THEORETICAL ASPECTS OF THE DRYING OF FORAGE WAFERS

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ABSTRACT

THEORETICAL ASPECTS OF THE DRYING OF FORAGE WAFERS

by Frederik Wilte Bakker-Arkema

The high labor requirement of existing forage harvesting systems is a major expense in the production of forages. Hay wafering is a recently developed forage harvesting process in which the complete mechanization of the hay crop may be more nearly realized. The drying behavior of forage wafers was investigated under different drying conditions. Rectangular shaped wafers of different densities and dimensions were utilized.

An apparatus was developed for weighing the test samples continuously in the drying oven. A recording potentiometer was used to obtain the wafer temperatures. The air velocities were measured with a hot-wire anemometer.

Diffusion is the physical mechanism controlling the drying behavior of forage wafers. The diffusion coefficients of wafers (92 percent alfalfa) in the density range from 0.30 to 1.0 g/cm^3 were determined for temperatures from 120° to 200°F.

Forage wafers, unlike many other hygroscopic agricultural products, display a constant diffusion coefficient in the moisture content range from 40 to 5 percent, d.b. The effect of temperature on the diffusion coefficients of forage wafers can be expressed by an Arrhenius plot. The empirical equation for the diffusion coefficient of non-cracked wafers with a density of 0.45 g/cm^3 is given by

$$D = 0.114 \exp(-10{,}810/RT)$$

where D is the diffusion coefficient in cm^2/sec, R the universal gas constant and T the absolute temperature in ^{o}R of the drying air.

The wafer temperature rather than the ambient air temperature should be used to analyze theoretically the drying behavior of forage wafers. Wafers dried in the temperature range from 120-200°F do not reach the ambient air temperature until they are dry enough for safe storage. A semi-theoretical method is utilized to account for the increasing temperatures and diffusion coefficients of drying forage wafers.

The second order differential equation for diffusion with constant initial and boundary conditions does not properly describe the drying behavior of forage wafers. The wafer drying surfaces do not reach the static moisture equilibrium instantaneously as usually assumed, but rather change with time. An exponential type function describes the changing boundary condition.

A forage wafer can be treated as a semi-infinite body during the major part of drying. A drying solid with one directional moisture gradient behaves as the sum of two semi-infinite bodies until 65 percent of the original moisture in the product has been removed.

Approved Carl W. Hall
Major Professor

THEORETICAL ASPECTS OF THE DRYING OF FORAGE WAFERS

By

Frederik Wilte Bakker-Arkema

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To

Paula, Peter, Irma and Erik

Mr. and Mrs. P. W. Bakker-Arkema

Mr. and Mrs. P. de Mol

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ABBREVIATIONS AND SYMBOLS

BTU	British Thermal Units
C	concentration, g per cm^3
cm	centimeter
D	diffusion coefficient, cm^2 per sec
D_o	a constant, cm^2 per sec, defined by equation (4.2.1)
d.b.	as a subscript, dry basis
eq	as a subscript, equilibrium
erf	error-function
erfc	error-function complement
exp(x)	e^x
E	energy of activation, BTU per mole
°F	degrees Fahrenheit
ft	feet
G	mass rate of flow, g per sec
g	gram
hr	hour
in	as a subscript, initial
j	dry weight density, g per cm^3
k	dimensionless coefficient defined by equation (4.4.1)
ℓ	distance, cm
L	infinite series defined by equation (4.1.3)
liq	as a subscript, liquid
M.C.	moisture content, percent dry basis
M.E.	moisture equilibrium content, percent dry basis
P	vapor pressure, g per cm^2

R	universal gas constant, BTU per mole per degree Rankine
$^{\circ}R$	degrees Rankine
Q	mass rate of flow, g per sec
sec	second
t	time or, as a subscript at time t
T	absolute temperature, degrees Rankine
vap	as a subscript, vapor
w.b.	as a subscript, wet basis
x, y, z	rectangular coordinates, cm
β	coefficient, sec^{-1}, defined by equation (4.4.2)
$\phi(t)$	boundary condition at time t
Re	Reynolds number

I. INTRODUCTION

Hay has always been and continues to be one of the most important agricultural crops grown in the United States of America. Zimmerman (1964) reported that hay ranks second in both acres (65 millions) and value (2.5 billion dollars) on the list of agricultural crops. Hay can be harvested in long-loose, chopped, baled or wafered forms. The United States Department of Agriculture (Agricultural Statistics, 1963) estimated that about 85 percent of the hay crop is baled, 4 percent chopped and 10 percent handled as long-loose hay. No data are available on the amount of wafered hay produced in 1963. However, it is known that the interest in wafering is growing rapidly (Zimmerman, 1964).

Hay wafering is a process whereby the hay is tightly compressed into small high density packages. The shape, size and density of the individual wafers varies according to the make of the wafering machine. The major benefit of wafering is the relatively low labor requirement of the system compared to other available hay harvesting methods. Wafers make a completely mechanized hay harvesting and feeding system possible, because, if properly made, they will flow through augers, conveyors and other types of transporting equipment. Additional advantages of wafers are the high bulk density and nutritional value of the product. Matthies (1963) reported that the bulk density of wafers is two and one-half times as large as that of bales and four times as large as that of loose hay. The animal consumption of wafers is significantly greater than of baled hay while the feeding losses are reduced and the selective eating by the animals is eliminated (Ross *et al.*, 1963).

Before wafers can be stored they must be dried. The maximum moisture content of long hay for safe storage is about 17 percent wet basis (Hall, 1957). However, the moisture content of forage wafers has to be reduced to 12 or 13 percent wet basis to prevent molding after three to four weeks (Lamp et al., 1963). It is not clear why long hay and forage wafers develop mold at different moisture contents. It seems most likely that the mechanical treatment of the hay during wafering changes the sorption characteristics of the forage, resulting in a shift of the moisture equilibrium curve.

Notwithstanding the rapid molding of forage wafers, no precise data are available for predicting the drying behavior of forage wafers under different conditions. Holdren et al. (1962) performed drying tests with alfalfa wafers stored in a round steel grain bin fourteen feet in diameter. The results of this investigation were inconclusive as far as the effect of the wafer density and ambient air temperature were concerned.

It can be expected that the necessity for drying wafers will become more pronounced in the future, because of the expected development of new wafering machines which will be able to process forages with an initial moisture content of over fifty percent wet basis (Farrall, 1963).

1.1 Objectives

The objectives of this study were to investigate the physical laws describing the drying behaviour of forage wafers and to relate wafer size, wafer density and ambient air temperature to the drying parameters of the individual forage wafers.

1.2 Topics Covered in the Dissertation

The work reported in this study may be divided into five parts:

1. The treatment of a forage wafer as a two dimensional body with constant boundary conditions.
2. The effect of drying air temperature and wafer density on the diffusion coefficients of forage wafers.
3. The treatment of a forage wafer as a three dimensional body with a changing diffusion coefficient.
4. The treatment of a forage wafer as a two dimensional body with changing boundary conditions.
5. The treatment of a forage wafer as a one dimensional body.

II. THEORY OF DRYING

The history of theoretical drying goes back to 1904 when the Russian scientist Kossowitsch published his thesis on the molecular mechanisms of moisture movement in capillary products (Toponitzki, 1949). The first serious attempt in the English speaking world to explain the mechanisms of drying did not come until almost twenty years later when Lewis (1921) published his now famous treatise on the rate of drying in solids.

One person who deserves more credit for the early development of the theory of drying than any other researcher is T. K. Sherwood. In the period from 1929 to 1936 Sherwood published a series of articles which have become the basis of the diffusion theory in drying (1929, 1930, 1931, 1932, 1933, 1936).

Although the economical importance of drying is well recognized, no textbooks have been published in the English language on the theoretical aspects of drying. One exception is the book by Van Ardsdel et al. (1963) in which some theoretical drying principles are briefly reviewed. In order to study the subject extensively a knowledge of German is essential.

Several books have been published rather recently in Germany dealing exclusively with the principles of drying (Lykov, 1955; Lebedev, 1960; Maltry et al., 1962; Krischer, 1963). Since these works are not available to the average English reader, they will be referred to rather frequently in this study.

The physical and chemical make-up of a product determines to a large extent the drying behavior of that product. Most materials which are dried in the chemical engineering industry are non-hygroscopic. On the other hand, most agricultural products are classified as hygroscopic. The difference between hygroscopic and non-hygroscopic materials is based on the final moisture content to which a material can be dried in a particular mass of air. Hygroscopic materials, in contrast with non-hygroscopic materials, do not dry to zero percent moisture content in air of which the vapor pressure is greater than zero, but approach an equilibrium condition at which the moisture content is larger than zero. At the moisture equilibrium content the vapor pressure of the product is equal to the vapor pressure of the surrounding air.

Several physical mechanisms, namely, osmosis, capillary forces and diffusion, can cause the flow of moisture in a capillary-hygroscopic product during drying. The importance of each mechanism is limited to a particular moisture content range of the drying product. Lykov (1955) showed that osmosis plays an important role in drying only when the moisture content of the product is above the hygroscopic point. The hygroscopic point is defined as the product moisture content corresponding to a relative humidity of 100 percent. Since the initial moisture content range of forage wafers is well below the hygroscopic point (Bakker-Arkema et al., 1962), the mechanism of osmosis does not have to be considered in this study.

The drying of sand and other similar non-hygroscopic products is controlled mainly by capillary forces due to the molecular attraction between the liquid and the solid. Ceaglske and Hougen (1937) maintained that all drying processes are based on capillary behavior. Krischer (1963), among others, pointed out that only during the constant rate drying period, when the flow of moisture away from the drying surface limits the rate of drying, capillary forces determine the rate of moisture transfer through a capillary-hygroscopic product.

Since no constant rate drying period has been observed in forage wafers, it can be assumed that capillary forces can not be the mechanism responsible for the drying behavior of forage wafers.

If neither osmosis nor capillary forces describes the drying behavior of forage wafers, the third of the three physical mechanisms, diffusion, must be the determining factor controlling the process. The exact nature of the potential causing drying has been the cause of much debate among researchers. Görling (1956) distinguishes between: (a) liquid diffusion due to a concentration gradient; and (b) vapor diffusion due to a vapor pressure gradient. Which mode of diffusion applies to the drying of a particular material is difficult to determine. In general, it can be stated that liquid diffusion only takes place in the moisture content range corresponding to relative humidity values between 85 and 100 percent, while vapor diffusion becomes important at lower moisture contents (Krischer, 1963).

Although it is of considerable academic interest which type of diffusion is the controlling factor in the drying of forage wafers, this knowledge is not required for writing the differential equation of the general process. The diffusion equation which describes the drying behavior has the form:

$$G = - D \frac{\partial c}{\partial x} \qquad (2.1.1)$$

For liquid diffusion equation (2.1.1) becomes:

$$G_{liquid} = - D_{liq} \frac{\partial c_{liq}}{\partial x} \qquad (2.1.2)$$

Since the liquid concentration is equal to the dry weight density times the moisture content dry basis, equation (2.1.2) can also be written as:

$$G_{liquid} = - D_{liq}\, j \frac{\partial (M.C.)}{\partial x} \qquad (2.1.3)$$

where j is the dry weight density of the material.

In the case of vapor diffusion equation (2.1.1) becomes:

$$G_{vapor} = - D_{vap} \frac{\partial C_{vap}}{\partial x} \qquad (2.1.4)$$

Treating water vapor as an ideal gas and applying the ideal gas law gives for equation (2.1.4):

$$G_{vapor} = - D_{vap} \frac{1}{R_{vap} T} \frac{\partial P_{vap}}{\partial x} \qquad (2.1.5)$$

$\partial P_{vap}/\partial x$ can be written as $[\partial P_{vap}/\partial (M.C.)]\,[\partial (M.C.)/\partial x]$ and so equation (2.1.5) becomes:

$$G_{vapor} = -D_{vap} \frac{1}{R_{vap} T} \frac{\partial P_{vap}}{\partial (M.C.)} \frac{\partial (M.C.)}{\partial x} \qquad (2.1.6)$$

As Lykov (1955) pointed out, the importance of equations (2.1.3) and (2.1.6) lies in the fact that both types of mass transfer, liquid diffusion and vapor diffusion, are due to the same driving potential, namely, the moisture content gradient of the drying material. This means that the use of the general diffusion equation for the drying of forage wafers is justified:

$$G = - D \frac{\partial (M.C.)}{\partial x} \qquad (2.1.7)$$

where

$$D = D_{liq}\, j + \frac{D_{vap}}{R_{vap} T} \frac{\partial P_{vap}}{\partial (M.C.)} \qquad (2.1.8)$$

It is possible to obtain values for D, the diffusion coefficient, by the use of equation (2.1.7). The disadvantage of this equation is that a static condition must be reached before the values of G and $\partial (M.C.)/\partial x$ can be measured and D be calculated. A static condition was never reached when the method was tried with forage wafers because the wafers started crumbling and molding long before a constant value for G was obtained.

If a mass balance is written for a cube using equations (2.1.1) and (2.1.7), respectively, the following second order differential equations result:

$$\frac{\partial C}{\partial t} = D\left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right] \qquad (2.1.9)$$

and

$$\frac{\partial (M.C.)}{\partial t} = D\left[\frac{\partial^2 (M.C.)}{\partial x^2} + \frac{\partial^2 (M.C.)}{\partial y^2} + \frac{\partial^2 (M.C.)}{\partial x^2}\right] \qquad (2.1.10)$$

In both equations (2.1.9) and (2.1.10) the assumptions are made that the diffusion process is isothermal and that the diffusion coefficient is constant. The equations are simplified if drying takes place from just two opposite surfaces. Only the concentration gradient in one direction has to be considered in that case.

The solutions to equations (2.1.9) and (2.1.10) of the concentration and the moisture content as a function of time and position, will depend on the initial and boundary conditions of the drying material under study. Newman (1931) derived solutions of the one dimensional diffusion equation for the drying of four cases:

a) initial moisture content uniform and surface moisture content constant

b) initial moisture content uniform and the rate of drying at the surface constant

c) initial moisture content uniform and the rate of evaporation at the surface a function of the surface moisture content

d) initial moisture content parabolic and the surface moisture content constant

Case b is only of interest in the constant rate drying period, which has not been observed for forage products. Case c is of importance for products of which the mass transfer coefficient is finite due to a thick boundary layer. This condition will only occur during natural convection air drying; forced convection air drying will soon make the mass transfer coefficient infinite.

The parabolic initial moisture content distribution of case d is of importance in products with an initial moisture content above the hygroscopic point. During the drying of such materials the moisture content distribution becomes parabolic when the hygroscopic moisture content is reached (Lykov, 1955). Due to the low initial moisture contents of forage wafers, this case does not have to be considered in this study.

Forced air drying of forage wafers is best represented by case a. The mass transfer coefficient in forced air drying is very large which accounts for the assumption that the surface of a product attains the moisture equilibrium content as soon as the drying has started (Lebedev, 1961).

Many solutions to equations (2.1.9) and (2.1.10), representing different initial and boundary conditions, can be found in the heat transfer literature. The book on conduction heat transfer by Carslaw and Jaeger (1959) is especially useful in this respect.

When the solutions for the concentration or moisture content distribution, obtained from equations (2.1.9) and (2.1.10), are integrated with respect to time, values for the amount of moisture diffused in or out of the product, are obtained. Crank (1956) gives such solutions for many different initial and boundary conditions.

Several investigators (Babbit, 1949; Becker et al., 1955; Hustrulid et al., 1959; Pabis et al., 1961), have used the solutions of the diffusion equation given by Lewis (1921), Sherwood (1929) and

Newman (1931) in the drying research of agricultural products. The predicted drying curves agreed reasonably well with the experimental data. However, when the experimental curves were examined closely, divergence from the theoretical curves can be observed in most cases. The divergence is usually most noticeable during the first and last stages of drying. The theoretical drying rate at small times is higher than is observed from the experimental data. Pabis *et al.* (1961) explained this phenomenon by taking the temperature change of the drying product into account.

The divergence of the theoretical drying curve from the experimental curve during the last stages of drying of many organic products, is probably a result of a change in the value of the diffusion coefficient (Van Arsdel, 1947). Becker *et al.* (1955) found that in the case of wheat the calculated and experimental values agreed well for moisture contents from 20.5 to 12 percent dry basis. In the moisture content range below 12 percent smaller diffusion coefficients had to be used to make the theoretical curve fit the experimental values. No extensive study has been made on agricultural products as to the exact behavior of the diffusion coefficient in the moisture content range from 5 to 10 percent wet basis as has been done for other products. King (1945) studying the drying behavior of keratin proteins found more than a hundred-fold change in the diffusion coefficient during the drying from 12 to 2 percent wet basis. Fish (1958) reported a decrease in the diffusion coefficient from a value of 1.23×10^{-7} cm^2/sec to a value of 0.05×10^{-7} cm^2/sec for starch gel in the moisture content range from 30 to 0 percent dry basis.

