THE MEANING OF TEACHING MATHEMATICS: TEACHER POSITIONINGS AS EMBEDDED IN ALGEBRA TEACHERS' GUIDES

By

Heejoo Suh

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

Curriculum, Instruction, and Teacher Education—Doctor of Philosophy

2017

ABSTRACT

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Teacher educators have been examining the professional status of teaching, including defining central practices of teaching, comparing teaching to other professions, and understanding teachers' own perspective via interviews, surveys, and observations. The present study intends to contribute to the discussion by examining the meaning of teaching as reflected in teachers' guides.

I chose to examine teachers' guides because they are known to have impact on teachers. Being a resource that is close to teachers' everyday practices, curriculum materials inspire teachers with what they could do in their classroom. Educators intending large-scale educational reform pay attention to curriculum materials. Careful use of linguistic features therein could enhance thus further support teachers' guides in effective communication with teachers.

I investigated four 8th grade Algebra teachers' guides, which were selectively chosen to represent variety. Drawing on positioning theory, I observed positionings regularly appearing in each of those four. Then I turned to idiosyncratically-observable positionings. Two research questions sought understanding of what teaching mathematics entails as presented in the TGs, hence how each guide constructs teaching mathematics. Last, I examined the guides' linguistic features. This was for understanding the degree of teacher professional judgment acknowledged by the guides.

My results indicated that, as constructed by each guide, teaching mathematics does not differ much across the guides. Those guides presented various types of knowledge as entailed in teaching mathematics. Compared to aspects on teacher professionalism in the

literature, the guides occasionally addressed teacher research, interaction with colleagues, utilizing knowledge, and acknowledging uncertainty. In addressing these aspects, the guides in most cases did so with insufficient support. Examining idiosyncratic positionings, I found two types: i) positionings occurred idiosyncratic only, and ii) positionings occurred idiosyncratically in addition to their regular appearance. The latter can be classified into two types: i) one that succeeds the context of the communication actions associated with the general positioning, and ii) one dissociated from the context. My linguistic examination of the guides suggested that they varied in their use of voice. Some are more directive, others are more suggestive, indicating different levels of acknowledgement of teachers' professional judgment.

In this dissertation's last chapter, I present ways in which this study contributes to understanding of curriculum materials and of teacher professionalism. I end this dissertation with implications for curriculum authors, teachers, teacher educators, and researchers.

Dedicated to **f**;
we have survived another day.

ACKNOWLEDGEMENTS

My sincerest gratitude to my Michigan State University professors, namely Dr. Alicia Alonzo for introducing me to critical issues in teacher education, Dr. Tonya Bartell for your support and insights into how I better could address prospective teachers' needs as an instructor, Drs. Kristine Bieda and Janine Certo for your support and advice on how I successfully could enact my duties as a field instructor, and Drs. Mary Juzwik and Ralph Putnam for your guidance during my early stage in the program.

I also sincerely thank leaders of the College of Education and the Department of Teacher Education: the current and former department chairs, Drs. Margaret Crocco and Avner Segall**;** the current and former program coordinators, Drs. Sandra Crespo and Peter Youngs; and staff members Linda Brandau, Ann Castle, Terry Edwards, Lisa Keller (from the PRIME program), Lindsay Metiva, and Amy Peebles.

Thank you also to professors at the University of Michigan and at Ewha Womans University. Thank you to Dr. Vilma Mesa for her patience and support when I was lost in the woods; to Dr. Edward Silver with whom I was honored to work in a research project and in the school of education at Dearborn; to Dr. Mary Schleppegrell for her guidance into the world of discourse analysis; and to 고응일교수님 (Dr. Ko, Eung Il), 윤정호교수님 (Dr. Yoon, Jungho), and 이향숙교수님 (Dr. Lee, Hyang-sook) for mentoring me in my Ewha time and beyond.

Drs. Jillian Cavanna, Kate Johnson, Alex Theakston Musselman, and Samuel Otten, thank you for the enjoyment you gave me in my being your office mate. Thank you to Dr. Heather Howell for your mentoring, research opportunities, and writing opportunities, in addition to emotional support and encouragement you gave me to enable me to complete my

dissertation. And thank you to 김성식교수님 (Dr. Kim, Sungsik), 김수철교수님 (Dr. Kim, Soochul), 양희준박사님 (Dr. Yang, Heejoon), 이성회박사님 (Dr. Lee, Sunghoe), 이현선생님 (Mr. Lee, Hyun), 최유림선생님 (Dr. Choi, Yourim), and 황지원교수님 (Dr. Hwang, Jiwon), for patiently supporting me in my dissertation work while I adjusted to the Korean educational research community. I also thank 유정애 센터장님 (Dr. You, Jeongae), the director of the Institute of Curriculum Excellence at Chung-Ang University, for believing that I could complete my dissertation. I am deeply grateful for the opportunity to become a part of your center.

I sincerely thank my dissertation committee due largely to whom I have come this far. Thank you to Dr. Beth Herbel-Eisenmann – my dissertation chair, academic advisor, researcher role model, and US mother – for her huge intellectual and emotional support; her close advising deeply impacted me as researcher and a writer and instilled in me the aspiration to be in future the kind of mentor she is. And for your questions, inspiration and support in my dissertation process, thank you Dean Robert Floden, Dr. Corey Drake, and Dr. Amy Parks. My dissertation's contributions are possible because of *you*. Its flaws are mine.

I thank those in the popular culture industry, thanks to whose hard work I could relax and refresh myself and my thoughts and keep on dissertating, *a la* "All work and no play makes Jack a dull boy!" Last, I am appreciative of my immediate family: 김상숙 (Kim, Sang Sook), 서정호 (Suh, Jung Ho), 서희연 (Suh, Heeyeon). And my lovely nephew 신윤호 (Shin, Peter Yoonho). You believed in me even when I did not. Thank you.

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KEY TO ABBREVIATIONS

TG	Teachers guide
IntTG	Introduction to a TG
IntCh	Introduction to a chapter
IntLe	Introduction to a lesson
BeLe	Beginning of a lesson
MiLe	Middle of a lesson
EnLe	Ending of a lesson
EnCh	Ending of a chapter

CHAPTER 1: INTRODUCTION

Teachers' beliefs about what their profession entails, or about their professional *selves*, can impact their practice (Zohar, Degani, & Vaaknin, 2001; Zohar & Dori, 2003). Therefore, understanding sources of teachers' formation of their professional selves is important to making productive changes to their practice. By understanding sources, I mean first to know what the sources are, and, second, what each source presents as to what teaching mathematics entails.

In this study, I conceptualize self as a dynamic construct combining internal and external factors (Harré, 2012; Hermans, 2013). I paid particular attention to external factors. I did so because people rely on "[their] personal background knowledge, and their attitudes toward each other, sociocultural assumptions concerning role and status relationships, as well as social values associated with various message components" (Gumperz, 1977, p. 191). Research about beliefs suggests similar points. A belief becomes stable when it is shared with others or is reinforced in multiple contexts (Leatham, 2006; Walkerdine, 1988). Teachers' beliefs are less an individual phenomena, but more a reflection of the perspectives that are shared in a professional community (Parks, 2010; Parks & Wager, 2015; Taylor, 1996).

Applying these insights to teaching, I see teacher professional self as what shaped by both individual self-reflection and interaction with the outside world (Cooper & Olson, 1996; Hand & Gresalfi, 2015; Parks, 2010; Parks & Wager, 2015). Outside-world factors include teachers' self-concepts, expectations by the professional community, and images society broadly shares on teaching (Beijaard, Meijer, & Verloop, 2004; Reyes & Rios, 2003). Also relevant is that teachers with innovative ideas likely would find it challenging to manifest those ideas if a majority of peers regard such innovations as only barely aligning with *their* perceptions of what teaching is and should be, or if the school system does not understand the value of innovative ideas teachers are hope to implement (Davis, 2002; Deemer, 2004).

Among a vast range of potential external factors, in this dissertation I attend to TGs. Research indicates that the external factor in teachers' landscapes impact teachers' development of their professional selves, regardless whether messages are explicit or implicit (Cooper & Olson, 1996; Rennert-Ariev, 2008; Reynolds, 1996; Sugrue, 1997). Mathematics education research, too, reports the on-reader impact of messages embedded in written texts (e.g., Dowling, 1996; Herbel-Eisenmann, 2007).

Building on this research, I explored externally-assigned professional selves by drawing on positioning theory as expounded by Harré and colleagues (see Harré, 2010). In particular, I gave attention to teacher positionings from four TGs, following the storyline of teacher professionalism. When identifying positionings, I adopted two approaches. First, I attended to those guides' range of duties presented for model teachers. In so doing, I aimed to demonstrate teaching mathematics as constructed by each guide, hence the kinds of duties a professional teacher should fulfill. My assumption was that the guides provide contents seeming relevant and important for teaching mathematics well, rather than random information that may or may not support teaching of mathematics. Then, I attended to the form of the language used in the guides. Drawing on the notion of *voice* (Herbel-Eisenmann, 2007; Remillard et al., 2011), I examined the imperatives, personal pronouns, and modality in each TG. Examining the tone used throughout the guides, I sought to understand the guides' suggested space in which teachers may maneuver. Such collective analysis allowed me to provide a multifaceted understanding of external positionings relative to teachers' professional selves.

Although I use the word *teacher*, this study is more about teaching *practices* than teachers' *personal qualities* (see Kennedy, 2010). This study paid attention to teaching as a profession and to what society expects of that profession, as constructed by TGs. In this study, I did not intend to examine the teacher professional self an individual teacher holds;

rather, this study focuses on the professional selves as presented in the surroundings of teachers' environments. Positioning theory was particularly useful for such analysis because it acknowledges self as reciprocally constructed. In other words, teachers develop their professional self not in a vacuum, but while interacting with the external world. The professional practice as constructed by external world may not be individualized, but it is a source that can influence individual teachers' sense of professional selves. In addition, positioning theory attends to "taken-for-granted practices" (Harré, 2010, p. 53) conceived as natural by the external world. Therefore, the theory allowed me to discuss teaching practices the guides assumed to be natural.

Organization

This dissertation comprises five chapters. The Introduction *supra* is the first, an overview of this study's context and *infra* this dissertation's organization. Chapter 2 provides this study's theoretical background and presents its research questions. First, I reviewed literature on curriculum materials, thereby seeking to unfold what is known about written curriculum materials as an external factor with a potential to impact teachers' construction of their professional selves. In addition, I attended to literature on teacher professionalism. In particular, I focused on five aspects, namely i) knowledge base, ii) teacher research, iii) interaction with colleagues, iv) utilizing knowledge, and v) acknowledging uncertainty.

In Chapter 3, I turned to *positioning theory*, the theoretical framework of this dissertation. In particular, I explained each of the three main concepts of the theory, namely i) positioning, ii) storyline and iii) communication act. Following the theoretical framework, I presented three research questions, two about the positionings embedded in the selected TGs and one about the voice the guides are using. In addition, I expanded on the notion of rights and duties, two terms widely used when defining positioning. In so doing, I explained why I chose to attend to duties only and not rights in this study. Then, I outlined and discussed my

method and my rationale for choosing Algebra TGs as my data source. For each of the three research questions, I gave a data analysis plan. In addition, I made explicit how I approached the guides when observing positionings, as a way to be explicit with the analysis process, hence enhance this study's rigor. Following my approach to the guides, I described who I am in the subsection named researcher subjectivity.

Chapter 4 describes my results. First I attended to generally-observable positionings. I compared these findings to the literature about teacher professionalism. Then, I discussed the seven positionings present in all the selected guides. Following this, I presented for each guide the positionings revisited multiple times. I closed my reporting of findings for the first research question by sharing the differences I observed in terms of the degree of support. With the idiosyncratic positionings, I presented idiosyncratic-only types followed by idiosyncratic-regular types of positionings to present how idiosyncratic positionings are occurring. I closed my examination of positionings by presenting pervasive positionings counting idiosyncratic cases. Following the positionings is the examination of voice, to examine which I attended to pronoun, modality, and imperatives. For an across-guides comparison, I provided the ratio of each of the three aspects (pronoun, modality, and imperatives) to the totals of sentences and of pages examined, instead of using the number counts.

Chapter 5, the last, presents my discussion, in which I revisited my results in light of the research questions. Answering the research questions, I found that the regularly occurring positionings were more similar than different across the teachers' guides I examined. In addition, I identified idiosyncratic positionings occurring in at least three different ways, which showed that idiosyncratic occurrence is key to understand the presence of a positioning. Further, I found that the teachers' guides are, in general, more likely to use linguistic features that restrict teachers from using their professional judgements. Following

this is a retrospective on what writing this dissertation taught me about research. I also offered suggestions looking forward to possible extensions of this study: first by considering implications for curriculum authors; second for teacher educators, and teachers. Last, I suggested possible future directions for research.

CHAPTER 2: RELEVANT LITERATURE

This chapter is my review of literature on curriculum materials and on teacher professionalism. Regarding curriculum materials, I focused on empirical research on teachers' engagement with those materials, their potential as an external factor that may impact teachers' sense of professional self, and ways to maximize those materials' potential as a resource for teacher education. Regarding teacher professionalism, I attended to five aspects: i) knowledge base, ii) teacher research, iii) interaction with colleagues, iv) utilizing students' knowledge, and v) acknowledging uncertainty. To be clear, these five aspects are by no means a comprehensive list of what the educational research field suggests in relation to teacher professionalism.

Review of the Literature on Curriculum Materials

In this section, I discuss how teachers engage with curriculum materials, and those materials' potential for impacting teachers' sense of professional self. Researchers suggest that teacher development can be effective i) when it is connected to teachers' classroom practices, ii) when it goes through several iterations over time, iii) when supports are ongoing, and iv) when teachers can experiment and reflect on what they learned (Collopy, 2003; Remillard, 2000). Curriculum materials satisfy these conditions, which means that those materials have strong potential to bring changes to teachers' sense of professional self. Collopy's (2003) empirical study well supports this point: the teacher in the study changed her practice as she adapted ideas from curriculum material she was using.

How the materials impact teachers' sense of professional self is a combination of the material-authors' intentions, and the teachers' readings of the materials. In other words, the impact that materials can have on practice depends on what teachers take from the materials (Collopy, 2003; Remillard, 2000; Remillard & Bryans, 2004; Sosniak & Stodolsky, 2007; Stein & Kim, 2009). Observing two mathematics teachers' use of curriculum materials,

Remillard (2000) concluded textbooks are unlikely to have direct impact on enacted curricula, because teachers modify contents within those books. Yet she also recognized curriculum materials' potential as a tool for teacher learning. Teachers appreciate textbooks because those texts save their planning time, and help them pace even when they did not follow exactly what the textbook asked them to do (Sosniak & Stodolsky, 2007). This variability is reflected well in Sherin's and Drake's *Curriculum Strategy Framework* (2009), a framework for understanding teachers' engagement with curriculum materials. The framework includes *reading*, *evaluating*, and *adapting*. When teachers engage with curriculum materials, they do their own reading of selected parts that address their needs. They then approach their reading with their own evaluation stances, and with their own orientation toward adaptation. The term adapting, instead of enacting it with fidelity, signifies existing variability in teachers' use of curriculum materials.

Despite that the materials' impact on teachers' sense of professional self depends on how teachers engage with them, some researchers attend to the substantial potential impact which curriculum materials have for teachers' sense of professional self. Curriculum materials are known to have large-scale influence on classroom practice (Ball & Cohen, 1996; Collopy, 2003; Stein & Kim, 2009). To implement reform-based instruction in a large number of classrooms, educators strategically chose to promote curriculum materials of innovative design (Ball & Cohen, 1996) with the expectation that teachers could teach themselves and develop teacher professional self as intended by the reform from reading the curriculum materials (Sherin & Drake, 2009). Curriculum materials' potential as tools for teacher professional development becomes more significant when considering beginning teachers. Because they newly are starting their careers, it is unrealistic to expect them to invent effective teaching materials. Although teachers' use of curriculum varies by individual teachers, and well-experienced teachers may choose not to use textbook materials (Sosniak &

Stodolsky, 2007), less-experienced teachers tend to rely more on their curriculum materials (Ball & Feiman-Nemser, 1988; Remillard & Bryans, 2004). A more realistic expectation might be, as Ball and Feiman-Nemser (1988) have suggested, orienting beginning teachers "toward learning *from* teacher's guides and other curriculum materials in a way that allows them to move toward building their own units of study, units that are responsible for subject matter goals and responsive to students" (p. 421). That is, teacher education programs should aim to encourage prospective teachers in flexibly adapting curriculum materials rather than ignoring or totally relying on them (Drake, Land, & Tyminski, 2014). Because beginning-teachers tend to rely on curriculum materials – and such reliance is reasonable to expect from them – curriculum materials' impact on beginning-teachers' development of professional self is not to be ignored.

Several researchers have given serious attention to maximizing the potential of curriculum materials as a resource for teacher development (Ball & Cohen, 1996; E. A. Davis & Krajcik, 2005; Drake et al., 2014). Using the term *educative curriculum materials*, Davis and Krajcik (2005) suggested that well-designed curriculum materials can "help to increase teachers' knowledge in specific instances of instructional decision making" while helping them "develop more general knowledge that they can apply flexibly in new situations" (p. 3). Ball and Cohen (1996) also explored the potential of curriculum materials and how these materials can contribute more to teacher learning. More recently, Drake, Land, and Tyminski (2014) pointed out that "educative curriculum materials can and should be utilized in teacher preparation to support PTs [prospective teachers] in developing not only knowledge and practices related to curriculum materials and their use, but also the broader knowledge bases needed for successful novice teaching" (p. 155). In other words, curriculum materials have potential to strongly impact teachers' sense of professional self. Curriculum authors can

achieve this by designing curriculum materials with a clear intent of making them tools for teacher learning.

Taken together, although curriculum materials do not have control over teachers' learning and practice, they can and do serve as a source of input for the formation of professional selves. In this dissertation, I pay particular attention to this potentiality that curriculum materials have. Among the range of curriculum materials, I chose to focus on the guides written specifically for teachers. By so doing I hope to make visible what teachers guides (hereafter, TGs) consider teaching mathematics to entail. I also hope to understand the amount of teacher professional judgment acknowledged by the guides. Thoroughly understanding teachers' professional selves is complex because many factors influence it, including teachers' personal experiences, their past experiences as prospective and/or practicing teachers, *common sense* shared in the community (of which the teachers are a part), policy expectations, school leadership, and more. By examining TGs assumed to be close to teachers' practices and learning, my dissertation will contribute to understanding a portion of so complex a matter.

Review of the Literature on Teacher Professionalism

Here, I discuss the literature addressing teacher professionalism in order to use it in the results section as a guide to interpret the positionings I observed from the TGs. The teacher professionalism aspects included in this review are that of i) knowledge base, ii) teacher research, iii) interaction with colleagues, iv) utilizing students' knowledge, and v) acknowledging uncertainty. This review's purpose is not to provide a comprehensive map of all aspects extant in this research area, nor on presenting the five most-widely accepted aspects. Instead, this dissertation's readers might consider the abovementioned aspects as five example contexts. I am making such suggestion because this review locates the results of this study.

Knowledge Base

One line of research on teacher professionalism attends to the multiple knowledge bases that successful teachers possess (Ball, Thames, & Phelps, 2008; L. Shulman, 1986). The expectation here is that in addition to their knowledge of mathematics, mathematics teachers have other knowledge bases. Three decades ago, Shulman (1986; 1987) questioned whether the then-current version of teacher evaluation standards were sufficiently comprehensive to address properly what teaching entails. Shulman suggested further specifying types of knowledge required when teaching. Instead of using "content knowledge" to cover multiple types of knowledge, Shulman subcategorized knowledge into i) subject-matter knowledge, ii) pedagogical-content knowledge, and iii) curricular knowledge.

Table 1. Subdomains of the Knowledge Base of Teaching Mathematics

Subdomain	What it is
Common content knowledge	"[T]he mathematical knowledge and skill used in settings other than teaching." (Ball, Thames, & Phelps, 2008, p. 399)
Specialized content knowledge	"[T]he mathematical knowledge and skill unique to teaching." (Ball, Thames, & Phelps, 2008, p. 400)
Horizon content knowledge	"[A]n awareness of how mathematical topics are related over the span of mathematics included in the curriculum." (Ball, Thames, & Phelps, 2008, p. 403)
Knowledge of content and students	"[K]nowledge that combines knowing about students and knowing about mathematics." (Ball, Thames, & Phelps, 2008, p. 401)
Knowledge of content and teaching	"[C]ombines knowing about teaching and knowing about mathematics." (Ball, Thames, & Phelps, 2008, p. 401)
Knowledge of content and curriculum	"[R]epresented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to these programs, and the set of particular curriculum or program materials in particular circumstances." (Shulman, 1986, as cited in Ball, Thames, & Phelps, 2008, p. 391)

Extending Shulman's idea, Ball and colleagues (2008) suggested that mathematical knowledge for teaching can be bisected into subject matter knowledge and pedagogical content knowledge. Subject matter knowledge consisted of common content knowledge, horizon content knowledge, and specialized content knowledge. Pedagogical content

knowledge consisted of knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum. Table 1 describes each of these subdomains. Since this groundbreaking work began, a number of researchers focused on mathematical knowledge for teaching in a variety of settings, or improvised the subdomains list (e.g., Bell, Wilson, Higgins, & McCoach, 2010; Kleickmann, Richter, Elsner, Besser, & Krauss, 2013; Kunter et al., 2010; Ryve, Nilsson, & Mason, 2011; Speer, King, & Howell, 2014; Steele & Rogers, 2012; Tatto et al., 2008). In my data analysis, I adopted Ball and colleagues' (2008) classification to understand which of the six subdomains are addressed in TGs, and which are not.

Teacher research

Another line of research related to teacher professionalism expects teachers to utilize their classroom practices as a source of professional development. Cochran-Smith and Lytle (1990) suggested that teacher research is likely to differ from research conducted by university researchers, but the difference is the main reason that makes teacher research a fruitful source of information. Teacher research is valuable because it focuses on current issues from the classroom. It also satisfies participating teachers. When university-generated knowledge is not seen to be superior to knowledge generated by teachers in their classrooms, and when the knowledge from two different sources is equally appreciated, teachers benefit more than when knowledge from one source is less appreciated (Cochran-Smith, 1991). At the heart of this approach is acknowledgement of the importance of the local knowledge produced by teachers within their own context (Lytle, 2006). Some may question the value of teacher research by pointing out that teachers' observations are not generalizable to other classrooms; their documentation process may not be scientific, and often their research is not grounded in theories. Yet these critiques cannot be a reason to refute teachers' roles as researchers: first, teacher research should not be treated as a subset of rigorous academic

research (Cochran-Smith & Lytle, 1990); second, teacher research provides valuable information to teachers so that they can enhance the quality of teaching (Cochran-Smith & Lytle, 2006). In addition, work on teacher-action research suggests it to be an effective tool for professional development, attesting to its pragmatic nature (Gore, 1991; Sales, Traver, & García, 2011). Examples of teacher research are journaling, writing essays with varied lengths, oral inquiries, classroom studies, and any activities contributing to teachers' systematic examination of local classroom context (Cochran-Smith & Lytle, 1990). Reading this line of research, I attended to the positionings that foregrounded the aspect of teacher research. Among the options for conducting a teacher research, most positionings I found were about collecting data from students via assessment. In the results section, I discuss this in more detail.

