DEVELOPMENT AND USE OF A VOLUMETRIC TRANSDUCER FOR STUDIES OF PARAMETERS UPON SOIL COMPACTION

By

Joseph Der Hovanesian

AN ABSTRACT

Submitted to the School for Advanced Graduate Studies of Michigan State University of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Department of Agricultural Engineering

Year 1958

Approved Wesley F. Buchele

JOSEPH DER HOVANESIAN AN ABSTRACT

Large machines and repeated traffic of modern farming are compacting our agricultural soils. The lack of information concerning the behavior of cultivated soils has hampered remedial efforts. In order to create a rational basis for the solution, an empirical study was conducted in which bulk density of soil was measured at various mean-stress loadings. According to a co-worker, bulk density is related to the mean-stress in soil. Physically, the mean-stress is found by taking the algebraic mean of soil stress in three mutually perpendicular directions at a point.

In experiments performed here, different mean-stress values were applied by means of hydrostatic pressure. Bulk density was measured in two different ways. First, soil was placed in a small rubber balloon which was connected to non-collapsible plastic tubing. Bulk density changes in soil were then accompanied by air displacement in the plastic tubing. The displacement was measured by movement of a mercury bubble in a calibrated capillary tube. Second, a strain-gage transducer was used which was designed and constructed for this purpose. The transducer was connected to the plastic tubing. Its ability to record small continuous volume changes was an important contribution to work presented in this thesis.

On the basis of tests performed with three soil types at various moisture contents, a general mathematical equation relating mean-stress with bulk density, was developed. The empirical equation, which is based on laboratory tests, depends on initial conditions and two constants. The initial conditions are: 1) The initial soil bulk density, 2) The initial mean-stress state in the soil. The two constants depend on soil parameters.

The empirically developed relationship connecting mean stress with bulk density was adapted for special uses which may aid in the design, selection and use of farm machinery for a given soil. Some of the applications of the adapted mean-stress versus bulk-density equations are: 1) To determine change in bulk density $\Delta\gamma$ occurring in a soil due to a mean-stress application from σ_o to σ_{max}, then relieved back to σ_o, 2) To determine the maximum mean-stress σ_{max} load that may be applied to a given soil if $\Delta\gamma / \gamma_{oc}$, the critical percent change in bulk density is given.

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P.O. Box 1346
Ann Arbor, MI 48106 - 1346

Joseph Der Hovanesian

candidate for the degree of

Doctor of Philosophy

Final examination, April 29, 1958, 8:00 A. M., Room 218 Agricultural Engineering Department

Dissertation: Development and Use of a Volumetric Transducer for Studies of Parameters Upon Soil Compaction

Outline of Studies

Major Subject: Agricultural Engineering
Minor Subject: Mathematics

Biographical Items

Born, August 14, 1930, Detroit, Michigan

Undergraduate Studies, Morningside College, Sioux City, 1949-50, Michigan State University 1950-53 (B.S.)

Major Subject: Agricultural Education

Minor Subjects: Physical Science and Shop

Graduate Studies, Michigan State University, 1953-54 (M.A. in Education) and 1954-58

Experience: Truck Mechanic, Reo Motor Company, Lansing, Michigan, 1951-54, Graduate Teaching Assistant, Michigan State University, 1954-56, Graduate Research Assistant, 1956-58.

Member of American Society of Agricultural Engineers; Sigma Pi Sigma, Physics Honor Society; Pi Mu Epsilon, Mathematics Honor Society; Society of the Sigma Xi.

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I. INTRODUCTION

The decline of virtually all historical nations of importance has coincided with problems of soil compaction. In many of the cases, for example, The Roman Empire, Eqypt, Mesopotamia, and others, the problems arose as a consequence of intensive irrigation without proper soil management practices. More recently, California has suffered the loss of 2,000,000 acres of Class I land because of excessive compaction, with another 2,000,000 acres destined to the same fate within a short period of time (5)*. An informed person can readily detect symptoms of excessive compaction in our own state of Michigan.

The causes of excessive compaction are somewhat different today from those which plagued historical nations. Of course, some of the same causes may still be factors, but because of more scientific knowledge about irrigation and drainage, particle-size distribution, energy of droplet impact, these practices are no longer the basic causes. The basic causes of modern-day compaction may be related to increased mechanization. While bigger tractors and implements appear to increase production efficiency, their long-run effect on soil compaction may be most disastrous. In Michigan for example, the trend toward more traffic and bigger machines of specialized farming has presented a serious problem. These problems are demonstrated by inadequate soil air move-

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*Numbers in parentheses refer to the "BIBLIOGRAPHY" on pages 76 to 77.

ment, reduced water infiltration, lowered cation exchange capacity and ultimately by reduced crop yields.

At the present time there are experimental tractors that weigh over 16,000 pounds. In the thumb area of Michigan, bean combines have been constructed by mounting old threshing machines on airplane wheels. These combines are pulled by large tractors and the combination applied concentrated loads on the soil. A reduction of dairy farming in this area has caused the removal of alfalfa, pasture and other soil restoring crops from the rotation, further adding to the problem. In another area of Michigan, a deficiency of nitrogen has been detected at locations where tractor wheels have repeatedly traveled on muck soils. Between the tractor tracks, no deficiency was observed. This indicates that among other things, compaction affects nutrient availability.

Many attempts have been made to alleviate compacted conditions. Some people have tried to throughly mix and pulverize soil, while others have resorted to deep tillage, chiseling and related practices. Again others have proposed limited or "minimum" tillage. They have all failed in their efforts because they lacked basic engineering information for the justification of their proposed solutions. The writer classifies these proposed solutions as of the "trial and error" type. It should be pointed out here however, that some aspects of the above practices may ultimately become accepted on the basis of scientific information. But first this information must be sought and disclosed.

The lack of adequate agricultural soil mechanics information has for some time handicapped workers who have attempted to find the solution (6). Civil engineers have used certain theoretical results, with some modifications, to predict and explain certain soil phenomena. Usually, these theoretical results were derived under the assumption that soil is an idealized continuous media, homogeneous, isotropic and elastic. Since civil engineers usually deal with highly compacted, coarse textured, relatively homogeneous and elastic soils, thus approaching the above idealized conditions, use of these theoretical results has been successful to some degree. Since agricultural soils are low in apparent density, varied in texture, non-homogeneous and inelastic, the same solutions used in civil engineering work are likely to lead to faulty reasoning and failure (12).

Indeed the ultimate solution depends on many aspects of research work. For example, a force applied to soils will set up a stress state at every point in the medium. The relationship between the force and the resulting stress state has not been clearly defined, nor has soil stress itself been defined completely. Further, if a certain stress state is applied to soil, it has not been possible to predict the resulting strain, which is a measure of soil compaction.

To see how forces are applied to soil by farm equipment, Lask (12) has instrumented a common tractor tire. With data to be collected from this tire, it is hoped that information can be obtained which may be used to predict what forces a tire applies to soil. With this kind of basic information, one may later be able to predict what stress states can be expected in soils due to tractor-traffic. Further instrumentation of other implements may render an accurate prediction of soil stress states that can be expected with essentially all field operations.

VandenBerg (20), using the model of a continuous medium for soils, was able to define soil stress. He pointed out that the stress of soil requires a set of quantities in the form of a stress tensor rather than a single value. The stress tensor can be separated into two components, namely, the mean normal stress tensor and the deviatoric stress tensor. The former is similar to hydrostatic pressure and can be easily measured in soil by taking the algebraic mean of stress in three mutually perpendicular directions at a point. This expression is an invariant being independent of coordinate-axis selection at the point. Mathematically it is called the "trace" of the stress tensor. VandenBerg (20) was able to show that volumetric strain in soil is related to the trace of the stress tensor.

In view of the importance of this relationship, it was decided to examine the bulk density versus mean-stress function carefully. Since this relationship can be expected to be a complicated function depending on soil type, soil moisture content, organic matter present in soil, rapidity at which a given mean stress is applied, and other factors, many instrumentation problems were involved. It was possible to apply various mean-stress states to a small sample of soil in a hydrostatic medium by means of a water-pressure chamber described in section IV. The next requirement was to find an accurate means for continuous measurement of bulk density in soil. Originally this was considered only as a minor problem, however, this phase grew into a major portion of the work presented in this dissertation.

Soil is a complex medium which is difficult to define. Its many physical properties are not well understood. It is generally recognized that it will be very difficult or even impossible to formulate adequate analytical laws relating bulk density with mean stress. Undoubtedly the description of bulk density versus mean stress in terms of a model will be just as difficult.

The first study was therefore an empirical approach. The bulk density was recorded for various mean-stress state applications for three different type soils at various moisture contents. The observed relationship of mean-stress versus bulk density was tried in various equations until a general expression was found which would be satisified by all the agricultural soils tested.

The writer hopes that the various aspects of work in soil compaction may soon be coordinated. The coordination of these results will be extremely important in an overall solution to this pressing problem. The effects of soil parameters on compaction can be more clearly understood and dealt with when basic information is presented. These results may serve not only as a guide to tractor-tire and farm-equipment design, but also may furnish a basis for renovation of already compacted soils.

Definitions of Terms and Symbols Used

AGRICULTURAL SOIL:	Natural medium for plant growth which consists primarily of the following fractions: 1. Sand (1mm-0.1mm dia. particles) 2. Silt (0.05mm-0.005mm dia. ") 3. Clay (0.002mm-0.00005mm dia. ")
SOIL TYPE:	Textural Classification based on ratio of above soil fractions.
BULK DENSITY (γ):	Apparent density of soil which includes air space in it's calculation. Bulk density is computed on a dry weight basis. The term is used in this work as an indication of "volumetric strain."
MEAN STRESS (σ) :	May be called the mean normal stress or the "trace" of the stress tensor. It is determined experimentally by averaging measured soil stress at a point in three mutually perpendicular direction. This value is an invariant being independent of coordinate-axis selection.
PARAMETRIC CONSTANTS:	Certain measurable constants which are characterized by specific soil parameters. A parametric constant as used in this work may be a function of one or several soil parameters.
INITIAL CONDITIONS:	Merely refers to values of mean stress (σ_o) and bulk density (γ_o) at instant when observation is begun.

TRANSDUCER: An instrument that is capable of converting a physical phenomenon into measurable electrical resistance or output.

SOIL COMPACTION: A widely accepted and somewhat inadequate word used to describe the many phases of "agricultural soil mechanics." The word must be translated in context since it may have varied meanings.

SOIL MOISTURE CONTENT: Percent water in soil calculated on a dry weight basis. Soil moisture was determined by oven drying and weighing for moisture loss.

1. Lower Plastic Limit: A widely recognized moisture content designation for soil. The lower plastic limit is the moisture content at which soil begins to lose its crumbly feel and shows a tendency to become plastic.
2. Air Dried: Equilibrum moisture level of soil with air. The condition of the air is taken as average room conditions and the soil is spread in such a manner to allow normal air movement throughout the soil medium.
3. Typical Field Conditions: A typical moisture content at which tillage operations may be performed.

II. REVIEW OF LITERATURE

The importance of the effects of soil parameters on compaction has been recognized by many people. The role of some of these parameters is described by Soehne in the following manner:

> "An area of compressive stress in the soil which has arisen from the rolling of tractor of field-wagon wheels over it depends upon the size of the load, the size of the contact surfaces between tires and soil, and the distribution of the surface pressure in these contact surfaces, as well as on the kind of soil, the moisture, and the density of the soil layers." (17)

According to Soehne, compressive soil stresses, resulting from an applied load, concentrate toward the load axis more with sandy soils than with the more pliable and cohesive soils.

Cohesive and adhesive properties of soil are dependent on moisture content. Several investigators made extensive studies dealing with properties of soil dependent on soil moisture (10, 13, 14). Other writers (2) have pointed out that soil plasticity is dependent on soil moisture in addition to being a function of particle size and shape. It was shown that soils generally become more plastic with increasing moisture, except for sand which possesses few plastic properties to begin with.

Atterberg(1) suggested three values be used to describe soil plasticity. These are: a) The upper plastic limit, the moisture content at which soil will barely flow under an applied load; b) The lower plastic limit, the moisture content at which the soil will barely roll out into a wire;

c) The plasticity number, the difference between the upper and lower plastic limits and it is used as an index of plasticity. These above values have gained widespread use and are known as the "Atterberg Limits." It is shown that the "plasticity number" as defined by Atterberg is itself dependent on many other factors besides soil moisture.

Russell (16) reported that a linear relationship exists between plasticity number and clay content (5 u particles). Other factors that have been given are: a) Nature of soil minerals, b) Chemical composition of the colloid, c) Nature of exchangeable cations and d) Organic matter. To the contrary, Baver (3) showed that the plasticity number decreased as soil organic matter is increased.

Of more direct interest to work presented herein, Terzaghi (19) reported that compression of soil increases rapidly with moisture above the lower plastic limit. He showed that the compression is low when dealing with soils at low moisture contents. In agreement with above findings, Bekker (4) , in doing research work dealing with military land locomotion, shows a very direct relationship between vehicle sinkage and moisture content. According to Bekker, Russian agricultural engineers (7) generalized the formula: $P = kz^n$. P in this expression represents "ground pressure," z sinkage, n is an empirical exponent reflecting the ratio of strain change with load and k is a constant which is a function of not only the soil but also the nature of load application and area.

According to R. R. Proctor (15), for a given applied compaction energy, soil bulk density will increase almost proportionally to moisture content up to a point. The point where this relationship deviates from proportionality is when the moisture content approaches a "saturation point." This above relationship is known as the "Proctor Curve." It is used as a guide to determine a soil moisture level that allows a given compaction energy to yield a <u>maximum</u> compaction state or soil density. From a civil engineering standpoint, this knowledge is important, but it is equally important from an agricultural engineering standpoint. However, <u>minimum</u> compaction is usually desired during a given field operation since excessive compaction is a problem in agriculture. Thus an agricultural engineer would seek a soil moisture level which will result in <u>minimum</u> compaction for a given energy input.

Spangler (18) reports that a logarithmic relationship was found to exist between a voids ratio (e) of soil versus applied pressure (p). He plotted the logarithm of pressure (abscissae) versus the voids ratio (ordinate). This curve yielded a straight line of negative slope. The two parametric constants that accompany any straight line are in this case dependent on initial conditions and soil parameters (% moisture content, soil type, organic matter in soil, soil structure, type of applied load area, etc.). In agricultural engineering, bulk density (γ) is used instead of voids ratio (e). (γ) and (e) are almost inversely related (i. e., $\gamma \propto 1/e$), thus if (γ) were plotted versus pressure (p), the resulting straight line reported by Spangler would be preserved but with different parametric constants. Spangler indicated that this relationship may not always conform perfectly to a logarithmic type. This, he said is due to prior loading, geological history and other factors.

According to Spangler, strain may be a function of time as well as stress. His time-compression curves showed that bulk density of soil increased with time for a given load. However this curve flattens if sufficient time elapses. For permeable coarse-grained soils, only 1 second is required in order for the curve to flatten. For impermeable fine-grained soils, several seconds may be necessary. His reason for this was that some of the water contained in the voids of soils has to be squeezed out before the volume of the voids can decrease. The rate of outflow of this pore water depends on the permeability of the soil.

Hoegentogler (8) in his discussion of structural properties of soil indicated that soils may be expected to perform differently when subject to moisture content variation. He further indicated that if an applied load to soil is released, the soil will rebound somewhat, but some permanent deformation will remain since soil is not perfectly elastic.

III. DESIGN AND DEVELOPMENT OF A RECORDING VOLUMETRIC TRANSDUCER AND AN INDICATING VOLUMETER

The Indicating Volumeter

The indicating volumeter Figure 1 and Figure 3 consists principally of a capillary tube connected to a spherical shaped rubber balloon (approximately 3 c.m. diameter). A non-collapsible plastic tubing is located between the tube and balloon and connects the two. When the balloon, filled with soil, is surrounded with a large quantity of the same type soil, the encompassed minute volume in the balloon represents a point. When the soil medium is subjected to stress, its decrease of volume at the point mass is easily determined by the displacement of a mercury bubble in the capillary tube since the volume/linear unit of displacement of the capillary tube is known.

The volume of the capillary tube/cm displacement was measured by two means. The first method, which proved unsatisfactory, consisted of determining the weight of water in the capillary tube, and thus obtaining volume/cm of the length. A more accurate means of calibration was employed by using a one milliliter pipette (accurate to 0.01-ml) connected to the capillary tube. A known volume of water ΔV was next transferred from the pipette into the capillary tube, at which time ΔL, the displacement of a column of water in

