STATIC ELECTRIFICATION
OF FILAMENTS

by

Robert Gail Cunningham

AN ABSTRACT

Submitted to the School for Advanced Graduate Studies of Michigan State University of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Department of Physics and Astronomy

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Approved
ABSTRACT

An apparatus has been constructed to rub filaments together under controlled mechanical and ambient conditions, and to measure the charge remaining after separation. The apparatus is similar to that of Hersh and Montgomery, with refinements to obtain better control of mechanical variables, and to allow variation of ambient pressure. A photomultiplier has been added near the filaments to detect the incidence of electrical breakdown of the atmosphere. Many of Hersh's findings for 45°-45° rubs have been confirmed, specifically those on reproducibility and triboelectric series. On the other hand, some findings have not been substantiated when the range of variables is extended. For nylon rubbed on polyethylene, the charge $q$ shows a square-root dependence on the normal force $F$, in place of the linear dependence found by Hersh; and the charge shows an inverse proportionality with diameter $d$, in contrast with the independence noted by Hersh. For tantalum rubbed on nylon, the charge is found to depend on a combination of velocity $v$, conductivity $\sigma$, and presumably dielectric constant $\varepsilon$, together with a characteristic distance $\rho_0$ to be determined empirically. If the charge dependence on length of rub $L$ is taken to be the proportionality found by Hersh, the combined relation

$$q = c \left( LF^{1/2} / d \right) \left( 1 - e^{-\frac{\rho}{\rho_0}} \right)$$

is suggested, where the sign and the magnitude of $c$ are in principle determined by the details of the band structure, but in practice are fixed experimentally. No detailed theoretical picture has been obtained to justify the rest of the expression, and it must be considered at present as an empirical relation whose generality and basis remain to be established.
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CHAPTER I
INTRODUCTION

The phenomenon of frictional electrification has become of increasing importance in recent years with the introduction of the new, poorly conducting, synthetic materials such as nylon, Dynel, Orlon, and Dacron. The manufacturer is plagued with such problems as how to make filaments and fiber assemblies pass along guide systems smoothly, how to fold charged fabrics properly into boxes, and how to reduce or eliminate attraction or repulsion and sparking of charged fabrics. The consumer is well acquainted with clinging garments, and the shock encountered after sliding across synthetic automobile seat covers. Moreover, the problem is of major importance in such places as explosive factories and flour mills where great care must be taken to avoid sparks. For instance, the workmen engaged in the manufacture of dynamite must wear special shoes with rubber soles; even their shoe laces are of a special kind, without metal tips. Each day they must draw surgically clean white cotton overalls, underwear, socks, and gloves from the plant laundry. The floors of the buildings are made of non-sparking lead, and all machinery must be made of non-sparking metals*. Thus the problem is of

* Reader's Digest, October 1956, p. 129
interest from a practical standpoint as well as from the standpoint of understanding the physics involved.

A. History

S. P. Hersh (1) has summarized the history of static electrification in his thesis presented for the doctorate at Princeton University. The treatment is broken into two parts: 1) qualitative investigations aimed at establishing a triboelectric series, and 2) quantitative investigations attempting to determine the factors involved and the manner in which they affect the charge transfer.

B. Theory

A theory based on modern solid state theory is outlined by Hersh and Montgomery (2). The assumptions of this theory are: 1) electrons only are transferred from one material to the other; 2) the direction of charge transfer is determined by the relative position of the Fermi levels in the two contacting materials, the transfer taking place so as to equalize the levels; 3) the amount of charge transferred during contact is determined by a) the energy band structure of the contacting materials, and b) the relaxation time for establishment of charge equilibrium between the materials; 4) the area of "true contact", that is, the area of interpenetration of atomic fields, depends only on the normal force between the materials rubbed; 5) after separation the transferred charge is localized at the place of contact in insulators, but not of course in metals; 6) upon separation of the materials after contact, the charge
initially transferred is reduced by tunneling through the gap formed
by the separating surfaces; 7) the charge remaining after separation
is not neutralized significantly by ions resulting from atmospheric
breakdown. The degree to which 5) in the theory is satisfied will
obviously depend on the resistivity and perhaps the dielectric con­
stant of the insulating material. The degree to which 3b) is satis­
fied will depend on the length of time the materials are in contact,
which in turn will depend on the velocity of the rub. The amount of
charge tunneling back through the gap will depend on 1) the potential
difference across the gap, which in turn will depend on the degree of
localization of the charge; and 2) the time available for charge to
flow back through the gap, which will depend on the velocity of rub.
Thus the charge measured in a rubbing experiment, which is actually
the charge retained upon separation, should depend on an interrela­
tionship between velocity of rubbing, the resistivity, and the dielectric
constant of the material or materials rubbed.

C. Objectives

The objectives of this work are three in number: 1) to assess
the validity of the conclusions of Hersh and Montgomery (2,3) by
repeating selected measurements while directly checking for electrical
breakdown; 2) to extend the studies of the effect of velocity of rub,
with the idea of finding the interrelationship between the effect of
velocity, resistivity, and dielectric constant of the materials on the
charge measured; 3) on the basis of these results, to get a more
detailed picture of the static electrification process.
CHAPTER II
APPARATUS

Hersh and Montgomery (3) found that the principal mechanical and ambient variables which control the amount of charge generated when two materials are rubbed together are the following: 1) length of rub, 2) velocity of rub, 3) nature of contact, 4) normal force between filaments, 5) temperature, and 6) relative humidity. The apparatus shown in Figures 1 to 10 is constructed in such a manner that these variables may be adjusted and controlled. It consists in three parts: 1) the rubbing apparatus, 2) the control apparatus, and 3) the measuring apparatus.

A. Rubbing Apparatus

1. Bottom Yoke

The materials to be investigated are mounted in the form of filaments in the yokes as shown in Figure 1. The bottom yoke, Figure 2, consists of a circular brass plate with an aluminum clamp mounted on one side, and a set of two pulleys mounted on the opposite side. The filament is mounted by clamping one end in the clamp and draping the other end over the pulleys. A loop is tied in the latter end, and weights are attached to provide a known tension in the filament. The length of filament which may be accommodated between the clamp and pulley is 10.4 centimeters. The brass plate
is marked in degrees in order that the azimuthal position of the filament may be specified. The yoke is mounted on four interchangeable polystyrene and Teflon stand-off insulators, which in turn are mounted on a double brass ring sliding over the cylindrical case as shown in Figure 1. The ring can be moved up or down by means of the knurled screws thus controlling the height of the filament above the base plate. Further, the ring can be rotated around the case thus changing the azimuthal position of the filament. A bare copper wire connected through a glass feed-through insulator connects the bottom yoke to the high terminal of a Keithley vacuum-tube electrometer, model 29, which measures the charge generated on the bottom filament.

