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AN ECONOMETRIC ANALYSIS OF ENGEL CURVES:
BASED ON M.S.U. CONSUMER PANEL DATA

By

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ABSTRACT

AN ECONOMETRIC ANALYSIS OF ENGEL CURVES:
BASED ON M.S.U. CONSUMER PANEL DATA

By

Phisit Setthawong

In the present study, the author made use of the M.S.U. Consumer Panel data of 1958 to (1) analyze the Engel curves, (2) modify the Engel curves, and (3) approximate utility functions by means of the Engel curves. The commodities under studies were dairy products; fats and oils; fruits; vegetables; and meat, poultry, fish, and eggs.

First, thirteen cross sectional studies on the Engel curves were set out for the five composite foods. Three mathematical forms were selected for the true functional form of the Engel curves. They were linear, semi-log, and double-log forms. The thirteen cross-sectional studies showed that no functional form uniformly gave a better "goodness of fit" to the observations. For each composite food, the semi-log form did consistently give the highest income elasticity estimates which were

widely different from those based on the linear and double-log forms. Surprisingly, the income elasticity estimates for each composite food based on the linear and double-log forms were very nearly equal over all the thirteen cross sectional studies. The mean of the income elasticity estimates for dairy products based on the semi-log, linear, and double-log forms were .2604, .1009, and .0938, respectively. As for fats and oils, the mean of the income elasticity estimates based on the semi-log, linear, and double-log forms were .5217, .2091, and .2209, respectively. The mean of the income elasticity estimates for fruits based on the semi-log, linear, and double-log forms were .7052, .2889, and .3388, respectively. The figures .4768, .1885, and .1988 were the mean of the income elasticity estimates for vegetables based on the semi-log, linear, and double-log forms, respectively. As for meat, etc., the mean of the income elasticity estimates based on the semi-log, linear, and double-log forms were .5428, .2259, and .2353, respectively. Generally, the income elasticity estimates for the five composite foods based on either functional forms were predominantly inelastic: Engel's law of consumption was confirmed. Elastic and negative income elasticity estimates based on the nonlinear forms were found, but they were relatively infrequent.

Secondly, the cross sectional and time series data were pooled. The modified Engel curves were formulated for estimating both income and price elasticities for the five composite foods. The linear form was selected as the first order approximation for the true functional form of the modified Engel curves. The M.S.U. price indices, based on the panel reported food prices, were used to represent the actual food price faced by the panel households. Based on the first order autoregressive scheme of disturbances, the autoregressive coefficients for the five composite foods were estimated. They were highly positive, close to but less than one: .8284 for dairy products; .6966 for fats and oils; .7100 for fruits; .6629 for vegatables; and .5031 for meat, etc. After eliminating the autoregressive effects, the income elasticity estimates for dairy products; fats and oils; fruits; vegetables; and meat, etc. were .0170, .0928, .1808, .0533, and .1472, respectively. As for the price elasticity estimates, they were -1.4543, -1.5712, -.8158, -.7424, and -.3580 for dairy products; fats and oils; fruits; vegetables; and meat, etc., respectively. Regarding the signs of the income and price elasticity estimates, the results of these combined studies were highly successful as they confirmed the demand theorem. The consistency of the panel data was probably the main reason for this success.

Lastly, based on Wald's theorem, some preliminary evidence on the approximate determination of utility functions by means of Engel curves was presented. Using the results of the thirteen cross sectional studies on the Engel curves and the M.S.U. price indices, two numerical illustrations were given. These illustrations, at least, indicated that, under certain circumstances, the empiriral utility functions could be approximated.

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INTRODUCTION

The functional relationship from disposable income to expenditure on (or quantity purchased of) a particular good is generally termed the Engel curve. Theoretically, the functional relationship of the Engel curve is derivable from the loci of tangencies between indifference and budget surfaces with fixed prices and variable income, provided the utility function is known. But, the utility function being normally unknown, a direct search on the functional relationship of the Engel curve from the observed data is inevitable. Fitting the Engel curve, in certain circumstances, the utility function might be approximately determined.

Most previous studies on the Engel curves classically used cross sectional data of a particular period of time to estimate Engel or income elasticities. A particular functional form was selected for the functional relationship of the Engel curves.

In the present study, the author will make use of the M.S.U. Consumer Panel data of 1958 to (1) analyze the Engel curves, (2) modify the Engel curves, and (3) approximate utility functions by means of Engel curves.

In analyzing the Engel curves, thirteen cross sectional studies on the Engel curves are set out for five composite foods. Three functional forms are selected for the functional relationship of the Engel curves. In this way, one can investigate if the income elasticity estimates based on three alternative functional forms are widely different. In addition, one can investigate if a particular functional form uniformly gives a better "goodness of fit" to the observations.

In modifying the Engel curves, cross sectional and time series data are pooled. Both income and price elasticities for five composite foods are estimated. Statistically, these income elasticity estimates are more reliable than those obtained from individual cross sectional studies, since more observations are used in the estimation procedure. In addition, the estimation of price elasticities is possible because of the continuous collection of the panel data.

Lastly, the present study attempts to show that the cross sectional studies on the Engel curves are not limited on the estimation of income elasticities. In certain circumstances, the fitted Engel curves might be used to approximate utility functions.

As for the outlines of the present study, the following is the order of discussion.

In Chapter I, a general survey of the empirical studies on the Engel curves is set out. The M.S.U. Consumer Panel Survey is briefly reviewed in Chapter II. In Chapter III, thirteen cross sectional studies on the Engel curves based on three alternative functional forms are set out for five composite foods. In Chapter IV, combined analyses of the modified Engel curves are set out for five composite foods. Some preliminary findings on approximating utility functions by means of Engel curves, and some areas for future research, are given in Chapter V.

CHAPTER I

REVIEW OF LITERATURE

1. Engel Curves

1.1. Historical Review

The empirical study of the functional relationships between household income and household expenditure on goods has a long history, certainly dating back as far as LePlay and Engel.[1] Perhaps the first and most famous study of these relationships was made by Ernst Engel in 1857.[2] The study was based on Ducpetiax's data for 153 Belgian families, which had been classified into three socio-economic groups: families dependent upon public assistance; families just

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[1] The early studies of family budgets are considered at length in C. C. Zimmerman, Consumption and Standard of Living (New York: D. Van Nestrand, 1936).

[2] Ernst Engel (1821-96), Director of the Prussian Bureau of Statistics, was an administrator who published many prominent articles. For a list of his works, see J. A. Schumpeter, History of Economic Analysis (New York: Oxford University Press, 1954), footnote 14, p. 961.

It is interesting to note that Engel's work was mainly influenced by two of his contemporaries. One was the French engineer F. LePlay, who had collected budgets from households all over Europe mostly, it seems, from humanitarian interest. The other was the Belgian Statistician Que'telet, who was a firm proponent of the idea that human characteristics, at least on the average, were governed by laws as definite as those which govern physical phenomena; see H. S. Houthakker, "An International Comparison of Expenditure Pattern, Commemorating the Centenary of Engel's Law," Econometrica (October, 1957), p. 532.

able to live without such assistance; and families in comfortable circumstances. On the basis of this study, Engel proposed a law of consumption: "The poorer a family, the greater the proportion of its total expenditure that must be devoted to the provision of food."[3] In other words, the law states that the proportion of expenditure devoted to food decreases as the standard of living of the household increases.[4]

Neither Engel himself, though he published his law originally in 1857, nor anyone else, seemed to have realized its importance from the viewpoint of economic theory.[5] Aside from Engel's works, the quantitative analysis of family budgets did not attract much attention among professional economists. It was not until 1935 that interest was revived by the work of Allen and Bowley.[6] Since then, economists have begun to understand the significance of the results obtained from the analysis of budget data.

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[3]G. J. Stigler, "The History of Empirical Studies of Consumer Behavior," Journal of Political Economy, Vol. 42 (April, 1954), p. 98.

Engel also asserted that the wealthier a nation, the smaller the proportion of food to total expenditure. Ibid., footnote 9.

[4]S. J. Prais and H. S. Houthakker, The Analysis of Family Budgets (Cambridge: University Press, 1971), p. 79.

[5]Schumpeter, op. cit., p. 961.

[6]The study was done by R. G. D. Allen and A. L. Bowley, Family Expenditure (London: Staple, 1935).

As time passed, Engel's law changed slightly. Now it is often stated as: "percentage expenditure on food is on the average a decreasing function of income";[7] or: "food expenditure increases with income, but at a lesser rate, i.e., that food demand is inelastic with respect to income."[8]

In the last 35 years, tremendous effort has been devoted to the measurement of the income elasticity of food consumption.[9] Engel's law[10] has been tested and repeatedly confirmed by numbers of studies.[11] The following are a few examples. The Wharton school survey, in 1950, indicated that, in the U.S., household food expenditures took

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[7]Schumpeter, op. cit., p. 961.

[8]J. S. Cramer, Empirical Econometrics (Amsterdam: North Holland, 1969), p. 135.

[9]M. C. Burk, Influences of Economic and Social Factors on U.S. Food Consumption (Minn.: Burgess, 1961), p. 70.
Recent studies also show interest in other household items, such as housing, clothing, house furnishings, and services. For a partial list of studies of these subjects, see R. Ferber, "Research on Household Behavior," in Surveys of Economic Theory, Vol. III, Survey XII (New York: St. Martin's Press, 1966), footnote 3, p. 138. These studies yield low income elasticities for housing, elasticities close to unity for clothing, and higher elasticities for various types of recreation, personal care, home operation, and other services; see Ferber, Ibid., p. 139.

[10]Similar laws have also been formulated for other items of expenditure. For example: Schwabe's law states that the per cent of income spent for housing declines as income rises; see Ferber, Ibid.

[11]For a list of studies, see the bibliography by J. N. Morgan, "A Review of Recent Research on Consumer Behavior," in Consumer Behavior: Research on Consumer Reactions, ed. by L. H. Clark (New York: Harper & Bros., 1958), pp. 93-219.

up about 30% of household disposable income when household income was $5,000, but only 21% when household income was $10,000.[12] Houthakker, based on regression analyses of about 40 surveys from about 30 countries, found that the income elasticities for food in these countries were all significantly less than one. They were similar, but not equal; the highest figure was 0.731 for Poland and the lowest 0.344 for Britain (middle class).[13] Recently, the National Food Survey report for 1965 published the following estimates of income elasticity of expenditure on food in various years in Britain: 0.30 in 1955; 0.28 in 1958; 0.25 in 1960; 0.27 in 1962, 0.23 in 1965; and 0.23 in 1966.[14]

In order to understand the empirical results of Engel curves, it is necessary to review some techniques and data that have been widely used.

1.2. Cross Section

The study of Engel curves frequently uses cross sectional data obtained from a sample of households for a

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[12]The U.S. Bureau of Labor Statistics for the Wharton School of Finance, University of Pennsylvania: Study of Consumer Expenditures, Incomes and Savings, Vol. 1-2 (University of Pennsylvania Press, 1956).

[13]H. S. Houthakker, "An International Comparison of Expenditure Patters, Commemorating the Centenary of Engel's Law," Econometrica, Vol. 25 (October, 1957), pp. 530-51.

[14]The National Food Survey Committee, Domestic Food Consumption and Expenditure (London: H.M.S.O., 1965), p. 136.

particular period of time.[15] The period may be a day, a week, a month, a year, or any other convenient interval.[16] The techniques of arc elasticity, single regression equation, and simultaneous system of equations have been used to estimate income elasticities.

Arc elasticity is the oldest technique.[17] West, using this technique, obtained the following income elasticities for food in the Lansing area: 0.26 for households receiving less than \$4,000; 0.10 for those with incomes of \$4,000-\$6,000; and -0.20 for households receiving more than \$6,000.[18]

Single regression equation is the most popular technique to investigate income-food relationships. In a cross sectional study, the Engel curve is generally stated as:[19]

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[15]There are at least three possible methods of collecting the data: personal interview, mail, and telephone. For a discussion of relative merits of these methods, see R. Ferber and P. J. Verdoorn, Research Methods in Economics and Business (New York: Macmillan, 1962), p. 209-13.

[16]K. A. Fox, Intermediate Economic Statistics (New York: John Wiley & Sons, 1968), p. 72.

[17]For a discussion of this technique, see W. C. Waite and H. C. Trelogan, Agricultural Market Prices (2nd ed.; New York: John Wiley & Sons, 1951), p. 41.

[18]J. G. West, "Estimates of Income Elasticity of Consumer Panel Data" (unpublished Ph.D. dissertation, Department of Agricultural Economics, Michigan State University, 1958).

[19]In a cross sectional study, prices are held reasonably constant. Expenditure on a particular good is proportional to the physical quantity. Either quantity or

$$Y_{ik} = f_k(M_i) + U_{ik}$$

where Y_{ik} is the i^{th} household expenditure on the k^{th} good,

M_i is the i^{th} household income,

U_{ik} is the stochastic error term representing both the effects of variables that are not explicitly introduced into the equation and errors of measurements in the dependent variable, and

f_k is a specific functional form of the Engel curve.

Regarding the function f_k, it is appropriate on the assumption that different households are homogeneous except for the differences in variables in the equation and the stochastic error term. The homogeneity assumption implies that household A would, on the average, spend as much on the k^{th} good as household B does if A's income were the same as B's.[20]

It is interesting to point out that cross sectional estimates of income elasticity for food in most studies fall mainly in the neighborhood of 0.50. For example: Tobin obtained 0.56 for the United States in 1941; Stone, et. al.,

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expenditure may be used as the dependent variable. This is the technique of scaling of variables. The regression coefficients and their standard errors are different by mere changes of units. For a full discussion of this technique, see A. S. Goldberger, Econometric Theory (New York: John Wiley & Sons, 1964), pp. 185-86.

[20] L. R. Klein, Introduction to Econometrics (Englewood Cliffs: Prentice Hall, 1962), p. 54.

estimated 0.53 for the United Kingdom in 1938; Wold and Jureen obtained 0.51-0.53 for Sweden in 1933; Clark, et al., estimated 0.40 for the United States in 1948; Burk obtained 0.58 for the United States in 1950; Snyder obtained an average income elasticity for food of 0.54 calculated from over three hundred cross section family budgets studies in the United States during 1880-1950 (of individual cities, at different dates).[21]

The single structural regression equation is appropriate upon the presupposition that household income is purely exogenous. If household income is not purely exogenous, the simultaneous system of equations would be more appropriate to estimate income elasticities. In practice, however, the simultaneous system of equations has, thus far, only rarely been used in the analysis of Engel

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[21] J. Tobin, "A Statistical Demand Function for Food in the U.S.A.," Journal of the Royal Statistical Society, Series A (1950), p. 119; R. Stone, et al., The Measurement of Consumer Expenditure and Behavior in the United Kingdom, 1920-38 (Cambridge: University Press, 1954), p. 327; H. Wold and L. Jureen, Demand Analysis (New York: John Wiley & Sons, 1953), p. 303; F. Clark, et al., "Food Consumption of Urban Families in the United States," Agricultural Information Bulletin No. 132 (Washington, D.C.: U.S. Department of Agriculture, 1954), p. 39; M. C. Burk, "Income-Food Relationships from Cross Section and Time Series Surveys," Proceedings of the American Statistical Association (B.E.S.S., 1957), p. 103; and E. M. Snyder, "Long-term Changes and Family Expenditure," in Consumer Behavior: Research on Consumer Reactions, ed. by L. H. Clark (New York: Harper & Bros., 1958), pp. 361-62.

curves mainly because of the complexity of estimation and specification of the structural behavior equations.[22]

1.3. Time Series

Besides cross sectional data, the aggregate time series data have also been used in the analysis of Engel curves. Data of this type are normally obtained by aggregating household expenditure and household income over periods of time. The Engel curve based on time series observations is generally stated as:

$$Y_{kt} = f_k(M_t) + U_{kt}$$

where Y_{kt} is the aggregate expenditure on the $k^{\underline{th}}$ good at the $t^{\underline{th}}$ period of time,

M_t is the aggregate income at the $t^{\underline{th}}$ period of time,

U_{kt} is the stochastic error term, and

f_k is a specific functional form.

The function f_k is appropriate on the assumption that different periods of time are homogeneous except for differences in the explicit variables and for differences in the stochastic error term.

Strictly speaking, in a time series study, expenditure should be converted to quantity since price varies

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[22] H. S. Houthakker and L. D. Taylor, Consumer Demand in the United States: Analyses and Projections (2nd ed.; Harvard: University Press, 1970), p. 7.

over periods of time. In the case of a composite commodity such as food, the food price index may be used as a deflator. That is, the quantity index is computed in the following manner: $Q_{kt} = (Y_{kt}/P_{kt})*100$; where P_{kt} is the price index of the k^{th} food.

Time series income-food relationships, where variables are deflated for price movements, give substantially lower values for income elasticity for food than cross sectional studies.[23] Nevertheless, time series estimates of income elasticity for food seem to confirm Engel's law, although the law was initially stated in terms of cross section rather than time series data. For example: using time series, Tobin got an estimate of income elasticity for food of 0.27 for the United States; Wold and Jureen, values of 0.23 and 0.28 (depending on the period covered) for Sweden; Burk obtained 0.21 and 0.23 (depending on the period covered) for the United States.[24]

1.4. Consumer Panel

Recently, a number of studies have made use of consumer panel data.[25] This type of data is obtained from

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[23] For an explanation of this phenomenon, see p. 90, footnote 17.

[24] Tobin, *op. cit.*, p. 134; Wold and Jureen, *op. cit.*, p. 303; Burk, "Income-Food Relationships," *op. cit.*, p. 103.

[25] A consumer panel is essentially a sample of people who are interviewed repeatedly over a period of time. Strictly speaking, a sample becomes a panel operation if its members are interviewed in at least two different points in time on the same general subject. If only two or three

the same group of households over periods of time. The data provide both cross sectional and time series information.

Ferber appraises the panel data by stating that

> Time series aggregates have serious disadvantages because of the frequently unstable estimates of income elasticities. . . . On the other hand, cross sectional data are essentially static . . . Hence, a combination of the two types of data would seem to offer a much more powerful technique for understanding consumer behavior.[26]

By keeping continuous records, a comprehensive picture can be obtained, not only of the factors influencing a household's purchasing behavior, but also of the manner in which decisions are made.[27]

Marschak notes that combining time series and cross sectional data increases the accuracy of the estimated parameters.[28] However, the combined studies face some

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"waves" of interviews are involved, there is a tendency to use the term "reinterview sample" rather than "panel." Then again, if the same people are interviewed several times on different aspects of the same subject, neither "panel" nor "reinterview sample" may be used to describe the operation. For more discussion of the consumer panel, see Ferber and Verdoorn, op. cit., pp. 267-76; G. G. Quackenbush and J. D. Shaffer, "Collecting Food Purchase Data by Consumer Panel, 1951-58," Technical Bulletin 279, M.S.U. Agricultural Experiment Station (August, 1960).

[26]Ferber, op. cit., p. 141.

[27]Ibid., p. 145.