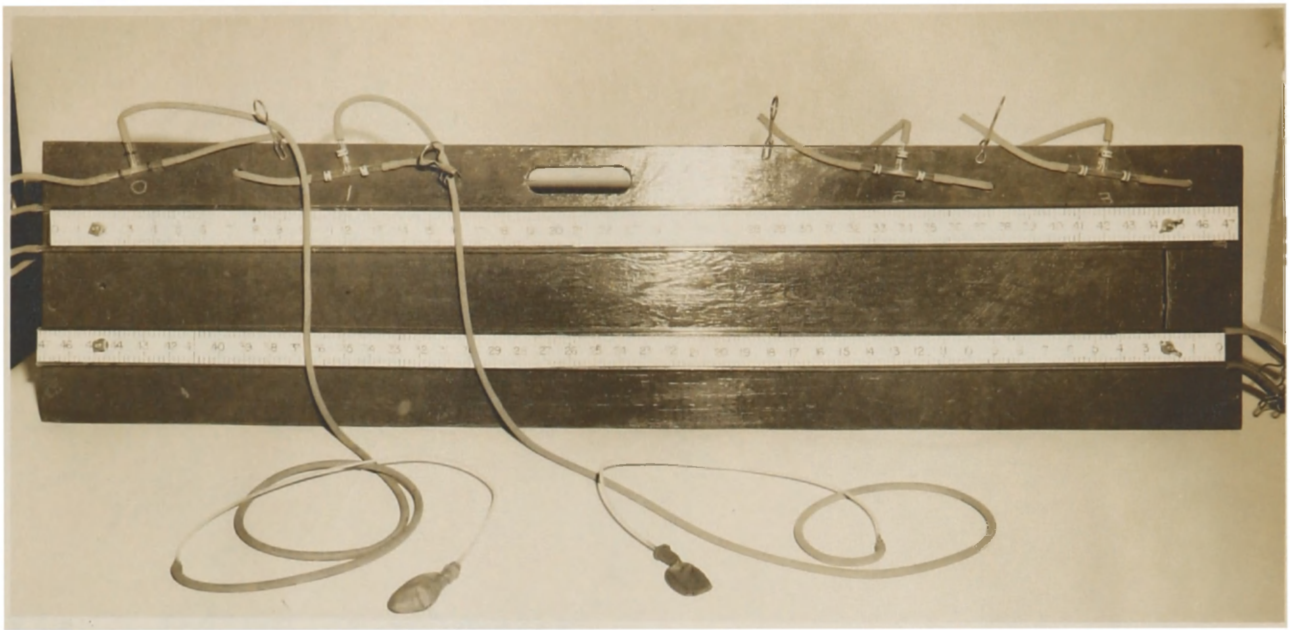


Fig. 1. Volumeter photo

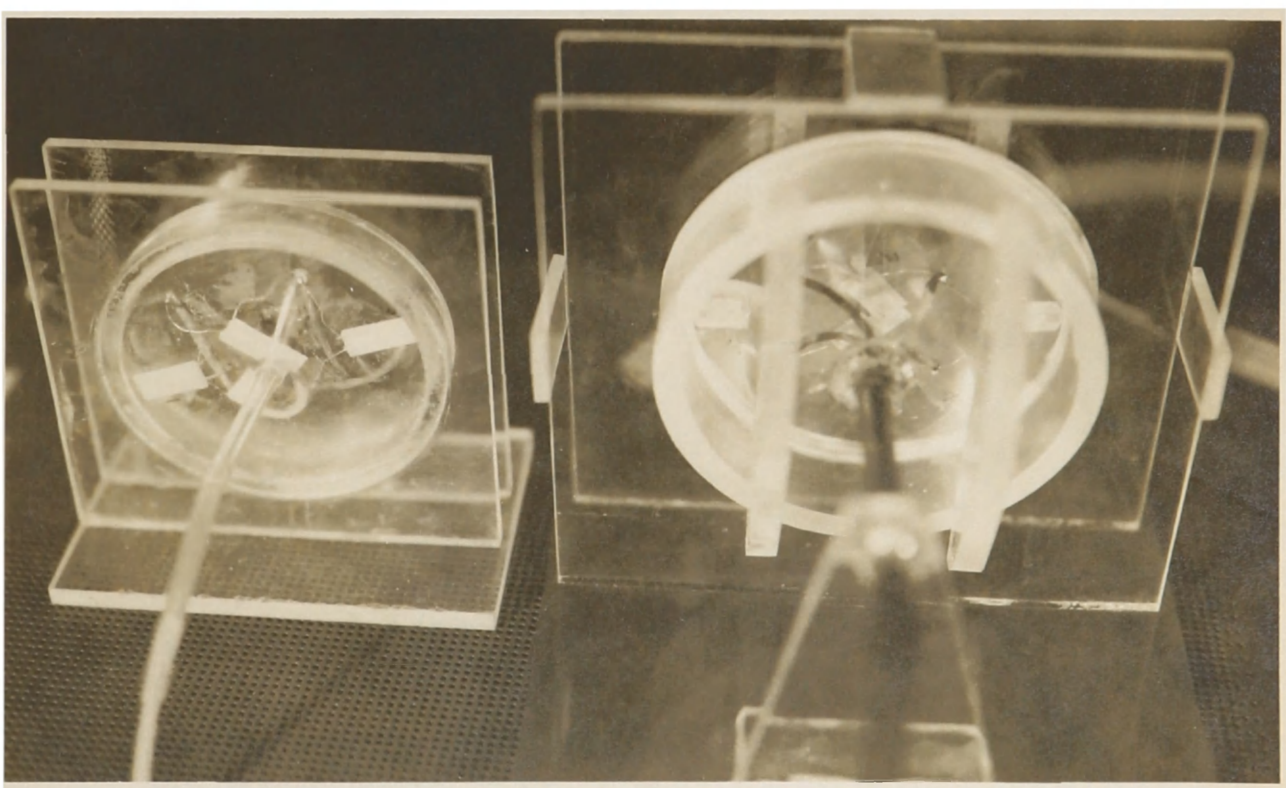


Fig. 2. Volumetric Transducer photo

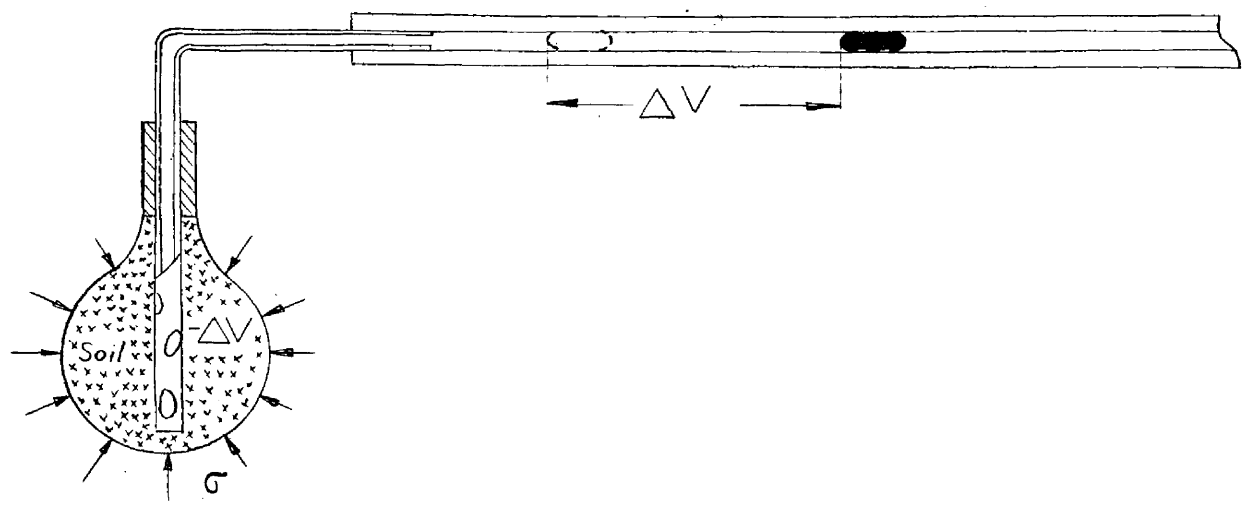


Fig. 3. Indicating volumeter

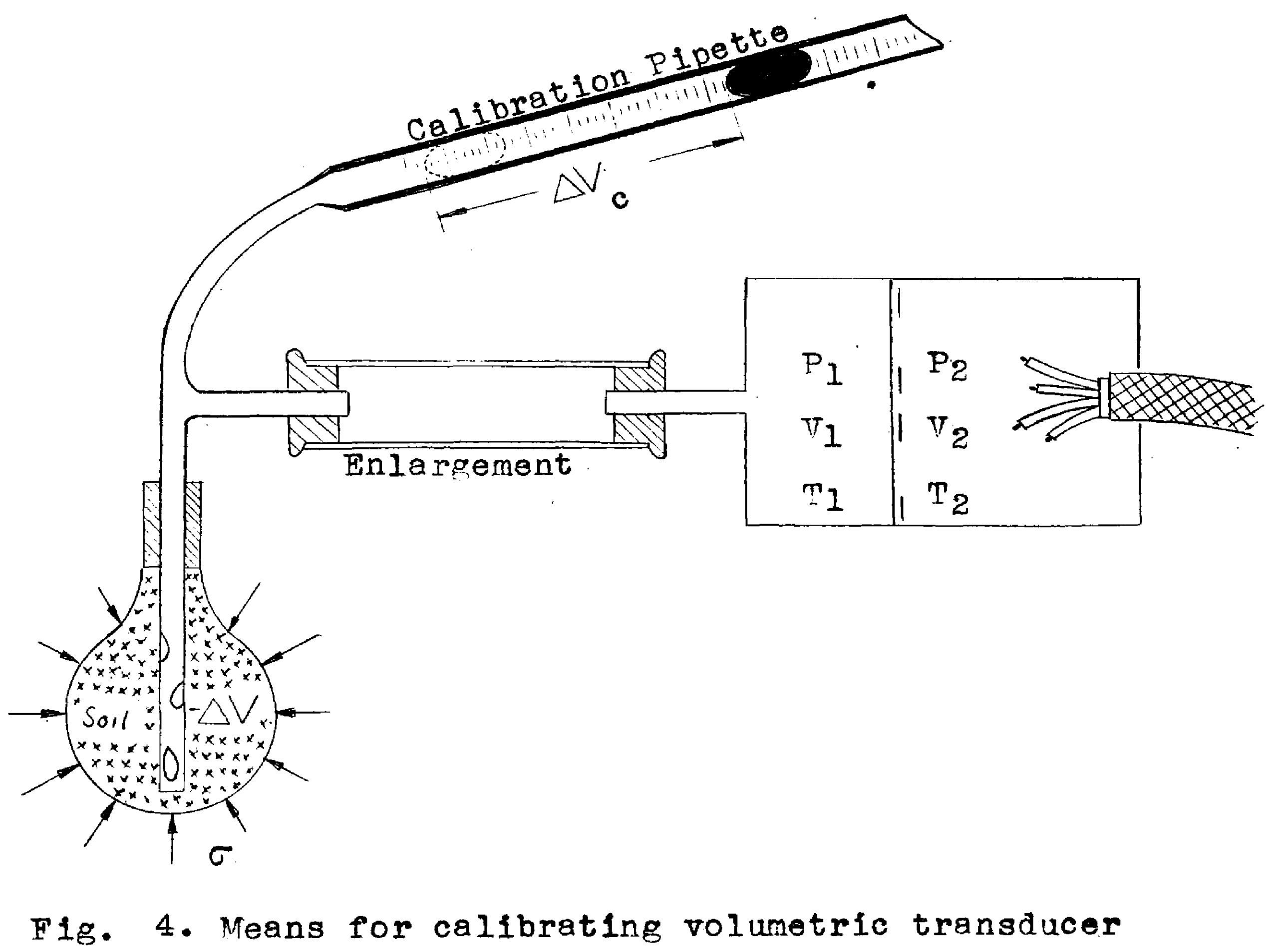


Fig. 4. Means for calibrating volumetric transducer

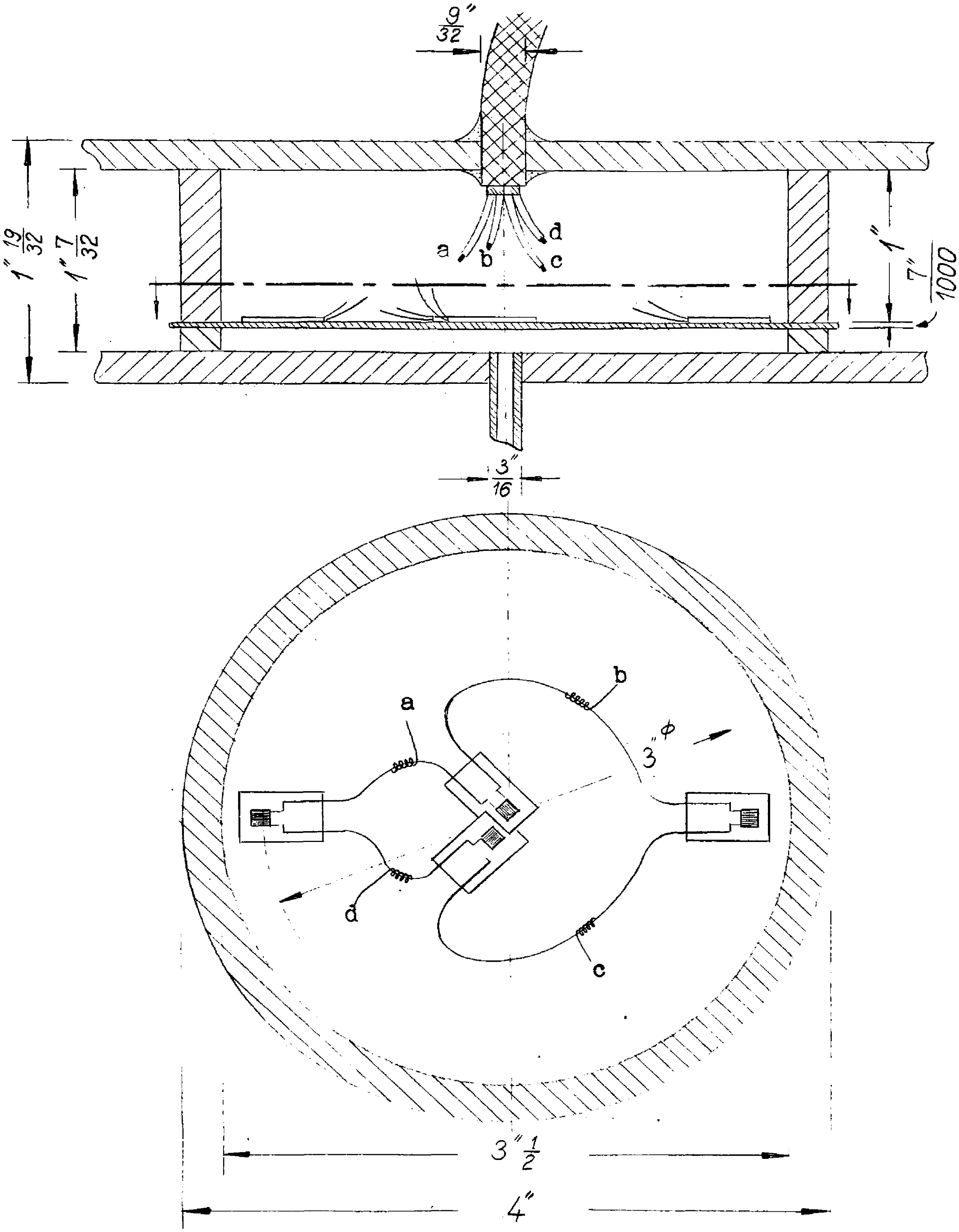


Fig. 5. The Volumetric Transducer

the tube, was measured and recorded. Now since ΔL and ΔV are known, the calibration ratio is simply $\Delta V/\Delta L$.

The Recording Volumetric Transducer

The recording volumetric transducer is an instrument which is capable of recording minute volume changes in soil. It is more accurate and sensitive than the volumeter, and it has the additional advantage of being adaptable for use with conventional strain-gage recording equipment.

The transducer is an extremely sensitive pressure-sensing element. Figure 5 shows how this element was constructed. The skeleton sketch of Figure 4 shows the working features of the sensing element and the soil sample under test.

Basic Principles of Transducer

The soil sample (encompassed with balloon) and the space in the pressure side of the transducer are represented by V, with V_1 representing the initial volume of the system as in Figure 4. Thus:

$$V = V_1 - \Delta V$$

When ΔV occurs in the soil as a result of σ

And

$$P = P_1 + \Delta P$$

Since

$$P_1 V_1 = PV = (P_1 + \Delta P)(V_1 - \Delta V), \text{ finally,}$$

$$\Delta V = V_1 \Delta P/(P_1 + \Delta P) = (V_1/P_1)\Delta P \qquad (1)$$

Hence, the above derivation shows a proportionality between volume change and pressure change in the transducer system. Since the strain resulting in a rigidly supported circular thin plate is proportional to a uniformily applied load (air pressure in this case), the resistance changes occurring in the SR-4 strain gages are directly proportional to ΔV. Since the ultimate goal was to measure ΔV, this transducer was used directly as a "volumetric transducer" for measurements in soil.

Two Models of Transducers

Two models of the transducer were constructed as shown in Figure 2. The model on the left is practically identical to the one on the right except that its sensing diaphragm is a Plexiglas element 0.04 inch thick. The other model employes a stainless-steel diaphragm 0.007 inch thick. The other portions of the transducer were constructed from Plexiglas.

Linearity of Transducers

The linearity of the transducers were tested by means of imposing known volume changes ΔV into the system. This was accomplished by use of a 1-ml pipette, accurate to 0.01-ml. By means of the pipette, an imposed ΔV was compared to the resulting ΔR of the strain gages. This was repeated several times for several values of ΔV ranging from 0 to 1-ml. Subsequent data were collected for volume changes from 0 to 5-ml by means of a larger pipette. The results of these tests showed excellent linearity for ΔV versus ΔR in the above range. It was possible to predict ΔV's within less than 0.01-ml error from readings of ΔR's for the entire range of interest. See APPENDIX B.

Calibration and Sensitivity of Transducers

The extremely high $\Delta R/\Delta V$ ratio of the transducers rendered possible the measurement of very small ΔV's. Such sensitivity was unnecessary for measurements of ΔV's in the range of 0.1-ml (as needed in this work), and it was reduced in two ways. The first way was to increase V_1, the initial air volume of the pressure side of the transducer, connecting tubes and soil. Since it is difficult to change the volumes of the transducer and soil, V_1 was increased by adding an enlargement to the connecting tube as shown in Figure 4.

Because:

$$\Delta P = (P_1/V_1)\ \Delta V$$

An increase in V_1 reduces the proportionality constant P_1/V_1. Since ΔR is linearly related to ΔP, the $\Delta R/\Delta V$ ratio (sensitivity) was reduced when V_1 was increased. The second method of reducing the sensitivity of the transducer system was to merely reduce the gain on the strain-gage amplifier.

Calibration was accomplished by a direct method. The procedure was as follows: The initial preload σ_o

was applied to the sample in order to eliminate void spaces between the balloon wall and the soil sample. (It may be again mentioned here that the same preload, $\sigma_o = 1$ psi was applied to all samples in order to standardize the tests) Next an imposed calibration volume change ΔV_c was introduced into the system by means of a 1-ml calibration pipette and a bubble of mercury as shown in Figure 4. Hence, the gain of the strain-gage amplifier was adjusted so that the pen deflection was some convenient multiple of ΔV_c. Since subsequent changes in volume will be recorded linearly, the calibration involved only these steps.

As a result of collecting data from several samples of soil, each varying somewhat in initial volume, it was discovered that very slight modification in amplifier gain adjustment was necessary from sample to sample. This was easily understood, since the initial air volume of the soil is very small compared with the volume of the tubes, enlargement, and pressure chamber of the transducer. This can be shown better if one looks at the proportionality constant P_1/V_1 which relates ΔP and ΔV. Since V_1 is almost a constant volume being affected very slightly by small variations between different soil samples, the ratio P_1/V_1 remains nearly unchanged. Use was made of this by adjusting the size of V_1 (by altering the tube enlargement) such that the same calibration factor could be used for the transducer independent of the small air volume of the soil. This meant that the pipette calibration method was used only once, subsequent calibrations were accomplished by adjusting the amplifier gain to the same position for all samples. This can be justified as follows:

$$\Delta P = (P_1/V_1)\,\Delta V \qquad (1)$$

The reader will be reminded that ΔP is the physical change indicated on the recorder as a result of ΔV. It is desired to minimize the error in ΔP resulting from δV_1. This is accomplished by selecting a V_1 sufficiently large in the following manner:

From equation (1)

$$\ln \Delta P = -\ln V_1 + \ln P_1 + \ln \Delta V \qquad (1a)$$

Thus:

$$\pm\,\delta(\Delta P)/\Delta P = -\,\delta(V_1)/V_1 \qquad (2)$$

If δV_1, the air volume variation expected between soil samples is known (usually much less than 4-ml), and it is desirable to keep the error in ΔP, i.e., $\delta(\Delta P)/\Delta P$ less than say 1% then:

$$\delta(V_1)/V_1 \leq 0.01$$

or

$$V_1 \geq \delta V_1/0.01 = 4/0.01 = 400\text{-ml}$$

Hence a constant calibration factor may be used. In practice, the gain was adjusted to the same value by making use of a shunt calibration resistor built into the strain-gage amplifier. For example, if originally the gain was adjusted to obtain 10 lines recorder deflection for a 1-ml

ΔV_c, one would depress the amplifier calibration switch (this places a 390-K ohm resistor in parallel with one of the strain gages in the bridge) to observe what deflection will result. This latter deflection can be considered as a "simulated volume change." Thereafter, one may calibrate the system on the basis of the simulated volume change as long as changes in V_1 do not exceed 4-ml. If one expects changes in V_1 to exceed 4-ml, a larger V_1 can be selected by the above means.

Temperature Compensation and Effects on Transducer

If the reader will again review Figure 5, he will notice that the strain gages are mounted on the transducer diaphragm in such a manner that all four gauges are active components of a bridge network. Of course this is advantageous from a sensitivity standpoint since four times more sensitivity is attained.

This type of circuit is additionally desirable because of its inherent electrical temperature compensation since $(\partial R/\partial T)\Delta T = \Delta R$ will be identical for each arm of the bridge thus leaving the balance unchanged when temperatures flucuate.

Another important consideration is the effect of temperature on air expansion in the system. This effect may be analyzed in the following way:

$$\Delta P_i = \frac{\partial P_i}{\partial V_i}\Delta V_i + \frac{\partial P_i}{\partial T_i}\Delta T_i \qquad ; \; i = 1, 2^*$$

*See Figure 4

Since:

$P_iV_i = n_iRT_i \quad ; i = 1, 2$

and if: $\Delta T_i = 0$, and $\Delta V_2 \simeq 0$; $i = 1, 2$

$\Delta P_{net} = \Delta P_1 - \Delta P_2 = (P_1/V_1)\Delta V_1$

when: $\Delta V_2 \simeq 0$, and $\Delta T_1 = \Delta T_2 = \Delta T \neq 0$

$\Delta P_{net} = (P_1/V_1)\Delta V_1 - R(n_2/V_2 - n_1/V_1)\Delta T$

If: $P_1 = P_2$, then $n_2/V_2 = n_1/V_1$ since $T_1 = T_2$

Thus:

$$\Delta P_{net} = (P_1/V_1)\ \Delta V_1 \qquad (3)$$

Equation (3) shows that ΔP_{net} is dependent only on the proportionality constant P_1/V_1 and the volume change ΔV_1 on the pressure side of the diaphragm and independent of temperature change ΔT. Of course this is only true if $P_1 = P_2$.

With data presented in this investigation, no special effort or provision was made to make $P_1 = P_2$, thus the term $R(n_1/V_1 - n_2/V_2)$ was not equal to zero. Since data were collected under laboratory conditions where temperature variations were negligible, temperature effects were not considered.

This section dealing with temperature effects and temperature compensation is included here to show that the volumetric transducer may be used successfully where larger temperature variations may be expected. This will involve few additional efforts as pointed out above (namely $P_1 = P_2$), however insufficient data dealing with temperature effects have been collected to verify this prediction experimentally.

Pressures Developed in Soil

Air pressure in natural soil is of course equal to the atmospheric value, namely about 14.7 psi. It is therefore desirable to maintain soil air pressure comparable to natural conditions so that there will be little interaction of the means of measurement (pressure change ΔP) with volume change ΔV. If ΔP is held less than those values expected from natural variations in atmospheric pressure, a serious error will not result in the measurement of ΔV.

$$\begin{aligned} \Delta P_{max} &= (P_1/V_1)\,\Delta V_{max} \\ &= (14.7/400)(9) = 1/3 \text{ psi} \end{aligned} \tag{4}$$

Experimentally, the ΔP_{max} was found to be only about 1/4 psi because V_1 was larger than 400-ml. Thus, pressures developed in the soil sample will probably not create any serious error. The largest % error is $\frac{\Delta P_{max}}{\sigma_{max}} \times 100 = 1.1$ %.

