A STUDY OF STRENGTH PROPERTIES OF PLYWOOD WEB WOODEN I BEAMS

Thesis for the Degree of M. S.

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Putnam S. Robbins

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by

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AN ABSTRACT

Submitted to the College of Agriculture
Michigan State University of Agriculture and
Applied Science in partial fulfillment of
the requirements for the degree of

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Department of Forest Products

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ABSTRACT

Design strength of wooden I beams is determined by applying certain well known engineering formulas to the physical properties and strengths of wood. Controlling factors in the design of plywood-wood I beams are the bending strength, shear stress, and deflection of the beam.

In this study the bending strength, horizontal shear stress, and deflection of plywood-web wooden I beams in depths of four, six, eight and ten inches was analyzed. Bending strength, represented by modulus of elasticity, was estimated from the modulus of elasticity of small bending samples. Deflection of sample I beams was measured under regulated loading tests and compared with theoretical deflection determined from engineering formulas. Horizontal shear stress of the plywood web was determined. In general empirical data agreed with theoretical data.

Horizontal shear stress and shear deflection were the most important limiting factors for the beam sizes studied. All beam failures were by horizontal shear of the plywood webs. As the depth of the beam increased the importance of shear deflection to total deflection also increased. The relationship of depth of beam to length of span, called span/depth ratio was the most important variable in determining the bending strength of the I beams tested.

Results and conclusions of this study should be applied only to I beams of similar construction and species, although the theory and design data may be generally applied to beams of I and box section.

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INTRODUCTION

History of Structural Plywood and I Beams

Structural type plywood did not appear on the construction scene until the 1920's, and then only in small volumes.[1] The advent of better adhesives during the 1930's greatly aided the structural use of plywood, starting it climbing to national importance as a construction material. World War II and the increasing stature of structural plywood coincided to make the use of wooden I section beams both possible and practical.[1]

The first extensive research and experimentation on the strength properties of plywood web I and Box type beams was done by the Forest Products Laboratory.[2,3] This work was done for the U. S. Government to determine the feasibility of using such sections as structural members of aircraft. This work was subsequently revised and adapted by the Douglas Fir Plywood Association for design and use in building construction.[4] Methods of construction of plywood web I beams have been much the same for many years, and revised design and fabrication specifications have been published just recently.[5,6]

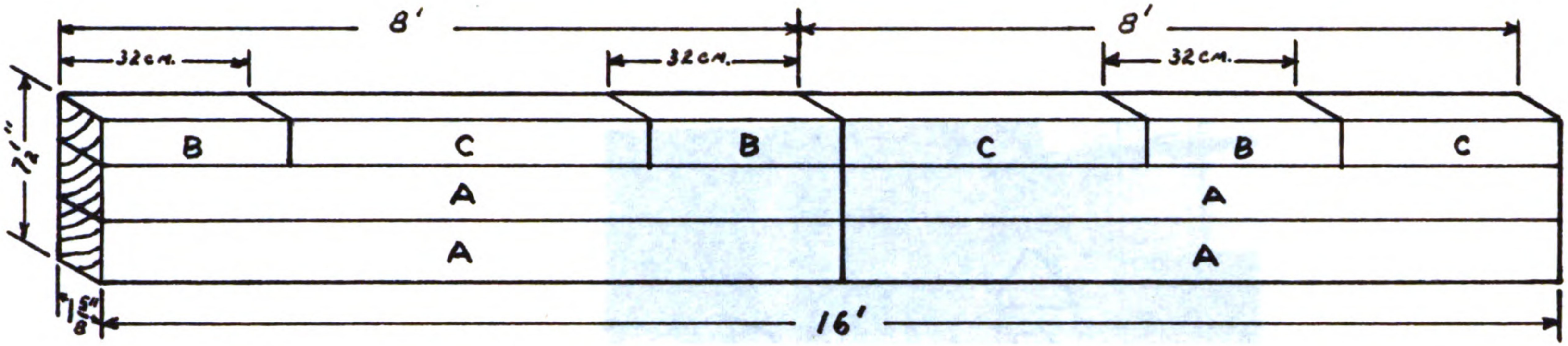
The present published reports on the strength properties of plywood-wood I section beams indicate that the design strength may be predicted by existing engineering formulas. Present experimentation at Michigan State University, recent work at Michigan State[7] and previous work done by Radcliffe[8] at Purdue University suggest that the recommended working stress for horizontal shear of plywood is conserative; and places plywood web structural constructions, such as I and Box beams, at a definate design disadvantage.

It is important to note that deflection in I beams has two main components, flexural deflection and shear deflection. Flexural deflection is caused by the lengthening of tension fibers and shortening of compression fibers and is generally considered the major component of total deflection. Shear deflection, resulting from horizontal shearing of fibers, is of considerable importance in I beams due to the small cross section of the web. This type of deflection is fundamental in the inability of the standard deflection formulas to give accurate predictions of actual observed deflection. It should not be assumed that shear deflection is present only in I type beams however, It is present in all wooden beams, but the section characteristics of an I beam serve to emphasize and amplify this deflection. Newlin and Trayer[16] developed formulas which enable one to calculate the theoretical shear deflection in I beams. When added to the deflection found by regular engineering formulas, the resulting total deflection correlated closely with measured deflection. The Douglas Fir Plywood Association used the data derived by Newlin and Trayer and adapted it for different standard conditions of loading, thereby decreasing the complexity of the original computations. The formulas for shear deflection found in Appendix A are the Douglas Fir Plywood Association revised formulas.[4]