It should be stressed that not all drying products show a continuous decrease in the diffusion coefficients at low moisture contents. Jason (1958) found that the analytical solution to the diffusion equation for the drying of fish muscle described the drying process if two constant diffusion coefficients were used; his data did not show a change in the diffusion coefficient in the range from 10 to 2 percent moisture content dry basis. Krischer (1963) reported that soap and paper are two products which appear to have constant diffusion coefficients over the full moisture content span in which diffusion is the rate controlling drying mechanism.

The drying of forage wafers reported in this study is limited to single wafer drying. This can be compared with single layer drying of grain. No attempt has been made to investigate multi-layer drying of forage wafers. The literature of deep bed drying, therefore, will not be reviewed.

III. EXPERIMENTAL

3.1 Forage Wafers

The forage wafers employed in this study were made with an experimental wafering machine manufactured by the Massey-Ferguson company. The machine is shown in Figure 1. The mechanics of the machine have been described by Lundell et al. (1961). The only difference between the Lundell and the experimental Massey-Ferguson machine is the position of the wafering dies. Figure 2 is a close-up of the horizontal wafering dies of the Massey-Ferguson machine.

The cross-sectional area of each die, with zero p.s.i. die pressure applied, is 6.0×6.5 cm = 39 cm^2. The length of the wafers varied from 1 to 8 cm. Schoedder et al. (1964) reported that the average size of the Massey-Ferguson wafers is $6.2 \times 6.3 \times 6.7$ cm.

The density of the wafers varied from 0.25 to 1.00 g/cm^3. Wafers with a density below 0.4 g/cm^3 were not tested due to their non-uniform character. The average density of alfalfa wafers made with the Massey-Ferguson machine is 0.435 g/cm^3 (Schoedder et al., 1964). The wafer densities are expressed on a dry matter basis.

The wafers used in this study were made from a chopped alfalfa-grass mixture (92 percent alfalfa, 8 percent grass) with an initial moisture content of 18 percent dry basis. The average length of the chopped forage was two inches. Moisture was added to the forage before it was wafered. The moisture content of the wafers was between 20 and 40 percent dry basis.

Some of the wafers employed in the drying tests were chemically analyzed. Table 1 gives the results.

Figure 1. Experimental Massey-Ferguson wafering machine.

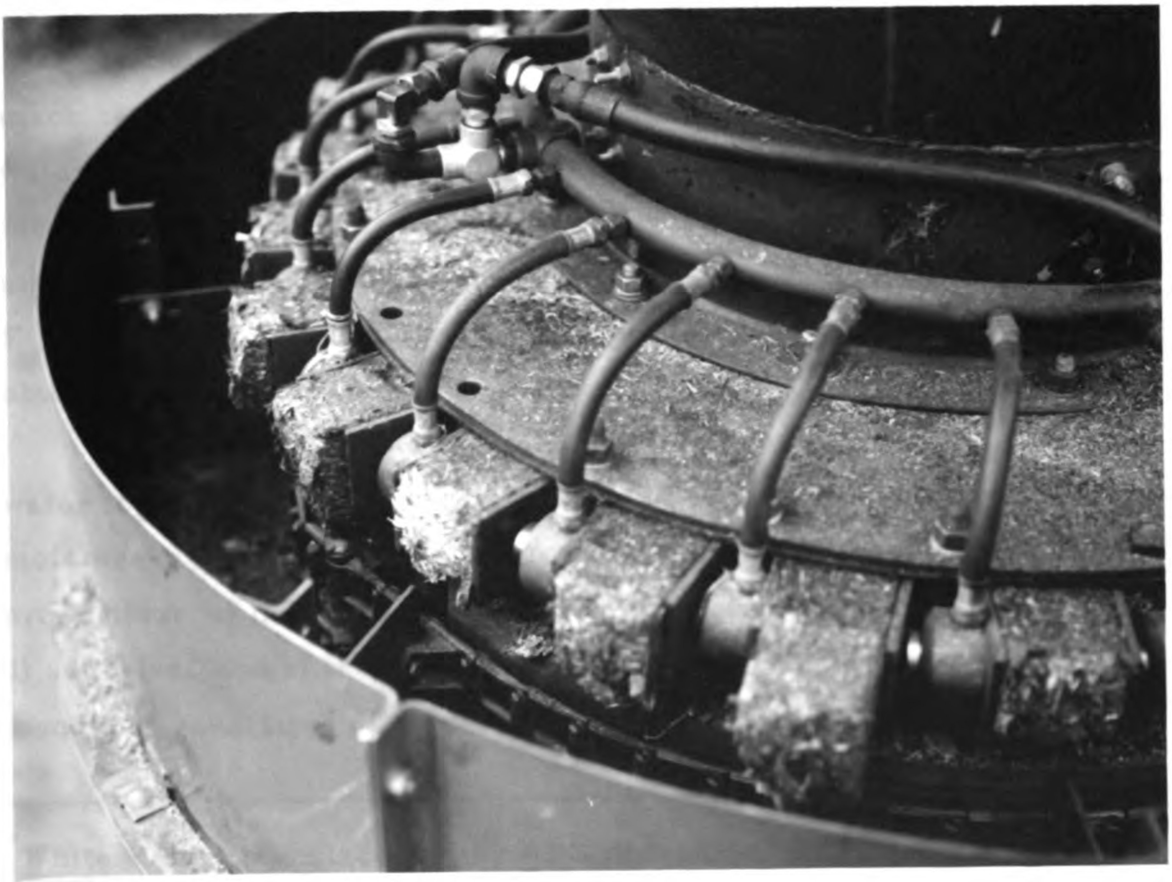

Figure 2. Close-up view of wafering dies.

Table 1. Feed analysis of forage wafers used in the drying tests, percent oven-dry basis.

Sample	Protein	Crude fiber	Ether extract	Ash	N-free extract
1	16.56	22.99	1.40	7.41	51.64
2	16.63	28.05	1.45	7.06	46.81
3	16.50	25.52	1.45	6.75	49.78

Immediately after the wafers were made they were placed in plastic bags and stored at 33°F. The wafers were left in storage for at least three months to assure an uniform moisture distribution in each wafer before it was used as a drying sample. A total of 216 wafers were dried and their drying curves analyzed.

The average wafer produced by the Massey-Ferguson wafering machine was evenly shaped except at the two ends. The non-flat wafer ends can be seen sticking out of the wafering dies in Figure 2. Since it is very difficult to treat an irregularly-shaped body mathematically, the wafers were cut in brick-shaped samples on a bandsaw. Figure 3 shows some wafers of different thicknesses which were obtained from irregularly-shaped parent wafers.

When it was desired to limit the moisture gradient in a drying wafer to one dimension, four sides of the wafer were coated with a moisture-proof material. This material had to have the following properties: a) moisture-proof; b) heat resistent; c) expandable; d) adhesive, and e) inert. After testing many materials a metal mender,* consisting of 50 percent polyester-resin and 50 percent

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*White Velvet manufactured by ABC Autoparts, Inc., Lansing, Michigan.

inert filler, was found to have the required properties. The only disadvantage of the material was that the weight decreased somewhat when placed in the oven at high temperatures. The weight loss of the material was accounted for in the wafer drying curves. Figure 4 shows some wafers coated with the metal mender. A full-size wafer along with its moisture-proof shell can be seen in Figure 5.

3.2 Drying Setup

An overall view of the instrumental setup used in obtaining the drying curves for forage wafers is shown in Figure 6. The equipment consists of a drying oven, analytical balance, automatic recording potentiometer and a hot-wire anemometer. The balance, located on top of the oven, was fitted with a brace on which a mesh-wire basket was suspended. A sample was placed in the basket during a drying test. This setup made it possible to weigh the sample without removing it from the oven. Figure 7 is a close-up view of the interior of the oven. The illustration shows, in addition to the drying basket and sample, the thermocouples and the hot-wire anemometer probe for measuring the wafer temperatures and oven air flows, respectively.

A Precision Scientific (cat. no. 31058) oven (range 100° to 500°F) served as the drying apparatus. The velocity of the oven air was adjustable between 75 and 250 feet per minute. The oven temperature was thermostatically controlled. A sensitive microswitch was added to limit the temperature span to 4°F. A Mettler balance (type k7, accuracy ± 0.03 g) was employed to obtain the drying wafer weights. The wafer temperatures were sensed by means of copper-constantan (1938 calibration, 24 B and S gage) thermocouples and recorded on a Brown (Type 153 Electronik) 12 point potentiometer. The oven air velocities were measured with a hot-wire anemometer (Flow Corporation, Model HWB3). The moisture contents of the forage wafers were determined after drying the samples for 48 hours at 200°F.

Figure 3. Different size forage wafer used in drying tests.

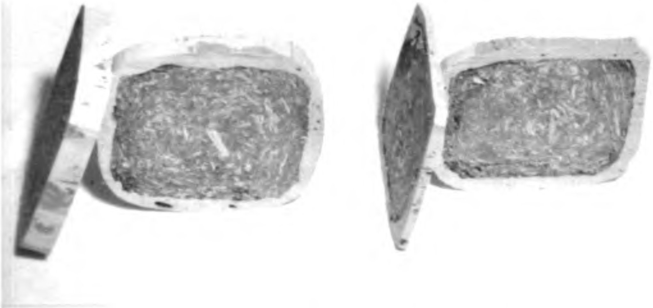


Figure 4. Wafers covered on four sides with moisture-proof material.

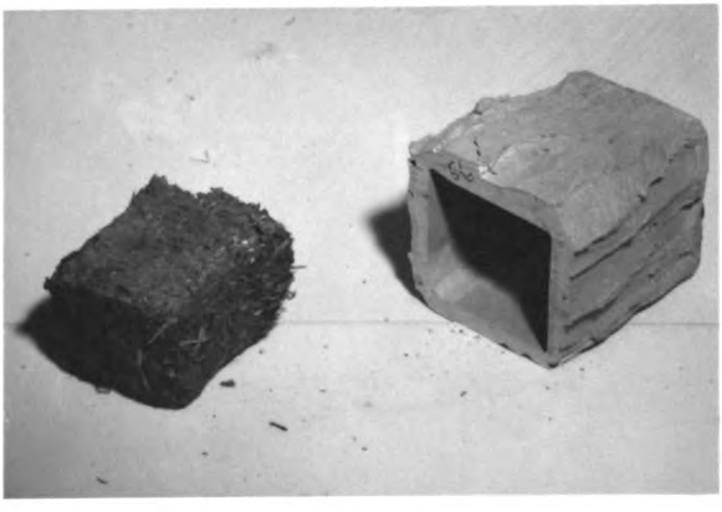


Figure 5. Moisture-proof shell plus forage wafer.

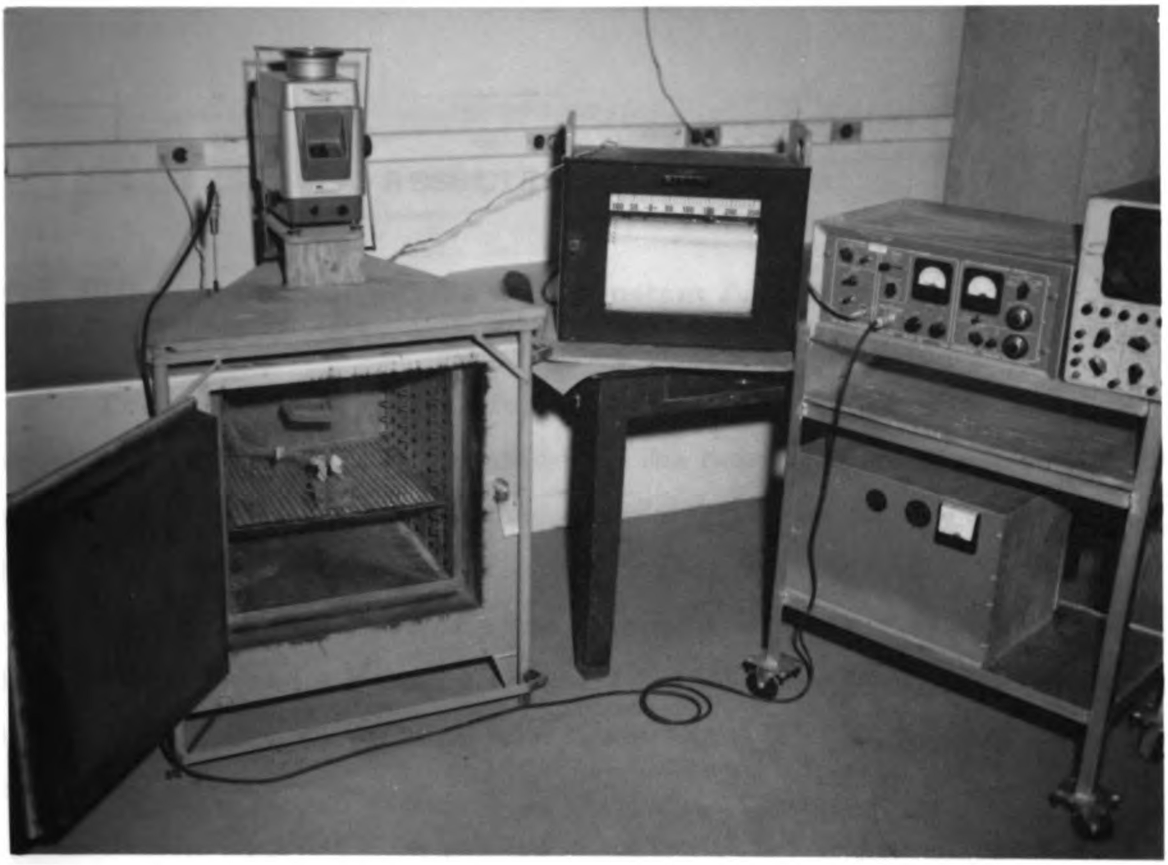


Figure 6. Experimental drying set-up.

Figure 7. Close-up view of drying oven.

IV. RESULTS AND DISCUSSION

4.1 Diffusion in Thin Wafers with Constant Boundary Conditions

The forage wafers used for the drying tests in the first two sections of this study were coated with the moisture-proof compound on all but two sides. This caused the moisture content gradients in the y and z directions, $\partial(\mathrm{M.C.})/\partial y$ and $\partial(\mathrm{M.C.})/\partial z$, to be zero. When the thickness of a wafer is 2ℓ, with the surfaces located at $+\ell$ and $-\ell$ and a constant diffusion constant D is assumed, the general diffusion equation describing the isothermal drying process becomes:

$$\frac{\partial C}{\partial t} = D\,\frac{\partial^2 C}{\partial x^2} \qquad (4.1.1)$$

If the boundary and initial conditions are $C_{x=\pm\ell} = C_{eq}$ at $t \geqslant o$ and $C_{t=o} = C_{in}$ for $-\ell \leqslant x \leqslant +\ell$, respectively, the solution to equation (4.1.1) becomes (Crank, 1956):

$$\frac{C_{t,x} - C_{in}}{C_{eq} - C_{in}} = 1 - \frac{4}{\pi}\sum_{n=o}^{\infty}\frac{(-1)^n}{2n+1}\exp\left[-D(2n+1)^2\pi^2 t/4\ell^2\right]\cos\frac{(2n+1)\pi x}{2\ell} \qquad (4.1.2)$$

The amount of moisture Q_t which has diffused out of the wafer per unit area in time t will then be (Crank, 1956):

$$Q_t = \int_o^t D\left(\frac{\partial C}{\partial x}\right)_{x=o} dt$$

$$= Q_\infty\left[1 - \sum_{n=o}^{\infty}\frac{8}{(2n+1)^2\pi^2}\exp\left[-D(2n+1)^2\pi^2 t/4\ell^2\right]\right] \qquad (4.1.3)$$

where Q_∞ denotes the amount of moisture removed from the wafer per unit area after an infinite time. McKay (1930) has tabulated the values of Q_t/Q_∞ for different values of $\pi^2 Dt/4\ell^2$. The values of the series on the right hand side of equation (4.1.3) were called L and have been reproduced in Table A.1 of the Appendix. McKay's table was used to calculate these values.

