A STUDY OF THE INTENSITY DISTRIBUTION OF THE LIGHT DIFFRACTED BY ULTRASONIC WAVES

by

Robert Bruce Miller

AN ABSTRACT

Submitted to the School for Advanced Graduate Studies of Michigan State University of Argiculture and Applied Science in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Department of Physics and Astronomy

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Approved E. A. Hiedemann

Robert Bruce Miller

ABSTRACT

The diffraction of light by an ultrasonic wave, predicted by L. Brillouin (1) and discovered independently by Debye and Sears (2) and by Lucas and Biquard (3), is an interesting phenomenon. The mathematical difficulties arising in any attempt to formulate an adequate theoretical explanation of the intensity distribution of the diffracted light has led to derivation of several theories.

The simple theory of Raman and Nath (4, 5 & 6) is outlined and the predicted region of useful application given. The somewhat more involved and mathematically rigorous theory of Mertens (7) is also outlined, and a procedure suggested whereby it may be experimentally checked. The rather detailed computations needed in the application of the Mertens' correction terms are carried out. The results of these are included in the appendix.

The usual optical method for the detection of the ultrasonic diffraction pattern is described, and methods for using a microphotometer for actual intensity measurements are outlined.

Results are presented for a frequency range of

2 - 7 Mc, and for sound field depths of ¾ and 1 inch. Distribution curves, relating the intensity of the diffracted light to the sound field intensity, are given. In the more interesting cases the first five diffraction orders are shown. These curves are compared to the theories of Raman and Nath and of Mertens. Suggestions are made as to the regions of usefulness of each.

Robert Bruce Miller

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INTRODUCTION

In 1921 it was predicted by L. Brillouin (1) that a light beam, upon passage through a transparent medium in which a sound beam of sufficiently short wavelength was present, would be diffracted. That this was the case was demonstrated experimentally in 1932 by Debye and Sears (2) in the United States and by Lucas and Biquard (3) in France.

Of great interest, however, was the observation, not of a single diffraction line as predicted, but of multiple diffraction orders which obeyed the simple grating formula,

$$n\lambda = d \sin\theta$$

where d becomes the wavelength of the ultrasonic wave.

This observed multiple diffraction was not in agreement with the original theory of Brillouin (1). He had by making use of the method of retarded potentials predicted only zero and plus and minus first orders. The intensities of which took on maximum values for angles of incidence satisfying a relation analogous with the formula established by Bragg for the diffraction of X-rays by crystals.

A similiar result was obtained by P. Debye (4).

After the experimental investigation a more rigorous treatment of the problem was undertaken by L. Brillouin (5), but mathematical difficulties restrict the application of this work to low ultrasonic energies. Debye (6) suggested that the multiple orders might arise from non-linear relationships between density and dielectric constant, or that the presence of harmonics might produce the observed effect. Lucas and Biquard (7, 8) pointed out the unlikelyhood of both proposals, the first, due to the relatively small pressure amplitudes involved, and the second due to the fact that a piezoelectric crystal will resonate only on odd harmonics. These latter men in the same work develop a theory based on a mirage effect. This theory predicts multiple orders the number and intensity of which increase with the path length of the beam in the medium, and the ultrasonic intensity. However, their work indicates that the relative intensity of the orders would decrease monotonically with increasing order number. That this was not always the case was shown experimentally by R. Bar (9).

If we now define a parameter,

$$\delta = \Delta \frac{\lambda^{*2}}{\lambda}$$

where Δ = the ratio of the maximum density change to the average density of the medium.

λ = wavelength of the light.

λ^* = wavelength of the sound.

it is possible to divide the theoretical treatments into two rather broad divisions. First, the case where

$\delta << 1$, this cooresponds to high ultrasonic frequencies and in general the theories here have been patterned after the original work of Brillioun. We shall discuss only briefly any theoretical treatment in this region, For $\delta \simeq 1$ no satisfactory theory exists, except perhaps that the work of Exterman and Wamier as extended by Nath (10) is applicable for intensities where only zero and plus and minus first orders appear. Our chief interest lies in the region $\delta > 1$.

The first theory having any real success in describing the observed phenomena for $\delta > 1$ was first published in 1935-36 in a series of three papers by Raman and Nath (11, 12, 13). Their work follows closely the method of Lord Rayleigh in his treatment of the diffraction of a plane wave incident normally on a periodically corrugated surface. The three papers are quite complete treating both progressive and standing waves for cases both of normal and oblique incidence. They not only give relative intensities of the diffraction orders, but also describe the observed angular dependance and the frequency variation effects in the several orders as observed by Bar (9). In two later papers (14, 15) Raman and Nath give a somewhat more general treatment. These start from a differential equation, but the final results are the same as for the simplier theory.

Experimental confirmation of the above theory was reported in 1936 by Sanders (16). His results show good agreement between theory and experiment, and this work is widely quoted and reproduced in many publications dealing with ultrasonic diffraction. While no claim is made to the contrary, it should be pointed out that the region chosen for this experimental work was in the range best adapted to fit the theory, and that the theory is not in general well suited to explain diffraction effects over the entire frequency range, the entire range of sound intensities or of sound beam widths. An exact theory must allow in some manner for the relationship between these three variables.

Various authors have attempted to do this, most notable among them have been Extermann and Wamier (1936), David (1937), Nath (1936, 1938), Van Cittert (1937), and Mertens (1951). It is the efforts of the last of these men that shall be the chief concern of this investigation.

In general all of the above mentioned theoretical treatments have been directed at improving the approximations of Raman and Nath made by assuming that terms of the type n^2/δ could be neglected, where n is the diffraction order and δ is the parameter previously defined. It is very difficult to treat this last term theoretically because of the number of variables involved.

Thus the success of a given theory and the region in which it is applicable can best be determined by direct experiment.

In this connection we note the validity limits of the elementary Raman-Nath theory. These are pointed out by Nath (10). It is shown that these conditions are either,

$$\rho v^2 < 1$$

if we accept the assumptions of Lucas and Biquard, or;

$$\tfrac{1}{2} \rho v^2 < 1$$

according to the work of Extermann and Wamier. Where

$$\rho = \frac{\lambda^2}{\mu_0 \mu_0 \lambda^{*2}}$$

and

$$v = \frac{2\pi \mu L}{\lambda}$$

where the following notation is used,

λ = wavelength of the light.

λ^* = wavelength of the sound.

μ_0 = index of refraction of the medium.

μ = maximum variation in μ_0.

L = thickness of the sound field.

It is now possible to calculate validity limits for this theory if we assume a given maximum value for v. We notice that the two conditions differ by a factor of two and will produce this difference in the calculated limits. If one takes the most rigorous of the restrictions, those of Lucas and Biquard, and assumes for the wavelength of light 5×10^{-5}cm the following results are obtained;

for maximum v = 8 upper limit = 1.8 Mc

for maximum v = 4 upper limit = 3.6 Mc

for maximum v = 2 upper limit = 7.2 Mc

It is the purpose of this investigation to recheck the actual intensity of the diffraction pattern for progressive waves over a wider range of frequency and field depth than that reported by Sanders (16). Special consideration will be given to the recent work of Mertens (17), both as to the region in which it applies and to the actual improvement it may offer to the work of Raman and Nath.

THEORY

In this section we shall outline the theories which shall be of interest in this discussion. Our purpose in doing this is several fold; first, to review the actual theoretical treatments and point out the assumptions that have been made; second, to put the theoretical results in a form which may be related to the experimentally measurable variables; and third, it is necessary that we use a uniform notation for the theoretical treatment. In this latter connection we shall follow rather closely the original notation of Raman and Nath, adapting it to cover the work of Mertens.

Theory of Raman and Nath.

This simple restricted theory bears a close analogy to the theory of diffraction of a plane wave (optical or acoustical) normally incident on a periodically corrugated surface, as given by Lord Rayleigh (19).

Figure I may be used to illustrate the physical set-up. Here P represents a point on a distant screen where it is desired to find the intensity of the diffracted light. The sound and the light are directed normal to each other along the x and y axis respectively. Δs indicates the difference in path length between the two indicated paths to P. It is equal to $x \cos\theta$. L is the distance the light travels through the sound

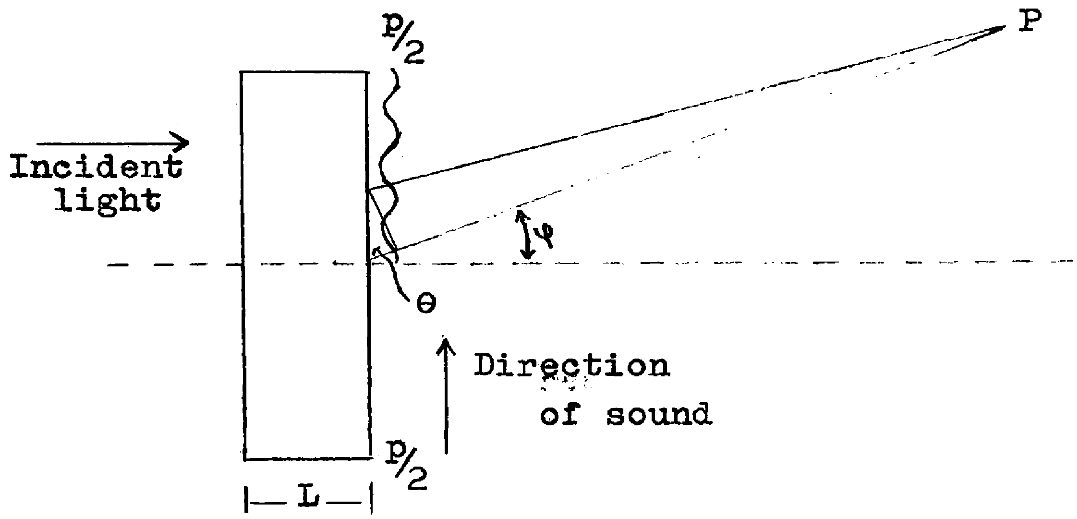


Figure I

field. p is the length of the sound field.

With no sound present a plane wave would pass directly through the medium and emerge as a plane wave. With the sound on it is assumed that the emergent wave will have a corrugated front as indicated in the figure, and that the phase change represented by this wavy front is merely the path length L multiplied by the index of refraction of the medium $\mu(x)$. Where

$$\mu(x) = \mu_0 - \mu \sin \frac{2\pi x}{\lambda^*} \qquad (1)$$

in which; μ_0 = index of refraction of the medium

μ = maximum variation in μ_0.

λ^* = wavelength of the sound wave.

The following assumptions have been made, that there is, first, no deflection of the beam by the medium carrying the sound; second, no amplitude change in the light wave; and third, the assumption is made that the variation in μ will be sinosiodal in nature, this assumption seems to be valid in many substances except for relatively high sound energies.

The amplitude of the incident wave can be represented by the expression;

$$Ae^{2\pi i \nu t} \tag{2}$$

and that of the emergent wave by;

$$Ae^{2\pi i \nu\left\{t - \frac{L\mu(x)}{c}\right\}} \tag{3}$$

where ν = frequency of the light

t = time

c = velocity of sound in the medium

Then the amplitude due to the corrugated wave at a point on a distant screen will be given by,

$$\int_{-p/2}^{p/2} e^{2\pi i\left\{lx + \mu L \sin\frac{2\pi x}{\lambda^*}\right\}/\lambda}\, dx \tag{4}$$

where $l = \cos\theta$

λ = wavelength of the light

The time dependance is dropped since the velocity of light is much greater than the velocity of sound. The sound field is assumed to be of uniform thickness and intensity.

Equation 4 is now broken into its real and imaginary parts and written in sine and cosine form. These can be expanded in a series of Bessel functions which can be integrated. This reduces the imaginary part to zero and the solution can be written in the following series form;

$$F(A) = P\sum_{0}^{\infty} J_n(v) \left\{ \frac{\sin(ul + nb)\frac{P}{2}}{(ul + nb)\frac{P}{2}} + \frac{\sin(ul - nb)\frac{P}{2}}{(ul - nb)\frac{P}{2}} \right\} +$$

$$P\sum_{0}^{\infty} J_{n+1}(v) \left\{ \frac{\sin[ul + (n+1)b]\frac{P}{2}}{[ul + (n+1)b]\frac{P}{2}} - \frac{\sin[ul - (n+1)b]\frac{P}{2}}{[ul - (n+1)b]\frac{P}{2}} \right\} \cdots (5)$$

where; $u = 2\pi/\lambda$

$b = 2\pi/\lambda^*$

$v = 2\pi\mu L/\lambda$

and F(A) is the amplitude at a point on a distant screen.

Examination of this series shows that for any value of n only one term in the series will give any significant contribution to F(A). This is true when

$$ul = nb \qquad (6)$$

in which case the denominator reduces to zero, but for all other terms the denominator is large compared to the numerator and so we drop all terms but this single term.

If we use equation 6 and Figure I we see that

$$l = \cos\theta = \sin\psi$$

combining with 6 gives

$$\sin\psi = n\lambda/\lambda^* \qquad (7)$$

this is the grating equation, and gives the direction of the light incident on the screen.

To get the relative intensity of the nth to the mth order we note that by using our approximation the bracket in equation 5 becomes one for both n and m. Thus the ratio of the intensities of any two components is simply the ratio of the square of the amplitude functions

$$J_n^2(v)/J_m^2(v) \qquad (8)$$

For experimental purposes the light for the zero order is taken as one, so a plot of the square of the nth Bessel function* for an arbitrary set of values for v gives the distribution curve for the nth order. These curves can then be fitted to the experimental curves without actually measuring the quanity μ.

