of average daily solar radiation to cloudless day radiation with percentage of possible sunshine. Percentage of possible sunshine is suggested in favor of average percentage of cloud cover because the photoelectric cell which is used to measure the minutes of sunshine only records when the intensity of solar radiation is more than 82 BTU per square foot per hour. Therefore, some of the very thin cirroform clouds will be ignored by the sunshine recorder as they reduce solar energy by relatively small amounts. United States Weather Bureau Technical Paper No. 12 (32) gives long time means, based on up to 58 years of data, of hours and percentage of possible sunshine for nearly 200 United States Stations. Table X shows a copy of this information for the stations used in this study.

Monthly means of the percentage of possible sunshine are currently recorded in the United States Weather Bureau Climatological Data, National Summary.

Procedure and Results

The mean daily recorded solar radiation on a horizontal surface and the mean percentage of possible sunshine were secured from Climatological Data, National Summary (31) for Albuquerque, Blue Hill, Madison, Lincoln and Lander for each month during the period, 1950 through 1954. Calculated total radiation incident upon a horizontal surface for cloudless days was calculated for each station for the 15th of each month using the equations in Figure 5 and interpolating for the latitude of the respective stations. The values secured from the computation for Lander and Albuquerque were corrected for the altitude of the respective stations by use of the equations in Figure 19.

Table XI shows a compilation of the percentage of possible sunshine (S), mean daily recorded radiation (I) and the ratio of recorded to calculated cloudless day radiation (I/I_o), for the stations mentioned. The ratio, I/I_o, for each month was plotted as a function of the percentage of possible sunshine in each case as shown in Figure 22. The method of least squares was used to determine the linear equation p . The coefficient of correlation between the ratio and percentage of possible sunshine is 0.82 and the standard error of estimate is 6.4 percent.

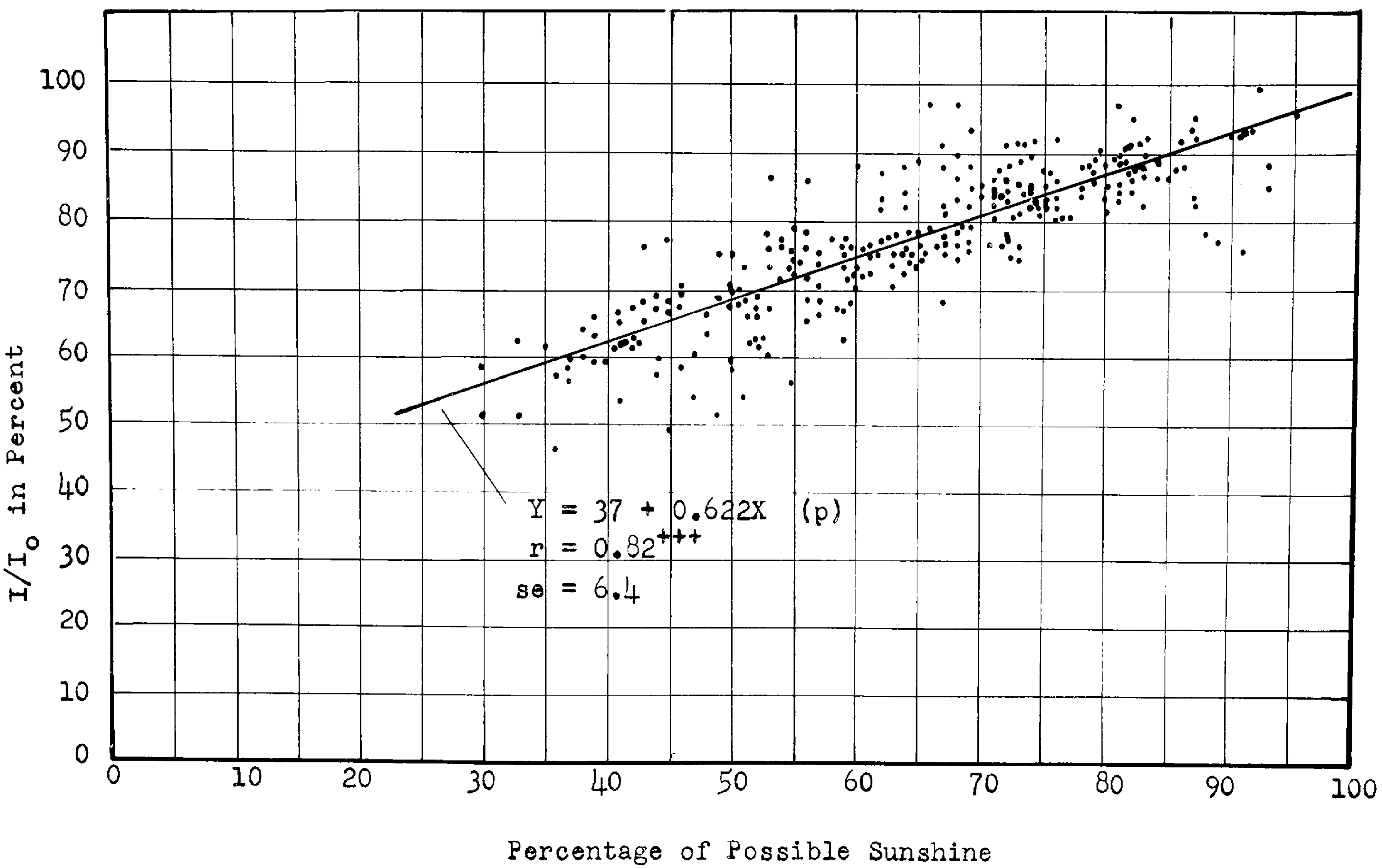


Fig. 22. Relationship between ratio of actual to calculated cloudless day radiation, I/I_o, and percentage of possible sunshine.

DISTRIBUTION OF DAYS WITH VARIOUS AMOUNTS OF SOLAR RADIATION

Average Number of Days in Various Categories for Madison, Wisconsin

The discussion so far presents a method for approximating the many-year average availability of solar radiation. Prediction of the day to day availability of radiation accurately is not possible because of the variation in cloudiness. The need for day to day predicting is great when energy is to be used for house heating, or for some other purpose, where solar energy is to be stored for use during night-time hours or during periods of cloudy weather, because the solar collector performance depends on solar intensity. Telkes (26) states that from the standpoint of solar house heating, the most important information to derive from solar statistics is the sequence of clear, partly cloudy and cloudy days.

The fact that there were relatively few stations in the United States with solar statistics extending over a very long period of time was pointed out in the introduction. In addition, until July of 1941, only the weekly mean of daily total solar radiation recorded by the Weather Burean were published; it has been only since that time that daily totals have been published.

