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A FORMATIVE EVALUATION OF THE MATHEMATICS EDUCATION  
COMPONENT OF THE EIGHTH CYCLE TEACHER CORPS  
PROGRAM AT MICHIGAN STATE UNIVERSITY.

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1974

A FORMATIVE EVALUATION OF THE MATHEMATICS  
EDUCATION COMPONENT OF THE EIGHTH CYCLE  
TEACHER CORPS PROGRAM AT MICHIGAN  
STATE UNIVERSITY

By

Ganiyu Ademola Badmus

A DISSERTATION

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## ABSTRACT

### A FORMATIVE EVALUATION OF THE MATHEMATICS EDUCATION COMPONENT OF THE EIGHTH CYCLE TEACHER CORPS PROGRAM AT MICHIGAN STATE UNIVERSITY

By

Ganiyu Ademola Badmus

The major purpose of the study was to provide both "intrinsic" and "pay-off" formative evaluations of the mathematics curriculum and instruction of the eighth cycle Teacher Corps program at Michigan State University using internal, external and contextual sources. The intrinsic aspect (1) analyzed and evaluated the mathematics content-method integrated component; (2) provided critical appraisal of instructional method and clinical experiences. The pay-off aspect evaluated (3) learning; (4) learners; (5) environments of learning; (6) compared the method of instruction with two similar methods on mathematics achievement and attitude.

Three groups of students were involved in the study. Students in the Teacher Corps mathematics education program (interns) met for six hours per week Fall term 1973, four hours per week Winter and Spring terms of 1974. They



studied five learning units which were laboratory-oriented mathematics content-method integrated taught under mastery-learning approach. All interns spent four hours daily in elementary school, where they were provided with clinical experience supervised by team leaders and faculty members. Twenty-four of thirty interns who originally entered the program were used in this study. The second group consisted of twenty-one students randomly selected from volunteers in Fall, 1973, and given content-method integrated instruction similar to that of interns but without mastery-approach. They met six hours per week in the laboratory and spent one hour per week on clinical experience in elementary schools. The third group of students, had the regular content and method separated mathematics education program. They were used for the study during the Fall term when they were having the methods course. The content course is a pre-requisite for the methods course. Eighteen students from this group were included in the study.

Five criterion-referenced achievement measures with reliability estimates ranging from 0.79 to 0.94 were developed and used in evaluation of learning; the method of construction of the measures insured their content validity. Other instruments used were Hicks and Perrodin's instrument for analysis of mathematical topics, Dossett's Test of Basic Mathematical Understandings, Dutton's Attitude Inventory, Attitude Scales Toward Different Aspects of Mathematics developed by the International Study of

Achievement in Mathematics, Aiken's Enjoyment and Value of Mathematics Scales.

Multivariate and univariate analysis of variance were used in assessing the effect of the content-method integrated course on interns' performance on the criterion-measures and basic mathematical understanding and attitude of the interns. Two-way analysis of covariance was used to compare the effect of the three instructions, entry attitude, and mathematical aptitude on terminal basic mathematical understandings and attitude. Two-factor by one-way repeated measures design was used to evaluate environments of learning. Stepwise regression techniques were used to (a) assess the contribution of learning units to the basic mathematical understanding of the interns, (b) determine the relationship between attitude toward mathematics and interns' perception of mathematics learning, enjoyment, value, and environments.

The results of the study indicated:

1. The interns made significant gains ( $p < .001$ ) on the criterion-referenced measures. The percentage of interns that reached mastery level ranged from 67 percent in Fractions to 96 percent in Numeration.
2. The interns showed significant gains ( $p < .0001$ ) on test of basic mathematical understanding and attitude toward mathematics.

3. Allowing for initial differences, the interns, after completing the mathematics education component of the program showed significantly better mathematical understanding than a group of students in the regular teacher education program.
4. The interns have better perceptions of their learning environments in many aspects than other two groups of students.
5. Three of the learning units accounted for more than 63 percent of the interns' basic mathematical understandings.
6. The interns' initial attitude toward mathematics and their enjoyment of the instruction accounted for more than 73 percent of their terminal attitude toward mathematics.

This Thesis is Dedicated  
to  
Gbadamosi Adebisi, my father;  
Alimotu Adufe "Alayo," my mother;  
to  
Salamotu Lateye, my late step-mother  
(my father's senior wife);  
to  
late Salamo Iya-Ake;  
to  
Salimotu Adetohun, my dear aunt;  
to  
Sulaiman A. Lawal, my dear friend;  
to  
Morenike, my dear wife;  
to  
Risikat Oyin-Ade, my sister;  
to  
All my "Brothers" and "Sisters"  
and to  
All my children.

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## CHAPTER I

### THE PROBLEM

#### Introduction

In the early nineteen sixties there was a tumult about the difficulty of training teachers for urban schools whose pupils were described as "disadvantaged" or "culturally different." McGeoch and Copp (1963) reported a pilot project at Teachers College, Columbia University called the "Teaching Corps," which was a program designed to train special teachers for the disadvantaged. Goldberg (1963) described a hypothetical model of a successful teacher of the disadvantaged. Haubrich (1965) gave guidelines for effective training of teachers for culturally disadvantaged children while Rivlin (1962) had earlier outlined a variety of desired modifications in existing modes of teacher preparation for large city schools. Ausubel (1964) discussed the reversibility of the cognitive and motivational effects of cultural deprivation and implications for teaching the culturally deprived child.

In a nation-wide effort to give children from "low income" families better educational opportunities and to

improve the quality of teacher education programs for both certified teachers and inexperienced teacher interns, Teacher Corps was established by Congress in 1965.

Today Teacher Corps projects exist in over 150 school districts, 5 prisons, and 17 juvenile institutions. They operate in cooperation with about 85 colleges and universities. The project gives school districts in low income areas, their communities, and nearby universities the chance to work together. Its philosophy embodies models and guidelines of McGeoch and Copp (1963), Goldberg (1963), Rivlin (1962), Haubrich (1965), and others. It is a competency-based, field-based, community-based, bilingual and bicultural program.

#### Purpose of the Study

The purpose of this study is to provide a formative evaluation of the mathematics education component of eighth cycle Teacher Corps Project at Michigan State University with the Lansing, Michigan school district.

Following the model of Scriven (1967), Sanders and Cunningham (1973), this study will focus on both the "intrinsic" and "pay-off" parts of formative evaluation of the process and product of the mathematics curriculum and instruction. Internal, external, and contextual sources are used.

Specifically, the intrinsic aspect of the investigation sought:

1. To analyze and evaluate the mathematics content in the mathematics education component of the program and to assess whether they meet the mathematical need of the interns.
2. To provide a critical appraisal of the instructional methods and the clinical experiences in the program.

The pay-off aspect of the study sought:

3. To evaluate the effect of the instruction as prescribed by the mathematics education component of the Teacher Corps Program on the interns in relation to specified competencies and to assess if the interns achieved a degree of mastery over these competencies.
4. To evaluate the effect of the instruction on the basic mathematical knowledge of the interns.
5. To evaluate the effect of the instruction on the interns' attitudes toward mathematics.
6. To assess the contribution of different units of mathematics instruction to the general mathematical knowledge of the interns.
7. To assess the contribution of different aspects of attitude toward mathematics and school learning to the general attitude of the interns toward mathematics.

8. To compare the mathematical understanding and attitude toward mathematics of the interns in this program with the mathematical understanding and attitudes of students enrolled in the regular teacher-education program. The purpose is to determine,
  - (a) the effect of method of instruction (three levels) and entry attitude (three levels) upon the mathematics achievement at the end of instruction;
  - (b) the effect of method of instruction (three levels and mathematical aptitude (three levels) on attitude toward mathematics at the end of instruction.
9. To compare the effect of three methods of instruction on different aspects of attitude (five levels).
10. To use the results of the investigation to make specific recommendations for replication, diffusion, and installation of a change of procedure, as well as refinement of its overall design.

#### Need for the Study

While "lower-class" and "disadvantaged" are not necessarily synonymous, the literature generally views the middle-class teacher in relation to lower-class and disadvantaged children, the results seem to coincide. Many

educators like Becker (1952), Davis (1948), Havighurst, Bowman, Liddle, Matthews, and Pierce (1962), Arnez (1966), Hickerson (1966), Riessman (1962), agree that the teachers middle-class attitudes and values are in conflict with those of lower-class or disadvantaged students, and therefore antithetical to the focal concern of the children and youth they serve. Becker (1952), Davidson and Lang (1960), and Rosenthal and Jacobson (1968) have shown that teachers' expectations influence the aspirational level and learning of the child and these expectations tend to vary inversely with the child's socio-economic class. Unfortunately children perceive and fulfill these lower expectations, confirmed Clark (1965). Davidson and Lang (1960), Friedenberg (1962), Vontress (1963), and Sexton (1964) maintain that most of the teachers of the disadvantaged dislike and/or distrust the children and teachers replace their main function of teaching by an emphasis on discipline. Clark (1965), Rivlin (1962, 1965), Landers (1964), Groff (1967) reported that many new teachers are unwilling to accept appointment to teach the disadvantaged while experienced teachers tend to seek transfers.

In response to these sensitivities, the National Science Foundation funded an SMSG conference on Mathematics Education in the Inner City Schools in March, 1970. One of the five position papers was prepared and presented by Woodby on "A Survey of Existing Projects Which Attempt to Attend to Innercity Problems in Mathematics Education."



This paper examined the forces and issues that led to the funding of the projects. There was also a panel focusing on pedagogy and the laboratory approach as possible partial solution to the problems.

In his reaction paper, Forbes observed that although the projects surveyed differ in many ways, there are common themes like individualization, diagnosis and prescription, objective-oriented programs, success for experiences of students, students involvement in learning, student self-image, teacher-training and development and these themes have relevance for education of all children. The participants almost unanimously agreed that priority in innercity education should be given to preschool, kindergarten and early primary years and that little value would be accomplished unless the program includes great emphasis on developing appropriate attitude, insights, and the understanding of different cultures among teachers in the innercity schools.

The last remark of the preceding paragraph coupled with what is already known about existing middle-class innercity teachers possibly led to change the goals of "special programs." The changed goals were directed toward giving young people from poverty backgrounds new opportunities to obtain a college education and to make higher education responsive and relevant to their special educational needs (Astin et al., 1972). The students recruited into these compensatory programs were considered

"disadvantaged" or "high risk" on the assumption that they lack the requisite motivation and academic skills to seek and successfully pursue a college education. Accordingly such programs have focused on correcting these motivational and academic deficits.

Considering the amount of money expended by the government and the amount of time, energy and human resources supplied by the universities in such programs, their existence raises some issues not only about the general educational experiences that lead to the successful development of latent talent but also about the basic premises underlying the admission criteria. In an attempt to pave the way for needed answers to general questions of the form:

Can these programs help the underprepared, specially admitted students to make educational and social adjustment necessary to complete a college education? To what extent do higher educational programs for the disadvantaged serve their clients? What types of programs' components show the greatest promise?

This study examines the following issues:

To what extent does the mathematics instruction for the disadvantaged interns serve the interns? Which of the college environments and experiences facilitate the mathematical growth of these disadvantaged interns? Assuming (i) Carroll's thesis, (ii) Bloom's theory of Mastery learning are valid, (iii) recent, Begle (1971) research report which seems to justify the theories and (iv) recent publication by Astin, et al. (1972) that being socio-economically disadvantaged is not in itself, a severe handicap to the student once he gets to the college, is it possible to design a mathematics instruction for these interns that will bring their basic mathematical understanding and their attitude toward mathematics to the same level as other prospective elementary teachers in other programs?

In the mastery learning model, all students are helped to achieve a criterion mastery of the learning at hand. The focus is not on separating students into grade classifications but rather on helping students reach the mastery level. Riessman (1963) contended that the disadvantaged child is typically a physical learner, and the physical learner is generally a slower learner and this slowness should not be equated with stupidity. Discussing the characteristics of the slow learner, Schulz (1972) remarked that cultural differences and deficient cognitive functioning are major influences on the behaviour and achievement of slow learners. In the words of Pikart and Wilson (1972), "Research on the development, use and validation of the mastery learning model for slow learners in mathematics is an obvious need."

#### Definition of Terms With Comments

1. Criterion-Referenced Measure: "One that is constructed to yield measurements that are directly interpretable in terms of specific performance standards."
2. Economically Disadvantaged--children and adults from home and/or community background where a majority of the residents lack adequate financial income thus resulting in substandard living conditions.
3. Educationally Disadvantaged--individuals from home and/or community background lacking cultural assets

necessary for normal school achievement thus placing the individuals at a disadvantage grade level-wise.

4. Experimental Groups:  $G_1$  was a group of thirty junior elementary education majors selected for the Teacher Corps program who participated in the eighth cycle program at Michigan State University. The members were usually referred to as "Interns." The group participated in a specially designed mathematics instructional program.  $G_2$  was a group of twenty-two students in the regular teacher education program randomly selected from students who registered for Mathematics 201 in 1973 fall. The group was given a mathematics instructional program similar to that of  $G_1$ .  $G_3$  was a group of students in the regular teacher education program exposed to the regular mathematics instruction which was different from those given to  $G_1$  and  $G_2$ .

5. Formative Evaluation--is the process of judging a fluid process or product that can be revised in form. The results of such evaluation studies are given to persons directly involved in the process or in developing the product.

6. Formative Process/Interim Evaluation--is the type of evaluation which provides periodic feedback to persons responsible for implementing plans, or procedures or developing a product that is not yet fully assembled. It

has three objectives: (1) to detect or predict defects in procedural design or its implementation during implementation stages, (2) to provide information for programmed decisions, and (3) to maintain a record of procedure as it occurs. It is, thus, concerned with program improvement.

7. Formative Product Evaluation--(following Sanders and Cunningham, 1973) is the evaluation of the product as it has been put together strictly for feedback to the developer. Descriptive and content analyses techniques as described under interim formative evaluation activities are extremely important at this point. Knowledge about the extent to which valued objectives are achieved with a plan/product are important. What Anderson (1969) termed "field test" is an excellent example of this type of evaluation. Borich (1971) has also suggested a conceptual model for formative product evaluation. Validation of a product with a sample of subjects from the target population or a feasibility study of a plan for educational change are the most frequently found formative product evaluation studies in the literature.

The sources of external and internal information listed under formative interim/process evaluation activities are applicable to formative product evaluation activities also. The object under scrutiny at this point will be the entire assembled product, however, rather than its components. Contextual information is of utmost importance at

this point. The formative product evaluation should test the product in the context within which it is intended to function. The collection of contextual information in interim/process formative evaluation cannot be accomplished in isolation from a particular set of objectives or a particular product since by definition the role of context is to specify the limits of the product.

"The task of the formative evaluator, therefore, is to establish whether predicted relationships between context, internal, and external information holds. Is it the case, for instance, that students with specified entry behaviours (context) learn more mathematics (external) from a programmed test using hierarchical sequencing (internal)? An analogous question in the proposed study is,

"Is it the case that prospective elementary school teachers with low socio-economic background--low-achievers in mathematics--(context) can learn mathematics to the same level of competency as students in regular program (external) from a mastery learning mathematics instruction conducted in a laboratory setting (internal)?"

The formatitive evaluator often is not satisfied if he observes sharp differences in the effectiveness of two programs, he likes to find out why the difference occurs if he is to give complete information concerning possible revisions of the material. Explanatory information is, though not always, needed for formative product

evaluation work, it can be critical and should not be overlooked.

8. Interim Intrinsic Evaluation--(following Scriven, 1967) is the evaluation of transactional or means-to-the-ends program characteristics. It is an interim/process formative evaluation which relies on internal information, both descriptive and critical appraisal. Examples of activities that fell into this category are the analysis of the content of the program components or the appraisal of instructional strategies which are well illustrated by Morrisett and Stevens (1968, 1971), Tyler and Klein (1968), and Eash (1970). Stake (1970) has provided an excellent discussion of the use of professionals in such evaluation studies.

9. Learning Unit: The mathematical objectives to be acquired under a single set of learning conditions.

10. Mastery Learning--is an instructional strategy which proposes that under appropriate instructional conditions virtually all students can learn most of what they are taught. The sequential steps in the strategy are specification of objectives, designation of the mastery level (score), unit teaching, formative tests, immediate feedback, diagnosis of areas of deficiency, alternate procedures, re-test. In Bloom's model, an attempt is made to alter the amount of time a student spends in studying a

task and thus bring about a level of learning determined to be mastery of the task.

11. Pay-off Evaluation--(following Scriven, 1967) is interim/process formative evaluation which relies on external information. This is the most common type of evaluation activity in instructional development; indeed, for some people, this type is the only "real" type of evaluation. The methods used to collect external information for pay-off evaluations are excellently illustrated by Metfessel and Michael (1967), Markle (1970), Abedor (1971), Goodwin and Sanders (1971).

12. Poverty-area School--elementary school in which a majority of the children are from educationally and economically disadvantaged environments.

13. The Disadvantaged--the individual who comes from a home and/or community environment that is lacking economically and educationally.

### Research Hypotheses

The following hypothesis will be tested to assess the effect of the instructional program on the achievement of interns on the prescribed mathematical competencies (integrated content and methods). In each case the .05 level of significance will be used.



- A. The post-test means score will exceed the pre-test means score of the interns ( $G_1$ ) on the criterion-referenced measures.

The univariate hypotheses associated with this multivariate hypothesis are:

The mean post-test score of the interns will be higher than the mean pre-test score on the criterion-referenced measures in:

- a. measurement,
- b. numeration,
- c. addition and subtraction of whole numbers,
- d. multiplication and division of whole numbers,
- e. fractions.

The following two hypotheses will be tested to assess changes in the interns' basic mathematical knowledge and attitude toward arithmetic:

- B 1. The post-test mean score of the interns will exceed their pre-test mean score on Dossett's test of mathematical understanding.
- B 2. The post-test mean score of the interns will exceed their pre-test mean score on Dutton's arithmetic attitude inventory.

The following hypotheses will be tested to compare the interns and students enrolled in the regular teacher education program. The purpose is to determine (1) the effect of method of instruction (three levels) and entry attitude toward mathematics (three levels) upon the

mathematics achievement at the end of instruction, and (2) the effect of method of instruction (three levels) and mathematical aptitude (three levels) on the attitude toward mathematics at the end of instruction.

C 1.

- (a) When a linear adjustment is made for the effect of variation due to differences in prior mathematical aptitude, as measured by Dossett's pre-test, there will be no significant difference in mathematics achievement, as measured by Dossett's post-test, between the methods of instruction.

That is, there will be no treatment effect.

- (b) When a linear adjustment is made for the effect variation due to differences in prior mathematical aptitude, as measured by Dossett's pre-test, there will be no significant difference in mathematics achievement, as measured by Dossett's post-test, between the entry attitudes.

That is, there will be no attitude effect.

- (c) When a linear adjustment is made for the effect of variation due to differences in prior mathematical aptitude, as measured by Dossett's pre-test, there will be a constant difference in mathematics achievement, as measured by Dossett's post-test, between the methods of instruction at all levels entry of attitude.

That is, there will be no treatment by attitude interaction.

C 2.

- (a) When a linear adjustment is made for the effect of variation due to differences in prior attitude toward mathematics, as measured by Dutton's pre-test, there will be no significant difference in attitude toward mathematics, as measured by Dutton's post-test between the methods of instruction.

That is there will be no treatment effect.

- (b) When a linear adjustment is made for the effect variation due to differences in prior attitude toward mathematics, as measured by Dulton's pre-test, there will be no significant difference in attitude toward mathematics as measured by Dulton's post-test, between the entry attitudes.

That is there will be no attitude effect.

- (c) When a linear adjustment is made for the effect of variation due to differences in prior attitude toward mathematics, as measured by Dulton's pre-test, there will be a constant difference in attitude toward mathematics, as measured by Dulton's post-test, between the methods of instruction at all levels entry of attitude.

That is, there will be no treatment by attitude interaction.

The following hypotheses will be tested to compare the interns and other two groups of students in the regular teacher education program on five different aspects of attitudes.

- D. (a) There will be no significant difference between the mean-scores of the three methods instruction (groups) on Husen's Attitude Scales.

That is, there will be no treatment main effect.

- (b) There will be no significant difference between the mean-scores of the three attitude levels groups on Husen's Attitude Scales.

That is, there will be no entry attitude main effect.

- (c) There will be a constant difference in attitudes, as measured by Husen's Attitude Scales, between the methods of instruction at all levels of entry attitude.

That is, there will be no treatment by attitude interaction.

### Assumptions of the Study

The mathematics education component of the Teacher Corps program at the Michigan State University includes the following assumptions (which are not necessarily unique to the disadvantaged):

1. That there are differences between the skills required to teach in low-income schools and

middle-class schools, but this does not imply that unique principles of learning are involved in the two different settings.

2. That the program for inner city education should include great emphasis on developing appropriate attitudes, insights, and understanding among teachers in inner city schools.
3. That low achievers in mathematics may be classified into three groups, viz: those who have intellectual deficiencies, those who have cultural deficiencies, and those who have both intellectual and cultural deficiencies (and perhaps a different group who are neither culturally nor intellectually deficient but are slower in learning). It is important to consider these differences among low achieving pupils in teaching, both for individualizing the curriculum and for developing self-instructional materials.
4. That many children from low-income areas are assumed to be low-achievers because they are culturally deficient and two factors are of importance in motivating such children, viz: (a) the need to develop a technique of changing children's behaviour in order for school learning to take place, and (b) the need for flexibility in approach to teaching depending upon the value systems of the homes from which these children come.

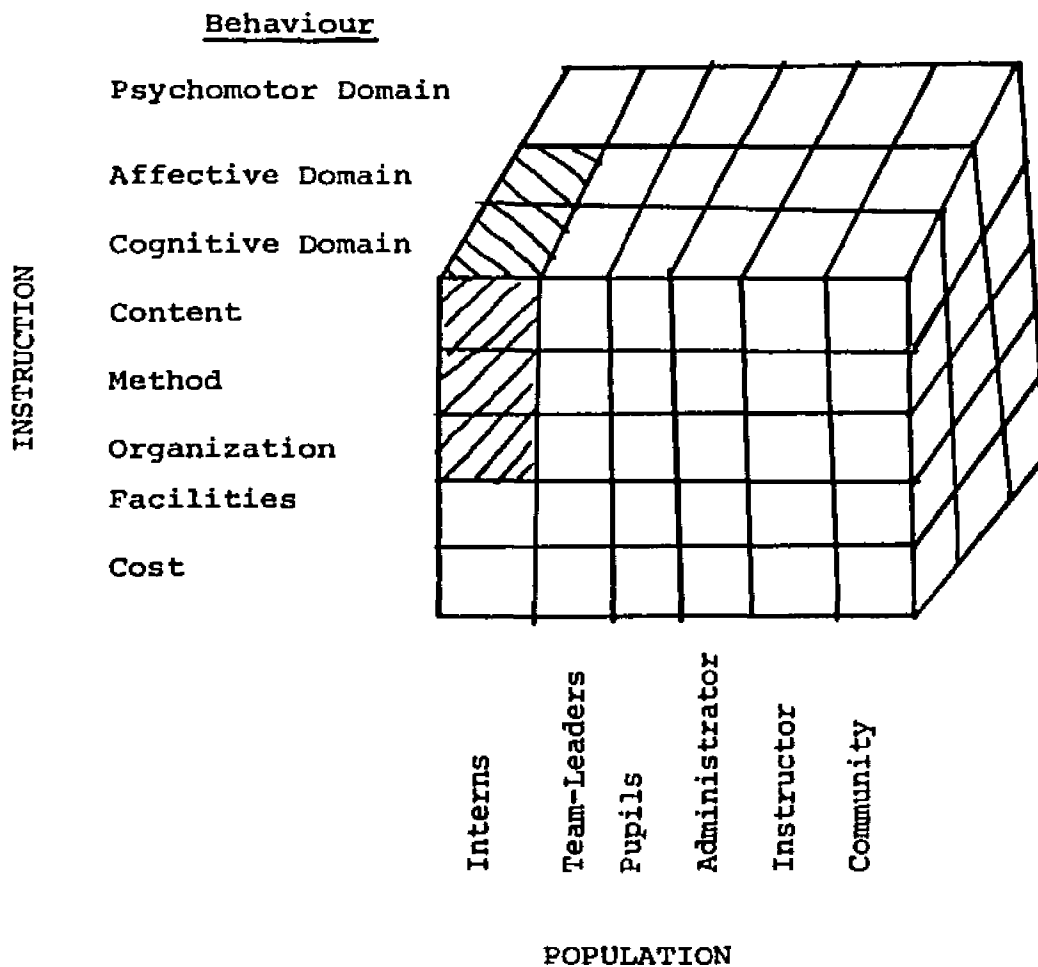
5. That differences in values, prior experiences and environments among children from various income, ethnic, and racial sub-groups are so great that, teachers need special training in order to apply the principles of teaching and fashion the instructional procedures for each group.
6. That the following three assumptions often made with respect to the pupil/adult of low ability must be rejected:
  - a. That the program for the pupil/adult of low ability should be formed on drill!
  - b. That the low-ability child/adult should not be required to think!
  - c. That any program for low-ability students should involve little or no reading.
7. That anyone who teaches mathematics should (a) know mathematics, (b) like mathematics, (c) continue to learn mathematics, (d) be able to communicate well with the learner, and (e) understand the learning process. In addition, teachers of low achievers need special knowledge of the psychological and sociological backgrounds of the children.
8. That particularly for the low-achievers, the need for mathematics comes from experiences in the physical world. The emphasis in elementary school

should be upon the development of the experiential background enabling symbolic activity and abstract reasoning.

9. That Ausubel's three-part teaching strategy of culturally disadvantaged children in a laboratory setting is both a theoretically and pedagogically sound approach.
10. That Bloom's model of mastery learning which emerged from the work of Carroll (1963), and supported by the ideas of Morrison (1926), Bruner (1966), Skinner (1954), Suppes (1966), Goodlad and Anderson (1959), and Glaser (1968) is a useful strategy in designing the instruction of the (disadvantaged) prospective teachers of the disadvantaged.
11. That little is known about disadvantaged children's capacity to learn, but the limits may be far beyond what they now learn and, if properly taught, can learn more mathematics than ordinarily.
12. That teachers with Poffenberg-Norton (1956) characteristics (in chapter II) affect students' attitude and achievement positively, moreover, teacher-initiated teacher-student personal interactions do significantly influence students' achievement.

### Scope and Delimitations of the Study

To provide information on the forces that influence a student's achievement, it is necessary to work within a framework that offers a wide range of potentially relevant variables which reflect theory and practice of teaching and learning. EPIC Evaluation Center in Tucson, Arizona has designed such framework. An adaptation of it to the proposed study is shown below:





The structure is composed of three sets of variables --instruction, population, and behaviour--and it has been most useful as a heuristic device to reveal combinations of variables leading to a more complete description and analysis of the instructional program. Analysis of variables is generally limited only by the nature and scope of the program and the desire for simple or complex analysis. The forces affecting programs results are obviously produced through the interaction of variables on each of the dimensions. Consequently, the major limitations of this study are:

1. The study intends to evaluate only the mathematics education component of the Teacher Corps program.
2. Evaluation is confined solely to:
  - a. Analysis and critical appraisal of the content, method and organization of instruction--internal information;
  - b. Cognitive and affective behaviour of the interns at interim/process stage--external information;
  - c. Cognitive and affective behaviours of the interns at the entry and product stages--contextual information.

([a] and [b] have been termed "intrinsic" and "pay-off" evaluations respectively by Scriven (1967) and [c] has been generally termed "contextual" evaluation.)

3. The study does not evaluate the effect of instruction on teaching behaviour of the interns.
4. The study does not evaluate the effect of the program on the behaviours of the pupils taught by the interns, neither does it evaluate the effect of the program on the remaining part of the population.
5. The facilities and cost aspects of the instruction and the psychomotor domain of behaviour shall not be investigated.
6. The instruments to be used in the study shall have the inherent limitations as discussed by Glennon (1949).

## CHAPTER II

### REVIEW OF LITERATURE

#### Introduction

Distinguishing between "curriculum static" and "curriculum developing," Wright, et al. (1971) stressed that modern concepts of curriculum favor dynamic curriculum development which is controlled by philosophical, psychological, and sociological forces. A decade before this, Foshay (1961) discussed the difficulty of having a "balanced" curriculum. In particular, three sources of elementary school mathematics have been identified:

1. the nature of the learner, which may be referred to as the expressed needs-of-the-child theory of curriculum. This provides psychological basis for curriculum theory,
2. the nature of his adult society, which may be referred to as the needs-of-adult society, social utility, instrumentalism or sociological basis for curriculum theory,
3. the nature of the cognitive area-mathematics, which may be referred to as the structural, the pure mathematical, or the logical theory of curriculum. This provides the logical, or pure mathematical basis for curriculum theory.

Each has the potential to contribute significantly to a well-designed curriculum. "Any unilateral authoritarian

view of the curricular basis of the program is an extremist view," argued Glennon and Callahan (1970). In order to have a clear perception of a balanced theory of curriculum, a clear perception of each of these extremist theories is important.

The major objectives of the Teacher Corps are to strengthen the educational opportunities available to children from low-income families and to assist colleges and universities and local school districts to bring about basic changes in the ways in which teachers are trained and used. The teacher interns are being trained to teach in the poverty-area schools. Many of the interns are themselves members of minority or low-income groups. These minority and "low-income" of interns can provide children with models of achievement and scholarship. Consequently, the first part of the review examines the type of mathematics curriculum and instruction which will meet the needs of the disadvantaged while the second part examines recent, important and substantive methodological developments which are helpful in designing the study in order to refine, revise, and extend what are already known in the field of mathematics education.

Specifically, the review of literature pertinent to the study has been organized under seven categories:

1. the influence of the school and the teacher on the disadvantaged child,

2. research on the nature of the disadvantaged and his learning process,
3. the theoretical foundations of the emerging mathematics curriculum, instructional procedures and goals for the disadvantaged,
4. recent empirical studies which seem to justify some of the curricular and instructional practices,
5. Bloom's model of mastery learning as an emerging method of instruction, its theory, practice and research findings,
6. promising, innovative/experimental field-experiences in teacher education,
7. Related (Research) Methodology.

Influence of the School and the Teacher  
on the Disadvantaged Child

"The essence of educational history," claims Gross\* "is to enable educators to learn from the mistakes of the past and use them intelligently to understand the development of the philosophy of present educational institutions." The literature cited under the need for the study together with the present discussion helps to illuminate the philosophy of the program and hence account for the philosophical forces on the curriculum.

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\*Professor Carl Gross made the statement at the introductory lecture on Educational History: Plato to Locke, Winter Term 1974, Michigan State University.

Reissman (1963) maintains that there is a great deal of evidence that the deprived children and their adults have a much more positive attitude toward education than is generally believed. One factor that obscures the recognition of this is that while deprived individuals value education, they dislike the school, they are alienated from the school and they resent the teachers. Reissman believes that it is not the disadvantaged who have capitulated to their environment, but the teachers and hence when considering attitude of the disadvantaged, attitude toward education and attitude toward the school must be considered separately. Deutsch (1967) claimed that the school only reinforces the negative responses of children from deprived backgrounds. Gordon and Wilkerson (1966) argued that two factors--low motivation and low self-esteem--handicap the disadvantaged child in his academic development and they cited studies which indicate that the motivation of disadvantaged children not only is "likely to be lower but is likely to be directed toward goals inconsistent with the demands and goal of formal education. . . ." They went further, that the typical curriculum is incongruent with the child's social experiences, and this incongruence, together with lack of motivation, makes "normal school achievement or success" unlikely. Fantini and Weinstein (1968) suggest that the condition of being disadvantaged cuts across all segments of society; the idea of human failure is erroneous, only institutions fail. Examining

the relation between a student's self-concept and his experiences in school, Deutsch recognized that the latter may "either reinforce invidious self-concepts acquired from the environment or help to develop--or even induce--a negative self-concept. The school contributes further to these negative self-images because it fails to stimulate or create interest in the child at the same time that it regulates his behaviour. Vane (1966) notes that few students improve, once they have established a poor achievement record early in their career. Oakland (1970) remarked that it seems likely that the lack of necessary antecedent experiences causes the child to fall further behind as the curriculum builds upon abilities he has not acquired. Deutsch (1967) maintains that,

as the age increases it becomes more and more difficult for these disadvantaged children to develop compensatory mechanisms, to respond to special programs, or to make the psychological re-adjustments required to overcome the cumulative effects on their deficits.

In a nation-wide study on disadvantaged students recently published by Astin, et al. (1972) it was reported:

. . . It was encouraging to find that the high school achievements and college progress of these disadvantaged students\* did not differ substantially from those students from more advantaged backgrounds. It would seem that being socio-economically disadvantaged is not, in itself, a severe handicap to students once they get to college.

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\*For the purpose of the analysis of data collected, they defined "disadvantaged" students as those whose family incomes was below \$6,000 per year and whose parents had not completed high school.

Goldberg (1967), Stodolsky and Lesser (1967), Wolf and Wolf (1962) pointed out that teachers often start with zeal and energy and wish to do an effective job, but are thwarted by "reality" and "cultural shock" and are unable to fulfill their professional responsibilities and therefore become frustrated, indifferent, angry and "learn" the wrong attitudes, added Ornstein (1968a).

### Nature of the Disadvantaged\* and His Learning Process

Roueche and Wheeler (1973) define the "disadvantaged" as "the social strata having least access to higher education." They remarked that in other context it may be an euphemism for low-achieving students or simply for economically poor students. They observed that this group is variously described as "socially disadvantaged," "high-risk educationally underprepared," "culturally deprived," "socio-economically deprived," "opportunity deprived," "developmental students," "socially and culturally disadvantaged," "chronically poor," "poverty-stricken," "culturally alienated."

The populations among the "disadvantaged" vary from each other in a number of ways, Gordon (1965) and Noar (1967) observed that they can have such common characteristics as low economic status, low social status, low

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\*The word "disadvantaged" will henceforth be taken to qualify both children and youth (young men and women) wherever no specification is made for distinction.



education achievement, marginal or no employment, limited participation in community organizations, limited immediate potential for upward mobility, too little food and sleep, too little personal attention, too little self-respect and self-confidence, too little reason to try and too little happiness. The populations consist primarily of American blacks, Mexicans, Puerto Ricans, American Indians, and Southern rural/mountain whites. Their children come to and leave school disadvantaged to the degree that their culture has failed to provide them with the experiences typical of children and youth that American schools and colleges are accustomed to teaching. Traditional methods tend to widen the gap as the deprived student stays in schools. There are some specific environmental factors, personal characteristics and particular experiences (all inter-related) that have a detrimental effect on the learning process (treated as the dependent variable) in studies reviewed. In the studies the overlapping antecedent variables are self-concept, motivation and the components of variable learning are language, cognition and perceptual style.

The socio-economic class in which a child is socialized provides experiences which may influence his academic achievement. Reviewing the results of numerous studies, Gordon (1965) concluded that homes of low SES fail to prepare the child for learning because they lack appropriate stimuli like books, toys, and instructional

equipment. The absence of visual stimuli, together with an excess of noise, limits concentration. This led Deutsch (1967) to report that a person for low SES lacks the experience that enables him to "manipulate and organize the visual properties of his environment and thus perpetually to organize and discriminate the nuances of his environment."

Among others, Montague (1964), Deutsch and Brown (1964) showed that SES correlates with intelligence which, according to Hunt (1961), is a function of the process of personal interaction with environment rather than a product of genetic factors. Ricsin (1961), Thompson and Schaefer (1961) describe how early stimulation affects the development of proper neural structures while Solomon, et al. (1961) contend that an impoverished environment reduces a person's discriminatory and manipulative abilities and his desire for exploratory behaviour which is thought to be necessary for problem solving. "Failure to acquire competence in various languages," reported Ausubel (1964), "has limited the disadvantaged child's ability to advance from concrete to abstract reasoning"; "hence he cannot handle symbolic language and concepts," added Slaughter (1969). Gordon (1965) went further to say that the inability to reason by induction and to apply or transfer knowledge through linking of concepts is characteristic of the thought process of disadvantaged youth.

Gordon (1965) suggested that lower-class homes are deficient in the necessary interaction with parents that activates the child's interest and motivation. Deutsch (1967) emphasized that the middle-class child receives greater intellectual stimulation, for which he is rewarded, than does the lower-class child, whose parents seldom subject him to the pressure of a formal adult-child learning situation.<sup>1</sup> It is not the status per se, argued Dave (1963), parent-behaviour is the critical variable (which does not make the home environment stimulating.) Fifer (1964) found that, within ethnic groups, children of different social classes differed in their performance on a number of tests. Epps (1970), Rosenberg (1965), Asbury (1968) found that parental background affects the child's self-concept and self-aspirations. While Goff (1954) reported that children of low SES feel more inadequate in school than others, Epps (1970) hypothesized that even at the time they enter school, such children have little self-confidence,<sup>2</sup> whereas Edwards and Webster (1963) found that positive self-concepts are related to academic achievement and aspiration. Coleman, et al. (1966) reported, that of all factors considered, the degree to which a person perceives himself as being able to control his environment and his future is the most critical to achievement. Hall (1969)

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<sup>1</sup>op. cit., p. 49.

<sup>2</sup>op. cit.

reported a study which supported Coleman's conclusion. Hall's study goes a step further to show that conversely, the sense of powerlessness and inability to control one's destiny, characteristic of persons from lower socio-economic groups and culturally deprived homes, reduces motivation and leads to unfavourable self-concepts, thereby inhibiting learning potential. Clift (1969) has summarized the characteristic traits of culturally disadvantaged youth under three categories--personality, cognitive functions and educational values.

The Theoretical Foundation of the Emerging  
Practices in Mathematics Curriculum,  
Instructional Procedures and Goals  
for the Disadvantaged

Gordon and Wilkerson (1966) remarked that despite all our current efforts (on educating the disadvantaged), effective approach to teaching the disadvantaged has not been found. Goldberg (1964), Haberman (1964), Passow (1963), Webster and Lund (1969) and Wilkerson (1964, 1966), all claim there is need for much study on teacher behaviour characteristics and the relationship between the teachers of the disadvantaged and their students. Stodolsky and Lesser (1967) point out that teachers of the disadvantaged want to succeed but fail because behaviour techniques have not been developed "which provide desirable outcome." Daniel (1967), Goldberg (1967), and Wilkerson (1964, 1966) contend that we must learn what works, with whom, with what

variables; without this knowledge, we cannot succeed and will jump from one approach to another.

