ADAPTING GLASS REINFORCED PLASTICS TO A RIGID FRAME DESIGN

by

Robert Adams Aldrich

AN ABSTRACT

Submitted to the school for Advanced Graduate Studies of
Michigan State University of Agriculture and
Applied Science in partial fulfillment of
the requirements for the degree of

DOCTOR OF PHILOSOPHY

Department of Agricultural Engineering
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Approved _______________

The primary objective of this investigation was to determine the feasibility of using fibrous glass reinforced plastics as a structural material in farm building design.

A review of literature indicated very few instances where glass reinforced plastics have been used to provide both structural member and covering layer for complete structures.

A rigid frame was chosen as the structural form to be used in the investigation. A modified T cross section was adopted to provide depth of section to resist bending moments and provide rigidity. A two foot width of section would allow convenient building construction.

Based on a survey of commercially available plastic resins and fibrous glass reinforcement, a modified epoxy and a polyester resin were used with 10 ounce plain weave glass cloth as reinforcing in the construction of the test structural elements. A structural analysis was completed to estimate stresses and deflections in the frame.

A model analysis of the frame was made to facilitate the investigation. Distorted models were used and prediction factors for stress and deflection established to test the hypotheses. Two models were built of each resin; one essentially a distorted model of the other. The layout scale between the models and the full scale frame was four, that is, the models were one-fourth the size of the full scale frame in layout. With a given resin, one model frame was built full scale in cross section, the other one-half scale.

The two model frames of each resin were tested by applying short term static loads perpendicular to the span through a system of hydraulic cylinders and loading shoes. Loads were applied in increments to establish definite load-deflection and load-stress relationships for each frame. The special test floor permitted testing the models in a hori-
Robert Adams Aldrich

zontal plane. Strain in the outer fiber at midspan was measured by SR-4 strain gages and deflection measured by dial indicators.

Moduli of elasticity and maximum strengths in tension and bending were established for each resin-glass cloth laminate from samples cut from the flange sections of the model frames. The moduli of elasticity in tension were used with the SR-4 strain gage strain readings to relate stress to load on the frame. The moduli of elasticity in bending were used to estimate the deflection of the frames.

Measured stresses and deflections were compared with estimated values for the phenomena for the four model frames to check the analysis of the structure. The prediction factors for stress and deflection for the distorted models were compared to check the validity of the model analysis. Reasonable agreement existed in all comparisons.

From the results of the investigation the following conclusions were made:

(1) Epoxy and polyester resins reinforced with glass cloth produce a laminate with excellent mechanical properties.

(2) The use of structural models permits investigation of hypotheses concerning mechanical phenomena in structures. Distorted models cause no increase in difficulty over true models in the use of model analysis.

(3) Fibrous glass reinforced plastic resins can be used with confidence in farm building design and construction.

(4) The structural test floor with the hydraulic loading system provides a convenient and accurate means for applying static loads to structural elements such as the model frames.

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VITA

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Dissertation: Adapting Glass Reinforced Plastics to a Rigid Frame Design

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INTRODUCTION

Farm buildings have traditionally been constructed of wood, metal, masonry, concrete, or combinations of these materials. This does not necessarily imply that the use of one or more of these materials will always produce the best farm building to fit a specific need. For many farm buildings a substance which combines some of the more favorable characteristics of the traditional materials would help the farmer in his search for better buildings. For example, a material which combines the impermeability of metal, the light weight of wood, and the decay resistance of concrete in addition to adequate mechanical strength and heat control, would be an excellent choice for milkhouse construction. A building composed of reinforced plastics for strength and covering with a foamed plastic for temperature control would display the above characteristics to a marked degree.

This investigation deals with the use of glass reinforced plastics to form structural units for farm buildings. The study may be divided into the following objectives:

1. to examine plastic resins commercially available to determine those suitable for use in structural laminates;

2. to establish a building form which would be acceptable from a management viewpoint and which would be structurally sound;

3. to determine a method of analysis and testing to prove the utility of the material;

4. to make a model analysis and from such analysis predict the behavior of the prototype.

LITERATURE REVIEW

Plastic Resins

Plastics are defined as "a large and varied group of materials which consists of, or contains as an essential ingredient, an organic substance of large molecular weight which, while solid in the finished state, at some stage in its manufacture is made liquid and thus capable of being formed into various shapes, most usually through the application, either singly or together, of heat and pressure"(22). Plastics are subdivided into two general categories, thermoplastic and thermosetting. Laminates and reinforced plastics are those materials in which the plastic, usually in a liquid form, is used as a binder and surfacing material in conjunction with reinforcing materials such as glass mat, glass cloth, paper, etc.

From a review of the thermosetting resins available two were chosen to be used in the study. Epoxy resin which is based generally upon epichlorohydrin and bisphenol-A, and polyester resin. No attempt is made to present the chemical formulations as they are considered to be outside the scope of the investigation.

Epoxy Resin

Commercial resins are a mixture of polymers. They contain both epoxide and hydroxyl groups capable of further reaction. Handling properties such as pot life, strength, chemical resistance, high-temperature strength retention, and electrical properties of the cured resins are directly dependent on the curing agents (29). Present applications of epoxy resins involve use of polyamines, as well as polyamide resins, acid anhydrides, and other resin types. They are generally 100 percent reactive, i.e., no gas or liquid is evolved during the curing process. Compared with other resins, epoxy resin laminates present the optimum mechanical strength properties, excellent thermal ins ulating characteristics, shatter and fungus resistance. They can be formulated to any viscosity and blended with a variety of reinforcing.agents and fillers. They have excellent adhesion qualities (28).

Polyester Resin

A polyester refers to an unsaturated polyester base resin dissolved in a polymerizable monomer. Polyester resins for reinforced plastics come generally from the polyfunctional unsaturated esters such as diallyl phtha late. They are polymerized or copolymerized through the use of peroxides. The base resin component can be prepared from innumerable chemical combinations (26, 27). Basic advantage of these materials is that they start as mobile liquids and can be converted quickly to solids. They are also 100 percent reactive.

Fibrous Glass Reinforcement

Glass fibers make excellent plastics reinforcement because:

1. they have strengths greater than steel,
2. they have flame, heat, and chemical resistance of ceramics,
3. there is no moisture absorption,
4. they are available in almost unlimited quantity,
5. they develop high interface shear strengths with many of the plastic resins.

Fibrous glass reinforcement is available in mat form, as glass cloth in several different weave patterns, and as rovings. Glass mat consists of random lengths of fibers with no particular orientation. Glass cloth has threads in two directions in the weave. Threads running parallel to the length of the cloth are known as "warp" threads. Those running across the cloth and perpendicular to the warp are referred to as "pick" threads. In plain weave the warps and the picks cross alternately. Rovings are produced essentially by winding a number of ends of sliver onto a core. They are not twisted or plied, but sometimes a binder is used to hold them together (10).

Special finishes are applied to glass to improve its wettability with the resins. One such finish is known as #114 in which methacrylate chromic chloride is applied to heat cleaned fabric from an aqueous solution. Volan A is an improved #114 where the chlorine is hydrolyzed off.

According to Brossy, et al (8), individual glass fibers have intrinsic strengths in excess of 500,000 psi for continuous loads and strengths half again as high for loads of short duration. Roughly one-half of this strength is lost by spinning and weaving the fibers. Additional strength is lost in use in glass reinforced plastics because the twisted fibers cannot equally share applied loads. The maximum strength of all the fibers is not realized simultaneously resulting in preferential failure. This accounts for the curved stress-strain curve for composite resin-fiber materials. The character of the glass surface is important in developing bond with plastic resins since all the load must be transferred through this bond by interface shear.

In the work reported by Brossy, et al (8), both epoxy and polyester resins were used with parallel glass fibers. Individual glass filaments were spun onto a rotating drum and sprayed with a binding resin to preserve their relative position. Flexural tests made with samples laminated from these sheets developed dry maximum ultimate strengths of 211,000 psi with special glass. The importance of interfacial bonds, glass surface neutrality and binder resin is established with parallel glass fiber reinforced laminated. This is supported by Trivisonno and Lee (30), who found the strength of adhesive bond between resin and glass can be greatly altered resulting in increased flexural strength of polyester laminates by the use of glass finishing agents.

Glass Reinforced Plastic Resins

Reinforcing Methods

Although there are available and in use several different reinforcing materials the discussion here will be limited to glass fibers since they will produce a laminate with the most desirable mechanical and chemical properties for farm buildings. Used in the mat form, there is random orientation of the fibers which produces an approximately isotropic material. Many of the pressure laminated products use glass mat for strength and filler as it is the least expensive of the forms of glass reinforcing. However, it requires about twice as much glass mat to produce the same strength as with glass in cloth form. The ratio of glass to resin varies from approximately 70% glass-30% resin to 50% glass-50% resin by volume (31).

One method of laminating is to use glass cloth which produces an orthotropic material. Laminates can be produced with no pressure hand-wet layup or with the addition, either singly or together, of heat and pressure. Directional reinforcement can be obtained by laying threads in a predetermined pattern such that maximum strength can be obtained in the direction of expected maximum stress. Donaldson and V elleu (16), developed directionally reinforced plastics with tensile strengths up to 120,000 psi at tension moduli up to 5.9×10^6 psi. They found the epoxies are not sensitive to environmental conditions, abrasions, notch effects, etc. The elastic limits of the laminates are very close to ultimate strengths.

Mechanical Properties

Tests have been conducted which have established a fairly definite range of values for the mechanical properties of glass cloth-reinforced plastics. Typical values for glass cloth-reinforced epoxy and polyester resin laminates are given in Table 1.

TABLE 1

MECHANICAL PROPERTIES OF TWO PLASTIC RESINS REINFORCED WITH GLASS CLOTH[1]

Mechanical Property	Resins	
	Epoxy	Polyester
Maximum strength in bending (psi x 10^{-3})	45-80	50-63
Maximum strength in tension (psi x 10^{-3})	33-50	40-50
Maximum strength in compression (psi x 10^{-3})	50-90	30-60
Modulus of elasticity in bending (psi x 10^{-6})	2.0-3.6	2.0-3.0
Modulus of elasticity in tension (psi x 10^{-6})	2.5-3.5	1.0-2.8

[1]data from Modern Plastics Encyclopedia Issue XXXlll (1955).

Investigations have shown that glass reinforced epoxy and polyester laminates serve exceptionally well over a wide range of temperature and humidity. Lower temperatures generally result in slightly higher static strength properties.

The Forest Products Laboratory, USDA, Madison, Wisconsin, has carried on an extensive program of testing and evaluation of plastic laminates. Werren (31, 32, 33, 34), presents the results of a number of these tests on polyester, epoxy and phenolic resin laminates. Values obtained for the mechanical properties substantiate those given in Table 1.

Della Rocca (12), reports that fiber glass reinforced polyester resin laminates exhibit very little creep at room temperatures. In relating thickness of laminate to strength it appears that one-eighth inch thickness gives the optimum strength properties. He considers reinforcement and molding method to be of primary importance to laminate properties and variations due to resin formulation, humidity, etc., as being secondary since their effect, while important, can be minimized with proper quality control.

Fatigue Characteristics

Although cyclic loading is generally not considered in farm building design, it is necessary to have some knowledge of fatigue characteristics of a new material before complete acceptance for design and construction purposes.

Using $5x10^6$ cycles as criterion for fatigue strength in flexure, Fried (17), found that the ratio of fatigue strength to ultimate static strength ranged from 0.20 to 0.30 for polyester resin with laminates using a combination of glass cloth and mat. No endurance limit was reached up to 10^7 cycles although the data appeared to be approaching an asymptote.

For fatigue tests, Nara (21), used rectangular cross section cantilever beams of both epoxy and polyester resin reinforced with longitudinal fibers continuous over the length of the beam. Failure was in shear in the form of delamination beginning at the neutral axis. He plotted shear stress versus the logarithm of the cycles and noted that the 'knee' of the S-N curve appeared between 10^4 to 10^5 cycles. Using the knee as a fatigue strength value, the ratio of fatigue strength to ultimate static strength is approximately 20 percent for polyester resin laminates and 32 percent for epoxy laminates.

Boller (7) determined fatigue strength based on axial loading. At a fatigue strength for ten million cycles the ratio of fatigue strength to ultimate static strength ranged from 20 to 25 percent for polyester laminates to 40 percent for epoxy laminates. Both were room temperature curing resins. He found that moisture effects the fatigue strength of standard polyester resins only slightly (2 percent of static tensile strength at ten million cycles); for epoxy resin laminates the change is insignificant. Both materials show a decrease in static and fatigue strengths at increased temperature.

In flexural fatigue tests of 32 different polyester, epoxy, and phenolic resins laminated with either glass cloth or glass mat, Pusey (24), found that the polyesters offer the weakest fatigue strengths. The choice of resin is significant within the epoxy group but choice of hardener within the same group has no significant effect. He also noted that glass cloth produces stronger laminates than glass mat and that cloth finish effects fatigue strength. Using the fatigue strength at ten million cycles, the ratio of fatigue strength of all laminates to static flexural strength is 24 percent.

Present Use of Reinforced Plastics

One of the many applications of reinforced plastics is in sandwich construction which consists generally of a low density core with glass reinforced plastic surfaces. Panels built in this manner are used primarily as non load bearing walls in structures, as refrigerator walls and other installations where their excellent insulating qualities are in demand and as panels in aircraft, truck trailer and boat hull design (11, 12).

There has been extensive use of reinforced plastic panels for lighting and decorative purposes in home and commercial construction. Recent uses of these panels or for similar laminates include; panels for geodesic dome construction, small boat design, and radome construction. Epoxy resin laminates are being used in Keller model construction for the automotive tool and die industry (19).

A 'new' house co-sponsored by MIT and Monsanto Chemical Company has been built in which the floor sections are hollow girders consisting of an outer molded portion and an inner flat section. Woven roving reinforced polyester resin laminate is used for the surface skins. Dietz, et al (14), in reporting the project, indicate a working strength of 50 percent of ultimate static strength was used in design of the house components. Since the design was fairly comples, full scale prototypes of portions of the house were built and tested. Full design load held on the floor for six weeks produced no appreciable creep. The structure reacted as design calculations had predicted.

Glass reinforced epoxy resin parabolic reflectors show no creep or distortion after three years service (20). Successful full scale tests on glass reinforced polyester resin spherical radomes of 21, 31, and 55 feet equatorial diameter have been conducted and the radomes are now in service under arctic conditions. The 21 foot radome is molded, the 31 foot and 55 foot radomes are built of stiffened triangular panels. The structures reacted within the acceptance limits established from structural analysis and preliminary tests (11).

Frame Analysis

Most texts on indeterminate structures present one or more methods for analyzing rigid frames. In all cases use is made of the displacement characteristics of the structure to provide the equations necessary for solution. The most basic method is that known as Virtual Work or Elastic Energy. In frame analysis, when the deformation is primarily flexural, the energy due to shear and axial stresses is generally ignored as its contribution is a very small percentage of the total elastic energy of the structure (4, 5).

Model Analysis

A model is a device which is so related to a physical system that observations on the model may be used to predict the future performance of the physical system in a given respect (2). In many engineering problems it becomes advantageous, even necessary at times, to use models to predict performance of a physical system. Accurate predictions can be made if the principles of model theory and similitude are employed correctly.

The Prediction Equation

A dependable prediction equation can best be obtained through the use of dimensional analysis. The development of the idea of dimensiohal analysis has been attributed generally to Buckingham and

Bridgeman. The theorem is developed from a consideration of the dimensions in which each of the pertinent quantities in a phenomena exist.

In general, given a set of physical quantities which describe a phenomena, a set of dimensionless and independent terms can be established by a combination of dimensional analysis and the Buckingham Pi theorem. The Buckingham Pi theorem states that the number of dimensionless and independent quantities required to express a relationship among the variables in any phenomena is equal to the number of quantities involved minus the number of fundamental dimensions in which these quantities may be expressed. In equation form: $r = n - a$, where r is the number of Pi terms, n is the number of quantities and a is the number of fundamental dimensions involved. A problem is generally set up such that one Pi term is expressed as a function of the remaining terms, i.e., $\pi_1 = F(\pi_2, \pi_3, \cdots \pi_r)$. The only restrictions placed on the Pi terms are that they be dimensionless and independent. A general prediction equation involves determining the relationship between the Pi terms as expressed above.

A general theory of models can be developed by extension of the equation given above. The equation for the prototype is

$$\pi_1 = F(\pi_2, \pi_3, \cdots \pi_r) \tag{1}$$

Since this equation is perfectly general it can be applied to any other similar physical system. Hence it applies in particular to a model of the prototype. Thus

$$\pi_{1m} = F(\pi_{2m}, \pi_{3m}, \cdots \pi_{rm}) \tag{2}$$

An equation for predicting π_1 from π_{1m} can be most simply obtained by dividing equation (1) by equation (2).

$$\frac{\pi_1}{\pi_{1m}} = \frac{F(\pi_2, \pi_3, \cdots \pi_r)}{F(\pi_{2m}, \pi_{3m}, \cdots \pi_{rm})}$$

Now, if the model is designed such that

$$\pi_2 = \pi_{2m}, \quad \pi_3 = \pi_{3m}, \quad \cdots \quad \pi_r = \pi_{rm} \tag{3}$$

then

$$F(\pi_2, \pi_3, \cdots \pi_r) = F(\pi_{2m}, \pi_{3m}, \cdots \pi_{rm}) \tag{4}$$

Therefore

$$\pi_1 = \pi_{1m} \tag{5}$$

Equation (5) becomes the prediction equation for the system and must be valid if equations (3) (known as the design and operating conditions) are satisfied.

If all the design conditions are satisfied, the model is said to be a 'true' model. If all the design conditions are not satisfied, the behavior of the model may be distorted with reference to the factors included in the corresponding Pi terms and the prediction equation may be affected materially.

In general, the design conditions will involve distances indicative of the size of model and prototype. The ratio of a given distance on the prototype to the corresponding distance on the model is known as the length scale and is designated as n. $L = nL_m$.

Structural models

Structural models are generally constructed for the purpose of studying behavior under load. The behavior is then evaluated in terms of resistance to failure. Therefore, the problem is one of evaluating deflection and stress.

Deflections and stresses are functions of (1) geometry, (2) loads and restraints, and (3) properties of the material. The effect of two resultant forces on a short length of a structural member is one or more of the following:

1. axial tension or compression,
2. cross shear,
3. torsion,
4. bending.

The response of the member to the action of these forces is a function of the geometry and properties of the material. For example, if a member is subjected to bending, the principal moments of inertia and distances from the neutral axis are the important geometric characteristics, while the moduli of elasticity and some measure of flexural strength are the required properties of material.

A true structural model is expected to give accurate information about stress and deformation.

Frequently it is not possible to satisfy all the design conditions for a true model. A model so designed that one or more of the design or operating conditions is not satisfied is known as a distorted model. Distortion may exist in any one or a combination of the three principal factors involved in structural analysis, vis., statics, geometry, and properties of the material. The prediction equation for a distorted model may be established by determining a prediction factor δ such that

$$\pi_1 = \delta \pi_{1m}$$

Murphy (2) gives prediction factors for stress and deflection based on the four basic types of loading. The prediction factor, δ , is given in terms of a distortion factor α . Values of δ are given for exact, partial, and no similarity of cross section between model and prototype.