Average distribution of days with various categories of solar radiation incident upon a horizontal surface at Madison, Wisconsin during the period July of 1941 through October of 1955 was determined as presented in Table II. Madison, Wisconsin was selected because it had the first solar station in the United States and because the records published for that station were fairly complete. Category I includes days with daily total radiation equal to, or greater than, the calculated cloudless day radiation from the formulas in Figure 5; category II is from the lower limit of category I to the calculated mean determined from the formulas in Figure 5 corrected for cloudiness by use of equation p and the 43-year average percentage of possible sunshine from Table X; category III is from the lower limit of category II to 25 percent of the calculated cloudless day radiation; and category IV includes days with radiation less than the lower limit of category III. The range of the number of days in each category during the period studied is also shown. The wide range of the number of days which fall into each category points out the large variability in the day-to-day availability of solar radiation, about which nothing can be done except to derive probabilities on the number of, and sequences of, days in various categories when sufficient recorded data are available to do so. Generally speaking, solar radiation statistics of the kind mentioned would

TABLE II

AVERAGE DISTRIBUTION OF DAYS WITH VARIOUS CATEGORIES* OF RADIATION AS RECEIVED ON A HORIZONTAL SURFACE FOR MADISON, WISCONSIN 1941 THROUGH 1955

Month	Category	Number of Days		Total Monthly Radiation BTU per (Ft^2)(Month)	Percentage of Total
		Mean	Range		
January	I	7.4	3 - 12	6,624	38.8
	II	10.1	7 - 14	6,582	38.6
	III	9.7	5 - 15	3,457	20.2
	IV	3.8	1 - 8	378	2.2
February	I	6.6	3 - 13	8,212	37.3
	II	9.5	7 - 14	8,957	40.7
	III	9.1	4 - 12	4,716	21.4
	IV	3.1	1 - 6	120	00.5
March	I	8.5	3 - 13	14,450	42.1
	II	9.6	6 - 16	12,924	37.6
	III	7.9	4 - 11	5,940	17.3
	IV	4.9	2 - 10	982	2.8
April	I	9.2	5 - 16	19,802	44.8
	II	8.2	4 - 13	14,284	32.3
	III	9.6	6 - 15	9,373	21.2
	IV	2.9	0 - 8	738	1.6
May	I	7.1	1 - 13	18,207	34.4
	II	9.5	3 - 15	19,981	37.7
	III	11.7	7 - 20	13,870	26.2
	IV	2.6	0 - 5	809	1.5
June	I	5.2	2 - 9	14,383	25.4
	II	11.7	7 - 15	27,003	47.7
	III	10.9	7 - 15	14,445	25.5
	IV	2.1	0 - 5	683	1.2
July	I	5.5	2 - 8	14,575	23.7
	II	13.2	9 - 17	30,733	50.1
	III	11.6	9 - 15	15,732	25.6
	IV	.7	0 - 1	229	00.3

TABLE II (Cont.)

Month	Category	Number of Days		Total Monthly Radiation (BTU per (Ft^2) (Month)	Percentage of Total
		Mean	Range		
August	I	4.8	1 - 9	11,280	21.2
	II	14.2	8 - 19	28,793	54.2
	III	10.8	8 - 14	12,653	23.8
	IV	1.1	0 - 5	325	00.6
September	I	6.3	3 - 10	12,027	30.2
	II	12.3	7 - 18	19,451	48.8
	III	8.5	5 - 13	7,548	18.9
	IV	3.3	0 - 7	757	1.9
October	I	4.9	1 - 11	6,804	22.8
	II	14.3	10 - 17	16,715	56.0
	III	8.8	5 - 12	5,725	19.1
	IV	3.4	0 - 7	587	1.9
November	I	2.1	0 - 5	1,966	13.9
	II	9.6	7 - 18	7,310	51.9
	III	9.8	4 - 14	4,008	28.4
	IV	6.7	3 - 10	786	5.5
December	I	3.5	1 - 10	2.800	22.7
	II	10.4	7 - 18	6,158	49.9
	III	9.3	7 - 14	2,884	23.3
	IV	5.3	0 - 11	485	3.9
Overall Mean	I				29.8
	II				45.5
	III				22.6
	IV				2.0

*Category I includes days with daily total radiation equal to or more than calculated cloudless day radiation from the formulas in Figure 5; Category II is from the lower limit of category I to the calculated mean determined from the formulas in Figure 5 corrected for cloudiness by use of equation p and percentage of possible sunshine from Table XI ; Category III is from the lower limit of category II to 25 percent of calculated cloudless day radiation and category IV includes radiation less than the lower limit of category III.

apply to a relatively small area immediately adjacent to the measuring station.

It is also possible that with advanced meteorological techniques, short range forecasts and relatively long range trends in solar radiation availability could be made by relating solar energy availability to some other meteorological factors which can be forecast.

Mean Total Solar Radiation in the Various Categories

The approximate means total radiation, in the various categories, mentioned in the last section, was determined for Madison, Wisconsin by multiplying the mean number of days by the most probable average radiation for each category. The percentage of the total radiation of each month in the various categories is also shown. The latter shows that, for the year, about 75 percent of the total radiation is available on days with above average solar radiation. Where solar energy is to be collected and stored at relatively high temperatures, the relative importance of the clear days, or days with above-average solar radiation, will be even greater. The importance of the latter was pointed out by Telkes (27) in a report on the Dover Solar House, where it was shown that almost 94 percent of the energy collected during the month of February 1949 occurred during days with above average radiation.

Sequence of Dark Days

The sequence of days with low amounts of solar radiation, during which little or no energy can be collected, is important, particularly where storage of energy is involved.

Table III shows the maximum sequence of days in category IV during the various months between July of 1941 and October of 1955 for Madison, Wisconsin. The importance of statistics of this kind is apparent from an examination of data for December which shows that the sequence of days in this category varies from none to five during the period of years studied. For solar collector and energy storage design, statistics of this kind would be needed, based on a threshold amount of solar energy below which the amount of energy collected and stored would be negligible.

TABLE III

MAXIMUM SEQUENCE OF DAYS IN CATEGORY IV*

Year	Months											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1941								0	2	3	2	2
1942	2	4	3	1	1	1	1	1	2	2	2	3
1943	1	1	1	1	1	1	1	2	2	1	4	0
1944	1	1	3	2	1	2	1	1	1	1	3	2
1945	1	1	1	0	1	1	0	1	1	1	4	3
1946	2	2	5	0	1	1	1	1	1	-	3	2
1947	2	0	1	3	1	1	0	0	0	3	-	-
1948	0	1	1	0	1	0	1	0	2	1	4	1
1949	1	1	2	1	1	1	1	0	2	1	2	3
1950	3	2	1	2	0	0	1	2	2	1	1	2
1951	1	1	2	2	1	1	1	2	2	2	1	2
1952	2	1	1	2	1	2	0	1	1	0	3	5
1953	2	2	2	1	1	1	1	0	0	0	-	-
1954	2	1	1	1	1	-	1	0	2	2	3	2
1955	2	1	2	1	1	1	0	0	1	3	-	-
Mean	1.57	1.28	1.86	1.20	0.93	1.00	0.71	0.73	1.40	1.75	2.67	2.25

*Category IV includes days with less than 25 percent of calculated cloudless day radiation.

CONCLUSIONS AND PROPOSED FUTURE RESEARCH

The amount of solar radiation available is very large but has the disadvantages of being intermittent and relatively low in intensity. Prediction of the day to day useful energy collection of a given solar collector is not possible owing to the random variation of meteorological factors, such as cloudiness, which are interrelated with solar radiation intensity.

The relative importance of cloudless day solar radiation is great because approximately 75 percent of the total solar radiation is available on days with above-average radiation intensity. In addition, the efficiency of collection with a solar collector is a function of solar intensity in such a way that even more importance is attached to the cloudless day solar radiation.

The cloudless day solar radiation intercepted by a horizontal surface is at a maximum and has little latitudinal variation during mid-summer in the United States. The amount of radiation incident upon a horizontal surface during mid-winter decreases to about 50 percent and 25 percent of the summer value at 30° and 45° north latitude, respectively.

The increase in solar radiation intensity with altitude above sea level as we progress from east to west in the United States is significant with the increase being greater in summer than in winter.

