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A FORMATIVE EVALUATION OF THE MATHEMATICS COMPONENT OF AN EXPERIMENTAL ELEMENTARY TEACHER EDUCATION PROGRAM AT MICHIGAN STATE UNIVERSITY

By

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ABSTRACT

A FORMATIVE EVALUATION OF THE MATHEMATICS
COMPONENT OF AN EXPERIMENTAL ELEMENTARY
TEACHER EDUCATION PROGRAM AT
MICHIGAN STATE UNIVERSITY

By

Mosen Sharif Shakrani

This study was a formative evaluation of the mathematics component of an experimental elementary teacher education program at Michigan State University. Specifically, this investigation sought to: (1) evaluate the adequacy of the mathematical content in meeting the need of the future elementary school teachers, (2) evaluate the effect of the instructional treatment on the participating students' performance on the prescribed mathematical competencies, (3) ascertain whether a specified degree of mastery (80 percent) had been attained on the prescribed competencies within each mathematical topic, (4) evaluate the effect of instruction on the students' mathematical understandings and attitudes, (5) compare students in the experimental program with students in the regular elementary teacher education program in relation to their mathematical understandings and attitudes, and (6) determine the relationship between selected variables and achievement in mathematics.

The experimental program is funded and staffed by the "Trainers of Teacher Trainers" (TTT) Project and is based on aspects of the Behavioral Science Teacher Education Program (BSTEP) developed at Michigan State University in 1968.

The mathematics component of the experimental program is composed of two courses that integrate the study of mathematics with the methodology of teaching mathematics in a laboratory setting. The first course, offered during the freshman year, emphasizes arithmetic. The second course, offered during the junior year, emphasizes algebra and geometry. Each course includes clinical experience where prospective teachers translate theory into practice by teaching mathematical concepts, taught at the University, to groups of elementary school children.

During the academic year 1971-1972, the first integrated content-methods course was developed by a team of mathematics educators and elementary school teachers. The course comprised the following mathematical topics: (1) Measurement, (2) Numeration Systems, (3) Sets and Set Relations, (4) Whole Numbers, (5) Fractions, (6) Decimals, (7) Relations and Functions, (8) Probability and Statistics, and (9) Mathematical Systems.

For each topic, mathematical competencies were specified; and to achieve these competencies, experiences utilizing manipulative and other instructional materials were prescribed. Each topic was covered in one week (eight class hours). At the end of each week the students, working in small groups, planned instructional designs to teach aspects of that topic. These designs were implemented in an elementary school in the succeeding week.

For each topic, two parallel forms of criterion-referenced tests were developed by this investigator to assess the students' performance on the prescribed mathematical competencies. The method by which these tests were constructed insured their content validity. Reliability estimates of the tests ranged from 0.77 to 0.93.

The following instruments were also selected for the collection of data: (1) Test of Basic Mathematical Understandings, (2) Dutton Arithmetic Attitude Inventory, and (3) Attitudes Toward Different Aspects of Mathematics developed by the International Study of Achievement in Mathematics.

The experimental group in this study were thirty-eight freshman elementary education majors who volunteered and were selected to participate in the experimental program. Evidence indicated that these volunteers did not differ from other freshman elementary education majors in their cognitive and affective behaviors toward mathematics.

Multivariate and univariate analysis of variance were used in assessing the effect of the integrated content-methods course upon students' performance on the criterion-referenced tests. The t-test for correlated means was used in testing changes in mathematical understandings and attitudes. Pearson Product Moment Correlation Coefficient was utilized in the relationship analysis reported in this study.

The level of significance was set at 0.05 for testing all hypotheses in this study.

Findings of the Study

1. The experimental group made significant gains ($p < .005$) in mean scores from pre- and post-tests during the integrated content-methods course on the criterion-referenced tests for all topics except Measurement.

2. The experimental group attained the mastery level (achieving at least 80 percent of the items correct on the post-test) on the criterion-referenced tests for all topics except Measurement and Mathematical Systems.

3. The experimental group showed significant improvement ($p < .001$) between pre- and post-test means on a test of basic mathematical understandings and on arithmetic attitude scale, while enrolled in the integrated content-methods course.

4. With initial differences allowed for, the experimental group, after completing only the first part of their mathematics education, showed significantly better understanding of basic mathematical concepts and more positive attitudes toward mathematics than a group of students in the regular teacher education program who had completed all the required mathematics education.

5. There were significant correlations between: (a) pre- and post-test scores on the criterion-referenced measures, (b) post-test scores on the test of mathematical understandings and the arithmetic attitude scale, (c) number of high school courses in mathematics and pre- and post-test scores on the test of mathematical understandings, (d) pre- and post-test scores on the mathematical understandings test and high school grade-point average.

6. The experimental group expressed desire for more participation in clinical experience concurrent with the laboratory oriented integrated content-methods courses.

Conclusions

The activity-oriented integrated content-methods course concurrent with clinical experience had a significant positive effect on prospective elementary teachers' cognitive and affective behaviors toward mathematics.

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CHAPTER I

INTRODUCTION

Our civilization is fast becoming more and more technological in nature. We only need look at the kinds of jobs that existed fifty, twenty, and even five years ago to see the extremely rapid change toward a technologically extremely complex environment which man is creating on this planet. Consequences of such a change are twofold: (1) less and less "unskilled" work will be required in the future as such jobs will be done increasingly by machines, and (2) a highly-trained manpower is needed to handle the new situation (95).

Since changes are becoming so rapid, it is clearly not practical to train personnel to handle clear-cut situations and specific problems, because the chances are that by the time such people have completed their education, their training will be out of date. What the manpower needs to be trained for is how to learn to adapt to new situations. This means a flexible approach in the training, an approach which is relatively new, since most of what goes by the word "education" up until quite recently has meant fact-learning and not learning to think.

This flexibility especially applies in mathematical training. Learning how to think mathematically, how to reason about abstract structures built into and around each other will soon be an imperative requirement of every citizen.

Background

In the last fifteen years, mathematics educators from all over the world have been increasingly concerned with the necessity of improving mathematics education in line with technological and sociological changes and in accord with viable research findings in the behavioral sciences. This concern has been evident especially at the elementary level, since learning and manipulation of abstract mathematical structures clearly begin at the elementary level.

In the United States, no less than 32 mathematics curriculum projects have been developed by mathematics educators and nationally-recognized advisory groups or organizations (20). Most of these projects stressed the need to change not only the content and approach to "modern mathematics" but also methods of teaching and teacher preparation in mathematics.

As a child's mathematics education begins at the elementary school, it follows that the teacher, in daily contact with children, forms an extremely important link in the chain of problems presented by the new necessity to spread mathematical education far wider than has been the case until now. However, available data suggest that the greatest bottleneck in obtaining sound mathematical education for children (and consequently for the future citizen) is the problem of educating the teachers who are to impart the revised mathematical curriculum to these children. One reason for the difficulty is the almost "total ignorance" on the part of the vast majority of elementary school teachers of what mathematics really is; another is that it seems almost impossible to introduce a new mathematical curriculum without considerably changing the conditions under which children learn. Having learned their mathematics in a mechanical way and often as a skill subject, today's elementary school teachers may have serious difficulties teaching mathematics meaningfully to children. Thus, meaningful teaching and meaningful learning are closely associated. Today's elementary school teachers must have a clear understanding of each new mathematical concept presented to children if they are to succeed in their teaching (46).

Conscious of these problems, the Committee on the Undergraduate Program in Mathematics (CUPM) published, in 1960, a list of recommendations for the mathematical preparation of elementary school teachers (8). Since then, nationwide conferences for mathematics educators from various educational institutions and background were conducted to discuss the problems inherent to teacher education and to explore possible avenues to improved mathematical preparatory programs.

At Michigan State University, an experimental program for the preparation of elementary school teachers was recently designed. The program is funded and staffed by the Michigan State Trainers of Teacher Trainers (TTT) Project, and is based on several aspects of the Behavioral Science Teacher Education Program (BSTEP) model developed at Michigan State University in 1968.

The TTT Program at Michigan State University

In the words of the specialists who conceived and developed the program:

> The basic purpose of the Michigan State University TTT (Trainers of Teacher Trainers) project is to bring about that type of institutional change at the University that has the greatest promise of re-designed teacher education programs that are far more relevant to the real world of local school and community than now is the case. . . (92:1).

The specific need to which the TTT project is addressed is the production of teachers who are competent and whose discipline knowledge and teaching behavior is more relevant to the real world of the school, the students who populate it, and the community which surrounds and supports it (92).

Among the major objectives of the TTT Project at Michigan State University are the following:

1. Involve the community, the school, and the university in the training of teachers and teacher trainers.
2. Include the best educational developments and focus on the most critical educational issues.
3. Institutionalize the improvements from TTT into the university and into the school.
4. Develop a competency-based teacher education program that incorporates several aspects of the Model Program BSTEP.

The Behavioral Science Teacher Education Program (BSTEP)

At Michigan State University, a comprehensive program for the preparation of elementary school teachers was designed in 1968. The program centers the professional foundations of the teacher education program model upon the behavioral sciences, that is "those inquiries . . . their methods and their findings . . . . which constitute reliable and valid sources of enlightenment about the human, his nature and his conditions" (22:A-3).

The program model emphasizes developmental clinical experiences which begin in a prospective teacher's freshman year and continue throughout his preservice education.

The program is organized into five major curricular areas and each area is divided into various components. Mathematics is one component of the five curricular areas. The five curricular areas as they relate to the mathematics component are:

1. General Liberal Education, whose objective is to relate to the general education acquired by the teacher-trainees a knowledge of the historical development of mathematics and of its place in our technological culture.
2. Scholarly Modes of Knowledge, whose purpose is to provide the teacher-trainees with the necessary background for teaching elementary school mathematics.
3. Professional Use of Knowledge, which provides the teacher-trainees with an opportunity to translate the mathematics learned in Scholarly Modes of Knowledge into instructional strategies for children. This area provides the teacher-trainee with an awareness of the instructional dimensions to be considered in planning for related clinical activities.

4. Human Learning, whose basic purpose is to introduce the teacher-trainees to the three basic behavioral areas brought into interaction in any planned educational experience; that is, exploring human capacity for learning, understanding environmental systems, and enquiring into the cognitive development.
5. Clinical Experience. The objective in this area is to develop and expand a prospective teacher's facility in employing the clinical behavior style into teaching mathematics. Pre-professional clinical procedures are analyzed and practiced through simulated and actual situations.

The experimental teacher education program as part of the TTT Project used many aspects of the BSTEP model. In 1971-1972, the major thrust of the TTT Project was directed toward the development of a new kind of elementary school teacher who is basically well-educated, engages in teaching as clinical practice, is an effective student of the capacities and environmental characteristics of human learning, and functions as a reasonable agent of social change (22).

Development of the first phase of the experimental program began Fall term 1971. Forty students chosen from among the fifty-two who manifested a desire to participate in the program, constituted the first group of freshmen involved in the new elementary teacher education curriculum.

Need for the Study

Viable teacher education programs to meet the challenge of the future must include three aspects: (1) objectives stated in performance terminology, (2) teaching-learning unit to implement these objectives, and (3) evaluative instruments to assess the extent to which the prospective elementary school teachers accomplish the program objectives. The latter two are contingent upon careful delineation of the first (22).

Conscious of these facts, the teams of scholars who worked closely together to integrate the program have repetitively stressed the need for constant evaluation and feedback into the program: "a carefully designed, extensive, and workable evaluation system which in turn supports program development" (22:II-37).

The need for such evaluation during the formative stage of the development of the experimental program was the basis for the present study.

Purpose of the Study

The general purpose of this investigation was the formative evaluation of the mathematics component of the experimental elementary teacher education program at Michigan State University.

More specifically, this investigation sought:

1. To analyze and evaluate those mathematical competencies specified by the program to assess whether they do in fact meet the basic mathematical need of the elementary teacher.

2. To evaluate the effect of the instruction as prescribed by the mathematics component of the experimental program on the students who participated in it, in relation to the specified competencies; and to assess if the students have achieved a degree of mastery over these competencies.

3. To evaluate the basic mathematical understanding of the students who participated in the program prior to and after the completion of instruction, in order to assess the effectiveness of the prescribed mathematics treatment on their general mathematical knowledge.

4. To assess the effect of the experimental program on the attitudes toward mathematics of the students who participated in the program.

5. To determine the relationship between selected variables and achievement in mathematics.

6. To compare the mathematical understanding and attitudes toward mathematics of prospective elementary school teachers participating in the experimental program with the mathematical understanding and attitudes of students enrolled in the regular teacher-education program.

7. To use the results of the investigation to make specific recommendations that may help rectify any weaknesses in the program.

Formative Evaluation and Curriculum Improvement

Formative evaluation involves the collection of appropriate evidence during the construction and trying out of a new curriculum in such a way that the revision of the curriculum can be based on this evidence. In all educational evaluation as in legal trials, the merits of the case rest primarily on the kind and quality of the evidence presented. Because formative evaluation of any new curriculum focuses on the statements of objectives of the program, the evidence gathered on these objectives needs to be valid. Unfortunately, most published or standardized tests do not meet this criterion, since they are designed to facilitate comparison among individuals, rather than assessing their attainment of specified curriculum objectives (4). Thus, development of valid tests that measure the specific objectives of the curriculum being evaluated is a major part of any formative evaluation. In this study, a set of criterion-referenced measures are developed to evaluate the students' achievement on the mathematical competencies prescribed in the program. These measures are based on the mathematical objectives specified in the topics included in the mathematics curriculum of the experimental program.

Hypotheses

The following two hypotheses will be tested to assess the effect of the experimental program on the achievement of the experimental group on the prescribed mathematical competencies:

A1. There will be a significant difference between the post-test means and the pre-test means of the experimental group on the criterion-referenced measures.

The univariate hypotheses associated with this multivariate hypothesis are:

The mean post-test score of the experimental group will be significantly higher than the mean pre-test score on the criterion-referenced measures in:

a. Measurement
b. Numeration
c. Sets and Set Relations
d. Whole Numbers
e. Fractions
f. Decimals
g. Relations and Functions
h. Probability and Statistics
i. Mathematical Systems.

A2. There will be no significant difference between the post-test means and the mastery level (80 percent) on the criterion-referenced measures.

The univariate hypotheses associated with this multivariate hypothesis are:

The mean post-test score of the experimental group will be at least equal to the mastery level (80 percent) on the criterion-referenced measures in:

a. Measurement
b. Numeration
c. Sets and Set Relations
d. Whole Numbers
e. Fractions
f. Decimals
g. Relations and Functions
h. Probability and Statistics
i. Mathematical Systems.

The following two hypotheses will be tested to assess changes in basic mathematical understanding and attitudes toward arithmetic in the experimental group:

B1. There will be a significant difference on a test of basic mathematical understanding between the post-test scores of the experimental group and their pre-test scores.

B2. There will be a significant difference on an arithmetic attitude inventory between the post-test scores of the experimental group and their pre-test scores.

The following two hypotheses will be tested to compare the experimental group and students enrolled in the regular teacher education program on basic mathematical understanding and attitudes toward arithmetic.

C1. The adjusted mean post-test scores of the experimental group will be at least equal to the adjusted mean post-test scores of a group of prospective elementary teachers enrolled in the regular teacher education program on a test of basic mathematical understanding.

C2. There will be a significant difference in an arithmetic attitude inventory between the adjusted post-test scores of the experimental group and the adjusted post-test scores of a group of prospective elementary teachers enrolled in the regular teacher education program.

Assumptions

The mathematics component of the experimental program is based on these assumptions:

1. That the needs of preservice teacher in elementary education are better served by professionalized subject-matter courses that blend both content and method.

2. That the content-method courses should be taught in a mathematical laboratory setting where well-planned activities utilizing manipulative materials will better facilitate the learning of mathematics and the learning of how to teach mathematics.
3. That the preservice teacher should study the theories of teaching and learning concurrently with laboratory and clinical experience and thus relate theory to practice.
4. That this combined study and experience should begin as early as the student's freshman year and continue throughout his education.
5. That the study and experience should integrate what the prospective teacher has learned about mathematics with what he has learned about humanistic and behavioral sciences.
6. That good mathematics teaching behaviors by prospective teachers is fostered by good mathematics instruction, and that teachers tend to teach as they are taught.

Limitations of the Study

The major limitations of this study were the following:

1. While there are several goals that pertain to the formative evaluation of an educational program, this

study evaluated only the mathematics component of the experimental teacher education program.

2. Evaluation of the program was confined solely to those prospective elementary teachers who volunteered and were selected to participate in the first year trial implementation of the program.
3. The study did not attempt to evaluate the effect of the integrated content-method course on the teaching behavior of its recipients in elementary school setting.
4. The study did not attempt to evaluate the effect of the experimental program on the mathematical competency of the school children who were taught by the experimental program participants.
5. The extent to which the evaluative instruments adequately measured the effects of the integrated content-method course and the clinical experience was also a limitation. The instruments used in this study had the inherent limitations of paper-and-pencil tests.[1]

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[1]As pointed out by Glennon (53:395), far superior to the paper-and-pencil test would be: "the study of the behaviors of each person individually through conversing with him and keeping anecdotal records of his performance on the test items."

Definition of Terms

This section provides a definition of the major terms used in this study.

Attitude: "A learned predisposition or tendency on the part of an individual to respond positively or negatively to some object, situation, or another person" (36:551).

Competency-Based Teacher Education Program: "Any training program that requires its trainees to demonstrate at a specified level of competence behaviors that have been explicitly described and prescribed as desirable and effective professional behaviors" (97:4).

Criterion-Referenced Measure: "One that is deliberately constructed to yield measurements that are directly interpretable in terms of specified performance standards" (13:43).

Curriculum Evaluation: "Collection, processing and interpretation of data pertaining to an educational program" (34:1).

Experimental Group (The): A group of thirty-eight freshmen elementary education majors who volunteered for the experimental program and participated in the first year trial implementation of the mathematics component of the program.

Formative Evaluation: The use of systematic evaluation in the process of curriculum construction, teaching, and learning for the purpose of improving any of these processes (4:117).

Learning Unit: The mathematical capabilities to be acquired under a single set of learning conditions.

Mathematics Achievement: The level of competency of the student in regard to the specified instructional objectives of the mathematics curriculum.

Mathematics Content: Description of the expected competencies of the student in mathematics activities.

Mathematics Curriculum: A set of prescribed mathematical competencies and the instructional designs to achieve these competencies.

Mathematical Understanding: The level of competency of the student in regard to the general mathematical knowledge needed for elementary school teaching.

CHAPTER II

REVIEW OF LITERATURE

The review of literature pertinent to this study has been organized under six categories: (1) curriculum evaluation, (2) rationale for the improvement of teacher education programs, (3) designs for programs of teacher education, (4) approaches to teacher training, (5) content of mathematics curriculum, and (6) research on attitudes toward mathematics.

Curriculum Evaluation

Baker (41:339) pointed out that, although it is possible to distinguish between curriculum evaluation and curriculum research, "it is not very useful to base the distinction on a review of literature dealing with curriculum research and curriculum evaluation . . . the terms used by authors in their titles and the operational definitions assigned to the terms in the articles is nearly impossible."

Walbesser and Carter (71) recognized the importance of defining curriculum by developing a sequenced set of instructional objectives. They experienced problems with discrepancy between the statement of the objectives or the proposed assessment and the perceived meaning and intention of the curriculum developers. They recommended pre-evaluation strategies that might help increase the efficiency and contribution of behavioral objectives to curriculum development and evaluation.

Atkin (40), on the other hand, warned that the rigid adherence to specifying the behavioral outcomes of all instructional activities tends to decrease their educational relevance and eliminates many other worthwhile experiences from the curriculum.

Defining Curriculum Evaluation

Evaluation is the gathering of information for the purpose of making decisions. Evaluation differs from basic research in its orientation to a specific program rather than to variables common to many programs. Evaluation is concerned with questions of utilities that involve value and judgment.

Curriculum evaluation requires the collection, processing and interpretation of data pertaining to an educational program. It serves two important functions: (1) it provides a means of obtaining information that can be used to improve a course, and (2) it provides a basis for decisions about curriculum adoption and effective use. The first function is generally called formative evaluation while the second is referred to as summative evaluation (34).

As defined by Scriven (31), formative evaluation involves the collection of appropriate evidence during the construction and trying out of a new curriculum in such a way that revisions of the curriculum can be based on this evidence. Bloom et al. (4:117) regard formative evaluation as useful not only for curriculum construction but also for instruction and student learning. They define this type of evaluation as: "The use of systematic evaluation in the process of curriculum construction, teaching and learning for the purpose of improving any of the three processes."

Dyer (50) maintains that formative evaluation is very important in the development of a teacher education program. It is not experimental in any formal sense; it cannot tell you much about the ultimate pay-off; and it is, in fact, purely descriptive. But it is absolutely vital as a means of finding out in detail how the new material is working, what kind of material is working for what kind of student, and what changes are needed to make it better. As the educator tries to fashion the individual components of a new course, he needs to know, as he goes along, how students are reacting to the new materials, "what is connecting and what is not" (50).

In formative evaluation, Dyer continues:

> you do not worry about experimental designs, control groups, and test of statistical significance; you do worry about the adequacy of

> the week-to-week and month-to-month feedback, so that when you bone the course down into its final shape, you will have some assurance that it will do the job you intend it to do (50:24).

Formative evaluation techniques are employed when one is interested in reviewing the curriculum while it is still in its developmental stages. This implies that evaluation activities must take place in predetermined stages in the development of the curriculum and that strategies must be included in the activities to permit changes to be made on the basis of a reliable and valid criterion reference evidence. It is also suggested that a trial-revision cycle based on preassigned standards be utilized (50).

According to Bloom et al. (4) formative evaluation requires a more microscopic and diagnostic analysis of the content. The information obtained from this type of evaluation is used as a feedback into the system in order to determine subsequent activities for the learner.

Commenting on formative evaluation in their _Taxonomy of Educational Objectives_ (3), Bloom et al. indicated that the taxonomy does not attempt to classify the instructional methods used by the teacher, or the ways in which teachers relate themselves to students, or the different kinds of instructional materials they use; in fact, the taxonomy attempts to classify the intended behavior of the learner, the way in which the individuals mentally act or think as a result of participating in some unit of instruction. It is possible that the actual behavior of the learners after they have completed the unit of instruction may differ in degree as well as in kind from the intended behavior specified by the objectives.

In recent years, many publicly funded mathematics curriculum groups or projects concentrated on developing curriculum materials for the elementary schools and teacher preparation. However, there was little provision in most of these projects for systematic evaluation either during development (formative stage) or of the final product (summative). Lockard (20) surveyed sixty-eight Science and Mathematics projects and found only nineteen (28 percent) which possessed research evidence of success in achieving stated objectives.

Curriculum evaluation is difficult to accomplish because both the unique and common features of various curricula must be evaluated both objectively (by measuring outcomes of instruction) and subjectively (by personal judgment) as to the quality and appropriateness of goals.

Cronbach (45) is more inclined to favor the utilization of both formative and summative, non-comparative evaluation. He feels that in an experiment where treatments differ in many respects, no valid conclusion can be drawn from the fact that the experiment shows a numerical advantage in favor of a new method. Guba (96) presented an overview of the curriculum evaluation problems and pointed out many of the deficiencies that currently exist in the field of evaluation. He suggested some taxonomy of evaluation designs. The sixty-eighth Yearbook of the National Society for the Study of Education (23) provides an excellent discussion of the changes that have been taking place and the many needed changes in curriculum evaluation.

Measurement of Achievement

Concerned by the importance of improving the measurement of achievement, a score of researchers have written extensively on the distinction between two kinds of measures that are used to assess subject matter proficiency; that is, to determine the characteristics of student performance with respect to specified standards. One is the relative ordering of individuals with respect to their test performance. Glaser (13) calls it the norm-referenced measure. It is defined by Popham and Husek (28:20) as: "those [measures] which are used to ascertain an individual's performance in relationship to the performance of other individuals on the same measuring device." According to that definition, most standardized tests of achievement or intellectual ability can be classified as norm-referenced measures.

The other type of measurement, called the criterion-referenced measure, depends upon an absolute standard of quality and it is used "to ascertain an individual's status with respect to some criterion, i.e., performance standard" (28:20). With this type of measurement, the individual is not compared with other individuals and his score is not dependent on those of other students in the class. Criterion-referenced measurement "provides information as to the degree of competence attained by a particular student" (28:8) along a "continuum of knowledge acquisition ranging from no proficiency at all to perfect performance" (28:7).

It is not always easy to make the distinction between the two types of measurement. Glaser (13:43) writes that "the distinction is found by examining (a) the purpose for which the test was constructed, (b) the manner in which it was constructed, (c) the specificity of the information yielded about the domain of instructionally relevant tasks, (d) the generalizability of test performance information to the domain, and (e) the use to be made of the obtained test information."

Criterion-referenced measurement is relatively new in education and has seldom been used to assess student's performance. But development of instructional technology and the recent emphasis on curriculum research and curriculum evaluation have stressed the need for the kind of information made available by the use of criterion-referenced measures.

With criterion-referenced tests, it is possible to make two types of decisions: (1) decisions about individuals, and (2) decisions about treatments, e.g., instructional programs. As to decisions regarding individuals, such tests help determine whether a student had mastered a criterion skill necessary to continue his program. This means that performance standards must be established prior to test construction and that the purpose of testing is to assess an individual's status with respect to these standards. In the case of decisions about treatments, the criterion-referenced measure designed to reflect a set of instructional objectives and administered to the students after they have completed a specified instructional sequence will provide the information necessary to reach a decision regarding the efficacy of the treatment.

Most educators have used norm-referenced measures to make decisions about individuals and instructional systems. But, as mentioned by the proponents of the criterion-referenced measurement, "norm-referenced measures were really designed to spread people out and . . . are best suited to that purpose" (28:22). On the other hand, criterion-referenced tests are "specifically constructed to support generalizations about an individual's performance relative to a specified domain of tasks" (13:42). They should be used, stresses Gavin (11:62) "to control entry to successive units in any instructional sequence where the content is inherently cumulative and the rigor progressively greater."

The use of criterion-referenced measurement has many implications for test construction and evaluation. Popham and Husek (28:17) have mentioned the following:

1. Variability is not a necessary condition for a good criterion-referenced test. Variability is irrelevant as the meaning of the score is only related to the connection between the items and the criterion and is not dependent on comparison with other scores.

2. Reliability is desirable but the authors point out that it is not obvious how to assess internal consistency since the classical procedures are dependent on score variability. According to the authors, it is sometimes possible for a criterion-referenced test to have a negative internal consistency index and still be a good test. What is needed are new indices and estimates appropriate to measure internal consistency of criterion-referenced tests.

3. Validity is also assessed by procedures based on correlations and thus on variability. Hence, the results of the procedures are useful if they are positive but should not be considered devastating if they are negative. In general, validity must depend upon the correspondence of the test items with the objectives to which the test is

referenced. Thus, the test items must be constructed for, or matched to, goals of instruction.

4. Item construction for criterion-referenced measurement requires that the items included in the test are an accurate reflection of the criterion behavior. When used to make decisions about individuals, criterion-referenced tests must be the same or an equivalent form for all students. On the other hand, if criterion-referenced tests are used to evaluate programs (treatments), many tests, each containing different criterion-referenced items, could be constructed.

5. Item analysis for criterion-referenced measurement is different than item analysis for norm-referenced measurement. In the case of criterion-referenced tests, more concern is given to identifying negative discriminators than non-discriminators. Consequently, new procedures for item analysis should be found. Cox and Vargas (7:71) tested two methods of analysis for the evaluation of items on tests administered both as pre- and post-tests. One index was computed using the common upper minus lower groups technique while the second index was computed by subtracting the percentage of pupils who passed the items on the pre-test from the percentage who passed the item on the post-test. In the first case, the index provided information on how well each item discriminated between

the groups, while in the second case the index provided discrimination information between pre- and post-test groups, indicating items useful for pre-test diagnosis. The authors of the study found that the pre- and post-test method of item analysis should be considered in criterion-referenced measurement, where score variability is not the concern.

6. Reporting and interpretation of an individual's performance on a criterion-referenced test does not require the same group-relative descriptors as when percentile ranking or standard scores are used to interpret results in norm-referenced tests. Scores obtained are essentially "on-off" in nature and we are interested in knowing if the individual has mastered the criterion or not. It is also possible to report the degree of the student's performance and to determine how far away he is from the criterion. Such reports on the degree of "less-than-criterion" performance exclusively depend on the use made of the data. If criterion-referenced measures are used for the evaluation of treatments, different procedures are possible: (1) report the number of individuals who achieved the pre-established criterion, (2) use traditional descriptive statistics such as "percentage correct," means and standard deviations, (3) if a criterion level has been set, report the proportion of the group which reached that level and

report the proportion and degree of the "better-than-criterion" performances. Popham and Husek (28) believe that the best course of action would be to use as many reporting procedures as possible in order to permit a better interpretation of the test results.

Rationale for the Improvement of Teacher Education Programs at the Elementary Level

For some years, continuing movement toward reform of mathematics curricula at the secondary level has been evident in the United States and in many countries of the world. Such changes have stressed the importance of the primary stage in a child's learning. Reform is now directed to the modification of basic teaching in the primary school in order to prepare the child adequately for the new patterns of further education. These changes in the content of curricula have been accompanied by experiments in the development of new teaching methods and in a new conception of preservice and in-service education for prospective elementary school teachers.

As the demands of an increasingly complex society for quality education continue to mount, demands upon the teacher accelerate wildly and there is general agreement from all segments of society that teachers and their education are the principal object behind any effort made anywhere for the ultimate improvement of educational systems.

According to Brousseau (42:265): "teachers must be better prepared for two reasons: (1) they are faced with an enlightened generation of children who, due to the impact of television, have accumulated much knowledge upon entrance into today's elementary schools, and (2) they are not adequately prepared to exploit this accumulated reservoir of knowledge."

The requirement to retrain elementary school teachers in mathematics presents many problems. A large proportion of elementary school teachers have had little or no mathematical training (21:108) and they have not studied mathematics beyond the age of 14. Some have had secondary training through rote-learning of solutions to certain kinds of problems and mathematicians and/or mathematics educators agree that such training does not result in a true knowledge of mathematics. Others have been exposed to mathematics carried out in a highly logical manner. These teachers may understand the structural properties of mathematics but they will have to learn how to use the inductive method in the teaching of mathematics to young children.

Considering such problems, what are the criteria for successful teacher education? How much mathematics should an elementary school teacher know? How much psychological knowledge of the development of children, and of the methods of thinking should he be familiar with?

Specialists in the education of mathematics teachers for the elementary school (6, 16, 21, 46, 59, 64) agree that the following characteristics are basic to a successful teacher education program:

1. Teachers should have a clearer picture of the objectives for a contemporary mathematics program.
2. They must have a high degree of mastery of the mathematics they need to teach.
3. They must also have a thorough mastery of the mathematics they need to give perspective to their teaching.
4. They must be knowledgeable about teaching steps and instructional materials involved.
5. They must be competent in developing in the elementary school classroom mathematical concepts learned on a sophisticated level.
6. They must know enough about how children learn mathematics to select and design appropriate activities for children.

Addressing the National Council of Teachers of Mathematics at a forum on Teacher Education, James Gray (16:8) spoke of the challenge faced by teacher educators to "provide the new teacher with as much of a science of methodology as it can. They (teacher educators) are challenged to attempt to get the future mathematics teacher as an artist, as one who has an independent confidence in his understanding of mathematics and of his students so that he can best arrange what is done in the classroom to the achievement of objectives." He added:

> It is only in the security of high competence that a teacher feels sufficiently free to himself proportion this mathematics to the need of his students without feeling bound by the particular packaging of the textbook, his teachers' note or other prepared materials which do not exactly fit the particular situation of his students. In the security of truly understanding what he is about, the teacher can consider his situation, his students' need and his mathematical objectives and go about his own strategies for the accomplishments of these objectives (16:4).

For Leblanc (59:606), the basic characteristic of the newly-trained elementary school teacher will be "his ability and competence in making wise decisions concerning programs and organizations in teaching mathematics." This implies not only the different competencies mentioned earlier, but also knowledge in behavioral terms of the goals of a contemporary mathematics program and the sequence in which they seem easiest and most appropriate to be attained. In addition, knowledge about tests and measurement in mathematics will help the teacher design evaluations of mathematical learning, an important component of educational programs.

The International Study Group for Mathematics Learning has formulated some questions that will help determine some criteria for the training of prospective teachers of mathematics at the elementary level (21:114-115).

1. If the content to be included in elementary school mathematics is the one suggested by the Cambridge Conference of 1963, then the questions must be asked, How well the teacher is conversant with these topics? and, Can he follow and be quite at home with such courses as the UICSM Course I or Suppes' Logic for Elementary Schools?
2. Another question would be: How well is the teacher prepared to help develop creative mathematical enquiry in his students? According to the authors, teachers "should have at their fingertips many avenues through the encouragement of which children can be shown at quite an early stage that mathematics is an inductive and creative enterprise" (21:115).
3. It is also relevant to ask: How well is the teacher prepared to obtain insight into how children learn? A good knowledge of the literature of educators, child psychologists and research mathematicians is a must to acquire comprehension on insight into how children learn.
4. Finally, the question should be asked: To what extent are teachers able to understand the purposes and methods of mathematical education and to what

extent can they explain these purposes and methods to other people: their own pupils, other teachers, parents, and administrators? This suggests an awareness of the kinds of applications that are being made of mathematics in the present society today.

Teacher educators have also been concerned with the negative attitudes toward mathematics of many prospective elementary teachers and they have repeatedly stressed the importance of developing a mathematics curriculum for teacher education that would get people to attend to mathematics, to respond to it, to value it, to enjoy and participate in their mathematical education experiences instead of merely submitting to or fighting them. Most teacher educators believe that interest in the objectives of the affective domain must parallel interest in the objectives of the cognitive domain.

Designs for Programs of Teacher Education

Many models and programs based on some or all the criteria mentioned in the previous section have been proposed for the education of elementary school teachers in general and the education of elementary school teachers in mathematics in particular. Some of the latter are familiar and have been functioning for a few years now: the SMSG, the UICSM, etc. Late in the 1960's, the United States Office of Education proposed to fund "models for teacher education" that would incorporate the most recent findings on the subject. Ten models were chosen to be funded, among them the Michigan State Model proposed by Michigan State University (the BSTEP).

S. C. T. Clarke (6) has listed a number of factors to be considered in the preparation of a teacher education program and he has evaluated the models funded by the United States Office of Education in terms of the way they dealt with these factors. Clarke states that designs for teacher education must deal with teaching in terms of presage, process, and product factors.

Presage Factors

Presage factors represent decisions made prior to the development of a program and which shape the direction of the program. They include: context, cybernation, extent of lead in terms of time, and boundaries in terms of integrating general education, subject matter, and related disciplines.

Decisions about the context for which teachers are being prepared must be made in advance of planning a program of teacher education. Most of the model teacher education programs developed in the last half of the 1960's are based upon predictions of what society and education will be like in 1980. The extent of lead varies, however.

Most of the programs also contain cybernetic self-correcting devices for periodical examination and updating of the program as well as the preparation of self-renewing teachers capable of shaping the changes that "seem certain in the future world of education."

In the models evaluated by Clarke, there seems to be a general tendency for the representatives of the institutions to determine context, cybernation, and lead, while at the same time professing the need for the involvement of many organizations, institutions, and agencies.

There is agreement that general education, subject matter to be taught to pupils, and command of related disciplines, are within the boundaries of teacher education. A Teacher Education Program can be viewed as the professional preparation of teachers, while the subject matter and general education portions are regarded as "givens." According to the Standards Evaluative Criteria of Teacher Education (6:124):

> [General education] should include the studies most widely generalizable to life and further learning. . . . The general studies component for prospective teachers requires that from one-third to one-half time be devoted to studies in the symbolics of information, basic physical and behavioral sciences, and humanities.

As to the professional component, it covers all requirements that are justified by the work of the specific vocation of teaching. B. O. Smith says (6:124):

> the subject matter preparation of the teacher should consist of two interrelated parts: first, command of the content of the disciplines constituting his teaching field and of the subject matter to be taught; and second, command of knowledge about knowledge.

The Michigan State Model is an outstanding example of how to deal with the important matter of boundaries. The five major areas of this program are: general liberal education, scholarly modes of knowledge, professional use of knowledge, human learning, and clinical studies. The program was developed by an interdisciplinary team and its continued direction was to be representative of the various interests. According to Smith, no other model has such a complete treatment of relationship with general education and academic disciplines.

Process Factors

The process factors include dimensions, extent of individualization, graduated conceptualization-practice, support systems, and task-centered curriculum.

In the model programs there is a movement away from the traditional dimensions (time, credits, courses) toward performance modules, that is, teaching tasks which can be mastered in few or more hours of instruction-practice, and whose end product is teaching behavior. According to Clarke, the Michigan State Model is the most completely developed example among new models of teacher education programs and includes over 2,700 modules. The standard format for these modules includes objectives, prerequisites, experience, setting, materials, level, hours, and evaluation.

The major trend in teacher education as exemplified by the models is individualization even if most programs recognized the institutional barriers to individualized programs, such as time required for a degree and course or credit requirements. The models proposed have stressed graduated exercises leading up to practice teaching such as simulation, analysis of teaching, tutoring, and microteaching. The Michigan State Model places considerable emphasis on the development of clinical behavior with the sequence starting during the first two years with tutorial experiences with children, continuing with a career decision seminar, analytical study of teaching using simulation and microteaching, through team teaching, internship, and teacher specialization.

The models vary greatly in their treatment of management systems. They all face the problems of recording and student accounting created by the process factors and by the multiple entrances and exits provided for selection.

In the models funded by the United States Office of Education, there is an emphasis on task analysis, task specification, required behaviors, treatments designed to develop these behaviors, and assessment of results in terms of the original task analysis. This program of experiences designed for teacher candidates stresses the idea that performance criteria formulation is the base of most models. The Michigan State Model starts from the clinical behavior style of teachers, including: (1) the reflecting phase: (a) describing, (b) analyzing; (2) the proposing phase: (a) hypothesizing, (b) prescribing; and (3) the doing phase: (a) treating, (b) seeking evidence on consequences.

Product Factors

The product factors refer to the features incorporated in a teacher education program to evaluate the program and the teacher behaviors produced. Most of the models studied have stressed the need for evaluation but, according to Smith, "designs to evaluate teacher behaviors were not, on the whole, well-developed, with the exception of one education program (the Michigan State Model)" (6:153).

Myths About Teacher Effectiveness

The question of teacher effectiveness, the problem of measuring it, and the problem of predicting it are extremely important. In any educational system a vast number of decisions are made which require some knowledge about teacher effectiveness. Among them, decisions about changes in the curriculum should be based, in part, on information about the effectiveness of the teachers who will be called on to implement the changes.

Because of the importance of this matter the SMSG (93), during the course of a rather extensive five year longitudinal study of mathematics achievement which started in the fall of 1962, gathered a considerable amount of information about a large number of teachers and completed an analysis of some of these data to find more about teacher effectiveness based on student achievement. Results showed significant, and in most cases, large variations in teacher effectiveness but the variation did not seem to be correlated with any of the extensive information about teachers: age, sex, teaching experience, amount of training beyond that minimally required for the job, recent inservice training, attitudes toward mathematics, attitudes toward teaching, attitudes toward students. In all cases, regression analysis showed that this amount of information about the teachers did not account for more than a small fraction of the variance in the teacher effectiveness scores, in most cases less than 10 percent.

The belief that mathematical ability, like intelligence, is not shared equally among individuals still influences most mathematics curriculum despite the fact that it has been challenged by many research mathematicians. The SMSG conducted an experiment to test Carroll's hypothesis[1] that all students could be brought to the same level of achievement in any particular scholastic topic, but the amount of instruction needed to bring a student to a particular level of achievement would vary from student to student. Results of the study provided evidence in favor of this hypothesis (108).

More recently, some researchers have taken the position that it is the teaching, not the teacher, that is the key to the learning of students. That is, not what teachers _like_ but what they _do_ in interacting with their students (36, 38).

Approaches to Teacher Training

Many methods may be used for the implementation of a successful teacher education program. Mathematics departments in a number of colleges and universities have

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[1]John Carroll, _A Model for School Learning_, Vol. 64, Teachers College Record, 1963, pp. 725-733.

instituted special courses in mathematics for elementary school teachers; a number of colleges and universities offer a subject-matter major for elementary school teachers in preparation; and several schools have experimented with laboratory courses in teaching for undergraduates.

From its beginning to the present, teacher education for elementary school mathematics has progressed from a rather formalized approach to attention to more professional problems through demonstrations, field work, projects, readings, laboratory work, and participation in elementary school mathematics classes. But, according to Mueller (62:434), "strong indication as to which type of course is best is still lacking: separate methods and content courses, combined content-methods course, CAI course, remedial course, course with or without discussions."

Leblanc (59) believes that prospective mathematics teachers need both a good content course and a well-structured methods course in mathematics. He points out that the content taught should be in closer alliance with the content that teachers will be teaching children and that the course needs to be carefully fashioned in terms of performance objectives. As to the methods course, it should prepare the teacher to:

a) be able to list some sequence of learning expressed in performance objectives;
b) be able to identify some "need-to-know" concepts and skills, or "need-to-know" objectives as opposed to "nice-to-know";

c) be able to use mathematical laboratories or learning-resource centers and know the reasons for mathematics resource centers (59:607).

