

THE APPLICATION OF NOMOGRAPHIC
CHARTS FOR THE SOLUTION OF
VARIOUS ENGINEERING PROBLEMS

THESIS FOR THE DEGREE OF E. E.

Harold C. Roberts

1934

THESIS

MICHIGAN STATE UNIVERSITY LIBRARIES



3 1293 00628 6037

25994790

Title nomographic charts

SUPPLEMENTARY
MATERIAL
IN BACK OF BOOK

Electrical engineering

**THE APPLICATION OF NOMOGRAPHIC CHARTS
FOR THE
SOLUTION OF VARIOUS ENGINEERING PROBLEMS**

**A THESIS SUBMITTED TO THE FACULTY
OF
MICHIGAN STATE COLLEGE
OF AGRICULTURE AND APPLIED SCIENCE**

**BY
HAROLD C ROBERTS**

**Candidate For The Professional Degree Of
Electrical Engineer**

June 1934

THESIS

**SUPPLEMENTARY
MATERIAL
IN BACK OF BOOK**

HISTORICAL DATA

103439

The history of the theory of equations takes us back to the year 300A.D. when Diophantos, who has been called the "Father of Algebra", developed the use of abbreviations and symbols for Algebra. However the history of graphical representation of functions goes back six hundred years earlier to the time of Euclid, who represented certain functions by constructing graphs in the form of rectangles.

Later in history, about 1500 A.D., in the writings of Bhaskara, a Hindoo scholar, we find the first steps taken toward constructing graphs of equations when positive and negative numbers were represented by opposite segments of a straight line. From this conception, it was but a step more to graph the relationship of one variable to another by measurements along two lines. This was done by Nicholas Horem, a French teacher, in the fourteenth century. At that time negative numbers were considered fictitious, so the graphs of Horem were constructed in one quadrant only.

At the beginning of the seventeenth century, Rene' Descartes laid the foundation for graphical representation of functions in all quadrants. Descartes was a French philosopher and mathematician, born at La Haye in 1596. He adapted the system of locating points in a plane by means of their distances from two fixed lines, and introduced the terms "ordinate" and "coordinate". The term "abscissa" was first used by an Italian writer about 1659.

The Nomograph, or alignment chart as it is sometimes called, is a special type of graph generally used to solve certain equations which would require considerable time if treated by mathematical analysis. This type of graph also originated with the French mathematicians. A most comprehensive treatise on the subject was published by Professor Maurice d' Ocagne of Paris.

In the field of Engineering a limited number of formulae are used so repeatedly that any means of conserving time and effort in their application is desirable. The nomograph is readily adaptable to this purpose since its scales are so easily constructed and read. Of course, the nomograph is not to be used as an exact solution, since the accuracy of the graphical solution depends upon the precision of the diagram; however, the results so found may be regarded as a very close approximation and generally are within the limits of accuracy required in practical engineering.

GENERAL APPLICATION OF GRAPHS

Graphs generally are constructed and used to present a picture of the variations of a given function, or the relationship of one function to that of another.

Functions of two variables are generally plotted as in Fig. 1 and functions of three variables, as in Fig. 2.

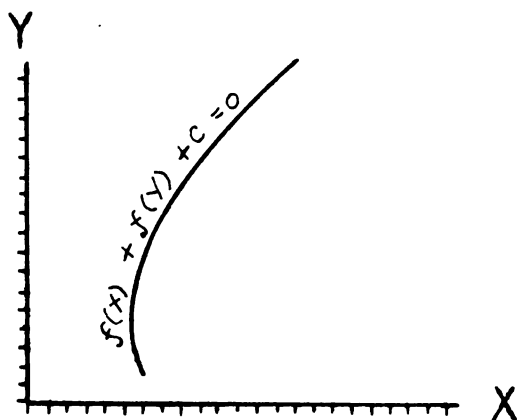


Fig. 1

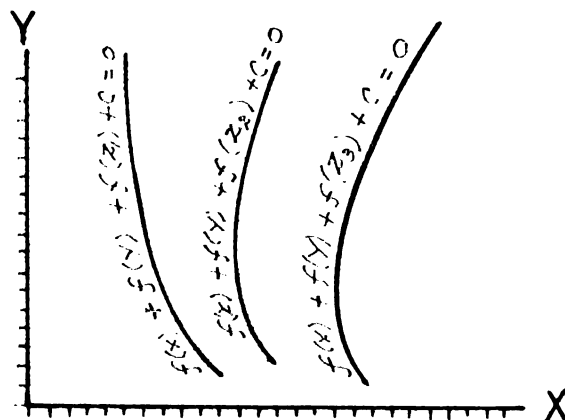
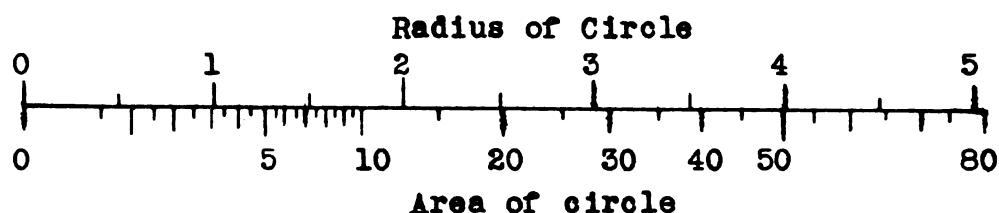


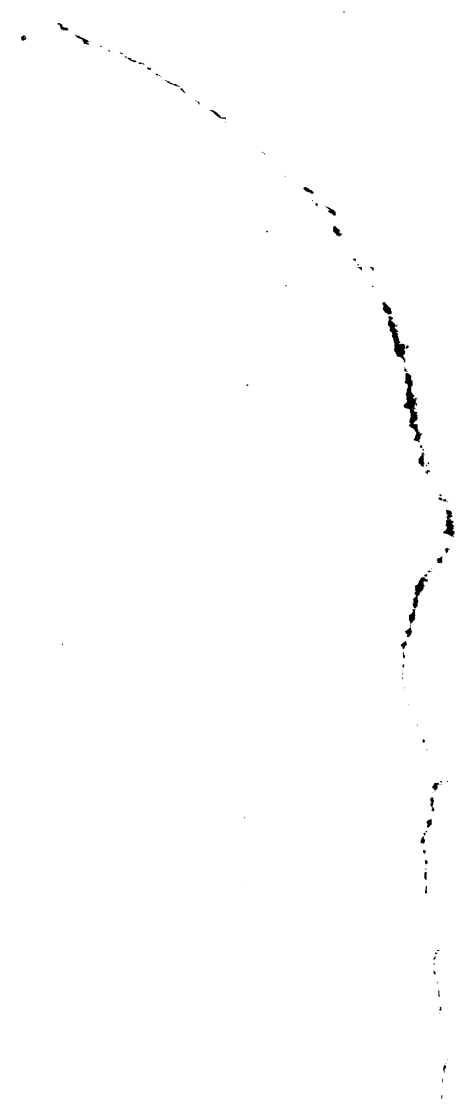
Fig. 2

To read the values of a point on these curves requires interpolation in two directions i.e. along the coordinate axes. This is generally mentally fatiguing especially if several readings are to be made and the divisions on the axes are small, in which case, errors are easily made.

A function of two variables may be graphed in scale form where interpolation is accomplished easily. As an example, we will construct a small scale showing the relationship of the radius of a circle and the area of the circle:



1000000



Such scales are quickly made, easily read, and where used often, are decidedly advantageous over the type of graphs represented by figures 1 and 2 (page 5).

SCALES

Throughout this thesis the term, SCALE, will be frequently used, and in each case it refers to a graphical scale. Lipka defines a graphical scale as "a curve or axis on which are marked a series of points or strokes corresponding in order to a set of numbers arranged in order of magnitude."

In the construction of nomographs, the scales are generally non-uniform, depending upon the type of equation and its degree. In the following work equations will be developed for plotting the scales and will be expressed in terms similar to a Cartesian coordinate equation with reference to a given origin.

A factor of major importance in constructing the scale is the scale modulus to be used for each variable. We may define the SCALE MODULUS as a number (or fraction), which, when applied to the $f(X)$ we wish to plot, will determine the distance along the ordinate or abscissa for plotting the various values of X . In the subsequent work this distance will be in inches.

For instance suppose:

$$f(X) = 6 X^2 + 10$$

Assume X varies from 0 to 5, and that the scale is to be 10 inches long.

The scale modulus would be the scale length divided by the largest value of $f(X)$, which in this case would be:

$$\frac{10}{f(5)} = \frac{1}{16}$$

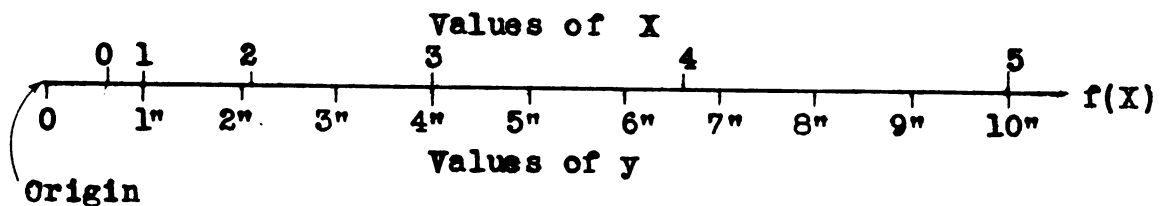
If we plot this $f(X)$ along the "Y" ordinate, the plotting equation is:

$$y = (\text{Modulus}) f(X) = \frac{1}{16} (6X^2 + 10) \text{ inches.}$$

Substituting values for X in $f(X)$, gives

$X =$	0	1	2	3	4	5
$y =$	$5/8"$	$1"$	$2 - \frac{1}{8}"$	$4"$	$6 - \frac{5}{8}"$	$10"$

Plotting these values produces a scale like the following:



The scale is calibrated much the same as a slide rule, i.e. instead of marking the value of the $f(X)$, the value of X is marked off at the point corresponding to the value of $f(X)$. NOTE that the origin of the scale does not always coincide with the zero value of the variable X , however, in most cases it will or can be made to correspond by rearranging the equation unless the scale is logarithmic.

THEORY OF THE
NOMOGRAPHIC CHART

At the present time there are several good text books published which treat of the construction of nomographs. Each text adapts a different geometrical solution for the plotting equations; however, the final results by each method are identical. Rather than attempt a new geometrical solution for these equations we shall apply a mathematical interpretation which has the virtue of flexibility in arranging the variables.

In an attempt to compare a graph constructed with Cartesian coordinates and a nomograph for the solution of the same equation, let us assume:

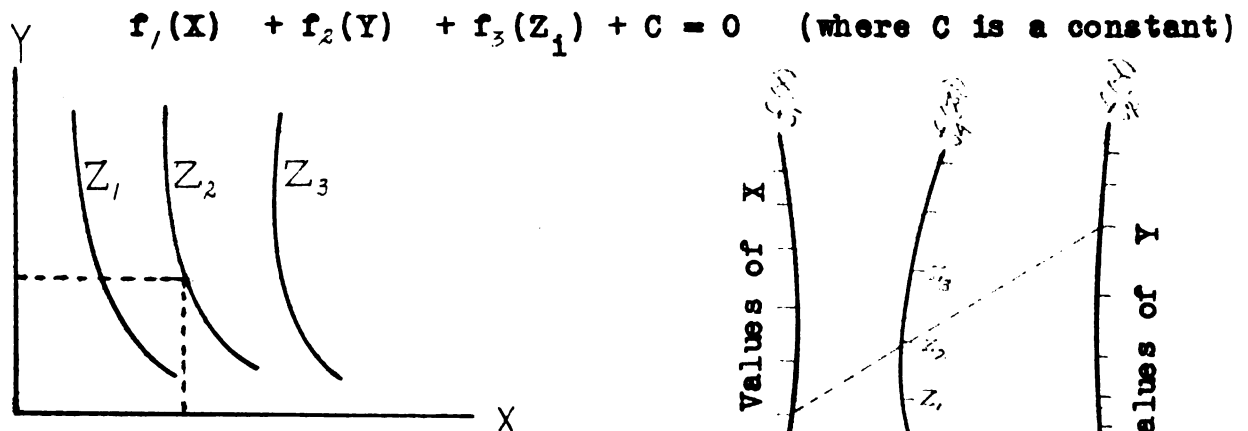


Fig. 1
Cartesian coordinates

Fig. 2
Nomograph

It appears that in Fig. 2 the scale of X and Y in Fig. 1 are rearranged and that the curves Z_1 , Z_2 and Z_3 become points on a third scale $f_4(Z_1)$ located somewhere between $f_1(X)$ and $f_2(Y)$. The plotting equations are developed to locate the position of these scales and to give their calibration. The scales $f_1(X)$ and $f_2(Y)$ are so calibrated that when a given value of X and Y on their respective scales are joined with a straight edge, the correct value of Z on the $f_4(Z_1)$ scale may be read.

Let us consider the solution of these plotting equations for the different scales by means of determinates.

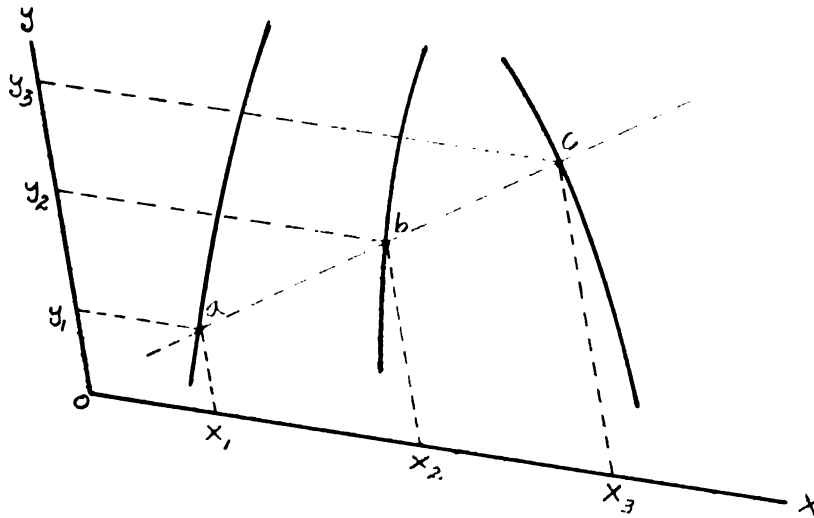


Fig. 3

Referring to the above Fig. 3; if a, b and c are in a straight line, then

$$1. \quad \frac{x_2 - x_1}{y_2 - y_1} = \frac{x_3 - x_2}{y_3 - y_2}$$

Or clearing fractions,

$$2. \quad x_1 y_2 - x_1 y_3 - x_2 y_1 + x_2 y_3 + x_3 y_1 - x_3 y_2 = 0$$

Equation 2 may be set up in determinate form, as

$$3. \quad \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} = 0$$

The solution of an equation by a nomograph requires that three specified points that satisfy the equation must be in a straight line. Examining this determinate (equation 3) we see that x_1 and y_1 refer to scalar distances. In a preceding part of this work, covering scales, it was shown how to construct a scale for the $f(X)$ expressed in scalar distances. So referring to the determinate (equation 3) we may replace x_1 and y_1 by the functions they represent. In other words,

$$x_1 = m_1 f(X) \quad \text{and} \quad y_1 = M_1 F(X)$$

where: X is any variable.

m_1 is the scale modulus along the x axis.

M_1 is the scale modulus along the y axis.

Referring to Fig. 3, we may consider "a" as a point in one scale, "b" a point in a second scale, and "c" a point in a third scale. Hence, if we can write the equation in determinate form of the third order so that only one of its variables will be in each row, a nomograph can be constructed for the solution of the equation, providing the value of the determinate is equal to zero. Throughout this discussion, only third order determinates are considered.

Assume the variables to be U , V and W . We may then write equation 3 in this manner:

$$4. \quad \begin{array}{c} (x) \\ 1 \quad m_U f(U) \\ 1 \quad m_V f(V) \\ 1 \quad m_W f(W) \end{array} \quad \begin{array}{c} (y) \\ M_U F(U) \\ M_V F(V) \\ M_W F(W) \end{array} \quad = 0$$

This is a general type of determinate which would produce plotting equations for curved scales. These equations would be:

$$U \begin{cases} y = M_u F(U) \\ x = m_u f(U) \end{cases} \quad V \begin{cases} y = M_v F(V) \\ x = m_v f(V) \end{cases} \quad W \begin{cases} y = M_w F(W) \\ x = m_w f(W) \end{cases}$$

Where: x is the distance from the origin along the x axis.
 y is the distance from the origin along the y axis.

If a determinate is equal to zero, we may perform the following operations without changing the value of the determinate.

1. Interchange any two columns.
2. Interchange any two rows.
3. Add any two columns or any two rows.
4. Multiply any column, or row, by a number other than zero.
5. Divide any column, or row, by a number other than zero.

Generally an equation of three variables, $f(U,V,W)$, can be expressed in one of the following forms:

$$5. \quad f_1(U) + f_2(V) + f_3(W) + C = 0$$

Where C = a constant.

$$6. \quad f_1(U) \cdot f_2(V) + f_3(W) + C = 0$$

If an equation contains more variables than three, it can be reduced to three variables by the introduction of a quantity that represents the sum of the excess variables. For instance

assume the equation to be:

$$f_1(U) + f_2(V) + f_3(W) + f_4(X) + C = 0$$

Let: $f_3(W) + f_4(X) = f_5(Y)$

We may then construct a nomograph for the solution of this equation, and a second nomograph for the solution of the equation:

$$f_1(U) + f_2(V) + f_5(Y) + C = 0$$

These nomographs generally can be combined so that $f_5(Y)$ scale is used as a TURNING SCALE and does not require calibrating. In like manner, equations containing several variables can be solved by using turning scales and combining the nomographs.

We will now proceed to develop the plotting equations for the scales of each variable. First consider the type equation:

5. $f_1(U) + f_2(V) + f_3(W) + C = 0$

This can be set up in determinate form, as

$$7. \quad \begin{array}{cc} (a) & (b) \\ \left| \begin{array}{ccc} 1 & 0 & -f_1(U) \\ 0 & 1 & -f_2(V) \\ 1 & 1 & f_3(W) + C \end{array} \right| & = 0 \end{array}$$

Let: the modulus of the scale for $f_1(U) = M_u$

" " " " " " $f_2(V) = M_v$

" abscissa distance between the zeros of the above two scales = D inches.

In determinate 7 :

Multiply column (a) by $1/M_u$

" " (b) " $1/M_v$

We then have: (by using the operations stated on page 14)

$$8. \begin{vmatrix} 1/M_u & 0 & -f_1(U) \\ 0 & 1/M_v & -f_2(V) \\ 1/M_u & 1/M_v & f_3(W) + C \end{vmatrix} = \begin{vmatrix} 1 & 0 & -M_u f_1(U) \\ 0 & 1 & -M_v f_2(V) \\ 1/M_u & 1/M_v & f_3(W) + C \end{vmatrix} =$$

$$\begin{vmatrix} 1 & (b) & 0 & -M_u f_1(U) \\ 1 & 1 & -M_v f_2(V) \\ 1 & \frac{M_u}{M_u + M_v} & \frac{M_u M_v}{M_u + M_v} [f_3(W) + C] \end{vmatrix} = 0$$

Now multiply column (b) of the above determinate by D , and we have:

$$9. \begin{vmatrix} 1 & (x) & 0 & (y) & -M_u f_1(U) \\ 1 & D & -M_v f_2(V) \\ 1 & \frac{M_u D}{M_u + M_v} & \frac{M_u M_v}{M_u + M_v} [f_3(W) + C] \end{vmatrix} = 0$$

The plotting equations are:

$$\text{For } U \begin{cases} y = -M_u f_1(U) \text{ inches.} \\ x = 0 \text{ inches.} \end{cases}$$

$$\text{For } V \begin{cases} y = -M_v f_2(V) \text{ inches.} \\ x = D \text{ inches.} \end{cases}$$

$$\text{For } W \begin{cases} y = \frac{M_u M_v}{M_u + M_v} [f_3(W) + C] \text{ inches.} \\ x = \frac{M_u D}{M_u + M_v} \text{ inches.} \end{cases}$$

This will produce a nomograph of three parallel scales, such as the following figure:

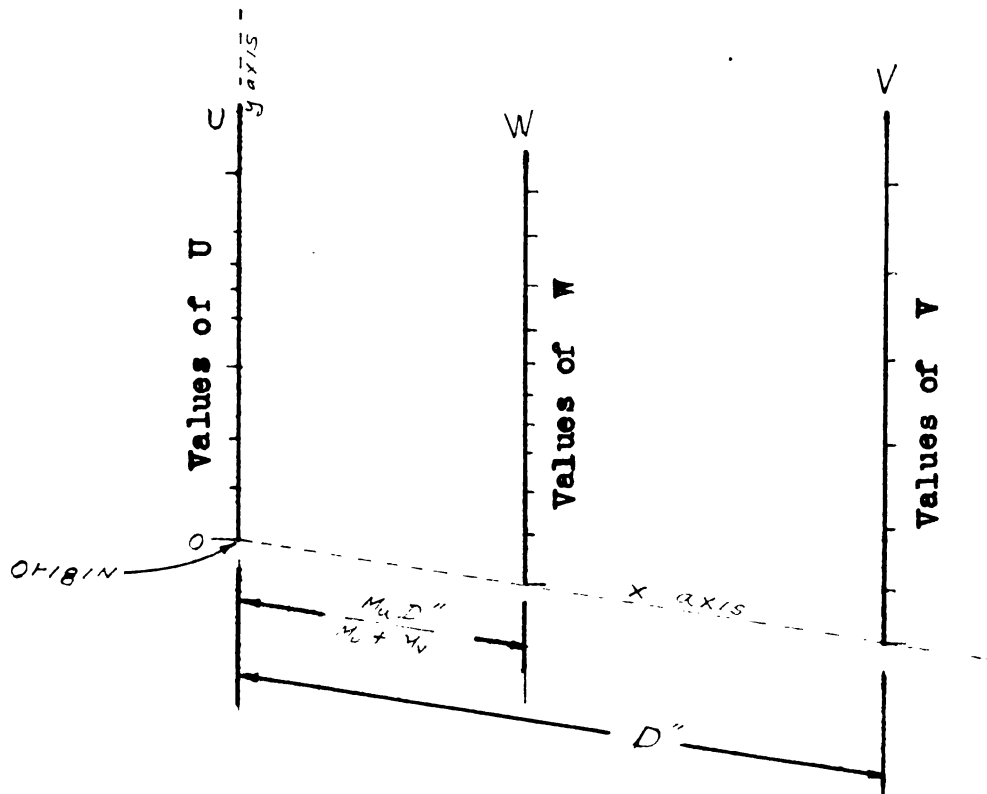


Fig. 4

We will now consider the type equation, as

$$6. \quad f_1(U) \cdot f_2(V) + f_3(W) + C = 0$$

Equation 6 can be set up in the following determinate form:

$$10. \quad \begin{array}{cc|c} (a) & (b) & \\ \hline 0 & 1 & -f_1(U) \\ 1 & 0 & -f_3(W) \\ 1 & f_2(V) & C \end{array} = 0$$

Let: the modulus of $f_1(U) = M_u$

" " " $f_3(W) = M_w$

" abscissa distance between the zeros of scales

$f_1(U)$ and $f_3(W) = D$ inches.