THE INVESTIGATION

Preliminary Consideration

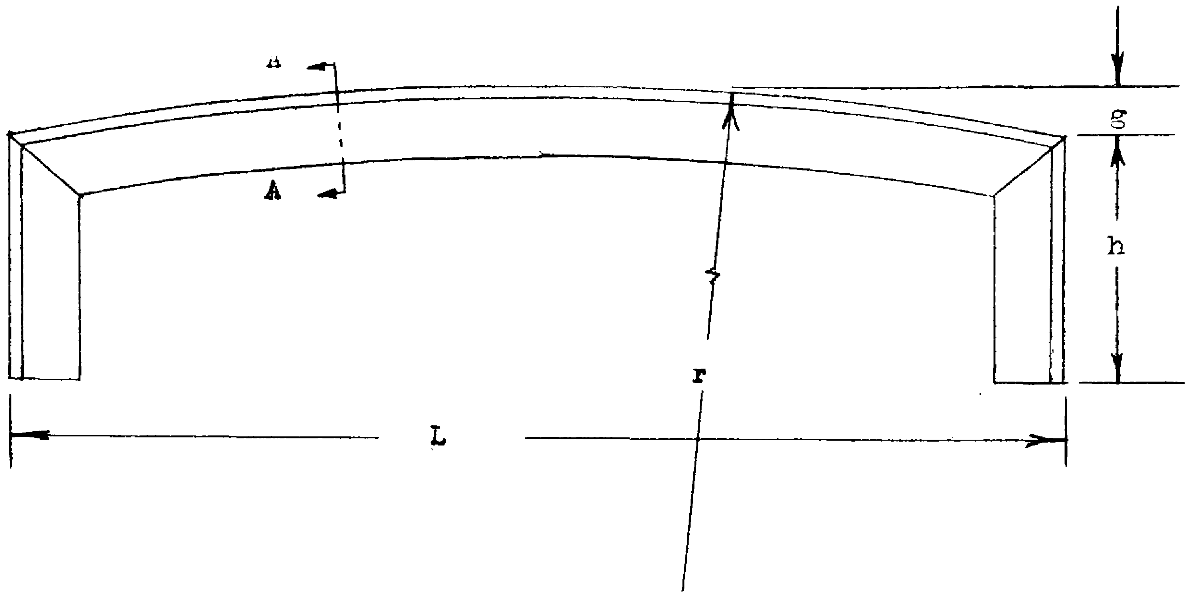
The review of literature indicates use of reinforced plastics in many areas with a great deal of success. However, with a few exceptions, their use in building construction has been limited to corrugated or flat sheets as walls and roof panels for light and/or architectural effect, or as skins in sandwich construction.

As a guide to a final choice of structural shape and cross-section, the following points were considered; The form should

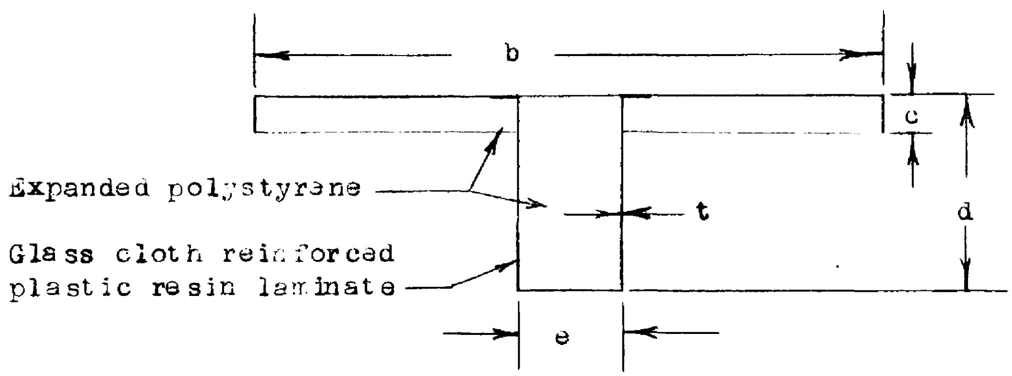
1. provide clear span for optimum floor and space utilization,
2. permit prefabrication,
3. provide skin and frame as an integral unit, and
4. allow convenient building expansion.

Using the guide points listed, a rigid frame was designed as shown in Figure 1. The forty foot span was chosen as being maximum for farm buildings. Ten foot sidewalls should be ample for most applications. A long radius was used to hold head room to a minimum and still allow water to move off the surface. The frame can be erected on concrete pad footings with simple anchor straps and backfill providing adequate foundation support.

The modified T cross-section permits the construction of the skin and frame as an integral unit. The adhesive qualities of the resins provide a strong weld between stem and flange, permitting the section to act as a unit. Expanded polystyrene acts in the dual role of stiffener and insulator. The 24 inch width of section will allow building



L A Y O U T



S E C T I O N A-A

Figure 1. Layout and cross-section of experimental rigid frame of glass cloth reinforced plastic resin laminates and expanded polystyrene. See Table 2 for dimensions of prototype and models.

erection in increments. Adjacent units can be fastened together either by use of a resin and glass cloth batten on the joint or by using mastic in the joint and fasteners (bolts, rivets, etc.) at intervals along the common edge. The resin-cloth batten would provide a permanent water-tight joint.

Frame Analysis

The frame analysis was made using the method of Virtual Work. Figure 2 is a definition sketch for establishing geometric relationships used in the analysis.

The frame is indeterminate to the first degree so the horizontal reaction component at A is chosen as the redundant. The problem becomes one of determining X_A such that the horizontal displacement at A is equal to zero.

With a virtual unit load acting horizontally at A in the direction of X_A, Figure 3c, the displacement due to the uniform load 'w' plus the displacement due to the unit load, is made equal to zero.

$$\int (mM_w/EI)ds + X_A \int m^2 ds/EI = 0 \tag{1}$$

in which the integrals are evaluated over the entire frame.

Referring to Figures 3c and 3d, the following moment equations apply:

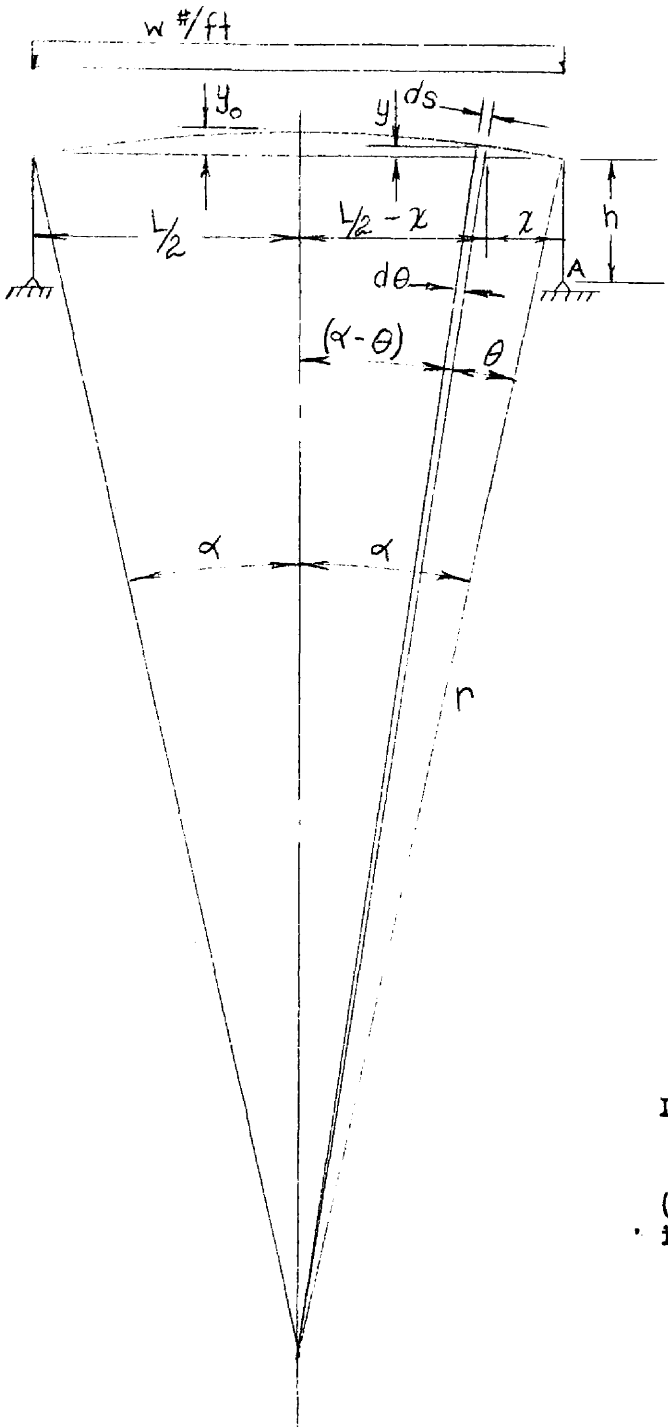
For the columns in the reduced structure;

$$M_w = 0, \quad m = y.$$

For the beam, from the end to the center; ($x = 0$ to $x = L/2$);

$$M_w = -wLx/2 + wx^2/2, \quad m = h + y.$$

Therefore, the integrals above become;



x = distance from the outside of the leg toward the center of the span.

$x = L/2 - r \sin\alpha \cos\theta + r \cos\alpha \sin\theta$

y = distance from the top of the column.

$y = r \cos\alpha \cos\theta + r \sin\alpha \sin\theta + D$

r = radius of curvature of the beam.

$D = y_o - r$

y_o = total rise of the roof.

h = height of the column.

α = one-half the central angle.

θ = variable angle.

$d\theta$ = increment in θ.

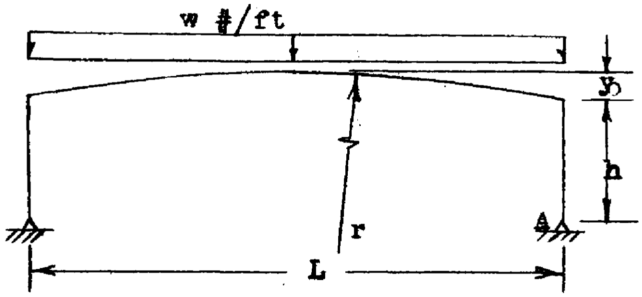
$ds = r\, d\theta$, arc length along the beam.

w = intensity of the uniformly distributed load on the beam in #/ft of beam length.

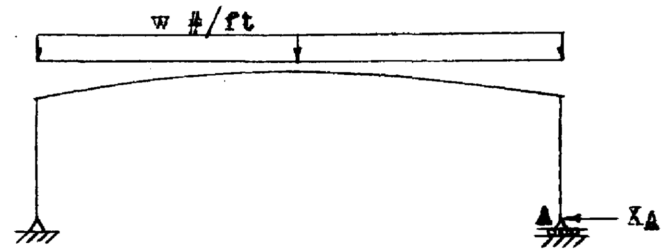
$L/2$ = one-half the span.

(All lengths are in feet, angles in radians)

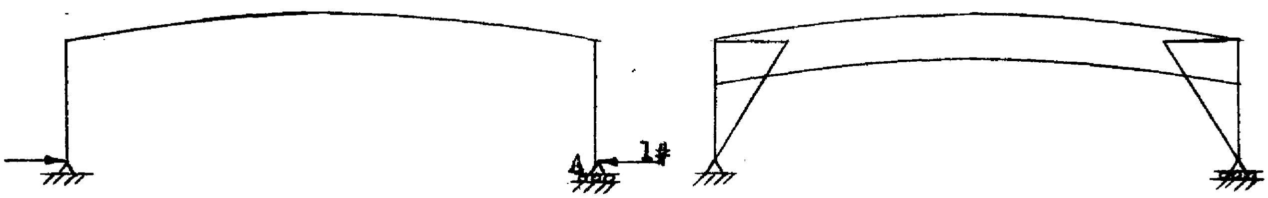
Figure 2. Definition sketch for the frame analysis.



Original structure
(a)

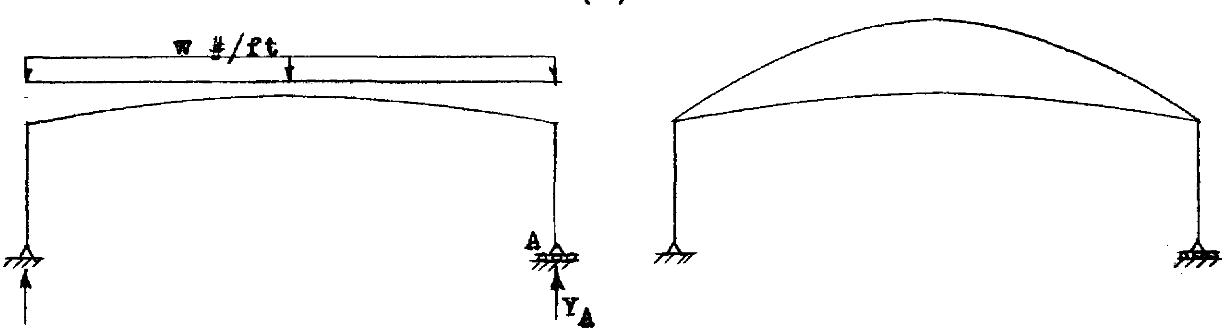


Reduced structure
(b)



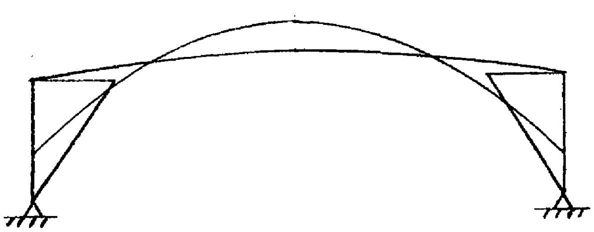
Moment distribution on the reduced structure due to a unit load applied at A.

(c)



Moment distribution on the reduced structure due to a uniformly distributed load.

(d)



Final moment distribution on the frame.

(e)

Figure 3. Load and moment diagrams for solution of the rigid frame.

$$2rw/EI \int_0^{\alpha} (-Lhx/2 + hx^2/2 - Lyx/2 + x^2y/2)d\theta + 2X_A/EI \int_0^{h} y^2 dy + 2X_A r/EI \int_0^{\alpha} (h^2 + 2hy + y^2)d\theta = 0 \qquad (2)$$

Substituting the proper expressions for x, y, h, and r (given in Figure 2 and Table 2) into equation (2) and performing the integration,

$$X_A = 11.2w\# \qquad (3)$$

The moment at the junction of beam and column is thus 112w#-ft, and at the crown, -65.6w#-ft. The maximum stress will be at the joint; therefore, with a design load of 60#/ft of beam and using the cross-section shown in Figure 1, (dimensions from Table 2), the stress in the outer fibers of the stem will be 12,500 psi. This is well below the strength values listed in Table 1 as typical for glass reinforced plastics. Assuming maximum compressive strength of 30,000 psi, this would provide a stress factor of safety of 2.5. The final moment distribution for the frame is shown in Figure 3e. In a frame such as this one, the energy due to shear and axial loading is very small compared to bending energy; therefore, their effects were ignored in the analysis.

From the preliminary analysis it became apparent that the construction of 40 foot span frames would require considerable amounts of materials and represented a large investment in funds for materials and construction labor. Therefore, to reduce the cost for test frames, a model analysis was completed which allowed testing the hypothesis at less expense even though two models were required to validate the analysis.

TABLE 2

LAYOUT AND CROSS-SECTION DIMENSIONS OF
THE FRAME AND THE MODELS

Structure	Dimension										
	L (ft)	g (ft)	h (ft)	r (ft)	b (in)	c (in)	d (in)	e (in)	t (in)	I (in^4)	c (in)
Frame	40	2	10	100	24	1.5	8	4	0.125	40	6.2
Model											
E025050	10	0.5	2.5	25	12	0.75	4.38	2	(1)	3.03	3.38
E025100	10	0.5	2.5	25	24	1.5	8	4		35.90	5.55
P025050	10	0.5	2.5	25	12	0.75	3.75	2.13		4.13	2.82
P025100	10	0.5	2.5	25	24	1.5	7.70	4.19		44.29	5.66

(1)See Figure 4 for laminate thickness of the models.

Model Analysis

The model analysis involves choice of layout and cross-section scales, material to be used, and developing prediction equations for the desired phenomenon.

The first step is to develop prediction equations for stress and deflection of the models, the two factors which best describe the action of a structural member under load. This is done using the development presented in the literature review.

Prediction Equation for Stress

The variables and their respective dimensions to be considered are:

Variables		Dimensions
s =	stress	$F-L^{-2}$
L =	length of span	L
λ =	any length (layout)	L
η =	any length (cross-section)	L
w =	load/foot of beam	$F-L^{-1}$
E =	modulus of elasticity in tension	$F-L^{-2}$

From this $s = f(L, \lambda, \eta, w, E)$

There are six variables and two fundamental quantities; therefore, there will be $6 - 2 = 4$ Pi terms.

$$\pi_1 = sL/w, \quad \pi_2 = \lambda/L, \quad \pi_3 = \eta/L, \quad \pi_4 = w/EL$$

or

$$\pi_1 = f(\pi_2, \pi_3, \pi_4)$$

The same terms apply to the model.

$$\pi_{1m} = s_m L_m/w_m, \quad \pi_{2m} = \lambda_m/L_m, \quad \pi_{3m} = \eta_m/L_m, \quad \pi_{4m} = w_m/E_m L_m$$

and

$$\pi_{1m} = f(\pi_{2m}, \pi_{3m}, \pi_{4m})$$

The design conditions become:

$$\lambda/L = \lambda_m/L_m, \quad \eta/L = \eta_m/L_m$$

The operating conditions are:

$$w/EL = w_m/E_m L_m$$

And the prediction equation for stress is

$$sL/w = s_m L_m/w_m$$

Prediction Equation for Deflection

Variables	Dimensions
d = deflection	L
L = length of span	L
λ = any length (layout)	L
η = any length (cross-section)	L
w = load/foot of beam	$F\text{-}L^{-1}$
E = modulus of elasticity in bending	$F\text{-}L^{-2}$

From this $d = f(L, \lambda, \eta, w, E)$

Again there are six variables and two fundamental quantities; therefore, there are $6 - 2 = 4$ Pi terms.

$$\pi_1 = d/L, \quad \pi_2 = \lambda/L, \quad \pi_3 = \eta/L, \quad \pi_4 = w/EL$$

and for the model,

$$\pi_{1m} = d_m/L_m, \quad \pi_{2m} = \lambda_m/L_m, \quad \pi_{3m} = \eta_m/L_m, \quad \pi_{4m} = w_m/E_mL_m$$

The design conditions are:

$$\lambda/L = \lambda_m/L_m, \qquad \eta/L = \eta_m/L_m$$

The operating conditions are;

$$w/EL = w_m/E_mL_m$$

And the prediction equation for deflection is

$$d/L = d_m/L_m$$

The design conditions indicate that the model and prototype should be geometrically similar and the operating conditions give the load relationship.

A length scale of four was used, or $L = 4L_m$. This scale was satisfactory for layout but if used for the cross-section would result in a very thin skin which would cause difficulty, both in construction and in behavior under load. Therefore, distorted models were used, that is, the cross-section scale differed from the layout scale.

When a distorted model is used, there have to be two models; essentially one a distorted model of the other. This is necessary in order to 'prove' the model. If the prediction equations for the distorted models are proven correct, then the information secured from the tests can be applied with confidence to the full scale structure.

For the distorted model, the design conditions are;

$$\lambda/L = \lambda_m/L_m, \qquad \beta\eta/L = \eta_m/L_m$$

where β is the distortion factor.

From Murphy (2), the prediction factors for stress and deflection, considering bending type loads, are; for displacement, α^4, for stress, α^3. α is defined $I_m = \alpha^4 I / n^4$, n is the length scale, I is the moment of inertia of the cross-section.