Interaction with Colleagues

Researchers suggested collegiality as what professional teachers recognize and practice to improve their teaching. Unlike the literature about multiple knowledge bases, which focuses on individual and cognitive aspects of teaching, Research expects teachers to be social and interactive. Teaching involves being a member of a learning community of teachers. Shulman and Shulman (2004) stated that "[an] accomplished teacher is a member of a professional community who is ready, willing, and able to teach and to learn from his or her teaching experiences" (p. 259), emphasizing the power of interaction in a collegial community. Mathematics-teacher study groups, in particular, are found useful because teachers can reflect on their own practices and perspectives through active engagements often involving dissonance (Crespo, 2006; Drake & Cirillo, 2009; Males, Otten, & Herbel-Eisenmann, 2010). In fact, researchers argue that such dissonance is what gives a teacher interaction productivity. The naïve approach of "every teacher has to find his or her own style" may contribute to a warm atmosphere, but the approach prevents teachers from

learning, and has them talk past and around each other (Ball & Cohen, 1999; 1994). Questioning what a teacher said is more likely to push the teacher to articulate her assumptions, and provides opportunities for teachers to think about how to deal with an issue together (Ball & Rundquist, 1993; Little, 1990; S. Wilson & Berne, 1999; Zech, Gause-Vega, Bray, Secules, & Goldman, 2000). According to Lord (1994),

> [k]ey features of professional development, in the light of national content standards, are to support teachers in their efforts to bring to the surface these questions and concerns, to help teachers expose their classroom practices to other teachers and educators, and to enable teachers to learn from constructive criticism (p. 184).

This set of key features reflects the expectation that teachers should interact mutually (i.e., with each other). When teachers openly talk about their concerns, question assumptions about teaching, and produce productive disequilibrium in traditional concepts and daily routines, collegial interaction becomes meaningful. Building on these studies, I examined which positionings encourage teachers to interact with other teachers.

Utilizing Emergent Bilinguals' Knowledge as a Resource

A number of researchers suggest that teachers with professionalism pay attention to what students *can* do, rather than what they can*not*, in order to support students from a range of backgrounds, including those from lower socioeconomic status, who are emergent bilinguals (i.e., second- or even multilingual), with cultural backgrounds different from school culture, and from under-represented racial groups (Aguirre et al., 2013; Battey, 2012; Civil & Andrade, 2002; Domínguez, 2010; Domínguez, LópezLeiva, & Khisty, 2013; Gutiérrez, 2002; Ladson-Billings, 1998; Turner, Domínguez, Empson, & Maldonado, 2013). These researchers consider students' background knowledge as a resource for having them engaged in learning school mathematics. Instead of focusing on the mismatch between students' home culture and school culture, these researchers question whose cultural contexts

are included and excluded when teaching and learning mathematics. By changing their beliefs to how students' home culture and home language can be a resource rather than an obstacle, in teaching mathematics, researchers and teachers can be open to search for better pedagogical supports. In their two-step empirical study with teachers, Planas and Civil (2009) found that teachers' beliefs about immigrant students can be altered through having a series of teacher study-group meetings. During the second phase of their research, the authors followed teachers to their classrooms to see to what extent their talk was reflected in their practice. Although teachers did not dramatically change their practice, they did provide more space to immigrant students than before, and were much more reflective on their interactions with those students, which may lead to change in teachers' practice in the future. In my analysis, I searched not only for the positionings asking teachers to make teacher moves to meet those students' needs, but also for whether any of those teacher moves are in fact drawing on what bilingual students bring in.

Acknowledging Uncertainty

The last type of expectation on professional teachers in this review is to acknowledge uncertainty. Multiple components contribute to uncertainty – from instructional content (Capobianco, 2010; Floden & Buchmann, 1993; Melville & Pilot, 2014), to pedagogy (Meister & Nolan, 2001; Midthassel, 2006; Wheatley, 2002; 2005), to student traits or school culture (Friedman, 1997; Labaree, 2003; Roeser, Skinner, Beers, & Jennings, 2012; Villaume, 2000). Consider students' emotional state as an example of uncertainty: teachers neither can control nor predict what emotionally impacts a student when she is at home. For the sake of improving classroom practice, acknowledging uncertainty is much more constructive than denying it. The acknowledgement is indeed a sign of expertise (Floden & Chang, 2007). This is so because the denial gives teachers the impression that everything is fine, hence prevents teachers from searching for alternatives (Cohen, 1988; Helsing, 2007). A

number of studies have shown that by acknowledging uncertainty, teachers were able to search proactively for effective pedagogical moves (Bennett, Woods, Wise, & Newton, 2007; Capobianco, 2010; Friedman, 1997; Meister & Nolan, 2001; Melville & Pilot, 2014; Midthassel, 2006; Villaume, 2000). To support teachers' professional development, Jordan, Kleinsasser, and Roe (2014) suggested, teacher educators should push teachers to confront uncertainty with critical thinking so that they can avoid settling down on simple but less effective pedagogical moves. During my data analysis, I looked for incidences where TGs attend to possible uncertainty or encourage teachers to be flexible in finding effective pedagogical moves.

So far, I have reviewed background literature explaining why I attended to TGs and the five aspects that have been proposed as duties to be fulfilled by professional teachers. In the following chapter, I present research questions and methods I used for this study.

CHAPTER 3: METHODS

In this chapter, I introduce *positioning theory*, the theoretical framework of this study. Before I explain my research methods, I present the three research questions I attended to in this study. Then, I examine the following: First, I discuss why I chose four algebra TGs as my data sources. Then, I provide specifics of my analysis processes to answer each of those three research questions. Following this, I provide a structural comparison of selected TGs, and a description of who I was as a reader of the guides during the analysis process. I end this chapter by explaining my effort in maintaining the rigor of this study, in that effort. I make explicit my researcher subjectivity.

Theoretical Framework: Positioning Theory

In my dissertation study, I used *positioning theory* to analyze how teaching and teachers are constructed in the chosen texts in order to better understand teachers' sense of professional self. Positioning theory has three mutually-constituted components: *positioning*, *storyline*, and *communication acts*, each of which I explain in more detail. In explaining positioning, I extend the relationship between the notions of *rights* and *duties* because positioning often is defined as a cluster of these two.

Positionings

Harré (2010) described positionings as "clusters of beliefs about how rights and duties, or expectations, are distributed in the course of an episode of personal interaction and the taken-for-granted practices in which most of these beliefs are concretely realized" (p. 53). Positioning is a cluster of beliefs because the theory is more interested in "what a person '*may* do and *may not* do'" than in "what he or she *can* do" (p. 52, emphasis added). The terms *rights* and *duties* depict the illocutionary nature of actions that are acceptable, expected, or natural at a moment. Rights and duties, or positionings, frame what a person can do or say. Different sets of rights and duties are associated with different positionings.

Coherency is not an essential characteristic of positionings, nor is intentionality (Wagner & Herbel-Eisenmann, 2009). Positionings may vary across settings. In addition, people can intentionally negotiate self and other's positionings, but the process of positioning can be unintentional. Positioning theory does not assume the existence of a *mental first person*, which is the consistent self that oversees and controls other selves (Louis, 2007). Instead, an individual's positionings change over time and space. Also, a person can be positioned in multiple ways by herself and others. Embracing incoherency, positioning theory sees positioning as ephemeral and negotiable (Harré, 2010). Because positioning is ephemeral, for example, my positioning as a daughter can fade when I interact with people other than my parents. In short, I do not carry a positioning with me, but let positionings emerge from the situation in which I am interacting. One can position oneself or a counterpart in an interaction (i.e., first-order positioning), refute a positioning imposed by others on oneself (i.e., second-order positioning), or position others who are not present at the scene of interaction (i.e., third-order positioning). Second-order positioning happens when people try to negotiate imposed positionings. Interaction here includes written communication in addition to spoken communication. For example, a TG construes first-order positionings for teachers by describing what teachers need to do to teach the content. When teachers read the guides, some might disagree with the rights and duties described therein, hence engage in second-order positioning. Because the guide is written for teachers, its description of students can be considered as third-order positionings.

Earlier, I presented literature suggesting that teachers engage with curriculum materials in varied ways. Using language from positioning theory, I focused on identifying first-order positionings from the TGs I chose for this study but this does not mean that teachers readily will accept those observed positionings. Rather, those observed positionings are open to refutation. Teachers can and do cast second-order positionings when they

disagree with first-order positionings from the guides. In this study, I attended to a smaller slice of this whole picture, which is the first-order positioning presented by the TGs I selected as my data sources.

Now I turn to the discussion of rights and duties. A positioning often is defined in terms of rights and duties. In this dissertation, I chose to use duties rather than rights. Here, I explain my rationale for such decision. In Harré's early work, a positioning was defined with rights, duties, and obligations. Examples of early definitions are these:

1. In so far as the content of a position is defined in terms of rights, duties and obligations of speaking, and these "moral" properties are locally and momentarily specified. (Harré & van Langenhove, 1991, pp. 404-405)
2. Technically a "position" is defined by a certain set of rights, duties and obligations as a speaker. (Sabat & Harré, 1999, p. 93).
3. To position someone or to be positioned is implicitly (and sometimes explicitly) to assign or remove contributing rights. (Harré, 2002, p. 618).

In Harré's recent contributions, however, a positioning is consistently described as a cluster of rights and duties. For example,

1. [Complaining] is embedded in a complex weave of 'positions,' that is of clusters of rights and duties with respect to what can legitimately be said and done by whom. (Harré, 2005, p. 186)
2. Positions, as clusters of right and duties, exist more in the practices of social actors than in structural demands. (Moghaddam, Harré, & Lee, 2007b, p. 9)
3. Positions are clusters of beliefs about how rights and duties are distributed in the course of an episode of personal interaction and the taken-for-granted practices in which most of these beliefs are concretely realized. (Harré, Moghaddam, Cairnie, Rothbart, & Sabat, 2009, p. 9)

4. [T]he application of positioning analysis—that is, how rights and duties are distributed among the actors in the course of complex discursive interactions, sometimes personal, sometimes in the newspapers and television media. (Moghaddam & Harré, 2010, p. 6)
5. [P]ositioning theory looks at what a person "may do and may not do." "Rights" and "duties" are shorthand terms for clusters of moral (normative) presuppositions that people believe or are told or slip into and to which they are momentarily bound in what they say and do. (Harré, 2010, p. 53).

These definitions show that the more refined version of definitions of positioning draw on the notions of rights and duties. Thus, to understand the meaning of positioning, an examination of rights and duties is central.

Although a position is described as a cluster of rights and duties, the relationship between rights and duties were not clearly explored by the leading scholars of positioning theory. This invites researchers using positioning theory to decide themselves whether to describe an act as an implementation of a right or of a duty. Some researchers did not use rights and duties to describe the positionings (de Freitas, Esmonde, Knipping, Lunney Borden, & Reid, 2012; Fahlgren & Sawyer, 2011; Glazier, 2005; Herbel-Eisenmann & Wagner, 2010; Langer-Osuna, 2011; Turner, Domínguez, Maldonado, & Empson, 2013). Other researchers used the language of rights and duties, without explaining their criteria for classifying as a right or a duty (Anderson, 2009; Barnes, 2004; Redman, 2007; 2013).

Fathali Moghaddam, who actively coauthored and coedited with Rom Harré about positioning theory (Harré et al., 2009; Harré & Moghaddam, 2013; Moghaddam, Harré, & Lee, 2007a), discussed rights and duties elsewhere (Moghaddam, 2000; 2004; 2011; Moghaddam & Riley, 2005; Moghaddam, Slocum, Finkel, Mor, & Harré, 2000). Although Moghaddam's work on rights and duties had no explicit connection to positioning theory at

the surface level, his work clarifies the uncertainty that researchers confront when attempting to explain a positioning in terms of rights and duties.

Moghaddam (2004) described rights as "a demand placed on others by the person who possesses it" and duties as "a demand placed by others on the person who owes it" (p. 126). Having a closer look at the relationship between these two, however, Moghaddam makes explicit that "rights and duties are almost always replaceable" (p. 126). In other words, "a right can be reinterpreted as a duty, just as a duty can be reinterpreted as a right:" (p. 126). Cultural conditions, or the situations of the person doing the interpreting of an act, are what impact the decision of whether an act is a right or a duty (Moghaddam & Riley, 2005). The law is one of a "few exceptions" that is clearly a duty (Moghaddam & Riley, 2005, p. 76). Regarding the law, there is "duty to obey the law" but it is absurd to say that one has a "right to obey the law" (Moghaddam, 2004, p. 126). Regardless of one's agreement to the law, one has no choice but to obey the law or one will be punished. Therefore, for an act to be a right or a duty mostly depends on the situation and the interpretation; there are a few exceptions that the replacement between the label rights and the label duties is more complicated.

In practice, people do act as though they feel obligated when the obligation is not a law. Moghaddam acknowledges rights and duties that are informal and implicit in everyday social life (Moghaddam & Riley, 2005). Following Finkel's (1995) work, Moghaddam names such rights and duties as "commonsense rights and duties" (Moghaddam & Riley, 2005, p. 75). Among the commonsense duties, some can be categorized as "supererogatory duties," meaning the "duties an individual will be praised for performing, but not morally blamed for omitting" (Moghaddam, 2004, p. 126). People performing supererogatory duties do so with conscious recognition and intention (Moghaddam et al., 2000). Performing supererogatory duties is voluntary and personal. In the imaginary world that all duties are supererogatory, all people have rights to obey the law (Moghaddam et al., 2000).

In this dissertation, I conceptualize the positionings from TGs as supererogatory duties. Behind this conceptualization is the understanding that the curriculum authors will include content that they think i) could be useful or even crucial for teachers to know to perform the work of teaching, and ii) teachers do not know well enough to perform without supports from TGs. It is supererogatory because teachers are not blamed by refusing to implement everything suggested in the guides. Rather, teachers are encouraged to make selections and modifications using their professional knowledge when reading the guides. In my data analysis, I first attend to positionings as supererogatory duties in the guides. Among the supererogatory duties, some are more strongly recommended than others. That is, there is a spectrum of supererogatory duties from mild suggestions to strong ones. This spectrum also will be addressed, especially by examining linguistic features of each guide.

Storyline

Storyline is a larger context in which positionings are located and the storyline at play relates to positionings possibly available to those interacting with spoken language or with written documents. Storyline "draws on knowledge of cultural structures and positions recognizably allocated to people within those structures" (Ritchie, 2002, p. 37). When one practices a certain act, the act may be welcomed in one context but discouraged in another, because the unfolded storylines are different in those contexts. Unfolded storylines are not necessarily exclusively revealed in interactions (Harré, 2010). Depending on what storyline to follow, the positioning of curriculum materials might vary. For example, within the storyline of mathematics being abstract and should be taught abstractly, project-based curriculum materials are unlikely to be considered as quality material.

For a given positioning in a given situation, multiple storylines can be involved while those also vary within the scales of interaction (Herbel-Eisenmann, Wagner, Johnson, Suh, & Figueras, 2015). For example, imagine a student who rarely participates during whole-group

discussions. A teacher can draw on the storyline that students might participate less when feeling unwell. Alternatively, the teacher can draw on the storyline that classroom norms or the interaction pattern among students influence students' participation. The scale of the former storyline, which lasts for a school day, is smaller than that of the latter storyline, which lasts for a semester or a school year. This often needs several months to develop.

When a positioning is challenged, refuted, and negotiated, new storylines not previously part of the scene may unfold. In other words, positionings and storylines are concepts that affect and are affected by each other. Henriksen (2007) explained this relationship by comparing positioning to role, and storyline to structure:

> …roles are constituted by structure, but structure is unaffected by the enactments of these roles. In comparison, the post-structuralist line of thinking employed by positioning theory considers the relationship between the positioning and the structuring storyline as mutually constituting. Whereas roles are given by structure, positions are created, occupied, held and abandoned through social interaction, including both the spoken and the written. Similarly, norms, values, rights and duties are structurally provided within role-theory, whereas they are negotiable within positioning theory. (p. 48)

The excerpt above addressed that positionings are not dominated or controlled by storylines but positionings and storylines are "mutually constituting" (p. 43). That is, changes in storylines can bring changes in positionings, as much as changes in positionings might bring changes in storylines. Therefore, suggesting alternative storylines can be a strategy to challenge imposed positionings.

Because positionings and storylines are closely interconnected, fixing a storyline to follow allows me to delve deeply into one changing variable (i.e., positionings) instead of chasing two changing variables (i.e., positionings and storylines). In this study, I chose to

pick a storyline – the storyline of teacher professionalism – with larger scale than smaller scale to reserve enough space to recognize a range of positionings from the four TGs, which are written with different goals and assumptions. In addition, this fixation allows me to consider teachers only. Considering the reciprocal aspect of a positioning, texts directed at teachers might contain positionings of students or mathematics. In the sentence "You may want to assign these tasks as homework," teachers are positioned to assign homework to students. Reciprocally, such a statement positions students as people who can be assigned homework from their teacher. Another possible read of the sentence is that mathematics is positioned as a subject matter that can be learned by solving tasks, individually. Although the storyline of my choice is broad, it is not too broad to drag my attention from the positionings of teachers.

Communication Act

In positioning theory, the communication act means meaningful practice of actions. Drawing on speech act theory, positioning theory distinguishes an act from an action. An action is "a meaningful, intended performance" such as speech or gesture (Harré, 2012, p. 198). An act is the meaning of action or how the action functions in the given context. For example, a teacher action of gazing at a student could be an act of enforcing behavior management. Both linguistic and nonlinguistic discourses including gestures and spatial distance are considered as sources of communication acts (Slocum & van Langenhove, 2003).

Although more work has been done with spoken communication acts, some researchers have used positioning theory to examine written communication acts. For example, Rothbart and Bartlett (2007) examined media reports on the Rwandan genocide of 1994. These communication acts are different from face-to-face spoken interaction in that the reports were prepared for a wide range of audiences rather than for the person in front of the

author. Additionally, there was a time gap between the initiator, or author, and receivers, or readers. The authors drew on systemic functional linguistics (Halliday & Hasan, 1985) and borrowed the notions of field, mode, and tenor to identify events and situations, their re-contextualized form, and speakers, accordingly. Rothbart and Bartlett's (2007) examination is relevant to my study in that curriculum authors write their TGs for a wide range of audiences, or teacher readers, rather than for a particular teacher in front of them.

Scheer (2013) also conducted media analyses by examining articles from news cites influential in the U.S., in addition to news articles the author had been surveying. The purpose of the study was to understand Wall Street's positioning after the U.S. economic collapse *circa* 2007-2008. Scheer's approach was thematic in that the author discussed the themes observed and supported the existence of such themes by citing the news articles. Allen and Moghaddam (2013) analyzed introductory psychology textbooks to understand their representation of hostility and friendships. The authors assembled a list of terms (e.g., enemy, friendship, aggression, violence, peace, etc.) and searched in which context and how often those terms were used. These examples show that positioning theory is a framework useful for investigating written texts. In this study, communication actions are texts written in the TGs. Then, the communication act of these actions is, roughly speaking, assigning duties, the specifics of which may vary depending on the content of the communication action. Thus far I explained positioning theory by examining each of the three inter-connected components of positioning, storyline, and communication act. Now I turn to the research questions that I used to guide this study.

Research Questions

In this dissertation study, I focused on external expectations others have of mathematics teachers in written texts close to teachers' daily practice and professional development. In particular, I analyzed texts, or communication actions in algebra TGs. I did

so to understand teaching of mathematics as constructed by the guides, and to what extent the guides acknowledge teachers' professional judgment to accept, revise, or refute the TGs' suggestions. Following the storyline of teacher professionalism, specific research questions I examined in this dissertation are:

1. What teacher positionings regularly are observable from the four selected mathematics TGs? How do these positionings compare with research on teacher professionalism? What teacher positionings are pervasive in the four guides?
2. What teacher positionings irregularly are observable from the four selected mathematics TGs?
3. What is the range of *voices* across the four TGs? How might these voices shape the degree of obligation to the teacher positionings as duties?

Examining the first and the second research questions, I explored the TGs' construction of what teaching mathematics entails. Examining the last research question, I described to what extent teachers' professional judgments are welcomed in each TG. That is, the first two questions are about the kinds of duties assigned to teachers, and the last question is about the degree of dutifulness of the assignment. If my examination of the first and the second research questions show that the TGs cover a range of different duties, this might be a sign that the TGs recognize teaching mathematics as a complex activity that only professionals could do. If my examination of the third research question suggests a low degree of dutifulness, this might indicate high appreciation of teachers' ability to make professional judgments rather than mindlessly follow what the TG tells them to do.

Data Sources and Data Analysis

Data Sources

In this dissertation study, I narrowed my focus to TGs for middle-school algebra. I compared four TGs from different group of authors. To control possible noise from mathematical content being different, I chose algebra as a subdomain. Algebra is known to have significant impact on students' futures. Algebra has been conceptualized as a gateway to later academic achievement (Bush & Karp, 2013; Gamoran & Hannigan, 2000; Liang, Heckman, & Abedi, 2012; Smith, 1996; Stein, Kaufman, & Sherman, 2011; National Mathematics Advisory Panel, 2008). From both educators and policymakers' perspectives, algebra significantly impacts students' advancement in future mathematics and science courses and, in turn, influences their academic success (Liang et al., 2012). According to the Mathematics Advisory Panel (2008), algebra drew extra attention from those interested in educational policy because students' mathematics achievements drop drastically when they begin algebra coursework. The Panel further pointed out that "completion of Algebra II correlates significantly with success in college and earnings from employment" (p. xiii). The chance for college graduation was more than twice higher for students who took Algebra II than for those who took fewer mathematics courses. Access to algebra is better when it is given at earlier grades, because early access positively impacts students' high-school mathematics performance (Smith, 1996). Spielhagen (2006) mentioned that students who began algebra in Grade 8 enrolled in more high-school mathematics courses than did those who began it in Grade 9. Further, the author found that students who began algebra in Grade 8 were more capable of taking advanced courses in high school.

Despite such importance of algebra on students' future, I could find only a small number of research studies focused on teachers teaching algebra (Stein, Kaufman, & Sherman, 2011). Among the few, some researchers focused on developing frameworks and

measures that could advance the field with understanding on the knowledge matters when teaching algebra (Buschang, Chung, Delacruz, & Baker, 2012; Izsák, Çağlayan, & Olive, 2009; McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012). Others attended to specific subdomains of algebra in relation to teaching (Even, 1993; Haimes, 1996; Nathan & Koedinger, 2000; Sánchez & Llinares, 2003; Stein & Baxter, 1990; Stump, 2001; M. R. Wilson, 1994). One observation from these efforts is that teachers' perception or knowledge on algebra and algebraic practice affect students' learning (Even, 1993; Nathan & Koedinger, 2000). With the potential of curriculum materials as a resource for teacher learning, the research so far reviewed suggests that teachers' sense of professional self, as represented in algebra TG, is a topic worthy of careful investigation. Echoing Doerr's (2004) point that "there is a significant shortage of research about how teachers learn to teach algebra, how they understand their own practice, and how they form and are formed by their own practice within their own specific cultural contexts" (p. 282), I paid specific attention to TGs as an external source of impact on teacher professionalism, rather than a container of a collection of algebraic contents with more impact on students than teachers (U.S. Department of Education, 2008). In so doing, I hope to suggest what authors can add or revise to better support algebra teaching by contributing to the understanding of teachers' sense of professional self.

For middle-school algebra TGs, I chose CMP (Lappen, Phillips, Fey, & Friel, 2014), Eureka (Great Minds, 2017), UCSMP (Brown et al., 2008), and Pearson (Charles et al., 2015). In choosing TGs, I not only considered market share, but also included curriculum materials with a range of design principles and structures, while keeping the number of materials manageable. To be clear, although I wanted to include a range of types of texts, I did not intend these four TGs to be a representative sample of any kind, but more of a purposeful sampling. This dissertation study's purpose is to explore what is being

communicated to prospective and practicing mathematics teachers because those positionings may contribute to teachers' professional selves. By exploring this set of TGs, I hope to provide an in-depth description of a smaller part of the whole picture.

Data Analysis Plan for Research Question 1: What teacher positionings regularly are observable from the four selected mathematics TGs? How do these positionings compare with research on teacher professionalism? What teacher positionings are pervasive in the four guides?