Substituting in equation (4.1.3)

$$Q_t = [(M.C.)_{in} - (M.C.)_t] \times \text{dry matter}$$

and $$Q_\infty = [(M.C.)_{in} - (M.C.)_{eq}] \times \text{dry matter}$$

yields:

$$\frac{(M.C.)_t - (M.C.)_{eq}}{(M.C.)_{in} - (M.C.)_{eq}} = \sum_{n=o}^{\infty} \frac{8}{(2n+1)^2\pi^2} \exp[-D(2n+1)^2\pi^2 t/4\ell^2] \tag{4.1.4}$$

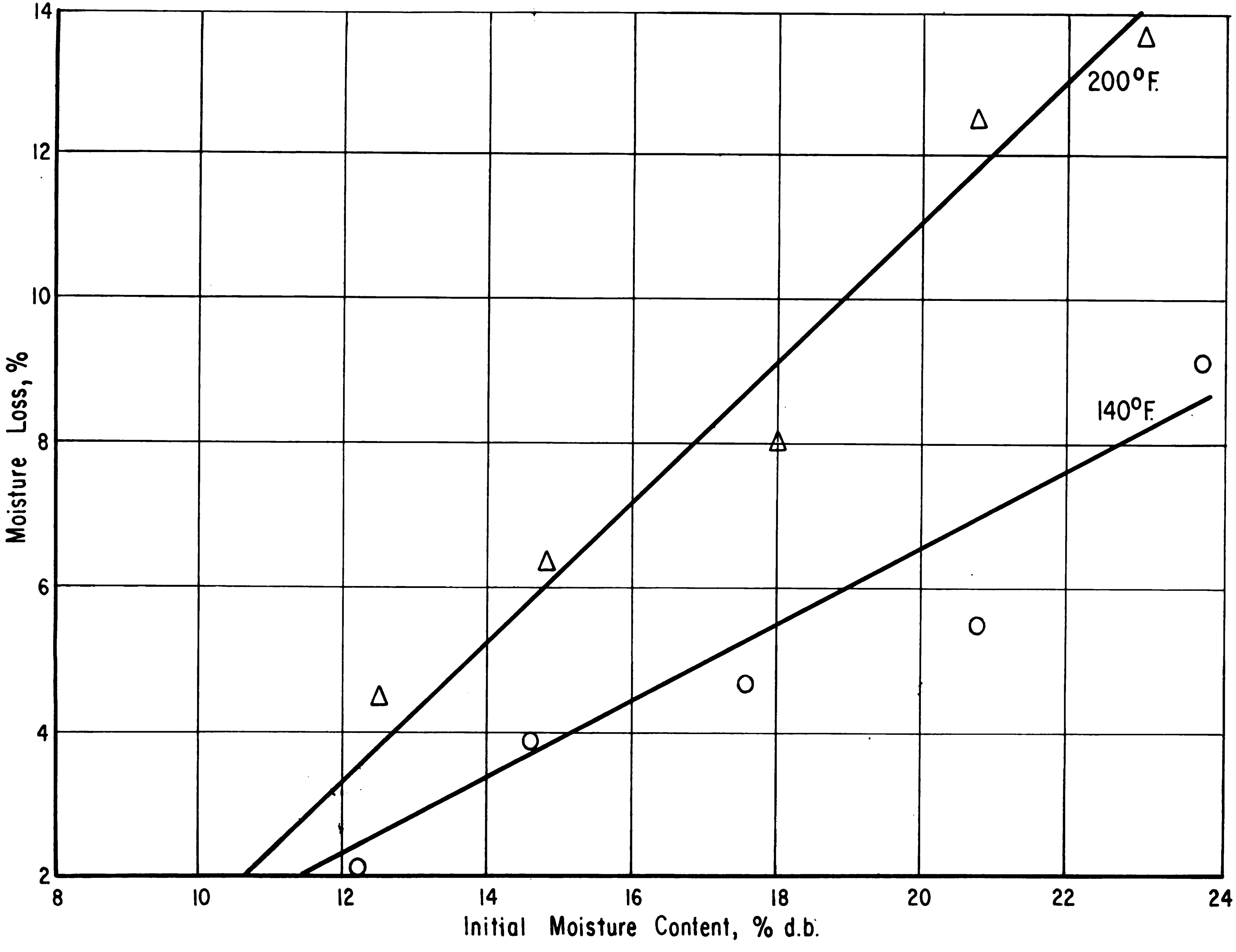
If equation (4.1.4) describes correctly the process of drying, a straight line will result for all but small times, when the left hand side of the equation, the moisture ratio, is plotted on semi-logarithmic paper versus time. This straight line character is due to the fact that for large times all but the first term of the series become insignificant. However, for small values of time the series solution cannot be expressed by the first term alone. This means that the initial drying period will not display a straight line on semi-logarithmic paper. It is for this reason that Newton's equation for drying (Hall, 1957) does not hold for the initial period of drying.

As noted in Section II, equation (4.1.4) has been employed by other researchers to explain the drying behavior of agricultural products. As boundary condition these investigators did not use the static moisture equilibrium content but the so-called dynamic moisture equilibrium content (also named the effective surface moisture content), whose value is considerably larger than that of the static value. Jones (1951), who originated the idea of the dynamic moisture equilibrium, stated that the surface moisture concentration of a hygroscopic product will remain above the static equilibrium moisture content during the falling rate drying period as long as the "more loosely" held water has not been removed. Jones obtained a value of 9.1 percent for the dynamic equilibrium moisture content of wheat by plotting the moisture content loss in 20 minutes at different temperatures versus the initial moisture content. Becker and Sallans (1955), employing Jones' experimental method, found a dynamic moisture equilibrium value for wheat of 10.3 percent, while Fan et al. (1961) reported a value between 7.0 and 9.0 percent for the same crop. Pabis et al. (1961) used values between 6.2 and 10.6 percent in the construction of the drying curves for corn.

Dynamic moisture equilibria have been published for grains only. It was therefore decided to obtain these values for forage wafers. One centimeter thick slices from four wafers of almost the same density, but with different initial moisture contents, were dried for 60 minutes at different temperatures. The moisture content loss was plotted versus the initial moisture content in Figure 8 for two temperatures. The best straight lines were drawn through the experimental points. It is clear from Figure 8 that the straight lines do not fit the experimental points. It should further be noted that the lines do not intersect the abscissa at one point as had been predicted by Jones.

The possibility exists that failure of Jones' method for obtaining the dynamic moisture equilibrium contents of forage wafers is caused by the fact that the wafers used for this experiment were not of the exact same density. Before accepting this as the definite reason for the failure of the appearance of a Jones' plot, it should be recognized

Figure 8. Moisture loss after one hour drying time at different drying temperatures as a function of initial wafer moisture content. Wafer density: 0.80 g/cm^3; wafer thickness: 1 cm.



that Jones' own data as well as that of Becker and Sallans (1955) and of Fan et al. (1961) did not show perfect straight lines. Especially the points at the lower initial moisture contents tended to fall off the straight lines.

It is not feasible to test directly the moisture content behavior of the drying surfaces of grain due to the small size of the kernels. With forage wafers, however, direct moisture content determinations can be made. In order to investigate the surface moisture concentration representing the boundary conditions in equation (4.1.1), thin surface layers were sliced from several 5 cm thick wafers, insulated on all but one surface, after different drying periods and the moisture contents of these slices determined. Some of these data are tabulated in Table 2. The data indicate that the surface layer moisture contents fell below 7 to 10 percent, the dynamic moisture equilibrium range, before the average moisture content of the whole wafer had reached this value. These results contradict the assumption made by Jones (1951) and other researchers (Becker and Sallans, 1955; Hustrulid et al., 1959; Pabis et al., 1961) of the existence of a dynamic moisture equilibrium content for all hygroscopic products.

Table 2. Moisture contents of surface layers of forage wafers after different drying periods.*

Time, hr	Surface layer thickness, cm	Surface layer M.C., % d.b.	Whole wafer M.C., % d.b.
5.0	0.25	8.48	18.2
11.1	0.28	4.33	11.1
24.2	0.24	2.65	7.5

*Drying temperature 140°F; wafer thickness 2.50 cm; wafer density 0.80g/cm^3; insulated surfaces 5.

Due to the results of Figure 8 and Table 2, the moisture equilibrium contents reached after an infinite time period, the so-called static equilibrium contents, were employed as boundary conditions in solving the diffusion equation (4.1.1) for the drying of forage wafers.

The static moisture content equilibrium values of an agricultural product are a function of the temperature, history and the chemical analysis of the product. It was shown in an earlier investigation (Bakker-Arkema et al., 1962) that high protein content alfalfa has higher moisture content equilibrium values than alfalfa with a lower protein content. Wafers made from the same forage may have different leaf and stem contents and therefore may not give the same feed analysis or moisture equilibrium content values (see also Table 1).

The history of the wafers used in this investigation is unique in that most of the moisture that had to be removed during the drying process had been added in the liquid form to low moisture content chopped alfalfa. It is likely that this affects the moisture equilibrium content of the forage material.

Since neither the exact chemical analysis of a particular forage wafer nor the effect of its previous history on the equilibrium moisture contents is known, the experimentally found moisture equilibria determined for wafers by other investigators should not be used for making the accurate diffusion coefficient calculations of this study. Instead the actual equilibrium moisture content, determined after a wafer has been dried for a long period of time, should furnish the boundary condition for solving equation (4.1.1) for that wafer. This procedure was followed in this section. The equilibrium moisture content values of wafers 34A and 34B, used for further analysis in this study, dried at 140°F, were 2.39 and 3.70 percent d.b., respectively, although the wafers were cut from the same parent wafer.

In Figure 9 the experimentally obtained values of the moisture ratio of wafer 34B, with an actual moisture equilibrium of 3.70 percent d.b. are plotted versus time. The points fall on a straight line after about 90 minutes. The straight line continues as long as the test lasted, in other words until the moisture equilibrium of 3.70 percent d.b. was reached. Two other lines are drawn in Figure 9. The upper line represents the experimental moisture ratio versus time obtained by assuming the lower moisture equilibrium content of 2.39 percent, d.b., which was obtained experimentally for wafer 34A. The lower line in Figure 9 represents points calculated by choosing 7.0 percent as the equilibrium moisture content, a value approximately equal to the dynamical moisture equilibrium of wheat as found by Fan et al. (1961). It can be seen from Figure 9 that only the experimental moisture ratios calculated from the actual moisture equilibrium of 3.70 percent, d.b. display a one-straight-line behavior after 90 minutes.

If equation (4.1.4) describes the mechanism of wafer drying correctly, the values of D, the diffusion coefficient, can be calculated once the moisture ratio values at the different times are known. Equation (4.1.4) predicts that one D-value will represent the full drying range. In Table 3 the D-values are tabulated for the three moisture ratio curves of Figure 9. The M.E. = 7.0 percent and M.E. = 2.39 percent columns never reach a constant value. The diffusion coefficients calculated from the 3.70 percent moisture equilibrium curve increase from an initial value of 0.44×10^{-5} cm^2/sec at 15 minutes and 22.71 percent moisture content to about 0.75×10^{-5} cm^2/sec at 90 minutes and 14.81 percent moisture content, but remain constant throughout the remainder of the drying period. This trend of a constant diffusion coefficient after a period of 90 minutes is not observed for the other moisture ratio lines of Figure 9.

Figure 9. Moisture ratio versus time for wafer 34B, assuming different moisture equilibrium values.

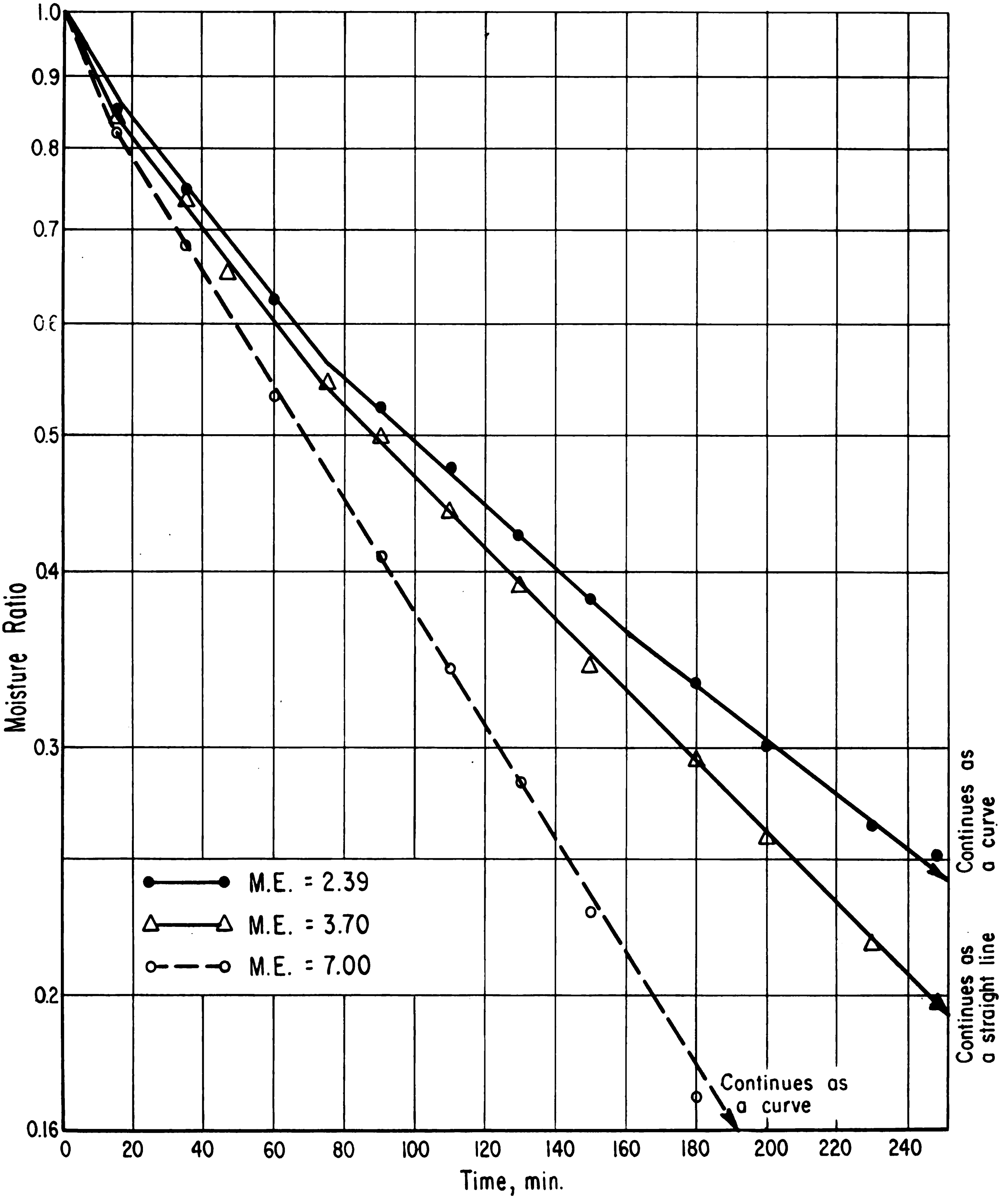


Table 3. Diffusion coefficients times 10^{-5} of wafer 34B* at different moisture contents and drying times, assuming different moisture equilibrium values.

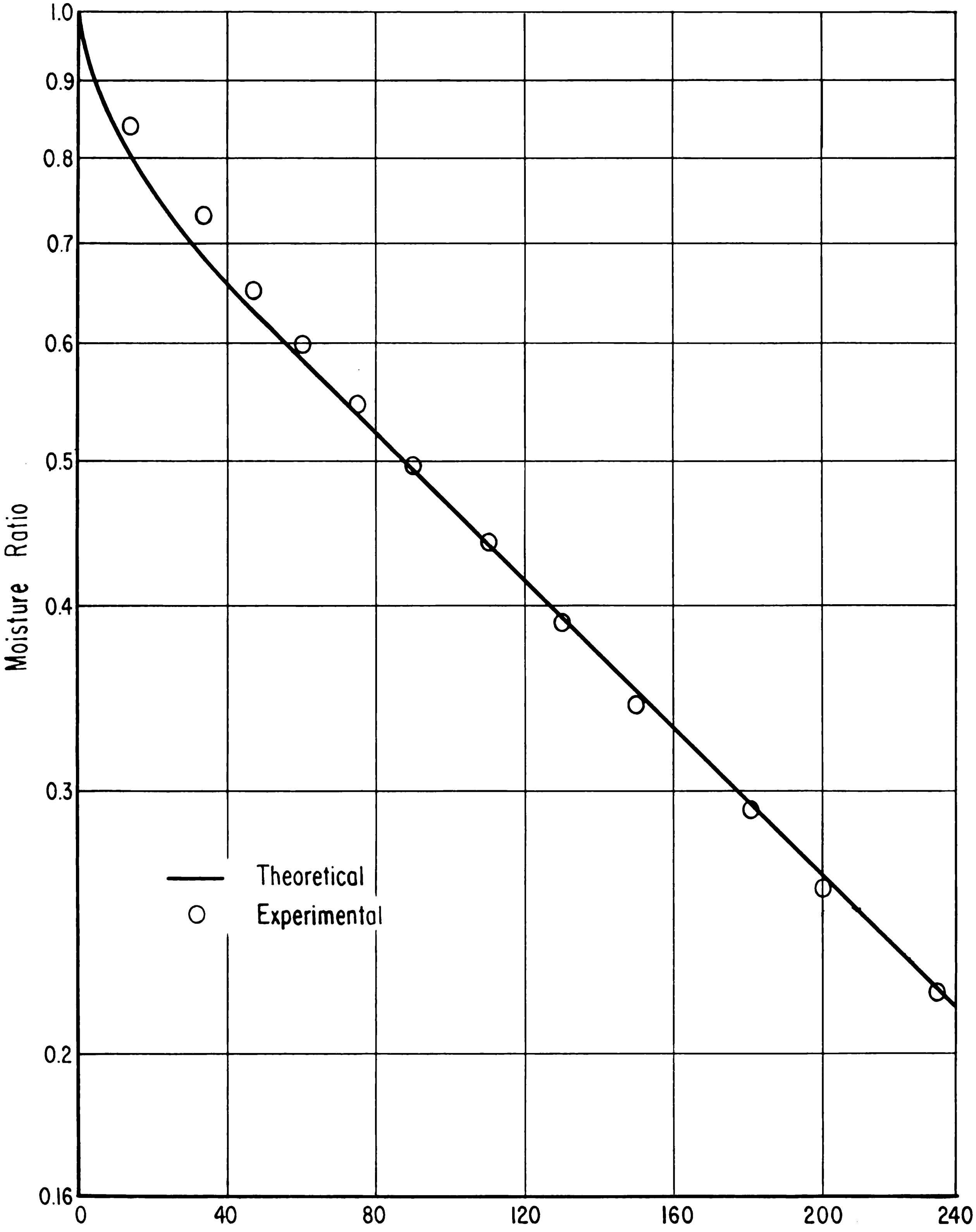
Time	M.C., %d.b.	M.E. = 7.00	M.E. = 3.70	M.E. = 2.39
0	26.16	----	----	----
15	22.71	0.58	0.44	0.38
35	20.14	0.80	0.48	0.50
45	18.91	0.85	0.64	0.54
60	17.18	0.94	0.67	0.60
90	14.82	1.01	0.75	0.65
120	12.79	1.09	0.76	0.67
150	11.37	1.15	0.76	0.68
180	10.24	1.14	0.75	0.65
250	8.10	1.46	0.76	0.66
430	5.17	----	0.76	0.60

*Drying temperature 140°F; wafer thickness 0.90 cm; wafer density 0.94g/cm^3; insulated surfaces 4.