[28]J. Marschak, "Review of Schultz, Theory and Measurement of Demand," Economic Journal, Vol. 49 (1939), p. 487.

difficulty in interpretation and specification of the function f_k and the disturbance.[29]

With the combined analysis, Sparks, using M.S.U. Consumer Panel Data of 1955-58, found that the estimate of income elasticity for food was about 0.25.[30] Similarly, Crockett, using the consumer panel data of the Market Research Corporation of America for 1951-53, obtained an estimated income elasticity for food of 0.23.[31]

1.5. Concepts of Variables

Besides using different techniques and data, different concepts of variables are also used in the single regression analysis. Some studies use expenditure as the dependent variable, others use quantity.[32] As for the independent variable, some studies use income, others use total expenditure. The use of observed or current income as the independent variable has been widely accepted. Nevertheless, several other concepts of income have been

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[29] For a full discussion of this problem, see E. Kuh, Capital Stock Growth: A Micro-Econometric Approach (Amsterdam: North Holland, 1963), pp. 116-40 and pp. 158-63; Also, J. Kmenta, Elements of Econometrics (New York: Macmillan, 1971), pp. 508-17.

[30] W. R. Sparks, "Estimates of the Demand for Food from Consumer Panel Data" (unpublished Ph.D. dissertation, Department of Agricultural Economics, Michigan State University, 1961).

[31] J. Crockett, "A New Type of Estimate of the Income Elasticity of the Demand for Food," Proceedings of the American Statistical Assocation (B.E.S.S., 1957), pp. 117-22.

[32] For a discussion of this problem, see p. 8 and p. 11.

proposed. Among them, the concepts of permanent income and relative income have been well known.

Briefly, the relative income hypothesis states that the i^{th} household expenditure on the k^{th} commodity at the t^{th} period of time would depend not on its observed income, but on the ratio between observed income and the mean income of the group.[33] As for the permanent income hypothesis, the i^{th} household expenditure on the k^{th} commodity depends on its permanent income and not on its transitory income, where the observed income equals the permanent income plus the transitory income.[34]

The use of total expenditure as the independent variable can be justified on the grounds that the data on income are unavailable, or that the available data of

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[33] The relative income hypothesis seems to have been first propounded by D. Brady and R. Friedman. Much additional theoretical and empirical support of this hypothesis was provided by the work of Modigliani and of Duesenburry. See D. S. Brady and R. Friedman, "Savings and the Income Distribution," N.B.E.R., Studies in Income and Wealth, Vol. 10 (New York, 1947), pp. 247-65; F. Modigliani, "Fluctuations in the Saving-Income Ratio: A Problem in Economic Forecasting," N.B.E.R., Studies in Income and Wealth, Vol. 11 (New York, 1949), pp. 371-443; and J. Duesenburry, Income, Saving and the Theory of Consumer Behavior (Cambridge: Harvard University Press, 1952).

[34] M. Friedman has greatly elaborated and tested the permanent income hypothesis. M. Dunsing and M. G. Reid, and M. Nerlove have explored the applications of this hypothesis to foods. See M. Friedman, A Theory of the Consumption Function (Princeton: National Bureau of Economic Research, 1957); M. Dunsing and M. G. Reid, "Effect of Varying Degree of Transitory Income on Income Elasticity of Expenditures," Journal of American Statistic Association, Vol. 53 (June, 1958), pp. 357-59; M. Nerlove, "The Implications of Friedman's Permanent Income Hypothesis for Demand Analysis," Agricultural Economic Research (January, 1958).

income are highly unreliable and distorted.[35] Nevertheless, if total expenditure is used as the independent variable, the elasticity refers, of course, to the total expenditure elasticity, not income elasticity.[36] The difference between the two is slight. For an empirical example, Stone, et al., realized that the total expenditure elasticities should be reduced by 10% in order to approximate income elasticities.[37]

2. Prices and Non-Economic Factors

Besides household income, other factors such as prices and non-economic factors might have significant influences on household expenditure behavior.

2.1. Prices

Prices vary over time. In a time series study on Engel curves, prices should be introduced as an explicit variable in the regression analysis. In a cross sectional

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[35] The use of income observed with error as the independent variable will lead to systematic underestimation of income coefficient by least squares estimates. For a full discussion of this problem, see Cramer, *op. cit.*, p. 139.

[36] The use of total expenditure as the independent variable may lead to inconsistent estimates. For a full discussion of this problem, see N. Liviatan, "Errors in Variables and Engel Curve Analysis," *Econometrica*, Vol. 29 (1961), pp. 336-62; Cramer, *op. cit.*, p. 140; and R. Summers, "A Note on Least Square Bias in Household Expenditure Analysis," *Econometrica*, 27 (January, 1959), p. 121.

[37] Stone, *et al.*, *op. cit.*, p. 312.

study, prices are reasonably presumed to be constant since households are likely to face the same set of market prices except, perhaps, for regional differentials or price discrimination.[38] Principally, cross sectional studies concentrate on the estimation of income elasticities. Among the households participating in a survey, there is not enough price variation to permit the analysis of price effects, particularly since much of the apparent price variation may be attributed to quality differences.[39]

2.2. Non-Economic Factors

Apart from income and prices, there are many non-economic factors that affect household consumption. The first, and probably the most obvious cause of variations is household size and household composition. By "household composition" is meant age and sex of the members of the household. Second, there are regional factors which reflect external conditions and social habits of the members of the household. And, third, social class and occupation may also influence household expenditure, as well as religion and various psychological characteristics of the members of the household.[40]

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[38] L. R. Klein, A Textbook of Econometrics (Evanston: Row, Peterson & Co., 1953), p. 213; Stone, et al., op. cit., p. 312; Prais and Houthakker, op. cit., p. 110.

[39] Houthakker and Taylor, op. cit., p. 254.

[40] There have been numbers of studies of non-economic factors. In order to limit the survey, this study will concentrate on household size and household composition. For

Of all the non-economic factors, household size and household composition have been studied most intensively.[41]

Household size and household composition are usually measured by adult equivalent scales, man values, or unit consumers. This scale expresses the food expenditure of each age-sex type as a proportion of some "standard" type, say, the adult male. Thus, the scale is constructed by regarding a child or an adult female as equivalent to some fraction of an adult male.[42] In fact, this scale was already used by Ernst Engel who labelled the unit a "quet"; where the quet was defined as the value of food that was consumed by a child less than one year old, an adult female 3.1 quets, and an adult male 3.5.[43]

In a cross sectional study, the Engel curve that includes household size and household composition may be stated as:

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more surveys and discussions of other non-economic factors, see R. O. Hermann, "Household Socio-Economic and Demographic Characteristics as Determinants of Food Expenditure Behavior" (unpublished Ph.D. dissertation, Department of Agricultural Economics, Michigan State University, 1964); Burk, U.S. Food Consumption, op. cit., pp. 53-64; and Ferber, op. cit., pp. 126-34.

[41]It should be pointed out that household size and composition vary from household to household, yet they vary very little from year to year. Thus, these factors might be omitted in a time series study. See Houthakker and Taylor, op. cit., p. 275.

[42]Prais and Houthakker, op. cit., p. 126.

[43]C. S. Bell, Consumer Choice in the American Economy (New York: Random House, 1967), p. 104.

$$Y_{ik} = f_k \left(M_i, \sum_j c_{jk} N_{ij}\right) + U_{ik}$$

where N_{ij} is the number of persons of the j^{th} age-sex type in the i^{th} household,

c_{jk} is the expenditure scale value of the j^{th} age-sex type of person for the k^{th} good, and

$\sum_j$ is the summation over j.

In this formulation, a different scale of equivalent adult is distinguished for each commodity, and the scale for the k^{th} commodity may be termed the k^{th} specific scale.[44]

The set of values c_{jk} may be the nutritional scale (the scale that is based on the nutritional requirements of varying age and sex), or the actual expenditure scale that is constructed from the actual observed data.[45] In practice, a set of nutritional scales for each type of person and for each commodity is chosen.

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[44] Some researchers expect that the scales of equivalent adults will be similar for all commodities, so that it will not be necessary in practice to distinguish a scale for each commodity.

[45] For a full discussion of computation and application of age-sex equivalent scales, see D. W. Price, "Age-Sex Equivalent Scales for United States Food Expenditures--Their Computation and Application" (unpublished Ph.D. dissertation, Department of Agricultural Economics, Michigan State University, 1965).

Traditionally, most studies assume $c_{jk} = 1$ for all j, k.[46] In most survey data, household size and household income are highly positive correlated.[47] The simplest hypothesis allowing for the effects of variations in household size is to suppose that consumption per person depends only on the level of income per person. The Engel curve may be stated as:

$$\frac{Y_{ik}}{N_i} = f_k \left(\frac{M_i}{N_i}\right) + U_{ik}$$

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[46] The assumption implies that there is no difference in the amount spent for any commodities consumed by persons of differing age and sex. In general, many differences exist in the amount spent for commodities consumed by individuals of different age-sex composition. However, this assumption is acceptable when information regarding age-sex composition of individual households is unavailable.

[47] It should be noted that the positive correlation is not due to a direct causal link between the two variables but to fortuitous characteristics of the existing social structure, as may be illustrated by the two extreme instances which largely determine the observed correlation. At one end of the scale we have households of one or two persons, which usually represent the very young--bachelors and young couples--or the old; both categories tend to have substantially lower incomes than the active adult population. At the other end of the scale, very large families of eight or more persons often include more than one wage earner, either because they are in fact composite households, or because of the natural age structure of families with six or more children. See Cramer, *op. cit.*, p. 162.

This hypothesis obviously corresponds to the assumption of constant returns to scale often made in the theory of production.[48]

3. Functional Forms of Engel Curves

Perhaps the most difficult part of the analysis of Engel curves is to express the function f_k in an appropriate

[48] The assumption of constant returns to scale is likely improper since a large household may be able to attain a higher level of per capita consumption than a smaller household. Particularly with food, economies may arise in purchasing, storage, and preparation of food. Several empirical studies have attempted to investigate the degree of economies of scale. For a classic example, Prais set up the following model:

$$\frac{Y_{ik}}{N_i} = f_k\left(\frac{M_i}{N_i}\right) + g_k(N_i) + U_{ik} ;$$

where f_k and g_k are undefined functional forms.

In Prais's model, if there were no economies of scale, g_k is zero. Nevertheless, the model faces two difficulties. They are: if f_k is not specified correctly, then part of the variance properly ascribable to M_i/N_i is ascribed to N_i, and values of M_i and N_i tend to be correlated so that the coefficients of the regression are imprecise because the standard errors of the regression coefficients become large. Prais used the semi-log form; with the British data of 1938, he found that the economies of scale appeared, but were very small. With a belief that the disadvantages of small households may today be not so great, the assumption of constant returns to scale may be taken as substantially correct in the formulation of Engel curves. For a fuller discussion of this problem, see S. J. Prais, "Non-Linear Estimates of the Engel Curve," Review of Economic Studies, Vol. 20 (1952-53).

algebraic functional form.[49] Theoretically speaking, if households were regarded as the fundamental consumer units, all households approximately had the same preference function and were faced by the same prices, it would be possible to derive functional forms of all Engel curves provided a particular form of the preference function were given.[50]

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[49] The problem of finding the most appropriate form of Engel curves is an old one in econometrics. As yet, no solution appears to have found general acceptance. Generally speaking, it is probably true that the investigation of the form of Engel curves has attracted less attention than have methods of estimating parameters for specified equations. See C. E. V. Leser, "Forms of Engel Functions," Econometrica, 31 (October, 1963), p. 694.

[50] Denote $q^1, \ldots, q^n$ be a set of n goods purchased by the representative consumer at a period of time; $p^1, \ldots, p^n$ be the corresponding set of prices; m is the disposable income; $u(q^1, \ldots, q^n)$ the utility indicator. Given p's and m at a period of time, the first condition for maximizing the utility subjected to the budget constraint is fulfilled if the consumer purchased the quantities such as

$$\frac{\partial u}{\partial q^1}/p^1 = \frac{\partial u}{\partial q^2}/p^2 = \ldots\ldots = \frac{\partial u}{\partial q^n}/p^n ,$$

$$\sum_{k=1}^{n} p^k q^k = m.$$

Solving the above equations, one gets the quantities purchased as functions of prices and incomes. In a cross section with constant prices, quantities purchased will depend only on the income; that is,

$$q^1 = f^1(m), \ldots\ldots, q^n = f^n(m) ,$$

or $q^k = f^k(m)$ for $k = 1, 2, \ldots, n$.

These are, in fact, the Engel curves.

By the technique of scaling of variables, the Engel curves can be rewritten as:

$$y^k = f^k(m) ,$$

Unfortunately, the functional form of the preference function is unknown. Needless to say, economic theory alone could not provide appropriate knowledge of the mathematical form of Engel curves. Both economic and statistical considerations influence the choice of the algebraic formulation. As Goldberger mentions, "The choice of an appropriate functional form, in practice, involves a compromise among . . . economic theory, goodness of fits, and simplicity."[51]

Some researchers believe that there is at least one element from economic theory that can be taken over for formulating the algebraic functional form of Engel curves.[52]

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where $y^k = p^k \cdot q^k$ for $k = 1, 2, \ldots, n$.

For a full discussion of this problem, see A. Wald, "The Approximate Determination of Indifference Surfaces by Means of Engel Curves," Econometrica, Vol. 8 (1940), pp. 144-46.

It is interesting to point out that, knowing the shapes of Engel curves, the functional form of indifference surfaces might be approximately determined. See Wold and Jureen, op. cit., pp. 130-31; Wald, op. cit., pp. 144-75; and H. T. Davis, The Theory of Econometrics (Bloomington, Ind.: Principia Press, 1941), pp. 165-68.

[51] Goldberger, op. cit., p. 217.

[52] If possible, there is another element: the saturation point or the point of zero marginal utility. Theoretically, the saturation point is very unlikely to be reached. In practice, however, most researchers believe that if a commodity is a specific item, for an individual household, the saturation point is likely to occur at a high level of income. On the other hand, if a number of commodities are aggregated as a composite commodity, or if a group of households rather than an individual household is investigated, the satiety level is not likely to be reached. See Prais and Houthakker, op. cit., pp. 16-17; and Cramer, op. cit., p. 149.

Assuming no saving, that element is the budget restriction or the adding criterion: $\sum_k Y_{ik} = M_i$ which implies $\sum_k \frac{dY_{ik}}{dM_i} = 1$; where dY_{ik}/dM_i is the slope of the Engel curve, on the average for the k^{th} commodity.

If the number of commodities in the budget restriction is allowed to vary, at low income consumption is restricted to small number commodities. As income rises, new commodities enter. Thus, starting with the most elementary situation when income is very low, only one commodity is bought, then the slope of the Engel curve for that commodity is unity. If income rises, successively additional commodities are bought, then, it is apparent that the slope for the first commodity will gradually diminish to make room, as it were, for new entrants. If the commodities are substitutes,[53] it is valid to suppose that the Engel curves for all commodities become less steep as income increases.[54]

As for statistical considerations (goodness of fit and simplicity), some researchers believe that the

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[53]But there is no necessary reason for all commodities to be substitutes. The introduction of a new commodity into the budget restriction might cause the slopes of its complements to rise. If severe, the successive new entrants might replace the already present commodities entirely, so that the shapes of Engel curves might be kinked or discontinuous over certain ranges of income.

[54]Prais and Houthakker, op. cit., pp. 15-17; Cramer, op. cit., pp. 147-49.

functional form of Engel curves should broadly fit the data and reproduce any marked curvature the observations may possess. The form can be made linear by a simple transformation of the data, so that linear regression can be applied to the correspondingly transformed observations.[55]

There are dozens of algebraic functional forms that have been proposed for Engel curves.[56] The following are some mathematical forms that have been used in most empirical studies of this subject. They are: linear, semi-log, double-log, and log-normal.

The justification of the linearity is that it is a first order approximation to any function which is undefined. The linearity of Engel curves is acceptable when

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[55]Cramer, op. cit., p. 147; Leser, op. cit., p. 694.

[56]It might be thought that the problem of searching for the appropriate form of Engel curves is trivial, since a polynomial of sufficiently high degree can assume any required shape. However, the flexibility of a polynomial is only an advantage if the degree of scatter of the observations is small enough to allow a precise determination of the parameters of the polynomial. The data provided by family budgets do not seem to have sufficient regularity to make this advantage possible. It is therefore necessary to choose a form which substantially represents the required form. See Prais and Houthakker, op. cit., p. 86.

For some researchers, the choice of the appropriate form is ignored. Generally, they believe that it will be more important to have relationships which are convenient for one or the other purpose. See C. E. V. Leser, "Family Budget Data and Price Elasticities of Demand," Review of Economic Studies, Vol. 9 (1941), p. 47.

the observations of income are confined to a relatively narrow interval where curvature matters little or not at all.[57]

The curvature of Engel curves cannot be neglected since survey data generally cover a considerable income range.[58] Among several forms, Prais and Houthakker, using the British data, found out that in the case of foods a semi-log relationship was preferable to other alternative functional forms.[59] Similarly, Liviatan found that the semi-log form of Engel curves gave the best "goodness of fit" to his Israeli household budgets.[60]

Ferber found that the functional form of Engel curves used in most studies was essentially the same as

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[57]Cramer, op. cit., pp. 146-47; Allen and Bowley, op. cit.

[58]Stuvel and James, in their study of household expenditure on food in Holland, showed that neither the linear nor the exponential forms generally used in estimating income elasticities were appropriate for the whole range of budgets in that collection. See G. Stuvel and S. F. James, "Household Expenditure on Food in Holland," Journal of Royal Statistic Society, Series A, 113 (1950), p. 59.

[59]Prais and Houthakker, op. cit., p. 166.

[60]N. Liviatan, Consumption Patterns in Israel (Jerusalem: Falk, 1964), pp. 29-30.
Besides its simplicity, the semi-log function has the following properties: the slope of the curve and the income elasticity decline with the rise of the income level; the curve has no satiety level; the curve does not pass through its origin but intercepts a positive level of income. See J. Johnston, Econometric Methods (New York: McGraw-Hill, 1963), p. 47; Goldberger, op. cit., p. 214.

used by Ernst Engel, namely, the double-log form.[61] The main reason for the popularity of the double-log form, as explained by Ezekiel and Fox, is that

> Though the income elasticity for a commodity may change from one income level to another, it is often more desirable to obtain an average elasticity over some specified range of incomes. In fact, this is equivalent to assume that the income elasticity is constant over the range in question.[62]

Another classic functional form of Engel curves is the integral log-normal curve or a sigmoid response curve proposed by Aitchison and Brown.[63] This curve has an upper asymptote and at the same time passes through the origin. Nevertheless, this curve is not easy to fit, in that it requires iterative methods, and nonconvergence is apparently possible, as Jorgensen found in his analysis of Danish budgets.[64]

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[61] Ferber, <u>op. cit.</u>, p. 138. The double-log form provides for a constant income elasticity. It passes through the origin, and it has an upward rather than a downward curvature when the elasticity is greater than one. See Johnston, <u>op. cit.</u>, p. 48; Goldberger, <u>op. cit.</u>, p. 215.

[62] M. Ezekiel and K. A. Fox, <u>Methods of Correlation and Regression Analysis</u> (3rd ed.; New York: John Wiley & Sons, 1959), p. 110.

[63] For a full discussion of this curve, see J. Aitchison and J. A. C. Brown, <u>The Log-Normal Distribution</u> (Cambridge: University Press, 1957).

[64] E. Jorgensen, <u>Income-Expenditure Relationships of Danish Wage and Salary Earner</u> (Copenhagen: Kobenhaven, 1965), p. 55.

CHAPTER II

REVIEW OF M.S.U. CONSUMER PANEL SURVEY

1. General Remarks

Since the present study will make use of the M.S.U. Consumer Panel data of 1958 in analyzing Engel curves, a brief review of this survey will first be given.[1]

The panel was composed of approximately 300 households selected as representative of about 25,000 to 30,000 households in Lansing, Michigan, a city of about 100,000 inhabitants. East Lansing was excluded.