The amount of solar radiation incident on a surface is affected to a very great extent by the orientation of the surface. The proper tilt angle of a south-facing surface is a function of the latitude and season of the year. Little increase in incident solar radiation is noted for an orientation other than horizontal during mid-summer while during mid-winter the distinct advantage of a surface which is nearly vertical is evident. A south-facing surface with optimum tilt angle will have approximately 5 percent and 15 percent more incident cloudless day radiation than a vertical surface on December 21 at 45° and 30° north latitude, respectively. The optimum tilt angle from vertical increases from December to June when the optimum surface would be very nearly horizontal.

The curves and polynomials developed for estimating: (1) the amount of solar radiation available on horizontal surfaces during cloudless days, (2) the ratio of radiation incident upon a south-facing tilted surface to that incident on a horizontal surface, and (3) the increase in radiation with altitude, show reasonably good correlation with actual recorded data except for the Great Lakes region where the calculated values are 10 percent to 25 percent high, depending on the season.

The regression equation relating the ratio of actual to calculated cloudless day radiation and percentage of possible sunshine, provides a method for estimating the average solar radiation for any season at a great many places. The percentage of possible sunshine is a parameter measured and recorded at many stations; long-time averages for the parameter are available. The estimates apply only to average values and are subject to large deviations in individual cases due to local variations in conditions such as, atmospheric pollution, ground reflection, snow cover and local variations due to large cities and industrial areas.

There is a need for verification of the ratio of solar radiation incident upon tilted surfaces to that incident on horizontal surfaces further than that presented in this thesis, as the only verification used was the ratio of vertical to horizontal incident radiation at one station. It is also conceivable that these ratios could be refined further by determining the affect of cloudiness upon them as it would be expected that the ratios would tend toward unity with increased cloudiness.

There is also a need for an intensive study of atmospheric moisture conditions and an extension of the data for direct solar radiation at direct incidence, as shown in Figure 1, to include data for a wider range of atmospheric moisture conditions. It would then be possible to predict solar radiation intensity more accurately.

Methods of describing the variability of solar radiation availability in terms of average distribution of, and probabilities of, numbers and sequences of days with various categories of incident radiation are needed. The categories of incident radiation would conceivably be based on the performance factors of a solar collector design as affected by solar radiation intensity. A method could be devised by using data from one of the older recording stations such as from Madison, Wisconsin. The method could then be used for other localities as more recorded data becomes available.

APPENDIX I

TABLES

TABLE IV

CALCULATED SOLAR RADIATION PER SQUARE FOOT OF SURFACE DURING CLOUDLESS DAYS

30° Latitude

Solar Time 21st day of month	Sun Azimuth Degrees	Sun Altitude Degrees	I_o BTU per hour	Horizontal Surface				
				cos i	I_h BTU per hour	I_{dh} BTU per hour	I_{th} BTU per hour	cos i
Dec								
8am 4pm	126	11	196	0.192	38	17	55	0.577
9am 3pm	136	21	263	.358	94	26	120	.672
10am 2pm	148	29	288	.485	140	30	170	.742
11am 1pm	163	35	300	.574	172	31	203	.783
12 noon	180	37	302	.602	182	32	214	.799
Total BTU /day							1268	
Jan Nov								
8am 4pm	123	14	230	0.242	56	20	76	0.529
9am 3pm	133	24	274	.407	112	28	140	.623
10am 2pm	146	32	295	.530	156	30	186	.703
11am 1pm	162	38	305	.616	188	32	220	.749
12 noon	180	40	308	.643	198	32	230	.766
Total BTU /day							1412	
Feb Oct								
7am 5pm	107	7	140	0.122	17	12	29	0.121
8am 4pm	116	19	256	.326	84	26	110	.412
9am 3pm	126	30	290	.500	145	30	175	.509
10am 2pm	140	40	309	.643	199	32	231	.587
11am 1pm	158	47	315	.731	230	33	263	.632
12 noon	180	49	316	.755	239	34	273	.656
Total BTU /day							1696	
Mar Sept								
7am 5pm	98	13	225	0.225	51	20	71	0.135
8am 4pm	106	26	280	.438	123	28	151	.248
9am 3pm	117	38	305	.616	188	32	220	.358
10am 2pm	131	49	316	.755	239	34	273	.430
11am 1pm	152	57	322	.839	270	35	305	.481
12 noon	180	60	324	.866	281	35	316	.500
Total BTU /day							2090	

TABLE IV (Cont.)

Vertical Surface			Surface Sloped 30° From Vertical				Surface Sloped 60° From Vertical			
I_v BTU per hour	I_{dv} BTU per hour	I_{tv} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour
113	12	125	0.596	117	14	131	0.455	89	15	104
177	26	204	.761	200	26	226	.646	169	26	195
214	32	246	.885	255	31	286	.791	228	31	259
235	43	278	.965	289	39	328	.889	267	35	302
241	43	284	.993	300	39	339	.921	278	36	314
		1886				2173				1950
122	16	138	0.579	133	17	150	0.474	109	19	128
171	29	200	.743	204	29	233	.664	182	28	210
207	37	244	.874	258	35	292	.811	239	32	271
228	45	273	.957	292	41	333	.908	277	36	313
236	45	281	.984	303	41	344	.940	289	36	325
		1867				2229				2059
17	4	21	0.166	23	10	33	0.166	23	6	29
105	19	124	.520	133	20	153	.488	125	23	148
148	33	181	.691	200	32	232	.688	199	31	230
181	42	223	.829	256	39	295	.850	263	35	298
199	46	245	.912	287	42	329	.949	299	37	336
207	46	253	.945	299	42	341	.982	310	38	348
		1677				2193				2197
30	11	41	0.229	51	14	65	0.263	59	17	76
69	24	93	.434	121	25	146	.503	140	27	167
109	34	143	.618	188	33	221	.712	217	33	250
136	41	174	.749	236	39	275	.869	274	36	310
155	46	201	.836	269	42	311	.967	311	39	350
162	47	219	.866	281	43	324	1.000	324	39	363
		1345				2092				2366

TABLE IV (Cont.)

Solar Time 21st day of month	Sun Azimuth Degrees	Sun Altitude Degrees	I_o BTU per hour	Horizontal Surface				
				cos i	I_h BTU per hour	I_{dh} BTU per hour	I_{th} BTU per hour	cos i
Apr Aug								
6am 6pm	81	6	80	0.105	8	5	13	-0.155
7am 5pm	89	19	195	.326	64	22	86	- .017
8am 4pm	94	32	240	.530	127	29	156	.059
9am 3pm	103	45	266	.707	188	32	220	.159
10am 2pm	116	57	281	.839	236	34	270	.239
11am 1pm	139	67	288	.920	265	35	300	.295
12 noon	180	72	290	.951	276	35	311	.309
Total BTU /day							2401	
May July								
6am 6pm	72	10	122	0.174	21	14	35	-0.304
7am 5pm	79	23	210	.391	82	25	107	- .176
8am 4pm	85	35	248	.574	142	30	172	- .071
9am 3pm	93	48	270	.743	201	32	233	.035
10am 2pm	103	61	284	.875	249	34	283	.109
11am 1pm	122	73	291	.956	278	35	313	.155
12 noon	180	80	293	.985	289	35	324	.174
Total BTU /day							2589	
June								
6am 6pm	70	11	140	0.191	27	15	42	-0.336
7am 5pm	75	24	215	.407	87	25	112	- .237
8am 4pm	82	37	251	.602	151	30	181	- .111
9am 3pm	88	49	271	.755	205	32	237	- .023
10am 2pm	97	62	284	.883	251	34	285	.057
11am 1pm	112	75	291	.966	281	35	316	.097
12 noon	180	83	293	.993	291	35	326	.122
Total BTU /day							2641	

TABLE IV (Cont.)

