AN EVALUATION OF DRIVER DECISIONS AND REACTIONS AT AN INTERSECTION CONTROLLED BY A STOP SIGN

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This is to certify that the

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AT AN INTERSECTION CONTROLLED BY A STOP SICH

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ABSTRACT

AN EVALUATION OF DRIVER DECISIONS AND REACTIONS AT AN INTERSECTION CONTROLLED BY A STOP SIGN

by Frederick Allan Wagner, Jr.

The problem of determining the most effective modes of controlling intersections of highways at grade is being studied with analytical and digital computer simulation models. Such models require detailed knowledge of the characteristics of driver decisions and reactions.

The objectives of this research were:

2. To evaluate the influence of the following traffic factors on the lag and gap acceptance distributions: vehicle type; pressure of traffic demand; direction of movement through the intersection; sequence of gap formation; and conditions on the opposite stop signed approach.

3. To determine the parameters of the distributions of starting delay times of vehicles accepting lags and gaps, and to evaluate the influence of certain traffic factors on these distributions.

4. To investigate the delays encountered by vehicles waiting on the stop signed approaches. The data were collected at an intersection of a four-lane, bidirectional, intermediate speed, undivided, major state highway with a two-lane, bi-directional, low speed city street. A total of 472 minutes of sample data were gathered during daylight hours on weekdays, in fair, dry weather, during the months of August, October, and November, 1961. The decisions and reactions of 1,203 side street vehicles were observed and recorded on a multiple pen event recorder. A total of 5,179 separate acceptance-rejection decisions were included in the sample.

The form and parameters of the lag and gap acceptance distributions were determined using a graphical curve fitting technique, and a special application of standard statistical difference tests permitted evaluation of the influence of traffic factors on the distributions.

The major findings of this research were:

1. The results strongly supported earlier findings which indicated that the relationship between the logarithmic transform of lag or gap size and the probability of acceptance follows a cumulative normal distribution curve. Lag acceptance differed significantly from gap acceptance.

2. Of the traffic factors studied, those which significantly influenced the parameters of the lag and gap acceptance distribution were: pressure of traffic demand; direction of traffic movement during periods of heavy traffic demand; and main street vehicle sequence during periods of heavy demand.

3. No evidence was found to indicate that either side street vehicle type or conditions on the opposing side street approach significantly affected the gap acceptance distribution. 4. The results of the study of starting delay times in accepting lags and gaps indicated that factors which had important influence on the driver decisions, namely pressure of traffic demand and main street vehicle sequence, also had similar and significant effects on starting delay time.

5. The components of delay to side street drivers were determined and are presented to enable those interested to validate the results of their analytical and simulation model work.

AN EVALUATION OF DRIVER DECISIONS AND REACTIONS *

AT AN INTERSECTION CONTROLLED BY A STOP SIGN

By

Frederick Allan Wagner, Jr.

A THESIS

Submitted to Michigan State University in partial fulfillment of the requirements for the degree of

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1. INTRODUCTION

Since the advent of automobile transportation, the problem of dealing with the conflict of vehicles traveling on roadways intersecting at grade has been a concern of those responsible for safe and efficient movement of persons and goods. Intersections at grade remain as critical elements of the highway system in that they are principal sources of accidents and delays; furthermore, their capacities restrain the entire system's ability to process traffic. This is particularly evident in and around urban areas where population and traffic demand are concentrated.

The control of traffic operation at intersections has been the subject of extensive study by traffic engineers in the past. The fact that interest in this topic remains on a high plane, and that the volume of related research work currently underway equals that at any point in history is evidence that much knowledge is yet to be created, and many problems have yet to be solved.

1.1 BACKGROUND

1.1.1 The Selection of the Control Mode at an Intersection at Grade

The modes of control which have been developed to facilitate traffic movement at intersections include:

- Traffic control signal or time sharing mode, in which the right to proceed alternates in time between the conflicting streams.
- 2. Stop sign or priority mode, in which one stream is assigned the right of way and vehicles on the conflicting

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stream are required to stop and yield right of way to vehicles in the priority stream which are near enough to constitute an immediate hazard.

- 3. Yield sign, a special case of the "priority" mode, which requires a stop only when necessary to yield right of way to the priority stream.
- 4. Four-way stop (or more correctly, all-way stop), which might be considered a hybrid of the time sharing and priority modes. Vehicles in both streams are required to stop and priority is generally taken by the vehicle which was first to arrive.

The specific wording and legal interpretation of traffic ordinances relating to the four items above vary between jurisdictions; model ordinances which reflect the nation wide consensus may be found in Reference (1).

At intersections along major arterial roadways, either the traffic control signal mode or the stop sign mode are usually applied. Consequently, it becomes the task of the traffic engineer to make an objective selection between the two principal alternatives. In the history of an intersection, stop sign control is usually applied first. A most common decision, therefore, is whether to retain the stop sign control or to install a traffic control signal.

Lewis and Michael (2) have pointed out that general warrants for intersection control methods have evolved which are based in part on empirical data, but which to some degree are merely rules of thumb. The most recent national standard warrants are contained in a manual (3) which was prepared through a cooperative effort of municipal, county,

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state, federal governments and professional associations. These warrants are based on measurements of vehicular traffic volume and speed, pedestrian volume, accident experience and other special considerations.

Gazis and Potts (4) have recently detected new impetus in the study of intersection operations due to developments in the application of mathematical theory and digital computer simulation. While their comments pertained specifically to optimization of traffic signal operation, it will be seen, from references cited later in this paper. that the comment pertains with equal validity to stop signed intersection operation, and to the problem of selecting one of the alternatives. Whether an analytical model or a simulation model is utilized, the approach is to apply the model over full ranges of geometric design, traffic variables, and, in the case of signalized intersections, timing of the traffic signal, solving for a measure of effectiveness or performance criterion. Then, for any specific intersection condition, the control mode which provides for the 'best' performance of traffic is implemented. Such techniques can provide answers which are only as good as the weaker of the two models, whether the stop sign model or the traffic signal model is weaker. Performance criteria which have been commonly proposed include: 1) measurements on the distribution of individual vehicular delay, such as mean delay or maximum individual delay; and 2) measurements of intersection throughput (vehicles served per unit of time).

Relatively speaking, research on operation under traffic signal control has been more intensive and has borne more fruit than research on the stop signed intersection. This is perhaps the case because at a

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signalized intersection performance criteria are highly dependent on signal timing, which can be precisely observed or stated in a model. At an intersection controlled by a stop sign, however, where delays are for the most part encountered by vehicles on the yielding street, performance is highly dependent on the decisions and reactions of the waiting driver. The content of the remainder of this paper is limited to the stop sign mode, and in particular, reports on research performed by the writer pertaining to driver decisions and reactions.

1.1.2 Analytical Models of Stop Signed Intersections

One of the earliest applications of theoretical techniques in the study of stop signed intersections was reported by Greenshields, et al. (5), in 1947. They presented a sample application of probability theory to determine the percentage of side street cars unnecessarily delayed by a stop sign in attempting to cross Poisson distributed uni-directional main street traffic. It was assumed that side street cars were unnecessarily delayed if, at their time of arrival at the intersection, no main street vehicle was nearer than six seconds from the intersection. Raff (6) carried forward the work of Greenshields by generalizing the theory and applying it to the development of warrants for stop signs. His general premise was that a stop sign is warranted if more than 50 percent of the side street traffic is delayed by main street traffic.

More recently, several theoreticians have applied their knowledge of queueing theory and stochastic processes to the highway crossing problem (7-15).