In the area of general psychology, Gordon and Wilkerson (1966) discussed the interactionist and projective theories of behaviour. The former sees behaviour patterns as being shaped by interaction of individual with his environment which acts to foster or impede the psychological and intellectual development necessary for learning. This interactionist theory implies that compensatory education and enrichment of the programs can affect, in some fashion, the disabilities associated with disadvantaged status. The projective theory assumed that individual behaviour is based on certain predetermined patterns which are activated by environmental stimuli; environmental forces may affect behavioural forms, but, in general, behaviour is controlled by drives that are "genetically established and bound." The theory thus emphasizes the intractability and permanence of early characteristics implying that no amount of social action can improve performance.

The proponents of the projective theory must have received a great shock from Bruner's (1962) hypothesis:

Any subject can be taught effectively in some intellectually honest form to any child at any stage of development.<sup>1</sup>

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<sup>1</sup>Jerome S. Bruner, The Process of Education (Cambridge: Harvard University Press, 1962), p. 33.

Carroll (1963) time-to-mastery model gave a second blow to proponents of projective theory. Describing Carroll's thesis Bloom (1969) said,

. . . Implicit in this formulation is the assumption that given enough time, all students can conceivably attain mastery of a learning task. . . .

Adler (1957) rejected projective theory in its entirety. He reaffirmed his position again in 1972. He described the theory as one of the principal causes of poor learning by those whom it stigmatizes as genetically inferior. He explained that pupil failure is taken for granted, and so neither teacher nor child is required to exert the effort that might prevent failure. Adler (1972) maintains that though the interactionist theory points to social influences that do exist and that can be corrected, it only tells the half truth and it has been harmful because it is generally combined with the tacit assumption that children who have been subjected to these retarding social influences cannot be expected to learn in school. He contends that any theory that reduces our expectation of what children can do inevitably reduces their level of achievement and becomes a self-fulfilling prophecy.

Silberman (1970) and Adler (1972) believe the I.Q. theory is harmful, it affects not only the teacher's expectation, as shown by Jacobson and Rosenthal (1968), but also the pupil's expectation of himself. It holds the teacher back from teaching, and holds the pupil back from learning. In the words of teachers:

We think the children are not capable of learning and we teach them less. Then they learn less and become less capable of learning.

Clearly by 1964 the USOE and the NCTM had lost confidence in the projective theory and they jointly sponsored a conference held in Washington, D.C. on March 25-27, 1964 to discuss the "low achievers" in mathematics. H. L. Phillips, specialist in mathematics at the Office of Education gave five reasons why the USOE and the NCTM were concerned about low achievers in mathematics. Five position papers were presented on sociological and psychological factors in low achievement and six other papers were invited on the then current promising practices. Among other recommendations that emerged from the conference was a guide-line for teaching mathematics to the low achiever (appendix C of the report), where it is recommended that a laboratory setting is especially effective for low achievers, Woodby (1965).

Again in April, 1964, through the interest of the Cooperative Research branch of the United States Office of Education funds were made available by the USOE to SMSG for an exploratory Conference on Mathematics Education For Below Average Achievers held in Chicago, Illinois, on April 10 and 11, 1964. Seven papers were presented at the conference. Two of the papers of particular relevance to disadvantaged are "Psychological Issues In the Development of Mathematics Curricula for Socially Disadvantaged Children" by H. Beilin and L. G. Gotkin and "Mental

Development and Learning of Mathematics in Slow-learning Children" by Gloria F. Leiderman.

The summary of the Conference General Discussion probably stimulated by these two papers put emphasis on four areas:

1. Mental Development: To proceed from concrete gradually to abstract, and habits of thought which makes mathematics instruction possible was suggested for research.
2. Distinction between low achievers: Four distinct classes were identified:
  - a. children who have intellectual deficiencies,
  - b. children who have cultural deficiencies,
  - c. children who have both deficiencies,
  - d. children who are neither culturally nor intellectually deficient but are slow learners, although they may not be less intelligent than the average group, they take longer time to process information.

Individualization of curriculum and development of self-instructional material necessitated the distinction among these groups. They emphasized that in order to motivate those classified as culturally deficient, we should (a) develop "docility" in order for school learning to take place, (b) be flexible in approach to teaching



depending upon the value systems of the homes from which these children come.

3. Action Programs were suggested for compensating for the deficiencies in culture. These are based upon our knowledge and hunches as to what and how to teach.
4. Demonstration and Research Programs on motivation, helping children to change goals and attitudes, strategies for teaching and what mathematics to teach. They rejected vigorously three assumptions often made with respect to pupil of low ability, viz:
  - a. that the program for the pupil of low-ability should be founded on drill,
  - b. that the low-ability child should not be required to think,
  - c. that any program for low-ability students should involve little or no reading.

The year 1964 appeared to be remarkable in the history of mathematics education of the disadvantaged. David P. Ausubel (1964) published a paper, "How reversible are the cognitive and motivational effects of cultural deprivation? Implications for teaching the culturally deprived child." Ausubel draws on research to aid in his assessment of the consequences of cultural deprivation on the development of verbal and abstract intelligence, as well as on motivation for academic achievement. He based

his argument on the "critical periods" hypothesis that "there are optimal periods of readiness for all kinds of cognitive development." The corollary to this hypothesis is that individuals who fail to acquire these skills at appropriate times are forever handicapped in attaining them. To Ausubel, the theory does not proclaim that a person cannot acquire these intellectual skills or subject matter contents at times other than the critical period; rather, he contends, there is a considerable loss of "years of opportunity when reasonably economical learning could have occurred if attempted, but did not." The consequences is a learning deficit which hampers both current and future intellectual development. Ausubel believes that the environmentally induced retardation in verbal intelligence is somewhat reversible. He examines theoretical bases and research evidence in this area and possibilities of reversing such retardation. In discussion of educational implications of this theory for the culturally deprived child, Ausubel concludes with a three-part teaching strategy that emphasizes,

1. selection of learning materials geared to the learner's readiness state,
2. consolidation of all ongoing learning tasks before introducing new ones, and
3. development and use of structured materials to facilitate sequential learning.

Jencks and Riesman (1968) have called "in dramatic language" for bold new approaches in meeting the needs of

the disadvantaged students. They believe it is a mistake to try to teach unsophisticated students traditional academic subjects by traditional academic methods. Such students must progress step by step from their natural culture in which they are immersed. They argued that many skills can be taught by using materials drawn from this popular culture, "assuming the teacher is familiar with it and has some appreciation of it." One of the consequences of this call was the SMSG conference on the Mathematics Education of the Inner City Schools held in Philadelphia in March, 1970. This conference was discussed in one of the preceding sections.

Bloom (1968) states the social and economic imperatives for extending higher education opportunity to all. He maintains that the basic problem is to determine how the largest proportion can learn effectively those skills and subject matter regarded as essential for their own development in a complex society. The Bloom (1968) Mastery model appears to be one of the most promising developments on the current scene. Commenting on Mastery learning and its implication for Curriculum Development, Cronbach (1969) expressed how enthusiastic he was once about the Carroll's model and how they (his group) had to abandon the whole notion of time to mastery or time to reach criterion or rate of learning. He reported that they did this because learning is multi-dimensional in a laboratory, and it is far more multi-dimensional in the school

for as one thing is being taught, a lot of things are happening. He concluded that things for which we can clearly use a training methodology designed to bring people to a performance criterion apparently are limited to static knowledge and algorithms.

To cause learning is the key concept in the Bloom's model. New programs will not be developed overnight. The individual instructor remains the key to any effective program for the disadvantaged. After studying compensatory educational programs at a variety of institutions, Gordon and Wilkerson (1966)<sup>1</sup> defined four general themes or objectives of such programs, the first two of which are humanitarian, the third is a research objective and the fourth is a variation of social lifting; viz.,

1. helping the disadvantaged to develop their potential and providing them with equal opportunities;
2. assisting in the elimination of academic deficiencies;
3. studying the effects of the programs; and
4. achieving a diversified student body.

The first three have been incorporated into the proposed study but restricted to the mathematics education component of this program.

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<sup>1</sup>op. cit., pp. 148-49.

Research and Evaluation Literature on Laboratory  
Approach as an Instructional Methodology  
for the Disadvantaged

As it has been pointed out several times in preceding sub-sections, the word "disadvantaged" refers to children as well as adolescents culturally and socio-economically disadvantaged. Research has shown that they are slow-learners and consequently low-achievers because they have cultural deficiencies.<sup>1</sup> Pikart and Wilson (1972) reviewed research on instructional programs for slow-learners in general. This review together with several reports on teaching of the disadvantaged indicate that the most promising practice in the mathematics teaching to the disadvantaged is the mastery learning model via laboratory approach. A summary of the approach was given by Allen C. Friebel.<sup>2</sup>

In 1971, the mathematics education community in the United States gave favorable publicity in support of the laboratory approach to teaching mathematics to elementary school children and prospective elementary school teachers. The December 1971 issue of the Arithmetic Teacher was

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<sup>1</sup>Richard W. Schulz, "Characteristics and Needs of the Slow Learner," The Slow Learner, 35th Yearbook, NCTM, 1972, pp. 1-25. Also Riessman (1963).

<sup>2</sup>Allen C. Friebel, "Mathematics Experience," in Teaching the Disadvantaged Child, edited by S. W. Tiedt (Oxford University Press, 1968), pp. 165-94.

devoted to mathematics laboratories. Three of these articles will be briefly discussed:

Discussing "The Mathematics Laboratory for Elementary and Middle Schools," Barson (1971) noted that it is impossible to give a universal definition of a "Math. Lab." due to its various uses and styles of organization. He gave seven characteristics of a "good" math-lab and distinguished between four types of math-lab--decentralized/classroom lab, centralized lab, team-room lab, roving/movable lab--and concluded with the major purposes/objectives of math-lab viz. motivation; enrichment; articulation with the regular mathematics program; and review, reinforcement and remediation. Embank (1971) attempted to answer seventeen questions on what? why? when? how? as related to the mathematics laboratory. Robert E. Reys (1971) discussed "Considerations for Teachers Using Manipulative Materials." He distinguished manipulative materials from teaching aids and cited references for selection and uses of manipulative materials and gave a list of nine, non-independent, non-exhaustive statements on learning theories which form the basic foundation underlying the rationale for using manipulative materials in learning mathematics. He concluded that though the rationale seems educationally sound, research in this area has not been conclusive in supporting/refuting the value of manipulative materials.

Feinstein (1972) observed that little is known about inner-city youth or how they learn mathematics, the characteristics of good teachers in urban schools and related educational problem. He reviewed the need to improve the preparation of teachers from CUPM point of view and emphasized that it would be most unwise to impose additional qualifications on prospective teachers of ghetto youth when it is already difficult to attract capable teachers to inner-city schools. He strongly recommended discovery-oriented activity for training of prospective teachers of inner-city youth using laboratory approach.

Rouse (1972) discussed the misconception of mathematics laboratory by some teachers. He emphasized that the term "math lab" is not totally descriptive of the instructional strategy that it represents, for the term suggests only the presence of special equipment (as some teachers take it), but provides no clue to the special methodology which comprises the essence of this approach to instruction. He distinguished Dienes' concept of it from that of Nuffield Mathematics Teaching Project to illustrate that there exists no single, universal concept of what constitute an ideal "math-lab" program. He then gave seven characterizing principles of the strategy (using some Dienes' terms) discussed the relationship between the principles and Piaget's Cognitive Theory and distinguished the laboratory approach from the well-known symbolic discovery method.

Vance (1969) investigated a laboratory program that might be typified as a separated program. It functioned as an adjunct to the regular curriculum in seventh and eighth grades. Tests of achievement, retention, and transfer revealed that the student did learn new mathematical ideas in the laboratory setting, although they learned slightly less than a second experimental group taught the whole-class situation. While attitudes in mathematics among the experimental groups were not significantly different, student reaction was more favourable to the laboratory setting than to the class setting. The group  $G_3$  in this study received their mathematics instruction (Math 201) separated from the methods instruction.

Wilkinson (1970), using an integrated approach, developed laboratory units to teach topics in metric geometry to sixth grade pupils. Analysis of data indicated that the students taught by the laboratory method did as well as on the geometry achievement post-test as students in the control class instructed by a conventional teacher-textbook approach. Wilkinson also reported that the laboratory approach did not significantly affect pupils' attitude toward mathematics but that the method appeared to be more effective with students of middle or low intelligence. Group  $G_2$  in the proposed study will use an integrated approach similar to this. Johnson (1970) in a year long study to determine the effectiveness of using activity-oriented lessons to teach number theory, geometry,



and measurement, and rational numbers in seventh grade mathematics found that (1) performance of students taught exclusively by the activity-approach was inferior to that of students receiving text-book-based or activity-enriched instruction, (2) laboratory lessons in the study of measurement and geometry were particularly effective for low and middle-ability students, and (3) attitude measures failed to reveal any significant differences among treatment groups.

Wasylyk (1970), using integrated program, organized a math-lab to teach measurement concepts and skills to low-ability ninth grade students. First the students worked in small groups using concrete materials, there were class discussions, problem laboratory sessions, each with a specific purpose. Results of the study indicated that the achievement of students in the laboratory group was significantly higher than those in the control group taught the same topics in teacher-directed setting. In addition, it was found that the students in the laboratory group exhibited significantly higher attitude towards mathematics than did control group. Students' preference for this laboratory method was strongly indicated. This seems to indicate a strong support for the use of this approach to disadvantaged students. The approach is almost identical with that of Group  $G_1$  in the proposed study. I see many of them as having a weak background.

Hollis (1971) reported a study which is noteworthy for the period of study, careful sampling and analysis. The investigator concluded that laboratories organized to provide personal and individualized assistance are most helpful to learners that are either academically or culturally disadvantaged. Wilderman and Krulik (1973), who have been described as the initiators and developers of mathematics laboratories at the university level and public schools in the United States, reported two studies carried out by Schippert (1965) and Howard (1970). Schippert found that the inner-city pupils who manipulated actual models or presentations of mathematical principles showed significantly higher achievement on measures of skills than did pupils taught by the discovery-oriented approach using verbal and written descriptions of those principles. Howard used mathematics laboratory experiences to facilitate a hierarchy of needed concepts with environmentally and academically disadvantaged rural children. Such experiences resulted in both achievement and attitudinal gains.

In summary, the research and evaluation literature suggests that laboratory approaches can be used practically and very effectively with culturally/environmentally/academically disadvantaged students. However, the degree of effectiveness of utilization depends on the organizations. Furthermore, laboratory approaches are not a

panacea, but appear to be an effective instructional methodology in a teacher's repertoire.

### Bloom's Model of Mastery Learning

Over the years educators have believed that only a few (between 20% and 30%) can learn, to any great extent, what the schools have to teach. One idea beginning to shape educational views and practices is mastery learning. The basic assumption is that all, or almost all, students can learn well (1) if instruction is systematically approached, (2) if students are provided with adequate help when and where they have learning difficulties, (3) if they are given sufficient time to achieve mastery, and (4) if there is some clear criterion of what constitutes mastery. This basic idea was emphasized by the Jesuit Schools before the seventeenth century, Comenius in the seventeenth century, Pestalozzi in the eighteenth century, Herbart in the nineteenth century, Washburne and his Winnetka Plan in the twentieth century and Morrison in 1920s and 30s (Washburne, 1922; Morrison, 1926). The idea did not resurface until the late 1950s and early 1960s as what is known as Programmed Instruction (Skinner, 1954; Suppes, 1966; Glasser, 1968; Atkinson, 1968). Bloom's model of Mastery learning (Bloom, 1968), developed from Carroll's model of school learning (Carroll, 1963), has proved to be very effective. It has been demonstrated in United States and Korea that the classroom procedures of this model is capable of making

75 percent of students achieve a high level (80% or higher) of mastery (Chung, et al., 1970; Bloom, 1968).

Theory:

Carroll's model (1963) is a conceptualized model based on five variables--aptitude for particular kinds of learning, quality of instruction, ability to understand instruction, perseverance, and opportunity. Carroll (1963) defines aptitude as the amount of time required by the learner to attain mastery of a learning task. Bloom (1971) argued that Carroll's definition of aptitude implies that given enough time, all students can conceivably attain mastery of a learning task and that if Carroll is right, then learning mastery is theoretically available to all, if we can find the means of helping every student. He supported Carroll's view of aptitude with the work of Glasser (1968), Atkinson (1967), Bloom (1964), and Hunt (1961). Carroll (1963) defines the quality of instruction in terms of the degree to which the presentation, explanation, and ordering of elements of the task to be learned approach the optimum for a given learner. Carroll (1963) explains how the ability to understand instruction interacts with the method and type of instruction. He defines the ability to understand instruction as the ability of the learner to understand the nature of the task he is to learn and the procedures he is to follow in learning of the task. Carroll defines perseverance as the time the

learner is willing to spend in learning. Finally, by "opportunity," Carroll means "time allowed for learning." Carroll (1963) combined the five variables to define the degree of learning of the  $i^{\text{th}}$  individual and the  $t^{\text{th}}$  task as a function of the ratio of the amount of time the individual (learner) actually spends on the learning task to the total amount of time he needs. He demonstrated (Carroll, 1962) that the numerator of this fraction will be equal to the smallest of the three quantities: opportunity, perseverance and aptitude (all defined in terms of time) and the denominator is the time needed to learn after adjustment for quality of instruction and ability to understand instruction.

Thus,

$$\text{Degree of learning} = f\left(\frac{\text{time allowed, perseverance, aptitude}}{\text{quality of instruction, ability to understand instruction}}\right)$$

The "function" is not used in mathematical sense. If the quality of instruction and the ability to understand instruction were optimal the time needed for instruction would be minimized. Implied in the definition of the degree of learning is the hypothesis:

Everybody can learn to mastery level if he spends the amount of time he needs to master the task.

Bloom's model (1968) is a transformation of Carroll's conceptual model into an effective working model. Having supported Carroll's view of aptitude with some

studies as discussed above, Bloom reasoned that if aptitudes were predictive of the rate at which, and not necessarily the level to which, a student could learn a given task, it should have been possible to fix the degree of learning expected of students at some mastery level and to systematically manipulate the relevant instructional variables in Carroll's model such that all or almost all students attained it. Bloom asserted that if students were normally distributed with respect to aptitude for a subject and if they were provided uniform instruction in terms of quality and learning time then achievement at the subject's completion would be normally distributed. Under such conditions the correlation between aptitude measured at the beginning of the instruction and achievement measured at the end of the instruction will be relatively high (typically about + .70). Conversely, if students are normally distributed with respect to aptitude, but the kind and quality of instruction and learning time allowed are made appropriate to the characteristics and needs of each learner, the majority of students will achieve mastery of the subject. The correlation between the aptitude and achievement should approach zero (Bloom, 1968). From Carroll's model Conant (1964) argued that the degree of success of a pupil faced with a task in school depends on four factors two of which reside in the individual and the other two stem from external condition. The two individual elements being the aptitude and perseverance while the

external element are the quality of instruction and the amount of time available. In Bloom's strategy an attempt is made to find ways of altering the time individual students need for learning as well as to find ways of providing whatever time is needed by each student. The strategy finds some way of solving the instructional problem as well as the school organizational problems, including that of time.

Practice:

The operating procedures for mastery learning are not static. Active research changes procedural strategies. The major operating procedures that have been found most useful in developing and carrying out mastery learning strategies will be described.

The success of the strategy rests on the acceptance of its basic assumption that almost all students can learn to a high level. The acceptance stimulates the teachers, administrators, as well as students. It provides a touchstone for the solution of most procedural problems encountered during a strategy's development and/or its implementation by searching for actions which are likely to promote the learning of all, not just some, students. Its acceptance also helps justify modification of grading policies and practices so that all students who attain mastery can be appropriately rewarded for their efforts.

Its cooperative instead of competitive effect will be discussed in chapter III.

Mastery learning approach produces best results in subjects possessing some and frequently all of the following characteristics:

1. Subjects that require either minimal prior learning or previous learning which most learners already possessed. For example, first grade arithmetic or a beginning algebra course. Clearly the success of learners in such a subject depends a great deal on the quality of instruction, and mastery approach provides instruction of optimal quality to each learner.
2. Subjects that are sequentially learned. Such subjects contained well-defined units whose learning is cumulative in that the learning of any units builds upon the learning of all prior units.
3. Subjects which tend to be closed and emphasize convergent thinking.

By "closed" subjects Bloom (1971) means subjects where there is a finite set of ideas and behaviours to be learned and about which there is a considerable agreement among curriculum makers and teachers. Examples of this are English, Mathematics, Sciences. Following Guilford (1959), Bloom (1971) defines subjects which emphasize convergent thinking as those in which students are taught to obtain



"right answers," or "good solutions" through "appropriate thought processes" or "accepted problem-solving modes."

Though mastery learning has demonstrated its relatively positive effects on subjects possessing the above characteristics (like early courses in Mathematics, English, Reading and Sciences), it has also worked for subjects like philosophy (Moore, Mahan, and Ritts, 1968) possessing other characteristics than those discussed above.

Basic in the practice is the problem of defining what is meant by mastery on the achievement (summative) test. Originally Bloom and his associates set the level required for a grade of A in a non-mastery class as the definition of mastery for the mastery classes. Recently, more objective empirical standard setting procedures have been developed (Block, 1970). The empirical work suggests that if students learn 80 to 85 percent of the skills in each unit, then they are likely to exhibit maximal positive cognitive and affective development as measured at the subject's completion. This work further suggests that besides being an unrealistic expectation in terms of student and teacher time and effort to require or even encourage students to learn all or nearly all (90 to 95%) of each unit it may have marked negative consequences for students interest in and attitudes toward the learning (Sherman, 1967; Bornmuth, 1969; Block, 1970). Milliman (1973) discussed five factors which can be considered in

setting a passing-score and test lengths on domain-referenced measures. This is very useful in designing summative tests.

Though summative evaluation can assess student achievement at the end of instruction, it cannot help guide the teaching-learning process. More central to the mastery learning strategies then is the development of feed-back and corrective procedures at various stages or parts of the learning process. While a variety of feedback processes like workbooks, quizzes, homework, etc., are possible, formative evaluation designed to be an integral part of the teaching-learning process has been most useful (Airasian, 1969). Such instruments are brief, diagnostic, and constructed to determine what each student had learned in a particular unit, chapter, or part of the course and what still needs to learn. The instruments not used to judge or guide the student but are of value in providing feedback to both student and teacher on what aspects or elements of learning unit still need to be mastered. The success or failure of mastery learning strategy depends on the degree of efficiency of these formative tests in pinpointing the learning needs of the student and supplementation of the original instruction (Bloom, et al., 1971). Since there are no known methods for going from a student's incorrect formative test responses to specific learning corrective he needs, a wide variety of instructional correctives is generally made available so that the student can discover

those best suited to his characteristics and needs. Such correctives are small group problem sessions, individual tutoring, and alternative learning materials (like alternative textbooks, workbooks, programmed instruction, audio-visual methods, tapes, academic games and puzzles and re-teaching).

#### Research Findings on Use of Mastery Learning Concepts and Strategies

The results from over 45 major studies carried under school conditions indicate that mastery learning has marked effects on student cognitive and affective development and their learning rates (Block, 1971; Peterson, 1972). Mastery learning procedure can enable four-fifths of students to reach a level of achievement which less than one-fifth attain under conventional, uniform, group-based instructional procedures. The additional time needed for this is 10 percent to 20 percent of the normal class time. The strategies seem to be especially effective for those students who typically have had problems in learning under ordinary instructional conditions. Research is repeatedly demonstrating that individual differences in achievement, time, or rate of learning is largely a function of the preparatory or prior instructional approaches and that for subjects where most of the students have achieved the pre-requisite learnings, mastery procedures appear to be able to almost eliminate the effects of individual differences on level

of achievement which indicates that under ideal conditions individual differences in school learning approach a vanishing point (Bloom, 1973). Mastery methods also produce markedly greater interest in and better attitudes toward the material learned than more conventional approaches. They seem to help most students overcome feelings of defeatism and passivism brought to learning. Their powerful affective consequences may be attributed to many factors, especially the cooperative rather than competitive learning conditions (Johnson and Johnson, 1974) personalized attention to each student's learning problem, successful and rewarding learning experiences and the use of certain correctives which add a personal-social aspect to the learning. Finally, mastery approaches of the earliest units in a school subject appears to facilitate the learning of the subsequent units, especially where the learning units are sequentially arranged (Block, 1970; Merril, Barton, and Wood, 1970).

#### Promising Innovative/Experimental Field Experiences in Teacher Education

Earlier in 1972 Hatfield acting for the National Council of Teachers of Mathematics Commission on Teacher Education requested over 100 mathematics teacher educators, teachers, and supervisors to identify a few of the major questions or issues they saw in mathematics teacher preparation. Here are six of the paraphrased report of their responses on some school based issues and problems:

1. There is a great push for more field experiences in teacher education today. I am sure at least certain aspects or kinds of field experiences are good. But if other institutions are like mine, their students too often get placed for field experiences in quite conventional, mediocre situations. Such experiences tend to develop teachers who adhere pretty much to the status quo; and they do not help much to develop innovative, original-type teachers. I think there is merit in some of the kinds of simulated experiences which have been developed over the past several years. Which ones and for what purposes are not really clear. There seems to be a diversity of opinion about them.
2. How can the student teaching experience be improved? Student teachers usually model their teaching after the critic teacher. This is fine when the critic is a superior teacher, but all too often teachers who are merely average serve as critic teachers, and it seems as if the cycle of mediocrity producing more mediocrity is perpetuated.
3. What structural changes can be instituted so that the mathematics educator working with preservice elementary education majors in a university-centered methods course can coordinate his efforts with the real life situations that exist out in the schools?
4. Providing early and somewhat consistent public school classroom involvement for preservice teachers.
5. How to facilitate interaction between school personnel and college staffs? Is it possible to get college teachers on a regular basis in the public schools? I surely think that school of education professors ought to be in that classroom teaching. We are planning on using a team approach for this but finding lots of opposition. It is not just in scheduling--many are simply afraid of a large class of unruly high school students with all the problems they have--especially in the inner city areas (Hatfield, 1972).
6. In spite of fewer jobs for mathematics teachers, there seems little evidence that the best people are being hired. Indeed, it seems that many people are still being hired to teach mathematics who are not qualified in any way. How can hiring practices be improved to help . . .

Though there are many rationales for the existing innovations in field experience, this report will focus on programs which use field experience, (1) as an exploration of teaching as a career (screening), (2) as part of method courses (and formation of style and philosophy), (3) in internships, and (4) to foster better understanding between the college and public school personnel with greatest emphasis on (2).

1. Classroom Experiences Before Admission to Teacher Education Programs.--Coupled with issues (2) and (4) above is the problem screening of future teachers in general. Conant (1963) believed that future teachers should be selected from the top 30 percent of high school graduates by 1974 and suggested a high school program that would help such prospective teachers study with profit and without an excessive demand on time and energy of the program. This is a way of screening for better academic preparation of teachers, if implemented. Analogous to this is screening for professional preparation of teachers. Many young men and women do not discover until their senior year that they could have concentrated on teaching in the elementary or the secondary school, or, do not go into teaching programs at all. Dougherty (1973) reported a program which provides experiences that will detect persons who will not be happy or successful in teaching. The Michigan State University has also initiated a selection procedure, prior

to admission to teacher education, that is helpful in this direction. The most important criterion used in this selection process is demonstrated ability, on the part of the applicant, to work effectively with children. Prior to admission to the program, each student is required to spend a minimum of 60 hours in a public school classroom service as a teacher's aide. An evaluation of his personal qualities in relation to the expectations of the teacher role, with special emphasis on the prospective teacher's ability to relate to youngsters, is made by both the public school teacher with whom the student has worked and the university representative who has observed him as a teacher aide.

The EXCEL project of Teachers College, Ball State University, Muncie, Indiana reported by the American Association of Colleges for Teacher Education (1973) also involves the student with classroom activities from his freshmen through his senior year. This allows him ample time to decide if teaching would be a suitable career for him. This project won the distinguished achievement in 1973. The Association (1974) also reported on the Indiana University Southeast, New Albany Teacher Education program which has early and continuing field experiences incorporated into the program.

2. Classroom Experiences as Part of Method Courses.---  
Del Popolo (1970) observed often the student teacher arrives upon the student teaching scene completely

unheralded; amidst the curious glances of pupils and staff alike, he gropingly attempts to find his classroom in a maze of corridors.

In attempt to alleviate this problem which other educators have also observed, Anderson and Boop (1972) reported a restructured secondary professional semester of student teaching of Butler University in Indianapolis which used to be essentially the same as the Michigan State University's conventional student teaching program. The principal change within the professional semester is the introduction of twelve full days of observation organized into five blocks of time with the sequential number of days within each progressive block being increased to provide more gradual introduction into school student teaching. The intent is to provide more gradual continuity between college classroom theory and application in the public school assignment by providing frames of reference to which the students could relate during the general methods segment of classroom work in the university classroom. The calendar for the professional semester has this form:



Week	Monday	Tuesday	Wednesday	Thursday	Friday
1	College Classroom				
2			College Classroom		
3	College Classroom		College Classroom		Public Schools Obs. #1
4	Public Schools Obs. #2	College Classroom		College Classroom	
5	College Classroom			Public Schools Obs. #3 and #4	
6	Public Schools Paraprofessional Activ.		College Classroom		
7	Public Schools Limited Participation			College Classroom	
8	College Classroom		Public Schools EXTENSIVE PARTICIPATION		
9-16	Full Time Student Teaching				

An evaluation of data, using Edwards Personal Preference Schedule, Minnesota Teacher Attitude Inventory and other constructed opinionnaires, revealed:

- a. A substantial increase in knowledge of students by the student teachers.
- b. A substantially greater professional involvement of the student teachers.
- c. A greater continuity of school and college experiences.
- d. A better provision for cooperation in supervisory activities between the university supervision and cooperating teachers (Anderson and Boop, 1972).

This design has been in existence for two years and the original findings have been substantiated, confirming that the results were not just due to Hawthorne's effect. Availability of a sufficient number of schools around (close enough) to the university is a necessary condition for the implementation of this program.

The Ohio State University undergraduate program in Science and Mathematics Education is one which involves teaching experiences in the public schools throughout the undergraduate student's junior and senior years (Blosser, 1972). It was first initiated in 1968-69 after the faculty and students have labelled the methods courses as lacking reality in that they are taught in a theoretical frame-work, most times, totally divorced from children and practical experiences. Its modifications in 1969-70, and 1970-71 were based on evaluations by students, instructors, observers, and formal research studies (Blosser, 1972). Science and mathematics undergraduates are actively involved in teaching at elementary, junior high school and senior high school levels during five quarters of their junior and senior years in college.

Learning about pupils as individuals is the emphasis of the program during the three quarters of junior year. This is accomplished by spending two half-days per week on one-to-one tutorial then broadened to that of the individual as a member of a small group in the second quarter of the junior year. They spend other times in

method-courses. They spend the third quarter on either mathematics laboratory course or laboratory activities in science education. The first quarter of the senior year is divided in time between two schools in contrasting context (inner-city, suburban) and the students are involved in a half-day teaching assignments, five days a week, for the quarter. In the second quarter the students participate in a full day teaching assignment in a single school, one of the two had in preceding quarter.

Staropoli and Heitzmann (1973) reported a similar program being implemented at University of Delaware, Newark for eight method courses.

Lancaster (1973) reported a program at Emerson College, Boston based on the rationale of a counterpoint theory that direct experience gives relevance to theory and that theory in turn gives meaning to experience. The program reflects an attempt to breakdown the learning into more manageable pieces. Students are exposed to teaching experiences as student aides from freshman to senior year in the Veterans Memorial Elementary School, a non-graded school which utilizes teaching-teams. They have formal teaching practicum in the junior year. The program has many things in common with those already described. The first cycle of it was just completed in 1973.

Sowell and Hodgins (1972) reported on a "Head Start" program in student teaching in mathematics. A more subjective evaluation procedure was used. Balka (1974)

reported on early experiences in the teaching of secondary school mathematics in which a survey of the attitudes and comments of the under-graduates preservice mathematics teachers response was highly positive.

Colles and Pagni (1973) reported a study which was a field-based method course which they designed because they believed that the methods taught in methods courses prior to theirs did not carry over to actual teaching and this has resulted in frustration, discouragement and eventual loss of a potentially good teacher from the profession. Their program is very similar to those describe above. They carried out an evaluation which utilizes a comprehensive feedback system containing three elements: (1) encourage high school students to react, which provides insight on the prospective teacher's behavior, (2) video-taped lessons were reviewed by the instructor and the student, which enabled the student to see his role in definition with his perspectives, and (3) modified Flanders Interaction Analysis category was used to obtain objective data about the teaching behaviors as related to elements of the student's personality.

Despite its obvious value and increasing reports of its observance, there are limited objective data in the educational literature about the effects of and reactions to early field experience for education students prior to student teaching. A study which represents a step toward objective appraisal of students' and cooperating teachers'

reactions to one pre-student teaching program of methods courses combined with field experiences was reported by Gantt and Davey (1973). The study involved 40 junior under-graduate elementary majors in the University of Maryland's College of Education, 5 methods faculty-members, and 18 teachers at 2 elementary schools. Four out of the 29 three-hour sessions were devoted to elementary school classroom experience. The announced objective of the school-based phase of the methods course was:

To provide observational and teaching experience for the student in elementary language arts, reading, and social studies: emphasis should not be focused on the requirement of teaching skills, but rather on familiarization with pupil and behaviour.

Gantt and Davey (1973) utilized a three-part evaluative form in this study: Part A reported how confident the student felt about his ability to apply ideas stressed in methods course to his forthcoming student teaching experience; part B measured the extent to which the field experience was perceived as a valuable part of methods course; part C offered the student opportunity to comment fully on any of his reactions to the program. It was reported that feelings of confidence about readiness for student teaching were expressed by students as a consequence of the combination of theoretical input, direct field experiences, and critical group discussions and a strong mandate in support of the pre-student teaching field-experience seems apparent.

3. Internships.--Two programs have been selected to illustrate this experience--one is described in detail while other is documented.

The Michigan State University in cooperation with over sixty different public school systems in Michigan State have established the Elementary Intern Program (EIP). Students preparing for elementary school teaching at Michigan State University are given a choice at the end of their sophomore year of either following the traditional on-campus route or enrolling in the EIP for their final two years of preparation. The student choosing EIP attends a ten-week summer session at Michigan State University during his sophomore and junior years. From September-March, he moves to an off-campus internship center. Elementary school teaching methods and student teaching are integrated during this six-month period. He returns to the campus for spring quarter and a five-week summer session which follows. During this time, he completes his work toward general education requirements and major and minor requirements in liberal arts areas.

During his fourth calendar year of study, the student becomes an intern teacher with a salary of approximately \$5,000 per year with the responsibility for a classroom. During this year, he is supervised by the intern consultant who has as a full-time assignment working with about five or six interns. One evening a week is spent in classroom study and at the end of the year, the

student qualifies for a bachelor's degree and a teaching certificate. The help provided for these beginning teachers is built into the program on a self-financing basis. Although intern teachers are not paid full salaries, the school system pays the same amount for their services as they would for a regularly certified beginning teacher. The salaries of the intern consultants are paid from the difference between the amount paid by the school district and the salary paid to each intern.

In each center, a member of the university faculty serves as program director and is permanently based in the off-campus center. He acts as general program coordinator, teaches some of the elementary methods courses, coordinates the student teaching experiences, and supervises the work of the intern consultants. Campus-based faculty share the responsibility for the methods instruction. Cooperating local districts furnish all necessary physical facilities such as office and classroom space, including utilities.

Since its inception, Michigan State University has engaged in a continuous evaluation of its Internship Program through the use of systematic depth interviews of the students themselves and of those working most closely with them. Some of the advantages which have been discovered in this approach to teacher preparation are as follows:

1. Educational theory and practice can be integrated much more easily. Methods courses are taught while

students are spending part of their time in the environment of the public school and thus rich opportunities for the immediate transfer of formal instruction in pedagogy to work in the classroom is possible.

2. This program makes possible frequent evaluations of the student, using as the basis for decisions and retention in the program evidence of growth in the student's ability to work effectively with children.
3. EIP's major contribution has been its development of a new dimension in teacher preparation, the intern consultant position. Instead of expecting a beginning teacher to perform well all of the tasks undertaken by an experienced teacher, the EIP student receives continuing individualized guidance when he assumes responsibility for a classroom. His introduction to teaching thus, is gradual and carefully directed.

The intern consultants, selected from among the most able teachers in the cooperating school districts, have developed in-service education of new teachers far beyond the initial expectation. The low ratio of interns to intern consultants and the closeness and continuity of the relationship over time has made it possible for very specific help to be offered and accepted. Most importantly, the consultants have helped "bridge the gap"



between the college course work and the public school classroom by helping the intern to relate theory and practice.

The active participation of public school staff members in certain phases of the program tends to guarantee a realism and practicality which may sometimes be lost as college professors work in isolation. At the present time, approximately 40 percent of Michigan State University elementary education majors are enrolled in the EIP Program.

Stiles (1973) reported that the entire state of Wisconsin with its State Department of Education, institutions of higher learning and public schools, has put into practice the intern-in-team plan. A key feature is the responsibility taken by school systems for organizing and supervising the clinical experience of prospective teachers. This method allows the schools (where the action is) to best determine what kinds of experiences a student teacher should be aware of in order to maximize his potential to become a teacher. Also, it gives the intern a chance to implement appropriate theory under a supervised situation; therefore, he minimizes the risk of being out in left field. "We have learned, for example, that experience for experience's sake may not be the most effective way to prepare teachers," observed Stiles (1973).

4. Public School Experiences for University Faculty Members.--In the last few years, Michigan State University

has made serious attempts to involve its College of Education faculty in significant teaching experiences in the public schools. It was particularly concerned that professional courses at the university level are often staffed by personnel who have not had recent experiences in inner-city classrooms.

Participants in Operation REFUEL (relevant experiences for urban educational leaders) serve on one of four instructional teams at the Allen Street School in Lansing, Michigan. Each team consists of two Lansing teachers, one Michigan State University professor, one or two graduate interns, and two to four student teachers. Each is responsible for the instruction of approximately fifty elementary students. The Michigan State University staff member is a team member half-time for twelve weeks. His role in the classroom is in the area of his specialty and involves active participation with children. Although his primary function is classroom instruction, a Michigan State University professor may be asked to consult with members of other instructional teams in his speciality area.

Similar secondary school opportunities are offered to Michigan State University faculty at Pattengill Junior High and Eastern High School in Lansing as a part of our TTT Project.

In the first three years, approximately sixty university faculty members were engaged in the direct instruction of pupils in the public schools. The

participating professors indicated that their experience in the schools helped them to improve their methods course teaching. It helped each to freshen his memory regarding the day-to-day difficulties encountered in public school teaching. It also helped him gain creditability among college students by his willingness to put his ideas "on the line" in a real classroom.

