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# BELIEFS AND PRACTICES: A STUDY OF MODERNNESS <br> IN MATHEMATICS TEACHING IN SELECTED MICHIGAN PUBLIC ELEMENTARY SCHOOL CLASSROOMS 

By
J. Thomas Murphy

## A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

College of Education

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## ABSTRACT

# BELIEFS AND PRACTICES: A STUDY OF MODERNNESS IN MATHEMATICS TEACHING IN SELECTED MICHIGAN PUBLIC ELEMENTARY SCHOOL CLASSROOMS 

By
J. Thomas Murphy

## Introduction

At this date, after many years of the "modern" approach in the teaching of elementary school mathematics, the extent to which modernness has been implemented by the classroom teacher appears to be unclear.

## Purpose

The purposes of this study were: (l) to develop a definition of the "modern mathematics approach" reflecting a pooling of recent writing and research and validated by expert opinion; (2) to determine the extent to which there is a difference between what the theorists say and what the teachers believe, as relates to elementary school mathematics; (3) to ascertain the practices of respondents and the occurrence rate of those practices in elementary classrooms; and (4) to analyze the data collected by a survey questionnaire concerning elementary classroom teachers'
utilization of validated modern mathematics teaching strategies, in light of specific variables.

## Design

Eight mathematics education experts were polled and asked to validate (as modern or non-modern) items on a list of mathematics teaching principles that were drawn from current research and writing. Based on this list of validated principles, a questionnaire instrument was developed that consisted of two parts. Part $I$ asked the respondent what he or she believed about mathematics ideology and Part II asked what the respondent actually did on a day-today basis in the classroom. The responses on the two parts of the instrument were analyzed statistically to measure the significance of the four research hypotheses that the study sought to investigate. A Jennrich two-way analysis of variance with repeated measures technique, as well as a one-way analysis of variance with unequal subclasses, was applied to the returned data. In both statistical procedures the level of significance was .025.

## Sample

Each of the mathematics experts utilized in this study was a faculty member of a different teacher certificate granting university in the State of Michigan. The random sample of classroom teachers used in the study were
a11 elementary classroom teachers who taught in one of sixteen southwestern Michigan school districts. These districts were chosen for inclusion in this study because of their size and per pupil expenditure. The questionnaire instrument was mailed to the home address of each respondent selected. The response rate was 93.3\%.

## Conclusions

On the basis of the data collected and analyzed in this study, the following conclusions seem warranted:

1. The responses of a group of mathematics experts and a randomly selected sample of one hundred eighty elementary school classroom teachers in Michigan to a list of contemporary mathematical principles of instruction, revealed a significant difference at the . 025 level between the reported beliefs of the two groups as to the consistency of a modern approach to teaching mathematics to elementary school learners.
2. The sample of one hundred eighty elementary classroom teachers were also asked to respond to a list of practices correlated to the set of contemporary mathematical principles of instruction. A significant difference was indicated between what the teachers indicated they believed to be a modern approach to teaching mathematics, and what they reported as actually practicing in their classrooms.
3. When a score of "modernness" was established for each of the respondents in the study, there was no significant difference indicated when the scores were grouped, a mean taken, and compared across the variable of sizes of school district populations in the districts included in this study.
4. When a score of "modernness" was established for each respondent in the study, there was no significant difference indicated when the scores were grouped, a mean taken, and compared across the variable of general fund expenditure per pupil in each of the districts included in this study.

## ACKNOWLEDGMENTS

This study would not have been executed or brought to fruition, without the assistance and guidance of numerous individuals, most of whom must necessarily go unnamed, but not unremembered.

The writer wishes to express his deepest appreciation to Dr. Calhoun C. Collier, chairman of the guidance committee. His suggestion brought me into the field of elementary education, and his tutorage has immensely facilitated the furtherance of my professional education, and the completion of this graduate program.

In addition, I would like to thank the other members of the guidance committee, Dr. Janet Alleman, Dr. Richard Marquard, and Dr. Donald olmsted for their encouragement, understanding, and assistance. Their help was extended in unstinted amounts, not only in the completion of this writing, but from the beginning of my doctoral program at Michigan State University.

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## CHAPTER I

## INTRODUCTION

The enigma which so often confronts those who attempt to improve educational practices is the wide divergence between theoretical ideas of effective teaching strategies, as included in the literature and current research, and the beliefs of classroom teachers relating to appropriate instruction with a particular group of learners. Perhaps in no other professional vocation, does there exist the volume of research as exists in education. Perhaps further, because of the fact that so many theories have been developed, they are sometimes often conflicting with one another, with the result being only a limited acceptance of any particular theory by classroom teachers in general. The complaint is often heard from teacher-educators and school administrators, that classroom teachers seldom utilize the findings in the literature available to them, which in reality could have dramatic impact upon the effectiveness of their teaching. Assuming that the problem is crucial in the improvement of education, there are direct implications for professionals of teacher-education, professionals in public school classrooms, and all other professionals in the field of
education. If education is going to continue to expend vast sums of money, time, and effort for the purpose of research, opinion or editorial writing, in-service education, workshops, client-institution relationships, university course work, etc., then it is imperative, in the researcher's opinion, that the findings of such research and the results of such workshops and classes become actualized in belief and practice among classroom teachers, to a far greater extent than currently exists.

The advent of "modern mathematics" upon the elementary education scene was initially, and still remains, one of the most misunderstood and unjustifiably attacked pedagogical developments of recent years. Unlike other new innovations in other disciplines of elementary education, which are recognized and utilized by only a few educators, (let alone those outside the profession-such as parents) modern math has become almost a household word. Much of the theoretical base from which current mathematics arose, was research oriented and corroborated by expert opinion. The typical elementary teacher was first made aware of the "change" through in-service education which in the researcher's experience was an ineffectual method of creating real change in classrooms. This "single-shot" indoctrination with little or no follow up procedures, coupled with an existing skepticism of research resulted in poor initial implementation.

At this date, after many years of the "modern" approach, the extent to which "modernness" has been accepted as it concerns elementary mathematics instruction appears to be unclear.

The researcher views the improvement of education as an important undertaking and with this criteria in mind this study assumes the following purposes:

1. To develop a definition of the "modern mathematics approach" reflecting a pooling of recent writing and research and validated by expert opinion.
2. To analyze the data collected by a survey questionnaire concerning elementary classroom teachers utilization of validated modern mathematics teaching strategies, in light of specific Variables.
3. To determine the extent to which there is a difference between what the theorists say and what the teachers believe, as relates to elementary school mathematics.
4. To ascertain the practices of respondents and the occurence rate of those practices in elementary classrooms.

## Conceptual Framework of the Study

The modern approach to the teaching of elementary mathematics is broad, generalized, and not very well defined.

In fact, this is part of the problem. It is necessary to limit the conceptual framework with which this study will deal to specific aspects of a modern approach to instruction. This study therefore, limits its emphasis to modern mathematics as it relates to (a) teacher-student attitude and interest; (b) discovery-inquiry versus expository learning; (c) influential factors of organization; (d) modern versus traditional organization; (e) techniques of modern versus traditional approaches; (f) media-materials of modern versus traditional instruction.

## Significance of the Study

There is often a gap between understandings of elementary mathematics principles, as developed and exposed by literature and research, and the daily practice taking place in the elementary classroom. It is a fair assumption that the practices of a given classroom are formulated by the teacher of that classroom, who has the greatest impact upon the scope of what is to be learned by that particular group of learners. In the case of elementary school mathematics, and in particular the recent development of "modern" mathematics techniques, the attitude of the classroom teacher was necessarily changed from one of dogmatic computational skill development, to a broader conceptual framework of mathematical understandings and applications on the part of the learner. A significant aspect of this
study is to ascertain the degree of skepticism and acceptance of modern approaches on the part of classroom teachers. This is not to say that all or even most of the literature and research is applicable to the majority of classroom teachers. It is important, however, that if in fact, modern mathematics is a valuable technique in facilitating academic growth in children to a fuller understanding of mathematics in particular and education in general, then some change should take place in the dissemination techniques of modern thought, as well as a different organization of implementation controls at the public school level. To this point, the significance of this study might be considered appropriate only to existing classroom teachers, who through many months and years of experience have developed techniques unique to themselves. The significance of this study also should have relevance to the beginning or entering teacher through new organizational methods in teacher training. Combs ${ }^{l}$ asserts that "it is at the source of the supply-our teacher preparation programs that review and innovation are most critically called for if we are to bring about the improvements we need in education." (P. v) He calls for a reexamination of teacher education in the context of a framework evolving from "our changing social needs and purposes on the one hand, and our new understandings about
$1_{\text {A. W. Combs, The Professional Education of Teachers }}$ (Boston: Allyn and Bacon, 1965).
behavior and learning on the other." (P. vi) His belief is particularly apropo to "modern mathematics" and its concept based understanding because he further believes an individual's learning is affected only in the degree to which he has discovered its personal meaning for him.

A number of other writers also see education as needing conceptual frameworks. (Goodlad, Howard, Smith, Wattenberg). 2 Wiles $^{3}$ expresses a similar opinion on dissemination of educational ideas by calling for an emphasis on theories of teaching, rather than on a particular model or pattern of teaching. If in fact, we create "thinking mathematical" instructors, the result should be more "mathematical thinking" on the part of the learner, as opposed to responding in a programmed manner following extensive formal analysis procedures of a particular skill.

Statistical Hypotheses to be Tested

Hypothesis I
Ho There is no significant difference between expert opinion on the consistancy of a modern mathematics program and the opinion of all the teachers included in this study.
${ }^{2}$ John I. Goodlad, "Next Steps in Cooperation in Teacher Education: By Who and for What?" Educational Research. 1962. 43, 223-27; E. Z. Howard, "Needed: A Conceptual Scheme for Teacher Education." School Review, 1963, 71, 12-26; E. R. Smith, (ed)., Teacher Education: A Reappraisal, (New York: Harper and Row, 1962); and W. W. Wattenberg, "Evidence and the Problems of Teacher Education," Teachers College Record, 1963, 64. 374-80.
${ }^{3}$ Kimball Wiles, "The Teacher Education We Need," Theory into Practice, 1967, 6, 260-65.

Ho There is no significant difference between what the respondents say they believe constitutes an effective mathematics program and what they say they do in their day to day activities in the classroom.

Hypothesis III

Hypothesis IV
Ho There is no significant difference in "modernness" between teachers of varying school district sizes.
$H_{0}$ There is no significant difference from school districts that have varying per pupil general fund expenditures.

## Assumptions Upon Which the Study is Based

1. It is assumed that the development of current pedagogical practices on the part of the classroom teacher is essential to effect maximal mathematical learning growth on the part of the student.
2. It is assumed that there exists a lack of implementation on the part of classroom teachers, of research, workshops, literature, etc. as they relate to elementary mathematics instruction.
3. It is assumed that the general fund per pupil expenditure of a school district is divided proportionally and equally across all academic areas within the curriculum of that school district.
4. It is assumed that the attitudes of teachers about mathematics affords a key to the indication of the type of pupil acceptance and achievement in mathematics.
5. It is assumed that the self-reports of teachers and experts are valid representations of their true feelings, belief systems, and practices within the delimitations of any self-report (Combs, ${ }^{4}$ Combs and Syngg ${ }^{5}$ ).
6. It is assumed that the sample selected from southwest Michigan is representative of classroom teachers within a similar population regardless of geographical location.
7. It is assumed that content of elementary mathematics can be considered dichotomous with instructional strategies of elementary mathematics.
8. It is assumed that the analysis of variance correlation coefficients are appropriate statistical treatments for this exploratory study.
${ }^{4}$ op. cit.
${ }^{5}$ A. W. Combs and D. Snygg, Individual Behavior: A Perceptual Approach to Behavior, (New York: Harper and Row, 19591.

## General Procedures of the Study

In order to investigate the preceding hypotheses, the researcher conducted a mailed questionnaire technique among a large number of public elementary classroom teachers. The items included on the questionnaire were drawn from recent literature and research as being part of a modern approach to mathematics instruction. The items were validated by a panel of mathematics educators from the various state institutions of higher learning in Michigan. After field testing the original instrument, so as to allow the researcher the opportunity to refine the inquiry technique, the final copy of the questionnaire was sent to randomly drawn classroom teachers, as well as back to the same panel of experts. This allowed the researcher the opportunity to compare classroom teacher perceptions and beliefs about modern approaches to elementary mathematics with expert's beliefs on the same principles.

The study also sought to ascertain if there was any difference in "modernness" when teachers in districts of varying wealth and size were compared. Each district selected for inclusion in the study was chosen on the basis of geographic location, per pupil expenditure, and enrollment size. This allowed for a population of districts to be chosen for statistical purposes.

Upon the complete collection of the data as returned by the questionnaire, statistical analysis was performed for the purpose of testing the hypotheses stated earlier.

## Definition of Terms Used

Attitude--is defined as the degree of positive or negative affect, or feeling, associated with some psychological or real object.

Base--is defined as selected items of mathematical instruction in elementary school mathematics, that are drawn from literature and research and validated by expert opinion.

Elementary Teacher--is defined as a classroom instructor in grades Kindergarten through six in any public school selected for inclusion in this study.

Expert--is defined as a member of a University staff in the field of elementary mathematics education in the State of Michigan, or other individual who has exhibited high competance in the field.

Client-institution relationship--is defined as the situation in which an individual teacher or a single school district utilize personnel from either public or private institutions, such as a University, for the purposes of consultation on a particular concern.

Modern Mathematics Approach--shall mean the approach to mathematics organization and understanding reflected in the base of this study.

Modernness-is defined as the numerical mean score of the means of beliefs and practices, as established by responses to the base.

Large--is defined as a school district with more than $10,000 \mathrm{~K}-12$ students.

Mid-size--is defined as a school district with more than 2,500 , but less than $10,000 \mathrm{~K}-12$ students.

Smal1--is defined as a school district with less than $2,500 \mathrm{~K}-12 \mathrm{students}$.