The Theory of Mertens.

The development by Mertens is similiar to that in the previous section, but embodies a more rigorous mathematical formulation and solution of the problem.

The sound and light again enter the medium at right angles to one another, see Figure II. The index of refraction is assumed to vary in the same manner as before and is given by;

$$\mu(x,y,z,t) = \mu_0 + \mu \sin\left[2\pi\nu^* t - (\vec{k}\cdot\vec{r})\right] \qquad (9)$$

where $\mu(x,y,z,t)$ the refractive index is a linear function of the density, and the following notation

* see appendix Table I for these values

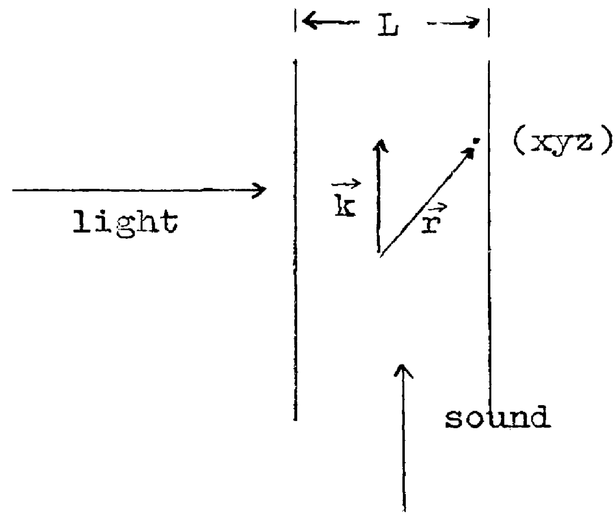


Figure II

applies;

μ_0 = refractive index of undistrubed medium.

μ = maximum variation of μ_0.

ν^* = frequency of the ultrasonic wave.

$\vec{k}$ = propagation vector.

λ^* = wavelength of sound in the medium.

$\vec{r}$ = position vector.

L = thickness of sound field.

The light waves entering the medium must satisfy

$$\text{I. Curl } \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \qquad \text{III. Div } \vec{H} = 0$$

$$\text{II. Curl } \vec{H} = \frac{1}{c}\frac{\partial(\mu^2\vec{E})}{\partial t} \qquad \text{IV. Div } \mu^2\vec{E} = 0 \tag{10}$$

If $\vec{H}$ is eliminated in the usual manner, we get

$$\nabla^2\vec{E} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\mu^2 E) + \text{grad}\,(\text{div}\,\vec{E})$$
$$\text{div}\,(\mu^2\vec{E}) = 0 \qquad (11)$$

a system of partial differential equations describing the light diffraction. We assume that since $\nu^* << \nu$ we may consider μ independent of t in the calculations and reestablish the time dependence in the final results. Then,

$$\nabla^2\vec{E} = \frac{\mu^2(x,y,z,t)}{c^2}\frac{\partial\vec{E}}{\partial t^2} + \text{grad}\,(\text{div}\,\vec{E})$$
$$\text{div}\,(\mu^2\vec{E}) = 0 \qquad (12)$$

now assume that the plane of the ultrasonic beam is parallel with the x-y plane. Equation 9 becomes,

$$\mu(x,t) = \mu_0 + \mu \sin 2\pi(\nu^* t - y/\lambda^*) \qquad (13)$$

and the second equation in 12 reduces to

$$\frac{\partial(\mu^2 E_y)}{\partial y} = 0 \qquad (14)$$

since μ is not a function of x or z. Solving this for the div $\vec{E}$, gives;

$$\text{div}\,\vec{E} = -\frac{1}{\mu^2}\frac{\partial\mu^2}{\partial y}E_y \qquad (15)$$

which may be substituted in the first equation of 12 giving the expression;

$$\nabla^2 E = \frac{\mu^2(x,t)}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} - \text{grad}\left(\frac{1}{\mu^2(x,t)}\frac{\partial\mu^2(x,t)}{\partial y}E_y\right) \qquad (16)$$

Brillouin (7) shows that the last term may be neglected leaving

$$\nabla^2 \vec{E} = \frac{\mu^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t} \qquad (17)$$

but since there is no variation of $\vec{E}$ in the z direction this reduces to,

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} = \frac{\mu^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \qquad (18)$$

Now taking

$$\vec{E} = e^{2\pi i \nu t}\, \Phi(x,y,t)$$

and substituting in the above equation we get, after terms containing a $1/c^2$ factor have been dropped,

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial \Phi}{\partial y^2} = \frac{\mu^2}{c^2}\left[\frac{\partial^2 \Phi}{\partial t^2} + 4\pi i \nu \frac{\partial \Phi}{\partial t} - \Phi 4\pi^2 \nu^2\right] \qquad (19)$$

Because of the periodicity of the sound wave along the y axis we can write,

$$\mu(y + p\lambda^*,\ t) = \mu(y,t)$$

$$\mu(y,\ t + \nu^*/q) = \mu(y,t)$$

where p and q are whole numbers.

Also since the sound and light waves are perpendicular to each other displacements of $p\lambda^*$ or ν^*/q are without influence and,

$$\Phi(y + p\lambda^*,\ x,t) = \Phi(x,y,t)$$

$$\Phi(y,x,t + \nu^*/q) = \Phi(x,y,t)$$

Thus Φ can be expanded in a double Fourier Series

$$\Phi = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} a_{nm}(x)\, e^{2\pi i n y/\lambda^*}\, e^{2\pi i m \nu^* t} \qquad (20)$$

Making this substitution for Φ gives

$$2\frac{d\Phi}{dv} - \left(\Phi_{n-1} - \Phi_{n+1}\right) = i \tag{21}$$

where $v = 2\pi\mu L/\lambda$

$\rho = \lambda^2/\mu_0\mu\lambda^{*2}$

For ρ equal to zero the Bessel functions satisfy the equation and this is just the solution of Raman and Nath. That is,

$$\Phi_n(v) \simeq J_n(v) \tag{22}$$

Mertens, however, writes his solution in the following form;

$$\Phi_n(v) = J_n(v) + \sum_{p=1}^{\infty} \rho^p \Psi_{np}(v) \tag{23}$$

where $\Psi_{np}(v)$ is a function to be determined which must satisfy the boundary conditions

$$J_o(0) = 1$$

$$J_n(0) = 0 \qquad n \neq 0$$

$$\psi_{np}(0) = 0$$

The desired function is found to consist of the sum of two series. The intensity may then be written as the square of equation 23, and has the following form,

$$I_n(v) = J_n^2(v) + \rho^2\left\{\left[\psi_{n1}(v)\right]^2 + 2J_n(v)\psi_{n2}(v)\right\} \tag{24}$$

where for small values of ρ the following expressions give adaquate values for the last terms in equation 24.

$$\psi_{n1}(v) = \frac{v^{n+1}}{6 \cdot 2^n} \sum_{m=0}^{\infty} \frac{(-1)^m [2m + n(2n+1)]}{m!\,(m+n)!} v^{2m} \tag{25}$$

$$\psi_{n2}(v) = \frac{v^{n+2}}{60 \cdot 2^n} \sum_{m=1}^{\infty} \frac{(-1)^m [4m + (n+1)(10n-7)][m + \frac{1}{4}(2n^2+3n-6)]}{2^{2m}(m-1)!\,(m+n-1)!} \tag{26}$$

Thus the intensity distribution in the nth order may be calculated from equations 24, 25 and 26. The problem presented then reduces itself to checking the contribution of these series terms which is simply added to original results of Raman and Nath.

The factor ρ is the only term in the correction expression involving experimental variables. Thus it is possible to evaluate the series for arbitrary values of v chosen as before and to find the correction term for a given experimental situation by simply multiplying by ρ^2. These series computations are tabultated in the appendix. We must bear in mind that for the correction to be of any value we must have $\rho < 1$.

Recalling that ρ is given by;

$$\rho = \frac{\lambda^2}{\mu_0 \mu \lambda^{*2}}$$

and v by the expression,

$$v = \frac{2\pi \mu L}{\lambda}$$

we see that the product ρv will eliminate the troublesome factor μ. When this product is formed we have

$$\rho v = \frac{2\pi L \lambda}{\mu_0 \lambda^{*2}} \qquad (27)$$

The terms on the right of this expression are all experimentally measurable, and since the v's have been arbitrarily chosen we can arrive at a cooresponding ρ for each v. This then allows us to compute the correction for each value of v and plot a theoretical curve which can be fitted as before.

EXPERIMENTAL APPARATUS

The experimental arrangement consisted of the usual optical set-up for the observation of the diffraction of light by an ultrasonic wave in a liquid. It is illustrated in Figure III. The light source S was a 100 watt General Electric type AH-4 mercury vapor lamp. It and the condensor lens L_1 were housed in a light tight box so that excessive scattering of the light was prevented. The condensor lens L_1 focused the light on the slit Sl_1. The latter was located at the focal point of lens L_2, this gave parallel light through the tank. A filter F was placed between L_1 and Sl_1. This was a Central Scientific Wratten filter No. 87310E designed to pass the mercury 5841 A line. Actually this filter was not necessary since a similiar filter was used in the photocell housing, it did, however, aid greatly in the optical alignment of the apparatus.

The plane wave from L_2 was then passed through the tank T and was focused by means of L_3 upon the second slit Sl_2. This slit, ahead of the photocell, permitted one to pick out and measure the intensity of each of the several orders. The photocell was an RCA 931A and was used in conjunction with a Photomultiplier Microphotometer Type 10-210 manufactured by the American Instrument Company. The readings from the microphotometer could either be observed visually and point by point obser-

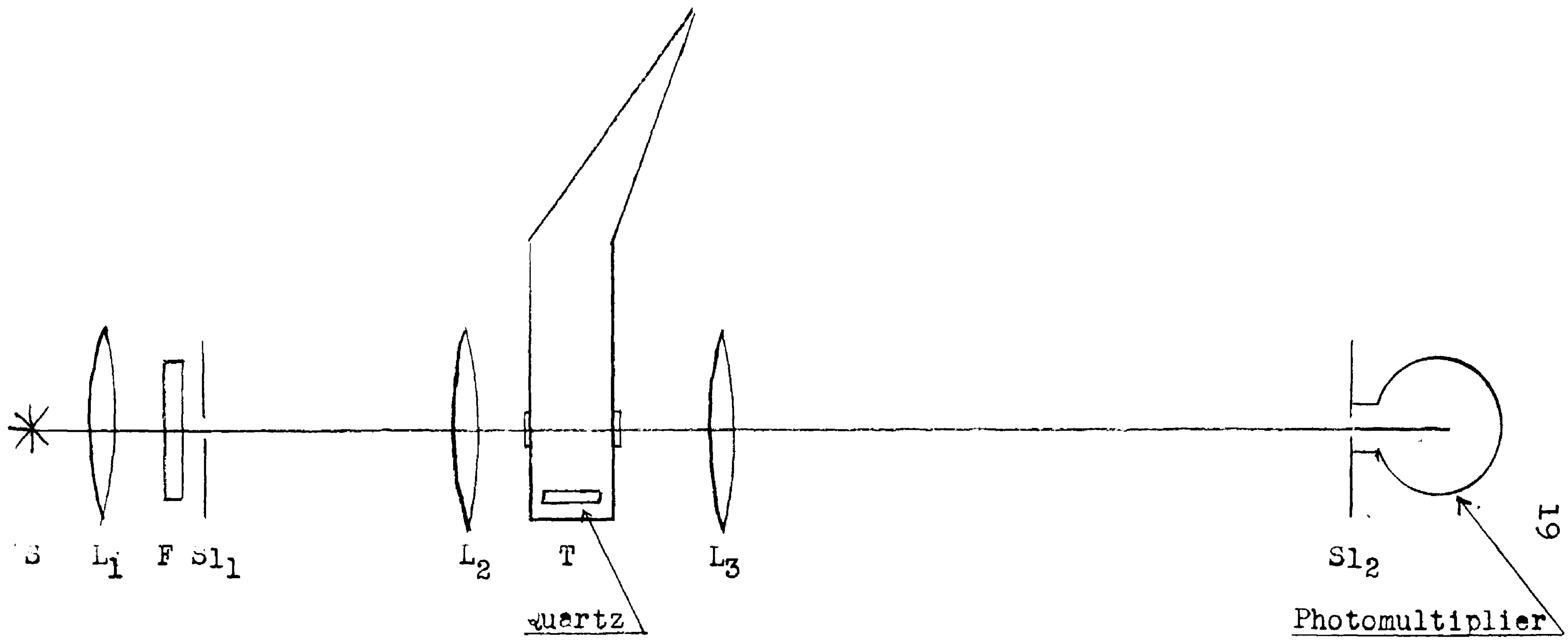


Figure III. Optical arrangement for obtaining the light diffraction pattern produced by an ultrasonic wave in liquids.

vations made or it could be used in connection with a Brown Recorder. Both methods were used, but point by point readings were found somewhat more desirable, since readings could be made directly in percent of light transmitted, and a constant check could be made that the original light intensity or the sensitivity of the photocell did not vary.

L_1 was combination lens of approximately 8 cm focal length. L_2 had a focal length of about 12 cm, and for L_3 a lens of 100 cm focal length was chosen so as to obtain greater seperation of the lines at the second slit.

The tank T presented the most serious problem. The difficulty was to get the light in and out of the tank without excessive scattering by multiple reflections, and also to prevent the establishment of standing waves in the tank. The first difficulty was overcome by using 1½ inch square plane parallel plates as windows on the tank. One side of each window contained an anti-reflection coating designed to transmit the 5841 A mercury green line. By using the coated side at the air-glass surface reflections here were largely eliminated. At the inner surface no problem was presented since glass and xylene (xylene being the liquid used throughout the experiment) have practically identical values for the index of refraction.