In my dissertation study, the analysis utilized a combination of thematic analysis (Braun & Clarke, 2006) and the analytic process described in the literature on positioning theory (Anderson, 2009; Barnes, 2004; Esmonde & Langer-Osuna, 2013; Harré & Moghaddam, 2013; Herbel-Eisenmann & Wagner, 2010; Moghaddam, Harré, & Lee, 2007b). Thematic analysis is "a method for identifying, analysing and reporting patterns (themes) within data" (Braun & Clarke, 2006, p. 79). It is extremely flexible and has been used in a number of studies, including cases when they did not explicitly claim to use thematic analysis. The flexibility is in fact the strength of the method. Grounded theory analysis is similar to thematic analysis in that both seek patterns. Grounded theory analysis, however, ultimately aims to develop a theory from data (Suddaby, 2006). Thematic analysis does not require researchers to develop a theory to explain data. Patterns or themes need not be determined by quantifiable measures such as how much it is present in data (Braun & Clarke, 2006). An example of thematic analysis process is described in Table 2 below.

Table 2. Phases of Thematic Analysis Table, from Braun and Clarke (2006, p. 87).

Phase	Description of the process
1. Familiarizing yourself with your data:	Transcribing data (if necessary), reading and re-reading the data, noting down initial ideas.
2. Generating initial codes:	Coding interesting features of the data in a systematic fashion across the entire data set, collating data relevant to each code.
3. Searching for themes:	Collating codes into potential themes, gathering all data relevant to each potential theme.
4. Reviewing themes:	Checking if the themes work in relation to the coded extracts (Level 1) and the entire data set (Level 2), generating a thematic 'map' of the analysis.
5. Defining and naming themes:	Ongoing analysis to refine the specifics of each theme, and the overall story the analysis tells, generating clear definitions and names for each theme.
6. Producing the report:	The final opportunity for analysis. Selection of vivid, compelling extract examples, final analysis of selected extracts, relating back of the analysis to the research question and literature, producing a scholarly report of the analysis.

To observe the positionings, I attended to written-for-teachers texts often provided at the margins of the guides. Following the storyline, I did not necessarily exclude the positionings that are contrary to teachers as professionals. In fact, my assumption was that the guides are providing such information because the curriculum authors think it is useful for teachers to know, and there is a possibility that authors include the information because they do not know whether teachers know it or not. What I planned to exclude are texts directed to others, for example, students. Positionings identified within the storyline have the potential to shape teachers' productive professional selves.

When observing positionings, I attended to explicit and pervasive communication actions. I acknowledge that teachers can be positioned in certain ways from what is absent in the TG. For example, the absence of solution paths (as opposed to a strong presence of answers to mathematical tasks), positions teachers in certain ways. Naming and explaining what those "certain ways" are requires an in-depth look at all aspects of the TGs. To make claims from what is not present in the TGs is an extremely complicated project, even when

additional sources of information (e.g., interviews with textbook authors) are provided. Because of this difficulty, I attended only to explicit and pervasive communication actions that are stated closely and revisited repeatedly in the TGs. To this end, I attended to the communication actions firmly embedded in the design of the guides. If a certain communication action is embedded in the design, it means such communication action appears in most, if not all, lessons in the guides. When answering this research question, I did not examine communication actions with sporadic presence. I focused on those communication actions when answering the second research question. Drawing on the thematic analysis, I revisited the data multiple times to generate a list of positionings per each of the four selected TGs. To make the analysis process manageable, I drew heavily on the first 19-28 lessons of the four guides, and reviewed the rest of the chapters to verify the consistency of my observation from the first 19-28 lessons. The number of lessons reflects the lengths of first three chapters of UCSMP (23 lessons) and Pearson (27 lessons). CMP has 19 Problems in the first additional unit for use with Algebra 1. Eureka presents 28 lessons in Module 1.

To keep data to a manageable size, and to enable my comparison across the guides, I grouped portions of TGs into seven phases, then attended to each phase individually. These phases are: introduction to a TG (IntTG), introduction to a chapter (IntCh), introduction to a lesson (IntLe), beginning of a lesson (BeLe), middle of a lesson (MiLe), end of a lesson (EnLe), and end of a chapter (EnCh). I considered structural features, or the headings, from the four TGs when deciding on which to group as a phase. I found the phases useful especially because each TG uses different headings for the portion that essentially serves a similar purpose. By using phases, I was able to compare TGs despite their varied wordings for the headings.

I attended to texts written for teachers, including those at the margins and those between lessons or chapters. Their placement varies, depending on publisher. My primary criterion is to attend to texts written *for teachers only*, hence ignore texts in *student versions*, often presented at the center of the guide per each lesson. For the same reason, I did not analyze texts in the exercise section or the worksheet, unless they contained there a significant amount of text to guide teachers. When identifying positionings, I planned to prioritize headings repetitively appearing across lessons in the first 19-28 lessons, or first three chapters. The goal of the first research question was to devise an overview of the types of positionings in four TGs, which reflect a range of design principles.

I paid attention to each heading individually and compared all texts under headings from the first three chapters. My goal was to generate one positioning per heading that described commonality among texts. For example, UCSMP has the heading *Warm-Up* in its introduction to the chapter (IntCh) phase. To examine the positioning observable from this heading, I reviewed all warm-ups from the first three chapters of UCSMP and looked for common features across the texts. A common feature was that the warm-up was presenting something for teachers to use to open a lesson in class. Some warm-ups presented mathematical tasks, others presented activities. Some drew on real-life contexts, others were without such contexts. Therefore, I described initially-observed positioning in the warm-up in the following way: teachers are duty-bound to know how to open each lesson with mathematical tasks or activities, either contextualized or not. After examining all the headings in the IntCh of UCSMP, I turned to the same phase of the other three TGs.

For each phase, I made a table with each column allocated to a TG and each row to an initially-observed positioning. In the cell was the TG heading from which I read the initial positioning. For example, the cell where the column UCSMP intersects with the row *teachers have a duty to know how to open each lesson with mathematical tasks or activities, either*

contextualized or not, I wrote the heading *Warm-Up*. Then, I examined the row for positionings to find those essentially addressing a similar aspect of teaching. For example, in Pearson I observed the initial positioning that *teachers have a duty to know questions to ask when facilitating the opening task*. Similarly, in CMP I observed that *teachers have a duty to know how to introduce lesson-content*. I combined these two initially-observed positionings with one from UCSMP to the following observed positioning: *teachers have a duty to know tasks/activities/questions for opening a chapter/lesson*. As a result, I observed 17 positionings from Introduction to a TG (IntTG), 30 from Introduction to a chapter (IntCh), 12 from Introduction to a lesson (IntLe), 6 from beginning of a lesson (BeLe), 8 from middle of a lesson (MiLe), 14 from end of a lesson (EnLe), and 12 from end of a chapter (EnCh).

Through the process of identifying positionings, I chose to use the phrase "teachers have a duty to know." I made such a decision to foreground the intentionality behind the presence of certain content in TGs. TG authors are more likely to examine carefully the contents they want the teacher readers to know. The authors, however, choose not to provide every detail needed when teaching the lessons. For example, TGs rarely explain how teachers properly could move hands and arms when handing out sheets to students. In other words, content present in the TGs reflects the authors' assumption on what teachers do not know, or at least what they need to be reminded of for successful teaching of the lessons.

My next step was to aggregate these tables into one comprehensive table. Columns were the TGs, and rows were positionings, but this time the cell was filled in with the phases instead of the headings, which are the labels in each of the phases. Headings might provide rich information about the TG and positioning, but similar richness could be reached with phases while causing the comparison to be more practical and manageable. In the aggregation process I again was able to refine descriptions of the positionings so they are not overly specific yet retain enough detail to articulate the duty implied.

Although my initial approach was to allocate one positioning per heading, some headings are coded multiple times in the aggregated table (see Appendix B). Pearson's heading *Teaching Resources* is one such case. The positioning I observed from this heading in the IntTG table was that teachers are duty-bound to know the range of available resources per type and purpose. In the aggregated table, I revised the positioning to: *teachers have a duty to know, generally, what supplementary resources are available*. I made such a revision so that the three other TGs could also be covered by the positioning. Later in the aggregated table, I found the positionings, observed from different headings, that teachers were duty-bound to know supplementary information to make accommodations, and to know how to support emergent bilingual students. These two aspects were observable from the texts under the heading Teaching Resources. Between obligating to the rule of one positioning per heading *versus* providing a rich picture of the data, I chose the latter. By so doing, I could avoid misrepresenting a TG as not addressing a positioning when it really *is* really addressing it. Therefore, I coded the heading to address these two positionings also. To identify those cases, I provided in the aggregated table headings in blue that are coded multiple times.

Data Analysis Plan for Research Question 2: What teacher positionings irregularly are observable from the four selected mathematics TGs?

The danger of giving so much attention to the headings is that I might miss positionings that appear sporadically through the guide. For example, if a TG provides tips for supporting tactile learners whenever such opportunity came but not in all lessons, it is misleading to ignore such communication action of the guide. To avoid misrepresentation of the guides, I attended to less pervasive positionings that are worth mentioning.

To answer the second research question, I began with the aggregated table from the first research question. The aggregated table represents positionings regularly observable. Cells left blank in this table mean that positionings are not regularly observable from the TG.

This does *not* mean, however, that such positionings are completely unobservable in the TG. From my reading of TGs, I observed positionings present in it, although their appearance was irregular. For example, the table might indicate that Positioning A is present in Guides 1, 2, and 4, but not in Guide 3. What is possible for Guide 3 is that it actually does not say anything about Positioning A. Alternatively, it is possible that Positioning A is construed in a couple of lessons only. In such a case, Positioning A is present in Guide 3, although its presence is not as strong as in the other three guides. Because of the possibility of the positioning to be present sporadically rather than consistently, the list from the first research question cannot be evidence of nonexistence of the positionings.

To fully understand a guide's potential as an external factor in developing professional selves, I paid attention to the blank cells of the aggregated table, and verified whether those positionings are not at all addressed, or addressed in an irregular manner in each TG. The irregularly-observed positionings are indicated by the phase using red in Appendix B.

Data Analysis Plan for Research Question 3: What is the range of *voices* across the four TGs? How might these voices shape the degree of obligation to the teacher positionings as duties?

In addition to the examination of less pervasive positionings, I examined the *voice* of the guides by taking a finer-grained linguistic approach. The dataset for this part of analysis is the texts from Chapter 1 of UCSMP (7 lessons) and Pearson (9 lessons), Investigation 1 of CMP (3 Problems), and Topic A (5 lessons) of Eureka. My initial informal examination of the guides told me they are consistent with their language use. For example, the guide using the pronoun "we" to refer to textbook authors (e.g., "We expect this task to help students with…") almost always use this pronoun when referring to authors in the later part of the TG. Similarly, the guide using sentences with low modality ("You might solve this task with

students.") used modality in other sentences, too. Analyzing linguistic features of sentences written in some number of lessons, I discussed the strength in which the duties are imposed on teachers.

This part of my analysis focused on the use of pronouns, imperatives, and modality, because these can be indicators of the voice (e.g., Herbel-Eisenmann, 2007), or "how the text communicates to the teacher, what it communicates about, and how the text positions the teacher as a reader and user of it" (Remillard et al., 2011, p. 3). Any text can have multiple voices that might even be contradictory (Herbel-Eisenmann, 2007). Examining voices of TGs, Remillard (2000) distinguished when a TG spoke through teachers *versus* when it spoke to teachers. Depending on what language a TG uses, it might end up speaking through teachers by offering "steps to follow, problems to give actual questions to ask, and answers to expect" (Remillard, 2000, p. 347). Such an approach emphasized the outcomes of teaching. When a TG speaks to teachers, it foregrounds "the rationales, assumptions, or agendas supporting them" (p. 347), which in turn encouraged teachers to engage with the guide's decisions and suggestions, in addition to its underlying ideas. The notion of voice allows researchers to consider external teacher positionings done via the form of the communication actions. My analysis for the first and the second research questions attended to the content of the texts. By conducting such analysis, I expected to understand external expectations on what teachers do when teaching mathematics. The analysis of voice provides information on the degree of teachers' autonomy assumed by the authors, or the degree of dutifulness the guides imposed on teachers. In other words, are the positionings in the guides presented as what teachers should think through using their professional knowledge and make productive modifications as needed, or are the positionings presented as strong duties that teachers should obey?

I drew on the notion of *speaking through* versus *speaking to*. Curriculum materials speak through teachers when they guide actions, offer steps to follow, and give actual questions to ask. Materials speak to teachers when they give rationales, assumptions or agendas supporting information provided (Remillard et al., 2011). My assumption is that a subtle difference exists between "Give manipulatives to students" *versus* "Consider providing manipulatives to students," and "This task connects mathematics and real-life context" *versus* "We designed this task to connect mathematics and real-life context." Teachers are likely to develop authoritative professional selves when exposed to the latter type because of the consistent invitation from the guides for teachers to recognize their authoritative selves. From this, what I saw as central is acknowledgment of the presence of teachers as those who can make professional judgments, rather than those who merely follow given directions. I paid attention to this particular aspect of the speaking through *versus* speaking to dichotomy. Therefore, I examined the presence of teachers and space available for them to either accept or refute suggestions from the guides.

I drew on Herbel-Eisenmann's (2007) work for a specific analytic process. In her work on the voice of mathematics textbooks, Herbel-Eisenmann drew on Morgan's (1996) articulation of systemic functional linguistics and attended to imperatives, personal pronouns, and modality to examine the interpersonal function of the books. According to Herbel-Eisenmann's use of Rotman (1988), imperatives can be either inclusive or exclusive. Inclusive imperatives, such as "consider" or "define," position readers as thinkers, whereas exclusive imperatives, such as "use" or "copy," position readers as those who perform actions. Because of such nature of imperatives, I examined imperatives in TGs to see when the guides were speaking to teachers with inclusive imperatives, and when they were speaking through teachers with exclusive imperatives. Pronouns are important because they portray the guides' (or guide-authors') personal involvement (Herbel-Eisenmann, 2007). By

attending to when and how *I*, *we*, and *you* were used, I expected to unveil whether the developers' design decisions are presented in the way those are being talked to teachers and if teachers are supported to make decisions while enacting the curriculum. Modality is another linguistic form that matters when examining voice of a text. Modality is to be found in the modal auxiliary verbs, adverbs, adjectives, and hedges (Herbel-Eisenmann, 2007). Modality reflects the degree of certainty of the voice. By attending to modality, I expected to uncover when the guides were using assertive voices and for what purpose.

Structural Comparison of the TGs

Before sharing my researcher reflexivity, I present a structural comparison of the four TGs, showing their structures and providing background information for the results section. Then I describe how I read the guides, an effort to maintain rigor of the study by being reflexive to what I did.

TGs structures were similar to each other but with subtle differences (Table 3). Both UCSMP and Pearson were two-volumes in hardcover; each volume of both had several chapters, each chapter had several lessons, and each lesson was designed to be taught in one day unless the lesson is blocked. CMP came with eight books, each representing a unit, each unit consisting of four to five investigations with a similar number of problems, and by design each problem was likely to be covered in one day. Eureka was provided digitally in five pdf files, each file a module, each module a couple of topics, each topic a number of lessons, and each lesson intended to be covered within a day. For ease of discussion, I used the label *lesson* to indicate lessons in UCSMP, Pearson, and Eureka, and problems in CMP.

Table 3. General Structure of the Four Guides

UCSMP	Pearson	CMP	Eureka
2 volumes (13 chapters total)	2 volumes (12 chapters total)	1 set (8 units)	5 pdf files (5 modules)
Chapter (7-10 lessons)	Chapter (6-10 lessons)	Unit (4-5 investigations)	Module (2-4 topics)
		Investigation (2-5 problems)	Topic (3-15 lessons)
Lesson	Lesson	Problem	Lesson

To compare TGs structurally, I followed a 7-phase classification based on my initial observation of the TGs: introduction to the TG, introduction to a chapter, introduction to a lesson, beginning of a lesson, middle of a lesson, ending of a lesson, and ending of a chapter. This classification covers every set of content across the four TGs, each of which has all seven phases, excepting CMP (for the introduction to the TG) and Eureka (for the beginning of a lesson). Introduction to the TG (IntTG) is texts placed at the very beginning of a TG; this phase communicates overall design and organization, main features, TG strength, and messages from curriculum authors to teacher readers. Structure and length varies across the four TGs. Introduction to a chapter (IntCh) is where the TG provides a general overview of the chapter; often included is chapter pacing, standards/claims/objectives addressed in the chapter, and relevant resources. Each lesson consists of introduction to the lesson (IntLe), its beginning (BeLe), its middle (MiLe), and its ending (EnLe). Ending of a chapter (EnCh) is a comprehensive review of the whole chapter; long-term projects, assessment tasks, and reflections are common features.

I show lesson phases in Table 4 below and describe in detail for each TG because a lesson is the most frequently repeated structure in the TGs.

Table 4. Phases of a Lesson

	IntLe	BeLe	MiLe	EnLe
UCSMP	GOAL; SPUR Objects; Materials/Resources; HOMEWORK; Local Standards; Background	1 Warm-up	2 Teaching	3 Assignment; 4 Wrap-up
Pearson	Preparing to Teach	1 Interactive Learning; PowerAlgebra.com	2 Guided Instruction; PowerAlgebra.com	3 Lesson Check; 4 Practice; Lesson Resources; PowerAlgebra.com
CMP	Problem Overview	Launch	Explore	Summarize
Eureka	Student Outcomes		Classwork	Closing; Exit ticket; Exit ticket Sample Solutions; Problem Set Sample Solutions

Lesson flow in each TG is similar but with minor variations; generally, they begin with an overview including goal, standards, and background information. Then follows the main teaching part where teachers interact with students; this comprises a lesson beginning, a middle, and an ending; in those, respectively, teachers invite students to the lesson (typically a warm-up activity), then a main part (one task or more), then an ending wherein i) via discussion or an exit ticket teachers assess students' understanding of what has been taught so far, and ii) homework is assigned. This structure of beginning, middle, and ending of a lesson somewhat aligns with the *exposition-examples-exercises model* common in textbooks (Love & Pimm, 1996).

Reflexivity and Expert Debriefing

To be explicit with my positionality I present myself first as a researcher and second as a reader of the TGs in a later part of this chapter. For a study to be accepted by the research community, it should be rigorous. Researcher reflexivity is a validity practice whereby researchers "report on personal beliefs, values, and biases that may shape their inquiry" (Creswell & Miller, 2000, p. 127). To practice reflexivity, researchers need to acknowledge their strengths and limitations when conducting the research, and present them to the readers (Tracy, 2010). A researcher can write a separate section on herself/himself, provide interpretive commentary in discussing the findings and in describing personal experiences, and so forth (Creswell & Miller, 2000). In any form, researchers are asked to make explicit their orientation to the readers.

For the expert debriefing, I consulted members of my dissertation committee. They had read my work from the early stage of proposal, and provided feedback and support until this study's completion, including the data analysis. Expert debriefing, or peer review, is "the review of the data and research process by someone who is familiar with the research or the phenomenon being explored" (Creswell & Miller, 2000, p. 129). Although experts are expected to be familiar with the research, they are not members of the research team. Their distance from the research is *key*, as such distance allows experts to provide with fresh eyes a wide range of feedback, including both positive and skeptical, with fresh eyes (Creswell & Miller, 2000; R. B. Johnson, 1997). Researchers can ask experts for written feedback or at the developing stage can consult with them on ideas, preferably through the whole process of the research than at the end of the analysis (Creswell & Miller, 2000).

Researcher Reflexivity: My Motivation to This Study

It was March of 2011. I was at the third annual Teacher Education Accreditation Council (TEAC) writing workshop. Other members from the TEAC preparation team at the

University of Michigan-Dearborn (UMD) were there with me. At that time, I was working as an administrative assistant at the School of Education at UMD, who worked preparing TEAC accreditation. The workshop was full of people from Michigan and Ohio. They, too, came to the workshop to prepare an audit of their teacher education programs. Representatives from TEAC facilitated the workshop. The engagement was unbelievable. People actively raised questions and responded to each other. They were eager to share their understanding of what makes teaching a real profession (rather than a para-profession) and what teacher educators could do to guide prospective teachers to meet those standards. Behind this engagement, I noticed a tension between protecting teachers and the field of education. Participants of the workshop had seen outsiders, who were not necessarily interested in education, trying to determine what defined teaching. Workshop participants were correct to be sensitive to such an assertion of power from outsiders because despite shallow evidence, the public bought into these outsiders' claims. Letting outsiders define what teaching entails can be more harmful than helpful. Often, it seemed threatening. Defining teaching based on insufficient experience and knowledge is indeed a threat to practicing teachers, to rigorous teacher education programs, to prospective teachers, to schools, to students, and to the future of the community at the local, national, and international level.

Defending one's stance is important, especially when the teaching profession is being attacked, from local interactions to interactions on a larger scale. At times, the assumptions underneath larger scale interactions are deeply embedded in the system, hence pervasive yet invisible. I learned this growing up as a woman in Korea, an east-Asian country that ranks 116th among 144 countries in terms of gender inequity.[1] I am lucky to have been born, given that in the 1980s, aborting female children was not unusual. Abortion was, and still is, illegal, but the social norm of boy preference forced women to clandestinely terminate their

[1] http://reports.weforum.org/global-gender-gap-report-2016/rankings/

pregnancies at a rate that was tantamount to female-infanticide, which is a distinct category of genocide. I remember very young males who sexually harassed me but were forgiven by adults because these boys liked me. *My* will, however, was not considered. Because adults forgave them, I had to do the same, because in Korean society obedience is expected. I attended an all-girls middle school and high school. There, I met male teachers who enjoyed verbally harassing students as well as female teachers who asserted that girls should behave like girls. Despite freezing temperatures, school regulation required us to wear black stockings under a knee-length skirt. Teachers punished girls who wore gym pants underneath their skirts to offset the cold. The university where I did my undergraduate studies is a famous (and prestigious) women's university. Because I enrolled there, many people assumed my only interest (or my duty) was to an affluent husband. All the time and energy I spent on intellectual activities were acknowledged only by the people in the university. Underlying these unpleasant and disrespectful attitudes is misogyny. No matter how often or how much I protested, nothing seemed to change. After much struggle and countless upsets, I gave up. I gave up speaking up for myself. Instead, I chose to escape. After all, Korea was never a country for women.

In May 2016, a man committed a random murder at the Gangnam station area. To be precise, the murder was random in the sense he killed a woman he did not know. Yet, it was not random, in the sense he killed the seventh person he met, letting six men go. In other words, he killed the first woman he saw. An undertone in pertinent press descriptions was that the victim was not as careful as she could have been. Some social media comments blamed her. However, others protested the victim-shaming by gathering at a Gangnam Station. Again, us women knew from past experience that we were lucky to be alive. I knew that as a woman in Korean society that my life was a matter of daily survival, as gender discrimination resulted not only in continuous acts of disrespect but also in the death of

countless women. There we stood, finally spelling out the violence we had to endure and the fears we live through. At the station, we recognized each other. Sharing our personal experiences that should not be as general as how it is then, we sensed support and oneness. It became clear that defending my gender equality is the matter of survival. Misogyny was profound at the level of actual physical murder. Despite of the dire situation, only a few women have spoken out against gender inequality in Korea. From now on, however, things will be different. This incident, the so-called *Gangnam murder case*, was a wakeup call for the society. Although misogyny did not disappear after the protest at Gangnam station, people became more proactive.

