

DESCRIPTION AND ANALYSIS
OF SOCIO-ECONOMIC SYSTEMS
BY PHYSICAL SYSTEMS TECHNIQUES

THESIS FOR THE DEGREE OF PH. D.

MICHIGAN STATE UNIVERSITY

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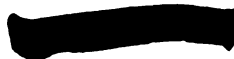
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ABSTRACT

THE DESCRIPTION AND ANALYSIS OF SOCIO-ECONOMIC SYSTEMS
BY PHYSICAL SYSTEMS TECHNIQUES

By Jack B. Ellis

Techniques of systems analysis which were originally evolved for electrical networks have previously been generalized to enable their use in analyzing physical systems of other types, such as mechanical, hydraulic and mixed types. This thesis shows that they may be further generalized to include the analysis of systems containing social and economic components. The advantage of physical systems analysis in this application is that it provides a consistent and rigorous procedure for formulating mathematical models of a wide range of social and economic phenomena, both of the static and dynamic types, as shown by the two selected examples in the text. These models can be formulated and solved on a digital computer, giving the analyst a wide range of experience with his model in a short time.

The requirements for successful application of systems analysis are:

1. The phenomenon must be identifiable as a collection of components with discrete interfaces with one another.
2. Two complementary variables must be found which satisfy the two generalized Kirchoff postulates.
3. Each component of the system must be modeled quantitatively in terms of these two complementary variables.

In the first example, the problem is to allocate the attendances of campers at all state parks in Michigan in 1964. The system is partitioned into three types of components; origin areas, transportation links, and parks. Each component is modeled in terms of the variable Y, the flow of campers, and the variable X, the propensity to camp or the demand pressure for camping. An algebraic model was evolved which successfully described the operation of the system for the year 1964, the only one for which complete data was available.

The second model attempts to describe the dynamic growth of a three-sector national economy by means of a discrete-time state-space model. Each component is modeled in terms of the variable Y, the flow of output, and the variable X, the stocks of accumulated capital goods. Representative parameters are chosen from historical United States data, and solutions are obtained for the growth of investment in each productive sector over a five-year period. Application of state-model theory in the field of economics enables a complete analysis of an economy at one time, without having to combine the many types of partial analysis usually employed. A new type of production function results from cutset relationships, which extends the Leontief inter-industry concepts to include the Marxian linear labor-capital production function. This enables the model to reflect all types of inputs to production processes.

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By

Jack B. ^{array}Ellis

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Electrical Engineering

1965

ACKNOWLEDGMENTS

The author would like to acknowledge the guidance and encouragement of his supervisor, Professor H.E. Koenig, during his entire program. His enthusiasm for and encouragement of research efforts in what appeared at the outset to be impenetrable and untrodden areas will be a source of inspiration to the author for many years to come. Thanks are also due to Professor D.N. Milstein for his helpful discussions and suggestions during the research leading to this thesis, and for the foresight to instigate systems study effort in the recreational field. The author would like to express his appreciation of the financial help received during his program from the Department of Electrical Engineering, The Ford Foundation, and from the Michigan Outdoor Recreation Demand Study in the Department of Resource Development, Michigan State University.

Commendation is also due to the fortitude of my wife, Barbara, without whose support and encouragement this research would have been impossible, and to my mother and father for their constant encouragement as well.

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I. INTRODUCTION

For the last several decades, scientific workers in a wide range of fields have addressed themselves to the problem of describing and analyzing the phenomena of the natural universe, involving both man and his environment. A comprehensive mathematical description usually has been the most desired goal. Considerable progress has been made in the mathematical analysis of man's physical environment, and most physical phenomena are today amenable to some method or other of physical systems analysis. However, when man enters a phenomenon as an integral component, as is the case when social or economic phenomena are considered, the analytical picture is still one of fragmentary quantitative results and a preponderance of qualitative theory, supported in some cases by certain empirical observations. In the most recent decade, several prominent workers in the social sciences [1, 2, 3, 4, 5, 6, 7] have expressed their desire to overcome this quantitative gap by employing the powerful conceptual tool of "system" and the body of analytical procedures which flow forth from it. Their goal, however, usually has been more than just a mathematical framework for various phenomena; they desire also to establish a systems analytical framework which is rigorous and consistent from discipline to discipline and from phenomenon to phenomenon [1, 2, 3, 4, 6]. Needless to say, there has been as yet no unqualified success in achieving this goal. This thesis does not purport to describe man's place in the universe, or to have fully achieved the aforementioned aims and desires. It does, however, present the foundations of a rigorous and

consistent formulation procedure for modeling certain classes of socio-economic systems, along with the first large scale results of the application of such models. The systems model formulation techniques are those recently generated for electrical network analysis [8], and generalized to include the analysis of other discrete physical systems [9,10]. Their application to systems containing social and economic components, and the prerequisite component modeling procedures, are developed in the body of the thesis.

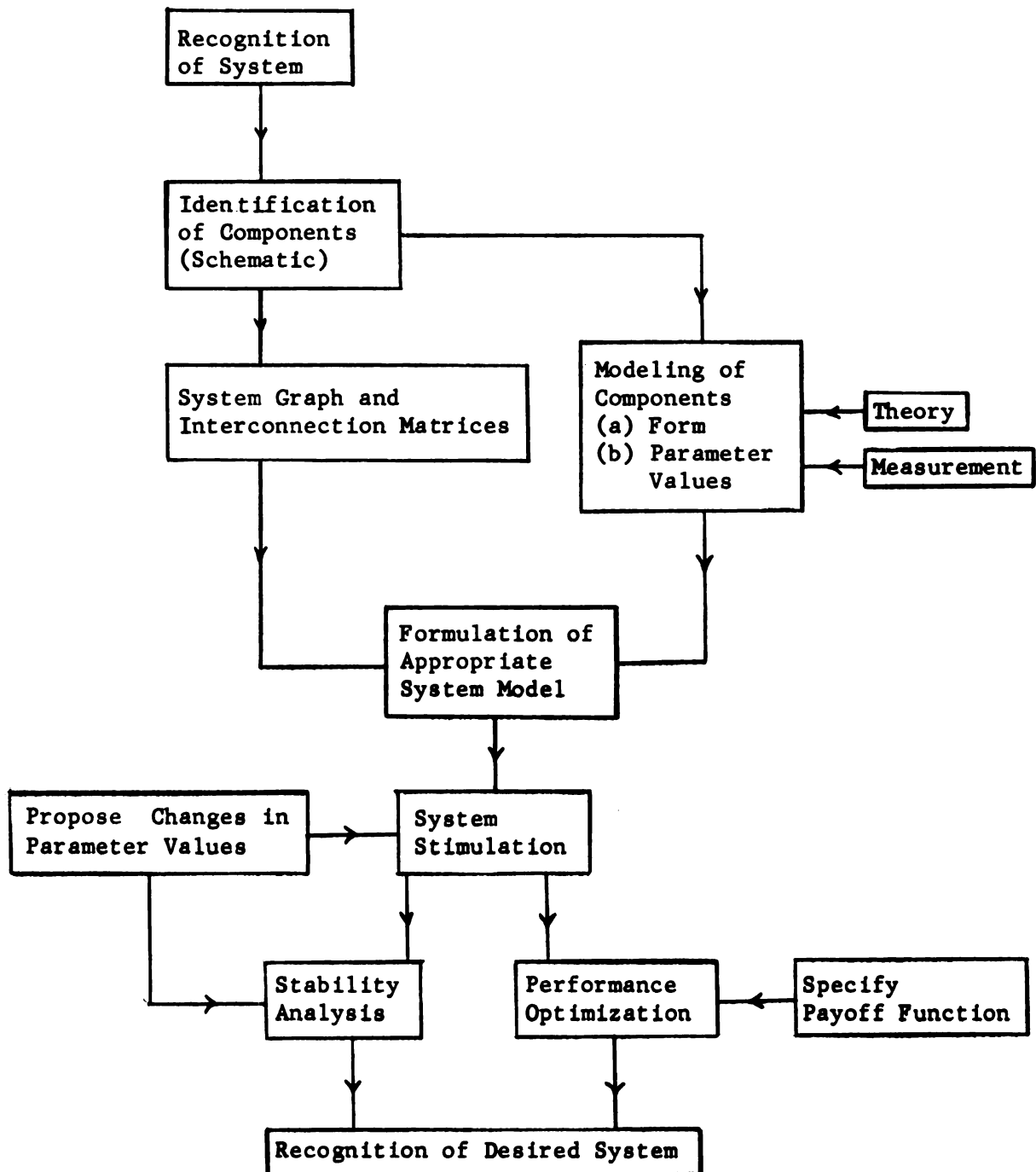
The aim of the research leading to this thesis was to describe and analyze in mathematical terms the behavior of selected non-physical complexes by application of the theory of physical systems. New methods were evolved to use existing and new data to obtain mathematical component models. The choice of examples was such that the application of physical systems analysis in both a static and a dynamic case could be shown, and also to illustrate the diversity of phenomena which are amenable to such analysis. The model discussed in the first example, a recreational travel model, is not paralleled in the literature of the field to date. The results of the second model, of the United State national economy, can be equalled by conventional methods existing in economics [6, 34, 36], but the contribution of this thesis lies in the explicit demonstration of the applicability of the rigorous and general methods of physical systems analysis in this field.

The following postulates must be satisfied by any complex before the methods of physical systems analysis are applicable [10]. Thus, any socio-economic system studied on this basis must also be amenable to these requirements:

1. The system must be identifiable as a collection of component parts or sub-phenomena.
2. The components must be discrete in nature. This implies that interactions between components must be considered as taking place only at points of interaction, called terminals. If in fact an interface is a line or surface boundary, it must be considered as collapsed to a single point.
3. Two complementary variables, X and Y, must be selected as a basis of modeling all the individual components. This pair of variables must satisfy postulates 5 and 6 below. An equation relating X and Y for a component is referred to as a component modeling equation.
4. Measurements of pairs of the complementary variables are referable to the edges of a linear graph, which has an edge to indicate the terminals of the component to which the measurements refer. An N-terminal component is completely specified as to performance by an arbitrarily chosen terminal graph of N-1 edges and a set of N-1 component modeling equations, relating the 2(N-1) complementary variables X_i and Y_i , $i = 1, 2, \dots, N-1$.
5. The algebraic sum of the variables Y_i corresponding to the directed edges of a cutset of a system linear graph must vanish.
6. The algebraic sum of the variables X_i corresponding to the directed edges of a circuit of a system linear graph must vanish.

When all of the above postulates are satisfied, a systems analysis may be accomplished by the analytical and formulation procedures presented in this thesis and in the literature [10]. The formulation procedure is consistent, irrespective of the social-science field of study to which the components "belong". Only the method of component modeling may vary from one field of study to another, where phenomenological-specific component behavior theories generally will be called upon. A schematic of the entire process of a systems analysis for a non-physical system is shown in Figure I-1, which suggests that the systems model may finally be used as a basis of stability, optimization and other analyses [11].

Figure I-1. Conceptual Stages of Systems Analysis



II. AN ANALYSIS OF A STATEWIDE PARK SYSTEM

II-1. Background

In recent years there has been a great awakening of interest in America regarding problems of outdoor recreation for all members of society. Over a quarter billion acres of public land are used in recreation, and perhaps as much additional private land. Over 90 percent of the population participates in some manner, patronizing a \$20 billion a year industry which receives an additional \$1 billion of government investment per year [16]. For decades, however, such entities as the National and State Park systems have been established and their growth planned under quite crude intuitive assumptions about the needs and desires of the population for features which these park systems offer. The origin of such parks historically was to protect a unique or especially attractive natural site from being despoiled by noxious commercial or industrial development. Later, in the 1930's, extensions to these systems often were made on the basis of make-work projects to alleviate conditions of severe unemployment. In these cases, most emphasis was placed upon the availability of publicly-owned land resources and a pool of unemployed workers in the locality, rather than upon specific development to provide demonstrably needed facilities.

In the latter part of the 1950's, mass availability of leisure time created the need for a much more rational pattern of development of outdoor recreation facilities. No longer is such development confined to preservation of "priceless" scenic sites which are part of

a "national natural heritage", although this is continuing, of course. Rather, the creation of sites of acceptable scenic attractiveness which are more accessible and therefore available to fill specific demands of large population centers for leisure facilities is now receiving considerable emphasis [12,13,14].

While considerable progress has been made in the analysis of demand and facility planning, there are still two broad questions which remain to be answered before a completely satisfactory quantitative analysis of outdoor recreation can be made. The first is; what is the true measure of the demands of society for outdoor recreation activities of various kinds, and what economic value is placed upon them by society? The second is; given a certain demand rate of the population, how do people behave with respect to the particular sets of alternative facilities open to them? In other words, given a demand rate and a set of facilities, how can one determine the intensiveness of use of each?

Answers to the first broad question are still fairly far off. Certain attempts have been made to measure demand [14,15,16], and an analytic framework for the economic criteria to be evaluated has been suggested [13]. However, there still has not been enough data gathered to provide a reasonably accurate picture of the behavioral parameters of society in this regard. For example, the main source of demand rates and preferences is the report entitled "National Recreation Survey" by the Outdoor Recreation Resources Review Commission [15], which is based upon interviews with 3,647 persons out of approximately 180 million. In a state the size of Michigan, this consists of only about 120 persons. While such a sample ordinarily is sufficient to overcome even the pro-

found regional, racial, ethnic and economic differences of groups of society for a simple determination such as the preference for Candidate X or Candidate Y for President of the United States, or television programs A, B, or C, the immense variety of possible outdoor recreation activities causes the results of such a survey to have disappointingly high standard deviations. For example, the participation rate for camping by females in the North-Central region of the United States during the summer season is given as 6 percent [15]. However, the table of standard error of estimated participation percentages([15], Table IV, pg. 106) shows that based on a sample of 213 persons (the number of females surveyed in the North-Central region), the 68 percent confidence interval for this percentage is the interval from 3 percent to 9 percent. Actual participation rates by subgroups of this group which vary by a factor of up to 3 may thus not be inconsistent with the estimation. Very little planning or future estimation may be meaningfully based upon such data. In the face of such uncertainty, then, this thesis can add little to the knowledge of the first question.