2. **Top Yoke**

The top yoke, shown in Figure 3, is made of aluminum. A spring is clamped at one end of the yoke, and one end of the filament to be mounted is tied to the spring. The filament is then led over an aluminum post and fastened, usually by gluing, at the opposite end to a brass post. This post is part of a screw which can be moved back and forth by means of a knurled nut in order to control the tension of the filament. The length of filament between posts may be adjusted up to a maximum of 5 centimeters. The yoke is fastened to a semicircular brass plate marked in degrees for specifying the position of the filament.

3. **Driving Mechanism**

The plate to which the top yoke is fastened is itself fastened
to the supporting arm as shown in Figure 1. The supporting arm slides along the guides driven by a pin attached to a bicycle chain. The pin rests in a slot cut in the arm support between the guides and moves the arm back and forth along the guides as the bicycle chain moves around its path. The bicycle chain is driven by a synchronous motor, called the follower, and the motor along with the guides is fastened by means of machine screws to a steel base plate 3/4 inch in thickness.

4. **Rotary Solenoids**

The arm supporting the top yoke is hinged at the end nearest the follower, in order that the opposite end may move up and down. The latter end is supported by a guide fastened to two 28-v d-c rotary solenoids. Since these solenoids are mounted in opposition to each other, when one is activated the guide is in one extreme position and when the other is activated the guide is in the opposite extreme position. The total rotation from one extreme to the other is approximately 30°, consequently the opposite extreme from that shown in Figure 1 leaves the arm in a horizontal position. Four 6-v Edison storage batteries are used to operate the solenoids. The guide is spring loaded for the purpose of keeping the arm in the "up" position when neither solenoid is activated. The solenoids are fastened to an aluminum support and the support is fastened to the base plate by means of machine screws. Constructed in this manner, the filament in the top yoke will come to the same height above the base plate each time it is lowered.
5 Normal Force

The normal force between the filaments is controlled solely by adjustments in the bottom yoke. By making the tension very high in the filament mounted in the top yoke, and controlling the tension and height of the bottom filament as described, the expression for the normal force can be derived under the assumption that the point of contact between filaments is at the center of the bottom filament.

Referring to Figure 1 Z, let

\[ T = \text{tension in bottom filament}, \]
\[ d = \text{displacement of bottom filament}, \]
\[ J = \text{length of bottom filament between supports}, \]
\[ \theta = \text{displacement angle}. \]

Then the upward force at the point of contact will be given by

\[ \text{Normal Force} = 4Td/J, \]

for the case where \( d \) is small compared to \( J \), as is the case in this work. This relation very nearly gives the normal force along the whole rub, since the length of rub is 2.3 centimeters, and \( J \) is 10.4 centimeters, which gives a variation in normal force of 25\% or less from the extremes to the center of the rubbed portion of the bottom filament.

6. Nature of Contact

The azimuthal positions of the top and bottom yoke determine the nature of the rubbing contact. In a polar coordinate system in
which the zero of this system is the direction in which the top yoke moves, the angle between the length of the filament and the direction of movement of the top yoke specifies the position of the filament. The nature of the rubbing contact is then given by two angles. If the first angle is the coordinate of the bottom filament, and the second is that of the top filament, the three major forms of rubbing contact recognized by Hersh may then be described as follows:

1) 90-0, in which the length of the top filament saws into one spot on the bottom filament;

2) 0-90, in which one spot on the top filament slides along the length of the bottom filament;

3) 45-45, in which the length of both filaments are rubbed together; fresh surface is always contacting fresh surface in this rub.

B. Control Apparatus

The control apparatus, shown in Figure 4, consists of another synchronous motor, hereafter called the driver, microswitches to activate the rotary solenoids, a thyratron-controlled constant-speed d-c motor, and a timing control.

1. Length of Rub Control

The microswitches are operated by means of cams connected to the driver through a gear train which adjusts the rotation of the cams to the rotation of the bicycle chain on the rubbing apparatus. One revolution of the cams corresponds to one revolution of the bicycle chain thus, by a proper adjustment of the cams, the activation
of the rotary solenoids can be controlled and the top yoke may be lowered and raised at any desired time during the cycle.

2. **Velocity of Rub Control**

The driver is connected electrically to the follower and is driven by the constant speed motor through a leather belt. The speed of the latter motor is controlled by means of a rheostat which varies the armature voltage of the motor. Thus the rub velocity and the length of rub can be controlled apart from the rubbing apparatus.

3. **Rubbing Cycle**

Before each rub the filaments are discharged with an Ionotron®. Thus a cycle begins with the yokes together but the filaments not touching, and the Ionotron in place near the filaments. The Ionotron is moved away and about 25 seconds is allowed for the ions to clear. The constant-speed motor is then started, and the top yoke moves away and returns. As it passes over the bottom yoke, one of the microswitches is closed activating the rotary solenoid which lowers the top yoke and the filaments make contact. It was found necessary to attach a brass wire to the guide in such a way that when the top yoke is down the wire presses against the arm to keep it from bouncing when it falls. The wire can be seen in Figure 1. The two filaments are then rubbed together and when the desired length of filament is rubbed, the first microswitch is opened.

* Obtained from United States Radium Corporation, 535 Pearl Street, New York 7, New York.
and the second is closed, raising the top yoke and breaking the con-
tact between the filaments. The top yoke moves away and then returns
to its original position over the bottom yoke where it is stopped.
The Ionotron is then moved near the filaments. After allowing about
25 seconds for complete discharge, the cycle is ready to be repeated.
The complete cycle takes about 1 minute from beginning to end.

4. Timing Control

The timing of the cycle is made automatic by the use of micro-
switches in the circuit shown in Figure 6. The microswitch which
stops the constant-speed motor is located near the microswitches
controlling the rotary solenoids and is operated by a lever attached
to the same axle as those microswitches. The other four microswitches
are operated by cams mounted on a shaft which is turned by a 1-rpm
clock motor.

C Measurement Apparatus

1. Photomultiplier

It was found desirable to have some direct means of detecting
the occurrence of electrical breakdown. This phenomenon was sus-
pected by Hersh and Montgomery (2, 3) but not verified.

For this purpose, an RCA type 5819 end-cathode photomulti-
plier is mounted inside the cylindrical brass case directly beneath
the bottom fiber. The case is closed at the end nearer the fiber
by cementing a piece of ordinary glass with Picein cement to a cir-
cular ring of brass approximately 1 cm. wide. This ring is silver
soldered to the end of the cylinder. The other end of the cylinder is silver soldered to a brass plate which contains a hole of the same size as the brass cylinder. The case is placed through a hole cut in the steel base plate, and the plate is fastened by means of machine screws to the underside of the base plate. A rubber O-ring between the brass plate and steel plate provides an air tight seal. A slot about 2 centimeters wide and 5 centimeters long is cut in the bottom yoke with the longer dimension in the direction of the bottom filament, in order that the photomultiplier can "see" the filament as it is being rubbed. The photomultiplier is encased in a mu-metal shield to protect it from stray magnetic fields. The opening between the base of the photomultiplier and the brass case is sealed with electrical tape in order to exclude stray light from below. The photomultiplier circuit is shown in Figure 5. The base plate is mounted on a wooden frame to bring it about 16 inches off the floor and allow room for adjustment of the photomultiplier and its case.