Comparison of Volumeter and Transducer

Figure 6 shows a comparison of measurements made with the volumeter and the volumetric transducer. During the compression cycle of mean-stress loading, the ΔV versus σ relationship was practically identical for both means of measurements as may well be expected. During the relaxation phase, a marked variation in readings between the volumeter and transducer was observed. This can be explained by noting

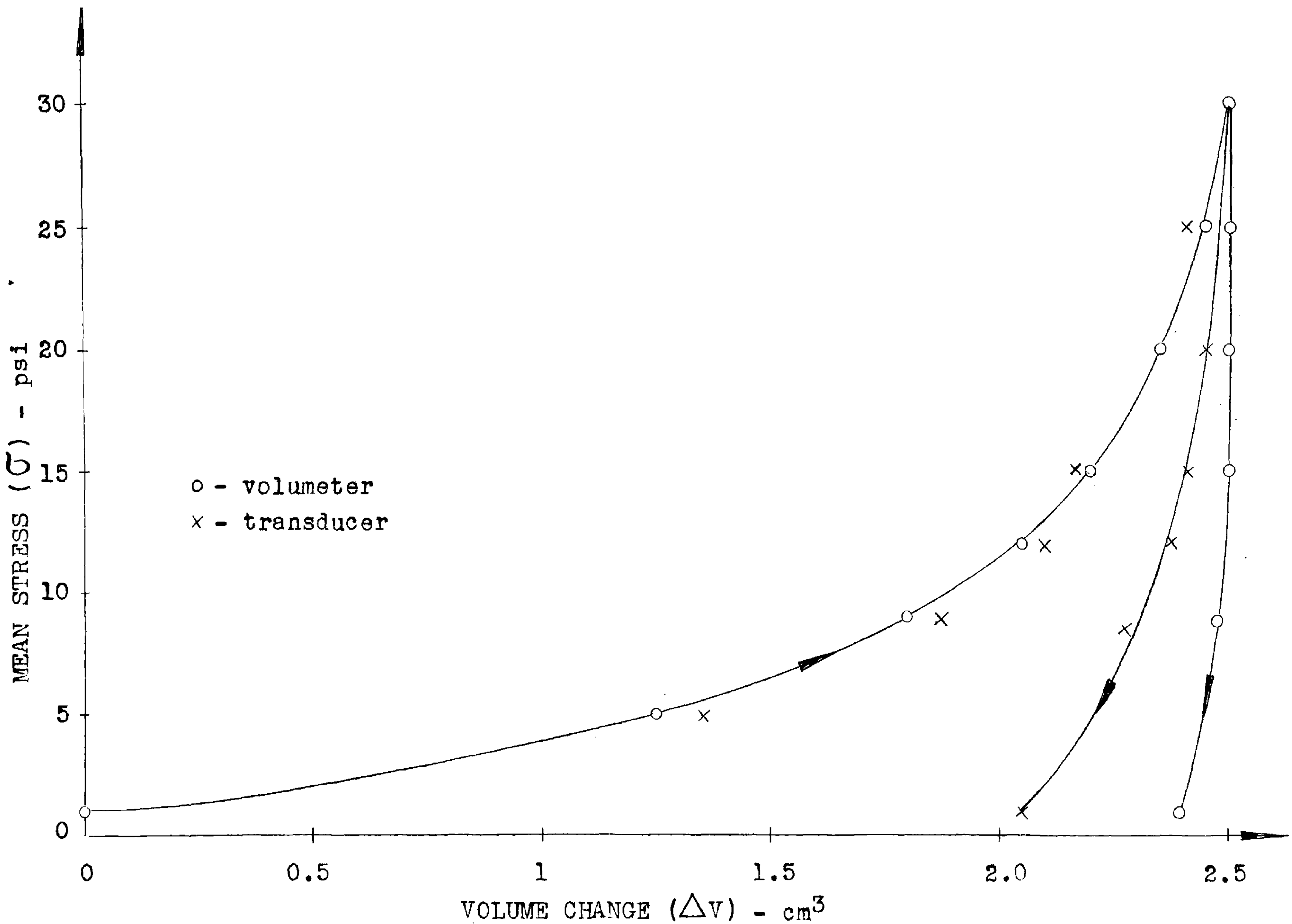


Fig. 6. Volume change measurements by volumeter and volumetric transducer

a characteristic of the volumeter. Despite the fact that the capillary tube of the volumeter was mounted horizontally, a slight constant pressure was necessary for the displacement of the mercury bubble. When the volume change has reached a maximum and reverses, the slight constant pressure also reverses which results in a lag as seen on Figure 6. After this lag, the volumeter indicates properly. Since the transducer does not possess this characteristic, this lag was absent and accurate measurements of end conditions were recorded.

The volumeter is considered a valuable instrument for soil compaction research work despite this one characteristic. Much of the work in soil compaction research deals only with the compression cycle, thus being well within the accurate operating range of the volumeter. The indicating volumeter offers the advantages of being simple, inexpensive, and easy to operate. Some dissadvantages of the volumeter are: Inaccuracy during relaxation measurements, unsuitability for dynamic measurements, non-recording, and somewhat bulky.

The volumetric transducer was an excellent instrument, particularily for work in this investigation. Its compactness, linearity, sensitivity and recording features made the work presented in this thesis possible.

IV. DEVELOPMENT OF A LABORATORY TECHNIQUE FOR THE STUDY OF THE EFFECTS OF PARAMETERS ON SOIL COMPACTION

The General Technique

Data were obtained with the recording volumetric transducer described in the previous section. Various mean-stress states were attained quite simply by applying hydrostatic pressure to a small sample of soil (about 15 gm) contained in a small rubber balloon as shown in Figure 2 and Figure 4. The hydrostatic pressures were controlled by means of a regulator valve and pressure indicating system. The pressure indicating system consisted of a mercury manometer for pressures from 1 to 10 psi, and a dead-weight calibrated bourdon tube gage from 10 to 30 psi.

It must be pointed out here that the soil samples were "disturbed samples." Of course it would have been extremely desirable if data could have been taken on undisturbed natural soil as in the field, but at the present time this is impossible since no adequate means of measuring specific volume at a point in undisturbed soil has been devised. Even recently plowed land cannot be considered as undisturbed because the action of the plow causes considerable disturbance. Since traffic over recently-plowed land contributes to excessive compaction, it was decided to approximate conditions of recently-plowed soil. Modern farming is based on the growth

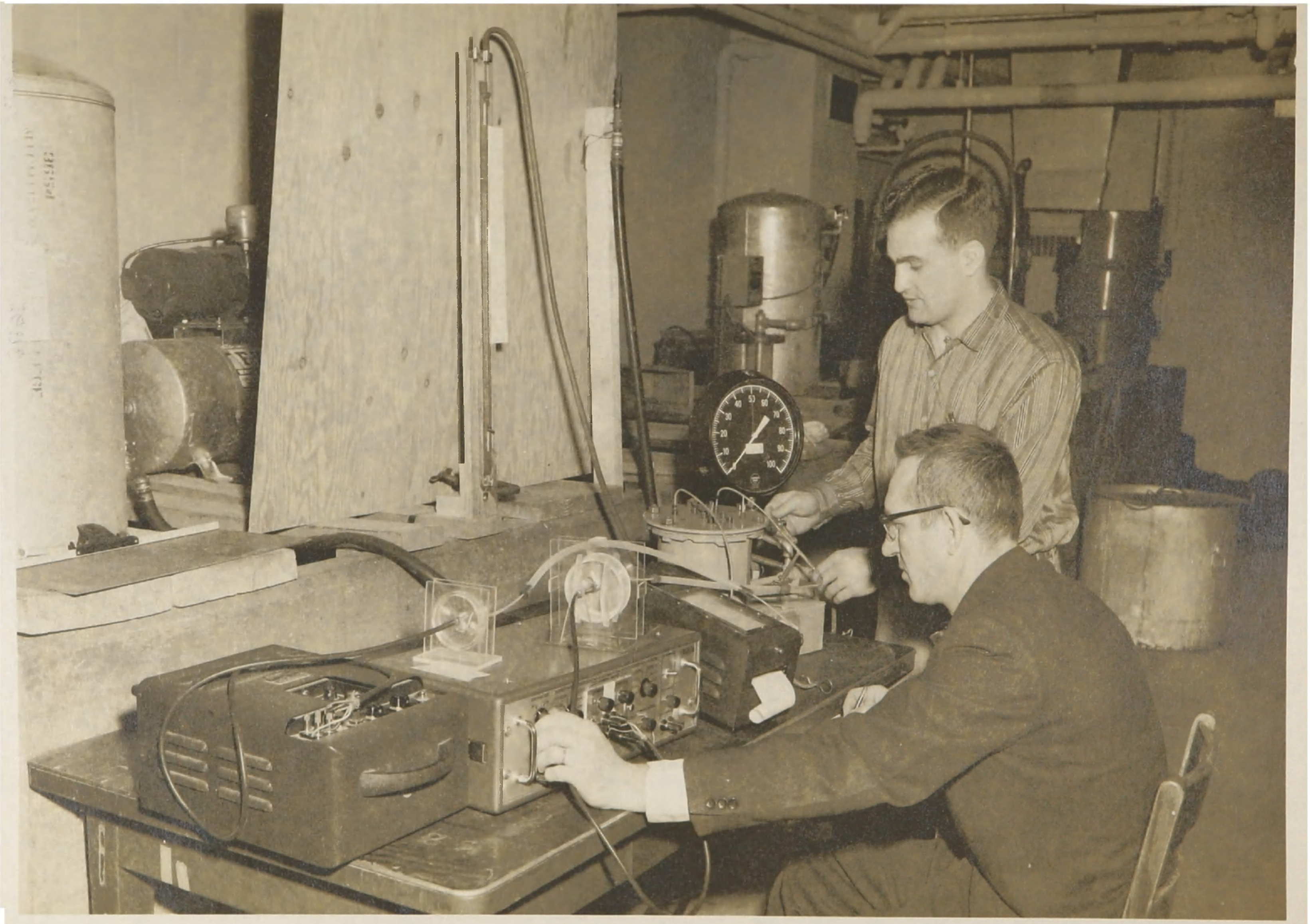


Fig. 7. General view of test equipment

of plants in disturbed (cultivated) soils.

Soils that were tested were crumbled to a density state approximately equal to freshly-plowed soil. The sample was then preloaded to 1 psi to standardize tests by bringing all samples to the same stress state. Large unnatural air pockets were eliminated by this procedure and the balloons took the shape of the enclosed soil mass. After the soil was preloaded to 1 psi, the sample was removed from the pressure chamber and the initial volume at 1 psi was determined. This was accomplished by a series of weighings in and out of water. Next, the sample was replaced into the chamber and tests were continued from 1 to 30 psi. An upper limit of 30 psi was selected since mean-stress states in actual agricultural soils have seldom been found to exceed this value (21).

Impact Loading

The apparatus used for impact loading consisted of many elements of the equipment ulilized for the general technique. With this phase of the work, it was desired to maintain a constant preload σ_0 of 1 psi before and after impact applications. This was done as shown above in Figure 8 by maintaining a constant water head (equivalent to 1 psi) above the pressure chamber containing the soil sample. Mean-stress impact-loading was accomplished by immediate application and removal of air pressure above the water head as shown in Figure 8.

Upon inspection of the data recorded by the oscillograph, it was seen that the average duration of load application from 1 to 27 psi took about 1/2 second. Likewise, 1/2 of a second was required to relieve the load from 27 to 1 psi. Hence, the total time required to apply and relieve the load (impact) was equal to 1 second.

Figure 9 shows how a tractor tire may apply impact loads to soil. The time ΔT required for the tire to rotate $\Delta\theta$ degrees may be taken as an approximate duration of loading applied by the tractor wheel when $\Delta\theta$ is the soil contact angle. Then ΔT can be expressed in terms $\Delta\theta$, V, the forward speed of the tractor, r, the radius of the wheel and h, the height of the tire lug:

$$\Delta\theta = (V/r)\Delta T = 2\cos^{-1} r/(r + h) \qquad (5)$$

For a typical case where r = 24", h= 1" and V = 5mph:

$$\Delta T = 0.15 \text{ second}$$

The above analysis shows that impact conditions as imposed by a tractor tire were approximated to some degree by the impact loading technique used in the laboratory.

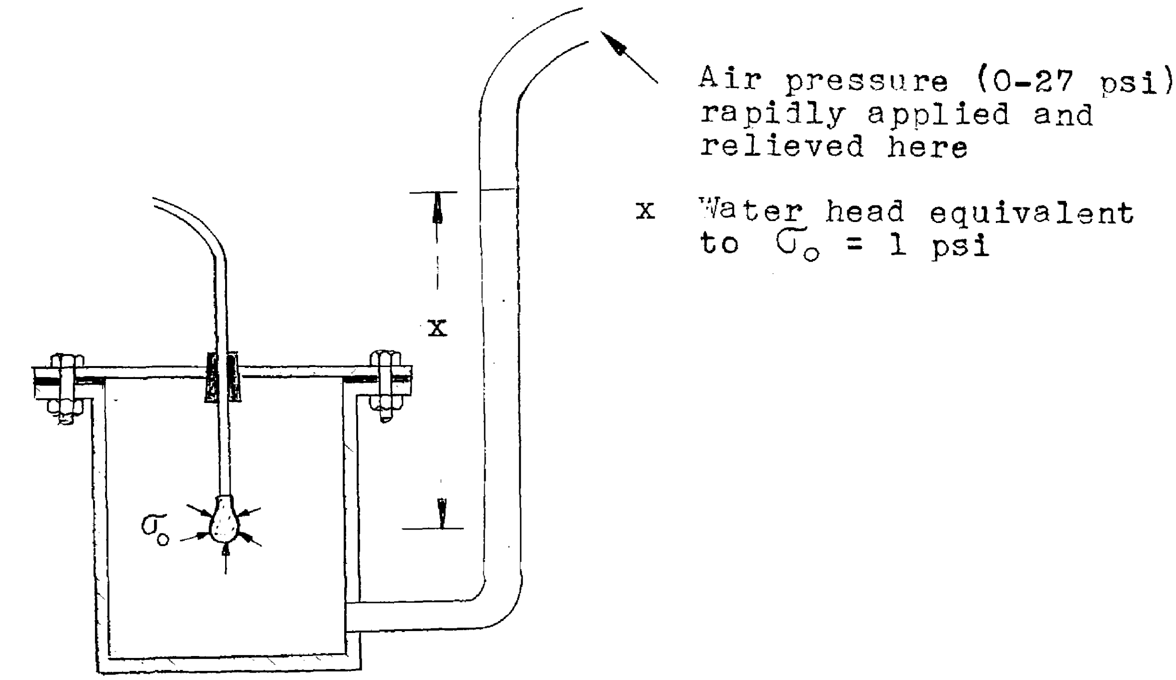


Fig. 8. Impact loading mechanism.

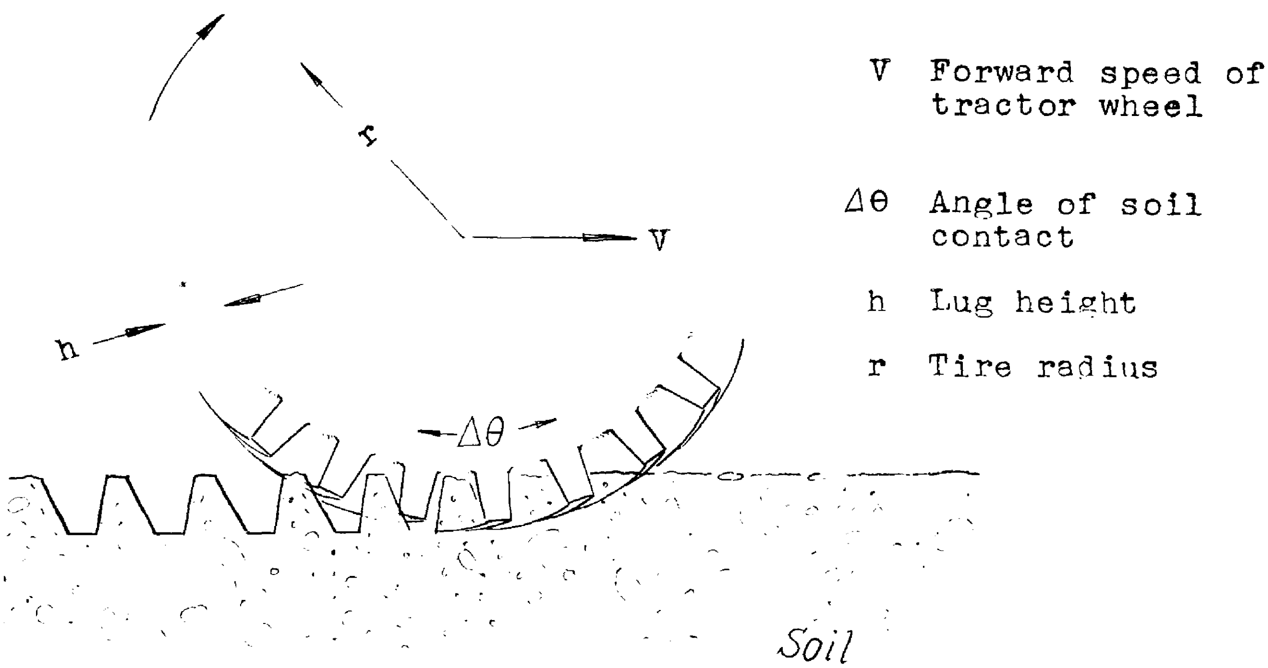


Fig. 9. Analysis of impact loading applied by a tractor tire.

V. PRESENTATION AND ANALYSIS OF LABORATORY DATA

Tests of Clay Soil

Table 1 shows how bulk density changes with a given mean-stress σ application from 1 to 30 psi. Data were compiled from a clay soil at two moisture content levels; 30.9%, the lower plastic limit; and 8.0%, the air-dry equilibrium point. The reader will note a remarkable similarity between Figure 10 and Figure 11. The former is a curve of γ versus σ for the air-dry soil, while the latter represents the same clay soil at its lower plastic limit.

This similarity can be more readily seen if the reader will look at Figure 12. Here γ versus σ was plotted for both moisture levels on semi-logarithmic paper. Since $\frac{\partial \gamma}{\partial (\ln \sigma)}$ is constant as shown on Figure 12, it is seen that γ is a logarithmic function of σ with certain parametric constants. Thus we may express γ in terms of σ in the following manner:

$$\gamma = \gamma_o + B \ln \sigma/\sigma_o \tag{6}$$

Where:

γ_o - The initial bulk density

σ_o - The initial mean-stress state for the particular soil

B - A parametric constant.

It will be noted that the values of γ_o and B also depend on whether σ is being increased or decreased, hence

TABLE 1

Applied load (σ) versus bulk density (γ) for clay soil at two moisture levels

Applied load	Moisture content of soil	
	8.0 % water*	30.95 % water**
(psi)	(gms/cm3)	(gms/cm3)
1	1.30	1.12
2	1.35	1.21
4	1.38	1.35
6	1.38	1.43
8	1.39	1.52
10	1.40	1.59
15	1.42	1.70
20	1.44	1.76
25	1.46	1.83
30	1.47	1.87
25	1.47	1.87
20	1.46	1.85
15	1.46	1.83
10	1.45	1.80
8	1.45	1.78
6	1.44	1.77
4	1.43	1.74
2	1.42	1.67
1	1.40	1.58

* Air-dry equilibrum point of soil
** Lower plastic limit

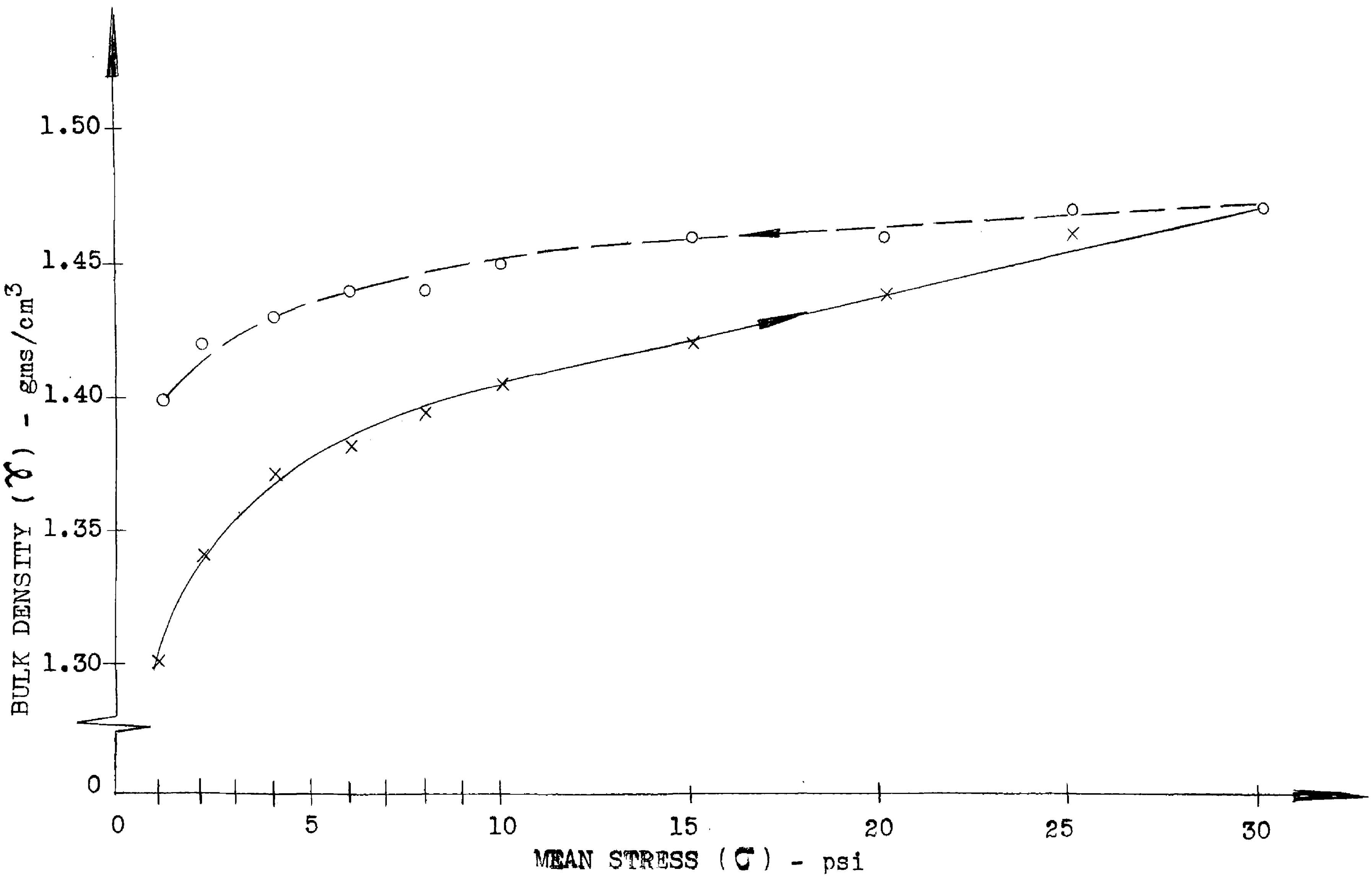


Fig. 10. Bulk density (γ) versus mean stress (σ) for 8 % moisture content clay soil (air dried).

