MODELLING AND CONTROL OF A TURBOCHARGED DIESEL ENGINE

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ABSTRACT

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The diesel engine is known for its high efficiency, performance, and durability. With stringent fuel economy and emission regulations, diesel engines face increasing challenges. To accommodate emission regulations, fuel economy and performance requirements, modern diesel engines are equipped with the variable geometry turbocharger (VGT) and exhaust gas recirculation (EGR) system. VGT extracts energy from exhaust gas to drive the compressor to improve transient response, steady-state performance, and fuel efficiency under wide range of engine flow conditions. Meanwhile, EGR dilutes fresh air with exhaust gas to reduce the formation of mono-nitrogen oxides NO and NO$_2$ (NOx). The VGT and EGR control design is complicated due to the natural coupling between VGT and EGR, and high nonlinearity of diesel engine air-path system. The extra assisted power and regenerative power on the turbocharger shaft further increase the control system complexity. In this dissertation, new approaches for turbocharger system modelling and multivariable control design for the coordinated actuation of the VGT-EGR system are investigated. The control design is further extended to hydraulic regenerative assisted turbocharger system.

New modelling approaches for turbocharger system are proposed based on turbomachinery physics. Proposed turbine and compressor models eliminate the interpolation error, and especially, allow smooth extrapolation outside the mapped region. A high fidelity reduced order mean value model of a diesel engine for automotive application is developed based on developed
turbocharger model. Further, new models for high-speed hydraulic turbines and centrifugal pumps are developed for hydraulic assisted and regenerative turbochargers.

A regenerative hydraulic assisted turbocharger (RHAT) system is investigated in this dissertation. A system level approach based on 1-D simulations is used to understand the assist benefits and design trade-offs. Simulation results show that 3-5% fuel economy improvement for FTP 75 driving cycle, depending on different sub-component sizing. The study also identifies technical challenges for optimal design and control of RHAT systems.

A linear controller design approach is proposed in this dissertation for regulating both boost pressure and EGR mass flow rate of the VGT-EGR system. The linear quadratic control with integral action is designed based on the linearized system. Local controllers are scheduled based on engine operational parameter: engine speed and fuel injection quantity. The gain scheduled liner controller is validated against baseline controller based on the nonlinear plant. Results show that designed multi-input and multi-output (MIMO) controller can well manage the trade-offs between boost pressure tracking and EGR mass flow tracking, compared to baseline controller (two single input single output (SISO) controllers). A novel approach is proposed for closed-loop control design with respect to engine performance and engine emission trade-offs. The controller design is further extended to assisted and regenerative turbocharger system with VGT and EGR. The results show that emission reduction, engine performance and fuel economy improvement can be achieved at the same time with external power applied to the turbocharger shaft.
To my grandparents, my parents, my wife Mengyan and my son Franklin
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KEY TO SYMBOLS AND ABBREVIATIONS

\( P_0 \): Environment pressure \( (P_{air}) \)

\( P_1 \): Compressor inlet pressure

\( P_2 \): Intake manifold pressure

\( P_3 \): Exhaust manifold pressure

\( P_4 \): Turbine downstream pressure

\( \omega \): Turbocharger speed

\( P_p \): Pipe pressure between pump and pump valve

\( P_t \): Pipe pressure between hydraulic turbine and turbine valve

\( P_{Acc} \): High pressure accumulator pressure

\( \chi \): Piston position

\( v \): Piston velocity

\( P_{return} \): Hydraulic tank return pressure

\( m \): Mass of piston

\( T_3 \): Engine exhaust manifold temperature

\( T_2 \): Engine intake manifold temperature

\( \eta_{vol} \): Volumetric efficiency

\( \dot{m}_c \): Compressor mass flow rate

\( \dot{m}_t \): Turbine mass flow rate

\( \dot{m}_{in} \): Engine intake mass flow rate

\( \dot{m}_{out} \): Engine exhaust mass flow rate
\( m_{egr} \): Exhaust gas recirculation mass flow rate

\( \dot{m}_{turbine} \): Hydraulic turbine mass flow rate

\( \dot{m}_{valve,t} \): Hydraulic turbine vale mass flow rate

\( \dot{m}_{pump} \): Hydraulic pump mass flow rate

\( \dot{m}_{valve,p} \): Hydraulic pump valve mass flow rate

\( k \): Spring coefficient

\( \rho_{fluid} \): Hydraulic fluid density

\( F_f \): Piston friction force

\( W_T \): Turbine power

\( W_C \): Compressor power

\( W_{Loss} \): Turbocharger mechanical loss power

\( w_{turbine} \): Hydraulic turbine power

\( \dot{w}_{pump} \): Hydraulic pump power

\( V_0 \): Initial tank volume

\( \beta \): Hydraulic fluid bulk modules

\( V_t \): Pipe volume between hydraulic turbine and turbine valve

\( A \): Piston area

\( V_p \): Pipe volume between hydraulic pump and pump valve

\( F_0 \): Spring preload

\( c \): Damping of friction coefficient associated with the piston

\( c_p^{air} \): Air constant pressure specific heat [J/kgK]

\( \eta \): Efficiency

\( \gamma_{air} \): Isentropic index for air
\( h: \) Specific flow enthalpy [J]
\( \dot{m}: \) Mass flow rate [kg/sec]
\( \tau: \) Torque [Nm]
\( \omega: \) TC angular velocity [rad/sec]
\( N_{TC}: \) TC rotational speed [RPM]
\( J_{TC}: \) TC shaft inertia [kg/m2]
\( \alpha: \) VGT vane angle [radians]
\( \rho: \) Gas density[kg/m3]
\( \mathfrak{R}: \) Universal gas constant
\( A: \) Geometric area[m2]

**VGT:** Variable geometry turbocharger

**EGR** Exhaust gas recirculation

**RHAT** Regenerative hydraulic assisted turbocharger

**REAT** Regenerative electric assisted turbocharger
CHAPTER 1: INTRODUCTION

1.1 Motivation

Modern diesel engines are widely equipped with variable geometry turbocharger (VGT) and exhaust gas recirculation (EGR). A control system for a diesel engine must meet driver’s torque demand, meanwhile satisfying constraints on emission and fuel economy. Two devices (VGT and EGR) can provide diesel engine with fresh air as well as required exhaust gas fraction. This helps to meet the requirement of engine emission standard, fuel economy and transient performance requirements. However, due to the natural coupling between VGT and EGR systems, and high nonlinearity of diesel engine air-path, designing a robust and optimal controller is challenging. Especially, regenerative and assisted turbocharger system adds another actuator on turbocharger (TC) shaft, leading to additional complexity for controller design.

Model-based control is popular with the automotive research community. Not only it reduces the design duration but also allows control engineers to gain insight into plant behaviour without running engine in the test cell. For turbocharged diesel engine modelling, turbocharger system is one of the critical subsystem models. Most of the previous modelling methods for turbocharger utilized manufacturer based maps, which leads to non-physical extrapolation and modelling errors. Especially for modelling variable geometry turbocharger, it involves 3-dimensional map extrapolation based on traditional method. To mitigate the requirement of accurate representation for the complex of physics system and simple structured control-oriented modelling, physics-based turbocharger modelling is needed for diesel engine air-path system.

Considering hydraulic assisted and regenerative power on the turbocharger shaft, it provides extra control input, which dramatically changes plant performance and controller design. In order
to understand the benefit and challenging for hydraulic assisted and regenerative turbocharger (RHAT), coordinated control of the VGT-EGR-RHAT system is needed for improving engine performance, reducing emissions and improving fuel economy. To design closed-loop controller, system level analysis for RHAT system is needed, beforehand.

For the traditional diesel engine controller development, first, a static map is developed that provides the desired steady fuel command as a function of engine speed and driver’s pedal position. By analysing the emission, performance and fuel economy trade-offs, optimal set-points for air-path actuators, such as VGT vane and EGR valve positions, are obtained with respected to engine fuel injection and engine speed. During transient operations, it is clear that the coupling effect between VGT and EGR is ignored, and the VGT vane position is used to control boost pressure and EGR valve position is utilized to control EGR mass flow rate. To achieve better tracking performance, coordinated control for VGT and EGR has enjoyed increased interest. However, most of the current multivariable controls fail to take account of the design target directly during the controller design process. There is no systematic approach of designing controller for different performance targets, namely, handling trade-offs between performance, emissions and fuel economy for closed-loop control design. Hence, a new coordinated control design approach is needed for diesel engine air-path control. The proposed approach in this thesis can also be easily extended to VGT-EGR-RHAT systems.

1.2 Research Overview

1.2.1 Turbine modelling

Control-oriented VGT models are required for model-based control design. Typically, the VGT turbine power is modelled using a fixed or a map-based turbocharger mechanical efficiency with isentropic assumptions. However, the fixed efficiency approach could be over simplified,
leading to large modelling errors; and on the other hand, the map-based approach may suffer the interpolation error between two VGT vane positions, especially with large extrapolation errors when the turbine is operated outside the mapped region. In this thesis, physics-based models of the turbine power and its power loss are modelled as functions of both VGT vane position and turbine shaft speed, where the mechanical efficiency is defined as a function of the vane position. As a result, the proposed model eliminates the interpolation errors, especially with smooth extrapolation outside the mapped region. The proposed model is validated against test data sets under both steady-state and transient operational conditions.

1.2.2 Compressor modelling

Control-oriented models for automotive turbocharger compressors typically describe the compressor power, assuming an isentropic thermodynamic process with fixed isentropic and mechanical efficiencies for power transmission between the turbine and compressor. Although these simplifications make the control-oriented model tractable, they also introduce additional errors due to un-modelled dynamics. This is especially true for map-based approaches since the manufacture-provided maps tend to be sparse and often incomplete at the operational boundaries, especially at operational conditions with low mass flow rate at low speed. Extrapolation scheme is often used when the compressor is operated outside the mapped region, which introduces additional errors. Furthermore, the manufacture-provided compressor maps, obtained using steady-flow bench test data, could be quite different from these under pulsating engine flow conditions. In this dissertation, a physics-based model of compressor power is developed using Euler equations for turbo-machinery, where the mass flow rate and compressor rotational speed are used as model inputs. Two new coefficients, speed and power coefficients, are defined. As a result, this makes it possible to directly estimate the compressor power over the entire
compressor operational range based on a single analytic function. The proposed modelling approach is validated against test data from standard turbocharger flow bench tests, standard supercharger tests, steady-state and transient engine dynamometer tests. Model validation results show that the proposed model has acceptable accuracy for model-based control design and also reduces the dimension of the parameter space typically needed to model compressor dynamics.

1.2.3 Hydraulic assisted turbocharger modelling for controller design

A systematic modelling approach for engine air-path system with hydraulic assisted and regenerative turbocharger system are proposed and presented in this thesis. Newly developed turbocharger sub-models are integrated with engine air-path and EGR systems. Furthermore, new modelling approaches for high speed hydraulic turbine and hydraulic centrifugal turbo-pump are proposed. This modelling approach also reduces the complexity of high speed hydraulic turbomachinery. System level model integration and plant behavior investigation are carried out for engine air-path system with hydraulic assisted turbocharger. The results show proposed reduced order engine model has adequate accuracy and can be used for model-based analysis and control design.

1.2.4 Hydraulic regenerative and assisted turbocharger system level investigation.

A regenerative hydraulic assisted turbocharger (RHAT) system is investigated in this dissertation. For this new system, a hydraulic turbine is used to spin the turbocharger shaft via high pressure supply hydraulic fluid; a turbo pump is used to absorb excessive power from the turbocharger shaft while pressurizing the fluid and pumping it back into the supply tank. A driveline pump is also used to recover vehicle kinetic energy during vehicle deceleration operations and pump the fluid into the high pressure supply tank. Both hydraulic turbine and the turbo pump are packaged inside the central house of the turbocharger assembly. Compared to
traditional electric assisted and regenerative turbocharger, RHAT has a much higher assist and regenerative capability due to its high power density, and is more durable and cost effective. The abundance hydraulic energy that is recovered during vehicle deceleration can be used to assist the turbine so that VGT can operate at most efficient position with large opening rather than at small opening (low efficient position) to meet the compressor power demand. With two additional actuators on turbocharger shaft, VGT could be potentially replaced by the fixed geometry turbocharger for reduced cost and improved durability with the same efficiency. A 1-D medium duty turbocharged diesel engine model for a production vehicle was used in the investigation. The hydraulic turbine, turbo pump, and driveline pump maps were obtained using the 1-D hydraulic model provided by suppliers, respectively. A baseline controller was developed and coupled with the 1-D model to control the engagement and disengagement of RHAT system and manage the energy stored in the hydraulic supply tank. The 1-D simulation demonstrates that the proposed RHAT turbocharger system can significantly improve engine transient responses. The 1-D vehicle level simulation shows that 3-5% fuel economy improvement for the FTP 75 driving cycle, depending on different sub-component sizes. The study also identifies technical challenges for optimal design and operation of RHAT, as well as additional fuel economy improvement opportunities from the RHAT system.

1.2.5 Gain-scheduling controller design for diesel engines with EGR-VGT system

Control design for a diesel engine air-path system equipped with variable geometry turbocharger (VGT) and exhaust gas recirculation (EGR) is critical for engine performance enhancement, emission reduction, and fuel efficiency improvement. The challenges for control design in this class of dynamic system lies in the inherent coupling between VGT and EGR
systems and high nonlinearity of engine air-path system. A linear controller design approach is proposed in this dissertation for simultaneously regulating boost pressure and EGR mass flow rate. Linear quadratic controllers with integral action are designed based on the linearized systems over the engine operational map. Controller is scheduled based on engine operational parameters: engine speed and fuel injection amount. The gain-scheduling linear controller is validated against the baseline controller using the nonlinear plant. Results show that designed MIMO gain-scheduling controller can manage the tracking trade-offs between boost pressure and EGR mass flow over the baseline controller (two individual loop SISO controllers). This validates the proposed novel approach for designing controllers with trade-offs between engine performance and emissions. This controller design approach is further extended to assisted and regenerative turbocharger system with VGT and EGR.

1.3 Dissertation contributions

The following is a list of major contributions:

1. A new control-oriented turbine power model based on the turbine vane position, using only turbine downstream conditions, is proposed, where the VGT vane position is a direct input for turbine power. A generalized approach of identifying friction loss for the proposed turbine model is proposed. The turbine power and its mechanical efficiency models are suitable for the model-based VGT control due to the analytic nature of the proposed models. The proposed model has adequate accuracy due to the introduction of turbocharger dynamic friction.

2. A new physics-based model of compressor power is developed using Euler equations for turbo-machinery, where the mass flow rate and compressor rotational speed are used as model inputs. Two new coefficients, speed and power coefficients, are defined. As a
result, this makes it possible to directly estimate the compressor power over the entire compressor operational range based on a single analytic relationship. Proposed model largely reduces the number of parameters to be identified, compared to traditional modelling approach. The reduced-order and reduced-complexity model is especially useful for the control applications.

3. A system modelling approach for a reduced-order diesel engine air-path system with a regenerative hydraulic assisted turbocharger is proposed. Newly developed turbocharger sub-models are integrated with engine air-path and EGR systems. Furthermore, new modelling approaches for high speed hydraulic turbine and hydraulic centrifugal turbo-pump are proposed. The proposed model can be used for model-based control design for VGT-EGR system as well as VGT-EGR-RHAT system.

4. A system level investigating for hydraulic assisted and regenerative turbocharger systems shows the benefits and challenges for the proposed system. The preliminary 1-D simulation results demonstrate that the proposed RHAT turbocharger system can significantly improve engine transient responses. The vehicle level simulations show that 3-5% fuel economy improvement for the FTP 75 driving cycle, depending on different sub-component sizes. The study also identifies technical challenges for optimal design and operation of RHAT systems, as well as additional fuel economy improvement opportunities.

5. A linear control design scheme for diesel engine air-path system is proposed to handle engine performance and emission trade-offs for closed-loop controller design. It can not only regulate the boost pressure and EGR mass flow rate to their desired values with the proposed coordinated control for EGR-VGT system but also design the closed-loop
controller to achieve different design targets, such as transient response performance and emission target by LQ weighting selection. Gain-scheduling based on engine speed and fuel injection quantity is used to extended linear controller design to the nonlinear engine plant. The simulation validation results show that the designed controller has high flexibility for different performance targets, compared to the baseline controller. The proposed control design process has potential to significantly reduce efforts for early prototyping control design and calibrations. Furthermore, a coordinated VGT-EGR-RHAT controller is designed for diesel air-path system based on the same scheme.

1.4 Dissertation outline

The organization of dissertation is shown in Error! Reference source not found.. The control-oriented VGT turbine model and the associated friction identification are investigated in Chapter 2; and the control-oriented compressor model is investigated in Chapter 3. The turbocharged diesel engine model with the regenerative hydraulic assisted turbocharger (RHAT) is developed in Chapter 4; and its system level simulation investigation results using the high fidelity 1-D engine and vehicle model is presented in Chapter 5. Finally, a multivariable gain-scheduling control design approach is developed for both traditional VGT-EGR and proposed VGT-EGR-RHAT systems in Chapter 6. The last Chapter adds some conclusions and future work.
Figure 1. Dissertation outline
CHAPTER 2: CONTROL ORIENTED VGT TURBINE POWER MODELS FOR TURBOCHARGED ENGINE

2.1 Abstract

Control-oriented models of Variable Geometry Turbochargers (VGT) are required for model-based VGT control. Typically, the VGT turbine power is modelled using a fixed or a map-based turbocharger mechanical efficiency with isentropic assumptions. However, the fixed efficiency approach could be over simplified, leading to large modelling error; and on the other hand, the map-based approach may suffer the interpolation error between two VGT vane positions, and especially the extrapolation inaccuracy when the turbine is operated outside the mapped region. In this chapter physics-based models of the turbine power and its power loss are modeled using both the VGT vane position and turbine shaft speed as inputs, where the mechanical efficiency is defined as a function of the vane position. As a result, the proposed model eliminates the interpolation error and especially allows smooth extrapolation outside the mapped region. The proposed model is validated against a few test data sets under both steady-state and transient operational conditions.

2.2 Introduction

Variable Geometry Turbocharger (VGT) is common in modern diesel engines. Benefits of using VGT over traditional Fixed Geometry Turbocharger (FGT) in diesel engines have also been long established [1]-[4]. Federally mandated emission standards on Nitrogen Oxides (NOx) have forced the utilization of the exhaust gas recirculation (EGR) in diesel engines. This introduces an additional coupling between the VGT vane position and EGR and it has been a topic of active research within the engine control community [5]-[8] for the last two decades.
Most of control-oriented turbocharger models describe using turbocharger efficiency maps; the associated air-path models of VGT-EGR diesel engines calculate the turbine power based upon the ideal isentropic expansion assumption; and the overall turbocharger (TC) efficiency, used to obtain the power available to the compressor, is based on a family of manufacture provided maps (one map corresponding to one VGT vane position). However, TC performance maps, provided by the manufactures, are often sparse or not available (see Figure 2) at light engine load. A comparison of turbine operational ranges obtained from turbine flow bench, engine steady-state and transient dynamometer tests is shown in Figure 2. It is clear that the operational range of the flow bench steady-state tests does not match with the actual steady-state and transient operational ranges. As a result, operating the TC outside its performance map requires extrapolation under the assumption of a convex hull using the base map, leading to several investigations in extrapolation methods; see [8]-[10],[16] and [18]. Usually, empirical fitted models (usually 2nd or 3rd order) as a function of blade speed ratio (BSR) are used for extrapolation when the turbocharger is operated outside the provided map. This could lead to physically impossible values and extremely large modelling errors. Note that the typical performance map or the data set, used to calibrate and validate the model, is provided by the TC manufacture and based on the data from hot gas flow bench tests. Since these tests are performed under steady flow conditions, the mapped data cannot match the pulsating flow operational conditions when the TC is coupled to an internal combustion (IC) engine [1],[2],[14],[15].
Also, the turbine efficiency maps provided by turbine manufactures combine the turbine efficiency with mechanical loss one. Note that it is difficult to measure mechanical loss [17],[29],[29] and it is not required to be measured (see the test code described in [26]) in the standard flow bench tests. As a result, enthalpy change across the compressor is calculated as the turbine output power. This makes the calculated turbine efficiency depend on the compressor characteristics and the measured turbine efficiency is not the actual turbine efficiency since it is a combination of both turbine and mechanical efficiencies. This makes turbine performance map obtained from flow bench different from the turbine characteristics when it is coupled with engine as show in Figure 2. In the automotive turbocharger system, turbine extracts energy from engine exhaust to drive compressor for increased boost pressure. Note that in this application the available turbine power due to the exhaust gas expansion process needs to overcome both bearing mechanical and heat transfer losses to drive the compressor; see Figure 3. Friction loss is due to both journal and thrust bearings in radial and axial directions; and heat transfer loss also
drives turbocharger operation away from the adiabatic behavior. Both losses affect turbine efficiency. However, under transient engine operations, friction loss dominates the turbocharger behaviors, and especially, under fast transient operations, thrust friction loss is determined by the balanced thrust force between turbine and compressor and it is not available from TC manufactures. Hence, turbine efficiency map provided by the manufacture is not suitable for transient operations. Most of the existing control-oriented turbocharger model assumes constant mechanical loss efficiency for simplicity [2],[5],[7]-[10]. Note that the axial friction due to the thrust bearing load and radial friction due to journal bearing cannot be directly measured on the hot gas flow bench, which makes it challenge to model the mechanical loss directly.

Figure 3. Turbocharger system structure

Another issue for map-based model is that the required interpolation and extrapolation of the manufacture TC performance map to obtain turbine efficiency at the current operational condition leads to multiple dimensional lookup tables with at least three inputs (pressure ratio of upstream and downstream, reduced mass flow rate, and VGT vane position). This could introduce additional computational burden for real-time control. Further issues with map based
traditional isentropic modelling; it needs both upstream and downstream conditions, either for pressure or temperature. This needs extra dynamics states for engine air-path modelling or extra sensors for real time application.

Finally, map-based model is not applicable to the assisted and regenerative turbocharger since the operation ranges of turbine and compressor are quite different from the turbocharger without assisted power. With the assisted power on the TC shaft, the turbine could operate at the condition with much lower pressure ratio and higher shaft speed, leading to operating the turbine operation outside of its traditional operation range. Also with assisted power on TC shaft, the thrust friction torque could change dramatically since the thrust force direction could vary during the transient operations. This is due to the fact that thrust force balance between turbine and compressor is a function of the assisted or regenerative load on the TC shaft. This results in different thrust forces for thrust bearing, and therefore the mechanical loss is different from the non-assisted turbocharger.

Hence, turbocharger modelling for turbine, compressor and mechanical loss need to be separated based on their own physics operation principles. In open literatures, some researchers [23] proposed a fluid dynamics based approach to model turbine power rather supplier’s map. The advantage of this model is compactness of its form. It needs only upstream conditions rather than map based approach, which needs both upstream and downstream conditions. However, this model needs rotor inlet conditions (after vane nozzle), which are difficult for measurements. Further, this model also needs mechanical loss model to fulfill mechanical efficiency modelling. No one has investigated the turbine downstream conditions for turbine modelling. Since, turbine downstream conditions are shared with both turbine and after-treatment system. Hence, turbine
downstream conditions would be a good model inputs candidate for turbine modelling, as well as serving as model inputs of after-treatment for model-based control.

With the turbine modelling issues addressed above, this chapter proposes to develop a physics-based control-oriented model for the turbine power based on turbine downstream conditions and a general method to identify turbocharger mechanical loss to accurately model the compressor power under both steady-state and transient conditions. The proposed modelling approach can be directly applied to power-assisted turbocharger. The fundamental Euler turbine equation is used to model the turbine power [1],[2],[4],[19],[20] by incorporating flow and rotor-dynamics. Three different shaft mechanical loss models (only friction loss in this study) are also evaluated. In order to improve the shaft speed transient dynamic model, shaft mechanical loss is modeled based on both thrust and axial friction torques. The turbine power and mechanical loss models are validated with given compressor power and the compressor power calculation is based on the standard isentropic compression assumption with measured compressor upstream and downstream conditions (temperature, pressure, compressor mass flow rate). Turbocharger rotor dynamic equation is used to couple the compressor, turbine, and mechanical loss powers during steady-state and transient operations. The main contributions of this dissertation are two-fold: a new control-oriented turbine power model as a function of input turbine vane position based on only turbine downstream conditions and a generalized approach of identifying friction loss for the proposed turbine model.

The rest of this chapter is organized as follows. Section II discusses the turbine modelling using the Euler equation with three mechanical loss candidate models and Section III provides both steady-state, transient validation results. The last section adds some conclusions.
2.3 VGT Turbine Power Model-based on Euler Turbine Equation

2.3.1 Euler turbine equation

The Euler turbine equation bridges the power added to (or removed from) the flow and the characteristics of rotating blades as described in literature [1],[19],[21],[27]. The Euler equation is based on the concepts of conservation of angular momentum and conservation of energy and it can be illustrated in Figure 4.

![Figure 4. Simplified turbine model [27]](image)

Applying conservation of angular momentum, it is noted that the turbine torque $\tau$ should equal to the rate of change of angular momentum in a stream tube that flows through the turbine:

$$\tau = \dot{m}(v_{in}r_{in} - v_{out}r_{out}) \tag{2.1}$$

Where, $v_{in}$ and $v_{out}$ are the inlet and outlet turbine flow velocities, respectively; $r_{in}$ and $r_{out}$ are the radius of the inlet and outlet flow with respect to the rotation axis, respectively; and $\dot{m}$ is the mass flow rate. The work per unit time $\dot{W}$, or power can be defined as

$$\dot{W} = \omega \tau = \omega \dot{m}(v_{in}r_{in} - v_{out}r_{out}) \tag{2.2}$$

where $\omega$ is the turbine speed. Now consider the steady flow energy equation

$$\dot{Q} - \dot{W} = \dot{m}\Delta h_T \tag{2.3}$$

where $\Delta h_T$ is the enthalpy change across the turbine, and $\dot{Q}$ is turbine heat transfer rate. Assuming an adiabatic process for the gas expansion process inside turbine [27] with $\dot{Q} = 0$ leads to
\[ W = \dot{m}(h_{in} - h_{out}) \]  
\[ (2.4) \]
Combining the above equation with the conservation (6) yields

\[ W = \dot{m}(h_{in} - h_{out}) = \dot{m} \omega (v_{in}r_{in} - v_{out}r_{out}) \]  
\[ (2.5) \]
Since under the ideal gas assumption that the constant pressure specific heat value \( c_p \) for the flow is a constant [22], we have

\[ W = \dot{m} c_p (T_{T1} - T_{T2}) = \dot{m} \omega (v_{in}r_{in} - v_{out}r_{out}) \]  
\[ (2.6) \]
Note that equation (2.6) is the well-known Euler turbine equation, that links the temperature ratio (and hence the pressure ratio) across a turbine to the rotational speed and the change of momentum per unit mass.

### 2.3.2 VGT turbine power model as a function of vane angle

For completeness the following turbine power equation is from [1] and [20]. A typical variable geometry turbine is shown in Figure 5 along with their gas flow velocity triangles at the turbine rotor inlet and outlet, see [1] and [21] for details. Using the Euler turbine equation yields:

\[ W = \omega \tau = \omega \dot{m}(v_{in}r_{in} - v_{out}r_{out}) = \dot{m}(v_{in} \omega r_{in} - v_{out} \omega r_{out}) = \dot{m}(u_1 c_{\theta 1} - u_2 c_{\theta 2}) \]  
\[ (2.7) \]
where \( \omega \) is the turbine rotational speed in radian per second; \( \tau \) is the turbine torque; and \( \dot{m} \) is the turbine mass flow rate. Note that the swirl at the turbine exit [21] can be ignored under full energy recovery assumption within the turbine, which leads to \( c_{\theta 2} = 0 \) the following power equation:

\[ W = \dot{m}u_1 c_{\theta 1} \]  
\[ (2.8) \]
Note that the turbine is normally designed such that its radial velocity at the vane nozzle outlet \(c_{r1}\) in Figure 5) is the same as the axial velocity at the rotor exit \(c_{a2}\) in Figure 5), i.e., \(c_{r1} = c_{a2} = c_1 \cos \alpha_1\). This assumption leads to the following velocity relationship:

\[
c_\theta_1 = c_1 \cdot \sin \alpha_1 = c_{a2} \cdot \frac{\sin \alpha_1}{\cos \alpha_1} = c_{a2} \cdot \tan \alpha_1
\]

(2.9)

where \(\alpha_1\) is the gas entry angle to the rotor and is determined by the vane guide blade angular position controlled by the VGT position actuator. Since the mass flow rate at the rotor outlet is a function of the exit geometric area \(A_2\), the gas velocity at the turbine outlet can be defined as:

\[
c_{a2} = \frac{m}{A_2 \rho} = \frac{\dot{m} W_{T2}}{A_2 P_{T2}}
\]

(2.10)

where \(T_{T2}\) and \(P_{T2}\) are turbine outlet temperature and pressure (see Figure 5). Note that the turbine outlet area of gas flow \(A_2\) can be calculated using the turbine outlet and nut diameters \(D_{t2}\) and \(D_{tn}\) as follows,
\[
A_2 = \frac{\pi}{4} (D_{t2}^2 - D_{tn}^2) \tag{2.11}
\]

and the blade tip speed \( u_1 \) in equation (8) can be calculated using the turbine wheel diameter \( D_{t1} \) below.

\[
u_1 = \omega \frac{D_{t1}}{2} \tag{2.12}\]

Combining equations (8) to (12), the turbine power can be defined as

\[
\dot{W}_T = \dot{m}\omega \frac{D_{t1}}{2} \frac{\dot{m}}{\frac{\pi}{4}(D_{t2}^2 - D_{tn}^2)} \frac{\varrho T_{T2}}{P_{T2}} \tan \alpha_1 = 2\omega \dot{m}^2 \frac{D_{t1}}{\pi(D_{t2}^2 - D_{tn}^2)} \frac{\varrho T_{T2}}{P_{T2}} \tan \alpha_1 \tag{2.13}\]

Now substituting the TC shaft speed, \( \omega = \frac{2\pi N_{TC}}{60} \), exhaust mass flow rate, \( \dot{m} = \dot{m}_{ex} \), the power and torque produced by the turbine can be expressed as:

\[
\dot{W}_T = \frac{1}{15} N_{TC} (\dot{m}_{ex})^2 \frac{D_{t1}}{D_{t2}^2 - D_{tn}^2} \frac{\varrho T_{T2}}{P_{T2}} \tan \alpha_1 \tag{2.14}\]

\[
\tau_T = \frac{2}{\pi} (\dot{m}_{ex})^2 \frac{D_{t1}}{D_{t2}^2 - D_{tn}^2} \frac{\varrho T_{T2}}{P_{T2}} \tan \alpha_1 \tag{2.15}\]

Using equations (14) and (15), turbine power and torque can be easily obtained. This model can be directly applied to turbocharger shaft rotor dynamics equation or turbocharger shaft speed differential equation. This finding is in line with model proposed in [23] as shown in (4).

\[
\dot{W}_T = \frac{1}{60} N_{TC} (\dot{m}_{ex})^2 \frac{1}{B} \frac{\varrho T^*}{P^*} \tan \alpha_1 \tag{2.16}\]

where \( B \) is the turbine inlet clearance as shown in Figure 5; and \( T^*, \) and \( P^* \), are temperature and pressure at rotor inlet (location 1) in Figure 5, respectively. The advantage of equations (14) and (15) is that it only depends on the downstream conditions of turbine. Note that, for traditional turbine efficiency model, at least one pair of upstream and downstream conditions are
(temperature or pressure) required for turbine efficiency model. However, rotor inlet conditions are often not available for measurement. Furthermore, when turbocharger is coupled with the after-treatment system, downstream conditions are critical for modelling and control of both turbocharger and after-treatment system. Hence, when the downstream measurements are available, equations (14) or (15) are more practical than (16) for both turbocharger and after-treatment system. In this case, turbine power is a direct function of control input-VGT vane position. This reduces the complexity for VGT turbocharger controller design.

2.3.3 Turbocharge friction model

Power transmission between turbine and compressor is coupled with turbocharger mechanical loss that is a combination of both heat transfer and friction losses. In this study, only friction loss is considered. The turbocharger friction consists of mainly two parts: radial and axial direction ones associated with radial and thrust bearings, respectively. The bearing system and the forces on the rotor are shown in Figure 3. Therefore, the associated mechanical loss $\dot{W}_{\text{loss}}$ (also called the shaft loss) can be expressed as

$$\dot{W}_{\text{loss}} = \dot{W}_{\text{journal}} + \dot{W}_{\text{thrust}}$$

where, $\dot{W}_{\text{journal}}$ and $\dot{W}_{\text{thrust}}$ are the losses associated with the journal and thrust bearings, respectively. Note that journal bearing friction depends on shaft angular speed, oil film thickness as well as oil viscosity; and thrust bearing friction depends on both thrust force from axial direction and shaft angular speed. Thrust force is due to the pressure difference between the compressor inlet pressure $P_{C1}$ and turbine outlet pressure $P_{T2}$ (see Figure 3). Note that the impulse force in the axial direction due to the axial direction flow inside both compressor and turbine wheels is normally ignored in the model. The compressor wheel action force $F_C$ due to
$P_{c1}$ and turbine wheel action force $F_T$ due to $P_{T2}$ (see Figure 3) will be used to determine the thrust bearing load. There are three existing modelling approaches from literature shown in Table 1, where Model 1 and Model 3 lump the thrust and journal bearing friction losses into a polynomial of turbocharger shaft speed; and Model 2 separates the thrust and journal bearing friction losses by considering the turbine upstream pressure ($P_{T1}$) and compressor downstream pressure ($P_{C2}$) (see Figure 3). However, in order to capture turbocharger transient behavior more accurate, in this study, another friction model is proposed for transient operations as in Model 4. A transient dynamic compensation term $c_3|\dot{N}_{TC}|$ for dynamic operation is added to Model 2. This term is set as a tuning parameter for unmodeled transient dynamics. For steady state operation, Model 2 and Model 4 are identical, since $\dot{N}_{TC} = 0$. All the four friction models will be investigated in next section for steady state and transient operation. For simplicity, all friction model coefficients are assumed to be constant in this study. The heat transfer loss, bearing oil film thickness, lubrication oil viscosity as well as oil temperature variations are not considered in this study.