The panel operation started in February, 1951, and continued through a period ending in December, 1958. Each household in the panel reported weekly on all food purchased for home use, giving the quantity, price, and expenditure for each item. There were about 500 food items or 14 composite foods in each report. The following were the fourteen food groups: dairy products; fats and oils; fruits; vegetables; meat; poultry, fish, and eggs; jam, etc.; prepared baby food; bakery and cereal products; sugar, sweets, and candy; nuts and nut products; beverages; vitamins and minerals; and cooking aids.

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[1]For more discussion of this survey, see Quackenbush and Shaffer, op. cit.

In addition, each household reported its income after federal income tax, change in household composition, number of meals served to guests, and expenditure for meals away from home on a weekly basis.

2. Methods of Selection

The sampling procedure involved periodic sample censuses of the City of Lansing. Sample censuses were done in 1950, 1954, and 1956. The number of sampled households was 1,885, 1,775, and 2,103, respectively. Each sample census was composed of a random sample of households, obtained by using every N^{th} residential address in the street and address section of the Lansing City Directory. East Lansing was excluded. The original M.S.U. panel sample of 323 was drawn from the 1,885 households in the 1950 sample census. Size of family, age of homemaker, education of homemaker, and income of the family were used as controls. All families were serialized by use of punch card sorting on the four controls, and then each N^{th} family was drawn from the listing as an original panel member.

Substitutes needed for replacements and refusals were selected from families with characteristics similar to those refusing or dropping out.

3. Collecting the Information

The panel survey was carried out through mailing questionnaires to the panel households. All households in the panel were asked to report each week by mailing in a food purchased diary which had been mailed to them the previous week.

There was no assurance that the panel members wouldn't forget, neglect, or refuse to enter some items in a diary. An even greater problem was the collection of the weekly current disposable income data. About half the panel households were visited annually in order to verify and check weekly reports against annual reports of income. These visits were generally well received. This rapport indicated a reasonably high accuracy in the data on observed income.

CHAPTER III

CROSS SECTIONAL STUDIES

1. Objectives and Some General Remarks

Most previous studies on Engel curves used cross sectional data of a particular period of time to estimate Engel or income elasticities.[1] A particular functional form was selected for the true functional form of the Engel curves.

In this chapter, thirteen successive cross sectional studies on the Engel curves are set out for five composite foods. Three alternative functional forms are selected for the true functional form of the Engel curves. In this way, the income elasticity estimates for the five composite foods, based on three alternative functional forms, are obtained. Thus, one can investigate if the income elasticity estimates based on three alternative functional forms are widely different. In addition, one

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[1]For pure economic theory, an income elasticity has long been used to indicate if a certain commodity is a luxury, a necessity, or an inferior good. For business firms, an income elasticity for a particular good may be used as an index of demand, or market potential sale. For economic policy, income elasticities might be used for adopting various possible policies. See J. M. Slater, "Regional Consumer Expenditure Studies Using National Food Survey Data," Journal of Agricultural Economics (May, 1969), p. 197; G. Tintner, Econometrics (New York: John Wiley & Sons, 1952), pp. 57-62.

can investigate if a particular functional form uniformly gives a better "goodness of fit" to the given observations.

2. Statistical Cross Sectional Models

The data used are taken from the M.S.U. Consumer Panel data of 1958. Four weekly reports are grouped together and treated as a period of time. Since there were thirteen periods of time in 1958, thirteen cross sectional studies on the Engel curves are able to be set out for five composite foods. These composite foods are: dairy products; fats and oils; fruits; vegetables; and meat, poultry, fish, and eggs.[2]

In each cross sectional study belonging to each particular period of time, all households in the panel are assumed to be homogeneous except for per capita expenditure, per capita disposable income, and stochastic error.[3]

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[2]Theoretically, different varieties of goods can be grouped as a single composite good if the relative prices remain fixed, or they are consumed in fixed proportions. For a full discussion of this problem, see D. Patinkin, Money, Interest, and Prices (2nd ed.; New York: Harper & Row, 1965), pp. 411-16.

The exact composition of these grouped goods is given in Appendices A, B, C, D, and E.

[3]In a cross sectional study, prices are held constant. They are omitted from the formulation of Engel curves. The per capita hypothesis is adopted to cope with the influence of household size on household expenditure behavior. The omission of household size in the formulation of Engel curves will result in biased and inconsistent estimates of income elasticity since household income and household size, in most survey data, are highly positive correlated. The per capita hypothesis may be taken as substantially correct in the formulation of Engel curves. See pp. 17-21 and footnote 48, Chapter I.

In other words, the Engel curve for the k^{th} composite food, in each cross sectional study, can be stated as:

$$Y_{ik}/N_i = f_k(M_i/N_i) + U_{ik}$$

where Y_{ik} is the i^{th} household aggregate expenditure on the k^{th} food,

M_i is the i^{th} household aggregate income after federal income taxes,

N_i is the average number of persons in the i^{th} household,

U_{ik} is the stochastic error term representing both the effects of either non-economic factors besides household size, and the error of measurement of the regressand, and

f_k is the undefined functional form.

In this chapter, the functional forms selected for f_k are linear, semi-log, and double-log forms.[4] Thus, the Engel curve for the k^{th} food is expressed as follows:

Linear: $$Y_{ik}/N_i = \alpha_{k1} + \beta_{k2}(M_i/N_i) + U_{ik}$$

Semi-log: $$Y_{ik}/N_i = \alpha_{k2} + \beta_{k2} \log (M_i/N_i) + U_{ik}$$

Double-log: $$\log (Y_{ik}/N_i) = \alpha_{k3} + \beta_{k3} \log (M_i/N_i) + U_{ik}$$

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[4]These three functional forms have been widely used in the analysis of Engel curves; see pp. 26-27, Chapter I.

where α_k's and β_k's are the parameters, i.e., the constant terms, and the income coefficients for the $k^{\underline{th}}$ food, based on alternative functional forms.

Regarding the probability distribution of the disturbance U_{ik}, and the values of the explanatory variable, the following assumptions are assumed:

(i) Normality: U_{ik} is normally distributed;

(ii) Zero mean: $E(U_{ik}) = 0$;

(iii) Homoskedasticity: $E(U_{ik}^2) = \sigma_k^2$;

(iv) No interdependence: $E(U_{ik}U_{jk}) = 0$ for all $i \neq j$;

(v) The exogenous variable, M_i/N_i, is measured without error.

Considering the above assumptions, the assumptions (i), (ii), and (iii) are assumed. Though the assumption (iii) is not fulfilled, the existence of heteroskedasticity does not affect the unbiased property of least squares estimates. The assumption (iv) is likely to be satisfied, since the panel households were randomly selected. As for the assumption (v), the present study, like many others, simply assumes the measured incomes are accurate. Based on the above assumptions, the ordinary least squares estimators are BLUE.[5]

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[5]The assumptions (ii) through (v) suffice to establish that the least squares estimates are BLUE. The assumptions (ii) and (v) assure the unbiasedness of least squares estimators. The assumption (i) serves to establish

2.1 Estimation Procedure

The ordinary least squares method is used to estimate the regression coefficients over all the one hundred and ninety-five regressions.

Before analyzing the main results, one difficulty of computation should be mentioned. It is the problem of zero values of household expenditure on a composite food. Since log 0 = $-\infty$, this creates a computational problem. In this chapter, those pairs of observations for which the values of household expenditure on that composite food are zero are excluded.[6] They give no information regarding the outcome of the experiment and should not be counted as part of the sample.[7]

2.2. Results of Cross Sectional Studies

The estimates of the constant terms and the income coefficients for the five composite foods, based on three alternative functional forms, over thirteen cross sectional

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an identity between least squares and maximum likelihood estimates and to justify strictly the use of t, F, and z test procedures. For a full discussion of this problem, see E. Malinvaud, Statistical Methods of Econometrics (2nd ed.; New York: American Elsevice, 1970), pp. 84-86; also E. J. Kane, Economic Statistics & Econometrics (New York: Harper & Row, 1968), pp. 355-57.

[6] Some studies assigned an arbitrarily low value for zero observations of household expenditure; see Prais and Houthakker, op. cit., p. 50.

[7] Kmenta, op. cit., p. 337.

studies are presented in Table 1 through Table 5. The figure in parentheses is the standard error of the estimated income coefficient. The mark @ indicates that the corresponding income coefficient is not significantly different from zero at 5% level of significance. In an economic sense, it has been well known that the income coefficient based on the linear form is the marginal propensity to consume (MPC); whereas the income coefficient based on the double-log form is the income elasticity.

At this point, some conclusions might be drawn from Table 1 through Table 5.[8]

(1) For dairy products, the mean of the estimates of MPC based on the linear form is .0024; the mean of the income elasticity estimates based on the double-log form is .0938.

(2) For fats and oils, the mean of the estimates of MPC based on the linear form is .0014; the mean of the income elasticity estimates based on the double-log form is .2209.

(3) For fruits, the mean of the estimates of MPC based on the linear form is .0043; the mean of the income elasticity estimates based on the double-log form is .3388.

(4) For vegetables, the mean of the estimates of MPC based on the linear is .0026; the mean of the income elasticity estimates based on the double-log form is .1988.

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[8] It should be noted that these results, from Table 1 through Table 5, provide the information needed to approximate utility functions by Wald's method. Some preliminary evidence on this subject is given in Chapter V.

TABLE 1.--Estimates of Regression Coefficients for Dairy Products Based on Alternative Functional Forms

Period of Time	Estimates of Constant Terms Based on			Estimates of Income Coefficients Based on		
	Linear	Semi-Log	Double-Log	Linear	Semi-Log	Double-Log
1	3.678	1.976	.407	.0011@	.8887	.0657@
				(.0008)	(.4013)	(.0423)
2	3.719	2.203	.373	.0005@	.7584	.0794@
				(.0005)	(.3769)	(.0420)
3	3.366	3.769	.533	.0025	.0039@	.0003@
				(.0008)	(.0058)	(.0006)
4	3.286	.722	.194	.0033	1.4550	.1576
				(.0011)	(.4471)	(.0542)
5	3.443	1.082	.257	.0025	1.3089	.1325
				(.0010)	(.4167)	(.0495)
6	3.114	-.541	.105	.0048	2.0850	.2041
				(.0012)	(.4618)	(.0533)
7	3.430	1.575	.366	.0015@	.9947	.0691@
				(.0011)	(.4234)	(.0517)
8	3.125	.457	.199	.0034	1.5176	.1451
				(.0012)	(.4528)	(.0561)
9	3.055	.684	.204	.0029	1.3456	.1323
				(.0011)	(.3945)	(.0569)
10	3.157	.729	.182	.0030	1.3760	.1538
				(.0010)	(.3890)	(.0494)
11	3.365	1.889	.349	.0020	.8589	.0803@
				(.0010)	(.3448)	(.0422)
12	3.350	3.763	.522	.0026	-.0002@	-.0001@
				(.0011)	(.0044)	(.0005)
13	3.355	3.611	.506	.0015@	.0024@	.0001@
				(.0009)	(.0060)	(.0007)
Mean				.0024	.9688	.0938

Remarks: The mark @ indicates that the income coefficient is not significantly different from zero at 5% level of significance.

The t^{th} period of time implies the t^{th} cross sectional study, since a cross sectional study belongs to a particular period of time.

TABLE 2.--Estimates of Regression Coefficients for Fats and Oils Based on Alternative Functional Forms

Period of Time	Estimates of Constant Terms Based on			Estimates of Income Coefficients Based on		
	Linear	Semi-Log	Double-Log	Linear	Semi-Log	Double-Log
1	.901	-.210	-.545	.0009	.5953	.2279
				(.0004)	(.1997)	(.0592)
2	1.020	-.691	-.692	.0007	.8667	.3169
				(.0002)	(.1807)	(.0583)
3	.904	1.027	-.072	.0007@	-.0020@	-.0009@
				(.0004)	(.0027)	(.0006)
4	.627	-1.034	-.795	.0028	.9920	.3485
				(.0005)	(.1925)	(.0627)
5	.855	-.704	-.682	.0017	.8683	.3039
				(.0004)	(.1963)	(.0652)
6	.712	-.706	-.857	.0020	.8174	.3628
				(.0005)	(.2032)	(.0734)
7	.830	.004	-.492	.0007@	.4490	.1739
				(.0004)	(.1854)	(.0750)
8	.773	-.509	-.756	.0017	.7334	.3202
				(.0005)	(.1885)	(.0733)
9	.719	-.305	-.851	.0016	.6037	.3441
				(.0004)	(.1709)	(.0730)
10	.835	-.025	-.484	.0011	.4900	.1899
				(.0004)	(.1706)	(.0617)
11	.750	-.028	-.690	.0015	.5728	.2812
				(.0004)	(.1542)	(.0591)
12	.760	1.052	-.078	.0018	.0021@	.0011@
				(.0004)	(.0020)	(.0007)
13	.852	1.037	-.082	.0010	-.0041@	-.0014@
				(.0004)	(.0028)	(.0009)
Mean				.0014	.5382	.2209

Remarks: The mark @ indicates that the income coefficient is not significantly different from zero at 5% level of significance.

The t^{th} period of time implies the t^{th} cross sectional study, since a cross sectional study belongs to a particular period of time.

TABLE 3.--Estimates of Regression Coefficients for Fruits Based on Alternative Functional Forms

Period of Time	Estimates of Constant Terms Based on			Estimates of Income Coefficients Based on		
	Linear	Semi-Log	Double-Log	Linear	Semi-Log	Double-Log
1	1.459	-1.649	-.616	.0029	1.6923	.3718
				(.0007)	(.3326)	(.0737)
2	1.867	-2.117	-.533	.0021	2.0468	.3605
				(.0005)	(.3724)	(.0756)
3	1.794	2.119	.229	.0020	.0057@	.0018@
				(.0007)	(.0048)	(.0010)
4	1.362	-1.800	-.609	.0046	1.8399	.3887
				(.0008)	(.3542)	(.0753)
5	1.517	-2.533	-.917	.0051	2.2980	.5518
				(.0009)	(.3611)	(.0774)
6	1.593	-2.251	-.703	.0050	2.1390	.4594
				(.0010)	(.3962)	(.0779)
7	1.958	-2.313	-.680	.0054	2.4292	.4775
				(.0012)	(.4546)	(.0814)
8	1.402	-3.776	-.896	.0075	2.9988	.5586
				(.0011)	(.4403)	(.0807)
9	1.531	-2.420	-.817	.0058	2.3043	.5058
				(.0010)	(.3863)	(.0858)
10	1.548	-1.221	-.506	.0048	1.6702	.3594
				(.0009)	(.3520)	(.0737)
11	1.389	-1.127	-.552	.0049	1.5710	.3675
				(.0009)	(.3218)	(.0737)
12	1.752	2.156	.230	.0025	.0023@	.0017
				(.0009)	(.0037)	(.0007)
13	1.694	2.279	.244	.0034	-.0018@	-.0006
				(.0009)	(.0061)	(.0010)
Mean				.0043	1.6153	.3388

Remarks: The mark @ indicates that the income coefficient is not significantly different from zero at 5% level of significance.

The t^{th} period of time implies the t^{th} cross sectional study, since a cross sectional study belongs to a particular period of time.

TABLE 4.--Estimates of Regression Coefficients for Vegetables Based on Alternative Functional Forms

Period of Time	Estimates of Constant Terms Based on Linear	Semi-Log	Double-Log	Estimates of Income Coefficients Based on Linear	Semi-Log	Double-Log
1	1.779	-.392	-.211	.0024	1.2070	.2257
				(.0006)	(.3058)	(.0562)
2	1.980	-.506	-.227	.0013	1.2793	.2371
				(.0004)	(.3061)	(.0551)
3	1.864	2.164	.260	.0019	.0024@	.0003@
				(.0006)	(.0045)	(.0009)
4	1.599	-1.243	-.313	.0042	1.6580	.2805
				(.0008)	(.3454)	(.0604)
5	2.069	-1.236	-.289	.0036	1.8400	.3034
				(.0009)	(.3594)	(.0578)
6	1.959	-.176	-.162	.0028	1.2149	.2277
				(.0008)	(.3252)	(.0560)
7	1.864	-.549	-.277	.0029	1.3650	.2699
				(.0010)	(.3545)	(.0603)
8	1.579	-1.328	-.416	.0039	1.6651	.3213
				(.0008)	(.3067)	(.0638)
9	1.432	-.629	-.354	.0027	1.1850	.2552
				(.0007)	(.2615)	(.0651)
10	1.548	-.049	-.232	.0021	.9197	.1982
				(.0007)	(.2667)	(.0664)
11	1.476	-.521	-.351	.0028	1.1742	.2639
				(.0007)	(.2289)	(.0569)
12	1.628	2.023	.224	.0024	.0042@	.0012@
				(.0007)	(.0030)	(.0006)
13	1.640	1.850	.180	.0012	.0019@	.0002@
				(.0006)	(.0042)	(.0009)
Mean				.0026	1.0397	.1988

Remarks: The mark @ indicates that the income coefficient is not significantly different from zero at 5% level of significance.

The t^{th} period of time implies the t^{th} cross sectional study, since a cross sectional study belongs to a particular period of time.

TABLE 5.--Estimates of Regression Coefficients for Meat, Etc. Based on Alternative Functional Forms

Period of Time	Estimates of Constant Terms Based on Linear	Semi-Log	Double-Log	Estimates of Income Coefficients Based on Linear	Semi-Log	Double-Log
1	5.993	-5.586	-.134	.0112	6.3161	.3285
				(.0022)	(1.0157)	(.0503)
2	5.984	-15.359	.133	.0125	11.0688	.3232
				(.0036)	(2.5591)	(.0566)
3	6.167	7.382	.806	.0077	.0032@	-.0000@
				(.0018)	(.0124)	(.0008)
4	5.792	-4.034	.148	.0125	5.5704	.3202
				(.0023)	(.9136)	(.0585)
5	5.621	-6.806	.050	.0157	7.0562	.3723
				(.0026)	(1.0665)	(.0568)
6	6.248	-.295	.308	.0075	3.6459	.2403
				(.0022)	(.8532)	(.0563)
7	5.011	-7.032	.142	.0173	1.9938	.3127
				(.0035)	(1.3370)	(.0614)
8	5.371	-3.654	.070	.0129	5.2173	.3446
				(.0023)	(.8841)	(.0628)
9	5.447	-2.720	.149	.0109	4.6853	.3021
				(.0021)	(.7671)	(.0612)
10	6.179	-.440	.330	.0095	3.8448	.2345
				(.0023)	(.8519)	(.0531)
11	5.499	-2.293	.235	.0138	4.7586	.2783
				(.0022)	(.7477)	(.0490)
12	6.793	8.128	.845	.0083	.0213	.0019
				(.0024)	(.0099)	(.0006)
13	6.781	7.799	.827	.0060	-.0009@	-.0002@
				(.0022)	(.0145)	(.0008)
Mean				.0112	4.1678	.2353

<u>Remarks</u>: The mark @ indicates that the income coefficient is not significantly different from zero at 5% level of significance.

The $t^{\underline{th}}$ period of time implies the $t^{\underline{th}}$ cross sectional study, since a cross sectional study belongs to a particular period of time.

(5) For meat, etc., the mean of the estimates of MPC based on the linear is .0112; the mean of the income elasticity estimates based on the double-log form is .2353.

