

EDUCATIVE SUPPORTS FOR TEACHERS IN MIDDLE SCHOOL MATHEMATICS
CURRICULUM MATERIALS: WHAT IS OFFERED AND HOW IS IT EXPRESSED?

By

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ABSTRACT

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Teaching can have a substantial impact on student learning (Darling-Hammond, 1999). However, teaching excellence depends on many factors, including the need for high quality teachers and their continued education, and high quality materials (Cohen, Raudenbush, & Ball, 2002; Putnam & Borko, 2000). This learning includes learning to plan and enact lessons that are appropriate for all students, which requires learning to interpret and understand student thinking and learning instructional routines and practices that will enable them to use student thinking productively. As we enter into the era of the Common Core State Standards for Mathematics this learning is even more critical, as the standards may require teachers to not only learn to understand and unpack the standards themselves, but may also require teachers to learn new content and learn to teach in different ways (Lappan, McCallum, Kepner, 2010).

Due to the complex nature of teaching and the myriad of demands placed on teachers, mathematics educators need to consider all possible venues for teacher learning. In this paper, I discuss my examination of the opportunities for teacher learning embedded within written curriculum materials. Research indicates that teachers can and do learn from curriculum materials. Curriculum materials, particularly educative ones, emerge as a potential source for opportunities for teacher learning in ways that set them apart from more traditional professional development, which is often criticized for being decontextualized, contrived, short-term, fragmented, discontinuous, and disconnected (Ball & Cohen, 1999; Little, 1994; Lord, 1994;

Wilson & Berne, 1999). *Educative curriculum materials* are materials for Grades K-12 that are “intended to promote teacher learning in addition to students’ ” (Davis & Krajcik, 2005, p. 3).

I investigated the opportunities to learn embedded in four middle school curricular series in the areas on introduction to variables and geometric transformations, by examining the content and its expression in the teachers' guides. I developed and used two analytical frameworks; one to code the content support derived from work in science education (Beyer et al., 2009) and a second framework to describe the expression of text developed by Morgan (1996) and augmented by Herbel-Esienmann (2007).

My results indicated that all four curricular series included opportunities for teacher learning (mostly related to *Pedagogical Content Support for Practices and Curricular Knowledge*, depending on the curriculum) in both the variable and the transformations units, but these opportunities were quite minimal and focused heavily on particular types of supports. This lack of support was particularly true for *Rationale Guidance* for teachers. In addition to the content support, my analysis of aspects of voice indicated that although these four series provided opportunities for teacher learning, they also may hinder teachers' learning by speaking "through" teachers rather than "to" teachers (Remillard, 2000), as evidenced by the ways in which personal pronouns were used and the frequencies of imperatives and modal verbs. I discuss implications for curriculum development, teacher education, and research.

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DEDICATION

I dedicate this dissertation to my husband Josh,
a fellow mathematics teacher, who was there for me
throughout this journey and never made me feel
like I was making the wrong decision

and

to all the curriculum developers out there,
particularly those who have "pushed the envelope"
when it came to what and how students should learn
mathematics - you have a hard job. I thank you for
making my work as a mathematics educator worth it

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CHAPTER 1: INTRODUCTION

Teaching can have a substantial impact on student learning (Darling-Hammond, 1999). However, teaching excellence depends on many factors, including the need for high quality teachers and their continued education, and high quality materials (Cohen, Raudenbush, & Ball, 2002; Putnam & Borko, 2000). In the last two decades there have been many efforts to reform mathematics teaching, but for the pedagogical change to be realized in the way envisioned by these reformers, there is the need for substantial teacher learning beyond prospective teacher education (Borko & Putnam, 1996; Remillard, 2000; Schneider & Krajcik, 2002). Teachers need not only to know subject matter, but also must know subject matter in ways that allow them to teach this subject matter to students in effective ways. This learning includes learning to plan and enact lessons that are appropriate for all students, which requires being able to interpret and understand student thinking and develop instructional routines and practices that will enable them to use student thinking productively. As we enter into the era of the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) this learning is even more critical, as the standards may require teachers to not only learn to understand and unpack the standards themselves, but may also require teachers to learn new content and learn to teach in different ways (Lappan, McCallum, Kepner, 2010).

Due to the complex nature of teaching and the myriad of demands placed on teachers, such as having to teach for understanding and fluency across domains and topics while also trying to make sure students meet the standards and raise their test scores, mathematics educators need to consider all possible venues for teacher learning. In this paper, I discuss my examination

of the opportunities for teacher learning embedded within written curriculum materials. Research indicates that teachers can and do learn from curriculum materials (Choppin, 2008; Collopy, 2003; Drake & Sherin, 2009; Lloyd, 2008a; 2008b; Remillard, 2000; Remillard & Bryans, 2004; Schneider, Krajcik, & Blumenfeld, 2005; Schneider & Krajcik, 2002; Schneider, 2006; Van Zoest & Bohl, 2002). Curriculum materials, particularly educative ones, emerge as a potential source for opportunities for teacher learning in ways that set them apart from more traditional professional development, which is often criticized for being decontextualized, contrived, short-term, fragmented, discontinuous, and disconnected (Ball & Cohen, 1999; Little, 1994; Lord, 1994; Wilson & Berne, 1999). *Educative curriculum materials* are materials for Grades K-12 that are “intended to promote teacher learning in addition to students’ ” (Davis & Krajcik, 2005, p. 3). Ball and Cohen (1996) advocated for the development of such materials, because unlike other frameworks or mechanisms for guiding curriculum and engaging reform practices, curriculum materials are used on a daily basis, affording curriculum materials a “uniquely intimate connection to teaching” (p. 6). Curriculum materials are an ideal source for opportunities for teacher learning, particularly learning that has the potential to impact practice, because curriculum materials are situated in the practice of teaching. Moreover, curriculum materials often serve as a dominant source of curricular knowledge (Grouws, Smith, & Sztajn, 2004; Jackson, 1968; Schmidt, et al., 1996). Teachers use student texts and teacher’s guides in their daily planning and in the enactment of lessons.

Despite the plethora of research on teachers use of curriculum materials in mathematics education, we know little about how the structure and features of written curriculum materials impact teacher learning (Stein, Remillard, & Smith, 2007).

Overview of Study

As a starting point to investigating the impact of curriculum materials for teacher learning, in this study, I examined the opportunities to learn for teachers in written middle school mathematics curriculum materials by examining what supports were available and how these supports were expressed. Specifically, I examined opportunities to learn by investigating the *content* and the *expression* of this content (i.e., how curriculum authors speak to teachers in the written text) in four middle school curriculum teachers' guides. By examining both the content supports available and how curriculum authors speak to teachers, I will describe the opportunities available for teacher learning in four middle school curriculum materials teachers' guides in the areas of introduction to variable and geometric transformations.

Organization of this Dissertation

This dissertation is organized into five chapters. This first chapter is meant to provide an introduction to the issues surrounding teacher learning and motivate my study of written curriculum materials.

In Chapter 2, I present the theoretical background of my study. I do this by presenting both a review of the literature and the conceptual framing for my study. I situate this study within the larger context of studies on curriculum, focusing specifically on studies of written curriculum materials and those of the enacted curriculum that have focused on teachers use of and learning from using curriculum materials. I then present my theoretical framework, beginning with a discussion of the role of the teacher in enacting curriculum. I then discuss issues related to teacher learning and the role of curriculum materials in teacher learning, specifically discussing content and expression in written curriculum materials. Finally, at the end of this chapter, I describe the focus of my study and present my research questions.

In Chapter 3, I describe my method. I begin by describing my choices of curricular series and mathematical content. I then describe my analytical frameworks and how these came to be. I begin first by introducing and describing the coding scheme I used to identify and capture the content supports for *Subject Matter Content Knowledge*, *Pedagogical Content Knowledge for Topics*, *Pedagogical Content Knowledge for Practices*, and *Mathematics Curricular Knowledge*. I then describe my second framework that captures aspects of the voice of the materials. I describe the major pieces of this framework; personal pronouns and “you”-forms, which include imperatives and modality. I then describe my coding procedures and analysis. Finally, I end this chapter with my percentages for inter-rater reliability and how I calculated these percentages.

In Chapters 4 and 5, I present my results. First, in chapter 4, I describe the structure of the teachers' guides and details on each of the units. In chapter 5, I present my results related to content, followed by my results for aspects of voice. For my content analysis I begin by providing details about each of the units including the total number of sentences and codes. I then provide frequencies for the types and locations of supports and discuss the content supports that appeared frequently and infrequently. I discuss differences across curriculum and units where appropriate. Following my content results, I begin the results related to aspects of voice with person pronoun use. I discuss the frequency of pronouns and the ways in which these pronouns were used. I then discuss "you"-forms and give special attention the most common "you"-forms, imperatives and "you" + modal verbs. As with the content results, I discuss differences in voice across curriculum and units where appropriate.

I found that, although teachers' guides provided some content support, this support often lacked *Rationale Guidance* and was focused heavily on particular types of supports, including those related to *Pedagogical Content Knowledge for Practices* and *Curricular Knowledge*. In

addition, although most support was located at the *Lesson Level*, I found that a significant amount of support, particularly for CMP and MiC, was located at the *Unit Level*. This may have implications for whether teachers use the supports. In addition to the content support, my analysis of voice indicated that, although some curriculum authors chose to include personal pronouns such as “we” and “you” in the written text, it is not clear that these pronouns served to construct a collegial relationship with teachers. Curriculum authors were not speaking *to* teachers. Instead, curriculum authors often chose to command teachers to perform actions as evidenced by their use of imperatives.

Finally, in Chapter 6, I present my discussion. I return to my research questions and discuss what my study indicates about teachers’ opportunities to learn from middle school curriculum materials. I begin by summarizing my results and then describing the relative frequencies of content supports, how curriculum authors spoke to teachers, and how the opportunities differed across the variable and transformations units. In addition, I provide my study’s contribution to the field and discuss limitations and future directions for my work.

CHAPTER 2: THEORETICAL BACKGROUND

In this chapter, I first present a review of the relevant literature, focusing specifically on studies of written curriculum materials and those of the enacted curriculum that have focused on teachers use of and learning from using curriculum materials. I then describe my theoretical framework by discussing the critical role of the teacher in shaping the mathematics curriculum in classrooms and the role of curriculum materials in teacher learning.

Review of the Literature

I begin my review of the literature by first discussing curriculum broadly, including issues of terminology and meaning. Second, I focus on curriculum research in mathematics education (and from science education, where appropriate) in the United States. In this section, I briefly address trends in curriculum research and then focus specifically on a) research on the written curriculum and b) research involving curriculum enactment and the interaction between teachers and curriculum materials, focusing specifically on studies that attempt to describe teacher learning from using curriculum materials.

Meanings of curriculum. First, as a way to frame the sections to come, it is necessary to describe what I mean by "curriculum" and "curriculum materials" (or textbooks) and to describe the ways in which educators have come to talk about curriculum. I discuss these issues before describing the relevant literature. First, with the terminology issue, when I looked at the research on "curriculum," not just in mathematics education, but in other disciplines as well, I saw that it was diverse and encompassed the work of many scholars in many fields. Jackson (1992) pointed out, that research on curriculum is "confused." Today, this confusion stems partly from what the term curriculum might be taken to mean. To some curriculum, stemming from the Latin word

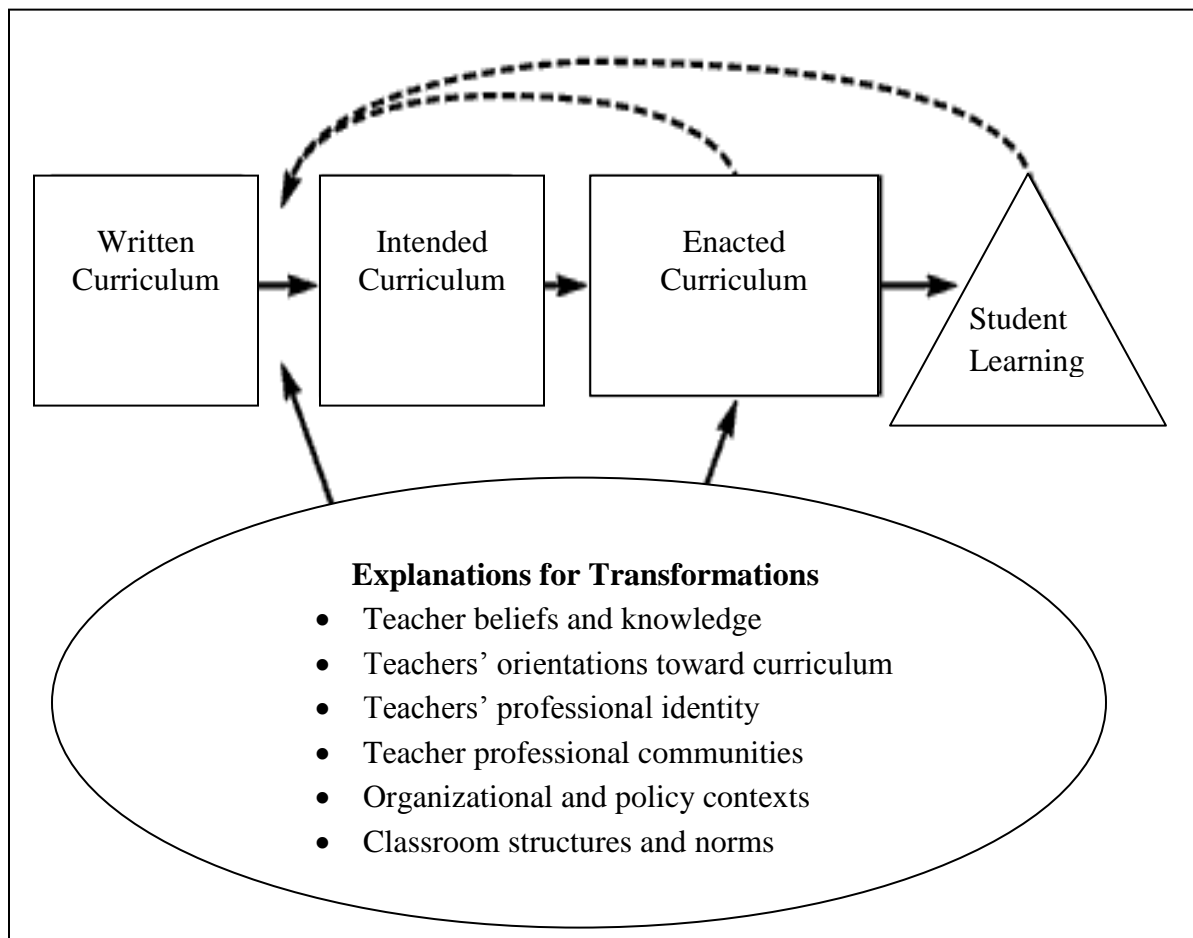
meaning race-course, includes the set of experiences that students undergo (Dewey, 1902) throughout the course of their lives, whereas, to others, curriculum is synonymous with textbook. More recently, particularly since the implementation of *The No Child Left Behind Act*, the term curriculum is used to refer to state frameworks and policy documents. While, like textbooks, these frameworks and policy documents may in fact be a part of what constitutes the curriculum, these artifacts are not the curriculum. According to Bobbitt (1918), “as applied to education, it [curriculum] is that series of things which children and youth must do and experience by way of developing abilities to do things well that makeup the affairs of adult life; and to be in all respects what adults should be” (p. 42).

With this in mind, it is essential to have another term, rather than “curriculum” to describe the materials used by teachers and students to enact the mathematics curriculum in classrooms. I have chosen to use, as others (Stein, Remillard, Smith, 2007) have as well, the terms *curriculum materials*, and *teachers' guides*. I use curriculum materials to refer to the main text that students and teachers have as part of a larger curriculum program. I specifically use teacher's guide to signify that I mean the teacher's text. Although some educators use the term curriculum materials to describe a particular kind of text, texts that focus more on the pedagogy of teaching (Stein, Remillard, & Smith, 2007), I do not make this distinction at this point.

The issues of the meaning of "curriculum" become even more critical when we wish to research and describe curriculum. When someone researches curriculum, what are they actually researching? Not only do we have terminology issues, we have conceptualization issues. Curriculum theorist have been using a number of different terms to describe the differences between curriculum as outlined in some form of text or as enacted in classrooms. For example Doyle (1992) used "formal curriculum," whereas Gehrke, Knapp, & Sirotnik (1992) used

"planned curriculum" to describe outlined goals, whether these be in policy documents, textbooks or created by teachers. Yet, others have used terms such as "intended curriculum" to describe this same thing. To describe the curriculum as it unfolds in classrooms, researchers have used terms such as "implemented curriculum" (Snyder, Bolin, & Zumwalt, 1992) or "enacted curriculum" (Gehrke et al., 1992; Stein, Remillard, & Smith). Based on years of research, Stein, Remillard, and Smith (2007) proposed that curriculum use "unfolds in a series of temporal phases" (p. 321) as presented in Figure 1.

Figure 1. The Temporal Phases of Curriculum Use (Stein, Remillard, & Smith, 2007, p. 322)



This unfolding begins with the printed page (*the written curriculum*), progresses to the teachers' plans for instruction (*the intended curriculum*), and ends with the actual implementation of

curricular-based tasks in the classroom (the *enacted* curriculum)" (p. 321), which hopefully produce some form of student learning. It is within these phases, beginning with the written curriculum, that transformations between written, intended, and enacted curriculum can occur.

Research on curriculum. In this section, I describe the literature pertaining to curriculum. I begin by discussing the general trends in curriculum research and then focus on research on written curriculum and research that has attended to issues of teachers' use of curriculum materials. I focus primarily on what was studied. I provide specific information about the findings of these studies when appropriate; I provide details of the results of these studies when the results are related to the framing of my study.

Overview of curriculum research. Curriculum research in mathematics education in the United States has focused on a) the content of curriculum materials, b) the engagement between teachers and curriculum materials, c) the enactment of curricula in classrooms, and d) student learning from curriculum materials (Stein, Remillard, Smith, 2007). In this review, I primarily focus on the first three categories because the content of materials, the interaction between teachers and materials, and curriculum enactment, provide valuable insight for this study in which I propose and use a framework for studying teachers' opportunities to learn from using curriculum materials. Particularly with the introduction of the Standards-based NSF-funded curricula, there has been a focus on the content of textbooks. These studies have primarily focused on what mathematical content is present in curriculum materials, rather than on how it is presented. However, a few studies have aimed to describe the latter as well. In addition, a few studies have focused primarily on what support, both mathematical and pedagogical, is available for teachers to enact lessons. I discuss studies of this nature in the next section on written materials. This section will be followed by my discussion of studies that describe the nature of

the interaction between teachers and curriculum materials and the enactment, with a focus on what and how teachers learn from materials. I include research from science education as well, because science researchers have also focused on curriculum and the teacher-curriculum relationship.

Research on written curriculum. Although in some respects the written materials influence all areas of curriculum research, the features of the written materials themselves have gotten little attention. According to Stein, Remillard, and Smith (2007), written curriculum has the potential to influence transformations between the written, intended, and enacted curriculum, and deserves our attention. The question they pose is "How do characteristics or features of the particular curriculum influence how a teacher uses it?" (p. 356). Although some curriculum research has focused on the written materials themselves, particularly studies investigating the content of curriculum materials, this is still an underdeveloped area of research. For a list of these studies, including a summary of a) the number of curricular series analyzed, b) the level (i.e., elementary, middle, high, college), c) what materials were analyzed (i.e., student's textbook, teacher's guide), d) the focus of the analysis (i.e., content, sequence, voice), and e) what countries' curriculum materials were analyzed, see Appendix A. Most studies focused primarily on the content of the student textbook and many included the examination of multiple curricular series, either textbooks from the same country (n = 23) or textbooks from different countries (n = 16); however, some studies (n=6) focused more centrally on the teacher materials. I first describe the studies that focused more, or equally on student materials, including those that focused on the mathematical content, the expression of content, and the alignment of content with standards documents. I then discuss studies that focused more centrally on teacher materials.

Content of curriculum materials. Some studies focused broadly on the topics or content present (or not) and its sequence in curriculum materials at both the elementary (Hook, Bishop, & Hook, 2007) and secondary level (Flanders, 1987). Larger-scale studies, such as the TIMSS (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997; Schmidt, Houang, & Cogan, 2002; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002) also focused on the content present (or not) when examining curriculum guides and/or textbooks from 45 countries.

Studies that focused on particular content, did so in various strands; eight studies focused on number and operations, three studies on algebra, five on geometry and measurement, and one on statistics and probability. Whereas some studies focused more broadly on a strand, such as Star, Herbel-Eisenmann, & Smith III (2000)'s report of algebra in both the first edition of the *Connected Mathematics Project* and Forrester's (1990) *Algebra I* textbook, published by Addison-Wesley, most focused more narrowly on specific mathematical content. These foci included area measurement (Smith, Gonulates, Males, & Mosier, in preparation), arithmetic average (Cai, Lo, & Watanabe, 2002), bar models (Hoven & Garelick, 2007), concatenations (Lee & Messner, 2000), the distributive property (Ding & Li, 2010), estimation of linear measurements (Chang, Males, Mosier, & Gonulates, 2011), fractions (Charalambous, Delaney, Hsu, & Mesa, 2010; Watanabe, 2003; Yang, Reys, & Wu, 2010), functions (Mesa, 2004), integers (Li, 2000), length measurement (Lee & Smith, 2011; Smith, Dietiker, Lee, Males, & Mosier, in preparation), patterning (Olsen, 2010), probability (Jones, 2004), and transformational geometry (Nissen, 2000).

In addition to studies that focused primarily on content, some focused on the integration of certain aspects in a text such as the processes or tools involved including problem-solving (Fan & Zhu, 2007), proof (Stylianides, 2005), and calculator use (Chval & Hicks, 2009).

Expression of content in curriculum materials. Some studies have described more than just the mathematical content of the text, but have also described the ways in which this content was expressed, or have attempted to describe how readers may interpret the text. These studies may begin to help in answering the question posed by Stein, Remillard, and Smith (2007), as to how the features of a textbook influence how a student or teacher uses it. Herbel-Eisenmann (2007) investigated whether the text of one unit of the *Connected Mathematics Project* was aligned with the ideas espoused in the NCTM Standards by investigating the *voice* of the text. Drawing on Morgan (1996) and Halliday's (1985) Systemic Functional Linguistics, Herbel-Eisenmann investigated linguistic forms used in the text. Other studies have also tried to describe the ways in which content is expressed, such as the work currently underway by the *Strengthening Tomorrow's Education in Measurement* project (Lee & Smith, 2011; Smith et al., in preparation) who have analyzed opportunities to learn spatial measurement in written curriculum materials by examining both what is present in the text (content) and how this content is present (textual forms). In addition to these studies, a recent study (Weinberg & Wiesner, 2011) attempted to provide a framework for describing a readers' response when reading written curriculum materials. Drawing on ideas from reader-oriented theory, Weinberg and Wiesner (2011) describe features of a calculus textbook that may impact ways in which students read textbooks.

Alignment with standards. Some studies have investigated curriculum materials with respect to the NCTM Standards. Nissen (2000), who specifically compared transformational geometry tasks in four elementary, three middle, and one integrated high school textbook to the NCTM Standards, found that whereas elementary and middle school textbooks met the requirements of the NCTM Standards, high school textbooks did not. Martin, Hunt, Lannin,

Leonard Jr, Marshall, and Wares (2001), compared all the National Science Foundation-funded high school curricula (*Mathematics: Modeling Our World*, *Core Plus Mathematics*, *Interactive Mathematics Program*, *MATH Connections*, and *SIMMS Integrated Mathematics*) to the NCTM Standards and identified features of the textbook that showed how each series aligned with the Standards.

Focus on Teacher Materials. Whereas most of the studies above focused primarily on describing student textbooks or opportunities for student learning (and in some cases using the teacher's guides to aid in this analysis), a handful of studies (n = 7) focused primarily on, or attended equally, to teacher materials in their analysis. These studies included investigations of elementary and middle school textbooks and the analysis of textbooks within and outside the United States.

In three studies on teacher materials, researchers examined curriculum materials used outside of the United States. First, Netwon & Netwon (2006) studied 18 teachers' guides from England. Specifically, these texts were investigated for their opportunities for teachers to teach with attention to reason. This study indicated that the teachers' guides did not draw teachers' attention to discourse or attention to reasoning, but instead focused on computational skills.

Second, in two of these studies, researchers compared teachers' guides used in the United States with those used in other countries. Li (2004) studied the opportunities in the teacher's guide of Scott-Foresman Addison Wesley's *Mathematics*, University of Chicago School Mathematics Project's *Everyday Mathematics* and People's Education Press *Mathematics*, used in China, whereas, Watanabe's (2001) study compared two U.S. texts, *Investigations* and *Math in My World* to two Japanese texts, *Shintei sansuu*, and *Shinhes atarashi Jansu*. These two studies indicated that the teachers' guides used in the United States differed from those used in

China and Japan. Both studies described teachers' guides in the United States as lacking attention to the mathematical big ideas, compared to the teachers' guides from these other countries. In addition, Li (2004) indicated that the Chinese text included specific focus on conceptual content and pedagogical approaches, including providing rationales. Watanabe (2001) pointed out that the Japanese texts he analyzed were actually much more prescriptive than the American texts; however, the text "appeared to encourage teachers to study the materials more carefully and polish their daily lessons" (Watanabe, 2001, p. 200). The Japanese teachers' guides aimed to help teachers learn mathematics and mathematics teaching by including discussions of the mathematics, rationales for the organization of the materials, and samples of instructional plans and student responses. Watanabe did not find these same features in the American teacher materials.

Other studies that focused on teacher materials used in the United States included those that focused on one text (Castro, 2006; Stylianides, 2007) and those that examined multiple texts (Kim, Achubang, Lewis, Hoe, Reinke, & Remillard, 2010; Stein & Kim, 2009). The former studies both investigated the teacher materials of the *Connected Mathematics Project*. Castro's (2006) study focused on the teacher's guide as a resource in planning lessons, whereas Stylianides (2007) specifically investigated the guidance provided in the teacher's guide for teaching proof. These studies indicated that more opportunities may be needed for teachers to learn from using the teacher's guide.