Evans, Herman, and Weiss (16) have commented on several of these papers, indicating the general approach and citing the limitations of such techniques. Generally speaking, assumptions are first made

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relating to: 1) the form of the distribution of arrivals on the main street and side street; and 2) the gap acceptance criterion. Then, expressions of performance values, such as waiting time distributions, queue length distributions, probability of zero delay, and highway transparency* are derived. Several interesting comments contained in Reference (16), pages 2-4, illustrate the rather severe limitations imposed on the theoretical approach. They are quoted below:

"... The problem of crossing a highway involving two lanes in opposite directions, for example, becomes extremely difficult to handle. ... In fact, these theoretical papers concern rather restricted models which are set up in order that the mathematics be tractable. It was quite clear from the start that the complexity of the merging or highway crossing problem is such that one cannot hope to take into account all of the detailed real effects that occur on the streets and highways in such theoretical studies. ... It is quite clear to us that a deep understanding of this problem will result from the interplay between theoretical work, machine simulation, and good observations and experiments in the field. ..."

1.1.3 Simulation Models of Stop Signed Intersections

General purpose digital computers have been in existence for little more than a decade. A relatively short period of time after their introduction, attempts were made to use them for the simulation of vehicular traffic. The Dictionary defines to <u>simulate</u> as "to assume the appearance of, without the reality; to feign." The use of a digital computer as a simulator has become commonplace in a broad variety of fields of endeavor. The foremost examples are, perhaps, in the area of telephone traffic and switching problems.

Pioneering efforts in vehicular traffic simulation were reported by Gerlough (18) in 1955, and Goode, Pollmar, and Wright (19) in 1956; the

Highway transparency is defined as the percentage of time during which a waiting driver would say that it is safe to cross the highway.

former involving a model of freeway-type traffic and the latter, a signalized intersection. Goode, et al. exposed certain advantages of simulation in a very succinct manner:

"... There is one aspect of the attack on a problem in which simulation is better than both analysis and trial. ... In analysis we may use only those criteria which are mathematically tractable (e.g., least squares, but not maximum absolute deviation), and in trial, we choose only one criterion because even one is costly to measure. In simulation, we may select any criterion and as many as we like, measuring them continually if necessary. ..."

later simulation models are especially pertinent, in that consideration of stop signed intersections was included. Stark (20) developed a program for the IBM-704 to simulate nine blocks of a city street; three of the crossing streets were controlled by stop signs. His gap acceptance rules were based on a constant acceptable gap in main street traffic of four seconds. Lewis and Michael (2) and Kell (21) have each developed models of a single intersection on which either stop sign control or traffic signal control can be imposed. The lewis-Michael program was written in IBM 709/7090 FORTRAN language. One of the stated objectives of their work was to establish a realistic set of traffic volume warrants to indicate when to go from stop sign control to semi-actuated traffic signal control. They made simulation runs using constant values for time gap in main street traffic acceptable to the waiting driver, 4.8 seconds and 5.8 seconds. Kell's undertaking has similar final objectives. He is developing two separate models, programmed for the IEM-701, first a stop sign model, and then a traffic control signal model, with the ability to operate the signal in three different manners -- fixed-time, semi-actuated, and full-actuated.* For gap

Definitions of these three types of traffic signal operation may be found in Reference (3)

acceptance criteria, Kell utilized a function in which the probability of accepting a given size time gap follows a log normal distribution.

1.1.4 Current Knowledge of Driver Decisions and Reactions at Stop

Signed Intersections

Several papers were found in the literature which pertained directly **v** or were analagous to driver decisions and reaction at stop signed intersections. Before proceeding with a review of this work, one should understand certain terminology which has come into common usage as related to this topic. A gap is considered as the time interval between arrival of successive main street vehicles at a point in the intersection. A <u>lag</u> is that portion of current gap remaining when an approaching side street vehicle stops or reaches its lowest speed. In other words, a lag is the time interval between arrival of a side street vehicle and arrival of the next main street vehicle. A lag or gap is either <u>accepted</u> or not accepted (<u>rejected</u>) by the side street vehicle. A lag is accepted if the side street vehicle crosses or enters the main street before the arrival of the main street vehicle. A gap is accepted if the side street vehicle crosses or enters the main street between the two main street vehicles comprising the gap.

Greenshields, et al. (5) were the first to study the acceptance and rejection of lags. They studied an intersection in New Haven, Connecticut to determine an 'acceptable average-minimum' lag (i.e., the one accepted by more than 50 percent of the drivers) of 6.1 seconds.

Raff (6) studied behavior at four intersections in New Haven to determine the 'critical lag,' which was defined as having the property that the number of accepted lags shorter than this value equals the number of rejected lags longer than this value. Critical lags at the

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four intersections ranged from 4.6 seconds to 6.0 seconds. Raff attempted to evaluate factors which affect the critical lag, and concluded that, of those factors investigated, sight distance was most significant. Contrary to what might be expected, intersections with poorer sight distances had lower critical lag values.

Robinson (22) in the study of an analagous situation of the decisions of pedestrians crossing a road, introduced to the traffic engineering profession the use of a probability distribution to describe data of this type. By transforming percentage acceptance to a probit scale and gap size to a logarithmic scale, Robinson was able to fit a straight line to his data.

More recently, Bissell (23) reported an analysis of driver decisions at two stop signed intersections in Oakland, California and in suburban Contra Costa County, California. He treated data in a manner similar to that used by Robinson. Using a graphical method, he found that by transforming gap size to a logarithmic scale, the probability of accepting a lag or gap followed a normal distribution. In addition, Bissell investigated the effects of some traffic factors on the acceptance functions, concluding that drivers proceeding left, straight and right from the side street behaved differently. He found no significant differences between the lag and gap functions, nor any difference between crossing in front of a main street vehicle approaching from the left (i.e., near side) and crossing in front of one from the right (i.e., far side).

A comparison of these strictly empirical investigations, including the present study, with the controlled experiments conducted by Herman and Weiss (12) at the General Motors Research Laboratories should prove interesting. Here, experimental subjects (drivers) waited at a stop sign

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while a single car approached on a crossing path at 30 mph. At an unspecified time, an observer, equipped with a stop watch, called 'mark' and the waiting driver then either accepted or rejected the lag.

No information was found in the literature pertaining to the measurement of 'starting delay time' for drivers accepting lags and gaps. <u>Starting delay times in accepting gaps and lags are defined by</u> this writer as follows:

- 1. Starting delay time in accepting gaps is the elapsed time between arrival of the first main street car comprising a gap and the complete entry into the intersection of the side street car.
- 2. Starting delay time in accepting lags is the elapsed time between arrival of a side street car (i.e., either when it stops or reaches its lowest speed) and its complete entry into the intersection.

Arrivals of main street cars are measured at a line which is the extension of the side street center line. A side street car is assumed to have completed its entry into the intersection when its rear bumper has crossed the line which is an extension of the near side edges of the main street. These reference lines are illustrated in Figure 1.

Starting delay time is an important parameter in both the empirical study and the simulation of stop signed intersections at any time more than one vehicle is waiting in line at the stop sign. The problem here is to determine from what point in time the lag presented to the second vehicle should be measured. Using the above definitions, the arrival of the second car can be considered as coinciding with the entry of the first waiting car, and the second car's lag can be measured from this reference.

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Figure 1. Intersection Reference Lines

Thus, both the central tendency and dispersion of starting delay time are of interest, and it is rather surprising that such measurements have not been reported in the past.