While considering separate courses (one on content and one on teaching) an appropriate means for achieving the goals set up for teacher preparation, Phillips (64) lists three advantages to the combined methods-content course over two separate courses: (1) the prospective teacher learns mathematical concepts on the abstract level and obtains in addition a functional knowledge, (2) the prospective teacher learns the teaching steps, and (3) the course offers efficiency in learning.

Little research is available as to the extent of the effect of the combined content-methods course on the cognitive and affective behaviors of the learner. Phillips conducted a research study at the University of Illinois (Urbana) in 1964-1965 with 73 prospective elementary school teachers enrolled in the first required mathematics course. Students were placed in three groups: two groups were taught a mathematics content course while a combined-content teaching approach was used with the third group. Students were tested and compared on operational skill in arithmetic and algebra, meaning and understanding in arithmetic, and vocabulary knowledge. Results indicated that students enrolled in the combined content-teaching course had higher mean score on all three tests than students in the other two sections. Phillips related his findings to those of another study at the University of Illinois which showed that 94 percent of forty-nine experienced teachers completing a combined content-teaching course in elementary mathematics by correspondence favored the combined content-teaching over the separate-course approach (64).

A research study by Max Bell and associates (103) on an activity-oriented mathematics content-methods course for preservice teachers was conducted during 1972. Preliminary results indicated the following:

1. learning with manipulative materials increases desire to use, increases ability to use, and changes teacher's behavior with respect to the use of manipulative materials for the teaching of mathematical concepts;
2. learning with manipulative materials increases the desire to teach and orient actual teaching behavior in a learner-focused way;
3. the activity-oriented combined course had a significant positive effect on the attitude of the preservice trainee toward teaching mathematics;
4. the preservice trainees showed substantial gains in understanding elementary mathematics. They also showed a sizeable increase in their mathematical self-image: after the course, they believed they could learn mathematics.

Analysis of the interview data gathered by Bell et al. indicated that utilization of manipulative materials in the combined course acts as "enablers" for trainees learning mathematics. Manipulative materials enable the trainee to "play around" with the concepts, to actually see the process that is entailed. They enable the trainee to discover a concept for himself. For some trainees, the insight received through the use of physical materials differs in kind from the one received in abstract, verbal teaching. Insight from physical materials is a more powerful, believable kind of insight for these trainees.

Past criticisms of methods courses both by teachers and non-teachers centered around two points of view: (1) they are too theoretical and unrealistic, and (2) they are too superficial and insulting to one's intelligence. However valid these criticisms may be, Zahovic (72) maintains that the real problem lies in the fact that "the topics often stressed by methods courses are not teaching but rather the planning and preparation that take place prior to teaching; in short, curriculum concerns. . . . They are largely void of matters dealing with teacher behavior in the interactive classroom situation where teachers confront learners in an effort to effect learner behavioral change" (72:198). According to this author, prospective teachers enrolled in a methods course should learn:

- what kinds of questions to ask learners;
- what kinds of directions to provide;
- what clues or promptings to give;
- how much structuring to do for the learner;
- what kinds of feedback to provide;
- how to lift levels of thought;
- how to extend and use learners' ideas;
- what kinds of praise to use;
- how to terminate discussion of a topic and make transition to a new one;
- how and when to employ convergent memory, classifying, associate, or affirmative-denying questions.

The methods course "should deal not only with what behaviors or acts teachers should employ in the interactive situation but also with the timing of a particular act and the sequence or pattern in which it should be used in order to provide maximum service to learners" (72:198). Zahovic concludes:

> The need for the inclusion of these aspects of teaching in methods course is clear. Teaching methods courses must indeed become teaching methods courses and not just curriculum courses (what they are now).

One way of improving the methods course is suggested by Kalik (57) who argues that the traditional one-semester methods course without an accompanying classroom experience is deficient in terms of time and reality to prepare the student adequately for the myriad problems he will face as a beginning teacher. Ideally, in the author's view, "a team of methods and foundations instructors should be assigned to a group of twenty to twenty-five students, with whom they should work within a nearby public school setting over a three-year period" (57:262).

This notion of a fully field-centered program where the prospective teacher in his training is placed early in the position of having to answer the whys of his students is shared by many teacher educators. Travers (69) expresses a similar point of view by saying that "an important end of student teaching is to assist the student teacher to become a student of his own teaching" (69:374). For so doing, he sees the need for a: "strong clinical component in the professional education of teachers, that is that element of training which is problem-centered and gives training in finding solutions within the context of actual situations" (69:375).

The philosophy of Zoltan Dienes on the subject has been very influential in shaping the modern programs of mathematics for prospective elementary school teachers. He wrote:

> A teacher will teach as he was taught himself. . . . If he was taught in school and even in teacher's college through lectures, he will tend to lecture to the children. In other words, he will tend to explain rather than set up situations through which the children can be led to understand.

> If we wish teachers to be able to set up concrete problem situations that the children can manipulate, then they must also learn to set up such concrete situations for themselves and to manipulate them themselves. They must feel in their own skins what it is like to start from scratch and learn something.
>
> Many demonstration classes with children (should be included in teacher education program) and, following those, many situations in which the teachers themselves can begin to handle groups of children.
>
> Any principles arrived at should be learned by the trainees themselves as the result of their experience with materials and with children and among themselves. Demonstration sessions or workshop laboratory sessions should be followed by seminar-type discussions between the teacher educator and the teacher trainees (46:268).

This notion negates the usual lecturing, reciting, and testing found in many formal university courses and replaces it with an active involvement (discussing, doing, sharing, evaluating, and redoing) as a method of learning.

Although the teacher has a somewhat different role in a laboratory setting from that in a more traditional classroom situation, he is still the key to a successful program. He must select or devise worthwhile activities that will be appropriate for his class. During a laboratory period, he acts as a guide or counselor. After the activity, he must evaluate and record pupil progress.

Laboratory activities may be used in three ways: separated from, integrated into, and correlated with the regular instructional program. Experimental studies of mathematics laboratory have been done by a few investigators.

Vance (87) evaluated a mathematics laboratory separated from the instructional program and he found that student reaction was more favorable to the laboratory setting than to class setting. Wilkinson (100) found that the mathematics laboratory integrated into the regular program appeared to be more effective with students of middle and low intelligence. Wasylyk (99) compared students taught in the traditional manner with students taught by the integrated approach (mathematics laboratory integrated into the regular program) and he indicated that the achievement of students in the laboratory groups was significantly higher than the achievement of the control group taught in a teacher-directed setting. On the other hand, Johnson (80) found that performance of students taught exclusively by the activity approach was inferior to that of students receiving textbook-based or activity-enriched instruction. In their analysis of research done on mathematics laboratories, Vance and Kieren (70) concluded that: (1) students can learn mathematical ideas from laboratory settings, (2) the approach is particularly effective for low-ability students, (3) there is limited evidence of attitude change, and (4) in maximizing achievement on cognitive variables, other meaningful instruction appears to work as well if not better.

Summarizing the theoretical arguments for use of manipulative activities and play-like activities in mathematics learning, Kieren says (58:231):

> They [manipulative activities] have a fundamental position in the sequence of expanded learning activities both on a macro- and micro-instructional basis; they can provide an information-seeking, non-authoritarian environment; they should best include a variety of concrete referents for a concept; they can contribute a readiness foundation for latter ideas.

Mathematical Competencies of Elementary School Teachers

What are the mathematical competencies needed by elementary school teachers? In 1961, the Committee on the Undergraduate Program in Mathematics (CUPM) made specific recommendations for the mathematics preparation of elementary school teachers. In 1966, the Cambridge Conference on Teacher Training advanced bold recommendations for the mathematics content to be included in elementary teacher education programs, these recommendations exceeding by far those made previously by the CUPM. Since then, other guidelines have been suggested and research conducted to evaluate the content of mathematics curriculum for elementary school teachers indicate that in general teacher preparation in the United States did not meet the minimum requirements of the CUPM (52).

In 1972, the Committee on Guidelines of the Commission on Pre-Service Teacher Education of the NCTM analyzed guidelines prepared by the CUPM, the Cambridge Conference, the American Association for the Advancement of Science

(AAAS), the Associated Organizations of Teacher Education (AOTE), and other groups. Based on the findings of the analysis, the Committee members prepared new guidelines for the preparation of elementary school teachers in mathematics. These guidelines are based on the premises that it is essential for teachers to know more than they are expected to teach and to be able to learn more than they already know. Specifically, the Committee recommended the following as minimal knowledge and competencies needed by the elementary school teacher (94:24).

1. Teachers of early childhood and primary grades (ages 4-8) should:

   a. Be able to use and explain base ten numeration system.

   b. Be able to distinguish between rational (meaningful) counting and rote counting.

   c. Be able to recognize stages in the conservation of number and quantity in activities of children.

   d. Be able to perform the four basic operations with whole numbers and with positive rationals with reasonable speed and accuracy.

   e. Be able to explain, at appropriate levels, why operations are performed as they are, and numerals processed as they are.

   f. Be able to use equality, greater than, and less than relations correctly.

   g. Be able to relate the number line to whole numbers and positive rational numbers.

   h. Be able to relate the number line to the concept of measure and describe and illustrate basic concepts of measuring such quantities as weight, volume, etc.

i. Be able to extract concepts of two- and three-dimensional geometry from the real world of the child, and be able to discuss the properties of simple geometric figures such as line, line segment, triangle, quadrilateral, circle, perpendicular and parallel lines, pyramid, cube, sphere, etc.

j. Be able to use a protractor, compass and straight edge.

k. Be able to use the metric system of weights and measures, and be able to estimate such measurements in metric units before actually measuring.

l. Be able to create and interpret simple bar and line graphs on two dimensional coordinate systems and understand the nature of scale changes.

m. Be able to use a calculator to help solve problems.

n. Be able to use all of the above competencies (a-m) to help create, recognize, and solve problems which are real to adults and children. (To "solve problems" in this context includes recognition of problems which have no solution and ability to estimate the expected magnitude of the solution of a problem.)

o. Be able to discuss the history, philosophy, nature and cultural significance of mathematics, both generally and specifically.

2. Teachers of upper elementary and middle school grades (ages 8-12) should:

   a. Have all competencies listed above.

   b. Be able to name large and small numbers and create their own physical examples of approximations for such numbers (e.g., one million is approximately the number of minutes in two years; one billion is approximately the number of seconds in 32 years, etc.) and distinguish between infinity and such numbers as a googleplex.

c. Be able to produce reasonable, consistent, and logical arguments (proofs) for elementary mathematical facts.

d. Be able to perform the four basic operations with positive and negative rational numbers using decimal notation and fractional notation and explain, at appropriate levels, why the operations are performed as they are.

e. Be able to develop new algorithms for operations and be able to test the effectiveness and correctness of algorithms.

f. Be able to solve practical and theoretical problems in two and three dimensional geometry relating to congruence, parallel and perpendicular lines, similarity, symmetry, incidence, areas, volumes, circles, spheres, polygons, and polyhedrons.

g. Be able to use the methods of probability and statistics to solve simple problems pertaining to measures of central tendency and dispersion, expectation, prediction, and reporting of data.

h. Be able to graph functions and relations related to polynomials and to make appropriate selection and use of such relations in the solution of practical problems.

i. Be able to write flow charts for simple mathematical operations and other activities.

j. Be able to use quantitative skills to help recognize, create and solve appropriate problems.

Mathematical Competencies of Elementary School Children

The mathematical preparation of the elementary school teachers must be highly correlated with that of elementary school pupils as the Committee on Guidelines suggested. The mathematical content of the curriculum for elementary school students should be a subset of the content of the mathematics curriculum for prospective elementary school teachers.

What are the mathematical competencies that should be acquired by the elementary school child? Among the most known studies made in this country in recent years concerning what mathematics should be at the elementary level is the Strand's Report published in 1968. The following statement of goals and objectives for the mathematics program, K-8 was proposed by those who contributed to the Strand's Report (105:34).

1. Numbers and Operations

   To use effectively the fundamental operations of arithmetic, computing with fractions and with decimals; to understand and utilize the properties of the operations, and the properties of order and absolute value; and to understand the structure of the several number systems and the special properties of each. To read and understand mathematical sentences involving operations, exponents, and letters, and to formulate and use such sentences in the analysis of mathematical problems.

2. Geometry

To recognize and use common geometric concepts and configurations; to utilize compass and straight edge for simple constructions; and to understand and to construct simple deductive proofs. To use the elementary quantitative geometric notions, such as measure of angle, area and volume; to utilize the concepts of similarity and congruence in applications such as plans and maps; and to utilize the coordinate plane.

3. Measurement

To make measurements; to understand the notion of unit of measurement, and to use and interpret various units; to understand the degree of accuracy of an approximate measurement; to estimate measurements and the results of simple calculations involving measurements; and to conceive and use forms of measurement as functions.

4. Applications

To analyze concrete problems by using an appropriate mathematical model; to employ graphs, scale drawings, sentences, formulae, computations and reasoning in studying the mathematics of such a model; to interpret mathematical consequences in concrete terms; and to examine the concrete results of such an analysis in terms of reasonableness and accuracy.

5. Statistics and Probability

To construct and read ordinary graphs. To collect and organize data by means of graphs and tables; to interpret data using concepts describing central tendency, such as mean, median and mode; and to understand statistical variance as a measure of central tendency. To understand, at a simple level, the idea of sampling, and to interpret and predict from data samples. To understand rudimentary notions of probability theory and of chance events.

6. Sets

   To understand and use routinely the basic set concepts, notations, and operations.

7. Functions and Graphs

   To use the coordinate plane to display relations and to organize data; to recognize and utilize the concept of function, or functional relation; and to use functions and the usual functional notation in analysis and problem solving.

8. Logical Thinking

   To understand, to appreciate, and to use precise statements; to understand and use correctly the simple logical connectives such as: "and," "if-then," etc.; to distinguish, conceptually and in operations, between the "for some" and "for all" quantifiers; and to follow and to construct simple deductive arguments.

9. Problem Solving

   To devise and apply strategies for analysis and solution of problems, and to use estimation and approximation to verify the reasonableness of the outcome.

It is obvious that there is high agreement between the content proposed by the Committee on Guidelines for the preparation of elementary school teachers and the Strand's Report. However, so far, no program has been implemented which incorporates the Committee's recommendations.

Research on Attitudes Toward Mathematics

It has been maintained repeatedly by professionals involved in the development of modern mathematical programs that students' attitudes toward mathematics would improve greatly with changes in the curriculum and in the methods of teaching the "new" mathematics.

Although many investigations have been conducted in the last fifteen years to determine whether modern curricula have fostered more positive attitudes toward the subject, the evidence to support this claim is still meager. This section summarizes the following research on attitudes toward mathematics: (1) techniques used to measure attitudes, (2) attitudes and personality characteristics, (3) teachers' attitudes toward mathematics, (4) attitudes and achievement, and (5) attitudes and the new mathematics curricula.

Techniques Used to Measure Attitudes

A number of techniques have been employed to measure attitudes[2] toward mathematics: behavioral observations, interviews, questionnaires, rank ordering of school subjects,

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[2]Attitude is defined by Aiken (36:551) as a "learned predisposition or tendency on the part of an individual to respond positively or negatively to some object, situation, concept, or another person."

attitude scales, sentence completions, picture preferences, content analysis of essays, and even apparatus indicating physiological states (36). Among these methods, the most popular have been the attitude scales developed by Thurstone and Likert and the semantic-differential techniques.

In view of the fact that the types of measuring instruments employed in the research should affect to some degree the interpretation of results, a few investigators have questioned the relative merits of the measuring techniques used. Morrisett and Vinsonhaler (109) stated a few years ago that there were no valid measures of attitudes toward mathematics. Aiken (36) concluded from a review of research that reliability and validity of the attitude scales vary somewhat with grade levels, being generally more reliable and valid in high school and college. One of the reasons he offered to explain this fact was the many problems of readibility and interpretability of self-report inventories encountered in the lower grades. Anttonen (73) mentioned the same problem in his doctoral dissertation and he pointed out the need for improving the readability of attitude measurements at the elementary school level.

Romberg (66) indicated that many problems of validity, internal consistency and score stability result from an operational definition of attitudes from scores on paper-and-pencil tests. He stressed the need for a theoretical formulation which would conceive of attitudes as "a set of moderator variables that affect the subject's response to mathematical situations in observable and predictable ways" (66:481). Romberg also pointed out that using a single, global measure of attitudes toward mathematics, which is what most investigators do, is not realistic "since there is probably a set of predispositions or feelings that vary from computation to problem-solving, etc." (66:481). Similar recommendations were also made by Aiken and Moss and Kagan (36, 61).

Attitudes and Personality Characteristics

Many studies have been done to investigate relationships between attitudes toward mathematics and a number of personality and social factors, namely anxiety, attitude toward school work in general, achievement, general ability to learn, attitude of one's peers, socioeconomic status, parental influences, sex differences, masculinity-feminity of interests, etc.

The correlations in most studies were found to be generally low. In most cases, the findings of studies relating personality variables to mathematics attitudes indicate that individuals with more positive attitudes tend to have better personal and social adjustment (36, 37, 38, 39). However, it is necessary to remember that personality variables are also affected by family, school and society.

Parents do have some effect on children's attitudes toward mathematics (38, 74, 101) but it appears that they might be more influential in more verbal subjects, such as language development. Socioeconomic status does not seem to be significant in developing attitudes toward mathematics (38, 43, 60, 81).

According to Aiken (38:231), attitudes toward mathematics are "positively related to both verbal and quantitative ability and with a masculine-interest pattern." It would seem that these attitudes and abilities are not only learned-response tendencies determined by social and cultural experiences but are dependent on a genetically determined temperamental and ability base.

Teachers' Attitudes and Effectiveness Toward Mathematics

Most educators view teacher's attitudes and effectiveness in mathematics as the prime determiners of students' attitudes and performance in the subject. As was pointed out by Banks (2:16-17):

> The teacher who feels insecure, who dreads and dislikes the subject, for whom arithmetic is largely rote manipulation, devoid of understanding, cannot avoid transmitting his feelings to the children. . . . On the other hand, the teacher who has confidence, understanding, interest and enthusiasm for arithmetic has gone a long way toward insuring success.

Data concerning the relationships between teacher attitudes and student attitudes support Bank's assertion. Torrance et al. (112) studied 127 sixth through twelfth grade mathematics teachers and he found that the teacher effectiveness had a positive effect on student attitudes. In a study concerning teachers' and pupils' attitudes toward algebra, Garner (77) found significant relations between teacher's attitude toward the subject and students' attitudes. Peskin (84) compared the attitude and understanding of teachers and students in nine junior high schools. The correlations between teachers' and students' understandings of algebra and geometry were significantly positive, as were the correlations between teachers' understanding scores and students' attitudes. The relationships of teacher understanding and attitude to student achievement and attitude were not so clear cut. Teachers with a "middle of the road" attitude and a "high" understanding had students achieve differently according to the mathematical topics. Correlations between teacher understanding and student attitude and achievement were also affected differently by students having very high or very low levels of achievement.

What are the reasons why teachers and prospective teachers like or dislike arithmetic? Dutton and others have conducted extensive studies on the subject (32, 48, 49, 65, 89), principally with prospective elementary teachers.

Reasons given for liking the subject were: its challenge, its practical application, its exactness, appreciation of specific skills, and solving problems. Those who disliked arithmetic gave reasons such as: word problems, boring work, long problems, lack of understanding, poor teaching, lack of teacher enthusiasm, failure or fear of failure. Dreger and Aiken (47) estimated that approximately one-third of prospective elementary school teachers and perhaps of college students in general have unfavorable attitudes toward arithmetic. Reys and Delon (65) reported that the majority of the prospective elementary school teachers in their study developed their attitude toward arithmetic during the seventh to ninth grades.

Investigations conducted by Dutton, Purcell, Reys and Delon, and Gee (48, 49, 65, 78) to find relationships between the attitudes and achievements of prospective teachers in teacher-training courses indicated that: (1) attitudes toward mathematics improved significantly after the students had completed the course (65, 78, 86), (2) post-test scores on an arithmetic comprehension test were significantly higher than pre-test scores (48, 78, 86), (3) nonsignificant correlation between changes in attitudes and changes in understanding of mathematics (48, 49, 86).

In general, the results of these investigations indicate that improving teacher attitude toward mathematics can result in more positive attitude and better understanding on the part of students. Proper training is also very likely to improve attitudes of prospective elementary school teachers toward mathematics.

Attitudes and Achievement

Results of investigations relating attitudes to achievement in mathematics are often contradictory. Aiken (36) believes that this is due to the fact that the majority of these investigations have employed experimental designs that were inadequate for answering the questions posed by the investigators. Harrington (79), at the University of Florida, studied the relationship between attitudes toward mathematics and grade obtained in a freshman mathematics course. He reported a statistically insignificant relationship between attitude and performance, although he found that the selection of an elective course in mathematics was significantly related to attitude. Whitnell (88), in a study done to determine mathematical understanding of college students, found that the best predictors of achievement were ability, attitude, and high school mathematical background. In his study of 160 students enrolled in three different sections of an upper division methods course dealing with the teaching of arithmetic, Dutton (49) reported that student attitudes toward arithmetic reflected a growing appreciation of the subject as they increased their understanding of arithmetic. The general attitude of about 75 percent of the students toward arithmetic was quite favorable--varying from 6.0 to 9.5 (value of the scale items ranged from a low of 1.5-dislike to 10.5-very favorable). The lowest 25 percent of the students in this study held unfavorable attitudes toward arithmetic.

Litwiller (82) investigated the change in attitudes of prospective elementary teachers resulting from a change in "method." Her sample consisted of 145 students enrolled in a content course at Indiana University, ninety-five of whom were in the experimental group and fifty in the control group. Results indicated the following: (1) the attitudes of the experimental group changed significantly relative to the attitudes of the control group, (2) there was a significant difference between the achievement scores of those students in the experimental group relative to the control group, and (3) there was a significant correlation between the post-test attitude score and achievement and SAT mathematical scores, respectively.

In somewhat different studies, Dreger and Aiken (47) found that scores on an inventory of anxiety was significantly related to the final grade of 704 freshmen enrolled in a mathematics course. In another study, the same investigators (39) reported that scores on the Mathematical Attitude Scale contributed significantly to the prediction of final grades in a mathematics course for sixty-seven college women, but not for the sixty men subjects of the same study.

An international study reported by Husén (17) to compare the mathematics achievement of secondary students in twelve countries provided data concerning the relation of attitudes and interests to mathematics achievement. Three of the five attitude scales administered were: measures of attitudes toward mathematics as a process, attitudes about the difficulties of learning mathematics, and attitudes about the place of mathematics in society. Correlational results of this investigation were: significant negative rank-order correlations between mean mathematics achievement and mean scores across countries on the attitude scales; small correlations between achievement and attitude within countries; moderate to high correlations between achievement and interest measures within countries.

Attitudes and the New Mathematics Curricula

Research designs used to compare attitudes and achievement of students enrolled in new mathematics programs with those of students enrolled in traditional programs resulted in findings that are not consistent. Investigators who have compared SMSG and traditional curricula in elementary and junior high school (55, 83, 85, 90, 101) found that, generally, the mean mathematics attitude scores of students taught by SMSG curriculum was not significantly greater than the mean attitude scores of students taught mathematics by the traditional curriculum. As to achievement, scores on conventional standardized tests tended to favor traditional programs while scores on more specialized tests favored the SMSG curriculum.

Similar designs were used to compare other mathematics programs with the traditional programs. Comley (75) compared the college mathematics achievement and attitudes of students enrolled in the University of Illinois Committee on School Mathematics (UICSM) program with those of students who had traditional high-school mathematics. There were few differences between the two groups in college mathematical achievement after the criterion scores of the UICSM and non-UICSM groups were adjusted on a number of variables, but the UICSM students had significantly more favorable mathematics attitudes than the non-UICSM group.

Yasui (91) compared a group of students exposed to a modern-mathematics program with a group not exposed to modern mathematics. After adjustment for individual differences, the investigator found that while the difference between the mean scores of the two groups on an inventory of attitudes toward mathematics was not significant, attitude scores were significantly correlated with achievement in both groups. Ryan (30) compared the effects of three experimental "modern" programs in secondary mathematics on the attitudes and interests developed in ninth-grade pupils. The results showed that the experimental programs had little differential effect on attitudes and interests. Investigating program-centered vs. teacher-centered teaching of first-year algebra, Devine (76) concluded that when an average or above average teacher is available, greater achievement is obtained in a conventional, teacher-centered classroom approach and attitudes toward mathematics are not affected significantly.

In his evaluation of the research done on attitudes toward mathematics, Aiken (38) indicated that one should be cautious in interpreting the results of the investigations on the subject. For one thing, available subjects were not always assigned at random to the two types of curricula. It is quite possible that students in special programs were initially attracted to or selected for the program because of their positive attitudes toward mathematics. Osborn (83) suggested that modern curricula are more abstract and demanding than the traditional curriculum with the result that students enrolled in modern mathematics programs fail to change their attitude toward mathematics or become more negative as the program develops.

For Aiken, who explored extensively the research on attitudes toward mathematics, it seems that "the teacher, rather than the curriculum, still appears to be the more influential variable as far as attitudes are concerned" (36:581).

Summary

This chapter was concerned with a search of the most recent and pertinent literature on curriculum evaluation, teacher education programs for elementary school teachers, approaches to teacher training, content of mathematics curriculum and research on attitudes toward and achievement in mathematics.

The increased concern of mathematics educators from all over the world with the necessity of improving mathematics education has been very influential in the development of new teacher education programs and in the improvement of methods of evaluation.

Since teachers and their education are the principal substance behind any effort made for the ultimate improvement of educational systems, educators have devoted a great deal of time to the improvement of teacher education programs, developing criteria for the training of prospective teachers of mathematics at both the primary and the secondary level. Changes in the content of curricula have been accompanied by experiments in the development of new teaching methods, and by new research on the mathematical competencies needed by elementary school teachers and elementary school children.

Curriculum research and evaluation has continued to progress. While summative evaluation is still regarded as an adequate and necessary method to make decisions about curriculum adoption and effective use, formative evaluation techniques are considered more and more important by most curriculum specialists during the development of a teacher education program and also for instruction and student learning.

Concerned by the importance of improving the measurement of achievement, a score of researchers have proposed a more extensive use of criterion-referenced measures in the assessment of the degree of competence attained by a particular student. This type of measurement is relatively new in education but the development of instructional technology and the recent emphasis on curriculum research and curriculum evaluation have stressed the need for the kind of information made available by the use of criterion-referenced measures.

Attitudes toward mathematics, an important element in the success of modern mathematics education programs, have also been investigated extensively in relation to personality characteristics, teacher's attitudes and effectiveness, students' achievement and the new mathematics curricula.

Materials discussed in the review of literature were used for the development of adequate procedures to be followed in the formative evaluation of the mathematics component of the Michigan State University experimental teacher education program. Chapter III describes the features of the study.

CHAPTER III

DESCRIPTIVE FEATURES OF THE STUDY

The formative evaluation of the mathematics component of the experimental elementary teacher education program at Michigan State University was conducted during the academic year 1971-1972. This chapter summarizes the different procedures which were followed in carrying out the present evaluation.

Students in the Study

The students in the experimental program were those freshmen elementary education majors who volunteered and were selected to participate in the program. Initially, fifty-two entering freshmen volunteered, forty of which were selected. At the beginning of the year, two students dropped from the program; thus, the remaining thirty-eight students comprised the group of prospective elementary teachers who participated in the first course of the mathematics curriculum of the program.

In addition to these students, other groups of students were utilized in this study:

1. A representative sample of three freshmen groups was used for comparison purposes. These "comparison groups" were selected by the TTT project evaluation team on the basis of a study of University records of number of freshmen with declared majors in various disciplines (98). The three "comparison groups" chosen were first term freshmen with declared majors in: (1) Elementary Education, (2) Mathematics and Secondary Education, and (3) Mathematics. The investigator was able to utilize the three comparison groups to assess whether the students who volunteered and were accepted in the experimental program were initially different in their cognitive and affective behaviors toward mathematics from other freshmen students who shared the same professional interests and a group of freshmen who showed a specific interest in mathematics.

2. A group of prospective elementary school teachers enrolled in the regular teacher education program at Michigan State University was also used for this investigation. These students had already completed the mathematics content course (Mathematics 201) and were enrolled in the methods course (Education 325E) at the same time that the experimental group was involved in the integrated content-methods course. These regular students differ substantially from the experimental group and the "comparison groups" in that they are second-year college

students (or higher). The regular students were used for two different purposes: (1) to evaluate the reliability of the achievement tests developed in this study, and (2) to assess their mathematical understanding and attitudes toward arithmetic after the completion of the methods course and to compare these results with those of the experimental group. The students in these groups are referred to in the study as the "regular methods course (325E) students."

Evaluation of the Experimental Program

Description of the Mathematics Component of the Experimental Teacher Education Program

At Michigan State University, undergraduate elementary education majors are required to complete a sequence of two mathematics education courses. The first, offered by the Department of Mathematics, is a four-quarter hour content course entitled Arithmetic for Elementary Teachers (Math. 201). During this course, prospective elementary teachers spend three hours a week in lecture rooms and two hours in a mathematics laboratory. This course is a prerequisite to the second required course in methods of teaching elementary mathematics (Education 325E) which is offered by the Department of Elementary School Education. These two courses are usually taken during the sophomore or junior year.

The total experience of the preservice elementary school teacher enrolled in the regular program in mathematics education thus consists of forty class hours of content and thirty class hours of methods.

The mathematics component of the experimental teacher education program, on the other hand, offers two combined mathematics content-methods courses which integrate the mathematical concepts introduced to the students with the methods to teach these concepts to elementary school children. Each course is accompanied by a clinical experience in which the students actually practice teaching the concepts covered in the course to elementary school children.

The first of these two courses is to be offered during the freshman year and the second, during the junior year. The major topics covered in the first course are: (1) Measurement, (2) Set Theory, (3) Numeration Systems, (4) Whole Number System, (5) Rational Number System, (6) Introduction to Relations and Functions, and (7) Probability and Statistics. The second course emphasizes the areas of the Real Number System, Algebra, and Geometry.

Upon completion of these two courses, the composite mathematical experience of the preservice elementary school teacher enrolled in the experimental program would consist of 160 class hours of content-methods and 80 clinical school hours.

During the 1971-1972 academic year, the first course was designed and tried with the first group of prospective elementary teachers who participated in the experimental program. It is this particular course that is the objective of the formative evaluation done by this investigator.

Assessment of the Mathematics Component of the Experimental Teacher Education Program: Development of a Criterion-Referenced List

In order to construct a scorecard of mathematical topics suggested for the preparation of elementary school teachers, the investigator sought the advice of mathematics educators at the University, who recommended a thorough review of the related literature such as the report of the Committee on the Undergraduate Program in Mathematics (CUPM), the Cambridge Conference Report "Goals for Mathematics Education of Elementary School Teachers," the Strand's Report, the publications of the National Council of Teachers of Mathematics, and various textbooks in the field of mathematics education for elementary school teachers.

The list of topics suggested in the publications reviewed served as a basis upon which to assess whether the content included in the experimental program at Michigan State University is sufficient in meeting the need of the prospective teacher in the field of mathematics.

The review of literature revealed a highly related survey study conducted by Hicks and Perrodin (54) which provided a sound base for the selection of topics appropriate for the preservice education in mathematics of elementary school teachers. Four types of sources were intensively reviewed by the authors to provide the necessary data. They were:

1. Review of forty-six selected research studies pointing out the mathematical competencies or weaknesses of elementary school teachers.
2. Review of thirty-two sets of recommendations of mathematics educators and nationally-recognized advisory groups or organizations.
3. Page-by-page analysis of sixteen recent textbooks designed for college courses in mathematics for elementary school teachers.
4. Analysis of eleven arithmetic series or teacher's guides for grades K-7 published since 1962.

A composite list of mathematical topics from the above sources was then compiled by Hicks and Perrodin (54) and a system of rating these topics was devised. Topics which appeared at least once in the composite list were categorized as Level I. To be categorized as Level II, topics had to meet one of the following conditions:

- appear in at least three of the research studies;
- appear in at least five of the recommendations of the mathematics educators or advisory groups;
- appear in at least eight of the sixteen college textbooks in mathematics for elementary school teachers;
- appear in at least six of the eleven arithmetic series or teacher's guides for grades K-7.

Finally, to be classified as Level III, a topic had to meet at least two of the four criteria listed above for Level II topics.

A total of ninety-eight topics were located in the four sources (nineteen in source one, fifty-four in source two, eighty-four in source three, and seventy-nine in source four). Of these topics, fifty-one were categorized as Level II and thirty-five were categorized as Level III.

Table 1 shows the topics in level three along with the sources in which they appeared. It is obvious from this table that the last three sources are in close agreement on what should be included in some manner in the mathematics curriculum of the elementary school teacher. The relatively low percentage in the first source does not indicate disagreement with the other sources; it only indicates the lack of experimental research done on the selection of mathematicl topics for the preparation of elementary school teachers.

Table 1

Suggested Topics for the Mathematical Preparation of Elementary School Teachers

Topic	Source 1	Source 2	Source 3	Source 4
1. Set Terminology		x	x	x
2. Set Operations		x	x	x
3. Relations & Functions		x		x
4. Whole Number Operations	x	x	x	x
5. Counting and One-to-One Correspondence		x	x	x
6. Order and Cardinality			x	x
7. Field Operations		x	x	x
8. Different Numeration Systems & Place Value	x	x	x	x
9. Ancient Numeration Systems			x	x
10. Roman Numeration	x		x	x
11. Primes and Composite		x	x	x
12. Factors and Multiples		x	x	x
13. Exponents & Exponential Notations		x	x	x
14. Divisibility Rules			x	x
15. The Number Line		x	x	x
16. Common Fractions	x	x	x	x
17. Decimal Fractions	x	x	x	x
18. Percentages	x	x	x	x
19. Ratio & Proportions	x	x	x	x
20. Real Numbers		x	x	
21. Square Root		x	x	
22. Measurement	x	x	x	x
23. Precision and Error		x	x	x
24. Formulae & Substitution		x	x	x
25. Basic Concepts of Geometry		x	x	x
26. Geometric Figures		x	x	x
27. Metric System & Conversion		x	x	x
28. Equations and Symbols	x	x	x	x
29. Inequations	x	x		x
30. Central Tendency	x	x		x
31. Statistical Graphs	x	x		x
32. Probability		x	x	
33. Problem Solving			x	x
34. Making Estimations			x	x
35. Rationalizing Algorithm	x		x	x
Total	13	28	31	32
% of Total No. of Topics	37	80	89	91

Review of the sources utilized in the Hicks and Perrodin survey suggested that almost all of the sources recommended to the investigator by the mathematics educators at Michigan State University were included. To further test the validity of this list, the investigator reviewed publications of similar sources for the years 1968-1972. The topics suggested in these sources are very consistent with the list described above except in the field of Geometry and in the field of Logic.

Analysis of the content of five textbooks for elementary mathematics for teachers (5, 18, 24, 26, 27) revealed that coordinate geometry and mathematical logic were not included in the list developed by Hicks and Perrodin (54). The Arithmetic Teacher, a publication of the National Council of Teachers of Mathematics, annually publishes a summary of research and articles on mathematics education conducted in the United States during the preceding year. Review of these summaries for the years 1968, 1969, 1970, and 1971 again pointed out that most research done on the content was in topics noted in the Level III list as defined by Hicks and Perrodin. However, two pieces of research, one by Shah (67) on the applicability of teaching geometry to elementary school children, the other by O'Brien and Shapiro (63) confirmed children's ability to learn mathematical logic. Research conducted by Suppes

(111) at Stanford University in teaching logic to elementary school children has not as yet provided conclusive evidence to the children's ability to learn and comprehend mathematical logic. Based on this review of recent literature, the investigator concluded that only the topic "Coordinate Geometry" met the qualifications of the Level III prescribed by Hicks and Perrodin, and therefore decided to include it as the thirty-sixth topic in the criteria list.

The Integrated Content-Methods Course in Mathematics

Designing the Curriculum of the Integrated Content-Methods Course in Mathematics

Among the objectives of the TTT project is the development of a competency-based elementary teacher education program that incorporates aspects of the Model Program BSTEP.

In the mathematics component of the program, competency-based teacher education program means a training program that requires its trainees to demonstrate, at a specified level of competence, mathematical behaviors that have been explicitly specified as effective professional behaviors.

These competencies (knowledge, skill, and behaviors) are determined by the program developers as specific statements of competencies needed by the future elementary school teacher for mathematics instruction. It is upon these statements that the instructional materials and designs were developed and implemented.

The task of determining what mathematical competencies (knowledge, skill, and behaviors) should be included in the program to make it effective in producing competent teachers was carried by an interdisciplinary team of professional people involved in education. The team consisted of two faculty members from the Department of Mathematics, three faculty members from the Department of Elementary Education, four doctoral students, and four elementary teachers from the school where the students had their clinical experience. The faculty members all had specific interest and background in mathematics education and have had recent experience working with elementary or junior high schools.

The team worked closely together on developing the mathematical experiences for the first year trial implementation of the experimental program. The product of their work consisted of a series of nine learning units, each one devoted to one of nine mathematical topics deemed viable and necessary for the future elementary school teacher. The topics were:

1. Measurement
2. Numeration Systems
3. Sets and Set Relations

4. The Whole Number System
5. Fractions
6. Decimals
7. Relations and Functions
8. Probability and Statistics
9. Mathematical Systems.

These learning units were all designed in accordance with guidelines proposed by the BSTEP and all have certain features in common:

1. Assessment tests
2. Goals and Objectives
    a. Required Activities
    b. Optional Activities
3. Strategies to achieve the objectives
4. Instructional design
5. Instructional design to be used with children at the elementary level
6. Instructional feedback
7. Comments.

A complete file on each of the nine topics used in the integrated content-methods course was prepared for each of the students enrolled in the experimental program. (A specimen of such a file is found in Appendix F.)

Experimenting with the Integrated Content-Methods Course: Procedure Followed

The integrated content-methods course was conducted in a Michigan State University mathematics laboratory. The laboratory was a large room equipped with spacious work tables, audio-visual materials, as well as numerous shelves and cabinets loaded with manipulative materials, e.g., Cuisenaire rods, attribute blocks, geoboards, Dienes blocks, Madison Project shoebox kits, balance beams, mirrorcards.

The students working in the laboratory also had at their disposal a sizeable collection of elementary school mathematics textbooks and numerous copies of The Arithmetic and Mathematics Teacher as well as publications of many mathematics education projects.

The basic motivation for conducting the course in a mathematics laboratory is that such a setting enhances not only the learning of mathematics and methods of teaching mathematics but also prepares the prospective teacher in using manipulative materials when teaching elementary pupils.

The experimental group met in the laboratory four days a week, Monday through Thursday from 3 to 5 p.m. during the Spring term of 1972. Each day of class, four or five instructors involved in the program were available to assist the students; and every week, one mathematical topic (learning unit) was covered. The weekly schedule for the course was as follows:

Monday 3:00-3:30	A pre-test specifically constructed to assess the student behaviors on the prescribed mathematical competencies for that week was administered (see Appendix A).
3:30-4:00	Files for that week learning unit were distributed to each student. The files (developed earlier by the unit designers) contained the goals and objectives and a description of the activities for each of these objectives.
4:00-5:00	Students divided into groups of four students worked on the prescribed activities utilizing the manipulative materials available to master the objectives set for that week.

Tuesday-Wednesday 3:00-5:00	Students continued working on the activities prescribed in the learning unit files. When one objective was completed, the student demonstrated his mastery over that objective to one of the instructors and had it checked on his file. When the instructors felt that some or all the students were having difficulties comprehending certain concepts, a short lecture, often utilizing manipulative materials, was conducted by one of the instructors.
Thursday 3:00-3:30	A post-test (parallel form of the pre-test) on the content learned on the previous week was administered (see Appendix A). The post-test on any particular topic was given only after the students had gone through one week of instruction in the laboratory and one week of self-teaching the pupils in the elementary school (see the schedule on the following page). Note: The only exception was for the last topic Mathematical Systems, which was not taught in the elementary school. In this case, the post-test was given at the end of the week of study in the MSU laboratory.
3:30-5:00	Each group of four students worked with an instructor or one of the elementary teachers present that day on designing a lesson which would incorporate assessment, goals and objectives, strategies, and evaluation. This lesson was taught the next week to the pupils of the elementary school by the four students of each group.
Monday through Thursday 8:00-12:00	Every day, nine or ten students went to the chosen elementary school in the Lansing area for clinical experience.

<u>Clinical Experience</u>

Students implemented the instructional designs developed at the university laboratory with the children at the elementary school mathematics laboratory.

INTEGRATED CONTENT-METHODS COURSE SCHEDULE

SPRING TERM 1972

Week Beginning	Unit Covered in the Lab.	Pre-Test	Unit Taught in School	Post-Test
April 3, 1972	Measurement	Measurement	---	---
April 10, 1972	Numeration	Numeration	Measurement	Measurement
April 17, 1972	Sets and Set Relations	Sets and Set Relations	Numeration	Numeration
April 24, 1972	Whole Numbers	Whole Numbers	Sets and Set Relations	Sets and Set Relations
May 1, 1972	Fractions	Fractions	Whole Numbers	Whole Numbers
May 8, 1972	Decimals	Decimals	Fractions	Fractions
May 15, 1972	Relations and Functions	Relations and Functions	Decimals	Decimals
May 22, 1972	Statistics and Probability	Statistics and Probability	Relations and Functions	Relations and Functions
May 29, 1972	Mathematical Systems	Mathematical Systems	Statistics and Probability	Statistics and Probability and Mathematical Systems

Each student spent one full morning (four hours) every week in the elementary school. Three hours were spent working in the classroom with the teacher and the remaining hour was split into two parts: (1) one-half hour of teaching four to six pupils the mathematical concepts designed at the university laboratory during the previous week; (2) one-half hour of meeting with the doctoral student or faculty member who observed the clinical practice to exchange comments and receive feedback on the methods or design used in teaching the pupils.