The latter studies have examined more than one text in order to describe the variation in terms of teacher learning opportunities. Drawing on Ball and Cohen (1996) and on the heuristics outlined by science educators (Davis & Krajcik, 2005), Stein and Kim (2009) investigated the "educativeness" of two elementary curriculum materials, University of Chicago School

Mathematics Project's (2004) *Everyday Mathematics* and TERC's (1998) *Investigations in Number, Data, and Space*. Using the ideas related to the cognitive demand of the tasks in each curriculum, Stein and Kim (2009) found that the support material in each curriculum was aligned with how cognitively demanding the tasks were for students and for teachers to enact. Stein and Kim (2009) focused on whether the curriculum materials made the developers' rationales for instructional tasks and pathways transparent and if the curriculum helped teachers to anticipate how students might approach these tasks. They found that these two curricula differed in terms of the demand for teacher learning and the opportunities provided for teacher learning. In their analysis of over 40 lessons of each curricula across grades 1-5, they found that *Everyday Mathematics* lessons tended to tell teachers what to do, but not why they were doing it, whereas the *Investigations* lessons were judged to be 80% transparent with respect to why a particular instructional activity was designed and how it represented worthwhile mathematics. Stein and Kim (2009) also found that while the authors of *Everyday Mathematics* provided less demanding tasks, they also provided fewer opportunities for teachers to learn about possible student responses. The demanding tasks in *Investigations* were accompanied by supports that included student responses, student work, examples of possible student difficulties, students' strategies for solving problems, and explanations for how students might make sense of content.

In addition to this published work, a cross-university group is currently working on investigating multiple elementary teachers' guides to describe the attempt to make "visible the mathematical and pedagogical features teachers encounter when reading curriculum resources in order to guide research on teaching and curriculum materials and improved materials design" (Kim, Achubang, Lewis, Hoe, Reinke, & Remillard, 2010, p. 38).

Research focused on teacher learning from curriculum use. In both mathematics and science education, curriculum researchers have investigated teacher learning from the use of curriculum materials (Choppin, 2008; Collopy, 2003; Drake & Sherin, 2009; Lloyd, 2008a; 2008b; Remillard, 2000; Remillard & Bryans, 2004; Schneider, Krajcik, & Blumenfeld, 2005; Schneider & Krajcik, 2002; Schneider, 2006; Van Zoest & Bohl, 2002). This research has indicated that elementary and secondary teachers can and do learn from using curriculum materials. In mathematics, this learning has been shown to be mediated by a variety of factors including those that teachers bring with them, such as teachers' prior knowledge, identity, beliefs, and orientations towards mathematics and towards curriculum materials (Collopy, 2003) and those that happen as a result of using a set of curriculum materials. The latter included engaging in the "reading process," or the constructive and dynamic process of making meaning through engaging with the written text in a textbook, and also engaging with students and tasks (Remillard, 2000; Remillard, 2005). In addition, struggling with new mathematics content (Van Zoest & Bohl, 2002) and enacting lessons multiple times (Choppin, 2008; Drake & Sherin, 2009) has enabled teachers to learn both about mathematics and pedagogy. Van Zoest and Bohl (2002) found that the unfamiliar mathematical content in the *Core Plus* curriculum materials forced an intern and mentor teacher to work through problems and have discussions about mathematics, resulting in the development of mathematics subject matter knowledge. Choppin (2008) and Drake and Sherin (2009) found that as teachers taught lesson multiple times they developed a sense of where the curriculum was going and were able to draw on previous experiences enacting lessons to improve them. These repeated enactments helped teachers to develop curricular knowledge that enabled them to become more skilled enactors. Teacher learning was also impacted by factors embedded in the written materials themselves, such as the structure of

the text, including how concise the text was (Lloyd, 2008a) and where support was located (i.e., at the lesson level or unit level) (Schneider, Krajcik, & Blumenfeld, 2005; Schneider & Krajcik, 2002; Schneider, 2006).

Learning from educative curriculum. Although research has rarely focused on how the features of written mathematics curriculum impact teacher learning, there has been some focus on this in science education. There has been a line of research that specifically investigated how features of educative curriculum materials (i.e., materials designed for teacher learning) supported teacher learning. Drawing on Shulman (1986), Ball and Cohen (1996), and Davis and Krajcik's (2005) design heuristics (or the precursors to these heuristics), the educative curriculum materials incorporated five principles: a) addressing content knowledge and pedagogical content knowledge, b) situating teacher learning by meshing the content of the support to lessons for students, c) linking different knowledge areas within lessons, d) making knowledge accessible to teachers by including short scenarios in the language of teachers or students involved in the lesson to illustrate or model the intended practice when possible, and e) addressing immediate needs for understanding as teachers plan lessons that will be enacted within a short time.

Three studies (Schneider, Krajcik, & Blumenfeld, 2005; Schneider & Krajcik, 2002; Schneider, 2006) examining the use of these educative materials indicated that teachers used the lesson-specific features, particularly when planning (Schneider & Krajcik, 2002). These included examples of student work such as student-generated graphs and notes about possible student misconceptions, strategies, and representations. According to Schneider and Krajcik (2002), while there was evidence that teachers read the content explanations for the teacher that went beyond what was needed by the students, observations indicated that the use of this content

knowledge was more effective in practice when it was related to the work of students, something that was attended to in the lesson-specific features. One teacher indicated that she learned physics content “from reading the notes about student misconceptions because she held some of those same misconceptions herself” (p. 238).

Schneider (2006) studied one science teacher, Ms. Shirley, over the course of two years teaching five units, all with different levels of educative support. Schneider found that Ms. Shirley used the materials purposefully to guide her teaching. At the beginning Ms. Shirley had difficulty guiding student thinking, particularly during group and whole-class discussion. Over time she improved in her ability to facilitate whole-class discussions, but still struggled doing this when students worked in groups. Ms. Shirley’s ability to facilitate thoughtful discussions happened particularly when the curriculum materials included questions and provided possible discussion scenarios. Ms. Shirley began teaching these units trying to “fit the new lessons into her previous teaching framework” (Schneider, 2006, p. 677). In the second year she began to think about teaching within a framework intended by the materials and began to question components of the materials. As she gained experience with the materials Ms. Shirley required “finer grained support” (p. 677). She also grew in her attention to student thinking, moving from being worried she might impart student misconceptions to thinking about how she could plan to support students’ developing ideas. Features of the materials, such as the scenarios and questions, provided scaffolding for Ms. Shirley’s efforts with discussion and enactments were most thoughtful when features included explicit attention to how a lesson fit in with the overall goals of the unit.

In all of these studies, lesson-specific educative features, such as descriptions of student talk, or explanations of possible misconceptions and where they may arise, had the greatest

impact on teachers' learning to enact inquiry teaching in science, contributing to teachers' pedagogical content knowledge. These lesson-specific features have also been linked to the improvement of content knowledge as reading about student misconceptions has resulted in teachers confronting their own misconceptions about science.

Towards a Framework for Studying the Educative Nature of Curriculum

Although some may believe that student achievement is independent of the teacher, some of the world's most respected educators remind us that teachers and teaching matter (Darling-Hammond, 1999). It is not just getting the right textbook into the hands of students, but it is about a combination of intricate factors, many of which rely on a student's teacher. In this section I describe my framework. I begin by discussing the role of the teacher in shaping the curriculum and how this role has been perceived in the American mathematics curriculum. I start here because it is first necessary to recognize the role of the teacher in shaping the curriculum. If one does not recognize the teacher's critical role, developing an educative curriculum and a framework for analyzing such curriculum will make little sense. Second, I discuss teacher learning, and finally I describe the role of curriculum materials in fostering teacher learning.

The role of the teacher in enacting the curriculum. The teacher plays an active role in designing and enacting the curriculum in their classroom. Furthermore, the curriculum is a guide not only for students, but for teachers. This notion of the curriculum being "for teachers" is not new. Dewey (1902) argued that the curriculum was in fact for the teacher, not the child.

Its [the curriculum's] primary value, its primary indication, is for the teacher, not for the child. It says to the teacher: Such and such are the capacities, the fulfillments, in truth and beauty and behavior, open to these children. Now see to it that day by day the conditions

are such that their own activities move inevitably in this direction, toward such culmination of themselves. (p. 39).

Dewey saw the role of the teacher as instrumental in shaping the conditions of the classroom in order for students to learn. He did not conceptualize the curriculum as something for students, but instead saw curriculum as a guide for teachers, a guide that helped teachers see the possibilities inherent in the culture. Using this curriculum, teachers could then design activities and an environment that would help students meet their potential, which according to Dewey, included being productive citizens in a democratic society.

However, over a century later, the role of teachers in the construction of curriculum was not recognized by curriculum developers. In the last half of the 20th century, while curriculum materials have exerted great influence over the mathematics teachers and their instruction (Jackson, 1968; Schmidt, et al., 1996), curriculum materials have done little to engage teachers in the active construction of the mathematics in their own classrooms. Mathematics curriculum materials have merely been a source of problems and explanations for students (Stein, Remillard, & Smith, 2007). Teachers relied heavily on textbooks as their primary source for what to teach (Grouws, et al., 2004), but the "how" and the "why" were virtually absent from curriculum materials. In mathematics, the textbook has had great influence over instruction, exerting so much control that the instruction of many teachers was merely aimed at giving the student the prerequisite knowledge to be able to complete the problems in the textbook (Jackson, 1968). This long history of reliance on curriculum materials, according to Remillard (2005), is due to the ways in which mathematics learning is viewed (e.g., math is learned by memorizing), the nature of the content itself (e.g., a stable body of knowledge), and teachers' knowledge of the content for teaching (e.g., lack of knowledge). For example, the same elementary teachers who adapted

textbook suggestions in language arts tended to stick to the textbook in mathematics (Sosniak & Stodolsky, 1993). Despite this reliance on curriculum materials in shaping instruction, curriculum developers underestimated the role of teachers when designing texts.

In the 1950's and 60's curriculum materials were a big part of the reform in mathematics sometimes labeled "New Math," garnering this label from one particular set of curriculum materials developed by the School Mathematics Study Group (Stein, Remillard, & Smith, 2007). In this wave of reform, subject matter experts were solicited to design curriculum materials to facilitate the development of scientifically productive citizens. Despite the unfamiliarity teachers had with the content and sequence of the mathematics in these newly developed materials, these materials were developed, like the texts before them, with the students as the intended audience. Although some developers recognized the role of the teacher in shaping the curriculum and attended to teacher learning (S. Senk, personal communication, April 2009), according to Cohen and Barnes (1993) these developers continued to underestimate the influence of the teacher and overestimate the educative power of the texts themselves. Curriculum materials from this era, in general, were not designed to communicate with teachers, but were designed to be "teacher proof."

This discounting of the role of teachers in enacting curriculum did not go unnoticed. Looking back on his work as head of the Woods Hole Conference, a conference commissioned by the National Academies of Sciences to engage the leading subject matter specialists in examining the American curriculum, Bruner (1977) argued, like Dewey, that curriculum is more for teachers than students.

A curriculum is more for teachers than it is for pupils. If it cannot change, move, perturb, inform teachers, it will have no effect on those whom they teach. It must first and

foremost be a curriculum for teachers. If it has any effect on pupils, it will have it by virtue of having an effect on teachers. The doctrine that a well-wrought curriculum is a way of “teacher-proofing” a body of knowledge in order to get it to the student uncontaminated is nonsense. (p. xv)

Bruner admits to brooding over this issue of curriculum since the first edition of this report in 1960. Like Dewey, in this preface, Bruner conceptualized curriculum as something for teachers. Bruner spoke out against the notion of a teacher-proof curriculum, one that was so popular in the 50’s and 60’s, stating that this idea was nonsense. In order for curriculum to have an effect on students, it must have an effect on teachers; it must change, move, perturb and inform teachers.

Despite Bruner’s claims, it was not until the 1990’s that developers began to recognize the critical role of the teacher and write materials not only with students, but also with the teacher in mind. The reformers of this new movement cast these curriculum materials as agents of change hoping that they would help to transform instruction from a focus on procedural skills to a focus on conceptual understanding that emphasized communication, reasoning and problem-solving (Senk & Thompson, 2003). This movement, heavily supported by The National Science Foundation (NSF), was focused on developing materials that were aligned to the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989). One of the greatest differences in these new Standards-based materials and the traditional texts or those developed in the “New Math” era, was the attention developers gave to the role of the teacher. The Standards-based materials acknowledged, some more explicitly than others, the role of the teacher in shaping the enacted curriculum. Unlike materials in the past, curriculum developers attempted to attend more to the teacher and issues of teacher learning. For example, in writing about the design of *The Connected Mathematics Project*

Lappan, Phillips, & Fey (2007) describe their desire to write with teachers in mind, "*It is for teachers as well as students*. The materials were written to support teachers' learning of both mathematical content and pedagogical strategies. The teachers' guides include extensive help with mathematics, pedagogy, and assessment" (p. 68). Although some curriculum developers began to consider the role of the teacher more in their design of materials, others did not. In addition, for those developers who considered the teacher, it is not clear that the types of learning opportunities they provided were sufficient. In some cases teachers were still positioned as passive receivers of the curriculum (Paris, 1993). In the next section I describe the issues related to teacher learning to gain insight into the type of learning opportunities needed by teachers.

Role of curriculum materials in teacher learning. As research indicates, teachers can and do learn from using curriculum materials (Choppin, 2008; Collopy, 2003; Drake & Sherin, 2009; Lloyd, 2008a; 2008b; Remillard, 2000; Remillard & Bryans, 2004; Schneider, Krajcik, & Blumenfeld, 2005; Schneider & Krajcik, 2002; Schneider, 2006; Van Zoest & Bohl, 2002). Although empirical work has rarely investigated the features of written curriculum materials and how these features promote learning, the research on teachers' use of materials and what and how they learn from using materials indicates that the written materials may play a role in this learning. According to Beyer et al. (2009), curriculum materials seem like a fruitful option for professional development as they are connected to teachers' everyday practice (Putnam & Borko, 2000) and provide ongoing support (Collopy, 2003). In addition, according to Schneider & Krajcik (2002), due to their wide-spread use, curriculum materials have a greater possibility of effecting large-scale reform compared to other efforts professional development efforts aimed at state and district level. Since curriculum materials are widely used and are intimately connected to what teachers do on a daily basis, they have a greater chance of impacting practice than efforts

that are geared towards the needs of an entire district, as these are often, by necessity, less connected to particular teachers' practices.

In order to serve as a source of professional development, curriculum materials need to be more than just textbooks for students. Curriculum materials must “speak to teachers, not merely through them” (Remillard, 2000, p. 347). Rather than being merely a set of directions for the teacher to follow (Remillard, 2000) or a source for problems to assign (Jackson, 1968), in order for curriculum materials to be educative, the materials must communicate with teachers directly about mathematics content, pedagogical practices, and curriculum. The authors of materials that communicate in this way recognize the influence teachers have in enacting mathematics curriculum with their students. Rather than treating teachers as passive receivers of the curriculum (Paris, 1993), authors of educative curriculum position teachers as legitimate readers of the curriculum (Remillard, 2000). This means that curriculum materials need to not only have content that is appropriate for teachers to engage with the teaching of mathematics, but also that materials need to be written in ways that include the teacher in the text and position the teacher as a legitimate reader and enactor of the mathematics curriculum. If the language choices made by curriculum developers do not include teachers in the text, the available content may do little to engage the teachers around important issues of subject matter, pedagogy, or curriculum.

Theoretical Framework

In this study I drew from two theoretical traditions. First, I draw on cognitive theories of knowledge that underlie the construction of the content supports and guidance available in teachers' guides. I specifically drew on the work of Shulman who described three types of knowledge (subject matter content knowledge, pedagogical content knowledge, and curricular knowledge) needed for teaching. Second, I drew on sociocultural theory, which focuses on social

and cultural practices as ways of knowing (not knowledge) and particularly those that examine discourse practices as ways of making shared meaning in dialogue. Specifically, I drew on a theory of language in social context, Systemic Functional Linguistics (Halliday, 1985), which proposes that by examining grammar and drawing from social theory one can say something about why clauses make the meanings they do. Since I am analyzing written text, I drew from these two traditions because they provide a way for me to describe what (i.e., content supports) is in teachers' guides and how it is expressed (i.e., aspects of voice). In the next sections I describe these theories in greater detail and characterize how they contribute to my analysis of content support and its expression.

Content Supports. My analysis of the content supports of teachers' guides required investigating the types of knowledge and guidance available. In the next two sections I describe Shulman's (1986) theory of knowledge for teaching and Beyer et al.'s (2009) descriptions of the types of guidance needed for teaching.

Knowledge. Learning to teach is complex. To develop expertise in teaching mathematics one must have many types of knowledge and be able to integrate these in ways that help one productively promote students' learning of mathematics. This requires more than just knowing subject matter. Teachers need to know subject matter in ways that are appropriate for teaching. In addition to *Subject Matter Content Knowledge*, teachers must have what Shulman (1986) called *Pedagogical Content Knowledge* and *Curricular Knowledge*. In essence, teachers require a specialized type of knowledge of their discipline, knowledge that allows them to teach, not just know their subject matter.

Shulman's (1986) framework distinguished between three types of content knowledge: *Subject Matter Content Knowledge*, *Pedagogical Content Knowledge*, and *Curricular*

Knowledge, all of which place special emphasis on the content knowledge needed for teaching. First, *Subject Matter Content Knowledge* involves having a robust understanding of the subject matter. According to Shulman (1986), “the ways of discussing the content structure of knowledge differ” (p. 9) in different disciplines. However, regardless of the discipline, it is essential that teachers go beyond just the facts or concepts and have an understanding of the structures of their respective subject matter. Teachers need understandings of their subject matter that goes beyond the “mere subject matter major” (p. 9). They must be able to understand that something is so and also why something is so. For example, teachers need to know not only how to compute $16 + 3$, but also know why the computation works. This requires knowing more than just a procedure, but also knowing something about place value and recognizing that if we added the 3 to the 1, we would in fact be adding 16 and 30 rather than 16 and 3. Second, *Pedagogical Content Knowledge* in its most simplified terms can be described as subject matter knowledge for teaching. This involves knowing more than subject matter, but knowing subject matter in a way that is related to the teaching of this subject matter. This includes knowing the most useful forms of representing content in ways that allow for its comprehensibility by others, knowing students and when and how students may excel or struggle, and knowing strategies for working with students’ ideas. For example, this would include knowing some of the most common mistakes students make. An example of this type of knowledge would include knowing that when students simplify rational expressions they often “cancel out” identical terms found in the numerator and denominator even when these terms are not common factors, such as canceling the x ’s in $\frac{x^2 - 2}{x}$. Finally, *Curricular Knowledge* is knowledge about the range of programs for the teaching of subject matter, the instructional materials available, and the knowledge related to making decisions about the fruitfulness of using particular materials in particular situations. At

the time his work was published Shulman indicated that teacher education programs were remiss in addressing curricular knowledge. More recently, drawing on Shulman's distinction, Choppin (2008) defined curricular content knowledge as "the knowledge of how a particular set of curriculum materials functions to engage students in a particular context" (p. 288).

Some researchers, such as Ball and colleagues, have come to call this type of knowledge for mathematics teachers *Mathematical Knowledge for Teaching (MKT)* to indicate that the knowledge described in their framework is a specialized kind of knowledge specifically for mathematics teachers. Ball's framework was built using Shulman's notion of pedagogical content knowledge as the basis (Ball, Thames, & Phelps, 2008).

Guidance. Unlike materials that merely described what to teach, educative curricula go beyond this and provide opportunities for teacher learning through two types of support: *Enactment*¹ *Guidance* and *Rationale Guidance* (Beyer, Delgado, Davis, & Krajcik, 2009). These are supports for knowing what to do and how to do it, and also for understanding why particular mathematical or pedagogical approaches are appropriate. Support in the form of *Enactment Guidance* includes more than just knowing *what* to teach, but also knowing *how* to teach it. According to Lappan and Phillips (2009), "The 'what to teach' and the 'how to teach it' are inextricably linked" (p. 9). *Enactment Guidance* includes detailed explanations for how to teach mathematics. For example, guidance of this type might include a sample of a class discussion in which the teacher asks specific questions to elicit students' justification for their reasoning or to evaluate the reasoning of their classmates. Such examples provide support (but not explicit

¹ Beyer et al. (2009) used the term *Implementation* for the guidance given to teachers about what and how to teach, rather than *Enactment*, however I chose to use the word *Enactment* as it acknowledges more the role of the teacher as a legitimate reader of curriculum materials and an active participant in constructing the classroom curriculum with students (Remillard, 2005), rather than as a passive implementer.

models) for how teachers might pose questions in related context to elicit similar student responses.

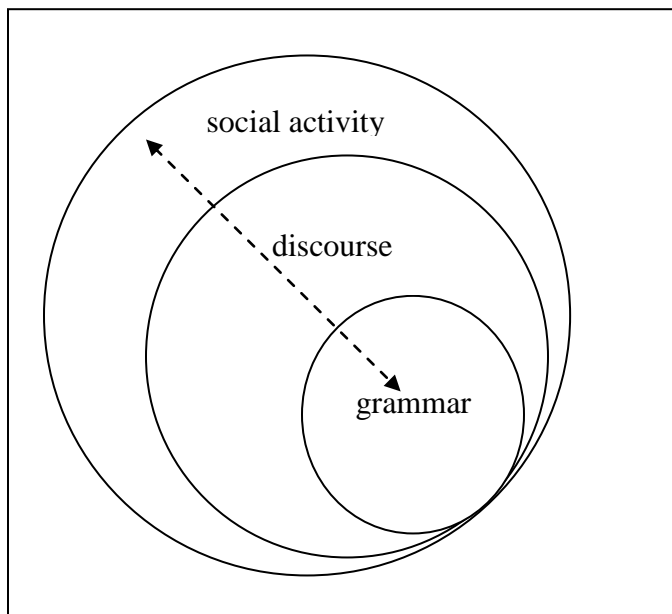
The second type of support, *Rationale Guidance*, enables teachers to know why particular mathematical or pedagogical approaches are appropriate and therefore opens up a dialogue between the authors and the teacher, where teachers have the opportunity to engage with the underlying conceptual ideas the curriculum is based on. Supports such as this allow teachers to make sense of their curriculum materials and develop what Drake and Sherin (2009) call “curriculum vision” or a sense of where the curriculum materials are going and an understanding of the “particular kinds of learning and teaching practices described in the curriculum materials” (p. 324). Teachers have many dimensions to deal with and in the midst of trying to decide how to address student thinking and lead discussions of the mathematical content they either “adapt, omit, or augment materials” (Ball & Cohen, 1996, p. 7). Rationales provide teachers with the opportunity to develop a curricular vision so they can adapt, omit, or augment materials appropriately. Teachers need to know the importance of particular content or approaches and how this content connects to previous and future content in order to make informed decisions about what they may want to adapt, omit, or augment. An example of such support might include a discussion of why having students create multiple representations for a particular situation (i.e., table, graph, equation) is important by describing how the facility between representations will help students develop a stronger concept of linearity.

Expression. Equally important to investigating *what* was in the teachers’ guides, was investigating *how* it was expressed. Language influences “the production, maintenance, and change of social relations of power” (Fairclough, 1989, p. 1). Therefore, curriculum authors’ language choices, even subconscious ones, can set up particular types of relationships with

teachers, which potentially impact the role of curriculum materials for teacher learning. I do not claim, nor does Fairclough (1989), that power is only a matter of language. I recognize that issues of power and authority in teaching are embedded in more than just the written text, as they are also involved in the social interactions and social institutions that teachers participate in each and every day. However, when conceptualizing the role of written curriculum materials in teacher learning, it seems clear that the linguistic features impact the reader in some way, whether minimally or extraordinarily I do not know. In addition, I suspect this differs from teacher to teacher, but nonetheless, this impact is a result of the words used in written materials. If curriculum materials are to play a role in teacher learning, written curriculum materials must be written to speak to teachers.

Halliday's (1985) Systemic Functional Linguistics (SFL) is a theory of language in social context. In SFL, language is defined as systemic; it is a network of interrelated choices for making meaning. Language is also functional; it fulfills various functions or purposes. According to Martin and Rose (2009), despite the complexities of language in social context, the basic principles of SFL make language issues relatively simple to manage. SFL is based on two perspectives. First, there are relevant levels of language (called strata) and relationships between these strata (realizations). Social contexts are realized as texts which are realized as a sequence of clauses. See Figure 2 from Martin and Rose (2007). Discourse is situated within grammar and social activity because discourse uses "the tools of the grammarians to identify the roles of wordings in passages of text, and employs the work of social theorists to explain why they make the meanings they do" (Martin & Rose, 2007, p. 4). This means that by looking at the grammar of a written text, one can say something about the social activity within which that text appears.

Figure 2. Points of View on Discourse: From Social Activity and From Grammar (recreated from Martin & Rose, 2007, p. 5)



Second, in SFL there are three general functions of language (called metafunctions). These metafunctions include the *interpersonal* (i.e., enacting relationships), the *ideational* (i.e., representing experiences), and the *textual* (i.e., organizing discourse in meaningful ways). For my study, I drew specifically on the framework developed by Morgan (1998) who examined students' mathematical texts. Morgan's overall goal was to develop ways of communicating with students so they could take control over their own writing and be assessed positively and she did this by describing the ways in which student text may be interpreted by readers. Morgan's (1998) framework was augmented by Herbel-Eisenmann (2007) to describe the voice of the student edition of one unit from *Connected Mathematics Project* in order to illuminate the construction of roles of the authors and readers. Herbel-Eisenmann's aim was to

“see whether the authors of the unit achieved the ideological goal (i.e., the intended curriculum) put forth by the NCTM's *Standards* (1991) to shift the locus of authority

away from the teacher and the textbook and toward student mathematical reasoning and justification. (p. 344).

Herbel-Eisenmann (2007) found that there may have been a mismatch between the goals set forth by NCTM and conventional textbook forms.

To describe relationships, one can examine both the *interpersonal* and *ideational* metafunctions, as these metafunctions describe how relationships are enacted and *who* is involved in doing *what* processes. In my study, I was concerned primarily with the roles and relationships that were constructed between curriculum authors and teachers and therefore, used the interpersonal metafunction because this is the function used to enact relationships.

Focus of Study and Research Questions

The focus of this study is to describe teachers' opportunities to learn *mathematics subject matter, pedagogical practices related to mathematics topics and practices, and mathematics curricular knowledge* in existing middle school mathematics curriculum materials related to introduction to variable and geometric transformations. In particular, I examine the opportunities for teacher learning by investigating the *content* and its *expression* in teachers' guides.