Vertical Surface			Surface Sloped 30° From Vertical				Surface Sloped 60° From Vertical			
I_v BTU per hour	I_{dv} BTU per hour	I_{tv} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour
	3				4	4	0.013	10	4	14
	10		-0.148	29	14	43	.273	54	18	72
14	23	37	.316	76	25	101	.489	117	27	144
42	34	76	.491	131	33	164	.692	184	33	217
67	39	106	.626	176	37	213	.846	238	36	274
85	43	128	.715	206	40	246	.944	272	38	310
89	45	133	.743	216	42	258	.977	283	38	321
		799				1802				2379
	5				8	8			11	11
	12		-0.043	9	16	25	-0.250	52	21	73
	16	16	.225	56	21	77	.461	114	25	139
9	32	41	.401	108	32	140	.661	178	32	210
31	37	68	.531	151	36	187	.813	231	35	266
45	43	88	.612	178	40	218	.906	264	38	302
51	44	95	.643	188	41	229	.940	275	38	313
		508				1532				2317
	5				8	8			12	12
	12				16	16	-0.233	50	21	71
	16		-0.205	51	21	72	.465	117	25	142
	20	20	.357	97	24	121	.642	174	28	202
16	30	53	.490	139	31	170	.793	225	33	258
28	42	70	.567	165	40	205	.885	257	37	294
36	44	80	.602	176	41	217	.921	270	38	308
		301				1385				2260

TABLE V

CALCULATED SOLAR RADIATION PER SQUARE FOOT OF SURFACE DURING CLOUDLESS DAYS

35° Latitude

Solar Time 21st day of month	Sun Azimuth Degrees	Sun Altitude Degrees	I_o BTU per hour	Horizontal Surface				
				cos i	I_h BTU per hour	I_{dh} BTU per hour	I_{th} BTU per hour	cos i
Dec.								
8am 4pm	126	8	160	0.140	22	14	36	0.582
9am 3pm	137	18	253	.310	78	23	101	.695
10am 2pm	150	25	278	.423	118	28	146	.785
11am 1pm	164	30	290	.500	145	29	174	.832
12 noon	180	32	295	.530	156	30	186	.848
Total BTU /day							1076	
Jan Nov								
8am 4pm	124	11	196	0.192	38	17	55	0.549
9am 3pm	135	22	261	.374	98	27	125	.655
10am 2pm	148	29	288	.485	140	31	171	.742
11am 1pm	163	33	298	.545	163	32	195	.802
12 noon	180	35	302	.575	174	33	207	.819
Total BTU /day							1258	
Feb Oct								
7am 5pm	108	6	120	0.105	13	10	23	0.129
8am 4pm	117	17	248	.293	74	23	97	.434
9am 3pm	128	27	283	.454	129	29	158	.544
10am 2pm	143	36	301	.588	177	31	208	.646
11am 1pm	160	42	310	.669	207	33	240	.698
12 noon	180	44	312	.695	217	34	251	.719
Total BTU /day							1527	
Mar Sept								
7am 5pm	98	12	208	0.209	43	18	61	0.136
8am 4pm	108	24	274	.407	112	28	140	.282
9am 3pm	120	35	300	.574	172	31	203	.410
10am 2pm	135	45	313	.707	221	33	254	.500
11am 1pm	155	52	319	.788	251	34	285	.558
12 noon	180	55	320	.819	262	34	296	.574
Total BTU /day							1940	

TABLE V (Cont.)

Vertical Surface			Surface Sloped 30° From Vertical				Surface Sloped 60° From Vertical			
I_v BTU per hour	I_{dv} BTU per hour	I_{tv} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour
93	9	102	0.574	92	11	103	0.412	66	12	78
176	25	201	.759	192	25	217	.620	157	24	181
218	33	251	.898	250	32	282	.771	214	29	243
241	41	282	.971	282	38	320	.849	246	34	280
250	41	291	.999	295	38	333	.882	260	34	294
		1885				2100				1802
108	12	120	0.571	112	14	126	0.441	86	15	101
171	29	200	.755	197	28	225	.635	166	27	193
215	39	254	.886	255	36	291	.791	228	33	261
238	42	280	.960	286	39	325	.900	268	35	303
247	44	291	.990	299	41	340	.939	283	36	319
		1894				2166				1952
16	3	19	0.173	21	5	26	0.156	18	7	25
108	17	125	.525	130	19	149	.474	118	21	139
154	32	186	.716	203	31	234	.695	197	30	227
194	40	234	.853	257	37	294	.832	250	34	284
216	45	261	.939	291	42	333	.928	288	37	325
224	45	269	.970	303	42	345	.962	300	38	338
		1746				2186				2107
28	10	38	0.223	46	13	59	0.249	52	15	67
77	24	101	.448	123	25	148	.493	135	27	162
123	31	154	.642	193	31	224	.702	211	31	242
157	41	198	.787	246	39	285	.862	270	35	305
178	46	224	.888	283	42	325	.961	307	38	345
184	47	231	.907	290	43	333	.996	318	38	356
		1481				2149				2311

TABLE V (Cont.)

Solar Time 21st day of month	Sun Azimuth Degrees	Sun Altitude Degrees	I_o BTU per hour	Horizontal Surface				
				cos i	I_h BTU per hour	I_{dh} BTU per hour	I_{th} BTU per hour	cos i
Apr Aug								
6am 6pm	82	7	80	0.122	10	8	18	-0.138
7am 5pm	88	19	195	.326	63	21	84	- .038
8am 4pm	97	31	236	.515	122	29	151	.105
9am 3pm	108	43	262	.682	179	31	210	.226
10am 2pm	123	54	278	.809	225	33	258	.320
11am 1pm	146	63	285	.891	254	34	288	.376
12 noon	180	67	288	.921	265	34	299	.391
Total BTU /day							2310	
May Jul								
6am 6pm	73	11	140	0.192	27	11	38	-0.286
7am 5pm	82	23	210	.391	82	25	107	- .128
8am 4pm	89	36	250	.588	147	30	177	0
9am 3pm	98	48	270	.743	201	32	233	.093
10am 2pm	111	60	283	.866	245	34	279	.179
11am 1pm	134	70	290	.940	273	35	308	.238
12 noon	180	75	291	.966	281	35	316	.259
Total BTU /day							2573	
June								
6am 6pm	71	13	155	0.225	35	16	51	-0.317
7am 5pm	79	25	220	.423	93	26	119	- .173
8am 4pm	85	37	251	.607	151	30	181	- .069
9am 3pm	94	49	271	.755	205	32	237	.046
10am 2pm	106	62	284	.883	251	34	285	.129
11am 1pm	127	73	291	.956	278	35	313	.176
12 noon	180	78	292	.978	286	35	321	.208
Total BTU /day							2655	

TABLE V (Cont.)

