THE INFLUENCE OF CHORDWISE FLEXIBILITY ON THE FLOW STRUCTURE AND STREAMWISE FORCE OF A SINUSOIDALLY PITCHING AIRFOIL

By

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A DISSERTATION

Submitted to Michigan State University in partial fulfillment of the requirements for the degree of

Mechanical Engineering – Doctor of Philosophy

2017

ABSTRACT

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Many natural flyers and swimmers need to exploit unsteady mechanisms in order to generate sufficient aerodynamic forces for sustained flight and propulsion. This is, in part, due to the low speed and length scales at which they typically operate. In this low Reynolds number regime, there are many unanswered questions on how existing aerodynamic theory for both steady and unsteady flows can be applied. Additionally, most of these natural flyers and swimmers have deformable wing/fin structures, three dimensional wing planforms, and exhibit complex kinematics during motion. While some biologically-inspired studies seek to replicate these complex structures and kinematics in the laboratory or in numerical simulations, it becomes difficult to draw explicit connections to the existing knowledge base of classical unsteady aerodynamic theory due to the complexity of the problems. In this experimental study, wing kinematics, structure, and planform are greatly simplified to investigate the effect of chordwise flexibility on the streamwise force (thrust) and wake behavior of a sinusoidally pitching airfoil.

The study of flexibility in the literature has typically utilized flat plates with varying thicknesses or lengths to change their chordwise flexibility. This choice introduces additional complexities when comparing to the wealth of knowledge originally developed on streamlined aerodynamic shapes. The current study capitalizes on the recent developments in 3D printer technology to create accurate shapes out of materials with varying degrees of flexibility by creating two standard NACA 0009 airfoils: one rigid and one flexible.

Each of the two airfoils are sinusoidally pitched about the quarter chord over a range of oscillation amplitudes and frequencies while monitoring the deformation of the airfoil. The oscillation amplitude is selected to be small enough such that leading edge vortices do not form, and the vortical structures in the wake are formed from the trailing edge. Two-component Molecular Tagging Velocimetry (MTV) is employed to measure the vortical flowfield over the first chord length behind the airfoil. A control volume method is used to estimate the mean thrust of the airfoil based on the mean and fluctuating velocity profiles from the MTV results.

The mean thrust results show chordwise flexibility increases the thrust produced by the airfoil over the range of motion parameters and the flexibility considered in this study. The flexible airfoil is also seen to experience the drag-to-thrust crossover at a lower oscillation frequency than its rigid counterpart. The relative change in thrust due to flexibility decreases with increasing amplitude. The increase in thrust can, however, be captured as an amplitude effect when the Strouhal number based on the actual trailing edge displacement, St_{te} , is used for scaling. Scaling based strictly on the prescribed motion, typically employed in the literature, is not sufficient for the data to collapse.

Motion trajectories which produced a classical von Kármán vortex street or a reverse von Kármán vortex street (depending on the arrangement of the vortices), are considered for further study. The vortices in the wake are characterized in terms of their strength, size, and spacing using phase-averaged MTV results. The circulation of the vortices are shown to collapse for both rigid and flexible airfoils when plotted against St_{te} . The actual trailing edge displacement is used as a length scale to normalize the transverse and streamwise spacing, and the vortex core size. These measurements also now collapse when plotted against St_{te} across oscillation amplitude for both the rigid and flexible airfoils.

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ACKNOWLEDGEMENTS

Few things in life are accomplished individually. There is nearly always someone or something that has significantly contributed or enabled one to succeed. I will make an attempt to express my gratitude to those individuals and entities. First and foremost, I'd like to humbly thank my wife, Olivia, for all of the love and support she gives me. Secondly, I'd like to thank all of my friends and family (near and far), for all their support over the many years.

I've had the opportunity to have two advisors during my entire time here at MSU. Dr. Ahmed Naguib has been an invaluable co-advisor; his ability to translate graduate student ramblings into coherent messages is remarkable, especially when I almost got the point across, but it needed a little tweaking to actually make sense. His work ethic and still-in-the-trenches style of advising is aspirational. Lastly, I need to thank one of the best advisors, Dr. Manoochehr Koochesfahani, I have had the opportunity to work with, meet, or even hear about. Sometimes, I would feel left out when graduate students get together and complain about their advisors, but then I would remember that I had it pretty good. I sincerely thank you for all of the guidance and support you have given me over the years.

Visitors to TMUAL commonly ask how is it that Dr. Patrick Hammer and I get along, with him being a computationalist and me an experimentalist working on the same projects. My response is always: Patrick has thick skin; and for that, I am grateful. Patrick's high fidelity simulations have been an invaluable resource, especially the numerous cases he ran specifically to compare with my experiments. For all of that, and more, Thanks! I'd like to thank all of the rest of the members (and escapes) of TMUAL over the years. I'll name a few: Dr. Rohit Nehe (for many years of friendship and camaraderie), Dr. Bruno Monnier (who designed and build the 3-DOF motion apparatus while I got to be the first one to use it), and Dr. Shahram Pouya (whose kind demeanor is only surpassed by his knowledge of TMUAL).

Lastly, this work was sponsored by AFOSR award numbers FA9550-10-1-0342 and FA9550-15-1-0224.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

There are numerous engineering applications for unmanned aerial vehicles such as remote hazardous material identification, search and rescue, and reconnaissance. Many of these applications require vehicles that are small and lightweight, maneuverable, and operate in complex environments. Collectively, these vehicles are typically referred to as Micro Air Vehicles (MAVs), and engineers have turned to nature for inspiration as these vehicles are the same physical size of small birds and/or fish (Spedding & Lissaman, 1998). At the low speeds and length scales in which these vehicles operate, viscous effects become increasingly important (Lissaman, 1983); necessitating many of the natural flyers and swimmers, and the biomimetic MAVs, to exploit unsteady mechanisms in order to generate sufficient aerodynamic forces for sustained flight and propulsion.

One challenge of investigating the performance and aerodynamics of complex biomimetic wings is generalizing the results to enhance our understanding of the underlying mechanisms of lift and thrust generation. The wings (or fins) of the natural flyers (or swimmers) are structurally intricate and undergo complex kinematics; one approach to studying the problem, from a direct biological perspective, utilizes replicas or the actual insect, bird, or bat wing and are experimentally or numerically studied with either realistic or simplified kinematics (Spedding & Lissaman, 1998; Shyy et al., 2010). While these types of studies have yielded tremendous insight into the flight and aerodynamics of the natural flyers, it is difficult to generalize the results to understand the fundamental unsteady aerodynamic and aeroelastic effects. Alternatively, another approach is to start with more traditional airfoils under 2D boundary conditions and simplified kinematics in order to parametrically investigate the lift and thrust generation. Systematically increasing the complexity of the airfoils, kinematics, boundary conditions, or a combination thereof, is seen as a path for improving our understanding of the fundamental unsteady aerodynamics.

1.2 Background

The initial theoretical explanations of the generation of thrust of flapping flight are credited to Knoller (1909) and Betz (1912), with Katzmayr (1922) being the first to experimentally observe the effect. The classical unsteady aerodynamic theory was developed typically under the assumptions of inviscid flow, low amplitude oscillations, thin and high aspect ratio (AR) wings, as motivated by the modeling of the forces associated with flutter (Theodorsen, 1934; von Kármán & Burgers, 1935; Garrick, 1937; von Kármán & Sears, 1938). The complex kinematics of flapping wings were reduced to several canonical motion types: pitching, plunging or heaving, and combined pitch-plunge motion. It is known from these classical unsteady theories that the generation of thrust can be accomplished by simple oscillatory pitching motion. While the current study focuses on pure harmonic pitching motion to simplify the kinematics to the minimum necessary to generate thrust, some background on harmonic plunging motion is required as both motion kinematics are studied throughout the relevant literature.

The generation of thrust from an oscillating airfoil was connected to an array of alternating-sign vortices shed from the airfoil by von Kármán & Burgers (1935) and Lighthill (1969). Koochesfahani (1989) showed experimentally for a pitching airfoil that the shed vorticity form these coherent structures as a von Kármán vortex street, indicative of a momentum deficit, and at a high enough frequency, a so-called reverse von Kármán street, indicative of a momentum excess or jet-like velocity profile. Ramamurti & Sandberg (2001) computed similar results for the pitching airfoil, while similar patterns were seen for a harmonically plunging airfoil experimentally by Freymuth (1988) and numerically by Platzer et al. (1993), who also later saw it experimentally in Jones et al. (1998) and Lai & Platzer (1999).

An array of alternating-sign vortices is shown schematically in Figure 1.1 where a pitching airfoil about its quarter chord (with a trailing edge displacement of h_p) generates a reverse von Kármán vortex street. The vortex pattern is quantified by its transverse spacing, b, and its wavelength, a. The transverse spacing (b) is the displacement from the center of the negative vortex ($y_{c,n}$) to the positive vortex ($y_{c,p}$). The induced velocity from the vortices, which may be visualized by



Figure 1.1: Schematic of a reverse von Kármán vortex street. An airfoil pitching about the quarter chord is depicted with the transverse spacing (b), wavelength (a), and the trailing edge displacement (h_p) indicated. The sign of the circulation of the vortex is indicated by the arrow and by the (+) or (-) sign.

the their sign in circulation (Γ) rotation direction shown in Figure 1.1, creates a velocity excess in the streamwise direction when *b* is positive. When the vortices are arranged in the opposite orientation (with the positive sign vortices below the negative sign vortices) the induced velocities of the vortices would create a velocity deficit in the streamwise direction.

Bohl & Koochesfahani (2009) found the switch from drag to thrust does not correspond with the frequency at which the vortex pattern changes, but they are close to one another; the switch in the vortex pattern occurs at a lower frequency than the drag-to-thrust crossover frequency. Bohl & Koochesfahani (2009) also studied the details of the vortex properties of a harmonically pitching NACA 0012 airfoil at a chord Reynolds number of $Re_c = 1.25 \times 10^4$ for an amplitude of $\alpha_0 = 2^\circ$, and $\alpha_0 = 4^\circ$ is included in Bohl (2002). The peak vorticity (ω_{pk}) and circulation (Γ) were used to quantify the strength, and the radius of gyration r_c , was used to quantify the size of the vortices. The wavelength (*a*) and transverse spacing (*b*), as depicted in Figure 1.1, were also quantified. Computationally, Hammer (2016) was able to closely replicate the experimental results of Bohl & Koochesfahani (2009) with high fidelity 2D simulations. Hammer (2016) investigated several additional pitching amplitudes, and Reynolds numbers while investigating the effect of these parameters on the wake vortex properties.

1.3 Flexibility

Many natural flyers and swimmers have extremely complex three dimensional deformable wing structures and undergo equally complex kinematics. Combes (2003) found that spanwise flexural stiffness was typically 1-2 orders of magnitude higher than chordwise flexural stiffness for many different insect species. A starting point for considering flexibility is to isolate the chordwise direction as it appears to be the more flexible than the spanwise direction in many insects in nature.

An inviscid model developed by Katz & Weihs (1978, 1979) was used to investigate passive chordwise flexibility for harmonically oscillating wings undergoing large amplitude motions along curved paths. The oscillating airfoil in Katz & Weihs (1978) is taken to be moving at high Reynolds number in an incompressible fluid such that potential theory developed earlier (Wu, 1961; Lighthill, 1975; James, 1973; Chopra, 1976) could be used. The airfoil motion along a curved path is such that while the transverse oscillation amplitude is large, the angle of attack (α) variation is low. The interaction between the chordwise flexible structure and the flow field was achieved by an elastic model iteratively interacting with the pressure field described by the potential flow model. The model utilizes the Kutta condition for shedding a discrete into the wake for the wake behavior; an assumption that is now known to be invalid for unsteady low amplitude oscillations (Bohl & Koochesfahani, 2009). Overall, Katz & Weihs (1978) observe that flexibility tends to improve efficiency but at a sacrifice to thrust production. In their subsequent paper, Katz & Weihs (1979) incorporated lifting line theory to account for the downwash of a 3D airfoil which resulted in slightly modified, but otherwise the same conclusions as their earlier work.

More recently, Moore (2014), used a torsional flexibility model where the leading edge is modeled as torsional spring, allowing the otherwise rigid airfoil to pitch about its leading edge. Moore (2014) extends the model of Wu (1961) using a torque balance between the fluid flow and the torsional spring that determines the pitch angle. Moore (2014) was able to capture the observed "ideal flexibilities" seen previously in different experiments (Heathcote & Gursul, 2007; Alben, 2008; Spagnolie et al., 2010; Dewey et al., 2013; Monnier et al., 2015). Moore (2014) shows in the limit of a massless compliant airfoil, thrust production is hindered compared to the rigid airfoil

but moderate flexibilities can increase thrust production. While this is not a new observation, the simplicity of the model enabling these features to be seen for a passive pitching (or flexing) airfoil is useful for further investigating these flows.

Moore (2015) extended his previous model by considering the wing as a bending beam with chordwise dependent rigidity. They investigated the optimal thrust production from airfoils modeled with a torsional spring at the leading edge, together with linear, quadratic, or cubic distributions of chordwise rigidity. All of the ideal conditions for stiffness distribution resulted in the airfoil closely resembling a rigid wing pitching about its leading edge. This is reminiscent of the "feathering effect" as described by Katz & Weihs (1978) and the the idea mentioned by Zhu (2007), that while complete feathering will produce almost no thrust, partial feathering would lead to a small amount of thrust reduction, but increased efficiency. Moore (2014, 2015) does not address efficiency in his models, and so, the efficiency of the "optimal" chordwise flexible airfoil remains unknown based on their model.

There is little consensus in the literature regarding the mechanism that drives the efficiency of passive chordwise flexible airfoils undergoing flapping motion (pitching, plunging or combined motion). Two competing theories are: (1) the motion oscillation frequency is tuned to the natural resonance frequency of the airfoil, and (2) the bending of the airfoil is timed such that its release of stored elastic energy is beneficial to the flow structures produced.

There have been many studies considering chordwise flexible wings undergoing plunging motion. Collectively, these studies (Heathcote & Gursul, 2007; Shyy et al., 2010; Kang et al., 2011; Cleaver et al., 2014, 2016) have shown there exists an optimal amount of chordwise flexibility to improve thrust production and propulsive efficiency. Cleaver et al. (2016) notes that the thrust improvement is observed in conjunction with stronger trailing edge vortices with a larger transverse spacing (which is known, based on vortex models such as Naguib et al. (2011), to correspond to an increase in thrust). Ramananarivo et al. (2011), based on the plunging, self-propelled flexible wing of Thiria & Godoy-Diana (2010), hypothesize that optimization of efficiency in unsteady flexible flyers is not achieved through structural resonance but through an optimal timing of the release

of the stored elastic energy into creating flow structures that are beneficial for thrust. Thiria & Godoy-Diana (2010) substantiate their hypothesis with a nonlinear oscillator model which shows that energy transfer may be achieved at significantly lower frequency than structural resonance. This is in contrast to the conclusions of Kang et al. (2011) which state that for a plunging airfoil, the maximum thrust is achieved near resonance while maximum efficiency is reached at approximately half of the natural frequency, which they found from numerical simulations and non-dimensional modeling.

Some closure on the debate regarding chordwise flexibility and efficiency/thrust production was presented by Dewey et al. (2013), albeit for a pitching panel compared to the previously mentioned plunging airfoils. Dewey et al. (2013) observed that chordwise flexibility can increase or decrease the thrust, depending on the amount of flexibility; consistent with other pitching airfoil studies of Prempraneerach et al. (2003), Marais et al. (2012), and Monnier et al. (2013, 2015). Dewey et al. (2013) also observed efficiency peaks below, at, and above the airfoil's structural natural frequency, depending on the flexibility. More importantly, Dewey et al. (2013) found that the peak in efficiency for each airfoil typically occurred when the Strouhal number was between 0.25 < St < 0.35, which was observed by Triantafyllou (1993) in a survey of Strouhal numbers seen in various fish species and through a stability analysis of the jet-like velocity profiles that flapping wings generate. Furthermore, Dewey et al. (2013) observed that when this oscillation frequency (falling within the 0.25 < St < 0.35 range) is also tuned to the first natural frequency, there exists a global maximum in efficiency for the optimally-flexible airfoil.

1.4 Current Study

In this experimental study, wing kinematics, structure, and planform are greatly simplified to investigate the effect of chordwise flexibility on the streamwise force (thrust) and vortical wake structure of a sinusoidally pitching airfoil.

Most of the previous studies discussed thus far on both pitching and plunging chordwise flexible airfoils focused primarily on thrust production and propulsive efficiency. Quantifying the vortical flow structures shed into the wake have been given less attention in the literature, even for rigid airfoils (Bohl & Koochesfahani, 2009; Hammer, 2016). The current study focuses on quantifying the vortex parameters and spacing in the wake of a harmonically-pitching chordwise flexible airfoil at a chord Reynolds number of nominally $Re_c = 1.2 \times 10^4$, which is chosen to allow direct comparisons to previous experiments (Bohl, 2002; Bohl & Koochesfahani, 2009) and simulations (Hammer, 2016) on wake vortex properties. Consistent with these studies, the pitching amplitudes considered at $\alpha_0 = 2^\circ$, 4° , and 6° . The motivation to study wake vortex properties is two-fold. The first is the potential use of low order discrete vortex models as an engineering tool for thrust/lift predictions, as discussed in Naguib et al. (2011) and Monnier et al. (2015). The second is the aforementioned general dearth of detailed studies into these flow fields. Detailed experimental studies on such dynamic and intricate flowfields are important for validating high fidelity numerical simulations.

Previous studies focusing on the vorticity in the wake of harmonically pitching flexible propulsors is limited. Cleaver et al. (2014) studied chordwise flexibility by considering a plunging NACA 0012 airfoil with an attached trailing edge flap of different lengths and thicknesses. While Cleaver et al. (2014) did not systematically study the wake vortex properties, they did observe, for a few selected cases, increased circulation and increased transverse spacing of the trailing edge vortices for the optimal flexible trailing edge flap compared to the rigid flap, albeit, for a plunging airfoil.

Monnier et al. (2013, 2015) considered the vortex properties behind rigid and flexible propulsors, but at a lower Reynolds number of $Re_c = 2.3 \times 10^3$. The model of Monnier et al. (2013, 2015) was not a standard airfoil shape, but a geometry similar to Heathcote & Gursul (2007) where the leading edge was the shape of a standard airfoil but had a long trailing edge attached to it. This geometry is a strong departure from standard airfoils where there is significant leading edge curvature that helps delay leading edge separation. The Reynolds number was also low enough to affect the results compared to $Re_c = 1.2 \times 10^4$ as discussed by Hammer (2016).

Prempraneerach et al. (2003) and Egan et al. (2016) created standard symmetric airfoils (by

creating urethane casts of different rigidities) to study chordwise flexibility. Their pitching amplitudes, $\alpha_0 = 10^{\circ}-30^{\circ}$, chord Reynolds numbers, $Re_c = 3 \times 10^4 - 3 \times 10^5$, are both considerably higher than the current study.

The current research utilizes a slightly different approach than most of the other studies looking at flexibility by studying one aerodynamic shape, specifically a NACA 0009, that is either rigid or flexible in the chordwise direction. Uniformity in the span, with regards to flexibility and shape, simplifies the boundary conditions and facilitates the direct comparisons to previous information that is known for rigid airfoils; this is important for comparing to existing literature as few other studies have investigated the vortex properties and their arrangement in the wake of the airfoil. The oscillation amplitudes are kept low enough to prevent leading edge vortices from forming and interacting with trailing edge vortices to simply the flowfield and subsequent discussion.

1.5 Outline

The remainder of this dissertation is broken into four chapters. Chapter 2 describes the experimental methods used to measure and quantify the airfoil's shape, shape deformation, and trajectory. The technique and data processing applied to quantify the velocity field is discussed. The method used to calculate the mean streamwise force (thrust) acting on the airfoil from the mean and fluctuating velocity field is outlined. Further, the parameters used to quantify the vortex properties and arrangement are described.

The primary results and discussion is broken into two chapters. The first, Chapter 3, begins with a comparison of the mean velocity profiles with other studies under the same conditions. Chapter 3 then compares the mean velocity profiles of the rigid and flexible airfoils, and lastly, compares the mean thrust of the airfoil based on a control volume method. The second results chapter, Chapter 4, initially compares the vorticity field characteristics of the rigid airfoil in this study to those in the literature. The influence of chordwise flexibility on the various vortex properties and spacing parameters is then compared and discussed. A primary focus is on the non-dimensional parameters that are useful for scaling the properties for overall collapse between the rigid and flexible airfoil.

Lastly, Chapter 5 concludes the dissertation with a brief summary of the important results and outlines future areas of research on the topic.

CHAPTER 2

EXPERIMENTAL METHODS

2.1 Flow Facility

2.1.1 Description

Experiments are conducted in a water tunnel located in the Turbulent Mixing and Unsteady Aerodynamics Laboratory (TMUAL) at Michigan State University (MSU). The water tunnel is a 10,000 liter closed-return facility with a 61 cm \times 61 cm \times 244 cm test section (shown in Figure 2.1). The flow management consists of two perforated plates in the initial diffuser followed by a 0.25" cell diameter honeycomb and fine mesh screen at the entrance of the settling chamber. After the settling chamber, a 6:1 contraction leads to the entrance of the test section where a 0.125" cell diameter honeycomb and fine mesh screen are located.

The test section has full optical access in the visible spectrum on both sides, the bottom, and a full height end-view window in the return plenum. Two quartz windows (41 cm \times 84 cm each which can be seen in the top of Figure 2.1) were installed on one side of the test section to extend the transmission range of the test section into the deep UV (ultraviolet) and near IR (infrared) ranges.

The impeller of the water tunnel is powered by a Toshiba 20 hp motor and controlled with a Toshiba VF-AS1 drive which ensures day-to-day repeatability of the same freestream velocity; a necessity since the experiments were conducted over a six month time period. The maximum freestream velocity (U_{∞}) of the facility is 1 m/s but is set to 10 cm/s in all of the current experiments. The water level was maintained at 63.5 cm in the test section to ensure all air pockets have been removed from the sealed sections upstream and downstream of the test section. This helps reduce some of the low frequency oscillations that are naturally present in these types of facilities. The freestream turbulence intensity of the facility (*FSTI* = $u_{\rm rms}/U_{\infty}$) consists of nearly spatially uniform low frequency (<0.2 Hz) oscillations in addition to broadband turbulence; the *FSTI* of the





Figure 2.1: Scaled drawing of the entire flow facility. The facility is 8.6 m in length, 1.9 m in height, and 2.5 m in width. The test section, translucent in the side profile view (top), has an internal cross section of 61 cm \times 61 cm and is 244 cm in length. The quartz windows are each 41 cm \times 84 cm installed on the back side of the test section, as shown by darker translucent rectangular sections in top drawing.

facility, most recently measured in Olson (2011), is 1.8% from all contributions and 0.5% from broadband turbulence.