Therefore, the prediction equations become

$$(sL/w)/(s_m L_m/w_m) = \alpha^3; \ (d/L)/(d_m/L_m) = \alpha^4.$$

Referring to the operating conditions,

$$w/EL = w_m/E_m L_m,$$

if the same material is used in model and prototype, then

$$w/w_m = EL/E_m L_m = L/L_m = n.$$

From the prediction equation for stress

$$(s/s_m)(L/L_m)(w_m/w) = \alpha^3, \qquad s = \alpha^3 s_m.$$

From the prediction equation for deflection

$$(d/d_m)(L_m/L) = \alpha^4, \quad d = n\ \alpha^4 d_m.$$

The relationship between model E025050[1] and the full scale frame

---

[1] The model frame designation. E indicates epoxy resin, the first 3 digits indicate layout scale, (e.g., 050 = one-half scale), the last 3 digits indicate cross-section scale, (e.g., 050 = one-half scale).

$n = L/L_m = 4, \qquad \eta = 2\eta_m$

$\alpha^4 = (4)^4(\frac{1}{2})^4 = 16$

$\alpha = 2$

Thus:

$s = 8s_m, \quad d = 64d_m$

The relationship between model E025100 and the full scale frame

$n = 4, \qquad \eta = \eta_m$

$\alpha^4 = (4)^4(1)^4 = 256$

$\alpha = 4$

Thus:

$s = 64s_m, \quad d = 1024d_m$

Frame Analysis for The Models

The general equations developed for the analysis of the full scale frame also apply to the models. Therefore, the expressions for L, h, x, y, and r for the model as indicated in Figure 2 and Table 2, were substituted in equation (2) giving

$$X_A = 2.7w \ \#$$

Thus the moment at midspan for the model is

$$X_A(h + g) - wL^2/8 = -4.4w \ \#\text{-ft}$$

The stress at midspan is

$$s = Mc/I = -52.8wc/I \ \#/in^2$$

The deflection at the crown (midspan) was estimated by the method of Virtual Work. A unit load was assumed to act at the crown in the direction of the expected displacement. The frame is indeterminant to the first degree so again the horizontal reaction at A was assumed as the redundant to take advantage of the previous analysis. The value of

X_A determined due to a unit vertical load acting at the crown is 0.445#.

The equation for deflection at the crown is

$$d_c = 2\int_0^h (mM_w/EI)ds + 2\int_0^{L/2} (mM_w/EI)ds$$

From 0 to h, $M_w = 2.7y$, $m = 0.445y$, $ds = dy$.

From 0 to L/2, $M_w = 2.7wh + 2.7wy - (wL/2)x + (w/2)x^2$,

$m = 0.445h + 0.445y - 0.5x$,

$ds = rd\theta$.

Carrying out the indicated operations, substituting expressions for x, y and ds as indicated in Figure 2 and given in Table 2, the deflection at the crown is

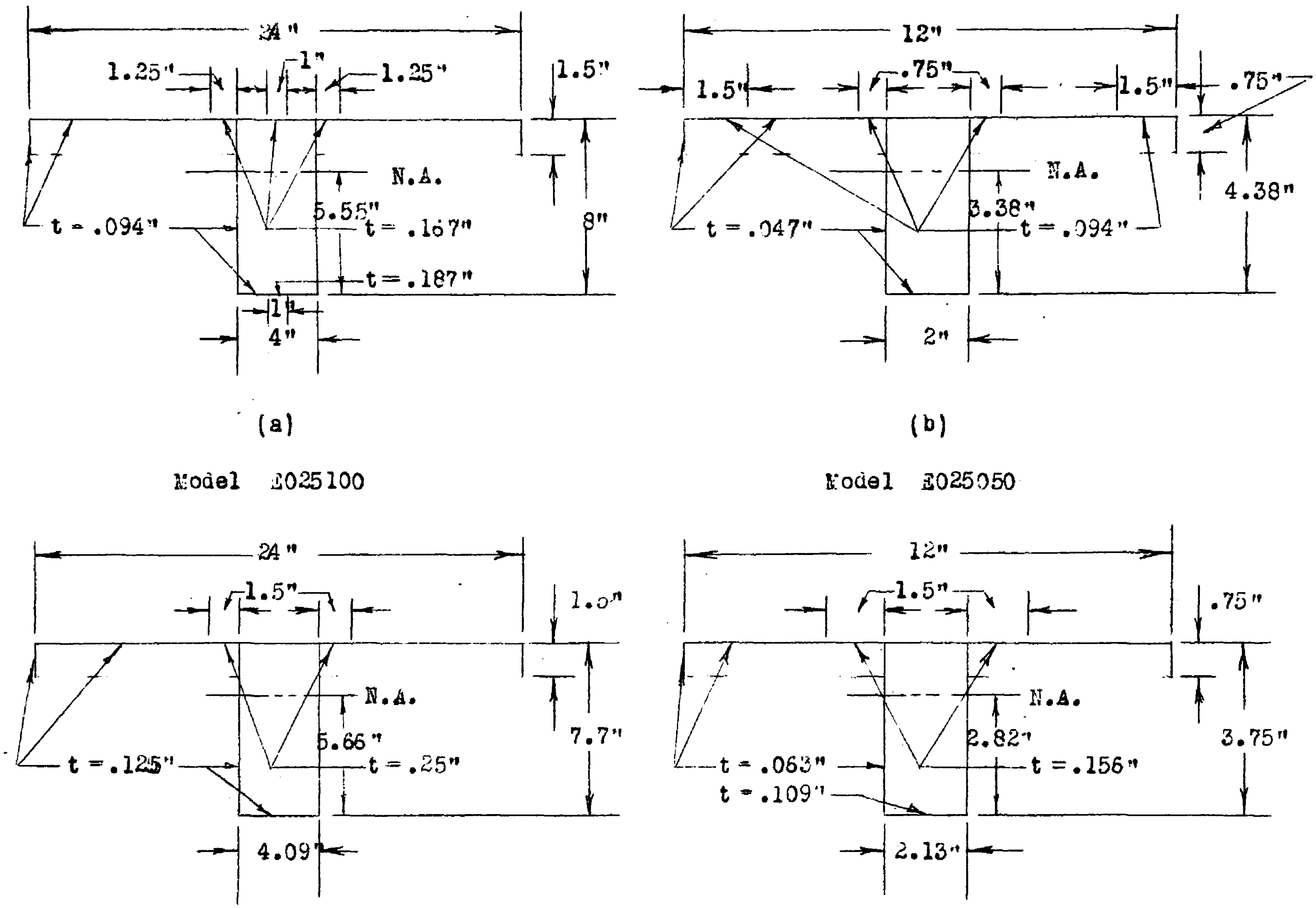
$$d_c = (0.12 \times 10^6)w/EI$$

The relationship between the models built with a given resin, using the cross section dimensions for the four models given in Table 2, and Figure 4, is given in Table 3. In the following discussion, the frame with the larger cross-section dimensions is considered the model, the other is the prototype.

TABLE 3

MODEL RELATIONSHIPS FOR THE DISTORTED MODELS

Epoxy resin	Polyester resin
$n = 1$	$n = 1$
$\alpha = 1.86$	$\alpha = 1.81$
$s = 6.47s_m$	$s = 5.96s_m$
$d = 11.83d_m$	$d = 10.8d_m$



(a) Model E025100

(b) Model E025050

(c) Model P025100

(d) Model P025050

Figure 4. Cross-section dimensions for the four model frames used in the study. See Table 2 for layout dimensions.

Experimental Investigation

Test Facilities

The first stage of the experimental work involved the construction of the test facilities. To facilitate testing, a special floor was constructed so that the frames could be loaded in a horizontal plane through a system of hydraulic cylinders and loading shoes. The floor as constructed is based on plans for a similar floor in use at Purdue University. Minor changes were made to fit the availability of space and materials. Figure 5 is a schematic diagram of the hydraulic system used to apply the loads. Rails built up from I beams are imbedded in the floor to provide reaction supports. Steel concrete inserts are set in a grid pattern in the floor to provide anchors for the hydraulic cylinders. Figure 7 shows model E025050 under test. The reaction braces are tied together, in addition to being bolted to the reaction rails, to resist the horizontal thrust of the frame. The whiffle tree arrangement of the loading shoes allows loading in a plane perpendicular to the span. The individual loading shoes are 6 inches on center; equivalent to 2 foot on center loading on the full scale frame. This is an acceptable approximation to a uniformly distributed load. The reaction braces are built heavy enough that there will be no displacement other than the deflection of the frame under test.

Model Construction

The model frames were constructed using a hand-wet-layup process. Wood female molds were built for the flange construction. After construction of the molds for the epoxy resin laminate, they were first given a coat of Johnson's paste wax. Then a layer of form release agent (a proprietary mix) was applied, allowed to dry, and a second

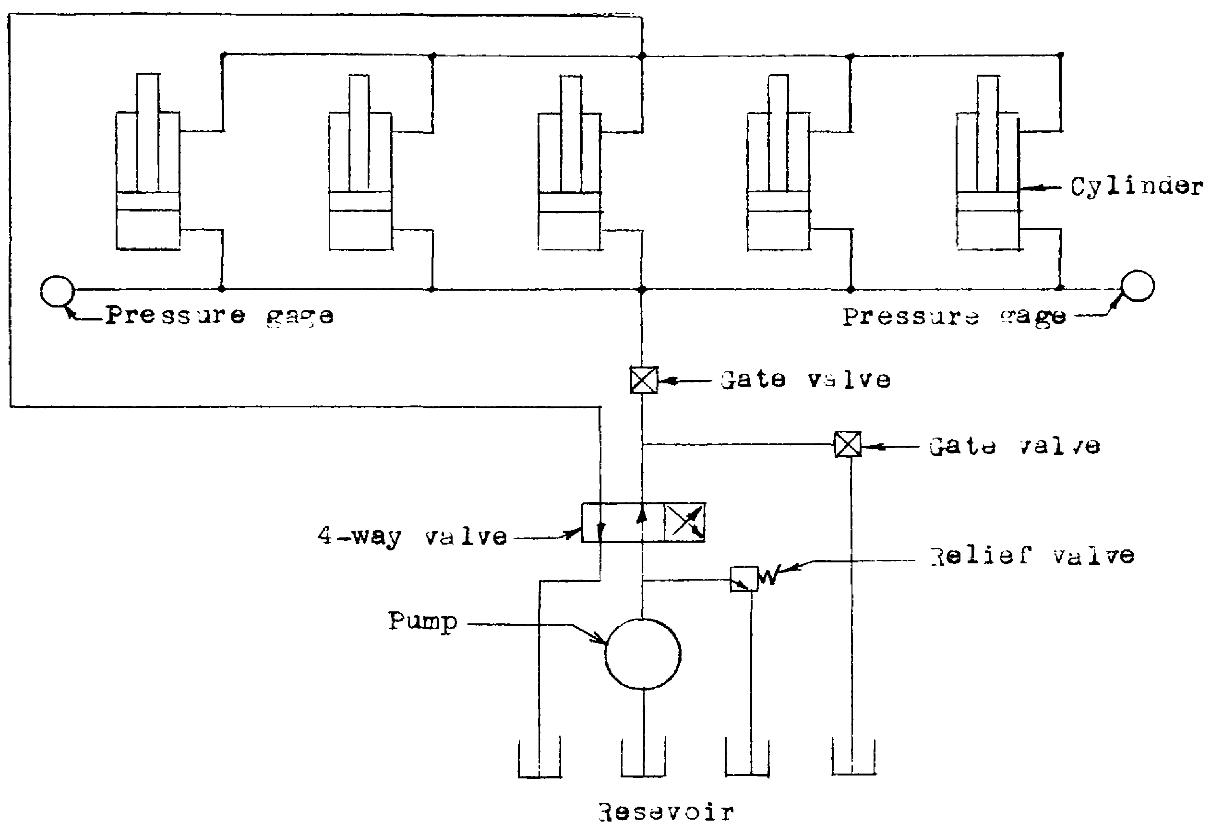


Figure 5. Schematic diagram of hydraulic system used in load tests of the model frames.

coat of Johnson's paste wax applied to complete the mold preparation.

The modified epoxy resin consisted of a laminating mix and hardener which were mixed and applied with a small paint brush. Since the pot life after mixing was only 30 minutes at 70°F, the resin was mixed in one pint batches and applied. Figure 6 shows the molds and materials used to build the flange sections of model E025050.

The laminating process consists of the following steps: (1) apply a coat of resin to the mold, (2) place a layer of glass cloth in the mold and work out all the air bubbles with a flat paddle, (this also assures adequate wetting), (3) apply another layer of resin, (4) place another layer of glass cloth, (5) apply a layer of resin. Continue the process until the desired number of laminations is obtained. As recommended by the manufacturer of the resin, an attempt was made to produce a laminate approximately 50% resin-50% cloth by volume. The resin reaches its peak exotherm a few minutes before beginning to gel. At this time it has the consistency of water so there is danger of resin starving if the laminating process is carried on too rapidly. This occurred in both of the epoxy resin models. Model E025050, being the first to be built, suffered the most. The resin cures to a hard mass in approximately $3\frac{1}{2}$ hours at 70°F and reaches full strength in about 12 hours. Precautions must be observed in handling epoxy resin as resin contact with the skin causes irritation and mild dermatitis.

In construction of the models, as soon as the flange was built up to the desired number laminations, the expanded polystyrene core for the stem was set in position and the skin laminated in place on it. Epoxy and polystyrene are non reactive so this could be done with ease. In fact, epoxy resin is a natural adhesive and produces a strong bond with polystyrene. The stem laminate was bonded directly to the flange

Figure 6. Materials used in laminating with epoxy resin. (1) female mold, (2) glass cloth, (3) modified epoxy resin, (4) resin hardener.

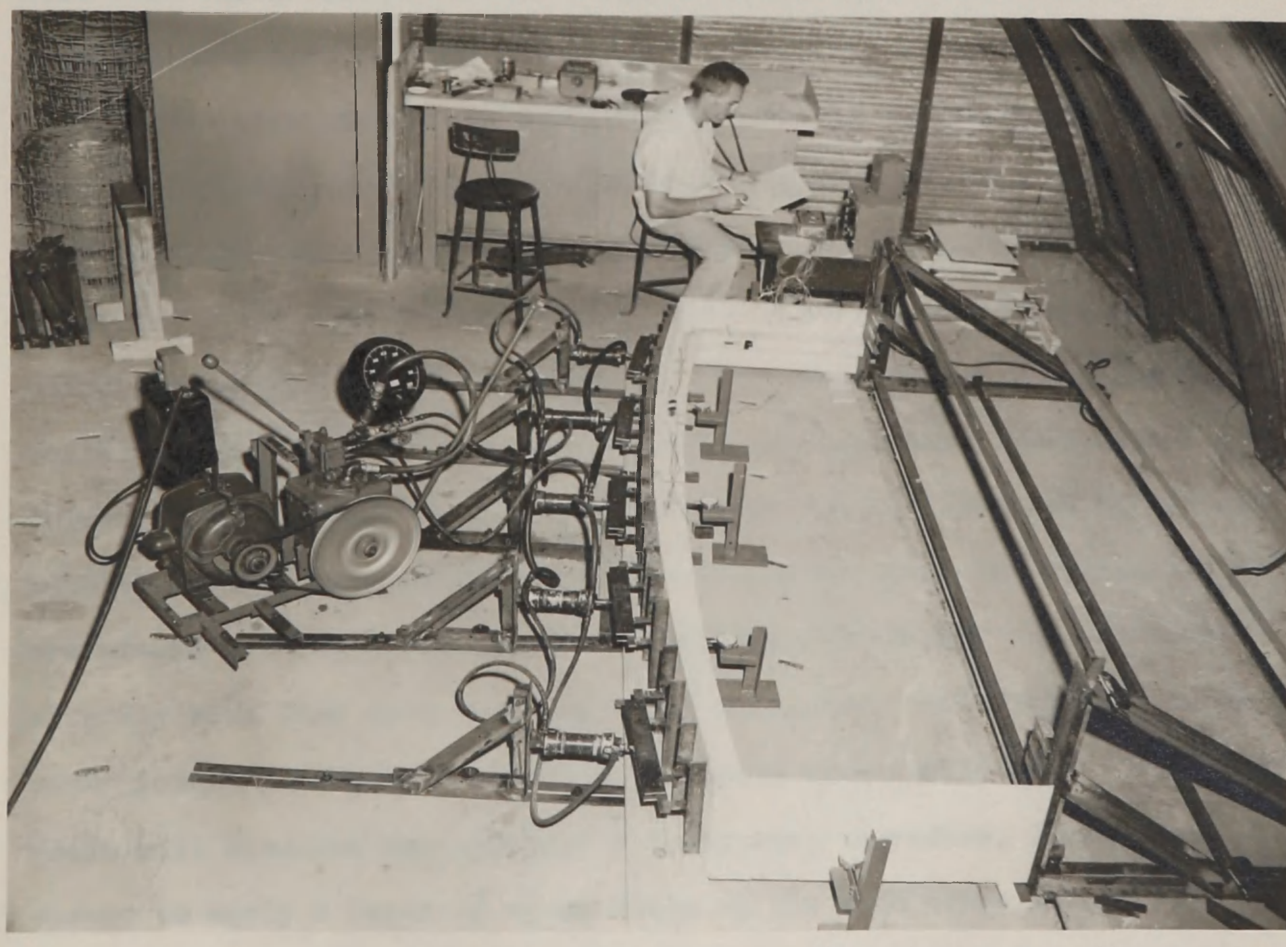


Figure 7. Model E025050 under short-term static test in the structures test laboratory.

laminate so the polystyrene serves only as a stiffener and form in the core. The polyestyrene board was set in place in the flange while the resin was still liquid to allow a bond to be produced to hold it securely in position.

The beam and columns were laid up separately, with an extra two foot section of the column flange produced from which samples were cut to provide data for establishing material constants.

After the beam and columns were cured, miter cuts were made on the ends of the beam and the upper end of the columns and the two parts joined by applying a resin-cloth scab to the joint along the stem and over the outside of the flange. A gusset plate was set in the joint to allow the entire width of the flange to carry load around the corner from the beam into the column.

The construction of the polyester models was essentially the same as for the epoxy resin except for the resin mixture. Figure 8 shows the various stages in the construction of model P025050. The polyester resin preparation involves more materials and thus, from that point, is not as convenient to use as the epoxy; however, it is non toxic to the skin. Another advantage to the particular formulation of polyester which was used is its translucency which allows one to see air bubbles in the laminate and therefore work them out. In welding the two pieces of cured laminate it is necessary to roughen the surfaces slightly with fine sand paper or emery cloth and clean them thoroughly with alcohol in order to insure development of adequate bond. Polyester resin will dissolve the expanded polystyrene; therefore, it was necessary to apply a layer of epoxy resin on the stem cores before covering them with the polyester resin-glass cloth laminate.

In all laminating work, care must be exercised to maintain clean

Figure 8. Materials and procedures used in construction of model P025050.

a. Materials used in laminating with polyester resin. (1) female mold, (2) glass cloth, (3) surface resin, (4) interior resin, (5) thixotropic agent, (6) cobalt napthanate accelerator, (7) MEK peroxide catalyst.

b. Finished flanges before removing from the mold.

c. Completed beam and column sections removed from the mold. The core of the stem is expanded polystyrene.

d. Frame sections with miter cuts fitted and held in position for welding.

e. Completed model P025050 under long-term load test.