Vertical Surface			Surface Sloped 30° From Vertical				Surface Sloped 60° From Vertical			
I_v BTU per hour	I_{dv} BTU per hour	I_{tv} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour
	4				5	5	0.037	3	7	10
	11	11	0.134	26	13	39	.263	51	17	68
25	24	49	.349	82	24	106	.499	118	27	145
59	33	92	.537	141	32	173	.704	184	31	215
89	41	130	.682	190	38	228	.861	239	35	274
107	45	152	.772	220	41	261	.960	274	37	311
113	46	159	.800	230	43	273	.994	286	38	324
		916				1893				2367
	4				6	6	0.022	3	9	12
	11		0.085	18	16	34	.275	58	20	78
	15	15	.294	74	20	94	.588	147	25	172
25	26	51	.453	122	28	150	.690	186	30	216
51	34	85	.588	166	33	199	.828	234	34	268
69	40	109	.676	196	38	234	.933	271	36	307
75	45	120	.707	206	42	248	.967	281	38	319
		630				1676				2411
	7				10	10	0.036	6	13	19
	12		0.062	14	13	27	.279	61	21	82
	15	15	.244	61	20	81	.491	123	25	148
12	33	45	.418	113	33	146	.677	183	32	215
37	39	76	.554	157	37	194	.830	236	36	272
51	43	94	.630	183	40	223	.916	267	38	305
61	45	106	.669	195	42	237	.951	278	38	316
		553				1588				2390

TABLE VI

CALCULATED SOLAR RADIATION PER SQUARE FOOT OF SURFACE DURING CLOUDLESS DAYS

40° Latitude

Solar Time 21st day of month	Sun Azimuth Degrees	Sun Altitude Degrees	I_o BTU per hour	Horizontal Surface				
				cos i	I_h BTU per hour	I_{dh} BTU per hour	I_{th} BTU per hour	cos i
Dec								
8am 4pm	127	5	110	0.087	10	8	18	0.600
9am 3pm	138	14	234	.242	56	13	69	.721
10am 2pm	151	21	263	.358	95	26	121	.817
11am 1pm	165	25	278	.423	118	28	146	.875
12 noon	180	26	280	.438	123	29	152	.899
Total BTU /day							848	
Jan Nov								
8am 4pm	125	8	160	0.139	22	15	37	0.568
9am 3pm	136	17	248	.292	72	23	95	.687
10am 2pm	149	24	274	.406	111	28	139	.783
11am 1pm	164	28	287	.470	135	30	165	.849
12 noon	180	30	290	.500	145	30	175	.866
Total BTU /day							1021	
Feb Oct								
8am 4pm	118	14	230	0.242	56	20	76	0.456
9am 3pm	130	24	274	.406	111	28	139	.587
10am 2pm	145	31	292	.515	150	30	180	.702
11am 1pm	161	37	302	.602	182	32	214	.756
12 noon	180	39	308	.629	194	32	226	.777
Total BTU /day							1383	
Mar Sept								
7am 5pm	100	11	196	0.191	37	17	54	0.171
8am 4pm	110	23	272	.391	106	27	133	.315
9am 3pm	123	33	297	.545	162	30	192	.457
10am 2pm	138	42	310	.669	207	33	240	.552
11am 1pm	157	48	316	.743	235	34	269	.615
12 noon	180	50	317	.766	243	34	277	.643
Total BTU /day							1831	

TABLE VI (Cont.)

Vertical Surface			Surface Sloped 30° From Vertical				Surface Sloped 60° From Vertical			
I_v BTU per hour	I_{dv} BTU per hour	I_{tv} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU's per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour
66	6	72	0.520	57	7	64	0.375	41	7	48
166	17	183	.745	174	16	190	.571	134	14	148
215	33	248	.886	233	31	264	.719	189	28	217
243	36	279	.968	269	34	303	.804	224	30	254
252	37	289	.998	279	35	314	.829	232	31	263
		1804				1920				1596
91	10	101	0.571	91	11	101	0.404	65	14	79
170	22	192	.751	186	22	208	.597	148	23	171
215	32	247	.881	241	31	272	.744	204	29	233
244	39	283	.970	278	36	314	.832	239	33	272
251	39	290	1.000	290	36	326	.866	251	33	284
		1858				2040				1736
105	16	121	0.516	119	17	136	0.437	101	19	120
161	29	190	.711	195	29	224	.646	176	28	204
205	37	242	.865	253	36	289	.797	233	32	265
228	42	270	.956	289	39	328	.899	271	35	306
239	42	281	.987	304	39	343	.934	288	35	323
		1818				2178				2010
34	7	41	0.243	48	10	58	0.251	49	14	63
86	21	107	.468	127	23	150	.497	135	25	160
136	34	170	.668	198	33	231	.700	208	31	239
171	41	212	.812	252	39	291	.855	265	35	300
194	45	239	.904	286	42	328	.951	300	37	337
204	46	250	.940	298	42	340	.985	312	38	340
		1590				2188				2264

TABLE VI (Cont.)

Solar Time 21st day of month	Sun Azimuth Degrees	Sun Altitude Degrees	I_o BTU per hour	Horizontal Surface				
				cos i	I_h BTU per hour	I_{dh} BTU per hour	I_{th} BTU per hour	cos i
April Aug								
6am 6pm	83	8	110	0.140	15	11	26	-0.121
7am 5pm	90	19	195	.326	64	21	85	.0
8am 4pm	100	30	234	.500	117	28	145	.151
9am 3pm	112	41	260	.656	171	31	202	.283
10am 2pm	128	51	275	.777	214	33	247	.387
11am 1pm	150	59	282	.857	242	34	276	.446
12 noon	180	62	284	.883	251	34	285	.470
Total BTU /day							2232	
May July								
6am 6pm	72	13	160	0.225	36	17	53	-0.301
7am 5pm	83	24	215	.407	88	25	113	-0.111
8am 4pm	92	36	250	.588	147	30	177	.028
9am 3pm	104	47	269	.731	197	32	229	.165
10am 2pm	119	58	281	.848	238	34	272	.257
11am 1pm	142	66	288	.913	263	35	298	.321
12 noon	180	70	290	.940	273	35	308	.342
Total BTU /day							2547	
June								
6am 6pm	72	15	170	0.259	44	18	62	-0.299
7am 5pm	80	26	220	.438	97	27	124	-0.156
8am 4pm	89	37	251	.602	151	28	179	.0
9am 3pm	100	49	271	.755	205	33	238	.114
10am 2pm	114	60	283	.866	245	34	279	.205
11am 1pm	138	69	289	.934	271	36	307	.266
12 noon	180	73	291	.956	279	36	315	.292
Total BTU /day							2647	

TABLE VI (Cont.)

Vertical Surface			Surface Sloped 30° From Vertical				Surface Sloped 60° From Vertical			
I_v BTU per hour	I_{dv} BTU per hour	I_{tv} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour
	4				7	7			9	9
	10		0.163	32	14	46	0.282	55	18	73
35	22	57	.381	89	24	113	.409	96	26	122
74	33	107	.573	149	33	182	.710	185	31	216
106	37	143	.723	199	36	235	.867	238	34	272
126	43	169	.814	230	40	270	.965	272	37	309
133	45	178	.848	241	42	283	1.000	284	38	322
		1084				1986				2342
	7	7			10	10			14	14
	12	12	0.107	23	16	39	0.296	64	21	85
7	24	31	.318	80	26	106	.523	131	28	169
44	34	78	.508	137	34	171	.716	192	32	224
72	40	112	.647	182	38	220	.863	243	36	279
92	44	136	.734	211	41	252	.952	274	38	312
99	47	146	.766	222	43	265	.985	286	39	325
		895				1850				2481
	8				11	11			17	17
	12		0.084	18	17	35	0.301	66	22	88
	17	17	.341	86	20	106	.521	131	24	155
31	35	66	.496	134	34	168	.711	193	33	226
58	38	96	.611	173	37	210	.853	241	35	276
77	41	118	.697	201	39	240	.942	272	37	309
85	44	129	.731	213	41	255	.974	283	38	321
		716				1781				2463