2.1.2 Flow Boundary Conditions

The scope of the current work has defined the desired boundary conditions to be those that approximate 2D flow conditions, i.e. large aspect ratio (AR = span/c). The span of the airfoil is limited to 33.6 cm due to manufacturing limitations described in Section 2.3, which yields an aspect ratio of AR = 2.8, since the chord was kept at 12 cm for consistency with previous studies

in the facility. With this *AR*, the current study will be effected by three dimensionality more than the previous studies in this facility, notably Bohl & Koochesfahani (2009) and Bohl (2002) who had an AR = 4. The impact of three dimensionality on the conclusions drawn from this study will be discussed in Chapter 5.

A false wall on the bottom of the test section was designed, fabricated, and installed in the test section to compensate for the span of the airfoil being less than the height of the test section. The assembly of the false walls and airfoil is shown in Figure 2.2. The walls are 91.4 cm \times 59.7 cm (7.6*c* \times 4.9*c*) in length and width, respectively, and are made from scratch resistant polycarbonate. The airfoil's leading edge is located in the center of the test section's width and 2*c* downstream from the leading edge of the plates. The airfoil is also centered vertically between the two false walls with a gap distance of < 2 mm between the wall and the airfoil. The lower plate's leading edge is rounded and the entire plate is offset 27.6 cm from the bottom of the test section. A 7.6 cm long flap is attached to the plate with a stainless steel piano hinge and was adjusted to an angle of $15^{\circ} \pm 1^{\circ}$ on the top (airfoil side) of the false wall. The upper wall has a single radius on the leading edge and is suspended approximately 15 cm from the top of the test section such that the water level was at the middle of the plate. The center portion of the top plate has two removable sections that allow the airfoil shaft assembly (a 3.81 cm diameter stainless steel cylinder) to pass through and connect to the motors.

The airfoil is considered to have stationary wall boundary conditions located at $\pm y=2.54c$ and at $\pm z=1.4c$ with a small rotating wall of diameter 0.31c centered around the pitch axis (0.25c) at the upper wall.



Figure 2.2: Airfoil and end walls assembly, as installed in the water tunnel. The translucent parts are made from chemical resistant polycarbonate and use aluminum L-brackets for added support on the top plate. The flap is attached using a stainless-steel piano hinge. The upper surface of the lower plate is positioned 27.6 cm above the bottom of the test section. The airfoil (black, centered between the plates) is positioned two chord lengths downstream of the leading edge of the plates. The plates extend 2*c* upstream from the leading edge and an additional 4.6*c* beyond the trailing edge of the airfoil before the flap begins. The shaded green area at the midspan of the airfoil, represents the wake region of interest. The freestream (U_{∞}) is shown in addition to the directions of the the (x, y, z) coordinate system. The origin is at the mid-span trailing edge of the stationary airfoil at $\alpha = 0^{\circ}$.

2.2 Airfoil Motion Trajectory

2.2.1 Equipment Description

The airfoil is mounted to a custom¹three degrees of freedom electromechanical system that is capable of providing arbitrary motion trajectories (e.g. pitch, plunge, pitch-plunge, and perching maneuvers) in the water tunnel. All of the components are from the Parker Hannifin Corporation and include: two Trilogy Ironless Positioners with 1 μ m resolution incremental encoders; a T4DB0576NPAMA4 and a T4DB0436NPAMA4 for streamwise and cross-stream directions respectively. A MPP Rotary Servo Motor (MPP1154A9D-KPSN) with a single turn, 2¹⁷ count, absolute encoder is used for the pitching axis. The entire assembly is mounted above the water tunnel (Figure 2.3) and raised and lowered into position using scissor jacks at each corner of the support structure. The current study requires only the use of the rotary motor, so once the airfoil was centered (to within ±0.3 cm) in the test section, the two linear motors' carriages were immobilized with clamps.

Each positioner/motor is driven using a 1300 Watt Aries Drive (AR-13AE) and controlled with an ACR9000 Controller (9000P1U4M1). A portable chassis, shown in Figure 2.4a, houses the three drives, controller, and interfacing electronics used for synchronization. Communication with the controller is over TCP/IP on a private Fast Ethernet connection, whose topology is also indicated in Figure 2.4b. The controller's Binary Host Interface is utilized for configuring the motion trajectory and additional program functionalities in addition to transferring the captured encoder positions back to the computer for analysis. The software on the central computer (shown in the figure) is a suite of custom MATLAB programs used for interacting with the controller.

The encoder positions of each axis may be either externally (up to 50 Hz) or internally triggered at the servo loop period and stored in the onboard memory with a maximum of over 140,000 values. The memory is read off of the controller and stored on a computer for further analysis. The position

¹The design, assembly, and initial testing of the system was done by Dr. Bruno Monnier concurrently with the work of Monnier et al. (2013).



Figure 2.3: *The three degree of freedom apparatus above the test section.* The entire system is attached to linear bearings on top of a separate (blue) support structure from the test section and may be positioned at different locations in the streamwise direction. The motor assembly is raised and lowered to move the airfoil into the test section using scissor jacks installed at each corner of the frame (not shown).

acquisition program typically records two sets of position data for each data set: (1) an externally triggered set synchronized with each MTV image pair, and (2) an internally triggered set recorded at 2 kHz for 5 seconds at the completion of the first set. The two data sets allow proper phase ordering of the MTV image pair data relative to the motion and for validation of motion trajectory of each data set. The accuracy of the motion trajectory is discussed in Section 2.2.3.

2.2.2 Prescribed Motion

The motion trajectory considered in the current work is pure sinusoidal pitching motion about the quarter chord as described by

$$\alpha = \alpha_0 \sin\left(2\pi f t + \phi_0\right) + \alpha_m,\tag{2.1}$$



Figure 2.4: *Electronics chassis and local network topology.* (a) Portable chassis that houses various components: high-powered drives (lower compartment); power switches, indicators, and emergency stop button (lower front panel); controller, DC power supplies, and I/O signal conditioning (middle compartment); I/O connectors (middle front panel). In the top compartment and on the top of the chassis two remotely accessed PCs used for image acquisition and controller configuration (not shown). (b) Simplified diagram indicating various interfacing done with the controller and computer on a local network.

where α is the instantaneous angle of attack, α_0 is the amplitude, f is the frequency, ϕ_0 is a phase angle, α_m is the mean angle of attack, and t is the time. The mean angle of attack is zero for all cases considered, and the phase angle is 180° (since the motion starts with *alpha* going negative). The prescribed motion is described by two non-dimensional parameters: the reduced frequency, $k = 2\pi f c/U_{\infty}$, and the Strouhal number, $St = f h_p/U_{\infty}$. The trailing edge deflection, h_p , is the peak-to-peak trailing edge amplitude and may be further classified between the actual and prescribed: $h_{p,m}$ and $h_{p,r}$, where the former must be measured experimentally and the latter is computed based on the simple geometric relation for a perfectly rigid airfoil: $h_{p,r} = 2 \cdot 0.75 \sin(\alpha_0)$.

For simplicity, h_p will be used from here on in place of $h_{p,m}$.

The single period of the motion trajectory is calculated and stored in a 1000-element-array on the controller. The time between each element is set based on desired period of the motion. The period of the motion is always an integer number of milliseconds, with no long-term deviation which the controller is able to repeat the motion cycle with no long-term deviations. The controller linearly interpolates the position array as needed at each servo loop period (0.5 ms in these experiments) and repeats the motion cycle until commanded to stop. Starting and stopping of the motion is ramped up and down by linearly adjusting the amplitude over 3 motion cycles. The motion, as indicated by the phase angle of 180° starts with the airfoil's trailing edge moving upwards (negative angle of attack).

2.2.3 Measured Trajectory

The deviation from sinusoidal motion has been characterized for all data sets used in this study. The instantaneous angle of attack, measured by the rotary motor encoder, was recorded for each image pair (synchronized to the first image of the pair) used in measuring the flow field. The dataset was phase ordered based on the known period of the motion and the image acquisition frequency (23.93 Hz). The phase ordered position data were fit, using the linear least squares method, to the sinusoidal function given by Equation 2.1 with α_0 , ϕ_0 , and α_m as unknowns. Each dataset was analyzed to determine the deviations from the best fit sine function in terms of the maximum deviation, amplitude, mean angle of attack, and the RMS deviation from the fitted trajectory as overall metrics of the accuracy of the motion.

The maximum deviation was less than 0.08° and the RMS deviation less than 0.02° for 95% of all motion trajectories considered. The maximum deviation is shown in Figure 2.5 and the RMS deviation is shown in Figure 2.6 for both the rigid and flexible airfoils. The average deviation from each of the datasets is plotted with the error bar indicating the 95% confidence interval of the entire data set which is comprised of approximately 36 sub-fields of view for the rigid airfoil and 40 sub-fields of view for the flexible airfoil. Overall, the prescribed motion trajectory was faithfully



Figure 2.5: *The average maximum deviation from the best fit sine wave of the motion trajectory.* Abbreviations for the rigid airfoil (R) and flexible airfoil (F) are used in the legend. The error bars indicate the 95% confidence interval of all of the independent datasets of each motion trajectory.

created experimentally.

In addition to the measured trajectory from the rotary motor's encoder, the shape of the airfoil at the bottom tip of the airfoil, closest to the false wall in the middle of the tunnel, was also recorded. The primary motivation of these measurements was to document the flexible airfoil's deformation, as described in Section 2.3.5, but these measurements were also used to measure the spanwise bending of the airfoil. The spanwise bending of the airfoil needs to be checked to verify the prescribed 2D motion satisfactory describes the actual experimental conditions since the airfoil is not being held at the other end. The cross-stream displacement of the quarter chord position was determined from the shape deformation images and displayed in Figure 2.7. The data show that with increasing reduced frequency, the amount of non-ideal deflection of the airfoil's tip increases, however, only up to a maximum of 0.4% of the chord (0.48 mm).



Figure 2.6: *The average RMS deviation from the best fit sine wave of the motion trajectory.* Abbreviations for the rigid airfoil (R) and flexible airfoil (F) are used in the legend. The error bars indicate the 95% confidence interval of all of the independent datasets of each motion trajectory.

2.3 Airfoil Properties

The manufacturing of chordwise flexible airfoil shapes was undertaken using a state of the art multi-material 3D printer. This technology enables single piece construction of parts with varying levels of flexibility. In addition to a baseline fully rigid airfoil, regions of the substructure can be modified to have a flexible leading or trailing edge (or any proportion) while maintaining the same geometric shape. This is a fundamentally different approach than previous studies, such as: Monnier et al. (2015), Heathcote & Gursul (2007), and Thiria & Godoy-Diana (2010) that relied on shape variations (typically by varying the thickness of a material) in order to modify the flexibility, or in the case of Marais et al. (2012) a rudimentary approximation of an airfoil profile. Utilizing the 3D printer is also simpler than the cast urethane models used by Prempraneerach et al. (2003) and

Figure 2.7: Spanwise bending measured at the tip of the airfoil. The cross-stream displacement of the quarter-chord (pitch axis) is plotted against the reduced frequency. The ordinate is in % of chord; for reference, 0.4%c is 0.48 mm. The error bars indicate the 95% confidence interval of the measurement.

Egan et al. (2016) and provides more flexibility in the varying the model's chordwise flexibility. The state of the art multi-material 3D printers are not without fault, as there are many nuances that need to be considered, that will be discussed in the rest of this section, to ensure the final output is of sufficient quality.

2.3.1 3D Printer Description

The printer, an Objet Connex350, is an inkjet-based 3D printer that uses photopolymers that cure to form materials ranging from rubber-like materials (with a selectable Shore Hardness values in the range of 27-95) and plastic-like material similar to PVC. While there are a wide array of different materials available for the printer (and the materials are under continual improvement by the company), the two materials used in this study are VeroWhitePlus (RG835), an opaque rigid material and Objet TangoBlackPlus (FLX980), a rubber-like material. The printer has a resolution

of 30 μ m in the vertical build direction while utilizing these materials and a resolution of 42 μ m in the horizontal directions. The maximum print size is 340 mm × 340 mm × 200 mm. In order to achieve uniform flexibility in the spanwise direction only a single part construction is considered; which limits the span of the airfoil to 33.7 cm.

2.3.2 Material Testing

The two materials utilized by the Objet Connex 350 printer in the current study are the rigid opaque Objet VeroWhitePlus (RG835) and the rubber-like Objet TangoBlackPlus (FLX980). While some material properties are documented and specified from the manufacturer, many of them are estimated and can vary greatly depending on printer configuration. Additionally, with no institutional knowledge with regards to their photochemical properties and material compatibility with our MTV triplex some basic material tests were required. These tests aimed to assess the feasibility of using these materials in this project by determining their compatibility with our MTV triplex, the effect of prolonged fluid submersion, and their ability to withstand exposure to high intensity UV laser light.

Initial tests were performed on a NACA 0012 airfoil profile whose first half of the chord was the rigid material and the latter half was the pure flexible material. The model had a 12 cm chord and a span of only 5.1 cm as shown in Figure 2.8. Immediately, several less than ideal 'features' of the printing process became apparent. The first was delamination at the rubber and plastic interfaces as shown in the image. The second was a considerable "drooping" of the rubber portion near the trailing edge (not visible in the image) in the downward direction relative to when it was printed; e.g. gravity appears to pull the flexible material down slightly during curing.

Initial structural response tests involved pitching the airfoil over a large range of frequencies and amplitudes to address its deformation relative to a rigid airfoil. These initial tests indicated that the NACA 0012 model's trailing edge only marginally (10-15%) deformed more than a rigid airfoil. Increasing the aeroelastic effects would require either a more flexible material, which is not available using this 3D printer, or a thinner airfoil; to this end, a NACA 0009 profile was designed

Figure 2.8: *Test specimen of 3D printer*. The airfoil has a chord of 12 cm and a span of 5.1 cm. A tapped 1/4-20 hole at the 0.25*c* location was used to pitch the airfoil. The dark rectangular marks on the plastic (white) surface are burn marks from an unfocused 308 nm laser beam. The small chuck of rubber (black) is detached and the interface plane between the rubber and plastic parts can be seen to have delaminated.

for use in this study.

The photochemical properties and the material's interaction with the MTV triplex was studied. The surface of the part was imaged as a pulsed Excimer laser beam (308 nm) was directed at the surface while it was in air. As shown in the image in Figure 2.8, the plastic material readily left burn marks after very few shots. The rubber material did not readily burn, although the surface of the material did heat up over time. No fluorescence or phosphorescent emission from the materials were visually observed. The model was submerged in the MTV triplex (see Section 2.4 for details on the composition of the solution) for a several minutes, wiped dry, and re-tested in air. The plastic material readily emitted a green phosphorescence, the rubber material did not have any noticeable emission.

To further quantify the emission from the material after its exposure to the MTV solution, the test was repeated but the results were quantified with a simple camera setup, recording images at different time delays relative to the laser pulse. The average intensity of the spatial region in which the laser beam was impacting the surface was used to estimate an exponential lifetime of the observed emission. The TangoBlackPlus material shows an exponential decay lifetime of τ =0.3 ms, whereas the VeroWhitePlus material had a lifetime of τ =3.5 ms (see Figure 2.9). The intensity between two tests were directly comparable, indicating that not only is there a considerable lifetime,

Figure 2.9: *Photochemical properties of 3D printer materials after exposure to MTV triplex.* The spatially averaged intensity of a region of the 3D printed airfoil excited by a 308 nm pulsed laser at different time delays relative to the laser pulse. Data has been normalized by the RGD835 (rigid opaque) material.

but the intensity is initially over 10x higher with the rigid material. A reasonable explanation to the behavior observed is that the tracer, 1-Bromonathalene, has been absorbed by the plastic material but not the rubber material. The implications are that for near-surface measurements, the airfoils should utilize a rubber coating on the surface.

An early observation in the material also indicated that the rubber material would increase in flexibility over time as it was continually submerged. The flexibility of the material tends to increase several percent (measured by comparing the trailing edge deflection for a particular motion trajectory) every few hours. This trend continues for the first couple of days of submersion and only begins to stabilize after a week or longer of continual submersion. This effect is shown in Figure 2.10, which was characterized early in the material testing phase of the NACA 0012 profile. The maximum change in h_p at k = 12 being only 12%, motivated the future use of the NACA 0009 profile to increase the effect of flexibility. Several of the test conditions (k=22, and 32) far exceed
the conditions that will be used in this study, but are included as they were the only conditions to be tested at the 21 day submersion test. The lower k cases all appear to behave the same, with regards to the h_p increasing with submersion time at approximately the same rate up until the 7 day submersion test, so the conclusions reached with this test should be transferable to the lower k regime that will be considered in this study.



Figure 2.10: *Trailing edge deflection as a function of the submersion time.* The ordinate is the percent change in the trailing edge deflection of the flexible airfoil (h_{p-flex}) compared to the computed trailing edge deflection of rigid airfoil $(h_{p-rigid})$. These experiments were conducted on the NACA 0012 airfoil shown in Figure 2.8

Within a couple days of submersion, the airfoil's rubber coating would begin to blister and delamination between the rubber and plastic parts would start to develop. With the duration required for achieving a constant level flexibility is over a week, an alternative method for conditioning the material to achieve a constant level of flexibility is required. The process developed started with precondition the airfoil in a saturated air environment for several weeks prior to the experiments, which softened the material without causing blistering. The airfoil was submerged in the water tunnel prior to the experiments for two hours before flowfield measurements were performed. The

flowfield measurements on each day consisted of two sub-fields-of-view (sFOV) for which the entire parameter space was repeated twice to increase the overall data resolution (see Section 2.4.3 for complete details). The airfoil was raised out of the water and wrapped in plastic to keep the airfoil wet at the end of the second sFOV approximately after 6 hours of continuously running experiments or 8 hours of total submersion time. The process was repeated daily until the entire flowfield was acquired.

Verification that this process produced a constant level of flexibility throughout the experiments is quantified by measuring the changes in the trailing edge deflection, h_p , which was measured throughout the experiments on the flexible airfoil. Complete details of the shape tracking methods and results are presented in Section 2.3.5. The first metric that will be considered is the change in flexibility between the two sFOVs acquired on a single day. Figure 2.11 shows the percent change in h_p from the first to the second sFOV. The average change between the two sFOVs is plotted against k with error bars indicating the 95% confidence interval of all of the FOVs. The average was computed from the 21 pairs of sFOVs used in mapping the entire flowfield. The second sFOV produces, on average across all k and α_0 , a 0.12% higher h_p than the first sFOV of the day. Overall, the deviations in h_p between the two sFOVs acquired on the same day is less than 1.5%. Figure 2.12 depicts the percent change in the average h_p measured for all (42) sFOVs as error bars representing the 95% confidence interval. The variation in h_p increases with increasing k but remains within 4% for all motion trajectories over the entire duration of the flexible airfoil experiments. The $\alpha_0 = 2^\circ$ cases vary more than $\alpha_0 = 4^\circ$ and $\alpha_0 = 6^\circ$, both of which see less than 3% variation for all cases. In physical units, the deviation in h_p by 3% for the largest h_p observed at k = 6 and $\alpha_0 = 6^\circ$ is 0.7 mm, while the maximum deviations of 4% for the highest h_p observed for $\alpha_0 = 2^\circ$ at k = 12is 0.4 mm. In summary, the change in flexibility throughout the course of the measurements has been well controlled with the procedure outlined and the change in flexibility, as measured by h_p has been documented. Further analysis and the full presentation of the measured h_p is located in Section 2.3.5.



Figure 2.11: *Change in the trailing edge deflection from the first sFOV of the day to the second sFOV.* Error bars indicate a 95% confidence interval of all of the FOVs in the flexible airfoil results (21 FOVs total).

2.3.3 Design and Shape Validation

The internal structure of the airfoil was designed to address several of the issues discovered during early testing of the 3D printer's materials. The design provisions included: (1) use of a rubber coating to prevent surface emission, and (2) a remedy for the observed drooping of the trailing edges. A cross sectional view of the rigid and flexible airfoils can be found in Figure 2.13. The chord length, c, of the airfoil is nominally 12 cm in length.

The first of the provisions listed above was implemented by including a 0.51 mm thick shell of a rubber coating on the entire outer surface of the airfoil; which was the minimum thickness that could cure properly to the plastic material without delamination issues. The second provision



Figure 2.12: *The change in the trailing edge deflection across all FOVs of the flexible airfoil.* The error bars indicate a 95% confidence interval of h_p from the average h_p of all FOVs (42 total sFOVs).



Figure 2.13: *Cross-sectional views of the 3D printed airfoils.* The rigid (top) and flexible (bottom) NACA 0009 airfoils printed on the 3D Printer. The flexible TangoBlackPlus (FLX980) material is shown in black and the rigid VeroWhitePlus (RG835) material is gray. The internal void in the part is where a stainless steel shaft (1.000 in \times 0.250 in with 0.080 in chamfers) is centered at 0.25x/c and is used throughout the span to interface the airfoil with the pitch motor.

entailed the introduction of a piece of solid plastic material in the last 14.2% of the chord. This added structure helps preserve the trailing edge shape, and since it runs the entire span, the structure also improves the two dimensionality of the chordwise flexing. However, the very tip of the trailing edge (0.01c or 1.2 mm) was still deformed in both the rigid and flexible airfoils and was removed, leaving the airfoil with a slightly blunt trailing edge of 0.4 mm, compared to the theoretical trailing edge thickness of 0.23 mm.

The airfoil cross sections shown in Figure 2.13 are colored to indicate the material used in each of the different regions. The void in the airfoils is centered at the quarter chord to make room for a 1.000 in \times 0.250 in stainless steel shaft with 0.080 in chamfers which runs through the entire span of the airfoil and connects, via several connectors and adapters to the pitching motor. The flexible material starts to make up the airfoil at 0.4*c*, which can be seen most easily in the top of Figure 2.14.

