STUDIES OF FILTRATION

THESIS FOR THE DEGREE OF M. S.

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Chemica

STUDIES OF FILTRATION

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Table of Contents.

I. Introduction.

Filtration is the process of separating the solids from a mixture of solids and liquids. This is done by allowing the liquid to flow through a porous medium whose orfices are not large enough to allow the passage of the solids. There must be some device to support the medium as well as a difference in pressure between the two surfaces so as to force the liquid through.

The most common of the various types, is the filter press. There are two general types of the filter press, the recessed, and the plate and frame press. The plate and frame press (fig. 1) which is the more common of the two types consists of a series of plates, (fig. 2), which are ribbed in one way or another and support the filter medium; and frames which are hollow and afford space in which the cake builds up. These plates and frames are so arranged that the feed channels on all frames are placed in the same line and the discharge channel of the plates are placed in line diagonally opposite the feed channels. One cloth is placed on each plate surface and holes are cut in the filler cloths corresponding to the feed and discharge channels. The slurry comes in the feed and the filtrate leaves through the discharge channels, the percipitate collecting on the filter cloth.

Other common types of filters are the centrifuge and suction filters (fig.3). Suction filters are of the filter bed, leaf, (fig.4) and rotary types and their various modifications.

The selection of any type depends upon the character of the slurry to be filtered. Only crystalling solids can be handled efficiently in centrifugal machines. Very collodial precipitates filter most readily on gravity filters, etc.

Generally speaking, slurries which filter at a high rate are filtered in large chambers. That is, the ratio of slurry to filter area is large. Where slurries are hard to filter the chambers are not thicker than one inch. That is, the ratio of slurry to filter area is small.

Washing of the filtered cake is also a big problem. Washing is carried on for two reasons, either to recover a valuable liquor from the solid particles which retain it or to free the precipitate of the mother liquor or of impurities in the mother liquor. There are two general methods of washing the direct wash and the counter flow wash. The direct wash consists of forcing the wash water through the feed channel. The disadvantage of this method of washing is danger of channeling, thus leaving parts of the cake that are not properly washed. In most cases the counter flow wash is used; but with some cakes there is danger os incomplete washing due to cracking of the cake. The success in washing a cake properly depends greatly upon the filtration technique.

"Given the filtration conditions and the design of a filter what volume of filtrate can be obtained in a definite length of time[1]?""

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[1] Badger and McCabe--Elements of Chem. Eng.---p 456

"What volume of wash water can be passed through the cake in a definite length of time? What will be the relationship between concentration of recovered material in the wash water and the amount of wash water used?"

"Theoritical solution of these questions is retarded by the facts that different sludges vary greatly in characteristics and the resistance to flow of any of its extremely sensitive to temperature, method of preparation, and age. It is very difficult, especially with collodial sludges, to duplicate results within several hundred per cent, even when different batches are prepared under apparently identical conditions."

These statements are taken from Elements of Chemical Engineering by Badger and McCabe (2). However, I am inclined to believe that results can be made to check if the filtration cycle is properly carried out. Filtration requires great care; and the temperature, pressure, and placing of filter medium should be carefully regulated.

In industrial practice the method of preparation of a slurry and age, in most cases is a minor problem in filtration. It is true that the characteristics of some sludges vary some from time to time. However, these slurries are a part of a certain chemical process which is carried on in a certain way from day to day under the same conditions, thus the variation in character should be very slight. Age does have a great effect upon filtration but industrally, the time slurries stand before filtering is generally about the same, from day to day.

If filtration is properly carried out it should not be hard to check results much closer than several hundred per cent.

The slurries used in this work were selected as representative slurries of the compressible non-homogeneous type. These slurries were made up in small batches of about fifteen gallons. Although the quantity was rather small, I was forced to do so because of my inability to check results where large quantities were made up, and one slurry stood longer than another.

All of this work pertaining to filtration theory has been done in connection with the equations of Walker, Lewis and Mc Adams (1) and Badger and McCabe (2). It has been my object to show the inefficiencies of these equations as well as the extent of involved theory whichtthey embody. It is almost impossible to get a thorough understanding of these equations without an unusual theoritical mind.

The real importance of a filtration equation is in affording a means of determining the area of filter surface required for any definite amount of sludge and the time required to carry out the filtration cycle. Although I realize that these equations could give a rough estimate in predicting filtering times, they do not meet the requirements of ordinary industrial practice. It is with this in mind that I have developed the equations which will follow in one of the chapters of this thesis.

II. Laboratory Experiment.

A. Preparation of Slurries.

The first slurry usediin this work and one which represents the non-homogeneous class of sludges, consisted of calcium carbonate and aluminum hydoxide.

This slurry was made by dissolving 12 calcium chloride and 3.3 aluminum chloride in 46.7 of water and adding this solution to a solution of 10.95 sodium carbonate in 53.1 of water. However, the calcium chloride used were not pure material and the slurry was made up according to the following table:

Table I

Solution A	Parts of Pure Compounds.	Actual Wt. in Pounds.
Calcium Chlordde	10.0	12.6
Aluminum Chloride	1.62	3.3
Water	48.7	48.7
Solution B		
Sodium Carbonate	10.95	10.95
Water	53.1	53.1
	Total Weight	128.65

Solution A was slowly added to solution B with constant stirring. The resulting slurry was allowed to stand for at least twelve hours Before using. After the slurry stood for this length of time, more uniform results could be obtained, upon filtering.

When ever possible or advisible simultaneous experiments were run with a second slurry. Slurry no. II consisted of sodium carbonate only as the precipitate, which represents the homogeneous class of slurries. This slurry was made up in a similar manner to that of slurry I, as follows:

Table II

Solution A	Parts of Pure Compounds.	Actual Wt. in Pounds.
Calcium Chloride	10.0	12.6
Water	45.0	45.0
Solution B		
Sodium Carbonate	9.6	9.6
Water	50.4	50.4
	Total Weight	117.6

In the experiments which are to follow these slurries will be referred to as slurry I and slurry II.

B. Constituents of each slurry.

1. Calculated

<u>Slurry I</u>

(1) $Cacl_2 + Na_2CO_3 \qquad 2Nacl + CaCO_3$

$\quad 110 \qquad 106 \qquad\qquad 116 \qquad 100$

(2) $3Na_2CO_3 + 2Alcl_3 + 3H_2O \qquad 2Al(OH)_3 + 6Nacl + 3CO_2$

$\qquad\qquad 264 \qquad\qquad\qquad 156 \qquad 348$

a. Per cent Solids.

From equation (1)

$110:100 = 10.0:x$

$x = 9.1 \ CaCO_3$

From equation (2)

264:156 = 1.62:x

x = 0.96 $Al(OH)_3$

Total Solids 9.1 + 9.6 = 10.06

Total Wt. of Slurry = 128.6

(10.06 : 128.6) X 100 = 7.87%

Solids

b. Per cent Sodium Chloride

From equation (1)

110:116 = 10:x

x = 10.5 Nacl.

From equation (2)

264:348 = 1.62:x

x = 2.13 Nacl

Total Nacl = 10.5 + 2.13 = 12.63

Total Wt. of Slurry = 128.6

Per cent of Nacl

12.63 : 128.6 X 100 = 9.98%

Slurry II

a. Per Cent Solids

From equation (1)

110:100 = 10:x

x = 9.1 $CaCO_3$

Total Wt. of Slurry = 117.6

Per cent Solids

9.1 : 117.6 X 100 = 7.87%

c. Per Cent Sodium Chloride

From equation (1)

110:116 = 10:x

x = 10.5 Nacl

Total Wt. of Slurry = 117.6

Per cent Nacl

10.5 : 117.6 X 100 = 9.1%

2. Actual

Slurry I

a. Per Cent Solids

Wt. of Sample as taken = 9.219 Gms.

Wt. of Sample Dry = 0.7089

Per cent Solids

0.7089 : 9.219 X 100 = 7.69 %

b. Per Cent Sodium Chloride

$AgNO_3$ Solution N/10

92.7 cc $AgNO_3$ N/10 neutralized a 5cc sample of filtrate.

Specific gravity of filtrate = 1.08

1 cc of N/10 $AgNO_3$ will neutralize .0058 grams of Nacl, therefore,

92.7 X .0058 = 0.53766 gr. Nacl

0.53766 : 5 = 0.10753 gms. Nacl in 1cc

0.10753 : 1.08 X 100 = 9.95% Nacl.

Slurry II

a. Per cent Solids.

Wt. of Sample taken = 8.29

Wt. of sample dry = .6234

Per cent Solids =

$.6234 : 8.29 \times 100 = 7.5$ °/o

b. Per Cent Sodium Chloride

$AgNO_3$ Solution N/10

86.3 cc $AgNO_3$ Neutralized a 5 cc sample of filtrate.

Specific gravity of filtrate = 1.08

1 cc of N/10 $AgNO_3$ will neutralize .0058 gms. of Nacl, therefore,

$86.3 \times .0058 = .49054$ gms. Nacl

$.4905 : 5 = 0.0981$ gms. Nacl in 1 cc.

$.0981 : 1.08 \times 100 = 9.08$ °/o

Table III

	Per Cents Calculated	Per Cents Actual
Slurry I		
Solids	7.87	7.69
Sodium Chloride	9.98	9.95
Slurry II		
Solids	7.87	7.5
Sodium Chloride	9.10	9.08

C. Experiments.

Test 1---Dewatering by continuous vacuum filtration.

Conditions---Using slurry I with 9.95 °/o Nacl and 7.69 °/o solids. Filtration carried on in a 6 inch bushner funnel with a two liter suction flask.

Table IV

Volume of Feed	1200	1000	750	500
Forming Time- Min.	23.75	17.75	11	5.25
Forming vacuum- in Hg.	25	25	25	25
Drying time- Min.	4	3.75	4	2.75
Drying vacuum- in Hg.	20	20	20	20
Cake Thickness- inches.	3/4	5/8	7/16	5/16
Wet Cake- Gms/	339	272	163.5	128
Per Cent Water	60.6	60.3	57.5	59.2
Dry Cake Gms.	133.5	108	80.5	52
Total Cycle- Minutes	27.75	21.5	15	7.75
Capacity- /Sg.ft./24 hs.	175.2	182.8	193.7	262.8

Test 2. Washing on continuous vacuum filters.

Problem: To determine the displacement of Nacl by wash water, using slurry I.

Procedure: The filter used was a 6" buchner funnel (.088 sq. ft.) with a two liter suction flask. The suction flask was connected to the vacuum line and 1000 cc of the slurry poured into the bushner funnel over filter cloth as the filter medium.

The vacuum was built up to 25 inches of mercury and continued until the cake was formed. That is, until all the water just disappeared above the cake. The filtrate was then removed and measured. Wash water was then poured over the cake and vacuum built up to 24 inches of mercury. The washing was continued until all the water had passed throught the cake.

Table V.

Test No.	1	2	3
Volume of Feed cc	1000	1000	1000
Forming time, Min.	16.75	17.5	17.75
Forming vacuum, inches Hg.	25	25	25
cc Wash Water	100	200	250
cc Filtrate	117	208	297
Ratio of wash water to dry solids	1:1	2.2:1	2.7:1
Washing time, Min.	6	11.75	12.75
Washing vacuum, inches Hg.	24	24	24
Cake thickness, inches	5/8	5/8	5/8
Wet cake in grams	276	266	269
°/o Water in cake	63.5	65.9	66.3
Dry cake in grams	100.5	90.5	90.5
Total cycle, Min.	27.75	34.5	35.5
Capacity, /sq. ft./ 24 hrs.	132	93	96
Gms. Nacl in cake after wash	10.3	3.04	0.01
Calc. gms. Nacl in cake before Washing	21.0	23.61	24.0
°/o Displacement	51.0	87.1	99.96
Drying time, Minutes	5	5.25	5

Test 3. Washing intermittently.

Procedure: This test was carried out with slurry I. A liter of the slurry was poured into the buchner funnel and dewatered as in test 1. After the filtrate was measured, 100 cc of wash water was poured over the cake and filtered through. As near as possible, the last 10 cc of filtrate was collected and tested for Nacl. This was then repeated until 600 cc of wash water had been added.

Results. Table VI

Time	cc Wash Water	Titration°
0	00	185.4
5	101	183.0
10	203	112.9
15	301	2.7
20	404	0.8
25	504	0.45
30	606	0.00

° Note: Titration is expressed in cc N/10 $AgNO_3$ Per 10 cc of Wash Water.

Discussion.

By referring to table III it is seen that the calculated per cents of solids and Nacl are practically the same as the actual. Any variation would be due to some of the materials used not being pure.

The use of a small laboratory apparatus for determining rate of filtration would not be accurate but an idea of a certain slurry's behavior could be observed. By referring to table IV a good comparison of the different rates of filtration using different cake thicknesses is found. According to these results the capacity increases as the cake thickness decreases. This does not necessarily mean that with a thinner cake more material could be filtered per twenty four hours, because of the time required to clean and recharge the filter.

III. Derivation of equations of Walker, Lewis, and McAdams and Badger and McCabe.

A. Discussion.

The equations which are to follow have been worked out for the most part by Walker, Lewis, and McAdams and Badger and McCabe. The books "Principles of Chemical Engineering" by Walker, Lewis, and McAdams and "Elements of Chemical Engineering" by Badger and McCabe, have been used as reference and the equation numbers will be the same as those in the respective books.

Although these equations have been derived by these respective authors, they have not given complete derivations of the fundamental filtration equations. In the following pages, I have attempted to bring out all the points in the derivation of these equations; especially those points which were not taken up by the respective authors.

B. Derivation of Filtration Equations.

The filter cake may be considered as equivalent to a series of capulary tubes. The passing of a liquid through a capillary tube is given by Poiseuille's equation:

$$P = \frac{32 \mu L u}{g D^2}$$

where, P = pressure drop
μ = absolute viscosity of liquid

L = length of tube.

u = linear velocity of tube.

D = diameter of capillary

For a filter cake of thickness L and area A with k capillaries,

$$P = \frac{32 \mu L u}{g D^2}$$

or

$$u = \frac{P g D^2}{32 \mu L}$$

However, , the velocity is equivalent to volume of filtrate per unit of time.

Then $$dv/d\theta = k \frac{gk D^2 D^2 PA}{4 X 32 \mu L}$$

or, $$= k \frac{k D^4 P g A}{128 \mu L}$$

"Although the filtration equations cannot by derived directly from this equation, it helps to explain certain things in developing a workable filtration equation[1]."

---

[1] Walker, Lewis, and McAdams--Principles of Chemical Engineering---Page 361.

The equation suggests that providing you have an incompressable sludge, the rate of flow is diredtly proportional to the pressure. It also suggests that for compressible cakes a direct proportion between pressure and rate of flow does not exist. The equation also suggests that the rate of flow of filtrate decreases with the increased thickness of cake.

<u>Differential Equations for Incompressible Homogeneous Sludges.</u>

"Since the pressure drop through a cake is proportional to the velocity of flow, we may consider filtration rate as equal to driving force. That is, pressure divided by resistance[1]."

[1] Principles of Chemical Engineering by Walker, Lewis, and McAdams---Page 361.

Nomenclature for following equations.

A = Total area of filtering surface in sq. ft.

V = Total volume of filtrate in cu. ft.

v = Volume of cake as it collects on cloth, in cu. ft. per unit volume of filtrate.

P = Pressure drop through cake.

P_i = Pressure drop through cake, filter, medium, press channels.

θ = Total time of operation.

R = Total resistance of cake.

r = Specific resistance of cake.

r' = Coefficient in equation: $r = r'P^s$

r'' = Coefficient in equation: $r = r''P^s(dv/d\theta)^t$

P/A = Resistance of press channels.

L = Thickness of cake at variable time.

S = Coefficient of compressibility.

t = Coefficient of velocity effect. (Scouring effect.) Zero for homogeneous sludges.

From above,

$$\frac{dv}{d\theta} = \frac{P_t}{R+P/A} \qquad \text{(1 - p.362 - W., L., and M.)}$$

In most cases the resistence of filter cloth and pure channels can be disregarded, then,

$$\frac{dv}{d\theta} = \frac{P}{B} \qquad \text{(1a - p.362 - W., L., and M.)}$$

Since resistance is proportional to length and inversely proportional to the area of a conductor, $R = \frac{r\,L}{A}$, and since, the volume of a cake is LA, while sludge brought in by liquor is vV, we have,

$$L = \frac{v\,V}{A}$$

Substituting,

$$R = \frac{rv\,V}{A}$$

then equation 1a becomes,

$$\frac{dv}{d\theta} = \frac{P\,A^2}{rvV} \qquad \text{(2a - p.262 - W., L., and M.)}$$

<u>Differential</u> <u>Equations</u> <u>for</u> <u>Compressible</u>, <u>Homogeneous</u> <u>Sludges</u>.

$$r = r'P^s$$

Substituting in 2a,

$$\frac{dv}{d\theta} = \frac{P^{1-s}\,A^2}{r'v\,V} \qquad \text{(3 - p.264 - W., L., and M.)}$$

<u>Differential</u> <u>Equations</u> <u>for</u> <u>Compressible</u>, <u>Non-Homogeneous</u> <u>Sludges</u>.

$$R = rL/A \qquad \text{(p.362 - W., L., and M.)}$$

and $\quad L = vV/A \quad$ or $\quad dL = v\,dV/A$

then

$$R = \frac{r\,v\,dV}{A^2}$$

but $\quad r = r''P^s(dV/Ad\theta)^t \qquad$ (p.365 - W., L., and M.)

Substituting we have,

$$R = \frac{r''vP^s}{A^{2+t}} \left(\frac{dV}{d\theta}\right)^t dV$$

but, $\quad dV/d\theta = P/R$

Substituting,

$$\frac{dV}{d\theta} = \frac{P}{\frac{r''}{A^2} + \frac{P^s}{t} \int \left(\frac{dV}{d\theta}\right)^t \, dV}$$

$$\frac{dV}{d\theta} = \frac{P^{1-s} \; A^{2+t}}{r'' \int \left(\frac{dV}{d\theta}\right)^t \; dV} \qquad \text{(4-p.368-W.L.\&M.)}$$

Integrated Equations.

Homogeneous Sludges.

Constant Pressure

$$\frac{V^2}{A^2} = \frac{2 \, P^{1-s} \, \theta}{r \; v} \qquad \text{(3a- W.L.\&M.)}$$

Constant Rate

$$\frac{dV}{d\theta} = \frac{V}{\theta} = \text{Constant} = \frac{P^{1-s} \; A^2}{r' \, v \, V} \qquad \text{(3b)}$$

Constant pressure Gradient through cake:

$$\frac{P}{V}^{\,1-s} = \text{Const.} = \frac{r' \quad V^{1+s}}{A^2 \; \theta \; (1+s)} \qquad \text{(3c)}$$

Sludges with Filter aids or equivalents.