To prevent the establishment of standing waves, the tank was constructed as shown in Figure IV. The main body of the tank was 8 inches long, 2½ inches wide and 3 inches deep. The wedge shaped tail used to absorb the sound beam by multiple reflections was also approximately 8 inches long and attached at about 30° to the main tank.

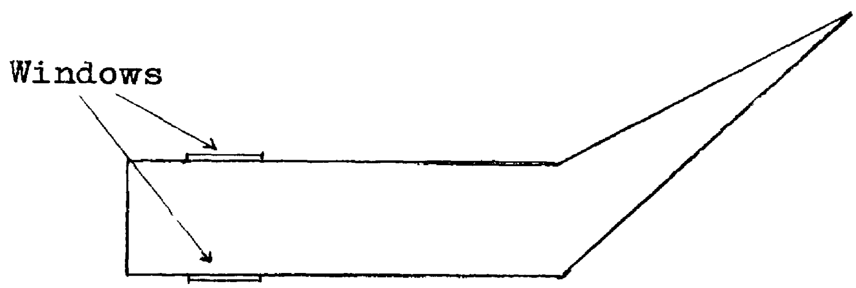


Figure IV

The wedge shaped tail was lined with cork as was the back and several other portions of the tank, from which waves might be reflected. Tests designed to show the presence of standing waves indicated that they had been eliminated by this construction.

The sound was produced by quartz crystals of various frequencies and of several sizes, so as to observe both effects of variation in L and frequency, where L is the depth of the sound field. The R.F. source used to drive the quartz was an oscillator designed and constructed in the laboratory. It could be made to cover the frequency range from 1-15 Mc. The final amplifier consisted of two 807 tubes connected in parallel. Maximum power output was about 100 watts.

The apparatus requirements were completed by the use of a surplus U.S. Army Singal Corps Frequency Meter Type BC-221-C manufactured by the Bendix Company, and of a General Radio Vacuum Tube Voltmeter Type 1800A for measurement of the R.F. voltage on the crystals.

Experimental Procedure.

In actually taking data the following method was found to give the best results, and the following precautions were observed.

The source slit was adjusted to ten microns. The lens L_2 was adjusted by means of a telescope. The latter was focused for parallel light, thus a sharp image in the field of the telescope indicated that we had parallel light coming through the sound field. Lens L_3 was then adjusted to focus the image on the slit Sl_2.

With the sound present the image was again viewed by means of the telescope, and a visual adjustment made to line the sound beam and the light beam normal to one another. This was done by observing when the number of orders on either sider of the zero order were equal in number and intensity. A final check on the intensity symmetry was made by means of the photocell. It should be noted that an absolute intensity symmetry about the zero order can only be approximated. This same observation was made by Sanders (16), and is probably due to the decrease in amplitude of the sound wave both from absorption and dispersion as it leaves the transducer.

After these adjustments had been made one was ready to make observations. It was often found necessary to allow both the microphotometer and the light source to "warm-up" for approximately an hour or one would observe a drift in intensity readings toward higher and higher values.

Other precautions included the following; it was found that unless all equiptment was properly grounded the microphotometer was affected by the R.F. source. Another source of error in the earlier work resulted when light slipped by in the fringe of the sound field. This was corrected by blocking out a part of the exit window, so that the vertical depth of the sound field was greater than the window. It was also found necessary that a stirrer be in constant operation in the tank to prevent local heating effects, and resultant disturbances in the light intensity. After these rather simple precautions had been taken, and after the coated windows had been mounted on the tank as discussed in the previous section, it was found that the intensity distribution curves could be readily duplicated.

In actually taking data, the zero order was checked first. The microphotometer was adjusted to read 100% transmission for no sound present, and then adjusted to read zero when the light was blocked out. Both of these adjustments were checked repeatly during the run and if appreciable drift was observed in either the run was started over.

The sound field intensity was then varied over a sufficient range of voltage so as to coorespond to a maximum intensity at least as great as 6v. Thus the curves could be plotted out to values of 6v.

Simultaneous readings were made for both voltage across the crystal and precent transmission. The crystal current was allowed to flow only long enough to make the necessary readings and adjustments and a short time lapse allowed between each reading. This together with the use of a blower on the tank and a stirrer in the tank prevented excessive heating. It was found that by this means temperature changes in the liquid could be kept to values of less than one degree centigrade.

Similiar runs were made on the plus and minus orders out to the plus and minus 4th order, using the same voltage steps. Temperature and frequency readings were made during each run.

PRESENTATION OF DATA

The information obtained in the manner described in the previous section was then plotted, percent intensity against voltage, and this curve fitted to a theoretical curve which was plotted percent intensity against v. The necessity of treating the curves in this manner was due to the inability to measure the variable μ as described in the section on theory.

The curves were actually fitted by assuming that several of the minimum points were correct on both curves. Two such points were present on the zero order curve and several others on the higher order curves. In this manner a multiplying factor was obtained which made it possible to convert the voltage readings plotted along the x axis into their cooresponding v values.

The curves shown on the graphs included in this section were obtained in the manner described above. In each case the theoretical Raman-Nath curve is showen together with the experimental curve. Also, in cases where the Mertens' correction proves applicable, and of sufficient order of magnitude so as to distinguish it from the Raman-Nath curve, it is plotted to the same scale and for the same values of v.

The curves are shown for several frequencies and for two different values for the thickness of the sound field.

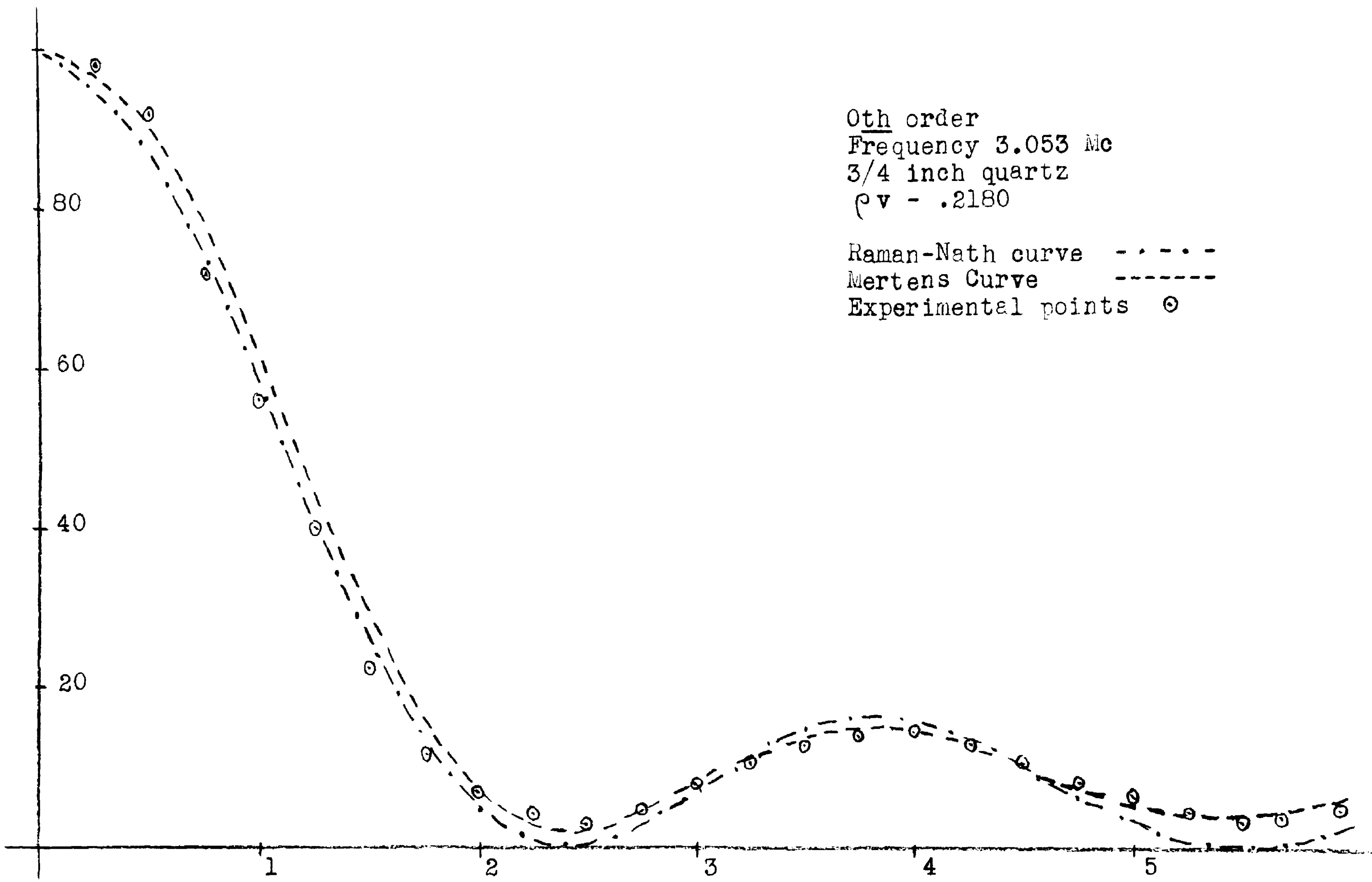
They are grouped in the following manner. We first show three frequencies of approximately 3, 4 and 5 Mc for the ¾ inch square quartz. Then five frequencies of approximately 2, 3, 4, 5, and 7 Mc for the one inch square quartz. In all cases the zero order distribution curve is showen. Higher order curves are reproduced only for the 3 and 5 Mc cases for the ¾ inch quartz and for the 3 and 4 Mc cases for the one inch square quartz.

These are sufficient to show the agreement with the Raman-Nath theory, and the region in which the Mertens' correction is of value.

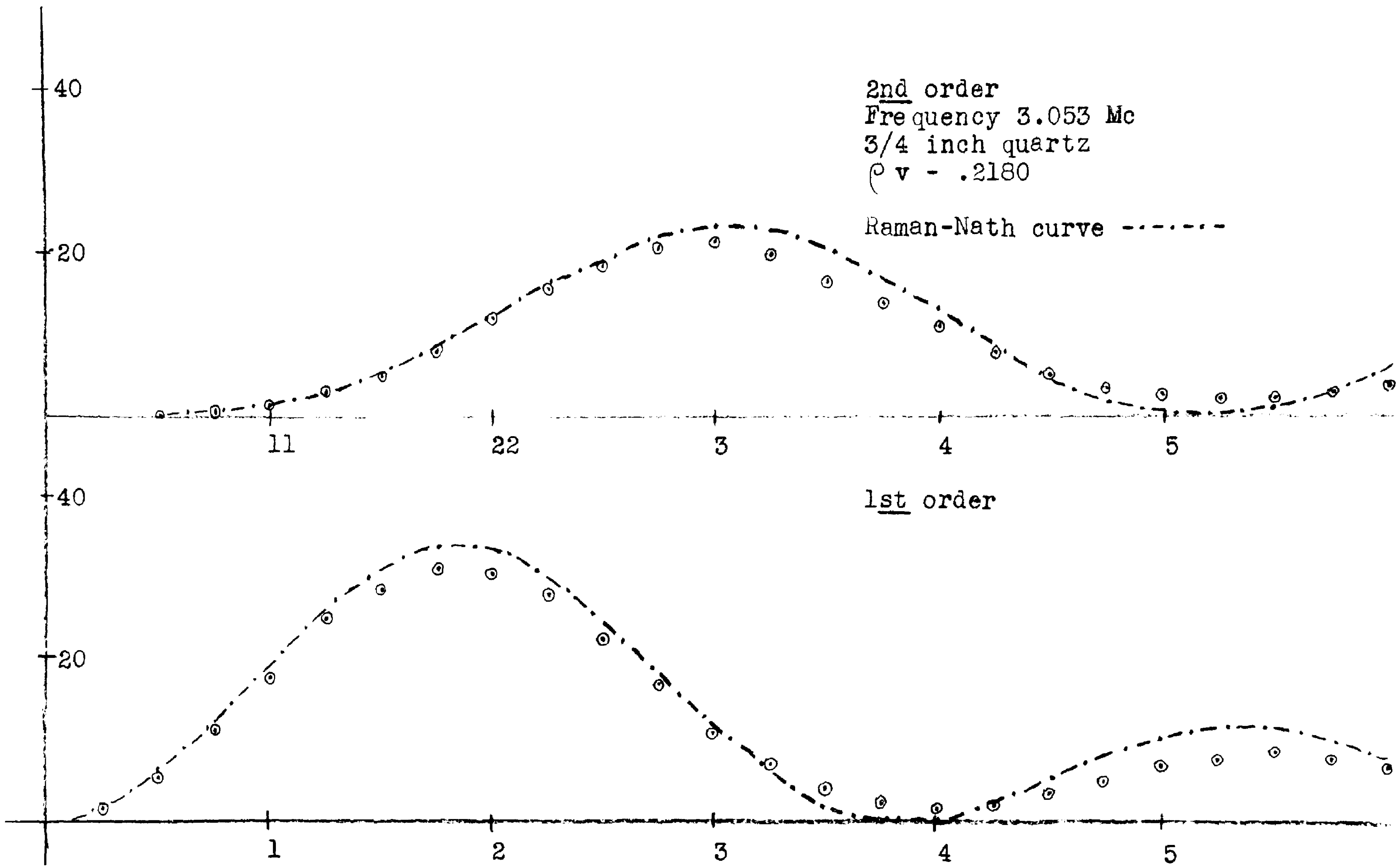
In all cases the exact frequency, quartz size, and the ρv product is indicated on the curve. This latter product enables one, by using it in connection with the correction multipliers listed in Table VI of the appendix, to see how the correction behaves in cases where it has not been plotted.

