DILEMMAS AND TEACHER DECISIONS
IN COLLABORATIVE MATHEMATICS DISCUSSIONS

By

Nicholas Joseph Gilbertson

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

Mathematics Education—Doctor of Philosophy

2017

ABSTRACT

DILEMMAS AND TEACHER DECISIONS
IN COLLABORATIVE MATHEMATICS DISCUSSIONS

By

Nicholas Joseph Gilbertson

Over the past few decades, organizations, researchers, and professional developers have noted the potential for using classroom discussions to improve student learning in mathematics classrooms. Yet, for teachers, managing the often-competing demands that arise while attempting to plan for and enact these discussions can be challenging. To better understand these challenges, this dissertation describes the results of a study characterizing the instructional decision-making of three experienced middle-grades mathematics teachers as they encountered instructional dilemmas while planning for and enacting collaborative mathematics discussions.

To study decision-making, I observed teachers across several lessons, interviewed them after lessons, and analyzed lesson plans. Studying decision-making as it appeared in teachers' lived classroom experiences provided an authentic and situated perspective on the complexities of their decision-making. In order to analyze their decisions, I attended to teachers' instructional goals, which consisted of lesson-specific content goals, broad content goals, mathematical practice goals, and goals for establishing a mathematical community. Additionally, I used Herbst & Chazan's (2011) theory of *professional obligations* to understand why teachers made certain decisions based on their goals, consistent patterns of instruction, and the particular dilemmas they encountered. This theory argues that teachers' decisions are a function of their obligations as representatives of the discipline of mathematics, in developing students as individuals, in mediating productive interaction between students, and as representatives of the school as an institution.

The main finding of this study was that all three teachers' decision-making was largely explained via their primary professional obligation – with important limitations. Along with this main finding, there were three subfindings. First, teachers' decision-making was largely explained via their primary orientation to their professional obligations. Second, all three teachers encountered scenarios in which they made decisions that deviated from their primary orientation to the professional obligations. Third, even though the teachers were experienced, used instructional resources (e.g., written curriculum materials) that supported implementing whole-class discussions, and were generally oriented philosophically with the importance of using discussions, this was not sufficient to avoid challenges one might expect to see while facilitating whole-class discussions. Most importantly, the types of challenges they encountered were, by-and-large, independent of the teachers and the nature of their dilemmas (i.e., how the dilemma was a conflict between two types of goals).

Understanding how the nature of a teacher's decision-making is influenced by attending to their professional obligations during instructional dilemmas contributes to multiple areas of mathematics education. First, this study contributes to the research on teacher decision-making by expanding teacher decision-making to include many more contributing factors beyond content-focused decisions. Second, this study contributes to the research on whole-class discussions by considering teachers' orientations (via their professional obligations) as a confounding variable in the nature of their decisions influencing the discussion. Third, this study contributes to professional development and teacher education by better understanding how teachers' professional obligations can influence the opportunities students have to learn mathematics via their interaction during whole-class discussions.

This dissertation is dedicated to my family that provided countless hours of support throughout its development. Thank you to Nikki, Josie, Gus, and Ovidia, for your dedication and encouragement throughout.

# ACKNOWLEDGMENTS

This dissertation study is a result of many individuals whom I wish to thank. I thank Dr. Kristen Bieda, my dissertation advisor, for her support throughout the development of this study. I also thank my committee -- Dr. Jack Smith, Dr. Corey Drake, and Dr. Jeanne Wald -- for their perspectives and input throughout the process. I also wish to thank Dr. Michael Steele for his early support in thinking through possible directions for the study. Thank you to Dr. Aaron Zimmerman and Rani Satyam for their help with data analysis and collection. Finally, this dissertation study was partially funded through fellowships from Michigan State University's College of Education and College of Natural Science. I wish to thank both colleges for supporting this work.

I would like to thank all those who supported me throughout the dissertation and in my development as a learner and teacher of mathematics. To my former teachers whom I have felt are with me every day in a classroom: Carol Hubbard, Dave Coutley, Vernon Daines, Rob Hepp, Ted Harris, Larry Krajewski, Mike Limberg, and Tom Thibodeau. For my early work in learning about whole-class discussions as a way to support students in communicating and thinking mathematically: Fred Rectanus, Nicole Rigelman, Sean Larsen, Luis Saldanha, and Karen Marrongelle. To those at Michigan State who supported me throughout this process: Kevin Lawrence, Betty Phillips, Yvonne Grant, and Billie Lozen. Most importantly, to my family: Nikki, Josie, Gus, and Ovidia, my parents Gary and Judy, and my grandparents. This dissertation is a direct result of their support and energy throughout the process.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

## KEY TO ABBREVIATIONS

| | |
|---|---|
| CCSSM | *Common Core State Standards in Mathematics* |
| CMP | *Connected Mathematics Project* |
| CMP2 | *Connected Mathematics* 2 |
| CMP3 | *Connected Mathematics* 3 |
| IQA | Instructional Quality Assessment |
| IRE | Initiate, Respond, Evaluate |
| LES | Launch, Explore, Summarize |
| MAD | mean absolute deviation |
| NCTM | National Council of Teachers of Mathematics |
| NGA-CCSSO | National Governor's Association and Council of Chief School State Officers |
| NSF | National Science Foundation |

# CHAPTER 1

# DISSERTATION OVERVIEW AND INTRODUCTION

With the recent widespread adoption of the *Common Core State Standards for Mathematics* (CCSS-M) in the vast majority of U.S. states, the *CCSS-M Standards for Mathematical Practice* could act as a much-needed catalyst to shift how mathematics has historically been taught and learned in the U.S. For example, if students are to "construct viable arguments and critique the reasoning of others" (NGA-CCSSO, 2010, p. 6), then students should learn mathematics in classrooms where social interaction around content is encouraged. However, many U.S. teachers still use direct instruction where teacher-to-student interactions often involve the teacher initiating a simple fact-based question, receiving a response from the students, and then providing feedback or evaluation (Cazden, 1988; Chapin, O'Connor & Anderson, 2009; Franke, Kazemi & Battey, 2007; Mehan, 1979; Sahin & Kulm, 2008; Stigler & Hiebert, 1999).

It is well documented that teaching using classroom discussions to serve the purpose of engaging students around rigorous content is quite challenging (Lampert et al., 2013; Tyminski, Zambak, Drake & Land, 2013). Unlike a more traditional mathematics lesson, which may involve large sections of teacher lecture with limited student talk, facilitating a classroom discussion involves being responsive to student thinking while continuing to move the conversation towards a mathematical goal (Sleep, 2012). The tension between being responsive to student thinking while simultaneously advancing the mathematical agenda in a lesson has the potential to create dilemmas for teachers as they grapple with how to attend to competing instructional goals.

If teachers are to teach with ambitious aims (Kazemi, Franke & Lampert, 2009; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010; Lampert, Boerst, & Graziani, 2011) such as being responsive to students' thinking while they work on authentic problems and engage in meaningful discourse, then this necessarily adds complexity to the nature of their decision-making. During discussions, teachers are faced with decisions related to the types of questions they ask students, which student ideas they choose to highlight, and how deeply to pursue a student response. All these decisions occur within the institutional constraints of schooling with finite resources, such as time and materials. As Lampert et al. (2010) argued, "Maintaining a coherent mathematical learning agenda while encouraging student talk about mathematics is perhaps the most challenging aspect of ambitious teaching" (p. 131).

This study aims to better understand how experienced teachers make decisions when faced with instructional dilemmas while planning and enacting whole-class discussions. Specifically, I investigated how teachers made decisions in whole-class discussions where various students are presenting their ideas while the teacher is managing several instructional goals in real time. A single decision can influence the direction of a discussion, and has the potential to send important (implicit and explicit) messages to students about their roles as learners and doers of mathematics. By studying how teachers manage instructional dilemmas when planning for and enacting whole-class discussions, this work contributes to the field's understanding of how teachers facilitate whole-class discussions (e.g., Chapin, O'Connor, & Anderson, 2009; Smith & Stein, 2011) by unpacking and explaining teachers' decisions related to facilitating discussions.

The primary research question guiding this study is: *What is the nature of instructional dilemmas that teachers face when planning for and enacting collaborative mathematics* *discussions, and how do teachers respond to those dilemmas through their decision-making?* In order to identify instructional dilemmas and analyze decision-making, I observed and video-recorded three middle grades mathematics teachers, conducted post-lesson and video-stimulated interviews, and collected lesson plans. To identify instructional dilemmas, I first conducted a preliminary analysis to identify teachers' instructional goals and their consistent patterns of instruction. This preliminary analysis allowed for identifying situations when goals came into conflict with one another, thus creating an instructional dilemma for the teacher. The decision-making related to the dilemma was analyzed using Herbst and Chazan's (2011) construct of *professional obligations*.

This dissertation consists of six chapters. Chapter Two consists of a review of the pertinent literature related to whole-class discussions and teacher decision-making. Chapter Three includes the theoretical perspective and analytic framework used in this study. Chapter Four describes the methods used in the dissertation and provides a description of each teacher's instructional practice. Chapter Five describes results pertinent to instructional dilemmas and teachers' decisions. Chapter Six includes a discussion of the results, implications of the work, and ideas for future work.

# CHAPTER 2

# LITERATURE REVIEW

Within the mathematics education research base, the word *discussion* is used frequently but is interpreted in different ways. This variance may be because the term connotes a vision of teachers and students actively talking together, which runs contrary to more historically prevalent modes of communication in mathematics classrooms, where students have been passive recipients of teacher talk. Researchers have used descriptors such as “whole-group,” “whole-class,” and “conceptual” to indicate these different types of discussion (e.g., Cengiz, Kline, & Grant, 2011; Crespo, Oslund & Parks, 2011; Henning, McKeny, Foley & Balong, 2012). Descriptions of discussions generally fall into two main categories: (a) *Discussions* are an approach to teaching allowing students and teachers to interact and talk about their mathematical ideas (e.g., Moschkovich, 1999); (b) *Discussion* is a specific portion of a lesson (e.g., Ellis, 2011; Jackson, Garrison, Wilson, Gibbons & Shahan, 2013). In the first case, discussions can occur at any point of a lesson with any grouping of students, and this type is more aligned with the view that discussions are a normative way of interacting in a mathematics class not tied to any particular portion of a lesson. The second type typically occurs towards the middle to end of a lesson after students have had an opportunity to work on a task. Although these two interpretations may be relatively unproblematic, for the purpose of this dissertation clarity about what is meant by discussion will help indicate which portions of the classroom interaction are included for data collection.

To provide a clear and consistent way to decide whether a discussion be included, I only chose those discussions focused on mathematical content or practice that involved the teacher and multiple students where the class had the opportunity to listen to the conversation (i.e., the teacher is not simply talking to a small group). I refer to the discussions that fit these criteria as *collaborative discussions* or, equivalently, *whole-class discussions* as a way to highlight the collaborative nature of the interactions that took place with the whole class. As a final note, this definition omits whole-class talk that may not be a "discussion," such as clarifying what to do on a task as well as during instances where a teacher lectured but had minimal student interaction, such as asking students a rhetorical question with responses that failed to impact the flow of the teacher's lecture.

## Talk in the Mathematics Classroom

Over time, we have come to recognize normative ways that teachers and students interact in mathematics classrooms. These ways of interacting within the classroom have notable differences from talk outside the classroom. Informal peer-to-peer conversations may be less goal-oriented than the conversations inside a classroom, where there is an explicit instructional goal by the teacher. As Cullen (1998) noted, while talk in the classroom and talk in the outside world are dictated by social norms and patterns of interaction, these two types of interactions are quite different because the main purpose of talking in the classroom is for pedagogical, not simply social, reasons.

In Western settings, traditional classroom environments rely upon the Socratic Method as a foundational approach to teaching through asking students questions. Particularly in mathematics classrooms, many teachers to this day still use the IRE format to talk with students during a lesson. In this format, teachers initiate a query, a student responds, and the teacher evaluates the response (Cazden, 1988; Mehan, 1979). An important characteristic of classrooms where the IRE method is predominant is that the teacher is not the only person talking during the lesson. This is unlike a lecture-based class where students may engage in little to no verbal interaction. Although researchers have noted the limitations of the IRE method, its historical past is so deeply rooted in U.S. mathematics education that it is not surprising that this method is still a predominant form of interaction in many U.S. classrooms, even when teaching for understanding is a focus of instruction (Franke, Kazemi & Battey, 2007). This predominance may also be the result of a more information-processing view of learning (e.g., Broadbent, 1958 as cited in Gardner, 1987), as opposed to a constructivist approach to learning mathematics (Schoenfeld, 1992; Smith, 1997).

This raises the question as to how this more traditional form of the IRE format is different than the types of collaborative discussions investigated in this study. Over the past few decades, organizations (e.g., NCTM, 2000) and researchers (e.g., Ball, 1993; Chapin, O'Connor, & Anderson, 2009; Kilpatrick, Swafford, & Findell, 2001; Lampert, 2001; Nathan & Knuth, 2003) have described a different vision for how discussions might take place in mathematics classrooms. There are two main differences between classrooms that predominantly make use of the IRE format and those that use a collaborative discussion. The first difference relates to the classroom environment and its normative modes of teacher and student interaction. The second difference relates to the format of discussion the teacher uses to advance lesson goals.

## Discourse-Oriented Teaching

The term *discourse-oriented teaching* provides a broad description of the type of classroom environment that supports collaborative discussions. According to Williams and Baxter (1996),

> We introduce the term *discourse-oriented teaching* [italics in original] to describe actions taken by a teacher that support the creation of mathematical knowledge through discourse among students. … [W]e see discourse-oriented teaching as involving not only the

> establishment of social norms that empower students to discuss mathematics but also a belief that properly orchestrated discourse among students will result in the production of useful knowledge. (p. 22)

There are a few key points to this approach in teaching mathematics that are similar and different from more normative approaches to teaching. One similarity is that both the traditional approach and discourse-oriented approach are focused on developing student knowledge. A key difference, however, is that in the discourse-oriented approach the role of the teacher is to provide opportunities and to support students with talking about mathematics with their peers. In a more traditional setting, student responses are often communicated through the teacher and are typically in response to a teacher query. In discourse-oriented classrooms, students are encouraged to ask questions to the teacher and to their peers, and they respond to these questions as a member of a mathematical community. Students are encouraged to engage in argumentation, justification, disagreement, and sense-making. Indeed, letting students talk and describe their understanding, along with their multitude of misconceptions and underdeveloped notions of mathematical ideas, makes teaching much more complex.

If teachers are to teach using a discourse-oriented approach, then this necessitates that teachers have a different type of knowledge in supporting students, such as the knowledge of student misconceptions and their possible causes, and how to act on those misconceptions in a productive way. Additionally, if students are to talk with their peers and their teacher about their thinking, social norms must be in place to help effectively support students’ willingness to share their thinking. In all, the demands on a teacher are much different in discourse-oriented teaching, compared to a traditional classroom setting where teaching focuses on procedural fluency, answering questions from students, and asking students predominantly simple fact-based questions with known answers (Chapin, O'Connor, & Anderson, 2009; Sahin & Kulm, 2008). In many cases, the "lecture" portion of a more traditional lesson is followed by individual seat-work, thus inhibiting students from having opportunities to engage deeply in the content with their peers.

For some students this traditional format may work, but for many students, talking through their underdeveloped ideas about mathematical concepts is essential for continuing to develop their mathematical knowledge. Perhaps more importantly, the teacher has better access to students' conceptions in real time through discourse-oriented teaching, because the students are describing their thinking at the time they are engaged in the lesson. Assessing student thinking by only looking at written work at the conclusion of a lesson provides only a glimpse of a student's thinking. The main implication for teachers attempting to use a discourse-oriented approach is that they must structure their lessons differently in order to support student-to-student interactions as a pedagogical approach to help produce useful mathematical knowledge.

## Collaborative Discussions

In contrast to the traditional form of mathematics instruction, a more *Standards*-based (NCTM, 1989; NCTM, 2000) approach to a lesson includes three main phases (Jackson & Cobb, 2010). A teacher starts the class by posing a high-demand task (Smith & Stein, 1998); students then work on the task in small groups, followed by a whole class discussion. These three phases are sometimes referred to as a teacher *Launching* the problem by setting up the task for students, followed by students *Exploring* in small groups, and then the class coming together as a whole group to discuss and then *Summarizing* the ideas in the lesson (Shroyer & Fitzgerald, 1986).

An inherent aspect of this approach to lesson design is that students learn from each other around rigorous mathematical content (Franke, Kazemi, & Battey, 2007) through collaboration instead of isolation (Lampert & Cobb, 2003). Aspects of high demand tasks include complex or non-algorithmic thinking, deeply understanding mathematical concepts, and fluently moving between representations (Smith & Stein, 1998). Students work together in small groups on tasks that focus on conceptually rich topics, providing opportunities for students to generate multiple solution strategies and to explore big ideas in mathematics.

In the latter part of the lesson, teachers facilitate students' work on the task in a collaborative discussion. During the discussion, students are actively engaged in listening to others' ideas about how they solved the task and how their strategies related to the strategies of their peers. When done effectively, teachers organize student solution strategies to make salient important ideas embedded in solution methods that correspond to the teachers' lesson goals. The teacher's role in this type of discussion is to be responsive to students while allowing students' ideas to take center stage, all the while advancing students' mathematical understanding.

This type of discourse-oriented teaching is quite challenging for experts and novices alike, because the teacher is constantly attending to ever-evolving ideas while also attending to individual students' needs and classroom interaction. Teachers often have several types of goals from content to communication (e.g., being precise in language use) to process/practice goals (e.g., justifying one's thinking). Even if a teacher is only concerned about the coherence of the mathematical ideas within a lesson; interpreting, connecting, and then building on students' responses in a way that promotes interaction between students can be daunting. A focus strictly on content does not take into account teachers' long-term, dispositional, or social goals that they may also have for students in the classroom. Teachers often have multiple purposes, both mathematical and nonmathematical, that they are attempting to satisfy during the same lesson (Sleep, 2012). As the discussion is occurring, teachers are monitoring the movement towards lesson goals; decisions about when to step in to expand on an idea, when to let students build off each other's ideas, and when to stop the discussion to summarize. These considerations occur in a setting that is time-sensitive, constraint-filled, and dilemma-ridden. The context of whole-class discussions is prime for studying how teachers make decisions in real time when they encounter instructional dilemmas requiring them to prioritize one aspect of their instruction over another. In this study, I investigated the nature of these dilemmas with special attention to teachers' decision-making during such dilemmas. In the next section, I provide a background of the teacher decision-making literature with specific attention to decision-making in whole-class discussions.

## Teacher Decision-Making Literature

Studying teacher decision-making dates back at least to the 1970s when different sets of researchers identified the centrality of decision-making to the improvement of teaching (Borko, Roberts, & Shavelson, 2008). Bishop and Whitfield (1972) identified a teacher's values and experience as contributing factors to a teacher's decision framework. Later, Shavelson and Stern (1981) identified other contributing factors such as a teacher's content knowledge, the nature of instructional tasks, and the influence of the institutional environment of the school. Recent work has included studies of science teachers viewing decisions as "argument-driven actions" (Lee & Lin, 2005, p. 429). Other teacher decision-making studies have focused on a variety of other aspects of teaching, from metacognition (e.g., Artzt & Thomas, 1998) to reflecting on one's teaching practice via video analysis (e.g., Rich & Hannafin, 2008). Put together, all of these studies in teacher decision-making focused on better understanding the nature of how teachers make decisions by understanding the underlying contributing factors that influence their decisions.

In mathematics education, two recent contributions to the teacher decision-making literature provide a foundation for this dissertation. First, Schoenfeld (2010) described modeling of decision-making and its role in mathematics teaching. He argued that "what people do is a function of their *resources* (their knowledge in the context of available material and other resources), *goals* (the conscious or unconscious aims they are trying to achieve), and *orientations* (their beliefs, values, biases, dispositions, etc.)" (p. xiv). Schoenfeld's (2010) perspective on teacher decision-making highlights the cognitive aspects of teacher thinking with strong emphasis on decisions in relation to mathematical content. He goes on to describe the importance of understanding teachers' goals and routines:

> *If you can understand (a) the teacher's agenda and the routine ways in which the teacher tries to meet the goals that are implicit or explicit in that agenda, and (b) the factors that shape the teacher's prioritizing and goal setting when potentially consequential unforeseen events arise, then you can explain how and why teachers make the moment-by-moment choices they make as they teach* [italics in original]. (Schoenfeld, 2010, p. 10).

Consistent with Schoenfeld's (2010) perspective on teacher decision-making, the main focus of this dissertation is to understand decision-making when teachers faced instructional dilemmas. By understanding their goals and consistent patterns of instruction, one can be better positioned to identify and explain the nature of their instructional dilemmas. Importantly, I deviate from Schoenfeld's (2010) orientation to teacher decision-making as a more rational nature and his perspective on instructional goals as more specific and content focused.

Others, such as Herbst and Chazan (2011), offer a different perspective on teacher decision-making. Their perspective considers teachers' decisions as a product of teachers attending to various *professional obligations.* Their contribution considers other external factors (such as the school setting) as being important factors in a teacher's decisions. These obligations provide a framework for understanding the reasons for the choices teachers make and for naming the influences that create instructional dilemmas for teachers. These obligations are described here:

> The *disciplinary obligation* [italics in original] says that the mathematics teacher is obligated to steward a valid representation of the discipline of mathematics. This may include the obligation to steward representations of mathematical knowledge, mathematical practices, and mathematical applications.
>
> The *individual obligation* [italics in original] says that a teacher is obligated to attend to the well-being of the individual student. This may include being obligated to attend to individual students' identities and to their behavioral, cognitive, emotional, or social needs.
>
> The *interpersonal obligation* [italics in original] says that the teacher is obligated to share and steward their medium of interaction with other human beings in the classroom. This may include attending to the needs and resources of shared discursive, physical, and social spaces within shared time.
>
> And the *institutional (schooling) obligation* [italics in original] says that the teacher is obligated to observe various aspects of the schooling regime. These include attending to school policies, calendars, schedules, examinations, curriculum, extracurricular activities, and so on. (pp. 450-451).

Much of Herbst and Chazan's previous work used simulated classrooms (e.g., Herbst, Chazan, Kosko, Dimmel, & Erickson, 2016), focusing primarily on understanding norms and deviations of those norms through simulated classroom scenarios. In these studies, they used simulated classroom environments to ask teachers what choices they would make in a particular situation. There are some limitations to this approach as it can constrain the teacher into choosing from only a small set of possible choices, which may or may not be plausible or realistic for that particular teacher. Additionally, it fails to address potential variables that may play out in the real world of the classroom. For example, a teacher may make two different decisions when posed with an interesting student question with five minutes left in class: if the question is asked the day before a chapter test versus the question being asked in the beginning of a unit. An exchange between teachers and students is inherently embedded within a broader sequence of lessons, so knowing whether the exchange aligns with a teacher's multiple instructional goals or not could make a huge difference as to the path the teacher takes. Finally, an important aspect of understanding how the interpersonal and individual obligations play out in the classroom is the teacher's own understanding of particular students and how they interact with each other.

Studying teachers in their own classrooms has the added benefit of understanding how teachers enact decisions in a more authentic environment. I deviate from Herbst et al.'s (2016) use of simulated classrooms by focusing on the inherent "situatedness" of teachers' cognition as the teaching occurs as a lived experience in their classrooms with students they encounter day after day. Consistent with Herbst and Chazan (2011), I use the construct of *professional obligations* to understand instructional dilemmas and teacher decision-making, because it affords the possibility of decisions being influenced by factors that are outside the domain of mathematical content that likely influence teacher decisions.

The professional obligations construct also provides a mechanism for understanding the inherent complexity in facilitating discussions while attempting to balance the needs of multiple goals (e.g., Sleep, 2012), the needs of various students, and the nature of classroom interaction. These conflicts were what made decisions meaningful, in that teachers decided whether to maintain their typical interaction-consistent patterns of instruction or consciously chose to deviate from these consistent patterns of instruction. Understanding what is typical and atypical of classroom behavior for teachers' facilitation of classroom discussions was important in understanding when, and eventually why, they made certain instructional decisions. Explaining teacher decision-making through the lens of professional obligations provided the opportunity to explain how teachers managed competing demands, based on teachers' views of who or what they felt primarily obligated to satisfy.

Balancing the demands of these different obligations may play out in different ways. Take for example the simple task of selecting which student to call on during a whole-class discussion. A teacher may choose to call on a student randomly to hold students accountable for paying attention in the class (institutional obligation). Another teacher might choose to call on a student because he has an interesting mathematical strategy to share with the class (disciplinary obligation). A different teacher might choose to call on a particular student because that student rarely participates, and the teacher wants to build the student's confidence by having him share his thinking with the class (individual obligation). Another teacher might choose to call on a student because she was able to summarize her group's thinking well, and she they wanted to talk about the importance of students listening to their peers (interpersonal obligation).

Each of these examples speaks to teachers' general orientation about their role in the discussion, and these orientations have important implications for how teachers enact decisions in the discussion. Experienced teachers particularly may also make decisions that attend to multiple obligations at the same time instead of prioritizing one over the other when the obligations come in conflict with each other. This construct of professional obligations can illuminate various challenges teachers face related to implementing whole class discussions. I describe some of these challenges from the literature as they relate to the four professional obligations.

**Institutional Challenges**

One set of challenges that many teachers face is the constraints from the school as an institution. Institutional challenges include issues that arise from school or district-level policies, schedules and calendars, mandatory assessments and their potential accountability for student achievement, pacing guides that require what lessons teachers will be teaching on certain days, and expectations for managing student behavior, to name a few. All of these challenges and others like them share one common feature: They typically act to constrain the teacher because the teacher is often not in a position to change these constraints. Acknowledging these constraints is important, because as Cobb and Smith (2008) argued, taking the position that teachers are fully autonomous in their classroom is “flying blind” with respect to reforming instruction (p. 3).

A main critique of teaching through implementing collaborative discussions is that it is less efficient. Content coverage is certainly a concern for many teachers given the adherence to pacing guides by some schools. While the potential usefulness of these guides is to help ensure that all students have opportunities to encounter all content at a certain grade level, it can also negatively influence teachers to focus more on simply covering content instead of being responsive to their students’ learning needs. Making use of collaborative discussions may inhibit coverage of content, and thus run in opposition to established expectations for pacing.

A second related challenge is that teachers are often held accountable for ensuring their students pass standardized tests, either statewide or department-level (e.g., Michigan Department of Education, 2012). If the tenet that "assessment drives instruction" is true, then the quality of these types of assessments likely play an important role in shaping the nature of instruction. There is evidence from the literature that this interaction between assessment and instruction is present. McGraw (2002), for example, described the constraints of instructional practice while facilitating whole-class discussions when she noted that the end-of-semester departmental exam was an important factor in teacher decision-making while considering ways to handle students' ideas during whole-class discussions.

**Disciplinary Challenges**

Students' learning opportunities to engage in mathematical content are largely constrained by instructional materials and their teachers' content knowledge. One might assume that a teacher's understanding of content strongly influences their ability to teach in discourse-oriented fashion; however, even in situations where the teacher was experienced and had strong content background, the shift in instructional demands was great. Speer and Wagner (2009), for example, studied an experienced teacher's implementation of an inquiry-oriented approach to teaching Differential Equations at the university level. They noted that even though the teacher had taught the course previously and had a strong content background, the teacher encountered challenges in realizing the mathematical potential and relevance of student contributions during a discussion.

Researchers have also noted that written curriculum materials, such as textbooks, play an important role in shaping students' opportunities to learn mathematics (e.g., Stein, Remillard, & Smith, 2007). A teacher who uses a worksheet of twenty procedural problems provides different opportunities for student interaction than a teacher who is using an open-ended task. This is not to say that the affordances of an open task are always capitalized. Teachers can readily diminish the demands of even the most challenging of tasks (e.g., Choppin, 2011). This being said, in a large-scale study of middle school students, Tarr et al. (2008) reported that students in classrooms with moderate to high implementation of a *Standards-Based Learning Environment* that also used NSF-funded curricula (which often focus on an inquiry approach) achieved significantly higher in problem solving, communication, and procedural fluency. Although studies comparing publisher-generated and NSF-funded curricula are not common, Tarr et al.'s findings are consistent with others that have shown that students using NSF-funded curricula typically do better on assessments that focus on problem solving or conceptual understanding (Stein & Smith, 2010). While the teacher certainly plays a vitally important role in implementing and shaping student's opportunities to learn, these opportunities are in large part shaped by the written materials themselves.

**Interpersonal and Individual Challenges**

Challenges related to Herbst and Chazan's (2011) interpersonal obligation focus primarily on the interactions between teachers and their students in the shared social space of the classroom. A major challenge for teachers using a discourse-oriented approach is that it requires students to engage and actively participate. In a discourse-oriented approach, students need to learn how to participate as part of a discourse community, which likely impacts how they view themselves as learners of mathematics. A teacher can make all the productive teaching moves they want to, but if it does not promote student interaction it will be difficult to teach using a discourse-oriented approach. A teacher's actions are important for the short term (e.g., calling on a student to clarify an idea), but also for the long term, in their role in supporting social norms and substantiating affective aspects of the classroom environment.

As one example, Jansen (2006) studied 15 middle school students and their motivation for engaging in classroom discussions. About half of the students in her study actively participated as speakers more consistently during classroom discussions than the other half, which tended to be quieter. She found that students who felt “threatened” to participate actively in the classroom discussions would still participate in the discussions if they felt that their participation helped their classmates. She later concluded, in Jansen (2008), that student participation was also related to whether the talk in the classroom was primarily procedural or conceptual. For a teacher, these realizations are important, because they point to the various reasons why students choose to participate (or not) in discourse-oriented classroom settings.

In addition to Jansen’s findings, others have noted the importance of student understanding in relation to how they participate. For example, Webb (1991) found a positive correlation between student achievement and students “giving content-related explanations” to their classmates (p. 377). She later argued that the ideal classroom “setting is one in which students freely admit what they do and do not understand, consistently give each other detailed explanations about how to solve the problems, and give each other opportunities to demonstrate their level of understanding” (p. 386).

More recently, Webb et al. (2014) argued that giving students the opportunity to build on each other’s ideas has the added benefits of encouraging students to monitor their own thinking, and they may be better positioned to “recognize gaps in their knowledge, misconceptions or contradictions between their own ideas and those they are hearing” (p. 80). In their study of over

100 children ages 8-10, they found a positive relationship between student achievement and students' engagement with other students' ideas.

**Summary**

Put together, Schoenfeld's (2010) focus on goals and Herbst and Chazan's (2011) construct of *professional obligations* frame the potential dilemmas teachers can face while enacting whole-class discussions, particularly when multiple obligations are needing to be satisfied or prioritized. It also points out the inherent issue when studying teacher decision-making as necessarily situated in teachers' classroom work (Lee & Lin, 2005). This work includes all the many factors and stimuli that teachers encounter as unique to the real work of teaching with students they interact with day after day. These factors vary from content specific (e.g., a student not understanding how to multiply) to those that are less content oriented (e.g., student affect).

# CHAPTER 3

# THEORETICAL PERSPECTIVE AND ANALYTIC FRAMEWORK

The guiding theoretical perspective for this study is *situated cognition* (Greeno, 1989). The situated perspective places strong emphasis on the interaction between individuals and their environment. As Greeno (1989) stated, "Cognition, including thinking, knowing, and learning, can be considered as a relation involving an agent in a situation, rather than as an activity in an individual's mind" (p. 134). In this study, teacher decision-making is best understood in the context of teacher activity in their classrooms; thus, I attended to teachers' interactions with their students over multiple lessons, observing behavior, hypothesizing about their decisions, and later interviewing them about possible decisions. This approach differs from simply interviewing teachers about a simulated classroom, or asking teachers to state what decisions they would make in a theoretical situation in their own classroom. The approach I used in this study aimed to provide a more authentic view of teachers' decision-making as a cognitive activity necessarily embedded in their environment.

## Main Analytic Constructs

To better understand teacher decision-making during whole class discussions I attend to five main constructs that individually and collectively support why they made particular decisions when they encountered a particular dilemma. I argue that a teacher's decision when faced with a particular dilemma is strongly influenced by their instructional goals and their professional obligations (Herbst & Chazan, 2011). Thus, to understand teacher decision-making, it was necessary to come to understand teachers' instructional goals, orientations to their professional obligations, consistent patterns of instruction that emerge when acting upon instructional goals, instructional dilemmas, and their resulting decisions that surfaced when instructional goals were enacted during whole-class discussions. In the next section, I first describe each of these constructs in detail, and then describe how these constructs connect.

## Professional Obligations

As described in the previous chapter, I use Herbst and Chazan's (2011) description of *professional obligations* as a way to understand teachers' general orientations to facilitating whole-class discussions, which in turn influence their decisions during whole-class discussions. Consistent with this view, I argue that teachers' decisions are influenced by their obligations to the institution of the school, to students as individuals, to supporting students in their social development, and to their role as a representative of the discipline of mathematics. A teacher's obligations and goals are connected in that they both orient a teacher's behavior and their decisions.

