THE USE OF LIGHT REFRACTION FOR THE STUDY OF PROGRESSIVE ULTRASONIC WAVES

by

MACK ALFRED BREAZEALE

AN ABSTRACT

Submitted to the School for Advanced Graduate Studies of
Michigan State University of Agriculture and
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Progressive ultrasonic waves at frequencies lower than one megacycle have been investigated in liquids by observing their refraction of light. Both the wave form and the sound pressure amplitudes were studied. For determining the wave form, a stroboscopic method and an oscillographic method were used. With either method, the deviation from a sinusoidal wave must be greater than 5 per cent to be detected. For the waves produced in this study the distortion was less than this limit.

For measuring the sound pressure amplitudes, two methods were used. The first is an extension to progressive waves of the method developed by Loeber and Hiedemann[1]. With this method one observes the decrease in intensity of the center of a light beam which has passed through an ultrasonic wave. This method is shown to give accurate values of sound pressure for light beam widths of a quarter sound wave length or less. The second method is based on a technique described by Hüter and Pohlman[2] who observed the broadening of a light beam by ultrasonic waves. This method can be used not only for narrow light beams but for ones a full sound wave length or even larger. Since both the optical methods give absolute amplitudes, they promise to be valuable for calibration of devices giving relative amplitudes.

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1. A. Loeber and E. Hiedemann, J. Acoust. Soc.Am.,28,27(1956).
2. T. Hüter and R. Pohlman, Z. Angew. Physik., 1,405 (1949).

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M.A.B.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The optical methods which have received most attention in research in ultrasonics have been those which depend upon diffraction of light by an ultrasonic grating. Since diffraction effects are increased by a decrease in the grating spacing, these methods have in general been limited to frequencies above one megacycle where the corresponding wave length is small. In the frequency range below one megacycle relatively few optical experiments have been performed. However, it was recognized already by Lucas and Biquard[1] that if one limits the cross section of the light passing through an ultrasonic beam to less than a half wave length, he finds, on imaging the light, that the image is broadened rather than diffracted. This condition is most easily satisfied at frequencies below one megacycle since the slit limiting the light cross section produces the usual slit diffraction effects if it is too narrow. Thus, this method is well suited for the lower ultrasonic frequencies.

Although in a later theoretical paper Lucas[2] pointed out that this effect might be used to measure absorption and reflection coefficients, the first practical use of this effect seems to have been made much later by Hueter and

Pohlman[3] who used it to measure the absorption of ultrasonic waves in animal tissues. In 1952, Porreca[4] measured the distribution of light intensity across the broadened slit image and attempted to relate it to the ultrasonic wave shape. Loeber and Hiedemann[5] used the method to study standing ultrasonic waves in liquids and showed that the method could be used to determine the sound velocity, the sound pressure amplitude, and the wave form of the standing wave. The present work is essentially an extension of the work of Loeber and Hiedemann to the case of progressive waves. This extension allows one to investigate a single wave, rather than the superposition of waves traveling in opposite directions.

Sound Waves of Finite Amplitude

In the usual derivation of the law of propagation of sound waves, one assumes that the particle velocity and the variation of the density are infinitely small. In this way one obtains linear equations which may be easily solved. The assumption of infinitely small amplitudes is equivalent to assuming that the particle displacements are small compared to a sound wave length and that the particle velocity is small compared to the sound velocity. While these assumptions are sufficiently good in the audible range of frequencies at ordinary intensities, they might be questionable at ultrasonic frequencies, particularly at high ultrasonic intensities. Thus, there have been a number of investigations in the ultrasonic region designed to establish the existence of a waveform distortion due to finite amplitudes.

The mathematical theory of sound waves of finite amplitudes has long been a subject of investigation. Probably the first important step in the solution of this problem was made by Poisson[6] who showed that the particle velocity is described by a function of the form

$$u = f\,[\,x - (a + u)\,t\,]$$

if one assumes Boyle's law $p = a^2\rho$. If the particle velocity u is very small, then this reduces to the usual form

$$u = f\,(x - at)$$

in which the wave is propagated at a velocity a. Earnshaw[7] used this relation to discuss the propagation of a wave from its generation through its propagation and consequent change of type.

Riemann[8] made a very basic study of the propagation of sound waves of finite amplitude. From his results it follows that the more dense portion of the wave travels faster than the less dense portion. This means that during propagation the wave is continually changing form. As the wave progresses the denser part continually gains on the less dense part until the front slope of the wave is vertical, at which time a discontinuity in the wave sets in and the equations no longer hold. This raises the question whether this wave is physically possible, since the wave form is continually changing. This would mean that any sound wave in air would become distorted and form a type of shock wave, which is physically not observed. Lord Rayleigh[9] showed that for real sound waves in air the viscosity and thermal conductivity of the medium must be taken into consideration. He showed that when one considers these, then it is possible to have a wave of a permanent type.

More recently, Fay[10] considered the propagation of finite amplitude waves in air. The principal object of his analysis was to find the change in type of plane finite amplitude sound waves propagated in free air. The solution of the exact equation of motion was obtained as a Fourier series. He found that, due to the non-linear relation between pressure and specific volume, energy is gradually transferred from the lower frequency components to the higher ones. Since the higher frequency components are attenuated more than the lower ones by the viscosity of the medium, the increase in magnitude of any component due to non-linearity is balanced by losses due to absorption and transfer of energy to other components, so that a stable wave form can exist. The conditions for stability vary with intensity, so that there is no stable wave form, only a "most stable wave form" for each intensity and wave length. Thus, for large distances it is found that the wave is attenuated and returns to the sinusoidal form found at infinitely small aplitudes.

A direct experimental verification of the effects of finite amplitude waves in air was achieved in 1935 by Thuras, Jenkins, and O'Neill.[11] They generated progressive sound waves in a tube and, by probing along the length of the tube with a microphone, they measured the amount of second harmonic for various distances along the tube. The existence of a second harmonic in the air when the sound is generated by a sinusoidally vibrating source is a direct verification of the existence of a distortion in the wave as it is propagated. This is a direct conclusion from the Fay theory. A distorted sound wave in air was observed by Hubbard, Fitzpatrick, Kankovsky, and Thaler.[12] The existence of a distorted wave form before a high intensity source in air was shown also by the staff of the Acoustics Laboratory at Pennsylvania State University.[13] They not only showed that a second harmonic was present, but also demonstrated the wave form distortion in air on an oscilloscope. The oscilloscope traces showed the distortion of the wave during propagation and the most stable wave form for various intensities at various distances from the source. They found also that there is apparently a maximum pressure amplitude which can be propagated as a periodic disturbance in air. They found that after reaching a certain value, which decreased with increasing distance from the source, the sound pressure amplitude registered by the probe no longer increased linearly with increasing amplitude of vibration of the source, but leveled off and approached a maximum asymptotically. This implies that the particle amplitudes were so great near the source that the restoring forces on the particle were severely changed.

The existence of a distorted wave form in liquids implies that there is a non-linear relation between the pressure and the density. Fox and Wallace[14] used this fact to explain the deviation from the expected exponential decay of sound intensity with distance in water and in carbon tetrachloride. Their measurements were made with a sound radiometer. Finite amplitude effects have also been considered by Eckart[15] in a theoretical discussion of the fluid flow near an intense source of sound (e.g. quartz wind). However, there has not been a direct experimental verification of the existence of finite amplitude effects in liquids. One aim of the present study is to establish whether wave form distortion due to finite amplitude effects exists to a measurable degree.

Measurement of Sound Pressure Amplitudes

The accurate measurement of sound pressure amplitudes, particularly in liquids, is a difficult problem. Methods of absolute measurement are not accurate, although relative measurements can be very good. The various types of radiometers, including the cavity radiometer, which measure the sound radiation pressure are examples of absolute measurement devices. However, the inherent difficulties such as streaming of the liquid under investigation and instability of the torsional fiber with regard to mechanical vibrations have made this method one which can mislead even a very experienced experimenter.

Probably the most trustworthy and least vexing method of measuring sound pressures is the probe hydrophone. This method requires an electro-mechanical transducer, usually a barium titanate ceramic, which is so small that it does not disturb the sound field under investigation to an appreciable extent. (This condition is increasingly difficult to fulfill as the frequency is increased.) The electrical output of such transducers can be calibrated to indicate the sound pressure amplitude directly. However, the accuracy of this method depends upon a previous calibration of the instrument. The most successful method of calibration of such instruments has been static measurement. Thus, one might question the use of this instrument in extremely accurate measurements, since it is possible that the theory relating its dynamic behavior to its static behavior is not sufficiently accurate. Another method which has been used successfully in aqueous solutions is the thermocouble probe[16]. This method, too, requires calibration by some other means. The optical method to be described in this thesis offers the possibility of absolute measurement of sound pressure amplitudes. An evaluation of this method, a determination of its accuracy as well as its inherent difficulties, is a second purpose of this study.

CHAPTER II

EXPERIMENTAL APPARATUS

The Oscillator

The oscillator used in these investigations was a commercial instrument (Hypersonic Generator, Model BU-204 manufactured by the Brush Development Company). The maximum power output was rated at 250 watts, although the experiments were usually performed at much lower power. The tuning drawer for the oscillator used in these experiments covered the frequency range of 300 kc to 1000 kc.

The Transducers

The barium titanate transducers used were discs two inches in diameter. One had a nominal frequency of 400 kc, the other a nominal frequency of 1000 kc. One of the quartz transducers used was a disc 6 centimeters in diameter having a resonant frequency of 300 kc; the others were square plates one inch on a side having resonant frequencies of approximately 420, 600, and 800 kc. All transducers had either silver or gold plated electrodes. The impedance of the barium titanate transducers was low enough to match the output impedance of the oscillator; however, the impedance of the quartzes was so high that, as is usual, an impedance matching transformer was used to match their impedance to the output impedance of the oscillator. In all cases the transducers were air backed for radiating in one direction only.

Optical Arrangment

The basic optical arrangement is shown in Figure 1. Light from the source S is imaged by lens l_1 on the slit Sl_1. The lens L_2 then forms an image of the slit Sl_1 on the plane of the slit Sl_3. Between the lens L_2 and the slit Sl_3 are placed the tank T, containing the liquid under investigation, and the slit Sl_2. The purpose of Sl_2 is to limit the cross section of light passing through the tank to less than a wave length of the sound in the liquid. Since the width of this slit enters into the calculations, it was a slit having a micrometer adjustment. (The least count of this micrometer was 0.01 mm.) The width of the slit Sl_2 is critical insofar as one must be careful to see that its width is small compared to a sound wave length and that it is not so narrow that diffraction effects broaden the image in the plane of Sl_3 enough to affect the measurements. In one case, however, it was easier to make measurements of sound pressure amplitudes with Sl_2 wider than a half wave length of sound. The possible error introduced by this width of Sl_2 is discussed later.