High Expenditure--is defined as a school district spending more than $\$ 895$ per pupil in general fund expenditures.

Middle Expenditure--is defined as a school district spending more than $\$ 800$ per pupil in general fund expenditures, but less than $\$ 895$.

Low Expenditure--is defined as a school district spending less than $\$ 800$ per pupil in general fund expenditures.

## Scope and Limitations

This study is designed to explore the beliefs and practices of elementary classroom teachers as they relate to mathematics instruction and organization, and to compare those beliefs and practices with a group of mathematics education experts. It further compares the beliefs and practices of the teachers among two selected variables--
size and wealth of district. This will allow the researcher to test the widely voiced belief that rich-large or rich-mid-size school districts are more "modern" in their approach to mathematics than poorer-small districts.

## Organization of the Study

The study is composed of five chapters. Chapter I presents an introduction and overview of the problem and the study, along with a rationale for undertaking the study.

The second chapter contains the related research on the topic of mathematics education, and it is from this research that the base was developed. The third chapter describes the research procedures of the study, with Chapter IV being devoted to a statistical analysis of the returned data. A summary, conclusion, implications and recommendations are presented in Chapter $V$.

REVIEW OF RESEARCH AND RELATED LITERATURE

The research and literature related to current elementary mathematics education techniques, strategies, and organizational patterns is extensive. In order to select the literature and research most pertinent to this study, careful attention was given to research that involved itself with the aspects of elementary mathematics contained in the conceptual framework of this study. The importance of this related research cannot be overemphasized, due to the unique nature of the way it was utilized in this study. The base of this study, as defined in Chapter $I$, was taken from a direct synthesis of this research, which was in turn validated by experts in mathematics education, and then submitted in a questionnaire format to the random sample of elementary classroom teachers.

This chapter presents both negative and positive research and literature on each of the aspects of elementary mathematics that were part of the study. Because the base of this study may be debatable among educators, it was felt that both pro and con arguments and points of view should be included in the study, so as to allow fair representation
of all approaches to the topic. The concepts for discussion in this chapter were selected because they represent the major concerns of current elementary mathematics thinking, as distinguished from organizational patterns and modes of thinking that are of a more traditional point of view. With this as the criteria in mind, the chapter is divided into concerns of modern mathematics and the research and literature of each discussed separately.

## Influential Factors on Organization and Techniques

Education remains a profession that demands and receives a variety of inputs for its effectiveness. The dissemination of academic principles, as well as sociological, moral, and philosophical ideas to a group of learners, each of whom is characterized by his/her uniqueness and individuality, over a period of years through formal school experiences, is a fine and difficult art. Inherent in that dissemination is the uniqueness and individuality of the educator, who is continually bombarded with mechanisms of change in the form of literature, in-service education, personal experience, etc. These factors of influence have an effect upon the educator as well as the learner, and for this reason the relationship between influential factors and achievement among learners is a most important consideration.

Reid ${ }^{l}$ in a study of sources of information used by teachers, cited professional publications, professional preparation, and colleagues in that order, as having the greatest input for teachers. Gibney ${ }^{2}$ looked at demographic factors and the influence they had on mathematics understandings of elementary school teachers, and concluded that there were no significant differences among teachers when they were grouped by size of community in which they were teaching; but that there was a difference over the variable of experience.

As will be demonstrated later in this chapter, the attitude of the teacher can be considered an important factor in pupil achievement; and the factors of influence will mold the attitude. As an example, Hollander ${ }^{3}$ in a study of incentives to arithmetic achievement among fifth and sixth grade learners, found that pupils worked faster when told they could earn a candy bar if they improved their scores on a second test, and with greater accuracy when told they had performed exceptionally well. on the other hand, those reproved by being told their scores were
${ }^{1}$ Isaih Reid, Diffusion of Innovations in Elementary Schools: A Study of the Primary Sources of Information used by Teachers, (Cornell University, 1969), DAO 30A: 3697-3698, March 1970.
${ }^{2}$ Thomas C. Gibney, et al.. What Influences the Mathematicsunderstanding of Elementary School Teachers, Elem. Sch. Journal 70:367-372, April 1970.
${ }^{3}$ Elaine Kind Hollander, The Effects of Various Incentives on Fifth and Sixth Grade Inner-City Children's Performance on an Arithmetic Test, (The American University, 1968) Diss Abst. 29A:1130, October 1968.
very low, attempted fewer items and made more errors than were made under any other condition.

Other writers and researchers have lessened the importance of the teacher upon mathematics achievement in elementary schools and concluded that other factors were of most significance. Wrigley ${ }^{4}$ for example, concluded that high intelligence was the most important single factor for success in mathematics. Unkel ${ }^{5}$ found that socioeconomic status has a significant effect on achievement in elementary mathematics, at all intelligence levels. . . as economic level increased so did the achievement in mathematics. In a similar study done later, Mckee ${ }^{6}$ found that elementary mathematics achievement wis not affected by the social level of community so much, as it was purely by the economic level. It appears obvious that there do exist factors that influence learning, that direct their attention to both teachers and learners, and that even though no consensus can be reached as to which ones are

[^0]most influential, it becomes necessary to acquaint oneself with the potential of these influences if mathematics learning is to be increased.

Much additional research has shown that age and intelligence are highly related to the ability to learn various specific mathematical ideas. Westbrook ${ }^{7}$ noted that the intellectual factors of reasoning and verbal meaning were related to achievement in mathematics in grades four, five, and six. Meconi ${ }^{8}$ found that pupils with high ability were able to learn under any method he investigated, while Ebert ${ }^{9}$ in an older study (1946) found large variations in generalization ability, depending on the mathematical concept involved, the intelligence level of the learner, and the visual pattern within which the concept was presented.

## Attitudes-Interests and Mathematical Learning

Perhaps due to the nature of attitude development and because attitude is a difficult entity to evaluate or
${ }^{7}$ Helen Rose Westbrook, Intellectual Processes Related to Mathematics Achievement at Grades Four, Five and Six, (University of Georgia, 1965) Diss. Abst. 26:6520, May 1966.
${ }^{8}$ L. J. Meconi, Concept Learning and Retention in Mathematics, Journal of Experimental Education 36:51-57, Fall, 1967.
${ }^{9}$ Rueben S. Ebert, Generalization Abilities in Mathematics, Journal of Educational Research 39:671-681, May 1966.
test, it is not completely accepted by writers and researchers that is has the effect upon achievement that it might be thought to have, especially in the area of elementary mathematics. Caezza and Wess ${ }^{10}$ reported that low correlations existed between pupil achievement in elementary mathematics and pupil or teacher attitude toward mathematics. In another study done later in the same year, Cox ${ }^{l l}$ found that pupil achievement was not found to be affected by the teachers knowledge of or interest in mathematics.

Pinetz, Cheikin and Bassham ${ }^{12}$ found no significant difference between teacher attitude and achievement among students. Many other authors have found similar results concerning teacher attitude and student performance. Some
${ }^{10}$ John F. Caezza, A Study of Teacher Experience, Knowledge of and Attitude Toward Mathematics and the Relationship of these Variables to Elementary School Pupils Attitudes Toward and Achievement in Mathematics, Diss. Abst. 31A: 921-922, September 1970; Roger G. Wess, An Analysis of the Relationship of Teachers Attitudes as Compared to Pupil Attitudes and Achievement in Mathematics, Diss. Abst. 30A: 3844-3845, March 1970.
$11_{1}$ Linda A. Cox, Study of Pupil Achievement in Mathematics and Teacher Competence in Mathematics, (University of Kansas, 1970), DIA 31A:2767-2768, December 1970.
${ }^{12}$ Mildred C. Pinetz, "The Relationship Between Teacher's Attitudes and Effectiveness in the Classroom", Diss. Abst. 24:2340, No. 6, 1963: Martin L. Cheikin, "An Investigation of the Effect of Measured Teacher Attitude on Selected Eighth Grade Students", Diss. Abst. 28:4042-A, No. 10, 1967; Harrell Clark Bassham, "Relationship of Pupil Gain in Arithmetic Achievement to Certain Teacher Characteristics", Diss Abst. 21:1839, No. 7, 1960.
researchers have accepted the findings of Pinetz and others and addressed themselves to pupil attitude and its relationship to achievement. In a publication entitled Using Research: A Key to Elementary School Mathematics, written in 1970 and edited by Marilyn Suydam at The Pennsylvania State University, the following was concluded:

Many people believe that mathematics is disliked by most pupils. . . It is true that in some surveys a significant proportion of pupils rated mathematics as the least liked subject. . . however, there is no consistent body of research evidence to support the popular belief that there is significant positive relationship between pupil attitudes toward mathematics and pupil achievement in mathematics. ${ }^{13}$

Relating this statement to the recent developments of modern mathematics, Abrego ${ }^{14}$ found no relationship between achievement and pupil attitude in either traditional or newer mathematics programs for elementary pupils. Though this seems to be a fairly large and representative sample of the research and literature that negates the causal relationship of attitude and achievement, there is equally large and representative literature and research of the opposite view.
${ }^{13}$ Mariiyn N , Suydam, Interpretive Study of Research and Development in Elementary School Mathematics. A Closer View. . Attitudes and Interests, Center for Cooperative Research with Schools, (The Pennsylvania State University, 1970), P. 4.

14 mildred B. Abrego, Children's Attitudes Toward Arithmetic, Arithmetic Teacher 13:206-208, March 1966.

Whipkey ${ }^{15}$ found admittedly a small, but in his opinion important relationship between mathematical attitude and achievement. Phillips ${ }^{16}$ found that the attitudes toward mathematics on the part of pupils was significantly related to the attitude of their previous grade teacher, and that in fact, the type of teacher attitude encountered by the pupil was significantly related to his present attitude and achievement.

Sex differences may play an important role in establishing attitude and achievement relationships. Boys seem to prefer mathematics slightly more than do girls, especially toward the upper elementary grades. Chase and wilson; Stright and Neale ${ }^{17}$ found that attitudes and arithemtic achievement are significantly correlated for boys only.
${ }^{15}$ Kenneth L. Whipkey, A study of the Interrelationship Between Mathematical Attitude and Mathematical Achievement, (Case Western Reserve University, 1969) DAI 30A:3803, March 1970.
${ }^{16}$ Robert B. Phillips, Teacher Attitude as Related to Student Attitude and Achievement in Elementary School Mathematics, Diss Abst. 30A:4316-4317, April 1970.
${ }^{17}$ W. Linwood Chase and Gilbert Wilson, Preference Studies in Elementary School, Journal of Education, 140:2-8, April 1968; Virginia M. Stright, A Study of the Attitudes Toward Arithmetic of Students and Teachers in the Third, Fourth and Sixth Grades, Arith. Teacher 7:280-286, October 1960; Daniel C. Neale, et al. Relationship Between Attitudes Toward School Subjects and School Achievement, Journal of Educational Research 63:232-237. January 1970.

Two authors address themselves to the question of pupil interest and the modern mathematics approach. Feldhake ${ }^{18}$ reported that high achieving pupils found new mathematics programs more interesting than did low achievers, and Dutton ${ }^{19}$ observed that fewer pupils were afraid of mathematics today, enjoyed it more, and achieved better on mathematical problems than ten years ago. Dutton went on however, to discover that fewer pupils today see the practical use of mathematics than ever before.

In summation, teacher and pupil attitude toward elementary mathematics and the resultant pupil achievement may or may not be significantly correlated on the basis of current research and literature.

> Modern Versus Traditional Organization and Learning

The research completed on the topic of comparisons between newer or modern mathematics and older or traditional methods of elementary mathematics instruction, is large and varied. The variables one researcher seeks to control in

18 Herbert J. Feldhake, Student Acceptance of New Mathematics Programs, Arithmetic Teacher 13:14-19, January 1966.
${ }^{19}$ Wilbur H. Dutton, Another Look at Attitudes of Pupils Toward Arithmetic, Elem. School Jour. 68:265-268, February 1968.
his/her study, are of little importance to another researcher, and the conditions in which a particular author or editor finds himself vary greatly oftentimes from conditions found by another. The literature and research included in this study reflects this dichotomy in research findings on the topic of modern versus traditional organizations and their effects.

Many authors and researchers have found that no differences exist in achievement and attitude on the part of the learner regardless of the type of instruction he receives. Hendrickson ${ }^{20}$ found no differences in achievement between groups using a mathematics laboratory, an enrichment problem technique, or a conventional approach, though all groups gained. The study further reports that attitude improvement was found in the conventional group more than any other. In an experimental study among fourth graders, Bartel ${ }^{2 l}$ established a control group in which instruction was traditional in nature, was not individualized, and did not contain "new mathematics" content. He also established an experimental group in which instruction was highly individualized and included "new mathematics"
${ }^{20}$ Arthur D. Hendrickson, A Study of the Relative Effectiveness of Three Methods of Teaching Mathematics to Prospective Elementary School Teachers (University of Minnesota, 1969) DAI 31A:1117, September 1970.
${ }^{21}$ Elaine $V$. Bartel. A Study of the Feasibility of an Individualized Instructional Program in Elementary School Mathematics (University of Wisconsin, 1965) Diss Abst. 26:5284, March 1966.
content. The finding was that no difference in achieve $\rightarrow$ ment was found when evaluation was measured by a standardized test.

The question of what the student learns as a result of modern programs as compared to traditional programs, is the confounding variable in many studies. In a review of 1iterature conducted by suydam ${ }^{22}$ it was found that generally, modern programs are as effective as traditional programs in developing traditional mathematical skills. In an earlier study. Payne ${ }^{23}$ used an experimental approach and reported similar findings. Wright ${ }^{24}$ went a step further and stated that he found no significant differences in learning traditional concepts between those using a traditional program and two separate modern programs, but that those individuals in the modern programs achieved higher scores on a test of modern concepts. Gardner ${ }^{25}$ made the assumption that standardized arithmetic tests could measure program effectiveness and on that basis concluded that there existed no
${ }^{22}$ Marilyn $N$. Suydam, Interpretive Study of Research and Development in Elementary School Mathematics, Overview. Planning and for Instruction. Center for Cooperative Research with Schools. (The Pennsylvania State University, 1970), p. 2.
${ }^{23}$ Holland Payne, What About Modern Programs in Mathematics? Mathematics Teacher 58:422-424, May 1965.
${ }^{24}$ R. E. Wright, Something old, Something New, School Science and Mathe 70:707-712, November 1970.