Most of the programs reported, especially under (2), are based on the assumption that laboratory-field-experience supplemented methods courses enables a student to translate theory into practice, thereby overcoming the inadequacies of a sterile, theory-based, non-participation methods course. A strong mandate, both from instructors of methods courses and from students, in support of the pre-student teaching field experience seems apparent. While a program of this type adds to the planning time of instructors, the overall effects definitely pay-off the extra effort involved. It also promotes the students' confidence about their potential functioning in student teaching. However, there is a danger if sufficient time is not allowed for in-class discussion and demonstration of methods and materials before sending the students to classrooms.

Selected Research and Evaluation Literature  
on Related (Research) Methodology  
of the Study

Herriot (1967) reported an exploratory study in which below average 7 and 9 grades students did as well as

(in some cases better than) the above average students in algebra covered by the first group (below average) in two years and by above average students in one year. The study seemed to support Carroll's hypothesis (1963). Carroll's hypothesis:

All, or almost all, students could be brought to the same level of achievement in any particular scholastic topic, but the amount of instruction that would be needed to bring a student to a particular level of achievement would vary from student to student.

Begle (1971) reported a similar study in which fourth grade classes were taught the same content dealing with base five numeration under three different plans devised to cover the same material in one class period, two class periods, and three class periods respectively thereby containing different increasing amount of teaching techniques, review of related materials, practice time and all the three treatments given to each group of students already classified as low, middle, and high ability using pretest of reasoning. Again the results are in accord with Carroll's hypothesis.

Bloom (1971) emphasized domain/criterion-referenced measured with high content-validity are needed for evaluation of instruction/curriculum. Milliman (1973) reviewed procedures for establishing standards and determining the number of items needed in "criterion-referenced" measures. He organized the discussion and procedure of setting a

passing score around five factors--performance of others, item content, educational consequences, psychological and financial costs and errors due to guessing and item sampling and believed that they require judgment. Classical test theory, binomial, and sequential models for determining test length were considered, the first was not viewed as useful, and the last was judged as most feasible when examiners interact with computers during testing.

Walbesser and Carter (1968) discussed the importance of defining curriculum by developing a sequenced set of instructional objectives while Atkin (1968) warned that the educational relevance of curriculum might be reduced by strict adherence to specification of behavioral outcomes of instructional activities. Bloom (1956), Stanley (1967), and Scriven (1967) all discussed the methodology of and distinctions between formative and summative evaluations. Recent writers like Reynolds and Light (1971), Abedor (1971), Tate (1971), Westbury (1970), and Weiss (1971) have noted the ambiguity in the definition of the term, formative evaluation, and the consequent paucity of well-defined procedures and techniques for conducting such evaluation. Technical terms in the study are used in Sanders and Cunningham's sense. Metfessel and Michael (1969) discussed methods used to collect external information for interim formative evaluation. Borich (1971) suggested a conceptual model for formative product evaluation. Contextual information is of utmost importance for formative product

evaluation emphasized Sanders and Cunningham (1973). Stufflebeam et al. (1971) discussed in detail forces that act upon programs designed to meet the needs of students in illustrative example of what they called "Contextual source of Information." Stufflebeam et al. (1971) also discussed steps taken to collect information for different kinds of evaluation. In Bloom et al. (1971), Wilson illustrated what Sanders and Cunningham termed Interim (Process) formative evaluation using internal source of information (which Interim intrinsic evaluation, following Scriven, 1967), on pp. 690-92. Collection of such information for all the units of a program put together (pp. 646-47) is what Sanders and Cunningham called "formative product evaluation" from internal source if used strictly for feedback of the developer but it will be a summative evaluation if used as consumer report-type of appraisal.

Hunsen (1967) showed that throughout the world student achievement in mathematics is related to parents' education socio-economic status. He suggested further multivariate studies of test scores and background factors affecting performance in mathematics.

On the relationship between mathematics achievement and attitude toward mathematics Neale (1969) wrote an article based on Cattell and Butcher's (1968) findings. Neale was of the opinion that improvement of attitude toward mathematics may not increase mathematics achievement. Aiken (1970) was of the opinion that Neale made too

definitive conclusions more than warranted from correlational studies. Aiken (1970) said the apparent contradictory/non-conclusive results on relationship between mathematics achievement and attitude is due to mis-use of attitude toward math. Aiken (1972) claimed that the Liker-like instrument is more reliable in high school and college than other types of instruments and defended his position. He reaffirmed his belief that improving teachers attitude towards mathematics can result in more positive attitude on the part of the students. He then recommended that attitude towards different aspects of mathematics will be more meaningful than just a single measure of attitude toward mathematics. Aiken (1972) reported a study carried out on three age groups (also classified by sex). His study shows that (a) there is a general variable of attitude toward mathematics that includes attitude toward routine computations, terms, symbols and word problems, (b) attitude toward mathematics is directly related to interest in problem-solving tasks in general, but inversely related to interest in language arts, social studies, and other "verbal" pursuits, (c) people with more positive attitude toward math tend to like detailed work and see themselves more perceiving or self-confident; they also tend to make higher marks in math and in school work in general, and (d) although there are age and sex differences in this regard, the reported attitude and achievement of the father (particularly in the case of male) and that of

mother (particularly in the case of female) are also associated with students' attitude toward math.

While an often stated objective in the preparation of elementary teachers of mathematics is "the development in these prospective teachers of favourable attitude toward mathematics" will influence students' attitude and achievement in mathematics, Peskin (1964) found no significant relation between teacher attitude and student attitude nor between teacher attitude and student achievement. Phillip (1973) claimed that Peskin's findings were consequential to his design. He then reported a study which showed that students achieve better in arithmetic if they had sequence of three teachers, all of whom had positive attitude toward arithmetic than if they had a sequence of three teachers having unfavourable attitude toward arithmetic. His study further show that type of teacher attitude toward arithmetic, student attitude toward arithmetic, and student intelligence do not interact in any way such as to produce a significant student achievement in arithmetic. This study shows that teachers' attitude toward arithmetic does not have significant effect on student's attitude and achievement unless student-teacher interaction lasts for a sufficiently long period, three years, say. Knaupp (1973) also reviewed literature on causal relationship between attitude and achievement in mathematics, he observed, like Aiken (1970), that in most of these studies instruments,



designs, method of analysis are defective and these account for apparently contradictory/inconclusive findings.

Todd (1966), Reys and Delon (1968) showed that prospective elementary school teachers who completed basic mathematics comparable to CUPM Level I recommendations demonstrated significant gains on their scores on the Dutton scale. Hunker and Quest (1972) noted that in none of the two cited studies was a comparison made between those students who had completed the course. Poffenberger and Norton (1956) found that teachers who,

1. display a strong interest in subject,
2. indicate a desire to have students understand the material, and
3. display a good control of the class without being overly strict tend to affect students' attitudes and achievement positively.

Noting this finding of Poffenberger and Norton, Hunker and Quast (1972) reported a study which compared attitude of prospective elementary school teachers,

- a. who have taken neither mathematics content nor mathematics-method course,
- b. who have taken mathematics content but not math-method course,
- c. who have taken both mathematics content and math-method, the latter being taught by an instructor who displayed Poffenberger-Norton characteristics.

The results of their study show,

1. the math-method designed for the prospective teachers did improve their mathematics attitude,
2. the math-content together with math-method courses, can probably be used to improve the mathematics attitude of prospective elementary school teachers.

Alexander, et al. (1971) reported a study in which the past grade points were used to classify students (by ranking) in High GPA and Low GPA. They were then randomly assigned to two treatments, I, II, in which Ss in I were politely treated by the instructor (but with no personal interest) while Ss in II were referred to by name and the instructor initiated discussion with them. The investigators reported that the teacher-initiated teacher-student personal inter-actions did significantly influence the achievement in favour of Ss II. They also found that Ss with High GPA achieved significantly higher scores than Ss with low GPA and contrary to previous study, by Means, et al. (1970) concerned with GPA the interaction was not significant although in the same direction as in the previous study. The investigators observed that as the semester progressed Ss II increasingly initiated interaction with the instructor.

Begle (1972) reported a study which sought the relationship between teachers understanding of modern algebra and their student achievement in ninth grade algebra. They found that:

1. the pretests given the students (math inventory from NLSMA and Reference Test for Cognitive Factors) turned out to be good predictors of success,
2. there was substantial variations in the effectiveness of teachers,
3. teachers effectiveness with male students was not significantly different from teachers effectiveness on female students,

4. teachers understanding of modern algebra has no significant correlation with student achievement in ninth grade algebra,
5. teachers understanding of the algebra of real numbers has no significant correlation with ninth grade algebraic skills,
6. teachers understanding of algebra of real numbers is significantly correlated with students achievement in understanding algebraic concepts but the correlation is so low that it is educationally insignificant.

CHAPTER III  
DESCRIPTIVE FEATURES AND DESIGN  
OF THE STUDY

Introduction

This chapter summarizes the evaluation activities carried out in the present study. It is presented in two parts.

The first part involves an appraisal of the mathematics education component of the program with reference to the content, goals, grading procedures, etc. This approach has been termed "intrinsic" evaluation. The criteria are usually not operationally formulated, it is, to some extent, an armchair affair (Scriven, 1967). It relies heavily on the (program) internal source of information. It provides information about the rationale, goals, and objectives of the program which contribute to an understanding of value positions taken by the developers and other persons involved in the program (Sanders and Cunningham, 1973). This is useful because of the use "objective" data alone is insufficient in the evaluation of learning under dissimilar systems of instruction (Brownell,

1966). Evaluation activities in the study under this approach are;

The general context and program description: An analysis of the mathematics content in the mathematics education component of the program;

A description of the mathematics method integrated with the mathematics content and clinical experience;

A critical appraisal of the instructional method.

They form what Wittrock (1968) described as evaluation of environments of learning.

The second part describes the procedure for examining the effects of the mathematics education component of the program on the interns. This approach has been termed "pay off" evaluation (Scriven, 1967). It involves the evaluation of learning, the evaluation of learners and the evaluation of instruction (Wittrock, 1968). The activities includes appraisal of the differences between pre- and post-tests, and between experimental and control groups tests on a number of criterial parameters. The evaluation relies heavily on external (samples) and contextual (entry behavior) sources. The samples, measures, research design, hypotheses tested, methodological assumptions and limitations are described.

## General Context and Program Description

### Its Origin

The Teacher Corps Program is a United States federally initiated reform effort created during the 1960s to improve the welfare of low-income people. The original purpose of the program as provided in authorizing legislation, were:

1. To strengthen educational opportunities for children in areas with concentrations of low-income families.
2. To attract and prepare persons to become teachers in such areas through coordinated work-study experiences.
3. To encourage colleges and universities, schools, and state departments of education to work together to broaden and improve teacher-education programs (Corwin, 1973).

The Corps resulted from the premise that there are critical differences between the skills required to teach in low-income schools and middle-class schools. But this premise does not imply that unique principles of learning are involved in the two different settings. It is believed that the differences in values, prior experiences, and environments among children from various income, ethnic, and racial subgroups are so great that the teachers need special training in order to apply the principles and fashion the procedures for each group.

The typical Corps program involves from 30 to 40 liberal arts graduates (interns) and five professional teachers who act as team leaders. The group receives about eight weeks of special preservice training at a college or university, after which it is divided into five teams, each composed of at least six interns and one team leader. Each team is assigned to a school that serve a poverty area, usually an elementary school, where the team spends at least 60 percent of its weekly time. In the beginning the team may work with small groups of students on specific lesson plans but, as the team gains experience, its tasks become more complex. It spends about 20 percent of its time in academic work at the university (some of this work is interdisciplinary and leads to teacher certification and a master's degree in two years). Finally, the interns also are expected to spend 20 percent of their time on community activities, learning as much as they can about the environment of their students.

Several thousand interns and experienced teachers have graduated from the program since the first cycle began in 1966. Corps teams serve in from 30 to 70 universities at one time, but they have served in more than 100 universities and 250 school systems in 37 states and Puerto Rico at various times. About half of the programs are in city school systems, including seventeen large cities, and about half are in small towns and rural areas. There have been programs in New York, Chicago, Detroit, Philadelphia,

Los Angeles, Kansas City, Miami, Atlanta, Seattle, and Dallas; in Appalachian towns, in the Ozarks, and in the rural South; in migrant communities, in Indian schools; and in Spanish-speaking communities in New York, Florida, and the Southwest.

The eighth cycle Teacher Corps program whose mathematics education component is being evaluated is cooperatively implemented by the Lansing School District, the Michigan State University, the Model Cities and the community. The Teacher Corps Advisory Board consists of representatives of these bodies.

### Rationale

In its effort to recruit teachers capable of working with disadvantaged children, the Lansing School District has endeavored to recruit teachers from ethnic groups representative of the student population. However, it has been realized that all teachers need training in working with disadvantaged children and in developing programs which benefit these and the other pupils of the district. Training of these teachers should be competency-based (in both the academic areas of the curriculum and in student attitudes), community-based, bilingual and bicultural (Lansing School District and Michigan State University, 1972).

A fundamental purpose of the Lansing School system is to sustain and nourish free society through transmission



of cultural and political heritage to children and youth. Recognizing that the free society is still the exceptional society, the schools plan to develop in pupils the necessary creative talent and intellectual vigor. Traditional teaching methods and personnel have difficulty realizing these educational objectives with many elements of the disadvantaged community, nor are they particularly suitable for teaching all children who will live and work in the year 2000.

Michigan State University has had a long-standing interest in developing competency-based teacher education. It has been heavily engaged in the Training of Teacher Educator Projects as well as in other programs which are competency-based or based on behavioral objectives.

Michigan State University and the Lansing School District have jointly sponsored a short term teacher education program which was oriented toward developing Clinical Teaching Strategies in reading and mathematics. Courses were offered on the undergraduate and graduate level in competency-based education. These courses were a joint offering of the College of Urban Development and the College of Education. New staffing patterns have been explored cooperatively by the university and the school system utilizing workshops and seminars on differentiated staffing and the open classroom.

The College of Education has established a council with members from all departments and programs involved in

competency-based education. The council's task was to coordinate efforts and disseminate information about the college's programs in this area. Finally, the College of Urban Development has, through the Project Development Specialist in the sixth cycle (who was a faculty member with the College), made a commitment to the development of competency-based programs as another viable instructional mode. Much of the curriculum being planned for the Urban Education component of the College would be competency-based.

Community based education and involvement, particularly that of parents of school children, was a central feature of the Teacher Corps strategy which was facilitated through the collaborative decision-making process outlined in the proposal. Developmental Community involvement programs of any kind, to be completely successful, require certain changes in the expressed attitudes and behavior of the people within the community concerned, otherwise no such program need be contemplated. It was realized that these changes could not be dictated by a few leaders within or outside the Teacher Corps structure operating at the city, county, or regional level. They could best be achieved when they are the result of the deliberations of the people working out their own problems in primary groups at the local level.

The community education component aimed at developing institutional understanding of the community

and community understanding of the institutions. This component functioned in a dual pattern. The individuals participating in this program were not only instructors but also listeners. The community had an opportunity to interact through seminars so that institutions had a clearer and better perspective of the community it was serving.

### Objectives

Teacher Corps Training Objectives for Interns, Cooperating Teachers, Team Leaders, and Community Volunteers:

1. To maintain and develop competency-based instructional programs which result in positive achievement of pupils in the Lansing School District with particular reference to children in grades Kindergarten-six from low-income families. At the conclusion of the program the participants should be able to meet the following objectives at a competency level established by the school district.
  - a. to use assessment and observational skills to diagnose learning strengths and weaknesses of all pupils including those with learning disabilities.
  - b. to use diagnostic data in developing behavioral objectives which speak to the needs of each pupil and to design effective strategies to attain those objectives.

- c. to evaluate pupil growth toward important educational objectives and plan curriculum revisions based on such evaluations.
- d. to equip the participants with the skills necessary to be able to assess his/her impact on students and to modify that impact by modifying their instructional approaches.
- e. to identify linguistic problems and conduct a language or dialectically adjusted reading skill developmental program so that children can improve their reading level.
- f. to conduct an inquiry-oriented multi-disciplinary program which, organized cross-culturally, will equip children to live in a pluralistic society.
- g. to conduct a mathematics program which emphasizes visual conceptualization of mathematical constructs without losing proficiency in computation, and relates mathematics lessons to the experiences of low-income children.
- h. to examine interpersonal relationships between staff members which impede the teaching-learning process.
- i. to effectively integrate instructional media with instructional modules developed in the previous cycle.

- j. to develop and involve community resources as integral parts of the teacher-learning process; and
  - k. to interpret the school's instructional program to the community and to encourage parents and patrons of the district to take part in the ongoing evaluations of goals and objectives within the framework established by the Board of Education.
2. To continually modify instructional programs to meet the changing educational needs of the students and to take advantage of new materials, techniques, and technology.
- a. to design curriculum models which relate to the multi-ethnic population of the Lansing School District and which provide ways for the development of positive self-images among low-income students,
  - b. to develop the use of video recorders and other related equipment as part of the instructional programs,
  - c. to move beyond the traditional team teaching model to a differentiated staff that would include community resource persons, professional and paraprofessional educators,

- d. to use modular design in development of training procedures and curriculum materials which permit individualized instruction.
3. To understand and use the "tasks of teaching model." This model includes the following:
- a. To assess the "givens" present in the unique instructional situation. This involves data-gathering, data-analysis, communication, and decision-making skills.
  - b. To set the goal by specifying the intended changes in student behaviour. This involves goal-identification, objective-specification, communications and/or negotiation skills.
  - c. To select, prepare, and implement strategies for producing intended changes. This involves decision-making, preparation and implementation skills.
  - d. To design, prepare, and implement evaluation instrument and procedures. This involves decision-making, data-analysis, and communication skills (Henderson, 1973).

Lansing School and Community Involvement:

To assess and articulate with the community those explicit and/or implicit needs that have been identified and to cooperate in implementing competency-based

educational programs which meet these needs and eventually eliminate them.

- a. to actively recruit interns, team leaders, and cooperating teachers primarily from, but not limited to, the Lansing area, for the Eighth-Cycle program.
- b. to educate parents and community residents to the concepts of competency-based education.
- c. to develop the mechanism through which participating parents and community residents can interact with principals, cooperating teachers, team leaders, and interns to facilitate genuine community-based education and involvement.
- d. to identify and recruit parents and community residents for the Lansing Teacher Corps.
- e. to organize specific parent and community residents at each participating school to address and articulate their unique problems.
- f. to organize parent and community residents at participating schools whose children are bused to non-neighborhood schools.
- g. to devise and structure an educational vehicle within the Teacher Corps framework that addresses itself to the communities and will upon termination of Teacher Corps involvement continue to function in a viable manner.

- h. to promote awareness of existing educational programs among instructional aides and cooperate with the Lansing School District in providing career mobility opportunities for low-income community persons.
- i. to establish a viable working relationship with various community groups which will facilitate community participation at all levels of decision-making of the program--i.e., needs assessment, program development, evaluation.

University Objectives:

- 1. To continue development of innovative competency-based teacher training programs which will result in effective teaching and learning among all pupils.
  - a. to systematically test and demonstrate components of the Teacher Corps training program which can become the basic units of an innovative competency-based teacher training program.
  - b. to provide time and support for university faculty to develop training modules for innovative competency-based teacher preparation programs.



- c. to develop empirical research and evaluation components which support innovative competency-based teaching training programs.
- d. to provide an improved competency-based teacher instructional program with emphasis on application in the field.
- e. to establish integrated competency-based teacher training sequences which involve other colleges in the University.
- f. to develop a competency-based bi-lingual, bi-cultural course which will become a part of the required experiences in the College of Education.
- g. to establish a process by which teacher training may be constantly monitored through community input so that programs reflect the changing needs of the community.
- h. to provide field settings where pre-service teachers can experience a variety of instructional, organizational patterns, e.g., team teaching, differentiated staffing, individualized instruction.
- i. to continue the development of competency-based teacher education toward teacher certification based on field demonstrated competencies.
- j. to develop a process which insures that teacher training programs involve school personnel in designing objectives and training strategies.

Teacher Education Program  
Philosophy

The Lansing Teacher Corps project in conjunction with the College of Education at Michigan State University has specifically focused its attention on the preparation of teachers who will have the competencies and sensitivity to meet the needs of low-income area children, including the needs of children who are considered culturally different.

It is no longer acceptable to permit prospective teachers to accumulate credit on the basis of attendance. It is imperative that student performance be tied in directly to the competencies and performances of the instructor; therefore, a prime objective of this program will be to strengthen the type of teacher training programs currently being offered at the university. Realizing that such a program cannot be administered in isolation, the Lansing School District, Michigan State University, Model Cities, Lansing Community College, and the State of Michigan Department of Education will pull together to develop guidelines that will provide for more flexible teacher training programs. The primary concerns at the consortium will be program formulation, implementation, and evaluation of teacher training as it specifically relates to the education of children in low-income areas.

A great deal was learned from the sixth cycle project and a continued effort to develop and integrate into

the system those aspects of the program that have provided for individual development in the areas of self-discipline, critical thinking, effective communication, and creativity will be made. Past experiences with the home, the classroom, the university, and the community indicate that a viable teacher training program must address itself to each of these factors.

Our society, as a whole, is undergoing dramatic changes; therefore, it is our responsibility as educators to explore new ways of responding to these changes as they affect our children's lives as well as our own. More specifically, educational institutions must take a leadership role in giving direction to these social changes. In dealing with these changes, flexibility, not stability, will be the most important catalyst in our teacher training program.

#### Competency-Based Education Philosophy

In the competency-based model of education, learning is defined in terms of planned behavior change. If a teacher is to postulate that learning has taken place, it must be objectively demonstrated that behavioral change, in the desired direction, has been manifested. The role of subjectivity, expectations, and value-based interpretations is minimized in determining existing learning, planning and implementing programs designed to teach new learning and evaluating achievement of learning objectives.

Further, the teacher is perceived as the manager of the learning environment, responsible and accountable for its condition and events arising therein. It is the teacher who must structure the milieu such that a predictable relationship between the student's classroom performance and the classroom environment is established. The traditional notion plaguing urban schools, which bases teaching methods and learning programs on the assumption that students, given an "average environment" will learn when they are "ready" is counterproductive to educational achievement. Too often in the urban setting, reference is made to undefined mechanisms called "intelligence" or "genetic endowment" if students fail to exhibit what is nebulously defined as learning, when in fact no adequate, explicit provision has been made for learning to take place.

In the competency-based model, faulty learning is perceived to be a product of the classroom environment rather than a product of postulated incompetencies and incapacities of a faulty student. This contrasts with the paradoxical, latent assumption implicit and prevailing in education that the schools cannot basically teach due to the fixed effects of genetic or early family experiences. This restrictive and unfounded notion is the converse of a basic tenet in competency-based education holding that behaviour not only can be, but is modified--for better or worse--in the school room, and that such behaviour change

(learning) may be positively accomplished through exposure to efficient and effective learning environments. Barring severe physiological impairment, no student can legitimately be deemed to be limited in what and how much he can learn. The competency-based model then does not assume that the educational achievement of students is determined by some relatively constant level of abilities, aptitudes, and characteristics identified, classified, and labeled by tests or perceptions of subjective judgment by the evaluator. Rather it perceives that the majority of overt behaviour is environmentally determined and subject to change. To say that behaviour is determined and fixed by forces beyond the school's influence is contrary to the principle thesis delineating the competency-based model of education.

#### Competency-Based Management of the Learning Environment

The competency-based trained teacher used behaviour management techniques to structure an appropriate learning environment and construct relevant learning strategies designed to promote planned behaviour change. Usually the change program is intended to promote the acquisition and maintenance of behaviours compatible with educational achievement. Learning activities may also be designed however, to prevent, decelerate, or eliminate behaviours incompatible with learning objectives of the classroom. In either instance, the object of attention and planned

manipulation is a clearly defined category of observable and measurable behaviour.

The assumptions underlying behavioural programs from which competency-based model issues are based upon empirically validated tenets of social learning. As described by Clark et al., (1972) these tenets hold that:

1. Individual behavior occurs in the context of a social environment and in interaction with the environment
2. Social behavior is learned in interaction with the environment
3. Behavior is taught and maintained by the social environment
4. Social learning is a process of reciprocal influence. Participants interacting in a social system mutually affect each others behaviors
5. The reciprocal influencing process may be explicit or implicit, planned or unplanned, but must be considered a factor in social systems.

In accordance with these empirically derived principles of learning theory, the competency-based model perceives individual student behaviour as being;

1. exhibit within the context of the social environment afforded by the classroom
2. malleable and amenable to change in the context of interaction with the classroom environment
3. taught, maintained, reduced or eliminated as a function of interaction with the classroom environment
4. reciprocally influenced in form and frequency by those with whom the student interacts in the classroom--the teacher and student
5. continually subject to conscious or unconscious influence by interactants in the classroom.

The competency-based model is centrally concerned with effectuating explicitly defined and carefully planned affects on the behaviour of students in the learning environment. To the extent that student behaviours are

unsystematically, randomly, and inexplicitly modified in the classroom, it is not a competency-based program of education. Learning rather, is left to chance and accident.

#### Faculty and Staff Orientation and Training

One of the main responsibilities in a teacher preparation program should be the orientation and training of faculty and staff members. Since this was the second cycle of Teacher Corps in which Michigan State University has been involved, a number of returning university staff members were on hand to form the nucleus of the eighth cycle project.

In order to facilitate understanding of the focus of the project and to contribute toward its growth, the new personnel was exposed to the philosophy of the program. Participants were involved in both the pre-service and in-service phases. As a whole, the College of Education was very involved with the sixth cycle program. Faculty from the areas of reading, children's literature, mathematics, and social studies and interpersonal development were directly responsible for classes for the Teacher Corps interns. During pre-service, Summer 1973, interaction groups were formed so that staff members took an active part in the developmental phase of the project.

Involvement with the project for faculty and staff members, as well as administrators, took place during both the pre-service and in-service phases of the program. The

period of greatest intensity toward orientation and training was the pre-service period; however, training and orientation was an on-going process throughout the program.

Some of the objectives of the orientation training phase were:

1. To orient those people directly or indirectly involved with the project with the Teacher Corps philosophy.
2. To explore avenues of integrating Teacher Corps objectives into the regular University teacher training program.
3. To seek ways to make Teacher Corps' objectives applicable to local educational conditions.
4. To develop new courses that will be jointly offered in Teacher Corps as well as in the regular teacher training program of the University.
5. To involve as closely as possible University staff in the development of the Teacher Corps curriculum.

The objectives of the faculty and staff orientation was to share and install sound educational objectives into the teacher training unit of the University. Orientation served a dual purpose in that it also enabled Teacher Corps to incorporate good programs already at Michigan State University into its overall objective. Workshops in the areas of competency-based education, bilingual/bicultural education, mathematics education, and community-based education were emphasized. Those included are:

1. Community-based Education.
2. Introduction to Competency-based Education  
Differentiated Staffing and Bilingual-Bicultural Education.
3. Four day Intern Retreat.



Orientation participants included Interns, Cooperating Teachers, Team Leaders, and selected Community Leaders.

Analysis of the Mathematics Content in the  
Mathematics Education Component  
of the Program

Michigan State University undergraduate elementary education majors are required to complete a sequence of two courses in the mathematics education component of their training. The first, offered by the Department of Mathematics, is a four-quarter hour content course entitled Foundations of Arithmetic (Mathematics 201). During this course, prospective elementary teachers spend three hours a week in lecture rooms and two hours in a mathematics laboratory. The second, offered by the Department of Elementary Education, is a three-quarter hour methods course entitled Teaching of Mathematics in Elementary Grades (Education 325E). Thus students in regular elementary education program earn a total of seven-quarter credit hours.

The mathematics education component of the Teacher Corps program being evaluated was an integrated content, methods and practice experience, seven-quarter hour credits being earned for the content and methods parts (the credits for practice experience aspect being earned under internship). The interns spent six hours per week in Fall term of 1973. However, an administrative problem arose in Winter and Spring terms which reduced the class-meeting to

four-hours per week. Thus the interns received seven-quarter hour credits for fourteen hour class-meeting. This will be explained later (under critical appraisal of instructional method).

#### Data Gathering Method

Educational reform can be considered to include two components--either or both of which may be present in new teaching practices: (1) new curricular content; (2) new teaching methods. In evaluating the effect of a change in either of these components many people believe that the task is generally easier if it is confined to evaluation of (2) alone. This is so because the teaching method might be compared with an alternative method in teaching the same curricular contents and hence use criteria that are fair to both methods. However, where we are comparing a new curriculum (content) with another we may well be trying to compare two teaching-conditions to each of which different criteria of success are appropriate. The teaching-aims that accompany the use of one curriculum are liable to differ greatly from those that accompany the use of the other (NCTM Committee on Analysis of Experimental Mathematics Programs, 1963, Williams (1967), and Brownell (1966)).

Hicks and Perrodin (1967) provided a base for the selection of topics appropriate for the pre-service education in mathematics of elementary school teachers. Four

types of sources were intensively reviewed by them to provide the necessary data. They were:

1. Review of forty-six selected research studies pointing out the mathematical competencies or weaknesses of elementary school teachers.
2. Review of thirty-two sets of recommendations of mathematics educators and nationally-recognized advisory groups or organizations.
3. Page-by-page analysis of sixteen recent textbooks designed for college courses in mathematics for elementary school teachers.
4. Analysis of eleven arithmetic series or teacher's guides for grades K-7 published since 1962.

A composite list of mathematical topics from the above sources was then compiled by Hicks and Perrodin (1967) and a system of rating these topics was devised. Topics which appeared at least once in the composite list were categorized as Level I. To be categorized as Level II, topics had to meet one of the following conditions:

- appear in at least three of the research studies;
- appear in at least five of the recommendations of the mathematics educators or advisory groups;
- appear in at least eight of the sixteen college textbooks in mathematics for elementary school teachers;
- appear in at least six of the eleven arithmetic series or teacher's guides for grades K-7.

Finally, to be classified as Level III, a topic had to meet at least two of the four criteria listed above for Level II topics.

A total of 98 topics were located in the 4 sources (19 in source one, 54 in source two, 84 in source three, and 79 in source four). Of these topics, fifty-one were categorized as Level II and thirty-five were categorized as Level III.

Table 1 shows the topics in level three along with the sources in which they appeared. It is obvious from this table that the last three sources are in close agreement on what should be included in some manner in the mathematics curriculum of the elementary school teacher. The relatively low percentage in the first source does not indicate disagreement with the other sources; it only indicates the lack of experimental research done on the selection of mathematical topics for the preparation of elementary school teachers. To test the validity of this we reviewed publications of similar sources for the years 1968-1973. The topics suggested in these sources are very consistent with the list described above except in the field of Geometry and in the field of Logic.

Analysis of the content of five textbooks for elementary mathematics for teachers revealed that coordinate geometry and mathematical logic were not included in the list developed by Hicks and Perrodin (1967). The Arithmetic Teacher, annually publishes a summary of research

Table 1.--Suggested Topics for the Mathematical Preparation of Elementary School Teachers.

Topic	Source 1	Source 2	Source 3	Source 4
1. Set Terminology		x	x	x
2. Set Operations		x	x	x
3. Relations & Functions		x		x
4. Whole Number Operations	x	x	x	x
5. Counting and One-to-One Correspondence		x	x	x
6. Order and Cardinality			x	x
7. Field Operations		x	x	x
8. Different Numeration Systems & Place Value	x	x	x	x
9. Ancient Numeration Systems			x	x
10. Roman Numeration	x		x	x
11. Primes and Composite		x	x	x
12. Factors and Multiples		x	x	x
13. Exponents & Exponential Notations		x	x	x
14. Divisibility Rules			x	x
15. The Number Line		x	x	x
16. Common Fractions	x	x	x	x
17. Decimal Fractions	x	x	x	x
18. Percentages	x	x	x	x
19. Ratio & Proportions	x	x	x	x
20. Real Numbers		x	x	
21. Square Root		x	x	
22. Measurement	x	x	x	x
23. Precision and Error		x	x	x
24. Formulae & Substitution		x	x	x
25. Basic Concepts of Geometry		x	x	x
26. Geometric Figures		x	x	x
27. Metric System & Conversion		x	x	x
28. Equations and Symbols	x	x		x
29. Inequations	x	x		x
30. Central Tendency	x	x		x
31. Statistical Graphs	x	x		x
32. Probability		x	x	
33. Problem Solving			x	x
34. Making Estimations			x	x
35. Rationalizing Algorithm	x		x	x
Total	13	28	31	32
% of Total No. of Topics	37	80	89	91

and articles on mathematics education conducted in the United States during the preceding year. Review of these summaries for the years 1968, 1969, 1970, and 1971 again pointed out that most research done on the content was in topics noted in the Level III list as defined by Hicks and Perrodin. However, two pieces of research, one by Shah (1969) on the applicability of teaching geometry to elementary school children, the other by O'Brien and Shapiro (1968) confirmed children's ability to learn mathematical logic. Research conducted by Suppes (1969) at Stanford University in teaching logic to elementary school children has not as yet provided conclusive evidence to the children's ability to learn and comprehend mathematical logic. Based on this review of recent literature, the investigator concluded that only the topic "Coordinate Geometry" met the qualifications of the Level III prescribed by Hicks and Perrodin, and therefore decided to include it as the thirty-sixth topic in the criteria list. It should be mentioned that the list contains thirty-six topics which is much less than the latest requirements in the "Guidelines for the Preparation of Teachers of Mathematics" published by NCTM Commission on Preservice Education of Teachers (1973).

The criteria list is then used to compare the mathematics contents in the mathematics education programs of the Teacher Corps, the regular elementary education and another experimental class of elementary education students

jointly taught by the Teacher Corps' mathematics instructor and another mathematics educator.

Bloom et al. (1971) noted that most fundamental to the use of formative evaluation is the selection of a unit of learning. Each course or educational program can be considered to have separable parts or divisions for analytic purposes. It is still possible to consider the parts separately, though these parts may be interrelated in various ways so that the learning (or level of learning) of one part has consequences for the learning of others.

The mathematics education component of the Teacher Corps program was originally planned to cover seven units. They are Measurement, Numeration, Addition and Subtraction of Whole Numbers, Multiplication and Division of Whole Numbers, Fractions, Geometry and Probability and Statistics. Three of these Measurement, Fractions, Probability and Statistics--were to be written by the instructor while the remaining units are Mathematics Methods Program, developed by the Mathematics Education Development Center of Indiana University. The first five of these seven units were well covered and mastery tests were taken on them. There was no time to look at the sixth and seventh units due to (administrative) circumstances beyond the control of the instructor. However, the sixth unit was given to the interns. There is no unanimous agreement among the mathematics educators about stating instructional objectives specifically in behavioral terms (Allendoerfer, 1971,

Forbes, 1971). The instructional objectives, though not specifically stated by the instructor, are very similar to those of the "Trainers of Teacher Trainers" (TTT) project (Shakrani, 1973). Findings will be discussed in chapter IV.

### Description of the Mathematics Methods Integrated with Mathematics Content and Clinical Experience

Five units were studied in the mathematics education component of the program. These were units of Measurement, Numeration, Addition and Subtraction of Whole Numbers, Multiplication and Division of Whole Numbers, and Fractions. The first and the last units, Measurement and Fractions, were prepared by the instructor while the remaining three were part of Mathematics Methods Program, a project of the Mathematics Education Development Center sponsored jointly by the Mathematics Department and the School of Education of Indiana University and funded through the UPSTEP program of the National Science Foundation. All the five units are content-methods integrated.

#### Measurement

The method part of this unit discussed major topics as measurement as a comparison, the arbitrary nature of measuring units, the approximate nature of measuring process, precision and accuracy, developing concepts of and skills in measuring, developing concepts of new units. All these ideas started with linear measurement and extended



to area, volume, weight, capacity and time. The use of manipulatives and experimentation was emphasized at the beginning and developed to the derivation and use of formula.

The implications of Piagetian research on measurement was extensively discussed. The interns came to realize that the necessary concepts will develop (1) when the child is old enough (eight to eight and one-half, according to Piaget), and (2) when he is allowed to operate on (experiment with, manipulation) objects used in measurement and that both conditions are necessary for the operational thought necessary to perform measurement. Further implications of Piagetian research were demonstrated and discussed: that before attempting systematic measurement the child (a) must be able to conserve the idea of length of an object, (b) must understand the concept of subdivisions since the object to be measured must be subdivided into sub-units of the same length as a measuring instrument or ruler, and (c) must realize that a distance between two objects is conserved when other objects are placed between them. Interns finally became aware of the fact that while children can understand the concept of area using intuitive methods (of super position) and conserve interior volume (by building) around the age of eight, the method of determining area and volume by formulas should not be expected to develop until eleven to twelve years of age (Copeland, 1973).

### Numeration

The main ideas presented in the unit were those of sets, number, and numeral, grouping, place value, and the use of these ideas. Informal learning was emphasized for kindergarten level. The development again preceded from the use of physical objects to pictures, to mere representations, and finally to the use of symbols, thus embracing the enactive, iconic and symbolic levels of representational thinking identified by Bruner (1966).

Activities that led to recognition of important characteristics of a good numeration systems were provided. More activities that led to distinction and relationship between grouping and place-value were provided, and importance of the latter in operation with numbers was emphasized. Exercises were provided in sequencing of numeration activities in elementary schools. Development of Numeration lesson plans on counting, numeral reading, ordering of numbers, rounding numbers, extension of numeration system to decimal, and exponential notation was encouraged. The diagnosis of common errors that children often make in elementary schools together with their remediation process were discussed.

Finally the psychological justification of each of the activities at different stages was established.

### Addition and Subtraction of Whole Numbers

In this unit the pedagogical relationship between mathematics and real world was stressed. The central theme was that mathematical learning of young children should flow from real world of experiences to symbols. Activities that could develop number readiness in children were provided, viz. activities that present pre-number concepts, development of numbers, and pre-addition concepts. Three approaches to teaching of addition--sets, measure (number line), and function (function machine)--and difficulties experienced by children in subtractions, reasons for their occurrence and methods of avoidance of their occurrence were discussed. Aids used in early addition activities were provided. Sequencing of addition and subtraction activities was practiced. Three models for subtraction--take-away, missing addend, and comparison--and writing lessons for addition and subtraction algorithms were extensively discussed.

The discussion of the reversibility of thought and the inclusion relation at Piaget's stages 1 and 2 in relation to addition and subtraction was used to appreciate the necessity for manipulation of concrete materials in those stages and see that children are not ready for systematic addition "facts" in abstract form until they are in stage 3. The primary implications of Piaget's work emphasized while teaching addition and subtraction of

whole numbers were on kinds of activities that should precede and be pre-requisite for such work.