In calculating the ρv values the following constants were used;

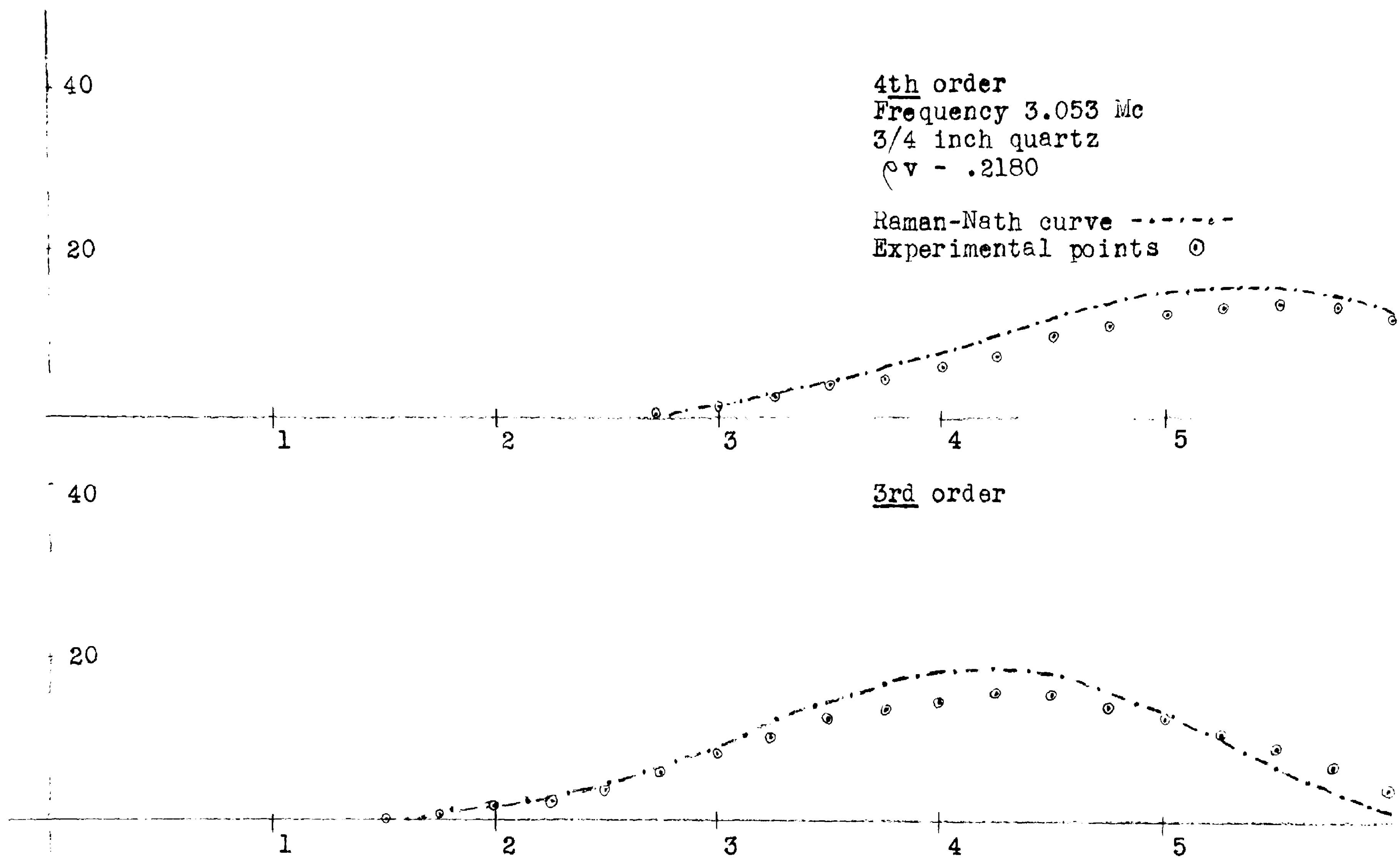
Velocity sound (Xylene 20° C)	1340 m/sec
Velocity temperature correction	4 m/sec $^{\circ}$C
Index of refraction (Xylene)	1.505
Wavelight light	5.461 X 10^{-5} cm
Field thickness	quartz width



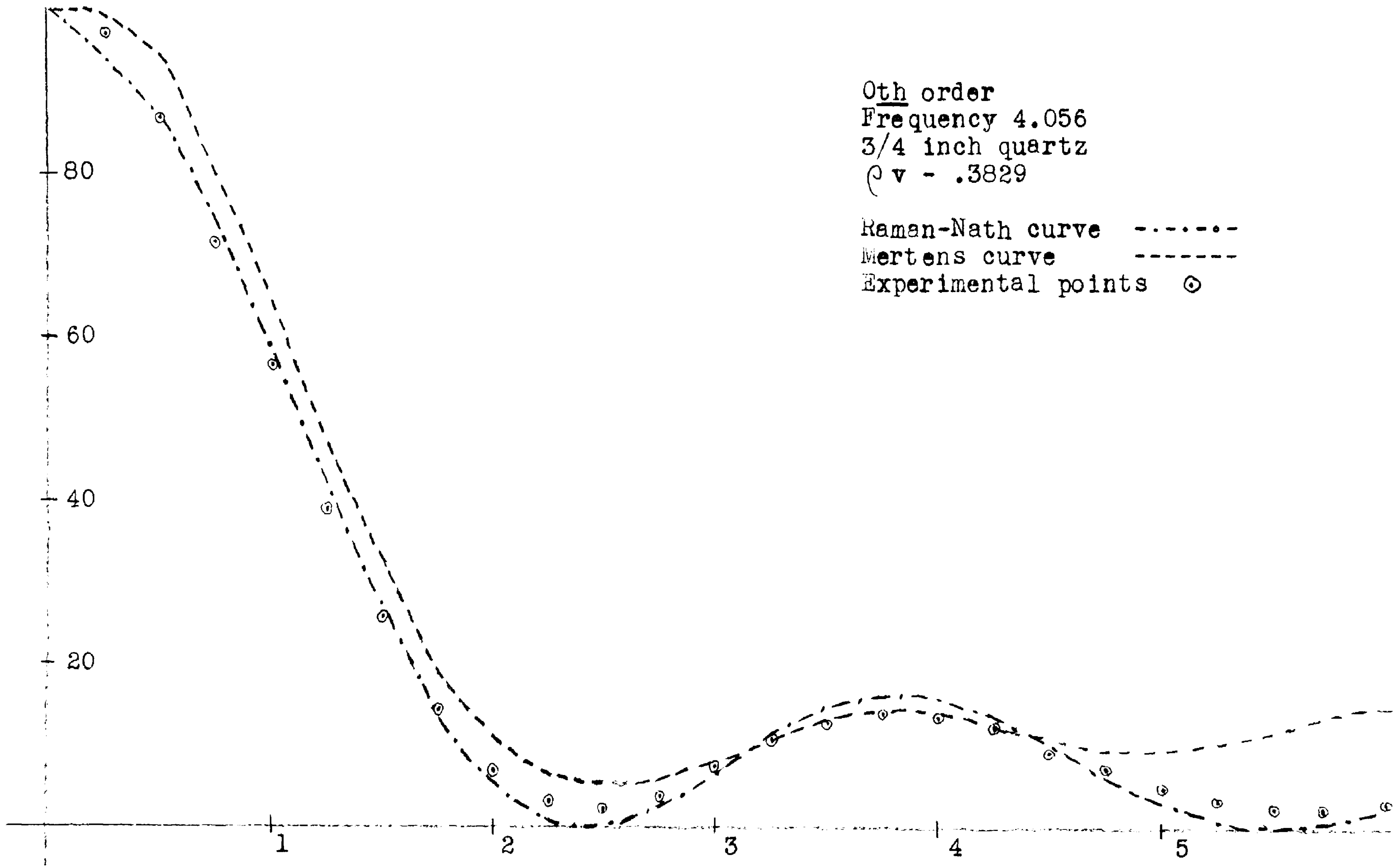
Graph No. 1 Percent Intensity vs v.



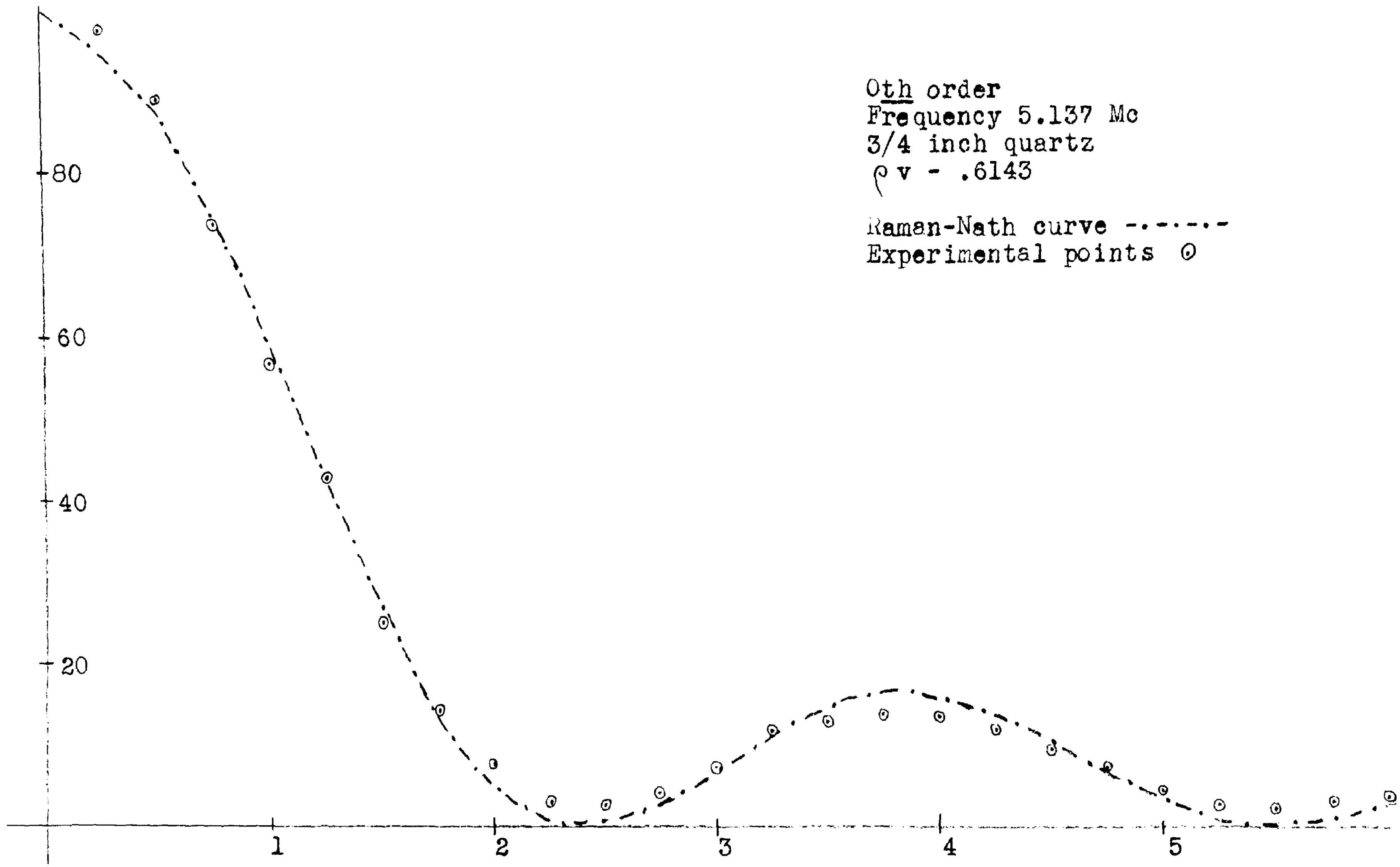
Graph No. 2 Percent intensity vs v. Mertens' correction less than ½ % at all points.