25
Harvey $F$. Gardner, A Comparison of the Modern and Traditional Curriculums and their Effects on Standardized Test Scores (Southern Illinois University, 1969) DAI 30A: 2847-2848, January 1970.
significant growth differences between modern and traditional mathematics curriculums in grades three and four. Hungerford ${ }^{26}$ and Graft ${ }^{27}$ studied fourth, fifth, and sixth graders, some of whom were taught utilizing Science Mathematics Study Group (SMSG) materials and some using conventional text materials and concluded that the SMSG group understood principles of multiplication better than the conventional groups, yet no significant difference was found in computation. It was shown that the data significantly favored the non-SMSG group on a test of conventional arithmetic and the SMSG group on a test of contemporary mathematics. In a more current study, Crowder ${ }^{28}$ worked with fourth, fifth, and sixth graders and concluded that pupils in the modern program achieved above the normal rate of achievement in arithmetic computation and concepts, but not in applications.

Questions often arise as to the necessary organizational patterns and classroom management procedures that are supposedly inherent in modern elementary mathematics instruction.

26 Ann D. Hungerman, Achievement and Attitude of SixthGrade Pupils in Conventional and Contemporary Mathematics Program Arith. Teacher 14:30-39, January 1967.
${ }^{27}$ William D. Graft and Arden K. Ruddell, Cognitive Outcomes of the SMSG Mathe Program in Grades Four, Five and Six, Arith Teacher 15:161-165, February 1968.
${ }^{28}$ Richard $P$. Crowder, The Effect of a Modern Mathematics Program on Achievement in Arith and on Intelligence Test Scores of Children in Grades Four thru Six, (University of Arkansas) Diss. Abs. 30A: 2901, January 1970.

As in the case of discussing the effects of modern and traditional mathematics instruction, organization of the instruction and the classroom, has been similarly researched with little or no concensus readily apparent to the reader. As an example, Sherer ${ }^{29}$ found that low achieving pupils in grades three through seven, taught by author developed materials, using instructional aids such as drawings, counters, number lines, and charts, showed significantly greater gains in achievement than those taught by a traditional approach. Lindgren ${ }^{30}$ on the other hand, reported that no significant differences in achievement existed for the learner between conventional teaching and any other organizational pattern.

Further research on modern versus traditional organization leaves the reader free to use his/her own belief system in accepting or rejecting modern or traditional ideologies. Ferney ${ }^{31}$ looked at the Program for
${ }^{29}$ Margaret T. Sherer, An Investigation of Remedial Procedures in Teaching Elementary School Mathematics to Low Achievers, Diss. Abst. 28A:4031-4032, April 1968.
${ }^{30}$ Richard Francis Lindgren, A Comparison of Team Learning with Learning Through Conventional Teaching as Methods in Teaching Arithmetic Reasoning in Grades Four and Five, (The University of Connecticut, 1965) Diss. Abst. 28A: 3369, March 1968.
${ }^{31}$ Gary A. Ferney, An Evaluation of a Program for Learning in Accordance with Needs, Diss. Abst. 30A:4327, April 1970.

Learning in Accordance with Needs (PLAN) organizational structure, and found that fifth graders not using PLAN achieved significantly higher on arithmetic reasoning tests than the PLAN group, and that girls scored higher than boys on PLAN organization. From this he concluded that PLAN is most appropriate to the learning styles of girls. Also on the topic of developmental reasoning abilities, Moore and Cain ${ }^{32}$ used a standardized test of logical reasoning and another of creative thinking, and claim that there is support for the idea that new approaches to teaching mathematics help develop reasoning, and that some of the creative abilities tested were fostered by new mathematics programs.

In 1969, Carry and Weaver ${ }^{33}$ reviewed the National Longitudinal Study of Mathematical Abilities (NLSMA) Which had completed extensive studies of mathematics textbooks and their relationship to the modern versus traditional dilemma. They compared achievement patterns based on thirty-eight measures over a three year period involving six textbook series--three of which were modern and three of which were conventional. The conjecture of the study
${ }^{32}$ William J. Moore and Ralph w. Cain, "The New Mathematics and Logical Reasoning and Creative Thinking Abilities", School Science and Mathematics 68:731-33. November 1968.
${ }^{33}$ Ray L. Carry and $J$. Fred Weaver. Patterns in Mathematics Achievement in Grades Four, Five and Six: X-Population NLSMA Reports, No. 10. (Stanford, California: The Leland Stanford Junior University, 1969).
was that achievement patterns would be more similar within textbook classifications than across. This hypothesis was not supported at the completion of the study. NLSMA found large differences in achievement among modern textbook groups and also among conventional textbook groups.

One of the major tenets of the modern program is that students should have options in their instruction and because they have a voice in the selection process, achievement should increase. Snyder ${ }^{34}$ however, found no significant difference in achievement between learners who were allowed to select the mathematical topics they would study and those who could choose from a limited assigned option list. Both groups gained more in reasoning tests, but less on skill tests than a third group receiving "regular" instruction. In a report on low achievers in mathematics, Devenney ${ }^{35}$ found that special programs developed for low achievers did not improve pupils' computational skills to as high a degree as a traditional program, but it did result in an attitudinal change among learners from negative to much more positive.
${ }^{34}$ Henry Duane Snyder, Jr., A Comparative-study of Two Self-Selection-Pacing Approaches to Individualized Instruction in Jr. High School Mathe, (University of Michigan, 1966) Diss Abst. 28A:159-160, July 1967.

35william S. DeVenney, Final Report on an Experiment with Junior High School Low Achievers in Mathematics SWSG Reports, No. 7 (Stanford: Stanford University Press, 1969), p. 53.

Hoover ${ }^{36}$ worked with learner independence among fourth graders and found no meaningful differences for pupils using programmed self-paced, write-in materials and those having a conventional hardbound text instructional mode.

Ward ${ }^{37}$ addressed himself to departmentalized versus self-contained organization. In a study of fourth, fifth, and sixth graders he found no difference in achievement between pupils in self-contained classes with teachers of limited in-service training, and in departmentalized classes where intensive in-service activities were conducted with the teacher.

The diversity of the literature and research as it relates to the effectiveness of modern versus traditional instruction, would seem to indicate a need for further exploration in this area. Although findings are conflicting, the literature seems to point to no significant difference in cognitive gain on the part of the learner regardless of the teaching method, or pattern of organization.

Gail L. Hoover, Learning to Learn Mathematics Independently: A Study at the Fourth Grade Level (Syracuse University, 1969) DAI 31A:293, July 1970.
${ }^{37}$ paul E. Ward, A study of Pupil Achievement on Departmentalized Grades Four, Five, and Six, (University of Georgia, 1969) DAI 30A:4749, May 1970.

## Technigues of Modern and Traditional <br> Organization and Achievement

For a long period of time it was doubted that children needed to understand what they learned in elementary mathematics. It was enough simply, if they developed high degrees of skill. It was believed that to take time to give detailed exploration and develop understanding was wasteful, as well as confounding to the learner. Then came the realization that if instruction were to be meaningful, and if the teacher was concerned about increasing the number of students who attained a high level of skill, then certain things were to be gained if the content of mathematics made sense to the learner. The problem remained one of defining meaningful instruction. In 1955, Dawson and Ruddell ${ }^{38}$ summarized many studies by other authors on the topic of meaningful mathematics instruction and concluded that meaningful instruction leads to greater retention, greater transfer, and an increased ability to solve problems independently.

Since the Dawson and Ruddell study of over fifteen years ago, many approaches or techniques have been attempted to aid in developing the meaningfulness of elementary mathematics instruction. One such technique involves
${ }^{38}$ Dan $_{\text {m. Daws on }}$ and Arden K. Ruddell, The Case of the Meaning Theory in Teaching Arithmetic, Elem. Sch. Jour. 55:393-399. March 1955.
improving pupil attitude and therefore increasing pupil achievement (it is assumed) by establishing classroom criteria for selection of the various mathematical concepts that are to be studied. Greathouse ${ }^{39}$ experimentally selected three groups of learners and taught one with a conventional drilil-computation method, a second with a group-selection or group-oriented meaningful method, and a third with an individually-selected or individually-oriented technique. His findings were that the group-oriented method resulted in significantly greater achievement than the individuallyselected, and that both the individual and group selection groups scored better than the drill-computation group.

Gleason ${ }^{41}$ conducted a study which revealed no significant difference in factual knowledge gain when pupils were either self-directed or teacher-led. Four self-directed study techniques were tried on one hundred twenty eight students enrolled in six fifth grade classes, while the control group consisted of one hundred thirty two fifth graders in seven classes of a more traditional nature.

[^1]The upper I.Q. subjects in the teacher-led group did better in general knowledge than the upper I.Q. students in the self-investigation group. On the other hand, students in the self-investigation group liked studies better than students in the teacher-led group.

The question of individualization in elementary mathematics has been widely researched and discussed. Wolff ${ }^{41}$ found that among third graders in oregon, an individualized approach to arithmetic resulted in no significant differences in achievement when compared to a non-individualized approach. In any discussion of individualization, careful attention must be paid to exactly how the concept of individual differences is being defined and exactly what it is that the researcher or author is attempting to do. What is considered as an individual technique with one researcher does not fit the definition of another. The clear understanding that can be drawn from research is however, that some type of differentiation be made in instruction between and among learners in a given classroom. The techniques can vary, but the concept is well defined.

41 Bernard Ryan Wolff, An analysis and Comparison of Individualized Instructional Practices in Arithmatic in Graded and Non-Graded Elementary Classrooms in selected Oregon School Districts (University of Oregon, 1968) Diss. Abst. 29A: 4397, June 1969.

In 1959 after extensive review of research, Bernstein ${ }^{42}$ concluded that differentiated (individualized) instruction was more effective than total class instruction, for the teaching of mathematics both of a general and remedial nature.

Singh ${ }^{43}$ suggested that the use of individualized enrichment homework resulted in significant achievement on most arithmetic subtests among fourth, fifth, and sixth graders. Looking at the idea of individualization in total, Suydam ${ }^{44}$ concluded that all in all, there is little substantial evidence to date that indicates that programs of individualized instruction will lead to higher levels of pupil achievement, when compared with non-individualized programs. She speculates that perhaps how each teacher teaches is the most significant factor and obscures any significant differences between the two types of programs.

It is this uniqueness of teacher instructional style that Suchman ${ }^{45}$ addressed himself to. Suchman conducted

42
Allen Bernstein, Library Research--A Study of Remedial Arithmetic, School Science and Mathematics, 59: 185-195, March 1959.

43 Jane M. Singh, An Investigation of the Effect of Individualized Enrichment Homework upon the Academic Achivement of Children in Grades 4 thru 6 (Arizona State University, 1969) DAI 30A: 3203-3204, February 1970.

44 Suydam, op. cit.
45 Richard Suchman, "The Illinois Studies in Inquiry Training," Journal of Research in Science Teaching, 2:320-32, 1964.
the Illinois Studies in Inquiry Training Project at the University of Illinois. He identified what he feels are necessary conditions for self-directed, individualized learning to occur in the elementary classroom. These conditions include:

1. The need for some kind of focus for the learners attention. The need to present a problem of stimulus that cannot be readily assimilated at that moment by the learner.
2. The need for a responsive environment. The child needs to have an environment where, when seeking data or information, he receives from the teacher only the data which he needs for testing the theory with which he is engaged.
3. The need for some condition of freedom. An external freedom which enables the child to seek out desired data and informstion and to acquire it at any rate, and in any sequence that child wishes. He can satisfy his own needs for gathering the kinds of information he wants and exercise freedom in trying out new ideas. In other words, the child becomes involved in inquiry and in an individualized approach to learning and is stimulated by the process.

Suchman envisions the teachers role as making the learner aware of an individual-inquiry process. Mathematics educators agree that such a role involves the presentation of material in such a way that the pupil gains skill in working independently in similar situations. The pupil needs to know what questions to ask, when to ask them, and where to find the answer. A teacher's question should not be designed to discover whether the pupil knows the answer, but to exemplify to the pupil the types of questions he must ask of the materials he studies and how to find the answers.

The individualization of mathematics instruction requires this type of sensitivity on the part of the teacher. Individualized instruction can lead to inquiry and further student motivation and as schwab ${ }^{46}$ hypothesized, "the art of conducting discussions is not easy, but it is through discussion that the skill of individualized inquiry can be conveyed."

Another technique commonly thought of as part of a modern mathematics program is the emphasis on grouping of learners for instruction. Koontz ${ }^{47}$ found that fourth graders who were heterogeneously grouped by achievement achieve significantly higher scores in mathematics than those homogeneously grouped by achievement. Dewar ${ }^{48}$ looked at the heterogeneous classroom and with an extensive treatment in sixth grade classes, concluded that providing three intraclass groups in elementary mathematics benefited high and low achieving learners more than did total-class instructions.
${ }^{46}$ Joseph J. Schwab, The Teaching of Science as Enquiry, (Cambridge: Harvard University Press, 1962) p. 62 .

47 william F. Koontz, A Study of Achievement as a Function of Homogenous Grouping, Journal of Experimental Ed, 30:249-253, December 1961.

48 John A. Dewar, Grouping for Arithmetic Instruction in Sixth Grade, Elementary School Journal, 63:266-269, February 1963.