The tank T was designed to reduce any reflection of the sound beam to a minimum. It was made of metal and had

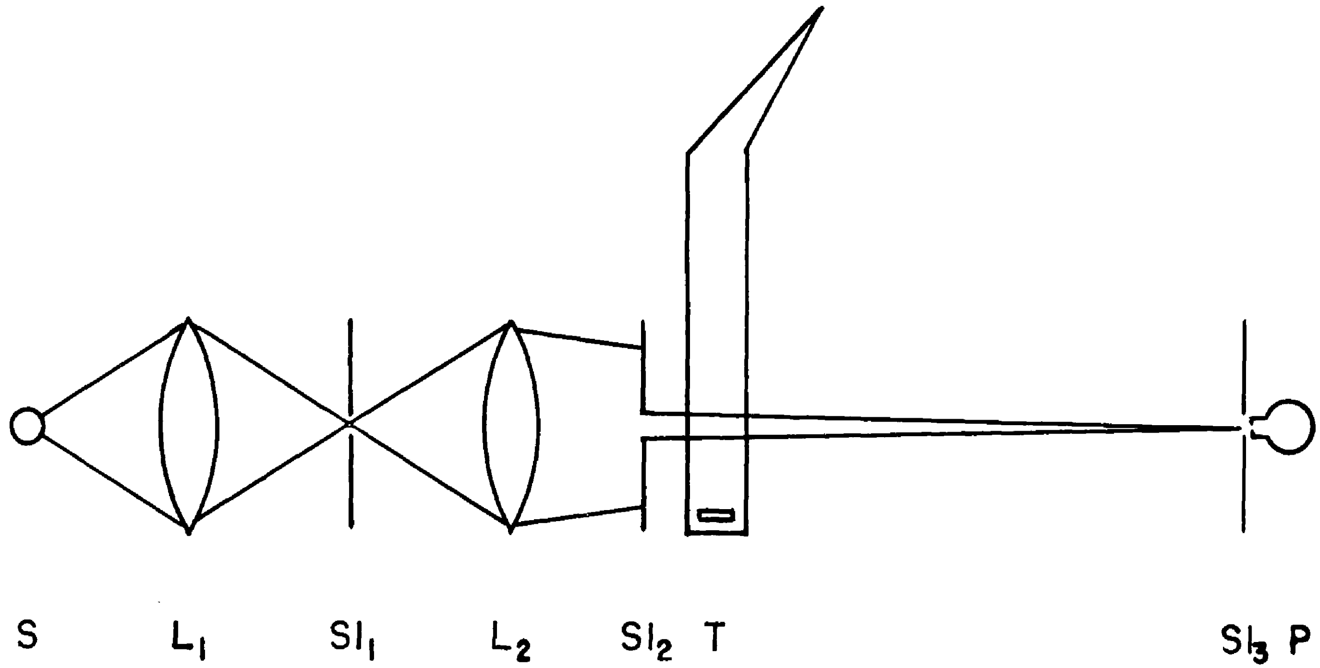

Figure 1. Basic optical arrangement for light refraction experiments

plane parallel glass windows. The total length was approximately 1.8 meters. One end was tapered, as indicated in Figure 1, and lined with cork. This reduced the reflected wave to a negligible value as was shown by the following experiment

Bär[17] has shown that with standing waves in the usual diffraction arrangement one finds on increasing the intensity of the sound that the central image (zero order) intensity decreases monotonously with increase of sound intensity. If, however, the sound wave is progressive, one finds that the intensity of the central image passes through maxima and minima as illustrated in Figure 2. These pictures were made by opening Sl_2 to approximately 1.3 sound wave lengths and placing the film in the plane of Sl_3. With the sound off picture (a) was taken. Then, increasing the sound intensity, picture (b) was taken when the central image passed through its first minimum of intensity. Picture (c) was taken at the first maximum of intensity of the central image, and so on. This behavior of the central image intensity was taken as indicating that the reflected waves in the tank were negligibly small.

Recording Instruments

When a progressive sound beam passes through the tank T, the light reaching the slit Sl_3 sweeps back and forth at the frequency of sound. The light passing through Sl_3, then,

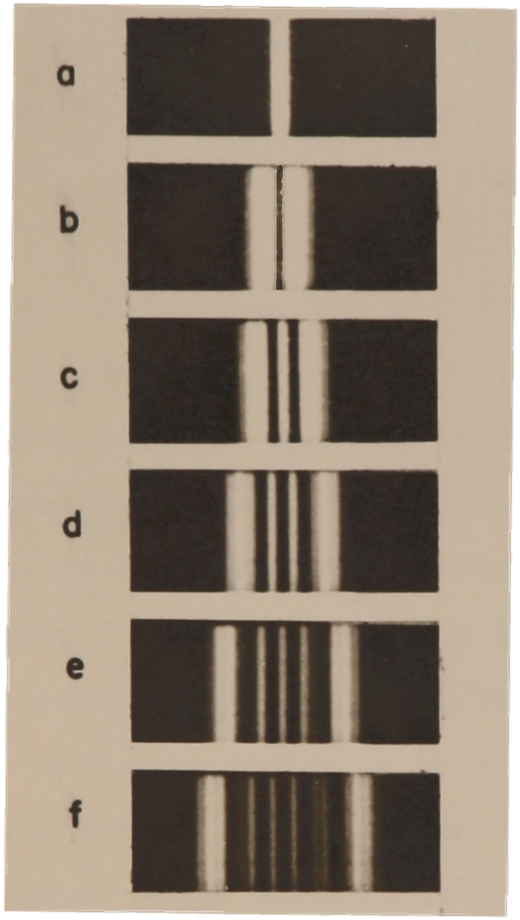

Figure 2. Diffraction patterns obtained for increasing sound intensities showing alternate maximum and minimum of central order characteristic of progressive waves.

is modulated. This modulation may be detected by placing a photomultiplier tube in the position P, amplifying the output signal and displaying it on an oscilloscope. In these experiments an RCA 1P21 photomultiplier tube was used. The signal was amplified by a high frequency amplifier made in this laboratory. The oscilloscope was a Hewlett-Packard model 150A.

Rather than detecting the modulated beam, it was important for some of the experiments to obtain a time average of the light intensity in various positions along the plane of Sl_3. This was done by using a photomultiplier microphotometer made by the American Instrument Company which used a 931A phototube. The phototube was again placed in the position P. In some of the experiments the phototube and Sl_3 were moved horizontally by a screw arrangement driven by a synchronous motor. On these occasions the readings of the microphotometer were continuously recorded by a recording potentiometer made by the Minneapolis-Honeywell Regulator Company. The paper in the recorder ran at a rate of three divisions (one inch) per minute. The synchronous motor, going at three revolutions per minute, drove the slit and phototube at three millimeters per minute. Thus, one division on the recorder paper corresponded to one millimeter of travel of the slit and phototube. It was found that the error incurred in measuring the distance traveled by the phototube by referring to the recorder chart was less than 0.1%.

CHAPTER III

EXPERIMENTAL PROCEDURE AND RESULTS

Waveform determination: Stroboscopic Method

The broadening of an image after light has passed through a sound beam offers the possibility for determination of the wave form of the sound in the liquid, and therefore the possibility of determining the extent of distortion of the wave due to a non-linear relationship between the pressure and the density of the liquid.

One method used in an attempt to record the wave form of sound in liquids is shown schematically in Figure 3. Neglecting the apparatus to the left of Sl_1 for the moment, let us consider what happens to the right of it. As has been indicated earlier, the light passing through the tank T_2 is refracted because of the gradient of index of refraction in the liquid caused by the sound emitted by the transducer Q_2. As the sound progresses, the light beam is refracted first in one direction, then the other as the sound wave fronts pass the slit Sl_2. Now, suppose the light, rather than being continuous, were intermittent such that the burst of light passed through the sound beam during the same phase of each wave length. This would mean that the image at Sl_3 would be broadened by an amount characteristic of that portion of the sound wave through which the light traveled.

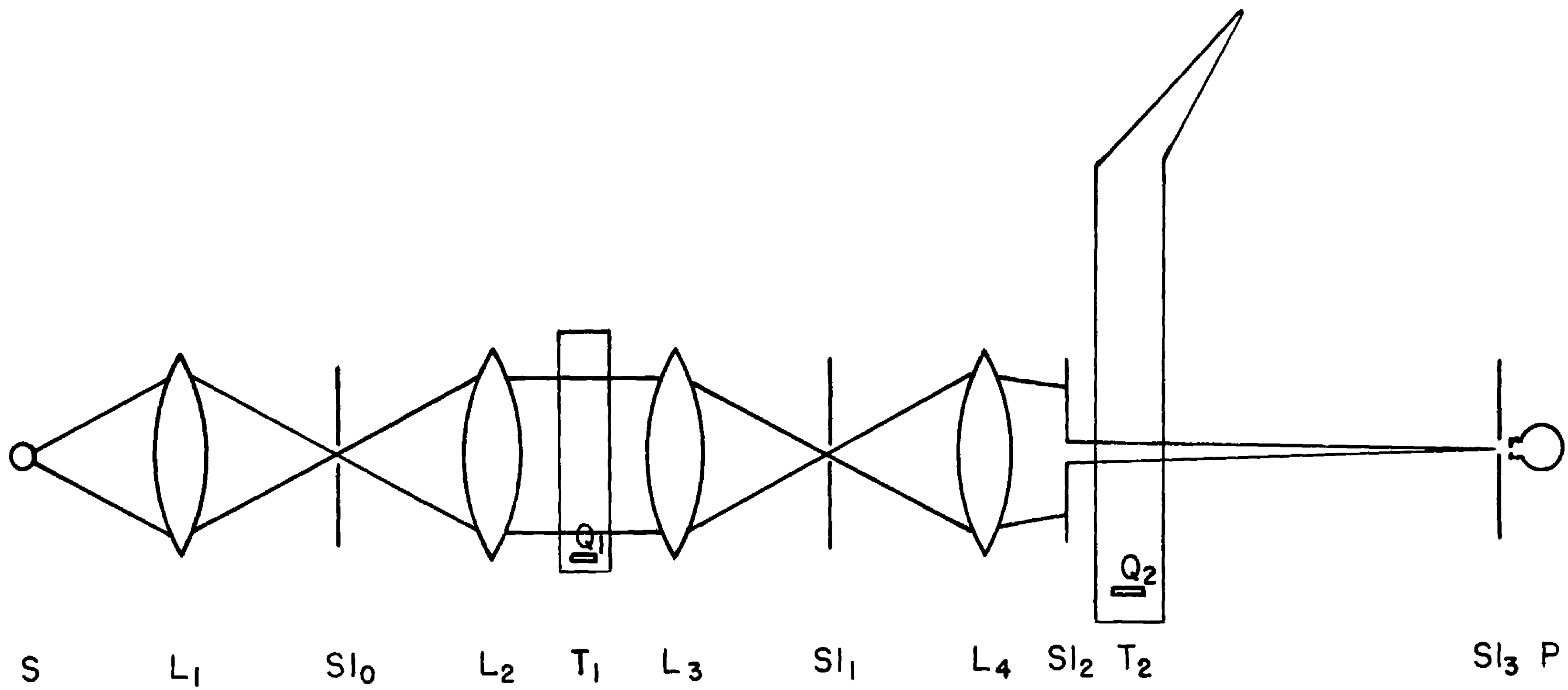

Figure 3. Optical arrangement used in stroboscopic method of waveform determination.