A teacher's professional obligations differ from their goals in that goal(s) are about accomplishing a task (e.g., supporting students in effectively communicating their thinking with the class), whereas the teacher's professional obligation(s) describe who or what they feel obliged to satisfy (e.g., the individual student, the school, etc.) during the discussion. For example, if two students get in a somewhat heated exchange disagreeing about the solution of a problem during a discussion the teacher could view this situation in very different ways. The teacher might see the situation as (a) a management issue needing to be addressed (institutional obligation), (b) an opportunity to re-phrase the crux of the argument and have the class investigate the mathematical issue further (disciplinary obligation), (c) an opportunity to re-focus the class on appropriate ways to interact with each other (interpersonal obligation), or (d) a need to intervene to protect each student's willingness to participate in the future (individual obligation).

## Instructional Goals

The *instructional goal* is what the teacher hopes to accomplish as a result of enacting a whole-class discussion. How teachers address their professional obligations influences the instructional goals they might have. For example, a teacher can have lesson-specific content goals (e.g., calculate the median of a data set) to address the disciplinary obligation of their role as teacher or goals for establishing a mathematical community (e.g., students will listen to their peer's thinking) to address their interpersonal obligation. For example, a teacher may choose to pursue an errant line of reasoning because it supports an individual student's willingness to participate in class, because it models an important classroom norm related to the value in sharing mistakes, or because it uncovers an important misconception with possibly long-lasting effect. These broader instructional goals may support student development but are not necessarily a content goal tied to a particular lesson. Understanding a teacher's instructional goals illuminates what they are intending to accomplish during the whole-class discussion.

I do not make the claim that teachers are always aware of their goals or, if they are aware of them, that they are able to articulate them clearly. In many cases, teachers may have instructional goals that are somewhat unknown to them (e.g., a new teacher knowing what they should be doing without a clear reason why they ought to be doing it). A teacher may be acting consistent with a goal, without knowing what their goal is. Some teachers, for example, might simply have a goal to make sure the class runs smoothly, and maintaining order is the most important aspect with very few mathematical goals. I take as an assumption that the nature of teaching falls in the category of "goal-oriented activity," but I am cautious in not assuming that any discussion necessarily includes all four types of goals. This being said, it is likely the case that a teacher's instructional goals are shaped in some capacity by their professional obligations.

For example, a teacher with a tendency to orient their instruction towards their individual obligation may enact lesson-specific content goals differently than a teacher with a tendency to orient their instruction to the disciplinary obligation.

**Consistent Patterns of Instruction**

The third construct in this study focuses on teachers' *consistent patterns of instruction*. These could be considered "routines," but given the small sampling of lessons observed in this study I use the term "consistent patterns of instruction" to describe characteristics of teachers' instruction that consistently occurred across several discussions. I am also cautious in not using the term "routine" to describe these consistent patterns of instruction because of the possibility of confusing my conception of routine with others in the field (e.g., Kazemi, Franke, & Lampert, 2009). For this study, I define a "consistent pattern of instruction" consistent with other mathematics education researchers' views on instructional routines (e.g., Leinhardt & Steele, 2005; Leinhardt, Weidman, & Hammond, 1987) as *a socially constructed script of behavior mutually understood and enacted by members of the classroom community*. This definition is also consistent with other descriptions of routines outside of mathematics education (e.g., Becker, 2005; Zisberg, Young, Schepp, & Zysberg, 2007).

A teacher may have particular goals and obligations they aim to attend to, but enact consistent patterns in their instruction that are incongruent with these actions. For example, a teacher may want to have students justify their reasoning during the discussion, but the teacher simply explains the reasons for the students and asks whether they agree with her or not. Thus, a teacher's consistent pattern of instruction can help describe how they enact (or fail to enact) their goals and obligations.

A second important aspect related to consistent patterns of instruction is that when teachers deviate from a reoccurring pattern, there may be greater likelihood that the teacher made a conscious decision. These deviations from the business-as-usual flow of the discussion can indicate a teacher's decision when faced with a conflict between goals, or a conflict between professional obligations (e.g., a teacher decides not to interrupt a student while describing their thinking when the bell rings to end class) (Parker, 1984; Sutcliffe & Whitfield, 1976).

I categorized the patterns into the following types: *management*, *instructional support*, *activity structure*, *interaction*, and *meta-routines*, derived from the research base on instructional routines (La Course, 2011; Leinhardt & Steele, 2005; Leinhardt, Weidman, & Hammond, 1987; Yinger, 1979). Given below in Table 1 is a list of the definitions and descriptions of each type of consistent behavior. Greater detail about identifying and categorizing consistent patterns of teacher behavior can be found in Appendix A. Within each set of patterns, I later identified finer-grained patterns of behavior and coded them after conducting several passes through the lesson observation data.

Table 1
*Category Types for Consistent Patterns of Teacher Behavior*

| Consistent Pattern | Description |
|---|---|
| Management | Any patterns of behavior related to classroom management, such as student discipline, grouping students, etc. |
| Instructional Support | Support teacher-student interaction, such as instructional resources to be used during interaction, and the location of where the interaction takes place |
| Activity Structure | These patterns consist of how the teacher typically starts and ends discussion, how long the discussion segments last, and transition points between group/individual work and whole-class discussion. |
| Interaction | These patterns specify the ways in which teachers and students should interact and communicate with each other. |

Table 1 (cont'd)

| | |
|---|---|
| Meta-Routine | These patterns involve the teacher explicitly referencing a consistent pattern of behavior with students. |

**Dilemmas**

As teachers enact their goals, they encounter situations where goals conflict with each other. I refer to this type of conflict as a *dilemma*. For example, a teacher may want students to explain their solution strategy to the whole class on how to find the slope of a line. This is done so (a) the student can engage in mathematical practice (communicating one's thinking) and (b) to advance lesson-specific content (calculating slope). But if the student finds the problem way too challenging to even begin to describe their thinking, the teacher is faced with the dilemma of managing both goals, which could be in competition with one another based on the teacher's interpretation of the situation.

Dilemmas can be observed as deviations from consistent patterns of instruction (described further in methods) or identified through teacher self-report about the dilemma. An important limitation of this study is that some data obtained about the dilemmas teachers encountered during whole-class discussion came from self-report during interviews, and, as such, some teachers may have been more forthcoming with their own understanding of the types of dilemmas they encountered. Although these self-reported dilemmas (i.e., a teacher stating, "This is a dilemma" during an interview that was not observed as a deviation to a consistent pattern of instruction) may have varied across teachers, omitting this data from the dissertation would essentially discount several dilemmas that teachers were fully aware of. These dilemmas were to be expected given the relatively conservative approach I took in identifying consistent patterns of instruction and using these patterns (and their deviations) to identify dilemmas. Although these self-reported dilemmas add a layer of variability likely not present in the observed dilemmas, they do provide a fuller picture of the dilemmas that teachers encountered while planning and enacting whole-class discussions.

### Decisions

A dilemma presents an instructional crossroads in the lesson where the teacher must make a decision about which goals and/or obligations take priority. By *decision*, I mean those consciously made choices that teachers make during a lesson. I take a much more restrictive view on decisions as only those with high likelihood of being consciously made, such as those that disrupted patterns of behavior that consistently occurred during a discussion or when a teacher acknowledged during an interview having made a particular decision. The decision is the end result of the other four constructs; that is, given a teacher's orientation to their obligations, goals, and consistent patterns of instruction, one can explain why the teacher made that decision when they encountered a particular dilemma.

### Connecting the Main Constructs

Figure 1 (below) shows a schematic diagram of how the five constructs interact with each other over the development of a lesson (with special attention to collaborative discussions). During each of the three phases, dilemmas and their decisions appeared in various ways. Although lesson planning is not a main emphasis of this study, some dilemmas did appear in lesson plans and during conversations about future lessons. During planning, teachers consider their goals for a particular lesson (as well as possibly broad instructional goals) and create a lesson plan (even if simply a mental script). During the planning phase, they may anticipate dilemmas that might arise as a result of goals conflicting with each other. Teachers make decisions based on how they expect the lesson to play out, framed by their goals and obligations.

The plan can include both teacher behavior that is consistently present across many lessons, or behavior that is idiosyncratic to a particular lesson.

While implementing collaborative discussions, teachers make decisions when faced with instructional dilemmas as a result of goals conflicting with each other. When teachers deviate from a consistent pattern of instruction, this deviation indicates a decision potentially linked to some instructional dilemma resulting from these conflicting goals. During the post-lesson phase (reflecting), teachers describe (via interview) dilemmas they faced during lesson enactment. At this stage, teachers may evaluate the extent to which they agreed or disagreed with their decisions related to how they implemented various instructional goals.

Put together, a teacher's goals and orientations to their professional obligations can help explain the particular decisions they made as a result of encountering a particular type of dilemma. It is this relationship between these five main constructs that provides the impetus for this study: that instructional decisions during whole-class discussions can be explained by their goals, obligations, and behavior.

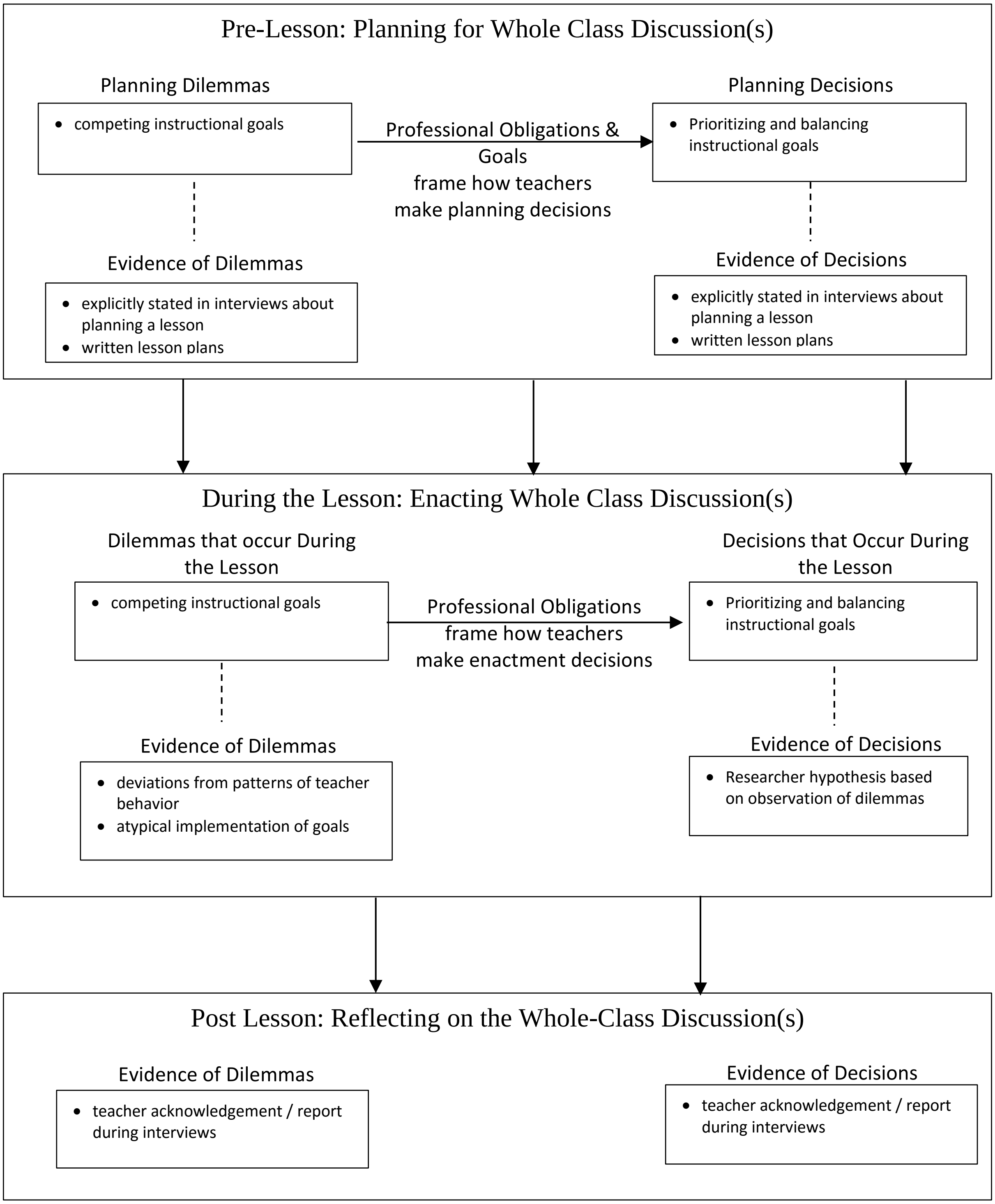


*Figure 1*. Phases of implementing a whole-class discussion

# CHAPTER 4

# METHODS

In this chapter, I describe the methods used to address the research questions for this dissertation. In order to address these questions, I collected data from teacher lesson plans, observations, and teacher interviews. The three teachers in the study were quite different in their approaches to facilitating discussions and had various professional experiences. At the end of this chapter, I provide a general overview of each teacher's instructional practice.

## Study Setting

This study took place with three grade seven mathematics teachers (pseudonyms are used for the teachers, students, and the name of the school). I chose to study middle grades teachers because they are in a unique position in their districts, in that they are directly affected by previous curricular experiences within the district (at the elementary level), and they are at least aware of the obligation to be responsive to the needs of the high school mathematics department. There is also evidence to suggest that students perceive differences in the nature of the middle school and the high school curricula, especially when the transition is from reform-oriented materials to more traditional ones (Star, Smith, & Jansen, 2008). The middle school mathematics curriculum includes new ideas and presents opportunities for students to think about previous knowledge in a new way, such as the shift from arithmetic to more complex algebraic thinking (e.g., Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005; Knuth, Stephens, McNeil, & Alibali, 2006). Put together, the complexities from the potential of competing obligations appearing is, at the very least, sufficiently possible at the middle grades level.

My initial goal was to select two school sites, with one teacher designated as an "expert" and one as a "novice." Expert and novice teachers have been identified in different ways (e.g., Berliner, 2001; Borko & Livingston, 1989; Sternberg & Horvath, 1995), for example, by the years of teaching experience or their role (e.g., student teacher or mentor). The literature on expert and novice teaching practice indicate that expert teachers focus more on student understanding, tend to plan lessons in relation to long-term plans, have more knowledge about teaching and content, have deeper connections between their types of knowledge, and are more adept at implementing their knowledge when faced with situations as they appear in the classroom.

In contrast, novice teachers tend to have less connected knowledge, tend to plan for the short-term, have difficulties implementing their knowledge during lesson enactment, and often fail to capitalize on important mathematical moments in the lesson (Borko & Lingston, 1989; Leinhardt, 1989; Stockero & Van Zoest, 2013; Westermain, 1991). Thus, one might expect to see more attention to a variety of instructional goals from expert teachers than novices, and therefore a greater likelihood of encountering various instructional dilemmas as expert teachers enact whole-class discussions. This means that if given a choice between studying either expert or novice teachers, expert teachers would potentially provide more interesting data in understanding the nature of their decisions while they encounter instructional dilemmas during whole-class discussions.

## Recruiting Teachers for the Study

As a first step in recruiting teachers, I identified schools that used the *Connected Mathematics* curriculum (Lappan, Phillips, Fey, & Friel, 2014; Lappan et al., 2005), a Standards-based (NCTM, 1989) curriculum that is consistent with the view of supporting students to engage in whole-class discussions to support mathematical understanding (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Jansen, 2008). Additionally, by selecting a single set of materials, this hopefully mitigated any curricular effect that might have influenced the nature of decision-making as embedded within whole-class discussions. Two teachers (Mrs. Stewart and Mr. Sandberg) were piloting the third edition of the materials (*CMP3*), while Ms. Mitchell taught using the second edition (*CMP2*).

To identify schools that used *CMP2 or CMP3*, I contacted the secretary of the *Connected Mathematics Project*, visited school district websites to identify their middle school mathematics curriculum, and contacted professional development leaders familiar with middle school mathematics implementation in their regions. I recruited 11 schools for the study, with the end goal of identifying a pair of grade 7 teachers from each school with various levels of expertise. Seventh grade was specifically chosen over grade 6 because of the potential issues with students being new to the curriculum. Additionally, I selected grade 7 because it had many opportunities for lessons with high-level discussions. Recruiting for school sites started in September 2014. Only one school agreed to take part in the study.

Highland Junior High School was very enthused about the study. I met with the principal and the department chair, and they expressed willingness to support the study. They had four grade seven teachers, and it was hoped at the beginning that all four teachers would be interested in participating. From our initial conversation, there appeared to be a good chance that the teachers would be classified with one pair as experts and one pair as novices. This could have helped with data collection in potentially restricting the study to one site. One teacher, however, chose not to participate because of other commitments. After unsuccessfully recruiting a second school site over several months, I decided to collect data with the three teachers at Highland. The remaining three teachers consented to participate in the study.

Due to recruitment issues (described in greater detail in Appendix B), the three teachers in the study can be best categorized as having different types of expertise. For example, two teachers (Mr. Sandberg and Mrs. Stewart) had multiple years of teaching the curriculum, whereas the third teacher (Ms. Mitchell) was in her first year teaching the curriculum. Mrs. Stewart and Ms. Mitchell had both taught at different grade levels (secondary and elementary, respectively), which afforded opportunities for understanding the trajectory of mathematical ideas. Mr. Sandberg had previously taught science before teaching mathematics. I describe later how I characterized and measured teaching expertise. This was especially important during the recruitment process of teachers to ensure that the teachers in the study included classroom discussions as a consistent aspect of their classroom interactions.

## Data Collection

I collected data for this study in four rounds. Round 0 consisted of an initial interview with the teachers (approximately 20-30 minutes) focusing on their beliefs about whole-class discussions, student learning, and planning. Additionally, there was one observation where I asked teachers to identify a "typical lesson" (as defined by the teacher) that involved discussion in the class. I later analyzed observation data using the *Instructional Quality Assessment Classroom Observation* [IQA] protocol (Boston, 2012) to determine the level of expertise of the teachers in facilitating whole-class discussions.

Rounds 1, 2, and 3 involved multiple classroom observations. For the most part, each observation included a lesson plan and a post-lesson reflection interview. I conducted a video-stimulated interview at the conclusion of each round, which was a set of consecutive lessons as taught by the teacher. There was a break in observed lessons between each round. Later in this section, I provide greater detail on each aspect of these observations. Table 2 below includes the lessons taught in each round by each teacher. Focus lessons are identified in the yellow cells. I observed at least one lesson before and at least one lesson after each focus lesson, with the exception of Ms. Mitchell's first round as this focus lesson spanned over three class periods. Lessons that corresponded in the textbook series are colored in blue, green, and red, respectively. Since Ms. Mitchell used *CMP2*, some of the lesson numbers as they appear in the materials are different than the lesson numbers in *CMP3*. Ms. Mitchell's third lesson in round 2 was a teacher-generated lesson on expected value.

Table 2
*Observed Lessons by Teacher and Round*

| Teacher | Rnd. | Unit | L1 | L2 | L3 | L4 | L5 | L6 | L7 |
|---|---|---|---|---|---|---|---|---|---|
| Ms. Mitchell | 1 | Moving Straight Ahead (*CMP2*) | 1-2 | 1-3 | 1-4 | 1-4 | 2-1 | 2-1 | 2-1 |
| Ms. Mitchell | 2 | What Do You Expect? (*CMP2*) | 1-3 | 2-1 | TG | | | | |
| Ms. Mitchell | 3 | Data Distributions (*CMP2*) | 1-1 | 2-1 | 4-4 | | | | |
| Mr. Sandberg | 1 | Moving Straight Ahead (*CMP3*) | 1-2 | 1-3 | 1-4 | 2-1 | 2-1 | 2-2 | |
| Mr. Sandberg | 2 | What Do You Expect? (*CMP3*) | 1-2 | 1-3 | 1-4 | | | | |
| Mr. Sandberg | 3 | Samples & Population (*CMP3*) | 1-2 | 1-3 | 1-4 | | | | |
| Mrs. Stewart | 1 | What Do You Expect? (*CMP3*) | 2-1 | 2-2 | 2-3 | | | | |
| Mrs. Stewart | 2 | What Do You Expect? (*CMP3*) | 4-1 | 4-2 | 4-3 | | | | |
| Mrs. Stewart | 3 | Samples & Population (*CMP3*) | 1-2 | 1-3 | 1-4 | | | | |

**Baseline Data Collection and Analysis**

Baseline data collection consisted of an initial interview and an observation of a typical classroom discussion. In contrast to the observed lessons in Rounds 1, 2, and 3; there was no post-lesson debrief between the researcher and the teacher. I gathered baseline data November – December 2014. To analyze the baseline data, I first transcribed the interviews, then looked for evidence of experience in facilitating whole-class discussions, overall teaching experience, beliefs about student learning and teaching, curricular experience, and how the teachers think about instructional dilemmas. Second, I used the *Instructional Quality Assessment* [IQA] (Boston, 2014) protocol to score the video-recorded lesson.

**Initial interview.** I conducted an initial interview with all three teachers with the purpose of gathering information about the teachers' beliefs about mathematics teaching and learning, their overall teaching experience, their experience using the curriculum materials, and their experience in facilitating whole-class discussions. The goal of gathering this data was (a) to ensure that teachers used whole-class discussions enough to participate in the study, and (b) to provide some baseline idea of potential factors (e.g., instructional goals, possible attention to professional obligations, instructional routines, types of dilemmas they encounter) that contribute to teachers' decision-making. The set of interview questions are listed here, with greater detail provided in Appendix C.

- Describe your experience with implementing CMP materials.
- Describe a typical lesson in your classroom.
- Do you think it's important that students are active in their math class?
- What are some challenges that you encounter when trying to facilitate classroom discussions?
- What do you think you have improved on during your teaching career in facilitating classroom discussions?

- Are there resources, organizational structures, or routines that you find helpful for supporting implementing discussions?

I also asked participants about dilemmas they faced while thinking about planning collaborative discussions. This data provided a general sense of issues that potentially constrain or impact how decisions were made during collaborative discussions. For example, teachers talked about district-mandated tests, concerns for students' social development, classroom management concerns, and concerns about preparing students for the next year's material. These topics provided insight for thinking about how dilemmas are managed during collaborative discussions.

**Curricular experience.** The three teachers varied in terms of their familiarity with the *CMP* materials. Ms. Mitchell was new to the materials, whereas Mrs. Stewart and Mr. Sandberg had previously used *CMP2* for at least four years. This being said, both Mr. Sandberg and Mrs. Stewart were piloting *CMP3*; therefore, there was some unfamiliarity due to some changes in lessons and sequencing between versions. Because all teachers used the same curriculum materials, instructional decisions were less dependent (at least theoretically) on curricular effects.

**Instructional Quality Assessment.** I used the *Instructional Quality Assessment Classroom Observation Tool* [IQA] (Boston, 2012) to determine each teacher's level of expertise in facilitating discussions. The IQA tool allows for measuring features of a discussion that include how the task was implemented as well as teacher and student talk during the discussion (e.g., the level of questions asked by the teacher). The rubrics on the IQA range from 0 to 4, with higher scores indicating greater expertise. During Round 0, each teacher chose a lesson that they deemed "typical" and that they thought had the potential for having a classroom discussion. The lessons were video-recorded, and both coders used the recordings to evaluate the teachers using the IQA rubrics *Student Discussion Following the Task* and the *Rigor of Teachers' Question*, as these two rubrics were most appropriate for analyzing teacher discussion practice for this dissertation. The scores were exchanged and any issues resolved between the two researchers. This analysis aided in classifying the teachers as expert or novice. There was 100% agreement on the sets of scores for Ms. Mitchell and Mr. Sandberg, and overall scores differed by 1 point each on the coding of Mrs. Stewart. Profiles of each teacher are given later in this chapter, based on the analysis of the interview data and the observed lesson.

**Lesson Observation Data**

Each set of observations consisted of observing teachers in consecutive lessons. Data collected from each lesson included a lesson plan, observation field notes taken during the lesson, a video recording of the lesson, and a post-lesson debrief interview. Additionally, the end of each set of lesson observations included a video-stimulated interview where I showed clips of lessons and asked each teacher about their decision-making processes. I observed all the lessons in person with the exception of two. Data collection took place during the spring semester of 2015. Data collection was planned to begin earlier in the school year, but several issues created delays (Appendix B).

Each set of lessons consisted of *consecutive* lessons. The term *consecutive lessons* used in this study refers to instructional lessons taught by the teacher that occurred in order Days where the teacher only gave a quiz, test, homework check, or some other planned non-instructional activity were not observed. When teachers were absent, these lessons were similarly excluded from the study. One of the teachers (Ms. Mitchell) had a student teacher who would typically teach in her absence, and again no observations took place on these days. As one example of how spaced out these observations could be, Mr. Sandberg's first round of 6 consecutive observations took 18 calendar days due to various teacher absences, assessments, and days when students were not present (e.g., parent-teacher conferences).

**Deviations in collecting lesson observation data.** There were very few deviations from "typical" observations in terms of collecting data. I was present for all observations with the exception of two in Mr. Sandberg's class. These lessons were recorded by another graduate student. During these lessons there were no post-lesson interviews, but I did later analyze the video-recordings of the lessons, and questions that seemed to relate to the next lesson were also included in the next lesson's post-lesson interview. Additionally, I missed one post-lesson interview by Ms. Mitchell because she had to leave immediately after class. Out of the 34 recorded lessons across the three rounds, I missed 3 post-lesson interviews.

The second round of all three teachers as well as the first round of Mrs. Stewart's observations fell within the state testing window. For Mrs. Stewart and Mr. Sandberg this impacted the number of students attending their class; in some cases there were as few as 11 students in one class during state testing (typical class size was around 20). Ms. Mitchell's class was during the last period of the day, so very few students were absent due to state testing. This likely had some effect on the types of interactions and planning for those class periods. All three teachers taught the unit *What Do You Expect?* (a unit on probability), which was purposefully chosen by the department to occur during the state testing window.

**Number of lessons per round.** The lesson observations for each teacher consisted of three rounds of consecutive lessons (see Table 2). Although most textbook sections were taught within one class period, some were not, which is the reason why there were more than five observations in a round in some cases. I originally planned to observe two rounds of five lessons per teacher. After collecting data for Ms. Mitchell and Mr. Sandberg in the first round, the number of rounds was changed to three rounds, each consisting of three lessons per round (Note: Mrs. Stewart had not yet been observed when the decision changed, so her observations consisted of three sets of three lessons). The decision to move to three sets of lessons was motivated by the following:

- There was not much observed lesson variation from day to day in terms of lesson structure, so adding an additional set of observations could control for content-domain effects by providing greater variety of topics being taught.
- The change to an extra set of lessons would allow for more variance in topics taught, which would control for decision-making related to specific content domains.
- All teachers approved adding an additional round, while shortening the number of lessons in each round.

**Identification of lessons**. An important choice in this study was selecting lessons with high potential for classroom discussions, which ideally could be observed for all three teachers in the study. Through inspecting the *CMP2* and *CMP3* materials, I determined several *focus lessons* that had the potential for rich discussion. Only lessons that included high cognitive demand tasks (Smith and Stein, 1998) were selected as focus lessons, because these lessons had the greatest potential for promoting discussions. Once I selected lessons, I verified their potential for collaborative discussions by talking with experienced users of the curriculum (who were not also research participants) and one of the authors of the *CMP* materials. For each focus lesson, I observed at least one class session prior to and at least one class session afterwards, in order to better understand the decision-making across lessons. I determined the focus lesson by the lesson as written in the materials (e.g., Lesson 2-1), not as the particular material covered in a single class period. In two cases, focus lessons spanned multiple class periods.

A similar reason why the same curriculum series was selected across teachers is that I had initially wanted to observe all teachers teaching the same lessons. Logistically, this became quite challenging for several reasons. One issue resulted from one teacher using *CMP2* materials while the other two were using *CMP3* materials. Finding lessons that satisfied the focus lesson criteria and appeared in both materials was also challenging. Second, the observations that took place at the end of the year consisted of different units that focused on data. Third, issues arose from scheduling issues, differences in pacing, and teacher absences.

Because observing identical lessons was so challenging, an alternative approach was used that created three sets of partially overlapping observations. Each teacher had one set of observations that overlapped with the second teacher, one set of observations that overlapped with the third teacher, and one set of observations that was unique to the teacher. This overlapping was done intentionally to limit the possibility that differences across teachers in instructional decision-making was not a result of highly different content being taught. No set of lesson observations was consistent across the three sets of teachers. The set of overlapping lessons can be seen in red, blue, and green in Table 2.

**Lesson plans.** Before each lesson, I collected lesson plans for the purpose of determining the extent to which unit-level or long-term goals are present in the plan (see Appendix D for lesson plan criteria). All teachers were asked to submit lesson plans ahead of time. I asked teachers to make sure their daily plan included what they would be doing in the class that day, along with the goals for the lesson and some rationale as to why they thought the lesson could help attain the goal(s). The hope was that by only asking for a few items, I could balance the needs of getting quality data without overly burdening the participants. I explained the expectations of what should be included in the lesson plans before data collection commenced.

Two of the three teachers were unable to consistently submit lesson plans ahead of their lessons. To accommodate this issue, I made some changes between the first and second rounds of lesson observations (see Appendices D and E for the original and revised versions), which included revising the lesson agenda and overview with more teacher-friendly language, contacting the teachers by email, and including questions about the next day's lesson in the debrief of the preceding lesson.

In terms of analyzing teacher lesson plans, I paid particular attention to how the teacher described the goals for the discussion, and whether the goal(s) seemed to be immediately attainable as well as whether the goals connected to longer-term or unit-level aims. I was interested in identifying the types of goals they mentioned; for example, whether the goals focused on content (e.g. students will understand), processes (e.g. students will justify their solutions), dispositions (e.g., students should be curious about…), or anything else, such as organizational or classroom norm-specific goals (e.g., students will effectively work in groups). I provide further detail in the data analysis section on how I coded for goal statements.

**Lesson observations.** With the exception of the two lessons missed, I arrived shortly before the start of each lesson to audio record and video record. My interaction with students was limited (such as when students would accidentally walk in front of the camera). In most cases, monitoring the video camera and taking field notes was non-problematic given the use of multiple cameras. Both cameras were in the back of the room, one on a fixed wide-angle shot, and I operated the second camera so that I could zoom in on work on the board and follow the teacher. I also used a microphone positioned in the back of the room to record audio.

Prior to lesson observations in Round 1, pilot data helped generate aspects of classroom interaction during whole-class discussions that could have been important to describe in relation to the research question (see Appendices F and G). These categories were determined through both identifying aspects that seemed relevant to the interaction during whole-class discussions via analyzing pilot data and by attending to the relevant literature base on whole-class discussions (e.g., teacher questioning and student talk). Although this list was not exhaustive (e.g., it did not include anything related to norms that could have affected the nature of class discussions, dilemmas, and decisions), it did prove sufficient in being able to identify several potential consistent patterns of instruction and instructional dilemmas.

During each lesson I wrote field notes for the purposes of noting segments with different types of interaction (e.g., group work, whole-class discussion, teacher lecture), and situations to be discussed in the post-lesson discussion. I paid particular attention to those situations that (1) appeared to deviate from consistent patterns within whole-class discussions across lessons, or (2) posed instructional dilemmas for the teachers. Identifying these deviations proved somewhat challenging, as the consistent patterns were necessarily emerging during data collection. The categories from the pilot data, and the framework for analyzing consistent patterns of instruction, provided a frame for identifying potential consistent patterns and possible deviations.  A list of situations that potentially showed instructional moments where active conscious decision-making occurred is listed in Appendix H.

In the first round of data collection, I took field notes using a classroom observation protocol (Appendix I). This document proved useful in capturing particular moments, but proved cumbersome in trying to follow along with the lesson and categorizing decisions in relation to any of the professional obligations. This issue, combined with the lack of receiving lesson plans consistently, prompted a change in the structure of collecting field notes. Appendix J shows the single page observation tool that I used for the final two rounds of data collection. I noted potentially interesting dilemmas, breaks in consistent patterns of instruction, and issues related to any of the professional obligations (along with a video time code) on the right side of the document. Additionally, the reflection questions were listed on the top of the document so that the reflection questions could be more specifically tailored to the content of that day's discussions.

While piloting the protocols, I noticed that the Classroom Observation Protocol was necessary but by itself insufficient to determine the actual decisions made by the teacher. This is because without the teacher being able to describe their thinking in that moment, there is a large amount of inference on my part as the researcher as to why they made the decision. Thus, the Classroom Observation Tool was used to mark moments that were later analyzed on the video and the video-stimulated interview at the conclusion of each round of lessons.

**Post-lesson interviews.** At the conclusion of most lessons, the teacher and I met to debrief the lesson. Whereas all of Mrs. Stewart's lessons included a post-lesson interview, there was one post-lesson interview that did not occur for Ms. Mitchell (Round 1, Observation 3), and two that did not occur for Mr. Sandberg (Round 1, Observations 4 and 5). The purpose of the post-lesson debriefs was to try to understand their decisions and the dilemmas they encountered during whole-class discussions while their recollection of the lesson was still fresh.

With this in mind, the field notes and reflection questions were starting points for a conversation aimed at uncovering the teachers' decision-making in the lesson and the possible dilemmas they felt they encountered. The debrief was structured on reflection questions (Appendices K and L) and field notes from the lesson. I video-recorded the debriefs, but the teacher and I did not watch any of the video-recorded data from that day's class period during the post-lesson debrief. Typically, the debriefs lasted between 5-15 minutes.