Another form of misogyny is not allowing women to earn enough money to live independently. In the past, these low wages pushed women into marriage. Divorce was impossible. Women had to endure verbal and physical abuses merely to survive. Today, more opportunities are given to women than before, but the gender wage gap is wide. By limiting women's professional and economic possibilities misogynists still control women. This approach, however, is no longer as powerful. Kim, a voice actress, is a perfect example. On July of 2016, Kim uploaded a photo on her personal Twitter account. The photo was her wearing a shirt with an imprint "Girls do not need a prince." Because of this, she received a notice from *Nexon*, a game company, that they would discontinue the contract with Kim. At that time, she was acting the voice of a character named *Tina*. Those who welcomed this decision argued that the shirt is evidence of Kim's participation to *Mersgall*, which they declared as a socially harmful and unjust online community for "feminazi." They considered the shirt as evidence of Kim's participation to the community. In fact, the shirt was a gift for the participants of the fundraising by *Mersgall4*. Mersgall4 was collecting funds for the civil suit against Facebook Korea. According to Mersgall4, Facebook Korea used their media power to discriminate women. Mersgall4 claimed that Facebook Korea repeatedly deleted

pages that introduced feminist issues in Korea and around the world, while leaving the pages posting images and videos with misogynic messages. The funding was successful – 4,103 people participated and it collected about $134,000. Misogynists started to call Mersgall4 feminazi because they posted feminist issues, even though no one member of Mersgall4 killed or physically harmed any Korean men. In addition, Kim said she was not a member of Mersgall4. She happened to have the shirt because she participated to the funding, being aware of the potential harm from misusing media power to the society. Of course, no evidence of her participation of the community was found, but Nexon did not take back their decision. People who recognized how outrageous Nexon's decision was actively voiced their support of Kim. This protest was difficult to imagine before the Gangnam station case. It sent a clear message that Kim's voice will always be welcomed and there are people waiting to hear her voice t. Although the game company is unlikely to ask Kim again to act their characters, we can hear Kim's voice on Cartoon Network Korea (e.g., *Powerpuff Girls*), Disney Channel Korea (e.g., *Star vs. the Forces of Evil*), and more.

We are not afraid of anymore. We know if someone has to resign due to sex scandal, it has to be the offender, not the victim. People on Twitter shared the sexual harassment they experienced in their workplace using the hashtag "ㅇㅇ_내_성폭력

(#sexualharassmentin____)." In particular, the hashtag "문단_내_성폭력

(#sexualharassmentinliteracy)" took several senior male writers to court, and ultimately, it led publishers to stop publishing these male writers' books. Female writers, inspired by the hashtag, established a crowdfunding to legally and medically support victimized writers. Their campaign reached its fundraising goal on its first day. Ultimately, 2,321 people donated, and the campaign reached 309% of its funding goal.

This fight is our fight. No one else can do it for us. More women are acknowledging the solidarity among us. In February 2017, a voice actress *Chae* posted on her blog that she would not act commercially sexualized female characters. As expected, while some people were disturbed by Chae's post, other supported her decision. I, too, support her decision. Now I can see that running away is not a solution. We will defend ourselves and we will win the fight. The world will become a better place for everyone. This is what I believe in. Yes, 우리는 서로의 용기야 (you are my courage as I am to you).

From these personal experiences, I learned two lessons. First, we can change society by proactively generating stories from our perspectives. Second, a sense of solidarity prevents exhaustion. Being a teacher is different from being a woman. To become a teacher, one has to go through some level of training or education. One can refuse to be recognized as a teacher by discontinuing teaching at schools. When it comes to biological sex, one is recognized as a female as soon as she is born. Once a person is recognized as a woman by others, it is challenging to ask people to see the person as a different gender. All this is to say that the identity as a teacher is not the same as the gendered identity. My gendered experience, however, has shown me that if people come together they can make changes to the world. I hope teachers and others in the education field to show the world that teaching is a serious profession requiring special kind of knowledge and skills. Also, I hope this dissertation could serve as a source of solidarity to those people fighting to persuade the world that teaching is a profession.

This dissertation contributes to the effort to understand the *potential* of curriculum materials in teachers' sense of professional self. Herbel-Eisenmann and Wagner (2007), in their study on how textbooks position students, suggested that "[l]anguage indirectly indexes particular dispositions, understandings, values, and beliefs" (p. 10). With such an approach to language, the authors explored language choices made in textbooks to unveil the constructed

model student reader. Following this approach, I examined how curriculum materials position teachers in their writing, and I want to investigate who the model teacher reader is.

Myself as a Reader of the Guides

For observation of teacher positioning, I built a reader persona. As reader, I was interested in learning from the TGs, whose texts I regarded as the minimum information I should know before teaching the lesson, and to whose suggestions and directions I was receptive. Further, I believed in TG authors' expertise and good intentions. Rather than raising questions, including productive questions, I inclined to accepting it as is. Because my reader persona was sensitive to visual representation, I by nature was attracted to larger-size and/or eye-catching color texts. I was more attentive to repetitive contents than to only occasional mention; this translates into my closer attention to contents with headings embedded in the design. For instance, if a tip for addressing potential student error is *given its own heading* and repeated in every lesson, I took it seriously, whereas if it was embedded in texts *under a different heading* – for instance 'Notes on the task' – I paid less attention to it. My reader perspective was that content possessed of its own heading means such content is more important than content without.

Another catalyst of my closer attention was multi-repetitive headings and associated contents in TGs. An interesting aspect of repetition is that teachers might develop positive attitudes to positioning that endures throughout a TG. Labeled *Mere Exposure Effect*, repeated exposure to a stimulus can engender a liking for that stimulus (Colman, 2015). In addition, frequent exposure to information contributes to considering the information to be valid or trustworthy (Rindfleisch & Inman, 1998). Accordingly, while not much work has been done on the effect of frequency of positioning on a person being positioned, repetition should not be disregarded.

An aspect of repetition is that it might reflect widely-shared positionings of teachers, the more so when different publishers present similar positionings in their TGs. Patterns observed from multiple sources "demonstrate evidence of particular hegemonic discourses or majority 'common-sense' ways of viewing the world" (Baker, 2006, p. 14) or "stereotypes" shared by the community (Hansson, 2015, p. 180). If a positioning is presented in four different TGs, it might indicate the positioning requires closer examination; it might be a shared stereotype – either welcomed or not – in the mathematics-education community.

To be clear, high number-count by itself cannot be used to claim the importance of a positioning or the likeliness of teachers' acceptance of that positioning. For a positioning to affect teachers, meaningful engagement is key (Gresalfi, 2009). Repetition, however, contributes to thickening of the position (Polman & Miller, 2010). Gresalfi (2009) echoed this point in an empirical study following four students for a year to examine shifts and lasting of their dispositions toward learning. Repetition reflects the number of opportunities presented to teachers, hence the higher chance of engagement and reinforcement. My intention was to discern where to find opportunities to engage with a positioning, because therein reside opportunities for teachers to engage. If a positioning is pervasive and repeatedly brought up in a TG, some might develop a positive attitude to the position due to mere exposure effect. Needless to say, teachers could refute the recognized positioning. In such case, the TG might end up communicating a limited amount of text (excluding texts with refuted positionings) or ask teachers to use considerable energy (to accept the information but separate out the positioning).

When observing positionings, I chose to use the "teachers have a duty to know" statement. This is a deliberate choice to acknowledge the possibility of modification when teachers enact the curriculum. Teachers read and make sense of the information and suggestions in the TG. After that, teachers enact a lesson, with or without modification of

what is in the TG. As this study pays attention to the moment when a teacher reads the TG, I tried to capture such a moment with "teachers have a duty to know" statement rather than "teachers do." Another reason for choosing "know" comes from my assumption about authors' situations, namely that TG authors choose content they deem meaningful for teachers to know, so as to teach well within the curriculum. Reality's conditions (e.g., page count, budget) are that authors may be blocked from including everything they hope teachers have a duty to know. In other words, TG texts are *selected* not *random* sets of information. Based on what has been discussed so far, I list headings I found to be repetitive per each phase and which represent parts on which I focus to observe positionings. See Appendix A for this list.

CHAPTER 4: RESULTS

In this chapter, I presented results for each of the three research questions. I began with my analysis for the first research question, which attended to positionings regularly occurring. Then, I provided results for the second research question that focused on idiosyncratically occurring positionings. Last, I described my examination of the three linguistic features, which is the third research question.

Results for Research Question 1: Regularly Occurring Positionings

I observed 44 positionings in total. In UCSMP, I observed 30 different positionings: in Pearson 27, in CMP 23, and in Eureka 24. The full list of positionings with phases I observed per each TG is in this dissertation's Appendix B. Here, instead of reviewing each positioning, I approached the analysis result from four directions. I began by discussing the identified positionings relative to the literature I reviewed regarding teacher professionalism. Then I focused on positionings all four TGs have in common. Following this, for each TG I presented positionings that are pervasive. Last, I provided cases where the TGs presented varied degree of support for teachers to go beyond the positionings of knowing, to doing. Based on my examination, I suggested that curriculum authors could enhance their TG's potential as sources of teacher education by proactively expanding the kinds of positionings they provide in the TG.

Narrow List of Positionings

By examining TGs, I found that aspects addressed by the positionings are not as varied as I had expected. In the literature review, I discussed five aspects related to teacher professionalism: i) knowledge base, ii) teacher research, iii) interaction with colleagues, iv) utilizing knowledge, and v) acknowledging uncertainty. These aspects suggested what is entailed in teaching mathematics. While these aspects are not mutually exclusive, all positionings I identified were about a teacher's knowledge base. By definition, all

positionings address at least one subdomain of the knowledge base. Some positionings address the other four aspects, although not as profoundly as in the literature. I now visit, in turn, each of the five aspects to show how TGs address them.

Knowledge base. Here I follow subdomains suggested by Ball, Thames, and Phelps (2008), among variations of classifications (e.g., Kunter et al., 2010; Tatto et al., 2008). Instead of classifying in which subdomain each positioning falls, I attended to whether each subdomain is addressed, by at least one positioning. From positionings I identified, it is clear that authors of the TGs are well aware that teaching mathematics requires more than knowledge of mathematics. Some subdomains were addressed in *all* four TGs; some others were addressed in *some* TGs.

Common content knowledge – i.e., mathematical knowledge required in a setting more general than teaching – is well reflected in the positioning that *teachers have a duty to know answers to tasks/questions*. By asking teachers to be aware of the answer, which falls in the general mathematical knowledge category, the TGs emphasized some common content knowledge.

Horizontal content knowledge – i.e., knowledge of the connection between mathematical topics in the curriculum – is not sufficiently visible to be elected as a positioning across all four TGs. Closest is the positioning that *teachers have a duty to know information of mathematical topics in addition to algebra.* This positioning attends to the connection between mathematical contents at the multiple-year level, rather than at a year. I identified this positioning from UCSMP and Pearson. For example, *An Articulated Curriculum Across All Grades*, from the IntTG phase of UCSMP, presented a diagram showing how the whole UCSMP program, including Algebra, can be used in Grades 5-12 classrooms, and by which group of students at each grade level. It showed the same information for Geometry; Advanced Algebra; Functions, Statistics, and Trigonometry;

Precalculus and Discrete Mathematics; and Calculus. Pearson presented big ideas in three topics namely Algebra, Statistics and Probability, and Geometry, to show how high school mathematics "fits together" (Pearson, p. T26). Both TGs are similar in that they attempted to show the placement of algebra to future mathematical topics. UCSMP does it via a diagram outlining when and to whom to teach each topic. Pearson does it by listing the big ideas.

Specialized content knowledge – i.e., mathematical knowledge specifically needed to teach mathematics – is emphasized by the following positioning: *Teachers have a duty to know supplementary information to implement a task*. This positioning highlights the need for teachers to have knowledge other than common content knowledge to successfully enact mathematical tasks in their classrooms. For instance, CMP's IntLe phase had *Problem Implementation* that informed teachers of a general set up for the lesson. This included whether the task was expected to be completed in a whole group, small group, or individual setting. Following the phases of MiLe, EnLe, and EnCh, the TG described what to attend to, when to call on students, how to support students needing more guidance, math facts, *etc*.

Knowledge of content and students – i.e., knowledge that attends to the intersection of content and student – is most-closely addressed by the two following positionings: *Teachers have a duty to know potential student errors*, and *teachers have a duty to know strategies students might use when solving tasks*. The IntCh phase of Pearson, for example, had three columns in the *Math Background* page, each with purple headings named after mathematical activities of the chapter (e.g., creating equivalent equations, solving proportions, working with percentages as proportions). Following the heading were general descriptions of strategies which students might apply when engaging with the math activity. Pearson often presented strategies using a simple math task as an example. Both these positionings regularly are observable in Pearson, whereas the other three TGs idiosyncratically present either one or both.

Knowledge of content and teaching – i.e., knowledge that attends to the intersection of content and teaching – is addressed by the positioning that *teachers have a duty to know mathematics for pedagogical purposes*. An example of this positioning is UCSMP's heading *Background* in the IntLe phase that explained the chapter's mathematics with particular reference to teaching it, including how students might react during the lesson, what to expect them to know already, and the aspect of the mathematical content related to the purpose of the lesson. This positioning is regularly observable from all TGs except for Eureka. In Eureka, this positioning is idiosyncratically observable.

Knowledge of content and curriculum – i.e., knowledge that attends to the intersection of content and curriculum – is addressed by the positioning that *teachers have a duty to know, in general, supplementary resources available*, among others; this positioning asks teachers to be aware of what instructional materials the TG provided either via supplementary student materials in reduced size (UCSMP), organizing available resources in a table (Pearson), listing resources needed to teach the lesson (CMP), and suggesting tools to prepare to solve tasks in the lesson (Eureka).

Apparent from this examination is that teaching mathematics as constructed by the TGs involves more than common content knowledge. All four TGs consistently suggested that teaching is about having specialized content knowledge and knowledge of content and curriculum, in addition to common content knowledge. Including idiosyncratic positionings, knowledge of content and students, and knowledge of content and teaching, are addressed in all TGs. Horizontal knowledge was the least visible subdomain across the four TGs.

Teacher research. Teacher research often involves collecting evidence from students and using it as data for analyzing one's own teaching. Teachers' analyses of such data can support them in determining possible changes in instructions that are specific to the classroom environment and which might enrich future teaching. Being as inclusive as

possible, I identified in Table 5 six positionings addressing teacher research. Not all positionings in Table 5 perfectly fit what the literature says about teacher research. I included those positionings to present the maximum potential of each TG so that this section could inform those interested in emphasizing teacher research to what they could begin with and extend on.

Table 5. Positionings Addressing Teacher research

Positionings	UCSMP	Pearson	CMP	Eureka
Teachers have a duty to know for each assessment type, when to administer, its format, and the standards.				IntCh
Teachers have a duty to know where students are at from student performance on the task/project/question/quiz using a rubric.	EnCh	EnLe		EnCh
Teachers have a duty to know questions and/or activities for students, to assess their learning of the lesson/chapter.	EnLe	EnLe EnCh	EnCh	EnLe
Teachers have a duty to know tasks for students, to assess their learning of the chapter.	EnCh	EnCh	EnLe	EnCh
Teachers have a duty to know what to consider when reflecting on their teaching to plan the next stage of the lesson accordingly.			MiLe EnLe	
Teachers have a duty to know they can make notes as needed.	IntCh EnCh		IntCh EnCh	

Among the six identified positionings, four were about assessing students (*teachers have a duty to know for each assessment type, when to administer, its format, and the standards*; *teachers have a duty to know where students are at from student performance on the task/project/question/quiz using a rubric*; *teachers have a duty to know questions and/or activities for students, to assess their learning of the lesson/chapter*; and *teachers have a duty to know tasks for students, to assess their learning of the chapter*). The four positionings I

identified have some aspect of teacher research because they expect teachers to collect data from students about their understanding of the mathematics taught in the lesson/chapter. To illustrate, I describe from where I observed the positioning that *teachers have a duty to know where students are at from student performance on the task/project/question/quiz using a rubric*. For specific locations of this and other positionings in each TG, see Appendix B.

Project Rubric, in the EnCh phase of UCSMP, is a guideline for teachers to determine where students are at, based on their performance on the project. The rubric description is general so that it can be copied and pasted into other chapters. According to the rubric, students are either advanced, proficient, partially proficient, not proficient, or no attempt. Pearson's *Lesson Quiz* in the EnLe phase is associated with *PRESCRIPTION FOR REMEDIATION.* The prescription is a simple rubric with three ranges of points, and into which level each range falls. Eureka's rubric is much more specific than those of UCSMP and Pearson. *A Progression Toward Mastery* in Eureka is a table showing for each assessment task, the CCSS content addressed and the rubric for assessing student learning. The TG defines four steps: Step 1 is for when the answer is missing or incorrect with little evidence of mathematical reasoning or application. Step 4 is when the answer is correct with rich evidence of mathematical reasoning or solid application. The table describes for each task what student performance would be from steps 1 to 4.

Communication actions associated with this and the other three positionings do not provide enough support for teachers regarding how they could use assessment data to make future plans, nor are explicit with the aspect of assessment results as a dataset for teachers. Therefore, these positionings address teacher research to some extent but less than sufficiently. Alternatively, communication actions associated with these positionings could be places for curriculum authors to extend if they are interested in promoting aspects of teacher research.

The fifth positioning in the table, *teachers have a duty to know what to consider when reflecting on their teaching to plan the next stage of the lesson accordingly*, more explicitly addressed teacher research as to what requires teacher professional judgment than do the four positionings discussed above. CMP had a notable feature providing two explicit opportunities for teachers to reflect on their interactions with students. In every MiLe of CMP, *Planning for the Summary* asked two questions: "What evidence will you use in the summary to clarify and deepen understanding of the Focus Question?" and "What will you do if you do not have evidence?" (CMP, p. 44). In EnLe, *Reflecting on Student Learning* presents questions directed at teachers to consider to "assess student understanding" (CMP, p. 46). Three main questions are: "What evidence do I have that students understand the Focus Question?" "How will I use this to plan for tomorrow? For the next time I teach this lesson?" and "Where will I have the opportunity to reinforce these ideas as I continue through this Unit? The next Unit?" (CMP, p. 46). CMP presented these questions, leaving it open for teachers to respond, instead of providing the authors' intended answers. By leaving questions open, CMP proactively invited teachers to collect their thoughts on classroom interaction, and to draw on their specific contexts so they effectively can enact their professional judgment in their next teaching move.

The last positioning, *teachers have a duty to know they can make notes as needed*, may seem to have the least connection to teacher research among the six identified positionings. I consider this positioning as somewhat addressing teacher research, however, because it provided space for teachers to express their own thinking. In particular, UCSMP and CMP presented spaces for teachers to make notes with no specific instructions. Pearson occasionally offered teachers some margin spaces wherein to jot notes, and teachers using Eureka could make digital notes on the pdf file, but such possibility is not explicitly suggested. In IntCh and EnCh of CMP, there was a lined-blank space with the heading *Notes*

and lines. IntCh and EnCh of UCSMP, too, regularly presented a lined-blank space with the heading *Notes* and lines. This simple feature offers teachers the possibility to record their thoughts/ideas as they wish. By having it, TGs invited teachers actively to utilize perspectives and conditions specific to them. In other words, teachers can add their teacher research observations in the TG. In so doing, teachers in effect become TG co-authors.

As to when these positionings are introduced, I identified all of them either in the introduction or end of a lesson or a chapter. The exception was CMP. Placing positionings addressing teacher research either at the introduction or the end phase of lesson or chapter is sensible because teacher research is about collecting data to plan future teaching practice. Yet teachers can collect evidence at various stages of a lesson. As CMP showed, it is not necessary to wait until the end of the lesson or chapter to collect such data. Findings so far support the fact that TGs acknowledged some aspects of teacher research, and that TGs could make more visible teacher research therein by attending to aspects of teacher research *other than* collecting evidence at *the end* of a lesson or a chapter.

Interaction with colleagues. Interaction with colleagues is another area for teachers to practice their professional judgment. By interacting with colleagues, a teacher becomes a participant of a collegial community. I identified one positioning to be relevant to this social engagement, namely *teachers have a duty to know tips from other teachers who have experience with using the curriculum*.

Table 6. A Positioning Addressing Interaction with Colleagues

Positionings	UCSMP	Pearson	CMP	Eureka
Teachers have a duty to know tips from other teachers who have experience with using the curriculum.	IntTG			

UCSMP listed tips from other teachers, suggesting a possibility of forming a community of professionals. *TEACHING MODES AND STRATEGIES*, within the IntTG of UCSMP, presents suggestions from practicing teachers who have used the curriculum. For example, suggestions included asking students to solve the questions before class, asking students to write steps they have taken, sharing additional examples from the TG with students, occasionally inviting students to read aloud. By presenting other teachers' voices, UCSMP established a community of professionals. Sense of community could have been stronger had teachers' voices been presented in the lessons, in addition to the introduction to the TG. Still, UCSMP is the only TG that aired insights from other teachers. Other TGs might have drawn on experiences of teachers who use a pilot version or a previous edition of the curriculum, but the TGs did not make explicit the presence of those teachers.

Utilizing emergent bilinguals' knowledge as a resource. Being responsible to students with diverse backgrounds is important. Teachers can achieve this by refusing deficit perspectives about these students and intentionally utilizing what the students already are familiar with. Pearson and CMP regularly provided information for teachers to support emergent bilingual students, although the supports are more around compensating students' deficiencies than utilizing the knowledge they bring in (Table 7).

Table 7. A Positioning Addressing Utilizing Emergent Bilinguals' Knowledge as a Resource

Positionings	UCSMP	Pearson	CMP	Eureka
Teachers have a duty to know how to support emergent bilingual students.		IntTG IntCh EnLe	IntCh	

In Pearson, *ELL Support* provides information for how teachers could support emergent bilingual students.[2] These supports include using graphic organizers, focusing on

[2] The term English Language Learner is not accurate because everyone, including those whose first language is English, is a learner of English all the time. Emergent bilingual student is a more accurate term for the student learning English as an additional language and attending an institution using English as its main language.

language, using role playing, focusing on communication, assessing understanding, using manipulatives, and connecting to prior knowledge. The communication action is crafted according to lesson content to provide supports specific to each lesson. In EnLe, the TG mentions the resource available for supporting emergent bilingual students at the intervention level. The table in IntTG allocates a column for to inform which resources could be used to support emergent bilingual students. The *Resources* table in IntCh is asterisked to indicate the resource is available in Spanish. In IntCh of CMP, it mentions sending English-language or Spanish-language letters to parents. The student version of CMP has an English-Spanish glossary, but the TG does not explain how teachers could use it for emergent bilingual students. UCSMP provides supports but they are idiosyncratic rather than regular.

In both Pearson and CMP, possible teacher moves to support emergent bilingual students are provided yet less from the direction for capturing their strengths. UCSMP and Eureka do not regularly attend to supporting emergent bilingual students, although UCSMP did provide suggestions on supporting emergent bilingual students when possible. This shows that efforts to utilize students' strengths is somewhat present in some TGs, but stronger attention from TG authors might be necessary.

Acknowledging uncertainty. Among the list of positionings I identified, four addressed uncertainty in teaching as described in the literature, namely that teachers are duty-bound to know i) additional tasks not in the student version; ii) how to extend tasks; iii) potential student errors; and iv) strategies students might use when solving tasks. As I illustrated in Table 8 and examples below, these positionings center mostly on uncertainty in pedagogical situations.

Table 8. Positionings Addressing Acknowledging Uncertainty

Positionings	UCSMP	Pearson	CMP	Eureka
Teachers have a duty to know additional tasks that are not in the student version.		MiLe		EnLe EnCh
Teachers have a duty to know how to extend tasks.	MiLe	MiLe		
Teachers have a duty to know potential student errors.		IntCh		
Teachers have a duty to know strategies students might use when solving tasks.		IntCh BeLe		

Pearson and Eureka presented additional tasks that are not available in the student version. Pearson presents additional tasks with answers at the bottom margin. The purpose thereof is unclear: Is it scaffolding? Or is it for students who are *faster* than others? Suggestions for implementation did not exist. Additional tasks, however, are provided consistently in the MiLe phase of all lessons. Eureka's hidden tasks were much more explicit regarding purpose. These are for assessing students' learning, either as an exit task in EnLe or as mid- and end-of-module test in EnCh. It seems that because tasks were intended as an assessment, they should not be readily available to students.

In terms of the positioning that teachers *have a duty to know how to extend tasks*, *Additional Examples* was available in all lessons of UCSMP except for five in the whole TG (108 lessons). These examples are not in the student version. Although the TG used the term *additional* in the heading, the examples here were more extensions of the existing examples than additional tasks. Answers were provided for each additional example. In the student version of Pearson, the main part of the lesson comprised a number of tasks. Each is named as *Problem #*. At the margin is a box with suggested questions followed by intended student answers. The questions asked students to explain their process, explain how they would check the correctness of their solution, describe the relationships, visualize a measure, *etc*.