1.2 RESEARCH OBJECTIVES

With an awareness of the importance and direct applicability of results of empirical studies of driver decisions and reactions at intersections controlled by a stop sign, and in consideration statements in the literature calling for verification and refinement of current knowledge in this area, the present study was undertaken. The objectives of this research were:

- 1. To perform a microscopic examination of an intersection controlled by a stop sign to determine and verify the parameters of the lag and gap acceptance distribution of the waiting vehicles.
- 2. To evaluate the influence of the following traffic factors on the lag and gap acceptance distributions:
 - a. Vehicle type: car vs. truck.
 - b. Pressure of traffic demand: peak vs. offpeak.
 - c. Direction of side street vehicle movement: left vs. straight vs. right.
 - d. Main street vehicle sequence: near vs. far for lags; and near-near vs. near-far vs. far-near vs. far-far for gaps.
 - e. Conditions on the opposite side street:approach: car . present vs. no.car present.
- 3. To determine the parameters of the distributions of starting delay times of vehicles accepting lags and gaps, and to evaluate the influence of certain traffic factors on these distributions.

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4. To investigate the delays encountered by vehicles waiting on the side street.

1.3 RESEARCH LIMITATIONS

The primary limitation of this work is that characteristics of behavior at only one intersection were studied. Thus, any direct applications of the results should be limited to intersections having generally similar geometric and traffic characteristics.

Since the resources with which to finance the study were rather limited, the installation of automatic traffic sensing and recording equipment which could expedite analysis was not possible. All data were sensed manually and recorded on an Esterline-Angus 20 pen event recorder. Thus, a practical limitation was placed on the size of the sample by the amount of time required in the tedious data reduction process. Only as much data as one could reasonably expect to subject to analysis was collected. As a result, certain of the stated objectives could not be fully met because of the lack of adequate sample data.

All sample data were collected during daylight hours in fair, dry weather during the months of August, October, and November. Any interpretation of this work should thereby be made in the light of these limitations.

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2. STUDY PROCEDURES

2.1 SELECTION AND DESCRIPTION OF THE STUDY SITE

Several factors influenced the selection of an intersection for study. A primary consideration was that the gaps in main street traffic should cover the full range of values which some drivers accept and some drivers reject. It had been determined from the review of earlier-workthat values between one and ten seconds should be contained in this range. The volume of traffic on the street controlled by the stop sign had to be substantial enough to make the question of alternative modes of control realistic.* Furthermore, if traffic were very sparse on the side street, an excessive amount of time would be required collecting an adequate sample. Other traffic factors of importance were that the side street carry both trucks and cars, and that these vehicles cross, make right turns and left turns into the main street in order that the effects of these variables could be evaluated.

Generally speaking, the intersection had to be fairly typical of those found throughout the country about which the question of alternatives of control is raised. Special characteristics, such as substantial horizontal or vertical curvature very near the intersection, an oblique crossing, severe sight distance restrictions, and one-way operation on either street were to be avoided.

The selection of the 'best' mode of control for the intersection studied was not one of the objectives of this research. No such decision was made.

The intersection selected for study, M-78 (Saginaw Road) and Abbott Road in East Lansing, Michigan, satisfied the important requirements reasonably well. M-78 is a major four-lane, bi-directional, undivided state highway, the arterial route connecting the cities of Flint and Lansing, Michigan. Abbott Road is an important two-lane, bi-directional city street which serves traffic to and from the business district of East Lansing and the campus of Michigan State University. which are situated approximately one mile south of the intersection. Figure 2 is a condition diagram of the intersection representing the characteristics existing at the time of the field study.

Pertinent features of the intersection are outlined below.

- <u>Control devices</u>: roadside and overhead illuminated stop signs facing Abbott. Flashing amber beacons warning M-78.
- 2. Speed limits: 25 mph on Abbott, 40 mph on M-78.
- 3. Iane widths: 10.5 feet on Abbott, 11 feet on M-78.
- 4. <u>Sight restrictions</u>: no major obstructions to vision across all four quadrants.
- 5. <u>Grade and curvature</u>: essentially flat and tangent within 400 feet of the intersection in all directions.
- 6. <u>Special right turn lanes</u>: pavement on M-78 widened eight feet on all but the northwest quadrant to accommodate decelerating vehicles turning right from M-78 and accelerating vehicles turning right from the northbound approach of Abbott. It was observed that very little use was made of these lanes except for eastbound N-78 vehicles turning right (south) onto Abbott. The other widened portions were evidently interpreted by most drivers as sections of paved shoulder.

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Condition Diagram of the Intersection Figure 2.

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- 7. <u>Traffic volume</u>: the average rates of traffic flow measured during the period of field study are presented in Table 1.
- 8. <u>Turning movements</u>: the approximate percentages of Abbott Road traffic proceeding left, straight and right are shown in Table 2. It will be noted that the directional movements are rather well balanced, particularly on the northbound approach. In comparison the turning movements on M-78 were minor, approximately seven percent left turns and nine percent right turns in the eastbound direction, and eight percent left turns and three percent right turns in the westbound direction.
- 9. <u>Traffic composition</u>: of the total sample of side street vehicles, 6.4 percent were trucks (not including one-half ton pickup trucks and panel delivery trucks which were considered as having performance characteristics similar to passenger cars). Approximately 14 percent of the M-78 traffic were trucks.

Another special consideration noted was that for several years, especially after the occurrence of a fatal accident on August 15, 1958, the residents of the surrounding community had been urging the installation of a traffic signal at this intersection.

2.2 DATA COLLECTION INSTRUMENTATION AND PROCEDURE

A special purpose survey device was devised especially for this project. The device was completely self-contained within a package measuring approximately one and one-half feet by one and one-half feet by three-quarters feet. In the top panel of the package were mounted

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Table 1. Average Rates of Traffic Flow During the Periods of Study

Period of	Main Stre	et (M-78)	Side Street (Abbott Road)
Study	Eastbound	Westbound	Northbound	Southbound
Offpeak	360	360	70	ጽ
Peak	815	310	125	55

Note: Average rates of flow are stated vehicles per hour.

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Table 2. Approximate Turning Percentages of Side Street (Abbott Road) Traffic During the Periods of Study

Period of	Nort]	hbound App.	roach	Sout	hbound App	roach
Btudy	left	Straight	Right	Fert	Straight	Right
Offpeak	304	376	33%	¥L.	264	37%
Ревк	K	414	37%	\$OT	524	38%

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ten normally-closed, push-button microswitches. Each of the switches was electrically connected to a separate pen of an Esterline-Angus 20 pen event recorder contained within the package. A sketch of the top panel is shown in Figure 3. Included on the sketch are the pen numbers to which the push-buttons were connected. In the following text the pushbuttons will be identified by these numbers. A portion of the front panel of the package was left open to permit easy access to the 20-pen recorder. Two 22.5 volt dry-cell batteries connected in series powered the system.

In the Esterline-Angus recorder a paper chart roll is transported, by a spring wound clock device, past a bank of 20 solenoid actuated pens. When one of the push-buttons was depressed, the circuit was closed and a solenoid activated, causing the corresponding pen to be displaced from its normal position. The pen remained in its displaced position until the push-button was released.

The survey device was operated by two observers seated in the front seat of a private automobile parked approximately 15 feet off Abbott Road in the northeast quadrant of the intersection (see Figure 2). The first observer: (1) depressed buttons 8, 6, and 4 upon the arrival of main street vehicles proceeding left, straight and right, respectively; (2) depressed button 1 upon the arrival of a northbound side street vehicle; and (3) depressed button 2 upon the complete entry of a side street vehicle into the intersection. In a similar manner the second observer operated buttons 12, 14, and 16 upon arrival of main street westbound vehicles, and buttons 20 and 19 upon arrival and entry of side street southbound vehicles. For car arrivals on both the main street and side street arrivals, the buttons were depressed and released immediately; whereas for trucks, the buttons were depressed for a longer period of

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Figure 3. Sketch of Top Panel of Survey Device



Note: Numbers in parentheses next to pushbuttons indicate the pen numbers of the event recorder to which the buttons were connected. time. For straight through, right turn, and left turn entries from the side street, the entry buttons were depressed once, twice in rapid succession, and three times in rapid succession, respectively. After a reasonably short period devoted to practice runs, the observers were able to perform their assigned tasks with a high degree of proficiency.