2. Electrometer

The charge generated on the bottom fiber is determined by observing the potential change on a system of known capacitance. A schematic diagram of the measuring circuit is shown in Figure 11. The symbols \( C_s \) and \( C_e \) represent the capacitance of the system and electrometer, respectively. \( R_s \) is the resistance of the insulators supporting the bottom yoke, and \( R_e \) is the leakage resistance of the electrometer. The combination of \( R_s \) and \( R_e \) has an equivalent resistance never less than \( 10^{14} \) ohms. The value of \( C_s + C_e \) is measured by observing the time constant of the system when it is
connected to ground through a known resistance. The value lies between 25 and 30 micromicrofarads depending on the distance between the bottom yoke and the case housing the photo-multiplier.

3. Recorder

The signal from the photomultiplier is amplified by means of a Keithley isolation amplifier, and then recorded on one channel of a Brush two-channel recorder. The other channel is used to record the magnitude of the charge generated as measured by the electrometer. It is necessary that the recorder be running for only a short time during the cycle, so a microswitch is incorporated in the automatic control as shown in Figure 6 for starting and stopping the recorder.

D. Test Chambers

1. Vacuum Tank

Two test chambers are used for the control of the ambient variables. The first chamber consists of a cylindrical steel shell 1/2 inch thick, 12 1/2 inches long, and 19 inches in inside diameter placed around the rubbing apparatus, with a rubber O-ring between the shell and the steel base plate providing an air-tight seal. A circular steel plate 1/2 inch thick and 25 inches in diameter is used to cover the shell with another rubber O-ring to make a seal. A 6 inch diameter hole is cut in the top plate, and a piece of metal or plate glass covers the hole depending on whether it is desired to watch the operation of the apparatus, or to make the enclosure light-tight in order to observe electrical discharge with the photo-
multiplier. A stopcock is inserted in the wall through which the air may be pumped from the system. Two feed-through insulators are mounted in reducing nipples, and the nipples are fastened to the wall in order that the proper electrical connections may be made. One of the insulators is a single feed-through for making the connection to the electrometer, and the other is a multiple feed-through insulator for making the connections between the synchronous motors and for connecting the rotary solenoids to their power supply.

Figure 7 shows the assembled apparatus. For this test chamber, several different mountings for the lonotron were attempted, but no completely satisfactory mounting was found. The best one consists of placing the lonotron in a hollow brass cylinder with a narrow slot, and then placing this cylinder in a slightly larger brass cylinder with a similar slot. The inner cylinder is fastened to a rotary solenoid in such a manner that when the solenoid is activated, the slots are in line and the ions can escape. The solenoid is spring loaded, consequently, when it is not activated the slots do not line up, and most of the ions are stopped by the walls of the brass cylinders.

The device is then mounted on the base plate close to the bottom yoke. In practice it is found that the device can not be mounted close enough to the bottom filament to discharge it in a reasonable length of time if a polonium lonotron is used. A radium lonotron is also available, but the amount of gamma radiation not stopped by the brass cylinders is enough to contribute a large leakage current from the bottom yoke.
2. Temperature and Humidity Control Chamber

The second test chamber consists of a Masonite box 3 feet square and 2 feet deep lined on the inside with copper screen to shield the apparatus electrostatically. Entry into the box can be made in two ways: 1) by removing the cover over a hole 10 inches by 6 inches cut in the front of the box, or 2) by removing the entire front of the box. The openings are made light-tight by gluing strips of sponge rubber around the edges of the covers and using snap fasteners and thumb screws to pull the covers up tight, thereby compressing the sponge rubber. Two heaters and a thermostat are mounted in the box at the same height as the base plate, to control the temperature in the box. Figure 8 is a schematic of the heater and thermostat circuit. Trays filled with a salt solution are placed on the floor of the box to control the humidity, and a fan is mounted in the top of the box to keep a constant stream of air flowing down past the heaters onto the trays. The electrical connections are made through a bakelite multiple feed-through insulator at the back of the box. The various connections are shown in Figure 9. Figure 10 shows the box with the front cover removed and the apparatus in place ready for rubbing. The discharge of the filaments in this test chamber is accomplished by mounting the polonium Ionotron on a brass rod inserted at right angles into a hole drilled in an aluminum rod of larger diameter. The aluminum rod rests in holes drilled in two pieces of aluminum of rectangular cross section which are then bolted to the base plate.
as shown in Figure 10. The Ionotron is counterbalanced by a lead weight fastened to the opposite end of the brass rod. The aluminum rod extends through a hole in the box, and a crank serves to rotate the rod thus moving the Ionotron close to the filaments or far away from the filaments. The opening for the aluminum rod is made light-tight by packing the opening with cloth. An enclosure made of aluminum foil serves to house the Ionotron when it is not being used to discharge the fibers, thus preventing ions from being carried to all parts of the box by air currents. A 24-v d-c motor attached to a gear reduction box raises and lowers the Ionotron by means of a cam and lever attached to the crank. The operation of the Ionotron is controlled by two of the microswitches shown in Figure 6, and by a third microswitch mounted beside the cam connected to the motor. The third microswitch is used to stop the Ionotron in the best position for discharging the filaments.
CHAPTER II

EXPERIMENTAL RESULTS

A. Reproducibility

Nylon yarns about 5 mils in diameter were taken from a piece of nylon cloth not treated with anti-static agent, and paired with 4 3/4-mil polyethylene monofilaments taken from the same spool. The pairs were not cleaned in any way before mounting, but care was taken to keep from contaminating them by touching with the fingers. These pairs were then rubbed together under controlled ambient conditions, and the charge generated at different normal forces was measured. The temperature was kept constant at 30°C to within 0.2°C. The relative humidity was kept constant at 33% to within an estimated ± 5% by placing trays containing a saturated solution of magnesium chloride on the floor of the test chamber. The length of rub was 2.3 cm, and the velocity of rub was 4.1 cm/sec. The normal force was changed both by changing the tension in the polyethylene and by changing the overlap.

1. Rub-to-Rub

Figure 13 shows the results for two different filament pairs, with the normal force kept constant at 5.5 grams. The dots refer to one pair and the crosses to the other. In general, the reproducibility for successive rubs with discharging between rubs is as
reported by Hersh; the charge measured usually lies within ± 5% of the mean. It is felt that the divergence of the three dots outside this range was due to a defect in the mechanical or electrical part of the apparatus, although this conjecture has never been confirmed. Later measurements with tantalum rubbed against nylon (see Figure 23) do not have points lying outside the range of ± 5%, but this neither confirms nor denies the existence of such a defect.