Table 3 shows that a higher moisture equilibrium results in a larger calculated diffusion coefficient and vice versa. At 90 minutes the calculated D-values for M.E. values of 7.00, 3.70 and 2.39 percent are 1.01, 0.75 and 0.65 x 10^{-5} cm^2/sec., respectively. In comparing diffusion coefficients for a particular product reported by different investigators, a difference in values might well be explained by taking the different moisture equilibria into consideration.

Although the diffusion coefficient calculated from equation (4.1.4), using the actual moisture equilibrium as boundary condition, is not constant during the initial drying period, a theoretical drying curve was constructed using for D the constant value of 0.75 x 10^{-5} cm^2/sec. Figure 10 shows good agreement between the theoretical and experimental points in the lower moisture ratio range. At the beginning of the drying period, however, the theoretical line falls

Figure 10. Drying curve for wafer 34B at 140°F.



below the experimental points. This could have been expected since too large a value for D was used to calculate the theoretical curve during the initial drying period.

Table 3 shows that no matter which moisture equilibrium is used in equation (4.1.4), the calculated diffusion coefficient increases in value during the initial drying period. This phenomenon will be explained in section 4.4. Other researchers (Babbit, 1949; Becker and Sallans, 1955; Hustrulid *et al.*, 1959) have reported constant diffusion coefficients during the beginning stages of drying while they observed that the diffusion coefficients at lower moisture contents decreased in value. It is not clear how to account for the constant diffusion coefficients during the initial drying period. The smaller diffusion coefficients at the lower moisture contents might have resulted from the improper choice of the moisture equilibria as boundary conditions.

This section has shown that the solution of the diffusion equation with constant boundary and initial conditions and constant diffusion coefficient gives the correct moisture ratio values for the drying of wafers except during the initial drying period. The diffusion coefficient remained constant to moisture contents as low as five percent dry basis. The proper choice of the moisture equilibrium was important. In the sections 4.3 and 4.4 several different solutions of the diffusion equation will be tested to see if the drying process of a wafer can be properly described over the complete drying range.

4.2. The Effect of Wafer Density and Drying Air Temperature on the Diffusion Coefficient

The diffusion coefficient of a drying wafer is primarily dependent upon the wafer density and the temperature of the drying air. The purpose of this section is to establish the nature of the relationships between the diffusion coefficient, the wafer density and the ambient air temperature.

The experimental procedure for obtaining the effect of the wafer density and the drying air temperature on the diffusion coefficient of forage wafers was similar to the procedure described in the previous section. Wafers of less than 1.2 cm thickness were coated on all but the two large sides with the moisture-proof material and dried at 120°, 140°, 160°, 180° and 200°F. Over 200 different wafers were dried. The diffusion coefficient of each wafer was calculated using the method described in section 4.1.

The physical make-up of a forage wafer will be discussed before evaluating the drying test results. A wafer is not a perfect product. The surface contains many cracks of different width and depth as can be seen from Figure 3. In addition to the numerous small surface cracks no wafer is completely homogeneous. In order to investigate wafer uniformity several wafers were cut in 1 cm thick slices, dried in a 200°F oven and the dry weight density of each slice determined. In Table 4 the results are tabulated.

Table 4. Densities, g/cm^3, of wafer slices cut from the same forage wafer.

	Slice				
Wafer	1	2	3	4	5
1	.278	.358	.289	.361	.501
2	.483	.457	.454	.441	.511
3	.465	.420	.408	.463	.520
4	.794	.790	.942	.929	.925
5	.605	.489	.493	.511	.519
6	.880	.963	.984	.970	.916
7	.588	.668	.528	.634	.671

The wafers used for the density tests were more homogeneous than the average wafer made with the Massey-Ferguson wafering machine. Still some of the wafers of Table 4 had a density difference of almost 200 percent between the individual slices cut from them.

The small surface cracks and uneven density along with the occasional development of large and deep cracks during the course of drying due to thermal stresses, make a theoretical drying analysis of forage wafers difficult. When a crack develops the surface area of the wafer is increased, resulting in an increase of the drying rate of the wafer. If the moisture ratios of such a wafer are plotted versus time, not a straight line results but a curve which is concave to the abscissa. The calculated diffusion coefficients appear to increase in value with increasing time while in actuality the surface area has become larger.

Three factors have to be considered before a proper comparison can be made of the diffusion coefficients of seemingly equal wafers: a) the surface cracks, b) the homogeniety, and c) the cracks developed during drying. An analysis of the experimental drying data revealed that the increase in the value of the diffusion coefficient due to a change in surface area from cracks or due to non-uniform density characteristics can be very significant. In Table 5 the minimum and maximum diffusion coefficients at different temperatures obtained in this study are tabulated for different wafer densities. In some cases the difference between the minimum and maximum diffusion coefficient values for wafers in the same density range is over 250 percent. It was seldom difficult to explain a large diffusion coefficient for a particular wafer. The sample was usually uneven in density and/or cracked. The minimum diffusion coefficient values were obtained with uniform wafers which had not developed any surface cracks. The condition of a wafer could be checked after drying since a photographic record was made of every drying sample.

Table 5. Minimum and maximum diffusion coefficients times 10^{-5} at different temperatures of forage wafers of different density.

	Temperature, °F									
Density,	120		140		160		180		200	
g/cm^3	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.
.40- .50	.89	1.76	1.40	4.65	1.72	4.50	2.40	5.80	2.84	4.75
.51- .65	.91	1.49	1.22	1.85	1.72	2.96	1.82	3.18	1.95	4.00
.66- .75	.75	1.40	.92	1.68	1.45	3.29	1.53	1.85	1.82	3.20
.76- .90	.75	.92	.87	.95	1.27	1.67	1.39	2.80	1.62	5.80
.91-1.00	.41	.80	.61	.83	.65	1.42	1.06	2.42	1.45	2.35

Considering the data in Table 5, it seems that the investigation into the effect of density and temperature on the diffusion coefficient of forage wafers has reached an impasse. Certainly, there is a trend that indicates that an increase in density and a decrease in drying air temperature decreases the value of the diffusion coefficient, but this trend is inconsistent especially if the maximum values are considered. The impasse can be overcome, however, if the objectives of the drying of forage wafers are considered.

An agricultural crop is dried before storage to keep the product from spoiling. It is necessary to dry every part of the material below a certain moisture content; one moist part may eventually cause molding of the total product. The old saying that one rotten apple will spoil the whole bunch, holds indeed for all agriculture crops. Thus work on drying should consider those samples of a product which show the slowest rate of drying. Therefore, the values in Table 5 which are of practical importance for the drying of forage wafers are the minimum values of the diffusion coefficients. It is for this reason that the relationships between density, temperature and the diffusion coefficient reported in the last part of this section, are those obtained from the drying curves of uniform, non-cracked wafers which had the minimum diffusion coefficient values for the particular condition under study. A disadvantage of reporting the minimum diffusion coefficient values at the different wafer densities and drying air temperatures is that there is only one minimum value for each condition. However, no other treatment of the experimental data seemed to offer the advantages obtained from following this procedure. In Figure 11 the minimum diffusion coefficients are plotted on semi-logarithmic paper versus the reciprocal of the absolute temperature for different wafer density ranges.

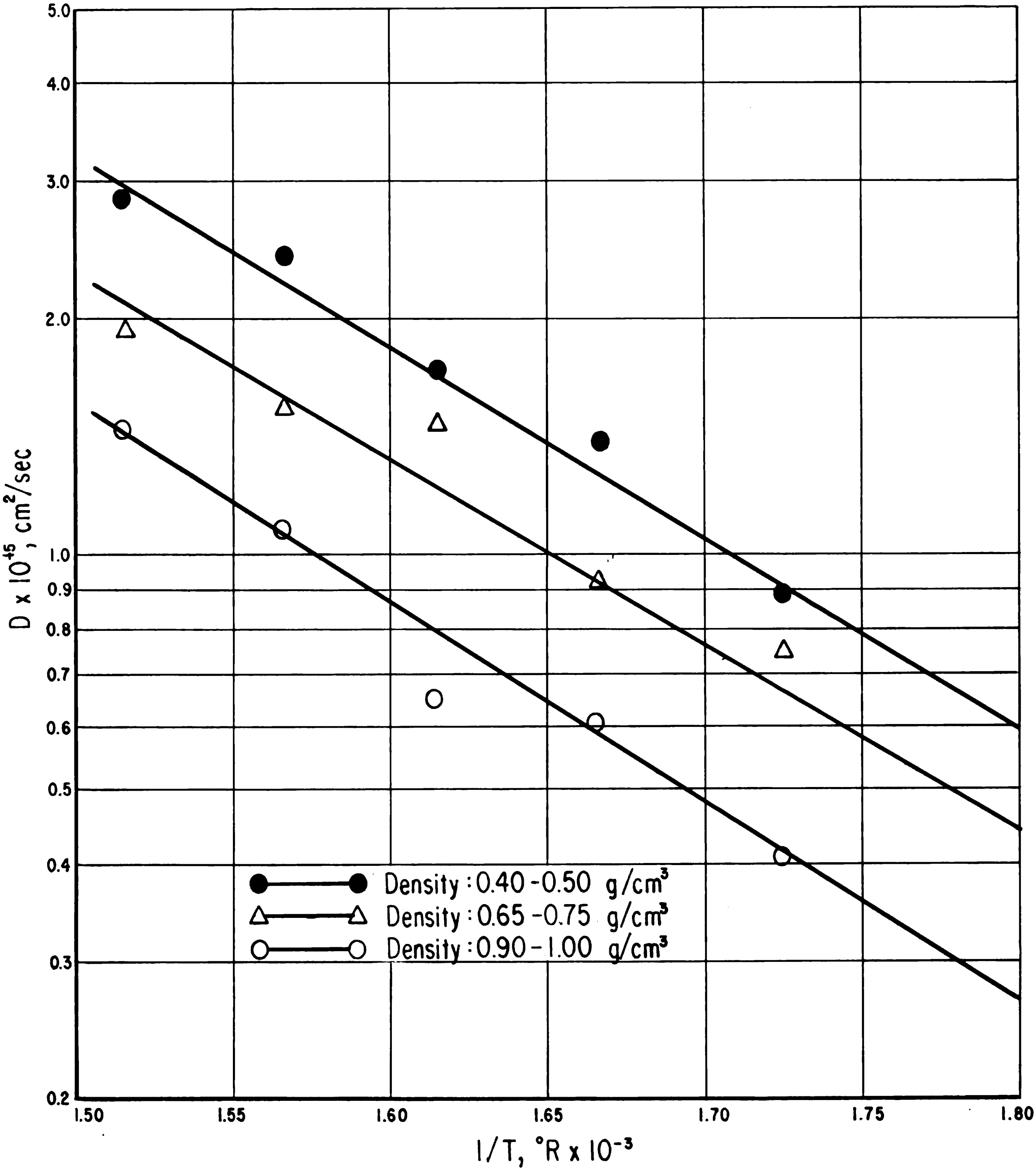
Figure 11 shows that a linear relationship exists between the logarithm of the diffusion coefficient and the reciprocal of the absolute temperature for each particular wafer density range. Each density range can therefore be represented by an Arrhenius type of equation which is of the form

$$D = D_o \exp(-E/RT) \tag{4.2.1}$$

where D_o is a constant, E the energy of activation and R the gas constant. For the density range 0.40 -0.50 g/cm^3 the activation energy is 10,810 BTU/mole; for the density range 0.65-0.75 g/cm^3, 9,960 BTU/mole and for the density range 0.90-1.00 g/cm^3, 10,240 BTU/mole. The values for the constants D_o are 0.114 cm^2/sec, 7.5 cm^2/sec, and 28 cm^2/sec, respectively. The values of the activation energy for the three densities are almost equal which could have been expected since all the wafers employed in this study were made from the same forage. The correlation coefficients of the three lines are about 0.92.

One of the important results of Figure 11 is that the three lines representing the three density ranges, are approximately parallel. In section 4.3 it will be shown that the temperature of a drying

Figure 11. Relationship between the diffusion coefficient and the reciprocal of the absolute drying air temperature for different wafer density ranges.



brick-shaped wafer is not constant, but will increase slowly during the course of drying. This means that the diffusion coefficient will also change. If the diffusion coefficient of a particular wafer is known at one temperature (this value can be found by analyzing a thin slice of the wafer), the point representing the wafer can be located on Figure 11. A straight line drawn through this point parallel to the other lines in Figure 11 will then make it possible to find the values of the diffusion coefficient at any time assuming that the wafer temperature history is known.

It is interesting to compare the values of the diffusion coefficients of forage wafers obtained in this study with the diffusion coefficients reported by other researchers for different hygroscopic products. In Table 6 some of the values available in the literature are tabulated. The diffusion coefficients for wood and potatoes are in the same range as those of forage wafers.

Table 6. Diffusion coefficients times 10^{+5} for several hygroscopic products.

Product	D, cm^2/sec	Temp. ^oF.	Investigator	Comments
Wheat	0.69	68	Becker *et al.* (1955)	M. E. =dynamic M. E.
Wheat	27.70	176	Becker *et al.* (1955)	M. E. =dynamic M. E.
Corn	2.78	115	Pabis *et al.* (1961)	M. E. =dynamic M. E.
Corn	5.56	152	Pabis *et al.* (1961)	M. E. =dynamic M. E.
Wood(pine)	0.47	122	Stamm *et al.* (1961)	tangential diffusion
Wood(pine)	1.22	190	Stamm *et al.* (1961)	tangential diffusion
Wood(pine)	0.53	122	Stamm *et al.* (1961)	radial diffusion
Wood(pine)	2.56	190	Stamm *et al.* (1961)	radial diffusion
Cottonbale	3.60	59	Henry (1939)	density 0.2 g/cm^3
Cottonbale	0.79	59	Henry (1939)	density 0.4 g/cm^3
Cottonbale	0.22	59	Henry (1939)	density 0.6 g/cm^3
Catfish	0.36	86	Jason (1958)	
Cod	0.34	86	Jason (1958)	
Potato	1.0	130	Saravacos *et al.* (1962)	potatoes were
Potato	2.4	156	Saravacos *et al.* (1962)	scalded

In this section it was shown that the diffusion coefficients of forage wafers are dependent upon the drying air temperature and the wafer density. Arrhenius plots relating these three variables were established for the slowest drying samples. An increase in the drying air temperature and a decrease in wafer density resulted in an increase of the diffusion coefficient.

4.3 Diffusion in Brick Shaped Wafers with Constant Boundary Conditions

In section 4.1 the drying of thin wafers with an initial temperature equal to the ambient air temperature was analyzed. It is questionable if the results of that section are of direct practical value since the average thickness of a Massey-Ferguson wafer is 6.2 cm while the initial temperature is well below that of the ambient drying air. In this part of the study the influence of these two factors on the drying behavior of forage wafers will be investigated.

Brick-shaped, non-insulated wafers of uniform density were selected and dried. An one cm slice was cut from each wafer, insulated on all but two sides and dried at the same temperature as its parent, the brick-shaped wafer. The initial temperature of the thin wafer was equal to that of the ambient drying air; the brick-shaped wafer temperature was equal to the room temperature.