Purpose of the Study

The purpose of this research was to evaluate the properties of modulus of elasticity and horizontal shear, as determined by experimentation with small scale plywood-wood I beams, in terms of existing theory and engineering formulas. Flexural behavior of the I beams was to be compared with theoretically predicated behavior by comparing total measured deflection with total deflection as calculated by engineering formulas. Modulus of elasticity values for the small I beams of this study were to be compared with modulus of elasticity values for larger scale I beams as determined by Luebs.[7]

SAMPLE SELECTION DETAIL



A - Flange stock - further machined to 1.81 x 0.81 inches

B - Small bending sample stock - machined to 2 cm x 2 cm

C - Stiffener stock - further machined to 1.81 x 0.81 inches

Figure 1: Sample Selection Detail

Figure 2: Test Beams

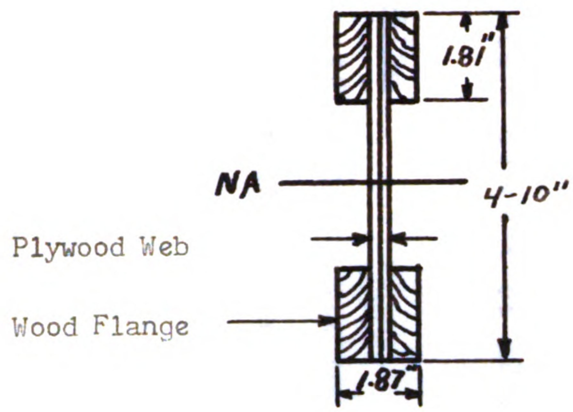
PROCEDURE

Description of Test Specimen Beams

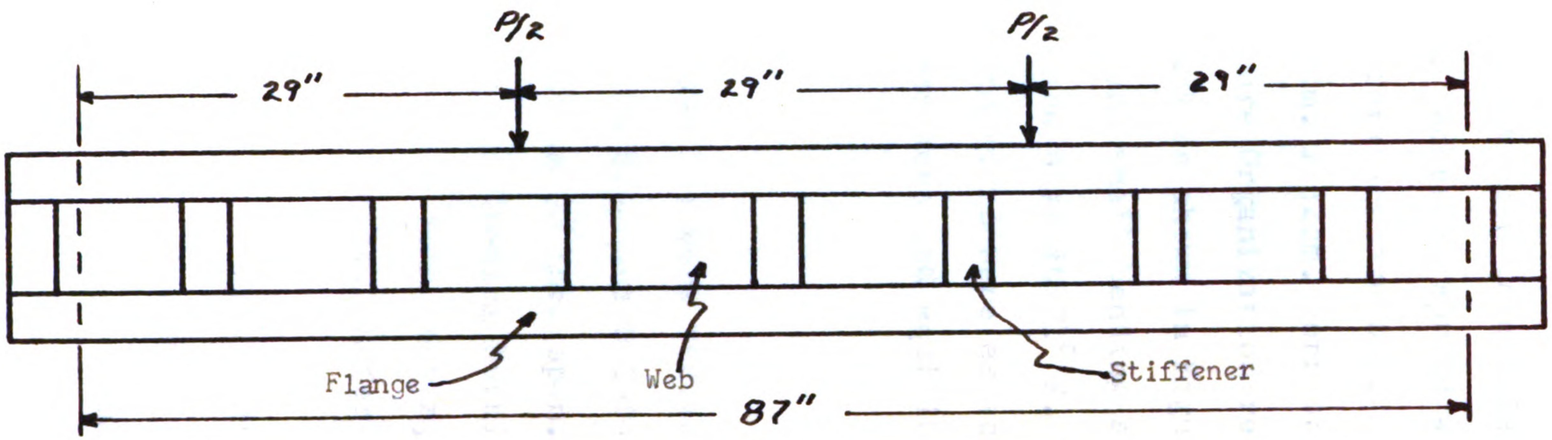
Test beams of I section were constructed in depths of four, six, eight and ten inches, with a constant flange width of 1.87 inches and flange depth of 1.81 inches. Four sample beams were constructed in each depth group. Beams of this size were made to be one-half scale models of the Luebs[7] beams except for their length dimension. Overall length of the I beams was eight feet, eliminating variables introduced by splicing of the web.[9] Flanges were made from selected Douglas fir planks of Construction grade. Web material was one-fourth inch sanded, three ply Douglas fir plywood, A-B interior grade. Flange stock for each individual sample beam was cut from a separate plank to provide for matching of strength properties and elimination of all possible variables, 16 planks being used to make the 16 sample I beams. Refer to Figure 1 for sample selection details.

Flange stock was machined to 1.81 x 0.81 inches. A flange member was glued to each side of the one-fourth inch plywood web, giving the 1.87 x 1.81 inch flange section as shown in Figure 3. Stiffeners of the same material and size were placed vertically between the flanges on each side of the web, at 12 inch spacing.

Aircraft type casein adhesive, conforming th U. S. Specification MMM A-125, was used in a mixing ratio of two



SECTION



LOADING DETAIL

Figure 3: Loading Detail and Dimensions for Test Beams

parts water to one part adhesive by weight. Pressure was applied by 4d box nails spaced at eight inch intervals and staggered to give uniform pressure. Adhesive spread was one side per joint at a rate giving uniform squeeze out.

Small bending samples 2cm. x 2cm. x 32cm. corresponding to United Nations Food and Agriculture Organization recommendations were taken from each plank as shown in Figure 1.