### Multiplication and Division of Whole Numbers

Pedagogical aspect of the unit began by developing an understanding of the models that could be used to interpret multiplication and division situations since these operations arise quite naturally from the child's real world. Throughout the unit it was demonstrated that mathematical properties can be used to help children in early multiplication and division. The use of number line and other pictorial models before symbolizing was highly recommended at initial stage. The role of using thinking patterns in helping children learn basic number facts was extensively discussed. This was followed by sequencing initial work in multiplication and division.

Numerous activities that could lead children from introductory concepts to memorization of facts was developed and these were followed by the use of properties and number patterns in learning the number facts. Practice was provided on writing an activity and outlining a lesson to achieve an objective in multiplication or division. Sequencing of objectives for developing multiplication and division (both standard and non-standard) algorithms was practiced. On the whole there was a long series of activities, mental and sensory, that led from the initial

ideas of multiplication to a mature concept and an efficient algorism.

From developmental point of view children are able to learn multiplication at the same time that they are able to learn addition, approximately seven years of age, yet multiplication is delayed. In fact children can multiply smaller numbers as readily as they add them, this may be due to close relationship of addition and multiplication--processes of putting together. This relationship was well emphasized. Piaget's work implies that multiplication at abstract or symbolic should be introduced at approximately the same time as addition.<sup>1</sup> Paradoxically, the natural situations for application of the concepts of addition and multiplication do not arise as often in young children's social environment as do division and subtraction situations. In reality the child probably begins with partition division before other operations, but the algorithms of the processes of division and subtraction are difficult and should be left until the child has used the concepts on a pre-number basis for a long time and has developed a deep understanding of their meaning (Crowder and Wheeler, 1972). The symbolism and paper work should come only after the inverse relationship between multiplication and division is understood using concrete material. Such problems should be done at the concrete material levels combining

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<sup>1</sup>Copeland, op. cit., p. 146.

and separating sets of objects. Multiplication facts should involve the corresponding division facts--as sets of objects are manipulated.

It was emphasized that the studies of Piaget indicate that paper work should not begin (on multiplication and division) until a child has the reversibility of thought characteristic of the third or "operational" thought level, and that for many youngsters the symbol or abstract work might best be done in systematized fashion toward the later part of the first grade or in the second grade since the necessary operational thought level does not occur in many children until around the age of seven. At this abstract level, it is recommended that the child should organize his multiplication facts into a table by using manipulative materials. Moreover, these basic multiplication facts (up to  $9 \times 9$ ) should be committed to memory with some kinds of reinforcement activities in form of enjoyable games which provide practice in the recall of these basic facts. The understanding of number properties and place value will then be necessary as they (children) move toward the conventional procedures involving multiplication and division problems. Since the learning involves discovery and relearning new facts, recalling all previous learning, having greater attention span, and being able to handle a more complex operation involving storage and recall of numbers during operation, it is recommended that the work be spread over a period of years (Fehr and Phillips, 19720).

### Fractions

The unit started with thorough examination of the meaning of  $1/a$  where  $a$  is a non-zero whole number. Following this was a set of activities that provided students with learning experiences that could help them grasp the fraction concept. Each experience was designed to bring the student into a personal encounter with fractions as they are represented by physical models or referents. The interns were exposed to experiences which they could later use periodically with either an entire class (of their pupils) in conjunction with a more comprehensive unit on fractions or with selected groups of pupils who appear to need additional practice with concrete representations in order to become familiar with basic concept. The interns were given the opportunity to work independent, to explore, to guess and to see if his/her guesses were correct. The activities required equipment and materials that were relatively easy to obtain and safe to use with a minimum of classroom supervision.

Though many manipulative and visual aids were used, three of them were predominantly used as models, viz. cuisenaire rods, number line and rectangular arrays. The interns at the end of the unit believed that the use of cuisenaire rods offered them a wealth of mathematically correct experience in fractions which could not be acquired through no other method (especially division of a fraction by a fraction). They believed it was both abstract and

concrete. The power of these coloured rods lies, mathematically, in their penetration to the core of relationships and structures, and, psychologically, in their stimulus to intuition and enquiry. They observed that fractions are mental structures extracted from straight forward, simple situations involving the rods, and made evident both through the colors and lengths, more important is the fact that instead of considering one fraction alone, they could consider those equivalent to it as well; rectangle-model is very helpful in this case.

### Clinical Experience

The interns started what was called the inservice part of their program in Fall 1973. Each intern spent full morning (four hours) daily in Fall and Spring terms in elementary school and spent the afternoon on the university campus. In Winter, 1974, the interns spent morning hours on university campus and afternoon hours in elementary school. The interns were under the supervision of experienced inservice teachers, called team leaders, in the elementary schools. These team leaders also attended the interns mathematics classes at the university throughout the academic year. The purpose of this was to solve some of the problems of (though professionally experienced but) mathematically incompetent supervising teachers raised by Hatfield (1972).



The interns after developing their lesson plans in cooperation with their team-leaders, taught these lessons to their pupils. It should be mentioned again that these interns were not fully responsible for their classes they were supposed to be involved in team-teaching with their cooperating teachers. Though they were to start by observation, they proceeded gradually to tutoring, teaching small groups, and eventually teaching the whole class.

The clinical experience provided the interns with:

1. The opportunity to relate theory to practice, by applying the knowledge gained at the university to actual teaching situations at the elementary school.
2. The opportunity to observe different classes, teachers, and teaching methods.
3. The opportunity to initiate their teaching experience starting by working with a small group of children, thus benefiting from closer individual relations and minimized problems of discipline and control, and eventually handling the entire class.
4. The opportunity to receive immediate feedback on the methods of teaching utilized from experienced in-service teachers and faculty members.

#### An Appraisal of the Instructional Method

The instructional strategy used by the instructor was Bloom's model of Mastery Learning. The procedure was

very similar to that described in chapter II under practice of mastery learning.

A look at the previous grades of the interns reflected that many of them were short of good background in mathematics or that they could not learn mathematics effectively in the traditional setting. A review of literature showed that laboratory approach in conjunction with mastery learning strategy would be very effective for these interns. More so this should be a useful method for them to teach their pupils (taking into consideration results of studies and recommendations of professional bodies on teaching of mathematics in the inner-city schools).

During the Fall 1973 two doctoral students (including the investigator) worked with the instructor to see that the needs of the individual intern was met. The team leaders also helped in this attempt.

On the basis of the formative evaluations carried out in Fall term of 1973, the interns were divided into two groups in Winter 1974, for the purpose of providing learning correctives. The group containing the average and below average interns were provided with remedial work while the above average was provided with enrichment. The instructor together with three doctoral students (including the investigator) were charged with this responsibility. An two-hour class-meeting per week was specially set aside for this though interns consulted with the doctoral students

and the instructor outside the class as well. Two doctoral students were working with interns who needed remedial work while the third doctoral student was working with interns who needed enrichment. The instructor worked with both groups. This arrangement gave the interns an opportunity not only to remove their deficiencies but also to explore. The activities included (for remediation) small group problem sessions, individual tutoring, and use of alternative learning materials while the enrichment group was introduced to Number Theory. In Spring term this special class-meeting could not hold due to some administrative problems in the schedule of the interns, however, help and learning correctives were provided in and outside the regular class-meetings.

In order to understand the overall goal of the mathematical instruction let us examine some of the common goals of instruction. Lewin's (1935) theory of motivation postulates that a state of tension within an individual motivates movement toward the accomplishments of desired goals. Three goals emerged from Lewin's notions: one where there is cooperative goal interdependence, one where there is competitive goal interdependence and one where a person has individualistic goals unrelated to anyone else's. Building a field theory of cooperation and competition, Deutsch (1949, 1962) defined (1) a social situation as one where the goals of the separate individuals are so linked together that there is a positive correlation between their

goal attainments, (2) a competitive social situation as one where the goals of the separate individuals are so linked that there is a negative correlation between their goals attainment, and (3) an individualistic situation as one where the goals of individuals are independent of each other. To Deutsch under the first an individual can obtain his goal if, and only if, the other person with whom he is linked can obtain his goal; under the second an individual can obtain his goal if, and only if, the others with whom he is linked cannot obtain their goals; under the third whether or not an individual accomplished his goal has no bearing upon whether other individuals accomplish their goals, in this situation the individual seeks an outcome that is best for himself, regardless of whether or not others achieve their goals. In a conceptualization based upon learning theory, Kelly and Thibaut (1969) defined a cooperative structure as one in which the individual's rewards are directly proportional to the quality of the group work; a competitive structure is one in which individuals are rewarded so that one receives a maximum reward and the other receives a minimum reward; an individualistic structure is one in which individuals are rewarded on the basis of the quality of their work independent of the quality of work of other students. Deutsch (1962) emphasizes that an individual will tend to facilitate the actions of others when he perceives that their actions will promote his chances of goal attainment and will tend to

obstruct their actions when he perceives that they will be detrimental to his goal attainment. For Kelley and Thibaut the reward distribution motivated individuals to behave cooperatively, competitively, individualistically depending upon the reward structure. For Deutsch it is the drive for goal accomplishment that motivates cooperative, competitive, or individualistic behaviour. When one is focusing upon extrinsic motivation, Kelley and Thibaut's definition is helpful; when focusing upon intrinsic motivation Deutsch's conceptualization is helpful.

The past success of programmed learning materials and mastery programs (Block, 1971) indicate that individualistic goal structures are appropriate for the learning of specific cognitive materials and skills. Due to lack of interaction among students and their independence from each other, feelings of loneliness and isolation may block the development of interpersonal and group skills which may lead to the suffering of affective outcomes and process variables. The work of Deutsch (1949a), Haines and McKeachie (1967), Hammond and Goldman (1961), Thomas (1957), Kogan and Wallach (1967), Johnson (1971, 1974a), show that cooperative goal structures should be used when instructional objectives focus upon such cognitive and affective outcomes as: problem solving effectiveness; group productivity; memorization and retrieval of information; competence in cooperative situations, cognitive development and its related areas of social adjustment,

communication effectiveness, autonomous moral judgment, and empathetic ability; positive attitude toward subject areas, instructional activities, teachers and students; reduction of prejudice and the appreciation of cultural and individual differences; development of positive self-attitudes and a belief in one's basic competence and worth; development of achievement and motivation; development of interpersonal skills; and development of behaviour based upon intrinsic motivation; learning processes which emphasizes moderate levels of anxiety, positive interpersonal relationships and related cohesion and psychological support and safety; the reduction of hostility and conflict among students; open and effective communication among students; trusts; mutual influence promoting achievement and task-orientation; sharing of ideas and materials and mutual helpfulness; involvement in instructional activities and tasks; coordination of efforts and division of labour; and divergent and risk-taking thinking. Crombag (1966), Deutsch (1949a, 1962), Deutsch and Krauss (1962), Johnson and Lewicki (1969) show that competitive situation produces the above results in negative direction.

In the purely academic (cognitive) areas of the mathematics education component of this program the instructor's goal structure could be described as cooperatively-individualistic in the sense that preparation for the post-test criterion-measures was cooperative in nature while the mastery approach made the outcome

individualistic. The professional aspect like preparation of lesson plans, units, journals were cooperative in some occasions and cooperatively-individualistic in others. The reason for using this approach is evident from the above review, background of the interns, and the nature of the job for which they are being prepared. The cooperatively-individualistic approach removed lack of interaction, and feelings of isolation and loneliness, blocking of interpersonal and group skills that could result under purely individualistic goal structures and still produced the learning of specific cognitive materials and skills. The interns also exhibited most of the positive outcomes of the purely competitive goal structure approach. Advantages of mastery approach have already been discussed in chapter II.

### Samples

Three groups of students were involved in the study.

The first and primary sample of interest comprised of thirty interns (forming group  $G_1$ ) selected for the eighth cycle Teacher Corps program at Michigan State University. The selection of the interns was based on the following criteria:<sup>1</sup>

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<sup>1</sup>Lansing School District Eighth Cycle Teacher Corps Proposal, VII--8 and 9.

Educational Requirement

1. Be a citizen/permanent resident of the United States of America.
2. Have no legal background that will hamper teacher certification.
3. Have a minimum of 60 semester hours or 90 quarter hours, from an accredited institution, that can be applied towards a Bachelors Degree.

Human Characteristics

1. Know the objectives of Teacher Corps as they apply to the Lansing project.
2. A willingness to work on a teaching team.
3. Be sensitive to the needs of low-income area children.
4. Willingness to deal with school personnel and administrations in an effort to implement new ideas.
5. Understand the three components of Teacher Corps (school, university, and community) and how they work together.
6. Understand and be willing to deal with the vigorous schedule demands of the Teacher Corps program.

Many of the interns are members of minority or low-income groups with weak educational background. They received a laboratory-oriented, content-methods integrated,



mathematics education program with mastery-learning approach in fourteen quarter-hours spread over three terms.

The second group,  $G_2$ , comprised of twenty-two students who were randomly selected out of volunteers from about 150 students who registered for mathematics 201 in the fall quarter of 1973 as students in the regular teacher education program. The group was given an instruction very similar to that of  $G_1$  and jointly taught by the instructor who taught  $G_1$  and another instructor in six quarter-hours in fall. They also had clinical experience of one hour per week. The main difference between the instructions given to  $G_1$  and  $G_2$  was that  $G_1$  received time-mastery-learning type. Moreover members of  $G_2$  were not necessarily from a particular social-background.

The third group,  $G_3$ , comprised of students in the regular program who were expected to have had mathematics 201 and who registered for education 325E, a methods course in mathematics, in fall, 1973. These students differ substantially from  $G_1$  and  $G_2$  in that most of them were juniors. This Group ( $G_3$ ) was used to compare the mathematical understanding and attitude toward arithmetic after the completion of the methods course which was separated from mathematics content.

#### Measures and Instrumentation

The following instruments were used in gathering data for the pay-off part of the study:

1. Five criterion-referenced achievement measures to assess the mathematical and pedagogical competencies on prescribed objectives (two parallel forms).
2. Dossett's test of Basic Mathematical Understanding of Prospective Elementary School Teachers (two parallel forms) (Dossett, 1964).
3. Revised form of Dutton Attitude Inventory Form C (Dutton, 1962).
4. Attitudes Scales toward different aspects of mathematics developed by the International Study of Achievement in Mathematics (Husen, 1967).
5. Aiken's Enjoyment and Value of Mathematics Scales (Aiken, 1974).

#### Development and Evaluation of Criterion-Referenced Achievement Measures

In order to assess the effectiveness of the mathematics education component of the Teacher Corps Program on the prospective elementary teachers participating in the program, it was necessary to develop a series of criterion-referenced tests designed specifically to test whether the prospective teacher could or could not exhibit the competency implied by the prescribed objectives in each learning unit (Glaser, 1963, 1971; Popham and Husek, 1969). It was also essential to develop two equivalent forms for each test in order to assess the entering behaviours and the terminal behaviours of the pre-service teacher toward the

prescribed objectives within each learning unit. Advantages for the choice of the "unit" as the convenient curriculum segment for analysis have been discussed by Hively, et al. (1973).

A review of the literature helped gain deeper insight on the methodology of constructing good tests. Much of the theory of achievement testing was outlined in the Educational Measurement (ed. by Lindquist, 1951), in which Lindquist recommends the following steps in the preparation of an educational achievement test: (1) planning the test, (2) writing the test items, (3) trying out the test form and assembling the finished test after try-out, (4) preparing the directions for administering and scoring the test, and (5) reproducing the test. Though the most common approach to construction of criterion-referenced measures has been to construct prototypical test items that are "keyed" to more generally stated or implied descriptions of the desired behavior (Mager, 1962; Gagne, 1967; Bloom, 1969; Merwin and Womer, 1969; Lindvall and Cox, 1970), Hively, et al. (1973) discussed the problem with this approach.

In this study the investigator, with assistance from mathematics educators at the Michigan State University, developed the items in each of the criterion-referenced measures bearing in mind the recommendations of Lindquist (1951), Popham and Husek (1969), Simmon (1969), Hambleton and Novick (1973). The passing scores and lengths were

determined by Milliman's (1973) criteria. Each test contained ten items. Two equivalent tests were prepared for each of the five units, one served as pre-test and the other as post-test. A second form of post-test for each unit was prepared and administered as suggested by Block (1972). Many of the items were not multiple-choice because of the nature of the content being tested. Since two of the test constructors administered the test, it was not necessary to prepare the detailed directions for the test examiner. There was a great flexibility on time allocated for testing, it varied between one and one-half hours to two hours. Partial credits were allowed in various parts which were non-multiple-choice.

The pre-test was administered prior to the instruction on the corresponding unit, and the post-test was administered between four to seven days after instruction to assess the effect of each unit. The investigator was present at all testing. When a student was absent during the the pre-test period, he was asked to take the test before starting on the activities for that unit. Though long-range or spiraling effects might also have been assessed after instruction in specified units, or at the end of academic year, or later, the evaluation focused primarily on the immediate effects of instruction in each unit.

Validity.--Criterion-referenced measures are validated primarily in terms of the adequacy with which they represent the criterion; therefore, content validity approaches are suited to such tests (Popham and Husek, 1971). The inherent method by which the set of tests were developed assured content validity, since the test items, in the judgment of the team of mathematics educators who developed and designed the learning units, did in fact reflect the specific objectives within the mathematical content of that unit.

Reliability.--Since each test was constructed to assess the instructional objectives within a specified topic, it was necessary to estimate the reliability of each test independently (Popham and Husek, 1971).

Students in five sections of the regular methods course (Education 325E) (three in Winter and two in Spring) were made available to test the reliability of the pre- and post-criterion measure achievement tests. There were about thirty students in each of these classes and the investigator was allowed approximately one and one-half hours for testing purposes.

Following Cook and Stufflebeam (1967) (who demonstrated empirically that group performance is more efficiently measured using small subsets of items distributed among large numbers of students than vice versa) and Hively, et al. (1973), it was decided to randomly select a 5-item

sample from each of the five pre- and post-tests (50%). When the selection of these particular items was completed, five ten-item tests were assembled:

1. Test I contained five items from pre- and five items from post-tests on Measurement.
2. Test II contained five items from pre- and five items from post-tests on Numeration.
3. Test III contained five items from pre- and five items from post-tests on Addition and Subtraction of Whole Numbers.
4. Test IV contained five items from pre- and five items from post-tests on Multiplication and Division of Whole Numbers.
5. Test V contained five items from pre- and five items from post-tests on Fractions.

Twenty copies of each of the first three tests were randomly distributed to the sixty students in the three sections of Education 325E (Teaching of Mathematics in Elementary Grades) in Winter term while twenty copies of each of the last two tests were randomly distributed to forty students in two sections of the class in Spring term. Based on the statistical results of these tests, reliability estimate for each test was obtained.

Estimate of the reliability of each of the item-sampled test was calculated using the Hoyt Reliability Coefficients (Hoyt, 1941) through an analysis of variance technique. Tables contain the statistics for the analysis

of variance for each test. The Spearman-Brown formula was applied to the Hoyt Reliability Coefficients to obtain the total test reliability. Table 2 shows the results obtained for each test from the statistical procedures described above.

The reliability coefficients for the tests varied from a low of .79 for the pre-test on fraction to a high of .94 for the post-test on Multiplication and Division of Whole Numbers.

These coefficients are considered to be acceptable for a criterion-referenced test (Gagne, 1967).

Table 2.--Reliability Coefficients for Pre- and Post-Criterion-Referenced Achievement Measures.

Measures	Pre-Test		Post-Test	
	(1) <sup>a</sup>	(2) <sup>b</sup>	(1)	(2)
Measurement	.7581	.8623	.8352	.9100
Numeration Systems	.8240	.9033	.8276	.9054
Addition and Subtraction of Whole Numbers	.7692	.8693	.8713	.9316
Multiplication and Division of Whole Numbers	.8475	.9172	.8922	.9432
Fractions	.6550	.7910	.7924	.8844

<sup>a</sup>Hoyt Reliability coefficients obtained from 50 per-cent item-sampled test.

<sup>b</sup>Reliability coefficients of total test after applying the Spearman-Brown formula to Hoyt Reliability coefficients.

$$R_{tt} = \frac{2R_{st}}{1 + R_{st}}$$

$R_{tt}$  = Reliability of total test.

$R_{st}$  = Reliability of sampled test.

Thorndike's guideline for preparation of equivalent pre- and post-tests was followed (Lindquist, 1951). The equivalence of the paired tests was further checked by computing Pearson-moment correlation coefficients on test scores of these 100 students. The correlation coefficients between pre- and post-test scores varied from a low of .72 for the test on Fractions to a high of .93 for the test on Multiplication and Division of Whole Numbers.

Table 3.--Correlation Coefficients Between Pre- and Post-Test Scores of the Students in Regular Methods Course (Education 325E) on Item-Sampled Criterion-Referenced Achievement.

Tests	N	Correlation-Coefficients
Measurement	20	.8857
Numeration	20	.7402
Addition and Subtraction of Whole Numbers	20	.8576
Multiplication and Division of Whole Numbers	20	.9324
Fractions	20	.7213



### Selection of a Test of Mathematical Understanding

This phase of the study began by searching for a well-documented instrument for measuring mathematical understanding. After a careful search of the literature, the investigator decided to use an instrument designed by Mildred J. Dossett (1964). The test was deemed most appropriate for the purpose of this investigation since the test items covered mathematical topics recommended by professional and advisory groups in mathematics education. Permission was granted by the author to use the test for the present study.

Dossett's instrument entitled "Test of Basic Mathematical Understanding" had a reliability coefficient of 0.87 obtained by correlating the scores made by fifty college students on the two equivalent forms of the test. Equivalency of the two forms was determined by using a t-test suggested by McNemar. The t-value obtained indicated no significant differences between the scores on the two forms of the test when administered to the fifty college students.

Form A of Dossett's test was administered to all groups at the beginning of Fall term, 1973 and post-test was administered to each group at the end of instruction.

### Selection of Attitude Inventory

The "Arithmetic Attitude Inventory," an attitude scale developed by Wilbur Dutton at the University of

California, was used in this study (Dutton, 1962). For this scale, Dutton utilized a technique perfected by Thurstone and Chave. He first selected a large number of written statements regarding attitudes toward arithmetic obtained from papers of six hundred university students over a period of five years. The statements were sorted by judges using a scale of one to eleven (extremely unfavorable to extremely favorable). The proportion of judges who placed each statement in the different categories constituted the basic data for computing the scale values of the statements. The instrument was used with over 289 students. A reliability of .94 was obtained through test-retest procedures (Dutton, 1965).

On the attitude instrument, the fifteen items have values that range from 1.0 to 10.5 representing extremely negative to extremely positive attitudes. The individual score is the average scale value of the statements which the individual checked.

The instrument was administered to each group the same day as (but preceding) Dossett's tests.

#### Selection of Attitudes Scales Toward Different Aspects of Mathematics

The attitude scale developed by the International Project for the Evaluation of Educational Achievement in Mathematics, Husen (1967), was used to measure subjects' attitude toward some aspects of mathematics in relation to

measure subjects' attitude toward some aspects of mathematics in relation to school and life in general. This test was administered toward the end of instruction period for various groups. The coefficient of reproductivity of the items obtained from Guttman Scale were generally above the .80 to .85 which are considered acceptable (Guilford, 1954) through slightly below the .90 recommended by Guttman (Stouffer, et al., 1950). Details for construction of the scales are in Hunsen (1967, Vol. I).

#### Selection of Enjoyment and Value of Mathematics Scales

Aiken (1974) designed two scales which were to measure both parts A and B of Objective IV, "Appreciation and use of mathematics," of the mathematics objectives of National Assessment of Educational Progress (1970). These sub-categories are:

- a. Recognizing the importance and relevance of mathematics to the individual and to society.
- b. Enjoyment of mathematics.

Several attitudes scales (including The Mathematics Attitude Scale by Aiken, 1972) measure objective VI-B fairly, while little attention has been given to VI-A. Aiken (1974) constructed 12 items, initially, on E-Scale (Enjoyment of Mathematics) and 11 items, initially, on V-Scale (Value of Mathematics) and randomly arranged these in a format of the Likert type together with 17 other items concerned with interests, achievement, and other biographical information.

The resulting 40-item opinionnaire was administered to 100 women and 90 men of a freshman class at a south-eastern college. Completed, usable opinionnaires were returned by 98 women and 87 men.

An analysis of these show that the final 11-item E-Scale used in the present study has a high internal-consistency reliability of .95 and the final 10-item V-Scale used in the present study has a moderately high internal-consistency reliability of .85. Information on the validity of the two scales were obtained by correlating the total scores on E and V with verbal and mathematical scores on the Scholastic Aptitude Tests and with T-score equivalents of rank in high school graduating class. The analyses of the correlation analysis indicate that the E-Scale is more highly related to measures of mathematical ability and interest, whereas the V-Scale is more highly correlated with measures of verbal and general-scholastic ability.

The E and V scales were administered to  $G_1$  toward the end of instruction. Their scores on these together with other scores were used, via stepwise regression method, to predict their scores on Dossett's post-test.

#### Design of Study

Wittrock (1969) maintained that to evaluate instruction one must first measure at least three components of instruction: (1) the environments of learning, (2) the

intellectual and social processes of learners, and (3) the learning. He went further stating that: (4) the relationship between these three parts of instruction must then be quantitatively estimated. Wittrock discussed the evaluative activities that could be carried out under each of these. The nonquantitative evaluation of the environment of learning has been discussed in various sections under the "intrinsic" part of this evaluation. The remaining three activities come under our "pay-off" evaluation.

#### A. The Evaluation of Learning

This component makes explicit the changes in students' behaviour to try to determine what had been learned during instruction. This evaluates relatively permanent change in behaviour occurring as a result of the experience. This instruction consisted of five units; Measurement; Numeration, Addition and Subtraction of Whole Numbers, Multiplication and Division of Whole Numbers, Fractions.

The research design over time, using Campbell and Stanley's notation,

$$\underline{O}_1 \quad \underline{X} \quad \underline{O}_2$$

was a one group pre-test-post-test design.  $\underline{O}_1$  was a set of five pre-tests corresponding the five units,  $\underline{O}_2$  was a set of post-tests corresponding to the units and  $\underline{X}$  was the set of five units. The pre-tests were used to modify the

dependent variable, post-test, to assess the effect of the instructional program. It provided a formative evaluation of the process using external sources (Sanders and Cunningham, 1973).

### B. The Evaluation of Learners

The second way, suggested by Wittrock (1969), to evaluate instruction is to make explicit the students' abilities, interests, and achievements to determine student performance at the end of instruction. This, thus, discuss the intellectual and social processes of learners. This was done in this study by assessing changes in the interns' basic mathematical knowledge and attitude toward mathematics.

The research design over time,

$$O_{11} \quad \times \quad O_{12} \qquad O_{21} \quad \times \quad O_{22}$$

was a one-group pre-test-post-test design,  $O_{11}$  and  $O_{12}$  were pre and post of Dossett's test of Basic Mathematical Understanding of prospective elementary school teachers, while  $O_{21}$  and  $O_{22}$  were pre and post of Dutton's Arithmetic Inventory. This provided a formative evaluation of the product using external source (Sanders and Cunningham, 1973).

### C. The Evaluation of Instruction

The purpose of this is to determine (1) the effect of method of instruction (three levels) and entry attitude

toward mathematics (three levels) upon the mathematics achievement at the end of instruction, (2) the effect of method of instruction (three-levels) and mathematical aptitude (three-levels) upon the attitude toward mathematics at the end of instruction. This provides formative evaluation using contextual and external sources (Sanders and Cunningham, 1973, p. 229).

The designs over time,

$$\begin{array}{ll}
 (1) \quad G_1: & O_1 \quad O_2 \quad X_1 \quad O_3 \\
 & G_2: \quad O_1 \quad O_2 \quad X_2 \quad O_3 \\
 & G_3: \quad O_1 \quad O_2 \quad X_3 \quad O_3 \\
 (2) \quad G_1: & O_1 \quad O_2 \quad X_1 \quad O_4 \\
 & G_2: \quad O_1 \quad O_2 \quad X_2 \quad O_4 \\
 & G_3: \quad O_1 \quad O_2 \quad X_3 \quad O_4
 \end{array}$$

were non-randomized control-group pre-test--post-test designs. The treatments (X's) are

$X_1$ --Teacher Corps Mathematics Education Program,

$X_2$ --Experimental Mathematics Content-Method-one weekly Field Experience Integrated Program,

$X_3$ --Regular Program,

and  $G_1$ ,  $G_2$ ,  $G_3$  were the three groups that received these treatments.  $O_1$  and  $O_2$  were pre-tests of Dossett's test of Basic Mathematical Understanding and Dutton's Attitude Inventory.  $O_3$  and  $O_4$  were post-tests of Dossett's test of Basic Mathematical Understanding and Dutton's Attitude Inventory. In (1)  $O_1$  was used as a covariate to modify the dependent variable (criterion)  $O_3$ , while  $O_2$  was used in assigning students to levels of an independent variable, attitude. In (2)  $O_2$  was used as a covariate to modify the

dependent variable (criterion)  $O_4$ , while  $O_1$  was used in assigning students to levels of an independent variable, mathematical aptitude.

The designs over variables were;

(1) Treatment Group

		$G_1$	$G_2$	$G_3$
Attitude	High			
	Med.			
	Low			

(2) Treatment Group

		$G_1$	$G_2$	$G_3$
Aptitude	High			
	Med.			
	Low			

In (2) the independent variables were attitude toward mathematics and treatment group, the dependent variable was student achievement on the post-test, form B, Dossett's test while the achievement on pre-test of Dossett's test was used as covariate. In (2) the independent variables were mathematical aptitude and treatment group, the



dependent variable was student achievement in Dutton's Attitude Inventory used as a post-test while the achievement on the same test used as pre-test was used as covariate. In each case the hypotheses were tested via a two-way Analysis of Covariance at the  $\alpha = .05$  level.

#### D. The Quantitative Evaluation of Environments of Learning

While non-quantitative evaluation of environments of learning has been done in various sections under "intrinsic" evaluation, an attempt is made here to quantify the students' perception of their learning environments. Specifically, the interns,  $G_1$ , and other two groups were compared on their perceptions of Mathematics as a process ( $B_1$ ), difficulties in learning mathematics ( $B_2$ ), the place of mathematics in the society ( $B_C$ ), attitude toward school and school learning ( $B_4$ ), and attitude toward man and his environment ( $B_5$ ).

The design over time,

$G_1:$	$O_1$	$X_1$	$O_2$
$G_2:$	$O_1$	$X_2$	$O_2$
$G_3:$	$O_1$	$X_3$	$O_2$

was a two-factor by one-way repeated measure design. The treatments ( $X$ 's) were group treatments as discussed above, the pre-test  $O_1$ , was Dutton's Attitude Inventory used in assigning students to levels one-factor (an independent variable), attitude.  $O_2$  was the dependent variable with

five components,  $B_1$ ,  $B_2$ ,  $B_3$ ,  $B_4$ ,  $B_5$  being scores on five measures in the preceding paragraph.

The design over variable was,

Two-way Multivariate Design

ATTITUDE

		HIGH					MEDIUM					LOW				
TREATMENT GROUP		$B_1$	$B_2$	$B_3$	$B_4$	$B_5$	$B_1$	$B_2$	$B_3$	$B_4$	$B_5$	$B_1$	$B_2$	$B_3$	$B_4$	$B_5$
	$G_1$															
	$G_2$															
	$G_3$															

The two factors were groups (three levels) and attitude (three levels).

Analysis of Data

The investigator selected several statistical procedures to analyze the data collected during the study.

A one-way multivariate analysis of variance technique as described by Winer (1971) was selected for testing the significance of the gain between the post-test scores

and the pre-test scores of the interns ( $G_1$ ) on the five criterion-referenced measures (Hypothesis A). Finn's program version 4 (1968) on analysis of gain scores was utilized.

To test the hypotheses related to the effect of the mathematics curriculum on the basic mathematical understanding (Hypothesis B1) and attitudes toward arithmetic (Hypothesis B2) one-way multivariate analysis technique (and the corresponding univariate techniques) on gain scores was used. Finn's (1968) program on analysis of gain scores was utilized.

To assess the relative performance of the interns ( $G_1$ ) as compared with two other groups ( $G_2$ ,  $G_3$ ) (1) on the basic mathematical understanding (Hypothesis C1), (2) on attitude toward arithmetic (Hypotheses C2), a  $3 \times 3$  Analysis of Covariance was used in each case.

To compare the interns,  $G_1$ , and the two other groups on (their perceptions of mathematics learning) five different aspects of attitudes (Hypothesis D) a three-factor analysis of variance (one-factor with repeated measures) technique was used as suggested by Winer (1971, 559-69). Finn's (1968) program on one-way repeated measures was utilized.

To determine how well the scores on post-tests of the criterion-referenced measures and pre-test on Dossett's test of mathematical understanding predict interns' ( $G_1$ ) scores on post-test of Dossett's test of mathematical

understanding, to estimate the proportion of variance in post-test score of Dossett's test accounted for by each of learning unit, and to assess the relative order of importance of the predictors, multiple regression and stepwise regression techniques were employed. Finn's (1968) program was again utilized.

Finally to determine how well the scores on Dutton's pre-test, five aspects of attitude and Aiken's E and V-Scales predict interns ( $G_1$ ) score on post-test of Dutton's Attitude Inventory, to estimate the proportion of variance in post-test score of Dutton's Attitude Inventory accounted for by each of the eight (measures) predictors and to assess the relative order of importance of the predictors, multiple regression and stepwise regression techniques were employed.

#### Significance Level Chosen

The .05 of significance for the rejection of statistical hypotheses being investigated was selected as being sufficiently rigorous for the conditions of this study. Thus, if the probability was at or less than five times in one hundred that the observed difference could be attributed to chance, the research hypothesis was accepted; if the observed difference was of such magnitude that it might arise more than five times in one hundred through the operation of chance factor, the research hypothesis was rejected.

Methodological Assumptions and Limitations  
of the Study

One Group Pre-Test--  
Post-Test Designs

In assessing the effect of the instructional program on the achievement of interns on the prescribed mathematical competencies, we wished to test the null hypothesis of equality of performance with respect to all dependent variables (five units) across the measures (pre- and post-tests) (Hypothesis A). In standard multivariate analysis parametric test (employed in the study) to test this hypothesis it is usually assumed that

$$X_{ij} = \mu + \alpha_j + \beta_i + e_{ij} \quad \begin{array}{ll} j = 1, 2, & (K) \\ i = 1, 2, \dots, 24. & (n) \end{array}$$

where  $\mu$ ,  $\alpha_j$ , and  $\beta_i$  are constant vectors of population means (elements of this vector are overall means on each dependent variable), treatment (measurement) effects, and block (subjects) effects. The vectors  $e_{ij}$  are 48 (nk) independent, normally distributed vectors with a common variance-covariance matrix. Frequently some or all of these assumptions cannot be justified especially block additivity and normality. In this study assumption of normality is inconsistent with the underlying theory of mastery-learning (chapter II). It is, therefore, illogical to use the parametric multivariate approach. A logical test would have been Gerig's multivariate extension of the Friedman

test (Gerig, 1969), a nonparametric test which relaxed the assumptions of block additivity and normality. The investigator attempted, but failed, to secure a computer program that could do this from the office of research consultation at the Michigan State University. The univariate parts of hypothesis A would have been tested by Friedman or Wilcoxon's tests (Conover, 1971). These remarks hold for testing hypotheses B as well if the gain scores were due to the effect of instruction (mastery learning).

A word is in order about the weaknesses of the one-group pre-test--post-test design used in assessing the effect of the instructional program. While this design controlled for "selection" and "mortality" variables (since the same subjects took tests  $O_1$  and  $O_2$ ) there is no assurance that the instruction (X) was the only or even major factor in  $O_2 - O_1$  difference. Plausible rival causes could have been history, maturation, testing effects, changing effects of instrumentation, statistical regression. An obvious way to control this is the use of control groups. Hively, et al. (1973, p. 35) argued that when one is primarily interested in finding out what curriculum can do and whether it satisfies its own objectives then control groups are not useful. Of these five plausible rival causes, maturation and testing effects seem to threaten the internal validity of the designs. Since, with the exception of the attitude inventory, regular classroom tests were used as  $O_1$ 's interaction of testing and treatment (X) which

usually threatens external validity of this design is not a threat in this study. The same remark holds for interaction of selection and treatment.

### Two-way ANCOVA Designs

In determining (1) the effect of method of instruction (three levels) and entry attitude toward mathematics (three levels) and mathematical aptitude (three levels) on the attitude toward mathematics at the end of instruction, it should be mentioned that while the investigator was aware of the differences in time spent by these groups on instruction, Kirk (1968, p. 457) showed that it is incorrect to adjust the dependent variable for the concomitant variable, time. Differences in learning ability or mathematical background or attitude toward mathematics might exist between these  $G_1$ ,  $G_2$ , and  $G_3$  prior to the introduction of the instructional methods, these extraneous variables would bias the evaluation. Since previous studies have indicated that these variables have some effects on the achievement of an instructional method, it was, therefore, necessary to control them statistically. Two methods have been suggested for this, provided that the concomitant variables are measurable.

First, measure the concomitant variables, called covariables, in addition to the variate of primary interest, termed the criterion, and in this case use analysis of covariance single-factor design. Winer (1971) suggested

the second method (p. 780). In this method, an experimenter, rather than using analysis of covariance (ANCOVA) might attempt to use covariate as a classification or stratification factor. If this were done in an experiment involving only one covariate, the experiment would be analyzed as a two-factor ANOVA. This might result in quite small cell frequencies and possibly no entries in some cells. If each of the resulting cell frequencies is relatively large, say five or more, Winer (1971), following Cochran (1957), is of the opinion that this type of stratification on the covariate is generally to be preferred to ANCOVA. Moreover, Cox (1957) found that randomized block design is better than ANCOVA if the correlation between criterion and concomitant variable,  $\rho$ , is less than 0.6 while ANCOVA is appreciably better than randomized-block design if  $\rho$  is greater than 0.8. According to Cox (1957) no preference between the two when  $.6 \leq \rho \leq .8$ . Cox (1957) pointed out that where treatment effect is not suspected to be entirely independent of the concomitant variable (that is where there might be treatment by concomitant variable interaction), such an interaction might give useful insight into the mechanism underlying the treatment effects and might also change any practical recommendations to be made in the study.

In this study, two covariates are of importance, mathematical aptitude and entry point attitude, and this will result in three-factor experiment--each of the



covariates having at least two levels (High, Low). Previous studies have indicated that it is better to consider three levels for each of these covariates, viz; High, Average, Low. This would force each group,  $G_1$ , to be distributed into nine cells. Since  $G_1$  had 24 members and  $G_2$  had only 21 members, this would not satisfy the conditions under which Winer prefers this stratification method to ANCOVA.