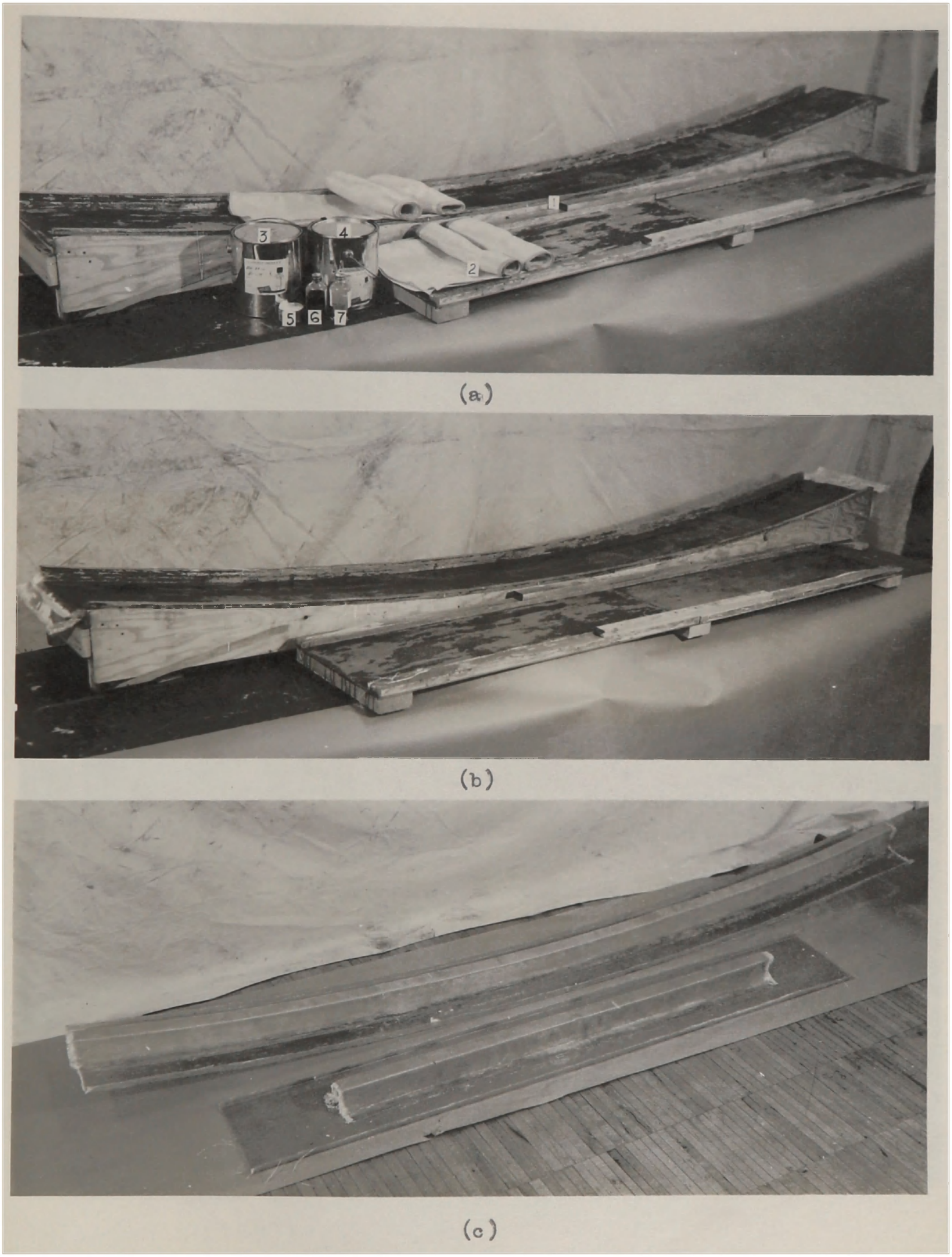
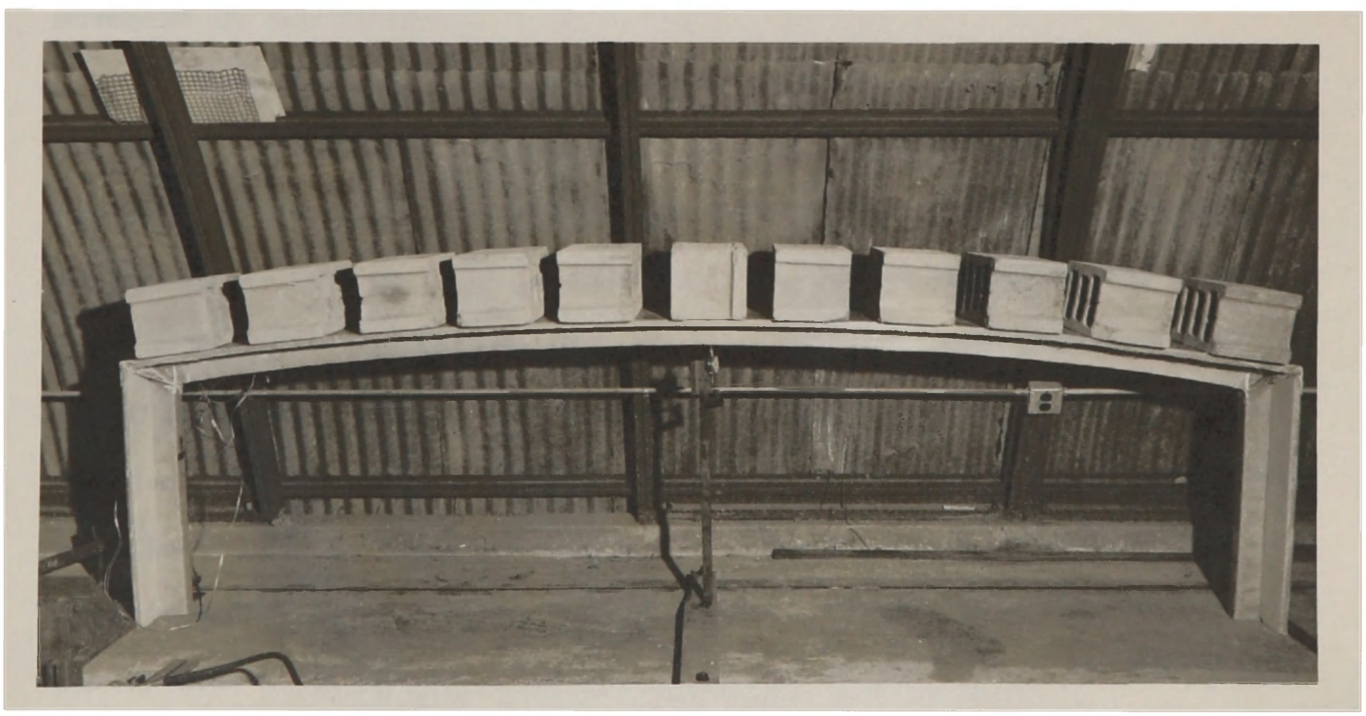


Figure 8. Materials and procedures used in constrction of model PO25050.

(d)



(e)

Figure 8. Materials and procedures used in construction of model P025050.

molds with a good covering of wax before applying any resin. With these precautions, the cured laminate can be stripped from the mold with ease.

Instrumentation

To check adequacy of a structural design some measurement of either stress or displacement or both should be made. As developed in the model analysis, measurements of both phenomena is required to check the analysis. Deflections are conveniently and accurately measured using dial indicators. Stress cannot be measured directly but must be obtained by measuring strain and relating strain to stress through known stress-strain relationships for the material. Electric resistance strain gages, sensitive to small changes in strain, are easily mounted, and for static strains, require only simple indicating instruments. A dial indicator was used in the tests to measure deflection at the crown, with an indicator on the outside of each column to indicate any lateral movement of the frame.

Type A-1, SR-4 electric resistance strain gages were mounted on the outer surface of the flange and stem at the crown and on one column to provide strain measurement at two positions on the frame. A Young strain indicator was used to indicate the strains sensed by the gages.

Two flexure samples (one from each resin laminate) were prepared for laboratory testing with Type A-1, SR-4 gages mounted on them to check the response of SR-4 gages on the laminates. The two specimens were loaded in the same manner as the samples used to determine the elastic moduli. A check of stress versus load indicated maximum difference of 2% between stresses indicated by SR-4 gages in the force transducer. The results of the tests are shown in Figure 9.

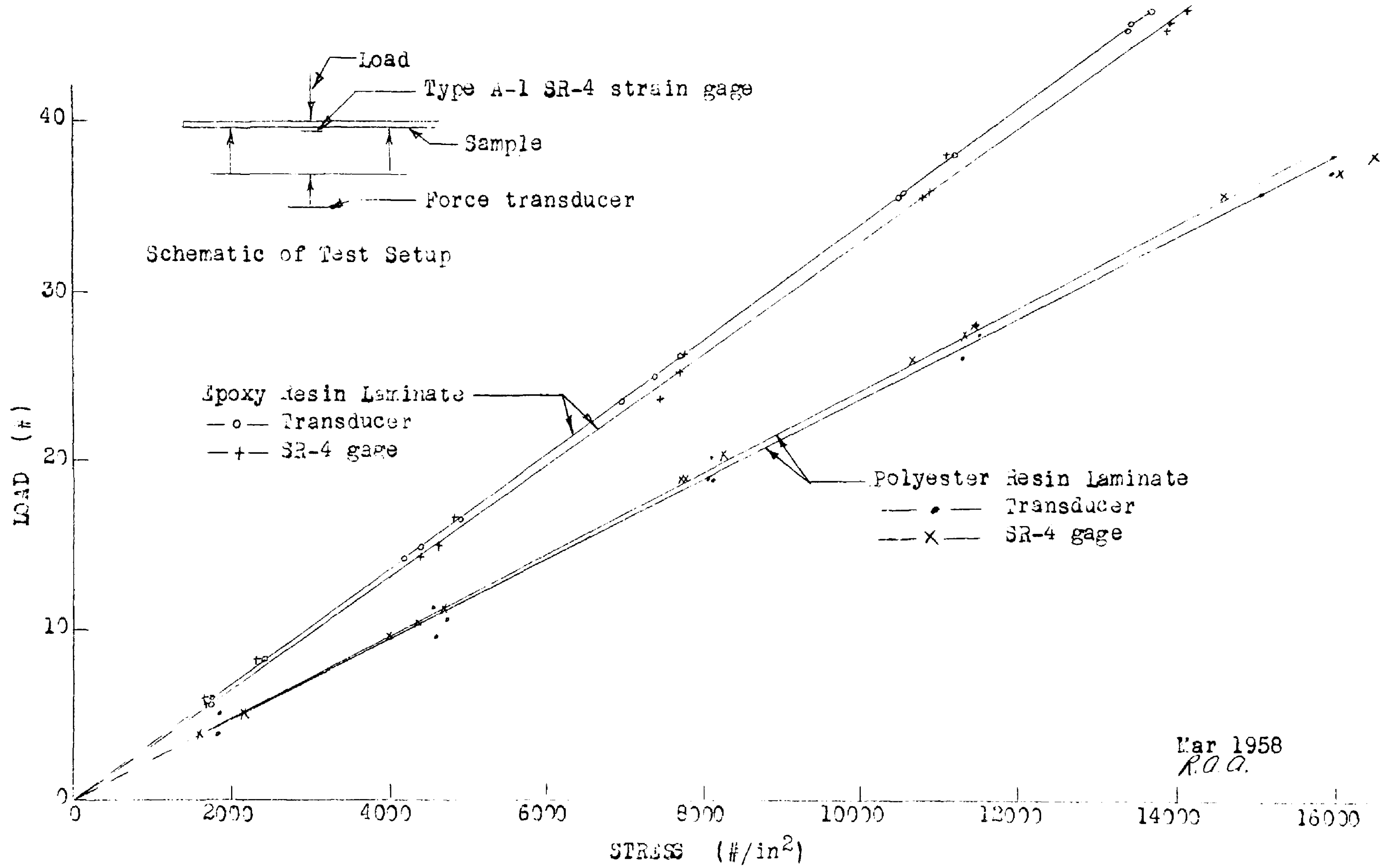


Figure 9. Load-stress curves in bending as determined by force transducer and SR-4 strain gage mounted on bending specimen.

Determining Material Constants

Values for the different moduli of elasticity and maximum strengths for the laminates listed in Table 1 were used in the original design calculations for the frame. To check both the quality of laminate produced and the design predictions, it was necessary to determine the material constants for each resin laminate used in the project. Therefore, samples for determining bending and tension moduli and strengths were cut from the excess flange laminate of each material. The test samples were prepared and tested according to ASTM designations (1). ASTM designation D638-52T was used for the tension specimens and ASTM designation D790-49T was used for the bending specimens.

The tension tests were carried out using a Baldwin-Emery SR-4, Model FGT, universal testing machine for applying the loads and a Tinius-Olsen subsize strain gage (one inch gage length) to measure strain. A total of twenty samples were tested from each resin laminate-10 from each model, with 5 parallel to the warp and 5 perpendicular to the warp. Results of the tests are shown in Figures 10, 11 and 12.

The flexural tests were made using an instrument developed for determining physical properties of grains (36). Figure 13a shows a test in progress on an epoxy resin specimen. Figure 13b shows sample data sheets from the Brush Oscillograph used to record the information from the tests. The graphs provide a record of both load and deflection from which modulus and maximum strength values can be readily computed. Table 4 lists the values of E and s_{max} for the epoxy and polyester resin laminates.

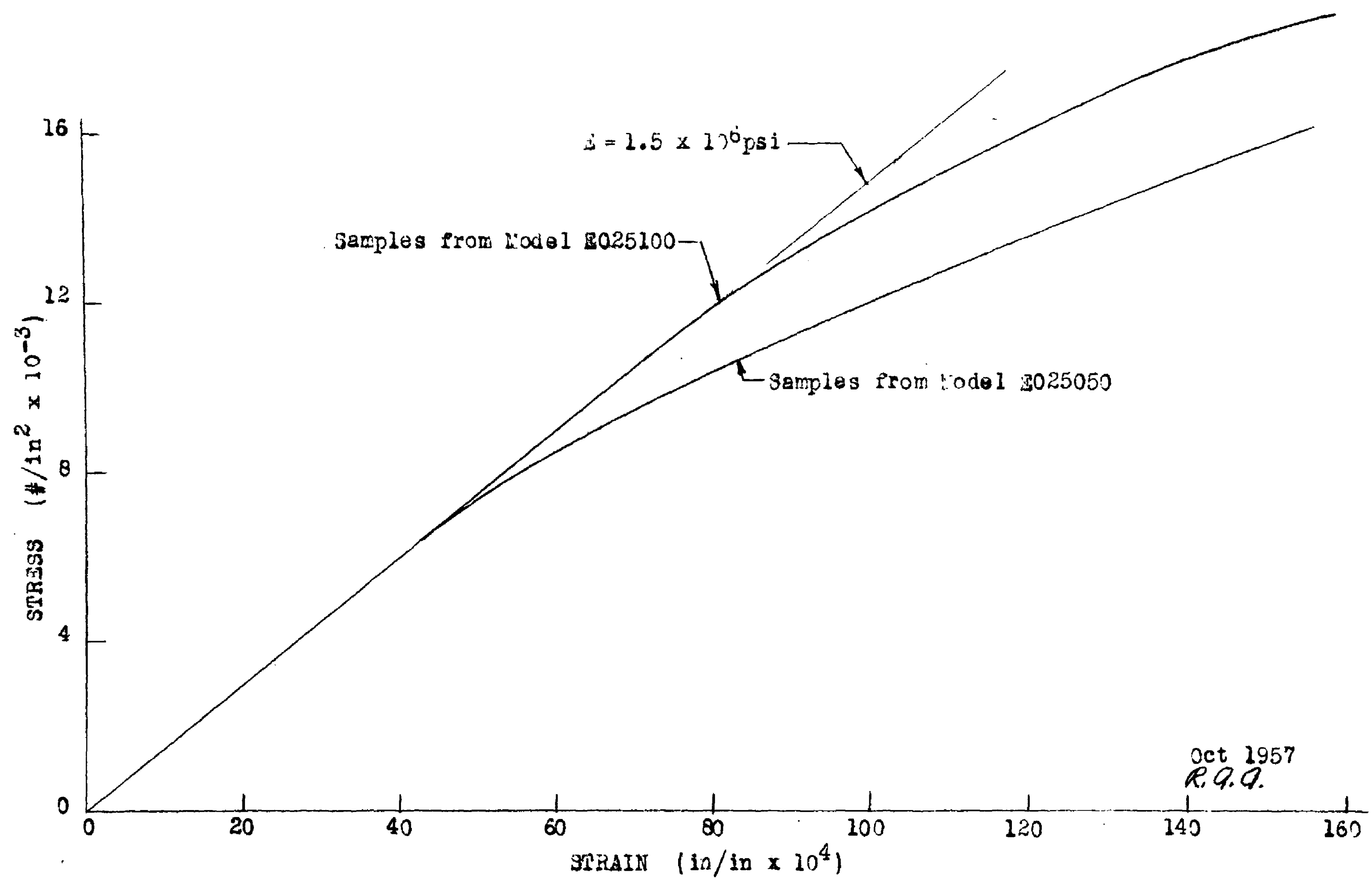


Figure 10. Average stress-strain curves in tension parallel to the warp for epoxy resin-glass cloth laminate.

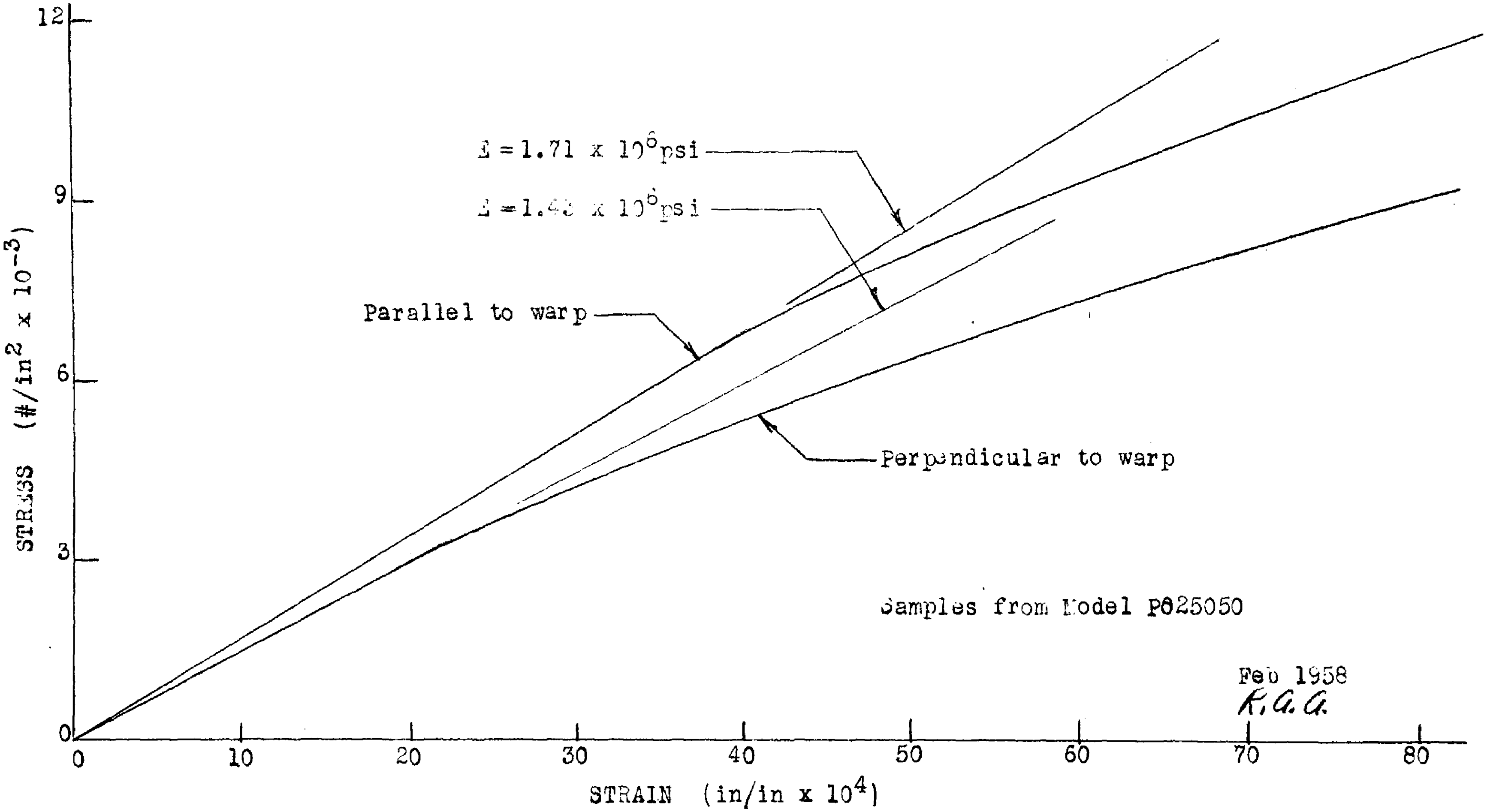


Figure 11. Average stress-strain curves in tension for polyester resin-glass cloth laminate.