Table 1. Friction model candidates

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 1 [2]</th>
<th>$W_{Loss} = c_1(N_{TC})^2 + c_2N_{TC} + c_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 2 [29]</td>
<td>$W_{Loss} = W_{journal} + W_{thrust}$, where $W_{journal} = c_1(N_{TC})^2$ and $W_{thrust} = c_2\sqrt{P_{T1} - P_{C2}(N_{TC})^2}$</td>
<td></td>
</tr>
<tr>
<td>Model 3 [23]</td>
<td>$W_{Loss} = c_1(N_{TC})^2$</td>
<td></td>
</tr>
<tr>
<td>Model 4 (this work)</td>
<td>$W_{Loss} = c_1(N_{TC})^2 + c_2\sqrt{P_{T1} - P_{C2}(N_{TC})^2} + c_3</td>
<td>\dot{N}_{TC}</td>
</tr>
</tbody>
</table>
2.4 Model Validation and Mechanical Loss Identification

2.4.1 Model validation using steady-state engine test data and mechanical loss estimation

After the turbine power and friction models are addressed, the fraction of the turbine power made available to the compressor can be determined. Note that under normal operating conditions the power balance between the turbine and compressor of a turbocharger is determined by the following governing equation, which includes shaft kinetic energy.

\[ \omega_T \frac{d\omega}{dt} = \eta_m W_T - W_C \]  \hspace{1cm} (2.18)

The mechanical loss is typically defined as a fraction of the source power (or turbine power) and modeled by the source efficiency \( \eta_m \), so the governing equation can be expressed as

\[ \omega_T \frac{d\omega}{dt} = W_T - W_{Loss} - W_C \text{ , where } W_{Loss} = (1 - \eta_m)W_T \]  \hspace{1cm} (2.19)

Under steady-state operation \( \frac{d\omega}{dt} = 0 \) leads to

\[ W_C = W_T - W_{Loss} = \eta_m W_T \]  \hspace{1cm} (2.20)

In this study, compressor power is assumed to be known by solving stand isentropic power equation using its upstream condition and downstream conditions. Standard compressor power (isentropic power corrected for isentropic efficiency) [1] can be expressed by

\[ W_C = \frac{1}{\eta_C} \dot{m}_{air} T_{C1} c_p^{\text{air}} \left( \frac{P_{C2}}{P_{C1}} \right)^{\gamma_{air} - 1} - 1 \]  \hspace{1cm} (2.21)

Substituting (24) and (18) into (23) yields:

\[ \eta_m \tan \alpha_1 = \frac{1}{\eta_C} \dot{m}_{air} T_{C1} c_p^{\text{air}} \left( \frac{P_{C2}}{P_{C1}} \right)^{\gamma_{air} - 1} r^{-1} \text{, where, } r = \frac{1}{15} N_{TC} \left( \dot{m}_{ex} \right)^2 \frac{\delta_{C1}}{D_{C2} - D_{in}} \frac{\delta_{TC}}{P_{T2}} \]  \hspace{1cm} (2.22)

From equation (22) it can be found that the mechanical efficiency, \( \eta_m \), depends on the turbocharger operational condition. It is not a constant and can be identified by varying VGT.
vane position $\alpha_1$, assuming that all parameters on the right hand side (RHS) of equation (22) are available. This also indicates that the turbocharger mechanical loss can be solved analytically using the proposed physics-based model. This could potentially separate actual turbine efficiency from the measured turbine efficiency using based on the supplied turbine map. The next step is to find an empirical relationship between $\eta_m$ and other turbocharger operational parameters so that the turbine mechanical efficiency can be predicted. Note that the left hand side (LHS) term $\eta_m \tan \alpha_1$ is a function varying VGT vane positions. A heavy-duty turbocharged diesel engine is used as an example to study the relationship, where the engine parameters and the experimental set-up can be found in [24] and [25]. The entire engine operating range is mapped via 195 steady-state speed and load points that provide enough data for conduction turbocharger power balancing study. The ideal gas parameters (such as isentropic index of air, constant pressure specific heat of air, and ideal gas constant) are from [26]. Vane position is based on turbocharger control signal between 0% to 100%. Figure 6 plots the 195 values ($\eta_m \tan \alpha_1$) calculated using the RHS term of equation (24) as a function of the VGT position $\alpha_1$, where 0% is associated with the fully opened position. Each point on this plot represents a unique steady-state operational condition for the power balanced turbocharger. Since $\tan \alpha_1$ is a constant for a given vane position, the variations observed in Figure 6 must be from $\eta_m$. This confirms that the turbocharger mechanical efficiency is not a constant, which is discussed in the introduction section.
From Figure 6, it is also clear that for vane positions in the range between fully open (or 0% closed) to about 60% open (or 40% closed) the variation in $\eta_m$ is small. This is an important observation since it indicates that with large vane openings (with unrestricted flow across turbine) the steady state mechanical efficiency of the TC (or the overall efficiency) is a constant for a given vane position regardless of the operational condition of the VGT-engine system. Note that constant $\eta_m$ was used by several researchers in literature [5], [7]. However, as the vane position is gradually closed to less than 60% open, flow restriction is introduced and as a result the TC mechanical efficiency shows large variability. Figure 6 shows large variability in $\eta_m$ with varying TC speed for a fixed vane position. Unfortunately this also clearly indicates that for more restricted flows the constant $\eta_m$ assumption may introduce large error. Therefore, for the region of restricted flow and high TC shaft speed, the loss is a function of shaft speed and a new model for the mechanical loss should be developed to include other sensitivities. Based on the results in Figure 7, where an obvious correlation between shaft speed and the variability can be seen, this is in line with findings in friction models shown in Table 1.
To determine the coefficients in both turbine and friction model (power loss) models defined in Table 1 a cost function is proposed in (23).

\[
J = \sum_{i=1}^{n} |\dot{W}_{T,calc}(i) - (\dot{W}_c(i) + \dot{W}_{loss}(i))|^2
\]  

(2.23)

where \(\dot{W}_{T,calc}\) is the calculated turbine power defined in (14) or (16); \(\dot{W}_c\) and \(\dot{W}_{loss}\) are defined in equation (21) and Table 1, respectively; and \(n\) is the number of turbine testing points.

The test data set used for optimization covers the entire VGT vane position. Ideally, the exact relationship between vane angle \(\alpha_1\) and the VGT actuator position can be obtained for a given turbocharger; see Figure 5. That is, \(\tan \alpha_1\) should be known for each given VGT vane position. However, for this study, the VGT vane position signal is not available and the VGT actuation duty cycle is used for estimating \(\alpha_1\) directly. Based on the kinematic model of the linkage mechanism from the VGT actuator to its vane (See Figure 8 for a typical VGT actuation mechanism), the following polynomial relationship(2.24) from the vane position control duty-cycle to the actual vane angle \(\alpha_1\) is used.
\[ \alpha_1 = f(u_{vgt}) = c_4 u_{vgt}^3 + c_5 u_{vgt}^2 + c_6 u_{vgt} + c_7 \]  \hspace{1cm} (2.24)

Therefore, the maximal number of optimization parameters is up to six, where up to three \((c_1 \text{ to } c_3)\) from the friction model (see Table 1) and four \((c_4 \text{ to } c_7)\) from vane position model in (24). A Least-Squares optimization method is used to find the optimal coefficients \((c_1 \text{ to } c_7)\) to minimize the cost function defined in (23). Since the vane position coefficients \((c_4 \text{ to } c_7)\) are independent of these for the friction model defined in Table 1, friction Model 1 is used to obtain optimal coefficients \((c_1 \text{ to } c_7)\) and the corresponding vane position coefficients \((c_4 \text{ to } c_7)\) will be used for initial conditions for all other three friction models. For steady state, derivative of TC speed is zero. This makes Model 2 and Model 4 identical for steady state operation. The optimal value of \((c_4 \text{ to } c_7)\) are shown in Table 2.

Table 2. Model validation results using steady-state engine test data for turbocharger 1

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2/Model 4</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction model coefficients</td>
<td>(c_1)</td>
<td>6.957\times10^{-10}</td>
<td>4.507\times10^{-10}</td>
</tr>
<tr>
<td></td>
<td>(c_2)</td>
<td>2.357\times10^{-14}</td>
<td>5.909\times10^{-11}</td>
</tr>
<tr>
<td></td>
<td>(c_3)</td>
<td>7.5023\times10^{-12}</td>
<td>0.00</td>
</tr>
<tr>
<td>Vane angle coefficients</td>
<td>(c_4)</td>
<td>1.011</td>
<td>0.961</td>
</tr>
<tr>
<td></td>
<td>(c_5)</td>
<td>0.171</td>
<td>0.206</td>
</tr>
<tr>
<td></td>
<td>(c_6)</td>
<td>0.349</td>
<td>0.344</td>
</tr>
<tr>
<td></td>
<td>(c_7)</td>
<td>0.572</td>
<td>0.572</td>
</tr>
<tr>
<td>Vane angle[rad]</td>
<td>Min</td>
<td>0.572</td>
<td>0.572</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>1.478</td>
<td>1.471</td>
</tr>
<tr>
<td>Model error[%]</td>
<td>-</td>
<td>4.38</td>
<td>4.23</td>
</tr>
</tbody>
</table>
Based on the calibrated vane angle model in Table 2, the VGT fully opened vane angle is 0.556-0.572 rad (31.85 degree to 32.77) that is corresponding to the zero position control command ($u_{vgt} = 0$), and the fully closed vane angle is 1.460-1.478 rad (83.65 degree to 84.68 degree) associated with the maximal (100 percent) control signal ($u_{vgt} = 0.8$). These calibrated model coefficients agree well with the designed ones. The TC mechanical loss is plotted in Figure 9, along with the corresponding VGT vane position. It is clear that the power loss increases as the TC speed does for all three friction models and the maximal mechanical loss over the entire engine operational range is 8.5 kW, which matches the test data in [29],[29]. The deviation between Models 1 and 3 are not significant for steady-state operations since the quadratic term in both models dominates friction loss. Since Model 2/Model 4 accounts for the pressure difference, the friction power variation between 40k and 100k rpm is due to the thrust load change. Note that, the friction variation is only for steady state operation for this validation. For transient operation, thrust friction will increase due to unbalanced thrust force from both turbine and compressor side.

As shown in Figure 2, max turbine pressure ratio for engine steady state is much smaller than max of that for transient operation. Thus, during transient operation, thrust forces would be higher due to increased pressure difference between turbine and compressor. Thus Model 2/Model 4 would have higher thrust friction due to higher pressure difference across turbocharger. Dynamics compensation term in Model 4 will be further investigated through transient test data in section D. Figure 9. Predicted turbine power subtracted by mechanical loss and compressor power at steady-state (power balanced conditions) is shown in the third plot of Figure 9. The error function is defined in (25). Admittedly, there are certain errors in this modelling approach due to unknown relationship between VGT position and vane angle as well
as unmolded physics with simplification. From Table 2, it shows that averaged errors for proposed models are about 4% error for steady state operation.

\[
\text{Error} = \left( \sum_{i=1}^{n} |W_{T,\text{calc}}(i) - (W_c(i) + W_{\text{loss}}(i))| \right) \left( \sum_{i=1}^{n} |W_c(i)| \right)^{-1}
\]

where \( n \) stands for total sample number. Model 2/ model 4 have smallest error. Model 1 and Model 3 are equivalent for model accuracy. For the selection of mechanical loss model to model or calculate friction loss, Model 2/ Model 4 should be used to capture accurate rotor dynamics related with thrust friction. However, Model 2/Model 4 needs the two extra two inputs compared to Model 1 and Model 3, which are turbine input pressure and compressor downstream pressure. Without these pressure measurement inputs or dynamics state models, Model 3 should be chosen for its simple structure. In summary, this section provides validation process of the proposed new turbine power model, as well as a general procedure for identifying mechanical loss for steady state engine operation.
2.4.2 Turbine model validation against standard hot gas flow bench test data

Secondly, the test data from the standard hot gas flow bench of a different turbocharger, called turbocharger 2, is used to validate the turbine model. In this case, the turbine inlet is under steady-state flow, which is quite different from the engine dynamometer test data under pulsation flow. The detailed test setup can be found in [26]. The test points over the VGT operational range are shown in Figure 10. The speed range for turbocharger 2 is between 5k and 150k rpm and the pressure is between 1.1 and 5.0 bar. The turbine model described in subsection III.A is used. Friction Model 1-3 were chosen for this study.
Table 3. Model validation results using steady-state flow bench test data for turbocharger 2

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2/Model 4</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction model</td>
<td>c1 6.988×10^-10</td>
<td>6.513×10^-10</td>
<td>6.990×10^-10</td>
</tr>
<tr>
<td>coefficients</td>
<td>c2 1.800×10^-10</td>
<td>-4.8812×10^-14</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>c3 1.000×10^-10</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>Vane angle</td>
<td>c4 0.8765</td>
<td>1.107</td>
<td>1.085</td>
</tr>
<tr>
<td>coefficients</td>
<td>c5 0.002476</td>
<td>-0.2495</td>
<td>-0.2191</td>
</tr>
<tr>
<td></td>
<td>c6 0.1581</td>
<td>0.2169</td>
<td>0.0121</td>
</tr>
<tr>
<td></td>
<td>c7 0.2541</td>
<td>0.2479</td>
<td>0.2508</td>
</tr>
<tr>
<td>Vane angle [rad]</td>
<td>Min 0.2541</td>
<td>0.2479</td>
<td>0.2508</td>
</tr>
<tr>
<td></td>
<td>Max 0.972</td>
<td>0.986</td>
<td>0.995</td>
</tr>
<tr>
<td>Error [%]</td>
<td>-</td>
<td>3.23</td>
<td>3.20</td>
</tr>
</tbody>
</table>

Note that for this study the vane angle equation (24) is used as a function of the percentage VGT control duty cycle. The optimized mechanical loss and vane angle coefficients are shown in Table 3 for four models, and the predicted mechanical loss and the predicted vane angle as functions of speed and VGT position are shown in Figure 11. Since this turbocharger has a maximum operational speed of 150k rpm, high friction loss (peaked at 16 kW) is expected. Error function is defined in (25). The results are comparable with these presented in the previous subsection. Is shows that the proposed turbine model is also suitable for the data set obtained.
from steady-state flow bench tests and can be used to extrapolate turbine efficiency based on the given turbine efficiency map.

2.4.3 Turbine power model validation using GT-Power transient simulation data

In order to understand the model characteristics during transient operations, the proposed model is calibrated using the transient responses obtained from the GT-Power model in this subsection. The GT-Power model is developed for the engine and turbocharger 1 described in subsection III.A, and the detailed GT-Power model and its simulation setup can be found in [25].

In the transient simulations, the engine follows the desired torque based acceleration pedal position. The VGT feedback control is used to track the target calibrated boost pressure. The purpose of this study is to study the model behavior under transient operational conditions.
First, the developed turbine model is calibrated using the FTP 75 simulation data to obtain the vane angle model with the VGT control duty cycle as input, and then, the calibrated model is used to predict the turbine power for US 06 driving cycle. Since the turbine map in GT-Power simulation is based on the steady-state flow bench map, friction loss is not modeled in the GT-Power model and the turbine power is an output in GT-Power simulations by interpolating or extrapolating the manufacture provided efficiency map [28]. The turbine vane angle can be calculated with equation (26):

\[
\tan \alpha_1 = \left( \frac{2n}{15} \frac{N_{TC}}{N_{TC}} \right)^2 \left( \frac{D_{t1}}{D_{t2} - D_{m}} \frac{\frac{\mu_{T} \mu_{T}}{P_{T2}^2}}{\mu_{T}} \right) W_{sim\_GT}^{-1} \tag{2.26}
\]

where, \(W_{sim\_GT}\) is GT-Power simulated turbine power. The relationship between vane angle and VGT control duty cycle is shown in Figure 12 and the following vane angle fitting is obtained

\[
\alpha_1 = -3.358 u_{vgt}^3 + 5.881 u_{vgt}^2 - 1.594 u_{vgt} + 0.58 \tag{2.27}
\]

where \(u_{vgt}\) is VGT map position. Using the proposed turbine model equation (14) and fitted vane relationship equation (27), the predicted turbine power can be obtained.

In order to show the consistence of the calibrated model, the model calibrated using the FTP 75 cycle is used to simulate turbine power under the US 06 driving cycle for the same engine. The simulation results and the associated errors are shown in Figure 13 and Table 4, where the power model error is defined in equation (28). Note that the error could be due to the map extrapolation inaccuracy based on the empirical equation used in the GT-Power model; and the unmodeled friction loss in the GT-Power model could also contribute to the error.

The above study shows that the proposed turbine power model can be used for modelling the turbine transient operations.
\[
\text{Error} = \left( \sum_{i=1}^{n} |W_{\text{sim,GT}}(i) - W_{\text{predicted}}(i)| \right) \left( \sum_{i=1}^{n} |W_{\text{sim,GT}}(i)| \right)^{-1} 
\]  
(2.28)

Table 4. Averaged Error between proposed model predicted turbine power and GT simulated turbine power

<table>
<thead>
<tr>
<th>Value</th>
<th>Driving cycle</th>
<th>FTP 75</th>
<th>US 06</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>[%]</td>
<td>6.6</td>
<td>8.2</td>
</tr>
</tbody>
</table>

Figure 12. Vane angle and the VGT map position with GT simulation

Figure 13. Model prediction results vs GT simulation results
Figure 14. Modelling error for FTP-75 in GT simulation

Figure 15. Modelling results for US_06 driving cycle
2.4.4 Model validation using vehicle test data

At last, vehicle test data is used to investigate the transient performance of the proposed turbine and friction models. The calibrated turbine and friction loss models of Turbocharge 1 in subsection III.A is used, where the turbine power is calculated based on (2.14) and the mechanical loss and the vane angle models are from (2.25) and Table 2. Note that for transient operations the turbocharger energy balance is defined in (2.18) and it is used to study the proposed model accuracy related to the mechanical loss and the relationship between VGT actuator position and the associated vane angle. For the model verification purpose, equation (2.18) is reorganized below in equation (2.29).

\[ \dot{W}_T = \omega J_{Te} \frac{d\omega}{dt} + \dot{W}_c + \dot{W}_{\text{Loss}} \]  

(2.29)

The turbine power defined in equation (2.29) is compared with that calculated using the proposed model in equation (2.14). Note that the compressor power, \( \dot{W}_c \), in equations (2.29) can be calculated based on change in flow enthalpy (see equation (2.18)) that can be measured directly, and four different mechanical loss (\( \dot{W}_{\text{Loss}} \)) models defined in Table 1 are used with the calibrations shown in Table 2, where the transient compensation coefficient \( c_4 \) in Model 4 is set to 0.001 to improve transient operation characteristics. In order to compare the performance of the proposed turbine model with the traditional map based one, the turbine model-based on map interpolation and extrapolation in [8] is used as the baseline in this study and the comparison results is shown in the top plot of Figure 16. Good agreements among turbine power calculated from the physics-based model (2.14) and the turbine powers calculated using equation (2.29) based on the experimental data with four friction models, while the turbine power from map-based model [8] deviates significantly from these calculated based on (2.29). This indicates that
the proposed turbine power and TC mechanical loss models are adequate. From the top plot of Figure 16, it clearly shows that the proposed physics-based turbine power model captures the TC transient operations better than the map-based model, and also note that the map-based model is reasonable when it operates within the map provided by flow bench test data and transient operation data shown in Figure 2. However, turbine transient operation is quite different from flow bench test range. Since the mapped data is not available at high TC speed with low pressure ratio and at low TC speed with light load, large extrapolation error over both regions leads fairly large model error (see the top plot of Figure 16), where the map-based turbine efficiency is significantly lower than the actual one. The study also shows that the proposed model is able to capture fast transient characteristics during tip-out and this is due to the fact that the proposed turbine and friction loss models depend on the TC speed.

From the fourth plot (from top) of Figure 16, the results also agree with previous discussion that the thrust friction increases as the pressure difference across turbine and compressor goes up. The modelling errors of the four friction models are similar when the turbocharger operates within the envelop of engine steady-state operations (see the second and third plots of Figure 16 for speed and pressure ratio); and the thrust friction increases significantly with high pressure difference across turbine and compressor during transient operations, which leads to about 10% mechanical efficiency. Also, the mechanical efficiency decreases similarly during the transient tip-outs, leading to increased compressor modelling error; see Table 5. The error for the physics-based model is defined in (2.30) and the error for the map-based model is the same as that in (2.30) by setting \( \dot{W}_{\text{Loss}}(i) = 0 \) since the mapped based model lumped the friction loss \( \dot{W}_{\text{Loss}} \) together with turbine power \( \dot{W}_T \). Note that large error of map-based model in Table 5 is due to the map extrapolation error. Although the physics-based model is able to capture the turbocharger
dynamics over the entire operating range, certain errors exists that are due to measurement errors (such as temperature sensor time constant effect for the transient measurement, flow measurement error under light compressor load) and unmolded physics (such as heat transfer, bearing friction loss due to oil viscosity variation).

\[
\text{Error} = \left( \sum_{i=1}^{n} |\dot{W}_T(i) - (\dot{W}_C(i) + \dot{W}_{\text{loss}}(i) + \dot{W}_{\text{kinetic}})| \right) \left( \sum_{i=1}^{n} |\dot{W}_C(i)| \right)^{-1} \quad (2.30)
\]

From Table 5 it can be observed that with the thrust load model (friction Models 2 and 4) the turbine power accuracy can be improved by 5% and 6% for Models 2 and 4 under the transient operations, respectively. Note that the steady-state operation study in Table 2 and Table 3 shows no significant error difference. This shows the importance of including the thrust friction in the turbine power model during transient operations. Even though the proposed turbine model is derived based on a power balanced (steady-state) TC, with the help of the friction model (especially Models 2 and 4) is can be also used under transient operations since the modelling error is reasonable (10%) under transient operations. As a conclusion, the proposed physics-based model with friction Model 4 reduce the model error under both steady-state and transient operations, comparing to the conventional map-based model.

<table>
<thead>
<tr>
<th>Table 5. Average error for different models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Map-based turbine model</td>
</tr>
<tr>
<td>Transient Error [%]</td>
</tr>
</tbody>
</table>
Figure 16. Model validation with transient vehicle test data
2.5 Conclusion

A physics-based turbine power model of a variable geometry turbocharger (VGT) is proposed in this chapter along with the thrust friction model. The turbine power model is derived based on the Euler turbine equation with the VGT vane position as the control parameter. Three existing friction loss models and one newly proposed model are also investigated. All four friction models have the potential of including the oil viscosity and heat transfer effect in the model. The proposed turbine power model, along with the friction models are investigated against two steady-state data sets (engine dynamometer test and flow bench test) and two transient data sets (1-D GT-Power transient simulations and vehicle transient test data). The steady-state data study shows that the proposed model is fairly accurate (with less than 4.5% modelling error) and the four friction models provides similar modelling accuracy; and for the transient data investigation, the proposed turbine power and acceleration based thrust friction models are able to reduce the transient modelling error from 22.8% (conventional map-based model) down to 10.1%. This indicates that thrust friction is a key to have an accurate transient model. Note that the proposed the turbine power and its mechanical efficiency models are suitable for the model-based VGT control due to the analytic nature of the proposed models as a function of the VGT vane angle.
CHAPTER 3: A REDUCED COMPLEXITY MODEL FOR THE COMPRESSOR POWER OF AN AUTOMOTIVE TURBOCHARGER

3.1 Abstract

Control-oriented models for automotive turbocharger compressors typically describe the compressor power assuming an isentropic thermodynamic process with fixed isentropic and mechanical efficiencies for power transmission between the turbine and compressor. Although these simplifications make the control-oriented model tractable, they also introduce additional errors due to un-modeled dynamics. This is especially true for map-based approaches since the manufacture-provided maps tend to be sparse and often incomplete at the operational boundaries, especially at operational conditions with low mass flow rate and low speed. Extrapolation scheme is often used when the compressor is operated outside the mapped regions, which introduces additional errors. Furthermore, the manufacture-provided compressor maps, based on steady-flow bench tests, could be quite different from these under pulsating engine flow. In this chapter, a physics-based model of compressor power is developed using Euler equations for turbo-machinery, where the mass flow rate and compressor rotational speed are used as model inputs. Two new coefficients, speed and power coefficients, are defined. As a result, this makes it possible to directly estimate the compressor power over the entire compressor operational range based on a single analytic relationship. The proposed modelling approach is validated against test data from standard turbocharger flow bench tests, standard supercharger tests, steady-state and transient engine dynamometer tests. Model validation results show that the proposed model has acceptable accuracy for model-based control design and also reduces the dimension of the parameter space typically needed to model compressor dynamics.
3.2 Introduction

It is common for plant models, used in the air-path control of turbocharged diesel engines, to assume that the ideal power consumed by the compressor is defined by an isentropic thermodynamic process. The actual power is then derived from either a compressor isentropic efficiency map [1] or an empirically fitted isentropic efficiency map [8]. A common alternative approach is to define the compressor power as a first order dynamics using an ad-hoc time constant with the turbine power as the input [5], [7]. On the other hand, map-based compressor power models are relied on the overall Turbo-Charger (TC) system efficiency maps as a function of the vane positions [32] applied to the calculated turbine power. Empirically fitted compressor efficiencies are typically 2nd or 3rd order polynomials of the Blade Speed Ratio (BSR). The polynomial coefficients are often dependent on the shaft speed [8]-[12] and are identified against the populated regions of the manufacturer-supplied maps. These polynomial models are, therefore, also subject to extrapolation when used under operational conditions outside the manufacturer-supplied test points. Note that the typical manufacturer-supplied compressor performance map is based on hot-gas flow-bench test data under steady flow conditions. Hence, actual maps could deviate from the manufacture-supplied ones under pulsating flow when the compressor is coupled to an Internal Combustion Engine (ICE) [1], [2], [33]. Another issue, often encountered when operating at light load conditions, is the sparsity of the manufacturer-supplied compressor performance maps in these operational areas, indicated in Figure 17 for a sample turbocharged engine. When operating the compressor outside the mapped region, extrapolation under some smoothness constraint becomes necessary. Several investigations into extrapolation based on map extension are reported in open literature; see [16], [18].
The traditional actual compressor power is computed from the isentropic efficiency in equation (3.1).

\[ W_c = \frac{1}{\eta_c} \dot{m}_c T_{in} c_{p \text{air}} \left( \frac{P_{out}}{P_{in}} \right)^{\frac{\gamma_{\text{air}} - 1}{\gamma_{\text{air}}} - 1} \]  

(3.1)

This compressor power model \( W_c \) relies on a number of measured inputs: the compressor inlet temperature \( T_{in} \), the mass flow rate \( \dot{m}_c \), and the inlet and outlet pressures \( P_{in} \) and \( P_{out} \), respectively. Additionally, the air properties, such as specific heat \( c_{p \text{air}} \) and isentropic index \( \gamma_{\text{air}} \), are assumed to be fixed. The isentropic efficiency \( \eta_c^{\text{is}} \) used in (1) is either an empirical model or is available from mapping data based flow bench tests. In a review of existing literature, there are various empirical models used to describe the isentropic efficiency. In [10], the efficiency is expressed as a third-order polynomial function of the non-dimensional mass flow rate, \( \phi_c \), that is a function of the mass flow rate and the blade tip speed. The polynomial coefficients \( a_i \) are three individual functions of inlet Mach number. In [37], the efficiency is modeled using an elliptical fit based on mass flow rate and pressure ratio. This model depends on
the maximum efficiency in terms of corrected mass flow rate and compressor pressure ratio. In [34], the efficiency model from [37] is further modified for pressure ratio variations. In [36], compressor work is defined as a polynomial function of corrected mass flow rate and the model is further fitted with experimental data. Table 6 summarizes these approaches and presents results from a previous performance assessment [39] of these models inside the manufacture-supplied maps. However, it has been well established that these models do not necessarily behave well under extrapolation, especially under light load conditions [11].

In this study, an alternative approach is investigated such that the model parameters are derived from compressor physics and therefore have clear physical interpretation. Further, a reduced-order model is proposed. Prediction capability of the proposed reduced-order model is investigated and is shown to be adequate for control design. Additionally this model also allows smooth extension to operational conditions beyond the typically mapped operational range. In the approach adopted in this study, the Euler equations for turbomachinery are used for developing a model for predicting compressor power. The proposed model uses the compressor mass flow rate and compressor angular velocity as inputs. Two new parameters, the speed and power coefficients, are defined. It is found that using a quadratic analytic function to model these two coefficients is adequate to characterize the compressor performance over the entire operational range. The model is validated using hot gas flow bench test data, steady-state engine dynamometer test data as well as transient simulation and test data. The validation results indicate that the proposed reduced-order model is suitable for both steady-state and transient operations under realistic pulsating exhaust flow conditions.

The rest of the chapter is organized as follows. Section II discusses the development of the proposed compressor model-based on the Euler equations with different slip factors. Section III
provides results from model identification based on flow bench tests. Section IV discusses results from model validation against transient data from GT Power simulations as well as from engine dynamometer tests. The last section adds some conclusions.

Table 6. Comparison among different efficiency modelling approaches (Assuming constant inlet condition)

<table>
<thead>
<tr>
<th>Reference</th>
<th>Compressor efficiency model</th>
<th>[Number of fitting coefficients]</th>
<th>[R²] [Error Mean Deviation (EMD)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jensen et al. [10]</td>
<td>[ \eta_c = \eta_{c,max} - \chi^T \Psi ] [ \chi^T = [\Psi - \Psi_{corr, max}] - \eta_c - \eta_{c,max} ]</td>
<td>( i = 1, 2, 3 )</td>
<td>( 9) [2] 0.971 [6.1]</td>
</tr>
<tr>
<td>Guzzella-Amstutz [37]</td>
<td>[ \eta_c = \eta_{c,max} - \chi^T \Psi ] [ \chi^T = [\Psi corr - \Psi corr, max] - \eta_c - \eta_{c,max} ]</td>
<td>( \frac{a_{12}}{a_{21}} )</td>
<td>( 4) [3] 0.927 [10.9]</td>
</tr>
<tr>
<td>Andersson [34]</td>
<td>[ \eta_c = \eta_{c,max} - \chi^T \Psi ] [ \chi^T = [\Psi corr - \Psi corr, max] - \eta_c - \eta_{c,max} ]</td>
<td>1 + \sqrt{\eta_c - 1 - \eta_{c,max}}</td>
<td>( 4) [3] 0.790 [22.2]</td>
</tr>
<tr>
<td>Canova et al. [36]</td>
<td>[ \eta_c = \eta_{c,max} - \chi^T \Psi ] [ \chi^T = [\Psi corr - \Psi corr, max] - \eta_c - \eta_{c,max} ]</td>
<td>( \frac{P_{ref}}{P_{in}} ) ( \frac{T_{ref}}{T_{in}} ) ( \omega_{corr} = \omega \sqrt{\frac{T_{ref}}{T_{in}}} )</td>
<td>( 4) [3] 0.947 [4.9]</td>
</tr>
<tr>
<td>Sieros et al.: Simple linear [38]</td>
<td>[ Y = a_1 + a_2 X ] [ Y = (\eta_c + \eta_0)^2 ] [ X = (\eta_c + \eta_0 + 1/\eta_{c})(\eta_c + \eta_0) ]</td>
<td>2 [3]</td>
<td>0.919 [11.6]</td>
</tr>
<tr>
<td>Sieros et al.: Generalized linear I [38]</td>
<td>[ \eta_c = A_1 + A_2 \pi_c + A_3 \lambda ] [ A_1 = a_1 + a_2 \omega_{max} + a_3 \omega_{corr} ] [ A_2 = a_4 + a_5 \omega_{max} ; A_3 = a_6 ]</td>
<td>( 6) [3]</td>
<td>0.978 [5.3]</td>
</tr>
<tr>
<td>Sieros et al.: Generalized linear 2 [38]</td>
<td>[ \eta_c = A_1 + A_2 \pi_c + A_3 \log(\pi_c) ] [ SSA_1 = a_1 A_2 = a_2 + a_3 \omega_{corr} A_3 = a_4 + a_5 \omega_{corr} ]</td>
<td>( 5) [2]</td>
<td>0.980 [4.8]</td>
</tr>
</tbody>
</table>
3.3 Compressor Power Modelling

3.3.1 Compressor power model-based on the Euler equations

The proposed model is derived, in part, using Euler equations of Turbomachinery. In dealing with the Euler equations, one must rely on the compressor geometry and velocity triangles associated with the gas flow at the impeller inlet and outlet. For completeness, figures of the compressor geometry and velocity triangle are reproduced from [1] and [21]. A typical centrifugal compressor geometry layout and the velocity triangles of the gas flow at the compressor impeller inlet and outlet are shown in Figure 18.

![Compressor geometry and velocity triangle](image)

**Figure 18.** Velocity triangles of a centrifugal compressor at the rotor inlet and outlet

The Euler equation provides the energy transfer to the fluid as a product of the angular velocity and torque shown in (3.2):

\[ \dot{W}_c = \omega \tau = \omega \dot{m}_c (v_{out}r_{out} - v_{in}r_{in}) = \dot{m}_c (U_2 C_\theta_2 - U_1 C_\theta_1) \]  \hspace{1cm} (3.2)

For a centrifugal impeller, it is assumed that the air enters the impeller eye in the axial direction so that the initial angular momentum of the air at the inlet of the compressor can be
assumed to be zero \((U_1 C_{\theta 1} \approx 0)\) [1], [21]. The ideal compressor power equation can then be reduced to:

\[
\dot{W}_{c,\text{ideal}} = \dot{m}_c U_2 C_{\theta 2}
\]  

or equivalently:

\[
\dot{W}_{c,\text{ideal}} = \dot{m}_c \omega R_2 C_{\theta 2}
\]

where, \(U_2 = \omega R_2\). Under nominal operation the flow exiting the blades will deviate from the ideal blade back-sweep angle \(\beta\), and exit at some angle \(\beta_2\). This deviation from the ideal is referred to as slip. Under the influence of slip, the corrected absolute flow velocity can be expressed as:

\[
C_{\theta 2}' = U_2 - C_{r2} \tan \beta_2'
\]

where \(C_{r2}\) is the impeller outlet flow radial velocity. The ratio of \(C_{\theta 2}\) to \(C_{\theta 2}'\) is defined as slip factor \(\sigma\):

\[
\sigma = \frac{C_{\theta 2}}{C_{\theta 2}'}
\]

The slip factor depends on a number of factors such as the number of impeller blades, the passage geometry, the ratio of impeller eye tip to impeller exit diameters, the mass flow rate and the compressor speed [21]. Introducing the slip factor in (3.4) and the expression for \(C_{\theta 2}\), the ideal compressor power is written as:

\[
\dot{W}_{c,\text{ideal}} = \dot{m}_c \omega R_2 C_{\theta 2}' \sigma = \dot{m}_c \omega R_2 (U_2 - C_{r2} \tan \beta_2') \sigma
\]

Using the conservation of mass flow rate yields:
Further assuming \( C_{r2} \approx C_{r1} \) (uniform radial flow with no radial flow losses) leads to \( C_{r2} = \frac{\dot{m}_c}{\rho_2 A_2} \approx C_{r1} = \frac{\dot{m}_c}{\rho_1 A_1} \). Substituting for \( C_{r2} = \frac{\dot{m}_c}{\rho_2 A_2} \) in (3.7), the compressor power can be re-formulated as:

\[
W_{c,ideal} = \sigma R_2^2 \dot{m}_c \omega^2 - \frac{\sigma R_2 \tan \beta'_2}{\rho_1 A_1} \omega \dot{m}_c^2 \tag{3.9}
\]

In order to account for flow and ‘windage’ losses, the actual required input work must be greater than the theoretical value necessary to achieve the target flow rate [45]. To account for this, a power loss factor \( \psi \) is used to modify the ideal power and a friction loss power term \( W_f \) is introduced into power model to define the actual, loss-compensated, power required to achieve a desired mass flow rate as:

\[
W_c = \psi W_{c, ideal} + W_f \tag{3.10}
\]

In this study, the factor \( \psi \) is assumed to be a design parameter and is therefore assumed fixed for a given compressor design. Ideally, this parameter must be established experimentally or identified through standard techniques (this work). In order to define the friction loss term, the loss models proposed in [21] and [46] are used here. The friction loss is modeled as a sum of the losses over the impeller and the diffuser and is defined as cubic functions of the mass flow rate:

\[
W_f = k_f \dot{m}_c^3 = (k_{fi} + k_{fd}) \dot{m}_c^3 \tag{3.11}
\]

where \( k_{fi} \) and \( k_{fd} \) are friction coefficients for the impeller and diffuser, respectively. Hence, the actual compressor power needed to achieve a desired mass flow rate can be written in its expanded form with all the loss modifiers as:
\[ W_c = \psi \sigma R_2^2 \dot{m}_c \omega^2 - \frac{\psi \sigma R_2 \tan \beta_2'}{\rho_1 A_1} \omega \dot{m}_c^2 + k_f \dot{m}_c^3 \]  \hspace{1cm} (3.12)

Two new parameters are defined: the power coefficient \( C_{\text{power}} \) and speed coefficient \( C_{\text{speed}} \), as follows:

\[ C_{\text{power}} = \frac{W_c}{\dot{m}_c^3} \]  \hspace{1cm} (3.13)

\[ C_{\text{speed}} = \frac{\omega}{\dot{m}_c} \]  \hspace{1cm} (3.14)

Using this notation, the compressor power in (3.12) can be rearranged in terms of the power and speed coefficients as follows:

\[ C_{\text{power}} = \psi \sigma R_2^2 \left( C_{\text{speed}} \right)^2 - \frac{\psi \sigma R_2 \tan \beta_2'}{\rho_1 A_1} \left( C_{\text{speed}} \right) + k_f \]  \hspace{1cm} (3.15)

It is clear from (3.15) that, \( C_{\text{power}} \) varies not only with compressor operating condition (such as \( C_{\text{speed}} \)) but also with compressor designs parameters. Different compressor designs could also impact the power law (3.15) through compressor specific geometry, power loss factor, and slip factor. A generalized power coefficient model can, however, be written for a specific compressor design. Since for a given compressor design, the parameters, \( \psi, k_f, R_2 \tan \beta_2', A_1 \) are constant. This allows the power coefficient to be expressed compactly as a function of the operating parameters, speed coefficient and the slip factor for a given operating condition \( i \) as:

\[ C_{\text{power}, i} = f \left( C_{\text{speed}, i}, \sigma_i \right), i = 1,2,\ldots,n \]  \hspace{1cm} (3.16)

One of the goals of this work was to derive a reduced order power model and investigate its prediction capability. The power and speed coefficients in conjunction with an appropriate slip factor model may be such a candidate model. It may be possible to further reduce the model
order by making the power coefficient a function of the speed coefficient only as indicated in (3.17).

\[ C_{\text{Power}} = f(C_{\text{Speed}}) \quad (3.17) \]

It is clear that in order to achieve the form shown in (3.17) the slip factor must be defined either as a parameter fixed by design or defined in terms of the speed and/or power coefficient to maintain the homogeneity of the compressor power model with respect to \( C_{\text{Power}} \) and \( C_{\text{Speed}} \). This leads us into an investigation of slip factor models.