TABLE VII

CALCULATED SOLAR RADIATION PER SQUARE FOOT OF SURFACE DURING CLOUDLESS DAYS

45° Latitude

Solar Time 21st day of month	Sun Azimuth Degrees	Sun Altitude Degrees	I_o BTU per hour	Horizontal Surface				
				cos i	I_h BTU per hour	I_{dh} BTU per hour	I_{th} BTU per hour	cos i
Dec								
9am 3pm	139	10	185	0.174	32	16	48	0.744
10am 2pm	151	16	241	.276	67	23	90	.841
11am 1pm	165	20	260	.349	91	26	117	.908
12 noon	180	22	266	.384	102	27	129	.927
Total BTU /day							646	
Jan Nov								
8am 4pm	125	5	110	0.087	10	8	18	0.586
9am 3pm	137	13	225	.225	51	19	70	.712
10am 2pm	150	19	256	.326	83	25	108	.818
11am 1pm	165	24	274	.419	115	28	143	.883
12 noon	180	25	278	.436	121	29	150	.906
Total BTU /day							824	
Feb Oct								
8am 4pm	119	12	208	0.208	43	18	61	0.474
9am 3pm	132	21	263	.366	96	26	122	.625
10am 2pm	146	28	287	.469	135	30	165	.732
11am 1pm	162	33	297	.545	162	31	193	.798
12 noon	180	34	298	.559	167	31	196	.829
Total BTU /day							1232	
Mar Sept								
7am 5pm	107	11	196	0.191	37	17	54	0.287
8am 4pm	112	21	263	.366	96	26	122	.359
9am 3pm	125	30	290	.500	145	30	175	.509
10am 2pm	141	38	305	.616	188	32	220	.612
11am 1pm	159	43	311	.682	212	33	245	.683
12 noon	180	45	313	.707	221	33	254	.707
Total BTU /day							1678	

TABLE VII (Cont.)

Vertical Surface			Surface Slcped 30° From Vertical				Surface Sloped 60° From Vertical			
I_v BTU per hour	I_{dv} BTU per hour	I_{tv} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour
137	13	150	0.731	135	14	149	0.522	97	15	112
203	23	226	.866	209	23	232	.714	172	23	195
236	31	267	.961	250	30	280	.756	197	28	225
247	33	280	.994	264	31	295	.976	212	29	241
		1601				1651				1320
64	4	68	0.551	61	5	66	0.368	40	7	47
160	18	178	.730	164	18	182	.551	124	19	143
209	27	236	.871	223	27	250	.691	177	26	203
242	37	279	.975	267	34	301	.804	220	31	251
252	37	289	.999	278	34	312	.891	248	31	279
		1772				1875				1539
99	12	111	0.514	107	14	121	0.415	86	16	102
164	26	190	.724	190	26	216	.629	165	26	191
210	36	246	.875	251	34	285	.772	222	32	254
237	41	278	.964	286	38	324	.871	259	34	293
247	41	288	.998	297	38	335	.898	268	34	302
		1843				2125				1900
56	6	62	0.345	68	9	77	0.308	60	13	73
94	16	110	.494	130	19	149	.496	130	23	153
148	31	179	.691	200	31	231	.687	199	30	229
187	40	227	.838	256	37	293	.839	256	34	290
212	45	257	.932	290	41	331	.932	290	37	327
221	45	265	.966	302	41	343	.965	302	37	339
		1698				2210				2198

TABLE VII (Cont.)

Solar Time 21st day of month	Sun Azimuth Degrees	Sun Altitude Degrees	I_o BTU per hour	Horizontal Surface				
				cos i	I_h BTU per hour	I_{dh} BTU per hour	I_{th} BTU per hour	cos i
Apr Aug								
6am 6pm	82	8	110	0.139	15	11	26	-0.138
7am 5pm	93	19	195	.326	64	22	86	.049
8am 4pm	103	30	234	.500	117	27	144	.195
9am 3pm	116	39	254	.629	160	30	190	.353
10am 2pm	133	48	270	.743	201	32	233	.456
11am 1pm	154	55	279	.819	229	33	262	.516
12 noon	180	57	281	.839	236	34	270	.545
Total BTU /day							2136	
May July								
6am 6pm	76	14	160	0.242	39	17	56	-0.235
7am 5pm	84	25	220	.423	93	26	119	-0.063
8am 4pm	96	35	248	.574	142	30	172	.056
9am 3pm	109	46	268	.719	193	32	225	.231
10am 2pm	125	55	279	.819	229	33	262	.338
11am 1pm	148	62	284	.883	251	34	285	.400
12 noon	180	65	287	.906	260	34	294	.423
Total BTU /day							2491	
June								
6am 6pm	76	20	200	0.342	68	22	90	-0.227
7am 5pm	85	27	220	.454	100	26	126	-0.078
8am 4pm	93	37	251	.602	151	30	181	.042
9am 3pm	105	48	270	.743	201	32	233	.173
10am 2pm	121	58	281	.848	238	34	272	.273
11am 1pm	145	65	287	.906	260	34	294	.346
12 noon	180	69	289	.934	270	35	305	.358
Total BTU /day							2610	

TABLE VII (Cont.)

Vertical Surface			Surface Sloped 30° From Vertical				Surface Sloped 60° From Vertical			
I_v BTU per hour	I_{dv} BTU per hour	I_{tv} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour	cos i	I_t BTU per hour	I_{dt} BTU per hour	I_{tt} BTU per hour
	4				6	6	0.091	10	9	19
10	10	20	0.205	40	13	53	.303	59	18	77
46	26	72	.419	98	26	124	.530	124	27	151
90	32	122	.621	158	31	189	.721	183	31	214
123	40	163	.767	207	37	244	.871	235	34	269
144	44	188	.857	239	41	280	.967	270	37	307
153	45	198	.892	251	42	293	.999	281	38	319
		1186				2083				2381
	8				11	11	0.093	15	14	29
	12		0.517	35	17	52	.334	74	21	95
14	27	41	.335	83	28	111	.525	130	29	159
62	34	96	.560	150	33	183	.738	198	33	231
94	40	134	.703	196	38	234	.878	245	35	280
114	44	158	.788	224	41	265	.965	274	37	311
121	45	166	.819	235	41	276	.996	286	38	324
		994				1986				2521
	10				14	14	0.183	37	18	55
	12		0.159	35	17	52	.359	79	21	100
11	20	30	.337	85	23	108	.542	136	27	163
47	37	84	.522	141	35	176	.729	197	34	231
77	38	115	.660	185	37	222	.870	244	35	279
99	43	142	.753	216	40	256	.958	275	37	312
103	45	148	.777	225	42	267	.988	286	38	324
		875				1919				2558

TABLE VIII

RATIO OF HORIZONTAL TO SOUTH-FACING TILTED SURFACE INCIDENT RADIATION

Date	Angle of Surface from Vertical Degrees	Latitude			
		30°	35°	40°	45°
Dec. 21	0	1.49	1.75	2.13	2.48
	30	1.71	1.95	2.26	2.55
	60	1.54	1.68	1.87	2.04
Jan. 21	0	1.32	1.51	1.82	2.15
and	30	1.58	1.72	2.02	2.28
Nov. 21	60	1.46	1.55	1.70	1.87
Feb. 21	0	0.99	1.14	1.31	1.50
and	30	1.29	1.43	1.57	1.73
Oct. 21	60	1.29	1.38	1.45	1.54
Mar. 21	0	0.64	0.77	0.87	1.01
and	30	1.00	1.10	1.19	1.32
Sept. 21	60	1.13	1.19	1.24	1.31
Apr. 21	0	0.32	0.40	0.49	0.57
and	30	.72	.82	.89	.98
Aug. 21	60	.99	1.02	1.05	1.12
May 21	0	0.19	0.25	0.35	0.40
and	30	.59	.65	.72	.80
July 21	60	.89	.94	.97	1.01
June 21	0	0.11	0.21	0.28	0.33
	30	.52	.60	.67	.73
	60	.85	.90	.93	.98