Specifically, I address the following research questions:

1. What is the relative frequency of educative content supports in middle school mathematics curriculum materials for teachers' *Subject Matter Content Knowledge, Pedagogical Content Knowledge for Mathematics Topics, Pedagogical Content Knowledge for Mathematical Practices, and Mathematics Curricular Knowledge* and where are these supports located?

2. How do middle school mathematics curriculum materials speak to teachers (i.e., what are some of the language choices they make) through the written text in the teachers' guides?
3. How does opportunity to learn (content and aspects of voice) differ for introduction to variable and geometric transformations?

To address my first research question, I coded for content supports and their location. I examined the teachers' guides of each unit and coded for educative content supports that addressed *Subject Matter Content Knowledge*, *Pedagogical Content Knowledge for Mathematics Topics*, *Pedagogical Content Knowledge for Mathematical Practices*, and *Mathematics Curricular Knowledge*. I then determined the relative frequency of each of these categories of supports and their location. To answer the second research question, since I analyzed a written text, I coded for linguistic features in the text. Specifically, I coded for personal pronouns and "you"-forms as these forms indicate how authors are constructing relationships with teachers through the written text. Finally, to address my last research question, I searched for differences in the results from my content and expression analysis for the variable and transformations units. In the next chapter, I describe my method for these analyses.

CHAPTER 3: METHOD

In this chapter, I describe the methodology I used in my study. I begin by describing my sample, including my choices of curricular series and mathematical content. I then describe my initial analysis which includes describing in detail my analytic frameworks. I begin first by introducing and describing the coding scheme I used to identify and capture the content supports for *Subject Matter Content Knowledge*, *Pedagogical Content Knowledge*, and *Curricular Knowledge*. I then describe my second framework that captures aspects of the voice of the materials. I describe the major pieces of this framework: a) personal pronouns and b) “you”-forms, which include imperatives and modality. I then describe my coding procedures and additional analyses. Finally, I end this chapter with my percentages for inter-rater reliability and how I calculated these percentages.

Sample

Choice of curricular series. In this study, I examined the opportunities for teacher learning provided in four middle school curricular series. Specifically, I analyzed a total of eight units; two each from four curricular series, one pertaining to the introduction to variable and the other pertaining to geometric transformations. I describe my reasons for choosing this mathematical content in the next section. It is important to note that middle schools in the United States vary considerably in terms of the types of mathematics courses offered. According to Dossey, Halvorsen, and McCrone (2008), the patterns and usage of textbooks in the United States for grades 6-8 are difficult to summarize because of the mixing of basal textbooks and the addition of supplemental algebraic content. In some middle schools, course options include Algebra I and therefore, textbooks might include Algebra textbooks typically used in high

school. For this study, I chose not to analyze textbooks such as these. I analyzed curriculum materials designed specifically for students in grades 6 – 8.

Since it would be impossible to analyze all middle school mathematics curriculum materials, I mindfully choose four series using the following criteria: a) market share in the United States and b) varied design principles. By varied design principles, I mean I included curriculum materials that are categorized as "Standards-based," or developed to be aligned with the recommendations set forth in the National Council of Teachers of Mathematics' (1989) *Curriculum and Evaluation Standards for School Mathematics* (Senk & Thompson, 2003), and those that are not, called "traditional" or "conventional" (Stein, Remillard, & Smith, 2007) curriculum materials. Although many curriculum materials claim to be standards-based and in fact are, as textbooks are often written to align with some set of standards, I distinguish between those written to align with the NCTM standards by using a capital "S." Therefore, I do not label curriculum not written specifically to align with the 1989 NCTM standards as "Standards-based." The Standards-based materials had a common set of design specifications. These curriculum materials included an expanded set of content, a de-emphasis of paper-and-pencil skills, and focus on "students' active construction of communication about solutions to challenging problems" (Stein, Remillard, & Smith, 2007, p. 320). I also purposefully chose multiple curricular series within the Standards-based category because we know little about the differences between curricula in the same category. In other words, although some mathematics educators may talk about "Standards-based curricula" as if they were one thing, not all Standards-based curricula are the same. The National Research Council (2004) argues that this categorization, in fact, masks some of the differences within categories difficult to see. In addition, I used information available about the design of materials (Hirsch, 2007) to ensure that

the inclusion of each curricular series would provide additional information about the nature of opportunities for teacher learning. For example, if two curricular series were designed with the same guiding principles or with the same structure (e.g., problem-centered, spiraling) it might be less likely that I would obtain something different regarding the nature of teachers' opportunities to learn from including both of these curricular series. I chose the *Connected Mathematics Project 2* (Lappan, Fey, Fitzgerald, Friel, Phillips, 2006) [CMP], Glencoe's *Math connects: Concepts, skills, and problem solving (Michigan Edition)* (Day, Frey, Howard, Hutchens, Luchin, McClain, et al., 2009) [Glencoe], *Mathematics in Context* (Wisconsin Center for Education & the Freudenthal Institute, 2010) [MiC], and UCSMP's *Transition Mathematics* (Viktora, Cheung, Highstone, Capuzzi, Heeres, Metcalf, et al., 2008) [UCSMP]. These curricular series were intended to be used with students in Grades 6 – 8, and in all but one case the series contained three books. For UCSMP, there were only two books in the series; however documentation from the publisher indicated that these two books were intended to be used with students in Grades 5 – 9, depending on student readiness at each grade level.

Choice of mathematical content. In my initial examination of teachers' guides it was apparent that for each curricular series (and others series not included in the final analysis), the structure and features of the teachers' guides repeated for each unit or chapter. For example, the *Connected Mathematics Project's* Teacher's Guide provided a section entitled "Mathematical Background" and this section had the same structure for all units. Since the structure and features were repeated throughout the texts at both the unit and lesson level, it was not necessary to analyze every page of every textbook. Therefore, I mindfully chose units to analyze; one algebra unit and one geometry unit, because algebra and geometry make up the dominant focus in the high school curriculum (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997), something that

students will encounter after middle school. Specifically, I analyzed units related to the introduction to variable and geometric transformations. I choose these particular topics because a) these topics are seen as important in the middle school mathematics curriculum as evidenced by the number of standards addressing them in standards documents (National Council of Teachers of Mathematics, 2000; Common Core Standards Initiative, 2010) and b) research indicates that these topics are typically problematic for middle school students or teachers (Clements, 2003; Kieran, 2007).

Introduction to variables. Algebra is the focus of much of the high school curriculum (most students take at least two algebra courses and it is also integral in precalculus and calculus courses as well) and has become increasingly integrated into and focused on in the middle grades, as it has been named as the gateway to higher mathematics (National Mathematics Advisory Panel, 2008). Investigating standards documents such as the NCTM Standards and the Common Core Standards, I found that in the middle grades there is an emphasis placed on *variables, expressions, and equations*, particularly related to linear expressions. Of the nine individual standards outlined by NCTM in algebra for grades 6-8, four of these involved symbolic algebra, including coming up with symbolic rules, developing an understanding of variables, and using symbolic algebra to represent and solve problems, including recognizing and generating equivalent expressions. See Table 1.

In the Common Core State Standards, this emphasis is even clearer. Across grades 6 – 8, more than 65% of the algebra-related standards involve variables and working with expressions and equations.

Table 1. Algebra Standards for Grades 6-8 (National Council of Teachers of Mathematics, 2000)

Instructional programs from prekindergarten through grade 12 should enable all students to—	In grades 6–8 all students should—
Understand patterns, relations, and functions	<ul style="list-style-type: none"> • represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules; • relate and compare different forms of representation for a relationship; • identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.
Represent and analyze mathematical situations and structures using algebraic symbols	<ul style="list-style-type: none"> • develop an initial conceptual understanding of different uses of variables; • explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope; • use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships; • recognize and generate equivalent forms for simple algebraic expressions and solve linear equations
Use mathematical models to represent and understand quantitative relationships Analyze change in various contexts	<ul style="list-style-type: none"> • model and solve contextualized problems using various representations, such as graphs, tables, and equations. • use graphs to analyze the nature of changes in quantities in linear relationships.

Specifically, 17 out of the 26 algebra-related standards focus on variables, expressions, and equations in some way. See Table 2 for frequencies of the Common Core Standards in Grade 6 – 8 pertaining to various algebraic topics. (Note: Some standards address more than one of these topics so the numbers will not add up to 26).

Table 2. Frequency of Topics within the Algebra Standards for Grades 6-8 from the Common Core State Standards for Mathematics

Topic	Number of Standards			Total
	Grade 6	Grade 7	Grade 8	
Expressions	8	1	2	11
Equations	4	2	6	12
Inequalities	6	1	5	12
Variable/Unknown	8	1	2	11
Systems of Equations			3	3

Table 2 cont'd

Linear Relationships	1	2	11	14
Functions			5	5
Graphing/Coordinate Plane	5		4	10

Since it was not possible to analyze all content relations to variables, expressions, and equations, I chose to focus specifically on the *introduction to variables*. Developing an understanding of variables is critical to understanding and being able to use algebra and understanding the concept of variable is difficult for students and teachers (Kieran, 2007). I also recognize that there are different conceptions or approaches to variable (e.g., Arcavi, 1994; Usiskin, 1988). Regardless of the conception of variable espoused by curriculum authors (e.g., variable as “unknown” or “true variable”), and maybe more importantly because there are various conceptions of algebra and variable, it seemed like an especially important topic to study.

Geometric transformations. In addition to algebra, geometry was named as being most missing from U.S. curriculum compared to other countries in the TIMSS (Schmidt et al., 1996) and more recently curricula have begun to put more emphasis on geometry to fill these gaps. I chose to focus on geometric transformations. In her review of the history of the geometry curriculum in the United States, Sinclair (2008) indicated that since the latter part of the twentieth century, geometry, which once had lost its place in the curriculum, has struggled to regain its place and has done this by adhering less to the strict Euclidean traditions. These less-strict Euclidean traditions include the transformational approach to geometry, and according to Sinclair transformations have recently figured prominently in middle school mathematics texts. This focus on transformations seemed evident in the NCTM Standards where transformations was included as a focal topic. See Table 3.

Table 3. Geometry and Measurement Standards for Grades 6-8 (National Council of Teachers of Mathematics, 2000)

Instructional programs from prekindergarten through grade 12 should enable all students to—	In grades 6–8 all students should—
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships	<ul style="list-style-type: none"> • precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties; • understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects; • create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.
Specify locations and describe spatial relationships using coordinate geometry and other representational systems	<ul style="list-style-type: none"> • use coordinate geometry to represent and examine the properties of geometric shapes; • use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides.
Apply transformations and use symmetry to analyze mathematical situations	<ul style="list-style-type: none"> • describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling; • examine the congruence, similarity, and line or rotational symmetry of objects using transformations.
Use visualization, spatial reasoning, and geometric modeling to solve problems	<ul style="list-style-type: none"> • draw geometric objects with specified properties, such as side lengths or angle measures; • use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; • use visual tools such as networks to represent and solve problems; • use geometric models to represent and explain numerical and algebraic relationships; • recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

In addition, geometric transformations received considerable attention in the Grade 8 Common Core Standards (see Table 4) and continues to be focused on in the high school standards as well. Furthermore, research indicated that students struggle with the geometry of motion (Clements, 2003).

Table 4. Frequency of Topics within the Geometry Standards for Grades 6-8 from the Common Core State Standards for Mathematics

Topic	Number of Standards			Total
	Grade 6	Grade 7	Grade 8	
Area	1	3		4
Surface Area	1	1		2
Volume	1	1	1	3
Length/Circumference		2		2
Scale Drawings		1		1
Angles		1	1	2
Relationships between 2-D & 3D Figures	1	1		2
Constructions		1		1
Coordinate Geometry	1			1
Similarity			2	2
Congruence			2	1
Transformations			4	4
Pythagorean Theorem			2	2

Units. Once I chose this mathematical content, I used the authors’ descriptions of the units to choose units that covered this content. The four curricular series, the units analyzed, and the total number of pages for each unit can be found in Table 5.

Table 5. Units and Total Pages per Unit for the Curricular Series

Curricular Series	Variable units	Transformations units
Connected Mathematics Project 2 (CMP)	Variables and Patterns (72)	Kaleidoscopes, Hubcaps, and Mirrors (88)
Math Connects (Glencoe)	Algebra: Number Patterns and Functions (77)	Geometry: Polygons (92)
Mathematics in Context (MiC)	Comparing Quantities (81)	Triangles and Beyond (121)
Transition Mathematics (UCSMP)	Using Variables (62)	Some Important Geometry Ideas (72)

Analytic Frameworks

Although often viewed as a subjective scheme, curriculum materials can be viewed as an objectively given structure (Otte, 1983). Analysis can be focused not only on the interaction between the reader and the text (subjective), but also on the structure of the written text itself and

the potential of this text to construct relationships between authors and readers (Herbel-Eisenmann, 2007). According to Remillard (2000), if texts are to be truly educative, authors must speak *to* teachers rather than *through* teachers. The way authors speak is construed in the written text by the content and language choices authors make and these choices, although not necessarily conscious ones, between alternative structures and content affect “the meaning that listeners or readers may construct from the utterances” (Morgan, 1998, p. 79). Therefore, choices of content and language in teachers’ guides, impact the opportunities for teachers to make sense of and learn from the written text.

To analyze the *content* and *expression* in the eight middle school teachers’ guides, I developed two frameworks. The first of these frameworks aimed to describe the supports for the development of *Subject Matter Knowledge*, *Pedagogical Content Knowledge for Topics*, *Pedagogical Content Knowledge for Practices*, and *Curricular Knowledge* in the teachers’ guides. I adapted a framework developed by Beyer et al. (2009) that included the types of knowledge and guidance needed by teachers. Since this was developed for coding science textbooks I adapted the codes to be appropriate for coding mathematics textbooks and I also included codes to describe curriculum knowledge, which was not part of Beyer et al.’s (2009) framework. The second framework I developed was aimed at describing some aspects of the language of the written text in the teacher’s guides. I drew on Herbel-Eisenmann’s (2007) *voice* framework which was developed using ideas from Morgan (1996) and Halliday (1985). Although research on voice more generally has been used to describe who is talking, Herbel-Eisenmann argues that voice can be used to investigate “how speakers shift positions, identities, and alignments toward the words they speak as well as toward one another” (p. 347). In the next sections I describe both of these frameworks in detail.

A framework for analyzing the content supports of teachers' guides. To analyze the content of the teachers' guides, I developed a coding scheme (see Table 6), adapted from Beyer, Delgado, Davis, & Krajcik (2009), based on the design heuristics outlined by Davis & Krajcik (2005) and proposals put forth by Ball and Cohen (1996). In addition, this scheme included codes derived from the literature on teacher learning from curriculum use (e.g., Choppin, 2008; Collopy, 2003; Drake & Sherin, 2009; Lloyd, 2008a; 2008b; Remillard, 2000; Remillard & Bryans, 2004; Schneider, Krajcik, & Blumenfeld, 2005; Schneider & Krajcik, 2002; Schneider, 2006; Van Zoest & Bohl, 2002). This scheme consisted of nine categories, which included codes for both *Enactment* and *Rationale Guidance*. These categories were further grouped into four domains: *Subject Matter Content Knowledge (SMK)*, *Pedagogical Content Knowledge for Topics (PCK-T)*, *Pedagogical Content Knowledge for Practices (PCK-P)*, and *Curricular Knowledge (CK)*.

Table 6. Content Coding Scheme

I. Mathematics Subject Matter Content Knowledge
Support Teachers in the Development of Subject Matter Knowledge
<u>Content</u> : Present mathematical content information [Con]

II. Pedagogical Content Knowledge for Mathematics Topics
Support Teachers in Engaging Students with Topic-Specific Mathematics
Enactment Guidance for:
<u>Experiences</u> : Engaging students in particular experiences. [E-Experiences]
<u>Possible Student Pitfalls</u> : Potential pitfalls with specific mathematical experiences (including describing where students may have difficulty or potential misconceptions) [E-Pitfalls]
<u>Engaging in Problematic Experiences</u> : Engaging students in experiences that may potentially be problematic [E-Problem]
<u>Activity Sequences</u> : Engaging in productive activity sequences [E-Sequence]
Rationale Guidance for:
<u>Experiences</u> : Why particular experiences are appropriate, including why a particular problem or activity is useful in a students' mathematical development. [R-Experiences]
<u>Activity Sequences</u> : Why particular activity sequences are appropriate or useful. [R-Sequence]

Table 6 (cont'd)

Support Teachers in Using Mathematical Instructional Representations

Enactment Guidance for:

Representations: Using representations (verbal, equations, tables, graphs, pictures) with students. [E-Representations]

Rationale Guidance for:

Representations: Why particular representations (verbal, equations, tables, graphs, pictures) are appropriate including the advantages and disadvantages for using particular representations. [R-Representations]

Support Teachers in Using Mathematical Tools

Enactment Guidance for:

Tools: Using particular tools (e.g., rulers, compasses, calculator)with students. [E-Tools]

Rationale Guidance for:

Tools: Why particular tools are appropriate including the advantages and disadvantages for using particular tools [R-Tools]

Support Teachers in Anticipating and Using Students' Mathematical Ideas

Enactment Guidance for:

Anticipating Student Ideas: Anticipate student ideas within a topic [I-Anticipate]

Using Student Ideas: Using students' ideas during the enactment of the lesson [I-Use]

III. Pedagogical Content Knowledge for Mathematics Practices

Support Teachers in Engaging Students in Questions

Enactment Guidance for:

Using Questions: Using questions with students (Suggested answers provided) [I-QuestionsA]

Using Questions: Using questions with students (Support for answers not provided) [I-QuestionsNA]

Helping Students Ask/Answer Questions: Having students ask/answer their own questions [I-Ask]

Rationale Guidance for:

Questions: Why particular questions or series of questions are appropriate or useful [I-Ask]

Support Teachers in Engaging Students in Justification, Reasoning, and Proof

Enactment Guidance for:

Justification, Reasoning, & Proof: Using approaches for engaging students in justification, reasoning and proof [E-Proof]

Rationale Guidance for:

Justification, Reasoning, & Proof: Why approaches to engaging in justification, reasoning and proof are appropriate or useful [R-Proof]

Table 6 (cont'd)

Support Teachers in Developing Students' Mathematical Vocabulary/Terminology

Enactment Guidance for:

Developing Mathematical Terminology: Developing students' mathematical terminology (including vocabulary and notation) and recognizing colloquial words that may help students understand context or promote/hinder students conceptions [E-Terminology]

Rationale Guidance for:

Developing Mathematical Terminology: Why developing mathematical vocabulary or terminology is important and why particular strategies are appropriate or useful [R-Terminology]

Support Teachers in Engaging Students in Appropriate Participation Structures

Enactment Guidance for:

Participation Structures: Using appropriate participation structures (e.g., individual, small-group, whole-class) [E-Participation]

Rationale Guidance for:

Participation Structures: Why particular participation structures are appropriate or useful [R-Participation]

IV. Mathematics Curricular Content Knowledge

Support Teachers in the development of Curricular Knowledge

Enactment Guidance for:

Curricular Overview: Using unit, section, or lesson content in the intended ways (by providing a description of what will unfold in the unit, section, or lesson) [E-Overview]

Curricular Features: Using features of the curriculum [E-Features]

Curricular Storyline: Connecting previous and future mathematics content, including within and across disciplines, courses, units, sections, and lessons [E-Storyline]

Curricular Goals: Using the goals of the unit, section or lesson (including knowing what the goals are and how to achieve them) [E-Goals]

Rationale Guidance for:

Curricular Philosophy: curricular philosophy. [R-Philosophy]

Curricular Features: Why curricular features are appropriate or useful (including explaining these features) [R-Features]

Curricular Storyline: Why particular pieces of content or processes are appropriate in the storyline [R-Storyline]

Curricular Goals: Why particular goals are appropriate or useful. [R-Goals]

Beyer et al.'s (2009) coding scheme was developed specifically for coding biology textbooks. Due to the frameworks intimate connection to the teaching of science, I needed to make adjustments in order to use it to code mathematics curriculum materials. That said, the categories pertaining to the types of knowledge (i.e., Subject- Matter Content Knowledge, Pedagogical Content Knowledge for Topics and Practices) and the types of guidance (Enactment

and Rationale) still seemed appropriate and served as organizing categories. Prior to generating my codes, I felt that this framework lacked attention to the very thing that it was being used to analyze, the curriculum and more specifically, curriculum materials. In addition to Subject-Matter Content Knowledge, and Pedagogical Content Knowledge, Shulman (1986) also described a need for Curricular Knowledge. Furthermore, when writing about this in 1986, Shulman indicated that this was the knowledge that teacher education was the most remiss in attending to. Therefore, including codes for describing supports for the development of Curricular Knowledge was something I decided I would do before generating my codes.

Generating codes. To generate codes I drew primarily on two sources: a) research literature, including theoretical pieces (Ball and Cohen, 1996; Davis & Krajcik, 2005; Shulman, 1986), and the research on mathematics and science teacher learning and curriculum use, including drawing heavily from the structure used by Beyer et al. (2009) and b) the curriculum materials themselves and my knowledge of a variety of materials (from my years of teaching and doing research). I used the latter as I developed my scheme in order to gauge the reasonableness of my codes. I did not want to have codes in my scheme that were so far removed from what could be expressed in written materials, or that were too specific to any one content or type of problem. To aid me in making these decisions, in addition to my own knowledge of materials, I also examined curricular series not included in my final study (i.e., Prentice Hall's *Mathematics*, *Math Themes*) and other units addressing different strands from the curriculum materials I chose for my study (i.e., units focusing on number and data analysis and statistics). In the next few sections, I describe the development of my codes organized by the four knowledge types. I provide representative examples for each of the codes, drawing from either the variable or geometric transformations unit from each curriculum. Since in this section my aim is to provide

examples of these codes, and not to compare the units, I include only one example for each code from one of the units, not both. For some examples I provide more than one sentence, as context may play a factor in understanding why I assigned particular codes. In these instances each of the sentences received that code. If I provided three sentences in my example for *Enactment Guidance for Anticipating Student Thinking*, all three sentences received this code.

Subject Matter Content Knowledge. The code that seemed the most obvious to include in my scheme concerned subject matter. Drawing on Beyer et al. (2009), I considered this support an “other educational feature” (p. 983), rather than a support providing *Enactment* or *Rationale Guidance*, although I recognize that having subject matter knowledge does impact the enactment of a lesson. An example of a support is a sentence that provides information about the subject matter, such as “The shape of a graph of a pattern of change over time shows the rate of change in the dependent variable as time passes” (CMP, Variables & Patterns, p. 5).

Pedagogical Content Knowledge for Topics. Drawing from Beyer et al. (2009), I included codes for *Enactment Guidance for Experiences, Pitfalls, Activity Sequences, Representations, Anticipating Student Thinking*, and *Using Student Thinking* and *Rationale Guidance for Experiences, Activity Sequences, and Representations*. These codes seemed appropriate for mathematics teachers as well as science teachers. In addition to these codes, I added three more. The first was *Enactment Guidance Engaging Students in Problematic Experiences*. My pilot analysis revealed instances that went beyond identifying potential pitfall, but provided information for teachers as to what to do when students found experiences difficult. The other two codes that seemed appropriate were *Enactment* and *Rationale Guidance for Using Tools*. In mathematics, tools such as calculators, rulers, and compasses, to name a few, are integral to engaging in mathematical activity. It seemed necessary, and appropriate that teachers

be provided with both *Enactment* and *Rationale Guidance for Using Tools* with their students.

Table 7 includes examples of these codes.

Table 7. Examples of Pedagogical Content Knowledge for Topics Codes

	Codes	Examples
Enactment Guidance	Experiences	"Have students determined the height of trees or other objects by using similar triangles" (Glencoe, <i>Geometry: Polygons</i> , p. 540a).
	Possible Student Pitfalls	"Watch for students who are so ruled by the words that they lose sight of their context. For example, students who add whenever they see the word total in the problem" "In the year 2000, the population of one town was 1,900 and in another town was 2,100. What was the total population of the towns?" (USCMP, <i>Using Variables</i> , p. 76-77).
	Engaging Students in Problematic Experiences	"If students are having difficulty, you might remind them that parallel arrows having the same length will produce the same change" (MiC, <i>Comparing Quantities</i> , p. 13).
	Activity Sequences	"After students work on problem 8, you may wish to discuss how exchanging relates to the combination chart to remind students what the numbers in the combination chart stands for" (MiC, <i>Comparing Quantities</i> , p. 18).
	Representation	"In the case of tables, one has to decide which values of the independent variable should be represented in the table to give most informative results" (CMP, <i>Variables & Patterns</i> , p. 3).
	Tools	"To draw a point at a certain distance from point A can be done using a centimeter ruler" (MiC, <i>Triangles and Beyond</i> , p. 8B).
	Anticipating Student Ideas	"Some students may trade for some corn, while others may trade until they have only corn" (MiC, <i>Comparing Quantities</i> , p. 5T).
	Using Student Ideas	"Some will talk about drawing lines from the center of the design to corresponding points of two adjacent parts of the pattern, thus making an angle that can be measured with an angle ruler or a protractor. Help students to understand that each design has a particular angle of rotation regardless of which two adjacent parts are chosen" (CMP, <i>Kaleidoscopes, Hubcaps, and Mirrors</i> , p. 24).
Rationale	Experiences	"We ask students to graph solutions to inequalities because such graphs convey a lot of information" (UCSMP, <i>Using Variables</i> , p. 115).

Table 7 (cont'd)

Activity Sequences	"We recommend going through Questions 1-10 in order. They will take students through the key points of the lesson" (UCSMP, Some Important Geometry Ideas, p. 410).
Representations	"Unlike the combination chart, notebook notation can be used to solve problems involving combinations of more than two kinds of items. In addition, notebook notation can be used to record the new combinations. Students should come to appreciate the advantages of notebook notation over guess-and-check strategies and combination Charts" (MiC, Comparing Quantities, p. 22B).
Tools	"This tool [calculator] allows students to look at many examples quickly and helps them observe patterns and make conjectures about functions" (CMP, Variables and Patterns, p. 11).