Consequently, the investigator decided to use one of the covariates--pre-test score on attitude toward arithmetic as a second factor, when the criterion was post-test score on Dossett's test while using pre-test score on the same test as covariate. The same reasoning was carried out in determining the effect of instruction on the affective behaviour of  $G_1$ ,  $G_2$ ,  $G_3$ . Thus a 3 x 3 ANCOVA was used in each case of hypotheses C1 and C2. Moreover the classical parametric ANCOVA test has been used. This test assumes linearity of regression of the criterion on covariate, normality, and others. While the assumption normality might hold for  $G_2$  and  $G_3$ , it is questionable for  $G_1$  if the achievement in the criterion was due to instruction (mastery-learning). Rank ANCOVA proposed by Puri and Sen (1969) which does not assume normality and linearity would have been used. The investigator could not do this for reason given earlier.

It should be mentioned that while interaction of selection and maturation, and interaction of testing and treatment which are normally sources of internal and

external invalidity to non-equivalent control-group, a quasi-experimental design, are not threatening in this study, it is not certain that regression is not a source of internal invalidity.

#### Non-Statistical Uncontrolled Sources

Among non-statistical factors that could affect the differences at the end of instructions (apart from the instructions) are instructors ability, instructors experience, class constancy, unplanned changes in instructions, the problem of measuring understanding, the problem of constructing or obtaining testing instrument that would favor no group and experimental stimulation (Hawthorne's effect). Most of these could not be controlled in the comparative parts of the study and should, therefore, be noted as weaknesses of the study.

#### Summary

This chapter described the mathematics component of the eighth cycle Teachers Corps program at Michigan State University and the procedures followed for its assessment.

The formative evaluation of the mathematics component of the eighth cycle Teacher Corps program at Michigan State University took place during the academic year 1973-1974. Twenty-four out of thirty interns who were originally admitted into the program were utilized for this evaluation. Two of the interns withdrew, two fell ill, and two were

randomly dropped to obtain an orthogonal design. In addition, samples of other student groups were used for comparison purposes.

The following steps were followed for the evaluation of the program:

1. General context and program description.
2. Analysis of the mathematics content in the program by means of a criterion-referenced list developed according to topics suggested by specialists in the preparation of elementary school teachers.
3. Appraisal of the mathematics methods integrated with mathematics content and clinical experience.
4. Appraisal of the instructional method used in the program.
5. Assessment of the content of the integrated content-methods course by means of the following instruments:
  - a. Five criterion-referenced achievement measures to assess mathematical competencies on prescribed objectives.
  - b. Test of Basic Mathematical Understanding.
  - c. Attitude Inventory and Attitude Scales.

The development and use of the test instruments as well as the statistical procedures used for the analysis of data were described in the last section of this chapter.

Results obtained from the different analyses and their interpretation are discussed in the following chapter.

## CHAPTER IV

### PRESENTATION AND ANALYSIS OF DATA

This chapter presents a summary of the data collected during this investigation, the analysis of data and findings based on this analysis. It consists of eight sections:

1. Analysis of the mathematics content and methods of the learning units;
2. the evaluation of learning obtained by assessing the interns performance on the criterion-referenced achievement measures;
3. the evaluation of learners which provides the effect of the teacher corps program on the basic mathematical understandings and attitude toward mathematics of the interns;
4. the effect of instruction and attitude on mathematical understandings;
5. the effect of instruction and mathematical aptitude on attitude toward mathematics;
6. the effect of instruction and attitude on students perception of mathematics learning and environment;

7. the contribution of the learning units to basic mathematical understandings of the interns;
8. the relationship between the attitude toward mathematics and the interns perception of mathematics learning, enjoyment, value and environment.

The chapter ends with a summary of results.

### Analysis of the Mathematics Content and Methods of the Learning Units

Like other curriculum projects, the most logical and convenient segment for analysis in the mathematics education component of the Teacher Corps program was the "unit" since this was treated by the (competency-based) project as the administrative and the theoretical building block. The use of the unit is therefore, fundamental to formative evaluation in general and helpful in providing learning correctives in particular (Keller, 1968, Bloom, et al., 1971).

There were five learning units in this study, each contained one mathematical topic and methods of teaching.

### Findings

Item by item comparison between Hicks and Perrodin criterion-referenced list (1967) and the mathematical contents of the five learning units covered shows that the following topics are not included, or covered to be precise, in the mathematics curriculum of the Teacher Corps program:

1. Cardinality
2. Divisibility Rules
3. Percentages
4. Ratio and Proportions
5. Square Root
6. Formulae and Substitution
7. Basic Concepts of Geometry
8. Equation and Symbols
9. Inequations
10. Central Tendency
11. Statistical Graphs
12. Probability
13. Coordinate Geometry

One should not rush to a conclusion that the mathematics content of this program is shallow because of the absence of thirteen out of the thirty-six topics in the criterion-list. It should be mentioned that every mathematical topic (not methods) in the five covered units is already contained in the criterion-list. The units contain both contents and methods of teaching them.

### Discussion

The Teacher Corps Training Objectives as viewed by Lansing School District includes,

To conduct a modern math program which emphasizes structure without losing proficiency in computation, and which can be applicable to low-income children's experiences with numerical variables. . . .

The success or failure in achievement of this objective cannot be fully analyzed here just by analyzing the mathematics content. The part of the objective that can be examined is ". . . without losing proficiency in computation." The question that might be raised here is "how high can the computational proficiency of these interns be without exposing them to those missing mathematical topics." This question cannot be answered in absolute terms. This is where Astin and Panos' (1970) view that,

the nature of evaluative research is that the impact of any educational practice or program can be assessed only by comparison with some alternative practices or programs. . . .

The purpose of the content analysis is to point to those missing elements, the information might be useful in explaining differences (success or failure) in magnitude of the impact of the program on the interns, an exercise which will be carried out below. At present a comparison of the mathematics content of the program with two or three other programs existing or that have existed on the campus might help in making one (a bit) comfortable with the Teacher Corps program (see the table below). It suffices to say that emphasis on mastery of mathematical content as opposed to coverage is still an issue in mathematics education. Ward (1970) discussed this as one of four major issues. At this stage it is necessary to examine further the mathematics education objective of the Teacher Corps program proposal and ask:

Table 4.--A Comparison of Mathematical Topics Covered by the Teacher Corps, the Regular Elementary Education Program, and the TTT Experimental Program.

Topic	Teacher Corps*	Regular Elem. Ed. Program	Another Elem. Ed. Expl. Class Taught by T.C. Instruction	TTT Project (1972)
1. Set Terminology	x	x	x	x
2. Set Operations	x	x	x	x
3. Relations & Functions	x	x		x
4. Whole Number Operations	x	x	x	x
5. Counting and One-to-One Correspondence	x	x	x	x
6. Order and Cardinality		x		
7. Field Operations	x	x	x	x
8. Different Numeration Systems & Place Value	x	x	x	x
9. Ancient Numeration Systems	x	x	x	x
10. Roman Numeration	x	x	x	x
11. Primes and Composite	x	x	x	x
12. Factors and Multiples	x	x	x	x
13. Exponents & Exponential Notations	x	x	x	x
14. Divisibility Rules				
15. The Number Line	x	x	x	x
16. Common Fractions	x	x	x	x
17. Decimal Fractions	x	x		
18. Percentages				
19. Ratio & Proportions		x	x	x
20. Real Numbers	x	x	x	
21. Square Root		x	x	
22. Measurement	x	x	x	x
23. Precision and Error	x	x	x	x
24. Formulae & Substitution				
25. Basic Concepts of Geometry		x	x	
26. Geometric Figures	x	x	x	x
27. Metric System & Conversion	x	x	x	x
28. Equations and Symbols		x	x	
29. Inequations		x	x	
30. Central Tendency				
31. Statistical Graphs				
32. Probability		x	x	
33. Problem Solving	x	x	x	x
34. Making Estimations	x	x	x	x
35. Rationalizing Algorithm	x	x	x	x
36. Coordinate Geometry				

\*x means the topic was taught.



Have these interns been taught all mathematical topics "applicable to low-income children experiences?"

This should be preceded by,

What mathematical topics are not applicable to low-income children's experiences?

as the last part of the project objective presupposes. This part of the objective is, in fact, contrary to popular view like those of Adler (1957), Bruner (1962), Carroll (1963), Bloom (1969), Silberman (1970), Adler (1972) and one of the specific recommendations made to SMSG by a Conference on Mathematics Education for Below Average Achievers in 1964. There are recommendations, in the literature, on methods of teaching mathematics to the disadvantaged children, none on what to teach these children.

### Evaluation of Learning

In this part of the study, the results of pre- and post-test scores were used to assess the effect of the instructional program on the achievement of interns in the prescribed mathematical competencies. The evaluation was carried out in two parts:

1. To determine the significance of gain in achievement on the prescribed mathematical competencies between pre- and first post-test scores.
2. To determine whether a specified degree of mastery over these competencies has been achieved by the interns at the first post-test or second post-test.

### Hypotheses Tested

The following multivariate hypothesis and associated univariate hypotheses were tested.

- A. There will be no significant differences between the post-test means and pre-test means of the interns on the criterion-referenced measures.

Symbolically:

$$\bar{\underline{Y}} - \bar{\underline{X}} = \bar{\underline{O}}$$

where  $\bar{\underline{Y}}$ , a 5 x 1 vector, is the post-test mean scores on the five measures and  $\bar{\underline{X}}$ , a 5 x 1 vector, is the pre-test mean scores on the five measures.

The associated univariate hypotheses also tested were:

The post-test mean of the experimental group will not significantly differ from their pre-test mean on the criterion-referenced measure in:

1. Measurement
2. Numeration
3. Addition and Subtraction of Whole Numbers
4. Multiplication and Division of Whole Numbers
5. Fractions

### Data Analysis

Data collected through the administration of pre- and post-test forms of the criterion-referenced measures developed as described in chapter III were used to test Hypothesis A.

The interns' scores on these measures are prescribed in appendix F. Data included in the tables in this section were drawn from appendix F.

Pre- and post-test means, standard deviations, and mean differences for the criterion-referenced measures are shown in Table 5.

Univariate and multivariate analysis of variance technique were used in the analysis of data related to Hypothesis A.

### Findings

Hypothesis A.--The data in Table 5 show gains made by interns on all criterion-referenced measures. The increase ranged from 8.42 to 21.54 points. When the vector of mean differences was tested against zero vector, the resulting multivariate F value was 29.38 which was highly significant ( $p < 0.0001$ ). Based on this result, the multivariate Hypothesis A which stated that there will be no significant difference between the post-test means and pre-test means of the interns on the criterion-referenced measures was rejected at .05 level of significance.

To examine the students response to each measure separately, the univariate hypotheses were tested at .01 level. Table 6 summarizes the findings for each univariate hypotheses that was tested. Results of the analysis indicated:

Table 5.--Means and Standard Deviations of Pre-test, Post-test, and Gain Scores on the Five Criterion Measures for the Interns.

Variable	Pre-test		Post-test		Gains	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
1. Measurement	28.91	11.29	37.63	9.57	8.42	11.06
2. Numeration	20.33	13.41	41.88	6.17	21.54	12.39
3. Addition and Subtraction of Whole Numbers	24.04	10.51	39.25	8.43	15.21	9.81
4. Multiplication and Division of Whole Numbers	19.83	9.23	33.67	10.90	13.83	9.07
5. Fractions	13.67	9.29	31.04	9.36	17.38	9.18

Table 6.--Multivariate Analysis of Interns on Differences Between Pre- and Post-test Scores on the Five Criterion Measures.

Variable	Between Mean Square	Univariate F	Significance Probability
1. Measurement	1700.1667	13.8871	.0012
2. Numeration	11137.0417	72.5241	.0001
3. Addition and Subtraction of Whole Numbers	5551.0417	57.6677	.0001
4. Multiplication and Division of Whole Numbers	4592.6667	55.7912	.0001
5. Fractions	7245.3750	86.0041	.0001

The instructional treatment of the integrated content-methods course had a positive effect on the interns performance on the five criterion-referenced measures. The univariate tests which stated that the post-test mean of the interns will not be significantly higher than their pre-test means on each of the criterion-referenced tested were rejected at .01 level for each of the univariate hypotheses.

The determination of achievement of the specified mastery level was not strictly statistical. The educational consequences discussed by Millman (1973) was used in setting the passing-score (mastery) at 80 percent for each criterion measure. Each intern was awarded mastery on each criterion-referenced measure if his raw score was not less than 95 percent of the passing score.<sup>1</sup> Table 7 shows number of interns that reached mastery level out of the twenty-four interns used in this analysis.

Second post-tests were administered to interns who could not reach mastery level in the first post-tests. Students who could not reach mastery at the second post-test were only given individual help outside classroom until they were competent in areas in which they were deficient. They were not given a third comprehensive post-test.

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<sup>1</sup>Passing score was 40 out of 50 points. Mastery was awarded if a student

Table 7.--Number and Percentage of Interns that Reached Mastery Level on Learning Units.

Criterion-Referenced Measure	Number of Interns that Attained Mastery Level		Total % of Interns (N=24)
	1st Post-test	2nd Post-test	
Measurement	14	8	91.67
Numeration	20	3	95.83
Addition and Subtraction of Whole Numbers	16	5	87.50
Multiplication and Division of Whole Numbers	10	10	83.33
Fractions	6	10	66.67

#### Analysis of Results

Table 7 shows the percentage all interns who attains mastery on each criterion-referenced measures by the end of second post-test. While about 93 percent reached mastery level on Measurement only 83 and 67 percents reached mastery level on the last two topics. This might be explained in two ways. First the Hawthorne's effect of laboratory approach turned them on at the beginning of the program. Secondly, the entire Teacher Corps program seemed to be well organized at that time. However, in Spring term there were administrative difficulties in scheduling the mathematics class. It was not certain, until very late, that mathematics class would be held, though this was in the original proposal. By the time this was

settled, another class has been scheduled to use the mathematics laboratory at the hours available for the interns. While every effort was made to bring the necessary materials to the classroom used this term, most of the interns felt that this was not truly a laboratory setting. This feeling might have some sort of negation of Hawthorne's effect--that they would not do well in non-laboratory setting--on the interns. This expectation of the interns (not the instructor) manifested into a modified form of Jacobson-Rosenthal effect on the interns' achievements this term.

Apart from the performance of interns on Fractions which might also be affected by the end of term's pressure on students from other courses in their schedule, the overall performance seems to follow findings on percentage of students that attain mastery under Bloom's model of mastery learning (Peterson, 1972).

### The Evaluation of Learners

In this part of the study, the effect the mathematics education component of Teacher Corps program upon the basic mathematical understandings and attitude toward mathematics of the interns were analyzed.

#### Hypothesis Tested

The following multivariate hypothesis and the associated univariate hypotheses were tested:

Hypothesis B. There will be no difference between pre- and post-test mean scores of the interns on basic mathematical understandings and attitude toward arithmetic as measured by Dossett's tests and Dutton's Attitude Inventory.

Symbolically,

$$\bar{\underline{Y}} - \bar{\underline{X}} = \underline{0}$$

where  $\bar{\underline{X}}$ , a 2 x 1 vector, is the pre-test mean scores of the interns on Dossett and Dutton's tests  
 $\bar{\underline{Y}}$ , a 2 x 1 vector, is the post-test means scores of the interns on Dossett and Dutton's tests.

The associated univariate hypotheses also tested were:

The post-test means of the interns' groups will not be different from their pre-test mean on

- B1. basic mathematical understanding as measured by the Dossett's tests,
- B2. attitude toward arithmetic as measured by the Dutton's Attitude Inventory.

### Findings

Hypothesis B.--The data in table 8 show gains made by the interns' group on both measures. The observed gain on the mean score on basic mathematical understanding was 7.375 while that on attitude toward arithmetic was 7.708. When the vector of mean differences was tested against zero vector, the resulting multivariate F was 15.9824 which was highly significant ( $p < .0001$ ). Based on this result, the



Table 8.--Means and Standard Deviations of Pre- and Post-test Scores of Interns on Dossett's and Dutton's Tests.

Variable	Pre-Test		Post-Test		Gains	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
1. Dossett's Test	27.56	8.23	36.12	9.68	7.375	6.69
2. Dutton's Inventory	5.52	1.94	6.29	1.79	7.708	11.76

multivariate hypotheses B which stated that there will be no significant difference between the post-test means and the pre-test means of the interns' group on Dossett's and Dutton's instruments was rejected at .05 level of significance.

On the univariate tests, the difference between pre- and post-test means of the interns on Dossett's test of basic mathematical understanding was significant at .025 level of confidence. The univariate null hypothesis which stated that the post-test mean score of the interns will be not different from their pre-test mean score on basic mathematical understanding as measured by Dossett's test was rejected ( $p < .0001$ ). The difference between pre- and post-test scores of the interns on Dutton Attitude Inventory was also significant at .025 level. The univariate null hypothesis that the post-test mean score of the interns will be not different from their pre-test mean score on attitude toward arithmetic as measured by Dutton's

Inventory was rejected ( $p < .0039$ ). Table 9 gives a summary of the results.

Table 9.--Multivariate and Univariate Analysis of Variance of Interns on Gains Dossett and Dutton's Tests.

---

Multivariate $F = 15.9824$			
D.F. 2 and 22 $p < .0001$			

---

Variable	Between Mean Square	Univariate F	Significance Probability $p <$
Gain on Dossett's Tests	1305.3750	29.1598	.0001
Gain on Dutton's Test	1426.0417	10.3046	.0039

---

#### Related Questions

In the above analysis average value score computed for each intern on Dutton's test was used. The instrument also asked questions pertaining to (1) grade where attitudes were developed, (2) aspects of arithmetic liked or disliked, (3) estimates of general feeling toward arithmetic, and (4) average grade in arithmetic.

#### Findings on Related Questions

1. Grades where attitudes were developed: Of the twenty-four interns, none indicated on the pre-test that he developed his attitude in any of grades 2, 4, 5, 12, 13, and 14. On the pre-test however, three indicated that they developed their attitudes in grade 4, one developed his

attitude in grade 5, one in grade 6 and again none in grades 2 and 13. A comparison of their responses on pre- and post-tests revealed some inconsistencies with a correlation of 0.58 between pre- and post- responses showing that many students could not clearly remember much about their attitudes in early age which is consistent with Poffenberger's study (Poffenberger, et al., 1956, 1959). However, the most crucial years for the interns were in the fourth through tenth grades, as reported in both the pre- and post-test (see table 10). This period overlaps with Dutton's findings (Dutton, 1962).

2. General feeling toward arithmetic: Each intern was asked to circle a number between 1 and 11 to show his overall feeling toward arithmetic (1 representing extreme dislike and 11 representing extreme likeness). On the pre-test, a comparison their estimation of their overall feeling with their average scores yielded a correlation of 0.73 while the same comparison on post-test yielded a correlation of 0.67. This shows that the interns have a good idea of overall feeling toward arithmetic. The difference between their overall feelings and their corresponding average score on items 1 to 15 was attributed to averaging of both favourable and unfavourable items checked on the scale by the individual to secure overall value of the inventory.

Table 10.--Grade Levels of Interns Where Attitudes Were Developed.

Pearson Correlation Coefficient between the Pre-  
Post-Test Responses of Interns is 0.5842

(N = 24)

Grade Level	Pre-Test No. of Interns	Post-Test No. of Interns
1	1	1
2	0	0
3	2	1
4	0	3
5	0	1
6	2	3
7	5	3
8	3	0
9	6	5
10	2	2
11	3	1
12	0	1
13	0	0
14	0	3

Table 11.--Interns' Feelings About Arithmetic in General.

N = 24

Correlation Between Pre-Test Feelings and Pre-Test  
Average Scores was 0.7256Correlation Between Post-Test Feelings and Post-Test  
Average Scores was 0.6681

---

Feeling About Arithmetic in General		Pre-Test	Post-Test
Extreme Dislike	1	0	0
	2	1	0
	3	3	1
	4	2	1
	5	2	0
	6	5	7
	7	2	4
	8	0	2
	9	6	5
	10	2	1
Extreme Like	11	1	3

---

3. Aspects of arithmetic liked or disliked: On the parts asking for aspects of arithmetic like or disliked. There were various responses. The aspects liked included challenge, application, satisfaction on correct solution of problems, mathematical games. The aspects disliked included story problems, teachers, memorization of rules, proofs, long division and boredom.

4. Average grade in arithmetic: No useful information was obtained.

### The Evaluation of Instruction

The evaluation of the Teacher Corps Mathematics Instructional program occurs in two stages: (1) The effect of method of instruction (three levels) and entry attitude toward mathematics (three levels) upon the mathematics achievement at the end of instruction was determined. This was an attempt to answer the question:

Is it the case that students with specified entry behaviour (context) learn more mathematics (external) from a particular method of instruction (internal)?

The answer to this question provides a collection of contextual information in process (interim) formative evaluation and this may help in specifying the limits of the products in terms of entry behaviour and terminal mathematics achievement. (2) The effect of method of instruction (three levels) and entry mathematical aptitude (three levels) upon the attitude towards mathematics at the end

of instruction was also determined. This again provides contextual information in the process formative evaluation in terms of entry mathematical aptitude and terminal attitude toward mathematics.

In addition to the Teacher Corps group two other groups (described in chapter III) were involved in this part of the study. The three groups having three different methods of instruction.

Data collected through the administration of the two equivalent forms of Dossett's test of mathematical understandings and the Dutton's Attitude Inventory were utilized in comparing the groups mathematical understandings and attitude toward arithmetic.

### Hypotheses Tested

#### Hypothesis C1

- a. When a linear adjustment is made for the effect of variation due to differences in prior mathematical aptitude, as measured by Dossett's pre-test, there will be no significant difference in mathematics achievement, as measured by Dossett's post-test, between the methods of instruction.

That is, there will be no treatment effect.

- b. When a linear adjustment is made for the effect variation due to differences in prior mathematical aptitude, as measured by Dossett's pre-test, there will be no significant difference in mathematics

achievement, as measured by Dossett's post-test, between the entry attitudes.

That is, there will be no attitude effect.

- c. When a linear adjustment is made for the effect of variation due to differences in prior mathematical aptitude, as measured by Dossett's pre-test, there will be a constant difference in mathematics achievement, as measured by Dossett's post-test, between the methods of instruction at all levels entry of attitude.

That is, there will be no treatment by attitude interaction.

### Findings

Hypothesis C1: The two-way analysis of covariance technique was utilized for the analysis of data. The assumptions and the limitations of the study were discussed in chapter III. The scores of twenty-four interns in  $G_1$ , twenty-one students in  $G_2$ , and eighteen students in  $G_3$  on a test of basic mathematical understandings were used in the analysis. Data are presented in table 13.

The analysis of covariance is summarized in table 12.

The F-tests for treatment effect, attitude effect and treatment by attitude interaction were 10.4406, 3.3821, and 0.6318 respectively. The first was highly significant ( $p < .0002$ ) and the second was marginally significant



Table 12.--Summary of Analysis of Covariance for the Groups on the Test of Basic Mathematical Understandings.

Source of Variation	D.F.	MS (Adjusted)	F	Significance Probability
Instruction (Group)	2	230.1307	10.4406*	$p < .0002$
Attitude	2	74.5476	3.3821*	$p < .0415$
Instruction x Attitude	4	13.9257	.6318	$p < .6421$
Error	53	22.041970		
Total	61			

\*Significant at  $\alpha = .05$ .

( $p < .0415$ ), while the third one was not significant at  $\alpha = .05$  level. Thus

Hypothesis C1(a) was rejected at .05 level of significance.

Hypothesis C1(b) was rejected at .05 level of significance.

Hypothesis C1(c) was not rejected.

The above results show that there are statistically significant differences between methods of instruction on one hand and attitude levels on the other hand. Scheffe's method was used to find the direction of the significance of differences.

Pairwise comparison among groups revealed that,

Table 13.--Groups Mean Scores on the Test of Basic Mathematical Understandings.

		Treatment Group											
Attitude		$G_1$				$G_2$				$G_3$			
		Pre-Test		Post-Test		Pre-Test		Post-Test		Pre-Test		Post-Test	
		Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
High		31.50	9.856	35.875	8.935	32.571	5.623	37.857	5.305	39.333	4.844	32.500	6.411
Medium		27.125	5.817	37.875	3.944	36.857	3.848	40.571	3.780	37.833	6.080	35.500	5.468
Low		23.75	7.544	30.750	7.285	28.000	6.952	32.429	4.504	31.500	5.01	30.000	3.794

1. the adjusted post-test mean score of  $G_1$  on basic mathematical understandings is significantly higher than that of  $G_3$ ;
2. the adjusted post-test mean score of  $G_2$  on basic mathematical understandings is significantly higher than that of  $G_3$ ;
3. the adjusted post-test mean scores of  $G_1$  and  $G_2$  on basic mathematical understandings are not significantly different.

Pairwise comparison among attitude levels revealed that

1. the adjusted post-test mean score of students with high entry attitude on basic mathematical understanding is not significantly higher than that of students with medium entry attitude;
2. the adjusted post-test mean score of students with high entry attitude on basic mathematical understanding is not significantly higher than that of student with low entry attitude;
3. the adjusted post-test mean score of students with medium entry attitude on basic mathematical understandings is not significantly higher than that of students with low entry attitude.

The apparent contradiction between the a posteriori results and Hypothesis C1(c) will be discussed below.

Hypotheses C2

- a. When a linear adjustment is made for the effect of variation due to differences in prior attitude toward mathematics, as measured by Dutton's pre-test, there will be no significant difference in attitude toward mathematics, as measured by Dutton's post-test, between the methods of instruction.

That is, no treatment main effect.

- b. When a linear adjustment is made for the effect of variation due to differences in prior attitude toward mathematics, as measured by Dutton's pre-test, there will be no significant difference in attitude toward mathematics, as measured by Dutton's post-test, between the entry mathematical aptitude.

That is, there will be no aptitude main effect.

- c. When a linear adjustment is made for the effect of variation due to differences in prior attitude toward mathematics, as measured by Dutton's pre-test, there will be a constant difference in attitude toward mathematics, as measured by Dutton's post-test, between the methods of instruction at all levels of entry mathematical aptitude.

## Findings

Hypotheses C2: The scores of 24 interns in  $G_1$ , 21 students in  $G_2$ , and 18 students in  $G_3$  on Dutton's Attitude Inventory (attitude toward arithmetic) were used in the analysis. Data are presented in table 15.

The analysis of covariance is summarized in table 14.

The F-test for treatment effect, aptitude effect and treatment by aptitude interaction were 0.5307, 3.5024, 0.1262 respectively. The second was significant ( $p < 0.0373$ ) while the first and the third were not significant at  $\alpha = 0.05$  level. Thus

Hypothesis C2(a) was not rejected.

Hypothesis C2(b) was rejected at .05 level of significance.

Hypothesis C2(c) was not rejected.

These results show that only entry mathematical aptitude effect is significant. Scheffe's method was used to detect the significant differences among pairs of aptitude (group) mean scores.

Pairwise comparison among the mathematical aptitude levels mean scores on attitude toward mathematics revealed that

1. the adjusted post-test mean score of the group with high entry mathematical aptitude on attitude toward mathematics is not significantly different

Table 14.--Summary of the Analysis of Covariance for the Scores of Groups on Attitude Toward Arithmetic.

Source of Variation	D.F.	MS (Adjusted)	F	Significance Probability
Instruction (Group)	2	92.0286	0.5307	$p < .5914$
Aptitude	2	607.3783	3.5024*	$p < .0373$
Instruction x Aptitude	4	21.8801	0.1262	$p < .9724$
Error	53	173.418232		
Total	61			

\*Significant at  $\alpha = .05$

from that of the group with low entry mathematical aptitude.

2. the adjusted post-test mean score of the group with medium entry mathematical aptitude on attitude toward mathematics is significantly different from that of the group with low entry mathematical aptitude.
3. the adjusted post-test mean score of the group with high entry mathematical aptitude on attitude toward mathematics is not significantly different from that of the group with medium entry mathematical aptitude (tables 14, 15).

Table 15.--Groups Means Scores on Attitude Toward Arithmetic.

				Treatment Group								
Aptitude	$G_1$				$G_2$				$G_3$			
	Pre-Test		Post-Test		Pre-Test		Post-Test		Pre-Test		Post-Test	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
High	64.00	13.918	68.875	14.730	55.00	17.981	63.143	10.777	73.667	3.615	67.667	10.652
Medium	53.50	17.246	66.625	16.707	60.714	15.713	69.714	14.997	62.833	16.315	71.167	5.154
Low	48.00	24.413	53.125	19.895	39.286	28.270	47.714	28.511	57.333	11.237	53.833	16.951

Quantitative Evaluation of Environments  
of Learning

The scores of the subjects on the attitude scales developed by the International Project for the Evaluation of Educational Achievement in Mathematics (Husen, 1967) were used. The instrument evaluates subjects' perceptions of,

1. learning mathematics as a process ( $B_1$ ),
2. the difficulties in learning mathematics ( $B_2$ ),
3. the place of mathematics in society ( $B_3$ ),
4. the school and school learning ( $B_4$ ),
5. man and his environment ( $B_5$ ).

This part of the study is based on the assumption that if the level of entry attitude toward mathematics does not affect their scores on the instrument, then their scores is a reflection of their learning environments.

#### Hypotheses Tested

- D. (a) There will be no significant difference on the mean scores on Husen's Attitude Scales due to methods of instruction.

That is, there will be no treatment main effect.

- (b) There will be no significant difference on mean scores on Husen's Attitude Scales due to entry attitude level.

That is, there will be no attitude main effect.



- (c) There will be a constant difference between the mean scores of  $G_1$ ,  $G_2$ ,  $G_3$  on Husen's Attitude Scales at all levels of entry attitude toward mathematics.

That is, there will be no treatment by attitude interaction.

### Findings

A 3 x 3 x 5 factorial design with repeated measures on the last factor was used. Multivariate analysis technique was employed in the analysis. A summary of the multivariate analysis of variance is given in table 16.

The multivariate F for method effect (row) was 2.2210 which was highly significant ( $p < .0223$ ). Based on the result of this analysis,

Hypotheses D(a) was rejected at .05 level of significance.

We might therefore conclude that there is strong evidence of non-chance differences due to the methods of instruction. Examination of the row-total vectors (table 17) indicates that, among the three methods of instructions, perceptions of environments of learning as measured by five sub-scores of Attitudes Scales developed by the International Project for the Evaluation of Educational Achievement in Mathematics (Husen, 1967), is highest in  $G_1$  apart from  $B_1$  sub-score where  $G_2$ 's score exceeds  $G_1$ 's score. However, the total mean score for  $G_1$  is highest, then followed by that of  $G_2$

Table 16.--Summary of Multivariate Analysis of Variance  
(Repeated Measures Design) on Perception of  
Learning Environments.

Sources	D.F.	Multivariate	
		F	P Less Than
Method (G)	2	2.2210	.0223
Attitude (A)	2	.8198	.6104
Group x Attitude (GxA)	4	1.2519	.2189
Subjects with Groups (S: GxA)	54		
(Measures) M	4	34.06	.0001
M x G	8	2.2637	.0286
M x A	8	.7568	.6414
M x G x A	16	1.1492	.3154
S x M : G x A			

(table 17). Application of Scheffe's method to the least-square estimates of effects revealed that the mean score for  $G_1$  is statistically different from each of the mean scores of  $G_2$  and  $G_3$  whereas the mean scores for  $G_2$  and  $G_3$  are not statistically different from each other. This interesting result will be interpreted later.

The multivariate F for entry attitude effect (column) was 0.8198 which was not significant ( $p < .6104$ ). Based on this analysis Hypothesis D(b) was not rejected. The implication is that the perception of the groups' environments of learning as measured by five sub-scales of

Table 17.--Mean Scores of Group on Perception on Learning Environments.

Treatment Groups	Measures (Dependent Variables)					Total
	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	
G <sub>1</sub>	8.95833	11.95873	0.208	13.6667	8.625	52.41636
G <sub>2</sub>	9.7143	10	8.8095	12.7619	6.4286	47.71429
G <sub>3</sub>	8.9444	9.7222	8.1111	12.0555	8.5	47.33326

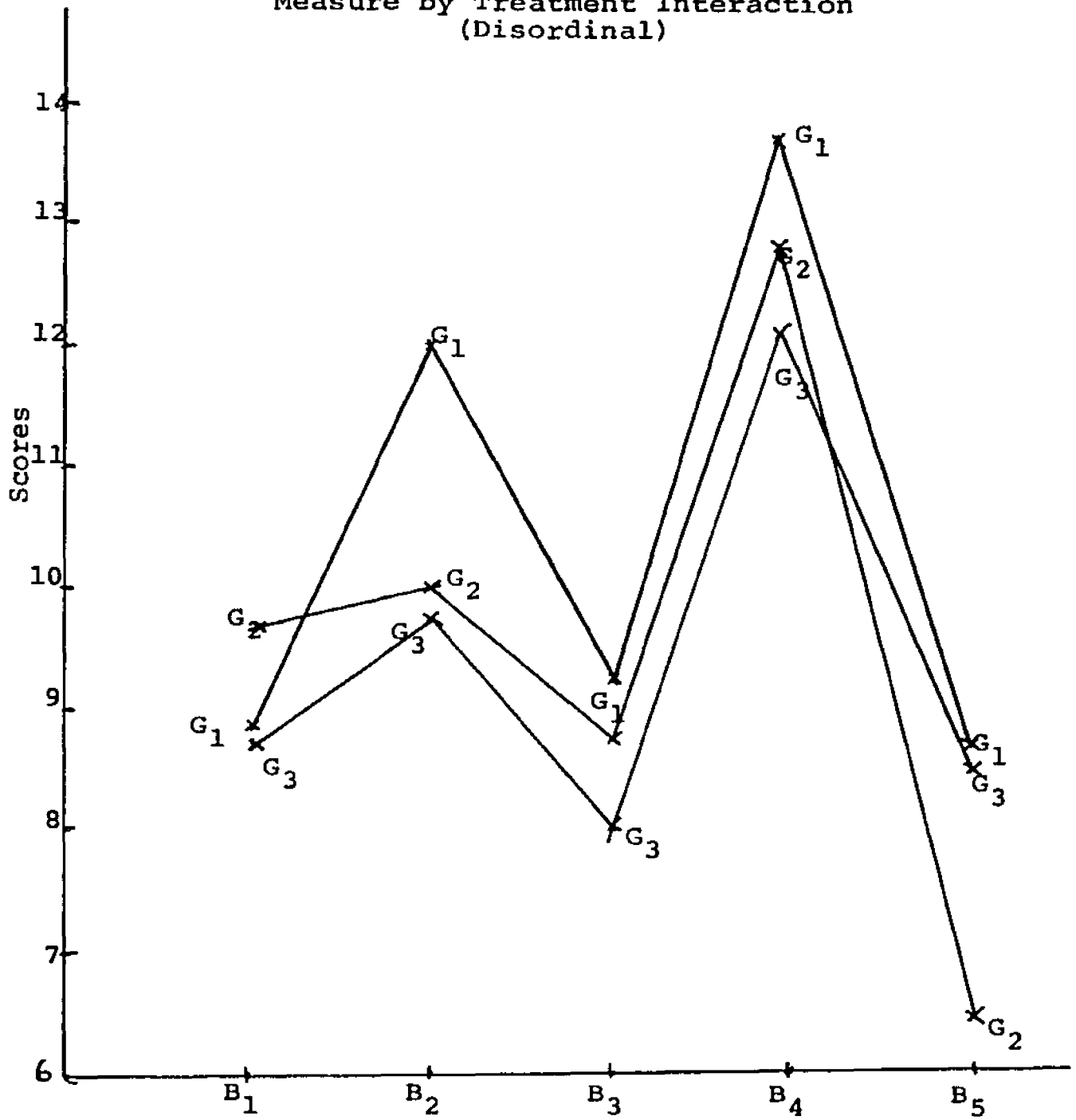
Hunsen's instrument is probably not affected by their entry attitude toward mathematics. This result is very useful in interpreting the preceding result which is the main result of interest in this part of the study, it is the assumption made at the beginning of this section.

The multivariate F for method of instruction by attitude (G x A) interaction was 1.2519 which was not significant at  $\alpha = 0.05$  ( $p < .2189$ ). Based on this result, Hypothesis D(c) that there will be a constant difference between the mean-scores of G<sub>1</sub>, G<sub>2</sub>, G<sub>3</sub> on Husen's Attitude Scales at all levels of entry attitude toward mathematics was not rejected. We may conclude that the perceptions of the environments of learning by the three groups is independent of the three entry attitude levels toward mathematics.

#### Interpretation and Discussion of Findings

Assuming that outcome of learning depends on the environments of learning it is logical to start the

Measure by Treatment Interaction  
(Disordinal)



B<sub>1</sub> is attitude toward mathematics as a process

B<sub>2</sub> is attitude toward the difficulties in learning mathematics

B<sub>3</sub> is attitude toward the place of mathematics in society

B<sub>4</sub> is attitude toward school and school learning

B<sub>5</sub> is attitude toward man and his environment

interpretation of results from evaluation of learning environments.

### Environments of Learning

Non rejection of Hypothesis D(c) indicates that the relationship between the perception of learning environments of subjects in groups  $G_1$ ,  $G_2$ ,  $G_3$  is the same across all levels of entry attitude toward mathematics. This suggests that the responses of subjects on Husen's Attitude Scale within each group were unaffected by their entry attitude toward mathematics. The probable implication of this is that their perceptions fairly estimates their environments of learning. This implication is however confirmed by acceptance of Hypotheses D(b) that there is no entry attitude effect. Results of Hypotheses D(b) and (c) imply that the subjects' perception of their environments of learning as measured by Hunsen's Attitude Scales is a fair evaluation of their learning environments.

The result of Hypothesis D(a) combined with implication of results of Hypotheses D(b) and (c) in the preceding paragraph show that the environment of learning of  $G_1$  is significantly superior than each of those of  $G_2$  and  $G_3$  while that of  $G_2$  is not statistically, appreciably better than that of  $G_3$ . In other words the environments of learning of  $G_1$  which was mastery approach, laboratory-oriented, content and method integrated together with substantial clinical experience provides a learning environment better

than that of laboratory-oriented, content-method integrated, together with one hour per week clinical experience; whereas the latter does not provide a learning environment better than the regular program which is partially laboratory-oriented mathematics content separated from methods course which is laboratory-oriented. The non-appreciable difference between the last two obtained from this study might be due to the fact that the pressure was too much for the students in  $G_2$ . This group,  $G_2$ , as pointed out in chapter III, had their mathematics education program in Fall quarter in six-quarter hour class meeting (and one hour clinical experience in school) what the regular program,  $G_3$ , had in eight-quarter hour class meetings. This pressure obscures the difference between non-mastery laboratory-content method integrated program for the regular content-method separated program. However, the findings agrees with Hollis (1971) study that laboratories organized to provide personal and individualized assistance are helpful to learners that are either culturally or academically disadvantaged.