The drying curves of the brick-shaped wafer 92A and the thin wafer 92B will be analyzed. The dimensions of wafer 92A and 92B were 6.2 x 5.7 x 6.1 cm and 6.2 x 5.7 x 1.0 cm, respectively; the densities, 0.518 g/cm^3 and 0.510 g/cm^3. The initial temperature of 92A was 79°F, the initial temperature of 92B, 138°F. The drying oven temperature was 140°F. The diffusion coefficients were calculated using the method of section 4.1. The results are tabulated in Table 7.

Table 7. Diffusion coefficients times 10^{+5} for wafers 92A and 92B* as calculated from equation (4.1.3).

M.C., % d.b.	92A	92B
42.22	.30	.89
39.13	.58	1.59
36.37	.65	1.65
31.60	.93	1.65
23.60	1.45	1.66
19.75	1.50	1.64
17.20	1.61	1.66
15.00	1.71	1.65
9.80	1.82	1.67
8.85	1.84	1.65

	92A	92B
*Drying temperature	140°F	140°F
Dimensions	6.2 x 5.7 x 6.1 cm	6.2 x 5.7 x 1.0 cm
Density	.518 g/cm^3	.510 g/cm^3
Insulated surfaces	none	four
Initial temperature	79°F	138°F

It is obvious from Table 7 that the initial temperature and the wafer thickness have a pronounced effect on the diffusion coefficients of forage wafers. The thin wafer displays an almost constant diffusion coefficient soon after the start of drying, while the diffusion coefficient for the brick-shaped wafer increases steadily in value. It will be shown that the difference in the drying behavior between wafers 92A and 92B, which were cut from the same parent wafer, is largely due to the different temperatures of the wafers during drying.

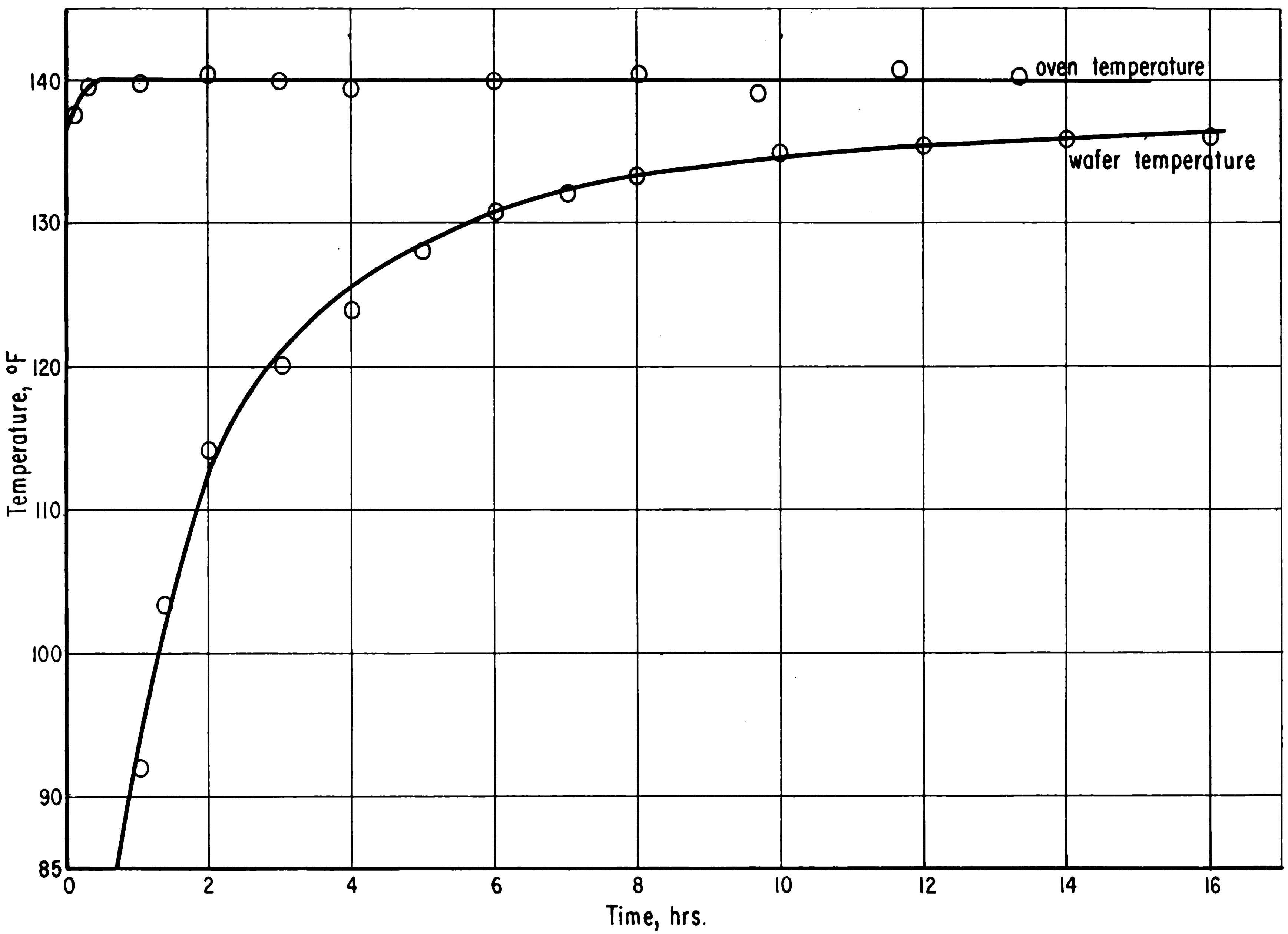
Pabis _et_ _al_. (1961) considered the change in temperature of corn kernels during drying. They assumed that the ratio of the external to the internal heat transfer was so large that the temperature gradients in the drying kernels could be neglected. The measurement of the temperature as the center of a corn kernel was therefore sufficient to obtain the temperature history of the drying product. The problem is more complicated for drying forage wafers. The temperature gradient in brick-shaped wafers is substantial and has to be taken into consideration in the analysis of the drying curves.

Before the moisture can evaporate from the surface of a drying solid, it must travel a certain distance through the internal parts of the solid. The distance covered by each moisture molecule is different, since their adsorption sites on the internal surfaces of the solid are different. Due to the temperature gradient in the drying brick-shaped wafer, each water molecule in the wafer will traverse through a different path-length and a different temperature differential. The overall effect of this condition on the drying behavior of forage wafers appeared to be accounted for best by considering the change in the average temperature during drying. This in turn leads to the consideration of the average diffusion coefficient of forage wafers.

To obtain the average temperatures of wafers 92A and 92B at different times during drying, nine thermocouples were connected in parallel and distributed through each wafer. The location of the thermocouples is calculated in Appendix A.1. The temperature readings showed that the average temperature of the thin wafer 92B remained almost constant at 138°F while the thicker wafer 92A did not reach this temperature until it had become practically dry. In Figure 12 the average temperature of wafer 92A is plotted versus time. The temperature increased very slowly after a fast initial rise. After twelve hours of drying the temperature difference between the wafer and the ambient drying air was still 4°F.

It was shown previously that the diffusion coefficient is a function of temperature, with an increase in temperature resulting in an increase of the diffusion coefficient. Therefore the increase in D of wafer 92A as reported in Table 7 is very likely caused by the slow increase of the wafer temperature.

Figure 12. Average temperature versus time for brick-shaped wafer 92A.



Although it is clear from Table 7 that the diffusion coefficient of wafer 92A is not constant due to the change in wafer temperature, the theoretical moisture ratios were calculated using one value for D as was done in section 4.1. Since the constant diffusion coefficient of wafer 92B was 1.65 x 10^{-5} cm^2/sec, this value was chosen to obtain the moisture ratio values of wafer 92A. The theoretical curves as well as the experimental values of both wafers are plotted in Figure 14. As could be expected, the experimental points of 92A fall well above the theoretical drying curve; wafer 92B follows the theoretical curve closely.

The solution of the basic diffusion equation for cases with a variable diffusion coefficient is generally rather complicated. However, in the case of drying wafers the diffusion coefficient is a function of time due to the change in the average wafer temperature in the course of drying. This makes the solution comparatively easy.

The diffusion equation for a brick-shaped wafer is:

$$\frac{\partial (M.C.)}{\partial t} = \frac{\partial}{\partial x}\left[D(t)\frac{\partial (M.C.)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D(t)\frac{\partial (M.C.)}{\partial y}\right] + \frac{\partial}{\partial z}\left[D(t)\frac{\partial (M.C.)}{\partial z}\right] \qquad (4.3.1)$$

or since D is not a function of x, y and z:

$$\frac{\partial (M.C.)}{\partial t} = D(t)\left[\frac{\partial^2 (M.C.)}{\partial x^2} + \frac{\partial^2 (M.C.)}{\partial y^2} + \frac{\partial^2 (M.C.)}{\partial z^2}\right] \qquad (4.3.2)$$

Making the transformation:

$$dT = D(t)\,dt \qquad (4.3.3)$$

equation (4.3.2) becomes:

$$\frac{\partial (M.C.)}{\partial T} = \frac{\partial^2 (M.C.)}{\partial x^2} + \frac{\partial^2 (M.C.)}{\partial y^2} + \frac{\partial^2 (M.C.)}{\partial z^2} \qquad (4.3.4)$$

The solution to equation (4.3.4) for the average moisture content, assuming a moisture gradient in one direction only, then becomes:

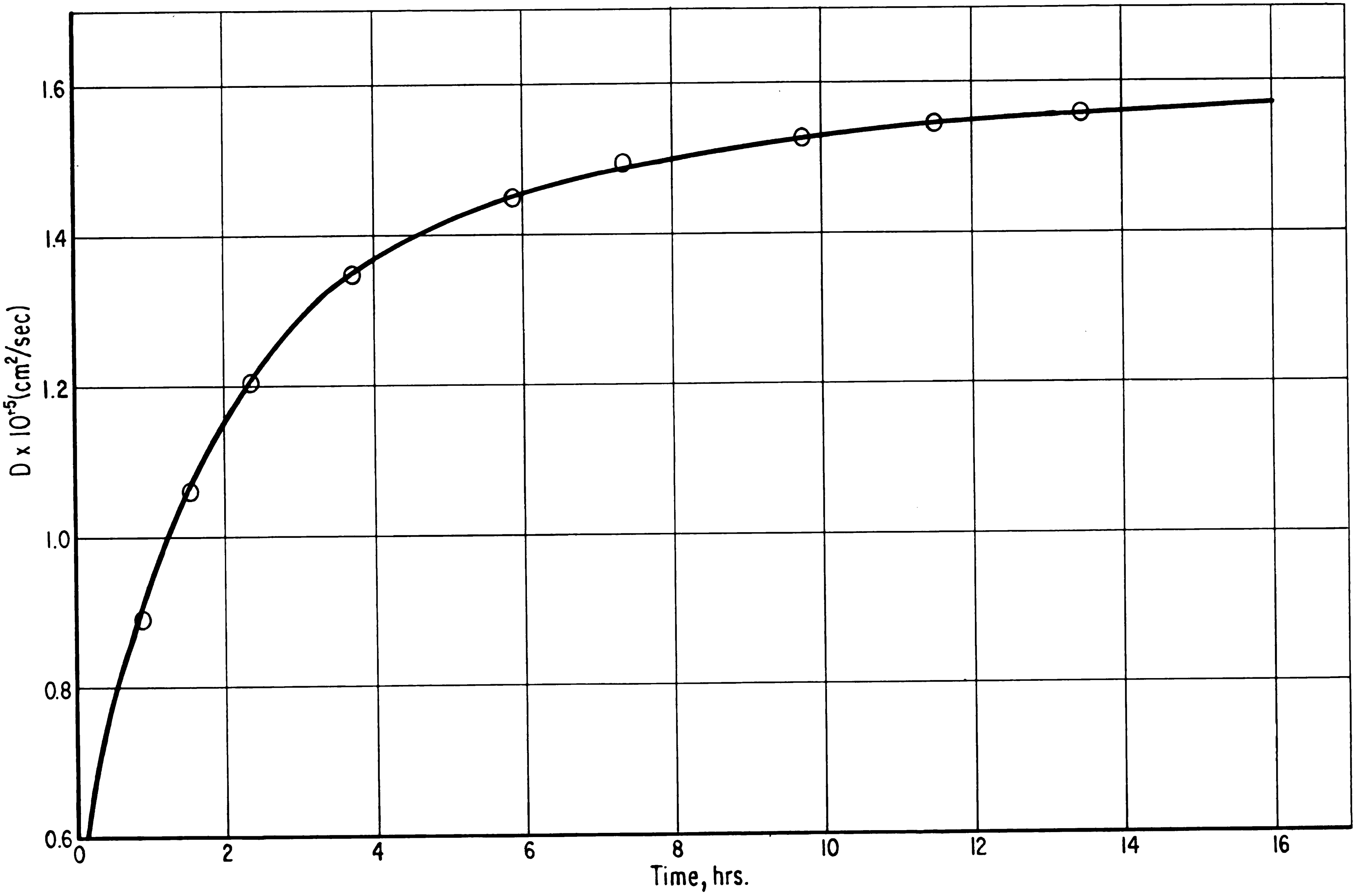
$$\frac{(M.C.)_T - (M.C.)_{eq}}{(M.C.)_{in} - (M.C.)_{eq}} = \sum_{n=o}^{\infty} \frac{8}{(2n+1)^2\pi^2} \exp[-(2n+1)^2\pi^2 T/4\ell^2] \tag{4.3.5}$$

Comparing equations (4.1.4) and (4.3.5), it can be seen that equation (4.3.5) can be obtained by replacing Dt in equation (4.1.4) by T. The moisture ratio values for the brick-shaped wafer can be calculated by multiplying three series of the type on the right hand side of equation (4.3.5), each series representing a value for ℓ (Newman, 1931). The table in Appendix A.1 can be used for the evaluation of each series.

Before equation (4.3.5) can be employed for calculating the theoretical moisture ratios, the relationship between T, D and time as expressed by equation (4.3.3) must be solved. Unfortunately the integral in equation (4.3.3) could not be evaluated formally for the case under study and so the relationship between T, D and time had to be obtained graphically. A similar procedure was followed by Pabis _et_ _al_. (1961) in analyzing the drying curve of shelled corn.

The relationship between the diffusion coefficient and the wafer temperature was obtained previously in section 4.2. Since the history of the average temperature of the brick-shaped wafer 92A is known (see Figure 12), a plot can be drawn of the average diffusion coefficient of the wafer versus time. Figure 13 shows this relationship. It is interesting to note that the diffusion coefficient has not reached the theoretical value of 1.65×10^{-5} cm^2/sec after 16 hours of drying. The actual value at that time is only 1.58×10^{-5} cm^2/sec due to the 4° difference between the ambient air and the average wafer temperature.

Figure 13. Diffusion coefficient versus time for brick-shaped wafer 92A.



After the average diffusion coefficients at different drying times are known, equation (4.3.3) can be evaluated graphically. The values for T are given in Table 8. The series on the right hand side of equation (4.3.5) can now be calculated for any values of ℓ with the aid of Appendix Table A.1; in the case of the brick-shaped wafer 92A $\ell_1 = 3.05$, $\ell_2 = 2.85$, and $\ell_3 = 3.1$ cm. The series are called L_1, L_2, and L_3 corresponding to the spatial dimensions ℓ_1, ℓ_2, and ℓ_3, respectively and are tabulated along with the theoretical and experimental moisture ratio values in Table 8. The theoretical moisture ratios are obtained by multiplying the three series L_1, L_2, and L_3. The theoretical drying curve for wafer 92A is drawn in Figure 15 along with the experimentally found points. The experimental data fit the theoretical curve much better in Figure 15 than in Figure 14. This could have been predicted since the temperature change during drying of the wafer is only accounted for in the construction of the theoretical drying curve of Figure 15.

The experimental points in Figure 15 fall above the theoretical drying curve during the initial drying period. This trend can also be observed in Figure 10, the theoretical drying curve for a thin wafer. An explanation of this phenomenon will be given in section 4.3.