Pedagogical Content Knowledge for Practices. The Beyer et al. (2009) framework did not contain a category for practices, but included a category for "Scientific Inquiry." I was not sure that I could translate scientific inquiry into one practice for mathematics teachers, but saw this as support for enacting and understanding the practices or processes relevant to mathematics teaching and learning. This would encompass some of what the Beyer et al. (2009) framework included for helping science teachers enact inquiry teaching, but would include other codes specific to mathematical teaching. From the Beyer et al. (2009) framework I included *Enactment Guidance for Using Questions* and *Helping Students Ask/Answer Their Own Questions* and *Rationale Guidance for Questions*. In addition to these codes, I generated six more codes; one *Enactment* and one *Rationale Guidance Support for Engaging in Justification, Reasoning, & Proof, Developing Mathematical Terminology, and Participation Structures*. The first two seemed to be critical as these are practices or processes that are integral to the teaching and learning of mathematics. Justification, reasoning, and proving are integral to the ways in which mathematics is done and communicated. Proving is often difficult for students and teachers (Harel & Sowder, 2007), particularly at the middle school level where students have difficulties

moving beyond examples-based reasoning (Knuth, Choppin, & Bieda, 2009) and teachers themselves often have a limited understanding of proof (Chazan, 1993; Knuth, 2002). The second addition, related to supports for developing mathematical terminology, seemed important because communicating about mathematics requires the use of mathematical terminology. Some may argue, like Lemke (1990) did about science that learning mathematics is learning to "talk mathematics." Finally, I included *Enactment* and *Rationale Guidance for Participation Structures*, as teachers are increasingly expected to have students interact in a variety of ways (i.e., individual, pairs, small-group, whole-class) in order to actively engage students in mathematical roles (Franke, Kazemi, & Battey; 2007; O'Connor & Michaels, 1996) and these structures are becoming integral to helping ELL students as well (Empson, Turner, Dominguez, & Maldonado, 2006). Table 8 includes examples of these codes.

Table 8. Examples of Pedagogical Content Knowledge for Topics Code

	Codes	Examples
Enactment Guidance	Using Questions (suggested answers provided)	"Ask: On which side of your reflection does your left arm appear? [answer provided] right side" (Glencoe, Geometry: Polygons, p. 58).
	Using Questions (suggested answers not provided)	"Ask them to tell you how they would solve it" (Glencoe, Algebra: Number Patterns and functions, p. 40).
	Helping Students Ask/Answer Questions:	"Through their work in this and other geometry units, students learn important questions to ask themselves about any situation that involves the principles explored, such as: <i>How can I use symmetry to describe the shapes and properties of figures in a design or a problem?</i> " (CMP, Kaleidoscopes, Hubcaps, and Mirrors, p. 2).
	Justification, Reasoning, & Proof	"Then you can show students an example of a deduction. Begin with a 360° as the sum of the measures of the angles around a point. Then it follows that 180° is the sum of the measures of the angles on one side of the line through that point. Then it follows...." (UCSMP, Some Important Geometry Ideas, p. 387).

Table 8 (cont'd)

	Developing Mathematical Terminology	“Help students understand that the term evaluate an expression involves two steps. The first is to replace the variables with their values. The second step is to perform the indicated arithmetic operations (UCMP, Using Variables, p. 85).
	Participation Structures	“Have students work Problem 1.5 individually or in pairs. If they work with a partner, make sure each student makes a table and a graph” (CMP, Variables and Patterns, p. 33).
Rationale Guidance	Questions	“These are important questions to discuss because they utilize properties of congruence” (UCSMP, Some Important Geometry Ideas, p. 399).
	Justification, Reasoning, & Proof	“It is likely that students will discover the angle congruence theorems that are usually taught and proved in high school geometry– Side-Side-Side, Side-Angle-Side, and Angle-Side-Angle. This engagement with the ideas in an informal way will help make the more proof oriented approach of high school geometry more understandable” (CMP, Kaleidoscopes, Hubcaps, and Mirrors, p. 8).
	Developing Mathematical Terminology	“The ideas of consecutive sides and consecutive angles apply to all polygons not just quadrilaterals. So this vocabulary is quite useful” (UCSMP, Some Important Ideas in Geometry, p. 402).
	Participation Structures	“Let the students work alone for a minute or two, and then move them into small groups of 2 to 4. This gives each student time to think individually before discussing ideas with a group” (CMP, Variables and Patterns, p. 55).

Mathematics Curricular Content Knowledge. As I previously stated, the Beyer et al. (2009) framework did not include codes that pertained to supports for the development of curricular knowledge. However, I included this knowledge because I was studying curriculum materials and Shulman (1986) described this knowledge as the knowledge teacher educators were the most remiss in teaching. Drawing on Shulman (1986), Ball and Cohen (1996), and the research on curriculum use, I generated a total of eight codes, four Enactment (i.e, overview, features, storyline, goals) and four Rationale Guidance supports (i.e., philosophy, features,

storyline, goals). Related to the idea that teachers should understand the resources that they use (Shulman, 1986), I created codes for *Enactment* and *Rationale Guidance for Curricular Features* and *Rationale Guidance for Curriculum Philosophy*. In order for teachers to understand their materials they must have knowledge of its features, including not only what the features are, but why these features were included in the textbook and an understanding of the philosophy of the materials. In order to capture ideas that related to helping teachers to develop “curriculum vision” or a sense of where the materials were going and how particular activities fit into this (Drake & Sherin, 2009), I generated codes for *Enactment* and *Rationale Guidance for Goals* and *Curricular Storyline*. Shulman (2009) advocated for this as a type of knowledge as well and called it “lateral” curriculum knowledge” (p. 10). In order to understand where a curriculum is going, it is necessary to understand the mathematical goals and to have a sense of what came before and after particular topics and lessons. Finally, the need for the *Enactment Guidance for Curricular Overview* arose during my pilot analysis. I found text in all the curriculum materials that described for teachers what the lesson or unit was about. At first I questioned whether this was in fact support for the teacher. I decided that this text provided teachers with an opportunity to learn about the activities that would unfold in a particular unit or lesson and so therefore, I wanted to capture this in my analysis. Table 9 includes examples of each of these codes. For *Rationale Guidance for Goals* I do not include an example from the curriculum materials because I did not find this in any of the curricula. Although the curriculum materials all provided goals for units and lessons, there was no support for understanding why these goals were chosen by curriculum authors as appropriate.

Table 9. Examples of Mathematics Curricular Knowledge Codes

	Codes	Examples
Enactment Guidance	Curricular Overview	“Students solve a more complex exchange problem involving the strength of horses, elephants, and oxen” (MiC, Comparing Quantities, p. 3T).
	Curricular Features	“ The Differentiated Homework Options provide suggestions for the exercises that are appropriate for basic, core, or advanced students” (Glencoe, Algebra: Number Patterns and Functions, p. 27).
	Curricular Storyline	“ For the teacher Students have encountered a variety of patterns in the tables and graphs of related variables. Many have been linear increasing or decreasing patterns. Some have.... The details of relating particular graph shapes to particular function types and symbolic expressions will develop over the next several years. (CMP, Variables and Patterns, p. 104).
	Curricular Goals	“The goal is for students to understand the use of algebra as a tool to solve problems that arise in the real world or in the world of mathematics, where symbolic representations can be temporarily freed of meaning to bring a deeper understanding of the problem” (MiC, Comparing Quantities, p. xx).
Rationale Guidance	Curricular Philosophy	" <i>Connected Mathematics</i> was developed with the belief that calculators should always be available and that students should learn when their use is appropriate" (CMP, Variable and Patterns, p. 11).
	Curricular Features	“The Reading Math features throughout the chapter help students understand the mathematical meaning by relating them to everyday use” (Glencoe, Geometry: Polygons, p. 508B).
	Curricular Storyline	“Rotations are critical in the study of congruence and tessellations used in this chapter to develop properties of angles” (UCSMP, Important Geometry Ideas, p. 374).
	Curricular Goals	“These goals are important and/or appropriate because....”

A framework for analyzing expression. The ways in which text is expressed in the teachers' guides indicates how curriculum authors are speaking to teachers. To investigate this expression, I drew on a framework developed by Morgan (1988) and augmented by Herbel-Eisenmann (2007), who called hers a framework for analyzing the *voice* of the textbook.

Although research on voice, more generally, has been focused on examining who is speaking, researchers argue that an examination of voice helps to identify “how speakers shift positions, identities, and alignments toward the words they speak as well as toward one another” (Herbel-Eisenmann, 2007, p. 347). To analyze voice, Herbel-Eisenmann (2007) used Halliday's (1985) SFL and described both the *interpersonal* and *ideational* functions since these functions describe the construction of personal relationships and who is speaking. According to Morgan (1998), although it is possible to separate the grammatical features related to the *ideational* and *interpersonal* functions, it is difficult to separate the effects that these forms have on readers. That said, in my analysis, I drew most heavily on the *interpersonal* metafunction of language as this function most closely describes the ways in which personal relationships are constructed. To examine the construction of interpersonal relationships between curriculum authors and teachers, I examined the use of *personal pronouns* and “*you*”-forms, particularly focusing on *imperatives* and “*you*”+*modal verbs* because these were the most prevalent forms. In the next sections I describe each of these grammatical features.

Personal pronouns. The first linguistic form I examined was personal pronouns because this is one of the most explicit ways in which relationships are expressed between authors of a text and their readers. First-person pronouns such as “I” and “we” indicate the authors direct involvement in the activity of the text (Morgan, 1996). According to Herbel-Eisenmann (2007) the use of the pronoun “we” could indicate the authors speaking with the authority of the discipline of mathematics, or it may be used to involve the readers in the mathematics. Researchers such as Pimm (1987) and Rowland (1999) have alluded to the ideological aspects embedded in the use of “we”, such as how using “we” assumes the reader’s agreement with the author. In my analysis, I coded all of the uses of “we.”

The second-person pronoun “you” directly involves the reader, and is of particular interest in my study because the readers of curriculum materials are the teachers using the materials. In the next section I describe in more detail my analysis of "you." In addition, the general absence of first-person pronouns, according to Morgan (1998), not only affects the picture of the nature of mathematics, but also has the potential to distance the author from the reader. Therefore, not finding personal pronouns in my analysis would also provide interesting evidence about the ways in which authors are communicating with teachers through the written text.

“You”-forms. Grammatically, the pronoun "you" serves to bring the reader into the text. Since the reader of curriculum materials are teachers, I chose to examine the use of this pronoun more fully to determine how it was used by curriculum authors. Although I captured all forms of "you" in the text (i.e., verb+"you", modal verb+"you" "you"+verb, modal verb+"you", imperatives), I specifically focus on imperatives and modality since these were the most common. In the imperative form "you" is implicit, whereas when used with a modal verb, "you" is explicit. I describe imperatives and modality below.

Imperatives. Imperatives are characteristic of academic mathematics texts (Morgan, 1998) and according to Herbel-Eisenmann (2007), imperatives “implicitly address the reader and involve him or her in the in the construction of mathematics” (p. 349) whether they want to be implicated or not (Pimm, 1987). According to Morgan (1998) the use of imperatives marks an author’s membership in the mathematical community and is to be expected in mathematical academic discourse. However, in the school context, Morgan questions the use of imperatives as there may be tensions between pupils and teachers as pupils and teachers strive to both show that they have familiarity with mathematical language, but at the same time explain their thinking.

The same may be the case between curriculum authors and teachers. This tension may arise as curriculum authors both need to exhibit their knowledge and authority, but at the same time explain their thinking.

According to Martin & Rose (2007), while there are different moods for realizing a command (i.e., declarative, interrogative, imperative) and these all produce the same result, these different grammatical moods differ in the kinds of social relationships they incur. Unlike other forms, the imperative assumes a position of authority and does not open up the possibilities of negotiation. Particularly useful is Rotman's (1988) distinction between two types of imperatives: *exclusive* and *inclusive*. According to Rotman (1988), although "the speaker of a clause which has chosen the imperative has selected himself the role of controller and for his hearer the role of controlled" (p. 8), different types of imperatives can include or exclude the hearer or reader. *Exclusive imperatives*, position the reader as a "scribbler," or someone who must perform some action. Some examples include "Multiply," "Add," and "Bisect the angle." *Inclusive imperatives*, on the other hand, address the reader as a "thinker" and these include imperatives such as "consider" and "suppose." These imperatives establish a "commonality between speaker and hearer" (p. 9). Although the reader is not positioned solely as either a "scribbler" or a "thinker," the choice of imperatives foregrounds one positioning over the other.

Modality. Modality refers to "the degree of likelihood, probability, weight or authority the speaker attaches to the utterance" (Hodge & Kress, 1993, p. 9 as cited in Morgan, 1996). There are two forms of modality, *relational modality* and expressive modality. Relational modality describes the authority of one participant in relation to another, whereas *expressive modality* describes the author's authority with respect to truth or probability (Fairclough, 2001).

In text, modality is established by the use of modal auxiliary verbs such as “may,” “must,” or “should,” or with *hedges* (Rowland, 1995). Hedges, or “linguistic pointers to uncertainty” (Rowland, 1995, p. 328) are words that modify the certainty of an utterance. As hedges and modal verbs get used together the strength of convictions of these utterances are modified. For example, “I think you might want to explain...” is less certain than “You might want to explain...” which is less certain than using the imperative “Explain...” According to Herbel-Eisenmann, Kristmanson, and Wagner (2011), “mathematics discourse that features modality can be associated with a fallibilist philosophy of mathematics” (p. 146). They propose that this underpins certain ways of teaching mathematics, including ways that involve students working on problems without being given solution methods in advance. The same may be said about the text in teachers' guides. Teachers' guides that do not feature modality may indicate that the methods of teaching are completely known and certain, and therefore, this may impact teachers' opportunities to engage around the ideas presented, hindering the potential for teacher learning. On the other hand, text that features modality may serve to open up a dialogue between curriculum authors and teachers, and teachers may feel empowered to make decisions about how to enact the curriculum. This openness may promote a more equitable relationship between authors and teachers, as teachers are being asked to take a greater role in making decisions about how to enact the curriculum in their classrooms. For example, there is a grammatical difference between “You might want to have students work in groups,” as opposed to “Have students work in groups.” These differences, although subtle to some readers, may be interpreted quite differently by teachers.

According to Fairclough (2001), there is an overlap between the modal verbs that express permission and obligation (i.e., relational) and those that express certainty or possibility

(i.e. expressive). For example, the modal verb "may" is used to express both permission and possibility. For example, in the expression, "You may multiply by four," it is possible that "may" indicates permission being granted by the speaker for the listener to multiply by four. However, it is also possible that "may" indicates the fact that it is mathematically possible to multiply by four. Therefore, in my study, I captured both types of modality as issues of permission and obligation seemed intimately related to how curriculum authors constructed relationships with the teachers reading their materials.

Procedures

Pilot analysis. After the initial development of my analytic frameworks, I used these frameworks to code teachers' guides from units that were not part of the final study. These units included a variety of mathematical content including algebra, geometry, and statistics. I refined my coding to account for additional aspects that were not initially part of my scheme. For example, the original Content Coding Scheme included a code for *Enactment Guidance for Questions*, but did not differentiate whether suggested answers were provided to teachers in the text. Therefore, before beginning coding of the included units I broke this code into QuestionsNA and QuestionsA to indicate whether suggested answers were provided in the text or not, since Ball and Cohen (1996) indicated that providing suggested answers is more supportive than the typical "Answers will Vary" (p. 7) or providing no answers at all. Throughout the coding of the final corpus, I continually revisited the definitions of my codes and either added codes or refined existing codes. When codes were added late into the analysis I revisited all previously coded pages to see if there were instances of these codes.

Coding Process. I coded each page of the teachers' guide of each of the variables and transformations units using both frameworks. The only exception to this page-by-page analysis

was in CMP, which included a condensed version of their teacher material for each lesson called “At a Glance.” I did not code any of the “At a Glance” pages as these included the same content as the non-condensed teacher support. My unit of analysis for my content coding was the sentence and for my expression coding was the clause. For my content coding I examined each sentence and assigned one or more content codes, if applicable, and also coded the location of the content support (i.e., unit, section, lesson). Sentences were coded for multiple supports if it was warranted. For example, in order to help teachers use a particular question with students, the text may also provide teachers with support for anticipating students' thinking, such as in the following excerpt from *Mathematics in Context*: “Some students may solve this problem without using squares. They reason: $25 + 64 = 89$, so the white square must have more than 89 tiles. So a square of 100 tiles is possible” (MiC, Triangles and Beyond, p. 26T). This excerpt, found at the lesson level, in “Comment About the Solutions,” a section which included text related to the questions in the student edition of the textbook, was coded for both *Enactment Guidance for Questions (suggested answers provided)* and *Anticipating Student Thinking* because these three sentences provided teachers with support in working with their students on a particular question, and provided teachers with some ideas as to what their students might say. This excerpt received three “E-QueA-L” codes and three “E-Ant-L” codes.

I also examined each clause coded all personal pronouns and "you"-forms and kept track of the location as well. I entered all codes into a spreadsheet for ease of calculating frequencies and percentages across all units and curriculum materials. This resulted in eight separate Excel workbook files (one per unit), each containing a sheet for all pages in my sample within each unit and two additional sheets that summed codes from all sheets in the workbook.

Inter-rater reliability. I used three additional coders to check the reliability of the coding process. The content coding was done by one coder, a mathematics education graduate student with curriculum analysis experience, who coded a random sample of 10% of the corpus, stratified by unit, meaning that he coded pages from all eight units. Percent agreement was calculated at the sentence level; agreement was achieved only when a sentence was coded in the exact same way by both coders. For example, for the content coding if a sentence received two codes from one coder and only one code was assigned by the second coder (even if this one code was in agreement with one of the codes assigned by the first coder), agreement was not reached. An agreement of at least 85% was reached for each unit. See Table 10 for the percentages of agreement for each unit by curriculum.

Table 10. Percentages of Agreement for content by Unit and Curriculum

Curricula	Variable	Transformations
CMP	85	86
Glencoe	88	91
MiC	85	85
UCSMP	91	95

For the coding related to the aspects of voice, I had two coders (both mathematics education graduate students) code personal pronouns and “you”-forms. One coder coded 10% of the algebra units and the other, 10% of the geometry units. I calculated a separate reliability percentage for personal pronouns, imperatives, and modal verbs used with the pronoun “you.” Agreement was reached only when both coders had identified the same personal pronouns, imperatives, and modal verbs used with the pronoun “you” in each sentence within the text. For example, if I coded a sentence as an imperative and the second coder did not, this was not counted as agreement. The same was true if we did not code the same personal pronouns and modal verbs in a sentence. See Table 11 for the percentages of agreement for personal pronouns, imperatives, and modal verbs.

Table 11. Percentages of Agreement for Personal Pronouns and "You"-forms by Unit and Curriculum

Curricula	Variable			Transformations		
	Personal Pronouns	Imperatives	Modal Verbs	Personal Pronouns	Imperatives	Modal Verbs
CMP	98	99	100	98	97	100
Glencoe	100	98	100	100	97	100
MiC	100	99	100	99	96	100
UCSMP	99	96	100	100	96	100

Analysis

Once my data was coded, I used the summary sheets in each workbook to determine the relative frequencies in each unit of : a) each individual content code, b) each content category (i.e., *Subject Matter Content Knowledge, Pedagogical Content Knowledge for Topics, Pedagogical Content Knowledge for Practices, Mathematics Curricular Knowledge*), c) personal pronouns, d) "you"-forms, e) imperatives (including frequencies of exclusive and inclusive imperatives) , and f) modal verbs. I used these relative frequencies to search for themes within units across curriculum materials and across the variable and transformations units.

In addition, for the voice analysis, I summed the frequencies of personal pronouns by location. I then did a deeper analysis of the types of imperatives, including determining the frequencies of imperatives that directed teachers versus those that directed students through the teacher. Finally, since the modal verbs "may" and "might" were so pervasive in all four curriculum materials, I investigated this further by determining the most frequent collocates (i.e., the words written directly after "may" or "might") in order to examine the types of processes that teachers were asked to engaged in.

CHAPTER 4: RESULTS –DIFFERENCES IN STRUTURE AND UNIT CONTENT

Before outlining the main results of my study, I describe the curriculum materials and the units in this chapter. This description provides an overall context for understanding the curriculum materials that differed in their structure. CMP and MiC had separate teachers' guides for each unit, whereas Glencoe and UCSMP each had a two-volume teachers' guide with each volume having half of the teacher support for that course. These curricular series also differed in terms of the ways in which support was broken up, with CMP and MiC having material separated into three levels (i.e., *Unit, Section, Lesson*) and Glencoe and UCMP separated into two (i.e., *Unit, Section, Lesson*). Finally, the curriculum materials varied in the "embeddedness" of their education supports (Beyer et al., 2009). Embedded supports are integrated within the directions and content for enacting activities found in the student text, rather than being close to but separate from student activities. CMP was the only teachers' guide that offered support in this way.

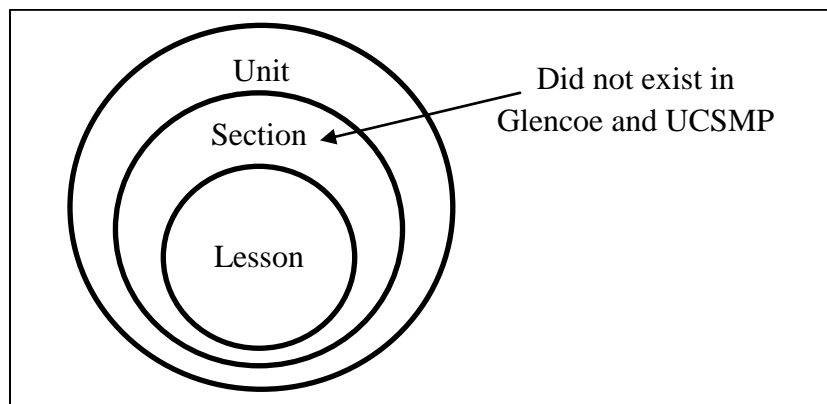
In addition to impacting the amount of content support, these structural differences may also impact the types of supports and the ways in which these were expressed. For example, the unit and two-volume structures may impact the total number of pages possible, therefore impacting the number of content supports and expression. With fewer pages there is less room for support and this may also require authors to be terser in their expression. In the next sections, I describe each of these structural differences.

Structural Differences

The curriculum materials varied in terms of the overall structures for their teachers' guides. First, CMP and MiC had a separate teacher's guide for each unit, whereas Glencoe and

UCSMP each had a two-volume teacher's guide. Each of these volumes contained half of the units for that course. Second, the ways in which the content in the teachers' guides was distributed varied. Just as a curriculum is broken into units, these units are broken up into smaller chunks of content. See Figure 3 for the structure of the units in this study.

Figure 3. Structure of the Units in this Study



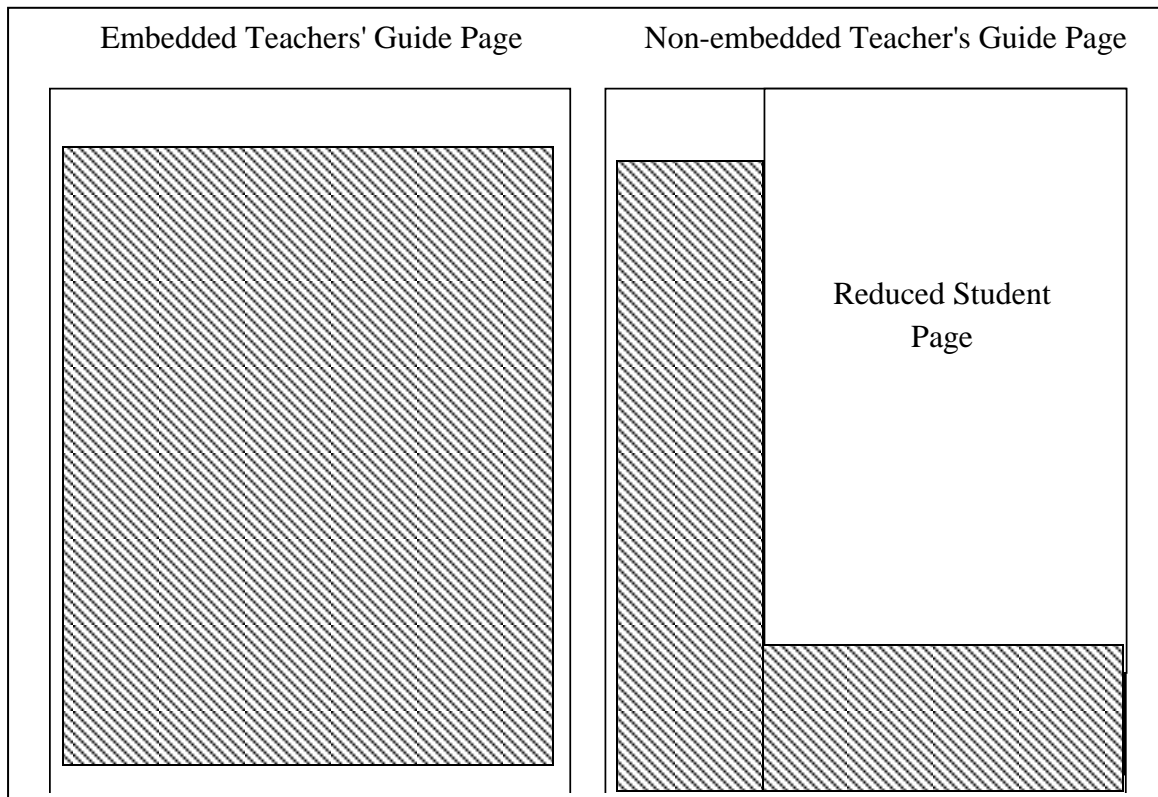
Although each curriculum used its own terminology to refer to the different levels of support, I will use *Unit*, *Section* and *Lesson* throughout this chapter. CMP and MiC was structured into three levels (i.e., *Unit*, *Section*, *Lesson*). *Unit* was the largest chunk of content (encompassing the *Section* and *Lesson* Levels) and *Lesson* being the smallest, the chunk to be covered in a typical class period. CMP had four sections (called *Investigations*) in their variable unit and five in their transformations unit with each section having between two and four lessons (called *Problems*) and MiC had five sections in their variable unit and six sections in their transformations unit (called *Sections*), with each section having between three and four lessons indicated by a specific topic name (e.g., *Combining Transformations: Using Models to Understand Transformations and Relationships Among Types of Two-Dimensional Objects*). Mathematics content in Glencoe and UCSMP was broken up into only two levels (i.e., *Unit*, *Lesson*), and similar to CMP and MiC, *Unit* encompassed *Lesson*. Including chapter reviews,

Glencoe and UCSMP had a total of 10 and 9 lessons in their variable units, respectively, and 11 and 10 lessons in their transformations units, respectively.