2 Sample-to-Sample

Sample-to-sample reproducibility was not as good as rub-to-rub. The charge measured usually lies within the ± 20% of the mean in agreement with the findings of Hersh. It was noted upon rubbing various pairs of filaments that two types of behavior were encountered, a sparking type, where electrical breakdown was detected even at the lowest normal force used, namely 1.60 grams; and a non-sparking type, where electrical breakdown was not detected until higher normal forces were used, on the order of 3 to 7 grams. Figure 13 shows a pair of each type, the crosses referring to a sparking pair, and the dots to a non-sparking pair. An interchange of nylon filaments among various pairs showed the behavior to be characteristic of the nylon. In general, it was found that samples taken from different yarns in the cloth showed this variation, but samples taken from the same yarn did not. If a sample taken from one yarn pulled from the cloth could be classed as a non-sparker, then another sample taken from the same yarn could also be classed as a non-sparker, although the normal force which was
necessary for onset of sparking would often vary. Similarly, if a sample taken from one yarn could be classed as a sparker, then another sample taken from the same yarn could always be classed as a sparker. It may be surmised that if electrical breakdown did not occur for the sparking pair of Figure 13, then the charge generated would be greater than is shown by the plotted points. Thus a larger variation in sample-to-sample reproducibility than the stated variation may be expected. However, the decrease in charge generated due to electrical breakdown is probably very little for the fiber pair shown in Figure 13, since a few of the many rubs on this pair gave no evidence of breakdown whatsoever, and some of the rubs gave only one small spark. The charge measured for these rubs is not significantly different from the charge measured for rubs where two or more sparks were detected. Thus it is felt that the breakdown which occurs, neutralizes very little of the charge transferred, and consequently the charge measured for different samples does not vary by more than 20% from the mean. It is perhaps worth while to note that the rub-to-rub reproducibility is better for the non-sparking pair than for the sparking pair, apart from divergence of the three points of widest variation which is believed to be due to instrumental malfunctioning.
3. **Day-to-Day**

One would perhaps expect the day-to-day reproducibility of charge generated between the same pair of filaments to be better than the sample-to-sample reproducibility if the mechanical and ambient conditions do not change. Such is usually not the case, however. The best that can be said is that the day-to-day reproducibility generally lies within ±20% of the mean, as reported by Hersh. Variations as large as this have been found and in one isolated case, a sparker became a non-sparker when it was remounted and rubbed under the same conditions as when it was first rubbed.

**B. Materials**

The materials used in this work are the same as those used by Hersh and Montgomery, with the following exceptions: (1) for the study of electrical breakdown and normal force, a pure tungsten wire was rubbed on a polyethylene monofilament; (2) for the study of the interrelationship between rubbing velocity, resistivity, and dielectric constant, a piece of tantalum wire obtained from the Fansteel Metallurgical Corporation was rubbed on a nylon monofilament; (3) for the study of different kinds of insulators, wires of various materials were rubbed on crystals of various alkali halides, and the crystals were rubbed against each other.
The rubbing of the metals and the alkali halide crystals was done in the hope of obtaining experimental verification of the theory with substances of fairly well known band structures. The experiment was not fruitful, however, owing to the fact that the ionic conductivity of these materials is so high that no appreciable charge is left on the crystal as it is rubbed. Perhaps if higher rubbing velocities or lower temperatures could be attained, the expected results would be obtained, but such velocities and temperatures were not attainable with the described apparatus.

C. Atmospheric Breakdown

1. Dependence of Charge on Ambient Pressure

Hersh and Montgomery found that a linear relationship exists between charge generated and normal force up to a point at which an increase in normal force no longer results in an increase in the charge (Figure 14). One cause of this behavior may be electrical breakdown of the atmosphere surrounding the charged filaments. The potential difference between two electrodes at a constant spacing decreases with decreasing ambient pressure until a minimum is reached at which time further decreases in ambient pressure cause an increase in the potential difference. A typical
curve, shown below, is reproduced from Meek and Craggs (5). If the product of pressure, p, times spacing, d,

![Graph](image)

exceeds that of the minimum for the curve shown, and if breakdown is limiting the charge, then the leveling off point of the curve of charge generated versus normal force should decrease with decreasing ambient pressure (Figure 15).

An indirect test based on these ideas was made prior to adding the photomultiplier to the apparatus. For this purpose, a bottom yoke similar in size and shape to the top yoke was mounted on the base plate, with a lucite insulator separating the yoke from the plate. The rubbing apparatus was then placed in the first test chamber where the ambient pressure could be varied. Figure 16 shows the results of rubbing a piece of 5-mil nylon filament against a piece of 4 3/4-mil polyethylene monofilament manufactured by Courtauld’s Ltd. Each plotted point is the average of ten successive rubs without
discharging, except at the beginning where it was usually necessary to make 50 to 100 rubs to reach a constant value for the charge. The arrows on the curves show the direction of variation of ambient pressure. Each curve is a separate set of measurements obtained from the same materials, but on different days. Before each run the system was pumped down to a pressure of about 4 mm Hg to rid the system of water. The air reentering the tank was dried by passing it through a drying column containing Drierite, and through one containing Anhydrous. The normal force between the filaments was about 5.5 grams, and the velocity of rub was about 4.1 cm/sec.

The results are qualitatively as expected, but the curves do not quantitatively follow the shape of the breakdown curve shown on page 21. This is not too serious, however, since it is practically impossible to determine d, the distance between points from which breakdown occurs. Furthermore, the nature of the materials and other factors tend to change the shape of the curve (5). On the other hand, there is a serious difficulty in the fact that neither the two curves at low normal force nor those at medium normal force coincide. This disparity is a real effect, as shown by sets of measurements taken with the starting point at some intermediate pressure and continued through nearly a complete cycle of raising and lowering the pressure. No explanation for this behavior has been found.
2. Detection of Sparks with Photomultiplier

To test the possibility of breakdown more directly, the photomultiplier was mounted and sparks occurring during breakdown were detected on an oscilloscope. Figure 17 is a set of photographs of the actual oscilloscope trace showing electrical breakdown from the same filaments when rubbed at atmospheric pressure, and at a pressure of 1 cm Hg. It can be seen that some breakdown is occurring even at atmospheric pressures, but the amount increases tremendously at lower pressures. The amount of breakdown becomes progressively larger, and the signals from the sparks get progressively larger, as the pressure is decreased.

An attempt was made to repeat the measurements for single rubs with discharging between rubs but, as mentioned previously, no satisfactory mounting for the source of ionization could be devised, so these attempts were given up.