Recognition of the fact that the temperature of large pieces of drying material changes only gradually, has important implications for the drying of at least one other agricultural commodity, namely, high density hay bales. It has been reported by the Aerovent Company (1964) that difficulties have been encountered in obtaining dry bales before molding starts. High density hay bales are made from the same basic material as forage wafers, but the density of the bales is lower than that of wafers. It is likely that the slope of the line on the Arrhenius plot for high density bales will be equal to the lines in

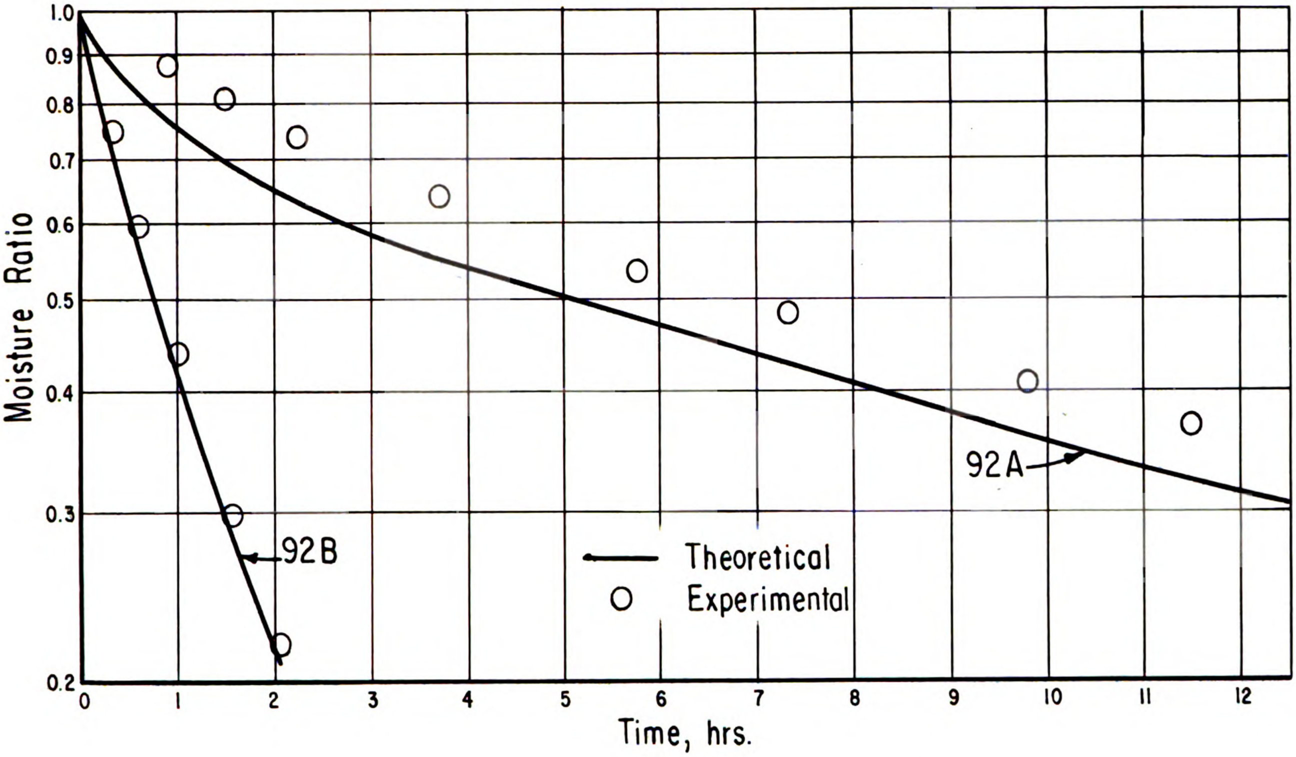


Figure 14. Drying curves for brick-shaped wafer 92A and thin wafer 92B assuming a $D = 1.65 \times 10^{-5}$ cm^2/sec.

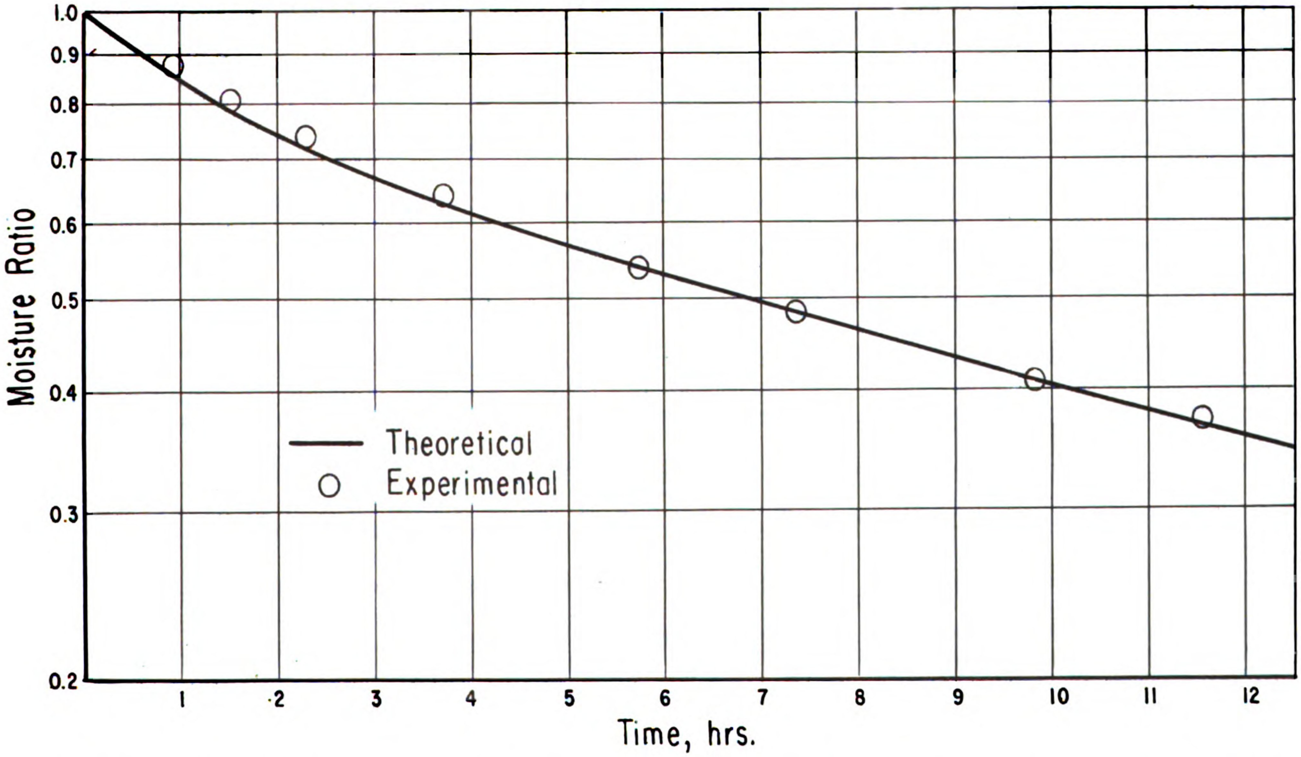


Figure 15. Drying curve for brick-shaped wafer 92A, assuming a changing diffusion coefficient.

Table 8. Drying parameters for the calculation of the theoretical moisture ratios of wafer 92A.

Time, min.	Temp., ^oF.	$Dx10^{+5}$, cm^2/sec	T, cm^2	L_1	L_2	L_3	$M.R_{th}$	$M.R._{exp}$
55	89	0.900	0.706	.951	.943	.953	.856	.879
92	105	0.106	1.307	.921	.916	.922	.780	.810
135	115	1.200	2.104	.896	.890	.897	.714	.739
225	124	1.350	4.034	.859	.850	.862	.632	.648
350	130	1.446	6.975	.815	.803	.815	.533	.540
440	133	1.495	9.214	.786	.773	.791	.481	.482
585	134	1.530	12.850	.748	.733	.753	.412	.410
690	135	1.550	15.574	.722	.704	.726	.369	.371
805	136	1.560	18.563	.697	.678	.698	.330	.321

Figure 11. When the location of this line in Figure 11 and the temperature history of the bales are known, it is possible to predict the time required for drying the bales to a particular moisture content. Also, if the maximum time to prevent molding is given, the required drying air temperature can be calculated.

It is somewhat surprising that only in two drying investigations concerning agricultural products (Becker and Sallans, 1955, and Pabis et al., 1961), the product temperature rather than the drying air temperature, is used as the drying variable. Simmonds et al. (1953), in studying the drying of wheat, eliminated the heating up period by making the initial grain temperature equal to the drying air temperature. No one, however, has seemingly recognized that large products with low thermal conductivities do not reach the drying air temperature before they are considered dry enough for storage.

A serious drawback of the theoretical treatment presented in this section is that it was not possible to calculate the average wafer temperatures during drying. The equations describing heat and mass transfer are basically the same and so the same solutions can be used for obtaining the average wafer temperatures and the average wafer moisture contents. Equation (4.3.4) was not solved for the average wafer temperatures at different times, because the relationship between the thermal conductivity and the moisture content for forage material is not known. If a plot of the thermal conductivity versus time, similar to Figure 13, could have been drawn, equation (4.3.5) would have furnished the temperature history of the drying wafer, which in this study had to be found experimentally (see Figure 12).

Although the relationship between the thermal conductivity and the moisture content was not known for forage wafers, a theoretical curve was plotted for the average temperature of a wafer versus time, assuming a constant value for the thermal conductivity of 0.30 BTU/hr ft^{o}F. The value of 0.30 BTU/hr ft^{o}F was reported by Krischer (1963) for paper which appears to have approximately the physical characteristics as forage wafers. The resulting theoretical temperature curve agreed rather well with the experimentally found points for the first two drying hours, but deviated substantially from the experimental points during the rest of the drying period. Two factors caused this deviation. In equation (4.3.5) the heat required for evaporation of the moisture is not taken into consideration. Furthermore the assumption of a constant thermal conductivity is incorrect. Krischer (1963) has reported the thermal conductivities of some materials in the dry and wet state and concluded that the thermal conductivity of a material during drying may change by as much as 150 percent. The wet material always showed a higher thermal conductivity value than the dry material.

It has been shown that the temperature of a drying product is an important factor in determining its drying behaviour. It cannot be assumed that forage wafers reach the ambient air temperature shortly after the drying starts. A method has been described to account for the changing wafer temperature in the analysis of a brick-shaped wafer. The theoretical curve agreed rather well with the experimental points. The temperature history of a drying wafer could not be determined theoretically because the relationships between the thermal conductivity and the wafer moisture content are not known.

4.4 Diffusion in Thin Wafers with Changing Boundary Conditions

In the previous section it was concluded that the diffusion equation with constant initial and boundary conditions describes the drying process correctly over all but the initial drying period. The explanation for the deviation from the theoretical drying curve for small drying times may be twofold. A slight temperature change of the wafer during drying will be caused by the fact that drying is not a true isothermal diffusion process. This is true even where the initial wafer and drying air temperature are the same. The result is a changing of the diffusion coefficient, although the effect appears to be insignificant. A more important fact which may cause a deviation of the experimental drying curve from the theoretical drying curve, is an improper choice of the boundary conditions used in solving the diffusion equation. It is this possibility which will be investigated in the following section.

The assumption has been made in section 4.1 that the surfaces of a forage wafer immediately attain the moisture content equilibrium after drying starts, or in mathematical terms, that the boundary condition for diffusion in a wafer is described by $C_{x=\pm \ell} = C_{eq}$ for $t \geq o$. Carslaw and Jeager (1959) reported that if a steel bar is suddenly dropped in a hot bath, the surfaces of the bar do not reach the temperature of the heating fluid instantaneously. Consequently the boundary condition used in the solution of equation (4.1.1), namely, that the moisture equilibrium is reached at $t \geq o$, may have been an approximation rather than a true mathematical fact.

There are several solutions available in the heat transfer literature for the temperature distribution in solids with variable boundary conditions. Carslaw and Jaeger (1959) list solutions for the temperature distribution in a plane with linearly and exponentially changing surface temperatures. Crank (1956) has adapted these solutions for diffusion problems and has calculated the total amount of diffusing substance in a plane sheet as a function of time by integrating Carslaw and Jaeger's solutions with respect to time. The experimental drying curves of forage wafers will be compared with the solutions obtained by Crank for diffusion in a plane sheet with variable surface concentrations.

In the case that the moisture concentration at the two surfaces of a plane wafer varies linearly with time, i.e., $\phi_1(t) = \phi_2(t) = kt$, the total amount, Q_t, of diffusing moisture is given by the expression (Crank, 1956):

$$\frac{DQ_t}{k\ell^3} = \frac{2Dt}{\ell^2} - \frac{2}{3} + \frac{64}{\pi^4} \sum_{n=o}^{\infty} \frac{\exp -D(2n+1)^2\pi^2 t/4\,\ell^2}{(2n+1)^4} \qquad (4.4.1)$$

In comparing the experimental drying curves of several wafers with the theoretical curves obtained from equation (4.4.1) for different values of k, the experimental values were found to differ very substantially from the theoretical drying points. Therefore it was concluded that a linearly changing boundary condition does not describe the behavior of a drying forage wafer surface correctly.

Much better success was obtained by assuming that the surface concentration varies exponentially with time. Crank (1956) gives a solution of the total amount of moisture absorbed in time t for a plane sheet of thickness 2ℓ, of which the concentration is initially zero and of which both the surfaces are approaching the equilibrium concentration C_{eq} exponentially. This boundary condition can be written as:

$$\phi_1'(t) = \phi_2(t) = C_{eq}\,(1-e^{-\beta t}) \qquad (4.4.2)$$

It was found that Crank's solution contains an error. The correct solution appeared to be:

$$\frac{Q_t}{\ell C_{eq}} = 1 - \exp\left[-(Dt/\ell^2)\,(\beta\ell^2/D)\,\tan\,(D/\beta\ell^2)^{\frac{1}{2}}\right] -$$

$$\frac{8}{\pi^2}\sum_{n=o}^{\infty}\frac{\exp\left[-(2n+1)^2\pi^2 Dt/4\ell^2\right]}{(2n+1)^2\left[1-(2n+1)^2\;D\pi^2/4\beta\ell^2\right]} \qquad (4.4.3)$$

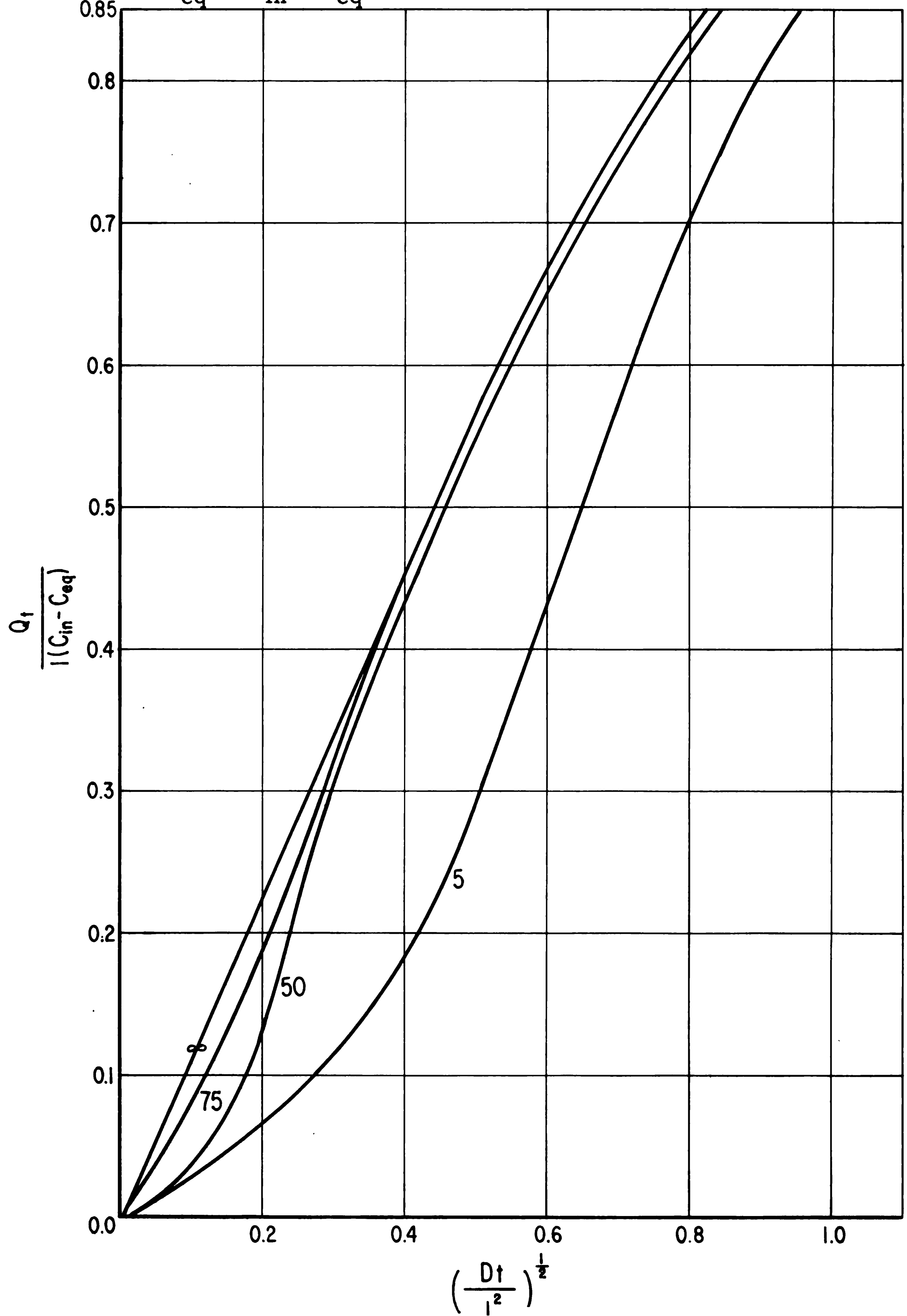
It is obvious that the boundary condition given in equation (4.4.2) applies to adsorption rather than to desorption. The proper boundary condition for a drying wafer is:

$$\phi_1(t) = \phi_2(t) = C_{eq} + (C_{in} - C_{eq})\,e^{-\beta t} \qquad (4.4.4)$$

The equation for the total amount of moisture diffused out of the wafer will, however, be similar to equation (4.4.3) except that the term C_{eq} has to be replaced by $(C_{in} - C_{eq})$.