The clinical experience provided the prospective elementary teacher with:

1. The opportunity to relate theory to practice, by applying the knowledge gained at the university to actual teaching situations at the elementary school.
2. The opportunity to observe different classes, teachers, and teaching methods.
3. The opportunity to initiate his teaching experience by working with a small group of children, thus benefiting from closer individual relations and minimized problems of discipline and control.
4. The opportunity to receive immediate feedback on the methods of teaching utilized from experienced in-service teachers or faculty members.

Assessing the Content of the Integrated Content-Methods Course

Instruments selected or developed for use in the collection of data were:

1. Nine criterion-referenced achievement measures to assess mathematical competencies on prescribed objectives (two parallel forms).
2. Test of Basic Mathematical Understanding (two parallel forms).
3. Revised form of the Dutton Attitude Inventory, Form C.
4. Attitudes Scales toward different aspects of mathematics developed by the International Study of Achievement in Mathematics.

Development of Criterion-Referenced Achievement Measures

Underlying the concept of achievement measurement is the notion of a continuum of knowledge acquisition ranging from no proficiency at all to perfect performance. A student's achievement level falls at some point in this continuum as indicated by the behaviors displayed during testing. The degree to which his achievement resembles desired performance at any specified level is assessed by criterion-referenced measures of achievement or proficiency (28). The term "criterion," when used in this way, does not necessarily refer to final end-of-course behavior. Criterion level can be, and informative evaluation should be, established at certain points in instruction when it is necessary to obtain information as to the adequacy of a student's performance with respect to some specified standard and to know whether learning is promoted by the presentation of the sequence of mathematical learning units.

A fairly straightforward method can be employed to test the effectiveness of the proposed curriculum. This consists in determining and administering tests which have been specifically constructed to yield information on the achievement of the students on each learning unit within the curriculum. The data from such tests are then analyzed to reveal the effect of the curriculum on the students who have been exposed to the instruction identified by the learning units (12).

In order to assess the effectiveness of the mathematics component of the experimental program on the prospective elementary teachers participating in the program, it was therefore necessary to develop a series of criterion-referenced tests designed specifically to test whether the preservice teacher could or could not exhibit the competency implied by the prescribed objectives in each learning unit. It was also essential to develop two equivalent forms for each test in order to assess the entering behaviors and the terminal behaviors of the preservice teacher toward the prescribed objectives within each learning unit.

Development of the Test Instruments

A review of the literature helped gain deeper insight on the methodology of constructing good tests. Much of the theory of achievement testing was outlined in the milestone volume Educational Measurement (ed. by Lindquist, 1951), in which Lindquist recommends the following steps in the preparation of an educational achievement test: (1) planning the test, (2) writing the test items, (3) trying out the test form and assembling the finished test after tryout, (4) preparing the directions for administering and scoring the test, and (5) reproducing the test (19:119).

In this study, the investigator used the following steps in the preparation of each criterion-referenced test for the sequence of nine learning units that make up the mathematics curriculum for the first-year trial implementation of the experimental teacher education program under investigation:

1. Identifying the objectives that the test is to measure.

   The specific statements of mathematical objectives for each learning unit as specified by the program developers were identified and listed. These objectives are the prescribed mathematical competencies that served as the criterion-reference for the achievement tests.

2. Developing the test instrument.

Lindquist made specific suggestions for writing items for achievement testing (19). The following were included among the list of suggestions he made:

a. Express the items as clearly as possible
b. Choose words that have precise meaning wherever possible
c. Avoid complex or awkward word arrangements
d. Include all qualifications needed to provide a reasonable basis for response selection
e. Avoid irrelevant inaccuracies in any part of the items
f. Avoid irrelevant clues to the correct responses
g. Avoid irrelevant sources of difficulties

There are many forms of test items in general use, such as essay, true-false, short answer, matching, and multiple choice. Most testing authorities indicate that the need for test items to be objectively and efficiently scored can be best attained through the utilization of multiple-choice type of items. The multiple-choice format, in which the answer choices are supplied and the student would choose the best or correct answer is the most generally applicable for mathematics achievement tests (19). The multiple-choice type of items were used extensively in test development for this study; however, the investigator recognized that, in many mathematical situations, producing the answer (rather than recognizing it) is an essential part of the ability being tested.

Among the specific principles suggested by Noll (25) for the construction of multiple-choice type test items were:

a. All options should be possible and plausible answers
b. Irrelevant grammatical clues should be avoided
c. The stem should not be loaded down with irrelevant materials
d. The number of choices should be at least four.

3. Preparing the test items.

Once the mathematical objectives within each learning unit had been specified and the plans for the test had been determined, the preparation of a supply of test exercises that conformed to the specifications listed above was initiated. The investigator utilized the following sources to assist in the construction and selection of test items:

a. Test exercises from previous mathematics content courses at Michigan State University (Mathematics 201)

b. Chapter exercises from recent textbooks on mathematics for elementary teachers

c. Tests developed by recent studies in mathematics education.

Most test items, however, were developed by the investigator with assistance from mathematics educators at Michigan State University. When it was felt that a sufficient number of items had been formulated for each specified mathematical objective within the learning units, attention was turned to the problem of selecting the best items which can be assembled into two equivalent forms. For each of the nine mathematical topics comprising the first year content, two equivalent tests were prepared: one was to serve as a pre-test; the second, as a post-test. Each test was to contain ten or eleven items[1] (an item could contain one or more questions). After the pool of test items were written for each learning unit, they were reviewed by individuals sensitive to common editorial shortcomings of test exercises. The items were also checked for mathematical correctness and precision of statement by independent mathematical educators.

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[1]Except for the test for Mathematical Systems, in which five items were included due to the limitations of instructional period and testing time.

For each learning unit, two forms of the test were assembled from the test items and were reviewed by the group of educators and teachers who developed and designed the activities for that particular learning unit. They were asked to review the test items and determine whether each item was a valid assessment of the objective it purported to evaluate. Comments and suggestions made by the unit designers were utilized in revising and for replacing items on the test forms. It must be noted that in selecting the items for the final forms of the final tests, an attempt was made to sample the behaviors under certain objectives. For example, if the instructional objective was stated as: "the student . . . be able to add two numerals in base other than ten," it would have been impractical to include items that assessed the student ability to add numerals in bases 2, 3, 4, 5, ... etc.; it was more appropriate to sample two or three bases and write items for the bases selected.[2]

4. Preparing the directions for administering and scoring the tests.

Since the test constructor administered the tests, it was not necessary to prepare detailed directions for

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[2]According to Rajaratnam, Cronbach, and Glaser (110), it is possible to generalize from such selection and still attain predictive validity for the curricular objective.

the test examiner. On the front page of each test, however, directions were given for taking the test (due to the nature of the test items, most were self-explanatory). The time allocated for testing was approximately 30 minutes but the students who needed more time were always allowed to complete their test.

In scoring the test, if an item contained only one question, ten points were allowed for the correct answer; if an item contained two questions, then five points were given for each correct answer, etc. No partial credit was allowed for incomplete answers. The score of a student on each test was determined in percentages.

5. Administering the tests.

After the two parallel forms of the tests for each learning unit were determined, copies were prepared for use with the experimental group. For each learning unit, the pre-test was administered prior to instruction, and the post-test was administered one week after instruction. All testing was supervised by the investigator. When a student was absent during the pre-test period, he was asked to take the test before starting on the activities for that unit.

Evaluation of the Criterion-Referenced Achievement Measures

Described above are the steps leading to the development of the set of criterion-referenced mathematics achievement tests used in this study. It is upon the soundness and appropriateness of these procedures that the claim of validity of the instruments must primarily rest. However, statistical evidence is central to establishing the reliability of the measures and does have some supplementary value in attesting to their validity.

Validity.--Criterion-referenced measures are validated primarily in terms of the adequacy with which they represent the criterion; therefore, content validity approaches are best suited to such tests (28:29). The inherent method by which the set of tests were developed assured content validity, since the test items, in the judgment of the team of mathematics educators who developed and designed the learning units, did in fact reflect the specific objectives within the mathematical content of that unit.

Reliability.--Since each test is constructed to assess the instructional objectives within a specified topic, it is necessary to estimate the reliability of each test independently (28:28).

Students in three sections of the regular methods course (Education 325E) were made available to test the reliability of the pre- and post-criterion measure achievement tests. There were respectively 19, 17, and 20 students in these classes and the investigator was allowed approximately one hour and a half for testing purposes. This implied that each student could complete one pre- and one post-test in the set period, but not more. Therefore, the investigator had the choice of giving one set of tests (pre- and post-test) to a sample of six students, or to sample the test items and give the sampled tests to a larger number of students.[3]

After consultation with faculty members knowledgeable in measurement theory, it was decided to randomly select a 5-item sample from each of the nine pre- and post-tests (about 50 percent). When the selection of these particular items was completed, three thirty-item tests were assembled:

1. Test I contained five items from pre- and five items from post-tests on Measurement, Numeration, and Sets and Set Relations.
2. Test II contained five items from pre- and five items from post-tests on Whole Numbers, Fractions, and Decimals.

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[3]Cook and Stufflebeam (44) and others demonstrated empirically that group performance is more efficiently measured using small subsets of items distributed among large numbers of students than vice versa.

3. Test III contained five items from pre- and five items from post-tests on Relations and Functions, Probability and Statistics, and Mathematical Systems.

Copies of these tests were randomly distributed to the 56 students in the three sections of Education 325E (Methods of Teaching Elementary School Mathematics). Based on the statistical results of these tests, reliability estimates for each test were obtained.

Estimates of the reliability of each of the item-sampled tests were calculated using the Hoyt Reliability Coefficients (19:570) through an analysis of variance technique (see Appendix Q). Tables 26-43 contain the statistics for the analysis of variance for each test. The Spearman-Brown formula was applied to the Hoyt Reliability coefficients to obtain the total test reliability. Table 2 (see Appendix M) shows the results obtained for each test from the statistical procedures described above.

The reliability coefficients for the tests varied from a low of 0.77 for the post-test on Measurement to a high of 0.93 for the pre-test on Relations and Functions. These coefficients are considered to be acceptable for a criterion-referenced test (12).

Table 2

Reliability Coefficients for Pre- and Post-Criterion-Referenced Achie"ement Tests

Tests	Pre-Test (1)	Pre-Test (2)[b]	Post-Test (1)	Post-Test (2)
Measurement	.6869	.8144	.6317	.7743
Numeration Systems	.7981	.8877	.7831	.8784
Sets and Set Relations	.8038	.8912	.7920	.8839
Whole Numbers	.6945	.8197	.7161	.8346
Fractions	.7212	.8380	.6840	.8124
Decimals	.7680	.8688	.7552	.8605
Relations and Functions	.8702	.9306	.8376	.9116
Probability and Statistics	.8412	.9138	.8041	.8914
Mathematical Systems	.8470	.8470	.8244	.8244

[a](1) Hoyt Reliability coefficients obtained from 50 percent item-sampled test.

[b](2) Reliability coefficients of total test after applying the Spearman-Brown formula to Hoyt Reliability coefficients.

$$R_{tt} = \frac{2R_{st}}{1 + R_{st}}$$

R_{tt} = Reliability of total test.

R_{st} = Reliability of sampled test.

Equivalency of the two forms of pre- and post-tests.--The problem of preparing truly equivalent forms of a test is, according to Thorndike (19:575): "a problem in the logic and practice of test construction. . . . The best guarantee of equivalence for two test forms would seem to be that a complete and detailed set of specifications for the test be prepared in advance of any final test construction." a further check on the degree of equivalency was made by examining correlation coefficients between the two test forms. Table 3 shows the Pearson-moment correlation coefficients obtained from the test results of the students enrolled in the regular methods course, Education 325E (Methods of Teaching Elementary School Mathematics).

The correlation coefficients between pre- and post-test scores varied from a low of .65 for the test on Measurement to a high of .90 for the test of Mathematical Systems. It was noted, however, that except for the test of Measurement, the lowest correlation coefficient was .77 (see Table 3).

Table 3

Correlation Coefficients Between Pre- and Post-Test Scores of the Students in Regular Methods Course (Education 325E) on Item-Sampled Criterion-Referenced Achievement

Tests	N	Correlation Coefficients
Measurement	19	.6452
Numeration	19	.8035
Sets and Set Relations	19	.8173
Whole Numbers	17	.7944
Fractions	17	.7781
Decimals	17	.8232
Relations and Functions	20	.8223
Statistics and Probability	20	.8615
Mathematical Systems	20	.9026

Selection of a Test of Mathematical Understanding

This phase of the study began by searching for a well-documented instrument for measuring mathematical understanding. It was hoped to find a standardized instrument that would test the mathematical topics covered in the recommended content for prospective elementary school teachers. It was also hoped to find a test with two equivalent forms to minimize the testing effect. After careful search of the literature on the subject, the investigator came across an instrument designed by Mildred J. Dossett as part of her doctoral dissertation at Michigan State University in 1964.[4] The test was deemed most appropriate for the purpose of this investigation since the test items covered mathematical topics recommended by professional and advisory groups in mathematics education. Permission was granted by the author to use the test for the present study.

Dossett's instrument entitled "Test of Basic Mathematical Understanding" had a reliability coefficient of 0.87 obtained by correlating the scores made by 50 college students on the two equivalent forms of the test. Equivalency of the two forms was determined by using a t-test suggested by McNemar. The t-value obtained indicated no significant

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[4]Mildred J. Dossett, "An Analysis of the Effectiveness of the Workshop as an In-Service Means for Mathematical Understanding of Elementary School Teachers" (unpublished doctoral dissertation, Michigan State University, 1964).

differences between the scores on the two forms of the test when administered to the 50 college students.

Selection of an Attitude Inventory

The "Arithmetic Attitude Inventory," an attitude scale developed by Wilbur Dutton at the University of California, was used in this study (48). For this scale, Dutton utilized a technique perfected by Thurstone and Chave (48). He first selected a large number of written statements regarding attitudes toward arithmetic obtained from papers of six hundred university students over a period of five years. The statements were sorted by judges using a scale of one to eleven (extremely unfavorable to extremely favorable). The proportion of judges who placed each statement in the different categories constituted the basic data for computing the scale values of the statements. The instrument was used with over two hundred eighty-nine students. A reliability of .94 was obtained through test-retest procedures (49).

On the attitude instrument, the fifteen items have values that range from 1.0 to 10.5 representing extremely negative to extremely positive attitudes. The individual score is the average scale value of the statements which the individual checked.

Construction of Student Questionnaire

The investigator sought to obtain from the students in the experimental group reactions to the experimental procedures of the mathematics curriculum. An eleven-item questionnaire was developed for the purpose of determining what the students thought or felt at the end of the school year toward particular aspects of the mathematics curriculum that they had encountered during the first year trial implementation of the experimental program. The first ten items of the questionnaire were statements each relating to one particular aspect of the program with five scaled-responses (strongly agree, agree, undecided, disagree, and strongly disagree). The eleventh question was a free response question that elicited the students' recommendations for methods of improving the integrated content-methods course. A copy of the questionnaire is included in Appendix E.

Statistical Procedures for the Analysis of Data

To analyze the data collected during the study, the investigator selected several statistical procedures for purposes of clarifying some aspects of the study and to test the hypotheses stated in Chapter I.

A one-way multivariate analysis of variance technique as described by Winer (35:332) was selected for use in analyzing the data relevant to the testing for significant differences between the post-test scores of the experimental group on the nine criterion-referenced tests and (1) their pre-test scores (Hypothesis A1), (2) the mastery level of 80 percent or higher completion of the post-test items (Hypothesis A2).

To test the hypotheses related to the effect of the mathematics curriculum on the basic mathematical understanding (Hypothesis B1) and attitudes toward arithmetic (Hypothesis B2), the analysis was done in two parts. The pre-test scores were subtracted from the post-test scores for each individual, resulting in differences scores. These differences were analyzed using t-test for the significance of difference between correlated means (9).

In order to assess the relative performance of the experimental group as compared with a group of prospective elementary teachers in the regular teacher education program on the basic mathematical understanding (Hypothesis C1) and attitudes toward arithmetic (Hypothesis C2), the analysis of covariance technique was utilized as suggested by Winer (35:753). The respective pre-test scores were covariate measures.

To study the degree of relationship between selected criteria under investigation, the product-moment correlation coefficients between all variables measured in this study were obtained for the experimental group. The resulting correlation matrix (Appendix J) was utilized to test for significant correlations between selected variables.

To compare the entering cognitive and affective behaviors toward mathematics of the experimental group with those of other freshmen groups, the Dunnett t-test (35) was used to determine whether significant differences existed between the experimental group and each of the freshmen comparison groups on tests of mathematics and arithmetic achievement, and tests of attitude toward different aspects of mathematics and mathematics learning in general.

Significance Level Chosen

The 5 percent level of acceptance or rejection of statistical hypotheses being investigated was selected as being sufficiently rigorous for the conditions of this study. Thus, if the probability was at or less than five times in one hundred that the observed difference could be attributed to chance, the research hypothesis was accepted; if the observed difference was of such magnitude that it might arise more than five times in one hundred through the operation of chance factor, the research hypothesis was rejected.

Summary

This chapter described the mathematics component of a new experimental teacher education program at Michigan State University and the procedures followed for its assessment.

The formative evaluation of the mathematics component of the experimental elementary teacher education program at Michigan State University took place during the academic year 1971-1972. The thirty-eight students forming the first group of prospective elementary teachers who participated in the experimental program were utilized for this evaluation. In addition, samples of other student groups were used for comparison purposes.

The following steps were followed for the evaluation of the experimental program:

1. Assessment of the mathematics component of the program by means of a criterion-referenced list developed according to topics suggested by specialists for the preparation of elementary school teachers.
2. Participation in the development of an integrated content-methods course in mathematics with an interdisciplinary team of professional people involved in education.

3. Implementation of the integrated content-methods course with the thirty-eight students participating in the experimental program.

4. Assessment of the content of the integrated content-methods course by means of the following instruments:

   a. Nine criterion-referenced achievement measures to assess mathematical competencies on prescribed objectives.

   b. Test of Basic Mathematical Understanding.

   c. Attitude Inventory and Attitude Scales.

The development and use of the test instruments as well as the statistical procedures used for the analysis of data were described in the last section of this chapter.

Results obtained from the different analyses and their interpretation are discussed in the following chapter.

CHAPTER IV

ANALYSIS OF DATA AND RESULTS

This chapter presents a summary of the data collected during this investigation, the analysis of data, and results based on this analysis. It consists of seven sections: (1) analysis of the mathematical content of the learning units, (2) comparison of the experimental group and other freshman groups on cognitive and affective behaviors toward mathematics, (3) evaluation of the experimental group performance on the criterion-referenced achievement measures, (4) effect of the experimental program on the basic mathematical understandings and attitudes toward mathematics, (5) comparison of the experimental group with a regular elementary teacher education group on mathematical understandings and attitudes toward mathematics, (6) correlation data, and (7) evaluation of student reaction to the mathematics component of the experimental program. A summary of results conclude Chapter IV.

Analysis of the Mathematical Content of the Learning Units

Most fundamental to the use of formative evaluation is the selection of a unit of learning. Within a course or education program there are parts or divisions which have a separable existence such that they can, at least for analytic purposes, be considered in relative isolation from other parts. While these parts may be interrelated in various ways so that the learning (or level of learning) of one part has consequence for the learning of others, it is still possible to consider the parts separately (4).

In this study each of the nine mathematical topics will be covered in one learning unit. Each learning unit, devoted to one mathematical topic, contains the mathematical competencies prescribed for that topic. The following is a list of the mathematical competencies introduced under each of the nine learning units.

1. Measurement

    1) To learn the concepts of volume and area of various geometric solids.

    2) To learn how to apply the concept of similarity to measurement.

    3) To learn the metric system of measurement and to be able to convert English to metric and vice versa.

    4) To learn the concepts of relative error and the greatest possible error in measurement.

5) To learn that measurement of areas and volume is approximate.

6) To learn about angles, and sum of angles of different geometric figures.

7) To learn some basic information about coordinate geometry and map reading.

8) To learn about linear measurement and scaling.

9) To be able to utilize the acquired knowledge in problem solving situations.

2. <u>Systems of Numerations</u>

1) To learn some motivation and history behind the study of numeration systems.

2) To learn to interpret a numeration system using different symbols.

3) To learn about the properties of positional systems of numeration.

4) To learn to write a numeral in expanded notation.

5) To learn about positional systems of numeration with base other than ten.

6) To learn to add, subtract, multiply two or more numerals in base other than ten.

7) To learn to convert numerals in base ten into numerals in other bases and vice versa.

8) To learn to add and subtract in base twelve.

9) To be able to apply the acquired knowledge in problem solving situations.

3. <u>Sets and Set Relations</u>

1) To learn to identify elements of a set.

2) To learn to identify subsets (proper and otherwise) of a given set as well as whether the subsets are disjoint or intersecting.

3) To learn about equivalent and equal sets.

4) To learn to identify and describe the intersection and/or union of sets.

5) To learn to identify the complement of a set.

6) To learn to identify finite and infinite sets.

7) To learn to utilize Venn diagrams to depict the relationship between two or more sets.

8) To learn to describe, using set notation, the union and/or intersection of two or more sets.

9) To learn about the concept of greater than and less than as they relate to set relation.

10) To learn to utilize correctly the symbolization commonly used to describe sets and set relations.

4. The Whole Number System

1) To learn the definitions and some properties of the whole number system.

2) To learn and apply a formal definition of addition of whole numbers and the basic properties of addition such as closure, commutative, and associative properties.

3) To learn about order relation for the whole numbers.

4) To learn the basic properties of the operation of multiplication such as the closure, commutative, associative and distributive properties.

5) To learn about additive and multiplicative identity.

6) To learn about the operations of subtraction and division in relation to addition and multiplication.

7) To learn and apply their knowledge of the addition, subtraction, multiplication and division algorithms for whole numbers.

8) To learn the definitions of prime and composite numbers and divisors and to learn the Fundamental Theorem of Arithmetic, and the prime factorization theorem.

9) To learn about number patterns by developing formulas for sums of number sequences.

5. <u>The Rational Number Systems-- Fractions</u>

1) To learn some motivation and history behind the construction of fractions.

2) To learn a formal definition of fractions (as ordered pairs of whole numbers belonging to the same equivalence set).

3) To learn the formal definition of addition and multiplication of fractions.

4) To learn about subtraction and division of fractions.

5) To learn how to apply the identity and inverse properties of addition and multiplication of fractions.

6) To learn some of the basic properties of the rational numbers such as the order, fractional representation, commutative, associative, and distributive properties.

7) To learn about the least common denominator and the greatest common factors and applying this knowledge in arithmetic operations.

8) To be able to apply the acquired knowledge of fractions in problem solving situations.

6. <u>The Rational Number System-- Decimals</u>

1) To learn some motivation and history behind construction of decimals.

2) To learn a formal definition of decimals in terms of place-value structure, and apply this knowledge in decimal expansion.

3) To learn about the four basic arithmetic operations with decimals.

4) To learn about scientific notation and its application.

5) To learn how to convert decimals into fractions and vice versa.

6) Estimate sums, differences, product and quotients of two or more decimal numbers to the nearest specified place.

7) To learn about rates and percents and be able to convert decimals into percents and vice versa.

8) To learn to convert decimals written in base other than 10 to their equivalent in base 10.

9) To be able to apply the acquired knowledge about decimals in problem solving situations.

7. Relations and Functions

1) To learn the definition of the reflexive, symmetric and transitive properties of a relation and to be able to determine whether a relation possesses any of them.

2) To learn the definition of equivalence relation on a set and to be able to determine if a given relation is an equivalence relation.

3) To learn and apply a formal definition of a relation and to identify function as special relation.

4) To learn and apply the definitions of domain, range and inverse of a relation and function.

5) To learn to plot a graph of a given relation.

6) To learn to sketch a non-linear function.

8. Statistics and Probability

1) To learn the definition of probability of an event in a sample space.

2) To learn about probability of independent and dependent events.

3) To learn to compute the probability of occurrence of at least one event.

4) To learn about some correct usage of sampling procedures.

5) To learn the definitions of and methods of computation of basic descriptive statistical data such as mean, mode, median and range.

6) To learn about the statistical measure of variability of data.

7) To learn some basic information about statistical inference.

8) To be able to apply the acquired knowledge about statistics and probability in problem solving situations.

9. <u>Mathematical Systems</u>

1) To learn the formal definition of a mathematical system and be able to identify examples of such system.

2) To learn the rudiments of clock arithmetic, the definition of congruence derived from it, and to observe that congruence is an equivalent relation.

3) To learn the definition of a mathematical field and some of the basic properties of a field, and be able to identify examples of a field.

4) To learn some of the basic properties of multiplicative and additive inverse of a field.

5) To learn to compute the addition and multiplication table in different modula systems.

Findings

Item by item comparison between the criterion-reference list and the topic listed above shows that the following topics are not included in the mathematics curriculum of the experimental program:

1. Divisibility rules
2. Exponents
3. Real Numbers
4. Square Roots
5. Basic concepts of Geometry.

However, as noted earlier, the real number system, which includes studies of square roots, and geometry will be covered in the second course during the junior year. The two topics left (exponent and divisibility rules) are usually covered under the study of the whole number system or mathematical systems. These topics could have been and should be incorporated into the first year course as they are highly related and also heavily emphasized in elementary mathematics textbooks. The prospective teacher could learn the utilization of manipulative materials as a method of instructing these two topics to elementary school children. As a whole, of the thirty-two topics included in the criteria-reference list that were applicable to the subjects covered in the first year course, thirty were included (94 percent). It is noted, however, that many topics are included in the experimental program that are not included in the criterion-reference list; these topics must be included as they facilitate the development of the required topics. For example, learning that measures of area and volume are approximate will facilitate understanding of the concepts of relative error and greatest possible error.

Based on the above, it is concluded that the mathematical competencies prescribed by the experimental program are sufficient in meeting the need of the future elementary school teacher in arithmetic.

Comparison of the Experimental Group and Other Freshman Groups on Cognitive and Affective Behaviors Toward Mathematics

The thirty-eight freshman elementary education majors who participated in the experimental program were selected from those students who volunteered to participate. It is reasonable, therefore, to question whether these students differed in their entry characteristics from the other freshman students at Michigan State University. The investigator was particularly interested in knowing if these students who volunteered and were selected differed from other freshman students on their cognitive and affective behaviors in mathematics.

As part of the overall evaluation of the TTT program at Michigan State University, the evaluation team selected three first-term freshman groups with declared majors in: (1) elementary education--but did not volunteer for the experimental program, (2) mathematics and secondary education, and (3) mathematics. From each of these groups, a simple random sample of students was selected and those who participated from each of the selected groups constituted the three "comparison groups."

In this investigation, the researcher was interested in determining whether: (1) the experimental group differed in their initial behaviors toward mathematics from other freshman groups with similar academic major--elementary education majors, (2) the experimental group differed from freshman groups with declared interest in mathematics--mathematics-secondary education majors and mathematics majors.

Instrumentation

The following instruments were administered during the fall term of 1971-1972 to the experimental group and the three "comparison groups":

1. MSU basic skill and placement tests in arithmetic and mathematics.
2. Dutton Attitude Scale.
3. Attitude scales developed by The International Study of Achievement in Mathematics.[1]

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[1]A copy of this scale is to be found in Appendix D.

The MSU basic skill and placement tests are used by the University to assess entering freshman ability in mathematics.

The Dutton Attitude Scale was discussed earlier.

The Attitudes Scales developed by The International Study of Achievement in Mathematics, were constructed to measure student attitudes toward:

1. mathematics as a process,
2. difficulties of learning mathematics,
3. place of mathematics in society,
4. school and school learning.

The coefficients of reproducibility obtained from the Guttman Scale Analysis for these scales ranged from a low of .88 to a high of .95 when tested on American preuniversity-year students. These coefficients were considered acceptable by Guttman (17:118).

Data Analysis

Summary of statistical data of the experimental group and the "comparison groups" on each test instrument are shown in Table 4.

Table 4

Means and Standard Deviations on Entry Data for the Experimental Group and Three Freshman Comparison Groups

Group		Experimental Group	Elementary Education Majors	Mathematics Secondary Education Majors	Mathematics Majors	Estimate of Pooled Variance
1)[a]	Mean	32.41	30.83	36.56	37.00	16.35
	S.D.	4.37	5.85	2.68	2.75	
2)	Mean	15.68	13.36	23.74	25.18	30.02
	S.D.	6.08	6.56	4.50	4.77	
3)	Mean	5.57	5.85	7.96	7.43	2.72
	S.D.	1.81	1.92	0.81	1.90	
4)	Mean	6.92	5.97	8.65	9.32	8.99
	S.D.	2.18	2.26	3.48	3.63	
5)	Mean	8.46	8.31	9.47	9.45	9.97
	S.D.	2.68	3.37	3.02	3.52	
6)	Mean	14.05	14.11	14.39	13.97	10.39
	S.D.	2.68	3.81	2.66	3.68	
7)	Mean	8.51	8.86	9.14	9.26	11.25
	S.D.	2.34	3.70	3.67	3.37	

[a]These numbers refer to the following:

1) = MSU Arithmetic Test
2) = MSU Mathematics Test
3) = Dutton Attitude Scale
4) = Attitudes Toward Mathematics as a Process
5) = Attitudes Toward Place of Mathematics in Society
6) = Attitudes Toward School and School Learning
7) = Attitudes Toward Difficulties of Learning Mathematics.

In order to determine the significance of the difference between the experimental group and each of the "comparison groups," the Dunnett t-test was used.[2]

To determine whether the observed t-ratio was significant at the 0.05 level of confidence, t-tables designed by Dunnett (35:873) were utilized.

Table 5 shows the t-ratios obtained from applying Dunnett t-test to the entry data of the experimental group and the "comparison groups."

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[2]The following formula was used in computing the t-ratios for each test.

$$t = \frac{M_{exp} - M_i}{\sqrt{2MS_{err}/\tilde{n}}}$$

Where M_{exp} is the mean score of the experimental group

M_i is the mean score of "comparison group" i

$\tilde{n}$ is the harmonic mean, which is equal to:

$$\frac{4}{1/n_1 + 1/n_2 + 1/n_3 + 1/n_4}$$

and MS_{err} is an unbiased estimator of the pooled variance. (It is in fact the value of the mean squares of error obtained from within-group data. Appendix P includes a summary of the analysis of variance for each test result.)

Table 5

t-Values for Mean Comparison of Experimental Group and the Three "Comparison Groups" on Entry Characteristics

	Elementary Education Majors	Mathematics Secondary Education Majors	Mathematics Majors
1)[a]	1.71	4.51*	4.99*
2)	1.50	5.20*	6.13*
3)	0.75	6.35*	4.95*
4)	1.38	2.50*	3.48*
5)	0.44	0.81	0.97
6)	0.29	1.94	1.90
7)	0.08	0.45	0.11

[a]These numbers refer to the following:
1) = MSU Arithmetic Test
2) = MSU Mathematics Test
3) = Dutton Attitude Scale
4) = Attitudes Toward Mathematics as a Process
5) = Attitudes Toward Difficulties of Learning Mathematics
6) = Attitudes Toward Place of Mathematics in Society
7) = Attitudes Toward School and School Learning.

*Significant beyond the .05 level.

Findings

1. The MSU Basic Skill Test in Arithmetic.--This test assessed the student knowledge in arithmetic in general. The mean score of the experimental group on this test was 32.41, which was not significantly different from the mean score of the elementary education majors ($M = 30.83$). However, when the experimental group was compared with the mathematics-secondary education majors ($M = 36.56$) and with mathematics majors ($M = 37.00$), the t-ratios were highly significant in favor of the secondary and mathematics majors ($p < .001$).

2. The MSU Basic Skill Test in Mathematics.--The content of this test is designed to assess general mathematical knowledge with emphasis on algebra and geometry. The mean score of the experimental group was 15.68, which was 2.32 higher than the score of the elementary education majors, a difference not significant at the 0.05 level. However, the experimental group scored significantly lower than the mathematics-secondary education majors ($p < .001$) and the mathematics majors ($p < .001$).

On the evidence provided by the two measures cited above, it is concluded that there are no significant differences between the experimental group and other freshman elementary education majors on their cognitive behaviors toward arithmetic and mathematics.

The arithmetic and mathematical knowledge of the students in the experimental group when entering college was significantly lower than the knowledge of students with specified interest in the subject (the mathematics-secondary education majors and the mathematics majors).

3. Arithmetic Attitude Scale.--The Dutton Attitude Scale was utilized to assess the students' feelings toward mathematics. Possible scores on this scale range from 1.0 to 10.5.

When the mean score of the experimental group ($M = 5.57$) was compared with the mean score of the elementary education majors group ($M = 5.85$), the t-ratio obtained ($t = 0.75$) was not significant at the 0.05 level. The experimental group scores, however, were significantly lower ($p < .001$) than those of the mathematics-secondary education majors ($M = 7.96$) and those of the mathematics majors ($M = 7.43$). The relatively high scores of these two groups were expected since they have an exhibited interest in the subject. On the other hand, the experimental group and the elementary education majors have relatively low scores when compared with the scores of third and fourth year elementary education majors in other studies (48, 49). This may be due to the fact that the two freshman groups base their attitudes solely on the experience of their pre-college education, while the attitudes of those students in other studies were influenced by their college training.

4. Attitudes Toward Mathematics as a Process (8 items).--This scale inquired about degree to which mathematics is viewed as a fixed and given, once for all times (a low score), or as something that is developing, and constantly changing (a high score). Possible scores on this scale range from 0 to 16.

Analysis of data pertaining to this scale revealed no significant difference between the mean score of the experimental group (M = 6.92) and the mean score of the elementary education group (M = 5.97). However, the experimental group had a significantly lower attitude toward mathematics as a process than did the secondary education majors (M = 8.65) and the mathematics majors (M = 9.32).

5. Attitudes Toward Difficulties of Learning Mathematics (7 items).--This scale referred to the perceived care of learning mathematics. A low score indicates that the student views mathematics as a difficult subject to comprehend, while a high score indicates that students view mathematics as a subject that can be learned by most. Possible scores range from 0 to 14.

Analysis of data revealed no significant differences between the mean score of the experimental group and the mean score of any of the three "comparison groups." The mean ranged from 8.51 for the experimental group to 9.26 for the mathematics majors.

6. Attitudes Toward Place of Mathematics in Society (8 items).--This scale represents an expression of the belief that mathematics has an important role in our society. A low score indicates a judgment that mathematics is of little value and a high score represents an expression of the belief that mathematics has a vital role. Possible scores range from 0 to 16.

Analysis of data relevant to this scale revealed no significant differences between the scores of the experimental group and the scores of any of the three "comparison groups." The mean scores ranged from a low of 8.31 for the elementary education group to a high of 9.26 for the mathematics majors.

7. Attitudes Toward School and School Learning (11 items).--This scale inquires into the feelings of the students toward school in general. A low score indicates dislike of school and general dissatisfaction with school learning, while a high score indicates enjoyment of school and feelings of challenge in learning. The range of possible scores for this scale is from 0 to 22.

The mean score of the experimental group on this scale was 14.05, which was not significantly different from the mean score of any of the three other "comparison groups." The relatively high scores on this scale indicate a high positive attitude by all groups toward the importance of school and the experience it provides.

Conclusions

On the evidence provided by the analysis of data of the seven attitude scales described above, it can be concluded that the freshman students who participated in the experimental program did not differ significantly from other freshman elementary education majors who did not volunteer for the experimental program on their attitudes toward: (1) arithmetic, (2) mathematics as a process, (3) difficulties of learning mathematics, (4) place of mathematics in society, and (5) school and school learning.

The experimental group, on the other hand, tended to have significantly less positive attitudes toward arithmetic than either the mathematics-secondary education majors or the mathematics majors. These two groups also tended to view mathematics as developing and constantly changing while the experimental group tended to view it as a rigid subject with rules to follow in solving problems.

On the tests of attitudes toward difficulties of learning mathematics, place of mathematics in society and school and school learning, there were no significant differences between the scores of the students in the experimental group and either the students in the mathematics-secondary education majors or the mathematics majors.

To summarize, the entering cognitive and affective behaviors toward mathematics of the students who volunteered to participate in the experimental program were similar to those of other freshmen with similar interest of becoming elementary school teachers. These entering behaviors, however, were significantly different from those of other freshman groups with specified interest in mathematics.

Evaluation of the Experimental Group Performance on the Criterion-Referenced Measures

In this part of the study, results of pre- and post-test scores on the criterion-referenced measures were analyzed to evaluate the extent of accomplishment of the experimental group on the prescribed mathematical competencies. The evaluation was carried out in two parts:

1. To determine the significance of gain in achievement on the prescribed mathematical competencies between pre- and post-test scores.
2. To determine whether a specified degree of mastery over these competencies has been achieved.

Hypotheses Tested

The following multivariate hypotheses and associated univariate hypotheses were tested:

A1. There will be a significant difference between the post-test means and the pre-test means of the experimental group on the criterion-referenced measures.

Symbolically:

$$\begin{pmatrix} \bar{Y}_1 - \bar{X}_1 \\ \bar{Y}_2 - \bar{X}_2 \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ \bar{Y}_9 - \bar{X}_9 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{pmatrix}$$

where $\bar{Y}_i$ is the post-test mean on measure i, and
$\bar{X}_i$ is the pre-test mean on measure i.

The associated univariate hypotheses also tested were: the post-test mean of the experimental group will significantly differ from their pre-test mean on the criterion-referenced measure in:

1. Measurement
2. Numeration Systems
3. Sets and Set Relations
4. Whole Numbers
5. Fractions
6. Decimals
7. Relations and Functions
8. Probability and Statistics
9. Mathematical Systems.

A2. *There will be no significant difference between the post-test means and the mastery level (80 percent) on the criterion-referenced measures.*

Symbolically:

$$\begin{pmatrix} \bar{Y}_1 - 80 \\ \bar{Y}_2 - 80 \\ . \\ . \\ . \\ . \\ \bar{Y}_9 - 80 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ . \\ . \\ . \\ . \\ 0 \end{pmatrix}$$

The associated univariate hypotheses also tested were: the post-test mean of the experimental group will be at least equal to the mastery level (80 percent) on the criterion-referenced measure in:

1. Measurement
2. Numeration Systems
3. Sets and Set Relations
4. Whole Numbers
5. Fractions
6. Decimals
7. Relations and Functions
8. Probability and Statistics
9. Mathematical Systems.

Data Analysis

Data collected through the administration of pre- and post-test forms of the criterion-referenced measures developed by this investigator for use in the present study were utilized to test Hypotheses A1 and A2.

The experimental group scores on these measures are presented in Appendix G. Data included in the tables in this section were drawn from Appendix G.

Pre- and post-test means, standard deviations, and mean differences for the criterion-referenced measures are shown in Table 6.

Univariate and multivariate analysis of variance techniques were utilized in the analysis of data related to Hypotheses A1 and A2.

In the multivariate analysis of variance, the effect of the instructional treatment on all criterion measures was observed simultaneously, taking into account the correlation between these measures. The multivariate test considered student's response to all measures as a single response, thus providing information about the total effect of the treatment. The univariate hypotheses, on the other hand, examined the student response to each measure separately.

Findings

Hypothesis A1.--The data in Table 6 show gains made by the experimental group on all criterion-referenced measures. The increase ranged from 2.35 to 32.01 points. When the column vector of mean differences was tested against zero column vector, the resulting multivariate F value was 26.83 which was highly significant ($p < 0.0001$). Based on this result, the multivariate Hypotheses A1 which stated that *there will be a significant difference between the post-test means and the pre-test means of the experimental group on the criterion-referenced measures* was accepted.

Table 6

Means and Standard Deviations of Pre- and Post-Test Scores on the Nine Criterion Measures for the Experimental Group

	Variable	Pre-Test Mean	Pre-Test S.D.	Post-Test Mean	Post-Test S.D.	Mean Differences
1)	Measurement	63.97	21.33	66.32	23.41	2.35
2)	Numeration Systems	64.79	24.59	75.45	22.54	10.66
3)	Sets and Set Relations	59.63	23.65	83.03	12.53	23.40
4)	Whole Numbers	55.37	17.18	79.00	16.54	23.63
5)	Fractions	62.00	18.92	80.97	16.22	18.97
6)	Decimals	68.45	18.82	81.53	15.12	13.08
7)	Relations and Functions	50.92	16.52	82.55	15.44	31.63
8)	Probability and Statistics	56.26	18.15	76.76	16.70	20.50
9)	Mathematical Systems	33.52	25.44	65.53	20.74	32.01

The obtained probability on the multivariate test prompted consideration of the univariate hypotheses. Table 7 summarizes the findings for each univariate hypothesis that was also tested. Results of the analysis indicated the following:

1. On the univariate test of measurement, the differences between pre- and post-test means on this criterion-measure were not significant at the 0.01 level of confidence. The univariate hypothesis associated with this test which stated that the post-test mean of the experimental group will be significantly different from their pre-test mean on the criterion-referenced test in measurement was rejected.