### 3.3.2 Investigation of slip factor models for flows over centrifugal compressor

Several slip factors models for centrifugal compressors are readily available in the open literature [17, 22, 29-32, 34]. The slip factor typically depends on the compressor design parameters as well as the operating conditions, such as: compressor rotational speed and mass flow rate. In order to preserve the impact of flow variations on slip, the slip factor models, as proposed in [27], [41], and [44] are investigated in this work:

**Slip 1:**
\[ \sigma = 1 + \frac{m \tan \beta_1^f}{\rho_1 A_1 \omega R_2} - 0.5 \ast \left( 1 - e^{-\frac{2\pi Z \cos \beta_1^f}{\omega R_2^2}} \right) (0 < \sigma < 1) \quad (\text{Reffstrup}) \quad (3.18) \]

**Slip 2:**
\[ \sigma = 1 - \frac{\frac{\pi Z}{2} \omega R_2 \cos \beta_2^f}{\omega R_2^2 - \frac{m \rho_1 A_1 \tan \beta_2^f}{\omega R_2^2}} \quad (0 < \sigma < 1) \quad (\text{Stodola}) \quad (3.19) \]

**Slip 3:**
\[ \sigma = 1 - a \frac{m}{\omega R_2^2} \quad (0 < \sigma < 1) \quad (\text{Stahler}) \quad (3.20) \]

where \( a \) is a design parameter in (3.20) related to the flow exit angle and is a constant for a given impeller design, and \( Z \) is the number of impeller blades. Integrating each of these models into the compressor power model (3.15), a generalized compressor power model can be established with the following structure:
\[
\frac{W_c}{m_c^3} = \varepsilon_1 \left( \frac{\omega}{m_c} \right)^2 - \varepsilon_2 \left( \frac{\omega}{m_c} \right) + \varepsilon_3 \Rightarrow C_{power} = \varepsilon_1 (C_{speed})^2 - \varepsilon_2 (C_{speed}) + \varepsilon_3
\]  

(3.21)

where, the coefficients \( \varepsilon_1, \varepsilon_2, \) and \( \varepsilon_3 \) depend on the selected slip factor model and are defined in Table 7. The derivations as below:

Table 7. Model coefficients for various slip models

<table>
<thead>
<tr>
<th>( \varepsilon_1 )</th>
<th>Slip model 1</th>
<th>Slip model 2</th>
<th>Slip model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi R_1^2 \left( 0.5 + 0.5 \frac{2\pi}{Z} \cos \beta_2' \right) )</td>
<td>( \psi R_2^2 \left( 1 - (\pi / Z) \cos \beta_2' \right) )</td>
<td>( \psi R_2^2 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{\psi R_2 \tan \beta_2' \rho_1 A_1}{0.5 \frac{2\pi}{Z} \cos \beta_2' - 0.5} )</td>
<td>( \frac{\psi R_2 \tan \beta_2' \rho_1 A_1}{0.5 \frac{2\pi}{Z} \cos \beta_2' - 0.5} )</td>
<td>( \frac{\psi R_2 \tan \beta_2' \rho_1 A_1}{0.5 \frac{2\pi}{Z} \cos \beta_2' - 0.5} )</td>
<td></td>
</tr>
<tr>
<td>( k_f - \psi \left( \frac{\tan \beta_2'}{\rho_1 A_1} \right)^2 )</td>
<td>( k_f )</td>
<td>( \psi \frac{\tan \beta_2' \rho_1 A_1}{0.5 \frac{2\pi}{Z} \cos \beta_2' - 0.5} + k_f )</td>
<td></td>
</tr>
</tbody>
</table>

Introduce slip factor model 1 into compressor power equation:

\[
\dot{W}_c = \psi \left\{ 1 + \frac{m \tan \beta_2' \rho_1 A_1}{\omega R_2} \right\} \left( 1 - \frac{2\pi}{Z} \cos \beta_2' \right) \left( \frac{\omega}{m} \right)^2 - \psi \left\{ 1 + \frac{m \tan \beta_2' \rho_1 A_1}{\omega R_2} \right\} \left( 1 - \frac{2\pi}{Z} \cos \beta_2' \right) \left( \frac{\omega}{m} \right)^2 + k_f
\]

\[
= \psi \left\{ 1 + \frac{m \tan \beta_2' \rho_1 A_1}{\omega R_2} \right\} \left( \frac{\omega}{m} \right)^2 - \psi \frac{R_2 \tan \beta_2' \rho_1 A_1}{\omega R_2} \left( \frac{\omega}{m} \right)^2 + k_f
\]

Introduce slip factor model 2 into compressor power equation:
\[ W_c = \dot{m} \omega R (U_2 - C_{r2} \tan \beta_2') \sigma \]
\[ = \dot{m} \omega R \left( \rho R - \frac{\dot{m}}{\rho_1 A_1} \tan \beta_2' \right) \left( 1 - \frac{(\pi / Z) \omega R \cos \beta_2'}{\rho R - \frac{\dot{m}}{\rho_1 A_1} \tan \beta_2'} \right) \]
\[ = \dot{m} \omega R \left( \rho R - \frac{\dot{m}}{\rho_1 A_1} \tan \beta_2' \right) \left( -\dot{m} \omega R (\pi / Z) \cos \beta_2' \right) \]
\[ = \frac{\dot{m} (\omega R)^2}{\rho_1 A_1} \frac{\dot{m}^2 \omega R \tan \beta_2' - (\pi / Z) \cos \beta_2' \dot{m} (\omega R)^2 + k \dot{m}^3}{R_2} \]
\[ = (1 - (\pi / Z) \cos \beta_2') R_2 \dot{m} (\omega R)^2 - \frac{R_2 \tan \beta_2'}{\rho_1 A_1} \dot{m}^3 \omega + k \dot{m}^3 \]

Introduce slip factor model 3 into compressor power equation:

\[ \dot{W}_c = \dot{m} \omega R (U_2 - C_{r2} \tan \beta_2') \sigma + k_f \]
\[ = \dot{m} \omega R \left( \rho R - \frac{\dot{m}}{\rho_1 A_1} \tan \beta_2' \right) \left( 1 - a \frac{\dot{m}}{\rho R} \right) + k_f \]
\[ = \dot{m} \omega R \left( \rho R - \frac{\dot{m}}{\rho_1 A_1} \tan \beta_2' \right) \left( -\dot{m} \omega R (\pi / Z) \cos \beta_2' \right) + k_f \]
\[ = \left( \frac{\dot{m}^2 R^2}{\rho_1 A_1} - a \frac{\dot{m}^2 R}{\rho_1 A_1} \tan \beta_2' \omega + \frac{\dot{m}^2 a}{\rho_1 A_1} \frac{1}{R} \right) + k_f \]
\[ = \frac{\dot{m} (\omega R)^2}{\rho_1 A_1} \frac{\dot{m}^2 \omega R \tan \beta_2' - (\pi / Z) \cos \beta_2' \dot{m} (\omega R)^2 + k \dot{m}^3}{R_2} \]
\[ = (1 - (\pi / Z) \cos \beta_2') R_2 \dot{m} (\omega R)^2 - \frac{R_2 \tan \beta_2'}{\rho_1 A_1} \dot{m}^3 \omega + k \dot{m}^3 \]

The model structure in (3.21) is referred to as the generalized compressor power model in the rest of this chapter. Note that the three coefficients in (3.21) take constant values and are fixed by compressor design under the assumption of constant or slowly varying inlet conditions. Once the relationship between the power and speed coefficients has been identified, the compressor power for a given operating point defined the pair \( \omega , \dot{m}_c \), can be established as in (3.22).

\[ W_c = \dot{m}_c^2 \left( \varepsilon_1 \left( \frac{\omega}{\dot{m}_c} \right)^2 - \varepsilon_2 \left( \frac{\omega}{\dot{m}_c} \right) + \varepsilon_3 \right) \]  

(3.22)

This approach shows that the compressor operation can be represented via an analytic quadratic function rather. The proposed model has only two inputs (the compressor mass flow rate and TC rotational speed) and three parameters to be identified. In practical applications,
while the mass flow rate is typically available as a measurement, the TC speed is not a standard measurement. In the absence of a TC speed measurement, the observed value of the TC speed via an appropriate observer design may be used. Alternately TC speed can also be obtained by solving the turbocharger rotor dynamic differential equations as in [3]-[7], and the compressor mass flow rate can be modeled with shaft speed and pressure ratio as inputs [10]. In this study, the compressor power model is validated under the former assumption that two parameters, mass flow rate and TC rotational speed, are available as measurements.

3.3.3 Compatibility with corrected mass flow rates

Corrected compressor mass flow rates $m_{\text{correct}} = m \sqrt{T_1 / P_1}$ and corrected TC angular velocity $\omega_{\text{correct}} = \omega / \sqrt{T_1}$ are typically used in compressor performance maps. So it was natural to investigate the structure of the power coefficient based compressor power model when using these corrected terms. It is found that the power coefficient with the corrected variables has the same structure as before and is scaled by the modifying term $\sqrt{T_1 / P_1}$ and $1 / \sqrt{T_1}$ as shown below. To show this, the corrected power coefficient is defined as: $C_{\text{power,correct}} = \frac{w_c}{(m_{\text{correct}})^3}$; and the corrected speed coefficient can be defined as $C_{\text{speed,correct}} = \frac{\omega_{\text{correct}}}{m_{\text{correct}}}$. Introducing these terms directly into the power coefficient model as in (3.23), the proposed model with corrected power coefficient is shown in (3.24):

$$\frac{w_c}{m^3} = \varepsilon_1 \left( \frac{\omega}{m} \right)^2 - \varepsilon_2 \left( \frac{\omega}{m} \right) + \varepsilon_3$$

\[
\begin{align*}
W_c &= \left( \frac{1}{\sqrt{T_1 / P_1}} \right)^3 \left( \frac{m\sqrt{T_1}}{P_1} \right)^3 \left( \frac{T_1}{P_1} \right)^2 \left( \frac{\omega}{\sqrt{T_1 / P_1}} \right)^2 - \varepsilon_2 \left( \frac{T_1}{P_1} \right)^2 \left( \frac{\omega_{\text{correct}}}{m_{\text{correct}}} \right) + \varepsilon_3 
\end{align*}
\]

(3.23)
\[
W_c = \left( \frac{1}{\sqrt{T_1 P_1}} \right)^3 (m_{\text{correct}})^3 \left( \varepsilon_1 \frac{T_1}{P_1} \left( \frac{\omega_{\text{correct}}}{m_{\text{correct}}} \right)^2 - \varepsilon_2 \left( \frac{T_1}{P_1} \right)^2 \left( \frac{\omega_{\text{correct}}}{m_{\text{correct}}} \right) + \varepsilon_3 \right)
\]

\[
\frac{W_c}{(m_{\text{correct}})^3} = \left( \frac{1}{\sqrt{T_1 P_1}} \right)^3 \left( \varepsilon_1 \frac{T_1}{P_1} \left( \frac{\omega_{\text{correct}}}{m_{\text{correct}}} \right)^2 - \varepsilon_2 \left( \frac{T_1}{P_1} \right)^2 \left( \frac{\omega_{\text{correct}}}{m_{\text{correct}}} \right) + \varepsilon_3 \right)
\]

\[
C_{\text{power, correct}} = \left( \frac{1}{\sqrt{T_1 P_1}} \right)^3 \left( \varepsilon_1 \frac{T_1}{P_1} \left( C_{\text{speed, correct}} \right)^2 - \varepsilon_2 \left( \frac{T_1}{P_1} \right)^2 \left( C_{\text{speed, correct}} \right) + \varepsilon_3 \right)
\]

Eq. (3.24) means that the model needs to be calibrated by reference temperature and pressure when compressor operates at different inlet conditions. In the rest of this chapter, all derivations and results are based on the model (3.22) without the corrected parameters, since all the measurements are under the atmospheric pressure.

3.4 Model Identification

3.4.1 Model identification using standard hot gas flow bench test data

Test data from standard hot gas flow bench tests was used to identify the three model parameters, \( \varepsilon_1 \), \( \varepsilon_2 \), and \( \varepsilon_3 \) for the generalized compressor model as in (3.21) for three different compressor designs, compressor-1, 2, 3. The inlet conditions of compressors are assumed fixed in line with standard test protocol [26]. The minimum test speed for Compressor-1,2 was 30k RPM, while the lowest speed for Compressor-3 was 46k RPM. Model parameters for the power coefficient model as in (3.21) are identified using a Least Squares optimization based on a model error cost function as in (3.25):

\[
J = \sum_{i=1}^{n} \left[ W_c^{\text{model}} (i) - W_c^{\text{mes}} (i) \right]^2
\]
In (3.25), \( n = 322 \), is the number of steady state compressor operating test points for the hot gas flow bench tests. \( W_{c\text{\_model}} \) is the calculated compressor power using (3.22); and \( W_{c\text{\_mes}} \) is the standard compressor power calculated from measured inputs as (3.26):

\[
W_{c\text{\_mes}} = \dot{m}_{c\text{\_mes}} c_p (T_{out \text{\_mes}} - T_{in \text{\_mes}})
\]  

(3.26)

Since the geometric design parameters \( R_2 \), \( A_1 \) and \( \beta_2' \) for Compressor-1 were available, Compressor-1 was selected as a candidate for identifying the parameters, \( \psi \), \( \beta_2' \) and \( a \), for all three slip models using the model structure (3.15). Note that the parameter “\( a \)” is relevant only for the Slip Model-3. The values of the identified parameters are shown in Table 8.

It is interesting to note that identified value of the power loss factor \( \psi \), is greater than unity, indicating that the compressor power necessary to achieve the desired flow rate must be larger than the ideal power calculated in (3.4) in order to account for losses. The order of magnitude of the identified parameter \( a \) (Slip Model-3) agrees with the result in [41]. Given the value for the friction coefficient \( k_f \), the maximum friction loss power identified was around 2.38kW over the entire compressor operating range. Modelling errors are investigated using four error metrics: the coefficient of determination \( R_2 \), error mean deviation (EMD), and \( PE_{B\pm5\%} \), and \( PE_{B\pm10\%} \) that are the percentage of total data points within the error bound \( \pm5\% \) and \( \pm10\% \), respectively. The error evaluation parameters used in these calculations are defined in (3.27), (3.28) and (3.29):

\[
\text{Error}(i) = \frac{W_{c\text{\_model}}(i) - W_{c\text{\_mes}}(i)}{W_{c\text{\_mes}}(i)}
\]  

(3.27)

\[
EMD = \frac{\text{Error} - \text{mean}(\text{Error})}{n}
\]  

(3.28)
\[ PEB_{\pm \%} = \frac{m}{n}, m = \sum_{i=1}^{n} N_k(i), \text{ where } |Error(N_k)| \leq k\% \]  

(3.29)

where, \( n \) is the total sample number. Based on model fitting error, as reported in Table 8, the Slip Model-3 has the best fit (least error) for Compressor-1. Since compressor design parameters were not available for Compressors-2, 3, we could only identify the 3 lumped parameters \( \varepsilon_1, \varepsilon_2, \) and \( \varepsilon_3 \) for the generalized power model (3.21) for these two compressor designs. The identified values for the coefficients of the generalized power coefficient are shown for all three compressor designs in Table 8.

**Table 8. Identified model coefficients for 3 model variants and 3 compressor design variants**

<table>
<thead>
<tr>
<th>Model (21) Compressor-1</th>
<th>( \varphi )</th>
<th>( k_f )</th>
<th>( a )</th>
<th>( R^2 )</th>
<th>EMD*</th>
<th>PEB** (±5%)</th>
<th>PEB (±10%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slip model 1</td>
<td>1.295</td>
<td>19040</td>
<td>NA</td>
<td>0.968</td>
<td>4.42</td>
<td>58.9%</td>
<td>83.52%</td>
</tr>
<tr>
<td>Slip model 2</td>
<td>1.323</td>
<td>6374</td>
<td>NA</td>
<td>0.968</td>
<td>4.56</td>
<td>66.44%</td>
<td>84.00%</td>
</tr>
<tr>
<td>Slip model 3</td>
<td>1.351</td>
<td>11390</td>
<td>1.334</td>
<td>0.989</td>
<td>3.24</td>
<td>79.15%</td>
<td>92.90%</td>
</tr>
<tr>
<td>Model (22)</td>
<td>( \varepsilon_1 )</td>
<td>( \varepsilon_2 )</td>
<td>( \varepsilon_3 )</td>
<td>( R^2 )</td>
<td>EMD</td>
<td>PEB (±5%)</td>
<td>PEB (±10%)</td>
</tr>
<tr>
<td>Compressor 1</td>
<td>3.52e-8</td>
<td>0.0123</td>
<td>1752</td>
<td>0.992</td>
<td>3.24</td>
<td>79.15%</td>
<td>92.90%</td>
</tr>
<tr>
<td>Compressor 2</td>
<td>2.98e-8</td>
<td>0.0162</td>
<td>3547</td>
<td>0.995</td>
<td>3.73</td>
<td>68.31%</td>
<td>90.22%</td>
</tr>
<tr>
<td>Compressor 3</td>
<td>1.68e-8</td>
<td>0.0086</td>
<td>1987</td>
<td>0.996</td>
<td>2.61</td>
<td>81.34%</td>
<td>91.69%</td>
</tr>
<tr>
<td>Model (30)</td>
<td>( p )</td>
<td>( q )</td>
<td>NA</td>
<td>( R^2 )</td>
<td>EMD</td>
<td>PEB (±5%)</td>
<td>PEB (±10%)</td>
</tr>
<tr>
<td>Compressor 1</td>
<td>2.439</td>
<td>-10.25</td>
<td>NA</td>
<td>0.994</td>
<td>5.97</td>
<td>70.07%</td>
<td>86.11%</td>
</tr>
<tr>
<td>Compressor 2</td>
<td>2.423</td>
<td>-10.31</td>
<td>NA</td>
<td>0.989</td>
<td>3.78</td>
<td>58.01%</td>
<td>85.83%</td>
</tr>
<tr>
<td>Compressor 3</td>
<td>2.262</td>
<td>-9.551</td>
<td>NA</td>
<td>0.996</td>
<td>2.37</td>
<td>86.63%</td>
<td>94.61%</td>
</tr>
</tbody>
</table>

*EMD: Error Mean Deviation  **PEB: Percentage of data points within Error Bound*
Figure 19. Identification Results for the generalized Compressor-Power model for Compressors-1,2,3.

Results from the fitted models for the three different compressor designs are shown in Figure 19 (b) the log-log plot is shown in Figure 19 (c) that each compressor design has a unique characteristic curve. This is expected and reflects the design differences among the different compressors. In fact, such plots allow a quick and easy comparison of different compressor designs. As an example, it is clear that for a given mass flow rate and TC shaft speed the compressor power follows the trend, Power1 > Power2 > Power3, indicating that Design-1 perhaps have a larger loss relative to the other designs. Also note that it is clear that Compressor-
3 may offer the widest operating range of the three designs considered in this study. The Log10-scale plots offer new insights. It is easy to see that the Log10-scale representation is a linear transformation of quadratic curves; that is, a power law relationship exists between the power and speed coefficients. The Log model is shown in (3.30).

\[
\log_{10}(C_{\text{Power}}) = p \log_{10}(C_{\text{Speed}}) + q
\]  

(3.30)

In (30), \( p \) is the slope of each characteristic line and \( q \) is the y-intercept for each design. From (30), we get the power law model, derived from a log10 plot as:

\[
C_{\text{Power}} = 10^q (C_{\text{Speed}})^p = \frac{W_c}{m_c^2}
\]  

(3.31)

The \( p \) and \( q \) values for the power-law plots were also identified and are noted in TABLE 3. The power law representation implies a scaling invariance and therefore provides a good measure of the sensitivity of power coefficient to changes in the speed coefficient. One important advantage of this is that (3.31) can, theoretically, be obtained from only two compressor operating conditions that span the high and low load operational conditions. This is in line with the two-point fitting method used for determining the power law exponent. Hence this method may significantly reduce the experimental burden during the early stages of compressor development.

In summary, model fitting results confirm that the generalized compressor power model proposed in this work is able to reproduce the compressor behavior. Proposed models only need two or three parameters to be identified. These results also demonstrate that for a centrifugal compressor the operating characteristics can be reduced to an analytic quadratic function in linear scale and a straight line in logarithmic scale. A summary of the relative errors of
prediction, for the various compressor designs and the model variants, is shown in Figure 20. It is clear that prediction error is well contained within a ± 5% relative error band.

Figure 20. Modelling error (27) for compressor power models

3.4.2 Model identification with ‘Supercharger Standard Test’

Supercharging Standard Tests (SST) are performed with an electric motor driving the compressor as opposed to the turbine. Given the wider speed control of electric motor, it is possible to perform more extremely light load (low speed, low mass flow rates) tests relative to standard hot-gas flow bench tests. Three different compressor designs, Compressor-4, 5, 6 were tested under this test protocol. The lowest compressor angular velocity achieved was 20k RPM compared to 30k RPM in the flow bench tests. The minimum mass flow rate was 0.01 kg/s in the compressor dynamometer, compared to 0.02 kg/s in the flow bench test.
Following the same procedure as for the previous set of compressors, we identified the three lumped parameters $\varepsilon_i$ for the generalized power coefficient model as in (3.22). The identified values are indicated in Table 9. The behavior of the fitted model is compared against SST data in Figure 21. Results indicate, as before, that the proposed model is able to reproduce the observed data. The Log scale plots also reproduce a similar power law behavior observed for the previous set of compressors. The parameters for the power law model are included in Table 9.

![Image](image_url)

Figure 21. Identification Results for the generalized Compressor-Power model for Compressors-4,5,6.
Table 9. Fitted model coefficients identified through supercharging test data

<table>
<thead>
<tr>
<th>Model (22)</th>
<th>$e_1$</th>
<th>$e_2$</th>
<th>$e_3$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor 4</td>
<td>1.928 e-06</td>
<td>0.0618</td>
<td>858.1</td>
<td>0.995</td>
</tr>
<tr>
<td>Compressor 5</td>
<td>1.965 e-06</td>
<td>0.0618</td>
<td>858.1</td>
<td>0.996</td>
</tr>
<tr>
<td>Compressor 6</td>
<td>1.967 e-06</td>
<td>0.0564</td>
<td>718.3</td>
<td>0.995</td>
</tr>
<tr>
<td>Model (29)</td>
<td>p</td>
<td>q</td>
<td>NA</td>
<td>$R^2$</td>
</tr>
<tr>
<td>Compressor 4</td>
<td>2.379</td>
<td>-10.09</td>
<td>NA</td>
<td>0.995</td>
</tr>
<tr>
<td>Compressor 5</td>
<td>2.378</td>
<td>-10.07</td>
<td>NA</td>
<td>0.989</td>
</tr>
<tr>
<td>Compressor 6</td>
<td>2.378</td>
<td>-10.06</td>
<td>NA</td>
<td>0.996</td>
</tr>
</tbody>
</table>

3.5 Model Validation

3.5.1 Model validation based on steady state engine dynamometer test data

The previously identified model for, $c_{power} = f(c_{speed})$, was validated against a, more realistic, data set from engine dynamometer steady-state tests. These tests were conducted on a heavy duty diesel engine. The turbocharger on this engine used a compressor equivalent to the design variant Compressor-1. Test details can be found from previously published work [24] and [25]. The steady-state test covers the entire engine operating range (185 testing points). The compressor operating range is defined by a mass flow rate range between 0.01 and 0.45 kg/s and compressor speed range between 5.9k and 109k RPM. Note that these tests achieve lighter operational conditions relative to the flow bench tests used for model identification for which the TC speed was limited, at the low end to, 30k rpm for the hot-gas tests, and 20k rpm for the SST tests. The engine operational range drives compressor operation beyond the flow-bench data and provides an opportunity to test the range extension properties of the proposed model. In Figure 22 (a), we show the engine test grid overlaid on the compressor mapping points (from hot-gas flow-bench tests). It is clear that the engine operation at light load conditions forces the compressor to operate in regions not covered by the flow-bench data.
The $c_{\text{power}}$ model, previously identified for Compressor-1, was used to reproduce the power–coefficient vs. speed-coefficient relationship for the engine test data. In Figure 22 (a), the predicted $c_{\text{power}}$ is compared against the calculated values for $c_{\text{power}}$ for both the flow-bench tests as well as the engine steady–state tests. Recall that the calculated values for $c_{\text{power}}$, are based on (26) and use measured inputs. Since the engine data set extends to lower load conditions (TC speed < 30k RPM), the characteristic curves for the engine data are plotted in two sets to span
operational regimes above and below the 30k RPM TC speed threshold. This was done primarily to assess the quality of the model prediction for light conditions that extend beyond the standard mapping domain. It is clear from Figure 22 (b) and (c), that, while the model adequately replicates the calculated values for $c_{power}$ for TC speeds $> 30K$ RPM, there is a significant error under operational conditions of TC speeds $< 30kRPM$, where the calculated values for $c_{power}$ lose monotonicity and appear to be quite random. This obviously anomalous behavior may have sources other than flow irregularities from extremely low flows. The actual compressor power (below 30K rpm) is investigated as show in

![Figure 22](image)

**Figure 22.** Model predicted power vs Measured power under 30K rpm TC speed

The compressor power error between model predicted value and measured value are between 0.1kW and 0.23 kW with an averaged value of 0.16 kW as shown in Figure 23. Measured
compressor power is higher than model predicted compressor power. In order to identify the error sources for both increased power coefficient and increased compressor power, the sensitivity analysis is conducted for both power coefficient $C_{power}$ and compressor power $\dot{W}_{c}$ as shown in Table 10. Sensitivity gain ranges are based on the value range from measurements.

Table 10. Sensitivity analysis for compressor power and power coefficient with TC speed below 30k RPM

<table>
<thead>
<tr>
<th>Sensitivity gain</th>
<th>$\Delta W_c$</th>
<th>$\Delta C_{power}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement (26)</td>
<td>$\Delta W_c = \frac{\partial W}{\partial \Delta T} \Delta T + \frac{\partial W}{\partial \Delta p} \Delta p + \frac{\partial W}{\partial \Delta m} \Delta m$</td>
<td>$\Delta C_{power} = \frac{\partial C_{power}}{\partial \Delta T} \Delta T + \frac{\partial C_{power}}{\partial \Delta p} \Delta p + \frac{\partial C_{power}}{\partial \Delta m} \Delta m$</td>
</tr>
<tr>
<td>Model (31)</td>
<td>$\Delta W_c = \frac{\partial W}{\partial \Delta T} \Delta T + \frac{\partial W}{\partial \Delta p} \Delta p + \frac{\partial W}{\partial \Delta m} \Delta m$</td>
<td>$\Delta C_{power} = \frac{\partial C_{power}}{\partial \Delta T} \Delta T + \frac{\partial C_{power}}{\partial \Delta p} \Delta p + \frac{\partial C_{power}}{\partial \Delta m} \Delta m$</td>
</tr>
<tr>
<td>Measurement (26)</td>
<td>$\Delta W_c = \frac{\partial W}{\partial \Delta T} \Delta T + \frac{\partial W}{\partial \Delta p} \Delta p + \frac{\partial W}{\partial \Delta m} \Delta m$</td>
<td>$\Delta C_{power} = \frac{\partial C_{power}}{\partial \Delta T} \Delta T + \frac{\partial C_{power}}{\partial \Delta p} \Delta p + \frac{\partial C_{power}}{\partial \Delta m} \Delta m$</td>
</tr>
</tbody>
</table>

$\Delta \dot{n}_c$ (kg/s) $C_{p}(T_{out} - T_{in}) \in [10.1, 20.2]$ $-10^3 \left( \frac{\partial \dot{m}}{\partial T} \right) \Delta T \in [-6.96e5, -1.05e5]$ $- \frac{C_{p}(T_{out} - T_{in})}{\dot{m}_c^3} \in [-4.81e6, -6.47e4]$ $\Delta \omega$ (rpm) NA $10^3 \left( \frac{\partial \omega}{\partial \dot{m}} \right) \Delta \dot{m} \in [0.18, 1.51]$ $\Delta \dot{T}_{out}$ (k) $C_p \dot{m}_c \in [0.012, 0.069]$ NA $\frac{C_p}{\dot{m}_c} \in [14.51, 78.90]$ $\Delta \dot{T}_{in}$ (k) $-C_p \dot{m}_c \in [-0.069, -0.012]$ NA $\frac{C_p}{\dot{m}_c} \in [-78.90, -14.51]$

The error in the measured power-coefficient is amplified by artificially low mass flow rates reported by the production mass flow sensor. Mass flow rate sensors are known to suffer from nonlinearity and loss of accuracy at low flow rates as observed for these tests. This is confirmed from the model behavior for the SST tests that used laboratory grade sensors and for which the $C_{power}$ model was able to reproduce the measured values quite adequately. An additional source of error may be attributed to a positive bias in the temperature downstream of the compressor due to heat transfer from the hot-end (turbine). This effect of heat transfer on the calculated compressor under light load operating conditions was also verified experimentally by the authors in [47][48]. However, this effect may not completely describe the error in the $C_{power}$ model observed here since the successive low load test conditions would result in a progressively cooler turbine.
housing leading to reduced heat transfer effects. In summary, since the error in the predicted power is less than 0.23kW in light load range and the model showed good predictive capability for the SST experiments for light load operation it is safe to project that the proposed model is adequate for light load extrapolation.

3.5.2 Model validation for US06 transient cycle based on GT-Power cycle simulations

Next, the model behavior is verified against transient data from GT-Power simulations for a US06 cycle. The TC speed varied between 14k and 105k RPM during the test. The model selected for evaluation was based on Compressor-1 design and Slip Model-3. Model prediction for $c_{power}$ is compared against calculated values as before. Figure 24 shows that the model prediction for $c_{power}$ agrees quite well with the expected values as calculated values over the test cycle.

![Graph](image)

Figure 24. Model validation over US-06 GT-Power transient simulation
3.5.3 Model validation over FTP cycle based on engine dynamometer tests.

Model prediction capability over an FTP test cycle with engine dynamometer data was investigated. The model candidate used is based on Compressor-1 design and slip Model-3. Since the calculated $c_{\text{power}}$ (for comparison) relies on the compressor downstream temperature, we assessed the signal behavior of the relevant sensor. This is because the location of the post compressor temperature sensor, in the test engine configuration, was expected to introduce measurement error from slow dynamics and delay as well as heat transfer losses. In Figure 25, it is clear that during this load step the mass flow rate and turbocharger speed has a much faster response compared to the temperature sensor. In order to address this, we corrected the measured temperature signal using a lead-lag filter. The lead-lag filter was designed to match the expected temperature profile for the same load step by inverting the compressor power model (3.26) shown in (3.32):

$$m c_p (T_{\text{out}} - T_{\text{in}}) = \dot{m}^3 f \left( \frac{\omega}{\dot{m}} \right) \Rightarrow \tilde{T}_{\text{out}} = \frac{\dot{m}^3 f \left( \frac{\omega}{\dot{m}} \right)}{m c_p} + T_{\text{in}}$$  \hspace{1cm} (3.32)

Figure 25. Normalized measurements for mass flow rate, TC speed and Compressor downstream temperature for a load step.
The corrected temperature, downstream of the compressor, is compared against the measured signal in Figure 26. The same filter was then applied to the measured temperature signal for the full cycle and the corrected temperature was used to evaluate the measured value of the compressor power using (3.26). The corrected measured temperature signals are shown in Figure 26. The predicted compressor power was established from the predicted $C_{\text{power}}$ and known mass flow rate. The predicted and calculated values are compared in Figure 26. The filter is seen to converge at 160secs, after which, it is clear that the predicted compressor power is capable of reproducing the actual compressor reasonably well. With this confidence, it is reasonable to claim that using the proposed method the compressor power can be predicted reasonably well based on measurements of compressor mass flow rate and TC speed.

Figure 26. Model validation against transient engine test data for a FTP 75 cycle
3.6 Conclusion

A compressor power model, based on the Euler turbomachinery equations with realistic assumptions, was developed. Two new performance coefficients, the power and speed coefficients were proposed as an alternative to multiple performance maps. The proposed correlation between $C_{\text{power}}$ and $C_{\text{speed}}$ is especially useful in defining the compressor power necessary for achieving a desired compressor mass flow rate. This compressor power demand can then be translated into a VGT (Variable Geometry Turbo) vane position or an assist demand for assisted boosting systems. This relationship can also be easily used to compare compressor design variants with respect to performance and range. The model is validated against data sets from standard turbocharger flow bench tests, steady-state engine dynamometer tests as well as transient engine simulations and test. Validation results indicate that the proposed model provides accurate compressor power prediction over a broad range of compressor operating conditions and provides for an easy and reliable extrapolation for operating conditions outside the standard mapping domain. Further, the proposed model reduces the dimensionality of the parameter space typically necessary for such applications. The reduced order, reduced complexity model is especially useful for the control applications. Future work will focus on improving prediction accuracy in the face of measurement noise as previously discussed. Model-based control design based upon proposed model will be investigated.
CHAPTER 4: MODELLING OF HYDRAULIC ASSISTED AND REGENERATIVE TURBOCHARGED D ENGINE

4.1 Abstract

In this chapter, a systematic modelling approach for engine air-path system and hydraulic assisted and regenerative turbocharger system are presented. New developed turbocharger sub-models are integrated with engine air-path and EGR system. Further new modelling approaches for high speed hydraulic turbine and hydraulic centrifugal turbo-pump are proposed. System level model integration and plant behavior investigation are carried out for engine air-path system and hydraulic assisted turbocharger. The results show proposed reduced order engine modelling has high fidelity. It could be used for model-based analysis and model-based controller design.