A sample section of a survey chart is shown in Figure 4, for the purpose of illustrating the manner in which pertinent data can be measured. For this survey, a chart speed of six inches per minute was used. The chart speed was checked with a stop watch periodically to insure the maintenance of an accurate time scale.

A total of 472 minutes of sample data were gathered on weekdays during the months of August, October, and November, 1961. The decisions and reactions of 1,203 side street vehicles were observed. A total of 5,179 acceptance-rejection decisions were made, 1,203 relating to lags and 3,976 relating to gaps.

2.3 DATA REDUCTION PROCEDURE

Extracting pertinent data from the graphical charts was the most time consuming phase of the project. The chart rolls were viewed on a special light table, which had been constructed for viewing rolls of aerial photographic film. Time measurements were made with the aid of a plastic template on which the appropriate time scale had been scribed. All time measurements were estimated to the nearest one-tenth second.

The charts were scanned for vehicle arrivals on the side street. Each arriving vehicle was numbered and its type and direction of movement through the intersection were determined. A check was made to determine if a vehicle was waiting at the same time on the opposite side street approach. A sequence of time measurements was then made in the manner previously illustrated in Figure 3:

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Figure 4. Sample Section of the Data Chart



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- Delay in queue, which applies only if more than one side street vehicle is waiting.
- 2. A near-side or a far-side lag. Near-side implies the direction of main street traffic nearest to the waiting driver, or in other words, traffic approaching from the driver's left. For side street right turning vehicles, only near-side lags are relevant.
- 3. A series of gaps which may have the following sequences: near-near, near-far, far-near, far-far (see Figure 5). For side street right turning vehicles, only near-near gaps are relevant.
- 4. Delay as queue leader (i.e., first in line).
- 5. Total delay, which equals the sum of delay in queue and delay as queue leader.

All of the above measurements were recorded in units of seconds on a single data form, a sample of which is found in Appendix. I ags and gaps which were accepted by side street drivers were circled. Starting delay times in accepting lags and gaps were measured and recorded separately during a second pass through the chart rolls.

2.4 STATISTICAL ANALYSES

The bulk of the statistical analyses performed related to the lag and gap acceptance data.

2.4.1 Stratification of the Data

To permit an effective evaluation of the effects of certain traffic factors, these data were first stratified in the manner illustrated by Figure 6. Such stratification is often undertaken to help safeguard against overlooking or misinterpreting the significance of a given

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Figure 5. Sequence of Main Street Gaps

NEAR-FAR GAP



FAR-NEAR GAP



FAR-FAR GAP





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factor caused by submerging the effects in a larger distribution affected by other important variables. Consideration was given to the possible use of analysis of variance techniques; however, with an interest in such a large number of levels of comparison, analysis of variance would soon have become unwieldy and difficult to interpret.

For each class of data shown in Figure 6, the parameters of the acceptance distribution were determined. Pertinent comparisons between classifications, as indicated by the dashed lines in the Figure, were then made.

2.4.2 Grouping Into Class Intervals

Lag and gap times ranging between zero and 15 seconds were grouped in one second intervals before proceeding with the analysis. For the sake of simplicity, the integer values (e.g., 1.0, 2.0, ...) were treated as the class boundaries, and any data falling exactly on a boundary was placed in the higher class interval. No lag or gap greater than 15 seconds was rejected, hence all values greater than 15 seconds were grouped together.

2.4.3 Confidence Interval Estimation

For each class interval the frequency of acceptances, A_i , and rejections, R_i ; and the interval sample size, $n_i = A_i + R_i$, were known. The true probability (or proportion) of acceptance for each interval, P_i , was estimated by the statistic

$$p_i = \frac{A_i}{n_i}$$

According to Dixon and Massey (24) the frequency, A_i , follows a binomial distribution. For relatively large samples, values of $n_i P_i$ and n_i (1 - P_i) greater than five, the sampling distribution of P_i

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should have mean P_i and variance $P_i (1 - P_i)/n_i$, and the statistic

$$\frac{P_i - P_i}{\sqrt{\frac{P_i (1 - P_i)}{n_i}}}$$

an approximately unit normal distribution.

For large samples, an interval estimate of P_i with confidence coefficient $(1 - \alpha)$ is

$$p_{i} + z_{1} \sqrt{\frac{p_{i} (1 - p_{i})}{n_{i}}} < P_{i} < p_{i} + z_{1 - \frac{1}{2} \alpha} \sqrt{\frac{p_{i} (1 - p_{i})}{n_{i}}}$$

where: Z_1 and Z_2 are ordinates of the unit normal distribution. $\frac{1}{2}\alpha$ $1-\frac{1}{2}\alpha$

The probability that the interval estimate computed from a given sample contains the population parameter, P_i , is $(1 - \alpha)$. For each class interval, an interval estimate of P_i for $(1 - \alpha) = .90$ was determined by referring to Table A-9b of Dixon and Massey (24). 2.4.4 Graphical Determination of Acceptance Distributions

Working with similar data, Bissell (23) plotted gap size, X_i , against sample proportions, p_i , on logarithmic-probability graph paper and fitted a straight line through the points. This indicated that the relationship between log X_i and p_i had a form approximating a normal distribution. He was thus able to estimate the mean and standard deviation of the transformed variate, log X_i , rather easily.

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Dixon and Massey (24) discussed the use of this type of graphical analysis for data from sensitivity experiments, such as submitting animals to different dosage levels to determine the lethality of a drug, or dropping explosive specimens from various heights to determine sensitivity. The analogy between such data and the lag and gap acceptance data at hand is evident.

This writer used a slightly modified version of this technique to determine the form of the acceptance distribution. Rather than plotting sample probabilities, interval estimates of P_1 were used, and a straight line was fitted to the data by eye which passed through most of the intervals. An example of this technique is presented in Figure 7. Also shown is a rectilinear graph which illustrates the relationship between gap size and probability of acceptance in their natural, untransformed state.

The mean and standard deviation of the distribution is estimated by noting where the line crosses the 50 percent point, $X_{.50}$, and the 15.87 percent line, $X_{.1587}$. Then

mean, $\log X = \log X_{50}$

standard deviation, $S_{\log X} = \log X_{.50} - \log X_{.1587}$

The transforms should always be used when working with the distributions, and the antilogs, or absolute values, only stated in general descriptions of the results.

An equation can be written for solving for the probability of accepting a certain gap size, X_i , in the form



Figure 7. Example of the Graphical Analysis Technique

*

$$P(\text{accepting } \mathbf{X}_{1}) = \frac{1}{\sqrt{2\pi}} \int_{\log \mathbf{X}}^{\mathbf{X}_{1}} e^{-\frac{1}{2}\left(\frac{\log \mathbf{X} - \overline{\log \mathbf{X}}}{8}\right)} d\mathbf{X}$$

However, the references indicate that this can only be solved by approximate methods. It is much more practical, therefore, to use the graph directly or to tabulate selected values read from the graph. 2.4.5 <u>Precision of the Estimates</u> and Tests of Hypotheses

The precision of the parameter estimates of the acceptance distributions, and testing of statistical hypotheses relating thereto, are complicated by difficulty in interpreting the effective sample size. Because the transforms of gap sizes are not random normal variates in the normal sense of the word, it is believed invalid to utilize the sum of the sample sizes within class intervals, $\sum n_i$, as the effective sample size used in estimating the parameters of the parent distribution.