The second question, that of the distribution of use of facilities, given a demand level, will be the subject of the remainder of this chapter. In the literature so far, various calls have been made for an analytical model which would consider a given demand level, road system and park facilities and allocate usage on some rational basis [16,17,18], but no comprehensive solution has as yet been offered. This thesis proposes an analytical framework for just such a model, which is developed in subsequent sections. The activity of camping in the state park system of Michigan was selected for a full-scale demonstration of such a model due to the availability of the largest amount

of data for any outdoor activity. For virtually all outdoor recreation activities there is only fragmentary data, if it exists at all, regarding the origin and destination of participants. For Michigan State Park camping, a one year sample (for 1964 attendees) of a time series of this data was available^{*}, and the example choice was made on this basis.

II-2. Definition and Modeling of Components

The system to be studied in this section consists of all state parks in Michigan having campgrounds which issued more than 1000 camping permits in the 1964 calendar year, the road system of the State of Michigan along with selected roads linking nearby states to Michigan, and the population centers of Michigan counties plus selected population centers in adjacent states. Three types of component are identified in the system:

1. Origin areas of campers,
2. Highway links,
3. Destination areas, or state parks.

Each component is modeled by means of an oriented line segment and an equation relating two variables, X and Y. The variable Y is taken

* A full determination of the origins and destinations of Michigan State Park campers was made under supervision of the author for the Michigan Outdoor Recreation Demand Study, Department of Resource Development, Michigan State University.

to be the flow or attendance of campers in each component, measured on a full-season basis in units of camper-days^{*}. It would be quite feasible to consider the measure of the Y variable to be camper-days per month or per week rather than per season, if more data were available to the analyst. The data required would have to include, among other things, the average weather conditions for each period considered, the exact timing of certain events such as the shut-down period of large manufacturing plants and schools, and the timing of such holidays as Independence Day and Labor Day. All of these considerations would cause certain of the component parameters to be time-varying. Since the methodology is new and unproven, and the quantitative effects of such influences remains largely unknown, a seasonal measure is used so that such effects may more reasonably be neglected.

The variable X is taken to be the propensity to camp, or the demand pressure for camping. Propensity to camp is considered to be annihilated by the process of making a trip to a campground and subsequently camping there.

The origin areas are modeled as two-terminal flow drivers of known magnitudes. Their component equations are taken as

$$Y_{oi} = \text{known} \qquad i = 1, 2, \dots, 77 \qquad (\text{II-1})$$

Their magnitude is taken to be the number of camper-days recorded for 1964 from that origin in the camper permit origin-destination

*One person camping over one night in a state park was considered to equal one camper-day of use. The season length coincides with the calendar year.

tabulations mentioned previously. Each origin considered is either the geographical center of population of a Michigan county, or a selected center of population in a nearby state. In order to reduce the final number of components and equations with as little sacrifice in accuracy as possible, in cases of counties with less than 10,000 population, the center of population of two or three counties is considered as one origin component. Appendix A gives a list, description and values of the 77 origin components used in the model. The line segment representing an origin area is considered to be connected in the system linear graph of Figure II-1 from an arbitrary reference point (not shown in Figure II-1, for clarity) and the appropriate center of population. The orientation is taken as away from the reference.

It is postulated that the highway links connecting the population centers and parks components could be modeled as

$$Y_{hj} = G_j X_{hj} \quad j = 1, 2, \dots, 208 \quad (\text{II-2a})$$

The parameter G_j represents the reciprocal of "deterrence to travel"; i.e., if $1/G_j$ is large, the flow of campers for a given propensity difference across the link is small. It is postulated that for the purposes of making a trip to a campground two factors of the route are influential; the time required to travel the route, and the direct out-of-pocket expenses incurred in so doing. Note that distance, as such, is considered to be a less representative measure of trip deterrence than time. It has been discovered in many studies that for many trip-making purposes people are willing to choose a longer route if they can save time by so doing. Cost of

gasoline and toll expenses only was considered, since most families seem not to consider incremental depreciation and repairs of their automobiles caused by relatively short recreational trips. The time-cost value of each link was raised to a power greater than unity in order to weight longer trips with proportionally more deterrence than shorter ones, in line with most traffic engineering findings [19,20,21].

A total of 208 links chosen from the Michigan State Highways Map represent the state highway system. All centers of population and state parks are connected by the most important road links on the map. The importance of road links is determined from a traffic-count map for 1963, provided through the courtesy of the Michigan Department of Highways. Roads showing estimated daily volumes of fewer than 300 vehicles are omitted, unless they are the only applicable links. Average driving times were estimated for each link with the assistance of Mr. Clifford Tiedemann of the Michigan Outdoor Recreation Demand Study staff. Average direct cost is arbitrarily estimated on the basis of 14 miles per gallon of 35¢ fuel, plus the toll charge for an automobile, plus one-half of the toll surcharge for a single-axle trailer, since roughly 50 percent of campers use trailers.

The parameter G_j is evaluated as

$$G_j = \frac{k_1}{(T_j + K_2 C_j)^{k_3}} \quad j = 1, 2, \dots, 208 \quad (\text{II-2b})$$

where

T_j = estimated average driving time in hours

C_j = estimated direct cost of gasoline plus tolls in dollars

k_1, k_2, k_3 = constants

The presence of the constants k_1 , k_2 , and k_3 , above, and k_4 in equation II-3a below is necessary to allow the model to be scaled when in operation. With no firm data to go on, it cannot be predicted in advance exactly how the propensity drop generated by a flow of campers in a circuit from an origin back to the reference point is distributed across the park and the highway link components. The constants are inserted at this stage to allow them to be manipulated for overall best fit of the model predictions at the solution stage.

A complete listing of the transportation links, with their estimated time and cost parameters is given in Appendix A. Their location is shown schematically in Figure II-1, with arbitrary orientations.

The parks are modeled as line segments connected from the park location to the arbitrary reference point (omitted for clarity in Figure II-1), with their orientations taken as towards the reference. It is postulated that the park components could be modeled as

$$Y_{pk} = k_4 A_k Y_k \quad k = 1, 2, \dots, 55 \quad (\text{II-3a})$$

where k_4 is a constant, and the parameter A_k is a measure of the "attraction" or the "attractiveness" of park k . The measurement of this parameter A_k in quantitative terms was the subject of considerable development. Such concepts as "attractiveness" are usually considered as being too subjective to be expressed in terms of hard numbers. There is a certain amount of data available, however, with which the problem of deriving a behaviorally-oriented attraction measure can be attacked. On the one hand, there is a certain amount of data on preferences of people for the activities they engage in and thus the facilities they

tend to demand when they go camping [15]. On the other, there is an inventory of the particular attributes and facilities possessed by each of the 55 state parks included in the study^{*}.

From a study of the desires of people, reported in [15], it is postulated that there are three main motivational factors which lead to any park being considered "attractive" to campers. These are:

1. Its endowment of water-oriented features,
2. Its scenic aspects,
3. Its provision of comfort and convenience facilities.

Preference data from the same source [15], further suggests that, if the water-oriented factor is given a relative importance weight unity, since nearly every camper wishes to participate in water-related activities, appropriate weights for the scenic aspects and the convenience aspects would be 1/3 and 2/3 respectively. The problem then is reduced to the following; find the relative quality ratings for each park for each of these three features, and establish the total park attraction rating as

$$A_k = k_4 C_k (Q_1 + 1/3Q_2 + 2/3Q_3) \quad k = 1, 2, \dots, 55$$

where

$$k_4 = \text{a constant}$$

$$C_k = \frac{\text{the ratio of campsites at park } k \text{ to the average}}{\text{number of campsites in 55 parks}}$$

^{*} This information was obtained by Mr. Carlton Van Doren of the Michigan Outdoor Recreation Demand Study staff from the Michigan Department of Conservation. Thirty-eight facilities and attributes were considered.

- Q_1 = quality rating of water facilities
- Q_2 = quality rating of scenic attributes
- Q_3 = quality rating of convenience facilities

For ready comparison of park attractiveness, it is desirable to have an attraction value distributed such that an "average" park has an attraction of unity. Thus we can let

- $\bar{Q}_1$ = mean of Q_1 for all parks
- $\bar{Q}_2$ = mean of Q_2 for all parks
- $\bar{Q}_3$ = mean of Q_3 for all parks

and then write

$$A_k = k_4 C_k (0.5 \times Q_1 / \bar{Q}_1 + 0.165 \times Q_2 / \bar{Q}_2 + 0.355 \times Q_3 / \bar{Q}_3)$$

$$k = 1, 2, \dots, 55 \quad (\text{II-3b})$$

It still remains to be established that the 38 facilities and attributes considered above actually do group into the three factors postulated and also what quantitative contribution is made by each particular attribute to the quality rating with which it is related.

At this stage, a factor analysis of the 38 facilities considered for the 55 state parks is made using the FANOD 3 library program on the CDC 3600 computer. The data is used in the analysis in dichotomized form; i.e., a particular park is scored 1 if it possessed facility i ($i = 1, 2, \dots, 38$) and 0 if it does not, yielding a 38×55 matrix, Z , of 0 or 1 scores. The program first produces mean values and standard deviations for all of the 38 variables. It then produces a matrix R , 38×38 , $R = \frac{ZZ^T}{55}$, of

intercorrelations between these variables. It then produces a principal-axis factor-loading matrix, A , 38×38 in size, such that $\bar{R} = AA^T$ is a close approximation to R [22]. The program then examines the eigenvalues of R , and selecting them in descending order, rotates the 2,3,...,k largest eigenvectors according to the varimax criterion [22]. This yields successive factor loading matrices, or k-factor solutions, A_k , $38 \times k$ in size, from which the correlation matrix can be reconstructed as $\bar{R}_k = A_k(A_k)^T$, and compared with the observed matrix, R . The proportion of variance "explained" by each of the k varimax-rotated factors at each stage is determined, and can be considered a measure of the goodness of fit to R .

The number of varimax factors considered necessary to "reproduce" R with $\bar{R}_k$ is usually measured by the behavior of the eigenvalues of R [22]. Usually, k varimax factors are considered sufficiently descriptive of the variables if the first k eigenvalues are large, and a relative drop in magnitude occurs with the (k+1)th eigenvalue.

The eigenvalues of R (given in full in Appendix B) are as follows:

7.53, 5.05, 3.84, 2.43, 2.35, 2.00, 1.84, ...

This shows that, indeed, there is a sharp drop after the 3rd eigenvalue, followed by a much slower decline. The hypothesis of only three significant factors is thus tentatively confirmed. Fuller confirmation comes from comparing the detailed structure of the three-factor solution with that of the four-factor solution. The first three of the four factors are virtually identical to the original factors,

but the fourth is loaded by a subset of the variables loading the second, or scenic, factor. Thus the three-factor solution is accepted, and used to compute the three relevant quality indices.

The contribution of each facility to the makeup of the quality score of the respective factors is taken as the factor loading coefficient from the three-factor solution. For the first or water factor, the highest value of loadings is obtained from the following ten facility variables, after which there is a noticeable gap before the remaining loading values:

<u>Variable</u>	<u>Factor 1 loading</u>
Great lakes shoreline	.8381
Inland lake shoreline	-.7573
Great lakes swimming	.8079
Inland lakes swimming	-.9304
Sand beach on great lakes	.6792
Sand beach on inland lake	-.9102
Water skiing	-.8448
Fishing	-.5229
Boat rental	-.5216
Lifeguard	-.5569

The positive and negative loadings for great lakes and inland lake variables, respectively, is considered to be the result of the special situation of Michigan State Parks. The correlation between inland lake and great lakes swimming, for example, is $-.7600$ since, for geographical reasons a great lakes park does not usually have an inland lake shoreline as well. The exceptions, however, are some of the

most popular parks in the system, partly for this reason. Thus, the quality score is taken on absolute values of the loadings:

$$Q_{1i} = \sum_k |a_{1k}| z_{ik} \quad i = 1, 2, \dots, 55 \quad (\text{II-4a})$$

where

z_{ik} = 0, 1; the score of the park i for variable k

a_{1k} = the factor loading of variable k on factor 1

k = 13, 14, 20-24, 25, 26, 33, 36 from the numbering of variables in Appendix B.

For the second or scenic factor, 8 variables are found to have significant loadings. They are:

<u>Variable</u>	<u>Factor 2 loading</u>
Virgin timber	.5155
Wilderness areas	.6502
Waterfalls	.6895
Springs	.5799
River frontage	.5299
Historical site	.6707
Interpretive program	.6579
Hiking trails	.6229

The quality score is taken as:

$$Q_{2i} = \sum_k a_{2k} z_{ik} \quad i = 1, 2, \dots, 55 \quad (\text{II-4b})$$

where

k = 7, 8, 10, 11, 12, 15, 16, 18, 19.

The third factor, representing comfort and convenience

facilities, is found to be highly loaded by the following 5 variables:

<u>Variable</u>	<u>Factor 3 loading</u>
Store within 1 mile	.5709
Shower facilities	.8381
Flush toilets	.8135
Laundry facilities	.8301
Electricity service	.7910

The quality score is taken as:

$$Q_{3i} = \sum_k a_{3k} Z_{ik} \quad i = 1, 2, \dots, 55 \quad (\text{II-4c})$$

where

$$k = 27, 28, \dots, 31.$$

The normalized scores are formed by dividing each Q_{ji} ($j = 1, 2, 3; i = 1, 2, \dots, 55$) by

$$\bar{Q}_{ji} = \sum_k a_{jk} \bar{Z}_k \quad \begin{array}{l} j = 1, 2, 3 \\ i = 1, 2, \dots, 55 \end{array} \quad (\text{II-4d})$$

where

$$\bar{Z}_k = \text{mean score of the 55 parks for variable } k.$$

The Appendix B contains a full listing of the mean scores of the variables, and the normalized factor scores of the parks, as well as the final attraction indices calculated from relation II-3b.

Thus the entire system is now described by the system linear graph of Figure II-1 and the component equations II-1, II-2a, and II-3a, with component values as given in the Appendices A and B. In the next section, these are used to formulate a set of system equations which,

when solved, establish the desired flows; i.e. the predicted attendances at the parks as measured in camper-days per season.

II-3. Formulation of the System Model

The system represented by the linear graph of Figure II-1 consists of a total of 340 elements and 146 vertices. The 340 edges in the graph identify 77 origins, 208 highway links and 55 parks.

The 140 vertices are associated with 77 origins, 55 parks, 13 highway intersections not at a park or origin and 1 reference point.