2.3. Income Elasticity Estimates

From Table 1 through Table 5, the income elasticity estimates for the five composite foods, based on three alternative functional forms, can easily be derived. Referring to the estimates of income coefficients (β_k's), the income elasticity estimates at mean values, based on these alternative functional forms, are computed as follows:[9]

Linear: $\hat{\beta}_{k1}(\bar{M}/\bar{Y}_k)$

Semi-log: $\hat{\beta}_{k2}/\bar{Y}_k$

Double-log: $\hat{\beta}_{k3}$

where $\bar{M}$ is the sample mean of the household per capita disposable income at a period of time,

$\bar{Y}_k$ is the sample mean of the household per capita expenditure on the k^{th} food at a period of time, and

$\hat{\beta}_k$'s are the estimates of income coefficients for the k^{th} food, based on alternative functional forms.

For each composite food, the income elasticity estimates at mean values, based on three alternative functional

[9]The values of $\bar{Y}_k$, $\bar{M}$, and $\bar{M}/\bar{Y}_k$ are set out in Appendix F.

forms, are widely different.[10] The semi-log form consistently gives the highest income elasticity estimates which are very different from those based on the linear and double-log forms. The income elasticity estimates based on the linear form are fairly stable over different cross sectional studies. In some cross sectional studies, the signs of the income elasticity estimates based on three alternative functional forms are different.

To clarify the analysis, the income elasticity estimates at mean values for each composite food, based on three alternative functional forms, are separately set out in Table 6 through Table 10. These income elasticity estimates are also graphically presented in Figure 1 through Figure 5.

2.3.1. Dairy Products.--The values and a graphical presentation of the income elasticity estimates for dairy products, based on three alternative functional forms, are given in Table 6 and Figure 1.

The first point to notice is that the semi-log form consistently gives the highest income elasticity estimates for each period of time except for the 3rd, 12th, and 13th periods. The values based on the semi-log form widely

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[10]The differences between the income elasticity estimates are greater when estimated at any point away from the mean, since each functional form makes different assumptions as to the way in which the elasticity varies. For a numerical illustration of this problem, see Prais and Houthakker, op. cit., p. 94.

TABLE 6.--Income Elasticity Estimates for Dairy Products Based on Alternative Functional Forms (at Mean Values)

Period of Time	Linear	Semi-Log	Double-Log
1	.0459@	.2302	.0657@
2	.0215@	.1995	.0794@
3	.1048	.0010@	.0003@
4	.1335	.3828	.1576
5	.0993	.3417	.1325
6	.1881	.5415	.2041
7	.0610@	.2725	.0691@
8	.1430	.4146	.1451
9	.1251	.3844	.1323
10	.1305	.3822	.1538
11	.0793	.2353	.0803@
12	.1097	-.0000@	-.0001@
13	.0702@	.0006	.0001@
Mean	.1009	.2604	.0938
S.D.	.0015	.0265	.0035

Remarks: The mark @ indicates that the income elasticity (or the corresponding income coefficient) is not significantly different from zero at 5% level of significance.

S.D. = Standard Deviation

The $t^{\underline{th}}$ period of time implies the $t^{\underline{th}}$ cross sectional study, since a cross sectional study belongs to a particular period of time.

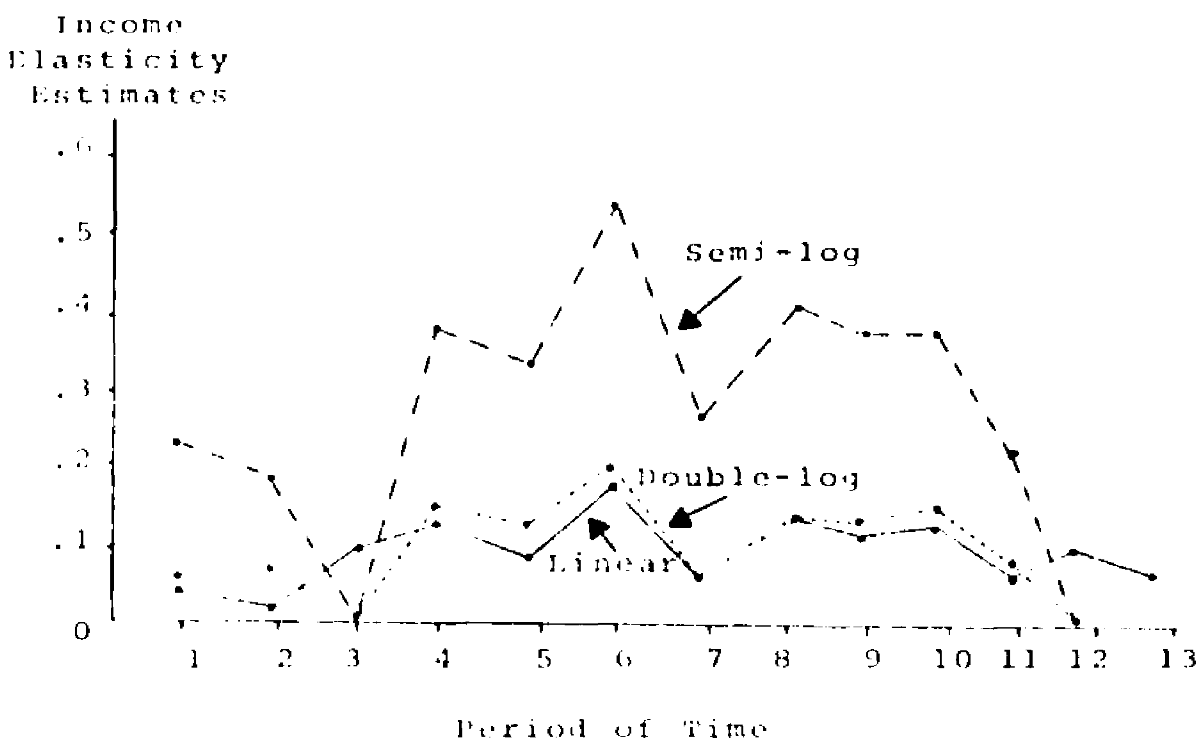

Figure 1.--A Graphic Presentation of Income Elasticity Estimates for Dairy Products Based on Alternative Functional Forms (at Mean Values)

fluctuate and are very different from those based on the other two forms. The mean of the income elasticity estimates based on the semi-log is .2604 and the standard deviation is .0265.

Secondly, the estimates based on the semi-log and double-log forms at the 12th period are negative, whereas the estimate based on the linear form is positive.

Thirdly, except for the 3rd, 12th, and 13th periods, the double-log and linear forms give the estimates that are very nearly equal over periods of time.

Fourthly, the income elasticity estimates based on the linear form are reasonably the same over different periods of time. The mean of the estimates based on the linear form is .1009 and the standard deviation is .0015.

2.3.2. Fats and Oils.--The values and a graphical presentation of income elasticity estimates for fats and oils, based on alternative functional forms at mean values, are shown in Table 7 and Figure 2.

It will be noticed that, except for the 3rd, 12th, and 13th periods, the semi-log form consistently gives the highest income elasticity estimates. The values based on the semi-log form widely fluctuate and are very different from those based on the other two forms. The values based on the semi-log and double-log forms at the 3rd and 12th periods are negative, whereas the estimates based on the linear form are positive. The linear and

TABLE 7.--Income Elasticity Estimates for Fats and Oils Based on Alternative Functional Forms (at Mean Values)

Period of Time	Linear	Semi-Log	Double-Log
1	.1365	.5724	.2279
2	.0997	.7669	.3169
3	.1079@	-.0019@	-.0009@
4	.4033	.9447	.3485
5	.2311	.7822	.3039
6	.2936	.8093	.3628
7	.1113@	.4776	.1739
8	.2528	.7120	.3202
9	.2502	.6354	.3441
10	.1725	.4900	.1899
11	.2223	.5844	.2812
12	.2733	.0020@	.0011@
13	.1638	-.0039@	-.0014@
Mean	.2091	.5217	.2209
S.D.	.0065	.0997	.0169

Remarks: The mark @ indicates that the income elasticity (or the corresponding income coefficient) is not significantly different from zero at 5% level of significance.

S.D. = Standard Deviation

The $t^{\underline{th}}$ period of time implies the $t^{\underline{th}}$ cross sectional study, since a cross sectional study belongs to a particular period of time.

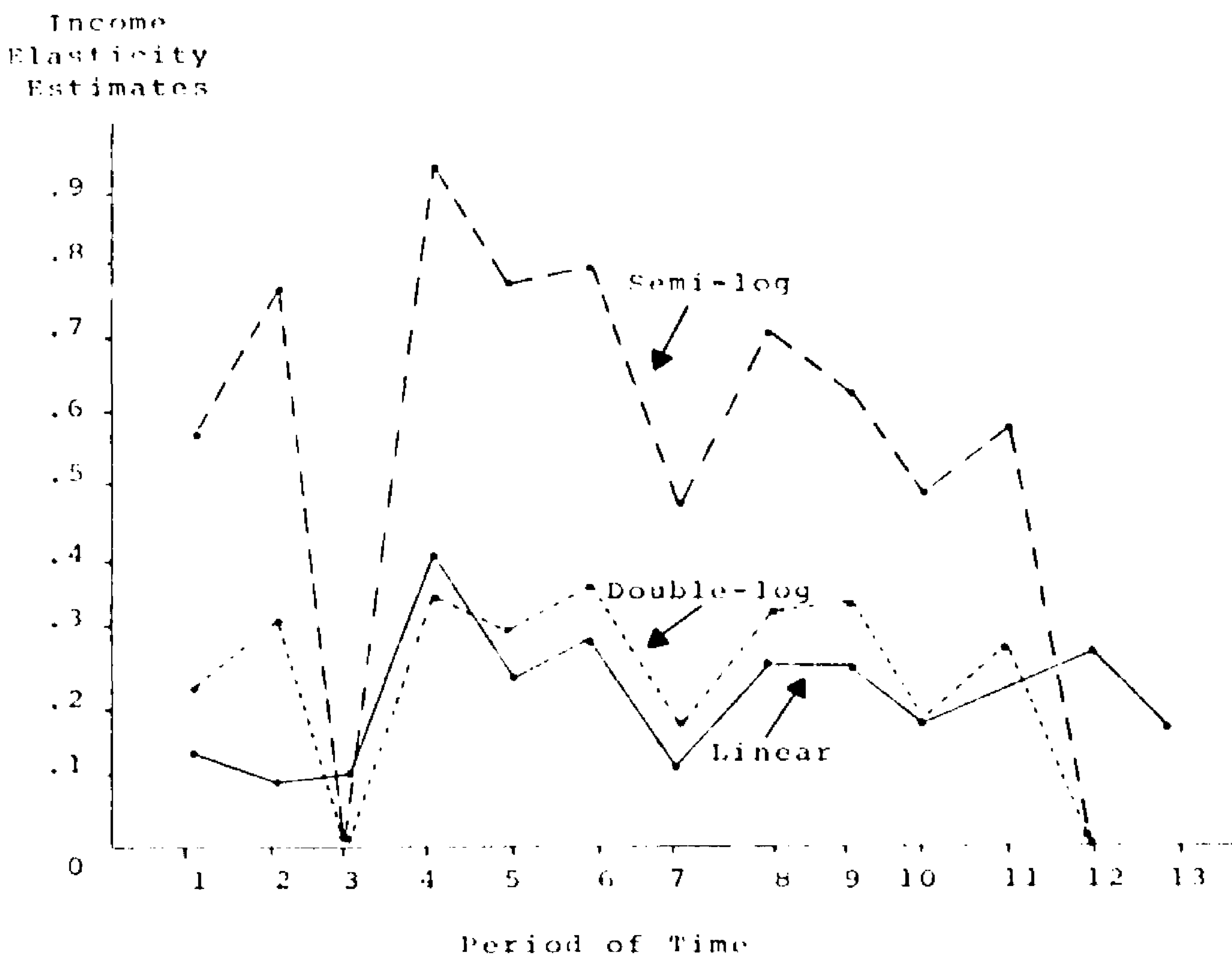

Figure 2.--A Graphic Presentation of Income Elasticity Estimates for Fats and Oils Based on Alternative Functional Forms (at Mean Values)

double-log forms give income elasticity estimates that are very nearly equal except for the 3rd, 12th, and 13th periods. The mean of the estimates based on the semi-log, linear, and double-log forms are .5217, .2091, and .2209, respectively. The standard deviation of income elasticity estimates based on the semi-log, linear, and double-log forms are .0997, .0065, and .0169, respectively. One would claim that the income elasticity estimates based on the linear form are almost the same over different periods of time.

2.3.3. Fruits.--The values and a graphical presentation of income elasticity estimates for fruits, based on alternative functional forms at mean values, are set out in Table 8 and Figure 3.

One can see that the semi-log form consistently gives the highest income elasticity estimates at each period of time except for the 3rd, 12th, and 13th periods. The values based on the semi-log form widely fluctuate and are quite different from those based on the other two forms. Both non-linear forms give negative estimates at the 13th period. Except for the 3rd, 12th, and 13th periods, the estimated income elasticities based on the double-log and linear forms are very close to each other.

In addition, at the 5th and 8th periods, the semi-log form gives income elasticity estimates that are higher than unity. The mean of the income elasticity

TABLE 8.--Income Elasticity Estimates for Fruits Based on Alternative Functional Forms (at Mean Values)

Period of Time	Linear	Semi-Log	Double-Log
1	.2396	.8768	.3718
2	.1554	.9261	.3605
3	.1499	.0026@	.0018@
4	.3389	.8845	.3887
5	.3384	1.1003	.5518
6	.3208	.9063	.4594
7	.2899	.8801	.4775
8	.4526	1.1668	.5586
9	.3665	.9561	.5058
10	.3287	.7230	.3594
11	.3381	.7445	.3675
12	.1856	.0010@	.0017
13	.2525	-.0007@	-.0006@
Mean	.2889	.7052	.3388
S.D.	.0041	.1753	.0376

Remarks: The mark @ indicates that the income elasticity (or the corresponding income coefficient) is not significantly different from zero at 5% level of significance.

S.D. = Standard Deviation

The t^{th} period of time implies the t^{th} cross sectional study, since a cross sectional study belongs to a particular period of time.

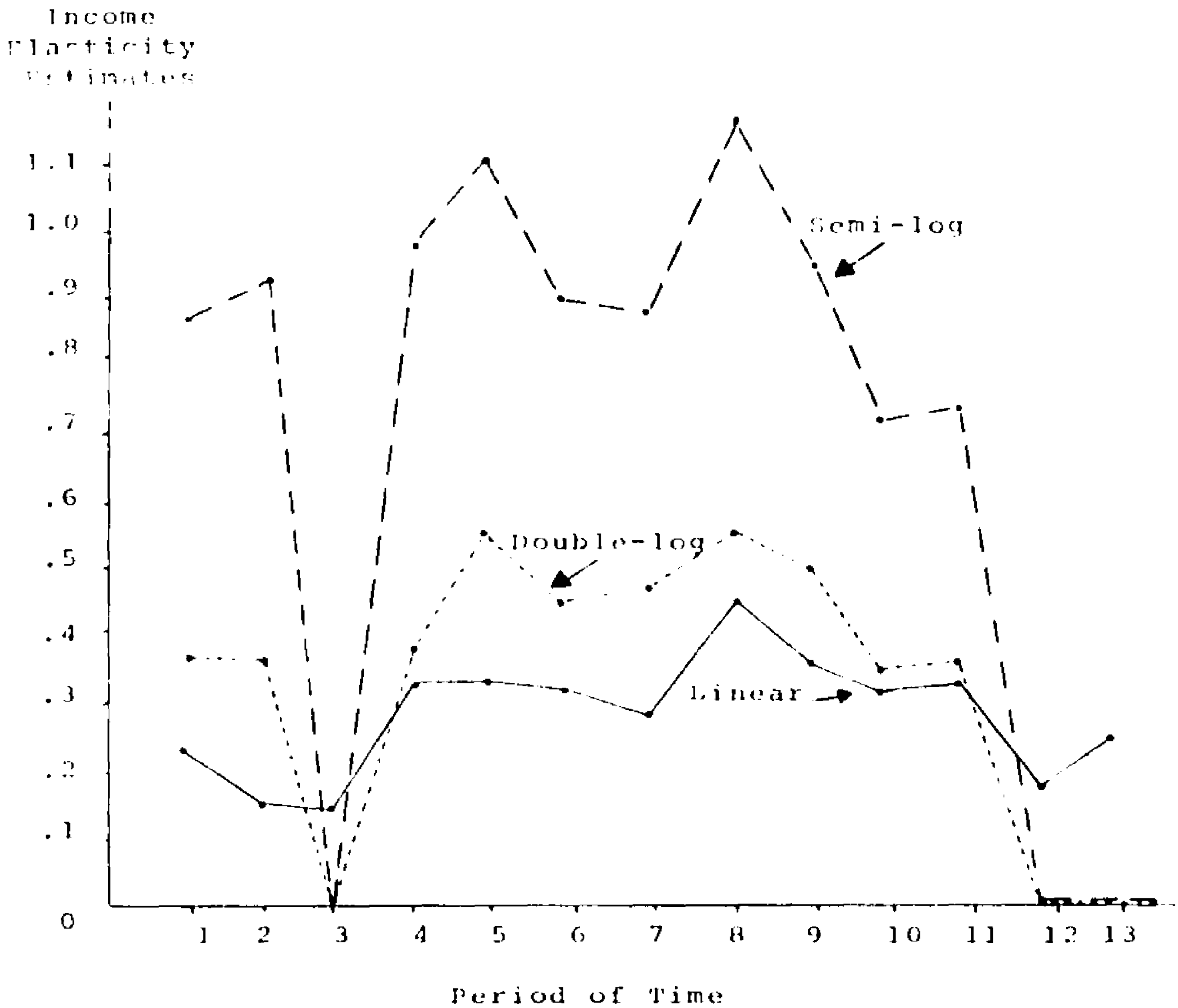

Figure 3.--A Graphic Presentation of Income Elasticity Estimates for Fruits Based on Alternative Functional Forms (at Mean Values)

estimates based on the semi-log, linear, and double-log forms are 0.7052, 0.2889, and 0.3388, respectively. The standard deviation of income elasticity estimates based on the semi-log, linear, and double-log forms are .1753, .0041, and .0376, respectively. The results indicate that the income elasticity estimates based on the linear form are fairly stable over periods of time.

2.3.4. Vegetables.--The values and a graphical presentation of income elasticity estimates for vegetables, based on alternative functional forms at mean values, are given in Table 9 and Figure 4.

The first point to notice is that the semi-log form consistently gives the highest estimates at each period of time except for the 3rd, 12th, and 13th periods. The estimates based on the semi-log form widely fluctuate and are considerably different from those based on the other two forms.

Secondly, except for the 3rd, 12th, and 13th periods, the double-log and linear forms give estimates that are very nearly equal.

Thirdly, the mean of income elasticity estimates based on the semi-log, linear, and double-log forms are 0.4768, 0.1885, and 0.1988, respectively.

Lastly, the standard deviation of income elasticity estimates based on the semi-log, linear, and double-log forms are .1064, .0003, and .0118, respectively. One

TABLE 9.--Income Elasticity Estimates for Vegetables Based on Alternative Functional Forms (at Mean Values)

Period of Time	Linear	Semi-Log	Double-Log
1	.1765	.5587	.2257
2	.0968	.5841	.2371
3	.1391	.0011@	.0003@
4	.2870	.7368	.2805
5	.2083	.6996	.3034
6	.1774	.5104	.2277
7	.1847	.5934	.2699
8	.2753	.7638	.3213
9	.2198	.6405	.2552
10	.1740	.4866	.1982
11	.2145	.6212	.2639
12	.1885	.0020@	.0012@
13	.1087	.0010@	.0002@
Mean	.1885	.4768	.1988
S.D.	.0003	.1064	.0118

Remarks: The mark @ indicates that the income elasticity (or the corresponding income coefficient) is not significantly different from zero at 5% level of significance.