Looking back on the discussion immediately after the lesson created both benefits and limitations. The main benefit was that the teachers had a fresh recollection of the decisions made during the lesson. This hopefully minimized the likelihood of a teacher trying to re-create the choices they made that led them to a decision were I to have interviewed them days later. The main limitation, however, was potentially missing an important interaction between the teacher and the students that was not observed during the class observation. To minimize this limitation, the video-stimulated interview took place after each round of observations. Lesson videos were watched multiple times for potential dilemmas and moments potentially influencing teacher decision-making in the discussions.

**Video stimulated interviews.** After each round of observations for each teacher, I conducted a video-stimulated interview. I watched all the video recordings of all the discussions for each particular round of observations. This included the recordings of the lessons and post-lesson debriefs, along with looking back at field notes. The video-stimulated interviews typically lasted between 20 and 35 minutes. The goal of the video-stimulated interview was similar to the post-lesson debrief: to better understand teacher's decision-making during collaborative discussions. Similar to the post-lesson debrief, I selected video clips based on their potential to uncover an instructional dilemma or to better understand instructional decisions, especially those related to deviations from consistent patterns of instruction. I asked teachers about their goals, the dilemmas they were managing, and their decisions to better understand how their professional obligations oriented their decisions (see Appendix M for the set of example questions).

There were other benefits to the video-stimulated interview as well. From an analytic perspective, watching the videos multiple times allowed me to generate hypotheses of instructional practice, and therefore possible reasons why a teacher made a particular decision. The video-stimulated interview allowed for checking my own hypothesis with the teacher's sense of why they made a particular decision. From a methodological perspective, the video-stimulated interview provided teachers with a different perspective on their teaching by watching an extended segment (anywhere from 30 seconds to 5 minutes).

**Data Analysis**

A main challenge with studying teacher decision-making during whole-class discussions is looking for moments when there is observable evidence of a decision. This is challenging because making decisions is inherently a cognitive process. An outside observer would at best be able to study the residue of such decisions and infer that the teacher made a decision (Sutcliffe & Whitfield, 1976). In this study, I take the more restrictive approach to identify those situations where there is strong likelihood that a teacher made a decision. For example, a teacher explicitly mentions they made a particular decision during an interview or in their lesson plan. Or, when a teacher breaks a highly consistent pattern of instruction, one can infer that the teacher made a decision to break from such a consistent instructional pattern (i.e., the assumption is that such deviations are less likely to be random occurrences as opposed to purposeful action). An important aspect of this analysis is describing what was considered typical instruction so that instruction that deviated from the norm could be identified. In the section that follows this one, I provide a background of each teacher's typical instruction in order to provide an analytic tool for identifying dilemmas and decisions. To determine the latter of these situations, I analyzed the data to determine consistent patterns of teachers' instructional practice related to whole-class discussions, then marked places where deviations from those patterns occurred. In this section, I describe how I analyzed goals, professional obligations, consistent patterns of instruction, dilemmas, and decisions.

**Goals analysis.** I identified goals that related to whole-class discussions across all sources of data: lesson plans, post-lesson interviews, video-stimulated interviews, baseline interviews, and lesson observations. Most goal statements occurred during interviews, where teachers were asked directly about their goals -- questions such as, "[W]ere there like a few big things you really wanted to come out of [this lesson]?" or "Were there other goals, or other things you wanted…?" The unit of analysis was at the sentence level where teachers talked about a single goal.

During the first pass through the data, I identified all potential goal statements. There were two types of goal statements analyzed. Some goals statements were stated explicitly by the teacher, where they clearly and explicitly identified a goal as such, using the word "goal." For example, during Ms. Mitchell's post-lesson interview (May 1, 2015) she stated, "[M]y goal as I told you was for them to get a better sense of what a sample space is." This is an illustrative example of when the teacher explicitly identified her instructional goal.

Other statements were interpreted as goal statements during analysis because they described a teaching goal even though the teacher did not explicitly use the word "goal." For example, Mr. Sandberg stated, "[T]hey need to be able to understand y-intercept, what that looks like on a table, graph, and equation" (post-lesson interview, February 25, 2015). I interpreted this statement as an instructional goal because he identified a topic that he wanted to have his students understand, and it acted functionally as a goal. Thus, some goals were stated explicitly, while others were interpreted as goals.

After identifying goal statements, I coded goals as to whether the goal was primarily focused on one of four categories. This first category, *lesson specific content goals*, focused on mathematical content to be learned in a particular lesson. The second category, *broad content goals*, included unit-level content goals that were broader in nature, such as understanding linear relationships, a topic taught over several lessons. This category also included long-term content goals that went beyond the unit, such as learning goals that connected to the high school curriculum. The third category, *mathematical practice goals,* are related to NCTM's (2000) Process Standards and the Common Core's Standards of Mathematical Practice (NGA-CCSSO, 2010). This goal type is related to the first two goal types in that it focuses on mathematics, but it differs in that it focuses not on content but on types of mathematical practices, such as justification and *critiquing the reasoning of others* (pp. 6-7). This third category included goals related to how students learn mathematics within a learning community, dispositions such as perseverance, and mathematical practices such as justification. The fourth category, *goals for establishing a mathematical community*, included statements about developing a classroom that would support shared exploration and expectations of having students learn in a social environment. As a final note about goals, these goals primarily focused on students from the perspective of their mathematical development in the classroom attending to practice, content (immediate and long-term), and social goals. Goals that were not primarily focused on mathematical development (e.g., developing into a productive citizen in a democracy) were not coded for, although these goals may have influenced the nature of students opportunities to learn.

**Consistent patterns of instruction analysis.** To determine each teacher's consistent patterns of instruction, I watched a minimum of five classroom videos in random order. The order was randomized in order to minimize the effects from watching successive lessons. I determined the maximum number of observations studied for each consistent pattern of behavior once *theoretical saturation* (Lewis-Beck, Bryman, and Liao, 2003) of the data occurred. For each teacher, at least half of their lessons were analyzed. I analyzed the video recording of each lesson twice. The first pass was focused on identifying interaction codes of teacher talk. This set of codes was at much greater detail and allowed both coders to attend to this fine-grained analysis. The second pass included coding for other macro-level consistent patterns of instruction such as activity structure, management, instructional support resources, and meta-routines, which was at a much larger grain size. After I completed coding, a second coder coded a minimum 20% sampling of the discussion sections in each set of teachers' lessons of the data for the purposes of inter-rater reliability (85.5% agreement). Once theoretical saturation occurred, I described the consistent patterns of instruction and deviations and watched the remaining lessons to determine if there were any major inconsistencies with the reported consistent patterns of instruction. In the event that inconsistencies had been found, more lessons would have been analyzed until the set of lesson observations was exhausted or a new consistent pattern of behavior became more strongly established.

**Professional obligations analysis.** To understand how a teacher's orientation to their professional obligations of mathematics teaching influences their decision-making, I analyzed teachers' interview data to determine which of the obligations they tended to prioritize across lessons. In some cases, the teachers talked about decisions with evidence of prioritizing their obligations. In other cases, I asked the teachers a question during the interview that posed a scenario where they may have chosen more than one obligation to satisfy. Based on the interview data, trends emerged showing tendencies to favor a particular orientation to their professional obligations for each teacher. Table 3 below provides greater detail as to how I coded this interview data in relation to the teachers' orientations to their professional obligations.

Table 3
*Professional Obligations Coding*

| Professional Obligation Type | Rationale for Decision | Example |
|---|---|---|
| Disciplinary | Appeal to mathematics | (When asked if having many students absent for testing factored into the discussion in any way): "No, I think I would have probably done it the same…My focus with that was I wanted them to realize that this still has to equal up to one…No, [the discussion] would have looked the same with the entire class" (Mrs. Stewart, post-lesson interview, May 11, 2015). |
| Individual | Appeal to attend to a particular student's needs | "…I was going to just kind of let that go, but then Larissa got really invested in that, and I was like, 'Well how would you do that?.' And then a couple other people figured that out, okay, we'll go with this thing too. So I didn't want to shut that down, I wanted to honor their thinking more so I was okay letting [that] go…" (Mrs. Stewart, post-lesson interview, May 14, 2015). |
| Interpersonal | Appeal to social goals / reasons | "…Something I need to think about, is giving the think time with the pairs and the communication time to help them buy in to a simpler social setting and then move it out" (Ms. Mitchell, post-lesson interview, March 2, 2015). |
| Institutional | Appeal to school expectations | (When asked about using popsicle sticks as a way to randomly call on students): "So there's times when I know everybody should be paying attention, and every person if they're paying attention can answer these types of questions when I use those" (Mr. Sandberg, video-stimulated interview, April 17, 2015). |

**Dilemmas and decisions analysis.** This study focused on those dilemmas that pertained to whole-class discussions. Dilemmas occurred when instructional goals came in conflict with one another. As mentioned previously, I identified dilemmas across the five data sources: lesson plans, baseline interviews, post-lesson interviews, video-stimulated interviews, and lesson observations. Some dilemmas occurred during planning (as evidenced by referencing dilemmas during interviews and/or lesson planning); some appeared during enactment (as potentially evidenced by a deviation in a consistent pattern of instruction); and others occurred after a lesson as teachers were looking back on their instruction (as mentioned by the teacher). Some dilemmas occurred independent of observation (i.e., there was no notable deviation to some pattern of behavior) because teachers described the dilemmas in either lesson plans or interviews; whereas other dilemmas were hypothesized during lesson observations (e.g., because a deviation to a consistent pattern of instruction occurred), and I inquired about these dilemmas during interviews.

As a first pass through the data, I looked for potential dilemmas by instances where teachers explicitly mentioned a potential dilemma by either stating a moment in a discussion as a "dilemma", "challenge", or "issue". I also looked for implicit situations that were instances where the teacher was describing a potential conflict between goals. In many cases, the teachers described their decision (and its rationale) and potential other decisions they could have made. It was not necessary for the teacher to come to resolution, however, as some dilemmas remain unresolved. It was important however, for the teacher to acknowledge the potential for making a different decision than the one they made.

After this first pass through the data, I completed a second pass through the potential dilemmas to identify those situations that involved a conflict between two goals types. I then used the goals analysis (looking for implicit and explicit references to what the teacher aimed t accomplish) to determine the specific types of goals that were in conflict. I only noted those potential dilemmas as “dilemmas” when they met the requirement that the dilemma was indeed a conflict between two of the four different goal types.

There are two important limitations to the dilemmas analysis. First, since a dilemma is defined as a conflict between goal types, the dilemmas were a function of the types of goals analyzed in this study, (i.e. those focusing primarily on students’ mathematical development). Second, because one might be able to determine dilemmas between two goals of the same type (e.g., a long-term mathematical content goal conflicting with some dispositional goal – both categorized as a broad content goal). While the first case was not analyzed (i.e., analyzing the data for goals outside the bounds of this dissertation’s analysis), there were very few instances of the second case which warranted only choosing dilemmas as a conflict between the two goal types.

## Background of Teachers’ Instructional Practice while Leading Discussions

In this section, I provide an overview of each teacher’s instruction, including a description of consistent patterns of instruction when facilitating a discussion; and an analysis of a baseline class discussion, their instructional goals, and their understanding of the professional obligations of mathematics teaching. This description and analysis is intended to frame the dilemmas and decisions analyzed in the next chapter. In the succeeding chapter, I describe the types of dilemmas that teachers encountered and the decisions they made as a result of these particular sets of goals, obligations, and patterns of instruction.

All three teachers included broad instructional goals in either their lesson plans or the interview data. They all used the four different goal types (long-term content, unit-level content, mathematical process/practice, being a member of a mathematical community). Additionally, all teachers incorporated consistent patterns of instruction, although they were quite varied from teacher to teacher.

### Mrs. Stewart's Instructional Practice

Mrs. Stewart taught using the *CMP2* materials for about four years and attended multiple professional development workshops for this curriculum series. She previously taught high school for about five years before teaching middle school. She had a strong sense of the curriculum materials across grade levels, having talked and worked with other math teachers beyond seventh grade. She identified two of her biggest challenges in teaching as (1) helping students connect their mathematical ideas during the discussion to their peers' thinking and (2) engaging all students in the discussion beyond the "five or six really confident kids" who always want to talk.

Mrs. Stewart typically used the *Launch-Explore-Summarize* structure for organizing lessons, a feature of the *CMP* materials. In this format, the teacher sets up the task for the students (the "Launch"); students then work in small groups (the "Explore"); and finally the teacher summarizes the main points through a class discussion at the end of the lesson. During the exploration time, Mrs. Stewart observed students and asked questions to later use in the whole-class discussion. Mrs. Stewart anticipated what strategies she was going to use before the lesson so that she could plan for how she was going to order solution methods during the discussion (e.g., from more concrete to more abstract). She viewed the *Summary* portion of the lesson as important in helping students see connections across different strategies.

**Baseline observation data.** The observed lesson for Mrs. Stewart was Problem 3.3 from *Comparing and Scaling* (*CMP3*) where students explored different ratios of fiber and protein in chimp food. The students were given two recipes for chimp food: one for babies (40% high-fiber and 60% high-protein) and one for adults (60% high-fiber and 40% high-protein). Students were asked to determine the number of scoops for each mix given three different batches: (a) 100 total scoops, (b) 1 scoop of high-protein, and (c) 1 scoop of high-fiber. The students' task was to determine the missing values in the ratio table (given in Figure 2) and to explain how they determined those values.

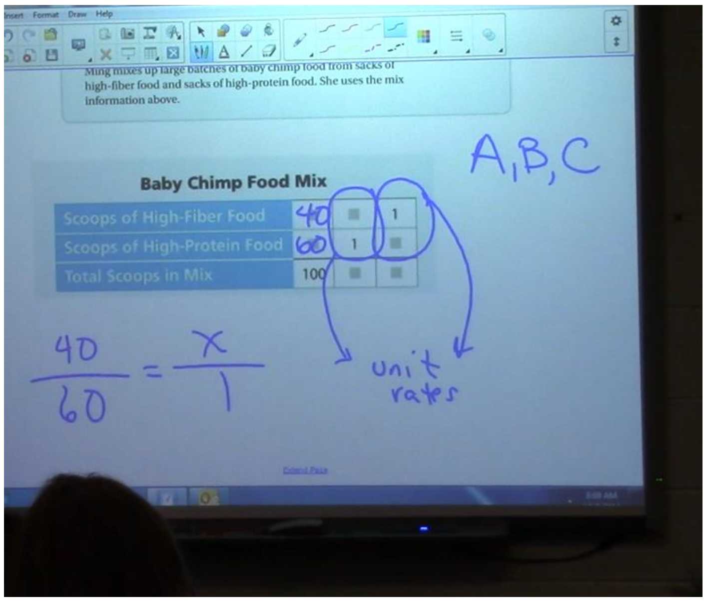


*Figure 2*. Students determining the unit rate for baby chimp food

Throughout the discussion Mrs. Stewart asked students to share multiple strategies, and at least three different strategies were given during the class. The connections across the strategies were primarily teacher-directed, with few opportunities for students to connect strategies during the discussion. She seemed to have a clear sense of which students she wanted to call on during the discussion, given that there was no hesitation in deciding who to invite to participate. Each strategy that was shared included something that had not yet been discussed in relation to unit rates. This said, after each strategy was shared Ms. Stewart took up the task of justifying and connecting, and there were few to no instances of probing for each strategy. Mrs. Stewart scored a 2 (out of 4) on the *Student Discussion Following the Task* rubric, and a 2 (out of 4) on the *Rigor of Teachers' Question* rubric.

**A typical class discussion**. Mrs. Stewart frequently made use of the Launch-Explore-Summarize (LES) format in her classes. In all lessons analyzed, Mrs. Stewart conducted a summary discussion regardless of how far the class got during that class period. During the initial interview, Mrs. Stewart mentioned that she tries to stick to the LES format as much as possible. In her view it is important to give students time to explore the problem and ensure that a summary occurs. In her words, "I don't want to just take the rest of the class and work through all of this and I'll summarize tomorrow because their brain doesn't hold on to that." The summary discussion, in her view, is where the teacher should be helping students make connections to previous content, standards, and strategies across problems.

Desks in Mrs. Stewart's classroom were arranged in groups of 4. In many cases (especially during state testing) there were fewer than 4 students in each grouping. There were no observed changes in the seating assignments of the students across all the lessons observed. Thus, there was no time wasted having students get into groups, since they were in groups the entire class period. The seating arrangement allowed for a more fluid lesson structure moving between group work and whole-class discussion throughout the lesson. As compared to the other teachers in the study, there were very few instances of "teacher lecture" where Mrs. Stewart was at the board simply talking or explaining an idea to students for a long period of time. Instead, students were either discussing or working on problems collaboratively in small groups, or talking/working in front of the whole class.

Discussions frequently shifted between a whole-class venue and small groups. When asked about one particular instance of an abrupt move from whole-class to small groups during the final video-stimulated interview, her response was as follows:

> So, for a question like that that I had posed that, I felt like it could have a variety of ways that kids could think about it or lots of different ways to look at, and lots of different information to look at. So I wanted to give them some time to discuss it with people in their group. And then the reason I did it at that point was because I had heard like maybe there had only been three or four voices sharing. And so often when I do that I can get more voices sharing, or I'll walk around I'll eavesdrop on groups and purposely call different kids out to either hold 'em accountable or because I want to make sure they have actually been listening to their group and can summarize in their own words. And so that's why I did it at that point. 'Cuz I thought it could have so much discussion behind it that I didn't want them to just have to either think through it on their own or to think through it quickly. So I wanted to give them the time to do that with each other. (Mrs. Stewart, video-stimulated interview, June 2, 2015)

Within each lesson there were between 3 and 5 discussion segments, with discussions ranging vastly from as short as 1:50 (one minute, 50 seconds) to 22:12. The average discussion segment was approximately 8 minutes long with the average duration per class period being 30 minutes (half a class period). Some breaks in between segments were lengthy, such as during the *Explore* part of the lessons; others were quite short. The shortest duration between discussion segments was 23 seconds, when students were asked to discuss whether reversing the order of letters in a permutation was allowable (see Figure 3). Discussions typically started within the first few minutes of class. There was one exception to this when the discussion did not start until about 15 minutes into the lesson. Typically, the beginning of each discussion involved individual students answering a question posed by the teacher.

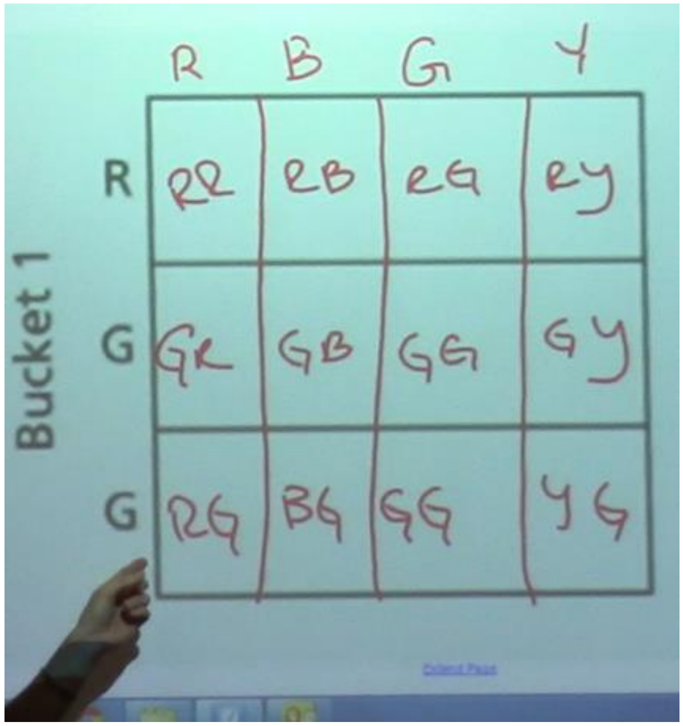


*Figure 3.* Reversed order of permutations in rows 2 and 3

**Goals**. Mrs. Stewart had all four types of instructional goals. She mentioned broad content goals such as supporting students in making sense of statistical summaries in comparing data sets. She also emphasized being a member of the mathematical community. She felt her role in a discussion was to maximize student-to-student interaction and not to be the focus of the discussion. She noted this during an interview: "[M]y goal every class is to have the kids just interact with each other the way they did and almost completely forget the fact that I'm up there, maybe just as a scribe." She placed strong emphasis on mathematical practice goals, since a major theme in Mrs. Stewart's classroom practice was the focus on justification. She frequently pressed students to explain their thinking and justify why a claim was true. She mentioned this repeatedly throughout her post-lesson interviews about the need to cite evidence when making a claim. This also played out in one of the lessons from the unit *Samples and Populations* as students were investigating survey data about roller coasters (see Figure 4). After a discussion about the data, students were given the task shown in Figure 5.

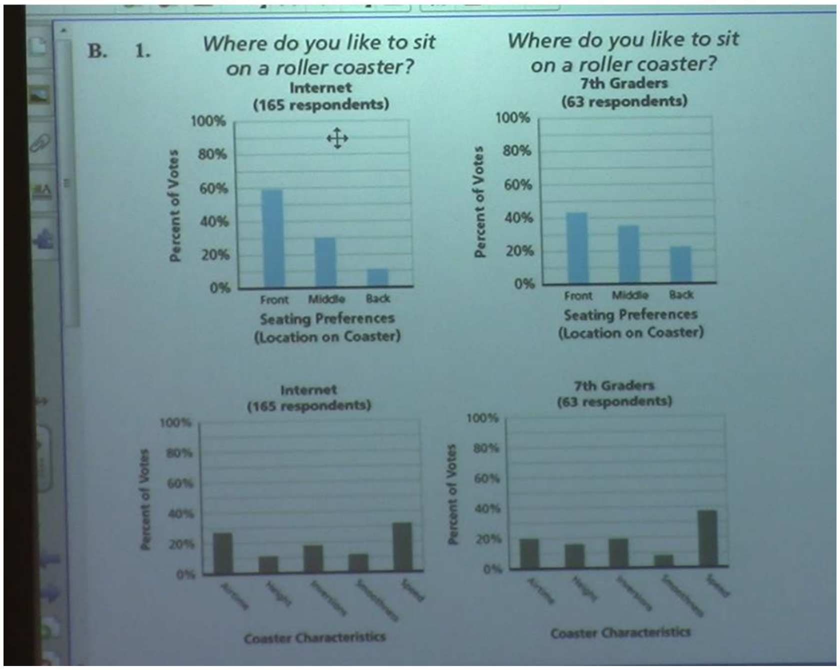


*Figure 4*. Survey data from Internet responses and a grade 7 class

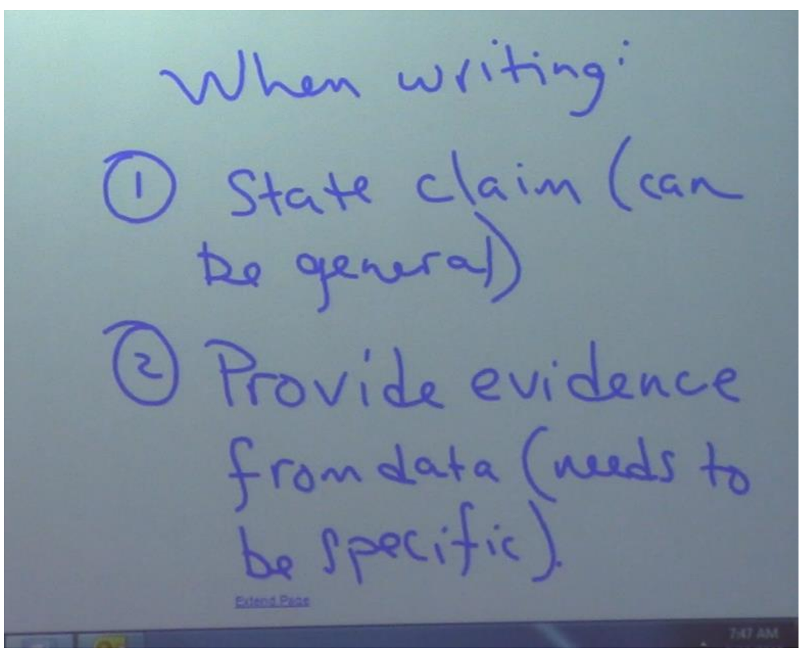


*Figure 5*. Instructions given to students to justify their thinking

In addition to mathematical practice goals, she also placed a strong emphasis on broad content goals, likely influenced by her experience as a high school teacher. She made multiple references to what students would need to be able to know when they were in high school. During a lesson on probability, students were asked to determine the probability of selecting different colored blocks from a bucket with 1 green block, 1 red block, and 2 yellow blocks. Mrs. Stewart noted in the post-lesson interview that many students used three as the denominator, "because of three colors." This stood out to her because as she put it, "And I remember as we get later in the unit, even when I taught high school, understanding what that outcome was, to put on a tree chart or a table, they just didn't have an idea." Mrs. Stewart noticed this issue of the denominator and the potentially long-lasting issues related to not understanding the idea of an outcome.

**Consistent patterns of instruction.** Three recurrent patterns of instructional practice occurred consistently across lessons. For Mrs. Stewart these three consistent patterns of instruction included *maintaining a rapid pace of instruction* (management), *emphasizing communication and process goals* (interaction), and *implementing fluid interaction structures* (activity structure). I provide some detail of these three patterns in the next paragraph, and provide greater detail on Mrs. Stewart's consistent patterns in Appendix N.

Mrs. Stewart's rapid pace of instruction was evidenced by the high volume of questions she asked students. During a typical 30-minute segment of discussion Mrs. Stewart asked about 83 questions, or nearly 3 per minute. On average, Mrs. Stewart made a statement, asked a question, or responded to students once every 10 seconds. This differed greatly from the other teachers in terms of the rapid exchange between her and her students. The second consistent pattern was *emphasizing communication and process goals.* A main feature of this orientation involved students making claims and providing evidence for their claims, both written and verbally. She often pressed students to justify or clarify their thinking when talking during whole-class discussions. The third consistent pattern, *implementing fluid interaction structures*, describes the back-and-forth nature of moving between small groups and the whole-class discussion space. Unlike the other teachers in the study, Mrs. Stewart often moved the discussion between small groups and the whole class several times during a lesson: when students seemed to struggle with an idea or when she felt the need to support their inquiry.

**Professional obligations**. The three consistent patterns of instruction in the previous section all point to a predominant orientation of prioritizing mathematical content as the ultimate deciding factor in making decisions for whole-class discussions (i.e., the disciplinary obligation). The rapid pace of interaction provided minimal management issues and therefore maximized the opportunities for students to engage in mathematical content. During the Caves problem, where the structure of interaction changed dramatically, Mrs. Stewart continued on managing the class as best she could. Students were highly engaged, and Mrs. Stewart changed some of her interaction patterns with the class to support their inquiry and discussion with their peers.

The second and third consistent patterns of instruction supported student understanding of mathematical content by providing opportunities for students to clearly describe their mathematical thinking in different interaction structures. On the rare occasion when the nature of a problem made it difficult for students to justify their thinking, Mrs. Stewart modeled this thinking for the class, maintaining the opportunity for students to deeply understand mathematical content. Mrs. Stewart also demonstrated flexibility in how she supported student inquiry based on the nature of problems and what students were saying in class.

In addition to the consistent patterns of instruction, interview data also supports this prioritizing of the disciplinary obligation. For example, when asked during the initial interview, "[A]re there ways of…organizing the class, or consistent patterns of instruction, or resources that you use so that you find helpful for implementing discussions?" (a question that could be answered attending to any of the professional obligations), Mrs. Stewart talked about how she tries to structure discussions by moving from concrete strategies to more abstract ones. This careful selection of student work based primarily on content ideas played out during the lessons as well. During a lesson where students made a listing of the sample space of flipping three coins, she was asked why she selected the two students to share their work on the document camera. The first student was chosen because she had an incomplete list that was un-ordered, and the second student had all eight permutations in an order. Both characteristics (completeness, ordering) were factors in her choices of sharing the work, showing strong attention to the disciplinary obligation.

**Ms. Mitchell's Instructional Practice**

Ms. Mitchell has taught over twenty years and was somewhat familiar with lessons in the *CMP* curriculum, although this was her first year teaching the curriculum and teaching middle school students (she previously taught elementary and high school). In her words, a typical lesson involved "an interesting hook" to pique students' engagement, followed by time for the students to explore a task. After the students worked on the task, the class engaged in a whole-class discussion.

Unlike the other two teachers in the study, Ms. Mitchell had a highly varied lesson structure. In contrast to the other two teachers, Ms. Mitchell's lesson structure seemed to be a dependent variable, depending on other factors to determine it instead of the other way around.

There were no consistent features across lessons in terms of inclusion or order of particular lesson segments. In one observation (Round 1, Observation 6), the class started with a discussion, moved to group work, then individual work, back to group work, and back to individual work. In another lesson (Round 1 Observation 4), the class started with group work for 33 minutes and was followed by a discussion of 23 minutes. Other observations included segments of teacher lecture about both mathematical content and non-mathematical content (i.e., a 13-minute segment talking to students about behavior). Ms. Mitchell was well aware of this lack of consistent discussion structure. When asked about this in the initial interview she explained:

> I wouldn’t say there’s too many typical lessons, but what I would hope for, is that there’s a meaningful, interesting hook with a launch, where students have either their own or some kind of inquiry that’s leading them to want to explore different mathematical phenomena either as a group, individually or with a pair. So that they’re thinking about things from a variety of perspectives and then bringing those together at the end of the lesson through a class discussion or student sharing either structured or unstructured with that. (Ms. Mitchell, initial interview, November 25, 2014)

**Baseline observation data**. In this lesson, the class was working on a modified version of Problem 3.1 in *Comparing and Scaling* (*CMP2*), which focused primarily on percent increase or decrease in the context of merchandise sales. After working on the task, Ms. Mitchell invited multiple students to go up to the board and describe their thinking to the class. Ms. Mitchell consistently asked questions that probed students’ thinking in order to have students provide more details of why their strategy worked. Based on the observed lesson, Ms. Mitchell scored a 3 (out of 4) on the *Student Discussion Following the Task* rubric, and a 4 (out of 4) on the *Rigor of Teachers' Questions* rubric.

**A typical class discussion.** Ms. Mitchell's class discussions were highly dependent on student engagement. Her lesson structure depended on the needs of the lesson. The number of discussion segments per class ranged from 0 to 3. About half the lessons included summary discussions, and about half the lessons included discussions in the middle of the lesson. The average duration of a single discussion segment was 10:37 (10 minutes, 37 seconds), and the average amount of class time devoted to whole-class discussion was 18:12. The only consistent feature of Ms. Mitchell's discussions was a description of the content and/or practice standards for each particular class period (Figure 6). These standards were stated explicitly at the beginning of class and sometimes referred to again during the discussion at some point.

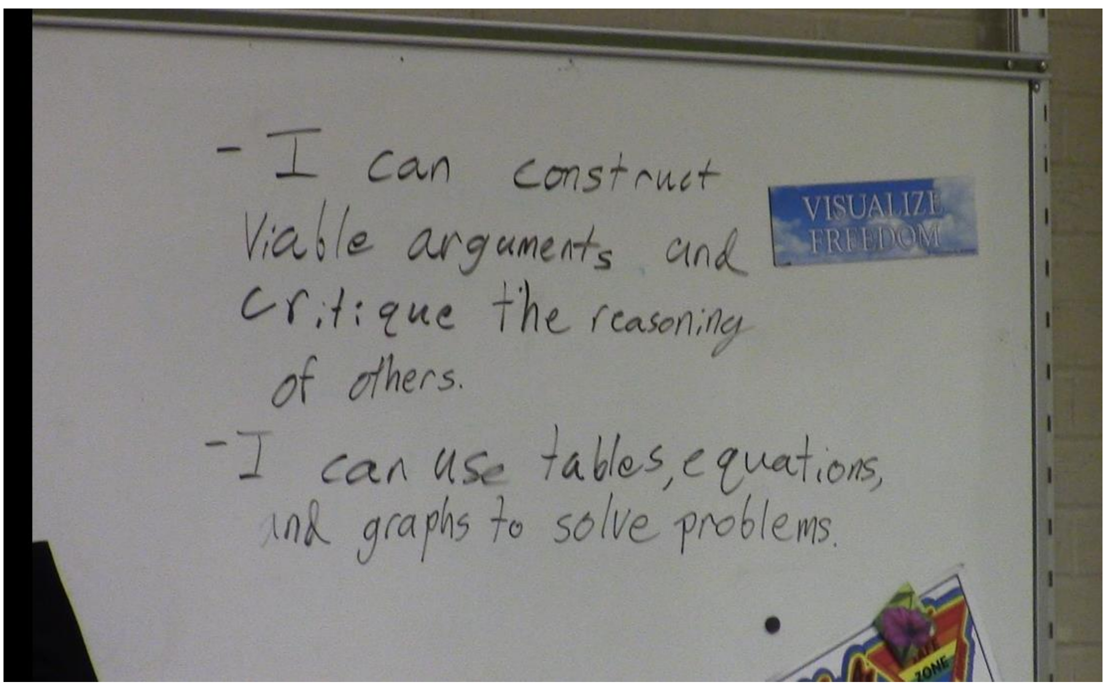


*Figure 6*. Statement of content and practice goals at the beginning of class

**Goals**. The analysis of Ms. Mitchell's instruction related to collaborative discussions included all four goal types. In one of Ms. Mitchell's lessons, a student was describing the *y*-

*intercept* as the "starting point" on the graph, and during the post-lesson interview she described how uncomfortable that made her feel. "When I first saw that [the y-intercept] I was like, oh, I don't like that one bit, 'cuz in high school I had to fight that for three years. It's not a starting point." Ms. Mitchell allowed the student to talk about the y-intercept as a starting point as they were just beginning to understand this important point across representations. This exchange highlighted Ms. Mitchell's attention to long-term content goals (*broad-content*), specifically related to her experience as a high school teacher.