After fabrication all I beams and small bending samples were conditioned for a minimum of nine days at 71° F. dry bulb and 64° F. wet bulb. These conditions were expected to give a relative humidity of 68 per cent and equilibrium moisture content of 12 per cent.

Testing

After conditioning the I beam samples were tested in static bending in accordance with ASTM Standard D 198-27.[9] One-third point loading was used over an 87 inch span. Load was applied by a 100,000 pound Universal Testing Machine with pressure points 14.5 inches from the beam center, giving the required pressure point span of 29 inches. Refer to Figure 3.

Deflection was measured through the use of a deflection yoke, supported at the neutral axis of the beam over the bearing points, and giving deflection at neutral axis at the center of the beam span as shown in Figure 4. Load was recorded at every 0.025 inches deflection to failure.

Figure 4: I Beam Test Showing Deflection Yoke

Figure 5: Testing I Beam

Rate of loading as calculated by the formula $N=\frac{za}{3d}(3L-4a)$ was determined for each beam depth as shown in Appendix A. The closest machine speed of the testing machine to the calculated optimum speeds was used. This was one-eighth inch per minute for the six, eight and ten inch beams. The calculated speed of loading for the four inch beams was found to be too fast for deflection and load observations so the next lower machine speed of one-eighth inch per minute was used. A constant machine speed of one-eighth inch per minute was therefore used for all tests of I beams. This loading rate was within the recommended loading speed variability limits for all but the four inch depth.[9] This fact that the loading speed for the four inch depth was not within recommended limits is not significant when it is considered that the four inch depth lies at the breaking point of recommended rate of fiber strain. If the lower rate of fiber strain, for beams of over four inch depth, is used the machine speed used closely approaches that recommended.

Small bending samples were tested as outlined in the FAO recommendations.[10] A Dillon dynamometer type Tensile Testing Machine employing a compression cage was used to apply the load, as can be seen in Figure 6. Rate of loading was 0.08 inches per minute. This gave maximum load in 4.5-5.5 minutes, with center point loading over a span of 28cm. Span/depth ratio was the recommended 14. Load was recorded at each 0.01 inch deflection to failure.