Finney's text (25) is principally devoted to the analysis of the same general form of data found in biological assay. However, the application of many of the techniques discussed depended on special sampling techniques common to the biological research field. General methods suggested were usually very complex and difficult. One method, however, which he attributed to Spearman and Karber could be reasonably applied to the data at hand to estimate the variance of the mean of the distribution. After working two lengthy samples, it was found that the technique yielded a variance of the mean which could be associated with a sample size slightly larger than the average sample within the one second class intervals. In order to get on with a reasonably practical analysis, therefore, it was decided that the average n_1 in each

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distribution would be utilized as a conservative estimate of the effective sample size.

For each of the comparisons of acceptance distributions, two null hypotheses were tested.

1. null hypothesis: $\log X_1 = \log X_2$ alternate hypothesis: $\log X_1 = \log X_2$ 2. null hypothesis: $S_{\log}^2 X_1 = S_{\log}^2 X_2$ alternate hypothesis: $S_{\log}^2 X_1 = S_{\log}^2 X_2$

Each hypothesis was tested at the level of significance, $\infty = .05$, and rejection of either caused the conclusion that the two distributions being compared were indeed different. The reader is referred to standard introductory texts in statistical analysis, such as Dixon and Massey (24) or Walker and Lev (26) for detailed descriptions of student's 't' test on means and the F ratio test on variances.

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3. RESULTS

3.1 DECISIONS OF MINOR STREET DRIVERS

Results of the statistical analysis of lag and gap acceptance distributions are presented in graphical form, and the influence of the traffic factors studied are discussed in the light of difference tests performed on the parameters of the distributions. For the sake of clarity, only the linear estimates are shown; however, each of these estimates resulted from the graphical analysis technique described and illustrated in Section 2.4.4. In performing each graphical analysis, it was found that a straight line could be drawn which passed through a great majority of the series of confidence bands plotted on the logarithmic-probability paper. This held up well for all levels of data stratification. Thus, the results gave rather strong verification of Bissell's (23) conclusion that the probability of accepting a lag or gap in main street traffic has a lognormal form.

3.1.1 The Composite Lag and Gap Acceptance Distribution

Illustrated in Figure 8 is an estimate of the acceptance distribution resulting from combining all driver decision data into one large sample. Also shown are estimates of the parameters of the distribution, i.e., the mean and standard deviation of the log transformed gap size distribution, $\overline{\log X}$ and $S_{\log X}$, along with the effective sample size, \overline{n}_i , associated with the estimates. The two points on the line which were used in computing the estimates are marked for reference. One can use the graph to estimate the probability of accepting a given lag or gap.

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It is interesting to note that the absolute value of the mean is 7.4 seconds, and that lags and gaps smaller than 4.3 seconds are accepted by fewer than ten percent of the side street vehicles, and gaps larger than 12.5 seconds are accepted by more than 90 percent. A total of 335 lags and gaps greater than 15 seconds were presented to side street drivers, all of which were accepted.

3.1.2 Comparison of Lags and Gaps

The results of separating the composite data into lags and gaps are shown in Figure 9. Two null hypotheses on the parameters of the two distributions were tested. First, the hypothesis that the mean of the lag distribution equals the mean of the gap distribution was tested, using a one-tailed student's 't' test at level of significance or = .05. Secondly, the hypothesis that the variances of the two distributions were equal was tested, using a one-tailed F ratio test at level of significance $\alpha = .05$. Both hypotheses were rejected, and it was concluded that the two samples are not members of a common distribution. In other words, the acceptance of lags and the acceptance of gaps should be treated separately. The rejection of either of the hypotheses would have caused the same conclusion to be drawn. The mean of the lag acceptance distribution, absolute value 8.00 seconds, and the mean of the gap acceptance distribution, absolute value 7.20, differed significantly. Also, the lag acceptance distribution is significantly more disperse than the gap acceptance distribution.

The sample was further segregated into cars and trucks, and a comparison of lag acceptance and gap acceptance for cars was made; results are shown in Figure 10. Once again, the hypothesis of equality for both means and variances was rejected, and it was concluded that

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lag acceptance and gap acceptance for cars are indeed different and should be treated separately. The absolute values of the means were 8.00 seconds and 7.10 seconds for lags and gaps, respectively.

3.1.3 Evaluation of the Influence of Traffic Factors on Driver Decisions

3.1.3.1 Vehicle Type

The lag and gap acceptance distributions for the two classifications of side street vehicles, trucks and cars, are presented in Figure 11. From a logical basis, considering the poorer acceleration capabilities of trucks, there was reason to expect that differences would be found. However, neither the hypothesis on the means nor the hypothesis on the variances was rejected. It was therefore concluded that there was no evidence to indicate that truck behavior and car behavior are significantly different. Until contrary evidence is found, trucks and cars need not be handled separately.

If the truck-car comparison had been made separately for lags and gaps, or for offpeak and peak periods, it is possible that significant differences would be found. However, the small size of the truck sample did not permit such further stratification. There was no way of knowing in advance the results of the truck-car comparison. In the original stratification plan, evaluation of the remaining traffic factors' influence on the acceptance distribution was to be based on car sample alone. This plan was not revised.

3.1.3.2 Pressure of Traffic Demand

A comparison of the offpeak period and peak period gap acceptance distribution for cars is presented in Figure 12. There was no evidence to indicate that the variances of the distribution were different.

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However, the null hypothesis on the means was rejected, and it was concluded that the two samples were not from a common distribution. Behavior during peak and offpeak periods was indeed different; data from the two periods should be treated separately.

The influence of pressure of traffic demand was more strikingly illustrated in the case of <u>lag</u> acceptance distribution (see Figure 13). The mean of the offpeak distribution differed significantly from the mean of the peak distributions. Once again, however, there was no evidence to indicate difference in the variances of the distributions. 3.1.3.3 Direction of Side Street Vehicle Movement

The gap acceptance distributions for side street cars waiting to proceed straight, turn left, or turn right into the intersection were compared.

Figure 14 illustrates this comparison for the peak period sample. Statistical tests were performed on the most widely separated means, and no evidence of significant difference was found. However, in testing the extreme values of variance estimates differences were found. The gap acceptance distribution for right turners was significantly more disperse than the distribution for left turners. This difference can be attributed to the right turning driver's willingness to accept a greater percentage of gaps in the low range of the distribution. No significant differences, either in the means or the variances, were evident in the comparison of left turns and straight through movements. It was concluded that in considering behavior during peak periods, the actions of right turners must be distinguished from left and straight movements.

An evaluation of the effects of the direction of side street movement was also made for the offpeak period, as illustrated by Figure 15.

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The hypotheses of equality tested for the extreme parameter estimates could not be rejected. For the offpeak period, in other words, there was no evidence to indicate that the left, straight, and right samples did not come from a common distribution.

The lack of an adequate sample of lags prevented the conduct of valid tests regarding the effect of direction of movement on the lag acceptance distributions.

3.1.3.4 Main Street Vehicle Sequence

Another factor investigated was the sequence of main street vehicles comprising the gaps presented to waiting drivers. Only the left and straight cars were included in this analysis, since for right turns only those gaps in the near side main street traffic are relevant.

During the peak period, the sequence of main street gaps had a significant influence on the gap acceptance distribution, as shown by Figure 16. Hypotheses were first tested on the parameter estimates of the most widely separated distributions (i.e., near-far and far-near). The mean of the near-far distribution was found to be significantly larger than the mean of the far-near distribution. There was no evidence indicating significant differences in the variance estimates. Next, the far-far and near-near distributions were compared, and no significant difference was found. Thus, in the consideration of peak period traffic, near-far and far-near gaps should be treated separately, but far-far and near-near gaps may be grouped.