2. The instructional treatment of the integrated content-methods course had a positive effect on the students performance on the other eight criterion-referenced measures. Significant differences in favor of the post-test means were noted between pre- and post-test means on the criterion-referenced measures in:

a. Numeration systems ($p < 0.005$)

b. Sets and Set Relations ($p < 0.0001$)

c. Whole Numbers ($p < 0.0001$)

d. Fractions ($p < 0.0001$)

e. Decimals ($p < 0.0001$)

f. Relations and Functions ($p < 0.0001$)

Table 7

Multivariate Analysis of Variance for the Experimental Group on Differences Between Pre- and Post-Test Scores on the Nine Criterion Measures

Multivariate $F = 26.8335$ $p < 0.0001$

	Variable	Between Mean Square	Univariate F	Significance Probability
1)	Measurement	208.4474	0.8454	0.3639
2)	Numeration Systems	4316.4474	8.8528	0.0052
3)	Sets and Set Relations	25740.0263	40.6567	0.0001
4)	Whole Numbers	21221.1579	99.7837	0.0001
5)	Fractions	13680.0263	54.7736	0.0001
6)	Decimals	6500.2368	35.5532	0.0001
7)	Relations and Functions	38021.1579	114.6630	0.0001
8)	Probability and Statistics	15969.5000	53.7865	0.0001
9)	Mathematical Systems	38912.0000	71.0263	0.0001

g. Probability and Statistics ($p < 0.0001$)

h. Mathematical Systems ($p < 0.0001$).

Based on these results, the univariate hypotheses, which stated that the post-test mean of the experimental group will be significantly different from their pre-test mean on the criterion-referenced measure in:

a. Numeration Systems

b. Sets and Set Relations

c. Whole Numbers

d. Fractions

e. Decimals

f. Relations and Functions

g. Probability and Statistics

were accepted.

Hypothesis A2.--The vector column of differences between post-test means on the criterion-referenced measures and the mastery level of 80 percent was tested against a zero column vector. The multivariate F value associated with this test was 12.68 which was highly significant ($p < 0.0001$). The multivariate Hypothesis A2 which stated that *the post-test means of the experimental group will not be significantly different from the mastery level (80 percent) on the criterion-referenced measures* was rejected.

The obtained probability on the multivariate test prompted consideration of the univariate hypotheses. Table 8 summarizes the findings for each univariate

Table 8

Multivariate Analysis of Variance for the Experimental Group on Differences Between Post-Test Scores and Mastery Level (80 percent) on the Nine Criterion Measures

Multivariate $F = 12.6829$ $p < 0.0001$

Variable	Between Mean Square	Univariate F	Significance Probability
1) Measurement	7115.7879	12.9798	0.0010
2) Numeration Systems	787.6053	1.5500	0.2210
3) Sets and Set Relations	348.0263	2.2167	0.1450
4) Whole Numbers	38.0000	0.1388	0.7117
5) Fractions	36.0263	0.1369	0.7135
6) Decimals	88.5263	0.3874	0.5378
7) Relations and Functions	247.6053	1.0390	0.3147
8) Probability and Statistics	398.1316	1.4284	0.2397
9) Mathematical Systems	7960.5263	18.5065	0.0002

hypothesis that was also tested. Results of these analyses indicated the following:

1. The post-test mean in Measurement ($M = 66.32$) was significantly ($p < 0.0001$) below the mastery level. The post-test mean in Mathematical Systems ($M = 65.53$) was also significantly below the mastery level. Based on these results the univariate hypotheses associated with testing the significance of difference between the post-test means and the mastery level on the criterion-referenced tests in (a) Measurement, and (b) Mathematical Systems were rejected.

2. At the 0.05 level of confidence, there were no significant difference between the post-test mean and the mastery level of 80 percent on each of the following criterion-referenced measures:

a. Numeration Systems ($p < 0.2210$)

b. Sets and Set Relations ($p < 0.1450$)

c. Whole Numbers ($p < 0.7117$)

d. Fractions ($p < 0.7135$)

e. Decimals ($p < 0.5378$)

f. Relations and Functions ($p < 0.3147$)

g. Probability and Statistics ($p < 0.2397$).

Based on these results the univariate hypotheses associated with testing the significance of difference between the post-test means and the mastery level on the criterion-referenced measure in the above seven topics were accepted.

Analysis of Test Results

Table 9 shows the percent of students in the experimental group who scored 80 or more (mastery level) on each of the criterion-referenced measures. Pre-test results show that students performed better on traditional topics with which they have had previous experience than on topics which were introduced for the first time such as Sets and Set Relations, Relations and Functions, Probability and Statistics, and Mathematical Systems. It was also noted that on the related topics, Whole Numbers, Fractions, and Decimals, many students became progressively more able as they learned the essentials on one unit to improve their performance on the next unit. Only 13 percent of the experimental group scored 80 or higher on the pre-test of Whole Numbers, while 21 percent scored 80 or higher on the pre-test of Fractions, and 37 percent attained the 80 percent or higher level on the pre-test of Decimals. The student performance on the pre-tests are obviously influenced by their performance on the post-tests of previous units. The experimental group showed significant improvement on achievement of the mathematical competencies prescribed by the program. However, it is not sufficient that these students improve their knowledge of mathematics, it is more important that they attain a certain level of achievement that would indicate mastery over that topic.

Table 9

Percentage of Students in the Experimental Group (N = 38) Attaining the Pre-Established Mastery Level

	Measure	Pre-Test (%)	Post-Test (%)
1)	Measurement	24	39
2)	Numeration Systems	37	50
3)	Sets and Set Relations	21	68
4)	Whole Numbers	13	61
5)	Fractions	21	79
6)	Decimals	37	74
7)	Relations and Functions	8	74
8)	Probability and Statistics	16	58
9)	Mathematical Systems	5	39

Bloom (4) suggested an accuracy level of 80 percent on each formative test as an indication of mastery. On the mathematical topics specified in the experimental program, students achieved mastery over all but two topics, Measurement and Mathematical Systems. Measurement was the first topic introduced and most students spent much time familiarizing themselves with the new surrounding and becoming acquainted with the manipulative materials in the mathematics laboratory. This, of course, minimized the amount of time spent on the mathematical activities associated with this topic. Mathematical Systems, on the other hand, was the last topic to be taught. Only three days were allocated for its instruction and the contents of this topic were completely new to most students (82 percent scored below 50 on the pre-test). Many students did not finish, in the limited time, all the activities in the unit file. Mathematical Systems was the only topic in which the instructional designs developed by the student were not implemented with the children at the elementary school mathematics laboratory due to the end of the academic year of the elementary school.

Overall, analysis of pre-test results indicated lack of understanding of basic mathematical concepts. While most students did comparatively well on computational problems, most had difficulties with problems dealing with the mathematical principles underlying the operations of addition, subtraction, multiplication, and division, as well as the basic properties of these operations.

Post-test results reflected the emphasis placed in this course upon insuring that the prospective teachers understand the basic mathematical concepts they are expected to teach children.

Effect of the Experimental Program on the Basic Mathematical Understandings and Attitudes Toward Mathematics

In this part of the study, the effect of the mathematics component of the experimental program upon the basic mathematical understandings and attitudes toward mathematics of the experimental group were analyzed.

These results are reported under two major headings: (1) growth in basic mathematical understandings, and (2) changes in attitudes toward mathematics. Within each heading related data were analyzed.

Growth in Basic Mathematical Understandings

The two forms of the test, "A Test of Mathematical Understanding," were utilized in an attempt to measure the basic mathematical understandings possessed by the experimental group prior to and after completing the integrated content-methods course in mathematics education.

Hypothesis B1

The hypothesis related to this aspect of the study was stated as: *There will be a significant difference on a test of basic mathematical understanding between the post-test scores of the experimental group and their pre-test scores.*

Data Analysis

Raw scores of the experimental group on both forms of the test of mathematical understandings are included in Appendix H.

Pre- and post-test means, standard deviations and changes (difference between means) on these tests were computed and are shown in Table 10.

In order to determine the significance of the difference between the pre- and post-test means (correlated), a t-test was used.[3]

The resulting t-ratio was compared with the "t" in a table designed for use in determining the significance of "t." These data are included in Table 10.

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[3]The following formulas were used to compute the significance of the difference between correlated means obtained from tests administered to the same group.

$$t = \frac{\overline{X}_1 - \overline{X}_2}{SE_{\overline{D}}}$$

Where $\overline{X}_1 - \overline{X}_2$ = difference between pre- and post-test means,

$SE_{\overline{D}}$ = standard error of the difference between correlated means

$SE_{\overline{D}}$ was computed by the following formula:

$$SE_{\overline{D}} = \sqrt{(SE_1)^2 + (SE_2)^2 - 2r_{12}\ (SE_1)\ (SE_2)}$$

Where SE_1 = standard error of pre-test,

SE_2 = standard error of post-test, and

r_{12} = correlation between pre- and post-tests.

Table 10

Pre- and Post-Test Results of the Experimental Group on the Test of Mathematical Understandings

	Pre-Test		Post-Test
Number of students	38		38
Mean	38.1316		44.0263
Standard deviation	4.7883		4.3027
Standard error	0.7872		0.7074
Correlation between pre- and post-test scores		0.7541	
Standard error of difference between means		0.5294	
Observed t-value		11.1347*	

*Significant at the 0.05 level ($t_{.05}$ (37) = 1.69).

Findings

The post-test mean was 5.90 points higher than the pre-test mean. When this mean difference was tested against the hypothetical zero gain, the resulting t-ratio was 11.13, which was highly significant ($p < .001$).

Based on the results of this criterion measure, it was concluded that the post-test scores were significantly higher than the pre-test scores. Hypothesis B1, which stated that *there will be significant difference on a test of basic mathematical understanding between the post-test scores of the experimental group and their pre-test scores*, was accepted.

Analysis of Pre- and Post-Test Results

Pre-test results.--The concepts which caused students most difficulties on the pre-test were: principles underlying number operations such as properties of addition and multiplication (items 17, 22, 29, 47), meaning of a partial product in multiplication and remainder in division (items 28, 38), converting decimals into fractions and vice versa (item 41), fundamental operations with bases other than 10 (items 7, 11), set vocabulary and set operations (item 55), and measurement of related geometric figures (items 33, 48). The incorrect responses selected by the students on the pre-test indicate previous teaching procedures which have emphasized computational aspects and drill procedures rather than understanding of basic arithmetic concepts. For example, when asked to choose the sentence that best tells why the answer in the division problem ($5 \div \frac{3}{4} = 6\frac{2}{3}$), is larger than 5, 82 percent of the students said because "inverting the division turned $\frac{3}{4}$ upside down" which indicates a memorization of rule rather than understanding of the concept that the divisor $\frac{3}{4}$ is less than 1.

On the fifty-five-item pre-test, the scores ranged from a low of 32 to a high of 49 with a mean score of 38.13.

Post-test results.--The majority of the students were able to improve their understanding of basic mathematical concepts during the integrated content-methods course.

The emphasis, during this course, on teaching mathematical concepts rather than on drill work and computational skills, increased the students capacity to analyze problems and to follow reasoning. Improvement was noted on problems related to operations with Whole Numbers, Fractions, and Decimals. About 30 percent of the students still had difficulties with problems related to Measurement of areas and volumes of geometric figures, and on Set Operations (items 50, 53, 55).

On the fifty-five-item post-test, the scores ranged from a low of 33 to a high of 51 with a mean score of 44.02.

Changes in Attitude Toward Mathematics

The 1962 Revised Dutton Arithmetic Attitude Inventory (Form C) was utilized in an attempt to evaluate changes, if any, in the attitudes of the prospective elementary school teachers in the experimental group toward mathematics which occurred during the academic year 1971-1972 during which the experimental program was implemented.

Hypothesis B2

The hypothesis related to this part of the study was stated as: *There will be significant differences on an arithmetic attitude inventory between the post-test scores of the experimental group and their pre-test scores.*

Treatment of Data

Responses to the fifteen statements on the first part of the attitude inventory were tabulated according to item. Each item on the scale was assigned a scale value (from 1.0 which represents an extremely negative attitude toward arithmetic to 10.5 which represents an extremely positive attitude). The individual score was obtained by finding the average or median scale of the statements which the student selected.

A composite report of the results from the administration of this arithmetic attitude inventory has been included in Appendix H. Data included in the tables in this section were drawn from Appendix H.

Findings

Pre- and post-test means, and standard deviations for scores on the attitude scale were computed. The mean difference for the experimental group was tested against hypothetical zero gain through the use of t-test for significance of difference between correlated means. These data are presented in Table 11.

The obtained t-value of 6.59 on the attitude scale was highly significant. The mean score of the experimental group ($M = 7.07$) was significantly higher than the pre-test mean ($M = 5.57$).

Table 11

Pre- and Post-Test Results of the Experimental Group on the Dutton Attitude Scale

	Pre-Test		Post-Test
Number of students	38		38
Mean	5.5684		7.0737
Standard deviation	1.8154		1.5206
Standard error	0.2985		0.2500
Correlation between pre- and post-test scores		0.6665	
Standard error of difference between means		0.2283	
Observed t-value		6.5935*	

*Significant beyond the 0.05 level ($t_{.05}(37) = 1.69$).

Based on the result of this criterion measure, Hypothesis B2 which stated that *there will be significant difference on an arithmetic attitude inventory between the post-test scores of the experimental group and their pre-test scores* was accepted.

Related Questions

In addition to the statements in the Attitude Inventory, other questions were included regarding attitudes toward arithmetic. Space was provided for the students to: (1) estimate their general feeling toward arithmetic, (2) indicate the grade level in which attitude toward arithmetic was influenced most, and (3) list two things liked and two things disliked about the subject.

Findings

1. General feeling toward arithmetic.--Each student was asked to circle a number between 1 and 11 to show his or her overall feeling toward arithmetic (1 representing extreme dislike and 11 representing extreme like). A summary of student judgment of individual attitude toward arithmetic on the pre- and post-tests is shown in Table 12.

Table 12

Students (N = 38) Feelings About Arithmetic in General

Feeling About Arithmetic in General		Pre-Test	Post-Test
Extreme Dislike	1	1	0
	2	0	1
	3	2	0
	4	0	3
	5	3	0
	6	7	5
	7	4	3
	8	4	11
	9	11	5
	10	3	4
Extreme Like	11	3	6

The mean score of the experimental group on the pre-test was 7.52 while it was 8.00 on the post-test. It was noted that the students' judgment was considerably higher than the scores obtained from the Attitude Inventory, where the mean pre-test score was 5.57 and the mean post-test score, 7.07. Dutton (48:420) attributes such a result to the averaging of both favorable and unfavorable items checked on the scale by each individual to secure the overall value of the inventory.

2. Grade where attitudes were developed.--Feeling toward arithmetic is developed in all grades. However, the most crucial years for the students in the experimental group seemed to be when the students were in the third through sixth grade, as reported in both the pre- and post-test data (see Table 13 below). These results are consistent with Dutton's findings (48).

Table 13

Grade Level Where Students' (N = 38) Attitudes Were Developed

Grade Level	Pre-Test	Post-Test
1	2	2
2	3	4
3	5	5
4	6	6
5	3	4
6	5	4
7	2	3
8	3	3
9	2	2
10	3	3
11	2	2
12	2	1

3. Aspects of arithmetic liked or disliked most.--Students in the experimental program were asked to list two aspects of arithmetic liked most and two aspects liked least. This technique was used to give equal treatment to favorable and unfavorable feelings.

In tabulating the data collected at the beginning and at the end of the academic year 1971-1972, it was noted that the challenge presented by arithmetic was the most frequent positive response given by the students both at the beginning and at the end of the school year. Story problems were the aspect of arithmetic disliked by most students (see Table 14).

Table 14

Aspects of Arithmetic Students (N = 38) Liked and Disliked Most

	Pre-Test	Post-Test
Aspects of Arithmetic Liked Most:		
1. The challenge presented by arithmetic.	21	18
2. Has practical application.	14	15
3. Stimulating, enjoy working with numbers.	6	7
4. Necessary for everyday life.	8	10
5. Satisfaction in working out problems.	4	7
6. Solving word problems.	3	5
7. Algebra.	3	0
8. Games about arithmetics.	1	4
Aspects of Arithmetic Disliked Most:		
1. Story problems.	17	12
2. Teachers.	9	4
3. Boredom and frustration.	6	3
4. Memorizing rules.	6	5
5. Drill and busy work.	5	2
6. Proofs.	5	1
7. Set theory	0	4
8. Long division.	4	1
9. Metric system.	0	3
10. Measurement.	0	2

It was noted by the investigator that on the pre-test, the students tended to choose one of the statements on the Attitude Inventory (first 15 statements) as the aspect of arithmetic liked or disliked most, while in the post-test, many students selected new aspects that they had confronted for the first time during the experimental course such as games about arithmetic, set theory, and metric system. In general, the results obtained in this study in relation to this particular question are similar to those obtained by Dutton in a study of attitudes of prospective elementary school teachers toward arithmetic (48).

Comparison of the Experimental Group with a Regular Elementary Teacher Education Group on Mathematical Understandings and Attitudes Toward Arithmetic

The two equivalent forms of the test "A Test of Mathematical Understandings" and the Dutton Arithmetic Attitude Scale were administered to a group of prospective elementary teachers in the regular teacher education program who are enrolled in the methods course (Education 325E). This particular class was chosen for three reasons: (1) it was taught during the same term as the experimental integrated content-methods course, (2) it was taught by a member of the TTT program also involved in the experimental course, and (3) all the students in that course had had the mathematics content course (Mathematics 201) within that school year. The pre-tests were administered on the first day of the term and the post-tests during the last day of the term.

Data collected through the administration of the two equivalent forms of the test of mathematical understandings and the attitude scale were utilized in an attempt to compare the basic mathematical understandings and attitudes toward arithmetic between the experimental group after completing the first-year trial of the mathematics curriculum of the experimental program and a group of prospective elementary teachers in the regular teacher education program after completing their required mathematics education training. Summary of the data collected is presented in Table 15.

Table 15

Pre- and Post-Test Results of the Experimental Group and the Regular Methods Course (Education 325E) Students on the Test of Mathematical Understandings (MU) and Dutton Arithmetic Attitude Scale (AA)

Group	Variable	Number	Pre-Test Mean	Pre-Test S.D.	Post-Test Mean	Post-Test S.D.
Experimental	MU	38	38.13	4.79	44.03	4.30
	AA	38	5.57	1.82	7.07	1.52
Regular Methods Course (Education 325E)	MU	21	40.90	5.51	41.29	4.78
	AA	21	6.10	2.14	6.25	2.03

Hypothesis C1

The first hypothesis related to this part of the study stated: *"The adjusted mean post-test scores of the experimental group will be at least equal to the adjusted mean post-test scores of a group of prospective elementary school teachers enrolled in the regular teacher education program on a test of basic mathematical understanding."*

Findings

The analysis of covariance was utilized for the analysis of data. This statistical technique, known to be particularly applicable to any experiment, such as the present one, in which groups could not be randomized or equated before treatment, made it possible for the investigator to adjust the outcomes of the experiment (gains in mathematical understandings) in terms of a source of variation (the pre-test). The scores of fifty-nine prospective elementary teachers, thirty-eight in the experimental group and twenty-one upper-classmen in the regular methods course (Education 325E) on a test of basic mathematical understandings were used for the analysis. Data are presented in Table 16.

When the F-ratio was applied to the adjusted "among groups" and "within groups" variance, F was highly significant ($p < 0.001$) in favor of the experimental group. It was concluded therefore, that the two final means, when initial difference was allowed for, did differ significantly in favor of the experimental group. Thus, Hypothesis C1, that *the adjusted mean post-test scores of the experimental group will be at least equal to the adjusted mean post-test scores of a group of prospective elementary teachers in the regular teacher education program on a test of basic mathematical understanding* was accepted.

Table 16

Summary of the Analysis of Covariance for the Scores of the Experimental Group and the Regular (Education 325E) Students on the Test of Mathematical Understandings

Source of Variation	D.F.	SS_y	SS_{xy}	SS_x	SS_y'	MS
Among groups	1	101.5881	-102.7957	104.0179	269.1336	269.1336
Within groups	56	1141.2594	985.4398	1456.1516	474.3703	8.4709
Total	57	1242.8475	882.6441	1560.1695	743.5039	

$$F = \frac{269.1336}{8.4709} = 31.7716$$

Critical value of F at .05 level = 4.02

at .01 level = 7.10

Hypothesis C2

The second hypothesis related to this part of the study stated: *There will be a significant difference in arithmetic attitude inventory between the adjusted post-test scores of the experimental group and the adjusted post-test scores of a group of prospective elementary teachers enrolled in the regular teacher education program.*

Findings

Analysis of covariance was utilized for the analysis of data. The outcomes of the experiment (changes in attitudes toward arithmetic) were adjusted in terms of the initial source of variation (the pre-test).

The scores of fifty-nine prospective elementary teachers, thirty-eight in the experimental group and twenty-one in the regular methods course (Education 325E), were used for this analysis. Data are presented in Table 17.

When the F-ratio was applied to the adjusted "among" and "within" variances, it was noted that the observed F ($F_{obs} = 13.65$) was highly significant in favor of the experimental group ($p < 0.001$). Based on this evidence, it was concluded that the two final means, when initial difference was allowed for, did differ significantly in favor of the experimental group. Thus, Hypothesis C2, that *there will be a significant difference on an arithmetic attitude inventory between the adjusted post-test scores of the* *experimental group and those of the students in the regular methods course (Education 325E)* was accepted.

Table 17

Summary of Analysis of Covariance for the Scores of the Experimental Group and the Students in the Regular Methods Course (Education 325E) on Dutton Arithmetic Attitude Inventory

Source of Variation	D.F.	SS_y	SS_{xy}	SS_x	SS_y'	MS
Among groups	1	9.1234	-5.9050	3.8220	18.2466	18.2466
Within groups	56	167.7461	140.8284	213.5621	74.8801	1.3371
Total	57	176.8695	134.9234	217.3841	93.1267	

$$F = \frac{18.2466}{1.3371} = 13.6464$$

Critical value of F at .05 level = 4.02

at .01 level = 7.10

Correlation Analysis

Thus far, the data collected from the experimental program have been utilized to determine the relative effect of the mathematics component on the mathematical achievement, understanding of basic mathematical concepts, and attitude toward arithmetic. In addition, attention has been given to the question of whether these effects are related or whether they are influenced by other factors such as the level of high school mathematics preparation and grade point averages.

For this phase of the study, scores from all pre- and post-tests as well as other background data of the experimental group were used to calculate an intercorrelation matrix. The resulting 36 by 36 matrix is included in Appendix J.

To determine whether the correlation coefficient between two variables is significantly different from zero at the 0.05 level, the following t - ratio was used

$$t = \frac{r_{xy}\sqrt{N-2}}{\sqrt{1-r_{xy}^2}}$$

where r_{xy} is the correlation coefficient between variables x and y. With thirty-eight subjects (df = 36), a coefficient which is more than 0.321 or less than -0.321 is considered to be sufficient for significance at the 0.05 level of confidence.

Analysis of the intercorrelation matrix revealed the following:

1. On the nine criterion-referenced tests, the following results were noted:

a. There were significant correlations between pre- and post-test scores on each measure.

b. There were significant correlations between all post-test scores except the tests on Sets and Set Relations, and on Relations and Functions.

c. There were significant correlations between the post-test scores on the test of mathematical understandings and each criterion-referenced post-test except the test on Relations and Functions ($r = 0.271$).

d. There were significant correlations between the post-test scores on the Dutton Attitude Scale and each criterion-referenced post-test.

2. There was significant correlation between pre-test scores on the test of mathematical understandings and the arithmetic attitude scale ($r = 0.454$). There were significant correlation between the post-test scores on these two measures ($r = 0.668$). The correlations between pre- and post-test scores on the test of mathematical understandings ($r = 0.754$), and the arithmetic attitude scale ($r = 0.667$) were also significant. As expected, the correlation

coefficient between scores on the arithmetic attitude scale and the student rating of his general feeling toward arithmetic were high (for pre-test, $r = 0.615$; for post-test, $r = 0.786$). The significance of these correlations are consistent with results obtained in similar studies by Dutton (49) and Litwiller (82).

3. On the attitude scales toward mathematics, negatively significant correlations were noted between student performance on the test of mathematical understandings and (a) attitudes toward mathematics as a process ($r = -0.561$), and (b) attitudes toward difficulties of learning mathematics ($r = -0.323$). Positive correlations were noted between performance on test of mathematical understandings and (a) attitudes toward school and school learning ($r = 0.439$), and (b) attitudes toward place of mathematics in society ($r = 0.669$).

4. Significant correlations were obtained between the number of mathematics courses taken in high school and (a) pre- and post-test scores on the nine criterion-referenced measures, (b) pre- and post-test scores on the test of mathematical understandings, (c) pre- and post-test scores on the arithmetic attitude scale, and (d) high school grade point average.

5. The final grade the students received on the integrated content-methods course was significantly correlated

to post-test scores on the criterion-referenced tests, test of mathematical understandings, and arithmetic attitude scale.

6. Scores on the MSU basic skill test in mathematics were significantly correlated to scores on the pre-test ($r = 0.48$) and post-test ($r = 0.521$) of mathematical understandings. Scores on the MSU arithmetic test were not significantly correlated to any of the criterion-referenced measures or the test of basic mathematical understandings.

In all, the correlation data indicated significantly positive relation among the criterion-referenced tests, the standardized test of basic mathematical understandings, and the Dutton Arithmetic Attitude Scale.

High school mathematics preparation and overall grade point average were significantly correlated to performance on pre-tests of mathematical understandings, and attitudes toward mathematics. Final grade on the integrated content-methods course, as assigned by the faculty members responsible for the course, was highly correlated with the post-test scores on the test of mathematical understandings ($r = 0.572$) and the Dutton Arithmetic Attitude Scale ($r = 0.657$).

Evaluation of Student Reaction to the Mathematics Component of the Experimental Program

Prospective elementary teachers' reactions to different aspects of the mathematics component of the experimental program are an important facet of the formative evaluation process. If the student reacts positively to new methods and procedures of instruction, he or she may be motivated to learn more under these methods, and may utilize similar procedures in future teaching strategies.

In this part of the study, the experimental group's responses to a questionnaire were analyzed to determine their reaction to specific aspects of the program. The eleven-item questionnaire was administered during the last day of classes after completing the integrated content-methods course.

A copy of the questionnaire is included in Appendix E. The first ten questionnaire items referred to particular aspects of the program with scaled responses (strongly agree, agree in general, undecided, disagree in general, and strongly disagree). The experimental group's response distribution is shown in Table 18.

Analysis of student responses to the questionnaire indicated the following:

1. When asked if more activities using manipulative materials should be used in the integrated content-methods

Table 18

Frequency Distribution of Experimental Group Response to Student Evaluation Questionnaire

Item	Strongly Agree	Agree in General	Undecided	Disagree in General	Strongly Disagree
1	0	16	12	10	0
2	9	21	4	4	0
3	8	23	5	2	0
4	4	11	12	10	1
5	3	8	7	17	3
6	4	18	6	9	1
7	8	16	12	2	0
8	0	5	2	26	5
9	1	1	4	25	7
10	13	18	3	3	1

course, 42 percent agreed with the statement, 32 percent were undecided, and 26 percent disagreed. This seems to indicate the group feels the amount of activities using manipulative materials was sufficient.

2. About 80 percent of the students suggested more time be spent on methods of teaching elementary school mathematics. This is probably due to the group realizing the need for such instruction during the clinical experience.

3. Thirty-one students (82 percent) thought more time should be spent on planning teaching strategies to be used at the elementary school (clinical experience). This response is consistent with their reaction that more time be spent on methods of teaching mathematics.

4. The group as a whole seemed undecided on whether more time should be spent in the mathematics laboratory at the elementary school. It was noted that the four students who "strongly agreed" with that statement had also scored very high on the mathematics tests and the attitude scale. A majority of the students seemed to feel that one-half hour per week of teaching experience is sufficient at this stage of their education.

5. When asked if there should be more lectures about mathematical content, 29 percent agreed with the statement, 53 percent disagreed, and 18 percent were undecided.

6. While the students did not want more lecture time on the mathematical content, the majority (58 percent) wanted more lectures on methods of teaching mathematics. Problems, encountered by the students at the clinical experience, may have prompted this reaction.

7. Students seemed to have enjoyed films related to the teaching of elementary mathematics. Only 5 percent of the students felt that the films were of no value.

8. A majority of students (82 percent) felt the number of weekly hours assigned to the integrated content-methods course should not be increased.

9. Only 5 percent of the students agreed there should be more time spent on paper and pencil problem solving activities, 84 percent disagreed with the statement, and 11 percent were undecided.

10. Eighty-two percent of the students liked the idea of working with elementary school teachers on planning of strategies for teaching mathematics at the elementary school. The students suggested that more time be spent on such activities.

Overall, the student reaction to the items on the questionnaire were quite consistent and seemed to indicate their satisfaction with the methods and procedures followed in the integrated content-methods course. They felt, however, that not enough emphasis is placed on planning strategies to be used with the elementary school pupils.

At the questionnaire's end, the students were asked to make suggestions that may help improve the mathematics component of the experimental program. Thirty-four (88 percent) students offered some comments about the program. The responses varied.

Many students wanted more time spent on preparing their lesson plan with the pupils in elementary school. Others stressed the need for more feedback from the faculty members on their work in the university and at the elementary school. Most students felt that the textbook assigned to the integrated content-methods course did not relate to the procedures followed in the mathematics laboratory, and was not clear. Following are some of the students' comments:

> Perhaps a question-and-answer sum-up of the week would be good on one of the last days. Just to be sure people know what's going on. Would also be nice for every person to get one of the conferences [with faculty members] around midterm if possible. Otherwise, I thought the math lab turned out pretty well.

> I really liked it when the Allen School teachers came and helped us with our strategies.

> The activities are usually pretty good, although some seem trivial at times. The method of presenting the materials, using folders, is superb! Whoever thought of that deserves a gold star. The textbook was useless, as far as I was

concerned. I never used it except for reference and when I used it for that, I usually found it inadequate.

I am strongly against having more paper and pencil work. All my years in Math, I have done written work. The math lab this term surprised me in showing me with manipulative materials how little I really understood math concepts.

My greatest problem came in thinking of strategies to use in teaching my math lesson; at times it was very difficult to think of activities that would hold the interest of the students [pupils at the elementary school].

More emphasis should be on the groups and their plans. More feedback from professors as to additional ideas for teaching from their experiences. The stress on individual work and creativity was very good.

Perhaps the course needs to be a little more structured at times.

More lectures or professors' explanations on certain subjects or materials.

More time at Allen Street School.

Before post-tests are given, a question and answer session would be helpful to those who did not understand some materials even after doing all activities.

Nuffield guides should be part of the Math Lab.

Summary

Analysis of the data collected during this investigation revealed the following results:

1. The mathematical content of the experimental program adequately meet the needs of the future elementary school teacher.

2. The mean post-test score of the experimental group was significantly higher than the mean pre-test score on the criterion-referenced measures in: (a) Numeration Systems, (b) Sets and Set Relations, (c) Whole Numbers, (d) Fractions, (e) Decimals, (f) Relations and Functions, (g) Probability and Statistics, and (h) Mathematical Systems.

3. The mean post-test score of the experimental group was not significantly different than the mean pre-test score on the criterion-referenced measure in Measurement.

4. The mean post-test score of the experimental group was not significantly different from the mastery level (score of 80 or higher) on the criterion-referenced measures in: (a) Numeration Systems, (b) Sets and Set Relations, (c) Whole Numbers, (d) Fractions, (e) Decimals, (f) Relations and Functions, and (g) Probability and Statistics.

5. The mean post-test score of the experimental group was significantly below the mastery level on the criterion-referenced measures in: (a) Measurement, and (b) Mathematical Systems.

6. There was significant difference on a test of mathematical understandings between the post-test scores of the experimental group and their pre-test scores.

7. There was a significant improvement on an arithmetic attitude inventory between the post-test scores of the experimental group and their pre-test scores.

8. The adjusted mean post-test score of the experimental group was significantly higher than the adjusted mean post-test score of a group of prospective teachers in the regular teacher education program on a test of mathematical understandings and on an arithmetic attitude inventory.

9. There were significant correlations between: (a) pre- and post-test scores on the criterion-referenced measures, (b) post-test scores on the test of mathematical understandings and the arithmetic attitude scale, (c) number of high school courses in mathematics and pre- and post-test scores on the test of mathematical understandings, (d) pre- and post-test scores on the mathematical understandings test and high school grade-point average.

10. The experimental group expressed desire for more participation in clinical experience concurrent with the laboratory oriented integrated content-methods courses.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The preceding chapters were devoted to a discussion on the current significance of the problem, a delineation of its purpose, and a description of the procedures followed in evaluating the effect of the mathematics curriculum of the experimental program upon cognitive and affective behaviors of a group of prospective elementary school teachers. Chapter V, the final chapter of this report, is devoted to: (1) a general summary of the study, (2) major conclusions, and (3) recommendations for future action and research.

Summary

This thesis reports the results of a formative evaluation of the mathematics component of an evolving elementary teacher education program at Michigan State University. This section contains a summary of this evaluation.

The Mathematics Component of the Experimental Program

An experimental elementary teacher education program was initiated at Michigan State University in the fall of 1971. The program is funded and staffed by the "Trainers of Teacher Trainers (TTT)" project, and is based on several aspects of the Behavioral Science Teacher Education Program (BSTEP) Model developed at Michigan State University in 1968. Although the program has many inovative facets in human and academic curricular areas, this evaluation is devoted to the mathematics component of the experimental program. The objectives of the experimental program in the field of mathematics education are to provide the prospective elementary school teacher with: (1) an adequate knowledge of the mathematics he or she would be required to teach, (2) an adequate knowledge and technique in teaching the mathematics to elementary pupils, (3) the opportunity to experiment with these teaching skills, and (4) understanding of human development and the nature of learning mathematics adequately well, as to adopt appropriate procedures to facilitate learning of mathematics.

Purpose

The major purpose of this investigation was the formative evaluation of the mathematics component of an experimental elementary teacher education program at

Michigan State University. Specifically, this investigation sought to (1) analyze and evaluate the adequacy of the mathematical content of the experimental program in meeting the future needs of the elementary school teacher in mathematics, (2) evaluate the effect of the instruction, as prescribed by the mathematics component of the experimental program, on the prospective elementary teachers who participated in the program, in relation to the specified competencies, (3) assess whether the students have achieved a degree of mastery over these competencies, (4) evaluate the basic mathematical understandings of the students who participated in the program prior to and after instruction, in order to assess the effectiveness of the prescribed mathematics treatment on their general mathematical knowledge, (5) assess the experimental program's effect on the attitudes toward mathematics of the students who participated in the program, (6) compare the students in the experimental group with students in the regular teacher education program in relation to their mathematical understandings and attitudes, and (7) determine the relationship between selected variables and achievement in mathematics.

Review of Literature

The increased concern of mathematics educators from all over the world with the necessity of improving mathematics education has been very influential in the development of new teacher education programs and in the improvement of methods of evaluation.

Since teachers and their education are the principal substance behind any effort made for the ultimate improvement of educational systems, educators have devoted a great deal of time to the improvement of teacher education programs, developing criteria for the training of prospective teachers of mathematics at both the primary and the secondary level. Changes in the content of curricula have been accompanied by experiments in the development of new teaching methods, and by new research on the mathematical competencies needed by elementary school teachers and elementary school children.

Curriculum research and evaluation has continued to progress. While summative evaluation is still regarded as an adequate and necessary method to make decisions about curriculum adoption and effective use, formative evaluation techniques are considered more and more important by most curriculum specialists during the development of a teacher education program and also for instruction and student learning.

Concerned by the importance of improving the measurement of achievement, many researchers and educators point out the need for criterion-referenced testing as a part of curriculum evaluation. They indicate that conventional testing instruments, although effective in differentiating among individual student's performance, are not always efficient, nor even valid, for assessing student's performance on specified learning objectives. Researchers have proposed a more extensive use of criterion-referenced measures in the assessment of the degree of competence attained by a particular student. This type of measurement is relatively new in education but the development of instructional technology and the recent emphasis on curriculum research and curriculum evaluation have stressed the need for the kind of information made available by the use of criterion-referenced measures.

The attitudes toward mathematics that the prospective teachers hold are almost as important as cognitive learning in mathematics, since mathematics, like any school instruction, is intended to form a base for future learning. If, while learning mathematics, the student acquires a dislike for the subject, further learning is unlikely, and part of the purpose of instruction is lost.

Attitudes toward mathematics have been investigated extensively in relation to personality characteristics, teacher's attitudes and effectiveness, students' achievement and the new mathematics curricula.

Hypotheses

The effect of the mathematics component of the experimental program on the cognitive and affective behavior of the experimental group were assessed by the following hypotheses:

A1. There will be a significant difference between the post-test means and the pre-test means of the experimental group on the criterion-referenced meaures.

The univariate hypotheses associated with this multivariate hypothesis were:

The mean post-test score of the experimental group will be significantly higher than the mean pre-test score on the criterion-referenced measures in:

a. Measurement
b. Numeration
c. Sets and Set Relations
d. Whole Numbers
e. Fractions
f. Decimals
g. Relations and Functions
h. Probability and Statistics
i. Mathematical Systems.

A2. There will be no significant difference between the post-test means and the mastery level (80 percent) on the criterion-referenced measures.

The univariate hypotheses associated with this multivariate hypothesis were:

The mean post-test score of the experimental group will be at least equal to the mastery level (80 percent) on the criterion-referenced measures in:

a. Measurement
b. Numeration
c. Sets and Set Relations
d. Whole Numbers
e. Fractions
f. Decimals
g. Relations and Functions
h. Probability and Statistics
i. Mathematical Systems.

B1. There will be a significant difference on a test of basic mathematical understanding between the post-test scores of the experimental group and their pre-test scores.

B2. There will be a significant difference on an arithmetic attitude inventory between the post-test scores of the experimental group and their pre-test scores.

The following two hypotheses were tested to compare the experimental group and students enrolled in the regular teacher education program on basic mathematical understanding and attitudes toward arithmetic.

C1. The adjusted mean post-test scores of the experimental group will be at least equal to the adjusted mean post-test scores of a group of prospective elementary teachers enrolled in the *regular teacher education program on a test of basic mathematical understanding.*

C2. There will be a significant difference in an arithmetic attitude inventory between the adjusted post-test scores of the experimental group and the adjusted post-test scores of a group of prospective elementary teachers enrolled in the regular teacher education program.

The Integrated Content-Methods Course

A team of mathematicians, mathematics educators, and elementary school teachers formulated the integrated mathematical experience for the first year of the experimental program. Nine learning units were designed in accordance with guidelines proposed by the BSTEP Model. Each learning unit was devoted to a mathematical topic deemed necessary for elementary school teacher education. The topics were:

1. Measurement
2. Numeration Systems
3. Sets and Set Relations
4. Whole Numbers
5. Fractions
6. Decimals
7. Relations and Functions
8. Probability and Statistics
9. Mathematical Systems

The learning units had the following common features:

1. Goals and objectives (mathematical competencies) for that topic.
2. Experiences and strategies utilizing manipulative, audio-visual and other instructional materials to achieve these objectives.
3. Criterion-referenced tests (two equivalent forms) to assess the students' pre- and post-treatment behaviors on the specified mathematical competencies.

The criterion-referenced tests, developed by this investigator, yielded measurements that were directly interpretable in terms of the specified mathematical competencies. The method in which these tests were constructed insured their content validity. The reliability coefficients for these tests varied from 0.77 to 0.93, which is acceptable for criterion-referenced tests.

The integrated content-methods course met eight hours a week for nine weeks. It was conducted in a mathematics laboratory equipped with manipulative and other instructional materials. The students worked in groups of four on the activities prescribed in the learning unit. Then at the end of each week they planned, with the assistance and supervision of instructors and elementary school teachers, instructional designs to be used with elementary school children the following week. Each member of the group was responsible for part of the teaching of four or five elementary school children. During the clinical experience, students spent one full morning per week in an elementary school. Three hours were spent working with or observing classroom teachers. The remaining hour was spent teaching mathematics to four or five pupils in a mathematics laboratory setting, and receiving feedback from teacher educators who observed the teaching experience.

Students in the Study

The students in the experimental program were freshmen elementary education majors who volunteered and were selected to participate in the program. Initially, fifty-two entering freshmen volunteered, forty of whom were selected. At the beginning of the school-year, two students dropped from the program. The remaining thirty-eight prospective elementary school teachers, who participated in the first trial implementation of the mathematics component, comprised the experimental group for this study. Other groups of students were utilized for comparison purposes and for testing the reliability of the measuring instruments developed in this study.

Instrumentation

The following instruments were developed or selected for the collection of data: (1) Nine criterion-referenced achievement measures (two parallel forms), (2) M. J. Dossett's Test of Mathematical Understandings (two parallel forms), (3) Dutton Arithmetic Attitude Inventory, (4) Attitude Scales Toward Different Aspects of Mathematics, developed by The International Study of Achievement in Mathematics, and (5) MSU Basic Skill Tests in Arithmetic and Mathematics.