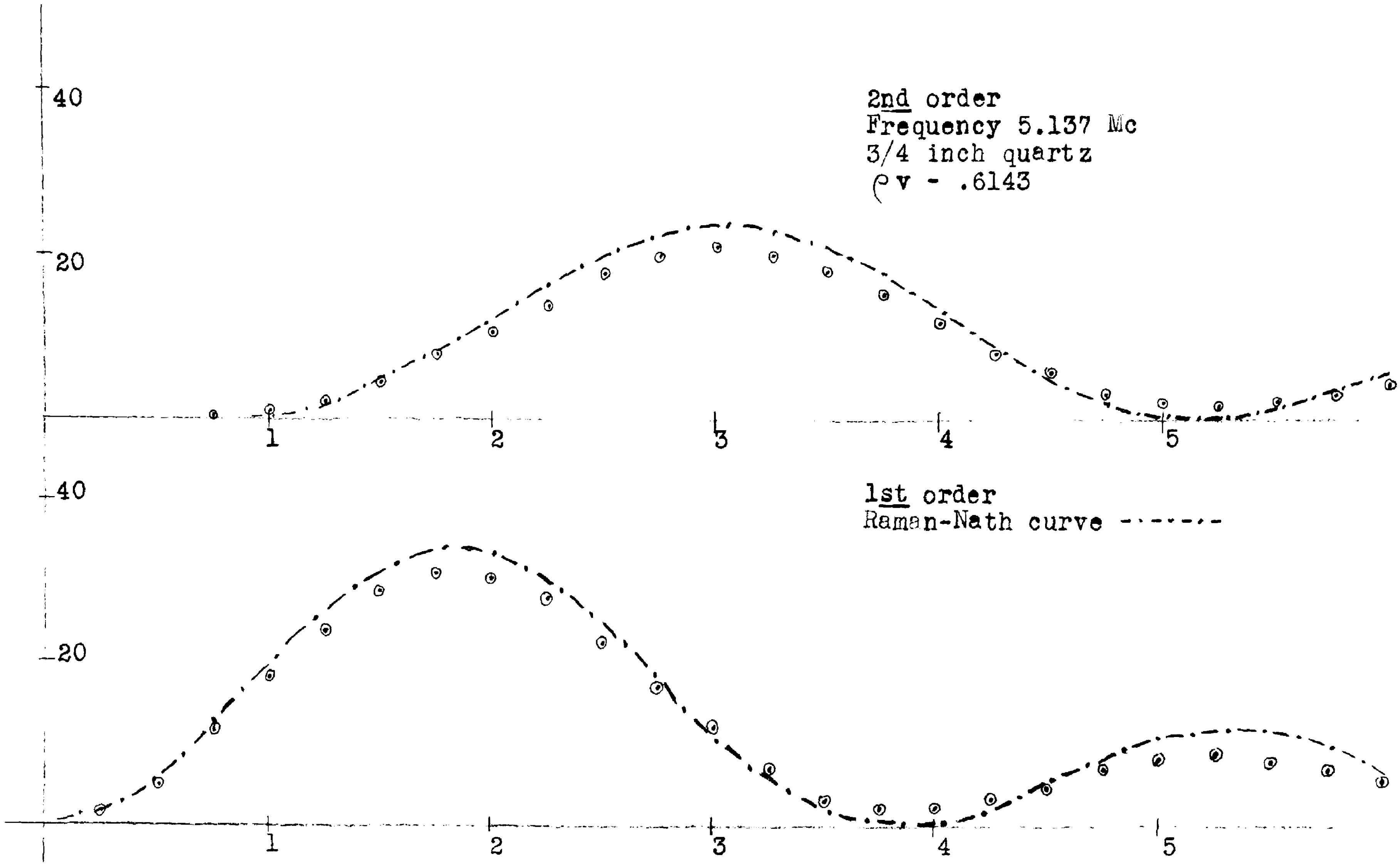
Graph No. 3 Percent Intensity vs v.



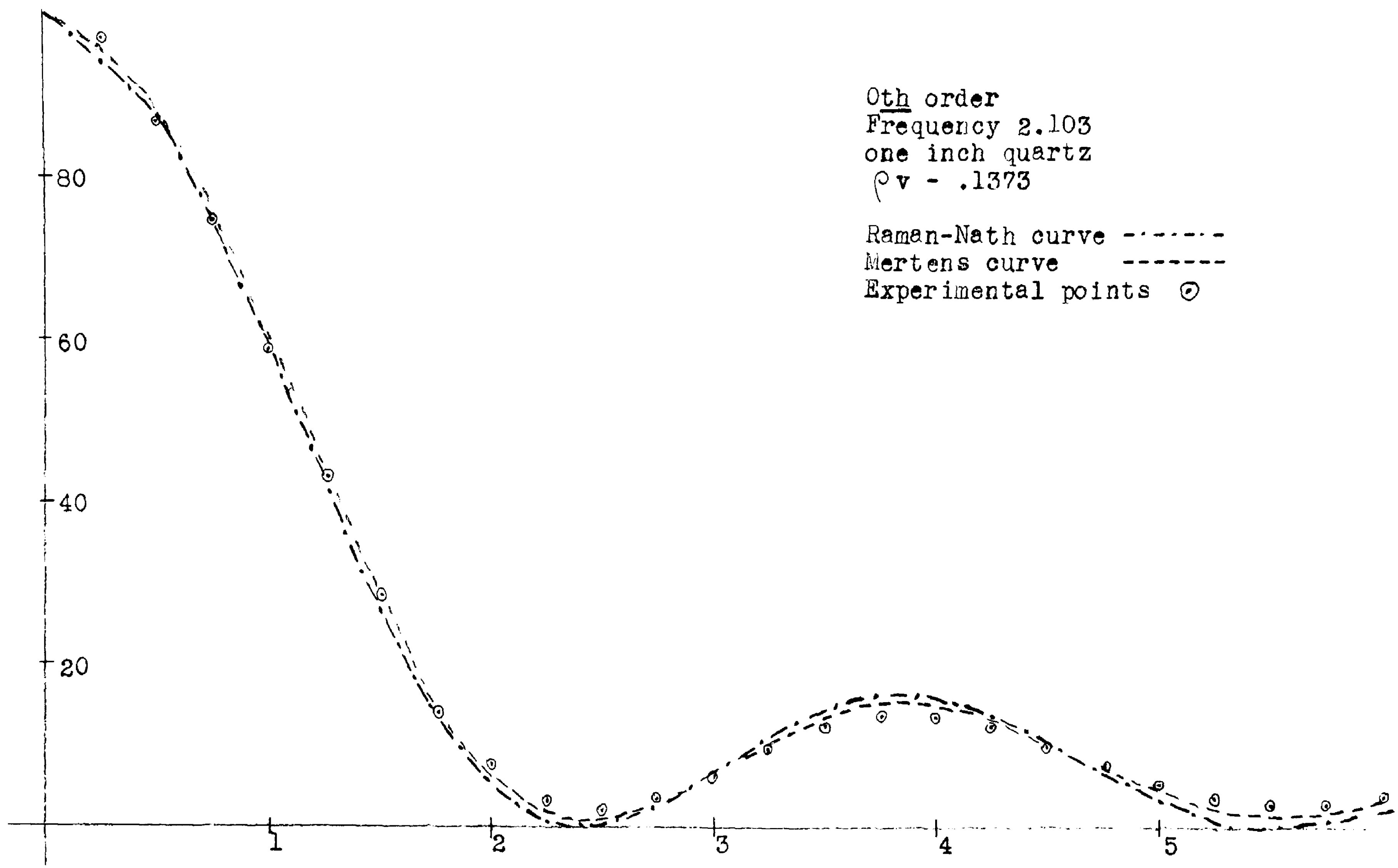
Graph No. 4 Percent Intensity vs v.



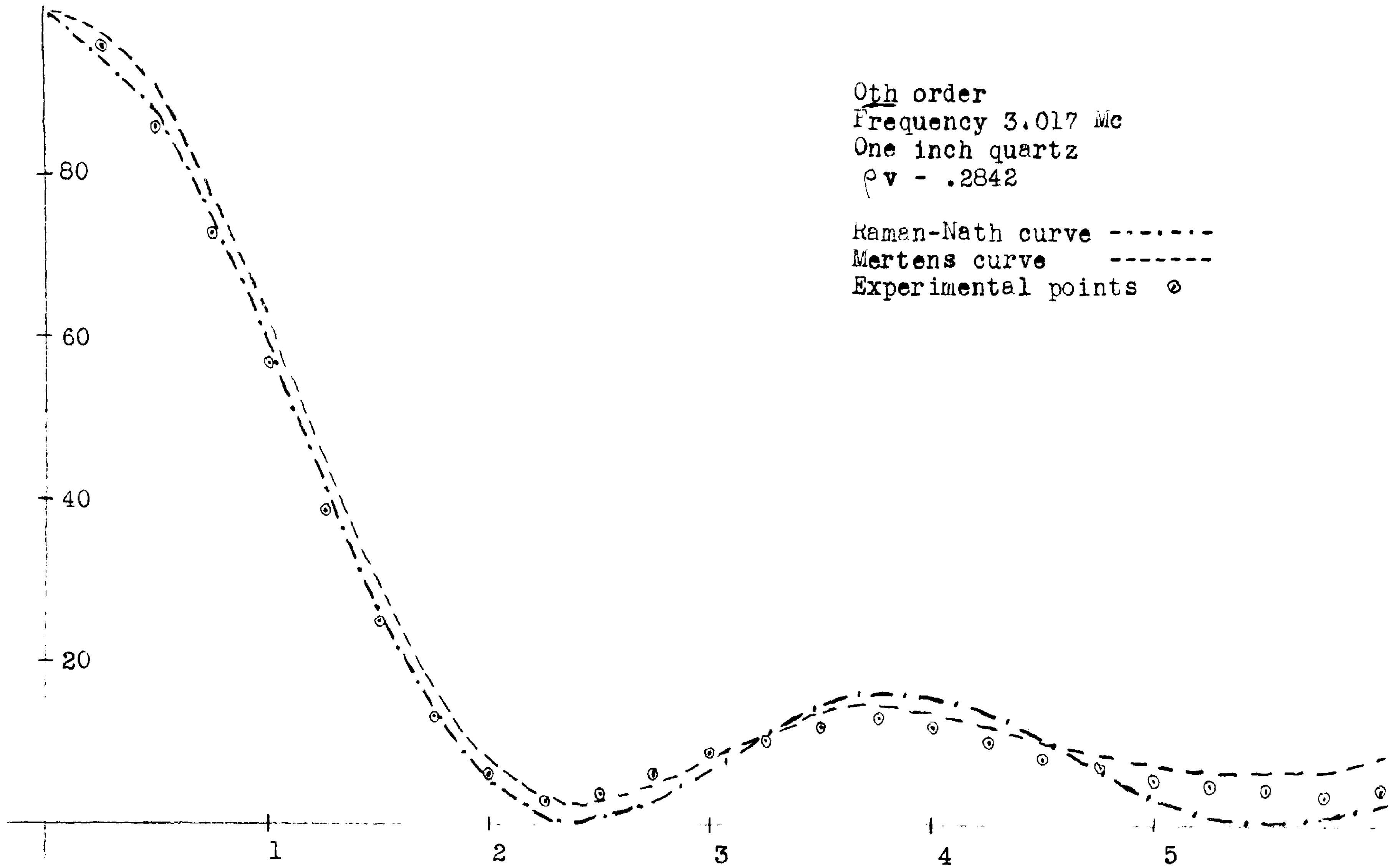
Graph No. 5 Percent Intensity vs v. Mertens' correction excessive. See Graph No 11 for behavior of v values of this order of magnitude.



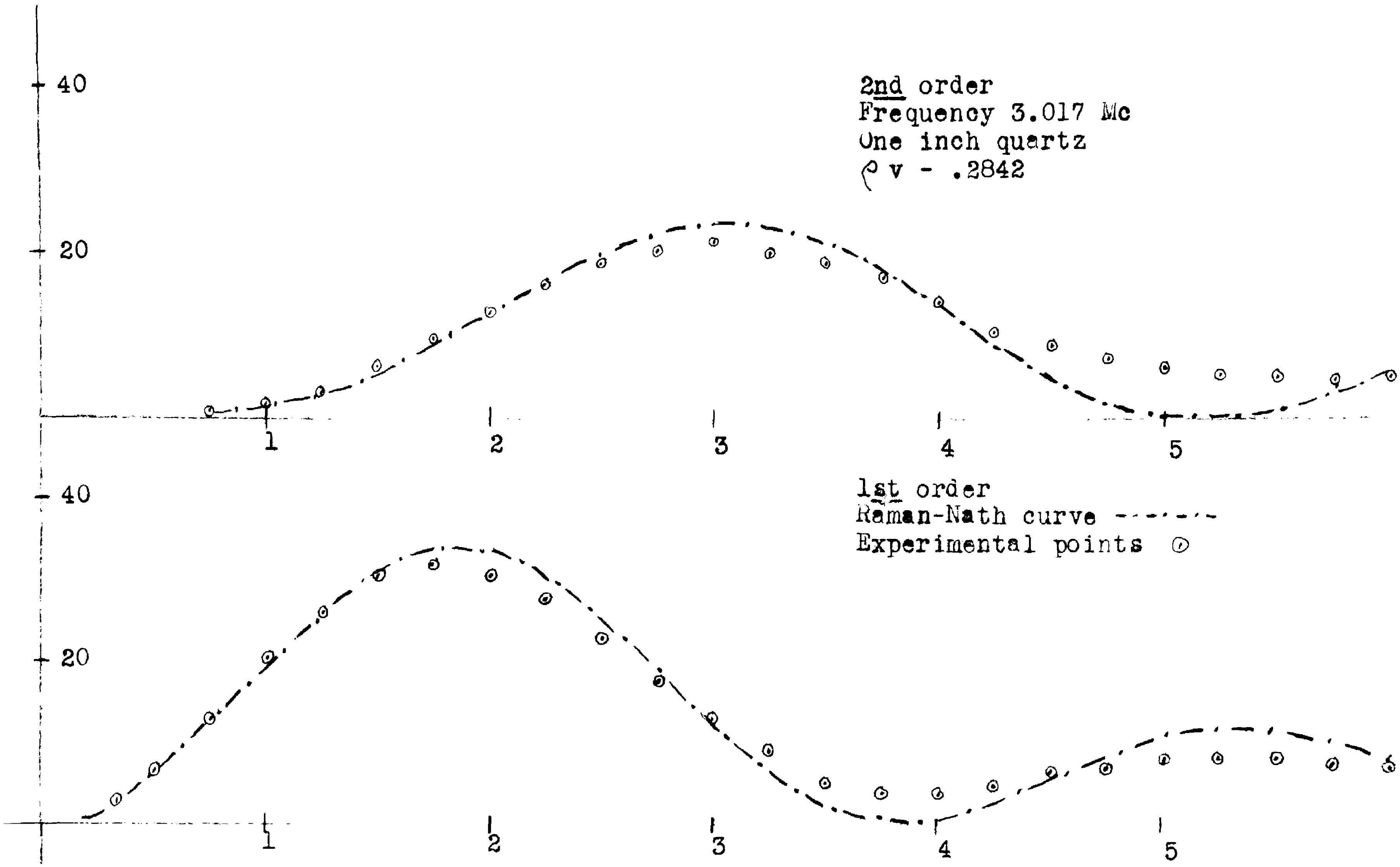
Graph No. 6 Percent Intensity vs v.



