THE ANALYSIS OF THE FRICTIONAL CHARACTERISTICS OF AND FORCES APPLIED TO STAIRWAY TREADS

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THE ANALYSIS OF THE FRICTIONAL CHARACTERISTICS OF AND FORCES APPLIED TO STAIRWAY TREADS

By

Larry Johnson Segerlind

AN ABSTRACT

Submitted to
Michigan State University
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Although the stairway has been used for many years it is still not a safe means of travel. In 1959 nearly 700 farm people died as a result of falls which occurred on steps and stairways. To reduce the injuries and deaths resulting from these falls an analysis of some of the factors affecting stairway safety was performed.

The investigation had two objectives. The first was to evaluate the coefficient of friction of different shoe sole and stair tread materials. The second was to evaluate and analyze the forces exerted by individuals when ascending and descending stairways of different designs.

The coefficient of friction was measured on a machine having a table which moved under a stationary holder. The holder was fastened to two vertical bars which contained strain gages. The strain gage bridge was connected to a Brush amplifier and inking oscillograph. The vertical load was applied by placing weights at the end of an arm which rested on the holder. The tread material was placed on the table and moved under the holder which contained the sole material. The horizontal force was recorded on the oscillograph.

New and worn specimens of six different tread materials and six different sole materials were used. The different sole materials were duplicated for two different sole sizes. The tread materials used were wood, linoleum, a non skid type paint, abrasive strip, varnish and rubber mat. The sole materials used were neoprene, neolite, crepe, leather, ripple and a hard surface sole made by B.F. Goodrich.

The investigation was designed so that it could be analyzed statistically and the order in which measurements were made was determined by randomization. All the measurements were made at one table speed and one vertical force.

The magnitude, direction and point of application of the forces exerted by a person ascending and descending a stairway were measured using a force plate. The force plate contains five separate strain gage circuits and is capable of measuring the vertical force, both horizontal forces and the point of application of the forces with respect to the front edge and side of the tread. The output of each circuit was recorded on an inking oscillograph.

To compare stairway designs nine different stairways were constructed. Stairways with 9, 10 and 11 inch treads were built for each of three riser heights, 6, 7 and 8 inches.

Two subjects, weighing 140 and 172 pounds, were used in the investigation. Each subject ascended and descended each stairway four times while traveling at his own natural speed.

The coefficient of friction for the abrasive strip and rubber mat materials was higher than the coefficient of friction of the other tread materials for most all of the soles investigated. Wood, varnish and paint generally showed a decrease in the coefficient of friction with use while linoleum increased with use. The ripple sole had the highest coefficient of friction values for any of the sole materials studied. The crepe sole was the only sole which had a higher coefficient of friction value on the new material than on the worn material.

The vertical force exerted on a tread had two maximum values when either ascending or descending the stairways. The initial rate of application of the vertical force varied linearly with time when ascending or descending the stairways.

The horizontal force varied considerably between both the stairways and the two subjects. The horizontal force was generally directed toward the front edge of the tread at the beginning of the step, when either ascending or descending the stairways.

The forces exerted by a person when ascending or descending a stairway normally are not large enough to cause slipping.

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INTRODUCTION

Ever since man has constructed buildings of more than one story he has been confronted with the problem of traveling from one floor to another. He has met this problem in several ways. He has built stairways, elevators and escalators. While elevators and escalators have received widespread usage in large commercial structures the stairway still remains the method used in his home.

Although the stairway has been used for many years man still has not made it a safe means of traveling from floor to floor. In 1959 more than 2200 farm people died as a result of falls. About a third of these accidents occurred on steps and stairways (National Safety Council 1959). A survey of 100 stairway accidents by Miller (12) showed that 38% of the accidents were caused by slipping. Miller also concluded that the non uniform dimensions of treads and risers was greater on the accident stairways than for a representative group of stairways reported by Velz (17).

Three factors must be considered in the analysis of stairway safety: 1) the tread material and its friction characteristics, 2) the actual forces exerted on the stair tread as a person ascends and descends the stairs, and 3) the point of application of the forces in relation to the front edge of the stair tread. The type of shoe sole may be considered a fourth factor; however, it is uncontrollable from the standpoint of stairway design.

Since tread materials are subject to wear, a high coefficient of friction under both new and used conditions is desirable. A tread material with a high coefficient of friction for a wide selection of shoe sole materials is also desirable. The measurement of the forces exerted by the foot of a person is necessary to determine how high the friction value of a tread has to be to prevent slipping. The forces exerted and their point of application may depend somewhat on the dimensions of the stairway. The wide variation in stairway dimensions shown by Miller (12) and Weaver and Pogue (21) suggest that no uniform construction pattern for stairways exists. While sets of design equations and specifications are available, they vary between "authorities".

The objective of this thesis is two fold. The first is to evaluate the friction characteristics of various shoe sole and stair tread materials. The second is to evaluate and analyze the forces exerted by individuals ascending and descending stairs of different designs.

REVIEW OF LITERATURE

Stairway Accidents

A survey of 100 stairway accidents by Miller (12) showed that most accidents happened in the mid-morning, mid-afternoon and early evening periods. The greatest number of accidents occurred around 10 A.M.. The fact that 81% of the accidents occurred to women and time of occurance suggests that many of the accidents occur during the process of doing household work. The rate of falls, however, did not increase with the person's age. Presumably older people were more cautious or decreased their use of the stairs.

Slipping was found to be the direct cause of 38% of the accidents. Missing a step was responsible for 18% while loosing balance accounted for 16%. Hurrying and having the arms full while using the stairs were contributing factors in 61% of the accidents. Twenty-four per cent of the falls caused by slipping occurred on rubber mats. Linoleum accounted for another 20%, varnish 18%, paint 16%, carpet 14% and 4% occurred on bare wood.

Miller (12) also found a wide variation in tread materials being used. The types of finishes for 135 stairways were as follow:

33	Paint
26	Rubber Mat
18	Varnish
16	Full length carpet
10	Bare wood
6	Other
13	Linoleum with no metal edge
7	Linoleum with edge type metal edge
6	Linoleum with surface type metal edge

Physiological Responses of Women

Weaver (20) investigated the physiological responses of women while ascending and descending stairways of different designs. Three stairways were constructed; one with a 33° slope and the other two with a 40° slope. Stairway #1, considered architecturally correct, had 7 inch risers and 11 inch treads. Stairway #2 had 7 inch risers and 9 inch treads while stairway #3 had 9 inch risers and 11 inch treads. Ten women subjects were observed ascending and descending the stairs at a rate of 2.2 m.p.h..

A statistical analysis of the investigation showed no significant difference between the three stairways. In ascending, the stairs with a 40° slope (2&3) used less energy but an increase in the heart rate and pulse pressure occurred. In descending, the stairs with a 40° slope used more energy and increased both the heart rate and pulse pressure.

Stairway Design

An attempt to improve the safety of stairways should include an analysis of stairway design. There is no definite set of dimensions for designing a stairway. In a survey of housing preferences by Weaver and Pogue (21) a wide variation in stairways was disclosed. The slope of the stairways varied from 31 to 47 degrees. The treads varied from 7 to 12.75 inches while the risers varied from 5.75 to 9.75 inches.

Parker, Gay and MacGuire (13) suggest that a good riser height is from 7 to 7.5 inches. They give as a set of design equations the following:

1) $T + R = 17''$ to $17.5''$
2) $T + 2R = 25''$ for interior stairs
3) $T \times R - 70$ in. to 75 in.

T is the tread width (less nosing) and R is the riser height.

Merrit (11) suggests that the second equation above should be $T + 2R = 24''$ to $25''$. The Forest Service of the U.S.D.A. (7) suggest that the first equation should be dropped and the last equation should read $T \times R = 75$ in.

The Forest Service states:

> There is a definite relation between the height of a riser and the width of a tread, and all stairs should be laid out to conform to well established rules governing these relations. If the combination of run to rise is too great, the steps are tiring; and if too short, the foot may kick the riser at each step and an attempt to shorten the stride may be tiring. Experience has proved that a riser 7 to 7½ inches high with appropriate tread combines both comfort and safety, and these limits therefore determine the standard height of risers commonly used for principal stairs. As the height of the riser is increased, the width of the tread must be decreased for comfortable results.

Coefficient of Friction of Stairway Materials

Hunter (10) reported that measurements on new or unworn specimens of walkway materials gave little indication of the true value of u (coefficient of friction). He states that test surfaces should be standardized with respect to smoothness, cleanliness and dryness to get reproducable results. However, he also states that there seems to be no possibility of selecting a single or even a limited number of surface conditions which can be defined or accurately produced. This is because the wear on a tread or walkway material will vary with its location in a building, thus making possible many different values of u for a single sole and tread combination.

Sigler, Geib and Boone (16) have conducted friction tests on a large number of materials. A pendulum type apparatus holding a test heel was used. The test method duplicated the way a heel hits a level walkway surface. Leather and rubber heels were tested in the experiment. In all tests on dry surfaces they found that the rubber heel gave a higher coefficient of friction than the leather heel. However, on wet surfaces both heels had low coefficients and could be classified as potential hazards. The only wet surfaces which gave good results were those containing asperities that projected through the film of water. The asperities prevented the water from acting as a lubricant.

A series of tests on waxed surfaces was also carried out. It was found that the coefficient of friction of a waxed surface was higher than that of a clean surface for a dry rubber heel. However, for the leather heel and both heels under wet conditions a decrease in the coefficient occurred on the waxed surfaces. Under continual use it took approximately one month to wear the wax off and return the coefficient of friction to that of a clean surface.

Friction of Non-Metals

The friction between shoe soles and stairway treads is the result of the contact of two elastic materials. Bowden and Tabor (2) report that for elastic materials the basic laws of friction do not apply. Instead of u being proportional it is inversely proportional to the weight applied. The coefficient of friction decreases as the weight is increased.

Bowden and Tabor (2) stated that all surfaces are rough on an atomic scale. As two materials are placed on one another the only contact occurs at the tips of the asperities. The actual contact area is very small making the resulting pressures extremely high. Over regions of intimate contact strong adhesion occurs and the two materials become a continuous body. The friction force is the force required to shear this junction. With metalic materials the area of true contact is proportional to the weight applied. However, for elastic materials the true area of contact can depend on the geometry of the surface, the load or the time of loading. For elastic materials

$$A \propto W^n$$

where $n < 1$ and A is the true contact area.

While the area of contact and weight play an important role in the value of u, Bowden and Tabor (2) state that the most important factor is the cleanliness of the surface. They point out that surface cleanliness is far more important than surface roughness. Gomer and Smith (9) show that the presence of certain types of films including water vapor will cause a decrease in the coefficient of friction. This decrease is due to a very thin film which is formed on the surface.

The physical adsorption of water vapor on clean surfaces amounts to only one or two molecular layers at the saturation pressure. However, a suitable trace of contamination will lead to the presence of large quantities of water on the surfaces at pressures well below saturation.

Germant (8) has given a qualitative picture of this boundary lubrication. Figure 1 shows two surfaces in contact, each covered with a thin film two molecules deep. When the lower surface is moved, layer 1 and layer 4 will still adhere to their respective surfaces. However, the forces of attraction between subsequent layers become weaker as their distance from the surface increases. There will be moderate slippage between layers 1 and 2 and between layers 3 and 4. A considerable amount of slippage will take place between layers 2 and 3.

The formation of the surface layers depends upon the attraction between the liquid and solid, as measured by the adhesion tension. The adhesion tension is particularly high for molecules containing polar groups such as OH and COOH.

Germant (8) also reported that where friction tests have been conducted the coefficient of friction drops according to a curve indicated in Figure 2. A definite

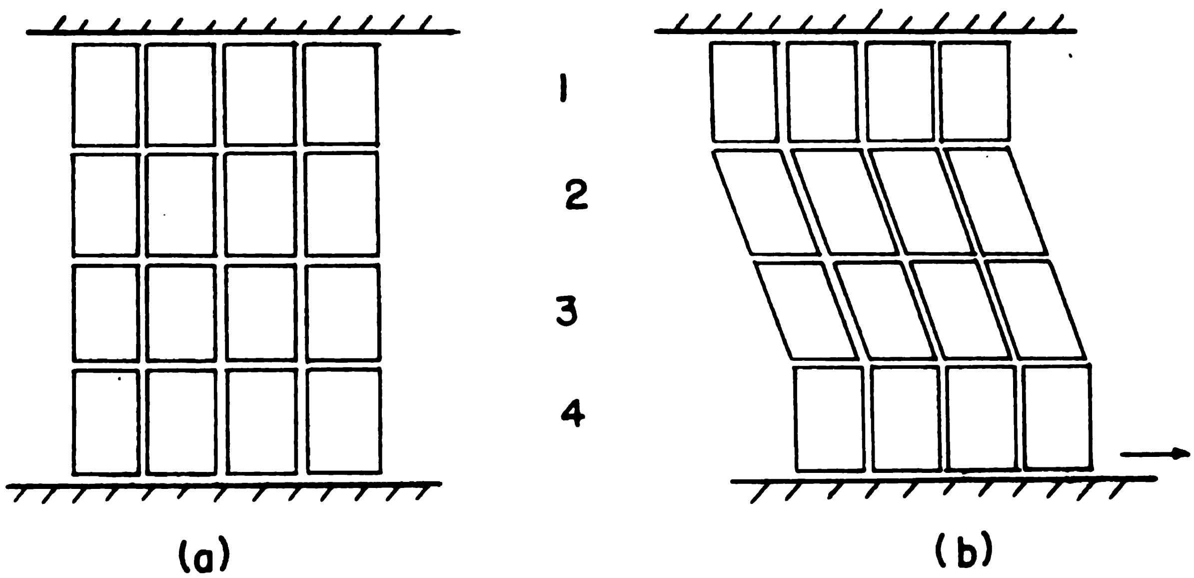


Figure 1: A schematic picture of molecules in boundary lubrication, a) surface at rest, b) surfaces in motion. Germant (8)

amount of lubrication is required to cause a change. When this has been reached the friction value dimishes rapidly and later slowly with an increasing thickness of the layer. It is believed that the point of sharp drop occurs after the first layer has been formed.

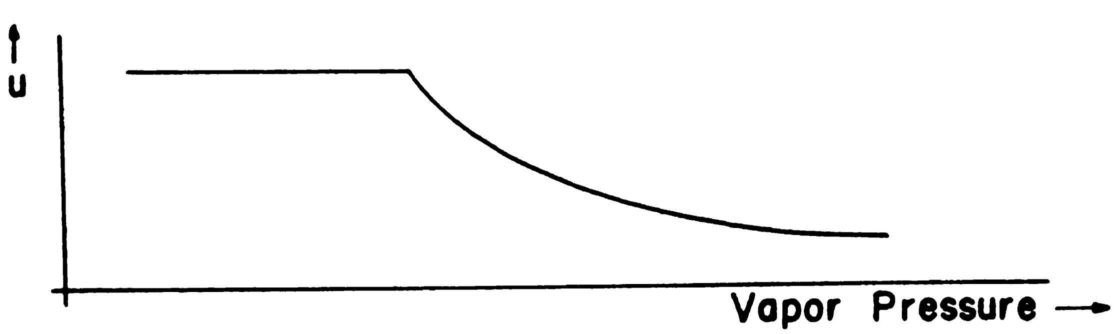


Figure 2: The relationship between u and the partial vapor pressure. Germant (8)

Human Locomotion

A study of human locomotion while ascending and descending stairs is an important part of any attempt to improve stairways. A considerable number of investigations on human locomotion have been reported. However, most of these have been carried out on a level surface.

A determination of the actual forces exerted by the foot while ascending and descending stairs would be very helpful. Eberhart and Inman (5) report that the most direct means of measuring all ground reactions is the force plate. With this plate records of the vertical force, both horizontal shears, torque, and the center of pressure of the foot can be obtained. Cunningham and Brown (3) discuss the design and operation of a force plate. They obtained the value of the forces by using strain gages on tubular columns.

With the use of a force plate Rehman, Patek and Gregson (14) determined the ground reactions which occur in level walking. They state that the maximum vertical, horizontal and lateral forces occur at 22.5% and 72.5% of the full stride. The maximum vertical force is greater than the body weight due to Newton's Law, $F = ma$. The first half of the horizontal force is directed posteriorly as the foot pushes forward. The last half of the reaction is forward while the foot pushes back. The maximum horizontal reaction amounted to about 15% of the total body weight. The maximum lateral forces amounted to only four to five percent of the body weight.

Barnett (1) measured the pressures of the foot with a pedograph and reported that the walking stride can be divided into five phases. The phases are as follows:

a) Heel Phase - In this phase only the heel touches the ground. The whole heel bears the weight evenly most of the time. This phase occurs in the first 15 to 20 percent of the total stance.

b) Standing Phase - The pressure is taken by the heel, the outer side of the sole and the heads of the metatarsal bones. The center of gravity of the body is directly over the foot. This phase lasts for 30 to 35 percent of the total stance.

c) Metatarsal Phase - All of the weight is borne by the metatarsal heads. This phase seldom exceeds 10 percent of the total stance.

d) Forefoot Phase - The toes and the metatarsal heads bear the weight. This phase occupies 30 percent of the total stance.

e) Step-off Phase - The weight of the foot is borne entirely on the toes, especially the hallax. However, most of the body weight is on the other foot. This phase will make up from 3 to 10 percent of the total stance.

EXPERIMENTAL INVESTIGATION

COEFFICIENT OF FRICTION FOR TREAD AND SHOE SOLE MATERIALS

Tread Covering Materials

The tread materials selected for this investigation were wood, inlaid linoleum, a non-skid type paint, varnish, marblized rubber mat and an abrasive strip. The first five materials were found most often in Miller's (12) study. The sixth material, the abrasive strip was studied for comparative purposes. New and worn specimens of each tread material were studied.

The surfaces for the new specimens of wood, linoleum, abrasive strip and rubber mat were left in their original condition. The linoleum, abrasive strip and rubber mat materials were cemented to pieces of plywood for the tests. A test surface for the abrasive strips, which were three quarters of an inch wide, consisted of two strips placed one half of an inch apart. The rubber mat, which contained surface grooves, was oriented such that the sole material crossed perpendicularly to the grooves. Varnish and paint were applied to regular wood stair tread material according to directions. A regular wooden stair tread was used for the tests on wood.

Because of the difficulty encountered in obtaining used tread materials, the worn tread materials were prepared in the laboratory. They were prepared by rubbing new tread materials with a very fine sand (50 mesh) as suggested by Hunter (10).

Shoe Sole Materials

The shoe sole materials used in this study were neoprene, neolite, leather, a neoprene base crepe, ripple and a hard surface sole made by B. F. Goodrich. The investigation was carried out using new and worn specimens for two different sole sizes for each of the six soles studied. The different sole sizes represented the worn area of an average size sole for a man and woman. The area of the two sole sizes was 16.4 and 10.7 square inches.

To perform the desired tests the shoe soles had to be mounted in a metal holder (see testing equipment). The soles were mounted as follows:

1) A sole was trimmed to the desired size.
2) A piece of plywood was cut to the same size.
3) The plywood was nailed to a wooden block approximately 8½ inches square.
4) The sole was cemented to the plywood and held in place by clamps until dry.

The sole holder was attached to the metal holder by four small bolts.

The total thickness of the shoe sole holder had to be constant because the arms connecting the metal holder to the strain gage transducer had to be level during the operation of the testing machine. The thickness of the shoe soles varied for the different materials so the total thickness was held constant by adjusting the thickness of the plywood. The sole was oriented so the vertical force applied to the metal holder was directly over its centroid.

The new soles were mounted to the holding block in their original condition. Worn soles of all the materials except the hard surface sole were obtained from old shoes purchased at second hand stores. These soles were removed from the shoes and mounted to the holder. The worn B. F. Goodrich sole was obtained by rubbing a new sole with the same type of sand used on the treads.

Testing Equipment

The coefficient of friction was measured with the machine shown in Figures 3 and 4. This machine consisted of a movable table under a stationary holder. The holder was fastened to two vertical bars containing strain gages. The strain gage bridge was connected to a Brush amplifier and inking oscillograph. The vertical load was applied by placing weights at the end of the arm resting on the holder. This arm had a 1 to 3 mechanical advantage (1 pound at the end of the bar places 3 pounds on the holder).

In this investigation the tread material was placed on the table and the shoe sole material was fastened to the holder. The tread materials moved under the holder at a uniform speed and the horizontal force was recorded on the oscillograph. From the horizontal and vertical forces the coefficient of friction was calculated (coefficient of friction equals the horizontal force divided by the vertical force). The dynamic coefficient of friction was used in

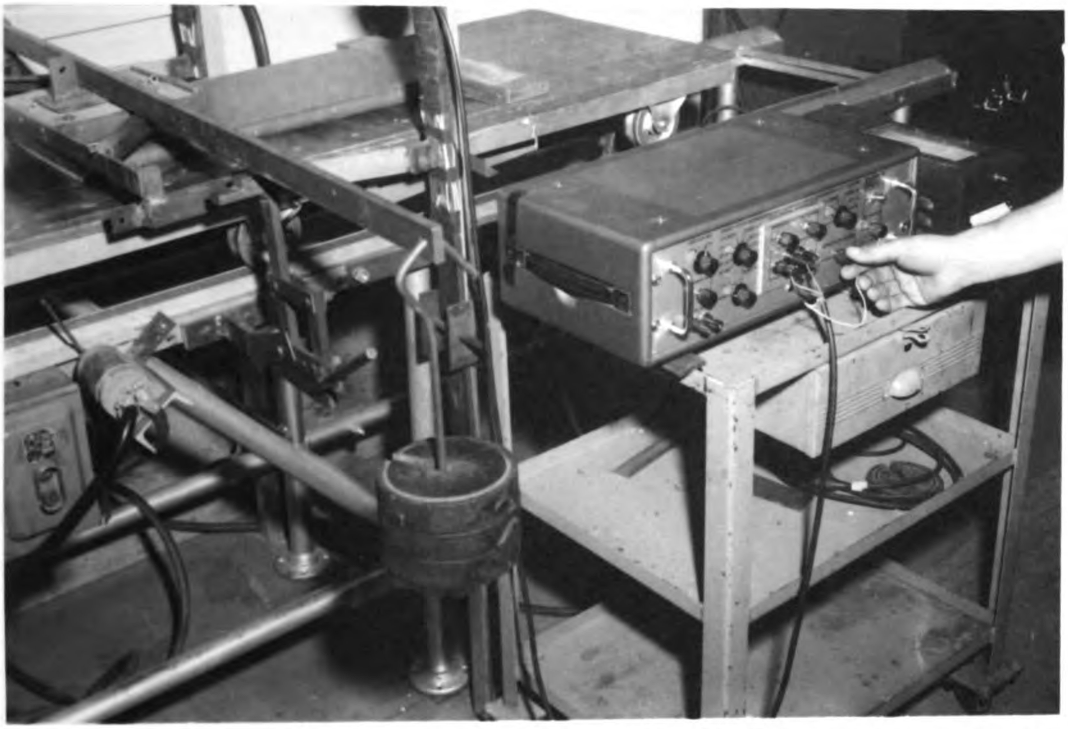


Figure 3: The machine used to measure the coefficient of friction.

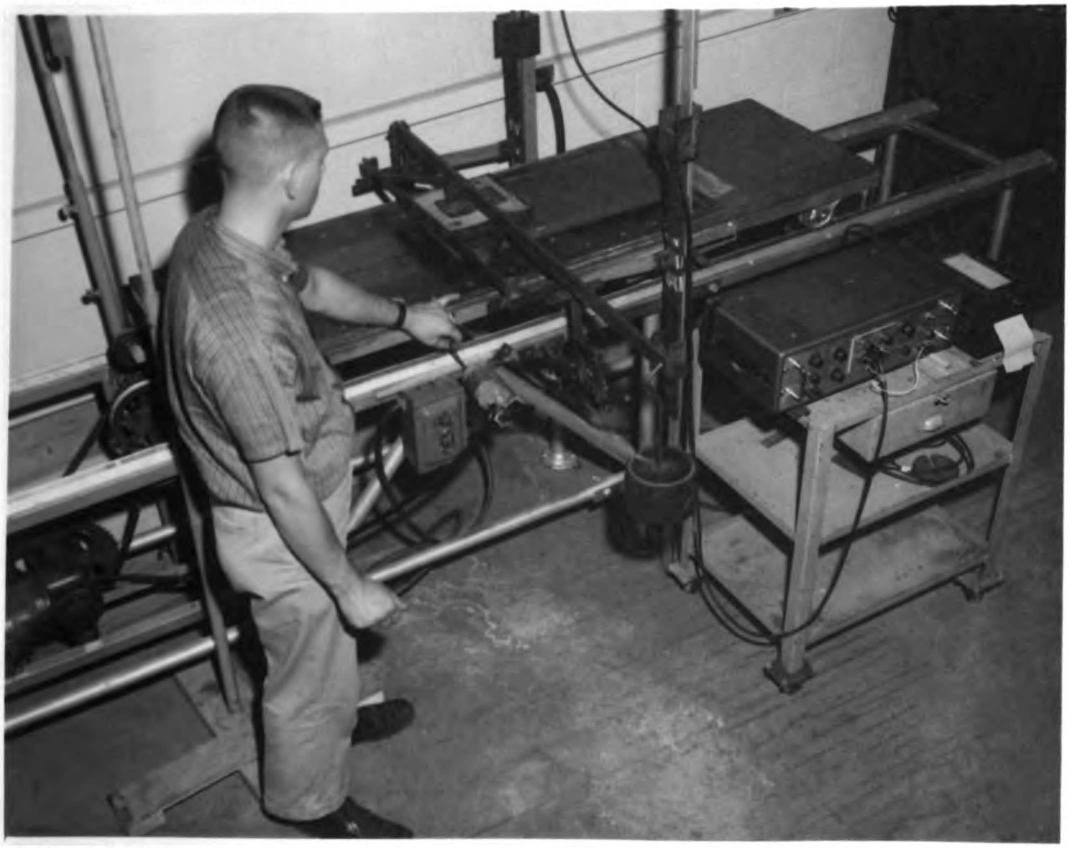


Figure 4: A close view of the machine used to measure the coefficient of friction showing the sole holder and the stair tread.

this study.

The strain gage circuit consisted of eight SR-4 (type 4-5) strain gages, two in each arm of the Wheatstone Bridge. The output of this circuit is given by the following formula:

$$\epsilon = \frac{6 P L}{E b H^2}$$

ϵ = Bridge output, in units of strain.
P = Horizontal force, pounds.
L = Distance from the end reaction to the center of the strain gage, inches.
E = Modulus of elasticity, psi.
b = Width of the bar, inches.
H = Thickness of the bar, inches.

A diagram of the strain gage circuit and the derivation of the above formula is in Appendix I.

The circuit was calibrated by applying known loads to the bars and adjusting the oscillograph to read pounds per line of deflection. It was possible to calibrate using this procedure because ϵ is directly proportional to P since L, E, b and H are all constant for any measurement. Once the strain gage circuit had been calibrated the internal calibration circuit of the amplifier was used before each test period to eliminate the process of manual calibration. The calibration curve for the circuit is shown in Figure 5.

Procedure of the Investigation

For this investigation the coefficient of friction was measured for only one vertical force and one table speed. A vertical force of 117 pounds was used. Loads

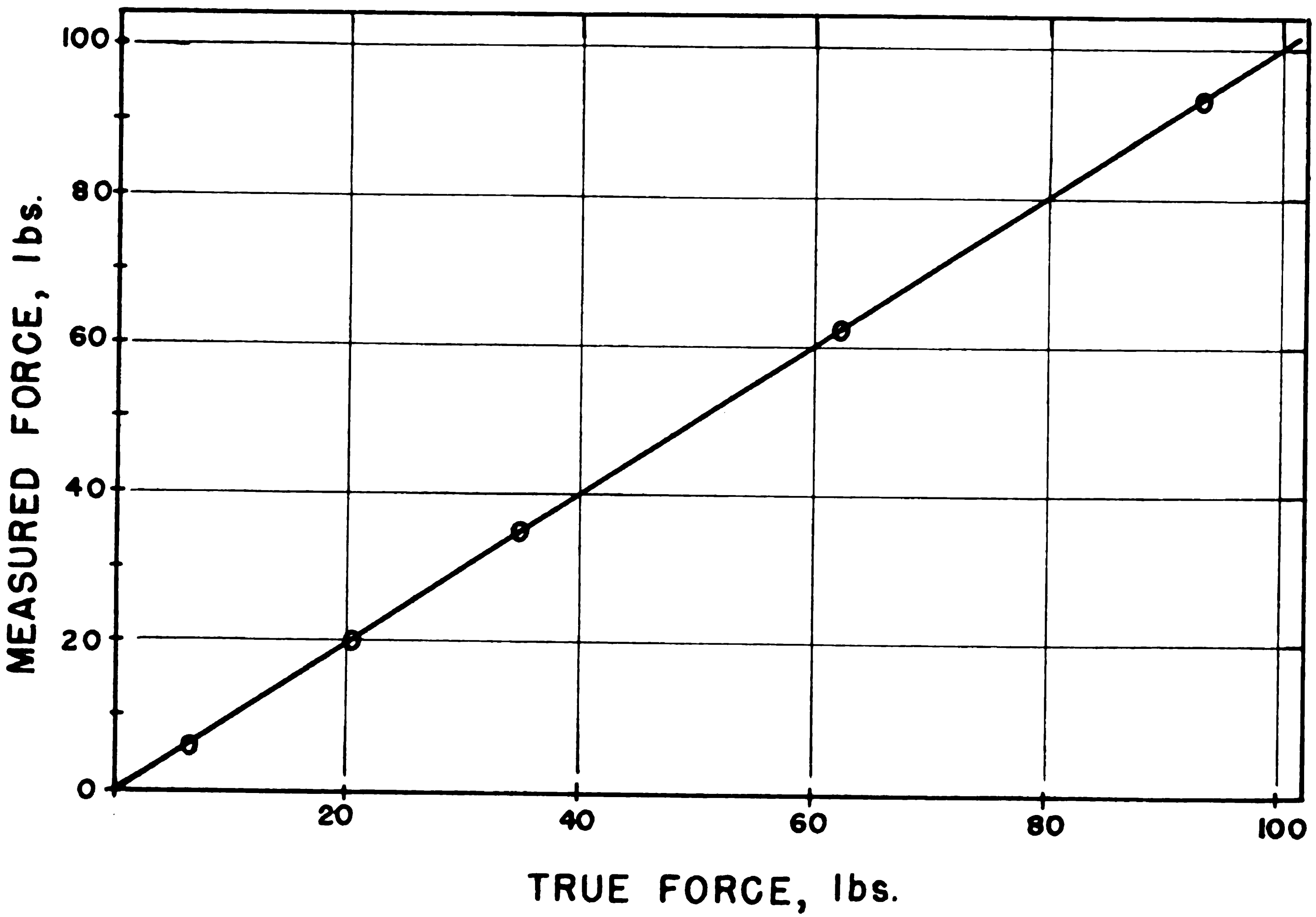


Figure 5: The calibration curve for the strain gage transducer of the friction machine.

greater than this caused one of the tread materials to tear apart. Higher loads also caused considerable wear on some of the soles. The table moved at a speed of 14.6 feet per minute. Preliminary tests indicated that faster table speeds caused the vertical bars to vibrate when the horizontal force was applied.

The investigation was designed so that it could be analyzed statistically. The study was run as a series of separate parts. Each part consisted of the new and used specimens of a single sole type and size. The coefficient of friction was measured using these soles and the twelve different tread materials. There were three duplications of each sole and tread material. The order in which the measurements were made was determined by randomization.

Each individual shoe sole was tested on twelve different tread materials during each single investigation. To prevent the sole and tread materials from becoming smooth they were brushed lightly with a piece of 3/0 sandpaper between each test. This procedure of keeping the materials in a condition similar to that of natural walking was suggested by Hunter (10).

MEASURING THE FORCES EXERTED ON STAIR STEPS

Description of the Equipment

The magnitude, direction and point of application of forces exerted by a person ascending and descending a stairway were measured by the force plate shown in Figure 6.

The force plate contains five separate strain gage circuits and is capable of measuring the vertical force, both horizontal forces and the point of application of these forces in relation to the front edge and side of the tread.

The frame holding the instrumented stair tread is supported at each corner by small columns made from Reynolds 618T6 Aluminum. This aluminum has a yield stress of 40,000 psi. The support is 5.62 inches high with an outside diameter of 0.876   0.008 inch. The inside diameter is 0.787   0.002 inch. The force plate was designed to be used in stairways with different riser height and tread width combinations. A detailed drawing of the corner support is shown in Figure 8.

Bridge Output

Each of the strain gage circuits consisted of eight SR-4 (type A-5) strain gages, two in each arm of the Wheatstone Bridge. The equations for the bridge output of the circuits are as follows:

Circuit #1
$$\epsilon_1 = \frac{P\ (2e_2-d)}{d\ A\ E} = \frac{P\ (d-2e_1)}{d\ A\ E} \qquad (1)$$

Circuit #2
$$\epsilon_2 = \frac{P\ (2e_3-L)}{L\ A\ E} = \frac{P\ (L-2e_4)}{L\ A\ E} \qquad (2)$$

Circuit #3
$$\epsilon_3 = \frac{P}{dLAE}\Big((u-1)(2e_1e_4) + (1-u)(e_4d-e_1L) + udL\Big) \qquad (3)$$

Circuit #4
$$\epsilon_4 = \frac{Hy\ t\ c}{EI} \qquad (4)$$

Circuit #5
$$\epsilon_5 = \frac{Hx\ t\ c}{EI} \qquad (5)$$

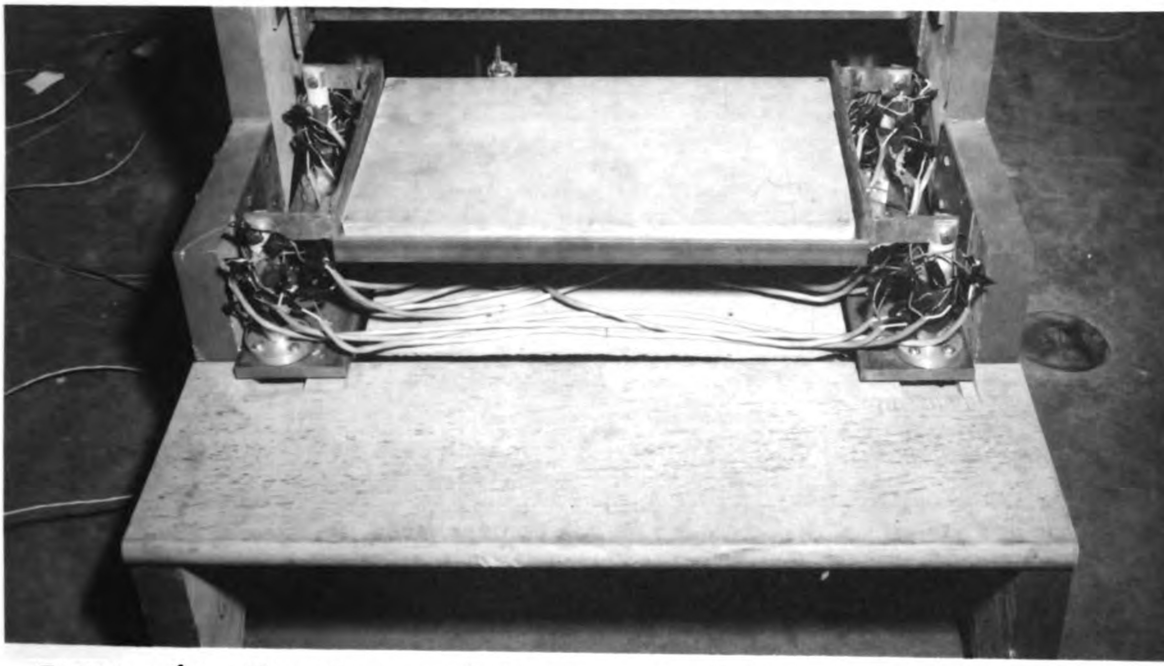


Figure 6: The force plate in one of the experimental stairways.