Constant Pressure.

$$\int \frac{dV}{d\theta}^{\,t} \, dV = \frac{P^{1-s} \; A^{2+t}}{r'' \quad (dV/d\theta)}$$

differentiate relative to V.

$$(dV/d\theta)^t = \frac{P^{1-s} \; A^{2+t} \; d(dV/d\theta)/dV}{r'' v \; (dV/d\theta)^2}$$

Solve for dV

$$dV = \frac{P^{1-s} \; A^{2+t} \; d(dV/d\theta)}{r'' v \; (dV/d\theta)^{2+t}}$$

Integrate,

$$V = \frac{P^{1-s} \; A^{2+t}}{r''^{V} \; (1+t)} \; \frac{1}{dV/d\theta}^{\,1+t}$$

Solve for $dV/d\theta$,

$$\frac{dV}{d\theta} = {}^{1+t}\ \frac{P^{1-s}\ A^{2+t}}{r''\ v\,(1+t)}\quad \frac{1}{V^{1/1+t}}$$

Integrate,

$$\frac{V}{A}^{2+t} = \frac{P^{1-s}}{r''(1+t)\,v}\ \frac{2+t}{1+t}\,\theta^{\,1+t} \qquad (4a)$$

<u>Derivation of equation 278 of Badger and McCabe.</u>

$$\frac{V}{A}^{2+t} = \frac{P^{1-s}}{r''(1+t)v}\quad \frac{2+t}{1+t}\ \theta^{\,1+t}$$

Taking logarithms,

$$2+t \log V/A = 1-s \log P - \log r''(1+t) + 1 + t \log (2+t/1+t) + 1 + t \log \theta$$

Deviding both sides od equation by 1 + t,

$$\frac{2+t}{1+t} \log \frac{V}{A} = \frac{1-s}{1+t} \log P - \log \frac{r''(1+t)}{1+t} + \log \frac{2+t}{(1+t)^2} + \log \theta$$

taking the anti-log,

$$\frac{V}{A}^{\frac{2+t}{1+t}} = P^{\frac{1-s}{1+t}} \quad \frac{2+t}{(1+t)^2}\,\left|r''(1+t)\,v\right|^{1+t}\,\theta$$

If log θ be plotted against log V, we will obtain a straight line with a slope of (2+t/1+t). (2+t/1+t) is a constant and for simplicity we may substitute m. In a similar manner if we plot log θ against log P we will have a straight line whose slope is (1-s/1+t). For (1-s/1+t) we may substitute n.

The value of $\dfrac{2+t}{(1+t)^2}\left|r''(1+t)\,v\right|^{1+t}$ is also a constant and k may be substituted in its place.

Then with these new constants, equation 4a becomes,

$$\frac{V}{A}^{m} = P^{n}\,k\,\theta \qquad (278 - \text{B. \& M.})$$

C. Conelusions:

The derivations of the above equations are very difficult and beyond the comprehension of the ordinary student. The application of such equations would then be difficult without some understanting of their origin.

The constant k cannot have any physical significance because it depends upon so many factors. The constant k which equals $\frac{2 + 1}{(1+t)^2} \left| r''(1+t)\, v \right|^{1+t}$. k is therefore dependent upon t, the coefficient of velocity effect, v the volume of cake as it collects upon the filter in cu. ft. per unit volume of filtrate and r'' which is a coefficient in equation $r = r''\, P^s\, (dV/Ad\theta)$.

IV. Application of equations of "Elements of Chemical Engineering" by Badger and McCabe, to the Filter press, Suction Filter and Leaf Filter and comparing the value of the constants.

A. Slurry I.

Procedure: The apparatus used in this work is shown in fig's. 1, 2, 3, and 4. The filter press consists of plates and frames and a filter medium. Two presses were used in my work. The larger press having fromes 8.5" X 8.5", or a filter area of 134 square inches. The other press is smaller having a filter area of 32 sq. in. (two surfaces). The auxillary equipment to the press, consists of a blow tank equipped with an air line, water line, steam line and a vacuum line for filling.

The filter press, (fig. 1), was first fitted with a filter cloth (one frame being used) and then the slurry to be filtered was drawn into the blow tank through the flexible rubber hose, by means of vacuum. It will be noticed that the air line extends nearly to the bottom of the blow tank, to keep the slurry aggitated during the filtrations.

A set of Toledo scales, which weigh accurately to 1/4 of an ounce, were set in such a position that the filtrate was allowed to run from the press into a container on the scales. This made it possible to record the exact quantity of filtrate at any given time.

The air in the blow tank was then turned on and the desired pressure regulated by means of the felief cock, When the desired pressure was obtained the valve into the press was opened, the pressure being kept constant by regulating the relief cock.

The filtration cycle was divided into periods and at the end of each period, the exact weight of filtrate was recorded.

1. Filter Press.

Data: Table VIII

Period	Time(Sec.)	Filtrate(lbs.)	Total Filtrate	Pressure
1	240	4 11/16	4 11/16	25
2	480	2 13/16	6 14/16	25
3	720	1 7/16	8 5/16	25
4	1280	1 15/16	10 4/16	25
1	240	5 7/16	5 7/16	35
2	480	2 7/16	7 14/16	35
3	720	1 12/16	9 10/16	35
4	1280	2 2/16	11 12/16	35

Calculations:

a. Determination of filter constants:

$$\left(\frac{V}{A}\right)^m = kP^n\theta$$

Where,

V = lbs. of Filtrate

A = in sq. inches.

θ = in seconds.

The value of m may be obtained by plotting log θ vs. log V for any given run and observing the slope of the line. (Fig. 5)

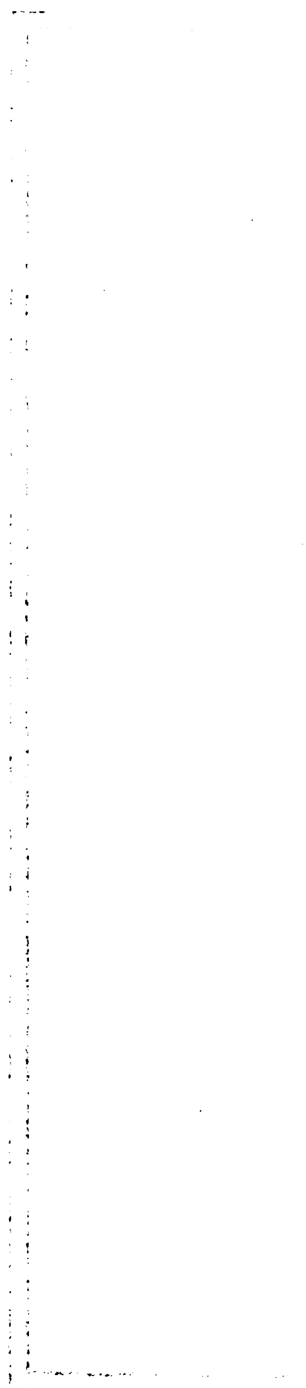


Figure 5

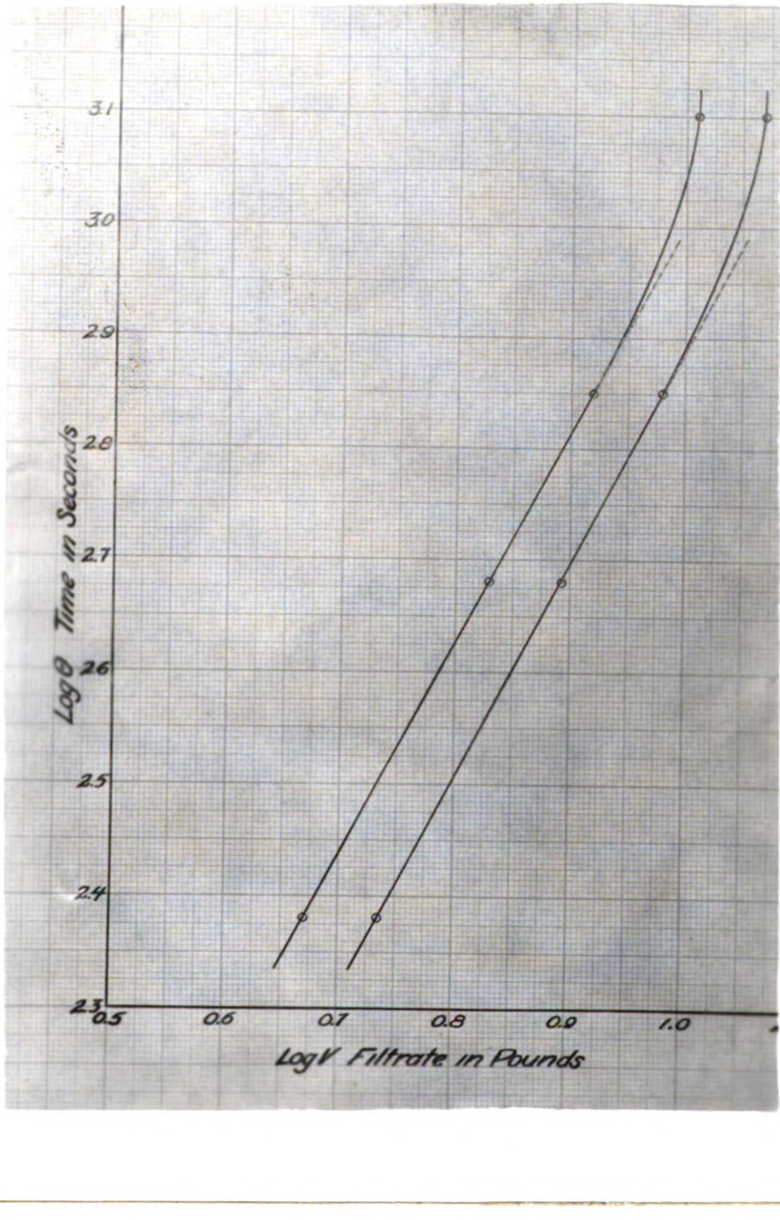

3.1
3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
Log θ Time in Seconds
0.5
0.6
0.7
0.8
0.9
1.0
Log V Filtrate in Pounds

Table IX Data for fig. 5

Point	θ	V	Log θ	Log V
1	240	4.688	2.3802	0.6710
2	480	6.876	2.6812	0.8374
3	720	8.312	2.8573	0.9197
4	1280	10.250	3.1072	1.0107
5	240	5.438	2.3802	0.7355
6	480	7.876	2.6812	0.8963
7	720	9.625	2.8573	0.9834
8	1280	11.750	3.1072	1.0700

The slope of line = 26/14 = 1.857 = m

Now we have,

$$\left(\frac{V}{134}\right)^{1.857} = k\ P^n = \theta$$

Applying this equation to the two runs for a convenient value of V and choosing as this value of V whose log = 0.9. V = 7.944

$$\left(\frac{7.944}{134}\right)^{1.857} = k\ P^n\ \theta = .00485$$

Where θ is the time at which V = 7.944

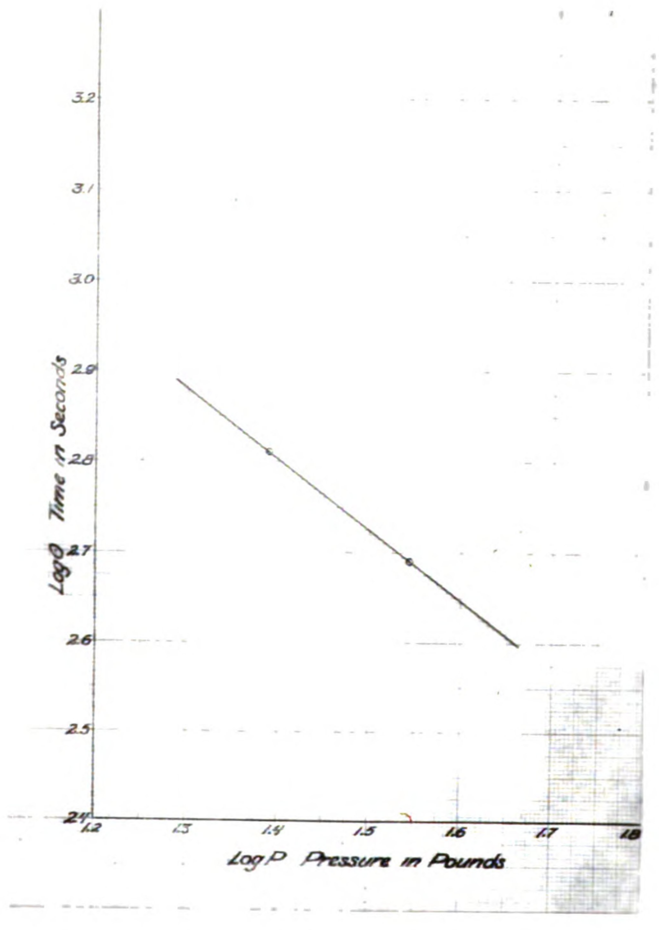
The value of θ at which log V = 0.9 are:

Table X Data for figure 6

Run	Press	Log P	Log θ at log V = 0.9	θ in sec.
1	25	1.3979	2.81	646
2	35	1.5441	2.69	490

Plotting log θ vs. log P of table X we have,

Figure 6



3.2
3.1
3.0
2.9
2.8
2.7
2.6
2.5
2.4
Log θ Time in Seconds
1.2
1.3
1.4
1.5
1.6
1.7
1.8
Log P Pressure in Pounds

n = slope of line = 15/19.5 = .77

Choosing convenient values of log P and log θ, fig. 8: log P = 1.3979 , log θ = 2.81. Substituting these values,

$$\text{Log } \frac{.00485}{k} = 1.3979 \times 77 + 2.81$$

$$k = \frac{.00485}{\text{Log } 3.885}$$

$$k = \frac{.00485}{7670}$$

$$k = .000000637$$

Rewriting equations,

$$\left(\frac{V}{A}\right)^{1.857} = .000000637\ P^{.77}\ \theta$$

Changing to engineering units

Units Used		Eng. Units	Factor
V	Filtrate	Cu. ft.	64.5
A	Sq. in.	Sq. ft.	144
θ	Seconds	hours	3600
P	/sq./in.	/sq.in.	1

$$\left(\frac{V}{A}\right)^{1.857} \cdot \left(\frac{64.5}{144}\right)^{1.857} = .000000637\ P^{.77}\ 3600\ \theta$$

$$\left(\frac{V}{A}\right)^{1.857} = \frac{.002295}{.225}\ P^{.77}\ \theta$$

$$\left(\frac{V}{A}\right)^{1.857} = .010185\ P^{.77}\ \theta$$

2. Calculation of filtering times:-

For 1" frame

$$\left(\frac{V}{A}\right)^{1.857} = .010185\ P^{.77}\ \theta$$

$$\left(\frac{.142}{.924}\right)^{1.857} = .010185\ (25)^{.77}\ \theta$$

$$.03755 = .010185 \times 11.95\,\theta$$

$$.1218\,\theta = .0314$$

$$\theta = .258 \text{ hours}$$

$$\theta = 15.5 \text{ minutes.}$$

For 2" frames.

Calculations same as for 1" frame.

$$\frac{.142 \times 2.125}{1.125} = .245$$

$$\left(\frac{.245}{.93}\right)^{1.857} = .1218\,\theta$$

$$\theta = .903 \text{ hours}$$

$$\theta = 55 \text{ minutes.}$$

For 3" frames.

$$\theta \doteq 1.69 \text{ hours}$$

$$\theta = 102 \text{ minutes}$$

For 4" frames.

$$\theta = 2.85 \text{ hours}$$

$$\theta = 172 \text{ minutes.}$$

2. Suction Filter

Procedure:

The apparatus used in this case is shown in fig. 3, which is an ordinary suction filter having a filter area of 154 sq. in.

The method of taking samples, using the suction filter is not as accurate as that used in the case of the filter press.

The outside diameter of the drain cock of the filter is about 1 1/4" and was fitted with a rubber stopper. A four liter bottle was attached to the stopper and supported to prevent its falling off the stopper under the weight of the filtrate. A second hole was put in the stopper in which was inserted a piece of ordinary glass tubing. This was then connected to the vacuum line on the opposite side of the filter by means of rubber tubing. This arrangement equilized the pressure in the bottle and filter after the bottle had been removed and put in position again. A pinch clamp was used on the rubber tubing when the bottle was removed, to prevent losing the vacuum.

The vacuum was built up in the vacuum line and the slurry poured over the filter cloth. The vacuum was then turned on and the drain cock on the filter opened. With the pressure equalized the filtrate runs directly into the bottle.

At the end of a certain period of time the drain valve was closed, the receiving bottle removed and filtrate weighed and the receiving bottle placed in position again. This process was repeated until the filtration cycle was completed.

Although this method is not as accurate as the sampling method of the filter press, satisfactory results may be obtained after some practice.

Data:

Table XI

Run I Period	Time(sec.)	Filtrate(lbs)	Per sq. ft. area Total Filtrate(lbs.)	ure Press-
1	120	4 1/16	4.06	12.5
2	240	1 15/16	6.02	12.5
3	360	1 9/13	7.58	12.5
4	720	3 15/16	11.50	12.5

Table XI (Continued)

Period	Time(sec)	Filtrate(lbs)	Per sq. ft. Area Total Filtrate(lbs)	Pressure
1	147	3 13/16	380	7.32
2	300	2 1/16	5.89	7.32
3	450	1 8/16	7.41	7.32
4	1080	4 10/16	12.00	7.32
1	200	3 12/16	3.75	4.9
2	400	1 12/16	5.50	4.9
3	600	1 7/16	6.91	4.9
4	1200	3 12/16	10.70	4.9

Weight of slurry charged	20 #
°/o Water in cake	63 °/o
Filtef Area (sq. in.)	154
Cake thickness (in.)	3/4
Volume of cake (cu. in.)	106
Temperature	19° C

Calculations:

All calculations same as in (1) above.

1. Det. of filter constants.

Table XII m = slope = 1.75

V	θ	Log V	Log θ
4.06	120	0.6085	2.0795
6.02	240	0.7800	2.3805
7.58	360	0.8800	2.5560
11.50	720	1.0610	2.8750

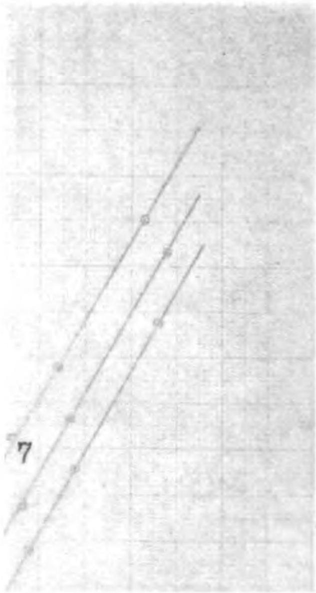
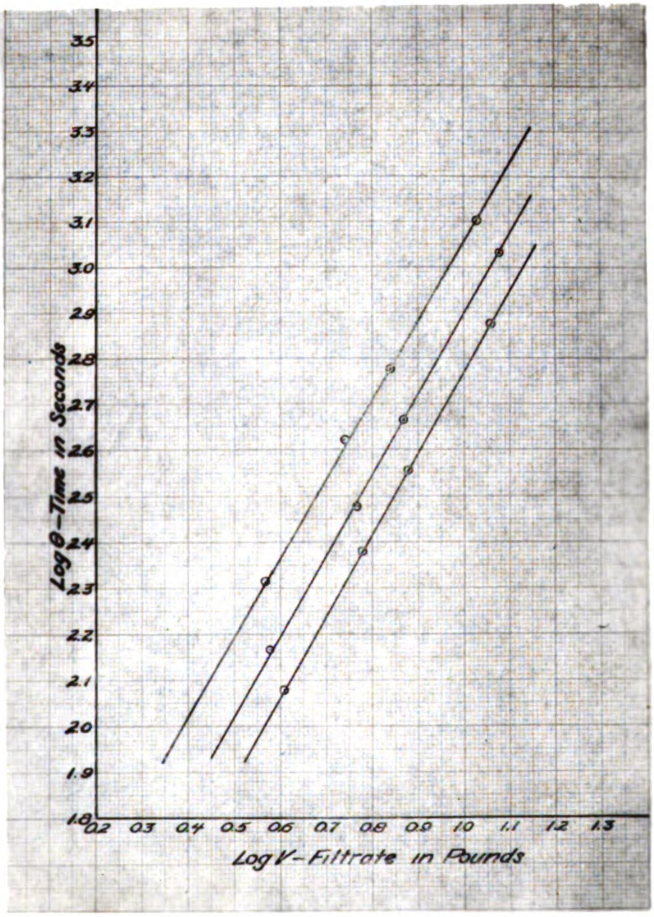


Figure 7



Log θ-Time in Seconds
Log V-Filtrate in Pounds

Table XII (Continued)

V	θ	Log V	Log θ
3.8	147	0.5800	2.1675
5.89	300	0.7705	2.4775
7.41	450	0.8700	2.6535
12.00	1080	0.7950	3.0335
3.75	200	0.5705	2.3150
5.50	400	0.7395	2.6205
6.91	600	0.8395	2.7785
10.70	1200	1.0295	3.0795

M = slope = 1.75

Data. Det. of n.