4.2 Regenerative Hydraulic Assisted Turbocharger With VGT-EGR Overview

Regenerative and assisted turbocharger system are used to assist turbocharger during engine acceleration and to recover exhaust energy during engine during engine deceleration. A most common studied actuation system for assisted and regenerative turbocharger are electric based and hydraulic based. Different actuation systems are reported in [53][54][63][65]. In this chapter, we introduce the modelling hydraulic assisted and regenerative turbocharger system. More details of system introduction can be found in [60]. A high speed hydraulic turbine and high speed hydraulic turbo-pump are added inside turbocharger center housing as shown in Figure 27.
Turbocharger turbine, compressor, hydraulic turbine and hydraulic turbo-pump share the same shaft. With adding two hydraulic actuators on TC shaft, this provides extra control inputs for the turbocharged air-path system. For traditional EGR-VGT system (Only HP EGR system), VGT vane control and EGR valve control are coupled for its sharing the same source to drive the exhaust flow and EGR flow. VGT vane position is normally used to track the target boost pressure when the engine needs higher boost command. For a transient tip-in, VGT is used to close further to build up exhaust pressure. To have demanded boost pressure, EGR valve is intended to close further to help to build up exhaust pressure, such that EGR flow capacity is reduced. To have the higher EGR mass flow rate, EGR valve open action is intended to drop exhaust pressure, which will lead to lower exhaust pressure and turbine flow. Then turbine power would drop. In such way, engine transient response would be compromised with lower compressor mass flow rate. This is a classic control problem for VGT-EGR system, which has been studied for almost three decades in control community. VGT-EGR control needs to be coordinated based on different engine operating condition.
When variable geometry turbocharger is interacting with high-pressure EGR loop, additional assisted and loaded power will serve as useful inputs to decouple this dynamic system. For instance, boost pressure might not need aggressive VGT and EGR closing with extra assisted during tip-in. Since the turbocharger can be driven by assisted power, VGT control can be used for partially boost control or EGR control. In such way, both EGR demanded and target boost pressure will be met. During transient deceleration without regenerative power, VGT is used to open to drop the pressure ratio across turbine wheel, reducing extracted turbine power. With this VGT open action, EGR valve cannot have the right high-pressure ratio to drive EGR. With additional loading power on TC shaft, TC shaft speed can be managed independently with VGT action. Hence, with external control inputs, EGR mass flow rate, and boost pressure can be well regulated.

Regenerative hydraulic assisted turbocharger system modelling consists of traditional engine air-path modelling and assisted and regenerative turbocharger modelling. The challenges are to identify a high fidelity mean value model for model-based analysis and controller design.

4.2.1 Engine air-path modelling overview

In this section, the subcomponents of engine block are discussed. To use mathematical equations to represent a complex physic dynamic process, subsystem introduction for the diesel engine system are presented. As shown in Figure 27, the diesel engine air-path system normally consists of air intake system and exhaust system. For turbocharged diesel engine, the turbine is driven by high temperature and high-pressure exhaust gas, which also drives the compressor to pump air into the intake manifold. To have turbine accommodate wide flow range, vane nozzle is used to adjust turbine inlet flow rate. Another approach is to size turbine smaller, bypassing turbocharger with waste-gate at higher mass flowrate avoiding turbocharger over speed. This
approach is widely used in gasoline engine. Part of exhaust gas is guided through high pressure EGR valve into intake manifold, then back into cylinder. Two coolers are normally deployed in the intake manifold system: turbocharger intercooler and EGR cooler, which cools high temperature compressor mass flow and EGR mass flow, respectively.

Typically mean value diesel engine model with VGT and EGR can be represented in equation as below (4.1) [52]. This is a seven states dynamics equation, which represents thermodynamics process of diesel engine air-path system. Further crankshaft rotational dynamics state and engine torque production can be added to make a comprehensive engine model, which can be coupled with transmission or driveline model. All the variables in the equation can be defined explicitly as a function of system states or other parameters. Experimental validation data can be used to calibrate these models.
\[
\begin{align*}
\frac{dm_1}{dt} &= W_{c1} + W_{21} - W_{te} - W_{12} \\
\frac{dm_2}{dt} &= W_{12} + W_{e2} - W_{21} - W_{2t} \\
\dot{p}_1 &= \frac{W_{21}(F_2 - F_1) - F_1W_{c1}}{m_1} \\
F_2 &= \frac{W_{e2}(F_{e2} - F_2) - W_{12}(F_1 - F_2)}{m_2} \\
\frac{dT_1}{dt} &= \frac{W_{21}(h_{21} - u_1) - W_{c1}(h_{c1} - u_1) - (W_{te} - W_{12})R_1T_1 - m_1\chi T_1\dot{p}_1}{c_v1m_1} - \frac{\dot{Q}_1}{c_v1m_1} \\
\frac{dT_2}{dt} &= \frac{W_{e2}(h_{21} - u_1) - W_{12}(h_{12} - u_2) - (W_{21} - W_{2t})R_2T_2 - m_2\chi T_2\dot{F}_2}{c_v2m_2} - \frac{\dot{Q}_2}{c_v2m_2} \\
\frac{dP_c}{dt} &= \frac{1}{\tau_{cc}}(-P_c + \eta_{tm}P_t)
\end{align*}
\]

But this high order system is difficult to do the controller design. Most of controller design framework for diesel engine air-path system used a simplified three states [5] [7] with the assumptions: 1. No thermodynamic dependence, all thermodynamic properties with respect to air; 2. Temperature dynamics are neglected.

Further simplification such as: air fraction states for the intake manifold and exhaust manifold are not measured directly, these parameters need estimation or can be removed by the assumption that emission control target can be met with designed target value. Then the seven dynamic states can be reduced into 3\textsuperscript{rd} system as follows:

\[
\begin{align*}
\dot{p}_3 &= \frac{RT_3}{V_3} (\dot{m}_{out} - \dot{m}_{egr} - \dot{m}_t) \\
\dot{p}_2 &= \frac{RT_2}{V_2} (\dot{m}_c - \dot{m}_{in} + \dot{m}_{egr}) \\
J_\omega \dot{\omega} &= W_T - \dot{W}_C - \dot{W}_{Loss}
\end{align*}
\]

Here we redefined boost pressure as \(P_2\) and exhaust pressure as \(P_3\). The three states diesel engine air-path system includes intake manifold pressure, exhaust manifold pressure and TC shaft speed. Based on previous researchers [5][7], three states model can well represent engine air-path dynamics and be used for controller design. Hence, in this investigation, we start with
three states model for modelling the diesel engine air-path system coupled with hydraulic assisted system.

### 4.2.2 Regenerative hydraulic assisted turbocharged engine modelling

Regenerative hydraulic assisted turbocharged engine modelling has three major subsystems, which are engine system, turbocharger and hydraulic system as shown in Figure 29. Simplified three state diesel engine air-path model structures are adopted from previous literatures. The model has eight states as indicated in Eq(1). The eight states are the engine intake and exhaust manifold pressures \( P_2, P_3 \), the turbocharger speed \( \omega \), the hydraulic accumulator pressure \( P_{\text{acc}} \), the pre-turbine hydraulic pressure \( P_t \), and the pump discharge pressure \( P_p \), and the hydraulic accumulator piston position \( x \) and piston speed \( v \). The 8 states are highlighted with green in Figure 29.

The control inputs of the system are the VGT vane position \( u_{\text{vgt}} \), the EGR valve position \( u_{\text{egr}} \), the hydraulic turbine inlet valve position \( u_{\text{turbine}} \) and the hydraulic pump discharge valve position \( u_{\text{pump}} \). The system modelling interaction and causality for each subcomponents are summarized in Figure 29. Each solid block represents a modelling sub-component. Each line delivers parameter from one block to another block. For instance, ‘VGT Turbine’ block needs temperature and pressure from exhaust pressure block ‘P3’ and turbocharger speed from ‘Inertia+Fric’ block to calculate turbine mass flow rate and turbine power. Figure 29 can be represented as the dynamic equations as in (4.3).

\[
\begin{align*}
\dot{P}_2 &= \frac{T_2}{V_2} (m_c - m_{\text{in}} + m_{\text{egr}}) \\
\dot{P}_3 &= \frac{T_3}{V_2} (m_{\text{out}} - m_{\text{egr}} - m_T) \\
J_\omega \dot{\omega} &= W_T - W_T - W_{\text{Loss}} + W_{\text{turbine}} - W_{\text{pump}} \\
\dot{P}_p &= \beta \frac{V_p}{V_2} (m_{\text{pump}} - m_{\text{valve,p}}) \\
\dot{P}_t &= \beta \frac{V_t}{V_2} (m_{\text{valve,t}} - m_{\text{turbine}})
\end{align*}
\]  

(4.3)
\[
\dot{p}_{\text{acc}} = \frac{\beta}{V_0 + xA} (\dot{m}_{\text{pump}} - \dot{m}_{\text{turbine}} - nA)
\]

\[
\dot{x} = v
\]

\[
\dot{v} = \frac{(P_{\text{acc}} - P_{\text{return}})A - F_0 - F_f(v) - cv - kx}{xPA}
\]

Figure 29. System modelling architecture and calculating loop.

Five control volumes are identified and they are engine intake manifold, engine exhaust manifold, pipe volume \((V_t)\) between the hydraulic turbine and hydraulic turbine inlet valve, pipe volume \((V_p)\) between the hydraulic pump and hydraulic pump valve, and high pressure accumulator displacement \((V_0 + xA)\), where \(V_0\) is the initial displacement, \(A\) is the piston area surface. The TC shaft dynamics model couples the hydraulic loop to the engine air-path loop. The shaft dynamics equation represents the power balance between the four components on the turbocharger shaft: turbine, compressor, hydraulic turbine and hydraulic pump. The power balance is adjusted for shaft friction. The hydraulic loop interacts with air-path loop directly through turbocharger shaft by transferring assist power from hydraulic turbine and extracting regeneration power by the hydraulic pump. The hydraulic turbine and pump are controlled by linear valves.
In summary, the system can be represented as in Figure 30. Four inputs and two outputs (compressor mass flow rate (or boost pressure) and EGR mass flow rate), which is a multi-input and multi-output (MIMO) system for the model-based analysis and controller design.

Modelling assumptions:

1. Engine intake and exhaust manifolds are modeled as thermodynamics control volumes with mass and flows in and out of these volumes.

2. For intake and exhaust manifolds volumes contains an ideal gas mixture of air and combustion gas. The ideal gas is assumed to be uniform across the volume.

3. No inverse flow through EGR valve.

4. The temperature in intake and exhaust manifold are assumed to be uniform across the controlled volume. \( T_i = T_{i1}, T_j = T_{j2} \).

5. No heat loss in air-path system is considered in this study.

6. The pressure of low pressure hydraulic accumulator is assumed as constant.

7. Parasitic loss of hydraulic turbine and hydraulic pump are not considered during inactivate mode.
8. Heat loss and pipe loss in the hydraulic system is not considered in this study.


10. Constant hydraulic fluid density.

11. Pressure of low pressure hydraulic tank is treated as constant.

As shown in the modelling governing equation (4.3), most of the modelling in this study involves mass flow rate modelling. The mass flow rate models can be overviewed in Table 11. The mass flow rate for each subcomponent depends on the pressure difference between subcomponents upstream and downstream. For engine air-path and hydraulic fluid path, control actuators directly impact the mass flow rate. Further, mass flow rate serves the outputs of control results, such as demanded fresh air amount, EGR mass flow rate or engine charged air mass flow rate.

<table>
<thead>
<tr>
<th>Table 11. Mass flow rate governing equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Governing equation</strong></td>
</tr>
<tr>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>Engine intake mass flow</td>
</tr>
<tr>
<td>Engine exhaust mass flow</td>
</tr>
<tr>
<td>EGR mass flow</td>
</tr>
<tr>
<td>Turbine mass flow</td>
</tr>
<tr>
<td>Compressor mass flow</td>
</tr>
<tr>
<td>Hydraulic pump mass flow</td>
</tr>
<tr>
<td>Hydraulic turbine mass flow</td>
</tr>
<tr>
<td>Hydraulic control valve flow</td>
</tr>
</tbody>
</table>

4.3 Engine Modelling and Validation

For simplified three states engine air-path modelling as shown in (4.2). Intake pressure, exhaust pressure and TC shaft speed are the three dynamic states. An expanded version for engine air-path system can be viewed as in (4.4). As shown in (4.4), all the parameters mainly depend on states and control inputs, with the assumption that turbine outlet pressure and compressor inlet pressure are set as constant. \( P_1 \approx P_4 \approx P_{\text{air}} \).
\begin{align*}
P_2 &= \frac{R f_T}{V_2} \left( \frac{\omega P_2}{P_1^2} \right) \left( f_{m_e}(\omega, P_2) - f_{m_{in}}(N_{\text{engine}}, P_2) + f_{hpegr}(u_{egr}, \frac{P_3}{P_2}) \right) \\
P_3 &= \frac{R f_T(P_3, \dot{m}_{fuel})}{V_3} \left( f_{m_{out}}(N_{\text{engine}}, P_2) + \dot{m}_{fuel} - f_{int} \left( \frac{P_3}{P_4} \right) - f_{hpegr} \left( u_{egr}, \frac{P_3}{P_2} \right) \right) \tag{4.4}
\end{align*}

\[ J \omega \dot{\omega} = f_{\dot{W}_{VT}} \left( u_{VT}, \frac{P_3}{P_4}, \omega, T_3 \right) - f_{\dot{W}_{c}} \left( \frac{P_2}{P_1}, \omega \right) - f_{\dot{W}_{loss}}(\omega) \]

Each of the nonlinear functions is investigated and validated in this chapter. Developed engine sub-models are identified from a medium duty diesel engine steady state mapping data. The 195 engine steady state operation points are used for model identification. Then engine air-path model is further validated through transient engine test.

### 4.3.1 Engine intake and exhaust mass flow rate

It is common to model the engine breathing process in four stroke engines with the speed density equation. The mass flow rate of intake charge \( \dot{m}_{\text{in}} \) in equation (4.2) can be modelled as:

\[ \dot{m}_{\text{in}} = \frac{\eta_{vol} P_2 N_e V_d}{120 R T_2} = \frac{\eta_{vol} P_2 N_e V_d}{120} \tag{4.5} \]

Where, \( \eta_{vol} \): volumetric efficiency of the engine

\( \frac{P_2}{RT_2} \): density of the gas mixture in the intake manifold.

\( V_d \): engine displacement volumes.

The volumetric efficiency is a function of engine speed and boost pressure. From [8], the volumetric efficiency can be taken as:

\[ \eta_{vol} = \left( coe_1 \cdot \sqrt{P_2} + coe_2 \cdot \sqrt{N_e} + coe_3 \right) \tag{4.6} \]

With knowing engine intake mass flow rate, density of gas mixture in the intake manifold, engine displacement volume, the volumetric efficiency can be easily identified. Results are shown in Figure 31.
The engine exhaust mass flow rate $\dot{m}_{out}$ can be approximated as the sum of charged air $\dot{m}_{in}$ and fuel flow rate $\dot{m}_{fuel}$. Obviously, this is based on the assumption of negligible and zero residual gas fractions in the cylinder. Thus:

$$\dot{m}_{out} = \dot{m}_{in} + \dot{m}_{fuel} \tag{4.7}$$

Fuel mass flow rate is based on the commanded fuel injection rate into each cylinder. For eight cylinder engine, the fuel mass flow rate is as in (4.8). Unit for fuel command is injected fuel quantity, which has the unit of mg/stroke.

$$\dot{m}_{fuel} = \frac{8 \times 10^{-6}}{120} u_{fuel} N_e \tag{4.8}$$

![Figure 31. Model identification results for volumetric efficiency](image)
4.3.2 Exhaust manifold temperature

The engine exhaust manifold temperature model is based on two sub-models, which are cylinder-out temperature and heat loss model for exhaust pipes. The cylinder out temperature is modelled as in (4.9) [55]. This approach is based upon ideal-gas Seliger cycle [56].

\[
T_{eout} = \left( \frac{p_{em}}{p_{im}} \right)^{1-1/\gamma_{air}} \cdot T_e^{1-\gamma_{air}} \cdot x_p^{1/\gamma_{air}-1} \cdot \left( m_{fuel}LHV \left( \frac{m_{air} + m_{fuel}}{C_{pa} + C_{va}} \right) + T_2 \frac{\gamma_{air} - 1}{\gamma_{air}} \right) \tag{4.9}
\]

This temperature doesn’t take account for temperature loss through exhaust pipe. With the same approach in [55], exhaust manifold temperature \( T_3 \) can be modelled as a function of the exhaust mass flow rate and the engine exhaust temperature.

\[
T_3 = T_{air} + \frac{h_{tot}d_{pipe}l_{pipe}n_{pipe}}{m_{air} + m_{fuel}} (T_3 - T_{air}) \exp \left( \frac{h_{tot}d_{pipe}l_{pipe}n_{pipe}}{m_{air} + m_{fuel}} \right) \tag{4.10}
\]

Where \( T_{air} \) is ambient temperature, \( h_{tot} \) the total heat transfer coefficient of exhaust manifold. The \( d_{pipe}, l_{pipe}, \) and \( n_{pipe} \) are pipe diameter, pipe length and number of pipes, respectively. The identified results can be viewed in Figure 32.
4.3.4 Intake manifold temperature

All turbocharged diesel engine includes an intercooler after the compressor, and EGR charger cooler. The temperature in the intake manifold is the combined temperature from both intake charger cooler and EGR cooler. These components can be treated as standard heat exchangers. Normally, a heat exchanger effectiveness $\epsilon$ is used to modelling the heat transfer effect through coolers. Then the downstream of cooler temperature can be taken as:

$$T_{\text{downstream}} = T_{\text{upstream}}(1 - \epsilon) + T_{\text{coolant}}\epsilon$$  \hspace{1cm} (4.11)

Assume temperature mixture based on mass flow mixture, then intake manifold temperature can be modelled as:

$$T'_{2} = \frac{\dot{m}_{\text{com}} T_{2}^* + \dot{m}_{\text{egr}} T_{3}^*}{\dot{m}_{\text{com}} + \dot{m}_{\text{egr}}}$$  \hspace{1cm} (4.12)

Where  \hspace{1cm} $T_{2}^* = T_{2}(1 - \epsilon) + T_{\text{coolant}}\epsilon$
\[ T_3' = T_3(1 - \epsilon) + T_{\text{coolant}}\epsilon \]

There are two ways to model temperature after compressor. One can be based on compressor power equation, which is discussed in Chapter 3. Another way is that the temperature after the compressor can be modeled as high order polynomial equation as below.

\[ T_3' = f(\omega, \dot{m}_{\text{compressor}}) = c_1 + c_2 \dot{m}_{\text{compressor}} + c_3 \omega + c_4 \dot{m}_{\text{compressor}}^2 + c_5 \dot{m}_{\text{compressor}} \omega + c_6 \dot{m}_{\text{compressor}}^2 \omega + c_7 \dot{m}_{\text{compressor}} \omega^2 + c_8 \dot{m}_{\text{compressor}}^2 \omega^2 \quad (4.13) \]

![Figure 33. Model identification results for temperature after compressor](image)

4.3.3 EGR mass flow rate modelling

EGR mass flow rate is controlled through EGR valve position. The EGR flow rate can be modeled by the conventional orifice flow equation, which has major two inputs: valve position \( u_{\text{egr}} \), upstream and downstream pressure ratio \( \frac{P_3}{P_2} \). Orifice flow equation for compressor
mass flow rate is usually modeled as a one dimensional, steady, compressible flow. Two different flows can be distinguished by using pressure ratio as below.

\[
m_{egr} = A_{eff} P_3 \frac{2}{\sqrt{\gamma T_3}} \psi
\]

If \( \frac{P_3}{P_2} > \left( \frac{2}{k + 1} \right)^{\frac{k}{k-1}} \), \( \psi = \sqrt{k - 1 \left[ \frac{(P_3)^{\frac{2}{k}}}{(P_2)^{\frac{k}{k-1}}} - \frac{(P_3^{\frac{k}{k-1}}}{(P_2)^{\frac{2}{k}}}} \right]} \) \hspace{1cm} (4.14)

If \( \frac{P_3}{P_2} > \left( \frac{2}{k + 1} \right)^{\frac{k}{k-1}} \), \( \psi = \sqrt{2k \left( \frac{2}{k + 1} \right)^{\frac{k+1}{k-1}}} \)

The EGR valve flow effective area \( A_{eff} \) is affected by engine RPM, pressure ratio and load as show in Figure 35. This can be explained with engine pulsation flow effect on EGR mass flow rate. In this modelling work, since this is not our primary interest, so a subsonic model with \( A_{eff} = f(u_{egr}) \) is adopted for this study. The identified results are shown in Figure 34.

Figure 34. Identified EGR effective area
4.4 Variable Geometry Turbocharger modelling and validation

The two major components are connected through turbine shaft for the turbocharger. Turbocharger speed dynamic equation has four terms for turbocharger itself, the exhaust gas driven turbine power, compressor power, friction power and kinetic energy term. Based on Newton's second law, power balance equation (without assisted and regenerative power) is:

\[ J \omega \dot{\omega} = W_T - W_C - W_{Loss} \]  \hspace{1cm} (4.15)

Energy balanced equation would serve as the link between assisted and regenerative devices. With external add-on power, turbine power will be balanced at higher speed region. For modelling of turbocharger power, most of the approaches are based turbine and compressor
efficiency map provided by manufactures. For integrity, some results from previous chapters are adopted here. But for details, please refer to previous chapters. Friction model and identification for turbocharger are presented in Chapter 2. New modelling approach for compressor mass flow rate modelling is discussed in details here.

### 4.4.1 VGT turbine model

The turbine utilizes the exhaust gas residual enthalpy to drive the compressor. Energy is extracted from engine exhaust, through expansion across turbine blades. Turbine power is used to overcome the compressor load and friction loss as well as acceleration inertia. Hence accurate turbine power is needed for modelling turbocharger rotational speed dynamics. The overall modelling structure of turbine block is as shown in Figure 36. The turbine model consists of sub-models for turbine mass flow rate and turbine power. The turbine modelling depends on inputs from the engine block, which are exhaust manifold pressure, exhaust manifold temperature.

![Figure 36. Turbine modelling layout](image)

#### 4.4.1.1 Turbine power

In this section, turbine operation depends on turbine inlet condition, turbine power based on inlet condition [23] are utilized here:

\[
\dot{W}_T = \frac{1}{60} N_{TC} \left( \dot{m}_{e,x} \right)^2 \frac{1}{B} \frac{1}{P_T} \tan \alpha_i
\]

(4.16)
Temperature $T^*$ and $P^*$ is temperature and pressure between the turbine blade and nozzle. In this study, these two parameters are approximated as temperature and pressure in the exhaust manifold. Model identification can be referred to Chapter 2.

### 4.4.1.2 Turbine mass flow rate

The mass flow rate through the turbine can be modeled using the similar flow equations in EGR section. The effective valve area is experimentally determined to be a polynomial function of VGT position. Two other variables which impact the flow rate are critical pressure and pressure ratio when flow rate is zero. From [1], the mass flow rate through turbine can be modelled as in (4.17). Turbine mass flow rate depends on pressure ratio across turbine as well as effective nozzle areas, the identified results can be found at Figure 37.

\[
\dot{m}_{turbine} = f(u_{vgt}, \frac{P_3}{P_4}) \\
\dot{m}_{turbine} = A_{eff}P_3^{\frac{2}{2\gamma-1}} \psi \\
\psi = \sqrt{\frac{k}{k-1} \left[ \left( \frac{P_4}{P_3} \right)^{\frac{2}{k}} - \left( \frac{P_4}{P_3} \right)^{\frac{k+1}{k}} \right]} = \left( \frac{P_4}{P_3} \right)^{\frac{1}{k}} \sqrt{\frac{c_p}{R}} \left[ 1 - \left( \frac{P_4}{P_3} \right)^{\frac{k-1}{k}} \right] \\
A_{eff} = f\left( \alpha(u_{vgt}) \right)
\]
4.4.2 Compressor modelling

Inputs for compressor modelling are intake manifold pressure and TC shaft speed. The output of compressor model is compressor power and compressor mass flow rate. In this section, we investigate different approaches for compressor mass flow rate modelling in details.

![Compressor model layout](image)

Figure 38. Compressor model layout

4.4.2.1 Compressor power

As shown in [84], compressor power can be expressed as a function of TC speed and mass flow rate. More detailed model development for compressor power can be found in chapter 3.
\[ \dot{W}_c = \dot{m}_c^2 \varepsilon_1 \left( \frac{\omega}{m_c} \right)^2 - \dot{m}_c \left( \frac{\omega}{m_c} \right) + \varepsilon \]  

(4.18)

### 4.4.2.2 Compressor mass flow rate

The compressor receives power from the turbine, driven by engine exhaust gas. High pressure is built by compression process from compressor inlet port to outlet port. High pressure in volute behaves as the resistance force to slow down compressor speed. Other resistance forces are compressor incidence losses (impeller wheel and diffuser) as well as aerodynamic friction losses (impeller wheel and diffuser). The resistance forces reduce compressor efficiency as well as mass flow rate for the compression process.

![Automotive compressor layout](image)

Figure 39. Automotive compressor layout

In this section, we redefine the \( P_3, P_4 \) as volute pressure and intake manifold pressure in this study. From compressor dynamic mass flow rate equation (4.19) [57], compressor mass flow rate dynamics depends on volute pressure and intake manifold pressure as well as the geometry of volute and intake pipe.

\[ \dot{m}_c = (P_3 - P_4) \frac{\Delta}{\bar{c}} \]  

(4.19)

Volute pressure \( P_3 \) can be calculated by pressure rise from the compressor. Based on forward flow, pressure rise can be modelled as:
From [84], increased enthalpy by impeller can be approximated with:

$$\eta \Delta h = \left( m_c^2 f \left( \frac{\omega}{m_c} \right) - \dot{m}_c^2 k_f \right) / \psi$$

(4.21)

Combining (4.19), (4.20) and (4.21), compressor mass flow rate dynamic equation is:

$$\dot{m}_c = \left( 1 + \left( \frac{m_c^2 f(\omega/m_c) - \dot{m}_c^2 k_f}{r_c c_p} \right)^{\gamma-1} P_1 - P_4 \right)^{\frac{A}{c}}$$

(4.22)

With constant inlet conditions, the dynamic mass flow rate can be expressed as a function of mass flow rate, turbocharger speed and intake manifold pressure:

$$\dot{m}_c = f(m_c, \omega, P_4)$$

(4.23)

This means the compressor mass flow rate dynamic system can take TC shaft speed and intake manifold pressure as inputs. Compressor mass flow rate is the dynamic state and output of this dynamic system. Hence, compressor mass flow rate can be expressed as the integration of derivative of mass flow rate with given initial condition and given time step.

$$\dot{m}_c = \int_0^t \dot{m}_c = \int_0^t f(m_c, \omega, P_4) + \dot{m}_{c,0}$$

(4.24)

In model (4.22), there are three parameters to be identified: $k_f, \psi, \frac{A}{c}$. $k_f, \psi$ identification can be found in chapter 3. $\frac{A}{c}$ is the geometry parameter from volute to the location (from 3 to 4 in Fig.1).

a. Hammer-Winner model for compressor mass flow rate

Based on (4.20), compressor mass flow rate can be taken as output of a transfer functions with TC speed and intake manifold pressure as inputs. Alternative approach is to use ‘Black box’ identification method to investigate the transfer function between inputs and outputs. Proposed ‘Black box’ dynamic system takes TC speed and intake manifold pressure as input as shown in (4.22) and Figure 40. It includes input nonlinearities, linear systems and output nonlinearities.
This is so called Hammer-Wiener (H-W) model. This system can also be expressed as state space representation as in (4.23).

\[
\dot{m}_c = f(\text{TF}_1(g_1(\omega)) + \text{TF}_2(g_2(P_4)))
\]  

(4.25)

![Figure 40. Hammer-Wiener Model](image)

\[
\dot{\hat{m}} = A\hat{m} + Bg(u) \quad \dot{m}_c = f(\hat{m})
\]  

(4.26)

Where, \( u = [\omega P_4] \), \( g(u) = [g_1(\omega) g_2(P_4)] \), \( g_1, g_2 \) and \( f \) are nonlinearity for inputs and output. \( \hat{m} \) is defined as the Intermediate variable. In this case, the coefficients need to be identified are \( A, B \) matrix and \( g_1, g_2 \) and \( f \) nonlinearity function. The system output value depends on input and output nonlinearities as well as linear systems.

b. Static nonlinear model compressor mass flow rate

Base on the power model from Chapter 3, the compressor mass flow rate can be obtained through turbomachinery flow coefficient equation. The speed coefficient is defined in Chapter 3. The flow coefficient is defined as in (4.27). The identified results can be shown in Figure 41.

\[
C_{\text{flow}} = \frac{\hat{m}_p^2}{\left(\frac{\gamma - 1}{P_r^{\gamma - 1}} \right)}
\]  

(4.26)
c. Compressor mass flow rate model validation

For physics based mass flow rate model, $k_f, \psi$ are identified through steady state flow bench test data [84]. Geometry parameter $A/L$ is tuned online in dynamic simulation to match with vehicle test results in this study. Geometry parameter $A/L$ could be further identified through experimental data. For H-W compressor mass flow rate model identified through vehicle transient test data. Identified linear transfer function is as below:

$$TF_1 = \frac{z^{-1} - 0.9993 z^{-2}}{1 - 1.437 z^{-1} + 0.7298 z^{-2} - 0.2922 z^{-3}}$$

$$TF_2 = \frac{z^{-1} - 0.9974 z^{-2}}{1 - 1.119 z^{-1} + 0.06416 z^{-2} + 0.05707 z^{-3}}$$

Let $v = g(u) = \begin{bmatrix} g_1(\omega) \\ g_2(P_s) \end{bmatrix}$. Hence, intermediate $\hat{m}$ can be obtained as:

$$\hat{m} = \begin{bmatrix} TF_1 \\ TF_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\hat{m} = \frac{z^{-1} - 0.9993 z^{-2}}{1 - 1.437 z^{-1} + 0.7298 z^{-2} - 0.2922 z^{-3} v_1}$$

$$+ \frac{z^{-1} - 0.9974 z^{-2}}{1 - 1.119 z^{-1} + 0.06416 z^{-2} + 0.05707 z^{-3} v_2}$$

Form above equation, it can be concluded that, current state of intermediate variable $\hat{m}$ is based on previous two steps of inputs $v$ and previous five steps of $\hat{m}$.
\[ \hat{m}_k = f(v_{1(k-1)}, v_{1(k-2)}, v_{2(k-1)}, v_{2(k-2)}, \hat{m}_{k-1}, \hat{m}_{k-2}, \hat{m}_{k-3}, \hat{m}_{k-4}, \hat{m}_{k-5}, \hat{m}_{k-6}) \]  

(4.30)

This is an interesting finding. Nonlinear physics based model can be converted to a ‘sandwich’ model; system dynamic is purely based on linear model, and input and output nonlinearity. Identified results can be shown in Figure 42 and Figure 43. It is clearly to show that compressor system has strong nonlinearity. Identified linear system has six states in this study. It varies with TC speed, intake manifold pressure, as well as mass flow rate dynamic system itself. The poles and zeros of linear systems show that this open loop system is not stable for some modes. It also shows non-minimum phase behaviors due to positive zeros for both input channels. This non-minimum phase could be potentially eliminated by the control technique of poles and zeros placement.

Figure 42. Nonlinearity for H-W compressor mass flow rate model
H-W compressor mass flow rate model has good accuracy compared to vehicle transient test data as shown in Figure 42 and Figure 45. The only small error happens at extreme light load and very fast transient operations. For model accuracy, proposed model shows 5% accuracy within 90.75% of the operating points. The error function is defined as in (9). This compressor model could be used as a subcomponent model for compression system (compressible flow), turbocharged internal combustion engine as well as a gas turbine engine, theoretically. Application for incompressible flow needs further investigation. For most turbocharged internal combustion engine air-path modelling methods, intake manifold pressure and TC speed are modelled as dynamic states. Hence, in those engines modelling approaches, proposed compressor mass flow rate sub-model can directly use intake manifold pressure and TC speed as model inputs to compute compressor mass flow rate.

$$error = \frac{|\omega_{predicted(i)} - \omega_{measured(i)}|}{\omega_{measured(i)}}$$  \hspace{1cm} (4.31)

Where, i is each sample data. By comparing with physics based model and H-W model as shown in Figure 44. Identification results for H-W compressor mass flow rate model. Both of these two models are simulated with same inputs, which are TC speed and intake manifold
pressure. Initial conditions for these two dynamic models are set as zero. The results for both models agree with test data, but physics based dynamic model has high error in some region. These could be due to unmodelled dynamics and simplification. For instance, for idling operation (480s-600s) in Figure 45, the high predicted compressor mass flow rate from physics-based model might be due to not considering the heat transfer effect from turbine side.

One potential application for these two models is to have more accurate compressor mass flow rate estimation by combining two models through sensor fusion techniques, such as using extended Kalman filter. By combining two models, the model output would have better accuracy as well as representing the physics of compression process. This approach can also be used as a virtual sensor for compressor mass flow rate.