Then, the amount of light passing through Sl_3 would also be characteristic of that portion of the sound beam. If the transducer Q_2 were slowly moved, then the amount of light reaching the photomultiplier would vary as the light beam passed through different portions of the sound wave length. A recorder trace of the photomultiplier output would then be an indication of the wave form of the sound in the liquid.

The appartus to the left of Sl_1 is a stroboscope for obtaining this intermittent illumination. Its action depends upon the fact that, due to the standing sound waves in T_1, the image of Sl_o on Sl_1 is modulated. This is the so-called modulation stroboscope. There are two deviations from the usual use of the ultrasonic stroboscope which are worthy of note. The first is that the frequency at which it is used is much lower than the usual range of application. The quarts Q_1 in Tank T_1 was excited at its fundamental frequency of 420 kc. Stroboscopes are most often used at frequencies above one megacycle. The second is that stroboscopic illumination is at the frequency of the quartz transducer Q_2 in T_2, and not twice this frequency. This was accomplished in the following way. It is well known that a stroboscope of this type produces light pulses which are at twice the frequency of the transducer in T_1 if one allows only the central order to pass through Sl_1. Thus, it was necessary to take a signal from the oscillator exciting the quartz Q_1, put it through a frequency doubler, amplify it and use it to excite the quartz Q_2. In this way the frequency of modulation of the light passing through T_2 was the same as that of the sound beam under study.

Although it was possible to obtain recorder traces corresponding to the sound wave in the liquid, this method offered a number of difficulties. These difficulties can best be brought out by describing the procedure necessary to make a single wave form determination.

After tuning the oscillator and doubler, the reflector in tank T_1 was carefully aligned and the distance between it and the transducer Q_1 was adjusted for standing waves. Ordinarily, one can determine the optimum position of the reflector for standing waves by simply observing the diffraction pattern in the plane of Sl_1; however, at the low frequencies used in this experiment the wave length was of the order of 1mm, which is too large to obtain noticeable separation of the diffracted orders. In order to prevent difficulties due to the heating effect of the sound, the sound intensity was kept to a relatively low value, which further limited the diffraction pattern. The most obvious method for determining the standing wave condition was an application of the visibility method. If one removes L_3 and places a screen in the region between T_1 and Sl_1, he finds that when standing waves are set up T_1 there will be various positions of this screen along the optic axis at which it will be illuminated by bright lines parallel to the standing wave fronts. Or, if Sl_1 is wide enough and Sl_2 is removed, with L_3 in place, one can observe the same phenomenon in the region between T_2 and Sl_3 since the light passing through T_2 is almost parallel. (The transducer Q_2 is, of course, disconnected for these observations.) This is the most easily accessible region since it does not require major changes in the optical arrangement. With a screen in place between T_2 and Sl_3, and with Sl_1 and Sl_2 very wide, the reflector in T_1 was adjusted until the bright lines on the screen were most intense. This meant that standing waves were set up in T_1. Relatively small motions of the reflector affected the brightness of these lines quite appreciably.

This screen was also used to indicate the optimum position and width of the slit Sl_1. On closing Sl_1, the bright lines disappeared, leaving an evenly illuminated field. On turning the sound off in T_1, the field became brighter if Sl_1 was on the optic axis and passed only the central order. Thus, by observing the difference in the illumination of the screen when the sound was on versus that when the sound was off, the optimum position and width of Sl_1 could be chosen. This optimum position of Sl_1 meant that the stroboscope would work properly. That the position and width of Sl_1 was extremely critical can be seen by considering the separation of the diffraction orders. Using the focal length of lens L_3 as 90 mm and the sound wave length as 1mm in the usual diffraction grating formula, one finds that the separation of the first order from the optic axis is 0.04 mm. The slit Sl_1 cannot be wider than twice this value, and must be positioned accurately on the optic axis. Also, the image of Sl_0 in the plane of Sl_1 must be very narrow and sharply defined. This was accomplished by using a percision slit manufactured in Germany by R. Fuess as the source slit Sl_0.

With these adjustments made, the screen was removed, L_4 was adjusted to give a sharp image of Sl_1 on Sl_3, and Sl_2 is put back in place. With the slit Sl_2 narrow compared to a sound wave length, usually approximately $\lambda/8$, the transducer Q_2 was moved slowly perpendicular to the optic axis by a screw arrangement driven by a synchronous motor. The different portions of the wave refracted the light differently, causing more or less light to pass through Sl_3 to the photomultiplier. The variations of the output of the photomultiplier recorded by a strip chart recorder were then interpreted in terms of the variation of the density gradient in the liquid throughout a wave length.

The recorder trace shown in Figure 4 is a typical trace obtained by this method. Trace (a) was obtained as the transducer moved toward the optic axis;(b) was obtained as it moved away. The curves indicate a distortion of some kind, but it is to be noted that the two curves are only approximately in agreement. The spurious distortions are explained in terms of line voltage variations and the like,

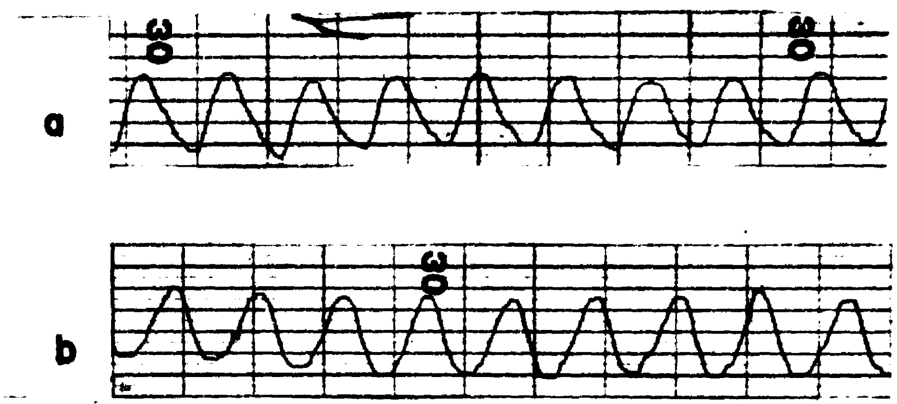

Figure 4. Recorder traces of sound wave form in glycerin by stroboscopic method. (a) Transducer moving toward optic axis; (b) Transducer moving away from optic axis.

but the consistent distortion, particularly in trace (a), might be cause for some interest. However, as will be shown in the next section, this distortion may be explained as due to a misalignment of the slit Sl_3. This points out the fact that this method is very difficult to use for reliable measurements. Among the difficulties encountered are: Line voltage fluctuations, change of standing wave pattern in tank T_1 due to heating of the liquid, slow recording of data, and difficulty in determining source of distortion, i.e., making sure that distortion is caused by distortion in sound wave.

Waveform Determination: Direct Observation

A more reliable and experimentally more amenable method was developed which allows one to observe the wave form directly on an oscilloscope screen. This method uses the fact that the light passing through the final slit is modulated at the frequency of the sound in the liquid. This modulation is detected by a photomultiplier, then amplified and presented on oscilloscope. An obvious advantage of this method is the fact that the wave form can be viewed on the oscilloscope screen directly and the effect of any adjustment can be immediately observed. This is to be contrasted with the previous method which required approximately a minute to record one complete wave.

The experimental arrangement used in this method is shown schematically in Figure 5. The light passing through

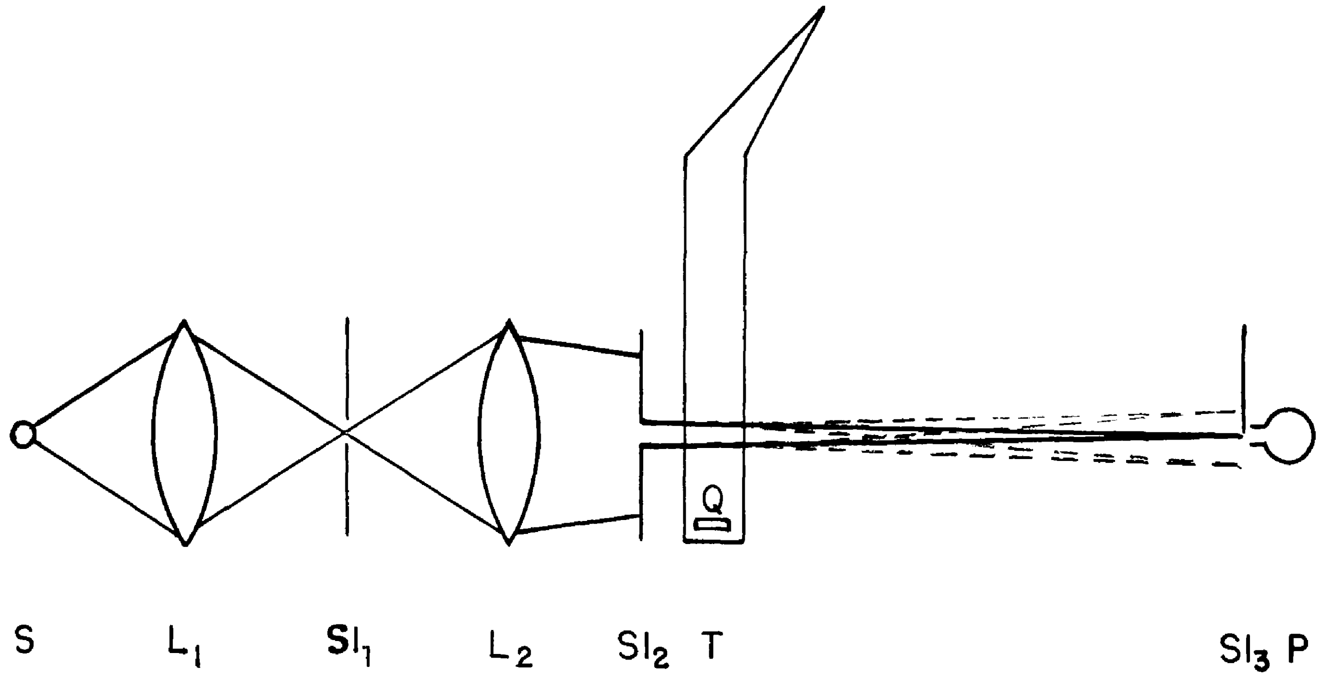

Figure 5. Optical arrangement used in direct observation of sound wave form. The output of photomultiplier P is observed on an oscilloscope.