When grouping is done on the basis of overall academic ability, a different set of findings is discovered. Provus ${ }^{49}$ and Balow and Ruddell ${ }^{50}$ found that ability grouping was especially effective for those learners with high I.Q.'s. Balow and Ruddell went further however, and concluded that the "decreased-range" grouping was more effective than either heterogeneous or homogeneous achievement grouping for all students. Savard ${ }^{51}$ found just the opposite from Provus and Balow-Ruddell. He found that ability grouping tended to be effective for lower ability pupils and of less advantage for upper ability pupils. To take the discussion full circle, Balow and Curtin ${ }^{52}$ reported that grouping by ability did not significantly reduce the range of achievement any more than any other type of grouping.
${ }^{49}$ Malcolm M. Provus, Ability Grouping in Arithmetic, Elem. Sch. Journal, 60:391-398, April 1960.
${ }^{50}$ Irving H . Balow and Arden K. Ruddell, The Effects of Three Types of Grouping on Achievement, California Journal of Educational Research, 14:108-117, May 1963.
${ }^{51}$ William G. Savard, An Evaluation of an Ability Grouping Program, California Journal of Educational Research, 11:56-60, March 1960.

52
Bruce Balow and James Curtin. Ability Grouping of Bright Pupils, Elementary School Journal, 66:32l-326, March 1966.

Davis and Tracy ${ }^{53}$ reported in their article that pupils in grades four, five and six who were taught in self-contained heterogeneous classrooms, scored significantly higher on factors such as verbal and quantitative ability, self-concept, anxiety, and attitude, than did those grouped by both ability and achievement across classrooms at each grade level.

Terry ${ }^{54}$ took the self-contained idea further and in a study of sixth grade students in New York, reported no significant differences in achievement was found between groups taught by specialists in mathematics education and those in a self-contained room with a regular teacher. As in the case of the effects of individualization, as well as various organizational schemes already mentioned earlier in this chapter, the effects of grouping are similarly hard to defend or reject due to the vast discrepancy of research findings.

Some other techniques of modern organization demand attention as well as grouping and individualization. The question of peer instruction, that is, allowing students to
${ }^{53}$ o. L. Davis and Neal H. Tracy, Arithmetic Achievement and Instructional Grouping, Arithmetic Teacher, 10:12-17, January 1963.

54Arthur F. Terry, An Investigation of the Relationship Between the Mathematics Achievement of Sixth Grade Students Under the Direction of the Teacher in the SelfContained Classroom and the Teacher Specialist, (The University of Rochester, 1969) DAI 30A: 3355-3356, February, 1970.
instruct other students, was experimentally tested among third graders by Ackerman ${ }^{55}$ in 1970. He found that pairing low achieving third graders with either high or low achieving sixth graders as tutors, resulted in computation scores which were significantly higher than the scores of those third graders who simply talked to sixth graders on nonmathematical activities or who had no tutor contact at all. Gray ${ }^{56}$ looked at the effects of discipline integration and its relationship to acquisition and retention of mathematical behaviors in grade five. His conclusion was that a higher achievement level was attained when unspecified mathematical content was integrated with unspecified science content, than when each was presented independentiy.

Morrel1 ${ }^{57}$ conducted an experimental study with fifth graders that compared four methods of teacher feedback from worksheets in arithmetic. His finding was that pupils who corrected errors or who were retaught frequently missed problems, either with or without written comments,
${ }^{55}$ Axthur P. Ackerman, The Effect of Pupil Tutors on Arithmetic Achievement of Third Grade Students, Diss. Abst. 31A:918, September 1970.
${ }^{56}$ William L. Gray, The Effects of an Integrated Learning Sequence on the Acquisition and Retention of Mathematics and Science Behaviors in Grade Five, (University of Maryland, 1970) DAI 31A:2004, November 1970.

57 James E. Morrell, A Comparison of Four Methods of Instructional Teacher Feedback from Practice Worksheets on Fifth Grade Arithmetic (LeHigh University, 1970) DAI 31A: 2794. December 1970.
retained more than pupils who only had written comments on their practice work. In addition, Morrell found that high achieving boys scored higher on the practice work than high achieving girls, but that girls retained better at medium and low achievement levels.

In summarizing the literature and research on techniques of modern and traditional mathematics for elementary school, the important variable is teacher uniqueness. All of the literature validity and all of the effectiveness of the research is contingent upon the ability of the classroom teacher to disseminate knowledge among learners in a manner that is compatable with individual learning styles as found then and only then in that classroom. Whether a technique is modern or non-modern is not the crucial issue. The manner of implementation by the classroom teacher is critical in the opinion of researchers and writers, either stated or implied.

Inquiry-Discovery Learning and Mathematical Growth

The significance of the inquiry-discovery learning style can be seen in the extent to which it is included in elementary mathematics textbooks and supplemental materials, in use in elementary classrooms today. The theory behind the inquiry method as opposed to more formal expository techniques is that, if the learner is interested
enough in a topic he will inquire into it further and hence "learn" more about it, than if he simply is told about a topic with little or no attention paid to whether or not the learner is genuinely interested. This cycle of interest breeding inquiry, which breeds greater academic growth, which then in turn produces further inquiry, is basic to the pedagogical belief in discovery learning.

Support for the inquiry-discovery technique comes from many sources. Fortson ${ }^{58}$ found that when working with youngsters of a very young age, those children experiencing an instructional approach that involved multiple-stimuli activities in a discovery mode, were significantly higher on tests of readiness than those using a non-discovery approach. It was further noted that creativity among discovery learners was much more prevalent than among learners of other groups.

In a well documented article on expository versus discovery modes and problem solving capabilities, scandura ${ }^{59}$ found that pupils taught by a discovery method were both better able to handle problem tasks and appeared much more self-reliant than groups taught by non-discovery techniques.

58
Laura R. Fortson, A Creative-Aestetic Approach to Readiness and Beginning Mathematics in the Kindergarten, (University of Georgia, 1969) DAI 30A:5339, June 1970.

59 Joseph M. Scandura, An Analysis of Exposition and Discovery Modes of Problem Solving Instruction, Journal of Experimental Education 33:148-159, December 1964 .

Other studies and articles dealing with the relative effects of inquiry-discovery teaching, as opposed to other modes of instruction, cover a wide realm of concerns. According to studies by Kellogg, Means and Parks, 60 as well as others, teachers can enhance their own problemsolving skills through practice and as a result they may become more proficient in setting the stage for inquirydiscovery learning in the classrooms.

With the interest in modern mathematics upon concept development as well as skill in computation, a study by Fleckman ${ }^{61}$ is particularly pertinent. It was reported that fifth and sixth graders taught by a "guided discovery" method learned more concepts than classes taught by conventional textbook procedures, at the same time that computation ability was equivalent in either method.

Perhaps the greatest test of any method of instruction is in the degree of retention the child possesses of

what he learned through the method. Worthen ${ }^{62}$ conducted a study of over five hundred fifth and sixth graders that supported the claim of discovery method advocates. He concluded that the inquiry and discovery techniques result in more learning and retention than expository methods, and that a discovery instructional method facilitated retention for a longer period of time. Bassler ${ }^{63}$ however, distinguished between two types of discovery and found differing retention abilities. He established an "intermediate guidance" technique in which pupils were led to a desired behavior through "guided discovery" with directed questions being asked by the teacher. The second type of instruction was termed "maximal guidance" in which the teacher specifically told students what they were to do and followed it by practice on the part of the pupil. His findings differ from Worthen in that he reports the "intermediate guidance" group had higher scores on a test immediately following instruction, while the "maximal guidance" group had higher scores on retention tests given much later.

Sowder ${ }^{64}$ worked in a study that was smaller than Worthen's or Bassler's and reported that "discovery techniques 62

Blaine R. Worthen, "A Study of Discovery and Expository Presentation", Implications for Teaching, Journal of Teacher Education 19:223-42, Summer 1968.
$6{ }^{3}$ otto $C$. Bassler, Intermediate Vs. Maximal Guidance-A Pilot Study, Arithmetic Teacher 15:357-362, April 1968.
${ }^{64}$ Larry Sowder, Discovery Learning: A Status Study, Grades 4-7 and an Examination of the Infleunce of Verbalizing Mode of Retention, Technical Report No. 99, Wisconsin Research and Development Center for Cognitive Learning, (Madison: The University of Wisconsin, 1969), xv, p. 140.
are most efficient when used after grade five and then only when an accuracy in computation has been developed by the learner."

It is this prior ability concern that gives rise to a negative attitude toward discovery and inquiry. Although Scandura ${ }^{65}$ reported positively on discovery, he did observe that it took longer for discovery taught learners to reach the desired level of facility, especially learners of high intellectual ability. Harmon ${ }^{66}$ went a step further and studied just the topic of ability among learners and the effects of discovery. In her study of sixth graders she concluded that "an expository approach was more effective than an inquiry approach for problems involving concepts, especially for children of average and high intellectual ability."
${ }^{65}$ scandura, op. cit.
${ }^{66}$ Adelaide T. Harmon, Problem Solving in Contemporary Mathematics: The Relative Merits of Two Methods of Teaching Problem Solving in the Elementary School, Diss. Abst. 30B: 3748, February 1970.

Heckman, Montague, Lisonbee and Fullerton, and Tanner ${ }^{67}$ among others, revealed that inquiry was no more effective than other approaches when achievement was the tested variable.

Tomlinson, May, and Adams ${ }^{68}$ studied the variable of retention as it relates to discovery techniques and unlike studies previously mentioned found that inquirydiscovery methods were not significantly better than other methodologies when retention was the criterion under investigation.

67 Maurice A. Heckman, "The Relative Merits of Two Methodologies for Teaching Verbal Arithmetic Problems to Undergraduate Elementary Education Majors," Diss. Abst. 23:3797, No. 10, 1962; Earl J. Montague, "Using the College Chemistry Laboratory to Develop an Understanding of Problem Solving in Science," Diss. Abst. 24:2815, No. 7, 1963: Lorenzo Lisonbee and William J. Fullerton, "The Comparative Effect of BSCS and Traditional Biology Ach," School Science and Mathematics 64:594-98, October 1964; Richard T. Tanner, "Expository-Deductive versus Discovery-Inductive Programming of Physical Science Principles," Diss. Abst. 29:1480-A, No. 5, 1968.

68 Robert M. Tomlinson, "A Comparison of Four Methods of Presentation for Teaching Complex Technical Material," Diss. Abst. 23:2813, No. 8, 1962; Lola J. May, "A Statistical Comparison of the Effectiveness of Teaching Per Cent by the Traditional, Ratio, and Discovery Method," Diss. Abst, 26:3109, No. 6, 1965; Barbara J. Adams, "A Study of the Retention of Information by SBCS Students and Traditional Biology Students," Diss. Abst. 29:2599-A, No. 8, 1968.

There is conflicting evidence as to whether inquirydiscovery methods foster more favorable attitudes on the part of the learner, than other techniques of instruction. Charen and Coulter ${ }^{69}$ were among those investigators who found that inquiry-discovery techniques fostered more favorable attitudes, while Butts and Rawn and Inventasch ${ }^{70}$ were among those who revealed that inquiry was no more effective than any other approaches with regard to attitude improvement.

In general, the conclusion may be made that although inquiry seems to be no more effective than other modes of instruction for most educational criteria, this method is at least equal to other methodologies for most criteria investigated. The key to the effect of discovery learning is dependent upon many variables, some of which appear to be controllable and some that are not. The essential criteria for its utilization is whether or not it appears to be working for a particular learner or not; and if not, something else should be used.

69
George Charen, "A Study of the Effect of openended Experiments in Chemistry on the Achievement of Certain Recognized objectives of Science Teaching," Diss. Abst. 23: 2815, No. 8, 1962; John C. Coulter, "The Effectiveness of Inductive Laboratory, Inductive Demonstration, and Deductive Laboratory in Instruction," Journal of Research in Science Teaching 4:185-86, 1966.
${ }^{70}$ David P. Butts and Chester E. Rawn, "The Relationship Between the Strategies of Inquiry in Science and Student Cognitive and Affective Behavioral Change," Journal of Research in Science Teaching 5:261-68, 1967-68; Harve Y Inventesch, WA Comparison of the Effects of Teacher Directed and Selfdirected Problem Solving on Attitudes and Understanding in Science, " Diss. Abst. 29:497-A, No. 2, 1968.

## Media-Materials of Traditional <br> and Modern Mathematics

A visable characteristic of the modern approach to elementary school mathematics, is the wide spread availability and usage of media and materials of instruction on nearly all elementary concepts of mathematics. This is not to say that the availability and the usage is entirely a positive thrust, but rather a statement of present conditions. In point of fact, a significant amount of research has belittled manipulative aids and other media to the point of their being ineffectual and a waste of valuable resources. In 1964, Jamieson ${ }^{71}$ conducted an experiment utilizing a multi-base abacus. He formulated three groups of learners each having a different mode of instruction on the same concept. One group watched the teacher demonstrate on a large variable-base abacus; a second group also watched the teacher, but in addition each learner had a small abacus at his desk. The thira group received instruction without any abacus in the room, having only the teacher and $a$ chalkboard for instructional purposes. Jamieson's finding was that there were no significant differences between mean per pupil gains across the three methods of instruction.
${ }^{71}$ King W. Jamieson, An Experiment with a Variable Base Abacus, Arith Teacher 11:81-84, February 1964.

Whether or not teacher demonstration with materials facilitates learning to a greater degree than pupil manipulation of materials is a question of debate. It becomes a very important discussion when a cost factor is considered. Trueblood ${ }^{72}$ concluded that pupils in elementary mathematics classes who watched the teacher demonstrate and manipulate materials scored higher on a posttest of skills than pupils who manipulated materials themselves. Toney ${ }^{73}$ on the other hand, studied the effectiveness of individual manipulation of instructional materials when compared to teacher demonstration and concluded that there were no significant differences among fourth graders using manipulative materials and those seeing only a teacher demonstration technique.