Statistical Analysis

Multivariate and univariate analysis of variance were used to determine the effect of the instructional treatment upon the experimental group performance on the criterion-referenced tests. The t-test for correlated means was used in comparing changes in the experimental group and a group of students in the regular teacher education program on their mathematical understanding and attitudes toward mathematics. The Dunnett t-test was used to assess significant differences between the experimental group and other freshman groups on their entering cognitive and affective behaviors toward mathematics. The Pearson product moment correlation coefficient was utilized in the relationship analysis reported in this study.

The 5 percent level of significance was chosen for accepting or rejecting the research hypotheses in this study.

Limitations of the Study

The present study contains several limitations which must be kept in mind when interpreting the results of this investigation.

1. While there are several goals that pertain to the formative evaluation of an educational program, this study evaluated only the mathematics component of the experimental teacher education program.
2. Evaluation of the program was confined solely to those prospective elementary teachers who volunteered and were selected to participate in the first year trial implementation of the program.
3. The study did not attempt to evaluate the effect of the integrated content-methods course on the teaching behavior of its recipients in elementary school setting.
4. The study did not attempt to evaluate the effect of the experimental program on the mathematical competency of the school children who were taught by the experimental program participants.
5. The extent to which the evaluative instruments adequately measured the effects of the integrated content-method course and the clinical experience

was also a limitation. The instruments used in this study had the inherent limitations of paper-and-pencil tests.

Findings of the Study

Analysis of the Mathematical Content of the Experimental Program

A scorecard of the mathematical topics suggested for the preparation of elementary school teachers was constructed based on the recommendations of mathematics educators, nationally-recognized advisory groups, research studies, and elementary school mathematics textbooks and teacher's guides. The score card served as a criteria-referenced list for assessing the adequacy of the mathematics content of the experimental program. The investigator noted that the mathematics component of the experimental program included 94 percent of the topics on the developed criteria-referenced list, and also other topics not suggested by specialists, but incorporated to facilitate the development of other required topics. It was concluded that the mathematical competencies prescribed by the experimental program were sufficient in meeting the needs of the future elementary school teacher in mathematics.

Comparisons of the Experimental Group with Other Freshman Groups on Entering Cognitive and Affective Behaviors Toward Mathematics

Students in the experimental group were compared with three first-term freshman groups with declared majors in: (1) elementary education--but did not volunteer for the experimental program, (2) mathematics and secondary education, and (3) mathematics. The following instruments were administered to all four groups during the Fall term of 1971-1972: (1) MSU basic skill and placement tests in arithmetic and mathematics, (2) Dutton Arithmetic Attitude Scale, and (3) Attitude Scales Toward Different Aspects of Mathematics (developed by the International Study of Achievement in Mathematics). Results obtained showed that the entering cognitive and affective behaviors toward mathematics of the students who volunteered to participate in the experimental program were not significantly different from those of other freshmen with similar interest of becoming elementary school teachers. These entering behaviors, however, were significantly different from those of other freshmen with specified interest in mathematics (the mathematics and secondary education majors, and mathematics majors).

Evaluation of the Experimental Group Performance on the Criterion-Referenced Measures

The effect of the instructional treatment was evaluated by analyzing the experimental group performance on the criterion-referenced measures.

Hypothesis A1 was tested to determine the significance of gain in achievement on the prescribed mathematical competencies between pre- and post-test scores on the criterion-referenced measures.

Findings.--The multivariate test indicated that the overall difference between pre- and post-test means was highly significant ($p < .0001$). Analysis of the univariate hypotheses associated with this test yielded probabilities that indicated significant gain between pre- and post-test scores on the criterion-referenced measures in:

1. Numeration ($p < .005$),
2. Sets and Set Relations ($p < .0001$),
3. Whole Numbers ($p < .0001$),
4. Fractions ($p < .0001$),
5. Decimals ($p < .0001$),
6. Relations and Functions ($p < .0001$),
7. Probability and Statistics ($p < .0001$), and
8. Mathematical Systems ($p < .0001$).

There was, however, no significant gain on the test of Measurement ($p < .3639$).

Hypothesis A2 was tested to determine whether a specified degree of mastery (a score of 80 or higher on the post-test) was achieved.

Findings.--The multivariate test indicated that the overall difference between post-test means and the mastery level was highly significant ($p < .0001$). Analysis of the univariate hypotheses associated with this test indicated that the experimental group's post-test scores were not significantly different from the mastery level on the criterion-referenced measures in:

1. Numeration Systems ($p < .2210$),
2. Set and Set Relations ($p < .1450$),
3. Whole Numbers ($p < .7117$),
4. Fractions ($p < .7135$),
5. Decimals ($p < .5378$),
6. Relations and Functions ($p < .3147$), and
7. Probability and Statistics ($p < .2397$).

The post-test scores of the experimental group were, however, significantly below the mastery level on the criterion-referenced measures in: (1) Measurement ($p < .001$), and (2) Mathematical Systems ($p < .0002$).

Based on these results, it was concluded that the instructional treatment produced positive results, that the experimental group achieved mastery over the mathematical competencies prescribed in seven of the nine mathematical topics of the integrated content-methods course.

The Effect of the Experimental Program on the Basic Mathematical Understandings and Attitudes Toward Mathematics

Two hypotheses were tested to evaluate the effect of the integrated content-methods course and the clinical experience on the basic mathematical understandings and attitudes toward mathematics. The first, Hypothesis B1 (related to the basic mathematical understandings), was stated as: *There will be a significant difference on a test of basic mathematical understanding between the post-test scores of the experimental group and their pre-test scores.*

Findings.--Testing this hypothesis, the investigator found that the mean post-test scores were numerically as well as significantly higher than the mean pre-test scores. Based on this data the hypothesis was accepted. It was concluded that the integrated content-methods course had a significant positive effect on the mathematical understanding of the experimental group. Analysis of pre-test results indicated lack of understanding of basic mathematical principles underlying the four arithmetic operations, and reflected the students' mathematics preparation which emphasized computational skill. Post-test results indicated a definite improvement in understanding number properties, multiplication and division algorithms, and operations with decimals and fractions. The emphasis placed on the structure of the numeration systems during the integrated content-methods course was clearly reflected on the student performance on related items in the post-test.

Hypothesis B2 (related to the attitude toward mathematics) was stated as: *There will be a significant difference on an arithmetic attitude inventory between the post-test scores of the experimental group and their pre-test scores.*

Findings.--The Dutton Arithmetic Attitude Scale was used to assess the change of attitude of the experimental group during the first year of their teacher education. Possible scores on this scale range from 1.0 (extreme dislike) to 10.5 (extreme like). On the pre-test the experimental group's mean score was 5.57 which reflects the negative attitudes most of these students had toward arithmetic. On the post-test the mean was 7.07. The difference was highly significant ($p < .001$). Based on this result Hypothesis B2 was accepted.

Related questions.--In connection with the attitude study, the following results were obtained:

1. Feelings toward arithmetic are formed in all grades. However, the most crucial years for the students seemed to be when they were in the third through the sixth grade.

2. The challenge presented by arithmetic and its practical application were the aspects most liked about arithmetic.

3. Story problems, teachers, and memorizing rules were the aspects most disliked about arithmetic.

In summary, the mathematics component of the experimental program had a positive effect on improving the experimental group's attitudes toward mathematics.

Comparison of the Experimental Group with a Regular Elementary Education Group on Mathematical Understandings and Attitudes Toward Mathematics

The experimental group was compared with a group of students in the regular elementary education program who had completed the mathematics content course and were enrolled in the methods course concurrently with the integrated content-methods course. Two hypotheses were tested. The first, Hypothesis C1, was stated as: *The adjusted mean post-test scores of the experimental group will be at least equal to the adjusted mean post-test scores of a group of prospective elementary teachers enrolled in the regular teacher education program on a test of basic mathematical understanding.*

Findings.--The analysis of covariance was used for the analysis of data, with the pre-test scores as the covariate variable. The obtained F ratio was highly significant ($p < .001$) in favor of the experimental group. It was therefore concluded that the two final means, when initial differences were allowed for, did differ significantly in favor of the experimental group. Thus, Hypothesis C1 was accepted.

Hypothesis C2 which stated that: *There will be a significant difference in an arithmetic attitude inventory between the adjusted post-test scores of the experimental group and the adjusted post-test scores of a group of prospective elementary teachers enrolled in the regular teacher education program.*

Findings.--The analysis of covariance was utilized to assess for significance of differences between adjusted final means. The F ratio obtained from the analysis of data related to this hypothesis was highly significant ($p < .001$). It was therefore concluded that the experimental group, with initial differences allowed for, had significantly more positive attitude toward arithmetic than did a group of students in the regular elementary teacher education program. Thus Hypothesis C2 was accepted.

Correlation Analysis

Data collected on mathematics achievement, basic mathematical understandings, and attitudes toward mathematics were studied to determine whether these effects were related or influenced by other factors, such as the level of high-school mathematics preparation and grade point averages. For this purpose scores from all pre- and post-tests, as well as other background data of the experimental group were used to calculate an intercorrelation matrix. In all, the correlation data indicated significantly positive relations between the criterion-referenced measures, the norm-referenced tests of mathematical understandings, and the attitude scale. High-school mathematics preparation and overall grade point average were significantly correlated to mathematical performance, understandings, and attitudes.

Student Reactions to the Mathematics Component of the Experimental Program

The thirty-eight students in the experimental group were asked to react to the methods and procedures used in the implementation of the integrated content-methods course, the mathematics laboratory and the clinical experience. Overall, reactions to the eleven-item questionnaire seemed to indicate a general satisfaction with the different aspects of the program. However, the students felt that not enough emphasis was placed on planning instructional strategies to be used with elementary school pupils. The majority of the students offered some comments about the improvement of the program.

Conclusions

The analysis of the data gathered in this study and presented in the preceding chapters appears to warrant a number of conclusions. These conclusions are based on evidence obtained from the findings of the present study and the investigator's observations and interpretations of these results.

1. Analysis of the mathematical content of the experimental program indicated a strong agreement with the present elementary school mathematics content and with recommendations of professional organizations of mathematics educators and research groups. It was concluded, therefore, that the mathematical competencies prescribed in the experimental program were sufficient in meeting the needs of future elementary teachers of arithmetic.

2. Since the group of freshman prospective elementary teachers who comprised the experimental group volunteered and were selected to the program, it was important to determine if they differ significantly in their entering characteristics from other freshman groups on cognitive and affective behaviors toward mathematics. Three groups of freshmen were used for comparison purposes: (1) a group of elementary education majors, (2) a group of mathematics-secondary education majors, and (3) a group of mathematics majors. Results of the analysis indicated that the

experimental group did not differ in their cognitive and affective behavior toward mathematics from a group with the same professional interests (e.g., elementary education majors). The experimental group's cognitive behavior toward mathematics was significantly lower than that of freshman students with manifested interest in mathematics (e.g., mathematics-secondary education majors, and mathematics majors). These two groups also had significantly higher attitudes toward arithmetic than did the experimental group.

3. On the criterion-referenced measures, the experimental group showed significant gains in achievement ($p < .005$) on the measures of Numeration Systems, Sets and Set Relations, Whole Numbers, Fractions, Decimals, Relation and Functions, Probability and Statistics, and Mathematical Systems. The experimental group displayed a positive, but not significant, gain on tests of Measurement.

4. When post-test scores were compared with the mastery level, which was a score of 80 or higher on the post-test, results showed that the experimental group attained the prescribed mastery level on measures of Numeration Systems, Sets and Set Relations, Whole Numbers, Fractions, Decimals, Relations and Functions, and Probability and Statistics. The experimental groups did not reach the level of mastery on the measures of Measurement and Mathematical Systems.

5. The experimental group showed significant gains on a test of basic mathematical understandings.

6. The experimental group showed significant positive gains in attitudes toward mathematics.

7. With initial differences allowed for, the experimental group scored significantly higher on a test of basic mathematical understandings than did a group of students in the regular elementary teacher education program.

8. With initial differences allowed for, the experimental group exhibited significantly more positive attitudes toward mathematics than did a group of students in the regular elementary teacher education program.

9. The experimental group exhibited desire for more participation in clinical experience concurrently with the laboratory oriented integrated content-methods course.

On the basis of changes in mathematical achievement on the criterion-referenced measures and the tests of basic mathematical understandings and attitudes, results of data analysis provided encouraging signs that: (1) combination of emphasis on mathematical content and commitment toward making mathematics understood by prospective teachers can be achieved, (2) unifying theory of teaching and learning mathematics concurrently with laboratory and clinical experience provide positive methods of improving the cognitive and affective behaviors of the prospective elementary teachers toward mathematics, (3) the use of manipulative materials is effective in teaching prospective elementary teachers the basic mathematical concepts they are expected to teach, and (4) the clinical experience provides a framework from which the prospective teachers could apply the theoretical content of their courses.

Discussion

On the criterion-referenced tests in Measurement, the experimental group's post-test mean ($M = 66.32$) was not significantly different from the pre-test mean ($M = 63.97$). The post-test mean remained significantly ($p < .0001$) below the mastery level (a score of 80 or higher). These results may be attributed to two reasons: (1) Measurement was the first topic to be taught in the integrated content-methods course in a setting (mathematics laboratory) unfamiliar to most if not all the students. The students spent much of the first week, as the instructors intended, becoming acquainted with the new surrounding and familiarizing themselves with the functions of manipulative and other instructional materials. This minimized the amount of time spent on mathematical activities associated with Measurement, and (2) A more plausible reason may have been that the post-test in Measurement was more difficult than the pre-test. Analysis of pre- and post-test items indicated that more emphasis on comprehension of mathematical concepts related to precision in measurement was placed on the post-test while the pre-test emphasized computations of problems related to the above concepts. Analysis of reliability estimates of the criterion-referenced tests revealed that the correlation between pre- and post-test results in Measurement ($r = 0.64$) was the lowest for the nine measures and that the reliability coefficient for post-test ($r = 0.77$) was also the lowest for all measures.

On the criterion-referenced test in Mathematical Systems, the experimental group exhibited a highly significant improvement ($p < .0001$) between their performance on the pre-test ($M = 33.52$) and the post-test ($M = 65.53$), however the post-test mean was significantly ($p < .0002$) below the mastery level. This was attributed to the following factor: only five class hours were allocated to this topic, most students did not finish the prescribed activities in this limited time, and the content of this topic was new to the majority of the students. Mathematical Systems was the last topic and the experimental group was not able to teach it to the elementary school pupils due to the end of the elementary school year. The students were aware that they would not have to teach this topic and it could be that they did not put the same level of effort into learning it as they did with other topics.

Observations

1. Achievement tests, attitude scales, and students questionnaires are important aspects of the formative evaluation process of an evolving educational program. These measures, however, suffer from the inherent limitations of paper-and-pencil tests. They do not explain the nature of the social interaction that takes place in the classroom which must be known if one is to place the effectiveness of the program in proper prospective. The following observations were made by this investigator as a result of observing the prospective elementary teachers in the integrated content-methods course and during the clinical experience at the elementary school's mathematics laboratory.

A. During the integrated content-methods course, it was noted that as the course progressed attendance improved. Frequently students could be found browsing through the mathematics laboratory looking at books or working with the manipulative materials before and after class. Often students would stay after class-hours to complete some instructional activities or to discusss their work with instructors. Many students would borrow some of the manipulative materials from the mathematics laboratory to use in preparing their instructional design. Students shared their experience at the elementary school with other members of their group and it was also noted that each small group tended to form a "leader" who would take the initiative in asking questions and preparing materials and act as a group leader in planning instructional designs. Students who were shy or had negative attitudes toward mathematics tended to become "followers" and took passive roles in the group activities, although such situations were minimized toward the end of the course.

Since the students in the experimental group worked in subgroups of four during the integrated content-methods course, it is expected that the students' performance on the criterion measures will not be influenced only by the instructional treatment but also by the interaction between members of each subgroup. In this study such interaction effect due to the subgroup formation was not evaluated for two reasons: (1) the students tended to change from one subgroup to another as the course progressed and they tended to know each other better, and (2) during the clinical experience the interaction between members of the same subgroup ceased to exist since on any particular day only one member of each group was present at the elementary school. During the clinical experience an interaction between students prevailed.

In future studies, if the subgroups remained constant, then interaction within subgroups must be explained in order to assess the overall effect of the instructional treatment.

B. At the elementary school, where the clinical experience took place, interaction between the prospective teacher and pupils at the mathematics laboratory clearly improved during the nine-week term. At the beginning, the prospective teacher was very nervous and tended to diverge from her lesson plan. This usually occurred when she tried to control the children's non-mathematical behaviors. Apparently she felt she could instruct these children only if they were quietly listening. She tended to show the pupils, with manipulative materials, the mathematical concepts the pupils were to learn but did not allow them to work with the materials. This, however, changed as the lessons went by and the prospective teacher let the pupils handle and play with the manipulative materials while acting more as a guide ready to assist the pupils in discovering mathematical concepts. In the short time the prospective teachers spent teaching, they showed a worthy change in their teaching abilities. The confidence they attained was built through the opportunities to try different techniques with children.

2. The Hawthorne effect virtually insures increased learning on the part of the students. The interest, enthusiasm, and capabilities of the project staff no doubt influence the students' performance as well as their attitudes toward mathematics. The presence of five to six mathematics educators to assist the thirty-eight

students in the experimental group was an important factor in the success of the program, as evidenced by the positive results of this study. This degree of success would have been doubtful had there been only one or two teacher educators in the mathematics laboratory. The presence of the elementary school teacher to assist the prospective teachers in planning their instructional strategies to be used with children contributed heavily to the change in affective behavior toward mathematics. The presence of teacher educators at the elementary school to provide an immediate feedback to each and every prospective teacher after the mathematics laboratory teaching session also influenced the students' future action. These observations lead to an important question: To what extent is such a program implementable? In a large university such as Michigan State University, if the number of prospective teachers involved in such a program swelled to five or six hundred, it would demand at least thirty mathematics educators to provide these prospective teachers with similar experiences as were described in this study. For such a large number of prospective teachers, no less than twelve elementary schools would be needed to provide these prospective teachers with the necessary clinical experience.

Recommendations

The following recommendations are based on the investigator's interpretations of the analysis of data collected in this study and his personal observations in carrying out the present evaluation. The recommendations have been organized in two categories: (1) recommendations for future action and (2) recommendations for future research.

Recommendations for Future Action

It is recommended that the mathematics preparation of prospective elementary school teachers should integrate the content and methods courses, thus teaching the prospective teacher the mathematical content in the same form he would be expected to teach it.

It is recommended that such integrated content-methods course be taught in a mathematical laboratory setting where manipulative, audio-visual, and instructional materials are available and utilized.

It is recommended that the prospective elementary school teacher study the theories of teaching and learning mathematics with laboratory and clinical experience so as to be able to relate theory to practice. This type of clinical experience should begin as early as practical in the education of the prospective teacher.

It is recommended that the prospective teacher's clinical experience provide him the opportunity to teach inner-city pupils as well as rural pupils, the highly motivated pupil as well as the unmotivated pupil, the hostile pupil as well as the cooperative pupil, the less able as well as the more able.

It is recommended that time be allocated for short lectures and/or demonstrations by a teacher educator for topics of mathematical concepts that the prospective teachers might not discover for themselves in the limited class time. These topics may include: division in bases other than ten, the concept of congruency in mathematical systems, probability of dependent events, and mathematical language and symbolization. Such lectures or demonstrations may facilitate faster and better understanding by the prospective teachers of related concepts.

It is recommended that for each learning unit (topic) a short film or video-tape recording of an example of instructional activities on this topic be shown. Such films or video-tapes would serve three major objectives: (a) enable the prospective teacher to observe experienced teachers working with children on mathematical activities, (b) enable the prospective teacher to observe children working on the learning situation, and (c) enable the teacher educator to demonstrate different methods of instruction.

It is recommended that the prospective teachers work in small groups of four or five on developing instructional designs to be used with children. Working in such groups, the prospective teacher must take an active role in planning the instructional design since he or she is responsible for teaching one part.

It is recommended that mathematical activities utilizing manipulative materials be used whenever possible by the prospective teachers in planning their instructional design.

It is recommended that instructional activities be added to the unit on Measurement to provide the prospective teacher with better knowledge and understanding of the metric system of measurement and its relation to the English system.

It is recommended that those prospective teachers who do not attain an acceptable degree of mastery over the prescribed mathematical competencies be given a deferred grade until they show evidence of attainment of the desired degree of competency.

It is recommended that the mathematics educators carefully identify and define the competencies that are essential for elementary school teaching of mathematics and that, for the prospective elementary teacher to complete his mathematical education, he must evidence attainment of no less than 80 percent of these competencies.

The relatively low number of freshmen prospective elementary school teachers (fifty-two) who volunteered to participate in the experimental program suggests the need for modifying the procedure by which the students are approached about participating in the program. For a new program to be successful in reaching prospective teachers, an aggressive recruitment procedure should be established. Instead of writing a letter to all incoming freshmen with declared interest in elementary education soliciting their participation in the program, it is recommended that members of the project staff contact these students individually or in groups during the students' visit to campus, orientation week, or the beginning of the school year. This would allow the student an opportunity to ask questions and receive an immediate response. It also would allow the project staff to make a special effort at recruiting male and minority students to participate in the program, thus obtaining a more representative sample of the population of elementary education majors.

Recommendations for Future Research

Because of the positive results obtained in this study, it is recommended that the study be replicated with another sample of prospective elementary school teachers who volunteer to participate in the experimental program.

It is recommended that a formative evaluation be conducted on the second integrated content-methods course (algebra and geometry) of the experimental program; and that criterion-referenced measures be developed to assess the effectiveness of that course on the students' achievement on the prescribed mathematical competencies.

It is recommended that an evaluation of the effect of the mathematical instruction by the prospective teachers on the cognitive behaviors of the elementary school children who were taught by these prospective teachers be conducted.

It is recommended that a longitudinal study of the thirty-eight prospective teachers who participated in the experimental program be initiated to provide answers to the following questions:

1. To what extent do these prospective teachers retain mathematical competencies, attitudes toward mathematics, and/or improve the teaching procedures developed as a result of this experimental program during their student teaching and during their teaching profession?
2. Are there any significant differences between the group of prospective elementary school teachers who participated in the experimental program and a group of prospective elementary school teachers in the regular teacher education program on their cognitive

and affective behaviors toward mathematics, immediately after both groups complete their undergraduate education?

3. Will this group of prospective teachers be more successful in: (a) teaching mathematics, (b) understanding of elementary school children, and (c) fitting in professionally, than regularly educated teachers?
4. Will this group of prospective teachers be more inclined toward adopting student-centered activities in their elementary school mathematics classes?

Finally, it is recommended that a research study be conducted to ascertain the effect of the clinical experience on the cognitive and affective behaviors toward mathematics of the elementary education majors who participate in the experimental program. The Post-test Only Control Group Design suggested by Campbell and Stanley[1] would be appropriate. Random assignment, of those who volunteer to participate in the experimental program, into two groups (thirty-five to forty students in each group would suffice) would provide an experimental and a control group. Both groups should receive the same treatment except that the

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[1]Donald T. Campbell and C. Stanley, Experimental and Quasi-Experimental Designs for Research (Chicago: Rand McNally, 1966).

experimental group would be involved in clinical practice of teaching mathematics once a week to a group of elementary school children. Concurrent with their integrated content-methods courses, criterion-referenced measures, and attitude scales would be administered to both groups. Any significant differences between the two groups on their cognitive or affective behaviors toward mathematics can be attributed mostly to the effect of the clinical experience. Such experimental design would control for most confounding variables that influence the source of internal validity of the experiment. The exclusion of pre-testing eliminates the effect of interaction between testings and treatment.

In conclusion, it seems that the present study has pointed out the need for objective and critical evaluation of elementary school teacher education programs. This is crucial if these programs are to train teachers who understand human development and the nature of mathematical learning sufficiently well so that they are able to recognize and foster conditions that facilitate learning of mathematics.

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APPENDIX A

SET OF NINE PRE- AND POST-TEST FORMS OF THE CRITERION-REFERENCED MEASURES

MEASUREMENT PRE-TEST

1. In one hour and a half, the minute hand of a clock rotates through an angle of:

   a) 60° b) 90° c) 360°

   d) 540° e) 720°

2. The distance between two towns on a map is 9 cm. (centimeters). If the scale of that map is: $\frac{3}{4}$ cm. = 10 km. (kilometers), the actual distance is:

   a) 50 km. b) 60 km. c) 75 km.

   d) 120 km. e) none of these

3. The measurement of a line segment was stated to be $(2\frac{1}{2} \pm \frac{1}{32})$ inches. This implies that the segment is:

   a) as long as 3 inches or as short as 2 inches

   b) as long as $2\frac{17}{32}$ inches or as short as $2\frac{15}{32}$ inches

   c) as long as $2\frac{17}{16}$ inches or as short as $2\frac{15}{16}$ inches

   d) exactly $2\frac{1}{2}$ inches long

   e) none of the above

4. A carpenter needs six wooden boards each 2 feet 8 inches long. If wood is sold by the foot, what is the least number of feet that must be purchased?

   a) 10 b) 12 c) 14

   d) 16 e) 18

5. A photograph measures 3 by 6 inches. It is enlarged so that the shorter measure will be 16 inches. The length of the enlarged longer measure will be:

   a) 8 inches b) 19 inches c) 32 inches

   d) 48 inches e) 80 inches

6. Of the following which is the shortest?

   a) 30 inches b) 20 centimeters c) one decimeter

   d) one yard e) one meter

7. If a box is 10 units high, 6 units wide, and 4 units deep, how many cubes will fill this box if each cube is 2 units on each side?

   a) 240 b) 120 c) 60

   d) 30 3) none of these

8. The surface of a cube is 150 square yards. What is the volume of this cube in cubic yards?

   a) 50 b) 100 c) 125

   d) 200 e) 250

9. There is a geometric figure of the shape and dimensions of the adjoining drawing. What is its area in square inches?

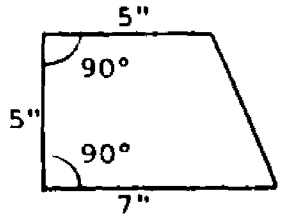
   a) 25

   b) 30

   c) 35

   d) 17.5

   e) none of these



10. What is the area in square centimeters of the shaded portion of the adjacent figure? (The circle is inscribed inside the square.)

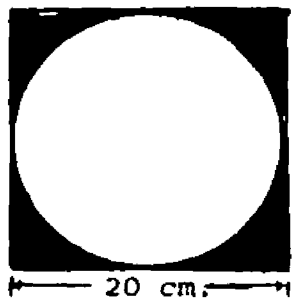
   a) 400 - 100 (3.14)

   b) 400 (3.14) - 4.00

   c) $\frac{1}{4}$ (400)

   d) 100 (3.14) - 100

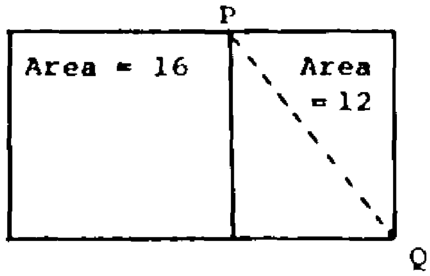
   e) none of these



MEASUREMENT POST-TEST

1. The rectangle below consists of a square of area 16, and a rectangle of area 12. What is the distance PQ?

    a) 3
    b) 4
    c) 5
    d) 6
    e) 7

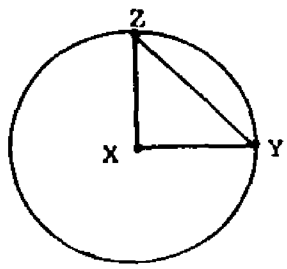


2. What is the height of a rectanble block that is 3 feet wide and 8 feet long, if its volume is equivalent to that of a cube with an edge of 6 feet?

    a) 12 feet   b) 9 feet   c) 6 feet
    d) 3 feet   e) none of these

3. In the figure below, X is the center of the circle and XY is perpendicular to XZ. If the area of the circle is 36π, what is the area of the triangle XYZ?

    a) 12
    b) 14
    c) 16
    d) 18
    e) 20



4. The sum of the measure of the four angles in a quadrilateral is:

    a) 90°   b) 180°   c) 270°
    d) 360°   e) depends on the size of the quadrilateral

5. The circumference of a circle is 8π meters. What is the area of the circle in square meters?

    a) 8π b) 16π c) 32π
    d) 64π e) 32

6. Which of the following is the nearest approximation of one yard?

    a) one kilometer   b) one centimeter   c) one millimeter
    d) one meter   e) 39 centimeters

7. How many cubic feet of water can fill a cylinder if the radius of its base is 2 feet and its height is 10 feet?

   a) 125.6 b) 62.8 c) 31.4

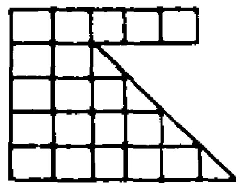
   d) 15.7 e) cannot be determined from the above information

8. If a scale of a map is 1 centimeter = 60 kilometers, what on a map would represent an actual distance of 720 kilometers?

   a) 12 centimeters b) 12 kilometers c) 18 centimeters

   d) 18 kilometers e) none of these

9. If  is equal to <u>ONE UNIT</u>, what is the area of the adjacent figure?



   a) 6 units

   b) 7 units

   c) 8 units

   d) 20 units

   e) 21 units

10. The "greatest possible error" in measurement is defined as:

   a) The smallest unit of measure used in the measurement

   b) One-half the smallest unit of measure used in the measurement

   c) One-tenth the smallest unit of measure used in the measurement

   d) Any fraction of a whole unit of measure used in the measurement

   e) None of the above.

SYSTEMS OF NUMERATION PRE-TEST

1. Suppose that in place of the number system, a symbol system was developed in which the following digits were used:

   Δ, L, Γ, μ, Σ, □, 7, V, Λ, & X

   representing respectively 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

   The digit Δ in the symbol system is used in the same fashion as 0 in the decimal system, e.g., ΣΔ = 40.

   A. Which of the following is equal to 10^2?

   a) LΔΔ   b) LΔΔΔ   c) LΔμ

   d) □□□X   e) XXXμ

   B. Which of the symbolic representation is equivalent to two-thirds.

   a) $\frac{\square}{L\Delta}$ b) $\frac{\Sigma}{7}$ c) $\frac{L7}{\Sigma}$

   d) $\frac{V}{L\Delta\Delta}$ e) none of these

   C. What is the value of LΓV <u>PLUS</u> Vμ?

   a) □Σ   b) ΓΔΣ   c) ΓΓΓ

   d) ΓΔΔ   e) XXΓ

2. The decimal expansion of the numeral 143.25 is?

   a) 1(100) + 4(10) + 3(1) - 2(10) - 5(1)

   b) 1(100) + 4(10) + 3(1) - 2(10) - 5(100)

   c) 1(1000) + 4(100) + 3(10) + 2(1) + 5(0)

   d) $1(10^2) + 4(10^1) + 3(10^0) + 2(10^{-1}) + 5(10^{-2})$

   e) none of the above is correct

3. In the numberal 7,698,500,000 (base 10), which of the following symbols is in the 10^9 place?

   a) 7 b) 6 c) 9
   d) 5 e) 8

4. Which of the following is the largest?

   a) 100,000 (base 2) b) 10,000 (base 3) c) 1,000 (base 4)
   d) 100 (base 5) e) 10 (base 10)

5. Which of the following numerals is not equal to the others?

   a) 100,000 (base 2) b) 210 (base 4) c) 36 (base 10)
   d) 51 (base 7) e) 121 (base 5)

6. In what base is 213 + 552 = 1205?

   a) ten b) nine c) eight
   d) seven e) six

7. In the following equation, what is the value of X?

   43 (base 5) - 24 (base 5) = X (base 5)

   a) 11 b) 12 c) 13
   d) 14 e) none of these

8. When working with base twelve, we need 12 symbols, so we will use 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, E, where T stands for ten and E stands for eleven.

   A. What is: 8T7 (base 12) + 319 (base 12) equal to?

   a) 1226 (base 12) b) 1004 (base 12) c) EE4 (base 12)
   d) TE6 (base 12) e) none of these

   B. In the numeral ET62 (base 12), the actual value of T in base 10 is:

   a) $10\ (12^2)$ b) $10\ (12^3)$ c) $10\ (10^2)$
   d) $10\ (10^3)$ e) none of these

9. If 12 (base 5) is an odd number (seven), which of the following is another example of an odd number?

   a) 101 (base 3) b) 100 (base 5) c) XVIII
   d) 121 (base 7) e) 101 (base 9)

10. The numbers 312 and 21 are in base 4. Their product (in base 4) is:

a) 20212 b) 13212 c) 6552

d) 1212120 e) none of these

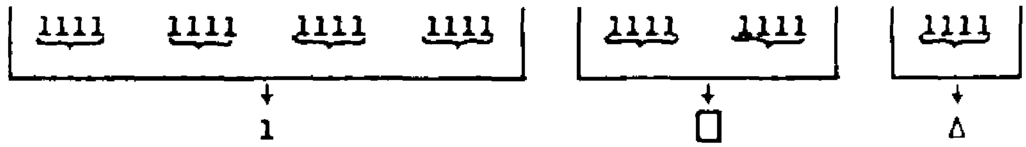
11. In what base is 15 ÷ 2 = 6?

a) ten b) nine c) seven

d) six e) four

SYSTEMS OF NUMERATION POST-TEST

1. What is the base of this numeration system?



a) 1 b) 4 c) □

d) Δ e) cannot be determined from the above information

2. The expanded notation:

$$5 \times 6^5 + 3 \times 6^4 + 4 \times 6^3 + 5 \times 6^2 + 1 \times 6^1 + 4 \times 6^0$$

is equivalent to which of the following numerals?

a) 534514 (base 10) b) 534514 (times 6) c) 30 + 18 + 24 + 30 + 24

d) 534514 (base 6) e) none of these

3. In the decimal number 8943.752, which of the following symbols is in the 10^2 place?

a) 2 b) 5 c) 3

d) 4 e) 9

4. The numeral 37 (base 10) is a different number than:

a) 41 (base 9) b) 52 (base 7) c) 211 (base 4)

d) 1,101 (base 3) e) 101,101 (base 2)

5. Jeff said there are 120 hours in a day. What numeration system was he working with?

a) base 9 b) base 8 c) base 4

d) base 2 e) none of these

6. In binary notation, what is the number which follows 11,011 (base 2)?

a) 11,010 b) 11,100 c) 11,111

d) 100,000 e) 11,110

7. What is the decimal equivalent of ET (base 12), where T stands for ten and E for eleven in the base 12 system?

   a) 120   b) 142   c) 132

   d) 122   e) none of these

8. If 302 (base 5) - 133 (base 5) = A (base 5), then A is:

   a) 440   b) 104   c) 114

   d) 124   e) 204

9. In which base does the numeral 35 represent an <u>even</u> number?

   a) twelve   b) ten   c) eight

   d) seven   e) six

10. The multiplication problem 16 x 4 = 60 has been computed in which base?

   a) thirteen   b) twelve   c) eleven

   d) nine   e) eight

11. Compute the quotient 41 (base 5) ÷ 3 (base 5) =

   a) 14   b) 13   c) 12

   d) 11   e) 10

SETS AND SET OPERATIONS PRE-TEST

1. If S (the universal set) = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and if A = {2, 4, 8}, B = {1, 2, 3, 4}, C = {7, 8}

   a) Enumerate $A \cup B$

      $A \cup B$ = { }

   b) Enumerate $A \cap B$

      $A \cap B$ = { }

   c) Enumerate $\overline{A \cup B}$ (complement of $A \cup B$)

      $\overline{A \cup B}$ = { }

   d) Enumerate $A \cap (B \cup C)$

      $A \cap (B \cup C)$ = { }

   e) Enumerate the set D which is described in set-builder notation as $\{x \in S \mid x$ is even <u>or</u> $x \in B\}$

      D = { }

2. Let A = {a, b, c}. Exactly how many subsets does A have?

   a) 8 b) 6 c) 4

   d) 3 e) 1

3. If a set A contains n distinct elements, which of the following formulas will always give the number of non-empty subsets of set A?

   a) n^2 b) $2(n)$ c) $2^n - 1$

   d) $n - 1$ e) cannot be determined from the information given

4. Let A = {3, 5, 9}, B = {9, 3} and C = {3, 5, 9, 4}. Then A B represents:

   a) A b) B c) {3, 5, 9, 4}

   d) {Ø} e) {5}

5. If set A = {a, b, c} and B = {1, 2, 3}, in how many ways could one establish a one-to-one correspondence between these two sets?

   a) 1 b) 3 c) 4

   d) 5 e) 6

6. Consider the set A = {(4, 2), (a, b)} and the set B = {4, 2}. Which of the following is true? (n(A) is the number of distinct elements in set A.)

    a) n(A) = n(B) b) n(A) > n(B) c) n(B) = 1

    d) n(A) < n(B) e) n(A) = 4

7. Let S = {x, y}. Then a complete listing of all possible subsets of S is:

    a) {x}, {y} b) {x}, {y}, {x, y} c) x, y

    d) ∅, {x}, {y}, {x, y} e) {∅}, {x} {y} {x, y}

8. If a set A has 10 subsets, how many proper subsets does A have?

    a) 10 b) 9 c) 5

    d) 1 e) 0

9. Let A be the set of all positive even numbers.
Let B be the set of all the letters in the English alphabet.
Which of the following statements is (are) true?

    a) A and B are matching sets.

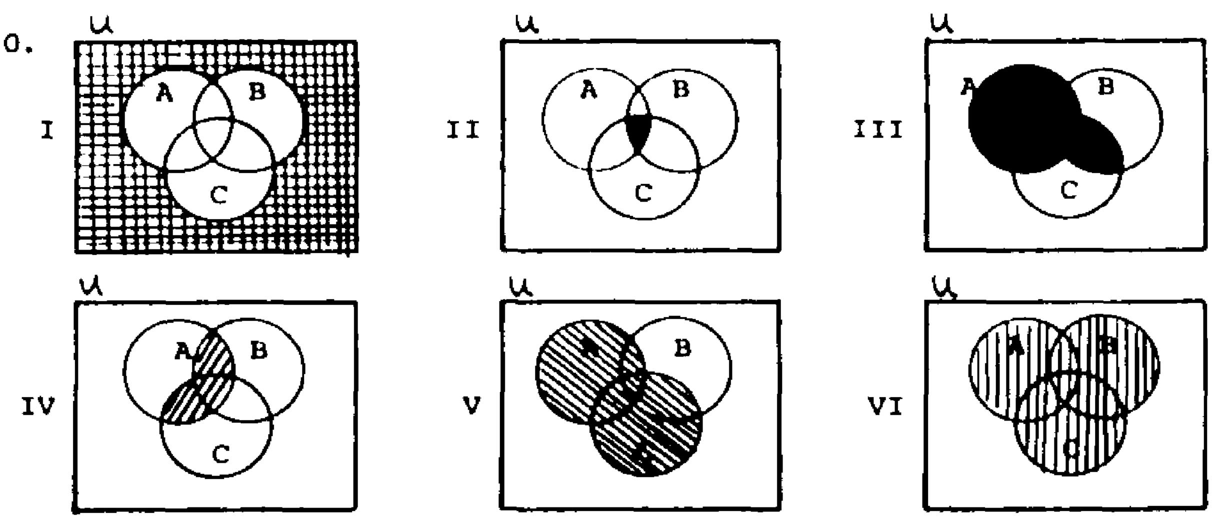
    b) B ⊂ A

    c) Both A and B are equivalent sets.

    d) Both A and B are infinite sets.

    e) A is an infinite set and B is a finite set.

10.



For each of the following sets, circle the Roman number which represents the shaded area of the Venn diagram above.

a) A ∪ (B ∩ C)	I	II	III	IV	V	VI
b) A ∩ (B ∪ C)	I	II	III	IV	V	VI
c) A ∪ C	I	II	III	IV	V	VI
d) (A ∪ B) ∪ C	I	II	III	IV	V	VI
e) A ∩ (B ∩ C)	I	II	III	IV	V	VI

SET AND SET OPERATIONS POST-TEST

1. If S (the universal set) = {a, b, c, d, e, f} and A = {a, c, d, e} B = {b, c, e} and C = {d, e, f}, list the elements of the following sets.

   a) $A \cup B = \{ \qquad \}$

   b) $A \cap B = \{ \qquad \}$

   c) $\overline{A \cup C}$ (complement of $A \cup C$) = { }

   d) $A \cup (B \cap C) = \{ \qquad \}$

   e) The set D which is described in the set-builder notation as

      $\{x \in S \mid x \in A$ <u>and</u> $x \in C\}$

      D = { }

2. If A = {x, y, z, w} , exactly how many subsets does A have?

   a) 4 b) 8 c) 16

   d) 32 e) none of these

3. What does the adjacent diagram illustrate?

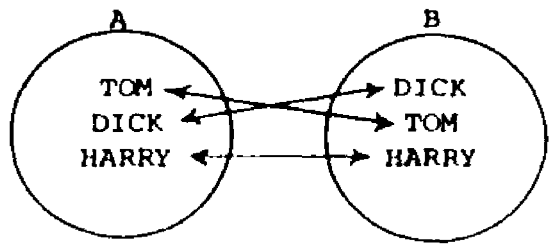
   a) a one-to-one correspondance

   b) matching sets

   c) set equality

   d) all of the above

   e) none of the above



4. Let A = {9, 7, 4, 2}, B = {2, 4, 7} and C = {4, 9}. Then B C =

   a) A b) B c) C

   d) {4} e) none of these

5. How many one-to-one correspondance are there between two two-number sets?

   a) 1 b) 2 c) 3

   d) 4 e) cannot be de·ermined from the above information

6. How many subsets are there in a four member set?

   a) 1 b) 4 c) 8

   d) 16 e) none of these

7. Which of the following statements about sets is (are) true?

   a) If A is a subset of B, then B is a subset of A.

   b) If A is a subset of B, then B is not a subset of A.

   c) If A is a proper subset of B, then B is not a proper subset of A.

   d) If A is a different set from B, then A is a proper subset of B or B is a proper subset of A.

   e) If set $A = \{(a, b)\}$ and $B = \{1, 2\}$, then A and B have the same number of elements.