Figure 44. Identification results for H-W compressor mass flow rate model
4.5 Hydraulic System Modelling

4.5.1 System overview

The RHAT system consists of two linear solenoid valves, a centrifugal hydraulic turbine, a centrifugal hydraulic turbo pump and a spring-loaded hydraulic accumulator as shown in Figure 46. In this study, both hydraulic pump and turbine are radial flow, centrifugal turbomachines. Two solenoid valves are used to control the mass flow rates of the hydraulic turbine and hydraulic pump. The Dynamic states are pressures between turbine and valve, the pressure between pump and pump valve, pressure in the high-pressure hydraulic tank, the piston position and velocity. For simplicity the turbine downstream pressure and pump upstream pressure are treated as a constant $P_0$ (pressure in the low-pressure accumulator).

\[
\dot{p}_p = \frac{\beta}{v_p}(\dot{m}_p - \dot{m}_{\text{valve,p}})
\]

\[
\dot{p}_T = \frac{\beta}{v_t}(\dot{m}_T - \dot{m}_{\text{valve,t}})
\]

\[
\dot{p}_{\text{Acc}} = \frac{\beta}{v_0 + sA}(\dot{m}_{\text{valve,p}} - \dot{m}_{\text{valve,t}} - vA)
\]

\[
\dot{x} = v
\]

\[
\dot{v} = \frac{1}{m}\left((P_{\text{Acc}} - P_{\text{return}})A - F_0 - F_f(v) - cv - kx\right)
\]
The governing equations of the hydraulic system, mass flow rates of turbine, pump, and valve are provided in (4.33):

\[
\begin{align*}
\dot{P}_t &= \frac{\beta}{V_t} \left( f_{\text{m}_{\text{valve}, t}} (P_{\text{Acc}} - P_t, u_{\text{valve}, t}) - f_{\text{m}_{\tau}} (P_t, \omega) \right) \\
\dot{P}_p &= \frac{\beta}{V_p} \left( f_{\text{m}_{\text{pump}, p}} (P_p, \omega) - f_{\text{m}_{\text{valve}, p}} (P_p - P_{\text{Acc}}, u_{\text{valve}, p}) \right) \\
\dot{P}_{\text{Acc}} &= \frac{\beta}{V_0 + xA} \left( f_{\text{m}_{\text{valve}, p}} (P_p - P_{\text{Acc}}, u_{\text{valve}, p}) - f_{\text{m}_{\text{valve}, t}} (P_{\text{Acc}} - P_t, u_{\text{valve}, t}) - vA \right) \\
\dot{x} &= v \\
\dot{v} &= \frac{1}{m} \left( (P_{\text{Acc}} - P_{\text{return}})A - F_0 - F_f (v) - cv - kx \right)
\end{align*}
\]  

Figure 46. Layout of hydraulic actuation system

The system dynamics depends on the mass flow rates through the subcomponents. For instance, pressure before hydraulic turbine \(P_t\) is based on the mass flow rate change in the small volume \(V_t\) between the hydraulic turbine and valve. The flow rate range is based on hydraulic turbine and valve mass flow rates. Furthermore, the turbine mass flow rate is based on the hydraulic turbine speed and the pressure difference across the turbine. For the control valve, mass flow rate is based on the pressure difference across control valve (or pressure difference between the hydraulic accumulator pressure \(P_{\text{Acc}}\) and pressure after the valve \(P_t\)). The dynamics for hydraulic accumulator pressure is mainly dependent on the valve mass flow rate, piston position \(x\), velocity, \(v\), and acceleration \(\dot{v}\). Energy stored in the pre-loaded spring pressurizes the
hydraulic fluid used to drive the hydraulic turbine. The hydraulic turbine output power is dependent on turbine mass flow rate $\dot{m}_T$ and speed $\omega$. Thus, with the known control input (turbine valve position), hydraulic turbine power can be calculated based on the flow rate and current TC shaft speed.

During the TC shaft deceleration process, the hydraulic turbine is deactivated by closing the turbine valve. The pump valve is opened to apply load onto the TC shaft. The pump drives the low-pressure fluid through the pump valve into the high-pressure accumulator and builds up pressure before and after the pump valve, causing fluid flow into the accumulator. The pump valve flow rate is based on the pressure difference across the valve (or the pressure difference between pre-valve pressure, $P_p$ and the accumulator pressure, $P_{Acc}$). The valve mass flow rate is regulated by the valve position, which is the control mechanism for the pump flow. Thus, the pump power can be regulated using the pump valve position. TC shaft kinetic energy can be stored in the form of the accumulator spring potential energy.

4.5.2 Hydraulic centrifugal pump modelling

4.5.2.1 Hydraulic centrifugal pump power model

Most of the early modeling approaches for hydraulic component power are based on efficiency map as shown in Figure 47. These modeling approaches are based on the utilization of the efficiency map by extrapolation and interpolation. Given the nonlinearity of these maps, it is typically difficult to obtain an analytical model describing these efficiency maps. However, since the hydraulic pump power is the target parameter for the turbocharger shaft speed dynamic equation, it is meaningful to model the hydraulic turbine power directly.
Based on the Euler turbomachinery equation in [84], the pump performance can be modeled using the defined speed coefficient \( \frac{\omega}{m_{\text{pump}}} \) and power coefficient \( \frac{W_{\text{pump}}}{(m_{\text{pump}})^3} \); as discussed in [84]. The power coefficient can be a quadratic function of speed coefficient (or linear function in the logarithmic scale). Based on the manufacture-provided performance map, these two variables are plotted in Figure 48.

\[
\frac{W_{\text{pump}}}{(m_{\text{pump}})^3} = a \left( \frac{\omega}{m_{\text{pump}}} \right)^2 + b \left( \frac{\omega}{m_{\text{pump}}} \right) + c \tag{4.34}
\]

\[
W_{\text{pump}} = \dot{m}_{\text{pump}}^3 \times 10^9 \left( \frac{\omega}{m_{\text{pump}}} \right)^q
\]
Coefficients in (4.34) can be determined by using the same approach in the previous compressor modeling section. The results in Figure 48 validate that turbo pump's operation characteristics agree with proposed centrifugal compression machine power model. With knowing pump speed and pump mass flow rate, pump power can be computed through proposed model.

4.5.2.1 Centrifugal pump flow rate model

Centrifugal pump flow rate modelling is based on TC shaft and pressure head of pump.

\[ \dot{m}_p = f(\omega, \Delta P) \]  

(4.35)

Based on turbomachinery fluid dynamics for the centrifugal pump, two dimensionless parameters are used to characterize pump operation, the dimensionless flow coefficients: flow coefficient \( C_q \) and head coefficient \( C_h \) [27], as in (4.36) and (4.37).

\[ C_q = \frac{V_x}{U \alpha} \left( \frac{\dot{m}}{\pi ND \alpha} \right) \frac{\pi (D^2 - d^2)}{4} \]  

(4.36)
\[ C_h = \frac{V_s}{U} \alpha \frac{gH}{\left( \frac{\pi ND}{60} \right)^2} \]  
(4.37)

These two parameters are related to the mass flow rate, angular speed, and the pressure difference across the pump:

\[ C_q \propto \frac{\dot{m}}{\omega} \]  
(4.38)

\[ C_h \propto \frac{\Delta P}{\dot{m}^2_p} \]  
(4.39)

Alternately two new flow coefficients can be defined for the centrifugal pump. A modified flow coefficient \( C_{\text{flow}} \) and a modified head coefficient, \( C_{\text{head}} \), are defined as shown in (4.40) and (4.41) respectively. Since the modified mass flow rate model depends only on two states (TC shaft speed and hydraulic accumulator pressure), this significantly reduces the model complexity.

\[ C_{\text{flow}} = \frac{\dot{m}}{\omega} \]  
(4.40)

\[ C_{\text{head}} = \frac{\dot{m}^2_p}{\Delta P} \]  
(4.41)

Based on flow coefficient and head coefficient, mass flow rate for centrifugal mass flow rate can be derived as in (4.42) and (4.43). The model calibration results are as shown in Figure 49. The model calibration results are shown in Figure 49 and it shows that this method looks promising for modeling the turbo pump mass flow rate.

\[ \dot{m}^2_p = \Delta P 10^r \left( \frac{\dot{m} p}{\omega} \right)^u \]  
(4.42)

\[ \dot{m}_p = \frac{\dot{m}^2_p}{\Delta P 10^r \left( \frac{1}{\omega} \right)^u} \]  
(4.43)
4.5.3 Hydraulic turbine modelling

4.5.3.1 Turbine power model

Hydraulic turbine is driven by high pressure fluid in high pressure accumulator. Hydraulic valve is used to control the mass flow rate through hydraulic turbine. A typical high speed hydraulic turbine efficiency is as shown in Figure 50.
Similar to the pump model, the hydraulic turbine is characterized using a map. The hydraulic turbine power model is investigated in this section. Turbine performance characteristics can be expressed in terms of coefficient of performance in [58].

\[
C_p = \frac{\dot{W}}{0.5\rho A_t V_\infty^3}
\]  

(4.44)

The coefficient of performance \( C_p \) is calibrated using the experiment data as a function of hydrofoil Reynolds number and tip speed ratio. Tip speed ratio is defined as the ratio of the rotor tangential velocity to flow velocity:

\[
\lambda = \frac{\omega R}{V_\infty}
\]  

(4.45)

Reynolds number is due to the sensitivity of hydrofoil lift and drag characteristics on blade Reynolds number [51]. This dependency is relatively small for a fixed turbine size. Thus, the coefficient of performance can be assumed as invariant with respect to Reynolds number. With fixed density assumption, the coefficient of performance can be taken as a function of turbine power and mass flow rate, which is analogous to power coefficient for compressor and pump in the previous discussion for the hydraulic pump and the centrifugal compressor. Further, tip speed ratio is also a function of shaft speed and mass flow rate, which is speed coefficient defined previously.

\[
C_p \propto \frac{\dot{W}}{V_\infty^3} \propto \frac{\dot{W}}{(m)^3}
\]  

(4.46)

\[
\lambda \propto \frac{\omega}{V_\infty} \propto \frac{\omega}{m}
\]  

(4.47)

With this simplification, the hydraulic turbine performance can be represented by the speed and power coefficients. Based on the data provided by manufacturers, the relationships for these two
coefficients are shown in Figure 51. In this case, a third order polynomial is used to approximate
the turbine performance.

\[ \frac{W_p}{m_T^3} = a \left( \frac{\omega}{m_T} \right)^3 + b \left( \frac{\omega}{m_T} \right)^2 + c \left( \frac{\omega}{m_T} \right) + d \]  \hspace{1cm} (4.48)

Figure 51. Identified hydraulic turbine model and test data

4.5.3.2 Turbine mass flow rate model

Investigating turbine flow using the same approach as a centrifugal turbo pump, the results are
shown in Figure 52. Using flow and head coefficients as discussed in hydraulic pump model
section, the mass flow rate of the turbine can be modeled in a similar way.

\[ C_{flow} = \frac{\dot{m}_T}{\omega} \]  \hspace{1cm} (4.49)

\[ C_{head} = \frac{\dot{m}_T^2}{\Delta P} \]  \hspace{1cm} (4.50)
4.5.3 Valve model

Valve flow is modelled based on flow factor $K_v$, which can be obtained using experimental data. Flow factor can be modelled as a nonlinear function of valve position (control current input) as shown in Figure 53. The mass flow rate $\dot{m}_{\text{valve}}$ through valve can be calculated by flow factor $K_v$ and pressure difference across valve $\Delta P$ [59]. In this modelling, valve loss is not modeled explicitly. However, the loss for the valve is already included in the valve performance map.

\begin{align*}
\dot{m}_{\text{valve}} &= K_v \sqrt{\Delta P} \quad (4.51) \\
K_v &= f(\theta_{\text{valve}}) \quad (4.52)
\end{align*}

In this study, the turbine and pump valves use the same actuator. The mass flow rate for both hydraulic turbine and pump valve can be calculated using the following equations:

\begin{align*}
\dot{m}_{\text{valve},P} &= f\left(\theta_{\text{valve},P}, P_{\text{Acc}}, P_2\right) \quad (4.53) \\
\dot{m}_{\text{valve},T} &= f\left(\theta_{\text{valve},T}, P_{\text{Acc}}, P_3\right) \quad (4.54)
\end{align*}
4.5.4 Modelling for piston accumulator

For the spring actuated accumulators the dynamics are governed by the spring constant and the combination of spring and piston masses. The natural frequency of a spring loaded accumulator is given by:

$$\omega_n = \sqrt{\frac{k_s}{m_p}}$$  \hspace{1cm} (4.55)

A dynamic model for a spring loaded accumulator consists of a first order differential equation of accumulator pressure and a second order differential equation of position mass, damper, and spring dynamics as shown in (4.56). This model is further intergraded with other subcomponents.

\begin{align*}
\dot{p}_{acc} &= \frac{\beta}{V_0 + xA} \left( \dot{m}_{pump} - \dot{m}_{turbine} - \dot{v}A \right) \\
\dot{x} &= \nu \\
\dot{v} &= \frac{(p_{acc} - p_{return})A - F_0 - F_f(v) - cv - kx}{xpA}
\end{align*}  \hspace{1cm} (4.56)
4.6 Model Validation and Plant Investigation

4.6.1 Model validation

A mean value diesel engine model coupled with regenerative assisted hydraulic assisted turbocharger was built in Simulink environment based on interconnected sub-models as shown in Figure 54. The governing equations and identification results have been presented in previous sections. The Simulink model includes engine air-path and hydraulic systems model. Since only transient test data for engine system without hydraulic system are available, only model validation results for engine air-path modelling are presented here.

![Engine system modeling in Simulink](image)

Figure 54. Engine system modeling in Simulink

First, a load step simulation is carried out. The integrated diesel engine air-path model is simulated with the inputs of engine speed, fuel injection, VGT vane position and EGR valve position with a transient load step in open loop sense. The simulation results are shown in Figure 55. Engine speed is kept at 2000 RPM. The initial increase of turbine mass flow rate is because
of VGT open action. Total turbine power is the sum of compressor power, friction power, and shaft power. Shaft power imbalance leads to TC shaft deceleration or acceleration action. During turbocharger acceleration, positive shaft power drives TC shaft to higher speed. Comparing with test data for three states (intake pressure, exhaust pressure, and TC speed), proposed model well represent system dynamics with reasonable agreement, some error happens for light load range. Further, an FTP cycle dynamometer test inputs (engine speed, fuel injection, VGT position, EGR position) are used to drive developed engine air-path model in simulation. The results are shown in Figure 57. Relative error distribution for intake pressure and exhaust pressure are presented in Figure 58. The relative errors for both states concentrate between ±10% bound. This provides confidence for future model-based analysis and controller design.

Figure 55. Open loop simulation with test inputs.
Figure 56. Model validation results for load step test

Figure 57. Model validation results for FTP 75 driving cycle
4.6.2 Plant behavior investigation

4.6.2.1 Engine air-path response with three actuators

To investigate plant dynamic behavior with three different actuator inputs, engine is simulated at 1200 RPM, 35 mg/cc fuel injection. Three different actuators change with different step variations. If one actuator has step action as shown in Table 12, the other two actuators are kept as the same as base value as shown in Table 12. The open loop simulation results are shown in Figure 59 Figure 60, and Figure 61.

<table>
<thead>
<tr>
<th>Base actuator position</th>
<th>Step change</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base VGT=50% opening</td>
<td>5%</td>
<td>100% =full close</td>
</tr>
<tr>
<td>Base EGR=15% opening</td>
<td>5%</td>
<td>100% =full open</td>
</tr>
<tr>
<td>Base RHAT=0kW</td>
<td>0.5kW</td>
<td>Negative=assist</td>
</tr>
</tbody>
</table>
The response of mass flow rate, pressure and TC speed are expected with respect to different actuators. With VGT vane close action in Figure 59, exhaust pressure dynamics is faster than intake pressure, which is due to small exhaust volume and fast change for turbine mass flow rate. Because EGR mass flow rate is dominated by pressure ratio across EGR valve with fixed valve position. With faster response of exhaust pressure, EGR mass flow rate increases, and then converges to steady state. The inverse response of compressor mass flow rate due to step change of VGT position shows the non-minimum phase behavior as in Figure 59. When the VGT vane open, an initially increased flow across the turbine results in increased compressor power $\dot{W}_c$, hence increased compressed air flow rate out of the compressor $\dot{m}_c$. This results in the emptying of the exhaust manifold at a much faster rate than the filling rate of the intake manifold. Eventually the exhaust manifold pressure $P_3$ will drop resulting in a reduction of the compressor power $\dot{W}_c$ and the compressor flow rate $\dot{m}_c$.

Similar behavior is found when the engine intake mass flow rate and intake manifold pressure response to step change of EGR valve position as in Figure 60. With a positive step change to the EGR valve, there is an initial flow, over the EGR valve, from the exhaust manifold (at pressure $P_3$) into the intake manifold. This results in an initial increase in the intake manifold pressure $P_2$, leading to an initial increase in the engine intake flow rate. However, flow of exhaust gas over the EGR valve implies a reduced flow across the turbine resulting in reduced turbine power and an eventual decrease in $P_2$ and $\dot{m}_{in}$ to the actual steady state values. The effect of the reduced boost on the overall intake manifold dynamics is delayed owing to the turbocharger dynamics. The asymmetry responses show highly nonlinearity of the engine air-path plant. This brings challenges for VGT-EGR system control. But, intake pressure, compressor mass flow rate well behaved response to step changes for VGT and EGR action.
With externally assisted power and regenerative power, there is no such non-minimum phase behavior as shown in Figure 61. With assisted power on TC shaft, compressor mass flow rate increases due to higher compressor power, hence, intake pressure increases. In the exhaust manifold side, increased exhaust pressure is due to increased engine exhaust mass flow rate, which is driven by intake pressure. However, the increased turbine power is not directly from VGT vane closing action, which results in the emptying of the exhaust manifold with similar filling rate of the intake manifold during transient operation. For steady state, since the VGT is kept at the same opening, the exhaust flow is not restricted by vane closing, which leads to exhaust manifold pressure is relatively lower than VGT closing case. Hence, pressure ratio across EGR valve decreases with assisted power. EGR mass flow rate decreases to its steady state flow. But this quit depends on how much assisted power on TC shaft. With high assisted power and wide open VGT position, pressure drops across EGR valve might not be able to drive demanded EGR mass flow rate anymore. In some extreme case, intake manifold pressure might be higher than exhaust manifold pressure. Then EGR valve will lose control authority of EGR mass flow rate. With regenerative power on TC shaft, exhaust energy extracted by turbine is used to drive both compressor and hydraulic pump. TC shaft speed drops because of reduced turbine power with reduced turbine mass flow rate. In extreme case, if constant TC shaft loading power exceeds the total turbine available power \( W_T - W_c - W_{\text{Loss}} \), then TC shaft speed intended to converge to zero, leading turbocharger to stall. With loading on TC shaft, turbine flow is restricted by slowing down turbine speed, leading to lower exhaust manifold pressure. Meanwhile, decreased compressor power leads to lower compressor mass flow rate. Hence, lower intake manifold pressure is observed. In this case, pressure ratio across EGR valve increases, which leads a higher EGR mass flow rate.
Table 13. Operating points for model linearization

<table>
<thead>
<tr>
<th>Engine speed [RPM]</th>
<th>Fuel injection [mg/stroke]</th>
<th>VGT position [%]</th>
<th>EGR valve position [%]</th>
<th>External power on TC shaft [kW]</th>
<th>Eigenvalue</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>25</td>
<td>55</td>
<td>6</td>
<td>0</td>
<td>-150.2801 -14.5672 -0.3581</td>
</tr>
<tr>
<td>1200</td>
<td>35</td>
<td>50</td>
<td>12</td>
<td>0</td>
<td>-93.0635 -9.5879 -0.8150</td>
</tr>
<tr>
<td>1500</td>
<td>45</td>
<td>45</td>
<td>10</td>
<td>0</td>
<td>-95.6492 -12.4370 -1.7939</td>
</tr>
<tr>
<td>2000</td>
<td>50</td>
<td>40</td>
<td>8</td>
<td>0</td>
<td>-86.3667 -5.6772 -5.6772</td>
</tr>
</tbody>
</table>

These behaviors further are confirmed by analyzing the linearized model. Three states model are linearized symbolically through using Matlab. Four equilibrium points were analyzed through linearization as shown in Table 13. Equilibrium points were simulation results from nonlinear plant simulation with steady state inputs. Since assisted hydraulic turbine or regenerative hydraulic pump only provides assisted or regenerative power for transient operation, base hydraulic power was set as zero. Results show real stable eigenvalue for all the equilibrium points as in Table 13.

Based on bode plot in Figure 62, at high load operating condition, VGT has more control authority over boost pressure relative to light load. It can be expected that EGR valve will have less control authority for boost pressure compared to light load. However, with assisted power on TC shaft, this shape the current VGT-EGR system control scenario; from Figure 62, it shows the benefit of assisted turbocharger for boost pressure and TC speed control for its higher DC gain and higher bandwidth with input unit of kW. It also shows higher control authority for EGR mass flow rate than VGT. Hence with assisted power, boost pressure might not depend on VGT action; VGT vane position might be used to control EGR mass flow rate. It also shows that VGT
actuator might be redundant for both EGR control and boost pressure control, since EGR valve dominates EGR mass flow rate regulation. Assisted power dominates boost pressure regulation. In this case, boost control and EGR mass flow rate control can be decoupled with assisted and regenerative power on TC shaft.

Figure 59. Engine air-path response respect to VGT position with step change
Figure 60. Engine air-path response with respect to EGR valve with step change
Figure 61. Engine air-path response with respect to assisted and regenerative power with step change
Figure 62. Frequency analysis for different engine operating points
4.7 Conclusion

In this chapter, a diesel engine air-path with regenerative hydraulic assisted turbocharger system is developed. The model (simulator) is developed by integrating engine system, variable geometry turbocharger system and hydraulic system. The exhaust and intake manifolds are modeled as volumes with ideal gas having constant specific heats. The EGR valve is modeled with the valve flow equations for flow through orifices. The effective area is determined experimentally. Volumetric efficiency, temperatures rise are modeled as static nonlinearities. Physics based compressor and turbine model are integrated with engine air-path system. For the turbine modelling, it uses VGT as direct input for turbine power and flow regulation. Compressor power uses speed and compressor mass flow rate. The hydraulic system is connected with turbocharger system through shaft speed dynamic equation. New sub-models for hydraulic turbine and centrifugal hydraulic pump are developed.

Engine air-path model is validated through engine transient test data. It shows proposed modelling approach has high fidelity with only three states. It could be used for model-based analysis and controller design. The complexity of the interactions between the VGT, EGR and hydraulic power has been illustrated through simulation of the nonlinear model. The model
CHAPTER 5: SYSTEM ANALYSIS FOR HYDRAULIC ASSISTED TURBOCHAED DIESEL ENGINE THROUGH 1-D SIMULATION

5.1 Abstract

Engine downsizing and down-speeding are essential to meet future fuel economy mandates. Further pushing the envelope for even better fuel economy improvement without compromising vehicle drivability via a turbocharged engine will run into a major constraint, i.e. transient response of the turbocharger. When high torque is demanded during acceleration, it is not available until a few seconds later with conventional turbocharger technologies, which is called "turbo lag." The key to the market acceptance of the downsized turbocharged engine to reduce petroleum consumption is to bring the torque to demand level without a noticeable delay. A regenerative hydraulic assisted turbocharger (RHAT) system is proposed in this chapter. In this new system, a hydraulic turbine is used to spin the turbocharger shaft via high-pressure fluid out of a tank; a turbo pump is used to absorb excessive power from the turbocharger shaft while pressurizing the fluid and pumping back into the tank. A driveline pump is also used to recover vehicle kinetic energy during vehicle deceleration mode and pump the fluid into the high-pressure tank. Both the hydraulic turbine and the turbo pump are packaged inside the turbocharger center housing. The RHAT concept itself will fundamentally change the operation format of a turbocharged engine, with reduced turbo lag, engine pumping loss as well as improved surge margin. Compared to traditional electric assisted and regenerative turbocharger, RHAT has a much higher assist and regenerative capability, except that it is more durable and cost effective. The abundance hydraulic energy that is recovered during vehicle deceleration can be used to assist the turbine so the variable geometry turbine (VGT) can operate at most efficient open positions rather than at small and inefficient positions to meet the compressor power
demand. With two extra actuators on turbocharger shaft, variable geometry turbocharger (VGT) could be potentially replaced with fixed geometry turbocharger for higher efficiency, lower cost, and better durability. A 1D production vehicle with medium duty turbocharged diesel engine model was used in the investigation. The hydraulic turbine and turbo pump maps and driveline pump were predicted out of 1-dimensional hydraulic model analysis from suppliers. A baseline controller was coupled with the 1D model and upgraded to control the engagement and disengagement of RHAT and energy management in the hydraulic energy storage tank. The preliminary 1-D simulation demonstrates that the proposed RHAT turbocharger system can significantly improve engine transient response. The 1D vehicle level simulation shows that 3-5% fuel economy improvement for FTP 75 driving cycle, depending on different sub-component sizing. The study also identifies technical challenges for optimal design and operation of RHAT, as well as additional fuel economy improvement opportunities that are enabled by the RHAT.

5.2 Introduction

5.2.1 Assisted turbocharger

Downsizing the engine for better fuel economy run into the turbo lag issue, among others. When high torque is demanded during acceleration, it is not readily available until a few seconds later for a typical turbocharged engine, which is called "turbo lag." Drivers would consider this turbo lag "lack of power." Conventional turbocharged engines have no effective synchronization between the engine and the turbocharger. That means that during the engine tip-in operation, the turbo lags behind the engine. This turbo lag results in sluggish air supply that is a root cause of transient smoke emission and lack of torque response. On the other hand, during the engine tip-out, i.e. sudden slowdown in engine speed and air requirement, the turbocharger, due to high
inertia, may continue to spin and pump the air more than required, driving the compressor into surge (typically high-pressure ratio and low mass flow) area.

For a turbocharged engine, diesel or gasoline, the instantaneous engine power or torque lug curve is different from the peak power or torque curve when the engine reaches steady state. Depending on the turbocharger system and the EGR purging time, a turbocharged engine may take 1 to 20 seconds to reach steady state torque or power. Figure 1 shows the instantaneous power of a light-duty diesel engine, measured out of load step response tests on an engine dynamometer. With the sluggish transient response of this engine, the vehicle transmission has to downshift to give the customer “crisp” response perception, thus pushes the engine to operate at a higher speed where the engine efficiency is much lower than the medium engine speed. Should the turbo have assisted power to accelerate to high boost/speed without turbo lag, the transmission downshift may be avoided, which will translate to 1-2% fuel economy improvement, besides the transient response improvement.

Figure 63. Instantaneous engine power lug curve based on a light duty diesel load step test
Figure 64. Instantaneous Engine EGR Rate, Soot and NOx Emissions during a light duty diesel FTP transient test

To address the impact of turbo lag on vehicle transient performance of turbocharged engines, typical measures taken by vehicle manufacturers include:

a. Transmission downshifting: shifting the transmission to a lower gear. Thus the engine operates at a higher speed to gain more power since the turbo lag prevents the engine from gaining torque instantly. The operation of the engine at a higher speed, thus higher friction and pumping loss, will result in fuel economy penalty (Figure 63);

b. Reduction of exhaust gas recirculation (EGR) during the tip-in operation replaces the exhaust gas with fresh air for better torque response. The momentary reduction of EGR will certainly lead to NOx emission spikes as shown in Figure 64, thus future NOx emission control challenges;

c. Reduction of smoke limited air-fuel ratio (AFR) allows the engine to momentarily run rich (thus higher soot emission) for better torque response. The smoke spike during this transient operation (Figure 64) may not be visible on diesel engine due to the wide application of diesel
particulate filters (DPF). However, accelerated soot loading on the DPF will result in DPF durability and fuel economy penalty concerns due to frequent DPF regenerations;

d. Increase in “air reserve” before the engine tip-in operation to allow more fuel to be injected into the cylinder and more exhaust energy to spin the turbine for more boost, more air, thus more torque at the moment of tip-in. This high “air reserve” at light load to ensure "crisp" transient response causes high pumping loss on diesel engines.

e. To address the tip-out surge issue, a high surge margin is needed for automotive compressors. The design requirement for wide operation range of the compressor also compromises the peak efficiency.

To ensure the right amount of air at any moment for the turbocharged engine during steady state or transient state, certain types of synchronization (like the mechanical connection between the engine and a supercharger) between the engine and the turbocharger, i.e. an assisted turbocharger, is needed. Since an assisted turbocharger will need substantial power during transient operation, in order to avoid FE penalty on the assisted turbocharger system, any assisted boost system has to be energy regenerative, i.e. it should be able to recover energy, e.g. during engine deceleration, engine or vehicle braking. With such a regenerative and responsive turbocharger system, even a downsized engine can work with a stiffer torque converter, more aggressive transmission shifting schedule and gear ratio, thus enabling substantial FE savings.

5.2.2 Hydraulic assisted and regenerative turbocharger

Different to other exhaust waste heat recovery technologies, such as turbocompound, which compete with reciprocal internal combustion engines for combustion energy, the regenerative assisted turbocharger proposed in this chapter, recovers waste energy, mainly during engine deceleration, throttling, or exhaust braking modes and vehicle braking events.
The hydraulic-assisted turbocharger has been studied since the early-1980's [64]. There have been publications and patents on using high-pressure, oil-driven hydraulic turbines on the turbocharger shaft to accelerate the turbocharger during acceleration [60]-[63]. A hydraulic turbine was added between the conventional compressor and the turbine wheels to assist the transient acceleration of the turbocharger, when energized by high-pressure fluid. The hydraulic wheel was very compact and could be integrated into the turbocharger center housing. However, all the previous arts require a stand-alone hydraulic pump to build up and maintain high pressure in the hydraulic accumulator.

One of the main reasons that the hydraulic-assisted turbo technology has not been widely accepted was that the stand-alone mechanical-driven hydraulic pump to pressurize the fluid had very high fuel economy penalty, which was not acceptable.

Two energy recovery devices are proposed for this hydraulic assisted turbocharger [59], including a turbo pump on turbocharger shaft and a driveline driven pump that is only engaged during vehicle deceleration. The primary energy recovery would be from during vehicle deceleration; The secondary energy recovery would be from turbo pump on the turbo shaft. The total system benefits come from the total amount of energy that can be recovered.

Every turbo acceleration will be followed by a deceleration. The exhaust gas energy should and could be recovered during engine deceleration, throttling, or exhaust braking mode rather than firing mode, as far as an energy storage device is available. Firing mode energy recovery without fuel penalty is only limited to very high engine load [66]. A hydraulic turbo pump on the same compressor/turbine wheel can recover part of the exhaust energy except it is much more compact, durable, and cost effective than the electric motor/electric energy storage system [59], [63]. The pressurized fluid out of the turbo pump can then be saved in the high pressure hydraulic/pneumatic
accumulator to be used later as a driving force on the hydraulic turbine that shares the same turbo shaft. The high-speed turbo pump, like the hydraulic turbine, is also a matured technology in the aerospace industry [67]. NASA in 1974 published a report indicating the efficiency of a hydraulic turbo pump to be around 73% for rocket applications [67]. Other studies (including research at Ford Motor Company a few years ago on a hydraulic assisted turbocharger), also indicated around 72% efficiency on large-size hydraulic turbines [59],[68],[69].

A combination of the hydraulic turbo pump and the hydraulic turbine on TC shaft can provide extra actuator for turbo speed control, i.e. to spin up the turbocharger for more air flow during engine acceleration or slow down the turbocharger to recover exhaust energy and to reduce air supply to the engine, avoiding surge during deceleration, as well as manipulating the turbo speed in steady state conditions to high efficiency areas of compressor and turbine to maximize turbocharger, thus engine system efficiency.

In order to harvest vehicle brake energy during vehicle tip-out, another hydraulic energy regeneration device (a driveline pump) is added on the driveline, which recovers vehicle brake energy during vehicle brake events. This idea is similar to the regeneration mode of the hydraulic hybrid vehicle However, for a hydraulic hybrid vehicle, a hydraulic tank with large volume is needed to store hydraulic energy, which is used to launch the whole vehicle. The tank size could be 112 liters with 420 bar designed pressure for a 15,000kg vehicle [73]. But in this study, a compact size tank would be enough to drive a hydraulic turbine for turbocharger assist due to the small inertia of a turbocharger. With the vehicle brake energy recovery, the assist capability of a hydraulic turbine would be further enhanced.

The regenerative hydraulic assisted turbo (RHAT) concept itself may fundamentally change the operation format of a turbocharged engine, i.e. to provide a means to have a "synchronized"
transient operation between the engine and turbocharger, thus addressing turbo lag and tip-out surge issues for turbocharged engines. The focus of this study was: using a hydraulic driveline pump and turbo pump (integrated in a turbocharger) to recover part of vehicle braking energy and engine exhaust energy to drive a hydraulic turbine (also inside the turbocharger) during engine acceleration. The energy from the hydraulic driveline pump and turbo pump is saved in a pressurized hydraulic/pneumatic storage. The details of this concept were included in a patent [59].

The main advantage of RHAT, compared to the electrically assisted turbocharger system, is power density and compactness of hydraulic wheels. Although the regenerative electric assisted turbocharger (REAT) is very attractive, it has the following concerns for light and medium duty automotive diesel applications:

1. As shown in Table 14, most of current electric assisted motor lies in the range of 1.5-10 kW. The desired power requirement to assist the acceleration of large turbochargers may be more than 10kW, depending on turbocharger size. This high assisted power is a challenge for vehicles with 12 volts electric infrastructure.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Designed speed</th>
<th>Max power</th>
<th>Supply voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ibaraki [75]</td>
<td>120,000 RPM</td>
<td>3 kW</td>
<td>72V</td>
</tr>
<tr>
<td>Pfister, P. D [76]</td>
<td>200,000 RPM</td>
<td>2 kW</td>
<td>65V</td>
</tr>
<tr>
<td>Takata [77]</td>
<td>220,000 RPM</td>
<td>2 kW</td>
<td>72V</td>
</tr>
<tr>
<td>Terdich [78]</td>
<td>120,000 RPM</td>
<td>3.5kW(motor) 5.4kW(generator)</td>
<td>650V</td>
</tr>
<tr>
<td>Noguchi [79]</td>
<td>150,000 RPM</td>
<td>1.5kW</td>
<td>12V</td>
</tr>
<tr>
<td>David [80]</td>
<td>75,000 RPM</td>
<td>10kW</td>
<td>200V</td>
</tr>
<tr>
<td>Winward [82]</td>
<td>180,000 RPM</td>
<td>5kW</td>
<td>-</td>
</tr>
</tbody>
</table>

2. The power capacity needed to recover exhaust energy during deceleration and braking/motoring should be in the range of 15-40 kW, since the turbo would be operating at
high speed with high kinetic energy when the excessive exhaust energy is available to be harvested. It is challenging that an electric motor of this power can be packaged and integrated into an automotive turbocharger center housing without a dramatic increase in inertia and mechanical stress when operating at high speed. That means the current production-ready 2kW electric motor supported by 12 volts infrastructure will not effectively recover exhaust energy when it is abundant.

3. For the 2 kW REATs that are available today, they typically have substantial increase in rotational inertia over conventional turbochargers [65], i.e. during acceleration, more energy will be used to rotate the motor itself.

4. A permanent magnetic electric motor is more efficient and compact than an induction motor, but the performance may deteriorate over time under high temperature; the material strength of the permanent magnet may limit its application to low operation speed, which is incompatible to smaller turbochargers.

The design target of the RHAT system will be 50% “round trip” efficiency, which may be slightly lower than a state-of-the-art regenerative electric-assisted turbocharger. But the advantages like lower inertia, cost, smaller packaging space, and potential better durability make the RHAT an attractive alternative option, especially for medium and heavy-duty turbocharged diesel applications.
Figure 65. System layout of diesel engine with hydraulic assisted and regenerative turbocharger

![System layout of diesel engine with hydraulic assisted and regenerative turbocharger](image1)

Figure 66. Hydraulic assisted turbocharger layout

Figure 65 illustrates the proposed Regenerative Hydraulic Assisted Turbocharger (RHAT) system. Whenever the vehicle or engine has ‘free’ energy (e.g. during vehicle or engine deceleration, exhaust braking, or steady state when the intake throttle is used for intake oxygen control, or when a wastegate is used, etc.), the driveline pump will be engaged to recover vehicle kinetic energy, while the hydraulic turbo pump is powered by gas turbine. The power collected by both driveline pump and the turbopump on TC shaft will pressurize the fluid and at the same time the turbo slows down to "synchronize" with the decelerating engine to avoid "tip-out" surge. The pressurized fluid out of the two pumps will pass a check valve and be saved in a high-pressure hydraulic accumulator. During the engine acceleration, the high-pressure fluid from the accumulator will be discharged into and drive the hydraulic turbine to accelerate the turbocharger.
When the TC turbine wheel gets external hydraulic energy, VGT can open up for better efficiency. Thus the enthalpy drop across the turbine will be reduced, thus engine pumping loss is reduced and engine net power output is increased. With higher enthalpy at TC turbine outlet, the temperature for after-treatment would be higher with hydraulic assist. Hence, after-treatment conversion efficiency would be improved due to increased temperature. With external energy input on the shaft, the turbine works in lower expansion ratios (i.e. at high turbine speed ratio, U/C as shown in Figure 67. Also, the turbine does not count on the small nozzle to collect sufficient exhaust energy to drive the compressor. Therefore, the turbine operates with lower expansion ratio, wider open nozzle, and higher speed, thus operating more efficiently (see Figure 67). Both the high U/C and relatively large nozzle may potentially make the turbine 30-50% more efficient than the turbocharger without external energy input. That means the RHAT converts exhaust energy into mechanical energy more efficiently, further improving the engine transient response with improved fuel economy, i.e. the improvement in transient response is partly from external hydraulic energy and partly from improved turbine and compressor efficiency. During very aggressive tip-out, the hydraulic loading on the turbo shaft will slow down the turbo to avoid trip-out surge and recover the aerodynamic and kinetic energy of the turbocharger.