Questions did not ask scaffolding questions that guide students through certain solution processes. No explanation is offered on the designers' intention, mathematical information, or what students know.

These examples show that uncertainty is somewhat addressed as what teaching mathematics entails in the TGs. The spectrum, however, of uncertainty represented in the TGs has potential to be extended to include other types of uncertainties. In addition, explicit presentation of the purpose of the communication actions as to what supports teachers dealing with uncertainty could contribute to enhancing clarity of communication between the TG and the readers.

Examination of observed positionings with the five aspects from the literature suggests that TGs narrowly construct what teaching mathematics entails. The construction is narrow because only a small number of positionings address each of the five aspects, excluding the knowledge base. In addition, in most cases the presence of those positionings is limited to one or two phases. Further, the positionings addressing these aspects cover a slice of each aspect, leaving various points of each perspective unshared with teacher readers. This indicates that the TGs do provide some positionings for teachers to engage with, but not as diverse as the literature suggests. In the literature review chapter, I discussed TGs' potential as sources for teacher professional developments. Despite such potential, my examination showed that TGs could expand on the kinds of positionings they provide to teacher-readers.

Common Positionings

Examining positionings with the five aspects, I found that teaching mathematics as constructed by the four TGs has much potential to be extended, so as to address aspects of teaching mathematics not actively dealt with so far. To better understand the positionings that exist in the TGs, I discuss the positionings commonly identified from the four TGs.

Among 44 positionings I identified, seven were in the four TGs. I grouped the seven positionings into three: 1) general information of the TG (*teachers have a duty to know, in general, supplementary resources available*; and *teachers have a duty to know the pacing the curriculum/chapter/lesson*); 2) specifics of the content (*teachers have a duty to know vocabulary*; *teachers have a duty to know supplementary information to implement a task*; and *teachers have a duty to know answers to tasks/questions*); and 3) assessment *(teachers have a duty to know questions and/or activities for students, to assess their learning of the lesson/chapter*; and *teachers have a duty to know tasks for students, to assess their learning of the chapter*). I give details of each of the positionings in Table 9 below.

Table 9. Common Positionings

Positionings	UCSMP	Pearson	CMP	Eureka
Teachers have a duty to know, in general, supplementary resources available.	IntCh	IntTG IntCh	IntCh	IntCh
Teachers have a duty to know the pacing of the curriculum/ chapter/ lesson.	IntCh	IntCh	IntCh	IntTG IntCh
Teachers have a duty to know vocabulary.	EnCh	IntCh	IntCh IntLe	IntCh
Teachers have a duty to know supplementary information to implement a task.	MiLe EnLe EnCh	IntCh BeLe EnLe EnCh	IntLe MiLe EnLe EnCh	MiLe
Teachers have a duty to know answers to tasks/questions.	IntTG BeLe MiLe EnLe EnCh	IntCh BeLe MiLe EnLe EnCh	MiLe EnLe EnCh	MiLe EnLe EnCh
Teachers have a duty to know questions and/or activities for students, to assess their learning of the lesson/chapter.	EnLe	EnLe EnCh	EnCh	EnLe
Teachers have a duty to know tasks for students, to assess their learning of the chapter.	EnCh	EnCh	EnLe	EnCh

General information of the TG. Two positionings, *teachers have a duty to know the pacing of the curriculum/ chapter/lesson.* and *teachers have a duty to know, in general, supplementary resources available* appeared in IntTG and/or IntCh. I describe the communication actions from each TG for the later positioning to illustrate (see Appendix B for information about other positionings). *Chapter Resource Masters*, from the IntCh phase of UCSMP, showed in reduced size supplementary student materials for each lesson. The size of each resource master was small – with font difficult to read but not impossible. This overview did not include electronic resources. IntTG of Pearson presented covers of hardcopy resources and screenshots of electronic resources; its following page had a table for each resource, whether for students at intervention level, on-grade level, enrichment, or emergent bilingual students. This table had no information specific to chapter or lesson level. The lesson-level specific information was in *Resources* of IntCh. The item *Resources* was a table showing resources available for each of the smaller parts of the lessons. Resources include web-based, printed, and in CD-ROM. Each resource type was annotated. An asterisk was used for resources available in Spanish. The *Planning Charts* of CMP had columns for *Materials* and *Resources* for informing about supplementary resources accompanying each chapter and lesson. Resources included *Labsheets* and *Teaching Aids.* In Eureka, *Suggested Tools and Representations* provided a list of tools (e.g., graphing calculator) and representations (e.g., coordinate plane, equations and inequalities) used in the chapter. Unclear is why Eureka decided to combine tools and representations. Except this, nothing was observed in terms of hardcopy resources. This may be because the curriculum is already an electronic version.

As the above description shows, these positionings are difficult to revisit elsewhere because they are more of an overview of particular aspects of the TGs. Among various options for an overview (e.g., mathematical concepts covered, general flow of each lesson),

those of pacing and of supplementary resources are pervasive and both are important to know for forward planning.

Specifics of the content. Three positionings were in the group of specifics of the content: t*eachers have a duty to know vocabulary*; *teachers have a duty to know supplementary information to implement a task*; and *teachers have a duty to know answers to tasks/questions*. Except for UCSMP, the TGs discussed providing vocabulary at IntCh or IntLe phases. These TGs asked teachers to remind students about vocabulary during introduction of a chapter or a lesson. UCSMP presented vocabulary in EnCh, indicating that vocabulary is to be reviewed before working on the end-of-chapter assessment tasks. All TGs except Pearson simply listed the terms.

The positioning that *teachers have a duty to know answers to tasks/questions* appeared in more than three phases of each of the four TGs. The positioning that *teachers have a duty to know supplementary information to implement a task* appeared in more than three phases of UCSMP, Pearson, and CMP, and in two phases of Eureka. In other words, these two positionings were not only pervasive *across* the TGs, but also pervasive *within* each TG. The TGs, however, placed relatively less attention on solution strategies. The positioning that *teachers have a duty to know strategies students might use when solving tasks* was observed idiosyncratically in UCSMP, CMP, and Eureka. Pearson addressed the positioning regularly in IntCh and BeLe, but not in MiLe, which was the main part of a lesson.

In general, teachers, at least those using one of the four TGs, are left alone to find strategies students might use to solve tasks. A possible explanation for this omission is that providing solution paths for all tasks requires significant space thus increases production cost. Another possibility is that omitting solution paths is so widespread as to have become the norm for curriculum authors. Alternatively, those authors might assume that teachers will

have no problem in devising multiple solution paths. Whatever the reason, as a result the positioning of knowing the answer appears more dutiful than the positioning of knowing solution paths. This imbalance in the degree of obligation might create in teachers the impression that, provided the answer is known, solution paths matter less. Further research is needed to understand how this imbalance impacts the practice of teachers aware that correctness of the answer does not guarantee correctness of the mathematical thinking involved in task solution, that solution paths are important evidence for formative assessment, and that attending to solution paths could support students' development of mathematical understanding.

Assessment. Two positionings about assessing students' learning of the lesson/chapter, *Teachers have a duty to know questions and/or activities for students, to assess their learning of the lesson/chapter.* and *Teachers have a duty to know tasks for students, to assess their learning of the chapter,* are observed from EnLe and EnCh phases of the TGs, suggesting that assessment is to be done at the end of each lesson and chapter. These two positionings together indicate that summative assessment is conducted not only with a set of tasks, but also with classroom interaction and small activities. Yet, this could mean also that the TGs foregrounded summative assessments and paid less attention to formative assessments.

The phase EnLe of UCSMP presented *Ongoing Assessment*, which suggests to teachers what to do to conclude the lesson. The suggestion included mathematical tasks for individual or small-group work, writing a definition or a paragraph of explanation, and using the graphic calculator. *Chapter # Self-Test*, from EnCh of UCSMP, presented texts accompanying Self-Test in the student version. The text read that this is an opportunity for students to receive feedback and correction, hence experienced an improvement on their performance after completing it. Pearson presented a set of mathematical tasks in EnLe and

pre-written questions in EnCh. In terms of the tasks, EnCh of Pearson had # *Chapter Test* and *Common Core Cumulative Standards Review*, both which supported teachers in conducting summative assessment of student learning. Two headings from EnCh phase of CMP, *Possible Answers to Mathematical Reflections* and *Possible Answers to Mathematical Practice,* provided ways to close a chapter by asking students to reflect on their learning. *ACE Assignment Guide* in EnLe of CMP listed tasks assignable to each lesson. *Closing*, from EnLe phase of Eureka, introduced a range of suggestions across the lessons. Those suggestions included discussing definitions of mathematical terms, revisiting mathematical processes used during the lesson, giving a set of tasks to students and discussing solutions, reminding of the goal of the lesson, and directing questions to students. Regarding tasks, Eureka had a *Mid-Module Assessment Task* and an *End of Module Assessment Task* in EnCh phase for teachers to give students.

To understand where students are at from their performance on the assessment, all TGs except CMP provided a rubric. These positionings might reflect the widely-spread perspective that teachers' duties include ensuring students' learning at the end of each lesson or chapter. Another perspective possibly reflected in this positioning is that assessment can be other than by paper-pencil tests. TGs support teachers practicing in such perspective by providing alternative ways to assess students' learning.

Pervasive Positionings from each TG

My examination so far showed positionings common to all TGs. In this section, I analyze in each TG, the positionings that are pervasive and particular to each TG. To address this, I examined from each TG the positionings identified in more than 3 phases. By so doing, I sought to understand for each TG the positionings repeatedly suggested as to what mathematics teaching entails. I organized those positionings in Figure 1 below. Underline indicates that the positioning is one of seven commonly appearing in the four TGs.

UCSMP
Teachers have a duty to know supplementary information to make accommodations.
Teachers have a duty to know answers to the tasks/questions.
Teachers have a duty to know supplementary information to implement a task.

Pearson
Teachers have a duty to know CCSS MP relevant to the TG content.
Teachers have a duty to know Big Ideas and essential questions/understandings.
Teachers have a duty to know what electronic resources are needed.
Teachers have a duty to know supplementary information to make accommodations.
Teachers have a duty to know how to support emergent bilingual students.
Teachers have a duty to know questions to ask.
Teachers have a duty to know answers to the tasks/questions.
Teachers have a duty to know supplementary information to implement a task.

CMP
Teachers have a duty to know answers to the tasks/questions.
Teachers have a duty to know supplementary information to implement a task.

Eureka
Teachers have a duty to know answers to the tasks/questions.

Figure 1. Positionings observed from more than three phases.

Across the four TGs, the positioning that *teachers have a duty to know answers to the tasks/questions* was pervasive. Except for Eureka, the positioning that *teachers have a duty to know supplementary information to implement a task* was pervasive also. This positioning about knowing supplementary information was present in Eureka, although not as pervasively as in three different phases. This mirrors a pervasive description of traditional pedagogy that the most central aspect of teaching mathematics is solving mathematical tasks and checking their answers.

In addition to the two positionings mentioned above, UCSMP frequently presented the positioning that *teachers have a duty to know supplementary information to make accommodations*. This positioning is mostly about supporting students at different levels. Example: *Differentiated Options* in IntCh of UCSMP is a table showing, for each lesson, on what page teachers can find making accommodations for learners, how to support vocabulary development, and ongoing assessment. The last column presents materials (e.g., graphing

calculator, Computer Algebra System, paper clips) needed in each lesson. The first two columns and the title of this table remind teachers that making accommodations is part of what teachers do. Regularly, MiLe of UCSMP presents what teachers could do, using the heading of *Accommodating the Learner.* Suggestions include questions, scaffolding mathematical tasks, and possible reactions from students. These are for "those learners who could benefit from more basic activities" (UCSMP, p. xxi). Occasionally the TG presents tips for students "who could benefit from more challenging activities" (UCSMP, p. xxi) under the heading *Extension*. *Recommended Assignment* of EnLe distinguishes tasks for all students and tasks for extra credits. Identical information is given in HOMEWORK in IntLe. Taken together, teachers using UCSMP are likely repeatedly to be exposed to the duty to make accommodations, as well as implementing tasks and knowing answers.

Reading Pearson, teachers are invited to engage at high frequency with eight different positionings. The three positionings pervasive in UCSMP also are pervasive in Pearson. Here, too, making accommodations meant differentiating instruction for students at *basic*, *average*, and *advanced* levels. The difference between the two is that Pearson's supplementary information is mostly about distributing worksheets to students at the end of each lesson (*i.e.,* EnLe), whereas UCSMP's is more is about practices teachers could try during the main part of each lesson (*i.e.,* MiLe).

Unique to Pearson is that the obligation to engage with CCSS MP and Big Ideas is heavy. Yet the TG is not necessarily consistent with presenting CCSS MP and Big Ideas. I illustrate via the instance of CCSS MP. In EnLe, *Mathematical Practices* lists the MP addressed by the exercise tasks. The TG first lists a selected set of MPs then places the task number next to the relevant MP. The MPs listed, according to the TG, is that of "supported in this lesson" (Pearson, p. 8). To be precise, the list is the MPs addressed by the tasks, not in the lesson. The MPs listed here do not match the MPs listed at the first page of the student

version where it provides CCSS content standard and MPs related to the lesson. Some lessons give more MPs than in the student version, while other lessons give fewer. An unexpected inconsistency is encountered with the TG's representation of the MPs addressed at the lesson level. Preparing to teach a lesson, teachers are asked to crosscheck texts about CCSS MP and Big Ideas and decide what to do with the inconsistency.

Apart from these positionings, Pearson users have obligations to attend to electronic resources, to predesigned questions to students in class, and to supporting emergent bilingual students. The positioning regarding emergent bilingual students is the only positioning pervasive in any of the four TGs addressing one of the five aspects I found from literature. As previously discussed, communication actions associated with this positioning focus more on providing additional support within the flow of dominant school culture than on actively utilizing students' knowledge. Yet, this positioning being pervasive, the TG constantly reminds teachers to consider emergent bilingual students when planning and teaching each lesson.

So far, I showed for each TG the positionings regularly occurred in more than three phases. The number of pervasive positionings varied across the TGs, as well as the kinds of positionings. I suggest two questions to consider. First, do most, if not all, of the pervasive positionings address the qualities that the TG seeks from a professional teacher? Second, are all the qualities of a professional teacher addressed by the pervasive positionings? Curriculum authors could intentionally place the positionings they would like to emphasize in multiple phases so that teacher-readers could gain sufficient opportunities to engage with those positionings.

Degree of Support

Until this point the examination I present did not necessarily attend to subtle differences within a positioning. Looking at seven different phases from four TGs, I was

more interested in the level of specificity that allowed me to make comparisons across the phases and the TGs. Yet I acknowledge that subtle differences exist within a positioning. The differences are due to variations in the range of support available from each TG, as mentioned briefly earlier. In other words, TGs may impose the same set of duties on teachers, but distinct variations exist in the degree of support those TGs individually provide to teachers acting on that set. Here I present those cases with different degrees of support in order to show that a positioning provided in two or more TGs does not mean that the teacher-readers will have the same experience with the positioning**.** Closer examination of this matter might require a separate study.

Teachers have a duty to know vocabulary. Knowing vocabulary is the positioning observed at an earlier stage of the chapter (i.e., IntCh) or the lesson (i.e., IntLe) of Pearson, CMP, and Eureka. UCSMP presented the positioning in EnCh so that students could review them before they work on the chapter assessment tasks. UCSMP offered the least support. Providing the vocabulary list in the student version only, UCSMP asked teachers to have students review the vocabulary. Pearson offered the most support. For each vocabulary word, Pearson explains how teachers could introduce it to students. This positioning exemplifies that although the same positioning is observable from the TGs, the degree of support for teachers to act on the positioning varies.

Teachers have a duty to know potential student errors. Pearson consistently attended to potential student errors. At the beginning of each chapter, the TG provided what students might do. Possible student errors discussed here include invalid application of a property, overgeneralization of linguistic features of mathematical sentences, steps students might forget, potential place of confusion, and more. Some chapters provide how teachers might address those errors by providing page numbers with potentially-helpful information or showing an example of a valid application of a property. In MiLe, not regularly but

occasionally the TG provides a brief paragraph for *error prevention*. Unlike errors discussed in IntCh, errors in MiLe were specific to the task. UCSMP and Eureka sometimes also discussed potential student errors but not regularly. CMP was relatively less proactive than the other three TGs in presenting student errors. IntCh of Pearson had three columns in the *Math Background* page, each with purple headings named after mathematical activities of the chapter (e.g., creating equivalent equations, solving proportions, and working with percentages as proportions). Following the heading are general descriptions of strategies students might apply when engaging with the mathematics activity. Often, TG presented strategies using a simple mathematics task as an example. PROCESS of BeLe describes the mathematically-valid process students might apply. Student strategies rarely were discussed for tasks in MiLe. UCSMP, CMP, and Eureka presented a valid student approach when needed, and are specific to the task, unlike IntCh of Pearson.

Teachers have a duty to know the pacing of the curriculum/chapter/lesson.

Support level varied also in a seemingly-straightforward positioning: *teachers have a duty to know the pacing of the curriculum/chapter/lesson*. All four TGs presented how many days are needed to cover the curriculum. UCSMP has a table with three different pacing options: for an average group of students, for an advanced group of students, and for a block-schedule class. For each option, the TG provided the number of days considered sufficient for students to learn each lesson. Pearson and CMP, too, showed numbers of days allocated for each lesson in both traditional and block schedule. Eureka is unique: in IntTG of Eureka, *Pacing and Preparation Guide* provided the number of instructional days needed to teach the whole curriculum; chapter-level pacing information is given in IntCh, but lesson-level pacing information not. Presumably, each lesson was supposed to be covered in one instructional day, but this was not communicated explicitly in the TG. In both IntTG and IntCh, Eureka did not indicate whether the days are for the traditional or the block-schedule.

Examples provided here are not exhaustive. Other positionings, too, have subtle differences in terms of the level of support they provide. My intention in providing these examples is partially an effort to strengthen this study's rigor. Acknowledging such subtlety, I aimed to provide a base for nuanced reading of the results. In addition, I suggest possible direction for future research by presenting examples.

Results for Research Question 2: Idiosyncratically Occurring Positionings

In this section, I share the positionings that idiosyncratically occurred in any of the four TGs. From my investigation, I found two different types of idiosyncratically occurring positionings: an idiosyncratic-only type and an idiosyncratic-regular type. I classified a positioning as an idiosyncratic-only type when it occurred sporadically not regularly in a TG. A positioning of idiosyncratic-regular type in a TG occurs regularly in certain phases and sporadically in other phases. Idiosyncratic-only type of positionings inform with the positionings that seemed absent from my investigation of the first research question. My investigation with idiosyncratic-regular type of positionings showed the following: some of the regularly occurred positionings were being reinforced by their idiosyncratic occurrences. From my examination, I suggested that idiosyncratic positionings have different level of presences and that idiosyncratic occurrence is what curriculum authors could use to foreground the positionings deemed significant.

General Descriptions

In total, I identified 17 positionings seeming to appear sporadically. Some of these were sporadic in some TGs but regularly observed in other TGs. No positionings were idiosyncratic in all four TGs, which was expected from the design of this research as explained in the Methods chapter. In the following, I discussed in turn the two types of idiosyncratic positionings. First, I attended to positionings that appeared, in any of the four TGs, idiosyncratically only. That is, I discuss the positioning *teachers have a duty to know*

potential student errors from UCSMP and Eureka (see Table 10). I name such positionings as idiosyncratic-only type. Second, I discuss idiosyncratic positionings that also regularly were observable. For example, I discuss the positioning in Table 10 from Pearson. I name these positionings as idiosyncratic-regular type. I close by presenting pervasive positionings in each TG when idiosyncratic ones were counted.

Table 10. Regularly and Idiosyncratically Occurring Positionings

Positionings	**UCSMP**	**Pearson**	**CMP**	**Eureka**
Teachers have a duty to know potential student errors.	MiLe	IntCh MiLe		MiLe

Notes. Green indicates phases where the positioning occurred regularly. Red indicates phases where the positioning occurred idiosyncratically.

Idiosyncratic-Only Type

In Table 11, each number indicates the number of phases in which the positioning was identified. The first column for each TG shows the number of phases where positioning occurred regularly. The second column shows the number of phases where positioning occurred idiosyncratically. Numbers with an underline are idiosyncratic-only types (i.e., the number in the first column is 0). Among 17 idiosyncratically occurring positionings, nine were of idiosyncratic-only type; four positionings in UCSMP, none in Pearson, one in CMP, and seven in Eureka.

Table 11. Idiosyncratic-Only Type Positionings

Positionings	UCSMP		Pearson		CMP		Eureka	
Teachers have a duty to know what electronic resources are needed.	2	0	5	0	6	1	0	1
Teachers have a duty to know how to support emergent bilingual students.	0	1	3	0	1	0	0	0
Teachers have a duty to know tasks/activities/questions for opening a chapter/lesson.	1	0	1	0	2	0	0	1
Teachers have a duty to know questions to ask.	0	0	5	0	1	2	0	2
Teachers have a duty to know potential student errors.	0	1	1	1	1	1	0	1
Teachers have a duty to know strategies students might use when solving tasks.	0	1	2	0	0	3	0	1
Teachers have a duty to know in general the mathematical contents of the chapter at the lesson level.	1	0	1	0	1	0	0	1
Teachers have a duty to know mathematics for pedagogical purposes.	1	1	1	1	2	0	0	1
Teachers have a duty to know to consider students' prior mathematical knowledge.	0	1	1	0	1	0	1	0

The idiosyncratic-only types in UCSMP are connected to students. These positionings address supporting emergent bilingual students, anticipating student responses (either mathematically valid or invalid), and incorporating mathematical knowledge that students are expected to know at the point of the lesson. Although these positionings are not built into the design (e.g., do not appear with regularity), UCSMP still acknowledges these positionings to be salient when teaching mathematics. These positionings were all placed in the phase of MiLe, except one about prior knowledge, which is in the phase of IntLe.

Pearson did not have idiosyncratic-only types. The six idiosyncratic positionings I observed from Pearson are of the second type – idiosyncratic yet regularly observable. I discuss these six later.

When regularly occurring positionings are considered, as in the first research question of this study, CMP seems not to suggest anticipating student strategy as a duty for teachers. My analysis for the second research question showed that this is a misunderstanding of the TG. CMP was in fact suggesting the positioning that *teachers have a duty to know strategies students might use when solving tasks*, idiosyncratically. I observed this positioning from three phases, namely beginning of a lesson (BeLe), middle of a lesson (MiLe), and ending of a lesson (EnLe). Such appearance of the positioning indicates that teachers using this TG do have ample opportunity to engage in anticipating student responses.

Eureka is the TG with the highest number of idiosyncratic-only types. Similar to the case of CMP, Eureka suggests more positionings than it seems from the list with regular positionings only. The two positionings, *teachers have a duty to know tasks/activities/questions for opening a chapter/lesson*, and *teachers have a duty to know in general the mathematical content of the chapter at the lesson level*, might be ones suggesting TG authors intentionally attend to the regularity of a positioning. Communication actions associated with positionings like the above might contain information beneficial for teachers in every lesson or chapter.

Idiosyncratic-Regular Type

Some communication actions associated with this positioning were embedded in paragraphs and sentences, and others were in headings. In Table 12, the underlined numbers indicate idiosyncratic-regular types. Similar to the idiosyncratic-only type table, numbers in cells indicate the number of phases where positioning was identified. Among 17 idiosyncratic

positionings, 12 were of this type; two positionings in UCSMP, six in Pearson, four in CMP, and three in Eureka.