Another experimental verification was attempted by making single rubs, with discharging between rubs, at constant ambient pressure and under controlled conditions of temperature and relative humidity. The charge generated was measured as a function of normal force.

The rubbing apparatus was placed in the second test chamber
and trays of a saturated solution of magnesium chloride were placed in the chamber to keep the relative humidity at 33%. The temperature inside the chamber was held constant to within ±0.2°C at 30°C.

The velocity of rub was 4.1 cm/sec. for all rubs. Figure 18 shows the results obtained from rubbing an 8-mil nylon monofilament against 4 3/4-mil, 10-mil, and 12-mil polyethylene monofilaments. Nylon and polyethylene were chosen because they generated high charge. The plotted points are the average of 5 rubs made after the filaments were rubbed 50 or 60 times to reach a constant value. The points with a "spark" beside them are for rubs where at least one spark was detected on some of the rubs, and points with no "sparks" beside them are for points where no sparks were detected on any rub.

The curves are to be compared with those obtained by Hersh (1) who used an 8-mil nylon monofilament rubbed against a 12-mil polyethylene monofilament. Our curves for materials of these same diameters agree quite well over the same range of normal forces used, about 3 to 11.5 grams, and no breakdown is observed out to the highest normal force used, 11.5 grams. But the curves obtained by using different diameter polyethylene filaments are different, especially the one for the 4 3/4-mil filament, where breakdown begins to occur at relatively low normal forces. This finding contradicts the conclusion of Hersh, who states that the charge
generated is independent of the diameter of the filaments.

The form of the curves shown in Figure 18 suggests a power law dependence. Accordingly the curves have been replotted on log-log paper. The plot for a fixed diameter shows an approximate square root dependence of charge generated on normal force. To treat the diameter effect at the same time, the normal force was divided by a different factor for each curve in order to achieve superposition of the curves. Figure 19 shows the results of this treatment, when the factor was numerically equal to the square of the diameter for each filament. A line of slope one-half is drawn through the plotted points. The falling of the experimental points below the curve at the top coincides with the appearance of sparks. The divergence of the points obtained using the 10-mil filament at the lower end can be attributed to the wearing effect. Not enough rubs were made to reach the flat portion of the curve of charge vs. rub number, before the plotted data were taken. This is illustrated by the three sets of two points each taken at the same normal force. The lower-lying points were all taken after obtaining a complete set of points, beginning with the lowest normal force. The divergence of the other points is probably due to the failure of the method of estimating normal force when weights of 5 grams or less are hung from the filament.
It is tempting to explain this dependence as purely a contact area effect, since the empirical law can be written as \( q = (F/d^2)^{1/2} \), that is, the charge varies as the square root of some kind of effective pressure. Unfortunately, the relation does not appear to be so simple. According to the work of Hertz, as given in say Timoshenko (6) the surface of contact between two elastically deformed, stiff, right-circular cylinders with axes at right angles to each other has the form of an ellipse with semi-axes given by

\[
a = \alpha (Fm/n)^{1/3}
\]

and

\[
b = \beta (Fm/n)^{1/3}
\]

where \( F = \) normal force

\[
m = 4 r_1 r_2 / (r_1 + r_2)
\]

\[
n = 3(1-\mu^2)/4E
\]

The factors \( r_1 \) and \( r_2 \) are the radii of the cylinders, \( E \) is Young's modulus, and \( \mu \) is Poisson's ratio. The factors \( \alpha \) and \( \beta \) depend on the radii of the cylinders, but their variation is quite small, and when they are multiplied together, are practically constant for the cylinders used in this work. The contact area then becomes

\[
A = \pi ab = k_1 (Fr_1 r_2 / r_1 + r_2)^{2/3}
\]
If we assume $q \alpha F/A$, we then obtain

$$q = kF^{1/3} \left( \frac{r_1 + r_2}{r_1 r_2} \right)^{2/3}$$

This gives a one-third power relationship between charge and normal force, instead of the one-half power obtained experimentally.

Experimentally it is found that to obtain identical charges generated on the 4 3/4-mil and 12-mil polyethylene filaments when rubbed with an 8-mil nylon filament, the normal force for the 12-mil filament must be 6.4 times the normal force for the 4 3/4-mil filament. On the basis of the theory the factor turns out to be 2.6, which is much too small to explain the observed results. Likewise, the experimental factor for the 10-mil and 12-mil filaments is 1.44, whereas the theoretical factor is 1.17. Thus this hypothesis fails to explain dependence of charge on either the normal force or the diameter. The explanation cannot be rejected absolutely, however, since it is likely that the materials under study are not perfectly elastic nor perfectly plastic under the conditions of the experiment, but lie in some intermediate region. Perhaps modification of the hypothesis would permit this interpretation to stand.
From the results of rubbing nylon on polyethylene monofilaments of different diameters, and from results of rubbing tungsten, platinum, and titanium wires on 4 3/4-mil polyethylene, some conclusions may be reached on the validity of the results of Hersh and Montgomery as far as limitation by atmospheric breakdown is concerned. It may be stated that their measured charges are not limited by breakdown in cases where the filament diameters are large and the normal forces are of moderate values; but in all likelihood their measured charges are limited by breakdown in some of their results at high normal forces and all their results with small diameter filaments.

D. Resistivity, Dielectric Constant, and Velocity of Rub

The phenomenon of static electrification can be broken into two parts; 1) the gain of charge upon making contact between the materials in question, and 2) the loss of charge upon breaking the contact. Van Ostenburg (7) has made some theoretical calculations of the charge gained upon contact, and has attempted some interpretation of the results of Hersh and Montgomery on this basis. The utility of his results is dependent on knowledge of the energy levels in insulators. Hersh and Montgomery have assigned energy levels to a few materials based on the experimental results of rubbing these materials together, but there is little independent confirmation of the values for insulators. Thus the work of Van Ostenburg will be of restricted value until independent information is available.
on the energy levels of insulators.

Only very crude conjectures on the loss of charge upon breaking contact are made by Hersh and Montgomery (2). One of the primary directions of extension of the work in static electrification is towards gaining more understanding of the loss of charge upon breaking the contact between materials. It is an exceedingly difficult problem to analyze exactly, and we have attempted to take ideas suggested from related phenomena of current flow in dielectrics in order to ascertain the controlling factors and the relation among them.