Table XIII n = slope = 0.741

Run	Pressure	Log P	Log θ at log V = 6.31	θ in sec.
1	12.5	1.097	2.41	257
2	7.32	0.864	2.545	350
3	4.90	0.690	2.700	501

m = 1.75

n = 0.741

k = .072 (Pressure = 12.5 # / sq. in.)

2. Calculation of filter times.

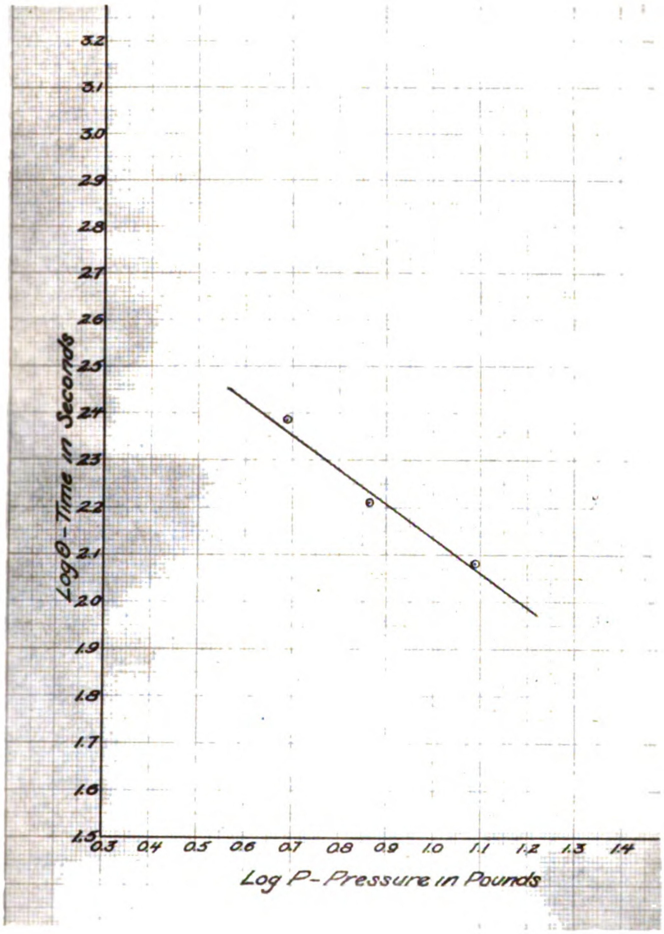
For 1" cake V = 18.00 #

A = 154 sq. in.

P = 12.5 / sq. in.

$$\left(\frac{V}{A}\right)^m = k\, P^n\, \theta$$

Figure 8



3.2
3.1
3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
Log θ - Time in Seconds
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
Log P - Pressure in Pounds

$$\left(\frac{.288}{1.07}\right)^{1.75} = .072 \text{ X } (12.5)^{.741} \quad \theta$$

$$(.269)^{1.75} = .072 \text{ X } 6.5 \quad \theta$$

$$\theta = .214 \text{ hours}$$

$$\theta = 13 \text{ minutes.}$$

For 2" cakes.

$$V = \frac{.288 \text{ X } 2.125}{1.125} = .541$$

$$\left(\frac{.541}{1.07}\right)^{1.75} = .4675 \quad \theta$$

$$\theta = .646 \text{ hours}$$

$$\theta = 39 \text{ minutes.}$$

For 3" cakes.

$$V = \frac{.288 \text{ X } 3.125}{1.125} = .80$$

$$\left(\frac{.80}{1.07}\right)^{1.75} = .072 \text{ X } 6.5 \text{ X } \theta$$

$$\theta = 1.29 \text{ hours}$$

$$\theta = 77 \text{ minutes.}$$

For 4" cakes.

$$V = \frac{.288 \text{ X } 4.125}{1.125} = 1.055$$

$$\left(\frac{1.055}{1.07}\right)^{1.75} = .4675 \quad \theta$$

$$\theta = 2.1 \text{ hours}$$

$$\theta = 127 \text{ minutes.}$$

3. Leaf Filter

Procedure:

The apparatus used in this experiment is shown in fig. 4.

The apparatus consists of a filtrate tank which is connected to the vacuum line and to the air line, and a frame which supports a filter cloth. The frame is a ½" pipe, welded in the shape of a square, 12" on a side, with holes on the inside of the pipe, through which the filtrate passes. Two pieces of 1/4" mesh wire screen are place between the filter clothes as a support to them. This prevents the cloths being pulled together and shutting of the vacuum.

The slurry to be filtered was placed in a 15 gallon crock on a large set of Toledo scales. A perforated pipe, 1/8" in diameter was laid in the bottom of the crock and connected to the compressed air line. The air was turned on and the precipitate was kept in suspension during the filtration cycle. The vacuum was then turned on in the filtrate tank and the leaf placed in the crock. The scales were then balanced and the vacuum in the leaf turned on, the time being noted at this instance.

At the end of a certain period of time the loss in weight of slurry was noted and tabulated. This was repeated until four losses in weight had been recorded.

Data: Table XIV

Period	Time(sec)	Filtrate(lbs)	Total Tiltrate(lbs)	Pressure
1	240	10	10	11
2	480	4.5	14.75	11
3	720	3.5	18.25	11
4	1080	4.75	23	11

Table XlV (Continued)

Period	Time(sec)	Filtrate(lbs)	Total Filtrate(lbs)	Pressure
1	240	9.25	9.25	64
2	480	4.5	13.75	64
3	720	3.5	17.25	64
4	1200	5.5	22.75	64

Temperature	20° C.
Cake thickness	2.25"
°/o Water in cake	63°/o
Filter area sq. in.	288

Calculations:

All calculations same as for filter press.

1. Det. of filter constants.

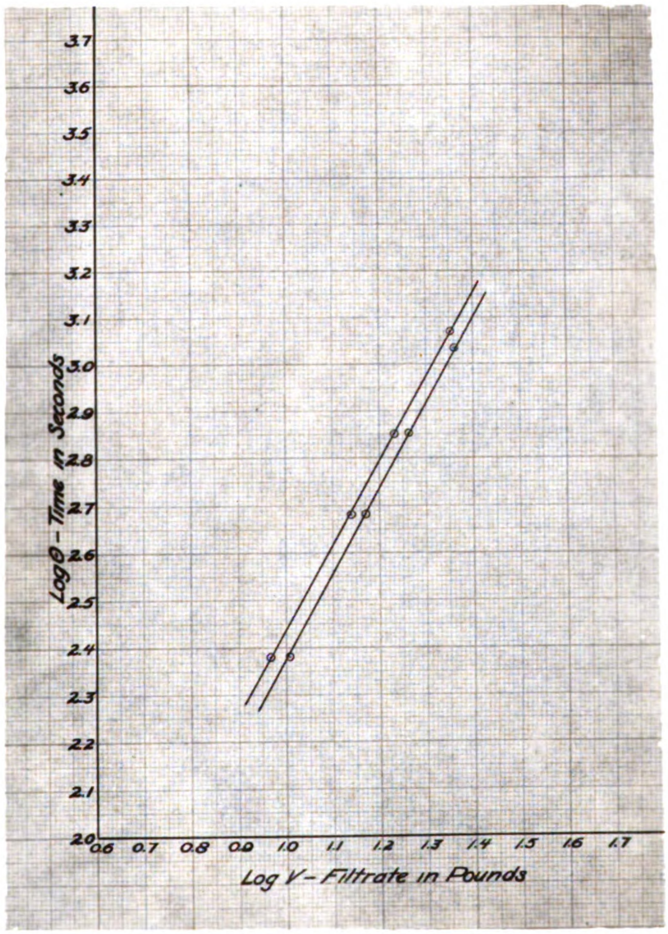
Data for Det. of m

Table XV

Point	θ	V	log θ	log V
1	240	10.0	2.3802	1.0050
2	480	14.75	2.6812	1.1688
3	720	18.25	2.8572	1.2613
4	1080	23.00	3.0334	1.3617
5	240	9.25	2.3802	0.9661
6	480	13.75	2.6812	1.1380
7.	720	17.25	2.8572	1.2368
8	1200	22.75	3.0792	1.3568

m = slope = 1.84 (fig. 9)

Figure 9



3.7
3.6
3.5
3.4
3.3
3.2
3.1
3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
Log θ - Time in Seconds
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
Log V - Filtrate in Pounds

Data for Det. of n

Table XVI

Run	Pressure	Log P	Log θ at log V = 1.3617	θ in sec.
1	11	1.0414	3.0334	1080
2	6.4	0.8062	3.085	1216

n = slope = .241 (fig. 10)

m = 1.84

n = .241

k = .0776

2. Calculations of the filtering times same as for filter press.

For 1" cake

$$\left(\frac{V}{A}\right)^{m} = k\,P^{n}\,\theta$$

P = 11 pounds /sq. in.

V = 0.368 cu. ft.

A = 2 sq. ft.

$$\left(\frac{.368}{2}\right)^{1.84} = .0776\ (11)^{.241}\ \theta$$

$$(.184)^{1.84} = .272\ \theta$$

$$.0442 = .272\ \theta$$

$$\theta = .162 \text{ hours}$$

$$\theta = 10 \text{ minutes}$$

For 2" cake.

$$V = \frac{.368 \times 2.125}{1.125} = 0.695$$

$$\left(\frac{.695}{2}\right)^{1.84} = .272\ \theta$$

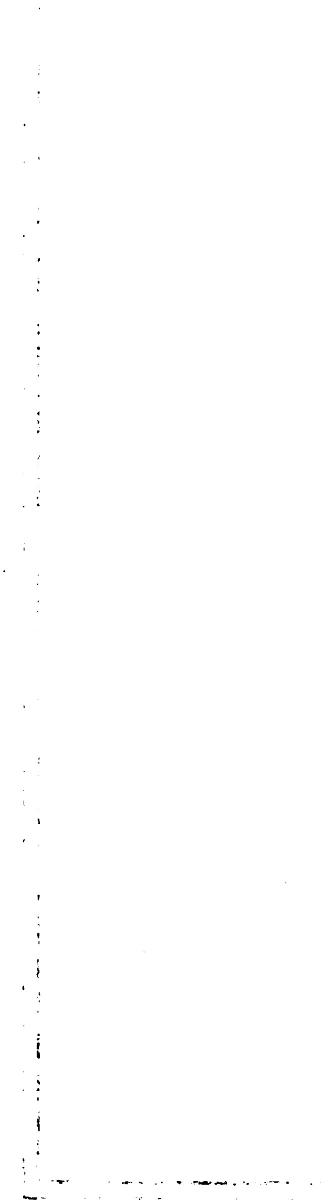
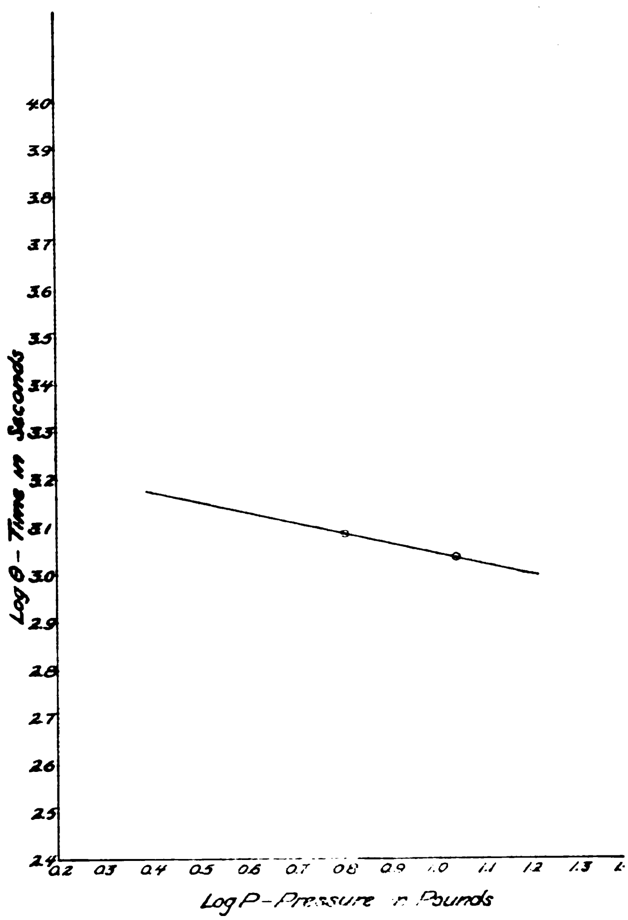


Figure 10



Log θ - Time in Seconds
4.0
3.9
3.8
3.7
3.6
3.5
3.4
3.3
3.2
3.1
3.0
2.9
2.8
2.7
2.6
2.5
2.4
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.1
1.2
1.3
Log P - Pressure in Pounds

$$.143 = .272\ \theta$$

$$\theta = .526 \text{ hours}$$

$$\theta = 32 \text{ minutes.}$$

For 3" cakes.

$$V = \frac{.368 \times 3.125}{1.125} = 1.02 \text{ cu. ft.}$$

$$\left(\frac{1.02}{2}\right)^{1.84} = 0.212\ \theta$$

$$\theta = 1.07 \text{ hours}$$

$$\theta = 64 \text{ minutes}$$

For 4" cakes.

$$V = \frac{.368 \times 4.125}{1.125} = 1.35 \text{ cu. ft.}$$

$$\left(\frac{1.35}{2}\right)^{1.84} = 0.272\ \theta$$

$$.485 = 0.272\ \theta$$

$$\theta = 1.78 \text{ hours}$$

$$\theta = 101 \text{ minutes.}$$

Table XVII Comparison of filter constants.

Filter	m	n	k
Filter press	1.85	.77	.01018
Suction Filter	1.75	.741	.072
Leaf Filter	1.84	.241	.0776

B. Slurry II

Procedure: The prodedure using slurry II was exactly the same as that of slurry I. The apparatus was also the same, except that with slutty II the leaf filter was not used.

1. Filter Press.

Data: Table XVIII

Period	Time(sec)	Filtrate(lbs)	Total Filtrate(lbs)	Pressure
1	60	11 13/16	11 13/16	35
2	120	5 13/16	17 10/16	35
3	240	4 6/16	22	35
1	70	9 1/16	9 1/16	25
2	140	4 13/16	13 14/16	25
3	300	6 14/16	20 12/16	25

	Run I	Run II
Average Pressure	35	25
Wet Cake (gms.)	1840	1922
Cake Thickness	1.375	1.375
Filter Area (sq. in.)	134	134
Temperature of slurry	19° C	19° C
°/o Water	41.0	46.5

Calculations:

1. Det. of filter constants.

$$\left(\frac{V}{A}\right)^2 = k' \theta$$

P = constant 25 / sq. in.

V = 20.75 = .332 cu. ft.

$$\left(\frac{.332}{.924}\right)^2 = k' \ .0834$$

$k = 1.56$

2. Det. of filtering times.

For 1" frame

$$\theta = 5 \text{ minutes.}$$

For 2" frame

$$V = \frac{.332 \times 2.125}{1.125} = .628$$

$$\left(\frac{.628}{.924}\right)^2 = 1.56 \times \theta$$

$$\theta = 16 \text{ minutes.}$$

For 3" frame

$$V = \frac{.332 \times 3.125}{1.125} = .924$$

$$\left(\frac{.814}{.924}\right)^2 = 1.56\ \theta$$

$$\theta = 38 \text{ minutes.}$$

For 4" frame

$$V = \frac{.332 \times 4.125}{1.125} = 1.22$$

$$\left(\frac{1.22}{.924}\right)^2 = 1.56\ \theta$$

$$1.74 = 1.56\ \theta$$

$$\theta = 67 \text{ minutes}$$

2. Suction Filter

Data: Table XIX

Period	Time(sec)	Filtrate(lbs)	Total Filtrate(lbs)	Pressure
1	60	7 ½	7 ½	9.82
2	120	6 9/16	14 ½	9.82
3	180	6 5/16	20 6/16	9.82
4	240	6	26 6/16	9.82
5	375	8 7/16	34 13/16	9.82

Table XIX (Continued)

Period	Time(sec)	Filtrate(lbs)	Total Filtrate(lbs)	Pressure
1	75	8 8/16	8 8/16	7.85
2	150	8 6/16	16 14/16	7.85
3	225	7 2/16	24	7.85
4	300	4 6/16	28 6/16	7.85
5	435	6 6/16	34 12/16	7/85

	Run I	Run II
Volume of feed (cuff)	0.72	0.72
°/o Water	43	45
Filter Area	154	154
Cake Thickness	15/16	15/16

Calculations:

Calculations same as in (1) for filter press.

1. Det. of constant.

$k = 2.63$ $V = 34.75$ at 13/16" cake

2. Det. of filtering times.

For 1" cake

$\theta = 13$ minutes.

For 2" cake

$\theta = 38$ minutes.

For 3" cake

$\theta = 76$ minutes.

For 4" cake

$\theta = 128$ minutes.

V Testing the Validity of Filtration equations of Badger and McCabe.

Procedure: The procedure in this case was the same as that for the filter press in part IV sec. 1, except that four runs were made instead of two.

After the data had been taken and the value of m was determined, the value of n was determined for three values. This was done by drawing a line through any three points as shown in figure 12.

Data: Table XX

Period	Time(sec)	Filtrate(lbs)	Total (lbs) Filtrate	Total Fil. Per sq. ft.	Pressure.
1	250	4 15/16	4 15/16	5.31	15
2	500	2 5/16	7 4/16	7.80	15
3	750	1 14/16	9 2/16	9.82	15
4	1080	1 12/16	10 14/16	11.70	15
1	240	5 9/16	5 9/16	6.00	20
2	480	2 8/16	8 1/16	8.67	20
3	720	1 12/16	9 13/16	10.55	20
4	1080	1 5/16	11 12/16	12.40	20
1	230	5 12/16	5 12/16	6.18	25
2	460	2 11/16	8 7/16	9.16	25
3	690	1 11/16	10 2/16	10.90	25
4	960	1 15/16	12 1/16	12.95	25

Table XX (Continued)

Period	Time(sec)	Filtrate(lbs)	Total(lbs) Filtrate	Total Filt. Per sq. ft.	Pressure
1	50	2 8/16	2 8/16	2.69	32
2	155	2 8/16	5	5.37	32
3	325	2 8/16	7 8/16	8.06	32
4	555	2 8/16	10	10.75	32
5	900	2 4/16	12 4/16	13.28	32

Filter Area (sq. in.)	134
Temperature	18° C
Calculations:	

$$\left(\frac{V}{A}\right)^m = k\,P^n\,\theta$$

Where, V = Filtrate in pounds.