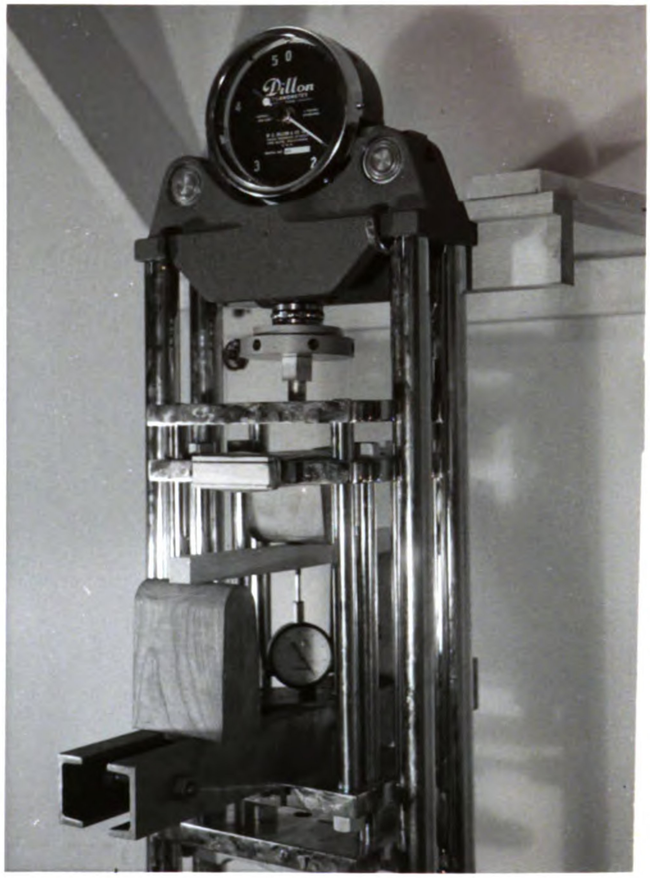


Figure 6: Testing Small Bending Sample

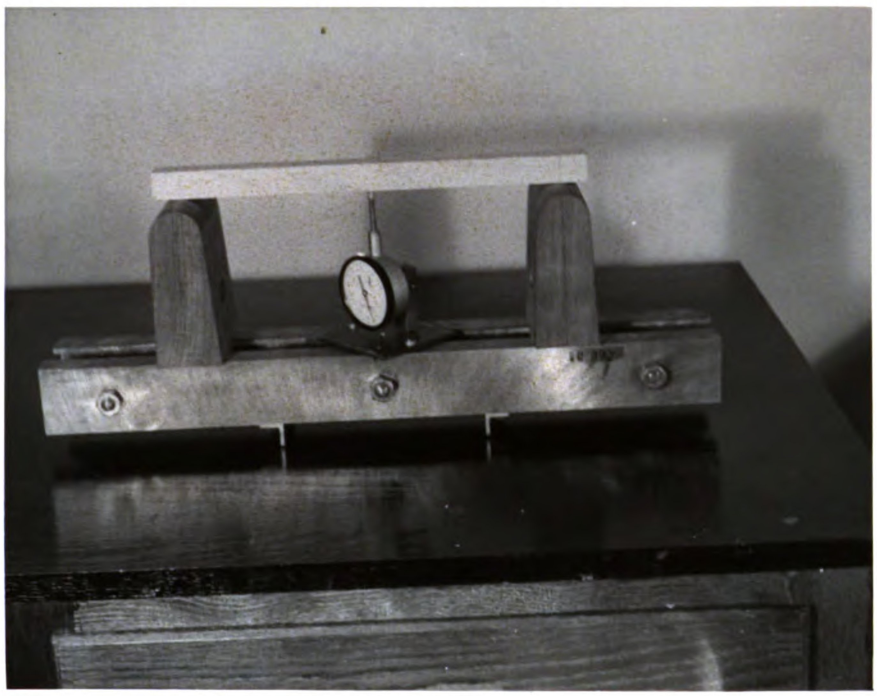


Figure 7: Small Bending Sample

Moisture content of I beam flanges and webs was checked by electrical resistance and power loss type moisture meters respectively. These results were substantiated by oven dry calculations with specimens from the small bending samples. Refer to Table I.

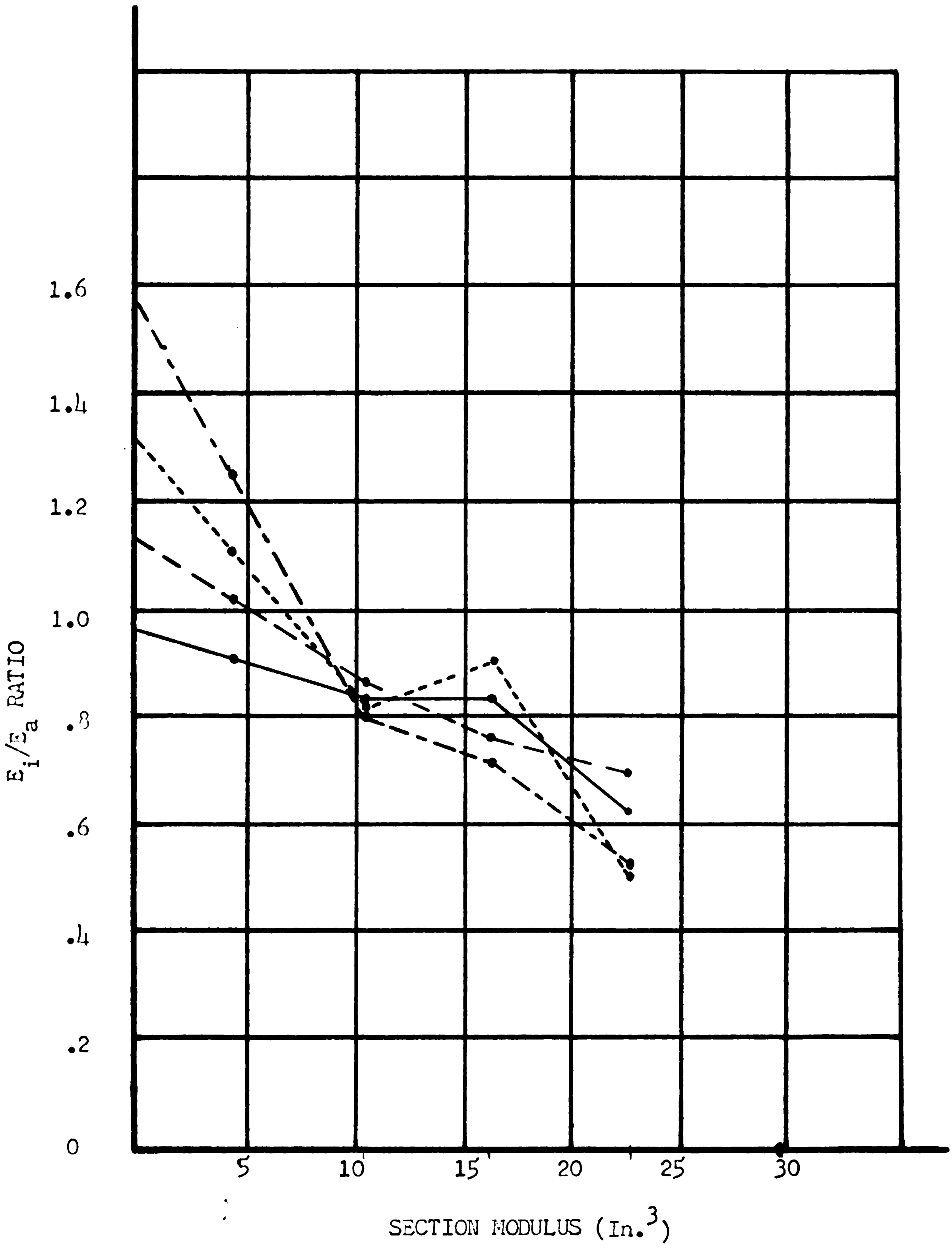


Figure 8: Modulus of Elasticity Ratios by Depth of Beam

TABLE I.