the tank T is small in cross section compared to a wave length of sound in the liquid. (This cross section was usually $\lambda/8$.) As the sound wave passes, the light is swept back and forth at the frequency of the sound. Thus, since Sl_3 is a semi-infinite plane, the photomultiplier P receives light whose intensity varies sinusoidally with time if the wave in the liquid is sinusoidal. After amplification, the output of the photomultiplier is put on the vertical gain of an oscilloscope and may be viewed directly. A signal from the oscillator could be used as a trigger to synchronize the oscilloscope trace with the signal from the photomultiplier. It was found, however, that at these frequencies the internal synchronization of the oscilloscope provided a stable trace so that an external trigger was unnecessary.

That the final slit Sl_3 must rather be a semi-infinite plane can be seen if one considers the intensity distribution in the image due to slit differation caused by Sl_2. This intensity distribution is shown in Figure 6.

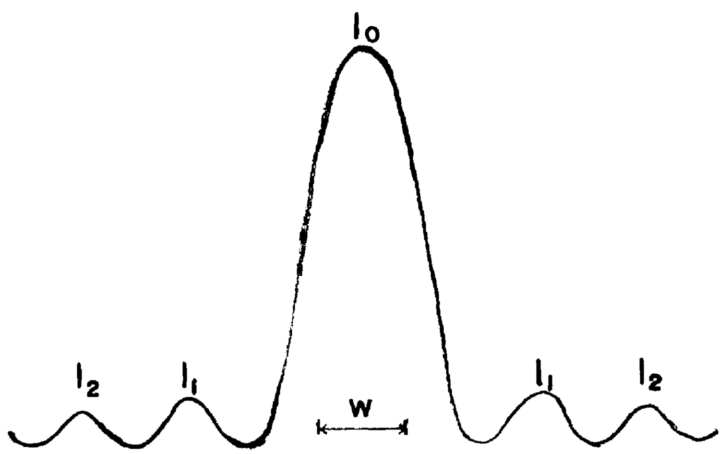

Figure 6. Single slit diffraction pattern caused by Sl_2.

Now, let us consider that Sl_3 is actually a slit of width w indicated in Figure 6. As the intensity distribution is swept back and forth across the slit, there will be a frequency doubling effect due to the central maximum's sweeping through the equilibrium position twice during each cycle, but more important, if the amplitude of motion of the image is large enough there will be contributions due to the diffracted images I_1,and possible I_2. The repetition rate for this contribution will be twice that of the contribution I_o since there are two images I_1. Thus, there will be a second harmonic component added to the photomultiplier output. This is the same type of distortion as is expected from the wave form distortion in the liquid. Therefore, it must be eliminated. This is done by placing one edge of the slit Sl_3 in the exact center of I_o and removing the other. Now, when the image is swept back and forth sinusoidally, the light passing one edge will be essentially sinusoidal if the amplitude is less than half the width of I_o.

The output of the photomultiplier was displayed on the screen of an oscilloscope. By observing the trace on the oscilloscope, then, one could determine the wave form in the liquid under investigation. It was found, however, that this method of direct observation was not accurate enough since in most cases the wave form was very nearly sinusoidal. In order to increase the ability to estimate the deviation from a sine wave, a sinusoidal signal from the oscillator was put on the vertical deflection plates of the oscilloscope and the signal from the photomultiplier was put on the horizontal, forming the familiar Lissajous figures. This method necessitated a graphical analysis in order to determine the change in the Lissajous figures introduced by any distortion of the sound wave in the liquid. This analysis was made as follows.

According to the Fay theory[10], one expects the wave form distortion to be such that the second harmonic term in the Fourier Expansion of the wave is the largest term other than the fundamental. Thus, assuming the second harmonic to be a certain fraction of the first and neglecting the higher harmonics, a drawing could be made of the composite wave which would approximate the expected distorted wave in the liquid. Figure 7 (a) shows the wave resulting from a fundamental and a second harmonic assumed to be 20% of the fundamental. (This value of 20% will be shown to be high. However, it will give an idea of the type of distortion one may expect.) Since the refraction of the light beam depends on the pressure gradient and not on the pressure itself, it was necessary next to take the slopes of the curve in Figure 7(a) and plot them, as is shown in Figure 7(b). Now, combining the curve 7(b) with a sinusoidal curve in the usual way, one obtains the Lissajous figure to be expected on the oscilloscope when the wave in the liquid is

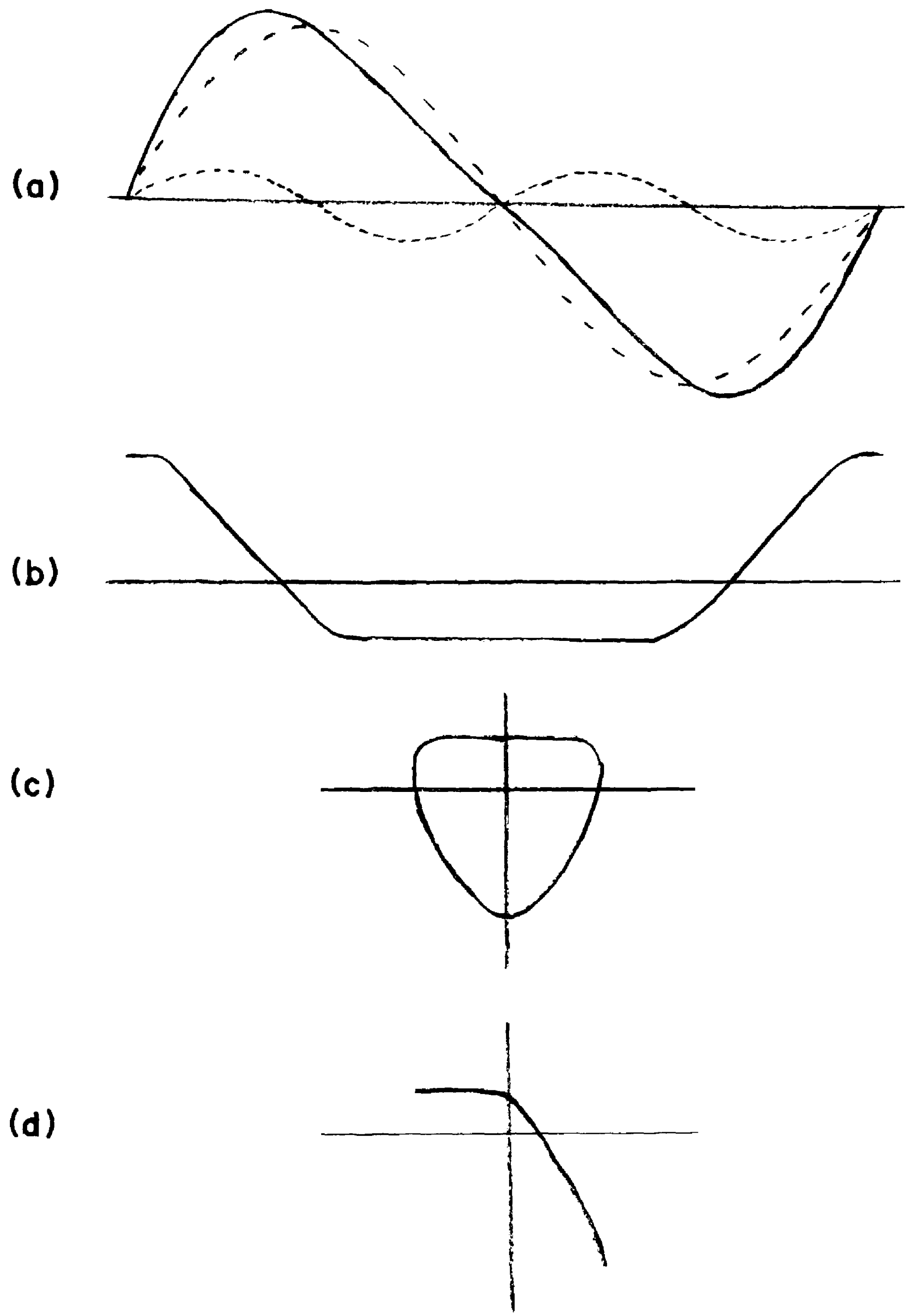

Figure 7. Graphical determination of Lissajous figure expected if second harmonic is present in sound wave. (a) Resultant of fundamental and 20% second harmonic. (b) Plot of slopes of (a). (c) Lissajous figure of (b) interacting with a sine wave in phase. (d) Lissajous figure for phase difference of $\pi/2$.

distorted. This is shown in Figures 7(c) and 7(d) for phase differences of 0 and $\pi/2$ respectively. If there were no distortion 7(c) would be a circle and 7(d) would be a straight line passing through the origin.

The Figures 7(c) and 7(d) indicate the type of Lissajous figure to expect on the oscilloscope. With these figures in mind, investigations were made in the following manner. In order to get the edge of the semi-infinite plane Sl_3 exactly in the center of the image of Sl_1, Sl_1 was made very long so that its image at Sl_3 was very long. The edge of Sl_3 was lined up parallel to the image visually, then the image was again shortened. Next, the photomultiplier P was moved aside and a photomultiplier microphotometer was put in its place. With the plane Sl_3 completely away from the image, the microphotometer scale was adjusted to read 100; with Sl_3 blocking out the image, the scale was zeroed. Now, the plane was moved until the microphotometer read 50. This was assumed to be the center of the image. The photomultiplier P was put back in place and the Lissajous figure was observed on the oscilloscope. No observations were made without first going through this procedure to position the plane Sl_3. That this was necessary can be seen from the following considerations.