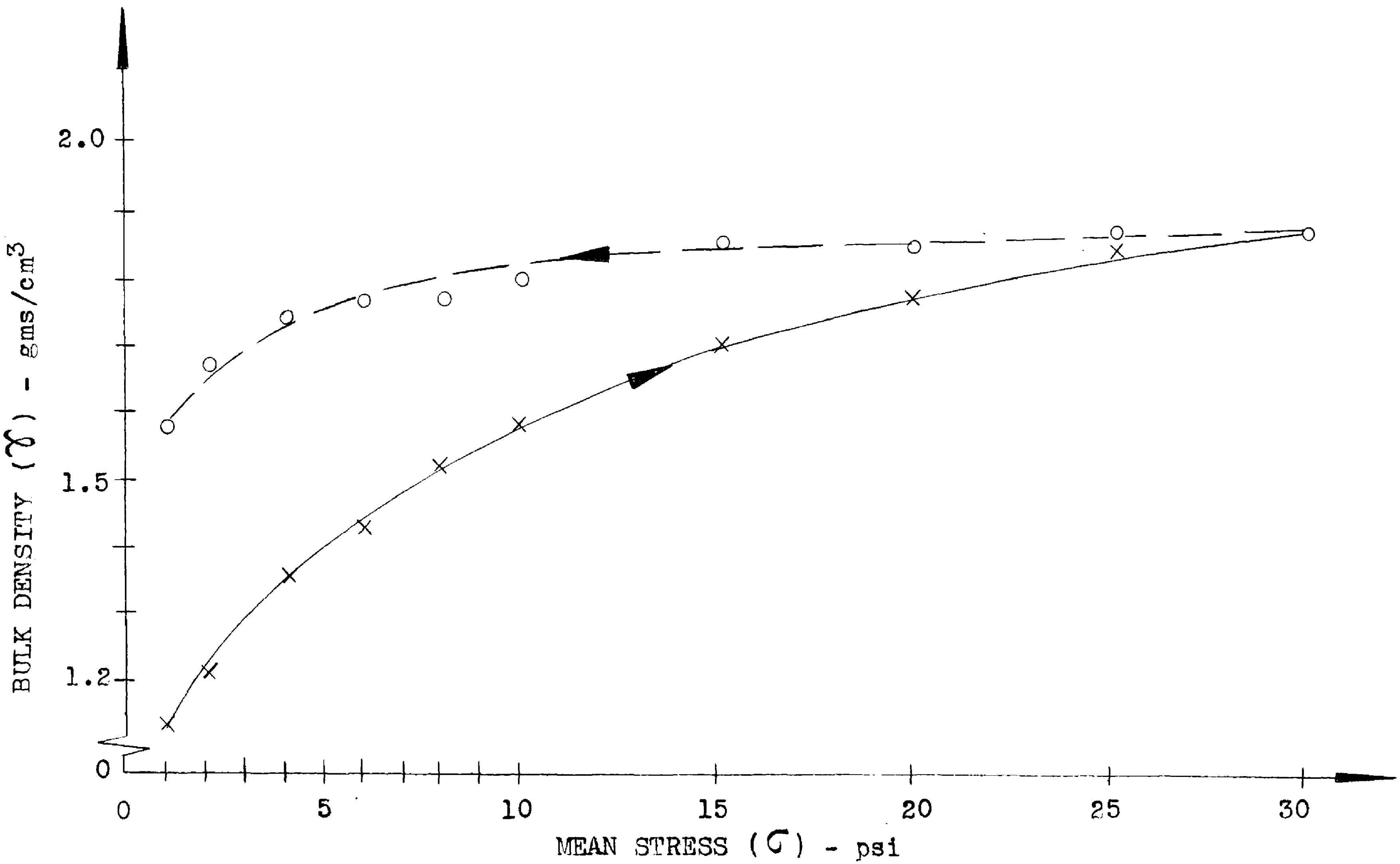


Fig. 11. Bulk density (γ) versus mean stress (σ) for 30.9 % moisture content clay soil (lower plastic limit)

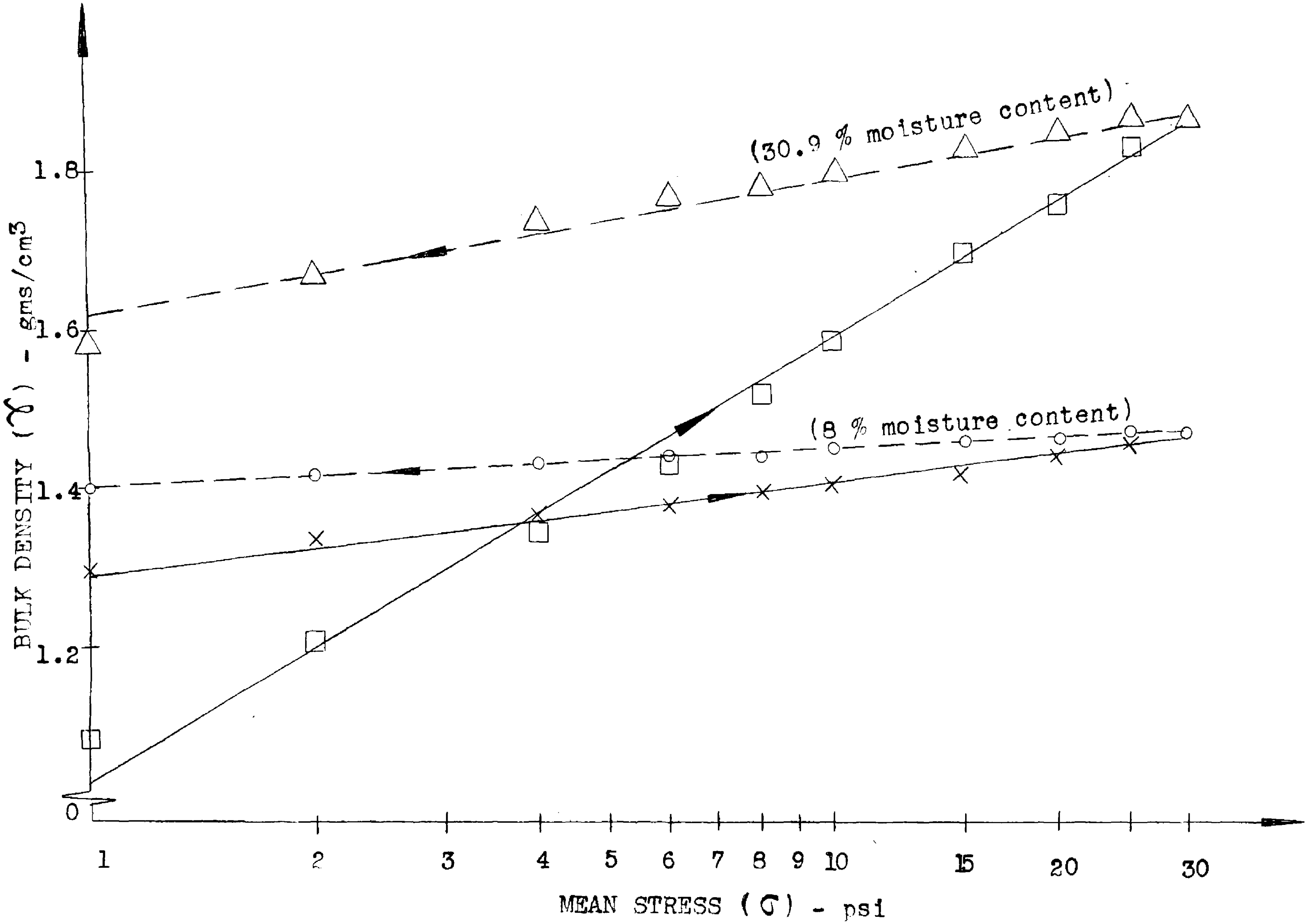


Fig. 12. Bulk density (γ) versus mean stress (σ) for clay soil at 30.9 % (lower plastic limit) and at 8.0 % (air dried moisture contents

values of γ_o and B were determined accordingly.

γ_{oc} - Refers to the initial bulk density prior to compression or application of σ

γ_{or} - Refers to the initial bulk density during relaxation, but it must be remembered that γ_{or} corresponds to σ_o, the mean-stress state before a load was applied.

γ versus σ for clay soil at air-dry moisture content

$$\gamma_r = 1.40 + 0.02 \ln \sigma / \sigma_o$$

$$\gamma_c = 1.29 + 0.053 \ln \sigma / \sigma_o$$

γ versus σ for clay soil at the lower plastic limit:

$$\gamma_r = 1.62 + 0.074 \ln \sigma / \sigma_o$$

$$\gamma_c = 1.04 + 0.23 \ln \sigma / \sigma_o$$

Tests of Silty Loam Soil

Table 2 shows how bulk density varies with different mean-stress applications from 1 to 30 psi. Data were taken from four different moisture levels. The first was at an air-dry equilibrium moisture content 13.3%, the second was wetted to the lower plastic limit 32.6%, while the third and fourth were moistened to typical field conditions 20% and 16% whereby fields might be tilled at such moisture contents.

Figure 13 and Figure 14 yielded curves which very closely resemble a γ versus σ function for clay soils. The reader will note that these curves are taken from the air-dry silty loam soil. On semi-log coordinate paper, the air-dry soil as shown in Figure 14, yielded a straight line thus behaving similarly to the clay soils.

Figure 15 for a 20% moisture content sample plotted somewhat differently than expected. $\frac{\partial \gamma}{\partial \sigma}$ did not decrease with σ as rapidly as with other soils, thus resulting in a curved line on semi-log paper as shown on Figure 16. Figure 16 also shows how this curve was straightened on semi-log paper by incorporation of a second parametric constant "K" in such a manner that the new expression for γ versus σ was changed to:

$$\gamma = \gamma_0 + B \ln \frac{(\sigma / \sigma_0 + K)}{(1 + K)} \tag{7}$$

It will be noted that $1 + K$ was added in equation (7) in order to satisfy initial conditions, thus if $\sigma = \sigma_0$, $\gamma = \gamma_0$. Obviously equation (7) is a generalized form of equation (6) since the latter will reduce to the former when $K = 0$. Therefore, the same equation can be used to describe the γ versus σ behavior of at least two general classes of soil by the proper selection of parametric constants.

Tests of silty loam soil at 16% moisture content yielded results very similar to those found for the same soil at 20% moisture content. Figure 17 and Figure 18 exemplify these results which obey equation (7), but of course the parametric constant are different since they are at least partially related to soil moisture.

TABLE 2

Applied load (σ) versus bulk density (γ) for silty loam soil at four moisture levels

Applied load	Moisture content of soil			
	13.3% water*	16% water**	20% water**	32.6% water***
(psi)	(gms/cm3)	(gms/cm3)	(gms/cm3)	(gms/cm3)
1	1.16	1.17	1.29	1.37
2	1.19	1.20	1.32	1.41
4	1.21	1.23	1.38	1.44
6	1.23	1.26	1.42	1.44
8	1.25	1.28	1.47	1.44
10	1.26	1.30	1.50	1.44
15	1.28	1.33	1.57	1.45
20	1.30	1.36	1.64	1.46
25	1.32	1.38	1.67	1.47
30	1.33	1.40	1.71	1.47
25	1.33	1.40	1.71	1.47
20	1.33	1.40	1.71	1.47
15	1.32	1.39	1.71	1.47
10	1.32	1.38	1.71	1.47
8	1.31	1.37	1.69	1.47
6	1.31	1.36	1.68	1.46
4	1.30	1.35	1.66	1.45
2	1.28	1.33	1.64	1.44
1	1.25	1.31	1.61	1.43

* Air dry equilibrum point of soil
** Typical tillage moisture contents
*** Lower plastic limit

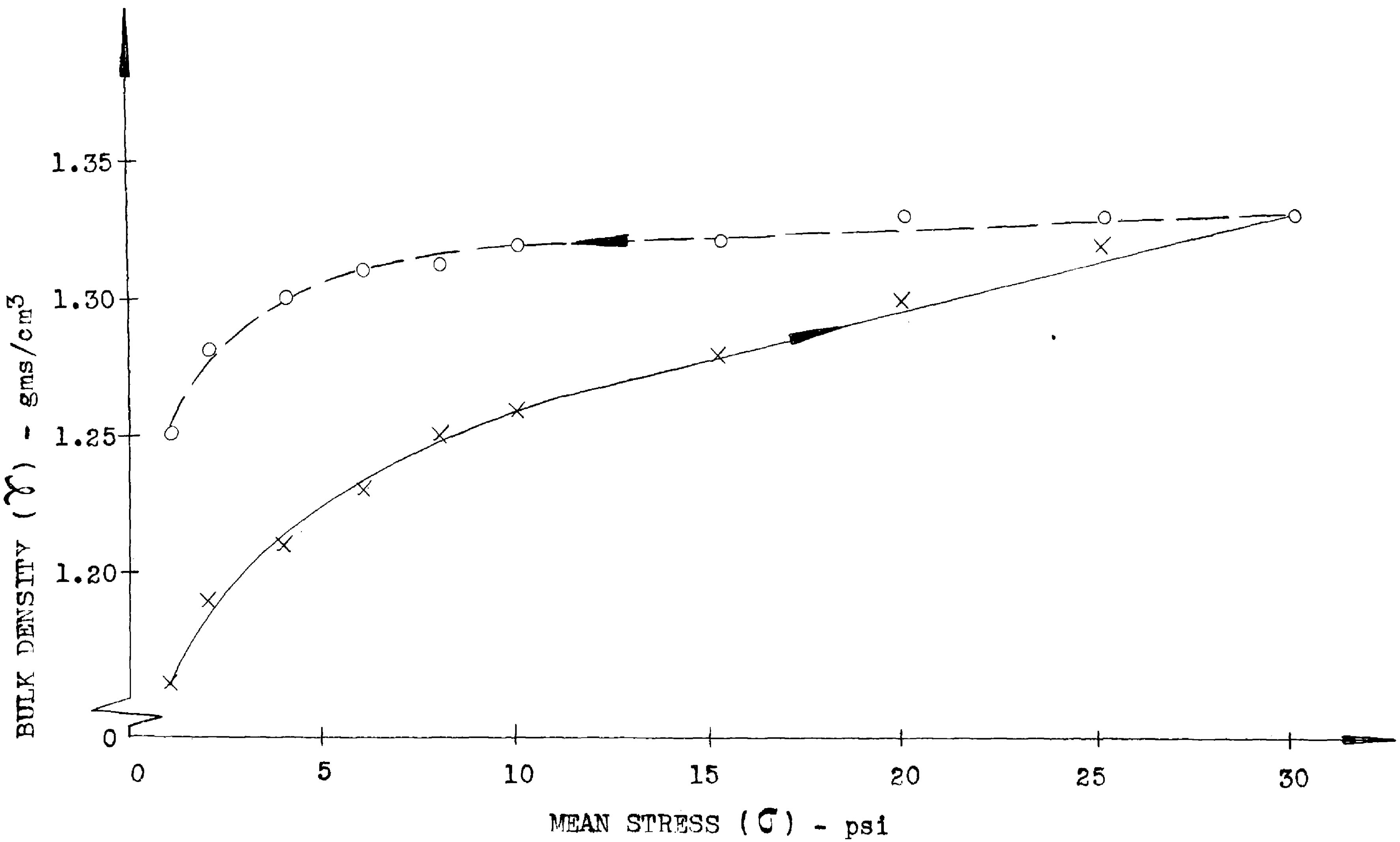


Fig. 13. Bulk density (γ) versus mean stress (σ) for 13.3 % moisture content silty loam soil (air dried) .

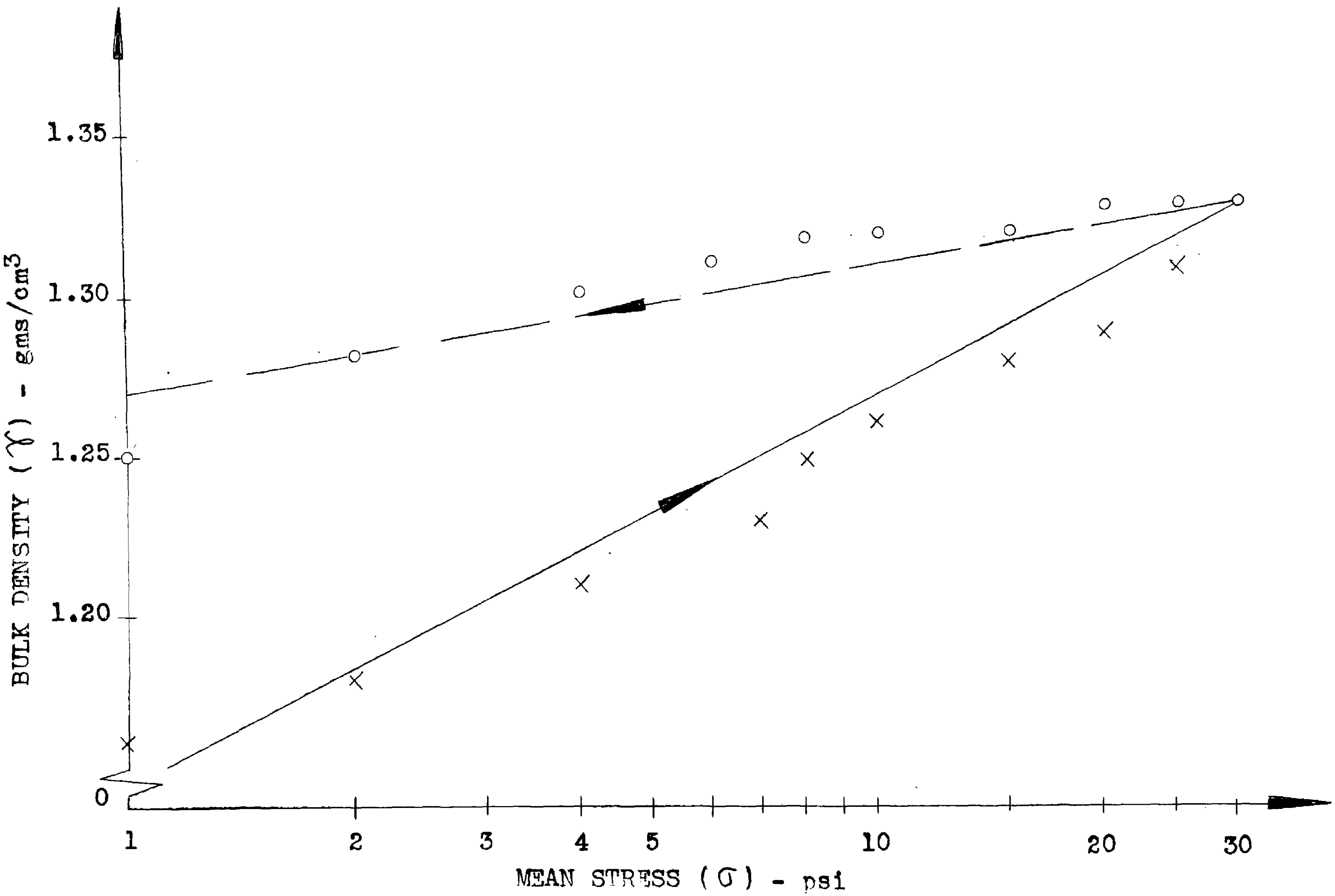


Fig. 14. Bulk density (γ) versus mean stress (σ) for 13.3 % moisture content (air dried) silty loam soil