The actual shape of the airfoils (when not moving) was measured by imaging the surface of the airfoil at midspan. A thin (2 mm) laser sheet illuminated the surface so it could be imaged and manually located with an accuracy of ± 0.08 mm ($\pm 0.07\%$ of the chord). The top and bottom surface were imaged in this fashion and combined together to estimate the airfoil thickness distribution shown in Figure 2.14. This is the thickness distribution of the airfoil relative to the chord line.

The thickness distribution of the rigid and flexible airfoils agree very well up until x/c=0.35 where the flexible airfoil starts to lose thickness compared to the rigid airfoil as the internal material starts to rapidly transition from the rigid to rubber material. The thickness of the two airfoils also agree very well again after x/c=0.75. Between x/c=0.35 and x/c=0.75 the difference in thickness between the rigid and flexible airfoil has a maximum value of 0.005c or 0.65 mm compared to the airfoil's maximum thickness of 10.8 mm. The overall shape of the both the rigid and flexible airfoils along with the theoretical profile, in an exaggerated scale, in Figure 2.15. Overall, the rigid and flexible airfoils compare well against the NACA 0009 airfoil, especially towards the leading edge, but both have slight deviations compared to the theoretical profile and to each other. Direct comparisons will be made in Section 3.1.1 between numerical simulations of a NACA 0009



Figure 2.14: *Measured thickness distribution of the rigid and flexible airfoils.* The thickness distribution measured at the mid-span. The internal structure of the flexible airfoil is shown above the plot for reference.

airfoil and the rigid airfoil measurements that will provide additional evidence that the measured differences in airfoils static shapes are not significant.



Figure 2.15: *Measured airfoil profile of the rigid and flexible airfoils.* The measured airfoil cross section (at mid-span) of the rigid and flexible airfoils compared to the theoretical NACA 0009 airfoil section

2.3.4 Estimated Flexibility

Non-dimensional analysis applied towards the effect of flexibility has been well discussed in the literature; with the typically two non-dimensional parameters surfacing, one related to the ratio of the densities between the airfoil and its working fluid, and the other relates the bending rigidity of the airfoil to the aerodynamic forces. Heathcote & Gursul (2007) define the bending stiffness parameter using a method they attribute to being similar to that of Murray (2000). Their bending stiffness parameter is

$$\lambda_x = \frac{(EI)_x}{0.5\rho U_{\infty}^2 Sc^2},\tag{2.2}$$

where $(EI)_x$ is the flexural stiffness in the chordwise direction, *S* is the planform area, and *c* is the chord. This is similar, as they discuss in Gursul et al. (2014) to the "effective stiffness", Π_1 , used by Shyy (Shyy et al., 2010; Kang et al., 2011). The primary difference between the two is Π_1 written explicitly for an isotropic plate using the classical thin plate bending theory for the flexural stiffness. The Π_1 term is given by

$$\Pi_1 = \frac{Et^3/(12(1-\nu^2))}{\rho U_\infty^2 c^3},\tag{2.3}$$

where *E* is again Young's modulus, *v* is Poisson's ratio, and *t* is the thickness of the plate. The density of the fluid (ρ), the freestream velocity (U_{∞}), and the chord length (*c*) are also used in the Π_1 term. In the current study, both terms will be estimated for comparison, but the first term, λ_x , is more directly applicable as the current study uses relatively thicker airfoils than the flat plates Π_1 was written for.

The flexible material used in this study, TangoBlackPlus (FLX980), has a manufacturer supplied data on the Shore Hardness value of 26-28 on the A scale (Stratasys, 2017). Converting between Shore Hardness and Young's Modulus (*E*) is not straightforward. A recent method for converting between Shore Hardness scales and *E* was developed in Mitchell et al. (2011), the details of which are excluded for brevity, leads to Young's Modulus estimates of $E=1.3\pm0.06$ MPa for the FLX980 material. The uncertainty range is based strictly on the Shore Hardness range specified by the manufacturer. The increase in flexibility of the material that was observed due to prolonged

	λ_x	П1		
Rigid	1.22×10^{3}	1.45×10^3		
Flexible	1.04	2.30		

Table 2.1: *Non-dimensional bending stiffness parameters of rigid and flexible airfoils*. Estimated non-dimensional bending stiffness of the rigid and flexible airfoils.

submersion (see Figure 2.10) most likely decreases this value of *E* by 10%-20%, but no further material testing were performed to improve the estimates of this value. Mitchell et al. (2011) state that the Poisson's ratio for rubber is well estimated by ν =0.5.

The manufacturer (Stratasys, 2017) provides the Young's Modulus of the rigid material used in this study, VeroWhitePlus (RGD835), as E=2 GPa-3 GPa. A value of 0.4 is used for Poisson's ratio, which is a common value for nylon. The average thickness of airfoil, 0.0411*c* or 4.93 mm, is used as the thickness of the plate for computing Π_1 . The chord length used for the flexible airfoil is 0.6*c*, which is the flexible portion of the airfoil (as shown in Section 2.3.3). When computing for the rigid airfoil, the entire chord length is used for *c* in the equations of λ_x and Π_1 . The average thickness of only the flexible portion is used when computing the area moment of inertia for the flexible airfoil, while the average thickness of the entire airfoil is used for the rigid airfoil. The average value in the range for *E* will be used in estimating λ_x and Π_1 .

The computed values for non-dimensional bending stiffness are summarized in Table 2.1. The Π_1 parameter for the rigid airfoil is estimated to be $\Pi_1 = 1.45 \times 10^3$ for the rigid airfoil, and $\Pi_1 = 2.3$ for the flexible airfoil. The rigid airfoil in the study by Monnier et al. (2015) was $\Pi_1 = 97$ and their flexible airfoils were in the ranged of $0.6 < \Pi_1 < 5.3$. In the study by Heathcote & Gursul (2007), λ_x varied from 0.07 to 260, compared to $\lambda_x = 1.04$ for the flexible and $\lambda_x = 1.22 \times 10^3$ for the rigid airfoil of the current study. This puts the current study within the range of "moderate" flexibility where it would be expected, based on these and other studies, to experience increased thrust and efficiency compared to the rigid airfoil.

2.3.5 Deformation Tracking

The shape of the flexible airfoil is monitored throughout the experiments by imaging the tip of the airfoil nearest to the bottom of the tunnel. This was accomplished by installing a Point Grey Grasshopper3 GS3-U3-28S4M camera with a 1928 x 1448 pixel CCD underneath the tunnel as pictured in Figure 2.16. This camera setup utilizes a mirror mounted at 45° relative to the bottom wall of the test section such that it imaged the end of the airfoil. The airfoil's tip itself was painted with a white rubberized coating (Rust-Oleum LeakSeal) to create contrast between the airfoil and the background as shown in the raw example image in Figure 2.16. The tip of the airfoil was illuminated by an LED flashlight that was externally focused and the beam was blocked such that it only illuminated the end of the airfoil and did not interfere with the velocimetry measurements. The camera was synchronized with the rest of the imaging system during the experiments.

The trailing edge displacement, h_p , was measured from the shape deformation images for each dataset to ensure the day to day repeatability of the flexible airfoil's flexibility, as discussed previously. The actual trailing edge deflection measured is compared to that based on the prescribed motion of the rigid airfoil in Figure 2.17. The trailing edge deflection normalized by the calculated (rigid) trailing edge deflection based on the prescribed motion $(h_{p,r})$ is shown in Figure 2.18. For all amplitudes the ratio of $(h_p - h_{p,r})/h_{p,r}$ increases with reduced frequency at the same rate until k=4, where the rate of $\alpha_0 = 6^\circ$ begins to decrease, while the cases with $\alpha_0 = 4^\circ$ stay with the $\alpha_0 = 2^\circ$ cases until k=6, where its rate of increase too begins to decrease. For the $\alpha_0 = 2^\circ$ trajectories, there appears to be a peak in the trailing edge deflection at k=12, after which the trailing edge deflection is reduced with increasing k.



Figure 2.16: Shape monitoring camera setup and sample image. The shape monitoring camera setup (top) is installed underneath the test section. The airfoil is not shown in the within the test section. The sample image (bottom) is for the flexible airfoil oscillating at k=8 with $\alpha_0 = 6^\circ$. The shadows and bright spots in the bottom image is primarily due to non-uniform illumination and the applied coating of Rust-Oleum LeakSeal bleeding slightly around the edges of the airfoil.



Figure 2.17: *Trailing edge displacement for all motion trajectories of the flexible airfoil.* The solid lines indicate the trailing edge displacement of the rigid airfoil, with the color indicating the α_0 . The error bars indicate a 95% confidence interval of h_p from all FOVs.



Figure 2.18: *Relative change in trailing edge displacement.* The percent change in the trailing edge deflection for all motions of the flexible airfoil. The error bars indicate a 95% confidence interval of h_p all FOVs.

2.4 Molecular Tagging Velocimetry

2.4.1 Background

The current experiments utilize molecular tagging velocimetry for measurements of the velocity field of the oscillating airfoils. Molecular Tagging Velocimetry (MTV) is a whole-field optical technique in which naturally occurring or premixed molecules (or atoms) in a flowing medium are turned into long lifetime tracers through photon excitation of an appropriate wavelength. Typically, a pulsed laser is used to excite or 'tag' regions of interest and the resulting emission is interrogated twice with a prescribed time delay to form an image pair. Spatial gradients in intensity form regions of interest in the image pair that are correlated with one another to yield a measurement of the Lagrangian displacement. Combined with the prescribed time delay between the images, the velocity field is determined as demonstrated in Figure 2.19. The reader is referred to Gendrich & Koochesfahani (1996) for details on the image processing procedures, Gendrich et al. (1997) for information on the phosphorescent supramolecule used in the current application, and Koochesfahani & Nocera (2007) for a comprehensive review of the development, photochemistry, and several applications of MTV.



Figure 2.19: *Example 2c-MTV image pair and resulting 2-component velocity vector field.* (a) The fluid flow of a vortex ring impinging on a flat plate with normal incidence is imaged 1 μ s after the laser pulse. (b) The same tagged fluid is interrogated again 8 ms after the first image. (c) The resulting 2-component velocity vector field is derived from correlating the undelayed (a) and delayed (b) images. (Gendrich & Koochesfahani (1996)) With permission of Springer

2.4.2 Current Implementation

In the current implementation, an excimer laser, a Coherent COMPexPro 205 XeCl (308 nm) is used as the excitation source. The optics and beam path used to create the tagging pattern are depicted in Figure 2.20. The optical assembly enables the tagging pattern to be placed in the test section along the streamwise, cross-stream and vertical directions independently. The current measurements are limited to a single horizontal plane at the midspan of the airfoils.



Figure 2.20: *Top view of optical beam path of excimer laser.* At the location of the red circle, indicated as the sheet forming optics, the beam is also directed vertically upwards from approximately 41 cm off of the floor, to the midspan of the airfoil, approximately 163 cm from the floor. As the beam travels vertically, it passes through the sheet forming optics. The airfoil depicted in the test section is 2.5 times larger than its actual size relative to the width of the test section width.

Referring to Figure 2.20, the beam path travels parallel to the test section before being directed vertically by a mirror that moves with the assembly only in the streamwise direction. The beam travels vertically to a set of sheet forming optics which move both in the streamwise and vertical direction with the rest of the optics platform. The sheet forming optics consist of a pair of lenses that form a very long focal length lens, creating a sheet thickness of <1 mm in the test section. The vertically traveling laser sheet passes through another mirror that turns the beam into the cross-stream direction on the measurement plane where the rest of the optics are located on a large wooden platform. The wooden platform moves with the overall system in the streamwise and vertical directions but is also moveable in the cross-stream direction to aid in repositioning the tagging pattern. While traveling on this platform the sheet first intersects a 50/50 beam splitter each of the two sheets are directed through 3:2 beam expanders in the horizontal plane (making the laser sheets $\sim 51 \text{ mm} \times 1 \text{ mm}$ in cross-section). Lastly, the sheets are directed into the test section and pass through brass beam blockers that have vertical slots to form the desired tagging pattern whose lines are approximately 0.6 mm in width and less than 1 mm thick in the spanwise (z) direction of the airfoil. Typically, there are 30 lines spaced approximately 1.4 mm apart from each laser sheet, which intersect to form approximately 800 intersections in each sFOV.

2.4.3 MTV Imaging System

The imaging system used for the current work utilizes two 1392×1024 pixel PCO.Pixelfly cameras which are mounted underneath the water tunnel as shown in Figure 2.21. The camera assembly is connected to the streamwise and vertical traverses such that it also moves with the entire optical assembly. It is on a separate cross-stream traverse since it must move at a different rate than the optical system due to the refraction of the UV beam passing through the quartz windows at an angle. A single 105 mm f/2.0 Nikon Nikkor Lens, a shortened F-mount to C-mount adaptor, and a standard 1" C-mount cube beam splitter from Edmund Optics completes the setup. The compact two-camera system relies on a calibration routine based on the Affine transform to compensate for large (greater than 10 pixel) sensor misalignments, which is described in detail in Appendix A, but

yields an uncertainty of less than 0.02 pixels RMS in calculating the displacement.



Figure 2.21: *Photograph of dual camera MTV imaging system.* Two PCO.Pixelfly cameras (blue) are flush mounted to a backing plate and a cube beam splitter (highlighted in red). A single lens is mounted to a shortened c-mount to Nikon F-mount adapter (highlighted green). The trigger boxes can be seen in the upper right of the image. The cameras are located underneath the water tunnel looking up towards the test section. The gray pipe in the bottom of the image is the return pipe of the water tunnel.

2.5 Experiments

2.5.1 Overview

The current study focuses on the wake structures of an oscillating rigid and flexible airfoil. The experimental study of Bohl & Koochesfahani (2009) will be used as a baseline comparison, and as such, the overall region of interest and required measurement resolution can be used as a guide for the current study. The measurement domain required to study the wake is nominally -0.1 < x/c < 1.5 in the streamwise direction, with the origin located at the location of the trailing edge of the stationary airfoil at $\alpha = 0^{\circ}$. In the transverse direction, the domain of interest is $\pm 0.3c$ over the entire streamwise extent of the domain but a larger extent of $\pm 0.6c$ is required at x/c = 1 where a control volume method for estimating the streamwise force (drag/thrust) will be applied. This region was shown previously in Figure 2.2. The required spatial resolution, based on the expected flowfield as measured by Bohl & Koochesfahani (2009), is approximately 0.01*c*, to accurately measure the expected vorticity field.

Multiple fields of view (FOVs) are required to perform the measurements at the desired resolution over the entire domain. The FOVs positions for the rigid airfoil measurements are shown in Figure 2.22, and Figure 2.23 for the flexible airfoil measurements. At each FOV displayed in the figures two sub-FOVs (sFOVs) are required to achieve the required spatial resolution. At these sFOVS the cameras do not move, but the laser tagging pattern is shifted by nominally one half of the grid spacing. The final velocity vector spacing is nominally 0.008*c* in both the streamwise and transverse directions. During the flexible airfoil experiments, a second set of FOVs centered at x/c=1 were acquired to have a complete redundant set of data for applying the control volume method (not shown on Figure 2.23). The camera's pixels were 2×2 binned on the sensor to improve image quality and acquisition rate; which was 23.93 Hz. Each FOV was approximately 5.5 cm × 4.1 cm (width × height) with a scale factor of 125 pixels/cm.

The experiments were carried out one field of view at a time. A calibration image was acquired for each field of view and is used to compute the Affine transformation for the two camera calibration,



Figure 2.22: *Fields of view for the rigid airfoil.* The dashed yellow lines are located at $\pm 0.6c$, the dashed red lines are located at $\pm 0.3c$ and the dashed vertical line is located at x/c = 1c. The airfoil is shown as black at an $\alpha = 0^{\circ}$, with its trailing edge located at (x, y) = (0, 0).

the image scale factor, and the sub-pixel tracking of the relative movement of one FOV to the next. To conduct the experiment, the airfoil starts from rest at $\alpha = 0^{\circ}$ and begins its motion. At least 50 convective times are allowed to pass after the start of each motion before measurements begin. After the conclusion of all measurements (described in the following paragraph) the motion is stopped and an additional 50 convective times are allowed to pass before the controller gains and settings are updated for the next set of motion parameters. The stop/start procedure was automated, but had to be monitored to ensure synchronization between the four separate acquisition systems. The laser grid pattern is manually shifted by approximately one half the grid spacing after all cases have been completed and the process is immediately repeated for the second sFOV.

The non-dimensional motion parameters for each of the airfoils are outlined in Figure 2.24,



Figure 2.23: *Fields of view for the flexible airfoils.* The dashed yellow lines are located at $\pm 0.6c$, the dashed red lines are located at $\pm 0.3c$ and the dashed vertical line is located at x/c = 1c. The airfoil is shown as black at an $\alpha = 0^{\circ}$, with its trailing edge located at (x, y) = (0, 0).

and represent 43 distinct trajectories for the rigid airfoil and 33 distinct trajectories for the flexible airfoil. The oscillation frequency ranged from 0.2 Hz to 4.8 Hz, and with amplitudes of $\alpha_0 = 2^\circ$, 4° , and 6° . This corresponds to a reduced frequency range of 0 < k < 18 and Strouhal number range of 0 < St < 0.6. The entire parameter space considered in this study is displayed in Figure 2.24, plotting *k* against the *St* of the prescribed motion. The mean angle of attack (α_m) was found to within $\pm 0.1^\circ$ by considering the symmetry of the velocity profile in the wake of the rigid airfoil.

The freestream velocity of the tunnel was set to 10 cm/s, as verified by independent 1c-MTV and 2c-MTV measurements at various parts of the test section (both with and without the airfoil present). The temperature of the water tunnel is monitored daily throughout all of the experiments and was in the range of 20.3° C and 23.3° C and is used in the computation of the viscosity based on



Figure 2.24: *Summary of all motion parameters.* The entire motion parameter space considered for the rigid and flexible airfoils.

the temperature dependence, which was measured to be the same as deionized water. Combined with a chord length of 12 cm yields a chord Reynolds number range of $Re_c = [11,900, 12,800]$ due to the viscosity change as a function of temperature.

The data set acquired for each case at each sFOV comprises of 2048 image pairs consisting of an undelayed image acquired < 5 μ s after the laser has fired and a delayed image acquired between 0.5 ms and 4 ms after the laser pulse (depending on the FOV and motion parameters). The exposure of the two cameras were identical, typically 1/10th of the delay time (select cases with very short delay times required longer exposures, but never exceeding an exposure to delay time ratio of 1/5). The airfoil's pitch angle is also recorded at the same time of each undelayed image. For the flexible airfoil experiments, the airfoil shape monitoring camera also records an image synchronized with each undelayed image.

The images are pre-processed by applying the Affine transform to the undelayed image se-

quence. The images are then processed using in-house cross-correlation program (Gendrich & Koochesfahani, 1996), the residual displacement field from the Affine transformation is subtracted, bad vectors are removed, and finally, the velocity field is phase-averaged relative to the airfoil oscillation cycle into 64 phase bins, such that there are 32 velocity samples per phase bin. The phase averaged velocity fields are combined from each sFOV and are remapped onto a regular grid and the vorticity is calculated using a central 4th-order finite difference method (Cohn & Koochesfahani, 2000). The processing requirements are as follows for a single motion trajectory at each sFOV: (1) pre-processing: 180 s, (2) cross-correlation: 2100 s, (3) two camera correction: 300 s, (4) data filtering: 120 s, and (5) phase averaging: 15 s, for a combined processing time of approximately 45 minutes. Combining all of the FOVs together and remapping onto a regular grid takes an additional 30 minutes or processing time per motion parameter.

2.5.2 Phase Averaged Data

Phase averaging is a useful measurement technique well suited to oscillatory flows where a single frequency (or its harmonics) dominate the flow. This is evident by considering the measured instantaneous velocity as

$$\mathbf{u}(x, y, t) = \overline{\mathbf{u}}(x, y) + \mathbf{u}'_{\phi}(x, y, \phi) + \mathbf{u}'_{1}(x, y, t) + \mathbf{u}'_{2}(x, y, t) + \mathbf{u}'_{3}(x, y, t)$$
(2.4)

where $\overline{\mathbf{u}}$ is the time average, \mathbf{u}'_{ϕ} is the oscillatory velocity fluctuation, \mathbf{u}'_1 is the cycle to cycle variation, \mathbf{u}'_2 is the random fluctuation from turbulence, and \mathbf{u}'_3 is the random fluctuation from measurement error. This is the same as the triple decomposition of Hussain & Reynolds (1970), but the third random term has been split into several distinct contributions. When \mathbf{u}'_{ϕ} is significantly larger than \mathbf{u}'_1 and \mathbf{u}'_2 then phase averaging will well represent $\mathbf{u}(x, y, t)$. Phase averaging will thereby also reduce the rest of the random velocity fluctuations; which includes the real cycle-to-cycle variations, turbulence, and measurement noise. Therefore, in oscillating airfoil experiments, phase averaging is commonly employed.

The phase of a particular velocity measurement may be computed by

$$\phi_i = i \times \frac{f}{f_{acq}}$$
, where $i = 0, 1, 2, ... (M - 1)$ (2.5)

where *M* is the number of samples in the acquisition (2048) and *f* is the frequency of the motion and f_{acq} is the acquisition rate of the data (23.93 Hz). The phase is reduced to the range of [0,1) by simply subtracting the integer number of cycles that have already occurred, i.e. $\phi = \phi_i - \lfloor \phi_i \rfloor$.

Phase averaging is the process of averaging the $\mathbf{u}(x, y, t)$ over a finite range of the the phase (ϕ_1, ϕ_2) where $\phi_1 = \phi - \Delta \phi/2$ and $\phi_2 = \phi + \Delta \phi/2$. Care must be taken with selecting f_{acq} and f to ensure each phase bin is equally populated. The size of the phase bin, $\Delta \phi$ is 1/64 for the current measurements, leading to having 64 phase bins whose center is $\phi_n = n(\Delta \phi) - (\Delta \phi)/2$ for n=1 to 64. The phase averaged streamwise velocity can be considered

$$\langle u \rangle (x, y, \phi_n) = \frac{1}{\Delta \phi} \int_{\phi_n - \Delta \phi/2}^{\phi_n + \Delta \phi/2} \left(\overline{u}(x, y) + u'_{\phi}(x, y, \phi) \right) d\phi,$$
(2.6)

in its integral form. The phase-bin-rms velocity, $\langle u \rangle_{rms}$ is defined as RMS of the instantaneous velocities that contributed to a single phase bin's $\langle u \rangle (x, y, \phi_n)$. It is a measurement of the overall fluctuations measured in the experiment, including cycle-to-cycle variation, turbulence, and measurement noise. If the phase bins are too large, the natural variation in the underlying value would contribute significantly to the phase-bin-rms as well.