Embeddedness of Supports

The curriculum materials also varied in terms of the "embeddedness of education supports" (Beyer et al., 2009). The embedded teachers' guide pages integrated supports into the student activities, whereas the non-embedded teachers' guides included supports close to, but separated from student activities. These non-embedded teacher guide pages included a reduced student page, which accounted for more than half of the space on the page. The text intended for teachers was wrapped around the reduced student page. For an example of pages from an embedded and non-embedded teacher's guide see Figure 4. The shaded portions indicate the part of the page containing text intended for the teacher.

Figure 4. Examples of Pages from Embedded and Non-Embedded Teachers' Guides



CMP was the only teachers' guide that provided embedded supports. Glencoe, MiC, and UCMP were not embedded, but were formatted as teacher editions of the student text with wrap-around text for the teacher. These guides had text close to, but separated from the activities in the student text, and support was often found in sections marked with specific headings (e.g., “Error Alert!,” “Reaching All Learners,” “About the Mathematics”). CMP supports were integrated within the directions and content for enacting activities found in the student text. For example,

“**Summarize 1.4** Question A raises the issue of connecting points on a graph—an idea introduced in Problem 1.3. Throughout this unit, students are asked to think about whether connecting the points on a graph makes sense. In this case, it does because the distance changes continuously over the 7.5-hour period. Students should understand that connecting points can help them see patterns of change more easily and make predictions or estimates about values between plotted points. However, they should also recognize that, connecting points with straight segments shows a constant rate of change between points, which may not accurately reflect the real situation” (CMP, Variables and Patterns, p.29).

This CMP example shows how the text included supports within the narrative, rather than in specially marked sections. This excerpt was intended to help teachers enact the *Summarize*, or final part, of the lesson and provided support for helping teachers understand the curricular storyline, details about the particular problem students were engaged with in the lesson, and anticipate student thinking.

Unit Details

Although roughly the same amount of content, the texts differed in terms of the number of lessons and pages in each unit. Table 12 provides the names of the units, the topics covered, and the total number of lessons and pages for each curriculum.

Table 12. Unit Details by Curricular Series

Curricular Series	Introduction to Variable		Geometric Transformations	
	Unit and Lesson, Page, & Sentence Totals	Topics	Unit and Lesson, Page, & Sentence Totals	Topics
<i>Connected Mathematics Project 2 (CMP)</i>	Variables and Patterns 14 lessons 72 pages 2,019 sentences	variables, representations (tables, graphs, words, symbols) of relationships	Kaleidoscopes, Hubcaps, and Mirrors 18 lessons 88 pages 2,249 sentences	transformations, symmetry, congruence (including a focus on triangle congruence)
<i>Math Connects (Glencoe)</i>	Algebra: Number Patterns and Functions 11 lessons 50 pages 631 sentences	variables, equations, prime factors, powers, exponents, order of operations, functions, formulas (including area)	Geometry: Polygons 12 lessons 92 pages 782 sentences	transformations, angles, quadrilaterals, polygons, tessellations, similar figures, displaying data in circle graphs
<i>Mathematics in Context (MiC)</i>	Comparing Quantities 15 lessons 81 pages 782 sentences	variables, equations, systems of equations (informal solving)	Triangles and Beyond 18 lessons 121 pages 1,258 sentences	transformations, congruence; constructions
<i>Transition Mathematics (UCSMP)</i>	Using Variables 8 lessons 62 pages 688 sentences	variables, expressions, formulas, graphing inequalities, open sentences, Pythagorean Theorem	Some Important Geometry Ideas 10 lessons 72 pages 721 sentences	transformations, symmetry, tessellations, angles and lines, parallelograms, distance, Triangle-Sum Property

Although the number of lessons, pages, and sentences varied, each covered relatively the same content for introducing variables and geometric transformations. Both of Glencoe's units included a lesson devoted to statistics content and this was not connected to the variable or transformations content. Since the main goal of this study was to examine opportunities for teacher learning and introduction to variable and geometric transformations served as a context for this examination, I took this small difference to be acceptable. Each curriculum defined and attended to the concept of variable and defined and attended to the basic transformations (i.e., translations, reflections, and rotations) in these units.

For each curriculum, the transformations units had more sentences in the teachers' guides than the variable unit. This may make sense as all transformations units had more lessons and pages in the teachers' guides than corresponding variable units. It is important to keep in mind that the number of pages in the teachers' guides is related to the number of student lessons in each of these units. In other words, if there are more student lessons in the unit with geometric transformations then it would make sense that there would be more pages in the teachers' guides for that unit and therefore, more sentences. I mention this because throughout this dissertation I will often use percentages. Therefore, knowing that the number of sentences in the units differed is important.

Other differences, such as the total number of pages and embeddedness of supports may have implications for the amount of support available in each unit. The transformations units all had more pages and sentences than the variable units. Therefore, one would expect to find more support in the transformations units. In addition, the embeddedness of supports in CMP might also impact the amount of support since each of these pages contained more space for text intended for the teacher.

In the next two chapters, I discuss the results for content and expression and I return to the issues of structure and how the differences in structure may impact the content and its expression.

CHAPTER 5: RESULTS – CONTENT SUPPORTS AND EXPRESSION

My aim in this study was to describe teachers' opportunities to learn mathematics subject matter, pedagogy, and to learn about mathematics curriculum from middle school mathematics curriculum materials. Characterizing the content and expression of the text in the teachers' guides provided me with insight into not only what was in the teachers' guide, but also how text was expressed to teachers in the written materials. Because my coding scheme generated a diverse set of numerical results (frequencies of different kinds of content supports and dimensions of their expression), I open this chapter by stating my clearest and most significant findings. The balance of the chapter provides support for each of these major claims. Although I could present the frequencies of all content supports and dimensions of their expression, this did not seem warranted since frequencies were often quite low. In addition, reporting such a diverse set of frequencies did not seem helpful in providing a description of the opportunities available in these curricula.

First and foremost, my content coding revealed very little *Rationale Guidance*, accounting for no more than 6% of the support in any curricular series. This was true for both the variables and transformations units. Second, the supports I found most often addressed *Pedagogical Content Knowledge for Practices*; these accounted for over 37% of the support in three of the four curricula (CMP, Glencoe, and UCSMP). MiC, on the other hand, focused more heavily on *Curricular Knowledge* and had an almost even split between this knowledge and *PCK for Topics* or *PCK for Practices*. *Subject Matter Content Knowledge* was attended to least across all curricula and units. When it was the focus, two curricula (CMP and MiC) located these supports at the *Unit* level of organization. Finally, although most of the content support was

located at the *Lesson* level, a substantial amount of support, particularly for CMP and MiC, was located at the *Unit* level. This may be a problem because previous research has shown that teachers are more likely to attend to supports located the *Lesson* level.

In addition, there were some noticeable content support differences between curricula and units. As previously stated, MiC attended more to *Curricular Knowledge* and *PCK for Topics*, whereas CMP, Glencoe, and UCMP focused most prominently on *PCK for Practices*. Within the variables units, CMP attended more to *Enactment Guidance for Representations*, than other curricula, providing support for teachers to help students interpret, create, and use tables, graphs, equations, and verbal representations. Glencoe attended more to *Enactment Guidance for Curricular Features* by providing information about the various features of their curriculum, including post-it notes that described what teachers would find in features such as *Tips for New Teachers* and *Error Alert!*. MiC and UCSMP focused more on *Enactment Guidance for Curricular Overviews* by providing descriptions of what each of their lessons would entail, such as

“Students find out what triangle, when rotated, can make an octagon and what triangle can make a hexagon. They investigate whether translation, rotation, or reflection can be used to make a 10-gon. Then they investigate whether or not they would get the same result if they started with a triangle that is not isosceles.(MiC, *Triangles and Beyond*, p. 50T)

In their transformations units, CMP and UCSMP attended more to *Subject Matter Knowledge* than other curricula, whereas UCMSP attended more to *Enactment Guidance for Terminology*. Although all curricula contained a large percentage of support devoted to *Enactment Guidance for Questions (suggested answers provided)*, this support was more

prominent in CMP. In addition, CMP focused more on *Enactment Guidance for Proof and Reasoning*. As they did in their variables unit, Glencoe devoted much more support to *Enactment Guidance for Engaging Students in Potentially Problematic Experiences* in their transformations units, and MiC included a higher percentage of *Enactment Guidance for Curricular Overviews*.

Differences in content supports were also noticeable between units for all or most curricula. The variables units frequently included *Enactment Guidance for Representations*, whereas the transformations units included *Enactment Guidance for Tools*, and *Justification, Reasoning and Proof Approaches*. Also, the Glencoe variables unit frequently included *Enactment Guidance for Curricular Features*; however this was not true for their transformations unit.

With respect to the linguistic expression, I first found that the first person plural pronoun “we” was used only by CMP and UCSMP, whereas the second-person pronoun “you” was used in all curricular series and was by far the most common pronoun. The use of these pronouns is important since they involve curriculum authors and teachers in the text. Second, when I looked more carefully at the use of “you,” I found that the most common “you”-forms were imperatives (commands to do something), a form in which the reader is implicitly implicated, and “you” + modal verb, a form in which the modal verb functions to provide the reader some freedom to choose what to do. Imperatives were much more common than all other “you”-forms, and at least 7% of the sentences in each curricular series were in the imperative form. Finally, a variety of modal verbs were used, but modal verbs “may” and “might,” verbs that carry a low level of obligation (Halliday, 1985), were the most common across all units and curricula. Modal verbs that carry a low level of obligation provide teachers with more freedom to choose to do (or not to do) what curriculum authors stated.

There were noticeable differences in the use of pronouns and “you”-forms between curricula, but not between units. In addition to the appearance of “we” only in CMP and UCSMP, Glencoe used imperatives much more frequently in their transformations unit than other curricula. Although the use of imperatives was more common in the transformations units for all curricula except for CMP, this difference was small. Finally, although the most common modals were similar across curriculum materials with “may” and “might” being the most pervasive, CMP and MiC, were the only curricula to use the high modal verb “need.”

In the next sections I provide descriptive statistics to substantiate these results. I begin first with the results related to content, including the knowledge addressed, the types of supports and the location of these supports. I then turn to issues of expression. I present the results related to pronouns and then “you”-forms. I discuss any differences I found between curricula or units where appropriate.

Content Supports

In this section I describe the findings related to content supports. I begin by describing the types of guidance provided, the knowledge addressed, and where the content support was located within the teachers’ guides. I then discuss differences in content supports between curricula and units.

Types of Content Support. For all curricula and units, content supports more often provided *Enactment Guidance*, or described what to teach and how to teach it. There was very little *Rationale Guidance*. Table 13 shows the percentages of each type of guidance. Since I did not code *Subject Matter Content Knowledge* as either *Enactment* or *Rationale Guidance* these percentages exclude these subject matter knowledge codes.

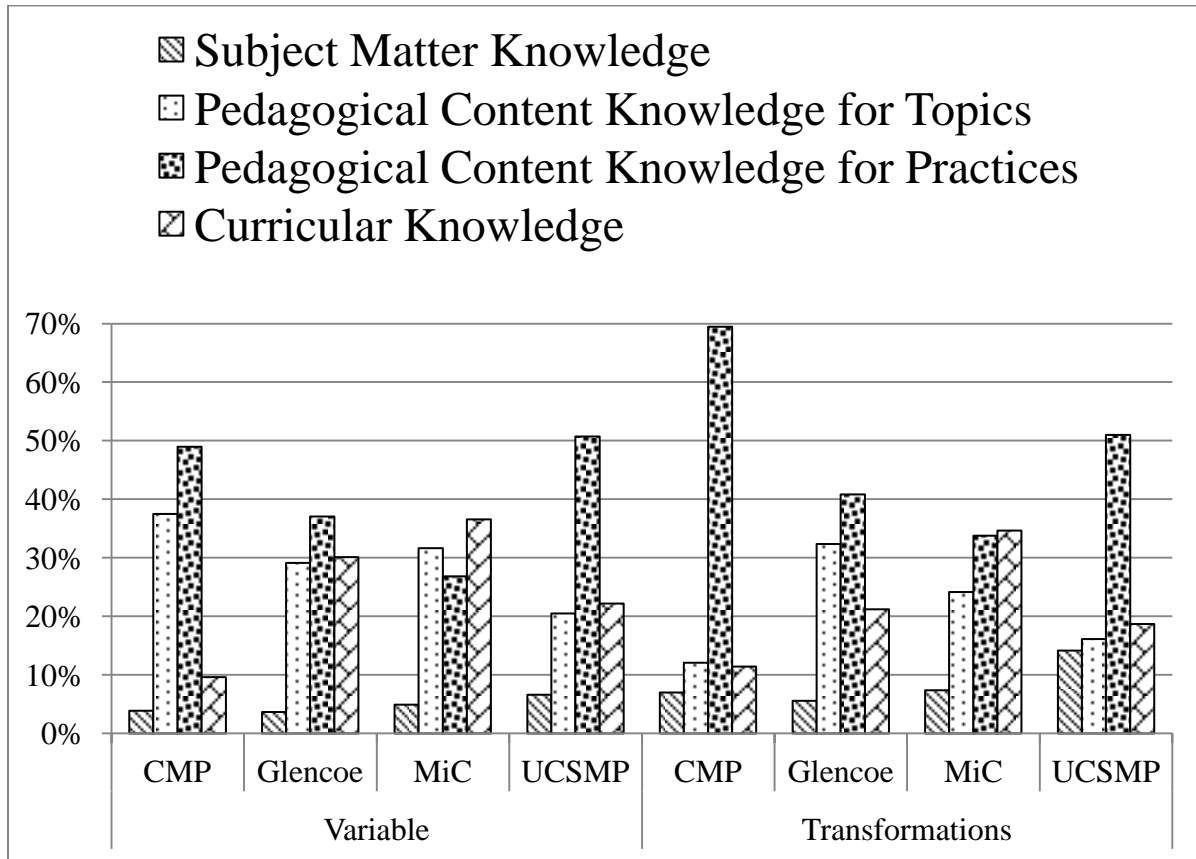
Table 13. Percentages of Enactment and Rationale Guidance by Unit and Curriculum

Curricula	Variable		Transformations	
	Enactment Guidance	Rationale Guidance	Enactment Guidance	Rationale Guidance
CMP	97	3	97	3
Glencoe	99	1	99	1
MiC	94	6	98	2
UCSMP	94	6	97	3

Enactment Guidance was much more prevalent than *Rationale Guidance*, which occurred for no more than 6% of any curriculum. This result is consistent with research in science education (Beyer et. al, 2009), and even more startling because 20% of supports in their Biology textbooks were found to provided *Rationale Guidance*. CMP and Glencoe were consistent in their distribution across the two units. MiC and UCSMP included a higher percentage of *Rationale Guidance* in their variable unit than in their transformations units, however this difference was modest (4% and 3% respectively). In all four curricula most of the *Rationale Guidance* occurred in the *PCK for Topics* category. The most frequent specific supports were *Rationale Guidance for Experiences*, which was common for both the variable and the transformations units, *Representations* which was more prevalent in the variable units, and *Tools*, which was more prevalent in the transformations units. Also, common in other knowledge categories was *Rationale Guidance for Questions, Philosophy, and Storyline*.

Frequency of Supports by Knowledge Type. *Subject Matter Knowledge, Pedagogical Content Knowledge for Topics and Practices, and Curricular Knowledge* were addressed by all four curricula in both units. Figure 5 shows the percentages of support for the four types of knowledge for both the variables and transformations units in each curriculum.

Figure 5. Percentages of Content Supports by Unit and Curriculum



The most prevalent type of content support for three of four curricula was *PCK for Practices*, accounting for at least 37% of all support in CMP, Glencoe, and UCSMP. These supports included mostly those designed to help teachers engage students in mathematical practices such as questioning, reasoning and proving, and using and terminology. MiC, on the other hand, split its attention evenly between *PCK for Practices* or *PCK for Topics* and *Curricular Knowledge*. The most prevalent *Curricular Knowledge* supports included those related to developing an understanding of the curricular features, and storyline, and provided a curricular overview, or description of what a particular lesson was about, which accounted for 21% of all content supports in MiC for both units. The least prevalent content support across all materials was in the domain of *Subject Matter Knowledge*, which accounted for no more than 14% for any curriculum, although in UCSMP's transformations unit support for *Subject Matter*

Knowledge was quite close to the percentage of support for *PCK for Topics*. It should be noted that this category had only one element (*Content Knowledge*) and therefore, it might make sense that it accounted for less support than the other categories which included far more elements. In future sections I will discuss the frequency of particular types of support (including *Content Knowledge*).

Location of Content Supports. Most content supports were located at the *Lesson Level*. However, particularly for CMP and MiC, a significant amount of support, sometimes nearly 50% of the total content support, was located at the *Unit* level. Table 14 shows the percentages of supports at the *Unit*, *Section*, and *Lesson* levels for each curriculum.

Table 14. Percentages of Content Supports by Location

Curricula	Introduction to Variable			Geometric Transformations		
	Unit	Section	Lesson	Unit	Section	Lesson
CMP	14	28	58	48	5	47
Glencoe	15	N/A	85	12	N/A	88
MiC	24	14	62	16	11	72
UCSMP	19	N/A	81	18	N/A	82

For the variables units each curriculum included most of their supports at the *Lesson Level*. For the transformations units three of four curricula included a majority of their support at the *Lesson Level*, accounting for more than 70% of the support in Glencoe, MiC, and UCSMP. CMP had a relatively even split between support that appeared at the *Unit* and *Lesson Levels*, with 48% and 47% respectively. These results for CMP’s transformations unit are important because research indicates that the location of educative supports impacts whether teachers use supports. For example, studies that investigated teachers’ use of educative curriculum (Schneider, Krajcik, & Blumenfeld, 2005; Schneider & Krajcik, 2002; Schneider, 2006) indicated that teachers used and learned from the support located at the lesson-level, rather than support located in other sections of the textbook and other books.

A deeper look into the location of supports indicated that supports for *Curricular Knowledge* often appeared at the *Unit Level*. Both CMP and MiC units and the UCSMP transformation unit all contained more *Curricular Knowledge* support at the *Unit Level*. Since many of these supports, such as *Rationale for Curricular Philosophy and Enactment* and *Rationale Guidance for the Curricular Storyline* involved content from the entire unit, finding this support at the *Unit Level* made sense. Supports for other knowledge types always appeared more often at the *Lesson Level*, except in CMP where in addition to *Curricular Knowledge*, *Subject Matter Knowledge* appeared more often at the *Unit Level* and *PCK for Practices* either appeared more often at the *Unit Level* or was split equally between the *Section* and *Lesson Levels*. It is important to note that support may also appear at the curriculum level (e.g., Enactment guides, teachers' guides front matter), however, since this study focused on two units in each curriculum, I do not discuss the support at the curricular level, but acknowledge that this is another location, further removed from the lesson level, for potential teacher support.

Some differences within units across curricula. Although, generally, the most prevalent content supports were the same for the units across curriculum materials, with *Enactment Guidance for Using Questions (suggested answers provided)* one of the most common supports, there were also some noticeable differences. First, I describe the differences within units, starting with the variables units. I then describe differences in supports between the two units.

Differences in variables units by curriculum. CMP included a much higher percentage of support for *Enactment Guidance for Representations* in their variables unit than other curriculum materials, accounting for 29% of support. In this CMP unit, one of the goals was to "Observe relationships between two quantitative variables as shown in a table, graph, or equation

and describe how the relationship can be seen in each of the other forms of representation" (CMP, Variables and Patterns, p. 2). Glencoe, on the other hand, focused more than other materials on *Enactment Guidance for Potentially Problematic Experiences* and *Curricular Features* in their variable unit. Glencoe included at the beginning of each lesson a section entitled "Options for Differentiated Instruction." Although, this section included alternative options for students at or above grade level, most of the supports (n = 73) were geared towards helping teachers engage struggling students or students with special needs in experiences that might be problematic for them. Glencoe also included more *Enactment Guidance for Curricular Features* in their variable unit than other curriculum materials. I discuss Glencoe's' inclusion of this support more in the section where I discuss differences across units. MiC, and to some extent UCSMP, included substantial percentages of *Enactment Guidance for Curricular Overviews*, accounting for 21% and 14%, respectively, of their total support.

Differences in transformations units by curriculum. In the transformations unit, there were differences in the attention to *Subject Matter Knowledge* in the curriculum materials. UCSMP and CMP both devoted over 17% of support to *Subject Matter Knowledge*. In addition, UCSMP devoted 15% to *Enactment Guidance for Terminology*, more than other curriculum materials, and often this support appeared on the same page as support for *Subject Matter Knowledge*. Of their 77 instances of *Enactment Guidance for Terminology*, 46 appeared on the same page and 23 on the page before or after *Subject Matter Knowledge* support. The close proximity of these supports suggest that UCSMP authors may see the use of appropriate terminology as related to learning subject matter. CMP contained a larger percentage of *Enactment Guidance for Questions (suggested answers not provided)* and *Enactment Guidance for Proof and Reasoning* than other curriculum materials. CMP included more suggested

questions and instances in which teachers were asked to have their students justify and explain their reasoning and begin to establish the notion of proof more than other curriculum materials. Finally, similar to the variables unit, Glencoe devoted much more support in their transformations units than other curricula, to *Enactment Guidance for Engaging Students in Potentially Problematic Experiences* than other curricula, accounting for 17% of its support, and MiC included a higher percentage of *Enactment Guidance for Curricular Overviews*, accounting for 21% of its total support.

Some overall differences across units. For the variables units, 13 content supports occurred more than 10 times in at least three units, whereas 14 content supports occurred frequently in at least three transformations units. The variables units frequently included *Enactment Guidance for Representations*; however this was not frequently included in the transformation unit for any curriculum. Instead, transformations units included *Enactment Guidance for Tools*, and *Justification, Reasoning and Proof Approaches*. Although learning to use representations, tools and engage in justification, proof, and reasoning are integral to the study of mathematics and require support for teachers in any content domain, it may seem appropriate that these supports were more prevalent in their respective units. In the variables units, particularly CMP's, which devoted 29% of its support to guidance for using representations, there was a larger focus than in the transformations unit on using different representations (i.e., situation, table, graph, equation). This is not to say that *Enactment Guidance for Representations* was completely absent from transformations units. In these units teachers were given guidance for using or having students use representations such as sketches or three-dimensional representations. The transformations units involved constructing and transforming geometric shapes and investigating symmetry, therefore incurring the use of tools

such as rulers and compasses. In addition, these units, particularly CMP and UCSMP introduced the ideas of using transformations to prove relationships such as similarity and congruence and therefore provided more support for teachers to engage their students in reasoning and proof.

Another difference between units was specific to Glencoe. As I mentioned earlier, Glencoe's variable unit had a high percentage of *Enactment Guidance for Curricular Features*; however this was not true for their transformations unit. A possible explanation for this is that the variable unit was the first chapter in the teacher's guide, whereas transformations was covered at the end of the book in chapter 10. The *Enactment Guidance for Curricular Features* supports, provided in the form of post-it notes, alerted teachers to the features of the curriculum materials, such as the following: "Web addresses, or URLs, are provided to online assets such as **Personal Tutor, Extra Examples, Self-Check Quizzes, and Concepts in Motion**" (Glencoe, Algebra: Number Patterns and Functions, p. 37). A cursory examination of other units indicated that these post-it note supports located in chapter 1 were absent in other units, not just the transformations unit. The authors may expect teachers to remember the information provided in Chapter 1 while reading other chapters.

Aspects of Voice

In this section, I describe the results regarding aspects of voice. I begin with findings related to personal pronoun use in the four curricula, particularly focusing on the use of "we" and "you." I begin with personal pronouns because the use of these words is one of the most obvious ways to see how curriculum authors are speaking to teachers, as personal pronouns explicitly express interpersonal relationships. I follow this with a more detailed look at how the pronoun "you" is used by discussing "you"-forms, specifically the use of imperatives and modal verbs, as

these were the most prevalent forms and also illuminate the roles that are being constructed between curriculum authors and teachers.

Personal Pronouns. The first linguistic form I examined was personal pronouns because these are used to express relationships between authors of a text and their readers. First-person pronouns such as “I” and “we” indicate the authors; direct involvement in the activity of the text (Morgan, 1996). According to Herbel-Eisenmann (2007) the use of the pronoun “we” could indicate the authors speaking with the authority of the discipline of mathematics, or it may be used to involve the readers in the mathematics. Researchers such as Pimm (1987) and Rowland (1999) have alluded to the ideological aspects embedded in the use of we, such as how using we assumes the readers agreement with the author. The second-person pronoun “you” directly involves the reader, and is of particular interest in my study because the readers of curriculum materials are the teachers using the materials.

In addition, the general absence of first-person pronouns, according to Morgan (1998), not only affects the picture of the nature of mathematics, but also has the potential to distance the author from the reader.

Personal pronouns were used by all four curricular series. Table 15 shows the frequencies of each of these pronouns by unit and curriculum.

Table 15. Frequencies of Personal Pronouns by Unit and Curriculum

Curricula	Variables			Transformations		
	I	We	You	I	We	You
CMP	0	38	52	0	102	42
Glencoe	0	0	19	0	0	25
MiC	0	0	61	0	3	129
UCSMP	0	20	33	0	17	38

As expected, I did not find any instances of the pronoun "I." As these texts were not written by one person, this makes sense. Unlike the pronoun "I" which indicates a single voice, the pronoun "we" indicates multiple voices. This pronoun seemed more likely to be found; however, I only found "we" in CMP and UCSMP. "We" was completely absent from Glencoe and almost completely absent from MiC, which had only three instances in the transformations unit; two of which occurred within sample solutions.

By far the most common pronoun was the second-person pronoun "you," which occurred at least 19 times in each unit and was very frequent in MiC's transformations unit which included 129 instances of "you." As a reminder, these teachers' guides did not contain the same number of sentences. It would make sense to have more pronouns in teachers' guides where there were more sentences.

Locations of personal pronouns. Generally, the personal pronouns "we" and "you" were used in a variety of sections. The pronoun "we" was used by CMP and UCMP within the narrative sections that described how to enact lessons, such as "Notes on Lesson" in UCSMP, and in CMP within the *Launch*, *Explore*, and *Summarize* sections, or within sections devoted most prominently to mathematical content support such as "Background" in UCSMP and "Mathematics of the Unit" in CMP. It was common for "we" to be used repeatedly in parts of the text where explicit steps were given for solving a problem, particularly in CMP. In the next section, I describe the uses and locations of "we" in more detail.