In determinate 10:

Divide column (a) by M_w

" " (b) " M_u

We then have:

$$11. \quad \begin{array}{cc|c} 0 & 1/M_u & -f_1(U) \\ 1/M_w & 0 & -f_3(W) \\ 1/M_w & \frac{1}{M_u} f_2(V) & C \end{array} = \begin{array}{cc|c} 0 & 1 & -M_u f_1(U) \\ 1 & 0 & -M_w f_3(W) \\ \frac{1}{M_w} & \frac{1}{M_u} f_2(V) & C \end{array} =$$

$$\begin{array}{cc|c} (a) & (b) & \\ \hline 1 & 1 & -M_u f_1(U) \\ 1 & 0 & -M_w f_3(W) \\ 1 & \frac{M_w f_2(V)}{M_u + M_w f_2(V)} & \frac{M_u M_w C}{M_u + M_w f_2(V)} \end{array} = 0$$

Now multiply column (b) of the preceding determinate (11) by D, and we have:

$$12. \quad \begin{array}{c} \begin{array}{cc} (x) & (y) \\ 1 & D & - M_u f_1(U) \\ 1 & 0 & - M_w f_3(W) \\ 1 & \frac{M_w D f_2(V)}{M_u + M_w f_2(V)} & \frac{M_u M_w C}{M_u + M_w f_2(V)} \end{array} \end{array} \quad = 0$$

The plotting equations are:

$$\text{For } U \quad \begin{cases} y = - M_u f_1(U) & \text{inches.} \\ x = D & \text{inches.} \end{cases}$$

$$\text{For } V \quad \begin{cases} y = \frac{M_u M_w C}{M_u + M_w f_2(V)} & \text{inches.} \\ x = \frac{M_w D f_2(V)}{M_u + M_w f_2(V)} & \text{inches.} \end{cases}$$

$$\text{For } W \quad \begin{cases} y = - M_w f_3(W) & \text{inches.} \\ x = 0 & \text{inches.} \end{cases}$$

If $C = 0$, the plotting equations will produce a nomograph commonly known as a "Z" chart, such as the following figure:

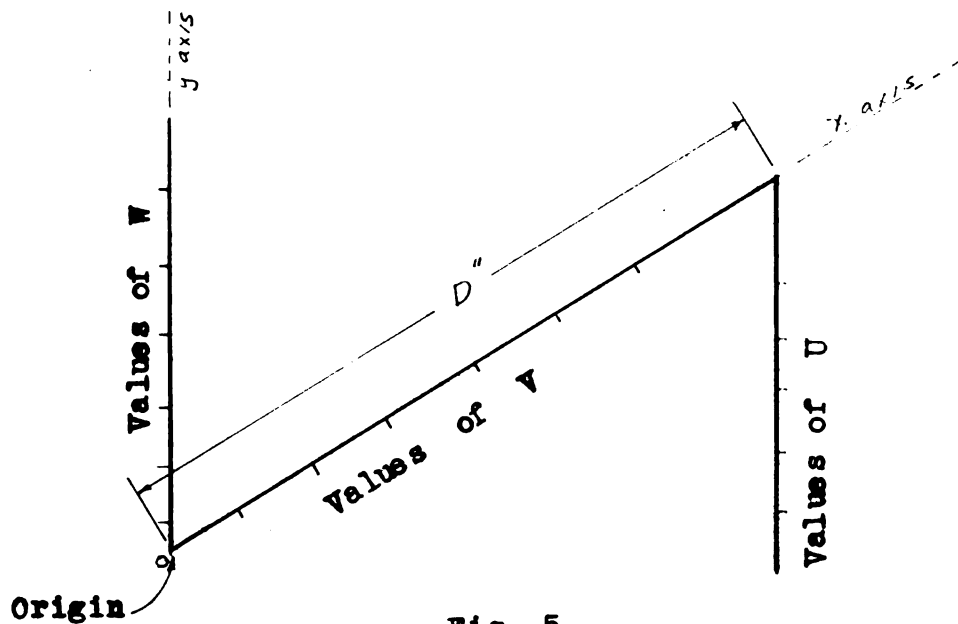


Fig. 5

If $C \neq 0$, the plotting equations will produce a nomograph of general configuration, such as the following figure:

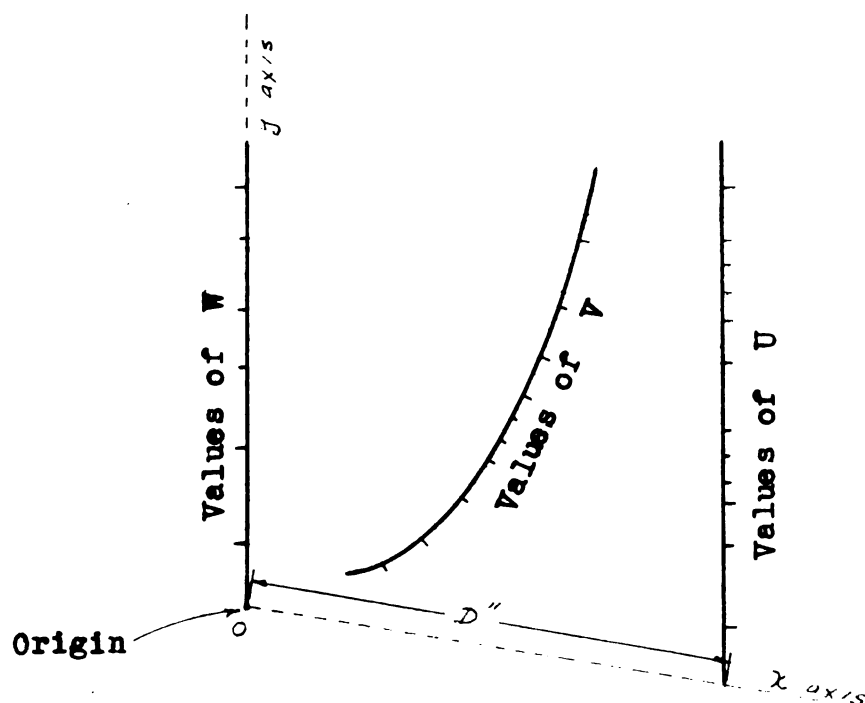


Fig. 6

As a special case of equation 4, let us consider the equation:

$$13. \quad \frac{1}{f_1(U)} + \frac{1}{f_2(V)} = \frac{1}{f_3(W)}$$

Setting equation 13 in determinate form, we have:

$$14. \quad \begin{array}{ccc} (a) & (b) & (c) \\ \left| \begin{array}{ccc} 1 & 0 & -\frac{1}{f_1(U)} \\ 0 & 1 & -\frac{1}{f_2(V)} \\ 1 & 1 & -\frac{1}{f_3(W)} \end{array} \right| & = & 0 \end{array}$$

Let the modulus of $f_1(U) = M_u$

" " " " $f_2(V) = M_v$

In determinate 14:

Multiply column (a) by M_u

" " (b) " M_v

" " (c) " (-1)

and we have:

$$15. \quad \begin{array}{ccc} & (y) & (x) \\ \left| \begin{array}{ccc} M_u & 0 & \frac{1}{f_1(U)} \\ 0 & M_v & \frac{1}{f_2(V)} \\ M_u & M_v & \frac{1}{f_3(W)} \end{array} \right| & = & \left| \begin{array}{ccc} M_u f_1(U) & 0 & 1 \\ 0 & M_v f_2(V) & 1 \\ M_u f_3(W) & M_v f_3(W) & 1 \end{array} \right| = 0 \end{array}$$

Hence the plotting equations are:

$$\text{For } U \begin{cases} y = M_u f_1(U) \text{ inches.} \\ x = 0 \text{ inches.} \end{cases}$$

$$\text{For } V \begin{cases} y = 0 \text{ inches.} \\ x = M_v f_2(V) \text{ inches.} \end{cases}$$

$$\text{For } W \begin{cases} y = M_u f_3(W) \text{ inches.} \\ x = M_v f_3(W) \text{ inches.} \end{cases}$$

Which would produce a nomograph like the following figure:

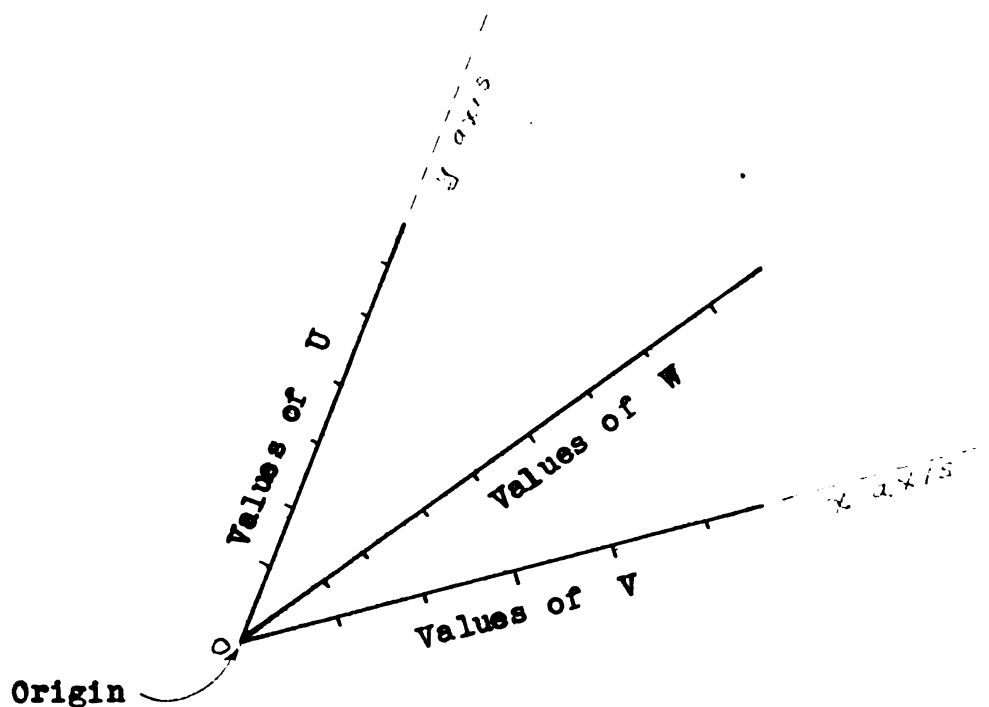


Fig. 7

Another method of plotting the $f_3(W)$, is to determine the modulus, M_W , for this scale. We proceed by locating any point, P , on the $f_3(W)$ scale by using the plotting equations developed on the preceding page.

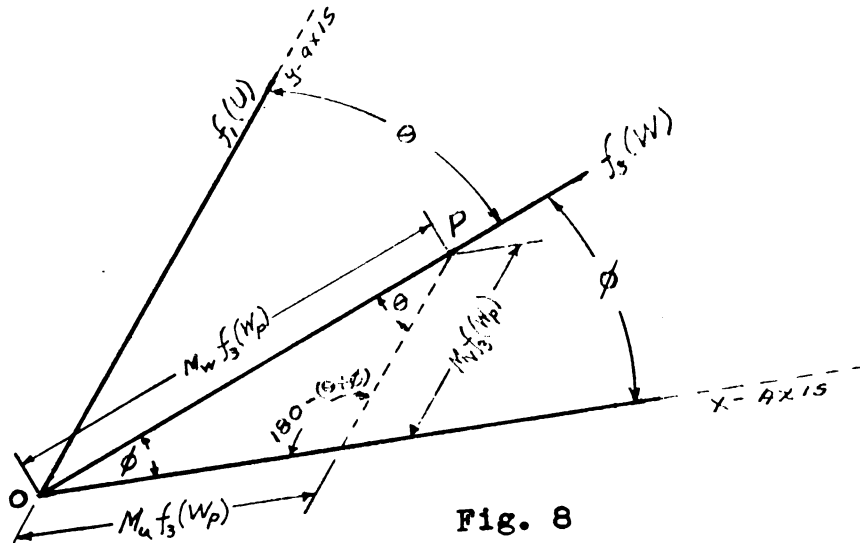


Fig. 8

Let the coordinates of P be: $y = M_V f_3(W_P)$ and $x = M_U f_3(W_P)$

Let the distance from the origin to P be: $M_W f_3(W_P)$

From trigonometry:

$$16. \quad \frac{\sin \theta}{\sin \phi} = \frac{M_U f_3(W_P)}{M_V f_3(W_P)} = \frac{M_U}{M_V}$$

$$17. \quad \frac{\sin 180 - (\theta + \phi)}{\sin \theta} = \frac{M_W f_3(W_P)}{M_U f_3(W_P)} = \frac{M_W}{M_U} = \frac{\sin (\theta + \phi)}{\sin \theta}$$

Or

$$18. \quad M_W = M_U \frac{\sin (\theta + \phi)}{\sin \theta} = M_V \frac{\sin (\theta + \phi)}{\sin \phi}$$

Having determined the scale moduli, M_U and M_V , one angle (say θ) may be assumed and the other angle (ϕ) may be determined from equation 16. Then the modulus M_W may be determined from equation 18.

As a special case of an equation containing four variables, let us assume the equation:

$$19. \quad f_1(U) + f_2(V) \cdot f_4(\phi) + f_3(W) \cdot f_5(\phi) + C = 0$$

Setting up this equation in determinate form, gives:

$$20. \quad \begin{array}{cc|c} (a) & (b) & \\ \hline 1 & 0 & -f_2(V) \\ 0 & 1 & -f_3(W) \\ f_4(\phi) & f_5(\phi) & f_1(U) + C \end{array} = 0$$

Let the modulus of the scale for $f_2(V) = M_V$

" " " " " " " $f_3(W) = M_W$

Let the abscissa distance between the zeros of the above scales = D inches.

In determinate 20:

Divide column (a) by M_V

" " (b) " M_W

and we have:

$$21. \quad \begin{array}{cc|c} \frac{1}{M_V} & 0 & -f_2(V) \\ 0 & \frac{1}{M_W} & -f_3(W) \\ \frac{1}{M_V} f_4(\phi) & \frac{1}{M_W} f_5(\phi) & f_1(U) + C \end{array} = 0$$

By performing some of the operations as stated on page 14 upon determinate 21, we arrive at the following determinate:

$$22. \begin{vmatrix} 1 & \begin{matrix} (x) \\ 0 \end{matrix} & \begin{matrix} (y) \\ -M_V f_2(V) \end{matrix} \\ 1 & D & -M_W f_3(W) \\ 1 & \frac{M_V f_5(\phi) D}{M_W f_4(\phi) + M_V f_5(\phi)} & \frac{M_V M_W [f_1(U) + C]}{M_W f_4(\phi) + M_V f_5(\phi)} \end{vmatrix} = 0$$

The plotting equations are:

$$\text{For } V \begin{cases} y = -M_V f_2(V) & \text{inches} \\ x = 0 & \text{inches.} \end{cases}$$

$$\text{For } W \begin{cases} y = -M_W f_3(W) & \text{inches.} \\ x = D & \text{inches.} \end{cases}$$

$$\text{For } U \text{ \& } \phi \begin{cases} y = \frac{M_V M_W [f_1(U) + C]}{M_W f_4(\phi) + M_V f_5(\phi)} \\ x = \frac{M_V D f_5(\phi)}{M_W f_4(\phi) + M_V f_5(\phi)} \end{cases}$$

To graph the scale for $f_1(U)$ and ϕ , assign desirable values for ϕ and plot the scales of $f_1(U)$ for each of the given values of ϕ .

These equations will generally produce a nomograph having a network of scales, as in the following figure.

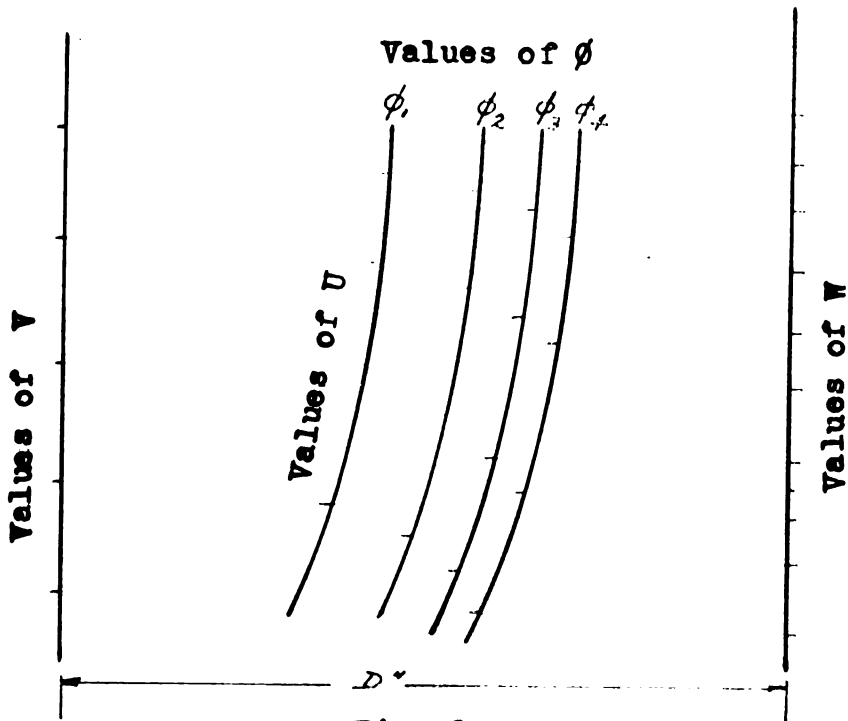


Fig. 9

SOLUTION

The solution of equations by means of a nomograph is accomplished by utilizing the main statement of the theory, i.e., that three specified points that satisfy the equation must be in a straight line. Hence by joining the value of U on the $f_1(U)$ scale with a straight edge to the value of V on the $f_2(V)$ scale, we read the value of W on the $f_3(W)$ scale that satisfies the equation.

**THE SOLUTION OF VARIOUS ENGINEERING PROBLEMS
BY THE APPLICATION OF
NOMOGRAPHIC CHARTS**

SECTION A

ELECTRICAL PROBLEMS

RELATIONSHIP OF
VOLTAGE - KVA. - CURRENT

Practically all computations for the solution of electrical circuits involve either the load in Kva., or the load current in amperes. A simple form of nomograph can be constructed to express the relationship of load in Kva., line current, and phase voltage.

The equations relating these quantities, are

1. $I = \frac{\text{Kva.}}{E}$ for single phase circuits.
2. $I = \frac{\text{Kva.}}{1.732 E}$ for balanced three phase circuits.

We may express the above equations in logarithmic form as:

3. $\log I - \log \text{Kva.} + \log E = 0$ for single phase current.
4. $\log I - \log \text{Kva.} + \log E + \log 1.732 = 0$ for three phase current.

Setting up equation 3 in determinate form, we have:

$$5. \quad \begin{vmatrix} 1 & 0 & \log \text{Kva.} \\ 0 & 1 & - \log E \\ 1 & 1 & \log I \end{vmatrix} = 0 \quad (\text{single phase circuit})$$

Setting up equation 4 in determinate form, gives:

$$6. \quad \begin{vmatrix} 1 & 0 & \log \text{Kva.} \\ 0 & 1 & - \log E \\ 1 & 1 & \log I + .23856 \end{vmatrix} = 0 \quad (\text{three phase circuit})$$

For limiting values, assume:

Kva. varies from 10 to 100.

I " " 1 " 1000.

E " " 100 " 10,000.

For scale lengths, let:

Kva. scale be 6 inches long.

E " " 6 " " .

Hence the scale modulus for $\log Kva.$ = 6

and " " " " $\log E$ = 3.

Assume an abscissa length equal to 5 inches.

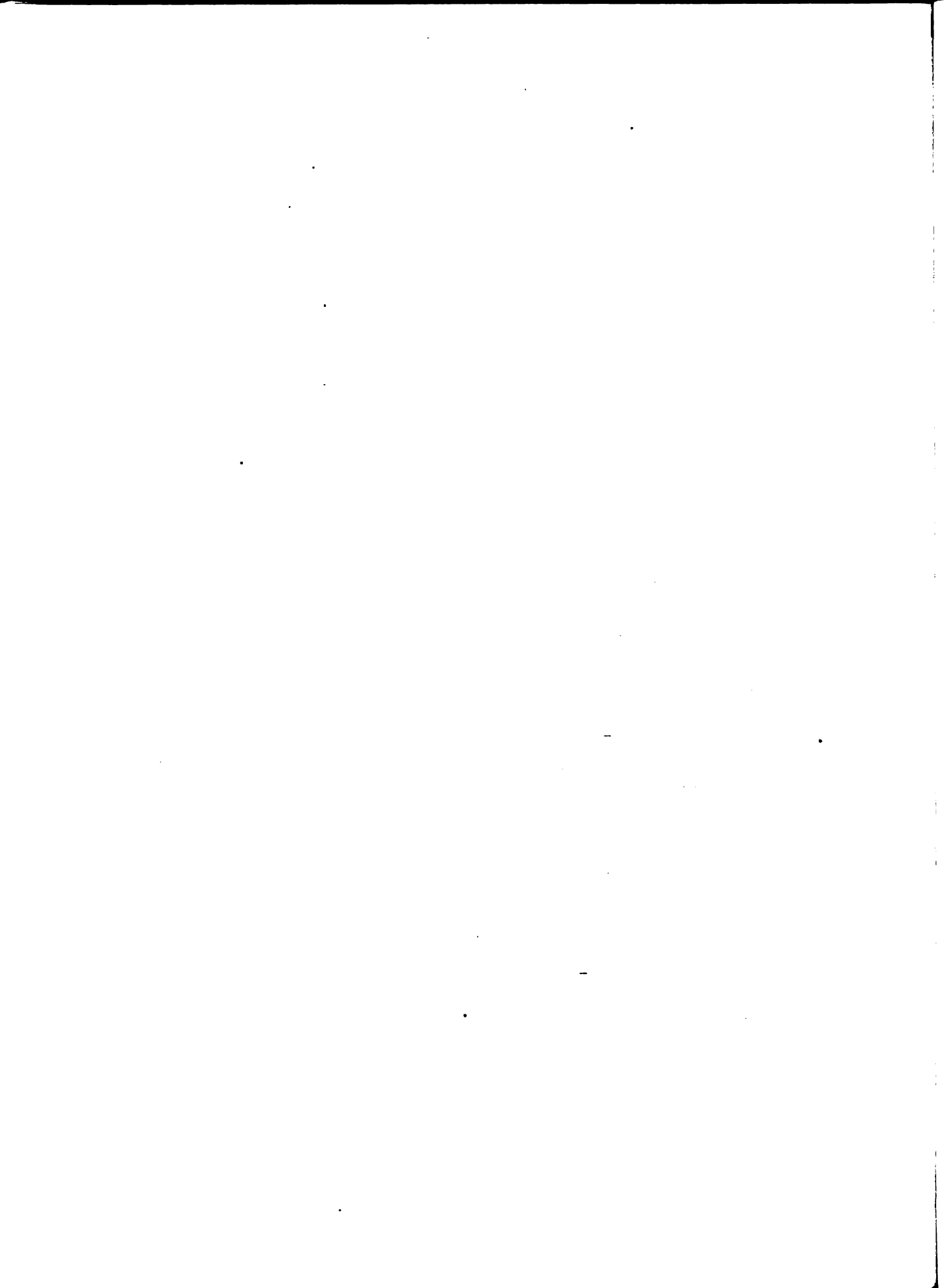
Applying these moduli and abscissa value to determinates 5 and 6, and reducing the determinate so that it contains a column of units, gives

From determinate 5:

$$7. \quad \begin{vmatrix} 1 & 0 & 6 \log Kva. \\ 1 & 5 & -3 \log E \\ 1 & \frac{10}{3} & 2 \log I \end{vmatrix} = 0 \quad (\text{single phase})$$

From determinate 6:

$$8. \quad \begin{vmatrix} 1 & 0 & 6 \log Kva. \\ 1 & 5 & -3 \log E \\ 1 & \frac{10}{3} & 2(\log I + .23856) \end{vmatrix} = 0 \quad (\text{three phase})$$



The plotting equations from determinate 7, are:

$$\text{For Kva. } \begin{cases} y = 6 \log \text{ Kva. inches} \\ x = 0 \text{ inches} \end{cases}$$

$$\text{For E } \begin{cases} y = - 3 \log E \text{ inches.} \\ x = 5 \text{ Inches.} \end{cases} \quad (\text{single phase})$$

$$\text{For I } \begin{cases} y = 2 \log I \text{ inches.} \\ x = 3.333 \text{ inches.} \end{cases}$$

The plotting equations from determinate 8, are:

$$\text{For Kva. } \begin{cases} y = 6 \log \text{ Kva. inches.} \\ x = 0 \text{ inches.} \end{cases}$$

$$\text{For E } \begin{cases} y = - 3 \log E \text{ inches.} \\ x = 5 \text{ inches.} \end{cases} \quad (\text{three phase})$$

$$\text{For I } \begin{cases} y = 2 \log I + .47712 \\ x = 3.333 \text{ inches.} \end{cases}$$

We may combine the two nomographs produced by these plotting equations. In doing this we calibrate the current scale on one side for single phase current and on the other side for balanced three phase current.

SOLUTION

Join the value of Kva. and E on their respective scales with a straight edge, and read the value of I on the current scale.