S.D. = Standard Deviation

The t^{th} period of time implies the t^{th} cross sectional study, since a cross sectional study belongs to a particular period of time.

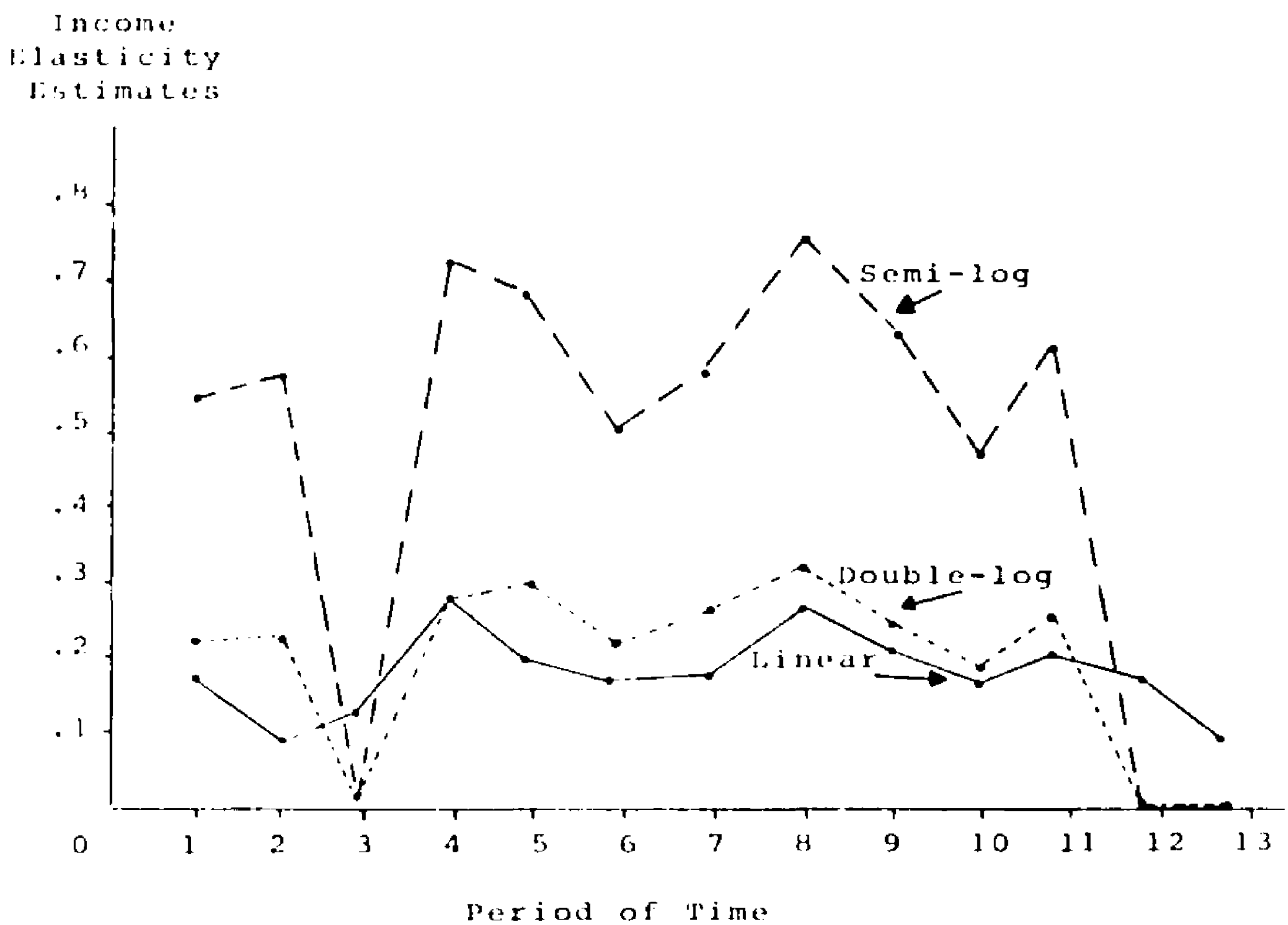

Figure 4.--A Graphic Presentation of Income Elasticity Estimates for <u>Vegetables</u> Based on Alternative Functional Forms (at Mean Values)

would notice that the income elasticity estimates based on the linear forms are highly stable over different periods of time.

2.3.5. Meat, Poultry, Fish, and Eggs.--The values and a graphical presentation of income elasticity estimates for meat, etc., based on alternative functional forms at mean values, are set out in Table 10 and Figure 5.

One would notice that the semi log form consistently gives the highest income elasticity estimates except for the 3rd, 12th, and 13th periods. The income elasticity estimates based on the semi-log form widely fluctuate and are very different from those based on the other two forms. At the 2nd and 13th periods, the semi-log form gives estimates that are greater than unity and negative, respectively. At the 3rd and 13th periods, the double-log form gives estimates that are negative. Except for the 3rd, 12th, and 13th periods, the linear form gives income elasticity estimates that are very close to those based on the double-log form. The mean of the income elasticity estimates based on the semi-log, linear, and double-log forms are .5428, .2259, and .2353, respectively. The standard deviation of income elasticity estimates based on the semi-log, linear, and double-log forms are .1385, .0011, and .0178, respectively. The results indicate that the income elasticity estimates

TABLE 10.--Income Elasticity Estimates for Meat, Etc. Based on Alternative Functional Forms (at Mean Values)

Period of Time	Linear	Semi-Log	Double-Log
1	.2299	.8097	.3285
2	.2526	1.3801	.3232
3	.1647	.0004@	-.0000@
4	.2478	.7224	.3203
5	.2978	.8798	.3723
6	.1532	.4940	.2403
7	.3382	.2630	.3127
8	.2694	.7079	.3446
9	.2321	.6608	.3021
10	.1938	.5012	.2345
11	.2663	.6353	.2783
12	.1620	.0026	.0019
13	.1293	-.0001@	-.0002@
Mean	.2259	.5428	.2353
S.D.	.0011	.1385	.0178

Remarks: The mark @ indicates that the income elasticity (or the corresponding income coefficient) is not significantly different from zero at 5% level of significance.

S.D. = Standard Deviation

The t^{th} period of time implies the t^{th} cross sectional study, since a cross sectional study belongs to a particular period of time.

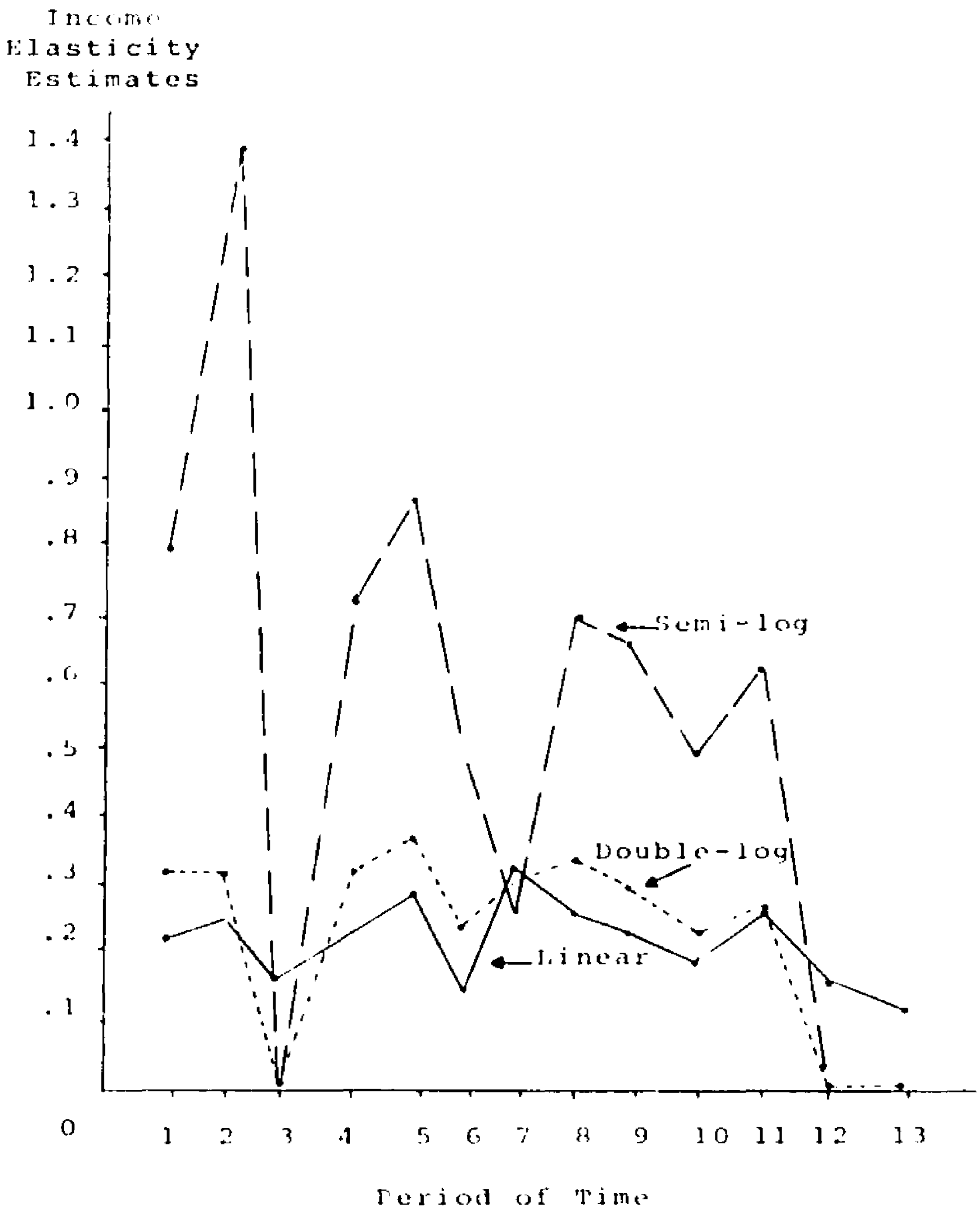

Figure 5.--A Graphic Presentation of Income Elasticity Estimates for <u>Meat, Poultry, Fish and Eggs</u> Based on Alternative Functional Forms (at Mean Values)

based on the linear form are nearly equal over different periods of time.

Before proceeding, one might ask why the income elasticity estimates for the five composite foods are considerably lower at the 3rd, 12th, and 13th periods. Perhaps one possible answer is that households' expenditures on foods are influenced by the holidays of the year. From Table 11, below, one can see that the 3rd, 12th, and 13th periods are the pre-Easter, Thanksgiving, and Christmas holidays.

TABLE 11.--The Actual Days and Holidays Included in the Four Weeks of each Period for the Year of 1958

Year	Period	Month and Date	Holidays
1958	1	1-1 to 1-29	New Years
	2	1-30 to 2-26	
	3	2-27 to 3-26	
	4	3-27 to 4-23	Easter
	5	4-24 to 5-21	
	6	5-22 to 6-18	Memorial Day
	7	6-19 to 7-16	Fourth of July
	8	7-17 to 8-13	
	9	8-14 to 9-10	Labor Day
	10	9-11 to 10-8	
	11	10-9 to 11-5	
	12	11-6 to 12-3	Thanksgiving
	13	12-4 to 12-31	Christmas

2.4. Distributions of Income Elasticity Estimates

From Table 6 through Table 10, one can derive frequency distributions of income elasticity estimates for foods as a whole, based on alternative functional forms. Such distributions are set out in Table 12.

TABLE 12.--Distributions of Income Elasticity Estimates for Foods Based on Alternative Functional Forms

Range of Elasticities	Income Elasticity Estimates: Linear	Semi-Log	Double-Log
-1 to 0		5	6
0 to .10	8	10	13
.10 to .20	27	1	8
.20 to .30	21	4	13
.30 to .40	7	4	20
.40 to .50	2	5	2
.50 to .60		8	3
.60 to .70		6	
.70 to .80		10	
.80 to .90		5	
.90 to 1.00		4	
1.00 to 2.00		3	
	65	65	65

One would notice that the semi-log form gives five income elasticity estimates that are negative, three that are greater than one, and the rest in the range 0 to 1.00. Of the sixty-five income elasticity estimates for foods based on the double-log form, six are negative and the rest lie in the range 0 to 0.60. All income elasticity estimates based on the linear form are positive and concentrate in the range 0 to 0.50.

In summary, the income elasticity estimates for foods based on either functional form are predominantly inelastic: Engel's law of consumption is confirmed. Elastic and negative income elasticity estimates based on the nonlinear forms are found, but they are comparatively infrequent.

2.5. A Comparison of "Goodness of Fit"

Up to this point, an attempt will be made to investigate if the semi-log or the double-log form uniformly gives a better "goodness of fit" to the observations. The values of $\bar{R}^2$, the corrected coefficient of determination, based on alternative functional forms are used for such determination.[11] Table 13 shows the values of $\bar{R}^2$.

It will be noticeable that all the values of $\bar{R}^2$ are small and close to zero. The values of $\bar{R}^2$ based on alternative functional forms differ only slightly except for the 3rd, 12th, and 13th periods, where the linear form obviously gives a better "goodness of fit" to the observations.

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[11] The corrected coefficient of determination $\bar{R}^2$ is used to describe how well the sample regression line fits the observed data. This measure takes into account the number of explanatory variables in relation to the number of observations. Needless to say, the purpose of $\bar{R}^2$ is to facilitate comparisons of the "goodness of fit" of several regression equations that vary with respect to the number of explanatory variables and the number of observations. For a full discussion of this problem, see Kmenta, *op. cit.*, pp. 229-35 and p. 365.

TABLE 13.--Values of $\bar{R}^2$ (the Corrected Coefficient of Determination) and n (the Number of Observations)

Period of Time	Composite Food	$\bar{R}^2$ Based on Linear	Semi-Log	Double-Log	n
1	Dairy Prod.	.002	.014	.005	275
	Fats & Oils	.011	.028	.048	269
	Fruits	.051	.084	.083	270
	Vegetables	.039	.050	.052	273
	Meat, etc.	.078	.120	.131	275
2	Dairy Prod.	.000	.011	.009	272
	Fats & Oils	.024	.076	.096	267
	Fruits	.052	.097	.074	270
	Vegetables	.030	.057	.060	271
	Meat, etc.	.040	.061	.105	270
3	Dairy Prod.	.026	.000	.000	268
	Fats & Oils	.008	.000	.000	263
	Fruits	.025	.001	.008	265
	Vegetables	.024	.000	.000	268
	Meat, etc.	.056	.000	.000	265
4	Dairy Prod.	.028	.034	.026	272
	Fats & Oils	.103	.087	.100	267
	Fruits	.091	.088	.087	268
	Vegetables	.079	.075	.070	272
	Meat, etc.	.095	.119	.097	268
5	Dairy Prod.	.018	.032	.022	269
	Fats & Oils	.041	.066	.073	263
	Fruits	.103	.129	.157	267
	Vegetables	.054	.085	.090	269
	Meat, etc.	.112	.139	.136	266
6	Dairy Prod.	.052	.067	.048	271
	Fats & Oils	.044	.054	.081	264
	Fruits	.076	.095	.112	268
	Vegetables	.034	.045	.054	271
	Meat, etc.	.036	.060	.060	268
7	Dairy Prod.	.003	.016	.003	264
	Fats & Oils	.005	.019	.017	248
	Fruits	.066	.096	.113	261
	Vegetables	.029	.050	.068	260
	Meat, etc.	.080	.092	.087	259

TABLE 13 (Continued)

Period of Time	Composite Food	$\bar{R}^2$ Based on Linear	Semi-Log	Double-Log	n
8	Dairy Prod.	.027	.037	.021	265
	Fats & Oils	.041	.052	.066	255
	Fruits	.135	.149	.153	259
	Vegetables	.077	.097	.084	265
	Meat, etc.	.100	.114	.100	262
9	Dairy Prod.	.023	.039	.016	262
	Fats & Oils	.036	.043	.076	256
	Fruits	.098	.119	.116	257
	Vegetables	.048	.069	.052	261
	Meat, etc.	.084	.123	.083	259
10	Dairy Prod.	.026	.042	.032	259
	Fats & Oils	.017	.027	.032	256
	Fruits	.090	.078	.082	253
	Vegetables	.030	.040	.029	259
	Meat, etc.	.058	.070	.067	256
11	Dairy Prod.	.010	.019	.010	257
	Fats & Oils	.042	.050	.080	247
	Fruits	.092	.083	.114	251
	Vegetables	.059	.090	.074	255
	Meat, etc.	.125	.135	.109	254
12	Dairy Prod.	.017	.000	.000	261
	Fats & Oils	.046	.000	.006	258
	Fruits	.024	.000	.017	255
	Vegetables	.037	.003	.008	261
	Meat, etc.	.038	.014	.035	257
13	Dairy Prod.	.006	.000	.000	260
	Fats & Oils	.019	.004	.004	250
	Fruits	.047	.000	.000	255
	Vegetables	.010	.000	.000	258
	Meat, etc.	.024	.000	.000	256

Remark: The $t^{\underline{th}}$ period of time implies the $t^{\underline{th}}$ cross sectional study, since a cross sectional study belongs to a particular period of time.

2.5.1. Distributions of Corrected Coefficient of Determination.--Before drawing any conclusions about a "goodness of fit," frequency distributions of $\bar{R}^2$, based on alternative functional forms, will be derived from Table 13. Such distributions are given in Table 14.

From Table 14, one would notice that the values of $\bar{R}^2$ based on alternative functional forms lie in the range 0 to .20. These distributions indicate that no functional form uniformly gives the highest values of $\bar{R}^2$. Obviously, one cannot claim that the nonlinear form, either the semi-log or the double-log form, uniformly gives a better "goodness of fit" to the observations.

TABLE 14.--Distributions of Corrected Coefficient of Determination ($\bar{R}^2$) Based on Alternative Functional Forms

Range of $\bar{R}^2$	Values of $\bar{R}^2$		
	Linear	Semi-Log	Double-Log
0 to .02	13	20	20
.02 to .04	19	7	7
.04 to .06	15	10	7
.06 to .08	5	8	8
.08 to .10	7	11	12
.10 to .20	6	9	11
	65	65	65

APPENDIX A

DAIRY PRODUCTS

Fresh Milk
- Homogenized--Vit. D.
- Multiple Vitamin Milk
- Homogenized--Plain
- Regular Pasteurized
- Jersey or Guernsey
- Buttermilk
- Chocolate
- Skim Milk
- Sour Milk
- Egg Nog, etc.
- Other Milk

Cream
- Cofee Cream
- Whipping Cream
- Sour Cream

Canned (Liquid)
- Evaporated--Unsweetened
- Condensed--Sweetened
- Canned--Baby Formulas

Dried
- Powdered--Skim Milk
- Powdered--Whole Milk
- Powdered--Baby Formulas
- Ice Cream Mix
- Sherbet Mix
- Malted Milk Powder

Ice Cream
- Hand Packed Ice Cream
- Pre-Packaged Ice Cream
- Other Ice Cream
- Sherbets and Ices
- Dairy Queen, Frostie, etc.