Pella and Sherman 74 used manipulative versus nonmanipulative procedures in a study conducted with ninth graders. Their approach was similar to Toney's in that individual manipulation of instructional materials was
${ }^{72}$ Cecil R. Trueblood, A Comparison of Two techniques for using Visual-tactial Devices to Teach Exponents and Non-decimal Bases in Elementary School Mathematics, Arithmatic Teacher 17:338-340, April 1970.
${ }^{73}$ Jo Anne $S$. Toney, The Effectiveness of Individual Manipulation of Instructional Materials as Compared to a Teacher Demonstration in Developing Understanding in Mathematics, (Indiana University, 1968) Diss. Abst. 29A: 1831-1832, December 1968.
${ }^{74}$ Milton O. Pella and Jack Sherman, "A Comparison of Two Methods of Utilizing Laboratory Activities in Teaching the Course IPS," School Science and Mathematics 69:303-14, April 1969.
compared to teacher demonstration without manipulation by pupils. In this investigation, the criteria were expanded to include critical thinking, understanding of concepts, interest, achievement, and laboratory skill. Again, there were no significant differences resulting from the treatments employed when understanding and achievement were the criteria. In addition, Pella and Sherman found similar results when critical thinking and subject interest were the criteria. The manipulative method did however, prove to be significantly better than the non-manipulative method for the development of selected laboratory skills. Sex and I.Q. were used as variables, but did not prove to be significant within the scope of the study.

This last fact of the relationship of sex to mathematics understanding and interest has long been a question of concern and debate among educators. Fogelman 75 reported that boys who manipulated materials demonstrated conservation more frequently than boys who watched the experimenter manipulate the materials. For girls, he found just the opposite was true.

Pella and Sherman's findings were corroborated later in a study by Yager, Engen, and Snider ${ }^{76}$ among high school science classes.
${ }^{75}$ E. R. Fogelman, Piagetian Tests and Sex DifferencesII, Educational Research 63:455-458, July/Aug. 1970.
${ }^{76}$ Robert G. Yager and Harold B. Engen, and William $C$. Snider, "Effects of the Laboratory and Demonstration Methods Upon the Outcomes of Instruction in Secondary Biology," Journal of Research in Science Teaching 6:76-86, 1969.

The question of the worth of materials in mathematics instruction is debatable due to the diverse opinions and findings on the topic in current literature and research. Harshman, Wells, and Payne ${ }^{77}$ in a study of manipulative materials in grade one, reported among other findings, that high expenditure for manipulative materials did not seem justified when inexpensive and/or teacher developed materials worked as well, especially when both were evaluated over the criteria of achievement.

In a one year study of one hundred students conducted in 1967, Devine ${ }^{78}$ found significant differences in achievement favoring a conventionally taught classroom when compared to another utilizing programmed materials. His findings went further to illustrate that the teacher attiude about a particular approach had significant impact upon student growth, and that teacher attitude about programmed materials was quite negative. In a follow up study by Sneider ${ }^{79}$
${ }^{77}$ Hardwick W. Harshman, David W. Wells and Joseph N. Payne, Manipulative Materials and Arithmetic Achievement in Grade One, Arithmatic Teacher 9:188-191, April 1962.
${ }^{78}$ Donald F. Devine, "Student Attitudes and Achievement: A Comparison Between the Effects of Programmed Instruction and Conventional Classroom Approach in Teaching Algebra, Mathematic Teacher 61:296-301, March 1968.
${ }^{79}$ Sr. Mary Joetta Sneider, "Achievement and Programmed Learning," Mathematics Teacher 61:162-64, February 1968.

Devine's findings were corroborated with a second group of students.

Fennema ${ }^{80}$ found no significant differences in overall
learning of a principle when learning was facilitated by either a meaningful concrete or a meaningful symbolic model and concluded that use of concrete materials in elementary school mathematics may not always be as essential to the development of meanings and skills as has been hypothesized.

In an older study, Sole ${ }^{81}$ concluded that the use of a variety of materials did not produce better results than the use of only one material, technique, or approach. He further concluded that the learning of mathematics depends more on the teacher than on the materials used.

There have been many studies that defend manipulative materials and their utilization in elementary mathematics. Golsby ${ }^{82}$ worked with first grade pupils using experimental manipulative materials designed to "promote readiness and enhance the curriculum" for low

80 Elizabeth H. Fennema, A Study of the Relative Effectiveness of a Meaningful Concrete and a Meaningful Symbolic Model in Learning a Selected Mathematical Principle, Diss. Abst. 30A:5338-5339, June 1970.
${ }^{81}$ David Sole, The Use of Materials in the Teaching of Arithmetic (Columbia University, 1957) Diss. Abst. 17: 1517-1518, July 1957.

82 Thomas M. Golsby, et al. Effect of Massive Educational Intervention on Achievement of First Grade Students, Journal of Experimental Education 39:46-52, Fall 1970.
achievers. His treatment resulted in significantly greater achievement than that attained by students using conventional non-manipulative materials.

Weber, ${ }^{83}$ though finding no significant differences between reinforcement of concepts through paper and pencil activities or with manipulative materials, did discover a trend which favored the use of manipulative materials, especially for students with a low socio-economic status. Goebel ${ }^{84}$ was concerned with materials and teacher time utilization. He reported that teachers using programmed instructional and manipulative materials, devoted sixtyeight per cent of their time to working with individuals, while teachers of conventional classes devoted less than five per cent of their time to individuals.

One of the most widely acceptable mathematical aids being used in classrooms and spoken of in literature and research is the Cuisenaire materials and method. There is very little negative research directly relatable to the Cuisenaire method specifically. All of the criticisms of
$83_{\text {Audra }}$ W. Weber, Introducing Mathematics to First Grade Children: Manipulative vs Paper and Pencil, (University of California, Berkeley, 1969) DAI 30A: 3372-3373, February 1970.
${ }^{84}$ Laurence G. Goebel, An Analysis of Teacher-Pupil Interaction when Programed Instructional Materials are Used (University of Maryland, 1966) Diss. Abst. 27A:982, October 1966.
manipulative materials in general are applicable to Cuisenaire, but any criticisms directly applied to this method are not easily found. Hollis ${ }^{85}$ found that children learned traditional subject matter with the Cuisenaire program as well as they did with the conventional method and that pupils taught by the Cuisenaire program acquired additional concepts and skills beyond the ones taught in the conventional program.

In a related study in 1966, Crowder ${ }^{86}$ found that Cuisenaire users learned more conventional subject matter and more mathematical concepts and skills than non users, at the first grade level. He reported that pupils with average and above average achievement patterns profited most from Cuisenaire programs, and that while sex was not a significant factor in relation to achievement, socioeconomic status was.

The studies and literature on materials in elementary mathematics are varied. The important fact that each stresses is the proper utilization of any material and that given improper use, no material usage would be most profitable.

85 Loye Y. Hollis, A study to Compare the Effect of Teaching First and Second Grade Mathe by the CuisenaireGattegno Me thod with a Traditional Method, School Science and Mathe 65:683-685, November 1965.
${ }^{86}$ Alex B. Crowder, A Comparative Study of Two Methods of Teaching Arithmetic in First Grade (North Texas State University, 1965) Diss. Abst. 26:3778, January 1966.

## A Summary

The purpose of Chapter II was to present both positive and negative research on the topic of elementary school mathematics instruction. Because of the breadth of the topic of modern approaches to instruction, the literature and research in the chapter was limited to six aspecta of current literature on mathematics approaches. The six aspects are from the conceptual framework established in Chapter I. A summarization of the research findings is as follows:

## Influential Factors of Mathematics Organization and

Achievement. --The research and literature is not in agreement as to the effect of any given influence upon mathematics achievement. Agreement can be found however, in the fact that specific internal and external factors such as age, attitude, literature, sex, socio-economic status, material rewards, and previous experience are some of the factors that can have dramatic affect upon the manner in which mathematics instruction is organized and disseminated to a group of learners. These, as well as others, can also have dramatic affect upon the degree of learning that takes place within a learner. The effective educator is one that is aware of the influential factors that are constantly at work in a classroom and works to make their influence positive for the learners in the class.

## Attitude-Interest and Mathematics Instruction and

Achievement. --As it relates to mathematics instruction and pupil achievement, the impact of student or teacher attitude upon academic growth, can be considered questionable. There is limited evidence to show that the modern approach to mathematics facilitates greater interest for the learner than did the traditional. Evidence can be found to support the commonly held belief that boys are more interested and have a more positive attitude about mathematics than do girls. Teacher and pupil attitude about elementary mathematics and the resultant pupil achievement may or may not be significantly corrolated on the basis of current research. Modern versus Traditional Organization of Mathematics

Instruction. - A significant body of knowledge was found that reported academic growth in elementary mathematics is not significantly different when either a modern or a traditional approach is utilized. It has been shown that modern programs are as effective as traditional in developing traditional skills. and perhaps more effective in developing modern concepts. It is a validated fact that no one specific pattern of organization for instruction in elementary school mathematics (such as a team approach, grouping schemes, text-no-text etc.) is any better in increasing academic growth than any other. The key to effective instruction is still the teacher, regardless of whether a modern or a traditional approach is being used.

Technigues of Modern versus Traditional Organization*-It is an accepted ideal of elementary mathematics instruction that some type of differentiated instruction should be established for each particular learner. fhe problem is that there is disagreement over the correct method of actualizing this belief. It has been illustrated that individualized programs of elementary mathematics will not, simply because they are individualized, lead to greater levels of pupil achievement when compared with non-individualized approaches. The single best technique that should be utilized is the one that works with a particular child on a particular concept or skill. The teacher is the critical variable and the art of asking probing questions and leading meaningful dialogue is essential for adequate elementary mathematics instruction. The idea of grouping for instruction is highly debatable. Ability and achievement grouping, or any other homogenous technique, results in achievement levels not significantly different in most cases, from heterogeneous organizations. This is particularly apropo in terms of low or below average learner abilities. Discovery-Inguiry versus Expository Learning.--There
is general consensus among the researchers and writers that expository methods of instruction alone will not result in pupil attitudes and interests of a positive nature, nor will achivement be as great, as will take place in a setting of some expository instruction supplemented by a discoveryinquiry mode. There remains considerable question as to
exactly what effect certain psycho, socio, and emotive variables have on a pupils capability in, or acceptability of, discovery-inquiry modes of instruction in elementary mathematics.

Media-Materials of Mathematics Instruction. --Although
elementary mathematics instruction today is heavily accented with media and manipulative materials, a large body of literature and research negates the essentialness or worthiness of the approach. There is basic question as to whether teacher demonstration with manipulative devices alone, or child manipulation by himself/herself results in increased academic retention and knowledge. It was noted that high expenditure for media might not be as important as once thought in curriculum planning, due to the fact that teacher developed materials were as effective as commercially prepared. Research has shown that manipulative materials may be of greatest benefit to the child of lower socio-economic status. Cuisenaire materials have proven to be very effective in the presentation of both modern and traditional concepts and skills.

# CHAPTER III <br> RESEARCH METHODOLOGY AND TECHNIQUES 

Introduction

This chapter addresses itself to the purposes for which this study was undertaken, and seeks to illustrate how each of those individual purposes were investigated. As stated in Chapter 1 , there are four purposes for this study. (1) To establish a working base that defines a "modern" mathematics ideology. (2) To ascertain the differences, if any, between what experts in mathematics education at the University level believe is current thinking in elementary mathematics, as compared to public school teachers. (3) To investigate whether a difference exists between what elementary school teachers believe about modern mathematics and what they do in their classrooms. (4) To determine whether or not the variables of school district size and wealth are correlated with a more "modern" concept of mathematics instruction among elementary classrooms in public school districts.

The techniques of investigating each of the above purposes are discussed in this chapter. Information is
presented as to how the sample was selected, how the questionnaire was developed, how the base was arrived at, and how the data was analyzed.

## Establishing a Base

The first purpose of this study was to adequately define what constituted "modern" thinking as it related to elementary mathematics instruction. To do this, careful and extensive review of the current literature and research was conducted. The research in Chapter II of this study encompasses the majority of the literature upon which the base was ultimately drawn. The researcher synthesized the pros and cons of the literature and research findings related to the topic, and arrived at a master list of thirty principles of current thinking in the field of elementary mathematics. This compiled list was mailed to a panel of eight mathematics experts in the $S$ tate of Michigan. Each of these experts represented a different major state institution of higher learning and was felt to be significantly aware of mathematics principles in education, to be considered an expert in the field. The experts (see Appendix A) were asked to agree or disagree with the principle as stated, disallowing any inherent and obvious questions they might have as to definitions, meanings, etc. It was believed that because of the number of experts used, and because each was asked to respond with an "off-the-top"
type of response, that the pooled responses would be a valid assessment of the "modernness" of the set of principles. Upon the return of all eight experts responses to the list of thirty principles of mathematics in elementary education, a tabulation was made on each item to determine the extent to which any one particular item could be considered acceptable and therefore modern in current thinking in the field. For the purposes of this study the following criteria were used to determine if an item was modern or non-modern thinking in the field of elementary mathematics instruction: if a particular item received more than five "agree" responses from the eight experts, it was recorded as "modern" thinking; if five or more of the experts checked the "disagree" response to an item, it was considered non-. modern ideology in today's mathematics. All items receiving either four or five checks in the "agree" column were eliminated entirely because it was believed that such responses indicated significant debate or indecision among the experts, therefore not allowing a clear determination of modernness to be made. The list of thirty principles submitted to the experts and the tabulation of responses by which modernness was determined, can be found in Appendix B.

The base of this study (as defined in Chapter 1) was developed by making a random selection of ten items from all of the items that were designated either modern
or non-modern by the above criteria. This list of ten items became Part $I$ on the questionnaire and included both modern and non-modern principles. A calibrated response scale was developed for this list of ten items. part $I$ of the instrument was subsequently sent back to the same panel of experts, and to a randomly selected sample of elementary classroom teachers. The responses of the experts on the calibrated scale was necessary to facilitate investigation of the first hypothesis concerning the difference between expert and classroom teacher beliefs about principles of elementary mathematics instruction. Originally, the experts simply agreed or disagreed with the principle, and this second inquiry from them sought to ascertain the degree of acceptance or non-acceptance on each item held by each expert. Part $I$ of the questionnaire became the measure that was utilized to investigate the first hypothesis concerning the difference between expert and classroom teacher beliefs about mathematics instruction.