Related Question: A related question is

Is the difference between groups' perceptions of learning environment the same across all measures (five subsets)? The multivariate F for this measure by method (treatment) interaction was 2.2637 which was significant ( $p < 0.0286$ ) in table 16. This indicates a significant

interaction, thus the difference in perception of environments of learning between the groups' is not the same for each aspect of environments. Mean score of the groups on each subset of Husen's Attitude Scales are in table 17. Figure shows that the interaction is disordinal. This disordinal interaction shows that only in some cases is the environments of learning as perceived by  $G_1$  superior to others. In fact  $G_1$  is superior in all cases except on the learning of mathematics as process ( $B_1$ ) where they were exceeded by  $G_2$ . The scores of  $G_1$  and  $G_3$  on this subset indicate that they viewed mathematics as fixed and given once and for all time (a low score) while group  $G_2$  viewed mathematics relatively more as something developing, growing, and changing (a high score) this might be a reflection of efforts of two faculty members on the group. The figure shows that,  $G_3$ 's perception of environment of learning is inferior in all cases except on attitude toward man and environment where they exceeded  $G_2$ .

It can be concluded that a mathematics education program for elementary teachers which uses Bloom's model of mastery learning, which is laboratory-oriented, content-method-integrated with substantial clinical experience provides an environment of learning where trainees' perception of mathematics learning, the role of mathematics in contemporary society, the school and school learning, the relationship of man to his environment is more favourable than the perceptions of trainee in either (1) a

program with everything except mastery approach and substantial clinical experience, or (2) a program similar to the present regular mathematics education component of elementary education at the Michigan State University.

### Evaluation of Instruction

The rejection of Hypothesis C1(a) and subsequent a posteriori comparisons suggested that there was strong evidence of nonchance differences due to method of instruction. Subsequently a posterioro comparisons, via least square estimates effects revealed a significant difference in mathematics achievement between  $G_1$  and  $G_3$  in favour of  $G_1$ , and between  $G_2$  and  $G_3$  in favour of  $G_2$  but not between  $G_1$  and  $G_2$ . Thus,

1. The interns basic mathematical understandings at the end of instruction is not significantly different from that of  $G_2$ . The implication of this is that with many of the interns from low-income background and poor mathematics background (test of fitness of model-ANCOVA-supports this), it is possible to bring them to the same level of basic mathematical understanding via Bloom's model of mastery learning, as students in the regular program who received some content-method integrated-laboratory-oriented instruction (except for mastery learning). This part agrees with research findings on Bloom's model of mastery learning (Block, 1971).



The result is also consistent with findings of Astin et al. (1972) and Ausubel's reversibility theory (1964) that academic deficiency associated with socio-economically disadvantaged children disappears at the college if the disadvantaged is in appropriate program (Astin, 1972) or taught by appropriate method (Ausubel, 1964).

2. The significant difference between  $G_1$  and  $G_3$  on one hand and  $G_2$  and  $G_3$  on the other hand extended the results of Waszly (1970) study to nondisadvantaged and show, in particular,
  - a. that the disadvantaged group, under mastery, content-method integrated approach, could reach a higher level of mathematical understandings for elementary teachers than a regular group in the present regular program at the Michigan State University;
  - b. that the content-method integrated course is capable of providing a better mathematical understandings for regular elementary teachers than the present regular mathematics education component of their program.

The rejection of Hypothesis C1(b) was marginal ( $p < .0415$ ). The subsequent a posteriori procedure using Scheffe's method failed to detect significance in pairwise comparison of the attitude levels. This might be due to two reasons: first Scheffe's technique is known to be weak

for pairwise comparison because of its wide confidence interval and especially when the significance is very marginal as in this case. Random dropping of six subjects out of twenty-four in  $G_1$  and three in  $G_2$  in order to use a more powerful a posteriori Turkey-technique is not desirable for we shall only be using 60 percent of the interns who originally signed up for the program. The second might be due to weak relationship between mathematics achievements and attitude toward mathematics that all research efforts have shown so far contrary to the feelings of mathematics educators.

The acceptance of Hypothesis C1(c) of no treatment by entry level interaction is a confirmation that the findings under Hypothesis C1(a) are independent of the entry attitude levels.

The acceptance of Hypothesis C2(a), that there is no significant difference among the groups on the attitude toward mathematics at the end of instruction is consistent with studies carried out by Vance (1969) and Johnson (1970) on the effect of laboratory approach on attitude measures though contrary to the claims of advocates of laboratory approach.

The rejection of Hypothesis C2(b) that there is no significant difference on attitude of groups with different entry mathematical aptitudes is an important result. A posteriori comparison shows there is significant difference only in the attitude of subjects with medium and low

entry mathematical aptitude in favour of the group with medium aptitude. The results indicate that students with high entry mathematical aptitude have medium attitude toward mathematics at the end of instruction, while with initial medium mathematical aptitude have high attitude toward mathematics, and that students with lowest mathematical aptitude at the beginning of instruction will have lowest attitude toward mathematics at the end of instruction. The acceptance of Hypothesis C2(c) of no treatment by aptitude interaction shows that the result of Hypothesis C2(b) holds for any method of instruction.

#### Evaluation of Learners

The rejection of the multivariate hypothesis and associated univariate Hypothesis B1, B2 that there will be no significant difference between the post-test means and pre-test means of interns on Dossett's and Dutton's instrument and the foregoing discussion show that the mathematics education component of the eight cycle Teacher Corps program meets the needs of the interns.

We shall now turn to an interesting test part of the study which is an attempt to assess contribution of different units and aspects of attitude of mathematics to their terminal basic mathematical and attitude toward arithmetic.

Assessment of the Contribution of the Learning  
Units to the Basic Mathematical  
Understandings of Interns

Results of Hypothesis C1 have shown that the basic mathematical understandings of the interns is significantly higher than that of students in the regular methods class while it is not significantly higher than that of students in the other experimental (content-method integrated) class. A number of possibilities arise: It might be the case that in fact  $G_1$  and  $G_2$  are significantly better than  $G_3$  because  $G_3$  received their mathematics content, at least, a term earlier than the methods class when their mathematical understandings was assessed. If this argument is tenable it would mean mathematics methods contains no mathematics and it would be unnecessary to make Foundation of Arithmetic (Mathematics 201) a pre-requisite for the methods course--Mathematics Methods for Elementary Grades (Education 325E). If this is not the case, the result of Hypothesis B1 shows that the post-test mean score of  $G_1$  is significantly higher than the pre-test mean score on the basic test of mathematical understanding, which indicates that probably this appreciable gain in mathematical understandings is due to the effect of instruction; and the difference shown in Hypothesis C1 is due to difference in instruction.

As it was pointed out under the methodological limitations and weakness of the design of the study that one-group pre-test--post-test design does not establish

cause and effect relation, following Punch (1971), correlation analysis and stepwise regression techniques were employed to assess how well the scores of interns on the learning units predict their score on the mathematics achievement and to estimate what proportion of the post-test score on basic mathematical understanding can be accounted for by the learning units.

### Analysis

Table 18 shows the simple correlations between scores on the learning units, the pre- and post-tests on basic mathematical understandings. The correlations of the measures with post-test on mathematical understandings in decreasing order of magnitude are .6695, .6675, .6636, .6377, .5809, .4153 for Numeration, Fractions, Multiplication and Division of Whole Numbers, Pre-test of Basic Mathematical Understandings, Measurement, Addition and Subtraction of Whole Numbers. It is interesting to note that of all instruments administered on the interns (including attitude scales) these six measures have the highest six correlations with the post-test score on Basic Mathematical understandings. Other correlations with post-test scores on basic mathematical understandings are as follows: .4138, .3583, .3288, .3218, .2510, .1162, .0829, -0.0717, -0.1224 for Enjoyment of Mathematics Scale, Post-test of Dutton's Attitude Inventory, Pre-test of Dutton's Attitude Inventory, Attitude on Difficulties of Learning

Table 18.--Correlation Coefficients Between Post-test, in  
Basic Mathematical Understanding, Attitude  
Toward Arithmetic and Other Variables (Measures).

Var No.				
POSTACH	1	1.00000		
POSTATT	2	.35834	1.00000	
PREACH	3	.63771	.32986	1.00000
PREATT	4	.32883	.80444	.41205
B1	5	.25100	-.03463	.04612
B2	6	.132176	.21419	.08472
B3	7	.11619	-.10894	.07443
B4	8	-.07170	-.20632	.03876
B5	9	-.12239	.42298	.12427
E	10	.41381	.79361	.49106
V	11	.08288	.05811	-.02241
PU1	12	.58094	.36795	.27834
PU2	13	.66948	.26431	.48262
PU3	14	.41532	-.00466	.29352
PU4	15	.56357	.25204	.29823
PU5	16	.66752	.28415	.44596
		1	2	2
		POSTACH	POSTATT	PREACH

Mathematics, Attitude Toward Mathematics as a Process, Attitude Toward the Place of Mathematics in Society, Value of Mathematics Scale, Attitude Toward School and School Learning, Attitude Toward Man and His Environment.

From these correlations, it can be concluded that apart from the learning units the attitude scale that has the greatest correlation with mathematics achievement is the "Enjoyment," and not Dutton's Attitude Toward Arithmetic. In fact the Enjoyment of Mathematics Scale was the last of all measures and the only attitude scale which has a statistically significant positive correlation with the post-test score on basic mathematical understandings at  $\alpha = .05$  level ( $N=24$ ). Though we shall return to this in the next chapter, it should be noted that the analysis of these correlations supports Aiken (1972b) contention that we should be talking about different aspects of attitude toward mathematics that contributes to mathematics achievement and not just general attitude toward mathematics.

The regression analysis showed that when Numeration and Addition and Subtraction of Whole Numbers were not deleted from the regression equation, with Fraction scores and a constant later added to the equation, the analysis of variance for overall regression gave an F value 11.5996 which was highly significant ( $p < 0.0005$ ) (table 19). This indicates scores on these three measures significantly predict the score on post-test of basic mathematical understanding. They have a multiple correlation of 0.7969 which

Table 19.--Analysis of Variance for Overall Regression of Post-test in Basic Mathematical Understandings and Other Variables.

	Sum of Squares	Deg of Freedom	Mean Square	F	Sig
Regression (About Mean)	797.17159623	3	265.723865	11.5996	0.0005
Error	458.16173710	20	22.90808687		
Total (About Mean)	1255.33333333	23			
Cases	Multiple Corr Coefs				S
24	R <sup>2</sup>	R	R Bar 2	R Bar	Standard Error of Estimate
	.6350	.7969	.5803	.7518	



was significantly different from zero, and that they jointly accounted for 63.50 percent of the variance in the Dossetts post-test. However Addition and Subtraction of Whole Numbers was found not to contribute significantly to the post-test score in mathematical understanding ( $p < .605$ ).

Assessment of the Contribution of Different  
Aspects of Attitude Toward Mathematics  
and School Learning to General  
Attitude Toward Arithmetic

The results of Hypothesis C2 indicates no significant difference on attitude toward arithmetic between the interns and other groups, yet test of the fitness of the model indicated that a significant precision was gained by using analysis of covariance model ( $F = 35.2510$ ,  $p < .0001$ ) implying that the groups were significantly different in entry attitude toward arithmetic. Results of Hypothesis B2 also showed that the interns attitude toward arithmetic significantly increased at the end of instruction. The result of the fitness of the model, Hypotheses C2 and B2 jointly imply that the attitudes of the interns which were different from those of other groups were brought to the same level as others. This part of the study attempts to find out what factors contributed to this gain.

By using the scores of the interns on all measures and attitude scales, the product-moment correlations between the post-test score on attitude toward arithmetic and each of the other measures and attitude scale were found to be .804, .794, .423, .368, .358, .330, .284, -.264,

.252, .214, 0.058, -0.005, -0.035, -0.109, -0.206. The first three correlation coefficients are found to be statistically different from zero and they are for correlations between the post-test score on attitude toward mathematics and pre-test score on attitude toward mathematics, Enjoyment of Mathematics Scale, Attitude Toward Man and His Environment respectively. Though the next two correlations .368 and .358 are not statistically significant it is interesting to note that they are for non-attitude measures (table 18).

Regression analysis showed that when attitude toward mathematics as a process score and the "Enjoyment" scale score were not to be deleted from the regression equation predicting the attitude at the end of instruction, analysis of variance for overall regression gave an F value of 18.4379 which was significantly high ( $p < .00005$ ) with a multiple correlation of .8570 significantly different from zero, and that they jointly accounted for 73.44 percent of the variation (table 20). The overall regression included pre-test score on attitude which contributed significantly ( $p < .011$ ), Enjoyment Attitude Scale which contributed significantly ( $p < .021$ ) and attitude toward mathematics as a process and a constant both of which did not contribute significantly ( $p < .693$ ,  $p < .203$  respectively).

Table 20.--Analysis of Variance for Overall Regression of Post-test Attitude Toward Arithmetic and Ohter Variables.

	Sum of Squares	Deg of Freedom	Mean Square	F	Sig
Regression (About Mean)	5438.28468099	3	1812.76156033	18.4379	0.0005
Error	1966.34031901	20	98.31701595		
Total (About Mean)	7404.62500000	23			
Cases	Multiple Corr Coefs				S
24	R2	R	R Bar 2	R Bar	Standard Error of Estimate
	.7344	.8570	.6946	.8334	9.91549373

### Summary of Findings

Analysis of the data collected during this study revealed the following results:

1. The mean post-test score of the interns was significantly higher than the mean pre-test score on the criterion-referenced measures in: (a) Measurement, (b) Numeration, (c) Addition and Subtraction of Whole Numbers, (d) Multiplication and Division of Whole Numbers, and (e) Fractions.
2. Of the 24 interns included in the analysis, 22 (92%) reached mastery level (80%) in Measurement, 23 (96%) reached mastery level (80%) in Numeration, 21 (87.5%) reached mastery level (80%) in Addition and Subtraction of Whole Numbers, 20 (83%) reached mastery level on Multiplication and Division of Whole Numbers, and 16 (67%) reached mastery level on Fractions.
3. There was significant difference on a test of mathematical understandings between the post-test scores of the interns and their pre-test scores.
4. There was a significant improvement on an arithmetic attitude inventory between the post-test scores of the interns and their pre-test scores.
5. The adjusted mean post-test score of the interns,  $G_1$  was significantly higher than the adjusted mean post-test score of a group of prospective teachers

in the regular teacher education program,  $G_3$ , on a test of mathematical understandings.

6. The adjusted mean post-test score of the group of prospective elementary teachers who had the experimental content-method integrated mathematics education,  $G_2$ , was significantly higher than the adjusted mean post-test score of prospective teacher in the regular teacher education program,  $G_3$ , on a test of mathematical understandings.
7. There was no significant difference between the adjusted mean post-test scores of the interns,  $G_1$ , and the group of prospective elementary teachers who had the experimental content-method integrated mathematics education,  $G_2$ , on a test of mathematical understandings.
8. There was a marginal significant difference between the adjusted mean post-test scores of three entry attitude levels. The direction of this significant could not be detected by Scheffe's method.
9. There was no method of instruction by attitude level interaction on basic mathematical understandings.
10. There was no significant difference on the adjusted, means of post-test scores of the three instructional groups on attitude toward arithmetic.
11. The adjusted mean post-test score of the group with medium entry mathematical aptitude on attitude

toward is significantly higher than the adjusted post-test score of the group with low entry mathematical aptitude on attitude toward mathematics. There was no significant difference between other pairs.

12. There was no significant interaction between methods of instruction and mathematical aptitude on attitude toward mathematics.
13. There was significant difference between the interns' perception of their learning environment and each of the other group's perception of their learning environments, mostly in favor of the Interns learning environments (see 16 below). There was no significant difference between other two groups.
14. There was no group (methods) by entry attitude interaction on perception of learning environment.
15. There was group (method) by measure of perception interaction, with interns consistently scoring higher except on view of mathematics as process where they were exceeded by the second experimental group,  $G_2$ .
16. There were significant correlations between (a) each of the post-test scores on the criterion-referenced measures and the post-test score on basic mathematical understanding, (b) pre-test and post-test scores on basic mathematical understandings, and

(c) score of Enjoyment of Mathematics Attitude Scale and post-test on basic mathematical understandings.

17. Scores on post-test criterion measures on Numeration, Addition and Subtraction of Whole Numbers and Fractions accounted for more than 63 percent of the score on basic mathematical understandings.
18. There was significant correlation between each of pre-test scores on attitude toward mathematics, scores of Enjoyment of Mathematics Scale and scores on attitude toward man and his environment and the post-test scores on attitude toward arithmetic.
19. Scores on pre-test scores on attitude toward mathematics and Enjoyment of Mathematics accounted for about 73 percent of the score on attitude toward mathematics.

## CHAPTER V

### SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

#### Summary

During the 1960s in the United States of America many educators felt the need for reform in preparation of teachers for inner-city schools. In response to this the Congress established the Teacher Corps in 1965. The original purposes of the program, as provided in the authorizing legislation, were (1) to strengthen educational opportunities for children in areas with concentrations of low-income families, (2) to attract and prepare persons to become teachers in such areas through coordinated work-study experiences, and (3) to encourage colleges and universities, schools, and state departments of education to work together to broaden and improve teacher-education programs.

This study was the initial evaluation of the mathematics education component the eighth cycle Teacher Corps program at the Michigan State University conducted during the 1973-74 academic year.



### Purpose

The major purpose of the study was to provide both the "intrinsic" and "pay-off" parts of the process and product of the mathematics curriculum and instruction using internal, external, and contextual sources of the eighth cycle. Specifically, the intrinsic aspect of the investigation sought to (1) analyze and evaluate the adequacies of mathematics content-method integrated component of the program in meeting the needs of the prospective elementary school teachers (interns); (2) provide a critical appraisal of instructional methods and the clinical experiences in the program. The pay-off aspect of the study sought to (3) evaluate-learning the effect of instruction as prescribed by the mathematics education component of the Teacher Corps program on the interns, in relation to specified competencies and assess of the interns achieved a degree of mastery over these competencies; (4) evaluate-learners the effect instruction on the basic mathematical understandings of the interns; (5) evaluate-learners-effect of instruction on the interns' attitudes toward mathematics; (6) evaluate instruction by comparing the mathematical understanding and attitude toward mathematics of the interns with the mathematical understandings and attitudes of other students enrolled [a] in another experimental program, [b] in the regular elementary teacher mathematics education program; (7) compare interns' and students' perceptions of their environments of learning; (8) assess the contribution of

different units of the mathematics instruction to the basic mathematical understandings of the interns; and (9) determine the relationship between the attitude toward mathematics and interns' perceptions of mathematics learning, enjoyment, value and environments.

### Students in the Study

Three groups of students were involved in the study. The students in the Teacher Corps programs (interns) met for six hours per week in Fall term 1973, four hours per week in the Winter and Spring terms of 1974. They covered five learning units which were mathematics content-method integrated taught under laboratory-oriented with mastery learning approach. They spent four hours daily in elementary school, where they were provided with clinical experience under their team leaders and faculty members. Twenty-four out of thirty who originally entered the program were used. The second group consisted of twenty-one students who were randomly selected in Fall, 1973 and given content-method integrated instruction similar to that of interns but without mastery-approach. They met for only six hours per week in the laboratory and spent one hour per week on clinical experience in elementary schools. The third group of students,  $G_3$ , had the regular content and method separated mathematics education program. They were used for the study during the Fall term when they were having the methods course. The content course is a

pre-requisite for the methods course. Only eighteen out of all the students that completed all the tests were used in the analysis.

### Literature Review

Research efforts on the nature of the disadvantaged, the difficulties encumbered by the inner-city schools, the root of the problems that led to establishment of Teacher Corps program were reviewed in chapter II. The work of Ausubel and others led to the premise that there are critical differences between the skills required to teach in low-income schools and middle-class schools but this premise does not imply that unique principles of training are involved in the two different settings. Results of research also show that the differences in values, prior experiences, and environments among children from various income, ethnic, and racial subgroups are so great that teachers need special training in order to apply the principles and fashion the procedures that will facilitate learning process.

Efforts of professional groups in exchanging ideas and disseminating information on learning and instructional theories finally led to theoretical foundation of the emerging practices in mathematics instructional procedures for the disadvantaged. Major part of this rests on the work of Bruner, Carroll, Ausubel and Bloom. The use of laboratory approach in the teaching of mathematics to

children and prospective school teachers in general and disadvantaged children in particular is beginning to have more supportive research evidence.

In the last seven years educators have realized, more than ever, the importance of using evaluation in making judgment and decisions about a program. This has led to the recognition of curriculum evaluation as an art, and a profession. Efforts are being made to establish a framework and build a theory of evaluation that would be universally accepted to practicing evaluators. A related problem currently receiving attention is the development of instruments that should be used in collecting data for such evaluation. Popham and others have collected readable papers in measurement literature that summarizes earlier efforts on the subject.

Concerned with the problem of teacher training, mathematics educators have not agreed on the mathematics content of both the elementary and secondary teachers education programs. Related issues still to be resolved include emphasis on mastery of mathematical content as opposed to coverage, the extent computational skill as opposed to conceptual understanding is sought, the use of concrete versus abstract in teaching mathematics, the geometry (kind and quantity) in elementary and secondary grades. Active research is going to resolve scanty contradictory research results available. Research findings on the relationship between the mathematical understandings

and attitude of elementary school teachers are still very disturbing. The effect of entry behaviour, both cognitive and affective, and method of instruction on achievement is still of interest to researchers.

### Instrumentations

The following instruments were developed or selected for the collection of data: (1) Five criterion-referenced achievement measures (two parallel forms), (2) M. J. Dossett's test of Basic Mathematical Understandings (two parallel forms), (3) Dutton's Attitude Inventory, (4) Attitude Scales Toward Different Aspects of Mathematics developed by the Internal Study of Achievement in Mathematics, and (5) Enjoyment and Value of Mathematics Scales developed by L. R. Aiken.

### Hypotheses and Findings

The following multivariate hypothesis and its associated univariate hypotheses were tested to assess the effect of the instructional program on the achievement of interns on the prescribed mathematical competencies:

- a. The post-test means score will exceed the pre-test means score of the interns ( $G_1$ ) on the criterion-referenced measures.

Hypothesis A and its associated univariate hypotheses were not rejected.

The following two hypotheses were tested to assess changes in the interns' basic mathematical knowledge and attitude toward arithmetic:

- B1. The post-test mean score of the interns will exceed their pre-test mean score on Dossett's test of mathematical understanding.
- B2. The post-test mean score of the interns will exceed their pre-test mean score on Dutton's arithmetic attitude inventory.

Hypotheses B1 and B2 were not rejected.

The following hypotheses were tested to compare the interns and students enrolled in the regular teacher education program. The purpose is to determine (1) the effect of method of instruction (three levels) and entry attitude toward mathematics (three levels) upon the mathematics achievement at the end of instruction, (2) the effect of method of instruction (three levels) and mathematical aptitude (three levels) on the attitude toward mathematics at the end of instruction.

- C1 (a) There will be no treatment main effect on mathematics achievement.
- (b) There will be no attitude main effect on mathematics achievement.
- (c) There will be no treatment by attitude interaction on mathematics achievement.

Hypotheses C1(a) and (b) were rejected at .05 level of significance. Hypothesis C1(c) was not rejected.

C2 (a) There will be no treatment main effect on attitude toward mathematics.

(b) There will be no aptitude main effect on attitude toward mathematics.

(c) There will be no treatment by aptitude interaction on attitude toward mathematics.

Hypothesis C2(a) was not rejected. Hypothesis C2(b) was rejected at .05 level of significance. Hypothesis C2(c) was not rejected.

The following hypothesis will be tested to compare the interns and other two groups of students in the regular teacher education program on five different aspects of attitudes.

D (a) There will be no treatment main effect on perception of learning environments.

(b) There will be no entry attitude main effect on perception of learning environments.

(c) There will be no treatment by entry attitude interaction on perception of learning environments.

Hypothesis D(a) was rejected at .05 level of significance. Hypothesis D(b) was not rejected. Hypothesis D(c) was rejected at .05 level of significance.

### Statistical Analysis

One way multivariate analysis and the corresponding univariate a analysis of variance were used for testing

Hypotheses A and B. A 3 x 3 Analysis of Covariance was used for each of Hypotheses C1 and C2. A 3 x 3 multivariate Analysis of Variance was used to test Hypotheses D. Stepwise regression method was used to estimate the contribution of learning units to the terminal mathematical understandings of the interns. Stepwise regression method was also used to assess the contributions of the interns perceptions of mathematics learning, school and school learning, enjoyment of mathematics, value of mathematics to the interns attitude toward mathematics.

Five percent level of significance was used in accepting or rejecting the research hypotheses.

### Conclusions

The following conclusions are based on the findings of this study.

1. Analysis of the mathematics education component of the interns indicated that the mathematics content is not meaningfully different from the content in the regular mathematics education for elementary teachers at the Michigan State University.
2. On the criterion-referenced measures the interns show significant gains in achievement ( $p < .0001$ ) on all measures.
3. Over 90 percent of the interns reached mastery level on Measurement and Numeration, the percentage



of students who reached mastery on the remaining units decreased gradually to 67 percent (minimum) on Fractions.

4. The interns showed significant gains on a test of basic mathematical understandings.
5. The interns showed a significant gain in attitude toward mathematics.
6. With initial differences allowed for, the interns group scored significantly higher on a test of basic mathematical understanding than did a group of students in the regular content, method separated mathematics program.
7. With initial differences allowed for the group of students in the content-method integrated mathematics program scored significantly higher than the group of students in the regular content-method separated mathematics program.
8. With initial differences allowed for, there was no significant difference in the basic mathematical understandings between the interns and the groups of students in the content-method integrated mathematics program.
9. With initial differences allowed for, there was statistically significant difference on a test of basic mathematical understandings of groups with different (three) entry levels of attitude toward

- mathematics but the difference is not meaningfully large to determine the direction of significance.
10. With initial differences allowed for, there was no method of instruction by entry attitude level interaction on terminal basic mathematical understandings.
  11. With initial differences allowed for, there was no significant difference between the attitude of all the groups toward mathematics.
  12. With initial differences allowed for, there was significant difference only between the groups with medium entry mathematical aptitude and low entry mathematical aptitude in favour of the former.
  13. With initial differences allowed for, there was no method of instruction by aptitude interaction on attitude toward mathematics.
  14. The interns' group perception of their environments of learning is more favourable than the perception of each of other groups. There was no significant difference between the other two groups' perceptions of their environments of learning.
  15. There was no group by entry attitude interaction on perception of learning environments.
  16. The learning units contributed significantly to the terminal basic mathematical understandings of the interns.

17. Entry attitude toward mathematics and the enjoyment of the mathematics class contributed significantly to the terminal attitude of the interns toward mathematics.

On the basis of changes in mathematical achievement on the criterion-referenced measures and the tests of basic mathematical understandings and attitudes the results indicate that most of the interns, though with poor (educational mathematical) background can reach a reasonable level of competency equal and probably higher than that of present regular students in mathematics for prospective elementary school teachers in laboratory-oriented, content-method integrated, program with substantial clinical experience if given sufficient time and help.

### Interpretation and Discussion of Findings

Assuming that outcome of learning depends on the environments of learning it is reasonable to start the interpretation of results from evaluation of learning environments.

### Environments of Learning

Non rejection of Hypothesis D(c) indicates that the relationship between the perception of learning environments of subjects in groups  $G_1$ ,  $G_2$ ,  $G_3$  might well be the same across all levels of entry attitude toward mathematics. This suggests that the responses of subjects on Husen's Attitude Scale within each group were unaffected by their

entry attitude toward mathematics. The probable implication of this is that their perceptions fairly estimates their environments of learning. This implication is however confirmed by acceptance of Hypotheses D(b) that there is no entry attitude effect. Results of Hypotheses D(b) and (c) imply that the subjects' perception of their environments of learning as measured by Husen's Attitude Scales is a fair evaluation of their learning environments.

The result of Hypothesis D(a) combined with implication of results of Hypotheses D(b) and (c) in the preceding paragraph show that the environment of learning of  $G_1$  is significantly superior than each of those of  $G_2$  and  $G_3$  while that of  $G_2$  is not statistically, better than that of  $G_3$ . In other words the evidence indicates that the environments of learning of  $G_1$  which was mastery approach, laboratory-oriented, content and method integrated together with substantial clinical experience provides a learning environment better than that of laboratory-oriented, content-method integrated, together with one hour per week clinical experience; whereas the latter does not provide a significantly better learning environment than the regular program which is partially laboratory-oriented mathematics content separated from methods course which is laboratory-oriented. The non-appreciable difference between the last two obtained from this study might be due to the fact that the pressure was too much for the students in  $G_2$ . This group,  $G_2$ , pointed out in chapter III, had their mathematics

education program in Fall quarter in six-quarter hour class meeting (and one hour clinical experience in school) what the regular program,  $G_3$ , had in eleven-quarter hour class meetings. This pressure obscures the difference between non-mastery laboratory-content method integrated program from the regular content-method separated program. However, the findings agrees with Hollis (1971) study that laboratories organized to provide personal and individualized assistance are helpful to learners that are either culturally or academically disadvantaged.

Related Question: A related question is:

Is the difference between groups' perceptions of learning environment the same across all measures (five subsets)? The multivariate  $F$  for this measure by method (treatment) interaction was 2.2637 which was significant ( $p < 0.0286$ ) in table 11. This indicates a significant interaction, thus the difference in perception of environments of learning between the groups' is not the same for each aspect of environments. Mean score of the groups on each subset of Husen's Attitude Scales are in table 17. Figure shows that the interaction is disordinal. This disordinal interaction shows that only in some cases is the environments of learning as perceived by  $G_1$  superior to others. In fact  $G_1$  is superior in all cases except on the learning of mathematics as process ( $B_1$ ) where they were exceeded by  $G_2$ . The scores of  $G_1$  and  $G_3$  on this subset

indicate that they viewed mathematics as fixed and given once and for all time (a low score) while group  $G_2$  viewed mathematics relatively more as something developing, growing, and changing (a high score) this might be a reflection of efforts of two faculty members on the group. The figure shows that,  $G_3$ 's perception of environment of learning is inferior in all cases except on attitude toward man and environment where they exceeded  $G_2$ .

It can be concluded that a mathematics education program for elementary teachers which uses Bloom's model of mastery learning, which is laboratory-oriented, content-method-integrated with substantial clinical experience provides an environment of learning where trainees' perception of mathematics learning, the role of mathematics in contemporary society, the school and school learning, the relationship of man to his environment is more favourable than the perceptions of trainee in either (1) a program with everything except mastery approach and substantial clinical experience, or (2) a program similar to the present regular mathematics education component of elementary education at the Michigan State University.

### Evaluation of Instruction

The rejection of Hypothesis C1(a) and subsequent a posteriori comparisons suggested that there was strong evidence of nonchance differences due to method of instruction. Subsequently a posterior comparisons, via

least square estimates effects revealed a significant difference in mathematics achievement between  $G_1$  and  $G_3$  in favour of  $G_1$ , and between  $G_2$  and  $G_3$  in favour of  $G_2$  but not between  $G_1$  and  $G_2$ . T-us,

1. The interns basic mathematical understandings at the end of instruction is not significantly different from that of  $G_2$ . The implication of this is that with many of the interns from low-income background and poor mathematics background (test of fitness of model-ANCOVA-support this) it is possible to bring them to the same level of basic mathematical understanding via Bloom's model of mastery learning, as students in the regular program who received some content-method integrated-laboratory-oriented instruction (except for mastery learning). This part agrees with research findings on Bloom's model of mastery learning (Block, 1971). The result is also consistent with findings of Astin et al. (1972) and Ausubel's reversibility theory (1964) that academic deficiency associated with socio-economically disadvantaged children disappears at the college if the disadvantaged is in appropriate program (Astin, 1972) or taught by appropriate method (Ausubel, 1964). The open, effective and average communications among students ( $G_1$ ) under cooperative goal structure instruction might have contributed to the performance of  $G_1$  in the study.

2. The significant difference between  $G_1$  and  $G_3$  on one hand the  $G_2$  and  $G_3$  on the other hand extended the results of Waszly (1970) study to nondisadvantaged and show, in particular,
  - a. that the disadvantaged group, under mastery, content-method integrated approach, could reach a higher level of mathematical understandings for elementary teachers than a regular group in the present regular program at the Michigan State University;
  - b. that the content-method integrated course is capable of providing a better mathematical understanding for regular elementary teachers than the present regular mathematics education component of their program.

The rejection of Hypothesis C1(b) was marginal ( $p < .0415$ ). The subsequent a posteriori procedure using Scheffe's method failed to detect significance in pairwise comparison of the attitude levels. This might be due to two reasons: first Scheffe's technique is known to be weak for pairwise comparison because of its wide confidence interval and especially when the significant is very marginal as in this case. Random dropping of six subjects out of twenty-four in  $G_1$  and three in  $G_2$  in order to use a more powerful a posteriori Turkey-technique is not desirable for we shall only be using 60 percent of the interns who originally signed up for the program. The second might be



due to weak relationship between mathematics achievements and attitude toward mathematics that all research efforts have shown so far contrary to the feelings of mathematics educators.

The acceptance of Hypothesis C1(c) of no treatment by entry level interaction is a confirmation that the findings under Hypothesis C1(a) are independent of the entry attitude levels.

The acceptance of Hypothesis C2(a), that there is no significant difference among the groups on the attitude toward mathematics at the end of instruction is consistent with studies carried out by Vance (1969) and Johnson (1970) on the effect of laboratory approach on attitude measures though contrary to the claims of advocates of laboratory approach. It might be the case as Phillips (1973) pointed out, that the periods of instruction were not sufficiently long enough to affect attitude of the student, especially  $G_2$ .

The rejection of Hypothesis C2(b) that there is no significant difference in attitude of groups with different entry mathematical aptitudes is an important result. A posteriori comparison shows there is significant difference only in the attitude of subjects with medium and low entry mathematical aptitude in favour of the group with medium aptitude. The results indicate that students with high entry mathematical aptitude have medium attitude toward mathematics at the end of instruction, while with

initial medium mathematical aptitude have high attitude toward mathematics, and that students with lowest mathematical aptitude at the beginning of instruction will have lowest attitude toward mathematics at the end of instruction. The acceptance of Hypothesis C2(c) of no treatment by aptitude interaction shows that the result of Hypothesis C2 (b) holds for any method of instruction.

### Evaluation of Learners

The rejection of the multivariate hypothesis and associated univariate Hypothesis B1, B2 that there will be no significant difference between the post-test means and pre-test means of interns on Dossett's and Dutton's instrument and the foregoing discussion show that the mathematics education component of the eight cycle Teacher Corps program meets the needs of the interns.

While 92 percent reached mastery level on Measurement only 83 and 67 percents reached mastery level on the last two topics. This might be explained in two ways. First the Hawthorne's effect of laboratory approach turned them on at the beginning of the program. Secondly, the entire Teacher Corps program seemed to be well organized at that time. However, in Spring term there were administrative difficulties in scheduling the mathematics class. It was not certain, until very late, that mathematics class would be held, though this was in the original proposal. By the time this was settled, another class has been scheduled

to use the mathematics laboratory at the hours available for the interns. While every effort was made to bring the necessary materials to the classroom used this term, most of the interns felt that this was not truly a laboratory setting. This feeling might have some sort of negation of Hawthorne's effect--that they would not do well in non-laboratory setting--on the interns. This expectation of the interns (not the instructor) manifested into a modified form of Jacobson-Rosenthal effect on the interns' achievements this term.

Apart from the performance of interns on Fractions which might also be affected by the end of term's pressure on the interns from other courses in their schedule, the overall performance seems to follow findings on percentage of students that attain mastery under Bloom's model of mastery learning (Peterson, 1972).

### Recommendations

The following recommendations are based on the investigator's interpretation of the results of this study and personal observations in carrying out the present evaluation.

#### Recommendations for Actions

##### a. Based on the Study

It is recommended that provision of enrichment activities be continued with the interns who demonstrate the desire and ability.

It is recommended that more Geometry be included in the Teacher Corps Mathematics Education Program.

It is recommended that the present method of instruction-mastery approach laboratory-oriented-content-method integrated with substantial time in clinical experience--be continued with increasing interjecting lectures.

It is recommended that elementary probability and statistics be included in the Teacher Corps program.

b. Based on Personal Observations

It is recommended that the faculty members involved with Teacher Corps spend more time in schools with the interns.

In order to achieve this type of involvement from college faculty people certain sacrifices seem eminent. Perhaps the most important of these is money. The cost of training teachers via field experienced based programs in which college faculty spend time in public schools would most likely be greater than the present nonfield based programs. It is also quite likely that this field-experience-based program would require additional personnel to be available for supervision.

It is recommended that the team leaders be required or encouraged to take some graduate courses in elementary mathematics education to improve their competency in the supervision of the interns.

It is recommended that interns be trained in questioning technique which will enable them to succeed in directing children's learning.

It is recommended that interns be encouraged to develop interest in reading the Arithmetic Teacher. It is recommended that this reading be required not just optional.

It is recommended that wherever the Indiana materials are to be used efforts should be made to secure the accompanying tapes.

It is recommended that class attendance be made compulsory for the interns.

#### Recommendations for Future Research

It is recommended that the relationship between the present regular mathematics education program and a well-structured laboratory-oriented mathematics content-method integrated (with interjecting lectures) program for prospective elementary teachers be examined in depth to determine differences in mathematics achievement and knowledge of teaching elementary school mathematics topics.

It is recommended that the effect of the mathematical instruction on the cognitive and affective behaviours of elementary school children taught by the present interns be studied.

It is recommended that more research should be conducted on treatment by aptitude interaction on mathematics achievement, treatment by attitude interaction on achievement.

It is recommended that more studies should be conducted on treatment by attitude interaction on attitude, treatment by aptitude interaction on attitude.