SUMMARY OF TEST DATA

Beam No.	Moisture content web	(%)	flange	Moment of Inertia in^4	Section Modulus in^3	Deflection (in.) meas. at P= 3000 lbs.	calc.	Modulus of Elasticity I beam x10^6 psi	small sample	Horiz. Shear psi
4-1	9.0	12.	12.0	9.54	4.77	2.790	2.90	1.3172	1.3299	1027
4-2	9.5		12.0			2.010	2.289	1.8283	1.6875	1269
4-3	8.0		11.5			2.280	2.920	1.6118	1.3202	1061
4-4	7.5		7.3			1.800	2.070	2.0415	1.8585	1425
6-1	7.0		9.5	30.41	10.13	.810	.840	1.4233	1.7200	1385
6-2	8.5		11.0			.650	.725	1.7736	2.0730	1513
6-3	8.3		10.0			.725	.730	1.5901	1.9784	1381
6-4	6.0		6.5			.850	.874	1.3563	1.6612	1376
8-1	7.2		9.3	65.03	16.25	.430	.538	1.2537	1.5056	1356
8-2	7.8		10.0			.365	.415	1.4765	1.9420	1349
8-3	9.5		11.3			.370	.345	1.4565	2.0366	1388
8-4	7.0		7.3			.420	.569	1.2831	1.4194	1532
10-1	6.4		6.7	114.12	22.82	.330	.346	.9307	1.4870	1351
10-2	9.3		12.6			.325	.346	.9450	1.3801	1258
10-3	6.4		7.0			.327	.330	.9392	1.8002	1189
10-4	6.0		5.6			.355	.296	.8775	1.7457	1398

RESULTS AND ANALYSIS

Results of all test data are presented in summary form in Table I. Detailed results are given throughout this chapter and formulas used in calculating results are found in Appendix A.

Modulus of Elasticity

Values for modulus of elasticity (E) were calculated using the standard engineering formulas for one-third point and center point loading, see Tables 1 and 2 and Appendix A. Average modulus of elasticity (Ea) for the wood flange material in each I beam was determined by averaging the E values for the three small bending samples from each plank. Theoretically the modulus of elasticity of the I beams (Ei) will closely approximate the average modulus Ea, as determined by the small bending samples, given proper consideration of flexural and shear deflection differences.

The results of Table II show that Ei only approximated Ea for the four inch beams when shear deflection was not taken into account. The variability between Ea and Ei increases as the depth of the beam increases. As will be seen later, this variability may be partially explained by the increasing influence of shear deflection as depth increases.[7]

While the variability within Ea may seem excessive, wood as a natural, orthotropic material has many factors

TABLE II

MODULUS OF ELASTICITY

I Beams and Small Bending Samples

Beam number	I Beam	Small Bending Sample
	x 10^6 psi	
4-1	1.3172	1.3299
4-2	1.8283	1.6875
4-3	1.6118	1.3202
4-4	2.0415	1.8585
6-1	1.4233	1.7200
6-2	1.7736	2.0730
6-3	1.5901	1.9784
6-4	1.3563	1.6612
8-1	1.2537	1.5056
8-2	1.4765	1.9420
8-3	1.4565	2.0366
8-4	1.2831	1.4194
10-1	.9307	1.4870
10-2	.9450	1.3801
10-3	.9392	1.8002
10-4	.8775	1.7457

which effect a larger variability than other construction materials. The Forest Products Laboratory has found wood to have a coefficient of variation of 22 per cent in modulus of elasticity.[11]

Shear Deflection

As it was mentioned earlier, shear deflection has a considerable effect on the stiffness of an I section beam as beam depth increases. Shear stresses in the sample I beams tested were of greatest magnitude in the plywood webs. All beams failed in horizontal shear of the web. This shear occurred in the outer one-third of the beam span without exception. It should be noted that shear in the center one-third, between the two load points, may be considered zero.[13]

Deflection as calculated by the standard engineering formulas for rectangular beams under symmetrical load, (see Appendix A) while applying to I section beams, does not account for all of the deflection in these type beams.

Table III shows measured and calculated deflection and the variation between them in terms of per cent of measured deflection. The measured deflections at a load of 3,000 pounds were made by extending the straight line portions of the load deflection curves. This difference between measured deflection (col. 3) and calculated deflection (col. 4) increases as the depth of beam increases given, as in this case, a constant span. More generally stated, the difference increases as the span/depth ratio decreases. This was as large as 50 per cent of measured deflection.

TABLE III.

COMPARISON OF MEASURED AND CALCULATED DEFLECTION IN I BEAMS

Depth of Beam	Span/Depth ratio	Deflection* Meas. by 1	Calc. by 2	Calc. by3	Error (per cent)** by 2	by 3
4	21.7	2.79	2.76	2.90	-1.0	+3.9
		2.01	2.18	2.29	+8.4	+13.4
		2.28	2.78	2.92	+21.9	+28.0
		1.80	1.97	2.07	+ 8.4	+15.0
6	14.5	.81	.67	.84	-17.3	+3.7
		.65	.55	.72	-14.6	+11.5
		.72	.58	.73	-20.0	+0.6
		.85	.69	.87	-18.3	+2.8
8	10.9	.43	.36	.54	-16.7	+25.1
		.36	.27	.41	-24.6	+13.7
		.37	.26	.34	-28.4	-6.7
		.42	.38	.57	-9.7	+35.4
10	8.7	.33	.21	.35	-37.5	+4.8
		.32	.22	.37	-31.6	+14.4
		.33	.22	.33	-32.7	+0.9
		.35	.18	.29	-50.4	-.6.6

*in inches at P=3000#

**in per cent of measured deflection

1. From initial straight portion of load-deflection curve.

2. $d = \frac{PL/3}{24EI}(3L^2 - 4L/3^2)$

3. $d = \frac{PL/3}{24EI}(3L^2 - 4L/3^2) + \frac{PLKh^2{}_c}{GI}$

Discounting the four inch depth, where shear deflection theoretically has little effect on total deflection, the average error introduced by not considering shear deflection is 25 per cent of measured deflection. By including shear deflection in calculations for total deflection (col. 5) the average error is reduced to 7.5 per cent, or by 17.5 per cent.