Cheese
- Natural American (Cheddar, etc.)
- Processed American (Velveeta, etc.)
- Swiss Cheese
- Cheese Spread
- Cream Cheese (Philadelphia, etc.)
- Cottage Cheese
- Other Cheese

APPENDIX B

FATS AND OILS

Fats
- Butter
- Oleomargarine
- Lard
- Swiftning
- Vegetable Shortening (Crisco, Spry, etc.)
- Other Fats

Oils
- Cooking Oils
- Mayonnaise
- Salad Dressing
- Roquefort Dressing
- Salad Oils, etc.
- French Dressing, etc.
- Sandwich Spread, Tartar Sauce
- Whips
- Other Oils

APPENDIX C

FRUITS

Berries
- Blueberries
- Cranberries
- Currants
- Dewberries and Blackberries
- Raspberries
- Strawberries
- Berry Juice
- Other Berries

Citrus
- Grapefruit
- Lemons
- Lemonade
- Lemon Juice
- Grapefruit Juice
- Limes
- Lime Juice
- Limeade
- Oranges
- Orange Juice
- Orange Drink (Hi-C, etc.)
- Tangerines
- Tangerine Juice
- Mixed Citrus Fruits
- Mixed Citrus Juice
- Other Citrus
- Other Citrus Juice

Other Fruits
- Apples
- Applesauce and Apple Butter
- Apple Cider
- Apple Juice
- Apricots
- Apricot Nectar
- Avocados
- Bananas
- Cherries--Maraschino
- Cherries--Sour
- Cherries--Sweet
- Dates
- Figs

Other Fruits (Continued)

- Grapes
- Grape Juice
- Cantaloupe and Muskmelon
- Watermelon
- Nectarines
- Olives
- Persimmons
- Peaches
- Pears
- Pineapple
- Pineapple Juice
- Plums
- Prunes
- Prune Juice
- Raisins
- Rhubarb
- Hawaiian Punch Base
- Mixed Fruits
- Fruit Cocktail
- Fruit Pie Mix
- Mixed Fruit Juice
- Fruit Gelatin Salad--Prepared
- Powdered Juice
- Candied Fruit
- Fruit Pickles
- Other Fruits
- Other Fruit Juice

APPENDIX D

VEGETABLES

Green Leafy Vegetables
- Brussel Sprouts
- Cabbage
- Cabbage Salad
- Sauerkraut
- Celery Cabbage
- Endive, Chicory, Escarole
- Greens--Beet, Mustard, etc.
- Lettuce--Head
- Lettuce--Leaf
- Lettuce--Bib
- Parsley, Swiss Chard, Water Cress
- Spinach
- Mixed Leafy Vegetables
- Other Leafy Vegetables

Green and Yellow Vegetables
- Artichokes
- Asparagus
- Beans--Lima
- Beans--Snap
- Bean Sprouts
- Broccoli
- Carrots
- Corn--Sweet
- Peas
- Peppers
- Pumpkin
- Squash
- Soy Steak and Choplets
- Mixed Green and Yellow Vegetables
- Others

All Other Vegetables
- Beans--Navy, Baked, White
- Pork and Beans
- Beans--Kidney
- Beets
- Cauliflower
- Cucumbers
- Cucumber Pickles
- Relish
- Egg Plant
- Garlic

All Other Vegetables (Continued)

Horseradish
Mushrooms
Onions--Mature
Onions--Green
Parsnips
Pimentoes
Michigan Potatoes
Maine Potatoes
Idaho Potatoes
California Potatoes
Other Potatoes
Potatoes--French Fries
Potato Chips
Potato Sticks
Potato Salad
Mashed Potatoes or Patties
Sweet Potatoes and Yams
Radishes
Tomatoes
Tomato Catsup
Tomato Juice
Turnips and Rutabagas
Prepared Vegetable Gelatin Salad
Mixed Vegetables
Chop Suey, Chow Mein, without Meat
Mixed Vegetable Juice
Other Vegetables

APPENDIX E

MEAT, POULTRY, FISH, AND EGGS

Beef
- Canned Beef
- Corned Beef
- Chipped Beef
- Ground Beef, Hamburger
- Ground Round Steak, Lean Ground Beef
- Beef Liver and Baby Beef Liver
- Heart, Tongue, other Organ Parts
- Chuck Roast (Pot Roast)
- Rib Roast
- Other Roast
- Round and Swiss Steak
- Sirloin Steak
- Porterhouse and T-Bone Steak
- Other Steak
- Stewing Beef (Boneless)
- Boiling Beef or Short Ribs
- All Other Beef

Pork
- Bacon
- Canadian Bacon
- Canned Pork
- Chops
- Steaks
- Ham--Center Slice
- Ham--Whole or Half
- Ham--Canned
- Ham--Other
- Picnic Ham, Cured Butts
- Pork Liver
- Heart, Tongue, other Organ Parts
- Roast--Fresh
- Sausage--Link
- Sausage
- Spareribs
- Side or Salt Pork
- Other Pork

Lamb-Mutton
- Chops, Steaks
- Roast (Leg, etc.)
- Other Lamb--Mutton

Veal
- Cutlets, Chops, Steaks
- Ground Veal
- Calf Liver
- City Chicken
- Roast
- Stewing, Soup Veal
- Other Veal

Other Meat and Meat Mixtures
- Wieners and Franks, etc.
- Bologna--Ring or Large Round
- Other Cold Cuts
- Prem, Spam, Treet, etc.
- Rabbit, Domestic
- Venison and Other Game Animals
- Chop Suey Meat and Kabobs
- Bouillon Cubes
- Beef Stew
- Chile Con Carne
- Hash
- Mincemeat
- Meat Balls and Spaghetti
- Ravioli and Tamales
- Chop Suey, Chow Mein with Meat
- Potted Meat
- Meat Spreads
- Pork and Beans
- Others

Chicken
- Broilers or Fryers
- Roasters
- Stewing
- Barbecued Chicken

Turkey

Duck

Other Poultry
- Game Birds

Mixtures--Chiefly Chicken
- Chicken Noodle Dinner
- Chicken a la King
- Chicken Chop Suey, etc.
- Others

Fish and Sea Food
- Tuna
- Salmon
- Fish Sticks
- Other Fish

Fish and Sea Food (Continued)

- Lobster, Lobstertail
- Oysters
- Oyster Stew
- Scallops
- Shrimps
- Tuna Pie or Casserole
- Sardines in Oil
- Sardines in Sauce

Eggs

APPENDIX F

VALUES OF $\overline{Y}_k$, $\overline{M}$, AND $\overline{M}/\overline{Y}_k$

Period of Time	$\overline{Y}_k$	$\overline{M}$	$\overline{M}/\overline{Y}_k$
Dairy Products			
1	3.86	161.09	41.73
2	3.80	163.74	43.08
3	3.77	158.17	41.95
4	3.80	153.79	40.47
5	3.83	152.24	39.74
6	3.85	150.89	39.19
7	3.65	148.63	40.72
8	3.66	153.95	42.06
9	3.50	151.10	43.17
10	3.60	156.63	43.50
11	3.65	144.80	39.67
12	3.76	158.70	42.20
13	3.61	168.99	46.81
Fats and Oils			
1	1.04	157.81	151.74
2	1.13	161.06	142.53
3	1.02	157.31	154.22
4	1.05	151.25	144.04
5	1.11	150.95	135.99
6	1.01	148.29	146.82
7	.94	149.50	159.04
8	1.03	153.23	148.76
9	.95	148.59	156.41
10	1.00	156.82	156.82
11	.98	145.30	148.26
12	1.05	159.47	151.87
13	1.03	168.81	163.89

Period of Time	$\bar{Y}_k$	$\bar{M}$	$\bar{M}/\bar{Y}_k$
Fruits			
1	1.93	159.49	82.63
2	2.21	163.63	74.04
3	2.12	158.94	74.97
4	2.08	153.29	73.69
5	2.29	151.52	66.16
6	2.36	151.44	64.16
7	2.76	148.19	53.69
8	2.57	155.11	60.35
9	2.41	152.33	63.20
10	2.31	158.20	68.48
11	2.11	145.65	69.02
12	2.15	159.64	74.25
13	2.27	168.64	74.29
Vegetables			
1	2.16	158.94	73.58
2	2.19	163.24	74.52
3	2.16	158.17	73.22
4	2.25	153.79	68.35
5	2.63	152.24	57.88
6	2.38	150.89	63.39
7	2.30	146.53	63.70
8	2.18	153.95	70.61
9	1.85	150.61	81.41
10	1.89	156.63	82.87
11	1.89	144.83	76.62
12	2.02	158.70	78.56
13	1.85	167.62	90.60

Period of Time	$\bar{Y}_k$	$\bar{M}$	$\bar{M}/\bar{Y}_k$
Meat, etc.			
1	7.80	161.09	20.53
2	8.02	162.12	20.21
3	7.38	158.00	21.40
4	7.71	152.96	19.83
5	8.02	152.16	18.97
6	7.38	150.80	20.43
7	7.58	148.23	19.55
8	7.37	154.00	20.89
9	7.09	151.03	21.30
10	7.67	156.59	20.41
11	7.49	144.56	19.30
12	8.12	158.62	19.53
13	7.79	167.90	21.55

Remark: The t^{th} period of time implies the t^{th} cross sectional study, since a cross sectional study belongs to a particular period of time.

CHAPTER IV

COMBINED STUDIES

1. Objectives and Some General Remarks

There are many reasons for studying Engel curves by combining cross sectional and time series data. As earlier mentioned, the combined regression will give more reliable income elasticity estimates than any individual cross sectional regression, since more observations are used in the estimation procedure.[1] Moreover, the combined model with several successive cross sections, or with different sets of prices, may be used to estimate price elasticities. As Prais and Houthakker mention:

> The derivation of price elasticities . . . has become possible following the collection of family budgets on a continuous basis for a length period. The analysis . . . is not different from that classically applied to time series, but the results recently achieved using family budget records appear more successful. The consistency of the data is probably the main reason for greater success, in that both prices and quantities are collected simultaneously, using precisely the same commodity-definitions and methods of observation, and over a lengthy period.[2]

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[1]See p. 13, Chapter I.

[2]Prais and Houthakker, op. cit., p. xxvi. The derivation of price elasticity estimates from the family budget records is also mentioned in Klein, Introduction to Econometrics, op. cit., p. 62, footnote 24.

Theoretically, the estimation of price elasticities is essential for explaining how household expenditure behavior changes according to variations in the price of a commodity. When the price elasticity estimate for a particular commodity is equal to unity in absolute value, the demand is neither elastic nor inelastic. That is, the same amount of money will be spent regardless of price changes. When the price elasticity estimate is numerically greater than unity, the demand is elastic, and the lower the price, the greater the total expenditures on the good. When it is less than unity, the demand is inelastic, and the lower the price, the smaller the total expenditures.[3]

The price elasticity estimates may also be interesting to economists who engage in economic policy. Assume, for instance, that the U.S. government decides to raise food prices. This may be done by price fixing. If the estimates of price parameters are reliable, the quantity

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[3]The above statement can be proved as follows: the total amount spent for a commodity is given by PQ; where P is the price, and Q is the quantity purchased of the good. Thus,

$$\frac{d(PQ)}{dP} = Q(1 + \eta);$$

where η is the price elasticity = $\frac{dQ}{dP}\ \frac{P}{Q}$.

This expression is negative for values of η between $-\infty$ and -1, zero for η equal to -1, and positive for values of between -1 and zero. That is, the total amount spent increases, remains constant, or decreases when price decreases; accordingly, the price elasticity of demand is numerically greater than, equal to, or less than, unity. See J. M. Henderson and R. E. Quandt, Microeconomic Theory (2nd ed.; New York: McGraw-Hill, 1971), p. 27.

of the products demanded can be expected. The results may be considered desirable or not depending on the social ends which are pursued in economic policy. Under certain circumstances, the decline in consumption of foodstuffs is negligible, compared with the benefit accruing to producers from the increase in prices. Various social ends pursued in economic policy may be in conflict. Yet, Tintner mentions:

> Econometrics can contribute nothing as far as the choice of a concrete policy based upon the social ends is concerned. But econometrics can perhaps contribute something in giving economists numerical estimates of the results of the adoption of various possible policies.[4]

For the reasons mentioned,[5] in this chapter, cross sectional and time series data will be pooled. The Engel curves will be modified for estimating both income and price parameters for the five composite foods. The data used are taken from the M.S.U. Consumer Panel data of 1958. As in the previous chapter, four weekly reports are grouped together and treated as a period of time. Those households that stayed and returned the panel reports all thirteen periods of time are selected for the sample of observations. There are 212* 13 = 2756 observations for each composite food.[6]

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[4]Tintner, op. cit., p. 12.

[5]To estimate price elasticities and to obtain more reliable estimates of income elasticities for the five composite foods.

[6]Two hundred twelve is the number of households that stayed and reported their expenditures over all thirteen periods of time in 1958.

2. Statistical Combined Models

By pooling cross sectional and time series data, the price of the food concerned becomes an important variable in determining household expenditure behavior. Different cross sections belong to different sets of prices. The M.S.U. retail food price indices constructed by Wang will be used in this chapter to represent the food prices faced by the panel households.[7] These price indices of the five composite foods are set out in Table 15.

In a cross sectional study, where prices are reasonably constant, household expenditure is usually used as a dependent variable in a cross sectional regression analysis. However, for the combined analyses, prices vary; expenditure must be converted to quantity purchased. The quantity purchased is simply computed by deflating expenditure by the proper price index.

The $i^{\underline{th}}$ household quantity purchased on the $k^{\underline{th}}$ food at the $t^{\underline{th}}$ period of time is computed as follows:

$$Q_{itk} = (Y_{itk} * 100)/P_{tk}$$

where Y_{itk} is the $i^{\underline{th}}$ household aggregate expenditure on the $k^{\underline{th}}$ food at the $t^{\underline{th}}$ period of time, and

P_{tk} is the price index of the $k^{\underline{th}}$ food at the $t^{\underline{th}}$ period of time.

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[7] H. F. Wang, "Retail Food Price Index Based on M.S.U. Consumer Panel" (unpublished Ph.D. dissertation, Michigan State University, 1960).

TABLE 15.--Price Indices of Five Composite Foods Based on M.S.U. Consumer Panel Data of 1958 (1955-57 = 100)

Period of Time	Dairy Prod.	Fats & Oils	Fruits	Vege-tables	Meat, etc.
1	100.9	101.3	93.4	115.9	105.0
2	98.8	98.3	105.8	117.5	107.4
3	100.2	95.9	100.5	129.1	106.4
4	98.8	100.0	115.4	136.6	110.2
5	98.1	100.7	112.5	130.4	111.4
6	97.4	100.1	119.8	119.6	113.7
7	97.9	98.8	114.5	112.2	114.7
8	97.6	97.4	91.1	92.2	113.2
9	99.2	97.3	82.5	73.5	111.7
10	98.2	96.4	76.1	71.2	111.8
11	98.2	97.6	85.2	79.5	109.6
12	97.7	95.0	83.1	93.8	111.6
13	96.1	96.4	82.3	101.0	110.0

Source: H. F. Wang, "Retail Food Price Index Based on M.S.U. Consumer Panel" (unpublished Ph.D. dissertation, Michigan State University, 1960), Table 10, pp. 146-47.

In this manner, one can notice that the identity of price multiplied by quantity equaling expenditure is preserved.

Based on the ceteris paribus assumption, the modified Engel curve for the k^{th} food can be stated as:[8]

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[8]This modified Engel curve is similar to the Marshallian demand function where only the price of the k^{th} food and per capita disposable income are allowed to vary and all other prices are held fixed. Under certain assumptions, Marshall deduced the so-called "law of demand," in which he stated that the slope of his demand curve with respect to price is always negative. For a full discussion of Marshall's law of demand, see D. W. Katzner, Static Demand Theory (New York: Macmillan, 1970), pp. 58-59.

$$\frac{Q_{itk}}{N_{it}} = f_k \left(\frac{M_{it}}{N_{it}}, P_{tk}\right) + U_{itk}$$

where Q_{itk}/N_{it} is the i^{th} household per capita consumption on the k^{th} food at the t^{th} period of time,

M_{it}/N_{it} is the i^{th} household per capita disposable income at the t^{th} period of time,

U_{itk} is the disturbance, and

f_k is the undefined functional form.

This modified Engel curve is consistent with the Engel curve defined in the previous chapter. As long as prices are held constant, the modified Engel curve is the Engel curve.

Regarding the functional form of the modified Engel curve, since neither the semi-log nor the double-log form gives a better "goodness of fit" to the observations based on the thirteen cross sectional studies in the previous chapter, the linear form is adopted in this chapter as the first order approximation.[9]

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[9]The adoption of linear relationships is a proper procedure. As applied to the measurement of demand of food q_k in terms of price p_k and income m, this involves the Taylor's series approximation around any given point $-q_k^0$, p_k^0, m_k^0) or

$$q_k = q_k^0 + \left(\frac{\partial q_k}{\partial m}\right)^0 (m-m^0) + \left(\frac{\partial q_k}{\partial p}\right)^0 (p_k-p^0) + \text{remainder.}$$

As long as the price and income changes were small, the remainder error term can be neglected. See P. A. Samuelson, "Some Implications of 'Linearity,'" The Review of Economic Studies, 1947-48, reprinted in The Collected Scientific Papers of P. A. Samuelson, ed. by J. E. Stiglitz (M.I.T. Press, 1966), p. 61.

Thus, the modified Engel curve for the $k^{\underline{th}}$ food is expressed as follows:

$$\frac{Q_{itk}}{N_{it}} = \alpha_k + \beta_k \frac{M_{it}}{N_{it}} + \gamma_k P_{tk} + U_{itk}$$

where α_k, β_k, and γ_k are parameters, i.e., constant term, income coefficient, and price coefficient, for the $k^{\underline{th}}$ food.

2.1. Estimation Procedure

Regarding the probability distribution of the disturbance U_{itk}, when various successive cross sections are pooled, autocorrelation obviously can arise.[10] The existence of autoregression implies that the disturbance occurring at one period of time is correlated with other disturbances at other periods of time. The common belief in the autocorrelation relies largely on the interpretation of the disturbance as a summary of a large number of random and independent factors that enter into the relationship under study, but which are not measurable.[11] Then one would suspect that the effect of these factors operating in one period would, in part, carry over to the following periods. As Professor Kmenta mentions:

> Autoregression of the disturbances can be compared with the sound effect of tapping a musical string: while the sound is loudest at the time of impact, it

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[10] Kuh, op. cit., p. 98.

[11] For a full discussion of autocorrelation, see Johnston, op. cit., pp. 177-99.

> does not stop immediately but lingers on for a time until it finally dies off. This may also be the characteristic of the disturbance, since its effect may linger for some time after its occurrence. But while the effect of one disturbance lingers on, other disturbances take place, as if the musical string were tapped over and over, sometimes harder than at other times. The shorter the time between the tappings, the greater the likelihood that the preceding sound can still be heard. Similarly, the shorter the periods of individual observations, the greater the likelihood of encountering autoregressive disturbances.[12]

In recent years, a substantial body of literature on how to cope with the autocorrelation has been accumulated. Most of the proposed corrections depend upon exact knowledge of the variance-covariance matrix of the disturbance which will seldom be known.[13]

This study, like many others, assumes that the autocorrelation has the first order autoregressive scheme. The disturbance U_{itk} and the values of the exogenous variables will be characterized as follows:

(i) Normality: U_{itk} is normally distributed;

(ii) Zero mean: $E(U_{itk}) = 0$;

(iii) Homoskedasticity: $E(U_{itk}^2) = \sigma_k^2$;

(iv) No interdependence: $E(U_{itk}U_{jtk}) = 0$ for $i \neq j$;

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[12] Kmenta, op. cit., p. 270.