The phase averaged variables are used to compute the overall mean velocity profiles $\overline{\langle u \rangle}$ and the fluctuating velocity profiles $\langle u \rangle'$. While $\overline{\langle u \rangle}$ is equal to \overline{u} , $\langle u \rangle'$ will be less than overall temporal RMS since the phase averaging process will remove some random noise (and some real components of the flow as discussed earlier).

2.5.3 Sample Measurements

A sample set of measurements of the rigid airfoil oscillating at a reduced frequency of k = 8, and a pitching amplitude of $\alpha_0 = 2^\circ$ (*St* = 0.13) are shown in Figures 2.25-2.27. The mean and fluctuating streamwise velocity are shown in Figure 2.25, while the transverse velocity component and its fluctuation are shown in Figure 2.26. A single vorticity field, extracted from $\phi = 0.5$ is shown in Figure 2.27 along with the uncertainty of the vorticity field. Further discussions of the uncertainties of the measured or calculated variables are included in the next sections and Appendix B.



Figure 2.25: *Representative mean and rms streamwise velocity field.* The calculated mean streamwise velocity (top) and its R.M.S fluctuation (bottom) for the rigid airfoil oscillating at k = 8 and $\alpha_0 = 2^\circ$ (*St* = 0.13). The black region in the lower left and lower center portion of the plots are regions where there was no velocimetry data.



Figure 2.26: *Representative mean and rms transverse velocity field.* The calculated mean transverse velocity (top) and its R.M.S fluctuation (bottom) for the rigid airfoil oscillating at k = 8 and $\alpha_0 = 2^{\circ}$ (*St* = 0.13). The black region in the lower left and lower center portion of the plots are regions where there was no velocimetry data.



Figure 2.27: *Representative vorticity field and its uncertainty.* The calculated vorticity (top) and its corresponding uncertainty (bottom) for the rigid airfoil oscillating at k = 8 and $\alpha_0 = 2^\circ (St = 0.13)$ extracted at $\phi = 0.5$. The cuttoff level in vorticity (top) is $-3 \le \langle \omega_z \rangle \le 3$ and there is no cutoff level used in the uncertainty contour (bottom). The black region in the lower left and lower center portion of the plots are regions where there was no velocimetry data.

2.5.4 Control volume method for estimating streamwise force

The control volume method for computing the streamwise force in the current study was described by Bohl & Koochesfahani (2009), whose inclusion of the fluctuating velocity profiles in the analysis demonstrated a marked improvement compared to the traditional analysis of using only the mean velocity profile. They found that only using the mean velocity profile would tend to overestimate the thrust produced by an oscillating airfoil. The control volume method is

$$C_{F} = \frac{2}{c} \int_{-H}^{+H} \left\{ \frac{\overline{\langle u \rangle}}{U_{\infty}} \left(\frac{\overline{\langle u \rangle}}{U_{\infty}} - 1 \right) + \varepsilon \left(\frac{\overline{\langle u \rangle}}{U_{\infty}} - 1 \right) + \left(\frac{\langle u \rangle'}{U_{\infty}} \right)^{2} - \left(\frac{\langle v \rangle'}{U_{\infty}} \right)^{2} + \frac{1}{2} \left(1 - \frac{U_{o}^{2}}{U_{\infty}^{2}} \right) \right\} \delta y,$$

$$(2.7)$$

where

$$\varepsilon = \frac{1}{2} \left(1 - \frac{U_0}{U_\infty} \right),\tag{2.8}$$

where U_0 is the average streamwise velocity at the extrema of the domain. In the current experiments U_0 is estimated by averaging 0.05*c* at the extrema of the streamwise velocity profile. The integral form of the equation is approximated using the numerical quadrature method with the current measurements at 1*c* downstream of the airfoil, with the the integration performed over the transverse region of $\pm H = \pm 0.6c$ from the center of the velocity profile.

A detailed analysis of the experimental uncertainty in C_T is relegated to Appendix B.4, where standard uncertainty analysis is used to propagate the experimental uncertainties of all of the measured quantities in Equation 2.7. Also included in this Appendix is a comparison of the C_T computed from each of the two independent sets of measurements acquired for the flexible airfoil. The uncertainty was, as expected, a function of the motion parameters, but the averaged 95% confidence interval for all cases was $C_T \pm 0.01$, with the uncertainty increasing with increasing St_{te} . From the repeated measurements for the flexible airfoil, all but 3 cases had differences in C_T less than $C_T = 0.01$.



Figure 2.28: Schematic of typical vortex pattern in wake of oscillating airfoil. A reverse von Kármán vortex street with the transverse spacing (*b*), wavelength (*a*) indicated. The trailing edge displacement (h_p) is also depicted of the airfoil pitching about its quarter chord. The sign of the vortex is indicated by the arrow and by the (+) or (-) sign.

2.5.5 Wake Vortex Properties

The properties of the vortices in the wake will be investigated in Chapter 4. The vortices of the rigid and flexible airfoils will be compared in terms of their strength, size, and spacing. In this section, these parameters will be defined and how they are calculated from the phase averaged experimental data will be described. These properties are extracted at x/c=0.5 downstream from the trailing edge of the airfoil. Only motion parameters producing a single pair of vortices, one of each sign, will be considered for analysis. An example of such a case was shown in Figure 2.27a and a schematic of this vortex configuration was shown previously in Figure 1.1, but is repeated in Figure 2.28 for convenience.

The vortices are located in the wake by monitoring the vorticity profile at extraction location, x/c = 0.5, throughout the entire phase of the motion. The maximum and minimum vorticity are found, recording their transverse location $(y_{c,p}, y_{c,n})$ and the phase $(\phi_p, \text{ and } \phi_n)$ at which these events occurr. The peak vorticity, $\langle \omega_{pk} \rangle$, are taken directly as these maximum and minimum vorticity values, respectively. The transverse location of the vortices are refined using a second order polynomial fit to the vorticity profile in the y-direction using ±2 grid points around the initial estimate of the peak location. The transverse spacing, *b*, is now computed as $b=y_{c,p} - y_{c,n}$. A conservative estimate of the uncertainty in b is $\pm 0.005c$, which is half the grid spacing.

The convective speed of the positive and negative vortices are computed by repeating the previous procedure at $0.25 \le x/c \le 0.75$. The phase at which the maximum (or minimum) vorticity arrives at the monitoring station is plotted against the monitoring station. A line is fitted to the extracted data and the inverse of the slope, $\Delta x/\Delta \phi$, is multiplied by the frequency of the motion, f, to yield the convective velocity, U_c . The streamwise spacing, or wavelength (*a*) is computed by

$$a = U_c/f. (2.9)$$

Further analysis of each (+) and (-) vortex is now performed. A vorticity cutoff is applied to the entire phase averaged vorticity field such that $|\langle \omega_{pk} \rangle| < 2s^{-1}$ is set to 0. This helps remove background noise in the vorticity field from effecting the integrated terms of the core size and circulation (to be defined below). The same vorticity cutoff limit was used by Bohl & Koochesfahani (2009) and Hammer (2016).

The circulation and core size, based on the radius of gyration, are computed from

$$\Gamma_0 = \iint \langle \omega \rangle \, dA,\tag{2.10}$$

and

$$r_c = \sqrt{\frac{\iint r^2 \langle \omega \rangle \, dA}{\iint \langle \omega \rangle \, dA}},\tag{2.11}$$

respectively. The integrals, estimated by numerical quadrature, are evaluated only within a radial distance of r=0.0833c from the center of the vortex to facilitate direct comparisons with previous studies (Bohl & Koochesfahani, 2009; Hammer, 2016). Additionally, only the same sign of vorticity as the vortex currently being analyzed is included in the integration. The effect of the radius of integration and vorticity cutoff on Γ_0 and r_c are discussed in Appendix C.

The core size, r_c , as estimated by the radius of gyration is more sensitive to the integration domain size and the vorticity cutoff level. It became necessary to include a secondary core size estimator to account for the wide range of parameters considered in this study compared to those of Bohl & Koochesfahani (2009). This alternative core size is the estimate of the gaussian core size, r_{γ} , computed from the best fit (in a least squares sense) to

$$\langle \omega \rangle (x, y) = \left\langle \omega_{pk} \right\rangle e^{-\left(\left(\frac{x - x_c}{r_{\gamma}} \right)^2 + \left(\frac{y - y_c}{r_{\gamma}} \right)^2 \right)}, \tag{2.12}$$

where the peak vorticity, vortex center (x_c, y_c) are also left as fit parameters. The same vorticity cutoff level is used but all vorticity within half a wavelength is included for determining the fit.

The positive and negative vortices are processed separately yielding individual measurements for Γ_0 , ω_{pk} , r_c , r_{γ} , a, U_c . The average value of the positive and negative vortices will be reported to focus the discussion of these properties and their dependence on k, St, and St_{te} . A brief comparison between the values obtained from the positive and negative vortices is included in Appendix F.

CHAPTER 3

STREAMWISE FORCE

In this chapter the phase-averaged results will be used to in conjunction with a control volume method to compute the mean streamwise force (drag/thrust) acting on oscillating airfoils. The chapter starts with a comparison of the rigid airfoil results to both previous computational and experimental data to establish a baseline for this study. The effect of using the thinner airfoil profile, a NACA 0009, rather than the NACA 0012 used in much of the literature on rigid airfoils is investigated. After validation of the baseline rigid airfoil, the effect of flexibility on the mean flow in the wake of the airfoils considered. Velocity profiles of the rigid and flexible airfoil are extracted at the same downstream location where the control volume method is applied for comparisons. The mean thrust of the rigid and flexible airfoils is compared and scaling based on k, St and St_{te} is considered.

3.1 Rigid Airfoil Results

The rigid airfoil serves as the baseline of comparison in this study. To this end, the rigid airfoil results will be compared against similar experiments and numerical simulations. Velocity profiles will be compared for a subset of trajectories considered in this study against the numerical simulations of Hammer (2017), which were performed with the NACA 0009 airfoil using the same approach employed in his extensive study of Hammer (2016) for the NACA 0012 airfoil. The experiments of Bohl & Koochesfahani (2009), a subset of those in Bohl (2002), are also used to for validation purposes. Lastly, the mean streamwise force is compared across a wider range of studies than mentioned above.

3.1.1 Velocity Profiles

The control volume method utilizes the mean and fluctuating velocity profiles in the wake of the airfoil to calculated the mean streamwise force acting on the airfoil. A starting point of comparison

	$\Delta\left(\overline{\langle u \rangle}/U_{\infty}\right)$		$\Delta\left(\overline{\langle v \rangle}/U_{\infty}\right)$		$\Delta\left(\langle u\rangle'/U_{\infty}\right)$		$\Delta\left(\langle v\rangle'/U_{\infty}\right)$	
k	Max	RMS	Max	RMS	Max	RMS	Max	RMS
4	0.1	0.02	0.03	0.02	0.05	0.01	0.06	0.01
6	0.05	0.01	0.02	0.01	0.05	0.01	0.04	0.01
8	0.03	0.01	0.01	< 0.005	0.04	0.01	0.04	0.01
10	0.04	0.01	0.01	< 0.005	0.06	0.01	0.02	0.01

Table 3.1: Deviations of rigid airfoil results and numerical simulations of NACA 0009. The maximum and RMS deviation between the velocity profiles extracted at x/c = 1 for the rigid airfoil and the numerical simulations of the NACA 0009 airfoil of Hammer (2017).

before calculating the thrust/drag, is the velocity profiles themselves. Figure 3.1 shows the velocity profiles extracted one chord length (1*c*) downstream of the trailing edge of the rigid airfoil and the simulations of Hammer (2017) which also used the NACA 0009 airfoil. The mean streamwise velocity profile, $\overline{\langle u \rangle}$, and its RMS fluctuation, $\langle u \rangle'$, the mean transverse velocity, $\overline{\langle v \rangle}$, and its RMS fluctuation, $\langle u \rangle'$, the mean transverse velocity, $\overline{\langle v \rangle}$, and its RMS fluctuation, $\langle v \rangle'$, are all compared at reduced frequencies of *k*=4, 6, 8, and 10. Overall there is excellent qualitative and quantitative agreement between the current study and the simulations; the details of which, are discussed below.

In comparison to the CFD, the measured mean streamwise velocity, shown in Figure 3.1a, has a maximum deviation of $0.10U_{\infty}$ at k = 4, while all other reduced frequencies have a maximum deviation of $\leq 0.05U_{\infty}$. The maximum deviation is less than 6% of U_{∞} for all of the other mean velocity and velocity fluctuation profiles. The average deviation, quantified by the RMS of the deviation of the entire profile is $\leq 0.02U_{\infty}$ for not only $\overline{\langle u \rangle}$, but also $\overline{\langle v \rangle}$, $\langle u \rangle'$, and $\langle v \rangle'$. These deviations are tabulated in Table 3.1 and indicate excellent quantitative agreement between the rigid airfoil and the numerical simulations at the conditions available for direct comparisons.

Overall, there is very good quantitative agreement between the simulations and experiments, the source of some of the deviations will be discussed next. The higher spatial resolution of the simulations is evident at the higher reduced frequencies in the streamwise fluctuation profile (shown in Figure 3.1b) near y/c = 0. The experiment's lower spatial resolution and inherent spatial smoothing makes the small localized peak in the profile just barely visible. The slight



Figure 3.1: Velocity profiles of the rigid NACA 0009 airfoil compared to the numerical simulations of Hammer (2017). The simulations have a solid line and the experiments have filled circles as markers with no connecting line with the color indicating the reduced frequency (k).

attenuation in the peaks in the RMS profiles of the experiment compared to the simulations is attributed to both the aforementioned spatial averaging of the experiment and phase averaging whereas the simulations have no phase averaging: the RMS profile is computed directly from the 96 instantaneous realizations available which were equally spaced in the phase of the motion.

Quantitatively, the experiments and the simulations deviate the most at a reduced frequency of k=4. Comparing the experiment to the simulation (whose results are nearly perfectly symmetric about the centerline) at k = 4 the experiment is seen to produce slightly asymmetric profiles. A number of experimental uncertainties can contribute to asymmetries, including the mean angle of attack and the asymmetry in the airfoil geometry itself. The flow itself can develop large asymmetries, which has been studied by Dong et al. (2006), Godoy-Diana et al. (2009), Marais et al. (2012), Calderon et al. (2014), and Wei & Zheng (2014). Most of these studies show that large asymmetries in the flow typically occur at high reduced frequency while the experiments of Bohl & Koochesfahani (2009) at $\alpha_0 = 2^\circ$ did observe a slight tilt in the flow field developing at k = 11.5. From the literature, most of the asymmetries in these flows are typically observed at high reduced frequency, so the small asymmetries observed in the current experiments compared to simulations are most likely due to the aforementioned angle of attack uncertainty and airfoil geometry.

While the maximum values of $\langle v \rangle$ are very small for all cases (less than 2.4% of freestream), there is a marked difference between the experiments and the simulations. The primary difference is the simulations have no value greater than 0.6% of freestream compared to the 2.4% seen in the experiments. However, the *v*-velocity profile is not used in the control volume method, so it will not directly impact the the estimates for drag/thrust. These differences can be linked to the simulations being 2D with a domain of over 100*c* in all directions compared to the *AR*=2.8 airfoil with sidewalls located at $y/c = \pm 2.5$ in the experiment. The true 3D nature of the experiments can lead to differences in *v*-velocity even with the *u*-velocity agreeing since there will also be a *w*-velocity.

Comparisons with the experiments of Bohl & Koochesfahani (2009) are slightly more difficult as the current study did not replicate the same exact reduced frequencies of the earlier measurements in addition to the difference in airfoil profile (NACA 0012 in Bohl & Koochesfahani (2009) compared to the current NACA 0009). The velocity profiles of the current rigid airfoil at k=0 (static airfoil), 5, 6, and 12 will be compared against k=0 (static airfoil), 5.2, 5.7, and 11.5 from Bohl & Koochesfahani (2009). While the reduced frequencies are close, the flow is highly sensitive to k and good quantitative agreement would not be expected. Figure 3.2 shows the comparison of $\overline{\langle u \rangle}$, $\overline{\langle v \rangle}$, $\langle u \rangle'$, and $\langle u \rangle'$ for these cases. A discussion of each k considered for comparison is discussed below.

The first observation of the differences between the current study and Bohl & Koochesfahani (2009) for the case of the static airfoil is the smaller width of the wake deficit region than the thicker airfoil; which can be observed in the profile of $\overline{\langle u \rangle}$ shown in Figure 3.2a. This an expected outcome due to the thinner airfoil section. The fluctuation in the mean velocity (Figure 3.2b) shows a qualitatively similar profile but is again thinner for the current experiment. The baseline freestream level of fluctuations is lower in the current experiments by approximately 20%-30% compared to the previous study. These differences are attributed to improvements in camera sensor noise between the studies and while the differences are only 2-3% of U_{∞} , the fluctuation profiles are used in estimating the streamwise force and hence fluctuation velocity errors would produce errors in the thrust/drag. However, this problem is unique to the static airfoil case as all other cases are phase averaged, which minimizes the influence of random noise on the measurements. As will be seen, for all of the moving airfoil cases, the baseline fluctuation levels between both studies are consistent with one another.

The next set of cases to compare are for a reduced frequency of k=5 for the current study and k=5.2 from Bohl & Koochesfahani (2009). Considering the mean streamwise velocity profile, both studies have a wake profile with a decreased maximum deficit compared the static case with a consistent value across both studies. Increasing the reduced frequency to k = 5.7, Bohl & Koochesfahani (2009) has captured the neutral wake, where the mean velocity profile is nearly U_{∞} across the entire transverse domain. The next case for the current study is k = 6, which has a peak mean streamwise velocity excess of $1.12U_{\infty}$, whose excess is the same as the velocity deficit seen at



Figure 3.2: Velocity profiles of the rigid airfoil compared to the experiments of Bohl & Koochesfahani (2009). The previous experiments have a solid line and the current experiments have filled circles as markers with no connecting line with the color indicating the reduced frequency (k). The previous experiments (B&K 2009) used the thicker NACA 0012 profile compared to the NACA 0009 airfoil of the current study.
k = 5 (0.88 U_{∞}), leading one to estimate, based on linear interpolation, the neutral wake condition of the current experiments to be approximately k = 5.5, slightly lower than what is observed by Bohl & Koochesfahani (2009) for the NACA 0012 airfoil. While Bohl & Koochesfahani (2009) did show that the neutral wake, which corresponds to the change in the vortex array pattern switching orientation from a von Kármán wake to a reverse von Kármán wake, is not sufficient for predicting drag-to-thrust crossover frequency, the decrease in k at which the neutral wake occurs in the current experiment indicates that there will be a difference in the drag-to-thrust crossover frequency. This is likely connected to the NACA 0009 airfoil section having a lower static drag coefficient than the NACA 0012 airfoil, which, will be considered in the next section.

At the highest k considered for comparison, k=11.5 of Bohl & Koochesfahani (2009) and k=12 for the current rigid airfoil continues to show deviations between the two cases. With the reduced frequencies differing by k=0.5, it is difficult to draw explicit conclusions regarding the differences between the two experiments. The same general trend, in both $\overline{\langle u \rangle}$ and $\langle v \rangle'$, is seen in each of the experiments as k is increased. The triple peak signature in the experiments of Bohl & Koochesfahani (2009) for $\langle u \rangle'$ for k=11.5 is not seen in the current experiments at k=12.

3.1.2 Streamwise forces

The streamwise force of the of the current experiments is computed using the control volume method described in Section 2.5.4. The drag/thrust of the rigid airfoil will be compared to the numerical simulations of the NACA 0009 airfoil by Hammer (2017) using both the control volume method and the values he obtained from the direct integration of the shear stress and pressure on the airfoil's surface. The discussion will shift to comparisons with the wider range of cases available in Bohl & Koochesfahani (2009) and Hammer (2016) in addition to several additional studies all of which used the NACA 0012 profile.

The mean streamwise force coefficient for the rigid airfoil at $\alpha_0 = 2^\circ$ is plotted against *k* in Figure 3.3 with the numerical simulations of the NACA 0009 airfoil from Hammer (2017) and the numerical simulations of the NACA 0012 from Hammer (2017). The static drag coefficient of the



Figure 3.3: *Mean* C_t *of the rigid airfoil and numerical simulations at* $\alpha_0 = 2^\circ$. The mean thrust coefficient (C_t) plotted against reduced frequency (k) for the rigid airfoil and compared to the numerical simulations of a NACA 0009 airfoil by Hammer (2017) and the simulations of a NACA 0012 airfoil by Hammer (2016). The control volume method is performed on the numerical simulations at k=4, 6, 8, and 10 for the NACA 0009, indicated by "C.V." in the legend, compared to "direct" which is the direct integration of the pressure and shear stress.

rigid airfoil is computed to be $C_d = 0.025 \pm 0.006$ compared to $C_d = 0.031$ of the simulations. At $k \le 6$, the current experiments are consistent with the numerical simulations of the NACA 0009 airfoil, while at higher k there is more deviation. At the reduced frequency of k=8, there is a large difference between the control volume method and the actual C_t than what is seen at other reduced frequencies. This shows that the control volume method may be responsible for a slight bias or shift in the estimated crossover frequency, but more further study is required. Looking at the curve as a whole, the crossover from drag to thrust of the experimental results appears to be shifted to a higher k than the numerical simulations beyond what is seen to be the influence of the control volume method. The numerical simulations (of the NACA 0009 airfoil) yield an estimate of the crossover

to be k=7.25 based on the actual C_t and k=8.25 based on the control volume method. The current experiments leads to an estimate of the crossover frequency at k=9. The crossover frequency of the simulations of the NACA 0012 airfoil is k=8.1. The NACA 0009 airfoil was expected (based on the simulations) to crossover at a lower k compared to what is seen in the experiment.