Figure 67. With assisted turbocharger, turbocharger can be regulated to its higher efficiency operation range.
During very aggressive tip-out, the hydraulic loading on the turbo shaft will slow down the turbo to avoid trip-out surge and to recover the aerodynamic and kinetic energy of the turbocharger, as shown in Figure 68.

Figure 68. The Hydraulic turbo pump can recover exhaust energy and avoid tip-out surge.

5.2.3 System causality

In order to mitigate the tradeoffs between fuel benefit, emission reduction and performance improvement, a system approach is investigated. System causality with hydraulic assisted turbocharger is shown in Figure 69. For a given turbocharger, engine and vehicle, there would be an optimal design solution to achieve different design target with given design constraints.
For performance improvement consideration, the hydraulic system should be designed to provide high assisted power as well fast response through a hydraulic turbine. To provide sufficient energy for the assist, tank volume should be designed as large as possible with high pressure. However, the largest tank volume size is limited by physical packaging size as well as the driveline pump and turbo pump energy recovery capability.

To meet stringent emission constraint, the air-fuel ratio (AFR) would be optimized with assisted turbocharger. For instance, the combustion air-fuel ratio set point can be dropped to a lower level with assisted turbocharger. An engine without assisted turbocharger needs higher AFR ratio setpoint to accommodate potential driver’s aggressive tip-in, by avoiding smoke limit. However, higher AFR set point will result in higher target boost pressure. In this case, higher exhaust pressure is needed to drive turbocharger with vane nozzle closing for VGT turbocharger. This will result in high pumping loss. With assisted turbocharger, air can be supplied to engine cylinder with a faster response by applying energy directly to turbocharger shaft.
With assisted turbocharger, variable geometry turbocharger can be replaced with fixed geometry turbocharger with waste-gate. But the system interaction depends on turbine sizing and hydraulic components sizing. With properly components sizing, the hydraulic assist can be used for engine light load tip-in. During engine medium and high load operations, the turbocharger is controlled through waste-gate and hydraulic components. This structure could also be applied to a turbocharged gasoline engine. For diesel engine application, this might eliminate nozzle vane for the turbocharger, which leads to potential cost reduction.

Driveline energy recovery capability depends on driveline pump size and vehicle weight. Hydraulic turbine and pump sizing on TC shaft depends on engine sizing and turbocharger sizing with considering the design target. Optimal designs of the hydraulic turbine and turbo pump that can deliver high efficiencies over wide operation range, regarding the pressure and turbo speed.

An energy storage management strategy should regulate the tank energy so that hydraulic pump and hydraulic turbine can work in high-efficiency areas at a wide range of flow and speed. The optimal control strategy is needed to best utilize the RHAT and operate the turbocharger in high-efficiency areas to achieve maximum engine system efficiency over entire customer driving cycles within the constraint of emission.

Governing the hydraulic pump and turbine wheels provides a means to "synchronize" the turbocharger with engine operation conditions to ensure the compressor and turbine working in a narrower but more efficient area, i.e. the hydraulically governed turbocharger will allow the compressor and turbine to be designed for higher efficiency, since they don't have to trade the efficiency for operation range. The hydraulic-governed turbocharger may control the airflow independently from engine operation conditions, thus eliminating "turbo lag" and the necessity of intake throttle and wastegate for turbocharged engines.
In this study, 1-D simulation approach is used to investigate the benefits and design trade-offs for regenerative hydraulic assisted turbocharger. The chapter is organized into three major parts:

1. Engine steady state operation with assisted and regenerative capability
2. Transient response improvement with hydraulic assisted turbocharger
3. RHAT design trade-off investigation through driving cycle study.

5.3 Vehicle Level Integrated Simulation in GT-Power / Simulink

5.3.1 Simulation platform and control algorithm

Vehicle level simulation is utilized to investigate hydraulic assisted turbocharger fuel benefit and design trade-offs. The 1-D model as shown in Figure 70 includes a vehicle model, driver, engine, torque converter, transmission, driveline, regenerative hydraulic-assisted turbocharger. The control includes a production version engine control, 6 speed transmission control, torque converter control, and designed RHAT control.

![Simulation platform and control structure](image)

Figure 70. Simulation platform and control structure
Figure 71. Modeling layout in GT-suites
In simulation, a production control algorithm for regular VGT turbocharger diesel engine control was modified with the following changes in the air handling and control strategies:

- Due to the energy input at the turbo shaft, the expansion ratio across the turbine may be too low to pump the conventional high-pressure exhaust gas recirculation (HP-EGR). Thus the low-pressure EGR (LP-EGR) is used, when necessary, in the analysis. In the LP-EGR, having EGR flowing through the compressor and turbine, instead of taking a short cut and bypassing the compressor and turbine, will have the turbine and compressor operate at more efficient areas at low engine speed and load conditions.

- RHAT control is designed to track the target boost pressure with a fixed gain PI controller.

- Driveline pump is controlled to recover vehicle brake energy within the constraint of max hydraulic tank pressure.

The base transmission shifting strategy, boost pressure set-point, fuel injection set-point and EGR fraction set-point are kept the same for all cases in this study. The engine air handling control is to track target engine boost pressure and EGR rate as shown in Figure 72.

![Figure 72. Engine air-path controller overview](image-url)
The vehicle simulation is to track the target vehicle speed. A driver model is used to control gas and brake pedal positions by feedback action. Target engine brake power is based on calibrated map with inputs of engine speed and gas pedal position. Base fuel injection is based on current engine speed and demanded engine torque. Fuel injection is controlled by both feedback and feedforward loops to achieve target torque output. VGT vane position is feedback controlled to track the required boost pressure, which is from pre-calibrated boost map. The transmission control is based on shift schedule map with inputs of engine speed and gas pedal position. Both HP-EGR and LP-EGR are used to achieve target EGR mass flow rate. Both VGT and EGR valve controllers are map based with gain scheduled PI controller. Detailed engine and turbocharger model validation can be found in [25]. The vehicle system model (including engine and turbocharger) validation is shown in Figure 73.

![Vehicle model validation](image)

Figure 73. Vehicle model validation through FTP_75 driving cycle
5.3.2 Hydraulic components

Preliminary meanline analyses of the hydraulic turbine and turbopump were conducted by a supplier. The regular engine oil at 100 deg C was assumed in the meanline analyses. From friction loss perspective, engine oil may not be the optimal choice, due to high viscosity. However, the fluids between the high-pressure RHAT and low-pressure lubricant in the turbo center housing can't be sealed 100%; the engine oil is the best choice at this point. The preliminary predicted hydraulic turbine map (Figure 74) shows that when the turbine power reaches above 10 kW, substantial operation area of the hydraulic turbine can have an efficiency above 70%, even though the high-efficiency operation area is not as large as a typical electric motor. When managed carefully between flow rate and pressure ratio, hydraulic turbine efficiency can be above 60% (Figure 74). is the preliminary meanline analysis of the performance of a turbo pump, indicating that the turbo pump can achieve 70% or better efficiency as far as the pressure in the hydraulic energy storage is managed to match the oil flow rate or the turbo speed.

The aforementioned analytical maps of a hydraulic turbine, turbo pumps and driveline pump out of meanline analyses (Figure 75, Figure 74, Figure 76) were integrated with GT-Power vehicle model for the engine system transient response investigation as well as fuel economy assessment over the FTP 75 driving cycle. The oil temperature was kept at 100 deg C throughout the cycle.

Other assumptions of the RHAT system include:

a. Hydraulic power 25 kW on RHAT_Turbine for assisted mode and 25 kW on RHAT_Pump in energy recovery mode; a 25 kW driveline pump to recovery vehicle kinetic energy during vehicle brake.
b. Hydraulic fluid pressures: 100-150 bar in Accum-HP tank, and 10-20 bar in Accum-LP tank; The pressure of low-pressure tank is set to avoid cavitation in the RHAT_Pump.

c. Hydraulic storage space varies for design trade-off investigation.

d. Hydraulic valve actuator response time<50 ms.

e. With assisted power on the shaft, the TC turbine will operate in lower expansion ratio and higher speed region, compared to operation without assisted power. Thus assisted and regenerative turbocharger needs further data extrapolation out of current turbine flow bench tested map. A further experimental test should be used for wide range turbine operation investigation. In this study, map extrapolations are based on 1-D simulation software [82].

A comparable study between hydraulic components and electric components for assisted turbocharger is illustrated in Figure 77. Hydraulic components (turbine and pump) have much higher power density than an electric motor or generator. It clearly shows that hydraulic turbine has higher torque than electric motor below 70K rpm, where the assist is needed for TC shaft for engine light load tip-ins. For the low-speed region, the assisted torque from the hydraulic turbine is three times of that from the electric generator. Hydraulic pump has much higher torque above 50K rpm, where high regeneration load is needed for energy recovery from TC shaft. These results clearly show the power density of regenerative hydraulic assisted turbocharger is greater than that of regenerative electric assisted turbocharger.
Figure 74. Hydraulic turbine efficiency

Figure 75. Hydraulic pump efficiency

Figure 76. Driveline pump efficiency
5.4 Simulation results and discussion

5.4.1 Engine alone steady state investigation for feedforward calibration

For a given engine speed and fixed fuel injection amount, with assisted power, exhaust pressure intends to decrease. Thus pumping mean effective pressure (PMEP) and engine brake specific fuel consumption (BSFC) will decrease. However, this improvement can only be maximized with optimal VGT vane position. This means VGT needs to be coordinately controlled with assisted power.

With loading power on TC shaft on the other hand, e.g. during engine throttling mode, TC speed intends to decrease. Lower TC speed will reduce compressor power. Thus compressor mass flow rate and intake manifold pressure decrease. Turbine mass flow rate decreases with lower TC speed. Hence, exhaust pressure intends to increase. Higher exhaust pressure and lower intake manifold pressure will lead to high pumping loss and lower engine efficiency. For different engine operations, thermal energy available to the TC turbine is dependent on engine operation point as well as VGT position. Thus regenerative power on TC shaft will be defined by engine load as well as VGT vane position.

Figure 77. Torque comparison between electric motor and hydraulic components
Based on turbocharger power balance equation, [83]:

\[
J\dot{\omega} = (\dot{m}_t)^2 \omega D \frac{R T_3}{P_3} \tan(f_1(VGT)) - (\dot{m}_c)^3 f\left(\frac{\omega}{\dot{m}_c}\right) + P_{\text{rhat}}
\]  

(5.1)

Where, \(f_1(u_{\text{VGT}})\) is vane angle; \(u_{\text{VGT}}\) is vane position control input. For steady state with assisted power, \(J\dot{\omega} = 0\), power balance equation becomes:

\[
(\dot{m}_t)^2 \omega D \frac{R T_3}{P_3} \tan(f_1(VGT)) + P^+_{\text{rhat}} = (\dot{m}_c)^3 f\left(\frac{\omega}{\dot{m}_c}\right)
\]

(5.2)

Max assisted power \(P^+_{\text{rhat, max}}\) can be defined by:

\[
P^+_{\text{rhat, max}} = (\dot{m}_c)^3 f\left(\frac{\omega}{\dot{m}_c}\right) - (\dot{m}_t)^2 \omega D \frac{R T_3}{P_3} \tan(f_1(VGT))
\]

(5.3)

Max hydraulic assisted power is defined by demanded boost requirement and VGT position, and is limited by compressor surge. For steady state TC shaft energy regeneration, power balance equation is:

\[
(\dot{m}_t)^2 \omega D \frac{R T_3}{P_3} \tan(f_1(VGT)) = (\dot{m}_c)^3 f\left(\frac{\omega}{\dot{m}_c}\right) + P^-_{\text{rhat}}
\]

(5.4)

Max regeneration \(P^-_{\text{rhat, max}}\) power is defined by

\[
P^-_{\text{rhat, max}} = (\dot{m}_t)^2 \omega D \frac{R T_3}{P_3} \tan(f_1(VGT)) - (\dot{m}_c)^3 f\left(\frac{\omega}{\dot{m}_c}\right)
\]

(5.5)

Max regeneration power from TC shaft depends on total energy availability to turbine as well as VGT position. Turbine mass flow rate can be represented as a function of VGT position and turbine pressure ratio \(P_{\text{rt}}\) as [84]:

\[
\dot{m}_t = f_2(VGT) \cdot f(P_{\text{rt}})
\]

(5.6)

Comparison of flow and power function, VGT position:
Hence, for a given turbine power demand without RHAT, there is an optimal VGT position for best system efficiency. With assisted power on TC shaft, compressor mass flow rate increases, and turbine mass flow rate increases. In order not to restrict turbine flow, VGT position should open with regenerative load on TC shaft. With opening of VGT position, turbine efficiency changes.

The aim of this study is to show that how assisted and regen power on TC shaft affects engine performance through steady state simulation. The engine is simulated at 2000 RPM with 60 mg/stroke fuel injection rate. Both EGR valves are fully closed for this case, VGT position varies from 0.15 to 1 (0.15 is fully close and 1 is fully open), and TC shaft loading power varies from -7kW to 7kW (positive is regenerative and negative is assisted power). It shows that engine BSFC changes with different VGT position and different assisted and regenerative power on TC shaft, as illustrated in Figure 79. For a typical system without assisted power or regenerative power, best turbine efficiency is located at VGT position=0.6. In order to have optimal (minimum) BSFC with assisted power on TC shaft at light engine speed and load, VGT should be open wider for maximum turbine efficiency. For example, best VGT position for 7kW assisted

![Figure 78. VGT impact on turbine power and mass flow rate](image-url)
power is VGT position=0.8. On the contrary, with loaded regenerative power on TC shaft at high engine speed and load, VGT should be closed down to achieve best turbine efficiency. In order to achieve optimal BSFC, both VGT position and assisted or regenerative power on TC shaft should be controlled coordinately as shown in Figure 79. If the flow is restricted too much such as VGT being closed to 0.4, this will lead to high fuel consumption.

The loading capacity on TC shaft depends on VGT position for a given engine operation. For this case in Figure 79, wider open VGT position decreases energy availability of turbine, leading to lower TC shaft loading capacity. For example, for VGT position=0.8, max TC shaft steady state loading power is 2kW. Power higher than 2kW will lead to turbocharger shaft stall, which means steady state turbine power cannot balance with compressor power.

![Figure 79. Engine BSFC under assist and regeneration](image)

For assisted mode and regeneration mode, turbine power and compressor power change with different level of input external shaft power. VGT fixed at 0.6 is investigated in this case. For baseline case, the compressor power is the same as turbine power without external power, as shown in Figure 80. During assisted mode, lower extracted turbine power leads to lower exhaust
pressure, and temperature. Compressor power is the sum of assisted hydraulic turbine power and turbine power. During regen mode, both compressor and TC turbine power drop because of lower TC speed and lower mass flow rate. The loss of engine power is mainly due to increased pumping loss thus lower engine efficiency.

![Power distribution between turbine and compressor](image_url)

Figure 80. Compressor and turbine power distribution under assist and regeneration

Engine air-fuel ratio (AFR) increases with higher assisted power as shown in Figure 81. reducing smoke emission during transient tip-in operation. This also indicates that more fuel can be injected into the cylinder to maintain the same AFR to achieve higher engine brake power with higher level of assisted power. This would be beneficial for engine performance for both steady state and transient operations. For steady state case as shown in Figure 82, the baseline case without assist and experimental case with 19kW TC shaft hydraulic assisted power have the same AFR. Engine max power can be significantly improved at light load with 19kW assisted power. Low-speed engine torque increases almost 4 times with the assisted power, compared to the case without assisted power. The results are based on the assumption that energy in the hydraulic tank is available to drive hydraulic turbine on TC shaft. For transient improvement,
engine operation during the transient process would avoid smoke limit with the external assisted power. This is discussed in next section.

![Air-Fuel Ratio](image)

Figure 81. Air fuel ratio under assist and regeneration

As mentioned previously, TC shaft loading capacity also changes with engine operation point. As shown in Figure 83, a case at multiple engine load points was studied with different assisted and regenerative power on TC shaft as well as varying VGT position. It is concluded that, for light engine load, it might not be possible to load any power on TC shaft to recover energy during steady state condition, such as engine speed=500 RPM. At engine speed= 1500 RPM, the TC shaft valid loading region expands with higher fuel injection level, which is due to higher engine exhaust energy. For medium engine speed from 1500 RPM to 2500 RPM, loading power on TC shaft will lead to high fuel consumption by increasing engine pumping loss and reducing engine efficiency. For high engine speed operations (3500 RPM), loading capacity on TC shaft is much wider than lower engine speed operations (<3500 RPM). Further, TC shaft energy can be recovered with minimum fuel penalty with high engine load region, which agrees with the finding in the literature [66], [70]. It can also be concluded that engine fuel consumption can be reduced with power regeneration at certain conditions, depending on engine load condition. Steady state optimization table for each engine operating points can serve as
feedforward calibration table for RHAT-VGT control. Note that, the investigation, in this case, does not include the impact of EGR (No EGR mass flow rate).

The hydraulic assisted and regenerative turbocharger introduces extra control inputs into engine system. It increases optimization dimension. With assisted power on TC shaft, engine BSFC will be improved. The proposed high power hydraulic turbine on TC shaft can significantly improve engine light load torque response. TC shaft loading capacity depends on engine speed and load condition. Coordinated control for VGT and RHAT should be implemented for optimal engine BSFC.

![Graph showing engine brake power and torque with and without assist at different engine speeds.]

Figure 82. Max engine power with 19 kW assist with same AFR
Figure 83. Loading power on TC shaft for engine BSFC impact (+: regeneration, -: assist)
5.4.2 Transient response improvement investigation

With assisted power from the hydraulic turbine, the transient response of engine would be significantly improved. In this study, transient behaviors under the different level of assisted power are investigated through simulation. Engine speed is kept the same for all cases while the torque increases. The control target is to track a same aggressive engine torque demand. Fuel injection is feedback controlled to meet the target torque, with smoke limit constraint. Air-fuel ratio (AFR) must be higher than 16 for avoiding rich burn during transient tip-in, to prevent excessive soot emission. VGT is used to meet the target boost pressure. HP and LP EGR valve controls are used to track the target EGR mass flow rate. Hydraulic turbine is feedforward controlled with different hydraulic valve position. Total 8 different hydraulic valve opening levels were studied, corresponding to different assisted power from 0kW to 14kW. The 0kW assisted power with fully closed valve position is the baseline case, which has only VGT to control the boost. With assisted power, VGT control also follows the same boost pressure tracking control algorithm.

5.4.2.1 Engine transient performance improvement

As shown in Figure 84, all the simulation cases start with 25kW engine power with the same engine speed; engine transient response is improved with different level of assisted power over the duration of 0.7 seconds. With 14 kW assisted power, engine power increased 60kW at 11.2s, compared to the baseline case. Without assisted power, base VGT case cannot achieve aggressive target torque set-point, due to insufficient air introduced into the cylinder.
The engine produces more power with the higher level of assisted power since more air is introduced into the cylinder and more fuel can be injected into the cylinder with respect to the smoke limit. With insufficient fresh air in the engine cylinder with traditional VGT, engine fuel injection is limited by air-fuel ratio limit to mitigate soot emission. Thus, less fuel results in less TC turbine extracted power to drive the compressor. This is one of the major reasons for so-called ‘turbo lag’. With assisted power on TC shaft, the power to drive compressor is not totally dependent on TC turbine power. Hence, with hydraulic assisted power on the shaft, engine transient response can be significantly improved with more air compressed by the turbocharger.

As show in Figure 85, a higher air-fuel ratio is achieved with assisted power. Note that smoke limit is used for rich fuel injection. The engine without assisted power has 50% of time operating below AFR=17 during this transient tip-in event. With increased assisted power, the operating time when AFR below 17 decreases. With 5.5 kW assisted power, AFR is higher than AFR=17 over the whole simulation duration. With less time operating below the smoke limit, less soot
emission is expected with assisted power. Note that, the fluctuation for AFR is due to different sampling rate for GT simulation and Simulink.

Figure 85. Air fuel ratio with different assist

As shown in Figure 85, the total time duration for engine acceleration from 28kW to 90kW is 0.3 second with 14 kW assisted power, compared to 0.55 second without assisted power. The acceleration time is reduced by 45% from baseline case. Although engine fuel injection rate increased with different level of assisted power, total fuel consumption for 14 kW is reduced by 42.7% (from 2.69g to 1.54g) achieving the same target engine power(90 kW) compared to the baseline case with no assisted power.

5.4.2.2 Turbocharger transient performance improvement

Turbocharger response can be significantly improved with different level assisted power as shown in Figure 86. With higher assisted power, turbocharger speed increases faster. The acceleration time from 20K rpm to 40K rpm can be reduced by 75% compared to non-assisted case. With higher TC shaft speed, both compressor and turbine operating efficiency are
improved. Associated with higher TC shaft speed, compressor and turbine achieves higher efficiency as shown in Figure 87, Figure 88.

Figure 86. Turbocharger speed transient profile with different hydraulic assisted power

Figure 87. TC turbine efficiency transient profile with different hydraulic assisted power.
5.4.2.3 Hydraulic components transient performance

Figure 89 shows the hydraulic turbine power and hydraulic tank energy profile for different level of assisted power. Since different hydraulic power is based on different hydraulic valve position, pressure difference across turbine as well as TC shaft speed. Although hydraulic valve position is fixed for each case, the hydraulic power changes according to tank pressure and TC shaft speed with respect to designed turbine efficiency. During these transient operations, hydraulic turbine could be controlled through hydraulic valve position to achieve higher operating efficiency.

In summary, hydraulic assisted turbocharger can significantly achieve faster engine response than non-assisted turbocharger. It can increase AFR, TC speed, TC turbine efficiency, TC
compressor efficiency and engine operating efficiency during transient engine tip-in. All these benefit quite depend on how energy can be recovered through driveline pump and turbo pump without fuel penalty. In order to evaluate the fuel benefit level for proposed hydraulic assisted turbocharger, driving cycle simulation is investigated in next section.

**Figure 89. Hydraulic turbine output power transient profile**

### 5.4.3 Design trade-offs for fuel benefit through driving cycle simulation

In the previous section, the benefit of hydraulic assist operation is discussed. However, in order to achieve sustainable power assisted operation, sufficient hydraulic energy needs to be recovered from both vehicle and exhaust gas. Total recovered energy depends on driveline pump sizing as well as hydraulic tank sizing for a given vehicle and an engine for a fixed driving cycle. Hydraulic tank size would be limited by physical packaging space for design considerations.
In this study, vehicle simulation for FTP 75 driving cycle was investigated to understand the design trade-offs for fuel benefit and hydraulic components sizing and turbine sizing with vehicle level simulation. In traditional VGT control, in order to achieve higher turbine power to drive compressor at engine light load, VGT vane position is closed to a small opening to increase pre-turbine pressure. In this case, pumping loss increases due to higher engine exhaust pressure, which leads to fuel economy penalty. With assisted power on TC shaft, VGT position can avoid operating at small opening, which increases turbocharger efficiency and reduces engine pumping loss. But the capability of assisted power depends on energy availability in hydraulic tank.

Baseline in this study is still the traditional VGT for boost control with both high-pressure exhaust gas recirculation (HP-EGR) and lower pressure exhaust gas recirculation (LP-EGR). For RHAT-VGT turbocharger, VGT position is fixed at 0.5, 0.65, 0.75 and 1 position. Position 1 is fully open. RHAT is used to control boost pressure to track target pressure. There are two advantages for simulating with fixed VGT positions: first, fuel saving can be easily evaluated without sophisticated control system dealing with interaction with VGT and RHAT control. Second, the result would be used to exam feasibility of fixed geometry turbocharger (FGT) with hydraulic assisted and regenerative turbocharger, compared to a VGT turbocharger. Increased inertia by hydraulic turbocharger is considered in this simulation.

Based on the distribution of VGT position on FTP driving cycle for traditional VGT control, vane positions 0.5-0.6 and 0.2-0.3 are the most frequent opening positions during the driving cycle. Vane position 0.2-0.3 is used to build up turbine back pressure to drive compressor at very light engine load. This results in high pumping loss and low turbine efficiency. Vane position 0.5-0.6 is the most efficient turbine operation range from design prospect. When the vane position is fixed at large openings, extra assisted power and regenerative power is needed to meet engine
boost target and torque demand. Note that, in this simulation study, performance map for FGT turbine inherits from VGT turbocharger with fixed vane position for comparison. But in a practical case, FGT turbine efficiency would be higher without losses across turbine vane nozzles.

![Figure 90. VGT position distribution for traditional VGT control over FTP 75 cycle.](image)

The simulation results are shown in Figure 91 and Table 16. All the simulation cases meet the driver’s demand by achieving the same target vehicle speed. In order to achieve the same vehicle speed, the engine needs to be provided with sufficient fresh air for combustion process to produce the right amount of torque. Thus, different level of energy is needed for a hydraulic turbine with different fixed geometry turbine. The results show that system fuel benefit mainly depends on tank size for a given VGT size. The tank could be sized for largest energy drop during the driving cycle which may be dictated by the highway portion of the FTP cycle if supplemental hydraulic energy input to the TC is required. Thus, small VGT opening position results in small tank size since small VGT open position requires less supplemental hydraulic energy. Note that only VGT=0.5 and VGT=0.65 can achieve tank energy balance with current
tank size and driveline pump size. In these two cases, RHAT application results in 0.8% and 4.0% fuel saving, respectively. In other two cases, the RHAT can't recover enough energy to balance tank pressure or the required supplemental hydraulic energy with current driveline pump size due to reduced turbine energy extraction with large VGT open positions.

![Graph showing vehicle speed, MPG, and tank energy balance for different FGT turbine sizes with and without RHAT.](image)

Figure 91. Fuel benefit for driving cycle with different FGT turbine

<table>
<thead>
<tr>
<th>Turbine size (VGT position)</th>
<th>0.5</th>
<th>0.65</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Saving</td>
<td>0.8%</td>
<td>4.0%</td>
<td>5.2%</td>
<td>6.6%</td>
</tr>
<tr>
<td>Tank energy end of cycle</td>
<td>100%</td>
<td>100%</td>
<td>75%</td>
<td>30%</td>
</tr>
<tr>
<td>Max tank energy drop</td>
<td>20%</td>
<td>38%</td>
<td>56%</td>
<td>83%</td>
</tr>
<tr>
<td>State of charge</td>
<td>balanced</td>
<td>balanced</td>
<td>unbalanced</td>
<td>unbalanced</td>
</tr>
</tbody>
</table>

Table 16. Vehicle fuel benefits and tank energy
Pumping loss reduction is shown in Figure 92. Vane position is closed aggressively for high boost pressure demand during vehicle tip-in without assisted power, leading to high pumping loss. With different fixed vane position and assisted power, pumping losses are reduced while meeting the same boost target. This is one of the main reasons for fuel consumption reduction over the driving cycle when assisted power is used. But the level of fuel savings depends on how much hydraulic energy that can be recovered during the whole driving cycle. Energy usage and recovery for the hydraulic turbine, TC shaft pump, and driveline pump are shown in Figure 95. It indicates that in order to achieve higher fuel benefit, more hydraulic energy is needed. With wider turbine vane open position, there are less energy recovery opportunities for the hydraulic pump on TC shaft. Total recovered energy from the hydraulic turbopump on TC shaft alone may not be enough to drive turbocharger to meet the same boost demand. Energy recovery mainly depends on the driveline pump. It is concluded that, energy recovery from hydraulic driveline pump itself would limit total system fuel benefit. Simulation results show that fuel saving rate is less than 0.5% without driveline energy recovery on the FTP cycle. Note that only turbine size 0.5 and turbine size 0.65 have balanced tank pressure over the cycle in this study.

Note that since dual EGR loop is utilized in this system to meet demanded EGR mass flow rate. Results show that total EGR mass fraction in the intake manifold is equivalent for all the VGT and FGT with RHAT cases. But, with hydraulic assisted power, more LP-EGR is needed to compensate insufficient HP-EGR, because the pre-turbine pressure is decreased with assisted power.

Engine system with RHAT needs to be optimally designed such that: system performance and fuel economy meets the design target within constraints of hydraulic tank packaging space and cost. Hence, with given hydraulic turbine and hydraulic pumps (turbo-pump and driveline
pump) as well as engine and vehicle size, the design matrices for RHAT system would be TC turbine size, hydraulic tank size as shown in Figure 96. Tank size is identified through largest tank energy drop during the driving cycle for four different cases. It clearly shows that, in order to have higher fuel benefit, larger hydraulic tank needed to be utilized to store hydraulic fluids.

Figure 92. Pumping loss reduction with hydraulic assisted power
With improved transient response, the engine operating trajectory can be moved to more efficient operating region for both transient tip-in, and steady state. The results for FTP driving cycle can be found as shown in Figure 93 and Figure 94. The results agree with the discussion above.

Figure 93. Improved engine efficiency during transient
Figure 94. Engine brake efficiency [%] distribution
In summary, hydraulic assisted turbocharger with driveline energy recovery is a feasible technology to improve engine transient response as well as improve vehicle fuel efficiency.
5.5 Conclusion

Combining a turbo pump and driveline pump with the hydraulic assisted turbocharger to recover the exhaust energy and vehicle brake energy to offset the fuel economy penalty as a result of hydraulic power input to the turbo shaft for turbocharger transient response improvement, is a novel idea.

The preliminary one-dimensional GT-Power analysis indicated that it is possible to gain 3-5% fuel economy improvement with the RHAT system, compared with the base turbocharged diesel engine, over the FTP 75 transient cycle. This FE improvement does not include the other FE benefits that may be enabled by RHAT technology, such as engine downsizing, transmission optimization, etc.

The uncertainties in the numerical study include: dynamic responses and flow losses on the hydraulic actuation system and energy storage tank, especially during the warm-up period. These uncertainties will be addressed as the component level technologies are studied and refined.

Like many other innovative technologies, there are some key technical challenges that have yet to be tackled before this RHAT system can see wide application in turbocharged engines, e.g.

- Development of fast-response hydraulic valves and actuation systems with high-flow capacity and low-flow losses;
- The reduction of parasitic or windage loss when the hydraulic wheels are not engaged;
- Management of energy storage tank pressure so the turbo pump can work in high-efficiency areas at different flow rate or turbo speed;
- Optimal designs of the hydraulic turbine and turbo pump that can deliver high efficiencies over wide operation range, in terms of pressure and turbo speed, without adding excessive thrust loading and excitation on the turbo rotor system.

- Optimal control strategy to best utilize the RHAT and operate the turbocharger in high-efficiency areas to achieve maximum engine system efficiency over entire customer driving cycles;
CHAPTER 6: LINEAR QUADRATIC CONTROLLER DESIGN FOR VGT-EGR

DIESEL ENGINE AIR-PATH

6.1 Abstract

This chapter is focused on control design for a diesel engine air-path system equipped with variable geometry turbocharger (VGT) and exhaust gas recirculation (EGR). The controller design is further extended to the assisted and regenerative turbocharger system with VGT and EGR.

Diesel engines are of great challenges due to stringent emission and fuel economy requirements. Compared with the conventional turbocharger system, regenerative assisted system provides additional degree of freedoms for the turbocharger speed control. Hence, it significantly improves control capability for exhaust-gas-recirculation (EGR) and boost pressure. This paper focuses on control design for the diesel engine air-path system equipped with an EGR subsystem and a variable geometry turbocharger (VGT) coupled with a regenerative hydraulic assisted turbocharger (RHAT). The challenges lie in the inherent coupling among EGR, turbocharger performance, and high nonlinearity of engine air-path system. A linear quadratic (LQ) controller design approach is proposed in this paper for regulating the EGR mass flow rate and boost pressure simultaneously and the resulting closed-loop system performance can be tuned by properly selecting the LQ weighting matrices. Multiple LQ controllers with integral action are designed based on the linearized system models over a gridded engine operational map and the final gain-scheduling controller for a given engine operational condition is obtained by interpreting the neighboring LQ controllers. The gain-scheduling LQ controllers for both traditional VGT-EGR and VGT-EGR-RHAT systems are validated against the in-house baseline controller, consisting of two single-input and single-output controllers, using the nonlinear plant.
The simulation results show that the designed multi-input and multi-output LQ gain-scheduling controller is able to manage the performance trade-offs between EGR mass flow and boost pressure tracking. With the additional assisted and regenerative power on turbocharger shaft for the RHAT system, engine transient boost pressure performance can be significantly improved without compromising the EGR tracking performance, compared with the baseline control.

6.2 Introduction

Control system for diesel engine must meet driver’s performance demand, meanwhile satisfying emission constraint. Two major emission pollutions for diesel engine are visible smoke or particle matter and NOx (nitric oxide and nitrogen dioxide). Visible smoke can be avoided by keeping the air-fuel ratio high for lean combustion. For reduction of NOx, commonly method is to use exhaust gas recirculation and NOx aftreatment system. Inside engine cylinder, recirculated exhaust gas acts as inert gas, it can increases the specific heat capacity and decreases the oxygen concentration of charged air. Hence it can reduces burn rate, lower peak flame temperature and reduce the formulation of NOx. In order to meet the driver’s demand and emission requirement, the air handling system for diesel engine must supply diesel engine with right amount of fresh air and desired EGR fraction in intake charge for a given engine operating condition.
To fulfill these purposes, modern diesel engines are normally equipped with Variable Geometry Turbocharger (VGT) and high-pressure Exhaust Gas Recirculation (EGR) as shown in Figure 97. The VGT turbocharger is driven by exhaust gas turbine. The gas turbine is used to drive the compressor to compress fresh air into engine intake manifold. Since, charged air quantity can be increased by the compressor, a larger amount of fuel can be burnt to provide higher engine torque, compared to the non-turbocharged engine. With variable geometry turbine, changing turbine vane nozzle position can accommodate both engine low flow and high flow conditions for better turbocharger system efficiency and transient response. The high-pressure EGR system connects exhaust gas manifold with the intake manifold, driving exhaust gas into the intake manifold. EGR valve position is adjusted to achieve desired EGR mass flow rate.

However, the nonlinear multivariable nature of the diesel engine VGT-EGR system and dual objectives make the close loop control design problem arduous. Traditional control strategy treat these two control actions as two single input and single output system by using only one actuator at one time. Reported in [53] is an overview of the control strategies used in heavy-duty Diesel
engines with EGR-VGT. During tip-ins, the control system closes the EGR valve and uses a Proportional Integral Derivative (PID) controller feedback on boost pressure to rapidly increase air supply to the intake using the VGT. In steady state operation, the controller fully opens the EGR valve regulates the EGR flow rate by open loop vane control of the VGT. This strategy is satisfactory for the vehicle with prolonged steady state operation as in commercial vehicles. For passenger car applications transient operation is more frequent and often more severe. For instance, the natural feedback established by the VGT can be compromised by the presence of high-pressure EGR valve action. The inverse response type behavior for the compressor mass flow by the step response of change of EGR increases the control design challenges. Therefore to fully exploit the potential of these devices, the control design needs to consider the problem in the context of multivariable control, with expectations of a better performance.