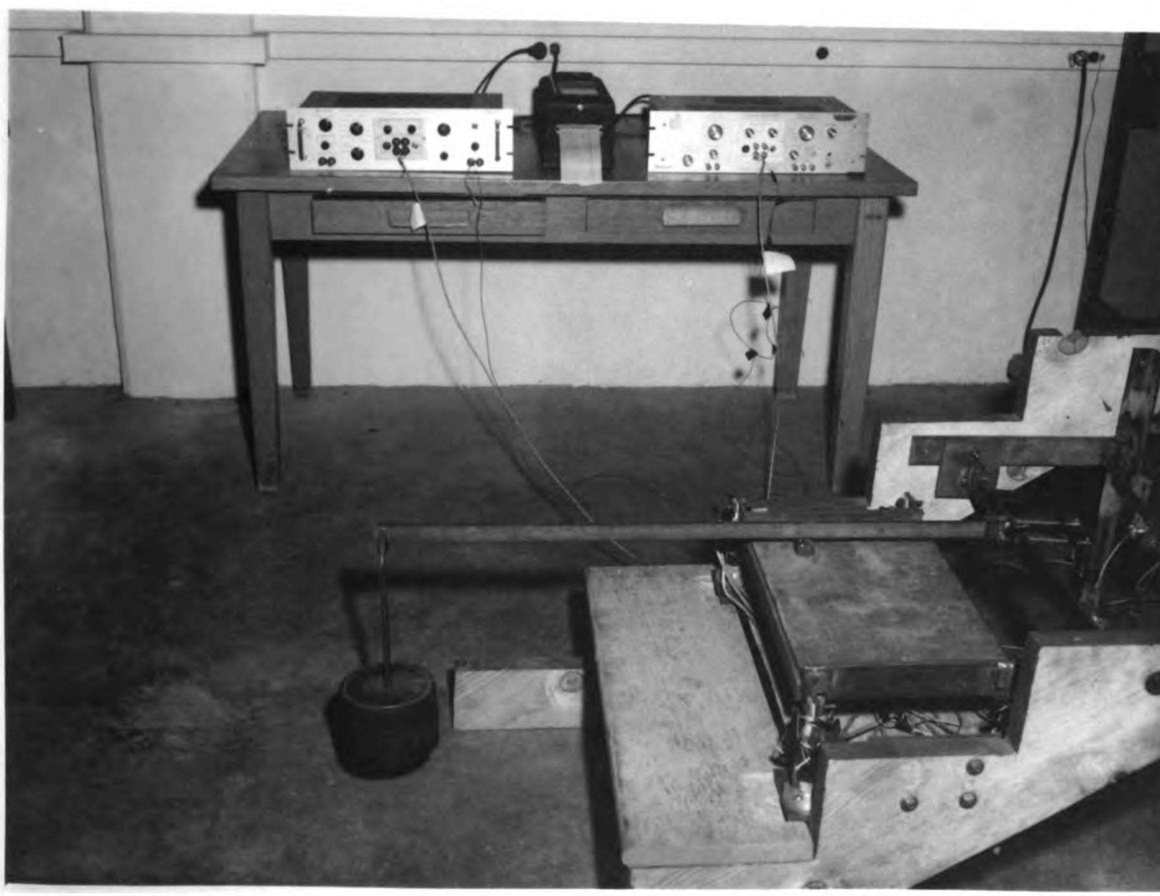
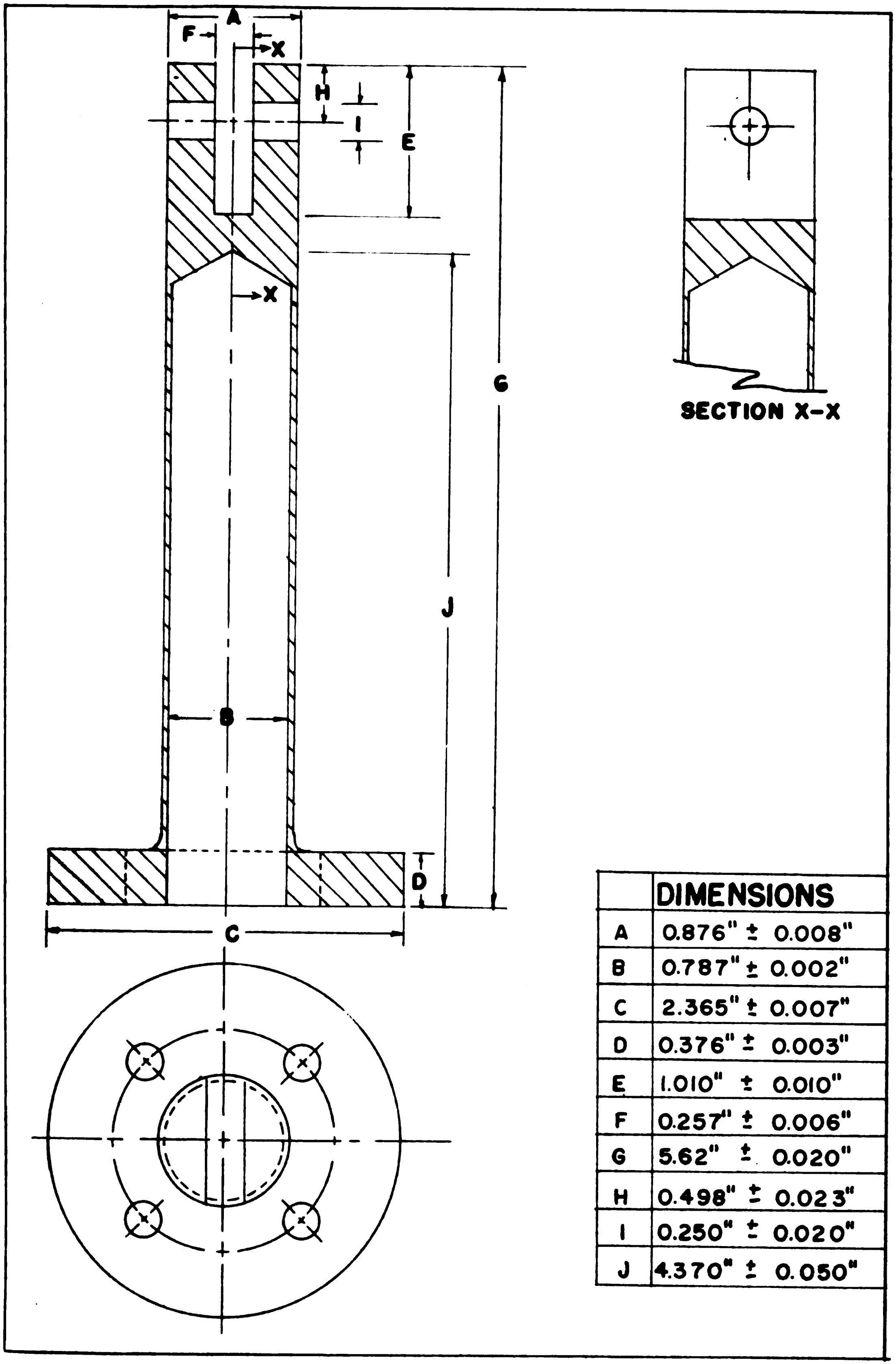


Figure 7: Calibrating the force plate for vertical forces and the coordinate system.



	DIMENSIONS
A	0.876" ± 0.008"
B	0.787" ± 0.002"
C	2.365" ± 0.007"
D	0.376" ± 0.003"
E	1.010" ± 0.010"
F	0.257" ± 0.006"
G	5.62" ± 0.020"
H	0.498" ± 0.023"
I	0.250" ± 0.020"
J	4.370" ± 0.050"

Figure 8: The corner support for the force plate.

ϵ_n = Bridge output in units of strain, n = circuit number
P = Vertical force, lbs.
A = Cross sectional area of one support, 0.116 in.
E = Modulus of elasticity of aluminum, 10^7 psi.
Hy = Horizontal force toward the front or rear edge of the tread, lbs.
Hx = Horizontal force toward the right or left side of the tread, lbs.
t = Distance from the end reaction of the supports to the center of the strain gages measuring the Hx and Hy forces, inches.
c = Radius of the support, inches.
u = Poisson's ratio, 0.33 for aluminum.
L, d, e , e , e and e are specific dimensions of the frame holding the stair tread. See Figure 9.

The orientation of the strain gages on the supports, a diagram of the circuits and the derivation of the equations are in Appendix II.

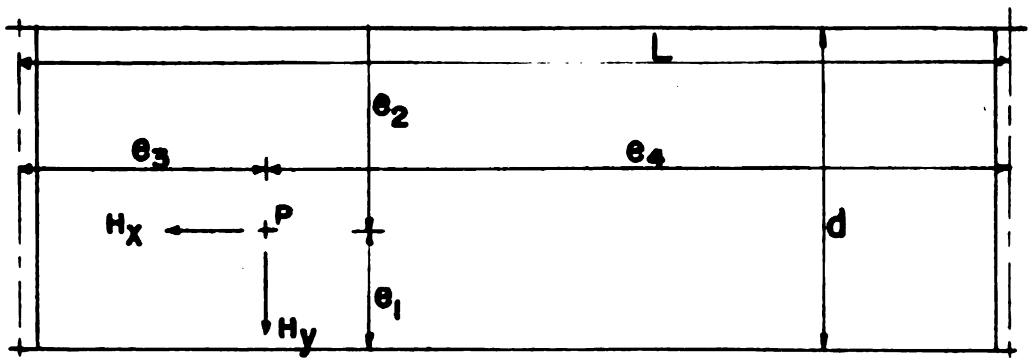


Figure 9: A schematic diagram of the tread holder showing the specific dimensions.

For any measurement the equations for circuits (1) and (2) contain two unknown quantities. The vertical force P and the location of the force with respect to the front <u>or</u> side of the tread. For any measurement the equation for circuit (3) contains three unknown quantities. The vertical force P and the location of the force with respect to both the front <u>and</u> side of the tread. To determine the magnitude of the vertical force, equations

(1) and (2) were solved in terms of e_1 and e_4 and substituted into equation (3). The resulting equation was:

$$\epsilon_3 = \frac{P}{A\,E}\left(\frac{1 + 4}{2}\right) + \left[\frac{A\,E\;\epsilon_1\;\epsilon_2\,(u - 1)}{2\,P}\right]$$

This equation can be put into the form of a quadratic equation and solved for P.

$$P = \frac{-\epsilon_3 \pm \sqrt{\epsilon_3^{\,2} - 4\left(\frac{1 + u}{2}\right)(1/AE)\left(\frac{A\,E\,\epsilon_1\,\epsilon_2\,(u-1)}{2}\right)}}{2(1/AE)\left(\frac{1 + u}{2}\right)}$$

Substituting in the numerical values for u, A and E reduces the equation to:

$$P = \frac{-\epsilon_3 \pm \sqrt{\epsilon_3^{\,2} + 0.90\;\epsilon_1\,\epsilon_2}}{1.146 \times 10^{-6}}$$

Calibration

Because equations (1), (2) and (3) contained more than 1 unknown quantity, circuits 1, 2 and 3 were calibrated so the oscillograph read 5 micro inches per inch of strain per line of deflection. The internal calibration circuit of the amplifier was used to achieve this reading. The calibration was accomplished by placing the attenuator on 20 and adjusting the oscillograph pen for 15.5 lines of deflection after the amplifier had been balanced in. The attenuator was then set at 5 and the measurements were made at this attenuator setting.

The relationship between the applied and measured vertical loads was determined by applying a known load at a known position as shown in Figure 7. This procedure was repeated for several different loads and positions. Calibration curves showing the relationship between the applied and measured vertical force are shown in Figures 10 and 11.

The vertical force could not be applied to a specific point but instead had to be applied over a small area. As shown in Figure 9, the vertical force was applied by a lever arm resting on a small block which was one inch square. The vertical force was applied as near as possible over the center of the block. The true position of e_1 and e_4 was the distance from their respective edges of the tread to the center of the block. However, due to the difficulty of determining the exact position of the vertical force an error of 0.125 and 0.25 inches were allowed for e_1 and e_4 respectively. These two limits allowed for errors in measuring the center line of the block, the placing of the lever arm on the block and the reading of the oscillograph chart. A larger error was allowed for e_4 because this measurement was affected more by a change in the placement of the lever arm. The relationship between the true and measured positions for e_1 and e_4 are shown in Figures 12 and 13. The band formed by the two lines indicate the region in which values were acceptable.

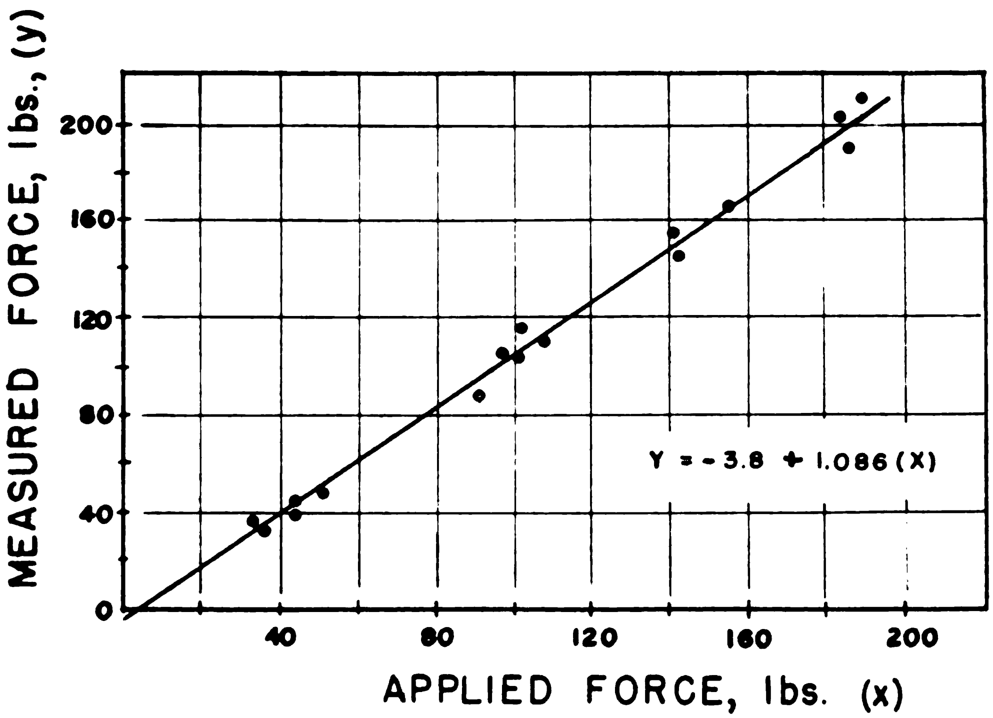


Figure 10: The calibration curve for the vertical force when e_1 is less than one half the tread width.

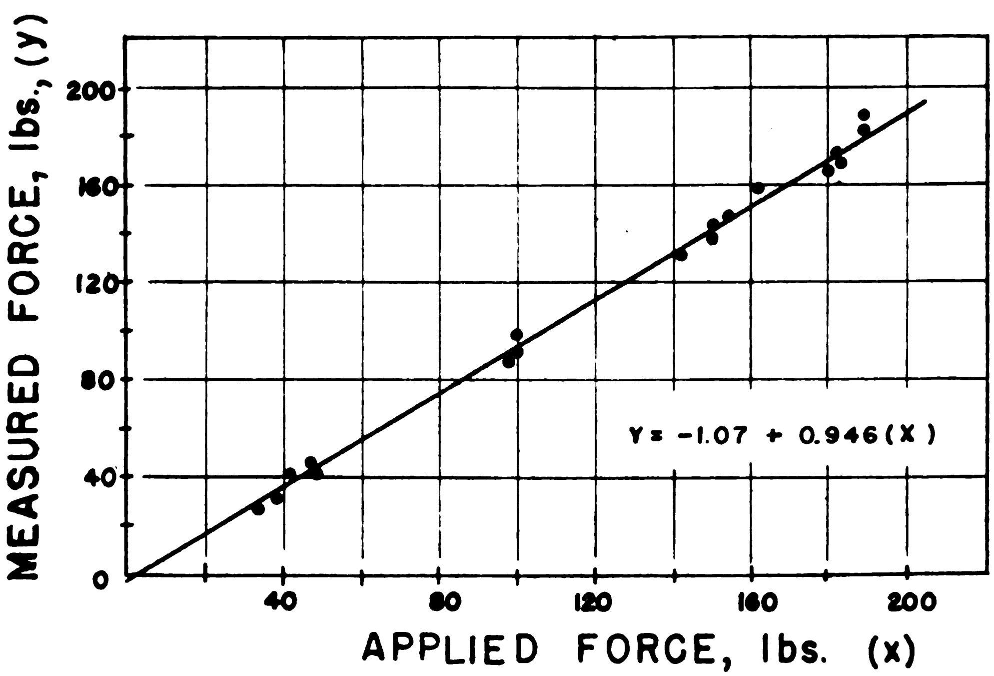


Figure 11: The calibration curve for the vertical force when e_1 is greater than one half the tread width.

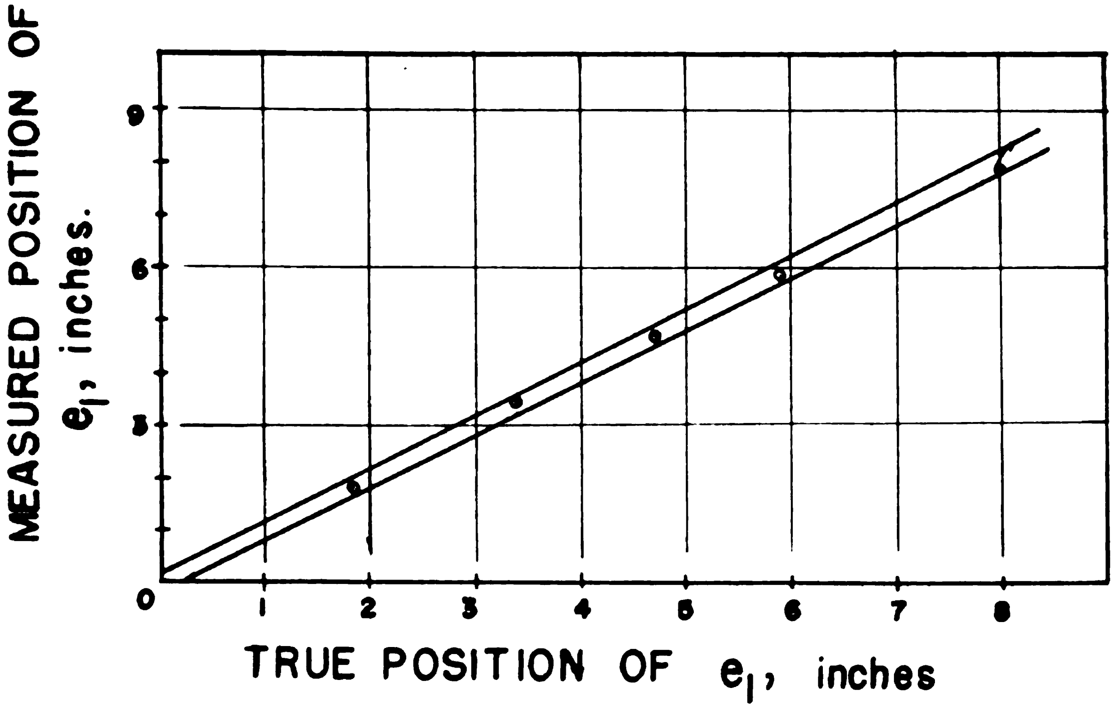


Figure 12: The calibration curve for e_1.

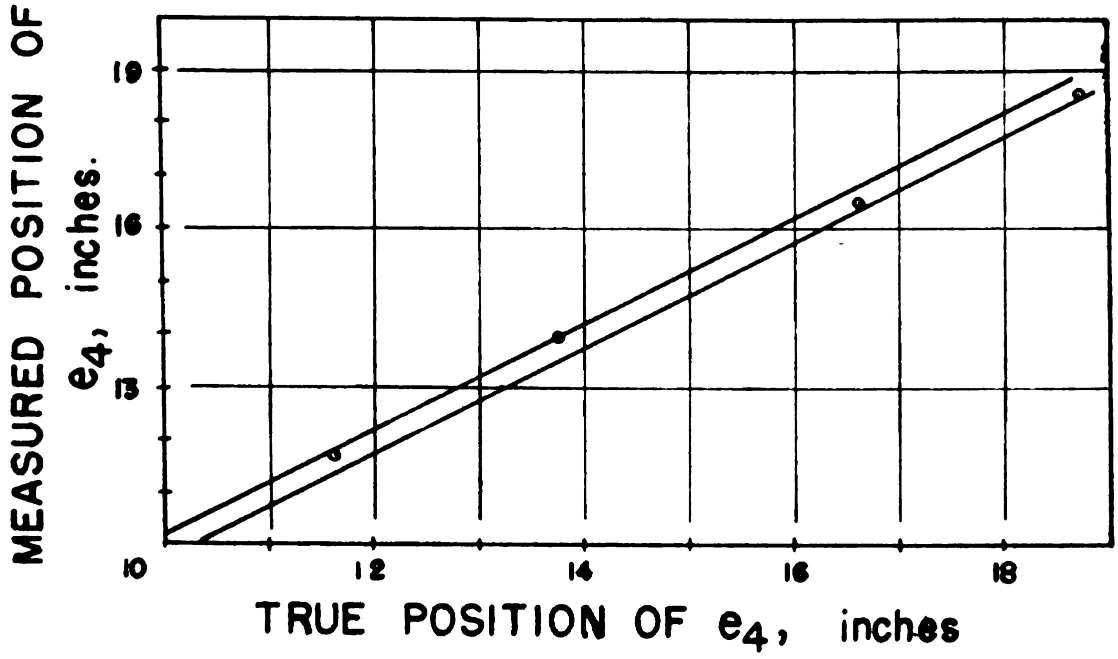


Figure 13: The calibration curve for e_4.

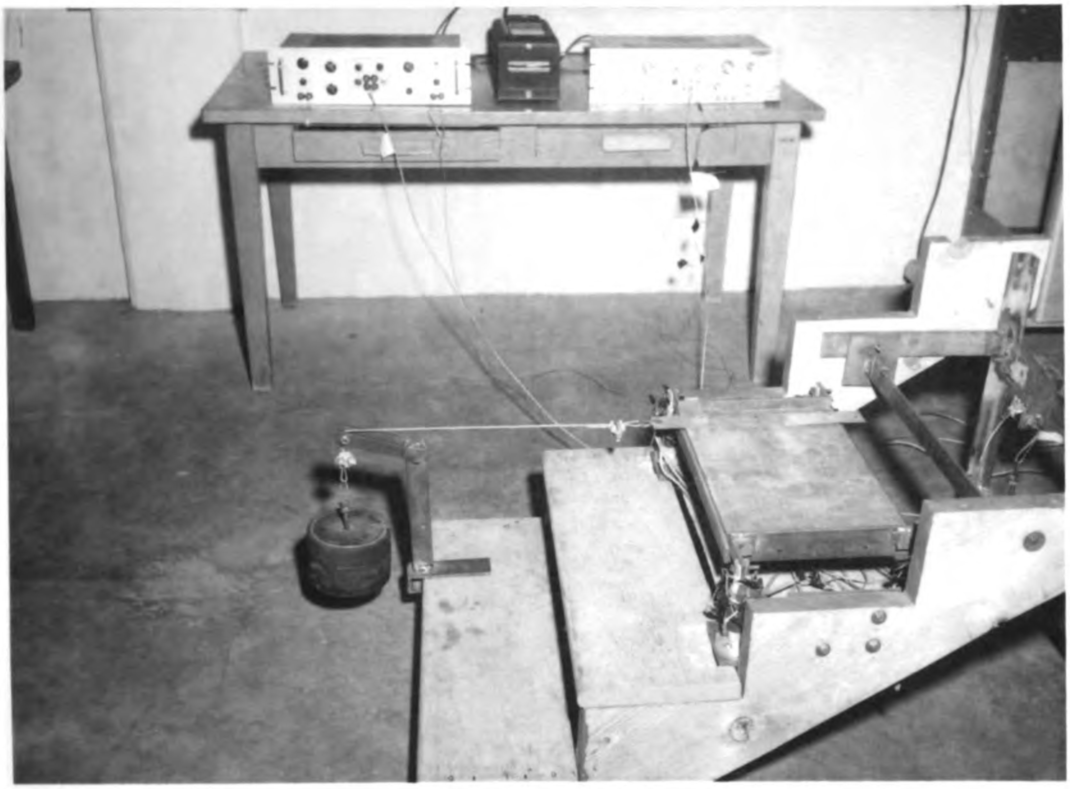


Figure 14: Calibrating the force plate for the horizontal forces.

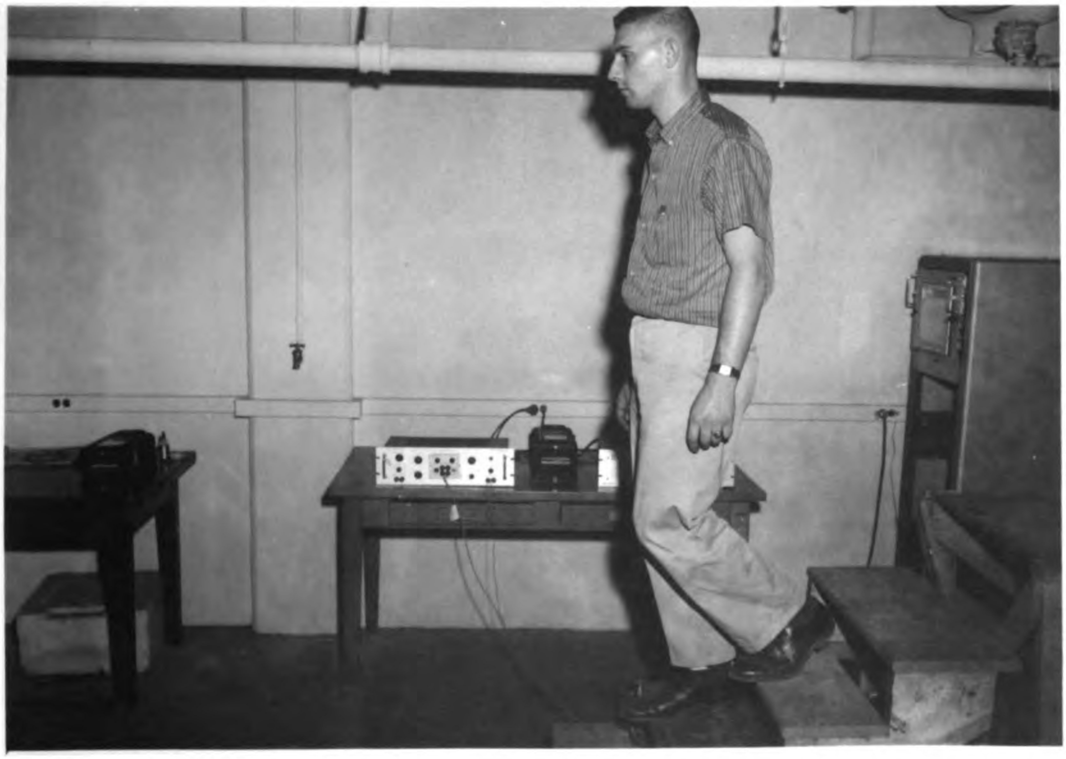


Figure 15: A person descending one of the experimental stairways.

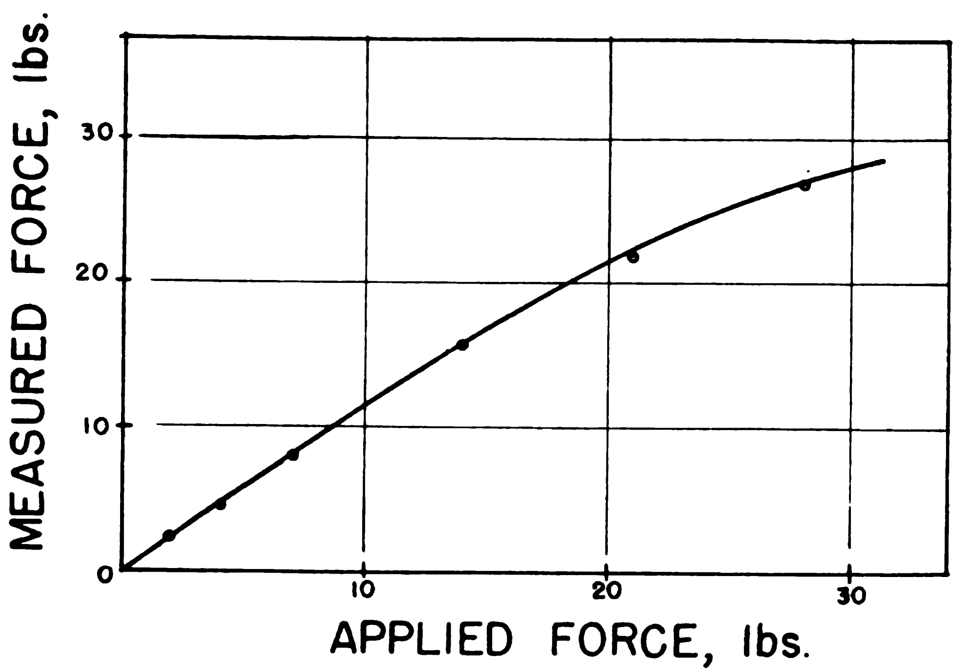


Figure 16: The calibration curve for the horizontal force directed toward the front edge of the tread.

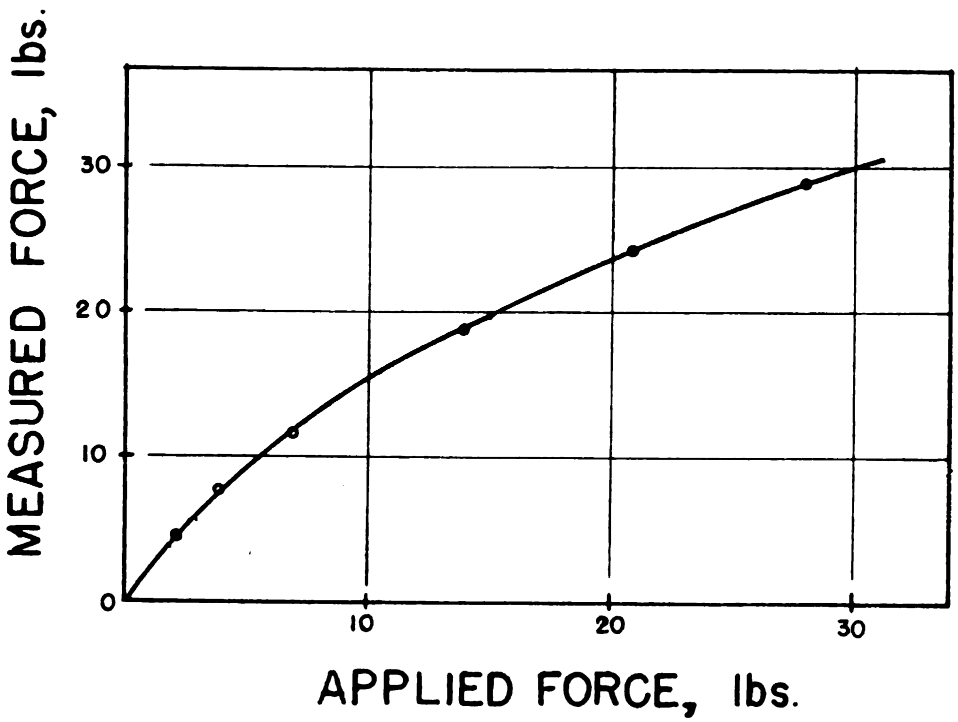


Figure 17: The calibration curve for the horizontal force directed toward the rear edge of the tread.

Equation 4 shows ϵ_4 to be directly related to the horizontal force. This allows the oscillograph to be calibrated directly in pounds of force. The horizontal force was calibrated by applying known loads at three different positions along the front and rear of the tread as shown in Figure 14. Figures 16 and 17 show the relation between the applied force and the measured force. The circuit that measured the horizontal force directed toward the end of the tread was not used during this experiment.

Stairways

To compare stairway designs, nine different stairways were constructed. Stairways with 9, 10 and 11 inch treads were built for each of three riser heights. The riser heights were 6, 7 and 8 inches. According to the standards set by Parker, Gay and MacGuire (13) the stairway with the 8 inch riser and 9 inch tread is the only stairway of the group which would be considered architecturally correct. Each of the nine stairways contained five treads with the force plate used as the third tread. Figure 15 shows a person descending one of the experimental stairways.

Procedure of the Investigation

The purpose of this investigation was to determine the vertical force, the horizontal force and the point of application of the vertical force as a person ascended and descended a stairway. Two male subjects were used for this study. Subject #1 weighed 140 pounds and subject #2 weighed 172 pounds.

Because the amplifiers failed to operate correctly the horizontal and vertical forces were not measured at the same time. The investigation was separated into two parts. The vertical force and its point of application were measured during the first part while the horizontal force was measured during the second part. Both subjects ascended and descended each stairway four times for each part of the investigation. Each subject walked up and down the stairway at his own natural speed.

The time spent on the force plate could be determined because the oscillograph chart paper moved at a constant speed (5.05 inches per second). To determine how the unknown quantities varied as a person moved across the step the measurement period (the time spent on the tread) was divided into eight parts. Each individual measurement was read at the eighth points and also at the first and last sixteenth points.

RESULTS

COEFFICIENT OF FRICTION

The surface conditions of a sole or tread material are dynamic properties changing from day to day or even from hour to hour. Although the surface conditions are continually changing there are some characteristics which would make a tread material desirable. These characteristics are: 1) a high coefficient of friction for new and worn surface conditions, 2) a high coefficient of friction for a wide selection of sole materials and 3) a small variation in the coefficient of friction between new and worn conditions of sole and tread. Because these characteristics would be desirable the data of this investigation will be analyzed with respect to the magnitude of the coefficient of friction and its variation between different surface conditions.

Analysis of the Experiment

The coefficient of friction data were analyzed statistically by the method of factorial analysis. Federer (6) states that,

> "A group of treatments which contains two or more levels of two or more factors or substances in all combinations is known as a factorial arrangement."

Federer also states that factorial experiments result in an unbiased conclusion even if trends or gradients are present in the experiment, thus making this design very favorable for this experiment because of small changes in the relative humidity.

The coefficient of friction data were divided into a number of 2x2 factorial arrangements. The two variables were sole and tread materials with two levels of each (new and worn). The factorial analysis was performed for each tread material individually making a total of 72 analyses. A diagram of the 2x2 analysis is shown in Figure 18. Each square indicates one of the four possible combinations in a 2x2 factorial design.

		TREADS (T) New	TREADS (T) Worn
SOLES (S)	New	N.S.N.T.	N.S.W.T.
	Worn	W.S.N.T.	W.S.W.T.

Figure 18: A schematic diagram of a 2x2 factoral analysis.

The Dixon and Massey (4) method of testing for significant differences between individual comparisons was used. For this investigation a worn sole on a new tread (W.S.N.T.) was compared to a worn sole on a worn tread (W.S.W.T.). This was a comparison of new and worn tread materials with the sole material held constant.

Presentation of Results

The results of this investigation are presented by dividing the analysis into four different phases: 1) general trends, 2) analysis of new and worn tread materials for a worn sole material, 3) worn sole materials on worn tread materials and 4) comparison of sole materials independent of tread materials.

The accuracy of the results for this investigation depended on how accurate the oscillograph charts could be read. The reading accuracy for the charts was plus or minus one pound. Because the vertical force was constant the percent error decreased with an increase in the horizontal force. The maximum error was approximately four percent.

The humidity and temperature were not controlled precisely during the investigation. All of the measurements were made at room temperature (72°F), however, the humidity varied some from day to day. The averages given in the table should not be considered absolute values because of the variation in the humidity and the fact that the values are for a limited number of surface conditions.