It has been shown [17] that one of the sources of error in optical methods of ultrasonics is the effect of local heating of the medium, particularly in the vicinity of the transmitting quartz. This local heating causes local gradients of index of refraction. Thus, any light passing through the medium will be refracted away from its original direction of travel. Let us consider that such gradients of index of refraction exist in the experiments under discussion. Light entering the medium will in general be refracted away from the optic axis. This means that the image at Sl_3 is no longer on the optic axis and that the edge of Sl_3 is thus no longer at its center. Thus, the variation with time of the light reaching the photocell resembles the curve shown in Figure 7(b), and the trace on the oscilloscope indicates an apparent wave form distortion when actually the wave may be sinusoidal. This effect was observed when the light beam passed very near the quartz face. Any misalignment of Sl_3 would cause the same type of effect. For these reasons extreme care was taken to see that the system was carefully aligned and that the effect of heat schlieren was negligible.

Some of the results of these experiments are shown in Figures 8, 9, and 10. In Figures 8(a) and 8(b) are presented the Lissajous figures obtained for phase differences of 0 and $\pi/2$ respectively at three sound intensity levels in water. The greatest intensity is of the order of two watts per square centimeter. By placing a straight edge along the figures one can observe the deviation of the figure from the straight line expected of perfectly sinusodial

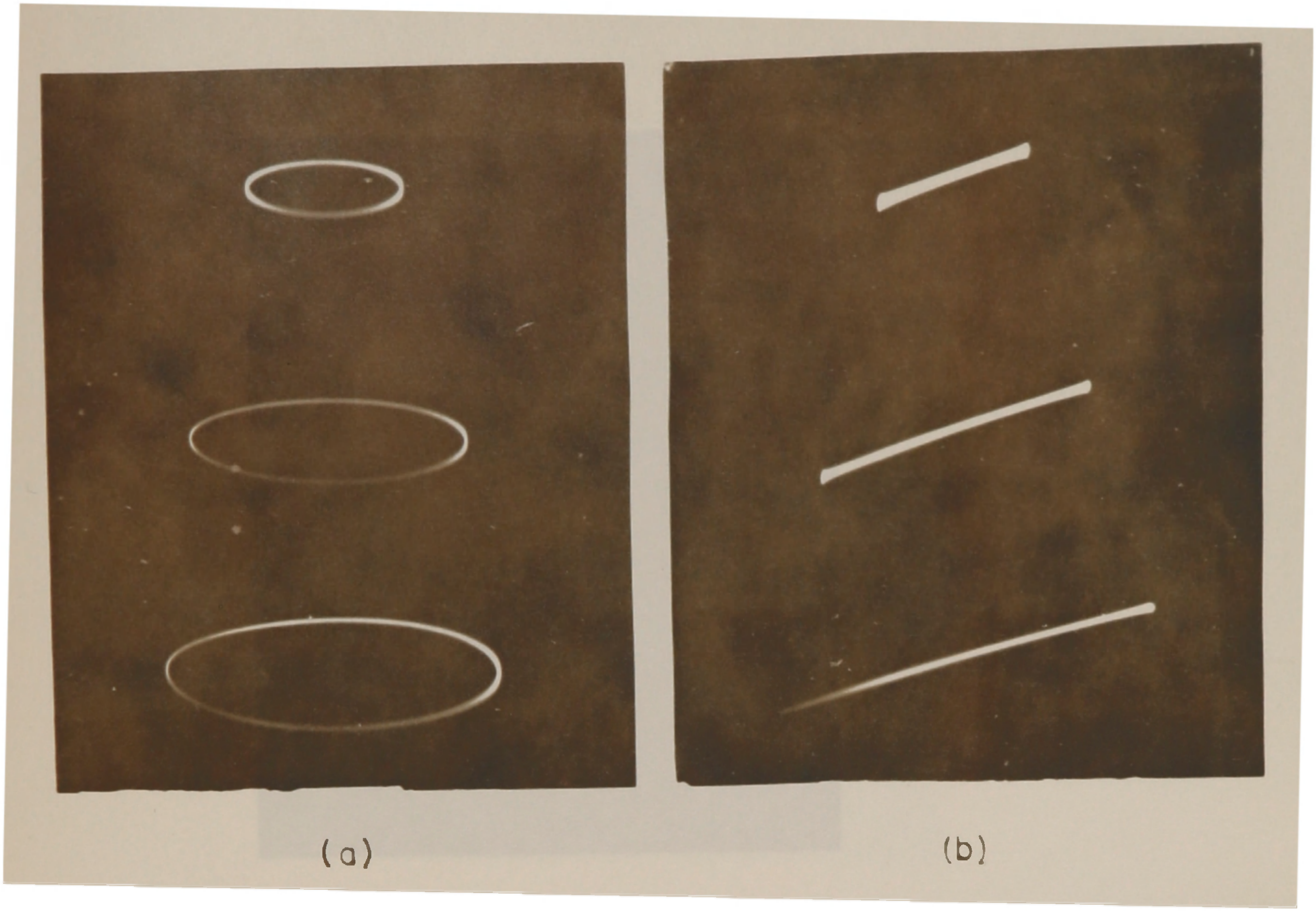

Figure 8. Lissajous figures obtained from water at three sound intensities. (a) In phase. (b) Phase difference of $\pi/2$.

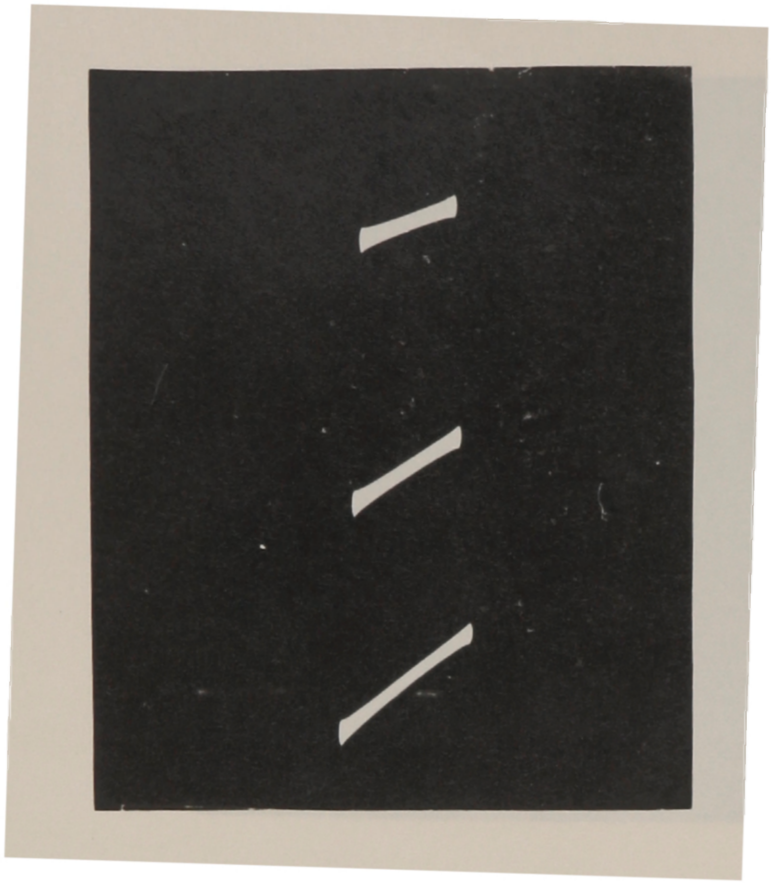

Figure 9. Lissajous figures obtained from glycerin at three sound intensities.

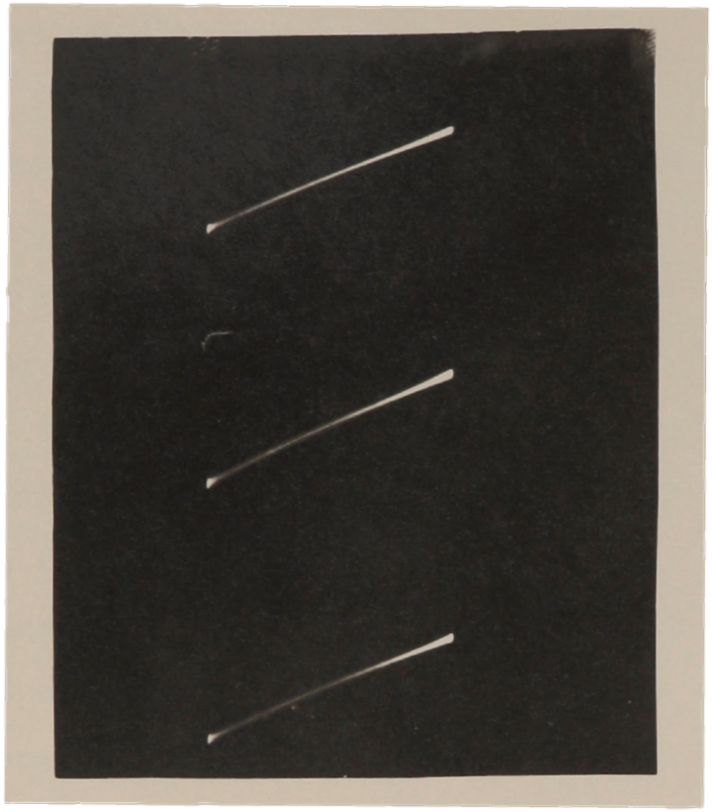

Figure 10. Lissajous figures obtained from water at distances of 10, 17, and 24 centimeters from the face of the transducer.

waves. The deviation shown in the last picture in 8(b) corresponds to a sound wave having a second harmonic component of approximately 5% of the fundamental.

Figure 9 is a series of three Lissajous figures in glycerin for approximately the same intensities as in Figure 8 for water. In this case one can see that the sound is very nearly sinusodial.

According to the theory, a sound wave originating at a sinusoidally vibrating source begins as a sinusoidal wave and travels a certain distance, which varies both with the liquid and the frequency, before reaching the "most stable wave form" characteristic of its intensity. It would be interesting to show the gradual change toward this "most stable wave form." This could be done by observing the wave shape at various distances from the source. Figure 10 represents an unsuccessful attempt to do this for water. The wave form was observed at distances of 10, 17, and 24 centimeters from the face of the transducer. The difference between the Lissajous figures is very slight, if it exists at all. This cannot, however, be taken as proof that the theory is wrong. For water at these frequencies the maximum distance required for stabilization, according to the Fox and Wallace theory, is approximately ten times the distance covered in these measurements. Therefore, the fact that these pictures do not show this change of wave form is not surprising. It might even be surprising that the distortion is observable at all at these distances, except for the fact that one would expect the energy transfer from the fundamental to the second harmonic to be greatest near the transducer and to be smaller and smaller with increasing distance.