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## APPENDICES



**APPENDIX A**

**SET OF FIVE PRE- AND POST-TEST FORMS OF THE  
CRITERION-REFERENCED MEASURES**

## APPENDIX A

### SET OF FIVE PRE- AND POST-TEST FORMS OF THE CRITERION-REFERENCED MEASURES

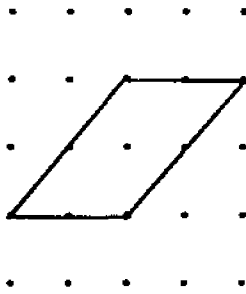
#### MEASUREMENT--PRE-TEST

Name \_\_\_\_\_

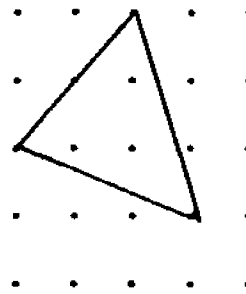
1. I need 45 square feet of a carpet to cover a floor. If it is sold in strips 24 inches wide, how long a strip do I need?
2. Select a grade level (K-2, 3-4, or 5-6). Name four manipulative aids that would be useful in teaching a measurement unit.
3. Which of the following is the shortest?
  - a. 20 centimeters
  - b. 30 inches
  - c. one meter
  - d. one yard
  - e. one decimeter
4. A wire is 20 centimeters long. It is bent to form a rectangle. What is the maximum area that can be completely enclosed by the wire?
5. Suppose the area of a triangle is fixed at 30 square units. Let its base be 'b' and its height be 'h'. Pick four different volumes of 'b' and calculate the corresponding volumes of 'h'. Graph the points. Use this to find the base of a triangle whose area is 30 square units and height is two--five units.

What is the slope of your graph?

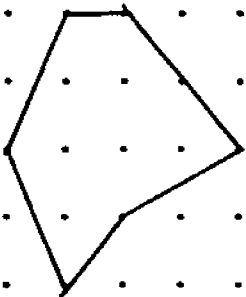
6. Find the area of the following figures:



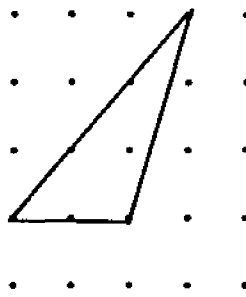
(a)



(b)



(c)



(d)

7. Two trees 50 meters apart are represented by two points which are four centimeters apart on a map. How far apart are two trees which are represented by two points six centimeters apart on the map?
  - a. 25 meters
  - b. 500 meters
  - c. 60 meters
  - d. 75 meters
  - e. 80 meters
8. A book is found to be as long as twenty-one paper clips. The same book is as long as nine equal pencils. If a paper clip is found to be two centimeters long, how long is the pencil?
9. If a car travelled at an average speed of twenty-five m.p.h., how long would a journey of 175 miles take?
  - a. 17 hours
  - b. 7 hours
  - c. 6 hours
  - d. 4 hours
  - e. 5 hours
10. It is now a belief that practical work should precede computational practice at early grades. With this in mind, outline a lesson plan for an introductory lesson on weight that will show the need for a standard unit.

## MEASUREMENT--POST-TEST

Name \_\_\_\_\_

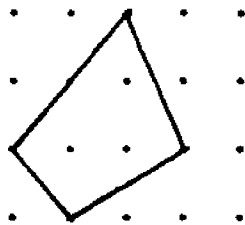
Student No. \_\_\_\_\_

1. Suppose the perimeter of a rectangle is fixed at 24 centimeters. Let  $x$  centimeters be its width and  $y$  centimeters be its length. Pick five different values of  $x$  and calculate the corresponding values of  $y$ . Tabulate your results and graph the points.

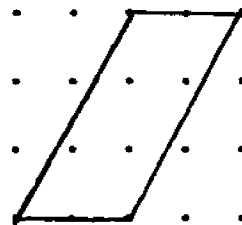
What is the shape of the graph?

What are the dimensions of the rectangle with the fixed perimeter of 24 that has the largest area?

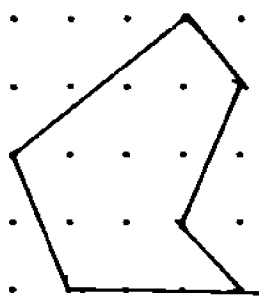
2. A book is found to be twenty-one paper clips long. The same book is seven (equal) pens in length. If a pen is two centimeters long, how long is a clip?
3. Design a lesson plan which will teach the formula  $L \times W = A$  to a group of three to five learners. Identify the pre-requisite mathematical concepts and/or skills for this lesson.
4. Which of the following is the longest?
- |              |                  |                   |
|--------------|------------------|-------------------|
| a. one meter | b. one yard      | c. one millimeter |
| d. 30 inches | e. one decameter |                   |
5. Find the area of the following figures:



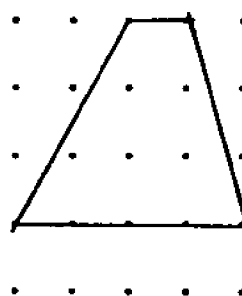
(a)



(b)

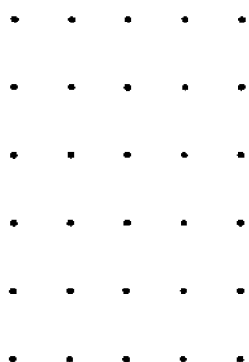


(c)



(d)

6. A triangle, on a geoboard, of area 7 units had 4 boundary nails. How many interior nails must it have? Represent such a triangle on these dots.



7. You are assigned the task of supplying the math lab with materials that would be useful in teaching a measurement unit. List all of the materials you would select, quantities of each, and how you would organize them in the lab. (Assume you want enough materials for at least three classrooms to be able to use at the same time).
8. Forty unequal pieces of stones are to be ordered by a scientist according to their weights. If the only thing available to him is just a balance (without any known weight), what is the minimum number of weighings he has to do to achieve his objective?
9. In a weighing exercise where a given block of bronze is the unit, it is found that two such blocks balanced three aluminum discs, two paper-clips balanced one aluminum disc. What weight should be assigned to a paper clip?
10. (a)  
Describe an activity that when graphed produces points which do not lie on a straight line. What is the slope of such graph.
- (b)  
Give an example of a measurement activity in which the transitivity property must be used to achieve the desired results. Explain how it must be used.

Name \_\_\_\_\_

## NUMERATION--PRE-TEST

Student No. \_\_\_\_\_

1. If you were judging a culture's numeration system (the way in which numbers were recorded), list two properties that would be considered important for the numeration system to have.
2. In a culture's numeration system the following symbols:  $\phi$ ,  $\alpha$ ,  $\beta$ ,  $\Gamma$ ,  $\Delta$ ,  $\nabla$ ,  $\overline{7}$ ,  $\pi$ ,  $\Lambda$ , and  $x$  represent respectively, 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 so that symbol " $\Gamma\phi$ " represents 30.
  - a. Which of the following is equal to  $7/23$ ?
 

A. $\Gamma/\beta\overline{7}$	B. $\pi/\Gamma\beta$	C. $\pi/\beta\overline{7}$
D. $\pi/\beta\Gamma$	E. $\overline{7}/\alpha\beta$	
  - b. What does  $\Delta\phi\Lambda$  represent?
 

A. 12	B. 48	C. 804
D. 408	E. 21	
3. Give an example of a grouping activity designed to develop a child's understanding of numeral 27.
4. The decimal expansion of the numeral 35.72 is:
 

A. $3 (10) + 5 (1) - 7 (10) - 2 (1)$
B. $3 (10) + 5 (1) - 7 (10) - 2 (100)$
C. $3 (10) + 5 (10^0) - 7 (10^1) - 2 (10^0)$
D. $3 (10^1) + 5 (10^0) + 7 (10^{-1}) + 2 (10^{-2})$
E. $3 (10^1) + 5 (10^0) + 7 (10^{-1}) + 2 (10^{-0})$
5. In base ten,  $421_{\text{five}}$  is:
 

A. 551	B. 111	C. 821
D. 100101	E. 35	
6. Which of the following is correct?
 

A. $23_{\text{ten}} = 32_{\text{seven}}$	D. $23_{\text{ten}} = 32_{\text{eight}}$
B. $23_{\text{ten}} = 32_{\text{five}}$	E. None of the above is correct
C. $23_{\text{ten}} = 32_{\text{six}}$	

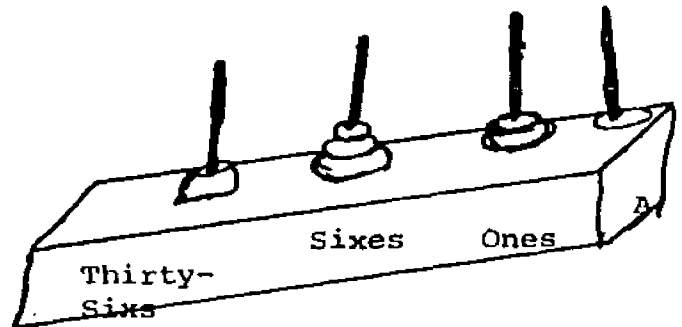
7. In which base is  $36 + 54 = 1127$

- A. five                      B. four                      C. eight                      D. seven  
E. none of the above

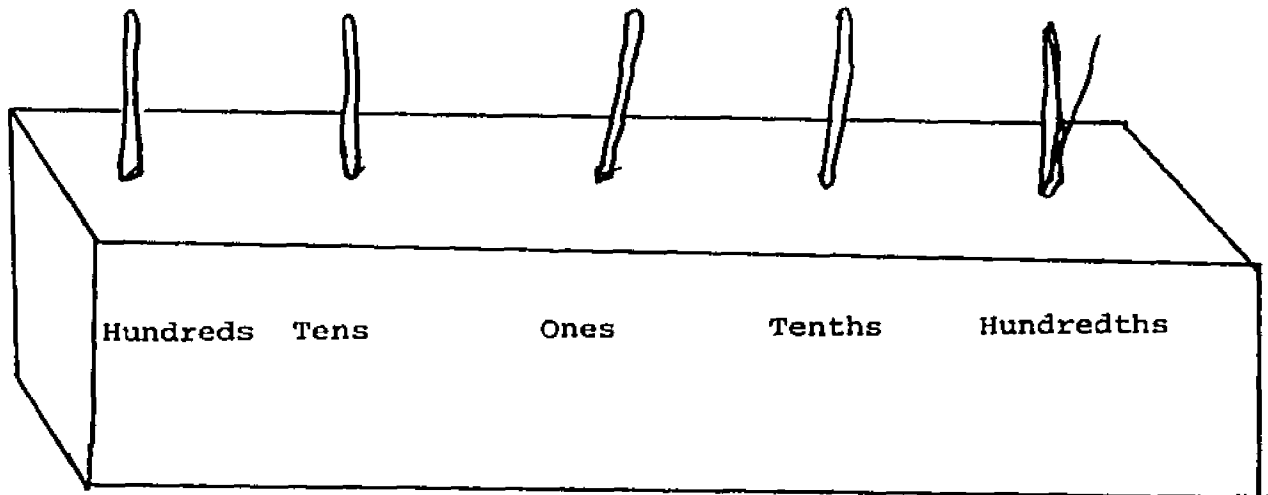
8. Name four manipulative aids you would use to teach a unit on numeration to children and arrange the materials in the order you would use them with your pupils.

9. The number represented on this instrument is

- A.  $132.1_{\text{six}}$   
B.  $1.231_{\text{six}}$   
C.  $1321_{\text{six}}$   
D.  $1321_{\text{thirty six}}$   
E.  $132.1_{\text{thirty six}}$



10. Using the labelling shown, represent  $427_{\text{ten}}$  on this instrument



Name \_\_\_\_\_

Student No. \_\_\_\_\_

## NUMERATION--POST-TEST

PLEASE SHOW ALL WORK

1. State three important characteristics of a good numeration system.
2. Jim stated that there are 44 hours in a day. He must be working in base
  - (a) four    (b) five    (c) six    (d) three    (e) none of these
3. What would you do with youngsters to enable them to understand the significance of the zero in 2035?
4. Describe an activity that would introduce the concept of decimals to elementary school youngsters.
5. (i)  $3(5^2) + 1(5^1) + 2(5^0) + 4(5^{-1})$  is the expanded notation of
  - (a)  $610.4_{\text{five}}$     (b)  $312.4_{\text{five}}$     (c)  $50_{\text{ten}}$     (d) 3124 times five
  - (e)  $75 + 5 + 2 - 20$
 (ii) In base ten,  $124_{\text{eight}}$  is:
  - (a)  $8 \times 1 + 8 \times 2 + 8 \times 4$     (b)  $(1 + 2 + 4) \times 8$
  - (c)  $1 \times 8 + 2 \times 8^2 + 4 \times 8^3$     (d)  $1 \times 10^{16} + 2 \times 10^8 + 4 \times 10^0$
  - (e)  $1 \times 8^2 + 2 \times 8^1 + 4 \times 8^0$
6. (i) The numeral  $32_{\text{ten}}$  is different from
  - (a)  $52_{\text{six}}$     (b)  $200_{\text{four}}$     (c)  $44_{\text{seven}}$     (d)  $62_{\text{five}}$
  - (e)  $40_{\text{eight}}$
 (ii)  $3211_4 = \underline{\hspace{2cm}}_{10}$   
 (iii)  $600_{10} = \underline{\hspace{2cm}}_5$
7. A number is represented using base 6 wood in the following way:  
 3 cubes    4 flats    5 longs    3 units  
 Using the smallest number of pieces of base 4 woods, represent the same number.



8. List four concrete materials you would suggest for use in numeration. List them in the order in which you would use them with elementary school children and defend your ordering.

9. Given that

$$\begin{array}{r}
 \square \square 2 \\
 + \\
 4 \quad 5 \quad \square \\
 \hline
 10 \square 3
 \end{array}$$

is an addition problem in which each addend has three digits and the numerals are base six representation. Find the missing numerals.

10. Summarize effective experiences for helping primary pupils learn the concept of place value.

Name \_\_\_\_\_

Student No. \_\_\_\_\_

## ADDITION AND SUBTRACTION--PRE-TEST

1. What property or properties of addition are you using when you check addition by adding from bottom to top after you have added from top to bottom?
2. Give one real world situation in the life of a child that would require subtraction for its solution.
3. How would you correctly regroup 432 in the following subtraction?

$$\begin{array}{r} 432 \\ -179 \\ \hline \end{array}$$

4. List three objectives that are important to attain in the formation of the concept of number which serve as a prelude to addition of whole numbers.
5. Give three models for addition with one example of each model.
6. List three manipulative aids that are helpful in developing the concepts of addition and subtraction.
7. Games can be used to help children master basic combinations of numbers. Describe one activity that can be used for such a purpose.
8. Give an example of a problem which employs the missing addend approach to subtraction.
9. List and give examples of 2 properties that hold true for the addition of whole numbers.
10. (a) The inverse of subtraction is division (T or F) \_\_\_\_\_
- (b) The set of whole numbers is closed with respect to subtraction (T or F) \_\_\_\_\_
- (c) The property that is illustrated in  $(17 + 24) + 13 = 17 + (24 + 13)$  is shown as the parentheses shift property (T or F) \_\_\_\_\_
- (d) Regrouping is needed in every addition problem (T or F) \_\_\_\_\_
- (e) The operations of addition and subtraction, on the set of whole numbers, are commutative (T or F) \_\_\_\_\_

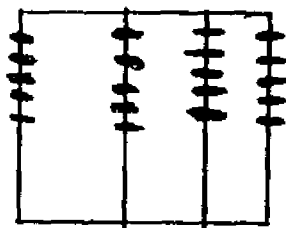
Name \_\_\_\_\_

Student No. \_\_\_\_\_

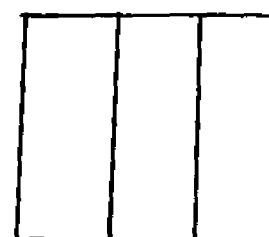
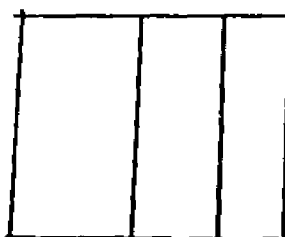
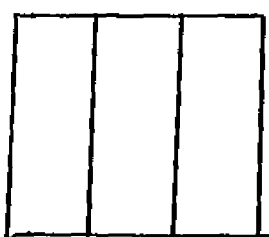
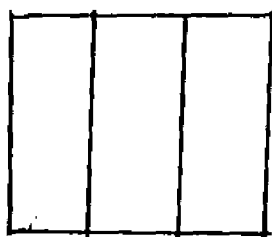
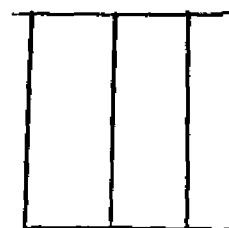
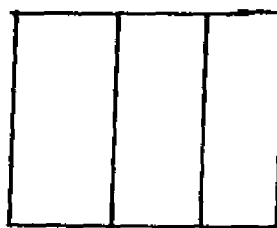
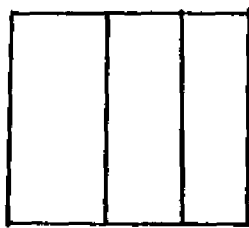
## ADDITION AND SUBTRACTION--POST-TEST

1. Name one visual aid and two manipulative-aids that can be used to develop the concepts of addition and subtraction. Choose one of the three and describe briefly how you would use it to help develop the concept of subtraction.
2. Show pictorially all steps involved in working the following problem on the abacus. Assume each place has 5 and only 5 beads and that each bead must be pictured somewhere. The problem should be worked as you would have the elementary school child work it.

$$43_5 - 14_5$$



Start



3. Give reasons in terms of the properties of whole numbers that justifies each step of the following:

$$\begin{aligned}
 (7 + 12) + 23 &= 7 + (12 + 23) \underline{\hspace{2cm}} \\
 &= 7 + (23 + 12) \underline{\hspace{2cm}} \\
 &= (7 + 23) + 12 \underline{\hspace{2cm}} \\
 &= (7 + 23) + 12 + 0 \underline{\hspace{2cm}} \\
 &= [(7 \times 1) + 23] + 12 + 0 \underline{\hspace{2cm}}
 \end{aligned}$$

4. Outline a lesson designed to introduce to a class of second or third graders the idea of regrouping or "carrying" in addition, using the problem

$$\begin{array}{r} 26 \\ +17 \\ \hline \end{array}$$

Describe briefly how you would use the appropriate concrete material and how you would make the transition from concrete to abstract (symbolic).

5. (a)  
Complete the table for the addition facts in base five

+	0	1	2	3	4
0					
1					
2					
3					
4					

- (b)  
Solve the following

$$\begin{array}{r} 23_5 \\ + 34_5 \\ \hline \end{array}$$

$$3_5 + \square = 12_5$$

$$13_5 - 4_5 = \square$$

6. Identify, with examples, four properties that hold for the following system. The given set is  $\{\square, \triangle, \circ\}$  and the operation is  $\oplus$ .

$\oplus$	$\square$	$\triangle$	$\circ$
$\square$	$\square$	$\triangle$	$\circ$
$\triangle$	$\triangle$	$\circ$	$\square$
$\circ$	$\circ$	$\square$	$\triangle$

1.

2.

3.

4.

7. Three approaches to subtraction were presented: comparison, take-away and missing addend. Give a real world example, from an elementary school child's world, and the symbolic representation of each.
8. Briefly outline the experiences involving addition activities you would give a child prior to the time when he would work the problem  $348 + 875$  in the following way:

$$\begin{array}{r} 348 \\ +875 \\ \hline 1223 \end{array}$$

9. The text lists 10 objectives for re-addition and subtraction skills (skills that you would want a child to have prior to beginning addition or subtraction). Give four of these skills. You may get credit for your own if they are valid pre-addition subtraction skills.
10. Show pictorially all steps involved in working the following base ten addition problem. Assume each place has 10 and only 10 beads and that each bead must be pictured somewhere. The problem should be worked as you would have the child work it.

$$385 + 468$$

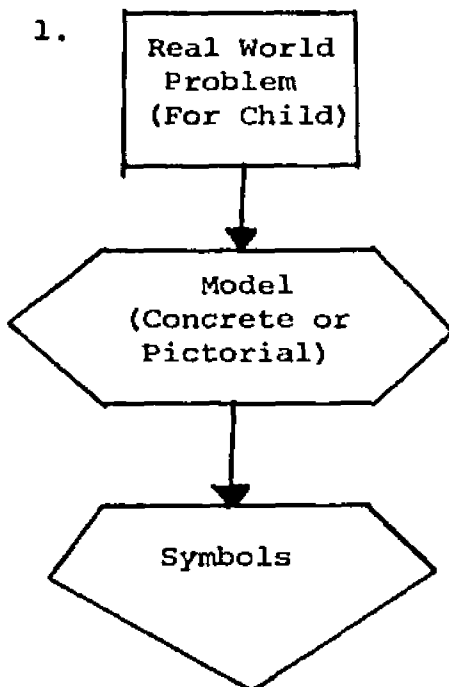
Name \_\_\_\_\_

Student No. \_\_\_\_\_

## MULTIPLICATION AND DIVISION--PRE-TEST

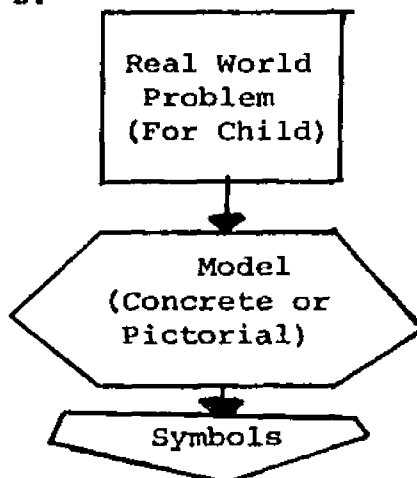
In relating the child's real world and mathematics, the teacher often starts with a problem which can initially be represented by concrete models (using real objects, counters, etc.) or pictorial models (using pictures, diagrams, etc.). Finally the model can be used to translate a real problem into mathematical symbols. In the first two problems one of the three parts of the process outlined above will be given. Your job will be to fill in the two missing parts.

1.



$$24 \div \square = 8$$

2.



There were 9 people in Jerry's club; if each person paid 3 cents for dues, how much would be paid all together?

3. Express the numbers 26 and 32 in expanded notation and then show how you apply the distributive law to solve the multiplication problem  $26 \times 32$ .
4. Explain how addition is related to multiplication and how you might use this relationship to introduce the idea of multiplication to kids.
5. Explain how subtraction is related to division.
6. What is the least common multiple of 24 and 36? What is the greatest common divisor of 24 and 36? (Show work)
7. Construct the multiplication table for single digit numbers in base five. Use this table to solve:
  - a. 
$$\begin{array}{r} 102 \\ \times 3 \\ \hline \end{array}$$
  - b.  $22 \div 3 = \square$
  - c.  $3 \times \square = 11$
8. Pick some manipulative aid and explain how you would use it to teach any concept you choose related to multiplication or division of whole numbers.
9. Define the following:
  2. a. The division algorithm
  2. b. Factor
  1. c. Multiple
10. After each property stated below write if the property holds for the set of whole numbers for the operation given. Support your answers with examples.
  - a. Multiplication--Commutative Property \_\_\_\_\_
  - b. Division--Commutative Property \_\_\_\_\_
  - c. Division--Identity Property \_\_\_\_\_
  - d. Multiplication--Inverse Property \_\_\_\_\_

Name \_\_\_\_\_

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## MULTIPLICATION AND DIVISION POST-TEST

1. Complete the following with TRUE or FALSE. If your answer is FALSE, explain why.
  - a.  $\{0, 1, 2\}$  is not closed under multiplication \_\_\_\_\_
  - b. The least common multiple of 84 and 126 is 42 \_\_\_\_\_
  - c. 21 is a multiple of 3 and a factor of 105 \_\_\_\_\_
  - d. The rectangular array, subtraction, the set, and the number line all provide models for multiplication of whole numbers \_\_\_\_\_
  - e.  $(3 \times a) \div 3 = a$  \_\_\_\_\_
2. Describe three plausible "thinking strategies" elementary school children might use in finding the answer to  $9 \times 7 = ?$
3. Describe how you would develop the concept of multiplication using a Real World Problem--Model-Symbol strategy. Be sure to include an example as you describe your strategy.
4. Pick two properties of whole numbers for the operation of multiplication and describe how they can be used to help children in early multiplication learning.
5. Multiply 123 by 38 using one of the following non-standard algorithms:

Lattice Method

Russian Method

Doubling

6. The following demonstrates one person's calculations using a "transitional" algorithm for division of whole numbers:

78	2267	10
	-780	
	1482	10
	-780	
	702	5
	-390	
	312	2
	-156	
	156	2
	-156	
	0	

therefore,  $2262 \div 78 = 29$



- a. Discuss the advantages of having elementary school children use this method.
  - b. Why does this method work? (give mathematical evidence)
7. a. The set of whole numbers is not closed under division. Why?
- b. If  $n$  and  $d$  are whole numbers with  $n$  greater than or equal to  $d$  and  $d$  not equal to zero then, there exists unique whole numbers  $q$  and  $r$  such that  $n = (q \times d) + r$ . What is the restriction on  $r$  for
- c. Define or describe the following:
1. Algorithm
  2. Least Common Multiple
  3. Greatest Common Factor

8.

Table 1

$\otimes$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Table 2

$\otimes$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

NOTE:  $\oslash$  is the inverse of  $\otimes$

- a. From TABLE 1, solve completely  $3 \otimes \underline{\hspace{1cm}} = 1$
- b. From TABLE 1, solve completely  $3 \oslash 2 = \underline{\hspace{1cm}}$
- c. From TABLE 1, solve completely  $\underline{\hspace{1cm}} \otimes 2 = 3$
- d. From TABLE 2, solve completely  $3 \oslash \underline{\hspace{1cm}} = 4$
- e. From TABLE 2, solve completely  $2 \otimes \underline{\hspace{1cm}} = 1$

9. Discuss the Weaver article "Big Dividends From Little Interviews" and include the following:
  - a. Brief summary of article
  - b. One fact that Miss Watkins (who is actually a red-hot Mama) found out about any of the six kids that helped her assess their needs.
  - c. One advantage and one disadvantage of using her technique.
10. Show, by example, 3 plausible errors that elementary school children might make in attempting to solve multiplication (or division) problems using the STANDARD ALGORITHM.

Name \_\_\_\_\_

Student No. \_\_\_\_\_

## FRACTION PRE-TEST

1. Briefly describe a strategy used to introduce fractions to elementary school children.
2. Explain how to develop the equivalence class  $\{2/3, 4/6, 6/9, \dots\}$  using a rectangle as an aid.
3. Explain how you could use the idea of equivalence classes of fractions to find the solution to  $1/3 + 3/4$ .
4. List five different types of manipulates that would be helpful in developing understandings in a fraction unit. State briefly how each might help.
5. Which is larger  $21/55$  or  $34/89$ ? Why?
6. When dividing fractions, say,  $2/5 \div 3/7$  the algorithm states: invert the divisor and multiply; that is,  $2/5 \div 3/7 = 2/5 \times 7/3$ . Explain why that produces the correct answer.
7. Using a rectangular model show how to represent the problem  $2/3 \times 3/4$ . Show also how the rectangular model shows the solution.
8. Given the set of fractions greater than zero (numbers of the form  $a/b$  where  $a$  and  $b$  are natural numbers) state all the properties, with examples, that are true for multiplication.
9.
  - a. State a number greater than  $2/3$  but less than  $3/5$ .
  - b. Exactly how many  $3/4$ 's are there in 1?
  - c. Express  $.412$  as a fraction.
  - d. Express  $.121212 \dots$  as a fraction.
  - e.  $(2/3 \div 3/4) \div 7/8 =$
10. Show  $1/5 + 2/3$  using the number line as a model.

Name \_\_\_\_\_

Student No. \_\_\_\_\_

## FRACTIONS POST-TEST

1. In beginning a unit on fractions one would usually start with examining the meaning of  $1/2$  where  $a$  is a non-zero whole number. Briefly describe the concepts that would follow and indicate the ORDER in which they should be taught. Be sure the list is a complete overview of those concepts (and their order) that would be included in a unit on fractions.
2. Explain how to develop the equivalence class  $\{3/5, 6/10, 9/15, 12/20, \dots\}$  using the rectangle as a model.
3. Explain how you would use the idea of equivalence classes to find the solution to  $1/3 + 2/5$ .
4. Solve the following ("reduce" answers to lowest terms) (show work)
  - a.  $13/756 + 17/504 =$
  - b.  $14 \frac{2}{3} \div 9 \frac{7}{8} =$
5. In adding  $2/7$  and  $1/3$  the phrase "selecting a proper form of one" can be used. Explain exactly what this means.
6. A child asks: "When you reduced  $6/8$  to  $3/4$ , you said that you divided both 6 and 8 by two to get the answer. What gives you the right (mathematically) to do this?" Answer his question.
7. Give an explanation why inverting the divisor and then multiplying produces the correct answer when dividing fractions.
8. Using a rectangular model show how to represent  $1 \frac{2}{3} \times 2 \frac{3}{4}$ . Use the model to solve the problem.
9. List three different types of manipulatives that would be helpful in developing a unit on fractions. State briefly how each might help.
10. Explain how to solve  $2/3 \div 3/4$  using Cuisenaire Rods.

## APPENDIX B

A TEST OF BASIC MATHEMATICAL UNDERSTANDINGS  
FORM A (PRE-TEST) AND FORM B (POST-TEST)

## APPENDIX B

### A TEST OF BASIC MATHEMATICAL UNDERSTANDINGS

#### FORM A (PRE-TEST) AND FORM B (POST-TEST)

### A Test of Basic Mathematical Understandings

Prepared By:

Dr. Mildred Jerline Dossett

Michigan State University

East Lansing, Michigan

1964

#### Directions:

This test is designed to measure your understanding of mathematics. Many of the items relate to the new content in present programs of mathematics for elementary pupils.

Each of the fifty-five questions is of multiple-choice type and includes four possible answers. Read each question carefully and decide which answer fulfills the requirements of the statement. Then circle the response on the answer sheet to indicate your choice.

Circle only one answer for each question. If you change your choice, erase your original mark and circle the correct one.

#### Sample Question:

1. Which of the following shows the decimal form of the fraction  $\frac{5}{4}$ ?
- |         |         |
|---------|---------|
| a. 125  | b. 12.5 |
| c. 1.25 | d. .125 |

Answer Sheet:

1. a      b      **(c)**      d

Since 1.25 is the correct answer, the letter (c) is circled.

## FORM A (PRE-TEST)

1. When you write the numeral "5" you are writing
  - a. the number 5
  - b. a pictorial expression
  - c. a symbol that stands for an idea
  - d. a Hindu-Babylonian symbol
2. Bill discovered that  $>$  means "is greater than" and  $<$  means "is less than." In which of the following are these symbols not used correctly?
  - a. The number of states in the United States  $<$  the number of United States Senators.
  - b. The number of states in the United States  $<$  the number of stripes in the flag.
  - c.  $2^3 > 3^2$
  - d.  $3 + a < 5 + a$
3. When two Roman numerals stand side by side in a symbol, their values are added.
  - a. always
  - b. sometimes
  - c. never
  - d. if the base is X

4. Which of the following describe/describes our own system of numeration?
- a. additive
  - b. positional
  - c. subtractive
  - d. introduces new digits for numbers larger than 10
- 1. a and b are correct
  - 2. a and c are correct
  - 3. a and d are correct
  - 4. a, b, and d are correct
5. Zero may be used
- a. as a place holder
  - b. as a point of origin
  - c. to represent the absence of quantity
  - d. in all of the above different ways
6. 2,200.02 is shown by
- a.  $2000 + 200 + 20$
  - b.  $2000 + 20 + 2/10$
  - c.  $2000 + 200 + 2/100$
  - d.  $2000 + 200 + 200$
7. 5840 rearranged so that the 8 is 200 times the size of 4 would be
- a. 5840
  - b. 8540
  - c. 5048
  - d. 5408



8. Which of the following does not show the meaning of  $423_{\text{ten}}$ ?

- a.  $(4 \times 100) + (2 \times 10) + 3(1) = 423$
- b.  $42 \text{ tens} + 3 \text{ ones} = 423$
- c.  $423 \text{ ones} = 423$
- d.  $4 \text{ hundreds} + 42 \text{ tens} + 23 \text{ ones} = 423$

9. A numeral for the X's in this example can be written in many different bases. Which numerals are correct?

a.  $100_{\text{four}}$

b.  $14_{\text{twelve}}$

c.  $16_{\text{ten}}$

d.  $31_{\text{five}}$

XX	X	XX	XX
X	X	X	X
X	XX	X	X

- 1. a and c are correct
- 2. b and c are correct
- 3. a, b, and c are correct
- 4. all four are correct

10. A "2" in the third place of a base twelve number would represent

- a.  $2 \times 12^3$
- b.  $12 \times 2^3$
- c.  $12 \times 2^{12}$
- d.  $2 \times 12^2$

11. In this addition example, in what base are the numerals written?

- a. base two  $120_?$
- b. base three  $+10_?$
- c. base four  $200_?$
- d. none of the above

12. About how many tens are there in 6542?
- a. 6540
  - b. 654
  - c.  $65 \frac{1}{2}$
  - d. 6.5
13. Place or order in a series is shown by
- a. book no. 7
  - b. three boxes of matches
  - c. a dozen cupcakes
  - d. two months
14. Which of the following indicates a group?
- a. 45 tickets
  - b. track 45
  - c. page 54
  - d. apartment No. 7
15. The sum of any two natural numbers
- a. is not a natural number
  - b. is sometimes a natural number
  - c. is always a natural number
  - d. is a natural number equal to one of the numbers being added
16. The counting numbers are closed under the operations of
- a. addition and subtraction
  - b. addition and multiplication
  - c. addition, subtraction, multiplication, and division
  - d. addition, subtraction, and multiplication

17. If  $a$  and  $b$  are natural numbers, then  $a + b = b + a$  is an example of
- a. commutative property
  - b. associative property
  - c. distributive property
  - d. closure
18. If  $a \times b = 0$  then
- a. must be zero
  - b.  $b$  must be zero
  - c. either  $a$  or  $b$  must be zero
  - d. neither  $a$  nor  $b$  must be zero
19. When a natural number is multiplied by a natural number other than 1, how does the answer compare with the natural number multiplied?
- a. larger
  - b. smaller
  - c. the same
  - d. can't tell from information given
20. Which of the following is the quickest way to find the sum of several numbers of the same size?
- a. counting
  - b. adding
  - c. subtracting
  - d. multiplication

21. How would the product in this example be affected if you put the 29 above the 4306 and multiplied the two numbers?

- a. The answer would be larger
- b. The answer would be smaller
- c. You cannot tell until you multiply both ways
- d. The answer would be the same

$$\begin{array}{r} 4306 \\ \times 26 \\ \hline \end{array}$$

22. An important mathematical principle can be helpful in solving the following example.

$$28 + 659 + 72 = \boxed{\phantom{000}}$$

What principle will be of most help?

- a. the associative principle
  - b. the commutative principle
  - c. the distributive principle
  - d. both the associative and distributive principles
23. The product of  $356 \times 7$  is equal to
- a.  $(300 \times 50) \times (6 + 7)$
  - b.  $(3 \times 7) + (5 \times 7) + (6 \times 7)$
  - c.  $300 \times 50 \times 6 \times 7$
  - d.  $(300 \times 7) + (50 \times 7) + (6 \times 7)$
24. Which of the following is not a prime number?
- a. 271
  - b. 277
  - c. 281
  - d. 282

25. Which of the following numbers is odd?

- a.  $18 \times 11$
- b.  $11 \times 20$
- c.  $99 \times 77$
- d. none of the above

26. The inverse operation generally used to check multiplication is

- a. addition
- b. subtraction
- c. multiplication
- d. division

27. The greatest common factor of 48 and 60 is

- a.  $2 \times 3$
- b.  $2 \times 2 \times 3$
- c.  $2 \times 2 \times 2 \times 2 \times 3 \times 5$
- d. none of the above

28. Look at the example at the right. Why is the "4" in the third partial product moved over to places and written under the 2 of the multiplier?

$$\begin{array}{r} 157 \\ \times 246 \\ \hline 942 \\ 628 \\ \hline 314 \end{array}$$

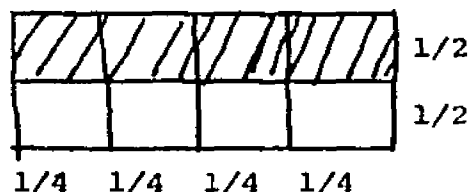
- a. If you put it directly under the other partial products, the answer would be wrong.
- b. You must move the third partial product two places to the left because there are three numbers in the multiplier.
- c. The number 2 is the hundreds column, so the third partial product must come under the hundreds column.
- d. You are really multiplying by 200.

29. Which of the fundamental properties of arithmetic would you employ in proving that  $(a + b) + (a + c) = 2a + b + c$ ?
- a. associative property
  - b. commutative property
  - c. associative and distributive properties
  - d. associative and commutative properties
30. If  $N$  represents an even number, the next larger even number can be represented by
- a.  $N + 1$
  - b.  $N + 2$
  - c.  $N + N$
  - d.  $2 \times N + 1$
31. Every natural number has at least the following factors:
- a. zero and one
  - b. zero and itself
  - c. one and itself
  - d. itself and two
32. It is said that the set of whole numbers has a natural order. To find the successor of a natural number, one must
- a. add 1
  - b. find a number that is greater
  - c. square the natural number
  - d. subtract 1 from the natural number

33.. The paper below has been divided into 6 pieces. It shows



- a. sixths
  - b. thirds
  - c. halves
  - d. parts
34. A fraction may be interpreted as:
- a. a quotient of two natural numbers
  - b. equal part/parts of a whole
  - c. a comparison between two numbers
  - d. all of the above
35. When a common (proper) fraction is divided by a common fraction, how does the answer compare with the fraction divided?
- a. it will be larger
  - b. it will be smaller
  - c. it will be twice as large
  - d. there will be no difference
36. Which algorithm is illustrated by the following sketch?