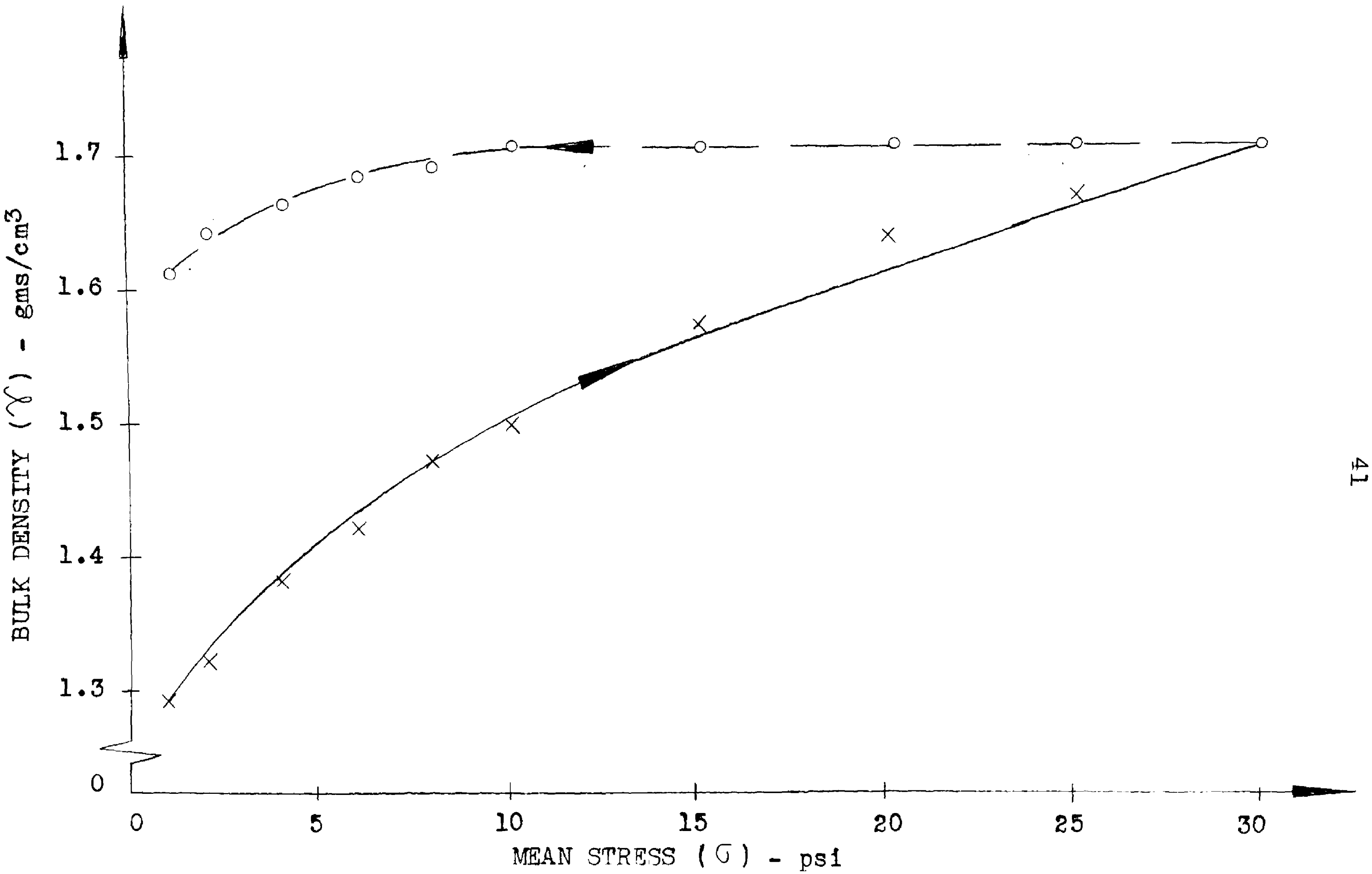


Fig. 15. Bulk density (γ) versus mean stress (σ) for 20% moisture content (typical field condition) silty loam soil

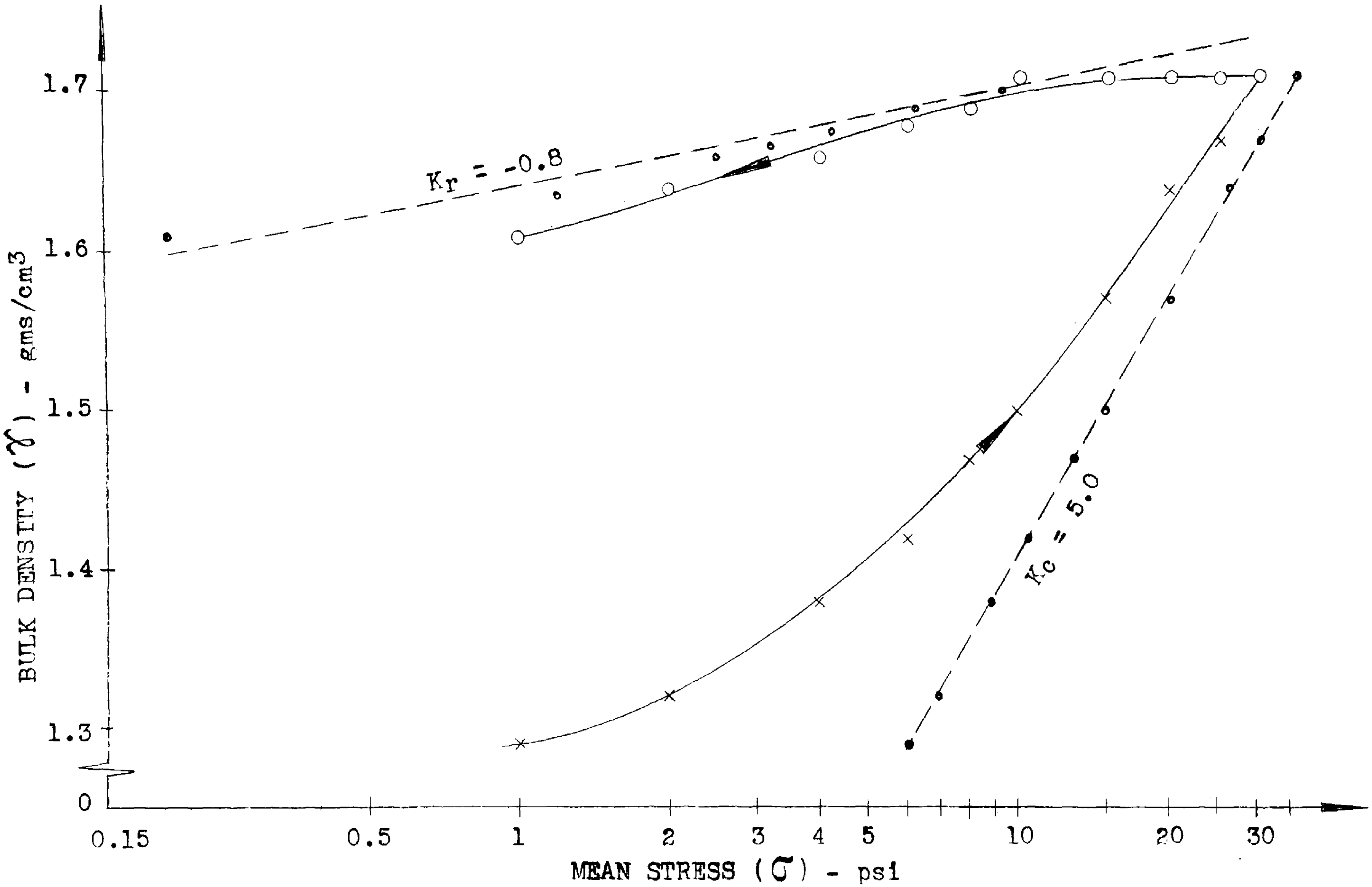


Fig. 16. Bulk density (γ) versus mean stress (σ) (corrected to straight line) for 20 % moisture content silty loam soil

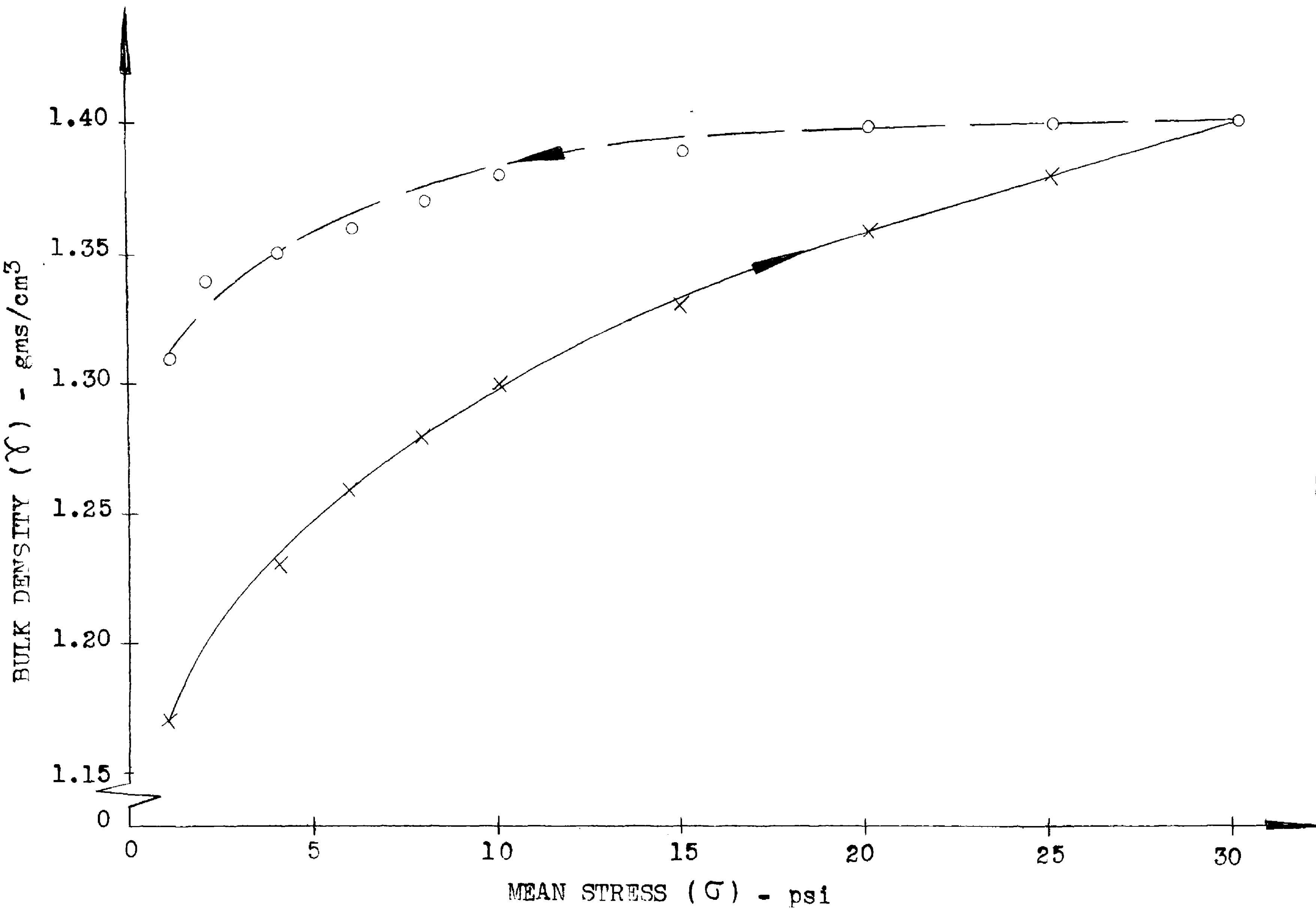


Fig. 17. Bulk density (γ) versus mean stress (σ) for 16 % moisture content (typical field conditions) silty loam soil

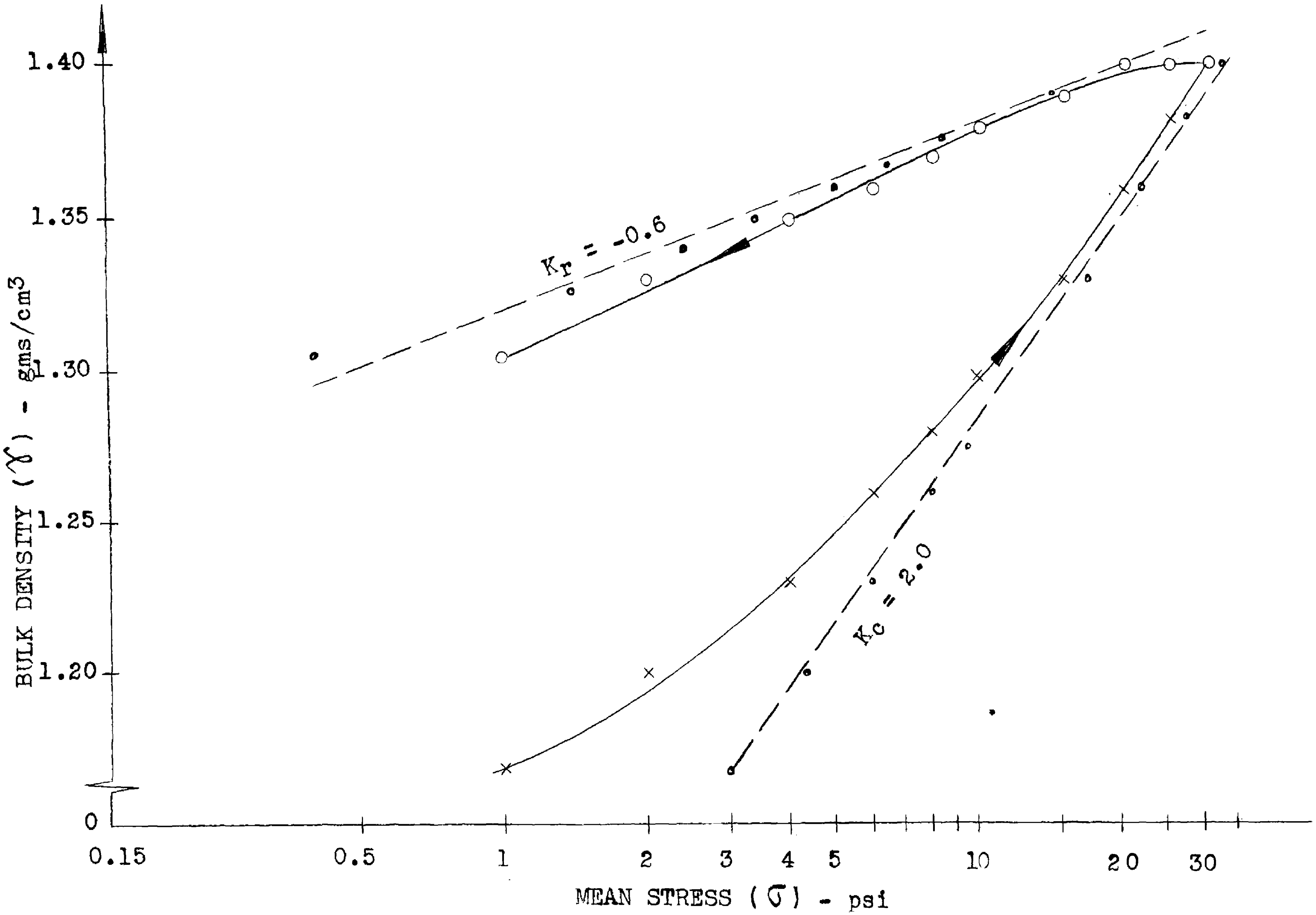


Fig. 18. Bulk density versus mean stress (corrected to straight line) for 16 % moisture content silty loam soil

γ versus σ for silty loam soil at air-dry moisture cont.

$$\gamma_c = 1.15 + 0.053 \ln \sigma/\sigma_o$$

$$\gamma_r = 1.27 + 0.018 \ln \sigma/\sigma_o$$

γ versus σ for silty loam soil at 16% moisture content.

$$\gamma_c = 1.17 + 0.098 \ln (\sigma/\sigma_o + 2)/3$$

$$\gamma_r = 1.31 + 0.023 \ln (\sigma/\sigma_o - 0.6)/0.4$$

γ versus σ for silty loam soil at 20% moisture content.

$$\gamma_c = 1.29 + 0.240 \ln (\sigma/\sigma_o + 5)/6$$

$$\gamma_r = 1.61 + 0.020 \ln (\sigma/\sigma_o - 0.8)/0.2$$

Tests of Sandy Loam Soil

Table 3 shows how the γ versus σ function for a sandy loam soil behaves when subjected to three moisture content levels; namely, 1.26% (air dried), 8.0% (a typical field working condition for sandy soil), and 20.6% (the lower plastic limit).

Figure 19 dealing with the air-dry soil yielded curves similar to those expected from clay or dry silty loam soils. However, Figure 22 illustrates a very interesting phenomenon when a soil is compressed to its "saturation limit." Physically this means that all the void space in the soil has been eliminated. Naturally this "saturation limit" is reduced when soil moisture is increased because excessive water occupies pore spaces in soil. With the sandy loam soil at the lower plastic limit was used, the "saturation limit" was found to be at 1.75 gms/cm^3 which occurred at σ equal to 6 psi.

TABLE 3

Applied load (σ) versus bulk density (γ) for sandy loam soil at three moisture levels

Applied load	Moisture content of soil		
	1.26 % water*	8.0 % water**	20.6 % water***
(psi)	(gms/cm3)	(gms/cm3)	(gms/cm3)
1	1.42	1.50	1.63
2	1.44	1.57	1.67
4	1.47	1.67	1.73
6	1.48	1.73	1.74
8	1.49	1.78	1.75
10	1.50	1.82	1.75
15	1.52	1.87	1.75
20	1.54	1.90	1.75
25	1.55	1.92	1.75
30	1.56	1.95	1.75
25	1.56	1.95	1.75
20	1.56	1.95	1.75
15	1.56	1.95	1.75
10	1.55	1.95	1.75
8	1.55	1.95	1.75
6	1.54	1.95	1.75
4	1.54	1.94	1.75
2	1.53	1.93	1.75
1	1.52	1.90	1.74

* Air dry equilibrum point of soil
** A typical tillage moisture content
*** Lower plastic limit

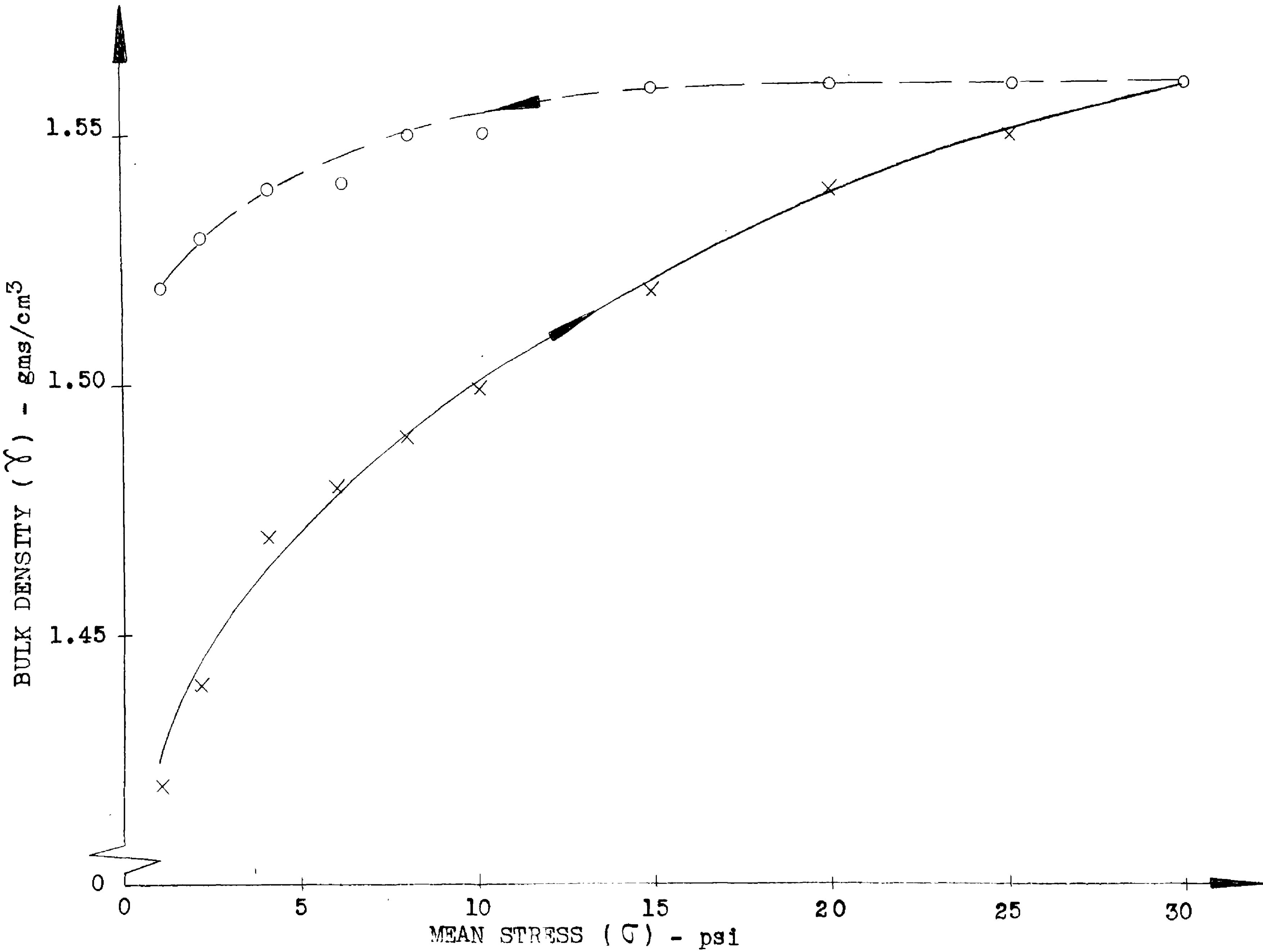


Fig. 19. Bulk density (γ) versus mean stress (σ) for 1.3 % moisture content (air dried) sandy loam soil