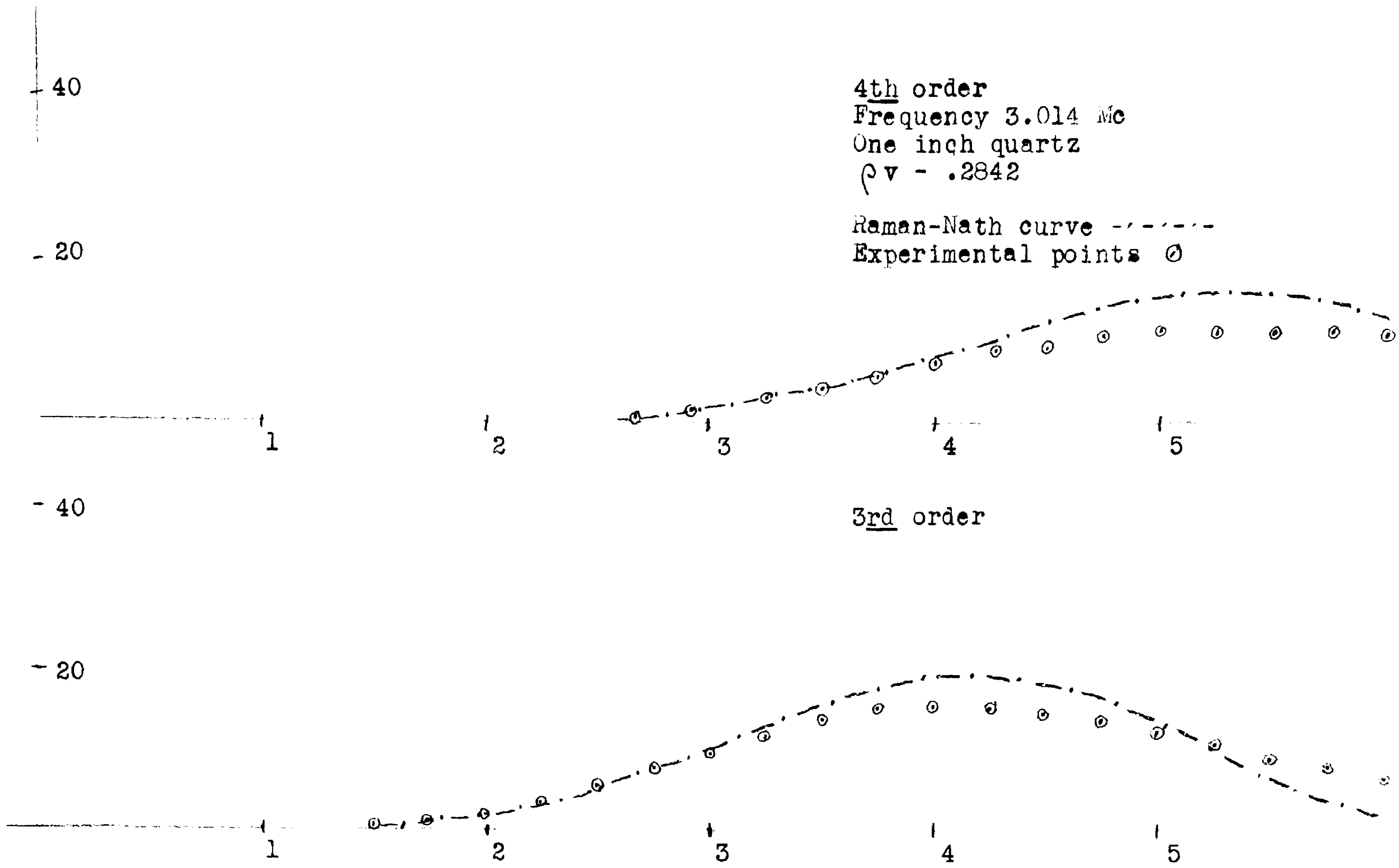
Graph No. 7 Percent Intensity vs v.



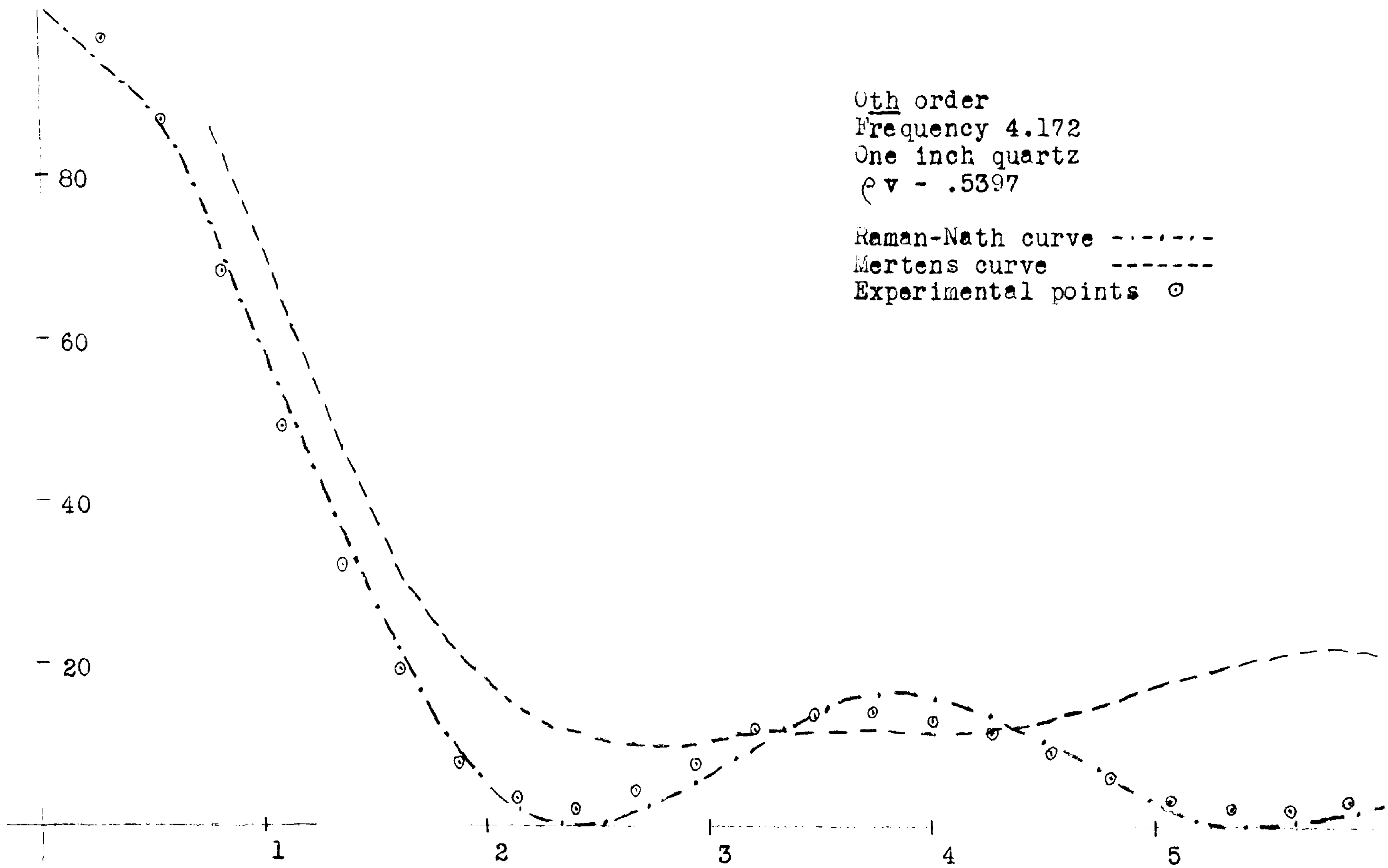
Graph No. 8 Percent Intensity vs v.



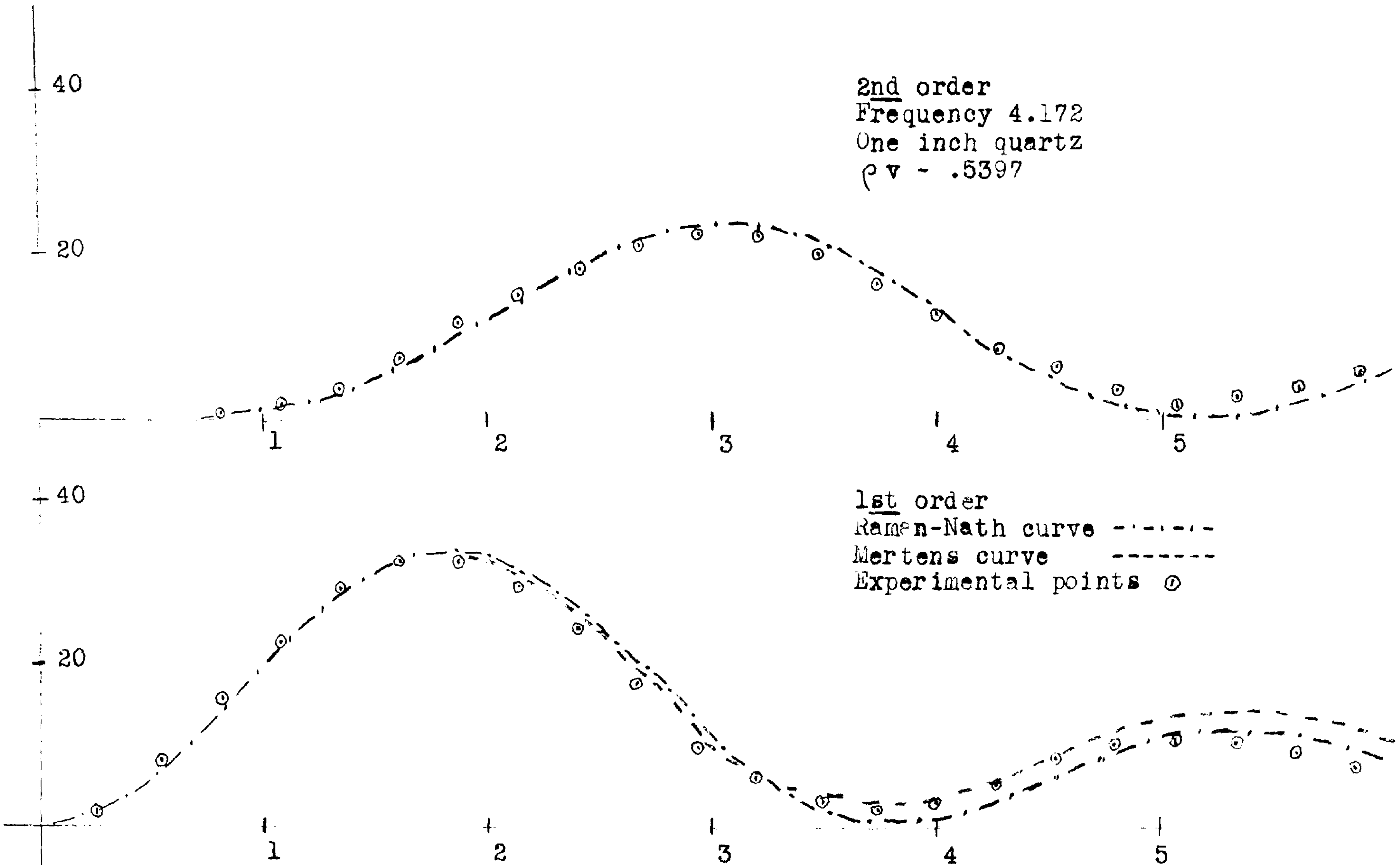
Graph No. 9   Percent intensity vs v.