- a.  $\frac{1}{2} \times \frac{3}{4} = ?$
- b.  $\frac{1}{2} + \frac{3}{3} = ?$
- c.  $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = ?$
- d.  $\frac{4}{4} - \frac{3}{2} = ?$

37. Another name for the inverse for multiplication of a rational number is the
- a. reciprocal
  - b. opposite
  - c. reverse
  - d. zero
38. Examine the division example on the right. Which sentence best tells why the answer is larger than the 5?  $5 \div \frac{3}{4} = 6 \frac{2}{3}$
- a. Inverting the divisor turned the  $\frac{3}{4}$  upside down.
  - b. Multiplying always makes the answer larger.
  - c. The divisor  $\frac{3}{4}$  is less than 1.
  - d. Dividing by proper and improper fractions makes the answer larger than the number divided.
39. The value of a common fraction will not be changed if
- a. We add the same number to both terms.
  - b. We multiply one term and divide the other term by that same number.
  - c. We subtract the same amount from both terms.
  - d. We multiply both terms by the same number.
40. The nearest to 45% is
- a. 44 out of 100
  - b. .435
  - c. 4.5
  - d. .405



41. The principal of a school said that 27 percent of the pupils went to the museum. Which statement best describes the expression "27 percent of the pupils went to the museum"?
- a. It means that 27 children out of every 100 children went to the museum.
  - b. It means that you must multiply the number of children in the school by  $27/100$  to find the number who went to the museum.
  - c. If the children were divided into groups of 100 and those who went to the museum were distributed evenly among them, there would be in each group 27 who went to the museum.
  - d. 27 percent is the same as .27--a decimal fraction written in percent form.
42. Sally completed  $2/3$  of the story in 12 minutes. At that rate how long will it take her to read the entire story?
- a. 18 minutes
  - b. 12 minutes
  - c. 6 minutes
  - d. 24 minutes
43. There were 400 students in the school. One hundred percent of the children had lunch in the cafeteria on the first day of school. On the second day 2 boys were absent and 88 children went home for lunch. Which of the following equations can be used to find the percent of the school enrollment who went home for lunch?
- a.  $400 - 88 = x$
  - b.  $x/100 = 88/400$
  - c.  $x/88 = 400$
  - d.  $400 - 90 = x$

44. What can be said about  $y$  in the following open sentence if  $x$  is a natural number?

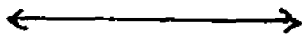
$$x + x + 1 = y$$

- a.  $x < y$
  - b.  $x > y$
  - c.  $x = y$
  - d.  $x \neq y$
45. Which one of the following fractions will give a repeating decimal?
- a.  $1/2$
  - b.  $3/4$
  - c.  $5/8$
  - d.  $6/11$
46. Which of the following is not an open sentence?
- a.  $7 + 2 = \square$
  - b.  $h - 5 = 9$
  - c.  $c/1 - 30 = 6$
  - d.  $n - 3$
47. For a mathematical system consisting of the set of odd numbers and the operation of multiplication.
- a. the system is closed
  - b. the system is commutative
  - c. the system has an identity element
  - d. all of the above are correct

48. Measurement is a process which

- a. compares an object with some known standard or accepted unit
  - b. tries to find the exact amount
  - c. is never an exact measure
  - d. chooses a unit and then gives a number which tells how many of that unit it would take
- 1. a and b are correct
  - 2. a and c are correct
  - 3. a, b, and d are correct
  - 4. a, c, and d are correct

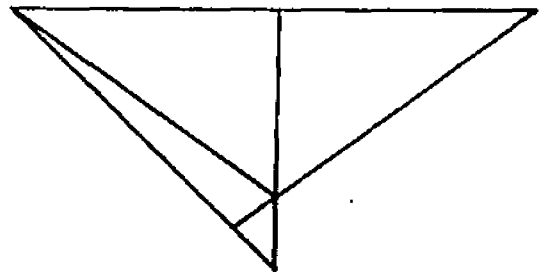
49. The set of points sketched below represents a



- a. line
- b. ray
- c. line segment
- d. none of the above

50. How many triangles does the figure contain?

- a. four
- b. six
- c. eight
- d. ten



51. The set of points on two rays with a common end-point is called

- a. a triangle
- b. an angle
- c. a vertex
- d. a side of a triangle

52. If a circle is drawn with the points of a compass 3 inches apart, what would be 3 inches in length?
- a. circumference
  - b. diameter
  - c. area
  - d. radius
53. The solution set of an open sentence may consist of
- a. two or more numbers
  - b. no numbers
  - c. only one number
  - d. any or all of these
54. Consider a set of three objects. How many sub-sets or groups can be arranged?
- a. nine
  - b. eight
  - c. seven
  - d. six
55. If two sets are said to be equivalent, then
- a. every element in the first set can be paired with one and only one element in the second set
  - b. every element in one set must also be an element in the second set
  - c. they are intersecting sets
  - d. one must be the null set

## FORM B (POST-TEST)

1. Which of the underlined words or signs in the following sentences refer to symbols rather than the things they represent?
  - a. 4 can be written on the blackboard
  - b. Regardless of what symbol we use, we are thinking about the number 2
  - c. A pencil is used for writing.
  - d. The number 16 is the same as the number  $7 + 9$ .
2. When we use the  $=$  symbol between two terms (as  $2 + 2 = 4$ ), we mean that both terms represent the same concept or idea. Which of the following is not correctly stated?
  - a.  $3 + 4 = 5 + 2$
  - b.  $5 + 2 = 7$  and  $7 = 5 + 2$
  - c.  $(5 + 2) \times 3 = 7 \times 3$
  - d.  $7 = 7$ 
    1. a and b are correct
    2. a and c are correct
    3. a, b, and c are correct
    4. a, b, c, and d are correct
3. If the Roman system of numeration were a "place value system" with the same value for the base as the Hindu-Arabic system, the first four base symbols would be
  - a. I, X, C, and M
  - b. I, V, X, and L
  - c. X, L, C, and M
  - d. X, C, L, and D

4. Which of the following does not describe a characteristic of our decimal system of numeration?
  - a. It uses zero to keep position when there is an absence of value.
  - b. It makes a ten a standard group for the organization of all numbers larger than nine.
  - c. It makes 12 the basis for organizing numbers larger than eleven.
  - d. It uses the additive concept in representing a number of several digits.
5. In the numeral 7,843, how does the value of the 4 compare with the value of the 8?
  - a. 2 times as great
  - b.  $1/2$  as great
  - c.  $1/10$  as great
  - d.  $1/20$  as great
6. In the numeral 6,666 the value of the 6 on the extreme left as compared with the 6 on the extreme right is
  - a. 6,000 times as great
  - b. 1,000 times as great
  - c. the same since both are sixes
  - d. six times as much
7. Which of the following statements best tells why we write a zero in the numeral 4,039 when we want it to represent "four thousand thirty-nine"?
  - a. Writing the zero helps to fill a place which would otherwise be empty and lead to misunderstanding.
  - b. The numeral would represent "four hundred thirty-nine" if we did not write the zero.
  - c. Writing the zero tells us not to read the hundreds' figure.

- d. Zero is used as a place-holder to show that there is no number to record in that place.
1. a and b are correct
  2. a and c are correct
  3. a and d are correct
  4. a, b, and d are correct
8. Below are four numerals written in expanded notation. Which one is not written correctly?
- a.  $4(\text{ten})^2 + 9(\text{ten})^1 + 3(\text{ones}) = 493_{\text{ten}}$
  - b.  $3(\text{seven})^3 + 6(\text{seven})^1 + 1(\text{one}) = 363_{\text{seven}}$
  - c.  $4(\text{twelve})^2 + 5(\text{twelve})^1 + e(\text{one}) = 45e_{\text{twelve}}$
  - d.  $2(\text{five})^2 + 2(\text{five})^1 + 4(\text{one}) = 224_{\text{five}}$
9. If you are permitted to use any or all of the symbols 0, 1, 2, 3, 4, and 5 for developing a system of numeration with a place value system of numeration similar to ours, a list of all possible bases would include:
- a. base one, two, three, four, five, and six.
  - b. base two, three, four, five, and six.
  - c. base two, three, four, and five.
  - d. base one, two, three, four, and five.
10. About how many hundreds are there in 34,870?
- a.  $3 \frac{1}{2}$
  - b. 35
  - c. 350
  - d. 3,500

11. In what base are the numerals in this multiplication example written?

- a. base five
- b. base eight
- c. base eleven
- d. you can't tell

$$\begin{array}{r} 34_? \\ 23_? \\ \hline 124_? \\ 70_? \\ \hline 1024_? \end{array}$$

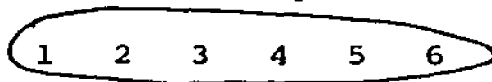
12. Which of the following are correct?

- a. In the symbol  $5^3$ , 5 is the base and 3 is the exponent.
  - b. In the symbol  $5^3$ , 3 is the base and 5 is the exponent.
  - c.  $5^3 = 5 \times 5 \times 5$
  - d.  $5^3 = 3 \times 3 \times 3 \times 3 \times 3$
- 1. a and d are correct
  - 2. b and c are correct
  - 3. a and c are correct
  - 4. b and d are correct

13. In the series of numerals 1,...,17, 18, 19, 20, 21,..., what term best applies to 19?

- a. nominal
- b. ordinal
- c. composite
- d. cardinal

14. Examine the following illustration:



Which of the following does the above best illustrate?

- a. The idea of a cardinal number.
- b. The use of an ordinal number.
- c. A means for determining the cardinal number of the set by counting with ordinal numbers.
- d. None of the above.



15. The quotient of any two whole numbers
- is not a natural number
  - is sometimes a natural number
  - is always a natural number
  - is a natural number less than one of the numbers being divided
16. The integers are closed under the operations of
- addition
  - subtraction
  - multiplication
  - division
- a and b are correct
  - a and c are correct
  - a, b, and c are correct
  - a, b, c, and d are correct
17. A student solved this example by adding down; then he checked his work by adding up.

Add 34		34
↓ 52	↑	52
86	Check	86

It could be classified as an example of

- the distributive principle
- the associative principle
- the commutative principle
- the law of compensation

18. The statement "the quotient obtained when zero is divided by a number is zero" is expressed as
- a.  $a/0 = 0$
  - b.  $0/a = 0$
  - c.  $0/0 = a$
  - d.  $a/a = 0$
19. When a natural number is divided by a natural number other than 1, how does the answer compare with the natural number divided?
- a. larger
  - b. smaller
  - c. one-half as large
  - d. can't tell from information given
20. If you had a bag of 350 marbles to be shared equally by 5 boys, which would be the quickest way to determine each boy's share?
- a. counting
  - b. adding
  - c. subtracting
  - d. dividing
21. If the multiplier is  $x$ , the largest possible number to carry is
- a.  $x$
  - b.  $x + 1$
  - c. 0
  - d.  $x - 1$

22. Which one of the following methods could be used to find the answer to this example?

$$17\overline{)612}$$

- a. Multiply 17 by the quotient.
  - b. Add 17 six hundred times.
  - c. The answer would be the sum.
  - d. Subtract 17 from 612 as many times as possible. The answer would be the number of times you were able to subtract.
23. Which one of the following would give the correct answer to this example?

$$\begin{array}{r} 2.1 \\ \times 21 \\ \hline \end{array}$$

- a. The sum of  $1 \times 2.1$  and  $21 \times 2.1$ .
  - b. The sum of  $10 \times 2.1$  and  $2 \times 2.1$ .
  - c. The sum of  $1 \times 2.1$  and  $20 \times 2.1$ .
  - d. The sum of  $1 \times 2.1$  and  $2 \times 2.1$ .
24. Which would give the correct answer to  $435 \times 563$ ?
- a. Multiply  $439 \times 3$ ,  $439 \times 60$ ,  $439 \times 5$  and then add the answer.
  - b. Multiply  $563 \times 9$ ,  $563 \times 3$ ,  $563 \times 4$  and then add the answer.
  - c. Multiply  $563 \times 9$ ,  $563 \times 39$ ,  $563 \times 439$  and then add the answer.
  - d. Multiply  $439 \times 3$ ,  $439 \times 60$ ,  $439 \times 500$  and then add the answer.
25. Which of these numerals are names for prime numbers?

- a. 3
- b.  $\frac{4}{2}$
- c.  $12_{\text{five}}$

- d. 9-2
1. a is correct
  2. a and c are correct
  3. a, b, and d are correct
  4. a, b, c, and d are correct
26. Let  $x$  represent an odd number; let  $y$  represent an even number. Then  $x + y$  must represent.
- a. an even number
  - b. a prime number
  - c. an odd number
  - d. a composite number
27. The inverse operation for addition is
- a. addition
  - b. subtraction
  - c. multiplication
  - d. division
28. The least common multiple of 8, 12, and 20 is
- a.  $2 \times 2$
  - b.  $2 \times 3 \times 5$
  - c.  $2 \times 2 \times 2 \times 3 \times 5$
  - d.  $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$
29. Which statement best tells why we carry 2 from the second column?
- |   |                          |
|---|--------------------------|
| a. If we do not carry the 2, the answer would be 20 less than the correct answer.       | 251<br>161<br>252        |
| b. Since the sum of the second column is more than 20, we put the 2 in the next column. | 271<br><u>271</u><br>935 |

- c. Since the sum of the second column is 23 (which has two figures in it), we have room for the 3 only, so we put 2 in the next column.
- d. Since the value represented by the figures in the second column is more than 9 tens, we must put the hundreds in the next column.

30. The operations which are associative are

- a. addition
  - b. subtraction
  - c. multiplication
  - d. division
- 1. a and b are correct
  - 2. a and c are correct
  - 3. a, b, and c are correct
  - 4. a and d are correct

31. Which of the following is an even number?

- a.  $(100)_3$  three
- b.  $(100)_5$  five
- c.  $(100)_7$  seven
- d.  $(200)_5$  five

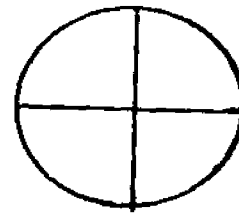
32. The fact that  $a + b (b + c)$  is exactly equal to  $(c + b) + a$  is an example of

- a. distributivity
- b. commutativity
- c. closure
- d. associativity

33. Observe the drawing on the right. When the circle is cut into equal pieces, the size of each piece

- a. decreases as the number of pieces increases
- b. increases as the number of pieces decreases
- c. increases as the number of pieces increases
- d. decreases as the number of pieces decreases

- 1. a and b are correct
- 2. a and c are correct
- 3. b and c are correct
- 4. b and d are correct



34. The symbol  $\frac{3}{4}$  may be used to represent the idea that

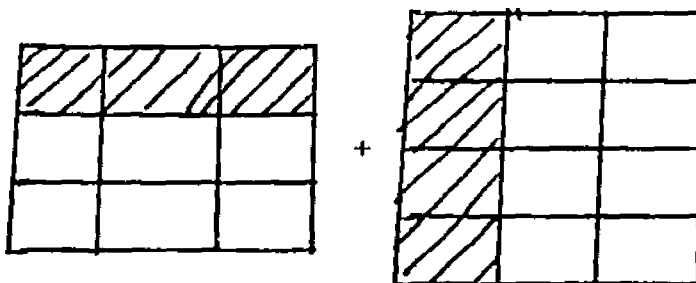
- a. 3 is to be divided by 4
- b. 3 of the 4 equal parts are being considered
- c. 3 objects are to be compared with 4 objects
- d. all of the above

35. When a whole number is multiplied by a common (proper) fraction other than one, how does the answer compare with the whole number?

- a. it will be larger
- b. it will be smaller
- c. there will be no difference
- d. you are not able to tell

36. Which of the addition examples is best represented by the shaded parts of the diagram below?

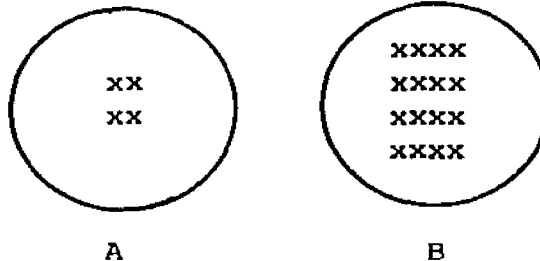
- a.  $\frac{1}{2} + \frac{1}{3}$
- b.  $\frac{2}{3} + \frac{3}{4}$
- c.  $\frac{2}{3} + \frac{1}{4}$
- d.  $\frac{1}{3} + \frac{1}{3}$



37. We can change the denominator of the fraction  $\frac{2}{3}$  to the number "1" without changing the values of the fraction by  $\frac{4}{5}$
- adding  $5/4$  to the numerator and denominator
  - subtracting  $5/4$  from the numerator and the denominator
  - multiplying both the numerator and the denominator by  $5/4$
  - dividing the numerator and the denominator by  $5/4$
38. What statement best tells why we "invert the divisor and multiply when dividing a fraction by a fraction?
- It is an easy method of finding a common denominator and arranging the numerators in multiplication form.
  - It is an easy method for dividing the denominators and multiplying the numerators of the two fractions.
  - It is a quick, easy, and accurate method of arranging two fractions in multiplication form.
  - Dividing by a fraction is the same as multiplying by the reciprocal of the fraction.
39. If the denominator of the fraction  $2/3$  is multiplied by 2, the value of the resulting fraction will be
- half as large
  - double in value
  - unchanged in value
  - a new symbol for the same number
40. 45% may also be written as
- .45
  - 45/100
  - 45 x 100%
  - .450
- a and b are correct
  - a and c are correct
  - a and d are correct
  - a, b, and d are correct


41. .5 and .27 are illustrations of "decimal fractions." They could be written as "common fractions" in the form of  $\frac{1}{2}$  and  $\frac{27}{100}$ , respectively. What is a decimal fraction?
- It is another way of writing percentage.
  - It is an extension of the decimal number system to the right of one's place.
  - A number like  $.37 \frac{1}{2}$  which has both a decimal and a fraction as parts of it.
  - A number like  $.2/.56$  which is a fraction and has a decimal as either the numerator or denominator or both.
42. The ratio of x's in Circle A to x's in Circle B can be shown by

- $\frac{16}{4}$
- $\frac{1}{4}$
- $\frac{1}{2}$
- $\frac{4}{16}$



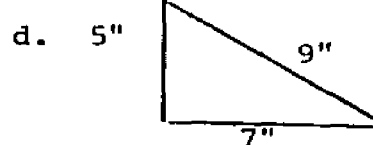
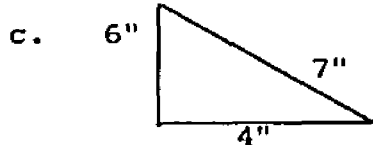
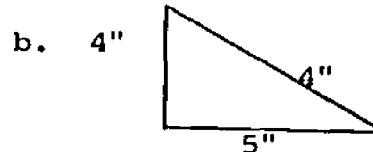
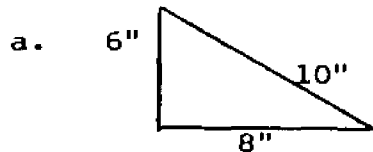
43. Sue paid 20¢ for 4 apples. Which of the equations below could be used to find the cost of 1 apple?
- $\frac{4}{20} = \frac{1}{x}$
  - $x + 4 = 20$
  - $\frac{x}{4} = 20$
  - $x - 4 = 20$
44. The decimal for the numeral  $\frac{6}{17}$  will
- be a repeating decimal
  - not repeat or end since 17 is prime
  - repeat in cycles of less than 23 digits
- a is correct
  - a and b are correct
  - a and c are correct
  - a, b, and c are correct



45. Which of the following statements is not correct?
- a.  $(-9) + 6 = -3$
  - b.  $(-5) + (-5) = -10$
  - c.  $-8 + 0 = -8$
  - d.  $(-8) + (9) = -1$
46. Which of the following is a list of all of the factors of 12?
- a. 1, 2, 3, 4, 8 & 12
  - b. 1, 2, 3, 4, 6 & 12
  - c. 1, 2, 3, 4, & 6
  - d. 2, 3, 4, 6 & 12
47. Modular arithmetic is
- a. cummutative
  - b. associative
  - c. distributive with respect to multiplication over addition
  - d. all of the above
48. Which of the following is an approximate measure?
- a. 35 farms
  - b. 12 buttons
  - c.  $7 \frac{1}{2}$  inches
  - d. 15 beads
49. Which of the following does the sketch below represent?
- 
- a. line
  - b. ray
  - c. line segment

- d. set of points
  - 1. a is correct
  - 2. a, b, and d are correct
  - 3. a, c, and d are correct
  - 4. b and d are correct

50. Which of these triangles are right triangles according to the length of the sides given?



51. A distinct point is
- a. a point you can see
  - b. a sharp object
  - c. the intersection of two lines
  - d. a dot
52. A clerk sold a lady a round tablecloth that had a radius of 14 inches. Which of the formulas can she use to determine the length around the cloth?
- a.  $A = \pi r$
  - b.  $C = \pi d$
  - c.  $C = 2\pi r$
  - d.  $A = C/d$

53. Which of the following best defines a solution set?

- a. A solution set is a set which includes each and every member that gives a true statement.
- b. A solution set is a single sentence which identifies a variable that will give a true statement.
- c. A solution set is a set containing all the positive integers, zero, and the negative integers.
- d. A solution set is a set containing rational numbers.

54. Examine the following illustration.

$$S = \{0, 1, (-1), 2, (-2), 3, \dots, 10\}$$

Which one of the following is not a subset of S?

- a.  $\{+9, +10\}$
- b.  $\{0, (-2), 5\}$
- c.  $\{3, (-3), 10\}$
- d.  $\{1, (-1), 6, 10\}$

55. If we use the set concept to define the operations for the counting numbers, addition would be defined in terms of

- a. the intersection of disjoint sets
- b. the union of intersecting sets
- c. the intersection of sets with common elements
- d. the union of disjoint sets

## APPENDIX C

### DUTTON ARITHMETIC ATTITUDE INVENTORY

APPENDIX C

DUTTON ARITHMETIC ATTITUDE INVENTORY

Name \_\_\_\_\_ Student Number \_\_\_\_\_

Place a check (✓) before those statements which tell how you feel about arithmetic. Select only the items which express your true feelings--probably not more than five items.

- \_\_\_ 1. I avoid arithmetic because I am not very good with figures.
- \_\_\_ 2. Arithmetic is very interesting.
- \_\_\_ 3. I am afraid of doing word problems.
- \_\_\_ 4. I have always been afraid of arithmetic.
- \_\_\_ 5. Working with numbers is fun.
- \_\_\_ 6. I would rather do anything else than do arithmetic.
- \_\_\_ 7. I like arithmetic because it is practical.
- \_\_\_ 8. I have never liked arithmetic.
- \_\_\_ 9. I don't feel sure of myself in arithmetic.
- \_\_\_ 10. Sometimes I enjoy the challenge presented by an arithmetic problem.
- \_\_\_ 11. I am completely indifferent to arithmetic.
- \_\_\_ 12. I think about arithmetic problems outside of school and like to work them out.
- \_\_\_ 13. Arithmetic thrills me and I like it better than any other subject.
- \_\_\_ 14. I like arithmetic, but I like other subjects just as well.
- \_\_\_ 15. I never get tired of working with numbers.
- 16. Place a circle around one number to show how you feel about arithmetic in general.  

1	2	3	4	5	6	7	8	9	10	11
Dislike										Like
- 17. My feelings toward arithmetic were developed in grades:  
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, other \_\_\_\_ (circle one).
- 18. My average grades made in arithmetic were: A B C D (circle one).

19. List two things you like about arithmetic.
  - A.
  - B.
20. List two things you dislike about arithmetic.
  - A.
  - B.

APPENDIX D

ATTITUDE SCALES TOWARD DIFFERENT  
ASPECTS OF MATHEMATICS

APPENDIX D

ATTITUDE SCALES TOWARD DIFFERENT  
ASPECTS OF MATHEMATICS

A Study of Attitude of Prospective  
Elementary School Teachers

Dear Student:

We are attempting to evaluate the attitudes of prospective elementary school teachers, such as yourself, toward some aspects of mathematics, school, and life in general.

Will you please read each statement and circle the response that reflects your feeling toward that statement? A, if you agree, D, if you disagree, or U, if you are undecided. Please be sure to circle only one letter for each statement.

The information obtained through this questionnaire will be kept strictly confidential. It will be used for research purposes only.

Student's Name \_\_\_\_\_

Student's Number \_\_\_\_\_



AGREE	DISAGREE	UNCERTAIN		
A	D	U	1.	Most school work is memorizing of information.
A	D	U	2.	In our school we got a great deal of practice and drill until we were almost perfect in our learning.
A	D	U	3.	The students spent most of their class time listening to the teachers and taking notes.
A	D	U	4.	My mathematics teacher showed us different ways of solving the same problem.
A	D	U	5.	Our teachers wanted us to do most of our learning from the textbook which is used in the course.
A	D	U	6.	My mathematics teacher did not like students to ask questions after he had given the explanation.
A	D	U	7.	My mathematics teacher wanted students to solve problems only by the procedures he taught.
A	D	U	8.	We were expected to learn and discover many ideas for ourselves.
A	D	U	9.	We were expected to develop a thorough understanding of ideas and not just to memorize information.
A	D	U	10.	Our teachers believed in strict discipline and each student did exactly what he was told to do.
A	D	U	11.	Students were encouraged to devise their own projects or experiments in order to learn on their own.
A	D	U	12.	My mathematics teacher expected us to learn how to solve problems by ourselves but helped when we had difficulties.
A	D	U	13.	In my mathematics classes, students who had original ideas got better grades than did students who were most careful and neat in their work.
A	D	U	14.	Most of our classroom work was listening to the teacher.
A	D	U	15.	My mathematics teacher required the students not only to master the steps in solving problems, but also to understand the reasoning involved.
A	D	U	16.	My mathematics teacher encouraged us to try to find several different methods of solving particular problems.

AGREE	DISAGREE	UNCERTAIN		
A	D	U	17.	My mathematics course required more thinking about methods of solving problems than memorization of rules and formulas.
A	D	U	18.	My mathematics teacher wanted us to discover mathematical principles and ideas for ourselves.
A	D	U	19.	My mathematics teacher explained the basic ideas; we were expected to develop the methods of solutions for ourselves.
A	D	U	20.	We did not use just one textbook for most of our subjects. Various sources and books from which we can learn were suggested to us.
A	D	U	21.	Most of the problems my mathematics teacher assigned are to give us practice in using a particular rule or formula.
A	D	U	22.	Much of our classroom work was discussing ideas and problems with the teacher and other pupils.
A	D	U	23.	In mathematics there is always a rule to follow in solving problems.
A	D	U	24.	I generally like my school work.
A	D	U	25.	It should be possible to eliminate war once and for all.
A	D	U	26.	Success depends to a large part on luck and fate.
A	D	U	27.	More of the most able people should be encouraged to become mathematicians and mathematics teachers.
A	D	U	28.	Someday most of the mysteries of the world will be revealed by science.
A	D	U	29.	Anyone can learn mathematics.
A	D	U	30.	Most school learning has little value for a person.
A	D	U	31.	By improving industrial and agricultural methods, poverty can be eliminated in the world.
A	D	U	32.	I dislike school and will leave just as soon as possible.
A	D	U	33.	With increased medical knowledge, it should be possible to lengthen the average life span to 100 years or more.

AGREE	DISAGREE	UNCERTAIN		
A	D	U	34.	Outside of science and engineering, there is little need for mathematics (algebra, geometry, etc.) in most jobs.
A	D	U	35.	Mathematics is of great importance to a country's development.
A	D	U	36.	The most important reason for studying arithmetic and secondary school mathematics is that they help people to take care of their own financial affairs.
A	D	U	37.	Very few people can learn mathematics.
A	D	U	38.	Mathematics help one to think according to strict rules.
A	D	U	39.	Mathematics (algebra, geometry, etc.) is not useful for the problems of everyday life.
A	D	U	40.	Someday the deserts will be converted into good farming land by the application of engineering and science.
A	D	U	41.	I am bored most of the time in school.
A	D	U	42.	Almost all of the present-day mathematics was known at least a century ago.
A	D	U	43.	Education can only help people develop their natural abilities; it cannot change people in a fundamental way.
A	D	U	44.	I enjoy everything about school.
A	D	U	45.	A thorough knowledge of advanced mathematics is the key to an understanding of our world in the twentieth century.
A	D	U	46.	School is not very enjoyable, but I can see value in getting a good education.
A	D	U	47.	It is important to know mathematics (algebra, geometry, etc.) in order to get a good job.
A	D	U	48.	Almost anyone can learn mathematics if he is willing to study.
A	D	U	49.	Mathematics is a very good field for creative people to enter.
A	D	U	50.	Unless one is planning to become a mathematician or a scientist the study of advanced mathematics is not very important.

AGREE	DISAGREE	UNCERTAIN		
A	D	U	51.	Any person of average intelligence can learn to understand a good deal of mathematics.
A	D	U	52.	The most enjoyable part of my life is the time I spend in school.
A	D	U	53.	Even complex mathematics can be made understandable and useful to every high school student.
A	D	U	54.	In the near future most jobs will require a knowledge of advanced mathematics.
A	D	U	55.	With hard work anyone can succeed.
A	D	U	56.	Almost every present human problem will be solved in the future.
A	D	U	57.	Almost all pupils can learn complex mathematics if it is properly taught.
A	D	U	58.	I like all school subjects.
A	D	U	59.	There is little place for originality in mathematics.
A	D	U	60.	I enjoy most of my school work and want to get as much additional education as possible.
A	D	U	61.	Only people with a very special talent can learn mathematics.
A	D	U	62.	Mathematics will change rapidly in the near future.
A	D	U	63.	Although school is difficult, I want as much education as I can get.
A	D	U	64.	In the study of mathematics, if the student misses a few lessons it is difficult to catch up.
A	D	U	65.	I find school interesting and challenging.

## APPENDIX E

### ENJOYMENT AND VALUE OF MATHEMATICS SCALES

## APPENDIX E

### ENJOYMENT AND VALUE OF MATHEMATICS SCALES

#### Two Scales of Attitude Toward Mathematics by Lewis R. Aiken

Directions: Draw a circle around the letter(s) that show(s) how closely you agree or disagree with each statement: SD (Strongly Disagree), D (Disagree), U (Undecided), A (Agree), SA (Strongly Agree).

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#### E SCALE: ENJOYMENT OF MATHEMATICS

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- |  |    |   |   |   |    |
|--|----|---|---|---|----|
| 1. I enjoy going beyond the assigned work and trying to solve new problems in mathematics. | SD | D | U | A | SA |
| 2. Mathematics is enjoyable and stimulating to me.   | SD | D | U | A | SA |
| 3. Mathematics makes me feel uneasy and confused.  | SD | D | U | A | SA |
| 4. I am interested and willing to use mathematics outside school and on the job.           | SD | D | U | A | SA |
| 5. I have never liked mathematics, and it is my most dreaded subject.                      | SD | D | U | A | SA |
| 6. I have always enjoyed studying mathematics in school.                                   | SD | D | U | A | SA |
| 7. I would like to develop my mathematical skills and study the subject more.              | SD | D | U | A | SA |
| 8. Mathematics makes me feel uncomfortable and nervous.                                    | SD | D | U | A | SA |
| 9. I am interested and willing to acquire further knowledge of mathematics.                | SD | D | U | A | SA |

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 E SCALE: ENJOYMENT OF MATHEMATICS
 

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- |   |             |
|---|-------------|
| 10. Mathematics is dull and boring because it leaves no room for personal opinion.      | SD D U A SA |
| 11. Mathematics is very interesting and I have usually enjoyed courses in this subject. | SD D U A SA |

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 V SCALE: VALUE OF MATHEMATICS
 

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- |  |             |
|--|-------------|
| 1. Mathematics has contributed greatly to science and other fields of knowledge.           | SD D U A SA |
| 2. Mathematics is less important to people than art or literature.                         | SD D U A SA |
| 3. Mathematics is not important for the advance of civilization and society.               | SD D U A SA |
| 4. Mathematics is a very worthwhile and necessary subject.                                 | SD D U A SA |
| 5. An understanding of mathematics is needed by artists and writers as well as scientists. | SD D U A SA |
| 6. Mathematics helps develop a person's mind and teaches him to think.                     | SD D U A SA |
| 7. Mathematics is not important in everyday life.  | SD D U A SA |
| 8. Mathematics is needed in designing practically everything.                              | SD D U A SA |
| 9. Mathematics is needed in order to keep the world running.                               | SD D U A SA |
| 10. There is nothing creative about mathematics; it's just memorizing formulas and things. | SD D U A SA |

APPENDIX F

RAW SCORES OF TEACHER CORPS INTERNS  
ON ALL MEASURES



# APPENDIX F

## RAW SCORES OF TEACHER CORPS INTERNS ON ALL MEASURES

Students I.D.	Dossett's Post-Test	Dutton's Post-Test	Dossett's Pre-Test	Dutton's Pre-Test	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	E-Scale	V-Scale	Pre-Test Measurement	Post-Test Measurement	Pre-Test Numeration	Post-Test Numeration	Pre-Test Add. & Sub.	Post-Test Add. & Sub.	Pre-Test Multi. & Div.	Post-Test Multi. & Div.	Pre-Test Fractions	Post-Test Fractions
1	40	6.8	20	5.3	10	12	8	15	8	31	30	40	49	10	46	16	48	20	45	12	42
2	52	7.8	44	6.9	8	11	11	14	10	44	36	46	38	50	41	42	42	35	43	35	47
3	41	6.3	42	7.4	13	13	8	16	10	33	29	30	44	43	46	35	47	31	32	25	28
4	36	4.7	22	3.3	12	14	15	14	8	23	39	25	40	10	39	34	47	32	36	13	35
5	21	6.2	13	2.3	6	12	11	14	11	23	33	20	28	16	39	19	38	6	31	8	28
6	41	8.3	36	7.1	11	12	8	12	7	31	31	35	45	43	47	39	45	34	44	37	45
7	38	5.5	21	6.8	10	14	10	16	5	23	31	35	36	33	49	25	43	20	40	3	36
8	29	7.1	28	5.3	6	12	10	15	10	27	26	30	31	10	38	13	36	8	15	5	29
9	36	7.9	37	6.8	10	14	6	18	10	38	35	30	50	23	49	45	49	36	46	13	42
10	23	7.4	22	7.6	8	7	11	15	11	31	27	20	31	3	29	15	28	16	25	9	18
11	41	7.1	30	5.8	4	14	10	13	8	38	29	40	45	18	43	35	43	17	45	20	42
12	34	8.6	23	8.6	11	10	6	7	7	38	38	25	44	30	45	19	38	25	35	9	22
13	32	7.8	32	7.6	10	14	12	13	16	44	40	35	45	15	46	7	43	15	29	18	17
14	34	4.9	33	4.5	7	10	12	16	7	21	35	45	39	5	46	16	48	23	46	8	29
15	35	3.8	21	3.5	8	9	10	12	6	15	29	25	33	9	44	33	44	22	35	7	18
16	28	7.4	17	8.0	6	14	6	12	10	33	31	15	27	26	29	18	26	10	14	12	28
17	27	2.6	16	2.0	12	13	6	17	10	13	29	15	39	0	34	25	36	21	28	10	16
18	39	7.1	23	5.5	10	12	8	15	8	29	30	20	36	30	45	17	31	16	43	18	31
19	36	7.6	36	7.0	5	12	9	11	8	15	21	50	26	23	43	23	41	24	36	21	35
20	39	7.1	32	6.3	11	12	8	13	11	28	25	20	43	18	47	19	19	12	19	4	26
21	41	8.0	26	4.8	12	14	9	13	8	30	35	35	47	13	42	23	40	9	41	20	33
22	29	3.6	30	3.2	7	11	7	15	6	27	30	5	11	31	39	19	24	14	14	7	25
23	42	4.5	33	3.9	9	12	12	10	7	23	31	30	50	23	40	32	50	25	46	14	45
24	22	2.8	22	2.9	9	9	8	12	5	18	33	30	26	6	30	8	36	5	17	0	28

B<sub>1</sub> = Attitude Toward Mathematics as Process  
 B<sub>2</sub> = Attitude Toward Difficulties of Learning Mathematics  
 B<sub>3</sub> = Attitude Toward Place of Mathematics in Society

B<sub>4</sub> = Attitude Toward School and School Learning  
 B<sub>5</sub> = Attitude Toward Man and His Environment  
 E-Scale = Enjoyment of Mathematics  
 V-Scale = Value of Mathematics

APPENDIX G

RAW SCORES OF THE "COMPARISON GROUPS"  
ON ALL MEASURES

# APPENDIX G

## RAW SCORES OF THE "COMPARISON GROUPS" ON ALL MEASURES

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Raw Scores of Students in the Content-Method Integrated Program, G <sub>2</sub>										Raw Scores of Students in the Regular Methods Class, G <sub>3</sub>									
Students I.D.	Dosssett's Post-Test	Dutton's Post-Test	Dosssett's Pre-Test	Dutton's Pre-Test	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	Students I.D.	Dosssett's Post-Test	Dutton's Post-Test	Dosssett's Pre-Test	Dutton's Pre-Test	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>
1	40	4.0	30	5.3	4	10	8	10	3	1	23	8.1	36	9.0	4	10	8	15	8
2	45	7.1	40	6.9	12	10	8	14	4	2	43	6.3	46	7.0	6	11	9	8	9
3	31	6.9	35	7.5	8	8	10	12	9	3	35	7.1	36	6.3	10	9	7	14	5
4	34	2.5	25	2.9	4	14	8	18	8	4	35	7.1	37	5.8	8	10	9	16	9
5	39	6.0	36	2.4	11	9	10	17	4	5	36	7.4	42	7.8	11	7	6	10	4
6	39	8.3	23	8.3	11	14	10	17	4	6	40	7.9	44	7.1	11	7	10	10	5
7	31	2.0	30	2.9	9	14	10	9	6	7	29	6.7	38	3.9	6	10	10	13	10
8	26	2.4	14	2.4	11	11	9	11	5	8	39	6.8	44	7.1	14	8	6	12	4
9	33	8.6	31	3.2	10	7	11	9	5	9	27	7.4	32	7.4	10	10	4	11	6
10	33	7.8	32	7.1	12	6	9	11	4	10	29	3.3	27	4.6	6	10	5	7	10
11	33	7.4	36	7.1	9	14	10	15	10	11	32	4.9	38	7.4	11	13	10	15	14
12	28	3.2	30	2.4	8	7	6	11	4	12	30	3.3	31	4.4	6	6	8	2	10
13	41	6.3	13	7.1	11	14	9	18	11	13	38	7.3	44	7.8	10	10	9	12	10
14	36	7.0	30	1.0	11	12	6	7	4	14	29	6.7	34	6.4	8	14	12	20	16
15	38	5.0	38	4.8	9	9	11	14	10	15	30	6.3	30	6.3	11	11	9	11	10
16	43	7.0	37	5.2	9	8	8	13	3	16	33	5.9	30	5.6	10	14	6	18	8
17	37	5.4	36	4.8	14	10	8	12	6	17	36	7.0	37	6.3	13	12	11	14	8
18	45	7.8	35	5.2	7	6	6	14	12	18	24	6.1	26	6.1	6	3	7	9	7
19	36	8.0	30	7.6	10	10	8	12	8										
20	36	5.7	42	6.5	12	8	10	11	8										
21	47	8.0	41	7.9	12	9	10	13	7										

B<sub>1</sub> = Attitude Toward Mathematics as Process  
 B<sub>2</sub> = Attitude Toward Difficulties Learning Mathematics  
 B<sub>3</sub> = Attitude Toward Place of Mathematics in Society  
 B<sub>4</sub> = Attitude Toward School and School Learning

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 E-Scale = Enjoyment of Mathematics  
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