Identification of the Sample

Once the base was established and a list of ten validated principles representing modern and non-modern thinking concerning elementary mathematics was determined, the researcher began to investigate the first hypothesis. Is there a difference between what experts and teachers believe constitutes a modern approach to mathematics in
elementary schools? The researcher wanted to explore this question on two levels. First, among school districts selected because of their varying size and wealth, and second, among a random sample of heterogeneous elementary classroom teachers from those same districts as compared to the panel of experts discussed earlier in this chapter. For this reason, some criteria of geography became essential. The five county region in Southwest Michigan comprised of Berrien, Calhoun, Kalamazoo, Kent, and Van Buren counties was chosen because this region, unlike other regions in the state, has many representative districts in categories of varying size and wealth. The southeastern region of Michigan is predominately urban and suburban, thus seriously limiting the possibility of securing responses from small size districts. The northern portion of the lower peninsula and the entire upper peninsula lack districts that can be considered large by the definition this study used of large in Chapter $I$.

Due to the large number of public school teachers trained in institutions of higher learning in the state of Michigan, and hence the large number of professional educators involved in teacher training in the state, it was believed that Michigan teachers and experts could be considered as representatives of all teachers and experts in elementary education.

Each individual respondent was an elementary classroom teacher in southwestern Michigan. Respondents were randomly chosen from the selected school districts within the five counties included in the study. The districts from which the respondents were drawn were not randomly selected, but rather constituted a total population of school districts in southwest Michigan given the two major variables in the study. The districts selected for inclusion in the study as diagrammed across the variables of size and per pupil expenditure, appear in Exhibit A. This information about size and per pupil expenditure was taken from the Michigan Department of Education Bulletin 1012 of December, 1970.

The selected school districts constitute a total population. As such, it can be argued that the randomly drawn teachers from these districts, are also representative of any other populations of similar characteristics, because within a population the distribution of the randomly drawn respondents scores can be considered as normally distributed. It can be noted from Exhibit A that the districts are grouped in cells and from those districts in each cell a list was made of all of the elementary classroom teachers. From this master list of all of the elementary teachers in the selected districts, a sample was

EXHIBIT A.--School District Variables: Size and Expenditures.

|  | Large | Mid-size | Small |
| :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { High } \\ (\$ 895+) \end{gathered}$ | Battle Creek Cty. Kalamazoo City Schs. | St. Joseph Cty Schs. Comstock Public Schs. Godwin Heights Pub. | Berrien Springs Pub. Schs. |
| $\begin{gathered} \text { Middle } \\ (\$ 800 \text { to } \\ 894) \end{gathered}$ | Grand Rapids Cty. | East Grand Rapids Pub. | Covert Public Sch. Springfield Cty. Sch. |
| $\begin{aligned} & \text { Low } \\ & (\$ 799-) \end{aligned}$ | Portage Pub. Schs. Benton Harbor Cty. | Gull Lake Comm. Schs. Niles Comm. Schs. | Godfrey-Lee Pub. Sch. Athens Area Schs. |

taken utilizing a table of random numbers. ${ }^{i}$ For stability and reliability of statistical tests, twenty respondents in each cell were utilized. This resulted in approximately an eight per cent sampling of the teachers in the population which has twenty-two hundred elementary classroom teachers and a K-12 population of one hundred twenty-one thousand students.

## Development of the Questionnaire Instrument

The questionnaire instrument was developed in two parts. Part I was the base of modernness as validated by expert opinion, and sought to elicit the beliefs of teachers and experts as discussed earlier. Part II of the instrument concerned itself with that which a teacher does in actual practice on a day to day basis in his/her classroom. It was this portion of the instrument that, when compared to Part I, allowed the researcher the opportunity to address himself to another of the stated purposes of this study-whether or not a difference existed between what teachers said they believed and what they actually did in their classroom, or as stated in null form: "There exists no significant difference between what teachers believe constitutes a modern mathematics program, and what they do in their day to day activities in their classroom."
${ }^{1}$ Paul Blommers and E. F. Lindquist, Elementary Statistical Methods (New York: Houghton Mifflin, 1960), pp. 512-517.

Each of the items in Part II of the instrument is correlated with the item in Part $I$ having the same number. As an example, item one in Part II is a question of actualization (what do you do) that relates to question one in Part I. This pattern continues throughout the instrument. As a test of its reliability, coherence, and validity, the instrument was field tested on seventy-five randomly chosen elementary teachers and necessary adjustments made.

The questionnaire was developed in a manner that attempted to insure a response rate that would allow stable statistical measures to be utilized, and that would give validity to the findings. Because questionnaires that are sent through the mail have had a history of immense nonresponse on the part of the sample, every effort was taken to control the variables that could cause such an occurrence. The questionnaire was anonymous. In no place did it call for the respondent to indicate any demographic or personal information. It was not important in the analysis of the data, who the respondent was, but rather, only where he or she taught. For this reason, the return envelopes, included in the original mailing, were coded so as to allow the researcher the ability to record the responses in the correct cell in Exhibit A. Besides being anonymous, the questionnaire was simple in response procedure, asking only for the circling of a word. It was also short and printed rather than duplicated. Finally, the questionnaire was mailed to home addresses and in addition was mailed at a
time just prior to the Easter break of the respondents, so that in most cases the respondent would have, and would take, the time to respond. A copy of the complete instrument and the cover letter are included in Appendix $C$.

## Size and Expenditure

The fourth purpose of this study was to ascertain whether or not size and per pupil expenditure in the district within which a person teaches, has any correlation with his/her modernness in mathematics instruction. As will be discussed in the next section of this chapter, the investigation of this question is done through an analysis of variance procedure. The analysis of variance with repeated measures techniques was chosen because of its assumption that the variance within any one cell is equal to the variance of any other cell and therefore will allow for valid comparisons to be made among all of the cells in Exhibit A. Given the instrument and the population discussed earlier in this chapter, this statistical procedure facilitated those statistical tests necessary to accept or reject the hypotheses.

## Analysis of the Data

To analyze the data collected during the investigation of this study, the researcher selected two statistical treatments for the purposes of clarifying some aspects of the study and to test the hypotheses as stated in Chapter I.

A two-way analysis of variance with repeated measures technique, as suggested by Winer, ${ }^{2}$ was utilized in determining whether or not significant differences existed between a teacher's beliefs and his/her actions, as well as the differences between school districts according to size and per pupil expenditure. This statistical treatment was chosen because of its ability to minimize the experimentwise error which has a cumulative effect within the alpha levels upon the results. Seven statistical tests were conducted with the alpha level being set at . 025 .

In the first hypothesis, dealing with expert versus teacher belief differences, a test procedure was utilized. This hypothesis could not be tested by an analysis of variance procedure because the panel of experts were not selected from the same population that the teachers were and were therefore separate from the statistical design as illustrated in Exhibit B. To test the hypothesis that no significant difference existed between expert and teacher beliefs as to the consistency of a modern program of mathematics instruction, a numerical value of "modernness" was established for each teacher. This was accomplished by computing the mean score for the teacher in Part $I$ of the instrument. This mean,

[^2]|  |  | Belief | Practice |
| :---: | :---: | :---: | :---: |
|  | Large |  |  |
| High per pupil | Mid-size | $\begin{gathered} \mathrm{T}-21 \\ \mathrm{~T}-22 \\ \prime \prime \\ " \\ \prime \prime \\ \prime \prime \\ " \\ \mathrm{~T}-40 \end{gathered}$ |  |
| expenditure | Smal1 | $\begin{gathered} T-41 \\ T-42 \\ " \\ " \\ " \\ " \\ " \\ T-60 \end{gathered}$ |  |
|  | Large | $\begin{gathered} T-61 \\ T-62 \\ " \\ " \\ " \\ " \\ " \\ T-80 \end{gathered}$ |  |
| Middle per pupil expenditure | Mid-size | $\begin{gathered} T-81 \\ T-82 \\ " \\ " \\ " \\ " \\ " \\ \hline \end{gathered}$ |  |

EXHIBIT B.--Continued.
Small
along with the other nineteen teacher means for Part I in that same cell, were sumed and a grand mean for the cell was computed. Comparisons with the experts mean score were then made between and across all cells.

Hypothesis two was tested by determining a cell mean for belief and a cell mean for practice within each cell and then by utilizing the analysis of variance procedure discussed previously, determining the validity of the hypothesis that no significant difference existed.

The format used for testing hypothesis three was also utilized with hypothesis four. For hypothesis three, a mean value was determined for each level of the variable of size--large, mid-size, and small. This was done by summing the teacher means for belief and practice and arriving at a cell mean for all respondents in that cell. This summing was carried out across the expenditure levels as well. Comparisons were then made through the analysis of variance procedure that determined the impact of size of school districts upon modernness.

Hypothesis four utilized the same format as hypothesis three except the summing was conducted for each level of the variable of per pupil expenditure--high, middle, and low. This was carried out across the other variables of size as well as belief and practices. (measures).

The statistical format for the analysis of the data in this study was as follows:

Sources of Variation

Levels d.f. Criteria of Significance

| $H_{0}^{2}$ Measures* (M) | 2 | 1 | if computed F>F (.025) $1,171 * *$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $H_{0}^{3}$ Size | (S) | 3 | 2 | if computed $F>F(.025)$ | 2,171 |
| $H_{0}^{4}$ Expenditure (E) | 3 | 2 | if computed F>F (.025) 2,171 |  |  |

* denotes the grand mean numerical value for individuals with the means for beliefs and practices added together.
**
171 degrees of freedom reflects twenty individuals per cell minus the total degrees of freedom for all sources of variation.

The following interaction effects are also tested:

Expenditure- ExS Size

Expenditure- ExM Measure

Size-Measure SXM
Expenditure-MeasureSize
significant if computed $F>F(.025) 4,171$ significant if computed F>F (.025) 2, 171 significant if computed $F>F(.025) 2,171$ significant if computed $F>F(.025) 4,171$

This technique of analysis of variance tests all the sources of possible variation to see if respondents vary
more due to these effects (size, expenditure, and belief vs. practice) than they vary within any given cell. It is actually a comparison of between cells versus within cells.

## Sumary

A panel of experts in the field of mathematics education was selected to validate as modern or non-modern, a list of specific items the researcher drew from current literature and research. From this validated list, a questionnaire was developed and mailed to a random sample of elementary classroom teachers in southwest Michigan. The responses of the teachers, as well as the experts, were converted into a numerical value of "modernness" as regarded elementary mathematics instruction. This "modernness" value was then compared across the independent variables of school district size, as well as per pupil expenditure. Tests were also conducted to determine the extent to which the teachers differed from the experts as to a modern mathematics ideology, and also to what extent there was a difference between what a particular teacher said he/she believed and what he/she actually did in class on a day to day basis.

## CHAPTER IV

## PRESENTATION AND ANALYSIS OF DATA

This chapter presents a statistical analysis of the scores on the questionnaire instrument for both the teacher and expert groups. An analysis of variancerepeated measures technique was utilized, as well as a standard $t$ test for the purposes of establishing the significance of differences among the relationships with which this study was concerned. A summary of the analysis of data collected for the study concludes the chapter.

## The Data Gathering Technigue

Using a mailed questionnaire technique to a large size sample of randomly selected school teachers, historically has resulted in minimal numbers of returns. In this study however, it should be noted that the technique utilized and discussed earlier concerning the process and development of the instrument, was most beneficial. In the investigation conducted in this study a total of two hundred twenty-five questionnaires were mailed and a total of two hundred eleven or 93.38 were returned. A random selection of one hundred eighty was made from the returned
instruments for validity, reliability, and equal cell size as discussed in Chapter III. This one hundred eighty constituted the random sample that was utilized in the investigation of all four of the research hypotheses investigated in this study. In the opinion of the researcher, the single most important factor contributing to the large size of the return was the utilization of the respondents home address for mailing purposes. This information was difficult to obtain, in many cases, due to the reluctance of certain school districts to furnish such information. Of the sixteen school districts in the study, as found in Exhibit A, thirteen responded to the researcher's request for a district directory or list. The Portage Public Schools, Springfield Public Schools, and the Gull Lake Community Schools did not respond to the request for information and for this reason, teachers in those school districts were not included in the final sample that was used in this study.

## Hypothesis Testing

A total of eight statistical tests were taken to ellicit the validity of the four major hypotheses investigated in this study. Hypothesis one was investigated using a single test, while seven tests were conducted in testing hypotheses two, three, and four. Table I presents a summary of the test results utilized for validating the final three hypotheses.

TABLE 1.--Summary of Statistical Tests Results Hypotheses Two, Three, Four.

| Sources of Variation | Degrees of Freedom | Mean Square | F Statistic | p |
| :---: | :---: | :---: | :---: | :---: |
| Expenditure (E) | 2 | . 054 | . 228 | n.s. |
| Size (S) | 2 | . 382 | 1.599 | n.s. |
| Measures (M) | 1 | 1.332 | 5.260 | >. 025 (sig) |
| Teachers: E x S | 171 | . 239 |  |  |
| E $\times \mathrm{S}$ | 4 | . 098 | . 411 | n.s. |
| E X M | 2 | . 254 | 1.004 | n.s. |
| $S \times \mathrm{M}$ | 2 | . 014 | . 054 | n.s. |
| E X S X M | 4 | . 234 | . 925 | n.s. |
| M x Teachers: E $\times$ S | 171 | . 253 |  |  |
| Total | 359 | . 246 |  |  |

Key--n.s. denotes no significance at $p<.025$
$p$ denotes the probability of a false rejection of the null typothesis
(M) denotes the hypothesis concerning beliefs versus practices
$\mathrm{Ho}_{1} \quad$ There is no significant difference between expert opinion on the consistency of a modern mathematics program, and the opinion of all the teachers included in this study.