A = Area in sq. in.

P = Press. in /sq. in.

θ = Time in seconds.

m.n. and k = constants.

The value of m is obtained by plotting log θ vs. log V and observing the slope of line. (fig. 11)

Table XXI

Point	θ	V	Log θ	Log V
1	250	5.31	2.3979	0.7251
2	500	7.80	2.6990	0.8921
3	750	9.82	2.8751	0.9921
4	1080	11.70	3.0334	1.0682

Figurė 11

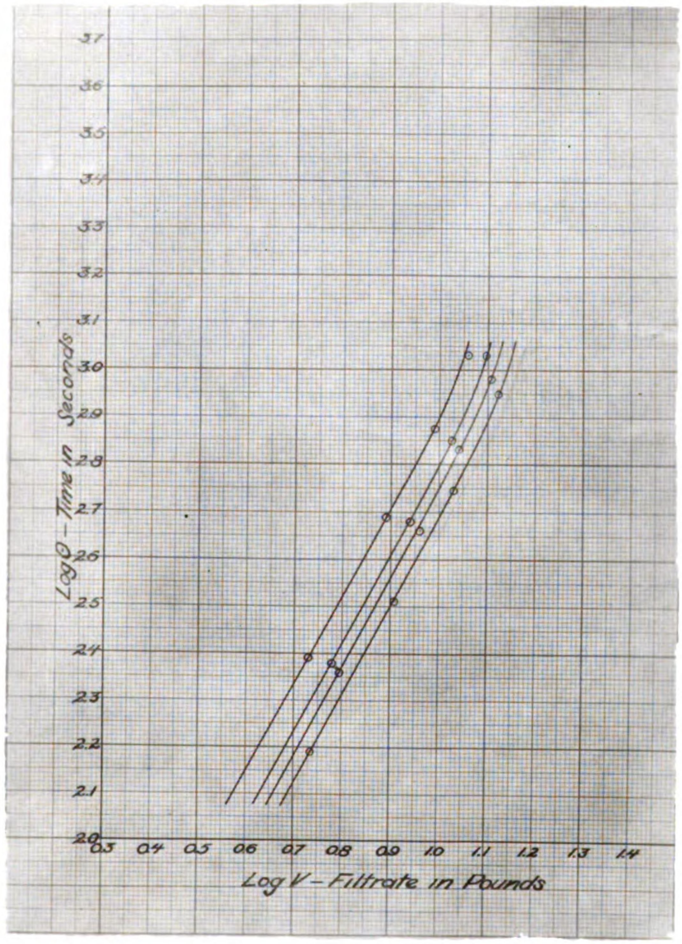

Log θ - Time in Seconds
Log V - Filtrate in Pounds
2.0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3.0
3.1
3.2
3.3
3.4
3.5
3.6
3.7
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4

Table XXI (Continued)

Point	θ	V	Log θ	Log V
5	240	6.00	2.3802	0.7782
6	480	8.69	2.6812	0.9390
7	720	10.55	2.8573	1.0232
8	1080	12.40	3.0334	1.0934
9	230	6.18	2.3617	0.7910
10	460	9.16	2.6628	0.9619
11	690	10.90	2.8388	1.0374
12	960	12.95	2.9823	1.1122
13	50	2.69	1.6990	0.4298
14	155	5.37	2.1903	0.7300
15	325	8.06	2.5119	0.9063
16	555	10.75	2.7443	1.0314
17	900	13.28	2.9542	1.1214

Plotting and observing slope, (fig. 11) $m = 1.85$

we now have, $\left(\frac{V}{A}\right)^{1.85} = k\,P^n\,\theta$

Applying this equation to the four runs for a convenient value of V, and choosing as this value of V whose log = 0.8; V = 6.31. Then,

$$\left(\frac{6.31}{134}\right)^{1.85} = k\,P^n\,\theta = .00035$$

where θ is the time at which V = 6.31 . The value of θ for the different pressures may be taken from fig. 11.

Figure 12

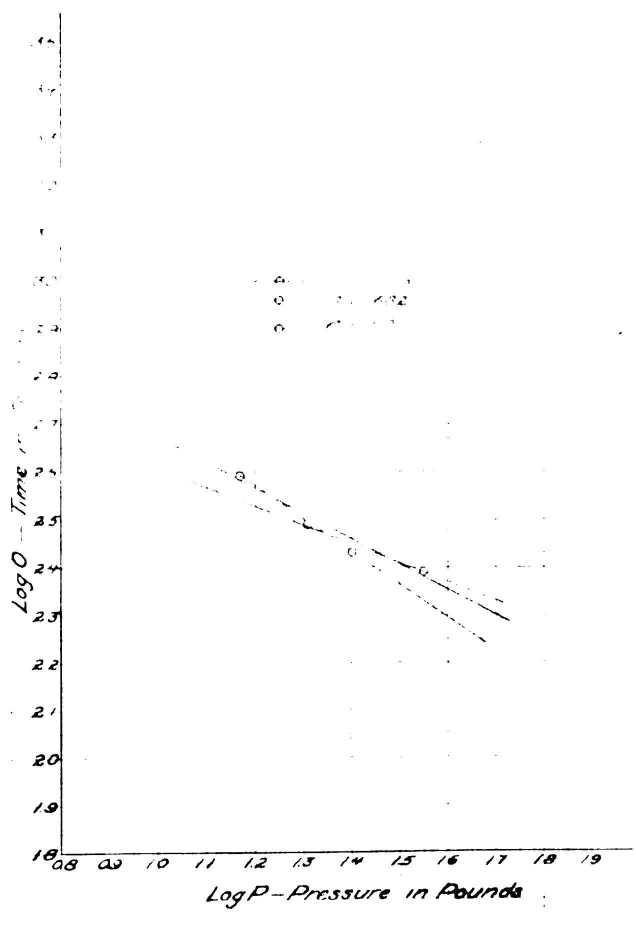

Log O - Time
Log P - Pressure in Pounds
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9

Table XXII

Run	Pressure	Log P	Log θ at log V= 0.8	θ in sec.
1	15	1.1761	2.525	335
2	20	1.3010	2.420	263
3	25	1.3979	2.370	233
4	32	1.5051	2.315	207

Plotting log θ against log P (fig. 12)[1] n = slope of line = .533

---

[1]Note: By referring to fig. 12 it may be seen that all the points do not fall in a straight line. If only three runs had been made the value of n might have been either of two other values, as illustrated.

By combining runs 1, 2, and 3, n = .425

By combining runs 2, 3, and 4, n = .682

Choosing convenient values of log P and log θ from fig. 12 log P = 1.3979; log θ = 2.370.

Substituting these values of log P and log θ we have,

$$.0035 = k(25)^{.533}\ 234.$$

Rearranging and taking logs we have,

$$2.37 = -.533(1.3979) + .0035/k$$

$$2.37 = -.746 + .0035/k$$

$$k = \frac{.0035}{3.116} = \frac{.0035}{1305} = .00000268$$

Changing to engineering units

Units use		Eng. units.	Factor
V	of filtrate	cu. ft.	62.5
A	Sq. in.	sq. ft.	144
θ	in seconds	hours	3600
P	/sq. in.	/sq. in.	1

$$\left(\frac{V}{A}\right)^{1.85} \times \left(\frac{62.5}{144}\right)^{1.85} = .00000268\ P^{.533}\ \theta\ 3600$$

$$\left(\frac{V}{A}\right)^{1.85} = \frac{.009648}{(.434)^{1.85}}\ P^{.533}\ \theta$$

$$\left(\frac{V}{A}\right)^{1.85} = .0446\ P^{.533}\ \theta$$

By substituting the two other values of n in the equation a different value of k is obtained for each value of n. The ultimate results are exactly the same, however, because n and k vary directly with each other.

Table XXIII

m	n	k
1.85	.533	0.0446
1.85	.425	0.0645
1.85	.682	0.0272

2. Calculation of filtering times.

$$V = 12.00$$

$$P = 25$$

$$A = .924 \text{ sq. ft.}$$

For 1" frames

$$\left(\frac{.192}{.924}\right)^{1.85} = .055$$

$$(25)^{.533} = 5.66$$

$$.055 = .0446 \times 5.66\ \theta$$

$$\theta = 0.218 \text{ hours}$$

$$\theta = 13 \text{ minutes.}$$

For 2" frames

$$V = \frac{.792 \times 2.125}{1.125} = .363 \text{ cu. ft.}$$

$$\left(\frac{.363}{.924}\right)^{1.85} = .0446 \times 5.66\ \theta$$

$$.183 = .0446 \times 5.66\ \theta$$

$$\theta = 0.723 \text{ hours}$$

$$\theta = 43 \text{ minutes.}$$

For 3" frames.

$$V = \frac{.192 \times 3.125}{1.125} = .533 \text{ cu. ft.}$$

$$\left(\frac{.533}{.924}\right)^{1.85} = .0446 \times 5.66\ \theta$$

$$.362 = .252\ \theta$$

$$\theta = 1.43 \text{ hours}$$

$$\theta = 87 \text{ minutes.}$$

For 4" frame.

$$V = \frac{.192 \times 4.125}{1.125} = .705 \text{ cu. ft.}$$

$$\left(\frac{.705}{.924}\right)^{1.85} = .0446 \times 5.66\ \theta$$

$$.606 = 0.252\ \theta$$

$$\theta = 2.41 \text{ hours}$$

$$\theta = 145 \text{ minutes.}$$

Table XXXIV

Comparison of actual and calculated filtering times.

Units use		Eng. units.	Factor
V	of filtrate	cu. ft.	62.5
A	Sq. in.	sq. ft.	144
θ	in seconds	hours	3600
P	/sq. in.	/sq. in.	1

$$\left(\frac{V}{A}\right)^{1.85} \times \left(\frac{62.5}{144}\right)^{1.85} = .00000268\ P^{.533}\ \theta\ 3600$$

$$\left(\frac{V}{A}\right)^{1.85} = \frac{.009648}{(.434)^{1.85}}\ P^{.533}\ \theta$$

$$\left(\frac{V}{A}\right)^{1.85} = .0446\ P^{.533}\ \theta$$

By substituting the two other values of n in the equation a different value of k is obtained for each value of n. The ultimate results are exactly the same, however, because n and k vary directly with each other.

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$$P = 25$$

$$A = .924 \text{ sq. ft.}$$

For 1" frames

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$$(25)^{.533} = 5.66$$

$$.055 = .0446 \times 5.66\ \theta$$

$$\theta = 0.218 \text{ hours}$$

$$\theta = 13 \text{ minutes.}$$

For 2" frames

$$V = \frac{.792 \times 2.125}{1.125} = .363 \text{ cu. ft.}$$

$$\left(\frac{.363}{.924}\right)^{1.85} = .0446 \times 5.66\ \theta$$

$$.182 = .0446 \times 5.66\ \theta$$

$$\theta = 0.723 \text{ hours}$$

$$\theta = 43 \text{ minutes.}$$

For 3" frames.

$$V = \frac{.192 \times 3.125}{1.125} = .533 \text{ cu. ft.}$$

$$\left(\frac{.533}{.924}\right)^{1.85} = .0446 \times 5.66\ \theta$$

$$.362 = .252\ \theta$$

$$\theta = 1.43 \text{ hours}$$

$$\theta = 87 \text{ minutes.}$$

For 4" frame.

$$V = \frac{.192 \times 4.125}{1.125} = .705 \text{ cu. ft.}$$

$$\left(\frac{.705}{.924}\right)^{1.85} = .0446 \times 5.66\ \theta$$

$$.606 = 0.252\ \theta$$

$$\theta = 2.41 \text{ hours}$$

$$\theta = 145 \text{ minutes.}$$

Table XXXIV

Comparison of actual and calculated filtering times.

Cake thickness	Calculated Filtering T.	Actual Filter. T.	°/₀ Variation from actual
1.375	13	16	18.75
2.375	43	54	20.40
3.375	87	100.5	13.40
4.375	145	185	21.65

Discussion:

According to Badger and McCabe (2) the values of the constants in their equation for non-homogeneous sludges, (278)[1] have a definite relation to the character of the sludge. "Since the value of m is not quite 2.0, the sludge is not quite homogeneous, but nearly so. On the other hand, n is far from 1.0, and hence the sludge is very highly compressible."[2]

The value of these constants (m and n) do not seem to give one and idea as to the character of the sludge according to my work. By referring to fig. 11, it may be seen that the value of m equals 1.84, and since the slurry was made up to be non-homogeneous there is agreement with the equation of Badger and McCabe (2). However, suppose instead of limiting the amount of collodial material, as was the case here, we increase it to say fifty per cent of the total solids.

In this case the filtration will be much more difficult and the time required to obtain a definite amount of filtrate,

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[1] Elements of Chem. Eng. by Badger and McCabe.

[2] Elements of Chem. Eng. page 464

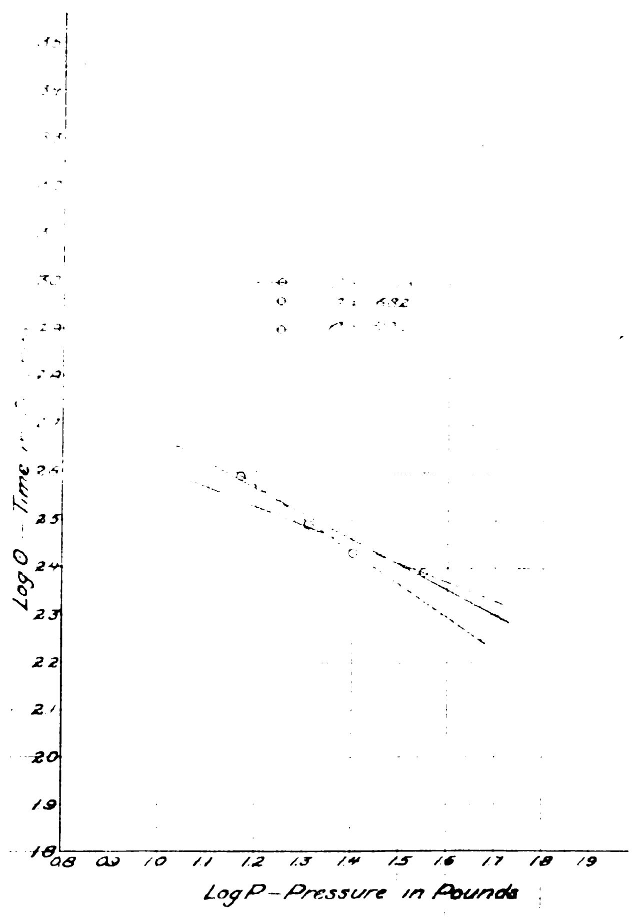
will be greater than was the time required for the same amount of filtrate, using slurry I. In other words the lines of the graph (fig. 11) will approach the vertical position with the greater amount of collodial material. Then the value of m would be greater than 2, (providing enough $Al(OH)_3$ had been used) which according to Badger and McCabe (2) would make the sludge homogeneous, although the slurry was originally made so as to be non-homogeneous.

On the other hand, when you do have a homogeneous sludge, it is true, except in case of collodial or finely divided particles, the rate of filtration is greater than in most cases of non-homogeneous sludges. According to this reasoning more filtrate will be obtained per unit time. The lines in the graph (fig. 11) will then flatten out and the value of m will become less. This will tend to show that the sludge is non-homogeneous. This is exactly what happened in the case of slurry II.

The value of n also apparently fails to ascertain the character of the sludge. Supposedly the closer the value of n approaches 1.0, the less compressible is the sludge. By referring to the graph (fig. 12) it may be seen that n may have different values for the same sludge. In this case the values of n vary from about 0.4 to nearly 0.7. It would be possible to even have greater variation in the value of n for the same sludge.

This could be done by making a greater number of runs and taking different combinations of points. These points of the

Figure 12



Log Θ – Time
Log P – Pressure in Pounds
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9

Table XXII

Run	Pressure	Log P	Log θ at log V= 0.8	θ in sec.
1	15	1.1761	2.525	335
2	20	1.3010	2.420	263
3	25	1.3979	2.370	233
4	32	1.5051	2.315	207

Plotting log θ against log P (fig. 12)[1] n = slope of line = .533

---

[1]Note: By referring to fig. 12 it may be seen that all the points do not fall in a straight line. If only three runs had been made the value of n might have been either of two other values, as illustrated.

By combining runs 1, 2, and 3, n = .425

By combining runs 2, 3, and 4, n = .682

Choosing convenient values of log P and log θ from fig. 12 log P = 1.3979; log θ = 2.370.

Substituting these values of log P and log θ we have,

$$.0035 = k(25)^{.533}\ 234.$$

Rearranging and taking logs we have,

$$2.37 = -.533(1.3979) + .0035/k$$

$$2.37 = -.746 + .0035/k$$

$$k = \frac{.0035}{3.116} = \frac{.0035}{1305} = .00000268$$

Changing to engineering units

Units use		Eng. units.	Factor
V	of filtrate	cu. ft.	62.5
A	Sq. in.	sq. ft.	144
θ	in seconds	hours	3600
P	/sq. in.	/sq. in.	1

$$\left(\frac{V}{A}\right)^{1.85} \times \left(\frac{62.5}{144}\right)^{1.85} = .00000268\ P^{.533}\ \theta\ 3600$$

$$\left(\frac{V}{A}\right)^{1.85} = \frac{.009648}{(.434)^{1.85}}\ P^{.533}\ \theta$$

$$\left(\frac{V}{A}\right)^{1.85} = .0446\ P^{.533}\ \theta$$

By substituting the two other values of n in the equation a different value of k is obtained for each value of n. The ultimate results are exactly the same, however, because n and k vary directly with each other.

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m	n	k
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$V = 12.00$

$P = 25$

$A = .924$ sq. ft.

For 1" frames

$$\left(\frac{.192}{.924}\right)^{1.85} = .055$$

$$(25)^{.533} = 5.66$$

$$.055 = .0446 \times 5.66\ \theta$$

$$\theta = 0.218 \text{ hours}$$

$$\theta = 13 \text{ minutes.}$$

For 2" frames

$$V = \frac{.792 \times 2.125}{1.125} = .363 \text{ cu. ft.}$$

$$\left(\frac{.363}{.924}\right)^{1.85} = .0446 \times 5.66\ \theta$$

$$.183 = .0446 \times 5.66\ \theta$$

$$\theta = 0.723 \text{ hours}$$

$$\theta = 43 \text{ minutes.}$$

For 3" frames.

$$V = \frac{.192 \times 3.125}{1.125} = .533 \text{ cu. ft.}$$

$$\left(\frac{.533}{.924}\right)^{1.85} = .0446 \times 5.66\ \theta$$

$$.362 = .252\ \theta$$

$$\theta = 1.43 \text{ hours}$$

$$\theta = 87 \text{ minutes.}$$

For 4" frame.

$$V = \frac{.192 \times 4.125}{1.125} = .705 \text{ cu. ft.}$$

$$\left(\frac{.705}{.924}\right)^{1.85} = .0446 \times 5.66\ \theta$$

$$.606 = 0.252\ \theta$$

$$\theta = 2.41 \text{ hours}$$

$$\theta = 145 \text{ minutes.}$$

Table XXXIV

Comparison of actual and calculated filtering times.