Table 12. Idiosyncratic-Regular Type Positionings

Positionings	UCSMP		Pearson		CMP		Eureka	
Teachers have a duty to know CCSS MP relevant to the content of the TG.	0	0	4	0	2	0	1	1
Teachers have a duty to know relevant CCSS content standards at the chapter/ lesson level.	0	0	2	0	1	0	1	2
Teachers have a duty to know what hardcopy resources and/or materials are needed.	2	0	1	0	1	3	0	0
Teachers have a duty to know what electronic resources are needed.	2	0	5	0	6	1	0	1
Teachers have a duty to know supplementary information to make accommodations.	4	0	3	1	0	0	1	0
*Teachers have a duty to know task**s**/project**s** that overarch a chapter, to be revisited multiple times.*	2	1	2	1	0	0	0	0
Teachers have a duty to know questions to ask.	0	0	5	0	1	2	0	2
Teachers have a duty to know answers to tasks/questions.	5	0	5	0	3	1	3	1
Teachers have a duty to know potential student errors.	0	1	1	1	0	0	0	1
Teachers have a duty to know how the curriculum contributes to students' college and career readiness.	0	0	1	1	0	0	0	0
Teachers have a duty to know mathematics for pedagogical purposes.	1	1	1	1	2	0	0	1
Teachers have a duty to know supplementary information to implement a task.	3	0	4	1	4	0	1	1

Within idiosyncratic-regular type of positionings, I identified two kinds of idiosyncratic-regular type. One is that of extending, the other is that of disconnected. This classification is based on the connection between communication actions associated with the

regular appearance of a positioning and communication actions associated with idiosyncratic appearance of the positioning. When a positioning occurred idiosyncratically, the contexts of those idiosyncratic occurrences either can align with, or extend on, the context of regular occurrence or be different, or disconnected. For each TG, I analyzed which idiosyncratic-regular positionings were in which category.

In UCSMP, the positioning that *teachers have a duty to know tasks/projects that overarch a chapter*, to be revisited multiple times being of an idiosyncratic-regular type was expected by its design. This is so because the TG introduced the overarching project at the beginning of the chapter and summarized at the end of the chapter. In addition to this regular introduction, the TG idiosyncratically reminded teachers of the projects in some of the relevant lessons. Therefore, there was a coherency between the contexts of regular occurrences and idiosyncratic occurrences of the positioning.

The positioning that *teachers have a duty to know mathematics for pedagogical purposes* is without such coherence. UCSMP, in its IntLe phase, regularly provided paragraphs for teachers to read about the mathematical content of the lesson. In the middle of each lesson, the TG idiosyncratically provided relevant mathematical knowledge that could support teachers in implementing the tasks of the lesson. These two kinds of communication actions are interrelated because they deal with mathematical content of the same lesson, or addressing the same positioning that *teachers have a duty to know mathematics for pedagogical purposes*. Yet, the communication actions associated with the idiosyncratic occurrences provided supportive information for a specific part of the lesson or a specific task. In other words, these communication actions did not extend on the contexts that regular occurrences are drawing on. Therefore, this positioning occurred regularly and was reinforced idiosyncratically in multiple contexts.

Pearson was dominated by those with disconnected contexts. All six idiosyncratic type positionings were more of idiosyncratic-general type than of idiosyncratic-only type. Five of these did not share contexts across regular and idiosyncratic occurrences. For example, *teachers have a duty to know potential student errors* occurred regularly in the phase of IntCh. At this phase, the TG discussed student errors with limited specificity so that the discussion can cover all lessons of the chapter. The TG compensated this lack of specificity by revisiting the positioning in the phase of MiLe. Here, Pearson described potential errors students can make, probably when the TG authors thought such information could be salient for teachers preparing their lessons. Positionings about knowing supplementary information for making accommodations, college and career readiness, and mathematics for pedagogical purposes are similar in that regular positionings were provided at the general level, and specific aspects of the positionings are revisited in the middle of or at the end of the lesson. The positioning that *teachers have a duty to know supplementary information to implement a task* had the strongest presence among the six idiosyncratic positionings. This positioning regularly appeared in four phases and idiosyncratically in the phase of the middle of a lesson. Only one positioning, *teachers have a duty to know tasks/projects that overarch a chapter to be revisited multiple times*, shared the context.

Among the four idiosyncratic-regular type of positionings in CMP, one positioning, *teachers have a duty to know what hardcopy resources and/or materials are needed,* had consistency across the contexts of regular and idiosyncratic occurrences. At the IntLe phase, the TG provided a comprehensive list of resources teachers should prepare for the whole lesson. In the BeLe, MiLe, or EnLe phase, the TG gave the name of the resource from the comprehensive list when the resource is needed. This is clearly the case when the idiosyncratic occurrences of a positioning extend on the context of regular occurrence. Such occurrence of idiosyncratic positionings provide teachers with supports at specific level,

which a regular occurrence by itself could not achieve. The other three positionings had disconnected contexts, meaning that the contexts of idiosyncratic occurrences and of regular occurrences differed. *Teachers have a duty to know what electronic resources are needed*, is a good example. In all phases except the IntTG, CMP regularly reminded teachers at the upper margin to visit a website for additional resources. Main support here was the address of the website. In BeLe phase, the TG idiosyncratically asked teachers to show a video. Those lessons provided the online address of the video with a brief description of what the video is about. The contexts on which the idiosyncratic occurrences drew were disconnected from the context of the regular occurrence. Still, both cases support teachers via the use of electronic resources.

In Eureka, both CCSS MP and CCSS content were revisited, regularly and idiosyncratically. Regularly in the phase of IntCh, the TG identified the standards addressed in the chapter. In the phases of MiLe and EnLe, the TG notified specific texts relevant to each standard. That is, when it comes to the two positionings about the standards, Eureka provided a general-level support at an earlier phase with a specific-level support at later phases. The other two positionings are of disconnected contexts.

Closing Remarks

From the examination of idiosyncratic-only type of positionings, I showed that UCSMP described what students know and ways to meet their needs, CMP did provide possible student strategies, and Eureka provided a range of information including where to find resources on possible student errors. Results from analysis of idiosyncratic-regular type of positionings suggested that UCSMP strongly emphasized teachers' duties of implementing projects and of knowing mathematics for pedagogical purposes. For Pearson, the analysis suggested that the TG reinforced the duties of knowing how to make accommodations, implementing projects, knowing possible student errors, considering students' college and

career, knowing mathematics for pedagogical purposes, and knowing supplementary information to implement a task. In terms of CMP, the TG highlighted the hardcopy and electronic resources needed to implement a lesson, knowing what questions to ask students, and knowing answers to tasks or questions. Eureka emphasized standards, answers to tasks or questions, and supplementary information to implement a task. Some of these idiosyncratic-regular positionings shared contexts across regular and idiosyncratic occurrences. Other positionings of this type did not share the contexts.

Idiosyncratic positionings are worth examining because these are the positionings the TG authors provided by breaking the structure of the genre of the TG. From my examination of idiosyncratic positionings, however, I claim that not all idiosyncratic occurrences are the same. Idiosyncratic-only type of positionings had rather vague presence while others had a much stronger presence, reinforced by regularity. In addition, among those with stronger presence, some positionings were placed in multiple contexts while others in singular context. Idiosyncratic-regular positionings with disconnected contexts might be those the TG authors wanted to emphasize the most, because of the multiplicity of contexts and their breaking of the TG's genre. In addition, those could be positionings teacher-readers might take up because of the support provided at both the general and specific levels in a variety of contexts. Therefore, curriculum authors can maximize the opportunities for teacher-readers to a certain positioning by placing it both regularly and idiosyncratically, and by drawing on multiple contexts. Without explicit attention to this matter, the authors might unintentionally overemphasize the positionings with mediocre importance.

Results for Research Question 3: Voices of the TGs

To understand the voice of the TGs, I examined the three following aspects: i) use of pronoun, ii) modality, and iii) imperatives. Regarding the pronoun, I showed that all three pronouns are used to acknowledge teacher-reader presence. Therefore, more pronoun use

means higher tendency of speaking *to* teachers. When it comes to modality, I grouped modalities into two: suggestive and assertive. With this grouping, I compared incidences of suggestive cases *versus* assertive cases to examine the degree of teachers' professional judgment acknowledged in each TG. Last, I analyzed the use of imperatives. My analysis showed that the TGs used imperatives directing teachers to follow TG suggestions rather than critically examine those suggestions. From this, I concluded that more use of imperatives in these four TGs indicates limited acknowledgement of teachers' professional judgment, hence closer to speaking through than speaking to. At the end of this section, I compared the TGs based on the results from the examination of pronoun, modality, and imperatives.

General Description

Before sharing the findings I outlined above, I provide general findings in Table 13. By total sentences directed at teachers, I mean the sum of sentences that include a pronoun referring to teachers plus sentences that begin with an imperative. Most pronouns were referring to teachers, either explicitly or implied. Only two sentences with the pronoun *we* clearly excluded teachers and were referring to the authors. I did not count these two sentences as referring to teachers. All imperative sentences are included because the sentences are from TGs, which are written for teachers to read. By total incidences directed at teachers, I mean the sum of sentences with pronoun directed at teachers and imperative incidences. Imperative incidences is the number gained by multiple counting of sentences with multiple imperatives (e.g., a sentence with two imperatives is counted twice, a sentence with three is counted thrice). In other words, I counted the number of commands directed at teachers.

Table 13. General Findings

	UCSMP	Pearson	CMP	Eureka
Sentences with Pronoun	70	17	45	13
Sentences with Pronoun Referring to Teachers	68	17	45	13
Imperative Sentences	135	219	38	110
Imperative Incidences	137	223	38	114
Imperative Incidences per Pronoun Referring to Teachers	2.01	13.12	0.84	8.77
Total Sentences Directed at Teachers	203	236	83	123
Total Incidences Directed at Teachers (IDT)	205	240	83	127
Total Pages	49	82	20	48

Simple counts may be inappropriate for cross TG comparison. One way to address this is by comparing the ratio of imperative incidences to the sentences with a pronoun referring to teachers; the higher the ratio, the less the sense of human existence. Imperative incidences do not use explicitly stated pronouns. Rather, the recipient of the message is implied. Therefore, these incidences have less human presence than the sentences with pronouns. Because this study's focus is on the TGs use of voice in relationship to teachers, I used the number counts of the sentences and incidences imposing duties on teachers. All imperative incidences were imposing duties on teachers. A small number of sentences with pronouns were exclusively referring to the curriculum authors or students. The number count of the sentences with pronouns referring to teachers only is in the row Sentences With Pronoun Referring to Teachers. The ratio in the row Imperative Incidences per Pronoun Referring to Teachers shows that Pearson, with a ratio of 13.12, rarely used a pronoun referring to teachers only rarely, compared to its use of imperatives. The ratio and the dominant use of imperatives might suggest Pearson's low personal involvement, hence the TG is speaking through teachers rather than *speaking to* teachers (Herbel-Eisenmann, 2007;

Remillard et al., 2011). CMP is the only TG that presented more sentences with pronouns than it presented imperative sentences. The next highest was UCSMP but it still was much lower than Eureka and Pearson, Eureka's ratio being more than thrice that of UCSMP and being slightly higher than half of Pearson. Below I discuss TGs' use of pronoun, modality, and imperatives in detail.

Pronoun: General Findings

In Table 14, I organized general findings regarding the use of pronoun.

Table 14. The Use of Pronoun: General

	UCSMP	Pearson	CMP	Eureka
Total Referring to Teachers (Explicit)	26	15	28	3
Total Referring to Teachers (Explicit or Implied)	68	17	45	13
Total Sentences with a Pronoun.	70	17	45	13
Pronouns referring to teachers per IDT	0.33	0.07	0.54	0.10
Pronouns referring to teachers per page	1.39	0.21	2.25	0.27

With the exception of two sentences from UCSMP, all sentences with a pronoun referred to teachers. Some sentences made explicit that the pronoun is directed at teachers. Two examples include "You might write or project the following so that students can answer these questions as they enter the classroom" (UCSMP) and "If you need to, show them or have a student demonstrate the answer to this question" (Eureka). Sentences Implied are those with pronouns open enough to refer to teachers and, at the same time, to others (e.g., students, curriculum authors). This often happened when the TG described mathematical facts or processes. Consider the following example. "Point out that as you round off to more

and more decimal places for your estimate, the estimate becomes more accurate." (Pearson). In this study, I counted this as a sentence with a pronoun implying teachers. The pronoun *you* in the previously given example could include students – seeing the students as those who were rounding the numbers and teachers were expected to almost read to the students the sentence excluding "Point out that." Yet, there is a possibility to include teachers in the pronoun – imagine a teacher rounding off numbers showing students the estimate is becoming more accurate. Two sentences from UCSMP were using the pronoun *we* to refer exclusively to the authors. These are the two sentences: "In this book, we have chosen to think of terms as added" (UCSMP, p. 16) and "We have purposely separated them in Lessons 1-1 and 1-2 so that students will see the associative properties as switching order of operations, while the commutative properties involve switching the order of addends (in addition) or factors (in multiplication)" (UCSMP, p. 17). Here, the pronoun "we" refers to the authors, with a clear purpose of sharing their design principle with teacher readers. Therefore, these are the incidences where the TG is speaking to teachers.

In the rate of sentences with pronouns per total incidences directing at teachers, CMP was the highest, followed by UCSMP, Eureka, and Pearson. This order remained the same when the ratio was examined via pages examined. This might suggest that CMP is the curriculum speaking *to* teacher readers, while Pearson is speaking through them.

Pronoun *You*

Table 15 is a summary of my findings regarding the TGs use of the pronoun *you*.

Table 15. The Use of Pronoun *You*

		UCSMP	Pearson	CMP	Eureka
You	Teachers (Explicit)	26	15	19	3
	Teachers (Implied)	13	2	4	0
	Not Referring to Teachers	0	0	0	0
	Total Referring to Teachers	39	17	23	3
	Total Referring to Teachers per Total Sentences with a Pronoun	0.56	1.00	0.51	0.23

All sentences with the pronoun *you* were referring to teachers. With such direct addressivity, more sentences were explicit than implying, which means the pronoun was open enough to refer to others in addition to teachers. In UCSMP and CMP, slightly more than half the sentences with a pronoun used *you*. Pearson had no sentence with a pronoun other than *you*. About a fifth of Eureka sentences with a pronoun used *you*. The table immediately above indicates that, across the four TGs, the pronoun *you* was used to communicate with teachers. This data indicates that all four TGs are acknowledging the teacher-reader presence, although with varied degree. The TGs more often explicitly referred to teachers. In addition, even when the sentences were not explicitly referring to teachers, they implied teacher presence. No use of the pronoun *you* excluded teacher presence. Drawing on my improvisation of speaking to versus speaking through (as explained in Chapter 3), this acknowledgement of teacher-reader presence indicated that the TGs are speaking to them. This result, however, does not guarantee that all TGs present teacher-reader with high visibility. Their use of other pronouns, modalities, and imperatives, which are discussed below, support this point. Considering that the TGs are written for teachers to read, all the sentences with the *you* pronoun referring to teachers is more of a natural phenomenon than anything else.

Pronoun *We*

I organized the TGs' use of the pronoun *we* in the table below.

Table 16. The Use of Pronoun *We*

		UCSMP	Pearson	CMP	Eureka
We	Teachers (Explicit)	0	0	0	0
	Teachers (Implied)	29	0	7	10
	Not Referring to Teachers	2	0	0	0
	Total Referring to Teachers	29	0	7	10
	Total Referring to Teachers per Total Sentences with a Pronoun	0.39	0.00	0.16	0.77

The TG with the highest use of the pronoun *we* is UCSMP, albeit that Eureka has the highest percentage of *we* pronoun sentences. CMP used *we* in less than a fifth of its sentences with pronouns directed at teachers; no sentence using *we* is directed explicitly at teachers; instead, the pronoun is used in describing mathematical activity in the context the authors assume is to be shared among teachers, students, and the textbook. Another use of *we* is when describing general mathematical facts. The exception was UCSMP, which twice used *we* to make the authors visible (e.g., "We have purposely separated them in Lessons 1-1 and 1-2 so that students will see the associative properties as switching order of operations" (UCSMP, p. 17).). Making authors visible is indeed a way to have the TG speak to teachers by acknowledging their presence. I observed such use of *we* from UCSMP only. Other TGs' use of *we* was to acknowledge the teacher-reader presence by referring to them.

Pronoun *I*

In addition to the pronoun *we*, I also examined how TGs are using the pronoun *I*. Table 17 is a summary of my findings.

Table 17. The Use of Pronoun *I*

		UCSMP	Pearson	CMP	Eureka
I	Teachers (Explicit)	0	0	9	0
	Teachers (Implied)	0	0	6	0
	Not Referring to Teachers	0	0	0	0
	Total Referring to Teachers	0	0	15	0
	Total Referring to Teachers per Total Sentences with a Pronoun	0.00	0.00	0.33	0.00

Only in CMP is the *I* pronoun used. The explicit *I* pronoun is in questions on which teachers should or may reflect, such as "How will I use this to plan for tomorrow?" (CMP, p. 46). In terms of implied *I* pronoun sentences, CMP placed those sentences immediately after providing suggested questions to ask students. Typically in CMP, what comes after suggested questions are intended answers or pedagogical information for teachers. The intended answers are where CMP used *I*. For example, in the sentence "I could do more chores around the house to increase my allowance" (CMP, p. 42), the TG used the pronoun *I,* which seems to refer to students for the most part. In other lessons, the TG gave pedagogical information without using a pronoun. In the sentence "Students should notice that the rate of change for the quadratic function is different from that for linear or exponential," (CMP, p. 52) the TG is making clear that the sentence is more for teachers than students. This brings complexity. The TG placed at least two different types of sentences (i.e., intended answers and pedagogical information) at the same location (i.e., after the suggested questions) across different lessons. Taking this into consideration, I interpreted the *I* pronoun as open enough to imply teachers in addition to students.

With or without counting the implied cases, CMP is the only TG to place itself as the teacher-reader. This proactive level of communication appears in one TG and in a third of the

total sentences with a pronoun. That is, the examination with *I* pronoun shows that CMP is most actively speaking to teachers while other TGs are not.

So far I examined three pronouns in the TGs. From examining *you*, I found all four TGs were foregrounding teacher-reader presence with the pronoun. Regarding the pronoun *we*, Eureka used it most frequently to refer to teachers. Particularly interesting was UCSMP's use of the pronoun *we*. UCSMP is the only TG that used the pronoun to refer to themselves as authors to explicitly discuss their design principles**.** With the pronoun *I*, CMP is the only TG which used it. All in all, these findings suggest that each TG has their own way to highlight teacher-reader presence, hence speak to teachers in various ways. Because most of the sentences with pronouns were acknowledging teacher presence, what becomes salient is how often the TGs used these three pronouns. For example, Pearson used the pronoun *you* to refer always to teachers. Focusing on this particular finding, it suggests that the TG is speaking to teachers. What if, however, only a few sentences in the TG had pronouns? Were this the case, it would be difficult to consider the TG as actually speaking to teachers. Therefore, the ratio of sentences with a pronoun to total sentences is worth examining. I presented results from such examination toward the end of this chapter.

Modality

To understand the use of modality, I examined sentences with pronouns referring to teachers. In other words, among the total sentences examined in this study, I excluded the sentences without pronouns, or imperatives. Table 18 shows the number of sentences per each modality used, explicitly or implicitly referring to teachers. [do] indicates sentences with no modality but a verb asking teachers to perform certain action. T indicates explicitly directed at teachers and (t) indicates implicitly directed at teachers. All modal verbs I found were positive, except for one case from Eureka that used *should not*. I count this as *should,* following Halliday and Matthiessen's (2004) classification.

Table 18. Modality per Explicit and Implying

	UCSMP		Pearson		CMP		Eureka	
	T	(t)	T	(t)	T	(t)	T	(t)
can	3	3	0	0	5	4	0	0
could	1	3	0	0	1	3	0	1
may	1	1	1	0	8	0	0	0
might	14	2	0	0	0	0	0	1
will	0	0	0	0	7	2	0	1
would	0	0	0	0	0	0	0	0
should	2	1	0	0	0	0	1	0
need	1	3	0	0	0	0	0	0
[do]	4	29	14	2	7	8	2	7
total	26	42	15	2	28	17	3	10

Halliday and Matthiessen (2004) classified modal operators as of low-, median-, or high-value. Table 19 below shows which modal verbs are in which group.

Table 19. Excerpts from Halliday and Matthiessen's (2014, p. 116) Table for Finite Verbal Operators

Modal operators:			
	low	median	high
positive	can, may, could, might, (dare)	will, would, should, is/was to	must, ought to, need, has/had to
negative	needn't, doesn't/didn't + need to, have to	won't, wouldn't, shouldn't (isn't/wasn't to)	mustn't, oughtn't to, can't, couldn't, (mayn't, mightn't, hasn't/hadn't to)

In addition to modal verbs in Halliday and Matthiessen's work. I paid attention to bald assertions, i.e., sentences with no modality (Herbel-Eisenmann, Kristmanson, & Wagner, 2011), or *root modality* (Rowland, 2005). Having no modality, bald assertions give no room for negotiation but rather present a sentence as a fact that does not reflect the speaker's stance (Herbel-Eisenmann et al., 2011; Rowland, 2005).

Table 20 presents the sum of the number of sentences for each modality.

Table 20. Modality with Explicit and Implying Combined

	UCSMP	Pearson	CMP	Eureka
can	6	0	9	0
could	4	0	4	1
may	2	1	8	0
might	16	0	0	1
will	0	0	9	1
would	0	0	0	0
should	3	0	0	1
need	4	0	0	0
[do]	33	16	15	9
total	68	17	45	13

In my analysis, I combined the modality with low and median values as *suggestive* (can, could, may, might, will, would, should), and high value and bald assertions as *assertive* (need, [do]). By so doing, I sought to compare the degree of obligation TGs are imposing on teachers. Table 20 presents the number counts of the sentences with suggestive voice *versus* those with assertive voice. Some modalities from Halliday and Matthiessen's (2004) work are not assigned to either group because none of the four TGs used them. Some of the imperatives I had under *suggestive* may seem stronger than suggestive. My rationale for such grouping is that I wanted to be conservative with my coding in order to highlight when the TGs strongly impose duties on teachers. Table 21 shows that despite the decision to conservatively code the TGs, three of the four TGs were using a more assertive voice than suggestive.

Table 21. Modality by Suggestive and Assertive

	UCSMP	Pearson	CMP	Eureka
Suggestive (can, could, may, might, will, would, should)	31	1	30	4
Assertive (need, [do])	37	16	15	9
Suggestive per Total Modality Sentences	0.46	0.06	0.67	0.31
Assertive per Total Modality Sentences	0.54	0.94	0.33	0.69
Suggestive per IDT	0.15	0.00	0.36	0.03
Assertive per IDT	0.18	0.07	0.18	0.07
Suggestive per page	0.63	0.01	1.5	0.08
Assertive per page	0.76	0.20	0.75	0.19

In UCSMP, suggestive and assertive voices are used with similar frequency. Pearson and Eureka used assertive voice more often than suggestive voice. CMP is the only TG using voice more suggestive than assertive. Suggestive voice in CMP appears about twice as often as the assertive voice in it. This result suggests that CMP is the TG most highly appreciative of teachers' ability to make professional judgment, followed by UCSMP, Eureka, and Pearson.

The ratio with IDT and page shows how frequently the two voices are used in the whole data set. In addition, the ratios make possible the comparison across TGs. Ratio per total incidences shows that overall modality is used rarely in Pearson and Eureka. Ratio per pages examined reinforces that CMP uses suggestive voice much more often than do other TGs, and that Pearson and Eureka rarely use modalities. That is, CMP is not only the TG that used more suggestive voice than assertive voice, but also the TG that used suggestive voice with the highest ratio – 36% – among the four TGs. The limited use of modality, combined with more frequent use of assertive than of suggestive, indicates that such TG might be forcefully imposing teaching duties on teachers, rather than communicating with teachers. The duty seems highly obligatory in such TG.

Imperatives

To find sentences with imperatives, I reviewed all sentences in the four TGs. In sum, I found in UCSMP 135, in Pearson 219, in CMP 38, and in Eureka 110. Some sentences had two or more imperatives, so my count is based on the imperatives used. For example, the sentence "Collect these papers, mix them up, and pass one out to each student" (Pearson, p. 28A), is counted as one imperative sentence and three imperative incidences. In Table 22, I organized number counts and ratios of imperatives.