Ms. Mitchell frequently mentioned Common Core standards (NGA-CCSSO, 2010) at the beginning of class – both content and practice – and her use of CCSSM practice standards was a theme throughout her instruction both in describing her decisions and in making these standards explicit to students during class time. In one interview she mentioned wanting students to engage in *repeated reasoning* and *thinking about structure*. Other practices she mentioned included having students provide evidence to support a claim and persevering when trying to solve a problem. These statements reflect *mathematical practice* goals.

Ms. Mitchell also talked about the importance of protecting students' think time, and what students can learn from each other in groups (*establishing a mathematical community*). She wanted her students to be able to interact with each other directly in the discussion and for her role to be minimized in the discussion. In Ms. Mitchell's classroom this involved a single lesson where she explicitly changed the norms of interaction by having students talk directly to each other, taking a less visible role in the classroom dialogue.

**Consistent patterns of instruction**. Two consistent patterns of instruction stood out in Ms. Mitchell's facilitation of class discussions: (a) emphasizing students' thinking (interaction pattern) and (b) using teacher mediated discussion structure to facilitate whole-class discussions (interaction pattern). This first pattern was highly consistent in that Ms. Mitchell frequently mentioned wanting to make sure that students' thinking was valorized during whole-class discussions. This played out in several ways and, as will be discussed later in results, created several dilemmas for Ms. Mitchell as she enacted content and practice goals. The second pattern focused on the talk structure that Ms. Mitchell typically used, which was for students to talk and then she responded. All student talk was essentially mediated through her. These patterns are described in further detail in Appendix O.

**Professional obligations.** Ms. Mitchell's instructional practice related to collaborative discussions was predominated by her obligations to her students, both as individuals and interpersonally. Throughout the interviews she repeatedly mentioned a multitude of specific students' needs from understanding content to more personal needs. When asked about why she called on particular students during a discussion, she responded that it was due to her wanting to get different voices heard in the discussion. When asked about consistent patterns of instruction that she uses, a recurring theme in her responses was the use of group work to help students learn from each other. Even when management issues made it challenging to continue on with group work, she found value in continuing with it. During one of the post-lesson interviews, Ms. Mitchell described her commitment to using group work despite the management issues that arose:

> Nicholas J. Gilbertson (NJG): "Yeah, so it's interesting you mentioned that this class is really productive, that causes some issues about like some teachers would say, like, well you can't have group work now because you lost the privilege of doing that…"
>
> Ms. Mitchell: "No, I wouldn't."

NJG: "...[B]ut there's that sense that you're not losing anything, you're just gaining better management. And you're not losing anything else, so, it seems to be something you're paying attention to."

Ms. Mitchell: "[W]hat worked out so well last time was they worked in pairs first and so I'm thinking that, that's something I need to think about, is giving the think time with the pairs and the communication time to help them buy in to a simpler social setting and then move it out, so. Or to be thoughtful about that, so part of that's how I set it up."

**Mr. Sandberg's Instructional Practice**

Mr. Sandberg has taught math for seven of his nine years teaching, all of which included the use of *CMP2*. He has attended multiple professional development workshops on using the curriculum. Mr. Sandberg noted challenges in getting students to talk to each other, including explaining their reasoning and connecting to prior knowledge. He attributes this mostly to student "buy-in" of the class atmosphere, which he described as being "comfortable" and team-focused. When asked about what it meant for students to be "active" in his classroom, he responded:

"They're [students] doing the thinking, connecting to prior knowledge, they're pulling those things out and really fending for themselves and wanting, wanting to solve the problems and not just regurgitating a formula or, or skill-and-drill not just doing the same thing over and over, but, but really just sticking with it and working through a hard problem, maybe asking a question of 'I tried this and I'm not quite sure here how to get from point A to point B' as opposed to 'I don't get this. I don't know how to do this, I'm not doing it because I don't know how.'"

According to Mr. Sandberg, during a typical class students start with bell work, which is usually a review of the previous day's work. This time is meant for review as well as time for Mr. Sandberg to take attendance and collect homework. He typically helps students through the first few problems of a lesson as a whole class before letting each student work with one partner that he assigns ahead of time. They work on the task for about twenty minutes before a whole-class summary and discussion takes place. He makes use of the Smart Board and document camera to help display student thinking during the discussion. More so than the other teachers, Mr. Sandberg had a highly consistent lesson format, listed below.

1. Bell work: Students work individually on a warm-up problem.
2. Discussion of bell work
3. A description of the task students will be working on for the day out of the textbook.
4. Work time – typically with an assigned partner
5. Reporting work from the day's mathematical task, with some discussion, or homework time or housekeeping

**Baseline observation data**. The lesson observed for his initial observation was Problem 1.1 of *Accentuate the Negative* (*CMP3*) where students explored adding positive and negative numbers based on correct and incorrect answers in a game show. The observed structure of this lesson was consistent with his description of a typical lesson. Students worked on introductory problems before going into partner-based group work. When the partner-work concluded, the whole class talked about what they had learned from the problem. Many of the opportunities for students to talk in the discussion were based on descriptions of their work. There were no instances of students making connections across strategies, nor were there explanations by the students of why their strategies worked. Although Mr. Sandberg did ask students to explain their thinking, he often took over the students' explanations, thereby limiting the opportunities to probe deeper into students' thinking. Based on the observed lesson, Mr. Sandberg scored a 2 (out of 4) on the *Student Discussion Following the Task* rubric, and a 2 (out of 4) on the *Rigor of Teachers' Question* rubric, which was similar to Mrs. Stewart's scores and below those of Ms. Mitchell's.

**A typical class discussion**. Within each of Mr. Sandberg's lessons, there were 1 or 2 segments coded as whole-class discussion. One of these segments usually occurred within the first five to ten minutes of the class. Discussion segments averaged about ten minutes, but ranged from as short as 2 minutes to as long as 21 minutes. The average amount of time devoted to discussion was approximately 16 minutes, and ranged from about 4 minutes to 24 minutes.

All lessons began with *bell work*, which served many purposes. He presented a problem to the class, and students worked individually on it for five to ten minutes. From a management perspective, Mr. Sandberg took care of various housekeeping tasks such as checking homework and attendance. There were also important content reasons for the bell work. The types of problems that Mr. Sandberg chose for the bell work usually connected the previous day's work with the next lesson. The problems were usually determined by common student questions from the previous day's work. Mr. Sandberg described having carefully chosen these problems, in many cases, so that a common mistake or misconception could be addressed collectively (*broad-content goal*). As one example (Figure 7), Mr. Sandberg asked the class to consider a relationship and determine if it was linear or not. The table presented in Figure 7 shows an inconsistent change in *x*. When asked during the post-lesson interview as to why he included this example in the discussion he stated,

> That's a big misconception I get later in the book…They always see them in the beginning go zero, one, two, three and then it starts to jump around later. I've seen in years past where they're completely thrown…So I wanted them to think about that it could still be linear…It's not just…see if it goes one, two, three or two, four, six, then yes it's linear. But I wanted them to think a little deeper than that. Seemed to work. (Mr. Sandberg, post-lesson interview, March 3, 2015)

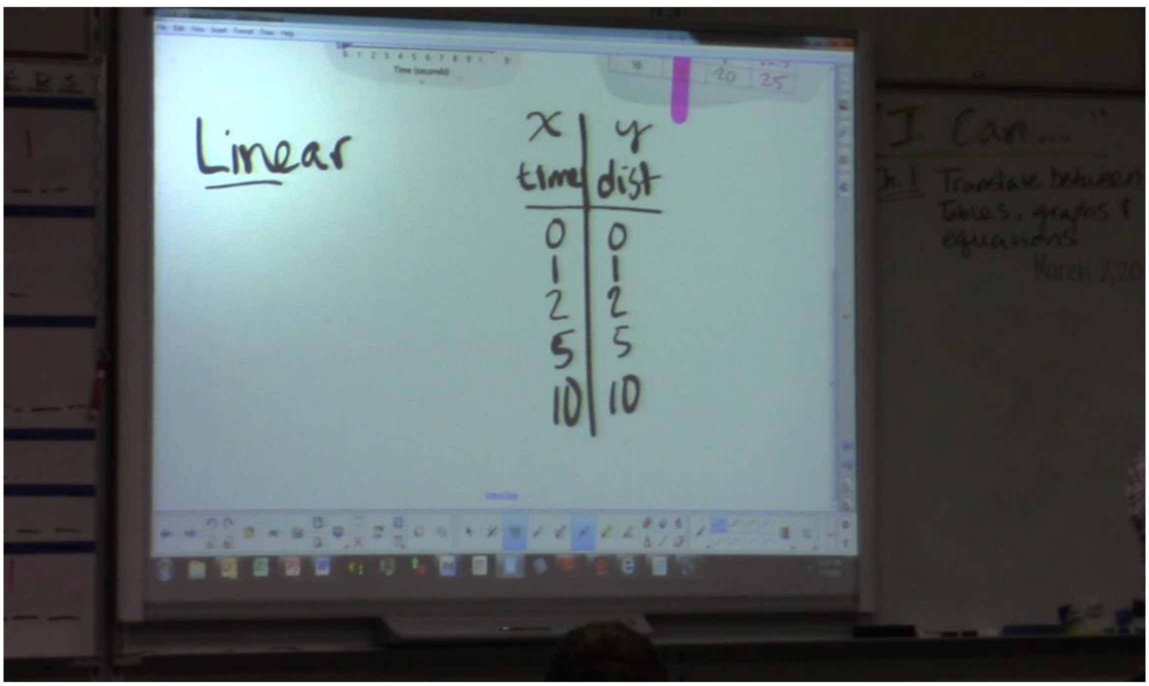


*Figure 7*. Mr. Sandberg asks if this table represents a linear relationship.

The bell work served as a way to connect the previous day's content with the new material. Discussion of the bell work occurred in all lessons but one, where Mr. Sandberg used the bell work as a way to introduce box-and-whisker plots, which was mostly new for students. Discussions of the bell work varied, ranging from as little as two minutes to as long as twenty minutes. In many respects, the bell work discussions replaced summary discussions from the previous day. This work focused on important ideas from the previous day's work and attempted to explicitly address student mistakes that were noticed across classes. During the final video-stimulated interview, Mr. Sandberg mentioned that he felt that he had student's attention more at the beginning of the class (compared to the end) and this was his reason for structuring a majority of the discussion time at the beginning rather than at the end during a summary.

After the discussion of the bell work, there was an often abrupt transition to starting the day's mathematical task. Mr. Sandberg would ask students to get out their notebooks and label their paper with the appropriate problem number from the *CMP* materials. This segment of the lesson could be categorized as "teacher lecture" as it typically involved Mr. Sandberg presenting new content to students without much dialogue. There was typically some student interaction, but it was very minimal both in the frequency of students talking and how they responded. These were generally not coded as *whole-class discussion* for this reason. In a typical *CMP* Problem, there are several parts (A, B, C, etc.). Mr. Sandberg usually worked on part A collectively with the whole class. When asked about this consistent feature of his lesson structure, he stated, "Yeah, I don't have anything set. Seems like this year with *CMP3* I'm doing *A* almost all of the time…I remember last year…they would just get into partners right away at *A*." As a reminder, Mr. Sandberg had been teaching using the *CMP* materials for several years, but he was currently field-testing the third edition materials, *CMP3*, and was using these particular curriculum materials for the first time.

After Mr. Sandberg described the mathematical task, students worked with a partner on the assigned problem. These partners were assigned by Mr. Sandberg, either through a list provided on the screen or by convenience (i.e., matching up students who were close to each other in their assigned seats). After partner work time, there was some variance in the final lesson segment. A majority of the lessons analyzed included some brief discussion with students reporting their work. There were also instances of partners moving on to work on homework, or other management related tasks such as handing out progress reports.

**Goals**. Mr. Sandberg had a variety of broad instructional goals. During warm-up problems he would at times focus on student misconceptions in hopes to bring to light issues that might not easily appear in the day's lesson but may impact students' long-term understanding of an idea. As one example, Mr. Sandberg asked students to consider which of the two equations had a greater rate of change: $y = 1.5x + 2$ or $y = 2x + 0.5$. The second equation exposed students to situations where the rate of change was an integer and the y-intercept of the graph was a non-integer, different than the more common first equation. This set of equations afforded students the opportunity to consider the meaning of those values in relation to a context they had previously investigated and to possibly avoid over-generalizing the types of numbers associated with the slope and y-intercept in a linear situation.

Similar to both Ms. Mitchell and Mrs. Stewart, Mr. Sandberg also had unit level goals (e.g., comparing data sets to make statements about them), and mathematical practice goals such as using technology strategically. In contrast to the other two teachers in this study, there were no instances of Mr. Sandberg explicitly referencing goals related to being a member of a mathematical community during interviews, lesson plans, or lesson observations. This may have been attributed to the fact that data collection did not start until February and these goals may have been present earlier in the semester.

**Consistent patterns of instruction.** Mr. Sandberg had two consistent patterns in his instruction: (a) the consistent use of technology as a lesson resource, and (b) the implementation of a consistent discussion structure in order to maintain classroom management. In regard to the first consistent pattern, he frequently used several types of instructional technology -- from clickers to graphing calculators to spreadsheets -- to support students in discussing their thinking during whole-class discussions. This varied use of technology differed greatly from the other two teachers' instruction. His second consistent pattern was the use of his lesson structure (described previously), which differed from the interaction structures that both Ms. Mitchell and Mrs. Stewart incorporated. His consistent patterns are described in greater detail in Appendix P.

**Professional obligations**. The predominant obligation influencing Mr. Sandberg's decision-making was attention to the institutional obligation because of his overwhelming emphasis on establishing a well-ordered classroom. Student work and talk were largely constrained by the need to have consistent interaction and activity patterns. These constraints allowed for students to have a clear sense of what was expected of them as learners of mathematics, but they also served as ways to limit the potential opportunities to pursue interesting ideas that were connected to but outside the exact scope of the lesson flow. In both consistent patterns of his behavior, there were situations where the need to maintain a consistent discussion flow and norms of interactions took priority over pursuing student ideas that could have led to interesting mathematical discussions. These instances, coupled with the lack of partner/group talk to the whole-class, support the notion that the institutional obligation was Mr. Sandberg's primary focus.

### Summary of Teachers' Instructional Practice

All three teachers in this study included the four goal types analyzed as features of their instruction. They all attended to lesson-specific content goals, broad content goals, mathematical practice goals, and goals for establishing a mathematical community. All four incorporated consistent patterns in their instruction, but they were quite varied. While all four had 1 or 2 predominant obligations in their practice, they all differed; Mrs. Stewart focused primarily on the disciplinary obligation, Mr. Sandberg on the institutional obligation, and Ms. Mitchell on the interpersonal and individual obligations. In the next chapter, I discuss the results of how these teachers' orientations to their professional obligations acted as a mechanism to frame how these teachers made decisions when faced with instructional dilemmas as a result of conflicting types of goals.

# CHAPTER 5

# RESULTS: INSTRUCTIONAL DILEMMAS AND DECISIONS

The results for this dissertation addresses the research question: *What is the nature of instructional dilemmas that teachers face when planning for and enacting collaborative mathematics discussions, and how do teachers respond to those dilemmas through their decision-making?* I remind the reader that dilemmas are defined as a conflict between instructional goals. Some dilemmas were observed (via a deviation from a consistent pattern of behavior), whereas others were self-reported (i.e., the teacher identified them as a dilemma during an interview or lesson plan). Some instructional dilemmas and their ensuing decisions occurred during planning for a class discussion, while most others occurred during the enactment of whole-class discussions.

The main finding of this study was that all three teachers' decision-making was explained via their primary professional obligation – with important limitations. Along with this main finding, there were three subfindings. First, teachers' decision-making was largely explained via their primary orientation to their professional obligations. Second, all three teachers encountered scenarios where they made decisions that deviated from their primary orientation to the professional obligations. Third, even though the teachers were experienced, used instructional resources (e.g., written curriculum materials) that supported implementing whole-class discussions, and were generally oriented philosophically with the importance of using discussions, this was not sufficient to avoid challenges one might expect to see while facilitating whole-class discussions. Most importantly, the types of challenges they encountered were, by and large, independent of the teachers' primary orientations to their professional obligations.

## Dilemma Type Results

Given the four different types of goals, a total of six different types of dilemmas were possible, considering the set of conflicts between two instructional goal types. This chapter is organized around these six dilemmas. Although the dilemma types are enumerated, the order is arbitrary. Within each dilemma, I describe one case from each of the teachers' practice. I chose to describe only one representative case per teacher, choosing the case that provided the most detail for the particular dilemma. Of the 18 possible cases that I describe in this chapter (6 dilemma types, 3 teachers per dilemma), 16 appeared in the analysis. There were 2 cases that did not occur, which are noted in the tables in each section. Across the 3 teachers, each teacher had at least 5 of the 6 dilemma types present. Table 4 provides the different dilemmas that occurred in the data for each of the teachers.

Table 4
*Dilemmas Teachers Encountered as a Conflict between Goal Types*

| | Dilemma Type | | | | | |
|---|---|---|---|---|---|---|
| Teacher | LS-BC | LS-MP | LS-MC | BC-MP | BC-MC | MP-MC |
| Mrs. Stewart | 2 | 2 | 1 | 1 | 0 | 1 |
| Mr. Sandberg | 2 | 1 | 3 | 0 | 2 | 1 |
| Ms. Mitchell | 2 | 5 | 1 | 1 | 2 | 2 |

Note: BC: broad content goals, LS: lesson-specific content goals, MP: mathematical practice goals, MC: establishing a mathematical community goals

I remind the reader that Mrs. Stewart's instructional decision-making in relation to collaborative discussions was primarily oriented to the disciplinary obligation. Mr. Sandberg's instructional decision-making in relation to collaborative discussions was primarily oriented to the institutional obligation. Ms. Mitchell's instructional decision-making in relation to collaborative discussions was primarily oriented to the interpersonal and individual obligations. Thus, each teacher represented a case of instruction oriented along a different professional obligation. These results provide insight into how teachers who typically favor a particular professional obligation make decisions when faced with particular dilemma types.

**Dilemma Type 1: Lesson Specific Content Goals Conflicting with Broad Content Goals**

This type of instructional dilemma took place when teachers encountered situations where they had to choose between immediate content goals in a lesson with broad content goals. As shown in Table 4, this dilemma type appeared at the same frequency across all teachers' observations. Given this dilemma type, one might anticipate a teacher primarily oriented towards the disciplinary obligation as trying to balance these two types of content goals, with perhaps more attention to broader content goals because of a possible understanding of the long-term storyline of certain mathematical ideas. A teacher with a primary orientation towards the institutional obligation could also favor broad content goals, if they were primarily concerned with such obligations as departmental or state-wide assessments. They might also focus on lesson-specific goals, if a deviation from the lesson goals would create more disruption to the class. A teacher with primary orientations to interpersonal and individual obligations may place greater emphasis on lesson goals being responsive to student thinking as it appears immediately in the classroom.

In the subsection below, I provide greater detail on one representative case from each of the teachers for Dilemma 1. As an overview of this dilemma, all three teachers made decisions when faced with instructional dilemmas that were consistent with their primary orientation to their professional obligations. Mrs. Stewart (disciplinary) was able to balance lesson specific content goals with broader content goals. When Mr. Sandberg was faced with what to do with a student's question that might disrupt the flow of the class by pursuing the student's idea, Mr. Sandberg (institutional) chose to quickly move on from the student's question to refocus the class on lesson-specific content. When Ms. Mitchell (interpersonal/individual) was faced with a similar situation, she too chose to refocus the class on the lesson goals, but not before acknowledging the importance of the student's insight to the whole class.  Table 5 provides an overview of this first dilemma.

Table 5
*Dilemmas between Lesson Specific Content Goals and Broad Content Goals*

| Teacher (typical orientation to professional obligations) | Dilemma | Decision |
|---|---|---|
| Mrs. Stewart (*disciplinary*) | Complete all parts of the lesson as written in the materials vs. Support students in connecting visual and numerical equivalence | Complete all parts of the lesson, but discuss in depth for a few minutes numerical and visual equivalence |
| Mr. Sandberg (*institutional*) | Continue on with lesson investigation vs. Pursue a student's idea that focuses on ideas later in the unit | Acknowledge student's idea, then redirect class to today's lesson |
| Ms. Mitchell (*interpersonal* and *individual*) | Understanding how to interpret a graph in a context vs. Pursuing a student's question about graphs of functions in general | State the importance of the student's question and tell the student the class will come back to that question later |

**Dilemma 1: Mrs. Stewart.** In Mrs. Stewart's class, students were working on representing an area model for a compound probability situation. They were given two buckets with colored marbles in each bucket. In the first bucket there were 3 marbles: red, green, and green. The second bucket had marbles colored red, blue, green, and yellow. The class discussed the question, "What is the probability of choosing a marble from each bucket and getting at least one blue?" A student described his thinking to the whole class, but Mrs. Stewart was unable to follow the student's explanation. Had Mrs. Stewart been primarily oriented to the institutional

obligation (focused on making sure the question was addressed in a short amount of time), she might have simply rephrased what the student was thinking. Concerned that the class was not really understanding where the one-fourth was represented in the area model, she posed a question: “If the probability of that is one-fourth, what size piece out of the whole are we actually counting?” She called on a student to come up to the board to show how they were thinking about where the 1/4 showed up in the figure. In Figure 8 the student correctly showed an area representation of 1/4 (in the lower right corner), but was unable to describe where 1/4 appeared in Mrs. Stewart’s model (on the left).

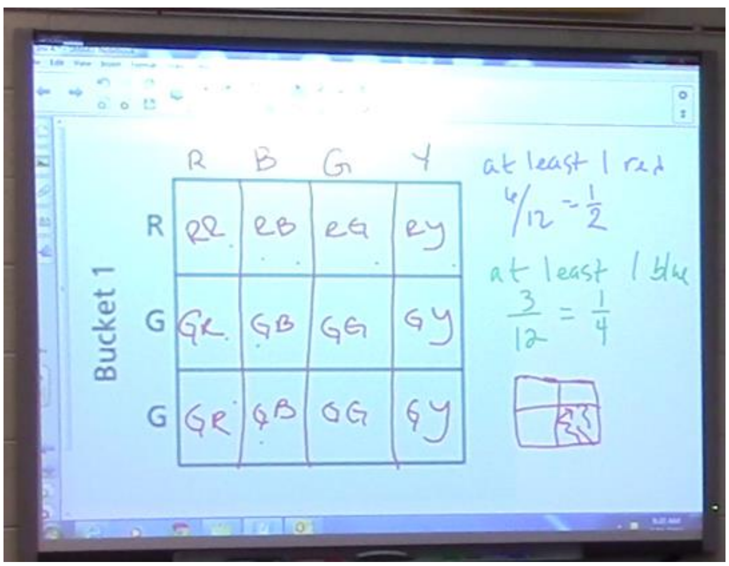


*Figure 8*. Area model for selecting marbles from two buckets

The deviation from the consistent pattern of her rapid pace of instruction prompted the question during the post-lesson interview about why she pressed students about this. If her goals had been oriented to the institutional obligation, where completing class on time and staying on pace with the curriculum is a key goal, an alternative decision could have been simply to state that the two values were equivalent and move on. Mrs. Stewart, however, chose to press students to see how the two values were equivalent with the area model. She indicated in the interview that her reason for doing this was that in a future problem students would be better prepared to reason through more complicated area models if they understood how the values were represented directly from the model. She also indicated that she had wanted to possibly go further with the explanation than she did, but, pressed for time at the end of class, she chose to press students on connecting the model to the representation on just this one problem.

Given the dilemma of meeting broad content goals with lesson-specific content goals, Mrs. Stewart found opportunities to support students in connecting symbolic and visual representations that could support their future work in subsequent lessons. Although the institutional obligation (time constraints) ultimately limited her ability to go deeper into broader content goals that address her disciplinary obligation, she was able to identify and facilitate discussion around fractional equivalence, a topic that shows up in later lessons. Being able to identify key areas that potentially cause problems in the future and being able to balance short-term content goals with broader content goals is consistent with Mrs. Stewart's propensity to favor the disciplinary obligation.

**Dilemma 1: Mr. Sandberg.** During the beginning of a unit on probability, Mr. Sandberg had asked the class to think about flipping one coin and then flipping multiple coins, and to consider the probabilities of getting all heads. One student had a longer explanation where he was trying to explain the difference between the probability of an individual coin flips versus multiple coin flips. The class appeared not to follow the student's explanation. Mr. Sandberg was faced with the decision of (a) pursuing the student's idea on compound probabilities with several coins or (b) moving on with the simpler lesson goals he had already planned.

Had Mr. Sandberg been primarily oriented to the disciplinary obligation, he might have spent time pursuing the student's idea or, at the very least, restating the student's idea so that the class could return to this idea in a future lesson. Mr. Sandberg's response to the student during the discussion was, "We have vocabulary for that," and then he quickly moved on with the lesson, not returning to the student's idea. During the post-lesson interview Mr. Sandberg commented on this exchange: "I had this sense that the other kids were not understanding him; I was having a hard time understanding him, and it would have taken us in a different direction." During the post-lesson interview Mr. Sandberg mentioned that the ideas that the student were sharing would be brought up later in the unit, but the class (in his opinion) was not ready yet to investigate those ideas. Mr. Sandberg chose the immediate lesson goals over broader content goals. Given the particulars of this exchange between Mr. Sandberg and the student, moving away from the student's ideas and back towards the flow of the original discussion was consistent with Mr. Sandberg's consistent pattern of instruction in line with his orientation to the institutional obligation.

**Dilemma 1: Ms. Mitchell.** In one of Ms. Mitchell's discussions, students were looking back on their previous day's work reviewing the graphs of three students who were fundraising in a walkathon (Figure 9). The problem in the textbook (Moving Straight Ahead, p. 8, *CMP2*) gave the fundraising as follows: Leanne $10 regardless of distance; Gilberto $2 per km; Alana $5 plus $0.50 per km. A student commented that the graphs were linear, and Ms. Mitchell asked the student why they were linear.

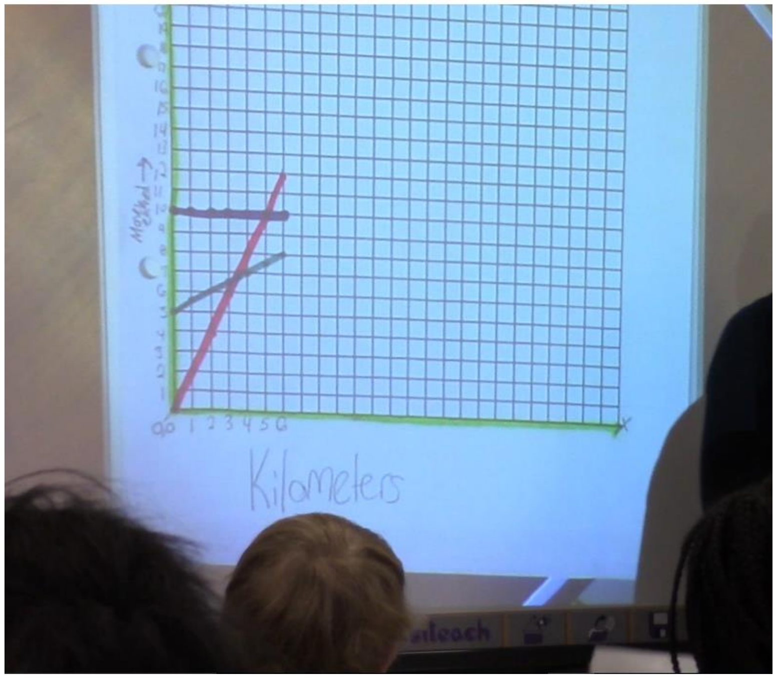


*Figure 9*. Ms. Mitchell displays walkathon results

Later in the discussion, the student was asked to come to the board and draw a graph of a non-linear relationship. Whereas the student may have simply drawn a non-linear function (e.g., a quadratic or exponential relationship), the student instead drew a graph that was not even a function (Figure 10). Given this strange graph, a different student asked, “Could a graph actually go backwards like that? Like could it go like this way and then eventually go back that way again?” This student was asking a question focused on an important relationship in understanding a mathematical function that went well beyond the scope of the individual lesson. Ms. Mitchell was faced with the dilemma of how to pursue this important mathematical insight, which was beyond the scope of the lesson.

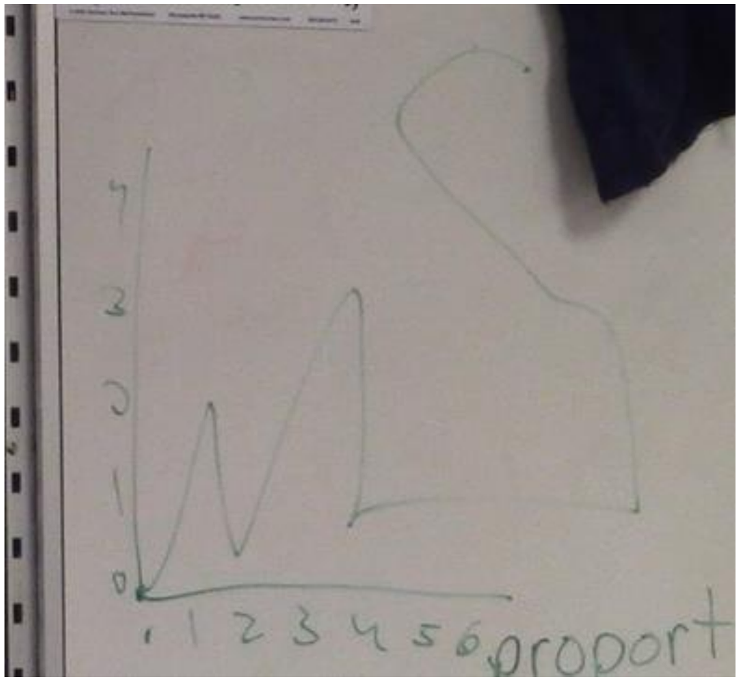


*Figure 10*. A student draws a graph of a non-linear function

Had Ms. Mitchell been oriented to the institutional obligation and been most concerned with maintaining a consistent discussion flow (as was the case with Mr. Sandberg), she too might have quickly moved on from this student’s thought, minimizing the opportunity for disruption. Instead, her decision was to write a note on the side of the board and to validate the importance of the student’s insight. “I’m going to write that right here Charlie because that’s outside what we want to talk about today. But that’s an extremely important question; could a graph actually go backwards? And we need to talk about that.”

During the video-stimulated interview after the first round of observations, Ms. Mitchell commented on this clip:

> Charlie’s question was so awesome as it usually is. That is an example where I don't want to shut down the conversation or, well I do want to shut down the conversation because the level of thinking it would take based on what they had had by one-two would be too

great…I do want to push a little bit…Now that thing about going back on itself I still have that written down and we will come [back] to that.

Although Ms. Mitchell's decision was to move on from the student's question, she did this only after valorizing the student's question. Ms. Mitchell could have chosen to pursue the student's idea directly, essentially moving away from her lesson goals for the discussion. Instead, she refocused the discussion in line with the lesson goals. Her decision to valorize the student's thinking was consistent with her professional obligation to students individually. By supporting the student's thinking and publicly acknowledging its importance, she continued to make explicit the importance of thinking and questioning in her classroom.

**Summary of Dilemma 1.** In all three cases, teachers encountered dilemmas on how to manage lesson specific content goals with broad content goals. In all three cases, the teachers made decisions that were consistent with their primary orientation to their professional obligations. Ms. Mitchell and Mr. Sandberg encountered very similar dilemmas: A student asked a question during the course of a discussion that had the potential to move the class beyond the scope of what the teacher had hoped to accomplish in the lesson. Although both teachers prioritized lesson specific content goals over broad content goals, they differed in their interaction with their students. Mr. Sandberg chose to quickly move on from the student's question because he felt the class might be confused, which could in turn cause disruption to accomplishing his lesson specific content goals. Ms. Mitchell chose to acknowledge the importance of the student's question before refocusing the discussion on the lesson specific content goals. These two scenarios contrasted with Mrs. Stewart, whose orientation on the disciplinary obligation was balanced between broad content goals and lesson specific content goals.

**Dilemma 2: Lesson Specific Content Goals Conflicting with Mathematical Practice Goals**

This second type of instructional dilemma was the most prevalent type of dilemma that occurred, but it happened more often in Ms. Mitchell's classroom than in the other teachers' classrooms. As I will explain in this section, Ms. Mitchell's orientations around the interpersonal/individual obligation likely played a role in the frequent emergence of this dilemma. In Table 6, I provide an overview of these dilemmas.