The results presented in Figures 8, 9, and 10 are not presented as proof that distortion exists at these sound intensity levels. More experimentation is necessary before such a claim can be made. In particular, it will be necessary to investigate other liquids which will exhibit the distortion to a greater extent than those investigated. One liquid which suggests itself is carbon tetrachloride. According to the Fox and Wallace theory[13], this liquid will be better in some respects. However, the wave length at the frequencies used will be shorter so that the ratio of the light cross section to the wave length will be less and the sensitivity of the method will be reduced accordingly. As the experiments were performed, the possible errors were of the same order of magnitude as the signal to be measured. This may also be true of carbon tetrachloride.

Considering the possible error, one cannot conclude from the foregoing experiments whether a distorted wave actually exists in liquids at the sound intensities used. One can, however, conclude that the distortion is probably less than 5% if it exists at all. Thus, it is safe to say that the distortion in the liquids used by Loeber and Hiedemann[5] was too small to account for their results.

The irregularities in the structures of the field of the stationary waves in some liquids revealed by their measurements must be explained in another way.

Sound Pressure Amplitude Measurement

Light refraction can be used to measure sound pressure amplitudes in two ways. These methods differ in the way in which the broadening of the image is determined. The first consists of a measurement of the decrease in the light intensity passing through the final slit Sl_3 (See Figure 1) occurring when the image at Sl_3 is broadened by a sound beam passing through the liquid under investigation; the second consists of a direct measurement of the angle of deflection of the light beam. With the first method one measures the ratio of the light intensity passing through Sl_3 when the image is broadened by sound to that when the sound is off and arrives at the sound pressure by use of the relation derived by Loeber and Hiedemann[5]. With the second he measures the distance the image at Sl_3 is displaced from the optic axis, determines the maximum angle of displacement, and arrives at the pressure by use of a very simple equation given by Hüter and Pohlman[3].

Method I: Intensity of the undeflected beam. In order to use this method to measure the sound pressure amplitude in progressive waves, it is necessary to make certain modifications of the theory for stationary waves given by Loeber and Hiedemann. They consider the superposition of two oppositely directed sound waves and how light is refracted on passing through them. They find that the amplitude of the pressure wave can be expressed in the form

$$P \sin \left({}^{\omega x}/_{u} \right) \approx A^{-1} \left(R^{-2} - .232 \right)^{1/2} \tag{1}$$

where

$$A = \frac{2 \pi a \omega g \delta k}{\lambda_L r u} \tag{2}$$

and

$$k = \chi \frac{(n^2 - 1) \quad (n^2 + 2)}{6n} \tag{3}$$

The quantities used in these equations are defined as follows (See Figure 1):

- P Sound pressure amplitude
- ω Angular frequency
- x Distance between light beam and face of transducer
- u Sound velocity in medium
- R Ratio of light passing through slit Sl_3 when sound is on to that passing through when sound is off
- a Width of slit Sl_2
- g Distance between sound beam and Sl_3
- δ Length of light path in sound field
- λ_L Light wave length
- r Distance between Sl_2 and Sl_3
- n Index of refraction of medium
- χ Compressibility of medium

The sound pressure is evaluated at the point in the stationary wave where $\sin (\omega x/u) = 1$, that is, where the ratio R is a minimum. Then, for the actual measurement one uses the formula

$$P \approx A^{-1} \left(R^{-2}_{min} - .232 \right)^{1/2} \tag{4}$$

which is more accurate for $R < 1/3$.

If one considers a single progressive wave and makes the same analysis made by Loeber and Hiedemann, he will find that the sine term will average out and that the pressure amplitude of a progressive wave may be expressed in the form

$$P_{prog.} = \frac{2\sqrt{2}}{A} \left(R^{-2} - .232\right)^{1/2}. \qquad (5)$$

This is the form which was used in all measurements made by this method. This formula is applicable under the same conditions for progressive waves as that derived by Loeber and Hiedemann for stationary waves.

Since all quantities except P in equation (5) are known or can be measured, it is possible to make an absolute measurement of the sound pressure amplitude in a progressive wave. This was done by setting up an optical system such as that shown in Figure 1. The actual measurement consisted simply of observing the ratio of the light passing through Sl_3 when the sound was on to that when the sound was off by taking the reading from the photomultiplier microphotometer scale. However, before actual measurements were made the system had to be carefully aligned. Especial care was taken to see that the sound beam was perpendicular to the light beam. This could be done by observing the minimum reading of the photometer scale as the transducer was rotated. When the necessary adjustments were made, a series of readings of the sound pressure amplitude was taken as the sound intensity was increased in steps. Without stopping, a series of readings was taken as the sound intensity was decreased by the same intervals. One such series of measurements is shown in Figure 11 where the pressure is plotted as a function of the voltage on the transducer. It can be seen that the points lie reasonably close to a straight line. It can also be seen that the points obtained for decreasing sound intensity deviate more from the straight line. This can be explained if one considers the heating of the medium by the sound beam. There appears to be a reasonable consistency in the data as shown in Figure 11; however, it is possible that the straight line drawn in Figure 11 deviates from the true curve relating the sound pressure amplitude to the transducer voltage. This possiblity is connected with the fact that the cross section of the light beam passing through the liquid is an appreciable part of a wave length.

The theory by Loeber and Hiedemann is restricted to stationary waves. In extending it to progressive waves the fact that the center of the broadened image passes through maxima and minima as the sound intensity is increased was not considered. In Figure 2 this passage through maxima and minima is shown for a light cross section of $4\lambda/3$. The same type of "fine structure" was observed for light cross sections as small as $\lambda/2$. Thus, if one were to use light cross sections greater than $\lambda/2$, the pressure would appear to pass through maxima and minima since the ratio R is always

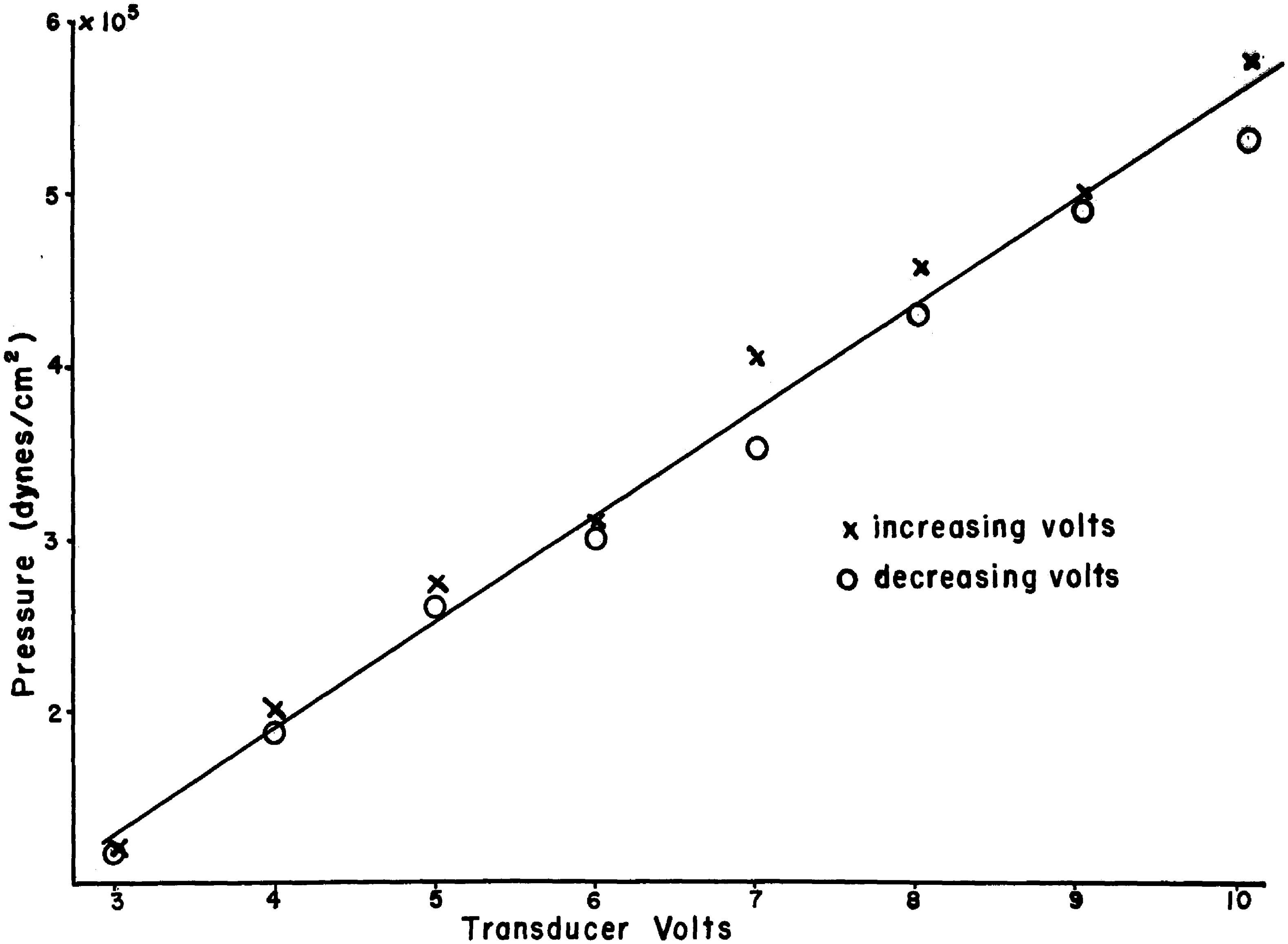

Figure 11. Sound pressure amplitude in water as a function of the transducer voltage as measured by Method I. Frequency: 1Mc. Light beam width: $\lambda/3$.

taken at the center of the image. This consideration rules out the use of this method for light cross sections greater than $\lambda/2$. However, it is important to know whether the method can be used for light beams which are an appreciable part of $\lambda/2$ since slit diffraction effects will set a lower limit on the light beam width. If the light beam is wider than $\lambda/2$, errors will be introduced by the presence of "fine structure"; if the beam is too narrow, errors will be introduced by the fact that the light is diffracted by the limiting slit and no longer passes through the liquid in a well-defined beam. It is thus necessary to determine whether there is a light beam width which will give the correct value of the sound pressure amplitude. A series of readings of the ratio R was made at a given sound intensity for various widths of the light beam. Curves for three intensities are shown in Figure 12. It can be seen that the ratio varies considerably with the changing light beam width. This means that the value given for the sound pressure will be greatly affected by the light beam width. The sharp increase in the value of R for very narrow light beams can be explained as resulting from slit diffraction. However, judging from observation, it is probably that these effects cease to be important somewhere between $\lambda/8$ and $\lambda/4$ (between approximately 0.5 and 1.0 mm). There should be a point along these curves below $\lambda/2$ which gives the correct value of sound pressure amplitude. This point was found in one case by a comparison of this method and Method II which will be presented later. It should be pointed