A good control solution for production application must provide a robust controller that does not use up many resources of the ECU and is simple to implement and calibrate. Most of the papers discussed the stability and robustness of controller design for diesel engine air flow regulation. A Control Lyapunov Function (CLF) based controller design is introduced in [7] This method is constructed for a simplified model by using input-output linearization. Robustness is achieved by using the domination redesign. In [5], a multivariable controller is designed based on input and output linearization with sliding model control. It provides a systematic method to regulate the EGR mass flow rate and intake manifold pressure with choosing different sliding surface. However, few papers discussed a systematic approach for close loop controller design with respect to engine performance, emission and fuel economy during transient operation, specifically, not only considering stability and robustness but also taking engine transient
performance into account during control design process. The engine transient performance here means engine response, emission, and fuel economy during transient operation.

The linear quadratic regulator (LQR) is a well-known design technique that provides practical steady feedback gains which are also known for its robustness which has been interpreted as gain margin \((1/2, +\infty)\) and \(60^\circ\) phase margin. LQR techniques has been widely investigated in other nonlinear system [85][86][87][88]. For diesel engine VGT-EGR air-path control, one of the challenges is to handle the trade-offs between engine performance, emission and fuel economy. In LQR design routine, weighting matrix can be directly used to design the controller for different purposes. It eventually provides a multi-input and multi-output (MIMO) controller to coordinate VGT vane position and EGR valve action. However, LQR-controller designs are only applicable for linear systems. Gain scheduling is a natural approach to extend the linear control design to a nonlinear system by using a family of linear controllers, which individually provides satisfactory control for a different local operating point of the system.

In this chapter, we show the development of gain scheduling control design for VGT-EGR system. Controller design is based on the high fidelity reduced order nonlinear diesel engine model developed in Chapter 4. Linearized models are obtained for the nonlinear system along with engine operating trajectory. A linear quadratic technique with integral action is used to design the local linear controller to regulate the boost pressure and EGR mass flow rate tracking error to zero and keep steady state error to zero. Controller design for different target scheme is proposed with weighting selection. Different performance indexes are used to evaluate the controller design. This provides flexibility for controller design with respect to different engine performance and emission trade-offs. Proposed MIMO linear controllers are scheduled by engine speed and fuel injection to be implemented with the nonlinear plant. By comparing with baseline
controller, the proposed controller shows its advantage of the multivariable controller over SISO controllers. It also shows a systematic method for engine performance and emission trade-offs mitigation during controller design. Further controller design is extended to assisted and regenerative turbocharger system with VGT and EGR system.

6.3 Controller design

6.3.1 Control objective and problem formulation

For diesel engine VGT-EGR air-path system, the control objective is to regulate the demanded fresh air and oxygen concentration in the intake manifold to the desired levels as determined from an optimized engine static calibration. These static maps are generated based on a trade-off between maximal fuel economy and minimal $NO_x$ generation, without violating the constraints on soot formation. Demanded fresh air and oxygen concentration can be transferred to air fuel ratio and EGR mass flow fraction. Hence, while the set-point for AFR determines the engine response and prevents smoke, the EGR flow fraction seeks to minimize in-cylinder $NO_x$ generation. If the fueling rate is known (from drivers pedal position), then the set-point for AFR pressure can be transformed into a set-point for compressor flow rate $\dot{m}_C^d$. Similarly, the set-point for EGR flow fraction can be expressed regarding the desired quantities $\dot{m}_{egr}^d$ and $\dot{m}_C^d$. Further set-points can be reformulated to intake pressure, exhaust pressure and TC speed and oxygen concentration intake manifold as shown in Figure 98. These are common states to be regulated to the desired valve in previous control design.
In order to compare with in-house baseline controller, the control objective of this study is to regulate the boost pressure \((P_2^d)\) and EGR mass flow rate \((\dot{m}_{egr})\) to their desired values. The reference trajectories are generated by calibration tables, which are not discussed in this study. With driver's pedal position input, desired engine torque is used to define desired fuel injection amount. Target boost pressure and target EGR mass flow rate are based on current engine speed and fuel injection. Meanwhile, optimal steady state set-point for VGT vane position \((u_{vgt})\) and EGR valve position \((u_{egr})\) are obtained based on steady state emission and performance requirement. Note that, for each pair of \((N_e, \dot{m}_{fuel}, u_{vgt}, u_{egr})\), there exists a unique equilibrium of diesel engine plant. Hence, with a given engine speed and fuel injection for steady state, engine plant will operate at a given operating point. For this reason, engine speed and fuel injection would be a natural candidate for gain scheduling.

Figure 98. Set-point for diesel engine air-path control

Figure 99. Tracking reference generation in baseline controller
In this study, the close loop control design is formulated as a regulation problem. Its target is to regulate the boost pressure tracking error and EGR mass flow rate tracking error to zero, meanwhile keeping the steady state errors to zero. For a regulation problem, the control design is based on error system. Linearization of the nonlinear plant is investigated first.

### 6.3.2 Linearization of Nonlinear System

Consider a nonlinear differential equation:

$$\dot{x}(t) = f(x(t), u(t))$$  \hspace{1cm} (6.1)

where $f$ is a function mapping $\mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$, a point $\bar{x} \in \mathbb{R}^n$ is called an equilibrium point, if there is a corresponding $\bar{u} \in \mathbb{R}^m$ such that:

$$zero(n) = f(\bar{x}(t), \bar{u}(t))$$  \hspace{1cm} (6.2)

System (6.1) starts from initial conditions $x(t_0) = \bar{x}$, with input $u(t) \equiv \bar{u}$ for all $t \geq t_0$. The resulting solution $x(t)$ satisfies:

$$x(t) = \bar{x} \quad \text{For all } t \geq t_0$$  \hspace{1cm} (6.3)

Suppose $(\bar{x}, \bar{u})$ is an equilibrium point and input for the nonlinear system (6.1). Considering states and control inputs deviate from the equilibrium point, the deviation variables can be defined as:

$$\delta x(t) = x(t) - \bar{x}$$  \hspace{1cm} (6.4)

$$\delta u(t) = u(t) - \bar{u}$$  \hspace{1cm} (6.5)

Variables $x(t)$ and $u(t)$ are governed by the system (6.1). Substitute equations (6.4) and (6.5) into dynamic system (6.1):
\[
\delta x(t) = f(\bar{x} + \delta x(t), \bar{u} + \delta u(t)) - f(\bar{x}, \bar{u}) \tag{6. 6}
\]

Applying Taylor expansion of the right hand side and neglect all higher order (>2) terms:

\[
\delta x(t) + f(\bar{x}, \bar{u}) \equiv f(\bar{x}, \bar{u}) + \frac{\partial f}{\partial x}|_{x=\bar{x}, u=\bar{u}} \delta x(t) + \frac{\partial f}{\partial u}|_{x=\bar{x}, u=\bar{u}} \delta u(t) \tag{6. 7}
\]

leads to

\[
\delta x(t) \equiv \frac{\partial f}{\partial x}|_{x=\bar{x}, u=\bar{u}} \delta x(t) + \frac{\partial f}{\partial u}|_{x=\bar{x}, u=\bar{u}} \delta u(t) \tag{6. 8}
\]

The differential equation governs system dynamics with the infinitely small deviations of \(\delta x(t)\) and \(\delta u(t)\). Note that the resulting dynamic system is linear time-invariant since derivative of \(\delta x(t)\) are the linear combination of \(\delta x(t)\) and \(\delta u(t)\). For the state-space realization, matrices A and B can be expressed as

\[
A = \frac{\partial f}{\partial x}|_{x=\bar{x}, u=\bar{u}} \in R^{n \times n}, \quad B = \frac{\partial f}{\partial u}|_{x=\bar{x}, u=\bar{u}} \in R^{n \times n} \tag{6. 9}
\]

They are constant matrices under the given equilibrium condition. Thus linear approximation for the nonlinear system (6.1), around equilibrium point \((\bar{x}, \bar{u})\), can be expressed as:

\[
\delta x(t) = A\delta x(t) + B\delta u(t) \tag{6. 10}
\]

This is called Jacobian linearization. For small values of \(\delta x(t)\) and \(\delta u(t)\), the linearized system approximately represents the dynamic relationship between \(\delta x(t)\) and \(\delta u(t)\).

6.3.3 Model linearization for diesel engine air-path system

Consider a three states diesel engine air-path model with exhaust pressure \((P_3)\), boost pressure \((P_2)\) and TC shaft speed \((\omega)\) as states. Control inputs are VGT vane position \((u_{vgt})\) and EGR valve position \((u_{egr})\); and control outputs are boost pressure \((P_2)\) and EGR mass flow rate.
(\(m_{egr}\)). The control target is to minimize the tracking errors of both boost pressure and EGR mass flow rate.

\[
\dot{p}_3 = \frac{RT_3}{V_3}(m_{out} - m_{egr} - \dot{m}_t)
\]

\[
\dot{p}_2 = \frac{RT_2}{V_2}(m_c - m_{in} + m_{egr})
\]

(6.11)

\[J\omega \dot{\omega} = W_T - \dot{W}_C - \dot{W}_{Loss}\]

The nonlinear plant (6.11) can be expanded as shown in (6.12) and is highly nonlinear. Plant dynamics varies under different engine operational conditions \((N_e, \dot{m}_{fuel}, u_{vgt}, u_{egr})\). More details about the nonlinear plan can be found in Chapter 4.

\[
\dot{p}_3 = Rf_{P_3}(P_2, \dot{m}_{fuel})\left(f_{\dot{m}_{in}}(N_e, P_2) + \dot{m}_{fuel} - \dot{m}_t\right) - f_{\dot{m}_{egr}}(u_{egr}, \frac{P_3}{P_2})
\]

\[
\dot{p}_2 = Rf_{P_2} \left(\omega, \frac{P_2}{P_1}\right)\left(f_{m_c} \left(\omega, \frac{P_2}{P_1}\right) - f_{\dot{m}_{in}}(N_{engine}, P_2) + f_{\dot{m}_{egr}}(u_{egr}, \frac{P_3}{P_2})\right)
\]

(6.12)

\[J\omega \dot{\omega} = f_{W_T} \left(u_{vgt}, \frac{P_3}{P_4}, \omega, T_3\right) - f_{W_c} \left(\frac{P_2}{P_1}, \omega\right) - f_{W_{Loss}}(\omega)\]

A linearized model about an equilibrium point of the nonlinear diesel engine air-path model (6.12) was developed using analytical methods. The linearized equations are shown below

\[
\delta \dot{x}_p(t) = A_p(t)\delta x_p(t) + B_p(t)\delta u(t)
\]

\[
y_p(t) = C_p(t)\delta x_p(t) + D_p(t)\delta u(t)
\]

(6.13)

\[z(t) = F\delta x_p(t) + W(t)\delta u(t)\]

where, \(x(t) \in R^3\) is the states of the plant \((P_3, P_2, \omega)\), \(u(t) \in R^2\) is the control inputs \((u_{vgt}, u_{egr})\); \(y_p \in R^2\) is the output available \((\dot{m}_{egr}, P_2)\) for feedback control; and \(z(t)\) is the performance output to be regulated to the desired reference value. In this study, \(y_p = z(t)\), assuming both outputs are measurable. The states, inputs, and outputs in (6.13) are all deviations of the
corresponding trajectories of the nonlinear system (6.12) from the equilibrium operation condition. For a typical diesel engine VGT-EGR air-path system, the linearized model in in this general form below in (6.14). More detailed plant linearization can be found in Appendix.

\[
\begin{bmatrix}
\delta \dot{P}_3 \\
\delta \dot{P}_2 \\
\delta \dot{\omega}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & 0 \\
A_{21} & A_{22} & A_{13} \\
A_{31} & A_{32} & A_{33}
\end{bmatrix}
\begin{bmatrix}
\delta P_3 \\
\delta P_2 \\
\delta \omega
\end{bmatrix} +
\begin{bmatrix}
B_{11} & B_{12} \\
0 & B_{22} \\
B_{31} & 0
\end{bmatrix}
\begin{bmatrix}
\delta u_{vgt} \\
\delta u_{egr}
\end{bmatrix}
\]  
(6.14)

\[
\delta y(t) =
\begin{bmatrix}
\delta \dot{m}_{egr} \\
\delta P_2
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & 0 \\
0 & C_{22} & 0
\end{bmatrix}
\begin{bmatrix}
\delta P_3 \\
\delta P_2 \\
\delta \omega
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & D_{21}
\end{bmatrix}
\begin{bmatrix}
\delta u_{vgt} \\
\delta u_{egr}
\end{bmatrix}
\]

Linear controllers are designed based on the linearized models at different equilibrium points, and the associated performances are evaluated through simulation studies based on the full-scale nonlinear model. The desired outcome of controller design is for the system to track the target EGR mass flow rate and boost pressure.

6.3.4 Augmented with actuator dynamics

For model accuracy, simple actuator dynamics are added to the plant model. The EGR valve actuator dynamics is modelled as:

\[
\delta \dot{u}_{egr}(t) = A_d(t) \delta \dot{u}(t) + B_d \delta u_{egr}(t)
\]  
(6.15)

In this case, only EGR actuator dynamics is modelled. The parameters of the actuator model are chosen to approximate values for an actual EGR valve. In this case, EGR actuator response time is about 0.03s. The model (6.15) is as follows:

\[
\delta \dot{u}_{egr}(t) = -\frac{1}{0.03} \delta \dot{u}_{egr}(t) + \frac{1}{0.03} \delta u_{egr}(t)
\]  
(6.16)

After augmenting the actuator dynamics, the new system is:

\[
\delta \dot{x}_1(t) = A_1(t) \delta x_1(t) + B_1 \delta u_1(t)
\]

\[
\delta y_1(t) = C_1(t) \delta x_1(t)
\]  
(6.17)
where
\[ x_1(t) = \begin{bmatrix} \delta x_p(t) \\ \delta u(t) \end{bmatrix}; \quad \delta u_1(t) = \begin{bmatrix} \delta u_{vgt} \\ \delta u_{egr} \end{bmatrix}; \quad A_1 = \begin{bmatrix} A_p & B_{pregr} \\ 0 & A_d \end{bmatrix}; \quad B_1 = \begin{bmatrix} B_{pregt} & 0 \\ 0 & B_d \end{bmatrix}; \quad C_1 = \begin{bmatrix} C_p & D_{pregr} \end{bmatrix} \]

Note that matrix \( D \) of (6.14) is only associated with the EGR control input that is the direct input for the EGR mass flow rate. With the actuator augmented actuator dynamics, matrix \( D \) in (6.17) is zero.

### 6.3.5 Integral action

It is desirable to include integral action into the state feedback control to eliminate the steady state errors. With model uncertainties presented in system, controller design must be able to compensate those uncertainties to eliminate steady state errors. Note that the standard LQR results in a proportional state feedback controller without integral action. This implies that at steady state the tracking errors of LQR are not zero. By augmenting the system with its integral errors it is possible to add integral action to the LQR with integral gains selected automatically.

The advantage of adding the integral action is that it eliminates the steady state tracking errors. Define a new system model with integral action that has the time derivative of output and states as states:

\[
\frac{d}{dt} \begin{bmatrix} \delta \dot{x}_1(t) \\ \delta y_1(t) \end{bmatrix} = \begin{bmatrix} A_1(t) & 0 \\ C_1(t) & 0 \end{bmatrix} \begin{bmatrix} \delta \dot{x}_1(t) \\ \delta y_1(t) \end{bmatrix} + \begin{bmatrix} B_1(t) \\ 0 \end{bmatrix} \delta v(t) \tag{6.18}
\]

The new augmented system is:

\[
\frac{d}{dt} (M(t)) = \dot{A}(t)M(t) + \dot{B}(t)U(t) \tag{6.19}
\]

where
\[
U(t) = \delta v(t) = \frac{d(\delta u_1(t))}{dt}, \quad M(t) = \begin{bmatrix} \delta \dot{x}_1(t) \\ \delta y_1(t) \end{bmatrix}, \quad \dot{A}(t) = \begin{bmatrix} A_1(t) & 0 \\ C_1(t) & 0 \end{bmatrix}, \quad \dot{B}(t) = \begin{bmatrix} B_1(t) \\ 0 \end{bmatrix}
\]

Based on equations (6.17), (6.18) and (6.19), the final augmented linear system have the following system matrices:
\[
\begin{align*}
\tilde{A} &= \begin{bmatrix}
A_{11} & A_{12} & 0 & B_{12} & 0 & 0 \\
A_{21} & A_{22} & A_{23} & B_{22} & 0 & 0 \\
A_{31} & A_{32} & A_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & A_d & 0 & 0 \\
C_{11} & C_{12} & 0 & D_{21} & 0 & 0 \\
0 & C_{22} & 0 & 0 & 0 & 0
\end{bmatrix}, \quad
\tilde{B} = \begin{bmatrix}
B_{11} & 0 \\
0 & 0 \\
B_{31} & 0 \\
0 & B_d \\
0 & 0 \\
0 & 0
\end{bmatrix}
\end{align*}
\] (6.20)

For instance, the linearized plant matrices at engine speed of \(N_e = 1200\) rpm with fuel injection quantity of \(\bar{m}_{fuel} = 35\)mg/cc are

\[
\begin{align*}
\tilde{A} &= \begin{bmatrix}
-73.94 & 77.13 & 0 & -1.61e5 & 0 & 0 \\
2.48 & -78.94 & 1.15e3 & 2.03e4 & 0 & 0 \\
0.04 & 0.023 & -5.1246 & 0 & 0 & 0 \\
0 & 0 & 0 & -33.33 & 0 & 0 \\
2.8e-07 & -2.6e-07 & 0 & -0.0024 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad
\tilde{B} = \begin{bmatrix}
1.129e5 & 0 \\
0 & 0 \\
-20 & 0 \\
0 & 0.0003 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\end{align*}
\] (6.21)

6.3.6 Linear quadratic regulator with integral action (LQI)

The target for this control design is to find a linear optimal controller that minimizes the tracking errors of both boost pressure and EGR mass flow rate for the given LQI cost function below

\[
J = \int_0^\infty \left( M(t)^T \tilde{Q} M(t) + U(t)^T RU(t) \right) dt, \quad \tilde{Q} = \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix} \in \mathbb{R}^6
\] (6.22)

To minimize the tracking errors, the integral of \(M(t)^T \tilde{Q} M(t)\) and \(U(t)^T R U(t)\) should be nonnegative and small. As a result, matrix \(Q\) must be positive semi-definite and matrix \(R\) must be positive definite to make sure all control channels are finite. In this case, \(Q\) is 2x2 and \(R\) is 2x2. Note that \(Q\) is the weighting matrix for tracking errors of both boost pressure and EGR mass flow rate; and \(R\) is the weighting matrix for the derivative control inputs, VGT vane and EGR valve positions. Now the problem is formulated as a standard LQR problem. The observability and controllability conditions required for the Riccati equation to have stationary positive definite solution must be checked for the matrix pairs.
\( \left( \bar{A}(t), Q^2[0_n, I_m] \right) \) and \( (\bar{A}(t), \bar{B}(t)) \) \hspace{1cm} (6.23)

Assume observability and controllability conditions are satisfied, then feedback law for \( \delta v(t) = U(t) \) is given by:

\[
\delta v(t) = U(t) = -R^{-1}\bar{B}(t)\bar{P} M(t) = -K_x\delta \dot{x}_1(t) - K_y \delta y_1(t) \hspace{1cm} (6.24)
\]

where, \( K_x, K_y \) and \( \bar{P} \) are given by:

\[
\begin{bmatrix}
K_x & K_y
\end{bmatrix} = [R^{-1}\bar{B}(t)\bar{P}_{xx} \hspace{0.5cm} R^{-1}\bar{B}(t)\bar{P}_{xy}]
\hspace{1cm} (6.25)
\]

and \( \bar{P} = \begin{bmatrix} \bar{P}_{xx} & \bar{P}_{xy} \\ \bar{P}_{yx} & \bar{P}_{yy} \end{bmatrix} \). \( \bar{P} \) is obtained by solving matrix algebraic Riccati equation:

\[
\bar{A}(t)^\top \bar{P} + \bar{P} \bar{A}(t) - \bar{P} \bar{B}(t)R^{-1}\bar{B}(t)^\top \bar{P} + \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix} = 0
\hspace{1cm} (6.26)
\]

Thus:

\[
\begin{bmatrix} A(t) & 0 \\ C(t) & 0 \end{bmatrix}^\top \begin{bmatrix} \bar{P}_{xx} & \bar{P}_{xy} \\ \bar{P}_{yx} & \bar{P}_{yy} \end{bmatrix} + \begin{bmatrix} \bar{P}_{xx} & \bar{P}_{xy} \\ \bar{P}_{yx} & \bar{P}_{yy} \end{bmatrix} \begin{bmatrix} A(t) & 0 \\ C(t) & 0 \end{bmatrix} - \begin{bmatrix} \bar{P}_{xx} & \bar{P}_{xy} \\ \bar{P}_{yx} & \bar{P}_{yy} \end{bmatrix} \begin{bmatrix} \bar{B}_1(t) & 0 \\ 0 & \bar{B}_1(t) \end{bmatrix} R^{-1} \begin{bmatrix} \bar{B}_1(t)^\top & 0 \\ 0 & \bar{B}_1(t)^\top \end{bmatrix} \begin{bmatrix} \bar{P}_{xx} & \bar{P}_{xy} \\ \bar{P}_{yx} & \bar{P}_{yy} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix} = 0
\hspace{1cm} \text{(6.27)}
\]

Expand each term for algebraic Riccati equation:

\[
2A\bar{P}_{xx} + 2C\bar{P}_{xy} - \bar{P}_{xx}BR^{-1}BP_{xx} = 0
\]

\[
A\bar{P}_{xy} + C\bar{P}_{yy} - \bar{P}_{xx}BR^{-1}BP_{xy} = 0
\hspace{1cm} (6.28)
\]

\[
Q - \bar{P}_{xy}BR^{-1}BP_{xy} = 0
\]

Thus, we can obtain control law by integrating from initial time 0 to \( t \). This control is a type of PI controller with state error and integration of output error.

\[
\delta u(t) = \int_0^t \delta v(t) \, dt = \int_0^t (-K_x\delta \dot{x}_1(t) - K_y \delta y_1(t)) \, dt = -K_x\delta x_1(t) - K_y \int_0^t (\delta y_1(t)) \, dt
\hspace{1cm} (6.29)
\]

where,

\[
\begin{bmatrix} K_x & K_y \end{bmatrix} = [R^{-1}\bar{B}(t)\bar{P}_{xx} \hspace{0.5cm} R^{-1}\bar{B}(t)\bar{P}_{xy}]
\]
To test the effectiveness of each controller developed in this study, their performance is evaluated through simulation studies using the nonlinear model (6.11). In order to implement the linear controller design for the nonlinear plant, the equilibrium states value must be subtracted from the states of the nonlinear model and the control inputs are fed by the outputs of the linear controller with equilibrium controls as in (6.28).

\[
\delta x_1(t) = x_1(t) - \bar{x}_1
\]
\[
u_1(t) = \bar{u}_1 + \delta u_1(t)
\]

Then the final control inputs for nonlinear plant are:

\[
u_1(t) = \bar{u}_1 - K_x(x_1(t) - \bar{x}_1) - K_y \int_0^t (\delta y_1(t)) \, d\tau
\]

where \(\bar{u}_1 = [\bar{u}_{vgt} \; \bar{u}_{egr}]^T \), \(\bar{x}_1 = [p_2^d \; p_3^d \; \omega^d \; u_{egr}^d]^T\). Specifically, the steady state control gain has the following structure

\[
K_x = \begin{bmatrix}
K_{x11} & K_{x12} & K_{x13} & K_{x14} \\
K_{x21} & K_{x22} & K_{x23} & K_{x24}
\end{bmatrix},
\]
\[
K_y = \begin{bmatrix}
K_{y11} & K_{y12} \\
K_{y21} & K_{y22}
\end{bmatrix}
\]

where \(K_x\) is the proportional gain; and \(K_y\) is the integral gain. First row of \(K_x\) and \(K_y\) are control gains for VGT vane position (\(\delta u_{vgt}\)). The second row of \(K_x\) and \(K_y\) are control gains for EGR valve position (\(\delta u_{egr}\)). The control actions for both VGT vane and EGR valve positions are coupled through state deviation and output error. In this study, the overall control architecture is shown in Figure 100, where \(x_1(t)\) is nonlinear plant states value; and \(\bar{x}_1\) are equilibrium values for each state that are desired value along the regulating trajectory \((p_2^d, p_3^d, \omega^d, u_{egr}^d)\). Boost tracking error (\(\delta P_2\)) and EGR mass flow rate tracking error (\(\delta \dot{m}_{egr}\)) are used to drive the integral control. The feedforward controls (\(\bar{u}_{vgt} \) and \(\bar{u}_{egr}\)) are generated from feedforward calibration map.
based on engine operating points \((N_e, \dot{m}_{\text{fuel}})\) that are used to drive nonlinear plant close to the steady state operating point.

Figure 100. Proposed linear quadratic regulator for Engine EGR-VGT air-path system

6.3.7 Observability and controllability analysis

Consider system described in (6.11), (6.12) and (6.13) with matrix \(Q\) as below:

\[
\tilde{Q} = \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix}
\]  \tag{6.33}

For the standard LQR controller design, the solution relies on solving the Riccati equation. In this case, we are dealing with infinite LQR design, in order to have a finite solution for the given cost function (6.22), \((\tilde{A}, \tilde{B})\) needs to be stabilizable; and \((\tilde{A}, \sqrt{\tilde{Q}})\) needs to be detectable. Consider the following observability Gramian

\[
O = \begin{bmatrix} \tilde{Q}^T \tilde{A}^2(t) & \tilde{Q}^T \tilde{A}(t)^3 & \tilde{Q}^T \tilde{A}(t)^4 & \tilde{Q}^T \tilde{A}(t)^5 \end{bmatrix}^T \in \mathbb{R}^{36 \times 6}
\]  \tag{6.34}

and controllability Gramian:
As shown in Table 17, the condition for controllability Gramian losing rank is when \( P_2 = P_3 \) or \( P_3 = P_4 \). For instance, considering \( B_{12}, B_{22} \) and \( D_{21} \), they are block parameters for EGR control inputs for exhaust pressure, boost pressure and EGR mass flow rate. When \( P_2 = P_3 \), \( B_{12} = B_{22} = D_{21} = 0 \), EGR valve lose control authority over EGR mass flow rate. And controllability Gramian rank \( \text{rank}(C) = 4 \). Further, the system is not fully controllable under this condition. This can be explained by physical meaning such that when HP EGR valve upstream pressure is equal to or smaller than EGR valve downstream pressure; EGR valve position would not have any effect for EGR control. For instance, \( B_{12} \) term is for EGR valve control input for EGR mass flow rate as shown as in (6.36). When \( P_2 = P_3 \), \( B_{12} = 0 \). When \( P_2 > P_3 \), inverse flow is not allowed, EGR valve needs to be fully closed. The same happens for the condition of \( P_3 < P_4 \). There would be turbine mass flow, hence system’s controllability Gramian losses rank.

\[
B_{12} = -\frac{287^3 \cdot T_2 \cdot P_3 \cdot 2 \cdot \left( \frac{P_2}{P_3} \right)^5 \cdot C_P \cdot \left( 1 - \left( \frac{P_2}{P_3} \right)^2 \right)^{\frac{1}{4}}}{T_3 V_{\text{im}}}
\]  

(6.36)

Based on physical limitation, such that TC shaft cannot be stalled \( (P_2 \leq P_1) \) or over speed\( (P_3 \leq P_4) \). Further, control target is to have exhaust pressure higher than intake manifold pressure\( (P_2 < P_3) \). This will avoid losing controllability as well as keeping EGR flow capability. In this case, operating space \( \psi = \{(P_1, P_2, P_3, P_4), \ P_3 > P_4, P_2 > P_1, P_3 > P_2\} \) will be guaranteed such that system will be fully observable and controllable. One thing needs to be noticed that, control inputs are bounded. VGT and EGR positions are bounded by their physical hardware actuator position. For instance, VGT position is [0-100] and EGR valve position is [0-
100]. However, if control design is targeted for engine performance, the EGR valve might need to be fully closed for aggressive boost demand during transient tip-in and tip-out.

Table 17. Controllability and observability analysis for VGT-EGR system

<table>
<thead>
<tr>
<th>Condition 1</th>
<th>Condition 2</th>
<th>Condition 3</th>
<th>Nominal condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1 \geq P_2$</td>
<td>$P_2 \geq P_3$</td>
<td>$P_3 \leq P_4$</td>
<td>$P_1 &lt; P_2$ $P_2 &lt; P_3$ $P_3 &gt; P_4$</td>
</tr>
<tr>
<td>Block value =0</td>
<td>None</td>
<td>$B_{12} B_{22} B_d A_d D_{21}$</td>
<td>$B_{31} B_{21}$ None</td>
</tr>
<tr>
<td>Singularity</td>
<td>$A_{11} A_{13}$ $A_{31} A_{33}$</td>
<td>$A_{12} A_{21} A_{22}$</td>
<td>$A_{22}$ None</td>
</tr>
<tr>
<td>Physical meaning</td>
<td>Leaking in the intake manifold</td>
<td>EGR valve closed to prevent inverse flow No EGR mass flow rate</td>
<td>No turbine mass flow rate None</td>
</tr>
<tr>
<td>Observability Gramian Rank with 3 states $(A_p(t), C_p(t))$</td>
<td>3(full)</td>
<td>3(full)</td>
<td>3(full) 3(full)</td>
</tr>
<tr>
<td>Controllability Gramian Rank with 3 states $(A_p(t), B_p(t))$</td>
<td>3(full)</td>
<td>3(full)</td>
<td>3(full) 3(full)</td>
</tr>
<tr>
<td>Observability Gramian Rank with 6 states(including actuator dynamics with integral action) $(\bar{A}(t), \frac{1}{2}Q)$</td>
<td>6(full)</td>
<td>5 (loss rank)</td>
<td>6(full) 6(full)</td>
</tr>
<tr>
<td>Controllability Gramian Rank with 6 states (including actuator dynamics with integral action) $(\bar{A}(t), B(t))$</td>
<td>6(full)</td>
<td>4 (loss rank)</td>
<td>3 (loss rank) 6(full)</td>
</tr>
</tbody>
</table>

6.3.8 Plant scaling

For the linearized plant, the output and input matrices are badly conditioned, which could lead to numeric issues when solving the Riccati equation. As shown in Table 18, the parameters for the linearized model has very different numerical range. Especially for the boost pressure and EGR mass flow rate. This results in a badly conditioned $C$ matrix since the tracking targets are
boost pressure and EGR mass flow rate. In order to compensate this issue, input and output scaling are carried out. The scaling factor for control input is 1000, scaling factor EGR mass flow rate is 3600*1000. After that, matrix C has proper scale for both boost pressure and EGR mass flow rate as shown in Table 19.

<table>
<thead>
<tr>
<th>Parameters numeric range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>States</td>
</tr>
<tr>
<td>Boost pressure</td>
</tr>
<tr>
<td>Exhaust pressure</td>
</tr>
<tr>
<td>TC speed</td>
</tr>
<tr>
<td>EGR valve dynamic</td>
</tr>
<tr>
<td>Control inputs</td>
</tr>
<tr>
<td>VGT position</td>
</tr>
<tr>
<td>EGR valve position</td>
</tr>
<tr>
<td>Plant outputs</td>
</tr>
<tr>
<td>Boost pressure</td>
</tr>
<tr>
<td>EGR mass flow rate</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C matrix reformulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>From</td>
</tr>
<tr>
<td>Boost pressure</td>
</tr>
<tr>
<td>Before scaling</td>
</tr>
<tr>
<td>Boost pressure</td>
</tr>
<tr>
<td>EGR mass flow rate</td>
</tr>
</tbody>
</table>

### 6.4 Controller Design Validation

#### 6.4.1 Baseline controller for control development

Gain-scheduling control of Proportional Integral Derivative (PID) is commonly used in industry to control a nonlinear system due to its advantages of simplicity and easy calibration. The current in-house baseline controller has two independent control loops for tracking both boost pressure and EGR mass flow rate as shown in Figure 101. EGR valve position is used to track target EGR mass flow rate. Note that EGR valve opening is directly related to EGR mass flow rate and pressure changes (exhaust pressure and boost). VGT vane position is feedback
controlled for boost pressure tracking. Each PID controller is well calibrated for given engine operating point. The controller gains are scheduled based on the engine speed and fuel injection quantity. The major drawback of this control structure is that the VGT and EGR actions are not controlled coordinately. This leads to non-optimal control of VGT and EGR systems. In certain extreme cases, the one controller performance (e.g., EGR) needs to be compromised for the other one (e.g., VGT). Because the boost pressure and EGR mass flow rate are strongly coupled since they share the same exhaust flow used to drive the turbine and EGR flow.

In order to understand the behavior of current baseline controller, a simulation study is carried out by integrating the baseline controller with the developed 1-D nonlinear diesel engine air-path model in Simulink environment. Although the baseline controller is not well tuned for this developed nonlinear engine model, the simulation results provide the benchmark performance for the LQI controller.

![Diagram](Figure 101. Baseline controller for control development)

Different engine operational conditions are studied and shown in Figure 102. The results show that the baseline controller can track the target boost pressure and EGR mass flow rate for the developed nonlinear engine model. However, during the aggressive tip-in and tip-out
operations, the EGR tracking is not satisfactory. One of the reasons behind this is that the VGT and EGR control is not coordinated for both targets. This is the typical disadvantage of using multiple single input single outputs (SISO) controllers for a multi-input and multi-output (MIMO) system. The detailed tip-in and tip-out investigation results can be found in Figure 103 and Figure 104.

For tip-in results shown in Figure 103, VGT vane position is closed to build up exhaust pressure and extract more engine exhaust energy to turbocharger to have enough compressor power so that fresh air can be charged into intake manifold. This action leads to the increased boost pressure. Meanwhile, exhaust manifold pressure \( P_3 \) increases rapidly because of flow restriction caused by the reducing effective turbine area. Because dynamics of exhaust pressure is much faster than that of boost pressure due to small exhaust manifold volume compared to that of intake manifold. This leads to the increased pressure ratio \( P_3/P_2 \); and high-pressure ratio \( P_3/P_2 \) results in closing EGR valve to reduce the EGR mass flow rate. The maximal pressure difference across EGR valve is determined by the VGT vane position. When VGT vane closes, the EGR flow rate increases; and meanwhile EGR valve opens to decrease EGR mass flow rate. The combined effect makes the overshoot of EGR mass flow rate overshot due to different dynamics of EGR and VGT systems. This results in unsatisfactory tracking results for EGR mass flow rate during tip-in.

For the tip-out case shown in Figure 104, the VGT vane position is fully opened to reduce boost pressure to target value. During this process, exhaust pressure drops much faster than boost pressure, which leads to negative pressure difference across the EGR valve and the EGR valve is fully closed to prevent the reversed EGR flow from intake to exhaust manifold. When boost pressure reduced down to below the exhaust pressure, EGR valve opens to track the target EGR
mass flow rate, leading to reduced exhaust pressure again. Hence the fluctuations in both exhaust pressure and EGR flow occur around 102 second. Fast exhaust pressure reduction leads to the EGR valve losing during aggressive transient tip-out.