The number of branch equations required to solve for the system variables is [9], $146 - 1 = 145$, and the number of chord equations $340 - 146 + 1 - 77 = 118$. In spite of the slightly fewer equations involved in chord formulation, a branch equation model is chosen, for the reason that the fundamental cutsets and circuits were formulated manually from the system graph. Since there are 145 fundamental cutsets and 195 fundamental circuits, there is a saving in manual error-prone work by using the branch equations. Further, formulation of the 118 chord equations requires the inverse of a matrix of order 118 with subsequent post-multiplication by a 118×77 matrix. Because of core memory limitations, multiplication of two such large arrays on the CDC 3600 computer can only be performed in steps, with intermediate tape read-out read-in. To solve the branch equations for the 55 park through variables, requires the inverse of a matrix of order 145 but the first 55 rows only are pre-multiplied by a diagonal matrix of order 55. This operation can be performed in

the memory capability of the computer with only a few precautions. For details of the compilation procedure, see Appendix C.

The nature of the graph is semi-Lagrangian, in the sense that all of the flow drivers, and all of the edges corresponding to parks share the reference point as a common vertex. Thus, all of the flow drivers can be classified as chords of some tree, Γ , and the edges corresponding to parks as branch elements. The tree thus selected, is shown in heavy lines in Figure II-1 and includes 90 highway links in addition to the parks.

The cutsets can be established by a self-checking procedure. The first 55 cutsets must contain each chord element exactly twice, and with opposite sign, since the first 55 branches correspond to parks and each defines a separate sub-tree.

The details of the constraint matrices and system model equations cannot be shown here because of their size, but in block form the branch equations describing the system are:

$$G \begin{bmatrix} X \\ -\frac{P}{X_{hb}} \end{bmatrix} = F Y_o = Y \quad (II-5)$$

where G is a 145 x 145 matrix with entries:

$$G_{jj} = A_j + \sum_{i=1}^{145} \delta_{ji}^2 G_i \quad j = 1, 2, \dots, 55$$

$$= \sum_{i=1}^{145} \delta_{ji}^2 G_i \quad j = 56, 57, \dots, 145$$

and $\delta_{ji} = 1$ if element i is in cutset j and has positive orientation,

$= -1$ if element i is in cutset j and has negative orientation,

= 0 otherwise.

$$G_{ji} = \sum_{k=1}^{145} \delta_{ik} \delta_{jk} G_k \quad i \neq j; j = 1, 2, \dots, 145$$

X_p is a 55x1 vector of park across variables

X_{hb} is a 90x1 vector of branch highway link across variables

F is a 145x77 matrix with entries:

$f_{ij} = 1$ if origin area j is in cutset i
= 0 otherwise.

Y_o is a 77x1 vector of known origin through variables

Y is a 145x1 vector, the product of F and Y_o .

It is a simple matter to perform the indicated matrix multiplication on the right-hand side of II-5 required to evaluate the 145x1 vector of known quantities Y .

If G^{-1} is partitioned in the form

$$G^{-1} = \begin{bmatrix} GI \ 1 \\ \hline GI \ 2 \end{bmatrix} \begin{matrix} 55 \times 145 \\ 90 \times 145 \end{matrix}$$

the park across variables are given by

$$X_p = [GI \ 1] Y \quad (II-6)$$

The park through variables are then

$$X_p = A X_p = A[GI \ 1] Y \quad (II-7)$$

where $A = \text{diag}(A_1, A_2, \dots, A_{55})$, a 55x55 diagonal matrix
of park attraction indices

The solutions indicated in equations II-7 and II-6 were carried out on the CDC 3600 computer, using Program PRKSYS shown in Appendix C. The results are given and compared with the actual attendances for 1964 in the next section.

II-4. Results of Solutions

The initial sets of solutions of the model are used to determine what values of the constants k_1, k_2, k_3 and k_4 yield the closest approximation to the actual 1964 attendances at the parks. Four measures are used to compare one set of solutions with another. These are:

1. The average percent error of the estimates, defined as:

$$\text{Average error} = \frac{1}{55} \sum_{i=1}^{55} | \text{percent error of park } i |$$

2. The root-mean-square error of the estimates, defined as:

$$\text{R.m.s. error} = \sqrt{\frac{1}{55} \sum_{i=1}^{55} (\text{percent error of park } i)^2}$$

3. The number of parks with percentage error equal or less than 20.

4. The number of parks with percentage error equal or greater than 50.

Percent error was defined as follows:

$$\text{Percent error of park } i \text{ attendance} = \frac{\text{predicted attendance at } i - \text{actual attendance}}{\text{actual attendance}} \times 100$$

The latter two measures provide quick means of rejecting obviously poor results. They are not nearly as discriminating when the results are even moderately close. The r.m.s. error provides the most acceptable test, since it is intolerant of large discrepancies. Table II-1 shows the error measures for a representative selection of the solutions obtained with the indicated constants.

Table II-1

Error measures for selected systems model runs:

<u>k_1, k_2, k_3, k_4</u>	<u>Avg. error</u>	<u>R.m.s. error</u>	<u># errors $\leq 20\%$</u>	<u># errors $\geq 50\%$</u>
	<u>%</u>	<u>%</u>		
1.0, 0.5, 1.3, .01	28	33.7	26	10
1.0, 0.5, 1.3, .02	29	36.5	24	11
1.0, 0.3, 1.3, .01	32	34.5	23	10
1.0, 0.1, 1.3, .01	33	35.3	22	10
1.0, 0.0, 1.3, .01	34	36.1	22	11
1.0, 0.5, 1.5, .01	32	34.3	22	11
1.0, 0.5, 1.1, .01	30	34.1	24	10

The solution accuracy is found to be extremely dependent upon the relative magnitudes of k_1 and k_4 . These two constants determine the relative magnitude of the propensity drop across the link and park components, respectively, for a given flow. Subjectively, one would

postulate that the parks should "dominate" the links in this respect. The solution obtained when k_1 and k_4 both equal unity yields interesting, but rather inaccurate results. Parks in portions of the state near large population centers are over-predicted by large percentages, while those in more remote northern areas are under-predicted by a factor of 5 to 10. Good prediction is obtained when k_4 is in the order of 0.01 when k_1 is unity. This result shows that, with respect to camping, a highway link with unit conductance* develops only one percent of the propensity drop across it for a given flow as an "average" park with unity attraction index.

The constant k_2 has the effect of including cost in addition to time in determining the link "resistance". This is very rarely done in the traffic engineering literature, and to the author's knowledge, has never been attempted for such a large-scale extra-urban system as this one**. Table II-1 indicates clearly that solution accuracy is improved by this cost addition.

The full set of predicted attendances is given in Table II-2 below, using constants as indicated. The errors and percent errors are expressed with respect to the 1964 attendance figures provided by the Department of Conservation:

* For example, a link having unit conductance could be one involving 0.6 hours of time and 0.8 dollars of direct cost, yielding $G_j = 1.0$ from relation II-2b.

** Cost has been included as a route determining factor in urban traffic flows in a study by Mr. Wallace McLaughlin in a forthcoming thesis for the Ph.D. degree at Purdue University, Department of Civil Engineering (from private communication).

Table II-2

Predicted Attendances and Error for 1964 Data:

($k_1 = 1.0$, $k_2 = 0.5$, $k_3 = 1.3$, $k_4 = 0.01$)

<u>Park</u>	<u>Predicted Attendance</u> (camper-days)	<u>Error</u>	<u>Percent Error</u>
1. Baraga	11,124	- 3,002	- 21
2. Brimley	32,010	- 38,177	- 54
3. Fayette	4,880	- 311	- 6
4. Fort Wilkins	18,342	- 6,479	- 26
5. Gogebic Lake	35,338	5,797	20
6. Indian Lake	36,418	- 19,009	- 34
7. McLain	9,869	- 12,604	- 56
8. Muskallonge Lake	16,804	5,557	49
9. Porcupine Mts.	27,423	- 3,700	- 12
10. Straits	22,769	- 28,214	- 55
11. Tahquamenon Falls	57,431	5,372	10
12. Van Riper	51,528	11,093	27
13. Wells	30,136	- 243	- 1
14. Aloha	57,038	2,189	4
15. Bay City	80,325	33,394	71
16. Burt Lake	87,384	3,649	4
17. D.H. Day	10,791	- 18,144	- 63
18. East Tawas	43,123	- 30,760	- 42
19. Harrisville	45,387	- 13,906	- 23
20. Hartwick Pines	11,581	- 10,085	- 47

21.	Higgins Lake	174,924	- 6,181	- 3
22.	Hoefft	30,586	9,393	44
23.	Interlochen	143,111	9,629	7
24.	Ludington	136,159	10,122	8
25.	Mears	21,718	- 3,119	- 13
26.	Mitchell	50,806	2,959	6
27.	Muskegon	86,942	7,787	10
28.	Onaway	24,840	4,574	23
29.	Orchard Beach	30,090	- 8,741	- 13
31.	Silver Lake	60,543	- 25,666	- 30
32.	Traverse City	79,365	- 4,562	- 5
33.	Wilderness	59,793	- 14,236	- 19
34.	Wilson	33,619	- 21,307	- 39
35.	Young	38,577	- 14,497	- 27
36.	Benzie	10,287	- 15,250	- 60
37.	Gladwin	7,834	528	7
38.	White Cloud	6,393	1,061	20
39.	Algonac	32,122	- 6,378	- 17
40.	Brighton	67,850	30,347	81
41.	Grand Haven	68,545	- 9,118	- 12
42.	Hayes	75,076	- 4,748	- 6
43.	Highland	9,724	- 2,246	- 19
44.	Holland	110,096	- 22,192	- 17
45.	Holly	51,172	10,266	25
46.	Island Lake	29,758	6,860	30

47. Lakeport	65,327	-15,598	-19
48. Metamora	39,321	-45,811	-54
49. Pinckney	110,354	36,990	50
50. Port Crescent	42,144	2,933	6
51. Proud Lake	36,552	-34,918	-49
52. Sleeper	100,891	34,320	52
53. Warren Dunes	52,905	-12,576	-19
54. Waterloo	90,839	-33,693	-27
55. Yankee Springs	86,228	-25,821	-23
			average error = 28%
			r.m.s. error = 33.7%

The final results do not show any particular bias as to geographical area, such as the vicinity of metropolitan areas. The positive and negative errors are reasonably uniformly distributed over the geographical area of the state. We find that there is a much greater propensity drop across the park components than the link components which verifies subjective considerations. A measure of this propensity drop, and some of its implications, can be found by solving for X for typical components:

for link 692 (see Figure II-1), with a flow of 100,000 on this 20 mile two-lane highway,

$$X_{692} = Y_{692}/G_{692} = 100,000/1.4535 = 68,799$$

for link 542, the Mackinac Bridge, with a flow of 100,000,

$$X_{542} = Y_{542}/G_{542} = 100,000/.2756 = 362,840$$

for park 21, Higgins Lake - the most popular park in Michigan,

$$X_{21} = Y_{21}/k_1 A_{21} = 174,924/.0391 = 4.47 \times 10^6$$

for park 31, Silver Lake - a "typical" Lake Michigan family park,

$$X_{31} = Y_{31}/k_1 A_{31} = 60,543/.0137 = 4.42 \times 10^6$$

for park 37, Gladwin - a small out-of-the-way park,

$$X_{37} = Y_{37}/k_1 A_{37} = 7,834/.0017 = 4.61 \times 10^6$$

The two examples of links indicate that the 20 mile link of two-lane State highway is less "deterrent" to camping trips by a factor of 53 than the Straits of Mackinac Bridge, with its present toll structure. Alternatively, one could say that the deterrence provided by a 106 mile stretch of two-lane State highway and the Mackinac Bridge are equal. Furthermore, it is at first astonishing, but upon reflection becomes intuitively obvious, that the propensity drop across such diverse types of parks is very nearly equal. Given that the camper has arrived at their gate, the wide differences in attendances (flows) is due solely to their attraction indices.

As a check on the sensitivity of the solution to the attraction indices (which will be the subject of further comment below), the model was solved with all of the attraction indices set equal to C_k ; i.e. all quality factors were set effectively to unity for all parks. The result

showed an average error of 38 percent, an r.m.s. error of 56 percent, with 17 parks having an error of ≤ 20 percent, and 13 parks having errors of ≥ 50 percent. These results are vastly inferior to those obtained with the indices set at their computed values.

In comparison with other types of models which might be constructed for such systems, the systems model as developed here shows superior results. Gravity models have found almost universal use by highway and traffic engineers for the study of many different types of traffic flows [19,44]. A gravity model of the same park system was made by Mr. Carlton Van Doren, a colleague of the author's, using the same travel time values and the same links. Certain additional out-of-state origins were included, since computer memory capacity was not critical. The results of the best model run to date^{*}, supplied by Mr. Van Doren, showed an average error for the same 55 parks of 31 percent, an r.m.s. error of 42.3 percent, with 24 parks having ≤ 20 percent error and 11 parks ≥ 50 percent.

Those accustomed to the precision of modeling possible in physical systems may find an r.m.s. error of 33.7 percent rather high in an absolute sense. To provide an evaluation of the absolute performance of the systems model, it is compared with a measure of the effects of purely random or "happenstance" fluctuations in such a phenomenon as camping. The r.m.s. value of the percentage attendance fluctuations from 1963 to 1964 was considered, for the same 55 parks. The total system camper-day increase was approximately 8 percent from

^{*} April 28, 1965

1963 to 1964. However, when one examines the figures for individual parks, a vastly different picture appears. Some parks show an increase of 158 percent (Aloha), 79.5 percent (McLain), and 74.5 percent (Van Riper) others show a decrease of 47 percent (Island Lake) and 26.3 percent (Baraga). The average value of the percent changes from 1963 to 1964 was 18.2 percent, but the r.m.s. value is 33.4 percent. Thus, it can be concluded that an overall r.m.s. error of 33.7 percent, as obtained in this investigation, is not excessive, and may, in fact, be extremely good.

Certain defects in the prediction of the systems model are fairly easily traceable. Let us examine in detail some of the 10 "worst" predictions of the model to ascertain what, if any, effects not accounted for in the component models used here could explain the large discrepancies.