To facilitate the use of equation (4.4.3) a graph was plotted of $Q_t/\ell(C_{in} - C_{eq})$ versus $(Dt/\ell^2)^{\frac{1}{2}}$ for different values of $\beta\ell^2/D$. The curves applicable to the drying of biological materials are plotted in Figure 16. The curves are of the same nature as those obtained by Crank for adsorption.

Figure 16. Calculated drying curves assuming the surface concentration $C_{eq} + (C_{in} - C_{eq}) e^{-\beta t}$ for different values of $\beta \ell^2/D$.

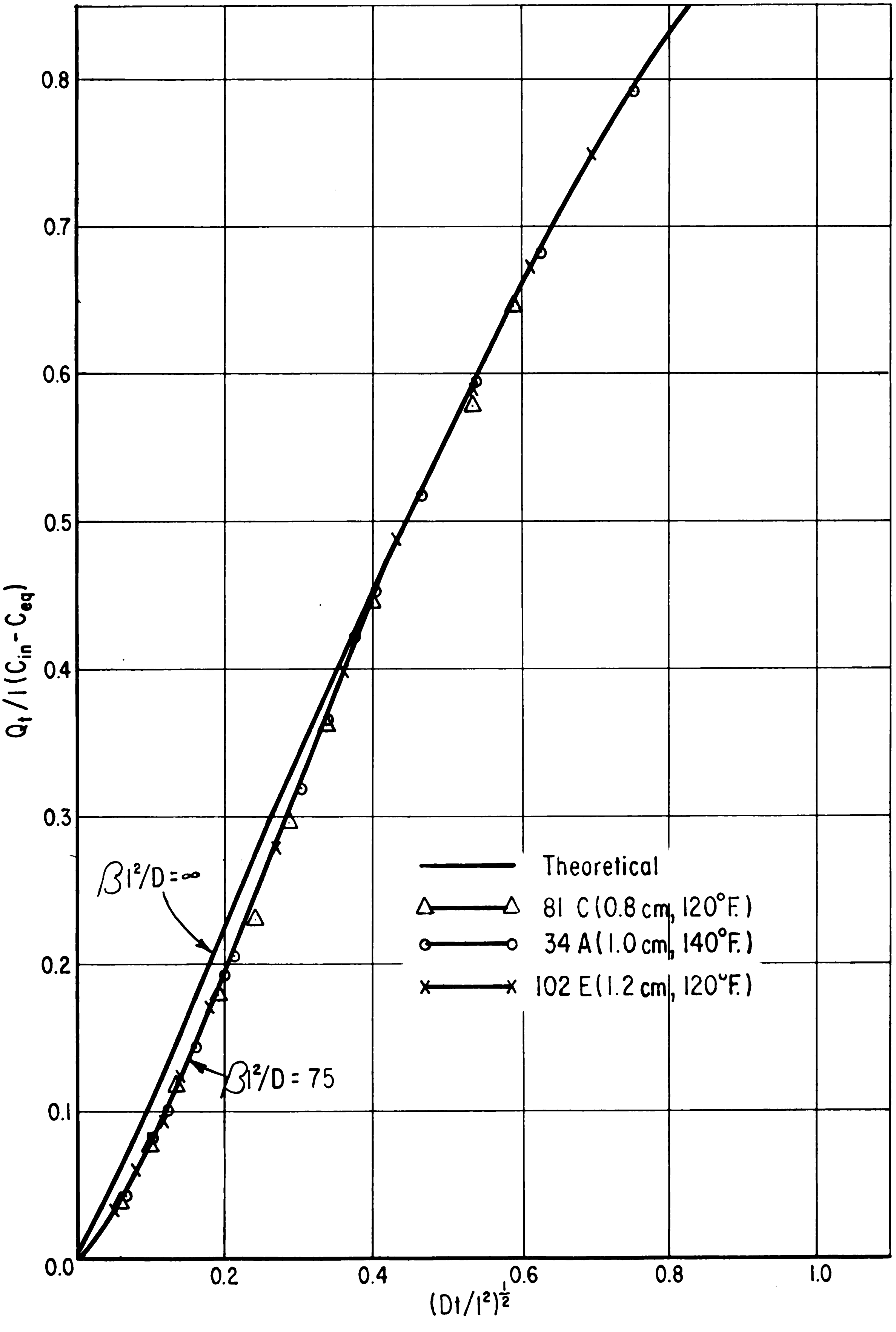


The use of equation (4.4.3) is not as direct as that of the solution to the diffusion equation with constant boundary conditions (equation 4.1.3), since equation (4.4.3) has two unknowns, β and D, while equation (4.1.3) has only one unknown, D. To obtain the magnitude of β, the value of D was assumed to be equal to that obtained from the diffusion equation with constant boundary conditions at large times. From Figure 16 it can be noted that this assumption is a proper one. The curve $\beta \ell^2/D = \infty$ corresponds to the boundary condition $C_{x=\pm\ell} = C_{eq}$ at $t \geq 0$, which was employed in the section 4.1. The $\beta\ell^2/D = \infty$ curve practically coincides with the $\beta\ {}^2/D = 75$ curve for large values of $(Dt/\ell^2)^{\frac{1}{2}}$. Since the diffusion coefficients calculated in the preceding section are obtained at relative large values of t, the diffusion coefficient calculated from equations (4.1.3) and (4.4.3) do not differ much. For wafer 34A this meant that a value for D of 0.75×10^{-5} cm^2/sec could be assumed in equation (4.4.3).

In Figure 17 the experimental values $Q_t/\ell(C_{in} - C_{eq})$ versus $(Dt/\ell^2)^{\frac{1}{2}}$ are plotted for several wafers of thicknesses between 0.8 to 1.2 cm dried at 120 and 140°F. Also drawn are the theoretical curves of equation (4.4.3) for values of $\beta\ell^2/D = \infty$ and 75. The important conclusion can be drawn from this figure that the drying curves of forage wafers do not coincide with the $\beta\ell^2/D = \infty$ curve but with the $\beta\ell^2/D = 75$ curve. This means that the surface concentrations of drying wafers do not drop instantaneously to the moisture equilibrium concentration C_{eq}, but that the value of C_{eq} is approached exponentially with time.

Since the experimental values for the drying of several thicknesses of thin forage wafers all fall on or closely to the $\beta\ell^2/D = 75$ curve (see Figure 17), it can be concluded that the value of $\beta\ell^2/D$ is constant for the particular conditions used in this study for the drying of forage wafers. This means that the value of β depends on the diffusion

Figure 17. Drying curves for thin forage wafers.



coefficient and the thickness of a particular wafer. In Figure 18 β has been plotted on log-log paper for a diffusion coefficient of 0.75 x 10^{-5} cm^2/sec as a function of the wafer half-thickness. The value of β decreases from a value of 8.20 x 10^{-3} sec^{-1} for a 0.5 cm thick wafer to a value of 1.41 x 10^{-4} sec^{-1} for a 4.0 cm thick wafer. In other words the surfaces of a thin wafer will approach their moisture equilibrium faster than the surfaces of thick wafers.

β has been plotted in Figure 19 as a function of the diffusion coefficient for a 2 cm thick wafer. An increase in the diffusion coefficient increases the value of β. The physical significance of this fact is that a wafer surface will approach its equilibrium moisture content sooner for a large than for a small D value. β changes from a value of 0.38 x $10^{-3}sec^{-1}$ at D = 0.5 x $10^{-5}cm^2/sec$ to a value of 3.0 x $10^{-3}sec^{-1}$ at D= 4.0 x $10^{-5}cm^2/sec$ for a 2 cm thick wafer.

At this point the question should be raised if the values of β obtained for forage wafers in this study are representative for all single wafer drying conditions. Since β is a value pertaining to a wafer surface, it could be possible that the velocity of the drying air, or the Reynolds number of the drying fluid, will affect the values of β. To investigate this possibility three wafers of the same thickness and density were dried at the same temperature but in air flows with different Reynolds numbers. The average free stream drying air velocities used in this part of the study were 1.15 ft/sec, and 3.96 ft/sec, corresponding to the Reynolds numbers 1252 and 4320, respectively. The experimental points of the three wafers fell on or close to the same theoretical drying curve for $\beta \ell^2/D = 75$. This means that the values of β are not dependent on the drying air velocity for Reynolds numbers above 1252. This result is not surprising. Lykov (1955), Krisher (1963), Simmonds _et_ _al_., (1953), and Jason (1958) all reported that air velocity has no influence on drying during the falling-rate period if the air

Figure 18. β as a function of wafer half thickness; $D = 0.75 \times 10^{-5}$ cm^2/sec.

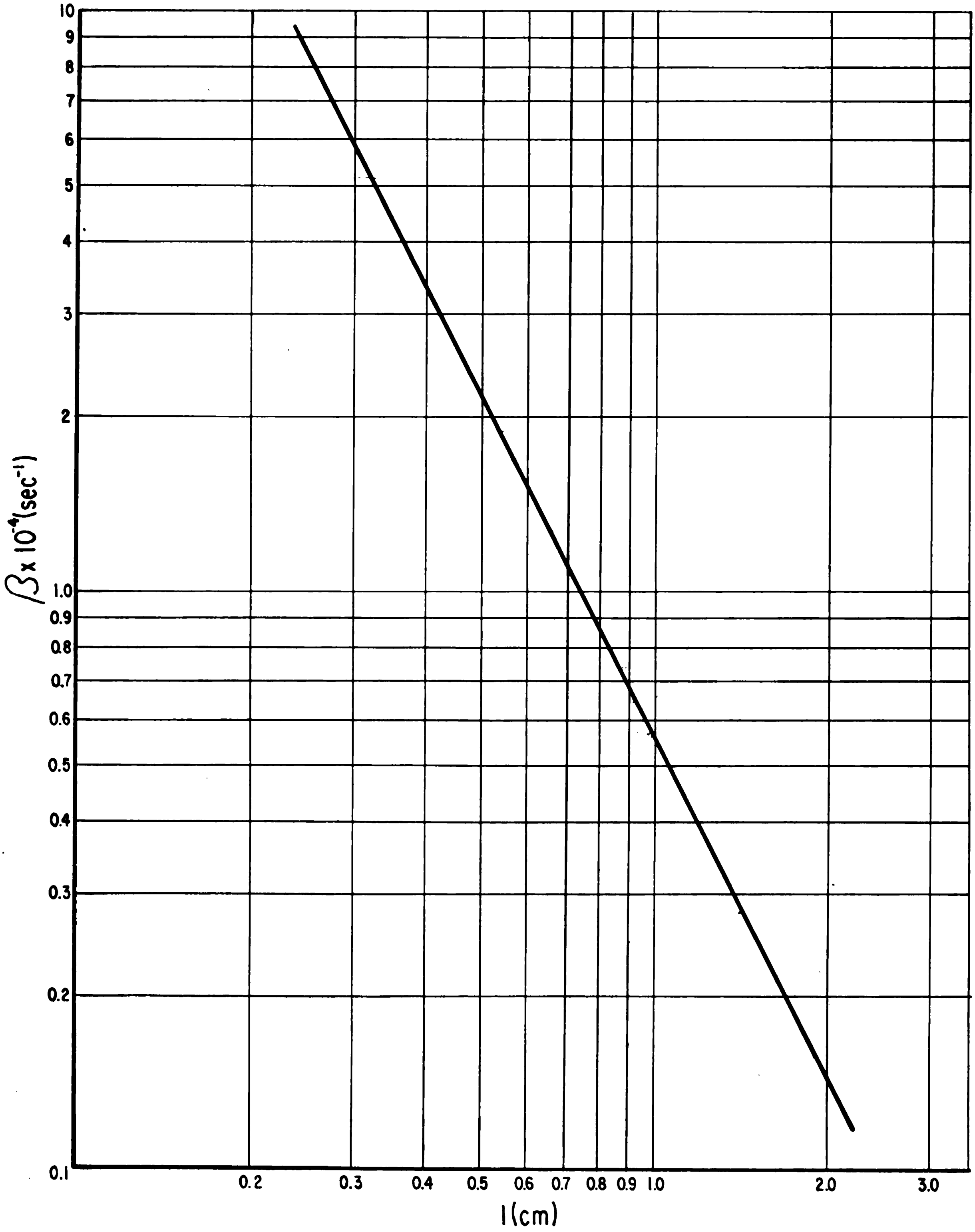
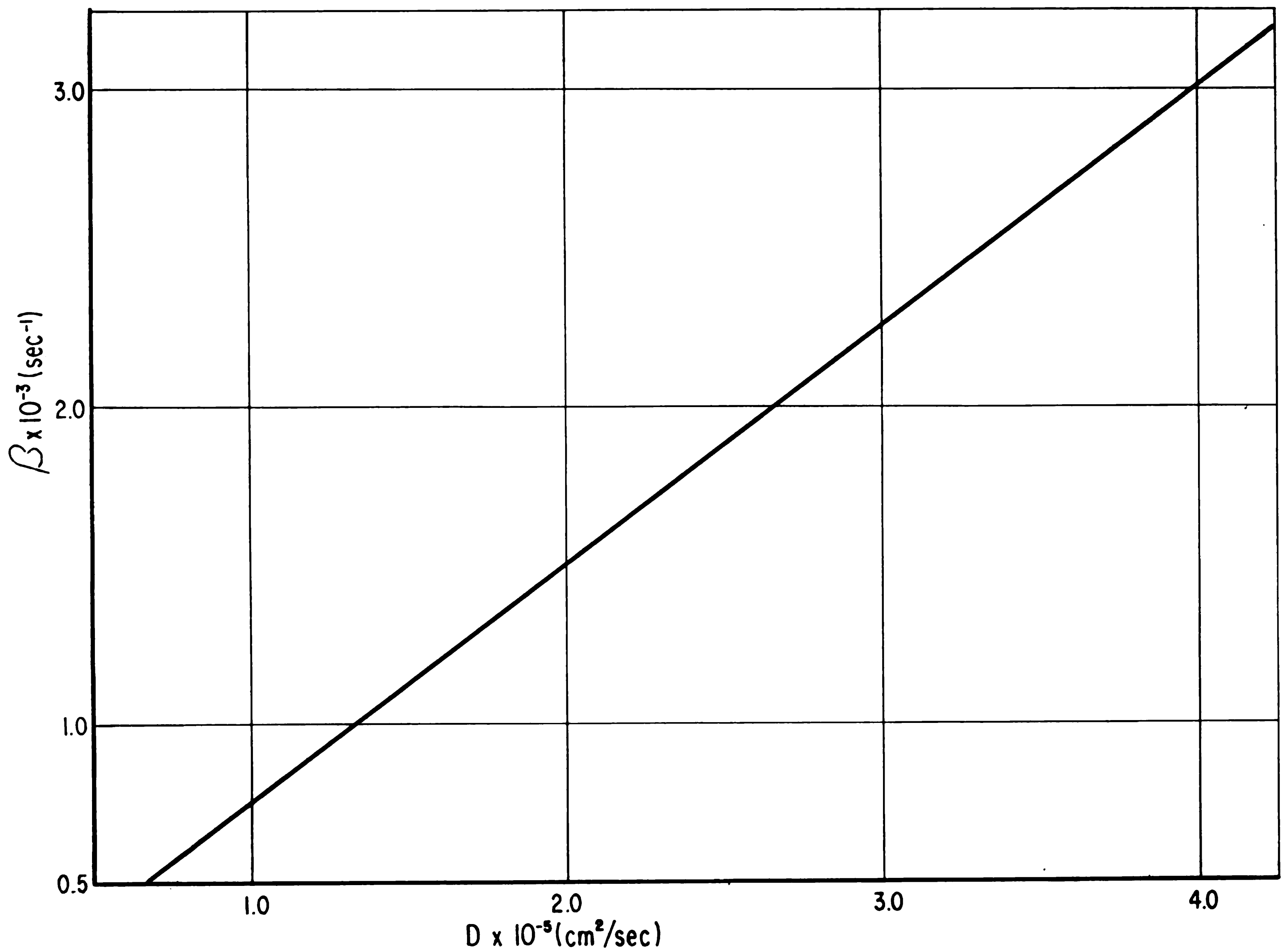


Figure 19. β as a function of the diffusion coefficient, wafer thickness 2 cm.



velocity is kept above a certain minimum value of about 12 ft/min. The minimum air velocity used in this study is well above that figure.