1. Theory

In a linear, isotropic, homogeneous medium, the following relations exist between \( \mathbf{D} \), the electric displacement vector; \( \mathbf{E} \), the electric field vector; and \( \mathbf{J} \), the current density vector:

\[
\mathbf{D} = \varepsilon \mathbf{E},
\]

\[
\mathbf{J} = \sigma \mathbf{E},
\]

where \( \varepsilon \) is the dielectric constant, and \( \sigma \) is the conductivity, or reciprocal of the resistivity. From these relations and Maxwell's First Equation, we find

\[
\nabla \cdot \mathbf{J} = \nabla \cdot \varepsilon \sigma \mathbf{D} = \frac{\varepsilon}{\varepsilon} \rho
\]

where \( \rho \) is the volume charge density. Using this relation, the equation for conservation of charge becomes

\[
\frac{\sigma}{\varepsilon} \rho + \frac{\partial \rho}{\partial t} = 0
\]
The solution of this equation in terms of the initial charge density, \( \rho_0 \), is

\[
\rho = \rho_0 e^{-\left(\sigma/\epsilon\right)t}
\]

Consequently, in a medium of finite resistivity, there can be no permanent distribution of free charge. If the charge density is zero initially, it remains zero for all time. If a charge is placed in the interior of the region it decays exponentially with time and, since it cannot reappear within the volume of the region, it must come to rest on the boundary or outer surface of the region. The time necessary for the charge to decay to \(1/e\) of its original value,

\[
\tau = \frac{\epsilon}{\sigma}
\]

is called the relaxation time (8). The relaxation time can vary over wide ranges, from values less than \(10^{-12}\) seconds for the case of fairly good conductors, up to values exceeding \(10^6\) seconds for good insulators.

The problem of static electrification, however, usually deals with charges placed on the surface of an insulator, not in the interior, so it is of interest to know how these surface charges decay in time. The problem of what happens to a point charge placed on the surface of a dielectric sphere is not excessively difficult, but the result is not easy to describe simply. The charge decay cannot be characterized by a unique relaxation time, but there will exist
some kind of average relaxation time not too different from that characterizing the relaxation of charge in the interior of the dielectric medium.

In our problem, we have a complicated distribution of charge in space and time. We have not been able to attack the problem exactly, but instead, we have tried to understand the gross effects by considering that the behavior can be partially analyzed in terms of the ratio of some time interval characteristic of the rub, to the relaxation time characteristic of the material. The former is given by the time necessary for some characteristic length, \( l_0 \), to be traversed at a velocity, \( v \), which is

\[ t_o = \frac{l_0}{v} \]

the latter will be approximated by the relaxation time for a homogeneous medium. Hence, the characteristic parameter is

\[ t_o / \tau = \frac{\sigma l_0}{\varepsilon v} \]

This same result could have been reached immediately by the standard methods of dimensional analysis, if the relevant variables are \( \sigma, l_0, \varepsilon \), and \( v \) (9).

2. Experimental Procedure

A piece of 8-mil tantalum wire was rubbed against an 8-mil nylon monofilament, and the temperature and the relative humidity were varied so as to vary the resistivity of the nylon. Figure 20
shows the relation between the resistivity of nylon and the temperature for various relative humidities, and Figure 2.1 shows the relation between the resistivity and the relative humidity for various temperatures. Both curves are derived from Hersh and Montgomery [10] who measured resistivity as a function of regain. The measuring circuit adopted by Hersh and Montgomery for the measurement of the resistance of various materials is shown in Figure 2.2. The same circuit was used in these studies to measure the resistivity of the nylon sample before and after rubbing. In this circuit, a high voltage supply impresses a known voltage $V_X$, across the unknown resistance of the nylon, $R_X$. The voltage, $V_S$, is adjusted until the electrometer shows zero potential difference across its terminals. The current through $R_X$ is then equal to the current through $R_S$, which is a standard resistor of about $10^{12}$ ohms, and $R_X$ is given by

$$R_X = \frac{V_X}{V_S}R_S.$$  

In practice, the wire from the high voltage supply is connected somewhere near the center of the nylon filament which is mounted in the bottom yoke. From the knowledge of the resistance of the filament, its length, and its cross-sectional area, the resistivity of the sample can then be determined. A continual check on the resistivity of the nylon can be made indirectly by monitoring the temperature and the relative humidity in the test chamber. The
relative humidity is monitored at three places in the chamber, about 6 inches to the right and left of the filament and at the top of the chamber directly above the filament, by the use of El-tronic humidity sensing elements*.

3 Results

Figures 25 and 26 show the results of measurements of charge generated on two separate nylon filaments. The charge generated is plotted as a function of velocity for three different temperatures, with the relative humidity held nearly constant at 75%. The normal force is kept constant at about 6.9 grams and the length of rub is 2.3 cm for all rubs. The points are obtained by taking the average of 5 rubs at each velocity and normalizing to rub number 45. Normalizing is necessary as can be seen from Figure 23, and is accomplished in the following manner: A smooth curve is drawn through all points obtained at 4.1 cm/sec. The average of the 10 rubs between 40 and 50 is then found, and a horizontal line drawn through the average (Figure 24). The charge at each rub is then multiplied by the ratio of the height of the horizontal line to the height of the smooth curve at that point and replotted. Then the average for the five rubs at each velocity is found. Since the five rubs at each velocity

* Manufactured by El-Tronics, Inc., Mayfield, Pennsylvania
are repeated at least once for each set of measurements, an average between these two sets of values is taken as the value of the charge generated at that velocity. The resistance of the nylon filament was measured before and after each set of measurements. Table 1 lists the experimental values obtained for each filament at the different temperatures.

<table>
<thead>
<tr>
<th>Filament no.</th>
<th>Temp. (°C)</th>
<th>$\frac{1}{\sigma} \times 10^{10} \Omega \cdot \text{cm}$ before rubbing</th>
<th>$\frac{1}{\sigma} \times 10^{10} \Omega \cdot \text{cm}$ after rubbing</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>4.03</td>
<td>5.38</td>
<td>4.00</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>2.00</td>
<td>1.97</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>1.02</td>
<td>0.82</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>4.00</td>
<td>3.65</td>
<td>5.50</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>3.35</td>
<td>1.79</td>
<td>3.65</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>0.83</td>
<td>0.87</td>
<td>1.17</td>
</tr>
</tbody>
</table>

Table 1

With the value of $1.0 \times 10^{10} \Omega \cdot \text{cm}$ found for filament no. 1 at 35°C taken as the correct value, all the other curves were normalized to this value by multiplying the velocity at each point by a normalizing factor, $\beta$. The values of $\beta$ necessary for normalization are also listed in Table 1. Figure 27 is a log-log plot of charge vs.
the product of $\beta$ and velocity for the curves of Figures 24 and 25.

It is felt by the author that the points lying off the curve around the points of 7 and 8 $\beta v$ may be explained by the fact that these points are obtained experimentally at high velocities for the rub. Under these circumstances the length of rubbed filament is less owing to the fact that a small time interval is necessary for lowering the top yoke. The top yoke actually has two components of velocity while it is being lowered, one in the downward direction and one in the direction of rub. As long as the downward component of velocity is large compared with the rubbing velocity, changes in the rubbing velocity will affect the point of contact between the top and bottom filaments very little. But when the rubbing velocity becomes comparable with the downward velocity of the top yoke, changes in the rubbing velocity will cause measurable changes in the contact point. The point at which the filaments break contact is virtually unaffected by these factors so it can be seen that the length of rub may decrease at high rubbing velocities.