Given a teacher with a primary orientation to the disciplinary obligation, the teacher may try to strike a balance between immediate content goals and mathematical practice goals (given that both focus on doing mathematics). In some cases, the teacher may privilege one goal over another given the need to co-develop both goals. Given a teacher with a primary orientation to the institutional obligation, one might expect to see more focus on immediate content goals, as those are often assessed more readily on standardized assessments, and supporting students in learning to engage in mathematical practice goals may cause unforeseen management issues. Given a teacher with a primary orientation to the interpersonal or individual obligation, the likely result is the teacher prioritizing mathematical practice goals, supporting students to develop their own understanding through reasoning, communicating, and working with their peers.

As an overview of this dilemma (details provided in Table 6 below), two of the teachers (Mrs. Stewart, Mr. Sandberg) made decisions when faced with instructional dilemmas that were consistent with their primary orientation to their professional obligations, while one (Ms. Mitchell) deviated from her primary orientation. Mrs. Stewart prioritized lesson specific content goals over mathematical practices when her students became unable to make progress on a problem consistent with her orientation to the disciplinary obligation. Mr. Sandberg considered the benefits and limitations of letting students work in groups to support the whole-class discussion. His choice to focus on structuring the group work more effectively was consistent with his institutional obligation. Ms. Mitchell prioritized departmental expectations for content coverage given the time constraints of teaching during the last few weeks of the year to take priority over a focus on mathematical practice. This decision was inconsistent with her general orientation to prioritize the individual and interpersonal obligations.

Table 6

*Dilemmas between Lesson Specific Content Goals and Mathematical Practice Goals*

| Teacher (typical orientation to professional obligations) | Dilemma | Decision |
|---|---|---|
| Mrs. Stewart (*disciplinary*) | Support students in engaging in mathematical practices or provide greater access to the content of the problem | Directly model the problem, show the class how to solve it, and explain how she went about solving the problem |
| Mr. Sandberg (*institutional*) | Use group work (allows space for students to discuss their thinking) or do not use group work (provides extra time for lesson goals) | Use group work in the lesson, but structure it more effectively |
| Ms. Mitchell (*interpersonal* and *individual*) | Extent to which you can emphasize mathematical practices when needing to cover content at the end of the school year | Cover what is expected by the department as best as possible during the last few weeks |

**Dilemma 2: Mrs. Stewart.** During a whole-class discussion in Mrs. Stewart's class, students were working on a problem from the *CMP3* materials, collectively trying to determine what a seventh test score would be in order for the average to be 85 (shown in Figure 11). Students were struggling significantly in terms of how they would even begin the problem. Typically, Mrs. Stewart would press her students to justify their thinking and they would respond with answers. This problem proved to be highly challenging to students (as evidenced by their lack of being able to really start the problem in any meaningful way).

Given the inability of students to describe their thinking in the whole-class discussion, Mrs. Stewart was faced with the dilemma of having to choose between the lesson specific content goals (i.e., understanding how to solve the problem) or pressing students in hopes they would be able to engage in mathematical practices. If Mrs. Stewart's decision had been focused primarily on the interpersonal obligation, she may have chosen to have students do a similar, easier problem that would have allowed them access to the more challenging problem presented. Usually, Mrs. Stewart provided time for her students to persist when challenged with a problem. In this instance, Mrs. Stewart directly modeled the procedure to the class in terms of how to solve the problem. The use of direct explanation (or modeling) was rare for Mrs. Stewart's instruction. This scenario highlighted that when Mrs. Stewart was compelled to decide between mathematical practice and lesson-specific content goals, Mrs. Stewart would choose lesson-specific content goals. This choice is consistent with Mrs. Stewart's general orientation to favor the disciplinary obligation.

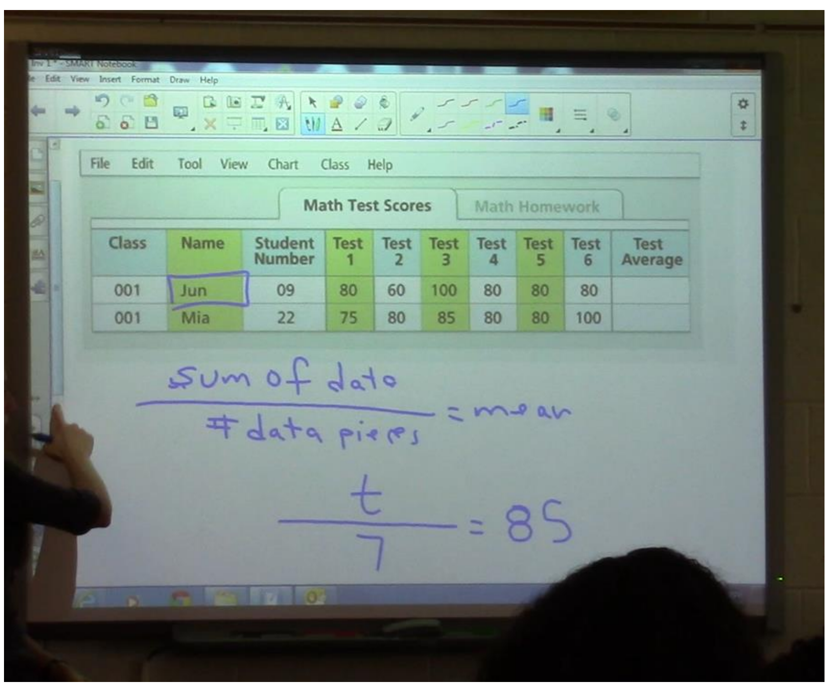


*Figure 11*. Students try to determine Jun's 7$^{th}$ test score for a resulting average of 85.

**Dilemma 2: Mr. Sandberg.** In Mr. Sandberg's class, students often had the opportunity to work with a partner on a problem to prepare for a whole-class discussion. Although this dilemma did not occur directly during the whole-class discussion aspect of the class, it did allow students to engage in discussing their thinking with each other in order to prepare their ideas for the ensuing discussion. During a post-lesson interview, Mr. Sandberg acknowledged this dilemma of whether allowing students to share their thinking with a partner prior to a discussion was productive or not for the discussion. Given the somewhat long sections of teacher explanation that frequently occurred during discussion segments, not allowing students to work in groups would have saved (in his words) about ten minutes of class time. The value of letting students work in groups is that they are potentially better prepared to discuss their thinking when the discussion begins. Both of these choices are focused on maintaining efficiency to support the flow of the lesson and discussion.

If Mr. Sandberg had been primarily oriented to the interpersonal obligation, he likely would have seen the value in students being able to share their thinking with a partner prior to sharing their thinking with the whole class. It would have been much less problematic than it was. His choice was to allow students to work in their groups but to be more structured in how he set up the groups. Creating more structure was consistent with his focus on management and structure and his orientation to the institutional obligation.

**Dilemma 2: Ms. Mitchell.** This second dilemma type for Ms. Mitchell was the most prevalent for any dilemma and teacher, appearing in five different situations. She frequently grappled with the need to meaningfully engage students in mathematical practice while also attending to lesson-specific content goals. This dilemma's frequency may have been a result of Ms. Mitchell's greater comfort with focusing on mathematical practice (given her general orientation to learning, her role as a teacher, and her experience in teaching several different grades) than her comfort with specific lessons (being in her first year teaching middle school and using the *CMP2* series).

Although many instances of this type of dilemma occurred during enactment of collaborative discussions, one of the most telling explanations of Ms. Mitchell's second dilemma type was described in her thinking about planning for discussions. During an interview she described the dilemma of how to fit in as much content as she is expected to (by the department) over the last few class periods remaining in the year while still trying to focus on mathematical practices.

> Well there's a dilemma, because on the one hand I would love to have all the ideas for them to have access to; on the other hand there's the bigger idea of the mathematical practices, like what do I do when I don't know what to do?...And, so I've got this treadmill keeping us moving toward the end at the same time as I'm trying to incorporate as much of the thinking stuff that I can. (Ms. Mitchell, post-lesson interview, May 19, 2015)

She later referred directly to how this dilemma impacted that day's lesson and the previous day's lesson, wondering if students saw the mathematical purpose of what they were doing. This dilemma mostly went unresolved, but she conceded the pressure of having to get through material as best she could before the end of the year. Although Ms. Mitchell's primary orientation to her obligations attended to students individually and interpersonally, in this case Ms. Mitchell's institutional obligation (being a member of mathematics department with expectations of coverage) seemed to override her primary orientation to the individual and interpersonal obligations. This is noteworthy because it shows the limitations of the implementation of her orientation to the interpersonal and individual obligations.

**Summary of Dilemma 2**. All three teachers encountered dilemmas where they had to make instructional decisions about lesson content goals and mathematical practice goals. The three teachers encountered very different scenarios in this second dilemma. In Mrs. Stewart's discussion, she encountered a scenario that compelled her either to continue pressing mathematical practice or to model directly and focus on lesson-specific content goals. Her choice was illustrative in showing her attention to mathematical content, when her students collectively seemed to be unable to access a problem. Mr. Sandberg's attention to maintaining an orderly and structured classroom environment presented itself in his second dilemma type. He grappled with the benefits and limitations of using group work as a mechanism to support students prior to a discussion. His choices all pointed to efficiency and structure, consistent with his orientation to the institutional obligation. Ms. Mitchell struggled to maintain goals for engaging students in mathematical practice, given the time constraints and coverage expectations at the end of the school year. Her decision, to subvert mathematical practice over content coverage, was the first instance of a teacher making a choice that was inconsistent with her primary orientation to her professional obligations. This choice showed the strength of the institutional obligation, and how time constraints can factor greatly into a teacher's decision.

### Dilemma 3: Lesson Specific Content Goals Conflicting with Establishing a Mathematical Community Goals

This third type of instructional dilemma occurred when teachers encountered situations where they had to choose between lesson specific content goals and goals focused on establishing a community of mathematical learners. Given a teacher with an orientation towards the disciplinary obligation, one might expect to see the teacher choosing lesson specific content goals over goals for establishing a mathematical community, taking into account this orientation's focus on mathematical content. In contrast, a teacher with obligations to the interpersonal or individual obligations may tend to prioritize establishing the community aspects of the classroom. A teacher who orients primarily toward the institutional obligation may prioritize lesson specific content, given the expectations of student learning and being assessed on content.

As an overview of this dilemma (details in Table 7 below), all three teachers made decisions when faced with instructional dilemmas that were consistent with their primary orientation to their professional obligations. Mrs. Stewart and Ms. Mitchell encountered this dilemma when a student asked an interesting question (in Mrs. Stewart's case) or posed an incorrect answer (in Ms. Mitchell's). In both cases, the teachers were careful about how they responded, mindful of how their responses might impact their goals for establishing a mathematical community. Mr. Sandberg encountered this dilemma during the last few minutes of class as he chose how to best use the few minutes remaining, balancing student participation and the need to summarize the lesson.

Table 7
*Dilemmas between Lesson Specific Content Goals and Establishing a Mathematical Community Goals*

| Teacher (typical orientation to professional obligations) | Dilemma | Decision |
|---|---|---|
| Mrs. Stewart (*disciplinary*) | How to appropriately respond to a question about representing probability with non-whole numbers | Describe how context matters (probability or general case) for whether that representation would be appropriate |
| Mr. Sandberg (*institutional*) | How to include student participation and having enough time to summarize | Limit participation to ensure adequate time to summarize |

Table 7 (cont'd)

| | | |
|---|---|---|
| Ms. Mitchell (*interpersonal* and *individual*) | How to respond to a student that gives an incorrect answer during the discussion | Have other students explain their thinking to the student and the class |

**Dilemma 3: Mrs. Stewart**. During one of the first problems involving probability, Mrs. Stewart asked her students to determine the probability of choosing an individual block given 3 blue blocks, 6 yellow blocks, and 9 red blocks. One student stated 1/18. Another student asked if she could state the probability as 0.5/9. One could imagine several choices about how to proceed when asked this question, ranging from a brief answer and moving on to deeply pursuing the student's question. Mrs. Stewart paused for a few seconds to think about this question before responding, deviating from the typical rapid pace of instruction that often occurred in her instruction.

What is interesting about this particular scenario is its similarity to the first dilemma that Mr. Sandberg and Ms. Mitchell encountered. Specifically, a student asks a question that has the potential to deviate the conversation into possibly interesting mathematics. A teacher who generally oriented to the institutional obligation may choose to quickly move the focus of the discussion back to lesson specific content goals to make sure that the flow of the discussion is not unduly interrupted. A teacher who focuses on the interpersonal or individual obligation may stop the class to have students discuss the question in their groups before resuming the discussion.

Given Mrs. Stewart's general orientation to the disciplinary obligation, her response was to tell the student she could write ${}^{0.5}/_{9}$ but that in the context of probability they should use integer values. Her response also included how they would use these values in the class versus how a mathematician might respond. This response attended to both the lesson specific goals of understanding how to appropriately represent probability numerically, while also communicating to the student the value in their thinking – an important aspect of establishing a mathematical community in the classroom. In the video-stimulated interview after the first round of observations Mrs. Stewart stated,

> There's still some weird things about that [the question]. And I don't know if she's thinking in terms of a probability 'Could that happen?' or just in general in math world, but I did want to make sure we tied it back to the probability of what that would mean. (Mrs. Stewart, video-stimulated interview, May 7, 2015)

Her reasoning for why she chose to handle the question during the discussion was consistent with her general orientation to focusing on the disciplinary obligation.

**Dilemma 3: Mr. Sandberg.** Mr. Sandberg encountered this dilemma during the end of a class period, when he was trying to get students to participate in sharing their solutions to a compound probability problem. The problem asked students to collect data to determine the likelihood of getting a "match" (heads-heads or tails-tails) when flipping two coins. Students had earlier in class predicted if a match was more, less, or about as likely as no-match (one each of heads and tails as the result). Mr. Sandberg was hoping to have students discuss their thinking about what they thought about their results compared to their predictions.

A teacher who prioritized the disciplinary obligation might not care how the data was presented on the board, leaving as much time as possible for a discussion of the results. Mr. Sandberg originally had students filling in the data on the board, but later called on students directly so that he could more efficiently put the values on the board for them. Running short on time, he described to the class why they might expect to see about 50% of the resulting coin flips as a match. He then quickly summarized the main points of their inquiry.

During the post-lesson interview I asked Mr. Sandberg about this exchange and he said, "I want them to be able to do that, just one tiny thing like that [putting numbers on the board]; they love it, and they buy in more, sometimes you have to cut 'em off, but it was simply taking too long." In this case, time constraints created the dilemma of whether to make an instructional decision that would support the community aspect of the classroom versus using the time to summarize what the class had determined and ask questions about the data. In this case, Mr. Sandberg chose to ensure he got through the end of the lesson as prescribed in the materials. This decision was consistent with his general orientation to prioritizing the institutional obligation, because of the need to complete the lesson and continue to stay on pace with the unit.

**Dilemma 3: Ms. Mitchell.** Similar to Mr. Sandberg's third dilemma, this dilemma took place as students in Ms. Mitchell's class explored a problem on compound probability. In this lesson, students rolled two six-sided dice and found the product of the two values. They did several trials. One student won if the product was even; the other student won if the product was odd.

Ms. Mitchell asked the class if they thought the game was fair. One student, Jonathon, said the game was fair because there are 3 odd outcomes and 3 even outcomes per die. This is incorrect since the outcome is based on the products of the two values, not just the individual values appearing. Ms. Mitchell was faced with how to appropriately respond to the student given that this particular student often has difficulties in the class. On the one hand, Ms. Mitchell needed to advance lesson-specific content goals, helping students see the inherent unfairness to this game. On the other hand, if her response was to be consistent with trying to establish a mathematical community, she would need to be careful about how she responded to a student who typically has difficulties in the class. A teacher with a general orientation to the disciplinary obligation may have pressed the class (or Jonathon) to see why he was incorrect. She also could have simply stated that Jonathon was incorrect, or called on a higher-status student in the class to explain why he was incorrect.

Instead, Ms. Mitchell supported Jonathon's thinking, and acknowledged how he was reasonable (in some way) in thinking that the game could be fair. During the discussion she had a student (Josh) sitting near by talk to Jonathon about the odd number times odd number case, and later asked for other students' thinking about the problem. She described this scenario during the post-lesson interview:

> Jonathon, you know, he flat out fails; he doesn't have a whole lot of perseverance intellectually. He thinks of things sometimes really in strong ways, but I was happy that he was participating, and is in fact that half the numbers are even and half the numbers are odd. So I wanted to, you know, honor that he said, and then, but also try to build on that and then I felt like Josh kind of jumped ahead and wasn't real clear about odd times odd. So rather than just go there, and have him or someone explain how he knew that, I felt like the whole group would kind of get lost, so I just stated it very simply after he said it so people could think about it. Seemed to me there was still a question about the odd times odd. And they were all just kind of letting it go… (Ms. Mitchell, video-stimulated interview, May 11, 2015)

The decision Ms. Mitchell made when faced with an incorrect student solution was consistent with her attention to the individual obligation when faced with how to attend to lesson-specific content goals and goals for establishing a mathematical community.

**Summary of Dilemma 3.** All three teachers encountered dilemmas where they had to navigate between lesson-specific content goals and goals for establishing a mathematical community. In all three cases, the teachers considered how they would continue to meet the needs of students to ensure that participation continued during whole-class discussions. This dilemma highlights the importance that teachers placed on establishing an environment where students felt welcome to contribute to the whole-class space, something that does not simply automatically occur but must be carefully cultivated. Mrs. Stewart was able to contextualize representing numbers by bringing attention to different communities and content ideas. Ms. Mitchell attended strongly to the individual and interpersonal obligations by allowing other students in the class to support a student with an incorrect answer. Mr. Sandberg ran into time constraints, and limited student interaction in order to summarize the main ideas from the lesson. All three teachers attended goals that were consistent with how they were each generally oriented to their professional obligations.

### Dilemma 4: Broad Content Goals Conflicting with Mathematical Practice Goals

This fourth type of instructional dilemma occurred when teachers encountered situations where they had to choose between broad content goals and mathematical practice goals. This dilemma only occurred with two of the three teachers. Given a teacher with a primary orientation to the disciplinary obligation, one might expect to see this teacher attempting to balance these two goals, with content possibly being favored over practice. Given a teacher with a primary orientation to the individual or interpersonal obligation, one might expect to see this teacher prioritize mathematical practice as these practices often include aspects of communication and students working with their peers. Given a teacher with a primary orientation to the institutional obligation, one might expect to see prioritization of broad content goals, given the institutional expectations of content based standardized assessments.

As an overview of this fourth dilemma (more details in Table 8 below), only two of the three teachers encountered this dilemma. Mrs. Stewart encountered this dilemma while she considered how much scaffolding to provide students via instructional resources in preparation for their collaborative discussion. Ms. Mitchell encountered this dilemma during planning while she considered how standardized assessments interfered with opportunities to support students in being more proficient in mathematical practice. Both teachers' decisions were consistent with their primary orientation to their professional obligations.

Table 8
*Dilemmas between Broad Content Goals and Mathematical Practice Goals*

| Teacher (typical orientation to professional obligations) | Dilemma | Decision |
|---|---|---|
| Mrs. Stewart (*disciplinary*) | How much scaffolding to provide students via instructional resources (handouts) prior to a discussion | Standardize scale & axes on graph paper so that students can communicate with each other more effectively |
| Mr. Sandberg (*institutional*) | No Dilemma Present | No Decision Present |
| Ms. Mitchell (*interpersonal* and *individual*) | How to balance practice goals with expectations to prepare students for standardized assessments | Focus on practice but try to prepare students for assessments as best as possible |

**Dilemma 4: Mrs. Stewart**. This fourth dilemma type occurred for Mrs. Stewart while she was planning for a class discussion on a problem where students analyzed data from different roller coasters. The lesson focused on the differences between categorical and numerical data, and measures of central tendency. Part of the data to be used in the discussion was a comparison

of different data sets (responses from Internet data and a fictitious seventh-grade class). Mrs. Stewart was faced with the dilemma of having students make their own bar graphs from scratch (i.e., just provide graph paper), or to provide more consistent scaling and axes on the graph. On the one hand, providing students with very little support provides students with the opportunity to ask important questions about graphing, such as which variable is independent (and dependent), and what scale to choose. Providing less information may prove useful to students' long-term understanding of how to create a graph on their own. On the other hand, providing consistent scales and axes to all students limits students' opportunity to consider these questions about graphing, but provides students with greater opportunity to clearly communicate their mathematical ideas with each other about the questions in the lesson. Choosing the graph paper with scales and axes would support students in engaging in mathematical practice.

A teacher who is generally oriented to the institutional obligation might have seen the value of attending to the broad-content goal of supporting students in knowing how to correctly set up a graph. In contrast, Mrs. Stewart chose the handout with the scale and axes provided. In the post-lesson interview, she noted that she had provided this handout because (a) her students are "notoriously slow" in making certain types of graphs, and (b) having consistent scales and labelled axes would allow students to compare and contrast solutions from different groups more readily. One of her goals in facilitating a discussion during this lesson was for students to understand the difference between categorical and numerical data – a topic that spans across several lessons (hence the categorization of this as a "broad content goal"). Mrs. Stewart rarely handed out supporting worksheets during her lessons, but felt that they were necessary for this lesson even though it effectively limited students engaging in thinking about how to best represent the data, how to label the axes of the graph, and which scales to use. Mrs. Stewart chose not to let students engage in the practice of modeling (representing) to serve a broad-content goal that she identified as problematic for students. Given her primary orientation to the disciplinary obligation, it was likely that any decision she made would have been consistent with this obligation. Her decision to focus on content over practice was consistent with her general orientation.

**Dilemma 4: Ms. Mitchell.** This fourth dilemma was discussed during a post-lesson interview with Ms. Mitchell, during the first of her observations. Thinking about her planning for discussions, she talked about the institutional expectations of preparing her students for standardized assessments (both departmental and state-wide). These assessments factor into broad-content goals because in many ways they describe the content that various stakeholders view as important for students to learn. Given the high value that Ms. Mitchell placed on mathematical practice, these standardized assessments created a recurring dilemma as she planned for collaborative discussions.

During the interview she talked about how her vision of effective collaborative learning differed from many other administrators whom she had known. In her view, students discussing mathematics and "putting up with lots of disequilibrium" and "being very patient" were positive traits, but other administrators viewed a teacher negatively for "not scaffolding appropriately." While she did not question her focus on process standards, she often felt constrained by not being able to pursue interesting student ideas as deeply because this choice might not prepare students adequately for departmental or state-wide tests.

Given absolute freedom to teach as she felt best, she would have tailored much more of her instruction (including discussions) to individual students' insights to further encourage their engagement in mathematical practice. This differed with a teacher who might have felt more oriented to the institutional obligation, as that teacher might feel the need to make decisions on how students will best meet expectations for standardized assessments. This dilemma highlighted Ms. Mitchell's primary orientation to the individual obligation, yet also showed how the institutional obligation played an important role in constraining her decision-making via the tension between teaching for content as mandated by outside voices and trying to encourage her students to engage in mathematical practice.

**Summary of Dilemma 4.** Overall, both Mrs. Stewart and Ms. Mitchell encountered this dilemma of how to ensure students are engaging in mathematical practices while also attending to either long-term or unit-level content goals. Whereas Mrs. Stewart's dilemma appeared in considering scaffolding via instructional resources, Ms. Mitchell's occurred more in planning, thinking about the broadness of assessment and its impact on being student-centered in class discussions. Both teachers navigated these dilemmas consistent with their typical orientations to their professional obligations.

### Dilemma 5: Broad Content Goals Conflicting with Establishing a Mathematical Community Goals

This fifth type of instructional dilemma occurred when teachers encountered situations where they had to choose between broad content goals and mathematical practice goals. Given a teacher with a primary orientation to the disciplinary obligation, one might expect to see this teacher prioritize broad content goals since the disciplinary obligation primarily focuses on content-based understanding. In contrast, given a teacher with a primary orientation to the individual or interpersonal obligation, one might expect to see this teacher prioritize goals for establishing a mathematical community considering the focus on social interaction. Given a teacher with a primary orientation to the institutional obligation, one might expect to see prioritization of broad content goals, taking into account the institutional expectations of content based standardized assessments.

As an overview of this dilemma (more detail provided in Table 9 below), only two of the three teachers faced this dilemma. Ms. Mitchell encountered this dilemma when her class failed to reach consensus on determining whether a set of graphs represented linear relationships. Mr. Sandberg encountered this dilemma as he wrestled with the benefits and limitations of supporting student talk by first letting them work in groups. Both teachers made choices consistent with their interpersonal obligation, which was consistent with Ms. Mitchell's primary tendency, but inconsistent with Mr. Sandberg's.

Table 9

*Dilemmas between Broad Content Goals and Establishing a Mathematical Community Goals*

| Teacher (typical orientation to professional obligations) | Dilemma | Decision |
|---|---|---|
| Mrs. Stewart (*disciplinary*) | No dilemma present | No dilemma present |
| Mr. Sandberg (*institutional*) | Use or not use group work to support student thinking prior to a discussion | Continue on with group work so that students feel better able to share their thinking in the discussion |
| Ms. Mitchell (*interpersonal* and *individual*) | What to do when a lack of consensus occurs on an important topic spanning multiple lessons (linearity) | Let students state their ideas, and press them to describe their ideas more clearly |

**Dilemma 5: Mr. Sandberg**. Mr. Sandberg identified this fifth dilemma during an interview, regarding how he grouped students to prepare for a whole-class discussion. He mentioned that when students get into a group for the first time, it often takes them a few days before they are willing to talk with each other about prior content knowledge. His overall goal with group work was to help establish a mathematical community by supporting students in

being able to discuss their thinking more effectively (both in group work and the discussion). He acknowledged, however, that an important aspect of group work that was missing was being able to connect back to prior work – an aspect of broad content goals. A teacher with a general orientation to the disciplinary obligation would likely see group work (in service of preparing students for a whole-class discussion) as supporting students in engaging in mathematical practice. Although Mr. Sandberg's primary orientation to the institutional obligation (namely maintaining a well-ordered classroom) likely would have resulted in not pursuing group work, his choice to structure group work as best he could is more consistent with a strong orientation to the interpersonal obligation.

**Dilemma 5: Ms. Mitchell.** This fifth dilemma occurred during a discussion in the walkathon problem, where Ms. Mitchell was asking students which of the three graphs were linear. The graphs (Figure 12) represent money that fictitious students (as provided in the textbook) earned in a fundraiser for how many miles they walked. Note that all three are linear; two have non-zero slopes, one of which intersects at the origin while the other does not. Students discussed which (if any) of the graphs represented linear relationships. Understanding linearity is a broad-content goal, one that spans multiple lessons. The dilemma took place when she asked the students whether the graphs were linear and very little consensus occurred.

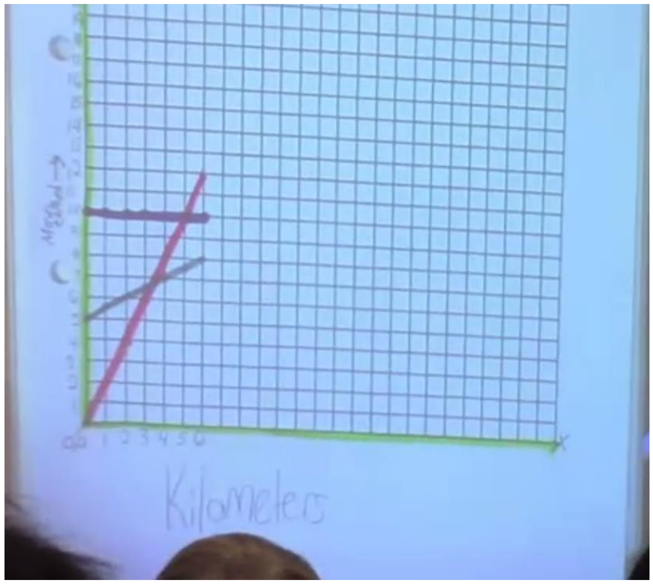


*Figure 12*. Students fail to come to consensus on determining which graphs are linear

A teacher primarily oriented to the institutional obligation (concerned primarily with preparing students for standardized assessments) may have chosen to be more direct in highlighting the differences between the graphs and focusing the conversation on what makes a graph linear. Instead, Ms. Mitchell chose to let students come to their own understanding of linearity by restating different students' ideas of whether they thought each of the graphs was linear or not. Although this choice may have confused some students during the lesson, it was consistent with her focus on mathematical practices and, by presenting non-evaluative comments, she supported her goals for establishing a mathematical community. During the video-stimulated interview, I asked Ms. Mitchell about this part of the discussion and she commented:

> Yeah, I recognize that moments like those are always difficult because what I want to do as a teacher of course is just clarify it. You know, just say, oh no these are all linear, see. And then, let's make this table and okay, let's make this table, let's make this one, and see they're all going up even; they're all linear. But I don't think, I mean I have multiple goals

> in the classroom; one of them is that they become a learning community and they listen to each other. And that one of the ways that they learn is by comparing their thinking against each other so really whether, I mean think, whether something's linear or not is very, very simple concept. And so I think I can easily kind of let them go with a lot of misconceptions about that for quite a while and practice this, well we have some uncertainty and I'm not always going to tell 'em which is important. (Ms. Mitchell, video-stimulated interview, April 1, 2015)

In this case, Ms. Mitchell chose to structure the discussion in a way that emphasized the class becoming more of a mathematical community rather than simply making the unit-level goal of understanding linear relationships be the main goal. Her decision was consistent with her general orientation to her interpersonal obligation.

**Summary of Dilemma 5.** Mr. Sandberg and Ms. Mitchell both encountered dilemmas related to broad content goals conflicting with goals for establishing a mathematical community. Both teachers made decisions consistent with their interpersonal obligation, highlighting the importance of establishing a community of learners as a somewhat necessary condition to be able to facilitate whole-class discussions. This tendency to favor the interpersonal obligation was consistent with Ms. Mitchell's general orientation, but was inconsistent with Mr. Sandberg's general orientation. This inconsistency in Mr. Sandberg's practice highlighted the limitations of his tendency to favor the institutional obligation over the others.

**Dilemma 6: Mathematical Practice Goals Conflicting with Establishing a Mathematical Community Goals**

This sixth and final type of instructional dilemma occurred when teachers encountered situations where they had to choose between mathematical practice goals and goals for establishing a mathematical community. All three teachers encountered this type of dilemma. Although all of the lessons observed took place during the second half of the school year, all three teachers continued to describe situations where establishing a mathematical community was important for enacting whole-class discussions. Given a teacher who is primarily oriented to the disciplinary obligation, one might expect to see this teacher focus on mathematical practice goals, as these goals are closer to mathematical content. Given a teacher who is primarily oriented to the interpersonal or individual obligations, one might expect this teacher to try to balance mathematical practice goals with establishing a mathematical community considering that both likely include the teacher focusing on students clearly communicating their mathematical thinking with their peers. Given a teacher who is primarily oriented to the institutional obligation, one might expect to see a focus on mathematical practice goals if these goals are an expectation of the school or department (which was the case at Highland Junior High School).

As an overview of this dilemma (more detail provided in Table 10 below), all three teachers encountered this dilemma. Mrs. Stewart was faced with the dilemma of what to do with an overly quiet class. Mr. Sandberg encountered a student who forgot what he was going to say and had to decide how to respond and when to move on with the discussion. Ms. Mitchell attempted a new talk structure that created management issues. Both Ms. Mitchell and Mr. Sandberg made decisions consistent with their primary orientation to the professional obligations, whereas Mrs. Stewart prioritized the interpersonal obligation over her normal tendency to the disciplinary obligation.

Table 10

*Dilemmas between Mathematical Practice Goals and Establishing a Mathematical Community Goals*

| Teacher (typical orientation to professional obligations) | Dilemma | Decision |
|---|---|---|
| Mrs. Stewart (*disciplinary*) | How to get a generally quiet class involved in a discussion | Avoid random selection as much as possible by calling on students purposefully during discussion |
| Mr. Sandberg (*institutional*) | How long to wait for a student when the student forgets his answer | Wait for a few seconds, then move on with the discussion as planned |
| Ms. Mitchell (*interpersonal* and *individual*) | How to support mathematical practice when a student receives harsh criticism from other students based on their contribution to the discussion | Stop the discussion and remind students about classroom norms |

**Dilemma 6: Mrs. Stewart.** This sixth dilemma occurred as Mrs. Stewart was planning for a class discussion. In one of her lesson plans, she described the difficulties she considered about how to make sure students stay involved in the class discussion given the general quietness in her first period class. It was important for Mrs. Stewart to get as many students as possible involved in the discussion, yet the overall quietness of the classroom created the dilemma of how best to call on students.

If she had been primarily oriented to the institutional obligation, she might have considered calling on students randomly as a management technique to ensure that students were paying attention in the discussion. She mentioned explicitly that she “waffled” on calling students randomly. She was hesitant in using this technique and talked about several other ways she might get students involved without resorting to random call. Her main goal was to make sure that students participated in communicating their mathematics, but she wanted this to come from the students' willingness to participate in a safe environment instead of feeling like she was putting them on the spot. Her decision to avoid putting students on the spot via random selection during discussions was more consistent with the individual obligation than her primary orientation to the disciplinary obligation.