The mean score of the eight elementary mathematics experts on Part $I$ of the questionnaire was 3.538, while the mean of the one hundred eighty randomly selected respondents was 2.903. Hypothesis One concerned only Part I (beliefs) of the questionnaire. The test of significance was conducted at the .025 level with one degree of freedom. The mean square difference was 3.156 and the computed $F$ value was 13.023 which was greater than the table value of $\underline{F}$ with one degree of freedom at the .025 level of confidence.

Finding
The hypothesis as stated $\mathrm{Ho}_{1}$ was rejected.
Summary

|  |  | Experts | Teachers |  |
| :--- | :---: | :---: | :---: | :---: |
| Mean Values: |  | 3.538 | 2.903 |  |
| Test Data: | d.f. | MS |  |  |
| Source of Variance | 1 | 3.156 | 13.023 | $>.025$ |
| Between Cells | 366 | .242 |  |  |
| Within Cells |  |  |  |  |


#### Abstract

Meaning Given the finding of this investigation and based upon the assumptions and procedures it has utilized-there is a significant difference between what experts in mathematics education believe is modern ideology concerning classroom instruction in mathematics, and what a randomly selected sample of elementary classroom teachers believe.


## Interpretation

Hypothesis One was rejected. The analyzation of the data clearly indicated that a significant difference existed concerning the beliefs of experts as compared to teachers. The large $F$ statistic was the result of two major factors: first, the items of Part $I$ of the questionnaire were originally sent to and validated by the same panel of experts whose scores were eventually used in establishing the expert's mean score. Therefore, the experts were agreeing and disagreeing to items they themselves had validated as being modern and non-modern. This resulted in a high mean value for experts. Second, the large difference between the size of the random sample and the size of the panel of experts affected the degrees of freedom and therefore created a tendency to inflate the mean square values from which the $\mathcal{F}$ statistic was directly computed. It perhaps should also be noted that experts in the field of mathematics education spend a significant
amount of their time involved with research and literature. Teachers in public school classrooms on the other hand, spend proportionally less time with research and literature. therefore, it could be expected that the two groups would react differently upon the base on this study (as defined in Chapter I) from each other.

This hypothesis was not developed specifically to determine the modernness of the teachers or the experts. To do so, would involve a subjective evaluation on the part of the researcher as to what mean score level would constitute modernness in a continuum from one to five. Such a task was not undertaken. The purpose of this investigation was only to ascertain if a difference existed between the two groups. A significant difference was discovered and the implications of that difference will be discussed in more detail in Chapter $V$.
$\mathrm{HO}_{2} \quad$ There is no significant difference between what the respondents say they believe constitutes an effective mathematics program, and what they say they do in their day-to-day activities in the classroom.

The mean score of the one hundred eighty respondents for Part $I$ of the questionnaire was 2.933 , while the mean value for the same sample on Part II of the questionnaire was 2.842. The test of significance was conducted at the . 025 level with one degree of freedom. The mean square of the difference was 1.332 which resulted in an $F$ value


#### Abstract

of 5.260 which was greater than the table value of $F$ given the aforementioned values.

\section*{Finding}


The hypothesis as stated in $\mathrm{HO}_{2}$ was rejected. Summary

|  | Experts | Teachers |
| :--- | :--- | :--- |
| Mean Values: | 2.963 | 2.842 |

## Test Data:

| Source of Variance | d.f. | MS | F | P |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Measures (M) | 1 | 1.332 | 5.260 | $>.025$ |

## Meaning

Given the findings of this study and based upon the assumptions and procedures it has utilized-there is a significant difference between what elementary classroom teachers believe is a modern mathematics program and that which they do in their classroom on a day to day basis.

## Interpretation

Hypothesis two was rejected. The evidence as presented in the data, clearly indicates that a significant difference existed between what a teacher says he or she believes is a modern approach to the teaching of mathematics in contemporary elementary education, and that which he or she actually does in the classroom. It was also
significant that the score of the respondents on Part II of the questionnaire was lower than on Part $I$, thereby indicating that teachers have beliefs that are more modern than the actual practices that they employ. Teachers in elementary grades do not use modern mathematical techniques, nor exhibit modern mathematical ideologies in their instruction, to the same degree that they say they believe in them. In effect, teachers think more modernly, than they function in the classroom.

The researcher believes that the nature of the construction of the questionnaire instrument allowed for a one-to-one mean relationship between Parts I and II to develop, if in fact that relationship actually existed among the randomly selected respondents. As stated earlier, each item of Part II was correlated with an item in Part I and the scoring scheme remained constant for both items. For this reason, the researcher interprets the statistical finding as valid and concludes that teachers function in a manner that is different from what they themselves believe to be modern, as related to elementary mathematics instruction. The implications of such a finding will be discussed in Chapter $V$.
$\mathrm{Ho}_{3} \quad$ There is no significant difference in "modernness" between teachers of varying school district sizes.

The mean score of the respondents who taught in the school districts listed in the column named "large" in Exhibit A was 2.837 when the scores from Part I and Part II of the questionnaire were combined. This compared with a total mean value of 2.938 for teachers in the "mid-size" column and 2.932 in the "small." The test of significance was conducted at the .025 level with two degrees of freedom. The mean square difference value between the column values was . 382 and the computed F statistic was 1.599 which is less than the table value of $F$ at the .025 with two degrees of freedom level.

Finding
The hypothesis as stated in $\mathrm{Ho}_{3}$ was not rejected. Summary

|  | Large | Mid-size | Small |  |
| :--- | :---: | :---: | :---: | :---: |
| Mean Values: | 2.837 | 2.938 | 2.932 |  |
| Test Data: |  |  |  |  |
| Source of variation | d.f. | MS | F | P |
| Size (S) | 2 | .328 | 1.599 | n.s. |

Meaning
Given the findings of this study and based upon the assumptions and procedures it has utilized-there is no difference in modernness between teachers working within

# districts of varying population size that can be said to be attributable to that varying size. 

## Interpretation

Hypothesis three was not rejected. The analyzation of the data indicated that statistically no significant difference existed among teachers of elementary school mathematics, when compared across the variable of the size of the school district population. It is significant however, (non-statistically) given the belief possessed by many that small school districts do not provide as modern an approach to school in general and mathematics in particular, as do mid-size or large school centers. Had the analysis in fact illustrated that modernness of instruction and belief concerning mathematics education was significantly different in rural areas as opposed to urban, then credence would be established for such an argument. Such was not the finding and the researcher accepts the validity of the analysis. The implications of this finding are many and will be discussed in Chapter v.
$\mathrm{HO}_{4} \quad$ There is no significant difference in "modernness" between teachers from school districts that have varying per pupil general fund expenditures.

The mean score of the respondents who taught in the school districts listed in the row named "high per pupil expenditure" in Exhibit A, had a total mean of 2.921 over both Part I and Part II of the questionnaire combined. This compared with a total mean value of 2.879 for teachers in the "middle per pupil expenditure" row and 2.907 for those in the "low" category. The test of significance was conducted at the .025 level with two degrees of freedom. The mean square difference value between the row values was . 054 and the computed F statistic was .228 which is less than the table value of the $F$ at the .025 with two degrees of freedom level.

Finding
The hypothesis as stated in $\mathrm{Ho}_{4}$ was not rejected. Summary

|  | High Exp. | Midale Exp. | Low Exp. |  |
| :--- | :---: | :---: | :---: | :---: |
| Mean Values: | 2.921 | 2.897 | 2.907 |  |
| Test Data: |  |  |  |  |
| Source of Variation | d.f. | MS | F | p |
| Expenditure (E) | 2 | .054 | .228 | n.s. |

## Meaning

Given the findings of this study and based upon the assumptions and procedures it has utilized-there is
no difference in modernness between teachers working within districts of varying per pupil expenditure that can be said to be attributable to that expenditure difference.

## Interpretation

Hypothesis four was not rejected. The analyzation of the data indicated that statistically no significant difference existed among teachers of elementary school mathematics when compared across the variable of per pupil expenditure within the various districts included in this study. As described earlier in this study, the researcher believes that the districts involved in the investigation of this hypothesis were representative of other districts of similar expenditure levels within Michigan and other locales. For this reason, the researcher believes that the implication of the findings is in the statistical non-significance that was found. Although statistically significance was not found, the researcher feels it very important that no difference in modernness according to expenditure amount was discovered. This finding negates the belief held by many educators that if a district, or an individual teacher within that district, simply had or spent more money on the program, his or her instructional approach could be "more modern." The statistical test of this hypothesis indicates that such a belief is not wellfounded. As with hypothesis three, the implications of this finding will be discussed in fuller detail in Chapter $V$.

## Summary

In this chapter the data collected irom the mailed questionnaire instrument was presented, analyzed, and interpreted. Consideration was also given to the data collection procedures that were utilized. It was concluded than an anonymous, short, printed, and home addressed technique of development and dispersion of the data collecting instrument, resulted in the high percent of return of questionnaires sent.

The chapter addressed itself to discussing each of the research hypotheses separately, by stating the findings, the statistical summaries, and a semantic interpretation of the results.

Hypothesis One concerning the experts beliefs as compared to the teacher beliefs, was rejected. A significant difference did exist between the beliefs or ideologies of these two groups concerning the consistency of a modern elementary mathematics program.

Hypothesis Two was similarly rejected. There was a clear significant difference established between what a teacher said he or she believed and what that same teacher actually did in his or her classroom. This difference existed across all of the independent variables.

Hypothesis Three was not rejected. As was the case of per pupil expenditure, the size of a school districts student population was not significantly correlated with the modernness of the elementary classroom teacher's mathematics instruction.

Hypothesis Four was not rejected. The per pupil expenditure within varying school districts could not be said to be correlated with the degree of modernness exhibited by the elementary classroom teachers in those districts. Though statistically the hypothesis was not rejected, it was significant to the researcher that wealth and modernness are not necessarily correlated.

## CHAPTER V

SUMMARIES, CONCLUSIONS, AND RECOMMENDATIONS

This chapter presents a summary of tha study, a summary of the research findings, conclusions, implications, and recommendations for possible action and future research.

## Summary of the Purpose and Design

The purpose of this study was to determine if a panel of mathematics experts, drawn from the major teacher preparation institutions in the State of Michigan, and a random sample of public elementary school classroom teachers agreed as to the consistency of a list of contemporary mathematical ideals. A second purpose of this study was to determine the consistency among teachers in public schools between what they believe about mathematics instruction and that which they actually perform in the classroom in the way of techniques. Finally, this paper sought to discover a correlation between an elementary school teacher's modernness in mathematics instruction and the student population size of the teacher's district, or the per pupil general fund expenditure of the district.

The literature and research contained in Chapter II of this investigation, reflects pro and con arguments on six major areas of interest in contemporary mathematics instruction.

1. Discovery-inquiry learning and mathematical growth.
2. Influential factors of organization in mathematics instruction.
3. Attitudes and interests and elementary mathematics learning.
4. The effects of modern versus traditional instruction and organization.
5. The techniques of traditional and modern mathematics approaches.
6. Media and materials of traditional versus modern mathematics.

Within these six areas, a synopsis of ideas was taken by the researcher, submitted to a panel of experts in mathematics education for validation, and recorded. From this validated list of principles, a questionnaire instrument was developed and mailed to a random sample of elementary classroom teachers in Michigan.

The response rate was $93.3 \%$ and the analysis of the returned data was carried out upon a random sample of one hundred eighty respondents of those returning the instrument. A Jennrich program of analysis of variancerepeated measures technique at the . 025 level of confidence,
and the $\operatorname{CDC} 6500$ computer were used in the analyzation procedure. A second statistical test called a one-way analysis of variance with unequal subclasses was utilized for $\mathrm{HO}_{1}$ and was carried out at the . 025 confidence level on the CDC 3600 computer.
$\frac{\text { Summary of the Results }}{\text { of Hypothesis Testing }}$

Hypothesis One: There is no significant difference between expert opinion on the consistency of a modern mathematics program and the opinion of all of the teachers in this study.

## Result:

The null hypothesis was rejected at the . 025 level of confidence on the basis of the one way analysis of variance ( $t$ test) discussed earlier. The computed $F$ statistic was 13.023. The direction of the difference indicated that experts could be considered more modern than elementary classroom teachers concerning mathematics principles.

Hypothesis Two: There is no significant difference between what the respondents say they believe constitutes an effective mathematics program and what they say they do in their day-to-day activities in the class room.

## Result:

The null hypothesis was rejected at the . 025 level of confidence using a two way analysis of variance-repeated measures technique. The computed $F$ statistic was 5.260.

It could therefore be inferred, that teachers function in a manner quite separate from a list of modern principles that they agree with.

Hypothesis Three: There is no significant difference in "modernness" between teachers of varying school district sizes.

Result:
The null hypothesis was not rejected because of a computed $\underset{F}{ }$ value of only 1.599 using a two way analysis of variance-repeated measures technique at the .025 level of confidence. Therefore, the student population size was not a significant variable when correlated to teacher modernness in elementary mathematics instruction.

Hypothesis Four: There is no significant difference in "modernness" between teachers from school districts that have varying per pupil general fund expenditures.

Result:
The null hypothesis was not rejected due to a computed $\underset{F}{ }$ value of only . 228 utilizing a two way analysis of variance-repeated measures technique at the .025 level of confidence. The inferrence can therefore be made that per pupil expenditure within a school district was not a significant variable when correlated to teacher modernness in elementary mathematics instruction.

## Summary of the Findings

1. A significant difference was fourd icimec. iow experts responded to a list of contemporary mathematics principles as taken from current research and literature, and how a randomly selected sample of public elementary classroom teachers responded. The direction of the difference indicated a higher mean score for experts than teachers.
2. A significant difference was found among elementary classroom teachers concerning their beliefs about the consistency of a modern elementary mathematics program, and the practices they utilized frequently in their classroom. The direction of the difference indicated that teachers believe more modernly than they actually perform in their class.
3. There was no significant difference in modernness among teachers randomly selected from school districts of varying student population size. The inference therefore, is that size of district and the modernness of instruction in elementary mathematics are not correlated.
4. There was no significant difference in modernness among teachers randomly selected from school districts of varying per pupil general fund expenditures. The inference thus becomes that per pupil spending and modernness in elementary mathematics instruction are not correlated.