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1.375	13	16	18.75
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According to Badger and McCabe (2) the values of the constants in their equation for non-homogeneous sludges, (278)[1]

[1] Elements of Chem. Eng. by Badger and McCabe.

have a definite relation to the character of the sludge. "Since the value of m is not quite 2.0, the sludge is not quite homogeneous, but nearly so. On the other hand, n is far from 1.0, and hence the sludge is very highly compressible."[2]

[2] Elements of Chem. Eng. page 464

The value of these constants (m and n) do not seem to give one and idea as to the character of the sludge according to my work. By referring to fig. 11, it may be seen that the value of m equals 1.84, and since the slurry was made up to be non-homogeneous there is agreement with the equation of Badger and McCabe (2). However, suppose instead of limiting the amount of collodial material, as was the case here, we increase it to say fifty per cent of the total solids.

In this case the filtration will be much more difficult and the time required to obtain a definite amount of filtrate, will be greater than was the time required for the same amount of filtrate, using slurry I. In other words the lines of the graph (fig. 11) will approach the vertical position with the greater amount of collodial material. Then the value of m would be greater than 2, (providing enough $Al(OH)_3$ had been used) which according to Badger and McCabe (2) would make the sludge homogeneous, although the slurry was originally made so as to be non-homogeneous.

On the other hand, when you do have a homogeneous sludge, it is true, except in case of collodial or finely divided particles, the rate of filtration is greater than in most cases of non-homogeneous sludges. According to this reasoning more filtrate will be obtained per unit time. The lines in the graph (fig. 11) will then flatten out and the value of m will become less. This will tend to show that the sludge is non-homogeneous. This is exactly what happened in the case of slurry II.

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This could be done by making a greater number of runs and taking different combinations of points. These points of the graph (fig. 12) do not lie in a straight line because the position of the points depend upon the distance apart of the lines in graph (fig. 11). The position of these lines depend upon the difference in pressures of the two runs. The greater the distance between these lines the greater the irregularity of the points in fig. 12.

As far as the different values of n are concerned they will not affect the workibility of the equation. The value of n and k are directly proportional to each other and as n and k vary the values of P and θ remain the same for a particular run.

The ultimate results obtained by this equation° are not

°1. Eq. 278 Elements of Chem. Eng. by Badger and McCabe. page 460

very satisfactory. By referring to table XXIII it may be seen that the calculated filtering times vary from the actual filtering times all the way from 18.75 °/₀ to 26.65 °/₀. The variation being greater with the increased thickness of cake.

VI. First attempt to develope a more accurate equation for predicting filtration times.

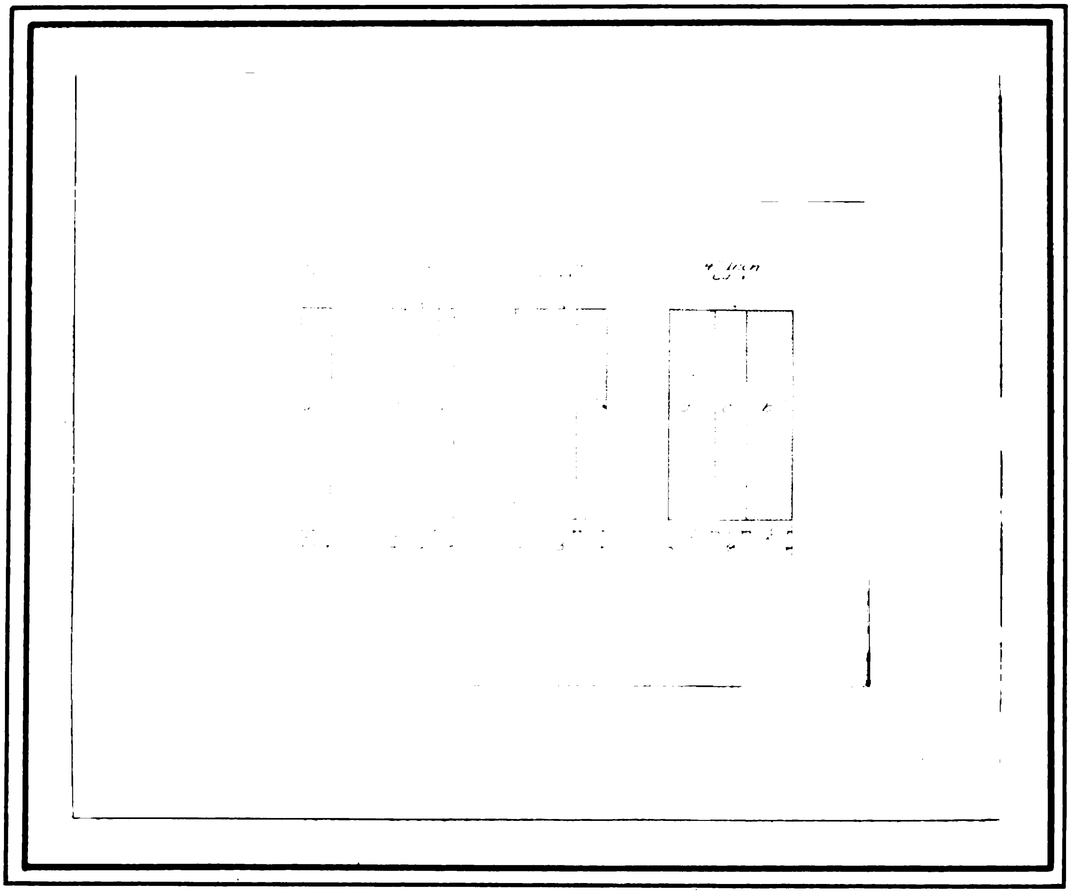
Procedure:

The equation which is to follow was not worked out from a mathematical standpoint but from actual experiments carried out in the laboratory. These equations have no theoritical backing but check up fairly well with actual experimental data. They check at least as good, and better in most cases, as the results obtained when applied to the equations of Walker, Lewis, and McAdams (1) as Badger and McCabe (2).

I first drew a picture of a 1, 2, 3, and 4 inch cake (fig. 13). The 2, 3, and 4 inch cakes being drawn representing cakes which had been filtered except for the final one inch in the center of the cake. The total time to filter a one inch cake was accurately determined. Then the time to filter a two inch cake will be equal to the time to filter a one inch cake plus the time to filter an additional one inch cake. The time to filter this final one inch cake will take longer than the first one inch cake because of the increased resistance due to the passage of filtrate through that cake which has already been filtered. To illustrate by referring to fig. 13 the time to filter II will be the time to filter a'+ b' + c'. But the time required to filter a'b' is the same as the time to filter a+b.

Then the time required to filter c' is dependent upon some power of the thickness. From many actual filtrations runs this

Figure 13



power was observed to be the first. Then the time to filter II will be,

$$\theta_2 = \theta_1 + n \times \theta_1 \qquad (1)$$

where, θ_2 = time to filter II

θ = time to filter I

n = thickness of II

The same thing holds for the 3 and 4 inch cakes. The time to filter a'' + b'' is the same as the time to filter a' + b' + c' and the time to filter a''' + b''' is the same as the time to filter a'' + b'' + c''. Also, the time required to filter c'' + c''' is dependent upon the first power of the thickness.

$$\theta_3 = \theta_2 + n_3 \times \theta_1$$

$$\theta_4 = \theta_3 + n_4 \times \theta_1$$

where, θ_3= time to filter a three inch cake

θ_4= time to filter a four inch cake

n_3= 3.125

n_4= 4.125

To test the workability of these equations, reference will be made to the data of part IV. The data for the 25 pressure run will be recopied for simplicity.

Table XXV

Period	Time(sec)	Filtrate(lbs)	Total Filtrate	Pressure
1	240	4 11/16	4 11/16	25
2	480	2 13/16	6 14/16	25
3	720	1 7/16	8 5/16	25
4	1280	1 15/16	10 4/16	25

By referring to fig. 5 it will be noted that the frame is full when log V = 0.95 or V = 8.9* . This point is determined by the sudden curvature of the line upward. That is, the amount of filtrate decreases per unit time. When the cake builds up on each cloth the frame gradually fills and finally the cakes build up to the point when they touch. The filter area is immediately reduced and it is at this point that the lines in the graph turn upward. The time at which this occurs is the time required to fill the cake. It was this time that was used as a basis for the following calculations, and found to be 16 minutes.

A. Application of equation to Slurry I.

Calculations:

$$\theta_L = \theta_{L-1} + n_L \times \theta_1 \qquad (1)$$

θ_1 = 16 minutes.

θ_{L-1} = Time req'd to filter the next less thickness of cake.

n_L = Thickness of cake being filtered.

θ_L = Filtering time.

For 2" frames.

$$\theta_2 = \theta_1 + n_2 \times \theta_1$$

$$\theta_2 = 16 + 2.125 \times 16$$

$$\theta_2 = 50 \text{ minutes.}$$

For 3" frames.

$$\theta_3 = \theta_2 + n_3 \times \theta_1$$

$$\theta_3 = 50 + 3.125 \text{ X } 16$$

$$\theta_3 = 99 \text{ minutes.}$$

For 4" frames.

$$\theta_4 = \theta_3 + n_4 \text{ X } \theta_1$$

$$\theta_4 = 99 + 4.125 \text{ X } 16$$

$$\theta_4 = 166 \text{ minutes.}$$

As a matter of comparison the following table was arranged. The values of calculated times by equation of Badger and McCabe were taken from part IV above. The actual filtration times werd arrived at by actual filtration. The technique being the same as in other runs with filter press except that runs were made for each size frame up to the 4 inches. The actual thickness of cake was measured and found to be frame thickness plus 0.125".

Table XXVI[1]

Cake Thickness Inches	Badger & McCabe Formula Filter. Times. Min.	Formula (1) Derived Times Min.	Actual Filter Times MIn.
1.125	15.5	16	16
2.125	55	50	49
3.125	102	99	92.5
4.125	172	165	150

°/o Variation from Actual of B&M For.	°/o Variation from Actual for For. (1)	Pressure
3.3	0	25
12.2	2.0	25

Table XXVI (continued)

°/o Variation from Actual of B&M For.	°/o Variation from Actual for For. (1)	Pressure
10.3	7.0	25
14.7	10.0	25

[1] It will be noted that the values of actual filtering time and calculated time of this table vary greatly from the same values of table XXIII. This is due to the fact that the slurry used in previous work was made up with chemically pure materials, while the slurry of the latter work was made up with commercial material. Although the slurries were made up exactly the same, they gave different results upon filtering.

Due to the fact that we were unable to obtain the C.P. materials, the commercial materials had to be used. In as much as comparative results were obtained, the commercial materials were satisfactory.

B. Application of equation (1) to Slurry II.

Procedure:

In using slurry II the same procedure was used as was used in the case of slurry I. The time to filter a 1" cake was accurately determined first. The times for filtering a 2", 3", and 4" frame were also determined by carrying out the filtration and found to be 4, 11½, 22½, 37 1/3 minutes respectively.

As a matter of comparison the filtering times as calculated in part II sec. B. will be tabulated in the table which is to follow.

Calculations:

$$\theta_L = \theta_{L-1} + n_L \times \theta_1 \qquad (1)$$

θ_1 = 4 minutes.

θ_{L-1} = Time required to filter the next less thickness of cake.

n_L = Thickness of cake being filtered.

For 2" frames.

$$\theta_2 = \theta_1 + n_2 \times \theta_1$$

$$\theta_2 = 4 + 2.125 \times 4$$

$$\theta_2 = 12.5 \text{ minutes.}$$

For 3" frames.

$$\theta_3 = \theta_2 + n_2 \times \theta_1$$

$$\theta_3 = 12.5 + 3.125 \times 4$$

$$\theta_3 = 25 \text{ minutes.}$$

For 4" frames.

$$\theta_4 = \theta_3 + n_4 \times \theta_1$$

$$\theta_4 = 25 + 4.125 \times 4$$

$$\theta_4 = 41.5 \text{ minutes.}$$

Table XXVII

Comparison of calculated and actual filtering times.

Cake thickness Inches.	Badger&McCabe Formuàà Time in minutes.	Derived For. (1) Min.	Actual T. in Min.
1.125	5	4	4
2.125	16	12.5	11.5

Table XXVll (Continued)

Cake thick-ness Inches.	Badger&McCabe Formula Time in minutes	Derived For. (1) Min.	Actual Filter. T. Min.
3.125	38	25	23 $\frac{1}{2}$
4.125	67	41.5	37 1/6

°/o Variation from Actual B & M For.	°/o Variation from Actual of For. (1)	Pressure
25	0	25
39.1	8.7	25
69	11.1	25
53.8	10.8	25

Discussion:

By referring to tables XXVI and XXVII a very good comparicon of the calculated filtering times by the equation of Badger and McCabe (2) and of the equation (1) above, is shown. In the case of slurry I the equations of Badger and McCabe hold fairly well, but in no case are the results as good as the calculated filtering times by equation (1).

When slurry II is used (table XXVII) neither equation holds as good as they do with slurry I (table XXVI). In either case it may be noted that equation (1) holds closer to the actual than does the equations of Badger and McCabe. Then too the per cent variation from actual of equation (1) is practically the same in either case and does not vary more than 11 °/o. These calculated values are greater than actual values in both cases. If these calculated values by equation (1) always give values greater than the actual times, regardless of the slurry used, and always by a certain per cent, and in this work all results tend to prove this, then a correction factor could be used which would give values very close to actual filtration values.

VII. Development of final equation for predicting the filtering times of a certain slurry for a certain constant pressure.

A. Derivation of equation.

Procēdure:

All the preliminary filtrations pretaining to this equation were carried on with slurry I. The first step in the procedure was to determine the exact time at which a frame was filled when filtering at a constant pressure of 25#/sq. in. using slurry I. The slurry was made up according to specifications and placed in the blow tank (fig. 1). The pressure was raised to 25# in the blow tank and filtration started. The filtrate was collected as in part IV, sec. A. That is, by allowing the filtrate to run in a container balanced on a Toledo scale.

The filtration cycle was divided into six periods and the weight of filtrate at the end of each of the periods was noted. This data is given in table XXVIII below.

Table XXVIII

Period	Time(sec)	Filtrate(lbs)	Total Filtrate	Pressure
1	240	1 12/16	1 12/16	25
2	480	12.5/16	2 8.5/16	25
3	720	10.5/16	3 3/16	25
4	1080	10/16	3 13/16	25
5	1500	7/16	4 4/16	25
6	2100	5/16	4 9/16	25

Figure 14

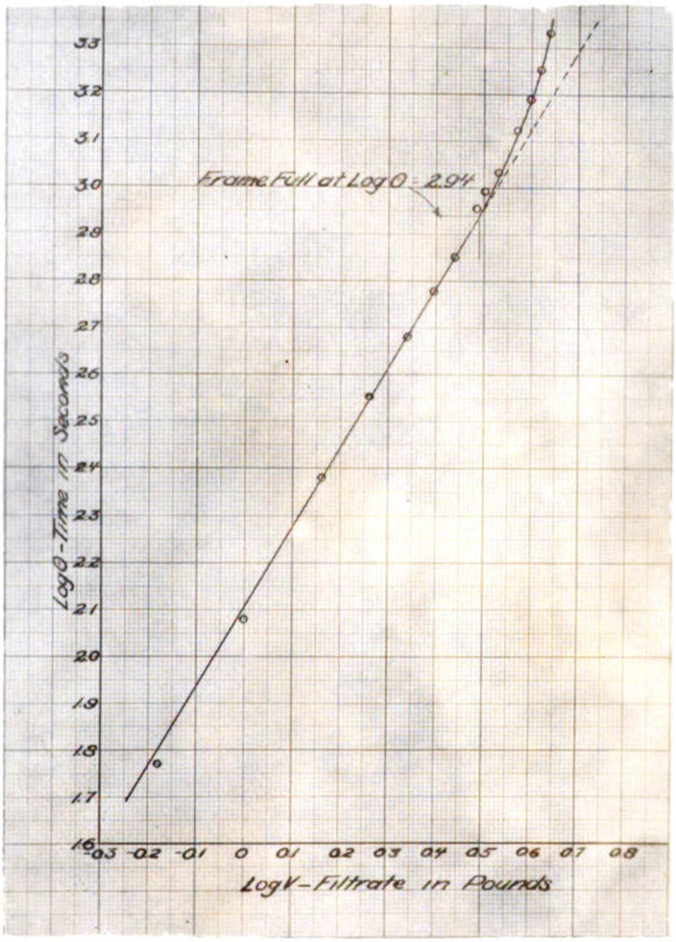

Frame Full at Log θ = 2.94
Log θ - Time in Seconds
LogV - Filtrate in Pounds
3.3
3.2
3.1
3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Table XXX (Continued)

Time (Min)	Wt. of filtrate	Cake Thickness	Pressure
46 ½	5.97	2.125	25
90	8.76	3.125	25
147	11.60	4.125	25

By plotting log of thickness against log of time it is seen that all the points follow the course of a straight line (fig. 15).

Table XXXI

Data for fig. 15

θ	L	Log θ	Log L
14.5	1.125	1.1395	0.207
46.5	2.125	1.2920	0.395
90	3.125	1.7950	0.752
147	4.125	2.1405	1.131

With all the points of the above graph (fig. 15) falling on a straight line, only two points would be necessary to draw the line. After the line had been determined, then the time for filtering any thickness cake could be determined from the graph.

The equation of a straight line is,

$$y = mx + k$$

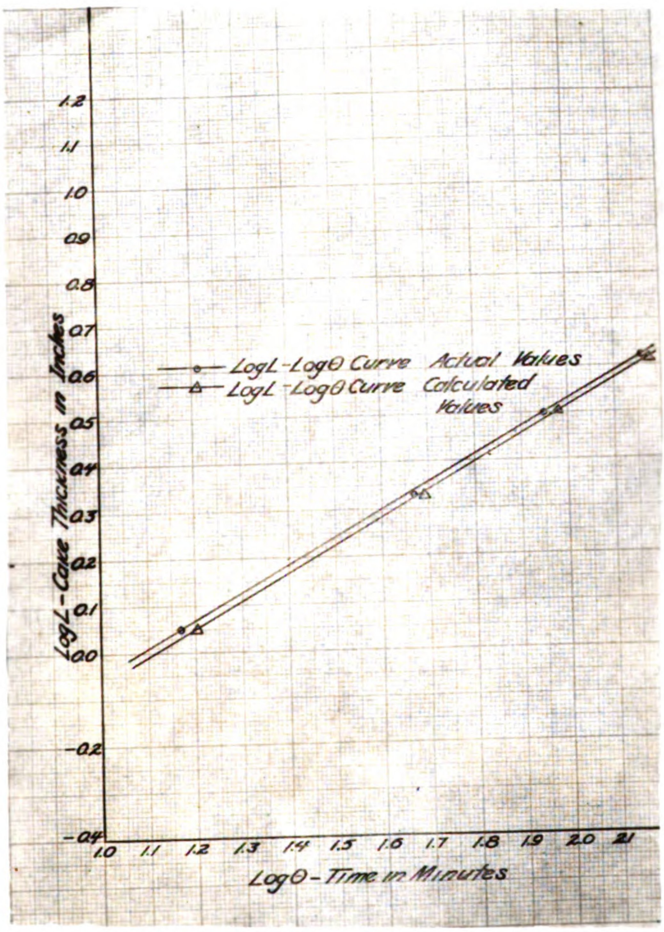
Then the equation of line of fig. 15 is,

$$y = m_1x + k_1 \qquad (1)$$

where,

y = Log L   Cake thickness in inches.

Figure 15



Log L - Log θ Curve Actual Values
Log L - Log θ Curve Calculated Values
Log L - Case Thickness in Inches
Log θ - Time in Minutes
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.2
-0.4
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1

$x = \log \theta$ Time to filter cake in minutes.

M_1 = slope of line.

k_1 = constant.