**Dilemma 6: Mr. Sandberg.** This final type of dilemma occurred in Mr. Sandberg's class during the discussion of his bell work exercises on distance-rate-time relationships. During the discussion, a student wanted to ask a question; he was called on and then forgot what he was going to say. The situation created the dilemma of how long to wait for the student to respond. On the one hand, giving the student ample time likely communicates to students that their thinking is valued (supporting the goal for establishing a mathematical community). On the other hand, waiting for the student to remember their thought can interrupt the flow of the discussion due to management issues, thereby limiting others' opportunity to engage in mathematical practices such as communicating their thinking.

Had Mr. Sandberg been primarily oriented to the individual obligation, he might have pressed the student by prompting him so that the student could remember what he was thinking. Instead, given his attention to the institutional obligation and maintaining a structured lesson flow, Mr. Sandberg responded, "We'll come back to you." There was no evidence of returning to the student's thought during subsequent discussions in that lesson. In the video-stimulated interview after Mr. Sandberg's first round of observations, he described this dilemma.

> I try to develop a good atmosphere to keep them, you know raising their hands and answering questions...You don't want to just cut him off and act like it wasn't important, because then he wouldn't raise his hand for a week, or ever; you know they're very

concerned about their social activities right now, it's huge in seventh grade. (M. Sandberg, video-stimulated interview, April 17, 2015)

His acknowledgment of the dilemma played into his decision to move on to the next question to the class. This choice was consistent with his general orientation to the institutional obligation of feeling the necessity to maintain the lesson structure (staying consistent with his management and interaction structures) as opposed to pressing the student further. This decision prioritized mathematical practice over establishing a mathematical community.

**Dilemma 6: Ms. Mitchell**. This last dilemma occurred in one of Ms. Mitchell's lessons where she purposely chose to change the structure of the discussion format to be more student-centered. She was generally dissatisfied with the teacher-to-student interaction pattern that occurred in many of her discussions. Before the discussion occurred, Ms. Mitchell explained to students that they would be interacting directly with each other and that they could talk after the speaker called on the next student or once the speaker had completed their turn. This was much different than the typical structure in her classroom up to this point, where most turns alternated between teacher and student. The reason for this was to support the class in being a more autonomous learning community, as well as for the class to better communicate with each other.

The dilemma took place when, according to Ms. Mitchell, this new structure became unproductive. She felt that two students were "ganging up" on another student by "using a mocking tone." She wanted to avoid the speaker being "fed to the sharks," as she put it, so she intervened in the discussion after being mostly silent. On the one hand, Ms. Mitchell felt the need to support students in being less dependent on her voice to have a discussion (goal for establishing a mathematical community). On the other hand, if students were unable to accomplish this goal it would hinder students' opportunity to engage in mathematical practice.

The dilemma was about how long to support students in learning to interact with each other directly as a mathematical community while ensuring that students felt protected individually in their thinking.

A teacher who is primarily oriented to the institutional obligation might have viewed the desire to help the class be less dependent on the teacher as inefficient, unnecessary, or problematic since this perspective views the teacher as a necessary component to a discussion. Ms. Mitchell's decision was to stop the discussion at the point where she felt the students were no longer talking to each other productively. She described this decision during the post-lesson interview: "[T]hat made me very uncomfortable, so that was what I was sorting through how long to let that go 'cuz the conversation was great; the tone of it was not." Later, she commented that "…if people were being kind I would have let that muck around for a long time…But I would really hold on as long as I could if I felt like everyone was being safe…emotionally and all that." Her decision to end the discussion was consistent with her primary orientation to the individual obligation, prioritizing the need to establish a mathematical community over her mathematical practice goals.

**Summary of Dilemma 6.** All three teachers faced dilemmas related to the tension between having students share their thinking with the class and feeling that they are a member of a mathematical community of learners. This tension shows up at different times based on the student and the type of response they provide. All three teachers tended to prioritize the importance of establishing a mathematical community based on their decisions. While Mr. Sandberg and Ms. Mitchell made decisions consistent with their primary orientation to their professional obligations, Mrs. Stewart did not.

### Subfinding 1

As evidenced by the dilemma results from the previous section, the first subfinding was that teachers' primary orientation to their professional obligations was highly influential in how they made decisions related to whole-class discussions. By-and-large teachers' decisions when faced with instructional dilemmas was consistent with their primary orientation to their professional obligations (see tables 11, 12 and 13 below). In some cases this involved teachers prioritizing one type of a goal over another, in other cases, the decision involved teachers balancing the needs of multiple goals.

### Subfinding 2

The second subfinding was that each teacher encountered one dilemma where they made a decision that deviated from their primary orientation to their professional obligations (see tables 11, 12 and 13 below). These exceptions are noteworthy as they highlight the complexity with which teachers managed their goals and obligations. The subfinding also shows the limitations to how the teachers managed and prioritized their professional obligations in relation to whole-class discussions.

Table 11
*Ms. Mitchell's Dilemmas and Orientations to Professional Obligations*

| Dilemma | Professional obligation emphasized in decision | Was the professional obligation consistent or inconsistent with teacher's primary orientation? |
|---|---|---|
| LS – BC | individual | Consistent |
| LS – MP | institutional | Inconsistent |
| LS – MC | individual | Consistent |
| BC – MP | individual | Consistent |
| BC – MC | interpersonal | Consistent |
| MP – MC | individual | Consistent |

Note: BC: broad content goals, LS: lesson-specific content goals, MP: mathematical practice goals, MC: establishing a mathematical community goals

Table 12

*Mr. Sandberg's Dilemmas and Orientations to Professional Obligations*

| Dilemma | Professional obligation emphasized in decision | Was the professional obligation consistent or inconsistent with teacher's primary orientation? |
|---|---|---|
| LS – BC | institutional | consistent |
| LS – MP | institutional | consistent |
| LS – MC | institutional | consistent |
| BC – MP | no dilemma present | no dilemma present |
| BC – MC | interpersonal | inconsistent |
| MP – MC | institutional | consistent |

Note: BC: broad content goals, LS: lesson-specific content goals, MP: mathematical practice goals, MC: establishing a mathematical community goals

Table 13

*Mrs. Stewart's Dilemmas and Orientations to Professional Obligations*

| Dilemma | Professional obligation emphasized in decision | Was the professional obligation consistent or inconsistent with teacher's primary orientation? |
|---|---|---|
| LS – BC | no dilemma present | no dilemma present |
| LS – MP | disciplinary | consistent |
| LS – MC | disciplinary | consistent |
| BC – MP | disciplinary | consistent |
| BC – MC | disciplinary | consistent |
| MP – MC | individual | inconsistent |

Note: BC: broad content goals, LS: lesson-specific content goals, MP: mathematical practice goals, MC: establishing a mathematical community goals

**Subfinding 3**

The third subfinding of this study was that teachers encountered several challenges that spanned across teachers and dilemma types. Although these challenges were identified more or less in the literature base provided earlier, this dissertation found that these challenges did not depend on a teacher's primary orientation to their professional obligations nor on their particular instructional goals. These three challenges for facilitating whole-class discussions included (a) considering appropriate scaffolding, (b) handling student responses, and (c) managing time constraints. In this section, I provide greater detail on each of these three challenges. Taken together, these three aspects of facilitating discussions provide insight into the various ways that teachers (with different primary orientations) made decisions and thereby provided different opportunities for their students when these challenges presented themselves in whole-class discussions.

**Considering Appropriate Scaffolding**

The first challenge that all three teachers encountered focused on how to appropriately scaffold the discussion. Indeed, learning to talk and listen for the purpose of understanding mathematics can be an unnatural experience for many students. Many students are unwilling to share their thinking with their classmates; and for those who are willing, understanding how to make sense of another student's ideas that differ from one's own ideas can be difficult.

All three teachers encountered various situations of how to appropriately scaffold the discussion. In one case this dealt with instructional resources. Mrs. Stewart considered what type of handout to use in her fourth dilemma (broad content goals vs. mathematical practice goals). She chose to support easier communication between students by providing graph paper already made with scales and axes. Teachers likely encounter this type of dilemma frequently, knowing that there are certain skills students should do proficiently (e.g., knowing how to determine the scale and axes on a graph given a set of data). Yet these skills might go undeveloped exactly for the reason that was viewed through Mrs. Stewart's choice: that there was a more compelling immediate goal that took precedence over the opportunity for students to develop a long-term skill.

Another type of scaffolding challenge took place as teachers considered the structural aspects of whole-class discussions – that is, how one goes about supporting student talk through various organizational structures. Mr. Sandberg encountered this while he considered broad content goals and mathematical community goals (Dilemmas 2 and 5). He wrestled with the benefits and consequences of using group work prior to a discussion. Mrs. Stewart's "quiet" class posed a dilemma (Dilemma 6) of how best to call on students to adequately engage them in mathematical practice while still supporting the development of a mathematical community. She also encountered this challenge (Dilemma 2) when students became perplexed with a problem and showed little ability to start it. Ms. Mitchell encountered this dilemma (Dilemma 6) when she grew dissatisfied with the discussion being filtered through her role as the teacher. She changed the talk structure to be more student-driven, yet ran into issues when students began talking negatively to another classmate.

In all these cases, the teachers considered how to most appropriately scaffold the learning to take place in a discussion. It is not surprising that all of these dilemmas involved mathematical community goals or mathematical practice goals (or both). This shows that while teachers are considering the mathematical content to be learned during a discussion, they are sensitive to helping students learn to talk and listen to others, whether this is done through group work prior to the discussion, through how they call on students, or through the talk structure of the discussion itself.

These scaffolding challenges highlight the fact that these teachers felt it necessary to support students prior to and during whole-class discussions independent of their goals and primary orientation to their professional obligations. This being said, all three teachers made choices that focused on their interpersonal obligation by supporting students in developing mathematical practice and the community aspects of the classroom. Scaffolding the discussion was a necessary component to enacting whole-class discussions.

**Handling Student Responses**

Another important component to facilitating whole-class discussions is how teachers make decisions when students respond during the discussion. Unlike more traditional math classes where the classroom discourse may involve large sections of teacher lecture with minimal student talk (i.e., possibly just one- or two-word responses), facilitating whole-class discussions requires the teacher to listen to students and use what they are saying in some productive way to move their agenda forward. This has the potential to create problems, such as when a student poses an interesting question that goes beyond the scope of the lesson, when a student states an incorrect answer, or when a student simply forgets what he is going to say. This challenge occurred across all three teachers, and across several dilemma types, with no particular pattern in the types of goals across dilemmas.

All three teachers encountered situations where they made decisions when students posed an interesting question. In Ms. Mitchell's class (Dilemma 1) a student considered the non-linearity of another student's graph where the graph was not even a function. In Mr. Sandberg's class (Dilemma 1), a student posed a question that the class would eventually be looking at during a lesson on compound probability. Mrs. Stewart encountered an interesting question as a student wondered whether a probability could be represented using non-whole numbers (Dilemma 3). All three teachers made decisions that were consistent with their primary orientation to the professional obligations, which presented different opportunities for students to engage in mathematics.

During Ms. Mitchell's discussions she encountered situations where students provided incorrect answers. In one situation (Dilemma 3), a student gave an incorrect answer during a compound probability lesson looking at the product of the values on two dice. In a different situation (Dilemma 5), the class failed to come to consensus on which graphs were linear. What is noteworthy about this set of data is that only Ms. Mitchell encountered dilemmas where students provided incorrect answers. This raises the question as to whether Ms. Mitchell's primary orientation to the interpersonal or individual obligation might have supported uncovering misconceptions and incorrect ideas in contrast to the other two teachers. A competing hypothesis might be that Ms. Mitchell's lack of experience with the curriculum and general focus on mathematical practice could also have contributed to the unmasking of student errors.

Finally, Mr. Sandberg encountered a situation (Dilemma 6) where a student simply forgot what he was going to say. Mr. Sandberg quickly moved on with the lesson, but not before considering the implications for moving on without adequate wait time. This highlighted both his tendency to maintain a consistent discussion structure while trying to make sure the student felt comfortable with participating in the future. This situation and the others described in this section all highlight that when teachers open up the discourse space of the classroom, they need to make decisions as students pose interesting questions, share incorrect answers, or simply forget what they are going to say. In all these situations, teachers made decisions that impacted the nature of how students learned mathematics in the classroom, in many ways consistent with the teachers' primary orientations to their professional obligations.

**Managing Time Constraints**

The third type of challenge that teachers encountered focused on how teachers managed time constraints. Mrs. Stewart (Dilemma 1), Ms. Mitchell (Dilemma 2), and Mr. Sandberg (Dilemma 3) all dealt with how to manage time constraints while attending to their instructional goals and their professional obligations. Mr. Sandberg and Mrs. Stewart encountered this challenge towards the end of lessons when they were running short on time. Ms. Mitchell encountered this challenge as she considered what content she needed to cover during the final weeks of the school year. In all three of these situations, one of the types of goals was lesson-specific content. All three teachers made decisions consistent with their primary orientation to their professional obligations.

This challenge highlights the importance of considering decision-making with the very real and constraint-filled dynamics of teachers in their classrooms. These challenges might not have been observed had this dissertation been focused on depictions of other people's classrooms, or if the decision-making had only focused on content-focused reasons. Indeed, the very real nature of these teachers' decision-making was necessarily embedded in their daily interaction with their students with all of the institutional constraints that go along with that interaction, such as managing time both within and across lessons.

## Results Summary

All three teachers had instructional goals of the four types analyzed, and all teachers had at least five of the six theoretical permutations of conflict between pairs of goal types. In some cases, teachers acknowledged a dilemma and had no resolution for it. In many other cases they acted to resolve the dilemma. As teachers encountered instructional dilemmas while planning for and enacting whole-class discussions, their decisions could be determined (for the most part) as a function of their professional obligations. However, each teacher made at least one decision that deviated from their primary orientation. Even though the three teachers had different primary orientations to their professional obligations, they all encountered challenges of how to handle student responses, how to appropriately scaffold the discussion, and how to manage time constraints. In the next chapter, I discuss the importance of these results and their value for research and practice. All teachers encountered various challenges to implementing discussions that, for the most part, were independent of the types of dilemmas. For example, all three teachers considered how to appropriately scaffold the discussion in a productive way. In the next chapter I discuss the relevance and implications of the findings of this dissertation.

# CHAPTER 6

# DISCUSSION AND CONCLUSION

For decades now, researchers, practitioners, and organizations have advocated for the potential benefit of using whole-class discussions in teaching mathematics, yet this vision has not been fully implemented. This raises the question as to why this is. In this dissertation, I analyzed the decision-making of three experienced middle grades teachers to better understand the nature of dilemmas they encountered while planning for and enacting whole-class discussions.

## Summary of Findings

The main finding of this study was that all three teachers' decision-making was explained via their primary professional obligation – with important limitations. This is important because the opportunities for students to learn mathematics in their classrooms are determined in large part by the decisions that teachers make. These decisions are influenced by their orientations to their professional obligations. Along with this main finding, there were also three important subfindings.

First, all three teachers' decision-making were mostly explained via their primary orientation to their professional obligations. In the vast majority of dilemmas, teachers made decisions which were consistent with their primary orientation to their professional obligations. This subfinding may not be surprising as one might have anticipated before this study took place that a teacher's orientations to the professional obligations would explain their decision-making, but the extent to which this was the case was insightful in understanding the relationship between decision-making and the multiple potential factors (e.g., teachers' professional obligations) that contributed to their decision-making.

Second, the three teachers in the study, at times, made decisions which were inconsistent from their primary orientation to their professional obligations (Subfinding 2). Each teacher encountered one dilemma where they actually deviated from their primary orientation. This deviation is noteworthy because it highlights the complexity with which teachers manage their goals and obligations, and because it also shows the limitations to how a teacher manages their professional obligations in the classroom. It also illuminates that while one of the obligations can generally explain how a teacher might make decisions in general, it cannot fully capture the complexity with which teachers may make decisions in their situated experience of classroom instruction. In general, these deviations might occur because atypical situations occur in the classroom (e.g. the class is interrupted multiple times) which may lead a teacher to atypically prioritize one of their obligations over another. More specifically, these deviations may occur for multiple reasons. In the next few paragraphs, I describe these inconsistencies and provide potential reasons why they may have occurred.

Ms. Mitchell was generally oriented to the interpersonal and individual obligations. Yet when she encountered a dilemma between lesson-specific content goals and mathematical practice goals (Dilemma 2), she made an instructional decision more consistent with the institutional obligation. In this scenario, Ms. Mitchell considered how she could incorporate mathematical practice given the content coverage expectations at the end of the year. This decision may have been attributed to (a) Ms. Mitchell's inexperience in teaching the curriculum ensuring that she met departmental expectations, and (b) as someone experienced in teaching within the district recognizing the importance of making sure her students have similar content exposure to their peers in other classes. Ms. Mitchell was in many ways an experienced novice in teaching the curriculum, having taught several years, but not at this specific grade level. Thus, we see in her teaching a confluence of her experience and inexperience in her decision-making. Overall, Ms. Mitchell's dilemma showed the importance of considering institutional constraints while studying decision-making; in this case, they became too great, and Ms. Mitchell found it necessary to cover what was expected instead of focusing more on mathematical practice.

Mr. Sandberg was generally oriented to the institutional obligation. He encountered a dilemma between a broad content goal and establishing a mathematical community (Dilemma 5). In this scenario, Mr. Sandberg was considering how group work was used to support students in the whole-class discussion. Mr. Sandberg felt it was necessary to have students talk in small groups prior to the whole-class discussion, which favored the interpersonal obligation. Even though Mr. Sandberg generally made decisions consistent with maintaining an orderly classroom and discussion flow, in this case, the potential disorder of group work was seen as necessary for students to be able to think through their ideas prior to a discussion.

Mrs. Stewart was generally oriented to the disciplinary obligation. She encountered a dilemma between mathematical practice goals and establishing a mathematical community goals (Dilemma 6). Her dilemma presented itself with how best to call on students, given a very quiet class. Her unwillingness to put students on the spot through random selection was consistent with supporting the individual obligation, making sure students are feeling supported in the classroom environment.

Both Mr. Sandberg and Mrs. Stewart primarily oriented themselves to obligations that were not the individual or interpersonal obligations, yet they made decisions aligned with those orientations. These decisions speak to the importance of establishing a mathematical community where students felt comfortable and capable in discussing their mathematical thinking. In both cases, the teachers may have made these decisions because they felt it was necessary to support the students more directly in supporting them learn mathematical content via whole-class discussions. One could not expect students to share their thinking openly in the class (especially at the middle school level) without feeling at least minimally supported to engage in the public discourse space. While the teacher may try to establish some norms explicitly, many other messages can be sent to students implicitly – based on how a teacher responds to questions or specific decisions they make in the class. Both teachers made decisions that supported developing a classroom atmosphere that would encourage students to this public discourse space.

In contrast, Ms. Mitchell primarily made decisions consistent with the interpersonal and individual obligations. Although her students appeared comfortable in talking with Ms. Mitchell and each other, she was not able to avoid dilemmas, which caused her to deviate from her primary orientation. In the dilemma described above, the institutional demands became too great and she chose to deviate from her typical student-centered decision-making approach. Thus, one conclusion from this subfinding is that attention to social norms for interacting during whole-class discussions was a necessary but not sufficient condition for ensuring that students were able to engage in mathematical thinking during whole-class discussions.

Third, even though the teachers were experienced, used instructional resources (e.g., written curriculum materials) that supported implementing whole-class discussions, and were generally oriented philosophically with the importance of using discussions, this was not sufficient to avoid challenges one might expect to see while facilitating whole-class discussions. The types of challenges these teachers encountered were, by-and-large, independent of the teachers' primary orientations to their professional obligations (Subfinding 3). One might have anticipated prior to this study that particular challenges would be more prevalent given a particular orientation to an obligation, or to a particular type of goal (e.g., that managing time constraints might be less challenging for someone primarily attending to their institutional obligation or lesson-specific content goals). Yet all three teachers encountered situations where they had to consider appropriate scaffolding, how to handle student responses, and how to manage time constraints. Put together, these findings point to how students' opportunities to learn mathematics -- and what it means for them to be thinkers and doers of mathematics -- were impacted to some degree by their teachers' decisions related to whole-class discussions. In the next sections, I describe the implications of these findings for research and practice.

## Discussion

Prevailing models of mathematics teachers' decision-making (e.g., Schoenfeld, 2010) have focused strong attention on teachers' content-based decisions and how they impact the flow of discussion. In this dissertation, I found that teachers make decisions for a multitude of reasons, but those reasons can be consistently described via the professional obligations (with some limitations) situated in the authentic experience of their classrooms. By understanding teachers' primary orientations to their professional obligations, one may be able to explain why certain behavior is occurring (or not) during instruction. This may be helpful for supporting teacher learning both pre-service and in professional development. This dissertation contributes to our understanding of teacher decision-making through the investigation of three different (with respect to their primary orientations to their professional obligations) teachers in the highly complex and situated environment of whole-class discussions.

This dissertation reinforced others' perspectives in the field that the teacher's role in facilitating discussions is centrally important for moving along mathematical content and motivating students (Chapin, O'Connor, & Anderson, 2009; Lampert, 2001). Similar to Lampert's (2001) explication of her teaching practice using whole-class discussions, the teachers in this study encountered many pedagogical demands simultaneously, such as keeping the discussion on track, engaging non-verbal students, deciding which students to call on, and choosing when to move on from an idea and when to press further.

As teachers facilitate discussions, the dilemmas they encounter and the decisions they make send implicit messages to students about what is valued as thinkers and doers of mathematics. These messages are important because as Jansen (2006, 2008) pointed out, students participate in discussions for various reasons, yet student participation is a necessary component of teaching using whole-class discussions. All three teachers in this study encountered dilemmas that required them to consider the importance of establishing a mathematical community as a necessary prerequisite to accomplishing other content-specific goals. Although some research (e.g., Sleep, 2012) has noted how non-mathematical aspects of instruction could potentially detract from accomplishing mathematical goals, in this study all three teachers found it necessary to support students in what Williams and Baxter (1996) referred to as *analytic* (scaffolding students' engagement in mathematical ideas) and *social scaffolding* (scaffolding students' expectations regarding norms of social behavior and talk).

Perhaps more importantly, Skott (2004) pointed out in reference to *reform*-minded instruction that there are "expected classroom practices and learning outcomes…but there is no set of well-defined methods for the teacher to carry out and only vague hints as to what kind of practice a certain situation may require" (p. 239). While the last decade has seen support grow (e.g., Smith and Stein's *5 Practices*) for this discourse-oriented approach to teaching, there is still much work to be done. The findings of this dissertation point out the largely unavoidable challenges that teachers are likely to encounter while planning for and enacting whole-class discussions. Filling the void that Skott pointed out may include methods for how best to support teachers in understanding the benefits and limitations in handling student responses, scaffolding the discussion, and managing time constraints while planning for and enacting whole-class discussions. Another well-defined method might include helping teachers understand their tendencies when making decisions during instructional dilemmas in favoring a particular professional obligation and goal type. Supporting teachers in making their practice explicit to themselves might result in both better teaching practice and better support for novice teachers should they be mentored by these teachers.

## Implications for Research and Practice

This study informs research on whole-class discussions and teacher decision-making. Although the results of this study are limited by the focus on teacher decisions related to whole-class discussions (and thus not the entire lesson cycle), there are some notable implications from this study for research. In relation to whole-class discussions, the findings of this study indicate that there were several challenges teachers encountered that were independent of their orientations to their obligations and their goals. Studying teachers as they navigate discussions in the realness of their classroom practice provided the opportunity to understand instructional dilemmas and the challenges they encountered. Research on whole-class discussions has yet to consider how teachers' various orientations to their obligations influence the learning experiences of their students in their classrooms. This dissertation found that the ways teachers scaffolded the discussion, structured the discussion, handled student responses, and managed time constraints were all generally explained via their orientations to their professional obligations. This study provides a first step in developing a theory of how teachers meet the demands of the challenges of facilitating whole-class discussions as influenced by their orientations to their professional obligations.

In relation to teacher decision-making, this dissertation highlighted the importance of teachers making decisions as necessarily embedded in the institution of the school with students they interact with daily. Prevailing models of teacher decision-making (e.g., Schoenfeld, 2010) have focused on the cognitive aspects of teachers' content-focused decisions. Current decision-making research and support for facilitating whole class-discussions ignore this aspect of teaching: that teaching is fundamentally an interaction between human beings in a social space over long periods of time, which therefore impacts the ways in which people interact.

Facilitating collaborative discussions is complex, even when experienced teachers are generally oriented to teaching using this method of instruction, they have classroom resources (e.g., curriculum materials) that support this type of instruction, and they are generally knowledgeable about this teaching approach. Yet if teachers are going to effectively implement collaborative discussions, the question then is how does one learn to do this?

For decades now, the NCTM and others have advocated for the use of whole-class discussions. Improvement has happened no doubt, but many teachers still teach in a direct-instruction way with limited student talk, or talk that is not consistent with the vision of discourse-oriented instruction. I argue that what is necessary to support this vision is a better understanding of the real constraints, dilemmas, challenges, and concerns of real teachers in the complex space of teaching mathematics in today's classrooms. This dissertation highlighted these characteristics of practice. Although these three teachers are not representative of the population of mathematics teachers across the United States, their stories provide some insight as to why supporting teachers in teaching this way is so challenging while suggesting potential pathways to move the field forward.

Before describing the implications of the results in terms of practice, I remind the reader that the purpose of this study was not in any way to measure effectiveness of implementation, or to make claims about how one teacher "got it right" and the others were somehow deficient. This perspective would be unproductive for understanding how different teachers make sense of their role in facilitating discussions. What this study did illuminate was that teachers' decisions could be understood via the types of dilemmas they encountered and their orientations to the professional obligations. Given any set of teaching practices that one would consider "productive" in supporting students to more deeply understand mathematics via a discussion, these teachers provide some insight as to the potential obstacles and challenges one might expect to see.

As one example, an outside observer may see the mathematical importance of pressing students to justify a mathematical claim they make during the discussion. Assuming this fails to happen, a teacher educator or professional development coach might wonder why this was. While a simple explanation is that the teacher missed the opportunity, this study illuminates several alternative reasons. Perhaps the teacher felt the student would be unable to make a justification, and they are considering the implications for that individual student's development (attending to the individual obligation). Perhaps the teacher wanted to leave the question open and they plan on circling back to the question later (attending to some broad-content goal). Perhaps the teacher felt pressed for time and unable to pursue this idea further (trying to manage the challenge of time constraints). Indeed, a teacher's decision not to press students during a lesson (even if they are convinced it is useful during planning) could be influenced by their goals, obligations, or one of the challenges described earlier in this chapter. By considering these other possibilities, and by trying to uncover the root of the decision, the field may be better positioned to support teachers in making effective decisions during whole-class discussions via understanding the reasons why they acted the way they did.

By better understanding teachers' these underlying reasons, teacher educators and professional development providers may be able to counteract any potential mismatch that happens as the teacher is learning in a professional setting. This mismatch may occur when a teacher is oriented primarily one way (e.g., to the institutional obligation) and the PD provider/teacher educator is primarily oriented another (e.g., to the disciplinary obligation). One could imagine a setting where a PD provider is describing the benefits of justification to teachers, while the teachers are considering when they would have time to incorporate this into their classroom. By understanding these different orientations, PD providers and teacher educators may be better positioned to effectively change teachers' practice and possibly orientations to their practice.

As one such specific example of how these results might be useful on a large scale, consider Smith and Stein's (2011) *Five Practices for Orchestrating Productive Discussions.* These practices include *anticipating* student strategies, *monitoring* students as they work in groups on a task, and finally *selecting, sequencing,* and *connecting* student strategies throughout the discussion. Taken together, these practices can support novice teachers in reducing the cognitive load of a lesson by shifting much of the thinking to the planning phases. However, these strategies are designed to support teachers in addressing their disciplinary (content-focused) obligations. For example, selecting and sequencing practices could involve having students consider multiple representations (e.g., table, graph, symbolic) and how one might order certain strategies to create a coherent storyline, or whether to select an underdeveloped strategy for presentation compared to a more abstract and generalizable one.

The results in this study highlight the different types of instructional goals that teachers had, and the inherent challenges that go along with facilitating such discussions. Thus, the choices teachers make when implementing the *5 Practices* (Smith & Stein, 2011) likely go beyond just addressing content-focused goals. For instance, a teacher's selecting and sequencing practices could be influenced by a teacher's orientations to the interpersonal and individual obligation, affecting who they called on and in what order.

## Future Research

In terms of future research, it is still an open question as to the extent to which teacher decision-making during whole class-discussions is different or not based on a teacher's experience. The teachers in this study were all experienced teachers who used instructional resources (i.e., written curriculum materials) that generally supported whole-class discussions, and who were generally aligned philosophically with this approach to instruction. This raises the question, however, about how this study might have been different had it included novice teachers. As one example, when Ms. Mitchell considered how to deal with meeting the expectations of content coverage during the final few weeks of the school year, she deviated from her primary orientation to the interpersonal obligation and instead made a decision consistent with her institutional obligation. As novice teachers are learning their role and professional identity in the classroom, they too might bow to institutional expectations.

APPENDICES

APPENDIX A

Iterations of Consistent Patterns of Instruction Coding

**First Pass through the Classroom Observation Data**

As a first pass through the data, I watched 1 set of videos from the three teachers with the goal of determining a general sense of their decision-making during whole-class discussions. To do this, I first watched the set of videos that were independent of the other teachers: Mrs. Stewart - Round 1, Ms. Mitchell – Round 3, Mr. Sandberg Round 2; that is, there was no overlap between any of the three teachers in these particular lessons. Although this means that the first pass through the data is not in chronological order (except for Mrs. Stewart), it is assumed that due to the years of experience of each teacher their decision-making processes did not evolve drastically from one set of observations to the next. Doing this also ensured that three videos each would be watched for this first set. Attention was only given to segments of the video that were considered "whole-class discussion." To help mitigate issues of analytic contamination across teachers, the lessons were watched in chronological order (lesson before focus lesson, focus lesson, lesson after focus lesson) but rotated across teachers. Given below is the order I analyzed this first set of videos:

1. Mrs. Stewart – Round 1, Lesson 1 of 3 – What Do You Expect (*CMP3*) Problem 2.1
2. Mr. Sandberg – Round 2, Lesson 1 of 3 – What Do You Expect (*CMP3*) Problem 1.2
3. Ms. Mitchell – Round 3, Lesson 1 of 3 – Data Distributions (*CMP2*) Problem 1.1
4. Mr. Sandberg – Round 2, Lesson 2 of 3 – What Do You Expect (*CMP3*) Problem 1.3
5. Ms. Mitchell – Round 3, Lesson 2 of 3 – Data Distributions (*CMP2*) Problem 2.1
6. Mrs. Stewart – Round 1, Lesson 2 of 3 – What Do You Expect (*CMP3*) Problem 2.2

7. Ms. Mitchell – Round 3, Lesson 3 of 3 – Data Distributions (*CMP2*) Problem 4.4
8. Mrs. Stewart – Round 1, Lesson 3 of 3 – What Do You Expect (*CMP3*) Problem 2.3
9. Mr. Sandberg – Round 2, Lesson 3 of 3 – What Do You Expect (*CMP3*) Problem 1.4

## Pacing and Lesson Structures

A consistent feature across the 3 teachers was that each used an instructional technique/consistent pattern of behavior to keep the flow of the discussion going (i.e., there was a productive "rhythm" to the lesson flow). These consistent patterns of behaviors differed in their form but served similar purposes – to keep the lesson moving while allowing students' thinking about the mathematics to be made visible for the whole class.

In Mr. Sandberg's classroom, there were four consistent major lesson segments. When students arrived to class they typically worked on "bell work," which consisted of problems related to the previous day's lesson. The problems were typically formative in nature. Problems often targeted common mistakes or misconceptions students would make related to the previous day's Problem. After students had an opportunity to work independently on these problems, the second lesson segment was a class discussion of the bell work. The next lesson segment was more focused on direct instruction that launched the problem. In contrast to Mrs. Stewart, Mr. Sandberg typically worked through the first part (i.e., "Part A") of the Problem with the class. The final segment of the class involved students working on the task in groups. In contrast to both Mrs. Stewart and Ms. Mitchell, Mr. Sandberg infrequently summarized the day's lesson during the same period. Instead, the summarizing of main ideas usually occurred during the next day's discussion of the bell work. Whole-class discussions almost exclusively occurred during the second lesson segment. Occasionally, a brief summary occurred at the end of class, but these discussions were generally shorter.

One main advantage to Mr. Sandberg's lesson structure was that it provided more processing time between identification of key issues and understandings and when these ideas were discussed in class. This approach also provided opportunities to share key insights across all classes investigating the same problem. Additionally, by waiting until the next day to discuss the previous day's work, the early discussion in the class served as a way to create coherence across lessons. The main limitation was that the discussion generally focused on problems from the bell work, which tended to be more procedural in nature. While Mr. Sandberg employed different techniques to get a variety of students involved in the discussion, the discussion format was generally in the scope of IRE or brief response to a procedural type problem. While Mr. Sandberg was able to get many students involved in the discussion, the involvement seemed to be more typically at a low level of conceptual engagement.

The class observed for Ms. Mitchell was the final class of the day. Ms. Mitchell mentioned how different this class was in terms of its makeup of students (more racial/ethnic and SES diversity and issues related to "fitting in" with school culture) and in relation to students' ability to stay on task – perhaps due to the class being the final class of the day. Ms. Mitchell was also in her first year teaching middle grades, which when combined with other factors may have contributed to her daily lesson structure being more varied (i.e., it appeared somewhat "experimental" at times) than Mr. Sandberg's and Mrs. Stewart's. This being said, discussions typically took part during two well-defined lesson segments, at the beginning of the lesson – either to wrap up or review the previous day's lesson -- and towards the end of the lesson to summarize key ideas from the lesson's exploration.