A similar evaluation of the effect of main street vehicle sequence was made for offpeak period traffic. It can be seen in Figure 17 that the distributions are in much closer proximity to one another. The hypotheses of equality applied to the extreme values of mean and variance

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could not be rejected. Therefore, it was concluded that during offpeak periods, data need not be segregated on the basis of the sequence of the main street gaps.

It had originally been planned to study acceptance of near lags as compared with far lags, but this was impossible due to an inadequate sample of lags.

3.1.3.5 Conditions on the Opposing Side Street Approach

The final traffic factor considered was the presence or absence of a vehicle waiting on the opposite side street approach. It was assumed that this factor is irrelevant to right turning vehicles, hence only left and straight vehicles were included in the analysis. In both the peak and offpeak samples, there was no evidence to indicate that the decision of the waiting driver was significantly affected by conditions on the opposing approach. Results are presented in Figures 18 and 19.

Once again, data were insufficient to evaluate the effect of this factor on the lag acceptance distribution.

3.1.4 Summary of Driver Decision Results

The results of the study of lag and gap acceptance distribution of side street drivers are summarized in Figures 20, 21, 22, and 23.

Figure 20 is a stratification diagram on which the effective sample sizes, \overline{n}_i , are entered for each item of classification. In a similar manner, in Figure 21, the absolute values of the estimates of the means of the acceptance distributions are summarized. In Figure 22, the standard deviations of the log transformed distributions are presented. Finally, the results of the tests of null hypotheses on the parameter estimates are summarized in Figure 23.

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The analysis of the lag and gap acceptance distributions resulted in the following conclusions:

1. The results of this study strongly supported Bissell's (23) findings which indicated that the relationship between the logarithmic transform of lag or gap size and probability of acceptance follows a normal distribution curve.

2. The <u>lag</u> acceptance distribution had a significantly higher mean and was significantly more disperse than the <u>gap</u> acceptance distribution.

3. There was no evidence to indicate that the lag and gap acceptance distribution for <u>trucks</u> differed significantly from that for <u>cars</u>.

4. Both the gap and lag acceptance distributions for cars had significantly higher means during <u>offpeak</u> periods than during <u>peak</u> periods.

6. During the peak period, the distribution for cars accepting <u>near-far</u> gaps had a significantly higher mean than the distribution for cars accepting <u>far-near</u> gaps. No significant effect of the sequence of main street vehicles on the gap acceptance distribution for cars was evident during the offpeak period.

7. In neither peak nor offpeak periods was there evidence to indicate that the presence of a vehicle on the opposing side street approach significantly affected the gap acceptance distribution for cars.

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3.2 REACTIONS OF MINOR STREET DRIVERS

3.2.1 Starting Delay Time Distributions for Lag Acceptances

It is recalled that in Section 1.1.4 starting delay time in accepting lags was defined as the elapsed time between the arrival of a side street car and its complete entry into the intersection. In the special case of more than one vehicle waiting on a side street approach, the arrival of the second, or succeeding, vehicle was defined as coinciding with the complete entry into the intersection of the first vehicle in the queue.

Frequency distributions of starting delay time for the first vehicle and for the succeeding vehicle are presented in Figure 24. One immediately notices that the two distributions appear to be different. Pertinent measurements on the sample distribution are tabulated below.

. <u>First</u>	Vehicle in Queue	Succeeding Vehicle
Sample Size	172	123
Mean	4.37	2.99
Median (50 Percentile)	4.4	2.7
15 Percentile	2.5	1.7
85 Percentile	6.2	4.8
Mode	5.25	1.75

Because the distribution for succeeding vehicles was rather highly skewed and the other approximately normal, the standard tests for statistical difference were not performed. However, it is evident that the starting delay time for succeeding vehicles has a lower central tendency and is less disperse than starting delay time for the first vehicle in the queue.

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3.2.2 Starting Delay Time Distributions for Gap Acceptances

Starting delay time in accepting a gap was defined in Section 1.1.4 as the elapsed time between arrival of the first main street car comprising the gap and the complete entry into the intersection of the side street car. The frequency distribution of this variable is shown in Figure 25. Pertinent measurements on this approximately normal sample distribution are tabulated below.

Sample Size	703
Mean	3.02
Median (50 Percentile)	2.8
15 Percentile	1.8
85 Percentile	4.4
Mode	2.75

It is interesting to note that, except for the one second difference in the mode, this sample distribution is very similar to the distribution of starting delay time in accepting lags for succeeding vehicles.

3.2.3 Influence of Traffic Factors on Starting Delay Time

3.2.3.1 Pressure of Traffic Demand

The sample of starting delay times of succeeding vehicles in accepting <u>lags</u> was segregated on the basis of period of the day to reflect different intensities of pressure of traffic demand. The frequency distributions for peak and offpeak periods are presented in Figure 26, and statistics are tabulated below.

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Figure 25. Starting Delay Time in Accepting Gaps

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Figure 26. Effect of Pressure of Traffic Demand on Starting Delay Time in Accepting Lags (Succeeding Vehicles)



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	Offpeak	Peak
Sample Size	66	57
Mean	3.30	2,62
Median (50 Percentile)	3.0	2.2
15 Percentile	1.9	1.6
85 Percentile	5.1	4.5
Mode	2.75	1.75

It is apparent that the central tendency of starting delay time for succeeding vehicles in accepting lags is lower during peak periods than during offpeak periods. On the other hand, little difference is seen in the dispersion of the two distributions.

The sample of first in queue vehicles accepting lags was too small to be segregated into peak and offpeak period samples.

The effect of pressure of traffic demand on starting delay time in accepting gaps was also considered. Offpeak and peak period frequency distributions are shown in Figure 27, and pertinent statistics are summarized below.

	Offpeak	Peak
Sample Size	442	261
Mean	3.11	2.86
Median (50 Percentile)	3.0	2.7
15 Percentile	1.9	1.7
85 Percentile	4.5	4.2
Mode	2.75	2.25
Standard Deviation	1.35	1.43

One was not able to determine by visual inspection whether the distributions differed significantly. However, since the distributions

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Figure 27. Effect of Pressure of Traffic Demand on Starting Delay Time in Accepting Gaps



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appeared approximately normal, standard deviations were computed and student's 't' was used to make a one-tailed test of the null hypothesis on the means. The hypothesis was rejected at level of significance $\alpha = .05$. It was therefore concluded that the mean starting delay time in accepting gaps was significantly smaller during peak traffic periods. 3.2.3.2 Main Street Vehicle Sequence

From a logical viewpoint, it was expected that starting delay time in accepting gaps might be affected to some degree by the sequence of main street vehicles comprising the accepted gap. In particular, if first car of the gap was on the <u>far</u> side, the side street car might commence motion earlier and complete entry into the intersection more quickly than if the first car of the gap was on the <u>near</u> side. The frequency distributions presented in Figure 28 illustrate the effects of this factor.

Tabulated below are pertinent statistics of the offpeak period sample distributions.

	First Car of Gap 'Near',	First Car of Gap 'Far'
Sample Size	248	194
Mean	3.27	2.91
Median (50 Percentile)	3.2	2.7
15 Percentile	2.1	1.7
85 Percentile	4.5	4.3
Mode	2.75	2.25
Standard Deviation	1.34	1.34

The distributions were approximately normal, so the null hypotheses on the means were tested with student's 't' at level of significance $\infty = .05$. It was concluded that starting delay time in accepting gaps,

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Effect of Sequence of Accepted Main Street Gap on Figure 28. Starting Delay Time in Accepting Gaps



when the first car of the gap was on the far side, was significantly smaller than when the first car of the gap was on the near side.