(See the following sketch)

EXAMPLE

Single phase :

$E = 2000 \text{ volts}$

$Kva. = 50$

$I = 25 \text{ amps. (from current scale)}$

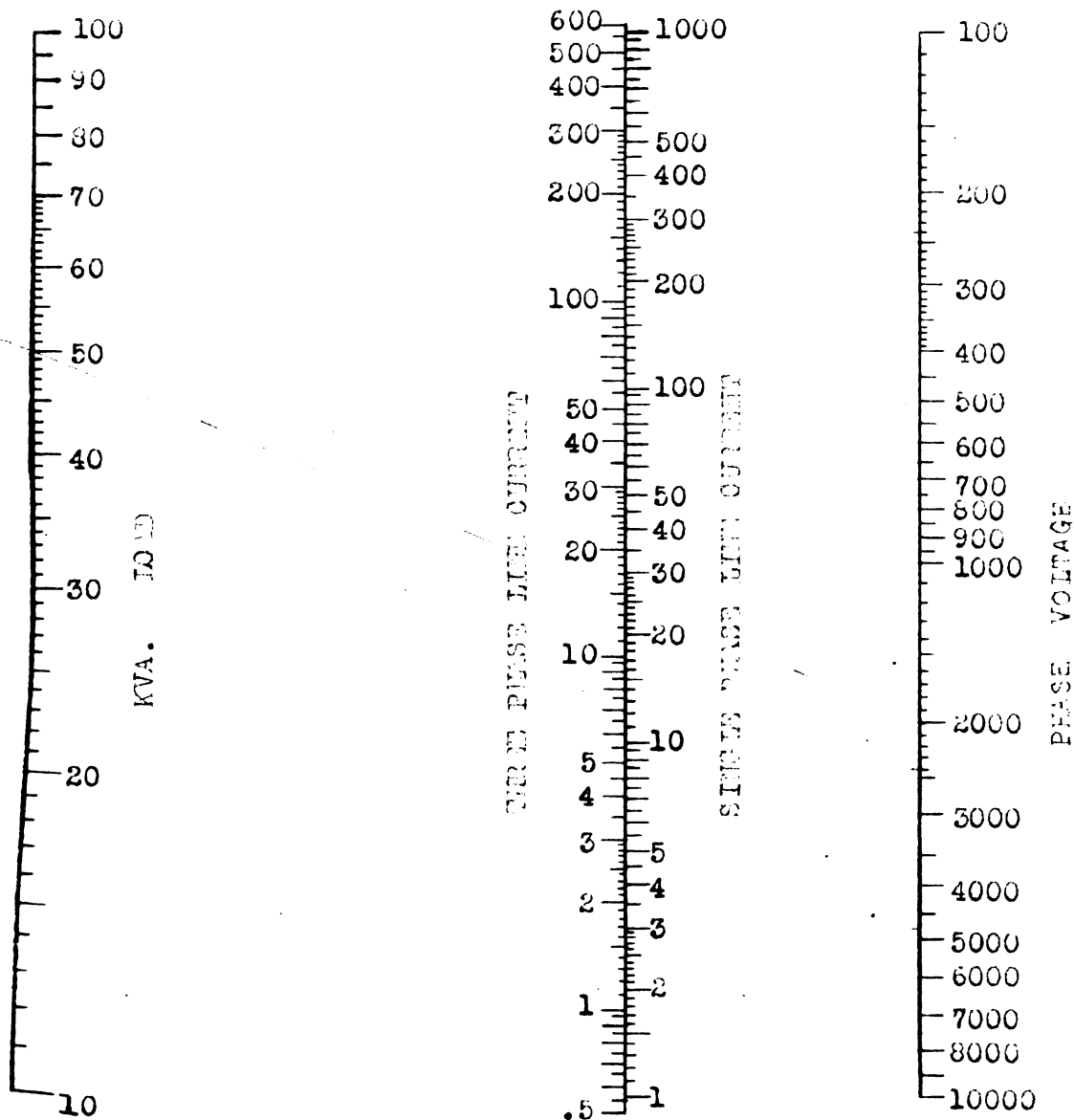
Three phase:

$E = 2000 \text{ volts (phase voltage)}$

$Kva. = 50$

$I = 14.4 \text{ amps. (from current scale)}$

LOAD - VOLTAGE - CURRENT CHART
For Balanced A.C. Circuits



ELECTRICAL SYSTEM
PLANNING DIVISION

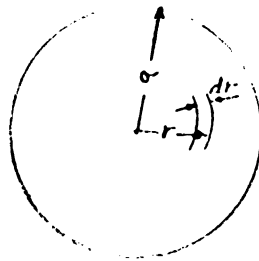
HCR - 3

10-23-33

INDUCTIVE REACTANCE

There have been several formulae developed for the computation of the inductance and inductive reactance of conductors. Such authors as E.B. Rosa and H.L. Curtis of the U.S. Bureau of Standards treat the subject very thoroughly, taking into account skin effect of the current and change in resistance of the conductor. However, for ordinary distribution work, such refinements are not required. In this particular case, we shall develop a modified formula from the basic relationships presented by E.B. Rosa in his paper # S - 80 of the U. S. Bureau of Standards.

INTERNAL SELF INDUCTANCE OF A CONDUCTOR



If I is the direct current flowing in the conductor of radius a , and uniformly distributed throughout, then the flux linkages may be expressed as:

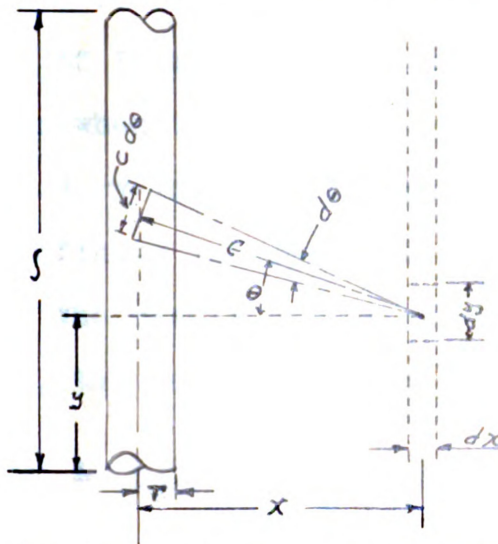
$$\begin{aligned}
 1. \quad I \phi &= \int_0^a (r^2/a^2) I (2 u I r/a^2) dr \\
 &= \frac{u I^2}{2} \quad \text{where } u = \text{the permeability of the conductor.}
 \end{aligned}$$

The internal self inductance then may be expressed as:

$$2. \quad L = \phi/I = \frac{u}{2} \quad \text{per centimeter length of conductor.}$$

EXTERNAL SELF INDUCTANCE OF A CONDUCTOR

Where the current is distributed uniformly over the cross section of the conductor.



The total flux, ϕ , outside the conductor may be expressed as:

$$3. \quad \phi = \int_r^\infty \int_0^l \int_{\sin^{-1} \frac{2-y}{c}}^{-\sin^{-1} \frac{y}{c}} \frac{1}{x} \cos \theta \, d\theta \, dy \, dx$$

$$= 2 I l \left(\log \frac{2l}{r} - 1 \right)$$

The external self inductance may be expressed as:

$$4. \quad L = \phi / I = 2 l \left(\log \frac{2l}{r} - 1 \right)$$

Hence the total inductance of the conductor of length l centimeters is the sum of the internal and external inductances which may be expressed as:

$$5. \quad L = 2 l \left(\log \frac{2l}{r} - 1 + \frac{\mu}{4} \right)$$

MUTUAL INDUCTANCE OF TWO PARALLEL CONDUCTORS

The mutual inductance of two parallel conductors of length l , radius r , and distance apart d , will be the number of lines of force due to unit current in one which will cut the other when the current goes to zero. This value may be determined by integrating equation 3 between the limits of d and infinity instead of between r and infinity, and dividing the result by the current I . Performing this operation we have, the mutual inductance M :

$$6. \quad M = 2l \left(\log \frac{2l}{d} - 1 \right) \quad \text{where } d \text{ is small compared to } l.$$

THE SELF INDUCTANCE OF A RETURN CIRCUIT

We may consider a return circuit formed by two parallel conductors similar to the case of two coils (of one turn each) connected in series so that their individual effects counteract the flow of magnetic flux. The effective inductance for such a condition may be expressed as the following equation:

$$7. \quad L_e = 2L - 2M$$

Substituting equation 5 for the value of L , and equation 6 for the value of M , we have:

$$8. \quad L_e = 4l \left(\log \frac{d}{r} + \frac{u}{4} \right) \text{ centimeters.}$$

$$= 4l \left(\log \frac{d}{r} + \frac{u}{4} \right) \times 10^{-9} \text{ henrys.}$$

It is assumed that the permeability, μ , of copper is unity. Hence for the inductance to neutral for one wire of length l centimeters, we have:

$$9. \quad L_{\bullet} = 2l \left(\log_{\bullet} \frac{d}{r} + \frac{1}{4} \right) \times 10^{-9} \text{ henrys.}$$

Changing the Napierian logarithm to the $\log_{10}$, the inductance per centimeter length is:

$$\begin{aligned} 10. \quad L_{\bullet}/\text{cm.} &= 2 \left(2.30259 \log_{10} \frac{d}{r} + \frac{1}{4} \right) \times 10^{-9} \text{ henrys.} \\ &= 2 \times 2.30259 \left(\log_{10} \frac{d}{r} + .10857 \right) \times 10^{-9} \text{ henrys.} \\ &= 2 \times 2.30259 \left(\log_{10} 1.284 \frac{d}{r} \right) \times 10^{-9} \text{ henrys.} \end{aligned}$$

The inductance per 1000 feet of conductor to neutral is:

$$11. \quad L_{\bullet}/1000' = .1404 \times 10^{-3} \log_{10} 1.284 \frac{d}{r} \text{ henrys.}$$

reactance

The inductive/per 1000 feet of conductor to neutral for 60 cycle frequency is:

$$12. \quad X = .052916 \log_{10} 1.284 \frac{d}{r} \text{ ohms. (logarithm to the base 10)}$$

To solve this equation requires the use of a log table. For general computations we may construct a nomograph to save time, and the accuracy of which is within the limits required for distribution work.

Setting up equation 12 in determinate form, we have:

$$13. \begin{vmatrix} 1 & 0 & \log d \\ 0 & 1 & -\log \frac{1.284}{r} \\ 1 & 1 & \frac{X}{.0529} \end{vmatrix} = 0$$

Assume: d varies from 2 inches to 200 inches.

r " " .10 " " 1.0 " .

a scale length of approximately 8 inches.
an abscissa of 5 inches.

The scale modulus for: d = 4 and for: r = 8.

Setting up the determinate to accomodate these scale moduli, gives: (from 13)

$$14. \begin{vmatrix} 1/4 & 0 & \log d \\ 0 & 1/8 & -\log \frac{1.284}{r} \\ 1/4 & 1/8 & \frac{X}{.0529} \end{vmatrix} =$$

$$15. \begin{vmatrix} 1 & 0 & 4 \log d \\ 1 & 5 & -8 \log \frac{1.284}{r} \\ 1 & \frac{5}{3} & \frac{4 X}{.0529} \end{vmatrix} = 0$$

From determinate 15, the plotting equations are:

$$\text{For: } d \begin{cases} y = 4 \log d \text{ inches.} \\ x = 0 \text{ inches.} \end{cases}$$

$$\text{For: } r \begin{cases} y = - 8 \log \frac{1.284}{r} \text{ inches.} \\ x = 5 \text{ inches.} \end{cases}$$

$$\text{For: } X \begin{cases} y = \frac{4 X}{.0529} \text{ inches.} \\ x = 1.666 \text{ inches.} \end{cases}$$

The above formula applies to solid conductor. For stranded conductor the inductance at 60 cycle frequency will be approximately 13×10^{-4} ohms less than for solid conductor of the same size. Hence on the reactance scale we may plot the inductance for stranded conductor by proportional readings.

On the radius scale, we may plot the wire size corresponding to the radius on one side of the scale and on the opposite side plot the radius in inches.

SOLUTION

Join the value of the spacing in inches in the d scale with a straight edge to the value of the radius (or wire size) on the radius scale, and read the value of the reactance on the X scale.

(See the following sketch)

EXAMPLE**Solid conductor:**

Conductor spacing = 30 inches

Conductor radius = .30 inches

Reactance /1000' = .1115 ohms.

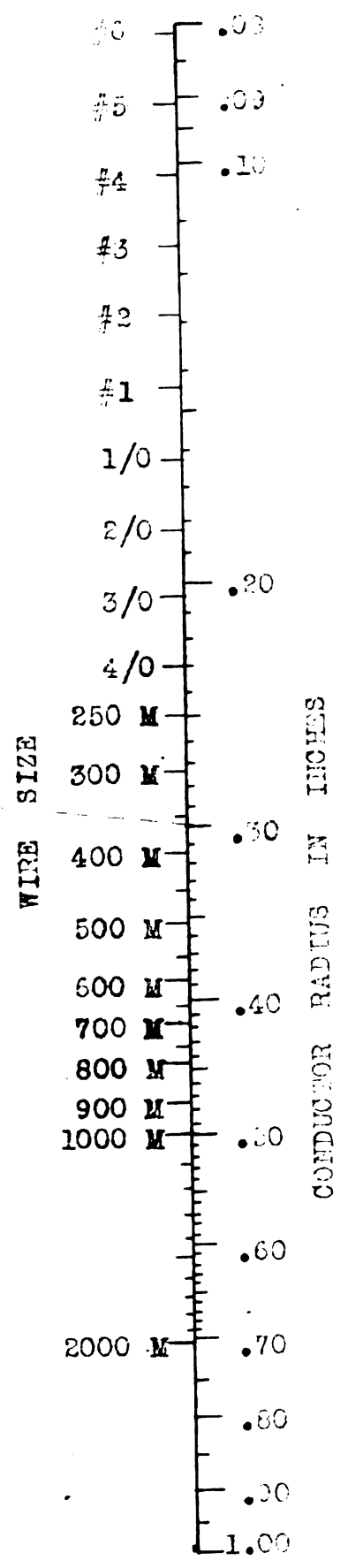
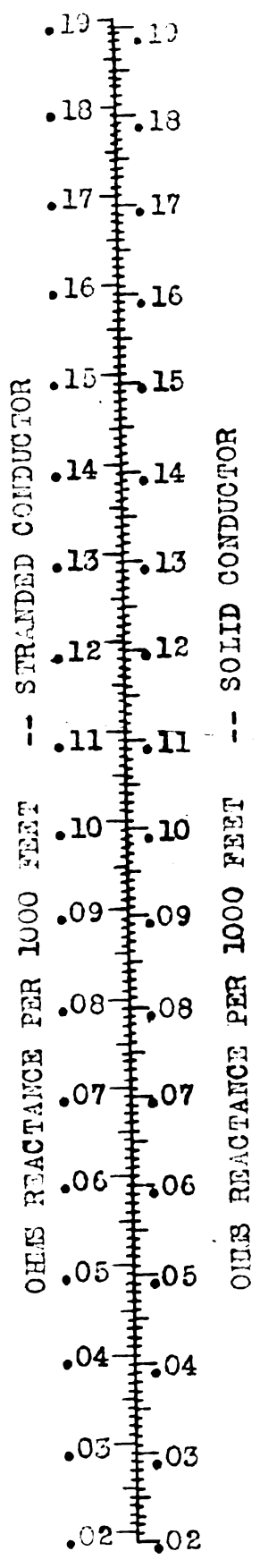
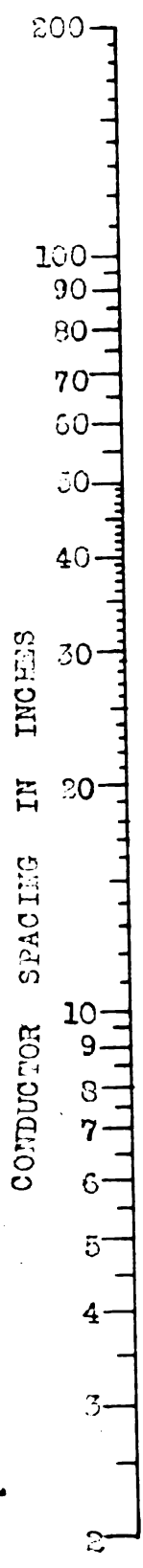
Stranded conductor:

Conductor spacing = 30 inches

Conductor radius = .30 inches

Reactance /1000' = .1102 ohms.

INDUCTIVE REACTANCE CHART
60 Cycle Frequency



ELECTRICAL SYSTEM
PLANNING DIVISION

IMPEDANCE CALCULATIONS

When solving electrical networks it is often desirable to express the impedance in polar coordinates rather than in rectangular coordinates to facilitate mathematical operations. However, it must be borne in mind that the impedance is a complex number and not a vector and due care must be exercised when using this expression in products containing vectors.

In a circuit under consideration:

Let: R = the resistance

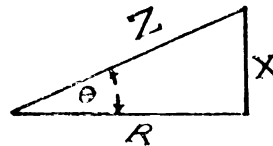
X = the reactance

Z = the impedance.

It can be shown by analysis that the following relations exist:

$$Z^2 = R^2 + X^2$$

$$\theta = \tan^{-1} X/R$$



The expression for the impedance in rectangular coordinates is generally given as:

$$1. \quad Z = R + jX$$

And in polar coordinates as:

$$2. \quad Z \angle \theta$$

The value of the impedance in polar coordinates is:

$$3. \quad Z \angle \theta = (R^2 + X^2)^{\frac{1}{2}} \angle \tan^{-1} X/R$$

Finding these values requires considerable slide rule work and a table of trigonometric functions. The solution of this equation is accomplished with ease and quickly by means of a nomograph. The nomograph may be constructed from the relationships stated on page 44, and may be expressed as:

$$4. \quad R^2 + X^2 - Z^2 = 0$$

$$5. \quad R^2 \tan^2 \theta - X^2 = 0$$

Setting up equation 4 in determinate form, we have:

$$6. \quad \begin{vmatrix} 1 & 0 & Z^2 \\ 0 & 1 & -R^2 \\ 1 & 1 & X^2 \end{vmatrix} = 0$$

Setting up equation 5 in determinate form, we have:

$$7. \quad \begin{vmatrix} 0 & 1 & -R^2 \\ 1 & 0 & X^2 \\ 1 & \tan^2 \theta & 0 \end{vmatrix} = 0$$

Examining the above determinates we note that determinate 6 will produce a nomograph like Fig. 1 below, and that determinate 7 will produce a nomograph like Fig. 2 below.

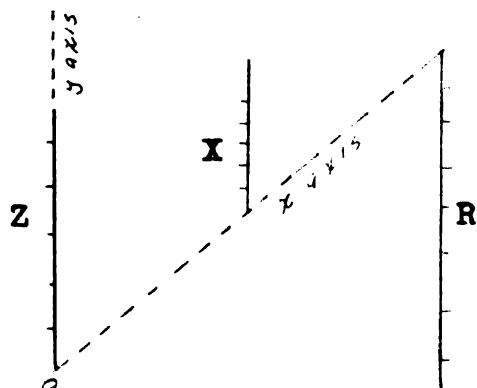


Fig. 1

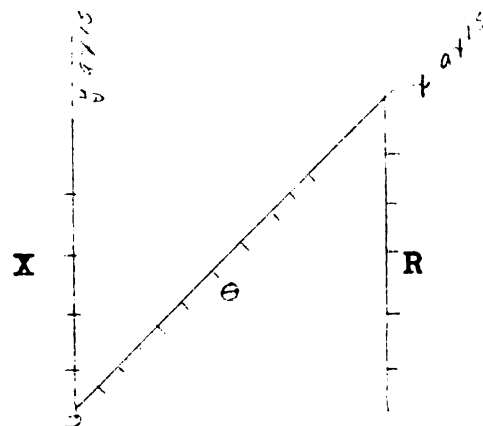


Fig. 2

If the correct abscissa lengths are chosen, the two nomographs (Fig. 1 and Fig. 2) may be combined. This may be accomplished by making the lengths of the abscissa between scales X and R in Fig. 1 equal to that between X and R in Fig. 2.

Assume limiting values of 10 ohms for the scales X, R, and Z. And assume scale moduli of $\frac{1}{11.75}$ for the scales of R and Z which will make these scales approximately 8.5 inches long. Assume an abscissa length equal to 10.2 inches. These odd figures were chosen in order to construct a suitable nomograph on $8\frac{1}{2}'' \times 11''$ paper.

Applying these scale moduli and abscissa length to determinate 6, gives:

$$8. \quad \begin{vmatrix} 11.75 & 0 & Z^2 \\ 0 & 11.75 & -R^2 \\ 11.75 & 11.75 & X^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & \frac{Z^2}{11.75} \\ 1 & 10.2 & \frac{-R^2}{11.75} \\ 1 & 5.1 & \frac{X^2}{23.50} \end{vmatrix} = 0$$

Hence the plotting equations are:

$$\text{For: } Z \begin{cases} y = Z^2/11.75 \text{ inches} \\ x = 0 \text{ inches} \end{cases}$$

$$\text{For: } R \begin{cases} y = -R^2/11.75 \text{ inches} \\ x = 10.2 \text{ inches} \end{cases}$$

$$\text{For: } X \begin{cases} y = X^2/23.50 \text{ inches} \\ x = 5.1 \text{ inches} \end{cases}$$

Using a scale modulus of $1/11.75$ for R , and a scale modulus of $1/23.50$ for X , and applying these moduli to determinate 7, with an abscissa length equal to 5.10 inches, we have:

$$9. \quad \begin{vmatrix} 0 & 11.75 & -R^2 \\ 23.50 & 0 & X^2 \\ 23.50 & 11.75 \tan^2 \theta & 0 \end{vmatrix} = \begin{vmatrix} 1 & 5.1 \\ 1 & 0 \\ 1 & \frac{5.1 \tan^2 \theta}{2 + \tan^2 \theta} \end{vmatrix} \begin{vmatrix} (x) & (y) \\ -\frac{R^2}{11.75} \\ \frac{X^2}{23.50} \\ 0 \end{vmatrix} = 0$$

From the above determinate, the plotting equations are:

For: $R \begin{cases} y = -R^2/11.75 & \text{inches.} \\ x = 5.10 & \text{inches.} \end{cases}$

For: $X \begin{cases} y = X^2/23.50 & \text{inches.} \\ x = 0 & \text{inches.} \end{cases}$

For: $\theta \begin{cases} y = 0 & \text{inches.} \\ x = \frac{5.1 \tan^2 \theta}{2 + \tan^2 \theta} & \text{inches.} \end{cases}$

Plotting these equations produces a nomograph as shown on page 48.

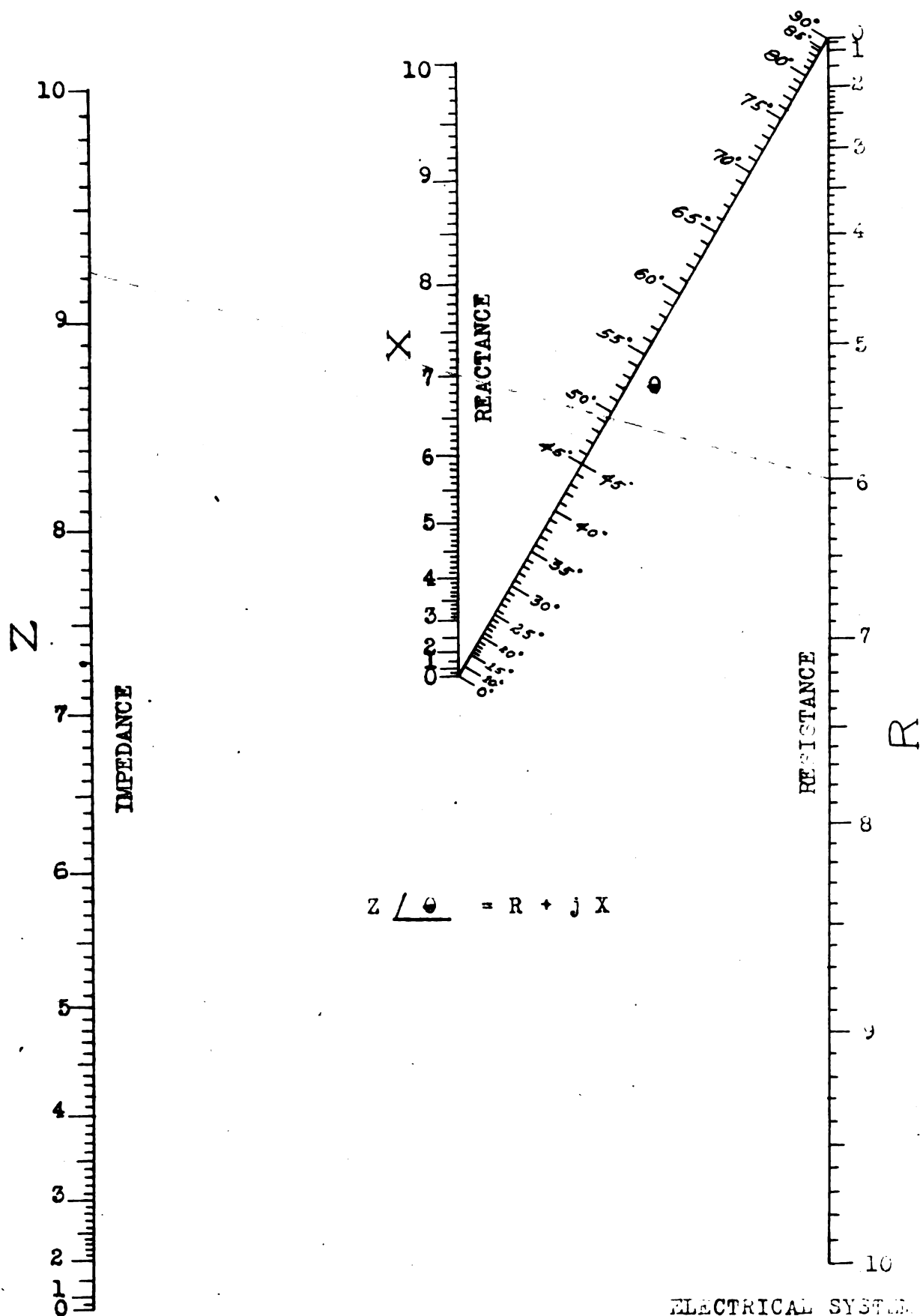
SOLUTION

Join the values of R and X on their respective scales with a straight edge and read the value of θ on the angle scale and the value of Z on the impedance scale.