But it is desired to get a relation between cake thickness, filtering time and pounds of filtrate. Then it will be necessaty to plot log of pounds of filtrate and log of time. (fig. 16)

Table XXXII

θ	V	Log θ	Log V
14.5	3.16	1.1662	0.500
46.5	5.97	1.668	0.776
90.0	8.76	1.954	0.943
147.0	11.60	2.168	1.065

Equation of line, fig. 16,

$$y = M_2 x + k_2 \text{------------------------} (2)$$

where,

$y = \text{Log } V$ Filtrate in pounds.

$x = \text{Log } \theta$ Filtering time in minutes.

M_2 = slope of line.

k_2 = constant.

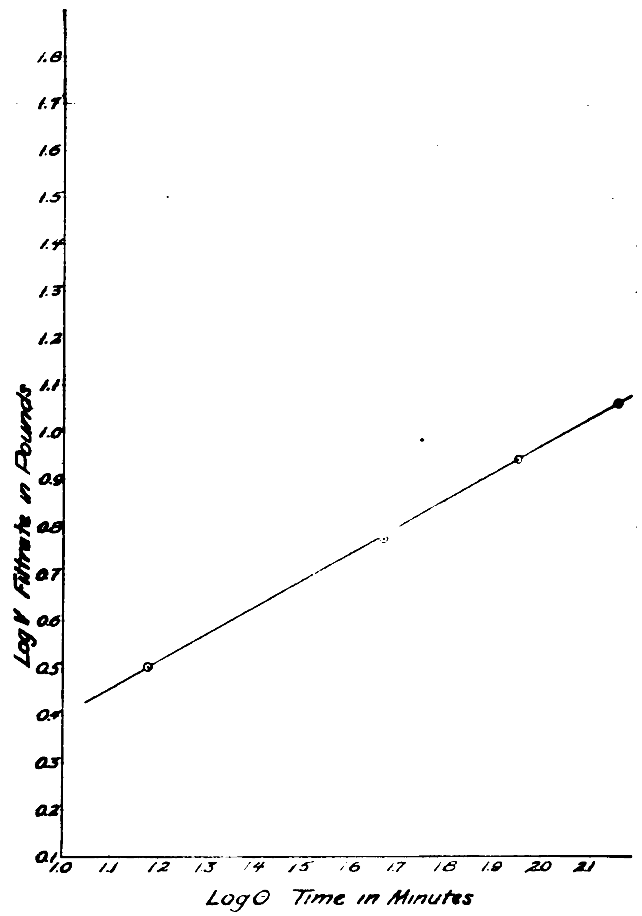
By substituting in (1), we have,

$$\text{Log } L = M_1 \text{ Log } \theta + \text{Log } k_1$$

Transposing,

$$\frac{M_1 \text{ Log } \theta}{\text{Log } L} = - \text{Log } k_1$$

Removing the log, we have,



1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Log V Filtrate in Pounds
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
Log Θ Time in Minutes

$$\frac{\theta}{L}^{M_1} = -k_1 \text{------------------------------------} (3)$$

By substituting in (2), we have,

$$\text{Log } V = M_2 \text{ Log } \theta + \text{Log } k_2$$

Transposing,

$$\frac{M_2 \text{ Log } \theta}{\text{Log } V} = - \text{log } k_2$$

or, $$\frac{\theta}{V}^{M_2} = -k_2 \text{------------------------------------} (4)$$

Multiplying (3) and (4), we have,

$$\frac{\theta^{M_1 + M_2}}{VL} = k_1 \ k_2$$

By combining the slopes of the two lines of fig. 15 (M_1) and fig. 16 (M_2) into a single slope M and the constants k_1 and k_2 into a single constant k, we have,

$$\frac{\theta^M}{VL} = k$$

rearranging,

$$\theta^M = kVL \text{------------------------------------} (5)$$

B. Application of eq ation (5) to Slurry I.

Procedure:

In as much as the testing when a one inch frame was filled; had been determined previously, (fig. 14) it will not be necessary to repeat it.

The one inch frame was inserted in the press and filtration started at 25# per. sq. in. pressure. When the required amount of filtrate was obtained the time was taken. This was then repeated for a two inch cake.

Data:

Table XXXIII

Run	Time(minutes)	Wt. of Filt. in pounds.	Cake Thick-ness in in.	Pressure
1	16	3.16	1.125	25
2	49	5.97	2.125	25

Temperature of slurry 19°C.

Calculations:

$$\theta^{M} = L\ V\ k$$

Apply this equation to the two runs at the constant pressure, by plotting log L vs. log θ and observing the slope of the line (fig. 15).

Table XXXIV

Run	θ	L	Log θ	Log L
1	16	1.125	1.204	0.051
2	49	2.125	1.690	0.328

This line shown in fig. 15.

$$M_1 = \text{slope of line} = \frac{57.3}{98.5} = .580$$

Plotting log V vs. log θ and observing the slope of line. (fig. 16)

Table XXXV

Run	θ	V	Log θ	Log V
1	16	3.16	1.204	0.580
2	49	5.97	1.690	0.776

In as much as the data of this table and that of table

XXXII are practically identical, the slope of the line of fig. 16 will be taken as the actual slope.

M_2 = slope of line = 47/82 = .57

$M = M_1 + M_2 = .58 + .57 = 1.15$

Applying this equation to run 1, we have,

$$\theta^{1.15} = L\ V\ k$$

Substituting we have,

$$(16)^{1.15} = 1.125 \cdot 3.16 \cdot k$$

$$24.3 = 3.56\ k$$

$$k = 6.64$$

Calculation of filtering times:

For 2" frames.

$$(\theta)^{1.15} = 6.64 \cdot 5.97 \cdot 2.125$$

$$(\theta)^{1.15} = 84.4$$

$$\theta = \sqrt[1.15]{84.4}$$

$$\theta = 47.5 \text{ minutes.}$$

For 3" frames.

$$(\theta)^{1.15} = 6.64 \cdot 8.76 \cdot 3.125$$

$$(\theta)^{1.15} = 182.$$

$$\theta = \sqrt[1.15]{182.}$$

$$\theta = 92.5 \text{ minutes.}$$

For 4" frames.

$$(\theta)^{1.15} = 6.64 \cdot 11.6 \cdot 4.125$$

$$(\theta)^{1.15} = 318.$$

$$\theta = \sqrt[1.15]{318.}$$

$$\theta = 150 \text{ minutes}$$

Table XXXVI

Comparison of Filtering Times.

Cake Thick-ness inches.	Badger & McCabe Form. T. in Min.	Form. (5) F. T. in Min.	Actual T. in Min.
1.125	15.5	16	16
2.125	55	47.5	46.5
3.125	102	92.5	90
4.125	172	150.0	147.

°/o Variation from actual of Badger & McCabe Form.	°/o Variation from Actual of Formual (5)
3.3	0.
18.5	2.15
13.3	2.78
17.0	2.00

Discussion:

The application of the above equation (5) as shown, is comparatively simple. It merely consists of making a run with a one inch cake, drawing a graph which is either a $V^2 - \theta$ or $\log V - \log \theta$ graph, to determine the point at which the frame is filled. After the pounds of filtrate have been determined for the one inch cake , the pounds of filtrate for a 2", 3", and 4" cake are determined. A second run is then made, filtering a two inch cake, the time being noted when the required amount of filtrate has been obtained.

With this data a $\log \theta - \log L$ and a $\log \theta \log V$ graph is constructed and the slopes of the lines noted. Using the data of the one inch cake, the values of V, L, and θ are substituted in the equation.

$$\theta^M = k\ L\ V$$

the value of M being equal to the combined slopes of the two lines. From this the value of k is determined. After k has been calculated, the filtering time for any thickness cake may be determined.

The results obtained from this equation depend greatly upon the accurateness of the initial run. That is, the run where the time and filtrate of a one inch cake are determined. Great care should be taken, then, in determining V and θ for the one inch cake.

By referring to table XXXVI it may be seen that the equation holds fairly well. It is seen that by equation (5) the calculated values vary only from 0 to 2.78 °/o from the actual filtration values. However, the filtration times as calculated from formula of Badger and McCabe vary from 3.3 to 18.5 °/o.

The correct method to use, in determining when the cake is filled, depends upon the slurry being filtered. If the slurry is filtered very easily, probably the best method would be the $V^2 - \theta$ graph. However, if the slurry is collodial or otherwise difficult to filter, it is best to use the log V - log θ graph. When interpreting these graphs, it must be understood that when the line starts to curve that the cake is not necessarially entirely filled. It does mean, however, that the half cakes on each filter cloth have touched each other and the cake is nearly filled.

If the graphs are read correctly a short time will be

allowed for a complete filling of the frame. The time allowed depending upon the total time of the cycle. In any case the time allowed will not be greater than one minute and in most cases not more than thirty seconds.

VIII. Comparison of the rate of filtration, using the several different kinds of filter cloths of Edward H. Best & Co.

Procedure:

Nine sets of filter cloths of the Edward H. Best & Co. were cut for use on the filter press. The same procedure was used in connection with the filter press as described in part III sec. A. Slurry I was filtered under the same conditions, with only the filter medium being changed. Filtration was carried on at a constant pressure of 32# per. sq. in.

Data:

Table XXXVII

Period	Time(sec.)	Filtrate(lbs)	Total Filtrate(lbs)	Pressure
1	50	2 ½	2 1/4	32
2	155	2 ½	5	32
3	325	2 ½	7 ½	32
4	555	2 ½	10	32
5	900	2 1/4	12 1/4	32
Regular Duck Cloth				
1	55	2 ½	2 ½	32
2	160	2 ½	5	32
3	330	2 ½	7 ½	32
4	560	2 ½	10	32
5	905	2 1/4	12 1/4	32
Cloth No. 3565---Edward H. Best & Co.				

Table XXXVII (Continued)

Period	Time(sec)	Filtrate(lbs)	Total Filtrate(lbs)	Pressure
1	50	2 ½	2 ½	32
2	150	2 ½	5	32
3	320	2 ½	7 ½	32
4	570	2 ½	10	32
5	920	2 ½	12 ½	32
Cloth No. 5085---Edward H. Best & Co.				
1	47	2 ½	2 ½	32
2	160	2 ½	5	32
3	345	2 ½	7 ½	32
4	600	2 ½	10	32
5	930	2 1/8	12 1/8	32
Cloth No. 8367---Edward H. Best & Co.				
1	20	2 ½	2 ½	32
2	50	2 ½	5	32
3	75	2 ½	7 ½	32
4	180	2 ½	10	32
5	525	4	14	32
Cloth No. 1642---Edward H. Best & Co.				
1	42	2 ½	2 ½	32
2	135	2 ½	5	32
3	300	2 ½	7 ½	32
4	540	2 ½	10	32
5	900	2 1/4	12 1/4	32
Heavy Weave Cloth---Edward H. Best & Co.				

Period	Time(sec)	Filtrate(lbs)	Total Filtrate(lbs)	Pressure
1	47	2 ½	2 ½	32
2	150	2 ½	5	32
3	320	2 ½	7 ½	32
4	550	2 ½	10	32
5	900	2 1/4	12 1/4	32
Cloth No. 1030---Edward H. Best & Co.				
1	47	2 ½	2 ½	32
2	155	2 ½	5	32
3	340	2 ½	7 ½	32
4	595	2 ½	10	32
5	910	2 1/4	12 1/4	32
Cloth No. 4555---Edward H. Best & Co.				
Temperature of slurry			19° C.	

Results:

Table XXXIII

Comparison of filtration rates of several kinds of filter cloths.

Cloth No.	Filtrate	# Filtrate	Time(sec)
Reg. Duck	Clear	12 1/4	900
3565	Cloudy at start.	12 1/4	905
5085	Clear	12 ½	920
8367	Cloudy ar start.	12 1/8	930
1642	Very cloudy.	14	525
Heavy Weave	Clear	12 1/4	900
1030	Clear	12 1/4	900
4555	Clear	12 1/4	910

Conclusions:

The idea in choosing a filter cloth is to select one which will allow the greatest amount of filtrate to pass through and at the same time be able to stand up under the chemical action of the filtrate and precipitate. The function of a filter cloth plays a very small part in the actual filtration cycle. The cloth affect enters only in the initial stages of filtration. The precipitate at the beginning of the cycle collects on the cloth and after a small amount has built up the precipitate acts as its own filter medium. The filter cloth is necessary through-out the filtration cycle, however, because it holds the precipitate in the chamber.

Concluding from table XXXVIII, we may say that the rate of filtration is affected very little by filter cloth used. In other words more attention could be spent to controling temperature and pressure than in selecting the filter medium. However, when the precipitate or filtrate affect the cloth chemically, then great care should be taken in the selection of the proper medium.

In only one case in the above table did the time vary greatly from the other filtering times. In this case the filter cloth was very porous and much of the precipitate passed through the cloth. It will be seen from table XXXVII that during the initial stages of filtration the filtering times varied somewhat. However, after the cake built up to where it was serving as the medium, filtration remained constant throughout the remainder of therun.

Probably the greatest variations occur, not because of the filter cloth, but because of slight differences in pressure or temperature control.

IX. Investigations of the washing characteristics of different slurries.

A. Slurry I.

Filter Press.

Procedure:

In carrying our the washing of a cake the filtration cycle was carried on as in all cases using the filter press (Part IV Sec. A). When the filtration cycle had been completed the valves on the water line were regulated so that the water was applied to one filter surface at the same pressure as that used in filtering. The water passed through the cake and filter cloths and into the grooves of the plate on the opposite side of the cake. The wash water then left the press through the discharge channel. At definite intervals of time samples of the wash water were taken and analyzed for NaCl. After the Na Cl had been reduced to 1 cc N/10 $AgNO_3$ per 10 cc of wash water the valves were closed.

Due to the cracking of these cakes, repeat runs were made, where, on the final few minutes of filtration the pressure was increased to 70#. Washing was then carried on at the same pressure as before.

Direct washing was also carried out, with apparently better results than those obtained by the counter flow wash. After the filtration cycle was complete the wash water was forced through the feed channel. Samples were taken as before.

Data:

Table XXXIX

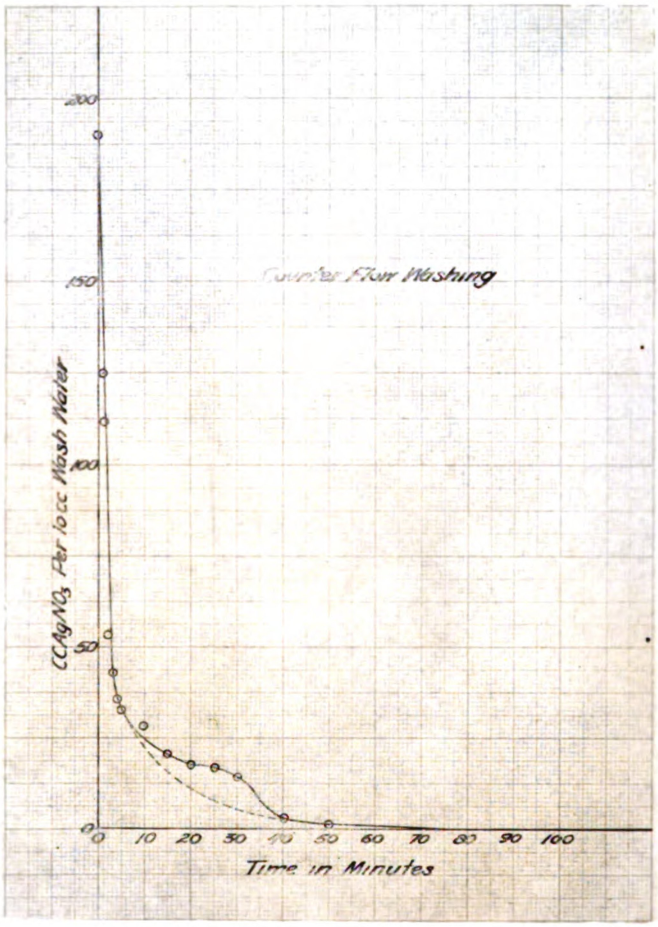
Counter Flow Wash.

Time (Minutes)	cc $AgNO_3$ per 10 cc of filtrate
0	194.
0.5	125.0
1.0	111.8
2	53.1
3	42.5
4	35.7
5	32.5
10	28.1
15	21.9
20	18.1
25	17.0
30	14.5
40	3.1
50	1.25
Temperature of water	19° C.
Weight of wash water	33 #
Weight of cake	3 12/16 #
Pressure	32 #/sq. in.
°/o Water	65 °/o

Plot time in minutes vs. cc N/10 $AgNO_3$ per 10 cc of wash water. (fig. 17)

Table XL

Figure 17



Counter Flow Washing
CC AgNO3 Per 10 cc Wash Water
200
150
100
50
0
0
10
20
30
40
50
60
70
80
90
100
Time in Minutes

Counter Flow Wash

Time (Minutes)	cc $AgNO_3$ per 10 cc Water.
0	184
5	105
10	73
15	61.0
20	43.0
25	37.5
30	33.0
40	26.5
50	19.0
60	14.2
70	9.0
80	6.8
90	5.6
100	4.3
110	3.7
150	1.0
Temperature of water	19° C.
Weight of wash water	28 7/16 #
Weight of cake	3 13/16 #
Pressure	32 #/sq. in.
°/o Water	62 °/o

Last five minutes of filtering cycle, the pressure was raised to 60 per. sq. in.