An identical analysis was made on the peak period data. Even by visual inspection of the frequency distributions, one can conclude that the effect of main street vehicle sequence on starting delay time is more marked during peak periods. The statistics for the peak period sample are tabulated below.

	First Car of Gap 'Near'	First Car of Gap 'Far'
Sample Size	170	91
Mean	3.15	2.33
Median (50 Percentile)	3.0	2.2
15 Percentile	2.0	1.1
85 Percentile	4.5	3.3
Mode	2.75	2.25
Standard Deviation	1.39	1.34

Once again, the null hypothesis on the means was rejected at $\infty = .05$. Thus for peak periods, as well as offpeak, starting delay times in accepting gaps were significantly smaller when the first car of the gap was on the far side than when the first car was on the near side.

3.2.4 Summary of Driver Reaction Results

1. The central tendency of starting delay time in accepting lags was greater for the <u>first</u> vehicle in the queue than for the <u>succeeding</u> vehicle.

2. The distribution of starting delay time in accepting lags for succeeding vehicles was very similar to the distribution of starting delay time in accepting gaps. Both of these distributions had lower central values during peak periods than during offpeak periods.

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3. During both peak and offpeak periods, the starting delay time distribution for gaps had a smaller mean when the first main street car comprising the accepted gap was on the <u>far</u> side than when the first main street car was on the near side.

3.3 DELAY IMPOSED ON MINOR STREET DRIVERS

The delay encountered by each minor street vehicle was measured from the 20-pen recorder charts. <u>Delay in queue</u> was defined as the elapsed time between the arrival of a vehicle at the back of the queue of waiting vehicles and its arrival as first in the queue. A vehicles arrival as first in queue was assumed to coincide with the completion of entry into the intersection of its leader. <u>Queue leader delay</u> was defined as the elapsed time between the arrival of a vehicle as first in queue and its completion of entry into the intersection. It may be recalled that when the side street approach was clear of other vehicles, a vehicle was assumed to have arrived (as first in queue) either when it stopped or reached its lowest speed. <u>Total delay</u> is the sum of in queue delay and queue leader delay.

The analysis of delays was much less exhaustive than was the case for driver decisions and reactions. The primary objective was to present sufficient information on delays so that a person who has developed an analytical or a simulation model may validate his results.

3.3.1 Total Delay

Frequency distributions of total side street delay are shown in Figure 29. Separate distributions are presented for the peak and offpeak periods and for the northbound and southbound side street approaches. It can be noted that during the offpeak period, the northbound and southbound distributions have a high degree of similarity. During the peak

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Figure 29. Total Delay on the Side Street Approaches

period, however, individual delays range up to 340 seconds on the northbound approach, nearly twice the maximum encountered on the southbound approach. This difference was attributed to heavier traffic demand and formation of longer queues on the northbound approach.

3.3.2 Delay in Queue

Frequency distributions of delay in queue are presented in a similar manner in Figure 30. Here again, the northbound and southbound offpeak period distributions are nearly identical; but during the peak period delay in queue on the northbound approach ranges up to 300 seconds as compared with a maximum of only 75 seconds on the southbound approach.

3.3.3 Queue Leader Delay

Frequency distributions of queue leader delays are presented in Figure 31. The distributions for the two side street approaches were very similar during offpeak periods. However, during peak periods, queue leader delay ranged up to 130 seconds on the southbound approach as compared with a maximum of only 60 seconds on the northbound approach. Furthermore, queue leader delays in the lowest class interval, zero to five seconds, were less than half as frequent on the southbound approach as on the northbound. Apparently, with lower traffic demand and shorter queues on the southbound approach, the driver is under much less pressure to cross or enter the main street as quickly as possible.

3.3.4 Summary of Delay Data

Table 3 summarizes the descriptive statistics of the in queue, queue leader and total delay distributions for side street vehicles. The mean, median, 15 percentile, 85 percentile, and maximum observation of the distributions are tabulated. Also shown are the average rates of flow on the main street and side street associated with the peak and offpeak period samples.

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Figure 31. Queue Leader Delay on the Side Street Approaches



Figure 31. Queue Leader Delay on the Side Street Approaches

Delay
Vehicle
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Summary
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Table

Period of	Main Street	Side Stude	Side Street	Component	Side i	Street D	elay Stat	ilstics	Seconds
tate tag	ITALILC FLOW EB & WB (Vph)	Approach	Approacn Traffic Flow (vph)	01 Jelay	Mean	Median	15 Fer- centile	85 Per- centile	Maximum
				In Queue	2.2	0	0	5	25
		NB	70	Queue Leader	10.6	8	2	19	<u>6</u>
() freek				Total	12.8	IO	٣	53	<u>6</u>
	<u>}</u>			In Queue	2.3	0	0	5	%
	_	SB	50	Queue Leader	12.6	6	m	23	සි
				Total	14.9	TO	2	25	75
				In Queue	45.0	74	ο.	115	300
		NB		Queue Leader	16.1	OL	ſ	ଝ	60
Back	1125			Total	61.1	õ	97	133	340
4	Ì			In Queue	6.3	ç	0	टा	. 51
		SB	55	Queue Leader	23.9	05 mi	7	55	130
·	_			Total	30.2	r r 1	Ŋ	65	135

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Merely to assist the reader in visualizing the statistics presented in the table, results are presented graphically in Figure 32. Here the statistics of the queue leader delay distributions are plotted against main street flow rates, and the in queue delay statistics are plotted against side street approach flow rates. The trends of the side street delay components as related to the flow rates, indicated by the dashed lines, should be considered only as very rough approximations. It is hoped that this figure will be viewed in its proper context.





4. DISCUSSION

4.1 INTERPRETATION OF RESULTS

Generally speaking, the results of this research tended to verify that which a professional traffic engineer might deduce on the basis of logical consideration of the factors involved.

For example, the fact that lag and gap acceptance differed significantly was not surprising. One might expect that a driver who has just arrived at a stop sign needs some time to orient his senses to the decision making process. Furthermore, when such a driver is nearing the stop sign, he is often not in as advantageous a position for the critical observation of main street traffic as if he had been waiting near the intersection entry line for some period of time. These factors may help explain why the mean of the lag acceptance distribution was significantly larger than the mean of the gap acceptance distribution.

With regard to another comparison, it was rather surprising to not find evidence indicating significant difference in truck and car decisions. One would normally expect that, on the average, trucks need to wait for longer gaps. However, the size of the truck sample prohibited stratified comparisons, and if true differences do exist they may be submerged among the effects of other important factors.

The factor which had the most striking effect on the lag and gap acceptance distributions was the pressure of traffic demand. The means of the acceptance distributions were significantly smaller during peak periods than during offpeak periods. Several factors might be of

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importance in explaining these differences. During the peak period, many drivers are traveling between work and home. Before they reach the position of queue leader, they most likely have spent a substantial period of time in the queue. Furthermore, when they do become the queue leader, it is likely that one or more vehicles are waiting behind. All of these factors might be expected to contribute some degree of impatience. Of possibly equal importance is the higher traffic volume, or in other words smaller average gaps, on the main street during the peak period. The driver who rejects a marginal lag or gap may have to wait a substantial time for another opportunity to cross or enter.

Reasonable explanations are also possible in the case of the effects of direction of side street vehicle movement. During the peak period, right turners had a greater probability of accepting gaps of a given size in the lower half of the distribution. This difference may be the reflection of three factors: (1) the right turners decision is less complicated since he needs to consider gaps in only one direction of the main street; (2) since there are two lanes in each direction on the main street studied, the main street vehicles may avoid conflict with an entering right turner by changing lanes; and (3) the few right turning drivers who did use the widened portion as an acceleration lane could accept smaller gaps. It was interesting that differences due to direction of movement did not appear in the offpeak period sample. Thus, an example of the appropriateness of a data stratification procedure was illustrated.