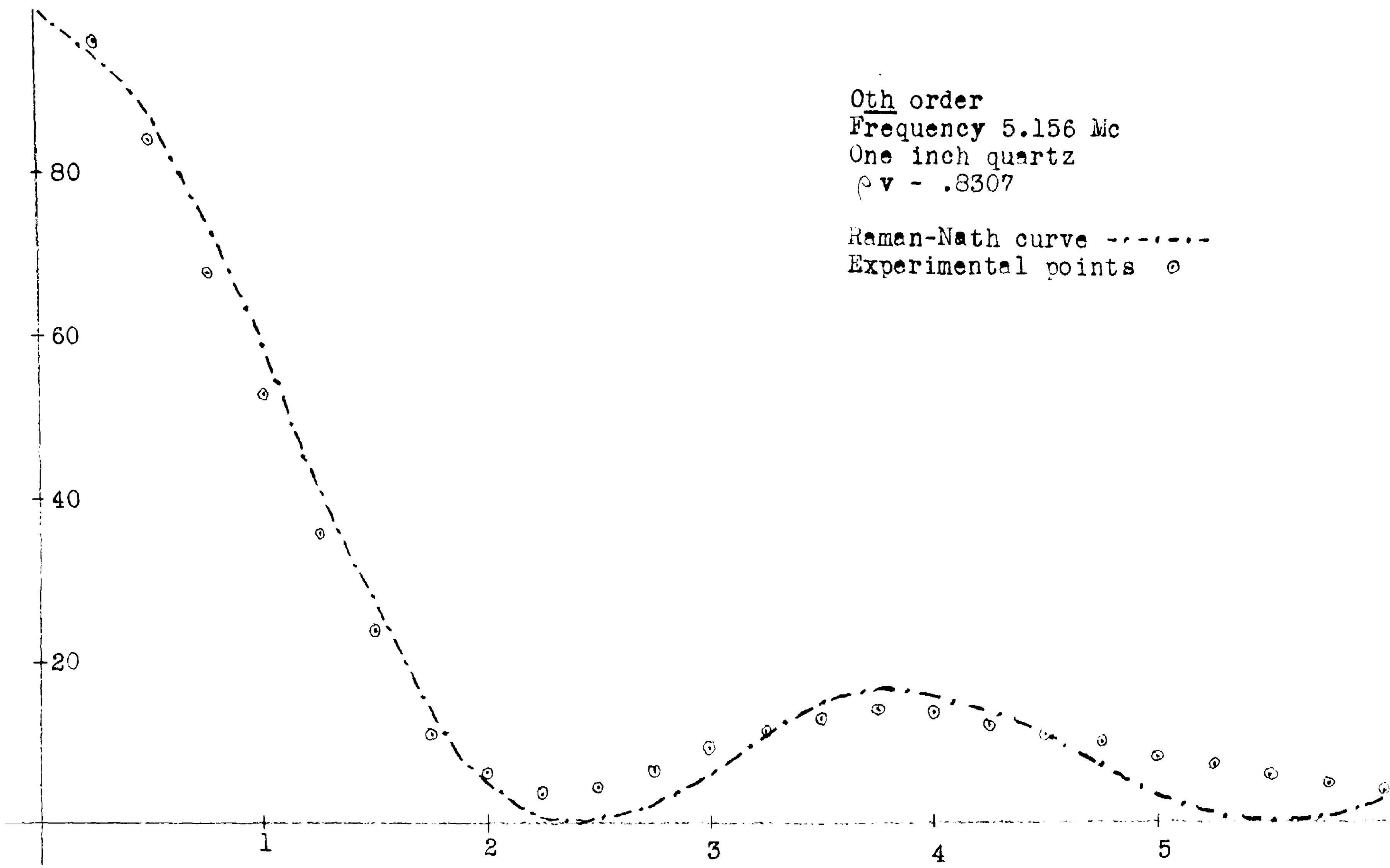
Graph No. 10 Percent intensity vs v.



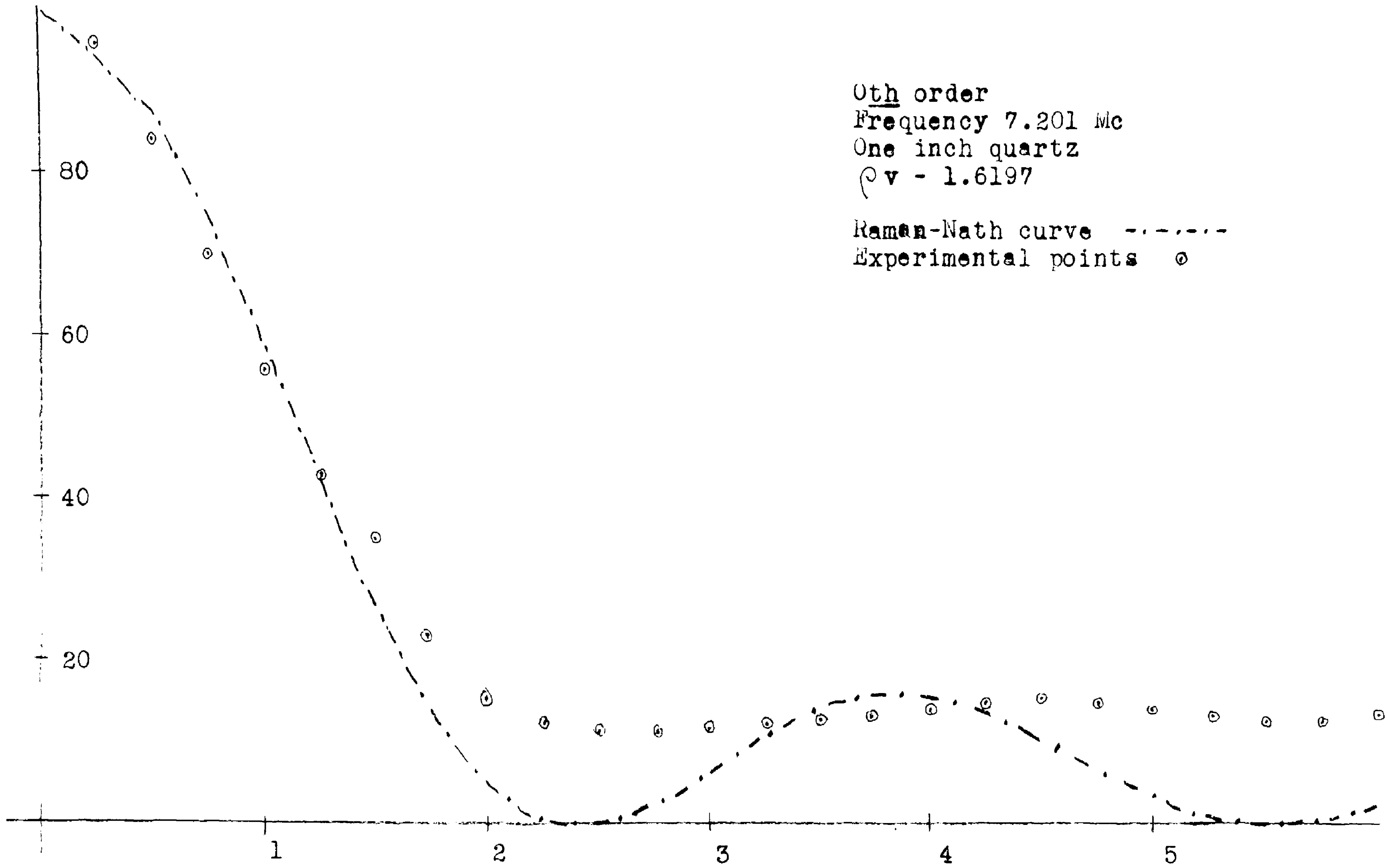
Graph No. 11. Percent intensity vs v. Note ρv value too high for Mertens' correction to be useful.



Graph No. 12 Percent intensity vs v. First order Mertens' correction shown.
Correction assumes wrong sign at about 4.5v.



Graph No. 13 Percent intensity vs v.



Graph No. 14 Percent intensity vs v.

ANALYSIS OF DATA

Raman-Nath Theory,

We can point out here that the simple Raman-Nath theory gives good results in the regions predicted. We note in the final curves that as the frequencies become higher and higher the fit becomes poorer and poorer for the high values of v. At 7 Mc the agreement becomes very poor for values of v greater than two. This is in accord with the prediction of Nath (10) as discussed in our introduction.

Mertens' Correction (zeroth order),

As indicated previously our chief interest is with the application of the work of Mertens rather than that of Raman and Nath. As noted in the theory this correction is limited to values of ρ smaller than one. This is also the condition on the Raman-Nath work but in this latter case the restriction seems to be less severe than in the former.

For the case of the zero order our results indicate the following; First, the correction is not useful for values of v less than two. This is to be expected, since v itself is a function of μ as is ρ, and since low values of μ give low values of v but high values of ρ the condition that ρ be less than one is less applicable in this region.

For values of v greater than two and for frequencies below 4 Mc the Mertens' correction offers some improvement to the original work of Raman and Nath. At or below 2 Mc the order of magnitude of the correction is so small as to be of little use. Above 4 Mc the correction tends to become an over correction for the lower values of v and pushes the region of usefulness toward higher and higher values for v. However, the usefulness of the correction for values of v above six is extremely limited, due first to mathematical difficulties encountered in calculating the correction terms, and also due to the fact that the original Raman-Nath theory becomes less applicable in this region.

Mertens' Correction (Higher Orders),

For orders above the zero order the Mertens' correction terms become less useful. At low frequencies and low values of v the terms are mathematically to small to be of much significance. At higher frequencies they give some correction for the low v range, but for the higher v values the correction tends to take on the wrong sign. Concerning this sign change the following observation may be made on the curves in general.

For high values of v there is a tendency for the intensity values of all orders to be lower than those predicted by the theories. It was noted during the experimental work, that the higher orders appeared with sufficient intensity to be observed at a faster rate than one would expect from the Bessel function relationship used in the theories.

OTHER RECENT THEORETICAL WORK

The other theoretical papers that came to our attention during the course of this work were directed at a frequency region above that in which our equipment was designed to operate.

We should, however, mention several, among them is a paper by Mertens (18). This work is an extension of that by Nath (10), and points out that for values of ρ much greater than one the first order intensities should be given by,

$$I_1 = 4/\rho^2 \quad \sin^2 \rho/4 \; (v)$$

for progressive waves, and by;

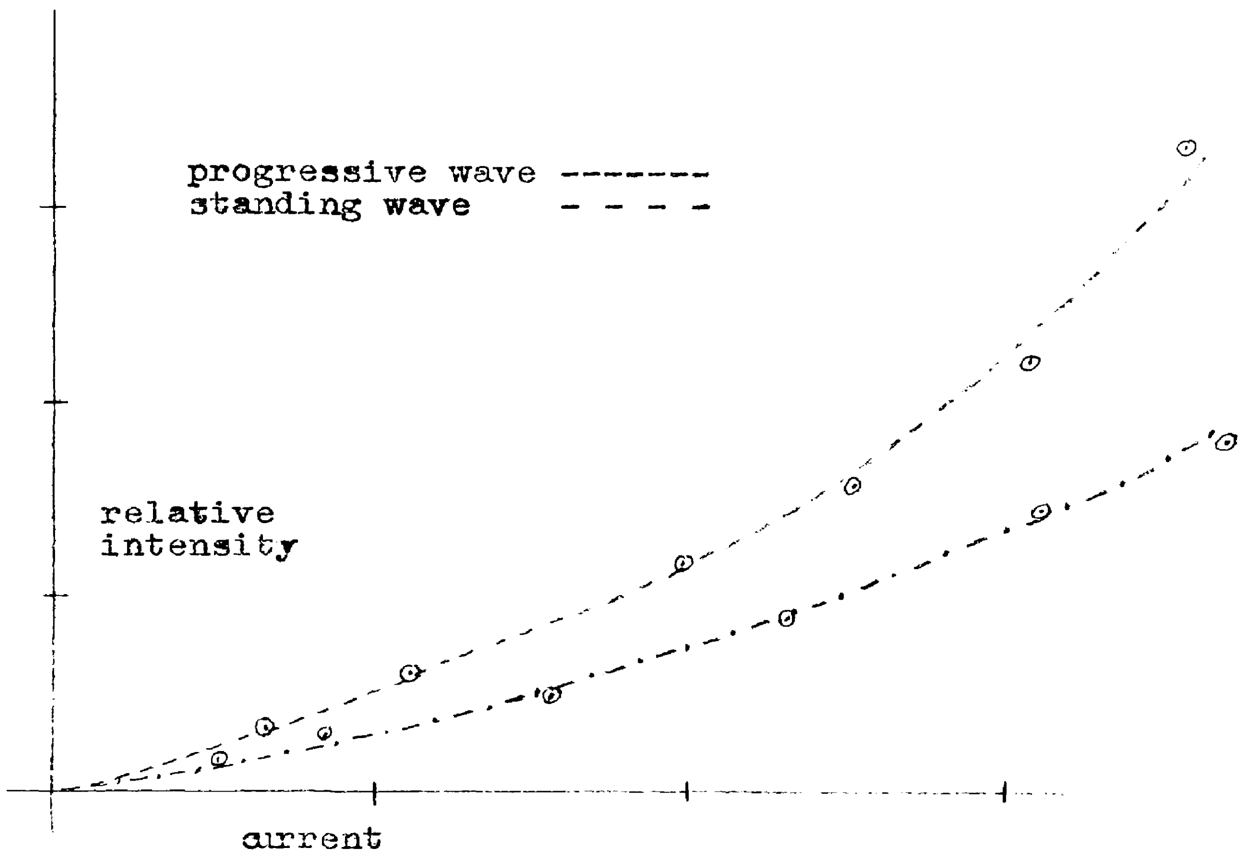
$$I_1 = 2/\rho^2 \quad \sin^2 \rho/4 \; (v)$$

for standing waves, where in both cases the sound intensity must be low enough so that we may assume that orders higher than the first are not present.

It would seem that the 1 to 2 ratio predicted here might be easy to check experimentally, and we looked for this result at a frequency of 15 Mc. While it is true that ρ is certainly greater than one at 15 Mc the requirement that ρ be much greater than one may not yet be too well satisfied.

Our efforts to check these results were carried out in the following manner; a reflector was placed in the tank and the quartz and reflector were adjusted to give the maximum effect for a standing wave pattern. The microphotometer was set to read the intensity of the first order diffraction line. Starting from a zero current reading the current was increased in very small steps until a previously determined value, at which the second order line was known to appear, was reached. For each current reading a microphotometer reading of the light intensity was also made.

The reflector was now carefully removed and a similiar set of readings made for progressive waves. The results obtained were plotted current vs light intensity and the results are shown on the accompanying graphs.



Graph No. 15 Standing and progressive waves in xylene at 15 Mc. Plots show relative intensities of 1st order diffraction line, for low ultrasonic intensities.

While the results shown here have only order of magnitude agreement, it should be pointed out that the light intensity in this region is so small that it was necessary to operate the microphotometer on a more sensitive scale than was the case for the previous work. This had the effect of greatly increasing the noise to signal ratio, and it is felt that we were overreaching the operational limits of our equipment.