Table 22. Imperatives: General Results

	UCSMP	Pearson	CMP	Eureka
Total Sentences w/ Imperatives	135	219	38	110
Total Incidences w/ Imperatives	137	223	38	114
Incidences w/ Imperatives per IDT	0.67	0.93	0.46	0.88
Incidences w/ Imperatives per page	2.80	2.72	1.90	2.38

Table 22 shows that Pearson and Eureka are quite heavy with imperative sentences. Although the per-page ratio of incidences with imperatives is higher in UCSMP than in Pearson, imperative sentences in UCSMP are lower than those of Pearson and Eureka when compared to the total incidences directed at teachers. CMP's ratio is the lowest in both per incidences directed at teachers and per-page. If all imperatives direct teachers in what to do and do not acknowledge teachers' professional judgment, which I unpack in the latter part of this subsection, the high number count and the high ratio indicates low degree of room for professional judgment. Pearson is the most obligatory among the four TGs, both in terms of its number counts and the ratio. The number count shows that UCSMP used more imperatives than Eureka. When the ratios are considered, Eureka is in fact using more imperatives than UCSMP. Slightly below half of all incidences in CMP were using imperatives. Therefore, if my assumption is reasonable – that all imperatives limit teachers'

professional judgment – Table 22 suggests that CMP is the most speaking to TG and Pearson is the most speaking through one.

Table 22 by itself does not present enough information about the degree of teachers' professional judgment acknowledged by the TGs. This is so because the number counts or the ratio is not a direct reflection of degree of obligation imposed on teachers. Rotman (1988) distinguished imperatives as *exclusive* or *inclusive*. Exclusive imperatives strongly impose duties on the readers. Inclusive imperatives acknowledge the judgment the readers have. Therefore, understanding which types of imperatives are dominant in which TG is important. In the following, I show that exclusive imperatives dominated inclusive ones in all four TGs.

To understand the type of imperatives each TG is using, I analyzed the kinds of imperatives. Ninety-seven imperative verbs appeared in at least one of the four TGs. UCSMP used 32 imperative verbs. Pearson used 45, CMP used 38, and Eureka used 48. Not all imperative verbs appeared with similar frequency. Table 23 presents the range, mean, median and mode of the imperatives.

Table 23. Range, Mean, Median, Mode of the Frequency of Imperative Incidences

	UCSMP	Pearson	CMP	Eureka
Range	39	53	5	21
Mean	4.28	4.85	1.9	2.38
Median	2	1	1	1
Mode	1	1	1	1

As Table 23 above shows, the common feature across the four TGs is that they used most of the imperatives once or twice. In UCSMP, the two most-used imperatives were *have* (39 x) and *ask* (35 x). Excluding those two, the frequency range becomes 9, which indicates a relatively low frequency of the 37 imperatives. In Pearson, *use* (53 x) and *have* (46 x) were the two most dominant imperatives; without those two, the frequency range becomes 19. Sentences in CMP used *ask* (5 x) and *tell* (5 x); again, not counting those two imperatives, the

frequency range is 3. Eureka used *have* (21 x) and *ask* (17 x) the most; not counting those two, the frequency range is 5. Overall the four TGs, the three most-used imperatives are *have* (107 x), *ask* (67 x) and *use* (60 x); excluding those three, the range is 19. Imperatives such as *have*, *ask*, *use*, and *tell* all direct teachers to perform such action, hence exclusive. This results shows that all TGs used more exclusive imperatives than inclusive ones. Therefore, the numbers and ratios I present in Table 22 is a reflection of the degree of obligation each TG is imposing on teachers.

Conclusion from the Three Results

In the subsections on pronouns, I argued that all pronoun use can be interpreted as cases of speaking to, based on my improvisation of Remillard's (2000) original definition. I showed that the use of assertive modality and imperatives are the cases of speaking through. In Table 24, I presented results combining the findings from pronoun, modality, and imperative. Based on the combined results, I discussed the degree of obligation imposed on teachers from each TG.

Table 24. Aggregated Table

		UCSMP	Pearson	CMP	Eureka
Total Sentences with a Pronoun Referring to Teachers		68	17	45	13
Modality	Suggestive	31	1	30	4
	Assertive	37	16	15	9
Imperatives		137	223	38	114
Pronouns Referring to Teachers per IDT		0.33	0.07	0.54	0.10
Suggestive per IDT		0.15	0.00	0.36	0.08
Assertive & Imperatives per IDT		0.85	1.00	0.64	0.97
Suggestive per page		0.63	0.01	1.5	0.08
Assertive & Imperatives per page		3.55	2.91	2.65	2.56
IDT per page		4.18	2.93	4.15	2.65

Considering the ratio of assertive voice and imperatives combined to the incidences directed at teachers shows that Pearson always use language with high obligation of duties (1.00). Eureka is second-highest at 0.97. CMP is lowest at 0.64. That order matches the IDT ratio for pronoun use. Based on numbers in the table, UCSMP and CMP actively communicate with teachers (IDT per page), being more open than directive than the other two. Accordingly, the degree of obligation to follow the TGs suggestions is higher in Pearson and Eureka than in UCSMP and CMP. From this aggregated result, I suggest that CMP and UCSMP are closer to the speaking to end of the spectrum while Pearson and Eureka are closer to the speaking through end, due to the different level of teacher professional judgement each TG is allowing. Yet, the per-page ratios show that teachers are likely to read sentences limiting their use of professional judgment more often than the sentences acknowledging it.

CHAPTER 5: DISCUSSION

In this chapter, I first revisit each of the three research questions in order to **locate my findings in the literature.** Then, in the section titled *Looking Backward*, I discuss what I learned from conducting and writing this dissertation study. I ended this chapter with the section *Looking Forward* to offer suggestions for practitioners and researchers.

Returning to the Research Questions

Research Question 1

Narrow list of positionings. In the Results chapter, I addressed RQ1 in four ways: i) by comparing the list of positionings to the literature; ii) by presenting positionings commonly observed across the four TGs; iii) by examining positionings revisited in more than three phases for each TG; and iv) by drawing attention to varied degrees of support.

Following the classification by Ball, Thames, and Phelps (2008), I found that TGs regularly presented three subdomains of what teaching mathematics entails: i) common content knowledge, ii) specialized content knowledge, and iii) knowledge of content and curriculum. Three TGs – UCSMP, Pearson, and CMP – regularly addressed knowledge of content and teaching. This subdomain was present in Eureka, although idiosyncratically. Knowledge of content and students was regularly attended in Pearson; the other three TGs did it idiosyncratically. No TG suggested horizontal knowledge as what teachers should know, although UCSMP and Pearson explained the place of the algebra curriculum within the context of secondary-school mathematics curriculum. This shows that each TG constructed teaching mathematics as entailing more than common content knowledge. Instead, mathematics teachers are asked to have knowledge in multiple subdomains.

The literature about knowledge base emphasized that knowledge required for teaching is well beyond common content knowledge (Shulman, 1986;1987) and that making visible various subdomains of knowledge contributes to understanding teaching as a full profession

(Ball, Thames, & Phelps, 2008). Considering this, having TGs pay attention to multiple subdomains is to be welcomed and encouraged. My findings suggest that while the research field's effort to redefine subdomains (e.g., Speer, King, & Howell, 2014; Tatto et al., 2008) are important contributions, the field also needs to examine the praxis of the subdomains. Curriculum materials, including TGs, are powerful sources with the potential to bring large-scale difference at the classroom-instruction level (Stein & Kim, 2009). More literature on using multiple knowledge bases as a design principle for TGs not only could contribute to enhancing the practicality of the subdomains, but also could support development of teachers' sense of professional self through developing their knowledge. This becomes even more significant to the teachers who had primarily learned mathematics procedurally. TG could provide opportunities for those teachers to develop a broader range of understanding of mathematics.

Comparison of the list of positionings to the other five aspects from the literature suggests that TGs could diversify their construction of teaching mathematics. Regarding teacher research, its most visible feature was, at the end of a lesson or a chapter and focused on collecting data from students about their learning. Teacher research was addressed in all four TGs with three or four positionings. This is encouraging because of the importance of teacher research (Cochran-Smith & Lytle, 2006). In addition, in all TGs the data collection is constructed as what could be done with verbal interaction or activities, in addition to a more traditional way of handing a set of mathematical tasks. UCSMP and CMP went a step further, inviting teachers to make notes in the TG. Yet in all TGs it is unclear how teachers might utilize data they collect. Further, data collection happens mostly at the end of a lesson or chapter. Teachers can conduct a teacher research through written assessment as well as journaling, writing essays, oral inquiries, and classroom studies (Cochran-Smith & Lytle, 1990). Literature emphasized the importance of teacher research because teachers such a

process can enhance their understanding of the local context in which they are teaching. Considering that curriculum authors design their TGs to a wider audience than [who?], teacher research is an important professional skill for teachers to render the lessons relevant to their students. My finding suggests that despite the importance of teacher research (Cochran-Smith & Lytle, 1990; Cochran-Smith & Lytle, 2006; Sales, Traver, & García, 2011), the TGs rarely presented sufficient support for teachers on this matter.

Results from my analysis on the aspects of *interaction with colleagues* and *utilizing knowledge* supports the aforementioned point. UCSMP was the only TG to address this aspect, although by no means extensively. From my review of literature about colleagueship, I found the research mostly to be done with a focus on teachers' spoken interactions (Crespo, 2006; Males, Otten, & Herbel-Eisenmann, 2010; L. Shulman & J. Shulman, 2004). Lack of attention to interaction with colleagues in TGs might indicate the need for closer investigation in the research field regarding the colleagueship built between teachers who contributed in the writing of a TG and teachers who are reading the TG.

Pearson and CMP were the TGs that addressed utilizing students' knowledge. Again, those two TGs could have made explicit what it would mean to utilize the knowledge students bring in. The literature has discussed ethical reasons with practical suggestions and evidence of effectiveness of utilizing students' knowledge (Aguirre et al., 2013; Civil & Andrade, 2002; Gutiérrez, 2002; Ladson-Billings, 1998; Turner, Domínguez, Empson, & Maldonado, 2013). Given this, curriculum authors need to add into their curriculum practical suggestions for teachers to utilize the knowledge emergent bilingual students bring in.

As to acknowledging uncertainty, TGs emphasized that uncertainty derives from pedagogical situations more than from other types of uncertainty. Except for CMP, all TGs regularly addressed pedagogical uncertainty (Meister & Nolan, 2001; Midthassel, 2006; Wheatley, 2002; 2005). Yet, similar to the two aspects above, TGs could have introduced

various other aspects of uncertainty, including, for example, those derived from mathematics itself and student traits or school culture (Friedman, 1997; Labaree, 2003; Roeser, Skinner, Beers, & Jennings, 2012; Villaume, 2000). Moreover, TGs could go a step deeper to present how teachers proactively could use situations with uncertainty as opportunities to refine their practice. Curriculum materials of quality help teachers build knowledge, both specific to the scene of instruction, *and* general enough to be applied to multiple situations (Davis & Krajcik, 2005). Making uncertainty visible, the TGs can achieve the latter.

Commonly-observed positionings. I selected four TGs with consideration of varied design approach. Despite their differences, I identified *seven* positionings observable in all four. The positionings were attending to general information (e.g., available supplementary resources, and numbers of days to cover portions of the curriculum), specifics of the content (e.g., vocabulary, supplementary information to implement a task, and answers to the tasks and questions), and assessment (e.g., questions, activities, or tasks to assess students' learning) of each TG. Given the literature's suggestion that curriculum materials can impact classroom practice on a large scale (Ball & Cohen, 1996; Collopy, 2003; Stein & Kim, 2009), these seven positionings likely would be valuable in shedding light on what teacher positionings the curriculum materials have reinforced and/or have neglected, regardless enormous effort and time spent to raise awareness of the positionings.

Pervasive positionings from each TG. With the positionings I observed, I sorted those, which occurred regularly from more than three phases of each TG. Notable from this examination is that in all TGs, the positioning that *teachers have a duty to know answers to the tasks/questions* (not solution strategies) was pervasive. In addition, except for Eureka, the positioning that *teachers have a duty to know supplementary information to implement a task* was observable from at least three phases of the TGs. In the Results chapter, I interpreted this as a possible widespread narrow understanding of teaching mathematics: teaching

mathematics is about solving tasks to find correct answers. In addition to these two positionings, UCSMP positioned teachers as having a duty to know supplementary information to make accommodations. This positioning was pervasive in Pearson also. Pearson had eight pervasive positionings. Teachers using Pearson are exposed consistently to a number of positionings yet, in some cases, communication actions associated with a positioning were *in*consistent across different phases. I question whether implementing a task and knowing the correct answer are the only aspects the authors would want to emphasize. If this is not the case, authors might want to consider the positionings they wish to emphasize, then seed those in relevant phases because being frequently exposed to a positioning could cause teachers to assume the positioning is important (Rindfleisch & Inman, 1998).

Degree of support. When a positioning is observable in TGs, this means that teacher readers have an opportunity to engage with the aspect of teaching addressed by the positioning. Whether a teacher enacts it, however, is that teachers' decision. Yet, if TGs do not provide enough support for teachers, they might refute the positioning in their practice, not because they disagree with it but because they do not know to try it in their classroom. In the Results chapter, I presented subtle differences within three positionings across TGs in terms of the level of support. Examination of the level of support requires careful inspection of communication actions associated with a positioning from each TG. The varied degree of support appears in all TGs, meaning that no TG always provides limited or ample support. Although I did not conduct such analyses, this is a topic worth examining because the degree of support of a positioning could positively impact the likelihood of teachers' enactment of the positioning. In addition, because prospective teachers are more likely to follow the suggestions from curriculum materials (Drake, Land, & Tyminski, 2014), providing sufficient support to the positionings deemed important could encourage teachers at an earlier stage to develop productive sense of professional self.

Summary. Attending to regular positionings, I found that all TGs construct teaching mathematics as more than having mathematical knowledge. Still, the construction could be refined by attending to other aspects (i.e., teacher research, interaction between colleagues, utilizing knowledge, and acknowledging uncertainty) with higher frequency and deeper connection. In addition, TGs might have been reflecting and strengthening the construction of teaching mathematics as solving tasks with correct answers. The four TGs showed differences across the areas I examined. There is, however, neither consistency nor patterns in the differences. In other words, no TG was superior or inferior to any other in many areas. This indicates that at least the selected TGs are considering numerous aspects as to what teaching mathematics entails, but not widely or deeply enough regarding the five aspects I reviewed from the literature. Curriculum authors might want to evaluate the positionings they use based on their ideally constructed model of teaching mathematics, and use such an evaluation to decide on the kinds and levels of supports they wish to provide in their TGs.

Research Question 2

Examining the second research question, which attended to idiosyncratic positionings, I found two types of idiosyncratically occurring positionings, namely idiosyncratic-only type and idiosyncratic-regular type. I again grouped the idiosyncratic-regular type positionings into two: those with consistency of contexts across regular and idiosyncratic occurrences, and those without consistency of contexts.

Idiosyncratic-only type. Pearson had no idiosyncratic-only type of positionings. Most positionings of this type in UCSMP were about students and appeared in the MiLe phase. CMP idiosyncratically presented anticipating student strategy as to what teaching mathematics entails. Eureka had as many as seven idiosyncratic-only type positionings. Among those, two positionings seemed to better support teachers when provided regularly rather than idiosyncratically. For example, teachers might benefit more from reading a

general description of mathematical contents of a lesson in each lesson, not in some selected lesson. All the idiosyncratic-only type positionings I identified appeared in one or two phases of the TGs, except one positioning in CMP that appeared in three different phases. This suggests that teachers are likely to have low exposure to the idiosyncratic-only type positionings due to their idiosyncratic and infrequent occurrence. Considering that repetition can thicken a positioning (Gresalfi, 2009; Polman & Miller, 2010), curriculum authors need to be careful not to have the positionings they deem as important to be that of idiosyncratic-only type.

Idiosyncratic-regular. Idiosyncratic-regular type positionings with coherency in the contexts are these: i) t*eachers have a duty to know CCSS MP relevant to the content of the TG*; ii) *teachers have a duty to know relevant CCSS content standards at the chapter/lesson level*; iii) *teachers have a duty to know what hardcopy resources and/or materials are needed*; and iv) *teachers have a duty to know tasks/projects that overarch a chapter*. These positionings are revisited multiple times, both regularly and idiosyncratically, within the same context. The positionings with no coherency in the contexts are as follows: i) *teachers have a duty to know what electronic resources are needed* (CMP); ii) *teachers have a duty to know supplementary information to make accommodations* (Pearson); iii) *teachers have a duty to know questions to ask* (CMP); iv) *teachers have a duty to know answers to tasks/questions* (CMP; Eureka); v) *teachers have a duty to know potential student errors* (Pearson); vi) *teachers have a duty to know how the curriculum contributes to students' college and career readiness* (Pearson); vii) *teachers have a duty to know mathematics for pedagogical purposes* (UCSMP; Pearson); and viii) *teachers have a duty to know supplementary information to implement a task* (Pearson; Eureka). The TGs presented these positionings both regularly and irregularly in multiple contexts. Literature suggests that repeated exposure to a positioning affects teachers' sense of professional self, and that the

multiplicity of contexts increases the strengths of influence (Baker, 2006; Gresalfi, 2009; Hansson, 2015; Parks, 2010; Parks & Wager, 2015). That is, idiosyncratic-regular type positionings with disconnected contexts could be most powerful among different types of positionings. When choosing among the four selected TGs, teachers might want to consider if the positionings listed above are those that align with their sense of professional self. Similarly, curriculum authors intentionally can place the positionings they deem important both regularly and idiosyncratically, drawing on multiple contexts.

I do not have access to the authors' intentions for choosing some positionings to be regular and others idiosyncratic. I even wonder if those choices were more unintentional than carefully designed, as many language choices tend to be. Yet the substance of idiosyncratic positionings cannot to be disregarded, because communication actions associated with the idiosyncratic positionings are crafted enough that the actions well address specifics of the TGs' contents. Contrarily, not all general positionings provide supports with high specificity. In some cases, communication actions are identical across lessons or chapters, suggesting that texts regularly are cut and pasted into multiple places in the TG. In addition to the specificity of the support, to be idiosyncratic a positioning means the authors provided it even when it is not required. The positioning is there because the author wanted to provide such support to teacher readers. A regularly occurring positioning, however, because of its regularity, has to appear at a certain part of the TG, even when there is no support to provide. For example, CMP has the heading *Vocabulary* at the beginning of each lesson. Some lessons listed vocabularies but some other lessons said there is no vocabulary to introduce. An examination of idiosyncratically occurring positionings showed that idiosyncrasy supports teachers by suggesting a positioning when it is highly relevant. *Regularity* of a positioning guarantees stable support from the TG to the positioning. *Irregularity* of a positioning provides focused support addressing specific parts of the TG. As I discussed in chapter 2,

well-designed curricula provide teachers with both specific and general support for teaching (Davis & Krajcik, 2005). By carefully designing which positionings to provide regularly or idiosyncratically, curriculum authors can maximize teacher-readers' opportunity to learn from curricula materials.

Research Question 3

To answer the third research question, which focused on teacher professional judgment as acknowledged by each TG, I examined sentences directed at teachers. Among these sentences, I analyzed those with pronouns to examine TGs' use of pronoun and modality. Then, I analyzed the rest of the sentences (i.e., sentences without pronouns) to examine TGs' use of imperatives. When analyzing the sentences, I drew on the notion of speaking *to* and speaking *through* (2011). Curriculum materials speak through teachers when they guide teachers' actions, offer steps to follow, and give pre-designed questions to ask students. Materials speak to teachers when they give rationales, assumptions, or agendas supporting them. What I see to be central from this work is acknowledgement of teachers' professional judgment by making explicit the presence of teacher-readers. I attended to this particular aspect of speaking through *versus* speaking to dichotomy, instead of following the original. By focusing on uses of pronoun, modality, and imperatives, I sought to understand the space available for teachers to enact their professional judgment.

Analysis suggests that, in general, the voices of UCSMP and CMP acknowledged teacher presence and their professional judgment more than did the voices of Pearson and Eureka. In other words, UCSMP and CMP spoke more *to* teachers than did Pearson and Eureka. In pronoun use, all but two were referring to teachers across the four TGs, either explicitly or implied. The two pronouns not referring to teachers were from UCSMP, referring to authors. Although these two incidences did not include teachers, by making explicit the UCSMP author's intentions, in fact, were speaking to teachers. For the purpose of

cross-TG comparison, I calculated two ratios: i) pronouns referring to teachers divided by total incidences directed at teachers (IDT), and ii) by total pages. Both ratios showed the same result that CMP is the TG that most-proactively used pronouns (0.54 and 2.25, respectively), followed by UCSMP (0.33 and 1.39), Eureka (0.10 and 0.27), and Pearson (0.07 and 0.21), Eureka and Pearson being close to each other. Although Morgan (1998) argued that the use of first-person pronouns of *I* and *we* could broaden the distance between the authors and the readers, my analysis showed that the first-person pronouns contributed to the acknowledgement of teachers' professional judgment. In particular, the pronoun *we* in a mathematical textbook places the author as an authority of mathematics or assumes agreement from the readers (Herbel-Eisenmann, 2007; Rowland, 1999), but this was not the case with the TGs.

To understand TGs' use of modality, I reexamined the sentences with pronouns. To understand the degree of obligation each TG was imposing on teachers, I classified the modalities and bald assertions as either suggestive or assertive. The result from the analysis supported that UCSMP and CMP spoke to teachers by using less assertive voice than that of the other two TGs. Regarding the ratio with IDT, CMP used twice-more suggestive modalities than assertive (0.36 and 0.18, respectively). UCSMP used more assertive voices than suggestive (0.18 and 0.15, respectively). The gap between suggestive modalities per IDT and assertive voice per IDT was smaller in Pearson (0.00 and 0.07, respectively) and Eureka (0.03 and 0.07, respectively) than in UCSMP and CMP, but this was because of the generally-low count in sentences with pronouns in Pearson and Eureka. CMP is the only TG with higher use of suggestive modalities than of assertive voice. Regarding ratio per page, CMP is still the only TG with higher use of suggestive modalities than assertive voice (1.5 and 0.75, respectively). In UCSMP more assertive voice was observed than suggestive modalities (0.76 and 0.63, respectively) and Eureka (0.19 and 0.08, respectively). Pearson's

use of assertive voice is *twentyfold more* than suggestive modalities (0.20 and 0.01, respectively). The high frequency of assertive voice to suggestive modalities is not to be overlooked. This is so because I conservatively defined assertive voice by grouping median modalities together with low modalities (Halliday & Matthiessen, 2014). Despite the conservative grouping, the result shows that three of the four TGs used assertive voice more often than suggestive modalities. Assertive voice could potentially restrict teacher-readers' enacting their professional judgment by suggesting that the content of the TG was true and not up for negotiation (see Herbel-Eisenmann, Kristmanson, & Wagner, 2011).

For analysis of the use of imperatives, I focused on the sentences without pronouns. From the use of imperatives, I confirmed that CMP most proactively acknowledged teachers' professional judgment. Results for UCSMP, Pearson and Eureka, on the other hand, were mixed. Frequent use of imperatives itself might not be sufficiently-strong evidence to show whether the TG is speaking to or speaking through teachers. This was so because although those sentences did not use pronouns, some imperatives, such as *consider*, are inclusive. The pattern I observed in all four TGs, however, showed that exclusive imperatives such as *have*, *ask*, and *use* were more dominant than inclusive imperatives. Accordingly, more use of imperatives suggested a higher degree of speaking through teachers. Compared to IDT, CMP was the lowest in its use of imperatives (0.46), followed by UCSMP (0.67), Eureka (0.88), and Pearson (0.93). In its use per page, CMP was the lowest (1.90), then Eureka (2.38), Pearson (2.72), and UCSMP (2.80). If the use of exclusive imperatives places authority on the authors only (Martin & Rose, 2007; Rotman, 1988), this frequent use of imperatives in the TGs implied low authority placed on the teacher-readers, hence limited opportunity for teachers to practice their professional judgment.