8. Let A be the set of all pupils in an elementary school.
   Let B be the set of first graders in that school.
   Let C be the set of teachers in that school.
   Which of the following statements is (are) true?

   a) C is a proper subset of A.

   b) B is a proper subset of A.

   c) $A \cap B$ is the empty set.

   d) $B \cap C$ is the empty set.

   e) $A \cap B = A$.

9. Which of the following statements is (are) correct?

   a) If A is a subset of the universal set S, then the complement of A is the set of elements in S that are not in A.

   b) If B is a proper subset of A, then the complement of A is the set of elements that are common to A and B.

   c) If A and B are two disjoint subsets of S such that $A \cup B = S$, then A is the complement of B and B is the complement of A.

   d) Only (a) and (b) are correct.

   e) Only (a) and (c) are correct.

10. Which of the following sets is represented by the shaded portion of the Venn diagram?

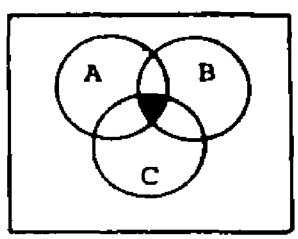
    a) $(A \cap B) \cap C$

    b) $(A \cup B) \cup C$

    c) $(A \cup B) \cap C$

    d) $(A \cap B) \cup C$

    e) $\overline{A \cap C}$ (complement of $A \cap C$)



THE WHOLE NUMBER SYSTEM — PRE-TEST

1. Match each statement with the property (ies) or definitions it illustrates.

_____ 1) $3 + (6 + 2) = (3 + 6) + 2$

_____ 2) $15 \times (8 + 5) = (15 \times 8) + (15 \times 5)$

_____ 3) $18 \times 1 = 1 \times 18$

_____ 4) $2 + (3 \times 8) = (2 + 3) \times (2 + 8)$

_____ 5) $18 + (7 + 5) = 5 + (18 + 7)$

_____ 6) $48 \times 86 = 86 \times 48$

_____ 7) $14 + 0 = 14$

_____ 8) $25 \div 0 = 0$

_____ 9) $(28 + 8) \div 7 = (28 \div 7) + (8 \div 7)$

_____ 10) $4 - 0 = 0 - 4$

a) commutative property
b) associative property
c) distributive property
d) identity property
e) false statement

2. Which of the following sets is (are) closed with respect to addition?

a) {Whole numbers less than 50}
b) {0, 1}
c) {2, 4, 6, 8, 10,.......}
d) {1, 3, 5, 7, 9}
e) {Whole numbers that are multiple of 3}

3. In the division algorithm: 10,998 ÷ 26, what multiple of 26 are used?

a) 10, 400, 520, 0
b) 4, 2, 3
c) 1200, 120, 2
d) 400, 20, 3
e) none of these

```
26)10,998
  -10,400
      598
     -520
       78
      -78
        0
```

4. In the adjacent algorithm, what does the number marked by the arrow actually represent?

a) 2 x 14
b) 20 x 14
c) 200 x 14
d) 14 x 2
e) none of these

```
     128
14)1792
   14
    39
    28 ←
    112
    112
      0
```

5. Which of the following statements is <u>false</u>? (f and g are whole numbers)

   a) $-f - g = -(f + g)$ b) $(-f)(-g) = fg$

   c) $-f - (-g) = (g - f)$ d) $(-f)(g) = (f)(-g)$

   e) if $f > g$, then $f - g = -(f + g)$

6. Which of these numbers is (are) <u>prime</u>?

   a) 51 b) 14 c) 1

   d) 43 e) 25

7. What is the highest prime to consider as a divisor in the factorization of 132?

   a) 131 b) 13 c) 11

   d) 9 e) 2

8. Which of the following statements is (are) true?

   a) All prime numbers are odd numbers.

   b) All composite numbers are even.

   c) If a prime number divides the product of two natural numbers, then it must divide at least one of the two numbers.

   d) Every natural number has at least two factors.

   e) There are finite number of prime.

9. The sum of the first 50 odd digits is: (i.e., the sum of 1, 3, 5,..........., 99)

   a) 1250 b) 2500 c) 5000

   d) 10,000 e) none of these

10. Which of these are the prime factorization of 60?

   a)
   2) 60
   2) 30   = 2 x 2 x 3 x 5
   3) 14
   5

   b) 60 ┌─30 ┌─10 ┌─5
   2 3 2   = 2 x 3 x 2 x 5

   c) both (a) and (b) d) neither (a) nor (b)

   e) $60 = 10 \times 30$

THE WHOLE NUMBER SYSTEM POST-TEST

1. Match each statement with the property (ies) of definitions it illustrates.

____	1)	$5 \times 12 = 12 \times 5$	a) commutative property of addition
____	2)	$(35 + 10) \div 5 = (35 \div 5) + (10 \div 5)$	b) commutative property of multiplication
____	3)	$5 + (9 + 11) = 11 + (5 + 9)$	c) associative property of addition
____	4)	$36 \times 92 = (36 \times 90) + (36 + 2)$	d) associative property of multiplication
____	5)	$18 \div 0 = 0$	e) distributive property
____	6)	$0 + 35 = 35$	f) identity property
____	7)	$5 + (9 \times 3) = (5 + 9) \times (5 + 3)$	g) false statement
____	8)	$13 - 0 = 0 - 13$	
____	9)	$11 \times 1 = 11$	
____	10)	$5 + (11 + 4) = (5 + 11) + 4$	

2. Which of the following sets is NOT closed under multiplication?

   a) {whole number}

   b) {odd natural numbers}

   c) {even natural numbers}

   d) {prime numbers}

   e) {whole numbers that are multiple of 5}

3. In the division algorithm 134,816 ÷ 32, what multiples of 32 are used?

   a) 4, 2, 1, 3

   b) 400, 20, 1, 3

   c) 128,000, 6,400, 320, 96

   d) 4,000, 200, 10, 3

   e) none of these

```
32)134,816
  -128,000
     6,816
     6,400
       416
       320
        96
        96
         0
```

4. In the adjacent multiplication algorithm, what does the partial product marked by the arrow represent?

   a) 6 x 342

   b) 3 x 342

   c) 60 x 342

   d) 30 x 342

   e) none of these

```
  342
 x 63
 1026
2052 ←
21546
```

5. Which of the following is (are) true for all m and $n \in W$?
(W is the set of whole numbers)

a) $3+n<5+n$ b) $2n>3n$ c) $2n+3\geqslant 5$

d) $(m \times n) \div n = m$ e) $(-3m) \div (-m) = -3$

6. Which of the following numbers has the greatest number of different prime factors?

a) 15   b) 16   c) 25

b) 27   e) 32

7. If p is a prime number, then $13+P$ is always:

a) a prime number   b) a composite number   c) an even number

d) an odd number   e) divisible by 13

8. Which of the following statements is NOT true?

a) 2 is the smallest prime number

b) If a and b are whole numbers and $b \neq 0$, there exist a unique whole number q and r such that

$$a = bq + r \quad \text{where} \quad 0 \leqslant r < b$$

c) Zero is a factor of every whole number

d) If p (a prime number) divides $m \times n$ (m and n are natural numbers), then p divides either m or n or both

e) The set of whole numbers is closed under substraction.

9. The sum of the first 100 even digits is:
(i.e., the sum of 2, 4, 6, ..., 200)

a) 20,200   b) 2,020   c) 10,100

d) 1,010   e) none of these

10. What does this diagram illustrate?

a) $4 \times 2 = 8$ b) $8 \div 4 = 2$ c) $8 < 9$

d) both a and b   e) none of these

FRACTIONS PRE-TEST

Note: r, s, t, u, w are whole numbers with s, u $\neq$ 0

1. Mark the correct statements below.

   a) If W is the set of whole numbers, and R is the set of fractions, then W R.

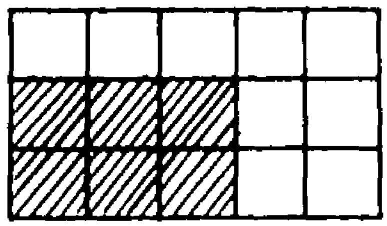
   b) The set of fractions is closed under division.

   c) The additive inverse of $\frac{1}{2}$ is $(\frac{1}{2})^{-1}$

   d) $\frac{r}{s} < \frac{r.u}{s.u}$

   e) If the product of two fractions is 1, then the two fractions are called reciprocals of each other.

2. The shaded portion of the rectangle below represents what part of the rectangle?



   a) $\frac{2}{3}$ of $\frac{3}{5}$ b) $\frac{3}{2}$ of $\frac{3}{5}$

   c) $\frac{6}{9}$ of $\frac{2}{5}$ d) $\frac{1}{6}$ of 1

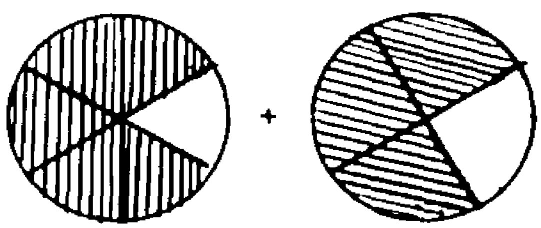
   e) none of these

3. The fraction $\frac{t}{s}$ is equivalent to:

   a) $\frac{t+u}{s+u}$ b) $\frac{t+t}{s+t}$ c) $\frac{t+s}{s+s}$

   d) n. $\frac{t}{u}$ e) $\frac{t/u}{s/u}$

4. What sum is represented by the shaded portions of this illustration?



   a) $\frac{8}{24}$ b) $\frac{19}{12}$ c) $\frac{8}{12}$

   d) $\frac{15}{24}$ e) none of these

5. The difference $\frac{r}{s} - \frac{t}{u}$ is equal to which of the following?

a) $\frac{r - t}{s - u}$ b) $\frac{r - t}{su}$ c) $\frac{ru - st}{su}$

d) $\frac{rs - tu}{us}$ e) none of these

6. The product $\frac{t}{u} \times \frac{r}{s}$ equals:

a) $t.r + u.s$ b) $\frac{t.s}{u.r}$ c) $(t.s) \cdot (u.r)$

d) $\frac{(t.r)}{(u.s)}$ e) none of these

7. Workmen have $\frac{3}{4}$ miles of road to build. If they build $\frac{1}{3}$ mile per day, how long will it take them to build the road?

a) $\frac{3}{4} \times \frac{1}{3}$ b) $\frac{3}{4} \div \frac{1}{3}$ c) $\frac{1}{3} \times \frac{3}{4}$

d) $\frac{1}{3} \div \frac{3}{4}$ e) $\frac{3}{4} - \frac{1}{3}$

8. Which one of the following fractions is in its lowest (reduced) terms?

a) $\frac{125}{126}$ b) $\frac{126}{128}$ c) $\frac{129}{132}$

d) $\frac{215}{330}$ e) $\frac{144}{153}$

9. What value replaces n in the equation:

$$\frac{n}{1 + \frac{1}{3}} = 1$$

a) $\frac{2}{3}$ b) $\frac{3}{4}$ c) $\frac{4}{3}$

d) 3 e) $\frac{1}{3}$

10. The least common denominator of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$ is:

a) 3 b) 6 c) 12

d) 13 e) 72

11. If the greatest common factor of p and q is 2, what is the least common multiple of p and q?

a) $\frac{pq}{4}$ b) $\frac{pq}{2}$ c) pq

d) 2 pq e) 4 pq

FRACTIONS POST-TEST

Note: t, u, v are whole numbers with v, t $\neq$ 0.

1. Which of the following statements is (are) false?

   a) The set of whole numbers is a proper subset of the fractions.

   b) The set of fractions is closed under multiplication and division.

   c) The additive inverse of $\frac{u}{v} = -\frac{v}{u}$

   d) If $\frac{u}{v} \cdot w = \frac{u}{v}$, then $w = 1$.

   e) The fraction $\frac{s}{u+v} = \frac{s}{u} + \frac{1}{v}$

2. The shaded portion of the rectangle represents what part of the rectangle?

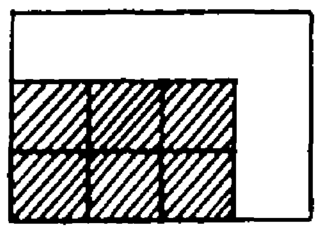
   a) $\frac{1}{2}$ of $\frac{3}{4}$

   b) $\frac{1}{3}$ of $\frac{3}{4}$

   c) $\frac{2}{3}$ of $\frac{3}{4}$

   d) $\frac{2}{3}$ of $\frac{3}{3}$

   e) none of these



3. If $u \neq v$, the fraction $\frac{u}{v}$ is not equivalent to:

   a) $\frac{t \cdot u}{t \cdot v}$ b) $\frac{u - 0}{v - 0}$ c) $\frac{0 - u}{0 - v}$

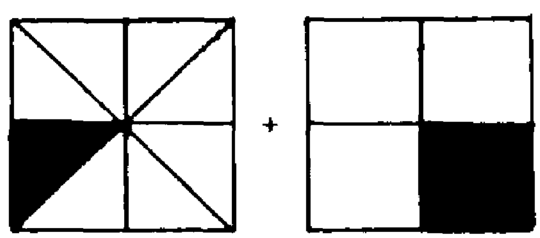
   d) $\frac{1 + u}{1 + v}$ e) $\frac{\frac{u}{t}}{\frac{v}{t}}$

4. What sum is represented by the shaded portion of this illustration?

   a) $\frac{2}{8}$ b) $\frac{2}{16}$

   c) $\frac{1}{12}$ d) $\frac{3}{8}$

   e) none of these



5. The difference $1 - \frac{u}{v}$ is equal to which of the following?

a) $\frac{-u}{v}$ b) $\frac{1-u}{v}$ c) $\frac{v-u}{v}$

d) $\frac{u-v}{u}$ e) $\frac{v-u}{uv}$

6. Mr. Farmer has $\frac{3}{4}$ acre of land and uses $\frac{2}{3}$ of it for a vegetable garden. What fraction of an acre is Mr. Farmer's vegetable garden?

a) $\frac{6}{9}$ b) $\frac{9}{8}$ c) $\frac{1}{2}$

d) $\frac{12}{15}$ e) none of these

7. Dave took a hike of 10 miles, walking an average of $2\frac{1}{2}$ miles per hour. How long did the hike take?

a) $\frac{28}{3}$ b) $\frac{25}{2}$ c) 4 hours

d) 5 hours e) none of these

8. Which of the following fractions is in its lowest (reduced) terms?

a) $\frac{75}{125}$ b) $\frac{1036}{2042}$ c) $\frac{123}{333}$

d) $\frac{29}{122}$ e) $\frac{144}{453}$

9. On the number line, what fraction is half-way between $\frac{11}{13}$ and $\frac{12}{12}$?

a) $\frac{23}{12}$ b) $\frac{132}{169}$ c) $\frac{23}{26}$

d) $11\frac{1}{2}$ e) none of these

10. The lowest common denominator of $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$ is:

a) 4 b) 8 c) 24

d) 48 e) none of these

11. Using the number line below, demonstrate the division algorithm $1\frac{1}{5} \div \frac{1}{5}$

DECIMALS AND PERCENTS PRE-TEST

1. Which of the following <u>are</u> correct?

   a) $2.17 \div 0.31 = 0.7$ b) $\frac{1}{40} = 0.25\%$ c) $5.701 - 0.37 = 5.664$

   d) $(30\%) \times (10\%) = 300\%$ e) $23.692 + 0.05 + 5 = 28.742$

2. What fraction is represented by 0.82828282..... ?

   a) $\frac{82}{100}$ b) $\frac{8}{9}$ c) $\frac{82}{99}$

   d) $\frac{8}{99}$ e) none of these

3. What is $\frac{0.15 \times 6.3}{5} =$ (rounded to <u>two</u> places)

   a) 189.00 b) 18.90 c) 1.89

   d) 0.19 e) none of these

4. What is $1.051 - 0.702 + 0.066 =$

   a) 0.283 b) 0.415 c) 1.009

   d) 1.819 e) none of these

5. Given the numbers 0.12, $\frac{1}{4}$, 0.125, $\frac{2}{11}$, and 0.0999, which is the smallest?

   a) 0.12 b) $\frac{1}{4}$ c) 0.125

   d) $\frac{2}{11}$ e) 0.0999

6. In an elementary school there were 220 girls. This was 55% of the school population. How many boys were in the school?

   a) 270 b) 220 c) 180

   d) 121 e) none of these

7. Which of the following IS false?

   a) $10^6 \div 10^2 = 10^4$ b) $10^{-4} = 0.0001$ c) $2.1 \times 10^{-2} = 0.021$

   d) $1.1 \times 10^3 = 11{,}000$ e) $(4.2 \times 10^4) \times (2.0 \times 10^4) = 8.4 \times 10^4$

8. Which number, on the number line, is half-way between 0.08 and 0.2?

   a) 0.14 b) 0.10 c) 0.40

   d) 0.50 e) 0.90

9. Which of the following is the expanded notation for the decimal 53.24?

   a) $5 \times 10^2 + 3 \times 10^1 + 2 \times 10^0 + 4 \times 10^{-1}$

   b) $5 \times 10^1 + 3 \times 10^0 \quad 2 \times 10^{-1} + 4 \times 10^{-2}$

   c) $5 \times 10^2 + 3 \times 10^1 - 2 \times 10^1 - 4 \times 10^2$

   d) $5 \times 10^1 + 3 \times 10^0 - 2 \times 10^1 - 4 \times 10^2$

   e) none of the above

10. The decimal 0.00074 written in scientific notation is:

   a) 74×10^5 b) 7.4×10^{-4} c) 74×10^4

   b) 74×10^{-4} e) none of these

11. The decimal 0.42 is written in base five. Its equivalent in base ten is:

   a) 0.88   b) 0.70   c) 0.24

   d) 0.21   e) none of these

DECIMALS AND PERCENTS POST-TEST

1. Which of the following <u>are</u> correct?

   a) $\frac{1.64}{0.4} = 0.41$ b) $53.005 - 0.28 = 25.005$

   c) $\frac{1}{5} = 20\%$ d) $(50\%) \times (20\%) = 10\%$

   e) $54.823 + 0.7 + 0.02 = 55.723$

2. What fraction is represented by 1.55555.....?

   a) $\frac{15}{9}$ b) $\frac{14}{9}$ c) $\frac{15}{10}$

   d) $\frac{15}{99}$ e) none of these

3. What is $\frac{0.014 \times 0.84}{.02} =$ (rounded to two places)?

   a) 0.59 b) 0.58 c) 5.888

   d) 0.5888 e) none of these

4. Given the five numbers: $\frac{1}{100}$, 7%, 0.1, 0.0199, and $\frac{2}{25}$, which is the largest?

   a) $\frac{1}{100}$ b) 7% c) 0.1

   d) 0.0199 e) $\frac{2}{25}$

5. What is: $0.407 - 0.32 + 0.076 = ?$

   a) 0.847 b) 0.651 c) 0.451

   d) 0.163 e) none of these

6. In a mathematics test, 85% of the students in a class of 60 passed. How many students did not pass?

   a) 15 b) 50 c) 51

   d) 9 e) none of these

7. Which of the following statements is <u>NOT</u> true?

   a) $10^8 \div 10^2 = 10^6$ b) $10^{-3} = 0.001$ c) $3.2 \times 10^{-2} = -320$

   d) $1.1 \times 10^4 = 11{,}000$ e) $(2 \times 10^2) \times (3.1 \times 10^3) = 6.2 \times 10^5$

8. Which number, on the number line, is half-way between 0.02 and 0.2?

   a) 0.22 b) 0.11 c) 0.40

   d) 0.18 e) none of these

9. The decimal 24.06 written in expanded notation is:

   a) $2 \times 10^2 + 4 \times 10^1 + 6 \times 10^0$

   b) $2 \times 10^2 + 4 \times 10^1 + 6 \times 10^2$

   c) $2 \times 10^1 + 4 \times 10^0 + 6 \times 10^{-2}$

   d) $2 \times 10^2 + 4 \times 10^1 + 6 \times 10^{-1}$

   e) none of these

10. The decimal 0.0031 written in scientific notation is:

   a) 31×10^{-3} b) 3.1×10^3 c) 3.1×10^{-3}

   d) 3.1×10^3 e) none of these

11. The number 2.3 is written in base four. Its equivalent in base ten is:

   a) 0.75 b) 2.30 c) 8.75

   d) 2.75 e) none of these

RELATIONS AND FUNCTIONS PRE-TEST

1. Let (R) be a relation (set of ordered pairs). Consider the following definitions.

   a) The set of all first members of the ordered pairs making up the relation (R).

   b) The set of all second members of the ordered pairs making up the relation (R).

   c) A relation with the reflexive, symmetric and transitive properties.

   d) A set of points in the plane corresponding to the ordered pairs of the relation (R).

   e) A relation in which no two ordered pairs have the same first element.

   Match the following terms with their definitions from above by circling the appropriate alphabetical representation.

1) An equivalence relation	a	b	c	d	e
2) The domain of relation (R)	a	b	c	d	e
3) A function	a	b	c	d	e
4) A graph of a relation (R)	a	b	c	d	e
5) The range of relation (R)	a	b	c	d	e

2. The relation "is greater than" is defined on the set of all natural numbers. Decide whether this relation is reflexive, symmetric, and/or transitive.

   a) reflexive

   b) symmetric

   c) transitive

   d) reflexive and symmetric

   e) reflexive, symmetric and transitive

3. On the set $\{1, 2, 3, 4\}$, we will define the relation (R) consisting of the following elements:

   $\{(1,1), (2,2), (3,3), (4,4), (1,3), (3,1), (2,4), (4,2)\}$

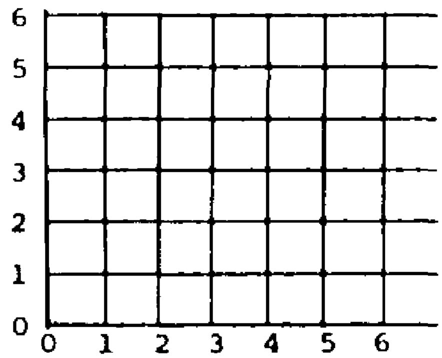
   Which of the following statements are TRUE?

   a) (R) is reflexive   b) (R) is symmetric

   c) (R) is transitive   d) (R) is an equivalent relation

   e) all of the above are correct

4. In the adjacent grid, graph the relation (R) described in Problem 3.

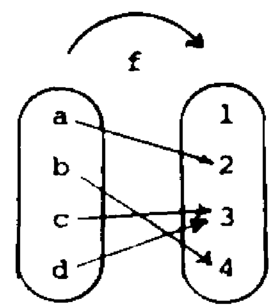


5. The accompanying figure describes a function (f).

a) The domain of f is: { }

b) The range of f is: { }

c) f(b) =



6. Suppose the adjacent function table was given. Which of the following pairs would also be on the table?

a) (0,-2)

b) (12,144)

c) (4,10)

d) (4,8)

e) none of these

□	△
1	1
3	9
5	25
8	64
.	.
.	.

7. Consider the accompanying diagram. Which of the following is (are) true?

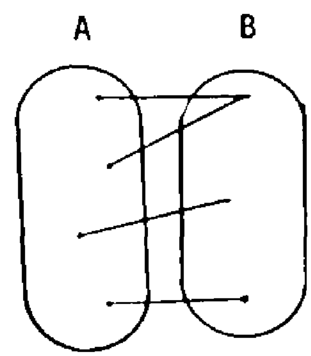
a) The diagram illustrates a function from A to B.

b) The diagram illustrates a function from B to A.

c) There exists a one-to-one correspondence between A and B.

d) All of the above.

e) None of the above.



8. The function g, from A to B is illustrated by the adjacent table. Which of the following defines the function g(A)?

   a) $g(A) = A + A$

   b) $g(A) = 2A + 2$

   c) $g(A) = A^2 + 2$

   d) $g(A) = 3A + 1$

   e) none of these

g →

A	B
0	1
1	4
2	7
3	10
4	13
.	.
.	.

9. Which of the following statements is correct?

   a) The inverse of a function is never a relation.

   b) The inverse of a relation is never a function.

   c) The inverse of a function is always a relation.

   d) The inverse of a function is always a function.

   e) none of these

10. Utilize the graph paper attached to graph the function f defined on set

$$A = \{1, 2, \ldots 10\} \text{ by } f(A) = 2A - 5.$$

1. Which of the following definitions is (are) false?

   a) A function is a relation in which no two ordered pairs have the same first element.

   b) An equivalence relation is a relation with the reflexive, symmetric, and transitive properties.

   c) The range of a function is the size of the object set of a set of ordered pairs.

   d) A relation is a set of ordered pairs.

   e) A relation R is symmetric if for all x and y given, xRy then yRx.

2. The relation "is the son of" is defined on the set of all men. Decide whether this relation is reflexive, symmetric, and/or transitive.

   a) reflexive

   b) symmetric

   c) transitive

   d) reflexive, symmetric and transitive

   e) none of these

3. On the set {a, b, c}, we will define the relation (R) consisting of the following elements:

   (a,a), (b,b), (c,c), (a,b), (b,a), (a,c), (c,a), (b,c), (c,b)

   Which of the following statements is (are) TRUE?

   a) R is reflexive but not symmetric.

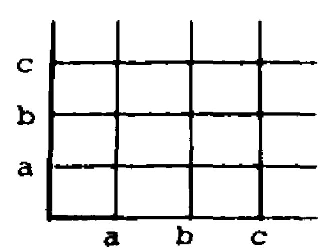
   b) R is symmetric but not transitive.

   c) R is transitive but not reflexive.

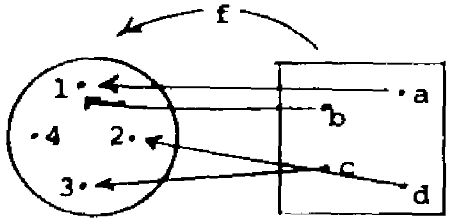
   d) R is reflexive and symmetric but not transitive.

   e) R is reflexive, symmetric, and transitive.

4. In the grid below, plot the relation (R) described in Problem 3.



5. The accompanying diagram describes a function (f) from A to B.



a) The domain of (f) is: { }

b) Thr range of (f) is: { }

c) f(c) =

6. Suppose the adjacent function table was given. Which of the following ordered pairs would also be on the table?

A	B
-2	-10
0	0
5	25
7	35
.	.
.	.

a) (-3,-20)

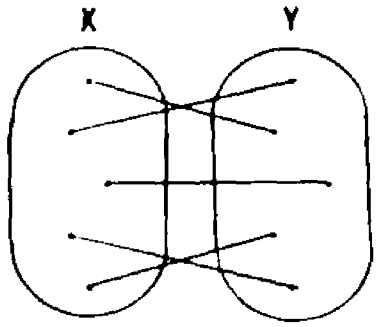
b) (-4,20)

c) (10,50)

d) (8,45)

e) none of these

7. Which of the following, does the adjacent diagram illustrate?



a) The diagram illustrates a function from X to Y.

b) The diagram illustrates a function from Y to X.

c) There exists a one-to-one correspondence between X and Y.

d) All of the above.

e) None of the above.

8. The function G, from # to $ is illustrated by the table below. Which of the following defines the function G(#)?

#	$
-1	2
1	2
-2	5
2	5
-3	10
3	10
.	.
.	.
.	.

a) G(#) = # + 1

b) G(#) = $\#^2$ + 1

c) G(#) = $\#^3$ + 1

d) G(#) = 2# + 2

e) none of the above

PROBABILITY AND STATISTICS PRE-TEST

1. A coin is tossed into the air. What is the probability that it will land "heads up"?

   a) 50 b) $\frac{2}{1}$ c) $\frac{1}{2}$

   d) 1.0 e) none of these

2. One ball is to be drawn at random from a box containing 3 red, 3 blue, and 4 green balls. What is the probability that the drawn ball is <u>red</u>?

   a) 0.1 b) 0.3 c) 10

   d) 30 e) none of these

3. If THREE coins are tossed simultaneously into the air, in how many ways could they land with:

A. Three heads up	a) 1	b) 2	c) 3	d) 4	e) none of these
B. Two heads up	a) 1	b) 2	c) 3	d) 4	e) none of these
C. One head up	a) 1	b) 2	c) 3	d) 4	e) none of these
D. 0 heads up	a) 1	b) 2	c) 3	d) 4	e) none of these
E. At least two heads up	a) 1	b) 2	c) 3	d) 4	e) none of these

4. A box contains five pieces of paper as shown below. What is the probability of drawing the two pieces with the numeral 1 on them in two successive draws without replacement?

   a) 0.50 b) 0.45 c) 0.40

   d) 0.05 e) none of these

5. A political committee of 10 is to be selected from a population of 60 Democrats and 40 Republicans. Which of the following provides the best representative sample?

   a) Select 10 names at random from the names of the 100 persons involved.
   b) Select at random 5 men and 5 women.
   c) Classify the population involved into 5 age groups; then select at random one Republican and one Democrat from each age group.
   d) From the Democrats select 6 at random and from the Republicans select four at random.
   e) Make a list of the last names of the 100 persons involved in alphabetical order and select every 10th name.

6. A journalist interested in knowing the attitude of his community toward a proposed increase in school millage sampled his population by questioning the first 20 customers of the local barber shop. From their responses he concluded that his community is against the millage increase. Which of the following best describes the sampling method used and the conclusion drawn?

   a) The conclusion is valid since the sample is representative of the community.
   b) The sample is an unbiased random sample of the male population and therefore the conclusion is valid for that population only.
   c) The sample is unbiased but the conclusion is biased.
   d) No valid conclusion can be drawn since the sample is biased.
   e) The sample is biased but the conclusion is not since most people do not like millage increases.

7. The IQ scores of any large group tends to be normally distributed about their mean. From a population of 10,000 college freshmen a sample of 100 is randomly drawn. They are tested and their IQ scores are found to have a mean of 108. What can be concluded from this experiment.

   a) The average IQ of the adult U.S. population is 108.
   b) The average IQ of the college students is 108.
   c) The college freshmen's IQ is 8 points higher than the average persons of the same age.
   d) Since the set of college freshmen is a subset of the total college students, then the average IQ of the college students is at least 108.
   e) The average IQ of the college freshman is about 108.

Questions 8, 9, and 10 ARE TO BE ANSWERED WITH REFERENCE TO THE FOLLOWING STATEMENT.

Suppose that the numbers below represent the scores of 15 students on a mathematical examination.

90	85	75	65	55
90	85	70	65	50
85	80	70	60	40

8. The mean score of this test is: __________

9. a) The median of the scores of this test is: __________

   b) The mode of the scores of this test is: __________

10. The range of the scores of this test is: __________

PROBABILITY AND STATISTICS POST-TEST

1. When tossing a coin, if the probability that it will land "heads up" is Pr (H), and the probability that it will land "tails up" is Pr (T), then:

   a) Pr (H) + Pr (T) = 100 b) Pr (H) + Pr (T) = 1

   c) Pr (H) x Pr (T) = 1 d) Pr (H) x Pr (T) = 0

   e) Pr (H) − Pr (T) = 1

2. A box contains 50 light bulbs, 10 of which are 50-watt, 15 are 75-watt and the remaining are 100-watt. What is the probability on one drawing a 75-watt bulb will come up?

   a) 0.10 b) 0.15 c) 0.25

   d) 0.50 e) none of these

3. If THREE coins are tossed simultaneously into the air, what is the probability that they will land with:

A. Three heads up	a) $\frac{1}{8}$	b) $\frac{2}{8}$	c) $\frac{3}{8}$	d) $\frac{4}{8}$	e) none of these
B. Two heads up	a) $\frac{1}{8}$	b) $\frac{2}{8}$	c) $\frac{3}{8}$	d) $\frac{4}{8}$	e) none of these
C. One head up	a) $\frac{1}{8}$	b) $\frac{2}{8}$	c) $\frac{3}{8}$	d) $\frac{4}{8}$	e) none of these
D. 0 heads up	a) $\frac{1}{8}$	b) $\frac{2}{8}$	c) $\frac{3}{8}$	d) $\frac{4}{8}$	e) none of these
E. At least one head up	a) $\frac{1}{8}$	b) $\frac{2}{8}$	c) $\frac{3}{8}$	d) $\frac{4}{8}$	e) none of these

4. A fair six-sided die (cube) is rolled three times. What is the probability of obtaining "five spots up" on each of the three rolls?

   a) $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ b) $\frac{1}{6+6+6}$ c) $3\frac{1}{6}$

   d) $\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$ e) none of these

5. From a group of 4 boys and 2 girls, how many ways are there of selecting a committee of 2 boys and 1 girl?

   a) 12 b) 8 c) 4

   d) 2 e) none of these

6. Suppose there are 1300 fifth-grade students in a school system. You are given the task of estimating their arithmetic achievement by testing a sample of the population. Which of the following would provide you with the best sample for this purpose?

   a) From the population, select one boy and one girl and test them.
   b) Select at random one elementary school in that system and test all the fifth graders in that particular school.
   c) In each school, ask the fifth-grade teacher to provide you with the names of 10 pupils in her class and test them.
   d) From each of the elementary schools select at random 10% of the fifth-grade students and test them.
   e) No representative sample can be obtained and therefore all 1300 students must be tested.

7. The measure of variability most used in testing is:

   a) the range b) the mode c) the mean

   d) the median e) the standard deviation

QUESTIONS 8, 9, and 10 ARE TO BE ANSWERED WITH REFERENCE TO THE FOLLOWING STATEMENT

The numbers below represent the ages of a sample of 20 pupils in an elementary school.

12	10	9	8	6
11	10	8	8	6
11	10	8	7	6
10	9	8	7	6

8. The mean age of the pupils in this sample is: __________

9. a) The median of the ages is: __________

   b) The mode of the ages is: __________

10. The range of the ages is: __________

MATHEMATICAL SYSTEMS PRE-TEST

1. Which of the following statements is (are) false?

   a) A mathematical system is a set of elements together with one or more binary operations defined on the set.

   b) The set of rational numbers with the operations of addition and multiplication is a field.

   c) In clock arithmetic (mod 12):

   $$(9 + 5) \bmod 12 = 14$$

   d) In clock arithmetic (mod 12):

   $$(A \times B) \bmod 12 = (B \times A) \bmod 12$$

   e) A numeral is divisible by q if the sum of the digits in its decimal representation is zero mod 9.

2. In clock arithmetic, 37 (mod 8) = 5. The result 5 is actually a:

   a) sum b) product c) quotient

   d) remainder e) none of these

3. What is (8 + 7) mod 4 equal to?

   a) 1 b) 3

   c) 8 (mod 4) + 7 (mod 4)

   d) a and c are correct

   e) b and c are correct

4. Complete the multiplication table (mod 4).

x	0	1	2	3
0	0	0	0	0
1	0			
2	0			
3	0			

5. Consider the following two tables.

Table 1

■	A	B	C
A	A	B	C
B	B	C	A
C	C	A	B

Table 2

●	A	B	C
A	A	A	A
B	A	B	C
C	A	C	B

A. Use the above tables to compute the following.

a) (A■B) C = ●

b) (A■B) A = ●

c) (C■C) C = ●

B. Does Table 1 describe a mathematical system?

yes _____ no _____

C. Does Table 2 describe a mathematical system?

yes _____ no _____

D. Does the set {A, B, C} and the two operations ■ and ● form a field?

yes _____ no _____

MATHEMATICAL SYSTEMS POST-TEST

1. Which of the following statements is (are) true?

   a) Clock arithmetic is closed under addition and multiplication.

   b) In clock arithmetic (mod 12):

      $A(B+C) = AB + C$

   c) The set of whole numbers and the operation of addition define a mathematical system.

   d) There is only one way of establishing a one-to-one correspondence between set $A = \{1, 2, 3\}$ and set $B = \{a, b, c\}$

   e) $(12 \times 6) \bmod 8 = 0$

2. In clock arithmetic (mod 9), 53 (mod 9) = 8. The result 8 is actually a:

   a) remainder b) partial quotient c) partial sum

   d) product e) none of these

3. In clock arithmetic (mod 8), what is: $(6 \times 3) \bmod 8 =$

   a) 6 (mod 8) x 3 (mod 8)

   b) 2

   c) 18

   d) a and b are correct

   e) a and c are correct

4. Complete the clock addition table (mod 4) below.

⊕	0	1	2	3
0	0	1	2	
1	1			
2	2			
3				

5. Consider the table below.

○	a	b	c	d
a	a	b	c	d
b	b	d	a	c
c	c	a	d	b
d	d	c	b	a

a) Does the table above describe a mathematical system?

yes _____ no _____

b) Does the operation ○ have the closure property?

yes _____ no _____

c) Does the operation ○ have the associative property?

yes _____ no _____

d) What (if any) is the identity element of the operation ○ ?

____

e) Pair the following elements with their inverse.

a_____, b_____, c_____, d_____

APPENDIX B

A TEST OF BASIC MATHEMATICAL UNDERSTANDINGS

FORM A (PRE-TEST) AND

FORM B (POST-TEST)

A TEST OF BASIC MATHEMATICAL UNDERSTANDINGS

PREPARED BY:

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MICHIGAN STATE UNIVERSITY

EAST LANSING, MICHIGAN

1964

Directions:

This test is designed to measure your understanding of mathematics. Many of the items relate to the new content in present programs of mathematics for elementary pupils.

Each of the fifty-five questions is of multiple-choice type and includes four possible answers. Read each question carefully and decide which answer fulfills the requirements of the statement. Then circle the response on the answer sheet to indicate your choice.

Circle only one answer for each question. If you change your choice, erase your original mark and circle the correct one.

Sample Question:

1. Which of the following shows the decimal form of the fraction $\frac{5}{4}$?

a. 125	b. 12.5
c. 1.25	d. .125

Answer Sheet:

1. a b (c) d

Since 1.25 is the correct answer, the letter (c) is circled.

FORM A (PRE-TEST)

1. When you write the numeral "5" you are writing

   a. the number 5.

   b. a pictorial expression.

   c. a symbol that stands for an idea.

   d. a Hindu-Babylonian symbol.

2. Bill discovered that > means "is greater than" and < means "is less than." In which of the following are these symbols not used correctly?

   a. The number of states in the United States < the number of United States Senators.

   b. The number of states in the United States > the number of stripes in the flag.

   c. $2^3 > 3^2$

   d. $3 + a < 5 + a$

3. When two Roman numerals stand side by side in a symbol, their values are added

   a. always.

   b. sometimes.

   c. never.

   d. if the base is X.

4. Which of the following describe/describes our own system of numeration?

   a. additive

   b. positional

   c. subtractive

   d. introduces new digits for numbers larger than 10

      1) a and b are correct

      2) a and c are correct

      3) a and d are correct

      4) a, b, and d are correct.

A.

5. Zero may be used

   a. as a place holder.

   b. as a point of origin.

   c. to represent the absence of quantity.

   d. in all of the above different ways.

6. 2,200.02 is shown by

   a. 2000 + 200 + 20.

   b. $2000 + 20 + \frac{2}{10}$

   c. $2000 + 200 + \frac{2}{100}$

   d. 2000 + 200 + 200.

7. 5840 rearranged so that the 8 is 200 times the size of 4 would be

   a. 5840.

   b. 8540.

   c. 5048.

   d. 5408.

8. Which of the following does <u>not</u> show the meaning of 423_{ten}?

   a. (4 x 100) + (2 x 10) + 3(1) = 423

   b. 42 tens + 3 ones = 423

   c. 423 ones = 423

   d. 4 hundreds + 42 tens + 23 ones = 423

9. A numeral for the X's in this example can be written in many different bases. Which numerals are correct?

XX	X	XX	XX
X	X	X	X
X	XX	X	X

   a. 100_{four}

   b. 14_{twelve}

   c. 16_{ten}

   d. 31_{five}

      1) a and c are correct.

      2) b and c are correct.

      3) a, b, and c are correct.

      4) all four are correct.

A.

10. A "2" in the third place of a base twelve number would represent

    a. 2×12^3

    b. 12×2^3

    c. 12×2^{12}

    d. 2×12^2

11. In this addition example, in what base are the numerals written?

    a. base two

    b. base three

    c. base four

    d. none of the above

$$\begin{array}{r} 120_? \\ +10_? \\ \hline 200_? \end{array}$$

12. About how many tens are there in 6542?

    a. 6540

    b. 654

    c. $65\frac{1}{2}$

    d. 6.5

13. Place or order in a series is shown by

    a. book no. 7.

    b. three boxes of matches.

    c. a dozen cupcakes.

    d. two months.

14. Which of the following indicates a group?

    a. 45 tickets

    b. track 45

    c. page 54

    d. apartment No. 7.

15. The sum of any two natural numbers

    a. is not a natural number.

    b. is sometimes a natural number.

    c. is always a natural number.

    d. is a natural number equal to one of the numbers being added.

A.