[13] Kuh, op. cit., p. 99. For a recent survey of literature on the specification of autoregressive scheme, see G. Tintner and J. K. Sengupta, Stochastic Economics (New York: Academic Press, 1972), pp. 12-21.

(v) First order autoregression: $U_{itk} = \rho_k U_{it-1k} + V_{itk}$;

where $V_{itk} \sim N(0, \sigma^2_{kv})$

$E(U_{it-1k} V_{jtk}) = 0$ for all i, j;

(vi) The exogenous variables are measured without error.

If there is serial correlation ($\rho_k \neq 0$), the conventional least squares estimates are unbiased and consistent, yet they are not efficient nor asymptotically efficient. And, the variances of the least squares estimates are biased.[14]

In order to obtain the estimates that, at least, have the desirable asymptotical properties (i.e., consistent, asymptotically efficient, and asymptotically normal), the two-stage estimation method suggested by Cochrane and Orcutt will be used.[15]

The procedure consists of the following two stages: (1) Apply the ordinary least squares method to the modified Engel curve for the k^{th} food. The resulting estimates of the regression coefficients are unbiased and consistent, and can be used to calculate the regression residuals $\hat{U}_{itk}$. From these residuals, one can obtain the estimate of ρ_k by

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[14]For the proofs, see Kmenta, op. cit., p. 269-97.

[15]See D. Cochrane and G. H. Orcutt, "Application of Least Squares Regressions to Relationships Containing Autocorrelated Error Terms," Journal of the American Statistical Association, Vol. 44 (March, 1949), pp. 32-61; Kmenta, op. cit., pp. 287-88 and pp. 509-12.

$$\hat{\rho}_k = \frac{\sum_{i=1}^{212} \sum_{t=2}^{13} \hat{U}_{itk} \hat{U}_{it-1k}}{\sum_{i=1}^{212} \sum_{t=2}^{13} \hat{U}_{it-1k}^2} ;$$

where 212 is the number of households, and 13 is the number of periods.

Obviously, $\hat{\rho}_k$ is a consistent estimator of ρ_k.

(2) Using the $\hat{\rho}_k$ to transform the observations:

$$Q_{itk}^* = \alpha_k^* + \beta_k M_{it}^* + \gamma_k P_{tk}^* + U_{itk}^* ;$$

where $Q_{itk}^* = Q_{itk}/N_{it} - \hat{\rho}_k \, Q_{it-1k}/N_{it-1}$

$M_{it}^* = M_{it}/N_{it} - \hat{\rho}_k \, M_{it-1}/N_{it-1}$

$P_{tk}^* = P_{tk} - \hat{\rho}_k \, P_{t-1k}$

$U_{itk}^* = U_{itk} - \hat{\rho}_k \, U_{it-1k}$

$i = 1, 2, \ldots, 212$

$t = 2, 3, \ldots, 13.$

The disturbance U_{itk}^* is asymptotically nonautoregressive. Applying the ordinary least squares method again, the estimators of α_k^*, β_k, and γ_k have the desirable asymptotic properties; i.e., consistent asymptotically efficient, and asymptotically normal.

2.2. Results of Combined Studies

At the first stage of computation, without eliminating the autoregressive effects, the least squares estimates of regression coefficients are obtained as follows:

Dairy Products: $$q_{it} = 4.0498 + \underset{(.0003)}{.0017}\, m_{it} - \underset{(.0217)}{.0058}\, P_t$$

$$\bar{R}^2 = .0112$$
$$S = 1.8163$$

Fats and Oils: $$q_{it} = 1.4148 + \underset{(.0001)}{.0011}\, m_{it} - \underset{(.0086)}{.0057}\, P_t$$

$$\bar{R}^2 = .0207$$
$$S = .8609$$

Fruits: $$q_{it} = 3.8473 + \underset{(.0002)}{.0039}\, m_{it} - \underset{(.0023)}{.0215}\, P_t$$

$$\bar{R}^2 = .0854$$
$$S = 1.8020$$

Vegetables: $$q_{it} = 3.1244 + \underset{(.0002)}{.0023}\, m_{it} - \underset{(.0011)}{.0133}\, P_t$$

$$\bar{R}^2 = .0787$$
$$S = 1.3354$$

Meat, etc.: $$q_{it} = 12.2750 + \underset{(.0006)}{.0089}\, m_{it} - \underset{(.0262)}{.0621}\, P_t$$

$$\bar{R}^2 = .0699$$
$$S = 3.7902$$

where $q_{it} = Q_{it}/N_{it}$;

$m_{it} = M_{it}/N_{it}$;

$\bar{R}^2$ is the corrected coefficient of determination; and

S is the standard error of estimate.

Using the resulting estimates of regression coefficients, the estimates of autocorrelated coefficients for the five composite foods are calculated. These estimates are presented in Table 16. They are all positive, close to but less than one. The figures .8284, .6966, .7100, .6629, and .5031 are the estimates of autocorrelated coefficients for dairy products; fats and oils; fruits; vegetables; and meat, etc., respectively. The highest estimate is .8284 for dairy products, and the lowest is .5031 for meat, etc.

TABLE 15. Estimates of Autocorrelated Coefficients for Five Composite Foods

Foods	Estimates of Autocorrelated Coefficients
Dairy Products	.8284
Fats and Oils	.6966
Fruits	.7100
Vegetables	.6629
Meat, etc.	.5031

Statistically, the positive estimate of autocorrelated coefficient for a particular food indicates that the disturbances for that food are positively correlated. The figure which is close to one indicates that the degree of the relationship between the disturbances is fairly high.

At the final stage of computation, the autoregressive effects are eliminated. The least squares estimates of regression coefficients for the five composite foods are obtained and shown in Table 17.

TABLE 17.--Estimates of Regression Coefficients for Five Composite Foods after Eliminating the Autoregressive Effects

Foods	Constant Term	Income Coeff.	Price Coeff.	$\bar{R}^2$	S
Dairy Products	1.5011	.0004 (.0002)	-.0553 (.0200)	.0038	1.0029
Fats and Oils	.7698	.0006 (.0001)	-.0165 (.0066)	.0104	.5589
Fruits	1.1656	.0027 (.0003)	-.0200 (.0025)	.0494	1.2784
Vegetables	1.1680	.0007 (.0002)	-.0147 (.0016)	.0318	.9872
Meat, etc.	4.0680	.0063 (.0007)	-.0221@ (.0395)	.0262	3.1263

Remarks: The mark @ indicates that the regression coefficient is not significantly different from zero at 5% level of significance.

$\bar{R}^2$ is the corrected coefficient of determination.

S is the standard error of estimate.

From Table 17, one would notice that all the estimates of income coefficients for the five composite foods are positive and significantly different from zero at 5% level of significance. Needless to say, these estimates of income coefficients are more reliable than those estimated from individual cross sectional studies, since more observations are used in the estimation procedure.

As for the estimates of price coefficients for the five composite foods, they are all negative. Except for meat, etc., the estimates of price coefficients are significantly different from zero at 5% level of significance.

2.2.1. Income and Price Elasticity Estimates.--When the estimates of income and price coefficients are obtained as shown in Table 17, the income and price elasticity estimates for the five composite foods can be easily derived. At the mean values,[16] for the k^th^ food, the income and price elasticity estimates are calculated as follows:

the income elasticity estimate = $\hat{\beta}_k \ (\bar{m}/\bar{q}_k)$,

the price elasticity estimate = $\hat{\gamma}_k \ (\bar{p}_k/\bar{q}_k)$;

where $\bar{q}_k$ is the average value of households' per capita quantity purchased on the k^{th} food,

$\bar{p}_k$ is the average value of price indices for the k^{th} food,

$\bar{m}$ is the average value of households' per capita disposable income, and

$\hat{\beta}_k$ and $\hat{\gamma}_k$ are the estimated income and price coefficients for the k^{th} food, respectively.

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[16]The values of $\bar{q}_k$, $\bar{p}_k$, $\bar{m}$, and $\bar{m}/\bar{q}_k$ are given in Appendix G.

The income and price elasticity estimates at mean values for the five composite foods are shown in Table 18, below.[17]

TABLE 18.--Income and Price Elasticity Estimates for Five Composite Foods after Eliminating the Autocorrelated Effects

Foods	Income Elasticity	Price Elasticity
Dairy Products	.0170	-1.4543
Fats and Oils	.0928	-1.5712
Fruits	.1808	-.8158
Vegetables	.0533	-.7424
Meat, etc.	.1472	-.3580

[17]These combined studies give substantially lower values of income elasticities than cross sectional studies in the previous chapter. One possibility to explain this phenomenon is the implications of Friedman's permanent income hypothesis. Friedman has demonstrated that the elasticity of consumption with respect to measured income separates into two elasticities: the elasticity of consumption with respect to permanent income and the elasticity of permanent income with respect to measured income. Given the assumptions of the permanent income hypothesis, Friedman demonstrates the equivalence of the elasticity of permanent income with respect to measured income and the elasticity of consumption on measured income. Thus, it would seem that the income elasticity estimated from cross sectional data is a reasonable approximation of the elasticity of the permanent income. As more time series data are introduced, the permanent income component of the measured income is reduced. This results in lower values of income elasticities derived from the combined studies. For more discussion of this problem, see Friedman, op. cit., Section 2, Chapter VIII, p. 206.

From Table 18, one would notice that the income elasticity estimates for the five composite foods are all inelastic. They confirm Engel's law. The income elasticity estimates are .0170, .0928, .1808, .0533, and .1808. The highest income elasticity estimate is .1472 for fruits, etc.,and the lowest is .0170 for dairy products. Theoretically, these figures imply that, other things being equal, if household per capita disposable income in the Lansing area rises by 1 per cent, on the average, the household per capita quantity purchased on dairy products; fats and oils; fruits; vegetables; and meat, etc. would increase by about .0170, .0928, .1808, .0533, and .1472 per cent, respectively.

As for the price elasticity estimates for the five composite foods, they are widely different. The figures are -1.4543, -1.5712, -.8158, -.7424, and -.3580 for dairy products; fats and oils; fruits; vegetables; and meat, etc., respectively. The price elasticity estimates for dairy products, and fats and oils are highly elastic. In an economic sense, an increase of 1 per cent in the price of dairy products or fats and oils, other things being equal, a decrease in demand for that product would be greater than 1 per cent. Alternatively, the price elasticity estimates for fruits, vegetables, and meat, etc. are inelastic. An increase of 1 per cent in the price of fruits or vegetables or meat, etc., other things being equal, a decrease in demand for that food would be less than 1 per cent.

In summary, regarding the signs of the estimates of income and price elasticities for the five composite foods, the results of these combined studies on the modified Engel curves are highly successful as they confirm the demand theorem. The consistency of the panel data is probably the main reason for this success.

APPENDIX G

VALUES OF $\bar{q}_k$, $\bar{p}_k$, $\bar{m}$, $\bar{m}/\bar{q}_k$, AND $\bar{p}_k/\bar{q}_k$

Foods	$\bar{q}_k$	$\bar{p}_k$	$\bar{m}$	$\bar{m}/\bar{q}_k$	$\bar{p}_k/\bar{q}_k$
Dairy Products	3.74	98.39	159.40	42.62	26.30
Fats and Oils	1.03	98.09	159.40	154.75	95.23
Fruits	2.38	97.09	159.40	66.97	40.79
Vegetables	2.09	105.57	159.40	76.26	50.51
Meat, etc.	6.82	110.51	159.40	23.37	16.20

CHAPTER V

SOME PRELIMINARY EVIDENCE ON APPROXIMATING EMPIRICAL UTILITY FUNCTIONS

1. Objectives

In this chapter, some preliminary evidence on approximating empirical utility functions by means of Engel curves, based on Wald's theorem, will be given.[1] In addition, some areas that were omitted from the present study will be proposed for future research.

2. Utility Functions and Engel Curves

In this section, some preliminary evidence on approximating empirical utility functions by means of Engel

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[1]For a full discussion of Wald's theorem, see A. Wald, *op. cit.*, pp. 144-55. This theorem is also mentioned in Z. Hellwig, *Linear Regression and Its Application to Economics* (New York: Macmillan, 1963), pp. 62-63; Tintner, *Econometrics*, *op. cit.*, pp. 60-61; G. Tintner, *Methodology of Mathematical Economics and Econometrics* (Chicago: University Press, 1968), pp. 21-23; and H. T. Davis, *op. cit.*, p. 168.

More research in this field of determining empirical utility functions and conditions of integrability are being undertaken at Michigan State University under Professor A. Y. C. Koo's leadership. For some of Professor Koo's works, see A. Y. C. Koo, "An Empirical Test of Revealed Preference Theory," *Econometrica*, 31 (October, 1963), pp. 646-64; A. Y. C. Koo, "Revealed Preference: A Structural Analysis," *Econometrica*, 39 (January, 1971), pp. 89-97; and A. Y. C. Koo and G. Hasenkamp, "Structure of Revealed Preference: Some Preliminary Evidence," *Journal of Political Economy*, Vol. 80 (July/August, 1972), pp. 724-44.

curves will be presented. Needless to say, it is of great theoretical and practical importance to know the empirical utility functions. One of the most important problems which can be solved if one knows the utility function is the determination of the demand functions for consumers' goods. Also, the determination of the utility function enables one to calculate the index of cost of living.[2]

It should be noted that the results of the thirteen cross sectional studies on the Engel curves as shown in Chapter III, and the given M.S.U. price indices as shown in Chapter IV, will provide all the information needed to approximate the utility functions by Wald's method.[3]

Before presenting Wald's theorem, some notations will be given first.

Denote $q = (q^1, \ldots, q^n)$ be a set of n goods purchased by a representative consumer at a period of time; $p = (p^1, \ldots, p^n)$ be the corresponding set of prices; m be the disposable income or the total expenditure; $u(q^1, \ldots, q^n)$ be an indicator of a well-defined total utility function. Given p and m at a period of time, the

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[2]For more discussion of the applications of the utility functions, see Wald, op. cit., p. 171-75; and G. J. Stigler, The Theory of Prices (3rd. ed.; New York: Macmillan, 1966), pp. 71-83.

[3]For a critical evaluation of the problem connected with the empirical derivatives of indifference surfaces, see W. A. Wallis and M. Friedman, "The Empirical Derivation of Indifference Functions," in Studies in Mathematical Economics and Econometrics, ed. by O. Lange, et al. (Chicago: University Press, 1942), pp. 175-89.

first or necessary condition for maximizing the utility subjected to the budget constraint is fulfilled if the consumer purchases the quantities such as:

$$\frac{\partial u}{\partial q^1}/p^1 = \ldots = \frac{\partial u}{\partial q^n}/p^n$$

$$\sum_{k=1}^{n} p^k q^k = m \; .$$

Solving the equations, one gets the quantities purchased as functions of prices and income. For a given period of time with constant prices, the quantities purchased will depend only on the income; that is,

$$q^1 = f^1(m), \; \ldots \; q^n = f^n(m) \; .$$

These functions are Engel curves and represent a loci in the n-dimensional quantity space which is called the consumption expansion path (C). To each system of prices belongs a certain set of Engel curves.

Assuming all consumers have the same preference function, thus, the Engel curves can be determined empirically in each period of time by observing the consumption of consumers belonging to different income levels, as shown in Chapter III.

Considering the consumption expansion paths $C_1, \ldots, C_T$ belonging to the periods $t_1, \ldots t_T$ or, more generally, to the different price situations. If all

Engel curves are linear, each consumption expansion path C_t ($t = 1, \ldots, T$) can be determined by two of its points; say,

$$\bar{q}_t = (\bar{q}_t^1, \ldots, \bar{q}_t^n) \text{ and } \bar{\bar{q}}_t = (\bar{\bar{q}}_t^1, \ldots, \bar{\bar{q}}_t^n) .$$

For the sake of simplification, it is advantageous to denote $\bar{q}_t$ by q_{2t-2} and $\bar{\bar{q}}_t$ by q_{2t-1}. For the set of prices $\bar{p}_t = \bar{\bar{p}}_t$, the symbols of p_{2t-2} and p_{2t-1} are also used.

Considering the vector $v_t = q_0q_t$ with the initial point q_0 and terminal point q_t ($t = 1, 2, \ldots, 2T-1$).

Wald's Theorem: If there exists an indicator $u(q^1, \ldots, q^n)$ which is a polynomial of the second degree in q over the (2T-1) dimensional linear space determined by the vectors $v_1, \ldots, v_{2T-1}$ associated with the consumption expansion paths $C_1, \ldots, C_T$, then this indicator is uniquely determined in S by the said consumption expansion paths, apart from an arbitrary proportionality factor and an arbitrary additive constant.

If $\lambda_1, \ldots, \lambda_{2T-1}$ are the vector coordinates of the points considered, and Λ is an arbitrary constant, the following formula is the indicator as a function of the vector coordinates $\lambda_1, \ldots, \lambda_{2T-1}$:[4]

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[4]For the proof, see Wald, op. cit., pp. 146-53.

$$f(\lambda_1, \ldots, \lambda_{2T-1}) = \frac{1}{2} \sum_{t=1}^{2T-1} \sum_{s=1}^{2T-1} \bar{\alpha}_{t,s} \lambda_t \lambda_s + \sum_{t=1}^{2T-1} \rho_{0,t} \lambda_t + A$$

where $\rho_{t,s} = \sum_{k=1}^{n} p_t^k v_s^k = \sum_{k=1}^{n} p_t^k (q_s^k - q_0^k)$

for $t = 0, 1, \ldots, 2T-1$; $s = 1, \ldots, 2T-1$.

$$w_1 = \frac{\rho_{0,2} - \rho_{0,3} + \rho_{2,3}(\rho_{0,1} - \rho_{0,2})/\rho_{2,1} - \rho_{3,2}(\rho_{0,1} - \rho_{0,3})/\rho_{3,1}}{\rho_{3,2}(\rho_{1,3}/\rho_{3,1}) - \rho_{2,3}(\rho_{1,2}/\rho_{2,1})}$$

$$w_t = (w_1 \rho_{1,t} - \rho_{0,t} + \rho_{0,1})/\rho_{t,1} \quad \text{for } t = 2, \ldots, 2T-1$$

$$\alpha_{t,s} = w_t \rho_{t,s} - \rho_{0,s} \quad \text{for } t,s = 1, 2, \ldots, 2T-1$$

$$\bar{\alpha}_{t,s} = \frac{1}{2} (\alpha_{t,s} + \alpha_{s,t})$$

w_t is the marginal utility of money at the point q_t under the system of prices p_t ($t = 0, 1, \ldots, 2T-1$).

Practically, one wants to have the indicator of the utility function as a function of quantities $q^1, q^2, \ldots, q^n$. The transformation of $f(\lambda_1, \ldots, \lambda_{2T-1})$ into the form $u(q^1, \ldots, q^n)$ can be made as follows:

$$\lambda_1 v_1^1 + \lambda_2 v_2^1 + \ldots + \lambda_{2T-1} v_{2T-1}^1 = q^1 - q_0^1$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\lambda_1 v_1^n + \lambda_2 v_2^n + \ldots + \lambda_{2T-1} v_{2T-1}^n = q^n - q_0^n$$

If the determinant of these equations is not equal to zero, one obtains $\lambda_t = g_t(q^1, q^2, \ldots, q^n)$ for

$t = 1, 2, \ldots, 2T-1$. Substituting these λ_t into $f(\lambda_1, \ldots, \lambda_{2T-1})$, one obtains $u(q^1, \ldots, q^n)$, the indicator of the utility function as a function of $q^1, \ldots, q^n$.