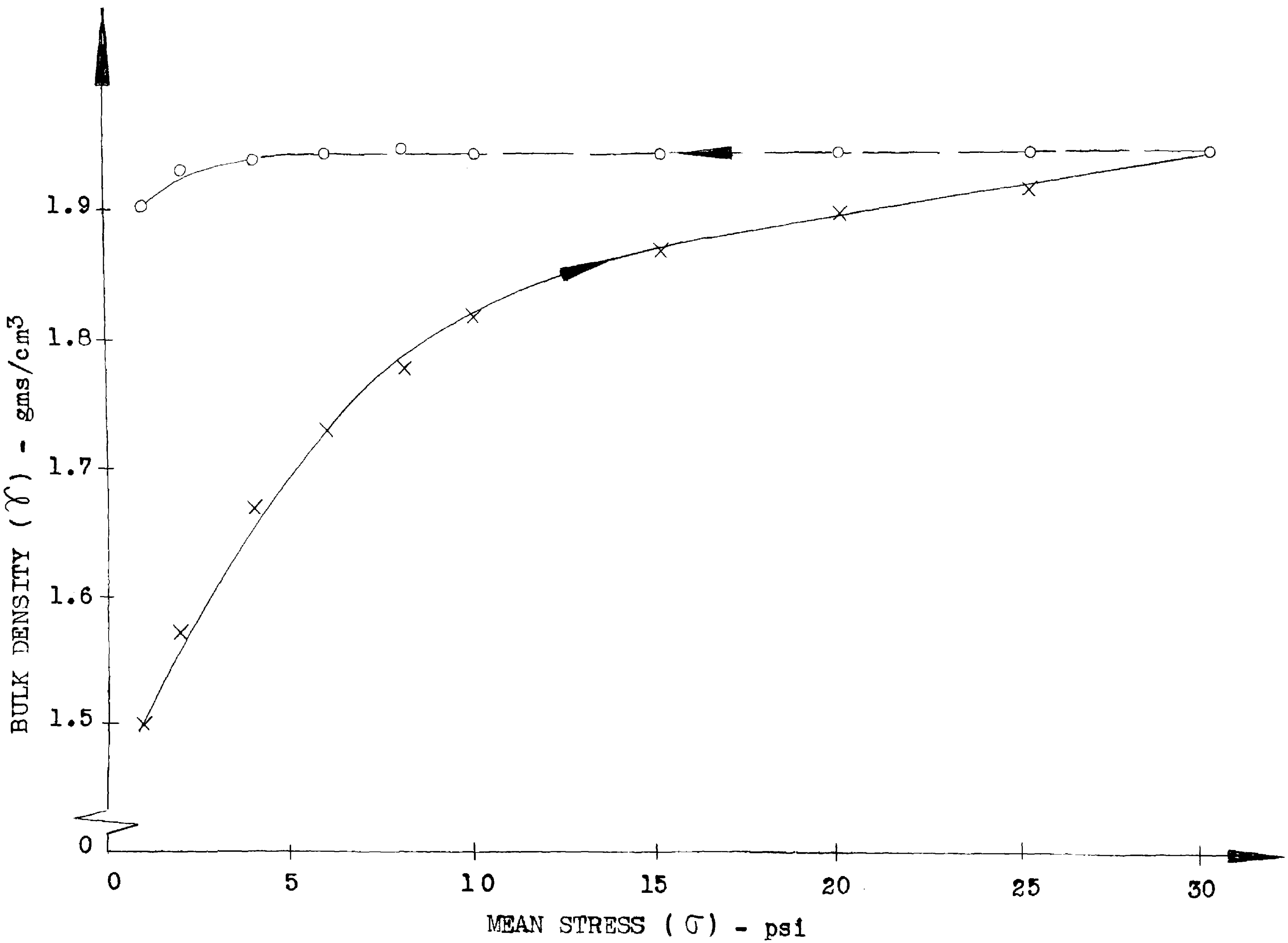


Fig. 20. Bulk density (γ) versus mean stress (σ) for 8 % moisture content sandy loam soil

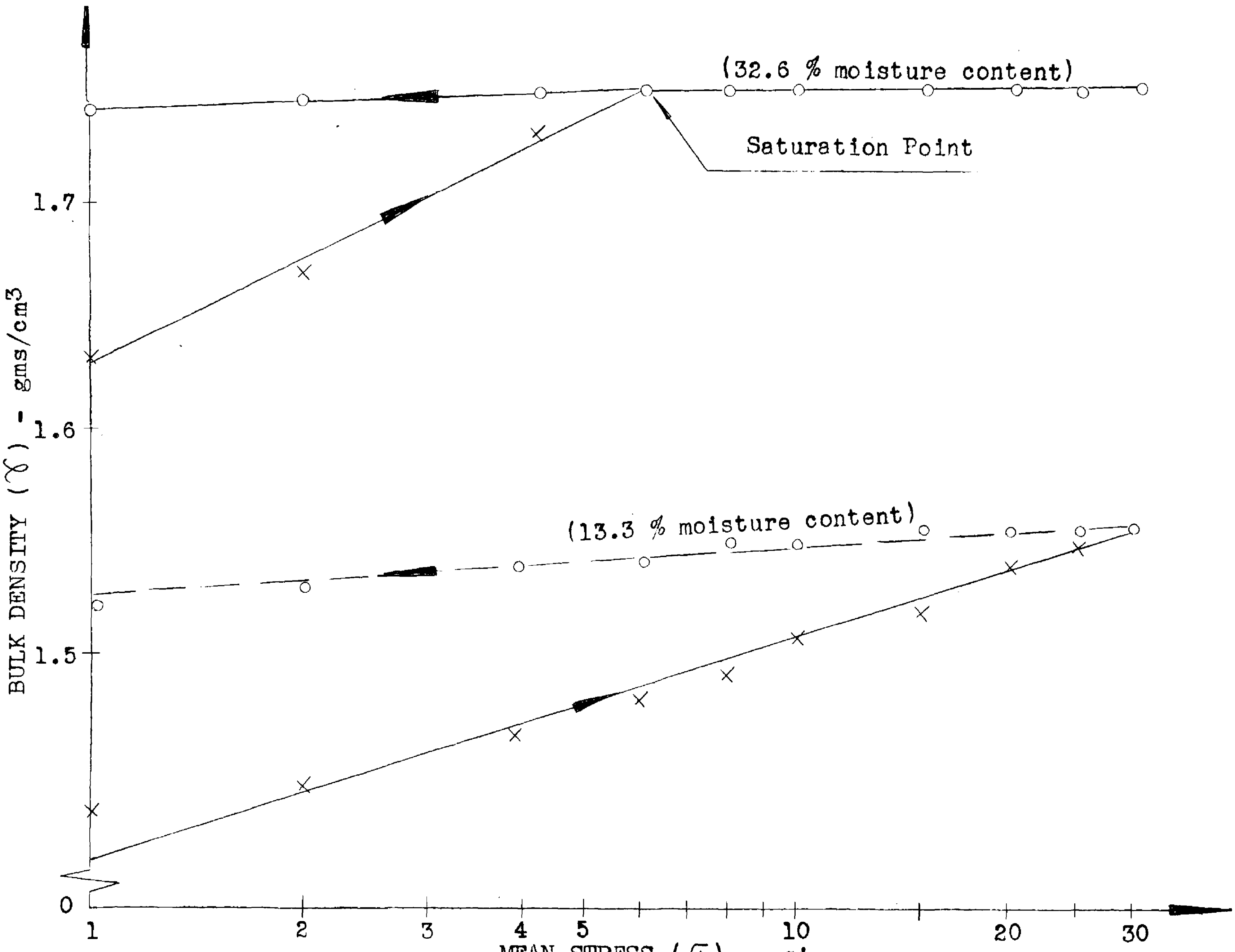


Fig. 21. Bulk density (γ) versus mean stress (σ) for sandy loam soil at 32.6 % (lower plastic limit) and 13.3 % (air dried) moisture contents

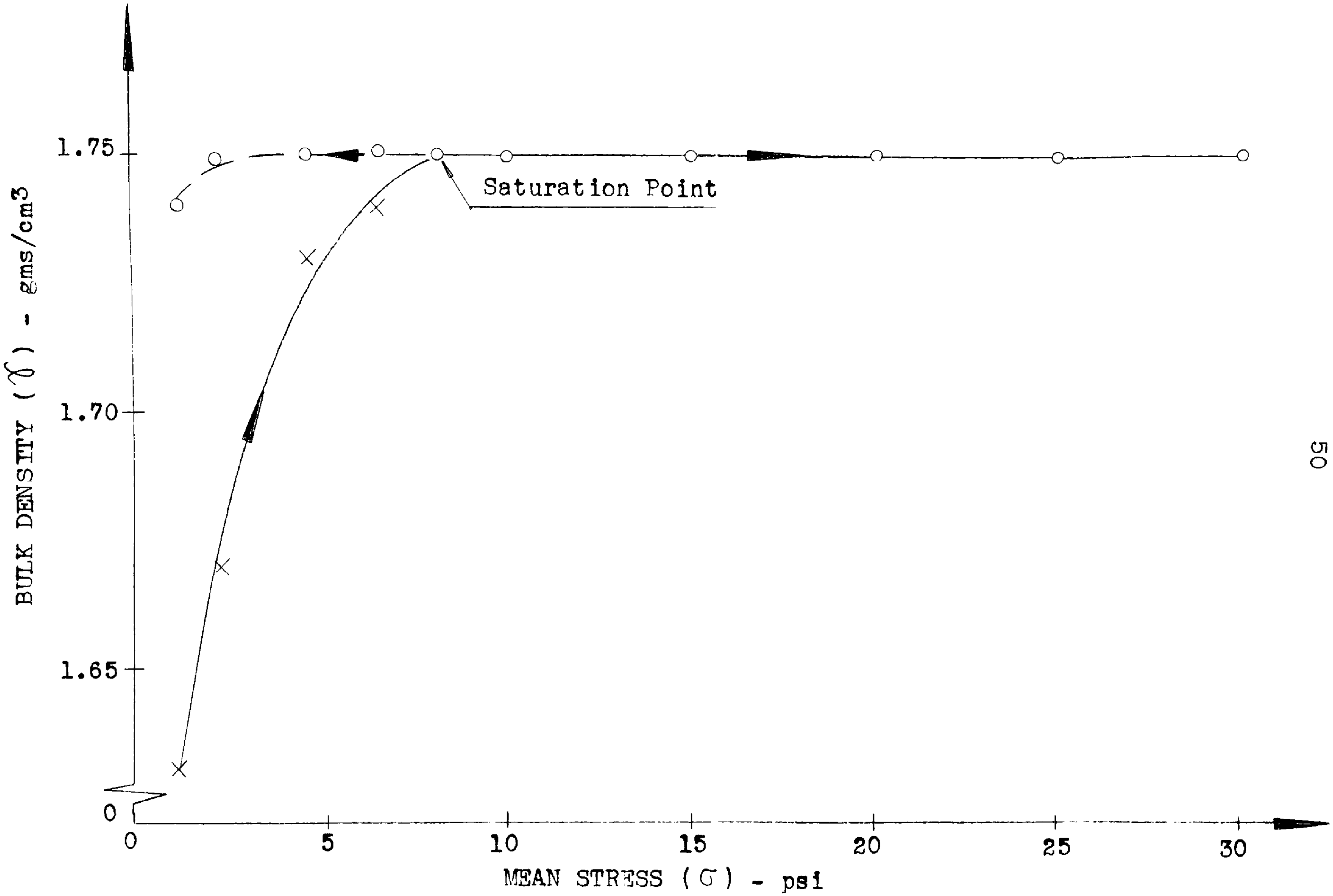


Fig. 22. Bulk density (γ) versus mean stress (σ) for 32.6 % moisture content (lower plastic limit) sandy loam soil

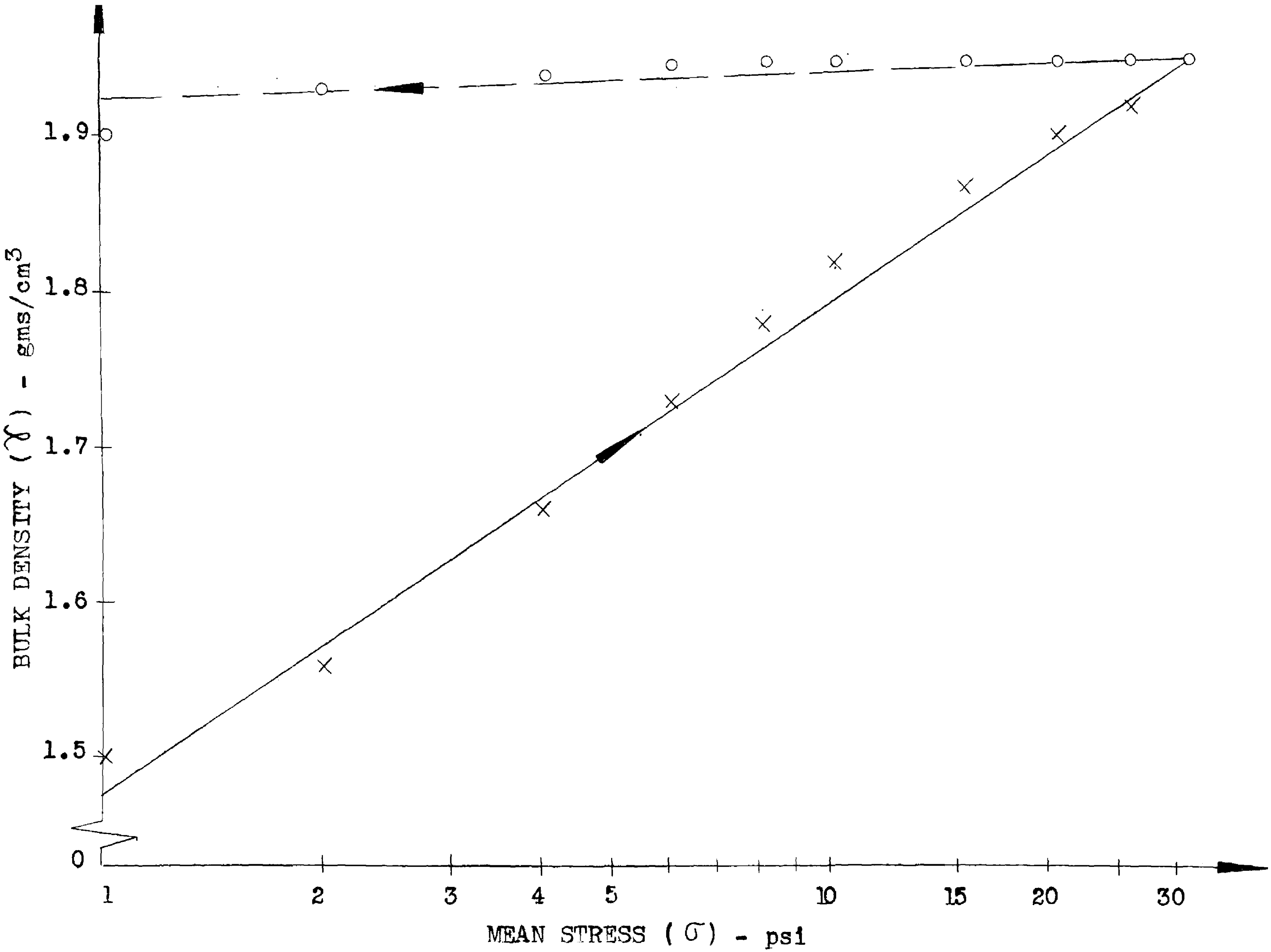


Fig. 23. Bulk density (γ) versus mean stress (σ) for 8 % moisture content sandy loam soil

Because the saturation limit was reached when the soil was moistened to the lower plastic limit, a third moisture content at 8.0% was selected. This represented the moisture level at which field operations could be performed. Figures 20 and 23 show that the γ versus σ function for sandy loam soil at 8% moisture content obeys equations (6) and/or (7). In the case of equation (7), it is obvious that K = 0 for this particular soil.

γ versus σ for air-dry sandy loam soil:

$$\gamma_c = 1.41 + 0.056 \ln \sigma/\sigma_o$$

$$\gamma_r = 1.53 + 0.020 \ln \sigma/\sigma_o$$

γ versus σ for sandy loam soil at 8% moisture cont.

$$\gamma_c = 1.48 + 0.138 \ln \sigma/\sigma_o$$

$$\gamma_r = 1.92 + 0.009 \ln \sigma/\sigma_o$$

Repeated Impact Tests

The purpose of these tests was to determine qualitatively how soils behave under impact loading conditions. Extensive data were not secured for this phase of the investigation since these tests were performed only incidentally. Reproducible data were secured from six replications taken from sandy loam and silty loam soils. It must be admitted nevertheless, that the apparatus by which impact loads were applied left something to be desired.

Figure 24 and Figure 25 shows how bulk density γ varied as impact loads (approximately from 1 to 27 psi) were repeated.