16. The counting numbers are closed under the operations of

    a. addition and subtraction.

    b. addition and multiplication.

    c. addition, subtraction, multiplication, and division.

    d. addition, subtraction, and multiplication.

17. If a and b are natural numbers, then $a + b = b + a$ is an example of

    a. commutative property.

    b. associative property.

    c. distributive property.

    d. closure.

18. If $a \times b = 0$ then

    a. a must be zero.

    b. b must be zero.

    c. either a or b must be zero.

    d. neither a nor b must be zero.

19. When a natural number is multiplied by a natural number other than 1, how does the answer compare with the natural number multiplied?

    a. larger

    b. smaller

    c. the same

    d. can't tell from information given

20. Which of the following is the quickest way to find the sum of several numbers of the same size?

    a. counting

    b. adding

    c. subtracting

    d. multiplication

21. How would the product in this example be affected if you put the 29 above the 4306 and multiplied the two numbers?

    a. The answer would be larger.

    b. The answer would be smaller.

    c. You cannot tell until you multiply both ways.

    d. The answer would be the same.

$$\begin{array}{r} 4306 \\ \underline{\times 29} \end{array}$$

A.

22. An important mathematical principle can be helpful in solving the following example.

$$28 + 659 + 72 = \square$$

What principle will be of most help?

a. the associative principle.

b. the commutative principle.

c. the distributive principle.

d. both the associative and distributive principles.

23. The product of 356 x 7 is equal to

a. (300 x 50) x (6 + 7).

b. (3 x 7) + (5 x 7) + (6 x 7).

c. 300 x 50 x 6 x 7.

d. (300 x 7) + (50 x 7) + (6 x 7).

24. Which of the following is <u>not</u> a prime number?

a. 271

b. 277

c. 281

d. 282

25. Which of the following numbers is odd?

a. 18 x 11

b. 11 x 20

c. 99 x 77

d. none of the above

26. The inverse operation generally used to check multiplication is

a. addition.

b. subtraction.

c. multiplication.

d. division.

27. The greatest common factor of 48 and 60 is

a. 2 x 3

b. 2 x 2 x 3

c. 2 x 2 x 2 x 2 x 3 x 5

d. none of the above

A.

28. Look at the example at the right. Why is the "4" in the third partial product moved over two places and written under the 2 of the multiplier?

```
   157
 x 246
   942
  628
 314
 38622
```

a. If you put it directly uner the other partial products, the answer would be wrong.

b. You must move the third partial product two places to the left because there are three numbers in the multiplier.

c. The number 2 is the hundreds column, so the third partial product must come under the hundreds column.

d. You are really multiplying by 200.

29. Which of the fundamental properties of arithmetic would you employ in proving that $(a + b) + (a + c) = 2a + b + c$?

a. Associative property.

b. Commutative property.

c. Associative and distributive properties.

d. Associative and commutative properties.

30. If N represents an even number, the next larger even number can be represented by

a. $N + 1$

b. $N + 2$

c. $N + N$

d. $2 \times N + 1$

31. Every natural number has at least the following factors:

a. zero and one.

b. zero and itself.

c. one and itself.

d. itself and two.

32. It is said that the set of whole numbers has a natural order. To find the successor of a natural number, one must

a. add 1.

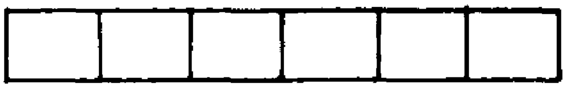
b. find a number that is greater.

c. square the natural number.

d. subtract 1 from the natural number.

A.

33. The paper below has been divided into 6 pieces. It shows



a. sixths.

b. thirds.

c. halves.

d. parts.

34. A fraction may be interpreted as:

a. a quotient of two natural numbers.

b. equal part/parts of a whole.

c. a comparison between two numbers.

d. all of the above.

35. When a common (proper) fraction is divided by a common fraction, how does the answer compare with the fraction divided?

a. It will be larger.

b. It will be smaller.

c. It will be twice as large.

d. There will be no difference.

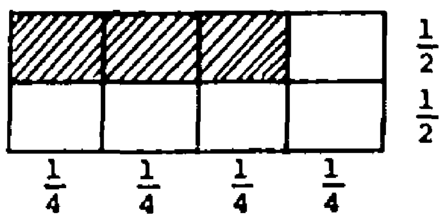
36. Which algorithm is illustrated by the following sketch?

a. $\frac{1}{2} \times \frac{3}{4} = ?$

b. $\frac{1}{2} + \frac{3}{3} = ?$

c. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = ?$

d. $\frac{4}{4} - \frac{3}{2} = ?$



37. Another name for the inverse for multiplication of a rational number is the

a. reciprocal.

b. opposite.

c. reverse.

d. zero.

A.

38. Examine the division example on the right. Which sentence best tells why the answer is larger than the 5? $5 \div \frac{3}{4} = 6\frac{2}{3}$

   a. Inverting the divisor turned the $\frac{3}{4}$ upside down.

   b. Multiplying always makes the answer larger.

   c. The divisor $\frac{3}{4}$ is less than 1.

   d. Dividing by proper and improper fractions makes the answer larger than the number divided.

39. The value of a common fraction will not be changed if

   a. we add the same number to both terms.

   b. we multiply one term and divide the other term by that same number.

   c. we subtract the same amount from both terms.

   d. we multiply both terms by the same number.

40. The nearest to 45% is

   a. 44 out of 100

   b. .435

   c. 4.5

   d. .405

41. The principal of a school said that 27 per cent of the pupils went to the museum. Which statement best describes the expression "27 per cent of the pupils went to the museum"?

   a. It means that 27 children out of every 100 children went to the museum.

   b. It means that you must multiply the number of children in the school by 27/100 to find the number who went to the museum.

   c. If the children were divided into groups of 100 and those who went to the museum were distributed evenly among them, there would be in each group 27 who went to the museum.

   d. 27 per cent is the same as .27--a decimal fraction written in per cent form.

42. Sally completed $\frac{2}{3}$ of the story in 12 minutes. At that rate how long will it take her to read the entire story?

   a. 18 minutes

   b. 12 minutes

   c. 6 minutes

   d. 24 minutes

A.

43. There were 400 students in the school. One hundred per cent of the children had lunch in the cafeteria on the first day of school. On the second day 2 boys were absent and 88 children went home for lunch. Which of the following equations can be used to find the per cent of the school enrollment who went home for lunch?

a. $400 - 88 = X$

b. $\frac{x}{100} = \frac{88}{400}$

c. $\frac{x}{88} = 400$

d. $400 - 90 = X$

44. What can be said about y in the following open sentence if x is a natural number?

$x + x + 1 = y$

a. $x < y$

b. $x > y$

c. $x = y$

d. $x \neq y$

45. Which one of the following fractions will give a repeating decimal?

a. $\frac{1}{2}$

b. $\frac{3}{4}$

c. $\frac{5}{8}$

d. $\frac{6}{11}$

46. Which of the following is <u>not</u> an open sentence?

a. $7 + 2 = \square$

b. $h - 5 = 9$

c. $\frac{c}{1} - 30 = 6$

d. $n - 3$

A.

47. For a mathematical system consisting of the set of odd numbers and the operation of multiplication.

    a. the system is closed.

    b. the system is commutative.

    c. the system has an identity element.

    d. all of the above are correct.

48. Measurement is a process which

    a. compares an object with some known standard or accepted unit.

    b. tries to find the exact amount.

    c. is never an exact measure.

    d. chooses a unit and then gives a number which tells how many of that unit it would take.

        1) a and b are correct.

        2) a and c are correct.

        3) a, b, and d are correct.

        4) a, c, and d are correct.

49. The set of points sketched below represents a

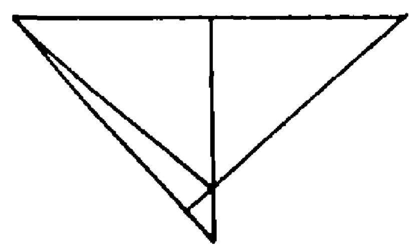
    a. line.

    b. ray.

    c. line segment.

    d. none of the above.

50. How many triangles does the figure contain?



    a. four

    b. six

    c. eight

    d. ten

51. The set of points on two rays with a common end-point is called

    a. a triangle.

    b. an angle.

    c. a vertex.

    d. a side of a triangle.

A.

52. If a circle is drawn with the points of a compass 3 inches apart, what would be 3 inches in length?

    a. circumference

    b. diameter

    c. area

    d. radius

53. The solution set of an open sentence may consist of

    a. two or more numbers.

    b. no numbers.

    c. only one number.

    d. any or all of these.

54. Consider a set of three objects. How many sub-sets or groups can be arranged?

    a. nine

    b. eight

    c. seven

    d. six

55. If two sets are said to be equivalent, then

    a. every element in the first set can be paired with one and only one element in the second set.

    b. every element in one set must also be an element in the second set.

    c. they are intersecting sets.

    d. one must be the null set.

FORM B (POST-TEST)

1. Which of the underlined words or signs in the following sentences refer to symbols rather than the things they represent?

   a. 4 can be written on the blackboard.

   b. Regardless of what symbol we use, we are thinking about the number 2.

   c. A pencil is used for writing.

   d. The number 16 is the same as the number 7 + 9.

2. When we use the = symbol between two terms (as 2 + 2 = 4) we mean that both terms represent the same concept or idea. Which of the following is not correctly stated?

   a. 3 + 4 = 5 + 2

   b. 5 + 2 = 7 and 7 = 5 + 2

   c. (5 + 2) x 3 = 7 x 3

   d. 7 = 7

      1) a and b are correct.

      2) a and c are correct.

      3) a, b, and c are correct.

      4) a, b, c, and d are correct.

3. If the Roman system of numeration were a "place value system" with the same value for the base as the Hindu-Arabic system, the first four base symbols would be

   a. I, X, C, and M.

   b. I, V, X, and L.

   c. X, L, C, and M.

   d. X, C, L, and D.

4. Which of the following does not describe a characteristic of our decimal system of numeration?

   a. It uses zero to keep position when there is an absence of value.

   b. It makes a ten a standard group for the organization of all numbers larger than nine.

   c. It makes 12 the basis for organizing numbers larger than eleven.

   d. It uses the additive concept in representing a number of several digits.

B.

5. In the numeral 7,843, how does the value of the 4 compare with the value of the 8?

   a. 2 times as great

   b. 1/2 as great

   c. 1/10 as great

   d. 1/20 as great

6. In the numeral 6,666 the value of the 6 on the extreme left as compared with the 6 on the extreme right is

   a. 6,000 times as great.

   b. 1,000 times as great.

   c. the same since both are sixes.

   d. six times as much.

7. Which of the following statements best tells why we write a zero in the numeral 4,039 when we want it to represent "four thousand thirty-nine"?

   a. Writing the zero helps to fill a place which would otherwise be empty and lead to misunderstanding.

   b. The numeral would represent "four hundred thirty-nine" if we did not write the zero.

   c. Writing the zero tells us not to read the hundreds' figure.

   d. Zero is used as a place-holder to show that there is no number to record in that place.

      1) a and b are correct.

      2) a and c are correct.

      3) a and d are correct.

      4) a, b, and d are correct.

8. Below are four numerals written in expanded notation. Which one is <u>not</u> written correctly?

   a. $4(\text{ten})^2 + 9(\text{ten})^1 + 3(\text{ones}) = 493_{\text{ten}}$

   b. $3(\text{seven})^3 + 6(\text{seven})^1 + 1(\text{one}) = 363_{\text{seven}}$

   c. $4(\text{twelve})^2 + 5(\text{twelve})^1 + e(\text{one}) = 45e_{\text{twelve}}$

   d. $2(\text{five})^2 + 2(\text{five})^1 + 4(\text{one}) = 224_{\text{five}}$

B.

9. If you are permitted to use any or all of the sumbols 0, 1, 2, 3, 4 and 5 for developing a system of numeration with a place value system of numeration similar to ours, a list of all possible bases would include:

   a. base one, two, three, four, five, and six.

   b. base two, three, four, five, and six.

   c. base two, three, four, and five.

   d. base one, two, three, four, and five.

10. About how many hundreds are there in 34,870?

   a. $3\frac{1}{2}$

   b. 35

   c. 350

   d. 3,500

11. In what base are the numerals in this multiplication example written?

   a. base five

   b. base eight

   c. base eleven

   d. you can't tell

$$\begin{array}{r} 34_? \\ \underline{23_?} \\ 124_? \\ \underline{70_?} \\ 1024_? \end{array}$$

12. Which of the following are correct?

   a. In the symbol 5^3, 5 is the base and 3 is the exponent.

   b. In the symbol 5^3, 3 is the base and 5 is the exponent.

   c. $5^3 = 5 \times 5 \times 5$

   d. $5^3 = 3 \times 3 \times 3 \times 3 \times 3$

      1) a and d are correct.

      2) b and c are correct.

      3) a and c are correct.

      4) b and d are correct.

13. In the series of numerals 1,...17, 18, 19, 20, 21,..., what term best applies to 19?

   a. nominal

   b. ordinal

   c. composite

   d. cardinal

B.

14. Examine the following illustration:

(1 2 3 4 5 6)

Which of the following does the above best illustrate?

a. The idea of a cardinal number.

b. The use of an ordinal number.

c. A means for determining the cardinal number of the set by counting with ordinal numbers.

d. None of the above.

15. The quotient of any two whole numbers

a. is not a natural number.

b. is sometimes a natural number.

c. is always a natural number.

d. is a natural number less than one of the numbers being divided.

16. The integers are closed under the operations of

a. addition.

b. subtraction.

c. multiplication.

d. division.

1) a and b are correct.

2) a and c are correct.

3) a, b, and c are correct.

4) a, b, c, and d are correct.

17. A student solved this example by adding down; then he checked his work by adding up.

Add ↓		Check ↑	
	34		34
	52		52
	86		86

It could be classified as an example of

a. the distributive principle.

b. the associative principle.

c. the commutative principle.

d. the law of compensation.

B.

18. The statement "the quotient obtained when zero is divided by a number is zero" is expressed as

    a. $\frac{a}{0} = 0$

    b. $\frac{0}{a} = 0$

    c. $\frac{0}{0} = a$

    d. $\frac{a}{a} = 0$

19. When a natural number is divided by a natural number other than 1, how does the answer compare with the natural number divided?

    a. larger

    b. smaller

    c. one-half as large

    d. can't tell from information given

20. If you had a bag of 350 marbles to be shared equally by 5 boys, which would be the quickest way to determine each boy's share?

    a. counting

    b. adding

    c. subtracting

    d. dividing

21. If the multiplier is x, the largest possible number to carry is

    a. x

    b. $x + 1$

    c. 0

    d. $x - 1$

22. Which one of the following methods could be used to find the answer to this example?

    $17\overline{)612}$

    a. Multiply 17 by the quotient.

    b. Add 17 six hundred times.

    c. The answer would be the sum.

    d. Subtract 17 from 612 as many times as possible. The answer would be the number of times you were able to subtract.

B.

23. Which one of the following would give the correct answer to this example?

$$\begin{array}{r} 2.1 \\ \underline{\times 21} \end{array}$$

a. The sum of 1 x 2.1 and 21 x 2.1.

b. The sum of 10 x 2.1 and 2 x 2.1.

c. The sum of 1 x 2.1 and 20 x 2.1.

d. The sum of 1 x 2.1 and 2 x 2.1.

24. Which would give the correct answer to 439 x 563?

a. Multiply 439 x 3, 439 x 60, 439 x 5 and then add the answer.

b. Multiply 563 x 9, 563 x 3, 563 x 4 and then add the answer.

c. Multiply 563 x 9, 563 x 39, 563 x 439 and then add the answer.

d. Multiply 439 x 3, 439 x 60, 439 x 500 and then add the answer.

25. Which of these numerals are names for prime numbers?

a. 3

b. $\frac{4}{2}$

c. 12_{five}

d. 9 - 2

1) a is correct.

2) a and c are correct.

3) a, b, and d are correct.

4) a, b, c, and d are correct.

26. Let x represent an odd number; let y represent an even number. Then x + y must represent

a. an even number.

b. a prime number.

c. an odd number.

d. a composite number.

B.

27. The inverse operation for addition is

    a. addition.

    b. subtraction.

    c. multiplication.

    d. division.

28. The least common multiple of 8, 12, and 20 is

    a. 2 x 2.

    b. 2 x 3 x 5.

    c. 2 x 2 x 2 x 3 x 5.

    d. 2 x 2 x 2 x 2 x 2 x 2 x 2 x 3 x 5.

29. Which statement best tells why we carry 2 from the second column?

$$\begin{array}{r} 251 \\ 161 \\ 252 \\ \underline{271} \\ 935 \end{array}$$

    a. If we do not carry the 2, the answer would be 20 less than the correct answer.

    b. Since the sum of the second column is more than 20, we put the 2 in the next column.

    c. Since the sum of the second column is 23 (which has two figures in it), we have room for the 3 only, so we put 2 in the next column.

    d. Since the value represented by the figures in the second column is more than 9 tens, we must put the hundreds in the next column.

30. The operations which are associative are

    a. addition.

    b. subtraction.

    c. multiplication.

    d. division.

        1) a and b are correct.

        2) a and c are correct.

        3) a, b, and c are correct.

        4) a and d are correct.

31. Which of the following is an even number?

    a. $(100)_{three}$

    b. $(100)_{five}$

    c. $(100)_{seven}$

    d. $(200)_{five}$

B.

32. The fact that a + b (b + c) is exactly equal to (c + b) + a is an example of

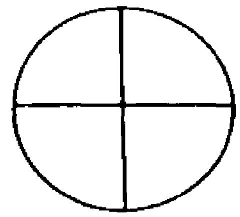
a. distributivity.

b. commutativity.

c. closure.

d. associativity.

33. Observe the drawing on the right. When the circle is cut into equal pieces, the size of each piece



a. decreases as the number of pieces increases.

b. increases as the number of pieces decreases.

c. increases as the number of pieces increases.

d. decrease as the number of pieces decreases.

1) a and b are correct.

2) a and c are correct.

3) b and c are correct.

4) b and d are correct.

34. The symbol $\frac{3}{4}$ may be used to represent the idea that

a. 3 is to be divided by 4.

b. 3 of the 4 equal parts are being considered.

c. 3 objects are to be compared with 4 objects.

d. all of the above.

35. When a whole number is multiplied by a common (proper) fraction other than one, how does the answer compare with the whole number?

a. It will be larger.

b. It will be smaller

c. There will be no difference.

d. You are not able to tell.

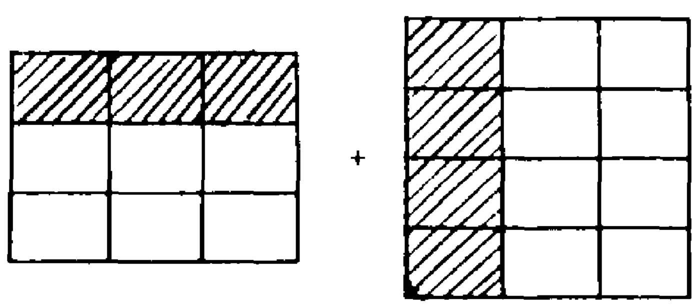
36. Which of the addition examples is best represented by the shaded parts of the diagram below?

a. $\frac{1}{2} + \frac{1}{3}$

b. $\frac{2}{3} + \frac{3}{4}$

c. $\frac{2}{3} + \frac{1}{4}$

d. $\frac{1}{3} + \frac{1}{3}$



B.

37. We can change the denominator of the fraction $\frac{\frac{2}{3}}{\frac{4}{5}}$ to the number "1" without changing the values of the fraction by

a. adding $\frac{5}{4}$ to the numerator and denominator.

b. subtracting $\frac{5}{4}$ from the numerator and the denominator.

c. multiplying both the numerator and the denominator by $\frac{5}{4}$.

d. dividing the numerator and the denominator by $\frac{5}{4}$.

38. What statement best tells why we "invert the divisor and multiply when dividing a fraction by a fraction?

a. It is an easy method of finding a common denominator and arranging the numerators in multiplication form.

b. It is an easy method for dividing the denominators and multiplying the numerators of the two fractions.

c. It is a quick, easy, and accurate method of arranging two fractions in multiplication form.

d. Dividing by a fraction is the same as multiplying by the reciprocal of the fraction.

39. If the denominator of the fraction $\frac{2}{3}$ is multiplied by 2, the value of the resulting fraction will be

a. half as large.

b. double in value.

c. unchanged in value.

d. a new symbol for the same number.

40. 45% may also be written as

a. .45

b. 45/100

c. 45 x 100%

d. .450

1) a and b are correct.

2) a and c are correct.

3) a and d are correct.

4) a, b, and d are correct.

B.

41. .5 and .27 are illustrations of "decimal fractions." They could be written as "common fractions" in the form of $\frac{1}{2}$ and $\frac{27}{100}$, respectively. What is a decimal fraction?

a. It is another way of writing percentage.

b. It is an extension of the decimal number system to the right of one's place.

c. A number like $.37\frac{1}{2}$ which has both a decimal and a fraction as parts of it.

d. A number like $\frac{.2}{.56}$ which is a fraction and has a decimal as either the numerator or denominator or both.

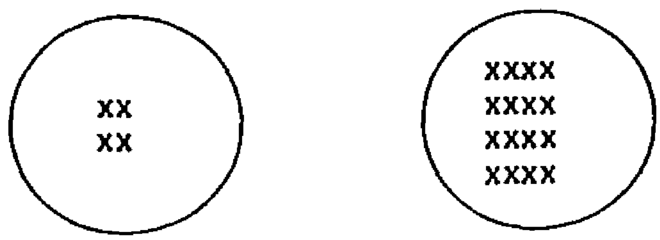
42. The ratio of x's in Circle A to x's in Circle B can be shown by

a. $\frac{16}{4}$

b. $\frac{1}{4}$

c. $\frac{1}{2}$

d. $\frac{4}{16}$



A B

43. Sue paid 20¢ for 4 apples. Which of the equations below could be used to find the cost of 1 apple?

a. $\frac{4}{20} = \frac{1}{x}$

b. $x + 4 = 20$

c. $\frac{x}{4} = 20$

d. $x - 4 = 20$

44. The decimal for the numeral $\frac{6}{17}$ will

a. be a repeating decimal.

b. not repeat or end since 17 is prime.

c. repeat in cycles of less than 23 digits.

1) a is correct.

2) a and b are correct.

3) a and c are correct.

4) a, b, and c are correct.

B.

45. Which of the following statements is <u>not</u> correct?

   a. (-9) + 6 = -3

   b. (-5) + (-5) = -10

   c. -8 + 0 = -8

   d. (-8) + (9) = -1

46. Which of the following is a list of all of the factors of 12?

   a. 1, 2, 3, 4, 8 & 12

   b. 1, 2, 3, 4, 6 & 12

   c. 1, 2, 3, 4 & 6

   d. 2, 3, 4, 6 & 12

47. Modular arithmetic is

   a. cumutative.

   b. associative.

   c. distributive with respect to multiplication over addition.

   d. all of the above.

48. Which of the following is an approximate measure?

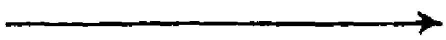
   a. 35 farms

   b. 12 buttons

   c. $7\frac{1}{2}$ inches

   d. 15 beads

49. Which of the following does the sketch below represent?



   a. line

   b. ray

   c. line segment

   d. set of points

      1) a is correct.

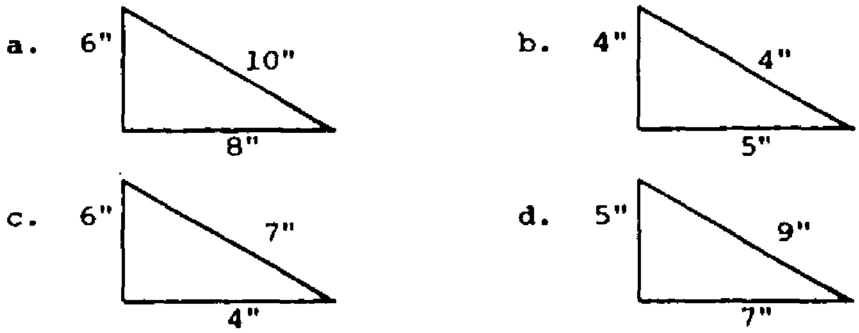
      2) a, b, and d are correct.

      3) a, c, and d are correct.

      4) b and d are correct.

B.

50. Which of these triangles are right triangles according to the length of the sides given?



51. A distinct point is

a. a point you can see.

b. a sharp object.

c. the intersection of two lines.

d. a dot.

52. A clerk sold a lady a round tablecloth that had a radius of 14 inches. Which of the formulas can she use to determine the length around the cloth?

a. $A = \pi r^2$

b. $C = \pi d$

c. $C = 2\pi r$

d. $A = C/d$

53. Which of the following best defines a solution set?

a. A solution set is a set which includes each and every member that gives a true statement.

b. A solution set is a single sentence which identifies a variable that will give a true statement.

c. A solution set is a set containing all the positive integers, zero, and the negative integers.

d. A solution set is a set containing rational numbers.

B.

54. Examine the following illustration.

    S = {0, 1, (-1), 2, (-2), 3,...10}

    Which one of the following is not a subset of S?

    a. {+9, + 10}

    b. {0, (-2), 5}

    c. {3, (-3), 10}

    d. {1, (-1), 6, 10}

55. If we use the set concept to define the operations for the counting numbers, addition would be defined in terms of

    a. the intersection of disjoint sets.

    b. the union of intersecting sets.

    c. the intersection of sets with common elements.

    d. the union of disjoint sets.

APPENDIX C

DUTTON ARITHMETIC ATTITUDE INVENTORY

APPENDIX C

DUTTON ARITHMETIC ATTITUDE INVENTORY

Name ______________________________ Student Number __________

Place a check (√) before those statements which tell how you feel about arithmetic. Select only the items which express your true feelings--probably not more than five items.

_____ 1. I avoid arithmetic because I am not very good with figures.
_____ 2. Arithmetic is very interesting.
_____ 3. I am afraid of doing word problems.
_____ 4. I have always been afraid of arithmetic.
_____ 5. Working with numbers if fun.
_____ 6. I would rather do anything else than do arithmetic.
_____ 7. I like arithmetic because it is practical.
_____ 8. I have never liked arithmetic.
_____ 9. I don't feel sure of myself in arithmetic.
_____ 10. Sometimes I enjoy the challenge presented by an arithmetic problem.
_____ 11. I am completely indifferent to arithmetic.
_____ 12. I think about arithmetic problems outside of school and like to work them out.
_____ 13. Arithmetic thrills me and I like it better than any other subject.
_____ 14. I like arithmetic, but I like other subjects just as well.
_____ 15. I never get tired of working with numbers.

16. Place a circle around one number to show how you feel about arithmetic in general.

1 2 3 4 5 6 7 8 9 10 11
Dislike Like

17. My feelings toward arithmetic were developed in grades: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, other _____ (circle one).

18. My average grades made in arithmetic were: A B C D (circle one).

19. List two things you like about arithmetic.
A.
B.

20. List two things you dislike about arithmetic.
A.
B.

APPENDIX D

ATTITUDE SCALES TOWARD DIFFERENT ASPECTS OF MATHEMATICS

A STUDY OF ATTITUDE OF PROSPECTIVE ELEMENTARY SCHOOL TEACHERS

Dear Student:

We are attempting to evaluate the attitudes of prospective elementary school teachers, such as yourself, toward some aspects of mathematics, school, and life in general.

Will you please read each statement and circle the response that reflects your feeling toward that statement? A, if you agree, D, if you disagree, or U, if you are undecided. Please be sure to circle only one letter for each statement.

The information obtained through this questionnaire will be kept strictly confidential. It will be used for research purposes only.

Student's Name ____________________

Student's Number ____________________

AGREE	DISAGREE	UNCERTAIN		
A	D	U	1.	Most school work is memorizing of information.
A	D	U	2.	In our school we got a great deal of practice and drill until we were almost perfect in our learning.
A	D	U	3.	The students spent most of their class time listening to the teachers and taking notes.
A	D	U	4.	My mathematics teacher showed us different ways of solving the same problem.
A	D	U	5.	Our teachers wanted us to do most of our learning from the textbook which is used in the course.
A	D	U	6.	My mathematics teacher did not like students to ask questions after he had given the explanation.
A	D	U	7.	My mathematics teacher wanted students to solve problems only by the procedures he taught.
A	D	U	8.	We were expected to learn and discover many ideas for ourselves.
A	D	U	9.	We were expected to develop a thorough understanding of ideas and not just to memorize information.
A	D	U	10.	Our teachers believed in strict discipline and each student did exactly what he was told to do.
A	D	U	11.	Students were encouraged to devise their own projects or experiments in order to learn on their own.
A	D	U	12.	My mathematics teacher expected us to learn how to solve problems by ourselves but helped when we had difficulties.
A	D	U	13.	In my mathematics classes, students who had original ideas got better grades than did students who were most careful and neat in their work.
A	D	U	14.	Most of our classroom work was listening to the teacher.
A	D	U	15.	My mathematics teacher required the students not only to master the steps in solving problems, but also to understand the reasoning involved.

AGREE DISAGREE UNCERTAIN

A D U 16. My mathematics teacher encouraged us to try to find several different methods of solving particular problems.

A D U 17. My mathematics course required more thinking about methods of solving problems than memorization of rules and formulas.

A D U 18. My mathematics teacher wanted us to discover mathematical principles and ideas for ourselves.

A D U 19. My mathematics teacher explained the basic ideas; we were expected to develop the methods of solutions for ourselves.

A D U 20. We did not use just one textbook for most of our subjects. Various sources and books from which we can learn were suggested to us.

A D U 21. Most of the problems my mathematics teacher assigned are to give us practice in using a particular rule or formula.

A D U 22. Much of our classroom work was discussing ideas and problems with the teacher and other pupils.

A D U 23. In mathematics there is always a rule to follow in solving problems.

A D U 24. I generally like my school work.

A D U 25. It should be possible to eliminate war once and for all.

A D U 26. Success depends to a large part on luck and fate.

A D U 27. More of the most able people should be encouraged to become mathematicians and mathematics teachers.

A D U 28. Someday most of the mysteries of the world will be revealed by science.

A D U 29. Anyone can learn mathematics.

A D U 30. Most school learning has little value for a person.

A D U 31. By improving industrial and agricultural methods, poverty can be eliminated in the world.

AGREE	DISAGREE	UNCERTAIN		
A	D	U	32.	I dislike school and will leave just as soon as possible.
A	D	U	33.	With increased medical knowledge, it should be possible to lengthen the average life span to 100 years or more.
A	D	U	34.	Outside of science and engineering, there is little need for mathematics (algebra, geometry, etc.) in most jobs.
A	D	U	35.	Mathematics is of great importance to a country's development.
A	D	U	36.	The most important reason for studying arithmetic and secondary school mathematics is that they help people to take care of their own financial affairs.
A	D	U	37.	Very few people can learn mathematics.
A	D	U	38.	Mathematics help one to think according to strict rules.
A	D	U	39.	Mathematics (algebra, geometry, etc.) is not useful for the problems of everyday life.
A	D	U	40.	Someday the deserts will be converted into good farming land by the application of engineering and science.
A	D	U	41.	I am bored most of the time in school.
A	D	U	42.	Almost all of the present-day mathematics was known at least a century ago.
A	D	U	43.	Education can only help people develop their natural abilities; it cannot change people in a fundamental way.
A	D	U	44.	I enjoy everything about school.
A	D	U	45.	A thorough knowledge of advanced mathematics is the key to an understanding of our world in the twentieth century.
A	D	U	46.	School is not very enjoyable, but I can see value in getting a good education.
A	D	U	47.	It is important to know mathematics (algebra, geometry, etc.) in order to get a good job.
A	D	U	48.	Almost anyone can learn mathematics if he is willing to study.

AGREE	DISAGREE	UNCERTAIN		
A	D	U	49.	Mathematics is a very good field for creative people to enter.
A	D	U	50.	Unless one is planning to become a mathematician or a scientist the study of advanced mathematics is not very important.
A	D	U	51.	Any person of average intelligence can learn to understand a good deal of mathematics.
A	D	U	52.	The most enjoyable part of my life is the time I spend in school.
A	D	U	53.	Even complex mathematics can be made understandable and useful to every high school student.
A	D	U	54.	In the near future most jobs will require a knowledge of advanced mathematics.
A	D	U	55.	With hard work anyone can succeed.
A	D	U	56.	Almost every present human problem will be solved in the future.
A	D	U	57.	Almost all pupils can learn complex mathematics if it is properly taught.
A	D	U	58.	I like all school subjects.
A	D	U	59.	There is little place for originality in mathematics.
A	D	U	60.	I enjoy most of my school work and want to get as much additional education as possible.
A	D	U	61.	Only people with a very special talent can learn mathematics.
A	D	U	62.	Mathematics will change rapidly in the near future.
A	D	U	63.	Although school is difficult, I want as much education as I can get.
A	D	U	64.	In the study of mathematics, if the student misses a few lessons it is difficult to catch up.
A	D	U	65.	I find school interesting and challenging.

APPENDIX E

THE EXPERIMENTAL GROUP EVALUATION OF DIFFERENT ASPECTS OF THE PROGRAM

APPENDIX E

STUDENT EVALUATION OF THE COURSE

Your evaluation of this course will be helpful in the future planning of similar courses in this program.

Consider each of the following statements separately and indicate the extent to which you agree or disagree with it by circling the appropriate symbol to the right of the statement.

The symbols used are:

SA--Strongly Agree
A--Agree in General
U--Undecided
D--Disagree in General
SD--Strongly Disagree

Please respond to all the items. Responses made to any items in these pages will have no bearing on your grade.

1. There should be more activities using manipulative materials in this course.	SA	A	U	D	SD
2. There should be more time spent on methods of teaching elementary school mathematics.	SA	A	U	D	SD
3. There should be more time spent on planning of teaching strategies to be used at the Allen Street School.	SA	A	U	D	SD
4. There should be more time spent teaching mathematics at the Allen Street School Laboratory.	SA	A	U	D	SD
5. There should be more lectures about mathematical content.	SA	A	U	D	SD
6. There should be more lectures about the methods of teaching mathematics.	SA	A	U	D	SD
7. There should be more films or video-tapes related to the teaching of elementary school mathematics.	SA	A	U	D	SD

8. There should be more hours assigned to this combined method-content course. SA A U D SD

9. There should be more time spent on paper and pencil problem solving activities. SA A U D SD

10. There should be more contacts with Allen Street School teachers in the planning of strategies for teaching mathematics at the school. SA A U D SD

What else would you suggest to improve the quality of this course in terms of content, method, materials, activities, etc? Write your comments below. They will be greatly appreciated.

Thank you.

APPENDIX F

A SPECIMEN OF STUDENT FILE FOR THE LEARNING UNIT ON NUMERATION SYSTEMS

Content Objectives		Activities	
1. Interpret a numeration system using different symbols.	☐	I. Make a set of beansticks and illustrate how to use them to explain some arithmetic problems.	☐
2. Write a number in expanded notation.	☐	II. Play with and become familiar with a number of chip trading games.	☐
3. Identify the place value of a numeral.	☐	III. Use the MA Blocks to illustrate arithmetic in different number bases.	☐
4. Interpret inequality of numbers in bases other than ten.	☐	IV. Explore other numeration systems.	☐
5. Interpret equality of numbers in bases other than ten.	☐		
6. Addition in other bases.	☐		
7. Subtraction in other bases.	☐		
8. Arithmetic in base twelve.	☐		
9. Odds and evens in other bases.	☐		
10. Multiplication in other bases.	☐		
11. Division in other bases	☐		

Activity I

Numeration Unit

Bean Sticks

Make some bean sticks for base ten with your partner.

Partner A--Explain to partner B how to solve the problem 23 - 17 = ? using the bean sticks.

Was that satisfactory to you Partner B? ______________
(If yes, go on)

Partner B--Explain to partner A how to solve the problem 47 ÷ 3 with the bean sticks.

Was that O.K., Partner A? __________

Optional Activity--Ia

Conduct a contest to see who can best guess the number of beans in a jar.

See E.M.I., Volume I.

Activity II

Numeration Unit

Chip Trading

Make a chip trading notebook (see card I-5).

Play at least one of the chip trading games from each of the sets I-V.

Tell someone about your favorite chip trading game and show them how to play it.

Activity III

Multi Base Arithmetic Blocks

Sketch how one hundred unit cubes look when represented with the minimum number of pieces of wood.

Base 6

Base 5

Base 4

Base 3

Optional Activities III

a. Make your own set of MA Blocks with sugar cubes and Elmer's glue. Spray them with plastic or they get sticky.

b. Make a set of activity cards for MA Blocks.

c. Make a binary computer.

Activity IV

Numeration

List as many ways you can write 1972 in other systems of numeration?

Optional Activities IV

a. Explore a numeration system with a negative base.

b. Invent a new numeration system and see if your friends can figure it out.

c. Read about and report on the Duodecimal Society.