TABLE IX

AVERAGE NUMBER OF CLEAR, PARTLY CLOUDY, AND CLOUDY DAYS[*]

	Boston	Lansing	Lincoln	Albuquerque	Madison	Lander
Yrs. of Data	63	37	51	53	69	56
Month						
Jan.						
Clear	9	4	11	18	8	12
Partly Cloudy	9	7	8	7	9	13
Cloudy	13	20	12	6	14	6
Feb.						
Clear	10	5	9	14	7	11
Partly Cloudy	8	8	8	8	8	12
Cloudy	10	15	11	6	13	5
Mar.						
Clear	10	7	10	16	8	10
Partly Cloudy	9	9	9	10	9	14
Cloudy	12	15	12	5	14	7
April						
Clear	9	8	9	15	7	8
Partly Cloudy	10	9	9	10	10	13
Cloudy	11	13	12	5	13	9
May						
Clear	9	9	8	17	8	9
Partly Cloudy	10	10	11	10	11	14
Cloudy	12	12	12	4	12	8
June						
Clear	10	9	10	18	7	12
Partly Cloudy	9	13	12	10	12	13
Cloudy	11	8	8	2	11	5
July						
Clear	9	12	13	12	10	13
Partly Cloudy	13	13	12	15	14	15
Cloudy	9	6	6	4	7	7
Aug.						
Clear	11	11	12	12	10	14
Partly Cloudy	11	13	12	15	13	13
Cloudy	9	7	7	4	8	4

TABLE IX (Cont.)

	Boston	Lansing	Lincoln	Albuquerque	Madison	Lander
Yrs. of data	63	37	51	53	69	56
Sept.						
Clear	12	10	14	17	10	14
Partly Cloudy	9	10	8	9	10	11
Cloudy	9	10	8	4	10	5
Oct.						
Clear	11	10	15	20	10	14
Partly Cloudy	10	9	7	7	9	11
Cloudy	10	12	9	4	12	6
Nov.						
Clear	9	5	11	20	7	11
Partly Cloudy	9	6	8	6	9	13
Cloudy	12	19	11	4	14	6
Dec.						
Clear	9	4	10	18	6	12
Partly Cloudy	9	6	9	7	8	13
Cloudy	13	21	12	6	17	6
Annual						
Clear	118	94	132	197	98	140
Partly Cloudy	118	113	113	114	122	155
Cloudy	129	158	120	54	145	170

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*Technical Paper No. 12, U. S. Department of Commerce Weather Bureau

TABLE X

PERCENTAGE OF POSSIBLE SUNSHINE*

Yrs. of Data	Boston 54	Lansing 37	Lincoln 43	Albuquerque 25	Madison 43	Lander 45
Month						
Jan.	49	33	57	71	43	66
Feb.	56	44	59	70	48	70
Mar.	58	53	60	74	51	71
Apr.	57	58	60	76	53	65
May	59	63	62	80	56	65
June	62	69	69	84	62	74
July	53	75	76	77	70	76
Aug.	63	68	70	77	64	75
Sept.	61	59	66	79	57	71
Oct.	58	52	64	80	52	66
Nov.	48	35	57	77	39	60
Dec.	48	28	54	72	36	62
Annual	57	53	63	77	53	69

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*From Technical Paper No. 12, U. S. Department of Commerce Weather Bureau

TABLE XI

RELATIONSHIP BETWEEN THE RATIO OF OBSERVED (I) TO CALCULATED CLOUDLESS DAY RADIATION (I_o) AND PERCENTAGE OF POSSIBLE SUNSHINE (S)

Date		Blue Hill				Lincoln				Madison	
		Radiation Ly./Day				Radiation Ly./Day				Radiation Ly./Day	
		I_o	I	I/I_o	S	I_o	I	I/I_o	S	I_o	I
Jan.	'50	236	120	.51	30	244	186	.76	54	217	136
	'51		150	.64	38		218	.89	73		168
	'52		142	.60	37		-	-	58		142
	'53		191	.81	40		188	.77	59		124
	'54		121	.51	33		185	.76	59		152
Feb.	'50	325	183	.56	37	344	258	.78	63	306	208
	'51		215	.66	39		241	.73	51		189
	'52		220	.68	51		-	-	50		206
	'53		226	.69	52		255	.77	58		210
	'54		192	.59	44		274	.83	71		204
Mar.	'50	442	380	.86	53	461	343	.74	55	434	314
	'51		257	.58	30		345	.75	55		285
	'52		274	.62	41		-	-	43		332
	'53		265	.60	38		338	.73	55		290
	'54		323	.73	57		338	.73	59		323
Apr.	'50	564	387	.69	44	576	426	.74	55	555	366
	'51		424	.75	50		350	.61	41		337
	'52		376	.67	44		-	-	51		431
	'53		354	.63	39		388	.67	52		363
	'54		394	.70	50		434	.75	67		380
May	'50	664	497	.75	49	669	504	.75	61	664	542
	'51		500	.75	57		502	.75	59		498
	'52		450	.68	50		-	-	68		414
	'53		456	.69	46		510	.76	64		482
	'54		390	.59	40		485	.72	64		468
June	'50	713	602	.84	64	713	657	.92	83	713	574
	'51		487	.68	43		440	.62	41		503
	'52		555	.78	65		-	-	80		498
	'53		655	.92	74		568	.80	76		589
	'54		479	.67	53		594	.83	81		548

TABLE XI (Cont.)

Madison		Lander				Albuquerque			
		Radiation Ly./Day				Radiation Ly./Day			
I/I_o	S	I_o	I	I/I_o	S	I_o	I	I/I_o	S
.63	48	257	220	.86	56	352	267	.76	68
.77	45	-	-	-	58		307	.87	75
.65	41	-	-	-	62		302	.85	69
.57	36		226	.88	69		341	.97	81
.70	46		-	-	64		312	.88	67
.68	56	368	324	.88	72	455	373	.82	71
.62	33		-	-	64		388	.85	73
.67	42		358	.97	66		388	.85	70
.69	50		-	-	66		409	.90	70
.67	59		320	.87	75		456	1.00	92
.72	55	496	442	.89	65	583	428	.74	73
.66	45		-	-	60		485	.83	68
.76	56		492	.97	68		505	.86	68
.67	46		-	-	78		534	.91	73
.74	68		437	.88	60		502	.86	72
.66	48	642	600	.93	69	700	601	.86	84
.61	35		-	-	64		600	.86	71
.78	64		582	.90	68		619	.88	72
.65	42		589	.91	67		639	.91	73
.68	51		559	.87	79		668	.95	87
.82	75	786	595	.76	65	789	615	.78	88
.75	59		-	-	63		674	.85	74
.62	59		595	.76	61		671	.85	75
.73	60		594	.76	61		724	.92	76
.70	57		632	.80	71		708	.90	79
.81	75	840	716	.85	78	840	683	.82	87
.71	54		-	-	76		778	.93	91
.70	60		747	.89	79		732	.88	86
.83	72		738	.88	86		745	.89	81
.77	69		679	.81	73		777	.93	90

TABLE

TABLE XI (Cont.)