General Trends

The average coefficient of friction for each simulated surface condition is given in Appendix III. Each value is an average of nine measurements except for the ripple sole. The values for the ripple sole are an average of six measurements.

The graphs in Figures 19, 20 and 21 show the average friction value of each simulated condition of the six tread materials for three of the soles investigated. The graphs are for the large neoprene, crepe and ripple soles.

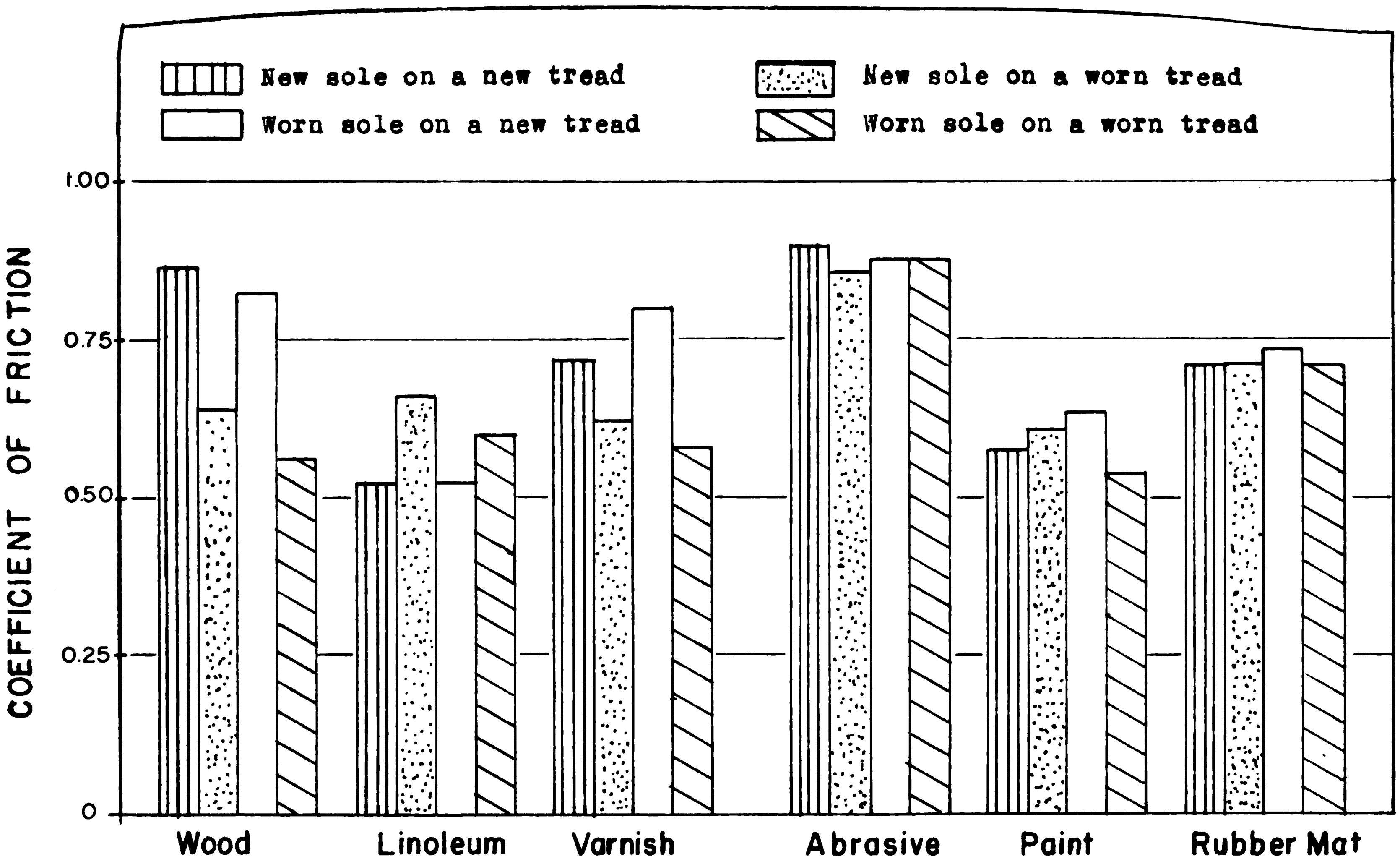


Figure 19: The average coefficient of friction for each of the simulated surface conditions and the large neoprene sole.

These graphs show some of the general trends and particular phenomenon of this investigation.

Figure 19, which is for the large neoprene sole, indicates that the abrasive strip had the highest coefficient of friction value for the six tread materials. The abrasive strip had the highest friction value (the four simulated conditions averaged together) of the six tread materials for nine of the twelve soles investigated. Only with the small leather and the large and small ripple soles were there other tread materials which had a higher coefficient of friction than the abrasive strip.

Another trend which is apparent from Figure 19 is that the worn linoleum material had a higher coefficient of friction than did the new linoleum. When the coefficient of friction was compared for the combination of a new sole material on new and worn tread materials the worn material had an equal or higher friction value for nine of the twelve soles investigated. Only for the large ripple and the large and small neolite soles was the friction value of the new linoleum higher than that for the worn linoleum. When the coefficient of friction for the worn soles on new and worn linoleum material was compared, the worn tread material had an equal or higher coefficient of friction for eight of the twelve soles investigated. The large neolite and the small neoprene, neolite and B.F. Goodrich soles had higher friction values on the new linoleum.

Another trend apparent from Figure 19 is the small variation between the means of the four simulated conditions of the abrasive and rubber mat tread materials. To determine the amount of dispersion in the data of each tread material the data for the four simulated conditions are combined and the mean and standard deviation of these data were computed. This analysis was performed for each of the twelve soles investigated. The standard deviation was computed from the following formula suggested by Walker and Lev (18).

$$S = \left(\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1} \right)^{\frac{1}{2}}$$

The mean ($\bar{X}$), the standard deviation (S) and the interval X S, which includes 68% of the data values, are given in Table 1. From this table it can be seen that the rubber mat material and the abrasive strip generally had the smallest standard deviation of the six tread materials investigated for the large and small neoprene, crepe and neolite soles and the small ripple sole. The abrasive strip had the smallest standard deviation for the large ripple and small leather soles. The painted tread had the smallest standard deviation for the large and small B.F. Goodrich soles while the worn tread had the lowest value for the large leather sole.

As a general rule the wood, linoleum and varnished tread materials had a larger standard deviation than did the abrasive, rubber mat and painted tread materials. For seven of the twelve soles investigated the rubber mat, abrasive and paint had the three smallest standard deviations. For four of the other five soles one of these three tread materials had the smallest deviation of the six tread materials.

Figure 20, which is for the large crepe sole, reveals a phenomenon which occurred primarily with the crepe sole. For this sole the combination of a new sole material on a new tread material gave the highest coefficient of friction value for the four simulated conditions. This was true for the wood, abrasive, varnish and painted treads for both the large and small crepe soles. This phenomenon occurred for other sole and tread combinations but not with the regularity that was observed with the crepe sole.

Figure 21, which is for the large ripple sole, shows the high coefficient of friction values obtained with the ripple sole. It should be noticed that the highest values occur on the smooth surfaced treads. The abrasive strip and rubber mat materials have the lowest values of the six tread materials. This was true for both the large and small ripple soles.

Analysis of New and Worn Tread Materials for a Worn Sole Material

When a stair tread material is placed on a stairway it is expected to last for a period of years. During this period of usage the surface condition of the tread material will change. It would be desirable, however, if the coefficient of friction of the tread material either

TABLE 1: The Mean ($\bar{X}$), the Standard Deviation (S) and the interval of $\bar{X} \pm S$ of the Coefficient of Friction Data of the Four Conditions involving a Single Sole and Tread Material.

Tread Material	Large Sole $\bar{X}$	Large Sole S	Large Sole $\bar{X} \pm S$	Small Sole $\bar{X}$	Small Sole S	Small Sole $\bar{X} \pm S$
Neoprene Sole						
Wood	0.72	0.14	0.58-0.86	0.63	0.14	0.49-0.77
Linoleum	0.58	0.08	0.50-0.66	0.54	0.11	0.43-0.65
Varnish	0.68	0.12	0.56-0.80	0.63	0.11	0.52-0.74
Abrasive	0.88	0.06	0.82-0.94	0.77	0.07	0.70-0.84
Paint	0.60	0.06	0.54-0.66	0.60	0.09	0.51-0.69
Rubber Mat	0.72	0.03	0.69-0.75	0.68	0.05	0.63-0.73
Crepe Sole						
Wood	0.58	0.08	0.50-0.66	0.56	0.12	0.44-0.68
Linoleum	0.46	0.10	0.36-0.56	0.44	0.11	0.33-0.55
Varnish	0.53	0.10	0.43-0.63	0.58	0.18	0.40-0.76
Abrasive	0.77	0.07	0.70-0.84	0.76	0.08	0.68-0.84
Paint	0.49	0.07	0.42-0.56	0.53	0.10	0.43-0.63
Rubber Mat	0.66	0.06	0.60-0.72	0.64	0.74	0.56-0.71
Leather Sole						
Wood	0.32	0.04	0.28-0.36	0.24	0.06	0.18-0.30
Linoleum	0.32	0.07	0.25-0.39	0.24	0.05	0.19-0.29
Varnish	0.39	0.06	0.33-0.45	0.31	0.10	0.21-0.41
Abrasive	0.79	0.10	0.69-0.89	0.47	0.04	0.43-0.51
Paint	0.48	0.12	0.36-0.60	0.32	0.07	0.24-0.39
Rubber Mat	0.72	0.07	0.64-0.79	0.49	0.11	0.38-0.60

TABLE 1: The Mean ($\bar{X}$), the Standard Deviation (S) and the interval of $\bar{X} \pm S$ of the Coefficient of Friction Data of the Four Conditions involving a Single Sole and Tread Material.

Tread Material	Large Sole $\bar{X}$	Large Sole S	Large Sole $\bar{X} \pm S$	Small Sole $\bar{X}$	Small Sole S	Small Sole $\bar{X} \pm S$
<u>Neolite Sole</u>						
Wood	0.60	0.08	0.52-0.68	0.58	0.14	0.44-0.72
Linoleum	0.55	0.09	0.46-0.64	0.55	0.08	0.57-0.63
Varnish	0.68	0.17	0.51-0.85	0.69	0.15	0.54-0.84
Abrasive	0.77	0.07	0.71-0.84	0.69	0.05	0.64-0.74
Paint	0.63	0.07	0.56-0.70	0.61	0.07	0.54-0.68
Rubber Mat	0.55	0.04	0.51-0.59	0.54	0.05	0.49-0.59
<u>B.F. Goodrich Sole</u>						
Wood	0.52	0.08	0.44-0.60	0.53	0.16	0.37-0.69
Linoleum	0.46	0.11	0.35-0.57	0.49	0.15	0.34-0.64
Varnish	0.58	0.03	0.50-0.66	0.58	0.13	0.45-0.71
Abrasive	0.58	0.09	0.49-0.67	0.60	0.14	0.46-0.74
Paint	0.56	0.02	0.54-0.58	0.56	0.10	0.46-0.66
Rubber Mat	0.51	0.12	0.39-0.63	0.55	0.12	0.43-0.67
<u>Ripple Sole</u>						
Wood	0.97	0.14	0.83-1.11	1.09	0.10	0.99-1.19
Linoleum	0.97	0.12	0.85-1.09	1.07	0.12	0.95-1.19
Varnish	1.08	0.15	0.93-1.23	1.19	0.10	1.09-1.29
Abrasive	0.92	0.04	0.88-0.96	1.03	0.09	0.94-1.12
Paint	0.98	0.14	0.84-1.12	1.14	0.08	1.06-1.22
Rubber Mat	0.87	0.07	0.80-0.94	0.89	0.06	0.83-0.95

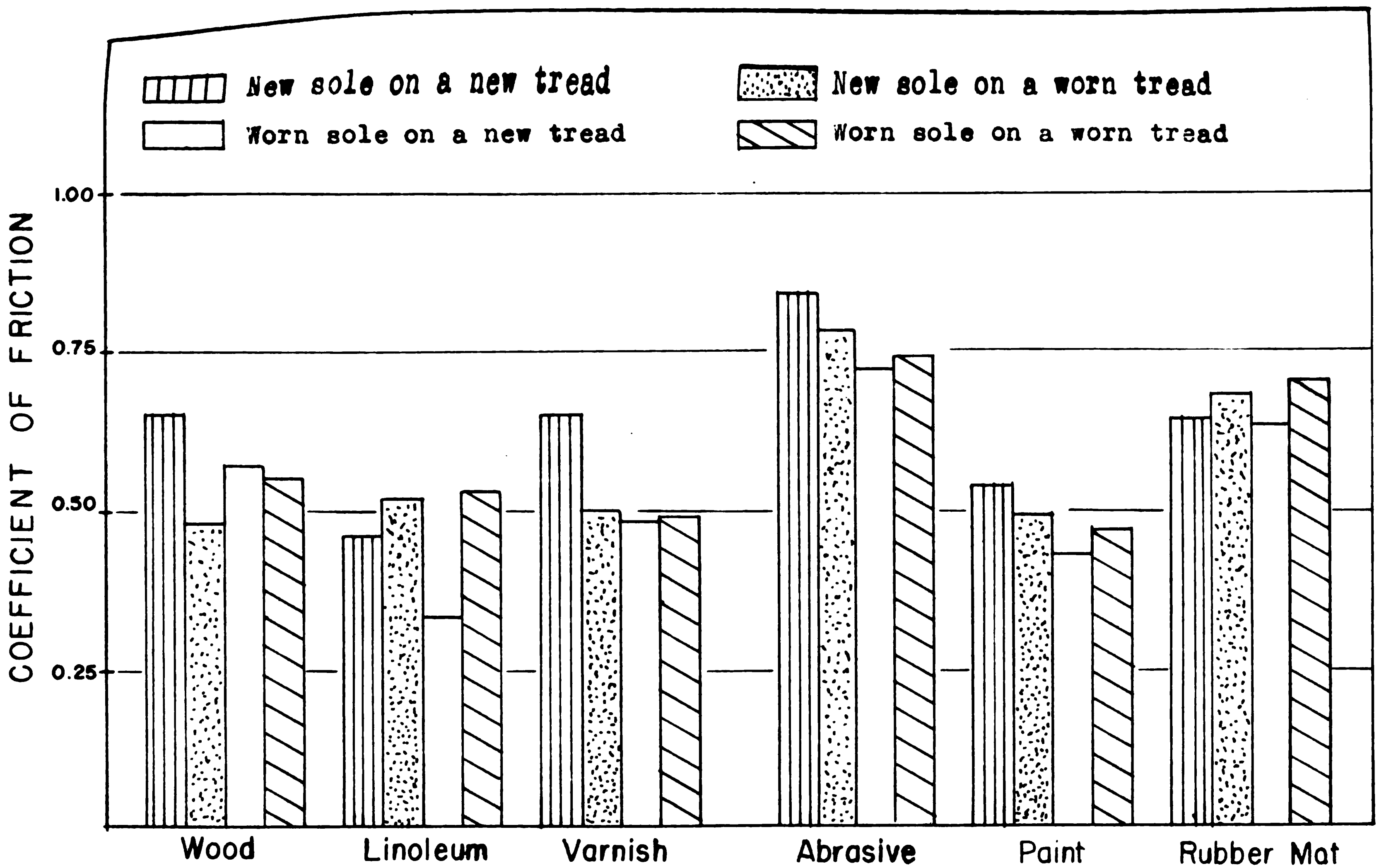


Figure 20: The average coefficient of friction for each of the simulated surface conditions and the large crepe sole.

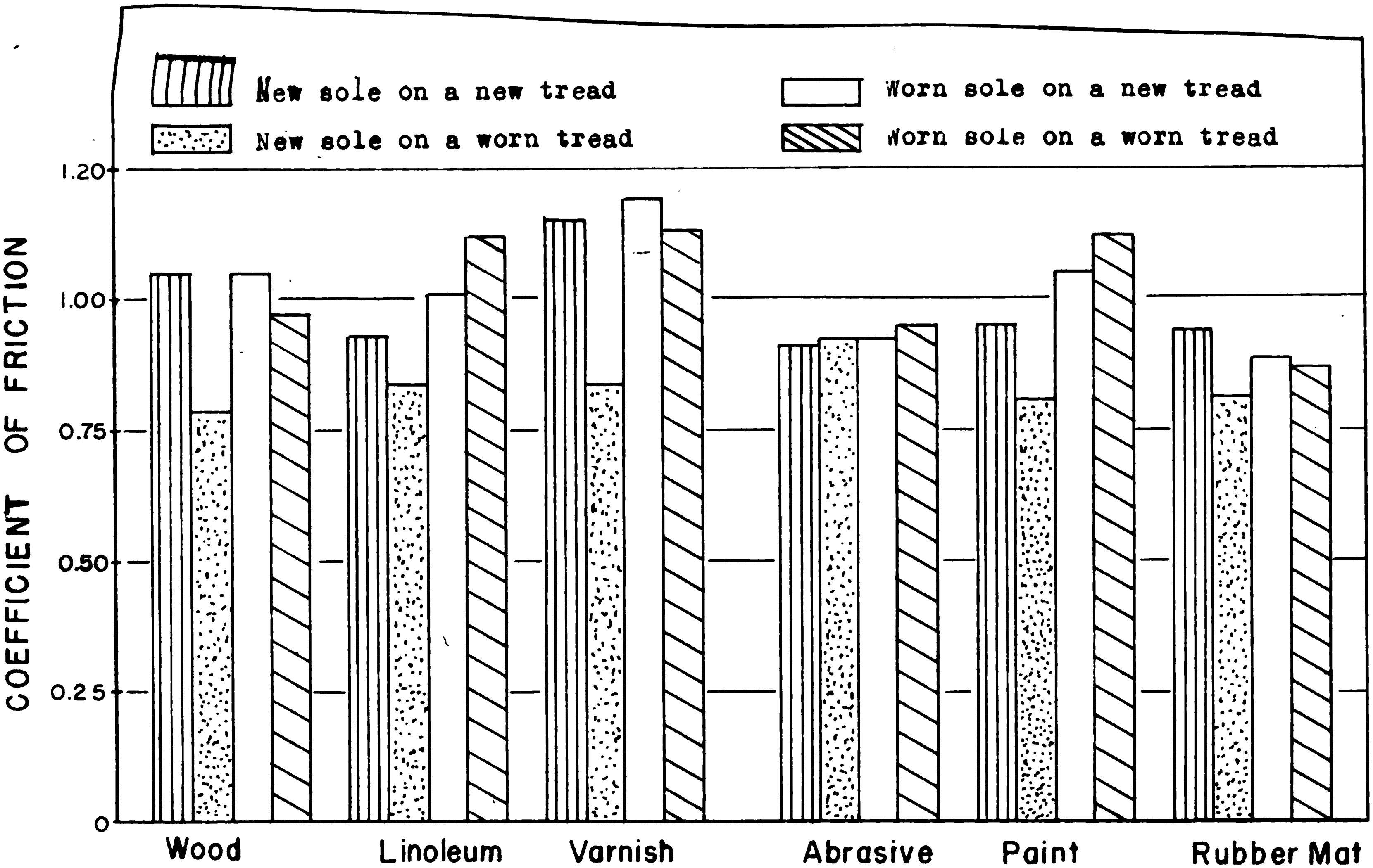


Figure 21: The average coefficient of friction for each of the simulated surface conditions and the large ripple sole.

remained constant or increased with use. To determine if there were significant changes in the coefficient of friction between new and worn conditions of a tread material the friction values for the conditions were compared using the factorial analysis. This comparison was performed only for the worn soles.

The results of the factorial analysis are in Table 2. The values given are the difference between the average coefficient of friction for the new tread material and the average value for the worn material. A negative sign indicates that the average friction value for the worn material was higher than the value of the new material. An asterisk indicates there was a significant difference between the two values at the 95% probability level.

For the large neoprene sole the new tread material had a significantly larger coefficient of friction than the worn material for the wood, varnish and painted treads. The two conditions of the linoleum tread were also significantly different except the worn tread had the highest coefficient of friction. For the small neoprene sole significant differences occurred with the varnished and painted treads. For both treads the new condition had the highest friction value.

For the large crepe sole the linoleum and rubber mat materials had a significant difference between their two surface conditions. In both cases the worn material had the highest coefficient of friction. For the small crepe sole there were significant differences between the two tread conditions of the linoleum varnish, paint and rubber mat materials. Except for the rubber mat material the highest coefficient of friction value was obtained on the surface.

The linoleum, varnish, paint and rubber mat materials had a significant difference between their two surface conditions for the large leather sole. Of these four materials, linoleum was the only one which had its highest coefficient of friction on the worn surface. The wood, varnish, paint and rubber mat materials had a significant difference between their new and worn surface condition for the small leather sole. The highest coefficient of friction was obtained on the new surface for each of the four materials.

For the large neolite sole the linoleum and varnish treads had significant differences between their two surface conditions. The linoleum had its highest value on the new material while the highest friction value for the varnish was obtained on the worn surface. The varnish also had a significant difference between its new and worn surface for the small neolite sole. However, its highest friction value was obtained on the new surface.

For the large ripple sole the linoleum material, which obtained its highest coefficient of friction on the surface, was the only material which had a significant difference between its new and worn surface. However, the

TABLE 2: The Results of the Factorial Analysis for the Two Surface Conditions of the Tread Materials with a Worn Sole.

	Wood	Linoleum	Varnish	Abras.	Paint	R. Mat
Large Neoprene	0.260*	-0.094*	0.224*	-0.010	0.076*	0.022
Small Neoprene	-0.035	0.095	0.154*	-0.024	0.091*	0.011
Large Crepe	0.024	-0.126*	-0.007	-0.022	-0.040	-0.074*
Small Crepe	0.014	0.107*	0.196*	0.020	0.094*	-0.116*
Large Leather	0.027	-0.161*	0.073*	0.034	0.157*	0.140*
Small Leather	0.060*	-0.027	0.106*	0.051	0.122*	0.146*
Large Neolite	-0.016	0.122*	-0.287*	0.027	0.031	0.030
Small Neolite	0.106	0.034	0.230*	0.000	0.011	0.021
Large B.F. Goodrich	0.074*	-0.003	0.042	-0.027	-0.069*	-0.028
Small B.F. Goodrich	0.111*	0.016	0.115*	0.013	0.020	0.010
Large Ripple	0.084	0.108*	0.057	0.031	-0.067	0.025
Small Ripple	-0.018	-0.164*	-0.031*	-0.133*	-0.125*	-0.013

linoleum, abrasive and painted treads had significant differences between their two surfaces for the small ripple sole. For each of these three materials its highest friction value was obtained on the worn surface.

The results given in Table 2 are summarized in Table 3. Table 3 gives the number of times a tread material had a significant difference between the two surface conditions (maximum of 12) and whether the highest coefficient of friction value was obtained on the new or worn surface. Table 3 shows that the varnished tread had the highest number of significant differences with eight and that on seven of these occasions the highest friction value was obtained on the new surface. This indicates that the varnish tread is the most susceptible to wear and that generally the coefficient of friction decreases with wear. The table also shows that the abrasive strip had a significant difference only once with the highest coefficient of friction being obtained on the worn surface. The fact of only one significant difference for the abrasive strip indicates that it is a very stable material and behaves the same under new or worn surface conditions for nearly all types of sole materials.

Analysis of Worn Soles on Worn Tread Materials

The combination of surface conditions most often encountered during the use of a stairway is a worn sole material on a worn tread material. The average coefficient of fr

Water

Wood

Linol

Thres

of friction for each sole and tread combination investigated under this condition is contained in Table 4.

TABLE 3: Summary of the Factorial Analysis

Material	No. of times significant	Highest friction value on New surface	Worn surface
Wood	4	4	0
Linoleum	7	3	4
Varnish	8	7	1
Abrasive	1	0	1
Paint	7	5	2
Rubber Mat	4	2	2

The six tread materials were compared with respect to a particular sole material. The worn abrasive strip had the highest coefficient of friction for the six tread materials on nine of the twelve soles. For the large worn ripple sole the worn varnish tread had the highest coefficient of friction value for the small ripple sole. For the small leather sole the rubber mat material gave the highest friction value.

The data for the six worn tread materials was grouped together for each worn sole. From these data the mean and the standard deviation was calculated for each of the soles. The mean, standard deviation and the interval of plus and minus one standard deviation from the mean, ($\overline{X}$ S), which includes 68% of the data, is given in Table 5.

All of the standard deviation values were in the range of 0.06 to 0.14 except for the large leather sole. This sole had a standard deviation of 0.21. The standard deviation obtained from the large leather sole is due to

TABLE 4: The Average Coefficient of Friction Value for Worn Sole Materials on Worn Tread Materials.

	Wood	Linoleum	Varnish	Abrasive	Paint	R. Mat
Large Neoprene	0.56	0.60	0.58	0.88	0.54	0.72
Small Neoprene	0.71	0.56	0.60	0.83	0.60	0.70
Large Crepe	0.55	0.53	0.49	0.74	0.47	0.70
Small Crepe	0.46	0.41	0.39	0.68	0.42	0.66
Large Leather	0.34	0.42	0.37	0.84	0.44	0.81
Small Leather	0.26	0.29	0.25	0.51	0.28	0.60
Large Neolite	0.63	0.58	0.63	0.74	0.68	0.56
Small Neolite	0.56	0.57	0.65	0.66	0.65	0.55
Large B.F. Goodrich	0.55	0.55	0.59	0.66	0.60	0.56
Small B.F.Goodrich	0.59	0.61	0.62	0.71	0.65	0.63
Large Ripple	0.97	1.12	1.13	0.95	1.12	0.87
Small Ripple	1.08	1.15	1.20	1.07	1.20	0.89

TABLE 5: The Mean ($\overline{X}$), Standard Deviation (S), and the interval of $\overline{X} \pm S$ for a Worn Sole Material with the Six Worn Tread Materials analyzed together.

Tread Material	Large Sole			Small Sole		
	$\overline{X}$	S	X S	$\overline{X}$	S	X S
Ripple	1.03	0.12	0.91-1.15	1.10	0.13	0.97-1.23
Neoprene	0.65	0.13	0.52-0.78	0.67	0.10	0.57-0.77
Neolite	0.63	0.08	0.55-0.71	0.61	0.06	0.55-0.67
Crepe	0.58	0.11	0.47-0.69	0.51	0.12	0.39-0.63
B.F.Goodrich	0.57	0.12	0.45-0.69	0.63	0.06	0.58-0.69
Leather	0.54	0.21	0.33-0.15	0.37	0.14	0.23-0.51

the wide difference between the coefficient of friction values on the tread materials. The large leather sole had friction values ranging from 0.34 on the wood tread to 0.84 for the abrasive strip. This is a range of 0.50 for the leather sole while the ranges for the other soles varied from 0.11 to 0.35. The large standard deviation for the leather sole would indicate that the frictional properties of this sole change considerably with the tread material. The other soles indicate less variation between tread materials which may make them safer.

Analysis of Sole Materials Independent of Tread Materials

The average coefficient of friction of the sole materials investigated is illustrated graphically in Figures 22 and 23 and given numerically in Table 6. Each of the values is an average of 108 measurements except for the ripple sole and includes measurements from each of the twelve tread conditions. The values for the ripple sole are an average of 72 measurements.

As a general rule the worn soles had a higher coefficient of friction than did the new soles when comparing the two conditions of a particular sole material. Of the six sole materials investigated the crepe was the only material in which the new sole had a higher coefficient of friction than the worn sole for both sizes of sole. The large neoprene and the small ripple soles both had a higher coefficient of friction for the new materials. However, there

condi

Sole

there was very little difference between the new and worn conditions, 0.01 and 0.02 respectively.

TABLE 6: The Average Coefficient of Friction for the New and Worn Condition of each Sole.

Sole Material	Large Sole		Small Sole	
	New	Worn	New	Worn
Ripple	0.91	1.02	1.08	1.06
Neoprene	0.70	0.69	0.59	0.69
Neolite	0.61	0.66	0.58	0.64
Crepe	0.60	0.55	0.66	0.51
B.F. Goodrich	0.48	0.59	0.45	0.65
Leather	0.47	0.54	0.32	0.37

An average coefficient of friction value was calculated for each sole size of each sole material. This value was for the twelve tread conditions. The coefficient of friction values were ranked from highest to lowest, see Table 7. Although the average friction value varied between the two sole sizes the order of arrangement was the same for both. The ripple sole had the highest friction value followed by the neoprene, neolite, crepe, B.F. Goodrich and leather soles.

TABLE 7: The Average Coefficient of Friction Values for the Shoe Sole Materials.

Sole Material	Large Sole	Small Sole
Ripple	0.97	1.07
Neoprene	0.70	0.64
Neolite	0.64	0.61
Crepe	0.58	0.59
B.F. Goodrich	0.54	0.55
Leather	0.51	0.35

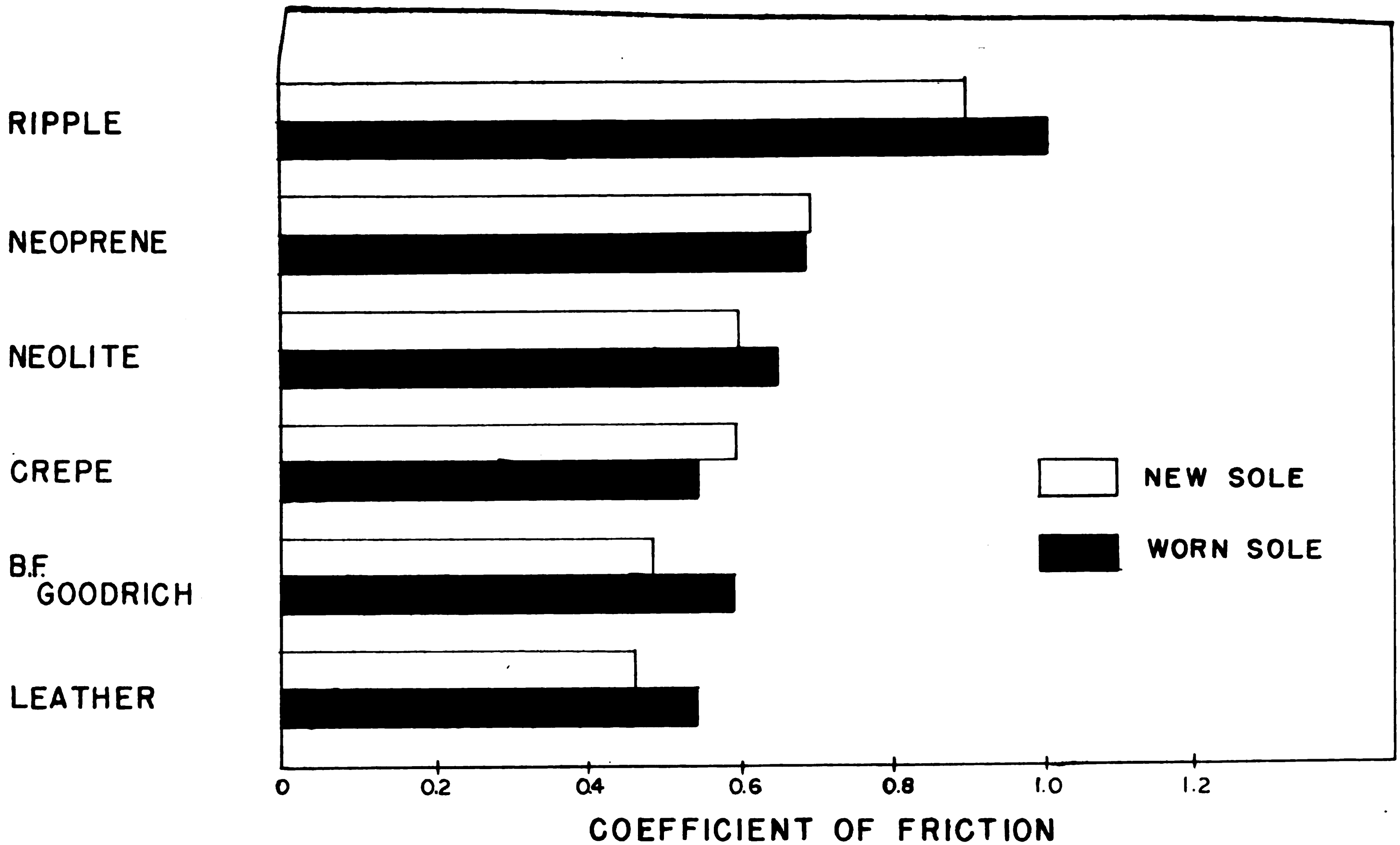


Figure 22: The average coefficient of friction for the large soles.

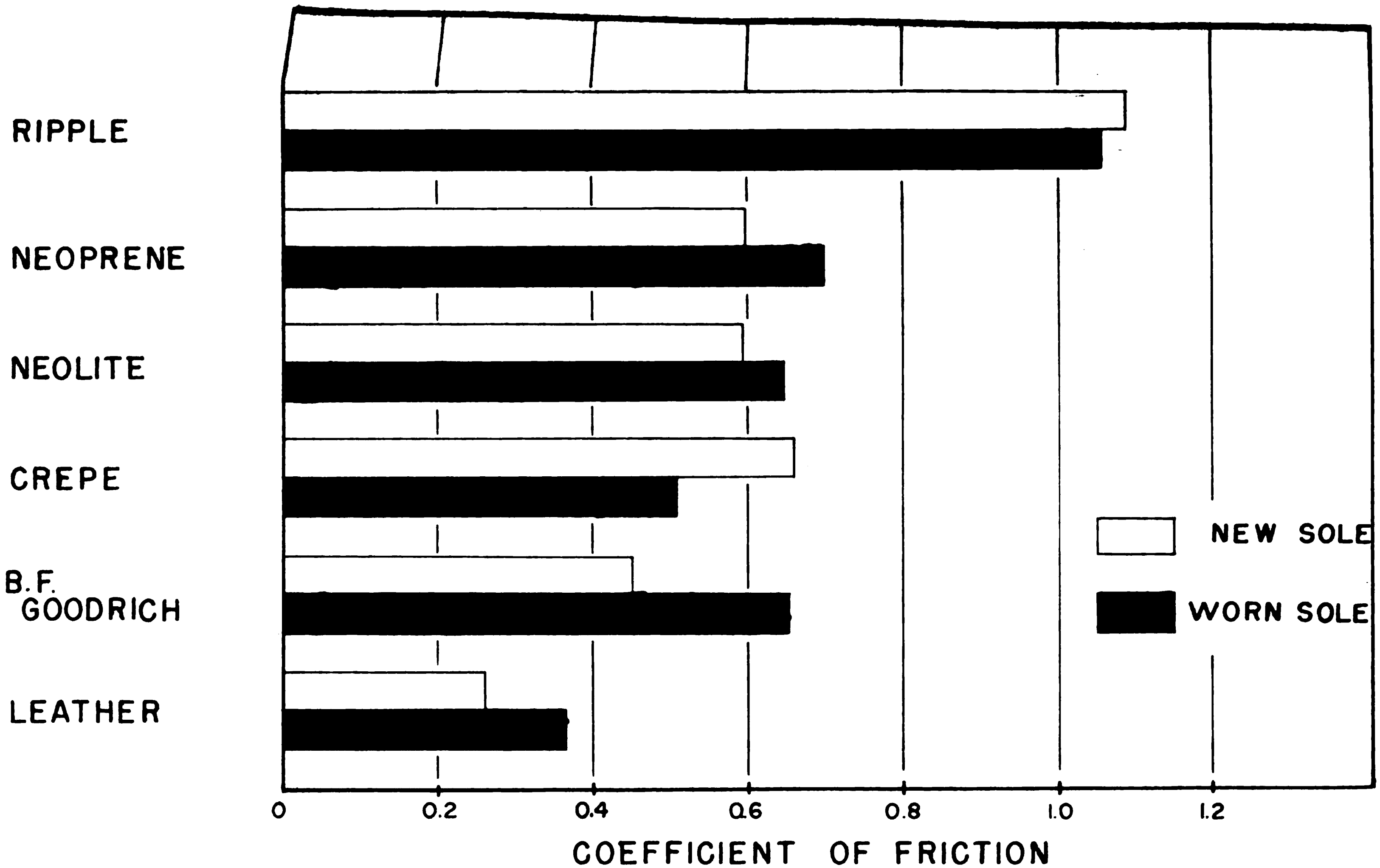


Figure 23: The average coefficient of friction for the small soles.