The curve has been fitted empirically with the equation

$$q = 20.0 \left(1 - e^{-\beta v/5.12}\right).$$

There are only two adjustable parameters in the equation: the saturation value of the charge, 20 $\mu\mu c/cm$ for the curve shown; and one controlling the rapidity of rise, 5.12 for the curve shown. If it is desired to identify the normalizing factor, $\beta$, with the product of resistivity and dielectric constant...
for the nylon sample, the constant characterizing the rate of rise of the curve has the dimensions of length. With a value of about 6.5 for the dielectric constant of nylon, this length turns out to be 2.9 x 10^{-2} cm. This value is quite rough, since the dielectric constant is probably not known to better than ±50%, and the resistivity may vary by as large a factor as 2 or 3 from the measured value as will be discussed later. Hence the stated characteristic length should probably be considered significant only in order of magnitude. More work is necessary before the value can be stated more precisely and before the nature of this characteristic length can be elucidated.

The comparison of $\beta$ with the measured values of $\varepsilon/\sigma$ can be made from Table 1. The comparison is disappointing. The disagreement may not be so serious, however, since the resistivity or dielectric constant or both may change with rubbing. Since the length of nylon rubbed is 2.3 cm and the length of nylon used for the resistivity measurement is 10.4 cm, a fairly large change in resistivity in the rubbed portion of the filament may not measurably change the resistivity of the nylon. A better technique for measuring the resistivity of the rubbed portion of the nylon is necessary before this problem can be resolved.
CHAPTER IV

DISCUSSION AND CONCLUSIONS

The theory put forth by Hersh and Montgomery is based on division of the process of static electrification into two steps, the direct flow of charge upon making contact, and the reverse flow upon breaking contact. The portion of the theory which they gave the most specific form was that in which the nature of the materials was taken into account. Detailed calculations on this point were carried out by Van Ostenburg, and no contradiction with experiment has been shown. It must be kept in mind, however, that there is little independent knowledge of the parameters appearing in the theoretical expressions for charge transferred. The work of Van Ostenburg proceeded from a one-dimensional model, and was not concerned with return of charge upon separation. Consequently his work has little to say about the effect of filament diameter or of normal force, and of course nothing at all on the effect of velocity and resistivity.

Hersh and Montgomery did not give very definite form to their ideas on return of charge. The reasons are two-fold. The experimental work was not precise enough or extensive enough to establish empirical laws, and the theoretical investigation is hampered by the high complexity of the phenomena. In the present work it has been attempted to find at least the empirical laws describing the effect
of filament diameter and normal force on charge gained upon making contact, and those describing the effect of filament resistivity and rubbing velocity on charge lost upon breaking contact. The hope for success in this attempt was founded on the construction of a better instrument and on the improved understanding of the phenomena achieved in the earlier work.

The first point to be established in applying the theory of Hersh and Montgomery is that the charge measured is indeed controlled by the solid-state processes, and not by electrical breakdown of the atmosphere. As reported in Chapter III, Section C-2, it appears that for the bulk of the experimental work described by Hersh, electrical breakdown did not occur. What is a more important fact, in the present work it was possible to know when the charge was limited by breakdown.

Hersh's experimental work had shown sufficient reproducibility to allow a rational attack to be made on the problem of static electrification. The present work shows about the same degree of reproducibility. So far as sign of charge is concerned, there is no disagreement. So far as magnitude is concerned, the comparison cannot be so clear cut. Ordinarily, some conditioning or aging is called for, and some averaging. One would like to be rid of these unsatisfactory features, but this is not yet possible.
Having ascertained the agreement in the large of the present results with the earlier ones of Hersh and Montgomery, we are in position to improve and extend the findings. The extension takes place in two directions: the effect of normal force and filament diameter on direct flow of charge, and the effect of filament resistivity and rubbing velocity on reverse flow of charge. Fortunately, the form of the dependence of charge measured on resistivity and velocity is such that the two effects mentioned can be separated. It will be recalled from Chapter III, Section D-3, that the dependence of charge on these variables can be expressed as

\[ q \propto \left(1 - e^{-\nu/v_0}\right) \]

Now if the magnitude of the exponent is large enough, its effect disappears, and the proportionality factor may be studied by itself. In practice this end may be achieved by working at relatively high velocities and low conductivities, the latter obtained by reducing the relative humidity or the ambient temperature. Then the dependence of the proportionality factor on type of material, type of rub, length of rub, diameter of filament, and normal force can be studied.

It will be recalled from Chapter III, Section C-2, that the dependence of charge on normal force \( F \) and filament diameter \( d \) can be expressed as

\[ q \propto F^{1/2} / d. \]
In this part of the work the relative humidity was maintained low enough that the resultant low conductivity suppressed the dependence of charge on the factors appearing in the equation in the preceding paragraph. In all likelihood the parameter $h_0$ in that equation depends on the filament diameter. In fact, for the particular specimens tested, the value of $h_0$ is very nearly equal to the diameter of the rubbed filament. This coincidence is undoubtedly fortuitous, but it may lend some support to the existence of an additional dependence on diameter.

In the present work, the dependence of charge on length of rub was not studied. It is difficult to see, however, that the relation can differ from the proportionality found by Hersh, except possibly under conditions where a strong velocity dependence occurs. We take then

$$q \propto L.$$  

The above equations may be combined to give

$$q = c (LF^{1/2}/d) \left( 1 - e^{-ev/\sigma h_0} \right),$$

where $q$ is the total charge produced on a rubbed length $L$ of a filament of diameter $d$, conductivity $\sigma$, and dielectric constant $\varepsilon$. This expression holds for $45^\circ-45^\circ$ rubs where fresh areas are being rubbed on both filaments pressed together with a normal force $F$. The characteristic distance $h_0$ is taken as an adjustable parameter, but it will be surprising if further work cannot elucidate its
nature. The proportionality constant $c$ is, in principle, determined by the details of the band structures of the contacting materials. In practice, so little is known of the band structures of the materials of interest that $c$ must be fixed experimentally.

The general equation above is an empirical relation connecting what may be considered the most important factors in static electrification, except that rub type has not been included. It may be, of course, that only the factor $c$ may change with rub type; but this is unlikely, since in some types local temperatures must become very high and thereby alter not merely the band structure but also the distribution of charge carriers in the bands and the mechanism of charge trapping. The elucidation of these phenomena constitutes a challenging and complex problem.