Benzie and D.H. Day parks were considerably under-predicted. D.H. Day has a rather low attraction index, which undoubtedly does not reflect the amenities immediately adjacent to the park, namely Glen Lake and the Sleeping Bear Dunes area. Furthermore, both of these parks are relatively exceptional in the Michigan state parks system in that they do not require a vehicle permit for entry, almost certainly adding to the "attraction".

Straits parks was also considerably under-predicted. This park lies immediately at the northern end of the Mackinac Bridge, thus providing perhaps a unique viewpoint from which to view the bridge itself and the considerable shipping traffic in the Straits, and is near the historic Mackinac Island. These features are perhaps inadequately

reflected in its relatively low attraction index of 0.740.

Two parks, Metamora and Sleeper, may have been influenced by the choice of links in the system. Metamora is not served by a road joining county centers of population, and since few other links were added to conserve computer space, this may introduce an accessibility barrier to this park which is not present in the real world. Sleeper park is very close to Port Crescent, and arbitrarily was chosen as the terminus of a road link. The over-prediction of Sleeper (52%) and Port Crescent(6%) represents a 26 percent over-prediction for the parks taken together as one unit, which they are functionally.

The over-prediction of Brighton (81%) and Pinckney (52%), may well be due to the effects of the dichotomized facility data. The fact that both parks have as high an overall quality index as some very "attractive" northern Michigan parks may be due to the fact that they possess many of the facilities loading the three quality factors - but that these facilities individually are not really equivalent in quality to those in some other parks. The dichotomized data cannot discriminate between an "inland lake" and a "beautiful inland lake", or between "fishing" and "good fishing".

While we cannot include exceptions such as those cited above in the index structure, it appears that further refinement of the park attraction indices would repay the effort - in particular, the gathering of data in other than 0,1 form and a more careful inclusion of nearby facilities commonly utilized by park users, even though technically they may be outside the park.

III. AN ANALYSIS OF A NATIONAL ECONOMY

III-1. Background

Economics as a separate discipline has been studied by various scholars over the last two centuries. The first studies were mainly of a deductive or social-philosophic nature, where postulates derived from common sense are set up as laws and the behavior of economic systems is inferred from them. Such studies were carried on by David Ricardo, Adam Smith, Thomas Malthus, Karl Marx, and many others. Their impact upon later thinking and even upon real events up until the present day attests to the widespread acceptance of their studies, at least among certain segments of mankind [46]. However deeply convincing these philosophical structures may have been, they do not embody the normal scientific precepts of hypothesis, test, verification or rejection, and re-hypothesis.

The truly scientific approach to economic analysis has probably come about only in the last half-century. In 1899, Karl Pearson published his Grammar of Science which introduced the discipline of statistics and statistical inference to a wide group of scientific fields. A few years earlier, Hollerith had introduced punched cards into the record keeping system of the U.S. census, and thus for the first time there existed both a large body of factual data regarding the actual functioning of an economy, plus a rigorous method for drawing inferences from it. Thus the stage was set for the economic philosophies to be tested against the cold facts.

Various workers proceeded to do just this. For instance,

Edward Chamberlin [23] observed the behavior of supply, demand and prices in actual marketplaces. He found that for a wide range of goods, there was not a "perfectly competitive" supply or demand side as had been postulated by the earlier social philosophers, and under such realistic conditions, prices do not always fall when demand drops. Thus the Ricardo-Smith "Law of Supply and Demand" was shown to hold only for a very limited range of economic goods. In the 1920's and 1930's the work of John Maynard Keynes [24,25] showed that the spending of income by consumers does not take place automatically in such a fashion as to tend constantly towards providing full employment, as had been postulated by the "laissez-faire" school of economists. He showed that a market economy often may balance itself by fluctuations in employment, rather than in prices, wages and interest rates. He introduced the hypothesis of an "investment multiplier", which states that an investment of X dollars in productive capacity causes a flow, mX dollars, of extra income, and that this effect obtains even when the investment is made by a government operating with a budgetary deficit.

The scientific analysis of the growth of an economy was attempted first by workers such as Colin Clark [26], Sir Roy Harrod [27] and Wassily Leontief [28,29]. Professor Clark made two important discoveries; firstly, that as of 1935 only seven countries in the world enjoyed a truly high real standard of living, and secondly; that a high percapita real income is almost always associated with an economy having a high proportion of workers in tertiary employment, or service industries, and a lower proportion in primary employment such as agriculture, forestry and fishing. Clark offered no system-theoretic

explanation for these observations, but they are arrived at in a scientifically sound manner.

Shortly thereafter, Harrod suggested a dynamic growth model using empirically observed parameters in an equation* which yields the growth-path or development of an economy over time. Harrod used the following variables:

$Y_1(t)$ = output or production at time t

$Y_2(t)$ = new investment at time t

$X(t)$ = stock of capital goods at time t

The relationships he postulated, and tested for several nations, are:

$$Y_1(t) = \frac{1}{k} X(t) \quad (\text{III-1})$$

$$Y_2(t) = \sigma Y_1(t) \quad (\text{III-2})$$

$$\frac{dX(t)}{dt} = Y_2(t) \quad (\text{III-3})$$

In the above, k is the "capital-output" ratio; i.e., the amount of total capital investment required to produce one unit of output, and σ is the "savings ratio"; i.e., the proportion of income which is saved. Equation III-2 states that investment equals savings. Equation III-3 states that the growth rate of total investment equals the amount of new investment.

The above equations are combined into a growth model as shown:

* Usually termed the "Harrod-Domar growth model" [36,39] due to the concurrent work of Domar.

$$\frac{dX(t)}{dt} = Y_2(t) \quad (\text{III-4a})$$

$$\frac{kdY_1(t)}{dt} = \sigma Y_1(t) \quad (\text{III-4b})$$

$$\frac{dY_1(t)}{dt} = \frac{\sigma}{k} Y_1(t) \quad (\text{III-4c})$$

The latter equation, III-4c, when solved for $Y_1(t)$, gives the time-path of output of the economy:

$$Y_1(t) = e^{\frac{\sigma}{k}t} Y_1(0) \quad (\text{III-4d})$$

Harrod estimated values of k between 2.5 and 3.5 for developed economies, and σ values of between 0.1 and 0.2, both on a per-year basis. Values of $k=3$ and $\sigma=0.15$ yield a $\frac{\sigma}{k}$ value of 0.05, implying that such an economy would enjoy a growth rate of 5 percent per year, which agrees quite well with the results for many industrialized nations.

However, such a model is useful only up to a certain point. It aggregates the entire production of an economy into a single "sector". It further assumes that only capital is required for production, and labor does not appear in his relationship. Many industrialized nations have shortages of labor, or of certain kinds of labor, thus making this model unrealistic. Also, no depreciation of capital goods is allowed for. This can be a serious omission in the case of a long-developed economy which has considerable investment in obsolete production facilities, or one in which technological obsolescence occurs rapidly.

The much more difficult problem of a structural analysis of economic growth was first attacked by Wassily Leontief before World

War II [28]. He considered a large number of industries as separate production sectors, and undertook the formidable job of estimating the amount of each industry's deliveries to consumers (as final end-products) and to other industries as intermediate inputs to their own productive processes. For instance, the amount and proportion of the output of the coal mining industry delivered to households for final consumption was shown separately from the amount delivered to the steel industry, electric power industry, railroads, etc., for use in producing their respective products.

Model construction of this type has been termed input-output or inter-industry analysis, and in recent years has led to a considerable sub-discipline of its own within economics. It has assumed such usefulness and importance for business and government purposes that the U.S. Department of Commerce is now charged with the responsibility of producing and up-dating just such an input-output table for the entire U.S. economy. An early effort, for the year 1947, based on the work of Leontief and colleagues was published in 1952, by the Bureau of Labor Statistics [30]. Unfortunately, this version was not based upon the same data as were the regular National Income Accounts for the year in question, and some direct comparisons with other government figures are not valid. The most recent work has been recently reported on and published [31]. It consists of a full input-output table plus supplementary data, evaluated for the year 1958 from the same data from which the industry total production figures, to be found in the National Income Accounts published by the U.S. Department of Commerce [32] are computed.

A brief look at the results of Leontief's analytical endeavors should now serve to clarify both what can and cannot be done with the input-output matrix alone, and also what some of the underlying assumptions implicit in its use are. Leontief allowed n different industries to each produce a unique economic good, or output. He traced the inputs of other goods needed to produce each output, as well as the amount of each output which is consumed in its final state. This latter amount was termed the final demand for a good. He postulated that a fixed amount, y_{ij} , of each good j would be needed to produce one unit of output, y_i . Thus, he wrote

$$y_i = \sum_{j=1}^n y_{ij} + y_{fi} \quad i = 1, 2, \dots, n \quad (\text{III-5})$$

where y_i = output of good i

y_{ij} = input of good j required to produce y_i of i

y_{fi} = final demand for i

As a further assumption, he considered that the quantity of input y_{ij} demanded would be a function only of the volume of output y_j of good j ; i.e. no internal scale economies are considered. Thus, he wrote:

$$y_{ij} = a_{ij} y_j \quad (\text{III-6})$$

where a_{ij} = a "technical" or "technological" coefficient

which gives us the relationship

$$y_i = \sum_{j=1}^n a_{ij} y_j + y_{fi} \quad i = 1, 2, \dots, n \quad (\text{III-7a})$$

or, in matrix form,

$$Y = AY + Y_f \quad (\text{III-7b})$$

The above relationship can now be used to solve what economists term an open static model. The term "open" implies that the model is driven by some externally-given quantity, usually called "exogenous variables". In this case, the variables required to be given exogenously are the entries of the vector of final demands. The relation III-7b can be used to solve for the production vector Y , the level of all industries' production, which is required to satisfy some given final demand vector, or "bill of goods", Y_f . The solution is

$$Y = (I - A)^{-1} Y_f \quad (\text{III-8})$$

There are two chief criticisms one can make of this simple model [36,39]. Firstly the assumption that the a_{ij} entries are constants is not valid if rapid technological change takes place, enabling one input to be substituted for another. For example, an extensive changeover from the use of steel to the use of aluminum for automobile engines would cause a noticeable distortion in the present input-output relations of the nation's economy. Leontief studied precisely this effect, the change of the matrix A over time, in one of his earliest works [33]. His conclusion was that the A matrix is largely stable over time periods shorter than a decade, during which time significant changes are likely to occur only in a few technologically-advancing industries.

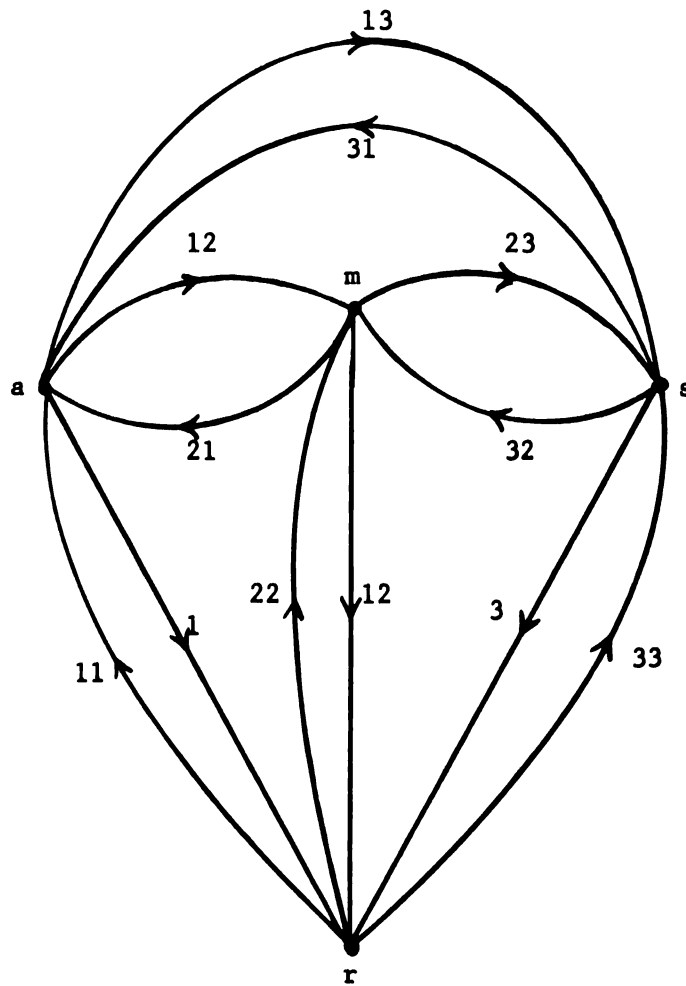
Secondly, the model requires a forecast or estimate of final

demand levels, which may be difficult to make. It would be preferable to "close" the model by making this exogenous variable a part of the model itself, i.e. to have demand endogenously derived from some other exogenous variable which is easier to predict, such as population. Various efforts have been made to close the basic input-output model, and all of them require theory in addition to the bare inter-industry matrix concept [34,35]. Generally speaking, concepts of a production function, a capital accumulation function, and an investment generating function are required. These concepts are clarified in the next section, which will use a rigorous systems analysis formulation to construct a closed model of economic growth. The latter two concepts will be seen to be equivalent to the behavioral relations governing component behavior; i.e., the component equations. The concept of a production function in the classic labor-capital value-added sense is a component equation as well. When cutset conditions are applied in the formulation procedure, inter-industry deliveries of intermediate inputs become a part of the production function as well.

III-2. A National Economic System

Consider a national economy wherein production of three kinds of goods takes place in three productive sectors. These sectors, respectively, represent agriculture, mining and manufacturing, and services, construction and government activities. Each sector component is represented as a line segment in the system linear graph of Figure III-1.

Figure III-1. The Linear Graph of the United States
National Economic System



These sectors are considered to be joined together by linking components, which are also shown as line segments in the system linear graph.

Each component is modeled in terms of an X and a Y variable, satisfying the postulates given in chapter I. The Y variable is considered to be a dollar-valued flow of income (or output, as will be seen in certain cases) on a per-year basis. The X variable is taken as the total dollar value of fixed capital investment.