Discussion of Results

For most of the conditions investigated during this study the abrasive material had the highest coefficient of friction values. This was most likely due to the fact that the abrasive strip had a rough surface. When a sole was placed on the abrasive strip and put under pressure the asperities of the abrasive material would embed themselves into the sole material. Thus a larger horizontal force was required to move the sole over the abrasive material.

During this investigation the linoleum tread material showed a tendency of having a higher coefficient of friction after it had become worn than when it was new. This occurred because of the increased roughness of the material once it had become worn. The new linoleum material was very smooth and as a result some sole materials did not make good contact with its surface. This was generally true with the large soles. Once the linoleum had become worn it contained more asperities which gripped the sole material, increasing the resistance to sliding.

Table 1 shows that there was a large difference between the standard deviations of the data for different tread materials. There appear to be two primary properties of the materials which accounted for a large share of the differences. These properties are the homogeneousness and the durability of the tread material. The tread materials which are made from ingredients that wear well and where each layer of material is like the layer preceding it, usually gave the lowest standard deviations. The abrasive strip and rubber mat materials are good indicators of this fact. For these two materials the condition of their surfaces changed little as they became worn.

The wood, varnish and linoleum treads underwent a large change in their surface condition once they became worn. The wood tread became smoother, the varnish wore off and also became smoother, while the linoleum roughened and contained more asperities. While this is not a fault for the linoleum it does account for the large standard deviation. For each of these three treads the change in their surface conditions caused a variation in the coefficient of friction. The standard deviation is a measure of this variation.

These same reasons also account for the fact that some tread materials had few significant differences between their new and worn surfaces while others had a larger number of significant differences between their two surface conditions.

A possible reason that the worn soles generally had a higher coefficient of friction than the new soles is the condition of the new soles. The new soles had a smooth hard surface and therefore made poor contact with the tread material. The sole material would slide across the tread with relatively little resistance. However, once the sole had become worn, the hard outside layer was replaced with a softer layer of material. This layer of material contained many small asperities which gripped to the tread material and increased the sliding resistance of the sole material.

The crepe material acted differently, however, because the new sole was somewhat flexible and did not have a hard surface covering. The new crepe material was able to take the shape of the tread when it was placed under a load. However, when the crepe sole became worn and/or was placed on a worn tread material the asperities did not allow as tight a contact with the tread material. The asperities acted much like miniature ball bearings and decreased the sliding resistance of the sole.

The high values which were obtained with the ripple sole could be the result of two conditions: 1) the contact area of the ripple sole was less than the contact area of the other soles. This resulted in a higher psi for the contact area. 2) The ripple sole was considerably more flexible than the other soles, which allowed the sole to take the shape of the tread. The ability of the ripple sole to make a good surface contact could have resulted in a seal formed between the sole and tread material giving the effect of a partial vacuum. If this were the case an additional vertical load would be produced by the pressure differential. If it were possible to measure this extra load and use it in calculating the coefficient, a smaller

value would have been obtained.

Sigler, et al. (15) while making coefficient of friction measurements on wet and oily walk-way surfaces found these surfaces to have a higher coefficient of friction than dry surfaces. Sigler stated that when walking on wet surfaces it was more difficult to obtain a foothold, but after a firm contact had been made there was less tendency to slip on the wet surface than on a dry surface of the same material. Sigler also observed that this was especially true on smoothface plane surfaces. He stated that the increase in the coefficient of friction was possibly due to a perfect contact giving the effect of a partial vacuum under the shoe.

When comparing the different tread materials it will be noticed that all of the smooth surface treads had higher coefficients of friction with the ripple sole than did the two tread materials with rough surfaces. The rubber mat, which had a grooved surface, had the lowest average coefficient of friction of the six tread materials investigated. The abrasive strip, which was moderately rough, had the next to the lowest friction value. Because of the similarity between this investigation and the work reported by Sigler, et al., the effect of a partial vacuum may have been a reason for the high frictional values of the ripple sole.

The variation in the coefficient of friction between the other sole materials was believed to be due primarily to the characteristics of the individual materials.

FORCES EXERTED ON A STAIR STEP

When a person ascends or descends a stairway he exerts a vertical force and a horizontal force toward either the front or rear edge of the tread. If the ratio of the horizontal force to the vertical force is greater than the coefficient of friction for the particular tread material the person's shoe will slide on the tread material. To prevent the shoe from slipping the coefficient of friction must be greater than the ratio of the two forces.

The magnitude of the vertical and horizontal force was measured to determine what coefficient of friction a material must have to prevent slipping. The point of application of the vertical force was also measured to determine where this force was applied.

Analysis of Experiment

The two variable classification method of analysis of variance was used to analyze most of the stairway data. The nine stairways were treated as one variable and the nine percent points were the other variable. The two variable classification contained four items per cell. A separate analysis was made for each of the eight combinations of subject, force and direction of travel. The analysis of variance table with degrees of freedom is shown in Table 8. The experimental error term (SxPP) was tested using the sampling error term (Error). If the SxPP term was found to be significant from the error term then the stairway and percent point terms were tested using the SxPP term, otherwise they were tested with the error term.

TABLE 8: The Analysis of Variance Table with Degrees of Freedom for the Two Variable Classifications involving Stairways and Percent Points.

	df
Total	323
Stairways	8
Percent Points	8
SxPP	64
Error	243

Because the vertical and horizontal forces were not measured at the same time and the rate of travel by a subject was not controlled it was possible for a subject to be moving at different rates when the two forces were measured. To determine if there was any difference in a subject's speed when the two forces were measured a two way classification analysis with four items per cell was performed. The nine stairways were treated as one variable and the two forces (vertical and horizontal) were the other variable. The length of time the foot was on the tread was the data analyzed. The error term was used to test the stairway and force terms unless the FxS term was significant from the error term. If the FxS term was significant it was used to test the two terms. The analysis of variance table with degrees of freedom for this analysis is given in Table 9. A separate analysis was made for each combination of subject and direction of travel.

TABLE 9: The Analysis of Variance Table with Degrees of Freedom for the Two Variable Classification involving Stairways and Forces.

	df
Total	71
Stairways	8
Forces	1
FxS	8
Error	54

The data for the initial point of application of the vertical force was analyzed by the two way classification method. However, for the analysis the replications were treated as a variable. The stairways were treated as the other variable. Using the replications as a variable resulted in an analysis with one value per cell. A separate analysis was performed for each combination of subject and direction of travel. The analysis of variance table with degrees of freedom is given in Table 10. The stairway and replication terms were tested with the error term (SXR).

TABLE 10: The Analysis of Variance Table with Degrees of Freedom for the Two Variable Classification involving Stairways and Replications.

	df
Total	35
Stairways	8
Replications	3
SxR	24

The initial point of application analysis for each subject while ascending or descending were combined to determine if there was a significant difference between the two subjects. The two variable classification was used, however, the two variables were subjects and stairways. The replications were treated as replications making four values per cell. Two analyses were performed, one for each direction of travel. The analysis of variance table with degrees of freedom is given in Table 11.

TABLE 11: The Analysis of Variance Table with Degrees of Freedom for the Two Variable Classification involving Stairways and Subjects.

	df
Total	71
Stairways	8
Subjects	1
S x S	8
Error	54

All of the tests for significant differences in the two variable classification analyses were made at the 95 percent probability level. All block and treatment averages were compared using the "Studentized Range Test" for the same probability level. For the comparison of block and treatment averages given in this thesis <u>all averages under the same line are not significantly different</u>.

Because of the difference in body weight it was difficult to compare the forces exerted by the two subjects using analysis of variance methods. Therefore, a comparison between the two subjects was made with the method of rank correlation. This method, as described by Walker and Lev (19) uses the ranking of a particular group of items by two people as a comparison of the two individuals. For this investigation the two subjects were compared by how they ranked the stairways or percent points for a particular force. The averages were usually ranked from highest to lowest. An example of the calculation of a rank correlation value is given in Appendix IV. It was not possible to check for significant differences between the two subjects because Walker and Lev did not give a method of testing.

The references used in the experimental design and analysis were Dixon and Massey (4), Steel and Torrie (17) and Walker and Lev (19).

Percent Error

All of the calculated values of the vertical force were within five percent of the true value. However, the percent error in the horizontal force ranged from five to greater than twenty percent. The wide range was due mainly to a reading error for the oscillograph charts. The charts could only be read accurately to plus or minus one pound. For small values of the horizontal force the reading error was nearly as large as the true value. However, as the horizontal force becomes larger the percent error is reduced. Although the percent error was high all of the values measured are accurate to plus or minus one pound. All of the values for the initial point of application of the vertical force were within the range of 0.125 inches, which was the initial control limits.

Time Spent on a Tread

It has been stated previously that the horizontal and vertical forces were measured separately and that the subjects ascended and descended the stairways at their own natural speed. Because the rate of travel was not controlled it was possible for a subject to be moving at different rates when the two forces were measured. A statistical analysis was performed to determine if there was any difference in a subject's speed when the two forces were measured. The same statistical analysis was used to determine if there was a difference in the rate of travel on the nine stairways. The length of time the foot was on the tread was used as a measure of the rate of travel.

The statistical analysis for the 140 pound subject, while ascending the stairways, indicated no significant difference between the average rate of travel while measuring the vertical force and the average rate of travel while measuring the horizontal force. However, the analysis did indicate a significant difference between the stairway averages. A comparison of the average time spent on a tread while ascending each stairway is given in Figure 24. Figure 24 indicates that the highest average, 0.79 seconds, occurred on the 8-10 (8 inch risers and 10 inch treads) stairway and was significantly different from only the averages of the 7-11 and 7-9 stairways. The lowest average, 0.68 seconds, was significantly different from all averages except for the 7-11 stairway.

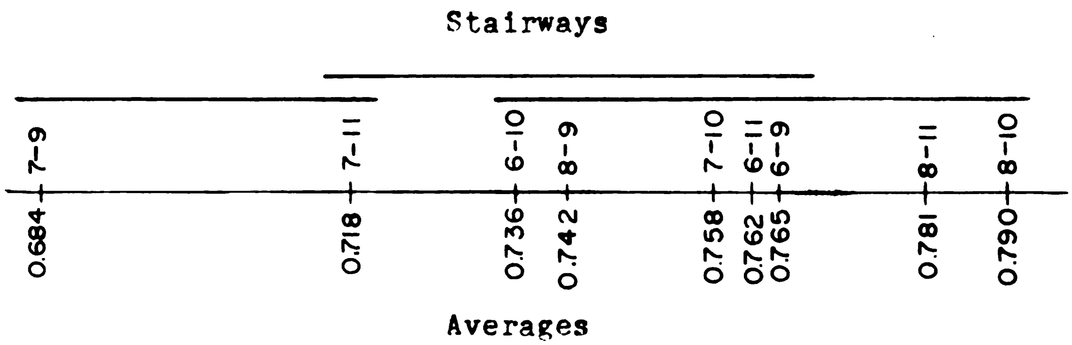


Figure 24: The comparison of the stairway averages for the time spent on a tread, 140 pound subject ascending the stairways.

The statistical analysis indicated no significant differences between either the stairway averages or the averages for the two forces for the 140 pound subject descending and the 172 pound subject ascending and descending the stairways.

The average time spent on a tread while measuring the vertical force and the horizontal force was given in Table 12. Each value in Table 12 is an average of thirty-six measurements. The average time spent on a tread by each subject while ascending and descending each stairway is given in Table 13. Each value in Table 13 is an average of four measurements. Table 12 indicates that the average time spent on a tread was nearly the same when ascending or descending. However, the 172 pound subject traveled at a faster rate than the 140 pound subject.

Ascending a Stairway

The event of ascending a stairway may be divided into three basic periods, the "set", "lift" and "swing" periods.

TABLE 12: The Average Time (seconds) spent on a Tread when measuring the Horizontal and Vertical Forces, average for the Nine Stairways.

	140 Pound Subject		172 Pound Subject	
	Ascending	Descending	Ascending	Descending
Vertical Force	0.73	0.69	0.62	0.63
Horizontal Force	0.72	0.67	0.61	0.61

TABLE 13: The Average Time (seconds) spent on a Tread by each Subject while Ascending and Descending each of the Stairways.

		STAIRWAYS								
		6-9	6-10	6-11	7-9	7-10	7-11	8-9	8-10	8-11
140 Pound Subject	Ascend.	0.77	0.74	0.76	0.68	0.76	0.72	0.74	0.79	0.78
	Descend.	0.68	0.70	0.67	0.66	0.64	0.70	0.67	0.69	0.68
172 Pound Subject	Ascend.	0.67	0.61	0.61	0.62	0.63	0.58	0.60	0.61	0.64
	Descend.	0.66	0.62	0.59	0.63	0.64	0.58	0.60	0.59	0.64

Suppose a person standing at the bottom of a stairway decides he wants to ascend it. The person's first move would be to lift one foot from the floor and place it on the first step. The instant at which the foot is placed on the step will be called the "set" period. The forefoot shall be defined as the foot which is either stationary or on the higher of any two steps. For this example the forefoot is on the first step. The rearfoot will be defined as the foot which is either moving or is on the lower of any two steps. For this example the rearfoot is on the floor.

The person's next movement would be to apply pressure on the tread with his forefoot and lift his rearfoot off the floor. The interval during which this movement occurs will be termed the "lift" period. The final phase is moving the rearfoot forward and placing it on the next step. The interval during which this movement occurs will be defined as the "swing" period. When the person's rearfoot is placed on the upper step he is again at the "set" period and the rearfoot now becomes the forefoot and vice versa.

The event of ascending a stairway is simply a continuous repetition of the "set", "lift" and "swing" periods. The three periods are illustrated in Figure 25.

Vertical Force, Ascending

The vertical force exerted by the 140 pound subject

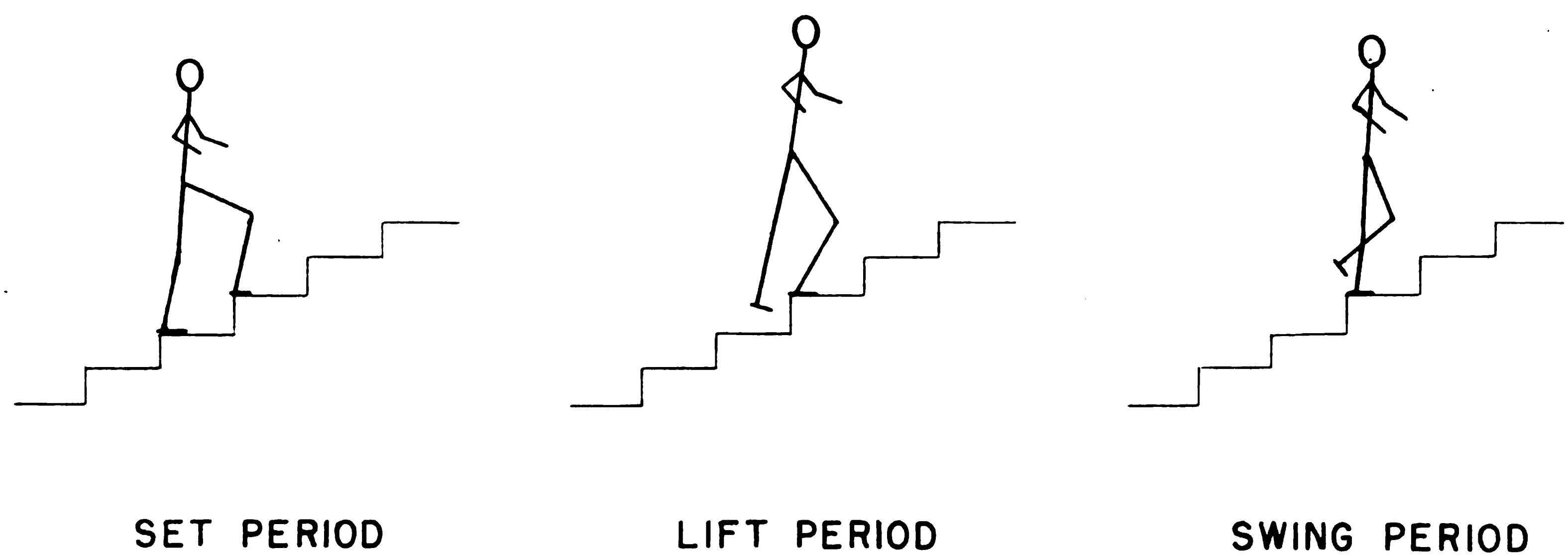


Figure 25: The basic divisions of one complete step while ascending a stairway.

while ascending a stairway with 6 inch risers and 11 inch treads is shown in Figure 26. Figure 26 illustrates the general pattern of the vertical force while ascending different stairways.

When a person ascends a stairway two relative maximum values in the vertical force occur. The first maximum takes place during the "lift" period and occurs when the forefoot is applying pressure on the force plate and lifting the rearfoot from the lower tread. The first maximum value was usually greater than the body weight. A force greater than body weight occurred because the body weight is supported entirely by the forefoot and this foot is also applying a force on the tread. The first maximum value occurred approximately when 25 percent of the step had been completed.

After the first maximum point was reached a depression occurs in the force curve. The depression occurs during the "swing" period. Although the total body weight was on the forefoot during this period, the total force measured was less than body weight. Since F = ma (Newton's Law) the smaller force may possibly be explained by the upward acceleration of different parts of the body which produce forces that partially offset the body weight.

The second maximum point occurs during the "set" period. The body is under little upward acceleration due to the completion of the "swing" period. The rearfoot which is now on the upper step supports little of the body

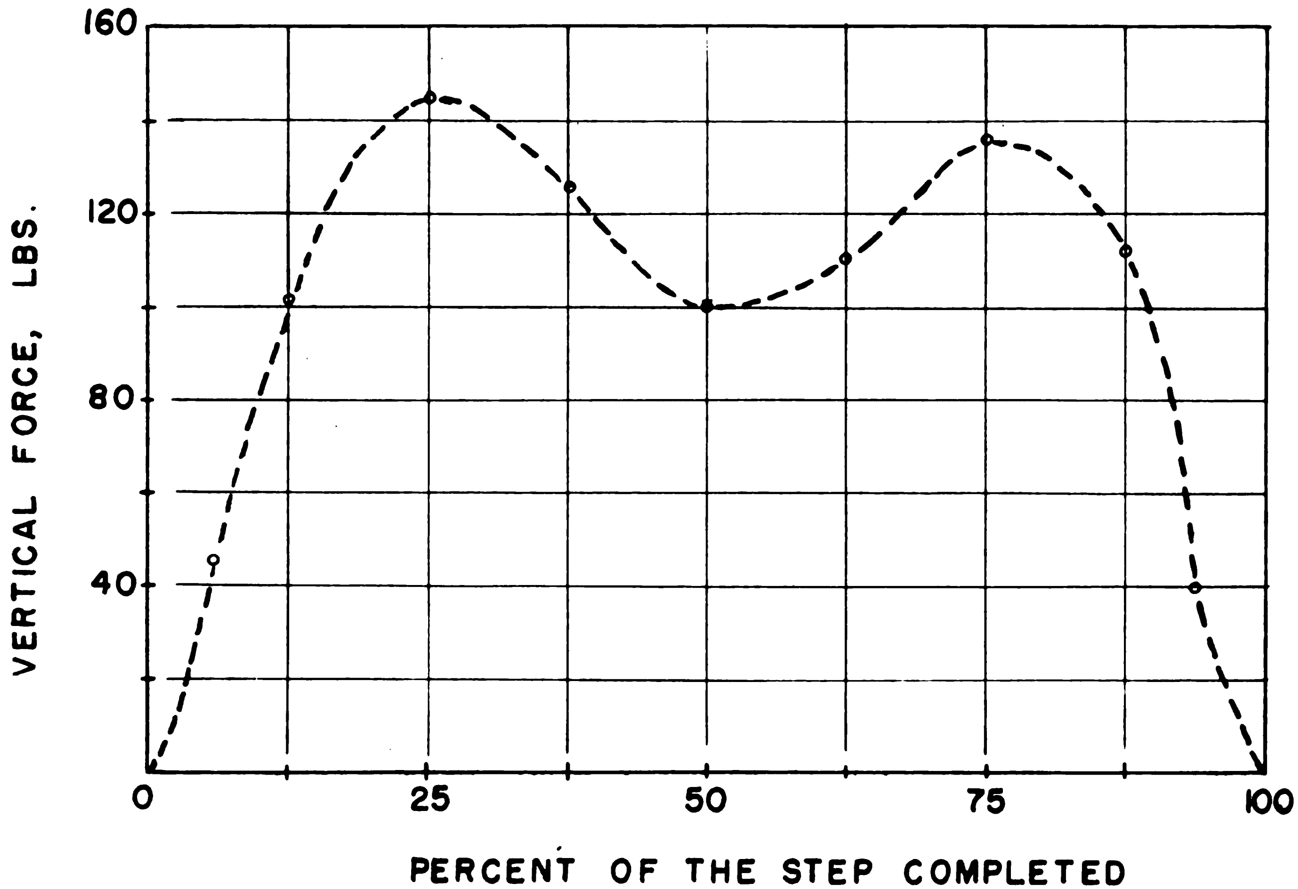


Figure 26: The vertical force exerted by the 140 pound subject while ascending a stairway with 6 inch risers and 11 inch treads.

weight. Thus most of the body weight is still on the force plate. The second maximum value is usually less than the total body weight and occurs approximately when 75 percent of the step has been completed.

Table 14 gives the value of the vertical force exerted on each stairway at the 25 percent point. This value was the largest value which was calculated; however, this value was not necessarily the maximum vertical force exerted on the step. The maximum force may have been exerted either just before or after the 25 percent point.

For the 140 pound subject there was little difference between the values at the 25 percent point. The values range from 139.7 pounds on the 6-9 stairway to 159.1 pounds on the 7-11 stairway. There was no definite relationship between the force exerted and the riser height. However, for each riser height the maximum value occurred on the stairway with an 11 inch tread.

For the 172 pound subject there was a greater variation in the values with a minimum of 174.9 pounds on the 8-9 stairway and a maximum of 215.6 pounds on the 8-11 stairway. Again there is no definite relationship between the force exerted and riser height. But for each riser height the maximum value occurred on the stairway with an 11 inch tread.

A statistical analysis of the vertical force for the 140 pound subject showed no significant difference between the average force exerted on each of the nine stairways.

TABLE 14: The Vertical Force exerted at the 25 Percent Point while ascending each Stairway.

		STAIRWAYS								
		6-9	6-10	6-11	7-9	7-10	7-11	8-9	8-10	8-11
140 lb. Subject	Force	139.7	143.6	144.5	148.1	139.2	159.1	149.1	145.5	154.2
	% of Body Weight	99.5	102.5	103.1	105.8	99.4	113.6	106.5	103.8	110.0
172 lb. Subject	Force	195.9	192.4	201.0	182.8	183.7	200.4	174.9	199.3	215.6
	% of Body Weight	114.0	112.0	116.8	106.1	106.8	116.7	101.5	116.0	125.0

However, the analysis did show significant differences between percent points (percent of the step completed.) The analysis indicated no significant difference between the 50.0, 87.5, 12.5 and 62.5 percent points. Each of the remaining points was significantly different from all the other points. The highest average, 147.0 pounds, occurred at the 25 percent point. The lowest average was 27.8 pounds for the 93.75 percent point. The comparison of percent averages is shown in Figure 27. All values under the same line are not significantly different from each other.

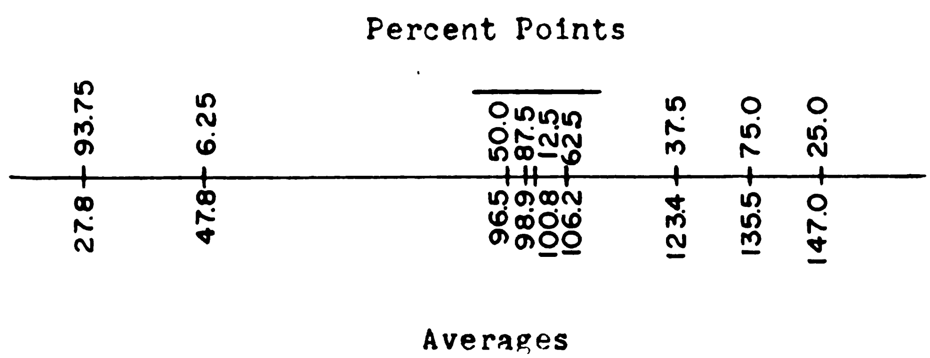


Figure 27: Comparison of the percent point averages for the vertical force, 140 pound subject ascending the stairways.

For the 172 pound subject the statistical analysis gave nearly the same results. There was no significant difference between the average force exerted on each of the nine stairways. However, there was significant difference between the averages of the percent points. The highest average, 194.0 pounds, occurred at the 25 percent point. This value was significantly different from all the other averages. The lowest average was 28.1 pounds for the 93.75 percent point. This value was also significantly different from all the other averages. The comparison of percent averages is given in Figure 28.

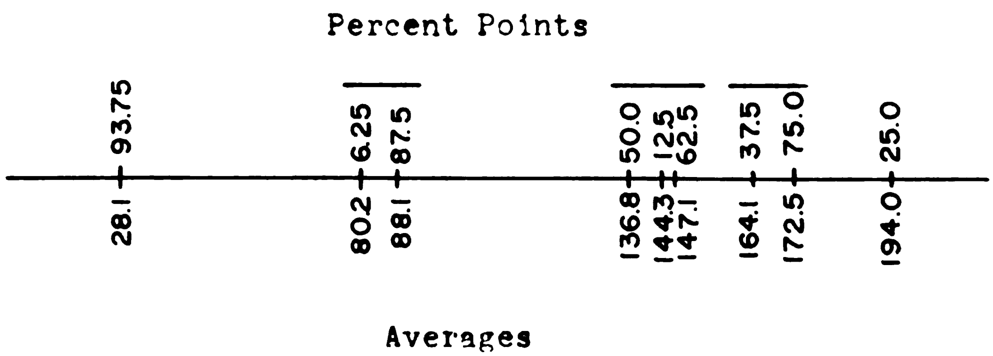


Figure 28: Comparison of the percent point averages for the vertical force, 172 pound subject ascending the stairways.

The percent point averages of the two subjects were compared using the method of rank correlation. The analysis gave a correlation value of R = 0.98. The correlation value indicates that the vertical force curve for each subject has the same pattern. Figure 29 indicates that the force curves do follow the same pattern. The difference in the two curves is due to a difference in body weight.

The average vertical force exerted by each subject while ascending each of the nine stairways is given in Table 15. The average vertical force exerted by each subject for each of the percent points is given in Table 16. Each of the averages in Tables 15 and 16 contains thirty-six measurements.

Figure 29 shows the average vertical force at each

TABLE 15: The Average Vertical Force exerted while ascending each Stairway.

		STAIRWAYS								
		6-9	6-10	6-11	7-9	7-10	7-11	8-9	8-10	8-11
140 lb. Subject	Force, lbs.	95.9	97.0	101.9	97.6	91.6	103.2	102.2	98.1	97.4
172 lb. Subject	Force, lbs.	128.1	126.5	127.4	123.9	124.8	127.8	127.6	134.7	134.7

TABLE 16: The Average Vertical Force exerted at each Percent Point while ascending the Nine Stairways.

		PERCENT POINTS								
		6.25	12.5	25.0	37.5	50.0	62.5	75.0	87.5	93.75
140 lb. Subject	Force, lbs.	47.8	100.8	147.0	123.4	96.5	106.2	135.5	98.9	27.8
	% of Body Weight	34.2	72.0	105.0	88.1	68.9	75.9	96.8	70.5	19.8
172 lb. Subject	Force, lbs.	80.2	144.3	194.0	164.1	136.8	147.1	172.5	88.1	28.1
	% of Body Weight	46.6	83.9	112.0	95.4	79.5	85.5	100.1	51.2	16.3

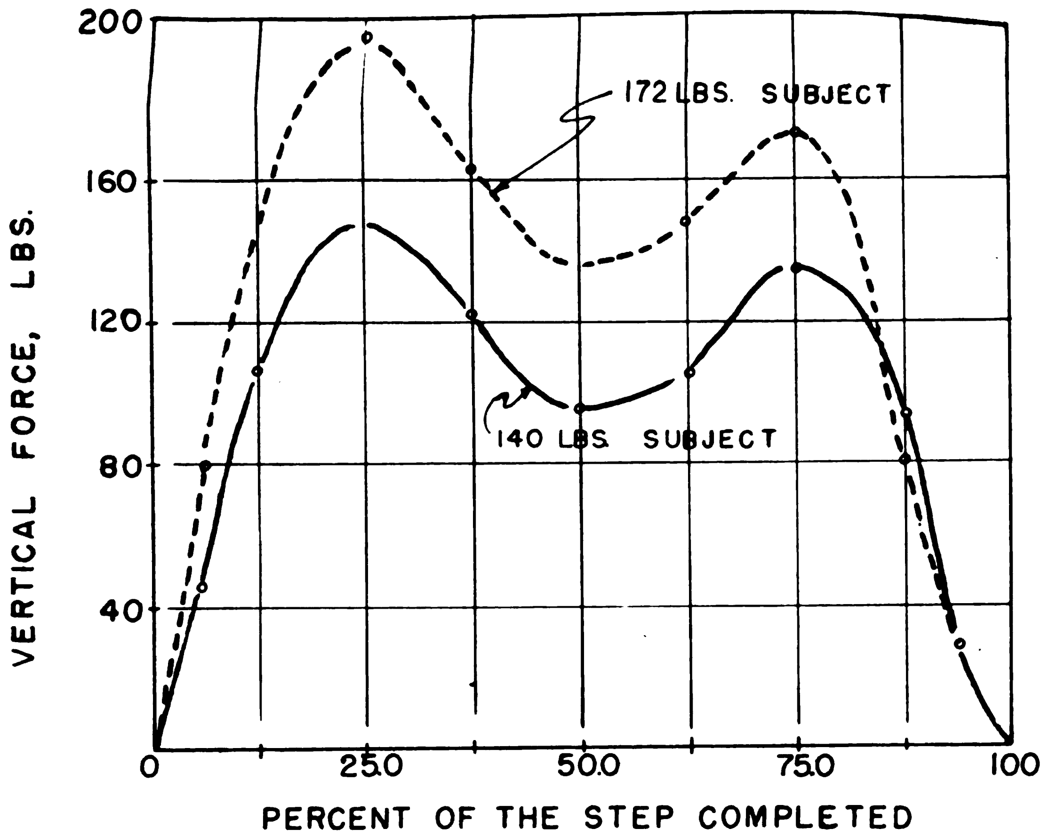


Figure 29: The vertical force exerted by the two subjects while ascending the stairways. Average for the nine stairways.

percent point for both subjects. It should be noted that the curves for both subjects were very similar. The rate of application of the vertical force was very rapid for each subject. The 140 pound subject applied a load that was 34.2 percent of his body weight in the first 6.25 percent of the step. The 140 pound subject had applied a load that was 72.0 percent of his body weight in the first 12.5 percent of the step. The 172 pound subject showed the same characteristic by applying loads that were 46.6 and 83.9 percent of his body weight in the first 6.25 and 12.5 percent of the step. The values for each subject indicate that the force exerted on the tread increases nearly linearly with time during the "lift" period.

The curves in Figure 29 indicate that the "lift" period occupies the first 25 percent of the step, the "swing" period the next 50 percent and the "lift" period the last 25 percent. The first "lift" period is the interval when the forefoot is applying pressure on the tread to lift the body. The second "lift" period is the interval when the rearfoot is being lifted from the tread. The "set" period, which is the short interval between the "swing" and the second "lift" period, occurs approximately at the 75 percent point. The length of the "set" period is negligible compared to the other two periods.

Figure 30 shows the average vertical force at each percent point as a percent of the body weight. The maximum values of 105.5 and 112.5 percent of body weight for the

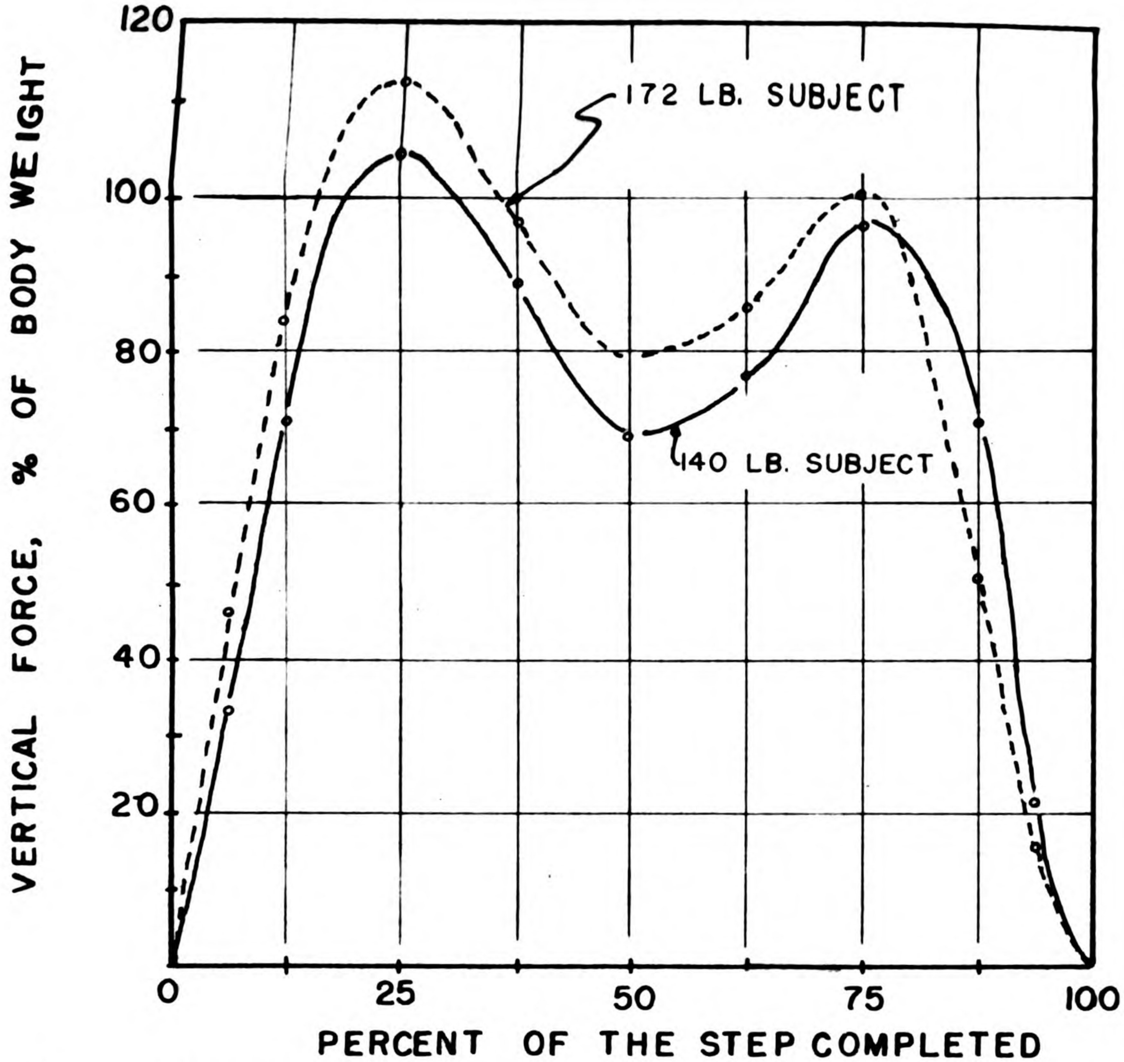


Figure 30: The vertical force, as a percent of body weight, exerted by the subjects while ascending the stairways. Average for the nine stairways.