Where the pronoun "we" was found frequently in sections in which mathematical subject matter was presented in CMP and UCSMP, the pronoun "you" was not very frequent. CMP did use the pronoun "you" in sections devoted solely to the mathematical subject matter, such as *Mathematics in this Unit*, but more often in the sections for the teacher that were devoted to the

enacting of individual lessons (called Problems), such as *Launch*, and *Summarize*, and in the *Possible Answers to the Mathematical Reflections* which occurred at the end of the unit. This is not to say that “you” was not used when mathematical subject matter was discussed, but it was ~~just~~ not frequent in sections devoted solely to discussions of subject matter. Similarly, for the other curriculum materials “you” was used most frequently in sections that were devoted to helping teachers enact lessons.

Uses of we. “We” can be a grammatical device for bringing the authors directly into the text. However, Pimm (1987) pointed out that often, particularly in classroom conversations, the referent for “we” is not clear or explicit. The question he posed is “To what *community* is the teacher appealing when using the word *we*?” (p. 65). The same question is appropriate when examining written text. When the referent is unclear, “we” may be used as a way of asserting one’s authority as a speaker for the discipline of mathematics (Herbel-Eisenmann, 2007) or including oneself in the mathematics classroom community. In either case, “we” may “have to do with both power and dominance, and attempts to enrol the (at least tacit) acquiescence of the reader in what is being expressed” (Pimm, 1987, p. 68). These different uses and functions make it necessary to look more closely at the instances of “we.” Table 16 shows the frequencies of “we” in which I determined that the referent was clear and those for which the referent was unclear in the curriculum materials that contained this personal pronoun, CMP and UCSMP. In the instances in which the referent was clear, “we” was used to refer to the group of authors of the curriculum materials, or at least the authors of that particular teacher’s guide (e.g., “We start with tables because many students are more comfortable working with tables than with graphs” (CMP, Variables and patterns, p. 87).). Unclear instances were uses of “we” for which I could not determine the referent. These instances went beyond the particular curriculum materials and

decisions made solely by the authors of these teachers' guides, such as marking mathematical definitions or working through mathematical deductions. It was difficult to tell who the "we" was in these cases.

Table 16. Uses of "We" by Unit and Curriculum

Curricula	Function	Variable	Transformations
CMP	Clear referent	9	14
	Unclear referent	25	89
UCSMP	Clear referent	20	10
	Unclear referent	0	7

UCSMP contained more instances of clear than unclear referents. There were no unclear referents in their variable unit, and fewer unclear referents in their transformations unit than clear referents. The clear instances included the authors directly in the text and served three functions: a) as a way of making suggestions or recommendations for teachers (e.g., "We recommend that...", "We suggest that...", "We encourage..."), b) as a way of stating assumptions or choices they made without rationale (e.g., "We introduce...", "We ask students...", "We assume..."), and c) as a way of stating assumptions or explaining choices with rationale ("We start with tables because...", "We have purposely chosen...because,"). CMP often chose to mark these instances with text that indicated that they were providing special information for the teachers (e.g., "Note," "For the teacher").

I also found instances in which the referent for "we" was unclear. These types were most often located at the *Lesson* or *Unit Level* when descriptions of solutions or mathematical subject matter content were provided. The instances were of two grammatical forms, the first being instances in which "we" appeared in a statement and the second in which "we" appeared in a question. Examples of the former, included phrases such as, "We say 'non-trivial'..." (UCSMP, *Some Important Geometry Ideas*, p. 375) or "Thus we know that the gap triangle is congruent to the original triangle?" (CMP, *Kaleidoscopes, Hubcaps, and Mirrors*, p. 105). Examples of the

latter included questions such as, "Can we be sure that last transformation exactly filled the gap?" (CMP, Kaleidoscopes, Hubcaps, and Mirrors, p. 105) or "Which variable should we put on each axis so we can best see the "story" the graph tells?" (CMP, Variables and Patterns, p. 21). I found questions such as these only in CMP. Although these appeared throughout both CMP units in various locations, I saw a combination of the two forms, "we" in questions and statements, more often in the excerpts in which there were explicit steps given for solving a problem.

"You"-Forms. "You"-forms used in the teachers' guides indicated how curriculum authors included teachers in the construction of their mathematics teaching². "You" was used in two ways, implicitly in the imperative form and explicitly with the personal pronoun "you."

Table 17 shows the frequencies of explicit and implicit "you"-forms.

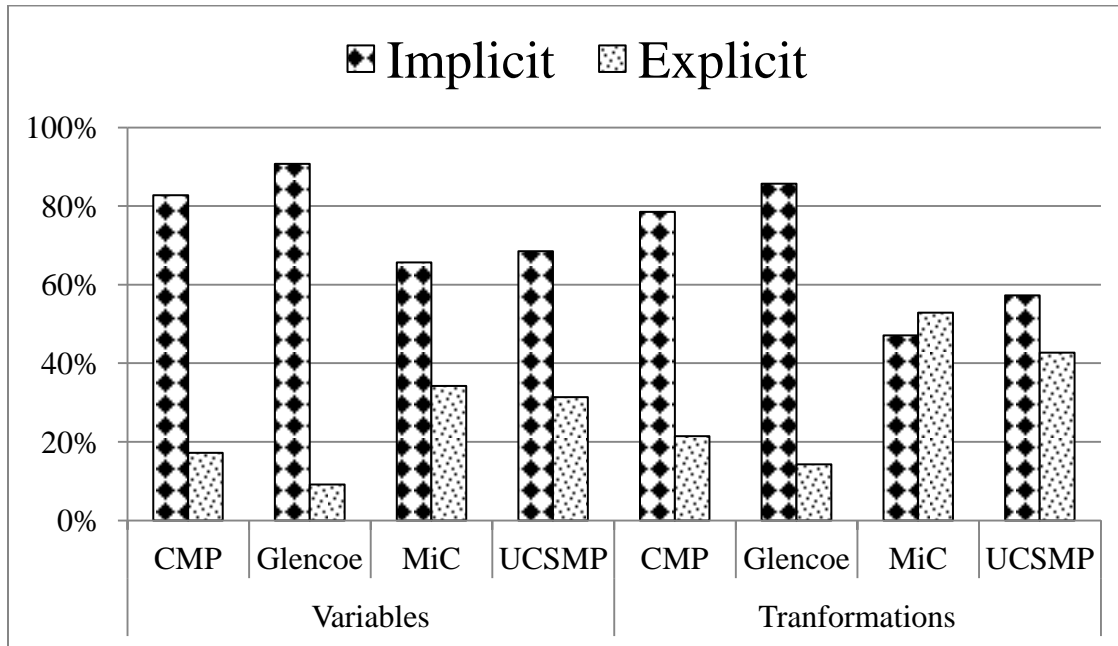
Table 17. Frequencies of Implicit and Explicit Use of "You" by Unit and Curriculum

Curricula	Variables		Transformations	
	Implicit (imperative)	Explicit (other "you"-forms)	Implicit (imperative)	Explicit (other "you"-forms)
CMP	250	52	154	42
Glencoe	188	19	150	25
MiC	117	61	115	129
UCSMP	72	33	51	38

Each curriculum used the imperative form much more frequently than explicitly using the personal pronoun "you" when addressing the teacher, with the exception of MiC. See Figure 6 for the percentage of each form by unit and curriculum.

² As a reminder, I did not analyze student pages of the textbook, even when these pages were found in the teachers' guides. Therefore, when I report details about particular words used, these details pertain only to the use of these words in the teachers' guides. These words may also have been used on student pages of the text. In addition, I did not include any text that was a copy of the student textbook within the narrative for the teacher.

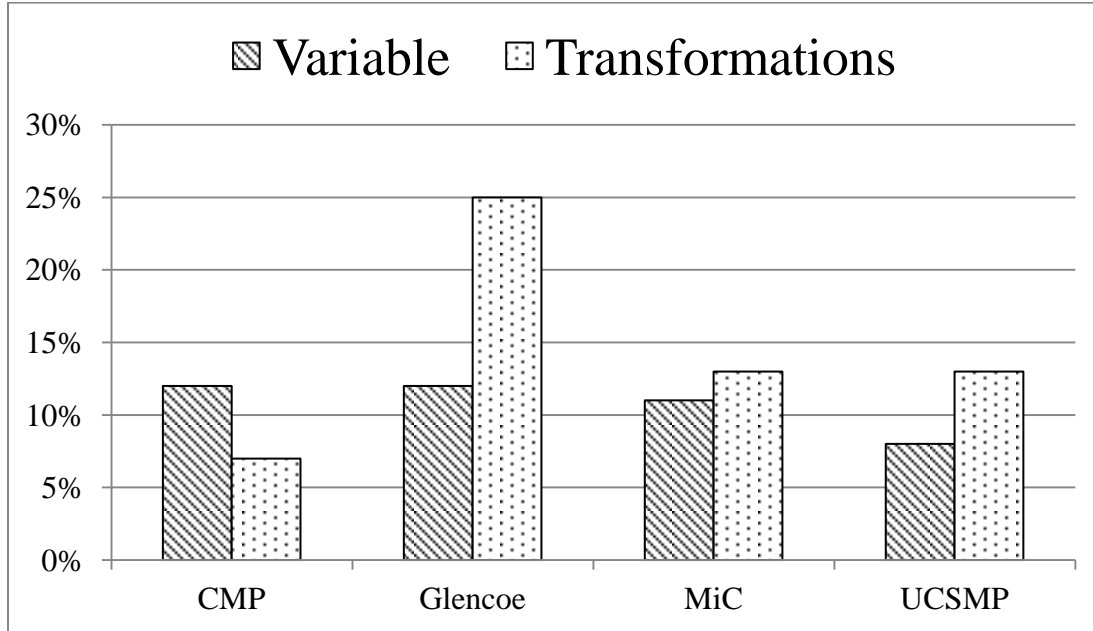
Figure 6. Percentages of Implicit and Explicit "You"-forms



In all units and curricula more than 57% of the uses of "you" were expressed as imperatives, except for MiC's transformations unit which had only 47% of "you"-forms expressed as imperatives. In their transformations unit when the teacher was addressed more often the word "you" was explicitly used. Glencoe had over 91% and 86% in the imperative form in their variables and transformations units, respectively. In the next two sections I describe the use of imperatives and explicit "you"-forms in more detail.

Imperative Form. An imperative expresses a command. A command positions the addressee (i.e., the controlled) as the one to carry out the service and the speaker (i.e., the controller) as the authority in the situation. Grammatically, this form does not require a subject, but the pronoun "you" is implied. Figure 7 indicates the percentages of total sentences that were in the imperative form.

Figure 7. Percentage of Sentences in the Imperative Form by Unit and Curriculum



Each curriculum had at least 7% of the sentences in each unit in the imperative form. The Glencoe transformations unit had by far the greatest proportion of sentences in the imperative form with 25%. Each curriculum had a greater percentage of sentences in the imperative form in their transformations unit, except in CMP where 5% more of the sentences in the variable unit were in the imperative form than in the transformation unit.

Types of imperatives. The types of imperatives that were used included those that directed teachers to perform certain actions themselves (e.g., "Ask...", "Discuss...", "Remind..."), and those that directed students through teachers (e.g., "Have students share...", "Have students create...", "Have students discuss...") or to ensure that students engaged in particular actions or processes (e.g., "Make sure students understand...", "Be sure students can explain..."). Table 18 illustrates the percentages of imperatives that directed teachers and students through teachers by unit and curriculum.

Table 18. Percentages of Imperatives Directing Teachers and Students by Unit and Curriculum

Curricula	Variables		Transformations	
	Directed teacher	Directed students through the teacher	Directed teacher	Directed students through the teacher
CMP	74	26	88	12
Glencoe	71	29	76	24
MiC	49	51	82	18
UCSMP	66	34	86	14

Only one unit, MiC's variable unit, contained more imperatives that directed students through the teacher than directed the teacher to perform actions themselves. It is necessary to look more closely at the specific imperatives used to see exactly what teachers were being commanded to do. Table 19 shows the frequencies of the common imperatives (those with a frequency of at least five), used in each of the units by curriculum. See Appendix B for the list of all imperatives.

Table 19. Commonly Used Imperatives by Unit and Curriculum

Curricula	Variable		Transformations	
	Imperative	Frequency	Imperative	Frequency
CMP	Ask	46	Ask	30
	Have students share	15	Encourage	11
	Use	15	Have students work	9
	Encourage	12	Use	8
	Discuss	10	Have students share	7
	Tell	10	Remind	7
	Let	9	Call	6
	Explain	7	Discuss	5
	Remind	6	Arrange	5
	Choose	5	Help	5
Glencoe	Use	42	Use	33
	Ask	29	Ask	17
	Remind	11	Remind	12
	Tell	9	Have students write	10
	Create	8	Make	9
	Write	6	Encourage	7
	Explain	5	Check	5

Table 19 (cont'd)

	Suggest	5	Draw	5
			Point out	5
MiC	Ask	16	Ask	26
	Have students share	10	Discuss	23
	Point out	8	Encourage	12
	Encourage	7	Remind	6
	Provide	7	Add	6
	Be sure students understand	6	Have students discuss	5
	Discuss	5		
UCSMP	Ask	15	Ask	13
	Give	7	Point out	7
			Emphasize	5

CMP, Glencoe, and MiC had a greater variety of imperatives that appeared at least five times in both units than UCMP. UCSMP's use is less varied as they used only two or three different imperatives (i.e., "Ask," "Give," "Point out," "Emphasize") more than five times. Most of the commonly used imperatives directed teachers to perform certain actions; however, all curricula, except UCMP, included imperatives that directed students through teachers at least five times in one of their units. I describe these two types of imperatives in the next two sections.

Imperatives that directed teachers. According to Rotmann (1988), *exclusive imperatives*, such as *tell*, *remind*, and *give*, position the reader as a "scribbler," or someone who must perform some action, whereas *inclusive imperatives*, such as *consider*, *think*, and *explain*, address the reader as a "thinker." The latter types of imperatives establish a "commonality between speaker and hearer" (p. 9). Although one is not positioned as only a "scribbler" or a "thinker," by choosing certain words, one can foreground these roles. Table 20 shows the percentage of inclusive and exclusive imperatives by unit and curriculum.

Table 20. Percentages of Inclusive and Exclusive Common Imperatives by Unit and Curriculum

Curricula	Variable		Transformations	
	Inclusive	Exclusive	Inclusive	Exclusive
CMP	28	72	27	73
Glencoe	31	69	17	83
MiC	43	57	48	52
UCSMP	0	100	0	100

The commonly used imperatives were most frequently *exclusive imperatives* rather than *inclusive imperatives*. The most common *exclusive imperative* across all curriculum materials was "Ask," except for in the Glencoe transformations unit in which "Use" was the most common, followed by "Ask." These two imperatives accounted for at least 35% of the commonly used imperatives in each curriculum. MiC and UCMP did not frequently use the imperative "Use," instead MiC's second and third most common imperatives were "Discuss" and "Encourage" whereas UCMP's included "Give," "Point Out" and "Emphasize."

Imperatives that directed students through teachers. The most common imperatives that directed students through the teacher included "Have students share," "Have students work," "Have students write," "Have students discuss," and "Be sure students understand." Although imperatives of this type still directed teachers, these imperatives positioned the teacher differently than those that directly commanded teachers. Rather than being positioned as controlled, in these instances, teachers were positioned as "instigators" (M. Schleppegrell, personal communication, October 28, 2011). This grammatical form assumes that a teacher has power over his or her students to make students do what he or she wants. Yet, since these instances were in the imperative form, and therefore commanded teachers to direct students to do certain things, a more apt interpretation would be that this positioned the teacher as an instigator or agent to make students do *what the curriculum authors want*. So, while these instances may have placed teachers in a more powerful position than other imperatives, this positioning was

quite subtle and narrow, as teacher may have power in these instances, but only over their students (not themselves). There was no indication in these instances that teachers had a choice as to what to make students do or even whether to make students perform these actions or not.

Explicit "You"- Forms. I now turn to describe the explicit "you"-forms I found in each curriculum. Instances in which "you" was explicitly used fell into the following four forms: a) verb + "you", such as "... tell you" or "...give you", b) modal verb + "you," such as "may need you..." or "might want you..." c) "you" + verb, such as "You have labeled..." or "You fill..." and d) "You" + modal verb, such as "You might want..." or "You should explain..." See Tables 21 and 22 for the frequencies of these you-forms in each unit by curriculum.

Table 21. Frequencies of "You"-forms in the Variable Units by Curriculum

Curricula	You + verb	You + modal verb	Verb + you	Modal verb + you
CMP	6	55	2	0
Glencoe	0	19	0	0
MiC	5	57	2	1
UCSMP	10	23	0	0

Table 22. Frequencies of "You"-forms in the Transformations Units by Curriculum

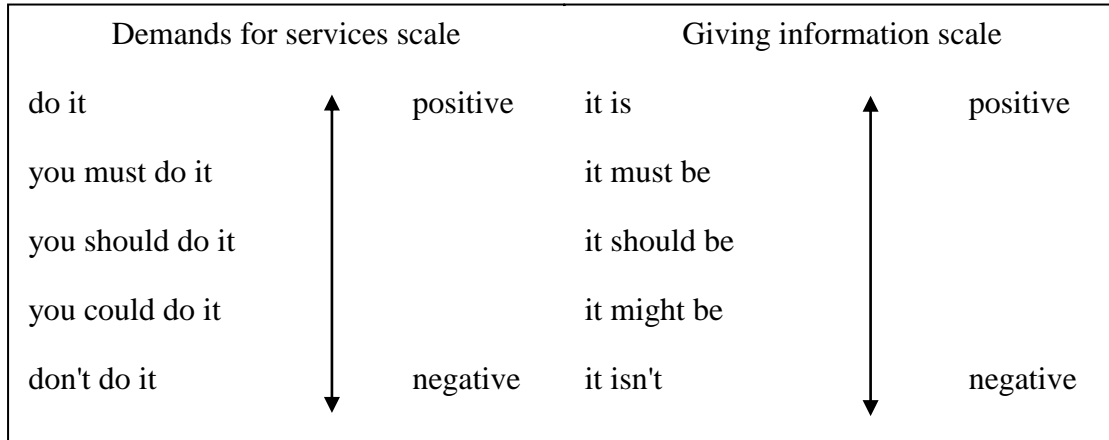
Curricula	You + verb	You + modal verb	Verb + you	Modal verb + you
CMP	2	47	0	0
Glencoe	3	23	0	0
MiC	3	126	0	0
UCSMP	3	35	1	0

The most pervasive "you"-form was "you" + modal verb. Other forms were quite uncommon across both units in all curriculum materials, although compared to other forms UCSMP did have a significant number of "you" +verb forms in their variable unit.

"You"+modal verb. Modal verbs are grammatical tools that allow the authors to provide readers with more freedom . Different modal verbs produce different levels of obligation or

probability. According to Martin & Rose (2008), modal verbs can be placed on a scale that describes "how obliged" you are to act or "how probable" a statement is. See Figure 8.

Figure 8. Scales for Negotiating Demands for Services and Giving Information (Martin & Rose, 2008, p. 53)



Demands for services can be negotiated on the left scale. This describes how likely one is to act. On the poles "do it" and "don't do it" indicate no alternatives, or complete obligation to do something or not; however, as one moves away from the poles there are varying levels of obligation. The same is true on the information scale. According to Halliday (1985), these varying levels of obligation or truth can be described as "low", "median", or "high." See Table 23 for the classification of modal operators. For example, "You must explain" is more obligatory than "You should explain," which is more obligatory than "You may explain," which is more obligatory than the imperative "Explain."

Table 23. Finite Verbal Operators (Halliday, 1985, p. 116)

Polarity	Levels of Obligations		
	Low	Median	High
positive	can, may, could, might	will, would, should, is/was to	must, ought to, need, has/had to
negative	needn't, doesn't/didn't + need to, have to	won't, wouldn't shouldn't, (isn't/wasn't to)	mustn't, oughtn't to, can't, couldn't (mayn't, mightn't, hasn't/hadn't to)

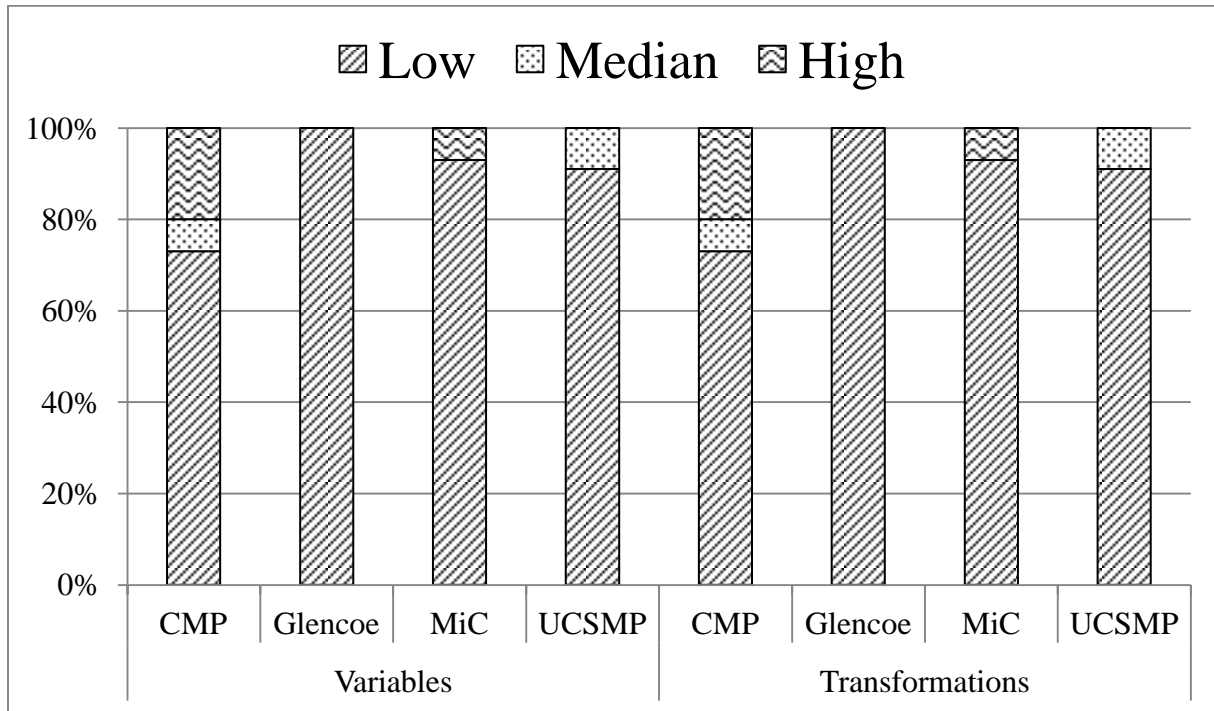
Although each curriculum used modal verbs, there was not much variety. Most of the modal verbs fell into Halliday’s (1985) low category. Table 24 indicates the frequencies of the modal verbs that appeared in each unit by curriculum.

Table 24. Frequencies of Modal Verbs by Unit and Curriculum

Units	Curricula	can	could	may	might	Should	will	need	Total
Variables	CMP	9	0	25	6	1	3	11	55
	Glencoe	2	0	16	1	0	0	0	19
	MiC	3	0	44	6	0	0	4	57
	UCSMP	1	1	4	15	2	0	0	23
	Total	15	1	89	28	3	3	15	154
Transformations	CMP	3	2	15	19	0	1	7	47
	Glencoe	3	0	18	2	0	0	0	23
	MiC	6	16	83	20	0	1	0	126
	UCSMP	2	2	4	19	5	3	0	35
	Total	14	20	120	60	5	5	7	231

The most pervasive modal verbs used by each curriculum were the modal verbs "may" and "might." "May" was always used more frequently than "might," except in UCMP, in which "might" was over three times as frequent as "may" in both units and in the CMP transformations unit, in which it was only slightly more frequent. Together these two modal verbs accounted for at least 56% of the modal verbs used in each curriculum, and for Glencoe and MiC accounted for over 80%. CMP had higher proportions of the modal “need” than other curricula, whereas UCMP had higher proportions of “should.” In addition to the modals “may” and “might” other low modals such as “can” and “could” and high modals, such as “should” and “need” were used, but in lower frequencies. Figure 9 indicates the percentage of low, median, and high modal verbs used in each unit by each curriculum.

Figure 9. Percentages of "Low," "Median," and "High" Modal Verbs Used



In each curriculum low modal verbs (i.e., can, could, may, might), or verbs that required the lowest level of obligation or probability of occurrence, were most common. In Glencoe, these were the only modals used. All other curriculum materials included median modal verbs (i.e., will, should), whereas CMP was the only curriculum to use a high modal verb (i.e., need). Since modal verbs were quite common, it seemed necessary to examine the verbs associated with these modals operators, particularly for the most common modals, “may” and “might.” The most commonly used verbs with the modal “may” were “want” and “wish,” (e.g., “You may want students to explain...,” “You may want to discuss...,” “You may wish to ask...”), accounting for at least 25% of all verbs used. See Table 25. “Might,” on the other hand, was most often used with “ask,” (e.g., “You might ask them to ...,” “You might ask some questions such as...,” “You might ask if a triangle...”), however, not nearly as often as “may” was paired with “want” or “wish.”

Table 25. Percentages of the Verbs "Want" and "Wish" Used with the Modal Verb "May" by Unit and Curriculum

Curricula	Variables		Transformations	
	want	wish	want	wish
CMP	26	29	41	0
Glencoe	0	88	5	70
MiC	24	0	42	7
UCSMP	21	32	9	9

I discuss possible issues related to using the words "want" and "wish" with modal verbs in the discussion.

CHAPTER 6: DISCUSSION

In this chapter, I return to the focus of my study, teachers' opportunities to learn from middle school mathematics curriculum materials. Specifically, I address my research questions. As a reminder, I aimed to answer the following questions:

1. What is the relative frequency of educative content supports in middle school mathematics curriculum materials for teachers' *Subject Matter Content Knowledge*, *Pedagogical Content Knowledge for Mathematics Topics*, *Pedagogical Content Knowledge for Mathematical Practices*, and *Mathematics Curricular Knowledge* and where are these supports located?
2. How do middle school mathematics curriculum materials speak to teachers (i.e., what are some of the language choices they make) through the written text in the teachers' guides?
3. How does opportunity to learn (content and aspects of voice) differ for introduction to variable and geometric transformations?

In the sections that follow, I address each of these research questions, followed by what these results indicated about teachers' opportunities to learn from middle school curriculum materials. I follow this with a discussion of the contributions of my study to the field and implications for curriculum development, teacher education, and research. I then discuss limitations of the current study and finally, I end with future directions for research.