Horizontal Shear Stress

Table IV shows the unit stress in horizontal shear calculated for the 16 test I beams. The average horizontal shear stress ($\bar{x}$) for the sample beams was 1329 p s i. Standard deviation (s) was found to be 140.

Statistical Data

Statistical analysis in this study was to accomplish three objectives. (1) Determination of the correlation between Ei for the I beams and Ea for the small bending samples, and if significant differences existed between Ea and Ei. (2) Comparison of the modulus of elasticity between the half scale I beams and the full scale I beams tested by Luebs.[7] (3) Analysis of the variation of measured deflection of the I beams between depths of beam.

Correlation between Ea and Ei as shown in Appendix B Table V varried from .9 for four inch depth of beam and 1.0 for the six inch depth to -.05 for the ten inch beam. This indicated a positive relationship between E values for the four and six inch depth I beams and the small bending samples, and no positive relationship for the ten inch depth.

TABLE IV

HORIZONTAL SHEAR STRESS

Beam	Stress p s i
4-1	1027.98
4-2	1269.20
4-3	1061.38
4-4	1425.07
6-1	1385.85
6-2	1512.67
6-3	1381.32
6-4	1376.80
8-1	1356.27
8-2	1349.88
8-3	1388.26
8-4	1532.21
10-1	1351.84
10-2	1258.10
10-3	1189.03
10-4	1398.71

$\bar{x} = 1329$

$s^2 = 19534$

$s = 140$

Analysis of variance[12] of the modulus of elasticity data, Appendix B Table VI, showed a significant difference in E between I beam depths and no significant difference in E between corresponding small bending samples. Throwing both I beam E data and small bending sample E data together showed no significant differences in E between depths or between I beam and small bending sample.

Comparison of the modulus of elasticity between half scale I beams and Luebs' full scale beams, in the 8-16 inch and 10-20 inch depths, gave correlation coefficients of .44 and .17 respectively. Refer to Appendix B Table VII.

This lack of correlation may be partially explained by variations between the two studies. Differing species of flange material, moisture content of beams, grade of plywood, temperature at test, and type of load were induced variables that undoubtedly effected the modulus of elasticity of the beams. Another factor effecting the variability was the different span/depth ratios of testing and the resulting effects upon shear deflection, discussed later in this study.

Table VIII Appendix B shows measured and calculated deflection for each I beam depth and the average deflection ($\bar{x}$) and standard deviation (s) of each group. Since sample size was very small the range method[14] for determining deviation was used.

Analysis of variance was not used in the analysis of deflection or the comparison of half scale and full scale I beam modulus of elasticity due to the large variations between sample groups. Variance ratio tests[15] were applied to test data in both cases. It was found that variance was large enough to indicate that data was not from the same population so could not be analyzed by analysis of variance methods.

DISCUSSION

In testing the sample I section beams in no case was there any sign of compression or tension failure. This indicates that shear failure occurred substantially before extreme compression fiber stress was reached.

Table II indicates that modulus of elasticity for the four inch I beams exceeds that of the small bending samples. Newlin and Trayer[16] explain that according to their tests the average real modulus of elasticity for solid and I beams is approached as the span/depth ratio increases above 20. Since the span/depth ratio of the small bending samples is 14 this may account for some of the difference between the two moduli. Another factor also enters into this difference. The flanges in the I beams subject to compressive stress may have actually acted as colums and tensil members. If this was the case then the modulus of elasticity in compression, averaging somewhat higher than bending modulus, would have increased the effective modulus of elasticity.

While the s/d ratio of the small bending samples was not the ultimate desired for determining real modulus of elasticity, it provided for close approximation and corresponded to the ratio recommended by FAO. According to Newlin and Trayer the average real modulus will not exceed the modulus as calculated by the small bending samples by more than ten per cent, on the average. The small bending sample method of arriving at Ea was a practical method giving values varying only slightly from the real average modulus. Due to the natural variability in wood strength it was felt that the error introduced by the testing method would be acceptable.

The previous discussion of results of deflection and shear deflection in this study indicated basic agreement between test results and theory that shear deflection is of considerable importance in I beams. From Table III one may note the degree of importance of shear deflection and the error produced by omitting this factor from deflection determination.

While it cannot be readily proved, a logical explanation of this error is that values used for average modulus of elasticity (Ea) were in error by a small percentage. Reasons for this error have been previously discussed. Since practically all of the error shown in col. 7, Table III is positive it follows that Ea was less than the real modulus. As was mentioned earlier Ea was suspected of being as much as ten per cent less than the real modulus.

The difference between measured and total calculated deflection may be further explained if the modulus of elasticity in compression was in fact the effective modulus. Use of a higher E value in the deflection formulas would have decreased the calculated values for total deflection and brought them closer to observed deflection values.

In either of the above two cases the effect of a larger value for modulus of elasticity would be the same, to bring theoretical, calculated data into closer agreement with empirical data.

It should be noted that the allowable shear modulus of elasticity as used in the calculations for shear deflection may not be a true value. Several sources recommended different values for shear modulus of elasticity. The value used in this study was 1/14 Ea. This was a compromise between the 1/14.5 - 1/15 E_t used by the Forest Products Laboratory and the constant 117,000 p s i recommended by the Douglas Fir Plywood Association. Varying this ratio changes considerably the calculated shear deflection.