To put in perspective, it is noted that the discrepancy between the experimental and computation regarding the crossover frequency is within the scatter in the greater body of literature, which is shown in Figure 3.4. In this figure the thrust coefficient, C_t is plotted against k for the numerical simulations of Ramamurti & Sandberg (2001), and Young & Lai (2004), joining those of Hammer (2016, 2017), in addition to the experiments of Bohl (2002), Bohl & Koochesfahani (2009), and Mackowski & Williamson (2015). The current results of the rigid airfoil fall well within the scatter in the data from all of these studies, and tend to agree better with the experiments of Bohl (2002) and Bohl & Koochesfahani (2009) compared to the experiments of Mackowski & Williamson (2015). Plotting C_t against Strouhal number, St, which is shown in Figure 3.5, shows decent collapse in the data across different oscillation amplitudes, with the current rigid airfoil results falling within the scatter of the results from all of these studies for all of the amplitudes in the current rigid airfoil experiments estimate the drag-to-thrust crossover frequency to be St = 0.13 - 0.15.

Additional observations can be made utilizing the additional data in the literature to form a hypothesis regarding the differences seen in the drag-to-thrust crossover frequency between the rigid airfoil of the current study, the previous experiments of Bohl & Koochesfahani (2009) and the simulations of Hammer (2016, 2017). The aspect ratio of the numerical simulations is effectively infinite, while Bohl & Koochesfahani (2009) has an AR = 4, and the current experiments has an AR = 2.8, and the experiments of Mackowski & Williamson (2015) have an AR = 2.4. Referring to Figure 3.4, the crossover frequency for of these studies are k=8, $k=7.7^{1}$, k=9, and k=10.1, respectively for each of the studies in decreasing aspect ratio. This indicates that the aspect ratio may be playing a significant role in shifting the drag-to-thrust crossover frequency; but this cannot

¹The crossover frequency listed in Bohl & Koochesfahani (2009) is 8.7 ± 0.2 , but has since been updated to 7.7 ± 0.1 (but this has not yet been published).



Figure 3.4: *Mean thrust coefficient* (C_t) *plotted against reduced frequency* (k) *for various different studies.* The current results (Rigid) and Hammer 2017 utilize a NACA 0009 airfoil and the rest of the studies utilize a NACA 0012 profile. Color represents a single study, marker shape represents pitching amplitude α_0 , and markers outlined in black are numerical simulations while non-outlined markers are experimental studies. Legend entries have been shorted for space: B&K - Bohl & Koochesfahani (2009), R&S - Ramamurti & Sandberg (2001), Y&L - Young & Lai (2004), M&W - Mackowski & Williamson (2015).

be concluded with the data available as the current rigid airfoil is a NACA 0009 compared to the NACA 0012 of the rest of the studies.

In summary, the current rigid airfoil results compare favorably with previous experiments and computations. The NACA 0009 airfoil does differ slightly in the wake velocity profiles compared to the NACA 0012 airfoil, but it is seen as a slight shift in *k* attributed to the differences in the drag of the static airfoil. The simulations of Hammer (2017) indicate the NACA 0009 airfoil has a lower crossover frequency compared to the NACA 0012 (k=7.2 versus k=8.2). The control volume method was shown, again using the simulations of Hammer (2017), to shift the crossover frequency for the NACA 0009 airfoil from k=7.2 to k = 8.2 for α_0 = 2°. The rigid NACA 0009 airfoil of the current study has a crossover frequency of k=8.9, slightly higher than the simulations, but within



Figure 3.5: *Mean thrust coefficient* (C_t) *plotted against Strouhal number* (*St*) *for various different studies.* The current results (Rigid) and Hammer 2017 utilize a NACA 0009 airfoil and the rest of the studies utilize a NACA 0012 profile. Color represents a single study, marker shape represents pitching amplitude α_0 , and markers outlined in black are numerical simulations while non-outlined markers are experimental studies. Legend entries have been shortened for space: B&K - Bohl & Koochesfahani (2009), R&S - Ramamurti & Sandberg (2001), Y&L - Young & Lai (2004), M&W - Mackowski & Williamson (2015).

the uncertainty of the current experiment. At the higher oscillation amplitudes of $\alpha_0 = 4^\circ$ and $\alpha_0 = 6^\circ$ the current results for mean C_t agree with the computations of Hammer (2016) and Bohl (2002) to within the experimental uncertainty. With the current baseline measurements validated, the effect of flexibility on the the mean streamwise force will be explored next.

3.2 Effect of Flexibility

The effect of flexibility on the mean velocity profiles and the mean streamwise force is examined here. The first subsection will examine the velocity profiles extracted at x/c = 1; which are the same velocity profiles used in computing the streamwise force using the control volume method. The velocity profiles are compared for the same prescribed motions between the rigid and flexible airfoils, which highlights the direct effect of flexibility on the velocity fields. The following subsection examines the effect of chordwise flexibility on the streamwise force in terms of the prescribed motion, and the actual trailing edge amplitude. The effect of chordwise flexibility on the drag/thrust of a pitching airfoil will be shown to collapse when plotted against the Strouhal number based on the actual trailing edge amplitude (St_{te}).

3.2.1 Velocity Profiles

Figure 3.6 shows the velocity profiles of $\overline{\langle u \rangle}$, and $\overline{\langle v \rangle}$ while Figure 3.7 plots the velocity profiles of $\langle u \rangle'$ and $\langle v \rangle'$ for the oscillation amplitude of $\alpha_0 = 2^\circ$ and reduced frequencies of k=2, 5, 6, and 10. The discussion below outlines the effect of flexibility on the mean and fluctuating velocity profiles as a function of the reduced frequency.

At the lowest reduced frequency shown, k=2, the rigid and flexible airfoil have nearly the same mean and fluctuating velocity profiles. At this reduced frequency, the trailing edge displacement of the flexible airfoil is only $5.7\pm2.2\%$ (0.36 mm) larger than the rigid airfoil; this small change in h_p does not modify the flow significantly. However, at a k=5, h_p of the flexible airfoil is $16.3\pm3.2\%$ higher than the rigid airfoil and the change in the flowfield is significant. The mean streamwise velocity for the rigid airfoil has a wake profile while the flexible airfoil has switched to a jet-like profile, closer to the jet-like profile of k=6 for the rigid airfoil. The fluctuating velocity profiles convey a similar message with both $\langle u \rangle'$, and $\langle v \rangle'$, of the flexible airfoil at k=5 exhibiting velocity profiles closer to the rigid airfoil case at k=6.

The highest reduced frequency plotted for comparison, k=10, shows a much more significant difference between the airfoils than at the lower reduced frequencies previously discussed. The $\overline{\langle u \rangle}$ profile for both rigid and flexible airfoils have a strong jet-like profile, but the flexible's centerline velocity magnitude is $1.95U_{\infty}$ compared to $1.6U_{\infty}$ of the rigid airfoil. The width (in y/c) of the velocity excess is much larger for the flexible airfoil as well. The increase in width of the velocity profiles is also observed in the fluctuating velocity profiles. The notable increase in width is consistent with h_p of the flexible airfoil being $48\pm5.7\%$ higher than the rigid airfoil; making the



Figure 3.6: Selected mean velocity profiles at $\alpha_0 = 2^\circ$. Mean velocity profiles extracted at x/c=1 of both the rigid and flexible airfoils at an oscillating amplitude of $\alpha_0 = 2^\circ$.



Figure 3.7: Selected fluctuating streamwise velocity profiles at $\alpha_0 = 2^\circ$. Fluctuating streamwise velocity profiles extracted at x/c=1 of both the rigid and flexible airfoils at an oscillating amplitude of $\alpha_0 = 2^\circ$.

effective α_0 nearly 3° compared to the prescribed amplitude of $\alpha_0 = 2^\circ$.

Similar effects as those described for $\alpha_0 = 2^\circ$ are seen at the higher pitch amplitudes of $\alpha_0 = 4^\circ$ and $\alpha_0 = 6^\circ$. For brevity, only the mean streamwise velocities of both of the higher amplitudes are compared in Figure 3.8 for the rigid and flexible airfoils. While it appears, by comparing $\alpha_0 = 4^\circ$ and $\alpha_0 = 6^\circ$ to $\alpha_0 = 2^\circ$, shown in Figure 3.6a, that the effect of flexibility is reduced at the higher amplitudes, this is not the case. Comparing the same reduced frequency from different oscillation amplitudes yields qualitatively similar changes in the velocity profiles between the rigid and flexible airfoils. This is consistent with the relative change in h_p (see Figure 2.18) being nearly the same for different α_0 as long as k < 6.

3.2.2 Streamwise force

The mean streamwise force is computed from the control volume method, described previously in Section 2.5.4, using the velocity profiles extracted 1*c* downstream of the airfoil trailing edge for both the rigid and flexible airfoils. The mean thrust, C_t , is plotted against reduced frequency in Figure 3.9. Not all cases studied are included in the figure to focus the discussion on the behavior near the drag-to-thrust crossover; all cases can be located in Appendix D.

The static airfoil measurements were repeated twice at each field of view (FOV), one before all of the motion cases, and one at the end. The measurements on the flexible airfoil were repeated twice with a separate set of FOVs to yield a second, independent set of measurements for the mean streamwise force. Therefore, there are four measurements of the static drag coefficient for the flexible airfoil. The drag coefficient of the stationary rigid airfoil is C_d =0.026±0.006 and C_d =0.039±0.008 for the stationary flexible airfoil. While the flexible airfoil does have, on average, more drag than the rigid (reasonable based on the extra drooping trailing edge of the flexible airfoil) the differences are within the experimental uncertainty.

The flexible airfoil is shown to produce more thrust (or less drag) under nearly all reduced frequencies and amplitudes considered in the current study with a few exceptions at $k \le 2$. This shifts the drag-to-thrust crossover frequency to a lower frequency for all amplitudes. The highest



Figure 3.8: Selected mean streamwise velocity profiles at $\alpha_0 = 4^\circ$ and $\alpha_0 = 6^\circ$. Mean streamwise velocity profiles extracted at x/c=1 for selected reduced frequencies of both the rigid and flexible airfoils at an oscillating amplitude of $\alpha_0 = 4^\circ$ (a) and $\alpha_0 = 6^\circ$ (b).



Figure 3.9: *Mean thrust plotted against reduced frequency for rigid and flexible airfoils.* The mean thrust computed from the control volume method using velocity profiles extracted at 1*c* downstream of the trailing edge. Error bars indicate a 95% confidence interval.

amplitude's ($\alpha_0 = 6^\circ$) crossover frequency is shifted the least by only $\Delta k \approx 0.1$, while $\alpha_0 = 4^\circ$ is shifted by $\Delta k \approx 0.5$ and $\alpha_0 = 2^\circ$ is shifted the most by $\Delta k = 2.5$. While the increase in thrust for a flexible airfoil compared to its rigid counterpart is well documented in the literature (Dewey et al., 2013; Gursul et al., 2014), this study shows the influence of chordwise flexibility in shifting the drag-to-thrust crossover to a lower frequency for the flexible relative to the rigid airfoil.

Figure 3.10 shows the mean thrust coefficient plotted against Strouhal number (*St*). The rigid airfoil results collapse well (within the experimental uncertainty) for all α_0 as shown previously in Section 3.1.2 but there is considerable scatter for the flexible airfoil results. Figure 3.11 shows C_t plotted against the Strouhal number based on the actual trailing edge displacement, i.e. $St_{te} = fh_P/U_{\infty}$, and much better collapse for both rigid and flexible airfoils across all α_0 is observed.

Triantafyllou et al. (1991); Triantafyllou (1993) first proposed the use of Strouhal number as the parameter to describe the wake dynamics of flapping airfoils. Triantafyllou (1993), more explicitly



Figure 3.10: *Mean thrust plotted against Strouhal number for rigid and flexible airfoils.* The mean thrust computed from the control volume method using velocity profiles extracted at 1*c* downstream of the trailing edge. Error bars indicate a 95% confidence interval.

stated that *St* should be taken as the maximum excursion of the trailing edge (i.e. h_p in the current measurements) but the direct application of this definition to chordwise flexibility has not been seen in the literature. Thiria & Godoy-Diana (2010) still used *St* based only on the prescribed motion, or *St* based on other fixed length scales such as *c* in Heathcote & Gursul (2007), or airfoil thickness in Godoy-Diana et al. (2008, 2009) and Marais et al. (2012).Dewey et al. (2013) defines the Strouhal number in this way (based on the actual h_p) but present their data based with *k*, pointing out that *St*_{te} is an output of the system and therefore, not known a priori. The current study reaffirms that it is necessary to use the actual trailing edge amplitude to properly characterize these types of flows. Recently, Monnier et al. (2015) showed *St*_{te} is better at describing the characteristics of the flowfield (in terms of various vortex array properties) compared to *St* for pitching flexible airfoils of the same teardrop based design of Heathcote & Gursul (2007) at a lower Reynolds than the current study of $Re_c = 2 \times 10^3$.

As previously discussed, the mean thrust coefficient, C_t , when plotted against St_{te} is shown



Figure 3.11: *Mean thrust coefficient plotted against Strouhal number based on the actual trailing edge displacement for rigid and flexible airfoils.* The mean thrust computed from the control volume method using velocity profiles extracted at 1*c* downstream of the trailing edge. Error bars indicate a 95% confidence interval.

to collapse the data in Figure 3.11. The scatter in the crossover Strouhal number (St_{te}) is well within the scatter shown previously in Figure 3.5 for several different pitching airfoil studies. Considering botht the rigid and flexible airfoils for amplitudes studied, the crossover frequency is St_{te} =0.13±0.02. This collapse based on St_{te} shows that, to first order, the effect of chordwise flexibility can be considered as an amplitude effect, i.e. the peak to peak excursion of the trailing edge is sufficient to capture the effect of chordwise flexibility on the mean thrust. The implications of this is the mean thrust is not effected by the shape change (camber) of the airfoil in this study. However, the current study's chordwise flexibility is not "very" flexible compared to many other studies, and only the first bending mode is observed, so the implication of mean thrust not being effected by the shape change of the airfoil may not apply to all other levels of flexibility exhibiting more complicated structural dynamics. Studies investigating the shape change of thin (typically flat plate or membrane wings) and their effect on forces and efficiency can be found in the analytical models of Katz & Weihs (1978, 1979) and Moore (2014, 2015) and the experimental based discussions in Ramananarivo et al. (2011) and Dewey et al. (2013).

3.3 Summary

In this chapter the effect of chordwise flexibility on the streamwise force was investigated. The measured thrust based on the control volume method was compared for the baseline rigid NACA 0009 airfoil against several different numerical and experimental studies. The effect of the NACA 0009 airfoil was shown, via a comparison of 2D numerical simulations, to lower the crossover frequency relative to the typically studied NACA 0012 airfoil, but the current data were seen to have a crossover frequency higher than the previous experiments of Bohl & Koochesfahani (2009) at $\alpha_0 = 2^\circ$. These differences are attributed to aspect ratio effects, but requires further study. The current experiments at higher amplitudes ($\alpha_0 = 4^\circ$ and $\alpha_0 = 6^\circ$) were observed to agree well with the available data in the literature in terms of mean C_t . Chordwise flexibility was seen to increase the thrust of the oscillating airfoil under nearly all conditions. While that observation is not a new finding, *the shift in the crossover frequency due to chordwise flexibility* has not been reported previously in the literature. This study finds that the crossover frequency, at a prescribed oscillation amplitude, decreases due to chordwise flexibility.

CHAPTER 4

WAKE VORTEX PROPERTIES

In this chapter the phase-averaged results will be used to explore the effect of chordwise flexibility on the vortical patterns in the wake of oscillating airfoils. The chapter focuses primarily on motion trajectories which produce a single pair of vortices, one of each sign, in the wake per oscillation cycle of the airfoil. These wakes are typically referred to as either the classical von Kármán vortex street or reverse von Kármán vortex street, depending on their arrangement. The chapter begins with a comparison of the rigid airfoil results to both computational and experimental data to establish a baseline for this study. The effect of using the thinner airfoil profile, a NACA 0009, rather than the NACA 0012 used in the comprehensive studies of Bohl (2002), Bohl & Koochesfahani (2009), and Hammer (2016) is investigated. The chapter continues with a detailed comparison between the rigid and flexible airfoils in terms of the vortex strength (peak vorticity, ω_{pk} , and circulation, Γ_0), size (radius of gyration, r_c), and arrangement (wavelength, *a*, and transverse spacing, *b*). The definitions of these terms and how they are computed for the current experiments was described in Section 2.5.5. The appropriate scaling of these wake vortex parameters are discussed in the context of flexibility. The data and scaling parameters are compared, and contrasted, to those seen in the literature when appropriate.

4.1 Validation of Rigid Airfoil Results

4.1.1 Comparison with 2D Simulations of the NACA 0009 airfoil

The current study utilizes a NACA 0009 airfoil profile while in the recent studies of Bohl (2002), Bohl & Koochesfahani (2009), and Hammer (2016), who also investigated the wake vortex properties in the wake of an sinusoidally pitching airfoil, used a NACA 0012 airfoil. In the previous chapter the effect of the thinner airfoil profile on the mean flowfield was described, with the primary effect seen to be the lower static drag coefficient, which causes, for the oscillating airfoil, the crossover frequency from drag to thrust to occur at a lower reduced frequency than the NACA 0012. The simulations of Hammer (2017), which were conducted as described in Hammer (2016), but on the NACA 0009 airfoil are again used in this section to compare the vorticity field both qualitatively and quantitatively in terms of the wake vortex parameters.

The phase-averaged vorticity of the experiment is compared (at the same phase of the motion) to the instantaneous vorticity of the simulation in Figure 4.1 at reduced frequencies of k=4, 6, 8, and 10 at an amplitude of $\alpha_0 = 2^\circ$. Qualitatively, the experiments and computations produce very similar flow structures at each reduced frequency.

At k=4, the shear layer in the experiment is seen to be rolling up into two positive and two negative vortices by x/c=0.5 while the one in the computation computation doesn't roll up as quickly. The braid vorticity is observed in the experiments to be thicker, or more diffuse. The experiment is phase-averaged over 1/64th of the cycle in contrast to the computation which is an instantaneous result. The phase-averaging process will result in a small amount of spatial averaging related to how far the flow structures convect during one phase bin. Cycle-to-cycle variations in the spatial location of the flow structures also contribute to a certain amount of spatial averaging. At k=4, the flow structures (vortices) convect 0.0125c in during one phase bin (1/64th of the cycle), just over one experimental grid point. The flow structures convect faster at higher k, which, counter-intuitively, means the effective spatial averaging due to the phase-averaging process will be negligible at higher reduced frequencies and this explains why some of the braid vorticity appears to be more diffuse in the experiment than in the simulation.

At k = 8 and k = 10 the vortex array pattern is seen to tilt downwards in the experiment but not in the computation. A slight tilt in the wake is commonly observed in experimental studies; it can be observed in the classic study by Koochesfahani (1989), and Bohl & Koochesfahani (2009) also observe a slight tilt in the wake pattern at k=11.5. In general however, the wake of a sinusoidally pitching airfoil is expected to remain mostly symmetrical at these reduced frequencies as seen in the studies of Koochesfahani (1989), Bohl & Koochesfahani (2009), Godoy-Diana et al. (2008), Godoy-Diana et al. (2009), and others. The slight asymmetry in the baseline results is contributed to Computation

Experiment



Figure 4.1: Comparison of experiment and simulations of phase-averaged vorticity fields. The phase-averaged vorticity at the same phase of oscillation: computations (left) and experiment (right). The reduced frequency increases from top to bottom, with the reduced frequency listed as the title of each plot. The experiment and simulation have identical colormaps for the same reduced frequency, but the colormap limits increase as *k* increases. The phase shown corresponds to $\alpha = 0^{\circ}$ while the airfoil is pitching down (trailing edge moving upwards).

either by a slight non-zero mean angle of attack (the current experiment determined $\alpha_m = 0^\circ \pm 0.1^\circ$) and/or the asymmetry in airfoil profile discussed in Section 2.3.3.

Another observation between the experiments and the computations is the reduction in the rates at which the peak vorticity decreases as the vortices convect downstream. In the study by Bohl & Koochesfahani (2009), the streamwise variation in the peak spanwise vorticity is attributed to the combined effects of vortex viscous diffusion and vortex stretching due to spanwise velocity gradients, with the latter being the result of the axial velocity. Since the computations are 2D, they do not replicate this feature of the real flows. Further analysis of this effect is beyond the scope of the current study.

The wake vortex properties of the computations were analyzed with the same post-processing tools used in the current study to ensure a 1:1 comparison in the computed parameters. The ω_{pk} , Γ_0 , r_c , and b/a were computed at x/c=0.5 for both the rigid airfoil and the simulations of the NACA 0009 airfoil of Hammer (2017). The results are shown in Figure 4.2 for the average value of the positive and negative vortices (except in the case of *b*, since it is the transverse distance between the positive and negative sign vortex). A discussion of the differences between the positive and negative sign vortex. A discussion of the differences are small. Overall, quantitative comparison shows that the experiments and computations agree favorably as discussed in detail below.

At a reduced frequency of k=6, 8, and 10 there is less than 3% difference between the experiments and the computations for Γ_0 and a/c (not shown in Figure), with less than 9% difference for r_c . The transverse spacing normalized by the wavelength (b/a), shown in Figure 4.2(d), agrees with the computations within the experimental uncertainty. The peak vorticity in the experiment however, is ~10-20% less than the computations. The accuracy in measuring vorticity is not expected to contribute to this discrepancy; the remapping procedure and subsequent finite difference method used for calculating the vorticity of the experimental data is expected, based on the analysis of Cohn & Koochesfahani (2000), to be less than 2% of the peak vorticity and therefore, is also not significant enough to explain the differences. A brief study in the effect of spatial resolution on



Figure 4.2: *Vortex Properties of Rigid Airfoil and Simulations of NACA 0009.* The peak vorticity (a), circulation (b), radius of gyration (c), and the transverse spacing normalized by the wavelength (d) are the average value of the (+) and (-) sign vortex. Both the experiments (Rigid) and the numerical simulations (Hammer 2017) use the NACA 0009 airfoil profile. For the experimental dataset invisible error bars (e.g. k < 12 for Γ_0/cU_∞) indicate the error bar is less than or equal to the symbol size.

the extracted vortex parameters was conducted with the simulation data. The resolution of the simulation was reduced from 0.0025c to 0.01c to match the current experiments and the vorticity was calculated with the same method as used in the experiments (an explicit fourth-order-central finite difference method). Additional details of this analysis can be found in Appendix G. When the resolution is reduced to that of the experiment, the peak vorticity decreases by 1.5% to 8%. The remainder of the difference in peak vorticity is attributed to the previously discussed vortex stretching mechanism that the computations do not capture. Koochesfahani (1989) observed that the magnitude of the axial flow increases with increasing k, which is consistent with the growing discrepancy between the experiments and computations as k increases.