EXAMPLE

$R = 6 \quad X = 7 \quad \text{From nomograph } Z = 9.22 \quad \text{and } \theta = 49.4^\circ$

IMPEDANCE CHART



$$Z \angle \theta = R + jX$$

ELECTRICAL SYSTEM
PLANNING DIVISION

VOLTAGE REGULATION

IN SHORT LINES

The distribution engineer of a Public Utility supplying electrical power is very much interested in voltage regulation. It is of great concern to him to keep voltage fluctuation within a very small limit, but while designing a distribution system to accommodate this limit, he must also be conservative in the investment and must anticipate the future growth of load. The years 1927, '28 and '29 rather upset his ability to prognosticate load growth, while 1930, '31, '32 and '33 left him with an excess of capacity.

Voltage fluctuation limits are generally between 2% and 5% for the general distribution system supplying municipalities. Where separate power lines are built for individual customers, this limit is much higher. However, for some types of loads that fluctuate so rapidly that the regulator at the substation cannot respond, the voltage regulation is generally limited to 1½% at the substation bus. A typical load of this classification is the starting current of a large motor which lasts only a few seconds, but which generally causes flickering lights on the system.

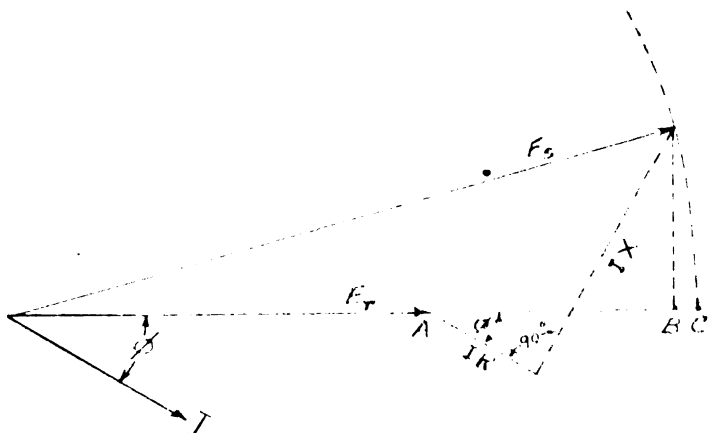
It is for this type of load that the engineer must compute the voltage drop on the system. The calculations are made assuming a constant voltage on the transmission system supplying the substation. The results are generally determined as a number of volts drop on some base voltage, rather than the percent regulation. In calculations of this type, the length of circuit is short enough that the conductance and susceptance effects of the line may be neglected.

81

METHOD I

FOR LAGGING POWER FACTOR

Theory



In the above figure:

1. $\overline{AC} = \overline{AB} + \overline{BC}$ = the voltage drop

From the geometry of the above figure:

2. $\overline{AB} = IR \cos \phi + IX \sin \phi$

3. $\overline{BC} = \frac{(IX \cos \phi - IR \sin \phi)^2}{2 E_s}$

Where: E_s = sending voltage

E_r = receiver voltage

I = load current in amperes

R = resistance of line

X = reactance of line

ϕ = power factor at receiver end of line.

Let:

$$4. \quad K = \frac{\overline{AC}}{I} = \text{voltage drop per ampere.}$$

$$= R \cos \phi + X \sin \phi + \frac{1}{2} \left(\frac{IX}{E_s} X \cos^2 \phi + \frac{IR}{E_s} \sin^2 \phi + \frac{IRX \sin 2\phi}{E_s} \right)$$

Generally the length, $\overline{BC}$, is extremely small compared to $\overline{AB}$ and may be neglected; in which case:

$$5. \quad K = R \cos \phi + X \sin \phi$$

However, if it is desired to use the complete equation (4) for K, It will simplify the construction of the nomograph to limit the working range of the nomograph within a given relationship of the resistance and reactance drops to the sending voltage.

Assume the IR and IX drops to be 5 % of E_s , the sending voltage. Substituting these values in equation 4, gives:

$$\begin{aligned} 6. \quad K &= R \cos \phi + X \sin \phi + .025 X \cos^2 \phi + .025 R \sin^2 \phi \\ &\quad - .05 X \sin \phi \cos \phi \\ &= R (\cos \phi + .025 \sin^2 \phi) + X (\sin \phi + .025 \cos^2 \phi - .025 \sin 2\phi) \end{aligned}$$

Setting up equation 6 in determinate form, gives:

$$7. \quad \begin{vmatrix} 1 & 0 & -R \\ 0 & 1 & -X \\ (\cos \phi + .025 \sin^2 \phi) & (\sin \phi + .025 \cos^2 \phi - .025 \sin 2\phi) & -K \end{vmatrix} = 0$$

Assume limiting values of 10 ohms for R and X, and a scale length of 6.66 inches. The scale modulus for R and X then will be 2/3. Assume an abscissa length equal to 5 inches. Applying these scale moduli and abscissa length to determinate 7, gives:

	(x)	(y)
	0	$\frac{2R}{3}$
8.	5	$\frac{2X}{3}$
	$1 \quad \frac{5(\sin \phi + .025 \cos^2 \phi - .025 \sin 2\phi)}{\sin \phi + \cos \phi - .025 \sin 2\phi + .025}$	$1 \quad \frac{2K}{3(\sin \phi + \cos \phi - .025 \sin 2\phi + .025)}$

= 0

From determinate 8, the plotting equations are:

For R $y = \frac{2R}{3}$ inches $\text{For X } y = \frac{2X}{3}$ inches
 $x = 0$ inches $x = 5$ inches

For K, ϕ $y = \frac{2K}{3(\sin \phi + \cos \phi - .025 \sin 2\phi + .025)}$ inches
 $x = \frac{5(\sin \phi + .025 \cos^2 \phi - .025 \sin 2\phi)}{3(\sin \phi + \cos \phi - .025 \sin 2\phi + .025)}$ inches

SOLUTION

Join the values of R and X on their respective scales with a straight edge, and read the value of K on the desired power factor scale. See sketch on following page.

For single phase circuits:

$$\text{Voltage drop} = 2 I K = 2 \frac{\text{Kva.}}{E} \times 10^3$$

For three phase circuits:

$$\text{Voltage drop} = 1.732 I K = \frac{\text{Kva.}}{E} \times 10^3$$

It may be seen from the above that the single phase voltage drop is twice the three phase voltage drop for the same Kva. load, providing the single phase and three phase circuit characteristics are the same.

EXAMPLE

$$R = 4.5 \text{ ohms} \quad X = 4.0 \text{ ohms} \quad \text{Power factor} = .70$$

$$\text{From nomograph } K = 6.0$$

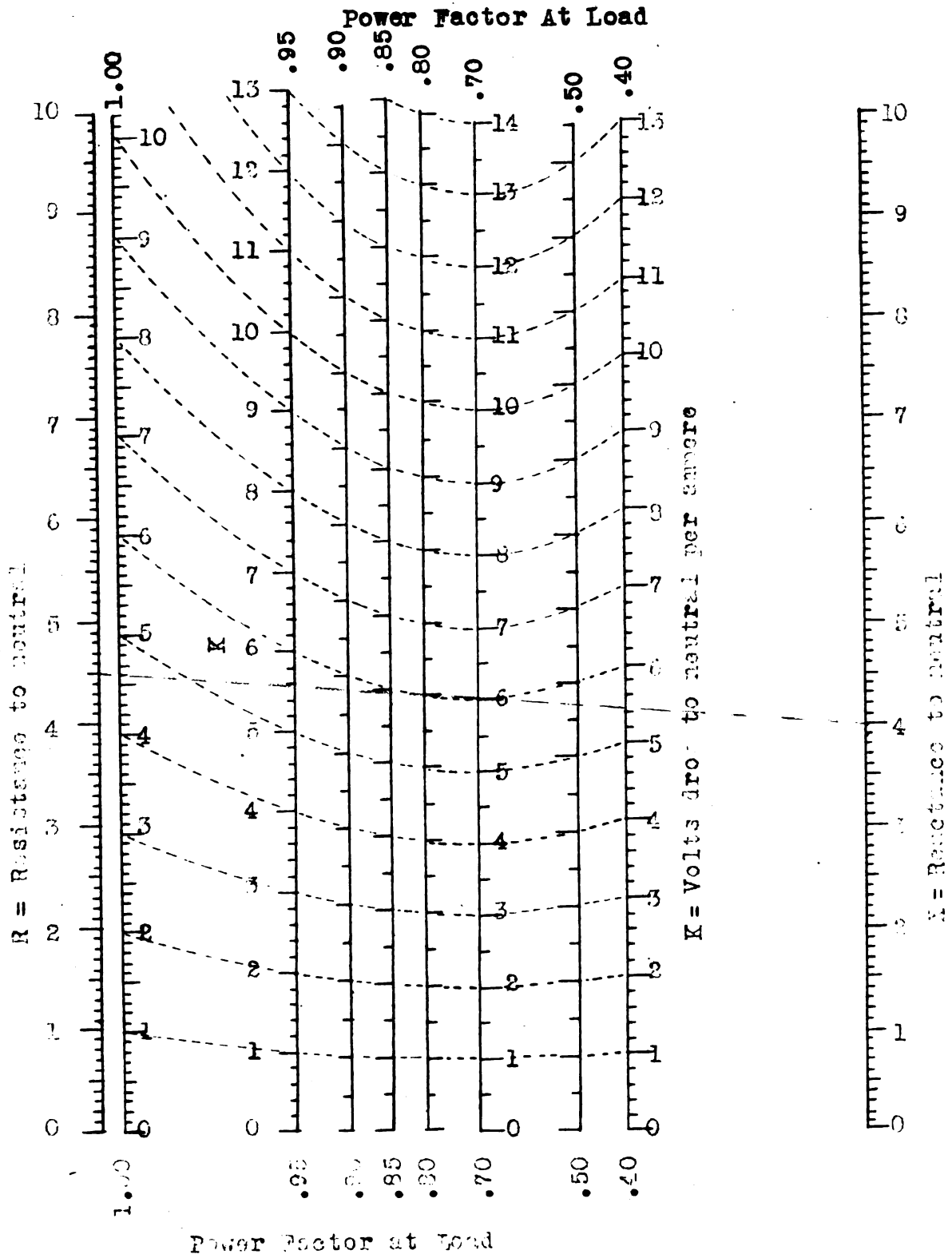
(a) Single phase current = 5 amps.

$$\text{Single phase voltage drop} = 2 \times 5 \times 6.0 = 60 \text{ volts.}$$

(b) Three phase current = 5 amps.

$$\text{Three phase voltage drop} = 1.732 \times 5 \times 6.0 = 51.9 \text{ volts.}$$

VOLTAGE REGULATION CHART for BALANCED A.C. CIRCUITS



Voltage drop for:

Single phase = $2 I K$

Three phase = $1.732 I K$

ELECTRICAL SYSTEM
PLANNING DIVISION

HCR -

METHOD II

FOR LAGGING POWER FACTORS

Where one works mostly with a given voltage, it is often desirable to compute the voltage drop using this voltage as the base voltage. Hence the values of resistance and reactance for transmission line, substation, distribution circuit, distribution transformer, and secondary circuit are expressed in relation to this voltage in terms of an equivalent circuit. A nomograph may be constructed with scales calibrated for resistance, reactance, power factor, length of circuit or a multiplying constant, current, percent voltage drop, and volts drop on some other base voltage upon which voltage flicker limits are based.

From Method I, we will use the formula

$$1. \quad \overline{AC} = E_s - E_r = IR \cos \phi + IX \sin \phi + \frac{1}{2} \frac{(IX \cos \phi - IR \sin \phi)^2}{E_s}$$

Since the last term of this equation has so little effect upon the result (except at unity power factor or where R is very large compared with X and the power factor is very poor) we may dispense with it for this nomograph. Generally the current and power factor are estimated in practice so that the inclusion of this term might assume accuracy which is not warranted. Hence we will use the abbreviated equation:

$$2. \quad E_s - E_r = IR \cos \phi + IX \sin \phi$$

Where: I = line current

R = line resistance

X = line reactance

$\phi = \cos^{-1}(\text{Power Factor})$

Let: E_s = sending voltage to neutral.

E_r = receiver voltage to neutral.

r = resistance to neutral per unit length of line.

x = reactance " " " " " " " " .

L = the number of units of length of line.

Substituting these elements in equation 2, gives:

$$3. \quad E_s - E_r = L I (r \cos \phi + x \sin \phi)$$

The percent voltage drop may be expressed as:

$$4. \quad D = 100 \frac{E_s - E_r}{E_s} = \frac{100}{E_s} L I (r \cos \phi + x \sin \phi)$$

Let equation 4 be broken up into parts, so that:

$$5. \quad A = \frac{100}{E_s} (r \cos \phi + x \sin \phi)$$

$$6. \quad B = L A$$

Then:

$$7. \quad D = I B$$

Equations 5, 6, and 7 will form three determinates for plotting three nomographs, and if we choose the scales correctly they may be combined into one nomograph having two turning scales which will not require calibrating.

Expressing equation 5 in determinate form, we have:

$$8. \begin{vmatrix} 1 & 0 & \frac{100 x}{E_s} \\ 0 & 1 & -A \\ \sin \phi & - & -\frac{100 r \cos \phi}{E_s} \end{vmatrix} = 0$$

Expressing equation 6 in determinate form, gives:

$$9. \begin{vmatrix} 1 & 0 & -A \\ 0 & 1 & B \\ L & 1 & 0 \end{vmatrix} = 0$$

Expressing equation 7 in determinate form, gives:

$$10. \begin{vmatrix} 1 & 0 & B \\ 0 & 1 & -D \\ I & 1 & 0 \end{vmatrix} = 0$$

To plot the nomograph, let us choose a length of 10 inches for the scales of x, A, B, and D. Assume the following conditions:

- $E_s = 24000/(3)^{\frac{1}{3}}$ volts to neutral, the base voltage.
- r varies from 0 to 10 ohms.
- x varies from 0 to 10 ohms (scale modulus = 120).
- A varies from 0 to .087 (scale modulus = 120).
- L varies from 0 to 100
- I varies from 0 to 10

Assuming an abscissa length of 10 inches and applying the given scale moduli to determinate 8, we have:

$$\begin{array}{rcl}
 & (x) & (y) \\
 11. & \left| \begin{array}{ccc} 1 & 0 & \frac{1}{2}(3)^{\frac{1}{2}} x \\ 1 & 10 & - 120 A \\ 1 & \frac{10}{1 + \sin \phi} & - \frac{\frac{1}{2}(3)^{\frac{1}{2}} r \cos \phi}{1 + \sin \phi} \end{array} \right| & = 0
 \end{array}$$

From the above determinate, the plotting equations are:

For: $x \begin{cases} y = .866 x \text{ inches.} \\ x = 0 \text{ inches} \end{cases}$

For $A \begin{cases} y = - 120 A \text{ inches (do not calibrate this scale)} \\ x = 10 \text{ inches.} \end{cases}$

For $r, \phi \begin{cases} y = - \frac{.866 r \cos \phi}{1 + \sin \phi} \text{ inches} \\ x = \frac{10}{1 + \sin \phi} \text{ inches.} \end{cases}$

Using the same scale modulus for A and a scale modulus of 2 for B, and applying these scale moduli to determinate 9 with an abscissa length of 10 inches, we have:

$$\begin{array}{rcl}
 & (x) & (y) \\
 12. & \left| \begin{array}{ccc} 1 & 0 & - 120 A \\ 1 & 10 & 2 B \\ 1 & \frac{600}{L + 60} & 0 \end{array} \right| & = 0
 \end{array}$$

From determinate 12, the plotting equations are:

For: A $\begin{cases} y = -120 \text{ A inches (do not calibrate this scale).} \\ x = 0 \text{ inches.} \end{cases}$

For B $\begin{cases} y = 2 \text{ B inches (do not calibrate this scale).} \\ x = 10 \text{ inches.} \end{cases}$

For: L $\begin{cases} y = 0 \text{ inches.} \\ x = \frac{600}{L + 60} \text{ inches.} \end{cases}$

Using the scale modulus of 2 for B and a scale modulus of unity for D, and applying these scale moduli to determinate 10 with an abscissa length of 10 inches, gives:

	(x)	(y)	
	0	2 B	
13. 1	10	- D	= 0
1	$\frac{20}{I + 2}$	0	

From determinate 13, the plotting equations are:

For: B $\begin{cases} y = 2 \text{ B inches (do not calibrate this scale)} \\ x = 0 \text{ inches} \end{cases}$

For: D $\begin{cases} y = -D \text{ inches.} \\ x = 10 \text{ inches.} \end{cases}$

For: I $\begin{cases} y = 0 \text{ inches.} \\ x = \frac{20}{I + 2} \text{ inches.} \end{cases}$

The plotting equations from determinates 11, 12, and 13, will produce three individual nomographs. Since we have chosen the ^{same}/scale moduli for the variables common to each individual nomograph, we may combine these three nomographs into one nomograph and use the scales A and B as turning scales. This is similar to constructing the three nomographs and then superimposing them so that the scales common to each coincide.

On page 60, we assumed the value of $24000/(3)^{\frac{1}{2}}$ for E_s , which is the voltage to neutral of a 24 Kv. three phase transmission system. Hence the current, I, in determinate 13 is to be considered the three phase line current. We could construct another nomograph for single phase voltage drop; however, we may accomplish the same thing by constructing a second scale for single phase current on the same nomograph. For a single phase system: (24 Kv.)

$$E_s = 24000/2 \quad \text{to neutral}$$

Hence for single phase line current, the plotting equations for the current I, from determinate 13, may be expressed as:

$$I \begin{cases} y = 0 \text{ inches.} \\ x = \frac{20}{2 I/(3)^{\frac{1}{2}} + 2} \text{ inches.} \end{cases}$$

See Sketch # HCR - 4 in rear of thesis.

SOLUTION

(1) Join the value of X with a straight edge to that of R on the desired power factor scale and locate the intersection with turning scale A . (2) Join this point with a straight edge to L on the length scale and locate the intersection with turning scale B . (3) Join this point with a straight edge to the value of I on the current scale and read the percent voltage drop.

EXAMPLE

$x = 4$ $r = 7$ Power factor = .80

$L = 40$ units $I = 2$ amps., three phase

From nomograph:

Percent voltage drop = 4.65

(See Sketch # HCR - 4)

METHOD III

FOR LAGGING POWER FACTORS

It is often desirable to compute the voltage drop on the overhead circuits only, where the impedance of the transmission line and substation may be neglected. For cases of small loads, or where a quick estimate of wire size is desired for an individual power customer, this calculation is sufficient.

Hence, we will attempt to construct a nomograph for close approximation using scales calibrated for conductor spacing, wire size, power factor, and volts drop per ampere to neutral. Using the approximate formula for voltage drop from Method II, page 59, we have:

$$1. \quad E_s - E_r = LI (r \cos \phi + x \sin \phi)$$

For this nomograph we will let:

L = length of line in 1000 ft. units.

I = line current in amperes.

r = resistance to neutral per 1000 ft.

x = reactance to neutral per 1000 ft.

for 60 cycle frequency.

From a preceding part of this thesis (page 38), the inductive reactance per 1000 ft. of conductor to neutral for 60 cycle frequency was developed, as:

$$2. \quad x = .0529 \log 1.284 \frac{d}{a}$$

Where: d = conductor spacing in inches.

a = conductor radius in inches.

Let equation 1 be broken up into two parts, so that:

$$3. \quad E_s - E_r = L I K$$

and:

$$4. \quad K = r \cos \phi + x \sin \phi$$

Substituting equation 2 into equation 4, gives:

$$\begin{aligned} 5. \quad K &= r \cos \phi + .053 \sin \phi \log \frac{d(1.284)}{a} \\ &= r \cos \phi + .053 \sin \phi (\log d + \log \frac{1.284}{a}) \\ &= \text{volts drop to neutral per amp. per 1000 ft. of conductor} \end{aligned}$$

We will construct the nomograph for the solution of equation 5 only, since the solution of the voltage drop is (from equation 3) just a matter of multiplication after the value of K is determined.

Examining equation 5, we see that the resistance, r , and the radius, a , of the conductor are directly related. Hence we will not require a scale for conductor radius if we construct a scale calibrated for wire size.

Setting up equation 5 in determinate form, gives:

$$6. \quad \begin{vmatrix} 1 & 0 & - .053 \log d \\ 0 & 1 & K \\ \sin \phi & 1 & r \cos \phi + .053 \sin \phi \log \frac{1.284}{a} \end{vmatrix} = 0$$

Assume a 3 inch scale for d and a 6 inch scale for K.

If d varies from 1 to 100, its scale modulus = 30.

If K varies from 0 to .60, its scale modulus = 10

Assume the abscissa length = 10 inches.

Applying the above moduli and abscissa length to the determinate 6, we have:

$$7. \left| \begin{array}{cc} (x) & (y) \\ 1 & 0 \\ & - 1.59 \log d \\ 1 & 10 K \\ 1 & \frac{30}{3 + \sin \phi} \end{array} \right| = 0$$

$$\frac{30 r \cos \phi + 1.59 \sin \phi \log \frac{1.284}{a}}{3 + \sin \phi}$$

From determinate 7, the plotting equations are:

For: $d \begin{cases} y = - 1.59 \log d \text{ inches} \\ x = 0 \text{ inches.} \end{cases}$

For: $K \begin{cases} y = 10 K \text{ inches.} \\ x = 10 \text{ inches.} \end{cases}$

For: Wire size and Power Factor $\begin{cases} y = \frac{30 r \cos \phi + 1.59 \sin \phi \log \frac{1.284}{a}}{3 + \sin \phi} \text{ ins.} \\ x = \frac{30}{3 + \sin \phi} \text{ inches.} \end{cases}$

SOLUTION

Join the conductor spacing, on the spacing scale, with a straight edge to the wire size on the desired power factor scale, and read the value of K on the K scale.

Then the:

$$\text{Three phase line drop} = (3)^{\frac{1}{2}} L I K = \frac{Kva.}{E_s} L K.$$

$$\text{Single phase line drop} = 2 L I K = \frac{2 Kva.}{E_s} L K.$$

(See sketch on next page 70)

EXAMPLE

Conductor spacing = 40 inches.

Wire size = #2 copper.

Power factor = .90

From nomograph, K = .21

I = 10 amps. line current.