CONCLUSIONS

Design strength of wooden I beams is determined by applying certain well known engineering formulas to the physical properties and strengths of wood. Because of the great natural variability of these physical properties large safety factors are incorporated in the design of wood structural members. These safety factors are used to adjust for the variation in strength properties within a given species of wood and to allow for variations in equilibrium moisture content, physical defects, type and span of loading, in use temperature fluxuations, and time length of load. An attempt was made in this study to eliminate most of these variables by matching and conditioning samples so that all variables except the basic strength of the wood material would be held constant.

Controlling factors in the structural use of plywood-wood I beams are the bending strength, shear strength and deflection of the beam. These factors were determined by test and studied in terms of existing data and theory. Test results agreed generally with expected results as determined by engineering formulas. Shear deflection proved to be of increasing importance to design as the span/depth ratio decreased.

When span/depth ratio was equal to or greater than 20, modulus of elasticity of the I beams could be approximated from modulus of elasticity values for small bending samples of the same material. As the span/depth ratio decreased from 20 the accuracy of approximation also decreased, due to the increasing importance of shear deflection.

Horizontal shear stress was the controlling strength factor in the test I beams. All beams failed in horizontal shear of the plywood web and at the outer one-third of the span.

Sample size of the study was too small for determination of positive statistical significance of results. The correlation between modulus of elasticity values for I beams and small bending samples did not quite equal the five per cent significance level coefficient due to the small sample size and few degrees of freedom. Further study of static bending tests by use of strain measuring devices would be helpful in making deductions of the strength behavior of I beams.

APPENDIX A

MODULUS OF ELASTICITY - DEFLECTION FORMULA

Symmetrical Load

$$E = \frac{PL}{24dI}(3L^2 - 4a^2)$$

d = deflection (inches) at beam center for load P

P = load in pounds at any point below proportional limit

L = total span, inches

E = modulus of elasticity, p s i

I = moment of inertia, inches^4, neglecting ply material not parallel to span

a = distance between support and load, inches

This formula can be simplified to give

$$E = \frac{23PL^3}{648dI}$$

Source: Popov, E. P. Mechanics of Materials. Prentice-Hall Inc., New York 1952. Table 11, p. 435.

SHEAR DEFLECTION

$$ds = \frac{PLKL^2C}{GI}$$

ds = shear deflection, inches

P = total load, pounds; at the same point of deflection used in the standard deflection formula and below proportional limit

L = span between supports, inches

C = loading coefficient, see note

G = sheaving modulus of web, $\frac{1}{14}E_t$

I = moment of inertia of cross section, neglecting plys not parallel to span, inches^4.

K = factor determined by beam cross section

$$K = \frac{1}{4} \quad 1 = \frac{12d^3 - 18hd^2 + 6dh^2}{h^3} \quad \frac{t_2}{t_1} - 1$$

d = flange depth, inches

h = depth of beam, inches

t_2 = total width of flange, including web, inches

t_1 = thickness of web, inches

Total deflection may be determined by a combination of the two given deflection formulas.

$$d = \frac{PL}{24EI}(3L^2 - 4a^2) + \frac{PLKL^2C}{GI}$$

Note: When determining the loading coefficient (C) it is important to note that the k used is not the same as the K factor, but rather a constant determined by length between support and load/total span.

Source: Technical Data Handbook. Douglas Fir Plywood Association. Tacoma, Washington. Section 9 p. 5 - 7.

APPENDIX B

TABLE V

Correlation Between Modulus of Elasticity of I Beams and Small Bending Samples.

4 inch Depth

I	SBS
1.3172	1.3299
1.8283	1.6875
1.6118	1.3203
2.0415	1.8585

= .92

6 inch Depth

I	SBS
1.4233	1.7200
1.7736	2.0730
1.5901	1.9784
1.3563	1.6612

= 1.00

8 inch Depth

I	SBS
1.2537	1.5056
1.4765	1.9420
1.4565	2.0366
1.2831	1.4194

= .58

10 inch Depth

I	SBS
.9307	1.4870
.9450	1.3801
.9392	1,8002
.8775	1.7457

= .05

TABLE VI

Analysis of Variance of Modulus of Elasticity

I Beams

Sample	Depths (inches) 4	6	8	10
1	1.3172	1.4233	1.2537	.9307
2	1.8283	1.7736	1.4765	.9450
3	1.6118	1.5901	1.4565	.9392
4	2.0415	1.3563	1.2831	.8775

Small Bending Samples

Sample	Depths (inches) 4	6	8	10
1	1.3299	1.7200	1.5056	1.4870
2	1.6875	2.0730	1.9420	1.3801
3	1.3202	1.9784	2.0366	1.8002
4	1.8585	1.6612	1.4194	1.7457

Individual Analysis

I

Source	D.F.	S. S.	M.Sq.	F.
Total	15	1.7761		
Depth	3	1.3415	.4471	12.35 **
Error	12	.4346	.0362	

** Significant at 1% level

SBS

Source	D.F.	S. S.	M.Sq.	F.	F.05
Total	15	.9693			
Depth	3	.2273	.9757	1.23	3.49
Error	12	.7420	.0618		

Final Analysis

Source	D.F.	S. S.	M.Sq.	F.	F.05
Total	31	3.4777			
Depth	3	.8658	.2886	1.23	9.28
Test	1	.7323	.7323	3.12	10.13
DxT	3	.7031	.2343		
Error	24	1.1765	.0490		

MOMENT OF INERTIA

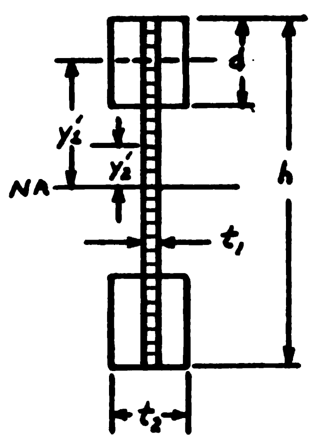
$$I = \frac{t_2 h^3}{12}$$

I = Moment of inertia, neglecting plys not parallel to span, inches^4.

t_2 = total width of flange, including web, inches

h = depth of beam, inches

t_1 = web width, neglecting non-parallel ply.