The modeling equations for the production sector components are:

$$X_k(n+1) = (1-d_k) X_k(n) + Y_{I_k}(n) \quad (\text{III-9a})$$

$$Y_{I_k}(n) = \sigma_k Y_k(n) \quad (\text{III-9b})$$

$$Y_k(n) = k_k X_k(n) + \lambda_k L_k(n) \quad (\text{III-9c})$$

$$k = 1, 2, 3$$

where X_k = accumulated dollar value of fixed investment
in capital goods in production sector k; $k = 1, 2, 3$
 n = time period (year)
 d_k = depreciation rate per year of capital goods in
sector k; $k = 1, 2, 3$
 Y_{I_k} = dollar amount of new investment per year in
sector k; $k = 1, 2, 3$
 σ_k = coefficient of investment in sector k; $k = 1, 2, 3$
 Y_k = dollar value of annual net income of sector k;
 $k = 1, 2, 3^*$

*At this point, Y_k represents the net income to sector k, taken as equal to the value added by sector k to its total output. When the cutset relations of the system graph are considered (see below), Y_k becomes the gross income of sector k, taken as equal to the total value of

k_k = capital-output ratio for investment in
sector k ; $k = 1, 2, 3$

L_k = number of workers employed in sector k in
a year; $k = 1, 2, 3$

λ_k = labor-output ratio for labor employed in
sector k ; $k = 1, 2, 3$

The significance of the relationship III-9a is in its description of the growth of investment in a productive sector. The relation states that the accumulated investment at the end of some time period $n+1$ is equal to the amount of investment accumulated at the end of the previous period n , less the amount of depreciation suffered during the period n , plus the amount of new investment received during the time period n (considering depreciation and new investment to occur in lumps at the end of period n). Such a function is quite widely used in the literature on economic growth [35,36], and is known as a capital accumulation function. Thus, capital accumulation is considered to be part of the behavioral process modeled by the component equations for a production sector.

Relation III-9b states that new investment in a sector will be a linear function of the income of that sector. This assumption is found in the Harrod-Domar type of rudimentary economic growth models [27,35], in which is σ_k the "savings ratio" of sector k . In the balance sheet of a company, this figure would be termed the proportion of retained earnings. Thus, the assumption in III-9b is twofold: firstly, that all earnings retained by industries in a

sector are invested entirely within that same sector; and secondly, that a ~~fixed~~ proportion of income is retained for investment by each sector, no other investment occurring. The work of Clark and Harrod [26,27] suggests that a constant σ_k value may be quite appropriate for a model which is highly aggregated, as a three-sector model is. When detailed investment patterns are considered, such as by each of 100 industries, or say by each of 500,000 firms and private investors, the picture is much more complex. In such a case, investment may be generated by expected rate of return, which is an extremely difficult quantity to predict with any great degree of reliability [37]. Thus, the Harrod-Domar investment function is used here to model the component.

The relationship III-9c is known as the production function of a sector. The equation states that income to sector k is generated because capital is invested and labor employed in the sector. Further, it states that amounts are attributable to labor and capital separately. The amount attributable to labor in a sector is taken as equal to the amount of wages and salaries paid to workers in that sector. The coefficient λ_k thus can be interpreted as the amount of output value added per worker employed in sector k. The amount attributable to capital invested in sector k is taken as the net sector income less wages and salaries paid. In the accounting world, this would appear as interest, dividends, and retained earnings. The coefficient k_k thus can be considered to represent the amount of net income per dollar invested in sector k - roughly, a gross rate of return to capital invested in k.

Other forms of production function also occur in the economics literature [24,26,27,29,30]. A common one is the so-called Cobb-Douglas production function [24], which states

$$Y = L^A X^B$$

where A,B are constants

This function is non-linear and, if used, would introduce serious difficulties into the solution of state equations. The production function III-9c is referred to as the linear labor-capital function, and was first proposed by Marx [39]. It is still used in economics today, although the Cobb-Douglas function is more prevalent for the following reason.

When differentiated with respect to X, the Cobb-Douglas function yields B, which can be interpreted in economic theory as a measure of the substitutability of capital for labor. Similarly, A is a measure of the substitutability of labor for capital. Since any production function can be regarded as a "recipe" for combining labor and capital, and a wide range of allowable choices of ingredient combinations will yield the same output, depending on technology, the degree of substitutability of capital for labor is of great interest to social scientists studying automation, for example. However, this feature of substitution of inputs is not allowed here, since it is not allowed in the Leontief input-output framework of inter-industry deliveries.

The linkage components in the system graph are modeled using Leontief's assumption of linear dependence of linkage flows upon output levels of the sectors. Thus, the component models are written as

$$Y_{jj}(n) = (1 + a_{jj})Y_j(n) \quad j = 1, 2, 3 \quad (\text{III-10a})$$

$$Y_{jk}(n) = a_{jk}Y_k(n) \quad j, k = 1, 2, 3; j \neq k \quad (\text{III-10b})$$

with the restriction that $Y_{jk} \geq 0$; $j, k = 1, 2, 3$.

In electrical systems terminology, the linkage components are dependent through drivers, whose value is dependent upon the value of some other through variable. By making use of Leontief's linearity assumption, we effectively cause subsequent models to be valid only about some actual given value of the Y vector. For the case of an economy undergoing profound development, the coefficients a_{jk} will be subject to change.

The state vector for the system is selected as the vector of X_k ; $k = 1, 2, 3$. Applying the cutset postulate at vertices a, m, and s respectively, we see that after the interconnection links are joined to the sectors, the flows Y_k become

$$\begin{bmatrix} Y_1(n) \\ Y_2(n) \\ Y_3(n) \end{bmatrix} = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix} \begin{bmatrix} X_1(n) \\ X_2(n) \\ X_3(n) \end{bmatrix} + \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} L_1(n) \\ L_2(n) \\ L_3(n) \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} Y_1(n) \\ Y_2(n) \\ Y_3(n) \end{bmatrix} \quad (\text{III-11a})$$

or, in matrix form,

$$Y(n) = KX(n) + \lambda L(n) + AY(n) \quad (\text{III-11b})$$

The relation III-11 can be considered to be the set of "complete" production functions of the sectors. They are complete in the sense that III-11 specifies all inputs to each sector, i.e. labor, capital, and intermediate goods, required for the total make-up of the gross value of its output. This notion of a complete production function is still very underdeveloped in the economics literature [36,39]. This may be due to the fact that most economists view the value-added portion (labor and capital) as more interesting and worthy of study, or to the magnitude of the task of estimating the inter-industry portion in great detail. A private discussion by the author* with Dr. Jan Kmenta of the Institute for Social Science Research, University of Wisconsin, revealed that some economists regard such functions as an ultimate ideal, not yet realized in a practical sense.

When equation III-11b is solved for Y, we obtain

$$Y(n) = (I-A)^{-1}[KX(n) + \Lambda L(n)] \quad (\text{III-11c})$$

which is needed for the derivation of the state model, shown below.

In deriving the state equation model of the system, we start with the equations

$$\begin{bmatrix} X_1(n+1) \\ X_2(n+1) \\ X_3(n+1) \end{bmatrix} = \begin{bmatrix} (1-d_1) & 0 & 0 \\ 0 & (1-d_2) & 0 \\ 0 & 0 & (1-d_3) \end{bmatrix} \begin{bmatrix} X_1(n) \\ X_2(n) \\ X_3(n) \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \begin{bmatrix} Y_1(n) \\ Y_2(n) \\ Y_3(n) \end{bmatrix}$$

(III-12a)

*At Michigan State University, Economic Workshop, February 19, 1965.

or, in matrix form,

$$X(n+1) = DX(n) + SY(n) \quad (\text{III-12b})$$

and eliminate the Y vector by substituting in equation III-11c. The system state model is then:

$$\begin{aligned} X(n+1) &= DX(n) + S(I-A)^{-1}[KX(n) + \Delta L(n)] \\ &= [D + S(I-A)^{-1}K] X(n) + S(I-A)^{-1}\Delta L(n) \end{aligned} \quad (\text{III-13})$$

The solution of this equation set is

$$\begin{aligned} X(n) &= [D + S(I-A)^{-1}K]^n X(0) \\ &\quad + \sum_{j=0}^{n-1} [D + S(I-A)^{-1}K]^{n-1-j} S(I-A)^{-1} \Delta L(j) \end{aligned} \quad (\text{III-14})$$

The relations III-14 are thus a set of time paths of fixed capital investment in the productive sectors, driven by a vector of labor-force numbers, $L(n)$, which must be known over time. It is acceptable in such an aggregated model as this to assume that the proportion of the total labor force employed in each sector is constant. This assumption is valid over perhaps even a slightly longer time span than the Leontief assumptions about constant interindustry delivery proportions [32]. Thus we may write

$$\begin{bmatrix} L_1(n) \\ L_2(n) \\ L_3(n) \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} L(n) = WL(n) \quad (\text{III-15})$$

where w_k = proportion of the total labor force employed in
sector k , $k = 1, 2, 3$
 L = total labor force.

Further, we can take the total labor force to be a fixed proportion of the total population of the country [32]; i.e., that

$$L(n) = \ell P(n) \quad (\text{III-16})$$

where ℓ = proportion of the total population in labor force

P = total population

When these relations are used in III-14, we have an expression for the growth paths of sector investment which depends upon our knowledge in addition to the parameters of the system, only of the initial investment levels, and a time series of population. The deduction of suitable parameters for such a model of the United States economy, as obtained from literature which is widely available, and its solution for the period 1947-1952 will be shown in the next section.

III-3. An Example: The U.S. Economy from 1947-1952

To provide a numerical example of the modeling concepts discussed in the preceeding section, data from the United States economy is used to provide the required parameters and to afford a check on the results produced by the model. It was decided to choose the period 1947-1952 for the analysis, since there was a 50-industry input-output table showing all transactions in dollar amounts for 1947* [30].

*A similar study [31] gives an 82-industry input-output table for the year 1958. However, the data is given only in A-matrix coefficient form, making it impossible to determine the dollar value of the transactions, which is necessary for re-aggregation of the matrix.

It is assumed that a five-year period would be short enough to validate the previously-stated linearity of parameters assumptions, yet would be long enough to provide an adequate test of the model.

An aggregation of the transactions between industries in 1947 is made from table 4 of [30], on the following basis:

Sector 1: Industries 1 and 2 (Agriculture)

Sector 2: Industries 3 through 29 (Mining and Manufacturing)

Sector 3: Industries 30 through 45, and 48

(Services, Construction and Government)

The rows and columns giving inventory changes, imports, and exports are neglected in the aggregation. The following table shows the results of this aggregation:

Table III - 1

Aggregated Inter-Industry Transactions for 1947

(Amounts in millions of 1947 dollars [30])

	Agriculture	Manufacturing	Services, etc.	Final Demand
Agriculture	33,192	5,635	5,408	33,149
Manufacturing	5,127	63,381	36,662	49,689
Services, etc.	12,399	37,944	73,422	150,187
Value Added	25,428	56,327	105,470	
Totals	$Y_1 = 76,146$	$Y_2 = 162,287$	$Y_3 = 220,992$	

The above table yields the following Leontief-type input-output matrix:

$$\begin{bmatrix} 0.436 & 0.035 & 0.024 \\ 0.067 & 0.391 & 0.166 \\ 0.163 & 0.234 & 0.332 \end{bmatrix}$$

The entries in each column represent the ratio of the corresponding entry in the columns of Table III-1 divided by the total at the foot of the same column in the table.

The system state model of equation III-13 requires a matrix A which is the transpose of the above matrix. Thus, we can write

$$A = \begin{bmatrix} 0.436 & 0.067 & 0.163 \\ 0.035 & 0.391 & 0.234 \\ 0.024 & 0.166 & 0.332 \end{bmatrix}$$

To determine the value of the σ parameters, data on investment in new capital equipment and structures purchased is taken from the data in [32] for manufacturing and services, and from [42] for agricultural investment. The figures are:

Investment in sector 1 = 3,203 (millions of 1947 dollars)

Investment in sector 2 = 9,394

Investment in sector 3 = 11,218

These yield σ values of

$$\sigma_1 = 3,203/76,146 = 0.042$$

$$\sigma_2 = 9,394/162,287 = 0.058$$

$$\sigma_3 = 11,218/220,992 = 0.051$$

Values of the labor-output coefficients and the labor-force vector are obtained by the use of the figures on the number of equivalent full-time employees by industry, from [32]. The data for the agricultural sector is supplemented by data from [42] on the number of family members "employed" on farms as proprietors or workers, but who are not reported in [32]. Their annual wage rate is estimated to be 1.33 times the average for the employees reported, to allow for managerial and entrepreneurial premiums. The values of λ are obtained by dividing the total sector wage bill (supplemented as described for agriculture) by the total number of employees in the sector. The L vector is taken as the number of reported employees and farm proprietors in the sectors.

Thus

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} 10,382 \\ 16,153 \\ 28,903 \end{bmatrix} \quad (\text{thousand workers})$$

$$\lambda_1 = \frac{(3097 \times 10^6 / 2392 \times 10^3) 2392 \times 10^3 + 1.33 \times 1290 \times 7,285 \times 10^3}{10,382 \times 10^3} = 1,725$$

$$\lambda_2 = 47,607 \times 10^6 / 16,153 \times 10^3 = 2,947$$

$$\lambda_3 = 78,036 \times 10^6 / 28,903 \times 10^3 = 2,700$$

The total labor force, L, is seen to be

$$L = L_1 + L_2 + L_3 = 47,448 \times 10^3$$

The using the population figure

$$P(1947) = 143,136 \times 10^3$$

we see that

$$\ell = L/P = 0.388$$

and we can write

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \ell P = \begin{bmatrix} 0.187 \\ 0.291 \\ 0.521 \end{bmatrix} 0.388 \times 143,136 \times 10^3$$

The next task is to determine the initial amounts of capital invested by sector, i.e. the $X(1947)$ vector. There are estimates of the value of real net capital (structures, equipment, and inventory) invested in manufacturing [32], which show

$$X_2(1947) = 88.3 \text{ (billion 1947 dollars)}$$

However, the other sectors are not explicitly covered in standard data sources. An estimate of the total investment in agriculture is made from data in [42], taking X_1 to be the sum of value of land and buildings on farms, value of machinery and equipment on farms, and value of livestock owned by farmers. This yields a valuation giving

$$X_1(1947) = 87.4 \text{ (billion 1947 dollars)}$$

The total investment in the service sector is the most difficult to determine. No estimate of it exists in the U.S. Government literature cited, and none could be found elsewhere. There is also no compatible time-series of new investment and depreciation charges which would allow an explicit determination to be made. Thus a very rough assumption is made that the total investment in service industries

is approximately the same proportion of the total investment in manufacturing that new construction plus new equipment expenditures in service industries is of new construction plus new equipment expenditures in manufacturing in 1946. This ratio is obtained from Table V-3 and V-7, pg. 190 and 193 of [32], which uses classifications not entirely compatible with the desired sector breakdowns, another caution to apply to the following estimate of X_2 :

$$X_2(1947) = 88.3 \times 0.6 = 53.0 \text{ (billion 1947 dollars)}$$

We can now determine the depreciation coefficients by making use of the depreciation charged to the sectors in the National Income Accounts [32]. These are as follows:

Depreciation charged to agriculture = 40 (million 1947 dollars)

Depreciation charged to manufacturing = 2,573

Depreciation charged to services, etc. = 2,667

whence

$$d_1 = 40/87,400 = 0.005$$

$$d_2 = 2,573/88,300 = 0.029$$

$$d_3 = 2,667/50,000 = 0.053$$

The only remaining parameters to be determined are the capital-output coefficients, k . These are determined by dividing the value added attributable to capital by the total invested capital, for each sector. The value added attributable to capital is taken to be the total value added per sector less the amounts attributable to labor in that sector, i.e. wages and salaries. For each sector, the wages paid to employees are determined from [32]. To the agricultural wage

bill an amount equivalent to wages of proprietors and their families is added, as explained previously. To the wages of the employees of the service sector is added the remuneration received by self-employed persons, such as professionals, which does not appear in the wage bill.