The current two SISO controllers cannot not well track EGR mass flow rate during transient tip-in and tip-out operations, which provides the improvement opportunities for the proposed coordinated control design. The main task for VGT and EGR control is to control the intake and exhaust pressures coordinately to achieve the desired boost pressure and EGR mass flow rate. Good boost pressure tracking leads to good engine transient response and combustion efficiency. Good EGR mass flow rate tracking leads to low NOx emissions during transient operations. To improve tracking performance, an MIMO controller needs to be designed for VGT and EGR.

Figure 102. Simulation results with baseline controller
Figure 103. Tip-in investigation for baseline controller

Figure 104. Tip-out investigation for baseline controller
6.4.2 LQI controller design for different design target

For the proposed LQI design, matrices $Q$ and $R$ are design parameters used to penalize the outputs and the control inputs. In this case, weighting coefficients $Q_{11}$ and $Q_{22}$ take account for the tracking errors of boost pressure and EGR mass flow rate. Meanwhile, $R_{11}$ and $R_{22}$ are weighting parameters for VGT vane and EGR valve positions. When $R \gg Q$, the cost function is dominated by the control effort $U$, and the controller minimizes the control action. When $R \ll Q$, the cost function is dominated by the output tracking errors, and there is almost no penalties for using large control efforts. It is challenging to tune the weighting matrices to achieve acceptable responses for all the performance outputs and keep the control inputs within their actuation limits.

In order to tune matrices $Q$ and $R$ for different controller design target, three different evaluation indexes are defined as Performance index, Emission Index, and Fuel Economy index as below.

$$
Performance_{index}(Q_{11}^i, Q_{22}^i) = 1 - \frac{\left(\int_{t_1}^{t_2} \left| p^d_2 - p_2 \right| dt \right)_i - \min_{i=1}^n \left(\int_{t_1}^{t_2} \left| p^d_2 - p_2 \right| dt \right)}{\max_{i=1}^n \left(\int_{t_1}^{t_2} \left| p^d_2 - p_2 \right| dt \right) - \min_{i=1}^n \left(\int_{t_1}^{t_2} \left| p^d_2 - p_2 \right| dt \right)} \quad (6.37)
$$

$$
Emission_{index}(Q_{11}^i, Q_{22}^i) = 1 - \frac{\left(\int_{t_1}^{t_2} \left| \dot{m}_{egr}^d - \dot{m}_{egr} \right| dt \right)_i - \min_{i=1}^n \left(\int_{t_1}^{t_2} \left| \dot{m}_{egr}^d - \dot{m}_{egr} \right| dt \right)}{\max_{i=1}^n \left(\int_{t_1}^{t_2} \left| \dot{m}_{egr}^d - \dot{m}_{egr} \right| dt \right) - \min_{i=1}^n \left(\int_{t_1}^{t_2} \left| \dot{m}_{egr}^d - \dot{m}_{egr} \right| dt \right)} \quad (6.38)
$$

$$
Fuel\_economy_{index}(Q_{11}^i, Q_{22}^i) = \frac{\left(\int_{t_1}^{t_2} \left| \tau_{engine} \right| dt \right)_i - \min_{i=1}^n \left(\int_{t_1}^{t_2} \left| \tau_{engine} \right| dt \right)}{\max_{i=1}^n \left(\int_{t_1}^{t_2} \left| \tau_{engine} \right| dt \right) - \min_{i=1}^n \left(\int_{t_1}^{t_2} \left| \tau_{engine} \right| dt \right)} \quad (6.39)
$$

for $i = 1 \ldots n$

The Performance index is defined as normalized boost pressure tracking error. Boost pressure tracking error is normalized to its maximum and minimum values of testing points for
different $Q_{11}$ and $Q_{22}$ combinations. The same normalized method is also applied to the Emission index that is used to evaluate the EGR mass flow rate tracking error. Note that, the index values vary from 0-1. For Performance and Emission indices, the highest value represents the best tracking results (minimum tracking error). For fuel economy evaluation, the Fuel Economy index is defined as the accumulated output engine torque. Since fuel injection quantity and engine speed are given for the gain-scheduling controller and engine model, engine output torque would be a good parameter to evaluate engine combustion efficiency and pumping loss. For the Fuel Economy index, the highest value is for the best fuel efficiency. The higher index, the better control performance.

For the control input weighting matrix $R$, the ratio between VGT vane and EGR valve positions is chosen to be 10:1. The reason is that, slow VGT action leads slow exhaust pressure dynamics; and selecting the weighting matrix in such a way gives more control authority for EGR mass flow rate tracking.

$$R = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix} \quad (6.40)$$

Weighting matrix for tracking boost pressure and EGR mass flow rate are tuned based on a sweep study. This sweep study is used to achieve different controller performance as defined previously in (6.37), (6.38), and (6.39). The range for the normalized block value of matrix $Q$ is shown in Table 20. The simulation studies under different $Q$ matrix are based on a small step perturbation around a base engine speed and fuel injection quantity. In this study, the small step change (100rpm and 2mg/cc) is simulated around a light load engine operational condition with 20mg/cc fuel injection quantity at 800 rpm. In the simulation study, the target engine boost pressure and EGR mass flow rate for each case are shown in Figure 105. The target tracking
values are generated using engine fuel injection quantity and engine speed as discussed previously. Each designed linear controller is simulated based on the nonlinear plant.

![Graphs showing Engine fuel injection, Engine speed, Target boost pressure, and Target EGR mass flow rate](image)

Figure 105. A load step test profile for engine operating at 800 RPM

<table>
<thead>
<tr>
<th>Table 20. Normalized range for Q matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Min value</td>
</tr>
<tr>
<td>Max value</td>
</tr>
</tbody>
</table>

With the defined performance indices in (6.37), (6.38) and (6.39) for controller design, simulation results for different weighting matrix Q are shown in Figure 106. The results clearly show the controller design trade-offs for different design targets. Note that $Q_{11}$ and $Q_{22}$ are penalty parameters for both tracking errors of boost pressure and EGR flow rate, respectively. With higher $Q_{11}$, the designed controller emphasizes more on the EGR flow tracking error, resulting a tight tracking error bound. The same works for $Q_{22}$. Larger $Q_{22}$ leads to smaller boost pressure tracking error than EGR mass flow rate. The different selection of $Q_{11}$ and $Q_{22}$ leads to different tracking error performances for the boost pressure and EGR flow rate.
Figure 106. Weighting selection for different controller design index for 800 rpm engine speed / 20 mg/cc fuel injection

Large weighting coefficients for both boost pressure and EGR mass flow rate errors leads to reduced boost pressure tracking error as shown in Figure 106 (a). Increasing weighting on boost pressure error ($Q_{22}$) leads to improved boost pressure performance index. Since, smaller EGR tracking error helps to increase exhaust pressure by improving controllability of EGR flow rate, and hence, increasing weighting $Q_{11}$ on EGR mass flow rate error will also reduce the boost pressure tracking bound. However, only large weighting $Q_{11}$ leads to improved EGR tracking (high Emission index) results as shown in Figure 106 (b). Since improved EGR flow rate and boost pressure tracking results lead to increased exhaust pressure; and engine output torque will decrease with higher pumping loss. Reduced engine output torque leads to worse Fuel Economy index as shown in Figure 106 (c). This also shows that the best boost pressure tracking, the EGR tracking and fuel economy cannot be achieved at the same time. From the fuel efficiency prospective, the best fuel efficiency controller design region is also opposite to the best performance design region.

Figure 107 shows the trade-offs for the VGT-EGR system; and it also shows that the design target for the performance, emission reduction, and fuel economy can be coordinated through weighting selection. It needs to compromise other two targets to improve the other. This also
shows that with proper selection of weighting parameters, the controller can be designed for the different targets. As shown in Figure 107, with coordinated weighting for $Q_{11}$ and $Q_{22}$, it is possible to achieve similar engine performance target (boost pressure tracking) with better emission target (EGR mass flow tracking) without degrading fuel economy. Same for the other two targets.

![Trade off between performance, emission, fuel efficiency for controller design for 800 rpm engine speed / 20 mg/cc fuel injection](image)

Figure 107. Trade-off between performance, emission and fuel efficiency for controller design for 800 rpm engine speed / 20 mg/cc fuel injection

![Trade off between performance, emission, fuel efficiency for controller design for 800 rpm engine speed / 20 mg/cc fuel injection](image)

Figure 108. Trade-off between performance, emission and fuel efficiency for controller design for 800 rpm engine speed / 20 mg/cc fuel injection
Since in this study, we focus on boost pressure and EGR mass flow rate tracking, the controller designed for the best performance (\(Q_{11} = 10\) and \(Q_{22} = 10\)) and for best emission (\(Q_{11} = 10\) and \(Q_{22} = 3\)) are simulated in the nonlinear engine plant. For simplicity, we call these two controllers performance and emission controllers, respectively. The simulation results are compared with the baseline controller shown in Figure 109, and it shows that the designed MIMO controllers can achieve different target performances. For instance, for the performance controller, it has faster boost pressure response and smallest boost tracking error under the transient tip-in operation. Since more weights are put for the boost pressure tracking error, the designed controller penalizes more on boost tracking error than EGR mass flow tracking error. As shown in Figure 109, EGR valve opens for the performance controller to build up boost pressure through EGR flow during the tip-in operation. Although the EGR tracking suffers for the performance controller during transient tip-in, EGR mass flow rate can be well regulated during tip-out, compared to the baseline controller. For the emission controller, the EGR flow rate has the smallest tracking error with a larger weighting on EGR tracking error used for controller design; and boost pressure response lags behind, compared with the other two controllers. During the tip-out, only the designed MIMO controllers can handle the aggressive exhaust pressure drop. The control actions for both MIMO controllers are to regulate both boost pressure and EGR mass flow rate at the same time. For the baseline controller, as mentioned previously, it cannot track EGR mass flow rate properly without coordinated control action of both VGT and EGR control during transient tip-out operation. As shown in Figure 110, pressure difference across EGR valve for the baseline controller drops down to zero, leading to no controllability for EGR mass flow rate through the EGR valve.
Figure 109. Comparison controller design with baseline controller.

Figure 110. Pressure difference across EGR valve for three different controllers
6.4.3 Gain-scheduling for linear controllers

In order to implement the linear controller for the nonlinear engine plant, gain-scheduling control approach is used in this study. With the same LQI control and weighting matrix selection approach, two sets of controllers are designed for four different engine operating points. One set is targeted for performance to minimize boost pressure tracking error; and the other is targeted for emissions to minimize EGR mass flow tracking error. Meanwhile, both set of controllers take account for the other tracking purpose. Designed controllers are shown in Table 21.

Table 21. Controller designs for different engine operation point

<table>
<thead>
<tr>
<th>Controller design for emission</th>
<th>Engine Speed [rpm]</th>
<th>Fuel Injection [mg/stroke]</th>
<th>Proportional gain $K_x$</th>
<th>Integral gain $K_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controller gain</td>
<td>800</td>
<td>20</td>
<td>$\begin{bmatrix} 2.19 \times 10^{-5} \ 4.34 \times 10^{-4} \ -2.43 \times 10^{-4} \ -7.5 \times 10^{-3} \end{bmatrix}$</td>
<td>$\begin{bmatrix} 5.06 \times 10^{-3} \ 11.06 \times 10^{-4} \ -1.60 \times 10^{-4} \end{bmatrix}$</td>
</tr>
<tr>
<td></td>
<td>1200</td>
<td>20</td>
<td>$\begin{bmatrix} 1.08 \times 10^{-5} \ 2.96 \times 10^{-4} \ -1.09 \times 10^{-4} \ -2.9 \times 10^{-3} \end{bmatrix}$</td>
<td>$\begin{bmatrix} 3.03 \times 10^{-5} \ 11.06 \times 10^{-4} \ -9.58 \times 10^{-4} \end{bmatrix}$</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>35</td>
<td>$\begin{bmatrix} 1.83 \times 10^{-5} \ 3.30 \times 10^{-4} \ -1.7154 \times 10^{-4} \ -5.6 \times 10^{-3} \end{bmatrix}$</td>
<td>$\begin{bmatrix} 4.74 \times 10^{-5} \ 1.10 \times 10^{-4} \ -1.49 \times 10^{-4} \end{bmatrix}$</td>
</tr>
<tr>
<td></td>
<td>1200</td>
<td>35</td>
<td>$\begin{bmatrix} 1.32 \times 10^{-5} \ 2 \times 10^{-4} \ -1 \times 10^{-4} \ -3 \times 10^{-4} \end{bmatrix}$</td>
<td>$\begin{bmatrix} 4.13 \times 10^{-5} \ 11.10 \times 10^{-4} \ -1.30 \times 10^{-4} \end{bmatrix}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Controller design for performance</th>
<th>Engine Speed [rpm]</th>
<th>Fuel Injection [mg/stroke]</th>
<th>Proportional gain $K_x$</th>
<th>Integral gain $K_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>800</td>
<td>20</td>
<td>$\begin{bmatrix} 3.44 \times 10^{-5} \ 4.29 \times 10^{-4} \ -2.7 \times 10^{-4} \ -1.71 \times 10^{-2} \end{bmatrix}$</td>
<td>$\begin{bmatrix} 5.07 \times 10^{-3} \ 11.10 \times 10^{-4} \ -1.1 \times 10^{-3} \end{bmatrix}$</td>
</tr>
<tr>
<td></td>
<td>1200</td>
<td>20</td>
<td>$\begin{bmatrix} 2.28 \times 10^{-5} \ 2.92 \times 10^{-4} \ -1.31 \times 10^{-4} \ -0.8 \times 10^{-2} \end{bmatrix}$</td>
<td>$\begin{bmatrix} 3.11 \times 10^{-3} \ 11.10 \times 10^{-4} \ -6.95 \times 10^{-4} \end{bmatrix}$</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>35</td>
<td>$\begin{bmatrix} 3.37 \times 10^{-5} \ 3.28 \times 10^{-4} \ -2.08 \times 10^{-4} \ -1.35 \times 10^{-2} \end{bmatrix}$</td>
<td>$\begin{bmatrix} 4.93 \times 10^{-5} \ 11.10 \times 10^{-4} \ -1.1 \times 10^{-3} \end{bmatrix}$</td>
</tr>
<tr>
<td></td>
<td>1200</td>
<td>35</td>
<td>$\begin{bmatrix} 2.61 \times 10^{-5} \ 2.7 \times 10^{-4} \ -1.4 \times 10^{-4} \ -1.05 \times 10^{-2} \end{bmatrix}$</td>
<td>$\begin{bmatrix} 4.21 \times 10^{-5} \ 11.10 \times 10^{-4} \ -9.41 \times 10^{-4} \end{bmatrix}$</td>
</tr>
</tbody>
</table>

A bilinear interpolation is used to schedule controller gain inside the operating envelop. The gain scheduling is based on engine speed and fuel injection quantity as shown in Figure 111.
With designed controller gains for the four boundary points in this study, each controller gain inside the operational range can be obtained as shown below:

\[
k^*(x^*, y^*) \approx \frac{1}{(x_2 - x_1)(y_2 - y_1)} [x_2 - x^* x' - x_1] \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix} [y_2 - y^*] - [y' - y_1]
\]

where, \( x \) is the engine speed (\( N_e \)), \( y \) is the fuel injection (\( \dot{m}_{fuel} \)), and \( k \) is the controller gain.

The final control law for the nonlinear plant is:

\[
u_1(t) = \bar{u}_1 - K_{x(N_e, \dot{m}_{fuel})}(x_1(t) - \bar{x}_1) - K_{y(N_e, \dot{m}_{fuel})} \int_0^t (\delta y_1(t)) \, dt
\]

The gain-scheduling for each controller can be found in Figure 113, Figure 114, Figure 115, Figure 116. The gain \( Kx_{12} \) is the proportional gain for the boost pressure error used for the VGT control. As shown in Figure 113 and Figure 115, the control gain \( Kx_{12} \) (from boost pressure to VGT control) for performance controller is higher than the emission controller. The gain \( Kx_{12} \) increases as engine load increases in terms of engine speed and fuel injection. Meanwhile, the gain \( Kx_{22} \) is negative, which is boost pressure error proportional gain for EGR valve control.
Negative $Kx_{22}$ gain means in order to have higher boost pressure, EGR value needs to be closed to build up exhaust pressure. Since large weighting for the boost pressure tracking are used during controller design to improve performance, the magnitude of $Kx_{22}$ for boost pressure tracking is larger than $Kx_{22}$ for emission controller. On the exhaust pressure side, the gain $Kx_{11}$ is the proportional gain for VGT to regulate exhaust pressure error. For the performance controller, $Kx_{11}$, $Kx_{12}$ and $Kx_{13}$ are much larger than that of emission controller to achieve better boost pressure tracking. For the emission controller, the magnitude of $Kx_{22}$ is smaller than that of performance controller, leading to less effort from EGR valve to boost pressure error. All the calculated gains distinguish the two different controller designs for different targets. Meanwhile, VGT and EGR actions are coordinated.

![Figure 112. Controller based on gain scheduling](image-url)
Figure 113. Gain-scheduling for VGT control (target for emission)

Figure 114. Gain-scheduling for EGR controller (target for emission)
Figure 115. Gain-scheduling for VGT (target for performance)

Figure 116. Gain-scheduling for EGR (target for performance)
A transient profile shown in **Error! Reference source not found.** within the envelop of light load operations is simulated for the three sets of controller: performance MIMO controller, emission MIMO controller and baseline controller (two SISO controllers). Both MIMO controllers are gain-scheduled based on engine speed and fuel injection quantity. The three different control algorithms, evaluated with the same set-point strategy, show comparable performance, emissions and fuel economy.

![Gain scheduling route for controller validation.](image)

Figure 117. Gain scheduling route for controller validation.

The simulation results are shown in Figure 118 and Figure 119. The simulation results of the gain-scheduling controller are shown in Figure 120. Performance controller shows more aggressive boost pressure tracking during tip-in and tip-out operations, compared to both emission and baseline controllers, and also has good EGR rate tracking performance. For the
emission controller, it has best EGR tracking performance among the three controllers. However, the transient response of boost pressure is compromised by reduced aggressiveness of the VGT vane control. As shown in Figure 120, the proportional gain of the performance controller for VGT vane position is higher than that of the emission controller. Also EGR control gain from TC speed channel $Kx_{23}$ is higher for the performance controller than that of the emission controller. This leads to higher VGT and higher EGR control gains for the performance controller. Both proportional and integral gains from boost pressure channel ($Kx_{22}$ and $Ky_{22}$) are negative for EGR valve control, which means the EGR valve is intended to be closed for reducing boost pressure error. However, the EGR valve position is still driven by associated tracking error and the VGT and EGR control is coordinated. The highest proportional gain for both EGR and VGT control is from TC speed, which dominates the closed-loop response. This perhaps could provide new thoughts for future gain-scheduling control design based on only TC speed. As engine load gets increased, VGT gain decreases and EGR control gain increases. Compared to the baseline controller, the two MIMO controllers is able to track both boost pressure and EGR mass flow.
Figure 118. Gain scheduling controller validation with baseline controller

As shown in Figure 119, different controller leads to the different pressure difference across EGR valve. Due to the aggressive VGT action of the performance controller, pressure difference between P_3 and P_2 is larger than the two other controllers, which results in different pumping losses. As shown in Figure 121, by comparing the boost pressure tracking, EGR mass flow rate tracking, and output torque, the performance controller has better boost tracking and EGR tracking than baseline controller. But it has lower output torque due to increased pumping loss. Emission controller has the smallest EGR tracking error with the best engine output torque. But the boost pressure tracking for the emission controller is the worst among the three cases. This agrees with the controller design trade-off shown in Figure 107 that the three design targets cannot be optimized at the same time.
The simulation results confirm the proposed gain-scheduling MIMO controller for the VGT-EGR system, and also demonstrate that the EGR-VGT control can be designed for different targets by selecting different weighting parameters. Comparing with the baseline controller, the designed performance controller has better tracking results with respect to both boost pressure and EGR mass flow rate. Furthermore, the proposed controller design method introduces flexibility for multi-targets closed-loop control design for the VGT-EGR system.
Figure 120. Gain-scheduling for VGT-EGR

Figure 121. Benchmarking with baseline controller
6.4.4 Extended controller design for assisted and regenerative turbocharger

The controller design scheme for the VGT-EGR system can be easily extended to the regenerative hydraulic assisted turbocharger system shown in Figure 122. In this study, the hydraulic power is only treated as control inputs for the diesel engine air-path system. The two actuators are treated as one control input \( u_{rh} \), where positive input of \( u_{rh} \) is for hydraulic pump power and negative power is for hydraulic turbine power. The hydraulic valve is assumed to have fast response time with neglecting actuator dynamics. Hydraulic energy in the hydraulic tank is not considered during the control design. The hydraulic system is only activated during transient operations to reduce the boost pressure and EGR mass flow rate errors. By analyzing both nonlinear and linearized plants for the hydraulic assisted and regenerative turbocharger, the difference between EGR-VGT and EGR-VGT-RHAT appears in the control input matrix \( \tilde{B} \). Since hydraulic power is a direct input to the TC shaft in the speed dynamic equation (6.34), \( \tilde{B} \) matrix can be reformatted into \( \tilde{B}^* \) by adding column \( B_{32} \) to take account of the assisted and regenerative power as shown in (6.35).
Figure 122. Regenerative hydraulic assisted turbocharged diesel engine

\[
\dot{\omega} = \dot{W}_T - \dot{W}_c - \dot{W}_{loss} + u_{rhat}
\]  \hspace{1cm} (6.43)

\[
\tilde{B}^* = \begin{bmatrix}
B_{11} & 0 & 0 \\
0 & 0 & 0 \\
0 & B_d & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]  \hspace{1cm} (6.44)

Table 22. Control input value range and physical interpretation

<table>
<thead>
<tr>
<th>Control Input</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>EGR valve ((u_{egr}))</td>
<td>0 %</td>
<td>100%</td>
<td>Valve control for EGR valve position</td>
</tr>
<tr>
<td>VGT vane position ((u_{vgt}))</td>
<td>0%</td>
<td>100%</td>
<td>VGT vane position</td>
</tr>
<tr>
<td>RHAT power ((u_{rhat}))</td>
<td>-25kW</td>
<td>25kW</td>
<td>Hydraulic system power range</td>
</tr>
</tbody>
</table>

As long as the operational space \(\psi = \{(P_1, P_2, P_3, P_4), \ P_3 > P_4, P_2 > P_1, P_3 > P_2\}\) is guaranteed for engine air path system, pairs \(\langle \tilde{A}, \tilde{B}^* \rangle\) and \(\langle \tilde{A}, \sqrt{Q} \rangle\) are fully controllable and observable for the VGT-EGR-RHAT system. The same gain-scheduling LQI approaches for the VGT-EGR system is used to design coordinated controller for the VGT-EGR-RHAT system shown in Figure 123. Note that there is no set-point for RHAT input and the RHAT control
is \( u_{\text{rhat}} = \delta u_{\text{rhat}} \). The final controller for the VGT-EGR-RHAT system is shown in (6.45). The RHAT control gain appears in the third row of control gain matrix.

\[
\begin{bmatrix}
    K_{x_{11}} & K_{x_{12}} & K_{x_{13}} & K_{x_{14}} \\
    K_{x_{21}} & K_{x_{22}} & K_{x_{23}} & K_{x_{24}} \\
    K_{x_{31}} & K_{x_{32}} & K_{x_{33}} & K_{x_{34}} \\
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
    K_{y_{11}} & K_{y_{12}} \\
    K_{y_{21}} & K_{y_{22}} \\
    K_{y_{31}} & K_{y_{32}} \\
\end{bmatrix}
\]

where

\[
\begin{align*}
    u(t) &= \bar{u} - K_{x(N_{e,m_{\text{fuel}}})}(x_1(t) - \bar{x}_1) - K_{y(N_{e,m_{\text{fuel}}})} \int_0^t (\delta y_1(t)) \, d\tau \\
\end{align*}
\]

With the assisted and regenerative power on TC shaft, extra energy is used to drive the compressor to achieve faster boost response. Based on previous analysis, assisted power on the TC shaft increases the bandwidth for boost pressure control, comparing with the traditional VGT, and the VGT vane position does not need to be closed tightly to build up high exhaust pressure for turbine power extraction. VGT can be used for both boost pressure control and EGR mass flow regulation. With three actuators, the VGT-EGR-RHAT system would have much better control for diesel air-path system. First control design trade-offs are investigated. Performance index, Emission index, and Fuel Economy index for the VGT-EGR system are extended to the
VGT-EGR-RHAT system. For the RHAT system, since assisted and regenerative power uses the external energy; additional RHAT energy index is defined for assisted and regenerative power in (6.46). The energy index is used to evaluate the RHAT control action. The higher energy index, the less hydraulic actuation energy is used. Frequent hydraulic actuation leads to low energy cost index.

\[
\text{RHAT energy index} (Q_{11}^i, Q_{22}^i) = 1 - \frac{\left(\int_{t_1}^{t_2} |u_{r\text{hat}}(t)| dt \right)_i - \min_{i=1}^n \left[\left(\int_{t_1}^{t_2} |u_{r\text{hat}}(t)| dt \right)_i\right]}{\max_{i=1}^n \left[\left(\int_{t_1}^{t_2} |u_{r\text{hat}}(t)| dt \right)_i\right] - \min_{i=1}^n \left[\left(\int_{t_1}^{t_2} |u_{r\text{hat}}(t)| dt \right)_i\right]} \tag{6.46}
\]

for \(i = 1 \ldots n\)

![Performance index](image1.png)
![Emission index](image2.png)
![Fuel economy index](image3.png)
![RHAT energy index](image4.png)

Figure 124. Normalized indexes with normalized Q matrix for controller design
The different indexes results for different combinations of weighting matrix are obtained from the same load step simulations for the same engine operated at 800RPM shown in Figure 105. As shown in Figure 124, compared with the VGT-EGR case, Performance index for the VGT-EGR-RHAT system is mainly dependent on the boost pressure weighting $Q_{22}$. This is because the external power can be used to control boost pressure instead of using VGT. The EGR weighting $Q_{11}$ has less influence on boost pressure tracking, compared to the VGT-EGR case. With larger $Q_{11}$, EGR tracking error reduction is expected. Higher exhaust pressure leads to high pumping losses and decreases the engine fuel efficiency. For the Fuel eEconomy index, since the boost control can be independent on the exhaust pressure with the RHAT system, improvement on the boost pressure tracking has less effect on EGR mass tracking. Hence, with the RHAT system, high energy usage (low RHAT energy index) leads to high Fuel Economy index as shown in Figure 124. Based on the analysis, fuel economy improvement is expected with extra assisted and regenerative power on TC shaft, assuming the hydraulic energy is mainly from the driving shaft during braking.

In this study, we pay more attention to the hydraulic actuation energy; hence two sets of controllers for the VGT-EGR-RHAT system are designed within the same operational range as the previous VGT-EGR case. The two sets of controllers are based on high and low energy indices. High energy index controller uses relative low RHAT energy for both hydraulic turbine and pump with weighting matrix ($Q_{11} = 2, Q_{22} = 2$). Low energy index controller uses relative high RHAT energy with ($Q_{11} = 10, Q_{22} = 10$). The same gain-scheduling approaches for the VGT-EGR system are used for the nonlinear engine plant with the same operating range as the VGT-EGR case. The simulation results, compared with the corresponding VGT-EGR cases, are shown in Figure 125, Figure 126, and Figure 127.
For the low energy index controller (high assisted and regenerative power), RHAT provides aggressive assistant during the initial tip-in operations as shown in Figure 126. Turbine mass flow rate increases with the increased TC speed, leading to reduced exhaust pressure. This leads to the aggressive VGT closing action to build up exhaust pressure; meanwhile, EGR valve closes to keep EGR mass flow rate error small. Next, exhaust manifold pressure increases due to the increased exhaust mass flow. Since less energy is needed from the turbo side for the low energy index case, then VGT vane opens to reduce the exhaust pressure. With the increased boost and exhaust pressures, both VGT vane and EGR valve open, leading to a constant pressure drops between exhaust and intake pressures shown in Figure 127. This small pressure different across EGR valve keeps EGR mass flow through EGR valve with reduced pumping loss. Hence, the low energy index controller has the best fuel efficiency due to reduced pumping loss. With less assisted power (high energy index), VGT action is still trying to build up exhaust pressure to drive the compressor for improving boost pressure tracking performance. With the increased exhaust pressure, EGR valve is closing to reduce the effect of higher exhaust pressure to keep the right amount of EGR mass flow rate during the tip-in operations. The coordinated control for the VGT-EGR-RHAT system improves engine tracking performances for both boost pressure and EGR mass flow rate.

The benefit of hydraulic assisted and regenerative turbocharger is obvious as shown in Figure 127 and Figure 129. The pressure difference between the exhaust and intake manifolds reduces with the RHAT system, leading to reduced pumping loss. Meanwhile, boost pressure and EGR mass flow rate tracking performance can be well maintained through the interaction among VGT vane and EGR valve positions, and RHAT turbine and pump power. As shown in Figure 129 and Table 23, RHAT system can reduce boost pressure tracking error, EGR mass flow rate tracking
error, and at the same time improve engine output torque, compared to baseline VGT-EGR system. By comparing with traditional VGT-EGR case, the performance, emissions, and fuel economy can be achieved at the same time with external assisted and regenerative power on TC shaft. However, the cost for these benefits is hydraulic energy.

In summary, the proposed control design approach is extended to the VGT-EGR-RHAT system successfully with promising results.

![Simulation results](image)

**Figure 125. Simulation results for different controller design**

![Hydraulic actuation inputs](image)

**Figure 126. Hydraulic actuation inputs for VGT-EGR-RHAT**
Figure 127. Pressure difference across EGR valve
Figure 128. EGR mass fraction, EGR mass flow rate and EGR valve position for different control design
Figure 129. Benchmarking with baseline controller

Figure 130. Comparison of different control designs

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Table 23. Benchmarking with baseline controller

<table>
<thead>
<tr>
<th>Controller</th>
<th>Boost pressure tracking improvement</th>
<th>EGR mass flow rate tracking improvement</th>
<th>Accumulated engine power improvement</th>
<th>Accumulated hydraulic actuation energy usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline controller</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>VGT-EGR controller designed for performance</td>
<td>14.6%</td>
<td>44.1%</td>
<td>-0.48%</td>
<td>0</td>
</tr>
<tr>
<td>VGT-EGR controller designed for emission</td>
<td>-92.9%</td>
<td>92.9%</td>
<td>0.89%</td>
<td>0</td>
</tr>
<tr>
<td>VGT-EGR-RHAT controller with low energy index</td>
<td>93.5%</td>
<td>29.4%</td>
<td>5.68%</td>
<td>4.6kJ</td>
</tr>
<tr>
<td>VGT-EGR-RHAT controller with high energy index</td>
<td>49.3%</td>
<td>31.3%</td>
<td>2.8%</td>
<td>2.8kJ</td>
</tr>
</tbody>
</table>

6.5 Conclusion

In this chapter, a systematic control design approach for a diesel engine air-path subsystem with an assisted and regenerative turbocharger is proposed. Linear quadratic integral (LQI) controllers are designed based on the linearized models over the gridded engine operational conditions for tracking both boost pressure and EGR flow rate, and gain-scheduling controller for a given operational condition is obtained by interpreting the LQI controllers around the operational conditions. The gain-scheduled control strategy is then used for evaluation using the nonlinear air-path engine system. Comparing to the baseline VGT-EGR controller, this approach provides a way to design the VGT and EGR controller with trade-off between boost pressure and EGR flow rate trade-offs by tuning the LQI weighting. Comparing with the dual-loop baseline single-input and single-output controller, the designed multi-input and multi-output controllers show improved tracking performance for both boost pressure and EGR mass flow rate, and nice trade-off characteristics between engine performance and emissions through weighting selection.

With the added regenerative hydraulic assisted turbine system, the VGT-EGR-RHAT controller further improves the transient engine performance without compromising EGR tracking performance due to additional power available on the turbocharger shaft.
CHAPTER 7. CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

A physics-based turbine power model of a variable geometry turbocharger (VGT) is proposed along with its thrust friction model. The turbine power model is derived based on the Euler turbine equations with the VGT vane position as the control input. A generalized method for identifying mechanical friction is also proposed. The proposed model has better accuracy than the traditional map-based model. The proposed the turbine power and its mechanical efficiency models are suitable for the model-based VGT control due to the analytic nature of the proposed models as a function of the VGT vane angle.

A compressor power model, based on the Euler turbomachinery equations with realistic assumptions, was developed. Two new performance coefficients, the power and speed coefficients ($C_{power}$ and $C_{speed}$), were proposed as an alternative to multiple performance maps. The proposed correlation between $C_{power}$ and $C_{speed}$ is especially useful to determine the compressor power necessary for a given compressor mass flow rate. This compressor power demand can then be translated into a VGT (Variable Geometry Turbo) vane position or an assist demand for an assisted boost systems. This relationship can also be easily used to compare compressor design variants with respect to performance and range. The reduced-order and reduced-complexity model is especially useful for the control applications. Developed compressor model is further extended to centrifugal hydraulic pump and hydraulic turbine system and show promising results.
A systematic approach for diesel engine air-path system modelling with regenerative hydraulic assisted turbocharger was developed. The model (simulator) is developed by integrating engine system, variable geometry turbocharger system, and hydraulic system. Engine air-path model is validated through engine transient test data. It shows that the proposed modelling approach has adequate accuracy with only three states for the engine air-path system. It could be used for model-based analysis and control design. The interactions between the VGT, EGR, and hydraulic actuators has been illustrated through validation simulations with the nonlinear engine model with good agreement.

Utilizing the combined efforts from the turbo and driveline pumps, used to recover both exhaust and vehicle braking energies, to offset the fuel economy penalty as a result of hydraulic power applied to the turbo shaft for improving turbocharger transient response is a novel idea. The 1-D GT-Power analysis indicates that it is possible to gain 3-5% fuel economy improvement with the RHAT system, compared with the baseline turbocharged diesel engine, over the FTP 75 transient cycle. This FE improvement does not include the other FE benefits that may be enabled by the RHAT technology such as engine downsizing, transmission optimization, etc.

A systematic approach for diesel engine air-path controller design is proposed. Gain-scheduling control is designed based on linear quadratic regulator with integral action. Gain-scheduling controller is implemented for the nonlinear VGT-EGR engine system. Comparing with the traditional VGT- EGR controller design, this approach provides a novel method to design the controller for different design targets by selecting different weighting matrices. By benchmarking with the baseline controller, the designed gain-scheduling multi-input and multi-output (MIMO) controller shows improved tracking results for both boost pressure and EGR mass flow rate, and it also provides design flexibility for the trade-offs between engine
performance and emissions. Further, this control design method is extended to the engine with an assisted and regenerative turbocharger. The simulation results show improved performance and fuel economy with reduced emissions at the same time with the additional hydraulic energy for the VGT-EGR-RHAT system. Note that the additional hydraulic energy can be recovered from driveline during vehicle braking.

7.2 Future work

1. Model turbocharger heat transfer effect to improve turbocharger system model accuracy.

2. Study the effect of operating the turbocharger outside its traditional region due to the utilization of assist or regeneration power. When a diesel engine is equipped with both low pressure EGR and high pressure EGR, the gas turbine could work as a compressor to have large amount of low pressure EGR mass; and compressor might work as a turbine when compressor upstream pressure is higher than the downstream one. Both lead to distinct physical characteristics, compared to traditional turbocharger. Both experimental and modelling approaches are needed to study the two new operation modes.

3. Validate the designed controller experimentally. The prototype hardware needs to be developed to assess this state of art technology.
REFERENCES
REFERENCES


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