Sample Beam Dimensions

d = 1.81"

t_2 = 1.79"

t_1 = .167"

STATICAL MOMENT OF INERTIA

$$Q = Ay'$$

Q = statical moment of inertia

A = area, $inches^2$

y' = distance between neutral axis of beam and centroid of flange, inches

I Beam Case:

$$Q = A_1y_1' + A_2y_2'$$

A_1 = area of flange, including web, $inches^2$

A_2 = area of web between NA and flange, $inches^2$

y_1' = distance NA to centroid of flange, inches

y_2' = distance to centroid of web area, inches

Beam Depth	I $inches^4$	S $inches^3$
4	9.54	4.77
6	30.41	10.13
8	65.03	16.25
10	114.12	22.82

$$S = \frac{I}{c}$$

c = distance NA to centroid of flange, inches

Source: Harris, C. O. Elementary Structural Design. American Technical Society. Chicago, Illinois 1951.

HORIZONTAL SHEAR STRESS

$$v = \frac{VQ}{It}$$

v = horizontal shear stress, p s i

V = total shear, p s i

Q = statical moment of inertia, inches^3

I = moment of inertia, inches^4

t = web thickness, inches

Source: Technical Date Handbook. Douglas Fir Plywood Association. Tacoma, Washington. Section 9.

TABLE VII

Correlation Between Full Scale and Half Scale I Beams

Modulus of Elasticity

18 = Span/Depth = 10.8		14.4 = Span/Depth 8.7	
16	8	20	10
1.31 *	1.25	1.39	.931
1.35	1.47	1.40	.945
1.59	1.45	1.29	.939
1.38	1.28	1.30	.878
= .44		= .17	

* Values are modulus of elasticity x 10^6.

TABLE VIII

DEFLECTION - ANALYSIS

4 inch Depth

Deflection (inches)

Measured	Calculated*
2.79	2.90
2.01	2.28
2.28	2.92
1.80	2.07

$\bar{x}$ = 2.22, 2.54

s = .48, .41

6 inch Depth

Deflection (inches)

Measured	Calculated*
.810	.840
.650	.725
.725	.730
.850	.874

$\bar{x}$ = .758, .792

s = .09, .07

8 inch Depth

Deflection (inches)

Measured	Calculated*
.430	.538
.365	.415
.370	.345
.420	.569

$\bar{x}$ = .396, .468

s = .03, .10

10 inch Depth

Deflection (inches)

Measured	Calculated*
.330	.346
.325	.372
.327	.330
.355	.296

$\bar{x}$ = .334, .333

s = .01, .03

$$s = \frac{R}{2.059}$$

* includes shear deflection

LITERATURE CITED

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1. Perry, Thomas D. Modern Plywood. Pitman Publishing Corporation, New York. 1948.

2. Munitions Board Aircraft Committee. ANC-18 Bulletin. 1951.

3. Forest Products Laboratory, Design of Plywood Webs in Box Beams. Forest Products Laboratory Report No. 1318. 1943.

4. Douglas Fir Plywood Association, Technical Data on Plywood. Tacoma, Washington. 1948.

5. General Design Specification No. 1., Design of Plywood-Lumber Structural Assemblies. Douglas Fir Plywood Association. Tacoma, Washington. 1959.

6. DFPA Specification No. BB-8., Fabrication of Plywood Beams. Douglas Fir Plywood Association. Tacoma, Washington. 1959.

7. Luebs, Donald F. , Nail-Glued Wood-Plywood I Beams for Residential Construction. Michigan State University. East Lansing, Michigan. 1959.

8. Radcliffe, B.M. and Suddarth, C.K., Notch Beam Shear Test. Purdue University. Lafayette, Indiana. 1955

9. ASTM Standards on Wood, Wood Preservatives and Related Materials , American Society for Testing Materials. March, 1959.

10. Annex 1., Third Conference on Wood Technology, May 1954. Food and Agriculture Organization, United Nations. 1955.

11. Forest Products Laboratory, Wood Handbook. U.S. Department of Agriculture, Washington, D.C. 1955.

12. Snedecor, George W., Statistical Methods. Iowa State College Press, Ames, Iowa. 1956.

13. Radok, J.R.M.; Silberstein, B.A.; Wells, H.A.; A New Theory for the Strength of Wooden Box Beams, Australian Aeronautical Research Report ACA-40. Council for Scientific and Industrial Research. Melbourne, Australia. 1948.

14. Duncan, A.J., Quality Control and Industrial Statistics, Richard D. Irwin, Inc. Homewood, Illinois. 1955.

15. Brownlee, K.A., Industrial Experimentation, Chemical Publishing Co., Inc. Brooklyn, New York. 1949. p.36.

16. Newlin, J.A. and Trayer, G.W., Deflection of Beams with Special Reference to Shear Deformations, Forest Products Laboratory Report No. 1309. March 1956.