Relative Frequency of Supports and their Location

In this section I remind the reader of the overall relative frequency of content supports and discuss possible reasons for these frequencies. In addition, I discuss differences in the four curricular series that my analysis illuminated.

As described in my results section, the most frequent content supports were *Pedagogical Content Knowledge for Practices* and *Curricular Knowledge*. See Table 26 for percentages for each knowledge type by unit and curriculum.

Table 26. Percentages of Each Knowledge Type by Curriculum

Curricula	Variable				Transformations			
	SMK	PCK- Topics	PCK- Practices	CK	SMK	PCK- Topics	PCK- Practices	CK
CMP	4	38	49	10	7	12	70	11
Glencoe	4	29	37	30	6	33	42	20
MiC	5	32	27	37	7	24	34	35
UCSMP	7	20	51	22	14	16	51	19

In all curricula, save MiC, the largest percentage of supports was supports for *Pedagogical Content Knowledge for Practices*. MiC, on the other hand was different. It contained mostly support for *Curricular Knowledge*, followed by *Pedagogical Content Knowledge for Topics* or *Pedagogical Content Knowledge for Practices*. In addition, across all curricula, *Subject Matter Content Knowledge* was least supported, accounting for no more than 7% of supports, except for in UCSMP’s transformations unit, in which it accounted for 14% of the content supports.

Why are certain supports more prevalent than others? What might explain the focus on PCK for Practices and Curricular Knowledge? First, as three of these curricula were Standards-Based, it might not seem surprising that there is a focus on Practices. The NCTM Standards included more than just a list of content expectations; it included “Process Standards” or a set of expectations that were about the mathematical processes students should engage in. These included Problem Solving, Reasoning and Proof, Communication, Connections, and Representations. See Table 27 for descriptions of the NCTM Process Standards.

Table 27. Process Standards form Principles and Standards for School Mathematics (NCTM, 2000)

Process Standard	Description
Problem Solving	<p>Instructional programs from prekindergarten through grade 12 should enable all students to—</p> <ul style="list-style-type: none"> • Build new mathematical knowledge through problem solving • Solve problems that arise in mathematics and in other contexts • Apply and adapt a variety of appropriate strategies to solve problems • Monitor and reflect on the process of mathematical problem solving
Reasoning and Proof	<p>Instructional programs from prekindergarten through grade 12 should enable all students to—</p> <ul style="list-style-type: none"> • Recognize reasoning and proof as fundamental aspects of mathematics • Make and investigate mathematical conjectures • Develop and evaluate mathematical arguments and proofs • Select and use various types of reasoning and methods of proof
Communication	<p>Instructional programs from prekindergarten through grade 12 should enable all students to—</p> <ul style="list-style-type: none"> • Organize and consolidate their mathematical thinking through communication • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others • Analyze and evaluate the mathematical thinking and strategies of others; • Use the language of mathematics to express mathematical ideas precisely.
Connections	<p>Instructional programs from prekindergarten through grade 12 should enable all students to—</p> <ul style="list-style-type: none"> • Recognize and use connections among mathematical ideas • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole • Recognize and apply mathematics in contexts outside of mathematics
Representation	<p>Instructional programs from prekindergarten through grade 12 should enable all students to—</p> <ul style="list-style-type: none"> • Create and use representations to organize, record, and communicate mathematical ideas • Select, apply, and translate among mathematical representations to solve problems • Use representations to model and interpret physical, social, and mathematical phenomena

CMP, MiC, and UCSMP, although originally designed to align with the 1989 NCTM Standards, were all also revised and were intended to align with the NCTM 2000 Standards as

well. I argue that although Glencoe was not originally designed as a Standards-based curriculum that its developers also attempted to align with the NCTM Standards, as evidenced by Standards alignment charts in the teachers' guides that lists which lessons are aligned to which specific NCTM Standards. Therefore, the curriculum I analyzed were designed, in part, to provide students experiences with these processes, many of which were included in my coding scheme as part of PCK for Practices. The most obvious of these is the Reasoning and Proof standard, as I had a code within PCK for Practices for both *Enactment* and *Rational Guidance for Reasoning and Proof*. However, my PCK-Practice codes, such as *Enactment* and *Rationale Guidance for Terminology and Participation Structures*, also seem to fit within NCTM's Communication Standard.

The same may be true for why *Curricular Knowledge* was prevalent in MiC, as the codes I included within this category include much of what NCTM's Connections Standard encompasses, such as understanding connections between mathematical ideas and how these build on each other and other disciplines. In my scheme these fall within the *Curricular Knowledge* category, as I see these supports are ones that help teachers understand how their particular curriculum is making connections within and across disciplines.

A note about Subject Matter Content Knowledge. Although I know that the numbers indicate that there was *Subject Matter Content Knowledge* I felt it necessary to draw readers attention to this because although the percentages were quite small for each curriculum, there were quite a few sentences devoted to subject matter content knowledge, especially in CMP, MiC, and UCSMP. This knowledge was dwarfed by other categories of knowledge. The focus on *Subject Matter Content Knowledge* may once again be attributed to the ways in which these curricula were designed, particularly in the case of CMP and MiC. These two curriculum

projects, and later UCSMP which received funding in the second wave, were funded by the National Science Foundation and at the time were qualitatively different than all other curriculum materials on the market. These materials contained different mathematical content and a different approach than traditional textbooks (Bradley, 2007, p. x) and therefore these materials required a deeper understanding of mathematics than some of the materials middle school teachers may have used previously.

Differences in frequencies across curricula. My study is not meant to be a horse race. I am not trying to determine the best existing curriculum or the one that provides the most or best supports. I am trying to describe the opportunities that currently exist so that we can learn from these and improve curriculum materials. That said, there were some differences in the total number of supports and in the relative frequencies of supports.

First, in terms of total number of content supports, CMP had a much larger number of supports for both units, having at least 150% more codes in both units than others, and often having close to or over three times as many codes. However, when taking into account total sentences, CMP is only slightly higher than the other four curricula in the variable unit with 1.3 codes per sentence, while Glencoe, MiC, and UCSMP, have 1.1, 1.2, and 1.3 codes per sentence, respectively. In the transformations unit, although CMP has more total supports than the other curricula, it only averaged 1.0 code per sentences which is the same as both Glencoe and UCSMP. For this unit, MiC actually had a larger codes per sentence ratio of 1.1. For both units, CMP and MiC had equal or higher codes per sentence. Possible reasons for this might be related to the demand of the tasks in the curriculum and the design principles or structure of these curriculum materials. I discuss these issues in the next few sections.

Cognitive demand of curriculum materials. As described above, CMP and MiC were two of the original NSF-funded curricula. The NSF-funded curricula were qualitatively different than other curricula and these materials placed "a greater cognitive demand on students and teachers" which made "the implementation of these materials difficult" (Bradley, 2007, p. x). Although UCSMP received funding from the NSF in the second wave, the UCSMP curriculum materials, although different from the traditional commercially-developed materials in arguably more subtle ways, did not seem as radically different as curricula such as CMP and MiC. The increased cognitive demand described by Bradley (2007) may be a contributing factor to the increased number of supports in CMP and MiC. Since these curricula are more demanding for teachers to teach with, teachers require more support. This is similar to what Stein and Kim (2009) proposed. That said, I did not do an analysis of the cognitive demand of the four curricular series I analyzed in this study. I merely propose this as a possible contributing factor based on Bradley's (2007) statement that the NSF-funded texts were more cognitively demanding for students and teachers than traditional curriculum materials.

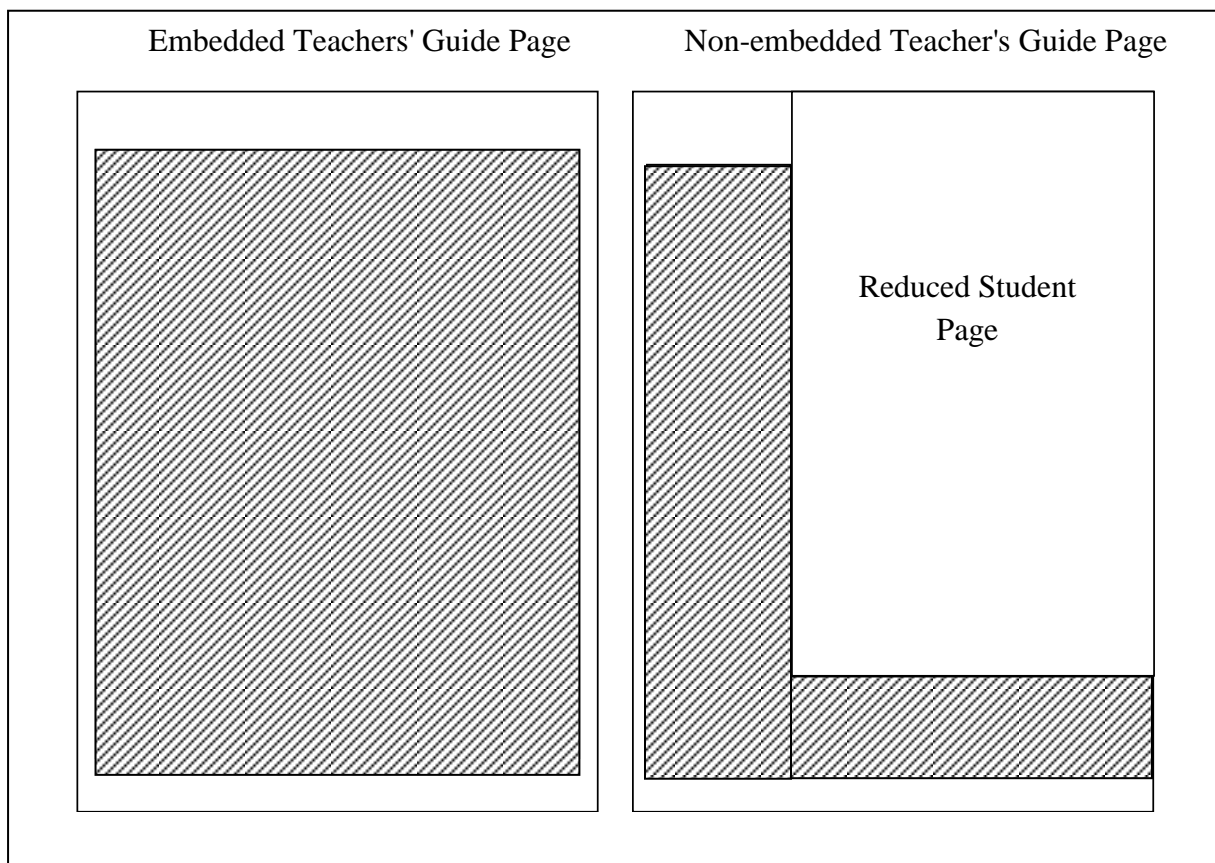
Design principles and structure. Another factor that might describe the differences in the curricula is the design principles and structure of the materials, particularly when it comes to particular support codes, such as *Enactment Guidance for Questions*.

Structure of teachers' guides. Overall structure of the teachers' guides may impact the total number of content supports. By structure I mean the ways in which the teachers' guides are packaged and organized. CMP and MiC teachers' guides are, like their student books, stand-alone guides for each unit, whereas Glencoe and UCSMP use a two-volume structure; each volume contains half of the teacher materials for that course. Although all curriculum developers are plagued by publishing demands to keep their materials as few pages as possible, the two-

volume structure might require this even more as the volumes include support for multiple units. An increased number of pages would result in a larger physical book, something that could potentially discourage teachers from wanting to carry it.

In addition, the "embeddedness" of the teacher's guide might also play a factor. CMP was the only embedded guide, a guide that included all teacher content within the narrative of the lesson, rather than a teacher edition of the student text with wrap-around information for the teacher. The wrap-around teachers guides might, by the nature of their structure include less room for teacher support. Figure 10 provides a representation of each of these types of teachers' guides. The shaded area indicates the space in which teacher support text is located.

Figure 10. Examples of Pages from Embedded and Non-Embedded Teachers' Guides



The CMP teachers' guide embedded the instructions to the teacher in a narrative that followed the unfolding of the lessons and provided detailed descriptions for enacting the major lesson pieces (i.e., Launch, Explore, and Summarize). Glencoe, MiC, and UCSMP, on the other hand, provided support text around the student page. Unlike the CMP structure, this structure leaves much less space for teacher text. For example, each Glencoe page was a 10 x 12 in. page, which provided 120 sq. inches of space, however since a reduced student page was included, this leaves only 53.5 sq inches or about 45% of the page for teacher text. This was typical for the non-embedded teachers' guides. All text on each CMP teacher's guide page was 100% teacher material. It is not clear whether curriculum developers have control over the structure of their teacher' guides. It could be that the publisher of the materials makes these decisions and the curriculum developers have little say in the matter. That said, it is an important issue for developers to think about. If curriculum authors were not constrained by the structure, or even if they were, thinking about ways in which they could include the types and amount of support necessary is critical in improving teachers' guides.

Design principles. In addition to the overall structure of the teachers' guides, the design principles of a curriculum may impact the types and frequencies of supports. First, if the curriculum was designed specifically to attend to teacher learning this would have an impact on the content supports. For example, CMP was specifically designed to be a curriculum for teachers as well as students. "The materials were written to support teachers' learning of both mathematical content and pedagogical strategies" (Lappan, Phillips, & Fey, 2007, p. 68). Although each curriculum created a guide for the teacher these differed and these differences may be attributed to the focus that the curriculum developers put on the teachers' guide and how much this focus involved teacher learning as opposed to directions for enacting or implementing

the curriculum. Whereas I can say that CMP stated teacher learning was a guiding principle in the design of their materials, I cannot say this about the other curricula. This does not mean that these curricula did not attend to teacher learning in the design of their materials, but this has not either been described by the developers as a focus, or at least, I could not find evidence of this.

Design principles may also impact the types and frequencies of particular supports. All curricula are designed with some set of principles. It would make sense that a curriculum's teacher support materials would be geared towards supporting teachers with the types of content, activities, and processes that teachers were expected to engage their students in. As I point out in the results, this might explain why CMP had such a large number of content supports for *Enactment Guidance for Questions*. CMP is "problem-centered" and the mathematics is "embedded in interesting problems to promote deeper engagement and learning for students" (Lappan, Phillips, & Fey, 2007, p. 68). When setting up a problem it seems natural to have questions and since students are expected to engage with these problems through questioning, it seems natural that the teacher support would warrant a focus on questions. This might also explain why CMP was the only curriculum to provide support for *Enactment Guidance for Helping Students Ask Their Own Questions*. Another instance in the data that indicated this possibility is the large number of *Enactment Guidance for Representations* in CMP's variable unit. Variables and Patterns places a special emphasis on different representations as evidenced by one of the goals of the unit, "Observe relationships between two quantitative variables as shown in a table, graph, or equation and describe how the relationship can be seen in each of the other forms of representation" (CMP, Variables and Patterns, p. 2) and this may be related to the large number of supports for teachers.

Constructing Relationships: Teacher as Agent

In this section, I describe issues related to the curriculum developers' construction of relationships with teachers. Specifically, I address how the curriculum authors spoke to teachers in the text and what this indicates about the relationships and positioning of the teachers reading these materials.

My results indicated that the curriculum authors did not speak "to" teachers, but instead this study showed that often curriculum authors speak "through" teachers. This was evidence by the large percentage of imperatives and the lack of *Rationale Guidance*. Although each curriculum developed a guide for teachers, the content and the language used in these guides may indicate that the curriculum authors, like writers in the early and mid- 1900's, underestimated the role of the teacher. Furthermore, some of the linguistic forms, whether used consciously or not, not only underestimated the teachers' role, but positioned the teacher in a diminished role, the role of agent doing the bidding of the more knowledgeable curriculum authors.

First, the large number of imperatives indicated that the curriculum authors often chose to command teachers to perform certain actions and in some cases directed students through commanding teachers. In addition, these imperatives often foregrounded the teacher's position as that of "scribbler," merely performing the actions the curriculum authors desired. Although the use of imperatives can be expected in academic discourse (Morgan, 1996), Morgan questions their use in the school context. Embedded in this linguistic form are issues related to power and authority. Although Morgan studied the written text of students, I argue that the same type of concern is warranted when investigating text in the teachers' guides. An imperative, or command, in the teachers' guides is a grammatical feature that allows for the curriculum authors

to demand teachers to carry out a service and positions the curriculum authors as the authority in the situation, the one (or in this case, the group) telling teachers what to do. The relationship constructed here has implications for agency. Based on the thoughts of philosopher and mathematician Charles Sanders Peirce, Rotman (1988) distinguished between two types of mathematical agency:

"the one who imagines (what Peirce simply calls the 'self' who conducts a reflective observation), which we shall call the *Mathematician*, and the one who is imagined (the skeleton diagram and surrogate of this self), which we shall call the *Agent*. In terms of the distinction between imperatives, it is the Mathematician who carries out inclusive demands to 'consider' and 'define' certain worlds and to 'prove' theorems in relation to these, and it is his Agent who executes the actions within such fabricated worlds, such as 'count', 'integrate', and so on, demanded by exclusive imperatives" (p. 11/106).

This distinction described by Rotman (1988), lends itself to describing the imperatives I saw in the teachers' guides. Where Rotman wrote about mathematician and agent, in my study, it is the curriculum authors that demand, and the teachers using the curriculum materials that execute the actions.

In addition to imperatives, each curriculum made use of modal verbs to open up the dialogue and provide the possibilities of negation. Modal verbs such as "may" and "might" provided room for teachers to decide whether to enact suggestions posed by curriculum authors. However, there was still a somewhat significant use of verbs such as "should" and "need." Although these verbs provide some room for negotiation, they embed a higher degree of obligation for teachers to act than the modals "may" and "might."

A more subtle indication of the teachers positioning as agent is illustrated by the lack of *Rationale Guidance* included in all the curriculum materials. *Enactment Guidance*, guidance that supported teachers in what and how to enact lessons, was much more prevalent than *Rationale Guidance*. *Rationale guidance*, which accounted for no more than 6% of the guidance in any unit, enables teachers to understand why particular mathematical, pedagogical, or curricular approaches are appropriate and creates a dialogue between authors and teachers. Supports such as this can allow teachers to understand the reasoning of the authors of the materials and develop what Drake and Sherin (2009) call “curriculum vision” or a sense of where the curriculum materials are going and an understanding of the “particular kinds of learning and teaching practices described in the curriculum materials” (p. 324). Furthermore, *Rationale Guidance* supports would allow teachers to understand the choices made by curriculum authors and could potentially help curriculum developers forge a different relationship with teachers; a more reciprocal relationship, one in which teachers and curriculum developers work together to construct the curriculum.

Another interesting finding from this study was that when curriculum authors opened up a space for teachers by using modal verbs, such as “may” and “might,” curriculum authors frequently used the verbs “want” and “wish.” It is possible that the use of the modal verbs “may” or “might” allowed curriculum authors not to force wants and wishes on teachers. On the other hand, the use of these words together may be a more subtle way for curriculum authors to influence what their teacher do. Rather than telling a teacher to *do* something, or telling a teacher he or she *may do* something, getting a teacher to *want to do* something might in fact be more influential because it indicates a personal desire. Although the choice of these words may be an unconscious one by curriculum authors, it is nonetheless a choice.

Opportunities to Learn in Variable and Transformations Units

Content. When I began this dissertation one thought I had was that the supports for the introduction to variables units and the geometric transformations units would differ. More specifically, I thought that there would be *more* content supports in the variables units than in the geometric transformations units because as a teacher I was always able to find more algebra support, whether this be in the form of professional development sessions, or resources from my district or the Internet. In addition, there is a greater emphasis on algebra than geometry since success in algebra is seen an indicator for success in higher mathematics (National Mathematics Advisory Panel, 2008). My results indicated that although the algebra units did have more content support, the difference was not as large as I expected. Three of four curricular series actually had a higher frequency of content supports for their geometric transformations unit, CMP was the only exception. This might make sense since for each curriculum there were more sentences in the transformations units. However, when I calculated a code per sentence ratio, the variable and transformations units were quite close, with the variable units being only slightly higher. See Table 28. This means that for each curriculum there were more supports in the variables units per unit of text than in the transformations units, but the difference was minimal. The transformations units may have included more sentences and supports because geometric transformations is a relatively new topic in middle school (Sinclair, 2008). Curriculum authors may have felt the need to include more teacher support for this potentially unfamiliar topic.

Table 28. Ratios of Codes per Sentence by Unit and Curriculum

Curricula	Variable	Transformations
	Codes per Sentence	Codes per Sentence
CMP	1.3	1.0
Glencoe	1.1	1.0
MiC	1.2	1.1
UCSMP	1.1	1.0

As described in the results section, although for both units the most prevalent supports were the same, there were some differences in the content supports for the two units. The transformations unit had a greater span of content supports than the variable unit; the transformations units contained more types of support than the variable units. In addition, there were differences in frequent content supports. Within the *PCK for Mathematics Topics* category, the variables units contained more instances and a larger percentage of *Enactment Guidance for Representations*, whereas the transformations unit included more *Enactment Guidance for Tools*. The transformation units also included in the *PCK for Practices* category, more instances of *Enactment Guidance for Approaches to Reasoning and Proof*. There were no noticeable differences in the frequencies of other content supports between the variable and transformations units.

Expression. Similarly, for the aspects of voice I analyzed the differences were quite subtle. In the next two sections I describe the differences in personal pronoun use and "you"-forms.

Differences in personal pronoun use by unit. As expected, since there were more sentences in the transformations units, generally there was a larger frequency of personal pronouns. The personal pronoun "I" was not found in either unit. The personal pronouns "we" and "you" were more frequent in the transformations units except for the use of "you" in CMP, which was more frequent in the variables you.

"We" was used differently in the units by CMP and UCSMP. In the transformations unit the referent for "we" was more often unclear. In these instances I was unable to determine who "we" referred to, unlike clear instances in which I could determine that "we" referred to the curriculum authors. Of all of the uses of "we" in CMP, 74% were unclear in their variables unit

and 86% were unclear in their transformations unit. Similarly for UCSMP, there were no unclear referents for "we" in the variable unit, whereas 41% were unclear in their transformations unit.

Why was this lack of clarity more common in the transformations unit? One possible explanation for this is that most of the instances in which the referent for "we" was unclear was when the authors provided details for a particular method or solution in a way that explicated the type of thinking that one might engage in, and these were more common, particularly for CMP, in transformations units. In these instances, "we" was used quite frequently and not in a way that clearly referred to the curriculum authors or a particular plural group.

Why is it that transformations units included more explanations such as these that detailed a particular method or solution? Two possible reasons come to mind. First, it could be that due to geometric transformations being perceived as "new" content, that curriculum authors thought it necessary to provide more teacher support. According to Sinclair (2008), up until the mid 1960s, in the U.S., geometric transformations were deemed too hard, even for high school students. Therefore, the inclusion of this new, hard content may have prompted curriculum developers to provide more detailed (often step-by-step) methods or solutions and model the types of thinking that performing transformations requires.

Another potential explanation for the differences in the two units, particularly related to the use of different language, is authorship. It is likely that units, particularly in the Standards-based curriculum materials, were written by different authors, or at least were primarily supervised by different authors. This might be one reason why the transformations units and variables units differ. Some authors may choose to use personal pronouns in particular ways, including not using them all. The possibility of different authors may have also impacted the

overall structure of the text including the inclusion of examples, such as the ones we see in the transformations units that provide detailed explanations.

Differences in "you"-forms by unit. In general, implicit “you”-forms (i.e., imperatives) were much more common across both units, except in MiC’s transformations unit which had more explicit than implicit “you”-forms. The ratios of implicit to explicit forms was higher in all cases for the variables units, meaning that the differences between the total number of each of these forms was greater relative to the total “you”-forms used in the variable units.

Imperatives. The use of imperatives in each curriculum was fairly consistent across units. Across both units, generally there was a greater relative frequency of imperatives in the transformations unit, with the Glencoe unit containing over 25% of the sentences in their unit in the imperative form. CMP was the only curriculum to have more imperatives in their variable unit. The locations and the types (i.e., inclusion, exclusive) of these imperatives were also fairly consistent across units. However, there was a greater difference in common imperatives in the transformations units between the percentage of imperatives that directed the teacher and those that directed students through the teacher.

Explicit “you”-forms. The relative frequencies of explicit “you”-forms were virtually the same across units. The most pervasive was “you” + modal verb in both units. I also did not find much of a difference in the types of modal verbs used in each unit. Across both units the common modals were the same, low modals “may” and “might,” with CMP and MiC using the median modal “should” and the high modal “need” and all curriculum materials including the low modals “can” and “could,” but at much lower frequencies than “may” and “might.”

Opportunities to Learn

What do these results indicate about teachers' opportunities to learn about mathematics, pedagogy, and curriculum, from middle school curriculum materials? Although each curriculum provided access to some content supports, this access might not be sufficient. Over 15 of 31 content supports were infrequent or unobserved across at least three of the curricular series. These absent supports were often *Rationale Guidance* supports, which not only impacted access to content, but also diminished the ways in which authors spoke to teachers. When curriculum authors discuss their rationale they open up a space in which teachers can engage with them around the underlying principles on which the curriculum is designed and allows for teachers to develop "curriculum vision" (Drake & Sherin, 2009). Generally, this space was not provided. Furthermore, the expression of the text, or the choice of words used by authors, as evidence by the aspects of voice I analyzed, often positioned teachers as agents doing the bidding of curriculum authors. This curriculum author – agent relationship may hinder teachers learning from the text as it is possible to see the teachers' guide as merely a set of directions to enact the curriculum.

As my discussion suggests, it is difficult to describe opportunities to learn in a diverse set of curriculum materials. I propose, like Stein and Kim (2009), that the types of support are intimately connected to the type of curriculum materials being analyzed. That said, regardless of the structure or philosophy of a set of curriculum materials, my study indicated that the content in these four series could be improved. All four series did not provide adequate access to content, particularly *Rationale Guidance* and used language that may hinder teachers' opportunities to engage with and learn from the text.

Contributions to the Field

This analysis is quite timely. Although professional development for some time has been deemed as a necessary piece in improving teaching, as we enter into the era of the Common Core Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), mathematics teacher educators and curriculum developers have already recognized that this need is even more critical (Lappan, McCallum, & Kepner, 2010). Teachers are going to be required to teach topics for which they have little or no background, such as geometric transformations, and may be required to drastically change the ways in which they approach particular topics or think more deeply about the ways in which they teach mathematics to incorporate the practices outlined in the Common Core Standards. Research indicates that teaching and curriculum matter and as the words of Bruner remind us, "A curriculum is more for teachers than it is for pupils. If it cannot change, move, perturb, inform teachers, it will have no effect on those whom they teach." As we enter into this new era, this idea from 35 years ago carries an important message. For any "curriculum", whether it is a set of written materials, or an outline (such as The Common Core Standards) for the types of things that should be taught, it must affect change in teachers for it to affect students.