140 pound and 172 pound subjects, occurred at the end of the "lift" period. The maximum values at the "set" period were 96.85 and 100.1 percent for the 140 and 172 pound subjects.

Horizontal Force, Ascending

The horizontal force exerted by the 140 pound subject on three different stairways is shown in Figure 31. Each curve illustrates the general pattern of the horizontal force while ascending a stairway. However, the curves are located differently on the coordinate scale for different riser heights.

The general pattern of the curve indicates that a positive horizontal force was applied at the beginning of the "lift" period. A positive horizontal force was directed toward the front edge of the tread. The horizontal force changed direction shortly after the start of the "lift" period and remained directed toward the rear of the tread until approximately a quarter of the "swing" period was completed. The force was directed toward the front edge for the remainder of the step.

A possible reason the horizontal force changed direction during the "lift" period was that the centroid of the body was moving toward the rear of the tread. A movement toward the rear of the tread may produce a negatively directed force which offsets any positive force produced by the forefoot. The body movement was also aided by a push

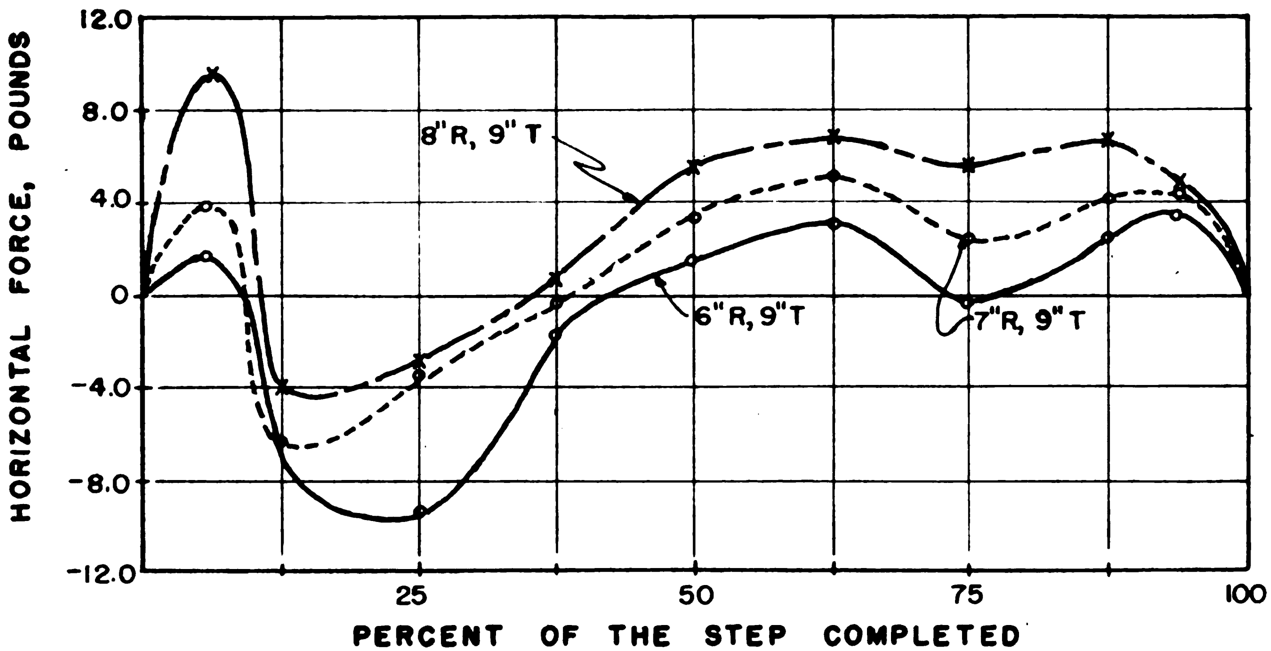


Figure 31: The horizontal force exerted by the 140 pound subject while ascending three different stairways.

from the rearfoot. The force exerted by the rearfoot may tend to reduce the amount of force exerted by the forefoot. During the last part of the "lift" period and the start of the "swing" period the centroid of the body was moving over the point where the horizontal force was applied. Thus the positive horizontal force may be small compared to the negative force exerted by the body during its movement.

The horizontal force shifts back to the positive direction during the "swing" period because the centroid of the body is moving away from the forefoot. The forefoot has to exert a positive force to give the body this movement.

The curves in Figure 31 indicate a slight dip at the 75 percent point. The "set" period occurs around this pount and some of the body weight was shifted to the foot on the upper tread. The added reaction may reduce some of the horizontal force needed for stability. The final rise in the curves is due to the push off by the rearfoot as the subject moves to the next step.

Although the curves in Figure 31 represent the general pattern of the horizontal force there were some stairways on which the horizontal force varied considerably from this pattern. The curves in Figure 32, for the 172 pound subject illustrate some of the variations. The horizontal force exerted on the 8-9 stairway was positive for the complete step. The 172 pound subject also showed this same characteristic on the 7-9 stairway. The horizontal

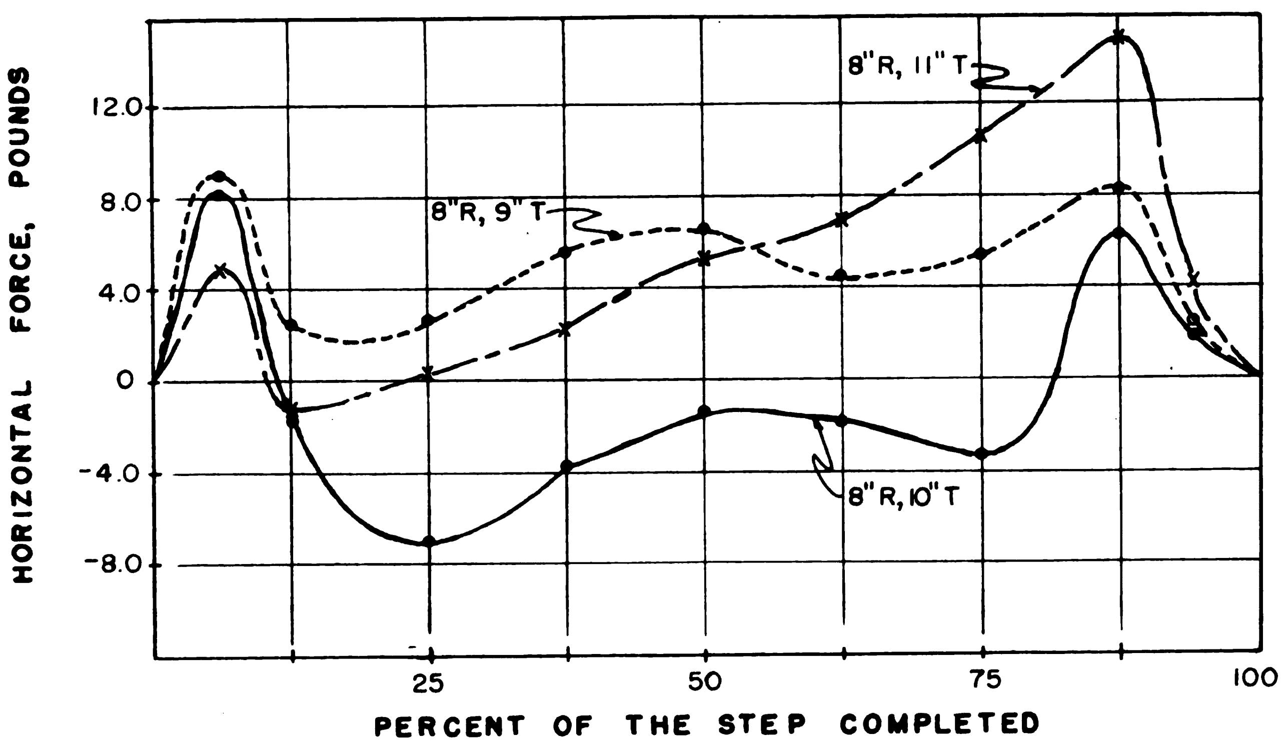


Figure 32: The horizontal force exerted by the 172 pound subject while ascending three different stairways.

force on the 8-10 stairway was completely positive except for the 12.5 percent point. However, the horizontal force on the 8-11 stairway was negative for nearly two thirds of the step.

Table 17 gives the value of the horizontal force exerted on each stairway at the 6.25 percent point. The horizontal force at the 6.25 percent point was directed toward the front edge of the tread and was the force which would cause a person to slip when ascending a stairway. The calculated value which was given was not necessarily the maximum horizontal force exerted. A higher value could have been exerted either just before or after the 6.25 percent point.

For the 140 pound subject there was little difference between the 6.25 percent point values on the stairways with 6 and 7 inch risers. However, the values for the stairway with 8 inch risers were considerably higher. The three highest values calculated occurred on the stairways with the 8 inch risers. The highest value, 9.5 pounds, occurred on the 8-9 stairway with 5.8 pounds on the 8-11 stairway. A force of 4.4 pounds was measured on the 8-10 stairway. For each set of stairways with a constant riser height, the lowest horizontal force in the set occurred on the stairway with a 10 inch tread width.

The 172 pound subject showed a greater variation in the 6.25 percent point values. The highest value, 9.2 pounds, occurred on the 8-9 stairway. However, the next

TABLE 17: The Horizontal Force exerted at the 6.25 Percent Point while ascending each stairway.

		STAIRWAYS								
		6-9	6-10	6-11	7-9	7-10	7-11	8-9	8-10	8-11
140 lb. Subject	Force, lbs.	4.1	2.7	3.7	2.2	1.9	2.8	9.5	4.4	5.8
172 lb. Subject	Force, lbs.	5.4	2.8	8.4	4.1	5.0	4.0	9.2	4.9	8.1

highest value, 8.4 pounds, occurred on the 6-11 stairway. For the stairways with 6 and 8 inch risers the lowest value occurred on the stairway with a 10 inch tread. However, in the set of stairways with a 7 inch riser the highest value occurred on the stairway with a 10 inch tread.

The statistical analysis of the horizontal force for the 140 pound subject indicated significant differences within both the stairway averages and the percent point averages. The comparison of the average horizontal force exerted on each of the nine stairways is shown in Figure 33. The average horizontal force for each stairway is also given in Table 18. Each average contains thirty-six measurements.

The highest average, 3.5 pounds, which occurred on the 8-9 stairway, was significantly different from all the other averages. The lowest average, -3.7 pounds, occurred on the 6-10 stairway. It was significantly different from all other averages. Six of the nine stairways had a negative average value. Thus the average horizontal force was directed toward the rear of the tread.

The comparison of the average horizontal force at each percent point is shown in Figure 34. The average horizontal force for each percent point is also given in Table 19. Each average contains thirty-six measurements.

The highest average, 4.3 pounds, occurred at the 6.25 percent point. However, the average for the 6.25 percent point was not significantly different from the average for the 87.5 or the 93.75 percent points. This fact indicates that the initial force exerted by the forefoot is approximately the same as the force exerted by the rearfoot when leaving the tread. The lowest average, -8.3 pounds, which occurred at the 25 percent point, was significantly different from all the other averages.

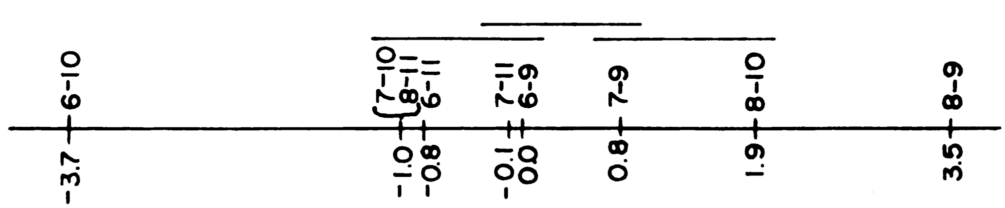


Figure 33: Comparison of the stairway averages for the horizontal force, 140 pound subject ascending the stairways.

The statistical analysis of the horizontal force for the 172 pound subject indicated significant differences within both the stairway averages and the percent point averages.

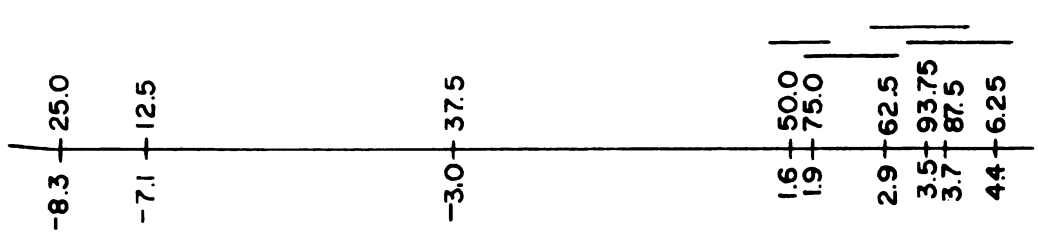


Figure 34: Comparison of the percent point averages for the horizontal force, 140 pound subject ascending the stairways.

TABLE 18: The Average Horizontal Force exerted while ascending each Stairway.

		STAIRWAYS								
		6-9	6-10	6-11	7-9	7-10	7-11	8-9	8-10	8-11
140 lb. Subject	Force, lbs.	0.0	-3.7	-0.8	0.8	-1.0	0.1	3.5	1.9	-1.0
172 lb. Subject	Force, lbs.	0.7	-2.9	2.2	3.7	1.0	0.8	5.1	5.3	0.1

TABLE 19: The Average Horizontal Force exerted at each Percent Point while ascending the Nine Stairways.

		PERCENT POINTS								
		6.25	12.5	25.0	37.5	50.0	62.5	75.0	87.5	93.75
140 lb. Subject	Force, lbs.	4.4	-7.1	-8.3	-3.0	1.6	2.9	1.9	3.7	3.5
172 lb. Subject	Force, lbs.	6.2	-0.8	-3.0	-0.8	1.0	1.3	2.7	7.1	2.3

A graphical comparison of the average horizontal force on each stairway is given in Figure 35. The average for each stairway is also given in Table 18. The highest average, 5.3 pounds, occurred on the 9-10 stairway. However, this value was not significantly different from the averages of 5.1 and 3.7 pounds which occurred on the 8-9 and 7-9 stairways. The lowest value, -2.9 pounds, was measured on the 6-10 stairway. This value was significantly different from all other averages and was the only stairway with a negative average.

The comparison of the percent point averages is given in Figure 36. The average for each percent point is also given in Table 19. The highest average, 7.1 pounds, occurred at the 87.5 percent point. This average was not significantly different from the average of 6.2 pounds for the 6.25 percent point. The lowest average, 3.0 pounds, occurred

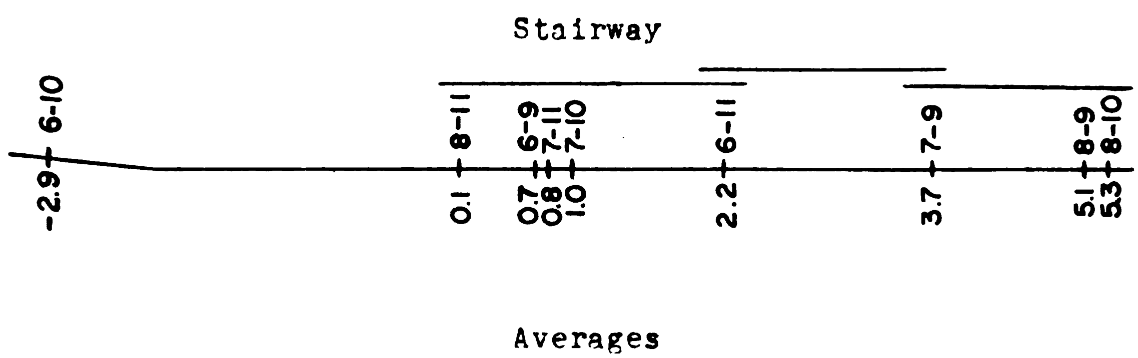


Figure 35: Comparison of the stairway averages for the horizontal force, 172 pound subject ascending the stairways.

at the 25 percent point. This average was significantly different from all other averages.

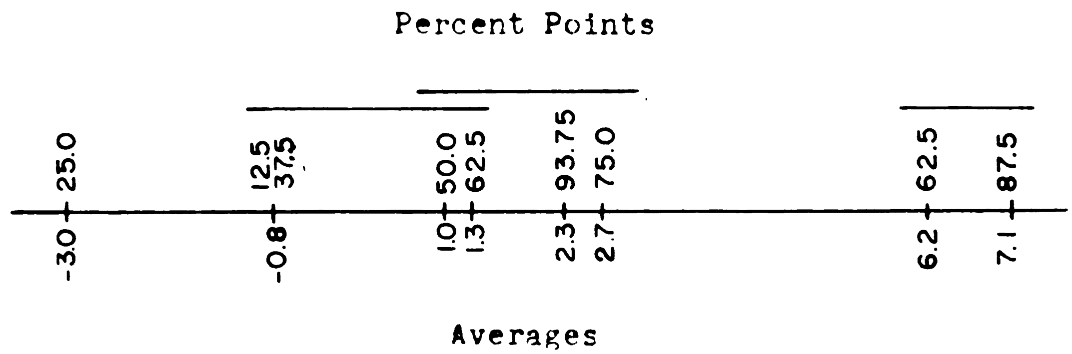


Figure 36: Comparison of the percent point averages for the horizontal force, 172 pound subject ascending the stairways.

The horizontal force exerted by the two subjects was compared using a rank correlation analysis. The analysis gave a correlation of R = 0.73. This value indicates a considerable degree of correlation between the two subjects. Thus when the averages for the stairways are ranked from the lowest to the highest the order in which the stairways occur is approximately the same. This fact can be observed by looking at Figures 33 and 35, starting from the left side and proceeding to the right.

Figure 37 shows the average horizontal force exerted by each subject while ascending the stairways. Each percent point is an average of the nine stairways. Figure 37 indicates that the horizontal force curve for the two subjects was very much alike. The 140 pound subject exerted a greater force at the beginning of the step than did the 172 pound subject. However, the 172 pound subject exerted a larger force at the end of the step than did the 140 pound subject.

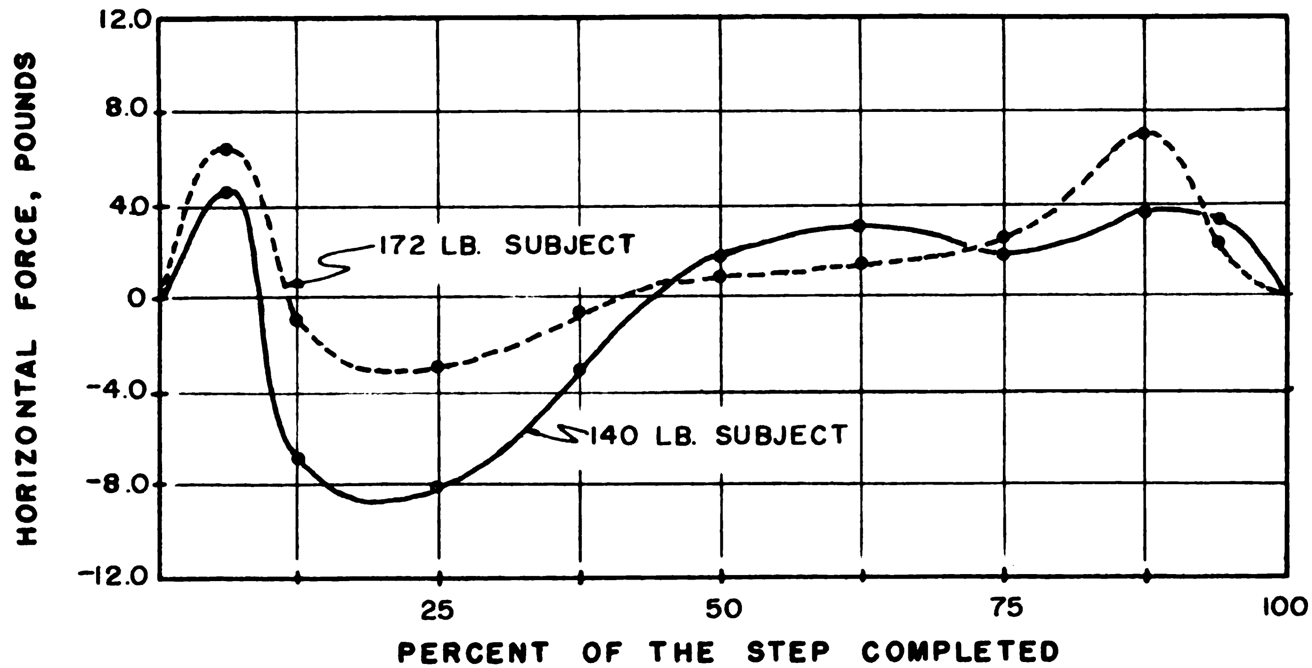


Figure 37: The horizontal force exerted by the two subjects while ascending the stairways. Average for the nine stairways.

Center of pressure, ascending

The force exerted on a stair tread is distributed over some proportion of the shoe sole area. The force applied on the stair tread can be replaced by a concentrated force, equal in magnitude, acting through the centroid of the distributed force. The force plate was designed so the point of application of the vertical force could be measured. This point, which is termed the center of pressure, is the centroid of the force applied on the stair tread.

The center of pressure was measured during this investigation to determine whether the dimensions of a stairway affected the initial point of application of the vertical force. Although the center of pressure was measured for the complete step only the measurement at the 6.25 percent point will be analyzed. The center of pressure at the 6.25 percent point is the initial point of application of the vertical force. If a person is to slip on a stairway because he is off-balance he will slip when he first places his foot on the tread. Once the person's foot is completely on the step there is very little chance of slipping. For this reason only the initial point of application will be analyzed.

The center of pressure for each subject while ascending a stairway with 6 inch risers and 10 inch treads is shown in Figure 38. The curves in Figure 38 illustrate the general pattern of the center of pressure while

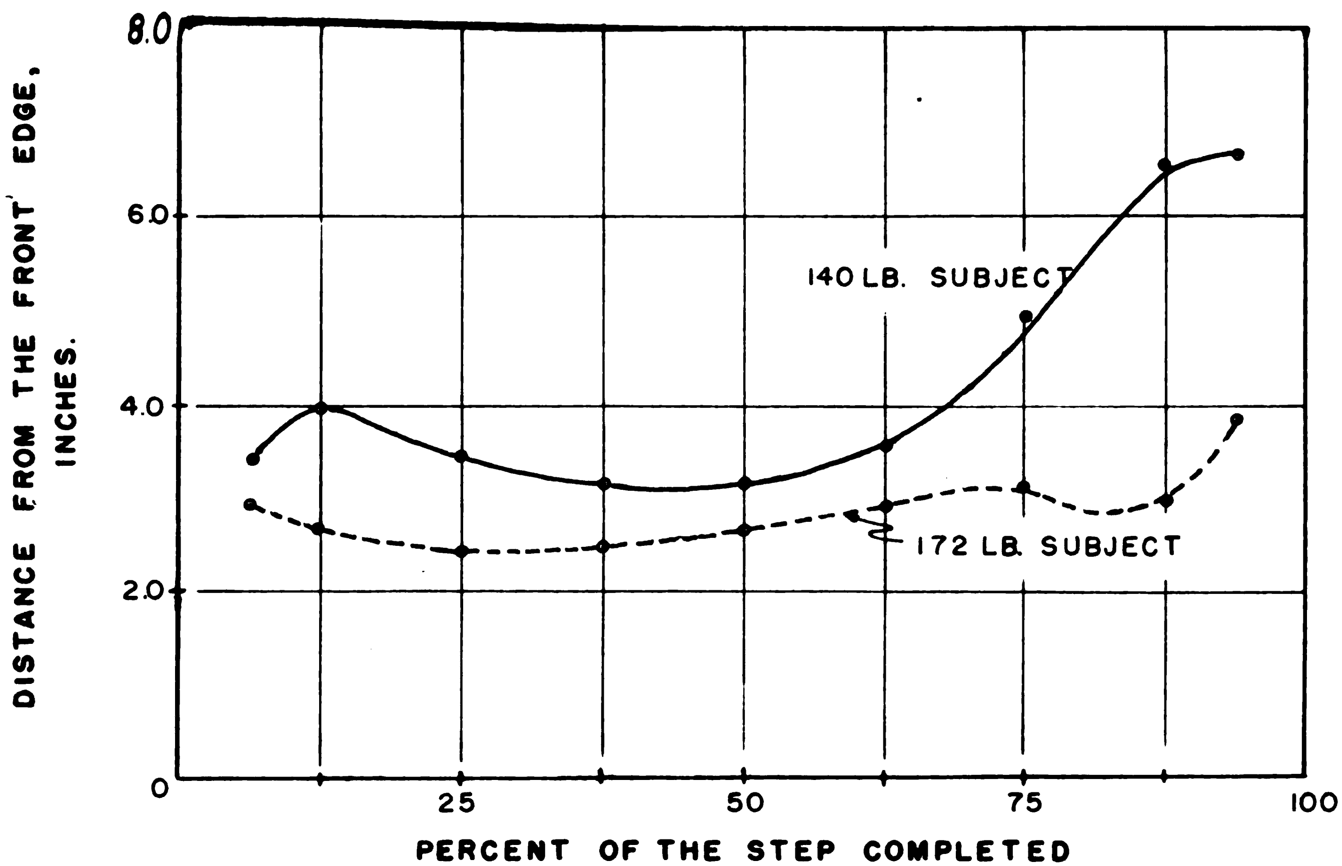


Figure 38: Center of pressure while ascending a stairway with 6 inch risers and 11 inch treads.

ascending the different stairways.

The basic differences indicated in Figure 38 can be accounted for in the way each subject ascended the stairway. The 140 pound subject placed his complete foot (sole and heel) on the tread when ascending. The second subject ascended the stairway placing only the sole of his shoe on the tread.

Figure 38 indicates that the center of pressure moves toward the front edge of the tread during part of the step. When the subject applies pressure on the tread to lift his body a larger portion of the shoe sole touches the tread. The vertical force is distributed over a larger area which causes the center of pressure to move toward the front edge of the tread.

Figure 38 indicates that at the completion of the step the center of pressure for the 140 pound subject was located a greater distance from the front edge of the tread. This occurred because the 140 pound subject placed his entire foot on the tread. Therefore, the front of his shoe was further from the front edge of the tread than was the shoe of the 172 pound subject.

The reasons for the sharp increase at the beginning of the step for the 140 pound subject and the dip near the end of the step for the 172 pound subject are not known. However, these features appear to be characteristic of each subject since these features occurred in most of the stairways analyzed.

- The values of the center of pressure at the 6.25 percent point are given in Table 20. The value for each of four repetitions on each stairway are given. Each value is the distance from the front edge of the tread.

A statistical analysis of the values in Table 20 indicated no significant differences between the stairway averages for either subject. A combined analysis indicated no significant difference between the two subjects. The statistical test was performed at the 95 percent probability level.

Descending a Stairway

The event of descending a stairway can be divided into three basic periods, the "swing", "set" and "support" periods.

Suppose a person standing at the top of a stairway decides he wants to descend the stairway. The person's first move would be to lift one foot from the floor and swing it forward. The time interval during which the foot is lifted from the floor and moved to a position in front of the body shall be termed the "swing" period. The forefoot shall be defined as the foot which is either stationary or on the lower of any two steps being considered. During the "swing" period the forefoot is the foot which is stationary. The rearfoot shall be defined as the foot which is either moving or on the higher of any two steps.

At the completion of the "swing" period the rearfoot

TABLE 20: The Initial Point of Application of the Vertical Force while ascending the Stairways. Distance from the Front Edge, inches.

		STAIRWAYS								
	Trial	6-9	6-10	6-11	7-9	7-10	7-11	8-9	8-10	8-11
140 lb. Subject	1	1.99	4.59	4.10	5.02	3.38	3.23	2.53	3.28	2.91
	2	3.48	3.97	2.46	3.92	3.57	4.24	3.92	3.55	3.16
	3	3.35	3.00	3.18	3.78	3.64	3.77	3.56	2.42	3.66
	4	4.01	4.07	3.91	3.19	4.23	3.93	4.47	2.43	4.11
	Average	3.21	3.91	3.41	3.98	3.71	3.79	3.62	2.92	3.46
172 lb. Subject	1	3.75	4.27	1.87	2.78	2.99	2.90	3.07	3.02	3.54
	2	3.00	3.22	3.34	3.34	3.49	4.55	2.81	3.13	3.88
	3	3.28	3.40	2.78	3.10	2.69	3.55	3.96	2.13	3.84
	4	2.04	4.71	3.54	2.91	3.94	3.56	3.53	3.18	3.78
	Average	3.02	3.90	2.88	3.03	3.27	3.64	3.34	2.87	3.76

is in front of the body at approximately the same level as the forefoot. The "set" period is the time interval during which the rearfoot is lowered to the next step. When the rearfoot is placed on the lower tread it becomes the forefoot and the "support" period is started. The "support" period extends from the instant the rearfoot becomes the forefoot until the time the foot on the upper tread (rearfoot) is lifted to start the "swing" period.

The event of descending a stairway is a continuous repetition of the "swing", "set" and "support" periods. Each of these periods is illustrated in Figure 39.

Vertical Force, Descending

The vertical force exerted by the 140 pound subject while descending a stairway with 6 inch risers and 10 inch treads is shown in Figure 40. Figure 40 also illustrates the general pattern of the vertical force while descending a stairway.

Figure 40 indicates that while descending a stairway the vertical force has two maximum values. The first maximum occurs near the end of the "support" period when the entire body weight is placed on the forefoot and the rearfoot has just been lifted from the upper tread. The first maximum value usually occurred at either the 12.5 or the 25 percent point. The second maximum value occurred near the end of the "set" period. At this time the entire body weight was still on the forefoot and the body was under

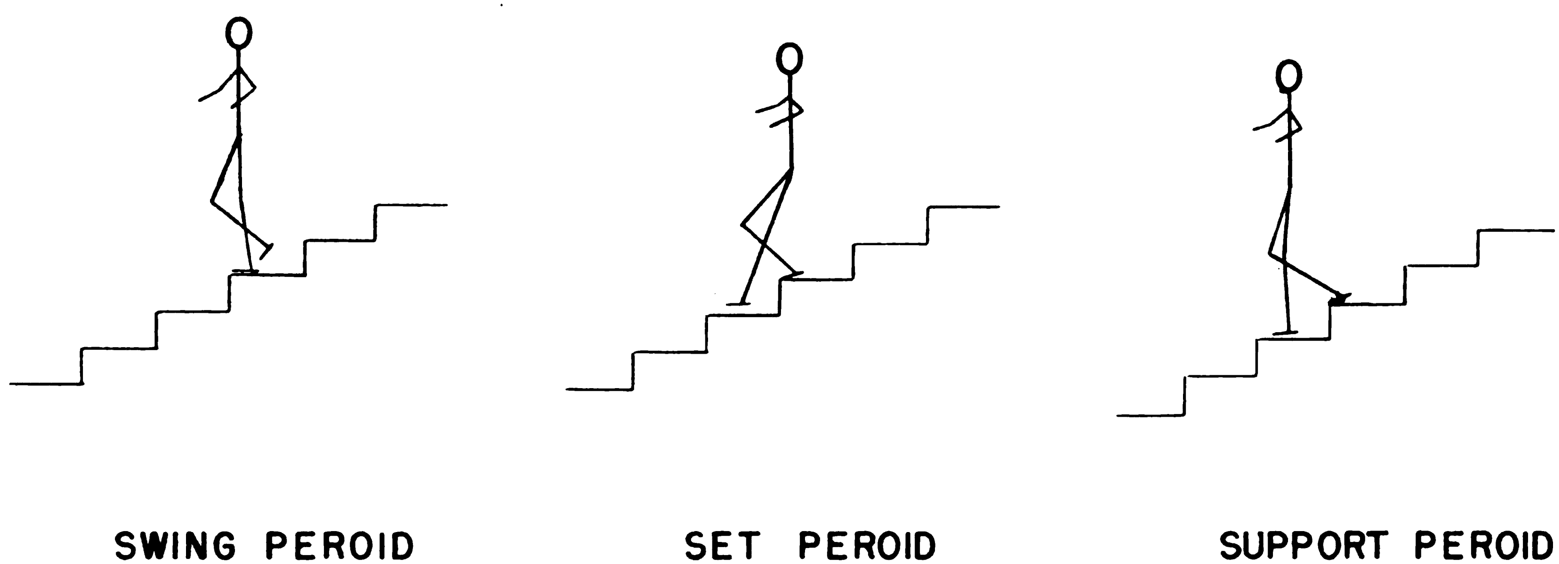


Figure 39: The basic divisions of one complete step while descending a stairway.

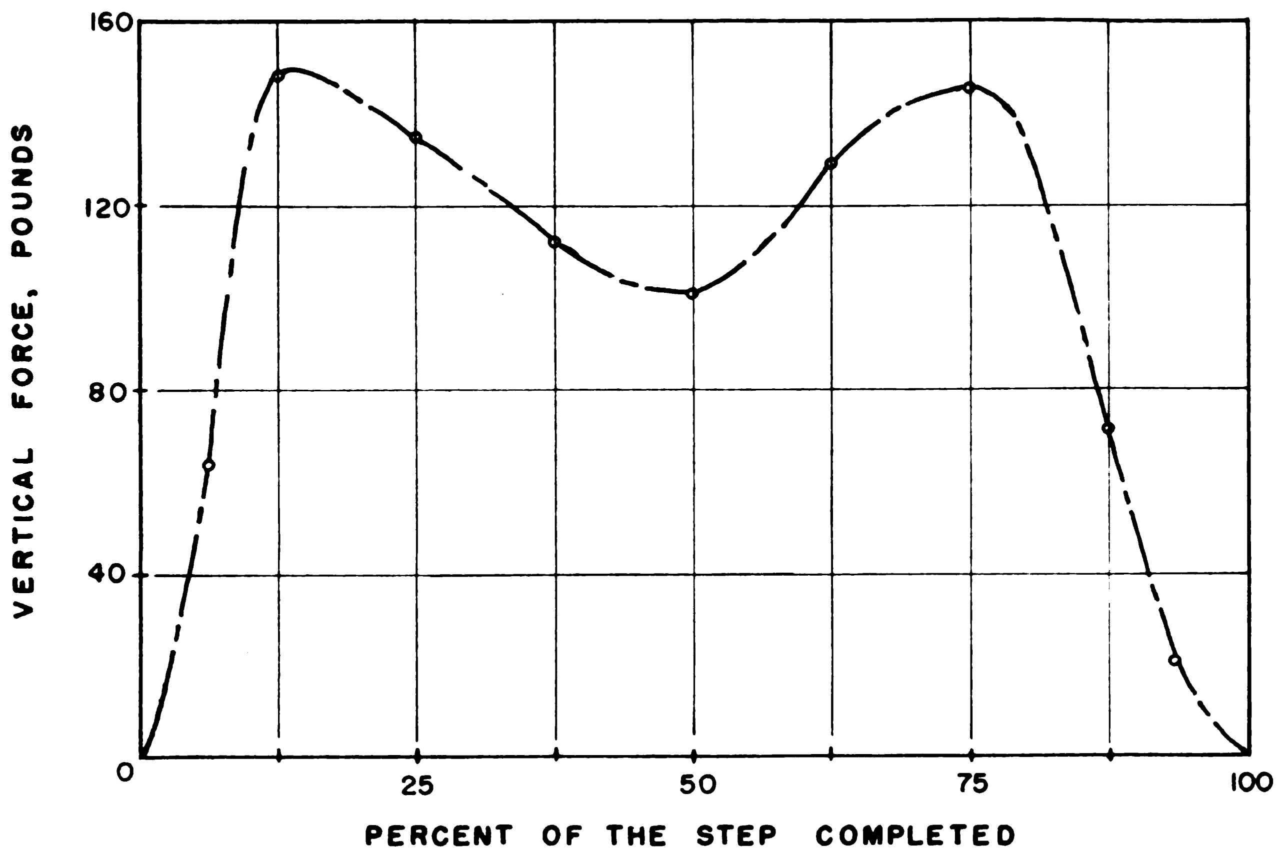


Figure 40: The vertical force exerted by the 140 pound subject while descending a stairway with 6 inch risers and 10 inch treads.

very little upward acceleration due to the completion of the "swing" period. The second maximum value usually occurred near the 75 percent point. Both maximum values were usually greater than body weight.