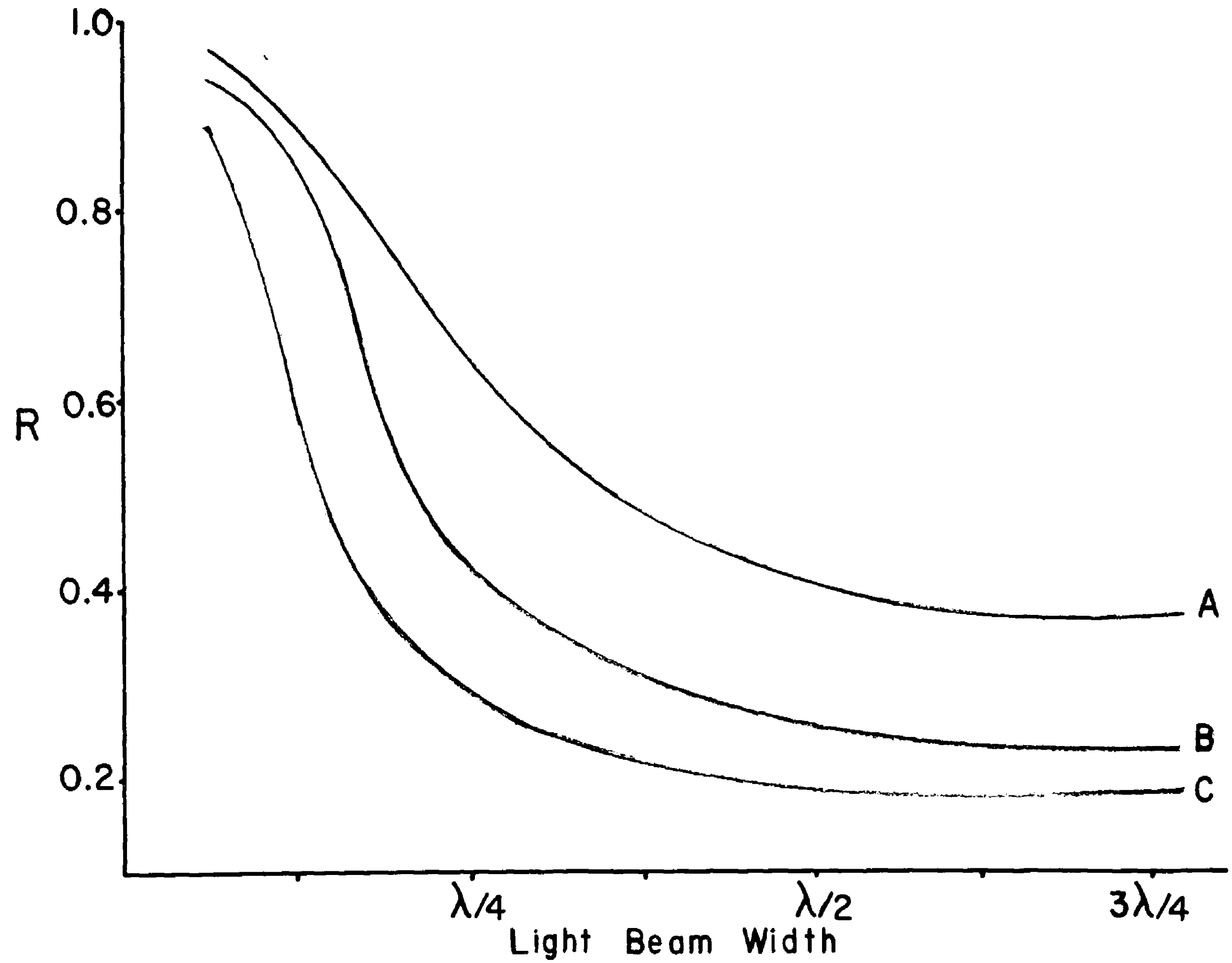

Figure 12. The variation of the ratio R with the width of the light beam through the sound field at three intensities. Curve A, 10 volts on transducer; B, 15 volts; C, 20 volts. Wave length of sound: 3.75 mm.

out that because of slit diffraction this method must be used with extreme care at wave lengths less than those used here (approximately 4 mm).

Actually, it would be expected that if there were no slit diffraction, the narrowest light beam which would give a reading on the photomultiplier would give the most correct value of the sound pressure amplitude. Thus, it is expected that an increase in the wave length (lower ultrasonic frequencies) would give an increase of accuracy. However, at lower frequencies it is increasingly difficult to get rid of reflected waves since the absorption varies as the square of the frequency.

Method II: Broadening of image. If one considers that the light in the tank, if its width is smaller than a half wave length, experiences a bending due to the gradient of index of refraction in the sound beam, he can calculate the maximum angle of deviation from geometrical optics considerations. Since the index of refraction gradient is proportional to the pressure gradient, one can arrive at the sound pressure amplitude expressed as a function of the maximum angle of deviation of the light beam and the parameters characterizing the liquid. This expression is given by Hüter and Pohlman[3] as

$$P = \frac{\gamma \lambda}{K L} \qquad (6)$$

where

$$K = \frac{2 \pi \chi (n^2 - 1) (n^2 + 2)}{6n^2} \qquad (7)$$

The quantities used in these equations are defined as follows:

- P Sound pressure amplitude
- γ Angle of maximum deviation of light beam
- λ Sound wave length
- L Path length of light in sound beam
- κ Compressibility of medium
- n Index of refraction of medium

(Equation (7) was not given by Hüter and Pohlman, but was derived from expressions given by Lucas and Biquard.)

Then, one can find the sound pressure amplitude by observing the maximum angle of deviation of the light beam. This can be done by observing the maximum displacement of the image at the position of Sl_3. There are in general two different ways to determine the amount of broadening of the image by the passage of sound (that is, the maximum angle of deviation). One may take a photograph of the image and make measurements directly on the negative, or one may place a photometer behind Sl_3, then, with Sl_3 very narrow, move both Sl_3 and the photometer perpendicular to the optic axis. With a method of making this motion very evenly and very accurately, one can record the output of the photomultiplier on a strip chart recorder and relate the distances on the strip chart directly to distances moved by Sl_3 and the photocell. The latter method was used exclusively in these measurements. The distances measured on the strip chart record differed from the distances measured by a micrometer screw attached to Sl_3 by less than 0.1%.

It has been mentioned[3] that, although the theory is derived for a light beam whose width a satisfies the relation

$a < \lambda/2$, one may use widths greater than this and find that the broadening is the same and that the edges of the broadened beam are better defined--even though the intensity distribution characteristic of a diffraction pattern is present. This is shown in Figure 13. The pictures (a) through (d) were taken for widths of Sl_2 equal to $\lambda/3$, $2\lambda/3$, λ, and $4\lambda/3$ respectively. Figure 13(a) is the picture usually obtained for a broadened image where the light cross section is less than a half wave length of sound. The succeeding pictures begin to show a diffraction pattern; however, it is to be noted that the bright bands near the edges are the same distance apart in all four pictures. Thus, it appears that the amount of broadening is independent of the cross section of the light passing through the medium. This can be much more accurately tested if one considers Figure 14, which is a series of recorder traces taken for the same situation. The heights of the peaks are relatively unimportant for these considerations. (The difference in heights was caused by changing the photomultiplier scale between recordings to keep the readings on scale.) What is more important is the fact that the two outer peaks in each picture are the same distance apart to within 4%. Thus, it is permissible to go to the larger light beam cross sections where the peaks are more sharply defined in measuring the sound pressure amplitude. The difference in these traces shows that the accuracy of this method is limited if one

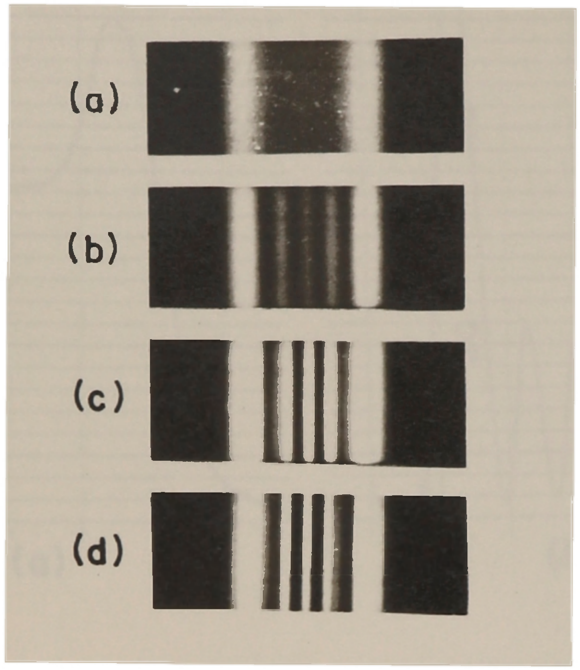

Figure 13. Broadened image caused by passage of light through sound beam showing changes in structure as light beam width is increased. Light beam widths are: (a) $\lambda/3$, (b) $2\lambda/3$, (c) λ, and (d) $4\lambda/3$.