L = 4000 feet.

$$\text{Three phase line drop} = (3)^{\frac{1}{2}} \times 4 \times 10 \times .21 = 14.6 \text{ volts}$$

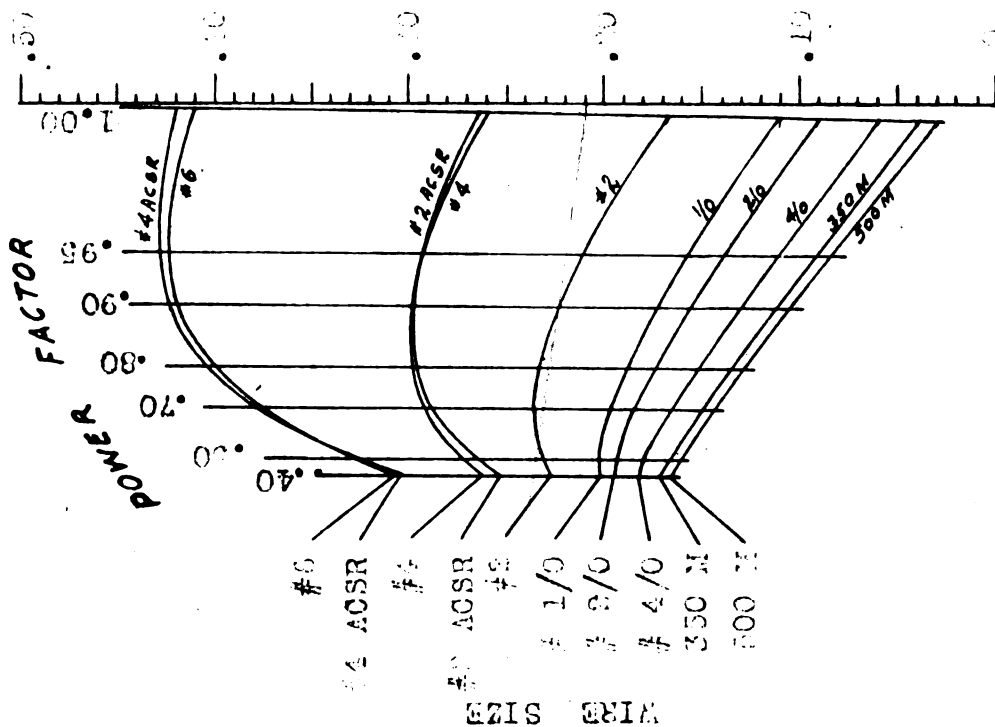
$$\text{Single phase line drop} = 2 \times 4 \times 10 \times .21 = 16.8 \text{ volts}$$

(See sketch on next page)

VOLTAGE DROP CHART

CONDENSATOR SPACING IN INCHES

1 2 3 4 5 6 7 8 9 10 20 30 40 50 100 200 300



Voltage drop for:
 Single phase = 2 I K
 Three phase = 1.732 I K

1000 VOLT 979.5 V
 1000 VOLT 979.5 V

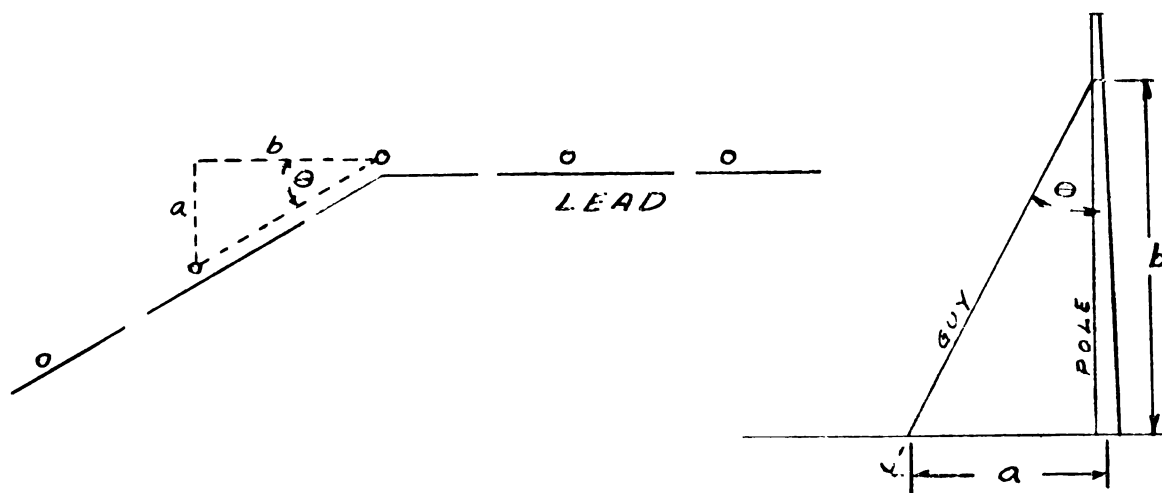
SECTION B

MECHANICAL PROBLEMS

STRESSES IN THE STRAND
OF ANCHOR GUYS

RELATIONSHIP BETWEEN THE ANGLE AND THE ELEMENTS OF ITS TANGENT

In rough field work, or where instrument accuracy is not required, angles in the lead are generally measured by the tangent method. This is done by pacing off the legs of a triangle; one side (b) of which is in the direction of the lead and the other side (a) in the direction of the offset. This method is also used to determine the steepness of pole guys, or anchor guys.



From the above figures, the following relation may be expressed:

$$1. \quad \tan \theta = a/b$$

or in logarithmic form:

$$2. \quad \log \tan \theta - \log a + \log b = 0$$

Equation 2 may be set up in determinate form, as:

$$3. \quad \begin{vmatrix} 1 & 0 & -\log b \\ 0 & 1 & \log a \\ 1 & 1 & \log \tan \theta \end{vmatrix} = 0$$

Let:

The scale modulus of $a = 3$

The scale modulus of $b = 6$

The abscissa length = 4.5 inches.

Applying these moduli and abscissa length to determinate 3, gives:

$$4. \quad \begin{array}{c|cc} & (x) & (y) \\ \hline 1 & 0 & -6 \log b \\ 1 & 4.5 & 3 \log a \\ 1 & 3 & 2 \log \tan \theta \end{array} = 0$$

From the above determinate the plotting equations are:

$$\text{For: } b \begin{cases} y = -6 \log b \text{ inches.} \\ x = 0 \text{ inches.} \end{cases}$$

$$\text{For: } a \begin{cases} y = 3 \log a \text{ inches.} \\ x = 4.5 \text{ inches.} \end{cases}$$

$$\text{For: } \begin{array}{l} \text{ratio } a/b \\ \text{or } \tan \theta \end{array} \begin{cases} y = 2 \log \tan \theta \text{ inches.} \\ x = 3 \text{ inches.} \end{cases}$$

SOLUTION

Join the values of a and b on their respective scales with a straight edge, and read the value of a/b or θ on the tangent scale.

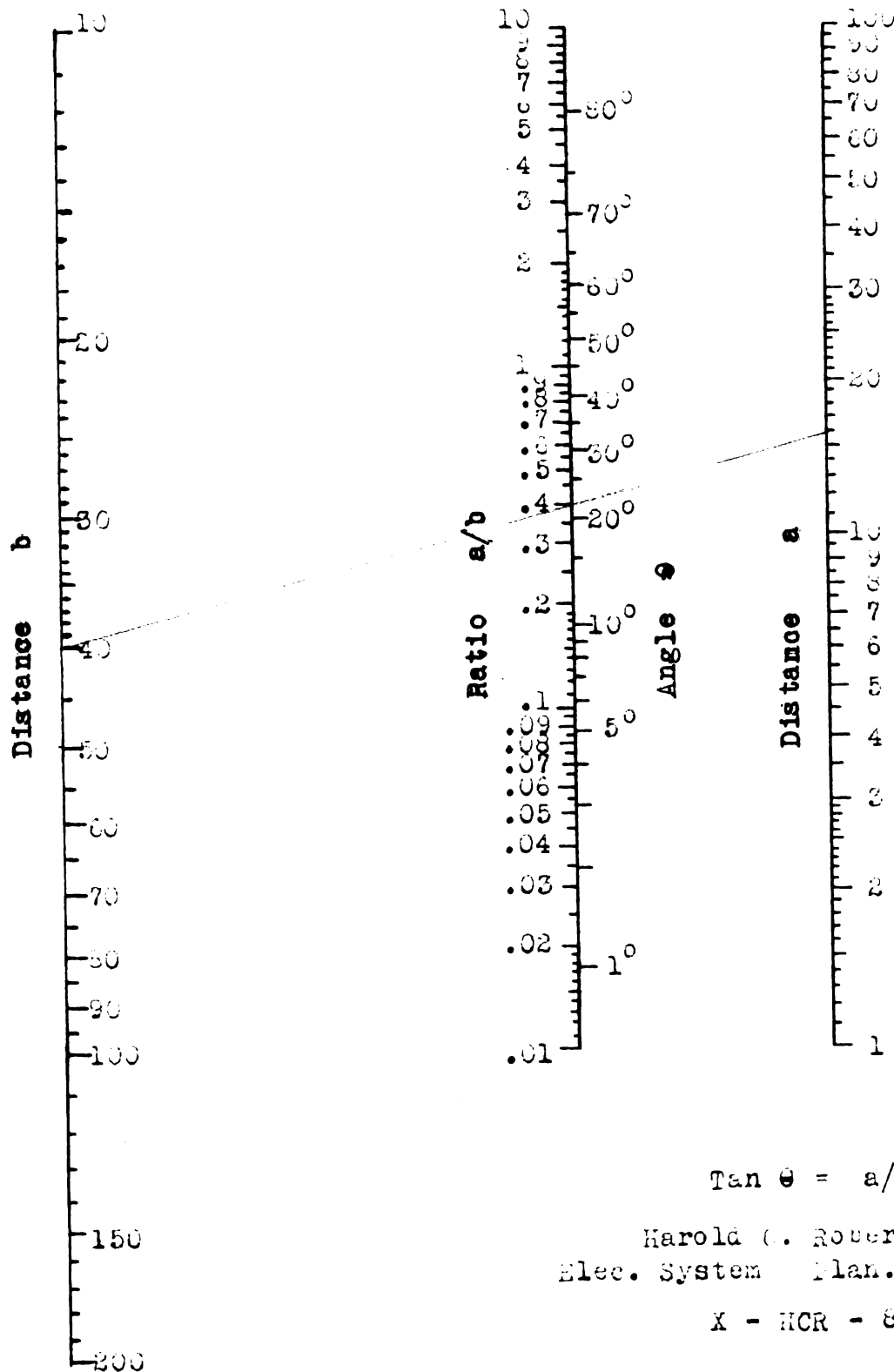
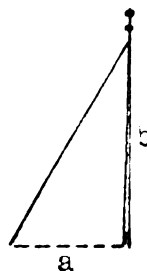
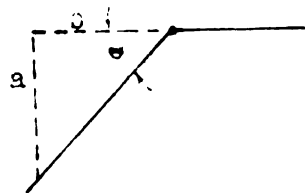
See sketch on next page (75).

EXAMPLE

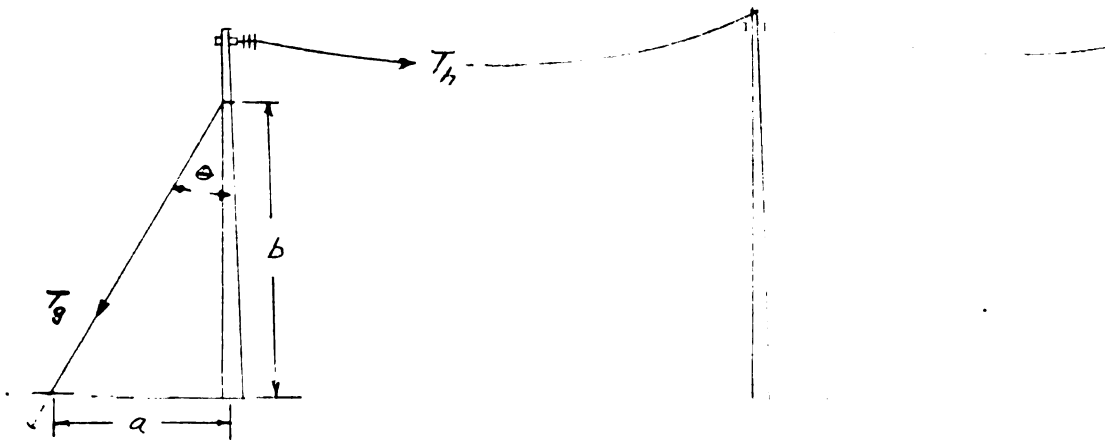
$$a = 16 \quad b = 40 \quad \text{From nomograph, } a/b = .40 \quad \theta = 22^\circ \pm$$

PTI CHART

giving ratio a/b and angle θ in degrees



TENSION IN A POLE GUY AT A DEAD END



From the above figure, the following relation may be expressed:

$$1. \quad T_g = T_h / \sin \theta = \frac{T_h}{\sin (\tan^{-1} a/b)}$$

OR in logarithmic form:

$$2. \quad \log T_g - \log T_h + \log \sin (\tan^{-1} a/b)$$

Expressing equation 2 in determinate form, gives:

$$3. \quad \begin{vmatrix} 1 & 0 & \log T_h \\ 0 & 1 & - \log \sin (\tan^{-1} a/b) \\ 1 & 1 & \log T_g \end{vmatrix} = 0$$

Let: The scale modulus for $T_h = 6$

The scale modulus for the ratio $a/b = 6$

The abscissa length = 5 inches.

NOTE: In the above figure, T_h is assumed to be the equivalent conductor tension acting at the point of guy attachment on the pole.

Applying these scale moduli and abscissa length to determinate 3 , gives:

$$4. \quad \left| \begin{array}{cc} (x) & (y) \\ 1 & 0 & 6 \log T_h \\ 1 & 5 & -6 \log \sin (\tan^{-1} a/b) \\ 1 & 2.5 & 3 \log T_g \end{array} \right| = 0$$

From the above determinate, the plotting equations are:

For: $T_h \begin{cases} y = -6 \log T_h \text{ inches.} \\ x = 0 \text{ inches.} \end{cases}$

For: Ratio $a/b \begin{cases} y = -6 \log \sin (\tan^{-1} a/b) \text{ inches.} \\ x = 5 \text{ inches.} \end{cases}$

For: $T_g \begin{cases} y = 3 \log T_g \text{ inches.} \\ x = 2.5 \text{ inches.} \end{cases}$

SOLUTION

Join with a straight edge the values of T_h and a/b on their respective scales and read the value of T_g on the T_g scale.

See sketch on next page (78).

EXAMPLE

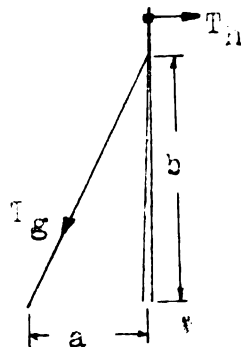
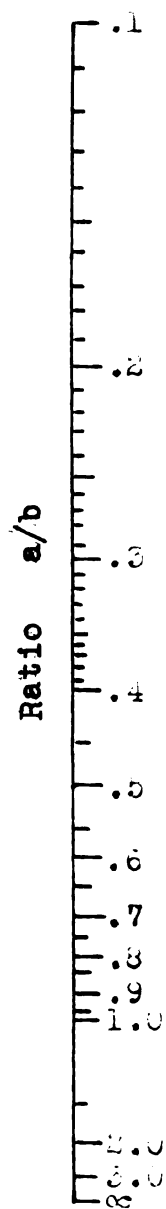
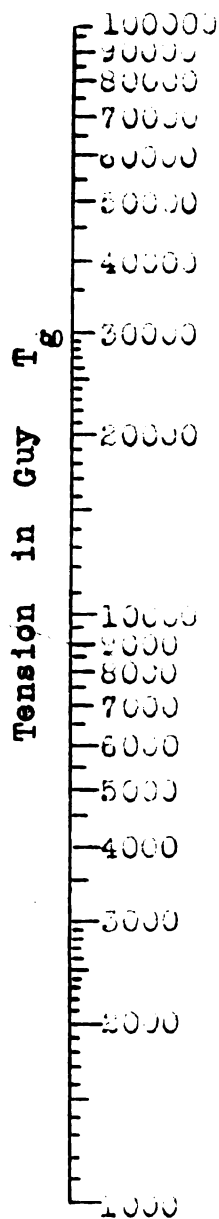
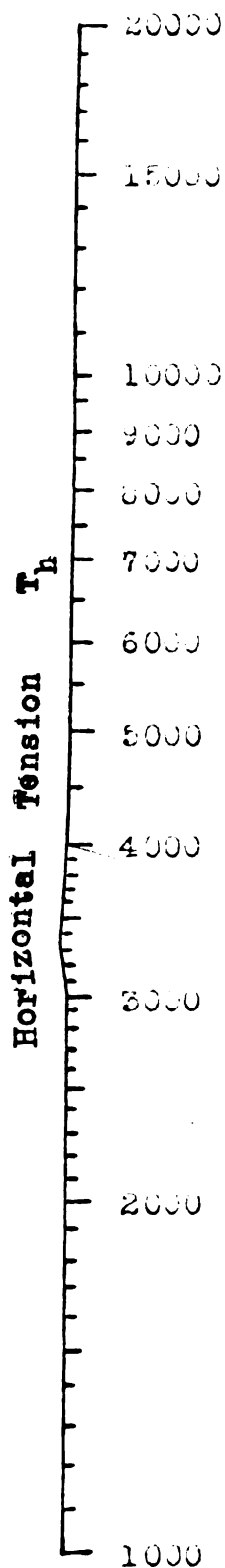
$a = 20$ $b = 40$ hence $a/b = .50$

$T_h = 4000$ pounds From nomograph, $T_g = 9000$ pounds $\pm$

See sketch on next page.

WUY CHART

Finding Tension in Pole Guy

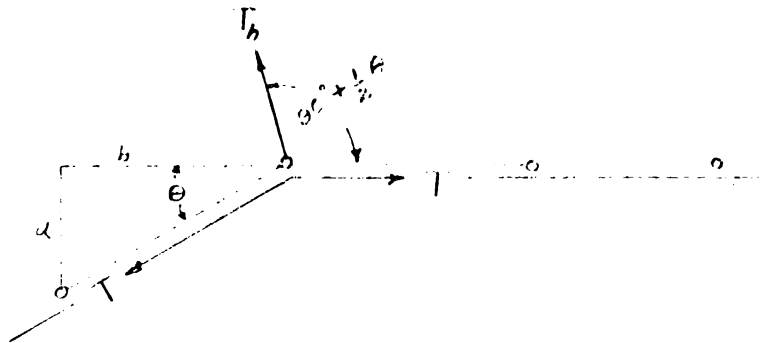


$$T_g = \frac{T_h}{\sin(\tan^{-1} a/b)}$$

Harold C. Roberts
Elec. System Plan. Division
X - MCR - 9

THE HORIZONTAL COMPONENT OF THE TENSION IN A GUY WHERE THE GUY SPLITS THE ANGLE OF THE CORNER

Where there is a small corner in a lead, it is often desirable to install the anchor guy so that it splits the angle of the corner and thus leaves the climbing space clear on the pole.



If the total conductor tension on each side of the corner pole is the same, the horizontal component of the tension in the anchor guy may be expressed as:

$$1. \quad T_h = 2 T \sin \frac{1}{2} \theta = 2 T \sin \frac{1}{2} (\tan^{-1} a/b)$$

Or in logarithmic form:

$$2. \quad \log T_h - \log 2T - \log \sin \frac{1}{2} \theta = 0$$

Setting up equation 2 in determinate form, gives:

$$3. \quad \begin{vmatrix} 1 & 0 & \log 2T \\ 0 & 1 & \log \sin \frac{1}{2} \theta \\ 1 & 1 & \log T_h \end{vmatrix} = 0$$

Let: The scale modulus of $2T = 6$

The scale modulus of $\theta = 3$

The abscissa length = 4.5 inches.

Applying these scale moduli and abscissa length to determinate 3 , gives:

$$4. \quad \begin{array}{c|cc} & (x) & (y) \\ \hline 1 & 0 & 6 \log 2T \\ 1 & 4.5 & 3 \log \sin \frac{1}{2}\theta \\ 1 & 3 & 2 \log T_h \end{array} \quad = 0$$

From the above determinate the plotting equations are:

$$\text{For: } 2T \quad \begin{cases} y = 6 \log 2T & \text{inches.} \\ x = 0 & \text{inches} \end{cases}$$

$$\text{For: Ratio } a/b \text{ or angle } \theta \quad \begin{cases} y = 3 \log \sin \frac{1}{2}\theta = 3 \log \sin \frac{1}{2}(\tan^{-1} a/b) & \text{ins.} \\ x = 4.5 & \text{inches.} \end{cases}$$

$$\text{For: } T_h \quad \begin{cases} y = 2 \log T_h & \text{inches.} \\ x = 3 & \text{inches} \end{cases}$$

SOLUTION

With a straight edge, join the values of $2T$ and the ratio a/b or θ on their respective scales and read the value of T_h on the T_h scale.

See sketch on next page - 81.

EXAMPLE

$2T = 5500$ pounds ratio $a/b = .3$

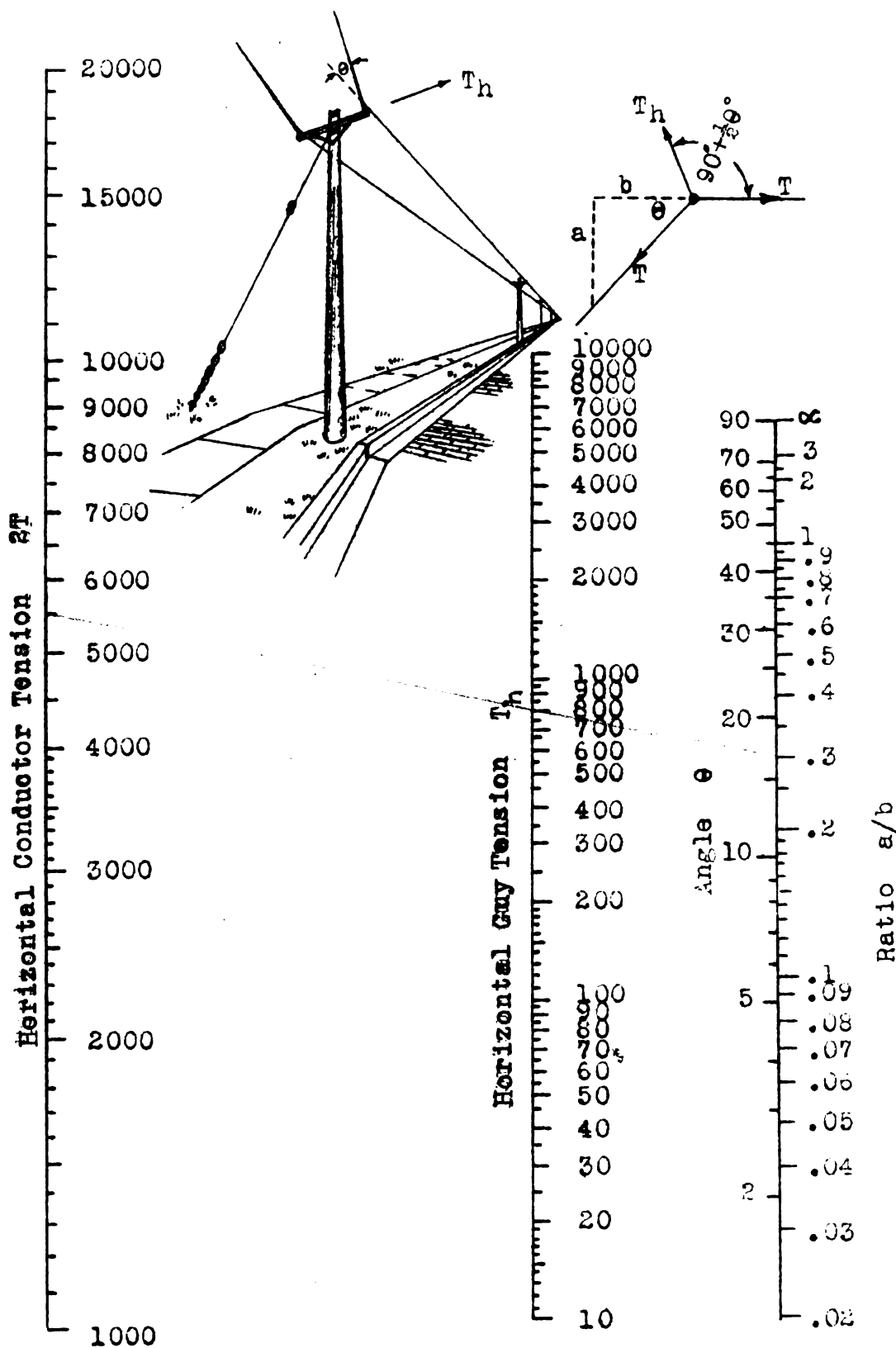
From nomograph, $T_h = 800$ pounds.

See sketch on next page.