It is interesting to speculate about the importance of the values of β obtained for wafers in this study. Do they hold for all hygroscopic products? Unfortunately this seems highly unlikely. The chemical and physical make-up of a product determines the adsorption and desorption rates of that product. Different individual fibers, for instance, do not have the same desorption rates (Nordon et al., 1960). One of the reasons that Becker and Sallans (1955) and other researchers (Hustrulid et al., 1959; Pabis et al., 1961) found constant diffusion coefficients during the initial drying period of grain may be due to the larger values of β and consequently to the higher desorption rates of the outer surface of the grain than of forage wafers. However, if some drying points had been determined sooner after the start of drying by these investigators (after minutes instead of hours as is usually done), the physical phenomenon of an exponential moisture content drop would probably have been observed.

It has been shown that the surfaces of a hygroscopic product do not obtain the moisture equilibrium content directly after the start of drying as has been assumed in prior drying investigations. The surface moisture equilibrium is reached exponentially.

4.5 Diffusion in Semi-Infinite Solids

When it became obvious that the diffusion equation with constant initial and boundary conditions did not describe the drying behavior of wafers during the initial drying period correctly, it was decided to investigate how long a plane wafer could be considered a semi-infinite solid. Since the solutions to most second order differential equations with changing boundary conditions or varying intrinsic properties are easier obtained for a semi-infinite solid than for a slab, it would be a real advantage if the in the literature available semi-infinite solid solutions to the diffusion equation could be used for a plane wafer.

When the same initial and boundary conditions are assumed for the drying of a wafer as in section 4.1, the solution to the diffusion equation for the concentration distribution in a semi-infinite solid, bounded by the plane x = o and extended to infinity in the direction of x positive, becomes (Crank, 1956):

$$\frac{C_{in} - C_{t,x}}{C_{in} - C_{eq}} = \operatorname{erfc} \frac{x}{2\sqrt{Dt}} \qquad (4.5.1)$$

The total amount of moisture which has diffused out of the semi-infinite solid per unit area will then be:

$$Q_t = 2(C_{in} - C_{eq}) \left(\frac{Dt}{\pi}\right)^{\frac{1}{2}} \qquad (4.5.2)$$

Equation (4.5.2) makes it possible to calculate the diffusion coefficient at different moisture contents or drying times. Wafer 34A was used to test how long equations (4.1.2) and (4.5.2) would give the same values for D. In Table 9 the D values calculated at different times with equations (4.1.3) and (4.5.2) are tabulated. The data show the surprising result that the solutions do not diverge until the moisture content of the wafer has decreased from 22.9 to at least 6.98 percent dry basis. It appears that for almost five hours a 1 cm thick wafer can therefore be treated as a semi-infinite solid; more than 65 percent of the original moisture had been diffused out of the slab by that time.

The experimental result obtained for wafer 34A was confirmed when other thin wafers were treated as a slab and semi-infinite solid and their solutions compared. The length of time that a wafer can be

Table 9. Diffusion coefficients times 10^{+5} of wafer 34A,* treating the wafer as a semi-infinite solid (eqn. 4.5.2) and as a slab (eqn. 4.1.3).

Time, min	M.C., %d.b.	D, eqn (18)	D, eqn (4)
10	20.80	.33	.32
30	18.02	.58	.58
60	15.28	.69	.70
90	13.23	.75	.74
150	10.38	.75	.75
210	8.44	.75	.75
270	6.98	.73	.74
300	6.47	.68	.75
430	4.33	.60	.75

*Drying temperature 140°F; wafer thickness 1 cm; wafer density 0.92 g/cm^3; insulated surfaces 4.

treated as a semi-infinite solid depends on its thickness. The time period was found to be equal to the time required for removing about 65 percent of the original moisture in the wafer.

In the following part of this section the mathematical reasons will be explored why a plane drying wafer can be treated as a semi-infinite solid during the major part of its drying period. It is clear that initially both sides of a drying plane will act as semi-infinite solids. This condition will continue at least as long as the concentration at the midplane has not significantly changed. The time in which the concentration ratio at the midplane in wafer 34A had decreased by the arbitrary small amount of 5 percent will be calculated with equation (4.5.1).

The solution to the diffusion equation for the concentration ratio of a plane wafer with one end at $x = 0$ and the other end at $x = 2\ell$ can be obtained from equation (4.5.1), by assuming that both ends behave as independent semi-infinite solids. Since the sum of two solutions of a linear differential equation is itself a solution, the concentration ratio of a plane wafer becomes:

$$\frac{C_{in} - C_{t,x}}{C_{in} - C_{eq}} = \operatorname{erfc} \frac{x}{2\sqrt{Dt}} + \operatorname{erfc} \frac{2\ell - x}{2\sqrt{Dt}} \qquad (4.5.3)$$

At the midplane, at $x = \ell$, the concentration ratio will be:

$$\frac{C_{in} - C_{t,x=\ell}}{C_{in} - C_{eq}} = 2 \operatorname{erfc} \frac{\ell}{2\sqrt{Dt}} \qquad (4.5.4)$$

The time at which the 5 percent change at the midplane moisture ratio occurs can now be calculated from equation (4.5.4):

$$0.05 = 2 \operatorname{erfc} \frac{\ell}{2\sqrt{Dt}} \qquad (4.5.5)$$

Using Jahnke and Ende's Tables on Error Functions, it is found that $\ell/2\sqrt{Dt} = 1.57$. Thus, the time at which a 5 percent change in the midplane concentration ratio of wafer 34A occurs is equal to 0.95 hours.

The fact that the concentration ratio at the midplane of the wafer has changed 5 percent after 0.95 hours might seem to indicate that equation (4.5.3) can be used for only a limited period of time. Therefore, the question remains to be answered on what theoretical grounds the treatment of the plane wafer 34A as a semi-infinite solid for 5 hours can be justified.

From equation (4.1.3) it is clear that the total amount of moisture which diffuses out of a wafer, is a function of the concentration gradient $\partial C/\partial x$ at the surface. This means that the concentration at the midplane does not serve as the criterion for the length of time a plane wafer can be treated as a semi-infinite solid, but the concentration at the surface. In other words, the concentration ratios at $x = 0$ and

$x = 2\ell$ will determine how long the plane wafer will behave as the sum of two semi-infinite solids. The concentration ratio of the surface at $x = 0$ will not be effected to any great extent by the surface at $x = 2\ell$ until the term erfc $(2\ell - x)/2\sqrt{Dt}$ at $x = 0$ becomes significant (see eqn. 4.5.3). Thus, as long as

$$\text{erfc}\,\frac{2}{2\sqrt{Dt}} << 1 \qquad (4.5.6)$$

a plane wafer will behave as a semi-infinite solid. Allowing for a 5 percent change in the concentration ratio at the surface $x = 0$ due to the surface at $x = 2\ell$, gives:

$$\text{erfc}\,\frac{2}{2\sqrt{Dt}} = 0.05 \qquad (4.5.7)$$

From equation (4.5.7) it is found that the time period in which the plane wafer can be treated as a semi-infinite solid is equal to 4.6 hours or in the moisture content range from 22.9 to at least 6.98 percent dry basis. This result is approximately the same that had been obtained experimentally (see Table 9).

In this section it has been shown that a plane wafer while drying can be treated as a semi-infinite solid over a large portion of its drying period. Since the differential equation describing diffusion often can be solved much easier for a semi-infinite solid than for a plane, this result has important applications for future drying investigations.

V. SUMMARY AND CONCLUSIONS

5.1 Summary

The proper drying of forages is one of the major problems encountered in modern hay harvesting systems. This is especially the case with forage wafers due to the high densities of the individual wafers. An effective solution to the drying problem demanded a more thorough understanding of the variables affecting the drying behaviour of forage wafers. Forage wafers (92 percent alfalfa), made with an experimental Massey-Ferguson wafering machine, were dried at five different temperatures between 120° and 200°F. The wafer densities varied from 0.30 to 1.0 g/cm^3; the wafer size from 6.2 x 6.0 x 5.7 cm to 6.2 x 6.0 x 0.6 cm.

An apparatus was developed for weighing the samples without removing them from the drying oven. The wafer temperatures were measured with copper-constantan thermocouples and recorded on an automatic recording potentiometer. The air velocities were measured with a hot-wire anemometer.

Three major points were studied in this investigation: (i) the effect of wafer density on the diffusion coefficient of forage wafers; (ii) the effect of temperature on the diffusion coefficient of forage wafers; and (iii) the physical laws describing the drying behaviour of forage wafers.

The effect of density and temperature on the diffusion coefficient of forage wafers was established. The minimum diffusion coefficient values for certain wafer density ranges were plotted versus the reciprocal of the absolute temperature on an Arrhenius plot. The minimum rather than the maximum values were reported because of the practical importance of the slowest drying samples in the design of drying apparatus. The equation for the Arrhenius line of the most common wafer density range (0.4 - 0.5 g/cm^3) was $D = 0.114 \exp(-10,810/RT)$. An increase in temperature and/or a decrease in wafer density resulted in an increase of the diffusion coefficient of forage wafers.

The solution to the differential equation for diffusion with constant initial and boundary conditions described the drying behaviour of forage wafers well in the latter stages of drying. The diffusion coefficients remained constant during this period until at least a wafer moisture content of five percent dry basis had been reached. The experimental moisture ratios fell above the theoretical drying curve during the initial drying period. In solving the diffusion equation it was imperative to use the experimentally found static moisture equilibrium values for the boundary condition. If the dynamic moisture equilibria were employed the diffusion equation did not describe the drying of forage wafers.

It was shown that the deviation between the experimental and theoretical drying curves during the initial drying stages was caused by the change of the wafer-surface moisture concentration. The surface concentration varied exponentially with time according to the equation $Q_{(t)} = C_{eq} + (C_{in} - C_{eq}) \exp(-\beta t)$. The value of β was a function of the wafer thickness and the diffusion coefficient. For a 4 cm thick wafer with a diffusion coefficient of $0.75 \times 10^{-5} cm^2/sec$ the value of β was equal to $1.41 \times 10^{-4} sec^{-1}$.

The temperature of the wafer rather than the ambient air temperature should be used to analyze theoretically the drying behaviour of the product. A wafer dried at an oven temperature of 140°F had reached a temperature of only 136°F by the time it had become dry enough for safe storage. A semi-theoretical method was developed which accounted for the changing wafer temperature during drying. The theoretical and experimental drying curves agreed well.

A forage wafer was treated as a semi-infinite solid during the major part of the drying operation. It was found, and mathematically substantiated, that a two dimensional drying solid behaves as the sum of two semi-infinite bodies until 65 percent of the original moisture in the product has been removed.

5.2 Conclusions

1. Diffusion is the physical mechanism controlling the drying behaviour of forage wafers. The diffusion coefficient is constant down to a wafer moisture content of at least 5 percent dry basis.

2. The wafer drying surfaces do not reach the static moisture equilibrium instantaneously, but exponentially with time. The boundary condition describing this physical phenomenon is of the form $Q_{(t)} = C_{eq} + (C_{in} - C_{eq}) \exp(-\beta t)$. β is a function of the wafer thickness and of the wafer diffusion coefficient.

3. The effect of temperature on the drying behaviour of forage wafers can be expressed by an Arrhenius plot. The equation for a.0.45 g/cm^3 wafer-density, relating the diffusion coefficient and temperature, is $D = 0.114 \exp(-10,810/RT)$. The diffusion coefficients of forage wafers decrease in value with an increase in density and decrease in ambient air temperature.

4. The temperature of forage wafers slowly rises during drying, resulting in an increase of the diffusion coefficient. By making a rather simple transformation the solution to the diffusion equation with constant D can be used for obtaining the theoretical moisture ratio values of forage wafers.

5. A forage wafer can be treated as a semi-infinite solid until 65 percent of the total initial moisture has been removed.

SUGGESTIONS FOR FURTHER STUDY

Further investigations should be conducted in the following areas:

1. Rewetted versus Non-Rewetted Wafers. The diffusion coefficients of rewetted and non-rewetted forage wafers should be compared when a wafering machine becomes available which is able to process high moisture forages.

2. Forage Properties. The influence of type of forage, length of cut and chemical analysis on the diffusion coefficients of forage wafers needs to be investigated.

3. Thermal Conductivity. The thermal conductivity of forage wafers as affected by density, moisture content and type of forage should be evaluated.

4. Heat of Vaporization. The heat of vaporization of forage wafers as a function of wafer moisture content is needed in order to study the analytical solution of the differential equation for diffusion with a negative heat sink.

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APPENDIX A

A. 1 Thermocouple Locations for Measuring the Average Temperature in a Cube-Shaped Forage Wafer.

Assumptions:
- (i) the wafer is uniform in density
- (ii) the physical properties of the forage wafer are the same at every point of the wafer
- (iii) the wafer can be divided into an arbitrary number of shells each having the same heat content

Solution:

Assume that the sides of the cube inside the innermost shell are equal to ℓ cm. The volume of this cube is ℓ^3cm. If the thickness of the first shell is called $\frac{1}{2}x$, the volume of the shell becomes:

$$(\ell + x)^3 - \ell^3 = x^3 + 3\ell x^2 + 3\ell^2 x \qquad (A.1.1)$$

Since the volume of the shell must be the same as the volume of the cube, equation (A. 1. 1) has to be equal to ℓ^3 or

$$x^3 + 3\ell x^2 + 3\ell^2 x = \ell^3 \qquad (A.1.2)$$

and $$x^3 + 3\ell x^2 + 3\ell^2 x - \ell^3 = 0 \qquad (A.1.3)$$

Equation (A. 1. 3) is a cubic equation of the general form $x^3 + px^2 + qx + z = 0$, and can be reduced to the form $y^3 + ay + b = 0$ by substituting for x the value $(y - p/3)$ or by setting a equal to $1/3(3q - p^2)$ and b equal to $1/27(2p^3 - 9pq + 27r)$ (Hudson, 1959).

Equation (A. 1. 3) then gives:

$$a = 1/3\ (9\ell^2 - 9\ell^2) = 0 \qquad (A.1.4)$$

and $$b = 1/27(54\ell^3 - 81\ell^3 - 27\ell^3) = -2\ell^3 \qquad (A.1.5)$$

Equation (A. 1. 3) then becomes:

$$y^3 - 2\ell^3 = 0 \qquad (A.1.6)$$

The real root of equation (A.1.6), which is of practical interest, is equal to $y = \ell \sqrt[3]{2}$. The solution to equation (A.1.3) is then:

$$x = y - \ell = \ell \sqrt[3]{2} - \ell = 0.259 \ell \qquad \text{(A.1.7)}$$

The locations of the parallel connected thermocouples, which will give the average temperature of a cube-shaped forage wafer, can be calculated from equation (A.1.7).

Table A. 1 Values of the function L(v) or $8/\pi^2(e^{-v} + 1/9\ e^{-9v} + 1/25\ e^{-25v} + ----)$ Adapted from A. T. McKay(1930).

v	0.00	0.02	0.04	0.06	0.08
0.00	0.0000	.8984	.8563	.8240	.7068
0.10	.7028	.6918	.7132	.7127	.6952
0.20	.6787	.6633	.6481	.6337	.6199
0.30	.6066	.5936	.5812	.5691	.5573
0.40	.5458	.5346	.5237	.5131	.5028
0.50	.4926	.4827	.4730	.4636	.4543
0.60	.4452	.4363	.4277	.4192	.4108
0.70	.4027	.3947	.3868	.3792	.3716
0.80	.3643	.3570	.3499	.3430	.3362
9.90	.3296	.3230	.3166	.3104	.3042
1.00	.2982	.2923	.2865	.2809	.2751
1.10	.2968	.2645	.2592	.2541	.2491
1.20	.2441	.2393	.2346	.2300	.2253
1.30	.2209	.2165	.2122	.2081	.2039
1.40	.1999	.1959	.1920	.1882	.1845
1.50	.1808	.1773	.1738	.1703	.1670
1.60	.1636	.1604	.1573	.1541	.1511
1.70	.1481	.1452	.1423	.1394	.1367
1.80	.1340	.1313	.1287	.1262	.1237
1.90	.1213	.1188	.1165	.1142	.1119
2.00	.1097	.1076	.1054	.1033	.1012
2.10	.0993	.0973	.0954	.0935	.0916
2.20	.0898	.0880	.0863	.0846	.0829
2.30	.0813	.0797	.0781	.0761	.0751
2.40	.0735	.0721	.0707	.0692	.0678
2.50	.0665	.0653	.0640	.0627	.0614
2.60	.0602	.0590	.0579	.0567	.0556
2.70	.0545	.0534	.0524	.0513	.0503
2.80	.0493	.0483	.0473	.0464	.0455
2.90	.0446	.0437	.0429	.0420	.0412
3.00	.0404	.0396	.0387	.0380	.0373
3.10	.0365	.0357	.0350	.0344	.0336
3.20	.0331	.0324	.0318	.0311	.0306
3.30	.0298	.0293	.0287	.0281	.0276
3.40	.0271	.0265	.0260	.0254	.0250
3.50	.0245	.0240	.0235	.0230	.0226