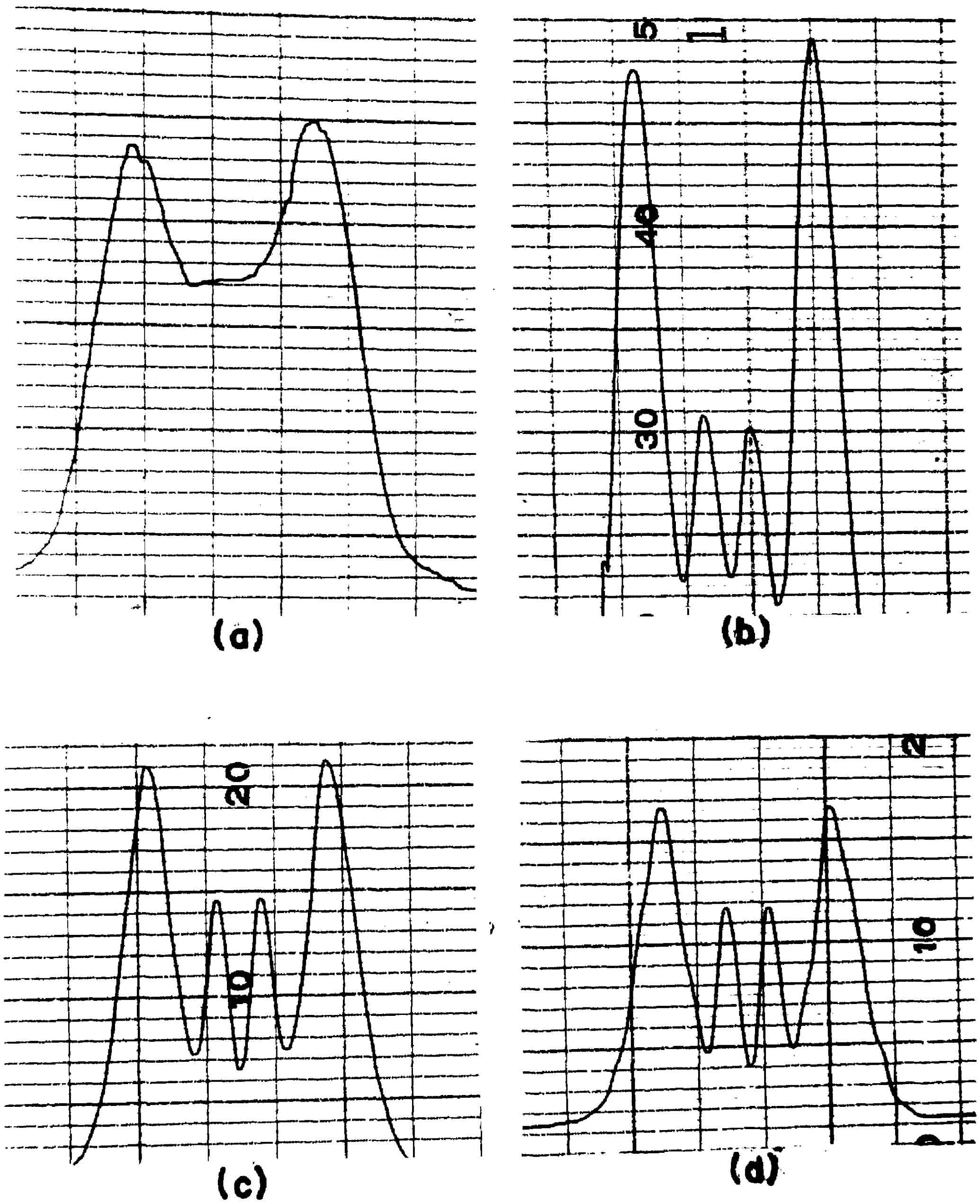

Figure 14. Recorder traces of broadened images for light beam widths of: (a) $\lambda/3$, (b) 2 $\lambda/3$, (c) λ, (d) 4 $\lambda/3$.

uses light beam cross sections greater than $\lambda/2$. However, going to smaller cross sections means that the peaks are not as sharp. This broadening of the peak will introduce an error into the measurements which will probably be of the same order of magnitude as that introduced by going to larger cross sections.

Comparison of the two methods. In order to compare the two methods the apparatus was set up and a recorder trace was made of the unbroadened images and the broadened images at a number of sound intensities and light beam cross sections. In this way the ratio of the height of the center of the broadened image to the height of the unbroadened image could be used in equation 4 to determine the pressure amplitude by Method I, and the distance between the peaks of the broadened image could be used in equation 5 to determine the pressure amplitude by Method II. This procedure allows the direct comparison of the two methods from the same data. The results of this procedure are shown in Figure 15. It can be seen that the points obtained by Method II lie reasonably close to the straight line drawn through them. The maximum deviation is of the order of 15%, but the average deviation is much lower than this. (This deviation is in part due to the use of a barium transducer whose output changes noticeably with temperature.) On the other hand, the points obtained by Method I are consistent within any particular run, i.e., they are consistent for any particular

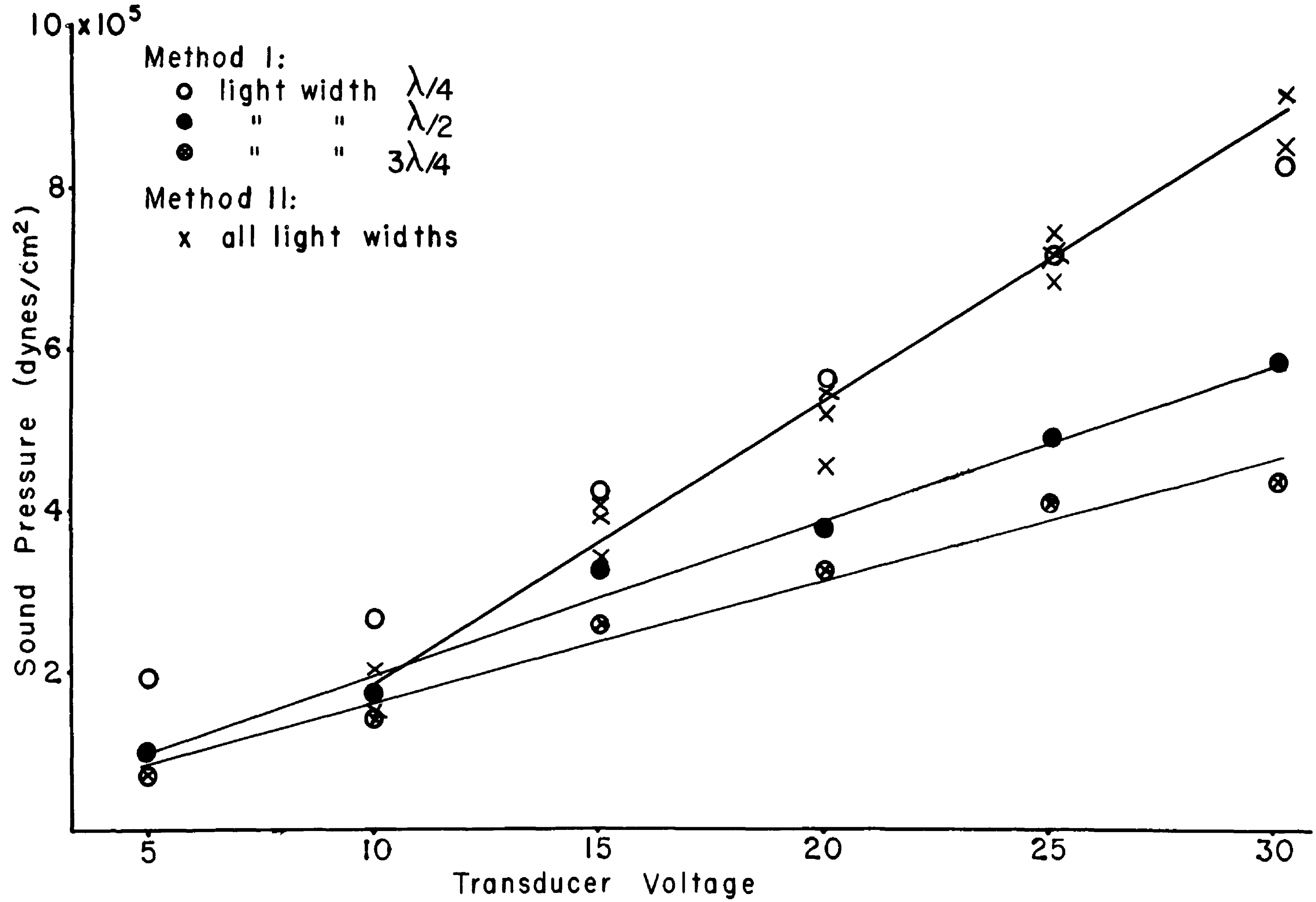

Figure 15. The variation of measured sound pressure with transducer voltage.

light beam cross section, but the differences between the data for different runs is quite appreciable. If one considers the three different runs, he sees that the curves get closer to the curve obtained by Method II as the light beam width is decreased. The points obtained for a light width of $\lambda/4$ agree with the curve obtained by Method II very well at the higher transducer voltages. The disagreement at the lower transducer voltages arises from the face that the ratio R is greater than 1/3. The condition $R < 1/3$ is satisfied for voltages greater than approximately 16 volts.

Figure 15 shows that Method II has a wider range of applicability than Method I. Method II also has the advantage that the value obtained for the sound pressure amplitude is independent of the light beam width--although Method I can be used just as successfully provided the sound wave length is large enough that slit diffraction becomes noticeable only at light beam widths less than $\lambda/4$. Method I has the advantage that readings can be taken instantly so that errors due to heating of the medium will be negligible; however, it has the disadvantage that the calculations are longer than with Method II.

As a result of the foregoing experiments and considerations it is concluded that Method II has a wider range of applicability, is less likely to give misleading values, and is more easily applied to measurements of sound pressure amplitudes in progressive waves. The value of this method can be realized more fully only after it has been compared directly with completely different methods.

CHAPTER IV

SUMMARY AND CONCLUSIONS

The wave form of sound in liquids has been studied by use of the light refraction method. Since the possible error was of the same order of magnitude as the signal to be measured, it was possible only to set an upper limit to the amount of distortion present. In water the distortion at the sound intensities and frequency used is 5 per cent or less. In glycerin it was unobservable.

Two methods were used to evaluate the sound pressure amplitude in a progressive wave in water. For accurate measurements, Method I, with which one measures the decrease in intensity of the undeviated light beam, must be used with very narrow light beams passing through the sound beam to be measured. The accuracy increases with decreasing width of the light beam until an optimum width is reached, after which the error due to diffraction ceases to be negligible.

It is believed that Method II, with which one measures the amount of broadening of the image due to the presence of the sound wave, will give values for the sound pressure amplitude which are accurate to within 10 per cent or less. This is not an unreasonable and useless accuracy if one considers that this is inherently an integration of the sound intensity across the width of the sound beam and that in most practical applications of the method one will be measuring the sound pressure amplitude in the Fresnel region where it is well known that the pressure amplitudes fluctuate between theoretically zero and the maximum pressure in the sound beam. It is also to be noted that this method is inherently an absolute method and that the absolute methods which have been used to date seldom have an accuracy which is greater than this.

Since these methods are absolute ones, they promise to be good methods for calibrating other methods for measuring sound pressure amplitudes. It is believed that these methods are more dependable than the sound radiometers which have been used heretofore.

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