GUY CHART

Giving Horizontal Tension in Guy

Where Guy Splits Angle of Corner

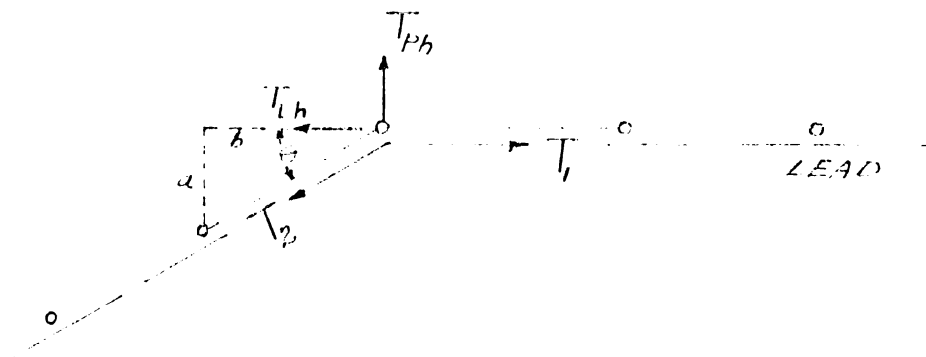


$$T_h = 2 T \sin \theta$$

Harold C. Roberts
Elec. System Plan. Division
X - ACR - 10

Often where there is a corner in the lead, it is not possible to install an anchor guy that splits the angle of the corner. In such cases, two guys are installed; one in the direction of the lead and the other perpendicular to the lead. Especially does this condition exist at street intersections where the line crosses the street in a diagonal manner.

HORIZONTAL COMPONENT OF THE TENSION IN A GUY
WHERE THE GUY IS IN THE DIRECTION OF THE LEAD



From the above figure, the following relation may be expressed:

$$1. \quad T_{Lh} = T_1 - T_2 \cos (\tan^{-1} a/b)$$

We shall break equation 1 into two parts, so that:

$$2. \quad K = T_2 \cos (\tan^{-1} a/b)$$

and

$$3. \quad T = T_1 - K$$

Setting up equation 2 in determinate form, gives:

$$4. \quad \begin{vmatrix} 1 & 0 & K \\ 0 & 1 & -T_2 \\ 1 & \cos (\tan^{-1} a/b) & 0 \end{vmatrix} = 0$$

Let: The modulus of $K = 1/2000$

The modulus of $T_2 = 1/2000$

The abscissa length = 3.33 inches.

Applying these scale moduli and abscissa length to determinate 4, gives:

$$5. \quad \begin{vmatrix} 1 & (x) & (y) \\ 1 & 0 & K/2000 \\ 1 & 3.33 & -T_2/2000 \\ 1 & \frac{3.33 \cos(\tan^{-1} a/b)}{1 + \cos(\tan^{-1} a/b)} & 0 \end{vmatrix} = 0$$

From the above determinate, the plotting equations are:

$$\text{For: } K \begin{cases} y = K/2000 \text{ inches. (do not plot this scale)} \\ x = 0 \text{ inches.} \end{cases}$$

$$\text{For: } T_2 \begin{cases} y = -T_2/2000 \text{ inches} \\ x = 3.33 \text{ inches.} \end{cases}$$

$$\text{For: Ratio } a/b \text{ or angle } \theta \begin{cases} y = 0 \text{ inches.} \\ x = \frac{3.33 \cos(\tan^{-1} a/b)}{1 + \cos(\tan^{-1} a/b)} \text{ inches.} \end{cases}$$

Setting up equation 3 in determinate form, gives:

$$6. \quad \begin{vmatrix} 1 & 0 & T_1 \\ 0 & 1 & -T_{Lh} \\ 1 & 1 & K \end{vmatrix} = 0$$

Let: The scale modulus of $T_1 = 1/1000$
The scale modulus of $T_{Lh} = 1/1000$
The abscissa length = 7.8 inches.

Applying these scale moduli and abscissa length to determinate 6, gives:

$$7. \quad \begin{vmatrix} & (x) & (y) \\ 1 & 0 & T_1/1000 \\ 1 & 7.8 & T_{Lh}/1000 \\ 1 & 3.9 & K/2000 \end{vmatrix} = 0$$

From the above determinate, the plotting equations are:

$$\text{For: } T_1 \begin{cases} y = T_1/1000 \text{ inches.} \\ x = 0 \text{ inches.} \end{cases}$$

$$\text{For: } T_{Lh} \begin{cases} y = -T_{Lh}/1000 \text{ inches.} \\ x = 7.8 \text{ inches.} \end{cases}$$

$$\text{For: } K \begin{cases} y = K/2000 \text{ inches. (do not calibrate this scale)} \\ x = 3.9 \text{ inches.} \end{cases}$$

We see that by choosing the correct scale moduli for T_1 and T_{Lh} , the scale modulus for K from determinate 7 is the same as that found in determinate 5, so that we may use the K scale for the turning scale and combine the two nomographs. Hence the K scale will not need to be calibrated.

The solution for this nomograph will be given later.

(See page 86)



HORIZONTAL COMPONENT OF THE TENSION IN THE GUY

where the guy is perpendicular to the lead

From the figure on page 82, we derive the following relation:

$$8. \quad T_{ph} = T_2 \sin (\tan^{-1} a/b)$$

Or in logarithmic form:

$$9. \quad \log T_{ph} - \log T_2 - \log \sin (\tan^{-1} a/b) = 0$$

Setting up equation 9 in determinate form, gives:

$$10. \quad \begin{vmatrix} 1 & 0 & \log \sin (\tan^{-1} a/b) \\ 0 & 1 & \log T_2 \\ 1 & 1 & \log T_{ph} \end{vmatrix} = 0$$

Let: The scale modulus for $T_2 = 6$

The scale modulus for ratio a/b or $\theta = 3$

The abscissa length = 4.5 inches.

Applying these scale moduli and abscissa length to determinate 10, gives:

$$11. \quad \begin{vmatrix} & (x) & (y) \\ 1 & 0 & 3 \log \sin (\tan^{-1} a/b) \\ 1 & 4.5 & 6 \log T_2 \\ 1 & 1.5 & 2 \log T_{ph} \end{vmatrix} = 0$$

11

From determinate 11, the plotting equations are:

$$\text{For Ratio } a/b \text{ or angle } \theta \begin{cases} y = 3 \log \sin (\tan^{-1} a/b) & \text{inches.} \\ x = & 0 \text{ inches.} \end{cases}$$

$$\text{For } T_2 \begin{cases} y = 6 \log T_2 & \text{inches.} \\ x = 4.5 & \text{inches.} \end{cases}$$

$$\text{For } T_{ph} \begin{cases} y = 2 \log T_{ph} & \text{inches.} \\ x = 1.5 & \text{inches.} \end{cases}$$

SOLUTIONS

A. For the horizontal component of the tension in the guy, where the guy is in the direction of the lead.

1. Join the values of T_2 and the ratio a/b on their respective scales with a straight edge and locate point K on the TURNING SCALE.

2. Join this point (K) with a straight edge to the value of T_1 on the T_1 scale and read the value of T_{Lh} on the T_{Lh} scale.

See sketch on page 88.

B. For the horizontal component of the tension in the guy, where the guy is perpendicular to the lead.

With a straight edge, join the values of the ratio a/b , or angle θ , and T_2 on their respective scales and read the value of T_{ph} on the T_{ph} scale.

See sketch on page 89.

EXAMPLE

$$T_1 = 4000 \text{ pounds.}$$

$$T_2 = 2500 \text{ pounds.}$$

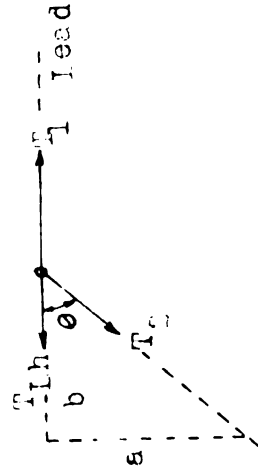
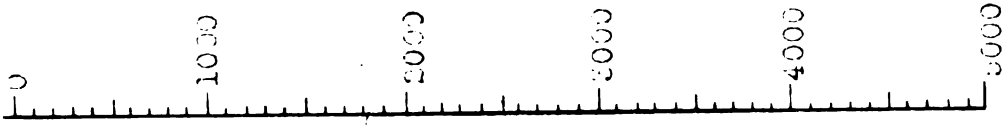
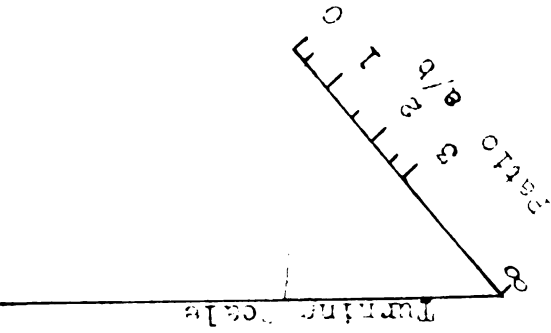
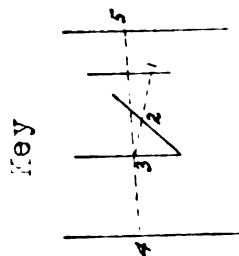
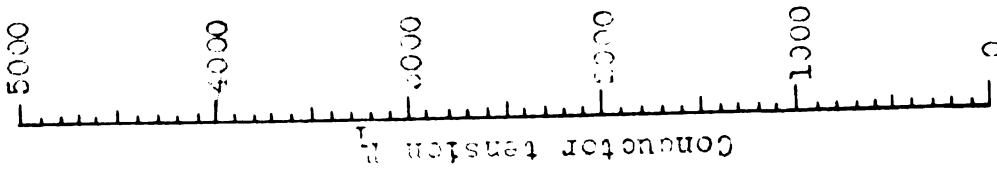
$$a/b = .50$$

From nomograph on page 88, $T_{Lh} = 2170 \pm$ pounds.

From nomographs on page 89, $T_{ph} = 1120 \pm$ pounds.

GUY CHART

Giving Horizontal Tension in Guy
where Guy is in direction of Lead



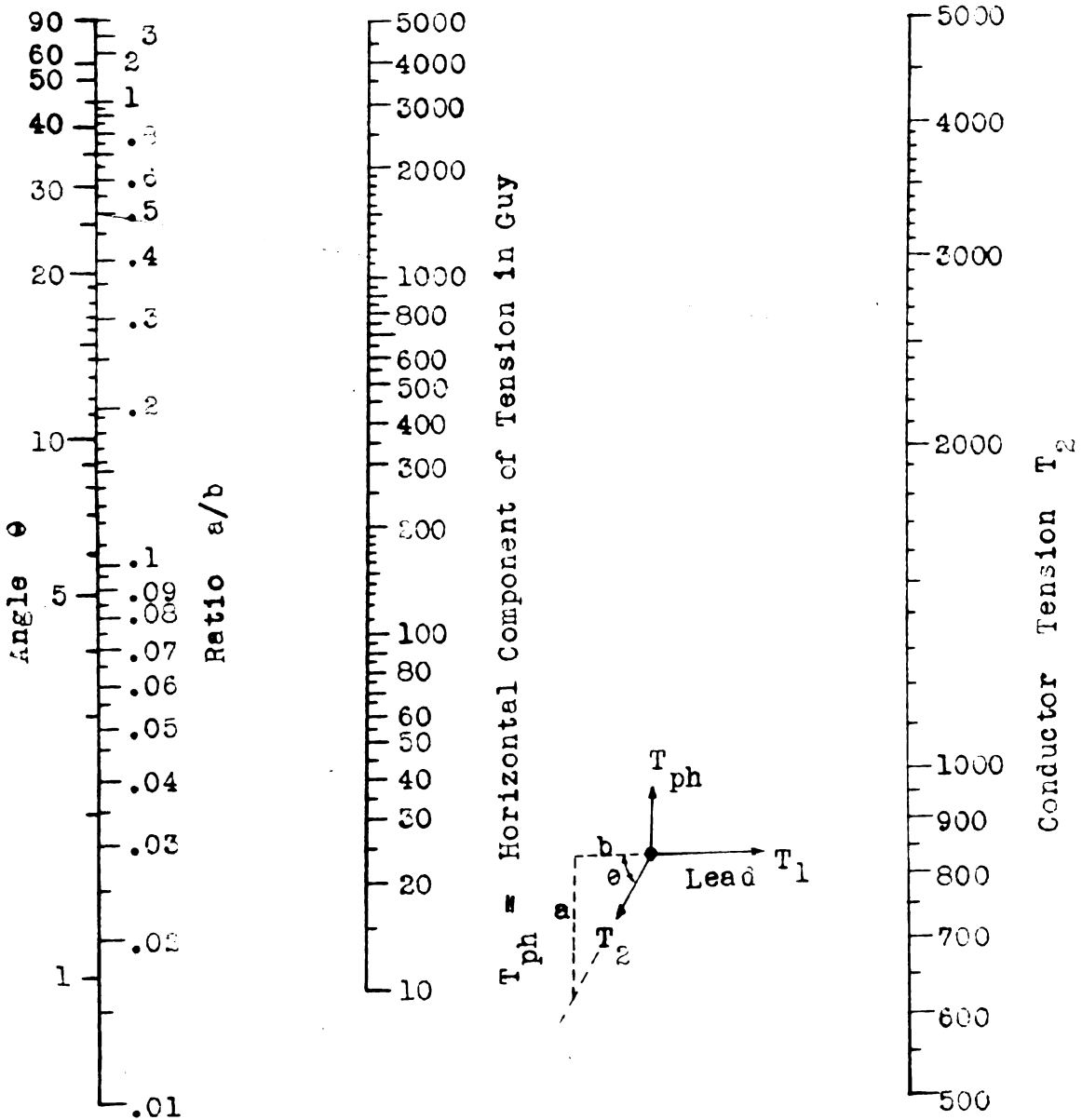
Electrical System
Planning Division
ECR - C
S - 11 - 21

$$T_{lh} = T_1 - T_2 \cos \theta$$

T_{lh} = Horizontal component of tension in Guy

GUY CHART

Giving Horizontal Tension in Guy
Where Guy is Perpendicular to Lead



Electrical System
Planning Division

HCR - 8

5 - 11 - 31

**SAG AND TENSION COMPUTATIONS
FOR SPANS ASSUMING PARABOLIC FORM
UNDER CHANGES IN LOADING AND TEMPERATURE**

It is often required that the planning engineer design special construction to accomodate certain field conditions. This occurs when the standard types of construction are not suitable due to various restrictions. Such cases under this classification are river crossings, lake crossings, and super highway constructions at complicated intersections.

After pole locations have been established, the next part of the design problem is the determination of conductor tensions and sags for stringing and loaded conditions so that the proper amount of guying may be obtained, and that proper clearances may be maintained.

For spans up to five hundred feet with the normal sags used, we will generally be within construction accuracy to consider the span assumes parabolic form. With this assumption we will proceed to develop the relationships of sag and tension.

SYMBOLS

w = weight per foot of conductor.

L = length of span in feet.

s = sag in feet.

$d = s/L$ = sag factor.

T = conductor tension in pounds (for which guying is computed)

E = modulus of elasticity of conductor in pounds.

A = area of conductor in square inches.

t = temperature in degrees Fahrenheit.

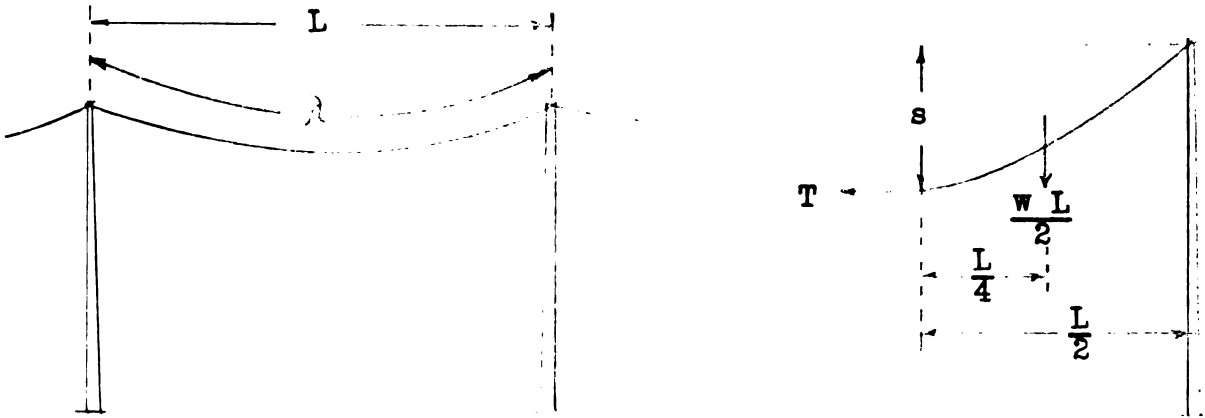
α = coefficient of thermal expansion of conductor per F°

l_0 = zero - stress length factor.

e = elongation per foot length of conductor.

λ = length of conductor in feet.

THE PARABOLIC CABLE



From the above figure the following relationships may be developed:

1. $T = \frac{w L}{8 d}$
2. $\lambda = L \left(1 + \frac{8 d^2}{3} - \frac{32 d^4}{5} + \dots \right)$

Since d is a very small number, all terms after the second in the above series (equation 2) may be disregarded. Hence we use the formula:

3. $\lambda = L \left(1 + \frac{8 d^2}{3} \right)$

In this development we are mainly interested in the horizontal tension T , since this is the value for which the guying must be designed to meet, and the pole foundations are designed to support the vertical load.

The following solution is based upon assumptions that are within the limits of accuracy required in practical overhead-lines construction, and is not to be considered a solution for precision work.

We may define the length factor as the length of conductor divided by the length of span, or, the length of conductor per foot length of span; and for a conductor under tension T , may be expressed as,

$$4. \quad \lambda = \frac{\lambda}{L} = 1 + \frac{8d^2}{3} \quad (\text{using equation 3, page 92})$$

The length factor for zero stress in the conductor may be expressed approximately, as:

$$5. \quad \lambda_0 = 1 + \frac{8d^2}{3} - \frac{T}{AE}$$

The elongation of conductor per foot length of span may be expressed as: (at temperature t)

$$6. \quad e_t = \lambda_0 - 1$$

Hence using equation 5 :

$$7. \quad e_t = \frac{8d^2}{3} - \frac{T}{AE}$$

Substituting the value of T from equation 1 into equation gives:

$$8. \quad e_t + \frac{WL}{8dAE} - \frac{8d^2}{3} = 0$$

Expressing equation 8 in determinate form, gives:

$$9. \quad \begin{vmatrix} 1 & 0 & \frac{WL}{AE} \\ 0 & 1 & e_t \\ \frac{1}{8d} & 1 & \frac{8d^2}{3} \end{vmatrix} = 0$$

For a suitable nomograph we will use:

$$\text{Scale modulus for } \frac{WL}{AE} = 4 \times 10^4$$

$$\text{Scale modulus for } e_t = 2 \times 10^3$$

$$\text{Abcissa length} = 10 \text{ inches.}$$

Applying the above moduli and abscissa length to determinate 9, and rearranging the determinate, we have:

$$10. \quad \begin{array}{c|cc} & (x) & (y) \\ \hline 1 & 0 & 4 \frac{WL}{AE} \times 10^4 \\ 1 & 10 & 2 e_t \times 10^3 \\ 1 & \frac{1600 d}{1 + 160 d} & \frac{256 d^3 \times 10^4}{3(1 + 160 d)} \end{array} = 0$$

From determinate 10, the plotting equations are:

$$\text{For: } \frac{WL}{AE} \begin{cases} y = 4 \frac{WL}{AE} \times 10^4 & \text{inches.} \\ x = 0 & \text{inches.} \end{cases}$$

$$\text{For: } e_t \begin{cases} y = 2 e_t \times 10^3 & \text{inches.} \\ x = 10 & \text{inches.} \end{cases}$$

$$\text{For: } d \begin{cases} y = \frac{256 d^3 \times 10^4}{3(1 + 160 d)} & \text{inches.} \\ x = \frac{1600 d}{1 + 160 d} & \text{inches.} \end{cases}$$

See sketch # X - HCR - 11 in rear of thesis.

The nomograph constructed from the plotting equations on page 94 will solve the condition of changes in loading providing the temperature remains constant. To make the solution applicable to changes in temperature, we use the following reasoning:

The zero-stress length factor for temperature t_1 is,

$$11. \quad \lambda'_0 = 1 + \frac{8 d^2}{3} - \frac{w L}{8 d A E}$$

Hence, the zero-stress length factor for temperature t_2 is,

$$12. \quad \lambda''_0 = \lambda'_0 + \alpha \lambda'_0 (t_2 - t_1)$$

Since λ'_0 is very nearly equal to unity and $\alpha(t_2 - t_1)$ is very small, we may assume:

$$13. \quad \lambda''_0 = \lambda'_0 + \alpha (t_2 - t_1)$$

Subtracting unity from each side of equation 13, gives:

$$14. \quad (\lambda''_0 - 1) = (\lambda'_0 - 1) + \alpha (t_2 - t_1)$$

Or, from equation 6, we have:

$$15. \quad e_{t_2} = e_{t_1} + \alpha (t_2 - t_1)$$

We shall now construct an auxiliary scale for $\alpha(t_2 - t_1)$ so that it may be used in conjunction with the e_t scale of the nomograph. It may be seen from the plotting equation of e_t , that a length of two inches on the e_t scale is equivalent to .001 foot change in length. Hence we will use the following equations for the auxiliary scales:

$$\text{For copper:} \quad y = (2 \times 10^3)(9.6 \times 10^{-6})(t_2 - t_1) \text{ inches.}$$

$$\text{For ACSR} \quad y = (2 \times 10^3)(10.1 \times 10^{-6})(t_2 - t_1) \text{ inches.}$$

$$\text{For Steel:} \quad y = (2 \times 10^3)(7.5 \times 10^{-6})(t_2 - t_1) \text{ inches.}$$

In order to construct a nomograph to a large enough scale for desirable accuracy, and yet not exceedingly large in size, the scale $\frac{WL}{AE}$ was folded on the value 5. Thus we have a Scale "A" for $\frac{WL}{AE}$ for values from 0 to 5, and a Scale "B" for $\frac{WL}{AE}$ for values from 5 to 10. Likewise the values of the sag factor "d" were plotted on two separate scales to accomodate Scales "A" and "B". Hence when solving a given condition of $\frac{WL}{AE}$ on scale "A" use "d" to scale "A", and for a given condition where $\frac{WL}{AE}$ is to scale "B", use "d" to scale "B".

SOLUTION

If the known conditions are: w_1 , d_1 , at temperature t_1 and we wish to find d_2 under conditions of w_2 and t_2 , proceed as follows:

1. Join the point w_1L/AE on the WL/AE scale with a straight edge to the point d_1 on the "d" scale and locate the point e_1 on the e_t scale. (For temperature t_1)
2. To the point e_1 add the length $\propto (t_2 - t_1)$ from the auxiliary scale to locate the point e_2 (For temp. t_2).
3. Locate the point w_2L/AE on the WL/AE scale.
4. Join the point e_2 with a straight edge to the point w_2L/AE on their respective scales and read the new value d_2 on the "d" scale.

See Sketch X - HCR - 11 in rear of thesis.

EXAMPLE

See Sketch # X - HCR - 11

Assume the following stringing conditions:

Using #4 TBWP Medium Hard conductor, solid.

$L = 150$ feet.

$w_1 = .164$ pounds per foot (no wind and no ice)/

$t_1 = 60^\circ$ Fahr.

$A = .033$ square inches.

$E = 14 \times 10^6$

$s_1 = 18$ inches. (1.5 feet)

$d_1 = s/L = .010$

$T_1 = w_1 L / 8d_1 = 307$ pounds.

Find the value of the sag, s_2 , and the tension, T_2 , for loaded conditions of an 8 pound wind, $\frac{1}{8}$ inch of ice, and a temperature, t_2 , of 0° Fahr. This involves a change in temperature of 60 degrees and a change in weight of wire per foot from .164 pounds to 1.16 pounds.

Proceed as follows:

$$w_1 L \times 10^4 / AE = .533$$

$$d_1 = .010$$

$$e_1 = -.0004 \text{ (not necessary to tabulate). (From nomograph)}$$

$$e_2 = e_1 - \propto 60 = -.00098 \text{ (not necessary to tabulate).}$$

$$w_2 L \times 10^4 / AE = 3.77$$

$$d_2 = .0215 \text{ (from nomograph)}$$

Hence:

$$s_2 = .0215 \times 150 = 3.225 \text{ ft} = 38.7 \text{ inches}$$

$$T_2 = 1.16 \times 150 / (8 \times .0215) = 1010 \text{ pounds.}$$

**ANALYSIS OF
A FLEXIBLE DEAD-END POLE**

It occurs sometimes, in subdivisions and on certain private property, that an anchor guy cannot be installed at the end of the lead. In such cases some means must be devised to hold the last pole, or as it is generally called, the "dead-end pole". However, the pole next to the dead-end pole in the lead generally can be anchored and treated as a rigid structure. With such construction, the last pole is called a FLEXIBLE DEAD-END POLE. The amount of tension that may be held by this pole depends mostly upon the pole foundation. It has been found from practice that a pole may be loaded up to one-half its ultimate strength if a log heel and breaster are used, two-thirds its ultimate strength if crushed stone is used around the pole, and full ultimate strength if the pole is set in concrete. However, consideration must be given to the type of soil and the grade of construction from which a suitable safety factor may be determined.