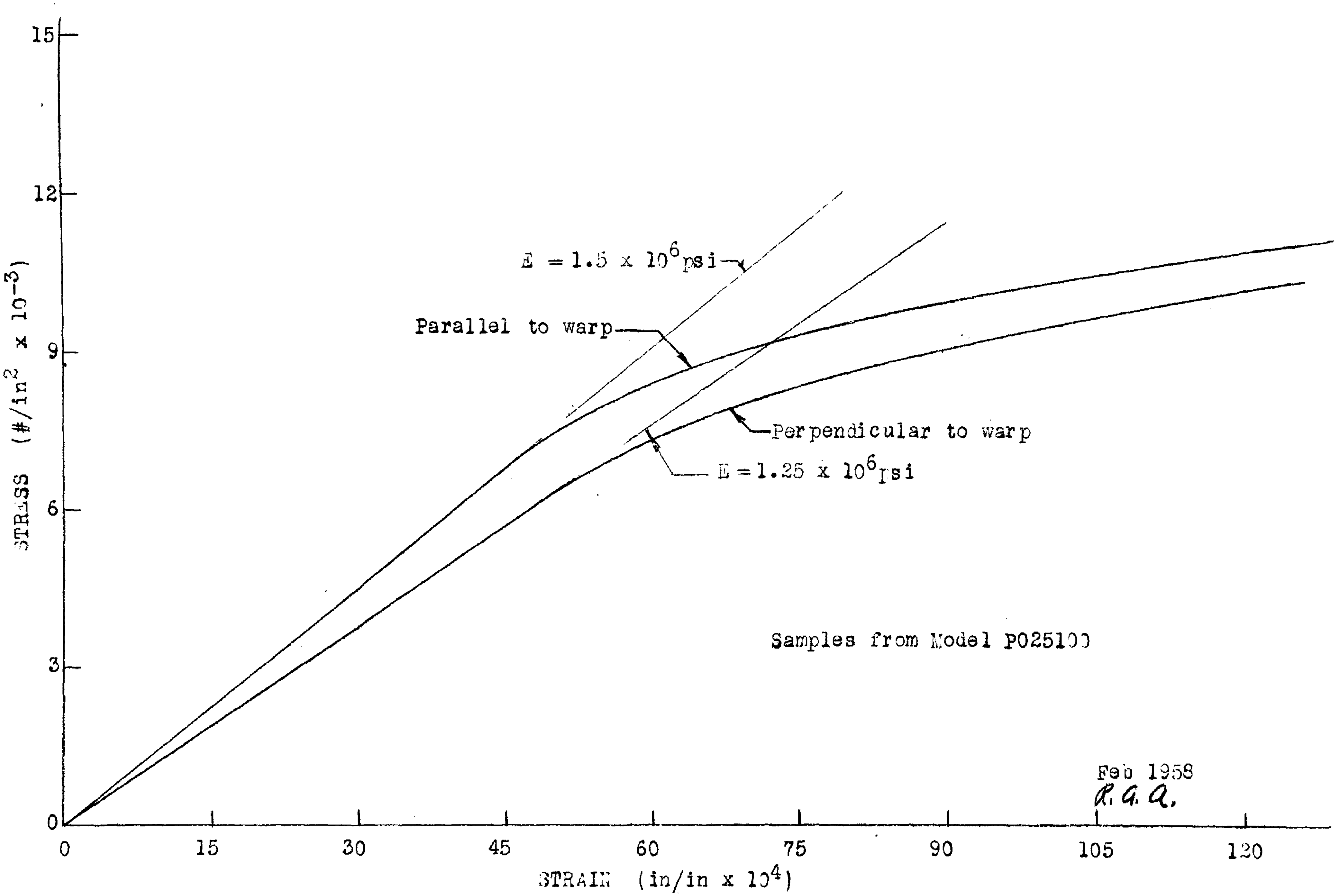
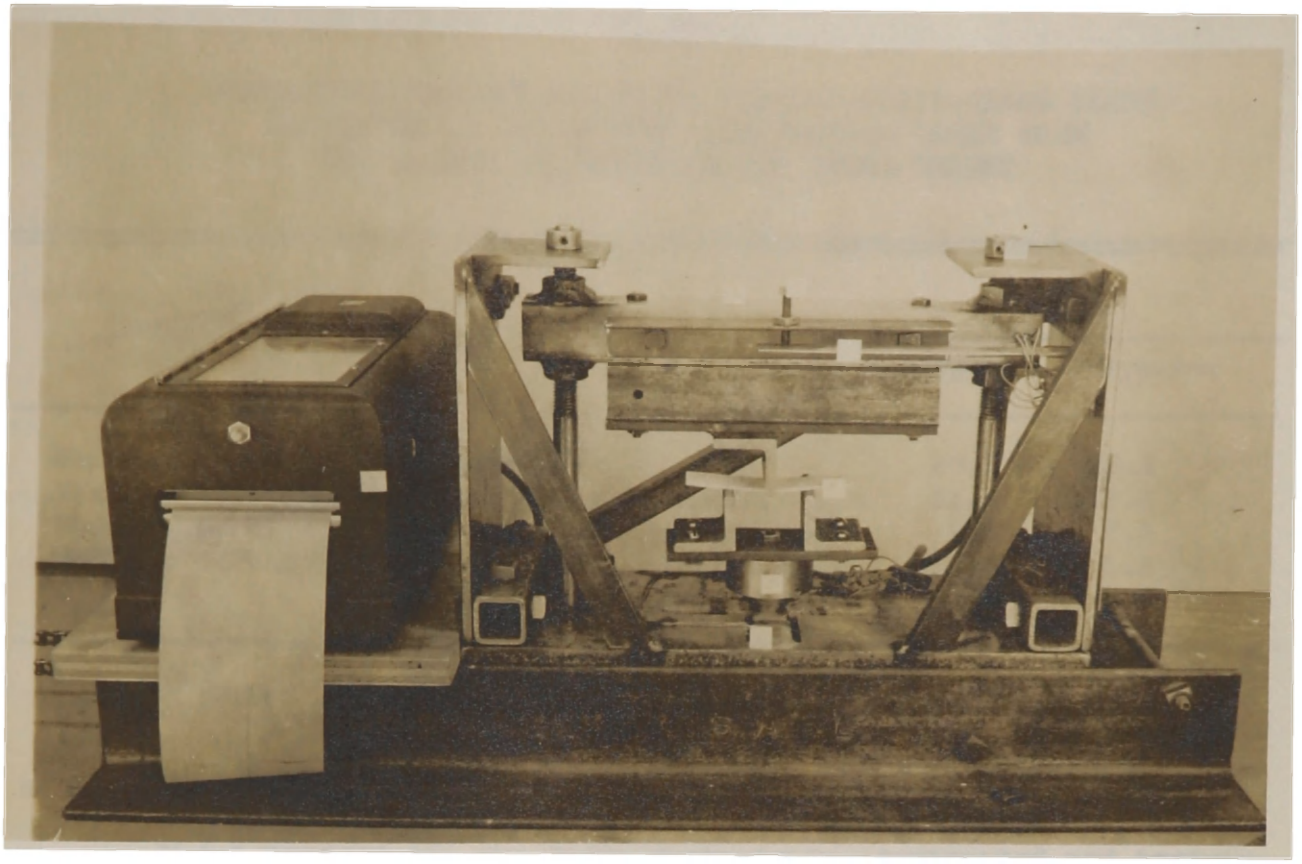
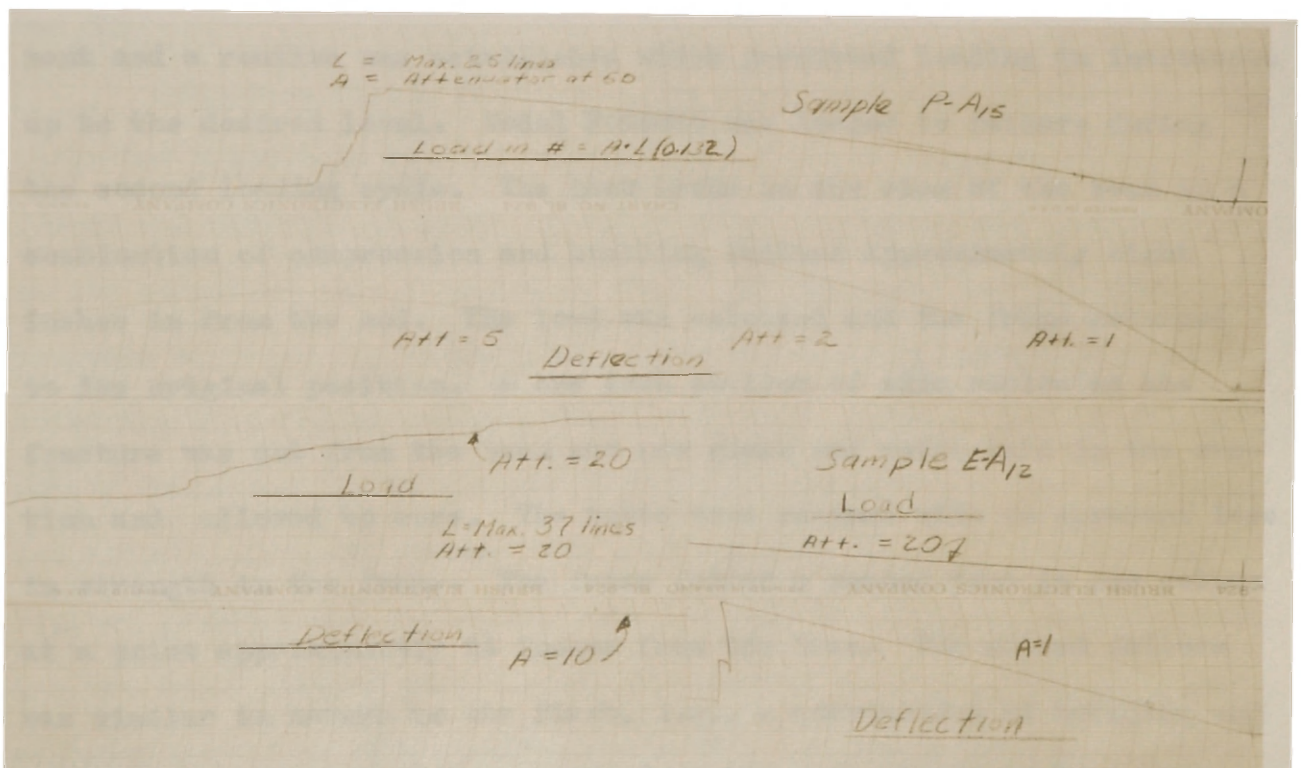


Figure 12. Average ~~stre~~ stress-strain curves in tension for polyester resin-glass cloth laminate.



(a)



(b)

Figure 13. (a) Flexure sample from model E025050 under test using the force transducer. (b) Typcial oscillograph chart record from the flexure tests.

TABLE 4

MECHANICAL PROPERTIES OF TWO PLASTIC RESIN-GLASS CLOTH LAMINATES AS DETERMINED FROM SAMPLES TAKEN FROM THE FLANGE LAMINATES OF THE MODEL FRAMES

Mechanical Property	Resins	
	Epoxy	Polyester
Maximum strength in bending (psi x 10^{-3})	20.8	23.3
Maximum strength in tension (psi x 10^{-3})	21.9	17.3
Modulus of elasticity in bending (psi x 10^{-6})	2.4	2.3
Modulus of elasticity in tension (psi x 10^{-6})	1.5	1.5(1) 1.7(2)

(1)Model P025100. (2)Model P025050.

Model Testing

Model E025050 was the first frame to be constructed and tested. No serious difficulties were experienced in the use of the test equipment and a routine was established which permitted loading in increments up to the desired level. Model E025050 was loaded to failure during the second loading cycle. The beam broke in the stem of the beam in a combination of compression and buckling failure approximately eight inches in from the end. The load was released and the frame returned to its original position. A one inch section of skin enclosing the fracture was cut from the beam and new glass and resin laid in the section and allowed to cure. The tests were resumed with no apparent loss in strength in the frame. The frame failed a second time in one column at a point approximately 24 inches from the base. The second failure was similar in nature to the first, i.e., a combination of buckling and compression. Both failures occurred at the same static load. Again a one inch section was cut from the laminated skin and new resin and cloth applied and allowed to cure. The tests were resumed with no apparent change in physical properties. The value for the modulus of elasticity in tension as determined by sample testing was used in computing the stress from indicated values of strain at the outer fiber of the stem at the crown. Results of the tests are shown in Figures 14 and 15.

Model E025100 was the second frame built and tested. Loads were applied in increments and the strain and deflection at the crown recorded for each load. At the upper limit of the loads, there was some crushing of the column flange due to the horizontal thrust developed in the frame. The results of the tests are shown in Figures 16 and 17.

Loading tests of the frames of polyester resin-glass cloth laminates were conducted in the same manner as those for the epoxy resin frames. The results of the tests are shown in Figures 18, 19, 20 and 21.

Long-term Static Loading. - The short-term static load tests as described previously do not provide information on behavior of the frames and material if loads of considerable magnitude are impressed on the structure over a period of time. To determine the possible presence of creep in the material, model E025050 was subjected to a uniformly distributed constant load of 50 #/ft of beam. (approximately 40% of the static failure load) Deflection measurements were observed and recorded from the crown. The load was left in place until an equilibrium position was definitely established. The load was then removed and the frame checked to determine if it would recover to its original unloaded position. The results of the test are shown in Figure 22.

Model P025050 was subjected to a long-term load test similar to the test of E025050. A uniformly distributed load of 42.5#/ft of beam

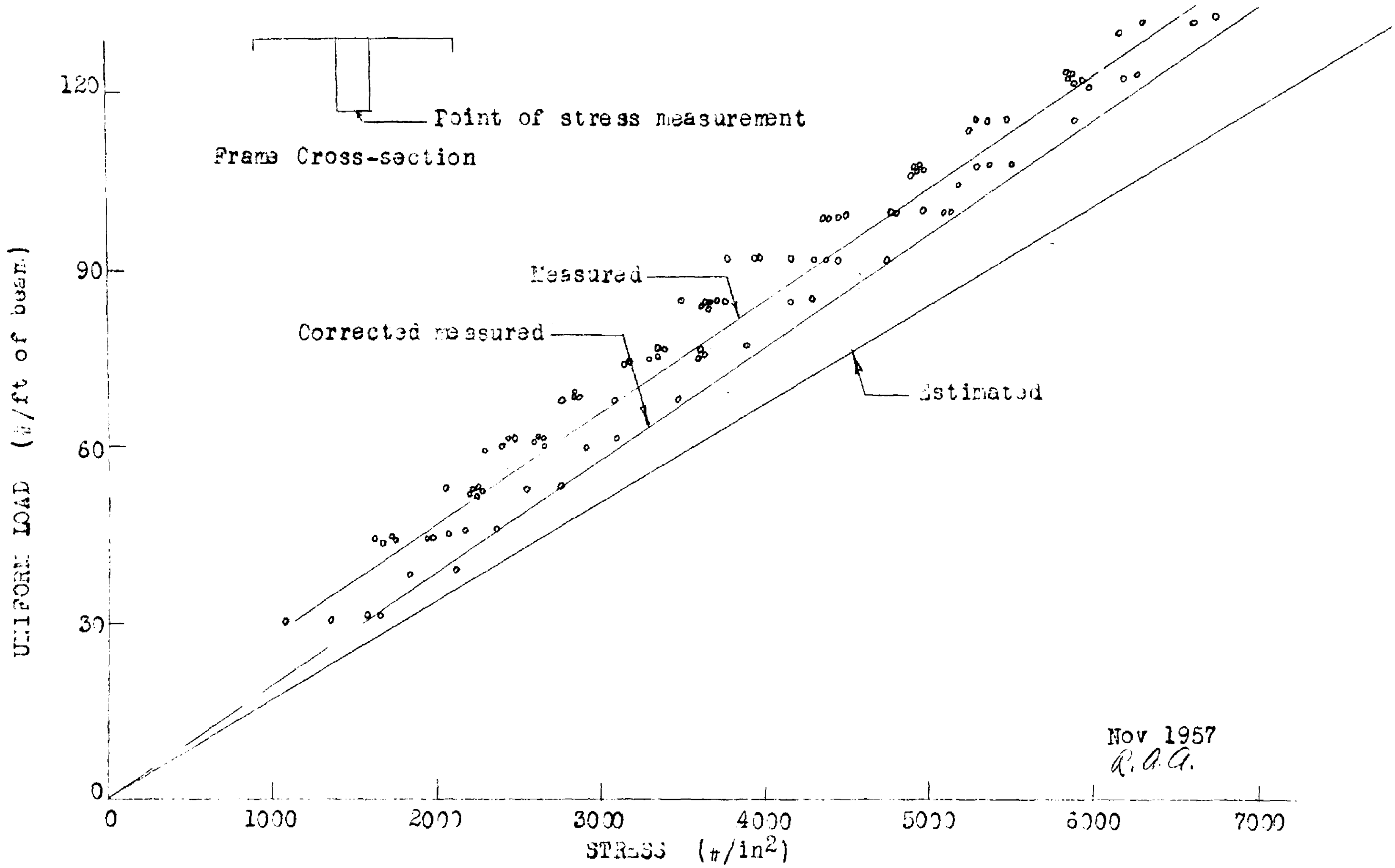


Figure **14**, Load-stress curves in bending for model E025050 (epoxy resin-glass cloth laminate).