In contrast to the other two teachers, Ms. Mitchell tended to rely on a small group of students to keep the conversation going across lessons. This core group of students engaged in conversations mediated through the teacher, as well as with each other in the whole-class setting. This is not to say that students were excluded from the conversation; rather, she capitalized on strong students who were willing to participate and take on a leadership role for the collective learning. This small group of students showed evidence of willingness to share their underdeveloped thinking with the class, and to defend their reasoning. She was generally supportive of students' thinking, whether this was by explicitly stating she valued their thinking or by using instructional moves that demonstrated this (e.g., using wait time while students were processing). This general orientation toward valuing thinking and relying on a small group of students as "discussion leaders" proved beneficial in maintaining the flow of whole-class discussions across lessons.

Whereas Ms. Mitchell and Mr. Sandberg both had more well defined lesson segments, many of Mrs. Stewart's lessons had discussions occurring throughout the class period. Students would work on parts of a problem individually or in small groups, and the class would discuss these portions. There usually was a "summary" portion of each lesson that occurred at the end of the lesson, but this differed only minimally from the discussions occurring throughout in terms of the types of student-to-student interaction. Often there were times when student responses seemed to occur simply to keep the conversation going in the class with minimal comment from Mrs. Stewart. For example, multiple students shared their strategy for solving a particular problem, and the teacher response was simply "Okay" before moving on the next student.

There was generally a quick pace to both her questions and what she was trying to accomplish during the lesson, perhaps due to wanting to get through as many subparts of each problem as possible – something that seemed less pressing for both Ms. Mitchell and Mr. Sandberg. There were cases where Mrs. Stewart would cut students off mid-thought and fill in their thinking for them. This being said, there were instances of Mrs. Stewart slowing down her pace, especially during key conceptual parts of the lesson. At these times, her comments were longer, questions were more specific and poignant, and the discussion seemed to be more a conversation and less of a rhetorical interchange. For example, during these points of the lesson Mrs. Stewart would ask for clarification by asking, “What do you mean by…?” and re-stating a student’s strategy or approach to thinking about the problem. Similar to Mr. Sandberg’s class, there were many students who spoke during discussions; however, in contrast to Mr. Sandberg’s class, the contributions tended to vary between procedural and conceptual aspects. Given the consistent discussion in her classroom, the variable pacing of discussion allowed Mrs. Stewart to move through many parts of the lesson, but this may have contributed to masking the big idea of the lesson.

Put together, the three sets of lesson videos (one from each teacher) were useful in providing a general sense of the nature of the discussions, but more importantly they highlighted the individual characteristics within and across the teachers’ discussions. This data provided the foundation for developing a coding scheme that aimed to more precisely describe the nature of teachers’ interactions and inevitably their decision-making. The next few sections describe how this process of developing a coding scheme was responsive to both the data and the research literature pertinent to decision-making as embedded in classroom discussions.

## Second Pass through the Classroom Observation Data

As a reminder, the first pass through the data was focused on all three teachers. The second pass of the data focused just on Mrs. Stewart’s videos because they were the longest discussions and seemed to be more structurally varied both in terms of showing up in different parts of the lesson as well as in having intermingled group work with whole-class work. The process of determining consistent patterns of behaviors for Mrs. Stewart was a primarily grounded process, although many categories were consistent with research on classroom discussions, consistent pattern of behaviors, and teacher decision-making. The process of creating a coding scheme was iterative; first constructing a coding scheme through watching observed lessons, refining the scheme, watching more lessons, and so on until the point of theoretical saturation occurred. The goal of developing a reliable coding scheme was for the researcher (me) to develop a complete coding scheme by the end of the 2nd round of observations so that reliability coding could be determined in the 3rd round by a second coder. This process of creating the coding scheme is described in greater detail in the next few paragraphs.

To start the second pass, I watched all (n = 9) of Mrs. Stewart's videos consecutively, noting consistent patterns of behaviors across the dimensions given in the bulleted list below. A short list was generated at the beginning, but as more aspects were found they were added to the list. As I progressed from one lesson's video to the next, I noted which consistent patterns of behaviors seemed to change from the previous lessons and which consistent patterns of behaviors seemed to be maintained. I only watched the observation videos for this second pass through the data (i.e., not looking at interview or lesson plan data) because the focus was on macro-level consistent patterns of behaviors at the lesson level. The following is a list of categories that were determined.

- Pacing
- How discussion was started
- How discussion ended
- The number of discussion segments
- Individual versus group participation in the discussion

- Role of content, specifically how do students show or explain their mathematical thinking and use mathematical resources?
- Explicit attention to establishing or maintaining norms
- Role of group talk in the discussion
- Pressing students for justification
- Connecting back to previous content
- Supporting students being careful about language
- Re-voicing students' work

Once these types of consistent patterns of behaviors were noted, the coding scheme was refined in order to code the first round (3 lessons) of Mrs. Stewart's observations. Coding included the following:

- Pacing
    - How interruptions were handled
    - Did the teacher quickly move on to other student comments quickly (with a possible press to the original student)?
    - Did the teacher get through all subparts of the problem?
    - Was the teacher overly patient in fully listening to a student?
- Question and Student Participation
    - Did the teacher call on a volunteer?
    - Did the teacher call on a non-volunteer?
    - Did the teacher probe the student response with questions?
- Teacher sense-making of Student talk
    - Did the teacher clarify what the student meant, including revoicing?

    - Did the teacher press for justification?
- Role of content
    - Did the teacher describe conceptual ideas?
    - Did the teacher clarify vocabulary?
    - How do students explain or show their thinking? (e.g., in their seats)
- Norms
    - What type of explicit talk of norms took place in the classroom?
- Group Talk
    - When does group talk occur during the lesson?
    - What tasks are students working on in groups?
    - Possible intention of group work
- How does the discussion start?
    - Did the discussion start by a volunteer?
    - Did the discussion start by a non-volunteer (e.g., directly after group work or by teacher monitoring)?
- How does the discussion end?
- Other
    - Notes about what was happening during the interaction in the classroom around the mathematical content
    - Questions and comments about coding
    - What part of the lesson was being taught (e.g., Launch 2.1 or Summary 2.2)
    - Possible deviations of consistent pattern of behaviors
    - Influences on disciplinary obligations

After coding these items, clusters of instances appeared around the types of activity that the teacher was engaged in. The following items were generated primarily from the data, but as a second check, the literature base was reviewed around classroom discussions, teacher decision-making, and instructional consistent pattern of behaviors. From the perspective of professional obligations, teacher decisions are a function of at least one of the following: *disciplinary* (content), *individual, interpersonal, institutional*. Because decision-making is inherently a cognitive process, one way to study decision-making is to look for activities that may indicate that decisions had been made. (Note: Another way is to interview individuals about their decision-making, which also took place). From the situative perspective (Greeno, 1989), studying decision-making as it occurs during the lesson is important; thus, it is necessary to determine the types of teacher activities that likely correspond to teacher decisions. In many cases decisions are a result of consistent patterns of behaviors; in other cases, teachers make carefully thought-out, conscious decisions during a discussion. The purpose of this revised coding scheme was to more effectively describe teacher activities related to decision-making and to better operationalize how these activities might relate to the influence of professional obligations (Herbst & Chazan, 2011) on a teacher's decision-making process.

Activity Codes

**Student Activity Decisions**

*How does the teacher start the discussion?*

General description

*How does the teacher end the discussion?*

General description

*Teacher chooses to change the structure of student interaction*

Types: Small group, individual, whole class

*Teacher gives students a task to work on*

Ex: A problem or question to work on in small groups or individually

**Type of Student Interaction Decisions**

*Is the student the teacher calls on during the discussion a volunteer or non-volunteer?*

*Is student discussion filtered through the teacher or student-to-student?*

*When does group talk occur during the discussion?*

Types: before, during, after

*How does group work support the discussion (i.e., possible intention for group work)?*

Ex: Less student interaction than typical; teacher asks students to discuss in groups.

*How do students discuss their thinking in a whole class?*

Types: Physical location (in seats, at board)

Resources: e.g., Using their own work on document camera, re-creating work at board

*When does the teacher use wait time?*

*Attending to interaction norms*

Ex: Teacher explicitly points out how to interact, talk, learn or think in the classroom.

*Pacing*

Types: Short answer - moving on quickly, giving students time to think

**Teacher Talk Moves** (These could be in the form of a teacher statement, a question, or a prompt for students to engage in this type of activity.)

(Pimm, 1987; Sfard 2008; Chapin, O'Connor, & Anderson, 2009)

*Re-voicing, (i.e., closely restating what a student said)*

*Clarifying / Expanding (restating what a student said with the same intended meaning, but said differently)*

*Pre-voicing (i.e., cutting a student off to describe their thinking for them)*

*Questioning*

*Connecting back*

Ex: Describing how an idea connects to previous work

*Connecting forward*

Ex: Describing how an idea connects to future work

*Connecting across*

Ex: Describing how two student strategies are similar or different

*Connecting beyond*

Ex: Describing how mathematics connects to contexts outside of the mathematics classroom

*Pressing for justification (could be a "how" or a "why")*

*Simple response*

This may be used to maintain the flow of the lesson. One example might be in having students share their answers with the class quickly.

*Talking about vocabulary*

*Providing an explanation*

*Evaluating a student response*

Ex: "I liked that you said…"

With thi new set of codes, I coded the third lesson of the first set of observations and created a description of Mrs. Stewart's consistent pattern of behaviors to be later refined in subsequent lessons. The goal was to continue using this iterative process; however, it was determined that the construct of what exactly is meant by the term *consistent pattern of behavior* was sufficiently problematic, as identifying deviations from a consistent pattern of behavior necessitates defining the existence of a consistent pattern of behavior to begin with.

Table 14

*Coding Scheme for Different Patterns of Instructional Behavior*

| Type of Pattern | Description | Guiding Questions to Identify Patterns |
|---|---|---|
| Management | Patterns in this category include student discipline routines, norms for interaction, and how the teacher handles disruptions. | • *How are disruptions typically managed by the teacher?* |
| Instructional Support Resources | Patterns in this category are connected to Leinhardt et al.'s (1987) *support routines*, with attention to the types of materials and technologies that students and teachers use to support their interactions. | • *What technologies does the teacher incorporate to support instruction?*<br>• *What instructional materials does the teacher use to support instruction?* |
| Activity Structure | Yinger (1979) identified 7 dimensions of instructional activity. For this category, the first four will be the focus as the others are described in other categories. These are:<br>1. Location<br>2. Structure and sequence<br>3. Duration (how long the activity lasts) | • *How does the teacher start the discussion?*<br>• *How does the teacher end the discussion?*<br>• *What is the task students are working on?*<br>• *Duration of activities*<br>• *How are students grouped when they work on the task (independent, size of groups)?*<br>• *Transition points when the teacher chooses to change the structure of* |

Table 14 (cont'd)

| | | |
|---|---|---|
| | 4. Participants (specifically how they are grouped)<br>5. Acceptable student behavior<br>6. Instructional moves<br>7. Content and materials | *student interaction (e.g., move from small group to whole class)* |
| Interaction | This category is based off Leinhardt et al.'s (1987) *exchange routine*, and Yinger's (1979) *instructional routines*. It describes the ways in which teachers and students interact with each other. It includes the way in which teachers and students ask questions and communicate with each other in the classroom. | Specific Codes:<br>• *Is the student the teacher calls on during the discussion a volunteer or non-volunteer?*<br>• *When does the teacher use wait time (Question they asked, possible purposes, duration)?*<br>• *What is going on when the teacher has the class move to group work (e.g., after Launching, during the discussion to clarify or further investigate, etc.)?*<br><br>General description<br>• *Pacing*<br>• *Is student discussion filtered through the teacher or student-to-student?*<br>• *How does group work support the discussion (i.e., possible intention for group work)?*<br>• *How do students discuss their thinking in a whole class (i.e., small group, partners, individually; where they present their work)?*<br><br>**Teacher Talk Moves** (These could be in the form of a teacher statement, a question, or a prompt for students to engage in this type of activity.)<br>(Pimm, 1987; Sfard 2008; Chapin, O'Connor, & Anderson, 2009)<br><br>**Restating moves:**<br>• **RR:** *Re-voicing, (i.e., closely restating what a student said)*<br>• **RC:** *Clarifying / Expanding (restating what a student said with the same intended meaning, but said differently)*<br>• **RP:** *Pre-voicing (i.e., cutting a student off to describe their thinking for them)* |

Table 14 (cont'd)

| | | |
|---|---|---|
| | | **<u>Connecting moves:</u>**<br>• **CK:** *Connecting back* (e.g., *describing how an idea connects to previous work*)<br>• **CF:** *Connecting forward* (e.g., *describing how an idea connects to future work*)<br>• **CA:** *Connecting across (e.g., describing how two student strategies are similar or different)*<br>• **CY:** *Connecting beyond* (e.g., *describing how mathematics connects to contexts outside of the mathematics classroom )*<br><br>**<u>Questioning</u>**<br>• **QG:** *Asks any type of question*<br>• **QJ:** *Pressing for justification (could be a "how" or a "why")*<br><br>**<u>General Discussion Moves</u>**<br>• **GS:** *Simple response to answer*<br>  ▪ This may be used to maintain the flow of the lesson. Ex: have students share their answers with the class quickly.<br>○ **GV:** *Talking about vocabulary*<br>○ **GX:** *Providing an explanation*<br>○ **GV:** *Evaluating a student response (e.g.,"I liked that you said…")* |
| Meta-Routine | This is connected to Yinger's (1979) *executive planning routine*. Meta-routines are when a teacher makes explicit the routine during instruction for students in order to reshape, reinforce, or break the routine. | • *The teacher explicitly refers to a routine or norm for the purpose of changing students' actions in relation to that routine* (e.g., *teacher explicitly points out how to interact, talk, learn, or think.*) |

APPENDIX B

Issues with Recruiting a Second School Site

At the same time that I was preparing for data collection at Site 1, I was trying to arrange a second site for the study. After several unanswered e-mails and non-returned phone calls, I simply showed up one day to talk to the principal at potential site 2A; but unfortunately, he was out of the building that day. I stopped in a few days later to talk with him and instead ended up talking to the assistant principal, which was quite productive. This school had three seventh-grade teachers. One was excluded as a potential participant because I knew the teacher through previous encounters during professional development. One teacher was quite active in *CMP* work and had been teaching the curriculum for a while. Due to other research obligations in which she was already taking part, she declined participation. The third teacher did not respond to inquiries via e-mail.

Potential Site 2B had used *CMP* previously, but after communicating with the principal via e-mail, I learned that the school was not currently using the program.

For Potential Site 2C, I contacted the department chair via e-mail after e-mailing the principal. They very quickly decided not to be included as a school site.

Potential Site 2D originally seemed promising. This site had a principal who was highly supportive of *CMP* and university research. There were two seventh grade teachers at this site, one of whom only taught grade 7 while the other taught one section of advanced seventh grade students. I met with the main seventh grade teacher, and although she was initially quite excited about participating in the study, she decided to not participate. It seemed like the two main issues were (1) being video recorded, (2) not being an ideal classroom, and (3) hosting a student-teacher in the spring. In regard to (1), I addressed this by telling her that if she did not want the classroom recordings used in future presentations I would ensure this would be the case. I found issue (2) to be the most interesting. She told me that the other teacher would be better to study because her students were more advanced. As with all the other teachers I talked with, the main thing I told this teacher was that my goal was to learn as much as I could from teachers in "typical" settings, and to study how these teachers go about thinking through the challenges that are presented to them. The study is not about evaluating the quality of instruction, but rather to help paint the picture (especially to PSTs) of the complexity of classroom interaction and to support new teachers in how to process this complexity. It seemed that the combination of the three factors were enough for the teacher to decline participation.

Potential Site 2E was chosen because they have four grade seven teachers, and three of them teach multiple subjects. Additionally, the entire department was new to teaching *CMP,* so many of them could be considered novices with the curriculum though they all had at least seven years of teaching experience. I met with the principal shortly before Winter Break. Teachers were contacted shortly after they returned from break in January. While this site seemed ideal, I was not able to visit any of the teachers before receiving an e-mail from the principal that indicated none of the teachers were interested in participating. During the meeting with the principal, I found out that all four of the grade 7 math teachers had at least five years of experience, which was mostly due to drastic budget cuts that the district had faced in the last few years. This may have contributed to a lack of willingness to do extra work given the pressures that these teachers most certainly face.

Potential Site 2F was a small, rural school district. The middle school has two grade 7 teachers, and both teachers co-teach with special education teachers for certain periods of the day. Again, I contacted the principal and had a face-to-face meeting. The principal was on-board with the study, appreciated having interest from the university in conducting a research study, and as a former middle school math teacher who had contact with people at the university she seemed very optimistic about including the school as a site. I met with both grade 7 teachers to talk about the study. Neither teacher had taken part in a research study before. After a few days, they let me know via e-mail that they would not be able to commit to the study given other professional development obligations that were currently taking place in the district.

Potential Site 2G was contacted multiple times via e-mail about the possibility of being used as a school site, but the principal never replied. During the second exchange, I offered to meet with an assistant principal or department chair as a first point of contact. Again, there was no reply. Given the experience at potential site 2A, I decided to move on to other districts.

In order to broaden the pool of potential *CMP* schools, I contacted a math specialist who is the coordinator of professional development at the middle school level in mathematics at a large Intermediate School District on the east side of the state of Michigan. She was quite helpful and sent along many contact people and potential school sites. Potential Site 2H was contacted because they currently have two grade 7 teachers. The principal forwarded me the teachers' contact information. After a few e-mails, both teachers decided not to participate.

The principal at Potential Site 2I talked to two of his teachers about the study. One was interested; one was not. Because the study requires two teachers at each site, I informed the principal that I would let him know if that requirement changed in the future, but that at this point I would not be interested in using that school as a potential school site.

The math coach for Potential Site 2J was contacted, and she checked with the appropriate people to see if their school could be a site for the study. The teachers she talked to in the building were not interested in receiving any information or being potential research participants.

The math coach for Potential Site 2K met with the curriculum director to see if this district could be used as a school site, but I was unable to reconnect with the people at this school site.

# APPENDIX C

## Initial Interview Questions

Table 15
*Initial Interview Questions*

| Question | Follow-up Questions | Notes |
|---|---|---|
| 1. Describe your experience with implementing *CMP* materials. | How long have you used these materials?<br><br>What grade levels have you taught?<br><br>Have you had experiences with attending or presenting professional development specifically focused on the curriculum materials? | The purpose of this question is to gauge the types of curricular experiences the teacher has had in enacting the curriculum, learning about the curriculum, and having a sense of how the materials are connected within and beyond a specific grade-level. |
| 2. Describe a typical lesson in your classroom. | In a typical lesson, what percent of class time is devoted to teacher-to-student interaction? …student-to-student interaction? …student work time? …going through homework? | The purpose of this question is to better understand to what extent teachers typically make use of classroom discussions in their instruction. This first question is purposefully open so that the teachers are able to talk about their instruction. If they do not mention interaction, the follow-up question will hopefully address that issue. |
| 3. Do you think it's important that students are active in their math class? | If no, have the teacher talk more about that position.<br>If yes…describe some ways you think students should be active as learners of mathematics. | This question will hopefully provide some insight to the teacher's orientations toward student talk and how they might use it in class. |
| 4. What are some challenges that you encounter when trying to facilitate classroom discussions? | What are some important constraints for your planning of discussions? What dilemmas do you typically encounter when you are enacting or planning to have a discussion? | This question is intended to have the teacher talk about factors that contribute to enacting discussion in their class. Some of these factors act as external constraints, and others relate to how the teacher thinks about the costs and benefits of their choices. |
| 5. What do you think you have improved on during your teaching career in facilitating classroom discussions? | For example, has your ability to implement or your beliefs about the role of discussions changed over your career? | This question may provide some insight into the differences between the experts and novices in terms of how their practice or beliefs have changed over time. |
| 6. Are there resources, structures, or routines that you find helpful for supporting implementing discussions? | If the teacher is having a difficult time with this, mention some examples, such as the use of technology (e.g., document camera, Smart board, etc.), grouping strategies, how they structure the lessons, etc. | This question provides some context for the classroom environment, specifically what resources the teacher is making use of to support the implementation of discussions. |

APPENDIX D

Lesson Planning Document

Teachers will be submitting lesson plans before each lesson. Listed below is the initial and revised set of questions used in the lessons.

**Original:**

## Problem 1.3 – What Do You Expect

**Lesson Goals/Objectives:**

**General flow of lesson** (best estimate of major pieces of the lesson, e.g., homework collection, review of yesterday's work, Launching the problem, Exploration time, Summarizing, etc.)

**What do you expect to happen in the whole-class discussion?**

- Are there ideas/strategies you anticipate showing up that are productive?
- Are there ideas/strategies you anticipate showing up that will be less productive?
- Are there misconceptions you anticipate showing up?
- Are there particular examples/solutions/strategies you want to make sure show up in the discussion?
- Are there any non-mathematical aspects of the discussion you are preparing for (e.g., participation, management, etc.)?
- Is there anything atypical about how you might structure the discussion (e.g., a new grouping strategy)? If yes, please list.
- Is there anything else about the discussion you are anticipating occurring that will be important for students interacting in the discussion?

APPENDIX E

Revised Lesson Plan

## Lesson Overview and Agenda

## Problem 1.2 – What Do You Expect

**What do you hope students understand/be able to do as a result of this lesson?**

What ideas/strategies do you anticipate showing up in the lesson? Are any of these particularly problematic?

Are there particular examples/solutions/strategies you want to make sure show up in the discussion?

Is there anything else about the discussion you are anticipating occurring that will be important for students interacting in the discussion (e.g., non-mathematical aspects such as participation, management issues, grouping issues)?

APPENDIX F

Examples of Descriptions for Teachers' Classrooms (Teacher A)

- Classroom arrangement
  - Students are working in small groups of approximately 4 students.
- Managing disruptions
  - There were very few if any noticeable disruptions in the class.
- Typical flow of lesson
  - Describe the problem to students as a whole class
  - Ask students for clarifying questions
  - Give students time to work on the problem in small groups, actively talk with groups of students about the problem in their groups
  - Whole-class discussion
- Teacher's Positioning in the classroom
  - During "Launch," teacher is in the front of the room.
  - During "Explore," teacher is actively talking with students in their groups sitting down with them.
  - During the whole-class discussion, teacher is on the outside of the room, sometimes in the front, but mostly behind students near the wall.
- Typical flow of the discussion
  - Have students introduce their strategy to the class, have them talk about why they solved the problem that way. Sometimes, students are addressing the class from their seats; other times they are in the front of the class showing their solution strategy. Many students share their strategies for solving the problem, while the teacher actively connects the strategies for the students.
- Types of Question being asked by the teacher during the discussion (e.g., Boston, 2014; Boaler and Brodie, 2004)
  - Teacher asks lots of clarifying and connecting questions/comments, such as
    - "Why did your group solve it this way?"
    - "If you look at what this group did, they solved it this way, which is similar to how this other group solved it."
- Student Talk during the discussion
  - The discussion seems very open and respectful. Students listen to the person who "has the floor," and if other students have questions for the speaker, the students can ask the speaker directly instead of asking the teacher. This shows some evidence that students have a certain level of authority in the classroom in relation to learning the content collaboratively.
- Types of student responses shared during the discussion (Boston, 2014)
  - Teacher seemed to have a strong sense of the different types of representations that she wanted to have come out during the discussion. She asked particular groups to share their strategies.

# APPENDIX G

## Examples of Descriptions for Teachers' Classrooms (Teacher B)

- Classroom arrangement
    - Students are working in small groups of approximately 4 students.
- Managing disruptions
    - Redirecting students individually using gestures and language, e.g., saying, "Hold on…" and putting a hand up.
    - Reminder to class to give students at the front of the room their attention while they are presenting.
- Typical flow of lesson
    - Describe the problem to students and give them a chance to talk with each other about some of the big ideas in small groups.
    - Reconvene as a whole group, occasionally ask students to come to the board to work on a problem from a previous day's lesson that will connect to the current lesson's work.
    - Describe the problem for the whole class and have the students start working.
    - Give students time to work on the problem in small groups, actively talk with groups of students about the problem in their groups.
    - Whole-class discussion
- Teacher's Positioning in the classroom
    - During "Launch," teacher is in the front of the room. A student gives their solution strategy to the class, teacher recounts the strategy by asking straightforward questions to the class, such as, "what fraction does this diagram represent?"
    - During "Explore," teacher is actively talking with students in their groups sitting down with them.
    - During the whole-class discussion, teacher is on the outside of the room, and frequently moved to the front to summarize students' arguments for the class.
- Typical flow of the discussion
    - Variety of strategies are shown to the class to help structure the discussion
    - Student presents their solution to the class
    - Questions are asked by students to the presenter
    - Teacher summarizes the presenter's mathematics in the front of the class
    - Next group presents
- Types of Questions being asked by the teacher during the discussion (e.g., Boston, 2014; Boaler and Brodie, 2004)
    - Ask whole class if they agree with a student's solution strategy before they sit down. The teacher frequently uses the question, "Does everyone agree with this?"
    - Teacher presses students for justification. When a claim is made, she asks, "Why?" to cue them to explain more.
    - Lots of recalling questions about what strategies had already been solved, e.g., "How did we get to the value here in the denominator?"
- Student Talk during the discussion
    - The discussion seems more teacher directed; however, there were multiple times when students had things they noticed, such as patterns, where they raised their hand and after being called on shared their thinking with the class.
    - Lots of redirecting the students while speaking to talk to their peers
    - Clarifying questions were asked by students to other students during whole class discussion.
- Description of the types of student responses shared during the discussion (Boston, 2014)
    - Teacher ordered the responses in a way to connect one idea to another, although many of the responses seemed quite similar.

# APPENDIX H

## Examples of Key Moments in the Lesson

BREAKING THE FLOW OF THE LESSON

- When a student asks an unanticipated question, which might be a key insight or something mostly unrelated to the problem being discussed
- When there is at least five seconds of silence before or after the teacher talks
- When the teacher talk moves beyond re-voicing/clarifying
    - E.g., the teacher is being more directive about a point, they are “planting” a point.
    - E.g., the teacher makes a connection to previous student work/conversations
- When the flow of the discussion is student-to-student for a period of exchanges, but then the teacher eventually joins back in the conversation after multiple turns; the reason why the teacher inserted herself at that point in the conversation.

MAINTAINING THE FLOW OF THE LESSON

- Why did the teacher choose particular students to share their thinking with the class…
    - During the Launch of the problem to clarify what was going to be solved in the problem?
    - During the Summary portion of the lesson, where certain examples of student work were put on display for the class to see at the onset of the discussion. What did the set of different strategies afford?
- When the teacher decides to summarize the lesson why was it done at that point?
    - What ideas/strategies to talk about during the summary?
    - What ideas/strategies that didn’t show up in the discussion to put into the summary?
- The teacher follows up a students’ response by asking a similar question.
- The teacher clarifies a students’ response in an atypical way. For example, a teacher typically lets students talk through their explanations, but for a response they go up to the board and show the class the students’ thinking.
- Press for justification, asking students to explain their reasoning (when this is a normative expectation).

**Key Moment Type 2: Talk that explicitly references the discussion or to a decision that must be made before moving on.**

Examples of these include:

- “I think we need to decide what to do with the equation before we move on…”
- When a student talks to the “wrong audience,” for example, the student is describing a solution strategy and they talk to the teacher, and the teacher tells them, “You need to talk to the class; they are the ones you are trying to convince.”

# APPENDIX I

## Classroom Observation Protocol

The purpose of this protocol is to capture key moments that occur during the discussion. The chart will be filled out initially during the classroom observation, and will be added to after watching the videos of the lesson. As many of the key moments as possible will be analyzed through the video, through follow-up discussions with the teacher (including after lesson discussions, post-lesson reflections, and the video-stimulated interview). As a first pass, the last four columns focusing on the four professional obligations will act as a running hypothesis of why each decision was made. For example, *interpersonal* decisions might include aspects of communication and participation; *individual* decisions would include aspects pertaining to student motivation; *institutional* decisions are related to classroom management and homework; and *disciplinary* decisions include content related decisions pertaining to lesson goals, the use of academic language, and developing productive dispositions as learners of mathematics. The next page shows an example chart of the Classroom Observation Protocol.

| Time / Part of Lesson | Teacher Action | Teacher Talk | Student Action | Key Moment: (Type #) Description | Professional Obligations Attending to |
|---|---|---|---|---|---|
| 5:03 Launch | Describing the task for students | "Today we are going to focus on parts A and B" | Listening, asking clarifying questions | | |
| 12:14 Explore | Monitoring work in small groups | "So how do you know this always works?" | Working on task in small group | | |
| 17:25 Summary / Whole Class Discussion | Redirecting class after multiple students off task | "I need to make sure that everyone is paying attention." | Listening to presenters | (2) Redirecting class's attention | Institutional: Teacher acted to redirect focus of the class. |
| 17:54 Summary / Whole Class Discussion | Teacher asks Susan to share her strategy | "Susan can you show your strategy?" | Susan presenting solution | (1) | Individual: Susan was eager to volunteer her solution with the class. |
| 47:00 Summary/Whole-Class Discussion | Asks class to think about difference across solutions | "Think about Joe and Susan's strategies" | Listening | (1) Having students look across strategies | Disciplinary: Having students look across strategies<br><br>Institutional: 2 minutes left in class tie<br><br>Individual: Validation of student solution strategies |

*Figure 13*. Classroom Observation Protocol

# APPENDIX J

## Combined Observation Document:

| | | | | | | |
|---|---|---|---|---|---|---|
| DATE | TEACHER | SCHOOL | UNIT | LESSON | ROUND OF OBS. | OBS NUMBER |

**Teacher Reflections on the Lesson**

*Listed below are some prompts to answer after each lesson.*

1) What was your general sense of where students are at with their understanding of _____?

2) Based on what you wanted students to learn from this lesson, how did you feel about that in relation to how the lesson went? Were there instances of student talk or work that showed evidence of them "getting it" or "not getting it?"

3) What (if any) changes did you make to the lesson for this class based on you teaching this lesson previously in the day? What changes (if any) would you make for the next time you teach this lesson based off of what happened in today's class? (e.g. were there misconceptions, management issues, questions or problems you handled differently with this class).

4) How do you feel about how the lesson set up the next day's lesson?

5) Were there questions you would have liked to ask, or student ideas you would have liked to pursue but did not get a chance to?

- What were some of those questions/ideas?
- What were some possible reasons why you were not able to pursue them as you had wished?

**LESSON SPECIFIC QUESTIONS:**

Video Sections
(dilemmas, routine breaks, etc.)
[management, social, content]

### Lesson Overview and Agenda

**What do you hope students understand/be able to do as a result of this lesson?**

What ideas/strategies do you anticipate showing up in the lesson? Are any of these particularly problematic?

Are there particular examples/solutions/strategies you want to make sure show up in the discussion?

Is there anything else about the discussion you are anticipating occurring that will be important for students interacting in the discussion (e.g. non-mathematical aspects such as participation, management issues, grouping issues)?

**Lesson Timeline**

| | | | | | | |
|---|---|---|---|---|---|---|
| Discussion<br>Group Work<br>Ind. Work | Discussion<br>Group Work<br>Ind. Work | Discussion<br>Group Work<br>Ind. Work | Discussion<br>Group Work<br>Ind. Work | Discussion<br>Group Work<br>Ind. Work | Discussion<br>Group Work<br>Ind. Work | Discussion<br>Group Work<br>Ind. Work |

*Figure 14*. Combined Observation Document

APPENDIX K

Teacher Reflections on the Lesson

*Listed below are some examples of prompts that teachers will be asked to answer after each lesson.*

- If you taught the lesson again, would there be anything you would have done differently?
- If you taught the lesson again, would you change anything in the discussion?
- Were there questions you would have liked to ask, or student ideas you would have liked to pursue but did not get a chance to?
    - What were some of those questions/ideas?
    - What were some possible reasons why you were not able to pursue them as you had wished?
- How do you feel about how the lesson set up the next day's lesson?
- How do you feel about the goals of the lesson in relation to how the lesson played out?
- Were there moments in the discussion that highlighted something important to you about student learning?
- Were there moments in the discussion that highlighted conflict in which direction to go with the discussion?

# APPENDIX L

## Potential Questions to Ask the Teacher Immediately After the Lesson

1. I noticed you called on [NAME] first in the discussion. Why did you think that student might be a good person to start off the discussion?

2. At the beginning of the lesson I saw that you put up groups 1, 4, and 5's work? Was there any reason for why you chose those three groups?

3. This student asked a really interesting question, and you said that the class was going to look into that later on in the unit. Were you thinking about pursuing his thinking further during this class?

    a. Alternative: Did you hear any interesting questions during the discussion today?

4. I noticed that there was a time in the discussion where you listened to students go back-and-forth with each other. Can you talk a little about when you decide to jump back into the conversation?

5. What helps you determine where you position yourself during the discussion?

6. I noticed that you asked [NAME] quite a few follow-up questions during the discussion. Do you have a sense of the types (and how many) questions you feel comfortable with asking [NAME]?