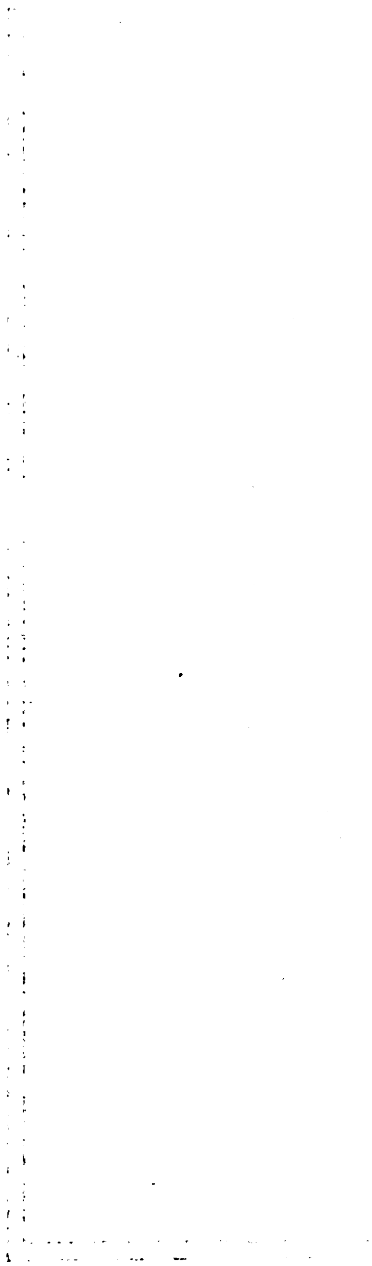
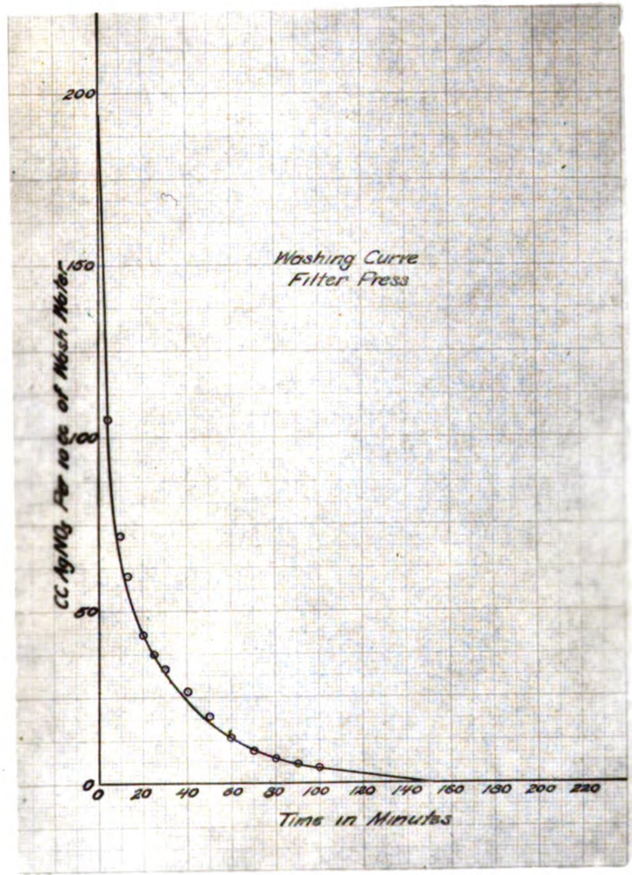


Figure 18


Washing Curve
Filter Press
CC AgNO3 Per 10cc of Wash Water
200
150
100
50
0
0
20
40
60
80
100
120
140
160
180
200
220
Time in Minutes

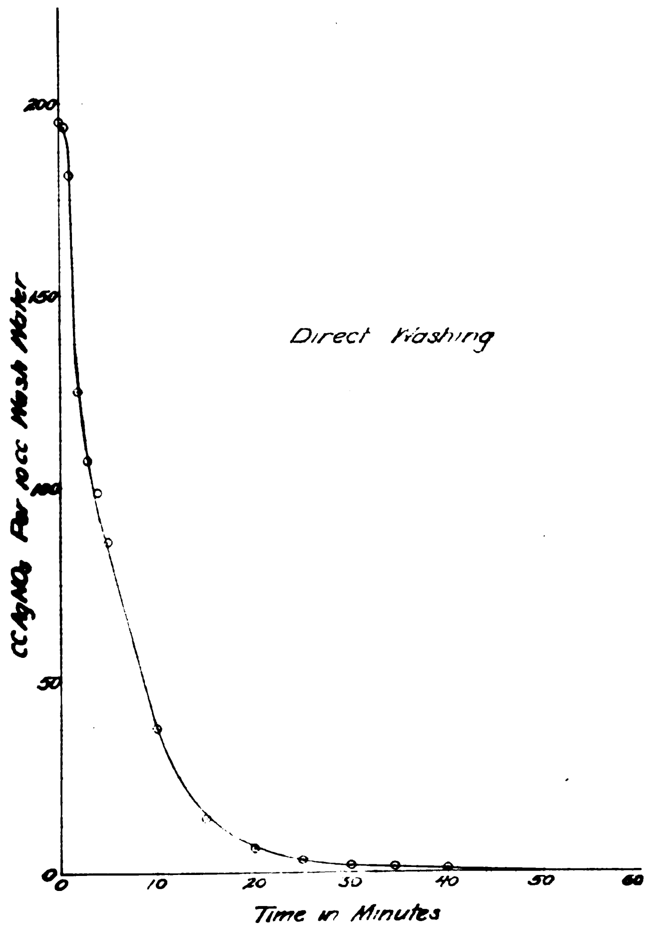
The data of table XXXVIII was then plotted. Time in minutes vs. cc$AgNO_3$ per cc 10 of wash water.

Table XLI Direct Washing

Time (minutes)	cc $AgNO_3$ per 10 cc Water
0	195
0.5	194
1	181
2	124
3	107
4	99
5	87
10	38
15	14
20	6.0
25	3.6
30	1.5
35	0.5
Temperature of water	18 ° C.
Weight of wash water	20#
Weight of cake	3 12/16#
Pressure - #/sq. in.	32
°/o Water	65 °/o

The data in table XLI was then plotted, time in minutes vs. cc N/10 $AgNO_3$ per 10 cc of wash water. (fig. 19)

Figure 19


Direct Washing
CC AgNO3 Per 10cc Wash Water
Time in Minutes
200
150
100
50
0
0
10
20
30
40
50
60

Suction Filter:

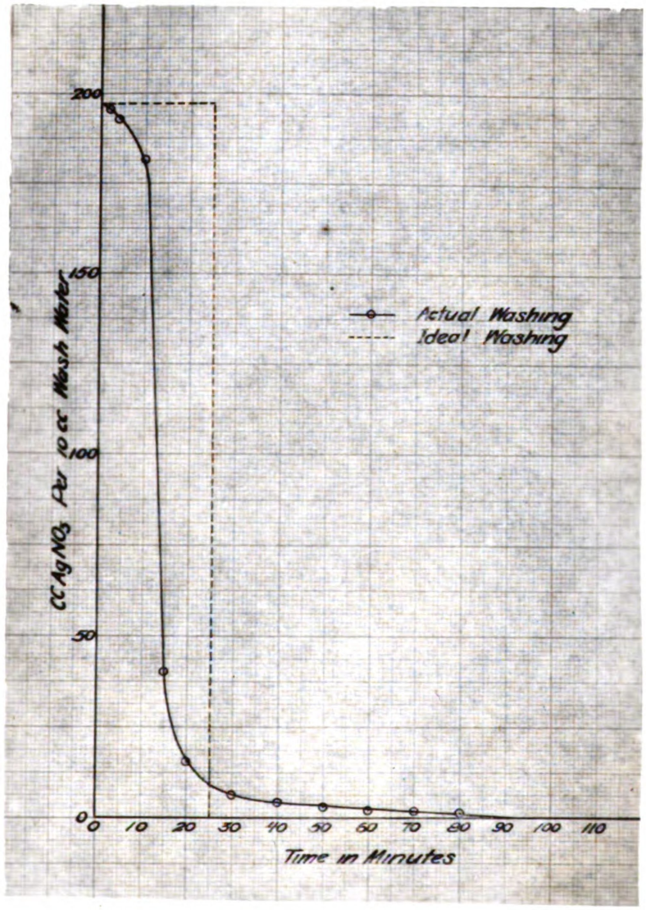
The procedure in filtering with a suction filter is described in part IV - sec. B. After the washing cycle was completed, a large quantity of wash water was poured over the cake and the vacuum kept the same as was used in filtering. At regular intervals of time the bottle which was attached to the drain valve was removed and wash water dumped. The bottle was immediately replaced, valved turned to take sample, removed and another bottle put in its place.

Taking samples in this way is rather inaccurate in as much as the pressure is affected by the removal of the bottle because the pressure has to be built up in the bottle each time it is removed. Then too, the samples collected will not be truly samples at the moment. Due to the time elapsed between the time of dumping the wash water and taking samples, the samples will be a little cumulative. However, for ordinary practice satisfactory results can be obtained.

Table XLII

Time(minutes)	cc $AgNO_3$ N/10 per 10 cc Water
0	197
1	197
2	191.5
3	190.5
4	189.5
5	187.0
10	179.5

Figure 20



CC AgNO3 Per 10 cc Wash Water
200
150
100
50
0
0
10
20
30
40
50
60
70
80
90
100
110
Time in Minutes
Actual Washing
Ideal Washing

Table XLII Continuted

Time (minutes)	cc $AgNO_3$ per 10 cc of Water
15	40
20	16
25	10
30	6.25
40	4.375
50	2.8
60	1.88
70	1.25
80	0.95
Temperature of water	19° C
Weight of wash water	32 1/8 #
Cake thickness	3/4"
Weight of cake	6 #
°/o Water	63 °/o

The data of table XLII was then, plotted; time in minutes vs. cc $AgNO_3$ per 10 cc of wash water. (fig. 20)

B. Slurry II

Filter Press.

Procedure:

Washing of a filter cake using slurry II was carried out in the same way as when slurry I was used; that is, section A of this part. The same difficulties were also encountered in washing, slurry II being almost impossible to wash without cracking.

Data:

Table XLIII Counter Flow Wash

Time (minutes)	cc $AgNO_3$ per cc (10) Wash
0	185.0
0.5	82.5
1	75.5
2	73.5
3	71.0
4	68.0
5	56.0
10	2.9
15	1.0
Temperature of slurry	19° C.
Weight of cake	3 3/4 #
Weight of wash water	14 5/16 #
Pressure	32 #/Sq in.
°/o Water	41.4 °/o

Plotting time in minutes vs. cc N/10 $AgNO_3$ per 10 cc of wash water. (fig. 21)

Suction Filter.

The procedure in this case is the same as in section A of this part.

Data:

Table XLIV Continuous Washing

Time (minutes)	cc N/10 $AgNO_3$ per 10 cc of wash
0	193.5

Figure 21

Figure 21

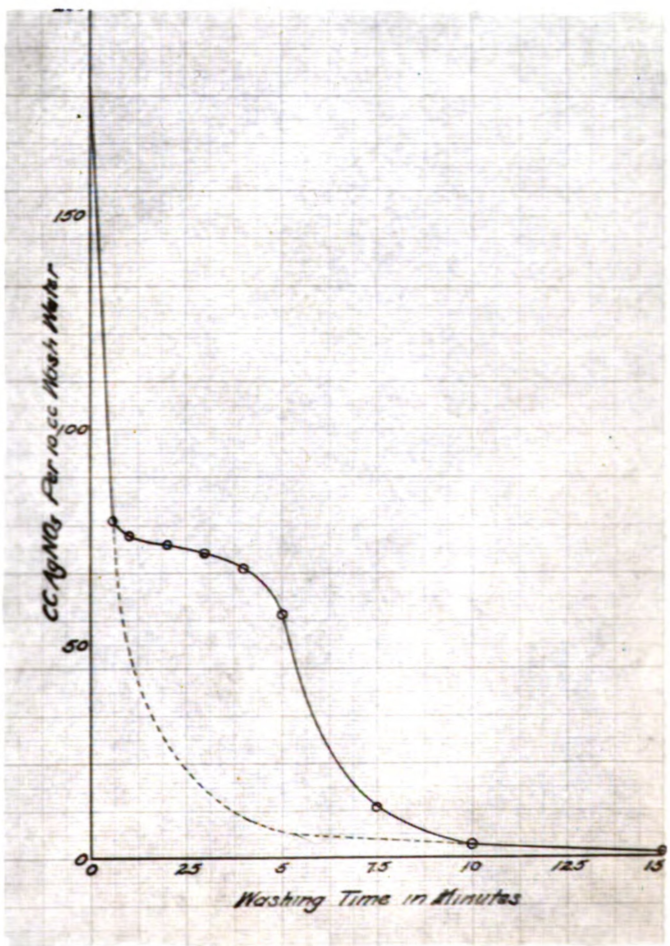

150
100
50
0
0
2.5
5
7.5
10
12.5
15
CC AgNO3 Per 10 cc Wash Water
Washing Time in Minutes

Table XLIV

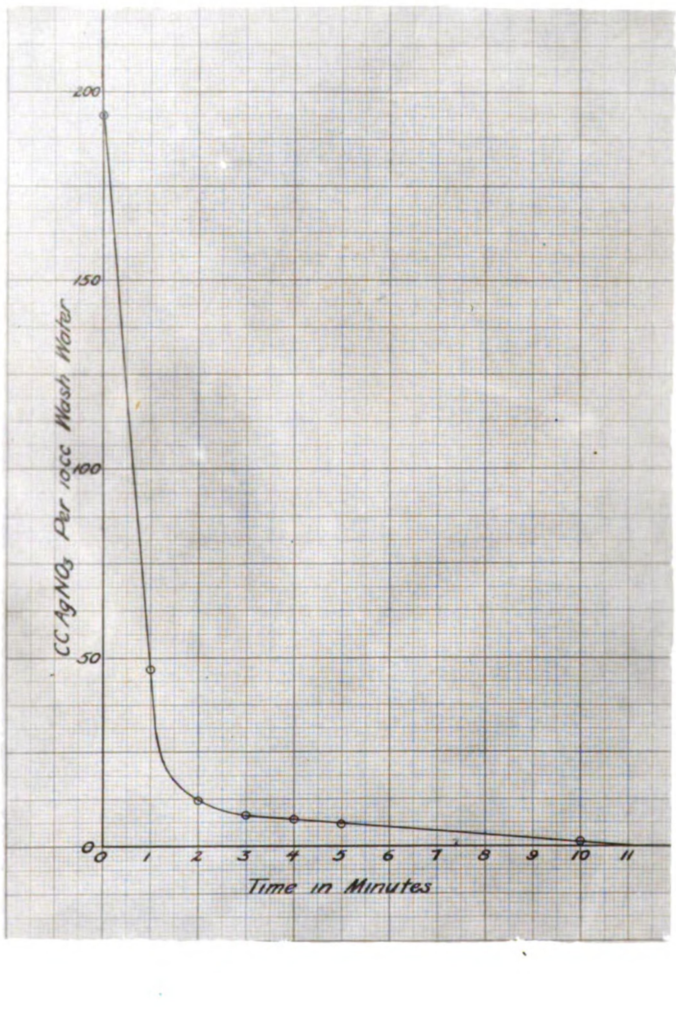
Time (minutes)	cc N/10 $AgNO_3$ per 10 cc of Wash
1	46.0
2	12.0
3	8.0
4	7.0
5	5.4
10	1.0
Temperature of Wash	18° C
Weight of cake in pounds	7 3/4#
Weight of wash water in pounds.	45#
Pressure	9.8 #/sq. in.
°/o Water	42.3

Plotting time in minutes vs. cc N/10 $AgNO_3$ per 10 cc of wash water. (fig. 22)

Discussion:

It was almost impossible to get satisfactory results by the counter flow method of washing, using the filter press. This is not surprising, however, because when the cake is being filtered the cake builds up on each side, from the center. When washing is started the flow of water throught half of the cake is in the opposite direction, thus there is a great strain upon that half of the cake. Unless the cake is quite solid and uniform this half of the cake is very likely to crack, allowing the water to reach the center of the cake and only properly washing one half of the cake. That is, the half throughh

Figure 22


200
150
100
50
0
CC AgNO3 Per 10cc Wash Water
0
1
2
3
4
5
6
7
8
9
10
11
Time in Minutes

which the wash water passes in the same direction as did the filtrate.

By referring to fig. 17 it will be seen that this very thing happened. When the water was first turned on the cake cracked and the filtrate in the pours of the opposite half of the cake was forced out. However, at the end of about five minutes the water seems to be gradually absorbing the Nacl from the cracked cake and the line flattened out some what. It is the absorbing of filtrate from the cracked half cake which prevents the graph from following the course of the dotted line. Finally all of the Nacl was removed.

Cakes washed as represented in fig. 17 were then tested for Nacl. In all cases very little Nacl was left in the cake. Although all the Nacl may be removed, it does not mean that satisfactory results have been obtained. The amount of wash water as well as the washing time are increased.

In an attempt to overcome this cracking of the cake many experiments were tun. Most of these trials were tried by differences in pressure of filtering and washing. Finally satisfactory results were obtained by increasing the pressure during the last five minutes of the filtering cycle to 60# /sq.in. Washing of the cake was then carried on at 25# to 33# .

Raising of the filtering pressure to 60# packed the cake so that it was more or less uniform throughout. When the washh water was gradually applied to the cake, displacement gradually took place, which gave a very smooth curve. (ffg. 18) Again, this method of washing is far from satisfactory. As will be seen in fig. 18; in order to wash the cake to 1 cc of N/10 $AgNO_3$ per 10 cc of wash water 145 minutes are required. The pours of the cake are reduced in size by the increased pressure of filtration and the washing rate is decreased.

There are also disadvantages in direct washing. That is, washing through the feed channel. When a cake is filtered, that portion of the cake near the inlet forms last. Then it is reasonable to expect that the cake will be less compact in this corner. When the wash water is turned on there will be a tendency for the wash water to pass through this corner of the cake. If the water finds no difficulty in passing through this portion of the cake, it will not reach the opposity corners of the cake which are more compact. In other words, channeling occurs and the cake is not properly washed.

Reference to fig. 19 shows peculiarities, which are probably due to channeling. The line of the graph comes down regularally, which shows proper washing for about three minutes. At this point the line takes a rather sharp jog which indicates channeling. That is, water seems to be coming through the cake some place which does not take up any Nacl. If channeling had not taken place, the curve would have been more regular like that of fig. 20.

Table XLV Comparison of Washing times

Type of Washing	Wt. of Wash Water(lbs)	Wash Time(min.)
Counter Flow	28 ½	55
Counter Flow (special wash)	33	150
Direct Flow	20	35

Washing with the suction filter is a much easier operation then that of the filter press, especially with this type of slurry (slurryI) The cake is washed in 80 minutes at a much lower pressure than that used with the filter press. The washing approaches that of ideal washing, as shown in fig. 20.

In general, washing consists of two operations, namely, first replacing filtrate from the pours of the cake, and second, gradual elimination of absorbed chloride from the cake during the last portion of the wash. This is illustrated very well in fig. 20. The Nacl is being removed from the pours of the cake for the first twenty or twenty two minutes. At this point the Nacl in the pours has been removed and the remainder of the wash water is spent in absorbing the Nacl from the cake.

The results obtained from slurry II were similar to those of slurry I. However, I was unable to obtain satisfactory data with the filter press on a cake which did not crack.

X. Conclusions:

Up to the present time, with possibly one exception, there is no mathematical equation for treating filter data which is satisfactory for industrial practice. This one possible exception is the work "Studies in Filtration" by B. F. Ruth which is not yet entirely published. Of the work which has been published (4),(5) I have been unable to make a thorough investigation.

Types of experiments to be carried out in filtration depends greatly upon the type of slurry. The general behavior of a certain slurry may be determined with an ordinary laboratory suction filter. However, it would not be wise to base calculations upon such data because of the smallness of equipment and small quantity of slurry, there is great chance for error.

The generally accepted filtration equations today are those proposed by Lewis (1), (2). However, D. R. Sperry has presented equations which constitute another group of filtration equations (3). and differ from the equations of Lewis only in the manner in which the filter base resistance is treated. Sperry's treatment leads to a more formidable equation then does Lewis'.

By referring to part II of this thesis it is seen that the equations of Lewis are very difficult to understand, and their derivations very complicated.

The validity of these equations is illustrated in part V above. The filtering times as calculated by the equations vary from 18 °/o to 21 °/o from the actual filtering times. Great care being used in carrying out the filtration while data was being collected. An equation to be of industrial use should not show calculated results varying as much as 20 °/o from the actual values.

Of the two equations I have suggested (part VI and part VII) probably the latter is more satisfactory. Equation (1) of part VI seems to hold fairly well for the slurries used in this work, but which might not hold for other slurries. The derivation of this equation is not based on mathematical calculations. Rather it is based on observations and results I have obtained in my work. In all cases I observed that the time to filter the final one inch of any thickness cake varied as the first power of the thickness.

Equation (5) of part VII proves very satisfactory. The calculated values by this equation.varying only from 2 °/o to nearly 3 °/o (table 38). This variation could easily be within experimental error.

The problem of selecting a filter medium is not so much selecting one which will allow the greatest amount of filtrate to pass through, but how long will a certain medium stand up using a particular slurry. The filter medium problem varies, however, depending upon the slurry being filtered.