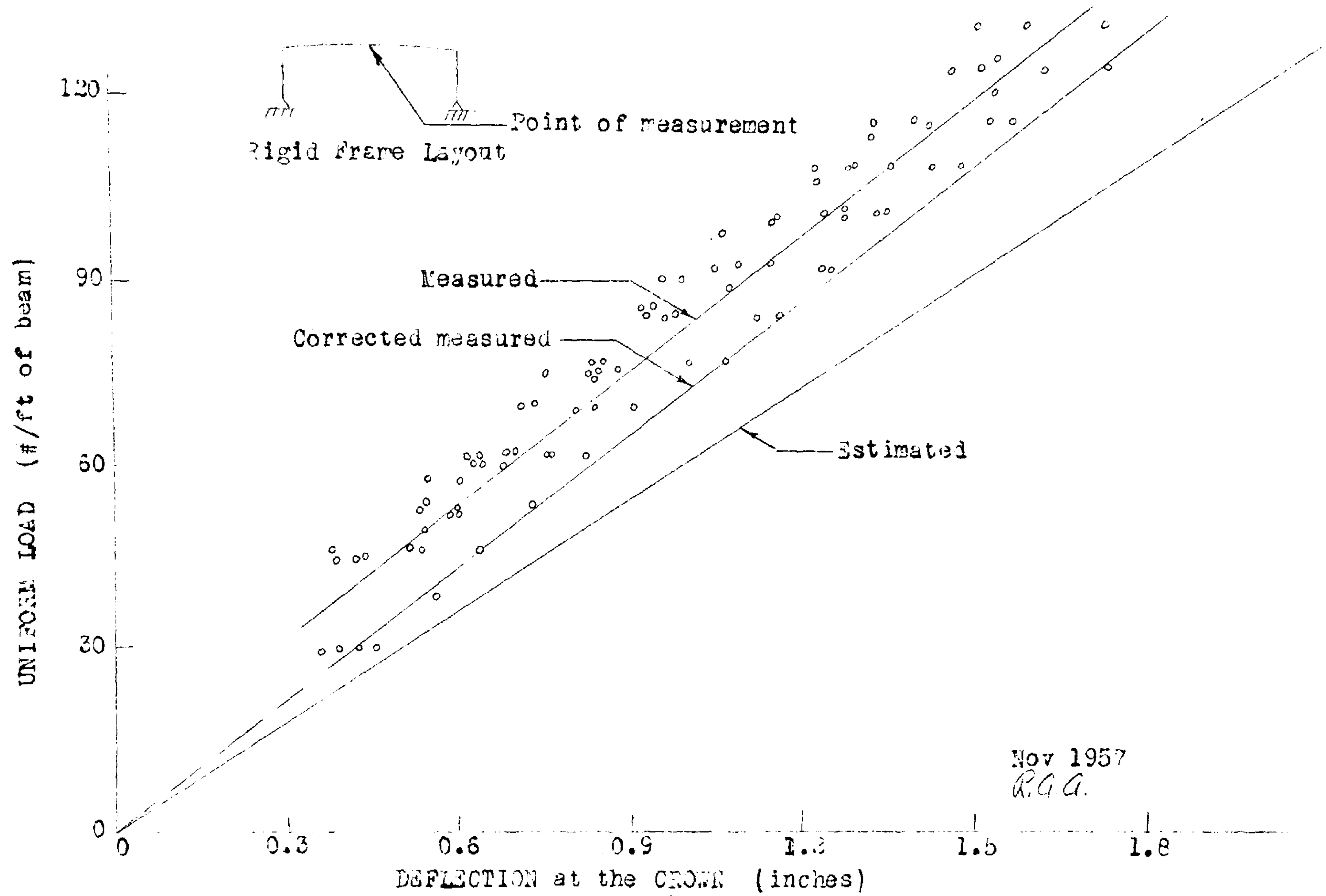


Figure 15. Deflection of model 1025050 at the crown due to a uniformly distributed load.

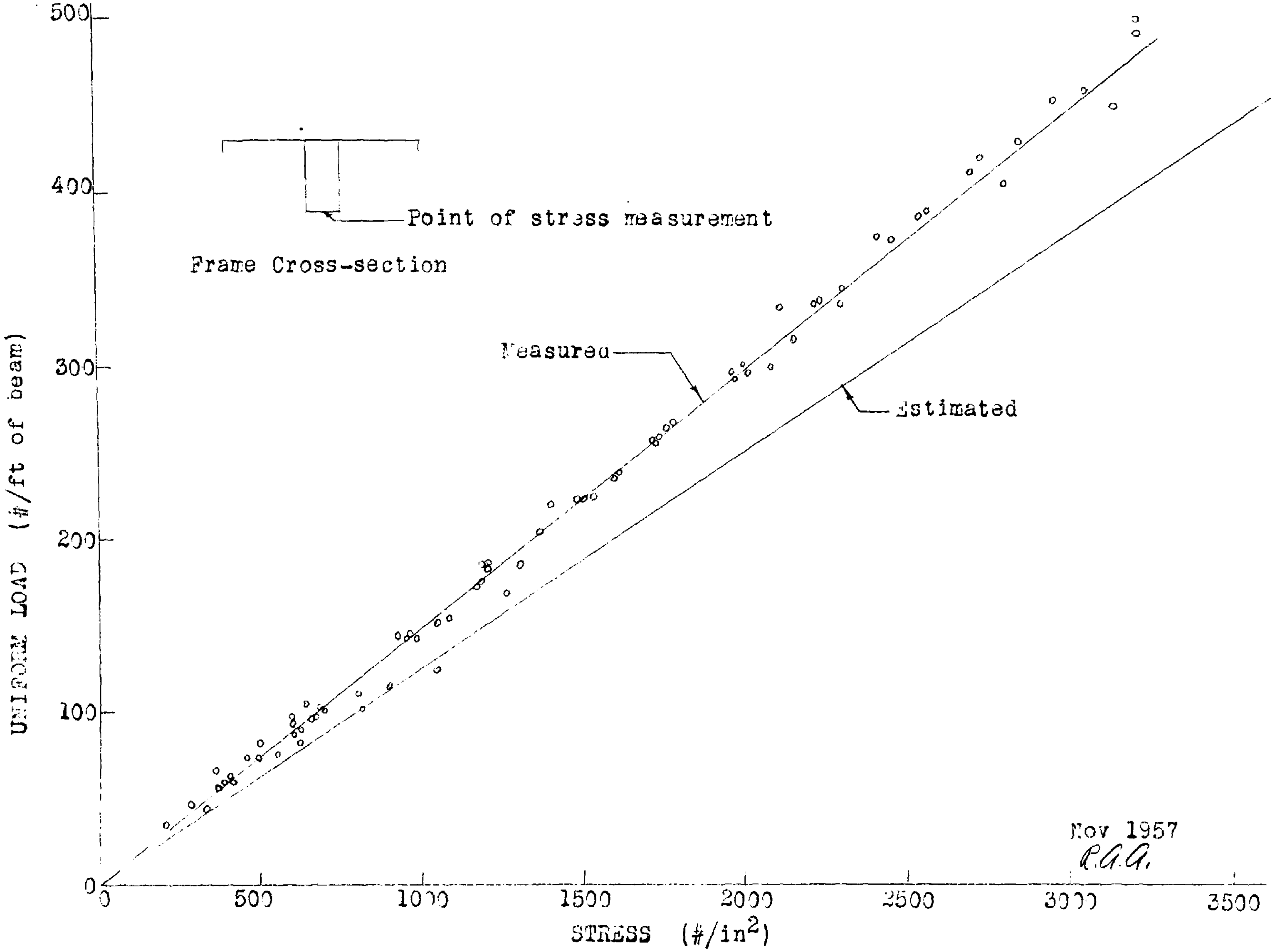


Figure 16. Load-stress curves in bending for model E025100 (epoxy resin-glass cloth laminate).

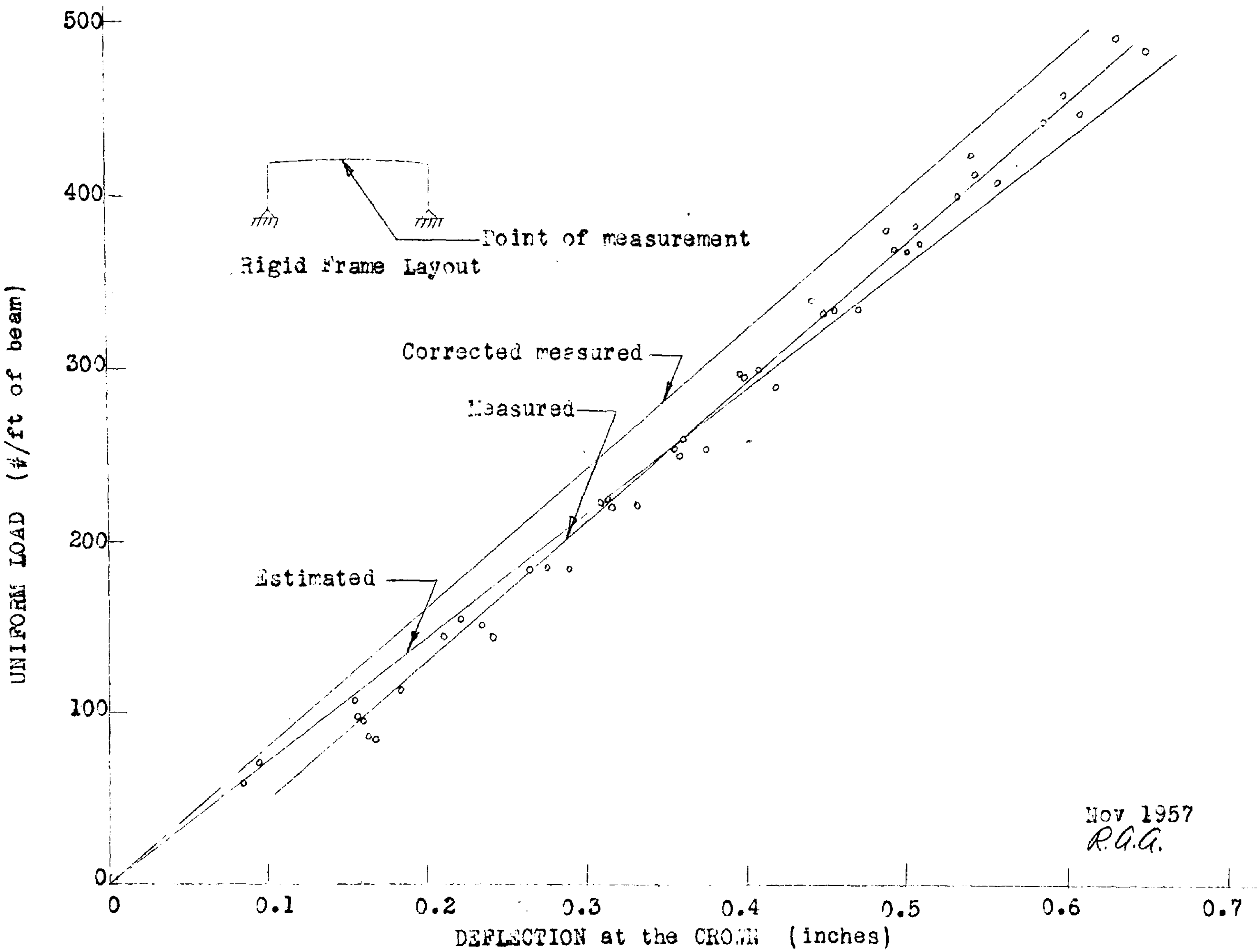


Figure 17. Deflection of model E025100 at the crown due to a uniformly distributed load.

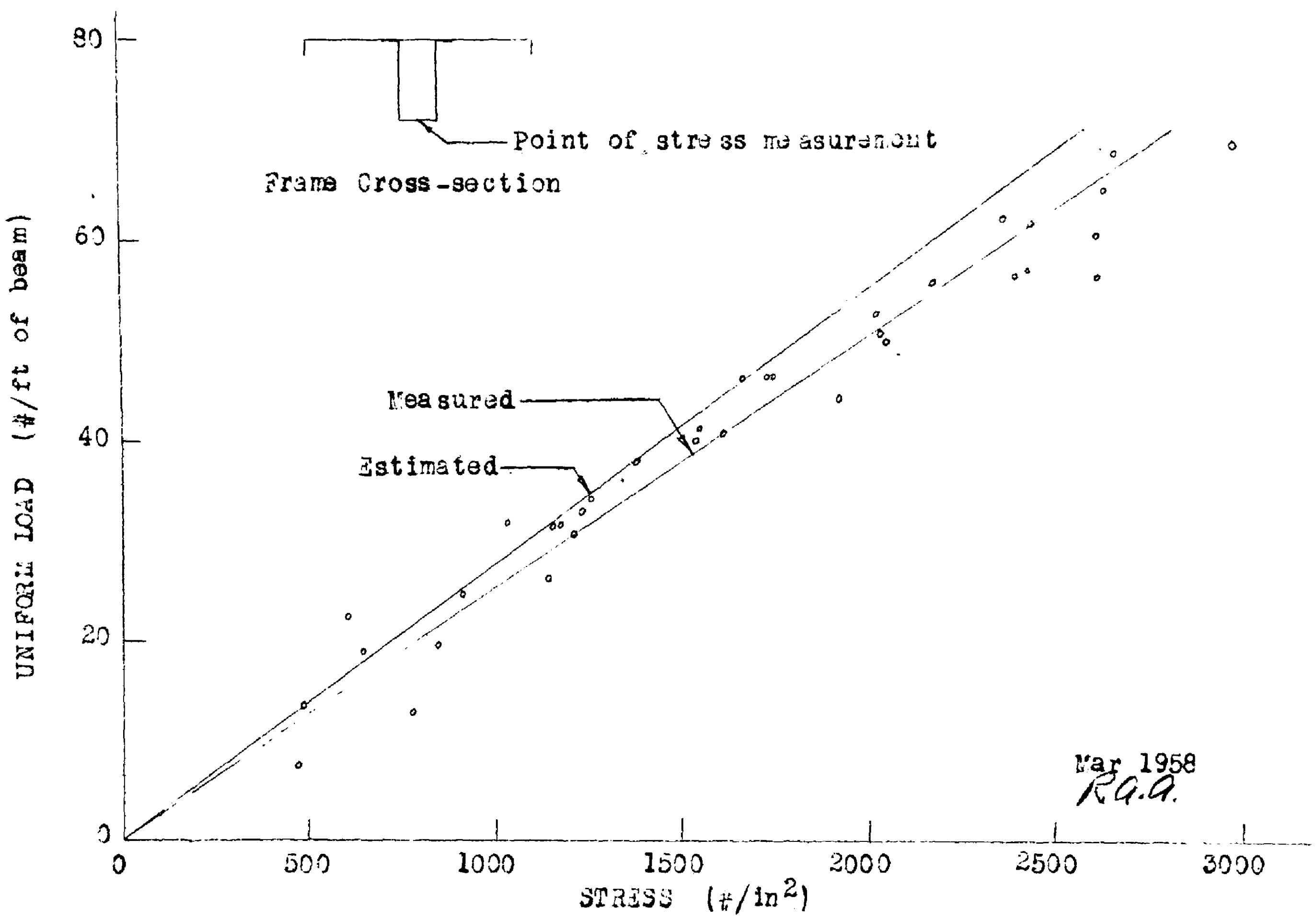


Figure 18. Load-stress curves in bending for model PO25050 (polyester resin-glass cloth laminate).

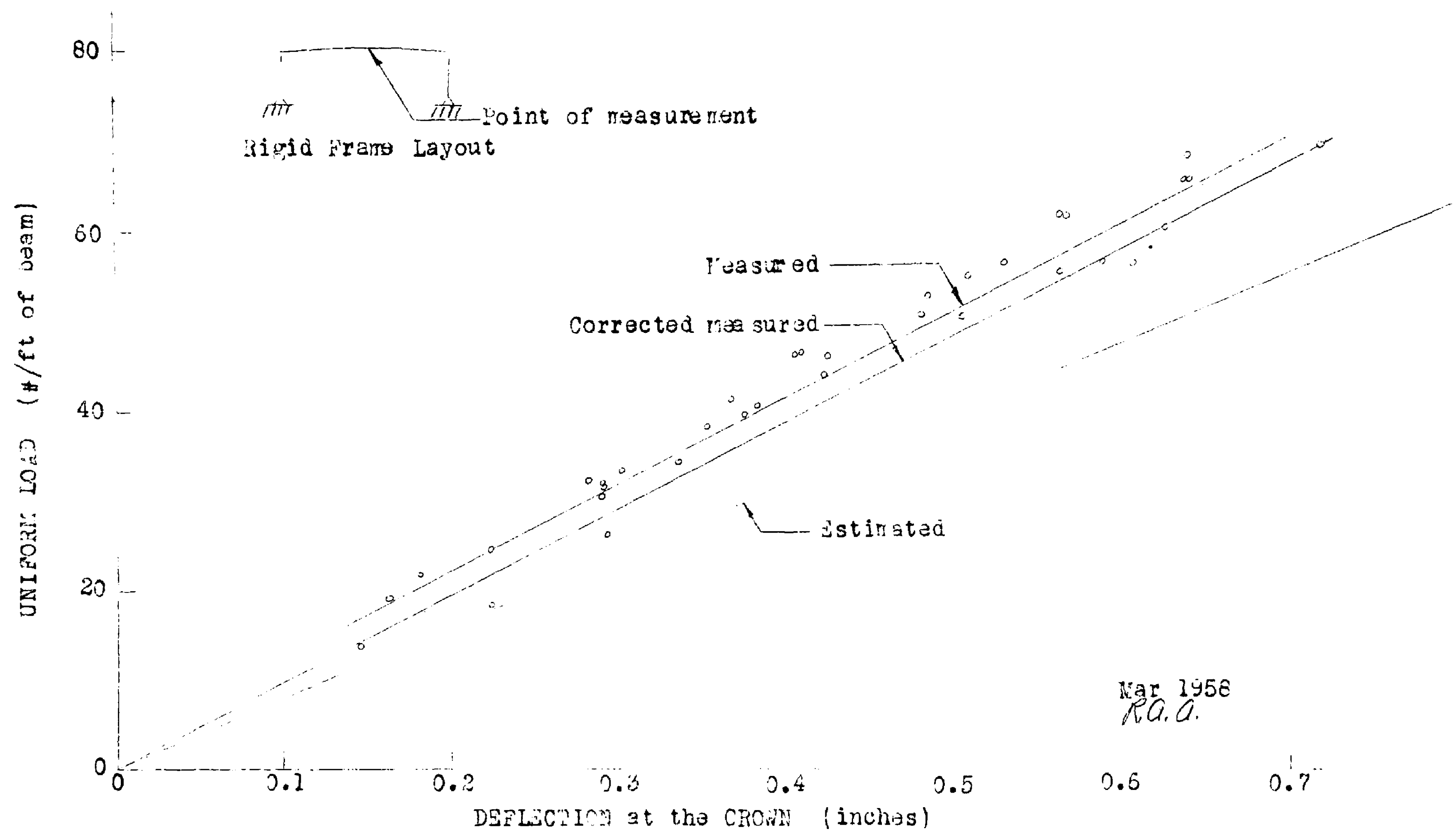


Figure 19. Deflection of model PO25050 at the crown due to a uniformly distributed load.