## Conclusions

Based on the findings of this scucy, the following conclusions would seem to be justified:

1. Mathematics education experts and elementary classroom teachers differ in their beliefs as to the consistency of a modern elementary mathematics program.
2. Teachers of elementary mathematics have belief systems concerning mathematics instruction, that differ from the practices they actually employ .
3. Student population size and modernness of elementary mathematics instruction are not correlated.
4. Per pupil expenditure and modernness of elementary mathematics instruction are not correlated.

## Implications from the Findings

Based on the data collected and the findings of this study, the following implications are presented for consideration:

1. Based upon the findings of the investigation of the first hypothesis, contemporary mathematics principles are not held by classroom
teachers to the extent that they are supported by mathematical education experts. Because these principles take their form from research and literature, and because mathematics experts have a large part to play in the writing of research and literature, it becomes necessary for newer and more effective methods of disseminating principles of mathematics to the teaching profession at large. It appears of questionable value to the teaching profession for researchers to continue vast and expensive research and writing endeavors, if and when the findings of that effort are not utilized or even discovered by the practitioners of the teaching profession to a greater degree than currently exists.
2. The implications of the finding concerning the beliefs of classroom teachers as opposed to their reported daily classroom practices, further points up the lag in research implementation. Clearly it was shown that teachers function in a manner quite different from their belief patterns concerning elementary mathematics. To the end that effective instruction instruction is built upon solid and reliable philosophical and psychological foundations,
careful attention in methoas tralning should be given to new teachers concerning the principles of mathematics instruction, as well as techniques. More importantiy, to the teacher already in the field, a stronger and more meaningful in-service or retraining program should be established, that is both on-going and evaluative. If modern mathematics is to continue to succeed, greater efforts need to be taken to insure that the pedagogical techniques with a class of learners correlates with modern principles of elementary mathematics to a greater degree than presently exists.
3. The finding of this study relative to school population size and teacher modernness in elementary mathematics, indicates a fallacy in the viewpoint, held by many, that mathematics education in smaller districts is somehow more "basic" or remedial in nature than in larger districts. Clearly, the teacher in the small consolidated rural school setting needs the same concerted effort to maintain contemporary ideals as much as any other elementary educator. This study indicates that he or she is at the present time functioning as "modernly" as any other teacher in an urban setting. For the researcher this points up the generally accepted belief that
the teacher is still the single most significant aspect in learning and the kev to the extent to which any subject, such as mathematice, is learned. By and large, rural districts cannot afford extensive in-service and retraining programs on a wide scale, and therefore it is necessary for mathematics experts and universities to develop methods of instruction and re-instruction that can aid the smaller districts as well as the larger.
4. The fourth hypothesis allowed for a difference to appear between the modernness of a teacher in an affluent school district and another from a poorer school district, if in fact such a difference actually existed among the respondents. Such a difference did not materialize and therefore, the commonly held ideal that instructional modernness and school district affluence are correlated can be considered questionnable. The implication from this is that similar mathematical proficiencies can be expected across the broad spectrum of all school districts, regardless of the relative wealth of any one district. The teacher, and the beliefs and practices of that teacher have greater impact on the achievement of a learner in elementary mathematics than how much money is
actually spent on the mathematics program. It would appear from the findings of this study that it is no longer an acceptable tenet to maintain; "give me more money for instructional purposes, and I'll do a better job."

## Recommendations for Future Investigations

Based on the findings, conclusiors, and implications from this study, the following recommendations would seem justified:

1. One of the findings in the related research in this study concerned the need for educators in elementary mathematios to possess positive attitudes toward mathematics and the teaching of mathematics. Further surveys of current practices in elementary mathematics instruction are needed to determine the validity of this finding.
2. Programs of pre-service for elementary teachers must concern themselves to a larger extent with the psychological principles upon which modern pedagogical practices in mathematics instruction are based. A need remains to develop this set of principles as they relate specifically to elementary mathematics, as compared to broader psychological principles that cut across all elementary disciplines.
3. The questionnaire instrument developed for and utilized in this investigation, could be expanded to include other demographic variables such as age, years of experience, major and minor etc. In this manner further isolation could be attempted to uncover the variables that may be significant in indicating a new approach to the in-service and pre-service of teachers. This study indicated that they were not size and/or expenditure, but what variables might serve as indicators are still unknown.
4. The researcher would suggest that a study similar in scope to this one be attempted in another geographical area. It is further suggested that any new geographical area of investigation include a "metropolitan" area or configuration. This would allow for differences to develop among neighboring contiguous school districts that would in all likelihood reflect nearly similar sizes and expenditures on the one hand, but allow for the ascertaining of other suspected variables of significance.
5. An in-depth study should be made of the effectiveness of elementary mathematics methods courses. Such a study should follow the development of an evaluative model that would enaile the researcher to maintain contact on a long range scale of time, with both the teacher and that teacher's methods instructor. Implementation of methods learned in the university methods courses in elementary mathematics, lag behind what could and should be expected.
6. Due to the large size of the volume of research conducted in mathematics education, and because concensus in that research is difficult to ascertain, a further research project should be conducted that isolated a particular aspect of mathematics instruction and developed a synopsis of all current research on the topic. Although this has been done many times over, the uniqueness of this study would be a new and innovative method of disseminating the findings of the project to the practioners of the profession. Developed concisely and with clarity of meaning, such a project would not necessarily need to take a written form, as in a piece of literature that only a relative few would see.

The innovativeness of this aissempnation would lend the project its needed credibility.
7. Because of the apparent difference between the practices of elementary ciassroom teachers in mathematics instruction and the beliefs those same teachers maintain, concerning an effective mathematics program, it is recommended that careful attention be given to development of a more adequate implementation model for preservice and in-service education. If this difference exists, as indicated in this study, then something must be done to eliminate the discrepancy between belief and practice if meaningful instruction in elementary mathematics is to take place.

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## APPENDICES

## APPENDIX A

## MATHEMATICS EDUCATION EXPERTS

## Panel of Mathematics Experts

Dr. Joseph Payne-----University of Michigan
Dr. William Fitzgerald-----Michigan State University Dr. Herbert Hannan-----Western Michigan University

Dr. William Swart-----Central Michigan University
Dr. Donald Buckeye-----Eastern Michigan University
Dr. Ted Eisenberg-----Northern Michigan University
Dr. Eugene Smith-----Wayne State University
Dr. David Wells-----Oakland University and Oakland Intermediate Schools

## APPENDIX B <br> PRINCIPLES OF ELEMENTARY MATHEMATICS AND TABULATION OF THE EXPERT'S RESPONSES

INSTRUCTICNS: Please check the appropriate blank to show whether you agree or disagree with each specific item as it relates to contemporary elementary school mathematics ingtruction.

AGREE
DISAGREE

1. There is a significant positive relationship between pupil 5 attitude toward mathematics and pupil achievement in mathematics.
2. No one single organizational pattern for mathematics instruction can be aaid to be better than any other in developing mathematical growth among elementary school learners.

| 7 | 1 |
| :---: | :---: |
| 8 | 0 |
| 6 | 2 |
| 2 | 6 |
| 3 | 5 |
| 7 | 1 |
| 5 | 3 |

9. A correct diagnosis procedure for individualizing in elementary mathematics is one in which the teacher ascertains the specific errors that a pupil is making, determines specifically how he works, and gives specific personalized remedial help.
10. Whether a learner is taught through a "guided discovery" technique or an "expository" technique, computational skill ability remains constant.
11. Grouping on the basis of ability in mathematics is of greatest benefit for pupils of higher ability.
12. Programs of individualized instruction in elementary mathematics do not lead to higher levels of pupil achievement, when compared with non-individualized programs.
13. Programmed materials are as effective in promoting mathematical growth as are textbooks, when either is used in a meaningful way.

5
$\square$
6
0

2
16. 'New mathematics' has more to offer the high achiever than the low achiever.
17. Having pupils manipulate materials is a much more effective method of facilitating mathematical growth, than merely watching teacher demonstrations with materials. $\qquad$ 0
18. Computer-assisted instruction is limited in its scope to drill and practice activities, and hence to computational concerns as opposed to concept development.
19. Providing children many opportunities to solve problems in a

| 4 |
| ---: |
| 7 |
| 2 | development of the ability of computational proficiency on the part of the learner.


| 2 |
| :--- |
| 2 |
| 7 |
| 6 |
| 2 |

6

6

1

2

6
25. Instruction in 'new mathematics', much more than in "traditional math', concerns itself with patterns and relationships of numbers and operations in a continuum of mathematical experiences.

| 8 | 0 |
| :---: | :---: |
| 5 | 3 |
| 5 | 3 |
| 5 | 3 |
| 6 | 2 |

## APPENDIX C

THE QUESTIONNAIRE AND COVER LETTER


March 23, 1972
Dear Educator:

I am a graduate assistant in the Department of Elementary and Special Education at Michigan State University and am in the process of completing my dissertation for a Ph. D. degree. My major field of interest in elementary mathematics education. I am in great need of your help and would appreciate it if you would take five to ten minutes to respond to the attached questionnaire.

This study is centered in southwest Michigan and your name tea selected at random from a compiled list of all elementary classroom teachers in the vast geographical area included in the study. In a few cases, some of your fellow workers may have also been chosen by the random process, and in other cases, your entire building because of its relatively mali size or location. The study simply seeks to ascertain what teacher f in the field believe and are experiencing in the realm of "modern" mathematics. I am concerned with whether or not mathematics instruction, as conceived at the University level, differs from what you, an experienced in the field educator, feel to be che situation. If so, the implications for the University are immense.

Please note that this questionnaire is completely anonymous. Therefore, I am not seeking any personal information about you. Hence, your responses to these questions will not be used in any personal evaluative sente, but will enable me to perceive how a large ample of randomly selected elementary public school classroom teachers feel about mathematics.

For economical reasons, please return the questionnaire even if you decide not to respond to it. I have enclosed an envelope for you to return your answered or unanswered form and certainly appreciate your help in this matter. Without your help it will not be possible.

Would you please try to complete this within the next week, ab am working under a deadline for completion of my degree requirements. Thank you again and feel free to contact me if you have any questions or concerns.

Sincerely,
Graduate Assistant


301 E Erickson Hall Michigan State University
(517) 353-4398

INSTHUCTINNS: Picas circla the appropriato word (s) that reflects your belief about rach or the following itoms, as they relate to contemporary elementary school mathematics instriction.

1. In an attempt to facilitate retention, intensive and specific review and practice should be provided in elementary mathematics.
Strongly agree Agree Undecided Disagree Strongly disagree
2. Grouping on the basis of ability in mathamatics is of greatest benefit for pupils of higher alility.
Strongly agree Agree Undecided Disagree Strongly disagree
3. Programmed materials are as effective in promoting growth in mathematics as are textbooks, when efiher is used in a meaningful way.
Strongly agree Agree Undecided Disagree Stronglydisagree
4. The learning of elementary mathematics is more dependent upon the possession of adequate materials, than any other single factor.
Strongly agree Agree Undecided Disagree Strongly disagree
5. Having pupils manipulate materials is a more effective means of creating mathematical growth, than watching the teacher demonstrate with matherials. Strongly agree Agree Undecided Disagree Strongly disagree
6. Providing children with many opportunities to solve problems in a variety of ways is more helpful than through formal analysis procedures.

Strongly agree Agree Undecided Disagree Strongly disagree
7. Pupils achieve more in mathematics when working independently than when working in groups of either small or large size.

Strongly agree Agree Undecided Disagree Strongly disagree
8. Verbal problem solving is as adequate a method of assessing mathematical growth as are paper and pencil techniques.

Strongly asroc Aproe Disagree Strongly disagre.
9. Instruction in 'now mathematics'much mere than in "traditional math' is concerned with patterns and relationships of numbers and operations in a continuum of mathematical experionces.

Strongly agree Agree Undecided Disagree Strongly disagree
0. The "new mathematics" results in greater concept development by the student. while the "traditional" technique results in greater skill or computational development.

Strongly agree Agree Undecided Disagree Stronglydisagree

INSTRUCTIONS: Circle the word (s) that most nearly answers the question for you.

1. How frequently is a typical learner in your classroom involved in review or practice activitios in mathematics?
```
Maily Twice a week Weekly Bi-weekly Monthly or rarely
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2. In the mathematics instruction in your classroom, how frequently are the students grouped by ability for instruction?
Daily Twice a week Weekly Bi-weekly Monthly or rarely
3. How often do you use programmed materials in your classroom in mathematics?
Daily Twice a week Weekly Bi-weekly Monthly or rarely
4. How frequencly do learners in your classroom use varied supplemental materials for mathematical instruction?
Daily Twice a week Weekly Bi-weekly Monthly or rarely
5. How frequently do learners in your classroom use manipulative materials in the acquisition of mathematical concepts and skilis?
Daily Twice a week Weekly Bi-weekly Monthly or rarely
6. How frequently do learners in your classroom solve problems utilizing more than one method?
Daily Twice a week Weekly Bi-weekly Monthly or rarcly
7. How frequently is a typical learner in your classroom working independently in mathematics, away from a group or the class?

> Daily Twice a week Weekly Bi-weekly Monthly or rarely
8. How frequently do vou formally assess mathematical growth with paper and pencil techoiques?
Daily Twice a week Weekly Bi-weekly Monthly or rarely
9. How of ten does your instruction im mathematics deal with number patterns and the relationship of operations?
Daily Twice a week Weekly Bi-weekly Monthly or rarely
10. How frequently does vour mathematical instruction concern itself with concept, as opposed to skill, development?

> Daily Twice a week Weekly Bi-weekly Monthly or rarely


[^0]:    ${ }^{4}$ Jack Wrigley, The Factorial Nature of Ability in Elementary Mathematics, British Journal of Educational Psychology, 28:61-78, February 1968.
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