My dissertation illustrates that these four curricula series can do more to move and inform teachers. I have not only provided the field with an analysis of the opportunities for the kinds of opportunities in four popular middle school mathematics curricular series, but I have also provided researchers with a framework for conducting similar analyses in other mathematical content areas. My study has implications for curriculum development, teacher education, and research. In the next section, I discuss these implications.

Implications

Curriculum development. Curriculum developers have an extremely difficult task. I recognize the pressures placed on curriculum authors from publishers to make their students' and teachers' guides as short as possible. This pressure is not something that curriculum researchers can ignore. I also recognize that what ends up in the final version of a textbook may not be identical to what a curriculum development team intended. Nonetheless, this work has some implications for curriculum developers. First and foremost, I hope that my study raises awareness about how the choice of content and its expression might impact teachers' opportunities to learn and that this encourages curriculum authors to critically examine their own texts. Before curriculum developers can exert change, they need to see that this change is necessary. I hope that this examination of text will enable curriculum developers to be more intentional, particularly in when it comes to discussing their rationale and their choice of words. In the next sections I outline a few recommendations for curriculum developers.

According to Remillard, for teachers to learn from their curriculum materials authors need to speak *to* teachers. This is something that all four curricular series I analyzed could do more of, particularly Glencoe, which addressed teachers through imperatives more than any other curriculum series (at least 25% of sentences were in imperative form in each unit). In addition, each curriculum suffered from the lack of *Rationale Guidance*. Including more guidance of this type would help to include teachers more in the construction of their mathematics teaching by opening up a dialogue with them and helping them understand why particular approaches or content is appropriate. This could help in allowing teachers to develop curriculum vision (Drake & Sherin, 2009) and enable teachers to become more skilled and informed enactors of the curriculum. An area that curriculum authors may want to focus their

attention on is providing more *Rationale Guidance for Goals*. This support was completely absent from all curriculum materials. Including information related to why particular goals are appropriate and why the curriculum authors chose particular goals for particular lessons may allow teachers to more fully understand the goals of the unit or lesson and how these fit together across the entire curricular series. This, in turn, may allow teachers to help their students achieve these goals more effectively.

Another area of attention for curriculum authors is in providing more *Implementation Guidance for Anticipating and Using Students Thinking* productively. If we expect teachers, which often the more reform-oriented curricula do, to engage in discussions with their students about the mathematics, more support is needed in anticipating and using student thinking (Star & Strickland, 2008). This is particularly important for new teachers who have less experience, as anticipating what students might do and say can be a challenge when you have limited experience to draw from.

Finally, curriculum developers may want to think more strategically about where support is located. For example, if all the subject matter support is at the unit level and teachers are not looking at this prior to or during the lesson, these valuable supports may not be providing the opportunities for learning that curriculum authors intended them to.

As I mentioned earlier, I recognize that curriculum developers are under pressure to make their teachers' guides even shorter, often requiring them to be terse and make tough decisions about what to include and what to exclude. With the increased availability and use of technology, however, these constraints may become a thing of the past to some extent. I hope that the findings from my study can influence the thinking related to the types of supports to include in digital textbooks. For example, some curricula already have websites for professional

development that include videos of lessons. With digital curriculum materials, images of classroom interactions (Remillard, 2000), such as transcripts from classroom discussions (similar to Dialogue Boxes in *Investigations*) or video could be embedded right into the lesson guidelines. I make no claims that writing this kind of teacher's guide is easy, but that it may help to eliminate some of the constraints that are placed on curriculum authors when developing a paper teachers' guide. As a mathematics educator I find the possibilities inherent in this technology quite liberating and potentially fruitful, but also think that as a field we need to think carefully about the ways in which we use this technology. For example, if electronic curriculum materials are just digital versions of paper teachers' guides, then these guides will suffer from many of the same issues I described in my study. I discuss electronic curriculum materials more in my section about future directions for design and research.

Teacher education. Although there have been many changes in the types of textbooks that are available to teachers, it is still true that some prospective teachers develop the impression that if they want "to be good teachers, they should avoid following textbooks and teachers' guides" (Ball & Feiman-Nemser, 1988) or more drastically, not just that they should not follow them, but that they should completely throw them out all together and create all of their own materials. This mentality, attributed to the need to be autonomous and to take control of every aspect of their teaching especially their lesson planning (Ben-Peretz, 1990), seems unrealistic and potentially problematic. Beginning teachers, while professionals in their own right, are not curriculum developers, nor should they be positioned as such. The fear that teachers become "text bound" has hindered some teacher education programs from addressing important issues related to curriculum interpretation. It is within teacher education programs that prospective teachers can begin to learn to read and interpret curriculum in meaningful and productive ways.

Without this education, teachers may be doomed to “remain ‘text bound,’ using textbooks or teacher guides because they ‘are there’ without attempting adaptation or enrichment of existing materials” (Ben-Peretz, 1990, p. 109). This study implies that, while teacher's guides are not replete with opportunities for learning, opportunities do exist and that it is essential for teacher educators to help prepare teachers that know how to use and learn from their textbooks.

First and foremost, this study implies that teacher educators need to help their students learn to read their curriculum materials since rich opportunities for learning were not frequently present. By reading, I mean the constructive process of meaning-making (Remillard, 2000). This reading process will enhance the opportunities available because reading can incorporate more than just what the supports present, such as thinking more deeply about student thinking. It takes more to enact the curriculum than just reading the words on the page and if teacher educators do not incorporate issues of written curriculum materials into their courses, teachers may remain text-bound since they will not develop the skills necessary to use their teachers’ guides. They may not know where to find supports or how to adapt materials to meet the needs of their students while still providing a coherent mathematical experience. This study provides valuable insight into the types of opportunities that are available in four commonly used curricular series. Although it is true that not all teachers will teach from these textbooks, teacher educators can use these insights to help student teachers develop the skills necessary to use textbooks productively.

Remillard (2005) found that teacher learning was related to the activities that were central in constructing the curriculum. These activities were wrapped up in the reading process which involved reading the text, but also reading their students and the tasks they were engaged in. For new teachers to be able to do this they will need assistance in learning how to read suggestions in the text and make educated decisions about how to proceed in their classrooms based on what

they know about their students (Kauffman, Johnson, Kardos, Liu, & Peske, 2002). This study indicated that a large percentage of the sentences were commands in the imperative form. For new teachers, particularly elementary and middle school teachers who may not feel empowered, learning to adapt and change what their curriculum says in order to best meet their students needs while still enacting a coherent and mathematically rich curriculum is difficult. Teacher education programs can prepare students to do this by critically examining mathematics curriculum materials. Assignments such as the ones given in mathematics methods courses (Lloyd & Behm, 2005; Lloyd & Pitt Bannister, 2010) where student were asked to compare lessons from two very different textbooks or use a teachers' guide to plan and enact a lesson can not only help students become more comfortable with the ways in which textbooks are designed, but these assignments can also help prospective teachers develop a sense of and learn to critique different approaches and content trajectories.

Another more simple, yet important implication of this study, is that opportunities for teacher learning occur in various locations of the teacher's guide and teachers will need to consult different locations to get this support. Directing prospective teachers to consult all parts of whatever teacher's guide they use is important. For example, teacher educators can help prospective teachers see that if they read the text at the unit level they may be able to gain insight into how the individual lessons fit together to develop the big picture, or find the subject matter support they needed.

Research. In my study I used use two frameworks for investigating opportunity to learn; one framework that focused on content and one on aspects of voice. As described in the background of my study, research on written curriculum materials often focuses more on content and less on the expression of this content, with a few recent exceptions. My study indicated the

importance of examining the expression of content in curriculum materials and the benefits of using a Systemic Functional Linguistics framework to do this. First, if I only studied the content of the teachers' guides I would not have gained insight into how curriculum authors speak to teachers, a critical factor in how teacher supports may be experienced. For example, although there were many supports for helping teachers use questions with students, my analysis indicated that, in many of the instances authors chose to use imperatives, which indicated that teachers were commanded to ask particular questions. The Systemic Functional Linguistics framework allowed me to achieve a more nuanced description of the learning opportunities in the middle school curriculum materials and illuminated the subtle shifts in the way language was used in different units and curriculum materials. I argue that unless researchers look at how content is expressed, we are not accurately describing opportunities to learn. Researchers should examine more than just the content, whether this be in teachers' guides or student textbooks. In addition, this means that researchers should employ other frameworks, such as ones that focus on forms of expression, as these frameworks provide a means for critically examining text in ways that previous frameworks that have focused on what is in a textbook do not. That said, it is equally important when analyzing expression to consider what is expressed. Without examining the content, the results tell us little about the social activity in which that expression occurs. Therefore, in my study it was imperative to look at both the content supports and the expression in order to be able to say something about the nature of the opportunities available for teachers in the teacher's guides.

In addition researchers should consider how the philosophy and structure of curriculum materials impact the types of supports provided in the text. Since I chose curriculum materials that were designed using different guiding principles, these issues arose in my study as I tried to

describe the difference I saw in the four curricular series. In my analysis I proposed that the frequency and types of content supports and the language may be related to the philosophy and structure of the curriculum itself. However, more analysis is needed in this area.

Limitations

No study is without limitations. In this section I discuss the limitations of my method. First, this dissertation was written by one person. Although it is the product of my thinking and countless conversations with colleagues (i.e., faculty, graduate students, teachers), the final words in this document are my own. This represents my perspective. I acknowledge that others using different frameworks (or developing different codes), or even the same frameworks, could possibly bring different meaning to the data. Second, although I chose the four curricular series in my study for particular reasons, I only analyzed four curricular series, all of which were designed for middle school. As a field, researchers need to use methods such as the ones I used in my study in order to make more nuanced claims about opportunities to learn. Finally, in my study I did not attempt to make links to what or how teachers learn or experience curriculum. This is an important aspect of this work, as more needs to be done in order to determine how the opportunities to learn, particularly the content and its expression, in curriculum materials actually promote teacher learning.

That said, despite these limitations, my study provided a way to describe a particular set of curriculum materials, those that are widely used throughout the United States and vary in design. In addition, my study provided a way to use two frameworks to obtain a nuanced description of teachers' opportunities to learn from curriculum materials; two frameworks that can be applied to other mathematics curriculum materials.

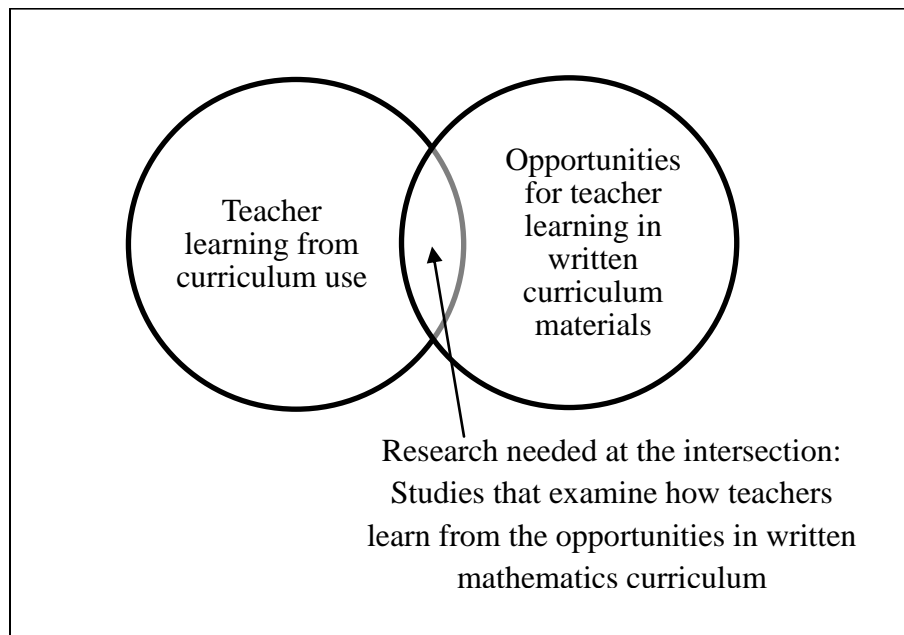
Future Directions for Research and Design

I see many directions for further research and design, all of which I am interested in pursuing myself. In this section, I describe three of these directions: a) examining curriculum materials at other grade levels, b) focusing research on studies that combine the research on curriculum use with the research in my study on opportunities in written curriculum materials, and c) exploring ideas for designing and researching the use of electronic curriculum materials.

Examining opportunities for teacher learning in written materials at other grade levels. In my study, I only investigated opportunities in middle school curriculum materials. I chose middle school curricular materials for particular reasons, such as the unique position of middle school teachers as those who bridge elementary and high school mathematics content and that not all middle school teachers have had adequate preparation to teach middle school, but have been former elementary or high school teachers (Hill, 2007). However, curriculum materials at all levels need to be examined. Although I think that my frameworks are appropriate for examining any mathematics curriculum materials, there may be level-specific issues that researchers need to consider. This may be particularly important when interpreting what is present in the materials. Most high school teachers are required to have college degrees in mathematics, whereas elementary school teachers are not, but are often required to take more child development courses and courses in other disciplines. These issues of prior coursework may have implications for the types of opportunities needed. For example, subject matter content knowledge may be more important for teachers who have had less exposure to mathematics courses, whereas supports for connecting mathematics to other disciplines, may be needed by those who have not had much exposure to the ideas of teaching literacy or science. If this is true, then the opportunities needed in curriculum materials for different levels would be different.

Examining teacher learning from opportunities embedded in written materials. I see my study as only the first step in investigating teacher learning from written curriculum materials. Although research indicated that teachers can and do learn from using curriculum materials, there has been minimal research that has looked at how the written curriculum impacts this learning. In my study, I described the opportunities for teacher learning within four middle school mathematics curricular series. However, I make no claims about whether or how teachers' make use of these opportunities and furthermore, actually learn from these opportunities. Further research must draw from both of these areas of research, the research on teacher learning from curriculum use and the research on the opportunities in written materials (See Figure 11), to investigate, whether or how teachers learn from the opportunities within written curriculum materials. Specifically, as called for by Stein, Remillard, and Smith (2007), research should investigate how the features or characteristics of particular curriculum materials influence how teachers use it, and more specifically how teachers learn from the materials. My study can serve as a starting point to address the following question: What and how do the features of curriculum materials impact teacher learning? The detailed description I provided in my this study can serve as the basis for studies of the enacted curriculum in which researchers observe teachers planning and teaching lessons from the four curriculum I analyzed. It may be possible to make links between the opportunities in these materials and what teachers are learning from using these materials.

Figure 11. Current Research and Future Needs



Designing electronic educative curriculum materials and researching their use.

Although electronic curriculum materials have already been conceptualized and designed, most of these materials have been merely digital versions of the old print curriculum materials. New curriculum materials will likely be electronic (Center for the Study of Mathematics Curriculum, 2009). My study indicated that teachers' guides lacked many of the supports needed by teachers, particularly those related to rationales and anticipating and using students thinking productively. In the traditional print teachers' guides, including support such as this would require more sentences, which would require more paper pages. Due to publishing demands, often curriculum developers do not have the space for such supports as they cost more money. This is where digital technologies provide a more publisher-friendly solution. First, there is not the same pressure to keep the number of pages down when creating a digital book. Second, digital media would allow for more than just sentences. Digital materials would allow elements such as interactive applets, discussion boards, and videos. Rather than just having a discussion scenario,

a teacher could click on a video that illustrates a conversation in a real classroom around the task that they are using that day.

Another advantage of electronic curriculum materials is that they can be made to be customizable and adaptable. First, teachers could have more control over the activities they do in their classroom by using a system that allowed them to easily choose from a menu of options, somewhat like a more sophisticated version of the Choose your Own Adventure books for kids. Brown (2009) indicates that materials such as these, like his Adaptive Instructional Materials (Brown, Pellegrino, Goldman, Nacu, Julian, Tarnoff, et al., 2004), can help teachers develop *Pedagogical Design Capacity*, the "capacity to perceive and mobilize existing resources in order to craft instructional episodes (Brown, 2002; Brown & Edelson, 2003)" (p. 29). Second, electronic materials are also more easily adaptable. As researchers learn how teachers use materials and what features promote learning, electronic curriculum materials can be adapted to meet the needs of teachers easily. We live in a world in which people download updates to the applications, or “apps” on their phones on a daily basis (or in many cases, their apps automatically update for them). The same could be true for curriculum materials. Designing materials like this is no small feat; these materials would require both the mathematical curriculum knowledge and the technical know-how, but I see this as a worthy direction for design work. Designers, particularly those in the university system, could develop relationships with those working in other departments such as computer science and engineering, or local or national companies that specialize in technology, such as Google. I see this as an exciting avenue for my own future as a researcher and possible designer.

APPENDICES

APPENDIX A

Table 29. Descriptions of Studies of Written Curriculum Materials

Authors	Number of Series Analyzed	Level	Resources Examined (i.e., student/teacher)	Focus of Analysis	Countries
Ashcraft, M. & Christy, K. (1995)	1	elementary	student	arithmetic facts (simple addition & multiplication)	U.S.
Cai, J., Lo, J. & Watanabe, T. (2002)	3	elementary/middle	student	arithmetic average	Asian countries, U.S.
Castro, A. M. (2006)	1	middle	teacher	teachers' guide as resource in planning	U.S.
Chang, K., Males, L. M., Mosier, A., & Gonulates, F. (2011)	3	elementary	student and teacher	estimation of linear measurement	U.S.
Charalambous, C., Delaney, S., Hsu, H. & Mesa, V. (2010)	5	elementary	student and teacher	fractions (addition and subtraction) - presentations of content and expectations	Cyprus, Ireland, Taiwan
Chval, K. & Hicks, S. (2009)		elementary		calculator use	U.S.

Table 29 (cont'd)

Ding, M. & Li, X. (2010)	3	elementary	student	distributive property - problem context, type, and variability	China, U.S.
Fan, L. & Zhu, Y. (2007)	9	high	student	problem-solving procedures (Polya)	China, Singapore, U.S.
Flanders (1987)	6	elementary/ middle	student	new content versus review of old	U.S.
Herbel-Eisenmann, B. (2007)	1	middle	student	voice	U.S.
Hook, W., Bishop, W. & Hook, J. (2007)	1	elementary	student	comparison to top TIMSS curriculum materials	U.S.
Hoven, J. & Garelick, B. (2007)	1	elementary	student	bar models for addition and subtraction	Singapore
Huntley, M. (2008).	2	middle	student	Framework for analyzing curriculum materials	U.S.

Table 29 (cont'd)

Jones, D. (2004).	8	middle	student	probability - historical (extent and nature)	U.S.
Lee, K., & Smith III, J. P. (2011)	4	elementary	student and teacher	length (content and presentation in text)	Singapore, U.S.
Lee, M. & Messner, S. (2000)	10	middle/high	student	concatenations and order of operations - instructional emphasis	U.S.
Leinwand, S. & Ginsburg, A. (2007)	1	elementary	student	content and organization (broad)	Singapore
Li, J. (2004)	2	elementary	teacher	whole number multiplication - comparing teachers' guides US and China	China, U.S.
Li, Y. (2000)	9	middle	student	integer addition and subtraction	China, U.S.
Martin, T., Hunt, C., Lannin, W., Leonard Jr., W., Marshall, G. & Wares, A. (2001)	5	high	student	Comparison to NCTM Standards	U.S.

Table 29 (cont'd)

Mesa, V. (2004)	24	middle	student	function - describe the practices associated with function notation	15 countries
Michalowicz, K. & Howard, A. (2003)	100s	elementary/middle/high	student and teacher	historical development of pedagogy and content in text	Canada, Mexico, U.S.
Newton, D. & Newton, L. (2006)	18	Elementary	teacher	teachers' attention to reason	England
Nissen, P. (2000).	9	elementary/middle/high	student	geometry/transformational geometry	U.S.
Olsen, T. (2010)	4	Middle	student	Patterning tasks - articulated learning trajectories	U.S.
Pickreign, J. & Capps, L. (2000).	5	Elementary	student	comparing to NCTM Standards - geometry	U.S.
Schmidt, W., Houang, R. & Cogan, L. (2002)	45	elementary/middle/high	student	content and organization	45 countries

Table 29 (cont'd)

Schmidt, W.H., McKnight, C.C., Valverde, G.A., Houang, R.T. & Wiley, D.E. (1997)	45	elementary/ middle/high	student	content and organization	45 countries
Smith III, J.P., Dietiker, L., Lee, K., Males, L.M., & Mosier, A. (in preparation)	3	Elementary	student and teacher	length (content and presentation in text)	U.S.
Smith III, J.P., Gonulates, F., Males, L.M., & Mosier, A. (in preparation)	3	Elementary	student and teacher	area (content and presentation in text)	U.S.
Star, J., Herbel-Eisenmann, B. & Smith III (2000)	2	middle/high	student	Algebra	U.S.
Stein & Kim (2009)	2	Elementary	teacher	"educativeness" of curriculum materials - supports for teachers	U.S.
Stylianides, G. (2005)	1	Middle	student and teacher	proof opportunities	U.S.
Stylianides, G. (2007)	1	Middle	teacher	Opportunities for teachers to learn about proof	U.S.

Table 29 (cont'd)

Valverde, G., Bianchi, L., Wolfe, R., Schmidt, W. & Houang, R. (2002)	45	elementary/ middle/high	student	content and organization	45 countries
Watanabe, T. (2001)	4	elementary	teacher	organization/cont ent	Japan, U.S.
Watanabe, T. (2003).	4	elementary	student	fraction	Japan, U.S.
Weinberg & Weisner. (2011).	1	college	students	framework for analyzing the reading of textbooks	U.S.
Yang, D., Reys, R. & Wu, L. (2010)	3	elementary/ middle	student	fractions	Singapore, Taiwan, U.S.

APPENDIX B

List of all imperatives used by CMP in Variables and Patterns

Ask	Be sure students understand	Make sure students
Have students share	Be sure use	discuss
Use	Call	Make sure students
Encourage	Collect	mention
Discuss	Compare	Make sure students points
Tell	Conduct	are made
Let	Connect	Make sure students can
Explain	Create	talk
Remind	Display	Make sure students
Choose	Do not discuss	explain
Allow	Elicit	Make sure you add
Emphasize	Find	Pair
Have students work	Go over	Put
Read	Go through	Refer
Continue	Highlight	Say
Describe	Have students adjust	Suggest
Have students look	Have students construct	Talk
Have students press	Have students draw	Translate
Make sure students understand	Have students examine	Verify
Move on	Have students explain	Write
Be sure s. use	Have students follow	
Check	Have students graph	
Determine	Have students make	
Do not worry	Have students perform	
Five	Have students present	
Help	Have students record	
Have students discuss	Have students refer	
Have students put	Have students set up	
Have students relate	Have students sketch	
Have students speculate	Have students summarize	
Look	Have students take	
Make	Have students talk	
Point out	Inform	
Repeat	Introduce	

List of all imperatives used by CMP in Kaleidoscopes, Hubcaps, and Mirrors

Ask
Encourage
Have students work
Use
Have students share
Remind
Call
Discuss
Arrange
Help
Choose
Give
Draw
Direct
Explain
Review
Look
Read
Let
Challenge
Focus
Set
Distribute
Point out
Talk
Collect
Circulate
Have students demonstrate
Pose
Check
Put

List of all imperatives used by Glencoe in Algebra: Number Patterns and Functions

Use	Have students simulate
Ask	Have students take notes
Remind	Include
Tell	Log on
Create	Make sure student read
Write	Make sure students fold
Explain	Make sure students understand
Suggest	Monitor
Check	Partner up
Encourage	Present
Have students complete	Recommend
Have students create	Refer
Point out	Repeat
Customize	Replace
Discuss	Review
Give	Roll
Have students work in groups	Search
Have students write	See
Post	Spot check
Provide	Start with
Allow	Stress
Assign	Urge
Bring	Watch
Challenge	
Consider	
Display	
Fill in	
Guide	
Have students add sticky notes	
Have students compare	
Have students count	
Have students discuss	
Have students draw	
Have students exchange	
Have students explain	
Have students find	
Have students look	
Have students pronounce	
Have students review	

List of all imperatives used by Glencoe in Geometry: Polygons

Use
Ask
Remind
Have students write
Make
Encourage
Check
Draw
Point out
Have students complete
Have students tell
Have students draw
Tell
Display
Have students create
Have students use
Separate
Suggest
Demonstrate
Discuss
Explain
Have students exchange
Have students make
Have students measure
Have students update
Write

List of all imperatives used by MiC in Comparing Quantities

Ask	Make sure students are aware
Have students share	Make sure students can find
Point out	Note
Encourage	Reinforce
Provide	Remember
Be sure students understand	Remind
Discuss	Say
Have parents review	See
Suggest	Tell
Copy	
Have students give	
Read	
Allow	
Be sure to discuss	
Have students find	
Have students try	
Add	
Be sure stud can explain	
Begin	
Challenge	
Do not expect	
Do not label one strategy	
Do not try	
Expect	
Give	
Have students act	
Have students add	
Have students begin	
Have students circle	
Have students create	
Have students cross	
Have students explain	
Have students play	
Have students put	
Have students read	
Have students use	
Have students work	
Leave	
Let	
Make	

List of all imperatives used by MiC in Triangles and Beyond

Ask
Discuss
Encourage
Remind
Add
Have students discuss
Have students reread
Be sure students know
Have students model
Point out
Have students explain
Let
Advise
Save
Use
Have students review
Involve
Show
Have students work
Have students share

List of all imperatives used by UCSMP in Using Variables

Ask
Give
Have students use
Point out
Encourage
Have students create
Have students work
Remind
Have students write
Emphasize
Administer
Have students look
Assure
Have students identify
Watch
Have students check
Help
ES evaluate
Look
Write
Compare
Relate
Call
Refer
Notice
Have students exchange
Have students draw
Display
Show
Have students record
Advise
Return
Assign

List of all imperatives used by UCSMP in Some Important Geometry Ideas

Ask
Point Out
Emphasize
Use
Be sure students include
Draw
Encourage
Explain
Extend
Advise
Have students trace
Administer
Have students solve
Assign

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