APPENDIX M

Video-Stimulated Interview

Listed below are some examples of the types of questions that will be asked during the video-stimulated interview. Clips will be selected for reviewing, and then I will ask the teacher to talk about their thinking at that moment during the lesson.

Example Questions:

- Did you have a sense of how the discussion was going to unfold before the lesson?
  - How well did it follow your vision?

- During this day's discussion, the first student to talk was Franklin. Was there any reason why you wanted to have Franklin start the discussion?

- Before Sally's description of how she solved the problem with the table, Joe's group used a graph. Did the ordering come up more naturally as part of the discussion, or was there anything purposeful about having Sally's group present before Joe's?

- I noticed at this point in the lesson, a student asks a question, and there's a long pause before asking the student to re-state their thinking. Were there other paths you were considering at this point?

- At the end of the discussion, you inserted a point about solving the problem using an equation. Was there any reason why you decided to put that in the discussion even though none of the students had really talked about that?

---

APPENDIX N

Mrs. Stewart's Consistent Patterns of Instruction

Although there were many aspects of Mrs. Stewart's instruction observed across the lessons, only a few rose to the threshold of being sufficiently consistent (i.e., they were present in either all, or all but one, lessons analyzed) to be deemed a consistent pattern of instructional behavior. These three patterns are (a) *maintaining a rapid pace of instruction*, (b) *emphasizing communication and process goals,* and (c) *implementing fluid interaction structures.* I provide greater detail of these three patterns in this section.

**Pattern 1: Maintaining a Rapid Pace of Instruction**

The first consistent pattern of instruction for Mrs. Stewart was how she maintained a rapid pace of instruction across lessons and was categorized as an *interaction* pattern. Maintaining a rapid pace of instruction may have been attributed to a need to get through as much of the lesson as possible within the class time. In many cases, Mrs. Stewart was able to get through the entire lesson in order to learn as much as possible from the piloting of the *CMP3* materials. Although this rapid pace limited the potential for disruptions and management issues, the consistent pattern of behavior was primarily about how students interacted with Mrs. Stewart during the class discussion. Most discussions could be described as a somewhat rapid exchange between the teacher and students individually, with not much idle time in between speakers.

During each discussion, Mrs. Stewart maintained a pace that supported a high volume of questions, statements, and responses by students. In a typical 30 minutes of discussion time in class, Mrs. Stewart asked about 83 questions, or nearly 3 per minute. These consisted of questions that probed students to justify or clarify their thinking, or for students to share their work or solution to a problem. On average, Mrs. Stewart made about 1 statement, question, or response to students every 10 seconds.

Wait time was occasionally present during discussions. In some cases, wait time appeared intentional, waiting for students to respond to a particular question or providing space in the discussion for students to think about a question. For example, in multiple instances, Mrs. Stewart asked, “Tell me what questions you have on…” about the particular topic they were studying before moving on to something slightly new. In other cases, the wait time seemed less intentional, as they occurred as a result of a student question.

With the exception of one lesson, Mrs. Stewart never had to refocus the class more than once. In some cases, this involved simply waiting and then continuing on with work at the board. Additionally, there was only one instance where Mrs. Stewart explicitly reminded a student to be on task during the discussion, which involved a student not taking notes when she asked the class to do so.

## Pattern 2: Emphasizing Communication and Process Goals

This second pattern of instructional behavior was categorized as a type of *interaction* pattern. A main feature of this orientation involved having students make claims and provide evidence for their claims. Throughout the lesson observations, Mrs. Stewart consistently pressed students to explain their thinking. At the surface, this could be noted by the amount of time and opportunities for students to share their thinking with other students. Mrs. Stewart described her own orientation towards focusing on communication and process goals during a video-stimulated interview.

> [W]hat seventh graders will explain versus write out is completely different. And they’re way better verbal. And I want to make sure in terms of our math practice standards, I

don’t want ‘em to be able to just write out and be precise with their details but I want ‘em to be precise verbally too. (Mrs. Stewart, video-stimulated interview, May 7, 2015)

In an average class period (with discussion being 30 minutes of that time), she averaged 11 questions asking students to justify their thinking (e.g., “Why is this the case?”) and 11 questions for students to clarify their thinking (e.g., “What do you mean by that?”). One reason for this orientation was described by Mrs. Stewart in the initial interview:

> [E]ven in your classwork you have to show me how you got there because when you raise your hand and tell me it’s thirty, and I say okay, how did you get it? It like takes you five minutes to figure it back out and it’s not going to serve you any purpose when you leave this room, like you’re not going to be able to tell what you did, you’re not going to be able to reproduce it, see how it applies to something else...” (Mrs. Stewart, initial interview, November 25, 2014)

During the initial interview Mrs. Stewart shared, “[I]deally I would love a kid to ask a question even if they’re looking at me and have four kids trying to answer that question, and me never having to initiate that.” During the observations, there were no cases of two students speaking consecutively, having one student build their response directly off another’s thinking in the whole-class space. Since students reported their work individually and not as a group this also limited the opportunity for students to build directly off each others’ work without first being mediated by the teacher. In an average 30-minute section of discussion (the average amount of time spent per lesson), there were approximately 31 restatements of student thinking, or about one per minute. Of those, 21 were a clarification, expansion, or exact restatement of a student’s idea; and 10 were simply a short restatement such as a student’s answer to a question.

While Mrs. Stewart devoted a large portion of class time to whole-class discussions, they were largely a product of students talking through the teacher instead of to each other.

## Pattern 3: Implementing Fluid Interaction Structures

Throughout her class discussions, she often prompted students to talk with the students in their groups as a way to scaffold the discussion. An overwhelming majority of class time involved students talking about their mathematical work either in small groups or whole-class. Mrs. Stewart fluidly moved between group talk and whole-class discussion, sometimes planned, other times based off of what was occurring at the moment in the discussion. In almost all instances, students shared their thinking and work from their desks individually (i.e., there was no “group reporting” or “group voice”), even when their work was displayed for the class via the document camera. Only one exception occurred to students staying in their seats to address the whole class, when one student came up to the board to draw a visual representation of an area model. These fluid shifts between small group work and whole-class discussion differed greatly from the other teachers’ interaction structures and provide the third and final consistent pattern in Mrs. Stewart’s instruction.

# APPENDIX O

## Ms. Mitchell's Consistent Patterns of Instruction

Of the few consistent patterns of instruction that were observed in Ms. Mitchell's class, two stood out for how consistently they occurred. The two consistent patterns of instruction were (a) emphasizing students' thinking (interaction pattern), and (b) using teacher mediated discussion structure to facilitate whole-class discussions (interaction pattern). These patterns are discussed in further detail below.

A main emphasis of Ms. Mitchell's teaching approach is on the interaction aspect of the classroom. One of her goals for classroom interaction was "that they become a learning community and they listen to each other." The use of partner work was tied to the Standard for Mathematical Practice #3 (NGA-CCSSO, 2010), generating and *critiquing the reasoning of others.* She felt that students should have a voice in the classroom and that she should protect students' think time to let them process their ideas. In her view, the discipline of mathematics was one that was mostly generated by human beings as a creative endeavor. She was the only teacher to share her broad view of mathematics as a field. This belief, along with wanting her classroom to avoid "social reproduction," created an environment that, on the one hand, ensured students' voices were heard and their thinking was honored, and, on the other hand, often created multiple disruptions.

### Consistent Pattern 1: Emphasizing Students' Thinking

A good example of this propensity to emphasize student thinking was evident in an interview question about a student's rather long response during a lesson observation. In such situations, a teacher could make multiple decisions, based on (1) how long to let the student go with their response, and (2) what to do after the response. The decision for (1) can indicate whether a teacher is worried about management issues (i.e., other students getting restless), the individual as a creator of mathematical knowledge, or whether the description accurately describes the mathematical idea. When asked about this type of situation -- “Do you have a sense of how long do you wait for a student to complete their thoughts before you move on?” -- Ms. Mitchell responded,

> I try to stay as long as I can…because I’m often surprised…I think I would be very frustrated if I was trying to say something and a teacher cut me off, even if I were wrong. Because then how do I know what I think and how do people respond to me if I even had my voice heard. (Ms. Mitchell, video-stimulated interview, May 11, 2015)

A main feature of Ms. Mitchell’s teaching was what she referred to as “honoring students’ thinking.” This involved many different aspects of her teaching practice. For example, Ms. Mitchell consistently used wait time after students described their thinking to the class to ensure that students had space to complete their thoughts, and when asking a question to the class she also consistently provided wait time. When disruptions did occur during the discussion, Ms. Mitchell would remind students of the expected norms that were typically focused on listening to each other’s thinking. This consistent pattern to allow students space to share their thinking by “honoring thinking” was Ms. Mitchell’s first consistent pattern of instruction.

**Consistent Pattern 2: Teacher Mediated Discussion Structure**

During whole-class discussions, students were typically seated at their desks describing their thinking to the whole class. During work time, students often worked with a partner, as she mentioned the difficulties of having the students work in larger groups during the last period of the day.

More so than the other teachers, Ms. Mitchell made attempts to make connections between the content they were learning and other sources. In an average class period, Ms. Mitchell made 2.6 connecting statements. These connections ranged from statements connecting back to previous knowledge, to those connecting forward (e.g., what they would be expected to do in high school), to those connecting their work to other classes' work. In one lesson in particular, Ms. Mitchell made multiple references to outside contexts. When trying to describe the meaning of a data cluster, she talked about why someone owning a hat store would care about data clusters.

At various times, students were invited to describe their work in front of the class individually. Ms. Mitchell freely allowed her students the opportunity to share their thinking at their desks or at the front of the room. A vast majority of class discussion was mediated by the teacher; that is, after a student talked Ms. Mitchell would either re-state the student's answer, provide an explanation, ask a question, or evaluate the student's response. More so than the other teachers, Ms. Mitchell evaluated student responses (generally positively), occurring about two and a half times per class period (18:12 of discussion). The interaction in the discussion typically involved Ms. Mitchell asking questions students had not yet had a chance to think about, students offering an explanation, and Ms. Mitchell either asking students to clarify their thinking or herself providing an explanation for the class (Table 16). Although Ms. Mitchell emphasized mathematical practice goals and generally attended to the individual and interpersonal obligations, the vast majority of her discussions were mediated by her (i.e., the turns in talk were teacher-student-teacher-student).

Table 16

*Frequency of Questions in Ms. Mitchell's Classroom Discussions*

| Question or Statement Type | Frequency per class period (18:12 of discussion time) |
|---|---|
| "New" Question – students had not had the opportunity to work on the specifics of the question. | 7.6 |
| Questions asking students to clarify their thinking | 3.9 |
| Questions asking students to justify their thinking | 1.7 |
| Questions asking students to report their work on a previously solved problem | 2.0 |

# APPENDIX P

## Mr. Sandberg's Consistent Patterns of Instruction

Mr. Sandberg had two consistent patterns of instruction noted in the data. These two patterns were (a) the consistent use of technology as a lesson resource (resource pattern), and (b) implementing a consistent discussion structure in order to maintain classroom management (interaction pattern). In regard to the first consistent pattern, he frequently used several types of instructional technology -- from clickers to graphing calculators to spreadsheets -- to support students in discussing their thinking during whole-class discussions. His second consistent pattern focused on his consistent use of his lesson structure, which differed from the interaction structures that both Ms. Mitchell and Mrs. Stewart incorporated.

### Consistent Pattern 1: Consistent Use of Technology as a Lesson Resource

Besides the use of *CMP3* curriculum materials, a consistent feature of Mr. Sandberg's lessons involved the use of various technologies to support the teaching and discussing of mathematical content. Mr. Sandberg used technology in various ways to support specific lesson goals. It was not used simply as a way to keep students engaged, but rather as a way to support students' thinking about mathematical problems. As one example, Mr. Sandberg used a spreadsheet during a lesson on mean absolute deviation. When asked during the post-lesson interview why he chose to use this technology, he stated that he had actually not used it during his first period class, but "it took too long and I felt like I already gave them sufficient amount of time to calculate it on their own…I wanted them to do it quicker so I just opened Excel and threw the numbers in there real quick." This was a good example of Mr. Sandberg's strategic use of technology, and his decision was influenced by both the need to efficiently use class time and to help students attend to the mathematical idea, not just calculations.

Mr. Sandberg used his Smart Board daily, both as an interactive white-board and as a display device from his computer and document camera. He used spreadsheets, an app-based graphing calculator, instructional videos (how to make a box-and-whisker plot), a cartoon video (presenting a scenario for students to think about), and clickers. Although the particular type of technology was lesson-dependent based on the specific demands of a lesson, he consistently implemented technology across several lessons. This consistent use of technology as a lesson resource was Mr. Sandberg's first identifiable consistent pattern of instruction.

**Consistent Pattern 2: Consistent Use of Lesson Structure during Discussion**

The interaction in whole-class discussions in Mr. Sandberg's class typically involved students sitting at their desks and individually responding to questions posed by Mr. Sandberg. There were no instances of a student being asked to share their thinking at the board or to share their thinking with their partner; all questions were answered by students in their seats. Long sections of teacher explanation often dominated whole-class discussions. On average in every two minutes of discussion Mr. Sandberg spoke three times, with student talk in between each of his turns. There were no instances of one student following another student's talk directly without first being called on by Mr. Sandberg. Students were typically prompted to respond to questions that provided short answers with little or no explanations required, such as collectively filling out a table or making an ordered list.

In an average discussion segment (i.e., the coding was averaged to represent a 9-minute, 31-second segment), there were just under 2 questions asked for students to justify their thinking, less than 1 question to have students clarify their thinking (with 80% of these occurring within one lesson), and no instances of questions asking students to explain their work in how they solved a problem. There were, however, over nine questions asking students to provide their answer for the class, and eleven instances of Mr. Sandberg providing an explanation of the mathematics for the class. There were only two questions across lessons that asked students to make a prediction.

In many lessons, there were potential opportunities for Mr. Sandberg to press students' thinking or justification, but often these situations involved simply getting student answers into the whole-class space. For example, in the last observation of Round 3, students analyzed roller coaster data (Figure 15) and were asked, "How do we find a fast speed?" Mr. Sandberg received several responses from averaging the data, finding the fastest speed overall, to creating a box-and-whisker plot. As was typical, students responded but were not asked to support their reasoning as to why they chose a particular strategy, or to explain possible benefits of their strategy over those of their classmates.

In a different lesson, Mr. Sandberg asked students what they thought the longer whisker represented in a box-and-whisker plot. After a student gave a short answer, Mr. Sandberg followed by providing an explanation of what it represented. This again was typical of a possible opportunity for a whole-class discussion that failed to transpire, and was instead replaced with direct explanation by Mr. Sandberg.

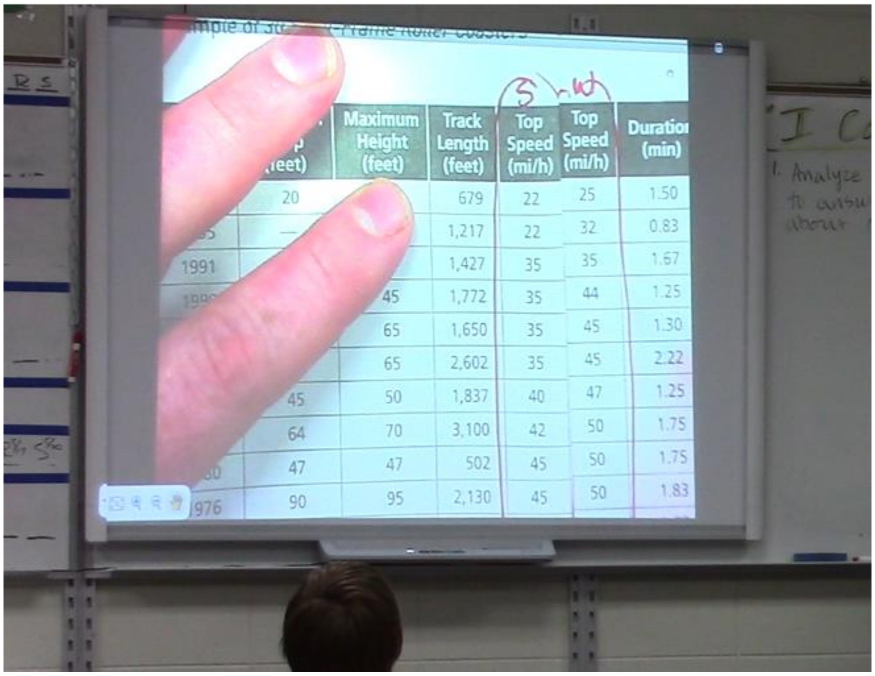


*Figure 15*. Determining a “fast speed” comparing wood and steel-framed roller coasters

There were very few disruptions in Mr. Sandberg’s class. When disruptions did occur, they were handled quickly and students remained on task. There were no instances of Mr. Sandberg having to explicitly describe expectations to students, which should not be surprising given the late nature of the lesson observations during the school year. After analyzing the lesson videos and interview data, the main reason for the lack of disruptions likely was the result of two aspects of his classroom management: first by solving problems through implementing consistent patterns of instruction, and second by constraining the nature of student interaction (especially in classroom discussions) to minimize the potential for disruptions.

Attention to managing interpersonal interactions and organizing work for a class of students seemed to be a high priority for Mr. Sandberg. For example, when asked in the initial interview about consistent patterns of instruction or resources that he finds productive or useful for facilitating discussions, he talked about how he groups students by partners, and how he ends class with having students put their calculators and desks back. What is notable of this response is that he could have answered the question by talking about consistent patterns of instruction of dialogue (e.g., how he structures classroom talk), or how resources could support discussion in the class as opposed to more logistical aspects of classroom interaction.

In a similar light, during the last observation in Round 2, Mr. Sandberg walked around and talked to students at the beginning of partner work time. When asked during the post-lesson interview about monitoring students working on the problem -- "When you were walking around…was there anything that stood out as unusual, or someone going in the wrong direction?" -- he responded, "No, not at all. I was just getting them started…giving them their own individual invitation to get pencil on paper." In this scenario, Mr. Sandberg was monitoring his student's progress, but it was more focused on management aspects rather than attending to individual students' needs or helping students with understanding mathematical content.

It was also during partner work that a Smart-Board app was displayed on the screen that kept track of how loud the class was working. If the noise-level increased over a certain threshold a certain number of times per class, then the class had to stay after. If the class worked quietly they earned a letter and were rewarded with a video at the end of the unit. This reward and consequence system was implemented consistently across lessons. In one lesson Mr. Sandberg was mid-conversation when the bell rang, and he let them go immediately. When asked about this in the video-stimulated interview as to why he did not continue on with his explanation he referred to the value of his tally-system and that the consequence of staying after worked only if he kept them after class for being too noisy.

In addition, the nature of student interactions in the discussion also allowed for maintaining a lesson flow that minimized the potential for disruptions. Not pursuing student ideas deeply, or opening up the conversation significantly, allowed the lesson to progress mostly as planned with minimal deviations. Having many students involved in providing short answers allowed for accountability to make sure students were doing their work by possibly needing to share their work with the class. Mr. Sandberg frequently used random calling on students via drawing sticks with their names on them for students to provide short answers. The predominance of Mr. Sandberg filtering the discussion through him meant that at any time any student might be called on to talk in front of the class without support from a partner or group.

The second consistent pattern in Mr. Sandberg's instructional was his systematic approach to classroom management, related to maintaining a strong lesson flow and structure. When asked about his classroom management, he shared that the location of his Smart Board, the direction the class faces, and the location of the pencil sharpener amongst other things, were all thoughtfully located to minimize disruptions. He also mentioned that he considered outside distractions (e.g., eighth-grade students walking by the door heading to lunch) as a factor in potentially changing his lesson structure to minimize potential interruptions.

Students appeared to have a strong sense of what was expected of their interaction in the discussions as there were very few disruptions during this time. The nature of Mr. Sandberg's questions was such that wait time was rarely used (i.e., students either had a short response or did not). When wait time was used for a students' response, generally Mr. Sandberg waited a few seconds and then he moved on to another student or to an explanation. Limiting student opportunities to provide in-depth answers allowed for a consistent flow between teacher and students, with little opportunity for deviations.

REFERENCES

REFERENCES


Artzt, A. F., & Armour-Thomas, E. (1998). Mathematics teaching as problem solving: A framework for studying teacher metacognition underlying instructional practice in mathematics. *Instructional Science*, *26*(1-2), 5-25.

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal*, 4, 373-397.

Becker, M. C. (2005). A framework for applying organizational routines in empirical research: Linking antecedents, characteristics and performance outcomes of recurrent interaction patterns. *Industrial and Corporate Change*, *14*(5), 817-846.

Berliner, D. C. (2001). Learning about and learning from expert teachers. *International journal of educational research*, *35*(5), 463-482.

Bishop, A. J., & Whitfield, R. C. (1972). *Situations in Teaching*. Maidenhead, UK: McGraw-Hill.

Boaler, J., & Brodie, K. (2004). The importance, nature and impact of teacher questions. In *Proceedings of the 26th annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 773-781).Boston, M. (2012). *IQA mathematics rubrics: Academic rigor training manual.*

Borko, H., & Livingston, C. (1989). Cognition and improvisation: Differences in mathematics instruction by expert and novice teachers. *American educational research journal*, *26*(4), 473-498.

Borko, H., Roberts, S. A., & Shavelson, R. (2008). Teachers' decision making: From Alan J. Bishop to today. In P. Clarkson and N. C. Presmeg, *Critical issues in mathematics education* (pp. 37-67). Springer, New York, NY.

Boston, M. (2012). *Instructional Quality Assessment Classroom Observation Tool*. Retrieved from http://peabody.vanderbilt.edu/docs/pdf/tl/IQA_RaterPacket_LessonObservations_Fall_12.pdf

Boston, M. D. (2014). Assessing instructional quality in mathematics classrooms through collections of students' work. In *Transforming mathematics instruction* (pp. 501-523). Springer International Publishing.

Broadbent, D. E. (1958). *Perception and communication.* Elmsford, NY: Pergamon Press.

Cazden, C. B. (1988). *Classroom discourse: The language of teaching and learning.* Portsmouth, NH: Heinemann.

Cengiz, N., Kline, K., & Grant, T. J. (2011). Extending students' mathematical thinking during whole-group discussions. *Journal of Mathematics Teacher Education*, *14*(5), 355-374.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn grades K-6* (2nd Ed.). Sausalito, CA: Math Solutions.

Choppin, J. (2011). The impact of professional noticing on teachers' adaptations of challenging tasks. *Mathematical Thinking and Learning*, *13*(3), 175-197.

Cobb, P., & Smith, T. (2008). District development as a means of improving mathematics teaching and learning at scale. *International handbook of mathematics teacher education*, *3*, 231-254.

Crespo, S., Oslund, J. A., & Parks, A. N. (2011). Imagining mathematics teaching practice: prospective teachers generate representations of a class discussion. *ZDM*, *43*(1), 119-131.

Cullen, R. (1998). Teacher talk and the classroom context. *ELT journal*, *52*(3), 179-187.

Ellis, A. B. (2011). Generalizing-promoting actions: How classroom collaborations can support students' mathematical generalizations. *Journal for Research in Mathematics Education, 42*(4), 308-345.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester, Jr. (Ed.) *Second handbook of research on mathematics teaching and learning* (pp. 225-256). Charlotte, NC: Information Age Publishing.

Gardner, H. (1987). *The mind's new science: A history of the cognitive revolution*. New York: Basic Books.

Greeno, J. G. (1989). A perspective on thinking. *American Psychologist*, *44*(2), 134.

Greeno, J. G., Collins, A. M., & Resnick, L. B. (1996). Cognition and learning. In D. Berliner & R. Calfee (Eds.), *Handbook of educational psychology* (pp. 15-46). New York: Macmillan.

Henning, J. E., McKeny, T., Foley, G. D., & Balong, M. (2012). Mathematics discussions by design: creating opportunities for purposeful participation. *Journal of Mathematics Teacher Education*, *15*(6), 453-479.

Herbst, P., & Chazan, D. (2011). Research on practical rationality: Studying the justification of actions in mathematics teaching. *Montana Mathematics Enthusiast*, *8*(3).

Herbst, P., Chazan, D., Kosko, K. W., Dimmel, J., & Erickson, A., (2016). Using multimedia questionnaires to study influences on the decisions mathematics teachers make in instructional situations. *ZDM Mathematics Education*, *48*, 167-183.

Jackson, K., & Cobb, P. (2010). Refining a vision of ambitious mathematics instruction to address issues of equity. Paper presented at the *Annual meeting of the American Educational Research Association, Denver, CO*. Retrieved from http://cadrek12.org/sites/default/files/Jackson%20%20Cobb%20Ambitious%20and%20equitable_vision_100702.pdf

Jackson, K., Garrison, A., Wilson, J., Gibbons, L., & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction. *Journal for Research in Mathematics Education*,*44*(4), 646-682.

Jansen, A. (2006). Seventh graders' motivations for participating in two discussion-oriented mathematics classrooms. *The Elementary School Journal*,*106*(5), 409-428.

Jansen, A. (2008). An investigation of relationships between seventh-grade students' beliefs and their participation during mathematics discussions in two classrooms. *Mathematical Thinking and Learning*, *10*(1), 68-100.

Kazemi, E., Franke, M., & Lampert, M. (2009). Developing pedagogies in teacher education to support novice teachers' ability to enact ambitious instruction. In *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 12-30).

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics.* Washington, D.C.: National Academy Press.

Knuth, E. J., Alibali, M. W., McNeil, N. M., Weinberg, A., & Stephens, A. C. (2005). Middle school students' understanding of core algebraic concepts: Equivalence & Variable *Zentralblatt für Didaktik der Mathematik*, *37*(1), 68-76.

Knuth, E. J., Stephens, A. C., McNeil, N. M., & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for research in mathematics education*, 297-312.

Lacourse, F. (2011). An Element of practical knowledge in education: Professional routines. *McGill Journal of Education/Revue des sciences de l'éducation de McGill*, *46*(1), 73-90.

Lampert, M. (2001). *Teaching problems and the problems of teaching.* New Haven, CN: Yale University Press.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In M. K. Stein and L. Kucan, *Instructional explanations in the disciplines* (pp. 129-141). Springer US.

Lampert, M., Boerst, T. A., & Graziani, F. (2011). Organizational resources in the service of school-wide ambitious teaching practice. *Teachers College Record*, *113*(7), 1361-1400.

Lampert, M., & Cobb, P. (2003). Communication and language. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 237-249). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., Cunard, A., & Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education 64*(3), 226-243.

Lappan, G., Fey, J. T., Fitzgerald, W., Friel, S. N., & Phillips, E. D. (2005). *Connected Mathematics 2*. Pearson Prentice Hall.

Lappan, G., Phillips, E. D., Fey, J. T., & Friel, S. N. (2014). *Connected Mathematics 3*. Boston, MA: Pearson.

Lee, S., & Lin, H. (2005). Using argumentation to investigate science teachers' teaching practices: The perspective of instructional decisions and justifications. *International Journal of Science and Mathematics Education, 3,* 429-461.

Leinhardt, G. (1989). Math lessons: A contrast of novice and expert competence. *Journal for Research in Mathematics Education*, *20*(1), 52-75.

Leinhardt, G., & Steele, M. D. (2005). Seeing the complexity of standing to the side: Instructional dialogues. *Cognition and Instruction*, *23*(1), 87-163.

Leinhardt, G., Weidman, C., & Hammond, K. M. (1987). Introduction and integration of classroom routines by expert teachers. *Curriculum Inquiry*, *17*(2), 135-176.

Lewis-Beck, M., Bryman, A. E., & Liao, T. F. (2003). *The Sage encyclopedia of social science research methods*. Sage Publications.

McGraw, R. H. (2002). Facilitating whole-class discussion in secondary mathematics classrooms. Unpublished doctoral dissertation, Indiana University, Bloomington, IN.

Mehan, H. (1979). *Learning lessons*. Cambridge, MA: Harvard University Press.

Michigan Department of Education (2012). Understanding educator evaluations in Michigan: Results from year 1 of implementation. Retrieved from http://www.michigan.gov/documents/mde/Educator_Effectiveness_Ratings_Policy_Brief_403184_7.pdf

Moschkovich, J. (1999). Supporting the participation of English Language Learners in mathematical discussions. *For the Learning of Mathematics*, *19*(1), 11-19.

Nathan, M. J., & Knuth, E. J. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and instruction*, *21*(2), 175-207.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

National Governors Association and Council of Chief School State Officers [NGA-CCSSO] (2010). *Common core state standards for mathematics.* Retrieved from http://corestandards.org/assets/CCSSI_Math%20Standards.pdf.

Parker, W. C. (1984). Developing teachers' decision making. *The Journal of experimental education*, *52*(4), 220-226.

Pimm, D. 1987. *Speaking mathematically*. London and New York: Routledge and Kegan Paul.

Rich, P. J., & Hannafin, M. J. (2008). Decisions and reasons: Examining preservice teacher decision-making through video self-analysis. *Journal of Computing in Higher Education*, *20*(1), 62-94.

Sahin, A., & Kulm, G., (2008). Sixth grade mathematics teachers' intentions and use of probing, guiding, and factual questions. *Journal of Mathematics Teacher Education, 11,* 221-241.

Schoenfeld, A. H. (1992). Radical constructivism and the pragmatics of instruction. *Journal for Research in Mathematics Education*, *23*(3), 290-295.

Schoenfeld, A. H. (2010). *How we think: A theory of goal-oriented decision making and its educational applications.* Routledge, Taylor & Francis Group, Florence, KY.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press.

Shavelson, R. J., & Stern, P. (1981). Research on teachers' pedagogical thoughts, judgments, decisions, and behavior. *Review of Educational Research, 51*, 455–498.

Shroyer, J., & Fitzgerald, W., (1986). *Mouse and elephant: Measuring growth.* Middle Grades Mathematics Project. Menlo Park, CA: Addison-Wesley Publishing Company.

Skott, J. (2004). The forced autonomy of mathematics teachers. *Educational Studies in Mathematics*, *55*(1-3), 227-257.

Sleep, L. (2012). The work of steering instruction toward the mathematical point: A decomposition of teaching practice. *American Educational Research Journal, 49*(5), 935-970.

Smith, E. (1997). Constructing the individual knower: Radical constructivism by Ernst Von Glasersfeld. *Journal for Research in Mathematics Education*, *28*(1), 106-111.

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics teaching in the middle school*,*3*(5), 344-350.

Smith, M. S. & Stein, M. K. (2011). *5 practices for orchestrating productive mathematical discussions.* Reston, VA: The National Council of Teachers of Mathematics.

Speer, N. M., & Wagner, J. F. (2009). Knowledge needed by a teacher to provide analytic scaffolding during undergraduate mathematics classroom discussions. *Journal for Research in Mathematics Education*, 530-562.

Star, J. R., Smith III, J. P., & Jansen, A. (2008). What students notice as different between reform and traditional mathematics programs. *Journal for Research in Mathematics Education*, 9-32.

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 319–369). Charlotte, NC: Information Age Publishing.

Stein, M. K., & Smith, M. S. (2010). The influence of curriculum on students' learning. *Mathematics curriculum. Issues, trends, and future directions*, 351-362.

Sternberg, R. J., & Horvath, J. A. (1995). A prototype view of expert teaching. *Educational researcher*, *24*(6), 9-17.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: Simon & Schuster.

Stockero, S. L., & Van Zoest, L. R. (2013). Characterizing pivotal teaching moments in beginning mathematics teachers' practice. *Journal of Mathematics Teacher Education*, *16*(2), 125-147.

Sutcliffe, J., & Whitfield, R. (1976). Decision making in the classroom: An initial report. *Research Intelligence*, *2*(1), 14-19.

Tarr, J. E., Reys, R. E., Reys, B. J., Chavez, O., Shih, J., & Osterlind, S. J. (2008). The impact of middle-grades mathematics curricula and the classroom learning environment on student achievement. *Journal for Research in Mathematics Education*, *39*(3), 247-280.

Tyminski, A. M., Zambak, V. S., Drake, C., & Land, T. J. Using representations, decomposition, and approximations of practices to support prospective elementary mathematics teachers' practice of organizing discussions. *Journal of Mathematics Teacher Education*, 1-25.

Webb, N. M. (1991). Task-related verbal interaction and mathematics learning in small groups. *Journal for Research in Mathematics Education*, *22*(5), 366-389.

Webb, N. M., Franke, M. L., Ing, M., Wong, J., Fernandez, C. H., Shin, N., & Turrou, A. C. (2014). Engaging with others' mathematical ideas: Interrelationships among student participation, teachers' instructional practices, and learning. *International Journal of Educational Research, 63*, 79-93.

Westerman, D. A. (1991). Expert and novice teacher decision making. *Journal of teacher education*, *42*(4), 292-305.

Williams, S. R., & Baxter, J. A. (1996). Dilemmas of discourse-oriented teaching in one middle school mathematics classroom. *The Elementary School Journal*, 21-38.

Yinger, R. (1979). Routines in teacher planning. *Theory into practice*, *18*(3), 163-169.

Zisberg, A., Young, H. M, Schepp, K., & Zysberg, L. (2007). A concept analysis of routine: Relevance to nursing. *Journal of Advanced Nursing, 57*(4), 442-453. a