The amounts of value added attributable to capital, per sector, are:

attributable to agriculture = 6,800 (million 1947 dollars)

attributable to manufacturing = 8,720

attributable to services, etc. = 7,500

which yields k coefficients of

$$k_1 = 6,800/87,400 = 0.078$$

$$k_2 = 8,720/88,300 = 0.099$$

$$k_3 = 7,500/50,000 = 0.150$$

The parameters determined above are all substituted into the state equation model, III-13, and the solution obtained for five cycles; i.e., until 1952 figures are obtained. At each stage, the appropriate yearly population figure is used for P . Also, in order that all of the results can be expressed in "constant 1947 dollars", an adjustment for dollar inflation is made to convert all values to 1947 dollars. The adjustment ratios used are the Gross National Product Deflators series from [32].

The results are as follows:

$$\begin{bmatrix} X_1(1952) \\ X_2(1952) \\ X_3(1952) \end{bmatrix} = \begin{bmatrix} 97.48 \\ 107.98 \\ 73.52 \end{bmatrix} \quad (\text{billion 1947 dollars})$$

The complete series of results are plotted in Figure III-2, which shows the changes in X over time.

In evaluating the results, the most soundly based comparison of the model with reality is in the X_2 variable, since [32] contains exactly this variable, plotted as a time-series. The actual value of X_2 obtained from [32], is

$$X_2(1952) = 104.1 \quad (\text{billion 1947 dollars})$$

The discrepancy here is $107.98 - 104.1 = 3.9$ billion dollars, or about 3.75 percent. The fact that the model prediction is on the high side may be the reflection of the fact that 1947 tended to be a somewhat singular year, with the immediate postwar industrial pressures yielding higher than normal returns to capital, for example. However, the fact that the error is not higher than it is, given that this was the case, may well be a result of the further impetus to the economy of the early 1950's given by the Korean war. The other values of X_1 and X_2 cannot be compared to any convenient data, as previously stated, but again a good estimate of X_1 can be made from the Agricultural Statistics [42]. The value of X_1 for 1952 from 20 is

$$X_1(1952) = 99.6 \quad (\text{billion 1947 dollars})$$

The X_1 value was deflated to 1947 dollars by applying the price-per-acre indices given in [42] to the farm land and buildings

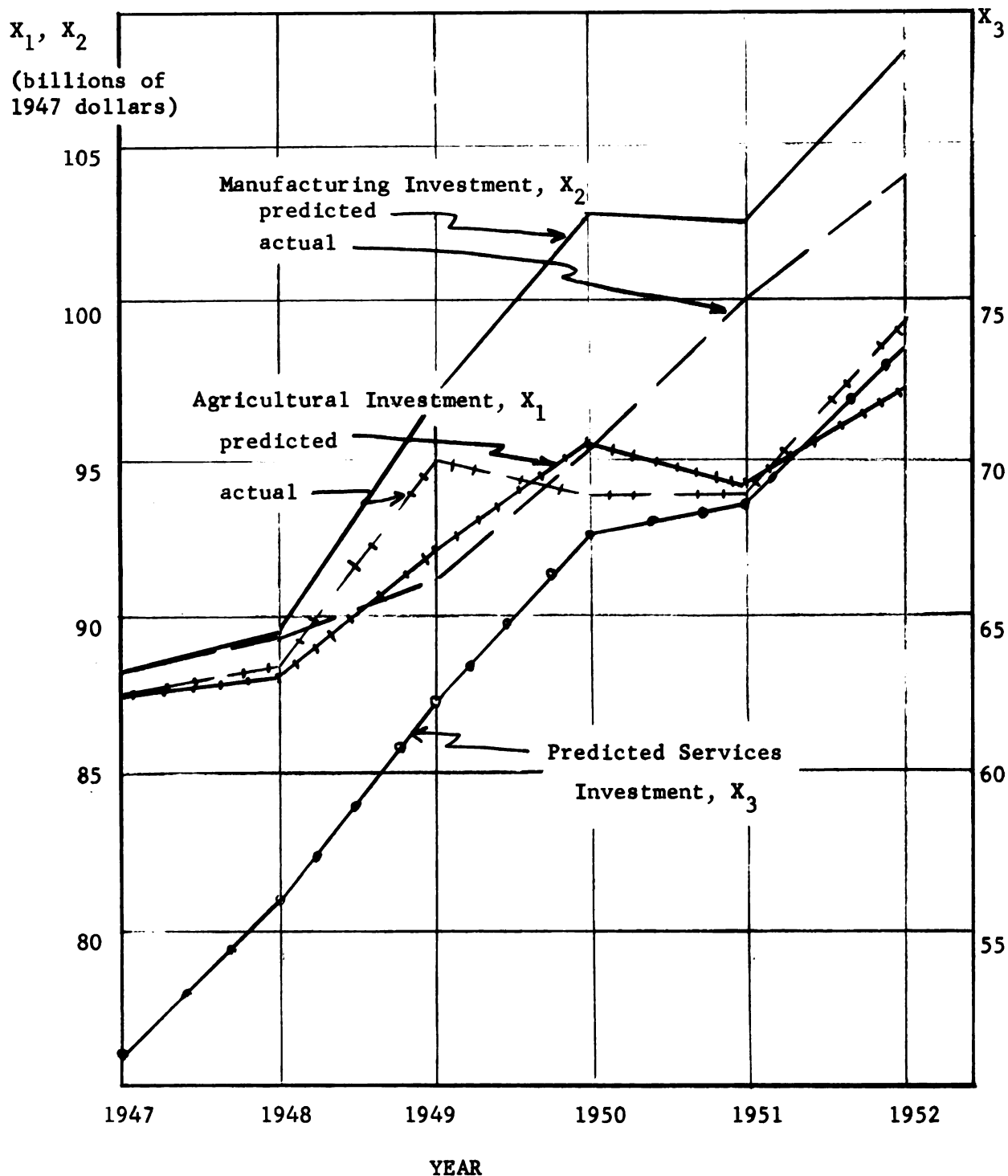


Figure III-2. Predicted and Actual Sector Investments for the United States Economy, 1947 - 1952

portion and the G.N.P. deflators of [30] to the remainder. The error of the model is small in this case, being $99.6 - 97.5 = 2.1$ billion low, or approximately 2 percent. This may well be due to the underestimation of the a_{21} or the σ_2 coefficients from 1947 data. At that early prewar stage, it is likely that farm machinery and equipment investment was at an artificially low level, due to its short supply so soon after the war. The 1952 total investment in farm machinery and equipment was more than 2.5 times the 1947 total investment [42], a very rapid rise which suggests that the σ_2 coefficient, in particular, may not have been constant over the period studied.

In any case, the solution results of the model confirm its reasonable accuracy in describing the workings of the economy. Undoubtedly, more work in this area is needed, in particular with more detailed models and for more recent time periods. The fact that a situation as complex as the national economy of the United States can be conceived of in systems terms, the components modeled with existing data, and that the dynamic model derived from rigorous application of physical systems modeling procedures yields good results when compared to actual figures, is considered to be one of the major contributions of this thesis.

IV. CONCLUSIONS

IV-1. Regarding the Park Systems Model

There now exists a logical, consistent quantitative framework for the analysis of spatial allocation of traffic flows, given a facilities configuration and specified origin flows. The requirements for component modeling make clear the sort of data which must be gathered in order to model the attendance patterns for other activities. Foremost, the origins of participants must be known. Facilities of the destination areas must be carefully considered as they pertain to the particular activity. The characteristics of the links may be considered fixed, but the model scaling constants, will, in general, assume new values. In all cases, the r.m.s. error criterion provides a suitable measure for evaluating the sensitivity of the solution to any parameter change which an analyst might wish to consider.

The existence of such a model for the Michigan state parks system will allow a considerable amount of experimenting with possible future configurations and parameters of the system. For instance, the Department of Conservation might wish to estimate the demand for and likely intensity of use of its state park camping facilities at some future date. They could use demographically determined population increases, with appropriate assumptions regarding leisure behavior, to scale up the present day origin flow drivers, and make estimates of the future highway link parameters based upon projected highway improvement programs.

The solutions found under these conditions could be examined

for "critical" areas; i.e., those in which the projected attendances were considerably above present capacity. They could thus select some criterion [45] for expansion of their facilities based upon indicated over-capacities. Presently used methods for determining expansion of facilities usually rely upon such things as population trends alone, and do not reflect the behavioral functioning of the system as a collection of three quite different components, of which population affects only one.

Future research might well apply the methods to some other geographical region to see if the model which works so well for the case of Michigan will also predict well in other circumstances. It must be remembered that the bi-peninsula landform of Michigan is unique in the United States, and this may have had some effect upon the model accuracy and consistency. However, the singular nature of the extensive water boundaries of Michigan can be considered to provide a more stringent test of a model of recreational travel [44], as opposed to a relatively homogeneous area with largely land boundaries, such as Iowa or Indiana.

Another area for future investigation is undoubtedly the extension of this modeling concept into other types of travel. The analysis of urban journey-to-work patterns is an obvious example. The modeling methods presented in this thesis can easily be used to solve for the flows on the highway links, in the same way that the flows in destination components are solved for. This would provide a one-step solution of two of the most important classical problems in traffic

analysis; that of distribution, the construction by a model of the destination flow values, given the origin flows, usually done by means of a gravity model; and that of assignment, the allocation of specific origin-destination flows to actual road links, usually done by a linear programming technique. In view of the many billions of dollars which our society expends on transportation facilities and the increasing pressures upon urban land space, this problem area is one of the most promising for new and advanced analytical techniques.

IV-2. Regarding the National Economic Model

It is fair to say that the model presented in this thesis is only the first very small step of many which must be taken before a comprehensive analytical model of the United States national economy is perfected. Certain progress is offered to the application of systems-type models in this area by the success of the relatively crude model discussed.

A logical extension of the immediate model framework would be to construct a detailed model which does not aggregate the data (in particular, the input-output matrix) so grossly. If validated on such a scale as a 50x50 economic model, this would surely provide a great impetus to further behavioral studies of components and to the more frequent gathering of basic input-output data.

A further, and most necessary step, is to include the effect of prices in the model. On the scale of aggregation of a three-sector

model, prices are probably meaningless in view of the great discrepancies in unit output values. However, on a truly inter-industry basis, price changes could be considered to cause substitution of one input for another, such as motor transport for rail transport, aluminum for steel, capital investment for labor input, etc. Since the possibility of substitution of inputs is not allowed by the fixed-coefficient concept of the Leontief input-output analysis, a new behaviorally-oriented framework of analysis may have to be developed.

APPENDIX A

VALUES OF THE ORIGIN AND LINK COMPONENTS OF THE MICHIGAN STATE PARK SYSTEM

A-1. The Origin Components

In connection with the Michigan Outdoor Recreation Demand Study, a set of approximately 296,000 I.B.M. data cards were prepared, one for each camper permit issued by Michigan State parks in 1964. These were used for many other analyses besides the origin and destination tabulations, such as length of stay studies, boat use by campers, type of camping equipment used, size of camping party, etc. For input to the park systems model, a listing was prepared showing the number of camper-days recorded for each origin considered in the study. These origin areas are described in the listing which follows. Counties in Michigan with less than 10,000 population were combined with neighboring counties to provide a reduction in the number of components in the system (to stay within the bounds of computer memory space) with a small sacrifice in accuracy. Nearby out-of-state origins were grouped as indicated in the listing.

There is a certain error introduced by the exclusion of out-of-state origins other than those given in the listing. The measure of this error can be judged from the fact that total camper-days recorded for the 55 parks considered was 3,050,000, whereas the total camper-days for the 77 origin areas considered was 2,840,000 - a difference of less than 6.9%. The areas not represented are:

North-eastern U.S.A.

Southern U.S.A.

Far West U.S.A.

Plains and Mountains U.S.A.

Iowa, Missouri, Kentucky, West Virginia

Canada, other than Ontario

Minnesota

Ontario

All of these origins with the exception of the latter two can be considered as causing a constant error of under-prediction, since their effect was nearly constant over the state. The latter two origins tend to become appreciable proportions of attendance (over 3%) at only a small number of parks in the Upper Peninsula, and are relatively constant and small elsewhere. The effect of these attendances is not included in the model results in Chapter II.

There was a consistent proportion of "unknown origin" cards in the whole number of approximately 17%, caused by omission or inadequacy of information on the original camper permit. To allow for this omission, it was postulated that the "unknown" campers were distributed with respect to origin in the same way as "known" campers, and the figures for each known origin were raised by 17% in each case.