We will now analyze this problem with an attempt to arrive at its solution.

SYMBOLS

n = number of conductors.

w_b = weight per foot of conductor under stringing conditions.

w_c = " " " " " " loaded " .

L = distance between poles at ground line in feet.

L_b = span length in feet under stringing conditions.

L_c = " " " " " " loaded " .

S_b = conductor sag in feet under stringing conditions.

S_c = " " " " " " loaded " .

$d_1 = S_1/L_1$ = sag factor.

T_b = conductor tension in pounds under stringing conditions.

T_c = " " " " " " loaded " .

P = pole deflection constant in pounds per inch deflection.

$\delta = T/P$ = pole deflection in inches.

A = conductor area in square inches. (cross-sectional area)

E = modulus of elasticity of conductor in pounds.

t_b = temperature in degrees Fahr. for stringing conditions.

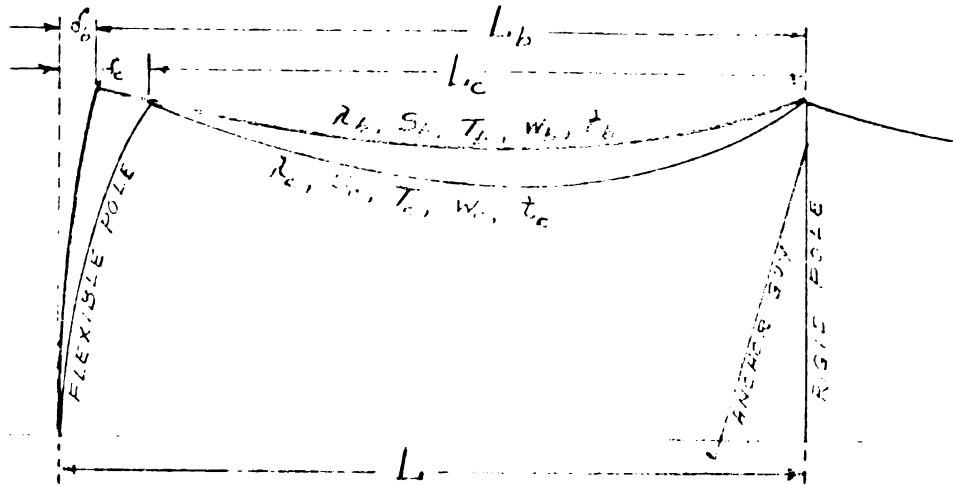
t_c = " " " " " " loaded " .

α = coefficient of thermal expansion of conductor per degree Fahr.

λ_b = conductor length in feet under stringing conditions.

λ_c = " " " " " " loaded " .

Theory



ASSUMPTION: The final tension in the wire will be equal to the resisting force offered by the pole, and that the conductor assumes parabolic form.

By analysis of conditions in the above figure, we have:

$$1. \quad d_b = \frac{n T_b}{12 P} \text{ feet.}$$

$$2. \quad d_c = \frac{n T_c}{12 P} \text{ feet.}$$

Subtracting d_b from d_c gives,

$$3. \quad d_c - d_b = \frac{n}{12 P} (T_c - T_b)$$

Analyzing the elongation of the conductor due to changes in temperature and tension, we have:

$$4. \quad r_c - r_b = r_b (t_c - t_b) + \frac{r_b}{AE} (T_c - T_b)$$

Likewise:

$$5. \quad r_b - r_c = r_b (t_b - t_c) + \frac{r_b}{AE} (T_b - T_c)$$

Expressing λ_b and λ_c in terms of L_b and L_c respectively, we have:

$$6. \quad \lambda_b = L_b + \frac{8 S_b^2}{3 L_b}$$

$$7. \quad \lambda_c = L_c + \frac{8 S_c^2}{3 L_c}$$

Substituting equation 7 in equation 4, we have:

$$8. \quad \lambda_c - \lambda_b = (L_c + \frac{8 S_c^2}{3 L_c}) \alpha (t_c - t_b) + (L_c + \frac{8 S_c^2}{3 L_c}) (\frac{T_c - T_b}{AE})$$

Substituting equation 6 in equation 5, we have:

$$9. \quad \lambda_b - \lambda_c = (L_b + \frac{8 S_b^2}{3 L_b}) \alpha (t_b - t_c) - (L_b + \frac{8 S_b^2}{3 L_b}) (\frac{T_c - T_b}{AE})$$

Subtracting equation 6 from equation 7, gives:

$$10. \quad \lambda_c - \lambda_b = L_c - L_b + \frac{8}{3} (\frac{S_c^2}{L_c} - \frac{S_b^2}{L_b})$$

Subtracting equation 7 from equation 6, gives:

$$11. \quad \lambda_b - \lambda_c = L_b - L_c + \frac{8}{3} (\frac{S_b^2}{L_b} - \frac{S_c^2}{L_c})$$

By examining the figure on page 101, we see that:

$$12. \quad L_b - L_c = \delta_c - \delta_b = \frac{n}{12 P} (T_c - T_b)$$

By ordinary comparison:

$$13. \quad T_b/T_c = (w_b d_c)/(w_c d_b)$$

Without introducing an appreciable degree of error, we may make the following assumption:

$$L_b L_c = L_b^2 = L_c^2$$

----- To find T_0 -----

When T_b , S_b , w_b , t_b , w_0 and t_0 are known

Setting equation 9 equal to equation 11, and substituting equation 12, gives;

when dividing the result by L_b , and letting $d_b = S_b/L_b$:

$$14. \quad T_0 \left[\frac{n}{12 PL_b} + \frac{(1 + \frac{8}{3} d_b^2)}{AE} \right] - T_b \left[\frac{n}{12 PL_b} + \frac{(1 + \frac{8}{3} d_b^2)}{AE} \right] - (1 + \frac{8}{3} d_b^2) \alpha (t_b - t_0) + \frac{8}{3} (d_b^2 - d_0^2) = 0$$

Substituting equation 13 into 14, and rearranging terms, gives:

$$15. \quad T_0^3 + T_0^2 \left[\frac{\frac{8}{3} d_b^2 AE - AE(1 + \frac{8}{3} d_b^2) \alpha (t_b - t_0)}{\frac{n AE}{12 PL_b} + (1 + \frac{8}{3} d_b^2)} \right] - T_b \left[\frac{\frac{8}{3} d_b^2 AE - AE(1 + \frac{8}{3} d_b^2) \alpha (t_b - t_0)}{\frac{n AE}{12 PL_b} + (1 + \frac{8}{3} d_b^2)} \right] - \frac{w_0^2 L_b^2 AE}{24 \left[\frac{n AE}{12 PL_b} + (1 + \frac{8}{3} d_b^2) \right]} = 0$$

With sufficient accuracy warranted, we may express equation 15, as:

$$16. \quad T_0^3 + T_0^2 \left[\frac{\frac{8}{3} d_b^2 AE - AE \alpha (t_b - t_0)}{\frac{n AE}{12 PL_b} + 1} \right] - T_b \left[\frac{\frac{8}{3} d_b^2 AE - AE \alpha (t_b - t_0)}{\frac{n AE}{12 PL_b} + 1} \right] - \frac{w_0^2 L_b^2 AE}{24 \left[\frac{n AE}{12 PL_b} + 1 \right]} = 0$$

Using equation 1, we may express:

$$17. \quad L_b = L - \frac{n T_b}{12 P}$$

----- To find T_b -----

When T_o , S_o , w_o , t_o , w_b and t_b are known

Setting equation 8 equal to equation 10 and substituting equation 12, gives;

when dividing the result by L_o , and letting $d_o = S_o/L_o$:

$$18. \quad T_b \left[\frac{n}{12 PL_o} + \left(1 + \frac{8}{3} d_o^2\right) \right] - T_o \left[\frac{n}{12 PL_o} + \left(1 + \frac{8}{3} d_o^2\right) \right] - (1 + \frac{8}{3} d_o^2)(t_o - t_b) + \frac{8}{3}(d_o^2 - d_b^2) = 0$$

Substituting equation 13 into 18, and rearranging terms, gives:

$$19. \quad T_b^3 + T_b^2 \left[\frac{\frac{8}{3} d_o^2 AE - AE(1 + \frac{8}{3} d_o^2)(t_o - t_b)}{\frac{n AE}{12 PL_o} + (1 + \frac{8}{3} d_o^2)} \right] - T_o \left[- \frac{w_b^2 L_o^2 AE}{\frac{n AE}{12 PL_o} + (1 + \frac{8}{3} d_o^2)} \right] = 0$$

With sufficient accuracy warranted, we may express equation 19, as:

$$20. \quad T_b^3 + T_b^2 \left[\frac{\frac{8}{3} d_o^2 AE - AE(t_o - t_b)}{\frac{n AE}{12 PL_o} + 1} \right] - T_o \left[- \frac{w_b^2 L_o^2 AE}{\frac{n AE}{12 PL_o} + 1} \right] = 0$$

Using equation 2, we may express:

$$21. \quad L_o = L - \frac{n T_o}{12 P}$$

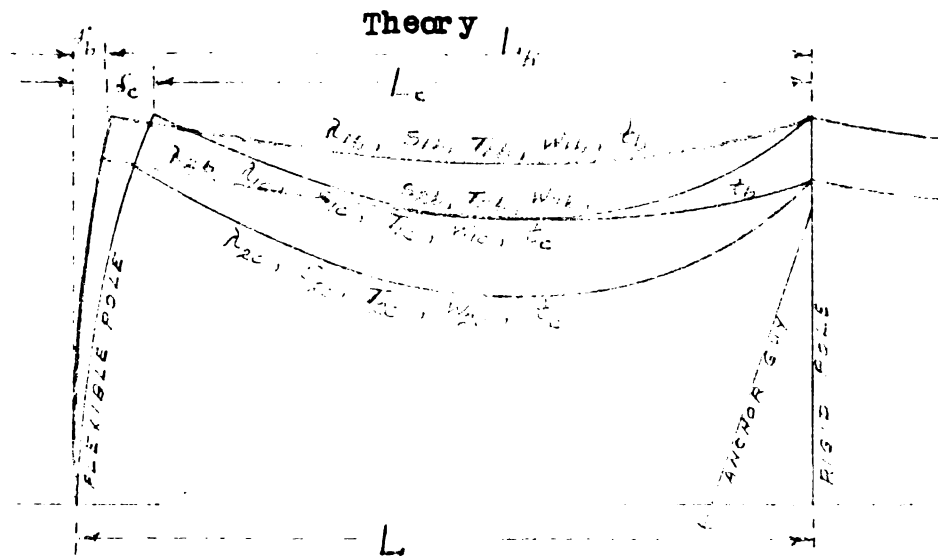
ANALYSIS OF A
FLEXIBLE DEAD-END POLE
USING
TWO DIFFERENT SIZES OF CONDUCTORS

SYMBOLS

All functions referring to the first conductor size have subscript "1", and all functions referring to the second conductor size have subscript "2".

The additional subscript "b" is used to denote stringing conditions at temperature t_b , and the additional subscript "c" is used to denote loaded conditions at the temperature t_o .

Other than the above subscript notations, the symbols used are the same as those in the previous problem.



ASSUMPTION: That the final tensions in the wires will be equal to the resisting force offered by the pole, and that the conductors assume parabolic form.

By analysis of conditions in the above figure, we have:

1. $s_b = \frac{n_1 T_{1b} + n_2 T_{2b}}{12 P}$ feet.
2. $s_c = \frac{n_1 T_{1c} + n_2 T_{2c}}{12 P}$ feet.

Analyzing the elongation of the conductors due to changes in temperature and tension, we have:

3. $\lambda_{1c} - \lambda_{1b} = \lambda_{1c} \alpha (t_c - t_b) + \frac{\lambda_{1c}}{(AE)_1} (T_{1c} - T_{1b})$
4. $\lambda_{2c} - \lambda_{2b} = \lambda_{2c} \alpha (t_c - t_b) + \frac{\lambda_{2c}}{(AE)_2} (T_{2c} - T_{2b})$
5. $\lambda_{1b} - \lambda_{1c} = \lambda_{1c} \alpha (t_b - t_c) + \frac{\lambda_{1b}}{(AE)_1} (T_{1b} - T_{1c})$
6. $\lambda_{2b} - \lambda_{2c} = \lambda_{2c} \alpha (t_b - t_c) + \frac{\lambda_{2b}}{(AE)_2} (T_{2b} - T_{2c})$

By applying the same methods as used in the previous problem, we arrive at the following:

----- To find T_{1c} and T_{2c} -----

When T_{1b} , S_{1b} , w_{1b} , T_{2b} , S_{2b} , w_{2b} , t_b , $(AE)_1$, $(AE)_2$, and w_{1c} , w_{2c} , t_c are known

$$7. \quad T_{1c}^3 + T_{1c}^2 \left[\frac{\frac{8(AE)_1}{3} \left[\frac{d_{1b}^2 + n_2 (AE)_2 (d_{1b}^2 - d_{2b}^2)}{12 P L_b} \right] - \alpha (AE)_1 (t_b - t_c)}{\frac{n_1 (AE)_1 + n_2 (AE)_2}{12 P L_b} + 1} - T_{1b} \right] - \frac{w_{1c}^2 L_b^2 (AE)_1}{24 \left[\frac{n_1 (AE)_1 + n_2 (AE)_2}{12 P L_b} + 1 \right]} = 0$$

$$8. \quad T_{2c}^3 + T_{2c}^2 \left[\frac{\frac{8(AE)_2}{3} \left[\frac{d_{2b}^2 + n_1 (AE)_1 (d_{2b}^2 - d_{1b}^2)}{12 P L_b} \right] - \alpha (AE)_2 (t_b - t_c)}{\frac{n_1 (AE)_1 + n_2 (AE)_2}{12 P L_b} + 1} - T_{2b} \right] - \frac{w_{2c}^2 L_b^2 (AE)_2}{24 \left[\frac{n_1 (AE)_1 + n_2 (AE)_2}{12 P L_b} + 1 \right]} = 0$$

From equation 1,

$$9. \quad L_b = L - \frac{n_1 T_{1b} + n_2 T_{2b}}{12 P}$$

----- To find T_{1b} and T_{2b} -----

When T_{1c} , S_{1c} , w_{1c} , T_{2c} , S_{2c} , w_{2c} , t_c , $(AE)_1$, $(AE)_2$, and w_{1b} , w_{2b} , t_b are known

$$10. \quad T_{1b}^3 + T_{1b}^2 \left[\frac{\frac{8(AE)_1}{3} \left[\frac{d_{1c}^2 + n_2(AE)_2(d_{1c}^2 - d_{2c}^2)}{12 P L_c} \right] - \alpha(AE)_1(t_c - t_b)}{n_1(AE)_1 + n_2(AE)_2 + 1} \right] - T_{1c} \left[\frac{w_{1b}^2 L_c^2 (AE)_1}{24 \left[\frac{n_1(AE)_1 + n_2(AE)_2}{12 P L_c} + 1 \right]} \right] = 0$$

$$11. \quad T_{2b}^3 + T_{2b}^2 \left[\frac{\frac{8(AE)_2}{3} \left[\frac{d_{2c}^2 + n_1(AE)_1(d_{2c}^2 - d_{1c}^2)}{12 P L_c} \right] - \alpha(AE)_2(t_c - t_b)}{n_1(AE)_1 + n_2(AE)_2 + 1} \right] - T_{2c} \left[\frac{w_{2b}^2 L_c^2 (AE)_2}{24 \left[\frac{n_1(AE)_1 + n_2(AE)_2}{12 P L_c} + 1 \right]} \right] = 0$$

From equation 2,

$$12. \quad L_c = L - \frac{n_1 T_{1c} + n_2 T_{2c}}{12 P}$$

NOMOGRAPHIC SOLUTION
OF THE CUBIC EQUATION

Examining equation 16 on page 103, equation 20 on page 104, equations 7 and 8 on page 107, and equations 10 and 11 on page 108, for the solution of flexible dead-end pole problems, we note that they are of a cubic type. This is rather a difficult type of equation to solve; hence we will construct a nomograph for the solution of this special form of equation. We note that the above mentioned formulae are all of the type:

$$1. \quad T^3 + K T^2 + C = 0$$

Expressing equation 1 in nomographic form, gives:

$$2. \quad \begin{vmatrix} 1 & 0 & -K \\ 0 & 1 & -C \\ T^2 & 1 & T^3 \end{vmatrix} = 0$$

Assume the scale moduli for K and $C = \text{unity}$
and assume an abscissa length = 10 inches.

Setting up determinate 2 to accomodate the above scale moduli and abscissa length, gives:

$$3. \quad \begin{vmatrix} & (x) & (y) \\ 1 & 0 & -K \\ 1 & 10 & -C \\ 1 & \frac{10}{T^2 + 1} & \frac{T^3}{T^2 + 1} \end{vmatrix} = 0$$

From determinate 3, the plotting equations are:

$$\text{For } K \begin{cases} y = -K & \text{inches.} \\ x = 0 & \text{inches} \end{cases}$$

$$\text{For } C \begin{cases} y = -C & \text{inches.} \\ x = 10 & \text{inches.} \end{cases}$$

$$\text{For } T \begin{cases} y = \frac{T^3}{T^2 + 1} & \text{inches.} \\ x = \frac{10}{T^2 + 1} & \text{inches.} \end{cases}$$

We will construct the nomograph for the following values:

K varying from + 5 to - 10

C " " 0 to - 15

T " " 0 to + 10

See Sketch # X - HCR - 6 in rear of thesis.

If the computed values of K and C do not lie within the plotted values on the chart, let:

$T = a T'$ where a is any desirable number.

Then, substituting (aT') for T in equation 1, we have:

$$(T')^3 + \frac{K}{a}(T')^2 + \frac{C}{a^3} = 0$$

Then:

For value of K on chart use $K' = \frac{K \text{ (computed)}}{a}$

For value of C on chart use $C' = \frac{C \text{ (computed)}}{a^3}$

Then value of T from nomograph = $T' = T/a$ or $T = a T'$

SOLUTION

With a straight edge, join the values of K and C on their respective scales and read the value of T on the tension scale.

See Sketch # X - HCR - 6 in the back of thesis.

EXAMPLE

Using a flexible dead-end pole, assume it is required to determine the stringing sag at 60° Fahr. for a 40 foot, class 2 Western cedar pole so that it will not be overstressed when holding three #0 bare copper conductors when subjected to a loading of an 8 pound wind and one-half inch of ice at 0° Fahr. Assume the soil conditions are such that a safety factor of 3 is chosen.

For such a case :

$$n = 3 \text{ (#0 bare copper conductors)}$$

$$w_b = .326 \text{ pounds (per foot, no wind and no ice)}$$

$$w_o = 1.262 \text{ pounds (per foot with an 8 \# wind and } \frac{1}{2}'' \text{ ice)}$$

$$L = 100 \text{ feet (assumed span length)}$$

$$t_b = 60^{\circ} \text{ Fahr. (stringing temperature)}$$

$$t_o = 0^{\circ} \text{ Fahr. (loaded temperature)}$$

$$AE = 1.1606 \times 10^6$$

$$= 9.6 \times 10^{-6}$$

40 foot, class 2 Western cedar pole

$$P = 60 \text{ pounds (per inch deflection)}$$

$$\text{Ultimate strength} = 3700 \text{ pounds}$$

$$\text{Safety factor} = 3$$

From these conditions:

$$T_o = \frac{3700}{3 \times 3} = 411 \text{ pounds tension per conductor (0° F., 8# wind, } \frac{1}{2} \text{ " ice)}$$

$$L_o = 100 - \frac{3 \times 411}{12 \times 60} = 98.29 \text{ feet (formula 21, page 104)}$$

$$S_o = \frac{1.262 \times 9.63 \times 10^3}{8 \times 411} = 3.70 \text{ feet (formula 1, page 92)}$$

$$d_o = \frac{3.70}{98.29} \quad (\text{sag factor})$$

Using equation 19, page 104, to show how little effect the portion $\frac{8}{3} d^2$ has in the term $(1 + \frac{8}{3} d^2)$:

Solving for the value of K from equation 19, page 104:

$$K = \frac{\frac{8}{3} \times 1.415 \times 1.1606 \times 10^3 + 1.1606 \times 1.0038 \times 9.6 \times 60}{\frac{3 \times 1.1606 \times 10^6}{12 \times 60 \times 98.29} + 1.0038} - 411$$

$$= - 310.5$$

Solving for the value of C from equation 19, page 104:

$$C = - \frac{.106 \times 9.63 \times 1.1606 \times 10^9}{24 \left(\frac{3 \times 1.1606 \times 10^6}{12 \times 60 \times 98.29} + 1.0038 \right)}$$

$$= - .981 \times 10^6$$

To solve this on the nomograph we will use the previously mentioned transformation (page 111) and let $a = 100$

Then:

$$K' = - 310.5/100 = - 3.105$$

$$C' = - .981 \times 10^6 / 10^6 = - .981$$

From the nomograph (Sketch # X - HCR - 6 in rear of thesis)

$$T' = 3.2$$

Hence:

$$T_b = 3.2 \times 100 = 320 \text{ pounds (tension per conductor for 60° Fahr. no ice and no wind)}$$

$$L_b = 100 - \frac{3 \times 320}{12 \times 60} = 98.67 \text{ feet (stringing condition)}$$

$$S_b = \frac{.326 \times 9.71 \times 10^3}{8 \times 320} = 1.23 \text{ feet (string. sag)}$$

CONCLUS ION

I have found in my work as a Planning Engineer for a utility company that these nomographs are practical, and sufficiently accurate, and that they provide economy of time and effort. Therefore with entire confidence in their value in the engineering field, I submit them to the members of the Graduate Council of Michigan State College.

BIBLIOGRAPHY

"The Construction of Graphical Charts" -- Peddle.

"Solid analytical Geometry and Determinates" -- Dresden.

"Bulletin of the Bureau of Standards, Volume 4" -- Dept. of Com.

"Overhead Systems Reference Book" --- Nat. Elec. Light Assoc.