4.1.2 Effect of NACA 0009 on vortex properties

In preparation of validating the current experiments with the experiments of Bohl & Koochesfahani (2009), the effect of the thinner airfoil on the vortex properties is investigated with the numerical simulations. The computations of Hammer (2016) and Hammer (2017) are used for this purpose as the he used the NACA 0012 and NACA 0009 airfoils in each of the respective data sets. The vortex properties of ω_{pk} , Γ_0 , r_c , and b/a are shown in Figure 4.3 for the two computations and the rigid airfoil of the current study for reference.

The following discussion is based on the comparison between the NACA 0009 and the NACA 0012 airfoil of the numerical simulations, and is limited to $\alpha_0 = 2^\circ$. The peak vorticity at k=6, 8, and 10 is higher by 4-8% for the thinner airfoil than the thicker airfoil. The circulation is 3.5% less for k=6, but 1.9% and 2.6% greater for k=8 and k=10, respectively, for the thinner airfoil. The core size is similar to circulation in that the thinner airfoil has a smaller vortex at k=6 (by 1.6%) but is larger at k=8 and k=10 (by 2.7% and 2.1% respectively). The b/a spacing tells a more important (but still small) difference between the two airfoils: the switch in the vortex pattern from a von Kármán to a reverse von Kármán wake structure occurs at a lower k for the thinner airfoil. This is consistent with thinner airfoil having a lower drag-to-thrust crossover frequency which was seen (for the numerical simulations) in Chapter 3.



Figure 4.3: *Vortex Properties of the rigid airfoil compared to numerical simulations of the NACA* 0009 and NACA 0012 airfoils. The peak vorticity (a), circulation (b), radius of gyration (c), and the transverse spacing normalized by the wavelength (d) are the average value of the (+) and (-) sign vortex. Both the experiments (Rigid) and the numerical simulations of Hammer 2017 use the NACA 0009 airfoil profile while the numerical simulations of Hammer 2016 utilize the NACA 0012 profile. For the experimental dataset invisible error bars (e.g. k < 12 for Γ_0/cU_{∞}) indicate the error bar is less than or equal to the symbol size.

There is conflicting observations of the current experiment's rigid NACA 0009 airfoil regarding the reduced frequency at which the vortex pattern switches and the drag-to-thrust crossover frequency for $\alpha_0 = 2^\circ$. In Chapter 3 the drag-to-thrust crossover frequency for the rigid airfoil was shown to be higher than the numerical simulations of both a NACA 0009 airfoil and a NACA 0012 airfoil, and the experiments of Bohl & Koochesfahani (2009) of a NACA 0012 airfoil. The switch in vortex pattern of the current experiment's NACA 0009 airfoil however, is consistent with the simulations of the NACA 0009, with the switch in the vortex pattern occurring at lower k compared to the thicker NACA 0012 airfoil. Bohl & Koochesfahani (2009) pointed out that the switch in the vortex pattern is not sufficient in determining the drag-to-thrust crossover frequency. This study confirms this observation and notes the difference between drag-to-thrust crossover and vortex pattern switch is also not strictly associated with the static drag coefficient of the airfoil. The drag coefficient of the rigid airfoil was measured to be less than the drag coefficient of the experiment of Bohl & Koochesfahani (2009) on the NACA 0012 airfoil (which is consistent with basic knowledge, see Abbot & von Doenhoff (1959), of the effect of airfoil thickness on drag). If the difference between in the frequency at which the vortex pattern switches and the drag-to-thrust crossover frequency was strictly a function of the static airfoil's drag, this drag-to-thrust crossover frequency should have been lower than what Bohl & Koochesfahani (2009) measured, but, this is was not observed.

The larger difference between the frequencies at which the vortex pattern switches and drag-tothrust occurs must be related to the few remaining differences between the current experiment, the experiments of Bohl & Koochesfahani (2009) and the numerical simulations. It is hypothesized that the likely reason is the aspect ratio, with the current study being AR=2.8 compared to AR=4of Bohl & Koochesfahani (2009) and the infinite AR of the simulations of Hammer (2016, 2017). Unfortunately, the limitations imposed on the current study's primary goal of studying flexibility prohibited the aspect ratio to be matched between the current experiments and those of Bohl & Koochesfahani (2009). Further study of this effect is recommended.

4.1.3 Comparison with Experiments and 2D Simulations of the NACA 0012

In Section 4.1.1 the instantaneous vorticity field of numerical simulations on the NACA 0009 airfoil were shown to agree well with those of the rigid airfoil. In the previous section the effect of the thickness of the oscillating airfoil, the NACA 0009 compared to a NACA 0012, was seen to shift the b/a curve to a slightly lower frequency, but otherwise, there was little impact on the rest of the quantified vortex properties. Moving forward, only the computational results of the NACA 0009 to reduce from Hammer (2016) will be used rather than including the results from the NACA 0009 to reduce clutter in the plots since there is little difference between the vortex properties of the two airfoils extracted from the numerical simulations. In this section, the experimental study reported in Bohl (2002) and Bohl & Koochesfahani (2009) will be used for comparison in addition to the aforementioned computational study. The vortex properties are computed in the same fashion and at the same downstream location of x/c = 0.5 as performed in Bohl & Koochesfahani (2009). Overall, quantitative comparisons between the current experiment and the previous experiment of Bohl & Koochesfahani (2009) and Bohl (2002) are in good agreement and will be discussed below.

Figure 4.4 shows the values of ω_{pk} , Γ_0 , r_c , and b/a for three different pitching amplitudes, $\alpha_0 = 2^\circ, 4^\circ$, and 6° . The peak vorticity (ω_{pk}), Figure 4.4(a), is consistent with, and agrees well overall with the other experiment and the computation for all oscillation amplitudes. At the lowest amplitude of $\alpha_0 = 2^\circ$, the peak vorticity agrees more with the other experiment than the computation, which supports the previous explanation of the differences between the computation and the current experiments being primarily due to the 2D computations not being able to capture the vortex stretching phenomena and the finite aspect ratio of the experiments. There is good agreement, albeit with some scatter, for both $\alpha_0 = 4^\circ$ and $\alpha_0 = 6^\circ$ in terms of the peak vorticity. Circulation, core size, and transverse spacing normalized by wavelength, Figure 4.4(b), (c), and (d), respectively, all agree very well across all of the studies. The overall quantitative agreement between the baseline measurements of the rigid airfoil and the literature gives a strong indication that the current study is faithfully reproducing, and measuring, the desired flow conditions. The rest of this chapter is focused on the influence of flexibility on these vortex properties.



Figure 4.4: Comparisons of vortex properties of the rigid airfoil and other experiments and simulations of a NACA 0012 airfoil. The peak vorticity (a), circulation (b), radius of gyration (c), and the transverse spacing normalized by wavelength (d). are the average value of the (+) and (-) sign vortex. The current experiments (Rigid) use a NACA 0009 airfoil profile while the other studies, Bohl (2002), Bohl & Koochesfahani (2009), and Hammer (2016), all utilize a NACA 0012 airfoil profile. For the experimental dataset, invisible error bars indicate the error bar is less than or equal to the symbol size.

4.2 Effect of flexibility on vorticity field

The phase-averaged vorticity field is shown for $\alpha_0 = 2^\circ$ in Figure 4.5 for several different reduced frequencies ($2 \le k \le 10$). The differences between the rigid and flexible flowfields are subtle, but will be described next. The peak vorticity of the flexible airfoil appears to be higher than the corresponding vortex of the rigid case (for all k). The transverse vortex spacing (b) is notably higher at k = 8 and 10 for the flexible airfoil compared to the rigid airfoil. The vortex array at k=10 also appears to tilt slightly downwards in the rigid case (as mentioned previously) and slightly upwards in the flexible case. This difference is attributed to the asymmetries of the airfoil profiles (discussed in Section 2.3.3) and when the airfoils were installed in the test section the slight curvatures of their trailing edges' were oriented in opposite directions. The current study does not have enough information to confirm this is the only issue that may be contributing to the different directions of the the wake deflection. However, the slight tilt of the vortex array is not significant near x/c=0.5 where the vortex properties will be extracted, so the difference in the slight upwards vs. slight downwards tilt should be negligible regarding the vortex properties under consideration.

Recalling the relative change in trailing edge amplitude is greatest at $\alpha_0 = 2^\circ$, the changes in the flowfield are expected to be even more subtle at $\alpha_0 = 4^\circ$ and $\alpha_0 = 6^\circ$, and therefore, comparisons at other amplitudes is relegated to Appendix E. The next section will make quantitative comparisons of the vortex properties to illicit the effect of flexibility on the vorticity field.

Rigid

Flexible



Figure 4.5: *Phase-averaged vorticity field of rigid (left) and flexible (right) airfoils.* The reduced frequency increases from top to bottom, with the reduced frequency listed as the title of each plot in addition to the Strouhal number (St) and the Strouhal number based on the measured trailing edge displacement (St_{te}). The rigid and flexible plots have identical colormaps for the same reduced frequency, but the colormap limits increase as *k* increases. The phase shown corresponds to $\alpha = 0^{\circ}$ while the airfoil is pitching down (trailing edge moving upwards).

4.3 Wake vortex properties of rigid and flexible airfoils

In this section the effect of chordwise flexibility on the wake vortex properties will be examined. The effect on peak vorticity (ω_{pk}), circulation (Γ_0) core size (r_c , r_γ), convective speed (U_c), wavelength (a), and transverse spacing (b) will be considered. The appropriate non-dimensionalization for each term will be evaluated and the appropriate scaling with motion trajectory parameters will investigated. Specifically, the current study will demonstrate that the influence of flexibility can be captured as an effective amplitude effect by using the actual trailing edge amplitude, (h_p).

4.3.1 Peak Vorticity

The peak vorticity nondimensionalized by c/U_{∞} is plotted against k for the rigid and flexible airfoils in Figure 4.6a. There are three distinct groupings visible for each α_0 , with the flexible airfoil data at $\alpha_0 = 2^\circ$ starting to deviate from the rigid airfoil more significantly as k increases. When plotted against St there remains three separate groupings for each amplitude (Figure 4.6b), and plotting against St_{te} makes the data collapse look even worse (Figure 4.6c). The effect of flexibility on the peak vorticity is not entirely clear; for all k at $\alpha_0 = 2^\circ$ the flexible airfoil has a higher peak vorticity, but at the higher pitching amplitudes, this is not always true. When plotted against St_{te}, it is clear that for all cases, the peak vorticity of the flexible airfoil is less than its rigid counterpart. Further investigation in scaling ω_{pk} is warranted to yield a consistent message and to at least, collapse the data with respect to α_0 .

In the literature there has been little success in scaling the peak vorticity. Akkala et al. (2015) scaled vorticity by the frequency (ω/f) and observed better scaling than with the convective time $(\omega c/U_{\infty})$ for the peak vorticity values. However, they reached this conclusion based on a visual assessment of the vorticity field, not by quantifying the actual peak vorticity for different cases. This scaling $(\omega c/U_{\infty})$ was tested with the current measurements, but no improvement was found. The alternative scalings of $\omega_{pk}h_p/U_{\infty}$, $\omega_{pk}h_p/(cf)$ were also evaluated but did not yield an overall collapse in the data for different oscillation amplitudes. It should be noted however, that the scaling



Figure 4.6: Non-dimensional peak vorticity for different oscillation amplitudes of the rigid and *flexible*. Non-dimensional peak vorticity plotted against k (a), St (b), and St_{te} (c).

of $\omega_{pk}h_p/U_{\infty}$ plotted against St_{te} did a decent job collapsing each oscillation amplitude between the rigid and flexible airfoils, as it is shown in Figure 4.7, but doesn't collapse the data for different oscillation amplitudes.



Figure 4.7: Scaling of the peak vorticity by h_p/U_{∞} . The peak vorticity of both the rigid and flexible airfoils are scaled as $\omega_{pk}h_p/U_{\infty}$ and plotted against St_{te} .

The peak vorticity non-dimensionalized as $\omega_{pk}c/U_{\infty}$ is plotted against kSt, and kSt_{te} in Figure 4.8. Plotting against kSt and kSt_{te} show excellent collapse of both rigid and flexible airfoils and for all oscillation amplitudes. While plotting against kSt appears to collapse the data better for the entire range, plotting against kSt_{te} is better at collapsing the data for $kSt_{te} < 1$. This was determined by comparing the norm of the residuals from the quadratic fit to the data, which is shown in the Figures as well.

Wu & Wu (1993) extended the analysis of Lighthill (1963) to describe the vorticity production at a moving wall. The analysis of Wu & Wu (1993), once sufficiently simplified for a 2D, incompressible flow, and for pitching motion, the vorticity production becomes a function of the wall acceleration and the pressure gradient. The term kSt_{te} is proportional to the maximum trailing edge acceleration of the pitching airfoils. From a dimensional analysis of flapping wing aerodynamics, Kang et al. (2011) also arrive at using kSt as their non-dimensional term for acceleration. Further analysis of this scaling term is warranted, but beyond the scope of the current investigation.



Figure 4.8: *Peak Vorticity scaling with trailing edge acceleration.* The non-dimensional peak vorticity for different oscillation amplitudes of the rigid and flexible airfoils are plotted against kSt (a) and kSt_{te} (b). The dashed lines shown are quadratic fits (in a least squares sense) to all of the cases.

4.3.2 Circulation

In Sections 4.1.1 and 4.1.3 the rigid airfoil measurements of the current study was shown to agree well across different studies for Γ_0/cU_∞ at each oscillation amplitudes of $\alpha_0 = 2^\circ$, $\alpha_0 = 4^\circ$, and $\alpha_0 = 6^\circ$. There was not however, good collapse across α_0 when plotted against the reduced frequency of k. Triantafyllou et al. (1991) proposed that the wake dynamics of oscillating airfoils is dominated by the Strouhal number, St. Ramamurti & Sandberg (2001), as discussed in the previous chapter, showed that St collapsed C_T for $\alpha_0 = 2^\circ$ and $\alpha_0 = 4^\circ$. The dependency on Strouhal number for wake vortex properties are discussed in the yet-to-be published follow up analysis from Bohl & Koochesfahani (2009), and the published works of Monnier et al. (2015) and Hammer (2016). These works show St to be effective in collapsing Γ_0 for different oscillation amplitudes. Monnier et al. (2013, 2015) showed St_{te} was better than k and St for collapsing circulation for chordwise flexible airfoils, albeit with a significant amount of scatter in the data.

Consistent with the previous discussion, there is no collapse across α_0 and between the rigid/flexible airfoils for Γ_0/cU_∞ when plotted against k, shown in Figure 4.9a. Better collapse is seen in Figure 4.9b where Γ_0/cU_∞ is plotted against St. There remains however, a systematic deviation in the flexible cases as St increases (when flexibility becomes more important). Figure 4.9c shows a complete collapse in Γ_0/cU_∞ when plotted against St_{te} which contrasts the result of Monnier et al. (2015) as they observed considerable scatter. Their study looked at a wide range of flexibilities including many cases where there was "too much flexibility", which may have contributed to the scatter. The scatter can also be explained, in party, by their large uncertainty in determining the trailing edge displacement, and therefore, a large uncertainty in St_{te} .

Monnier et al. (2015) found Γ_0/a to be a parameter that characterized the mean velocity excess from their extended analysis of the vortex array model of Naguib et al. (2011). In an effort to show better collapse in Γ_0 , they considered scaling the circulation as Γ_0/aU_{∞} and observed slightly better collapse in their data (but there was still significant scatter). This scaling was tested with the current measurements, shown in Figure 4.9d, but no such collapse in the data was found.



Figure 4.9: Nondimensional circulation plotted against k and St, St_{te} . Circulation in the nondimensional form of Γ_0/cU_∞ (a-c), and circulation nondimensionalized as Γ_0/aU_∞ plotted against St_{te} (d).

Buchholz et al. (2011) non-dimensionalized circulation as

$$\Gamma_{PG}^* = \frac{\Gamma}{fh_p^2},\tag{4.1}$$

and combined it with the similarity parameter for the pressure coefficient they developed for small aspect ratio pitching panels to arrive at a scaling of circulation to be

$$\Gamma_{GS}^* = \Gamma_{PG}^* \left(1 + \beta \frac{h_p}{S} \right). \tag{4.2}$$

They found this scaling is effective at collapsing their low aspect ratio cases using $\beta = 2$, while their higher aspect ratio (AR = 2.4) collapses, within their experimental uncertainty, for $\beta = 7$. This scaling is investigated, with a $\beta = 7$, with the current data but does not collapse the data as shown in Figure 4.10 for different pitching amplitudes or between rigid/flexible airfoils. In the experiments of Buchholz et al. (2011), they saw not only collapse in the data for different α_0 but the Γ_{GS}^* was nearly a constant value. Their efforts were focused on scaling circulation for pitching 3D panels, a large departure from the current experiment that strives for two dimensionality. They also compute circulation differently seeking to capture all of the shed vorticity from one half of a cycle compared to the current study which focuses on isolated vortices. The scaling parameter does however show a reduced variation in circulation as a function as St_{te} compared to Γ_0/cU_{∞} .

4.3.3 Vortex core size

The size of the vortices is determined by calculating the radius of gyration of the vortex when the ω_{pk} is located at x/c=0.5. The core sizes of the rigid and flexible airfoils are plotted against k and St in Figure 4.11 and against St_{te} in Figure 4.12a. For pitching amplitude of $\alpha_0 = 2^\circ$ and $\alpha_0 = 4^\circ$ the flexible airfoil is seen to increase the size of the vortex core size for k>6. At lower k there isn't any significant difference between rigid and flexible airfoils (except for the one outlier at k=5 where the core size of the flexible airfoil is 10% smaller than the rigid airfoil). The core size collapses only slightly better when plotted against St_{te} for each α_0 but there is still quite a bit of scatter and no overall collapse for different pitching amplitudes is observed.



Figure 4.10: *Circulation normalized following the scaling of Buchholz et al. (2011).* Circulation scaling using a $\beta = 7$ plotted against St_{te}

The measurement of r_c is very sensitive to the domain of integration. In the current study, the domain was fixed at 0.083*c* for comparison with the previous experiments of Bohl (2002), Bohl & Koochesfahani (2009), and the computations of Hammer (2016) (which was discussed previously in Section 4.1.3). The fixed domain size may be biasing the measurement for higher amplitude cases as r_c is shown to increase with increasing α_0 (see Figure 4.11a). A less sensitive metric of the core size is one based on a direct fit to a 2D Gaussian function of the vorticity field, designated at r_{γ} . The two different core size metrics are plotted against St_{te} in Figure 4.12a-b for r_c and r_{γ} . The core size estimated by r_{γ} consistently computes a smaller core size compared to r_c . While neither r_c nor r_{γ} collapse the data when plotted against St_{te} , when the core size is scaled by $r_{\gamma}/\sqrt{h_p/c}$, better collapse of the data across α_0 is observed when plotted against St_{te} for both the rigid and flexible airfoils. Figure 4.12c shows the original r_c (radius of gyration) with the $1/\sqrt{h_p/c}$ scaling plotted against St_{te} but it does not collapse the data well, again supporting the notion that the radius of gyration with a fixed integration domain is a very sensitive parameter and fails to capture all of the details of the vortex core size in experiments with different oscillation amplitudes. The influence

of flexibility is again captured by considering the actual trailing edge amplitude as an important scaling parameter to to collapse the vortex core size across all pitching amplitudes (α_0) considered.



Figure 4.11: *Vortex core size plotted against k and St*. The vortex core size, r_c , plotted against *k* (a), and *St* (b) for both rigid and flexible airfoils.



Figure 4.12: Core size of rigid and flexible airfoils plotted against St_{te} . The vortex core size calculated by the radius of gyration, r_c , (a,c), and calculated from a 2D Gaussian fit to vorticity field, r_{γ} , (b,d), all plotted against St_{te} . The core size is scaled by $1/(c\sqrt{h_p/c})$ in (c) and (d). The legend shown in (d) is common to all plots (a-d).
4.3.4 Convective Speed and Wavelength

The convective speed, U_c , of the vortices is calculated from tracking the location of ω_{pk} over $x/c = 0.5 \pm 0.25$ and finding a linear fit the peak's location vs. phase. The wavelength is computed from $a = U_c/f$. Estimating the wavelength in this way is warranted for two reasons: (1) it provides a localized estimate of the wavelength, and (2) removes the ambiguity that arises when wanting to extract the vortex properties close to the airfoil. If the vortex formation location (x_f) is farther downstream than $x_m - a/2$ where x_m is the extraction location, it would be impossible to measure *a* centered at x_m . Estimating the wavelength by $a = U_c/f$ circumvents this issue and has been used previously in Monnier et al. (2015).

The convective speed is plotted against k, St, and St_{te} in Figure 4.13 and acceptable collapse in the data is observed for U_c plotted against St_{te} , but the collapse is not as good as what was observed for Γ_0 and ω_{pk} . Monnier et al. (2015) observed a similar amount of scatter in their data for U_c/U_{∞} plotted against St_{te} , which can be seen in Figure 4.14. However there is a more significant discrepancy in U_c in the current study compared to Monnier et al. (2015), with the current study agreeing more closely with Bohl & Koochesfahani (2009), and Hammer (2016) which are also plotted in Figure 4.14. The discrepancy between the studies may be due to Reynolds number and airfoil shape differences. The airfoil used in Monnier et al. (2015) differs significantly from a the NACA 0009 airfoil used in the current study; its geometry was similar to Heathcote & Gursul (2007) with a rigid "head" and a flexible "tail". The Reynolds number in the study of Monnier et al. (2015) also was much lower than the current study, $Re_c=2.3\times10^3$ for compared to $Re_c=1.2\times10^4$ for the current study.