To present Wald's method of approximating utility functions by means of Engel curves, Figure 6 roughly shows such methodology for the case of three-dimensional commodity space and two consumption expansion paths C_1 and C_2 belonging to two periods of time.[5]

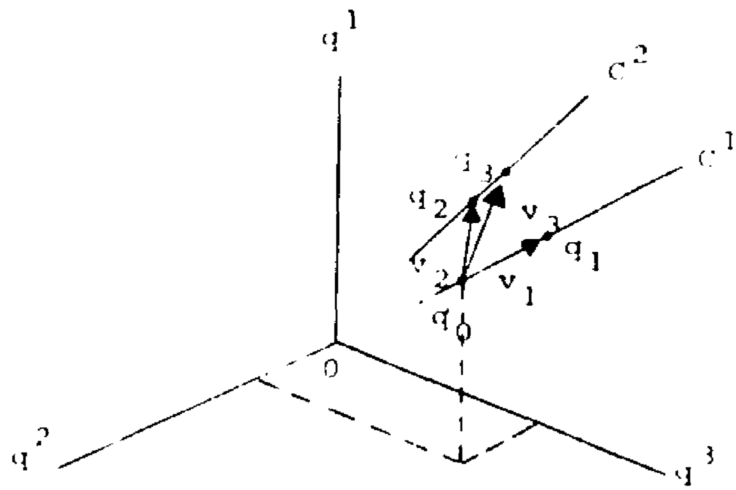

Figure 6.--A Diagram Showing Wald's Method, with Three-Dimensional Commodity Space and Two Consumption Expansion Paths

For a numerical illustration, consider the three commodities, no. 1 = dairy products, no. 2 = fats and oils,

[5]For numerical illustrations of Wald's method of approximating utility functions by means of Engel curves, see Wald, op. cit., pp. 153-55; and Tintner, Econometrics, op. cit., pp. 60-61.

no. 3 = fruits, and the two periods of time, t_1 = period 1, t_2 = period 2.

The price indices of these three commodities in the two periods are given as follows:[6]

$p_1^1 = 1,\ p_1^2 = 1,\ p_1^3 = 1;$

$p_2^1 = .979,\ p_2^2 = .970,\ p_2^3 = 1.132;$

where p_t^k is the price index of the $k^{\underline{th}}$ good at the $t^{\underline{th}}$ period of time.

At period 1, the consumption expansion path C_1 is given by the following set of Engel curves:[7]

$q^1 = 3.678 + .0011\ m_1$

$q^2 = \ .901 + .0009\ m_1$

$q^3 = 1.459 + .0029\ m_1$.

And at period 2, the consumption expansion path C_2 is given by the following set of Engel curves:

$q^1 = 3.798 + .0005\ m_2$

$q^2 = 1.051 + .0007\ m_2$

$q^3 = 1.649 + .0018\ m_2$;

---

[6]For the price indices, see Table 15, p. 80. Assume that period 1 is the based period.

[7]For the Engel curves, see Table 1 through Table 5, pp. 37-41. In order to obtain the quantities purchased q^k as a function of m, the estimates of the regression coefficients are deflated by the proper price indices.

where q^k is the per capita quantity purchased of the k^{th} good, and

m_t is the per capita disposable income at period t.

To approximate the utility function by Wald's method, two points on each C_t (t = 1,2) have to be chosen. On C_1, the points q_0 and q_1 corresponding to the disposable income \$150 and \$160, and on C_2 the points q_2 and q_3 corresponding to the disposable income \$160 and \$170 are chosen.[8]

Hence, one obtains

$$p_0 = p_1 = (1, \quad 1, \quad 1)$$
$$p_2 = p_3 = (.979, \ .970, \ 1.132)$$
$$q_0 = (3.8430, \ 1.0360, \ 1.8940)$$
$$q_1 = (3.8540, \ 1.0450, \ 1.9230)$$
$$q_2 = (3.8797, \ 1.1665, \ 1.9448)$$
$$q_3 = (3.8849, \ 1.1738, \ 1.9633)$$

The following figures are the values of $\rho_{t,s}$'s:

t \ s	1	2	3
0	.0490	.2181	.2490
1	.0490	.2181	.2490
2	.0524	.2202	.2532
3	.0524	.2202	.2532

[8]These levels of income are around the mean of the observed per capita disposable income; see Appendix F, pp. 73-75.

As for the values of w_t, the marginal utility of money at the point q_t under the system of prices p_t, one obtains:

$w_1 = 1.0065$

$w_2 = .9213$

$w_3 = .9193$

The following figures are the values of $\alpha_{t,s}$'s:

t s	1	2	3
1	-.0002	-.0008	-.0009
2	-.0008	-.0152	-.0157
3	-.0009	-.0157	-.0162

One would notice that $\alpha_{t,s} = \alpha_{s,t}$ for all t,s = 1,2,3. Thus, the values of $\bar{\alpha}_{t,s} = \frac{1}{2}(\alpha_{t,s} + \alpha_{s,t}) = \alpha_{t,s}$.

Hence, one can obtain the indicator of the utility function as a function of λ_1, λ_2, λ_3 as follows:

$$\begin{aligned} f(\lambda_1,\lambda_2,\lambda_3) = {} & .5(-.0002\ \lambda_1^2 - .0152\ \lambda_2^2 - .0162\ \lambda_3^2 - .0008\ \lambda_1\lambda_2 \\ & - .0009\ \lambda_1\lambda_3 - .0008\ \lambda_2\lambda_1 - .0157\ \lambda_2\lambda_3 \\ & - .0009\ \lambda_3\lambda_1 - .0157\ \lambda_3\lambda_2) + .0490\ \lambda_1 \\ & + .2181\ \lambda_2 + .2490\ \lambda_3 + A\ . \end{aligned}$$

In order to obtain the indicator of the utility function as a function of quantities purchased q^1, q^2, q^3, the following transformation is made:

$$.0110\ \lambda_1 + .0367\ \lambda_2 + .0419\ \lambda_3 = q^1 - 3.8430$$

$$.0090\ \lambda_1 + .1305\ \lambda_2 + .1378\ \lambda_3 = q^2 - 1.0360$$

$$.0290\ \lambda_1 + .0508\ \lambda_2 + .0693\ \lambda_3 = q^3 - 1.8940$$

Since the determinant of these equations is not equal to zero, the values of λ_1, λ_2, and λ_3 are calculated as:

$$\lambda_1 = 287.2330\ q^1 - 58.9996\ q^2 - 56.1575\ q^3 - 661.292$$

$$\lambda_2 = 471.4688\ q^1 - 63.0785\ q^2 - 159.2569\ q^3 - 212.207$$

$$\lambda_3 = -465.5462\ q^1 + 70.8890\ q^2 + 154.5865\ q^3 + 454.696$$

Substituting in $f(\lambda_1, \lambda_2, \lambda_3)$ for λ_1, λ_2, λ_3, one gets the indicator of the utility function as a function of q^1, q^2, q^3.

With an electronic computer, the utility functions can be approximated by extending the number of commodities and periods of time to n-dimensional commodity space and T-periods of time.

As for another numerical illustration, five commodities, no. 1 = dairy products, no. 2 = fats and oils, no. 3 = fruits, no. 4 = vegetables, no. 5 = meat, etc., and three periods of time, t_1 = period 1, t_2 = period 2, t_3 = period 3, are considered.

The price indices of these five commodities in the three periods of time are given as follows: $p_1^1 = 1$, $p_1^2 = 1$, $p_1^3 = 1$, $p_1^4 = 1$, $p_1^5 = 1$; $p_2^1 = .979$, $p_2^2 = .970$, $p_2^3 = 1.132$, $p_2^4 = 1.013$, $p_2^5 = 1.022$; $p_3^1 = .993$, $p_3^2 = .946$, $p_3^3 = 1.076$, $p_3^4 = 1.113$, $p_3^5 = 1.013$; where p_t^k is the price index of the $k^{\underline{th}}$ good at period t.

At period 1, the consumption expansion path C_1 is given by the following set of Engel curves:

$q^1 = 3.678 + .0011\ m_1$, $q^2 = .901 + .0009\ m_1$, $q^3 = 1.459 + .0029\ m_1$, $q^4 = 1.779 + .0024\ m_1$, $q^5 = 5.993 + .0112\ m_1$.

The C_2 at period 2 is given by the set of Engel curves:

$q^1 = 3.798 + .0005\ m_2$, $q^2 = 1.051 + .0007\ m_2$, $q^3 = 1.649 + .0018\ m_2$, $q^4 = 1.954 + .0012\ m_2$, $q^5 = 5.855 + .0122\ m_2$.

And the C_3 is given by the following set of Engel curves:

$q^1 = 3.389 + .0025\ m_3$, $q^2 = .955 + .0007\ m_3$, $q^3 = 1.667 + .0018\ m_3$, $q^4 = 1.674 + .0017\ m_3$, $q^5 = 6.087 + .0076\ m_3$;

where q^k is the per capita quantity purchased on the $k^{\underline{th}}$ good, and m_t is the per capita disposable income at period t.

On C_1, the points q_0 and q_1 corresponding to the disposable income \$150 and \$160; on C_2, the points q_2 and

q_3 corresponding to the disposable income \$160 and \$170; and on C_3, the points q_4 and q_5 corresponding to the disposable income \$170 and \$180, are chosen.

Hence, one obtains

$$p_0 = p_1 = (\quad 1, \quad 1, \quad 1, \quad 1, \quad 1)$$

$$p_2 = p_3 = (.979, \quad .970, \quad 1.132, \quad 1.013, \quad 1.022)$$

$$p_4 = p_5 = (.993, \quad .946, \quad 1.076, \quad 1.113, \quad 1.013)$$

$$q_0 = (3.8430, \quad 1.0360, \quad 1.8940, \quad 2.1390, \quad 7.6730)$$

$$q_1 = (3.8540, \quad 1.0450, \quad 1.9230, \quad 2.1630, \quad 7.7850)$$

$$q_2 = (3.8797, \quad 1.1665, \quad 1.9448, \quad 2.1582, \quad 7.8056)$$

$$q_3 = (3.8849, \quad 1.1738, \quad 1.9633, \quad 2.1710, \quad 7.9278)$$

$$q_4 = (3.8175, \quad 1.0806, \quad 1.9832, \quad 1.9634, \quad 7.3776)$$

$$q_5 = (3.8427, \quad 1.0880, \quad 2.0018, \quad 1.9804, \quad 7.4536)$$

The following figures are the values of $\rho_{t,s}$'s:

t s	1	2	3	4	5
0	.1850	.3699	.5358	-.3627	-.2185
1	.1850	.3699	.5358	-.3627	-.2185
2	.1912	.3753	.5463	-.3608	-.2129
3	.1912	.3753	.5463	-.3608	-.2129
4	.1909	.3705	.5405	-.3820	-.2340
5	.1909	.3705	.5405	-.3820	-.2340

As for the values of w_t, the marginal utility of money at the point q_t under the system of prices p_t, one obtains:

$w_1 = 1.0911$

$w_2 = 1.1435$

$w_3 = 1.2225$

$w_4 = .7962$

$w_5 = .8650$

These figures tend to indicate that the marginal utility of money declines as the per capita disposable income increases.

The values of $\alpha_{t,s}$'s are as follows:

t s	1	2	3	4	5
1	.0168	.0337	.0488	-.0330	-.0199
2	.0337	.0593	.0889	-.0499	-.0250
3	.0488	.0889	.1321	-.0784	-.0418
4	-.0330	-.0749	-.1055	.0585	.0321
5	-.0199	-.0495	-.0683	.0322	.0160

The following figures are the values of $\bar{\alpha}_{t,s}$'s:

t s	1	2	3	4	5
1	.0168	.0337	.0488	-.0330	-.0199
2	.0337	.0593	.0889	-.0624	-.0372
3	.0488	.0889	.1321	-.0919	-.0550
4	-.0330	-.0624	-.0919	.0585	.0322
5	-.0199	-.0372	-.0550	.0322	.0160

Thus, one can obtain the indicator of the utility function as a function of λ_1, λ_2, λ_3, λ_4, λ_5 as follows:

$$f(\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5) = .5(.0168\ \lambda_1^2 + .0593\ \lambda_2^2 + .1321\ \lambda_3^2$$
$$+ .0585\ \lambda_4^2 + .0160\ \lambda_5^2 + .0337\ \lambda_1\lambda_2$$
$$+ .0488\ \lambda_1\lambda_3 - .0330\ \lambda_1\lambda_4 - .0199\ \lambda_1\lambda_5$$
$$+ .0337\ \lambda_2\lambda_1 + .0889\ \lambda_2\lambda_3 - .0624\ \lambda_2\lambda_4$$
$$- .0372\ \lambda_2\lambda_5 + .0488\lambda_3\lambda_1 + .0889\ \lambda_3\lambda_2$$
$$- .0919\ \lambda_3\lambda_4 - .0550\ \lambda_3\lambda_5 - .0330\ \lambda_4\lambda_1$$
$$- .0624\ \lambda_4\lambda_2 - .0919\ \lambda_4\lambda_3 + .0322\ \lambda_4\lambda_5$$
$$- .0199\ \lambda_5\lambda_1 - .0372\ \lambda_5\lambda_2 - .0550\ \lambda_5\lambda_3$$
$$+ .0322\ \lambda_5\lambda_4) + .1850\ \lambda_1 + .3699\ \lambda_2$$
$$+ .5358\ \lambda_3 - .3627\ \lambda_4 - .2185\ \lambda_5 + A$$

In order to obtain the indicator of the utility function as a function of quantities purchased q^1, q^2, q^3, q^4, and q^5, the following transformation is made:

$$.0110\ \lambda_1 + .0367\ \lambda_2 + .0419\ \lambda_3 - .0255\ \lambda_4 - .0003\ \lambda_5 = q^1 - 3.8430$$

$$.0090\ \lambda_1 + .1305\ \lambda_2 + .1375\ \lambda_3 + .0446\ \lambda_4 + .0520\ \lambda_5 = q^2 - 1.0360$$

$$.0290\ \lambda_1 + .0508\ \lambda_2 + .0693\ \lambda_3 + .0892\ \lambda_4 + .1078\ \lambda_5 = q^3 - 1.8940$$

$$.0240\ \lambda_1 + .0192\ \lambda_2 + .0320\ \lambda_3 - .1756\ \lambda_4 - .1586\ \lambda_5 = q^4 - 2.1390$$

$$.1120\ \lambda_1 + .1326\ \lambda_2 + .2548\ \lambda_3 - .2954\ \lambda_4 - .2194\ \lambda_5 = q^5 - 7.6730$$

Since the determinant of these equations is not equal to zero, the values of λ_1, λ_2, λ_3, λ_4, λ_5 are calculated as:

$$\lambda_1 = -36.7916\ q^1 - 6.0435\ q^2 + 52.9775\ q^3 + 49.8635\ q^4 - 11.3748\ q^5 - 27.9322$$

$$\lambda_2 = -.9020\ q^1 + 8.2268\ q^2 + 27.9770\ q^3 + 46.2554\ q^4 - 17.7284\ q^5 + 20.9552$$

$$\lambda_3 = .0177\ q^1 + .7735\ q^2 - 30.7245\ q^3 - 44.8605\ q^4 + 17.5045\ q^5 - 18.9673$$

$$\lambda_4 = -57.0921\ q^1 + 10.666\ q^2 + 12.8541\ q^3 + 14.7763\ q^4 - 1.7445\ q^5 - 165.7879$$

$$\lambda_5 = 57.5593\ q^1 - 11.5761\ q^2 - 9.0355\ q^3 - 18.5843\ q^4 + 1.5990\ q^5 + 164.6116$$

Substituting in $f(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ for $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$, one gets the indicator of the utility function as a function of q^1, q^2, q^3, q^4, q^5. Of course, this utility function is more complicated than the previous one.

The above numerical illustrations indicate, at least, that the approximate determination of the utility functions by means of Engel curves could be made, if Wald's theorem is adopted. More research on this field of empirical utility functions is needed to be undertaken. The conditons of integrability, and the sufficient condition of equilibrium are needed for further empirical test before any economic policy recommendation can be drawn.[9]

3. Areas for Future Research

Besides some areas mentioned earlier, an area will be proposed in this section for future research.[10] This area, in particular, could be undertaken and lead directly to supplement the present study. It is the use of the Box-Cox model to test the linearity of Engel curves.

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[9]The fulfillment of the sufficient condition of equilibrium implies that the equilibrium position is the maximum one. The fulfillment of the integrability conditions means that there exists one and only one indicator of utility such that along the given consumption expansion paths the necessary conditions for the equilibrium position are fulfilled. For the empirical tests of these areas, see Wald, op. cit.

[10]Other possible areas of research are the simultaneous equations, the dynamic models, the projections of households' expenditure behavior, the estimation of coefficients of economies of scale, etc. For more specific areas of research, see Quackenbush and Shaffer, op. cit., pp. 46-51.

Consider the following function:

$$\frac{y_{ik}^{\lambda} - 1}{\lambda} = \alpha_k + \beta_k \left(\frac{m_i^{\lambda} - 1}{\lambda}\right) + U_{ik}$$

where y_{ik} is the $i^{\underline{th}}$ household per capita expenditure on the $k^{\underline{th}}$ food at a period of time,

m_i is the $i^{\underline{th}}$ household per capita disposable income at a period of time,

α_k, β_k, and λ are parameters, and

U_{ik} is the stochastic error term.

For $\lambda = 1$, one obtains

$$(y_{ik} - 1) = \alpha_k + \beta_k (m_i - 1) + U_{ik}$$

or $$y_{ik} = \alpha^*_k + \beta_k m_i + U_{ik}$$

where $$\alpha^*_k = \beta_k - \beta_k + 1$$

which is a simple linear regression model.

For $\lambda = 0$, one obtains

$$\log y_{ik} = \alpha_k + \beta_k \log m_i + U_{ik}$$

which is a double-log regression model.

In general, different values of λ lead to different functional specification of the regression equation.

This allows one to test the linear hypothesis against the alternative hypothesis.[11] Formally,

H_0: $\lambda = 1$

H_a: $\lambda \neq 1$

To carry out the test, one needs an estimate of λ and its standard error. Obviously, λ can be estimated along with the other parameters by the maximum likelihood method.

The likelihood function for $y_{ik}, \ldots, y_{nk}$ is

$$L = (\lambda-1)\sum_i \log y_{ik} - \frac{n}{2} \log 2\Pi - \frac{n}{2} \log \sigma^2 - \frac{1}{2\sigma^2} \sum_i [(\frac{y_{ik}^{\lambda}-1}{\lambda})$$

$$- \alpha_k - \beta_k (\frac{m_i^{\lambda}-1}{\lambda})]^2 \quad .$$

The maximizing values of λ, α_k, β_k, and σ^2 can be found with an electronic computer, and the respective standard errors can be estimated by reference to the appropriate information matrix.

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[11]For more discussion of this problem, see G. E. P. Box and D. R. Cox, "An Analysis of Transformations," Journal of the Royal Statistical Society, Series B, Vol. 26 (1964), pp. 211-43; also, Kmenta, op. cit., pp. 467-68.

It should be noted that the maintained hypothesis could be the double-log form, or H_0: $\lambda = 0$ against H_2: $\lambda \neq 0$.

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