It is theorized that during offpeak periods drivers feel no special compulsion to attempt to attain the maximum performance capabilities of themselves or their vehicles. Then during peak periods, when some degree

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of compulsion is working, reasonable lower boundaries on gap size corresponding to maximized performance are lower for right turns than for the other movements. More succinctly, there is more room to maximize right turn performance.

There is also a logical basis for explaining the results relating to the sequence of main street vehicles. At least three factors are believed to be important: (1) if the first car of the gap is on the near side, it blocks the waiting driver's vision of far side main street vehicles; (2) if the first car in the gap is on the near side, the waiting driver cannot normally begin his entry into the intersection until the near side car has passed; (3) if the second car of the gap is on the far side, the crossing driver must travel a longer distance to clear the area of conflict. In considering a far-near gap, the favorable condition of all three of these factors are met, whereas in the case of a near-far gap, all of the unfavorable conditions are working. Near-near and far-far gaps have a mixture of favorable and unfavorable conditions.

It is also interesting to note that differences in gap acceptance due to main street vehicle sequence were evident only during the peak period. The previously discussed hypothesis concerning differences in motivation and differences in ability to maximize performance could again be applied.

The results gave no evidence that the presence or absence of a vehicle on the opposite side street approach significantly affected gap acceptance. However, due to inadequacies in the sample size, the comparisons were made on distributions which included both straight and left turning vehicles. It would be desirable to test the effects of conditions on the opposite approach separately for left turns and straight movements.

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The study of starting delay times of side street vehicles in accepting lags and gaps yielded results which can be closely correlated with the driver decision data. It was readily apparent that the traffic factors which had important effects on the acceptance-rejection decision also influenced the distributions of starting delay times -- and in the same direction. For example, the average starting delay time in accepting gaps was lower during periods of heavy traffic demand. It is impossible to state with assurance that lower starting delays <u>enable</u> shorter gaps to be accepted, or alternatively, that the decisions to accept shorter gaps <u>cause</u> the lower starting delays. Rather, it is probably more accurate to say that both behavior characteristics are affected similarly by common factors, such as impatience, degree of motivation, and the reduced size of main street gaps presented to waiting drivers.

Regarding another important factor, main street vehicle sequence, which similarly affects starting delay and gap acceptance, there is some reason to note a causal relationship. It is believed that a partial reason for far-near gaps being more readily acceptable is that the position of the first car in the gap enables the side street car to start into the intersection sooner.

Another interesting parallel was that during offpeak periods the starting delay in accepting lags for succeeding vehicles averaged over one second lower than the starting delay for first in queue lag acceptances. To clarify the point, a first in queue lag acceptance was when a vehicle arrived on a clear side street approach, stopped, and then entered the intersection before the arrival of the first main street car. A succeeding vehicle lag acceptance was when the second car in a waiting

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queue followed the queue leader into the intersection, in effect, accepting the remaining portion of the gap which the leader accepted. In most cases, the succeeding vehicle has been waiting second in line long enough to become oriented to the decision process, and in effect is observing the gaps in a manner similar to the queue leader. The reader can verify this point if he has ever had the common experience of waiting second in line and commenting to himself or to a passenger, "he should have taken that gap."

4.2 COMPARISON WITH RELATED RESEARCH

The results of this study are compared with those Bissell (23), Greenshields, et al. (5), and Raff (6) in Figure 33, on a logarithmicprobability graph. Both Greenshields and Raff estimated central values for lag acceptance; their results are plotted on the 50 percent line. Bissell's straight line estimate of the lag and gap acceptance is shown.

It is evident that the acceptance distributions determined in this study had significantly higher central tendency than those found in any of the other studies. The variance of Bissell's lag and gap acceptance distribution, however, did not differ significantly from the present findings.

It is believed that an explanation of the differences is related to differences in the nature of the intersections studied. In particular, the main street of this study was wider and carried higher speed traffic than the main streets studied by the others. If one were to extend Raff's conclusion that better sight distance is associated with higher critical lags to the comparison at hand, the differences might be attributed to the better sight distance at the intersection studied by the writer. It is difficult, however, to expect that Raff's conclusion

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would hold for intersections with substantially different geometrics and traffic characteristics, such as the one studied.

Of special interest is a comparison of the studies of actual traffic intersections and the controlled experimentation done by Herman and Weiss (12); their experimental data is also shown in Figure 33. It can be seen that these data differ markedly from the rest. No lag smaller than 3.2 seconds was accepted and none larger than 4.2 seconds was rejected. The point where their line crosses 50 percent corresponds to only three to ten percent acceptance in the distributions of Bissell and this study. Herman and Weiss state ... "these experiments were rather artificial in that the drivers were highly motivated and quickly adapt to the situation...". However, their results are especially interesting and useful in that they represent 'maximized' performance characteristics. 4.3 FUTURE RESEARCH

The conclusions drawn from this research were based on observations at only one intersection. It would seem interesting and important, therefore, to make similar studies of driver decisions and reactions, and the effects of variable traffic factors thereon, at other intersections. If such studies were to give strong verification of the conclusions pertaining to the influences of traffic factors, an interesting topic of study could be presented to the applied psychologist. One might approach the problem of determining why one intersection has different gap acceptance characteristics than another from a study of the underlying human factors, rather than attempting to attribute differences to engineering features.

Certain specific items which could not be adequately handled in this study might be of interest. For example, the effects of direction of movement, gap sequence, and conditions on the opposite approach on lag

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acceptance could not be studied here due to inadequate sampling. Studies might be undertaken to correlate the effects of variables on lag and gap acceptance. Another inadequacy was the inability to make a really detailed study of the effects of conditions on the opposing approach, particularly as related to the direction of movement of the car in question and the car opposite. Further study of this factor might prove fruitful. Although the effects of different types of vehicles on the side street were studied, no consideration was given to vehicle type on the main street. Studies to determine whether acceptance of a gap in which the second vehicle is a heavy truck differs from the case in which the conflicting vehicle is a car might be of value.

4.4 CONCLUSION

The major findings of this research were:

1. The results strongly supported earlier findings which indicated that the relationship between the logarithmic transform of lag or gap size and the probability of acceptance follows a cumulative normal distribution curve. Lag acceptance differed significantly from gap acceptance.

2. Of the traffic factors studied, those which significantly influenced the parameters of the lag and gap acceptance distribution were: pressure of traffic demand; direction of traffic movement during periods of heavy traffic demand; and main street vehicle sequence during periods of heavy demand.

3. No evidence was found to indicate that either side street vehicle type or conditions on the opposing side street approach significantly affected the gap acceptance distribution.

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4. The results of the study of starting delay times in accepting lags and gaps indicated that factors which had important influence on the driver decisions, namely pressure of traffic demand and main street vehicle sequence, also had similar and significant effects on starting delay time.

5. The components of delay to side street drivers were determined and are presented to enable those interested to validate the results of their analytical and simulation model work.

In the final analysis, these efforts are wasted unless the findings can be applied. It is recalled that theoretical treatment and simulation both require the application of driver decision and reaction parameters. In using either of these approaches to arrive at improved warrants for intersection control methods, broad ranges of traffic variables such as turning percentages, truck percentages, directional splits, and traffic volumes must be studied. Realistic models would take into account significant changes in driver decisions and reactions associated with these variables.

In conclusion, therefore, it is hoped that this work will come to the attention of others, be subjected to verification, and finally, if verified, applied to the solution of the problem at hand.

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SAMPLE DATA FORM

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