Washing of a filter cake is a greater problem than is generally believed. The washing technique changes every time the filtering technique changes, Cracking of cake is a common difficulty in washing, as is channeling of a cake. Whether or not counter flow or direct washing is used, depends upon two factors. When cracking occurs direct washing is some times used. However, in case channeling occurs with direct washing the cake is compressed by special equipment, and then washed by counter flow.

Filtration as applied to most slurries is a large problem. There are so many variables involved that results which check are difficult to obtain. For experimental work the technique of filtering can be developed by practice and great care.

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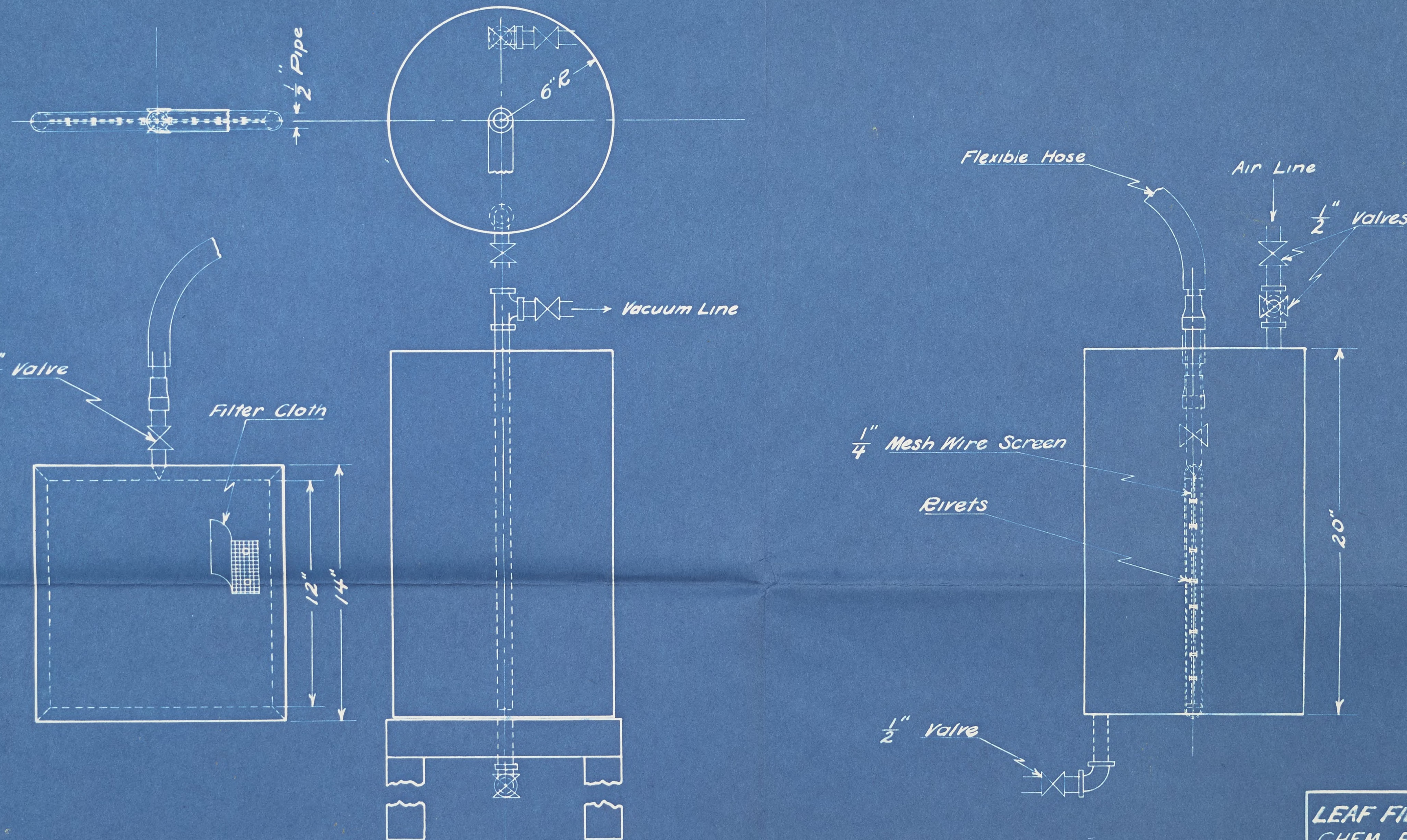

fig. 4

LEAF FILTER APPARATUS
CHEM. ENG. LABORATORY
MICH. STATE COLLEGE
EAST LANSING

DRAWN BY: H.T.W.	
TRACED BY: H.W.	SCALE: 3/16" = 1"
CHECKED BY: H.S.R.	DATE: MAY 1933

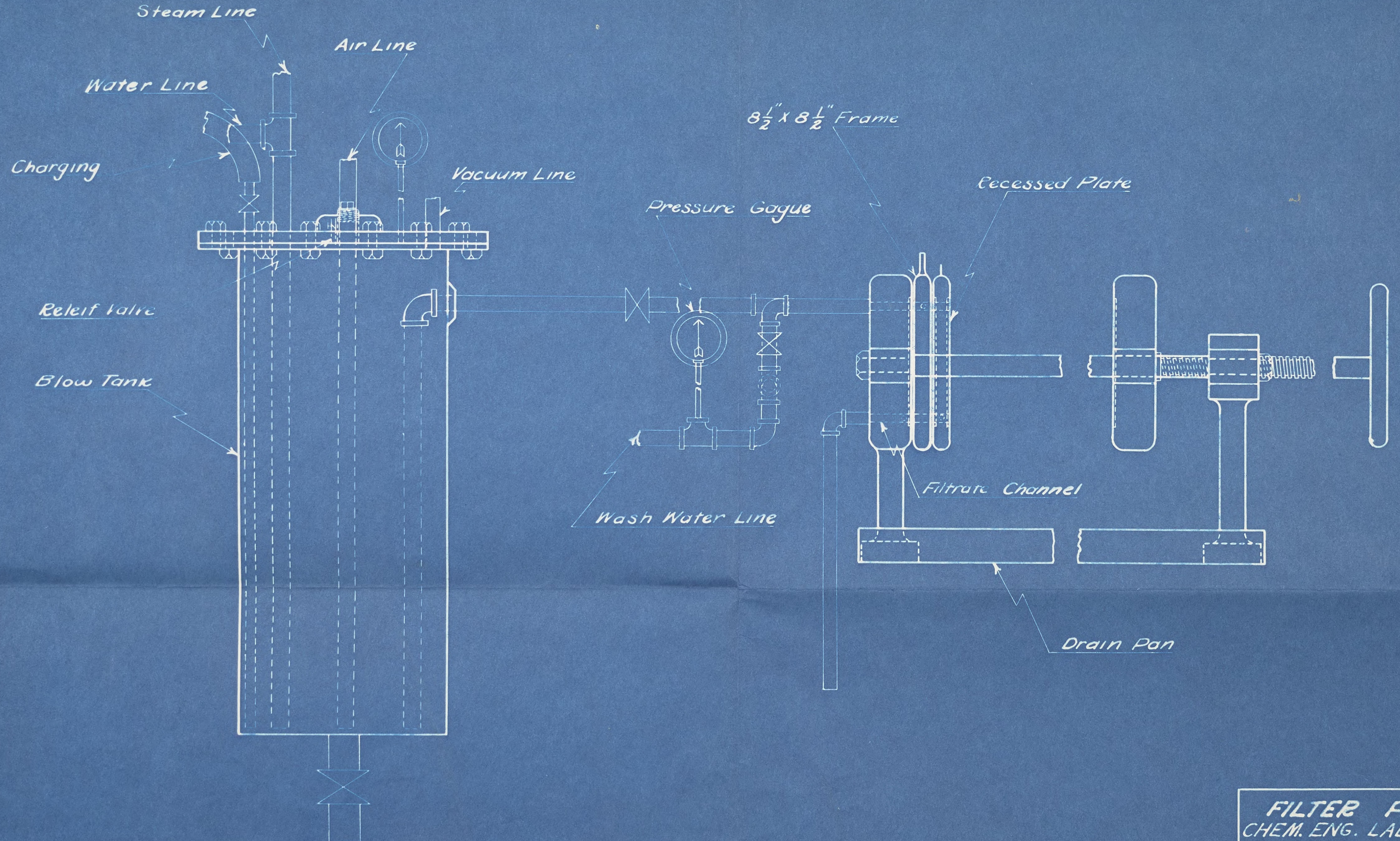

fig. 1

FILTER PRESS CHEM. ENG. LABORATORY MICH. STATE COLLEGE EAST LANSING	
DRAWN BY: H.T.W.	
TRACED BY: H.W.	SCALE 3/16" = 1"
CHECKED BY: A.S.E.	DATE - 5/15/33

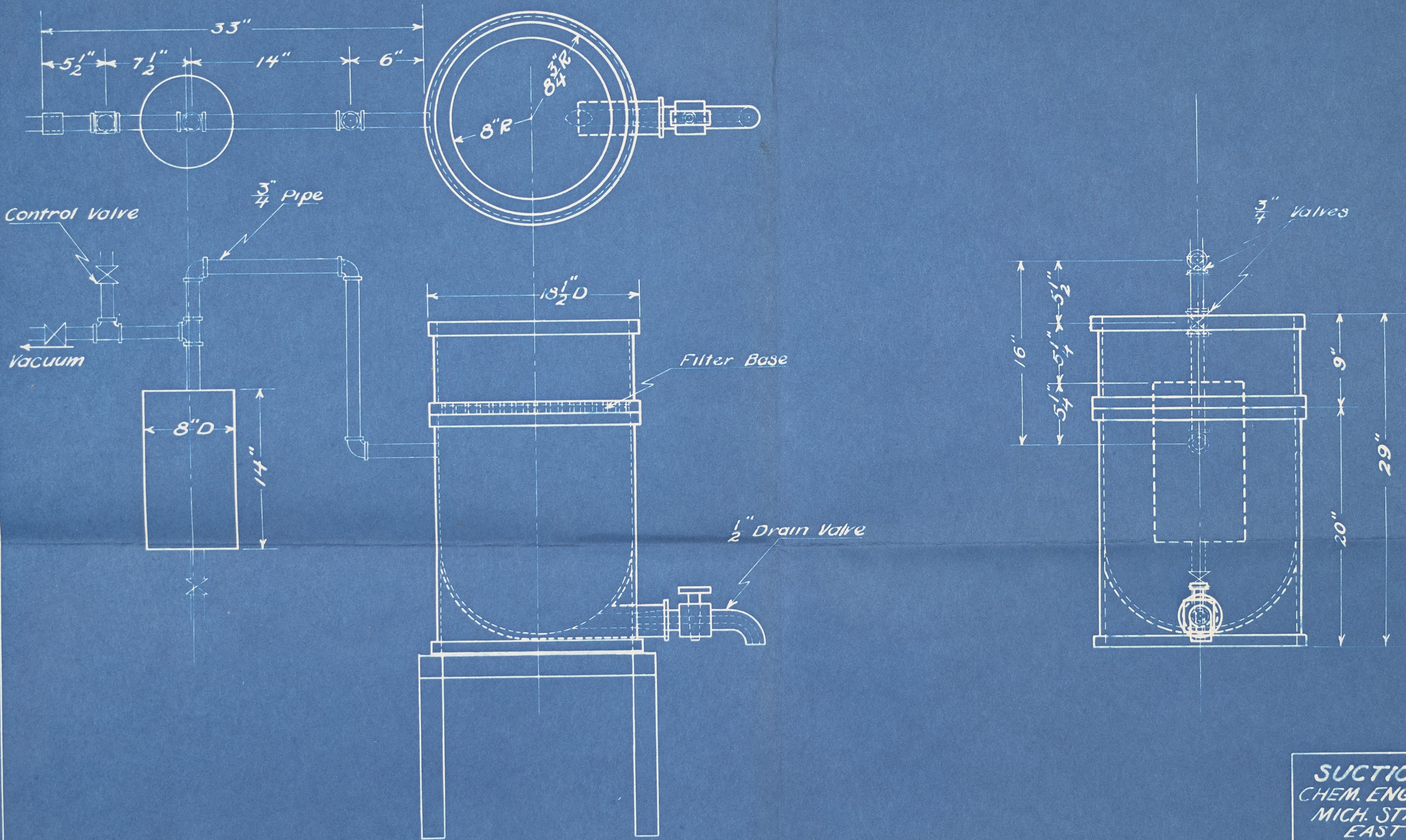

fig. 3

SUCTION FILTER
CHEM. ENG. LABORATORY
MICH. STATE COLLEGE
EAST LANSING

DRAWN BY: H.T.W.	
TRACED BY: H.W.	SCALE: $\frac{1}{8}$" = 1"
CHECKED: H.S.R.	DATE: 5/15/33

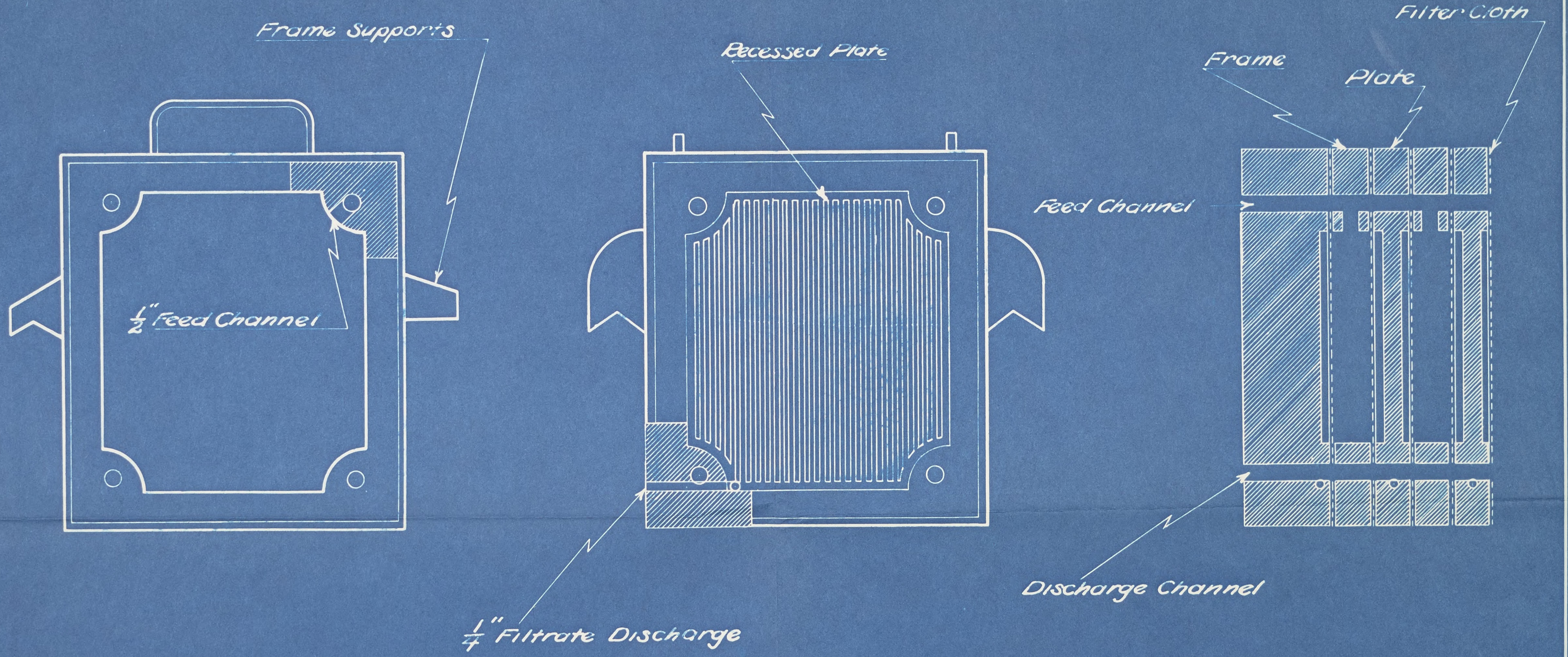

fig. 2

PLATE and FRAME
Chem. Eng. Laboratory
Mich. State College
East Lansing

DRAWN BY - H.T.H.	
TRACED BY - H.T.H.	SCALE - $\frac{3}{8}$" = 1"
CHECKED BY: H.S.R.	DATE - 5/15/33

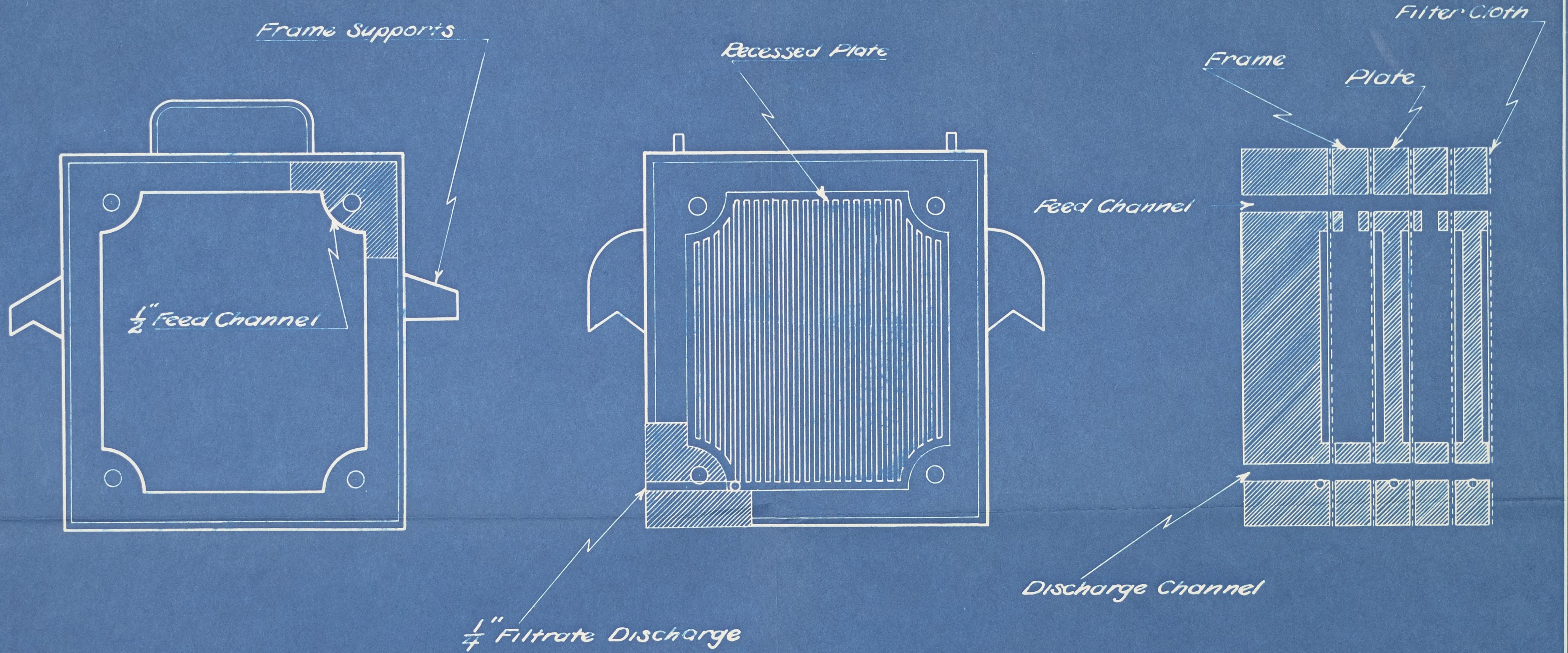

fig. 2

PLATE and FRAME
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CHECKED BY: M.S.R.	DATE - 5/15/33