A disturbing feature that appears from the results of the present experiment is that some doubt is thrown on a basic assumption of the theory, namely, that the equilibrium charge distribution is attained during contact. The dependence of measured charge on velocity and resistivity does not seem to be interpretable in a simple manner in terms of reverse flow of charge. Rather there is some suggestion that the direct charge flow is limited by the inability of electrons to be freed from traps in the better insulator. The results of Hersh and Montgomery with asymmetric rubs, in
particular where one spot on an insulating material is sawed into by another filament of the same material, give evidence of a similar phenomenon. The hot spot generated in the sawed-into filaments virtually always loses electrons. On the other hand, we can equally well speculate that the Fermi level rises in an insulator when it is heated, as suggested by Hersh and Montgomery. Until further experimentation and analysis have been carried out, such considerations must remain in the realm of conjecture.
CHAPTER V
SUMMARY

An apparatus has been constructed to rub together filaments of various materials and to measure the charges generated, while controlling the various relevant mechanical and ambient conditions. The apparatus is similar in principle to that used by Hersh and Montgomery, but it has been refined in order to obtain better control of the mechanical variables, and to allow variation of the ambient pressure. A spark detector has been incorporated to ascertain the incidence of electrical breakdown of the atmosphere.

With this apparatus, selected results of Hersh and Montgomery have been reexamined. These results, hereinafter referred to as "earlier," are as follows:

1. Reproducibility - Essentially the same results as reported earlier were obtained.

2. Triboelectric series - The positions of the materials tested were the same as those reported earlier.

3. Length dependence - No verification of the earlier results was attempted; these results showed a proportionality of charge on length of rub. It is unlikely that this relation would fail to hold in the present measurements, except perhaps in the region where a strong velocity dependence exists.
4. Effect of nature of rub - No verification was attempted of the earlier results.

5. Normal force - The normal force dependence found earlier, namely, a linear dependence of charge on normal force independent of filament diameter, has not been substantiated. For relatively large normal forces and large diameter filaments, the dependence may be approximately linear, but for small diameter filaments and relatively low normal forces, the dependence is definitely non-linear. Results of nylon rubbed on polyethylene over the whole range of normal forces and for all diameters of polyethylene filaments used, show that

\[ q \propto F^{1/2}/d \]

so long as the charge is not limited by breakdown.

6. Temperature and relative humidity - The temperature and relative humidity control the electrical resistance of the insulating materials studied. It is found with tantalum rubbed on nylon that if the resistivity of the nylon lies in a range of $10^9$ to $10^{11}$ ohm-cm, an increase in the resistivity will produce an increase in the charge generated as long as moderate velocities of rub are used.

7. Velocity of rub - When the resistivity of the nylon lies in the range $10^9$ to $10^{11}$ ohm-cm, a velocity dependence for the range of velocities available is encountered. The charge increases as the
Velocity of rub is increased up to a point where a further increase in velocity increases the charge very little. The region where this leveling off occurs is dependent on the resistivity of the better insulator. For resistivities lying outside the range mentioned, very slight velocity dependence is observed, as found in most of the earlier results.

The effect of velocity of rub and resistivity can be tied together by use of the experimentally determined relation

\[ q \propto \left( 1 - e^{-\frac{v}{\sigma L_0}} \right) \]

found from the results of rubbing tantalum on nylon of various resistivities at different velocities of rub.

The effect of normal force, diameter of rubbed filament, resistivity of the better insulator, length of rub, and velocity of rub on the charge generated may be brought together in one relation, namely,

\[ q = c \left( LF^{1/2}/d \right) \left( 1 - e^{-\frac{v}{\sigma L_0}} \right) \]

for rubs of the 45°-45° type. It must be remembered, however, that the experimentally determined relation for charge dependence on normal force and diameter was found for a pair of materials different from the pair used to obtain the experimental dependence of charge on velocity and resistivity. Thus the generality of the above relation remains to be established.
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Photomultiplier circuit.

Photomultiplier circuit diagram with labels and connections.
Figure 6
Automatic timing control.

Note. RC combinations for spark suppression not shown.
Temperature control circuit.
### Table: Feed-through connections for test chamber.

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1. **Ground**
2. **1 1/2 volt flashlight bulb**
3. **Heaters (common)**
4. **Heater (70 ohms)**
5. **Photomultiplier voltage supply**
6. **Spare**
7. **Heater (20 ohms)**
8. **Spare**
9. **Rotary solenoid (lowering)**
10. **Rotary solenoid (raising)**
11. **Wire No. 4**
12. **Wire No. 1**
13. **Wire No. 2**
14. **Wire No. 3**
15. **Wire No. 5**

**Figure 9**

Feed-through connections for test chamber.
Charge measuring circuit (See text for meaning of symbols).
Figure 12

Position of Bottom Filament when in contact with top filament.

(See text for meaning of symbols).
Charge generated for successive rubs (discharging between rubs).
Figure 14

Charge vs. normal force for acetate on nylon (after Hersh).
Expected behavior of charge vs. normal force for different ambient pressures if atmospheric breakdown is occurring.
Charge vs. ambient pressure for low and medium normal forces.
Figure 17

Pictures of breakdown as detected by photomultiplier.

$p \sim 1 \text{ atmosphere}$
(discharging between rubs)

$p \sim 1 \text{ atmosphere}$
(without discharging)

$p \sim 10^{-2} \text{ atmospheres}$
(without discharging)

$p \sim 10^{-2} \text{ atmospheres}$
(without discharging)
Figure 18

Charge vs. normal force for an 8-mil nylon monofilament rubbed on polyethylene monofilaments of the diameters shown.
Figure 19

Charge vs normal force x \((12.0/d)^2\) for 8-mil nylon rubbed on different diameter polyethylene monofilaments.
Figure 20

Resistivity of drawn nylon vs. temperature for various relative humidities.
Figure 21

Resistivity of drawn nylon vs relative humidity for various temperatures.
Figure 22
High resistance measuring circuit.
Figure 23

Charge generated at different velocities of rub.

(For 8-mil tantalum rubbed on 8-mil nylon.)
Figure 24
Charge generated at different velocities of rub illustrating method of averaging.

KEY
See Figure 23

RUB NUMBER

30 20 10 0

CHARGE (pC/cm)

0 20 40 60 80
Relative Humidity $\simeq 75\%$

![Graph showing charge generated vs. velocity for different ambient temperatures. 8-mil tantalum rubbed on 8-mil nylon monofilament No. 1.](image)

**Figure 25**

Charge generated vs. velocity for different ambient temperatures. 8-mil tantalum rubbed on 8-mil nylon monofilament No. 1.
Charge generated vs. velocity for different ambient temperatures.

8-mil tantalum rubbed on 8-mil nylon monofilament No. 2.
Figure 27

Charge vs $\beta v$ for 8-mil tantalum rubbed on 8-mil nylon.

KEY

- $\times$ Nylon filament No. 1
- $\circ$ Nylon filament No. 2