ROOM 602 UNIT

May 4 '37

Mar 29 '47

May 1948

Mar 20 1948

Dec 9 '53

SAG - FACTOR CHART
FOR
SPANS ASSUMING PARABOLIC FORM

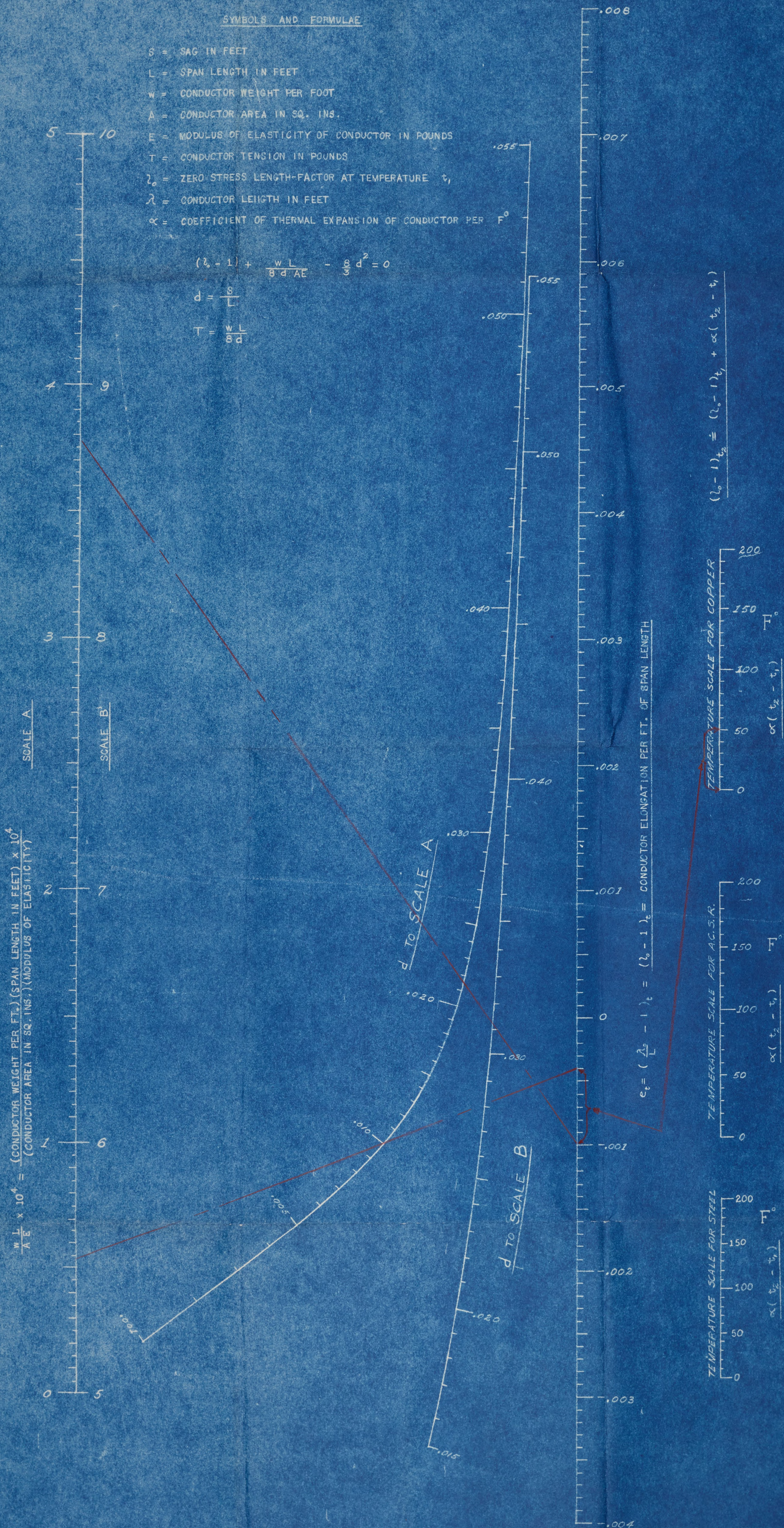
SYMBOLS AND FORMULAE

- S = SAG IN FEET
L = SPAN LENGTH IN FEET
W = CONDUCTOR WEIGHT PER FOOT
A = CONDUCTOR AREA IN SQ. INS.
E = MODULUS OF ELASTICITY OF CONDUCTOR IN POUNDS
T = CONDUCTOR TENSION IN POUNDS
 λ_0 = ZERO STRESS LENGTH-FACTOR AT TEMPERATURE t_1
 λ = CONDUCTOR LENGTH IN FEET
 α = COEFFICIENT OF THERMAL EXPANSION OF CONDUCTOR PER F°

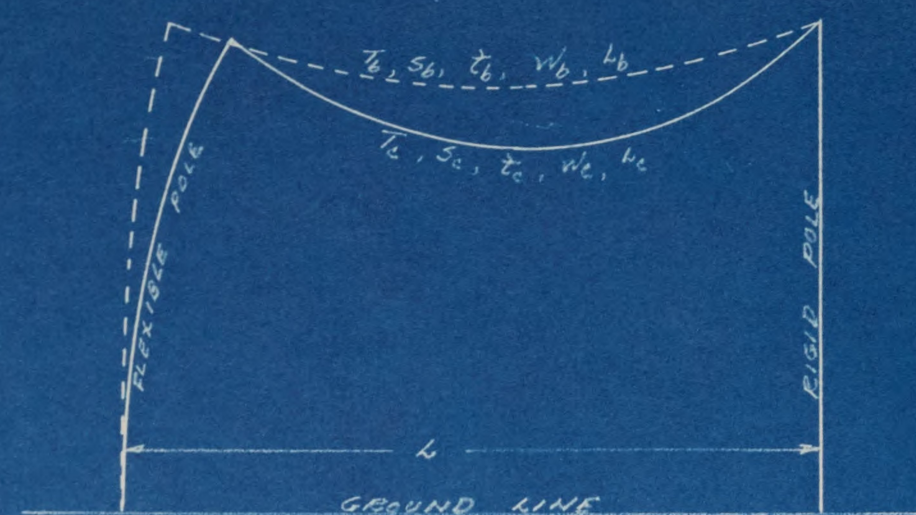
$$(\lambda_0 - 1) + \frac{W L}{8 d A E} - \frac{8}{9} d^2 = 0$$

$$d = \frac{8}{L}$$

$$T = \frac{W L}{8 d}$$



FLEXIBLE DEAD-END FOR SPANS ASSUMING PARABOLIC FORM



FORMULAE

W = WEIGHT PER FOOT OF CONDUCTOR
L = LENGTH OF SPAN IN FEET
S = SAG OF CONDUCTOR IN FEET
T = TENSION OF CONDUCTOR IN POUNDS
 $d = \frac{S}{L}$ = SAG FACTOR
A = AREA OF CONDUCTOR IN SQ. INS.
E = MODULUS OF ELASTICITY OF CONDUCTOR
P = POLE DEFLECTION CONSTANT IN IN./INCH
 $\delta = \frac{T}{P}$ = POLE DEFLECTION IN INCHES.
 t = TEMPERATURE IN DEGREES FAHR.
 α = COEFFICIENT OF THERMAL EXPANSION OF CONDUCTOR
N = NUMBER OF CONDUCTORS

METHOD #1

T_0 = STRINGING TENSION WHEN $\delta = \frac{NT_0}{P}$
 T_c = LOADED TENSION WHEN $\delta = \frac{NT_c}{P}$

I

WHEN T_0, S_0, t_0, w_0, l_0 ARE KNOWN - TO FIND T_c

$$d_0 = \frac{S_0}{L_0} \text{ WHERE } L_0 = L - \frac{NT_0}{P}$$

$$K = \frac{8}{3} d_0^2 HE - HE \alpha (t_0 - t_c) - \frac{T_0}{L_0} - \frac{NHE}{12PL_0} + 1$$

$$C = - \frac{W_0^2 L_0^2 AE}{24 \left(\frac{NHE}{12PL_0} + 1 \right)}$$

$T_c = T$ ON CHART

II

WHEN T_c, S_c, t_c, w_c, l_c ARE KNOWN - TO FIND T_0

$$d_c = \frac{S_c}{L_c} \text{ WHERE } L_c = L - \frac{NT_c}{P}$$

$$K = \frac{8}{3} d_c^2 HE - HE \alpha (t_c - t_0) - \frac{T_c}{L_c} - \frac{NHE}{12PL_c} + 1$$

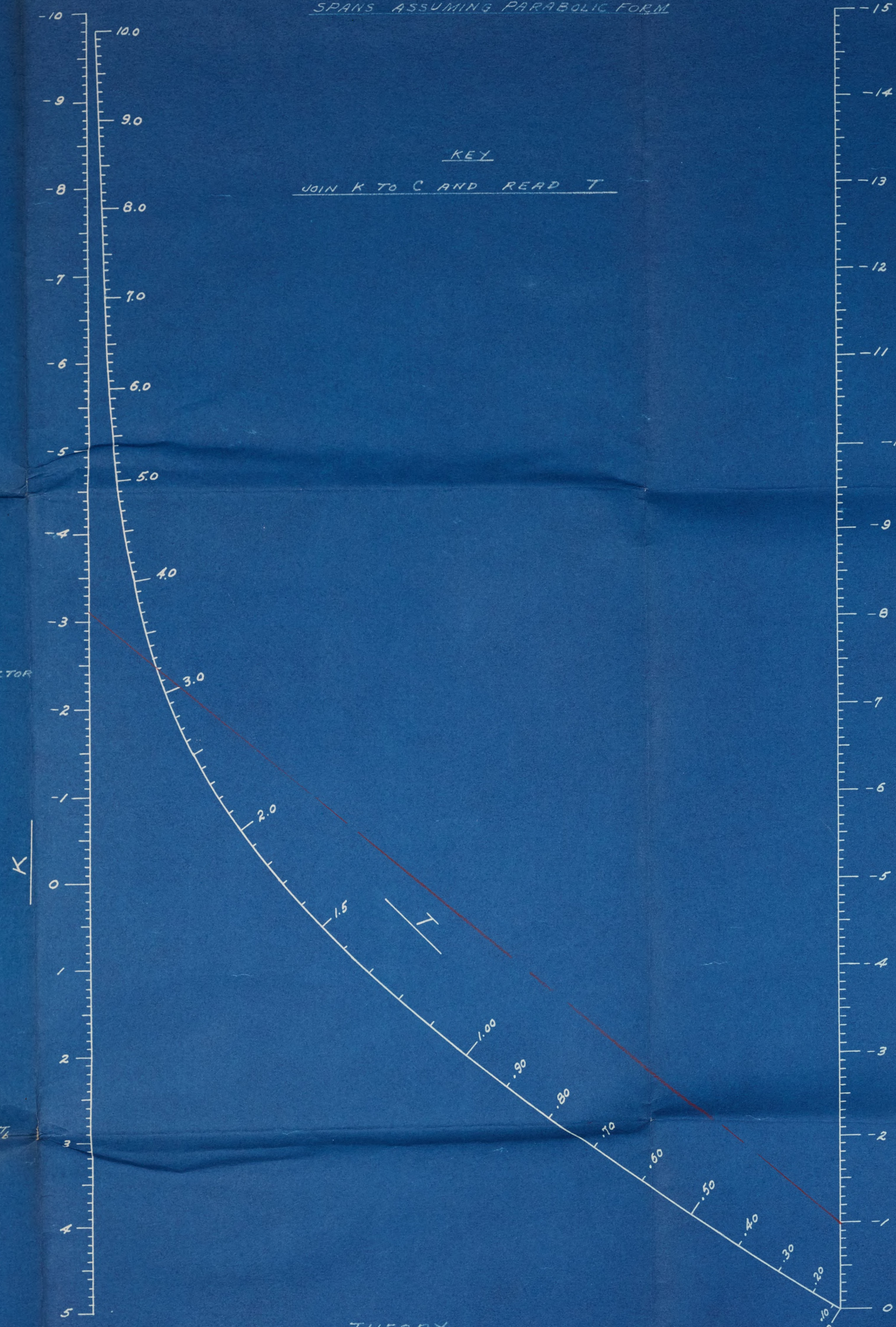
$$C = - \frac{W_c^2 L_c^2 AE}{24 \left(\frac{NHE}{12PL_c} + 1 \right)}$$

$T_0 = T$ ON CHART

SOLUTION USING METHOD #1

1. WHEN T_0 IS KNOWN, DETERMINE T_c BY MEANS OF FORMULAE I.
2. WHEN T_c IS KNOWN DETERMINE T_0 BY MEANS OF FORMULAE II

KEY
JOIN K TO C AND READ T



THEORY

$$T^3 + KT^2 + C = 0$$

IF COMPUTED VALUES OF K AND C DO NOT LIE WITHIN PLOTTED VALUES ON THE CHART LET $T = a(T')$ WHERE a IS ANY NUMBER, THEN

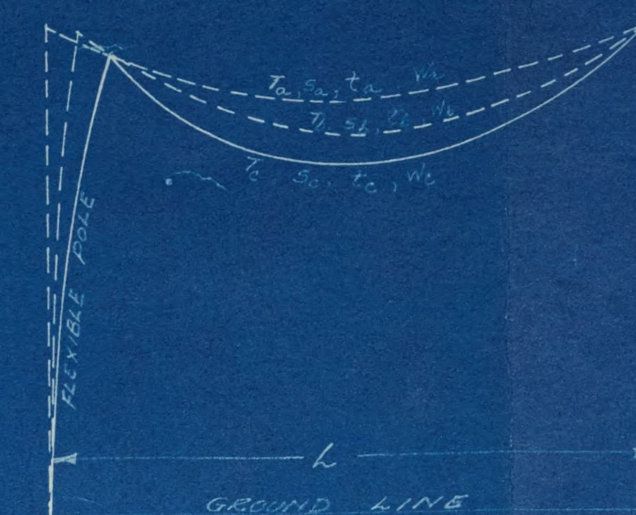
$$(T')^3 + \frac{K}{a}(T')^2 + \frac{C}{a^3} = 0$$

THEN K ON CHART = $K' = \frac{K}{a}$ (COMPUTED)

AND C ON CHART = $C' = \frac{C}{a^3}$ (COMPUTED)

AND T ON CHART = $T' = \frac{T}{a}$

$$\begin{vmatrix} 1 & 0 & -K' \\ 0 & 1 & -C' \\ T'^2 & T' & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & -K \\ 0 & 1 & -C \\ T^2 & T & 1 \end{vmatrix} = 0$$



FORMULAE

W = WEIGHT PER FOOT OF CONDUCTOR
L = LENGTH OF SPAN IN FEET
S = SAG OF CONDUCTOR IN FEET
T = TENSION IN POUNDS PER CONDUCTOR
 $d = \frac{S}{L}$ = SAG FACTOR
A = AREA OF CONDUCTOR IN SQ. INS.
E = MODULUS OF ELASTICITY OF CONDUCTOR
P = POLE DEFLECTION CONSTANT - POUNDS PER INCH.
 $\delta = \frac{T}{P}$ = POLE DEFLECTION IN INCHES.
 t = TEMPERATURE - FAHR.
 α = COEFFICIENT OF THERMAL EXPANSION OF CONDUCTOR
N = NUMBER OF CONDUCTORS

METHOD #2

T_0 = TENSION WHEN $\delta = 0$
 T_c = STRINGING TENSION WHEN $\delta = \frac{NT_c}{P}$
 T_c = LOADED TENSION WHEN $\delta = \frac{NT_c}{P}$

WHEN T_0, S_0 AND t_0 ARE KNOWN - TO FIND T_c

$$d_0 = \frac{S_0}{L_0}$$

$$K = \frac{8}{3} d_0^2 HE - \left(\frac{NHE}{12PL_0} + 1 \right) T_0$$

$$C = - \frac{8}{3} d_0^2 T_0^2 HE$$

$$T_c = T \text{ ON CHART}$$

WHEN T_c, S_c, t_c ARE KNOWN - TO FIND T_0

$$d_c = \frac{S_c}{L_c}$$

$$K = \frac{8}{3} d_c^2 HE - \left(\frac{NHE}{12PL_c} + 1 \right) T_c$$

$$C = - \frac{8}{3} d_c^2 T_c^2 HE$$

$$T_0 = T \text{ ON CHART}$$

WHEN T_0, S_0, t_0 ARE KNOWN - TO FIND T_c OR T_0

$$d_0 = \frac{S_0}{L_0}$$

$$K = \frac{8}{3} d_0^2 HE - \frac{T_0}{L_0} - \frac{NHE}{12PL_0} + 1$$

$$C = - \frac{8}{3} d_0^2 T_0^2 HE$$

$$T_c \text{ OR } T_0 = T \text{ ON CHART}$$

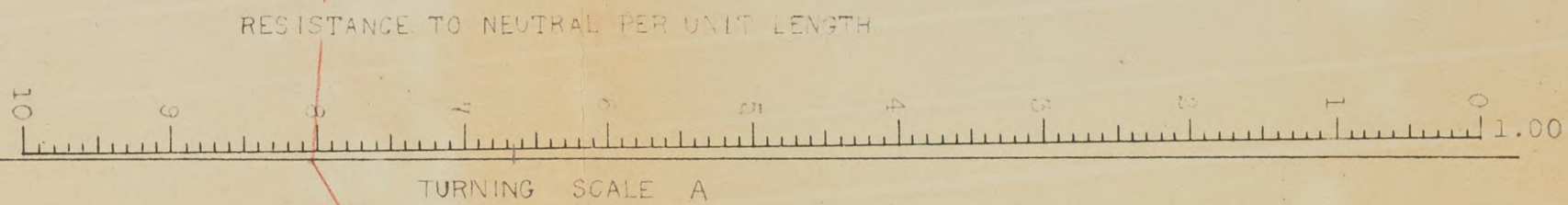
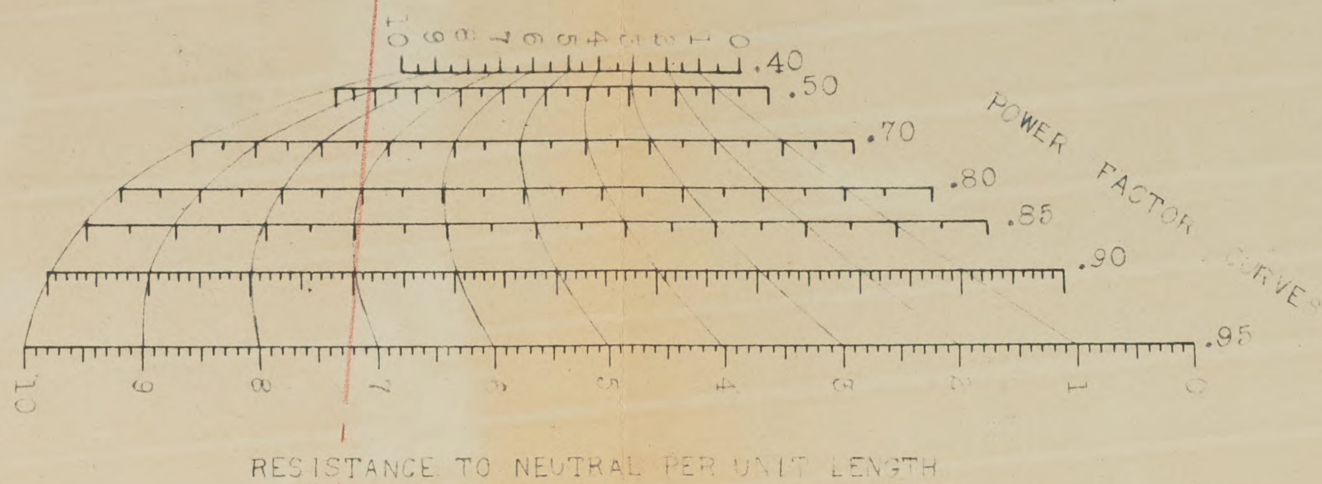
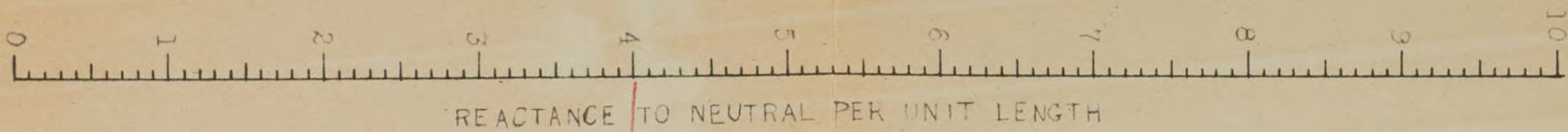
SOLUTION USING METHOD #2

1. DETERMINE T_0 BY MEANS OF FORMULAE I OR II.
2. DETERMINE NEW VALUE OF T_0 BY MEANS OF THOMAS CHART FOR CONDITIONS t_0 AND w_0 OR t_0 AND w_c .
3. DETERMINE T_c OR T_0 BY MEANS OF FORMULA III USING NEW VALUE OF T_0 FOUND FROM NOTE #2 ABOVE.

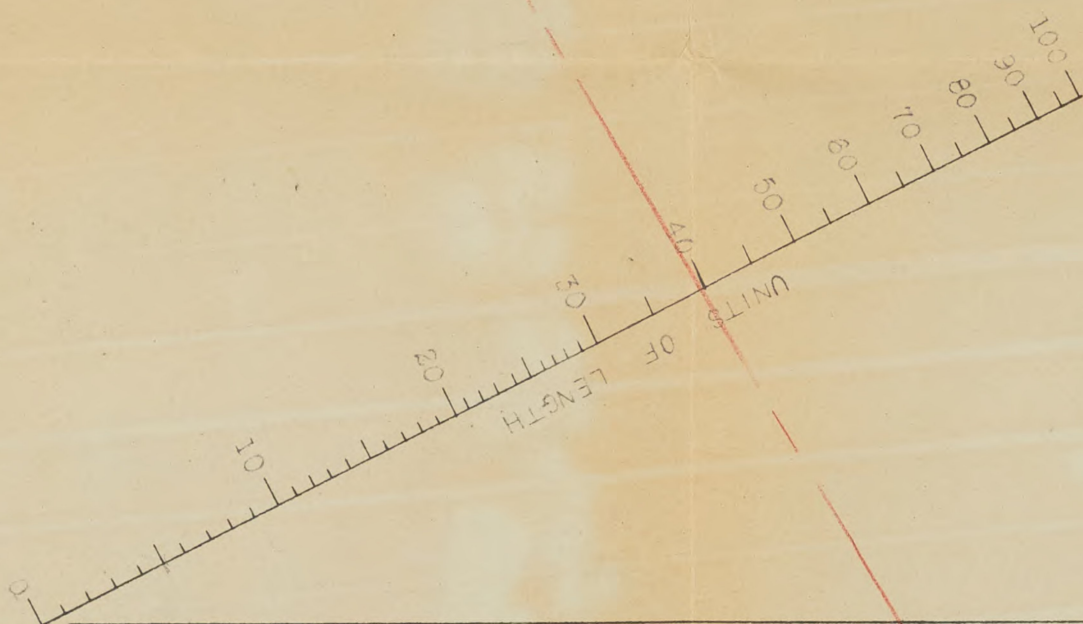
Harold C. Roberts

2-23-32

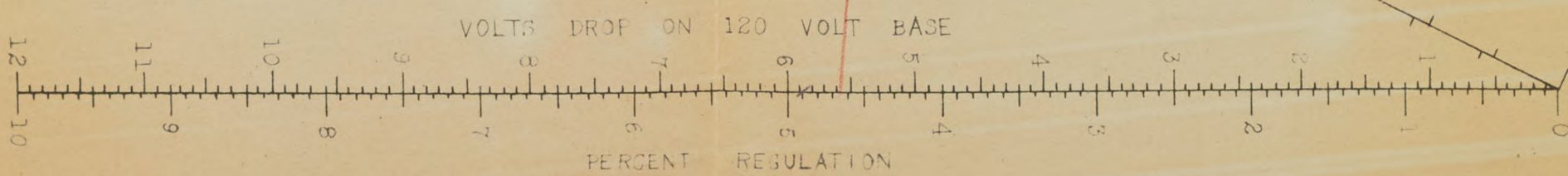
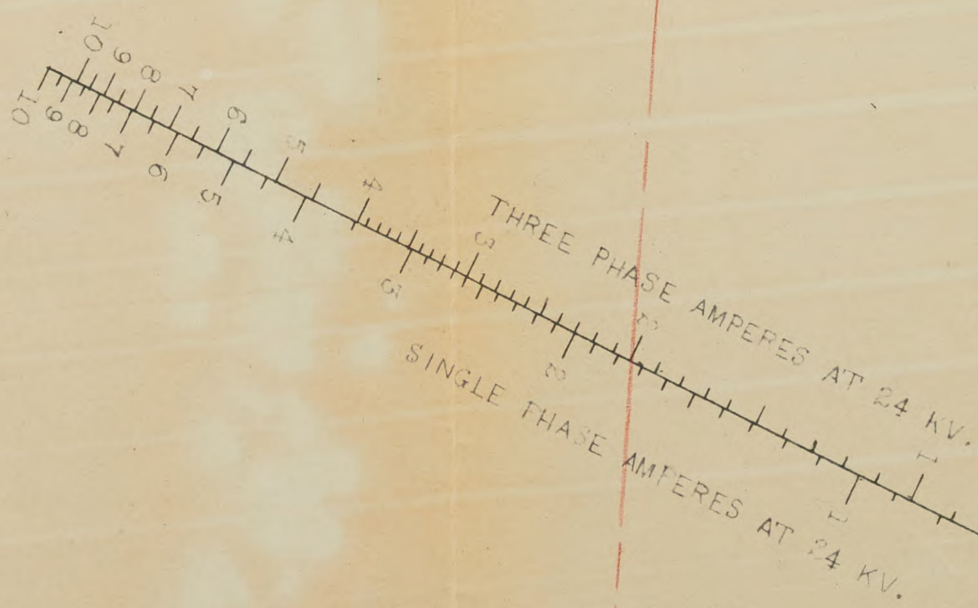
X - HCR - 6



TURNING SCALE A



TURNING SCALE B



VOLTAGE REGULATION CHART
ON 2400 VOLT BASE

ELECTRICAL SYSTEM
PLANNING DIVISION

HCR - 4
10 - 31 - 33

Pocket has: 3 Charts

SUPPLEMENTARY
MATERIAL



123
487
775
Chart 1

MICHIGAN STATE UNIV. LIBRARIES



31293006286037