Numeration Vocabulary List

1. abacus
2. additive principle
3. binary
4. decimal
5. digit
6. duodecimal
7. expanded notation
8. exponent
9. numeral
10. place value
11. power
12. Roman numeral
13. subtractive principle

OPTIONAL ACTIVITIES	INSTRUCTIONAL DESIGN	INSTRUCTIONAL DESIGN TO BE USED WITH CHILDREN	
		Tasks	Completed
Ia ☐	Having identified a group of 5 to 10 students...	Assessment	☐
IIa ☐	The TTT freshman student will demonstrate his ability to use his knowledge about the "Tasks of Teaching," by designing a lesson which incorporates assessment goals/objectives, strategies and evaluation. The academic content of the instruction design will be the topic of the week. The instructional design will be evaluated on the basis of:	Goals/Objectives	☐
IIIa ☐		Strategies	☐
IIIb ☐		Materials	☐
IIIc ☐		Evaluation	☐
IVa ☐		ADDITIONAL LESSONS DEVELOPED (Simulation or No Instruction)	
IVb ☐	1. Inclusion of an assessment instrument (pre-test).	Assessment	☐
IVc ☐	2. Goals for the week as developed by the four member team.	Goals/Objectives	☐
	3. Specific objectives for the lesson including terminal behavior, conditions and criteria.	Strategies	☐
	4. Strategies and the necessary materials which are appropriate for: the readiness and chronological level of the child, the content to be taught, and employ the use of concrete objects.	Materials	☐
	5. Inclusion of an evaluation instrument which tests specifically for the lesson objectives.	Evaluation	☐
		INSTRUCTIONAL FEEDBACK Comments/Questions	

APPENDIX G

SCORES OF STUDENTS IN THE EXPERIMENTAL GROUP ON THE PRE- AND POST-CRITERION REFERENCED TESTS

SCORES OF STUDENTS IN THE EXPERIMENTAL GROUP ON THE PRE- AND POST-CRITERION REFERENCED TESTS

	MEASUREMENT		NUMERATION		SETS & SET RELATIONS		WHOLE NUMBER SYSTEM		FRACTIONS		DECIMALS		RELATION & FUNCTION		PROBABILITY & STATISTICS		MATHEMATICAL SYSTEMS	
STU. I.D.	Pre-Test	Post-Test	Pre-Test	Post-Test	Pre-Test	Post-Test	Pre-Test	Post-Test	Pre-Test	Post-Test	Pre-Test	Post-Test	Pre-Test	Post-Test	Pre-Test	Post-Test	Pre-Test	Post-Test
1	67	50	76	55	78	80	36	75	64	73	64	82	70	90	35	45	0	44
2	64	70	76	91	34	100	46	100	64	91	64	82	50	95	50	96	33	80
3	58	70	82	46	32	80	46	88	55	82	64	82	30	92	60	88	0	80
4	88	100	100	100	98	100	82	90	82	100	100	91	80	90	80	90	67	90
5	66	20	18	10	64	58	46	82	45	45	73	64	40	50	40	54	0	30
6	70	60	48	73	64	86	72	84	45	91	45	73	50	84	20	82	0	70
7	16	20	70	82	24	66	64	77	36	82	70	91	40	57	50	77	40	60
8	36	50	20	73	56	68	36	61	82	55	64	82	50	95	30	57	33	26
9	57	80	78	74	48	80	54	66	70	91	91	91	60	95	60	90	0	32
10	70	60	85	100	26	70	36	61	73	82	82	82	50	85	60	85	33	38
11	42	30	60	60	66	72	46	60	55	82	55	73	50	71	40	20	33	40
12	33	40	10	92	11	58	28	44	36	55	55	55	30	87	30	47	0	30
13	85	80	80	92	46	90	92	90	82	82	82	82	40	94	50	90	60	80
14	85	80	73	74	76	100	36	89	64	100	82	82	40	70	70	90	43	52
15	57	70	73	74	54	76	46	62	55	55	64	91	50	87	50	60	0	80
16	55	90	86	82	54	90	54	88	82	82	73	100	30	92	70	88	67	100
17	50	70	80	92	86	86	72	89	45	91	82	100	50	90	68	80	50	68
18	92	80	60	82	86	88	64	88	73	91	82	82	60	90	50	90	38	100
19	71	80	86	100	66	70	72	90	73	82	64	100	50	85	50	86	50	62
20	43	60	46	50	30	90	46	89	73	91	55	73	40	90	80	80	33	68
21	92	100	80	82	100	100	72	99	91	91	64	91	60	89	80	89	83	90
22	85	90	74	92	88	100	54	88	73	91	91	91	50	90	40	85	50	80
23	92	90	33	100	86	98	46	89	45	82	73	82	55	60	90	86	28	80
24	92	100	100	100	78	100	92	91	73	91	91	100	70	97	70	90	83	100
25	42	60	33	46	72	90	54	79	82	82	64	100	60	80	60	70	33	80
26	71	50	80	100	72	90	46	81	73	100	54	82	70	97	60	70	33	50
27	100	100	93	100	77	90	92	100	82	100	100	100	90	87	80	100	67	80
28	64	90	93	100	58	90	46	77	36	82	82	91	40	100	60	80	50	72
29	29	50	53	75	17	76	46	57	27	55	45	55	20	47	70	65	10	56
30	71	80	86	100	83	90	64	99	64	100	45	82	90	100	60	83	50	46
31	43	30	27	30	68	88	46	56	64	82	36	64	40	42	50	55	17	50
32	36	40	60	64	60	64	36	42	18	36	18	55	40	62	40	71	10	52
33	43	30	40	46	34	68	64	76	73	82	64	65	30	97	55	86	17	80
34	88	100	100	82	70	90	82	99	73	91	91	91	80	84	100	94	70	80
35	45	50	60	40	46	65	36	43	18	54	36	36	40	67	50	66	43	50
36	75	70	40	64	26	80	54	98	64	82	91	100	40	92	20	78	0	60
37	79	70	64	70	88	90	54	75	82	91	81	81	50	70	50	76	50	78
38	79	60	39	74	44	78	54	75	64	82	64	74	50	87	60	78	0	76

APPENDIX H

SCORES OF STUDENTS IN THE EXPERIMENTAL GROUP ON THE TEST OF MATHEMATICAL UNDERSTANDINGS AND DUTTON ARITHMETIC ATTITUDE INVENTORY

APPENDIX H

SCORES OF STUDENTS IN THE EXPERIMENTAL GROUP ON THE TEST OF MATHEMATICAL UNDERSTANDINGS AND DUTTON ARITHMETIC ATTITUDE INVENTORY

STUDENT	TEST OF MATHEMATICAL UNDERSTANDINGS		DUTTON ATTITUDE SCALE		FEELING TOWARD ARITHMETIC	
I.D.	Pre-Test	Post-Test	Pre-Test	Post-Test	Pre-Test	Post-Test
1	35	41	56	57	8	8
2	40	46	63	75	9	9
3	35	43	54	63	6	9
4	49	50	78	86	10	11
5	33	36	24	42	6	8
6	35	46	19	71	5	6
7	39	45	21	78	5	8
8	34	41	29	52	10	9
9	35	38	48	79	7	6
10	42	45	54	74	7	8
11	33	38	57	74	6	7
12	33	35	56	26	8	4
13	45	47	68	82	9	10
14	34	43	48	56	6	6
15	38	46	69	74	9	8
16	35	47	54	79	6	7
17	33	41	68	73	9	8
18	40	47	90	91	11	11
19	48	47	54	80	10	8
20	38	47	65	78	7	10
21	41	48	81	90	11	11
22	41	47	68	84	9	11
23	37	49	74	74	9	8
24	42	51	74	86	9	10
25	37	43	54	67	7	8
26	37	45	71	79	3	9
27	48	49	74	84	11	11
28	38	44	57	78	9	11
29	33	42	21	27	1	4
30	39	48	54	79	5	8
31	41	45	26	57	8	4
32	34	39	54	54	6	6
33	34	40	26	31	3	2
34	46	48	74	81	9	10
35	32	33	63	68	6	6
36	39	45	58	71	9	8
37	44	47	63	80	8	9
38	32	41	49	71	6	7

APPENDIX I

THE EXPERIMENTAL GROUP HIGH SCHOOL BACKGROUND FACTORS AND FINAL GRADE ON THE COMBINED CONTENT-METHODS COURSE

APPENDIX I

THE EXPERIMENTAL GROUP HIGH SCHOOL BACKGROUND FACTORS AND FINAL GRADE ON THE COMBINED CONTENT-METHODS COURSE

Student I.D.	Number of Mathematics Courses Taken in High School	High School Grade Point Average	Final Grade on the Combined Content-Methods Course
1	4	3.00	3.0
2	3	3.78	4.0
3	4	3.50	4.0
4	5	3.78	4.0
5	2	3.44	2.5
6	3	3.11	4.0
7	3	3.00	2.5
8	3	2.65	3.5
9	3	2.84	3.0
10	3	2.80	3.0
11	3	3.57	3.0
12	2	3.29	3.0
13	5	3.50	4.0
14	3	2.87	4.0
15	3	2.68	3.0
16	2	3.36	4.0
17	4	3.28	4.0
18	4	3.71	4.0
19	4	2.90	3.5
20	3	3.72	4.0
21	6	3.67	4.0
22	4	3.52	3.5
23	4	3.71	4.0
24	6	4.00	4.0
25	3	3.31	3.0
26	4	3.33	3.5
27	6	3.81	4.0
28	4	3.35	4.0
29	1	2.68	1.5
30	4	3.82	4.0
31	3	3.00	3.0
32	2	2.73	3.0
33	3	2.80	2.5
34	5	3.68	4.0
35	2	3.08	3.0
36	3	3.55	4.0
37	4	3.33	4.0
38	3	3.29	4.0

APPENDIX J

CORRELATION MATRIX

APPENDIX J

CORRELATION MATRIX

*The symbolic notations on the correlation matrix indicate the following:

X_1 = Pre-Test on Measurement
Y_1 = Post-Test on Measurement
X_2 = Pre-Test on Numeration Systems
Y_2 = Post-Test on Numeration Systems
X_3 = Pre-Test on Sets and Set Relations
Y_3 = Post-Test on Sets and Set Relations
X_4 = Pre-Test on Whole Numbers
Y_4 = Post-Test on Whole Numbers
X_5 = Pre-Test on Fractions
Y_5 = Post-Test on Fractions
X_6 = Pre-Test on Decimals
Y_6 = Post-Test on Decimals
X_7 = Pre-Test on Relations and Functions
Y_7 = Post-Test on Relations and Functions
X_8 = Pre-Test on Probability and Statistics
Y_8 = Post-Test on Probability and Statistics
X_9 = Pre-Test on Mathematical Systems
Y_9 = Post-Test on Mathematical Systems
BIG X = Test of Mathematical Understanding--Pre-Test
BIG Y = Test of Mathematical Understanding--Post-Test
DUT X = Dutton Arithmetic Attitude Scale--Pre-Test
DUT Y = Dutton Arithmetic Attitude Scale--Post-Test
FEEL X = General Feeling Toward Mathematics--Pre-Test
FEEL Y = General Feeling Toward Mathematics--Post-Test
PROCESS = Attitudes Toward Mathematics as a Process
DIFFIC = Attitudes Toward Difficulties of Learning Mathematics
PLACE = Attitudes Toward Place of Mathematics in Society

SCH & LEAR = Attitudes Toward School and School Learning

ENVIRON = Attitudes Toward Man and His Environment

METHODS = Attitudes Toward Different Methods of Teaching Mathematics

HS GPA = High School Grade Point Average

READ = MSU Reading Test

ARITH = MSU Arithmetic Test

MATH = MSU Mathematics Test

COURSES = Number of Mathematics Courses Taken in High School

FIN GR = Final Grade Received in the Combined Content-Methods Course

CORRELATION MATRIX

		* X 1	Y 1	X 2	Y 2	X 3	Y 3	X 4	Y 4	X 5	Y 5	X 6	Y 6	X 7	Y 7
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
X1	(1)	1.000													
Y1	(2)	0.757	1.000												
X2	(3)	0.447	0.645	1.000											
Y2	(4)	0.422	0.623	0.564	1.000										
X3	(5)	0.599	0.479	0.321	0.199	1.000									
Y3	(6)	0.642	0.743	0.467	0.415	0.572	1.000								
X4	(7)	0.503	0.530	0.501	0.382	0.360	0.415	1.000							
Y4	(8)	0.669	0.632	0.439	0.333	0.399	0.683	0.627	1.000						
X5	(9)	0.513	0.449	0.261	0.204	0.370	0.511	0.423	0.564	1.000					
Y5	(10)	0.557	0.554	0.512	0.448	0.404	0.752	0.516	0.726	0.605	1.000				
X6	(11)	0.602	0.607	0.421	0.399	0.250	0.417	0.480	0.578	0.525	0.513	1.000			
Y6	(12)	0.443	0.695	0.521	0.488	0.368	0.521	0.520	0.690	0.605	0.590	0.703	1.000		
X7	(13)	0.579	0.491	0.470	0.409	0.643	0.438	0.489	0.446	0.444	0.515	0.385	0.449	1.000	
Y7	(14)	0.334	0.505	0.423	0.468	0.062	0.301	0.299	0.423	0.497	0.438	0.387	0.495	0.352	1.000

	X 1 (1)	Y 1 (2)	X 2 (3)	Y 2 (4)	X 3 (5)	Y 3 (6)	X 4 (7)	Y 4 (8)	X 5 (9)	Y 5 (10)	X 6 (11)	Y 6 (12)	X 7 (13)	Y 7 (14)
X8 (15)	0.347	0.552	0.461	0.298	0.282	0.512	0.371	0.395	0.260	0.423	0.319	0.275	0.341	0.021
Y8 (16)	0.555	0.703	0.523	0.466	0.187	0.555	0.557	0.655	0.365	0.564	0.490	0.459	0.248	0.384
X9 (17)	0.442	0.694	0.615	0.508	0.512	0.559	0.594	0.486	0.456	0.496	0.413	0.449	0.429	0.242
Y9 (18)	0.485	0.690	0.409	0.281	0.323	0.633	0.585	0.573	0.367	0.435	0.367	0.450	0.152	0.285
RIGX (19)	0.567	0.569	0.535	0.464	0.347	0.449	0.631	0.509	0.559	0.489	0.505	0.511	0.523	0.189
RIGY (20)	0.605	0.680	0.509	0.552	0.412	0.745	0.595	0.710	0.566	0.680	0.426	0.648	0.465	0.271
OUT X (21)	0.622	0.686	0.494	0.475	0.468	0.528	0.294	0.382	0.347	0.374	0.409	0.376	0.503	0.445
OUT Y (22)	0.620	0.661	0.622	0.456	0.300	0.606	0.536	0.572	0.440	0.635	0.458	0.698	0.606	0.389
FEEL X (23)	0.518	0.573	0.208	0.320	0.436	0.345	0.330	0.346	0.412	0.196	0.517	0.501	0.391	0.307
FEEL Y (24)	0.598	0.630	0.521	0.414	0.469	0.531	0.417	0.603	0.446	0.409	0.556	0.590	0.531	0.418
PROCESS (25)	-0.468	-0.459	-0.253	-0.380	-0.287	-0.300	-0.362	-0.278	-0.256	-0.357	-0.398	-0.343	-0.452	-0.145
DIFFIC (26)	-0.273	-0.348	-0.060	0.032	-0.234	-0.411	-0.394	-0.425	-0.327	-0.323	-0.121	-0.258	-0.193	-0.277
PLACE (27)	0.585	0.724	0.750	0.629	0.450	0.595	0.671	0.540	0.524	0.600	0.504	0.590	0.628	0.492
SCHLFAR (28)	0.320	0.442	0.515	0.350	0.286	0.340	0.454	0.253	0.524	0.367	0.223	0.335	0.524	0.534

		X8	Y8	X9	Y9	BIGX	BIGY	DUT X	DUT Y	FEEL X	FEEL Y	PROCESS	DIFFIC	PLACE	SCHELFAR
		(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
X8	(15)	1.000													
Y8	(16)	0.513	1.000												
X9	(17)	0.561	0.465	1.000											
Y9	(18)	0.450	0.610	0.502	1.000										
BIGX	(19)	0.337	0.502	0.607	0.431	1.000									
BIGY	(20)	0.452	0.608	0.577	0.673	0.754	1.000								
DUT X	(21)	0.424	0.334	0.520	0.505	0.454	0.425	1.000							
DUT Y	(22)	0.328	0.546	0.569	0.498	0.640	0.668	0.6	1.000						
FEEL X	(23)	0.099	0.250	0.417	0.322	0.573	0.399	0.615	0.553	1.000					
FEEL Y	(24)	0.295	0.386	0.577	0.443	0.588	0.598	0.667	0.786	0.646	1.000				
PROCESS	(25)	-0.256	-0.274	-0.290	-0.176	-0.561	-0.453	-0.255	-0.327	-0.388	-0.362	1.000			
DIFFIC	(26)	-0.177	-0.357	-0.213	-0.452	-0.323	-0.437	-0.128	-0.363	-0.233	-0.348	0.204	1.000		
PLACE	(27)	0.458	0.542	0.740	0.544	0.669	0.675	0.671	0.843	0.479	0.747	-0.366	-0.284	1.000	
SCHELFAR	(28)	0.328	0.257	0.655	0.305	0.439	0.379	0.511	0.542	0.343	0.439	-0.214	-0.246	0.740	1.000

	X 1	Y 1	X 2	Y 2	X 3	Y 3	X 4	Y 4	X 5	Y 5	X 6	Y 6	X 7	Y 7
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
ENVIRON (29)	0.410	0.442	0.443	0.260	0.298	0.440	0.445	0.370	0.359	0.395	0.292	0.230	0.341	0.222
METHODS (30)	0.479	0.433	0.195	0.193	0.335	0.559	0.242	0.347	0.081	0.342	0.294	0.151	0.347	0.084
HS GPA (31)	0.192	0.154	0.105	0.087	0.041	0.148	0.296	0.316	0.105	0.220	0.053	0.101	0.287	0.107
READ (32)	0.363	0.200	0.151	0.151	0.083	0.161	0.285	0.281	0.060	0.252	0.106	0.163	0.394	-0.017
ARITH (33)	0.196	0.203	0.324	0.260	-0.012	0.145	0.348	0.250	0.185	0.301	0.205	0.336	0.262	0.129
MATH (34)	0.461	0.449	0.435	0.338	0.382	0.390	0.552	0.453	0.415	0.370	0.177	0.392	0.440	0.292
COURSES (35)	0.573	0.526	0.501	0.367	0.445	0.436	0.653	0.536	0.467	0.514	0.399	0.486	0.626	0.366
FIN GR (36)	0.672	0.710	0.377	0.424	0.434	0.663	0.387	0.673	0.443	0.616	0.404	0.500	0.399	0.547

	X8 (15)	Y8 (16)	X9 (17)	Y9 (18)	BIG X (19)	BIG Y (20)	OUT X (21)	OUT Y (22)	FEEL X (23)	FEEL Y (24)	PROCESS (25)	DIFFIC (26)	PLACE (27)	SCHRLEAR (28)
ENVIRON (29)	0.423	0.439	0.595	0.468	0.525	0.458	0.522	0.590	0.490	0.510	-0.329	-0.299	0.584	0.558
METHODS (30)	0.304	0.337	0.297	0.274	0.185	0.352	0.351	0.301	0.235	0.414	-0.353	-0.397	0.342	0.114
HS GPA (31)	0.341	0.120	0.197	0.194	0.151	0.192	0.260	0.173	0.134	0.170	-0.044	0.060	0.198	0.266
READ (32)	0.269	0.130	0.086	0.120	0.215	0.271	0.141	0.194	0.103	0.113	-0.405	0.067	0.199	0.155
ARITH (33)	0.343	0.148	0.207	0.136	0.306	0.343	0.168	0.237	0.092	0.147	-0.066	0.069	0.277	0.274
MATH (34)	0.382	0.342	0.539	0.405	0.481	0.521	0.451	0.526	0.415	0.450	-0.213	-0.346	0.590	0.530
COURSES (35)	0.463	0.386	0.537	0.399	0.576	0.541	0.505	0.561	0.492	0.522	-0.461	-0.209	0.646	0.540
FIN GR (36)	0.265	0.550	0.466	0.510	0.388	0.572	0.597	0.657	0.558	0.613	-0.360	-0.405	0.569	0.442

		ENVIRON (29)	METHODS (30)	HS GPA (31)	READ (32)	ARITH (33)	MATH (34)	COURSES (35)	FIN GR (36)
ENVIRON	(29)	1.000							
METHODS	(30)	0.413	1.000						
HS GPA	(31)	0.002	-0.074	1.000					
READ	(32)	0.011	0.094	0.745	1.000				
ARITH	(33)	-0.023	-0.219	0.763	0.678	1.000			
MATH	(34)	0.421	0.114	0.436	0.317	0.543	1.000		
COURSES	(35)	0.357	0.175	0.568	0.607	0.667	0.771	1.000	
FIN GR	(36)	0.452	0.390	0.371	0.372	0.260	0.405	0.554	1.000

APPENDIX K

NUMBER OF STUDENTS IN THE EXPERIMENTAL GROUP WHO ANSWERED THE TEST ITEMS CORRECTLY ON THE NINE PRE- AND POST-CRITERION REFERENCED MEASURES

NUMBER OF STUDENTS IN THE EXPERIMENTAL GROUP (N = 38) WHO ANSWERED THE TEST ITEMS CORRECTLY ON THE NINE PRE- AND POST-CRITERION REFERENCED MEASURES

MEASUREMENT		
Item	Pre-Test	Post-Test
1	33	32
2	29	26
3	28	29
4	30	32
5	33	32
6	21	29
7	11	13
8	17	20
9	26	22
10	15	17

NUMERATIONS		
Item	Pre-Test	Post-Test
1a	34	23
1b	33	23
1c	26	23
2	31	27
3	28	31
4	15	32
5	23	27
6	29	30
7	25	37
8	15	25
9	25	33
10	24	21
11	25	29

SETS AND SET RELATIONS		
Item	Pre-Test	Post-Test
1a	31	36
1b	32	35
1c	25	31
1d	14	35
1e	6	31
2	20	34
3	15	37
4	35	37
5	13	21
6	17	25
7	23	35
8	19	33
9	34	36
10a	33	24
10b	18	24
10c	34	24
10d	29	24
10e	31	24

WHOLE NUMBER SYSTEM		
Item	Pre-Test	Post-Test
1-1	26	35
1-2	24	34
1-3	21	33
1-4	13	26
1-5	26	36
1-6	22	32
1-7	12	28
1-8	14	24
1-9	17	31
1-10	17	32
2	14	34
3-3	25	28
4-4	29	36
5	28	34
6	12	31
7	29	27
8	10	27
9	11	18
10	33	34

FRACTIONS		
Item	Pre-Test	Post-Test
1	16	36
2	19	24
3	22	29
4	26	35
5	16	24
6	33	26
7	27	36
8	26	31
9	25	33
10	33	36
11	17	29

DECIMALS		
Item	Pre-Test	Post-Test
1	20	35
2	18	26
3	29	30
4	34	33
5	34	35
6	28	31
7	19	34
8	24	25
9	30	35
10	33	34
11	17	23

APPENDIX K--Continued

RELATION AND FUNCTION			PROBABILITY AND STATISTICS			MATHEMATICAL SYSTEMS		
Item	Pre-Test	Post-Test	Item	Pre-Test	Post-Test	Item	Pre-Test	Post-Test
1a	8	29	1	26	29	1	12	24
1b	12	20	2	24	26	2	14	33
1c	20	26	3a	20	31	3	9	19
1d	10	34	3b	17	28	4	9	28
1e	18	25	3c	16	25	5a	16	30
2	14	27	3d	19	23	5b	24	24
3	14	26	3e	12	24	5c	23	15
4	33	36	4	8	25	5d	16	18
5	3	31	5	29	27	5e	--	16
6	33	33	6	31	34			
7	13	38	7	9	24			
8	32	35	8	14	31			
9	11	27	9a	20	35			
10	26	35	9b	24	32			
			10	34	36			

APPENDIX L

RAW SCORES OF STUDENTS IN THE EXPERIMENTAL GROUP OF ENTRY CHARACTERISTICS

RAW SCORES OF STUDENTS IN THE EXPERIMENTAL GROUP ON ENTRY CHARACTERISTICS

STUDENT I.D.	ATT. 1[a]	ATT. 2[b]	ATT. 3[c]	ATT. 4[d]	ARITH.[e]	MATH.[f]	COURS.[g]
1	11	5	5	11	32	18	4
2	8	9	9	13	29	10	3
3	6	8	8	10	28	11	4
4	6	12	12	16	37	24	5
5	8	4	8	12	28	3	2
6	4	8	9	11	33	16	3
7	9	10	11	12	35	18	3
8	7	7	8	15	24	10	3
9	9	9	10	14	30	9	3
10	10	11	12	12	36	14	3
11	10	8	6	15	30	9	3
12	10	8	11	16	30	8	2
13	7	11	8	16	36	24	5
14	9	6	3	14	35	14	3
15	5	12	9	15	36	20	3
16	9	8	8	16	34	11	2
17	7	8	5	8	35	16	4
18	8	12	12	15	32	20	4
19	8	8	6	13	35	23	4
20	3	7	5	15	35	17	3
21	3	9	11	17	35	26	6
22	NT	NT	NT	NT	NT	NT	3
23	9	12	13	17	29	12	4
24	7	11	12	18	39	28	6
25	5	8	8	16	29	12	4
26	8	9	11	16	39	19	4
27	5	8	8	16	38	24	6
28	4	9	9	13	36	17	4
29	9	8	8	6	31	8	1
30	5	10	12	15	32	22	4
31	5	9	8	17	31	12	3
32	8	8	5	17	24	11	2
33	5	7	5	13	25	11	3
34	4	9	9	16	38	25	5
35	9	8	6	16	22	9	2
36	5	9	9	12	37	19	3
37	6	6	3	15	33	15	4
38	5	10	11	11	31	15	3

[a]ATT. 1 = ATTITUDES TOWARD MATHEMATICS AS A PROCESS.

[b]ATT. 2 = ATTITUDES TOWARD DIFFICULTIES OF LEARNING MATHEMATICS.

[c]ATT. 3 = ATTITUDES TOWARD PLACE OF MATHEMATICS IN SOCIETY.

[d]ATT. 4 = ATTITUDES TOWARD SCHOOL AND SCHOOL LEARNING.

[e]ARITH. = MSU ARITHMETICS TEST.

[f]COURS. = NUMBER OF MATHEMATICS COURSES TAKEN IN HIGH SCHOOL.

APPENDIX M

TEST SCORES OF STUDENTS IN 325E ON 50 PERCENT ITEM SAMPLE OF CRITERION-REFERENCED TESTS

TEST SCORES OF STUDENTS IN 325E ON 50 PERCENT ITEM SAMPLE OF CRITERION-REFERENCED TESTS

STU. I.D.	MEASUREMENT		NUMERATION		SETS & SET RELATIONS	
	Pre-Test	Post-Test	Pre-Test	Post-Test	Pre-Test	Post-Test
1	4	2	2	2	4	5
2	3	3	2	1	1	2
3	5	3	4	3	5	3
4	4	4	5	5	4	5
5	4	3	5	4	5	5
6	1	0	0	1	1	0
7	5	3	4	3	5	4
8	5	2	5	5	4	4
9	4	2	3	3	2	2
10	1	1	1	1	2	3
11	3	4	2	3	3	4
12	5	4	5	4	4	4
13	2	2	1	1	1	1
14	3	5	2	4	3	5
15	5	5	3	5	4	3
16	2	1	1	1	1	2
17	2	1	3	1	2	2
18	1	2	2	1	0	1
19	5	5	5	4	5	5

STU. I.D.	WHOLE NUMBERS		FRACTIONS		DECIMALS	
	Pre-Test	Post-Test	Pre-Test	Post-Test	Pre-Test	Post-Test
20	3	5	5	5	4	4
21	2	1	1	1	0	1
22	5	4	5	3	3	3
23	4	5	4	4	3	5
24	4	4	3	2	5	4
25	5	3	4	5	3	4
26	5	5	4	5	5	5
27	1	1	0	1	1	1
28	4	4	2	3	2	2
29	5	4	2	2	4	5
30	3	3	5	4	4	5
31	1	1	1	2	1	1
32	2	2	4	3	2	2
33	5	5	3	5	5	3
34	2	3	3	4	3	3
35	2	2	2	2	2	2
36	4	3	5	5	4	4

STU. I.D.	PROBABILITY & STATISTICS		RELATIONS & FUNCTIONS		MATHEMATICAL SYSTEMS	
	Pre-Test	Post-Test	Pre-Test	Post-Test	Pre-Test	Post-Test
37	1	2	3	4	2	1
38	1	1	1	0	1	0
39	4	3	4	4	3	3
40	5	5	4	5	4	2
41	4	2	4	5	3	2
42	1	1	0	1	0	0
43	5	5	5	4	5	5
44	5	3	4	3	4	4
45	1	1	0	1	0	0
46	3	4	3	3	3	3
47	3	2	2	2	2	1
48	5	5	5	5	5	4
49	1	1	1	2	1	0
50	1	3	2	2	1	0
51	5	4	5	3	5	4
52	1	2	1	2	1	1
53	2	1	2	1	2	3
54	0	0	0	1	0	1
55	4	5	4	5	3	5
56	0	0	0	1	0	0

APPENDIX N

SCORES OF THE "COMPARISON GROUPS" ON ENTRY DATA

SCORES OF THE FRESHMAN ELEMENTARY EDUCATION MAJORS (COMPARISON GROUP) ON ENTRY DATA

STU. I.D.	MSU BASIC SKILLS AND PLACEMENT TEST IN ARITHMETIC	MSU BASIC SKILLS AND PLACEMENT TEST IN MATHEMATICS	DUTTON ARITHMETIC ATTITUDE SCALE	ATTITUDES TOWARD MATHEMATICS AS A PROCESS	ATTITUDES TOWARD DIFFICULTIES OF LEARNING MATHEMATICS	ATTITUDES TOWARD PLACE OF MATHEMATICS IN SOCIETY	ATTITUDES TOWARD SCHOOL AND SCHOOL LEARNING
1	23	10	6.9	4	3	5	12
2	37	18	7.3	9	6	10	5
3	28	17	7.4	4	10	12	14
4	34	15	6.4	6	8	13	14
5	21	3	5.2	5	4	4	12
6	36	21	7.4	6	10	9	16
7	17	7	2.3	2	5	7	5
8	27	13	8.0	9	14	9	16
9	24	9	7.4	7	4	4	22
10	36	13	8.8	9	2	7	13
11	31	11	7.4	4	14	7	19
12	31	6	2.4	8	6	2	14
13	29	21	8.6	5	8	12	14
14	34	15	2.5	4	6	2	14
15	31	18	7.9	8	14	8	10
16	36	18	5.4	10	8	11	15
17	30	5	6.0	8	11	7	10
18	32	17	3.8	2	4	10	13
19	31	17	6.1	4	11	12	14
20	27	6	5.4	6	14	11	16
21	20	3	7.8	4	12	11	14
22	34	22	6.3	6	13	7	14
23	39	28	2.6	6	10	10	16
24	31	8	4.1	6	4	4	14
25	39	14	1.9	6	11	7	16
26	23	3	4.5	4	14	13	15
27	37	9	4.6	7	6	9	16
28	32	13	5.1	5	7	3	13
29	35	17	6.8	4	10	2	11
30	29	10	4.9	4	7	10	17
31	35	22	6.5	3	10	10	6
32	39	28	8.0	10	14	10	20
33	40	15	7.8	8	9	8	18
34	23	8	6.8	10	7	13	20
35	28	12	4.5	6	14	12	16
36	31	9	NT	NT	NT	NT	NT

SCORES OF THE FRESHMAN MATHEMATICS-SECONDARY EDUCATION MAJORS (COMPARISON GROUP) ON ENTRY DATA

STU. I.D.	MSU BASIC SKILLS AND PLACEMENT TEST IN ARITHMETIC	MSU BASIC SKILLS AND PLACEMENT TEST IN MATHEMATICS	DUTTON ARITHMETIC ATTITUDE SCALE	ATTITUDES TOWARD MATHEMATICS AS A PROCESS	ATTITUDES TOWARD DIFFICULTIES OF LEARNING MATHEMATICS	ATTITUDES TOWARD PLACE OF MATHEMATICS IN SOCIETY	ATTITUDES TOWARD SCHOOL AND SCHOOL LEARNING
1	38	30	8.8	7	14	7	14
2	39	28	7.7	15	5	11	15
3	35	26	8.8	8	12	14	15
4	36	22	8.4	3	4	6	14
5	36	28	9.0	16	14	10	18
6	39	27	7.1	12	12	6	16
7	38	22	5.7	10	12	12	18
8	36	23	9.0	11	9	12	15
9	40	28	7.3	8	4	4	12
10	39	28	7.3	6	6	6	12
11	39	28	8.1	16	14	8	16
12	40	30	7.7	12	14	16	20
13	37	15	9.3	14	8	10	14
14	31	23	7.2	7	12	10	16
15	35	12	7.2	4	3	8	16
16	32	21	8.8	10	13	8	10
17	37	24	8.8	9	10	12	18
18	39	24	8.7	10	0	10	16
19	37	21	7.4	8	4	9	10
20	39	28	7.8	12	12	9	14
21	38	16	6.9	8	8	15	12
22	38	22	8.5	11	12	11	14
23	39	24	8.8	9	8	12	16
24	32	20	8.6	3	5	10	10
25	40	26	7.7	3	10	6	13
26	36	18	8.3	10	12	14	18
27	31	20	7.6	5	14	14	14
28	38	29	7.9	8	10	8	16
29	40	28	7.1	8	5	12	15
30	38	28	8.3	4	8	10	14
31	33	23	8.2	12	6	7	14
32	36	28	8.7	6	6	6	16
33	31	21	8.2	8	9	9	13
34	36	16	6.9	4	7	3	13
35	38	23	6.8	9	13	10	11
36	38	22	7.9	12	5	12	14
37	34	17	7.3	5	11	13	16
38	37	28	8.3	8	4	6	14
39	39	27	8.3	12	12	9	15
40	38	24	8.3	11	12	10	18
41	37	30	8.8	8	12	6	15
42	36	27	6.3	6	12	10	17
43	35	25	8.6	4	10	6	12

SCORES OF THE FRESHMAN MATHEMATICS MAJORS (COMPARISON GROUP) ON ENTRY DATA

STD. I.D.	MSU BASIC SKILLS AND PLACEMENT TEST IN ARITHMETIC	MSU BASIC SKILLS AND PLACEMENT TEST IN MATHEMATICS	DUTTON ARITHMETIC ATTITUDE SCALE	ATTITUDES TOWARD MATHEMATICS AS A PROCESS	ATTITUDES TOWARD DIFFICULTIES OF LEARNING MATHEMATICS	ATTITUDES TOWARD PLACE OF MATHEMATICS IN SOCIETY	ATTITUDES TOWARD SCHOOL AND SCHOOL LEARNING
1	39	28	6.8	3	14	8	10
2	39	20	7.6	10	13	16	20
3	36	28	8.1	13	9	10	15
4	37	26	8.7	11	14	15	13
5	33	26	2.6	9	5	6	11
6	39	24	7.8	8	10	10	15
7	38	30	7.4	7	11	10	17
8	39	29	2.6	10	14	7	14
9	39	28	9.0	12	8	7	16
10	37	27	4.8	16	6	7	9
11	38	26	7.4	4	10	9	16
12	39	25	8.8	16	11	9	12
13	36	17	8.8	9	13	12	16
14	36	21	8.0	9	8	6	8
15	36	21	9.1	11	6	14	18
16	39	30	9.0	9	6	7	10
17	35	27	5.8	4	10	8	16
18	35	27	6.6	14	12	8	14
19	40	29	9.0	13	8	8	10
20	38	23	9.8	10	12	12	10
21	40	30	8.8	16	9	9	20
22	39	26	7.1	12	8	14	8
23	36	27	8.2	12	12	12	19
24	38	28	6.0	7	5	6	18
25	38	26	6.5	8	6	6	3
26	36	28	7.8	10	6	9	12
27	36	17	6.3	3	5	10	16
28	37	26	7.4	12	7	8	16
29	24	2	9.3	5	14	9	18
30	34	20	8.8	7	4	7	12
31	37	30	7.9	14	13	8	16
32	39	28	7.8	8	8	16	16
33	36	28	8.8	5	12	16	14
34	36	23	8.6	3	8	3	13
35	37	25	7.4	7	3	2	14
36	40	29	7.7	10	4	11	16
37	39	29	4.5	15	14	8	14
38	39	23	9.0	12	14	16	16
39	37	23	NT	NT	NT	NT	NT
40	34	27	NT	NT	NT	NT	NT

APPENDIX O

PRE- AND POST-TEST SCORES OF STUDENTS IN REGULAR METHODS COURSE (325E) ON DUTTON ATTITUDE SCALE AND TEST OF MATHEMATICAL UNDERSTANDINGS

APPENDIX O

PRE- AND POST-TEST SCORES OF STUDENTS IN REGULAR METHODS COURSE (325E) ON DUTTON ATTITUDE SCALE AND TEST OF MATHEMATICAL UNDERSTANDINGS

STUDENT I.D.	TEST OF MATHEMATICAL UNDERSTANDINGS		DUTTON ARITHMETIC ATTITUDE INVENTORY	
	Pre-Test	Post-Test	Pre-Test	Post-Test
1	7.3	7.3	46	44
2	1.9	1.9	30	29
3	1.4	7.8	40	48
4	5.1	7.4	43	42
5	5.5	5.2	38	40
6	5.7	6.9	35	37
7	7.9	8.8	45	44
8	7.4	5.2	46	45
9	4.9	5.1	39	35
10	7.1	6.3	45	44
11	8.2	7.9	36	41
12	7.4	8.4	48	47
13	2.4	2.4	37	39
14	6.4	8.2	44	40
15	7.8	8.0	48	43
16	8.6	7.4	42	39
17	2.8	5.4	31	38
18	5.1	5.1	37	38
19	7.9	6.7	46	45
20	2.7	2.5	36	39
21	8.6	7.4	47	50

APPENDIX P

ONE-WAY ANALYSIS OF VARIANCE RELATIVE TO TESTING DIFFERENCES BETWEEN THE EXPERIMENTAL GROUP AND THE "COMPARISON GROUPS"

APPENDIX P

ONE-WAY ANALYSIS OF VARIANCE RELATIVE TO TESTING DIFFERENCES BETWEEN THE EXPERIMENTAL GROUP AND THE "COMPARISON GROUPS"

Table 19

Summary of Analysis of Variance for MSU Arithmetic Test

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Treatment	3	1075.65	358.55	21.93
Error	152	2484.52	16.35	
Total	155	3560.17		
				$F_{.95}(3, 152) = 2.66$

Table 20

Summary of Analysis of Variance for MSU Mathematics Test

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Treatment	3	3946.47	1315.49	43.67
Error	152	4578.37	30.12	
Total	155	8524.85		
				$F_{.95}(3, 152) = 2.66$

Table 21

Summary of Analysis of Variance on the Scale of Attitudes Toward Mathematics as a Process

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Treatment	3	264.16	88.05	9.79
Error	149	1339.72	8.99	
Total	152	1603.88		

$F_{.95}(3, 149) = 2.66$

Table 22

Summary of Analysis of Variance on the Scale of Attitudes Toward Difficulties of Learning Mathematics

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Treatment	3	12.62	4.20	0.37
Error	149	1676.06	11.25	
Total	152	1688.68		

$F_{.95}(3, 149) = 2.66$

Table 23

Summary of Analysis of Variance on the Scale of Attitudes Toward School and School Learning

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Treatment	3	4.16	1.39	0.13
Error	149	1548.69	10.39	
Total	152	1552.84		

$F_{.95}(3, 149) = 2.66$

Table 24

Summary of Analysis of Variance on the Scale of Attitudes Toward Place of Mathematics in Society

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Treatment	3	43.85	14.62	1.47
Error	149	1484.83	9.97	
Total	152	1526.68		

$F_{.95}(3, 149) = 2.66$

Table 25

Summary of Analysis of Variance on the
Dutton Arithmetic Attitude Scale

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Treatment	3	162.32	54.11	19.89
Error	150	408.34	2.72	
Total	153	570.66		

$F_{.95}(3, 150) = 2.66$

APPENDIX Q

HOYT RELIABILITY COEFFICIENT FOR CRITERION-REFERENCED ACHIEVEMENT MEASURES

APPENDIX Q

HOYT RELIABILITY COEFFICIENT FOR CRITERION-REFERENCED ACHIEVEMENT MEASURES

Table 26

Summary of Analysis of Variance for Pre-Test in Measurement

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	18	8.0842	0.4491	3.1942
Items	4	2.6737	0.6684	4.7539
Error	72	10.1263	0.1406	
Total	94	20.8842		
		Hoyt Reliability Coefficient = 0.6869		

Table 27

Summary of Analysis of Variance for Post-Test in Measurement

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	18	7.9368	0.4409	2.7149
Items	4	5.9052	1.4763	9.0905
Error	72	11.6948	0.1624	
Total	94	25.5368		
		Hoyt Reliability Coefficient = 0.6317		

Table 28

Summary of Analysis of Variance for Pre-Test in Numeration

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	18	9.9579	0.5532	4.9526
Items	4	5.1579	1.2894	11.5434
Error	72	8.0420	0.1117	
Total	94	23.1578		

Hoyt Reliability Coefficient = 0.7981

Table 29

Summary of Analysis of Variance for Post-Test in Numeration

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	18	8.7368	0.4854	4.6097
Items	4	7.2210	1.8053	17.1443
Error	72	7.5790	0.1053	
Total	94	23.5368		

Hoyt Reliability Coefficient = 0.7831

Table 30

Summary of Analysis of Variance for Pre-Test of Sets and Set Relation

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	18	9.7895	0.5439	5.0975
Items	4	5.5158	1.3790	12.9241
Error	72	7.6842	0.1067	
Total	94	22.9895		

Hoyt Reliability Coefficient = 0.8038

Table 31

Summary of Analysis of Variance for Post-Test of Sets and Set Relation

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	18	8.9053	0.4947	4.8076
Items	4	5.7895	1.4474	14.0661
Error	72	7.4105	0.1029	
Total	94	22.1053		

Hoyt Reliability Coefficient = 0.7920

Table 32

Summary of Analysis of Variance for Pre-Test in Whole Numbers

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	16	6.7765	0.4235	3.2728
Items	4	3.7177	0.9294	7.1824
Error	64	8.2823	0.1294	
Total	84	18.7765		

Hoyt Reliability Coefficient = 0.6945

Table 33

Summary of Analysis of Variance for Post-Test in Whole Numbers

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	16	6.6118	0.4132	3.5226
Items	4	5.2942	1.3236	11.2839
Error	64	7.5058	0.1173	
Total	84	19.4118		

Hoyt Reliability Coefficient = 0.6945

Table 34

Summary of Analysis of Variance for Pre-Test in Fractions

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	16	7.9529	0.4971	3.5866
Items	4	3.1294	0.7824	5.6450
Error	64	8.8705	0.1386	
Total	84	19.9529		

Hoyt Reliability Coefficient = 0.7212

Table 35

Summary of Analysis of Variance for Post-Test in Fractions

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	16	6.7059	0.4191	3.1654
Items	4	3.9294	0.9824	7.4199
Error	64	8.4706	0.1324	
Total	84	19.1059		

Hoyt Reliability Coefficient = 0.7212

Table 36

Summary of Analysis of Variance for Pre-Test in Decimals

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	16	7.2000	0.4500	4.3103
Items	4	6.5176	1.6294	15.6073
Error	64	6.6824	0.1044	
Total	84	20.4000		

Hoyt Reliability Coefficient = 0.7572

Table 37

Summary of Analysis of Variance for Post-Test in Decimals

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	16	6.8941	0.4309	4.0844
Items	4	6.0470	1.5118	14.3299
Error	64	6.7530	0.1055	
Total	84	19.6941		

Hoyt Reliability Coefficient = 0.7572

Table 38

Summary of Analysis of Variance for Pre-Test in Relations and Functions

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	19	13.3600	0.7032	7.7021
Items	4	4.6600	1.1650	12.7601
Error	76	6.9400	0.0913	
Total	99	24.9600		

Hoyt Reliability Coefficient = 0.8702

Table 39

Summary of Analysis of Variance for Post-Test in Relations and Functions

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	19	11.7900	0.6205	6.1558
Items	4	5.5400	1.3850	13.7401
Error	76	7.6600	0.1008	
Total	99	24.9900		

Hoyt Reliability Coefficient = 0.8376

Table 40

Summary of Analysis of Variance for Pre-Test in Probability and Statistics

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	19	12.6000	0.6632	6.2982
Items	4	4.4000	1.1000	6.2982
Error	76	8.0000	0.1053	
Total	99	25.0000		

Hoyt Reliability Coefficient = 0.8412

Table 41

Summary of Analysis of Variance for Post-Test in Probability and Statistics

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	19	10.5100	0.5532	5.1033
Items	4	6.1600	1.5400	14.2066
Error	76	8.2400	0.1084	
Total	99	24.9100		

Hoyt Reliability Coefficient = 0.8041

Table 42

Summary of Analysis of Variance for Pre-Test in Mathematical Systems

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	19	11.5500	0.6079	6.5365
Items	4	5.5000	1.3750	14.7850
Error	76	7.0700	0.0930	
Total	99	24.7500		
		Hoyt Reliability Coefficient = 0.8470		

Table 43

Summary of Analysis of Variance for Post-Test in Mathematical Systems

Source of Variance	D.F.	Sum of Squares	Mean Square	F-Value
Individuals	19	12.5100	0.6584	5.6956
Items	4	3.2100	0.8025	6.9420
Error	76	8.7900	0.1156	
Total	99	24.5100		
		Hoyt Reliability Coefficient = 0.8244		