The wavelength normalized by the chord, a/c, is plotted against k, St, and St_{te} in Figure 4.15a-c. The wavelength shows better collapse with k than St and scatters even more with St_{te} (since $St_{te} \ge St$ for all cases in the current study). However, referring to Figure 4.15b, a/c is seen to group well with α_0 with the flexible airfoil deviating systematically away from the rigid airfoil as St increases. This motivates seeking a more appropriate scaling of a, and h_p is certainly an appropriate length scale. Figure 4.15d shows the a/h_p collapses the data when plotted against St_{te}



Figure 4.13: *Vortex convective speed of the rigid and flexible airfoils.* The convective speed, U_c , normalized by the freestream, U_{∞} , and plotted against k (a), St (b), and St_{te} (c).



Figure 4.14: Vortex convective speed of the rigid and flexible airfoils compared to selected *literature*. Vortex convective speed of the rigid and flexible airfoils compared to selected literature. The convective speed normalized by the freestream, U_c/U_{∞} , and plotted against St_{te} . The data from Bohl & Koochesfahani (2009), Monnier et al. (2015), and Hammer (2016) are included. The Re_c of Monnier et al. (2015) is 2.3×10^3 compared to nominally 1.2×10^4 for the rest of the studies.

for all α_0 for both rigid and flexible airfoils. With very good collapse seen for a/h_p , it is reasonable to go back to convective speed and re-scale it. Scaling the convective speed as $U_c/(U_{\infty}St_{te})$ would show considerably better collapse in data compared to what was observed previously in Figure 4.13 however, this scaling simply reduces $U_c/U_{\infty}St_{te}$ to a/h_p since $U_c = af$, resulting in no new information.

This scaling of a/h_p is tested with the results from Bohl (2002), Bohl & Koochesfahani (2009), and Hammer (2016). Figure 4.16 shows a/h_p plotted against St_{te} for these studies and the current rigid and flexible airfoil measurements. The scaling of wavelength by h_p collapses all the data from all studies. Overall, this scaling is not a surprising outcome since b/h_p will be shown in the next section to quickly approach a constant value of 1 and, in their yet-to-be published follow-up study, Bohl & Koochesfahani (2009) show b/a, the vortex array aspect ratio, tends to collapse to a single value as well. However, this study demonstrates that yet again, the effect of chordwise flexibility is captured as an amplitude effect that is quantified by measuring the actual trailing edge



Figure 4.15: *Wavelength scaling*. The wavelength is normalized by *c* and plotted against *k* (a), *St* (b), and *St*_{*te*} (c). The wavelength normalized by h_p and plotted against *St*_{*te*} (d).



Figure 4.16: Wavelength normalized by the trailing edge displacement. Nondimensional wavelength, a/h_p , plotted against St_{te} including the data from Bohl (2002), Bohl & Koochesfahani (2009), and Hammer (2016).

displacement for scaling both U_c and a (since $a = U_c f$).

4.3.5 Transverse Spacing

Two different scalings for the transverse spacing (*b*) of the vortices will be considered. The first, b/h_p is plotted against *k*, *St*, and *St_{te}* in Figure 4.17. Considering $\alpha_0 = 2^\circ$, b/h_p in Figure 4.17a is shown to increase with increasing *k* up until $b/h_p \approx 1$, where it appears to asymptote to that value. As mentioned previously, this effect was observed in the yet-to-be-published paper by Bohl and Koochesfahani where they investigate the effect of amplitude using $\alpha_0 = 2^\circ$ and $\alpha_0 = 4^\circ$ data from Bohl (2002). This asymptotic behavior is shown more clearly when the data is plotted against *St* and *St_{te}*, as it is in Figure 4.17b-c. The higher amplitude cases fall onto one curve with the $\alpha_0 = 2^\circ$ data better for *St_{te}* as the abscissa compared to *St*. The limit of $b/h_p = 1$ is reasonable from the simplistic viewpoint of the vortex formation being connected with the extrema of transverse position of the trailing edge. However, at low *St_{te}*, the vortex spacing is nearly $-2h_p$, indicating that at these cases, the transverse spacing is dependent on a more complicated mechanism related

to the when in the oscillation cycle the vortex actually begins to roll up.

In terms of flexibility, the flexible airfoil's b/h_p at k = 4,5 and 6 is higher than its rigid counterpart when oscillating with $\alpha_0 = 2^\circ$. This is consistent with the flexible airfoil having a lower thrust crossover than the rigid airfoil, which was shown in Chapter 3. When b/h_p is plotted against St_{te} this shift is no longer observed as both rigid and flexible airfoils appear to fall on the same (steep) curve for $St_{te} < 0.1$ before reaching the asymptotic value.



Figure 4.17: *Transverse spacing normalized by trailing edge displacement of the rigid and flexible airfoils.* Nondimensional transverse spacing, b/h_p is plotted against k (a), St (b), and St_{te} (c).

Figure 4.18 shows b/h_p plotted against St_{te} for both the rigid and flexible airfoils of the current

study, the experiments of Bohl (2002) and Bohl & Koochesfahani (2009), and the simulations of Hammer (2017). The transverse spacing normalized by the trailing edge displacement, b/h_p , is consistent across all of the studies. Considering all of the studies presented, b/h_p appears to approach 1 around $St_{te} = 0.15$, but as St_{te} increases beyond this, b/h_p appears to gradually decrease to $b/h_p = 0.9$ as $St_{te} \rightarrow 0.4$.



Figure 4.18: *Transverse spacing normalized by trailing edge displacement compared with selected literature.* The transverse spacing normalized by trailing edge displacement b/h_p plotted against *St_{te}* of the rigid and flexible airfoils are plotted with the additional studies of Bohl (2002), Bohl & Koochesfahani (2009), and Hammer (2016).

The second scaling that is considered for the transverse spacing is b/a, shown in Figure 4.19, which is the vortex array aspect ratio. When *b* is normalized by *a*, the data still remain collapsed across α_0 and between rigid/flexible airfoils, but does not asymptote as quickly as b/h_p . When plotted against St_{te} , b/h_p asymptotes near $St_{te} = 0.13$ while b/a asymptotes at a higher range of $0.2 < St_{te} < 0.25$ (the scatter in the data at higher St_{te} makes selecting this value difficult). The two are not necessarily related either as the first relates to the trailing edge displacement, and the latter is the aspect ratio of the free vortex array. It is however, interesting that b/h_p asymptotes close to the thrust to drag crossover St_{te} while the aspect ratio of the vortex array, b/a doesn't reach its asymptotic value until a much higher St_{te} .



Figure 4.19: *Transverse spacing normalized by wavelength for the rigid and flexible airfoils.* The transverse spacing normalized by the wavelength, b/a plotted against k (a), St (b), and St_{te} (c).



Figure 4.20: *Transverse spacing normalized by wavelength compared with selected literature.* The vortex array aspect ratio, b/a, is plotted against St_{te} with the additional studies of Bohl (2002), Bohl & Koochesfahani (2009), Hammer (2016), Hammer (2017).

The asymptotic behavior of the transverse spacing normalized by the wavelength was shown previously by Bohl & Koochesfahani (2009) to be $b/a \rightarrow 0.2$ for $\alpha_0 = 2^\circ$ and Hammer (2016) also observed the same asymptotic value of 0.2 for an extended range of $\alpha_0 = 2^\circ$, 4°, and 6°. The current study also observes this behavior, but over the extended St_{te} range the asymptotic value of 0.2 is less clear; this can be seen by considering Figure 4.19c for $St_{te} > 0.2$, which shows the asymptotic value is greater than 0.2. Results from both of these previous studies and Hammer (2017) are plotted together in Figure 4.20 in addition to the current experiments. Considering all of these studies, the asymptotic value for b/a does appear to be greater than 0.2 when considering the larger range of St_{te} of the current experiments and the simulations of Hammer (2016) than those of Bohl & Koochesfahani (2009).

4.4 Summary

In this chapter, the baseline (rigid airfoil) cases of the current study were validated against previous experiments and computations, showing excellent overall agreement both qualitatively, in terms the vorticity fields, and quantitatively, in terms of the extracted vortex parameters of cases where a single pair of vortices were shed per oscillation. The effect of chordwise flexibility was shown to be captured as an amplitude effect measured by the actual trailing edge displacement (h_p) . As a length scale, h_p was shown to be useful for normalizing b, a, and r_{γ} , and in the Strouhal number based on the actual trailing edge displacement, St_{te} . When St_{te} is used as the abscissa, nearly all of the individual vortex properties and the vortex array properties collapse for all oscillation amplitudes for both rigid and flexible airfoils. Table 4.1 summaries all of the vortex configuration parameters and their dimensionless forms found to collapse the data for both rigid and flexible airfoils across all pitching motions considered in this study.

Base Parameter	ordinate	abscissa
Γ_0	$\Gamma_0/(cU_\infty)$	St_{te}
ω_{pk}	$\omega_{pk}c/U_\infty$	kSt_{te}
r_{γ}	$(r_{\gamma}/c)/\sqrt{h_p/c}$	St_{te}
b	b/h_p	St_{te}
b	b/a	St_{te}
а	a/h_p	St_{te}

Table 4.1: *Vortex parameter nondimensionalizations and scaling variable.* A summary of vortex parameters and the corresponding parameters used to scale and collapse the data.

CHAPTER 5

CONCLUSIONS

In this work, the influence of chordwise flexibility on the thrust and vortical wake structure produced by a harmonically pitching airfoil at a chord Reynolds number of $Re_c = 1.24 \times 10^4$ was investigated experimentally. While this effect has received renewed interested recently, this study takes a different approach from what is commonly seen in the literature (flat plates, membranes, flaps) and studies a single geometric shape, a standard NACA 0009 airfoil, that is either rigid or flexible. Molecular tagging velocimetry was used for planar two-component measurements of the velocity field at the mid-span in the wake of the oscillating airfoil. A rigid and a chordwise flexible airfoil were oscillated at multiple amplitudes and frequencies. The range of oscillation frequencies and amplitudes considered in this study were such that the trailing edge displacement increased with increasing k, and α_0 was kept low enough such that leading edge vortices were not formed. The wake vortex properties were quantified in terms of their strength (peak vorticity, ω_{pk} , and circulation, Γ_0), core size (radius of gyration, r_c , and 2d-gaussian core size, r_γ), and spacing (wavelength a, and transverse spacing b) for oscillation frequencies and amplitudes combinations that resulted in a von Kármám or reverse von Kármán vortex pattern.

The rigid airfoil results were compared against numerical simulations of a NACA 0009 and both experiments and simulations of a NACA 0012 airfoil to establish a baseline of comparison to previous studies. Both the individual vortex properties and mean velocity profiles were compared. While there are subtle differences between the thinner NACA 0009 airfoil compared to the NACA 0012 used more extensively in the literature, the differences were not significant compared to the effect of chordwise flexibility, the primary effect under consideration in this study.

Chordwise flexibility was shown to increase thrust production at a given oscillation frequency and amplitude under the conditions considered in this study. The greater body of literature shows that chordwise flexibility can increase or decrease the thrust production, depending on the oscillation frequency and the flexibility of the airfoil. It was found that when plotted against the Strouhal number based on the *actual* trailing edge displacement, St_{te} , that thrust collapses equally well for both rigid and flexible airfoils for the range of parameters studied.

Little attention in the literature is given to the wake vortex properties of chordwise flexible airfoils. Unique to this study, the properties of the vortices and their arrangement was studied in detail for a chordwise flexible airfoil at a chord Reynolds number of $Re_c = 1.25 \times 10^4$. The scalings and non-dimensionalizations that collapsed the rigid airfoil data for different oscillation amplitudes were shown, when plotted against St_{te} , to not only collapse the flexible airfoil, but to collapse the data to the rigid airfoil data as well. The measured trailing edge displacement (h_p) was found to be the proper length scale for non-dimensionalizing, except in the case of circulation (Γ_0) . When trailing edge displacement was used in scaling the transverse spacing *b*, wavelength *a*, and core size (r_γ) the data collapse when plotted against St_{te} for both rigid and flexible airfoils for all oscillation amplitudes studied ($\alpha_0 = 2^\circ$, 4°, and 6°). The peak vorticity was also found to collapse for both rigid and flexible airfoils for all α_0 considered when plotted against kSt_{te} , a non-dimensional term proportional to the maximum trailing edge acceleration. Overall, chordwise flexibility was shown to act primarily through alteration of the trailing edge amplitude for the range of parameters investigated through. APPENDICES

APPENDIX A

TWO CAMERA CALIBRATION

The imaging system used for the current work utilizes two 1392×1024 pixel PCO.Pixelfly cameras which are mounted underneath the water tunnel as shown in Figure A.1. The camera assembly is connected to the streamwise and vertical traverses such that it also moves with the entire optical assembly. It is on a separate cross-stream traverse since it must move at a different rate than the optical system due to the refraction of the UV beam passing through the quartz windows at an angle. A single 105 mm f/2.0 Nikon Nikkor Lens, a shortened F-mount to C-mount adaptor, and a standard 1" C-mount cube beam splitter from Edmund Optics completes the setup. The compact two-camera system relies on a calibration routine based on the Affine transform to compensate for large (greater than 10 pixel) sensor misalignments, which is described in detail in below.

An affine transformation is defined as any transformation that preserves collinearity and ratios of distances; in general an affine transformation is a composition of rotations, translations, dilations, and shears (Shapiro & Stockman (2001)). The use of affine transformations is very common in image processing and is generalized by the Equation A.1 where x' is the transformed coordinates and x is the original coordinates.

$$x' = T(x) = Ax + B \tag{A.1}$$

Interpolation is typically employed after computing the transformed coordinates to create output a new image with the original spacing. The most common interpolation methods are either nearest neighbor, bi-linear, and bi-cubic.

A single affine transformation needs to be determined to remap the image taken from one camera into the pixel coordinate system of the second camera. The transform is found by the linear least squares method using a large number of known control points that are imaged from both cameras simultaneously (as seen in Figure A.2), and determining the displacement of each control point (grid intersection) using our 2c-MTV correlation program. The displacement field, shown in Figure A.3, indicates a slight translation, but is dominated by angular rotation between the



Figure A.1: *MTV Imaging System.* Photograph of dual camera MTV imaging system. Two PCO.Pixelfly cameras (blue) are flush mounted to a backing plate and a cube beam splitter (highlighted in red). A single lens is mounted to a shortened c-mount to Nikon F-mount adapter (highlighted green). The trigger boxes can be seen in the upper right of the image. The cameras are located underneath the water tunnel looking up towards the test section. The gray pipe in the bottom of the image is the return pipe of the water tunnel.

two sensors. The traditional multi-camera approach would have used mechanically corrected these errors using translation and rotational stages such that there was less than 1 px misalignment for all control points. However, in this application, the computed displacements of the control points are used to determine the optimal affine transformation using the built-in functions in MATLAB's image processing toolbox.

The application of the affine transform to overcome the misalignment of the two camera's sensors is assessed by re-computing the displacement field shown in Figure A.3 with the original image of Camera #1 (Figure A.2 left) and the transformed image of Camera #2 (not shown). If the true transformation between the imaging planes were Affine (they aren't due to higher order



Figure A.2: *Targets of each camera*. (left) Camera #1 target image (right) Camera #2 target image. The red line is straight relative to the paper; the relative shift may be seen by comparing the distance of an intersection to the red line on both images.

distortions) and there was no error in the displacement calculations of the control points (there isn't due to imaging and processing noise), a perfect transformation should achieve a mean of zero displacement across the entire image. A mean zero displacement to the 1st decimal place is achieved as shown in Figure A.4 with an overall RMS (of 256 images and 357 intersections) of 0.04 and 0.01 pixels in the horizontal and vertical directions respectively. The displacement field still has a distinct pattern, indicating there may be room for improvement in either the affine transform parameters or by using a higher order transform to drive the error further to zero. The perspective transform was also tested but did not perform significantly better than the simpler affine transform. However, this residual displacement field is now a known bias error and can be subtracted from other results.

In order to demonstrated the feasibility to use this technique to correct for the sensor misalignment a known target is displacement and imaged by both cameras simultaneously. The true displacement of the target can be computed from correlating the target at location 1 to location 2 independently from each camera. When this is performed, the displacement of the target is: $\pm x =$ 0.75 pixels and $\Delta y = 15.23$ pixels. Now, the affine transformation computed previously is used to transform one set of images and the target image is correlated at location 1 from one camera to the



Figure A.3: Uncorrected displacement field between the two cameras. Initial displacement field. The vector field shows the displacement from Camera #1 (c1) at Location 2 (L2) to Camera #2 (c2) also at Location 2 (L2). The displacement field is computed by correlating the two images together that were looking at the same target as shown in Figure A.2. Reference vector shown in upper right of plot.

transformed images of camera 2 at location 2. The result yields an error in the mean displacement field of 0.07% in x (0.01 pixels), and 0.4% in y (0.61 pixels), with RMS values of 0.047 pixels in both directions. Now, the residual displacement field due to the not-perfect image transformation, as shown in Figure A.4, is subtracted. The results are more than satisfactory: -0.060% in x (<0.01 pixels), and -0.024% in y (<0.04 pixels), with 0.019 pixel and 0.031 pixel RMS in spatial variation, respectively.

These results demonstrate that the sensor misalignment in the current setup can be completely corrected for by applying the affine transform determined from a calibration target to one camera's images prior to correlation and subtracting the residual misalignment to the displacement fields with a subpixel accuracy of 0.01 ± 0.02 pixels RMS. The results can be improved with a higher spatial density calibration target such that residual displacement field is more finely known; which is done in all further experiments where the number of control points is increased by over a factor of 2 of the those computed here.



Figure A.4: *Corrected displacement field using affine transform.* Corrected displacement field using affine transform. A reference vector, 0.1 px in length, is displayed in the upper right of the plot. The vector field shows the displacement from Camera #1 at Location 2 to the transformed image of Camera #2 also at Location 2. This displacement field is the residual error left after applying the affine transformation. Reference vector shown in upper right of plot.

APPENDIX B

UNCERTAINTY ESTIMATES

B.1 Vorticity

The uncertainty analysis outlined here for the phase averaged spanwise vorticity extends what was presented in Bohl & Koochesfahani (2009) based on the fourth order accurate finite difference scheme discussed in Cohn & Koochesfahani (2000). The current analysis includes the spatial dependence of the uncertainty of the velocities. The spanwise vorticity, ω_z , is calculated using a fourth order accurate central finite difference scheme, given by:

$$(\omega_z)_{i,j} = \left(\frac{\partial v}{\partial x}\right)_{i,j} - \left(\frac{\partial u}{\partial y}\right)_{i,j}$$
(B.1)

where

$$\left(\frac{\partial v}{\partial x}\right)_{i,j} = \frac{-v_{i+2,j} + 8v_{i+1,j} - 8v_{i-1,j} + v_{i-2,j}}{12h},$$
(B.2)

and

$$\left(\frac{\partial u}{\partial y}\right)_{i,j} = \frac{-u_{i,j+2} + 8u_{i,j+1} - 8u_{i,j-1} + u_{i,j-2}}{12h}.$$
(B.3)

The data spacing, h, is a known constant; therefore, the vorticity is a function of four transverse velocities and four streamwise velocities; each with uncertainty ϵ_v and ϵ_u , respectively. The uncertainty of each of the transverse and streamwise velocity components are a function of both the subpixel accuracy of the velocimetry and the variation in the phase averaging process. Standard uncertainty analysis allows the determination of the uncertainty in spanwise vorticity, $\epsilon\omega_z$, according to

$$\left(\epsilon_{\omega_{z}}\right)^{2} = \left(\frac{\partial\omega_{z}}{\partial\chi_{n}}\epsilon\chi_{n}\right)^{2} \tag{B.4}$$

where χ_n and ϵ_{χ_n} represent the *n*th independent variable and its uncertainty, respectively. Carrying out the uncertainty analysis using the vorticity definition and the finite difference formulation of

velocity derivatives given above leads to the final result

$$(\epsilon_{\omega_z})^2_{i,j} = \left(\frac{-1}{12h}\epsilon_{\nu_{i+2,j}}\right)^2 + \left(\frac{2}{3h}\epsilon_{\nu_{i+1,j}}\right)^2 + \left(\frac{-2}{3h}\epsilon_{\nu_{i-1,j}}\right)^2 + \left(\frac{1}{12h}\epsilon_{\nu_{i-2,j}}\right)^2 + \left(\frac{-1}{12h}\epsilon_{u_{i,j+2}}\right)^2 + \left(\frac{2}{3h}\epsilon_{u_{i,j+1}}\right)^2 + \left(\frac{-2}{3h}\epsilon_{u_{i,j-1}}\right)^2 + \left(\frac{1}{12h}\epsilon_{u_{i,j-2}}\right)^2$$
(B.5)

B.2 Circulation

The circulation of a vortex, Γ_0 , is calculated by numerical integration (for a single sign of vorticity at a time), given by:

$$\Gamma_0 = \sum_i \sum_j \omega_{i,j} dA, \tag{B.6}$$

where the differential area, dA, is a known constant, and $\omega_{i,j}$ is summed only for the region of interest, typically a radial distance of *R* from the center of the vortex. The uncertainty in circulation is given by:

$$(\epsilon \Gamma_0)^2 = \sum_i \sum_j \left(\omega_{i,j} dA\left(\epsilon \omega_{i,j}\right) \right)^2, \tag{B.7}$$

where the uncertainty of each measurement of $\epsilon \omega_{i,j}$ is given by Equation B.5. The quadrature summation is derived from standard uncertainty analysis assuming each measurement of $\omega_{i,j}$ is independent.