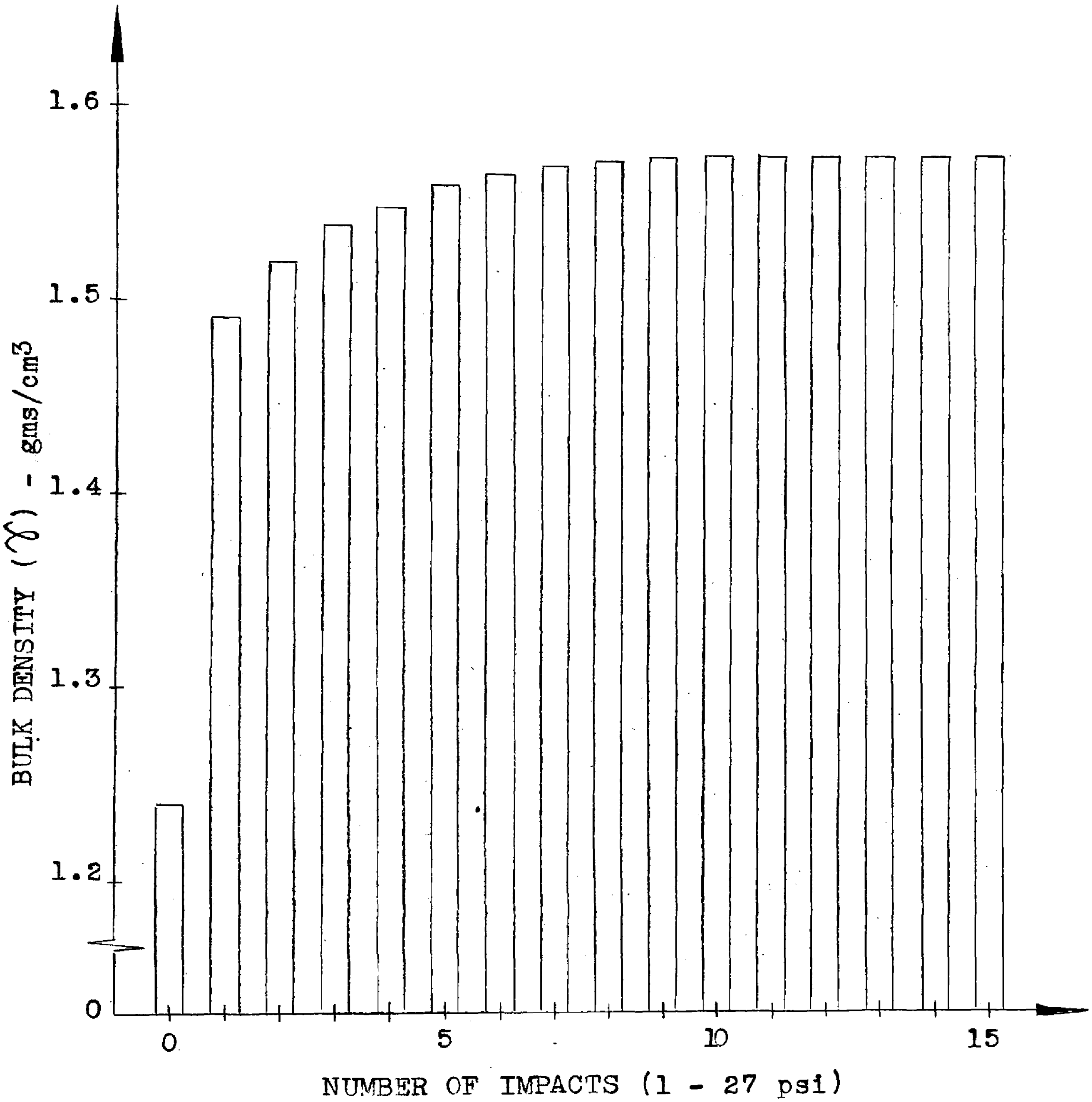


Fig. 24. Repeated impact applications of mean stress (σ) versus bulk density (γ) for 20 % moisture content (field conditions) silty loam soil

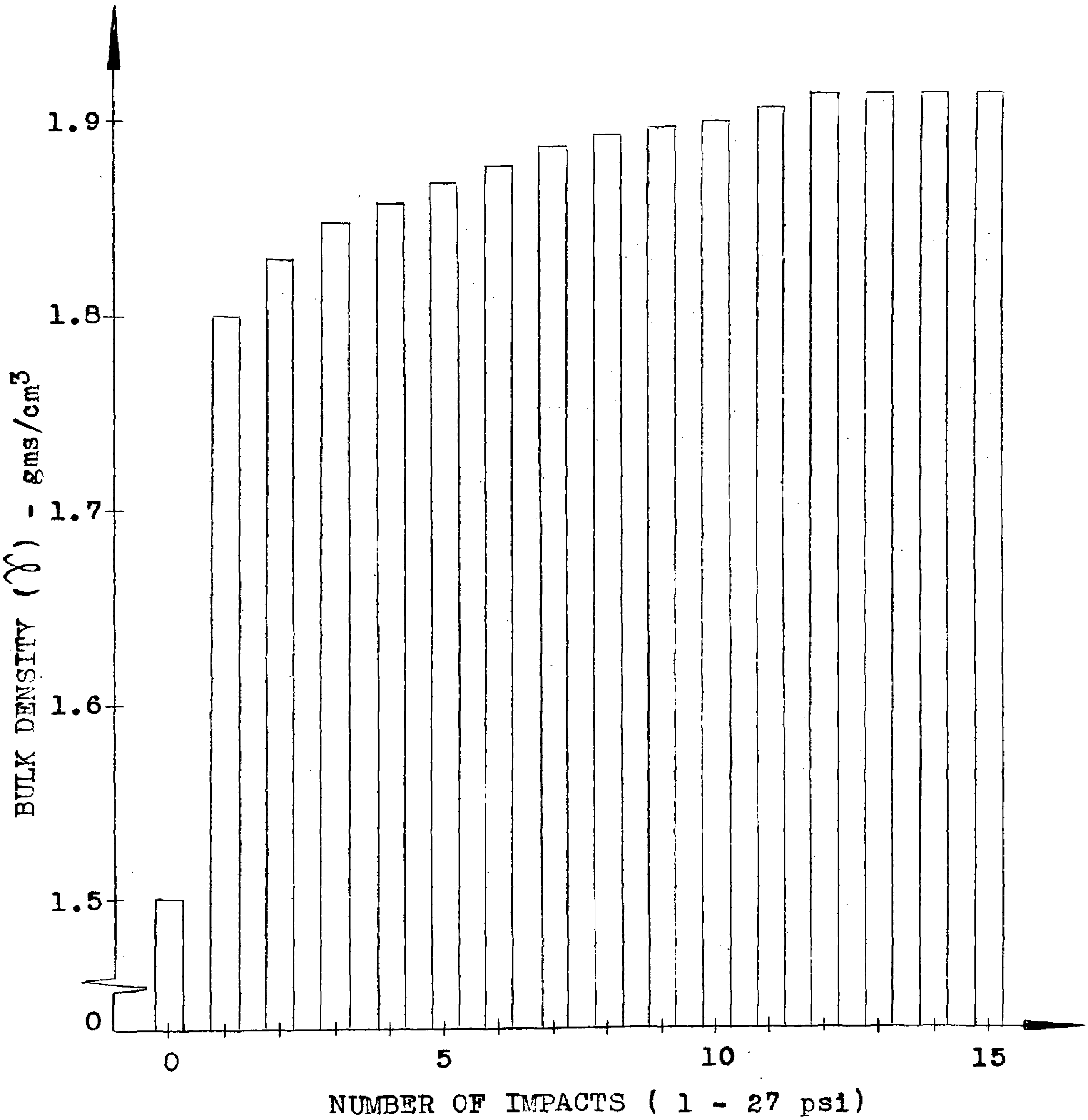


Fig. 25. Repeated impact applications of mean stress (σ) versus bulk density (γ) for 8 % moisture content (field conditions) sandy loam soil

It can be seen from these bar-graphs that from 70 to 90 percent of the final bulk density occurred during the first impact application. Subsequent changes in bulk density diminished rapidly as the number of repetitions were increased.

With the two soil samples tested, no additional increase in bulk density was noted when the number of repetitions exceeded 15. Further, the final bulk density attained as a result of 15 or more impacts approached that which would result from a gradually-applied load from 1 to 27 psi and relieved back to 1 psi. Thus, an impact load created only 70 to 90 percent of the volumetric strain expected from a gradually applied and relieved load. It should be further emphasized that when repeated loads are gradually applied, the final bulk density occurs with the first load application. Further repetition of a gradually-applied load will cause no further change in bulk density. Stated another way, beyond the first load application, the γ versus σ function will vary along the "relaxation" curve as previously described.

Adaptation of Equations Relating γ and σ to Field Problems and Special Uses

The preceding pages show that γ and σ can be related by means of a mathematical equation if initial conditions and parametric constants of a particular soil are known. More specifically, it can be said that the same general mathematical equation (Equation (7) and/or (6)) will be obeyed by virtually all agricultural soils. The reader should be reminded here that Spangler (18) reported a similar relationship with soils dealt with in civil engineering work. This general mathematical equation was also satisfied by other data collected by VandenBerg (21) and Hovanesian (9).

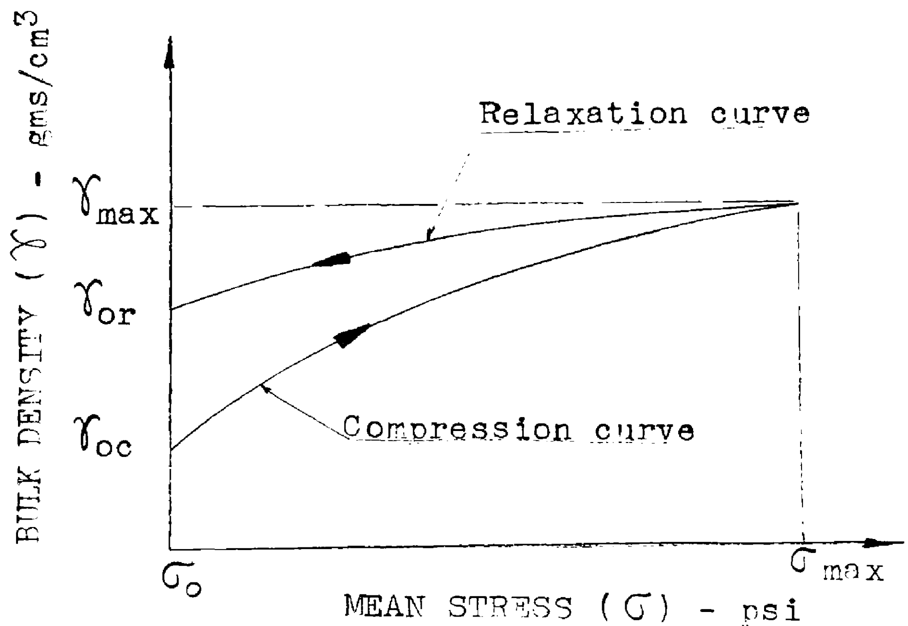


Fig. 26. A typical compression and relaxation curve for agricultural soils relating σ and γ

We know that:

$$\gamma_c = \gamma_{oc} + B_c \ln[(\sigma/\sigma_o + K_c)/(1 + K_c)] \quad (7a)$$

$$\gamma_r = \gamma_{or} + B_r \ln[(\sigma/\sigma_o + K_r)/(1 + K_r)] \quad (7b)$$

Where γ_c, γ_r, γ_{oc}, γ_{or}, B_c, B_r, σ, σ_o have been defined previously on pages 31 and 36.

K_c - Parametric constant during compression

K_r - Parametric constant during relaxation

σ_{max} - Final (max) mean-stress application

γ_{max} - Final (max) bulk density resulting from σ_{max}

Equations (7a) and (7b) of above have been adapted for the following cases:

CASE 1: Final bulk density change $\Delta\gamma$ occurring in a soil due to a load application σ_o to σ_{max} then the load relieved back to σ_o.

since

$$\gamma_{max} = \gamma_{oc} + B_c \ln\left[(\sigma_{max}/\sigma_o + K_c)/(1 + K_c)\right] \qquad (7c)$$

and

$$\gamma_{max} = \gamma_{or} + B_r \ln\left[(\sigma_{max}/\sigma_o + K_r)/(1 + K_r)\right] \qquad (7d)$$

subtracting Equation (7c) from (7d) we get:

$$0 = (\gamma_{or}-\gamma_{oc}) + B_r \ln\left[(\sigma_{max}/\sigma_o + K_r)/(1 + K_r)\right] - B_c \ln\left[(\sigma_{max}/\sigma_o + K_c)/(1 + K_c)\right]$$

or

$$\Delta\gamma = (\gamma_{or}-\gamma_{oc}) = B_c \ln\left[(\sigma_{max}/\sigma_o + K_c)/(1 + K_c)\right] - B_r \ln\left[(\sigma_{max}/\sigma_o + K_r)/(1 + K_r)\right] \qquad (8)$$

Equivalently:

$$\Delta\gamma = B_c \ln\left[\left(\frac{\sigma_{max}/\sigma_o + K_c}{1 + K_c}\right)\left(\frac{1 + K_r}{\sigma_{max}/\sigma_o - K_r}\right)^{B_r/B_c}\right] \qquad (9)$$

if; $K_c = K_r = 0$, then:

$$\Delta\gamma = (B_r - B_c) \ln \sigma_{max}/\sigma_o \qquad (10)$$

CASE 2: γ_r in terms of σ when γ_{or} is not known

since

$$\gamma_{or} = \gamma_{oc} + B_c \ln\left[(\sigma_{max}/\sigma_o + K_c)/(1 + K_c)\right] - B_r \ln\left[(\sigma_{max}/\sigma_o + K_r)/(1 + K_r)\right]$$

then

$$\gamma_r = \gamma_{oc} - B_c \ln\left[(\sigma_{max}/\sigma_o + K_c)/(1 + K_c)\right] + B_r \ln\left[(\sigma/\sigma_o + K_r)/(\sigma_{max}/\sigma_o + K_r)\right] \quad (11)$$

CASE 3: Determination of σ_{max} for a prescribed $\Delta\gamma/\gamma_{oc} = p$ and when $K_r = K_c = 0$

dividing Equation (10) by γ_{oc}, we get:

$$p = \left[(B_r - B_c)/\gamma_{oc}\right] \ln \sigma_{max}/\sigma_o$$

$$\sigma_{max} = \sigma_o \; e^{p\gamma_{oc}/(B_c - B_r)} \quad (12)$$

The reader will immediately realize problems to which the revised equations of cases 1 through 3 can be applied. For example, equations (9) and (10) of Case 1 can be used to predict a bulk density change resulting from a change in mean-stress $\Delta\sigma = (\sigma_{max} - \sigma_o)$ that may be caused by implement traffic. See Example 1.

Equation (11) of Case 2 will enable one to predict a bulk density value resulting from an applied load not totally relieved to its inital value σ_o, thus Equation (11) is particularly useful under this circumstance.

Equation (12) is a useful relationship which can be used to prescribe σ_{max} for certain permissible values of $\Delta\gamma/\gamma_{oc}$. For example, if one knows the critical value $\Delta\gamma/\gamma_{oc}$ for a given soil, the resulting critical value of σ_{max} may be determined for that particular soil. This critical value of σ_{max} can consequently be used as a guide for implement selection and/or design. See Example 2.

Examples of Adapted Equations

Example 1

Known:

$\sigma_o = 1$ psi, $\sigma_{max} = 20$ psi (then relieved to 1 psi)

$B_c = 0.098$, $K_c = 2$, $K_r = -0.6$, $\gamma_{oc} = 1.17$, $B_r = 0.023$

Required:

$\Delta\gamma$ resulting from a load application from 1 psi to 20 psi, then relieved to 1 psi

Solution:

From Equation (9)

$$\Delta\gamma = 0.098 \ln\left[\left(\frac{20 + 2}{3}\right)\left(\frac{1 - 0.6}{20 - 0.6}\right)^{0.023/0.098}\right]$$

$$= 0.11 \text{ gms/cm}^3$$

Example 2

Known:

$\sigma_o = 1$ psi, $B_c = 0.138$, $B_r = 0.009$, $K_c = K_r = 0$

$\gamma_{oc} = 1.48$

Required:

σ_{max} such that $\Delta\gamma/\gamma_{oc} \leq 0.2$

Solution: (From Equation (12)

$$\sigma_{max} \leq 1 \cdot e^{(0.2)(1.48)/(0.138 - 0.009)} = 10 \text{ psi}$$

VI. SUMMARY AND CONCLUSIONS

Summary

While larger machines and more intensive cultivation have increased production markedly, long-run consequences of excessive implement traffic have not been completely evaluated. The coincidence of the trend toward increased implement weight and traffic with excessive compaction indicates a need to scrutinize this trend.

Attempts to solve compaction problems in the past have been of a trial-and-error nature and have failed to meet any great success. Most of the attempted solutions could not be based on sound engineering information since this information was not available. Other research workers have tried to adapt formulae accepted in civil engineering to agricultural soils, but measured results failed to agree with predicted values.

VandenBerg (20) showed that the mean normal stress at a point in soil is simply related to bulk density. His theory was based on the mechanics of a continuous medium. The study presented here dealt with the relationship between bulk density and mean stress. It involved an important instrumentation problem of the accurate measurement of specific volume changes occurring at a point in soil. The problem was satisfactorily solved by the development of a recording volumetric transducer. This instrument

TABLE 4

Summary of parametric and other constants for soils studied

Constants	Clay Loam Soil		Silt Loam Soil			Sandy Loam Soil		
	8% moist. content*	30.9% moist. content**	13.3% *	16% ***	20% ***	1.3% *	8.0% ***	20.6% ** x
B_c	0.053	0.23	0.053	0.098	0.24	0.056	0.138	-
K_c	0	0	0	2	5	0	0	-
γ_{oc}	1.29	1.04	1.15	1.17	1.29	1.41	1.48	-
B_r	0.02	0.074	0.018	0.023	0.62	0.02	0.009	-
K_r	0	0	0	-0.6	-0.8	0	0	-
γ_{or}	1.40	1.62	1.27	1.31	1.61	1.53	1.92	-
$\gamma_{or}-\gamma_{oc}$	0.11	0.58	0.12	0.14	0.32	0.12	0.44	-

* Air dried soil
** Lower plastic limit
*** Typical field conditions
x Saturation point at 6 psi

was capable of measuring volume changes as small as 0.01-ml. Excellent dynamic response, linearity and sensitivity coupled with the ease of calibration and use, made this instrument desirable.

Data on bulk density as a function of mean-stress were taken with three different agricultural soils which represented extreme textural categories as shown in Appendix A.

Mean stress was applied to the soil by hydrostatic pressure regulated through a control system. Each sample was initially crumbled to approximately a freshly-plowed state, then preloaded to 1 psi. The preload was applied in order to standardize the testing procedure.

On the basis of tests, bulk density γ was found to depend on mean-stress σ according to the empirical relation $\gamma = \gamma_0 + B \ln[(\sigma/\sigma_0 + K)/(1 + K)]$. This equation was obeyed by all soils tested in this study. The values of the parametric constants B and K and the initial condition γ_0 and σ_0 depend on at least soil type and on moisture content. These constants took on different values when the load was released, but still the general relationship held. The equation has been adapted for various cases of field applications. Table 4 is a summary of the parametric constants and initial conditions found for the soils studied here.

With soils tested in this study, 15 or more repeated-impact loads of a given mean-stress value will result in approximately the same soil strain attained with a gradually applied and identical mean-stress value. The greatest strain, amounting from 70 to 90 percent of the final strain, results from the first impact. Repetition will increase the bulk density at a decreasing rate, finally leveling to a constant value which would result if the load was gradually applied.

Conclusions

1. The volumetric transducer successfully measured continuous volume changes in the soil.

2. The volumetric transducer can be adapted for use in field work as well as in the laboratory.

3. The calibration of the transducer is independent of the soil-sample size, provided that a sufficiently large volume enlargement is added in series with the balloon.

4. The electrical aspects of the transducer are not affected by temperature variations.

5. Air expansion in the transducer due to temperature variations will not cause errors in readings provided that initial pressures are equal on both sides of the sensing diaphragm.

6. With agricultural soils, bulk density γ is related to mean-stress σ by the following general formula:

$$\gamma = \gamma_o + B \ln\left[(\sigma/\sigma_o + K)/(1 + K)\right]$$

7. Soil will become permanently strained, that is the bulk density γ will increase as a result of a load app-

lication and load release. With knowledge of soil parametric constants and initial conditions, this permanent strain can be predicted for a given mean-stress application and release.

8. If critical bulk-density values are known for a given soil, critical mean-stress values can be found in order to avoid excessive compaction.

9. For a given mean-stress value, impact loads will cause less soil strain than that created by a gradually applied and released load.

10. Fifteen or more repeated impact loads of a given mean-stress value will cause the same soil strain attained with a gradually applied and released load.

11. The bulk density versus mean stress function will obey the relaxation formulae if the load is repeated after permanent deformation has occurred.