Thus it is felt that in general the results do indicate that the predicted 1 to 2 ratio is probably correct, especially when one also recalls that we were forced to work near the limit of the region in which one might expect to find agreement.

Other recent theoretical works included two other papers by Mertens (19, 20) and two by Bhatia and Noble (21, 22). In both cases the intensity distribution due to both progressive and standing waves is treated, and from quite different mathematical approaches. But again, due to our limited frequency range it is not possible to compare our results to the predictions of these papers.

SUMMARY AND CONCLUSIONS

The limits on the usefulness of the Raman-Nath theory are in agreement with the table shown in the introduction.

The usefulness of the Mertens' correction term is, however, somewhat more limited. Regarding frequency there is no lower limit, but at frequencies below two Mc the magnitude of the correction is so small as to be of doubtful value. The size of the correction increases rapidly with increasing frequency and tends to become an over correction at 3.5 - 4 Mc depending upon the value of L. Judging from our experimental results an arbitrary upper limit might be a ρv product of 0.4. In this product v can take on values that follow very closely the Raman-Nath limits. In the frequency range mentioned the best results are obtained for values of ranging between 2 as a lower limit up to 5 or 6 as the upper limit. Outside of this rather limited region the much simpler and more easily applied Raman-Nath theory gives just as acceptable results.

Concerning our equipment we might state that the stability of the light source and the sensitiveness of the microphotometer are still well within their useful limits as far as measurements of this kind are concerned. The limiting factor was the scattered light in the tank and in the medium. For further extensions of this study this source of error must in some manner be reduced. At present there is now in the testing stage in the laboratory a modulated R.F. source, by which it is hoped to decrease the light scattering problem. A frequency sensitive detector tuned to the modulation frequency of the source will be used.

APPENDIX

In constructing the curves presented in this thesis it was necessary to carry out rather extensive mathematical computations. With the view in mind of preserving these, since they would be of some assistance to anyone interested in the intensity distribution of a diffraction pattern in the 1 - 10 Mc range, the following tables have been constructed.

TABLE I
Squared Bessel Functions

In this table are reproduced the square of the Bessel functions of order 0 - 4 with arguments (representing in our work the value of v) chosen in steps of one quarter. These squared Bessel functions represent, according to the simple Raman-Nath theory, the relative intensity of the light in the various orders when the diffraction is produced by progressive waves (see equation 8). They would also be of value in the consideration of standing waves as treated by Raman and Nath. It will also be noted that they are needed in the more detailed work of Mertens (see equation 24).

TABLE I

v	$J_0^2(v)$	$J_1^2(v)$	$J_2^2(v)$	$J_3^2(v)$	$J_4^2(v)$
0.25	.9390	.0154	-	-	-
0.50	.8808	.0587	.0009	-	-
0.75	.7468	.1219	.0045	-	-
1.00	.5855	.1937	.0132	.0004	-
1.25	.4172	.2607	.0293	.0014	-
1.50	.2619	.3113	.0539	.0037	.0001
1.75	.1362	.3366	.0864	.0084	.0004
2.00	.0501	.3326	.1245	.0166	.0012
2.25	.0069	.3007	.1638	.0293	.0027
2.50	.0023	.2471	.1990	.0469	.0054
2.75	.0269	.1815	.2245	.0694	.0101
3.00	.0677	.1150	.2363	.0955	.0174
3.25	.1108	.0581	.2315	.1232	.0279
3.50	.1445	.0189	.2103	.1496	.0418
3.75	.1611	.0011	.1756	.1712	.0590
4.00	.1577	.0044	.1326	.1851	.0790
4.25	.1363	.0242	.0876	.1884	.1004
4.50	.1027	.0534	.0474	.1804	.1214
4.75	.0651	.0836	.0178	.1612	.1397
5.00	.0315	.1073	.0022	.1331	.1530
5.25	.0087	.1190	.0015	.0997	.1594
5.50	.0000	.1166	.0138	.0656	.1574
5.75	.0058	.1011	.0349	.0354	.1466
6.00	.0227	.0766	.0590	.0132	.1279

TABLE II

In the correction of Mertens, equation 24, it is necessary to sum two series. The first of these ψ_{nl} is given by the expression;

$$\psi_{n1}(v) = \frac{v^{n+1}}{6.2^n} \sum_{m=0}^{\infty} \frac{(-1)^m [2m + n(2n+1)]}{2^{2m+1}\, m!\,(m+n)!}\, v^{2m}$$

In this series each successive higher order of m must be multiplied by a different power of v. We reproduce here the multipliers for the zeroth and the first orders.

m	0th ($n = 0$)	1th ($n = 1$)
0	0	$2.41935 \times 10^{-1}\ v^{2}$
1	$-2.50000 \times 10^{-1}\ v^{3}$	$-5.04032 \times 10^{-2}\ v^{4}$
2	$3.12500 \times 10^{-2}\ v^{5}$	$2.94019 \times 10^{-3}\ v^{6}$
3	$-1.30208 \times 10^{-3}\ v^{7}$	$-7.87550 \times 10^{-5}\ v^{8}$
4	$2.71267 \times 10^{-5}\ v^{9}$	$1.20320 \times 10^{-6}\ v^{10}$
5	$-3.39084 \times 10^{-7}\ v^{11}$	$-1.18497 \times 10^{-8}\ v^{12}$
6	$2.82570 \times 10^{-9}\ v^{13}$	$7.59597 \times 10^{-11} v^{14}$
7	$-1.68197 \times 10^{-11} v^{15}$	$-3.84432 \times 10^{-14} v^{16}$
8	$7.50878 \times 10^{-14} v^{17}$	-

TABLE III

Here we are concerned with the second of the two series ψ_{n2} in the correction expression. Where

$$\psi_{n2}(v) = \frac{v^{n+2}}{60.2^n} \sum_{m=1}^{\infty} \frac{(-1)^m \left[6m+(n+1)(18n-7)\right]\left[m+\frac{1}{6}(2n^2+3n-6)\right]}{2^{2m}(m-1)!\,(m+n-1)!}$$

This differs from the previous one in that the multiplying factor involves only a single power of v for each n. We reproduce below the multipliers for orders zero and one. Note that for the zero order the summation starts for m = 2.

m	0th (n = 0)	1th (n = 1)
1	-	-0.03654
2	0.31250	0.01713
3	-0.08593	-0.00147
4	0.00553	0.00005
5	-0.00015	-
sum	0.23195 X v^2	-0.02083 X v^3

TABLE IV
Even Powers of v

v	v^2	v^4	v^6	v^8	v^{10}
0.25	0.0625	0.0039	0.0002	-	-
0.50	0.2500	0.0625	0.0156	0.0039	0.0012
0.75	0.5625	0.3164	0.1780	0.1001	0.0563
1.00	1.0000	1.0000	1.0000	1.0000	1.0000
1.25	1.5625	2.4414	3.8147	5.9604	9.3132
1.50	2.2500	5.0625	11.3906	25.6289	57.6649
1.75	3.0625	9.3789	28.7229	87.9638	269.676
2.00	4.0000	16.0000	64.0000	256.000	1024.00
2.25	5.0625	25.6289	129.746	656.840	3325.25
2.50	6.2500	39.0625	244.141	1525.87	9536.74
2.75	7.5625	57.1914	432.510	3270.85	24735.8
3.00	9.0000	81.0000	729.000	6561.00	59049.0
3.25	10.5625	111.566	1178.42	12447.1	131,472
3.50	12.2500	150.062	1838.27	22518.8	275,855
3.75	14.0625	197.754	2780.91	39106.6	549,936
4.00	16.0000	256.000	4096.00	65536.0	1048576
4.25	18.0625	326.254	5892.96	106,441	1922601
4.50	20.2500	410.062	8303.77	168,151	3405063
4.75	22.5625	509.066	11485.8	259,149	5847040
5.00	25.0000	625.000	15625.0	390,625	9765625
5.25	27.5625	759.691	20938.9	577,131	15907174
5.50	30.2500	915.062	27682.9	837,339	25331610
5.75	33.0625	1093.13	36141.6	1194931	39507399
6.00	36.0000	1296.00	46656.0	1679616	60466176

TABLE IV (cont.)

v	v^{12}	v^{14}	v^{16}
0.25	-	-	-
0.50	0.0002	-	-
0.75	0.0317	0.0178	0.0100
1.00	1.0000	1.0000	1.0000
1.25	14.5519	22.7373	35.5270
1.50	129.746	291.928	656.838
1.75	825.005	2526.57	7737.62
2.00	4096.00	16384.0	65536.0
2.25	16834.1	85226.3	431,458
2.50	59604.6	372,529	2328306
2.75	187,065	1414679	10698510
3.00	531,441	4782970	43046800
3.25	1388673	14667800	154929000
3.50	3379220	41395500	507094000
3.75	7733480	95250200	1339456000
4.00	16777200	268435000	4294970000
4.25	34726900	627256000	11329800000
4.50	68952500	1396290000	28274800000
4.75	131924000	2976530000	67158000000
5.00	244140×10^3	610352×10^4	152588×10^6
5.25	438441×10^3	120834×10^5	333050×10^6
5.50	766345×10^3	231819×10^5	701253×10^6
5.75	130621×10^4	431867×10^5	142786×10^7
6.00	217678×10^4	783642×10^5	282111×10^7

TABLE V
Odd Powers of v

v	v^3	v^5	v^7	v^9	v^{11}
0.25	0.0156	0.0009	-	-	-
0.50	0.1250	0.0312	0.0078	0.0019	0.0004
0.75	0.4219	0.2373	0.1335	0.0751	0.0422
1.00	1.0000	1.0000	1.0000	1.0000	1.0000
1.25	1.9531	3.0517	4.7683	7.4505	11.6414
1.50	3.3750	7.5937	17.0858	38.4430	86.4967
1.75	5.3594	16.4132	50.2654	153.938	471.435
2.00	8.0000	32.0000	128.000	512.000	2048.00
2.25	11.3906	57.6649	291.929	1477.89	7481.81
2.50	15.6250	97.6562	610.351	3814.69	23841.8
2.75	20.7968	157.276	1189.39	8994.76	68022.8
3.00	37.0000	243.000	2187.00	19683.0	177,147
3.25	34.3281	362.591	3829.87	40453.1	427,286
3.50	42.8750	525.219	6433.93	78815.6	965,491
3.75	52.7344	741.577	10428.4	146,649	2062250
4.00	64.0000	1024.00	16384.0	262,144	4194300
4.25	76.7656	1386.58	25045.1	452,377	8171060
4.50	91.1250	1845.28	37366.9	756,680	15322800
4.75	107.172	2418.07	54557.7	1230960	27773500
5.00	125.000	3125.00	78125.0	1953120	48828100
5.25	144.703	3988.37	109,929	3029920	83512200
5.50	166.375	5032.84	152,243	4605350	139312000
5.75	190.109	6285.49	207,814	6870850	227167000
6.00	216.000	7776.00	279,936	10077700	362797000

TABLE V (cont.)

v	v^{13}	v^{15}	v^{17}
0.25	-	-	-
0.50	0.0001	-	-
0.75	0.0249	0.0140	0.0079
1.00	1.0000	1.0000	1.0000
1.25	18.1899	28.4217	44.4089
1.50	194.618	437.890	985.252
1.75	1443.77	4421.55	13541.0
2.00	8192.00	32768.0	131,072
2.25	37876.7	191,751	970,739
2.50	149,011	931,319	5870740
2.75	514,422	3890300	29423900
3.00	1594320	14348900	129140000
3.25	4513210	49670800	524648000
3.50	11827300	144884000	1774830000
3.75	29000400	407818000	5734940000
4.00	67108800	1073740000	17179800000
4.25	147590×10^3	266584×10^4	481517×10^5
4.50	310287×10^3	628331×10^4	127237×10^6
4.75	626640×10^3	141386×10^5	319002×10^6
5.00	122070×10^4	305175×10^5	762937×10^6
5.25	230180×10^4	634434×10^5	174866×10^7
5.50	421419×10^4	127479×10^6	385624×10^7
5.75	751071×10^4	248323×10^6	821018×10^7
6.00	130607×10^5	470185×10^6	169267×10^8

TABLE VI

Here we reproduce the actual correction multipliers, or the term inside the bracket in the equation below equation 23.

$$I_n(v) = J_n^2(v) + \rho^2 \left\{ \left[\psi_{n1}(v) \right]^2 + 2 J_n(v)\, \psi_{n2}(v) \right\}$$

This term when multiplied by ρ^2 gives the correction term that we want. ρ is a function of λ, λ^*, μ_0, and μ and must be obtained for each individual case.

v	0th order	1st order
0.25	0.0285	0.0001
0.50	0.1097	0.0020
0.75	0.2352	0.0085
1.00	0.4033	0.0195
1.25	0.6271	0.0289
1.50	0.9280	0.0245
1.75	1.3132	-0.0098
2.00	1.7460	-0.0831
2.25	2.1209	-0.1892
2.50	2.2724	-0.3006
2.75	2.0184	-0.3691
3.00	1.2421	-0.3359
3.25	-0.0090	-0.1508
3.50	-1.4516	0.2082
3.75	-2.5634	0.7220
4.00	-2.6676	1.2843
4.25	-1.9743	1.8941
4.50	2.4669	2.3481
4.75	7.9875	2.5989
5.00	14.7458	2.6323
5.25	21.5323	2.6404
5.50	26.8667	2.4203
5.75	27.1508	2.6362
6.00	28.7801	3.6764

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