Comparing the use of suggestive modality *versus* the use of assertive voice and imperatives combined, CMP had the highest rate of incidences with suggestive voice (0.36

for suggestive and 0.64 for assertive or imperatives), followed by UCSMP (0.15 and 0.85, respectively), Eureka (0.03 and 0.97, respectively), and Pearson (0.00 and 1.00, respectively). This combined result with the other results discussed until now (i.e., examination of voice with TGs' use of pronoun, modality, and imperative) support that, in general, the voice of UCSMP and CMP were more open than the other TGs. In other words, these TGs acknowledge teacher professional judgment more pervasively than do the other two. Such use of language indicates that when compared to Pearson and Eureka, UCSMP and CMP were closer to the speaking to side of the spectrum than to the speaking through side. Attending to the high IDT per page ratio of CMP and UCSMP, an interesting topic for future study might be examining the relationship between talkativeness of a TG and the tendency of speaking to/through.

All in all, I found no extreme differences across TGs in their construction of what teaching mathematics entails. Subtle differences existed, but, in general, they place at the center solving mathematical tasks. Regarding less-conventional positionings, such as that about making notes from UCSMP and CMP or supporting emergent bilingual students from Pearson and CMP, TGs could have provided more support in their communication actions than they now do. The voice TGs use differs. UCSMP and CMP consistently invite teachers to use their professional judgment. In other words, these two TGs acknowledge that teachers could choose to refute positionings imposed on them by the TG based on their knowledge and experience.

Looking Backward

In writing this dissertation, I learned several things about conducting research. First, I learned to find proper grounding of the study. Initially when searching the literature I was hoping to find laid-out, essential knowledge or central practices all teachers – either in mathematics or in general – should have. In such literature, I hoped for an initial list of

positionings I could or should expect from the TGs. Unexpectedly, though, my literature search led me to explore a much broader range of topics than I originally had expected and, although different from my expectation, nonetheless provided my groundings.

In addition, I had the opportunity to develop deeper understanding of positioning theory. In this dissertation, I used that theory to examine the moment when TGs were being read. My reading of the literature on positioning theory indicated that more research has been done on spoken interaction than on written interaction. Exploration of rights and duties, the terms frequently used to define positioning, also led me to a more-precise understanding of the theory than before.

Regarding data, I have learned to find the balance between the amount of data and the amount of detail for the analysis. Because I wanted to compare the four TGs, I had to maintain a certain generosity to make the comparison possible. Because of the vastness of data, I had to choose to attend to the headings rather than to every minute detail in the texts below those headings. Were the focus of the study narrower than it now is, I could have provided results with denser description and lesser data. But from the data analysis process, I learned to make decisions so that the results best could address the research questions.

Looking Forward

Implications for Curriculum Authors

Each TG provides similar yet different sets of positionings with which teachers could engage. When curriculum authors decide the content to include, they might first seriously consider beginning the TG writing process with generating a list of positionings they wish teachers to take up. In doing so, one productive approach is to make explicit the storylines they want to enact or the values and the beliefs they have about teachers – their primary audience for TGs. The list itself and the comparison of the list to their storylines and beliefs could inspire the authors during the curriculum design and writing, including what headings

to use and what content to present at which phases. For example, authors might decide to consistently present tips for supporting emergent bilingual students in MiLe so that teachers are repeatedly positioned to address emergent bilingual student needs via TG supports closely connected to classroom interaction. Alternatively, authors could decide to provide general support via tips in IntCh or IntLe. In addition, this study's results showed that some positionings have a strong presence by being addressed in multiple phases. To go a step further, authors could make the positioning more visible by assigning a heading pinpointing the positioning. By carefully attending to their desired positionings, authors efficiently can use curriculum space to convey content they want, to control that content's visibility, and to position teachers in the ways they hope to do. Curriculum authors might consider using pronoun, modality, and imperatives as tools for regulating the voice of the TG. By using more suggestive modalities, fewer imperatives, and pronouns making visible their presence (i.e., the authors'), they can speak to teachers.

Implications for Teachers and Teacher Educators

This study's findings suggest to teachers that they could consider teacher positionings as factors to consider when choosing a curriculum. If positionings embedded in the curriculum rarely align with the teacher's sense of professional self, it could be wise not to pick that curriculum because the curriculum does not offer the kind of information the teacher considers to be relevant to her teaching. A teacher who are interested in expanding her perspective on teaching might benefit from the extreme mismatch between the positionings suggested by a curriculum and a teacher's sense of professional self. This, however, is not the case when the teacher is looking for a material that she could rely on to save planning time. From the very construct of positioning theory, and from my deliberate choice of "teachers have a duty to know" statement, instead of "teachers have a duty to do"

or “teachers have a duty to enact,” I wanted to stress that teachers can decide to accept or reject the positioning the TG is imposing.

Teachers fully could reject the positioning by ignoring texts conveying that positioning. Alternatively, teachers have the option to accept the positioning only partially, by modifications when enacting the curriculum. If, however, a TG with more positionings for teachers to accept than reject or make modifications, this might indicate that the TG has more information useful to their teaching context than the TGs with positionings that do not align with their self-positioning. Teacher educators could design an activity for teachers to recognize the positionings suggested by TGs. In addition to recognizing the kinds of positionings, attending to the voice of the TG could permit teachers to see the degree of obligation the TGs are imposing on them.

I hope teachers can raise awareness of the limitations of positionings in the TGs, can proactively demystify embedded positionings, and can refute positionings with which they cannot agree based on their own experiences as teachers. Women’s voices too often have been regarded as less than a full opinion. Considering that teaching is a female-dominant occupation, teachers’ active and critical engagement in discussion of teacher positionings is important in ensuring their voice is heard and that their professionalism is acknowledged by others *including curriculum authors*. Teacher educators might support teachers by encouraging their participation in the discussion space of teacher positionings.

Implications for Future Research

Several ideas for future research were discussed earlier in this dissertation. First, the field could benefit from much closer examination of some of the positionings provided in this study. Because this study looked at seven phases from four TGs, the observations could not be too specific. Additional examination of a phase or a positioning across the TGs could result in a much more nuanced understanding of this matter, in particular with the degree of

support TGs provide for teachers to act out the positioning. The positioning that *teachers have a duty to know supplementary information to implement a task* is an example of such case. I had to use language as vague as *supplementary information* because the kind of support provided in that part of the text was inconsistent – possible student errors, possible strategies, factual information of the real-world context of the task, mathematical knowledge from previous lessons, ways to support visual learners, etcetera. The range of information covered varies across the TGs. For example, from chapters examined for this study, Pearson did not elaborate on the factual information of the real-world context but UCSMP did. What this study can tell is that, at least from the four selected TGs, it is challenging to pinpoint the major duty involved in task-implementation, besides that *teachers have a duty to know the answer to the task*. Considering that this positioning is observed regularly from three phases of UCSMP, four of Pearson, four of CMP, and one of Eureka, *supplementary information* could be unpacked based on much more careful examination of this positioning in and of itself. Another example is the positioning of knowing vocabulary: such positioning was observable from the four selected TGs, with varying level of support, as scarce as simply asking teachers to have students review the vocabulary, or as sufficient as providing suggestions on each vocabulary how teachers could introduce it to students. For example, TGs could ask teachers to assign students a community-based research project about mathematical terminologies (Staats, 2009). This level of specificity was not addressed in this study, as it is outside the scope of this dissertation. Investigations at a smaller grain size than that of this study will result in insightful findings and discussions.

Another area for further research concerns TG features. Increasingly, scholars are paying attention to visual images in mathematics (Lemke, 2003; 2017; O'Halloran, 2003; 2005). Examining visual images is as important as examining language use in mathematics because meaning emerges also from visual images, both mathematical and in everyday

contexts. Not only visual images, but also layout designs might impact differently on the degree of emphasis of positionings. Related to layout, a possible extension of this study is analyzing electronic versions of TGs. In this dissertation, I used three hardcopy TGs and one electronic TG formatted as print-ready. Becoming more popular are electronic versions of curriculum materials with features ranging from as *close* to printed materials as pdf files, to as *far* from printed materials as interactive links or embedded videos. This has possible complications, in that features available only on electronic versions might bring teacher positionings difficult to observe from printed materials.

Last, I want to suggest examining teachers' engagement with their TGs. The motivation of this study was that of respecting teachers' professional judgement. As a teacher education researcher, I often heard prospective and practicing teachers be receptive to external positionings that see teaching as, at most, a non-profession or a para-profession. I expect the perspective and analysis from this study to encourage teachers with acknowledging teaching as a full profession and advocating such status of teaching. Casting alternative storylines might be an effective strategy, as positioning theory suggests. Therefore, a possible extension of this study is: When teachers engage with TGs, what positionings do teachers accept and refute and under which storylines?

APPENDICES

APPENDIX A

LIST OF THE REGULARLLY APPEARING HEADINGS

Table 25. Headings Examined from the Phase of IntTG

Phase	UCSMP	Pearson	CMP	Eureka
IntTG	• UCSMP Program Organization ▸ UCSMP Program Overview ▸ Teaching Modes and Strategies ([Suggestions from teachers]; An Articulated Curriculum Across All Grades) ▸ UCSMP Research Results ▸ UCSMP INSTRUCTIONAL PLANNING AND SUPPORT (Hardcover Student Edition; Teachers' Edition)	• A New Way to Experience Math • Problem Solving • Solving Rich Problems • Visual Learning • Big Ideas • Interactive Learning • Differentiated Instruction • Assessment • Big Ideas and the Common Core • Efficacy Research • Standards for Mathematical Practice • Pearson Algebra 1 Common Core • What is STEM Education? • College and Career Readiness • Teaching Resources • Common Core State Standards for Mathematical Content • Algebra 1 Pacing Guide	• [Not Applicable]	• Pacing and Preparation Guide • Preparing to Teach a Module • Preparing to Teach a Lesson

Table 26. Headings Examined from the Phase of IntCh

Phase	UCSMP	Pearson	CMP	Eureka
IntCh	• Chapter # overview ▸ Chapter Overview ▸ Differentiated Options ▸ Objectives ▸ Technology Resources ▸ Chapter Resource Masters • Chapter # opener ▸ Pacing ▸ Chapter # Projects ▸ Chapter # Overview ▸ Notes	• Get Ready! ▸ [the table with lesson and skill] ▸ Why Students Need These Skills ▸ Looking Ahead Vocabulary • Chapter # Overview ▸ BIG idea ▸ Content Standards ▸ Overview of the Performance Task ▸ Introducing the Performance Task ▸ PARCC CLAIMS ▸ SBAC CLAIMS • PowerAlgebra.com • Chapter# Math Background ▸ Math concept (BIG idea; ESSENTIAL UNDERST-ANDING; Common Errors With [Math Concept]; Mathematical Practices) • Pacing and Assignment Guide • Resources	• Unit Planning ▸ Unit Overview (Unit Description; Summary of Investigations; Unit Vocabulary; Planning Charts; Block Pacing; Parent Letter; Goals and Standards) • Mathematics Background • Unit Introduction ▸ Using the Unit Opener ▸ Using the Mathematic-al Highlights • Investigation Overview ▸ Investigation Description ▸ Investigation Vocabulary ▸ Mathematics Background ▸ Planning Chart ▸ Goals and Standards (Goals; Common Core Content Standards; Facilitating the Mathematic-al Practices)	• Module # ▸ OVERVIEW ▸ Focus Standards ▸ Foundational Standards ▸ Focus Standards for Mathematical Practice ▸ Terminology (New or Recently Introduced Terms; Familiar Terms and Symbols ▸ Suggested Tools and Representati-ons ▸ Preparing to Teach a Module ▸ Preparing to Teach a Lesson ▸ Assessment Summary • Topic @ ▸ Focus Standards ▸ Instructional Days ▸ [A description]

Table 27. Headings Examined from the Phase of IntLe

Phase	UCSMP	Pearson	CMP	Eureka
IntLe	• GOAL • SPUR Objects • Materials/Resources • HOMEWORK • Local Standards • Background	• #-# Preparing to Teach ▸ BIG idea ▸ ESSENTIAL UNDERSTANDINGS ▸ Math Background ▸ Mathematical Practice	• Problem Overview ▸ Focus Question ▸ Problem Description ▸ Problem Implementation (Materials) ▸ Vocabulary ▸ Mathematics Background ▸ At a Glance Lesson Plan	• Student Outcomes

Table 28. Headings Examined from the Phase of BeLe

Phase	UCSMP	Pearson	CMP	Eureka
BeLe	• 1 Warm-Up	• 1 Interactive Learning ▸ Solve It! (PURPOSE; PROCESS; FACILITATE; ANSWER; CONNECT THE MATH) • PowerAlgebra.com	• Launch ▸ Connecting to Prior Knowledge ▸ Presenting the Challenge	• [Not applicable]

Table 29. Headings Examined from the Phase of MiLe

Phase	UCSMP	Pearson	CMP	Eureka
MiLe	• 2 Teaching ▸ Notes on the Lesson/Activity ▸ Additional Examples ▸ Accommoda-ting the Learner	• 2 Guided Instruction ▸ Problem # ▸ Got It? ▸ Additional Problems • PowerAlgebra.com	• Explore ▸ Providing for Individual Needs ▸ Planning for the Summary	• Classwork (Example/Exercise)

Table 30. Headings Examined from the Phase of EnLe

Phase	UCSMP	Pearson	CMP	Eureka
EnLe	• 3 Assignment ▸ Recommended Assignment ▸ Notes on the Questions • 4 Wrap-Up ▸ Ongoing Assessment	• 3 Lesson Check ▸ Do you Know HOW? ▸ Do you UNDERST-AND? ▸ Close • 4 Practice ▸ ASSIGNMENT GUIDE ▸ Mathematical Practices ▸ Applications ▸ EXERCISE # ▸ HOMEWO-RK QUICK CHECK • #-# Lesson Resources ▸ Additional Instructional Support (Algebra 1 Companion; ELL Support; ▸ 5 Assess & Remediate (Lesson Quiz; PRESCRIPT-ION FOR REMEDIAT-ION) ▸ Differentiated Remediation • PowerAlgebra.com	• Summarize ▸ Orchestrating the Discussion (Suggested Questions) ▸ Reflecting on Student Learning ▸ ACE Assignment Guide	• Closing • Exit Ticket • Exit Ticket Sample Solutions • Problem Set Sample Solutions

Table 31. Headings Examined from the Phase of EnCh

Phase	UCSMP	Pearson	CMP	Eureka
EnCh	• Projects ▸ Project Rubric ▸ [a table presenting Project # and associated lesson #] ▸ [# Project Title] ▸ Notes • Summary and Vocabulary • Chapter # Self-Test • Chapter # Review ▸ Chapter Review ▸ Resources ▸ Technology Resources ▸ Assessment (Evaluation; Feedback)	• # Pull It All Together (Completing the Performance Task; [Questions]; FOSTERING MATHEMATI-CAL DISCOURSE; ANSWERS; On Your Own) • # Chapter Review ▸ Essential Questions (BIG idea; ESSENTIAL QUESTION) ▸ Summative Questions • Chapter Test • Common Core Cumulative Standards Review • PowerAlgebra.com	• Mathematics Reflections ▸ Possible Answers to Mathematic-al Reflections ▸ Possible Answers to Mathematic-al Practices Reflections	• Mid-Module Assessment Task ▸ [the tasks] ▸ A Progression Toward Mastery ▸ [answer to the tasks] • End-of-Module Assessment Task

APPENDIX B

TABLES OF REGULAR AND IDIOSYNCRATIC POSITIONINGS

The observed positionings presented below are grouped into eight topics: Standards and goals, resources, accommodation, tasks, assessment, general information: features and structures of the TG, general information: mathematics, and additional features. This grouping is for the purpose of breaking the results into smaller pieces to avoid bulkiness. That is, the grouping should not be understood as representing the significant themes that emerged from the data. Green indicates regular positionings or the heading coded once. Light blue indicates regular positionings with the heading coded multiple times, and hence appeared at least twice across the six tables. Red indicates phases with idiosyncratic positionings.

Table 32. Standards and Goals

Positionings	UCSMP	Pearson	CMP	Eureka
Teachers have a duty to know CCSS MP relevant to the content of the TG.		IntTG IntCh (Math Background) IntLe EnLe	IntCh EnCh (Possible Answers to Mathemati-cal Practices Reflections)	IntCh MiLe
Teachers have a duty to know relevant CCSS content standards at the chapter/ lesson level.		IntTG (Big Ideas and the Common Core; Common Core State Standards for Mathematical Content; Algebra 1 Pacing Guide) IntCh	IntCh	IntCh (OVERVI-EW) MiLe EnLe

Table 32 (cont'd)

Teachers have a duty to know to identify local standards relevant to each lesson.	IntLe			
Teachers have a duty to know SPUR standards addressed in the chapter/lesson.	IntCh (Chapter Overview; Objectives) IntLe			
Teachers have a duty to know Big Ideas and essential questions/understandings.		IntTG (Big Ideas and the Common Core; BIG ideas) IntCh (Chapter # Overview; Math Background) IntLe (BIG idea; ESSENTIAL UNDERST-ANDINGS) EnCh (Essential Questions)		
Teachers have a duty to know the aim of the chapter/lesson	IntLe		IntCh IntLe	IntCh (OVERVI-EW) IntLe
Teachers have a duty to know claims and skills relevant to the task.		IntTG (Algebra1 Pacing Guide) IntCh		
Teachers have a duty to know why students need the skills introduced in the chapter.		IntCh		

Table 33. Resources

Positionings	UCSMP	Pearson	CMP	Eureka
Teachers have a duty to know, in general, supplementary resources available.	IntCh	IntTG (Teaching Resources) IntCh (Resources)	IntCh (Planning Charts)	IntCh
Teachers have a duty to know what hardcopy resources and/or materials are needed.	IntLe EnCh	EnLe (EXERCISE #; Additional Instructional Support; Differentiat-ed Remediation)	IntLe BeLe MiLe EnLe	
Teachers have a duty to know what electronic resources are needed.	IntCh EnCh	IntCh BeLe MiLe EnLe EnCh	IntCh-EnCh* IntLe	MiLe

Table 34. Accommodation

Positionings	UCSMP	Pearson	CMP	Eureka
Teachers have a duty to know supplementary information to make accommodations.	IntCh IntLe MiLe EnLe	IntTG (Teaching Resources) IntCh (Get Ready!; Pacing and Assignment Guide) EnLe (ASSIGNMENT GUIDE; Differentiated Remediation) MiLe		IntTG
Teachers have a duty to know how to support emergent bilingual students.	MiLe	IntTG (Teaching Resources) IntCh (Resources) EnLe (ELL Support; Differentiated Remediation)	IntCh (Parent Letter)	

Table 35. Tasks

Positionings	UCSMP	Pearson	CMP	Eureka
Teachers have a duty to know tasks/activities/questions for opening a chapter/lesson.	BeLe (1 Warm-Up)	BeLe (FACILIT-ATE)	IntCh BeLe	MiLe
Teachers have a duty to know additional tasks that are not in the student version.		MiLe		EnLe (Exit Ticket) EnCh
Teachers have a duty to know how to extend tasks.	MiLe	MiLe (Problem #)		

Table 35 (cont'd)

Teachers have a duty to know tasks/projects that overarch a chapter, to be revisited multiple times.	IntCh EnCh EnLe	IntCh (Overview of the Performance Task; Introducing the Performance Task) EnCh (Completing the Performance Task) EnLe		
Teachers have a duty to know questions to ask.		IntCh (Introducing the Performance Task) BeLe (FACILIT-ATE) MiLe (Problem #) EnLe EnCh (Completing the Performance Task)	BeLe MiLe EnLe (Suggested Questions)	MiLe EnLe
Teachers have a duty to know answers to tasks/questions.	IntTG BeLe (1 Warm-Up) MiLe EnLe EnCh	IntCh BeLe MiLe (Problem #; Answers; Additional Problems) EnLe EnCh	BeLe MiLe (Providing for Individual Needs) EnLe (Suggested Questions) EnCh (Possible Answers to Mathematical Practices Reflections)	BeLe MiLe (Classwork) EnLe (Closing; Exit Ticket Sample Solutions; Problem Set Sample Solutions) EnCh
Teachers have a duty to know potential student errors.	MiLe	IntCh MiLe		MiLe

Table 35 (cont'd)

Teachers have a duty to know strategies students might use when solving tasks.	MiLe	IntCh (Math Background) BeLe	BeLe MiLe EnLe	MiLe

Table 36. Assessment.

Positionings	UCSMP	Pearson	CMP	Eureka
Teachers have a duty to know for each assessment type, when to administer, its format, and the standards.				IntCh
Teachers have a duty to know to selectively review tasks.	EnCh	EnLe		
Teachers have a duty to know where students are at from student performance on the task/project/question/quiz using a rubric.	EnCh	EnLe (Lesson Quiz)		EnCh
Teachers have a duty to know questions and/or activities for students, to assess their learning of the lesson/chapter.	EnLe	EnLe (Lesson Quiz) EnCh (Essential Questions; Summative Questions)	EnCh (Possible Answers to Mathemati-cal Reflections; Possible Answers to Mathematic al Practice Reflections)	EnLe (Closing; Exit Ticket)
Teachers have a duty to know tasks for students, to assess their learning of the chapter.	EnCh	EnCh	EnLe	EnCh (End-of-Module Assessment Task)

Table 37. General Information: Features and Structures of the TG

Positionings	UCSMP	Pearson	CMP	Eureka
Teachers have a duty to know the merits and significant features of the curriculum.	IntTG	IntTG		
Teachers have a duty to know information of mathematical topics in addition to algebra.	IntTG	IntTG (BIG ideas)		

Table 37 (cont'd)

Teachers have a duty to know the excellence of the curriculum proved by research.	IntTG	IntTG		
Teachers have a duty to know how the curriculum contributes to students' college and career readiness.		IntTG EnLe		
Teachers have a duty to know steps to follow when preparing to teach a chapter and a lesson.				IntTG (Preparing to Teach a Module; Preparing to Teach a Lesson)
Teachers have a duty to know the Lesson Structure of each lesson.				IntCh
Teachers have a duty to know general structure of the lessons in the curriculum, both student and teacher versions.	IntTG	IntTG		
Teachers have a duty to know the pacing of the curriculum/chapter/lesson.	IntCh (Chapter Overview)	IntCh (Pacing and Assignment Guide)	IntCh (Planning Charts; Block Pacing)	IntTG IntCh

Table 38. General Information: Mathematics

Positionings	UCSMP	Pearson	CMP	Eureka
Teachers have a duty to know in general the mathematical contents of the chapter at the lesson level.	IntCh	IntCh (Math Background)	IntCh	IntCh
Teachers have a duty to know mathematics for pedagogical purposes.	IntLe MiLe	IntLe MiLe	IntCh IntLe	MiLe
Teachers have a duty to know vocabulary.	EnCh	IntCh (Math Background)	IntCh IntLe	IntCh
Teachers have a duty to know to consider students' prior mathematical knowledge.	IntLe	IntCh (Get Ready!)	BeLe	IntCh
Teachers have a duty to know supplementary information to implement a task.	MiLe EnLe EnCh ([# Project Title])	IntCh (Overview of the Performance Task; Introducing the Performance Task) BeLe (FACILITATE) EnLe EnCh (Completing the Performance Task; FOSTERING MATHEMATICAL DISCOURSE; On Your Own) MiLe	IntLe MiLe (Providing for Individual Needs) EnLe EnCh (Possible Answers to Mathematical Reflections)	MiLe (Classwork) EnLe

Table 39. Additional Features

Positionings	UCSMP	Pearson	CMP	Eureka
Teachers have a duty to know what to consider when reflecting on their teaching to plan next stage of the lesson accordingly.			MiLe EnLe	
Teachers have a duty to know that they communicate with households.			IntCh (Using the Mathematic-al Highlights; Parent Letter)	
Teachers have a duty to know tips from other teachers who have experience with using the curriculum.	IntTG			
Teachers have a duty to know they can make notes as needed.	IntCh EnCh		IntCh EnCh	
Teachers have a duty to know which tasks are addressing CCSS MP Modeling.		EnLe		

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