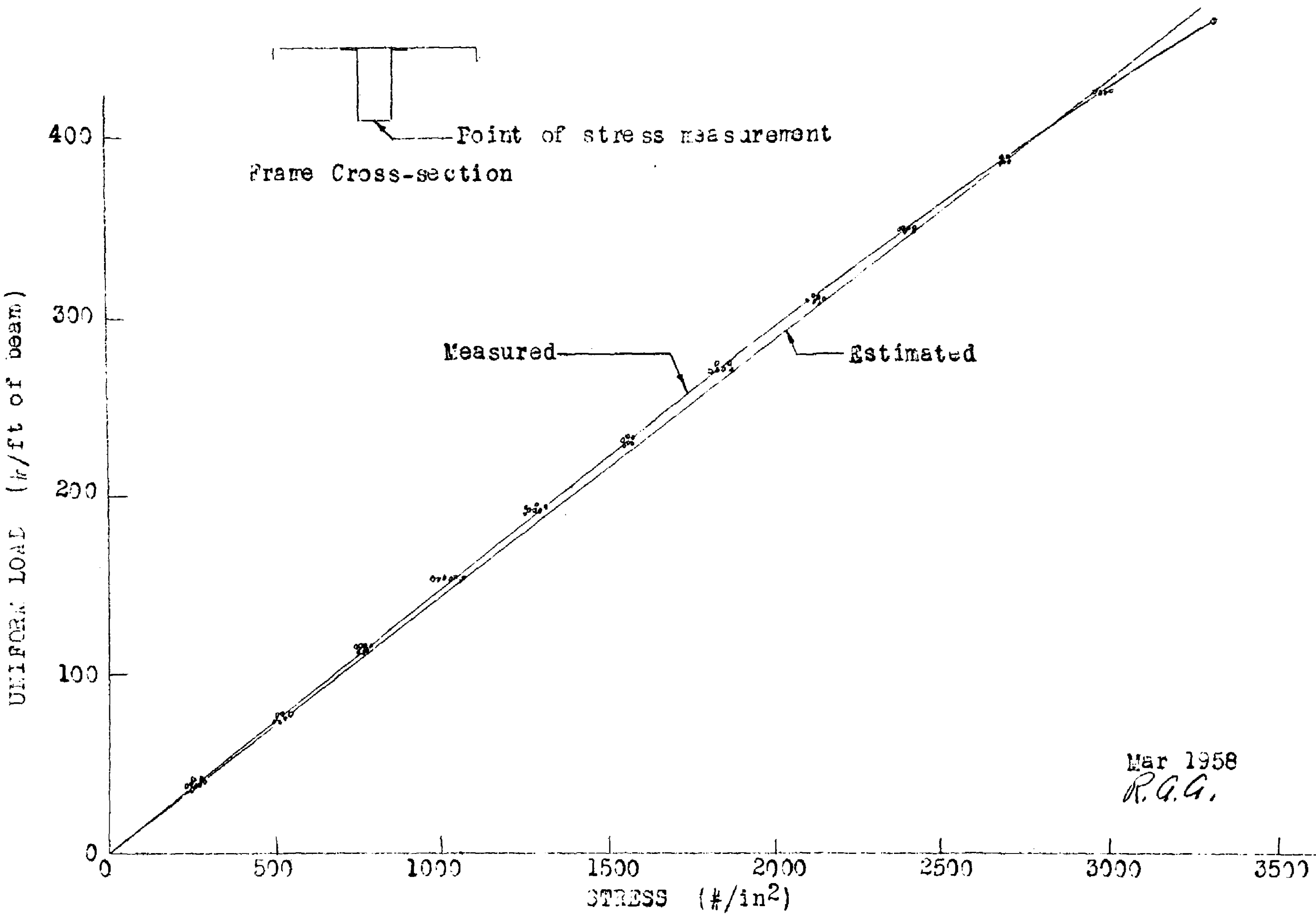


Figure 20. Load-stress curves in bending for model P025100 (polyester resin-glass cloth laminate).

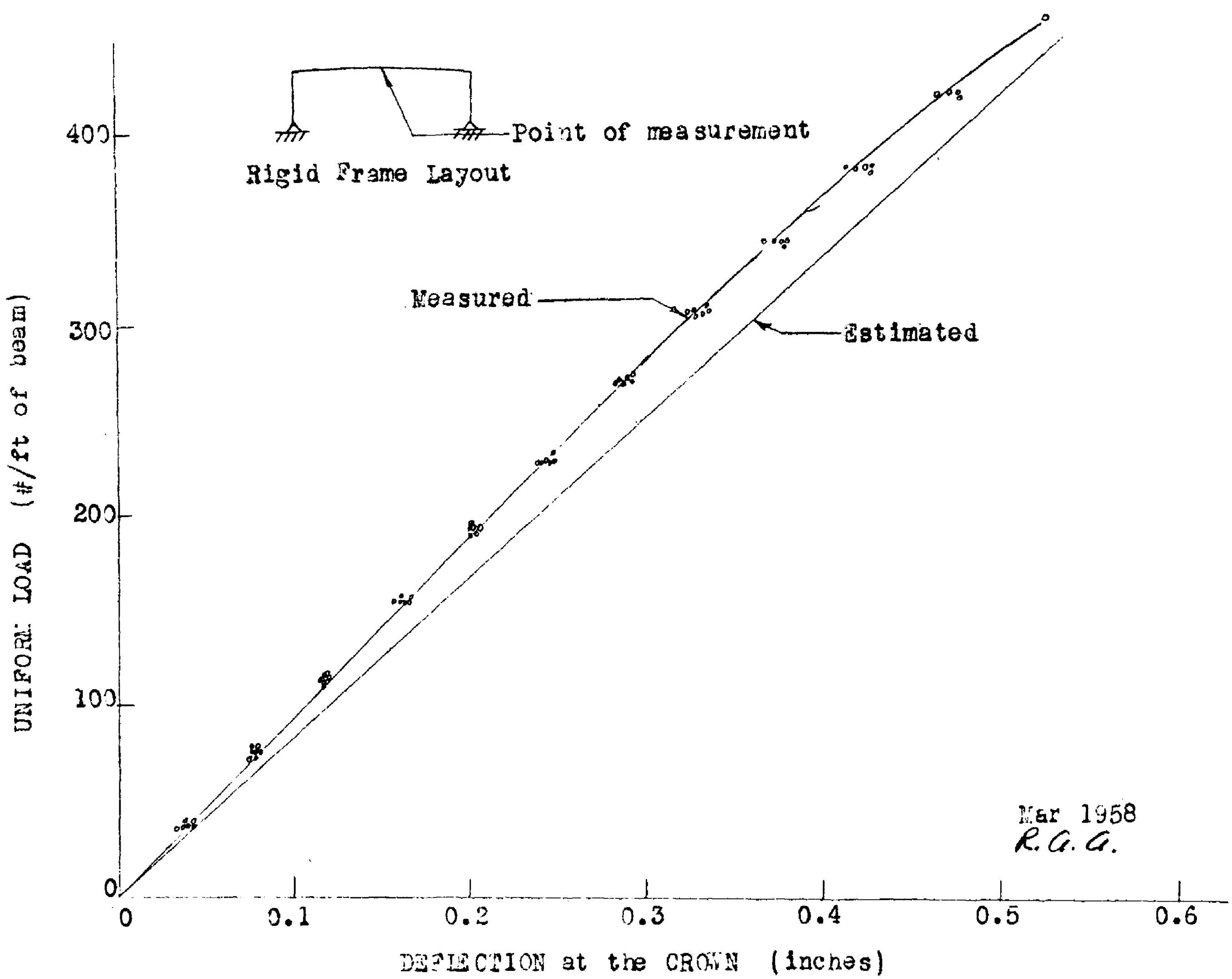


Figure 21. Deflection of model PO25100 at the crown due to a uniformly distributed load.

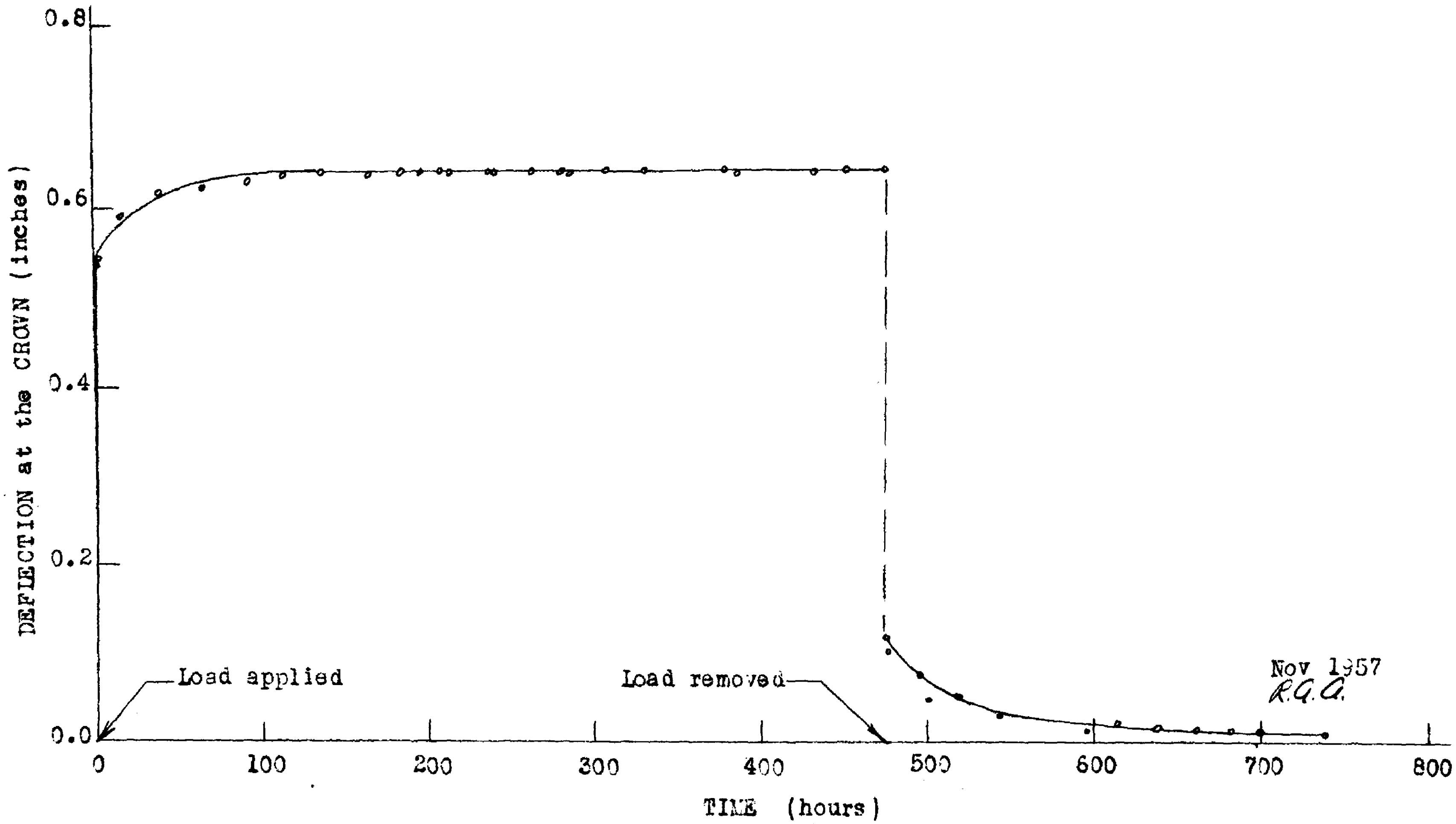


Figure 22. Deflection of Model E025050 at the crown with a constant uniform load of 50 #/ft of beam applied.

was placed on the frame and observations of the deflection at the crown were made at intervals until the frame had reached an equilibrium position. The load was then removed and observations continued until the frame had again reached equilibrium. The load of 42.5#/ft of beam on the model is equivalent to a load of 170#/ft of beam on the full scale frame. The results of the test are shown in Figure 23.

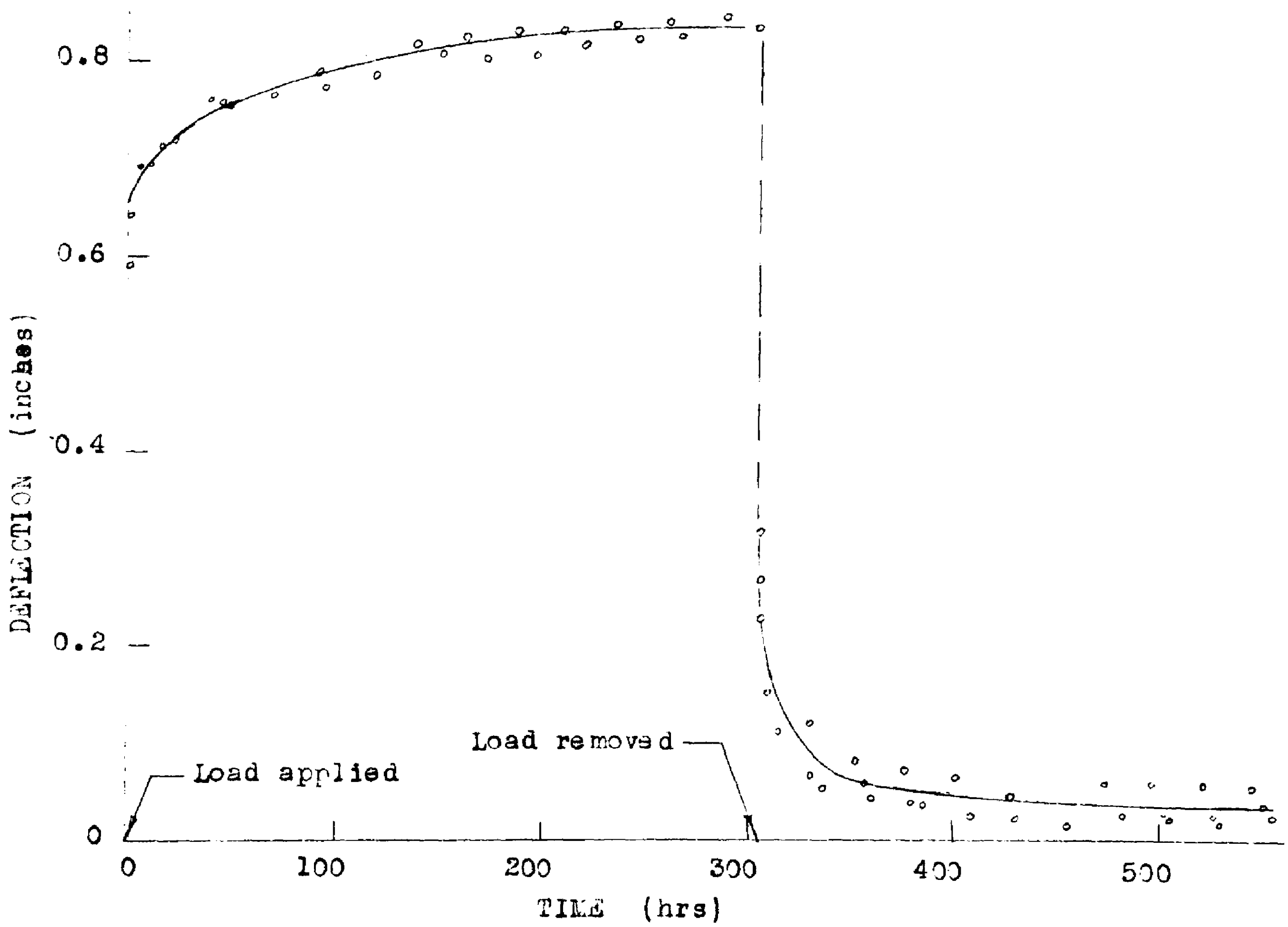


Figure 23. Deflection of model P025050 at the crown with a constant uniform load of 42.5#/ft of beam applied.

RESULTS AND DISCUSSION

Use of The Construction Materials

Successful use of the plastic resins for hand-wet layup molding depends on resin formulation, catalyst, hardener, mold preparation, and operator experience. Some of these factors are determined by the material manufacturer and some are controlled by the fabricator. The two resins employed in this project are relatively easy to work although some experience is helpful, if not necessary, to insure a uniform product.

Particular care must be taken to protect against air inclusion. Even with the translucent polyester resin bubbles are difficult to see unless the background and lighting are right. A light colored background with direct lighting aids in detection of the air pockets. A common error (experienced by the author) is to resin starve areas of cloth. This is generally due to ignoring, or not being aware of, the change in viscosity of the resin at peak exotherm - the heat generated chemically during the curing process. For a very short period of time, as the resin approaches peak exotherm, the viscosity reduces markedly, resulting in a runout of resin. If laminate thickness, and thus resin, is built up too quickly, the runout at peak exotherm will result in the upper cloth fibers being without sufficient resin to produce their maximum strengths. This can be prevented by laminating such that the resin has begun to gel before the succeeding layer is applied. Figure 24 illustrates an adequate resin-cloth ratio and a resin starved laminate. Too rapid resin buildup will also result in the cloth 'floating', resulting in uneven distribution of cloth across the section.

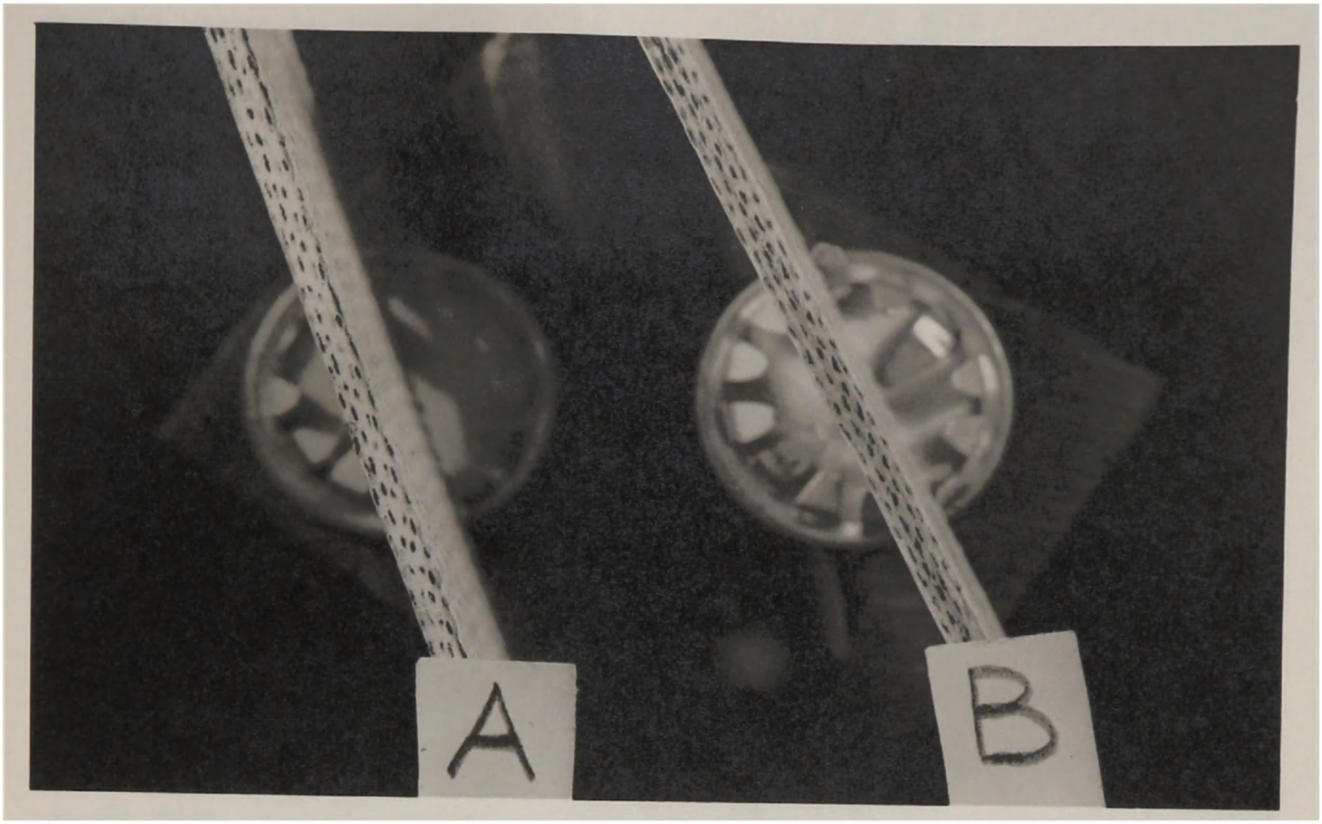
It is necessary to use a thixotropic agent if the laminate is to be constructed on a steeply sloping or vertical surface. The thixotropic agent prevents sagging without appreciably altering the mechanical properties of the final product.