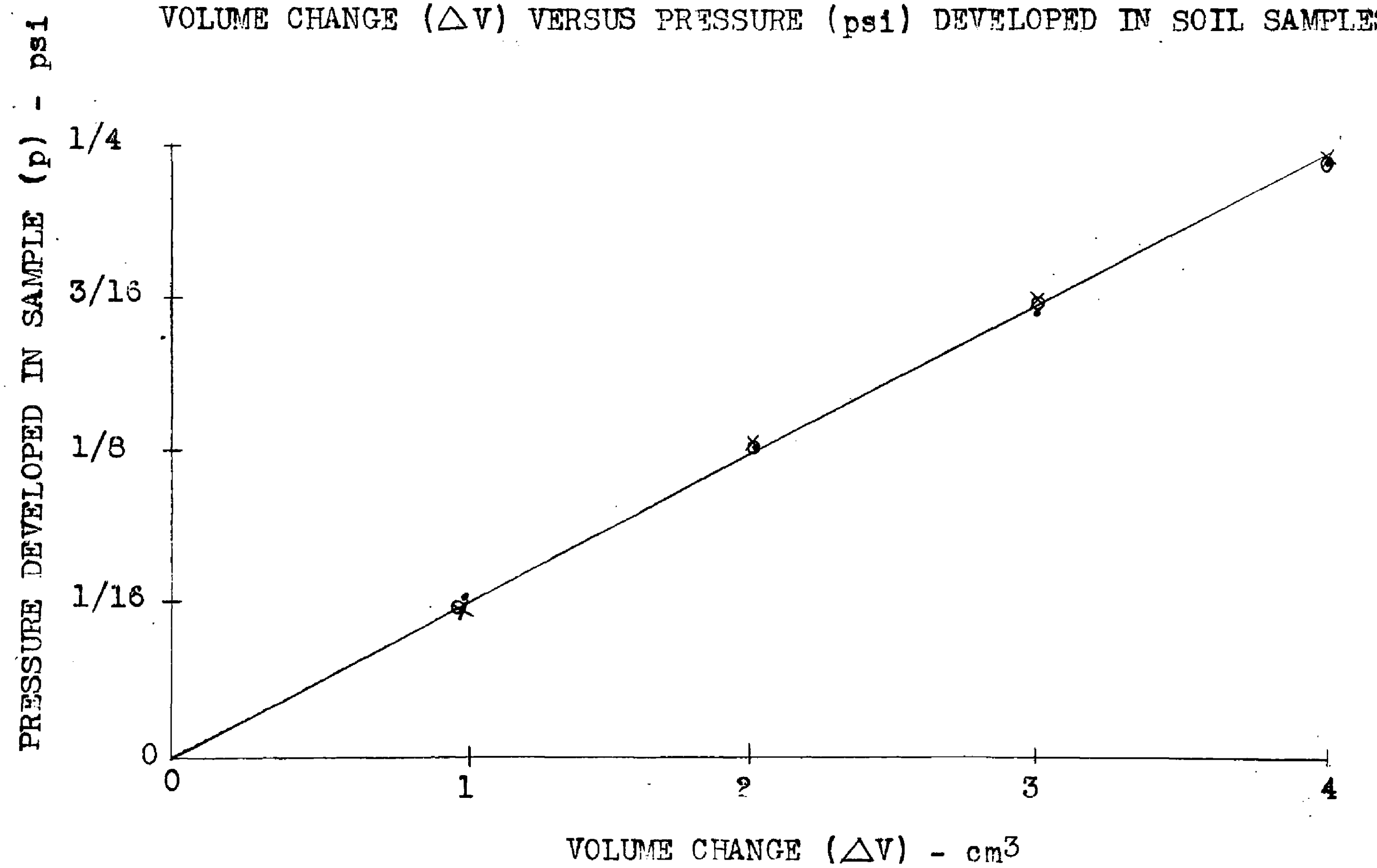
APPENDIX A

SOIL DATA OF THREE TYPES USED IN EXPERIMENTS

Soil	Moisture Contents				Composition			
	Air Dried	Moist. Equiv.	15 Atmos.	Lower Plastic Limit	Organic Matter	Clay	Silt	Sand
	%	%	%	%	%	%	%	%
(326) Pickford "C" Horizon -Clay-	8	50	28	30.9	0.3	89	9	5
(353) Iron River Surf. Soil - Silty Loam -	13.3	20.6	6.8	32.6	2	9	53	38
(225) Ensley Surface Soil -Sandy Loam-	1.26	18.2	7.7	20.6	3	8	24	68

APPENDIX B

VOLUME CHANGE ($\triangle V$) VERSUS PRESSURE (psi) DEVELOPED IN SOIL SAMPLES



APPENDIX C

TEST DATA FROM 32.6% SILT LOAM SOIL

Mean Stress σ	Volume Change ΔV Rep 1	Rep 2	Rep 3	AveΔV	Final Vol. V $V = V_o - \Delta V_{ave}$ $= 6.67 - \Delta V_{ave}$	Bulk Density γ $\gamma = W/V$ $= 9.16/V$
psi	cm^3					gms/cm^3
1	0	0	0	0	6.67	1.37
2	0.1	0.2	0.2	0.17	6.50	1.41
4	0.3	0.2	0.4	0.30	6.37	1.44
6	0.3	0.2	0.4	0.30	6.37	1.44
8	0.3	0.2	0.4	0.30	6.37	1.44
10	0.35	0.2	0.4	0.32	6.35	1.44
15	0.4	0.25	0.4	0.35	6.32	1.45
20	0.45	0.3	0.45	0.40	6.27	1.46
25	0.5	0.3	0.45	0.42	6.25	1.47
30	0.5	0.3	0.45	0.42	6.25	1.47
25	0.5	0.3	0.45	0.42	6.25	1.47
20	0.5	0.3	0.45	0.42	6.25	1.47
15	0.5	0.3	0.45	0.42	6.25	1.47
10	0.5	0.3	0.45	0.42	6.25	1.47
8	0.5	0.3	0.45	0.42	6.25	1.47
6	0.4	0.3	0.45	0.38	6.29	1.46
4	0.4	0.25	0.45	0.36	6.30	1.45
2	0.3	0.25	0.40	0.32	6.35	1.44
1	0.25	0.2	0.3	0.25	6.42	1.43

APPENDIX D

TEST DATA FROM 13.3 % SILT LOAM SOIL

Mean Stress σ	Volume Change ΔV Rep 1	Rep 2	Rep 3	Rep 4	Rep 5	Rep 6	Ave ΔV	Final Vol. V $V = V_o - \Delta V_{ave}$ $= 9.98 - \Delta V_{ave}$	Bulk Density γ $\gamma = W/V$ $= 11.53/V$
psi	cm^3								gms/cm^3
1	0	0	0	0	0	0	0	9.98	1.16
2	0.2	0.5	0.1	0.3	0.1	0.2	0.23	9.75	1.19
4	0.4	0.7	0.4	0.6	0.3	0.4	0.46	9.52	1.21
6	0.5	0.8	0.5	0.8	0.4	0.5	0.56	9.42	1.23
8	0.6	1.0	0.6	1.0	0.5	0.6	0.72	9.26	1.25
10	0.7	1.0	0.7	1.1	0.6	0.7	0.80	9.18	1.26
15	0.9	1.2	0.9	1.2	0.8	0.9	0.98	9.00	1.28
20	1.0	1.3	1.1	1.3	1.0	1.0	1.11	8.87	1.30
25	1.1	1.4	1.2	1.4	1.1	1.0	1.20	8.78	1.32
30	1.2	1.5	1.3	1.5	1.1	1.1	1.28	8.70	1.33
25	1.2	1.5	1.4	1.5	1.2	1.1	1.31	8.67	1.33
20	1.2	1.5	1.4	1.5	1.2	1.1	1.31	8.67	1.33
15	1.1	1.4	1.3	1.5	1.1	1.0	1.23	8.75	1.32
10	1.1	1.3	1.3	1.4	1.1	1.0	1.20	8.78	1.32
8	1.1	1.3	1.25	1.35	1.05	0.95	1.17	8.81	1.31
6	1.1	1.2	1.2	1.3	1.0	0.9	1.12	8.86	1.31
4	1.1	1.1	1.1	1.2	1.0	0.9	1.07	8.91	1.30
2	1.0	1.0	1.0	1.0	0.8	0.7	0.91	9.07	1.28
1	0.8	0.7	0.8	0.8	0.6	0.5	0.70	9.28	1.25

APPENDIX E

TEST DATA FROM 20 % SILT LOAM SOIL

Mean Stress σ	Volume Change ΔV Rep 1	Rep 2	Rep 3	Rep 4	Rep 5	Rep 6	Ave ΔV	Final Vol. V $V = V_o - \Delta V_{ave}$ $= 8.28 - \Delta V_{ave}$	Bulk Density γ $\gamma = W/V$ $= 10.07/V$
psi				cm^3					gms/cm^3
1	0	0	0	0	0	0	0	8.28	1.29
2	0.2	0.2	0.2	0.2	0.2	0.2	0.20	8.08	1.32
4	0.4	0.5	0.5	0.6	0.5	0.6	0.52	7.76	1.38
6	0.6	0.8	0.7	0.9	0.7	0.9	0.76	7.52	1.42
8	0.7	1.0	1.0	1.1	1.1	1.1	1.00	7.28	1.47
10	0.8	1.3	1.1	1.2	1.2	1.3	1.15	7.13	1.50
15	1.0	1.5	1.5	1.6	1.5	1.6	1.45	6.83	1.57
20	1.2	1.7	1.7	1.8	1.7	1.9	1.66	6.62	1.64
25	1.4	1.9	1.8	2.0	1.9	2.1	1.85	6.43	1.67
30	1.5	2.1	2.0	2.2	2.1	2.3	2.03	6.25	1.71
25	1.5	2.1	2.0	2.2	2.1	2.3	2.03	6.25	1.71
20	1.5	2.1	2.0	2.2	2.1	2.3	2.03	6.25	1.71
15	1.5	2.1	2.0	2.2	2.1	2.3	2.03	6.25	1.71
10	1.5	2.0	2.0	2.2	2.1	2.2	2.00	6.28	1.71
8	1.5	2.0	1.9	2.1	2.0	2.2	1.95	6.33	1.69
6	1.4	1.9	1.9	2.1	2.0	2.1	1.90	6.38	1.68
4	1.3	1.9	1.8	2.1	1.9	2.1	1.85	6.43	1.66
2	1.2	1.8	1.8	2.0	1.8	2.0	1.77	6.51	1.64
1	1.1	1.7	1.7	1.9	1.6	1.9	1.65	6.63	1.61

APPENDIX F

TEST DATA FROM 16 % SILT LOAM SOIL

Mean Stress σ	Volume Change ΔV Rep 1	Rep 2	Rep 3	Rep 4	Rep 5	Rep 6	Ave ΔV	Final Vol. V $V = V_o - \Delta V_{ave}$ $= 9.50 - \Delta V_{ave}$	Bulk Density γ $\gamma = W/V$ $= 11.1/V$
psi	cm^3								gms/cm^3
1	0	0	0	0	0	0	0	9.50	1.17
2	0.2	0.3	0.2	0.2	0.2	0.2	0.23	9.27	1.20
4	0.4	0.5	0.5	0.5	0.4	0.5	0.47	9.03	1.23
6	0.6	0.8	0.7	0.7	0.7	0.7	0.70	8.80	1.26
8	0.8	0.9	0.9	0.9	0.8	0.8	0.85	8.65	1.28
10	0.9	1.0	1.0	1.0	1.0	0.9	0.97	8.53	1.30
15	1.2	1.2	1.3	1.2	1.3	1.0	1.20	8.30	1.33
20	1.4	1.4	1.5	1.3	1.5	1.1	1.37	8.13	1.36
25	1.5	1.4	1.6	1.4	1.6	1.2	1.45	8.05	1.38
30	1.6	1.5	1.8	1.4	1.8	1.3	1.57	7.93	1.40
25	1.6	1.5	1.8	1.4	1.8	1.3	1.57	7.93	1.40
20	1.6	1.5	1.8	1.4	1.8	1.2	1.55	7.95	1.40
15	1.6	1.4	1.8	1.3	1.8	1.2	1.51	7.99	1.39
10	1.5	1.3	1.8	1.2	1.7	1.1	1.43	8.07	1.38
8	1.5	1.3	1.7	1.2	1.7	1.1	1.41	8.09	1.37
6	1.5	1.2	1.7	1.1	1.7	1.0	1.37	8.13	1.36
4	1.4	1.1	1.7	1.0	1.7	0.9	1.30	8.20	1.35
2	1.3	1.0	1.6	1.0	1.6	0.9	1.23	8.27	1.34
1	1.2	0.8	1.3	0.7	1.4	0.8	1.03	8.47	1.31

APPENDIX G

TEST DATA FROM 8 % CLAY SOIL

Mean Stress	Volume Change ΔV Rep 1	Rep 2	Rep 3	Rep 4	Rep 5	Rep 6	Ave ΔV	Final Vol. V $V = V_o - \Delta V_{ave}$ $= 10.51 - \Delta V_{ave}$	Bulk Density γ $\gamma = W/V$ $= 13.71/V$
psi	cm^3								gms/cm^3
1	0	0	0	0	0	0	0	10.51	1.30
2	0.2	0.1	0.3	0.2	0.2	0.12	0.19	10.32	1.34
4	0.4	0.2	0.9	0.45	0.5	0.35	0.47	10.04	1.37
6	0.5	0.3	0.75	0.55	0.6	0.5	0.53	9.98	1.38
8	0.6	0.3	1.0	0.75	0.75	0.6	0.67	9.84	1.39
10	0.6	0.4	1.1	0.8	0.8	0.65	0.72	9.79	1.40
15	0.7	0.5	1.3	1.0	1.0	0.8	0.88	9.63	1.42
20	0.8	0.6	1.45	1.2	1.1	0.9	1.01	9.50	1.44
25	0.9	0.7	1.5	1.3	1.2	1.0	1.10	9.41	1.46
20	0.9	0.8	1.5	1.35	1.2	1.1	1.14	9.37	1.46
15	0.9	0.8	1.45	1.2	1.15	1.05	1.09	9.42	1.46
10	0.8	0.7	1.4	1.2	1.1	1.05	1.04	9.47	1.45
8	0.8	0.7	1.35	1.2	1.1	1.0	1.02	9.49	1.44
6	0.7	0.7	1.3	1.2	1.05	0.95	0.98	9.53	1.44
4	0.7	0.6	1.25	1.1	1.0	0.9	0.92	9.59	1.43
2	0.6	0.6	1.1	1.0	0.85	0.8	0.82	9.69	1.42
1	0.5	0.5	1.0	0.85	0.75	0.7	0.72	9.79	1.40

APPENDIX H

TEST DATA FROM 30.9 % CLAY SOIL

Mean Stress σ	Volume Change ΔV Rep 1	Rep 2	Rep 3	Rep 4	Rep 5	Rep 6	Ave ΔV	Final Vol. V $V = V_o - \Delta V_{ave}$ $= 7.67 - \Delta V_{ave}$	Bulk Density γ $\gamma = W/V$ $= 8.58/V$
psi	cm^3								gms/cm^3
1	0	0	0	0	0	0	0	7.67	1.12
2	0.7	0.4	0.5	0.5	0.7	0.8	0.60	7.07	1.21
4	1.3	1.2	1.1	1.4	1.3	1.6	1.32	6.35	1.35
6	1.7	1.6	1.4	1.8	1.5	2.2	1.70	5.97	1.43
8	1.9	2.1	1.6	2.2	1.8	2.6	2.03	5.64	1.52
10	2.1	2.4	1.8	2.6	1.9	2.9	2.28	5.39	1.59
15	2.4	2.9	2.0	3.0	2.1	3.3	2.62	5.05	1.70
20	2.5	3.2	2.1	3.3	2.2	3.6	2.81	4.86	1.76
25	2.6	3.4	2.2	3.5	2.4	3.8	2.98	4.69	1.83
30	2.7	3.5	2.3	3.6	2.4	4.0	3.08	4.59	1.87
25	2.7	3.5	2.3	3.6	2.4	4.0	3.08	4.59	1.87
20	2.7	3.5	2.3	3.5	2.4	3.9	3.05	4.62	1.85
15	2.7	3.4	2.2	3.5	2.3	3.8	2.98	4.69	1.83
10	2.6	3.3	2.2	3.4	2.3	3.7	2.92	4.75	1.80
8	2.6	3.3	2.2	3.3	2.2	3.6	2.86	4.81	1.78
6	2.6	3.2	2.1	3.3	2.2	3.6	2.83	4.84	1.77
4	2.5	3.1	2.1	3.2	2.2	3.4	2.75	4.92	1.74
2	2.4	2.9	1.9	2.9	2.0	3.1	2.53	5.14	1.67
1	2.1	2.6	1.7	2.6	1.8	2.7	2.25	5.42	1.58

APPENDIX I

TEST DATA FROM 1.3 % SANDY LOAM SOIL

Mean Stress σ	Volume Change ΔV Rep 1	Rep 2	Rep 3	Rep 4	Rep 5	Rep 6	Ave ΔV	Final Vol. V $V = V_o - \Delta V_{ave}$ $= 10.85 - V_{ave}$	Bulk Density γ $\gamma = W/V$ $= 15.37/V$
psi	cm^3								gms/cm^3
1	0	0	0	0	0	0	0	10.83	1.42
2	0.3	0.1	0.2	0.2	0.4	0.3	0.16	10.67	1.44
4	0.5	0.3	0.4	0.2	0.4	0.3	0.35	10.48	1.47
6	0.6	0.4	0.5	0.3	0.6	0.4	0.47	10.36	1.48
8	0.7	0.4	0.6	0.4	0.7	0.5	0.55	10.28	1.49
10	0.8	0.5	0.7	0.4	0.7	0.55	0.60	10.23	1.50
15	1.0	0.7	0.8	0.5	0.85	0.7	0.75	10.08	1.52
20	1.1	0.8	0.9	0.7	0.9	0.75	0.86	9.97	1.54
25	1.2	0.8	1.0	0.7	1.0	0.8	0.92	9.91	1.55
30	1.2	0.9	1.1	0.8	1.05	0.9	0.99	9.84	1.56
25	1.2	0.9	1.1	0.8	1.05	0.9	0.99	9.84	1.56
20	1.2	0.9	1.1	0.8	1.05	0.85	0.98	9.85	1.56
15	1.2	0.9	1.1	0.75	1.0	0.85	0.97	9.86	1.56
10	1.2	0.8	1.0	0.7	1.0	0.8	0.92	9.91	1.55
8	1.2	0.8	1.0	0.7	1.0	0.75	0.91	9.92	1.55
6	1.1	0.7	1.0	0.7	0.95	0.75	0.87	9.96	1.54
4	1.1	0.7	1.0	0.65	0.95	0.7	0.85	9.98	1.54
2	1.1	0.7	0.9	0.6	0.9	0.6	0.8	10.03	1.53
1	1.1	0.6	0.85	0.5	0.85	0.5	0.73	10.10	1.52

APPENDIX J

TEST DATA FROM 8% SANDY LOAM SOIL

Mean Stress σ	Volume Change ΔV Rep 1	Rep 2	Rep 3	Rep 4	Rep 5	Ave ΔV	Final Vol. V $V = V_o - \Delta V_{ave}$ $= 8.9 - \Delta V_{ave}$	Bulk Density γ $\gamma = W/V$ $= 13.37/V$
psi	cm^3							gms/cm^3
1	0	0	0	0	0	0	8.90	1.50
2	0.4	0.4	0.3	0.4	0.4	0.38	8.52	1.57
4	0.7	1.1	0.9	0.9	0.9	0.90	8.00	1.67
6	1.0	1.4	1.2	1.2	1.1	1.18	7.72	1.73
8	1.2	1.7	1.4	1.4	1.3	1.40	7.50	1.78
10	1.3	1.9	1.6	1.5	1.4	1.54	7.36	1.82
15	1.5	2.1	1.9	1.7	1.5	1.74	7.16	1.87
20	1.7	2.3	2.0	1.8	1.6	1.88	7.02	1.90
25	1.8	2.4	2.1	1.8	1.6	1.94	6.96	1.92
30	2.0	2.5	2.2	1.9	1.7	2.06	6.84	1.95
25	2.0	2.5	2.2	1.9	1.7	2.06	6.84	1.95
20	2.0	2.5	2.2	1.9	1.7	2.06	6.84	1.95
15	2.0	2.5	2.2	1.9	1.7	2.06	6.84	1.95
10	2.0	2.5	2.2	1.9	1.7	2.06	6.84	1.95
8	2.0	2.5	2.2	1.9	1.7	2.06	6.84	1.95
6	1.9	2.5	2.2	1.9	1.7	2.06	6.84	1.95
4	1.9	2.4	2.2	1.9	1.7	2.02	6.88	1.94
2	1.8	2.4	2.1	1.8	1.7	1.96	6.94	1.93
1	1.7	2.3	2.1	1.7	1.6	1.88	7.02	1.90

APPENDIX K

TEST DATA FROM 20 % SANDY LOAM SOIL

Mean Stress σ	Volume Change ΔV Rep 1	Rep 2	Rep 3	Rep 4	Rep 5	Rep 6	Ave V	Final Vol. V $V = V_o - \Delta V_{ave}$ $= 10.31 - \Delta V_{ave}$	Bulk Density γ $\gamma = W/V$ $= 16.9/V$
psi	cm^3								gms/cm^3
1	0	0	0	0	0	0	0	10.31	1.63
2	0.2	0.2	0.2	0.4	0.1	0.2	0.22	10.09	1.67
$4\frac{1}{2}$	0.6	0.5	0.6	0.8	0.5	0.4	0.56	9.75	1.73
$6\frac{1}{2}$	0.6	0.5	0.7	0.8	0.7	0.4	0.62	9.69	1.74
8	0.7	0.5	0.7	0.8	0.75	0.4	0.64	9.67	1.75
10	0.7	0.5	0.7	0.8	0.75	0.4	0.64	9.67	1.75
15	0.7	0.5	0.7	0.8	0.8	0.4	0.65	9.66	1.75
20	0.7	0.5	0.7	0.8	0.8	0.4	0.65	9.66	1.75
25	0.7	0.5	0.7	0.8	0.8	0.4	0.65	9.66	1.75
30	0.7	0.5	0.7	0.8	0.8	0.45	0.66	9.65	1.75
25	0.7	0.5	0.7	0.8	0.8	0.45	0.66	9.65	1.75
20	0.7	0.5	0.7	0.8	0.8	0.45	0.66	9.65	1.75
15	0.7	0.5	0.7	0.8	0.8	0.45	0.66	9.65	1.75
10	0.7	0.5	0.7	0.8	0.8	0.45	0.66	9.65	1.75
8	0.7	0.5	0.7	9.8	0.8	0.45	0.66	9.65	1.75
6	0.7	0.5	0.7	0.8	0.8	0.45	0.66	9.65	1.75
4	0.7	0.5	0.7	0/8	0.8	0.45	0.66	9.65	1.75
2	0.7	0.4	0.7	0.8	0.8	0.45	0.64	9.67	1.75
1	0.6	0.4	0.6	0.8	0.75	0.4	0.59	9.72	1.74

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