The following is a description of the origin through driver components, with their values (the element and node numbers correspond to the numbering of Figure II-1):

<u>Origin Area i</u>	<u>Node #</u>	<u>Element #</u>	<u>Y_{oi}</u>
(country in Michigan, if not otherwise specified)	(geographical center of pop- ulation)	(reference point to node i)	(camper-days in 1964 cal- endar year)
1. Alcona and Oscoda	001	401	4,121
2. Alger	002	402	420
3. Allegan	003	403	17,383
4. Alpena and Montmorency	004	404	4,072
5. Antrim and Otsego	005	405	2,369
6. Arenac	006	406	911
7. Baraga	007	407	475
8. Barry	008	408	8,703
9. Bay	009	409	33,998
10. Benzie and Leelanau	010	410	2,066
11. Berrien	011	411	21,264
12. Branch	012	412	5,223
13. Calhoun	013	413	39,758
14. Cass	014	414	5,996
15. Cheboygan	016	416	1,719
16. Chippawa, Luce and Mackinac	017	417	3,331
17. Clare	018	418	3,153
18. Clinton	019	419	11,699
19. Crawford and Roscommon	020	420	2,364
20. Delta	021	421	4,443

21.	Dickinson	022	422	1,932
22.	Eaton	023	423	18,060
23.	Emmet and Charlevoix	024	424	2,517
24.	Genesee	025	425	157,352
25.	Gladwin	026	426	4,008
26.	Gogebic	027	427	795
27.	Grand Traverse and Kalkaska	028	428	8,108
28.	Gratiot	029	429	10,966
29.	Hillsdale	030	430	4,947
30.	Houghton and Keweenaw	031	431	1,901
31.	Huron	032	432	3,382
32.	Ingham	033	433	80,934
33.	Ionia	034	434	11,513
34.	Iosco and Ogemaw	035	435	4,599
35.	Iron	036	436	811
36.	Isabella	037	437	9,116
37.	Jackson	038	438	39,502
38.	Kalamazoo	039	439	53,605
39.	Kent	041	441	241,626
40.	Lapeer	044	444	12,802
41.	Lenawee	046	446	14,192
42.	Livingston	047	447	10,737
43.	Macomb	050	450	150,233
44.	Manistee	051	451	2,283
45.	Marquette	052	452	6,964

46.	Mason	053	453	3,337
47.	Mecosta	054	454	2,514
48.	Menominee	055	455	2,010
49.	Midland	056	456	36,140
50.	Monroe	058	458	26,159
51.	Montcalm	059	459	8,149
52.	Muskegon	061	461	41,729
53.	Newaygo	062	462	6,206
54.	Oakland	063	463	249,868
55.	Oceana	064	464	2,762
56.	Ontonagon	066	466	768
57.	Osceola and Lake	067	467	2,971
58.	Ottawa	070	470	67,123
59.	Presque Isle	071	471	1,669
60.	Saginaw	073	473	65,363
61.	St. Clair	074	474	30,485
62.	St. Joseph	075	475	5,996
63.	Sanilac	076	476	4,896
64.	Schoolcraft	077	477	739
65.	Shiawassee	078	478	21,545
66.	Tuscola	079	479	9,862
67.	Van Buren	080	480	5,794
68.	Washtenaw	081	481	44,195
69.	Wayne	082	482	512,035
70.	Wexford and Missaukee	083	483	3,955
71.	Chicago Illinois and Hammond-Gary Indiana	301	301	41,609

72. Rest of Illinois	301	306	149,526
73. South Bend, Indiana	302	302	13,565
74. Rest of Indiana	303	303	146,663
75. Toledo, Ohio	304	304	42,661
76. Rest of Ohio	305	305	228,770
77. Wisconsin	307	307	57,810

A-2. The Highway Links

The following list shows the values of estimated driving time and estimated direct cost of gasoline plus tolls for the highway links selected for the system study. The link numbers refer to the element numbers on the graph of Figure II-1.

<u>Link Number</u>	<u>Time</u>	<u>Direct Cost</u>
(from Figure II-1)	(estimated, in hours)	(estimated, in dollars)
501	0.533	0.600
502	0.711	0.800
503	0.925	0.925
504	0.844	0.950
505	1.075	1.075
506	0.225	0.225
507	1.263	1.200
508	0.825	0.825
509	0.120	0.075
510	0.711	0.800

511	1.524	1.600
512	0.267	0.300
513	1.260	1.575
514	0.938	1.125
515	0.575	0.575
516	0.578	1.650
517	0.750	0.900
518	0.666	0.750
519	0.625	0.625
520	0.657	0.625
521	0.500	0.625
522	1.375	1.375
523	0.600	0.750
524	0.553	0.525
525	1.400	1.575
526	1.119	1.175
527	1.100	1.100
528	0.225	0.225
529	0.750	0.750
530	0.820	1.025
531	1.160	1.450
532	0.300	0.375
533	0.750	0.750
534	1.000	0.750
535	0.800	0.800
536	0.480	0.600
537	0.875	1.000

538	0.889	0.750
539	0.200	0.250
540	0.700	1.050
541	0.067	1.200
542	0.120	5.150 (Mackinac Bridge)
543	0.400	0.300
544	0.417	0.625
545	0.300	0.375
546	0.356	0.400
547	0.250	0.250
548	0.167	0.250
549	0.600	0.900
550	0.571	0.500
551	0.450	0.450
552	0.533	0.600
553	0.889	1.000
554	0.100	0.075
555	0.560	0.700
556	0.428	0.375
557	0.778	0.875
558	1.750	1.750
559	0.500	0.500
560	0.800	0.900
561	0.143	0.125
562	0.700	0.700
563	0.450	0.450

564	0.300	0.300
565	0.450	0.450
566	0.250	0.250
567	0.933	1.050
568	0.200	0.175
569	0.920	1.150
570	0.167	0.125
571	0.933	1.050
572	0.200	0.300
573	0.200	0.300
574	0.277	0.450
575	0.375	0.375
576	0.369	0.600
577	0.143	0.125
578	0.300	0.375
579	0.060	0.050
580	0.800	0.900
581	0.800	0.900
582	0.250	0.250
583	0.711	0.800
584	0.440	0.550
585	0.900	1.125
586	0.500	0.625
587	0.200	0.200
588	0.367	0.550
589	0.533	0.800

590	0.183	0.275
591	0.940	1.175
595	0.600	0.675
593	0.325	0.325
594	0.550	0.550
595	0.800	0.900
596	0.400	0.450
597	0.250	0.250
598	0.600	0.600
599	0.809	0.850
600	0.250	0.250
601	0.620	0.775
602	0.250	0.250
603	0.300	0.375
604	0.320	0.400
605	0.175	0.175
606	0.600	0.750
607	0.333	0.375
608	0.333	0.375
609	0.333	0.375
610	0.720	0.900
611	0.333	0.375
612	0.850	1.275
613	0.327	0.450
614	0.450	0.675
615	0.383	0.575

616	0.750	0.750
617	0.500	0.500
618	0.460	0.575
619	0.250	0.375
620	0.222	0.250
621	0.488	0.550
622	0.333	0.500
623	0.416	0.625
624	0.667	0.750
625	0.644	0.725
626	0.538	0.875
627	0.578	0.650
628	0.300	0.300
629	0.650	0.650
630	0.521	0.625
631	0.366	0.550
632	0.555	0.525
633	0.825	0.825
634	0.550	0.550
635	0.711	0.800
636	0.583	0.875
637	0.500	0.750
638	0.400	0.500
639	0.633	0.950
640	0.400	0.500
641	0.581	0.800
642	0.880	1.100

643	0.333	0.500
644	0.217	0.325
645	0.755	0.850
646	0.267	0.300
647	0.400	0.500
648	1.125	1.125
649	0.200	0.200
650	0.444	0.500
651	0.778	0.875
652	0.952	1.000
653	0.250	0.250
654	1.000	1.125
655	1.444	1.625
656	0.500	0.500
657	0.636	0.875
658	0.440	0.550
659	0.489	0.550
660	0.450	0.675
661	0.517	0.775
662	0.625	0.625
663	0.300	0.300
664	0.375	0.375
665	0.330	0.500
666	0.625	0.625
667	0.857	0.900
668	0.375	0.375
669	0.675	0.675
670	0.350	0.525

671	0.375	0.375
672	0.200	0.200
673	0.375	0.375
674	0.400	0.600
675	0.109	0.150
676	0.100	0.150
677	0.240	0.300
678	0.133	0.200
679	0.433	0.625
680	0.333	0.500
681	0.291	0.400
682	0.440	0.550
683	0.222	0.250
684	0.625	0.625
685	0.727	1.000
686	0.750	0.750
687	0.675	0.675
688	0.500	0.500
689	0.625	0.625
690	0.667	0.750
691	0.133	0.200
692	0.500	0.500
693	0.375	0.375
694	0.260	0.325
695	0.667	0.750
696	0.720	0.900
697	0.760	0.950

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698	0.622	0.700
699	0.428	0.375
801	3.000	3.750
802	0.400	0.500
803	0.818	1.125
804	3.777	4.250
805	1.000	1.125
806	1.778	2.000
807	1.454	3.350
808	1.833	2.750
809	4.889	5.550

APPENDIX B

RESULTS OF THE FACTOR ANALYSIS AND ATTRACTION INDEX CONSTRUCTION FOR THE PARKS

The descriptions of the 38 variables representing facilities and attributes of each of the 55 parks, along with their mean values (with 55 observations of each) were as follows:

<u>Variable</u>	<u>Mean Score</u>
1. Rolling terrain	.5636
2. Mountainous or hilly terrain	.4545
3. Evergreen vegetation	.0909
4. Deciduous vegetation	.4364
5. Mixed vegetation	.4182
6. Barren terrain	.0727
7. Virgin timber stand	.0364
8. Wilderness areas	.0727
9. Shade for over 50% of campsites	.7636
10. Cliffs and overlooks	.2545
11. Waterfalls	.0909
12. Springs	.1636
13. Great lakes shoreline	.5455
14. Inland lake shoreline	.6182
15. River frontage	.4365
16. Historical site	.1636

17.	Contemporary interest site	.0909
18.	Interpretive program	.4727
19.	Hiking trails	.4545
20.	Swimming, great lakes	.4545
21.	Swimming, inland lakes	.4545
22.	Sand beach, great lakes	.3818
23.	Sand beach, inland lakes	.4364
24.	Boat launching facility	.7636
25.	Water skiing area	.4545
26.	Fishing	.8000
27.	Store within 1 mile	.4545
28.	Showers	.7273
29.	Flush toilets	.8364
30.	Laundry facilities	.7091
31.	Electricity service	.8545
32.	Pier	.0727
33.	Boat rental within 1 mile	.2727
34.	Horse rental within 1 mile	.0364
35.	Bathhouse	.5273
36.	Lifeguard	.5091
37.	Sports playground area	.6909
38.	Sand dunes	.0909

The eigenvalues of the matrix of intercorrelations observed between these 38 variables in the 55 cases were as follows (listed in order of size):

7.5276,	5.0503,	3.8350,	2.4281,
2.3485,	1.9994,	1.8394,	1.3594,
1.2372,	1.1232,	1.0836,	0.9453,
0.8689,	0.8479,	0.6842,	0.6676,
0.5462,	0.4774,	0.4216,	0.3484,
0.3213,	0.3179,	0.2957,	0.2253,
0.2136,	0.1932,	0.1509,	0.1459,
0.1127,	0.0979,	0.0755,	0.0666,
0.0483,	0.0320	0.0279,	0.0205,
0.0085,	0.0069		

The factor analysis solution containing three varimax rotated factors was accepted for computation of the park attraction indices, A_k . The rotated factor loadings are given below for the 38 variables. The proportions of variance of the observed correlation matrix, R , were for factor 1, .1768; for factor 2, .1319; for factor 3, .1232; making a total of .4319, which is considered to be reasonably high for studies of this type with such diversified data observations^{*}. The underlined values were the loadings used in the computation of the indices:

	<u>Variable</u>	<u>Factor 1 Loading</u>	<u>Factor 2 Loading</u>	<u>Factor 3 Loading</u>
1.	Rolling terrain	-.2662	.3540	-.1566
2.	Mountainous terrain	-.1819	.3728	-.1497
3.	Evergreens	.1738	.1156	.2147
4.	Deciduous	-.2960	-.1688	-.3831
5.	Mixed vegetation	.0767	.2841	.4108
6.	Barren	.2942	-.3591	.1582

^{*}Private discussion of the author with Mr. Thomas Danbury, Department of Communications, Michigan State University, Dec. 4, 1964

7. Virgin timber	.1228	<u>.5155</u>	.0744
8. Wilderness	.2538	<u>.6502</u>	.1602
9. Shaded camps	-.1354	.3635	.1818
10. Cliffs, overlooks	-.0688	.4504	-.0184
11. Waterfalls	.1031	<u>.6895</u>	.3183
12. Springs	.2947	<u>.5799</u>	.1601
13. Great lakes shoreline	<u>.8381</u>	.0256	.1517
14. Inland lake shoreline	-. <u>7573</u>	.1774	-.0711
15. River frontage	.0253	.5299	-.1644
16. Historical site	.1258	<u>.6707</u>	.0378
17. Contemporary interest	.1852	.0483	.0592
18. Interpretive program	-.1203	<u>.6579</u>	-.1077
19. Hiking trails	-.1522	<u>.6229</u>	-.2146
20. Swimming, great lake	<u>.8079</u>	-.0355	.2572
21. Swimming, inland lake	-. <u>9304</u>	-.0345	-.1762
22. Beach, great lake	<u>.6762</u>	-.1362	.3259
23. Beach, inland lake	-. <u>9102</u>	-.0340	-.1934
24. Boat launching	-.3911	.4728	-.1293
25. Water skiing	-. <u>8448</u>	.1380	-.0206
26. Fishing	-. <u>5229</u>	.1987	.0950
27. Store	-.1255	.0150	.5709
28. Showers	.0669	-.2354	.8381
29. Flush toilets	.0454	-.0976	<u>.8135</u>
30. Laundry	.1186	-.2272	<u>.8301</u>
31. Electricity	.0843	-.0901	<u>.7910</u>
32. Pier	.2416	-2741	.1932

33. Boat rental	- . <u>5216</u>	.0310	.2510
34. Horse rental	- .1933	.0870	- .3371
35. Bath house	- .3938	- .3351	.3021
36. Lifeguard	- . <u>5569</u>	- .3148	.2712
37. Sports grounds	- .1365	- .2709	.2295
38. Sand dunes	.0237	.2145	.2091

The four-factor solution was not as satisfactory. The fourth factor was loaded highly only by the variables for mixed vegetation, sand dunes, and rolling terrain; a set which amounts to perhaps a "mild scenery" factor. However, the increase in total variance of R represented by the four factors was only .0639 over the three-factor solution. Thus, the fourth factor contributes barely half as much to the "explanation" of the variances as did the third.