B.3 Radius of Gyration

The vortex core sized is calculated by the radius of gyration, r_c , given by:

$$r_c = \frac{\sum_i \sum_j r^2_{i,j} \omega_{i,j}}{\sum_i \sum_j \omega_{i,j} dA},$$
(B.8)

where *r* is the distance from the center of the vortex and the integration domain is a finite size of radius *R*. The uncertainty ϵr_c is derived from standard uncertainty analysis:

$$(\epsilon r_c)^2 = \left(\frac{\partial r_c}{\partial \chi_n} \epsilon \chi_n\right)^2,\tag{B.9}$$

where χ_n and $\epsilon \chi_n$ are the *n*th independent variable and its corresponding uncertainty. In the case of the radius of gyration the independent variables are $\omega_{i,j}$ located within the integration radius *R*

surrounding the center of the vortex. The differentiation of Equation B.8 with respect to each $\omega_{i,j}$ is:

$$\frac{\partial r_c}{\partial \omega_{i,j}} = \frac{dA}{2\sqrt{r_c}} \left(\frac{\Gamma_0 r_{i,j}^2 - 1}{\Gamma_0^2} \right),\tag{B.10}$$

therefore, the uncertainty in radius of gyration is:

$$(\epsilon r_c)^2 = \sum_i \sum_j \left(\left(\frac{dA}{2\sqrt{r_c}} \left(\frac{\Gamma_0 r_{i,j}^2 - 1}{\Gamma_0^2} \right) \right) \epsilon \omega_{i,j} \right)^2.$$
(B.11)

B.4 Control volume method for estimating the streamwise force

The control volume method for computing the streamwise force is described by Bohl & Koochesfahani (2009), whose improvements in using the fluctuating velocity profiles demonstrated a marked improvement compared to the traditional analysis of only using the mean velocity profile. In the context of the current measurements, the integral form of the equation is approximated using the numerical quadrature method at a specific downstream location of i = I and over the transverse region of $j \mid y = \pm H$ denoted by j = [0, M]. This leads to the expression:

$$C_{t} = \frac{2}{c} \sum_{j=1}^{M} \left\{ \frac{\overline{\langle u \rangle}_{I,j}}{U_{\infty}} \left(\frac{\overline{\langle u \rangle}_{I,j}}{U_{\infty}} - 1 \right) + \xi \left(\frac{\overline{\langle u \rangle}_{I,j}}{U_{\infty}} - 1 \right) + \left(\frac{\langle u \rangle'_{I,j}}{U_{\infty}} \right)^{2} - \left(\frac{\langle v \rangle'_{I,j}}{U_{\infty}} \right)^{2} + \frac{1}{2} \left(1 - \frac{U_{o}^{2}}{U_{\infty}^{2}} \right) \right\} \Delta y,$$
(B.12)

where

$$\xi = \frac{1}{2} \left(1 - \frac{U_0}{U_{\infty}} \right).$$
 (B.13)

Following standard uncertainty analysis the uncertainty of C_t is expressed as

$$(\epsilon C_t)^2 = \left(\frac{\partial C_t}{\partial \chi_n} \epsilon \chi_n\right)^2,\tag{B.14}$$

where the partial derivatives of C_t with respect to each of the independent variables (χ_n) needs to be computed. The independent variables are: $\overline{\langle u \rangle}_{I,j}$, $\langle u \rangle'_{I,j}$, $\langle v \rangle'_{I,j}$, and U_0 . The partial derivatives of C_t with respect each variable are:

$$\frac{\partial C_t}{\partial \overline{\langle u \rangle}_{i,j}} = \left(\frac{4\overline{\langle u \rangle}_{i,j} - U_0}{U_{\infty}^2} - \frac{1}{2U_{\infty}}\right) \Delta y \tag{B.15}$$

$$\frac{\partial C_t}{\partial \langle u \rangle'_{i,j}} = \left(\frac{2 \langle u \rangle'_{i,j}}{U_{\infty}^2}\right) \Delta y \tag{B.16}$$

$$\frac{\partial C_t}{\partial \langle v \rangle'_{i,j}} = \left(-\frac{2 \langle v \rangle'_{i,j}}{U_{\infty}^2} \right) \Delta y \tag{B.17}$$

$$\frac{\partial C_t}{\partial U_0} = \left(-\frac{\overline{\langle u \rangle}}{2U_\infty^2} + \frac{1}{2U_\infty} \right) \Delta y \tag{B.18}$$

The uncertainty of the mean streamwise velocity, $\overline{\langle u \rangle_{i,j}}$, can be estimated by the mean standard error,

$$\epsilon \overline{\langle u \rangle_{i,j}} = \frac{1.96 \langle u \rangle_{i,j}'}{\sqrt{n_{\phi}}},\tag{B.19}$$

where n_{ϕ} is the number of phase bins used in creating the phase averaged time series (64 in the current study). The factor of 1.96 is used for the error to represent the 95% confidence interval, which assumes the variable has a normal distribution. Similarly, the error in U_0 is estimated by the mean standard error since it it is estimated from averaging $\overline{\langle u \rangle}$ over *p* samples at the extent of the mean velocity profile as given by

$$U_0 = \frac{1}{2p} \left(\sum_{j=0}^p \overline{\langle u \rangle}_I, j + \sum_{j=M-p+1}^M \overline{\langle u \rangle}_I, j \right).$$
(B.20)

In the current study, p = 5, and the standard deviation of U_0 is the standard deviation of $\langle u \rangle$ over the region described above, denoted as $U_{0,rms}$. This leads to the uncertainty in U_0 being

$$\epsilon U_0 = \frac{(1.96)U_{0,rms}}{\sqrt{2p}}$$
 (B.21)

for a 95% confidence interval.

The uncertainties in $\langle u \rangle'_{I,j}$ and $\langle v \rangle'_{I,j}$ required a more detailed examination of the underlying flow and its measurement. The measured velocity is

$$u(x, y, t) = \overline{u}(x, y) + u_{\phi}(x, y, \phi) + u'_{1}(x, y, t) + u'_{2}(x, y, t) + u'_{3}(x, y, t)$$
(B.22)

where \overline{u} is the time average, u_{ϕ} is the oscillatory velocity fluctuation, u'_1 is the cycle to cycle variation, u'_2 is the random fluctuations from turbulence, and u'_3 is the fluctuations due to measurement error. Phase averaging this by

$$\langle u \rangle (x, y, \phi) = \int_{\phi_1}^{\phi_2} (\overline{u}(x, y)) d\phi$$
 (B.23)

yields, at each phase bin ϕ ,

$$u_{\phi}(x, y, \phi) = \langle u \rangle (x, y, \phi) + \langle u \rangle_{\phi}', \qquad (B.24)$$

where $\langle u \rangle'_{\phi}$ is the phase-bin-rms. The uncertainty in $\langle u \rangle'_{i,j}$ should be computed using the average variance from all of the individual phase-bin-rms results,

$$S_{\langle u \rangle_{i,j}'} = \sqrt{\frac{1}{n} \sum \left(\langle u \rangle_{\phi,i}' \right)^2}$$
(B.25)

where the actual confidence interval is based on the precision interval of the sample variance. The following expression is given in Figliola & Beasley (2006):

$$\frac{\nu S_x^2}{\chi_{0.025}^2} \le \sigma^2 \le \frac{\nu S_x^2}{\chi_{0.975}^2}$$
(B.26)

where v is the degrees of freedom, S_x^2 is the measured sample variance of measured variable x (in this case either $\langle u \rangle'_{I,j}$ or $\langle v \rangle'_{I,j}$), and σ^2 is the true variance of the population. The average difference between the upper and lower bounds and the measured S_x^2 is used such that it can be cast into the the uncertainty analysis being carried out, which utilizes a single (±) uncertainty value. The final expressions for uncertainty in $\langle u \rangle'_{I,j}$ and $\langle v \rangle'_{I,j}$ are therefore:

$$\epsilon \langle u \rangle_{I,j}' = \frac{1}{2} \left(\frac{\nu * S_{\langle u \rangle_{I,j}'}}{\chi_{0.025}^2} - \langle u \rangle_{I,j}' \right) + \frac{1}{2} \left(\langle u \rangle_{I,j}' - \frac{\nu * S_{\langle u \rangle_{I,j}'}}{\chi_{0.975}^2} \right)$$
(B.27)

$$\epsilon \langle v \rangle_{I,j}' = \frac{1}{2} \left(\frac{v * S_{\langle v \rangle_{I,j}'}}{\chi_{0.025}^2} - \langle v \rangle_{I,j}' \right) + \frac{1}{2} \left(\langle v \rangle_{I,j}' - \frac{v * S_{\langle v \rangle_{I,j}'}}{\chi_{0.975}^2} \right)$$
(B.28)

where $v = (n_{\phi} - 1)$, which is 63 in the current study.

The final uncertainty (95% confidence interval) of C_t is therefore the summation of each of the partial derivatives multiplied by each of their corresponding uncertainties, as previously indicated by Equation B.4. It should be noted that this indicates that each term is summed over the extent of the profile (i.e. summation over *j* for a single value of i = I), which in the current study is 121 elements, for a total of 484 total terms for the four flow variables used in computing C_t .

Overall, the uncertainty estimates computed here are compared to two sets of mean thrust coefficient measurements which were acquired from independent data sets. Figure B.1 shows the

two sets of measurements plotted against reduced frequency. The two sets of measurements, agree with each other within the computed 95% confidence intervals.



Figure B.1: Independent measurements of C_t plotted against reduced frequency (k). Comparison of independent measurements of C_t plotted against reduced frequency.

APPENDIX C

EFFECT OF VORTICITY CUTOFF LEVEL

A vorticity cutoff level (ω_{cut}) is used throughout this study to minimize the background noise in the vorticity from impacting parameters calculated from the vorticity fields such as circulation and the vortex core size. The cutoff functions as:

$$\omega_z(x, y) = 0$$
, where: $|\omega_z(x, y)| \le \omega_{\text{cut}}$. (C.1)

A convenient starting point for investigating the sensitivity of the vorticity cutoff is considering an isolated Gaussian vortex whose radial (r) vorticity distribution is given by:

$$\omega(r) = \omega_{pk} e^{-(r/r_c)^2}, r \ge 0,$$
 (C.2)

where ω_{pk} , is the peak vorticity and r_c , is the core radius. The core radius in this context is defined as the radial distance at which the vorticity has reduced to e^{-1} of its initial value (nominally 37% of ω_{pk}). In general, circulation is defined as the line integral around a closed curve of the velocity and application of Stokes' Theorem yields the relationship between circulation and vorticity as the area integral of the vorticity.

$$\Gamma = \oint_{\partial S} V \cdot dl = \iint_{S} \omega \cdot dS \tag{C.3}$$

The circulation of the isolated Gaussian vortex is computed from integrating Equation C.2 as $r \rightarrow \infty$ to yield

$$\Gamma_0 = \pi \omega_{pk} r_c^2, \tag{C.4}$$

and the radial distribution of circulation is

$$\Gamma(r) = \Gamma_0 \left(1 - e^{-(r/r_c)^2} \right),$$
 (C.5)

which may also be expressed as

$$\Gamma(r) = \pi \omega_{pk} r_c^2 \left(1 - e^{-(r/r_c)^2} \right).$$
(C.6)

These relationships show the interconnectedness of the peak vorticity, core radius, and circulation of a Gaussian vortex, and are useful in relating more complicated vortical flow fields to idealized vortices. Currently, these relationships are exploited to examine the sensitivity to a cutoff level of vorticity and its impact on the calculation of circulation and core size. Any vorticity cutoff is shown (Figure C.1) to reduce the calculated values of Γ and r_c , with a greater impact on r_c . The reduction in circulation is linear with a slope of 1 over the extent examined such that $\omega_{cut} = 0.04\omega_{pk}$ yields a 4% error in Γ_0 and a 6% error in r_c .



Figure C.1: *Effect of vorticity cutoff on* Γ *and* r_c . Effect of vorticity cutoff on calculated circulation and radius of gyration of a model Gaussian vortex. The cuttoff vorticity level always decreases both circulation and radius of gyration. The circulation will be reduced by 1:1 and radius of gyration 1:1.4 based on the vorticity cutoff's percentage of the peak vorticity over for up to 10% of the peak vorticity.

The effect of the vorticity cutoff is considered for three different vorticity cutoff levels on the rigid airfoil data and summarized in Table C.1. The change in Γ_0 for three different levels (0, 3, and 5 non-dimensional vorticity) is consistent with the relationship established above for an isolated Gaussian vortex based on the cutoff level and the peak vorticity. The change in r_c is more sensitive

than predicted for the highest cutoff level (5), but closer for the lower level of (3). The increased sensitivity to r_c is contributed to the spatial resolution of the measurements (1%) compared to r_c varying between 3% and 4% of the chord. Overall for the cutoff level of 3 used in this study r_c varies less than 12% for $\alpha_0 = 2^\circ$ and less than 5% for both $\alpha_0 = 4^\circ$ and $\alpha_0 = 6^\circ$ compared to using either no cutoff or a level of 5. Similarly, Γ varies less than 7% for $\alpha_0 = 2^\circ$ and less than 4% for both $\alpha_0 = 4^\circ$ and $\alpha_0 = 6^\circ$.

				r _c			Γ		
	k	St	ω_{pk}	$\omega_{\rm cut} = 0$	$\omega_{\rm cut} = 3$	$\omega_{\rm cut} = 5$	$\omega_{\rm cut} = 0$	$\omega_{\rm cut} = 3$	$\omega_{\rm cut} = 5$
$\alpha_0 = 2^\circ$	5	0.08	43	0.037	0.035	0.033	0.15	0.14	0.13
	6	0.10	68	0.036	0.035	0.033	0.18	0.18	0.17
	7	0.12	80	0.034	0.031	0.028	0.21	0.20	0.18
	8	0.13	111	0.033	0.031	0.028	0.25	0.24	0.23
	9	0.15	122	0.034	0.031	0.029	0.29	0.28	0.27
	10	0.17	143	0.034	0.032	0.029	0.33	0.32	0.31
	12	0.20	192	0.033	0.032	0.030	0.42	0.41	0.40
	15	0.25	253	0.033	0.032	0.031	0.56	0.56	0.55
$\alpha_0 = 4^\circ$	5	0.17	90	0.041	0.040	0.038	0.34	0.33	0.32
	6	0.20	134	0.039	0.039	0.037	0.42	0.41	0.40
	7	0.25	167	0.040	0.040	0.039	0.57	0.57	0.57
	9	0.30	223	0.040	0.040	0.039	0.73	0.73	0.73
	10	0.33	250	0.040	0.040	0.040	0.84	0.84	0.84
$\alpha_0 = 6^{\circ}$	4	0.20	96	0.044	0.043	0.042	0.40	0.40	0.39
	5	0.25	145	0.044	0.044	0.043	0.55	0.55	0.55
	6	0.30	166	0.044	0.044	0.044	0.71	0.71	0.71
	8	0.40	243	0.044	0.044	0.044	1.03	1.02	1.02

Table C.1: Effect of vorticity cutoff level on r_c and Γ . Circulation and radius of gyration for different vorticity cutoff levels. The peak vorticity for each case is also shown for comparison to the cutoff level. The average of the positive and negative vortex is displayed and all parameters have been non-dimensionalized (i.e. $\omega_{pk}c/U_{\infty}$, $\Gamma/(cU_{\infty})$, r_c/c).

APPENDIX D

STREAMWISE FORCES

Included in this Appendix are several figures that show the mean streamwise thrust coefficient plotted for k, St, and St_{te} for either the full range of the data or for a selected range of the data. These plots supplement the range of the parameters that were included in Chapter 3.



Figure D.1: *Mean thrust coefficient plotted against k, full range.* Mean thrust coefficient plotted against *k* for the full range of the data.



Figure D.2: Mean thrust coefficient plotted against k, selected range. Mean thrust coefficient plotted against k for a selected range of data.



Figure D.3: *Mean thrust plotted against St, full range*. Mean thrust coefficient plotted against *St* for the full range of the data.



Figure D.4: *Mean thrust coefficient plotted against St, selected range.* Mean thrust coefficient plotted against *St* for a selected range of data.



Figure D.5: *Mean thrust coefficient plotted against* St_{te} , *full range*. Mean thrust coefficient plotted against St_{te} for the full range of the data.



Figure D.6: *Mean thrust coefficient plotted against* St_{te} , *selected range*. Mean thrust coefficient plotted against St_{te} for a selected range of data.

APPENDIX E

VORTICITY FIELDS FOR RIGID AND FLEXIBLE AIRFOILS

The vorticity fields for the rigid and flexible airfoils are compared at selected reduced frequencies for each of the different oscillation amplitudes considered. Figure E.1 compares the phase averaged vorticity field of the rigid and flexible airfoils for four different reduced frequencies at $\alpha_0 = 2^\circ$. Figure E.2 compares the phase averaged vorticity field of the rigid and flexible airfoils for four different reduced frequencies at $\alpha_0 = 4^\circ$. Figure E.3 compares the phase averaged vorticity field of the rigid and flexible airfoils for four different reduced frequencies at $\alpha_0 = 6^\circ$.



Rigid

Flexible



Figure E.1: *Phase averaged vorticity field of rigid and flexible airfoil for* $\alpha_0 = 2^\circ$. The phase averaged vorticity at the same phase of the rigid (left) and flexible (right) airfoils. The reduced frequency increases from top to bottom, with the reduced frequency listed as the title of each plot in addition to the Strouhal number (St) and the Strouhal number based on the measured trailing edge displacement (St_{te}). The rigid and flexible plots have identical colormaps for the same reduced frequency, but the colormap limits increase as *k* increases.



Rigid

Flexible



Figure E.2: *Phase averaged vorticity field of rigid and flexible airfoil for* $\alpha_0 = 4^\circ$. The phase averaged vorticity at the same phase of the rigid (left) and flexible (right) airfoils. The reduced frequency increases from top to bottom, with the reduced frequency listed as the title of each plot in addition to the Strouhal number (St) and the Strouhal number based on the measured trailing edge displacement (St_{te}). The rigid and flexible plots have identical colormaps for the same reduced frequency, but the colormap limits increase as *k* increases.


Rigid

Flexible



Figure E.3: *Phase averaged vorticity field of rigid and flexible airfoil for* $\alpha_0 = 6^\circ$. The phase averaged vorticity at the same phase of the rigid (left) and flexible (right) airfoils. The reduced frequency increases from top to bottom, with the reduced frequency listed as the title of each plot in addition to the Strouhal number (St) and the Strouhal number based on the measured trailing edge displacement (St_{te}). The rigid and flexible plots have identical colormaps for the same reduced frequency, but the colormap limits increase as *k* increases.

APPENDIX F

POSITIVE AND NEGATIVE VORTEX PROPERTIES

The average value of the vortex properties of Γ_0 , ω_{pk} , *a*, U_c , and r_c were reported in this study rather than the values for each individual positive and negative vortex. For most vortex properties the differences between the positive and negative vortex are less than the experimental uncertainty. When the differences do exceed the experimental uncertainty, the differences are small relative to the change of the variable seen in the parameter space. Overall, the use of the average compared to the individual positive and negative vortices do not impact the interpretation of the data of this study.

The peak vorticity extracted from the both the positive and negative vortices are plotted against k in Figure F.1a for the rigid airfoil and Figure F.1b for the flexible airfoil. Overall, 10 out of 16 cases for the rigid airfoil and 11 out of 15 cases, 67% of all cases studied had no significant difference between the positive and negative vortices. The peak vorticity did not vary by more than 10% for 87% of all of the cases.

Similarly, Figure F.2 shows the circulation extracted for both positive and negative vortices plotted against k in for both the rigid and flexible airfoils. There is less difference between the pair of vortices in terms of their circulation than what is observed in was observed for the peak vorticity. Over 80% of all cases have less than a 4% difference in circulation between the pair of opposite sign vortices. The maximum deviation was 18%, which appears relatively high due to how circulation is scaled.



Figure F.1: *Peak vorticity of both positive and negative sign vortices.* The magnitude of the peak vorticity, $|\langle \omega_{pk} \rangle|$, for both the positive and negative vortices are plotted against *k* for the rigid (a) and flexible (b) airfoils.



Figure F.2: *Circulation of both positive and negative sign vortices.* The magnitude Circulation, $|\Gamma_0|$, for both the positive and negative vortices are plotted against *k* for the rigid (a) and flexible (b) airfoils.

APPENDIX G

SPATIAL RESOLUTION AFFECT ON VORTEX PROPERTIES

The numerical simulations of Hammer (2017) on the NACA 0009 airfoil are used to examine the impact of spatial resolution on the extracted vortex parameters of peak vorticity, ω_{pk} , circulation, Γ_0 , core size r_c , transverse spacing *b*, wavelength, *a*, and the vortex array aspect ratio, b/a. The resolution of the numerical simulations is 0.0025*c* in both the *x* and *y* directions in the wake of the airfoil. The spatial resolution of the experiment is 0.01*c* in both *x* and *y* directions. The vortex properties are computed, using the same methods outlined in Section 2.5.5 for the full resolution numerical simulation results, and also for a reduced resolution of the numerical simulations. The reduced resolution of the numerical simulations was achieved by keeping only every 4th data grid point, and computing the vorticity on this new grid, the same data spacing as the experiments.

The percent change in the vortex properties between the full resolution and lower resolution results are summarized in Table G.1. Peak vorticity will be attenuated due to the lower spatial resolution, as well as the circulation. The change in *b*, *a*, and r_c is not deterministic due to the lower spatial resolution. Overall, ω_{pk} and *b* are affected the most by spatial resolution, leading to errors of 1%-8% for the reduced frequencies considered. The error in circulation is less than 3% for *k*=4 and less than 1% for the rest of the cases. The error in wavelength, *a*, is less than 1% for all cases. The error in *b/a* is driven by the error in *b*, and is between 1% and 8%. The effect on both the (+) and (-) sign vortices is included in the table but don't significantly differ from one another.

k	$\Delta b/c$ (%)	Δ <i>a</i> (%)	$\Delta b/a$ (%)	$\stackrel{(+)}{\overset{\Delta\omega_{pk}}{(\%)}}$	(-) Δω (%)	$(+) \Delta \Gamma_0$ (%)	(-) ΔΓ (%)	$(+) \Delta r_c$ (%)	(-) Δ <i>r</i> _c (%)
4	4.72	0.86	3.83	-1.53	-1.65	-2.78	-2.93	-4.06	-3.93
6	7.70	0.08	7.61	-4.36	-4.47	-0.39	-0.34	0.09	0.16
8	3.38	0.35	3.01	-7.77	-7.88	-0.74	-0.41	-1.80	-0.96
10	-1.08	0.08	-1.16	-3.85	-3.51	-0.27	-0.38	-0.51	-0.89

Table G.1: Change in the vortex properties due to reduced spatial resolution. Change in the vortex properties due to reduced spatial resolution. Using the numerical simulations of Hammer (2017), the % change in the vortex parameters between the full resolution (0.00025x/c) and reduced resolution equal to the experiments (0.001x/c).

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