Date	Blue Hill				Lincoln				Madison	
	Radiation Ly./Day				Radiation Ly./Day				Radiation Ly./Day	
	I_o	I	I/I_o	S	I_o	I	I/I_c	S	I_o	I
July '50	699	553	.79	55	699	477	.68	49	699	599
'51		501	.72	56		525	.75	64		529
'52		577	.83	75		-	-	80		576
'53		-	-	61		569	.81	73		523
'54		504	.72	60		580	.83	80		514
Aug. '50	623	464	.74	53	626	518	.83	62	623	463
'51		413	.66	52		479	.77	62		407
'52		423	.68	57		471	.75	63		471
'53		452	.73	63		552	.88	84		504
'54		389	.62	52		449	.72	62		473
Sept. '50	507	343	.68	45	515	386	.75	65	496	356
'51		366	.72	60		315	.61	42		370
'52		378	.75	69		474	.92	83		411
'53		403	.79	69		482	.94	87		442
'54		258	.51	49		437	.85	81		331
Oct. '50	385	302	.78	63	396	349	.88	82	384	274
'51		224	.58	46		-	-	51		232
'52		262	.68	67		360	.91	82		312
'53		240	.62	52		351	.89	83		291
'54		225	.58	50		237	.60	47		211
Nov. '50	274	173	.63	42	279	218	.78	53	257	171
'51		158	.58	50		-	-	59		170
'52		156	.57	44		213	.76	71		161
'53		148	.54	47		185	.66	57		-
'54		148	.54	51		229	.82	76		127
Dec. '50	206	127	.62	42	217	176	.81	67	201	153
'51		143	.69	50		-	-	54		117
'52		86	.42	41		169	.78	66		93
'53		124	.60	53		166	.77	67		118
'54		110	.53	41		160	.74	65		-

TABLE XI (Cont.)

Madison		Lander				Albuquerque			
		Radiation Ly./Day				Radiation Ly./Day			
I/I_o	S	I_o	I	I/I_o	S	I_o	I	I/I_o	S
.86	76	818	637	.78	72	826	679	.82	67
.76	72		-	-	75		691	.84	74
.82	74		719	.88	79		688	.83	76
.75	64		670	.82	83		690	.84	70
.74	73		702	.86	85		724	.88	81
.74	65	732	611	.83	74	759	649	.86	79
.65	56		-	-	69		620	.82	69
.76	67		573	.78	68		640	.84	72
.81	80		585	.80	77		671	.88	78
.76	67		610	.83	87		645	.85	80
.72	61	593	415	.70	63	645	491	.76	73
.75	61		-	-	69		600	.93	91
.83	78		561	.95	82		555	.86	82
.89	81		528	.89	84		619	.96	95
.67	59		-	-	85		568	.88	82
.73	65	434	365	.84	82	512	451	.88	93
.62	52		-	-	55		452	.88	81
.84	81		405	.93	82		435	.85	93
.78	75		344	.79	68		471	.92	86
.56	55		-	-	80		471	.92	86
.66	41	298	234	.78	56	396	299	.76	91
.66	52		-	-	61		335	.84	74
.62	52		231	.77	54		336	.85	71
-	52		246	.82	62		348	.88	80
.49	45		239	.80	77		374	.94	93
.76	43	225	171	.76	53	325	250	.77	89
.58	37		-	-	55		269	.82	64
.46	36		199	.88	64		284	.87	62
.59	39		190	.84	71		296	.91	72
-	40		195	.87	83		295	.90	74

TABLE XII

MONTHLY MEAN PRECIPITABLE WATER IN THE UNITED STATES

Month	Mean Precipitable Water Centimeters	
	Recorded* for All Days	Cloudless Day Value Used in Computation
January	1.12	1.05
February	1.14	1.05
March	1.24	1.05
April	1.62	2.00
May	2.14	2.00
June	2.83	2.00
July	3.32	2.00
August	3.32	2.00
September	2.78	1.05
October	2.08	1.05
November	1.50	1.05
December	1.35	1.05

*From Technical Paper No. 10, United States Department of Commerce Weather Bureau

APPENDIX II

SAMPLE COMPUTATION

Sample Computation of Fourth Degree Polynomial for Solar Radiation Curves of Figure 5

For the sample computation shown below, the data from Table V for 35° Latitude is used. X is equal to the number of months from June 21 and Y is equal to the calculated cloudless daily total solar radiation incident upon a horizontal surface. Y' is the value of Y computed from the fourth degree polynomial.

Date	X	Y	Y'
Dec. 21	-6	1076	1091
Jan. 21	-5	1258	1218
Feb. 21	-4	1527	1550
Mar. 21	-3	1940	1954
Apr. 21	-2	2310	2319
May 21	-1	2573	2566
June 21	0	2655	2657
July 21	1	2573	2566
Aug. 21	2	2310	2319
Sept. 21	3	1940	1954
Oct. 21	4	1527	1550
Nov. 21	5	1258	1218
Dec. 21	6	1076	1091

For the above data:

$SX = 0$
$SY = 24{,}054$
$SXY = 0$
$SX^2Y = 247{,}825$
$SX^3Y = 0$
$SX^4Y = 5{,}537{,}449$
$SX^2 = 182$

$SX^3 = 0$
$SX^4 = 4{,}550$
$SX^5 = 0$
$SX^6 = 134{,}342$
$SX^7 = 0$
$SX^8 = 4{,}285{,}190$

The following normal equations are secured from the above data:

$$\begin{array}{rcrcrcrcrcr}
13a & + & 0b & + & 182c & + & 0d & + & 4,550f & = & 24,054 \\
0a & + & 182b & + & 0c & + & 4,550d & + & 0f & = & 0 \\
182a & + & 0b & + & 4,550c & + & 0d & + & 134,342f & = & 247,825 \\
0a & + & 4,550b & + & 0c & + & 134,352d & + & 0f & = & 0 \\
4,550a & + & 0b & + & 134,342c & + & 0d & + & 4,285,190f & = & 5,537,449
\end{array}$$

The solution of the above normal equations yields:

$$\begin{array}{rcr}
a & = & 2656.9 \\
b & = & 0 \\
c & = & 89.671 \\
d & = & 0 \\
e & = & 1.2824
\end{array}$$

Therefore the polynomial is:

$$Y = 2656.9 - 89.671X^2 + 1.2824X^4 \quad \ldots\ldots \qquad (j)$$

The average deviation between the values calculated by the polynomial (Y') and the corresponding Y value is 16.

GLOSSARY

Definition of Terms

Air Mass, m. Path length of light through the atmosphere, considering the vertical path at sea level as unity. The air mass is approximately the secant of the angle of incidence for a horizontal surface.

Angle of Incidence, i. Number of degrees between actual direction of the sun's rays and a normal to the surface.

Radius Vector. Actual distance between earth and sun, considering the mean distance between earth and sun as unity.

Solar Altitude. The angle, in a vertical plane, between the sun's rays and the horizontal.

Solar Azimuth. The angle, in a horizontal plane, from north to the horizontal projection of the sun's ray.

Solar Constant. The energy incident upon a unit area located at mean distance of the earth from the sun and oriented perpendicular to the sun's rays outside the atmosphere. The value of the solar constant is 2.00 ± 0.04 calories per minute per square centimeter or 440 BTU per square foot per hour.

Solar Declination. The angular distance of the sun north or south of the celestial equator.

Solar Noon. The time for any day when the sun reaches its maximum altitude for that day.

Transmission Coefficient. Portion of the solar energy incident at the top of the atmosphere which reaches the surface of the earth.

Wall Azimuth. The angle, in a horizontal plane, between a normal to the surface and north.

Wall Solar Azimuth. The angle, in a horizontal plane, between the sun's rays and a normal to the surface.

Zenith Angle. Number of degrees between actual direction of the sun's rays and a normal to a horizontal surface.

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