The depression in the curve is believed to be due to an upward acceleration of different parts of the body. The upward acceleration produces an upward force which cancels some of the force due to body weight.

The percent of the step taken by each basic period is not clearly designated. The "support" period occupied the first 12.5 to 25 percent and the last 25 percent of the step. The second portion of the "support" period was the time during which the foot was removed from the tread. The "swing" period usually covered the interval from the end of the first "support" period to the 62.5 percent point. The "set" period occurred between the 62.5 and 75 percent points.

Table 21 gives the value of the vertical force exerted on each stairway at the 12.5 and 25 percent points. The values are the calculated values and not necessarily the maximum force which occurred on the step. A larger force may have been exerted between the two points.

The maximum calculated force for the 140 pound subject ranged from 141.9 pounds on the 7-10 stairway to 192.9 pounds on the 8-9 stairway. The forces exerted on the stairways with 8 inch risers were considerably higher than the forces exerted on the stairways with 6 or 7 inch

TABLE 21: The Vertical Force (Pounds) exerted at the 12.5 and 25 Percent Points while descending each Stairway.

		STAIRWAYS								
	% point	6-9	6-10	6-11	7-9	7-10	7-11	8-9	8-10	8-11
140 lb. Subject	12.5	125.2	249.4	165.5	156.4	114.5	155.5	192.9	186.6	165.7
	25.0	145.1	135.5	150.1	143.0	141.9	144.1	153.1	146.8	162.3
172 lb. Subject	12.5	132.9	132.9	159.2	108.2	166.1	122.3	126.3	170.8	181.4
	25.0	201.3	211.2	239.4	187.8	201.3	206.6	194.7	232.2	232.3

risers. The maximum force on the stairways with 6 inch risers ranged from 145.1 to 165.5 pounds while a range of 141.9 to 156.4 pounds was obtained on the 7 inch stairways. However, the stairways with 8 inch risers had maximum forces ranging from 165.7 to 192.9 pounds. There was no relationship between the maximum force and the tread width for the 140 pound subject.

The maximum force for the 172 pound subject varied from 187.6 pounds on the 7-9 stairway to 239.4 pounds on the 6-11 stairway. The stairways with 7 inch treads had the lowest forces with a range from 187.8 to 206.6 pounds. Most of the forces on the stairways with 6 and 8 inch risers were above 200 pounds. For the 172 pound subject there seemed to be a relationship between the maximum force and the tread width. For each set of stairways with a constant riser height the maximum force for the set occurred on the stairway with an 11 inch tread.

A statistical analysis of the vertical force for the 140 pound subject indicated there were significant differences between the averages of the nine stairways. The analysis also indicated significant differences between the percent point averages. The comparison of the stairway averages is given in Figure 41. The average vertical force exerted on each stairway is given in Table 22. Each average contains thirty-six measurements.

The stairway with 8 inch risers and 9 inch treads had the largest average vertical force, 117.6 pounds. The stairways with 8 inch risers had the highest averages; however, there was no significant difference between these averages. The 7-10 stairway had the lowest average, 94.6 pounds; however, this average was not significantly different from the averages of the 6-9, 6-10 and 7-9 stairways.

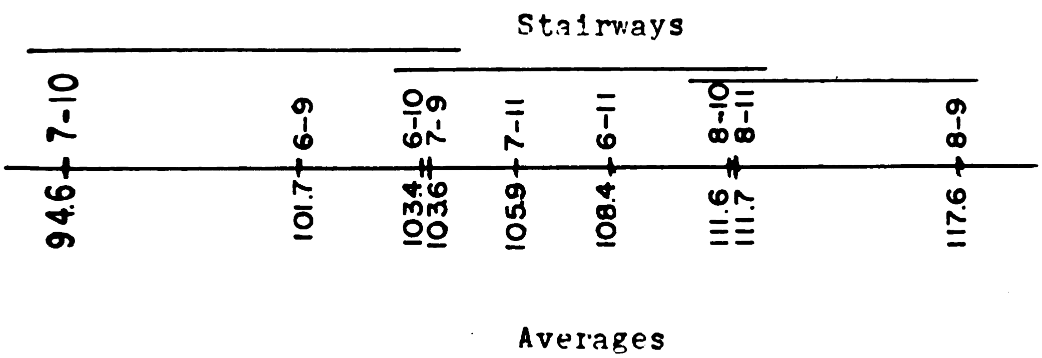


Figure 41: Comparison of the stairway averages for the vertical force, 140 pound subject ascending the stairways.

The comparison of the percent point averages is given in Figure 42. The average vertical force exerted at each percent point is given in Table 23. Each average contains thirty-six measurements.

The comparison of percent point averages indicates that the 12.5 percent point had the highest average, 156.9 pounds. However, there was no significant difference between the average for the 12.5 percent point and the average of 148.9 pounds for the 75 percent point. The 25 percent point with an average of 146.8 pounds was significantly different from the 12.5 percent point but was not significantly different from the 75 percent point. The lowest average, 25.8 pounds, occurred at the 93.75 percent point and was significantly different from all the other averages.

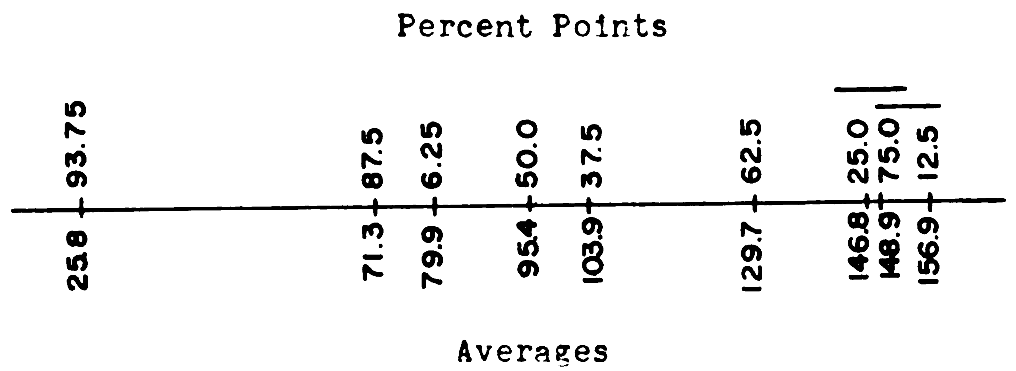


Figure 42: Comparison of the percent point averages for the vertical force, 140 pound subject descending the stairways.

A statistical analysis for the 172 pound subject indicated significant differences within both the stairway averages and the percent point averages. The comparison of the stairway averages is given in Figure 43 and the comparison of the percent averages is given in Figure 44. The average vertical force exerted on each stairway is also given in Table 22 and the average vertical force at each percent point is given in Table 23.

Figure 43 indicates the stairway with 6 inch risers and 11 inch treads had the highest average, 136.7 pounds; however, the average for the 6-11 stairway was significantly different from only the 8-9 and 7-9 stairways. The lowest average, 113.9 pounds, occurred on the 7-9 stairway and was significantly different from the 8-11, 8-10 and 6-11 stairways.

A comparison of the percent point averages indicated the 25 percent point had the largest average, 211.9 pounds.

TABLE 22: The Average Vertical Force (Pounds) exerted while descending each Stairway.

	STAIRWAYS								
	6-9	6-10	6-11	7-9	7-10	7-11	8-9	8-10	8-11
140 lb. Subject	101.7	103.4	108.4	103.6	94.6	105.9	117.6	111.6	111.7
172 lb. Subject	129.1	124.6	136.7	113.9	124.7	130.5	117.7	136.0	134.1

TABLE 23: The Vertical Force (Pounds) exerted at each Percent Point while descending the nine stairways. Average for the nine stairways.

		PERCENT POINTS								
		6.25	12.5	25.0	37.5	50.0	62.5	75.0	87.5	93.75
140 lb. Subject	Force	79.9	156.9	146.8	103.9	95.4	129.7	148.9	71.3	25.8
	% of Body Weight	57.0	112.0	104.8	74.2	68.0	92.5	106.2	50.8	18.4
172 lb. Subject	Force	67.9	144.5	211.9	169.4	137.4	147.0	165.6	75.6	26.5
	% of Body Weight	39.4	84.0	123.0	98.5	80.0	85.5	96.2	44.0	15.4

The value at the 85 percent point was significantly different from all of the other averages. The lowest average, 26.5 pounds, occurred at the 93.75 percent point and was significantly different from all the other values.

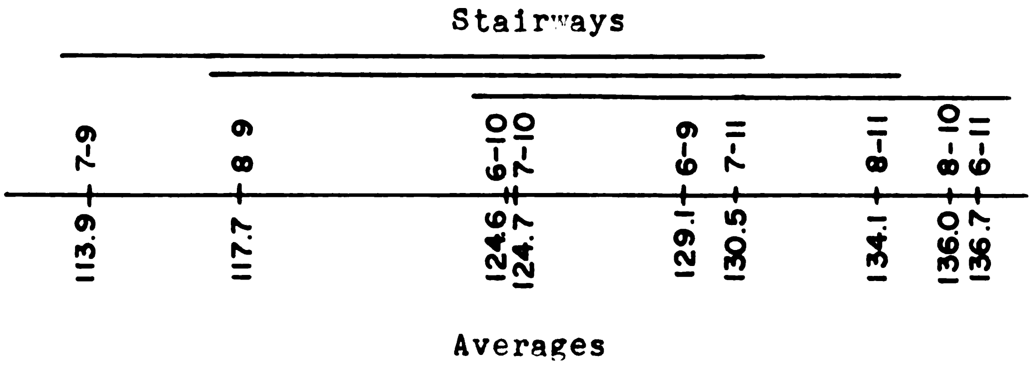


Figure 43: Comparison of the stairway averages for the vertical force, 172 pound subject descending the stairways.

The stairway averages of the two subjects were compared using rank correlation. For the stairway averages a rank correlation value of R = 0.27 was obtained. This value indicates very little correlation between the two subjects. When the stairway averages for each subject were ranked from highest to lowest the arrangement of the stairways was not the same for each subject.

The percent points were also compared using the rank correlation. For the percent points a correlation value of R = 0.67 was obtained. The correlation value indicated that the two subjects did not descend the stairways alike. When the averages for the percent points were ranked the order of ranking for the two subjects did not agree. The difference in the patterns of the two subjects may be seen in Figures 42 and 44 and Figure 45.

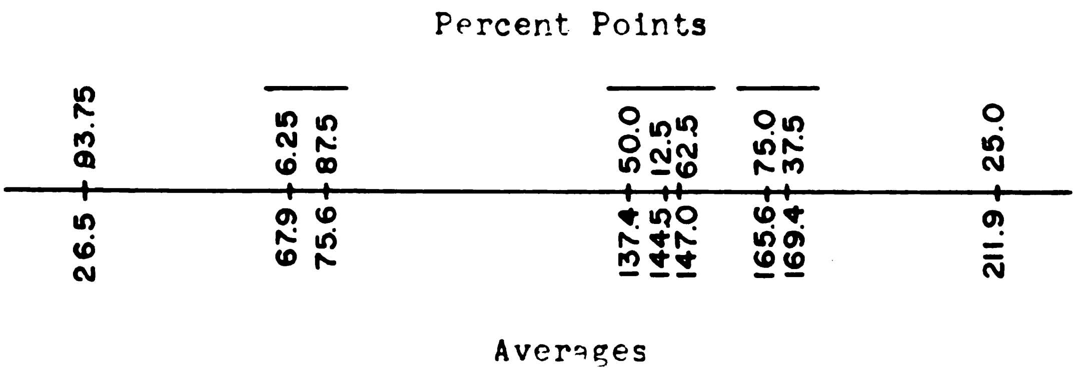


Figure 44. Comparison of the percent point averages for the vertical force, 172 pound subject descending the stairways.

Figure 45 shows the average vertical force at each percent point for the two subjects. Figure 45 indicates that the first maximum value for the 172 pound subject occurred approximately at the 25 percent point. However, for the 140 pound subject the first maximum value occurred somewhere between the 12.5 and 25 percent points. Figure 45 also indicates that the application of the vertical force during the "support" period varied linearly with time for both subjects. The second maximum value for each subject occurred at the 75 percent point.

Figure 46 shows the average vertical force at each percent point as a percent of body weight. Figure 46 indicates that the first maximum value was greater than body weight for both subjects. However, the second maximum was greater than body weight for the 140 pound subject and less than body weight for the 172 pound subject.

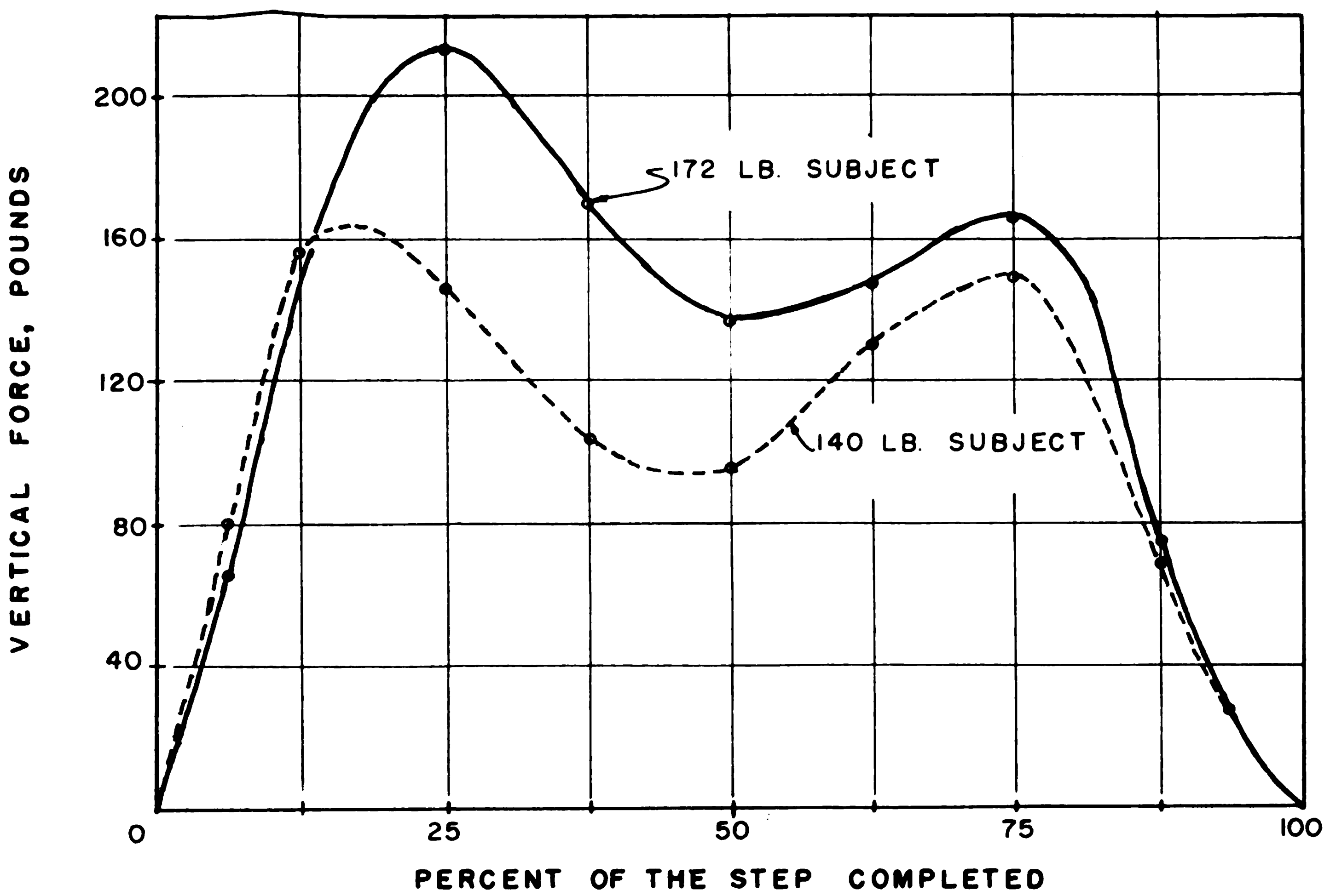


Figure 45: The vertical force exerted by two subjects while descending the stairways. Average for the nine stairways.

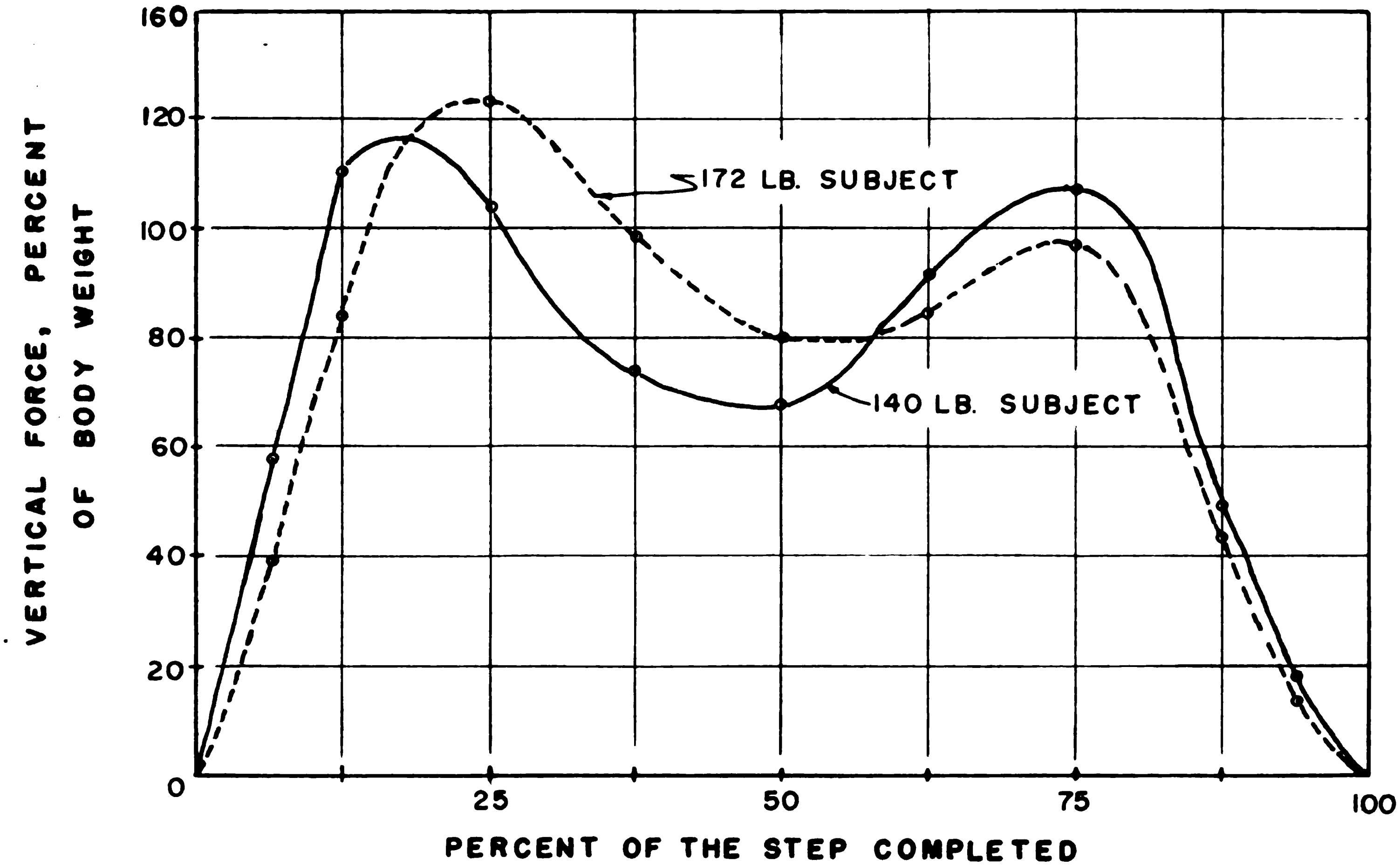


Figure 46: The vertical force, as a percent of body weight, exerted by each subject while descending the stairways. Average for the nine stairways.

Horizontal Force, Descending

The horizontal force exerted by each subject while descending a stairway with 6 inch risers and 10 inch treads is shown in Figure 47. The two curves illustrate the general pattern of the horizontal force exerted by each subject while descending the stairways.

The general pattern of the curve for the 140 pound subject indicates that a positive horizontal force (toward the front edge of the tread) was exerted during the first portion of the step. The maximum value for the 140 pound subject occurred between the 12.5 and 25 percent points which was approximately the same place as the first maximum value of the vertical force. The horizontal force remained fairly constant during the middle portion on the step. A large negative force was produced when the foot pushed off the tread. The large negative force usually occurred at the 87.5 percent point. For most of the stairways the absolute values of the maximum negative and positive forces were approximately the same.

The general pattern of the horizontal force for the 172 pound subject was somewhat different than the curve for the 140 pound subject. The 172 pound subject exerted a negative force at the beginning of his step. Apparently the subject's foot was moving toward the rear when it touched the tread. However, the horizontal force changed to the positive direction and reached a maximum value

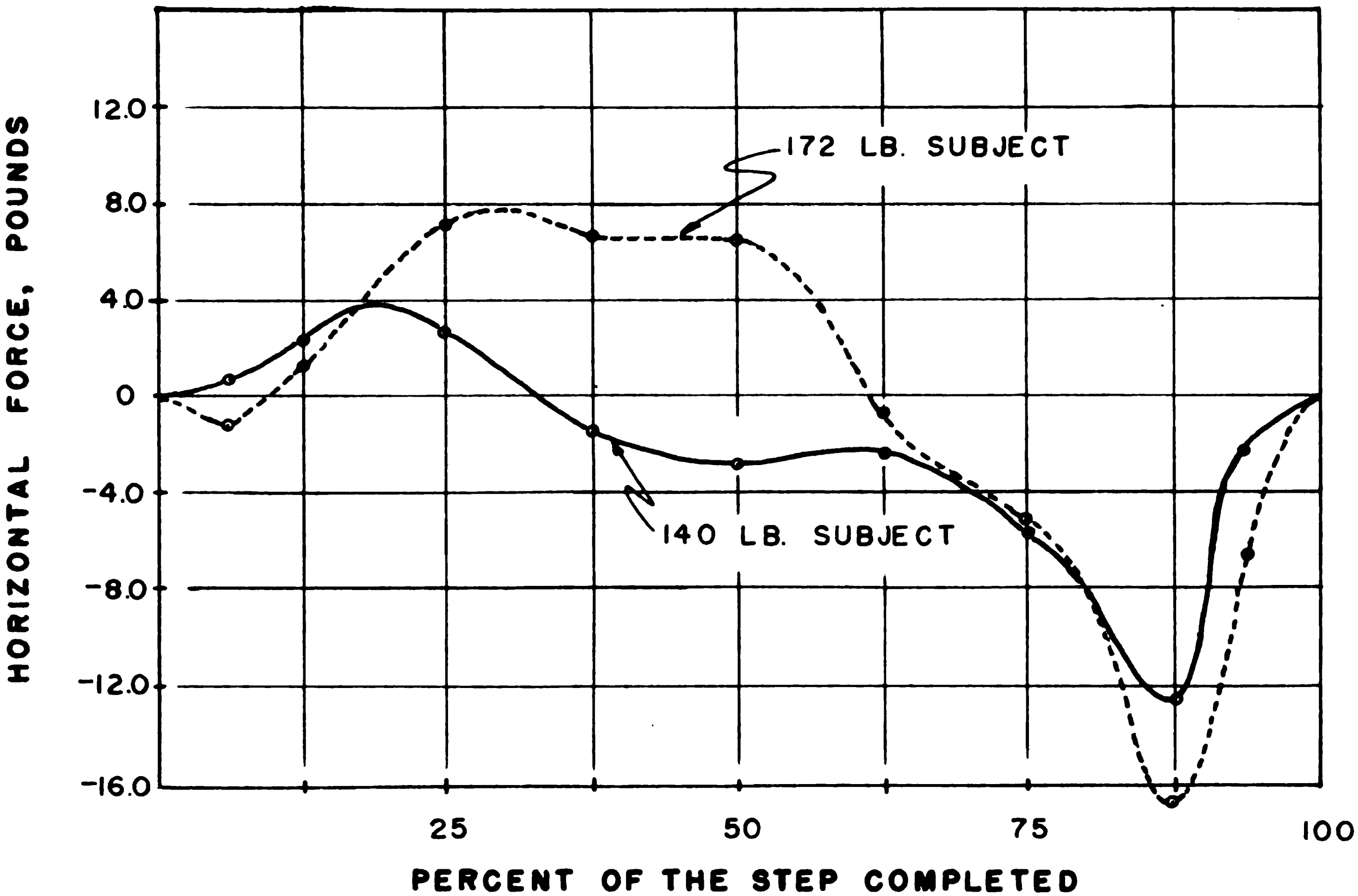


Figure 47: The horizontal force exerted by each subject while descending a stairway with 6 inch risers and 11 inch treads.

around the 25 percent point which was approximately the same time as the first maximum of the vertical force for this subject. The 172 pound subject maintained a positive force much longer than the 140 pound subject. On many of the stairways the horizontal force at the 37.5 percent point was nearly the same value as the force at the 25 percent point. The horizontal force for the 172 pound subject did not change directions until some time between the 62.5 and 75 percent points. The maximum negative force was exerted at the 87.5 percent point.

Table 24 gives the value of the horizontal force at the 12.5 and 25 percent points. The horizontal force at these points, which is directed toward the front edge of the tread, would be the force which would cause a person to slip when descending a stairway. The force given is not necessarily the maximum force. A larger force could have occurred between the 12.5 and 25 percent points.

A statistical analysis for the 140 pound subject indicated a significant difference within both the stairway averages and the percent point averages. The statistical comparison of the stairway averages is shown in Figure 48. The stairway averages are also given in Table 28. Each average contains thirty-six measurements.

For the 140 pound subject the stairway with 8 inch risers and 9 inch treads had the highest average horizontal force, 3.5 pounds. However, the average for the 6-9 stairway was not significantly different from the

TABLE 24: The Horizontal Force (Pounds) exerted at the 12.5 and 25 Percent Points while descending each Stairway.

		STAIRWAYS								
	% point	6-9	6-10	6-11	7-9	7-10	7-11	8-9	8-10	8-11
140 lb. Subject	12.5	6.8	2.3	7.7	11.5	6.7	6.3	12.6	8.2	6.0
	25.0	6.7	2.5	9.0	15.1	6.1	5.7	14.4	7.8	5.0
172 lb. Subject	12.5	1.6	1.7	5.7	3.6	3.8	2.3	5.9	11.9	3.5
	25.0	9.7	7.3	13.8	13.6	11.7	8.3	17.3	25.6	7.7

average of 3.2 pounds for the 7-9 stairway or the 8-10 stairway with an average of 2.0 pounds. The lowest average, -2.3 pounds, occurred on the 6-10 stairway but was not significantly different from the average of -0.8 pounds on the 8-11 stairway. The 6-10, 8-11, 6-9 and 6-11 stairways each had a negative average horizontal force.

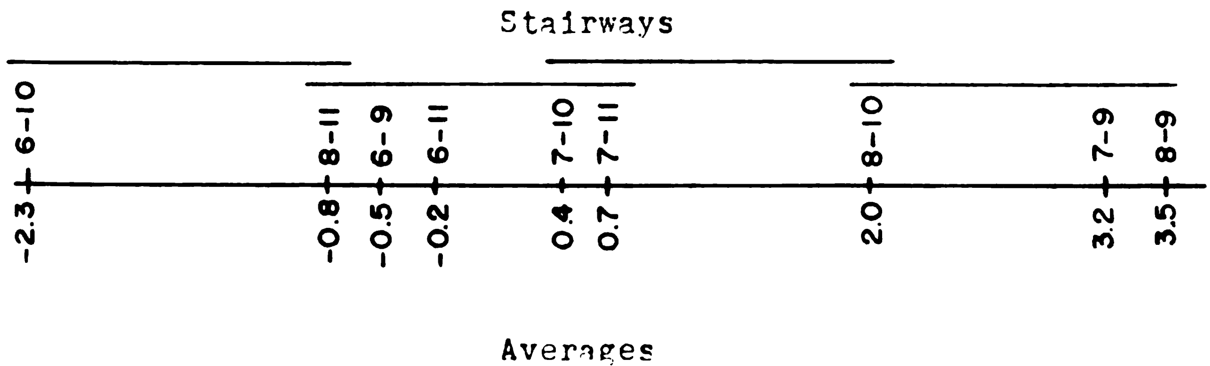


Figure 48: Comparison of the stairway averages for the horizontal force, 140 pound subject descending the stairways.

A comparison of the percent point averages is shown in Figure 49. The percent point averages for the 140 pound subject are also given in Table 26. Each average contains thirty-six measurements.

For the 140 pound subject the 25 percent point had the highest average horizontal force, 8.1 pounds. However, the average for the 25 percent point was not significantly different from the average of 7.6 pounds for the 12.5 percent point. The lowest average, -10.1 pounds, was at the 87.5 percent point and was significantly different from all the other averages. It should be noted that the absolute value of this force was greater than the value at the 25 percent point.

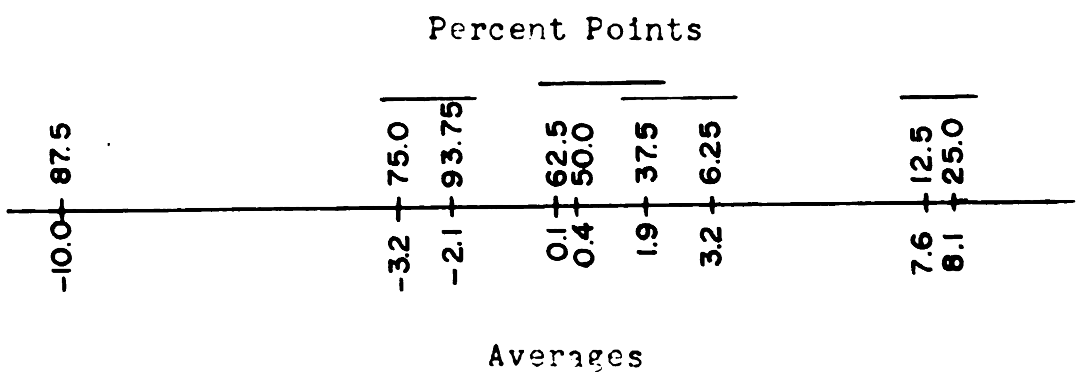


Figure 49: Comparison of the percent point averages for the horizontal force, 140 pound subject descending the stairways.

The statistical analysis for the 172 pound subject indicated significant differences within the stairway averages and within the percent point averages. A comparison of the stairway averages is given in Figure 50 and the values are also given in Table 25. Each average contains thirty-six measurements.

For the 172 pound subject the 8-10 stairway had the highest average, 8.5 pounds, and was significantly different from all the other stairways. The lowest average -0.9 pounds, was obtained on the 6-10 stairway. However, the average on the 6-10 stairway was not significantly different from four other stairways, the 8-11, 7-10, 7-11, and 6-9. The 6-10 and 8-11 stairways were the only ones with a negative average.

A comparison of the percent point averages for the

TABLE 25: The Average Horizontal Force (Pounds) exerted while descending a Stairway.

	STAIRWAYS								
	6-9	6-10	6-11	7-9	7-10	7-11	8-9	8-10	8-11
140 lb. Subject	0.9	-0.9	2.2	4.2	0.0	0.0	4.9	8.5	-0.7
172 lb. Subject	-0.5	-2.3	-0.2	3.2	0.4	0.7	3.5	2.0	-0.8

TABLE 26: The Horizontal Force, (Pounds) exerted at each Percent Point while descending the Stairways. Average for the nine stairways.

	PERCENT POINTS								
	6.25	12.5	25.0	37.5	50.0	62.5	75.0	87.5	93.75
140 lb. Subject	3.2	7.6	8.1	1.9	0.4	0.1	-3.2	-10.1	-2.1
172 lb. Subject	0.2	4.5	12.5	11.5	8.0	2.5	-5.7	-12.1	-2.2

172 pound subject is given in Figure 51. The percent point averages are also given in Table 26. Each average contains thirty-six measurements.

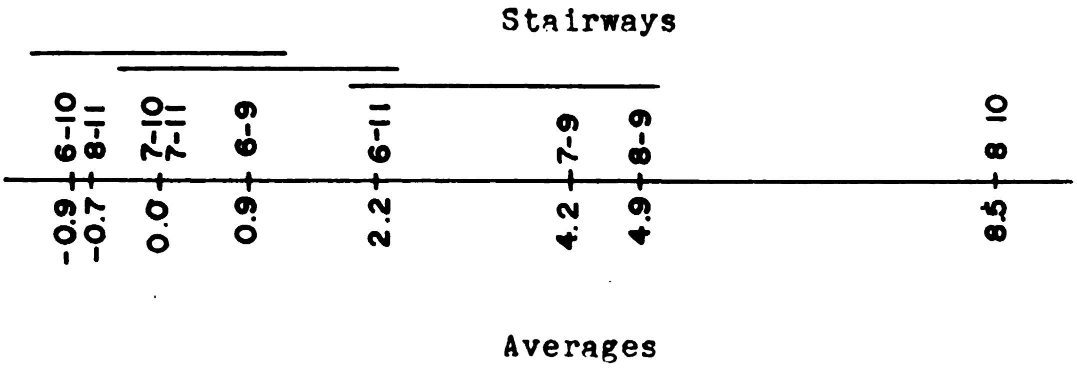


Figure 50: Comparison of the stairway averages for the horizontal force, 172 pound subject descending the stairways.

For the 172 pound subject the highest average, 12.5 pounds, occurred at the 25 percent point. However, the average at the 25 percent point was not significantly different from the average of 11.5 pounds which occurred at the 37.5 percent point. The lowest average, -12.3 pounds, which occurred at the 87.5 percent point was significantly different from all the other averages.

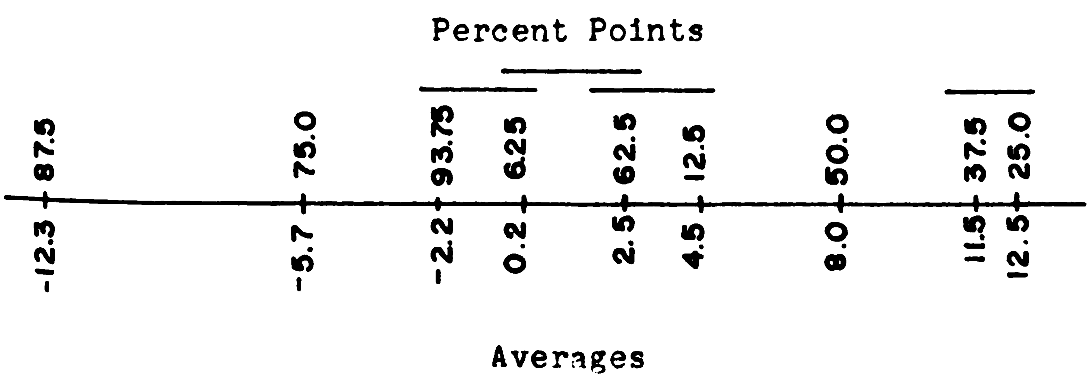


Figure 51: Comparison of the percent point averages for the horizontal force, 172 pound subject descending the stairways.

The stairway averages of the two subjects were compared using the rank correlation analysis and a value of R = 0.82 was obtained. This value indicates that a correlation between the two subjects exists. Figures 48 and 50 indicate that the two lowest and the three highest averages for each subject occurred on the same stairways.

The percent point averages were also compared using the rank correlation analysis and a value of R = 0.83 was obtained. A correlation exists between the two subjects for the percent points and may be seen by comparing Figures 49 and 51. For each subject the highest and three lowest averages occurred at the same percent points.