The following are the normalized factor scores of the parks, computed from relations II-4a, II-4b, II-4c, and II-4d. The attraction indices computed using relation II-3b are also given. In each column a value of unity represents an "average" park, in the sense that unity is the score computed using the mean values of the variables for the 55 parks.

<u>Park</u>	<u>$Q_1/\bar{Q}_1$</u>	<u>$Q_2/\bar{Q}_2$</u>	<u>$Q_3/\bar{Q}_3$</u>	<u>A</u>
1. Baraga	.796	.483	1.206	0.48
2. Brimley	.796	.643	1.206	1.03
3. Fayette	.796	1.535	0.0	0.14
4. Fort Wilkins	.828	2.537	1.272	0.56

5.	Gogebic Lake	1.250	.834	1.206	1.16
6.	Indian Lake	1.106	1.658	1.396	1.18
7.	McLain	.668	0.0	.719	0.30
8.	Muskallonge Lake	1.104	.662	0.0	0.54
9.	Porcupine Mountains	.817	3.362	.530	0.82
10.	Straits	.608	1.041	1.272	0.74
11.	Tahquamenon Falls	.940	2.569	1.272	1.65
12.	Van Riper	1.398	1.642	.843	1.54
13.	Wells	.796	1.658	1.396	0.95
14.	Aloha	1.254	.285	1.083	1.75
15.	Bay City	1.005	1.007	1.396	1.90
16.	Burt Lake	1.398	.965	1.396	1.70
17.	D.H. Day	.668	.197	0.0	0.25
18.	East Tawas	1.088	0.0	1.206	0.97
19.	Harrisville	.647	.475	1.083	0.96
20.	Hartwick Pines	.359	2.460	1.272	0.25
21.	Higgins Lake	1.398	.285	1.396	3.91
22.	Hoeffft	.796	1.304	1.206	0.67
23.	Interlochen	1.398	.475	1.396	3.35
24.	Ludington	2.045	2.380	1.272	3.04
25.	Mears	.944	0.0	1.396	0.47
26.	Mitchell	1.254	.285	1.206	1.08
27.	Muskegon	1.348	2.441	1.396	1.82
28.	Onaway	.853	1.012	1.206	0.50
29.	Orchard Beach	.647	.190	1.083	0.72
30.	Otsego Lake	1.348	.285	1.396	1.26

31.	Silver Lake	1.631	.285	1.083	1.37
32.	Traverse City	1.032	.285	1.083	1.67
33.	Wilderness	.796	2.392	1.083	1.35
34.	Wilson	1.018	0.0	1.396	0.81
35.	Young	1.162	.475	1.396	0.79
36.	Benzie	.382	.936	0.0	0.24
37.	Gladwin	.149	.293	1.083	0.17
38.	White Cloud	.149	.490	1.083	0.13
39.	Algonac	.382	.936	.530	0.63
40.	Brighton	1.348	1.495	.313	1.24
41.	Grand Haven	.944	.190	1.396	1.27
42.	Hayes	1.348	1.041	1.396	1.47
43.	Highland	1.106	1.399	.109	0.18
44.	Holland	1.015	0.0	1.396	2.03
45.	Holly	1.254	1.495	.124	0.99
46.	Island Lake	1.348	1.519	.124	0.55
47.	Lakeport	.795	.285	1.272	1.21
48.	Metamora	.869	.475	1.083	0.70
49.	Pinckney	1.348	1.692	.763	2.01
50.	Port Crescent	.647	.578	1.083	0.78
51.	Proud Lake	1.106	1.692	0.0	0.57
52.	Sleeper	1.149	.727	1.396	1.87
53.	Warren Dunes	1.005	.858	1.396	0.85
54.	Waterloo	1.254	1.692	.530	1.72
55.	Yankee Springs	1.348	1.012	.932	1.60

APPENDIX C

PROGRAM PRKSYS

The computer program given at the end of this appendix was used to construct the branch equation matrix for the system of parks and then to solve for the park through variables. In addition to the program, the following data cards were used:

1 card, constants k_1 , k_2 , k_3 , k_4

55 cards, park attraction indices

208 cards, time and cost values for highway links

145 cards, a list of passive elements in each fundamental cutset, with signs; first entry number of elements in cutset for control of DO loops.

145 cards, a list of origin camper-day entries in Y vector

55 cards, a list of 1964 park attendances in camper-days

The routine to construct the G matrix was prepared with the assistance of Mr. C. Hart. At first, a simple Gauss-Seidel method [43] was used to solve for $G^{-1} Y$. This was found to introduce considerable numerical error in the last 35 parks, and a more accurate matrix inversion routine SUBROUTINE MATINV, was adopted from the Computer Center Library. This routine was more accurate since it searched in each case for the maximum diagonal entry as pivot element, and transposed rows and columns accordingly.

Two measures of numerical accuracy were used, both rule-

of-thumb, since it was not feasible to compute GG^{-1} and compare it to the unit matrix of order 145 due to the size of the matrices involved. The first check was the size of the determinant of G. This was typically in the order of 10^{60} for the constants chosen, a large value which precludes the likelihood of calling for a division by a small number. The second check was the application of the cut-set postulate at the reference point of the graph. Here, all origins "leave" and all parks "enter", and thus the attendances predicted should equal the total of the origins entered. The total origins entered was 2,863,227. The total attendance predicted varied between 2,806,000 and 2,820,000 approximately, giving a possible error of from 1.5 to 2.0 percent. While it is not possible to state conclusively that the numerical error of any particular entry of the inverse or in any particular attendance computation is this low, it seems reasonable to expect any such error to be within, say, 3 to 4 percent.

The arrays and routines called in the program could not be accommodated in the 32,768 word memory of the CDC 3600 computer in the presence of either the SCOPE control unit programs or the normal full FORTRAN compiler. Thus, at the suggestion of Mr. P. Bintner of the Computer Center Staff, the FORTRAN compilation was called by program cards (*C job card) to be done to tape and then from tape to core storage. This enabled the use of more of the memory for the PRKSYS arrays and routines than is normally available with an * job card, which compiles FORTBIN to core. Average running time on the CDC 3600 computer was five minutes and forty-eight seconds.

```
*C550098 ELLIS
FORTBIN,C,L,4,P,P,P.
  PROGRAM PRKSYS
    DIMENSION G (145,146),KUTSET(145,16),E(263),CONS(4),ATT(55),DIF(55)
    1,PCT(55)
    COMMON G,KUTSET
1000 FORMAT (11X,F4.1,2X,F3.2)
1001 FORMAT(15X,2F4.3)
1003 FORMAT(11X,1614)
1004 FORMAT(15X,F8.0)
2000 FORMAT(6H PARK,I2,14H ATTRACTION = ,F7.4,14H ATTENDANCE = ,F10.0,
  110H ERROR IS ,F10.0,11H PCT OUT = ,F5.0/)
1005 FORMAT(15X,4F5.0)
  READ 1005, (CONS(I) ,I=1,4)
  PRINT 3000,(CONS(I),I=1,4)
3000 FORMAT(15H CONSTANTS ARE ,4F7.3///)
9001 FORMAT(15H1ERROR ON CARD ,I3,27H OF CUTSET. ELEMENT INDEX =,I2,
  1 9H IS ZERO.///)
9002 FORMAT(16H1ERROR ON CARDS ,I3,5H AND ,I3,31H OF CUTSET. SIGNS DO N
  10T AGREE.///)
C    READ MATRIX E FROM CARDS
    DO 10 I = 1,55
      READ 1000,ATTR,CSR
    10 E(I)=CONS(1)*ATTR*CSR
      PRINT 3001,(I,E(I),I=1,55)
3001 FORMAT (20H ATTRACTION OF PARK ,I2,4H IS ,F7.4/)
    DO 11 I = 56,263
      READ 1001, T, C
    11 E(I) = CONS(2)/((T+CONS(3)*C)**CONS(4))
      PRINT 4000,(I,E(I),I=56,263)
4000 FORMAT(6H LINK ,I3,10H VALUE IS ,F7.4/)
C    READ IN CUTSETS
    READ 1003,((KUTSET(I,J),J=1,16),I=1,145)
C    BUILD DIAGONAL ELEMENT OF MATRIX G FROM ARRAY E
    DO 40 I = 1,145
      DO 20 J = 1,145
    20 G(I,J) = 0.
      NI= KUTSET(I,1)+1
      DO 23 J = 2, NI
        M = KUTSET(I,J)
        IF(M)21,991,22
    21 G(I,I) = G(I,I)+E(-M)
        GO TO 23
    22 G(I,I) = G(I,I) + E(M)
    23 CONTINUE
C    BUILT OFF-DIAGONAL ELEMENTS OF MATRIX - UPPER TRIANGLE ONLY
    JJ = I+1
    DO 40 J = JJ, 145
      N2 = KUTSET(J,1)+1
      KSW = 0
      DO 40 K = 2, NI
        DO 40 L = 2, N2
          IF(KUTSET(I,K)-KUTSET(J,L))30,32,30
```

```
30 IF(KUTSET(I,K)+KUTSET(J,L))40,32,40
32 KPROD1 = KUTSET(I,K)*KUTSET(J,L)
   IF(KSW)33,34,33
33 IF(KPROD1*KPROD)992,992,35
34 KSW = 1
   KPROD=KPROD1
35 M = KUTSET(I,K)
   IF(M)36,37,37
36 M = -M
37 IF(KPROD)38,39,39
38 G(I,J) = G(I,J) - E(M)
   GO TO 40
39 G(I,J) = G(I,J)+E(M)
40 CONTINUE
C      FILL IN LOWER TRIANGLE OF MATRIX G
   DO 41 I = 1, 145
   N1= I+1
   DO 41 J = N1,145
41 G(J,I) = G(I,J)
   PRINT 2001,((G(I,J),J=1,5),I=1,5)
2001 FORMAT (5(5E17.10/))
   READ 1004, (G(I,146),I=1,145)
   CALL MATINV (G,145,G(1,146),1,DETERM)
   PRINT 2001,((G(I,J),J=1,5),I=1,5)
C      PREMULTIPLY BY MATRIX D
   SUM = 0.
   DISC = 0.
   PCTO = 0.
   DO 70 I=1,55
70 DIF(I)=0.
   DO 60 I = 1,55
   READ 6000, ATT(I)
6000 FORMAT(15X,F8.0)
   G(I,146)=G(I,146)*E(I)
   SUM = SUM+G(I,146)
   DIF(I)=G(I,146)- ATT(I)
   PCT(I) = 100.*DIF(I)/ATT(I)
   DISC = DISC + DIF(I)
   PCTO = PCTO + PCT(I)
60 PRINT 2000, I , E(I), G(I,146), DIF(I) ,PCT(I)
   PRINT 5000, SUM , DISC
5000 FORMAT(7H SUM = ,2(E20.10)///)
   PRINT 7000, PCTO
7000 FORMAT(8H PCTO = ,F8.0//)
   PRINT 2002,DETERM
2002 FORMAT (12HODETERMINANT,,E20.10)
   GO TO 999
C      ERROR ROUTINES
991 PRINT 9001, I, J
   TO TO 999
992 PRINT 9002, I, J
999 CONTINUE
END
```

```
      SUBROUTINE MATINV(A,N,B,M,DETERM)
      DIMENSION A(145,145),B(145,1),IPIVOT(145),INDEX(145,2),PIVOT(145)
C      INITIALIZATION
      10 DETERM=1.0
      15 DO 20 J=1,N
      20 IPIVOT(J)=0
      30 DO 550 I=1,N
C      SEARCH FOR PIVOT ELEMENT
      40 AMAX=0.0
      45 DO 105 J=1,N
      50 IF (IPIVOT(J)-1) 60, 105, 60
      60 DO 100 K=1,N
      70 IF (IPIVOT(K)-1) 80, 100, 740
      80 IF (ABSF(AMAX)-ABSF(A(J,K))) 85, 100, 100
      85 IROW=J
      90 ICOLUM=K
      95 AMAX=A(J,K)
      100 CONTINUE
      105 CONTINUE
      110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C      INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
      130 IF (IROW- ICOLUM) 140, 260, 140
      140 DETERM=- DETERM
      150 DO 200 L=1,N
      160 SWAP=A(IROW,L)
      170 A(IROW,L)=A(ICOLUM,L)
      200 A(ICOLUM,L)=SWAP
      205 IF(M) 260, 260, 210
      210 DO 250 L=1, M
      220 SWAP=B(IROW,L)
      230 B(IROW,L)=B(ICOLUM,L)
      250 B(ICOLUM,L)=SWAP
      260 INDEX(I,1)=IROW
      270 INDEX(I,2)=ICOLUM
      310 PIVOT(I)=A(ICOLUM, ICOLUM)
      320 DETERM=DETERM*PIVOT(I)
C      DIVIDE PIVOT ROW BY PIVOT ELEMENT
      330 A(ICOLUM, ICOLUM)=1.0
      340 DO 350 L=1,N
      350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
      355 IF(M) 380, 380, 360
      360 DO 370 L=1,M
      370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)
C      REDUCE NON- PIVOT ROWS
      380 DO 550 L1=1,N
      390 IF (L1- ICOLUM) 400, 550, 400
      400 T=A(L1, ICOLUM)
      420 A(L1, ICOLUM)=0.0
      430 DO 450 L=1,N
      450 A(L1,L)=A(L1,L)- A(ICOLUM,L)*T
      455 IF(M) 550, 550, 460
      460 DO 500 L=1,M
      500 B(L1,L)=B(L1,L)- B(ICOLUM,L)*T
```

```
550 CONTINUE
C    INTERCHANGE COLUMNS
600 DO 710 I=1,N
610 L=N+1- I
620 IF (INDEX(L,1)- INDEX(L,2)) 630, 710, 630
630 JROW=INDEX(L,1)
640 JCOLUM=INDEX(L,2)
650 DO 705 K=1,N
660 SWAP=A(K,JROW)
670 A(K,JROW)=A(K,JCOLUM)
700 A(K,JCOLUM)=SWAP
705 CONTINUE
710 CONTINUE
740 RETURN
750 END
    END
    (BLANK)
CALL,4.
RUN.
```

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