Repairs to the laminates made with either resin can be made very simply by removing the fractured portion and laying in new material with no apparent loss in strength. This is a small but important point in maintenance of buildings.

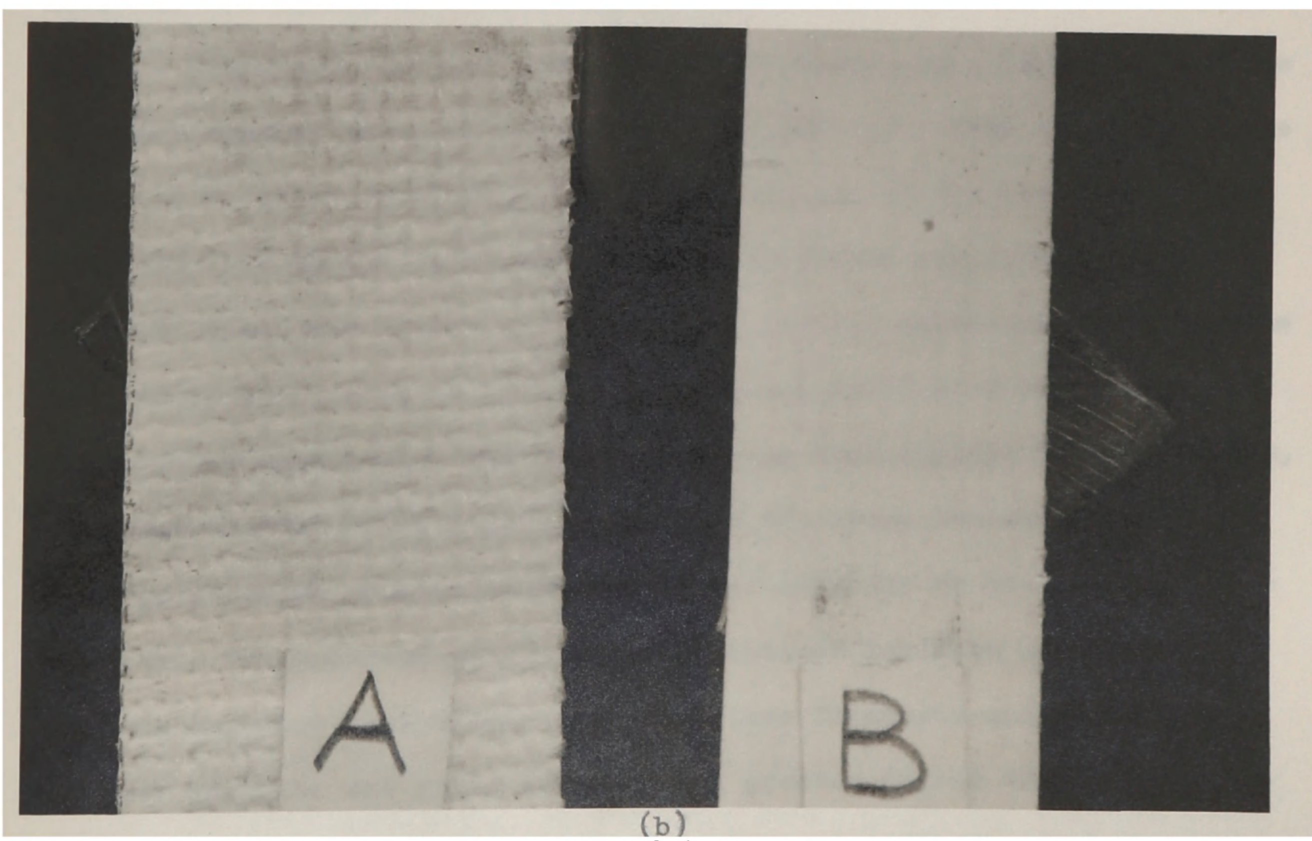
The plain weave glass cloth used in the laminates is very easy to handle although it is not possible to bend it into abrupt 90 degree corners. This characteristic is not important in the fabrication of relatively simple frame cross sections.

Type A-1, SR-4 strain gages are easily mounted on either resin laminate. The adjustment necessary to account for differences in modulus of elasticity between laminate and gage material is less than one per cent. This correction was considered negligible in the present work.

Figure 24 illustrates the difference between a satisfactory laminate and one that has been resin starved due to runout. Both are from laminates of four layers of glass cloth. Figure 24a is an edge view of the two samples with the glass threads blackened for contrast with the resin. Notice how the one layer of glass in sample A is at the edge of the laminate with very little resin covering, even though the other glass layers appear to be fairly evenly distributed through the sample. Sample B shows very clearly the uniform distribution of glass across the laminate with ample resin covering on both sides. Figure 24b is a plan view of the same samples again showing the contrast between the two laminates. The weave pattern of the cloth is



(a)



(b)

Figure 24. (a) Edge view of two epoxy resin-glass cloth laminates showing glass distribution through the cross section. (b) Plan view of the laminates showing the surface appearance.

clearly shown in sample A indicating the lack of resin cover. Sample B shows how a laminate surface should appear if proper resin-glass distribution is maintained.

Mechanical Properties of The Materials

Tension

The stress-strain curves for both resin laminates are quite similar in that they exhibit the general characteristics of a typical brittle material. This is a demonstration of what Brossy, et al, (8), call 'preferential' failure. This was audibly apparent during the tests as the individual fibers and strands would break with a pronounced snap as the load increased up to the point where the total remaining strands would fail suddenly and dramatically. There was no noticeable 'necking down' of the specimen before failure.

One set of curves is shown for the epoxy resin laminates - those for load applied parallel to the warp (Figure 10). This is done because there is no apparent difference in the behavior of the laminate loaded perpendicular to the warp until the stress-strain relationship has progressed beyond the linear portion of the curve. Since the straight line segment is the important part in determining modulus of elasticity and the modulus parallel to the warp is the one that applies to the problem, the other curves are not presented. The two epoxy laminates are uniform in strength properties as the curves coincide on the straight line portion. The apparent weakness of the thinner laminate can be attributed to the fact that there are fewer glass filaments and also the thinner laminate was resin starved to a greater degree than the heavier one. The modulus of elasticity and maximum strength in tension determined for the epoxy resin laminates from the samples are low compared to the values given in Table 1. The differences are most likely due to lower quality laminate plus the fact that the resin used is a modified epoxy with the manufacturer(1) indicating possible strengths slightly lower than those given in Table 1.

There is a difference in the stress-strain curves for the polyester resin laminates (Figures 11 and 12), both according to the model from which the samples were taken and according to direction of the sample with respect to the warp of the cloth. Samples from model P025050 show the highest modulus of elasticity in tension even though they represent the thinner laminate thickness. The difference here might be due again to better quality workmanship as model P025050 was constructed after completion of P025100. Another factor which could contribute to the differences is the ambient temperature during the curing period. The ambient temperature was approximately 50°F throughout the curing time for P025100 while it was approximately 75°F throughout the curing period for P025050.

It is natural to expect a lower modulus perpendicular to the warp as the cross threads, or pick, are forced to bend around the warp during weaving, thus when a pure-tension load is placed on the laminate in the direction of the pick the glass filaments must be pulled straight before they can develop maximum load carrying capacity. Fewer filaments are equally loaded at any one time during the tests.

The values for E in tension lie within the range given in Table 1 as typical for polyester resin laminates. The maximum strength in tension is low, but the comparison cannot be made too strictly since

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(1)The manufacturer of the epoxy resin gives the following values as representative for the laminating mix used in the models: maximum strength in tension - 27,400psi, in bending - 28,000psi.

the table values apply to low pressure laminates. No values are available for no pressure hand layup laminates.

The moduli of elasticity determined were used to relate the strain sensed by the SR-4 strain gages to the stress in the outer fibers of the frame cross section allowing a relationship to be established between stress in the frame and the applied load.

Flexure

Because of the small size of the bending specimens (one inch by four inches by the thickness of the laminate) it is difficult to develop equipment which will accurately apply the load and measure the displacement. A force transducer developed by G. C. Zoerb (36) used in determining the mechanical properties of grain provided the best means for accurately and conveniently conducting the bending tests. The test machine produces a constant rate of deformation in the test specimen as specified by ASTM designations. Through the use of SR-4 gages with a Brush oscillograph and amplifiers, a permanent record of continuous load-deflection relationship is obtained from which the maximum strength and modulus of elasticity can be computed. Figure 13a shows one of the epoxy resin laminate specimens being tested.

Figure 13b shows typical data from the two resin laminates. The upper chart is from the test of sample P-A_{15} (polyester resin, model P025100, parallel to the warp, fifth sample of the lot). The lower chart is for sample E-A_{12}. A similar description applies to this sample.

It is of interest to compare the failure pattern of the two laminates. The load on the polyester laminate increased at nearly constant rate to the maximum and then the specimen failed abruptly with nearly complete release of load. The epoxy laminate did not break completely at the maximum load but failed in clearly defined steps indicating discrete cloth fiber failure as the deflection was increased.

The maximum stress was calculated as the stress at maximum load before failure. With sample P-A_{15}, the maximum load in pounds, $P = A \cdot L(0.132) = 165$ pounds. A is the attenuator setting and L is the maximum number of lines of pen deflection. The constant, 0.132, is true for a particular calibration of the oscillograph. Since the moment at the center of a simply supported beam with a concentrated load at midspan is $PL/4$, the maximum stress is $3PL/2bh^2$. L for the sample is 2 inches; therefore, $s_m = 3P/bh^2$. For sample P-A_{15}, $s_m = 22{,}500$psi.

The modulus of elasticity in bending is determined from the relationship for midspan deflection of the simple beam, i.e., $d = PL^3/48EI$. Thus $E = PL^3/48Id = (2/bh^3)(P/d)$, where P/d is the load per inch of deflection. The load-deflection relationship is determined from the initial linear portion of the curves. E for P-A_{15} is, 2.61×10^6 psi.

The value for maximum stress in bending as given in Table 4 for the epoxy resin laminates is low as compared to the figures given by the resin manufacturers. No value was indicated for E but the figure 2.4×10^6psi as determined from the tests lies within the range given in Table 1. The value of E for the polyester resin (2.3×10^6psi) also lies within the range of values given in Table 1. However, the maximum strength in bending for the polyester resin laminate is low compared to the indicated normal range. The differences between normally expected values and those computed using samples from this work are probably due to the same influences as given for the tensile test results.

A maximum stress of approximately 30,000psi in tension, compression and bending was assumed in the original design as giving a stress safety factor of 2.5 (page 20). The maximum stress for the laminates produced varies from 17,300 to 23,300 psi; approximately 2/3 of the assumed design value. Thus, the stress factor of safety is reduced from 2.5 to about 1.4. This would still be acceptable for farm building construction if assurance could be had that the material would be uniform in quality. A commercial production of the laminates would insure better quality control so that stress factors of safety could be reduced allowing for more economical use of the material. With complete knowledge of the resin being used, its strength characteristics, etc., designs could be developed which would allow the farmer to take full advantage of the inherent strength of the material.

Evaluation of The Model Analysis

For the proposed solution of the frame to be valid, the test results on the models should show reasonable agreement both from a model-prototype comparison and from a comparison of test results with the estimated values for the phenomena being measured. A decision as to what constitutes reasonable agreement is conditioned by personal opinion based on experience with, and knowledge of, a given situation. The limits for agreement versus disagreement cannot be set rigidly but shift to fit the situation with reason controlling the final choice.

Examination of Figures 14 through 21 will provide a clear picture of the comparison of test results with estimated values for stress and deflection at the crown. There is quite a large spread in values for stress and deflection in the prototypes (E025050 and P025050) at any given load. This is due partly to observation and instrumentation difficulties. It was difficult to hold the load constant at any given point in the low range in which the tests were run and the bourdon type pressure gages were sometimes sluggish in response at these pressures. However, enough replicates were run to establish definite relationships between the variables concerned.

This difficulty was not present in testing models E025100 and P025100, primarily because of the magnitude of the applied loads which resulted in better pressure control and better gage response, but also because of the increased rigidity of the larger sections which tended to stabilize the frame action.

The best straight line was drawn through the plotted points and, if necessary, corrected to pass through the origin. Since a linear stress-strain relationship was assumed, the estimated values for stress and deflection were determined as functions of load per foot of beam and plotted on the curve sheets with the measured values. In all cases except one, the measured values are less than estimated. The one exception is P025050 in which the measured stress is approximately 9 per cent above the estimated value. These results are shown in Table 5. The greatest difference in stress occurred with E025100, 16.4 per cent, and the least difference with P025100, 1.5 per cent. The greatest difference in deflection occurred with P025050, 17.5 per cent, and the least difference with E025100, 9.3 per cent. The prototypes exhibited the greatest differences in deflection. Although the differences in some instances appear to be rather high it is felt that this does not detract from the validity of the analysis. Inaccuracies in frame construction and frame measurements plus any errors in the testing facilities could easily combine to produce the differences shown between estimated and observed values. Efforts were made to produce a uniform laminate on the frames and maintain uniform dimensions in layout and cross section but it is difficult to control these items without using large, precision made molds with skilled technicians doing the laminating. A small error in measuring skin thickness for determining the moment of inertia for the frame cross sections would cause a relatively large difference in the final estimate of stress and deflection.

TABLE 5

ESTIMATED AND MEASURED VALUES OF DEFLECTION (d) AND LOWER FIBER STRESS (s) AT THE CROWN OF THE FOUR MODEL FRAMES.

Resin	Epoxy				Polyester			
Model	E025100		E025050		P025100		P025050	
	d (in)	s (#/in^2)	d (in)	s (#/in^2)	d (in)	s (#/in^2)	d (in)	s (#/in^2)
Estimated	.00139w	8.08w	.0165w	59.0w	.00118w	6.75w	.0126w	36.1w
Measured	.00126w	6.75w	.0139w	51.8w	.00105w	6.65w	.0104w	39.4w
% diff.	9.3	16.4	15.8	12.2	11.0	1.5	17.5	9.1

TABLE 6

ESTIMATED AND MEASURED VALUES OF DEFLECTION AND STRESS FOR THE MODEL AND PROTOTYPE(1) FOR TWO RESIN-GLASS CLOTH LAMINATES.

Resin	Epoxy		Polyester	
Ratio	d/d_m	s/s_m	d/d_m	s/s_m
Estimated	11.83	7.3	10.8	5.35
Measured	11.03	7.63	9.9	5.84
% diff.	6.8	5.1	8.3	9.1

(1)Prototypes are E025050, P025050; models are E025100, P025100.

Examination of Table 6 will provide information of the validity of the model analysis. The agreement between estimated and measured values of the ratios for stress and deflection between model and prototype indicate whether or not the analysis can be accepted and the results of the tests projected to a full scale frame. The ratios are consistent for both resins. The measured ratios are below estimated ratios for deflection, above for stress. However, the differences between measured and estimated values are all below 10 per cent. Agreement within this range supports the decision that the analysis can be accepted and the results used in the design of a full scale structure.

It is necessary to examine the curves to get an estimate of the magnitude of stress and deflection which might be expected in a full scale frame based on the results of the model analysis. Referring to the original model analysis, the prediction equations for stress and deflection are; for model P025100, $s = 64s_m$, $d = 1024d_m$, if the length and cross section scales are used as given. Therefore, if we want to use measured stress and deflection values to predict frame behavior, we find, for a design load of 60#/ft of beam, that the stress in lower fibers at the crown will be 64(101.25) = 6480psi, and the deflection at the crown will be 1024(0.01575) = 16.1 inches. It should be remembered that in making the projection a 15#/ft load on the model is equivalent to a 60#/ft load on the frame, i.e., $w = 4w_m$. It is immediately apparent that the original frame design is extremely flexible and because of this would probably not be acceptable, even though the stress is quite low. This does not present any problem, however, as the design is easily modified to correct this defect. The point to be emphasized is that the model analysis is valid and information from the problem can be used to support future design estimates.

Long-term Loads

A study of Figures 22 and 23 supports the statement of Dietz, et al (14), that the materials do not exhibit any creep under long-term loads. The curves are representative of the elastic aftereffect as shown by the increase in strain with time after load application and the corresponding time dependent release of strain after unloading. The effect of temperature on material behavior is demonstrated by the cyclic nature of the values for deflection of P025050 after the rate of straining had reduced to almost zero. The observations were made at 0800 and at 1700 daily with the temperature difference between the two times about 20°F. This effect, though noticeable, is not of sufficient magnitude to be of any consequence in the design of farm buildings.

CONCLUSIONS

1. Epoxy and polyester resins reinforced with glass fibers produce a laminate uniform in mechanical properties suitable for use as a structural material. Hand wet layup of the laminate is possible although it is difficult to maintain good quality control unless the laminator has had considerable experience and the molds are carefully constructed. In addition to their advantages such as, rot resistance, water proofness, strength, etc., the reinforced plastics can be molded into an infinite number of shapes with ease.

SR-4 electric resistance type strain gages are very easily mounted on the laminates providing a very convenient means for measuring strain, and thus stress, in a structure built of the glass reinforced plastics.

2. The use of structural models permits investigation of hypotheses concerning mechanical phenomena in structures without the expense and difficulties encountered with a full scale unit. Distorted models cause no increase in difficulty over true models when considering the use of model analysis.

3. Agreement between model and prototype is within acceptable differences between measured and estimated values for the phenomena observed. The results of the model study indicate fibrous glass reinforced plastic resins can be used with confidence in farm building design and construction.

4. The structural test floor with the hydraulic loading system permits convenience, accurate control and rapid variation in load

application to any structure that is relatively narrow in dimension perpendicular to the floor and the direction of the applied loads. With the whiffle-tree type loading shoes, no difficulty was experienced in testing a section of two foot depth.

SUMMARY

The overall objective of the investigation was to determine the feasibility of using fibrous glass reinforced plastic resins in farm building construction.

A literature review was conducted to determine the suitability of the commercially available plastic resins for such uses. Two resins were chosen for use in the project - a modified epoxy and polyester resin. Plain weave glass cloth was used as reinforcement.

A rigid frame with a modified T cross section was developed as a building form which would provide clear span and allow use of the reinforced plastics as both skin and frame. The two foot depth of the unit would allow convenient building expansion with adjacent units easily welded together with resin and glass cloth.

A model analysis was made and distorted structural models were used to check the original design. Two models of different cross section scale were built of each resin, thus providing a method of 'proving' the model. The models were tested by applying short term uniformly distributed static loads through a system of hydraulic cylinders and special loading shoes, with deflection and strain measured at the crown.

Material constants were determined from samples cut from the flange laminate of each model frame. E values in bending determined from these samples were used to estimate deflections in the frames. Strains were sensed by SR-4 strain gages mounted on the outer surfaces of the stem and flange. Stresses were determined by multiplying the strain thus measured by the value of E determined from the samples.

Estimated and measured values of stress and deflection for the four models are shown in Table 5. Satisfactory agreement was obtained between measured and estimated values of both phenomena. Reasonable agreement was obtained between estimated and measured prediction factors for the model analysis; thus, the results of the tests on the model frames can be extrapolated with confidence.

SUGGESTIONS FOR FURTHER STUDY

1. Investigate the use of glass reinforced plastics for particular farm building applications such as tower silos.

2. Investigate building element cross section and layout to determine the most economical unit of reinforced plastics suitable for clear span construction for farm buildings.

3. Investigate the use of glass reinforced plastics in thin shell construction.

4. Investigate the possibilities of developing standard size prefabricated building elements of reinforced plastics which would permit flexible building arrangement.

5. Investigate the use of different forms of fibrous glass as plastic resin reinforcing materials.

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