Figure 52 shows the average horizontal force at each percent point for each subject. The curves represent the averages for the nine stairways. Figure 52 indicates that the 172 pound subject exerted a larger positive horizontal force for a much greater time than the 140 pound subject. The 140 pound subject exerted a maximum force and then quickly reduced the force to a value near zero for the 37.5, 50 and 62.5 percent points. However, the 172 pound subject exerted a maximum horizontal force near the 25 percent point and slowly reduced this force to zero. The negative portion of the horizontal force was very similar for the two subjects.

Center of Pressure, Descending

The center of pressure for each subject while

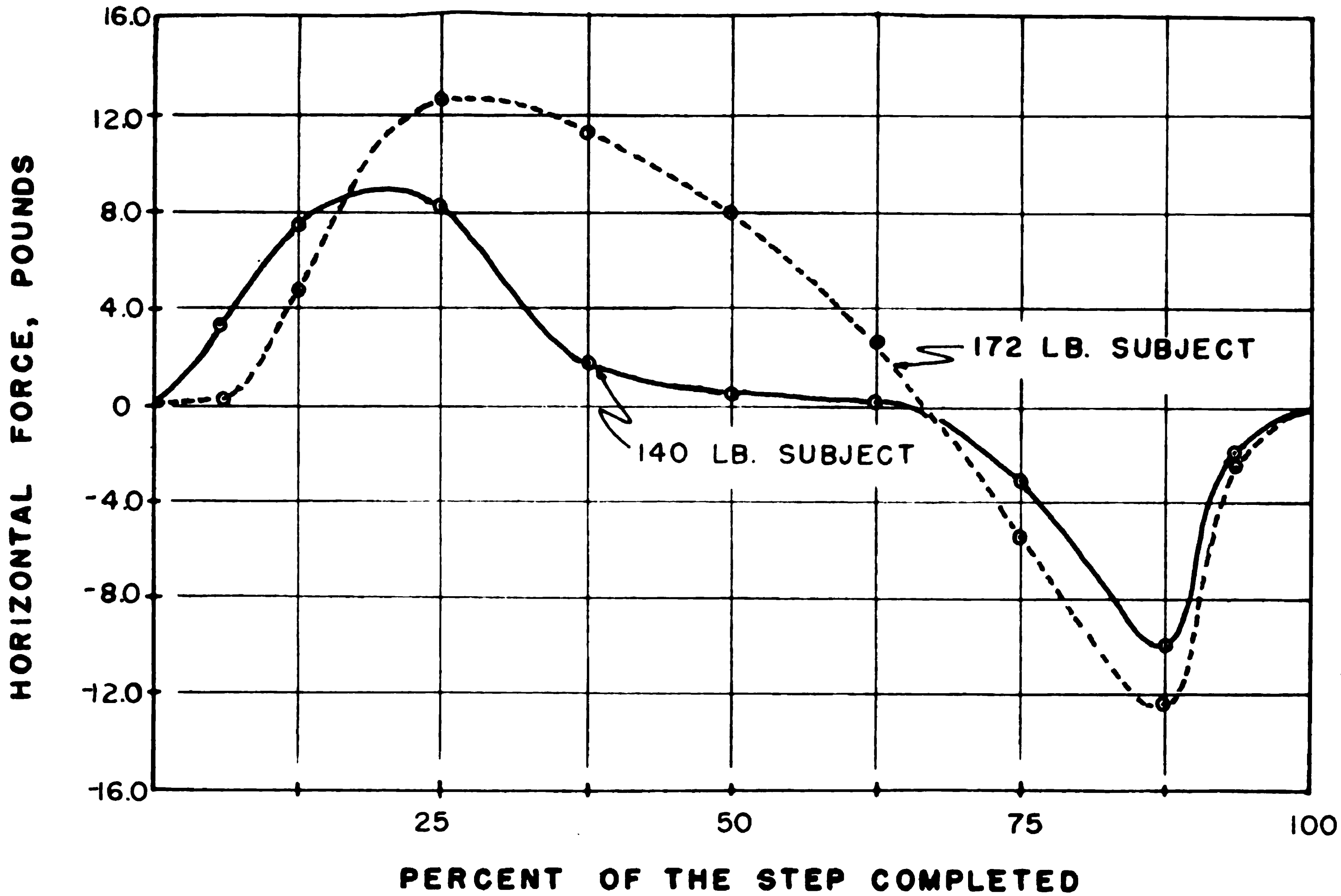


Figure 52: The horizontal force exerted by each subject while descending the stairways. Average for the nine stairways.

descending a stairway with 6 inch risers and 10 inch treads is shown in Figure 53. The curves in Figure 53 illustrate the general pattern of the center of pressure while ascending the different stairways. Figure 53 indicates that the characteristics of the center of pressure for the two subjects differed considerably.

The basic differences in the two curves can be accounted for in the way each subject descended the stairways. The 140 pound subject placed his entire foot on the tread while the 172 pound subject descended the stairway on his tip-toes and did not place his heel on the tread.

When the 140 pound subject descended the stairway he placed his sole on the tread and then gradually set his heel down. This method of descending accounts for the movement of the centroid away from the front edge of the tread. As the 140 pound subject leaves the tread the foot is lifted in a rolling fashion. Thus the centroid of the force moves toward the front edge.

When the 172 pound subject descended the stairway he placed only the sole of his foot on the tread. Because all the body weight is on the sole the center of pressure moves to an equilibrium point and remains nearly constant for a portion of the step. Figure 53 indicates that the center of pressure moved toward the front edge. This characteristic occurs because the foot makes its initial contact with the tread on the rear part of the sole. The

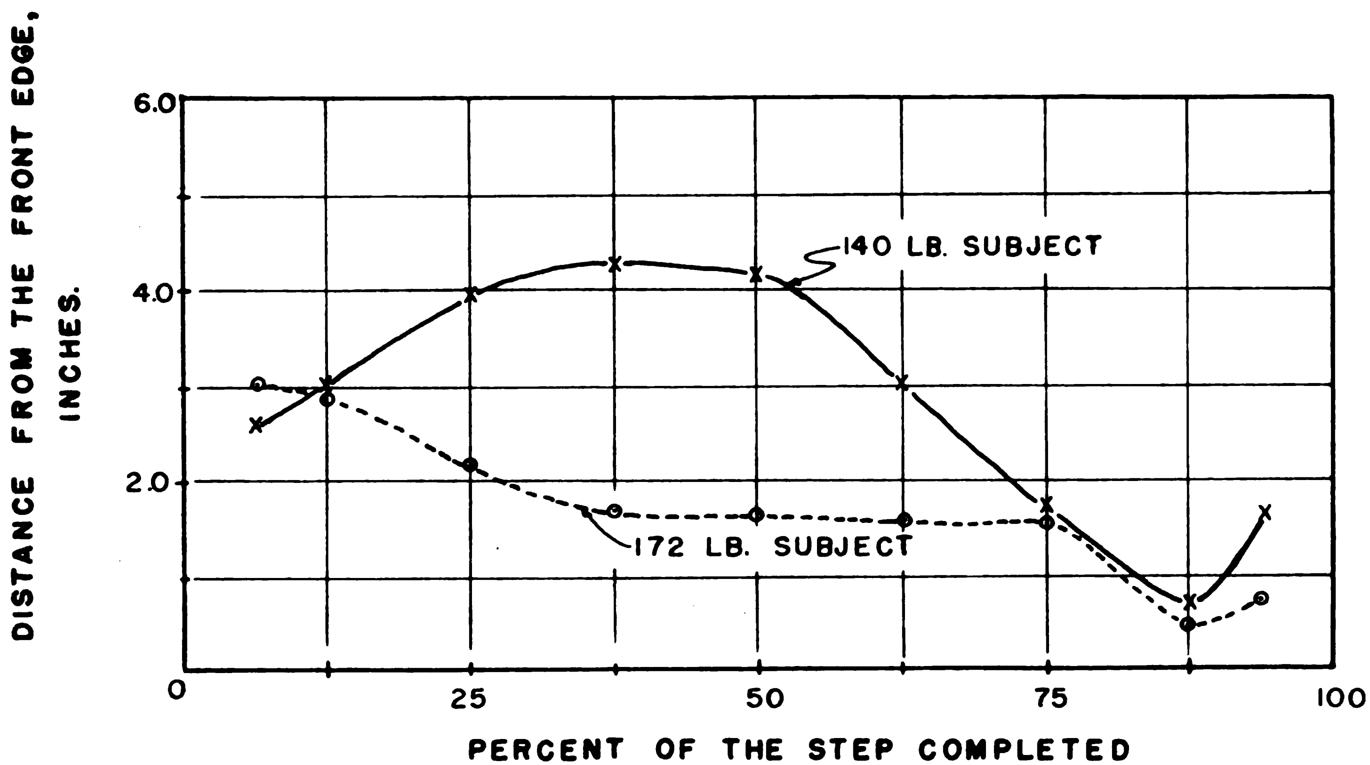


Figure 53: Center of pressure for each subject while descending a stairway with 6 inch risers and 11 inch treads.

foot then rolls slightly forward as the body moves forward.

The movement of the center of pressure toward the rear edge as the foot left the tread cannot be accounted for. This characteristic is definitely a part of each subject's step since it occurred on nearly all of the stairways. Slow motion pictures of a person descending a stairway would probably be needed to explain the movement. Because this movement does not affect the person's chances of slipping no further attention will be given to it.

A statistical analysis of the center of pressure data was performed. However, only the value at the 6.25 percent point, which is the initial point of application of the vertical force, was used. If a person is to slip on a stairway because he is off-balance, he will slip when he first places his foot on the tread. All values used were the distance from the front edge of the tread.

The analysis for the 140 pound subject indicated that significant differences did exist between the stairway averages. A comparison of the averages is shown in Figure 54 while the values for each of the four repetitions are given in Table 27. Figure 54 indicates that the 8-11 stairway had the highest average, 3.64 inches. However, the average of the 8-11 stairway was not significantly different from the averages of 3.40 and 3.21 inches which occurred on the 7-11 and 6-10 stairways, respectively. The 7-9 stairway, with an average of 2.36 inches, was significantly different from the 7-11 and 8-11 stairways. Although the 8-11 stairway had the highest average, 3.64 inches, the other stairways appear to have averages which would eliminate any chance of slipping when off-balanced.

The analysis for the 172 pound subject also indicated significant differences between the averages for each stairway. A comparison of the averages is shown in Figure 55. The value for each of the four repetitions is given in Table 27. The largest average, 3.61 inches, occurred

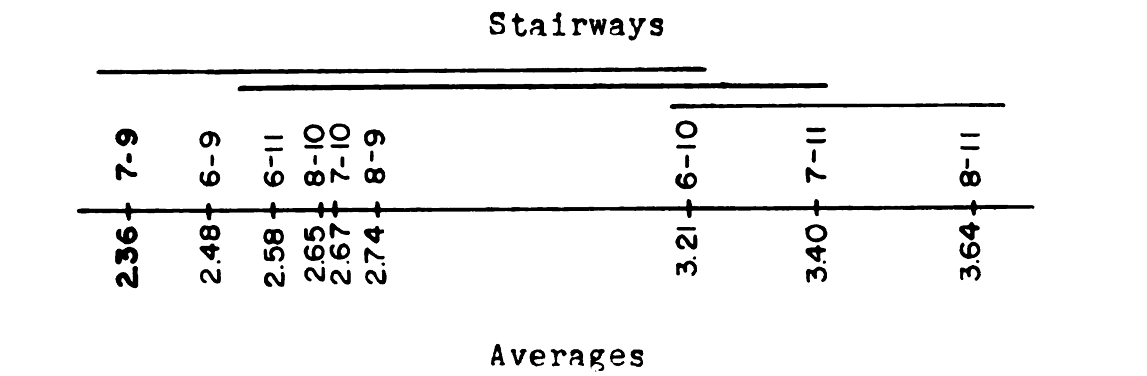


Figure 54: Comparison of the center of pressure averages, 140 pound subject descending the stairways.

on the 8-9 stairway but was not significantly different from the average of 3.33 inches which occurred on the 8-11 stairway. The lowest average, 1.58 inches, occurred on the 7-9 stairway, but was not significantly different from the average of 1.72 inches for the 7-10 stairway. While the highest averages for the 172 pound subject

TABLE 27: The Initial Point of Application of the Vertical Force while descending each Stairway. Distance from the Front Edge, inches.

		STAIRWAYS								
	Trial	6-9	6-10	6-11	7-9	7-10	7-11	8-9	8-10	8-11
140 lb. Subject	1	2.27	2.12	2.62	2.46	2.30	2.64	2.79	1.85	2.82
	2	1.78	3.87	2.28	2.33	2.82	4.39	2.72	2.83	3.27
	3	2.64	4.19	3.17	2.35	2.79	3.62	2.61	2.96	2.81
	4	3.22	2.64	2.24	2.30	2.76	2.93	2.85	2.96	4.65
	Average	2.48	3.21	2.58	2.36	2.67	3.40	2.74	2.65	3.64
172 lb. Subject	1	4.30	2.70	3.11	2.26	2.03	3.53	3.15	3.98	3.25
	2	1.11	2.69	3.03	1.69	2.15	3.10	3.54	3.64	3.50
	3	2.63	3.30	3.80	1.41	1.18	1.71	4.05	2.62	3.42
	4	2.48	2.63	2.10	0.97	1.50	4.29	3.70	2.73	3.15
	Average	2.63	2.83	3.01	1.58	1.72	3.16	3.61	3.24	3.33

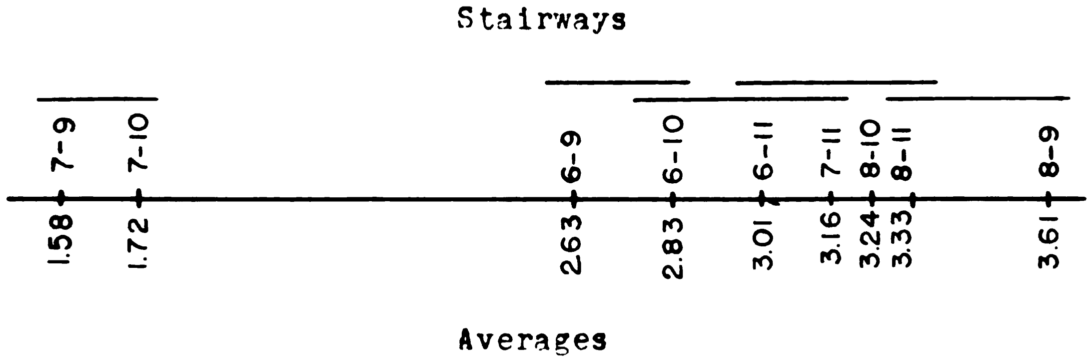


Figure 55: Comparison of the center of pressure averages, 172 pound subject descending the stairways.

appear safe with regard to slipping the lowest values are questionable.

The combined analysis of the data indicated no significant difference between the two subjects.

CONCLUSIONS

The following conclusions may be drawn from this investigation:

1) The abrasive strip and the rubber mat showed the best frictional properties of the materials studied. The coefficient of friction for these two tread materials was higher than the other tread materials for most all of the sole materials studied. For these two treads, there was little difference between the new and used materials. Wood, varnish and paint generally showed a decrease in the coefficient of friction with use while linoleum increased with use.

2) The ripple sole had the highest average coefficient of friction values of the sole materials studied. The highest coefficients of friction for the ripple sole occurred on the smooth surface tread materials. The highest coefficients of friction for the other soles occurred on the rough surface tread materials (abrasive and rubber mat). The crepe sole was the only sole for which the new material had a higher coefficient of friction than did the worn material.

3) The vertical force exerted by a person while either ascending or descending a stairway contained two maximum values. When ascending the stairway the first maximum value occurred near the 25 percent point and ranged from 100 to 125 percent of body weight.

When descending the stairways the first maximum value occurred between the 12.5 and 25 percent points and ranged from 100 to 140 percent of body weight. The second maximum value occurred near the 75 percent point for both directions and was approximately equal to the body weight. The initial rate of application of the vertical force varied linearly with time when ascending or descending the stairways.

4) The horizontal force exerted by a person while either ascending or descending a stairway varied considerably with different stairways and between subjects. When ascending a stairway the horizontal force had two maximum values and a minimum value. The maximum values occurred near the 6.25 and 87.5 percent points. The minimum value occurred near the 25 percent point. When descending a stairway the horizontal force had a maximum value which occurred between the 12.5 and 25 percent points and a minimum value which occurred at the 87.5 percent point. All maximum values were directed toward the front edge of the tread, the minimum values were directed toward the rear edge of the tread.

5) The initial point of application of the vertical force was independent of both stairways and subjects when ascending. When descending the initial point of application was independent of subject but varied some with the stairways. The initial point of application of the vertical force did not appear to be critical

with respect to slipping for this investigation.

6) The forces exerted by the subjects in this investigation normally were not large enough to cause a person to slip on a stair tread. However, each subject knew the objective os this investigation. Because each subject had some knowledge of the investigation it is possible that they were more careful when ascending or descending than they normally would be.

7) Although the vertical force tended to increase with an increase in the riser height none of the stairways may be regarded to be critical with respect to slipping under the conditions investigated.

SUGGESTIONS FOR FUTURE INVESTIGATIONS

The following list contains areas for future investigation and suggested changes in experimental procedure.

1) Evaluating the coefficient of friction for dry sole materials on wet tread materials, wet soles on dry treads and wet soles on wet treads. This investigation would determine the effect of surface wetness on the coefficient of friction.

2) Measuring the forces exerted on a stair step when carrying different loads or when moving at different speeds.

3) Measuring the forces exerted on a stair step when people are not aware of the investigation. An investigation of this type would give a truer evaluation of the ratio of the horizontal and vertical forces.

4) Enclosing the experimental stairways and adding more steps to make them more like a regular stairway.

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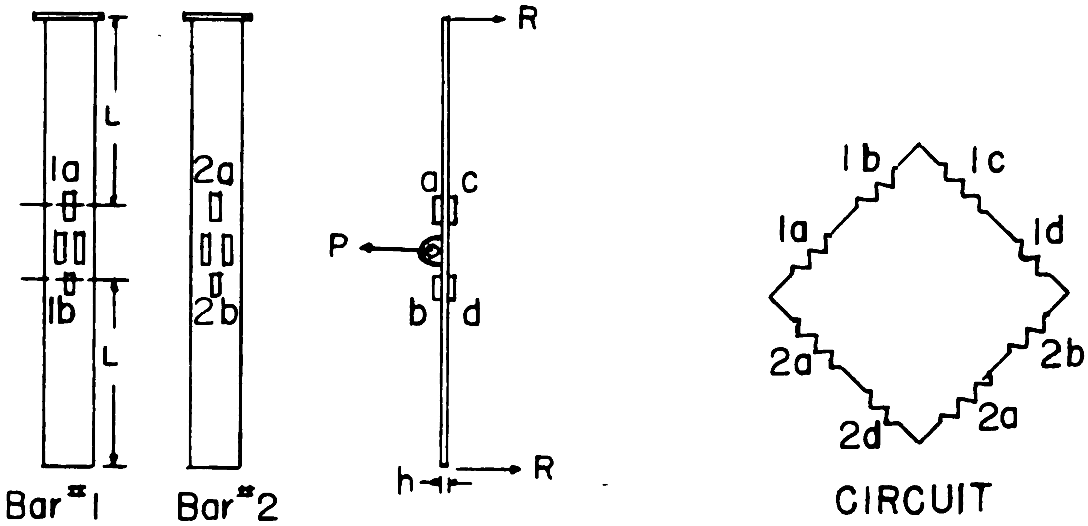
APPENDIX I. The strain gage circuit and bridge output for the transducer of the friction measuring machine.



Gages a and b are in tension.
Gages c and d are in compression.

I bar #1 receives $\frac{P}{2}$ amount of the total load, bar #2 will receive $\frac{PZ-P}{Z}$ amount of the total load. The reactions at the supports will be:

Bar #1 $\quad R = \frac{P}{2Z}$ $\qquad$ Bar #2 $\quad R = \frac{PZ-P}{2Z}$

$\epsilon = \frac{M\,c}{EI}$ where $M = RL$

For Bar #1 $\quad M = \frac{PL}{2Z}$, $\qquad$ for Bar #2 $\quad M = \frac{L(PZ-P)}{2Z}$

The indicated strain for each arm of the Wheatstone Bridge is as follows: (Gages connected in series in an arm will average)

$$\text{Arm 1} = \frac{1}{2}\left[\frac{P\,L\,c}{2Z\,EI} + \frac{P\,L\,c}{2Z\,EI}\right] = \frac{P\,L\,c}{2Z\,EI}$$

$$\text{Arm 2} = \frac{1}{2}\left[-\frac{P\,L\,c}{2Z\,EI} - \frac{P\,L\,c}{2Z\,EI}\right] = \frac{P\,L\,c}{2Z\,EI}$$

APPENDIX I (Continued)

$$\text{Arm } 3 = \frac{1}{2}\left[\frac{(PZ-P)L\,c}{2Z\,EI} + \frac{(PZ-P)L\,c}{2Z\,EI}\right] = \frac{(PZ-P)L\,c}{2Z\,EI}$$

$$\text{Arm } 4 = \frac{1}{2}\left[\frac{-(PZ-P)L\,c}{2Z\,EI} + \frac{-(PZ-P)L\,c}{2Z\,EI}\right] = \frac{(PZ-P)L\,c}{2Z\,EI}$$

The indicated strain of arms 1 and 3 of the Wheatstone Bridge is positive while the indicated strain of arm 2 and 4 is negative. Therefore the bridge output is the sum of the strain in each arm without respect to their signs.

$$= 2\left[\frac{P\,L\,c}{2Z\,EI}\right] + 2\left[\frac{(PZ-P)L\,c}{2Z\,EI}\right] = \frac{P\,L\,c}{E\,I}$$

Since c = the distance from the center to the extreme fibers and $I = \frac{b\,h^3}{12}$ the above formula can be simplified to the following:

$$\epsilon = \frac{6\,P\,L}{E\,b\,h^2}$$

APPENDIX II. Force Plate Design and Operating Circuits.

When a person ascends and descends a stairway a force F is exerted on the stair tread. This force is generally three dimensional and can be broken down into three component forces which lie along the coordinate axes. To determine the magnitude of these three components an instrumented stair tread was constructed using strain gages as the sensing element.

The top view of the instrumented stair tread is shown in Figure 56. This stair tread is under the influence of a force F. The two horizontal components are shown acting in their respective directions while the vertical force P is shown as a point. The tread is supported at the four corners with supports 2 and 4 being along the front edge.

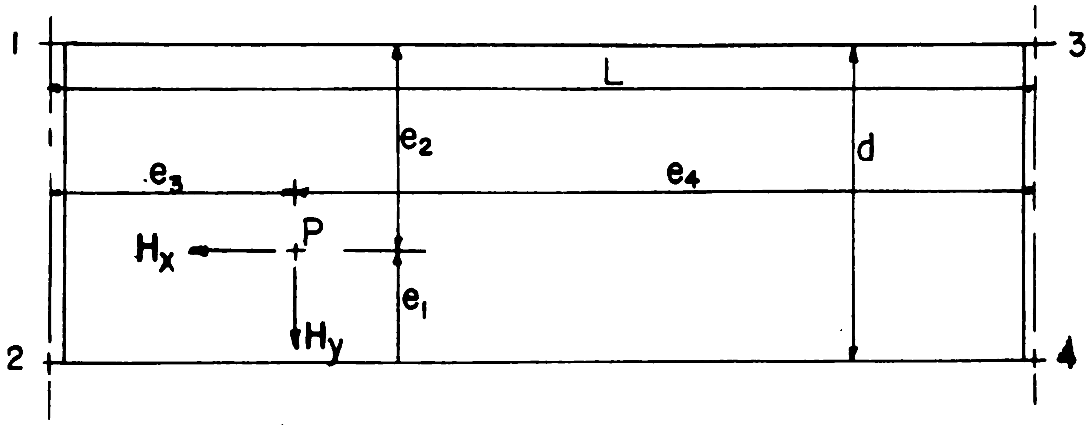


Figure 56: A Schematic Diagram of the Instrumented Stair Tread. A Top View Showing the Specific Dimensions.

Two basic assumptions were made with regard to the design and construction of the stair tread. The first was that the three forces were considered to be concentrated at a point rather than applied over an area. This assumption makes it easier to compute the forces and strains at the supports. This assumption has no effect on the accuracy of the instrument because a series of forces may be replaced by a single force acting through their centroid.

The second assumption was that the instrumented tread underwent negligible deflections when it was stepped on. Thus the length L and depth d of the tread remained constant. This assumption is valid because the tread was constructed using heavy pieces of angle iron to prevent any deflections.

The vertical reactions at each support can be determined by applying the elementary laws of statics. The first step is to determine the proportion of P taken by supports 1 and 2 (call this P_{12}) and the proportion taken by supports 3 and 4 (call this P_{34}). The next step is to determine the proportion of P_{12} taken by support 1 and the amount that is taken by support 2. The same procedure is followed for supports 3 and 4. The reactions at each support are given by the following formulas.

$$R_{1p} = \frac{Pe_1e_4}{dL} \qquad R_{2p} = \frac{Pe_2e_4}{dL} \qquad R_{3p} = \frac{Pe_1e_3}{dL} \qquad R_{4p} = \frac{Pe_2e_3}{dL}$$

R_{np} is the vertical reaction at the support n.

Looking at the tread from the top it is possible to apply the laws of statics and determine the proportion of Hy taken by supports 1 and 2 and the portion taken by supports 3 and 4. Because supports 1 and 2 are fixed at one end and a constant distance is maintained between them, where they connect to the tread, each support will receive one-half of the Hy component proportioned to them. The same follows for supports 3 and 4. The reactions due to Hy at each support are:

$$R_{1y} = R_{2y} = \frac{H_y e_4}{2}, \quad R_{3y} = R_{4y} = \frac{H_y e_3}{2}$$

R_{ny} is the horizontal reaction at support n due to Hy.

The same procedure can be followed for the horizontal component Hx. This force is divided between supports 1 and 3 and supports 2 and 4. The reactions due to Hx at each support are:

$$R_{1x} = R_{3x} = \frac{H_x e_1}{2d}, \quad R_{2x} = R_{4x} = \frac{H_x e_2}{2d}$$

R_{nx} is the horizontal reaction at support n due to Hx.

Strain Gage Techniques

Since strain is the fundamental quantity measured by strain gages the equations for the forces at the reactions must be changed to units of strain. The fundamental formula for changing stress to strain is

$$\epsilon = \frac{\sigma}{E}$$

where ϵ equals strain, micro inches/inch.

σ = stress, psi and E = modulous of elasticity. For the vertical force $\sigma = \frac{R_{np}}{A}$ and for bending $\sigma = \frac{M c}{I}$ where

$M = R_{ny}$ (t) for the Hy force and $M = R_{nx}(t)$ for the Hx force.

t - Distance from the reaction to the strain gages, inches.
c = Radius of the support, inches.
A = Crossectional area of the supports, sq. inches.
I = Moment of inertia of the support, (inches)

Perry and Lissner (14) state that the Wheatstone Bridge will be unbalanced only in proportional to the difference of resistance changes in any two adjacent arms, or in proportional to the sum of the resistance changes in any two opposite arms.

During this analysis the Wheatstone Bridge will be labeled as shown in Figure 57. The strain in any particular arm will be indicated by the notation ϵ_{An} where n is one of the arms.

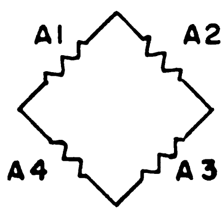


Figure 57: The Labeling of the Wheatstone Bridge.

Since strain is the fundamental quantity measured by strain gages units of strain can be used to determine the amount of unbalance in the Wheatstone Bridge. Using the above statement by Perry and Lissner and starting with arm A1, the unbalance for the Wheatstone Bridge can be written

$$\epsilon_B = \epsilon_{A1} - \epsilon_{A2} + \epsilon_{A3} - \epsilon_{A4} \qquad (1)$$

where ϵ_B is the strain indicated by the unbalanced bridge.

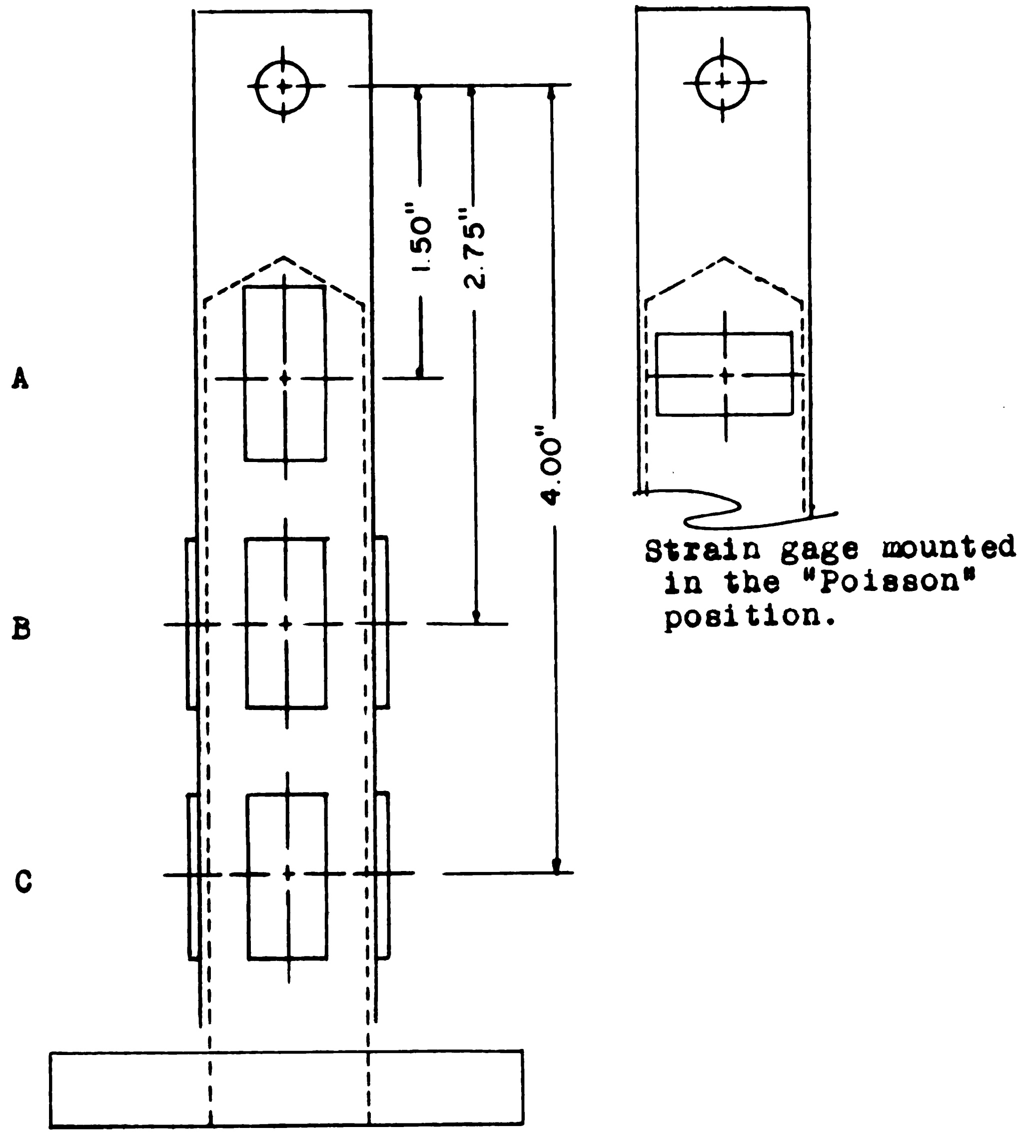
The placement of the strain gages on the supports is shown in Figure 58. The top row of gages was used to determine the vertical force. The middle row was used for determining the coordinate systems. The bottom row was used to determine the horizontal reactions. These gages were placed at the bottom to increase the lever arm thus increasing the sensitivity of the circuit.

Circuit No. 1
Strain Gage Circuit for Determining e_1 and e_2 .

The strain gage circuit and the placement of strain gages for determining e_1 and e_2 is given in Figure 59. There are only two forces measured by the gages in this circuit. The strain due to the vertical force and to the horizontal component Hx. The horizontal component Hy is canceled out because of symmetry of the gages. The strain due to the vertical force will be considered negative. The bending stress on the left hand side of the supports will also be considered negative. Assume the horizontal component Hx is acting to the left.

Particular combinations of the different quantities appear in all equations derived for each circuit. To make the presentation easier the following substitutions will be made.

$$\frac{t\,c}{I} = B, \qquad \frac{1}{2E} = D, \qquad \frac{1}{A} = A^*$$



A - Location of the strain gages measuring ϵ_3 .
B - Location of the strain gages measuring ϵ_1 and ϵ_2 .
C - Location of the strain gages measuring ϵ_4 and ϵ_5 .

Figure 58: Location of the strain gages on the corner supports.

APPENDIX II. (Continued)

Using these substitutions the following equations can be written for the Wheatstone Bridge.

$$\epsilon_{A1} = D\left[-A^{*}R_{1p} + R_{1x}B - A^{*}R_{3p} - R_{3x}B\right] = \frac{1}{2E}\left[\frac{-e_1 P}{d\ A}\right]$$

$$\epsilon_{A2} = D\left[-A^{*}R_{2p} - BR_{2x} - A^{*}R_{4p} + R_{4x}B\right] = \frac{1}{2E}\left[\frac{-e_2 P}{d\ A}\right]$$

$$\epsilon_{A3} = D\left[-R_{1p}A^{*} - BR_{1x} - A^{*}R_{3p} + BR_{3x}\right] = \frac{1}{2E}\left[\frac{-e_1 P}{A\ d}\right]$$

$$\epsilon_{A4} = D\left[-R_{4p}A^{*} - BR_{4x} - A^{*}R_{2p} + BR_{2x}\right] = \frac{1}{2E}\left[\frac{-e_2 P}{A\ d}\right]$$

Using equation (1) the bridge output is

$$\epsilon_1 = \frac{P(e_2 - e_1)}{EdA}$$

Using the relationship $e_1 + e_2 = d$ the following formulas may be solved for.

$$e_1 = \frac{d}{2}\left[1 - \frac{AE\epsilon_1}{P}\right] , \qquad e_2 = \frac{d}{2}\left[1 + \frac{AE\epsilon_1}{P}\right]$$

Circuit No. 2
Strain Gage Circuit for Determining e_3 and e_4 .

The strain gage circuit and the placement of the strain gages for determining e_3 and e_4 is given in Figure 59. The strain due to the vertical force and the horizontal component Hy are measured by the circuit. The strain due to the horizontal component Hx is canceled

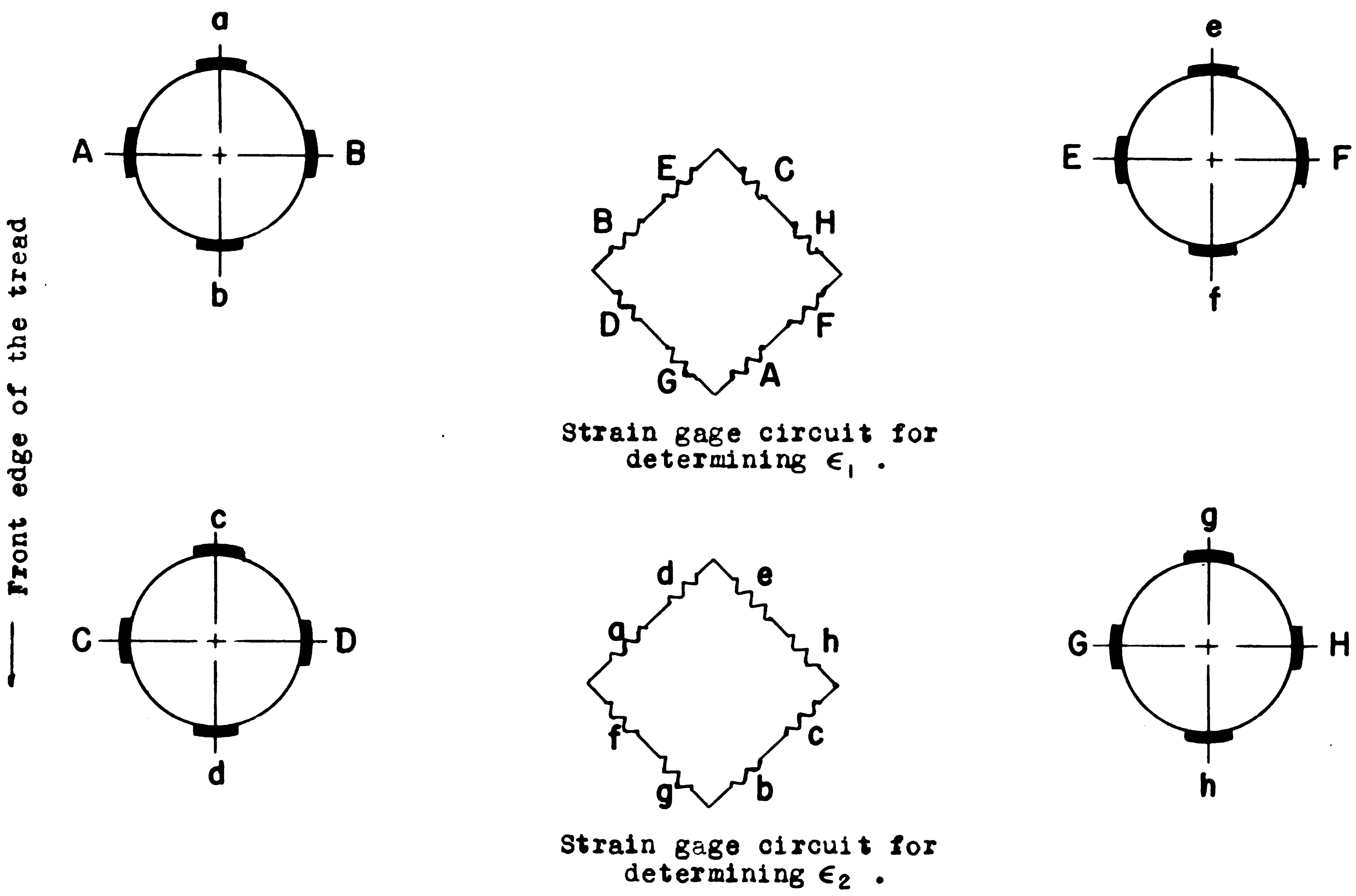


Figure 59: The strain gage circuits for determining ϵ_1 and ϵ_2 .

of the symmetry of the gages. Assume the horizontal component Hy is acting toward the front edge of the tread. This puts the front side of the supports in compression thus a negative strain results. The strain due to the vertical force is also negative.

The following equations may be written for the Wheatstone Bridge.

$$\epsilon_{A1} = D\left[-A^*R_{1p} + BR_{1y} - A^*R_{2p} - R_{2y}B\right] = \frac{1}{2E}\left[\frac{-e_4P}{AL}\right]$$

$$\epsilon_{A2} = D\left[-A^*R_{3p} + BR_{3y} - A^*R_{4p} - R_{4y}B\right] = \frac{1}{2E}\left[\frac{-e_3P}{AL}\right]$$

$$\epsilon_{A3} = D\left[-A^*R_{1p} - BR_{1y} - A^*R_{2p} + BR_{2y}\right] = \frac{1}{2E}\left[\frac{-e_4P}{AL}\right]$$

$$\epsilon_{A4} = D\left[-A^*R_{3p} - BR_{3y} - A^*R_{4p} + BR_{4y}\right] = \frac{1}{2E}\left[\frac{-e_3P}{AL}\right]$$

Using equation (1) the bridge output is

$$\epsilon_2 = \frac{P(e_3 - e_4)}{EAL}$$

Using the relationship $e_3 + e_4 = L$ the following formulas may be solved for.

$$e_3 = \frac{L}{2}\left[1 + \frac{AE\epsilon_2}{P}\right] \qquad e_4 = \frac{L}{2}\left[1 - \frac{AE\epsilon_2}{P}\right]$$

Circuit No. 3
Strain Gage Circuit for Determining the Vertical Force

The strain gage circuit and the placement of the strain gages for determining the vertical force P is given in Figure 60. The strain due to the vertical force and the horizontal component Hy are measured by the circuit. The horizontal component Hx is canceled because of the symmetry of the gages. Four of the strain gages, A, B, G and H, were mounted in the transverse direction of the support. If this had not been done the addition of the strains for the vertical force would have been zero. Assume the horizontal component Hy is acting toward the front edge of the tread. Thus the strains on the front side of the support are negative. The strain due to the vertical force is also negative.

The following equations may be written for the Wheatstone Bridge.

$$A1 = D\left[+ uA^*R_{1p} + BR_{1y} + uA^*R_{1p} - BR_{1y}\right] = \frac{1}{E}\left[\frac{ue_1 e_4 P}{A\ d\ L}\right]$$

$$A2 = D\left[- A^*R_{2p} + BR_{2y} - A^*R_{2p} - BR_{2y}\right] = \frac{1}{E}\left[\frac{-e_1 e_3 P}{d\ L\ A}\right]$$

$$A3 = D\left[+ uA^*R_{4p} + BR_{4y} + uR_{4pA^*} - BR_{4y}\right] = \frac{1}{E}\left[\frac{ue_2 e_3 P}{A\ d\ L}\right]$$

$$A4 = D\left[- A^*R_{3p} + BR_{3y} - A^*R_{3p} - BR_{3y}\right] = \frac{1}{E}\left[\frac{-e_2 e_4 P}{A\ d\ L}\right]$$

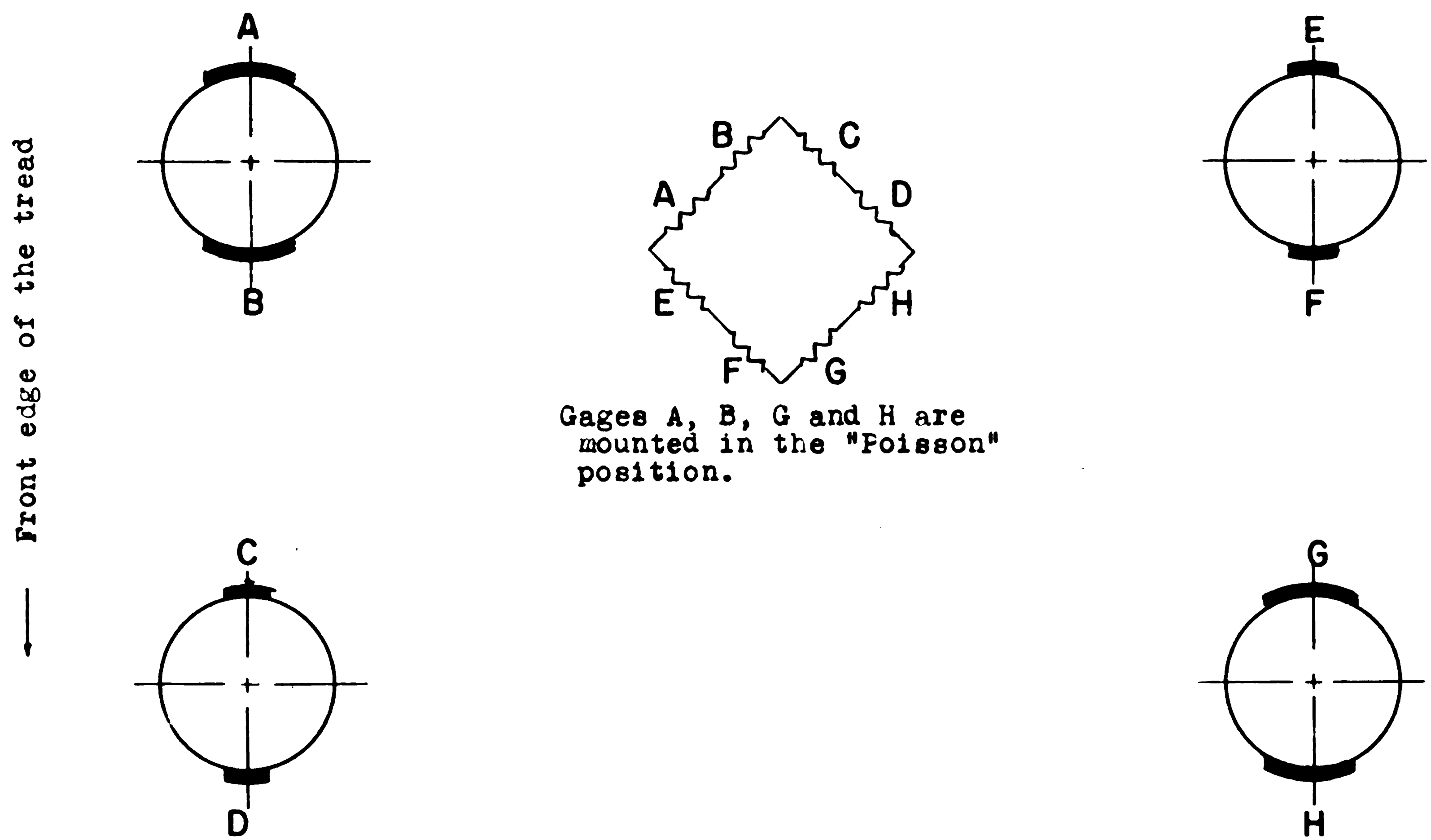


Figure 60: The strain gage circuit for determining ϵ_3 .

APPENDIX II. (Continued)

Using equation (1) the bridge output is

$$\epsilon_3 = \frac{P}{EAdL}\left[+ (ue_1e_4)(-e_1e_3) + (ue_2e_3) - (-e_2e_4)\right]$$

Using the relationships $e_2 = d-e_1$ and $e_3 = L-e_4$ the above equation becomes

$$\epsilon_3 = \frac{P}{EAdL}\left[(u-1)(2e_1e_4) + (1-u)(e_4d + e_1L) + udL\right]$$

To determine the magnitude of the vertical force P the equations of circuits 1 and 2 were solved in terms of e_1 and e_4 and substituted into the above equation. Upon substitution the above equation takes the form $aP^2 + bP + c = 0$ which can be solved by using the quadratic formula. The equation for P becomes

$$P = \frac{\epsilon_3 \pm \sqrt{\epsilon_3^2 - 4\left(\frac{1+u}{2}\right)\left(\frac{1}{AE}\right)\left(\frac{AE\epsilon_1\epsilon_2(u-1)}{2}\right)}}{2\left(\frac{1}{AE}\right)\left(\frac{1+u}{2}\right)}$$

Circuit No. 4
Strain Gage Circuit for Determining Hy.

The strain gage circuit and the placement of the strain gages for determining the horizontal component Hy is given in Figure 61. The strain due to the vertical force and the horizontal component Hy are measured by the circuit while the horizontal force Hx is canceled out. Assume the horizontal force Hy is acting toward the front edge.

The following equations may be written for the

Wheatstone Bridge.

$$\epsilon_{A1} = D\left[-A^*R_{1p} + BR_{1y} - A^*R_{3p} + BR_{3y}\right] = \frac{1}{2E}\left[\frac{-e_1P}{dA} + \frac{BHy}{2}\right]$$

$$\epsilon_{A2} = D\left[-A^*R_{2p} - BR_{2y} - A^*R_{4p} - BR_{4y}\right] = \frac{1}{2E}\left[\frac{-e_2P}{dA} - \frac{BHy}{2}\right]$$

$$\epsilon_{A3} = D\left[-A^*R_{2p} + BR_{2y} - A^*R_{4p} + BR_{4y}\right] = \frac{1}{2E}\left[\frac{-e_2P}{dA} + \frac{BHy}{2}\right]$$

$$\epsilon_{A4} = D\left[-A^*R_{1p} - BR_{1y} - A^*R_{3p} - BR_{3y}\right] = \frac{1}{2E}\left[\frac{-e_1P}{dA} - \frac{BHy}{2}\right]$$

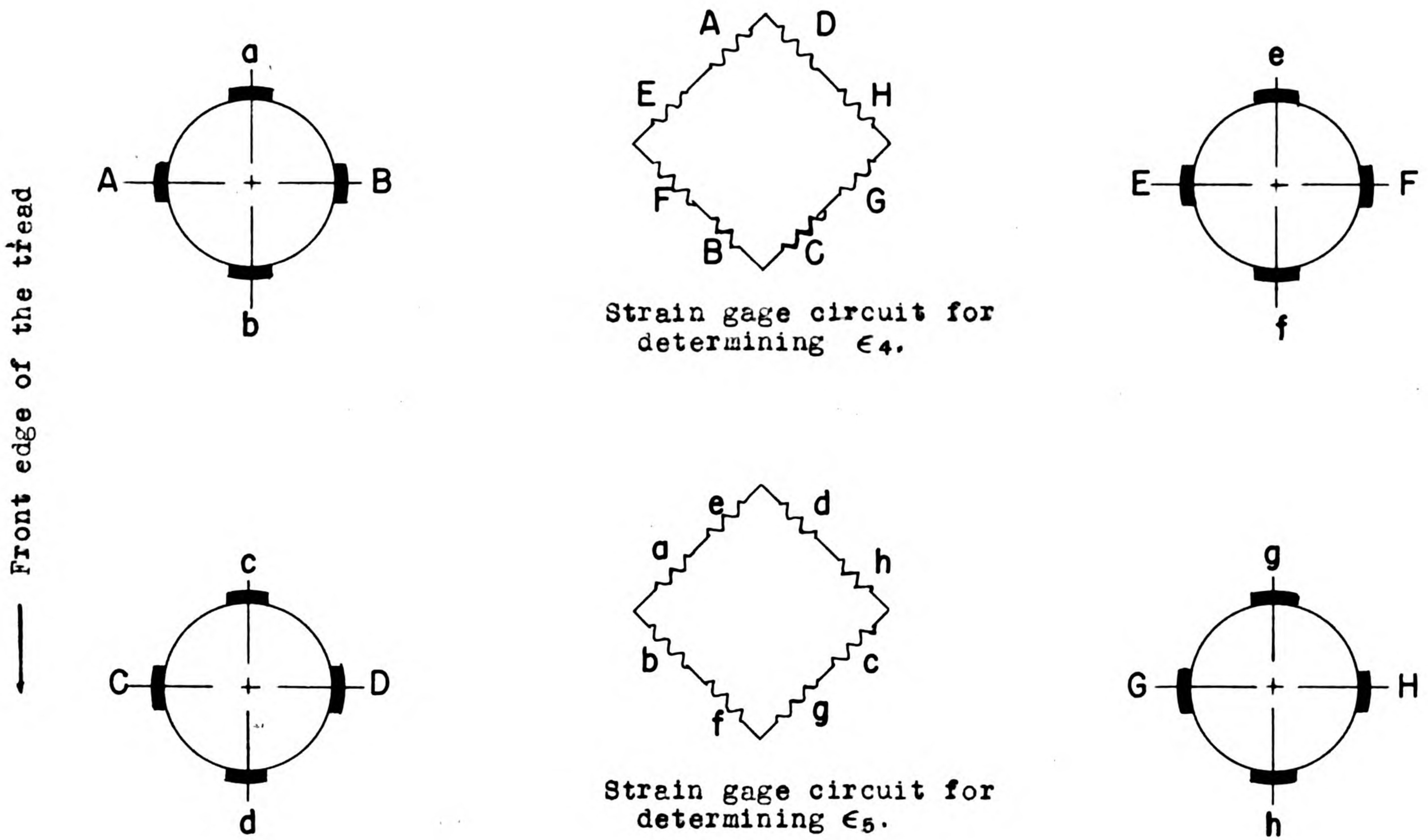
Using equation (1) the bridge output is

$$\epsilon_B = \frac{Hy\ t\ c}{E\ I} \qquad Hy = \frac{E\ I\ \epsilon_4}{t\ c}$$

Circuit No. 5
Strain Gage Circuit for Determining Hx.

The strain gage circuit and the placement of the strain gages for determining the horizontal component Hx is given in Figure 61. The strain due to the vertical force and the horizontal component Hx are measured by the circuit while the horizontal force Hy is canceled. Assume the horizontal force Hx is acting toward the left side of the tread.

The following equations may be written for the Wheatstone Bridge.



Strain gage circuit for determining ϵ_4.

Strain gage circuit for determining ϵ_5.

Figure 61: The strain gage circuits for determining ϵ_4 and ϵ_5 .

APPENDIX II. (Continued)

$$\epsilon_{A2} = D\left[-A^{*}R_{1p} - BR_{1x} - A^{*}R_{3p} - BR_{3x}\right] = \frac{1}{2\,E}\left[-\frac{e_1 P}{A\,d} - \frac{HxBe_1}{d}\right]$$

$$\epsilon_{A1} = D\left[-A^{*}R_{2p} + BR_{2x} - A^{*}R_{4p} + BR_{4x}\right] = \frac{1}{2\,E}\left[-\frac{e_2 P}{d\,A} + \frac{HxBe_2}{d}\right]$$

$$\epsilon_{A3} = D\left[-A^{*}R_{1p} + BR_{1x} - A^{*}R_{3p} + BR_{3x}\right] = \frac{1}{2\,E}\left[-\frac{e_1 P}{d\,A} + \frac{HxBe_1}{d}\right]$$

$$\epsilon_{A4} = D\left[-A^{*}R_{2p} - BR_{2x} - A^{*}R_{4p} - BR_{4x}\right] = \frac{1}{2\,E}\left[-\frac{e_2 P}{d\,A} - \frac{HxBe_2}{d}\right]$$

Using Equation (1) the bridge output is

$$\epsilon_5 = \frac{Hx\ t\ c}{E\ I} \qquad\qquad Hx = \frac{E\ I\ \epsilon_5}{t\ c}$$

APPENDIX III. The Coefficient of Friction Values for the Sole and Tread Materials.

Large Soles	Wood		Linoleum		Varnish	
	New	Worn	New	Worn	New	Worn
New Neoprene	0.86	0.64	0.52	0.66	0.72	0.62
Worn Neoprene	0.82	0.56	0.52	0.60	0.80	0.58
New Crepe	0.65	0.48	0.46	0.52	0.65	0.50
Worn Crepe	0.57	0.55	0.33	0.53	0.48	0.49
New Leather	0.31	0.28	0.29	0.32	0.42	0.31
Worn Leather	0.37	0.34	0.25	0.42	0.46	0.37
New Neolite	0.60	0.56	0.55	0.46	0.76	0.49
Worn Neolite	0.61	0.63	0.63	0.58	0.87	0.63
New B.F. Goodrich	0.48	0.45	0.31	0.44	0.62	0.47
Worn B.F. Goodrich	0.62	0.55	0.55	0.55	0.63	0.59
New Ripple	1.05	0.79	0.93	0.84	1.15	0.84
Worn Ripple	1.05	0.97	1.01	1.12	1.19	1.13

Large Soles	Abrasive		Paint		Rubber Mat	
	New	Worn	New	Worn	New	Worn
New Neoprene	0.90	0.86	0.58	0.62	0.72	0.72
Worn Neoprene	0.88	0.88	0.64	0.54	0.74	0.72
New Crepe	0.84	0.78	0.54	0.49	0.64	0.68
Worn Crepe	0.72	0.74	0.43	0.47	0.63	0.70
New Leather	0.75	0.69	0.56	0.33	0.67	0.72
Worn Leather	0.88	0.84	0.60	0.44	0.67	0.81
New Neolite	0.86	0.74	0.62	0.55	0.56	0.55
Worn Neolite	0.76	0.74	0.65	0.68	0.53	0.56
New B.F. Goodrich	0.49	0.54	0.56	0.53	0.32	0.59
Worn B.F. Goodrich	0.63	0.66	0.55	0.60	0.55	0.56
New Ripple	0.91	0.92	0.95	0.81	0.94	0.79
Worn Ripple	0.92	0.95	1.05	1.12	0.89	0.87

APPENDIX III. (Continued)

Small Soles	Wood New	Wood Worn	Linoleum New	Linoleum Worn	Varnish New	Varnish Worn
New Neoprene	0.59	0.54	0.44	0.52	0.60	0.55
Worn Neoprene	0.68	0.71	0.65	0.56	0.76	0.66
New Crepe	0.72	0.60	0.51	0.54	0.80	0.53
Worn Crepe	0.45	0.46	0.30	0.41	0.59	0.39
New Leather	0.21	0.19	0.21	0.21	0.43	0.21
Worn Leather	0.32	0.26	0.26	0.29	0.36	0.25
New Neolite	0.56	0.51	0.55	0.47	0.67	0.54
Worn Leolite	0.67	0.56	0.60	0.57	0.88	0.65
New B.F. Goodrich	0.39	0.41	0.28	0.45	0.52	0.46
Worn B.F. Goodrich	0.70	0.59	0.62	0.61	0.73	0.62
New Ripple	1.16	1.05	1.07	1.10	1.24	1.14
Worn Ripple	1.06	1.08	0.99	1.15	1.17	1.20

Small Soles	Abrasive New	Abrasive Worn	Paint New	Paint Worn	Rubber Mat New	Rubber Mat Worn
New Neoprene	0.71	0.73	0.54	0.57	0.62	0.62
Worn Neoprene	0.80	0.83	0.69	0.60	0.71	0.70
New Crepe	0.84	0.81	0.63	0.55	0.64	0.70
Worn Crepe	0.70	0.68	0.51	0.42	0.55	0.66
New Leather	0.39	0.46	0.35	0.23	0.36	0.56
Worn Leather	0.49	0.51	0.40	0.28	0.45	0.60
New Neolite	0.74	0.70	0.55	0.57	0.56	0.54
Worn Neolite	0.66	0.66	0.66	0.65	0.53	0.55
New B.F. Goodrich	0.46	0.54	0.51	0.47	0.38	0.54
Worn B.F. Goodrich	0.69	0.71	0.63	0.65	0.64	0.63
New Ripple	1.02	1.07	1.12	1.16	0.93	0.84
Worn Ripple	0.94	1.07	1.08	1.20	0.88	0.89

APPENDIX IV: The Calculation of a Rank Correlation Value.

The rank correlation value is a measure of the correlation between two people while performing the same task. For this investigation the two subjects were compared by how they ranked the different stairways or percent points with respect to the average force exerted on the stairways or percent points.

The correlation value will be calculated for the percent points while the two subjects ascended the stairways. The percent points for each subject are ranked from highest to lowest. For this example each subject exerted the largest force at the 25 percent point and the smallest force at the 93.75 percent point. The correlation value is calculated from the following formula:

$$R = 1 - \frac{6 \sum \Delta^2}{N(N^2 - 1)}$$

Ranking of the Percent Points

Percent Points	140 lb. Subject	172 lb. Subject	Δ	Δ^2
6.25	8	8	0	0
12.5	5	5	0	0
25.0	1	1	0	0
37.5	3	3	0	0
50.0	7	6	1	1
62.5	4	4	0	0
75.0	2	2	0	0
87.5	6	7	1	1
93.75	9	9	0	0
				2

For this example the correlation value is

$$R = 1 - \frac{6(2)}{9(81-1)} = 1 - \frac{1}{60} = 0.983$$

APPENDIX V: The Magnitude of the Vertical Force, Horizontal Force and the Point of Application of the Vertical Force.

Note: The vertical and horizontal forces were not measured at the same time. e (the point of application of the vertical force) is the distance from the front edge of the tread. For the horizontal force, positive values are directed toward the front edge of the tread; negative values toward the rear edge. Each value is an average of four replications.

140 POUND SUBJECT

Stairway: 6" risers, 9" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	40.1	3.21	4.1	58.2	2.48	1.3
12.5	91.2	4.15	-7.3	125.2	2.61	6.8
25.0	139.7	3.40	-9.6	145.1	3.81	6.7
37.5	121.9	3.24	-1.6	105.0	3.91	-0.7
50.0	101.9	3.49	1.7	97.3	3.59	-1.6
62.5	111.2	3.70	3.3	127.4	2.71	-1.1
75.0	123.3	4.75	0.4	154.3	1.60	-2.1
87.5	102.9	6.21	2.6	77.0	0.65	-11.9
93.75	30.1	6.77	3.6	26.1	1.59	-4.0

Stairway: 6" risers, 10" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	60.3	3.91	2.7	63.1	3.21	1.1
12.5	101.5	4.22	-10.7	149.4	3.58	2.3
25.0	143.6	3.62	-16.8	135.5	3.38	2.5
37.5	117.1	3.39	-7.4	110.7	3.82	-1.5
50.0	99.2	3.48	-1.4	100.7	3.36	-2.6
62.5	112.3	3.82	-0.8	130.9	3.02	-2.1
75.0	139.6	4.78	-2.6	146.3	2.06	-5.6
87.5	84.2	6.20	-0.2	72.3	1.87	-12.5
93.75	16.5	8.26	2.8	21.9	1.81	-2.4

Stairway: 6" risers, 11" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	45.9	3.41	3.7	84.2	2.58	2.5
12.5	101.2	4.00	-8.8	165.5	3.05	7.7
25.0	144.5	3.38	-10.2	150.1	3.98	9.0
37.5	125.1	3.22	-2.1	102.1	4.28	2.2
50.0	101.7	3.34	1.6	90.1	4.22	-0.6
62.5	110.2	3.51	2.2	118.8	3.01	-0.8
75.0	136.6	5.00	0.8	155.5	1.73	-4.5
87.5	112.2	6.57	3.5	80.9	0.73	-14.6
93.75	39.7	6.71	3.1	29.0	1.63	-2.8

APPENDIX V. (Continued)

Stairway: 7" risers, 9" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	45.2	3.98	2.2	78.5	2.36	4.8
12.5	98.1	3.97	-6.9	156.4	2.64	11.5
25.0	148.1	3.01	-3.8	143.0	2.91	15.1
37.5	123.1	2.65	-1.5	98.5	2.93	5.0
50.0	92.3	2.49	3.7	91.5	2.66	3.1
62.5	99.9	2.62	3.1	127.4	1.57	2.9
75.0	135.4	4.04	2.7	142.3	0.74	-1.2
87.5	101.5	5.04	4.4	68.1	0.90	-9.6
93.75	34.0	5.35	4.2	25.8	2.11	2.7

Stairway: 7" risers, 10" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	40.1	3.70	1.9	43.0	2.67	3.3
12.5	85.5	4.16	-9.3	114.5	3.23	6.7
25.0	139.2	3.20	-9.1	141.9	3.68	6.1
37.5	119.2	2.79	-2.8	108.4	4.03	3.2
50.0	92.3	2.47	-1.5	90.3	3.29	-0.3
62.5	102.6	3.16	1.5	112.9	2.27	-0.2
75.0	129.9	4.94	1.9	138.5	1.50	-2.6
87.5	91.5	6.10	4.7	74.2	0.72	-10.5
93.75	23.0	6.85	3.7	27.6	2.97	-2.1

Stairway: 7" risers, 11" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	62.0	3.79	2.8	88.6	2.74	2.0
12.5	121.1	4.16	-3.3	155.5	2.89	6.3
25.0	159.1	3.62	-6.0	144.1	2.93	5.7
37.5	122.1	3.50	-2.1	104.7	2.83	2.1
50.0	92.0	3.87	0.2	97.2	2.61	0.5
62.5	97.5	4.50	1.2	137.8	2.06	-0.2
75.0	141.2	5.58	1.0	146.8	1.55	-2.5
87.5	102.5	6.52	2.8	56.8	1.70	-6.7
93.75	30.3	7.51	2.3	22.0	2.37	-1.2

APPENDIX V. (Continued)

Stairway: 8" risers, 9" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	70.4	3.62	9.5	109.3	2.74	5.5
12.5	115.0	3.65	-4.5	192.9	2.89	12.6
25.0	149.1	2.94	-3.2	153.1	2.93	14.4
37.5	123.3	2.40	8.7	103.0	2.83	4.6
50.0	94.6	2.24	5.6	99.1	2.61	4.2
62.5	111.3	2.61	6.8	142.5	2.06	3.3
75.0	144.1	4.03	5.5	156.4	1.55	0.1
87.5	88.2	5.21	6.4	73.7	1.70	-10.7
93.75	22.3	5.99	4.9	27.7	2.37	-2.6

Stairway: 8" risers, 10" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	38.2	2.92	4.4	104.6	2.65	4.9
12.5	103.1	3.48	-5.3	186.6	3.10	8.2
25.0	145.5	2.71	-7.5	146.8	3.55	7.8
37.5	124.2	2.71	-1.2	99.2	2.89	3.2
50.0	94.5	2.58	4.0	98.7	2.37	2.0
62.5	105.8	2.63	6.4	135.5	1.78	1.4
75.0	131.3	3.84	6.8	144.8	1.34	-3.1
87.5	104.7	5.44	5.0	64.4	1.37	-6.0
93.75	26.7	6.34	4.3	24.5	2.27	-0.6

Stairway: 8" risers, 11" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	28.4	3.46	5.8	88.6	3.64	3.6
12.5	90.7	4.12	-7.2	165.7	3.72	6.0
25.0	154.2	3.30	-11.3	162.3	4.02	5.0
37.5	134.5	3.05	-4.6	104.4	3.80	0.1
50.0	99.6	3.09	0.6	93.7	3.62	-0.7
62.5	105.0	3.31	0.5	134.2	3.12	-2.1
75.0	137.6	4.79	0.8	154.6	2.63	-9.4
87.5	100.8	5.94	4.3	73.7	2.39	-11.4
93.75	26.4	6.55	2.7	28.0	3.11	-0.3

APPENDIX V: (Continued)

172 POUND SUBJECT

Stairway: 6" risers, 9" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	66.9	3.02	5.4	771.0	2.63	-0.6
12.5	141.5	3.06	-2.1	132.9	2.29	1.6
25.0	195.9	2.60	-4.2	201.3	2.41	9.7
37.5	151.9	2.35	-1.4	174.1	2.21	12.1
50.0	131.4	2.52	0.0	139.6	1.99	5.6
62.5	149.5	3.07	-0.3	151.0	1.71	10.3
75.0	176.5	3.31	1.6	168.1	1.59	-5.5
87.5	105.5	2.88	6.3	98.2	1.72	-15.6
93.75	33.0	3.14	1.8	35.2	3.25	-0.3

Stairway: 6" risers, 10" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	86.2	3.90	2.8	54.0	2.83	-1.2
12.5	147.9	3.42	-5.0	132.9	2.54	1.7
25.0	192.4	2.86	-11.0	211.2	2.01	7.3
37.5	150.5	2.72	-8.1	175.8	1.50	6.4
50.0	121.0	2.83	-3.9	139.2	1.61	6.4
62.5	143.5	3.12	-3.1	142.8	1.68	-0.7
75.0	174.2	3.12	-2.4	156.2	1.53	-5.0
87.5	93.7	2.52	2.6	77.7	0.84	-17.0
93.75	28.6	3.21	2.1	30.5	1.91	-6.3

Stairway: 6" risers, 11" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	63.2	2.88	8.4	66.0	3.01	2.1
12.5	125.8	2.68	-0.4	159.2	2.90	5.7
25.0	200.0	2.49	-2.7	239.4	2.21	13.8
37.5	182.5	2.36	0.4	189.6	1.66	10.5
50.0	145.4	2.36	0.4	130.8	1.64	9.3
62.5	146.5	2.97	1.7	136.0	1.53	3.2
75.0	168.1	3.10	4.5	165.8	1.58	-1.7
87.5	83.4	2.97	8.2	85.2	0.45	-15.8
93.75	30.0	3.82	2.0	32.2	0.76	-1.7

APPENDIX V. (Continued)

Stairway: 7" risers, 9" treads

% of Step	Ascending P	Ascending e	Ascending Hv	Descending P	Descending e	Descending Hv
6.25	74.2	3.03	4.1	53.9	1.58	-1.0
12.5	137.1	2.68	1.1	108.2	1.60	3.6
25.0	182.8	2.42	0.4	187.8	2.16	13.6
37.5	165.6	2.42	2.7	155.9	2.38	13.4
50.0	136.8	2.62	4.2	130.6	2.44	12.0
62.5	132.4	2.79	4.8	135.1	1.73	6.6
75.0	157.4	2.98	5.5	155.0	1.44	-3.5
87.5	97.7	3.37	8.1	73.7	1.45	-5.9
93.75	31.0	3.77	2.2	24.7	2.69	1.1

Stairway: 7" risers, 10" treads

% of Step	Ascending P	Ascending e	Ascending Hv	Descending P	Descending e	Descending Hv
6.25	89.5	3.28	5.0	80.6	1.72	-0.7
12.5	150.8	2.54	0.7	166.1	2.06	3.8
25.0	183.7	1.61	-2.6	201.3	1.60	11.7
37.5	137.1	1.17	-1.5	156.8	1.64	12.6
50.0	117.1	1.56	-0.6	121.6	0.72	9.3
62.5	147.8	2.26	-0.2	142.5	0.75	3.2
75.0	175.5	2.40	1.5	150.2	0.50	-18.0
87.5	90.8	1.26	5.2	74.9	0.61	-13.8
93.75	29.2	1.95	2.1	28.4	1.06	-3.7

Stairway: 7" risers, 11" treads

% of Step	Ascending P	Ascending e	Ascending Hv	Descending P	Descending e	Descending Hv
6.25	90.3	3.64	4.0	48.3	3.16	3.7
12.5	152.1	3.58	-0.8	122.3	2.55	2.3
25.0	200.4	3.37	-2.5	206.6	2.30	8.3
37.5	182.4	3.20	-1.6	187.9	2.36	5.8
50.0	155.3	3.36	-0.1	159.3	1.96	3.2
62.5	155.9	3.70	0.3	164.3	1.41	-0.1
75.0	143.6	3.87	1.4	184.8	1.17	-3.0
87.5	54.4	4.85	4.7	75.9	2.92	-11.8
93.75	15.4	5.72	5.8	25.7	4.24	-17.2

APPENDIX V. (Continued)

Stairway: 8" risers, 9" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	70.5	3.34	9.2	44.2	3.63	0.4
12.5	128.2	3.13	2.4	126.3	2.27	5.9
25.0	174.9	2.48	2.3	194.7	2.11	17.3
37.5	168.6	2.22	5.7	160.7	1.70	16.9
50.0	152.9	2.12	6.3	143.1	1.71	13.2
62.5	143.4	2.35	4.2	146.8	1.47	6.4
75.0	182.7	3.12	5.3	166.6	1.20	-7.8
87.5	88.1	3.35	8.2	61.0	1.20	-7.9
93.75	38.4	4.05	2.4	16.0	2.20	-0.5

Stairway: 8" risers, 10" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	107.1	2.86	4.9	95.8	3.24	7.2
12.5	159.3	2.70	-1.2	170.8	2.78	11.9
25.0	199.3	2.55	0.5	232.2	2.31	25.6
37.5	169.1	2.25	2.2	162.0	2.15	17.9
50.0	147.1	2.38	5.6	155.0	2.11	10.1
62.5	164.5	2.83	6.6	155.8	1.77	5.6
75.0	175.8	2.62	10.7	166.9	1.42	0.8
87.5	68.6	2.52	14.7	63.9	0.70	-4.8
93.75	21.2	3.38	3.8	21.8	2.47	2.0

Stairway: 8" risers, 11" treads

% of Step	Ascending P	Ascending e	Ascending Hy	Descending P	Descending e	Descending Hy
6.25	74.4	3.76	8.1	96.6	3.33	-0.5
12.5	155.8	3.32	-1.3	181.4	2.98	3.5
25.0	215.6	2.53	-7.2	232.3	3.09	7.7
37.5	168.3	2.20	-3.9	161.4	2.55	7.4
50.0	124.2	2.18	-1.6	116.7	2.35	4.9
62.5	139.7	2.61	-1.8	148.3	2.26	-2.7
75.0	198.0	2.86	-3.4	175.8	2.11	-11.6
87.5	110.1	3.70	6.2	70.9	2.15	-18.1
93.75	26.2	4.37	2.2	23.5	3.64	-3.4